Analysis of a Dynamic Voluntary Contribution Mechanism Public Good Game

23rd IPE Conference

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Why Solving the game is important?

To know optimal balance between investing in productive capacity and contributing to provision.

Examples of public goods

- Good environment
- National Defense

The Game

- 4 people in group for 10 periods
- Each period has two stages:
  1. investment stage
  2. contribution stage
- Endowments of 10 for each player in each period

Investment Stage

- Players can increase their contribution productivity from the starting value of 0.30
- Vote (median rule) to determine the amount each player in the group will invest in increasing contribution productivity
- Contribution productivity increases by 0.01 multiplied by the investment

$$\text{Contribution productivity} = M_t = M_{t-1} + 0.01 \cdot I_t$$

$$\text{for } t = [1..10]$$

$$M_0 = 0.3$$

Contribution Stage

Players decide how to allocate their remaining money between private consumption and public good.

Payoff:

$$\pi_{it} = \omega - I_t - c_{it} + M_t \sum c_{jt}$$

Example

Example ($M_0 = 0.3$):

Table

Players	ω	I_t	M_t	C_{it}	$M_t \sum c_{jt}$	π_{it}
1	10	3	0.33	7	4.95	4.95
2	10			5		6.95
3	10			3		8.95
4	10			0		11.95

Potential Outcomes

- **The Lowest Payoff outcome.** How would the players act to get the lowest possible payoffs? What are the lowest possible payoffs?
- **The Nash Equilibrium.** What would happen if each player acted in his own interest?
- **The Socially Optimal outcome.** How should the players act so that the sum of payoffs is maximized? What is this sum of payoffs?

Lowest payoff outcome

- Lowest possible payoff is 0
- Occurs if the group invests everything in every period and never contributes anything

Payoffs are 0 in every period and 0 at the end of 10 periods.

$$\pi_{it} = \omega - I_t - c_{it} + M_t \sum c_{jt}$$

Nash equilibrium

- Think of the last period
- Player maximizes his payoff. If he contributes anything he reduces his payoff. Decides not to contribute.
- All players follow the same strategy
- If nobody contributes, then nobody invests

Nash equilibrium

- Everyone is left with his endowment
- Occurs for all previous periods up to the first one
- All players follow the same strategy
- Nash equilibrium is for everyone to keep his money

Each person's payoff is $10 \cdot 10 = 100$.

The mathematical model

$$f(I, C) = [\omega - C - I] + [4 \cdot M_t \cdot C]$$

$$M_t = M_{t-1} \cdot (1 + 0.01 \cdot I)$$

$$M_0 = 0.3$$

$4 \cdot M_t \cdot C$ is payoff and $\omega - C - I$ is the amount left after both stages.

Assumption

Assumption: the optimal result requires contributing all that is left after the investment.
We can eliminate one of the two variables - C or I.

Now $I = p \cdot \omega$ and $C = (1 - p) \cdot \omega$.

$$f(p) = 4 \cdot M_t \cdot \omega \cdot (1 - p)$$

$$M_t = M_{t-1} \cdot (1 + 0.01 \cdot \omega \cdot p)$$

$$M_0 = 0.3$$

where:

- p is the *proportion* of investment
- ω is the endowment (10)
- M_t is the t_{th} multiplier

Final model

From now, let us solve it specifically for our case, when endowment is 10.

$$f(p) = 40 \cdot M_t \cdot (1 - p)$$

$$M_t = M_{t-1} \cdot \left(1 + \frac{p}{10}\right)$$

$$M_0 = 0.3$$

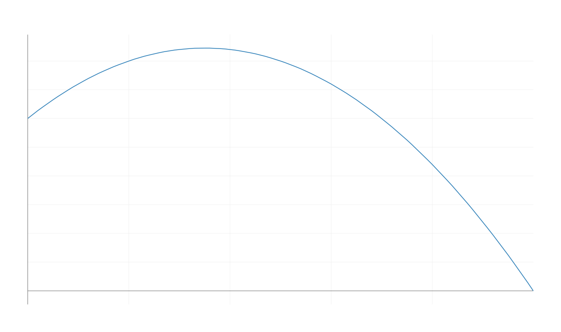
Approximation

- Ran the simulation for 10 periods with step 0.1
- Time complexity of the algorithm would be $O(n^a)$

Assumption: The optimal solution requires that players first only invest then only contribute.

Graphical representation of computational result

Plotted by https://plot.ly/plot



Regression analysis result

$$f(x) = 400 \cdot \left[-m \cdot \omega \cdot x^2 + (m \cdot \omega \cdot T - M_0) \cdot x + M_0 \cdot T\right]$$

$$x_{\max} = \frac{T}{2} - \frac{M_0}{2 \cdot m \cdot \omega}$$

$$f_{\max} = f(x_{\max}) = f\left(\frac{T}{2} - \frac{M_0}{2 \cdot m \cdot \omega}\right)$$

where:

- m is the increase in contribution productivity (0.01)
- T is the number of periods
- x is the stage when players switch to contributing. The number before the decimal point defines a period. The number after the decimal point defines an investment in that period.

In our specific case

$$f(x) = -0.1x^2 + 0.7x + 3$$

$$x_{\text{optimal}} = 3.5$$

which indicates investment until the 4_{th} period and in that period investment of 5

$$f_{\text{optimal}} = f(x_{\text{optimal}}) = 169$$

which implies the payoff of 169.

Thank you!

Questions?

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