Mini Project 1
David McCormick (DTM190000)
I am in a solo group.

Section 1.

a.

$$f_T(t) = .2e^{-.1t} - .2e^{-.2t},\ 0 \le t < \infty$$

$$P(T > 15) = 1 - P(T <= 15)$$

$$= 1 - \int_0^{15} .2e^{-.1t} - .2e^{-.2t} dt$$

$$= 1 - .2(\int_0^{15} e^{-.1t} - e^{-.2t} dt$$

$$= 1 - .2(-10e^{-.1t} + 5e^{-.2t})|_0^{15}$$

$$= 1 - .2(-10(e^{-.1(15)} - e^{-.1(0)}) + 5(e^{-.2(15)} - e^{-.2(0)}))$$

$$= 1 - .2(-10(e^{-1.5} - 1) + 5(e^{-3} - 1)$$

$$= 1 - .2(7.76870 - 4.75106)$$

$$= .396472$$

b.

i.

$$PDF <- function(x)\{return\,(.2 * exp(-.1 * x) - .2 * exp(-.2 * x))\}$$

ii.

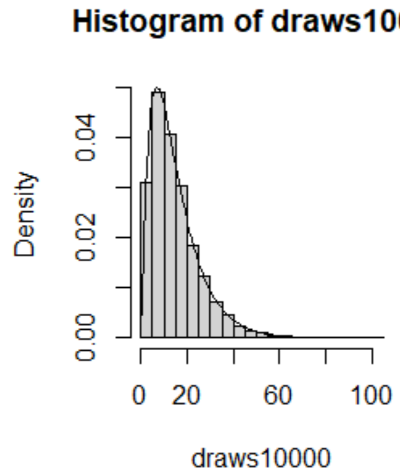
The satellite block lifetimes have exponential models. The satellite only fails when both of the blocks fail.

$$draws10000 = replicate(10000,\ max(rexp(n = 1,\ rate = .1),\ rexp(n = 1,\ rate = .1)))$$

iii.

$$hist(draws10000,\ prob = TRUE)$$

$$curve((.2 * exp(-.1 * x) - .2 * exp(-.2 * x)),\ xlim = range(0, 100),\ add = TRUE)$$



iv.

$E(T) = 15\ years$

$mean(draws10000) = 14.87924\ years$

The calculated mean is only about .12 years different. If we use more draws, the calculated mean will likely converge to 15 years.

v.

$P(T > 15) = .396472$

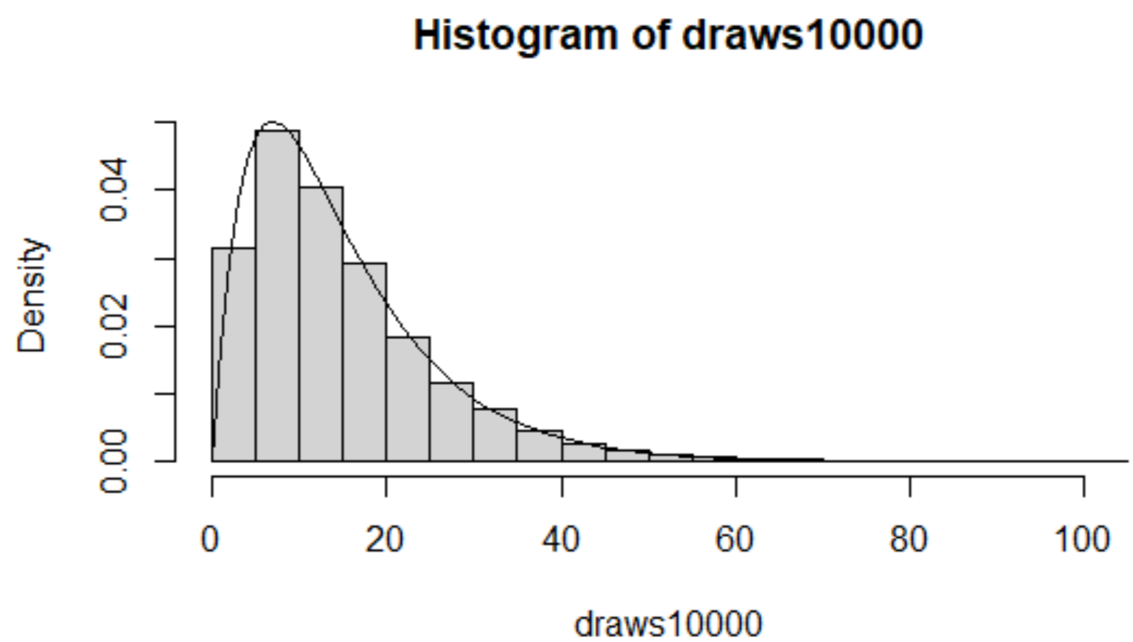
$sum(draws10000 > 15/length(draws10000) = .3971$

The calculated probability that the satellite will survive for more than 15 years when compared to the one calculated from the sample 10,000 trials is .000628 off. These probabilities are quite similar.

vi.

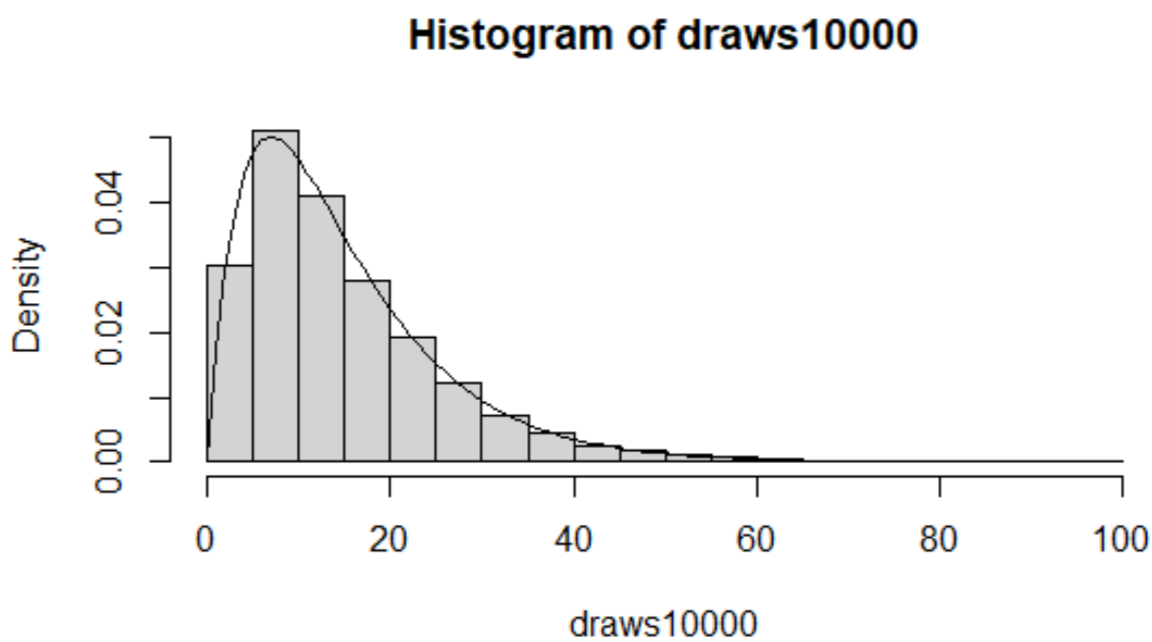
Test 2

```
> draws10000 = replicate(10000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws10000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws10000)
[1] 15.05093
> sum(draws10000>15)/length(draws10000)
[1] 0.3971
```



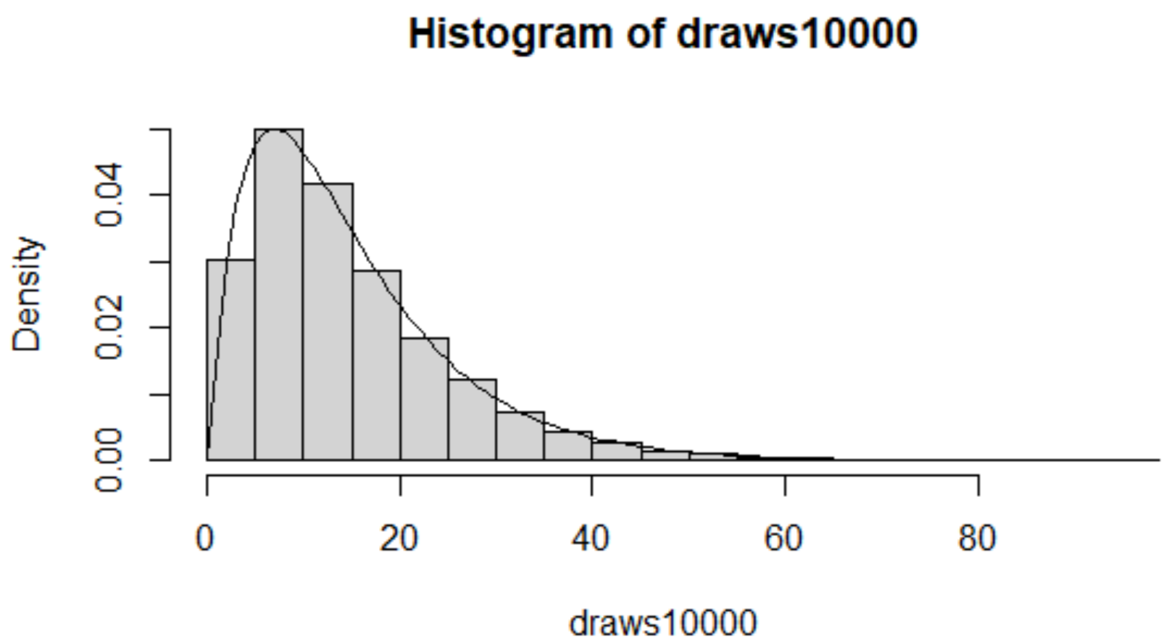
Test 3

```
> draws10000 = replicate(10000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws10000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws10000)
[1] 14.98981
> sum(draws10000>15)/length(draws10000)
[1] 0.3896
```


Histogram of draws10000
Density
0.00
0.02
0.04
0
20
40
60
80
100
draws10000

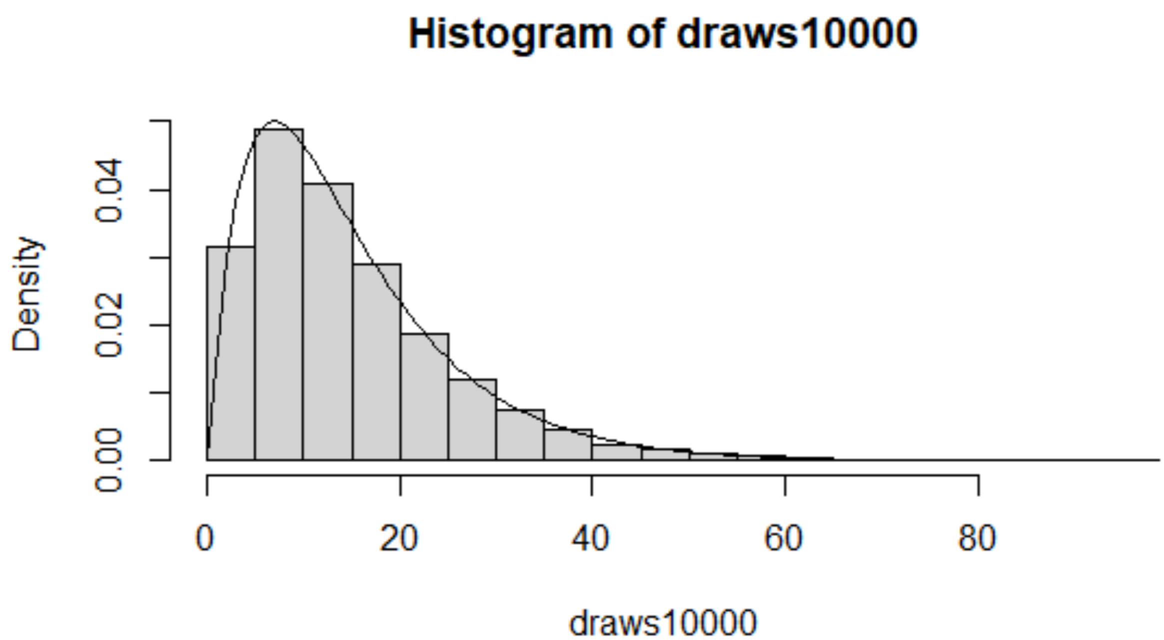
Test4

```
> draws10000 = replicate(10000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws10000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws10000)
[1] 14.91096
> sum(draws10000>15)/length(draws10000)
[1] 0.3907
```



Test 5

```
> draws10000 = replicate(10000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws10000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws10000)
[1] 14.9923
> sum(draws10000>15)/length(draws10000)
[1] 0.3945
```



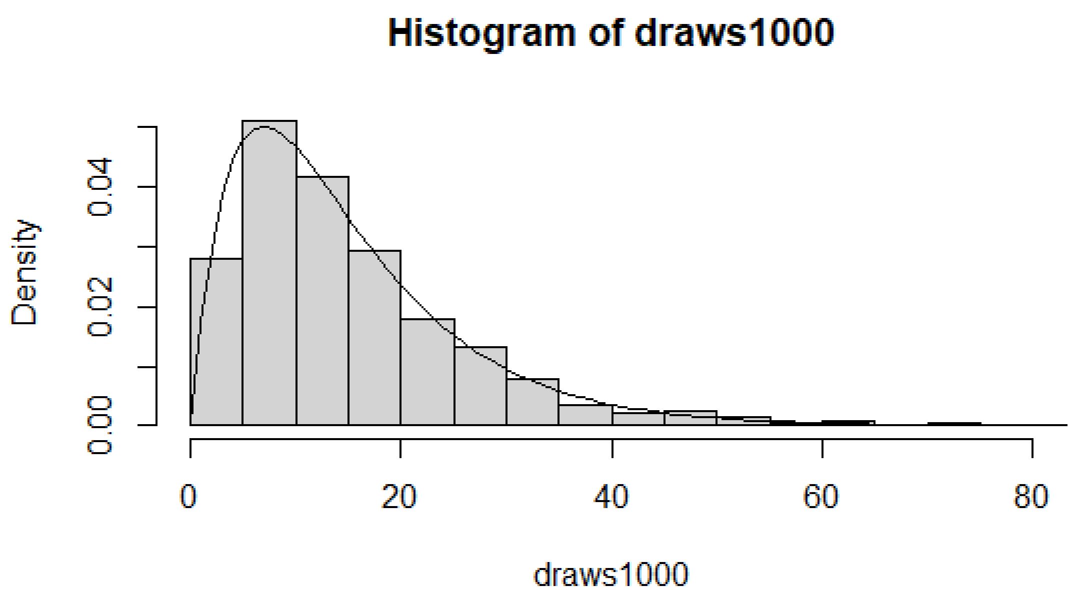
Test Number (10,000 Trials)	E(T)	P(T>15)
1	14.87924	.3971
2	15.05093	.3971
3	14.98981	.3896
4	14.91096	.3907
5	14.9923	.3945

With 10,000 trials, the calculated expected value and probability that the satellite survives for more than 15 years are both quite similar to their respective values when calculated with the formulas. They both exhibit some variance that would likely go down if we increased the number of trials due to the Central Limit Theorem. Interestingly enough, the probability that the satellite survives for more than 15 years was the same for test number 1 and 2.

c.

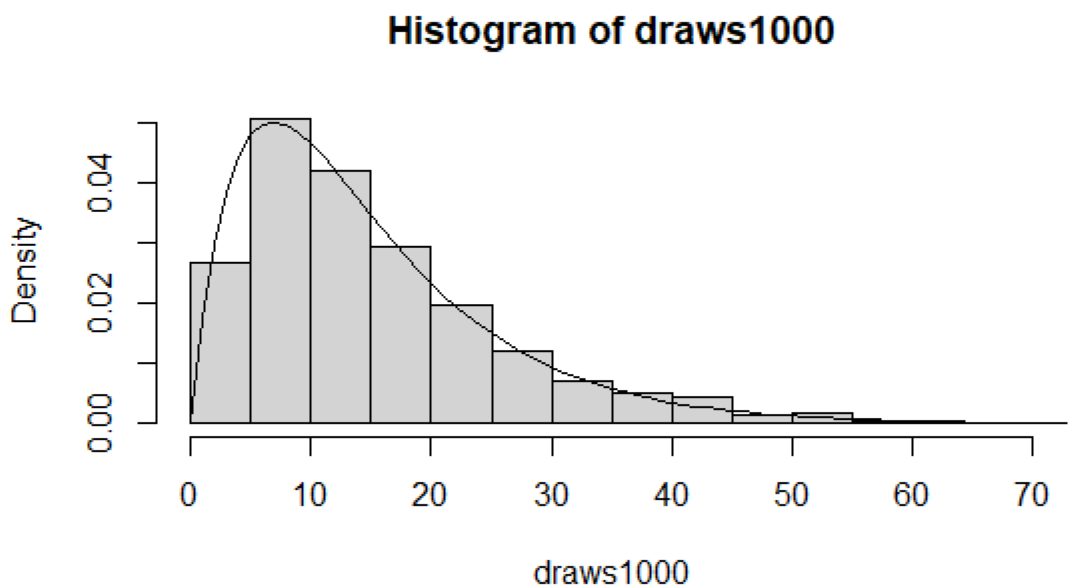
Test 1

```
> draws1000 = replicate(1000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws1000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws1000)
[1] 15.219
> sum(draws1000>15)/length(draws1000)
[1] 0.399
```



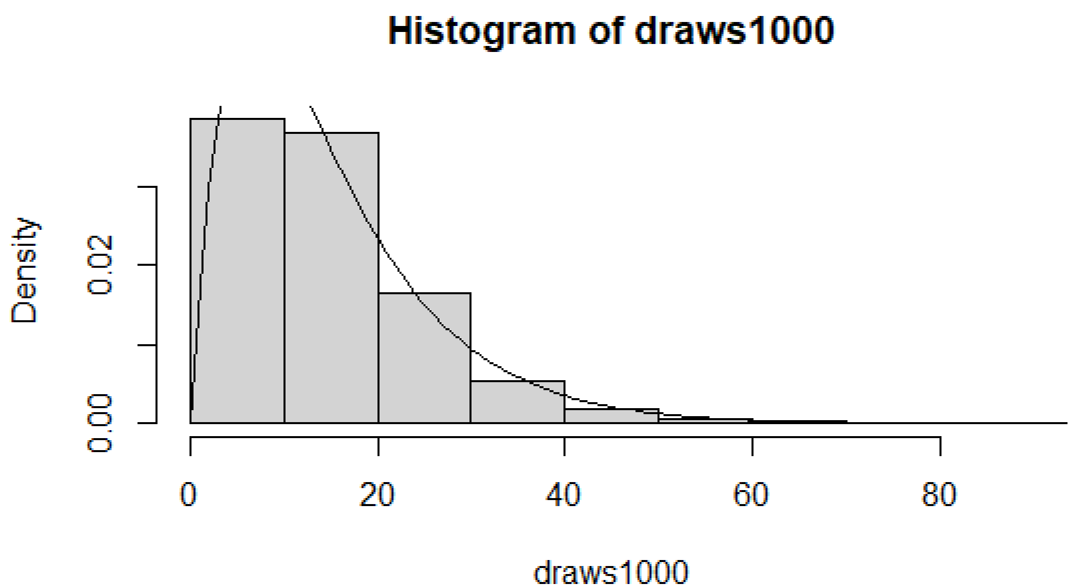
Test 2

```
> draws1000 = replicate(1000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws1000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws1000)
[1] 15.21197
> sum(draws1000>15)/length(draws1000)
[1] 0.406
```



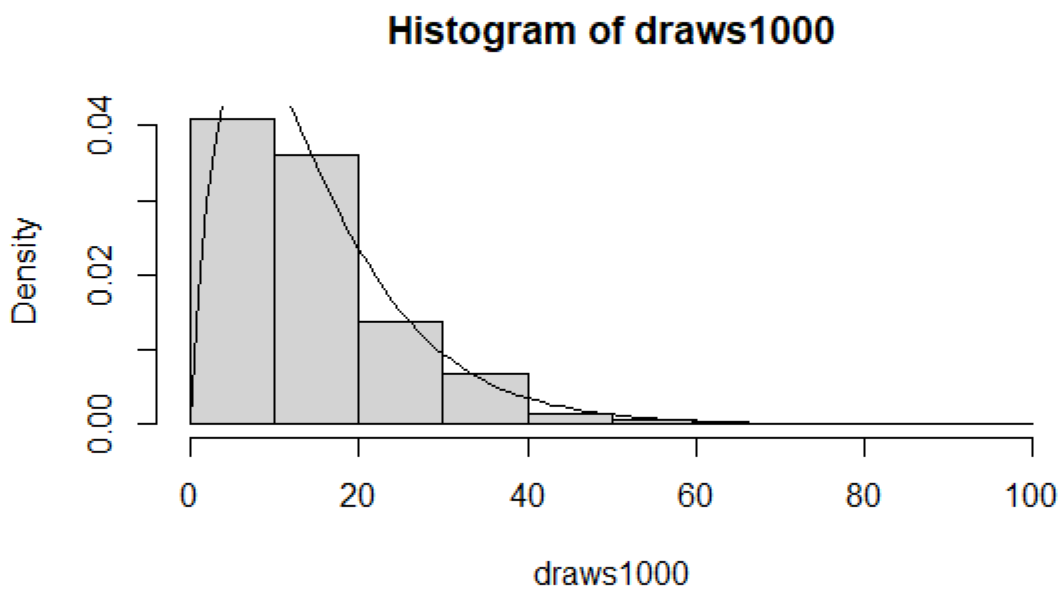
Test 3

```
> draws1000 = replicate(1000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws1000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws1000)
[1] 14.79328
> sum(draws1000>15)/length(draws1000)
[1] 0.399
```



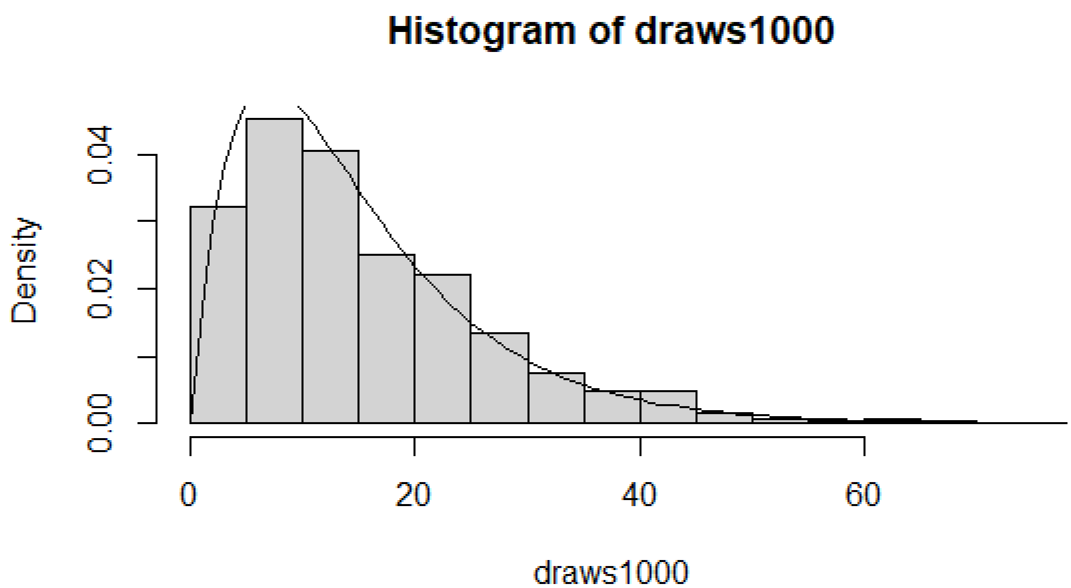
Test 4

```
> draws1000 = replicate(1000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws1000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws1000)
[1] 14.39823
> sum(draws1000>15)/length(draws1000)
[1] 0.38
```



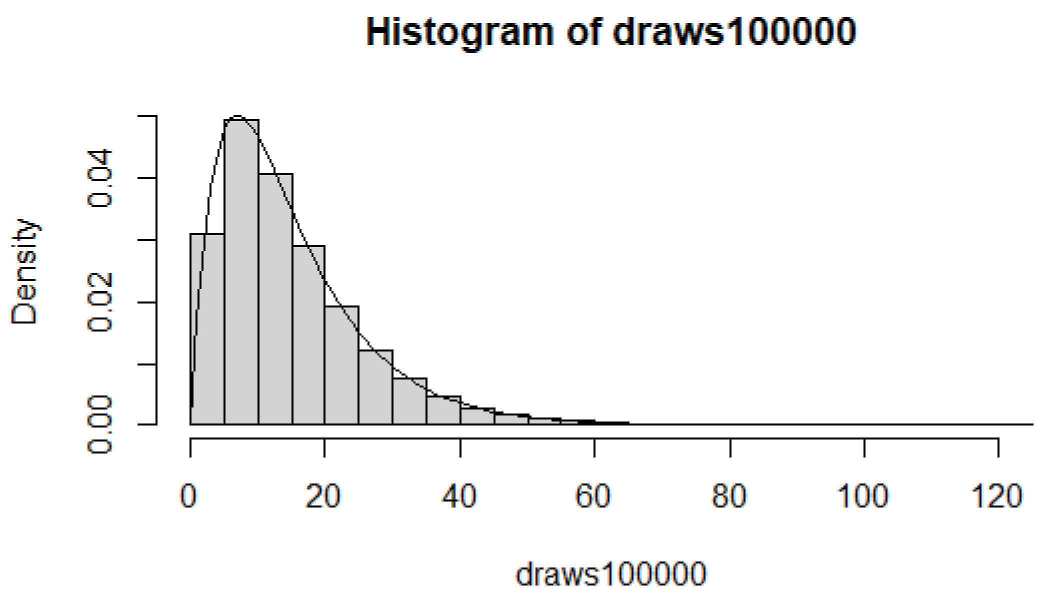
Test 5

```
> draws1000 = replicate(1000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws1000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws1000)
[1] 15.45206
> sum(draws1000>15)/length(draws1000)
[1] 0.41
```



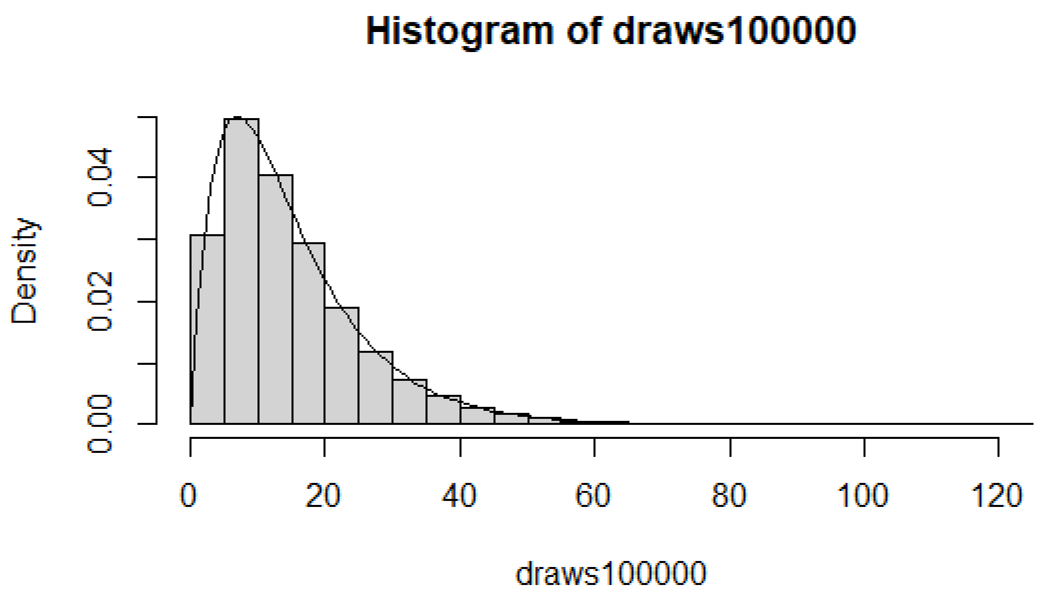
Test 1

```
> draws100000 = replicate(100000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws100000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws100000)
[1] 15.04434
> sum(draws100000>15)/length(draws100000)
[1] 0.39758
```



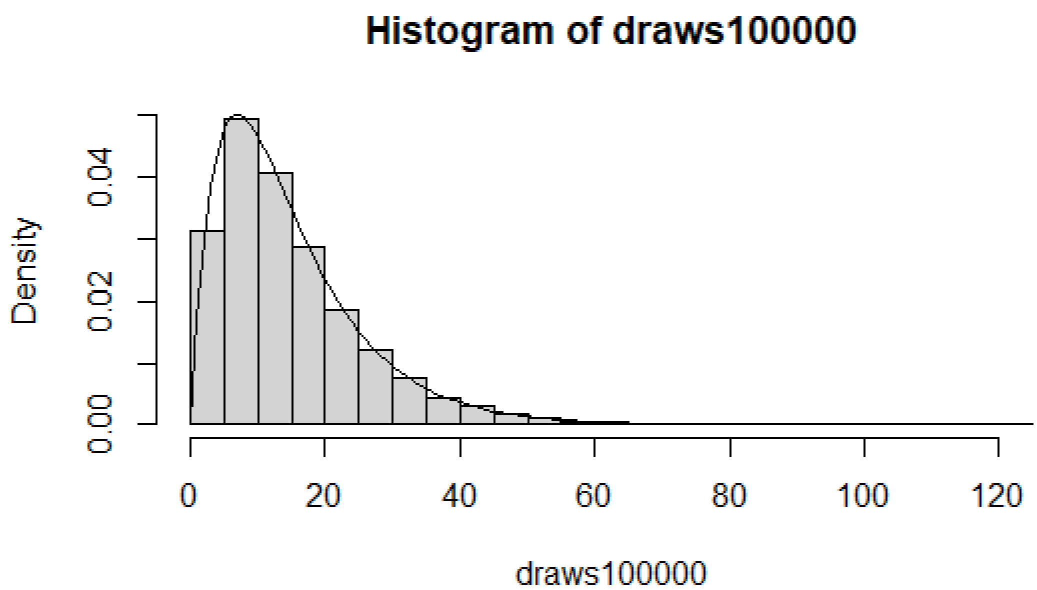
Test 2

```
> draws100000 = replicate(100000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws100000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws100000)
[1] 14.9846
> sum(draws100000>15)/length(draws100000)
[1] 0.39767
```



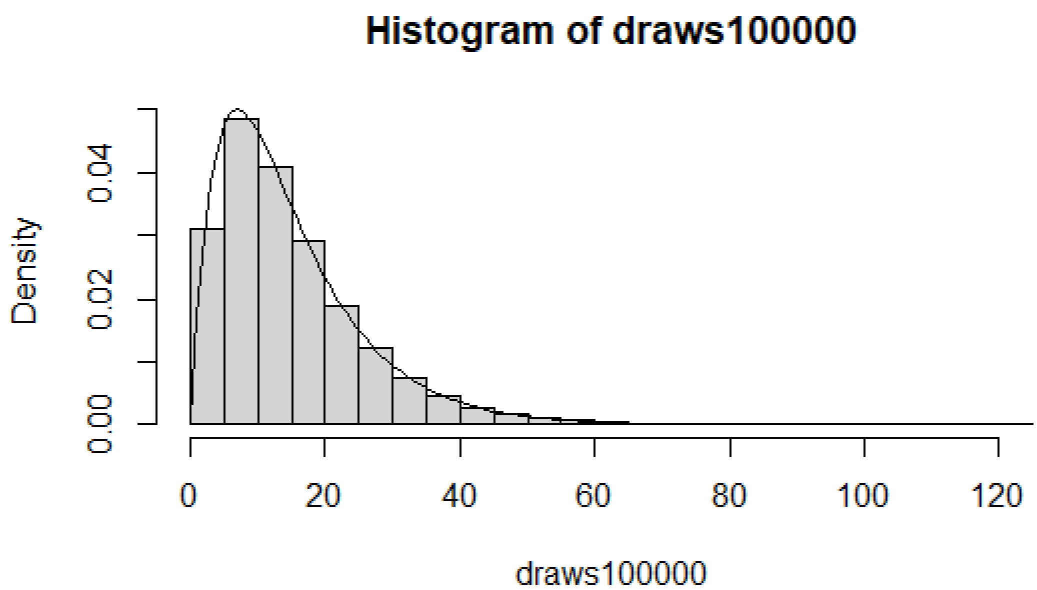
Test 3

```
> draws100000 = replicate(100000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws100000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws100000)
[1] 14.99973
> sum(draws100000>15)/length(draws100000)
[1] 0.3951
```



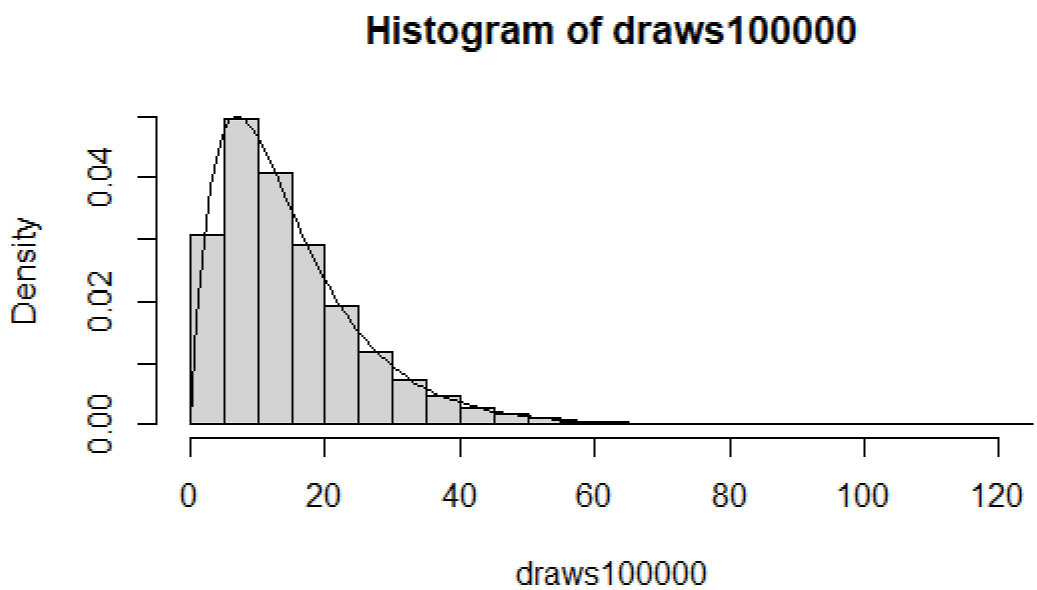
Test 4

```
> draws100000 = replicate(100000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws100000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws100000)
[1] 14.96031
> sum(draws100000>15)/length(draws100000)
[1] 0.39417
```



Test 5

```
> draws100000 = replicate(100000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1)))
> hist(draws100000, prob=TRUE)
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE)
> mean(draws100000)
[1] 14.97747
> sum(draws100000>15)/length(draws100000)
[1] 0.39712
```



Test Number (1,000 Trials)	E(T)	P(T>15)
1	15.219	.399
2	15.21197	.406
3	14.79328	.399
4	14.39823	.38
5	15.45206	.41

Test Number (10,000 Trials)	E(T)	P(T>15)
1	14.87924	.3971
2	15.05093	.3971
3	14.98981	.3896
4	14.91096	.3907
5	14.9923	.3945

Test Number (100,000 Trials)	E(T)	P(T>15)
1	15.04434	.39758
2	14.9846	.39767
3	14.99973	.3951
4	14.96031	.39417
5	14.97747	.39712

With 1,000 trials, the calculated results show more variance than with 10,000 trials. For instance, in Test 4, the calculated expected value was off by a maximum of about .6 years. The maximum difference in the tests conducted with 100,000 trials is .04434 in test 1. In the tests conducted with 100,000 trials, the calculated results were the closest to the expected values, which agrees with the Central Limit Theorem.

2.

```
> x = runif(10000, min=0, max=1)
> y = runif(10000, min=0, max=1)
> z = (x-.5)^2 + (y-.5)^2 <= .5^2
> sum(z == TRUE) * 4/ 10000
[1] 3.1404
```

The principle behind the solution to this problem is that we can approximate π by generating points in an area and then calculating the ratio of points in the circle to the total number of points generated. The R code creates 10,000 points measured by x and y between the area encompassing (0,0), (1,0), (1,1), and (0,1). Afterwards, it checks how many of those points are within the area of a circle. It sums up how many successes there were, multiplied by 4 because

$$\pi r^2/1 = number\ of\ points\ in\ the\ circle\ /\ total\ number\ of\ points$$
$$\pi(1/2^2)/1 = number\ of\ points\ in\ the\ circle\ /\ total\ number\ of\ points$$
$$\pi = 4 * number\ of\ points\ in\ the\ circle\ /\ total\ number\ of\ points$$

The approximated value with this method is 3.1404, which is close to the actual value of 3.1415.

Section 2.

1.

```
> draws10000 = replicate(10000, max(rexp(n=1, rate=.1), rexp(n=1, rate=.1))) # Performs the equation (second argument) 10,000 (first argument) times.
> hist(draws10000, prob=TRUE) # Draws the histogram, but with probabilities so the curve will fit properly
> curve((.2*exp(-.1*x)-.2*exp(-.2*x)),xlim=range(0,100), add=TRUE) # Adds the curve onto the current plot
> mean(draws10000) # Calculates the mean of the 10,000 trials
[1] 15.05093
> sum(draws10000>15)/length(draws10000) # Calculates the average of successes, where the satellite survives for more than 15 years,
[1] 0.3971

# Other trials were performed where n = 1,000 and n = 100,000. The variable was changed and n was substituted for in arguments.
```

2.

```
x = runif(10000, min=0, max=1) # Generates 10,000 random numbers between 0 and 1 for an x coordinate
y = runif(10000, min=0, max=1) # Generates 10,000 random numbers between 0 and 1 for a y coordinate
z = (x-.5)^2 + (y-.5)^2 <= .5^2 # Determines if the point is inside the circle
sum(z == TRUE)* 4/ 10000 # Performs the other necessary calculations to approximate pi.
```