On Bouncing Oil Drops

A Thesis
Presented to
The Division of Mathematics and Natural Sciences
Reed College

In Partial Fulfillment
of the Requirements for the Degree
Bachelor of Arts

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Approved for the Division
(Physics)

Daniel Borrero

Blog

This is the portion of the thesis that I will update regularly with rough notes, lit reviews, results, etc. some of which will be worked in to the real document after some polishing.

0.1 Goals

- 10/14: Define specific problem.
- Fall Break: Experimental Setup
- Winter Break: Lit Review Complete

0.2 To Do

- Obtain flashdrives. Miguel 9/30/14
- Learn Bohmian Mechanics. Miguel 11/4/14
- Find walking regime. Miguel 11/4/14
- Set up Accelerometer (should arrive around 11/19/14). Miguel 11/6/14
- Learn de Broglie's interpretation of QM. Miguel 11/6/14

0.2.1 Done

- Order Accelerometer. Miguel 11/6/14
- Learn Basics of Bohmian Mechanics. Miguel 10/28/14
- Make tray. Miguel 10/20/14
- Sort out camera situation. Miguel 10/20/14
- Learn how to use the new LaTeX and Github setup. Miguel 9/30/14

0.3 Literature Reviews

(Important: Particle-wave association on a fluid interface (Protiere 2006)).

In this annual review, Bush describes some of the characteristics of the bouncing oil drop experiment that are analagous to effects witnessed in the quantum mechanical world (single-particle diffraction, tunneling, quantized orbits, orbital level splitting, and spin states). Dynamics of the walker are described mathematically. Finally, comparisons to de Broglie's orginal formulation of QM (and not Bohm's) and Stochastic Electrodynamics (?) are made. (Most coming from Pilot-Wave Hydrodynamics (Bush 2015))

Basic Parameters

Consider a fluid of density ρ, viscosity ν, and surface tension σ, in a bath of depth H driven vertically at an amplitude A_0 at frequency $f = \omega/2\pi$. By defining $\gamma = A_0\omega^2$, the effective gravity in the frame of reference of the bath is $g + \gamma \sin(\omega t)$. The oil droplet of diameter D bounces in the regime $\gamma < \gamma_F$, where γ_F is the Faraday threshold (at this point, Fraday waves appear). The important experimental limits are outlined in Table 1.

Table 1: Approximate Limits for Bouncing Drop Behavior

Parameter	Lower Limit	Upper Limit
Viscocity ν (cSt)	10	100
Bath Depth H (mm)	4	10
Frequency f (Hz)	20	150
Amplitude A_0 (mm)	0.1	1
Drop Diameter D (mm)	0.6	1.0

For certain parameters, the bouncing drop will behave differently. The vibration number describes "the relative magnitude of the forcing frequency and the drop's natural oscillation frequency," and is given by:

$$V_i = \frac{\omega}{2}\sqrt{\frac{\rho D^3}{2\sigma}} \tag{1}$$

The natural frequency of the droplet occurs around $V_i = 0.65$, where the droplet can exhibit both walking and bouncing behaviors. Setting up a plot with V_i on the y axis and (dimensionless) γ/g on the x axis can help in showing the behavior of the droplet, shown in Fig. 1.

The various modes seen in Fig. 1 can be described by (m,n), where n is the number to times the droplet contacts the surface over period m/f. For example, in the (1,1) mode, the droplet hits the oil bath once per driving period. In the (2,2) mode, the drop makes two bounces of differing heights.

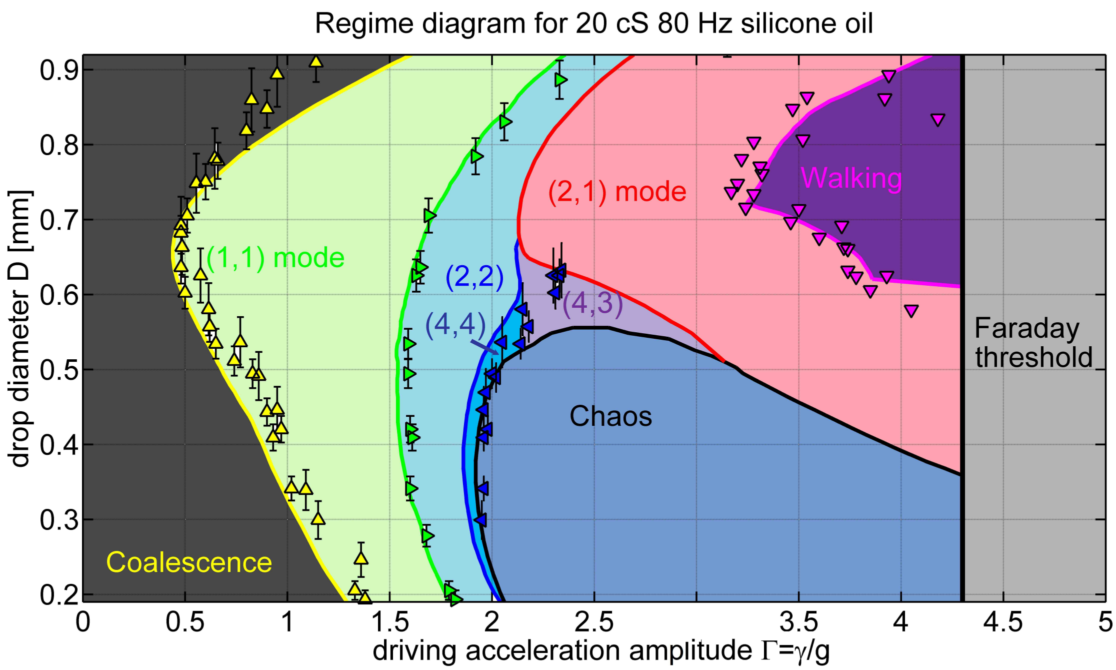

Figure 1: The different bouncing regimes for the oil drops of 20 cS silicon oil and at $f = \omega/2\pi = 80$ Hz. The parameters (m,n) describe the droplet that bounces n times in m forcing periods.

0.3.1 Path Memory

Path memory is a parameter that can be varied in this setup. Every time the droplet impacts the bath, it creates a radial traveling wave. Over the course of many bounces, a wavefield composed of a superposition of the many waves arises. If the bouncing droplet impacts the wavefield in such a way that it recives a lateral force from the slope of the wave, then it will be pushed to the side slightly. The next time the droplet makes contanct with the bath, it will again be pushed to the side. This propels the bouncing droplet, causing it to walk across the surface of the bath. These "walkers" are pushed in a direction dictated by previous impacts, and so it is said that the oil bath "remembers" the previous bounce.

Damping the waves results in a low-memory limit, since the walker is affected by only the previous impact. In an undamped system (approaching the Faraday threshold), the system becomes high-memory because walker can be affected by waves from many impacts ago. The quantum like features of this experiment arise in the high-memory limit. (For more, Eddi et. al, 2011b: Information stored in Faraday waves.)

0.3.2 Single-Particle Diffraction

In 2006 Couder and Fort showed that the system had properties that were strikinly similar to two famously controversial quantum experiments (Couder and Fort, 2006). They were able to demonstrate that a single walker travelling through one slit seemed to have its direction altered seemingly randomly, before continuing forward on its new path. By statistically analyzing many trials, Couder and Fort showed that the histogram of the "diffraction" actually resulted in a diffraction pattern strikingly similar to the single photon diffraction experiment performed by Taylor in 1909.

Next, Couder and Fort added a second slit. They again passed a single walker at a time through either slit, and discovered that a histomgram of this data returned another diffraction pattern. This result is of course reminiscent of one of the most famous experiments in physics: Young's double slit diffraction with photons and electrons.

Using a numerical simulation, Couder and Fort were able to reproduce similar results.

As Couder and Fort mention in their paper: "A discussion of the relation between these single-particle experiments and those concerning elementary particles is unavoidable." Important differences and similarities are then described between the quantum system and the quantum-like system. For the differences: we have a dissapative system, where energy is continually put in through the vibration of the tray; the particle can be followed;[1] it's really effectively moving in two dimensions; the velocity is measurable; and the probability distributuion is linked with the wave amplitude (rather than it's intensity). And then of course, the similarities: an uncertainty priciple arises from the statistical data (and without knowledge of the actual paths followed by the walkers); and some others that were unclear...

0.3.3 Tunneling

The guiding wave field can be partially reflected off of an edge or even a change in depth of the oil bath. This effect can be seen when a walker is pushed back from a under-the-surface step, seemingly without any contact. In rare cases, the walker will actually tunnel across the step; that is, it will continue to walk along the surface of the oil bath and pass over the step without reflection. In the first experiment done by Eddi et al., they demonstrated tunneling by by building square "corrals" of varying thicknesses. In the second experiment, they built a rhombus shape which forced the walker across the center of a rhombus. The barrier was placed perpendicular to the direction of travel of the walker, so that it would hit the wall directly rather than at an angle (as in the square corral). "The tunneling probability decreases exponentially with the barrier width and increases as the Faraday threshold is approached." Eddi et. al also found that the probability of tunneling increased with the velocity of the walker. (For more, Eddi et. al 2009b: Unpredictable Tunneling of a classical wave-particle association.)

The unpredictability of the tunneling comes from the complex interatction between the drop and its guiding wave.

[1] C and F note that it'd be impossible to dectect the particle without disturbing it "by any means at its scale," like a bouy, for example. As the bouy floated it would interfere with the system by altering the wave pattern on the surface.

0.3.4 Motion in a Confined Geormetry

By tracking the droplet as it bounces around the tray over a period of time, one can look at the overall statistical behavior of the droplet. Two experiments tracked walkers in an experimental coral (Harris and Bush, 2013, Harris et al. 2013) in the high-memory, chaotic motion regime. A histogram of the statistical data shows that the "probability of finding a walker at a given point in the corral is roughly prescribed by the amplitude of the Faraday wave mode of the cavity at the prescribed forcing frequency."

Quantum corral experiments performed by Crommie et al. (Crommie et al. 1993 a b) present similar findings. In the experiment, electrons were confined in a Cu(III) substrate using barriers of iron adatoms. Using tunneling spectroscopy, the electrons were found to have a specific resonances depending on the corral shape. As in the case of Harris' circular corral experiment where the corral and the Faraday wavelength, λ_F, dictate the wavelike statistical patter, in the quantum experiment the corral and the *de Broglie* wavelength, λ_{dB}, dictate the form of the wavelike statistical pattern.

0.3.5 Bouncing Mechanics

In the regime of walkers we have $R_e \sim 20$, $B_0 \sim 0.1$, and $W_e \sim 0.1$. For the millimeric walkers, the dominant force comes from impact of the curvature of the surface. Gilet and Bush (2009: Chaotic bouncing of a drop on a soap film, and the fluid trampline: droplets bouncing on a soap film) show that the surface of the vibrating oil can be modeled with a soap film, where the soap film acts like a linear spring.

As the oil bath is forced up and down, a tiny droplet of oil will "walk" across the surface. Moláček and Bush have developed an equation of a droplet that describes the trajectory of the walking droplet, ignoring the verical dynamics by time averaging them out (cite: J. Moláček and J. W. M. Bush, "Drops walking on a vibrating bath: towards a hydrodynamic pilot-wave theory" J. FluidMech. 727, 612-647 (2013).). The trajectory of the walking droplet of mass m at position $\mathbf{x}(t) = (x(t), y(t))$ is given by

$$m\ddot{\mathbf{x}} + D\dot{\mathbf{x}} = -mg\nabla h(\mathbf{x}, t) \tag{2}$$

where D describes the drag coefficient and $h(\mathbf{x}, t)$ describes the shape of the wavefield. Thus the second term describes the time averaged drag from both the flight and the impact of the droplet (as usual, depends on the velocity), and the third term describes the propulsive wave force resulting from drops landing on the inclined wave surface.

The wavefield is quite complicated because it depends on the memory. For a single impact of a droplet, Oza et al. argue the surface wave can be approximated with an integral of a monochromatic radial Bessel function of the first kind

$$h(\mathbf{x}, t) = \frac{F}{T_F}\int_{-\infty}^{t} J_0 \frac{(k_F|\mathbf{x}(t) - \mathbf{x}(s)|)}{|\mathbf{x}(t) - \mathbf{x}(s)|}(\mathbf{x}(t) - \mathbf{x}(s))e^{-(t-s)/(T_F M_e)}ds \tag{3}$$

with F giving the wave force coefficient (estimated in the above source), T_F describing the Faraday period, and k_F describing the Faraday wave number determined by the Faraday wavelength $\lambda_F = 2/k_F$ (integral from A. U. Oza, D. M. Harris, R. R. Rosales, and J. W. M. Bush, "Pilot-wave dynamics in a rotating frame: on the emergence of orbital quantization" J. Fluid Mech. 744, 404-429 (2014).). (Faraday was a popular guy.) Finally, that last term M_e is the nondimensional memory parameter $M_e = T_d/[T_F(1-\gamma/\gamma_F)]$ (with T_d being the unforced decay time).

0.4 Experimental Setup

0.5 Bohmian Mechanics

0.5.1 Formalism

The Schroedinger Equation and ψ

We begin with the Schoedinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi \tag{4}$$

where $\hat{H}$ is the Hamiltonian and ψ is the wavefunction. The Hamiltonian can be expanded (assuming there is no electric field) to give

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{x},t)\right]\psi(\mathbf{x},t) \tag{5}$$

where $V(\mathbf{x},t)$ is the potential energy of the system. The solution ψ is of the form:

$$\psi(\mathbf{x},t) = R(\mathbf{x},t)e^{iS(\mathbf{x},t)/\hbar} \tag{6}$$

where S and R are real. Plugging in our equation for ψ into the Schoedinger equation (Eq. (5)) will produce two separate equations: one giving the time derivative of R and the second giving the time derivative of S. From these equations, a Hamilton-Jacobi equation can be written for a quantum system. Let's begin by computing the left hand side of Eq. (5) in terms of R and S.

$$\begin{aligned}
i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) &= i\hbar\left(\frac{\partial R}{\partial t}e^{iS/\hbar} + R\frac{i}{\hbar}\frac{\partial S}{\partial t}e^{iS/\hbar}\right) \\
&= i\hbar\left(\frac{1}{R}\frac{\partial R}{\partial t} + \frac{i}{\hbar}\frac{\partial S}{\partial t}\right)\psi(\mathbf{x},t) \\
&= \left(i\hbar\frac{1}{R}\frac{\partial R}{\partial t} - \frac{\partial S}{\partial t}\right)\psi(\mathbf{x},t)
\end{aligned}$$

Let's leave that alone for a little bit, while we focus on the right hand side of Eq. (5). Since it's a little more complicated, we will start with one term of the right hand side:

$$\nabla^2\psi(\mathbf{x},t) = e^{iS/\hbar}\nabla^2 R + \left(\frac{i}{\hbar}\right)^2 (\nabla S)^2 Re^{iS/\hbar} + \left(\frac{i}{\hbar}\right) Re^{iS/\hbar}(\nabla^2 S) + \left(\frac{2i}{\hbar}\right)(\nabla R\cdot\nabla S)Re^{iS/\hbar}$$
$$= \left(\frac{\nabla^2 R}{R} - \left(\frac{\nabla S}{\hbar}\right)^2 + \left(\frac{i\nabla^2 S}{\hbar}\right) + 2i\left(\frac{\nabla R\cdot\nabla S}{\hbar}\right)\right)\psi(\mathbf{x},t)$$

Now the hard part is done, and we can say that the right hand side of Eq. (5) is given by

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{x},t)\right]\psi(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m}\right) - i\hbar\left(\frac{\nabla^2 S}{2m}\right) - i\hbar\left(\frac{\nabla R\cdot\nabla S}{m}\right) + V(\mathbf{x},t)\right]\psi($$

Ok, now that we've got that done, the next part will be super easy. Starting with Schrodinger's equation and plugging in left and right hand sides we calulated seperately.

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{x},t)\right]\psi(\mathbf{x},t)$$
$$\left(i\hbar\frac{1}{R}\frac{\partial R}{\partial t} - \frac{\partial S}{\partial t}\right)\psi(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m}\right) - i\hbar\left(\frac{\nabla^2 S}{2m}\right) - i\hbar\left(\frac{\nabla R\cdot\nabla S}{m}\right) + V(\mathbf{x},t)\right]\psi(\mathbf{x},$$

Now we can divide out ψ from both sides

$$i\hbar\frac{1}{R}\frac{\partial R}{\partial t} - \frac{\partial S}{\partial t} = -\frac{\hbar^2\nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m}\right) - i\hbar\left(\frac{\nabla^2 S}{2m}\right) - i\hbar\left(\frac{\nabla R\cdot\nabla S}{m}\right) + V(\mathbf{x},t)$$

and group the imaginary numbers on the left side and the real numbers on the right side

$$i\hbar\frac{1}{R}\frac{\partial R}{\partial t} + i\hbar\left(\frac{\nabla^2 S}{2m}\right) + i\hbar\left(\frac{\nabla R\cdot\nabla S}{m}\right) = \frac{\partial S}{\partial t} - \frac{\hbar^2\nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m}\right) + V(\mathbf{x},t)$$

Recall that both S and R are real. Note that the only way for all of the imaginary terms to equal all of the real terms is if they both equaled zero.

$$i\hbar\left(\frac{1}{R}\frac{\partial R}{\partial t} + \frac{\nabla^2 S}{2m} + \frac{\nabla R\cdot\nabla S}{m}\right) = \left(\frac{\partial S}{\partial t} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + \frac{(\nabla S)^2}{2m} + V(\mathbf{x},t)\right) = 0$$

This then gives us two sepeate equations, one for the time derivative of R and another for the time derivative of S.

$$\frac{\partial R}{\partial t} = -\frac{R}{2m}\left(\frac{\nabla^2 S}{m} - 2\nabla R\cdot\nabla S\right) \tag{7}$$
$$\frac{\partial S}{\partial t} = \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} - \frac{(\nabla S)^2}{2m} - V(\mathbf{x},t) \tag{8}$$

What does this do for us? Both equations will provide helpful descriptions of our system.

The Quantum Potential

We can rewrite Eq. (8) in a provokative way

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V(\mathbf{x}, t) + \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{9}$$

which should look suspiciously familiar. If I were to tell you that ∇S had units of momentum and $\partial S/\partial t$ units of energy, then this equation would look a lot like a Hamiltonian! The first term takes care of the kinetic energy, the second is the potential energy term, but we have this mysterious third term which we haven't ever encountered in classical mechanics. If we define this term as our "quantum potential"

$$U(\mathbf{x}) = \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = \frac{\hbar^2}{4m}\left[\frac{1}{2}\frac{\nabla^2 P}{P} - \frac{(\nabla P)^2}{P^2}\right] \tag{10}$$

then we really *can* think of Eq. (9) as a Hamiltonian with an extra potential term thrown in to make it "quantum." Note that in cases where $\hbar$ is much smaller than the rest of the terms (i.e. not the quantum realm), then this quantum potential term goes away, and we are left with the regular Hamilton equation from classical mechanics.

Recall that when writing a Hamiltonian, the potential terms govern the forces on the particle. For a conservative system, the force is given by F(x) = -$\partial U/\partial x$. If we include a quantum mechanical potential in our Hamiltonian, then this potential must cause a force on the paticle in addition to the one supplied by the $V(x)$ term.

Continuity Equation

Plugging in the probability density $P(\mathbf{x}, t) = R^2(\mathbf{x}, t)$ into Eq. (7) also gives us something quite interesting.

MATH?

which we can finally express as

$$\frac{\partial P}{\partial t} + \nabla \cdot \left(P\frac{\nabla S}{m}\right) \tag{11}$$

In describing the quantum potential term it was mentioned that ∇S can be thought of as momentum, so then from our classical relationship between momentum and velocity $\mathbf{v}(\mathbf{x}, t) = \nabla S/m$ can be thought of as velocity. Then by defining the probability current as $j(\mathbf{x}, t) = P\nabla S/m$ then we recover

$$\frac{\partial P}{\partial t} + \nabla \cdot j(\mathbf{x}, t) \tag{12}$$

known as the continuity equation! This tells us that P is conserved over time.

Finding R and S

Chapter 1

Mathematics and Science

1.1 Math

TEX is the best way to typeset mathematics. Donald Knuth designed TEX when he got frustrated at how long it was taking the typesetters to finish his book, which contained a lot of mathematics.

If you are doing a thesis that will involve lots of math, you will want to read the following section which has been commented out. If you're not going to use math, skip over this next big red section. (It's red in the .tex file but does not show up in the .pdf.)

$$\sum_{j=1}^{n}(\delta\theta_j)^2 \leq \frac{\beta_i^2}{\delta_i^2+\rho_i^2}\left[2\rho_i^2+\frac{\delta_i^2\beta_i^2}{\delta_i^2+\rho_i^2}\right] \equiv \omega_i^2$$

From Informational Dynamics, we have the following (Dave Braden):

After n such encounters the posterior density for θ is

$$\pi(\theta|X_1 < y_1, \ldots, X_n < y_n) \propto \pi(\theta)\prod_{i=1}^{n}\int_{-\infty}^{y_i}\exp\left(-\frac{(x-\theta)^2}{2\sigma^2}\right)\,dx$$

Another equation:

$$\det\begin{vmatrix} c_0 & c_1 & c_2 & \dots & c_n \\ c_1 & c_2 & c_3 & \dots & c_{n+1} \\ c_2 & c_3 & c_4 & \dots & c_{n+2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_n & c_{n+1} & c_{n+2} & \dots & c_{2n} \end{vmatrix} > 0$$

Lapidus and Pindar, Numerical Solution of Partial Differential Equations in Science and Engineering. Page 54

$$\int_t\left\{\sum_{j=1}^{3}T_j\left(\frac{d\phi_j}{dt}+k\phi_j\right)-kT_e\right\}w_i(t)\,dt=0, \qquad i=1,2,3.$$

L&P Galerkin method weighting functions. Page 55

$$\sum_{j=1}^{3} T_j \int_0^1 \left\{ \frac{d\phi_j}{dt} + k\phi_j \right\} \phi_i \; dt = \int_0^1 k\, T_e \phi_i dt, \qquad i = 1, 2, 3$$

Another L&P (p145)

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\big(\xi,\eta,\zeta\big) = \sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n} w_i w_j w_k f\big(\xi,\eta,\zeta\big).$$

Another L&P (p126)

$$\int_{A_e} (\,\cdot\,) dx dy = \int_{-1}^{1}\int_{-1}^{1} (\,\cdot\,) \det[J] d\xi d\eta.$$

1.2 Chemistry 101: Symbols

Chemical formulas will look best if they are not italicized. Get around math mode's automatic italicizing by using the argument `$\mathrm{formula here}$`, with your formula inside the curly brackets.

So, $\mathrm{Fe_2^{2+}Cr_2O_4}$ is written `$\mathrm{Fe_2^{2+}Cr_2O_4}$`
Exponent or Superscript: $\mathrm{O^-}$
Subscript: $\mathrm{CH_4}$

To stack numbers or letters as in $\mathrm{Fe_2^{2+}}$, the subscript is defined first, and then the superscript is defined.
Angstrom: Å
Bullet: $\mathrm{CuCl \bullet 7H_2O}$
Double Dagger: ‡
Delta: Δ
Reaction Arrows: $\longrightarrow$ or $\xrightarrow{solution}$
Resonance Arrows: $\leftrightarrow$
Reversible Reaction Arrows: $\rightleftharpoons$ or $\xrightleftharpoons{solution}$ (the latter requires the chemarr package)

1.2.1 Typesetting reactions

You may wish to put your reaction in a figure environment, which means that LaTeX will place the reaction where it fits and you can have a figure legend if desired:

$$\mathrm{C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O}$$

Figure 1.1: Combustion of glucose

1.2.2 Other examples of reactions

$NH_4Cl_{(s)} \rightleftharpoons NH_{3(g)} + HCl_{(g)}$
$MeCH_2Br + Mg \xrightarrow[below]{above} MeCH_2 \bullet Mg \bullet Br$

1.3 Physics

Many of the symbols you will need can be found on the math page (`http://web.reed.edu/cis/help/latex/math.html`) and the Comprehensive LaTeX Symbol Guide (enclosed in this template download). You may wish to create custom commands for commonly used symbols, phrases or equations, as described in Chapter **??**.

1.4 Biology

You will probably find the resources at `http://www.lecb.ncifcrf.gov/~toms/latex.html` helpful, particularly the links to bsts for various journals. You may also be interested in TeXShade for nucleotide typesetting (`http://homepages.uni-tuebingen.de/beitz/txe.html`). Be sure to read the proceeding chapter on graphics and tables, and remember that the thesis template has versions of Ecology and Science bsts which support webpage citation formats.

Chapter 2

Tables and Graphics

2.1 Tables

The following section contains examples of tables, most of which have been commented out for brevity. (They will show up in the .tex document in red, but not at all in the .pdf). For more help in constructing a table (or anything else in this document), please see the LaTeX pages on the CUS site.

Table 2.1: A Basic Table: Correlation of Factors between Parents and Child, Showing Inheritance

Factors	Correlation between Parents & Child	Inherited
Education	-0.49	Yes
Socio-Economic Status	0.28	Slight
Income	0.08	No
Family Size	0.19	Slight
Occupational Prestige	0.21	Slight

If you want to make a table that is longer than a page, you will want to use the longtable environment. Uncomment the table below to see an example, or see our online documentation.

Table 2.2: An example of a long table, with headers that repeat on each subsequent page: Results from the summers of 1998 and 1999 work at Reed College done by Grace Brannigan, Robert Holiday and Lien Ngo in 1998 and Kate Brown and Christina Inman in 1999.

Chromium Hexacarbonyl			
State	Laser wavelength	Buffer gas	Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$
$z^7P_4^\circ$	266 nm	Argon	1.5
$z^7P_2^\circ$	355 nm	Argon	0.57
$y^7P_3^\circ$	266 nm	Argon	1
$y^7P_3^\circ$	355 nm	Argon	0.14
$y^7P_2^\circ$	355 nm	Argon	0.14
$z^5P_3^\circ$	266 nm	Argon	1.2
$z^5P_3^\circ$	355 nm	Argon	0.04
$z^5P_3^\circ$	355 nm	Helium	0.02
$z^5P_2^\circ$	355 nm	Argon	0.07
$z^5P_1^\circ$	355 nm	Argon	0.05
$y^5P_3^\circ$	355 nm	Argon	0.05, 0.4
$y^5P_3^\circ$	355 nm	Helium	0.25
$z^5F_4^\circ$	266 nm	Argon	1.4
$z^5F_4^\circ$	355 nm	Argon	0.29
$z^5F_4^\circ$	355 nm	Helium	1.02
$z^5D_4^\circ$	355 nm	Argon	0.3
$z^5D_4^\circ$	355 nm	Helium	0.65
$y^5H_7^\circ$	266 nm	Argon	0.17
$y^5H_7^\circ$	355 nm	Argon	0.13
$y^5H_7^\circ$	355 nm	Helium	0.11
a^5D_3	266 nm	Argon	0.71
a^5D_2	266 nm	Argon	0.77
a^5D_2	355 nm	Argon	0.63
a^3D_3	355 nm	Argon	0.05
a^5S_2	266 nm	Argon	2
a^5S_2	355 nm	Argon	1.5
a^5G_6	355 nm	Argon	0.91
a^3G_4	355 nm	Argon	0.08
e^7D_5	355 nm	Helium	3.5
e^7D_3	355 nm	Helium	3
f^7D_5	355 nm	Helium	0.25
f^7D_5	355 nm	Argon	0.25

State	Laser wavelength	Buffer gas	Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$
f^7D_4	355 nm	Argon	0.2
f^7D_4	355 nm	Helium	0.3
Propyl-ACT			
$z^7P_4^\circ$	355 nm	Argon	1.5
$z^7P_3^\circ$	355 nm	Argon	1.5
$z^7P_2^\circ$	355 nm	Argon	1.25
$z^7F_5^\circ$	355 nm	Argon	2.85
$y^7P_4^\circ$	355 nm	Argon	0.07
$y^7P_3^\circ$	355 nm	Argon	0.06
$z^5P_3^\circ$	355 nm	Argon	0.12
$z^5P_2^\circ$	355 nm	Argon	0.13
$z^5P_1^\circ$	355 nm	Argon	0.14
Methyl-ACT			
$z^7P_4^\circ$	355 nm	Argon	1.6, 2.5
$z^7P_4^\circ$	355 nm	Helium	3
$z^7P_4^\circ$	266 nm	Argon	1.33
$z^7P_3^\circ$	355 nm	Argon	1.5
$z^7P_2^\circ$	355 nm	Argon	1.25, 1.3
$z^7F_5^\circ$	355 nm	Argon	3
$y^7P_4^\circ$	355 nm	Argon	0.07, 0.08
$y^7P_4^\circ$	355 nm	Helium	0.2
$y^7P_3^\circ$	266 nm	Argon	1.22
$y^7P_3^\circ$	355 nm	Argon	0.08
$y^7P_2^\circ$	355 nm	Argon	0.1
$z^5P_3^\circ$	266 nm	Argon	0.67
$z^5P_3^\circ$	355 nm	Argon	0.08, 0.17
$z^5P_3^\circ$	355 nm	Helium	0.12
$z^5P_2^\circ$	355 nm	Argon	0.13
$z^5P_1^\circ$	355 nm	Argon	0.09
$y^5H_7^\circ$	355 nm	Argon	0.06, 0.05
a^5D_3	266 nm	Argon	2.5
a^5D_2	266 nm	Argon	1.9
a^5D_2	355 nm	Argon	1.17
a^5S_2	266 nm	Argon	2.3
a^5S_2	355 nm	Argon	1.11
a^5G_6	355 nm	Argon	1.6
e^7D_5	355 nm	Argon	1

2.2 Figures

If your thesis has a lot of figures, LaTeX might behave better for you than that other word processor. One thing that may be annoying is the way it handles "floats" like tables and figures. LaTeX will try to find the best place to put your object based on the text around it and until you're really, truly done writing you should just leave it where it lies. There are some optional arguments to the figure and table environments to specify where you want it to appear; see the comments in the first figure.

If you need a graphic or tabular material to be part of the text, you can just put it inline. If you need it to appear in the list of figures or tables, it should be placed in the floating environment.

To get a figure from StatView, JMP, SPSS or other statistics program into a figure, you can print to pdf or save the image as a jpg or png. Precisely how you will do this depends on the program: you may need to copy-paste figures into Photoshop or other graphic program, then save in the appropriate format.

Below we have put a few examples of figures. For more help using graphics and the float environment, see our online documentation.

And this is how you add a figure with a graphic:

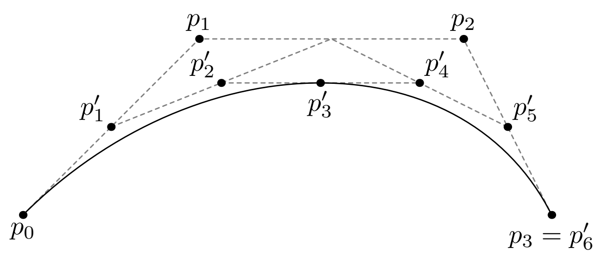

Figure 2.1: A Figure

2.3 More Figure Stuff

You can also scale and rotate figures.

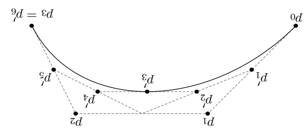

Figure 2.2: A Smaller Figure, Flipped Upside Down

2.4 Even More Figure Stuff

With some clever work you can crop a figure, which is handy if (for instance) your EPS or PDF is a little graphic on a whole sheet of paper. The viewport arguments are the lower-left and upper-right coordinates for the area you want to crop.

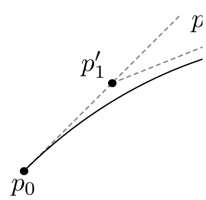

Figure 2.3: A Cropped Figure

2.4.1 Common Modifications

The following figure features the more popular changes thesis students want to their figures. This information is also on the web at `web.reed.edu/cis/help/latex/graphics.html`.

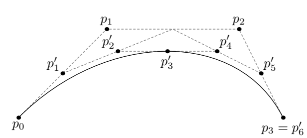

Figure 0.8: Interaction bar plot showing the degree of specialization for each flower type.

Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in LaTeX does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things LaTeX would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

Appendix A

The First Appendix

An appendix full of awesome

Appendix B

The Second Appendix, for Fun

An appendix full of win

References

Angel, E. (2000). *Interactive Computer Graphics : A Top-Down Approach with OpenGL*. Boston, MA: Addison Wesley Longman.

Angel, E. (2001a). *Batch-file Computer Graphics : A Bottom-Up Approach with QuickTime*. Boston, MA: Wesley Addison Longman.

Angel, E. (2001b). *test second book by angel*. Boston, MA: Wesley Addison Longman.

Deussen, O., & Strothotte, T. (2000). Computer-generated pen-and-ink illustration of trees. *"Proceedings of" SIGGRAPH 2000*, (pp. 13–18).

Fisher, R., Perkins, S., Walker, A., & Wolfart, E. (1997). *Hypermedia Image Processing Reference*. New York, NY: John Wiley & Sons.

Gooch, B., & Gooch, A. (2001a). *Non-Photorealistic Rendering*. Natick, Massachusetts: A K Peters.

Gooch, B., & Gooch, A. (2001b). *Test second book by gooches*. Natick, Massachusetts: A K Peters.

Hertzmann, A., & Zorin, D. (2000). Illustrating smooth surfaces. *Proceedings of SIGGRAPH 2000*, *5*(17), 517–526.

Jain, A. K. (1989). *Fundamentals of Digital Image Processing*. Englewood Cliffs, New Jersey: Prentice-Hall.

Molina, S. T., & Borkovec, T. D. (1994). The Penn State worry questionnaire: Psychometric properties and associated characteristics. In G. C. L. Davey, & F. Tallis (Eds.), *Worrying: Perspectives on theory, assessment and treatment*, (pp. 265–283). New York: Wiley.

Noble, S. G. (2002). *Turning images into simple line-art*. Undergraduate thesis, Reed College.

Reed College (2007). Latex your document. `http://web.reed.edu/cis/help/LaTeX/index.html`

Russ, J. C. (1995). *The Image Processing Handbook, Second Edition*. Boca Raton, Florida: CRC Press.

Salisbury, M. P., Wong, M. T., Hughes, J. F., & Salesin, D. H. (1997). Orientable textures for image-based pen-and-ink illustration. *"Proceedings of" SIGGRAPH 97*, (pp. 401–406).

Savitch, W. (2001). *JAVA: An Introduction to Computer Science & Programming*. Upper Saddle River, New Jersey: Prentice Hall.

Wong, E. (1999). *Artistic Rendering of Portrait Photographs*. Master's thesis, Cornell University.