

On Bouncing Oil Drops

A Thesis
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Bachelor of Arts

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Blog

This is the portion of the thesis that I will update regularly with rough notes, lit reviews, results, etc. some of which will be worked in to the real document after some polishing.

0.1 To Do

- Analyze data. Miguel 4/6/15
- Write Intro, Ch 1, Ch 3, and Conclusion. Miguel 4/6/15
- Learn de Broglie's interpretation of QM. Miguel 11/6/14

0.1.1 Done

- Take data. Miguel 12/2/14
- Figure out citations. 4/6/15
- Set up Accelerometer (Finally–ugh). Miguel 2/20/15
- Get New Silicone Oil. Miguel 1/10/15
- Send copy of Lit Rev to Daniel to proofread. 12/23/14
- Find walking regime. Miguel 11/20/14
- Order Accelerometer. Miguel 11/6/14
- Learn Basics of Bohmian Mechanics. Miguel 10/28/14
- Make tray. Miguel 10/20/14
- Sort out camera situation. Miguel 10/20/14
- Obtain flashdrives. Miguel 9/30/14
- Learn how to use the new L^AT_EX and Github setup. Miguel 9/30/14

0.2 Literature Review

(Important: Particle-wave association on a fluid interface (Protiere 2006)).

In 2005, Yves Couder showed that bouncing oil drops on vertically vibrating fluid bath exhibited properties analogous to the paradoxical properties seen only at the quantum scale (CITE: Dynamical phenomena: Walking and orbiting droplets?). Couder, John Bush, and others have shown that this system can reproduce double-slit single-particle interference, orbiting, tunneling, quantized orbits, spin, and more. The trajectory of the droplet can be modeled mathematically, and the dynamics of the walker have similarities to de Broglie's theory of quantum mechanics (CITE: Bush 2015).

The literature review will begin with a description of Faraday waves and the basic dynamics of a bouncing droplet and a walking droplet. Then we will describe in detail a few of the important quantum-like properties of this system.

0.2.1 Walking Droplets

It was Couder who then showed that an oil droplet could live for much longer. Long lifetimes meant that the focus could shift from how the droplet bounced (short time scale) to its interactions with other droplets and its motion (longer time scale).

Every time the droplet impacts the bath, it creates a radial traveling wave. If the bouncing droplet impacts the wavefield in such a way that it receives a lateral force from the slope of the wave, then it will be pushed to the side slightly. The next time the droplet makes contact with the bath, it will again make contact with a slope, and be pushed to the side. This propels the bouncing droplet, causing it to “walk” across the surface of the bath. These “walkers” turned out to have particularly interesting behaviours. Indeed, in 2006 Yves Couder and Emmanuel Fort showed that these droplets mimicked the behavior of electrons in the hallmark experiment of quantum weirdness: the double slit experiment. This was the first time that microscopic scale behavior had ever been seen at a macroscopic level, and it sparked interest in the experiment.

0.2.2 Macroscopic Quantum Scale Behaviors

Basic Parameters

Consider a fluid of density ρ , viscosity ν , and surface tension σ , in a bath of depth H driven vertically at an amplitude A_0 at frequency $f = \omega/2\pi$. By defining $\gamma = A_0\omega^2$, the effective gravity in the frame of reference of the bath is $g + \gamma \sin(\omega t)$.

The oil droplet of diameter D bounces in the regime $\gamma < \gamma_F$, where γ_F is the Faraday threshold (at this point, Faraday waves appear). The important experimental limits are outlined in Table 1.

For certain parameters, the bouncing drop will behave differently. The vibration number describes “the relative magnitude of the forcing frequency and the drop’s natural oscillation frequency,” and is given by:

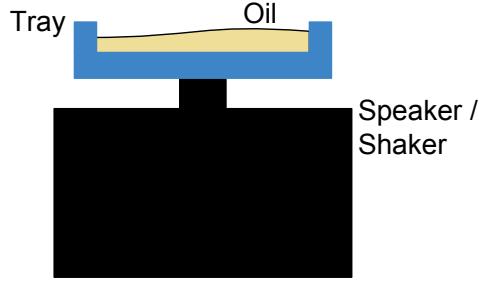


Figure 1: The experimental setup. The tray vibrates with an amplitude A_0 .

Table 1: Approximate Limits for Bouncing Drop Behavior

Parameter	Lower Limit	Upper Limit
Viscosity ν (cSt)	10	100
Bath Depth H (mm)	4	10
Frequency f (Hz)	20	150
Amplitude A_0 (mm)	0.1	1
Drop Diameter D (mm)	0.6	1.0

$$V_i = \frac{\omega}{2} \sqrt{\frac{\rho D^3}{2\sigma}} \quad (1)$$

The natural frequency of the droplet occurs around $V_i = 0.65$, where the droplet can exhibit both walking and bouncing behaviors. Setting up a plot with V_i on the y axis and (dimensionless) γ/g on the x axis can help in showing the behavior of the droplet, shown in Fig. 2.

The various modes seen in Fig. 2 can be described by (m,n) , where n is the number of times the droplet contacts the surface over period m/f . For example, in the $(1,1)$ mode, the droplet hits the oil bath once per driving period. In the $(2,2)$ mode, the drop makes two bounces of differing heights.

0.2.3 Path Memory

Path memory is a parameter that can be varied in this setup; it is essentially the damping of the system. Every time the droplet impacts the bath, it creates a radial traveling wave. Over the course of many bounces, a wavefield composed of a superposition of the many waves arises. In this way the wavefield “remembers” previous interactions. Because droplet motion is influenced by the wavefield, controlling the damping of the wavefield will influence the path of the walker. The heavily damped system has a low memory, while undamped system corresponds to higher memory.

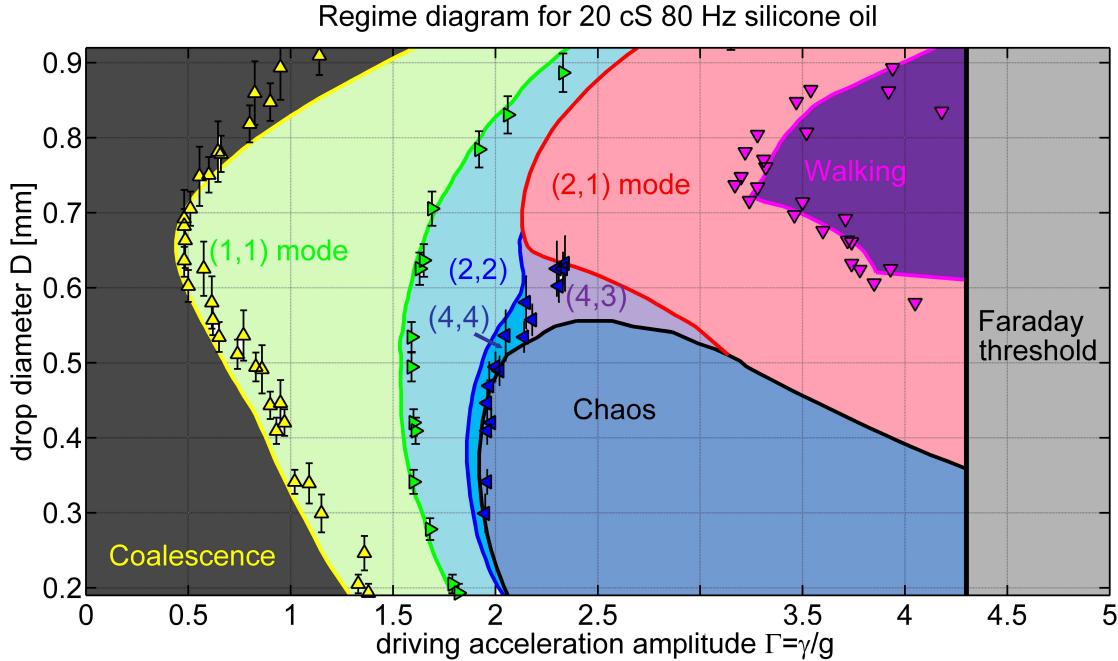


Figure 2: The different bouncing regimes for the oil drops of 20 cS silicon oil and at $f = \omega/2\pi = 80$ Hz. The parameters (m,n) describe the droplet that bounces n times in m forcing periods.

As one gets closer to the Faraday threshold, one achieves higher and higher memory because waves last for longer. The quantum like features of this experiment arise in the high-memory limit. (For more, Eddi et. al, 2011b: Information stored in Faraday waves.)

0.2.4 Bound States

A bouncing droplet creates a damped wave that depends on the driving acceleration (γ_m/g) CITE: Protiere 2005. A periodic damped wave allows for two bouncers to form a “bound state”. Starting far away from one another, the two droplets drift towards one another until a fixed distance d_0^{bd} . Increasing driving acceleration will decrease the value of d_0^{bd} . These bound bouncers form triangular lattices, and their periodicity is highly sensitive to the mass of the droplet. (LOOK AT EDDI ET ALL 2008).

Walkers can also form bound states. Two walkers of the same size that are approaching one another can form an orbit around their center of mass. Between the two droplets is the fixed distance d_n^{orb} given by

$$d_n^{orb} = (n - \epsilon_0)\lambda_F \quad (2)$$

where ϵ_0 is a fixed distance which is the same for all orbitals of these walkers (usually in the range $0.15 < \epsilon_0 < 0.25$ depending on droplet diameter), and $n = 1, 2, 3\dots$ for drops that are in phase or $n = 1/2, 3/2, 5/2\dots$ for drops out of phase. Orbiting periods

are approximately porportional to d_n^{orb} , which ends up meaning that the velocity of the orbiting walkers is a little less than the velocity of a free walker CITE: Protiere 2006. Orbiting of different sized droplets can also arise.

0.2.5 Scattering States

Two identical walkers headed towards each other can form fixed orbits, or they can scatter. The droplets are deflected through their wavefields (they never actually make contact with one another)

0.2.6 Single-Particle Diffraction

In 2006 Couder and Fort showed that the system had properties that were strikingly similar to two famous and controversial quantum experiments (Couder and Fort, 2006). They were able to demonstrate that a single walker travelling through one slit seemed to have its direction altered seemingly randomly, before continuing forward on its new path. By statistically analyzing many trials, Couder and Fort showed that the histogram of the “diffraction” actually resulted in a diffraction pattern strikingly similar to the single photon diffraction experiment performed by Taylor in 1909.

Next, Couder and Fort added a second slit next to the first one. Now a single walker could pass through one of two slits, and it was discovered that a histogram of this data returned another diffraction pattern. This result is of course reminiscent of one of the most famous experiments in physics: Young’s double slit diffraction with photons and electrons.

Using a numerical simulation, Couder and Fort were able to reproduce similar results.

As Couder and Fort mention in their paper: “A discussion of the relation between these single-particle experiments and those concerning elementary particles is unavoidable.” Important differences and similarities are then described between the quantum system and the quantum-like system. For the differences: we have a dissipative system, where energy is continually put in through the vibration of the tray; the particle can be followed;¹ it’s really effectively moving in two dimensions; the velocity is measurable; and the probability distribution is linked with the wave amplitude (rather than its intensity). And then of course, the similarities: an uncertainty principle arises from the statistical data (and without knowledge of the actual paths followed by the walkers); and some others that were unclear...

Recently, Harris attempted to reproduce single slit interference. With better technology, new results were found. Using a looping guiding bat, trajectories were found to follow the same loop without deviating. Only at a very high memory were there chaotic paths.

¹C and F note that it’d be impossible to detect the particle without disturbing it ”by any means at its scale,” like a buoy, for example. As the buoy floated it would interfere with the system by altering the wave pattern on the surface.

0.2.7 Tunneling

The guiding wave field can be partially reflected off of an edge or even a change in depth of the oil bath. This effect can be seen when a walker is pushed back from a under-the-surface step, seemingly without any contact. In rare cases, the walker will actually tunnel across the step; that is, it will continue to walk along the surface of the oil bath and pass over the step without reflection. In the first experiment done by Eddi et al., they demonstrated tunneling by building square “corrals” of varying thicknesses. In the second experiment, they built a rhombus shape which forced the walker across the center of a rhombus. The barrier was placed perpendicular to the direction of travel of the walker, so that it would hit the wall directly rather than at an angle (as in the square corral). “The tunneling probability decreases exponentially with the barrier width and increases as the Faraday threshold is approached.” Eddi et. al also found that the probability of tunneling increased with the velocity of the walker. (For more, Eddi et. al 2009b: Unpredictable Tunneling of a classical wave-particle association.)

The unpredictability of the tunneling comes from the complex interaction between the drop and its guiding wave.

0.2.8 Motion in a Confined Geometry

By tracking the droplet as it bounces around the tray over a period of time, one can look at the overall statistical behavior of the droplet. Two experiments tracked walkers in an experimental coral (Harris and Bush, 2013, Harris et al. 2013) in the high-memory, chaotic motion regime. A histogram of the statistical data shows that the ”probability of finding a walker at a given point in the corral is roughly prescribed by the amplitude of the Faraday wave mode of the cavity at the prescribed forcing frequency.”

Quantum corral experiments performed by Crommie et al. (Crommie et al. 1993 a b) present similar findings. In the experiment, electrons were confined in a Cu(III) substrate using barriers of iron adatoms. Using tunneling spectroscopy, the electrons were found to have a specific resonances depending on the corral shape. As in the case of Harris’ circular corral experiment where the corral and the Faraday wavelength, λ_F , dictate the wavelike statistical pattern, in the quantum experiment the corral and the *de Broglie* wavelength, λ_{dB} , dictate the form of the wavelike statistical pattern.

0.2.9 Walker Trajectories

In the regime of walkers we have $R_e \sim 20$, $B_0 \sim 0.1$, and $W_e \sim 0.1$. For the millimeric walkers, the dominant force comes from impact of the curvature of the surface. Gilet and Bush (2009: Chaotic bouncing of a drop on a soap film, and the fluid trampoline: droplets bouncing on a soap film) show that the surface of the vibrating oil can be modeled with a soap film, where the soap film acts like a linear spring.

As the oil bath is forced up and down, a tiny droplet of oil will “walk” across the surface. Moláček and Bush have developed an equation of a droplet that describes

the trajectory of the walking droplet, ignoring the vertical dynamics by time averaging them out (cite: J. Moláček and J. W. M. Bush, “Drops walking on a vibrating bath: towards a hydrodynamic pilot-wave theory” *J. FluidMech.* 727, 612-647 (2013).). The trajectory of the walking droplet of mass m at position $\mathbf{x}(t) = (x(t), y(t))$ is given by

$$m\ddot{\mathbf{x}} + D\dot{\mathbf{x}} = -mg\nabla h(\mathbf{x}, t) \quad (3)$$

where D describes the drag coefficient and $h(\mathbf{x}, t)$ describes the shape of the wavefield. Thus the second term describes the time averaged drag from both the flight and the impact of the droplet (as usual, depends on the velocity), and the third term describes the propulsive wave force resulting from drops landing on the inclined wave surface.

The wavefield is quite complicated because it depends on the memory. For a single impact of a droplet, Oza et al. argue the surface wave can be approximated with an integral of a monochromatic radial Bessel function of the first kind

$$h(\mathbf{x}, t) = \frac{F}{T_F} \int_{-\infty}^t J_0 \frac{(k_F |\mathbf{x}(t) - \mathbf{x}(s)|)}{|\mathbf{x}(t) - \mathbf{x}(s)|} (\mathbf{x}(t) - \mathbf{x}(s)) e^{-(t-s)/(T_F M_e)} ds \quad (4)$$

with F giving the wave force coefficient (estimated in the above source), T_F describing the Faraday period, and k_F describing the Faraday wave number determined by the Faraday wavelength $\lambda_F = 2/k_F$ (integral from A. U. Oza, D. M. Harris, R. R. Rosales, and J. W. M. Bush, ”Pilot-wave dynamics in a rotating frame: on the emergence of orbital quantization” *J. Fluid Mech.* 744, 404-429 (2014).). (Faraday was a popular guy.) Finally, that last term M_e is the nondimensional memory parameter $M_e = T_d/[T_F(1 - \gamma/\gamma_F)]$ (with T_d being the unforced decay time).

0.3 Experimental Setup

0.4 Bohmian Mechanics

0.4.1 Formalism

The Schrödinger Equation and ψ

We begin with the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi = \hat{H}\psi \quad (5)$$

where \hat{H} is the Hamiltonian and ψ is the wavefunction. The Hamiltonian can be expanded (assuming there is no electric field) to give

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) = \left[\frac{-\hbar^2}{2m} \nabla^2 + V(\mathbf{x}, t) \right] \psi(\mathbf{x}, t) \quad (6)$$

where $V(\mathbf{x}, t)$ is the potential energy of the system. The solution ψ is of the form:

$$\psi(\mathbf{x}, t) = R(\mathbf{x}, t) e^{iS(\mathbf{x}, t)/\hbar} \quad (7)$$

where S and R are real. Plugging in our equation for ψ into the Schrödinger equation (Eq. (6)) will produce two separate equations: one giving the time derivative of R and the second giving the time derivative of S . From these equations, a Hamilton-Jacobi equation can be written for a quantum system. Let's begin by computing the left hand side of Eq. (6) in terms of R and S .

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) &= i\hbar \left(\frac{\partial R}{\partial t} e^{iS/\hbar} + R \frac{i}{\hbar} \frac{\partial S}{\partial t} e^{iS/\hbar} \right) \\ &= i\hbar \left(\frac{1}{R} \frac{\partial R}{\partial t} + \frac{i}{\hbar} \frac{\partial S}{\partial t} \right) \psi(\mathbf{x}, t) \\ &= \left(i\hbar \frac{1}{R} \frac{\partial R}{\partial t} - \frac{\partial S}{\partial t} \right) \psi(\mathbf{x}, t) \end{aligned}$$

Let's leave that alone for a little bit, while we focus on the right hand side of Eq. (6). Since it's a little more complicated, we will start with one term of the right hand side:

$$\begin{aligned} \nabla^2 \psi(\mathbf{x}, t) &= e^{iS/\hbar} \nabla^2 R + \left(\frac{i}{\hbar} \right)^2 (\nabla S)^2 R e^{iS/\hbar} + \left(\frac{i}{\hbar} \right) R e^{iS/\hbar} (\nabla^2 S) + \left(\frac{2i}{\hbar} \right) (\nabla R \cdot \nabla S) R e^{iS/\hbar} \\ &= \left(\frac{\nabla^2 R}{R} - \left(\frac{\nabla S}{\hbar} \right)^2 + \left(\frac{i \nabla^2 S}{\hbar} \right) + 2i \left(\frac{\nabla R \cdot \nabla S}{\hbar} \right) \right) \psi(\mathbf{x}, t) \end{aligned}$$

Now the hard part is done, and we can say that the right hand side of Eq. (6) is given by

$$\left[\frac{-\hbar^2}{2m} \nabla^2 + V(\mathbf{x}, t) \right] \psi(\mathbf{x}, t) = \left[-\frac{\hbar^2 \nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m} \right) - i\hbar \left(\frac{\nabla^2 S}{2m} \right) - i\hbar \left(\frac{\nabla R \cdot \nabla S}{m} \right) + V(\mathbf{x}, t) \right] \psi(\mathbf{x}, t)$$

Ok, now that we've got that done, the next part will be super easy. Starting with Schrödinger's equation and plugging in left and right hand sides we calculated separately.

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) &= \left[\frac{-\hbar^2}{2m} \nabla^2 + V(\mathbf{x}, t) \right] \psi(\mathbf{x}, t) \\ \left(i\hbar \frac{1}{R} \frac{\partial R}{\partial t} - \frac{\partial S}{\partial t} \right) \psi(\mathbf{x}, t) &= \left[-\frac{\hbar^2 \nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m} \right) - i\hbar \left(\frac{\nabla^2 S}{2m} \right) - i\hbar \left(\frac{\nabla R \cdot \nabla S}{m} \right) + V(\mathbf{x}, t) \right] \psi(\mathbf{x}, t) \end{aligned}$$

Now we can divide out ψ from both sides

$$i\hbar \frac{1}{R} \frac{\partial R}{\partial t} - \frac{\partial S}{\partial t} = -\frac{\hbar^2 \nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m} \right) - i\hbar \left(\frac{\nabla^2 S}{2m} \right) - i\hbar \left(\frac{\nabla R \cdot \nabla S}{m} \right) + V(\mathbf{x}, t)$$

and group the imaginary numbers on the left side and the real numbers on the right side

$$i\hbar \frac{1}{R} \frac{\partial R}{\partial t} + i\hbar \left(\frac{\nabla^2 S}{2m} \right) + i\hbar \left(\frac{\nabla R \cdot \nabla S}{m} \right) = \frac{\partial S}{\partial t} - \frac{\hbar^2 \nabla^2 R}{2mR} + \left(\frac{(\nabla S)^2}{2m} \right) + V(\mathbf{x}, t)$$

Recall that both S and R are real. Note that the only way for all of the imaginary terms to equal all of the real terms is if they both equaled zero.

$$i\hbar \left(\frac{1}{R} \frac{\partial R}{\partial t} + \frac{\nabla^2 S}{2m} + \frac{\nabla R \cdot \nabla S}{m} \right) = \left(\frac{\partial S}{\partial t} - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{(\nabla S)^2}{2m} + V(\mathbf{x}, t) \right) = 0$$

This then gives us two separate equations, one for the time derivative of R and another for the time derivative of S .

$$\frac{\partial R}{\partial t} = -\frac{R}{2m} \left(\frac{\nabla^2 S}{m} - 2\nabla R \cdot \nabla S \right) \quad (8)$$

$$\frac{\partial S}{\partial t} = \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} - \frac{(\nabla S)^2}{2m} - V(\mathbf{x}, t) \quad (9)$$

What does this do for us? Both equations will provide helpful descriptions of our system.

The Quantum Potential

We can rewrite Eq. (9) in a provocative way

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V(\mathbf{x}, t) + \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} \quad (10)$$

which should look suspiciously familiar. If I were to tell you that ∇S had units of momentum and $\partial S/\partial t$ units of energy, then this equation would look a lot like a Hamiltonian! The first term takes care of the kinetic energy, the second is the potential energy term, but we have this mysterious third term which we haven't ever encountered in classical mechanics. If we define this term as our "quantum potential"

$$U(\mathbf{x}) = \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} = \frac{\hbar^2}{4m} \left[\frac{1}{2} \frac{\nabla^2 P}{P} - \frac{(\nabla P)^2}{P^2} \right] \quad (11)$$

then we really *can* think of Eq. (10) as a Hamiltonian with an extra potential term thrown in to make it "quantum." Note that in cases where \hbar is much smaller than the rest of the terms (i.e. not the quantum realm), then this quantum potential term goes away, and we are left with the regular Hamilton equation from classical mechanics.

Recall that when writing a Hamiltonian, the potential terms govern the forces on the particle. For a conservative system, the force is given by $\mathbf{F}(\mathbf{x}) = -\partial U/\partial \mathbf{x}$. If we include a quantum mechanical potential in our Hamiltonian, then this potential must cause a force on the particle in addition to the one supplied by the $V(\mathbf{x})$ term.

Continuity Equation

Plugging in the probability density $P(\mathbf{x}, t) = R^2(\mathbf{x}, t)$ into Eq. (8) also gives us something quite interesting.

MATH?

which we can finally express as

$$\frac{\partial P}{\partial t} + \nabla \cdot \left(P \frac{\nabla S}{m} \right) \quad (12)$$

In describing the quantum potential term it was mentioned that ∇S can be thought of as momentum, so then from our classical relationship between momentum and velocity $\mathbf{v}(\mathbf{x}, t) = \nabla S/m$ can be thought of as velocity. Then by defining the probability current as $j(\mathbf{x}, t) = P \nabla S/m$ then we recover

$$\frac{\partial P}{\partial t} + \nabla \cdot j(\mathbf{x}, t) \quad (13)$$

known as the continuity equation! This tells us that P is conserved over time.

Finding R and S

Introduction

Quantum is crazy. There is nothing like it. Particle-wave duality has incredible implications, documented by experiment.

(Important: Particle-wave association on a fluid interface (Protiere 2006)).

In 2005, Yves Couder showed that bouncing oil drops on vertically vibrating fluid bath exhibited properties analogous to the paradoxical properties seen only at the quantum scale (CITE: Dynamical phenomena: Walking and orbiting droplets?). Couder, John Bush, and others have shown that this system can reproduce double-slit single-particle interference, orbiting, tunneling, quantized orbits, spin, and more.

0.4.2 Quantum Motivation

0.4.3 Bouncing Droplets

Though it had been seen for at least a century, the phenomena of droplets bouncing on a fluid bath was first explained by Jearl Walker in 1978 [1]. The investigations began with a simple droplet of water falling onto a bath of water, and remaining just a second too long before coalescence.² It was then discovered that adding detergent to the water and then vibrating the bath would extend the lifetime of these droplets from fractions of a second to several minutes. Because these droplets are bouncing at frequencies of around 50 Hz (50 bounces per second) and the droplets are very small to begin with (with a diameter of a millimeter or less), it can be difficult to observe even the main mechanisms that drive the behaviour. A key insight by Walker was to flash a strobelight at a frequency slightly slower than the rate of vibration of the fluid bath, that way he could observe the droplet bouncing as if in slow motion.

Walker found that a trapped film of air kept the droplet and the bath from touching, as shown in Fig. 3. That is, the droplet is bouncing on a layer of air that's struggling to get out of the way but because the bounce happens so quickly, the fluid droplet and the fluid bath never touch. Walker concluded that the leakage rate of this trapped pocket of air depends on three factors: the nature of surface tension of the fluid bath, the viscosity of the droplet and the fluid bath, and the viscoicity of the air. The bath must be of uniform surface tension and free from particulate matter floating atop the bath, since both will lead to coalescence. Higher viscosity fluids translate to longer droplet lifetimes, since more viscous fluids keep air from escaping

²I don't drink coffee so I haven't seen this, but everyone seems to cite that this occurs with coffeemakers, as the coffee drips into the pot.

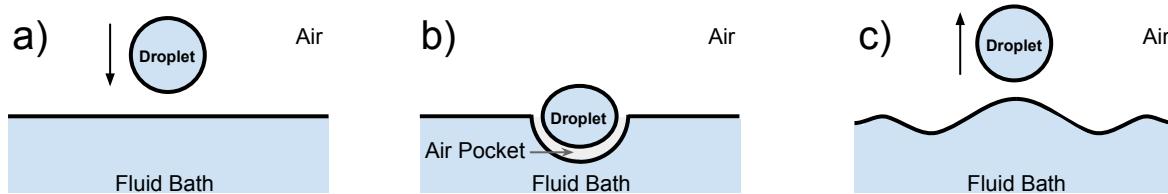


Figure 3: A depiction of a droplet bouncing on a bath of the same fluid. (a) A droplet falls onto a fluid bath. (b) An air pocket gets trapped underneath the droplet. (c) The droplet bounces back up off the cushion of air.

the pocket. Finally, adjusting the frequency and the amplitudes of the vibrations also affects droplet lifetime.³

More recent research suggests that a droplet fluid like silicone oil could bounce indefinitely on a vibrating bath [2]. The long lifetime occurs not only because silicone oil has a high viscosity, but also because it has a *low* surface tension. A low surface tension is beneficial because it makes the oil bath relatively immune to surfactants (surface acting agents) or contaminations which would otherwise make the surface tension nonuniform, and thus create a coalescence event.

0.4.4 Faraday Waves

For a vertically vibrated fluid bath, controlling the amplitude and the frequency of the motion will affect the behaviour of the fluid. Depending on a variety of factors (size of bath, fluid in bath, amplitude, etc.) each system has a specific amplitude (given a specific frequency), which if surpassed, will produce standing surface waves called Faraday waves [3].⁴⁵ A vibrating bath below this critical amplitude, also known as the Faraday instability, will be flat and motionless. A bath with a amplitude greater than the Faraday instability will have a turbulent surface with ripples and waves. An example of Faraday waves is shown in Fig. 4. Adjusting the frequency above the Faraday instability will change the size and shape of the Faraday waves. Note that Faraday waves can be created either by increasing amplitude to a critical level, or increasing frequency to a critical level.

0.4.5 Walking Droplets

A bouncing droplet will bounce differently depending on the frequency and amplitude of the vertical vibrations. If the parameters are set just a hair below the Faraday instability, then a very curious motion arises: the droplet seems to “walk” accross the

³Reedie Andrew Case ('92) wrote his thesis “Oil on Troubled Water: The Extension of Floating Drop Lifetimes Due to Interface Vibration” where he looked at droplet lifetime by the frequency of vibration.

⁴Faraday waves weren't actually discovered by Michael Faraday, they were originally discovered by X.

⁵Another Reed thesis, this one titled “Good Vibrations: A Visual Exploration of Faraday Waves” by Alison Saunders, compares mathematical formulation to experimental results.



Figure 4: A picture of Faraday waves in a dish of water at 80 Hz.

surface of the oil. As this **walker** bounces steadily in a straight line, it encounters a wall and pushes off in another direction, as if from a Newtonian collision. The droplet is being pushed by its own ripples, a dual effort in which neither can exist without the other. In essence, the walker is both a particle and a wave; a conjunction which has only ever been seen at the quantum scale.

0.4.6 Road Map

Recently, two main groups have been investigating this unique system. A group at MSC in Paris, France, headed by Yves Couder was the first to uncover some of the inherently “quantum” behaviours of these droplets, in 2005 [7]. Since 2010 John Bush’s team at MIT have been synthesizing previous experiments, creating a mathematical model, and performing their own investigations of the walker system. A large portion of the literature used in this investigation is drawn from, or at the very least, has its roots in these two groups.

The thesis documents an experimental investigation into a “tunneling” behaviour of this bouncing droplet system. While only one other paper really looks at this aspect [4], it falls short of completely examining the tunneling behaviour, focusing on barrier width and not examining barrier height. I hope to add the body of work in this subfield by studying how barrier height affects probability of tunneling.

This thesis is divided into three main chapters. **Chapter 1** gives a background of the hydrodynamic quantum analogs along with a brief survey of the relevant literature. In **Chapter 2**, I lay out the experimental design and explain the setup and the data taking procedures. Finally, the results and conclusions are presented in **Chapter 3**.

Chapter 1

Introduction to Tunneling in QM

1.1 Wavefunction

In classical mechanics, one can describe a particle with six variables: three position indicators (x, y, z), and three momentum indicators (p_x, p_y, p_z). One can find a function to represent variable by making multiple measurements across time, and discover an expression $x(t)$ that describes the particles location in space along the x dimension at time t . When all six variables can be described by a function, one can describe the position of the particle at any time.

In the standard, Copenhagen interpretation of quantum mechanics however, Heisenberg's uncertainty principle limits the knowledge of position and momentum:

$$\sigma_x \sigma_p \geq \hbar/2.$$

This equation states that as one knows more about the position of the system, then one loses knowledge about the momentum of the system, and vis versa. It's a tradeoff inherent to the nature of the system, not due to experimental deficiencies. And it means that the strategy of finding equations for position and momentum of a system are no longer possible. If a particle has no exact position, can one represent where it might be? One can use a wave describing the probability of finding the particle at that position. The square root of this wave is called the wavefunction, and is represented by $\Psi(x, t)$.

1.2 Schroedinger's Equation

The wavefunction $\Psi(x, t)$ is governed by the Schroedinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \Psi(x, t)$$

where $V(x, t)$ is the given potential, m is the mass of the particle, and \hbar is the reduced Planck's constant. For potentials that do not change in time, one can use separation

of variables to arrive at the time-independent Schrödinger equation

$$E\Psi(x, t) = \left[\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x, t)$$

where E is the total energy. The goal is to find $\Psi(x, t)$ knowing the potential $V(x)$.

1.3 Barrier Potential

Given a potential barrier of width $2a$ and height V_0 , we can find a probability of a particle reflecting back from the barrier and also the probability of the particle tunneling through the barrier. Since there are only two options, we know that the probability of reflection R and the probability of transmission T must sum to 1.

$$T + R = 1$$

This is a time independent situation, so we can get away with using the time independent schrödinger equation. We can find R by solving this equation in three separate regions ($x < 0$, $0 < x < 2a$, $2a < x$), and piecing them together using continuity of ψ and the first derivative of ψ at the two boundaries. Using the value of R , we can easily compute the value of T .

1.3.1 Continuity of ψ

We will go ahead and assume that the energy of the system E is greater than the potential energy of the barrier (otherwise, we would not see any transmission). Starting with the continuity of ψ at 0, we write that $\psi_I(0) = \psi_{II}(0)$ which gives us:

$$A + B = D, \quad (1.1)$$

while at the other end of the barrier we write $\psi_{II}(2a) = \psi_{III}(2a)$ giving:

$$C\sin(2k_2a) + D\cos(2k_2a) = F e^{2aik_1}. \quad (1.2)$$

1.3.2 Continuity of ψ'

At the front of the barrier, our expression $\psi'_I(0) = \psi'_{II}(0)$ simplifies to:

$$C = \frac{ik_1}{k_2}(A - B), \quad (1.3)$$

and finally, our last continuity equation is written as $\psi'_{II}(2a) = \psi'_{III}(2a)$ and gives us:

$$C\cos(2k_2a) - D\sin(2k_2a) = \frac{ik_1}{k_2} F e^{2aik_1}. \quad (1.4)$$

1.3.3 Finding R

We know that the probability of reflection is the wave being reflected (represented by our coefficient B) divided by the total incoming wave (the coefficient A).

$$R = \left| \frac{B}{A} \right|^2$$

Finding R requires that we solve for A in terms of B . We can do this by plugging Eq. (1.1) into Eq. (1.3) to eliminate C and D and make a new equation. Plug this equation into Eq. (1.2), and then again into Eq. (1.4) to find two new equations. Using these two new equations, we find an expression:

$$A \frac{k_1^2 - k_2^2}{k_2^2} \sin(2k_2 a) = B \left[\frac{k_1^2 + k_2^2}{k_2^2} \sin(2k_2 a) + \frac{2ik_1}{k_2} \cos(2k_2 a) \right]$$

We can rearrange this expression to solve for $\frac{B}{A}$, and squaring that to find R . Using the relation $T = 1 - R$ we find that:

$$T^{-1} = 1 + \frac{V_0^2}{4E(E - V_0)} \sin^2 \left(\frac{2a}{\hbar} \sqrt{2m(E - V_0)} \right) \quad (1.5)$$

1.4 Physics

Many of the symbols you will need can be found on the math page (<http://web.reed.edu/cis/help/latex/math.html>) and the Comprehensive L^AT_EX Symbol Guide (enclosed in this template download). You may wish to create custom commands for commonly used symbols, phrases or equations, as described in Chapter ??.

Chapter 2

Experimental Design

The bouncing droplet system allows the observation of surface waves guiding a droplet, and we're interested in learning more about the droplet-wave system in different settings. In the experiments discussed herein, we will look at how features *underneath* the surface of the oil (i.e. on the “floor” of the tray) affect the motion of the droplet.

A raised object on the floor of the tray (but still underneath the surface of the oil) can have an effect on height of the surface waves, and thus, on the motion of the walker [4]. Sometimes a droplet headed towards a raised object will be reflected backwards, as if from a collision with the object. For this reason, we refer to a raised object as a barrier. Oftentimes however, the droplet continues on and crosses the barrier without a collision. This is analogous to “transmission” in the quantum mechanical process of tunneling. In other words, for a given barrier we will see a probability of tunneling unique to that barrier. Other studies have shown that increasing barrier width decreases probability of tunneling [4]. This study looks at how height of the barrier affects the tunneling probability.

To test the effect of a barrier's height on the probability of tunneling, I used a combination of procedures from the investigations of Bush [5], Couder [?], and specifically, Eddi et al. [4]. These were slightly modified to fit some of the unique features of my experiment. In this section, I aim to give some of the reasoning behind the design of the experimental apparatus and data collection techniques, both of which are not well described in the literature.

2.1 Setup

The schematic of the experimental setup is shown in Fig. 2.1, and a picture of the actual setup is shown in Fig. 2.3(a). A waveform generator creates a sinusoidal signal which is amplified and fed into a shaker. The shaker vibrates the tray vertically. Both the frequency and the amplitude of the vertical oscillations can be controlled. An accelerometer records the vertical acceleration of the tray and is read by an oscilloscope. A camera records the droplet as it bounces along the surface of the oil.

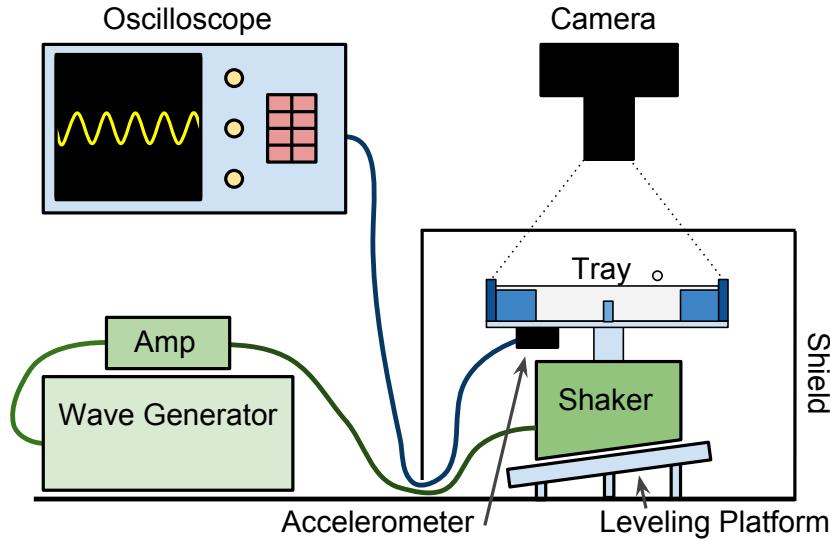


Figure 2.1: The experimental setup. The amplified signal from the wave generator drives the shaker, which shakes the oil-filled tray. The accelerometer generates a signal which is read by the oscilloscope. The shield blocks disturbances to the experiment, while allowing the camera to document the trials.

2.2 Materials

The key components of this experiment are the shaker, the oil, and the tray. In this section I will describe the specifics of this holy trinity, as well as some of the additional components used in data collection.

2.2.1 Tray

The tray was fabricated from acrylic plastic that were cut on the Trotek Rayjet 300 laser cutter. They were then glued together with Scigrip Weld-On 3 assembly adhesive. The tray's design, which was based off of the tray in the tunneling experiment done by Eddi et al. [4], naturally guides the droplet into a perpendicular collision with the barrier. The tray schematic is shown in Fig. 2.2.

A thin layer of oil spills over the constraining rhombus shape. As long as the layer is thin enough, the droplet will remain in the rhombus container, but the waves will continue to propagate unimpeded. This gives the waves time to decay, and means that the droplets motion is not affected by reflections of previous waves from the sidewalls, and is instead guided only by the unreflected waves.

The rhombus shape serves to steer the droplet into a perpendicular collision with the barrier. This forces the droplet to pin-ball into the acute corner of the rhombus and shoot out towards the barrier as shown in Fig. 2.3(c).

I designed my experiment to test barriers of three different heights: 2.75 mm, 3.0 mm, and 3.25 mm, measured from the bottom of the rhombus. A thin barrier of plastic made by the laser cutter has the tendency to bend and warp over time.

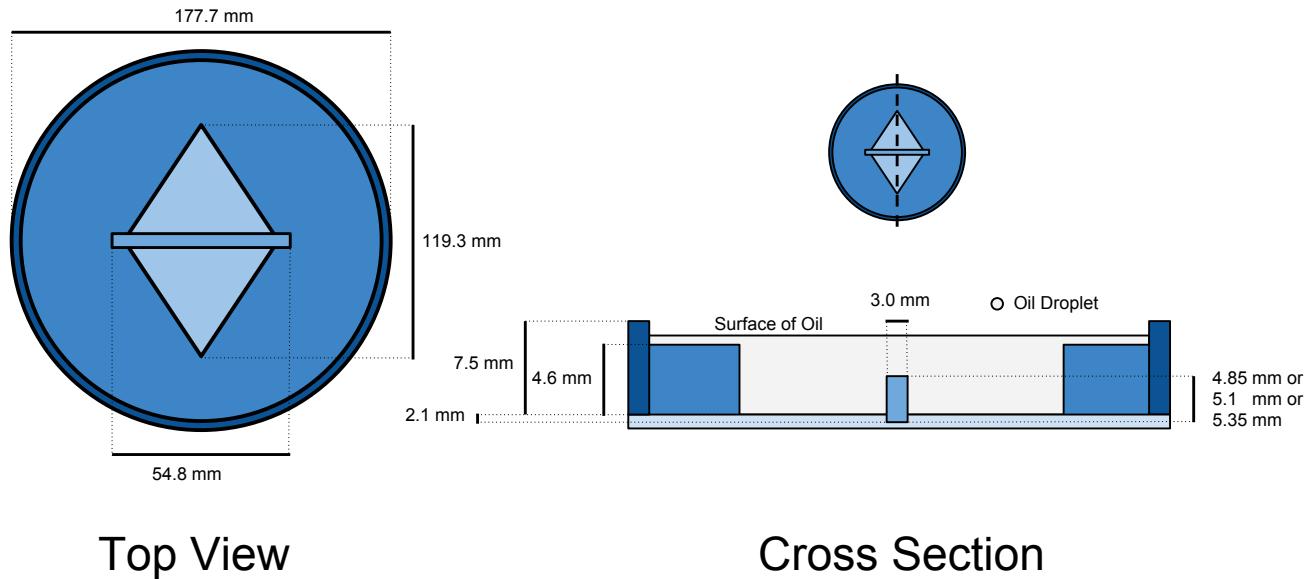


Figure 2.2: The specifications of the tray design. The top view (left) highlights the main elements in the tray, while the cross section (right) illustrates the topography of the tray. Depth is represented by the shading; darker shading is shallower.

The solution to this problem was to make these barriers taller than the specified heights, and create an cut-out in the rhombus so they could be inserted. The barrier cut-outs were deep enough to exactly counter the added height of the barrier, so the barriers still had (when measured from the surface of the rhombus) heights of 2.75 mm, 3.0 mm, and 3.25 mm. This also solved the problem of fixing the barriers in place, while still allowing them to be easily removed. The particular heights of the barriers were chosen because they allowed for both passage over and blockage by the barriers depending on the parameters. Other barriers that were too tall blocked all of the droplets, while barriers that were too short did nothing to prevent the droplet from crossing over.

The bottom of the tray was painted black in order to improve contrast, allowing the droplet to be more easily tracked by eye and using a camera.

2.2.2 Silicone Oil

Silicone oil is the ideal choice of fluid for this experiment because it remains clean, it has a low vapor pressure so it does not evaporate, and it can be purchased at specific viscosities. The silicone oil used in this experiment had a viscosity of 20 cSt (its viscosity is a little closer to water than olive oil) and was purchased from Clearco Products Co. Inc., Bensalem PA (CAS No: 63148-62-9). 20 cSt silicone oil, like the one used by Bush et al. [5] was chosen because it exhibits walking behaviour under a wider range of parameters [5] than more viscous oil, such as the 50 cSt viscosity oil used by Couder [?]. Depending on the height of the barrier and the desired height of oil above the barrier, the tray requires between 17 to 18 mL of fluid. The volume

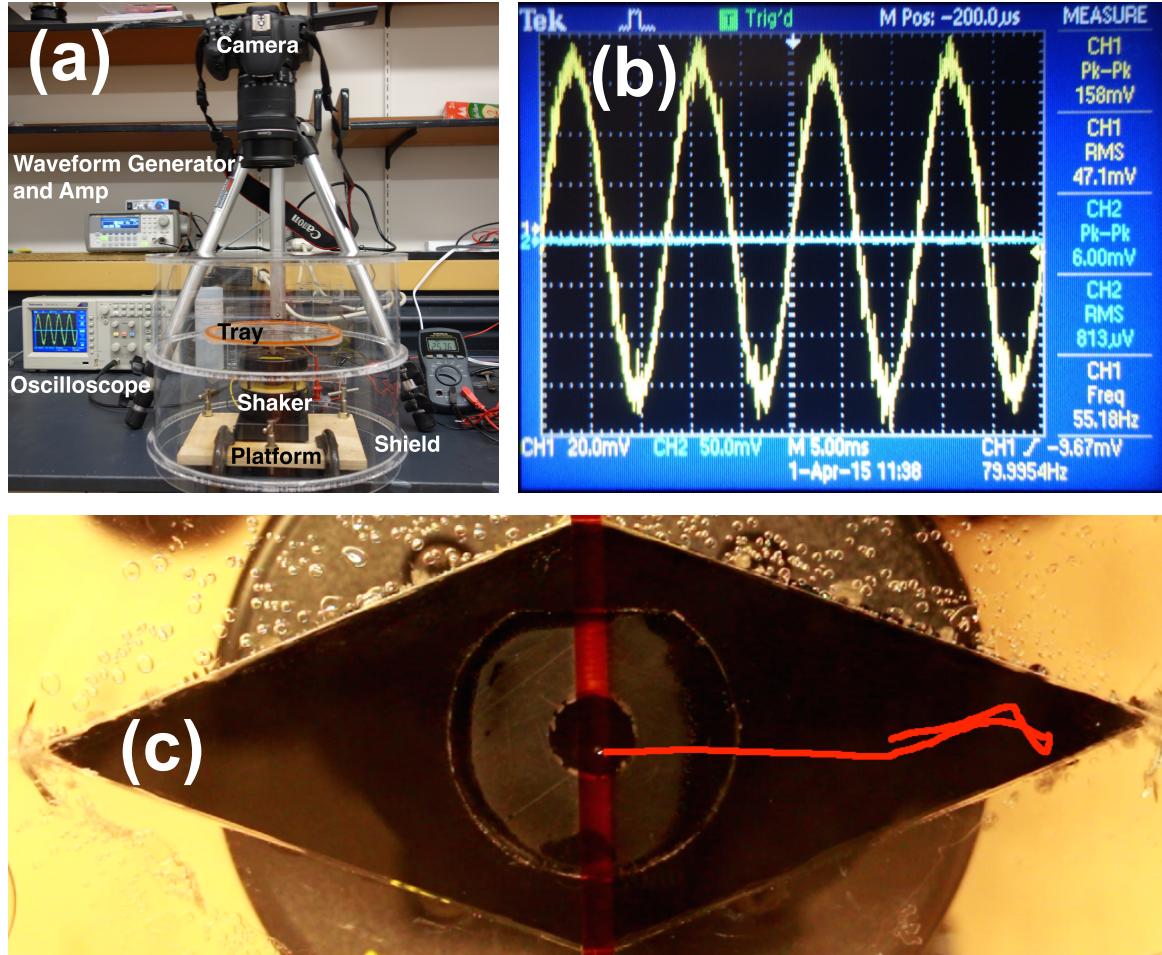


Figure 2.3: (a) The actual experimental setup. (b) The screen of the oscilloscope, showing the output from the accelerometer. The sine wave is the acceleration of the tray, and records a frequency 79.9954 Hz and a peak to peak voltage of 158 mV. (c) The path of a droplet of width 0.87 mm, highlighted in red, shows the droplet's motion as it walks into a corner before shooting out directly towards the barrier (orange).

of oil required for a total fluid height of 4.25 mm from the bottom surface of the tray (that is, 0.25 mm above height of the rhombus cut-out) was calculated for each height, using the dimensions of the components in the tray, and were found to be: 17.907 mL for the 2.75 mm barrier, 17.940 mL for the 3.0 mm barrier, and 17.974 mL for the 3.25 mm barrier. This means that the effective height of oil above each barrier is 1.5, 1.25, and 1.0 mm, respectively, while remaining unchanged with respect to the rest of the tray.

It is of vital importance to keep the oil as clean as possible since surface contamination can lead to droplet coalescence. This means protecting from particulate matter that is already in the tray. Contamination can be minimized by cleaning the tray before filling it and shielding it from the ambient dust.

2.2.3 Shaker

To shake the tray, we used a mechanical wave driver made by Pasco Scientific, Roseville CA, model SF-9324. This shaker was designed to drive a string or an elastic cord, not a 200 gram tray with oil inside, which was probably at the limit of what the shaker can handle.

2.2.4 Waveform Generator and Amplifier

The shaker was driven with an Agilent Arbitrary Waveform Generator model 33210. This was controlled digitally and was found to be more stable than other available frequency generators. The waveform generator was usually set to produce a sine wave of 80 Hz. The signal from the waveform generator was amplified using a Lepai LP2020A+ digital amplifier. This precise control of the the amplitude of the tray. This signal, which was also measured with a multimeter, was then fed into the shaker.

2.2.5 Accelerometer

Knowing the tray's acceleration allows us to characterize the behavior of our system. To measure acceleration, we attached an ADXL 326 triple axis accelerometer (made by Adafruit, New York City NY) to the bottom of the tray. Using screws, provided a much firmer hold than tape or glue while allowing for removal. The ADXL 326 has a range of $\pm 16g$, which is ideal for measuring the accelerations in our setup, usually below $5g$'s.

The signal from the z-axis of the accelerometer was measured directly on an oscilloscope, a sample output can be seen in Fig. 2.3(b). For the vibrating tray, the output was approximately sinusoidal (as expected). The spec sheet for the accelerometer indicates that the sensitivity can be translated to 57 ± 6 mV/g.

2.2.6 Shield

A large, see-through cylinder (covered at one end) was manufactured using the laser cutter. When placed over the tray, it served the purpose of keeping the oil clean from particulate matter and preventing air currents from influencing the motion of the walker.

2.2.7 Leveling Platform

A wooden leveling platform supported the shaker. Three adjustment screws allowed for precise adjustment of tilt of the apparatus. The tray was tuned using a level placed inside the center of the tray (before the oil was added).

2.2.8 Camera

To document trials we used a Canon EOS Rebel T5i DSLR camera supported on a tripod and aimed directly down at the tray. Attached to the camera was a Canon

18-135 mm lens. Set with a Tv configuration (Time Value – allows for shutter control) and in video mode, the image could be zoomed and manually focused on the bouncing droplet.

2.3 Procedure

Once the exact walking parameters are established (frequency and driving amplitude), tunneling measurements at a few different barrier heights can be made.

2.3.1 Finding the Walking Regime

Before investigating the rate of tunneling using different barriers, a rough estimate of the walking regime at a frequency of 80 Hz must be made. A “map” similar to the one in Fig. 2 will be sketched out, but rather than looking at all of the different kinds of bouncing we will limit ourselves to only the walking regime. Reproducing this figure allows us to find the parameters that are specific to our unique setup, which could have slightly different height, tray, oil, and shaker configurations than those used in the literature.

Droplet size is measured using a recorded video of the walking droplet in motion. By comparing the number of pixels making up the diameter of the droplet (unknown measurement) to the number of pixels making up the diameter of the tray (known measurement), we can estimate the length associated with each pixel, and thus find the diameter of the droplet in centimeters.

Driving acceleration values are measured by the accelerometer and displayed on the oscilloscope.

To ensure that every trial has the same oil depth, we must measure the volume of the oil before filling the tray. Knowing the volume of the tray and of each barrier, we can get a value for the oil depth without interfering with the system. In this way, oil height can be kept constant.

2.3.2 The Experiment

Tunneling was examined for three different barrier heights. At each height (and at a constant frequency of 80 Hz and constant driving acceleration), a string of continuous collisions were filmed with the camera. From this data, a basic tunneling probability was calculated, which provides the most simplistic analysis of this system.

The tray is designed such that most of the droplet’s collisions with the barrier occur “head on” (i.e. perpendicular to the length of the barrier), but not all collisions unfold ideally. A more involved analysis in *Tracker* requires looking at the component of velocity of the droplet perpendicular to the length of the barrier, and determining the probability of tunneling given this value. Since not all collisions in the simplistic analysis occur at the same velocity, this method allows for a more methodical analysis of the phenomena.

Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in L^AT_EX does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things L^AT_EX would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't* be indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

Appendix A

The First Appendix

An appendix full of awesome

Appendix B

The Second Appendix, for Fun

An appendix full of win

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