

On Bouncing Oil Drops

A Thesis
Presented to
The Division of Mathematics and Natural Sciences
Reed College

In Partial Fulfillment
of the Requirements for the Degree
Bachelor of Arts

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May 2015

Approved for the Division
(Physics)

Daniel Borrero

Blog

This is the portion of the thesis that I will update regularly with rough notes, lit reviews, results, etc. some of which will be worked in to the real document after some polishing.

0.1 Goals

- 10/14: Define specific problem.
- Fall Break: Experimental Setup
- Winter Break: Lit Review Complete

0.2 To Do

- Obtain flashdrives. Miguel 9/30/14
- Set up digital camera. Miguel 9/30/14
- Double Sided Tape. Miguel 10/7/14
- Conference Funds. Miguel 10/7/14
- Accelerometer. Miguel 10/7/14
- Tray geometry for aiming droplets. Miguel 10/7/14

0.2.1 Done

- Learn how to use the new L^AT_EX and Github setup. Miguel 9/30/14

0.3 Literature Reviews

0.3.1 Particle-wave association on a fluid interface (Protiere 2006)

0.3.2 Pilot-Wave Hydrodynamics (Bush 2015)

In this annual review, Bush describes some of the characteristics of the bouncing oil drop experiment that are analagous to effects witnessed in the quantum mechanical world (single-particle diffraction, tunneling, quantized orbits, orbital level splitting, and spin states). Dynamics of the walker are described mathematically. Finally, comparisons to de Broglie's original formulation of QM (and not Bohm's) and Stochastic Electrodynamics (?) are made.

Basic Parameters

Consider a fluid of density ρ , viscosity ν , and surface tension σ , in a bath of depth H driven vertically at an amplitude A_0 at frequency $f = \omega/2\pi$. By defining $\gamma = A_0\omega^2$, the effective gravity in the frame of reference of the bath is $g + \gamma \sin(\omega t)$. The oil droplet of diameter D bounces in the regime $\gamma < \gamma_F$, where γ_F is the Faraday threshold (at this point, Fraday waves appear). The important experimental limits are outlined in Table 1.

Table 1: Approximate Limits for Bouncing Drop Behavior

Parameter	Lower Limit	Upper Limit
Viscosity ν (cSt)	10	100
Bath Depth H (mm)	4	10
Frequency f (Hz)	20	150
Amplitude A_0 (mm)	0.1	1
Drop Diameter D (mm)	0.6	1.0

For certain parameters, the bouncing drop will behave differently. The vibration number describes “the relative magnitude of the forcing frequency and the drop’s natural oscillation frequency,” and is given by:

$$V_i = \frac{\omega}{2} \sqrt{\frac{\rho D^3}{2\sigma}} \quad (1)$$

The natural frequency of the droplet occurs around $V_i = 0.65$, where the droplet can exhibit both walking and bouncing behaviors. Setting up a plot with V_i on the y axis and (dimensionless) $\frac{\gamma}{g}$ on the x axis can help in showing the behavior of the droplet, shown in Fig. 1.

The various modes seen in Fig. 1 can be described by (m,n) , where n is the number to times the droplet contacts the surface over period m/f . For example, in the (1,1)

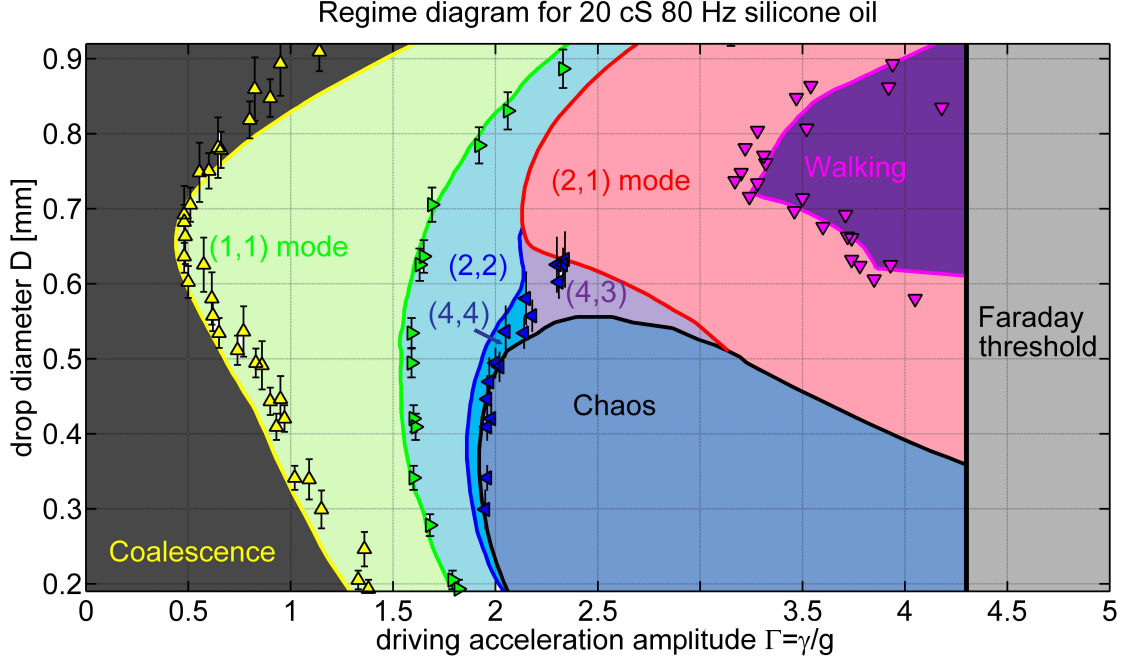


Figure 1: The different bouncing regimes for the oil drops of 20 cS silicon oil and at $f = \omega/2\pi = 80$ Hz. The parameters (m,n) describe the droplet that bounces n times in m forcing periods.

mode, the droplet hits the oil bath once per driving period. In the (2,2) mode, the drop makes two bounces of differing heights.

0.3.3 Path Memory

Path memory is a parameter that can be varied in this setup. Every time the droplet impacts the bath, it creates a radial traveling wave. Over the course of many bounces, a wavefield composed of a superposition of the many waves arises. If the bouncing droplet impacts the wavefield in such a way that it receives a lateral force from the slope of the wave, then it will be pushed to the side slightly. The next time the droplet makes contact with the bath, it will again be pushed to the side. This propels the bouncing droplet, causing it to walk across the surface of the bath. These “walkers” are pushed in a direction dictated by previous impacts, and so it is said that the oil bath “remembers” the previous bounce.

Damping the waves results in a low-memory limit, since the walker is affected by only the previous impact. In an undamped system (approaching the Faraday threshold), the system becomes high-memory because walker can be affected by waves from many impacts ago. The quantum like features of this experiment arise in the high-memory limit. (For more, Eddi et. al, 2011b: Information stored in Faraday waves.)

0.3.4 Single-Particle Diffraction

0.3.5 Tunneling

The guiding wave field can be partially reflected off of an edge or even a change in depth of the oil bath. This effect can be seen when a walker is pushed back from a under-the-surface step, seemingly without any contact. In rare cases, the walker will actually tunnel across the step; that is, it will continue to walk along the surface of the oil bath and pass over the step without reflection. In the first experiment done by Eddi et al., they demonstrated tunneling by building square "corrals" of varying thicknesses. In the second experiment, they built a rhombus shape which forced the walker across the center of a rhombus. The barrier was placed perpendicular to the direction of travel of the walker, so that it would hit the wall directly rather than at an angle (as in the square corral). "The tunneling probability decreases exponentially with the barrier width and increases as the Faraday threshold is approached." Eddi et. al also found that the probability of tunneling increased with the velocity of the walker. (For more, Eddi et. al 2009b: Unpredictable Tunneling of a classical wave-particle association)

The unpredictability of the tunneling comes from the complex interaction between the drop and its guiding wave.

0.3.6 Camera Modification

Chapter 1

Mathematics and Science

1.1 Math

T_EX is the best way to typeset mathematics. Donald Knuth designed T_EX when he got frustrated at how long it was taking the typesetters to finish his book, which contained a lot of mathematics.

If you are doing a thesis that will involve lots of math, you will want to read the following section which has been commented out. If you're not going to use math, skip over this next big red section. (It's red in the .tex file but does not show up in the .pdf.)

$$\sum_{j=1}^n (\delta\theta_j)^2 \leq \frac{\beta_i^2}{\delta_i^2 + \rho_i^2} \left[2\rho_i^2 + \frac{\delta_i^2 \beta_i^2}{\delta_i^2 + \rho_i^2} \right] \equiv \omega_i^2$$

From Informational Dynamics, we have the following (Dave Braden):
After n such encounters the posterior density for θ is

$$\pi(\theta|X_1 < y_1, \dots, X_n < y_n) \propto \pi(\theta) \prod_{i=1}^n \int_{-\infty}^{y_i} \exp\left(-\frac{(x-\theta)^2}{2\sigma^2}\right) dx$$

Another equation:

$$\det \begin{vmatrix} c_0 & c_1 & c_2 & \dots & c_n \\ c_1 & c_2 & c_3 & \dots & c_{n+1} \\ c_2 & c_3 & c_4 & \dots & c_{n+2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_n & c_{n+1} & c_{n+2} & \dots & c_{2n} \end{vmatrix} > 0$$

Lapidus and Pindar, Numerical Solution of Partial Differential Equations in Science and Engineering. Page 54

$$\int_t \left\{ \sum_{j=1}^3 T_j \left(\frac{d\phi_j}{dt} + k\phi_j \right) - kT_e \right\} w_i(t) dt = 0, \quad i = 1, 2, 3.$$

L&P Galerkin method weighting functions. Page 55

$$\sum_{j=1}^3 T_j \int_0^1 \left\{ \frac{d\phi_j}{dt} + k\phi_j \right\} \phi_i dt = \int_0^1 k T_e \phi_i dt, \quad i = 1, 2, 3$$

Another L&P (p145)

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 f(\xi, \eta, \zeta) = \sum_{k=1}^n \sum_{j=1}^n \sum_{i=1}^n w_i w_j w_k f(\xi, \eta, \zeta).$$

Another L&P (p126)

$$\int_{A_e} (\cdot) dx dy = \int_{-1}^1 \int_{-1}^1 (\cdot) \det[J] d\xi d\eta.$$

1.2 Chemistry 101: Symbols

Chemical formulas will look best if they are not italicized. Get around math mode's automatic italicizing by using the argument `$\mathrm{formula here}$` , with your formula inside the curly brackets.

So, $\mathrm{Fe}_2^{2+}\mathrm{Cr}_2\mathrm{O}_4$ is written `$\mathrm{Fe}_2^{\sim\{2+\}\mathrm{Cr}_{20}_4}$`

Exponent or Superscript: O^-

Subscript: CH_4

To stack numbers or letters as in Fe_2^{2+} , the subscript is defined first, and then the superscript is defined.

Angstrom: \AA

Bullet: $\mathrm{CuCl} \bullet 7\mathrm{H}_2\mathrm{O}$

Double Dagger: \ddagger

Delta: Δ

Reaction Arrows: \longrightarrow or $\xrightarrow{\text{solution}}$

Resonance Arrows: \leftrightarrow

Reversible Reaction Arrows: \rightleftharpoons or $\xrightleftharpoons{\text{solution}}$ (the latter requires the chemarr package)

1.2.1 Typesetting reactions

You may wish to put your reaction in a figure environment, which means that LaTeX will place the reaction where it fits and you can have a figure legend if desired:

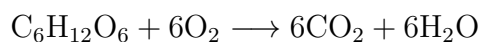
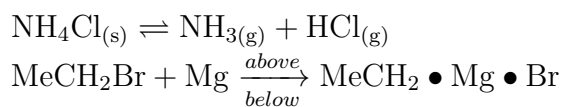


Figure 1.1: Combustion of glucose

1.2.2 Other examples of reactions



1.3 Physics

Many of the symbols you will need can be found on the math page (<http://web.reed.edu/cis/help/latex/math.html>) and the Comprehensive L^AT_EX Symbol Guide (enclosed in this template download). You may wish to create custom commands for commonly used symbols, phrases or equations, as described in Chapter ??.

1.4 Biology

You will probably find the resources at <http://www.lecb.ncifcrf.gov/~toms/latex.html> helpful, particularly the links to bst's for various journals. You may also be interested in TeXShade for nucleotide typesetting (<http://homepages.uni-tuebingen.de/beitz/txe.html>). Be sure to read the proceeding chapter on graphics and tables, and remember that the thesis template has versions of Ecology and Science bst's which support webpage citation formats.

Chapter 2

Tables and Graphics

2.1 Tables

The following section contains examples of tables, most of which have been commented out for brevity. (They will show up in the .tex document in red, but not at all in the .pdf). For more help in constructing a table (or anything else in this document), please see the LaTeX pages on the CUS site.

Table 2.1: A Basic Table: Correlation of Factors between Parents and Child, Showing Inheritance

Factors	Correlation between Parents & Child	Inherited
Education	-0.49	Yes
Socio-Economic Status	0.28	Slight
Income	0.08	No
Family Size	0.19	Slight
Occupational Prestige	0.21	Slight

If you want to make a table that is longer than a page, you will want to use the longtable environment. Uncomment the table below to see an example, or see our online documentation.

Table 2.2: An example of a long table, with headers that repeat on each subsequent page: Results from the summers of 1998 and 1999 work at Reed College done by Grace Brannigan, Robert Holiday and Lien Ngo in 1998 and Kate Brown and Christina Inman in 1999.

Chromium Hexacarbonyl			
State	Laser wavelength	Buffer gas	Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$
$z^7P_4^\circ$	266 nm	Argon	1.5
$z^7P_2^\circ$	355 nm	Argon	0.57
$y^7P_3^\circ$	266 nm	Argon	1
$y^7P_3^\circ$	355 nm	Argon	0.14
$y^7P_2^\circ$	355 nm	Argon	0.14
$z^5P_3^\circ$	266 nm	Argon	1.2
$z^5P_3^\circ$	355 nm	Argon	0.04
$z^5P_3^\circ$	355 nm	Helium	0.02
$z^5P_2^\circ$	355 nm	Argon	0.07
$z^5P_1^\circ$	355 nm	Argon	0.05
$y^5P_3^\circ$	355 nm	Argon	0.05, 0.4
$y^5P_3^\circ$	355 nm	Helium	0.25
$z^5F_4^\circ$	266 nm	Argon	1.4
$z^5F_4^\circ$	355 nm	Argon	0.29
$z^5F_4^\circ$	355 nm	Helium	1.02
$z^5D_4^\circ$	355 nm	Argon	0.3
$z^5D_4^\circ$	355 nm	Helium	0.65
$y^5H_7^\circ$	266 nm	Argon	0.17
$y^5H_7^\circ$	355 nm	Argon	0.13
$y^5H_7^\circ$	355 nm	Helium	0.11
a^5D_3	266 nm	Argon	0.71
a^5D_2	266 nm	Argon	0.77
a^5D_2	355 nm	Argon	0.63
a^3D_3	355 nm	Argon	0.05
a^5S_2	266 nm	Argon	2
a^5S_2	355 nm	Argon	1.5
a^5G_6	355 nm	Argon	0.91
a^3G_4	355 nm	Argon	0.08
e^7D_5	355 nm	Helium	3.5
e^7D_3	355 nm	Helium	3
f^7D_5	355 nm	Helium	0.25
f^7D_5	355 nm	Argon	0.25

State	Laser wavelength	Buffer gas	Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$
f^7D_4	355 nm	Argon	0.2
f^7D_4	355 nm	Helium	0.3
Propyl-ACT			
$z^7P_4^\circ$	355 nm	Argon	1.5
$z^7P_3^\circ$	355 nm	Argon	1.5
$z^7P_2^\circ$	355 nm	Argon	1.25
$z^7F_5^\circ$	355 nm	Argon	2.85
$y^7P_4^\circ$	355 nm	Argon	0.07
$y^7P_3^\circ$	355 nm	Argon	0.06
$z^5P_3^\circ$	355 nm	Argon	0.12
$z^5P_2^\circ$	355 nm	Argon	0.13
$z^5P_1^\circ$	355 nm	Argon	0.14
Methyl-ACT			
$z^7P_4^\circ$	355 nm	Argon	1.6, 2.5
$z^7P_4^\circ$	355 nm	Helium	3
$z^7P_4^\circ$	266 nm	Argon	1.33
$z^7P_3^\circ$	355 nm	Argon	1.5
$z^7P_2^\circ$	355 nm	Argon	1.25, 1.3
$z^7F_5^\circ$	355 nm	Argon	3
$y^7P_4^\circ$	355 nm	Argon	0.07, 0.08
$y^7P_4^\circ$	355 nm	Helium	0.2
$y^7P_3^\circ$	266 nm	Argon	1.22
$y^7P_3^\circ$	355 nm	Argon	0.08
$y^7P_2^\circ$	355 nm	Argon	0.1
$z^5P_3^\circ$	266 nm	Argon	0.67
$z^5P_3^\circ$	355 nm	Argon	0.08, 0.17
$z^5P_3^\circ$	355 nm	Helium	0.12
$z^5P_2^\circ$	355 nm	Argon	0.13
$z^5P_1^\circ$	355 nm	Argon	0.09
$y^5H_7^\circ$	355 nm	Argon	0.06, 0.05
a^5D_3	266 nm	Argon	2.5
a^5D_2	266 nm	Argon	1.9
a^5D_2	355 nm	Argon	1.17
a^5S_2	266 nm	Argon	2.3
a^5S_2	355 nm	Argon	1.11
a^5G_6	355 nm	Argon	1.6
e^7D_5	355 nm	Argon	1

2.2 Figures

If your thesis has a lot of figures, L^AT_EX might behave better for you than that other word processor. One thing that may be annoying is the way it handles “floats” like

tables and figures. \LaTeX will try to find the best place to put your object based on the text around it and until you're really, truly done writing you should just leave it where it lies. There are some optional arguments to the figure and table environments to specify where you want it to appear; see the comments in the first figure.

If you need a graphic or tabular material to be part of the text, you can just put it inline. If you need it to appear in the list of figures or tables, it should be placed in the floating environment.

To get a figure from StatView, JMP, SPSS or other statistics program into a figure, you can print to pdf or save the image as a jpg or png. Precisely how you will do this depends on the program: you may need to copy-paste figures into Photoshop or other graphic program, then save in the appropriate format.

Below we have put a few examples of figures. For more help using graphics and the float environment, see our online documentation.

And this is how you add a figure with a graphic:

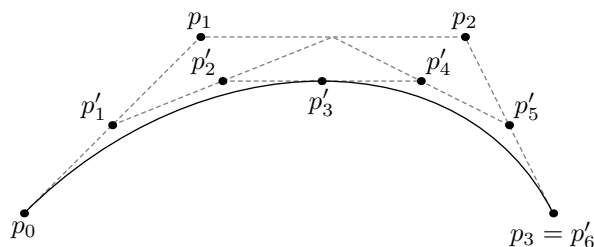


Figure 2.1: A Figure

2.3 More Figure Stuff

You can also scale and rotate figures.

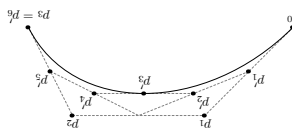


Figure 2.2: A Smaller Figure, Flipped Upside Down

2.4 Even More Figure Stuff

With some clever work you can crop a figure, which is handy if (for instance) your EPS or PDF is a little graphic on a whole sheet of paper. The viewport arguments are the lower-left and upper-right coordinates for the area you want to crop.

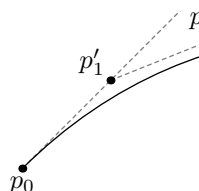


Figure 2.3: A Cropped Figure

2.4.1 Common Modifications

The following figure features the more popular changes thesis students want to their figures. This information is also on the web at web.reed.edu/cis/help/latex/graphics.html.

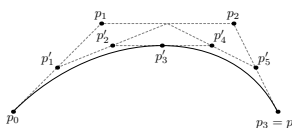


Figure 0.8: Interaction bar plot showing the degree of specialization for each flower type.

Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in L^AT_EX does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things L^AT_EX would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

Appendix A

The First Appendix

An appendix full of awesome

Appendix B

The Second Appendix, for Fun

An appendix full of win

References

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