

## **ABSTRACT**

### **STUDY OF THE SYSTEMATIC UNCERTAINTIES ASSOCIATED WITH THE TRACKING DETECTORS**

Daniel W. Boyden, MS.  
Department of Physics  
Northern Illinois University, 2019  
Michael Eads, Director

The muon g-2 experiment at Fermilab looks to measure the anomalous magnetic moment of the muon to an unprecedented precision of 0.14 ppm to verify results from the E821 experiment at Brookhaven National Laboratory. In addition to the anomalous magnetic moment of the muon the project also seeks to measure the muon electric dipole moment to a better precision than has previously has been achieved. These measurement along with standard model predictions may result in a discrepancy between the standard model and experimental results which may give some insight to beyond standard model physics.

This thesis I will present the results for systematics studies from the straw tube trackers in this experiment; mainly the resolution and cross talk in the detectors. I will also present how this will vary with a few of the final measurements for this experiment.

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**STUDY OF THE SYSTEMATIC UNCERTAINTIES ASSOCIATED WITH  
THE TRACKING DETECTORS**

BY

DANIEL W. BOYDEN  
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# **CHAPTER 1**

## **INTRODUCTION TO THE MUON G-2 EXPERIMENT**

In the following chapter there will be an explanation of the history of the experiment, why the experiment is being performed, along with the fundamental aspects of GM2. Furthermore, there will be an explanation of the results from the previous experiment E821 and how this relates to standard model predictions. This gives the fundamental reasoning for performing E989, the current G-2 experiment.

### **1.1 History and Motivation**

The muon anomalous magnetic moment has been measured in a series of experiments at CERN and more recently in the E821 experiment at the Brookhaven National Laboratory. In the first CERN measurements, muons were injected into a 6-m long straight magnet where they followed a drifting spiral path slowly traversing the magnet because of a small gradient introduced in the field. The muons were topped in a polarimeter outside the magnet and the measurement of their net spin precession determined with an uncertainty of 4300 ppm. The results agreed with the prediction of QED for a structureless particle.[1] The second CERN experiment used a magnetic ring to extend the muon storage time. The muon precession frequency was measured by the number of decayed positrons versus time and observing the precession frequency of the decayed positrons. They were able to determine the value of  $a_\mu$  to an uncertainty of 270 ppm, which agreed with theory only after the three-photon exchange contribution to the sixth-order anomalous magnetic moment were included.[2] Furthermore,

the third CERN experiment also used a magnetic storage ring with a much higher rate and was able to determine the value to within an uncertainty of 10 ppm, which also agreed with theory.[3] The most recent Brookhaven Experiment, E821, followed the same principles as the third CERN experiment but having multiple improvements in the field uniformity and increased storage efficiency. E821 was able to determine to an uncertainty of 0.54 ppm. Ref.[4]

### 1.1.1 Magnetic and Electric Dipole Moments

The study of magnetic moments started with the development of quantum mechanics. For fermions it is related to the spin by the Eq.1.6. This is derived from the fact that if there is circular loop of a wire carrying a current  $I$  with a cross sectional area  $A$ , it produces a magnetic moment  $\vec{\mu}$ . The current is given by Eq.1.2 where  $q$  is the charge,  $r$  is the radius of the loop, and  $Q = \pm 1$  taking account of the charge. The area is given by Eq.1.3 where  $v$  would be the velocity of the particle.

$$\vec{\mu} = I\vec{A} \quad (1.1)$$

$$I = \frac{Qqv}{2\pi r} \quad (1.2)$$

$$A = \pi r^2 \quad (1.3)$$

By plugging in Eq.1.2 and Eq.1.3 into Eq.1.1 yields Eq.1.4.

$$\vec{\mu} = \frac{Qq}{2m}\vec{L} \quad (1.4)$$

Using the relation of angular momentum  $L = mvr$  and knowing the only contribution to angular momentum of a free particle is due only to its spin this gives Eq.1.5. In the modern interpretation of the Stern Gerlach experiment which observed that the spin is quantized and 1.5 is off by a factor of 2.[5] To correct for this there was added a fudge factor called the g-factor giving 1.6. Although, coming to this conclusion required the discovery of the spin, quantum mechanics, and Thomas' relativistic correction. Where Thomas' correction applies to the spin of an elementary particle and relates the angular velocity of the spin to the angular velocity of the particles motion.

$$\vec{\mu} = \frac{Qq}{2m} \vec{s} \quad (1.5)$$

$$\vec{\mu} = g \frac{Qq}{2m} \vec{s} \quad (1.6)$$

In 1928 Dirac applied special relativity to Schrodinger's equation. Dirac's relativistic theory predicted that  $g = 2$ . In 1933 Stern showed that that the g-factor of the proton was approximately 5.5 which proved that the proton was not a pure Dirac particle.[8] In addition, Alvarez and Bloch discovered the neutron had a large magnetic moment which was not expected.[6] In 1947, motivated by measurements of the hyperfine structure in hydrogen, Schwinger showed that from a theoretical standpoint this splitting can be accounted for by adding in a term for the electron spin magnetic moment for the lowest radiative correction to the Dirac moment.[7]

$$\frac{\delta\mu}{\mu} = \frac{1}{2\pi} \frac{e^2}{\hbar c} \quad (1.7)$$

It has been found useful to break the magnetic dipole moment into two terms as follows with  $a$  being the anomalous magnetic moment. Ref.[4]

$$\mu = (1 + a) \frac{e\hbar}{2m} \quad (1.8)$$

Where

$$a = \frac{g - 2}{2} \quad (1.9)$$

### 1.1.2 The Muon

The muon was first observed in a Wilson cloud chamber by Kunze in 1933.[9] In 1936, Anderson and Neddermeyer reported that the muon is less massive than the proton but more penetrating than the electron.[10] Muon decays through the force  $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ . With the muons long lifetime of approximately  $2.2 \mu s$  this permits precession measurements of its mass, lifetime, and magnetic moment which makes the muon a good candidate for experiments.[4]

### 1.1.3 The Muon Magnetic Moment

The muon magnetic moment played an important role in the discover of the generation structure of the standard model. The muon spin experiment at Nevis Cyclotron witnessed parity violation in muon decays, in addition to showing that  $g_\mu$  is consistent with 2. Later experiments showed that in a magnetic field the muon behaves like a heavy electron. Ref.[4]

### 1.1.4 The Muon Electric Dipole Moment

In his relativistic theory Dirac discovered an EDM and like the magnetic dipole moment, the EDM must be in the direction of the spin. The equation for the Electric dipole moment is given by Eq.1.10.

$$\vec{d} = \eta \left( \frac{Qe}{2mc} \right) \vec{s} \quad (1.10)$$

Where  $\eta$  is dimensionless and analogous to  $g$  in Eq. 1.6. An EDM is forbidden by parity and by time reversal which was first pointed out by Landau and Ramsey by examining the Hamiltonian given by Eq.1.11. [11]

$$H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E} \quad (1.11)$$

Therefore, searches for a permanent EDM of electrons, neutrons, and atomic nucleus have become an important search for physics beyond the standard model. Ref.[4]

## 1.2 Quick Summary of Experimental Technique

In the modern G-2 experiment E989, a polarized beam of muons is produced and injected into a storage ring from the Fermilab accelerator complex. The magnetic field is a dipole field in the storage ring with vertical focusing being provided by the electrostatic quadrupoles. There are two frequencies are measured experimentally, the first frequency  $\omega_a$  is the rate at which the muon polarization turns relative to the momentum of the muon, and the second frequency  $\omega_p$  is the value of the magnetic field normalized to the Lamour frequency.

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C \quad (1.12)$$

Where S denotes the spin and C denotes the cyclotron which the individual terms are given by Eq.1.13 and Eq.1.14

$$\omega_S = -g \frac{Qe}{2m} B - (1 - \gamma) \frac{Qe}{\gamma m} B \quad (1.13)$$

$$\omega_C = -\frac{Qe}{m\gamma} B \quad (1.14)$$

It is worth noting that  $\omega_a$  depends only on the anomaly  $a_\mu$  and depends linearly on the applied magnetic field. In the presence of an electric field we get 1.27

$$\vec{\omega}_a = -\frac{Qe}{m} [a_\mu \vec{B} + (a_\mu - (\frac{m}{p})^2) \frac{\vec{\beta} \times \vec{E}}{c}] \quad (1.15)$$

If  $p_{magic} = \frac{m}{\sqrt{a_\mu}} \equiv 3.09 \frac{GeV}{c}$  the electric field contribution in Eq. 1.27 cancels to the first order which only requires higher order corrections to the term. The factor  $a_\mu$  from the two frequencies we use the relation 1.16.

$$a_\mu = \frac{\omega_a/\omega_p}{\lambda_+ - \omega_a/\omega_p} \quad (1.16)$$

The necessary steps in the experiment consist of the following. Ref.[4]

1. Production of an appropriate pulsed proton beam from accelerator complex.
2. Production of pions using this proton beam.
3. Collection of the polarized muons from pion decay.
4. Injection of the muon beam into the storage ring.
5. Kicking the muon beam onto stored orbits

6. Measuring the arrival time and energy of positrons from the muon decays.

### 1.2.1 Quick Introduction to Muon Beam Dynamics

Here is given a quick introduction to the physics of the beam dynamics of a weakly focused betatron which is used in the current E989 experiment. This section is mainly used to give an introduction to the pitch correction and the radial field correction which will be discussed later in the systematics study.

Beam dynamics of the storage ring directly affects the measurement of  $a_\mu$  since the detector acceptance for the decay electrons depends on the radial coordinate of the muon at the point where it decays. Resonances in the storage ring also can cause particle losses which affect the measurement. Care is taken in the experiment in setting the frequency of the coherent betatron oscillation which lies close to the second harmonic  $f_a = \omega_a/2\pi$ . If  $f_{cbo}$  is too close to  $2 \times f_a$ , which is the beat frequency, this complicates the extraction of  $f_a$  from the data and can introduce significant systematic uncertainty.

A pure quadrupole electric field supplies a linear restoring force in the vertical direction and the combination of the electric field. The central magnetic field provides a linear restoring force in the radial direction. The G-2 storage ring is a weak focusing ring with the field index given by Eq.1.17.

$$n = \frac{\kappa R_0}{\beta B_0} \quad (1.17)$$

Where  $\kappa$  is the electric quadrupole gradient,  $B_0$  is the magnetic field strength,  $R_0$  is the magic radius, and  $\beta$  is the relativistic velocity of the muon beam. For a ring with a uniform



vertical dipole magnetic field and a uniform quadrupole field the horizontal and vertical motion is given by Eq.1.18 and Eq.1.19 respectively.

$$x = x_e + A_x \cos\left(\nu_x \frac{s}{R_0} + \delta_x\right) \quad (1.18)$$

$$y = A_y \cos\left(\nu_y \frac{s}{R_0} + \delta_y\right) \quad (1.19)$$

Where  $s$  is the arc length along the trajectory of the ring. The horizontal and vertical tunes are given by Eq.1.20 and Eq.1.21 respectively.

$$\nu_x = \sqrt{1 - n} \quad (1.20)$$

$$\nu_y = \sqrt{n} \quad (1.21)$$

In E821 there were several tunes used for in the data acquisition  $n = 0.137, 0.142, 0.122$ . The horizontal and vertical betatron frequencies are given by Eq.1.22 and Eq.1.23 respectively.

$$f_x = f_c \sqrt{1 - n} \equiv 0.929 f_c \quad (1.22)$$

$$f_y = f_c \sqrt{n} \equiv 0.37 f_c \quad (1.23)$$

Where  $f_c$  is the cyclotron frequency and  $n = 0.137$ . The field index also determines the angular acceptance of the ring where the maximum horizontal and vertical angle of the muon momentum are given by Eq.1.24 and Eq.1.25 respectively.

$$\theta_{max}^x = \frac{x_{max} \sqrt{1 - n}}{R_0} \quad (1.24)$$

$$\theta_{max}^y = \frac{y_{max}\sqrt{n}}{R_0} \quad (1.25)$$

Where  $x_{max}, y_{max} = 45mm$  which is the radius of the storage ring aperture.

For a ring with discrete quads the focusing strength changes as a function of azimuth and the equation of motion looks like an oscillator where the spring constant constantly changes as a function of azimuth  $s$ .

$$X(s) = x_e + A\sqrt{\beta(s)}\cos(\phi(s) + \delta) \quad (1.26)$$

The presence of the coherent betatron oscillation was first discovered in the E821 experiment from a plot that showed there was in azimuthal variation in  $a_\mu$  so this led to a correction factor.

In the simplest case with the absence of an electric field and when the velocity is perpendicular to the magnetic field, the rate at which the spin precesses relative to the momentum is given by Eq.1.27.

$$\omega_a = -a\frac{Qe}{m}B \quad (1.27)$$

However, in the real experiment not all the muons are at the magic momentum, therefore a precision knowledge of the trajectories required. To look at this effect we calculate the effect on the electric field due to muon not directly at  $\gamma_{magic}$ , for the moment neglecting the  $\vec{\beta} \cdot \vec{B}$  terms.

$$\omega'_a = \omega_a[1 - \beta\frac{E_r}{B_y}(1 - \frac{1}{a_\mu\beta^2\gamma^2})] \quad (1.28)$$

Where  $\omega_a = -a\frac{Qe}{m}B$ . Now using the fact that  $p = \beta\gamma m = p_m + \Delta p$  we get Eq.1.29.

$$\frac{\Delta\omega_a}{\omega_a} = -2\frac{\beta E_r}{B_y}\left(\frac{\Delta P}{P_m}\right) \quad (1.29)$$

Where the fractional change in momentum in Eq.1.29 can be represented as Eq.1.30

$$\frac{\Delta P}{P_m} = (1 - n)\frac{\Delta R}{R_0} = (1 - n)\frac{X_e}{R_0} \quad (1.30)$$

Where the  $X_e$  is the muons beam equilibrium radius. Then the electric quadrupole field is given by Eq.1.31

$$E = \kappa X = \frac{n\beta B_y}{R_0}x \quad (1.31)$$

Therefore we obtain the final result to the change in frequency Eq.1.32.

$$\frac{\Delta\omega}{\omega} = -2n(1 - n)\beta^2 \frac{xx_e}{R_0^2 B_y} \quad (1.32)$$

Therefore, the effect of muons not at the magic momentum is to lower the measured  $\omega_a$  frequency. For a quadrupole focusing field plus a uniform magnetic field the time average of  $\langle x \rangle = x_e$ , so the electric field correction is given by Eq.1.33. The value  $\langle x_e^2 \rangle$  is determined in the fast rotation analysis which will not be covered in this thesis. In the Brookhaven experiment it was found for a low  $n$  that  $C_E = 0.47 \pm 0.054 ppm$ .

$$C_E = \frac{\Delta\omega}{\omega} = -2n(1 - n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2 B_y} \quad (1.33)$$

The betatron oscillations of the muon beam lead to the terms  $\vec{\beta} \cdot \vec{B} \neq 0$ . Since the  $\vec{\beta} \cdot \vec{B}$  term is quadratic in the component of  $\vec{\beta}$ , its contribution to  $\vec{\omega}_s$  will not generally average out to zero. The spin precession frequency has a small dependence on the betatron motion of the beam. In addition, it turns out the only significant correction comes from the vertical

betatron oscillations which is called the pitch correction. The pitch angle varies harmonically as in Eq.1.34

$$\psi = \psi_0 \cos(\omega_y t) \quad (1.34)$$

Where  $\omega_y$  is the vertical betatron frequency. If we set  $a_\mu - \frac{1}{\gamma^2 - 1} = 0$  we obtain 1.35

$$\vec{\omega}_{diff} = -\frac{Qe}{m} [a_\mu \vec{B} - a_\mu (\frac{\gamma}{\gamma + 1}) (\vec{\beta} \cdot \vec{B}) \vec{\beta}] \quad (1.35)$$

Now if we assume that the pitch angles are small,  $\vec{B} = \hat{y} B_y$ ,  $\beta = \hat{z} \beta_z + \hat{y} \beta_y$  then the difference in frequency is given by Eq.1.36.

$$\omega'_{ay} = -\omega_a [1 - (\frac{\gamma - 1}{\gamma}) \psi^2] \quad (1.36)$$

This then gives the pitch correction found in Eq.1.37.

$$C_p = -\frac{\langle \psi^2 \rangle}{2} = -\frac{n \langle y^2 \rangle}{4 R_0^2} \quad (1.37)$$

The result that the pitch correction and the radial field corrections are dependent on the average positions is what makes straw trackers important in E989 since these detectors have the best resolution for position measurements.

### 1.3 Results from Brookhaven Experiment

The Brookhaven based experiment E821 which was completed in 2001 was a successful experiment in its achievements. As previously mentioned, the experiment was able to measure the precession frequency to approximately 14 times better than what was ever achieved

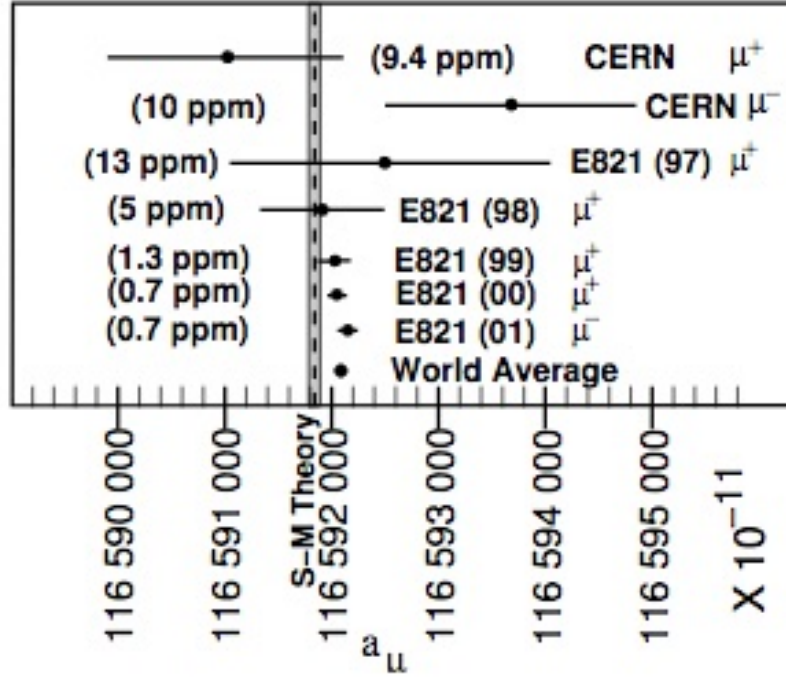


Figure 1.1: Measurement of  $a_\mu$  from CERN and BNL E821. The vertical band in the SM value using the hadronic contribution. [4]

before in the previous CERN experiments. Steady improvements in the theory resulted in a present measurement of  $a_\mu$  with an uncertainty of 0.42 ppm. The E821 experiment was able to reach a field uniformity to  $\pm 1$  ppm uniformity on average. The main systematical uncertainties of this experiment were caused from anything that caused the extracted frequency to vary the true fit value. Uncertainties in the  $\omega_a$  measurement mainly consist of gain instability in the detectors, lost muons, spin tracking, coherent betatron oscillations, differential decays, and pitch correction uncertainties. The main source of uncertainty from the  $\omega_p$  measurement consist of errors due to non-uniformity in the magnetic field. The results for E821 are given in Fig. 1.1 which show previous measurements and the Standard Model Predictions which shows that there was a discrepancy from standard model values for  $a_\mu$  and the Brookhaven National Laboratory experiment measured values up to 3.5 standard deviations. Ref.[4]

## **CHAPTER 2**

### **E989 EXPERIMENT AT FERMILAB**

This Chapter will explain the experimental apparatus used for the E989 experiment at Fermilab. It will also cover the improvements from the previous E821 experiment at BrookHaven National laboratory and the current status of this experiment.

#### **2.1 Explanation of Experimental Apparatus**

The GM2 experiment is a complicated project with many different subsystems to improve the results from previous experiments.[4] The experiment is re-using the storage magnet and the superconducting inflector that was used in E821, with a refurbished NMR trolley and magnetic field measurement system. Mainly, the new features to E989 are that a pure muon beam has been designed and commissioned minimizing hadronic components, segmented calorimeters consisting of a  $6 \times 9$  array of lead fluoride crystals to minimize ionizing particle such as lost muons, a new fast muon kicker, improved magnetic shimming to improve field variations by a factor of two, two straw tracker arrays located in the vacuum and behind the calorimeters, and a more rapid rate of data taking provided by Fermi Lab accelerator facility. The goal is to generate 21 times more data than Brookhaven and to improve the systematic errors by a factor of three and the overall uncertainty of the measurement. [4]

To produce muons in the experiment an 8 GeV pulsed proton beam coming from the recycler ring at Fermi Lab is collided with a target, producing pions. The pions are the selected by their energy and the beam is then transported into the delivery ring. During



Figure 2.1: Illustration of the Muon Campus at Fermi Lab Ref. [14]

this transportation the pions then decay into a polarized muon beam. These muons are then transported to the storage ring in the experimental hall on the muon campus. Refer to Fig.2.1 for a visual representation. [12]

The muon are injected through the inflector magnet which provides a region free of magnetic fields for the muon to pass through and into the storage ring at the MC1 building housing the storage ring. The muons are then centered in the ring using a kicker. The kicker is a series of three long plate that are able to produce a pulsed magnetic field. The magnetic field in the ring is produced by super conducting coils surrounded by iron yokes as to maintain uniformity of the magnetic field. Refer to Fig. 2.2 for a cross section schematic of the ring. As the polarized muons pass through this magnetic field they precess according to the  $\omega_a$  frequency as covered in previous sections. The muon then decays into a positron preferentially in the direction of it's spin and is detected by the calorimeters and the tracker

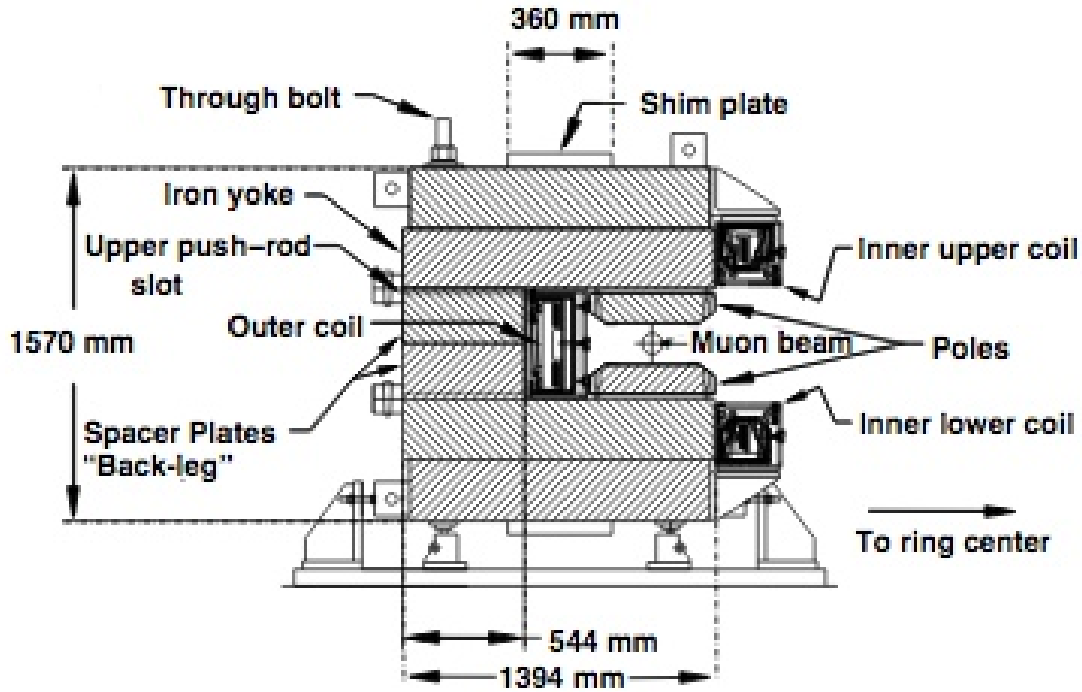


Figure 2.2: Illustration of the cross section of the storage ring Ref. [4]

stations on the inside of the ring. Shown in Fig. 2.3 is a birds eye view representation of the storage ring and the position of the kickers, inflector, tracker stations, and calorimeter stations. The detectors consist of 24 calorimeters and two tracker stations that each consist of 8 trackers.

Since uncertainties are extremely important in this experiment measurement and evaluation of the uniformity of the magnetic field is necessary. One system used for this are fixed NMR probes at several locations around the ring. A trolley system was developed that can travel throughout the ring and measure the magnetic field.



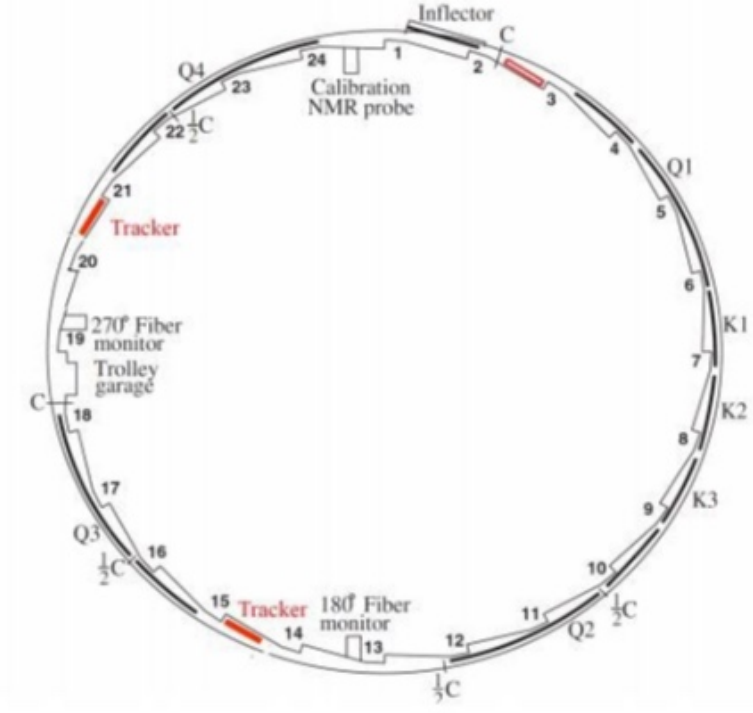


Figure 2.3: Birds Eye View of the Ring Ref. [4]

## 2.2 Requirements of the E989 Experiment

To meet the goal of an experimental precision of 140 ppb, 21 times more data needs to be collected than the E821 experiment. This will result in a 100 ppb statistical uncertainty in the  $a_\mu$  measurement. The measurement of  $\omega_a$  and  $\omega_p$  have a total proposed uncertainty of 70 ppb. The proposed limits for the expected uncertainty of the experiment can be found in Fig. 2.4 and Fig.2.5.

Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]
Gain changes	120	Better laser calibration	
		low-energy threshold	20
Pileup	80	Low-energy samples recorded	
		calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher $n$ value (frequency)	
		Better match of beamline to ring	< 30
$E$ and pitch	50	Improved tracker	
		Precise storage ring simulations	30
Total	180	Quadrature sum	70

Figure 2.4: Expected Improvement in the systematic uncertainties for  $\omega_a$  . Ref [4]

Category	E821 [ppb]	Main E989 Improvement Plans	Goal [ppb]
Absolute field calibration	50	Special 1.45 T calibration magnet with thermal enclosure; additional probes; better electronics	35
Trolley probe calibrations	90	Plunging probes that can cross calibrate off-central probes; better position accuracy by physical stops and/or optical survey; more frequent calibrations	30
Trolley measurements of $B_0$	50	Reduced position uncertainty by factor of 2; improved rail irregularities; stabilized magnet field during measurements*	30
Fixed probe interpolation	70	Better temperature stability of the magnet; more frequent trolley runs	30
Muon distribution	30	Additional probes at larger radii; improved field uniformity; improved muon tracking	10
Time-dependent external magnetic fields	–	Direct measurement of external fields; simulations of impact; active feedback	5
Others †	100	Improved trolley power supply; trolley probes extended to larger radii; reduced temperature effects on trolley; measure kicker field transients	30
Total systematic error on $\omega_p$	170		70

Figure 2.5: Expected Improvement in the systematic uncertainties for  $\omega_p$ . Ref. [4]

## **CHAPTER 3**

### **STRAW TUBE TRACKING IN E989**

This chapter supplies a brief introduction to the straw tube tracking fundamentals, the design of the trackers, and the terminology that will be used in later sections. Furthermore, an explanation of the tracking algorithms and how the software used is composed so that there is a further understanding of the motivation and how specifically the goals of the study have been achieved.

### **3.1 Straw Tube Trackers**

The straw tube trackers in E989 are a fundamental aspect in the measurement. In addition, the straws are an important part in measuring the beam profile in the experiment and can serve as a double check of the calorimeters. These measurements can be done purely by using the calorimeters in the experiment; however, the spatial resolution of the straw trackers will greatly reduce the uncertainty in the measurement.

#### **3.1.1 Basic Design of the Straw Tube Trackers**

A straw tube tracker consists of a series of what are essentially drift chambers. In general, a drift chamber consists of an enclosure containing an anode and cathode separated by a region containing a gas. Ref.[13] The straws in the detectors consist of aluminized tubes which are then filled with an argon ethane gas and have an anode wire consisting of gold

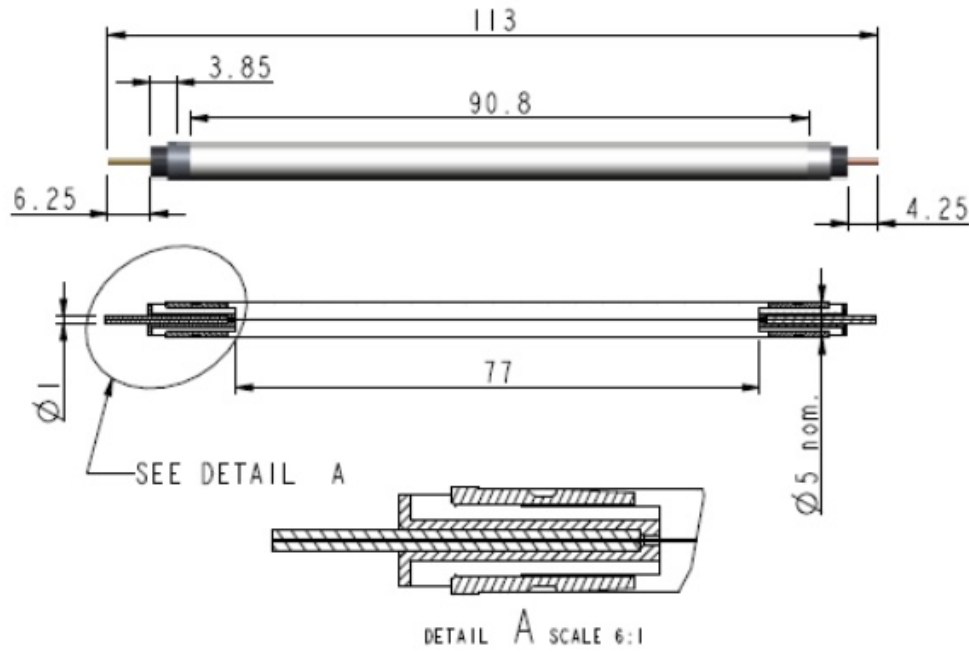


Figure 3.1: A Schematic showing the dimensions of a straw tube Ref. [12]

plated tungsten going through the middle of the straws. A diagram of the straw tube and its dimensions is found in Fig.3.1. For further information of the properties of the straws are given in Fig.3.2.

As a particle passes through the straw the particle interacts with the gas causing the ions to go to the cathode of the straw creating a detectable current. The ions are able to recover electrons from the cathode and the system and returns to a neutral state. This current can then be measured by the electronics designed for the experiment and can be given to the computer code which will extrapolate the decay position of the particle being detected in the experiment. For further improvement on spatial resolution, the amount of time for the ions to reach the cathode is dependent on where radially in the wire that the particle passes through, this is called the drift time. This drift time can then be extrapolated and added into the track fitting which an explanation of this will be in a later section.

Straw material	Aluminized Mylar
Straw wall thickness	15 $\mu\text{m}$
Wire	25 $\mu\text{m}$ gold-plated tungsten
Straw length	10 cm
Stereo angle	$\pm 7.5^\circ$ from vertical
Gas	50:50 Argon:Ethane
Pressure	1 Atm

Figure 3.2: A table show the properties of a straw Ref. [12]

One straw tracker consists of 128 of these straws which are each 5 mm in diameter. These 128 straws are then divided up into 4 layers which are oriented  $7.5^\circ$  off the vertical axis with two layers being in one direction off the vertical axis and the other layers are in the other direction. This  $7.5^\circ$  tilt of the straws allow for the tracking to take this into account a measure the vertical position that the particle hits the detector. This capability is especially important for the EDM measurement.

The next part of the straw tracker includes the manifold and the flobber. The straws are then connected to the manifolds which these allow gas to pass through them from one direction and out the other direction. In addition to this the manifold contains the cooling system and the first set of readout electronics. Attached to this manifold are two flanges referred to as the snouts which contain the electronic connections, the cooling system, and allows the gas to flow. Connected to these snouts is the flobber. The flobber houses the majority of the tracker electronics. A diagram of the whole straw tracker module is found in Fig.3.3.

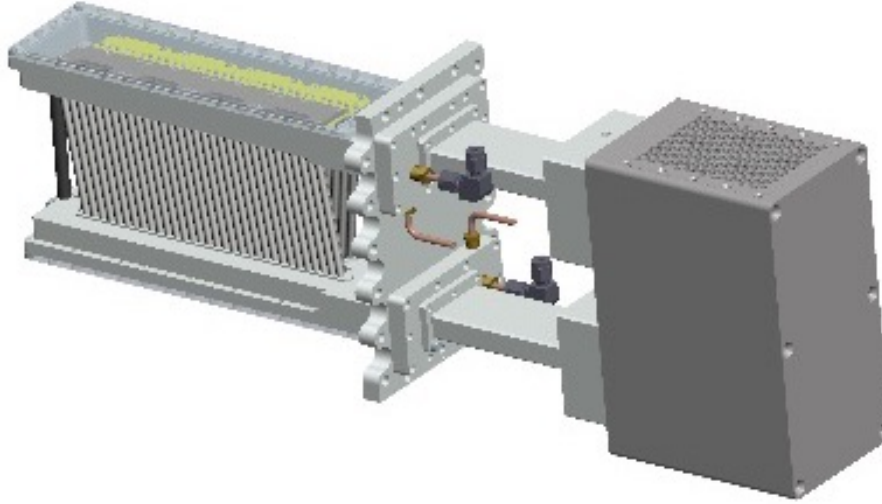


Figure 3.3: An example of a straw detector Ref. [4]

### 3.1.2 Hardware of the Straw Trackers

Now that the basic design, reasoning, and the terminology of the straw trackers themselves is established it is necessary for the systematic uncertainty discussion to give a basic introduction to the hardware that comprises the tracking system. In the tracker the straws are connected directly to the ASDQ's which is there to digitize the signal coming from the straw. Fig. 3.4 shows an image of one of these ASDQ boards. Every ASDQ board connects to sixteen straws so therefore there are eight ASDQ boards serving each tracker. Flexi-cables are attached directly to the top of the ASDQ boards and route the signal through the snouts to the flobber. A top view of the tracker showing the electronics can be found in Fig. 3.5 Inside the flobber the signal from the ASDQ boards is then passed to TDC motherboards. Each motherboard consists of two TDC's which connects to one ASDQ. The TDC boards

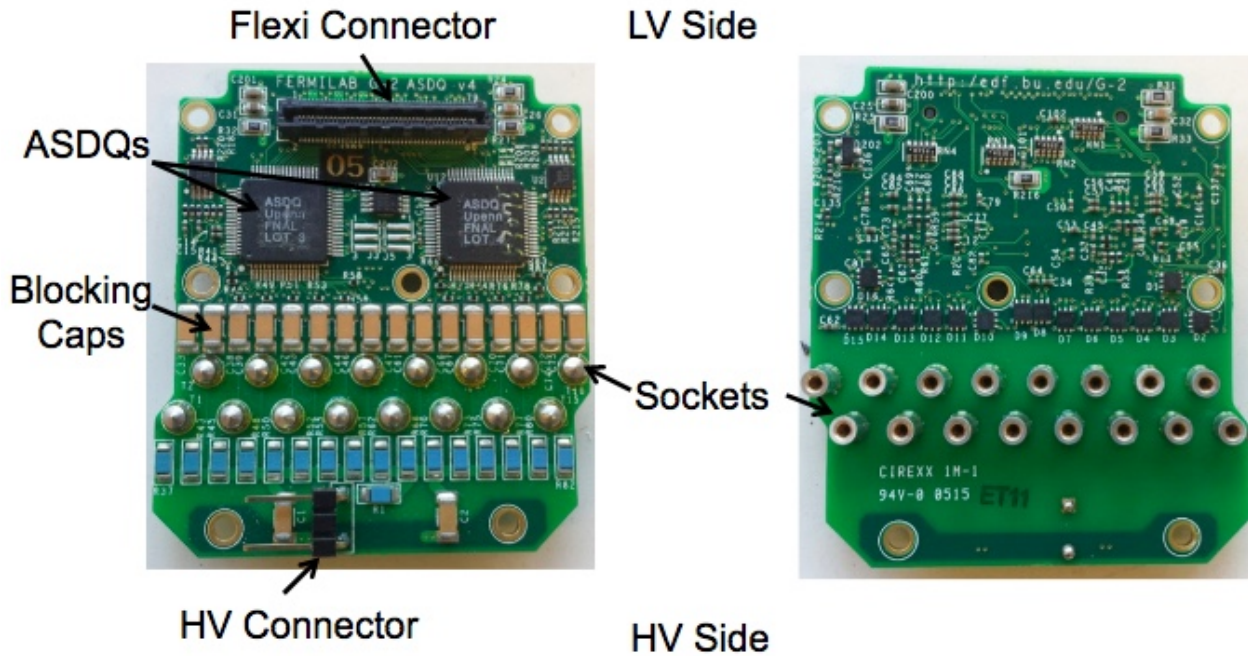


Figure 3.4: An example of a ASDQ board in the trackers Ref.[15]

buffer the signal before giving it to the logic boards. Each logic board takes the signal from two TDC's which means that there are four logic boards in a tracker. The logic boards serve as a further buffer for the data, control the clock, label the data, and incorporate data from the low voltage systems. The signal from the logic boards is then passed onto the external electronics located in the flobber.

The signal from the logic boards is passed to an FC7 board housed in an electronics rack and it serves to process the signals coming from the eight trackers at one station. Furthermore, the FC7 board controls the logic board clocks, converts the signal to a readable format for the next electronic board, and passes parts of the signal to a computer which can quickly identify data corruption. The FC7s communicate their data to AMC13 boards which control the clock of the FC7s and communicate the data to the data acquisition system. A single AMC13 can control all the tracking stations.



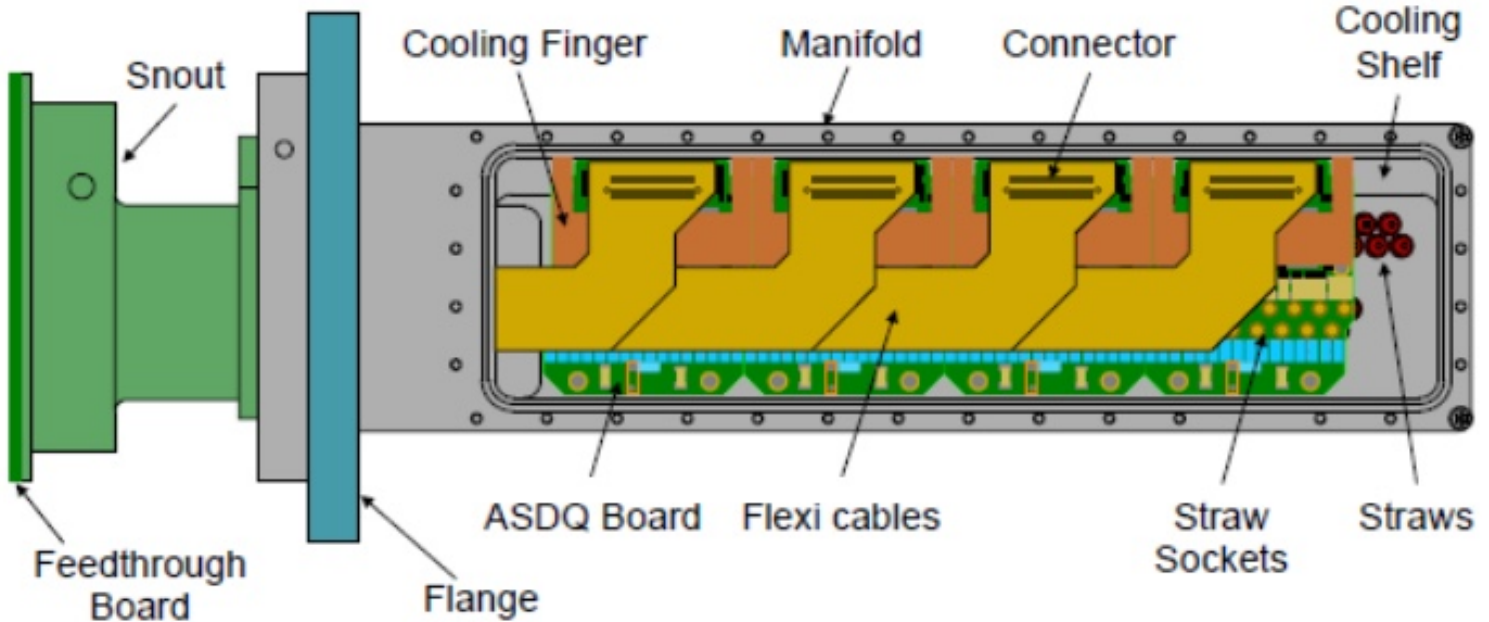


Figure 3.5: Cross Section Top View Showing Tracker Electronics Ref.[16]

## 3.2 Straw Tube Tracking Algorithms

An important part in evaluating the systematic uncertainties of the straw tube tracking is to understand what the tracking algorithm consists of and how the track fitting process works. In addition, in this section will be an explanation of the data products, what the components of the data products used physically mean, and how those data products are then derived from the tracker data.

### 3.2.1 Introduction to the Straw Tube Track Finding

For each process that the tracking algorithm follows it uses what is called an Art module. Art is an event processing framework used in particle physics.[17] Art is C++ framework that



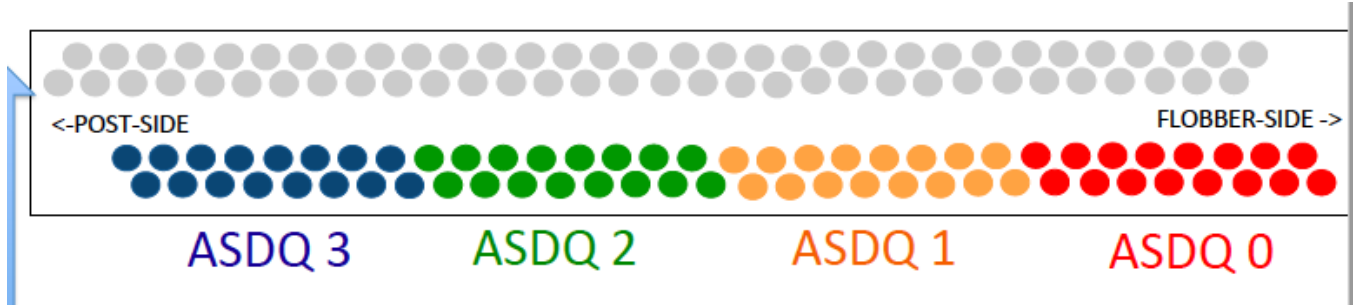


Figure 3.6: Illustration that shows how the channel ID's are split up for each wire. Ref.[18]

allows different components of code to be used interchangeably, as long as the appropriate data products are being used. This is the main framework for the whole project and all the code that is discussed further is written in this framework.

The straw tube track finding algorithm consists of the most processing steps in analyzing the data. Although, more complicated and robust than what will be presented the basic idea of the tracking finding and fitting is covered. From the electronics the computer collects the data and makes compressed data file. For tracking these files only consist of raw Digits. These raw digits contain all the information that will be used further in the code and consists of the timing, and where the signal comes from in the electronics. How the channel ID's are split up can be found in Fig.3.6. Each channel corresponds to the same ASDQ and the same logic board. Every station has the same AMC13 ID. There are two TDC's per channel which one TDC would correspond to one layer in a channel. Furthermore, this data product contains the width. The width that is the time difference between the rising edge in the wire when current is above a threshold and the falling edge where the current falls below the threshold.

From this raw data there is a module the makes a more user-friendly data product called straw digits. From the raw digits, and a geometry file converts the ID's to the specific 3d coordinate where the wire is positioned. In addition, from these Raw Digit ID's there is generated a wire ID, which gives an easier format to label wires. The wire ID gives the wire

that was hit and tells what station, tracker, layer, view, and wire position of the hit. The convention for number these value goes as follows, there are two stations 12 and 18, there are 8 trackers per station which the lowest number is labeled closest to the ring, there are 2 layers (0,1) looking at Fig.3.6 the first layer is 0 and the second layer is 1, the third layer is 0 and the fourth layer is 1. There are 2 views in a tracker (0,1) which this corresponds to u or v layer respectively. The wires are labeled from 0-31 with the lowest number being the side that is closest to the beam. Lastly this module, also calibrates the TDC times so that they are all in time with one another. Lastly, in the straw digits there consists several other parameters created in this module although they are just filled in as empty to be used later in the code consists of the drift time, error on the drift time, reconstructed distance of closest approach, and the distance of closest approach error. An explanation of these parameters will be given when these parameters are filled with data in the code.

From the straw digits the next module in the algorithm splits these straw digits into 100 ns time intervals which are called time islands to make the straw digits more manageable. Typically, these time island consist between one and four tracks. In addition, this module also extracts what is called the  $t_0$ , which is the time at which the track hits the first straw in the detector. The  $t_0$  has multiple different calculation methods. However, it has been found that the best working method for calculation is the average time method where you take the average between the minimum time and the maximum time found in the time island. This value is then adjusted to extrapolate back to where the tracks hits the first layer, telling you at what time the track hits the detector. The time islands data product consists of a unique island ID, the mean time, minimum time, maximum time,  $t_0$ ,  $t_0$  algorithm ID,  $t_0$  success, and the station where the time island occurs. In addition to this, there is a parameter called the upstream digit which stores the wire id of the wire used in the  $t_0$  calculation.

Next, the time islands are then passed onto the clustering module which groups the digits in the time islands by digits that are neighbor each other. In addition to this from the UV

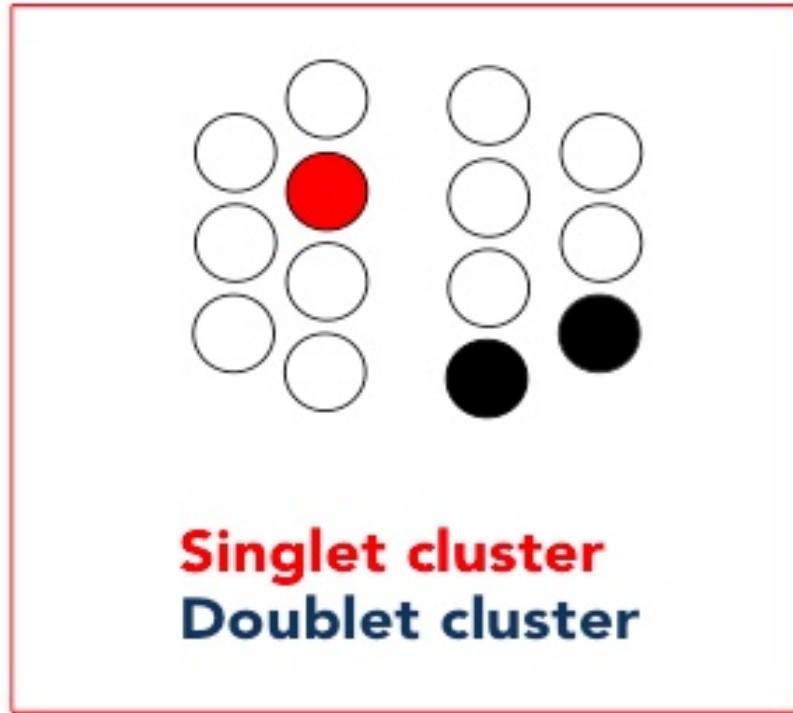


Figure 3.7: Examples of Clusters. Ref.[19]

angles and the geometry of the detectors the horizontal position of the track is measured. An example of a cluster pictorially is shown in Fig.3.7 . This data product that is passed on has the following parameters to use: mean time,  $t_0$  (initial time of cluster),  $t_0$  error, the coordinate position of the cluster, horizontal position, horizontal position error, the station number, module, view, and tells if the clusters either overlap with another cluster or if they have shared digits between the cluster. This information is useful for the next module in the process.

The clusters of digits are then processed by the next module which forms what are called seeds. Seeds are derived from clusters which give possible combinations of digits in a particular layer that would belong to a track, i.e. a track is composed of a multitude of seeds. Fig.3.8 Is a good example of how a cluster could form two seeds. The information that is given then from this module consists of: The type of seed (how many seeds belong

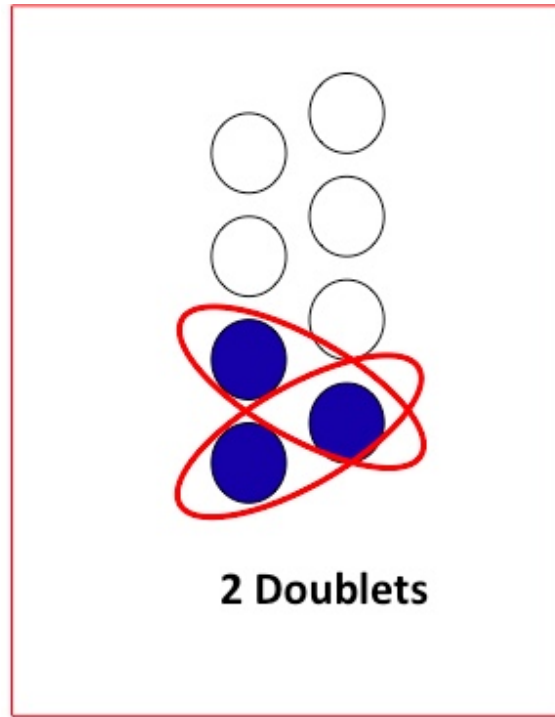


Figure 3.8: An example of two neighboring seeds from one cluster. Ref.[19]

to that cluster), time, 3d coordinate of the seed, the line slope of the digits on the seed, the line intercept of digits on the seed, if it has shared clusters, station, and whether the first digit is in the front view or the back view. The line slope of the digits on the seed, the line intercept of digits on the seed, if it has shared clusters, station, and whether the first digit is in the front view or the back view.

Probably the most complicated part of the code consists of the next module where this forms track candidates. A track candidate is a single particle track that is derived from the seeds previously established. This is a robust program that can pick out multiple tracks in a time island and forms every possible combination of the seeds and picks out which combinations make physical sense. Shown in Fig. 3.9 would explain how this algorithm forms different track candidates.

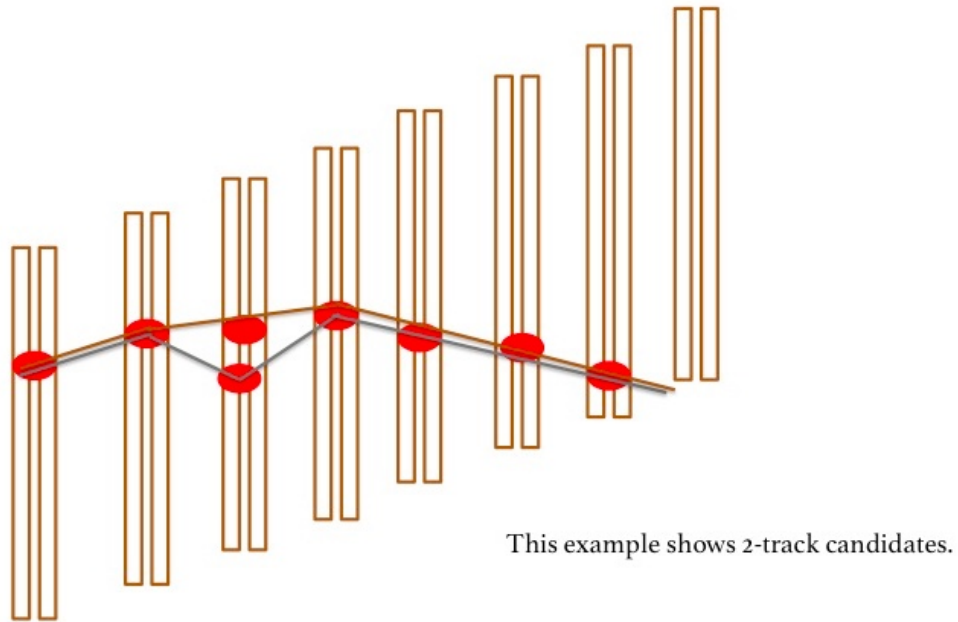


Figure 3.9: An example of how a track candidates would be formed. Ref.[19]

Now that the track candidates are created it is necessary to derive the track  $t_0$ , drift time, and to reconstruct the distance of closest approach. Similar to finding the  $t_0$  in the time island, the  $t_0$  is calculated by finding the average time from the first digit and the last digit in the track candidate and then extrapolating to the time that the first digit in time hits the track. The  $t_0$  is an important part to determining the drift time of the straw digit and therefore the distance of closest approach. The drift time of the straw hit is determined as the difference between the hit time of the straw (which is the average time between rising and falling edge of the current) and the  $t_0$ . Refer to Fig.3.10. An example drift time distribution that is normally seen is found in Fig.3.11 From this drift time now can be determined the distance of closest approach. The distance of closest approach is determined from a physical model that was determined from Ref.[21]. From an experimental method a 2d histogram of drift time vs. the distance of closest approach (from the track fitting algorithm not explained

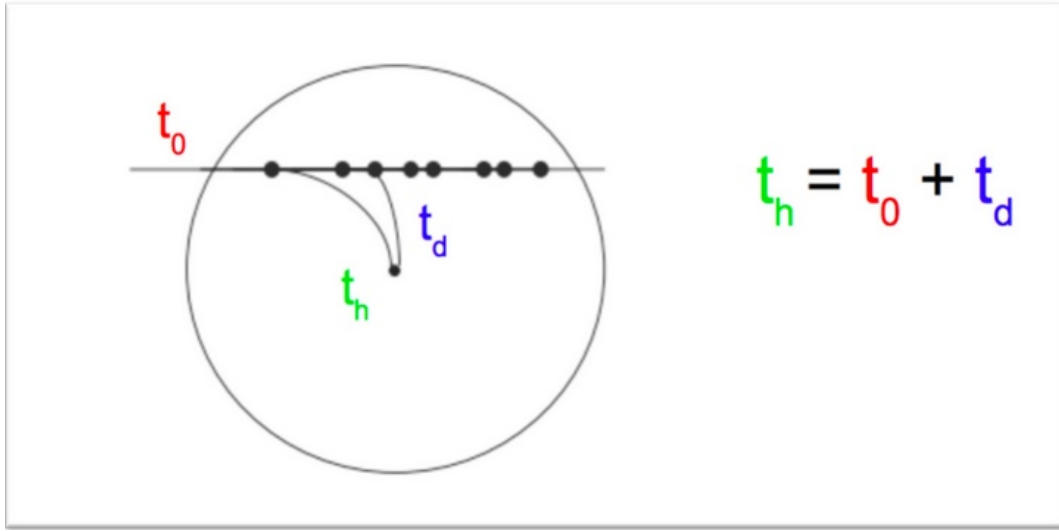


Figure 3.10: This shows how the drift time distribution by the given  $t_0$  and Hit Time Ref.[19]

yet) as shown in Fig.3.12. In Ref.[21] he produced a histogram by splitting it up in several regions dependent on the distance of closest approach. Then these functions were fitted with a gaussian convoluted with an exponential. Then generated was the mean value of the drift time versus the distance of closest approach. This model then directly converts a drift time to the distance of closest approach with some uncertainty.

### 3.2.2 Track Fitting and Reconstruction

The track fitting and the track reconstruction is a complicated process that will be explained briefly. The fitting algorithm used is a standard  $\chi^2$  fitting algorithm where there is a starting  $\vec{p}$  and then the  $\chi^2$  value is minimized to the data. In order to do the fit, it is necessary to determine whether the track goes to the left or right of each wire. This is handled by first doing a simple linear fitter between seeds that are doublets. For each doublet the algorithm goes between every different combination of left and right sides of

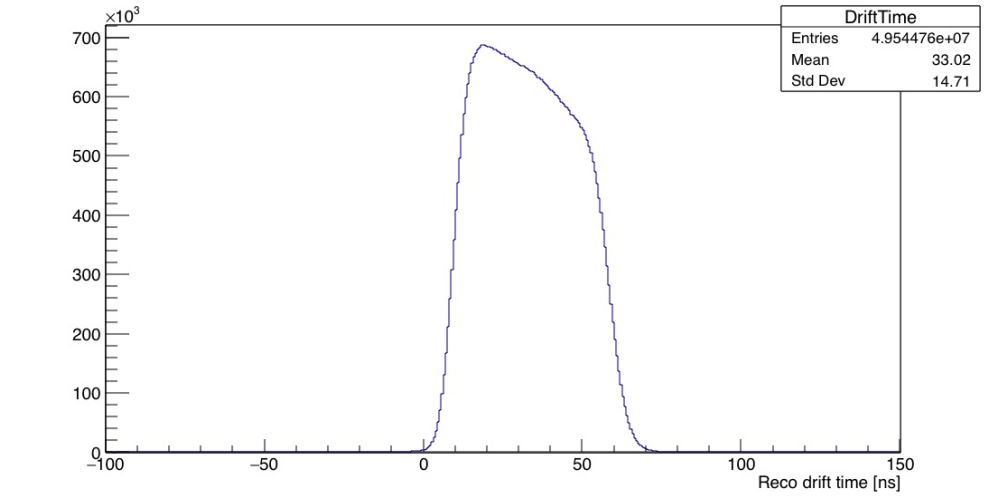


Figure 3.11: This show what a typical drift time distribution looks like

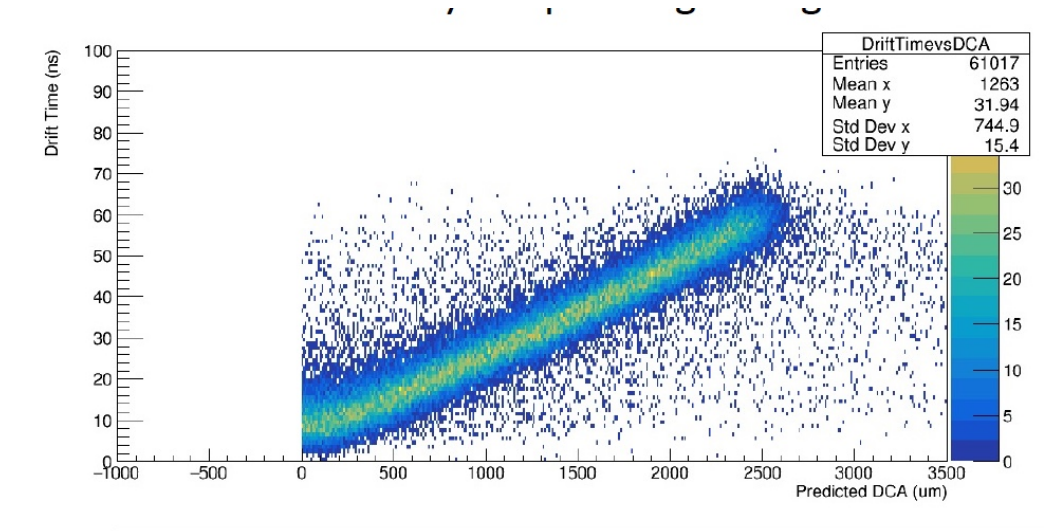


Figure 3.12: Example histogram of how the distance of closest approach is modeled

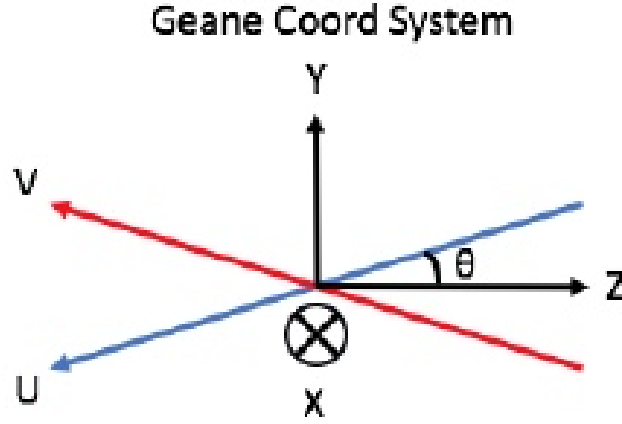


Figure 3.13: How the xy coordinates system relates to uv coordinates system, z is in direction of the trackers, x is parallel to the tracking planes, z is horizontal Ref.[?]

the wire. Then the angle between the lines between neighboring seeds are minimized to one another and those selections of left or right side of the wire is chosen. Now there is a five parameter  $\chi^2$  fit described by the variables  $(\frac{1}{p}, \lambda, \phi, y_{\perp}, z_{\perp})$ . The variable are then transformed using a Jacobian to  $(\frac{1}{p}, \frac{pu}{px}, \frac{pv}{px}, u, v)$ . Then they can be converted from uv coordinates to xy coordinates using another Jacobian. Ref.[?]. Fig.3.13 describes how the uv coordinates are related to the xy coordinates in the fitting.

The track extrapolation algorithm uses a Runge-Kutta Nystron algorithm. The algorithm begins with a state vector  $\mathbf{S}_i$ , positions and momentum of the fitted track. These parameters are propagated along a step  $\mathbf{d}_o$  by evaluating the equation of motion at four immediate stages. The magnetic field information is queried back at each one of these stages. The angles of the track at each stage are weighted to obtain the final state vector at the end of the step. Refer to Fig.3.14 for a pictorial representation. After each step the geometry of the ring is queried and flags each track that hits a material in the ring. Also, at each step the radial momentum component is evaluated. The extrapolation stops when the radial momentum is zero or in other words when the extrapolated track is parallel with the storage ring.



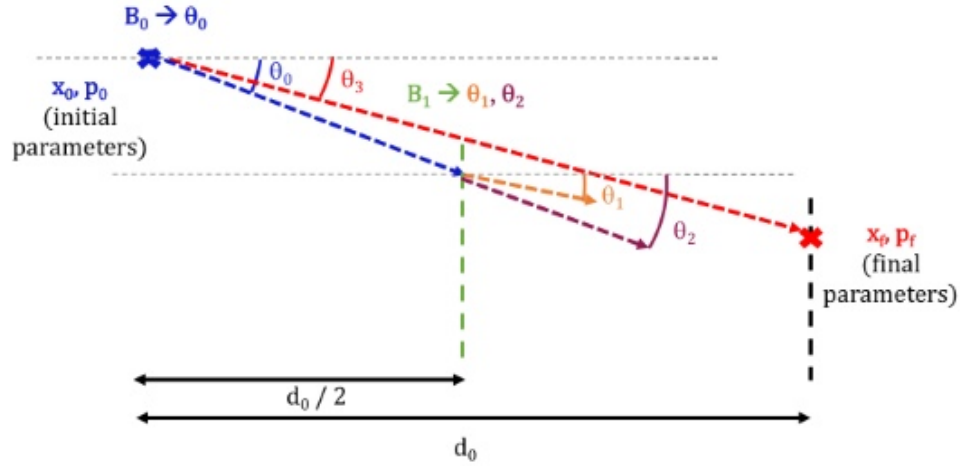


Figure 3.14: A pictorial explanation of an extrapolation step [23]

### 3.3 Uncertainties in the Straw Tube Detectors

After the tracking methods have been established it was necessary to understand exactly where the uncertainties and how much they contribute to the final measurements.

#### 3.3.1 Explanation of Uncertainties

Uncertainties are an important aspect of the E989 experiment. Significant care must be taken to ensure that we do not have more than the planned errors, or the experiment will not succeed. Therefore, since the development of the trackers and the tracking algorithms the tracking team has spent a significant amount of time trying to address and characterize different systematic uncertainties that may be present in the experiment. This section gives a brief explanation of the uncertainties that are associated with specifically the tracking analysis.

As shown before the main ability of the trackers is to have good spatial resolution as compared to the calorimeters. Therefore, in studying the systematics the main extrapolated quantities of interest are the extrapolated  $y$  position (horizontal position) and the radial position (decay point in the ring). All the important quantities for the experiment that are on a tight error budget are derived from these quantities. Furthermore, for brevity I am only going to cover the final errors contributed to the pitch correction and the electric field correction which use the  $y$  and  $r$  position respectively.

For the pitch correction refer to Table 3.1. The single largest uncertainty is from the acceptance effects of the trackers which is 16 ppb. Looking at the largest uncertainty found in Table 3.1 which is the straw stereo angle. The straw stereo angle is the deviation of the straws from the idealistic  $7.5^\circ$ . On a similar scale of this uncertainty is the ability to find the Left and Right side of the wire in the track fitting. The next largest error consists of lost muon contamination which are from the muons hitting the detectors. Internal alignment is the deviation of the individual straw trackers from the idealistic geometry. Close to this is our ability to determine the  $t_0$  of the track which was explained previously.

The following uncertainties contribute all on the same level. The hit resolution which will be explained in chapter 4. The external alignment uncertainty comes from the deviation of the tracker station from the ideal geometry. The track finding errors are from when we pick the wrong track candidate or a wrong digit. Material effects are from ignoring the fact that the positron is passing through an aluminized straw. Lastly, the smallest contribution is the contribution from electronic cross talk which will be discussed in a later section. The other errors in Table 3.1 are known to exist, however they have not been currently determined. This consists of variances in the magnetic fringe field near the trackers since this is a derived quantity not a measured quantity. The R to T error is the error being introduced from our distance of closest approach and drift time model variations. Ref.[24]

Table 3.1: Systematic Error Contribution of the Pitch Correction

Systematical Error	Error (ppb)
Stereo Angle	6.9
L/R	6.9
Lost Muon Contamination	2.9
Internal Alignment	1.5
t0	0.6
Hit Resolution	0.3
External Alignment	0.3
Track Finding	0.3
Material Effects	0.3
R to T	?
Magnetic Fringe Field	?
Cross-Talk	<0.005

In conclusion currently for the pitch correction the total systematic uncertainty  $19.3+?$ . For the first run the trackers found that the measured pitch correction is given by  $C_p = 164.7 \pm 0.1(stat) \pm 21.1+?(syst)$  Ref.[24]. Although a bit large for the total error budget of 30 ppb for both the E-field and pitch corrections this is only the first run in the experiment and in the future a lot of these systematics will get much smaller. The radial error is found in Fig.3.2. Note that the errors are not given in ppb and are given in terms of (mm). Here it looks like the internal alignment is the greatest contributor to the errors for the E-field correction.

Table 3.2: Systematic Error Contribution to Radial Distribution

Systematical Error	Error (mm)
Stereo Angle	?
L/R	0.2
Lost Muon Contamination	0.055
Internal Alignment	0.162
t0	0.035
Hit Resolution	0.03
External Alignment	0.01
Track Finding	0.03
Material Effects	0.02
R to T	0.026
Magnetic Fringe Field	?
Cross-Talk	<0.005

## CHAPTER 4

### STRAW TUBE TRACKER SYSTEMATICS

With the straw tube trackers being an important part to a variety of measurement that need to be made it is important that the systematic uncertainties of the system are investigated and understood. The measurements that are being made by the trackers are pitch correction, beam dynamics, electric field corrections, along with the electric dipole moments measurement. This chapter will explain specifically the errors of cross talk and hit resolution in the straw detectors and how these errors were characterized and their relative effect on the measurements.

#### 4.1 Cross Talk in the Straw Trackers

Cross Talk is an effect when there is a particle signal in one wire which causes a signal in another wire. This could be catastrophic for the measurement especially in the pitch correction and the EDM measurement as this would throw off the extrapolated  $y$  position due to the bias in the track fitting. Furthermore, If I add in another hit in a straw when there is not supposed to be a hit then this could shift the particular momentum at a plane and cause an error in the fit or an extra deviation in the fitting algorithm. Refer to Fig.4.1, if a digit that occurs at position 1 and 2 this could show that the particle trajectory in the right direction. Alternatively, if a digit occurs in position 2 and 3 this would show the particle trajectory would be moving to the left direction throwing of the results at this plane. This section will explain the process how I characterized and added this effect into simulation to

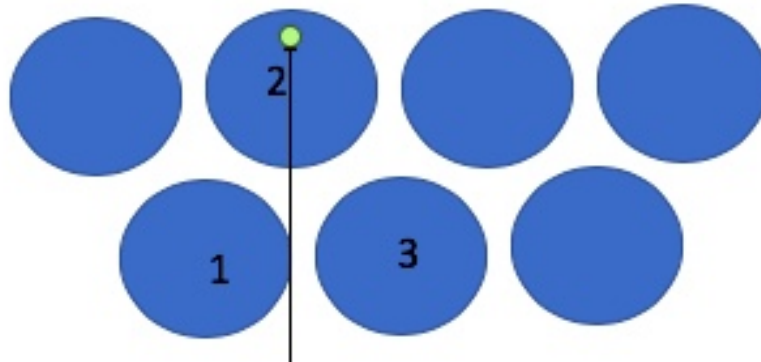


Figure 4.1: A pictorial representation of how Cross Talk was picked out

see how much of an effect this will cause on the extrapolation y positions and radial positions in the detector.

#### 4.1.1 General Explanation of Cross Talk

Cross talk, or what we are calling cross talk, in the detectors could be caused by multiple different processes. Firstly, because the aluminized drift tubes are conductive and these tubes are neighboring each other this create a capacitive coupling between straws, therefore allowing charge to exchange between the wires. The next source of cross talk in the detector could be from within the electronic boards themselves. When traces on a printed circuit boards are neighboring this creates an inductive or capacitive couple between the traces causing a signal from one trace to bleed into another trace. It is unknown which of these are the greatest effect, but they should have similar properties, and this is what we would consider cross talk in the detectors. Thirdly, although not technically cross talk we could have a case when there is a muon in the detectors, and it decays to a positron therefore causing a signal in another wire and confusing the tracking results. In addition, we could radiative effects coming from the positrons also creating neighboring hits. All these properties are

lumped together in this analysis since it is only needed to evaluate the properties of these effects and add it into simulation to observe these effects on the tracking analysis.

### 4.1.2 Evaluation of the properties of Cross Talk

In order to add the effects of cross talk into the simulation and to understand the true effects that cross talk has on our track finding and fitting we needed to ask the questions when does cross talk occur after the first digit, is there any correlations between the data products of the cross talk hit or the primary hit (The hit that causes the cross talk), how much cross talk occurs in our detectors...etc. To do this it is necessary to establish the known properties of cross talk just from the very definition of it being cross talk. Firstly, we know that cross talk is caused by something else so it has to occur after another hit in the detector. Secondly, we know that from the sources of cross talk established earlier we are most likely going to find cross talk in the neighboring wire. Lastly, since the cross talk signal again is caused by charge in another wire the primary signal charge cannot be larger than the cross talk signal charge. These properties will be important for selecting out particular tracks and will allow us to pick out cross talk signals.

Firstly, it was necessary to get a known cross talk sample. To get these known cross talk signals we looked at a case where there is only one particle that was going through the detector at a time. The next criterion that was necessary is that we required there to be two hits in the same layer of the detector without any hits being in the opposing layer. To take out the case of decaying muons in the detector there is a momentum cut at 2300MeV. Now there is only the case where there is a primary digit and a cross talk digit, with the primary digit occurring first in time. The time distribution between the primary digit and the cross talk digit is found in Fig.4.2, which has an odd shape mainly due to the fact that there

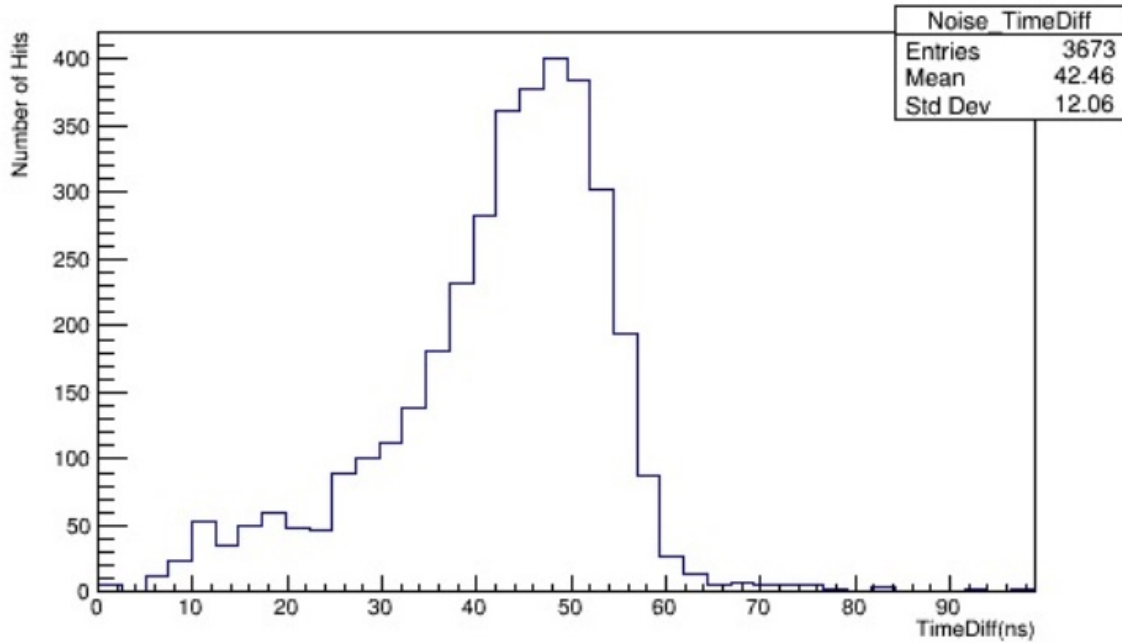


Figure 4.2: The observed cross talk time difference distribution from data at 1550V.

are multiple different situations of cross talk occurring. Next, if we look at the drift time distribution for cross talk, found in Fig.4.3, it is shown that generally the drift time for cross talk is fairly large. Furthermore, looking at the distance of closest approach distribution of cross talk in Fig.4.4, due to the large drift time, results in most of the cross talk looking like a particle hit on the edge of the wire.

In order to add the effects of cross talk into the simulation and to understand the true effects that cross talk has on our track finding and fitting we needed to ask the questions when does cross talk occur after the first digit, is there any correlations between the data products of the cross talk hit or the primary hit (The hit that causes the cross talk), how much cross talk occurs in our detectors...etc. To do this it is necessary to show the known properties of cross talk just from the very definition of it being cross talk.



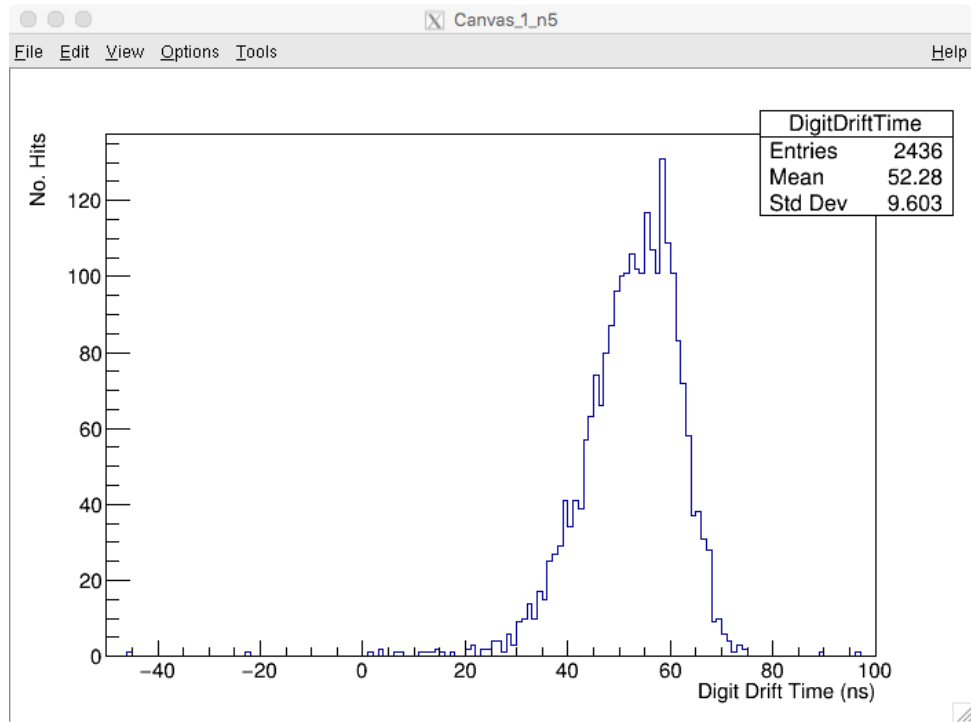


Figure 4.3: The observed cross talk drift time distribution from data at 1550V.

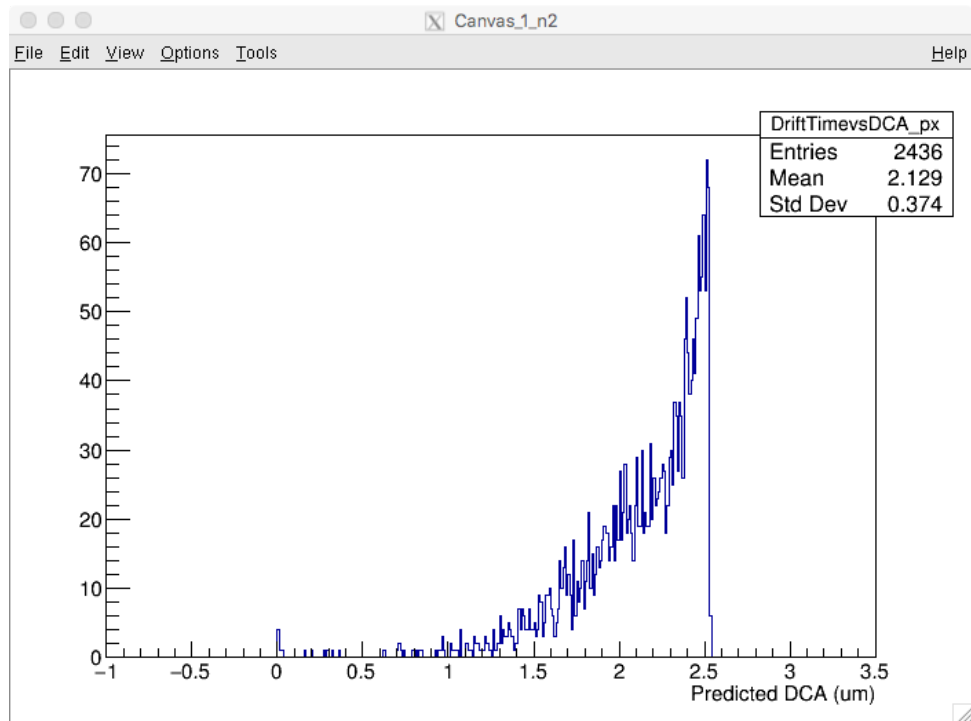


Figure 4.4: The observed distance of closest approach distribution from data at 1550V.

Firstly, we know that cross talk is caused by something else, therefore it must occur after another hit in the detector. Secondly, we know that from the sources of cross talk we are most likely going to find cross talk in the neighboring wire. Lastly, since the cross-talk signal again is caused by charge in another wire the primary signal charge cannot be larger than the cross-talk signal charge. These properties will be important for selecting the tracks and will allow us to pick out cross talk signals.

Next it was necessary to evaluate the general properties of cross talk, they are not necessarily important for the simulation; However, it is nice to know and document the properties as a quality control standard. In addition, this gives a good double check that the method to select the cross-talk digits are valid. It was found that the hit width distribution, explained previously, had an odd shape with one little hump on the small hit width side and a large overlaying hump as shown in Fig. 4.5. It seems that this would be the cross talk since we would expect the cross-talk signal to be small, at least smaller than the primary digit. To prove this, the earlier method to select out cross talk and the primaries method was used. Putting these into the hit width distribution Fig. 4.6 it is found that the assumption that digits that are larger are more likely to cause cross talk digits that are small is valid.

The cross talk in the detector should be dependent on the gain in the straws and therefore the cross talk will be dependent on external weather effects due to the properties of the argon ethane gas changing. To investigate this, a data set which was ran over 60 hours was used. Then the average temperatures and pressures over the series of runs was observed and the number of cross talk digits versus the number of tracks found in that layer. Fig.4.7 shows how the cross talk increases as a function of temperature and Fig.4.8 shows how the cross talk I selected decreases as a function of pressure. These results are consistent with what I would expect with cross talk. In Fig.4.9 shows how the cross talk varied over multiple detectors over that same 60 hour period which shows that there is a fairly large variance in cross talk from run to run. Note, the percentage on the y-axis is arbitrary since it's the

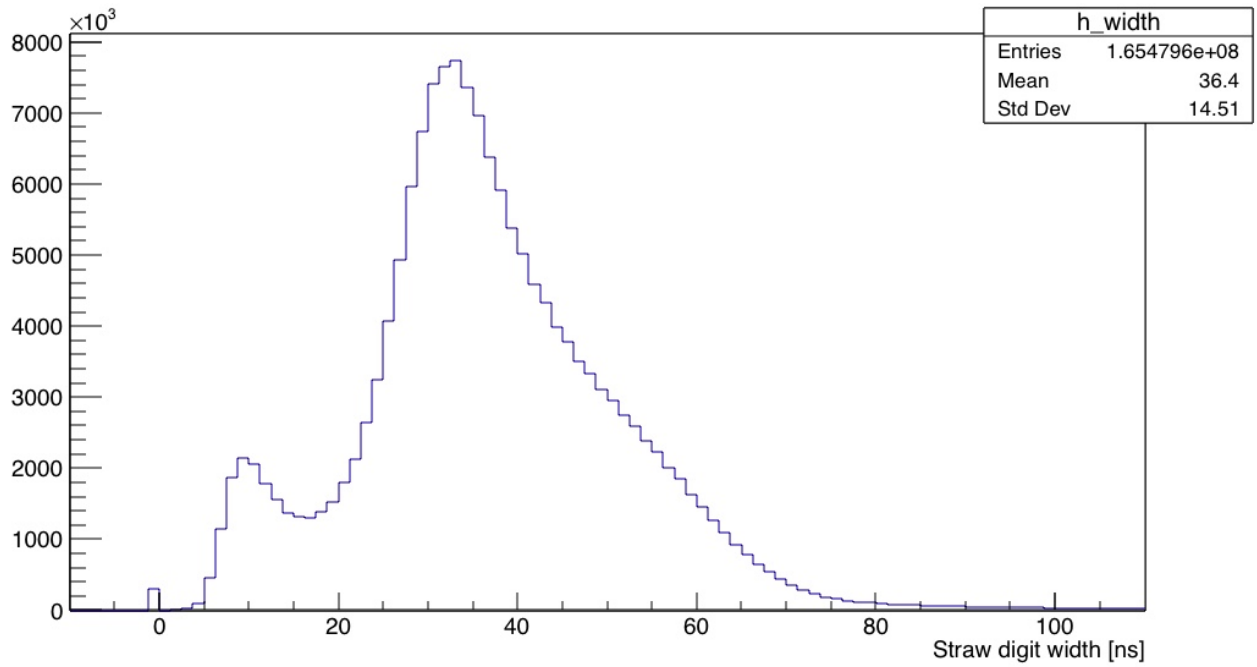


Figure 4.5: The Normal Hit width distribution without track selection

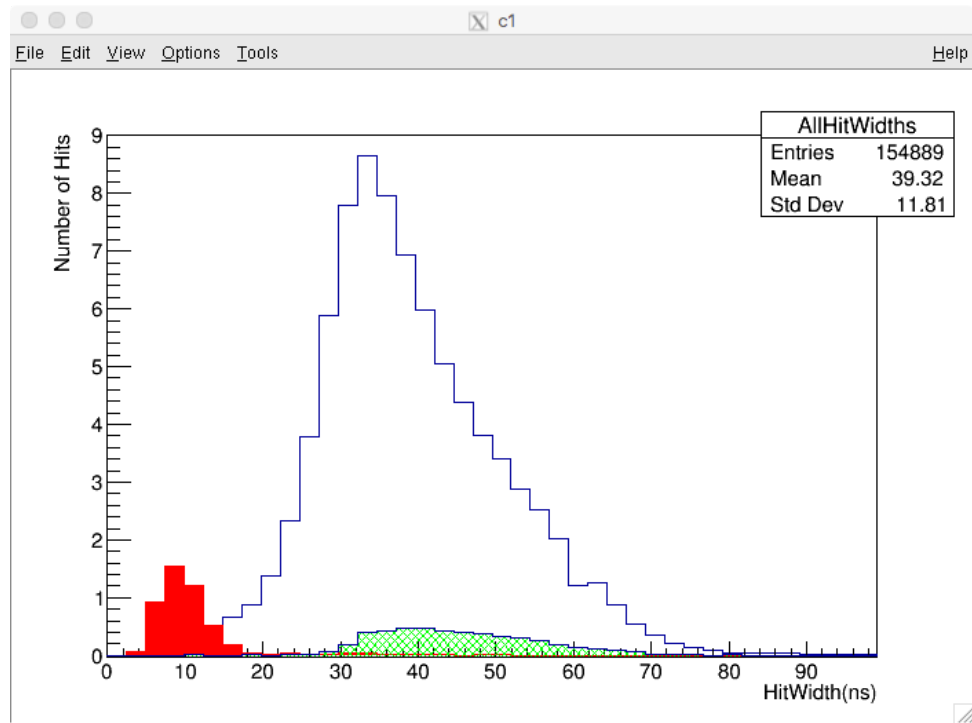


Figure 4.6: The Normal Hit width distribution overlaid with primary digits

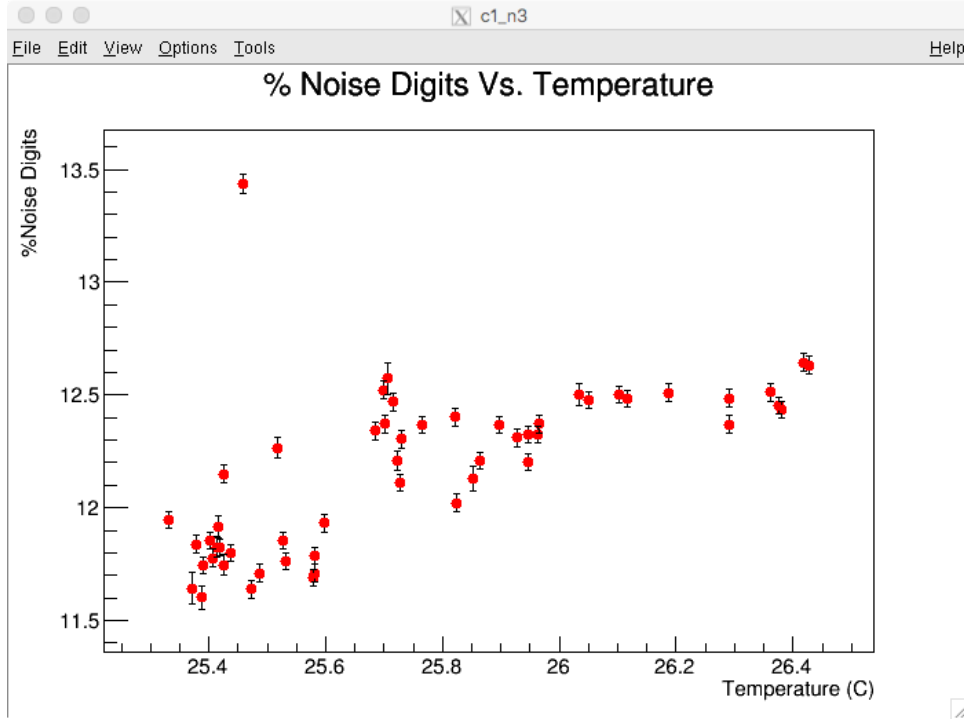


Figure 4.7: The observed correlation between cross talk and temperature. Note: Not true cross talk probability.

percentage of tracks, the point of these is to confirm that the properties of the digits that were collected as cross talk have the properties of cross talk. As another double check, the gain is also dependent on the voltage on the trackers. Therefore, we should expect that the cross talk versus voltage should give an exponential curve until we get to saturation. In Fig.4.10 shows how the gain is dependent on the voltage. Using data from different voltages in the detectors it is found that this is true given in Fig.4.11.

### 4.1.3 Introduction of Cross Talk into Simulation

Now that the properties of known cross talk signals are characterized it is now necessary to add the cross talk into a simulation to observe the effects that the cross talk has on the

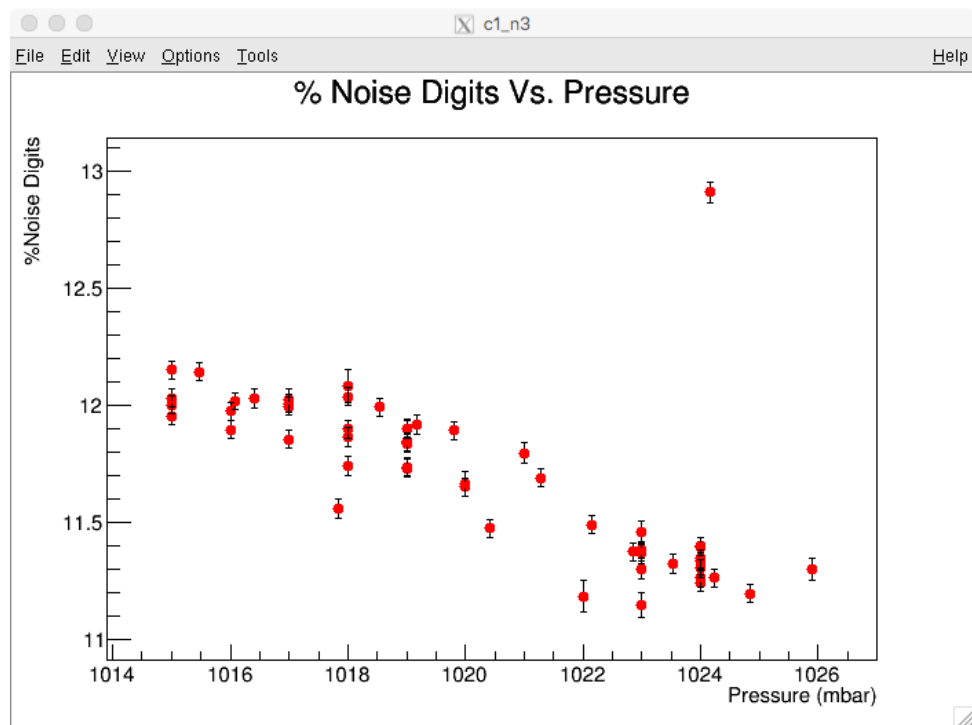


Figure 4.8: The observed correlation between cross talk and Pressure. Note: Not true cross talk probability.

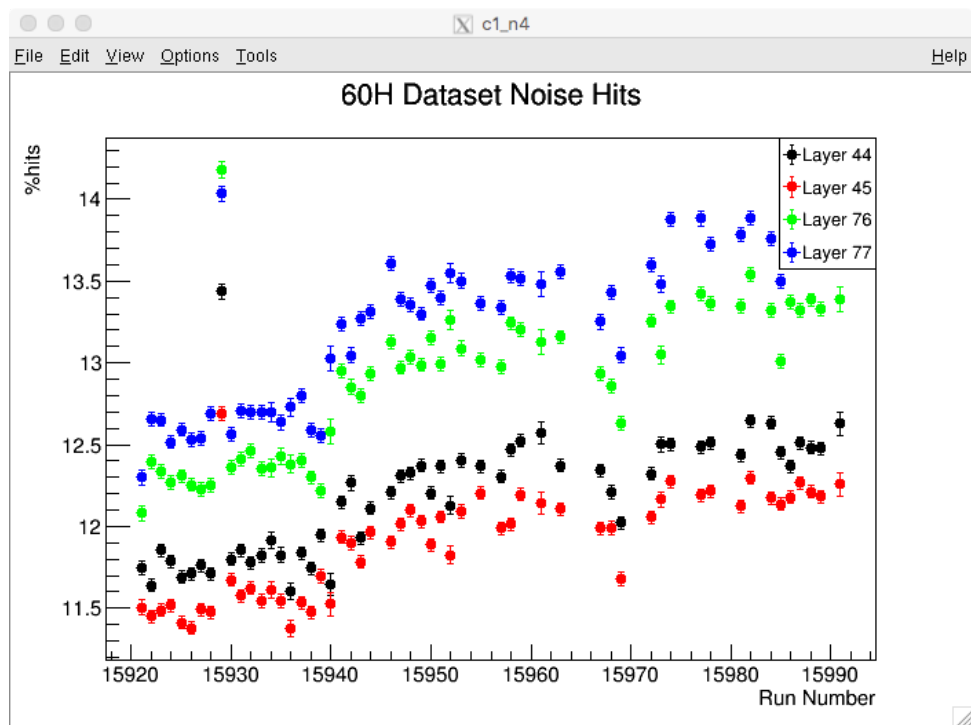


Figure 4.9: The observed correlation between cross talk over the course of time. Note: Not true cross talk probability.

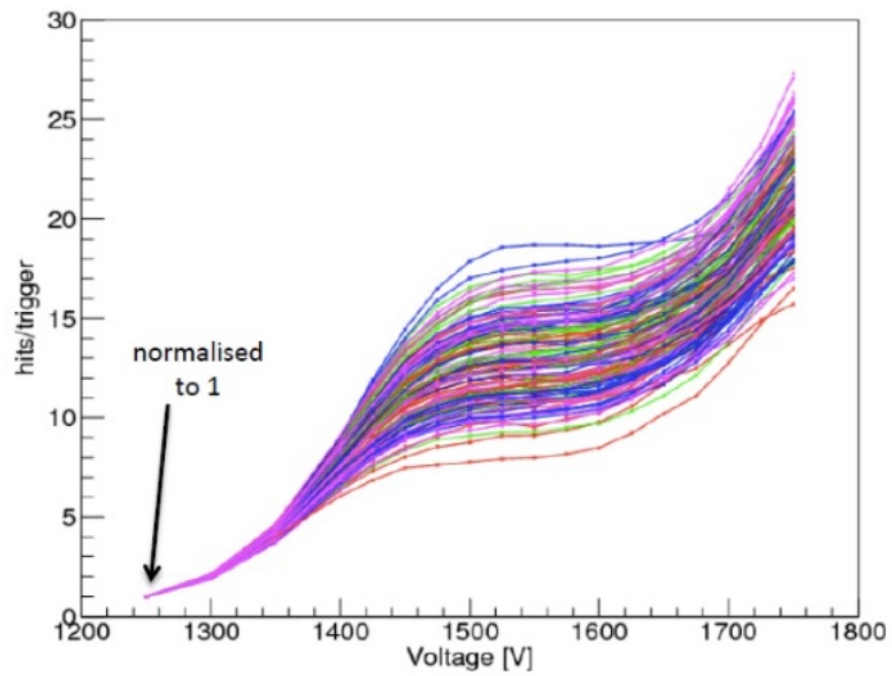


Figure 4.10: The plateau plot of 32 straws in the straw tracker. Y axis is essentially proportional to current in detector.

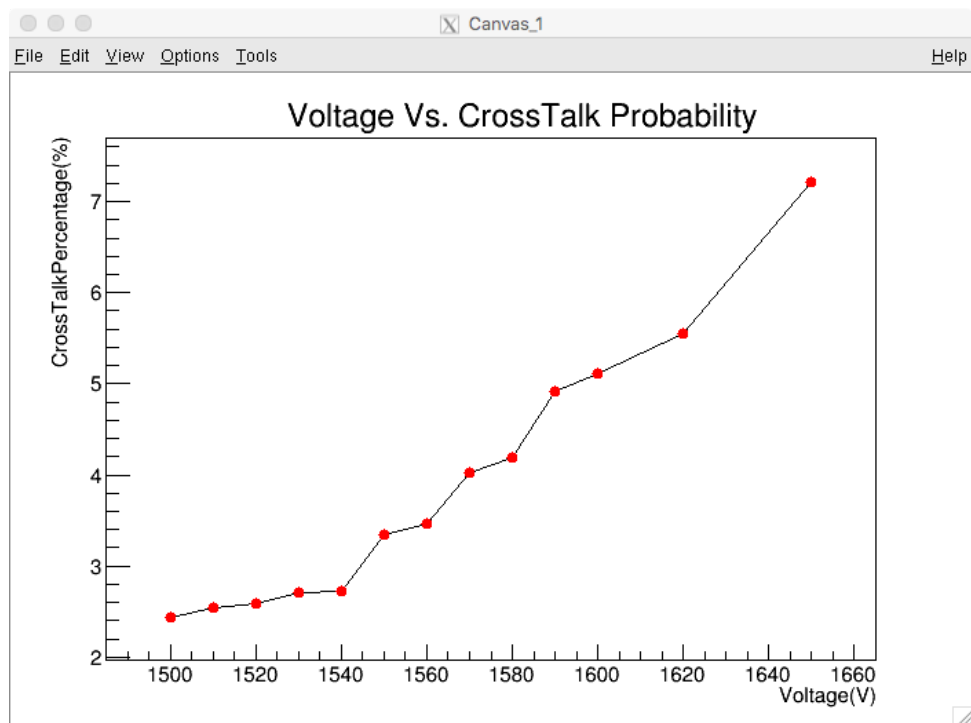


Figure 4.11: The observed correlation between cross talk with voltage. Note: Not true cross talk probability.



tracking. Firstly, it is necessary to establish at which step in the code that it is required to add in cross-talk and what properties are required so that we can randomly add this into the data.

To explain the reasoning behind this step it is necessary to explain a little bit about how the simulation that is being used works and what is being added in for data along the way. The simulation that we used is what is called a gas gun simulation where particles are randomly positioned in the ring at their decay points with some initial momentum defined by the simulation. The particle then decays to anywhere in the ring and then we can look at the decays that hit the detectors. The next step then picks out the specific straws that are hit and the distance from the center of the straw and then creates the digits. These digits only have the properties of what is called the wire id, which just labels the specific straw that is hit, and the time at which the wire was hit. Then the tracking algorithm uses this data to do the normal tracking algorithm that is shown in earlier sections. With it was found that the best place to add in the effect of cross talk is just after the digits for the actual track are being added. The only properties that are needed is that of those digits, such as where the cross-talk digits will occur and at what time the digits will occur.

To add the effect of cross talk into the simulation there was randomly picked some digits according to a defined percentage which is defined as a variable in my code. Then added in is another digit into the neighboring wire to the random time distribution that was found Fig.4.12. Next, there was regenerated all the digits along with the cross-talk digits and it was passed on to the tracking algorithm defined in earlier sections.

To confirm that the coding was correct it is satisfactory to compare the time distribution as given in the simulation, Fig.4.12, and the observed time distribution Fig.4.2, since this is the only defined parameter in the code. This shows that the data and the mathematical model agree so this confirms that the programming was correct in the simulation.

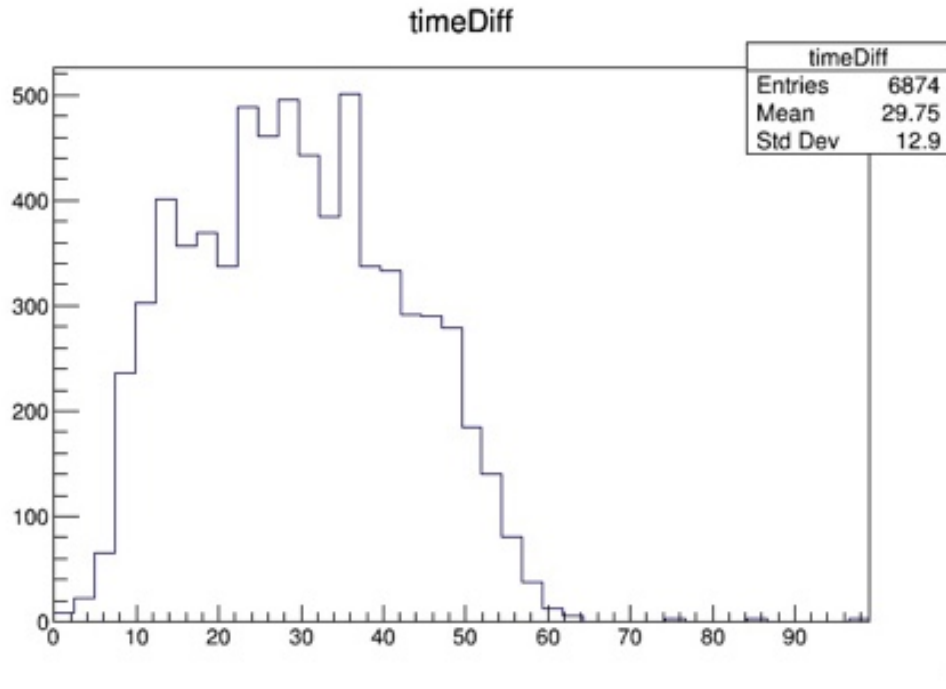


Figure 4.12: The time distribution of simulation

Comparing the vertical positions and the radial positions for different cross talk probabilities found in Fig. 4.13 and Fig.4.14 respectively. Here black is 3%, red is 5%, green is 10%, and Blue is 15%. These plots show that for a significant amount of data that the cross talk will not have a significant effect on the vertical or radial distributions. However, I need to get a number for error associated with the cross-talk probability and we know it is not zero. Furthermore, it is necessary to find out how many tracks are affected by the cross talk and specifically how the cross-talk effects the extrapolated vertical and radial positions. 4.15 and shows the percentage of tracks with cross talk found in them vs. the cross talk probability. Here the y axis is in percentage of tracks which shows that only 0.5% of the tracks make it past the track finding algorithm which shows how robust this algorithm is. Fig.4.16 shows the difference between a track with and without cross talk for the vertical distribution. Using

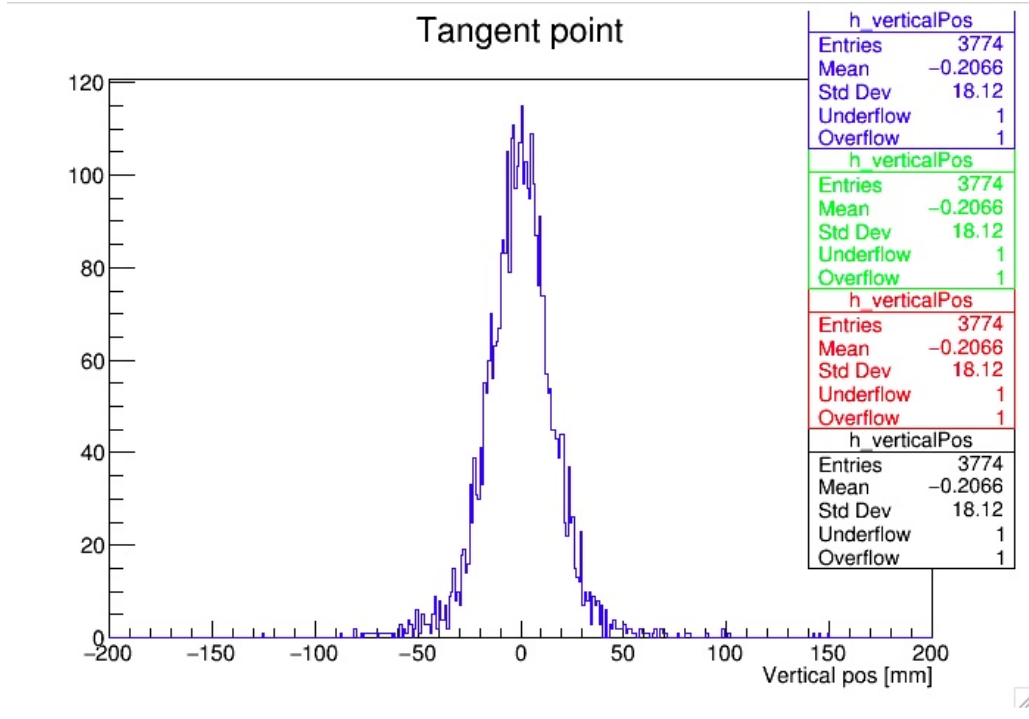


Figure 4.13: Comparison of vertical Distribution with different amount of cross talk 3% (black) 5%(Red) 10%(Green) 15%(Blue)

these numbers of the standard deviations and the means and the percentage of these tracks that are found in the final data gives an error of  $< 0.005mm$ .

## 4.2 Hit Resolution in the Straw Trackers

Hit resolution in the straw trackers is the spatial resolution at which we are able to determine the distance of closest approach to the wire in question from the found drift time of that hit. This is an important parameter since this establishes how well we are able to fit the track in the end. The resolution can change throughout the course of data collection and therefore the properties of the resolution need to be established and the effect of the changes of the resolution need to be properly evaluated.

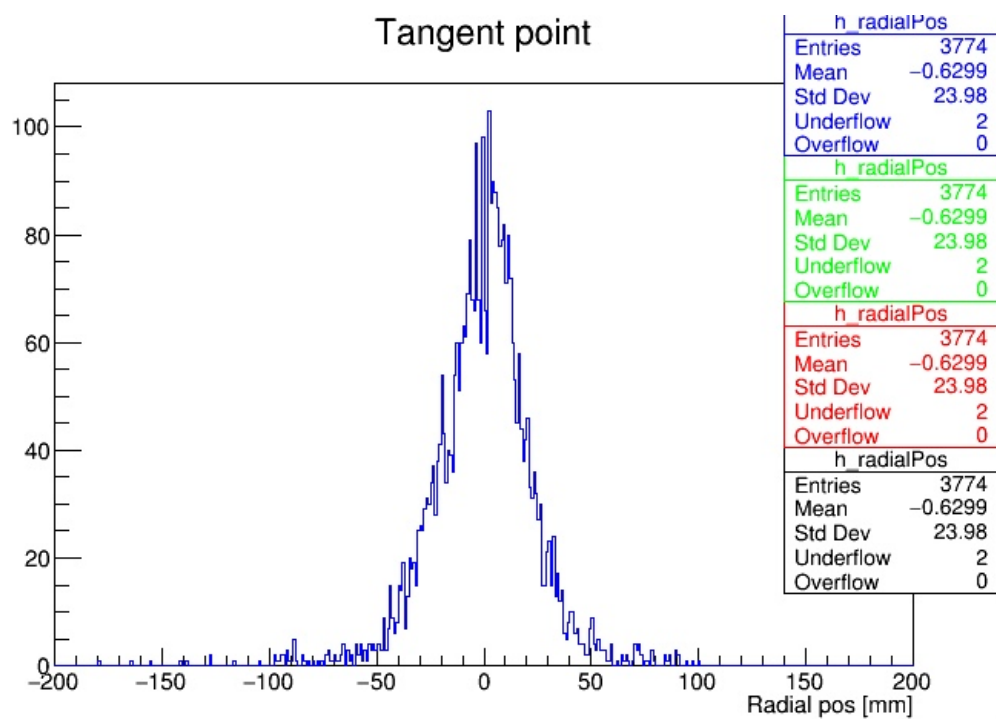


Figure 4.14: Comparison of radial Distribution with different amount of cross talk 3% (black) 5%(Red) 10%(Green) 15%(Blue)

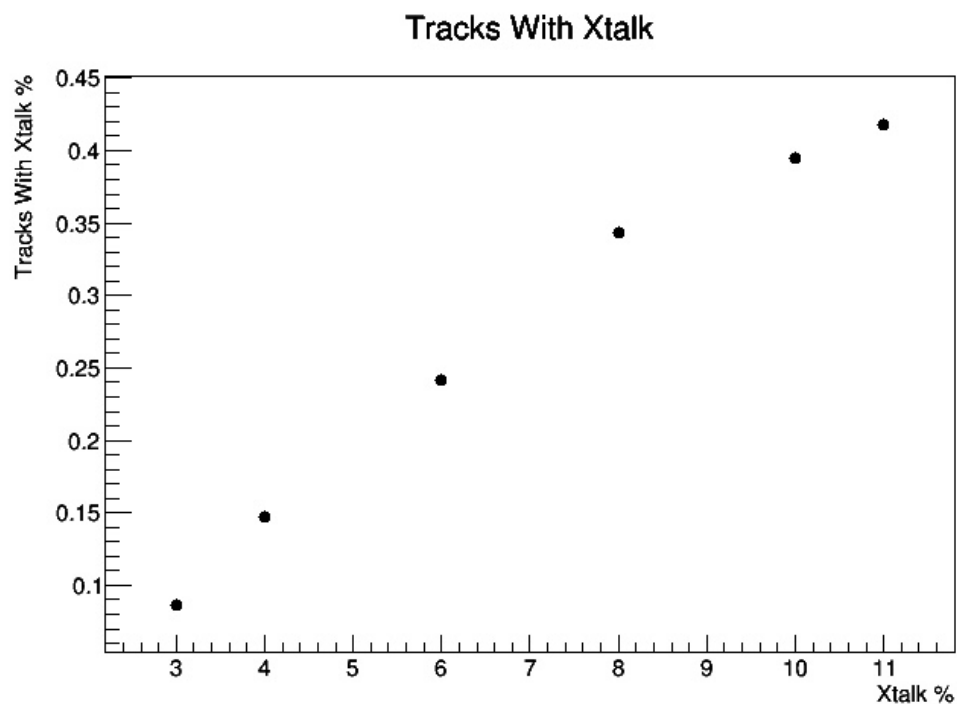


Figure 4.15: Number of tracks with cross talk that are found anywhere in them

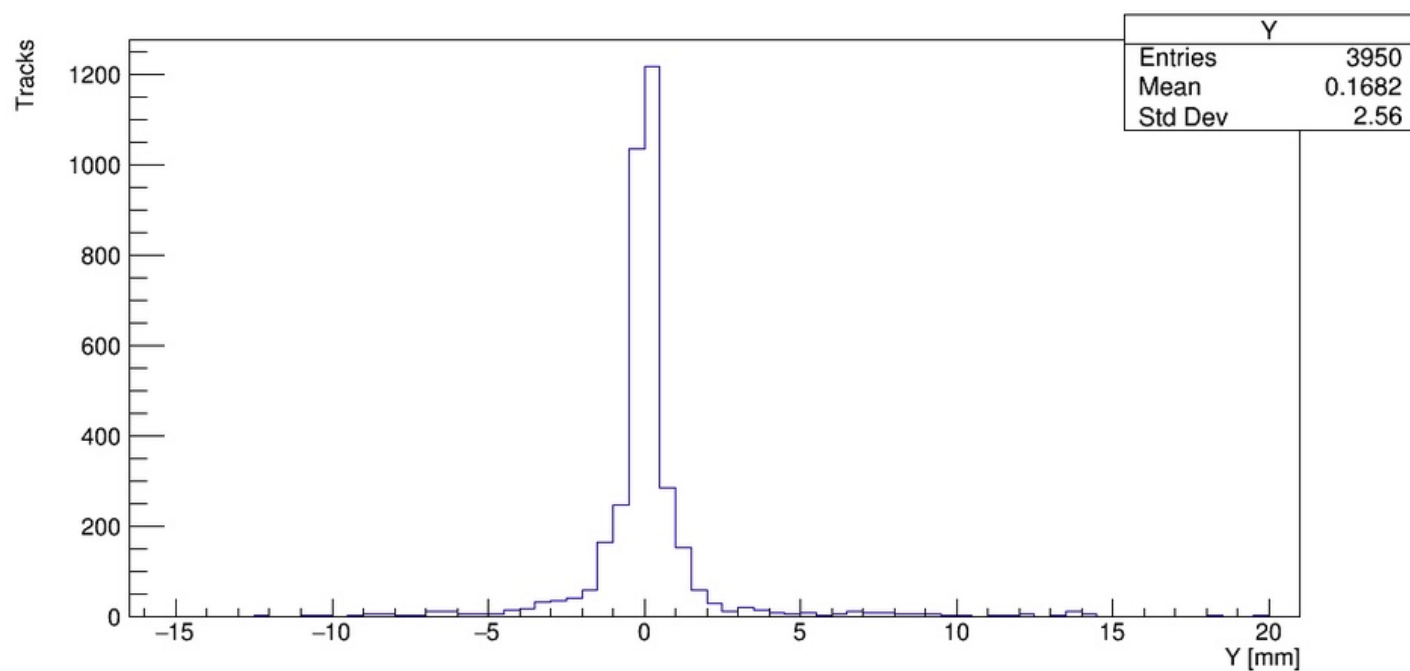


Figure 4.16: The Difference in Vertical positions in tracks with Cross Talk

### 4.2.1 General Explanation of Hit Resolution

The hit resolution in the straw trackers is determined from our ability to model the distance of closest approach to the corresponding drift times. The resolution is defined by the model of the radius to time determination of the straws, which is our modeled drift time vs the distance of closest approach. This model can change with different gasses, pressures temperatures of the surrounding environment. In addition, our model can change with variances in the voltage applied to the detectors.

### 4.2.2 Evaluation of Changes in Hit Resolution

To evaluate of time to radius distribution it is needed to find a way to observe and model the time to radius distribution in the straw tracker. To do this there was a method developed in Ref.[25] which was called the missing layers experiment. In the missing layers experiment there were a particular selection of tracks. Then there was removed a particular layer from the tracking algorithm so that we can look at where the fitted track lands on a wire without having the effects of that layer on fitting process. This would be the actual distance of closest approach, and then there was observed the corresponding drift times. Please refer to Fig.4.17 which gives a pictorial representation of the experiment.

If we look at this data with no track selections, found in Fig.4.18 , we can see that there are tails associated with this semi-gaussian distribution. These tails make it difficult to fit the distribution and skews our results for our characterization. It was necessary to look at these tracks that fall on the edge of the distribution as to improve our characterization and results. To do so we looked at a small subsection of these tracks with a plotting program that showed these tracks so that we could see what was going on. It was found that there

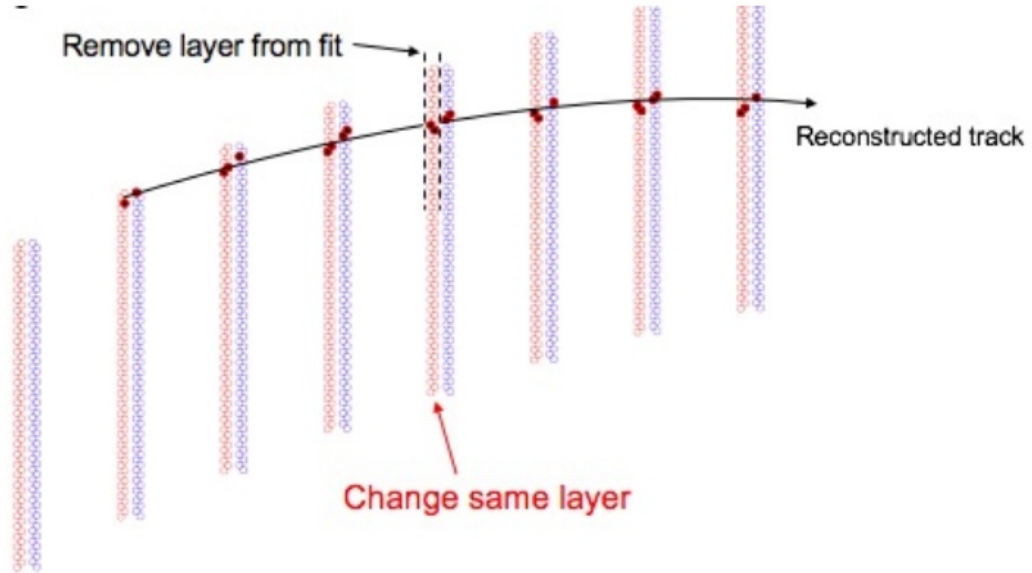


Figure 4.17: A pictorial representation of the missing layers experiment [25]

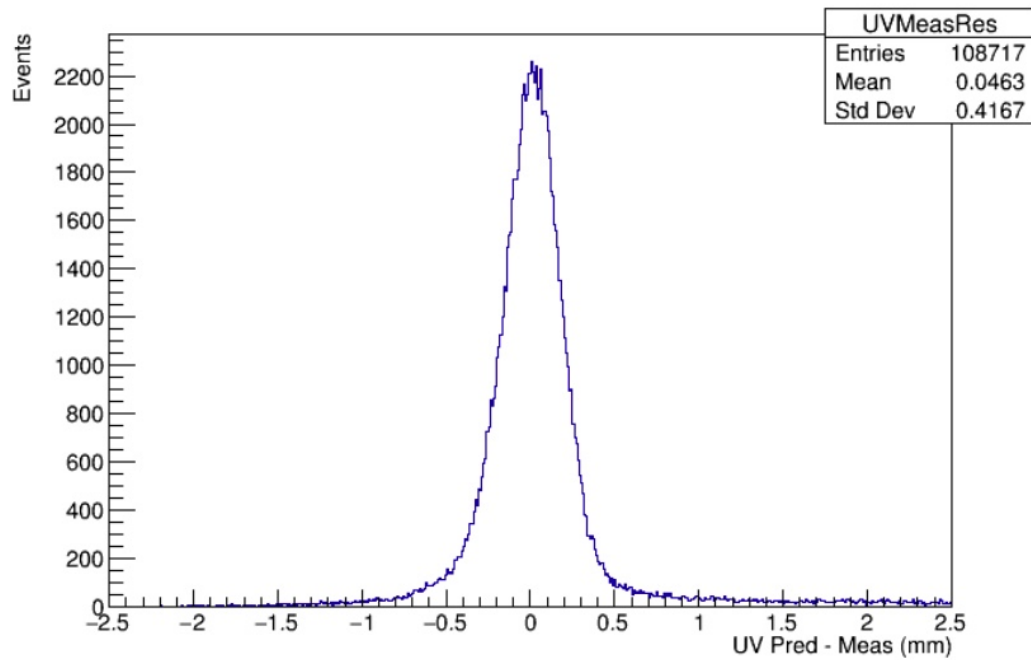


Figure 4.18: Resolution with no track cuts

were multiple different cases where the tracks were not accurately representing the data that we wanted to look at. Firstly, corresponding to the earlier section we have some cases where there might be cross talk or other similar effects. Therefore, it was required that there is one digit per layer for each track so that we are not looking at the variances added in by cross-talk. Secondly, to simplify the situation, it was required that for a time interval that there is only one track going through the detectors at a time. By looking at the tracks it was found that there were a couple cases that happened often. One case, it was found that a good portion of the tracks in the outliers were tracks that ended or began on the particular layer that we are studying. Here the track fitting with this missing layer caused the track fitting to not represent the true trajectory of the tracks and skewing our results. This was fixed by requiring that the tracks do not begin a couple of layers before and cannot end a couple of layers after the track. The next case that was found was tracks that were extremely short and they only hit a couple of layers. This causes the track fitting to be less precise due just to the sheer number of data points given in the track. To evaluate how many digits that we needed the tracks to get rid of the tracks that are not being properly represented, we looked at the standard deviations of the resolution versus the number of digits in the tracks found in Fig.4.19. Here notice that there is large tail that starts at when there is about 12 digits in the tracks. Therefore, it was required that there are more than 12 digits in the track for this study. Another common case of the outliers that was found was the case where the particle in the track reflected off of something at the layer that we were trying to study, found in Fig.4.20. Here this is a problem because before we removed the layer this track would normally fail, however when removing the layer this causes the chi squared value to not deviate out of range causing the tracks to not fail. To take care of the majority of these tracks we just put a more stringent cut on where the track begins and ends, further improving our results.



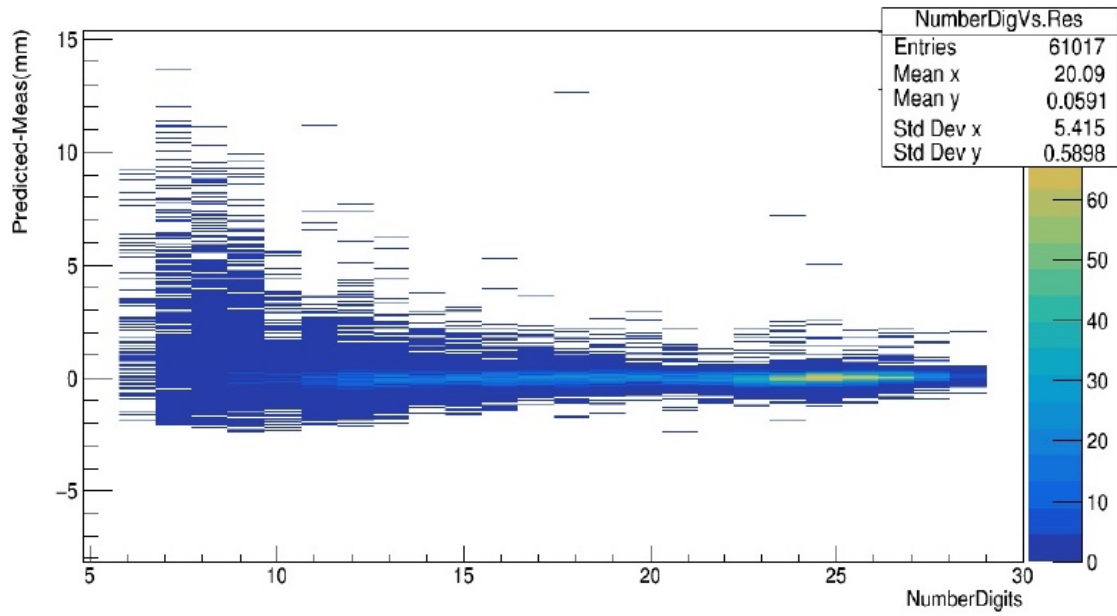


Figure 4.19: The Number of digits associated with a track vs. the Resolution

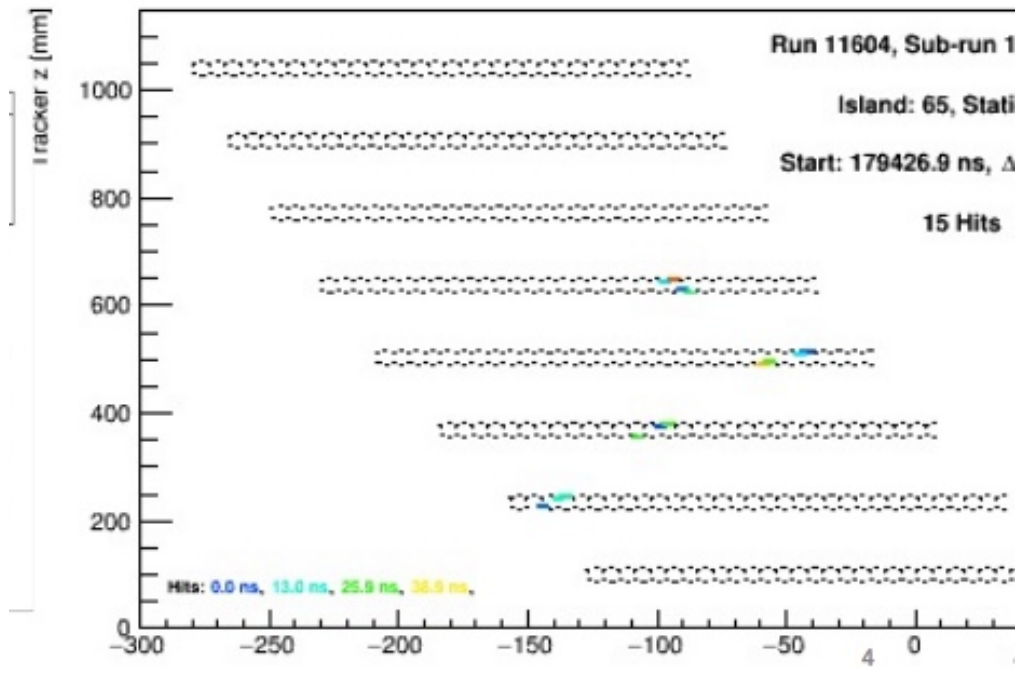


Figure 4.20: An example of a track being reflected off a boundary

To show the effects of the previously established track selection, what was looked at was the distance of closest approach efficiency at the layer that was being studied. This tells how effectively the prediction of the distance of closest approach is which would show if there were any improper track selections.

To make this plot, first there was made a histogram of all the predicted distance of closest approach, as found in Fig.4.21, and there was made a histogram of the predicted track position minus the position of the wires in the layer, as found in Fig.4.22. With the predicted distance of closest approach histogram is then divided by tracks positions histogram and normalized to the number of wires found in that layer. This will give what we call the distance of closest approach efficiency as found in Fig.4.23. This figure gives an initial result before the tracks selection which establishes that if the tracks hit a straw in the layer that only 90% of the tracks are being properly predicted. This result is not correct since if a particle hits a straw, it should always be able to detect this. Now, by looking at Fig.4.24 with the current tracks selection it is seen that a distance of closest approach efficiency is 99%, which is a more reasonable number and shows that the resolution is more effectively being measured. Now by looking at the resolution distribution before, Fig.4.18, and the resolutions distribution after, Fig. 4.26, most of the tails of this distribution are now pretty much gone and the distribution looks more gaussian with a smaller standard deviation.

To effectively measure the property of the resolution it was needed to fit this resolution distribution with an appropriate function. Although, this function looks fairly gaussian, which is what should happen when measuring the resolution, fitting to a gaussian it is seen the histogram is not being fit properly, Fig. 4.25. This deviation in the histogram from being a true gaussian occurs because what is actually being measured is a couple of different resolutions occurring at the same time, there is the resolution of our track fitting algorithm and the resolution of our distance of closest approach model. In addition, the effects of the gas that is being used along with timing differences in the detectors...etc. Knowing this

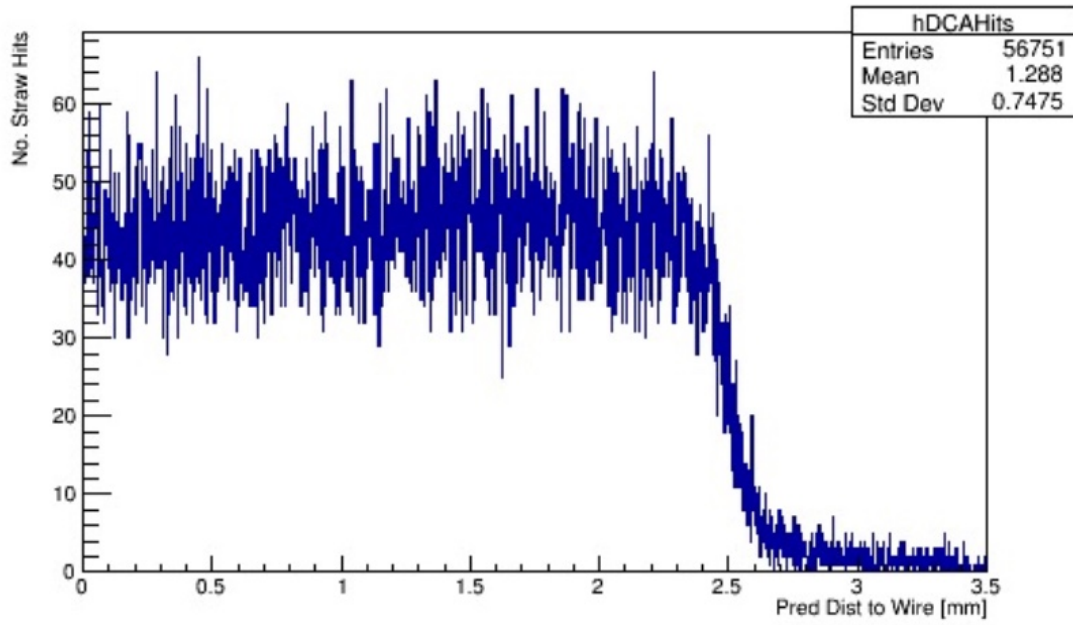


Figure 4.21: All the predicted DCA Hits

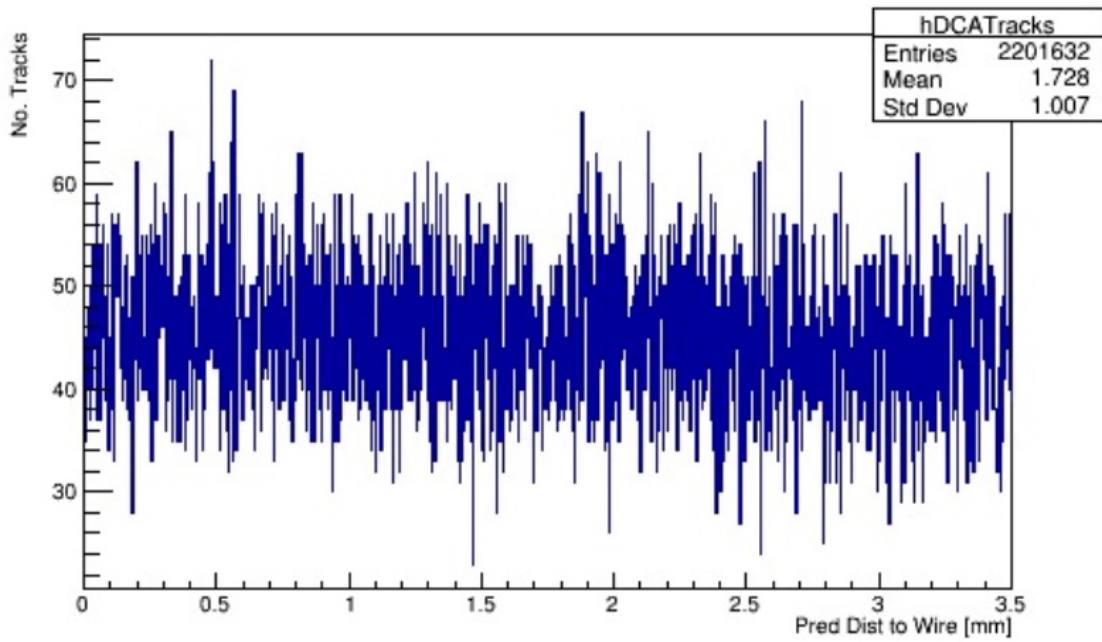


Figure 4.22: All the DCA's determined by the track position

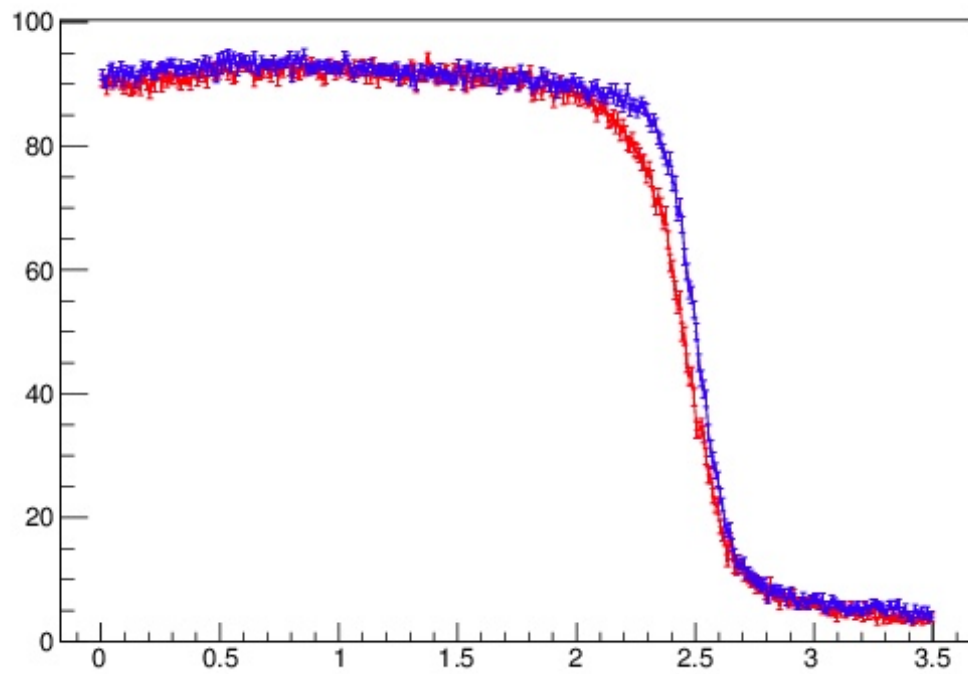


Figure 4.23: The distance of closest approach efficiency with poor track selection black is 1550V red is 1660V

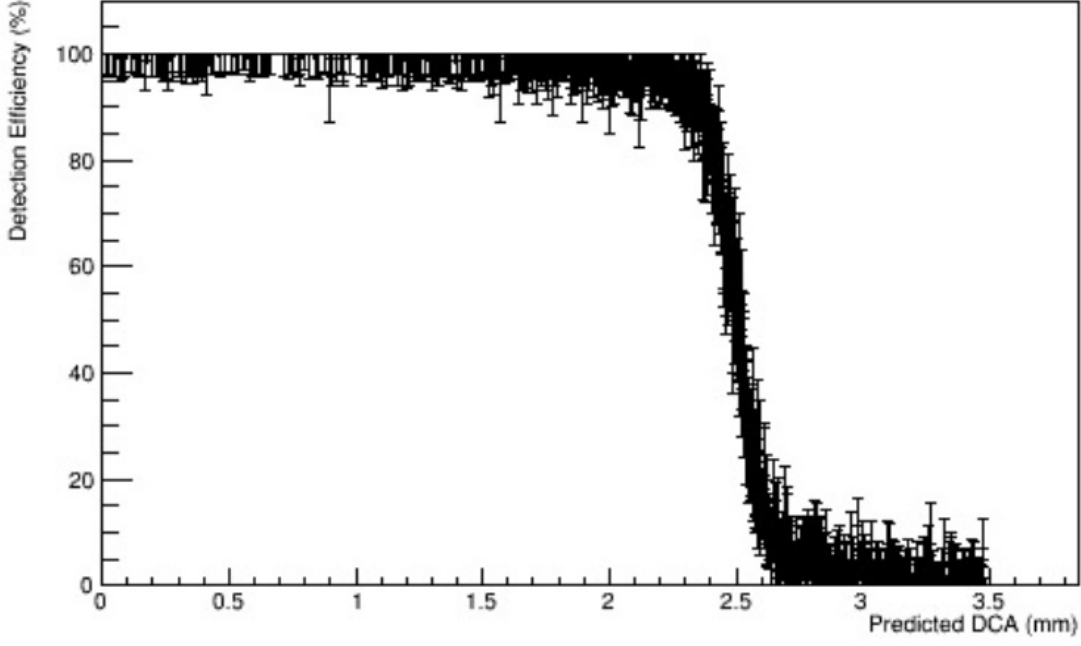


Figure 4.24: The distance of closest approach efficiency

there is a motivation to fit this function to multiple gaussians, one fixed at the core and another one to broadly overly to take in account the tails in the distribution. In Fig.4.26 is an example of the new fitting method which shows that the fitting function for the resolution plot is more effectively being fit. Then the overall distribution standard deviation is found by adding the standard deviations in quadrature based on their mean found in Eq.4.1. Here  $P_n$  is the integral of the gaussians,  $\sigma_n$  is the standard deviations, and  $\mu_n$  are the means.

$$var = P_1\sigma_1^2 + P_2\sigma_2^2 + [P_1\mu_1^2 + P_2\mu_2^2 + (P_1\mu_1 + P_2\mu_2)^2] \quad (4.1)$$

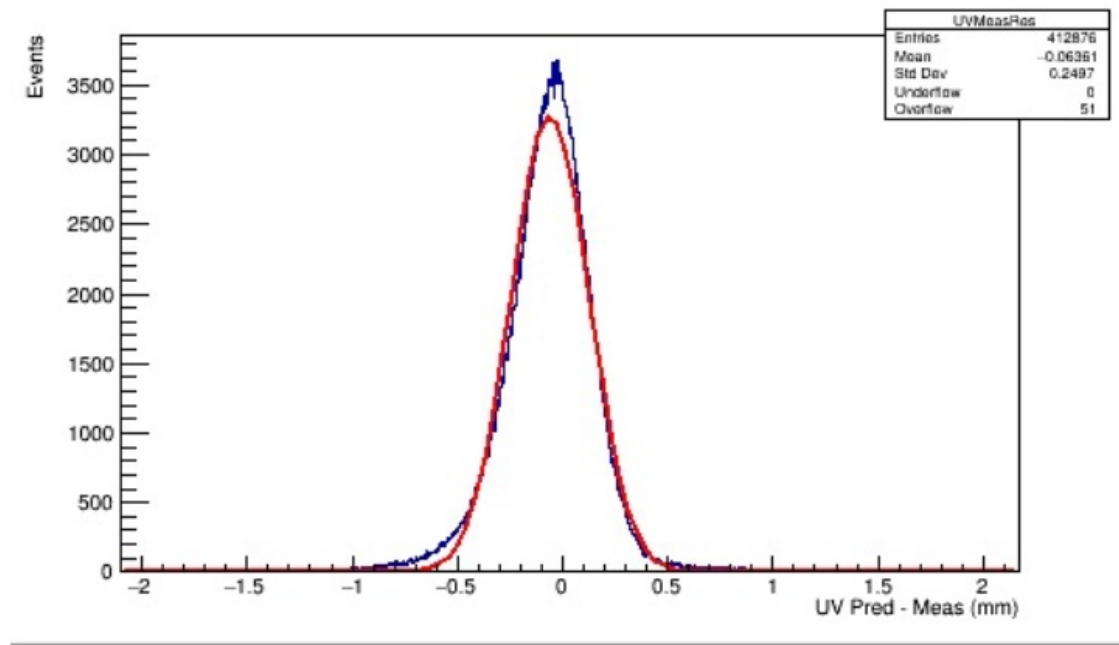


Figure 4.25: Resolution with poor fitting

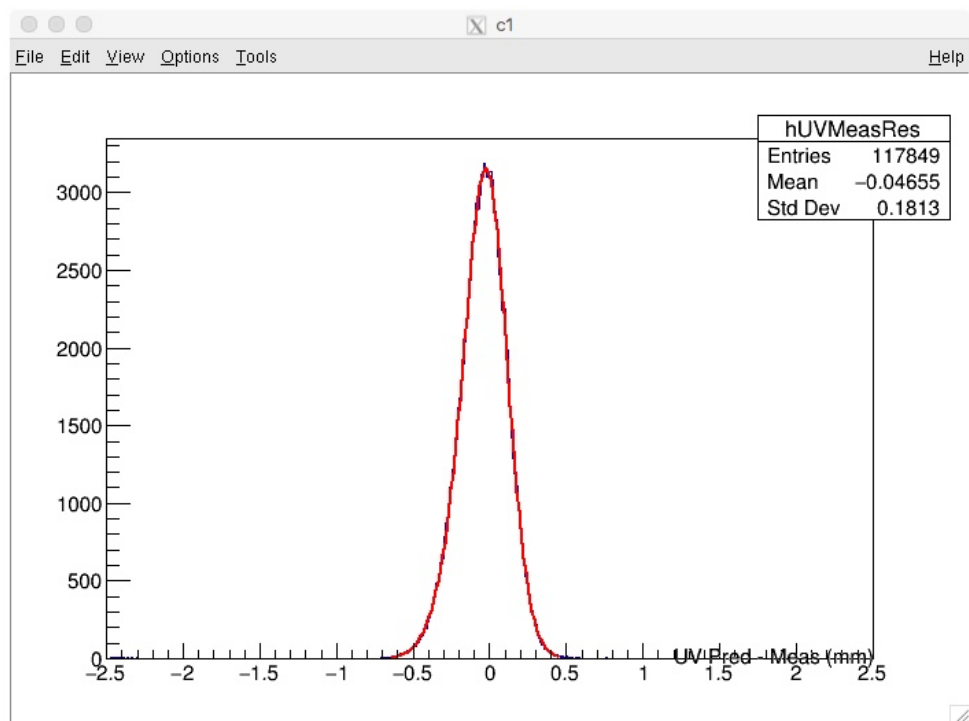


Figure 4.26: Resolution with the proper track cuts and good fit

### 4.2.3 Evaluation of Changes of Resolution

Using the methods established in the previous section we now want to look at how the resolution varies for changes in voltages, temperatures, pressures, and from run to run. This is important to understand how to model the changes to study how much of an effect these variances have on our data. In addition, this gives information on how improvements of the resolution of the detectors can be made.

The first study that was done was the HV Scans experiment. With the beam being on there were tracks collected while the voltage of the tracker for the layers that were being studied were varied, while keeping all the other tracker voltages the same. Now this tells how much the model deviates versus changes in voltages. Studied was both stations 12 and 18. The detectors that were being looked at was the third tracker in the stations, which gives a good example of tracks in the middle of the detector. The layers were the first two layers in the tracker. In Fig.4.27 it is seen that we definitely get a variance due to voltage changes which is expected however, it is seen from the plot that we only get a variance of 0.04mm over the course of 160 Volts change. This is a large voltage variation as compared to the variation that we would expect under normal conditions. Notice, that the resolution gets better as we increase the voltage, this is because at higher voltages the larger the deviations in the drift times and the distance of closest approach is approximated better. In addition, noticing the large deviations in the plot, away from the linear relationship that this seems to follow, this comes from the fact that the data was collected over a period of 12 hours. This could be due to large deviations in other parameters that were not recorded, therefore these deviations are not out of line. In addition, notice that there is a difference between the two tracking stations. This difference is due to the fact the two detectors being looked at have two different radii of straws in the tracker.

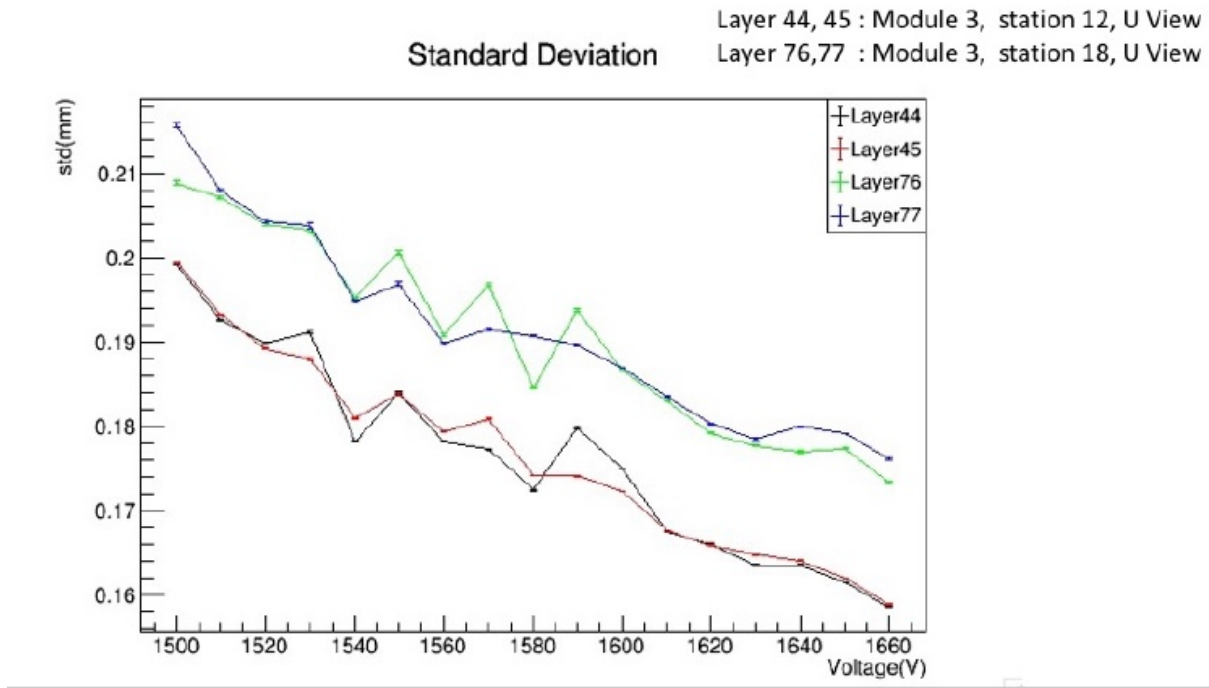


Figure 4.27: Resolution vs. Voltage

For the rest of the studies of the resolution it is necessary to have significant amount of data collected over a large amount of time. The data set being used is called the 60-hour data set. As it sounds this was data that was collected over a continuous 60-hour period. In Fig.4.28 there is the resolution for layers 44, 45, 76, and 77. From run to run it is observed that there is roughly a 10% deviation in the resolution over the course of the data set. Here the error bars are given by the fitting errors. To investigate how much the resolution varies over the course of a run two runs were taken and split into subsets of 10 sub-runs. The variance of the resolution is found in Fig.4.31 which shows that there is approximately 13% variance over the course of the run.

Now with this data the average pressures, temperatures, and humidity over the course of these runs can be observed. In Fig.4.30 shows how the temperature in the experiment room varies over the course of data collection. By looking at the average temperature over



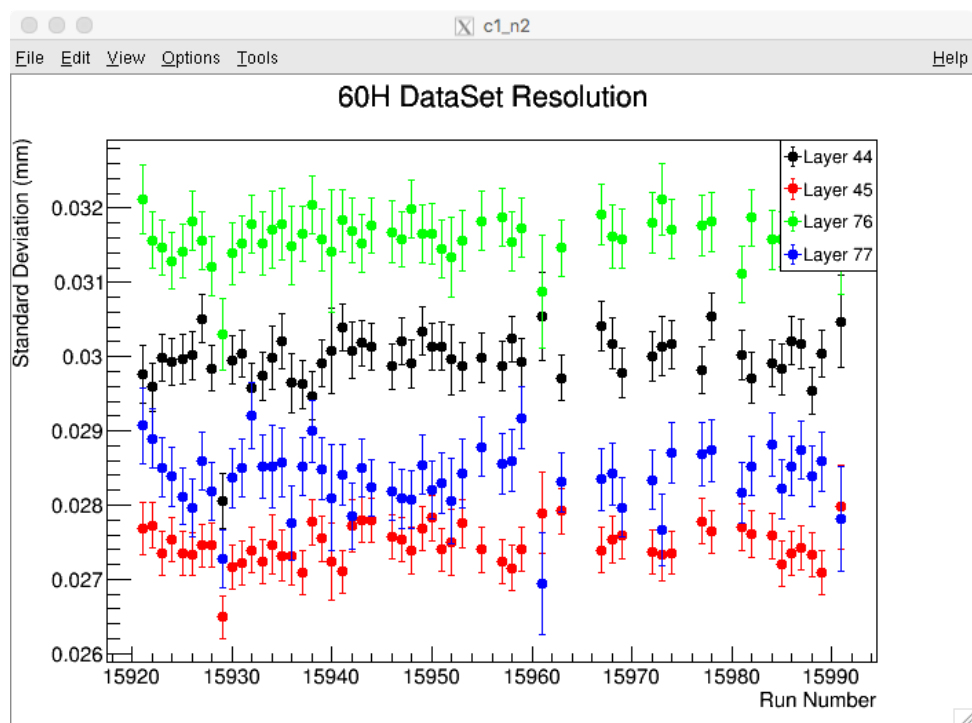


Figure 4.28: Resolution Vs. Run Number

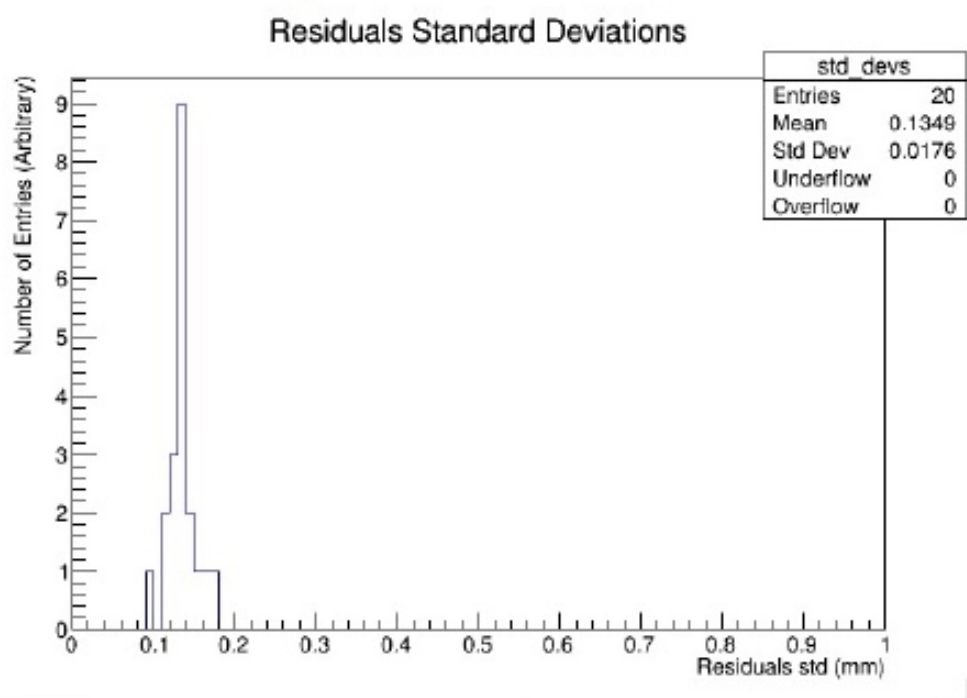


Figure 4.29: Resolution measurement given over two runs

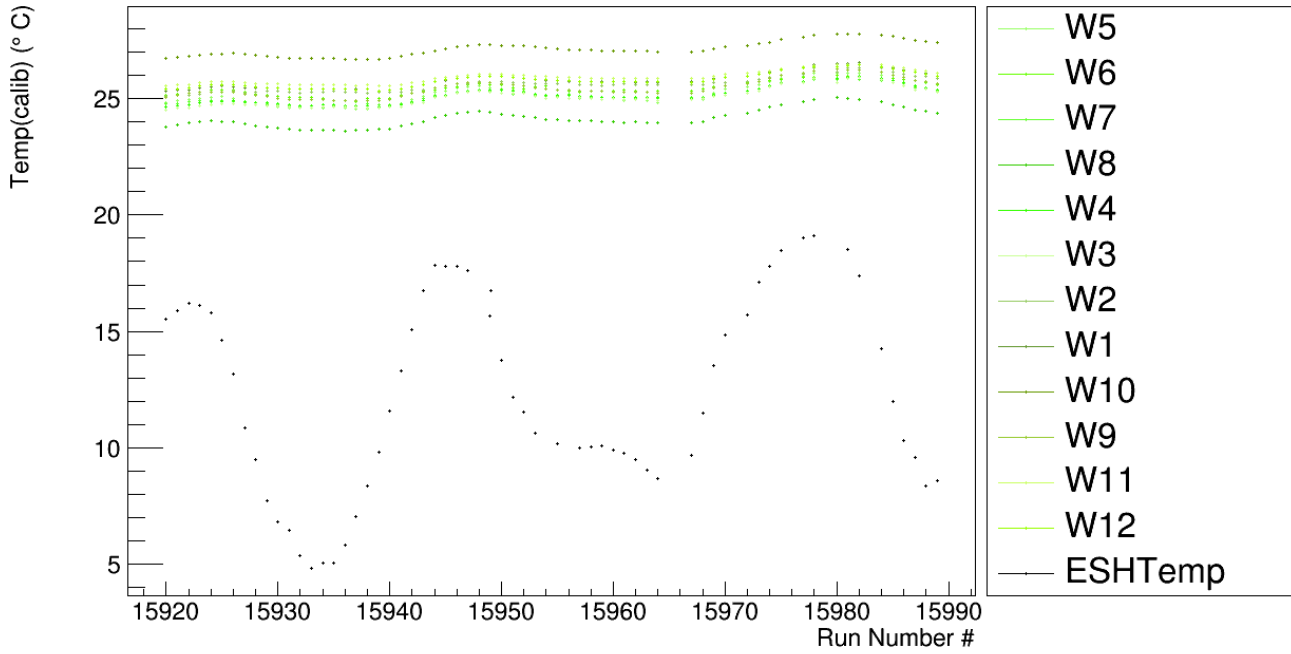


Figure 4.30: Temperature Vs Run Number

the course of a run versus the resolutions from run by run, Fig.4.30, it is seen that there is no significant correlation between temperature and resolution changes with a Pearson correlation coefficient of 0.06. It was shown by a colleague that the track fitting changed with temperature by plotting the chi squared value versus the temperature as shown in Fig.4.32 Ref.[26] . Which was expected to be from the resolution changes, but it was shown that was not the case. To make sure this is the case it was investigated that the chi squared values versus temperature didn't change. Fig.4.33 and Fig.4.34 shows the mean and standard deviations of the chi squared values do not vary with the temperature. Therefore, this reaffirmed the previous statement in saying the resolution was not significantly dependent on temperature in addition to saying that the deviation in the chi squared values versus temperature was caused by something else.

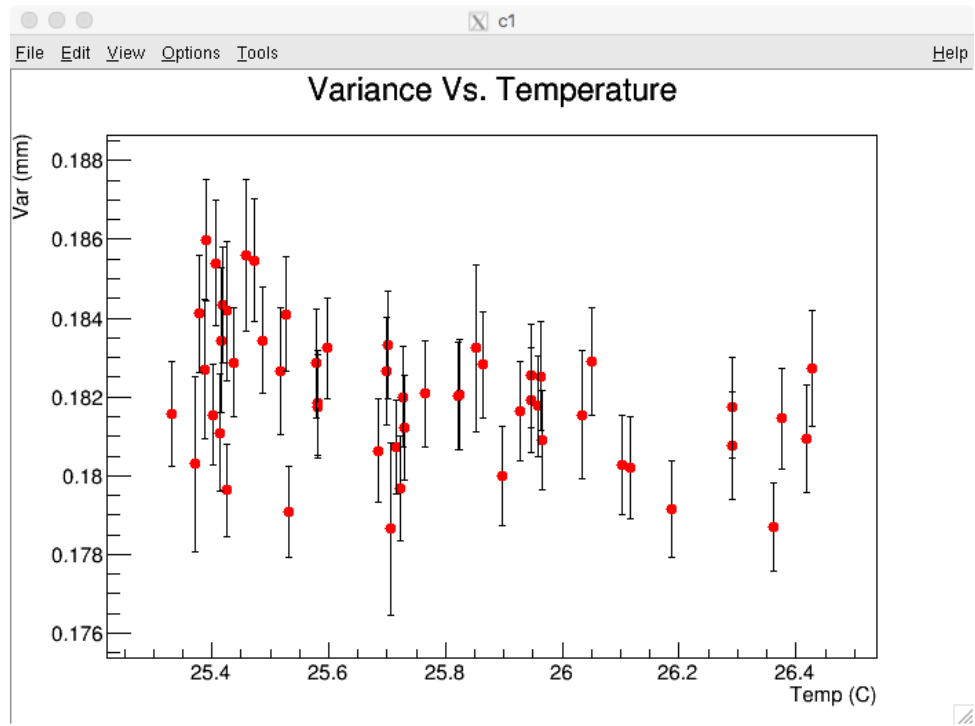


Figure 4.31: Resolution Vs. Temperature

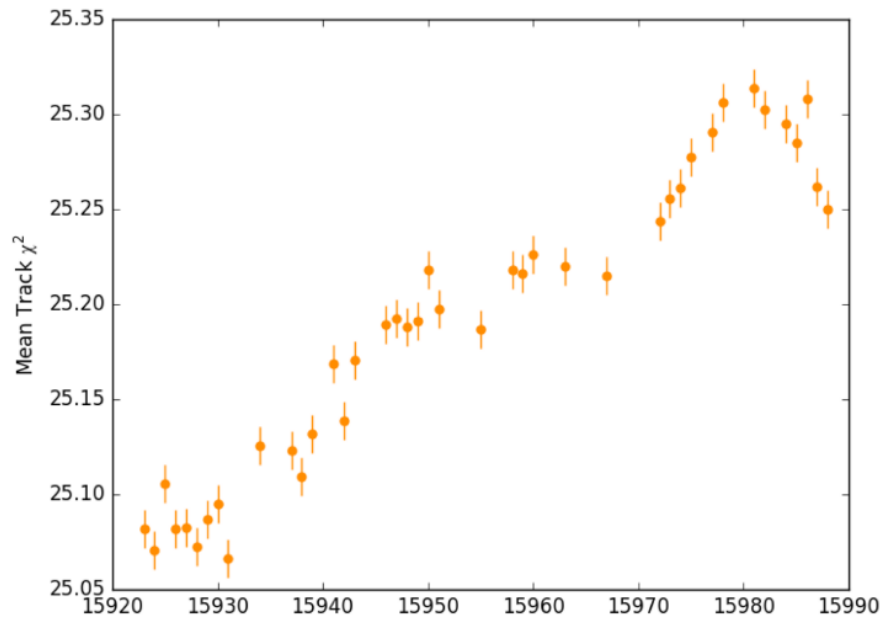


Figure 4.32: Mean Chi2 over 60H Data Set [26]

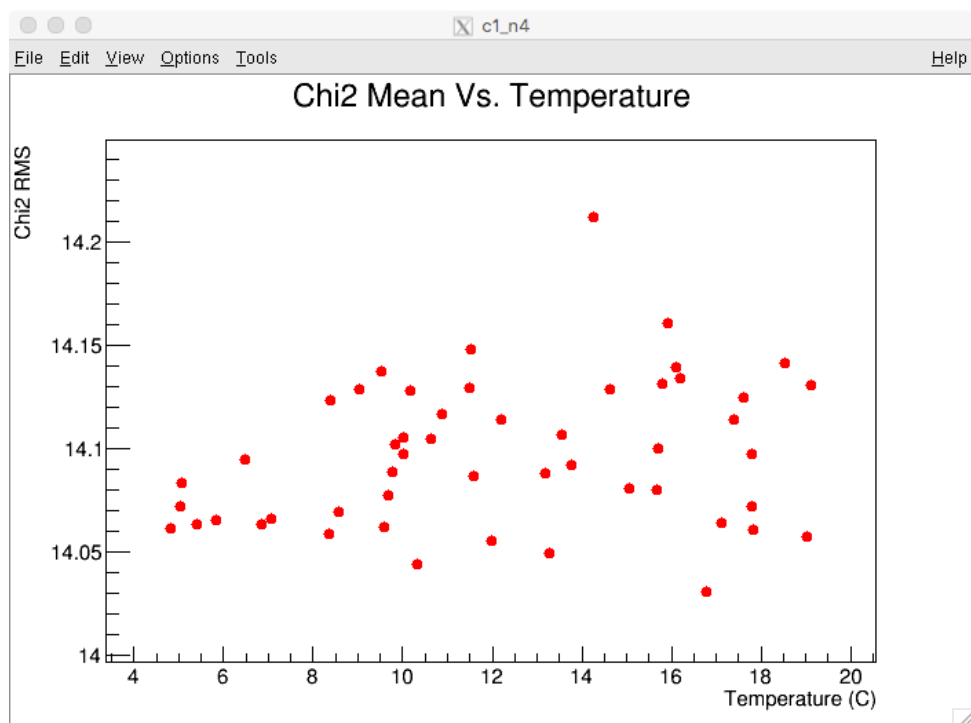


Figure 4.33: Chi2 Mean Vs. Temperature

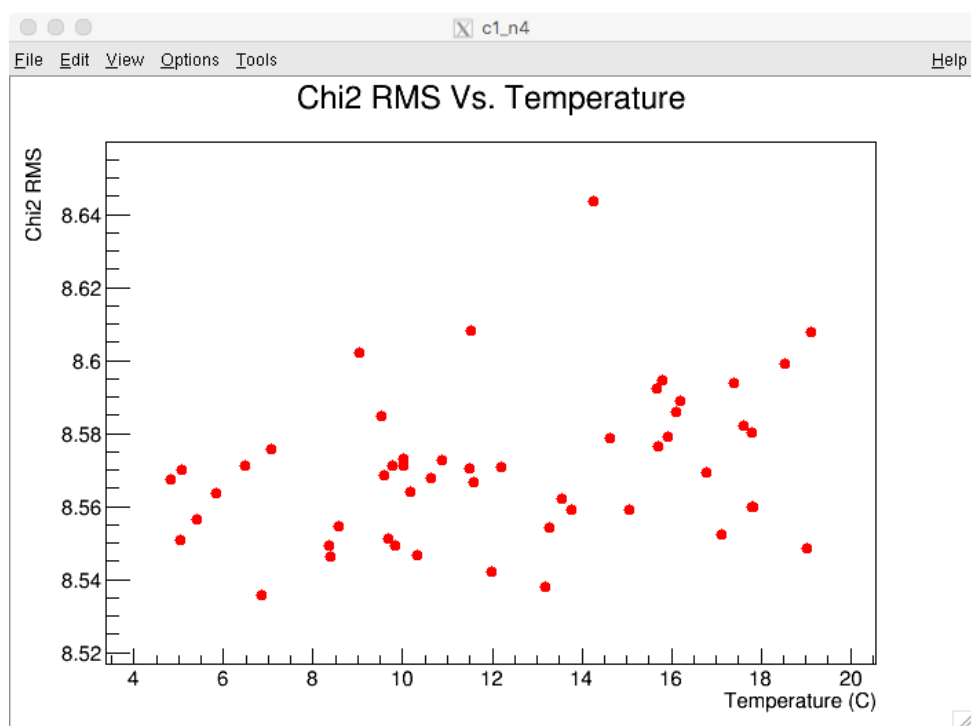


Figure 4.34: Chi2 RMS Vs. Temperature

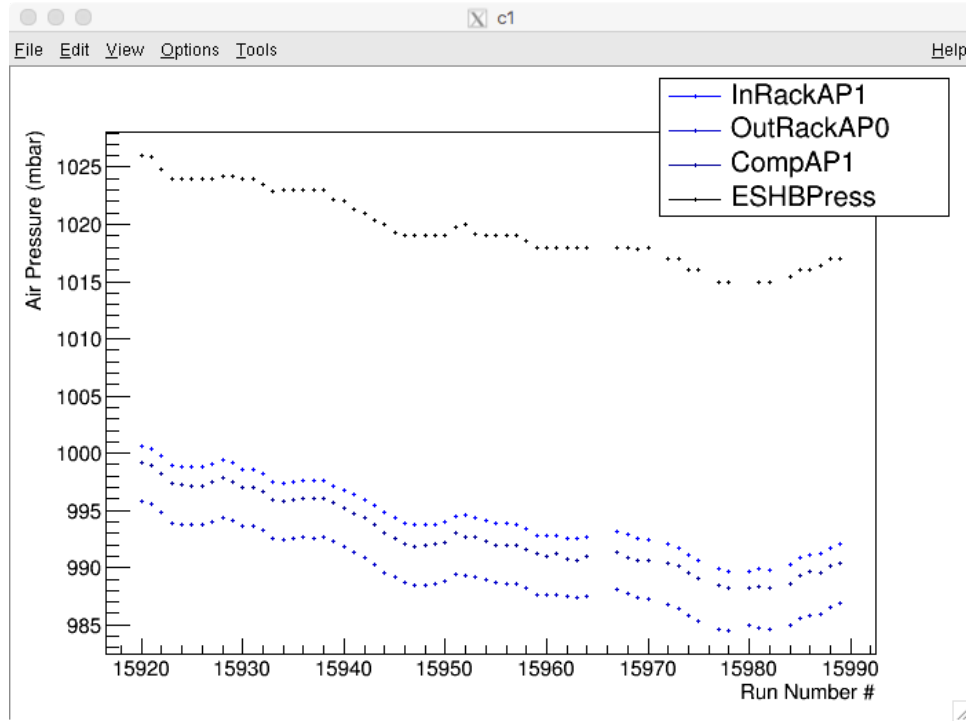


Figure 4.35: Pressure Vs. Run Number

Next the correlation between pressures and resolutions were investigated in a similar way to the temperatures. Fig.4.35 show how the pressure varied over the course of the runs that were being investigated. Looking at the average pressure versus the resolution averaged from run to run, Fig.4.35, again shows that there is no significant correlation between pressure and the resolution with no significant correlation factor. Similarly, it was shown that the chi squared value varied versus pressure. Again, we wanted to make sure that this was not due to the resolution. Shown in Fig.4.36 and Fig.4.37 shows the mean and standard deviations of the chi squared values do not vary and the variance is caused by some other changes in the tracks vs. pressure.

It has been shown that the weather effects have no significant effects on the resolution of the trackers. In addition, it has been observed the amount that we would expect the tracker

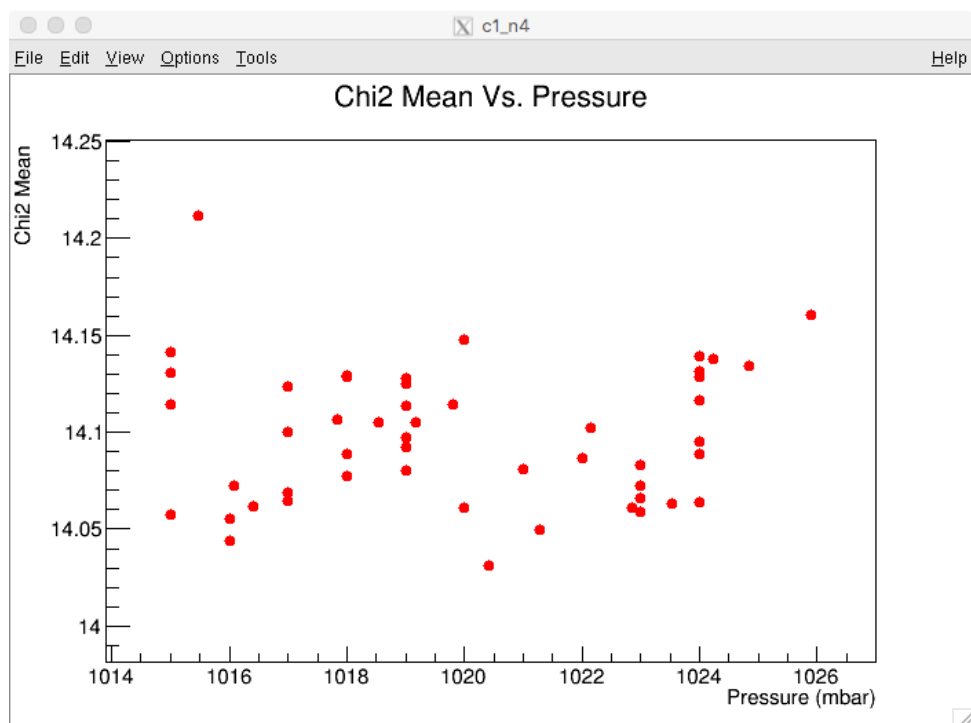


Figure 4.36: Chi2 Mean Vs. Pressure

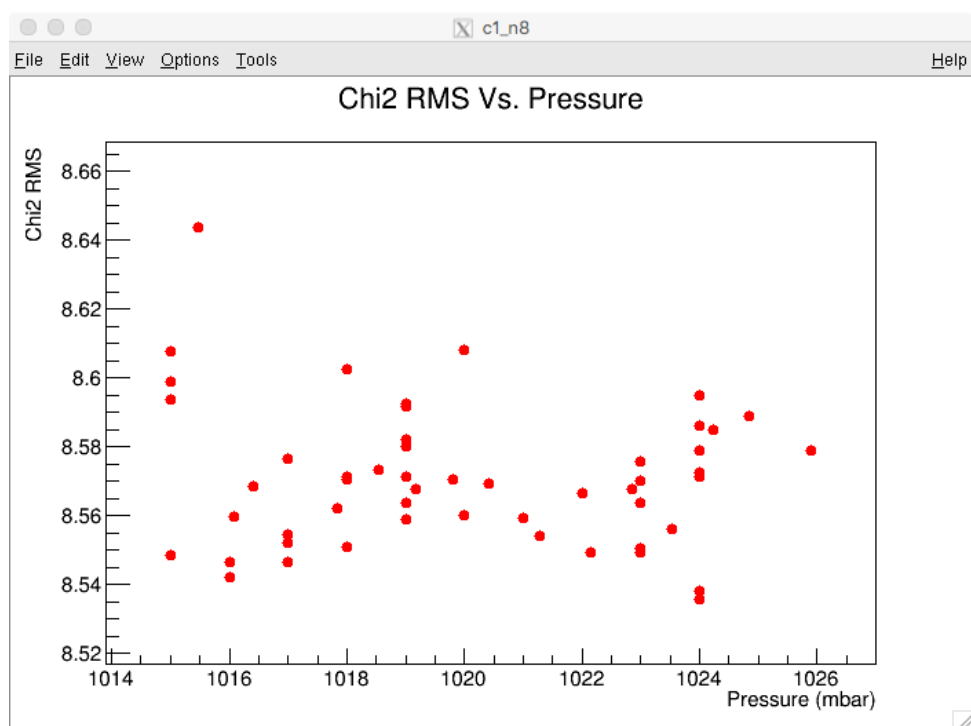


Figure 4.37: Chi2 RMS Vs. Pressure

resolution to change over the course of the experiment. This allows us to investigate how much of an effect this has on our final results.

#### 4.2.4 Changing Hit Resolution in Data

Now that it has been established what causes changes and by how much in the hit resolution it is necessary to evaluate how much the changes affect our final results. To do this one run from the 60-hour data set was looked at, and the distance of closest approach was randomly varied according to a gaussian distribution. This gives the same effect as the resolution varying and it can be looked at how the extrapolation changes the vertical and radial distributions. Practically, to do this experiment there was directly added in a section of code that randomly selects a value according to a gaussian distribution with a defined standard deviation and mean of zero. Then this randomly generated value was added to the distance of closest approach and passed on to the rest of the tracking algorithm.

Previously, it was shown that over the course of a run that there is a 13% variance in resolution. Looking at the extrapolated y positions versus different smearing percentages, Fig.4.38 , it finds that the average y position at 13% only varies by a small amount. In this figure the red line is at 13%. Therefore, even if the resolution is varied by 100% the average y position changes by 3mm which is small compared to the other error contributions covered. Similarly, by looking at the standard deviation of the extrapolated vertical position versus smearing Fig.4.40, it is observed that a change of 0.05mm at 13% and 5mm at 100%. Again, this shows that the standard deviation of the y position is not greatly affected by the resolution changes. Looking at the same plots except for the radial distribution, Fig.4.41 and Fig.4.42, observed is only a change of 0.05mm in the average radial position and 0.2mm

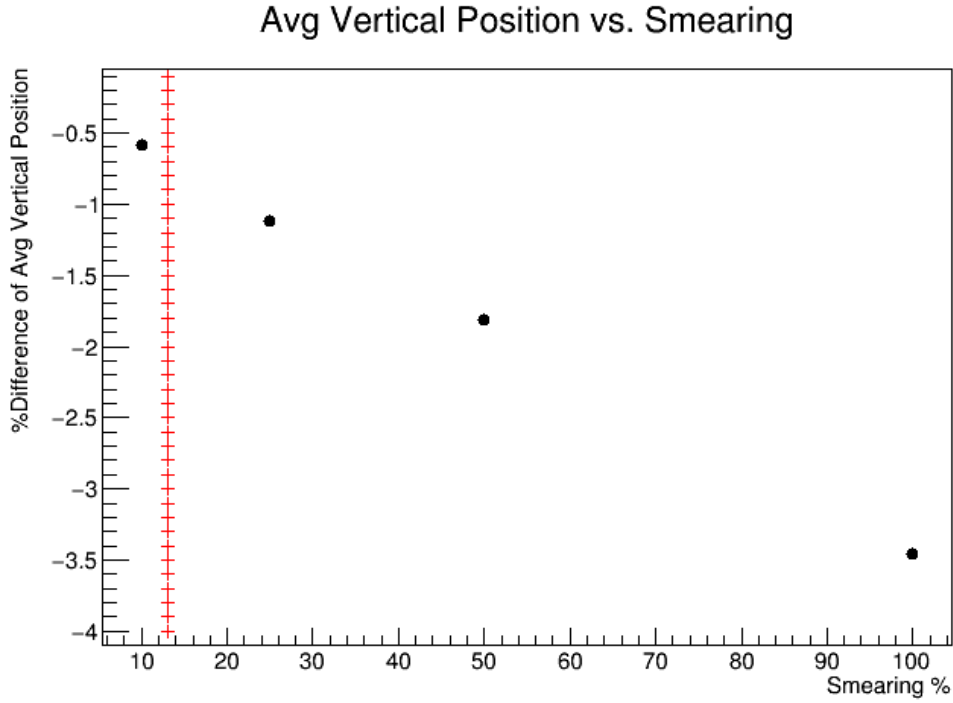


Figure 4.38: Difference in Vertical Position Vs. Smearing

change in radial standard deviation at the 13% mark, which again is a small contribution to the errors.

### 4.3 Results and Conclusions

In this chapter it has been explained the work that has been done towards trying to understand the systematics and their contributions to the final numbers. In other contributors work it has been found that there are many corrections to the tracking algorithms that can be worked towards to improve the results of the analysis. With the work that I have done it has been shown that these two effects that have been studied contribute in a small fashion. I showed that cross talk will only affect the final numbers to the  $<0.005$ ppb level in which the largest effects mainly the tracker alignment contributes to the 0.3 ppb level.



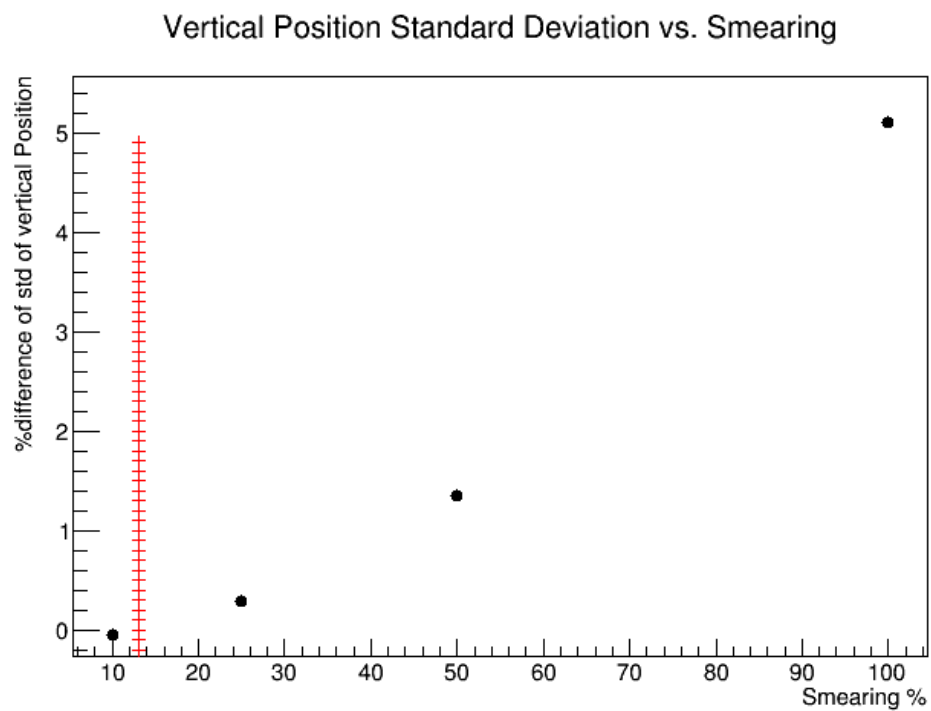


Figure 4.39: Difference in Vertical Position Standard Deviation Vs. Smearing

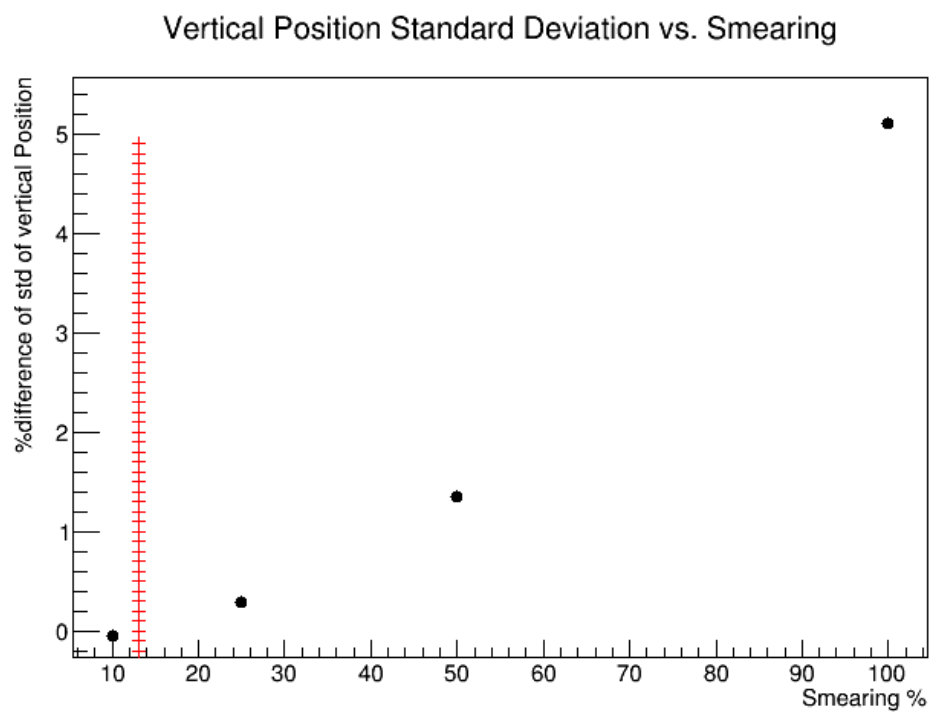


Figure 4.40: Difference in Vertical Position Standard Deviation Vs. Smearing

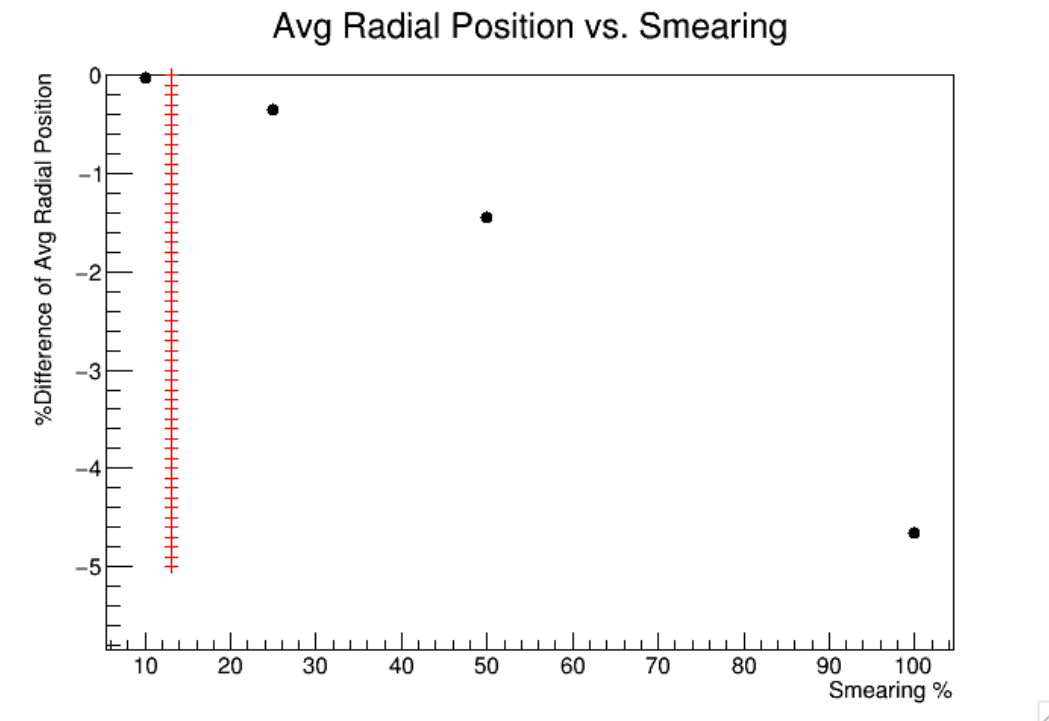


Figure 4.41: Difference in Radial Position Vs. Smearing

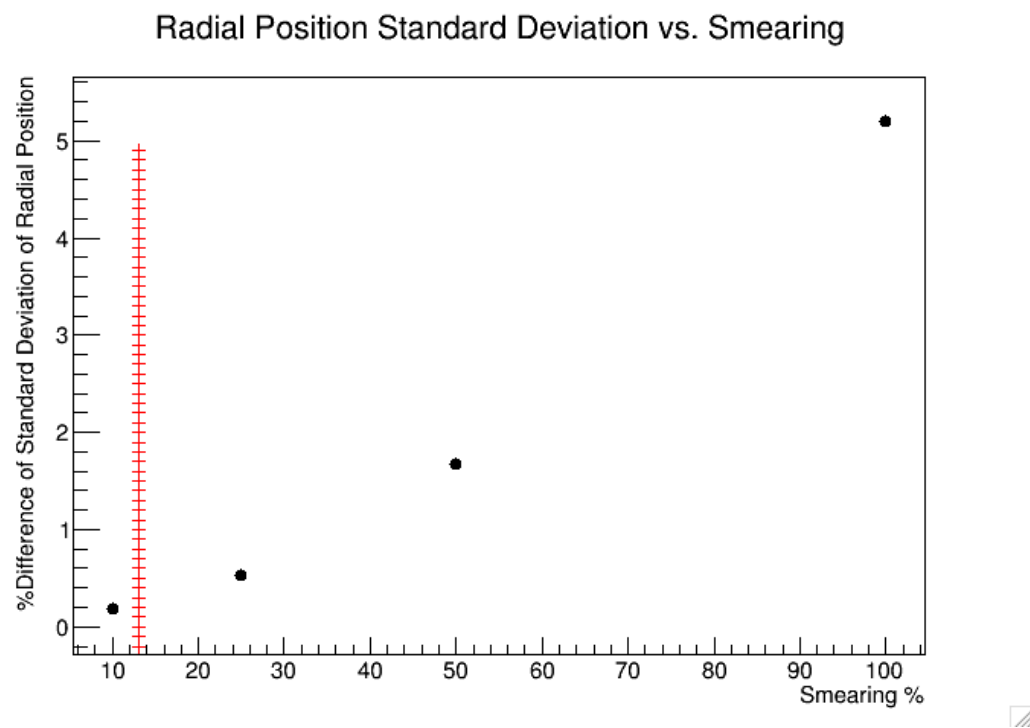


Figure 4.42: Difference in Radial Position Standard Deviation Vs. Smearing

In addition, I have found that due to the resolution changes that we would expect over the course of the experiment is also small and it effects the final measurement to the 0.03 ppb level. In conclusion, it has been found that there are other areas in the tracking which would contribute more than these uncertainties.

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