Section 2.7 Exercises

David Piper

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2.47 For two statements P and Q we want to show that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ and $(P \implies \sim Q) \wedge (P \wedge Q)$ are contradictions. First let us observe that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ has the following truth table.

P	Q	$\sim Q$	$P \wedge (\sim Q)$	$P \wedge Q$	$(P \wedge (\sim Q)) \wedge (P \wedge Q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	F	F
F	F	T	F	F	F

From this truth table we can observe that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ is false for all possible values of P and Q, thus it is a contradiction.

Next let's consider the statement $(P \implies \sim Q) \wedge (P \wedge Q)$. We can observe that it has the following truth table.

P	Q	$\sim Q$	$P \implies \sim Q$	$P \wedge Q$	$(P \implies \sim Q) \wedge (P \wedge Q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

From this truth table we can observe that $(P \implies \sim Q) \wedge (P \wedge Q)$ is false for all possible values of P and Q, thus it is a contradiction.

2.49 For the statements P, Q and R we want to show that $((P \implies Q) \wedge (Q \implies R)) \implies (P \implies R)$ is a tautology. First let us observe that these statements

have the following truth table.

P	Q	R	$P \implies Q$	$Q \implies R$	$(P \implies Q) \wedge (Q \implies R)$	$P \implies R$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$((P \implies Q) \wedge (Q \implies R)) \implies (P \implies R)$
T
T
T
T
T
T
T

From this truth table we can observe that $((P \implies Q) \wedge (Q \implies R)) \implies (P \implies R)$ is true for all possible values of P, Q and R; thus it is a tautology. If we restate this compound statement in words we have the following: If P implies Q and Q implies R then it follows that P implies R.

2.51 For the statements P and Q we want to find out if $(P \vee Q) \vee (Q \implies P)$ is a tautology, contradiction, or neither. First let us observe that these statements have the following truth table.

P	Q	$P \vee Q$	$Q \implies P$	$(P \vee Q) \vee (Q \implies P)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

From this truth table we can observe that $(P \vee Q) \vee (Q \implies P)$ is true for all possible values of P and Q, thus it is a tautology.