Section 2.5 Exercises

David Piper

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2.31 (a) Given the open sentences $P(x) : |x| = 4$ and $Q(x) : x = 4$ over the domain $S = \{ -4, -3, 1, 4, 5 \}$. We have the following truth values for $P(x) \implies Q(x)$ for each $x \in S$.

$$\begin{aligned} P(-4) &\implies Q(-4) \\ T &\implies F \end{aligned}$$

False.

$$\begin{aligned} P(-3) &\implies Q(-3) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(1) &\implies Q(1) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(4) &\implies Q(4) \\ T &\implies T \end{aligned}$$

True.

$$\begin{aligned} P(5) &\implies Q(5) \\ F &\implies F \end{aligned}$$

True.

(b) Given the open sentences $P(x) : x^2 = 16$ and $Q(x) : |x| = 4$ over the domain $S = \{ -6, -4, 0, 3, 4, 8 \}$. We have the following truth

values for $P(x) \implies Q(x)$ for each $x \in S$.

$$\begin{aligned} P(-6) &\implies Q(-6) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(-4) &\implies Q(-4) \\ T &\implies T \end{aligned}$$

True.

$$\begin{aligned} P(0) &\implies Q(0) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(3) &\implies Q(3) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(4) &\implies Q(4) \\ T &\implies T \end{aligned}$$

True.

$$\begin{aligned} P(8) &\implies Q(8) \\ F &\implies F \end{aligned}$$

True.

(c) Given the open sentences $P(x) : x > 3$ and $Q(x) : 4x - 1 > 12$ over the domain $S = \{\, 0, 2, 3, 4, 6 \,\}$. We have the following truth

values for $P(x) \implies Q(x)$ for each $x \in S$.

$$\begin{aligned} P(0) &\implies Q(0) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(2) &\implies Q(2) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(3) &\implies Q(3) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(4) &\implies Q(4) \\ T &\implies T \end{aligned}$$

True.

$$\begin{aligned} P(6) &\implies Q(6) \\ T &\implies T \end{aligned}$$

True.

2.33 (a) Given the open sentences $P(x, y) : x^2 - y^2 = 0$ and $Q(x, y) : x = y$ over the domain $S = \{ (1, -1), (3, 4), (5, 5) \}$. We have the following truth values for $P(x, y) \implies Q(x, y)$ for each given values of x and y.

$$\begin{aligned} P(1, -1) &\implies Q(1, -1) \\ T &\implies F \end{aligned}$$

False.

$$\begin{aligned} P(3, 4) &\implies Q(3, 4) \\ F &\implies F \end{aligned}$$

True.

$$\begin{aligned} P(5, 5) &\implies Q(5, 5) \\ T &\implies T \end{aligned}$$

True.

(b) Given the open sentences $P(x, y) : |x| = |y|$ and $Q(x, y) : x = y$ over the domain $S = \{ (1, 2), (2, -2), (6, 6) \}$. We have the following truth values for $P(x, y) \implies Q(x, y)$ for each given values of x and y.

$$\begin{aligned} P(1, 2) &\implies Q(1, 2) \\ F &\implies F \\ &\text{True.} \\ P(2, -2) &\implies Q(2, -2) \\ T &\implies F \\ &\text{False.} \\ P(6, 6) &\implies Q(6, 6) \\ T &\implies T \\ &\text{True.} \end{aligned}$$

(c) Given the open sentences $P(x, y) : x^2 + y^2 = 1$ and $Q(x, y) : x + y = 1$ over the domain $S = \{ (1, -1), (-3, 4), (0, -1), (1, 0) \}$. We have the following truth values for $P(x, y) \implies Q(x, y)$ for each given values of x and y.

$$\begin{aligned} P(1, -1) &\implies Q(1, -1) \\ F &\implies F \\ &\text{True.} \\ P(-3, 4) &\implies Q(-3, 4) \\ F &\implies T \\ &\text{True.} \\ P(0, -1) &\implies Q(0, -1) \\ T &\implies F \\ &\text{False.} \\ P(1, 0) &\implies Q(1, 0) \\ T &\implies T \\ &\text{True.} \end{aligned}$$