Section 2.1 Exercises

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2.1 (a) The sentence: the integer 123 is prime, is a valid statement and it has a truth value of False.

(b) The sentence: the integer 0 is even, is a valid statement and it has a truth value of True.

(c) The sentence: Is $5 \times 2 = 10$? is not a valid statement since it is a question and is not declarative.

(d) The sentence: $x^2 - 4 = 0$ is an open sentence, however we are not given the domain for the variable x. If we assume that x can be anything in the real numbers then this statement has a truth value of True when $x = -2$ or when $x = 2$ and it has a truth value of False otherwise.

(e) The sentence: multiply $5x + 2$ by 3, is not a valid statement as it is an imperative command.

(f) The sentence: $5x + 3$ is an odd integer is an open statement and if assume that the domain of x is the integers, as seems appropriate given that the sentence refers to the integers for the range of the function, then it is an open statement over the domain of $\mathbb{Z}$. This statement happens to have a truth value of True when x is even and a value of False when x is odd.

Proof. $5x + 3$ is an odd integer when x is an even integer. Since x is even it can be rewritten as $x = 2k$ for some $k \in \mathbb{Z}$. So we have $5(2k) + 3 = 10k + 3 = 2(5k) + 3 = 2(5k) + (2(1) + 1) = 2(5k) + 2 + 1 = 2(5k + 2) + 1$. If we let $l = 5k + 2$ we can rewrite $2(5k + 2) + 1$ as $2l + 1$. Since the definition of an odd integer is any integer that can be expressed as 2 times some arbitrary

integer plus 1 we must conclude that the statement $5x + 3$ is an odd integer *is* true when x is an even integer. Q.E.D.

Proof. $5x + 3$ is not an odd integer when x is an odd integer. For the sake of simplicity and time we can simply prove this by finding a single counter example; it is a relatively simple matter to establish a more long-form proof, but doing so is unnecessary in this case. If we let $x = 1$ then we have that $5x + 3 = 8$ which is not an odd integer, since we cannot write 8 as 2 times some integer $k + 1$. In fact we can write 8 as 2 times 4 which is an even integer not an odd integer. Q.E.D.

(g) The sentence: "What an impossible question!" is not a valid statement as it is simply an exclamatory sentence.