# Section 2.6 Exercises

David Piper

October 3, 2020

2.35 Let $P$ : 18 is odd and $Q$ : 25 is even. Then $P \iff Q$ can be written as 18 is odd if and only if 25 is even. Furthermore $P \iff Q$ is true.

2.37 For the open sentences $P(x) : |x - 3| < 1$ and $Q(x) : x \in (2, 4)$ over the domain $\mathbb{R}$ the biconditional $P(x) \iff Q(x)$ can be stated in a few different ways including the following.

1. $|x - 3| < 1$ if and only if $x \in (2, 4)$.
2. $|x - 3| < 1$ is equivalent to $x \in (2, 4)$.

2.39 For the following open sentences $P(x)$ and $Q(x)$ over a domain $S$, determine all values of $x \in S$ for which the biconditional $P(x) \iff Q(x)$ is true.

(a) $P(x) : |x| = 4; Q(x) : x = 4; S = \{ -4, -3, 1, 4, 5 \}$. This has the following corresponding truth table.

| x | -4 | 3 | 1 | 4 | 5 |
|---|---|---|---|---|---|
| P(x) | T | F | F | T | F |
| Q(x) | F | F | F | T | F |

From this truth table we can observe that $P(x) \iff Q(x)$ is false when $x = -4$ and true for all other vales of $x \in S$.

(b) $P(x) : x \geq 3; Q(x) : 4x - 1 > 12; S = \{ 0, 2, 3, 4, 6 \}$. This has the following corresponding truth table.

| x | 0 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| P(x) | F | F | T | T | T |
| Q(x) | F | F | F | T | T |

From this truth table we can observe that $P(x) \iff Q(x)$ is false when $x = 3$ and true for all other vales of $x \in S$.

(c) $P(x) : x^2 = 16; Q(x) : x^2 - 4x = 0; S = \{-6, -4, 0, 3, 4, 8\}$. This has the following corresponding truth table.

| x | -6 | -4 | 0 | 3 | 4 | 8 |
|---|---|---|---|---|---|---|
| P(x) | F | T | F | F | T | F |
| Q(x) | F | F | T | F | T | F |

From this truth table we can observe that $P(x) \iff Q(x)$ is false when $x = -4$ and when $x = 0$ and true for all other vales of $x \in S$.

2.41 The task is to determine all values of $n$ in the domain $S = \{1, 2, 3\}$ for which the following is a true statement: A necessary and sufficient condition for $\frac{n^3+n}{2}$ to be even is that $\frac{n^2+n}{2}$ is odd.

First we may observe that this question is really a biconditional question with two open sentences over the domain $S$. Given this, we can rewrite the problem by saying that we are given two sentences $P(n) : \frac{n^3+n}{2}$ is even and $Q(n) : \frac{n^2+n}{2}$ is odd over the domain $S = \{1, 2, 3\}$. This has the following truth table.

| n | 1 | 2 | 3 |
|---|---|---|---|
| P(n) | F | F | F |
| Q(n) | T | T | F |

From this truth table we can observe that $P(n) \iff Q(n)$ is true when $n = 3$ and is false for all other vales of $n \in S$.

2.43 Given two open sentences $P(n) : \frac{(n+4)(n+5)}{2}$ is odd and $Q(n) : 2^{n-2}+3^{n-2}+6^{n-2} > (2.5)^{n-1}$ over the domain $S = \{1, 2, 3\}$. We need to determine three distinct elements $a, b, c$ in $S$ such that $P(a) \implies Q(a)$ is false, $Q(b) \implies P(b)$ is false and $P(c) \iff Q(c)$ is true.

First let us observe that these statements have the following truth table.

| n | 1 | 2 | 3 |
|---|---|---|---|
| P(n) | T | T | F |
| Q(n) | F | T | T |

From this truth table we can observe that $P(a) \implies Q(a)$ is false when $a = 1$, $Q(b) \implies P(b)$ is false when $b = 3$, and $P(c) \iff Q(c)$ is true when $c = 2$.

2.45 Given two open sentences $P(n) : 2^n - 1$ is prime and $Q(n) : n$ is prime over the domain $S = \{2, 3, 4, 5, 6, 11\}$. We need to determine all values of $n \in S$ for which $P(n) \iff Q(n)$ is a true statement.

First let us observe that these statements have the following truth table.

| n | 2 | 3 | 4 | 5 | 6 | 11 |
|---|---|---|---|---|---|---|
| P(n) | T | T | F | T | F | F |
| Q(n) | T | T | F | T | F | T |

From this truth table we can observe that $P(n) \iff Q(n)$ is false when $n = 11$ and true for all other values of $n \in S$.