Section 2.7 Exercises

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2.47 For two statements P and Q we want to show that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ and $(P \implies \sim Q) \wedge (P \wedge Q)$ are contradictions. First let us observe that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ has the following truth table.

P	Q	$\sim Q$	$P \wedge (\sim Q)$	$P \wedge Q$	$(P \wedge (\sim Q)) \wedge (P \wedge Q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	F	F
F	F	T	F	F	F

From this truth table we can observe that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ is false for all possible values of P and Q, thus it is a contradiction.

Next let's consider the statement $(P \implies \sim Q) \wedge (P \wedge Q)$. We can observe that it has the following truth table.

P	Q	$\sim Q$	$P \implies \sim Q$	$P \wedge Q$	$(P \implies \sim Q) \wedge (P \wedge Q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

From this truth table we can observe that $(P \implies \sim Q) \wedge (P \wedge Q)$ is false for all possible values of P and Q, thus it is a contradiction.

2.49 For the statements P, Q and R we want to show that $((P \implies Q) \wedge (Q \implies R)) \implies (P \implies R)$ is a tautology. First let us observe that these statements

have the following truth table.

From this truth table we can observe that $(P \wedge (\sim Q)) \wedge (P \wedge Q)$ is false for all possible values of P and Q, thus it is a contradiction.