Section 2.9 Exercises

David Piper

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2.61 (a) The negation of the open sentence "Either $x = 0$ or $y = 0$." is "Both $x \neq 0$ and $y \neq 0$." by DeMorgan's Law.

(b) The negation of the open sentence "The integers a and b are both even." is "At least one of the integers a and b are odd." by DeMorgan's Law.

2.63 For a real number x, let $P(x) : x^2 = 2$ and $Q(x) : x = \sqrt{2}$. We want to state the negation of the biconditional $P \iff Q$ in words. First we need to find a logically equivalent version of the negation of this biconditional, which we can do using Theorem 2.25(b). This theorem says that $\sim (P \iff Q) \equiv (P \wedge (\sim Q)) \vee (Q \wedge (\sim P))$.

If we state this logically equivalent version in words then we have the statement "Either $x^2 = 2$ and $x \neq \sqrt{2}$ or $x = \sqrt{2}$ and $x^2 \neq 2$."

2.65 For a natural number n, we are given the compound statement 3n + 4 is odd and 5n - 6 is even. We want to find an implication such that its negation is this given compound statement. From Theorem 2.25(a) we know that $\sim (P \implies Q) \equiv P \wedge (\sim Q)$. So to make our given statements match this form we should first take the first statement to be $P(n)$ and then take the negation of the second statement to be $Q(n)$.

Now we have two statements $P(n) : 3n + 4$ is odd and $Q(n) : 5n - 6$ is odd and we can restate our given compound statement as $P \wedge (\sim Q)$. Therefore, by Theorem 2.25(a) we can restate this statement in its logically equivalent form of $\sim (P \implies Q)$. Looking at this we can observe that the implication whose negation gives us the original statement is simply $P \implies Q$ which we can write in words as "If $3n+4$ is odd then $5n - 6$ is odd."