Section 2.8 Exercises

David Piper

October 4, 2020

2.53 (a) For the statements P and Q we want to show that $(\sim P) \implies (\sim Q)$ and $P \implies Q$ are not logically equivalent. We start by observing that these statements have the following truth table.

P	Q	$(\sim P) \implies (\sim Q)$	$P \implies Q$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	T	T

From this truth table we can observe that the values for the statements $(\sim P) \implies (\sim Q)$ and $P \implies Q$ are not the same in all of their respective rows, which means that they are not logically equivalent.

(b) Another implication that is logically equivalent to $(\sim P) \implies (\sim Q)$ is $Q \implies P$. To verify this observe that these statements have the following truth table.

P	Q	$(\sim P) \implies (\sim Q)$	$Q \implies P$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

From this truth table we can observe that the values for the statements $(\sim P) \implies (\sim Q)$ and $Q \implies P$ are the same in all of their respective rows, which means that they are logically equivalent.

2.55 (a) For the statements P, Q and R we want to show that $(P \wedge Q) \iff P$ and $P \implies Q$ are logically equivalent. We start by observing that these statements have the following truth table.

P	Q	$(P \wedge Q) \iff P$	$P \implies Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

From this truth table we can observe that the values for the statements $(P \wedge Q) \iff P$ and $P \implies Q$ are the same in all of their respective rows, which means that they are logically equivalent.

(b) For the statements P, Q and R we want to show that $P \implies (Q \vee R)$ and $(\sim Q) \implies ((\sim P) \vee R)$ are logically equivalent. We start by observing that these statements have the following truth table.

P	Q	R	$Q \vee R$	$P \implies (Q \vee R)$	$(\sim P) \vee R$	$(\sim Q) \implies ((\sim P) \vee R)$
T	T	T	T	T	T	T
T	T	F	T	T	F	T
T	F	T	T	T	T	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

From this truth table we can observe that the values for the statements $P \implies (Q \vee R)$ and $(\sim Q) \implies ((\sim P) \vee R)$ are the same in all of their respective rows, which means that they are logically equivalent.

2.57 For the statements P, Q and R we want to show that $(P \vee Q) \implies R$ and $(P \implies R) \wedge (Q \implies R)$ are logically equivalent. We start by observing that these statements have the following truth table.

P	Q	R	$P \vee Q \implies R$	$P \implies R$	$Q \implies R$	$(P \implies R) \wedge (Q \implies R)$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

From this truth table we can observe that the values for the statements $(P \vee Q) \implies R$ and $(P \implies R) \wedge (Q \implies R)$ are the same in all of their respective rows, which means that they are logically equivalent.

2.59 *Proof.* We are given five compound statements: S_1, S_2, S_3, S_4 and S_5; which we are also told are all comprised of the same statements P and Q and they have truth tables which have the same values on the first and fourth rows.

We want to show that at least two of these five statements must be logically equivalent given these facts.

Since the statements have identical truth values in the first and fourth rows then we may observe that this acts as a selection of the possible values for those rows. That is, for each row generally we may choose any possible value of True or False for each cell, but since we know that two rows are identical these are already chosen for us.

Since these are already chosen, we have reduced the possible combinations from 2^4 to 2^2 for the remaining rows. Since we have five compound statements and only 4 possible different values for the remaining rows then we know that at least one of these five must have the same value as one of the others. Q.E.D.