Homework 1

Due January 21, 2022

1. Complete exercises 1,2,3 and 5 in Section 2.1
2. Complete exercises 1,11, and 12 in Section 2.2
3. Complete exercises 1,4, and 5 in Section 2.3
4. Complete exercises 1 and 4 in Section 2.4
5. A common question in epidemiology is the rate at which a virus or disease will spread throughout a population. The simplest model of this is known as the SI model, which contains two groups: a susceptible population S and an infected population I. When we consider the susceptible population interacts with the infected population at some r and the infected population recovers back to the susceptible population at some rate λ, then we can model the change in population according to the set of differential equations

$$\frac{dS}{dt} = -rSI + \gamma I$$
$$\frac{dI}{dt} = rSI - \gamma I$$

where r and λ are positive parameters that depend on the physical properties of the specific virus or disease.

(a) Show that $S + I = N$, i.e. that the total population remains constant where N is the size of the population. Further, show that this instead can be written as a single differential equation in terms of the infected population I.

(b) Using part (a), show the set of differential equations can be written as the single differential equation,

$$\frac{dI}{dt} = rI(N - I) - \gamma I$$

(c) We typically define the parameter R_0 to be the condition for an epidemic. If $R_0 > 1$, the infected population will dominate. If $R_0 < 1$, the infection will die out. Equivalently, for an epidemic, we require $\frac{dI}{dt} > 0$. Using this inequality, derive the condition for R_0 in terms of the system parameters. Verify this against what you find for the stability of the fixed points in the system.