Name: ________________________________

Instructions: Attach this page to the front of your written solutions. Show all work and sketch figures or plots if necessary for each problem. Solutions should be written clearly and in a well-ordered manner. Solutions that are unclear or unreadable will receive a zero. Collaboration is encouraged, but be sure to submit your own work. Homework will be assessed by a random sampling of questions. Please hand-in your work no later than 9/28.

1. In section 13.3 of the textbook, complete problems 18, 20, and 25.

2. In section 13.4 of the textbook, complete problems 10, 12, and 26.

3. In section 14.1 of the textbook, complete problems 12, 32, and 39.

4. In section 14.2 of the textbook, complete problems 11, 17, and 42.

   Note: As a hint, the limit does not exist for question 11. Show this through counterexamples. For question 17, the limit *does* exist, which you must show with the $\epsilon - \delta$ definition of the limit. Make use of the conjugate, then play a bit with inequalities.

5. In section 14.3 of the textbook, complete problems 16, 20, 39, and 53.

6. There is a field of mathematics called *partial differential equations*, which relates partial derivatives together through an equation. Many of these have physical applications.

   (a) A function is *harmonic* if the sum of its second partials is zero, i.e.

   $$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \qquad \text{in 2D or} \qquad \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \quad \text{in 3D.}$$

   This is equation is called *Laplace's equation*. Show the functions

   $$f(x,y) = \frac{x-y}{x^2+y^2} \quad \text{and} \quad f(x,y) = \arctan\left(\frac{y}{x}\right)$$

   are harmonic, and as a result, satisfy Laplace's equation.

(b) Consider temperature $T(x,t)$ as a function of space and time. Show that if we define T as

$$T(x,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-x^2/(4Dt)},$$

then T satisfies the equation

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}.$$

This is known as the *heat equation* and the function given for T is the *fundamental solution* of the heat equation.

7. A triangle has sides 10.1 and 19.9 cm which include an angle of 61°. Find an approximate area of the triangle use differentials. Compare with the exact results. Give reasoning for the error value.

   Hint: Construct a function using law of sines for the area of a triangle using the sides and an angle as variables.