Lecture 1.2: Limit Properties. Techniques of Limit Computation

Professor Leonard

February 17, 2023

1 Basics

1. **Limit of a constant**
$$\lim_{x \to a} C = C$$

The limit of a constant is always the constant (think because the plot is just a horizontal line).

2. **Limit of x**
$$\lim_{x \to a} x = a$$

Think because $f(x) = x$ is just identity, so point a on x is also a on y.

3. **Zero limit**

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

2 Properties

Given two functions with limits that exist:

$$\lim x \to a f(x) = L_1$$
$$\lim x \to a g(x) = L_2$$

1. **Function joining and separation**

$$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

This operation works both ways (can also join).

2. **Same thing for multiplication**

$$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

3. **For division**

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \lim_{x \to a} g(x) \neq 0$$

4. **Exponents**

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x) \right]^n \to \lim_{x \to a} \sqrt[n]{f(x)} \to \sqrt[n]{\lim_{x \to a} f(x)}$$