3D detector with highly resistive electrodes: An electrical model.

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Electrical model for 3D column detector

Let's consider a 3D detector with columns with radius $r = 3.5\mu m$, distance $d = 55\mu m$ and lenght $l = 500\mu m$. In this condition for a diamond detector, the capacitance would be around $C_D \sim 30fF$. Normally the effect of the detector on signal shape is model considering only the capacitance of such detector and assuming ideal electrodes with no resistance. The voltage at the input of a front-end connected to the detector in this case, would be given by the convolution of the current with a transfer function that, at first order, is a RC low pass filter where the resistance is the input resistance of the electronics R_{in}, while the capacitance is the sum of the one of the detector C_D and the input capacitance of the front-end C_{in} (Figure 1). If the

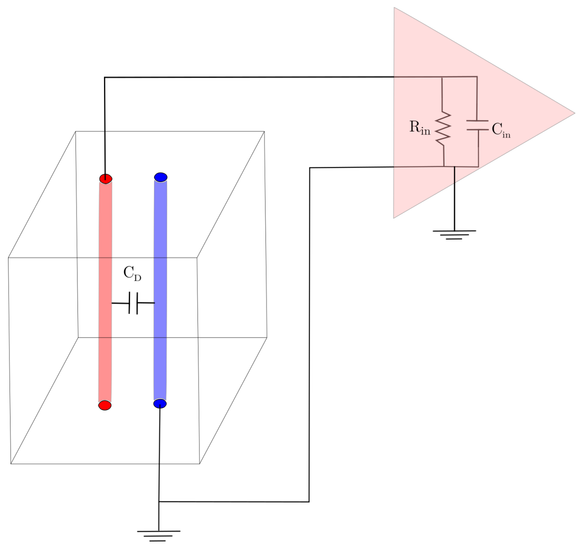

Figure 1. Simplest descrition of a detector connected to a front-end electronics

detector has highly resistive column this model has to change considering the shaping effect of such electrodes. If we consider a particle with a track parallel to the columns (Figure 2), we can assume that charges produced at different z would "see" a different resistance in series depending on whether they are electrons or holes. The resistance can be written as a function of z:

$$R_n(z) = R \cdot \frac{z}{l} \tag{0.1}$$

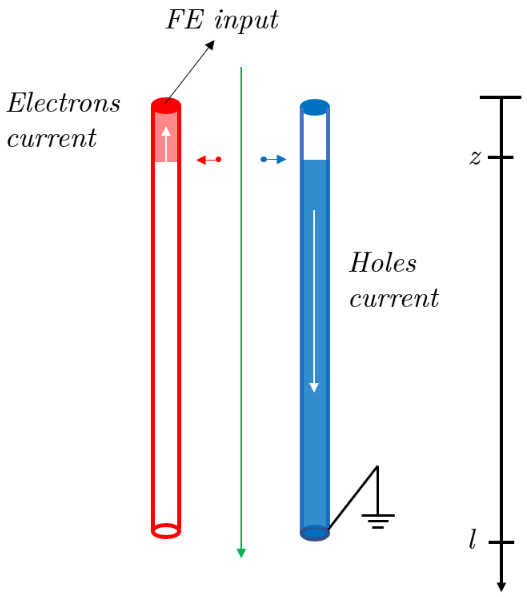

Figure 2. Model for 3D detector with highly resistive columns .

$$R_p(z) = R \cdot \frac{l - z}{l} \tag{0.2}$$

where $R_n(z)$ is the resistance seen by the electron at z and $R_p(z)$ is the one seen by the hole. The electrical model for the electron in position at z is shown in (Figure 3). With ideal electrodes the transfer function would be:

$$V_{in-e}(s) = I_e(s)\frac{R_{in}}{1 + s\tau^*} \tag{0.3}$$

where $\tau^* = R_{in}(C_{in} + C_D)$.

If the resistance is considered, the transfer function becomes:

$$V_{in-e}(s, z) = I_e(s)\frac{R_{in}}{sR_{in}C_D + (1 + s\tau)(1 + s\tau_N(z))} \tag{0.4}$$

where we have introduced $\tau_N(z) = R_n(z)C_D$.
Electrons at different highs z would induce the same current $I_e(t)$ but the shaping would be different because of the different resistance in series. We

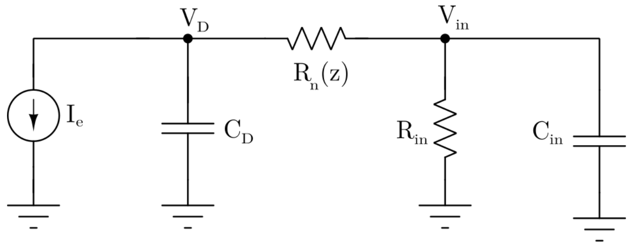

Figure 3. Equivalent circuit for the voltage induced by an electron at z.

can factorize the transfer function as the following:

$$V_{in-e}(s,z) = I_e(s)\frac{R_{in}}{(1+s\tau)}\frac{1}{\frac{sR_{in}C_D}{(1+s\tau)} + (1+s\tau_N(z))} \tag{0.5}$$

We assume as negligible the term:

$$\frac{sR_{in}C_D}{(1+s\tau)} \tag{0.6}$$

(at low frequency tends to zero while at high frequency tends to the ratio C_D/C_{in}, so for $C_{in} > C_D$ the assumption can be reasonable or a factor can be introduced to adapt to the specific case .)

The voltage at the input produces by the induction of the single electron at z can be written:

$$V_{in-e}(s,z) = \frac{I_e(s)}{(1+s\tau_N(z))}\frac{R_{in}}{(1+s\tau)} \tag{0.7}$$

so it can be expressed as the current:

$$I_e(s,z) = \frac{I_e(s)}{(1+s\tau_N(z))} \tag{0.8}$$

that has to be convoluted with the input impedance of the front-end without contribution from the detector. The total current induced by all the electrons at different z is:

$$I_{e,TOT}(s) = I_e(s) \sum_{z=0}^{z=l} \frac{1}{(1 + s\tau_N(z))} \tag{0.9}$$

considering also the holes contribution:

$$I_{TOT}(s) = (I_e(s) + I_e(s)) \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_N(z))} + \frac{1}{(1 + s\tau_P(z))} \right) \tag{0.10}$$

the transfer function that models the effect of the highly resistive column in the 3D detector is then:

$$H(s) = \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_N(z))} + \frac{1}{(1 + s\tau_P(z))} \right) \tag{0.11}$$

This can be calculated in the time domain as:

$$H(t) = \frac{1}{k} \left[\frac{e^{-\frac{t}{\tau_{N(z_1)}}}}{\tau_{N(z_1)}} + \frac{e^{-\frac{t}{\tau_{P(z_1)}}}}{\tau_{P(z_1)}} + \ldots + \frac{e^{-\frac{t}{\tau_{N(l)}}}}{\tau_{N(l)}} + \frac{e^{-\frac{t}{\tau_{P(l)}}}}{\tau_{P(l)}} \right] \tag{0.12}$$

where k is the number of slices used in the sum. As an example we can consider the effect of such transfer function on a rectangular current with duration $t_c = 100ps$ and magnitude $I_0 = 40\mu A$ for a total $4fC$ of charge Figure 4 left. Let's suppose the detector has $R_N = 100k\Omega$, $R_P = 100k\Omega$ and $C_D = 30fF$. The transfer function in the time domain is shown in Figure 4 right. The effect of such transfer function on the rectangular pulse is shown in Figure 5 left. The signal becomes smaller in amplitude and longer in duration. Figure 5 right, shows a comparison with a current obtained doing a convolution of the ideal current with a low pass filter with time constant $\tau_{LP} = 1ns$, which is three times smaller of the product $R_N C_D = 3ns$. In the specific case the resistance of the electrodes could be different, (if for example we have multiple columns connected in parallel in the return path) producing a different shaping effect.

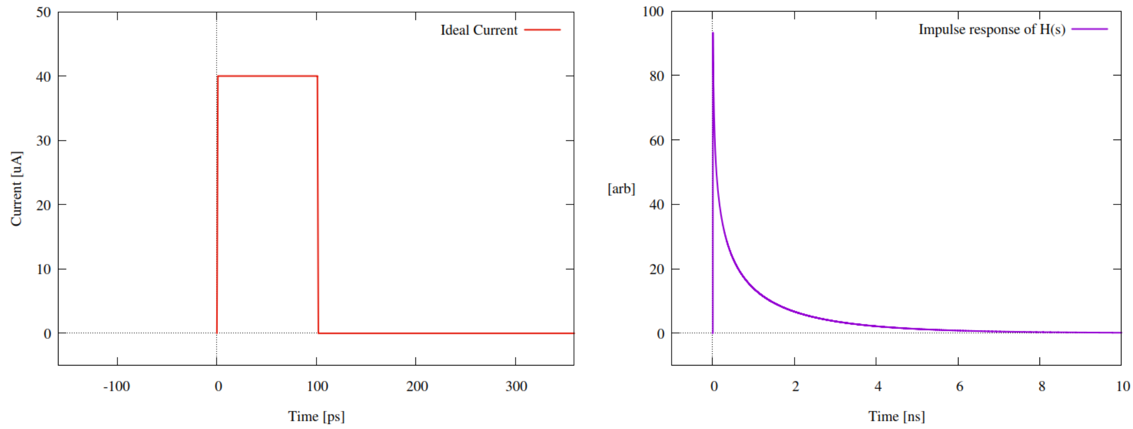

Figure 4. Ideal rectangular current (left) and pulse response of the detector with highly resistive columns (right)

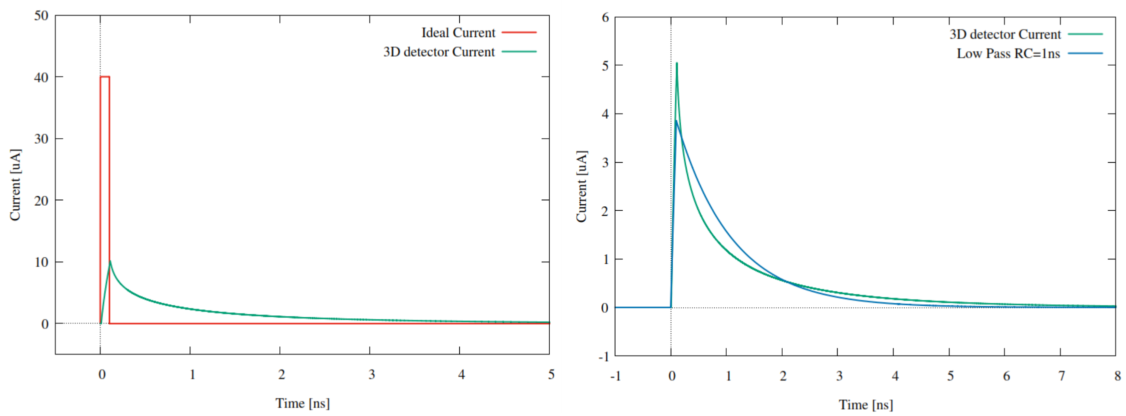

Figure 5. Comparison between the ideal current and the one of the 3D detector with highly resistive electrodes (left), Comparison of the 3D detector current with the convolution of the ideal current with a low pass filter with $1ns$ time constant (right).