3D detector with highly resistive electrodes: An electrical model.

Gian-Matteo Cossu[a]

E-mail: gmatteo.cossu@gmail.com

Electrical model for 3D column detector

Let's consider a 3D detector with columns with radius $r = 3.5\mu m$, distance $d = 55\mu m$ and lenght $l = 500\mu m$. In this condition for a diamond detector, the capacitance would be around $C_D \sim 30fF$. Normally the effect of the detector on signal shape is model considering only the capacitance of such detector and assuming ideal electrodes with no resistance. The voltage at the input of a front-end connected to the detector in this case, would be given by the convolution of the current with a transfer function that, at first order, is a RC low pass filter where the resistance is the input resistance of the electronics R_{in}, while the capacitance is the sum of the one of the detector C_D and the input capacitance of the front-end C_{in} (Figure 1).

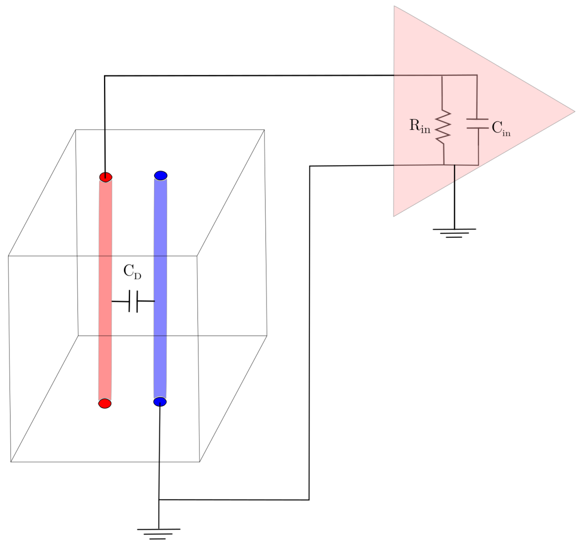

Figure 1. Simplest descrition of a detector connected to a front-end electronics

If the detector has highly resistive column this model has to change considering the shaping effect of such electrodes. If we consider a particle with a track parallel to the columns (Figure 2), we can assume that charges produced at different z induce a current that "sees" a different resistance in series depending on whether they are electrons or holes. The resistance can be written as a function of z:

$$R_n(z) = R_N \cdot \frac{z}{l} \tag{1.1}$$

$$R_p(z) = R_P \cdot \frac{l - z}{l} \tag{1.2}$$

where $R_n(z)$ is the resistance seen by the electron at z and $R_p(z)$ is the one seen by the hole. The electrical model for the electron in position at z is shown in (Figure 3). With

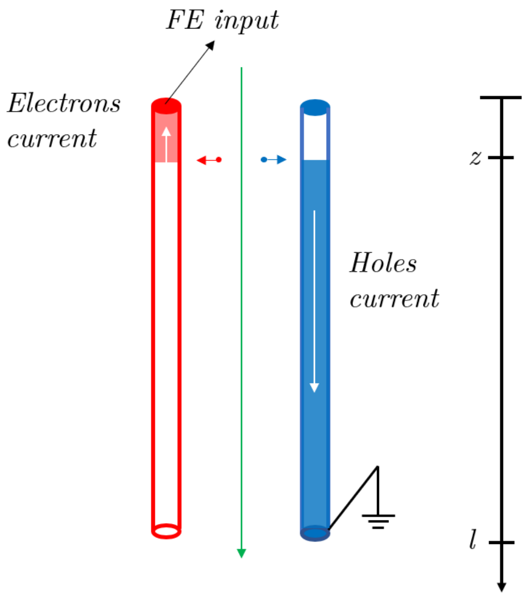

Figure 2. Model for 3D detector with highly resistive columns .

ideal electrodes the transfer function would be:

$$V_{in-e}(s) = I_e(s)\frac{R_{in}}{1+s\tau^*} \tag{1.3}$$

where $\tau^* = R_{in}(C_{in} + C_D)$.

If the resistance is considered, the transfer function becomes:

$$V_{in-e}(s,z) = I_e(s)\frac{R_{in}}{sR_{in}C_D + (1+s\tau)(1+s\tau_N(z))} \tag{1.4}$$

where we have introduced $\tau_N(z) = R_n(z)C_D$.

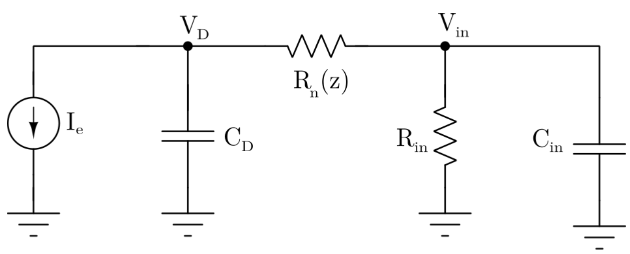

Figure 3. Equivalent circuit for the voltage induced by an electron at z.

Electrons at different highs z would induce the same current $I_e(t)$ but the shaping would be different because of the different resistance in series.
We can factorize the transfer function as the following:

$$V_{in-e}(s,z) = I_e(s)\frac{R_{in}}{(1+s\tau)}\frac{1}{\frac{sR_{in}C_D}{(1+s\tau)} + (1+s\tau_N(z))} \tag{1.5}$$

if we assume as negligible the term:

$$\frac{sR_{in}C_D}{(1+s\tau)} \tag{1.6}$$

(at low frequency tends to zero while at high frequency tends to the ratio C_D/C_{in}, so for $C_{in} > C_D$ the assumption can be reasonable or a factor can be introduced to adapt to the specific case .)

The voltage at the input produced by the induction of the single electron at z can be written:

$$V_{in-e}(s,z) = \frac{I_e(s)}{(1+s\tau_N(z))}\frac{R_{in}}{(1+s\tau)} \tag{1.7}$$

so it can be expressed as the current:

$$I_e(s,z) = \frac{I_e(s)}{(1+s\tau_N(z))} \tag{1.8}$$

that has to be convoluted with the input impedance of the front-end with no contribution from the detector (the influence of the capacitance C_D is already taken into account in the current).
The total current induced by all the electrons at different z is then:

$$I_{e,TOT}(s) = I_e(s)\sum_{z=0}^{z=l}\frac{1}{(1+s\tau_N(z))} \tag{1.9}$$

considering also the holes contribution:

$$I_{TOT}(s) = (I_e(s) + I_e(s))\sum_{z=0}^{z=l}\left(\frac{1}{(1+s\tau_R(z))}\right) \tag{1.10}$$

Where we have assumed that the same total resistance R for both columns.
The transfer function that models the effect of the highly resistive column in the 3D detector is then:

$$H(s) = \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_R(z))} \right) \tag{1.11}$$

This can be calculated in the time domain as:

$$H(t) = \frac{1}{k} \left[\frac{e^{-\frac{t}{\tau_{R(z_1)}}}}{\tau_{R(z_1)}} + \frac{e^{-\frac{t}{\tau_{R(z_2)}}}}{\tau_{R(z_2)}} + \ldots + \frac{e^{-\frac{t}{\tau_{R(l)}}}}{\tau_{R(l)}} \right] \tag{1.12}$$

where k is the number of slices used in the sum. As an example we can consider the effect of such transfer function on a rectangular current with duration $t_c = 100ps$ and magnitude $I_0 = 40\mu A$ for a total $4fC$ of charge (Figure 4 left). Let's suppose the detector has $R_N = R_P = 100k\Omega$ and $C_D = 30fF$. The transfer function in the time domain is shown in Figure 4 right. The effect of such transfer function on the rectangular pulse is shown in Figure 5 left. The signal becomes smaller in amplitude and longer in duration. Figure 5 right, shows a comparison with a current obtained doing a convolution of the ideal current with a low pass filter with time constant $\tau_{LP} = 1ns$, which is three times smaller of the product $RC_D = 3ns$.

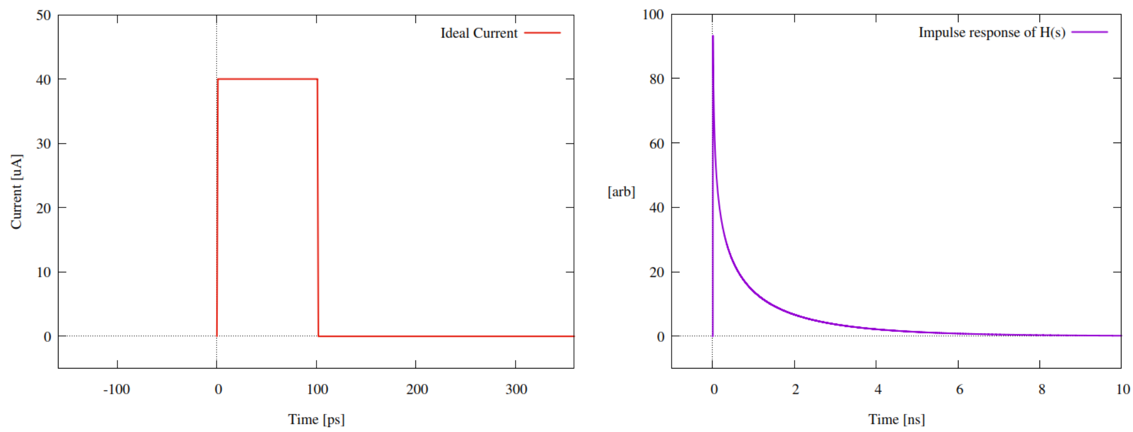

Figure 4. Ideal rectangular current (left) and pulse response of the detector with highly resistive columns (right)

In the specific case the resistances of the electrodes could be different, (if for example we have multiple columns connected in parallel in the return path). In this case the correct shape of the total current can be obtained considering separately the electrons current and the holes current. There would be two different transfer function $H(s)_e$ and $H(s)_h$ and the total current would be given by:

$$I_{e,TOT}(s) = I_e(s) \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_N(z))} \right) \tag{1.13}$$

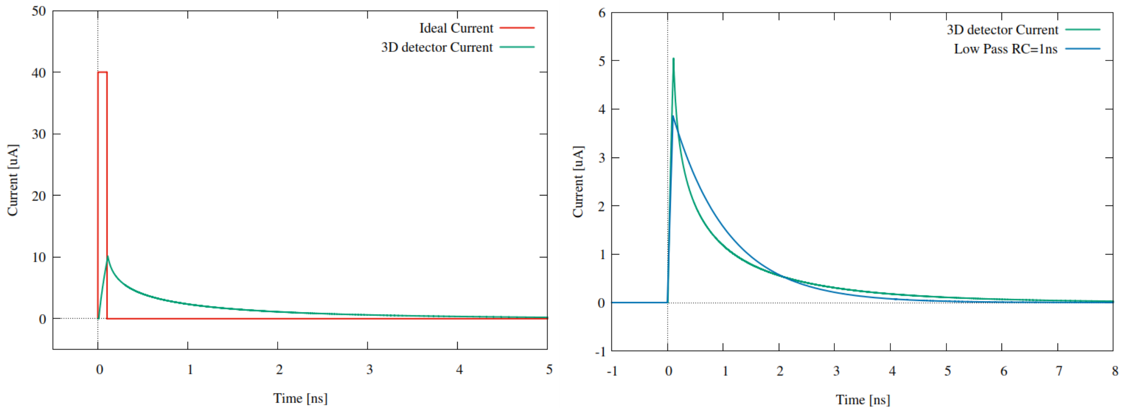

Figure 5. Comparison between the ideal current and the one of the 3D detector with highly resistive electrodes (left), Comparison of the 3D detector current with the convolution of the ideal current with a low pass filter with $1ns$ time constant (right).

$$I_{h,TOT}(s) = I_h(s) \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_P(z))} \right) \tag{1.14}$$

$$I_{TOT}(s) = I_e(s) \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_N(z))} \right) + I_h(s) \sum_{z=0}^{z=l} \left(\frac{1}{(1 + s\tau_P(z))} \right) \tag{1.15}$$

$$\begin{split} I_{TOT}(t) = I_e(t) \circledast \sum_{z=0}^{z=l} \left(\frac{e^{-\frac{t}{\tau_{N(z_1)}}}}{\tau_{N(z_1)}} + \ldots + \frac{e^{-\frac{t}{\tau_{N(l)}}}}{\tau_{N(l)}} \right) + \\ + I_h(t) \circledast \sum_{z=0}^{z=l} \left(\frac{e^{-\frac{t}{\tau_{P(z_1)}}}}{\tau_{P(z_1)}} + \ldots + \frac{e^{-\frac{t}{\tau_{P(l)}}}}{\tau_{P(l)}} \right) \end{split} \tag{1.16}$$

The most general case is when we have tracks not parallel to the columns but with an arbitrary direction. We could also have not homogeneous deposition of charge along z. Considering figure 6, the purple region of the track would have $dz = \frac{l}{k}$ and position $z = z_j$ so that the current $I_{eh}(z_j)$ can be written:

$$I_{eh}(t, z_j) = \left(I_e(t, z_j) \circledast \frac{e^{-\frac{t}{\tau_{N(z_j)}}}}{\tau_{N(z_j)}} \right) + \left(I_h(t, z_j) \circledast \frac{e^{-\frac{t}{\tau_{P(z_j)}}}}{\tau_{P(z_j)}} \right) \tag{1.17}$$

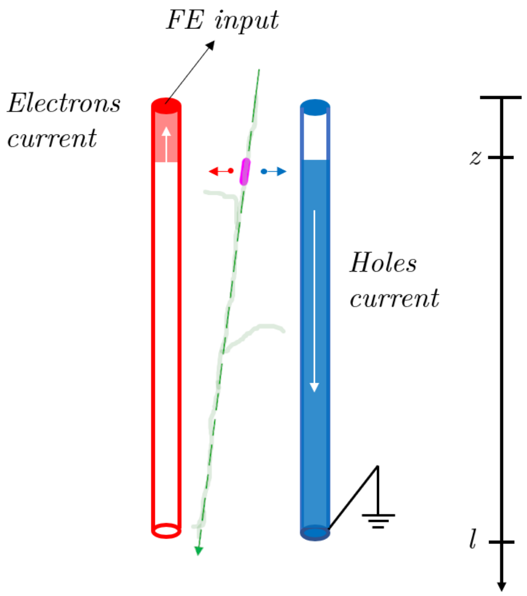

Figure 6. Model for 3D detector with highly resistive columns with arbitrary directory and shape of the tracks.

The total current would have to be calculated making $2k$ convolutions for every track, where k is the number of slices with which we divided the height of the columns:

$$I_{TOT}(t) = \left(I_e(t, z_1) \circledast \frac{e^{-\frac{t}{\tau_{N(z_1)}}}}{\tau_{N(z_1)}}\right) + \ldots + \left(I_e(t, l) \circledast \frac{e^{-\frac{t}{\tau_{N(l)}}}}{\tau_{N(l)}}\right)$$
$$\left(I_h(t, z_1) \circledast \frac{e^{-\frac{t}{\tau_{P(z_1)}}}}{\tau_{P(z_1)}}\right) + \ldots + \left(I_h(t, l) \circledast \frac{e^{-\frac{t}{\tau_{P(l)}}}}{\tau_{P(l)}}\right) \quad (1.18)$$

A fast convolution algorithm is needed to produce reasonable statistics. Considering $k = 20$ as the number of slices, if we want a set of $10k$ waveforms to characterize the hole detector we would have to make about $400k$ convolutions.

Complete solution

To consider the case without the approximation made neglecting the term in equation 1.4, we can modify the transfer function considering a generic input impedance $Z_{in}(s)$ instead of the low pass filter with time constant $\tau = R_{in}C_{in}$ so that we have:

$$I_{in}(s, z_j) = \frac{I_e(s, z_j)}{1 + sC_D(R(z_j) + Z_{in}(s))} \quad (1.19)$$

if the input impedance is known, for example from a simulation of the circuit, we can write:

$$H(\omega, z_j) = \frac{1}{1 + j\omega C_D\big(R(z_j) + \mathbb{R}\mathrm{e}[Z_{in}(\omega)] + j\,\mathbb{I}\mathrm{m}[Z_{in}(\omega)]\big)} \tag{1.20}$$

$$H(\omega, z_j) = \mathbb{R}\mathrm{e}[H(\omega, z_j)] + j\,\mathbb{I}\mathrm{m}[H(\omega, z_j)] \tag{1.21}$$

$$\mathbb{R}\mathrm{e}[H(\omega, z_j)] = \frac{1 - \omega C_D\,\mathbb{I}\mathrm{m}[Z_{in}(\omega)]}{\Big(1 - \omega C_D\,\mathbb{I}\mathrm{m}[Z_{in}(\omega)]\Big)^2 + \Big(\omega C_D(R(z_j) + \mathbb{R}\mathrm{e}[Z_{in}(\omega)])\Big)^2} \tag{1.22}$$

$$\mathbb{I}\mathrm{m}[H(\omega, z_j)] = -\frac{\omega C_D(R(z_j) + \mathbb{R}\mathrm{e}[Z_{in}(\omega)])}{\Big(1 - \omega C_D\,\mathbb{I}\mathrm{m}[Z_{in}(\omega)]\Big)^2 + \Big(\omega C_D(R(z_j) + \mathbb{R}\mathrm{e}[Z_{in}(\omega)])\Big)^2} \tag{1.23}$$

with a Fast Fourier Transform we can then compute the pulse response in the time domain for each transfer function that later can be used to make the convolutions and find the track current.

Calculation of the currents in TFBoost

The calculation of the currents with the effect of the resistive columns can be directly done with **TFBoost** to take advantage of its fast convolution algorithms. The currents simulated not considering the resistance of the columns have to be saved in a folder with a name: *prefix###_zXX.txt*, where ### indicates the track number while XX is the z slice number. First, all the $2k$ transfer functions $H(t, z_k)$ are calculated and then the ideal currents $I_D(t, z_k)$ are convoluted with them. After all the convolutions are done, the total current of each track is computed summing all the currents with different z_k. After the sum is done, the currents corresponding to different z_k can be deleted to save space. Each current file needs to have four columns:

1)time 2)Total Current 3)Electrons Current 4)Holes Current