

Competitive Programming

Dynamic Programming on a Tree

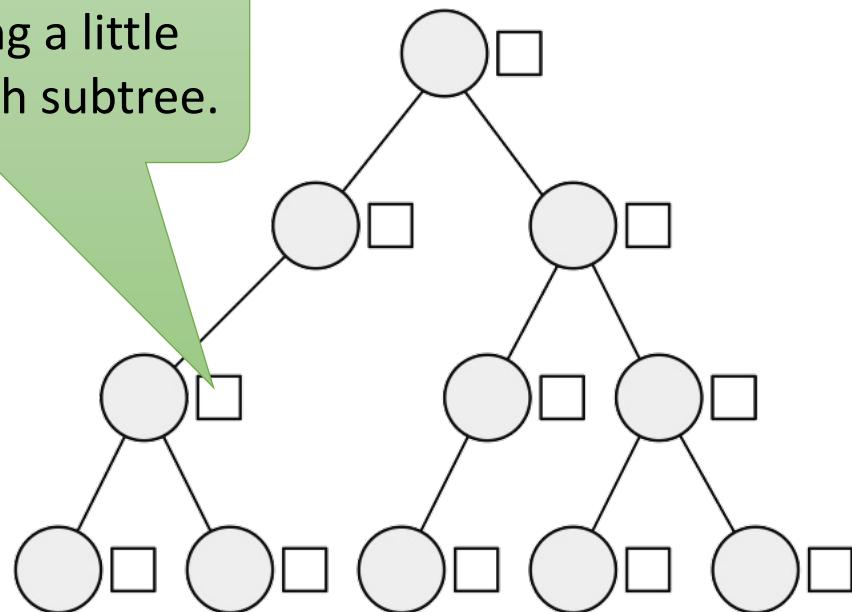
Overview

- Dynamic programming often involves filling in an array of solutions
 - 1-dimensional
 - 2-dimensional
 - ... 5-dimensional

DP on a Tree

- Subproblems may exhibit some other structure.
- Trees are common.

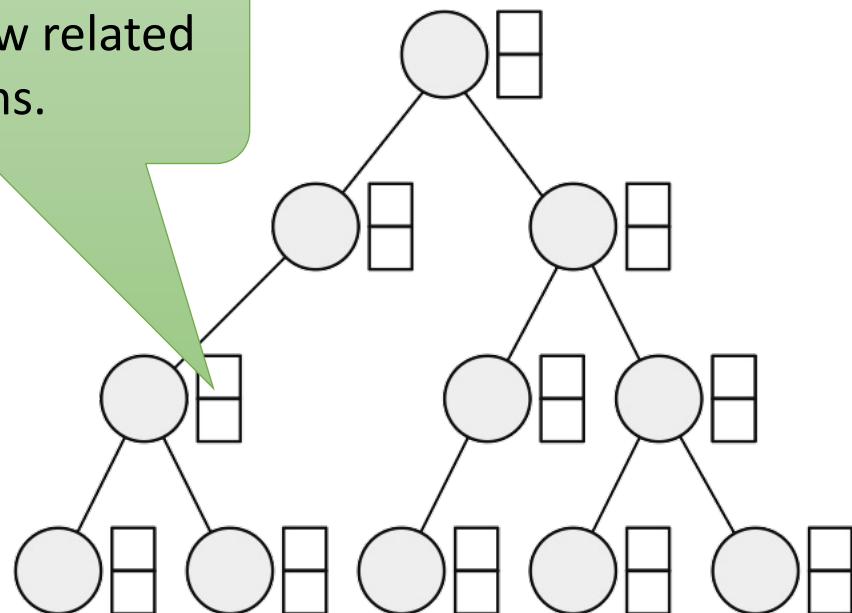
Imagine solving a little problem for each subtree.



DP on a Tree

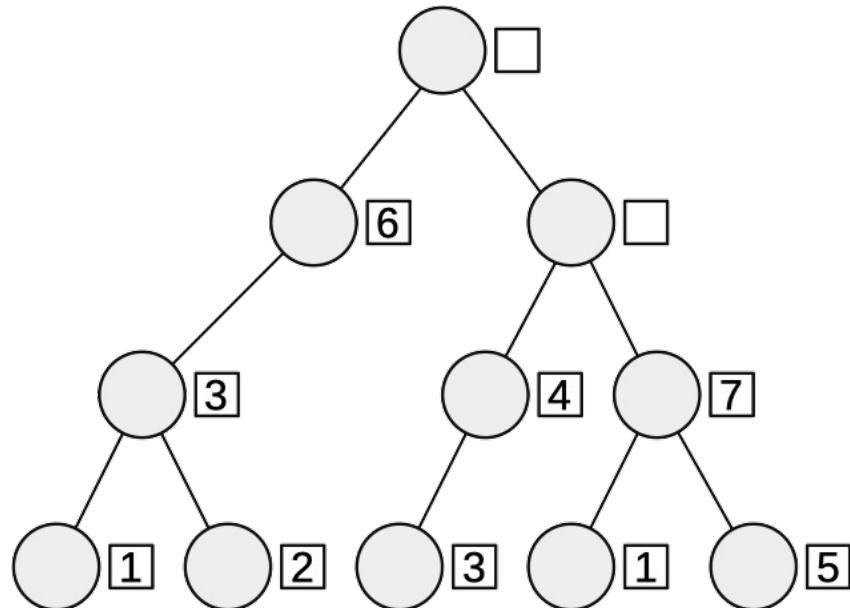
- Subproblems may exhibit some other structure.
- Trees are common.

Or, maybe a few related problems.



DP on a Tree

- After solving problems within a subtree
- The solutions can be used to solve larger problems at their parent.
- Eventually, at the root.



Marbles On A Tree

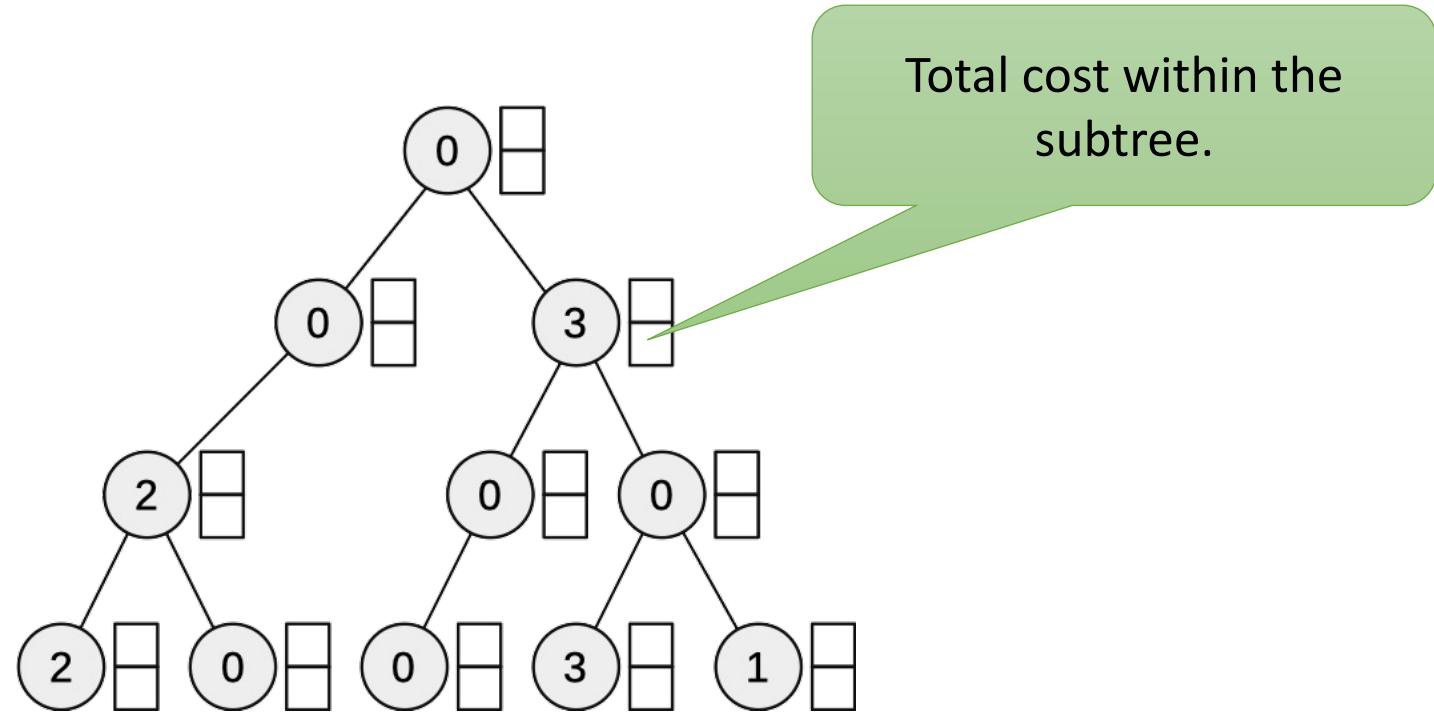
n boxes are placed on the vertices of a rooted tree, which are numbered from 1 to n , $1 \leq n \leq 10\,000$. Each box is either empty or contains a number of marbles; the total number of marbles is n .

The task is to move the marbles such that each box contains exactly one marble. This is to be accomplished by a sequence of moves; each move consists of moving one marble to a box at an adjacent vertex. What is the minimum number of moves required to achieve the goal?

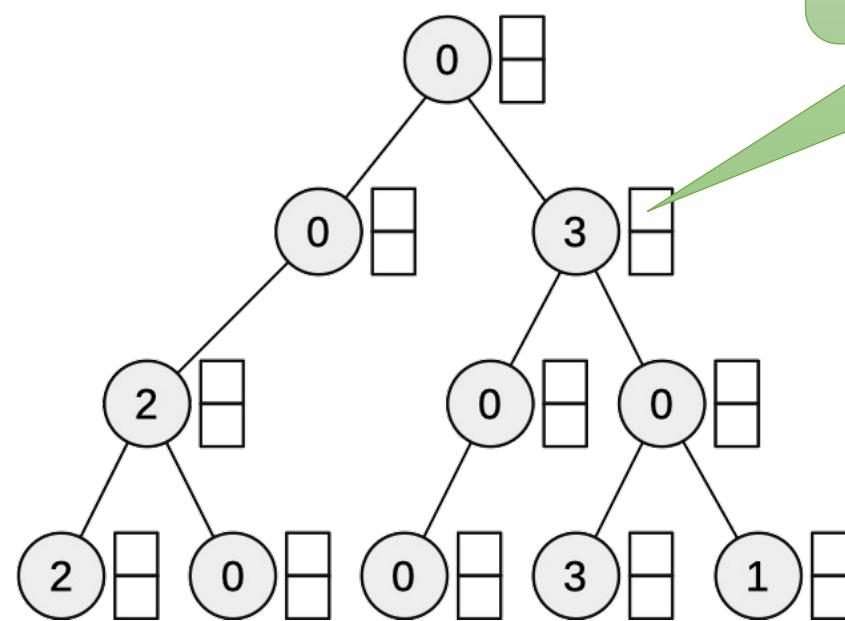


*Photo by chefranden on
Flickr*

Defining the Subproblems



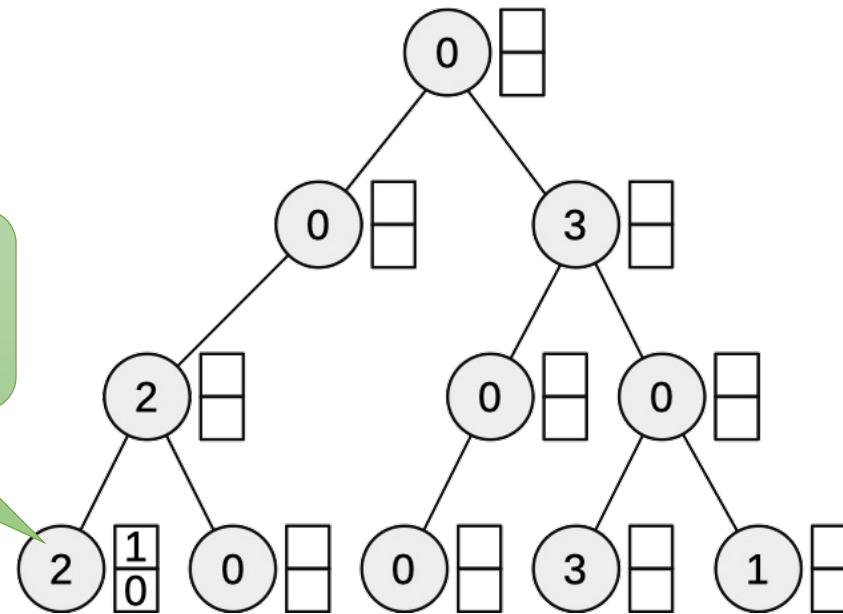
Defining the Subproblems



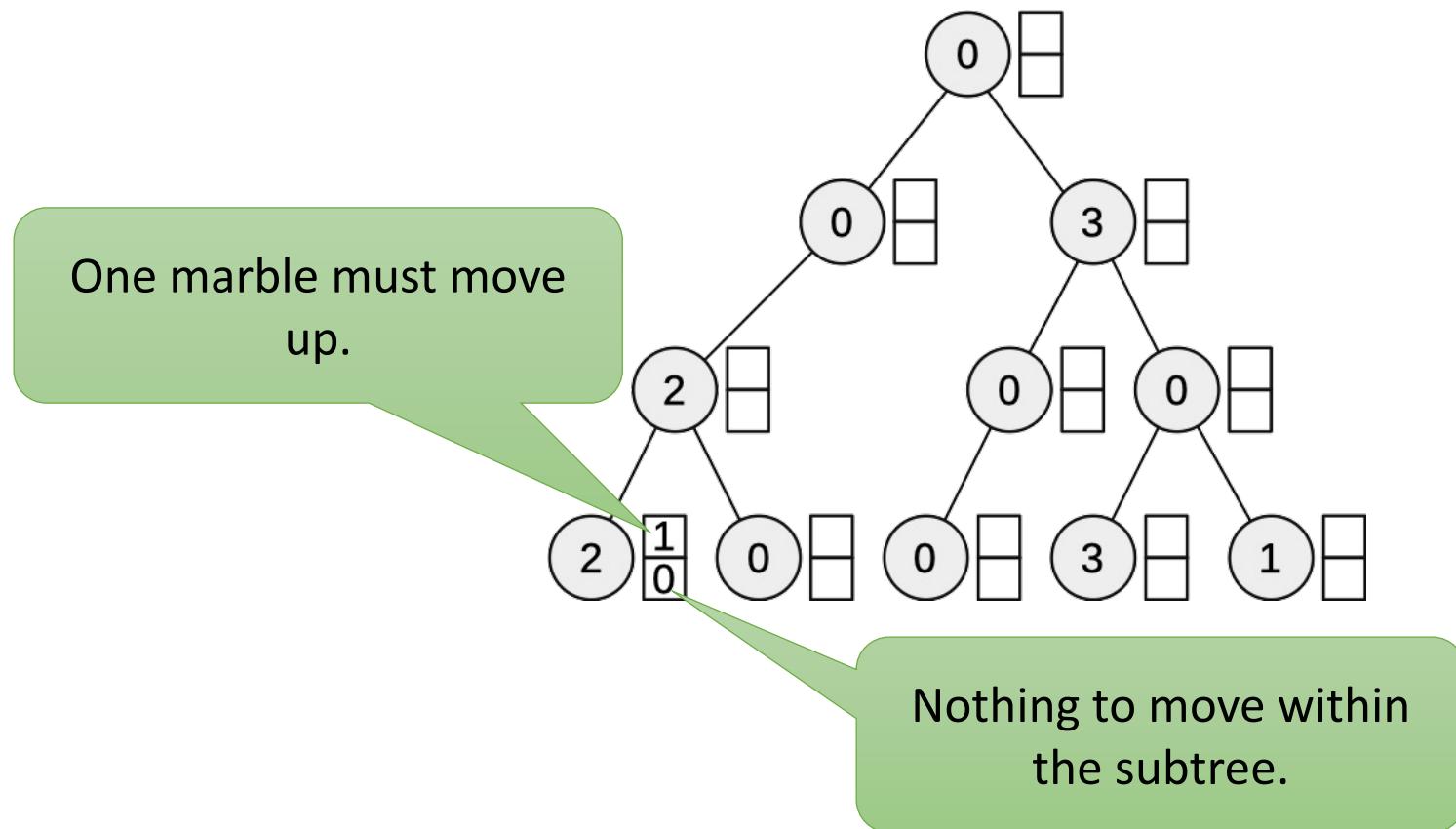
Number of marbles that must move in or out of the subtree.

Solving the Subproblems

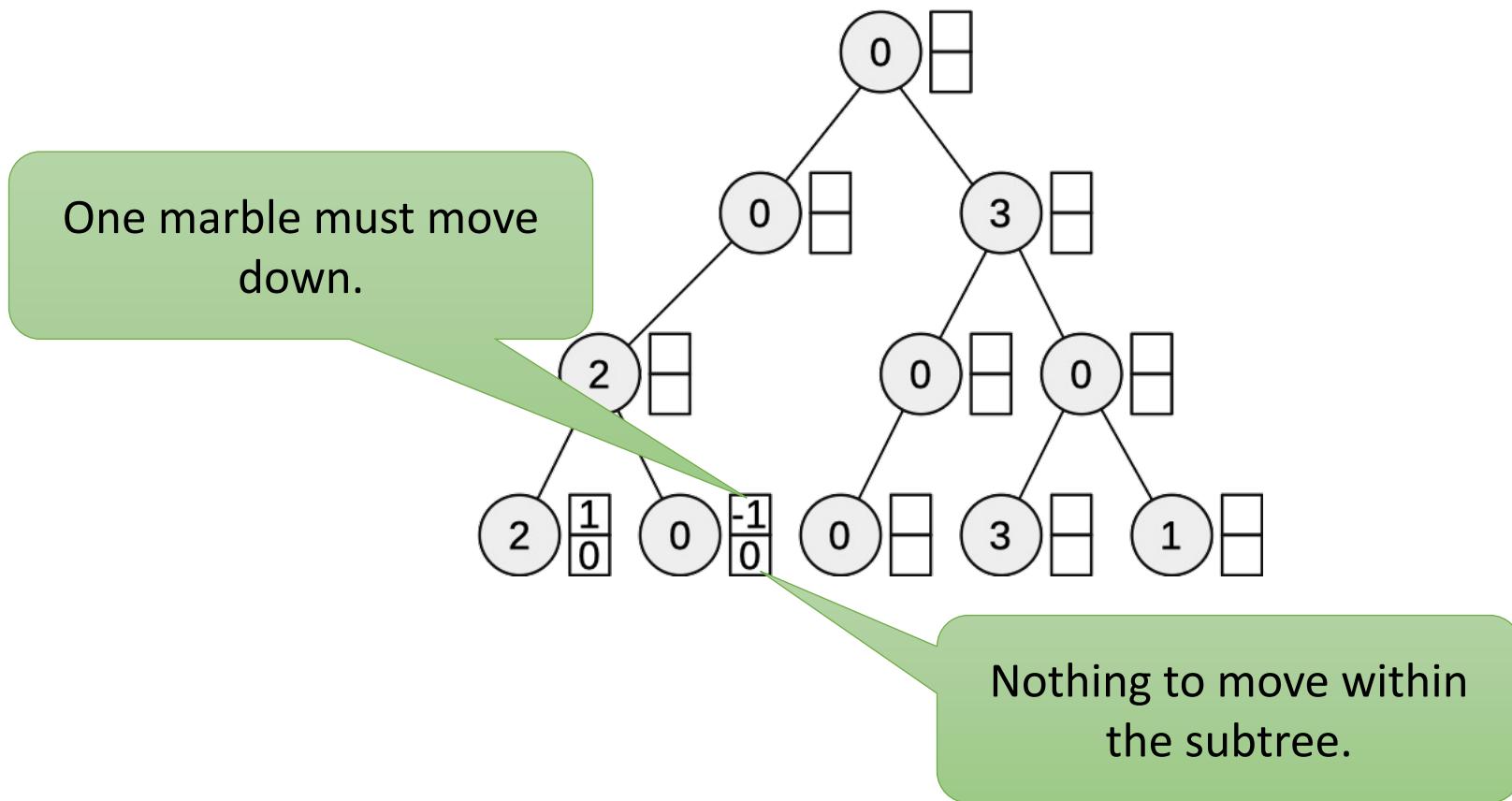
Solve during a postorder traversal.



Solving the Subproblems

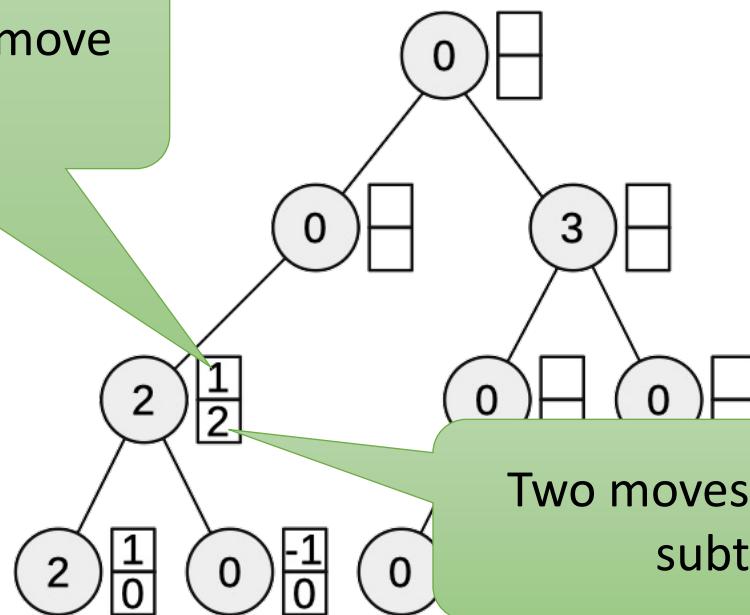


Solving the Subproblems



Solving the Subproblems

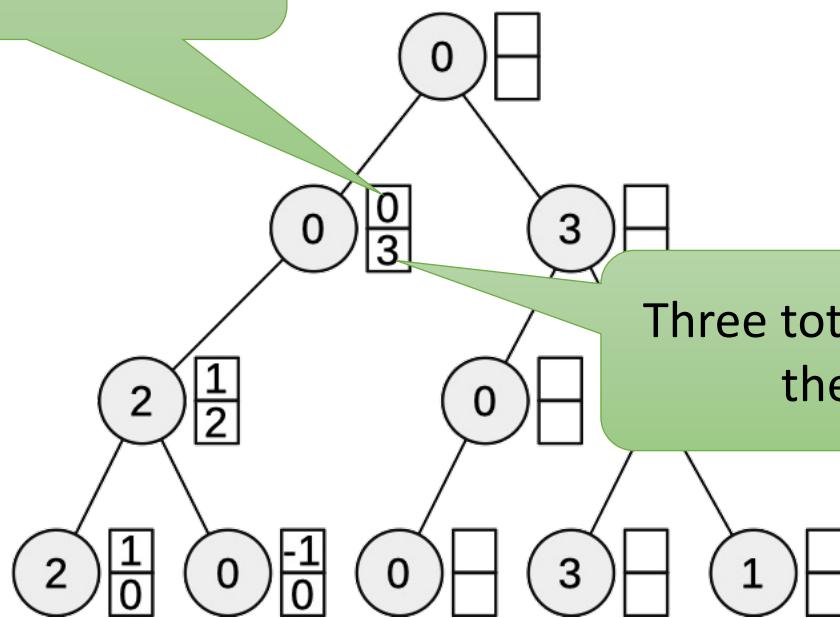
One marble must move up.



Two moves within the subtree.

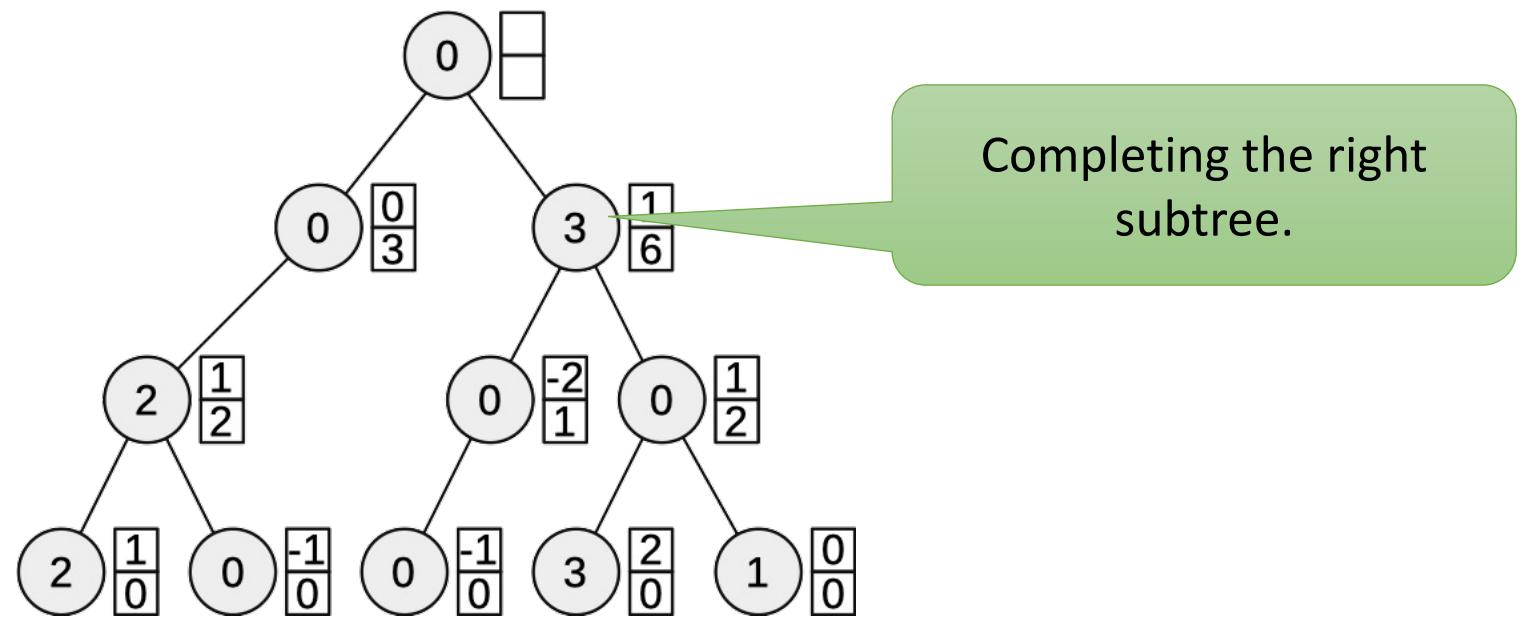
Solving the Subproblems

Nothing in or out of the subtree.

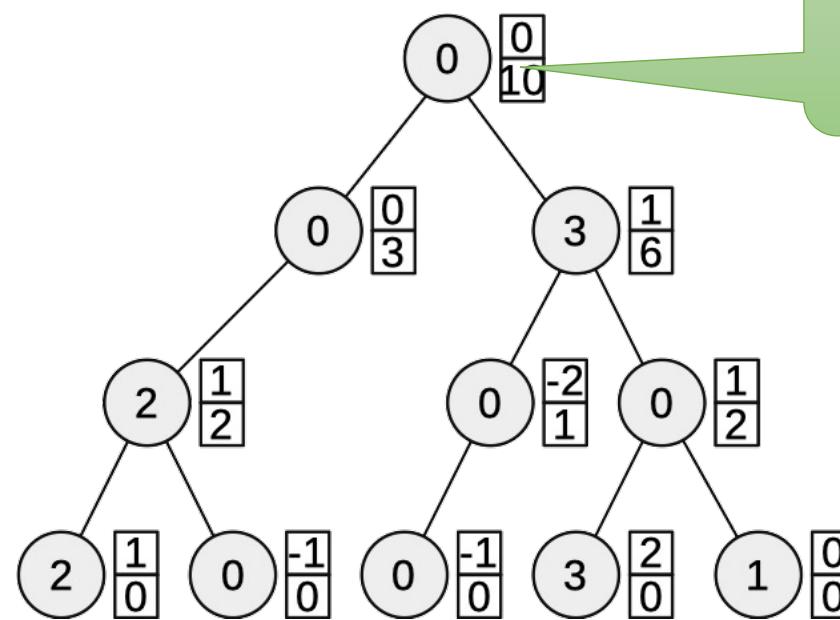


Three total moves within the subtree.

Solving the Subproblems



Solving the Subproblems



Sum of the subtree costs,
plus the movement in
and out.