Competitive Programming

Graph Representation

and

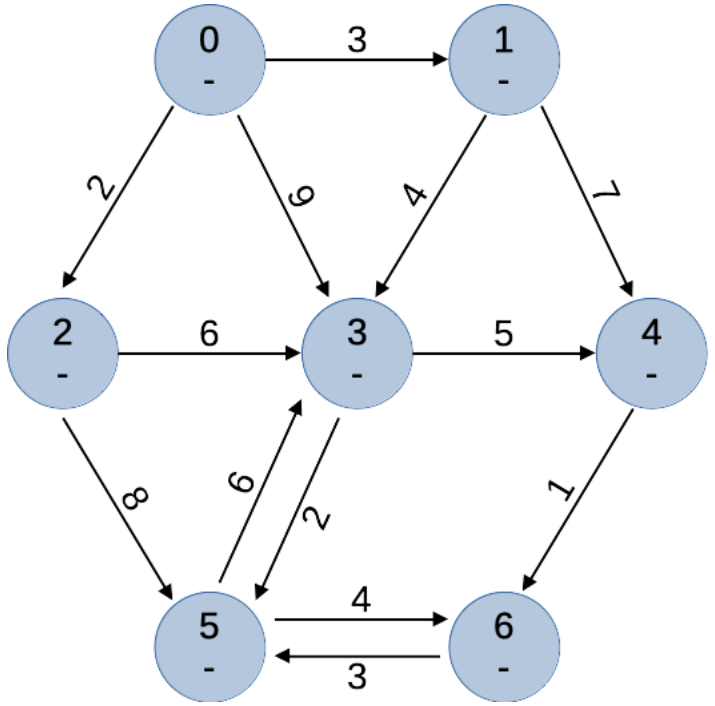
Dijkstra’s Algorithm

Graph Problems

- There are lots of standard algorithms for working with graphs.
- Lots of problems in competitive program depend on applying one or more of these techniques.
- You need to be able to quickly read in an work with graphs
- There are a few common representations.

Adjacency Matrix

- Build a square 2D array
- Each element representing a possible edge between vertices.
- Quick for checking a pair of vertices.
- Expensive for large, sparse graphs.



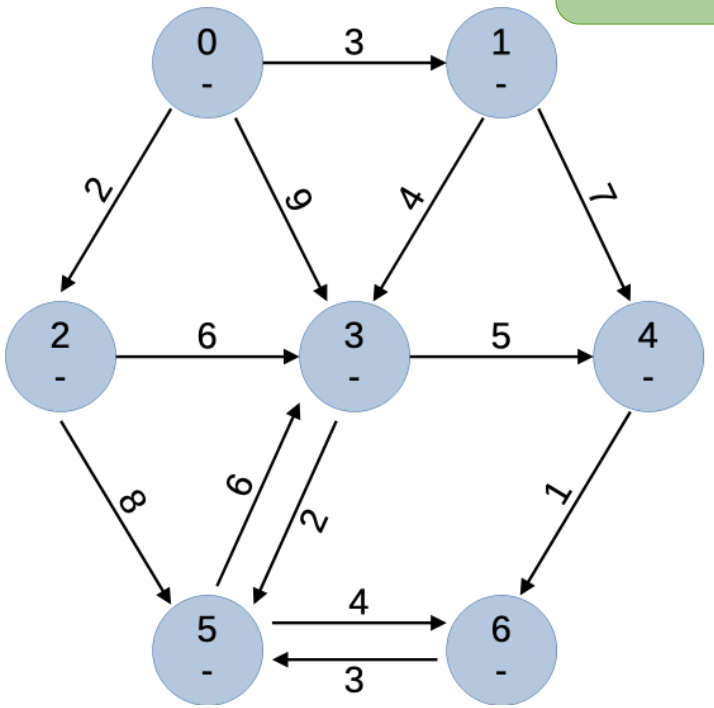
	0	1	2	3	4	5	6
0		3	2	9			
1				4	7		
2				6		8	
3					5	2	
4							1
5				6			4
6						3	

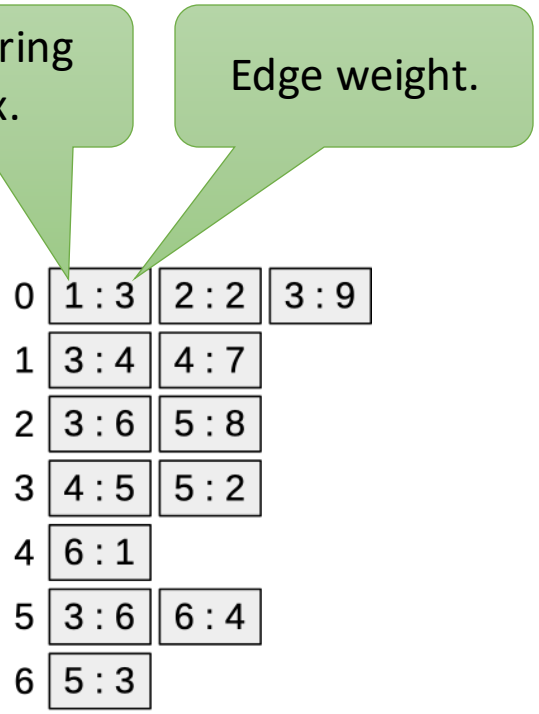
Maybe a nonsense value to indicate no edge.

	0	1	2	3	4	5	6
0	-1	3	2	9	-1	-1	-1
1	-1	-1	-1	4	7	-1	-1
2	-1	-1	-1	6	-1	8	-1
3	-1	-1	-1	-1	5	2	-1
4	-1	-1	-1	-1	-1	-1	1
5	-1	-1	-1	6	-1	-1	4
6	-1	-1	-1	-1	-1	3	-1

Adjacency List

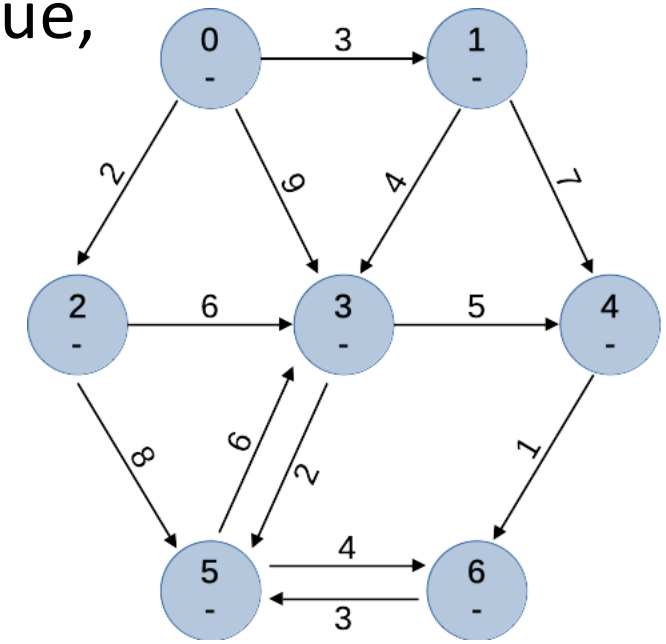
- Array or lists, one for each vertex.
- … listing neighbors of each vertex
  … maybe along with edge weight.
- Memory linear in the size of the graph.
- You want this to be really easy to code.

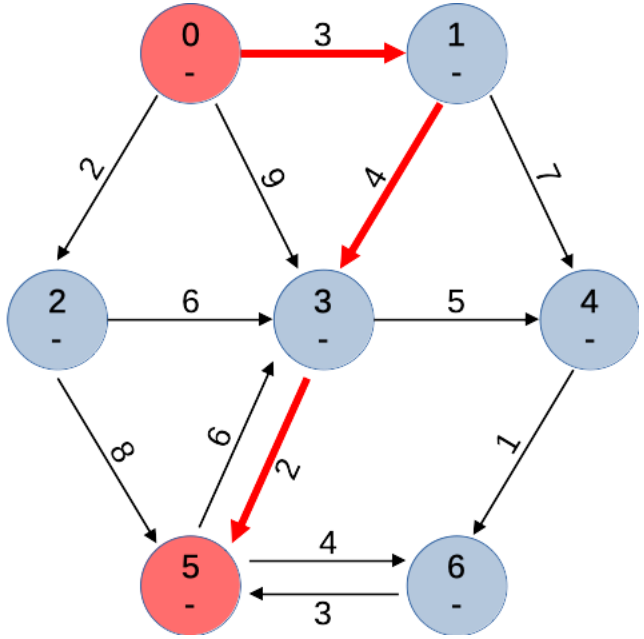




Shortest Path Problem

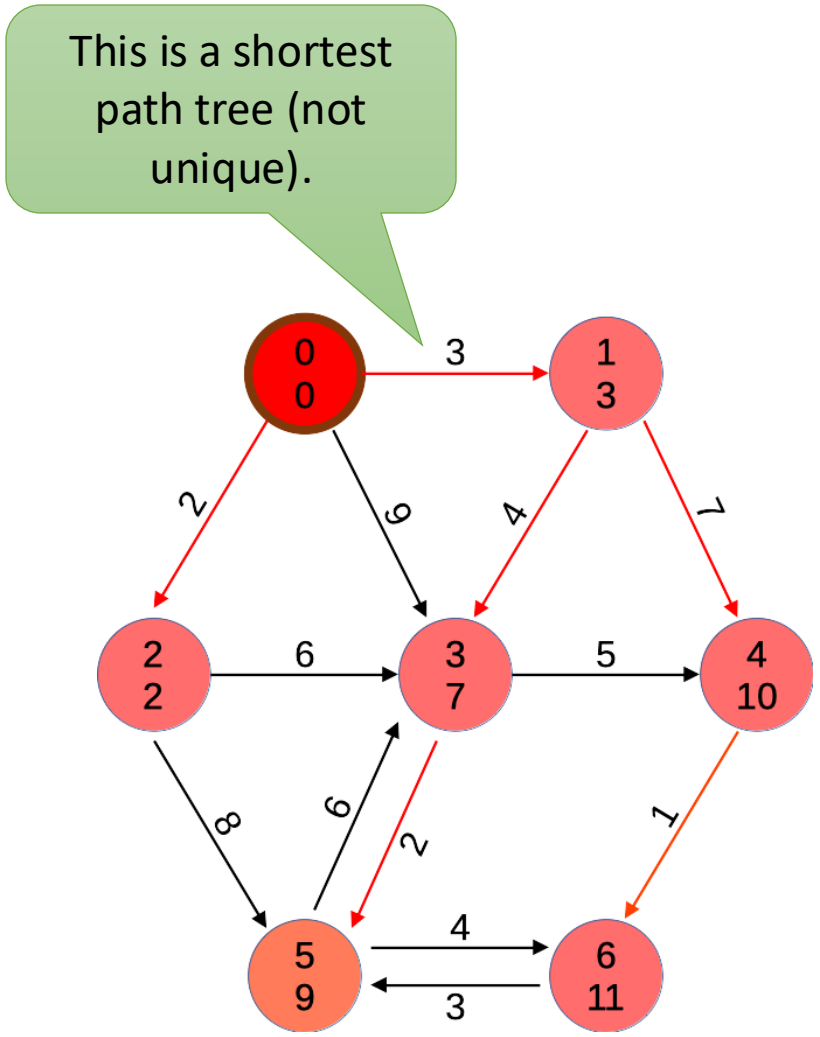
- Given a weighted (directed) graph.
- What's the shortest path between a pair of vertices.
- The path may not be unique,
- … but its length is.



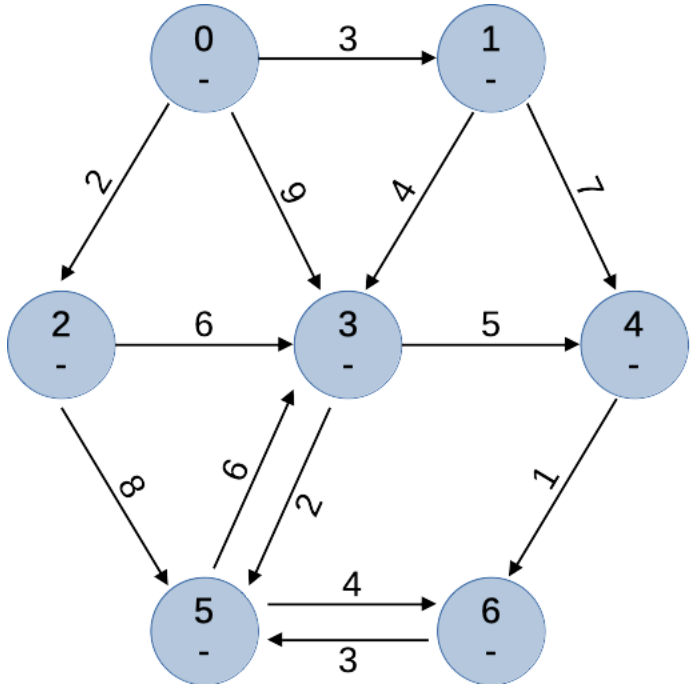


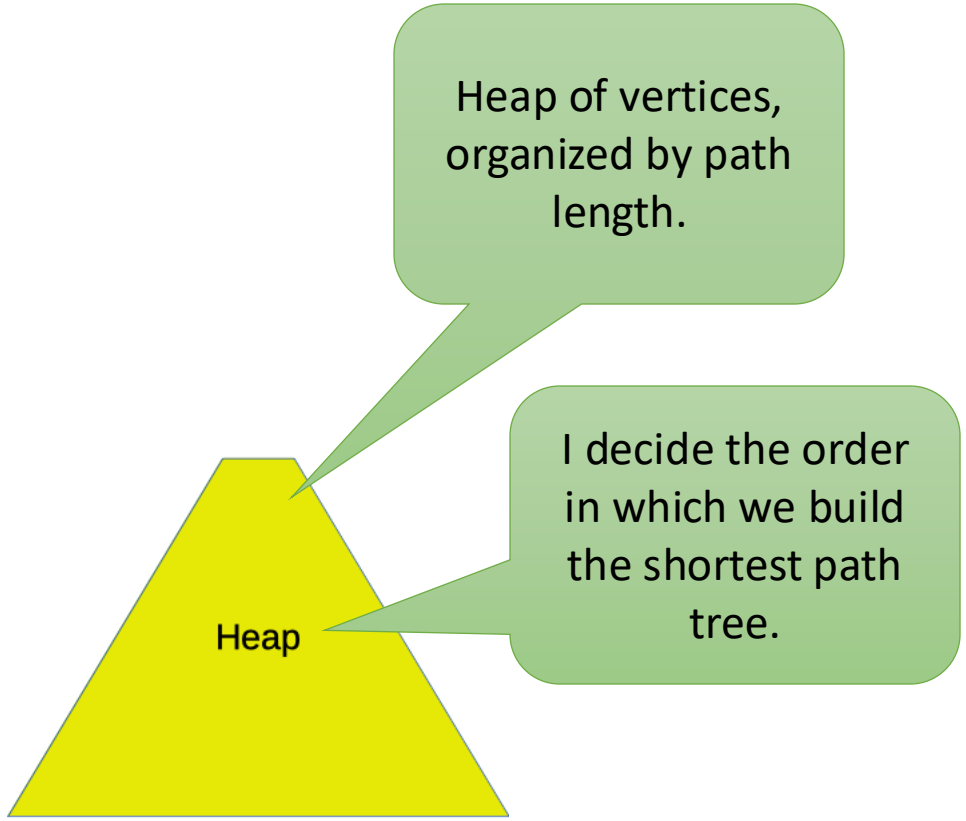
(Single Source) Shortest Path Problem

- Commonly solved by finding **all** shortest paths from a source vertex.
- That's what Dijkstra's algorithm does.
- O(n log(n)) for an n-vertex graph
  really, more like O(E log(V)) or O(E log(E)) depending on how it's implemented.

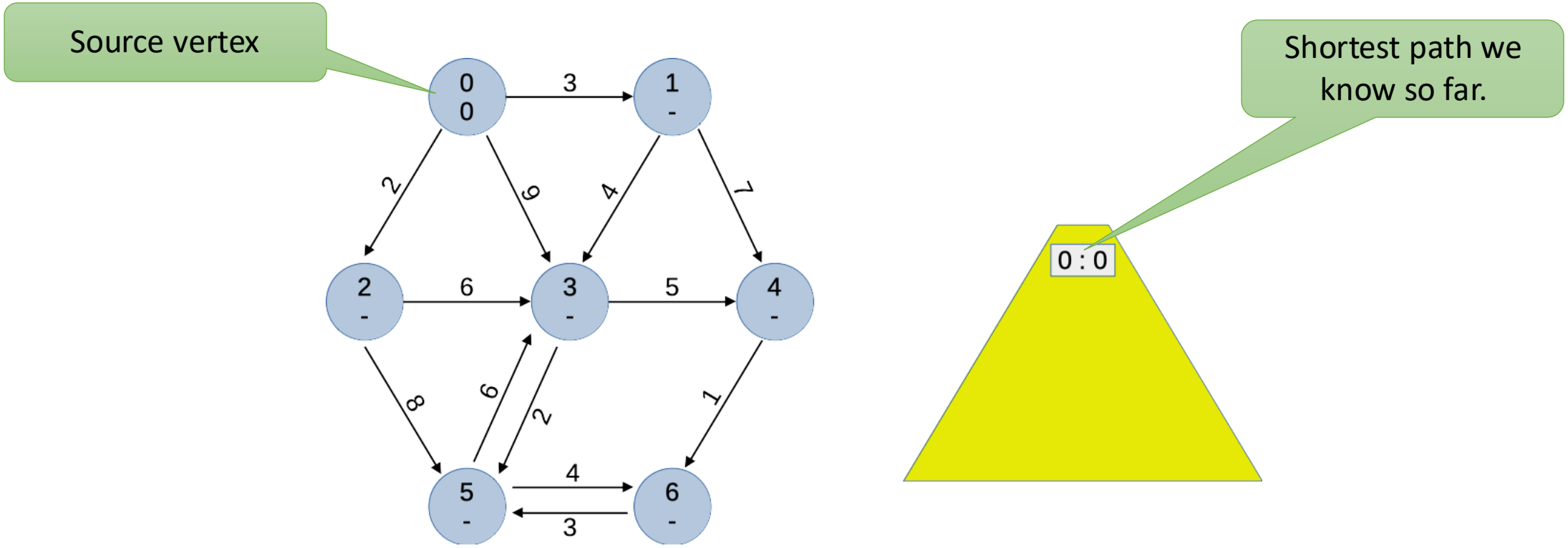


Dijkstra's Algorithm





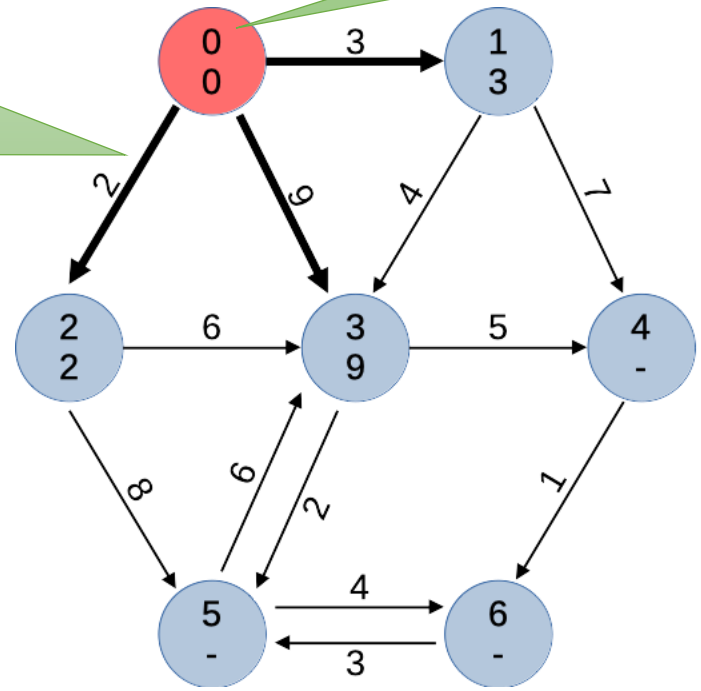
Dijkstra's Algorithm

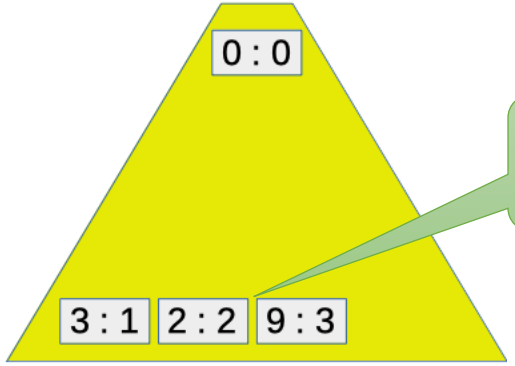


Dijkstra's Algorithm

Vertex 0 is done. We will never find a shorter path to here.

Update shortest paths to neighbors (relax)

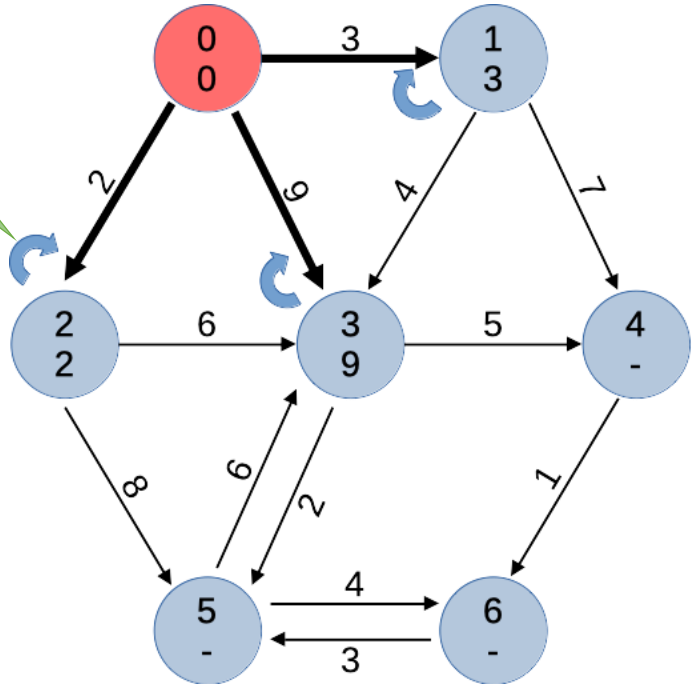


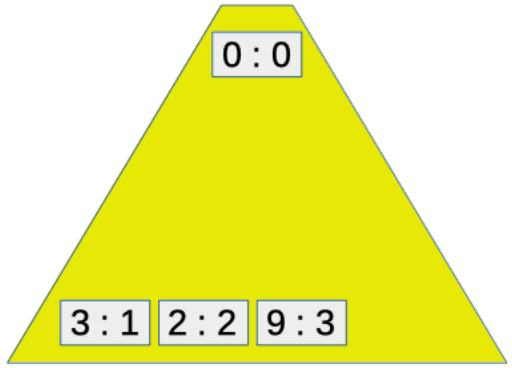


New paths in the heap.

Dijkstra's Algorithm

Each vertex remembers its predecessor.

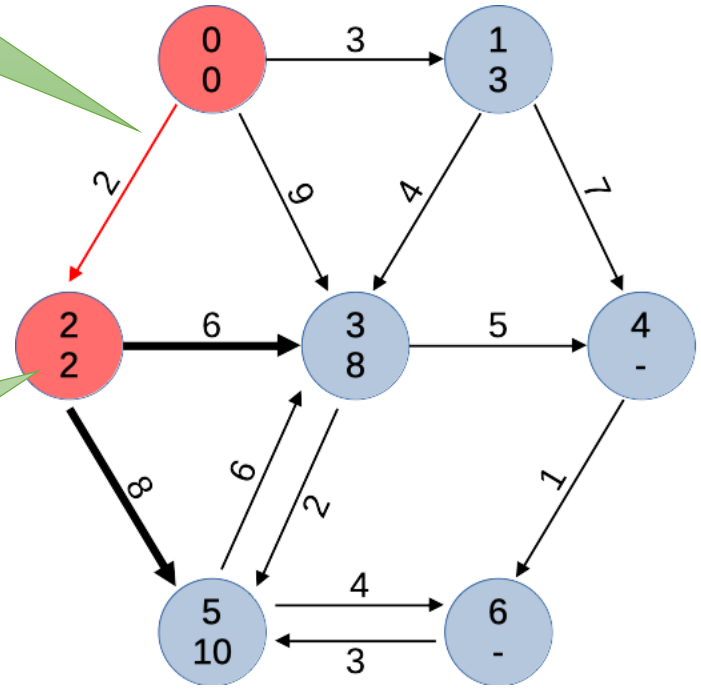




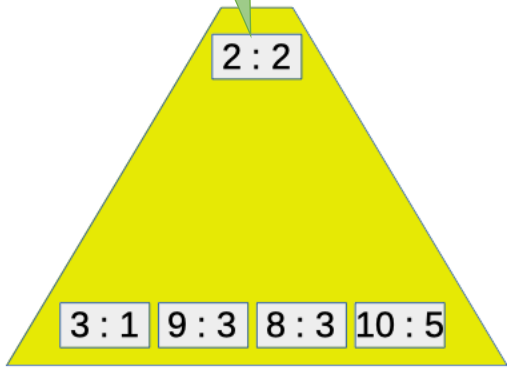
Dijkstra's Algorithm

We're selecting edges for a shortest-path tree.

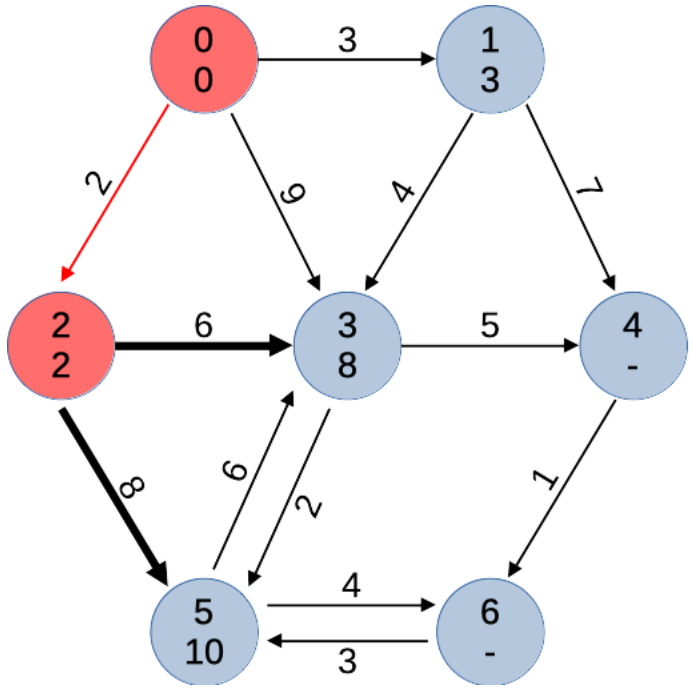
We will never find a shortest path to vertex 2.

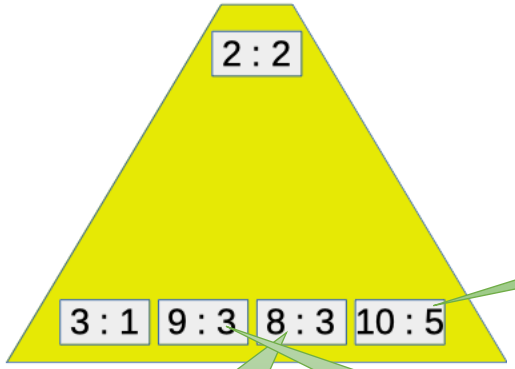


Shortest remaining path.



Dijkstra's Algorithm



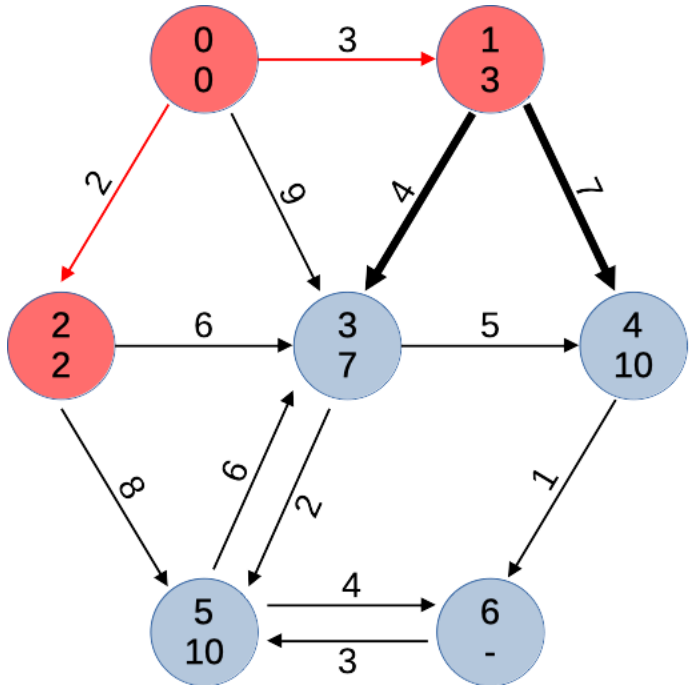


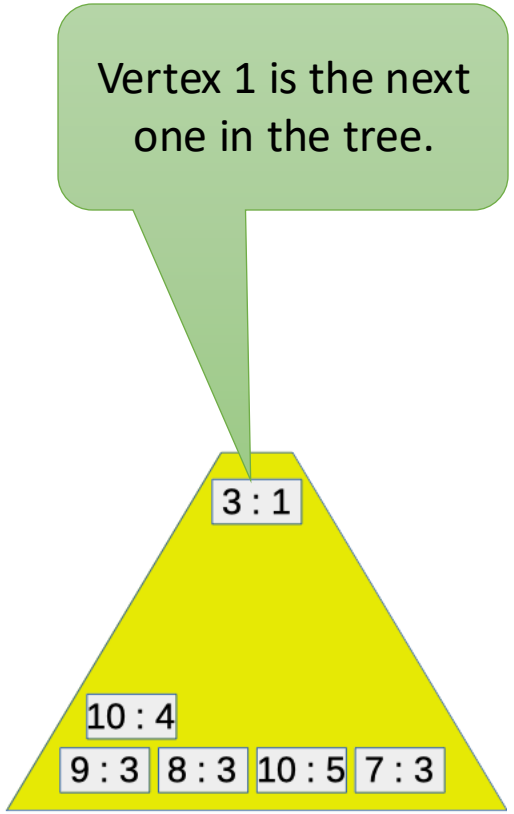
Neighbors of vertex 2 go into the heap.

Including some improved paths.

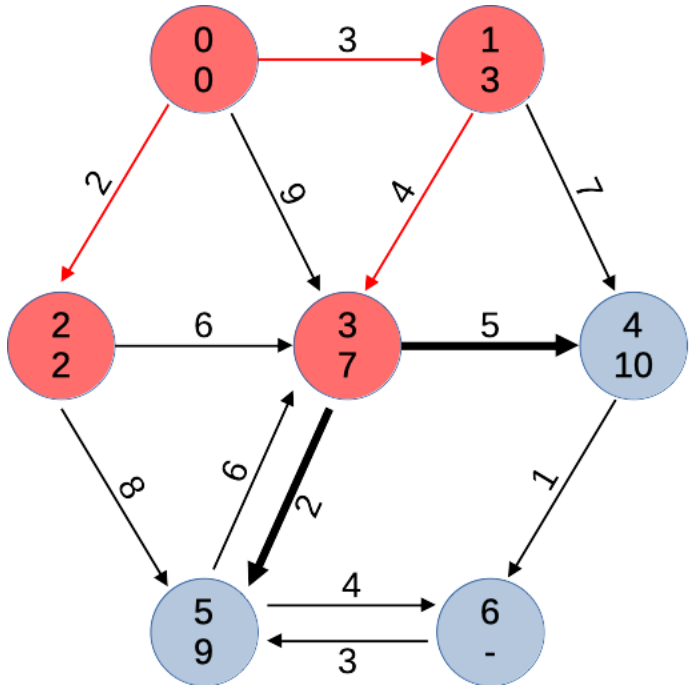
Would be better (but harder) to update this entry.

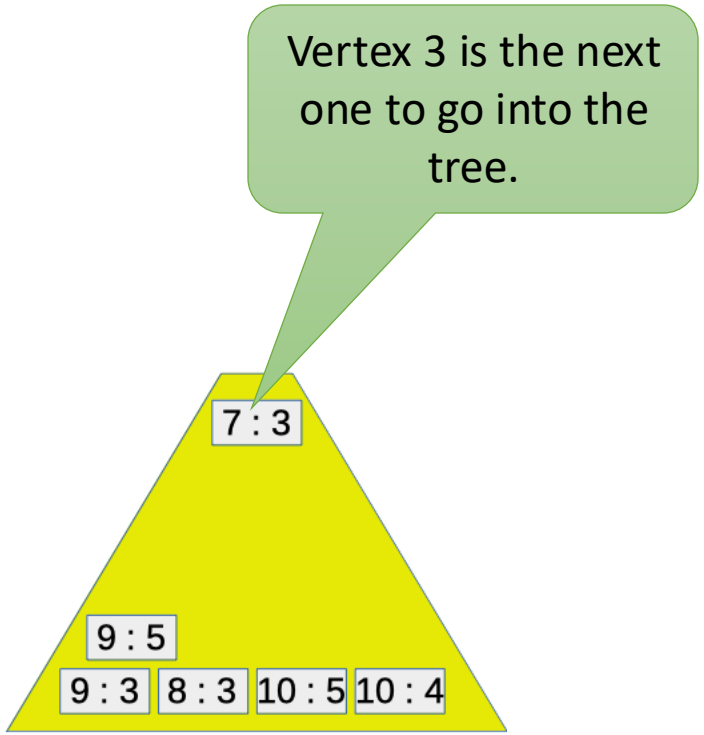
Dijkstra's Algorithm





Dijkstra's Algorithm





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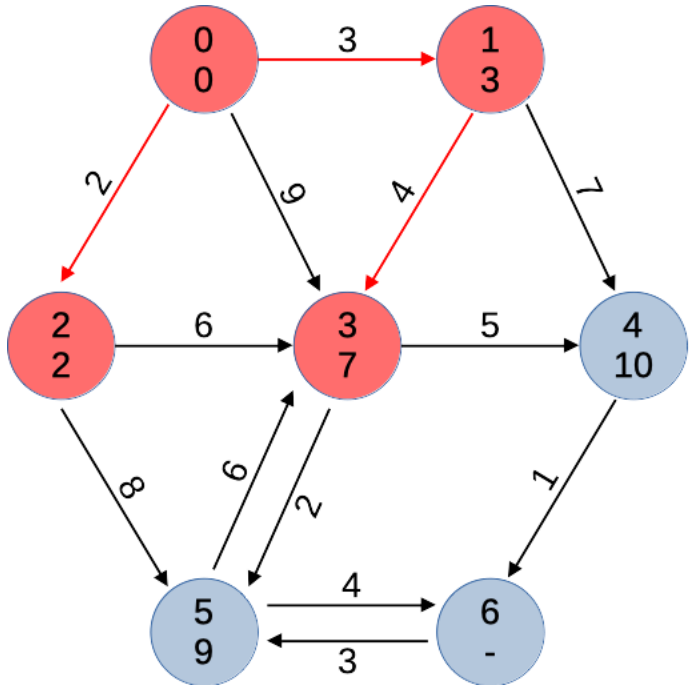

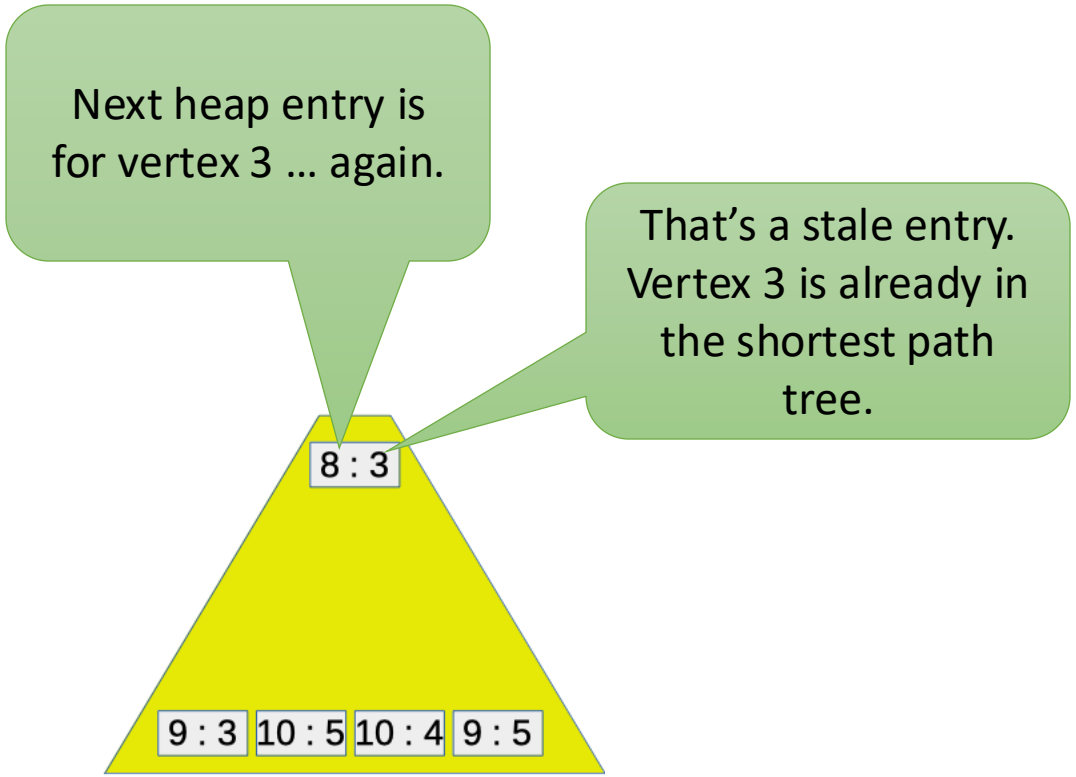

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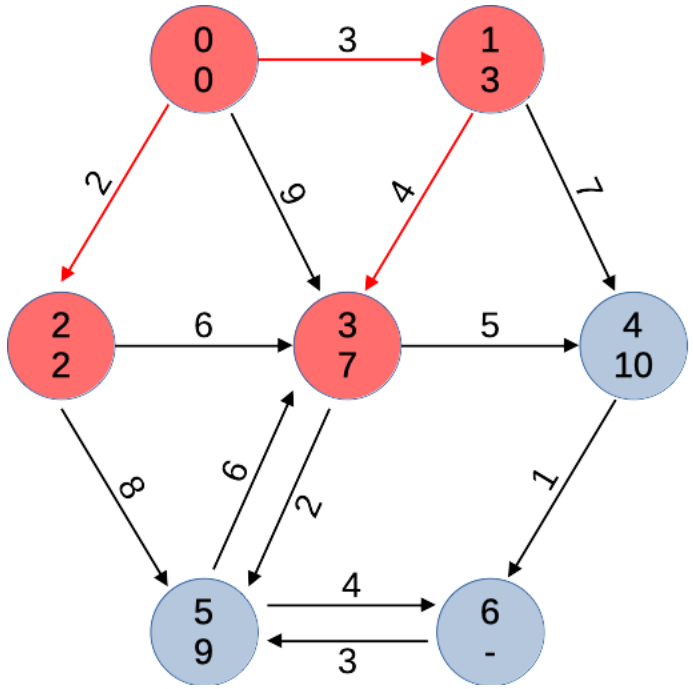

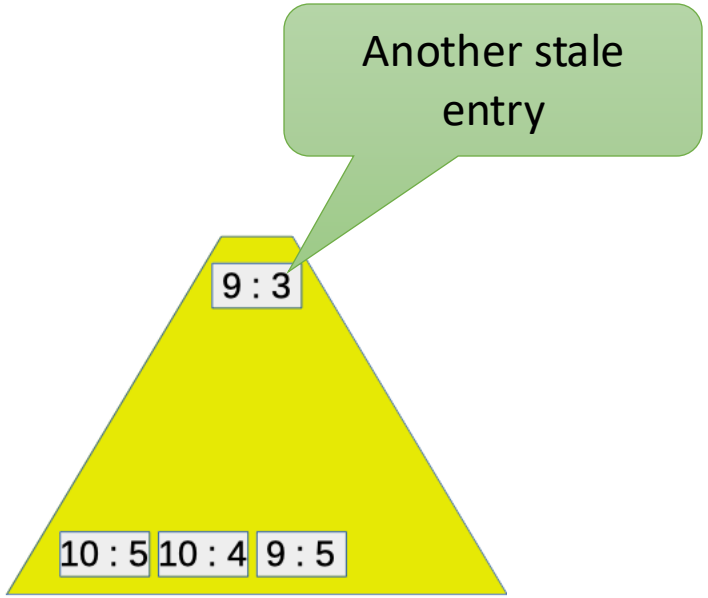

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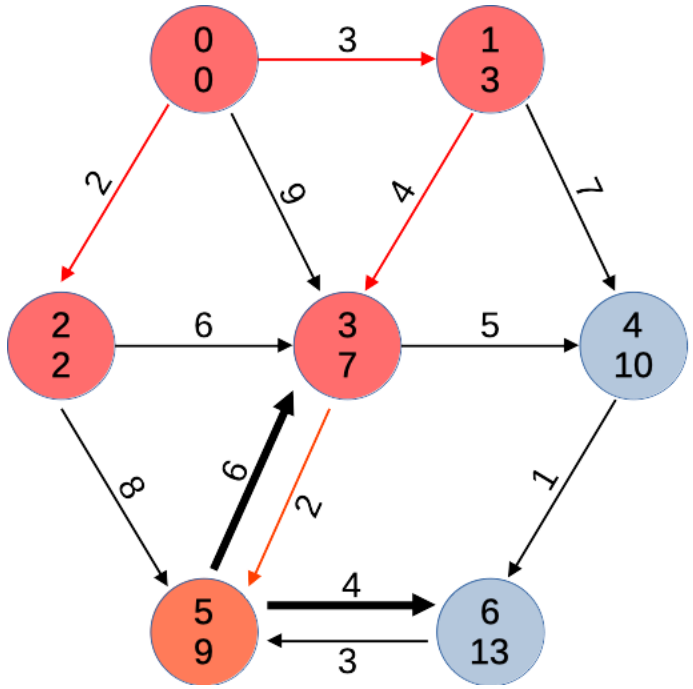

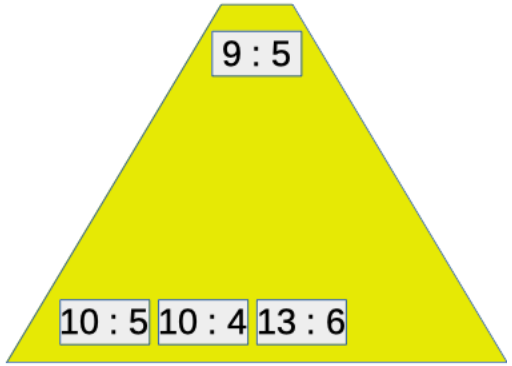

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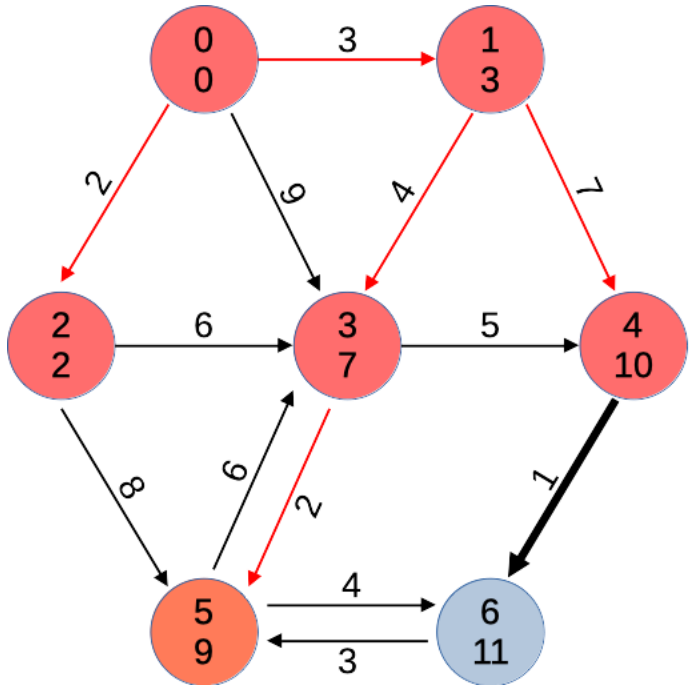

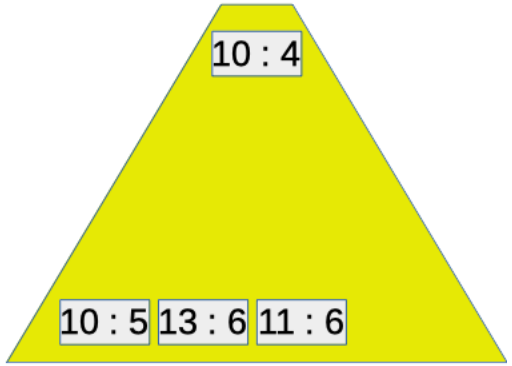

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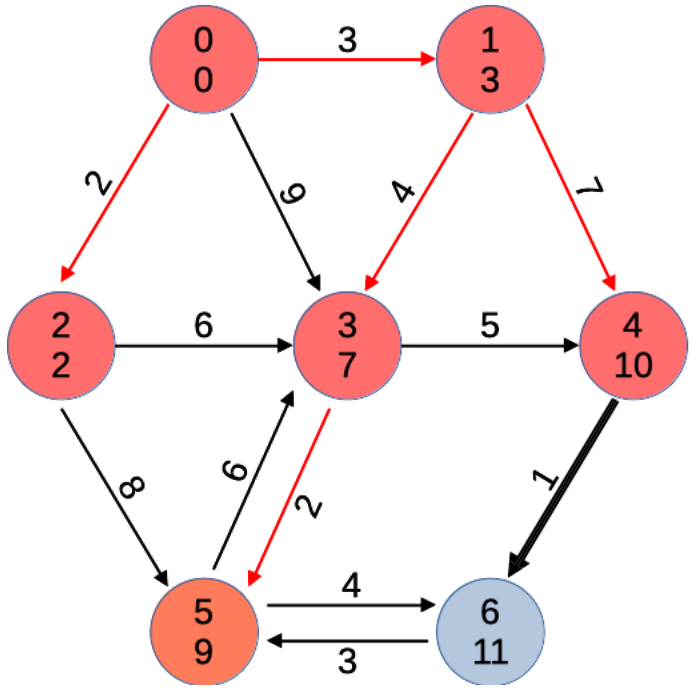

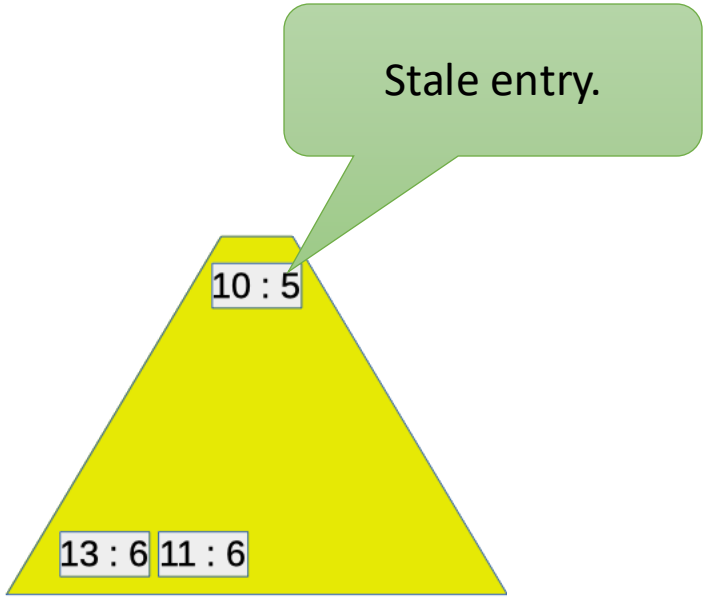

Dijkstra’s Algorithm