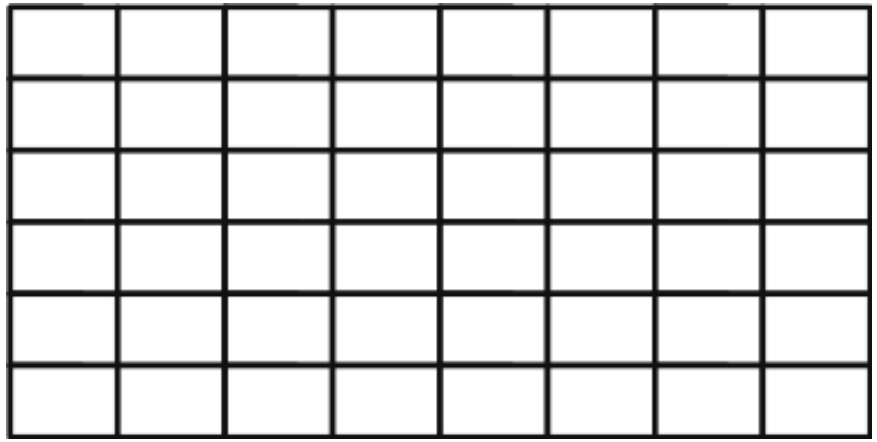
Competitive Programming

Dynamic Programming on a Tree

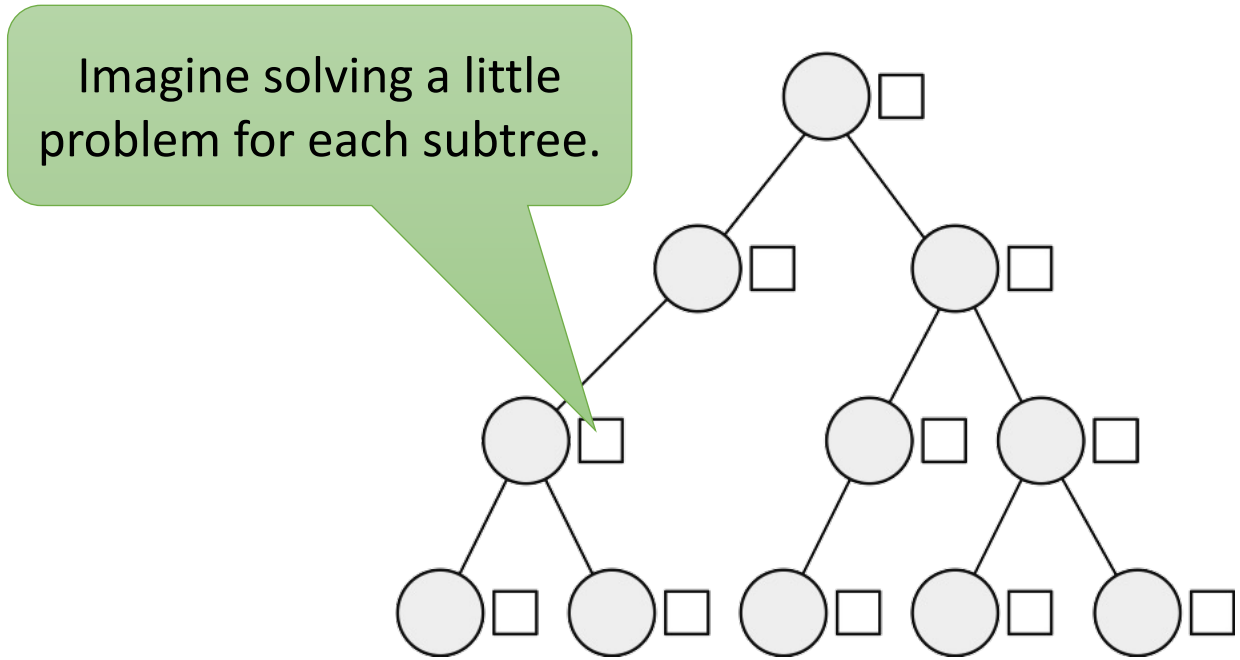
Overview

- Dynamic programming often involves filling in an array of solutions
  - 1-dimensional
  - 2-dimensional
  - … 5-dimensional



DP on a Tree

- Subproblems may exhibit some other structure.
- Trees are common.



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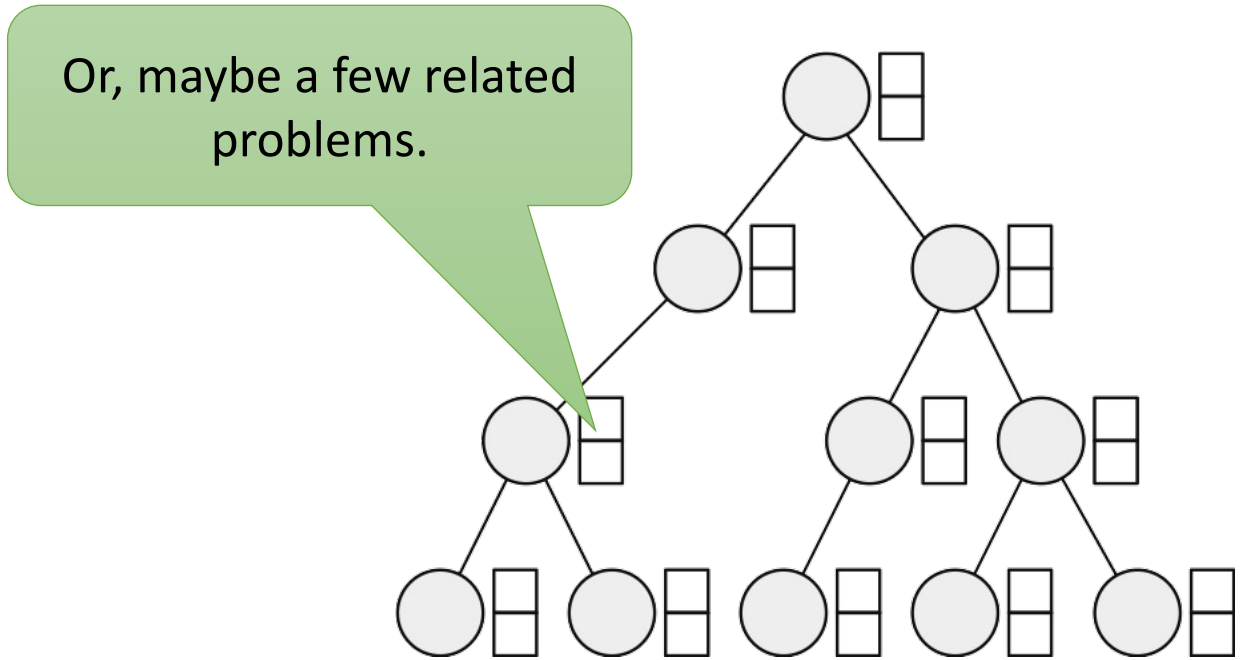
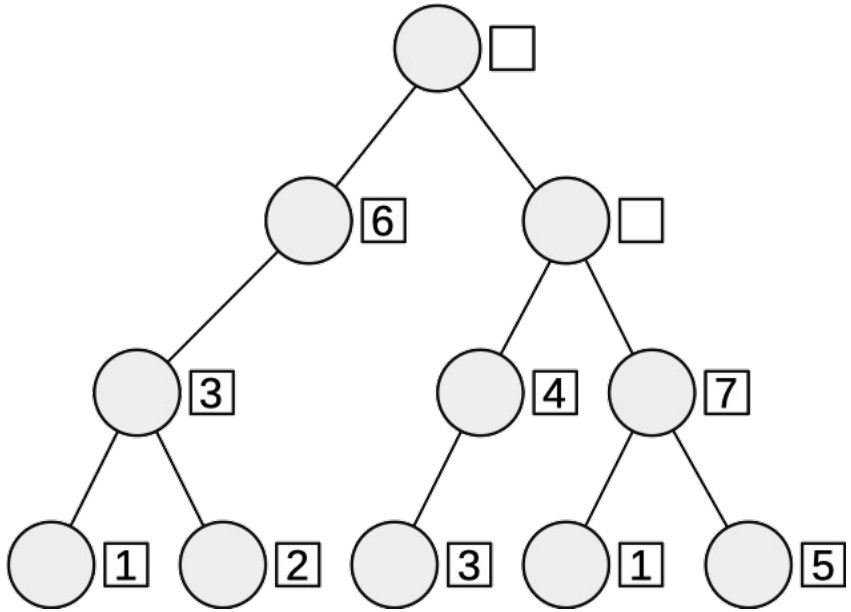


DP on a Tree

- After solving problems within a subtree
- The solutions can be used to solve larger problems at their parent.
- Eventually, at the root.



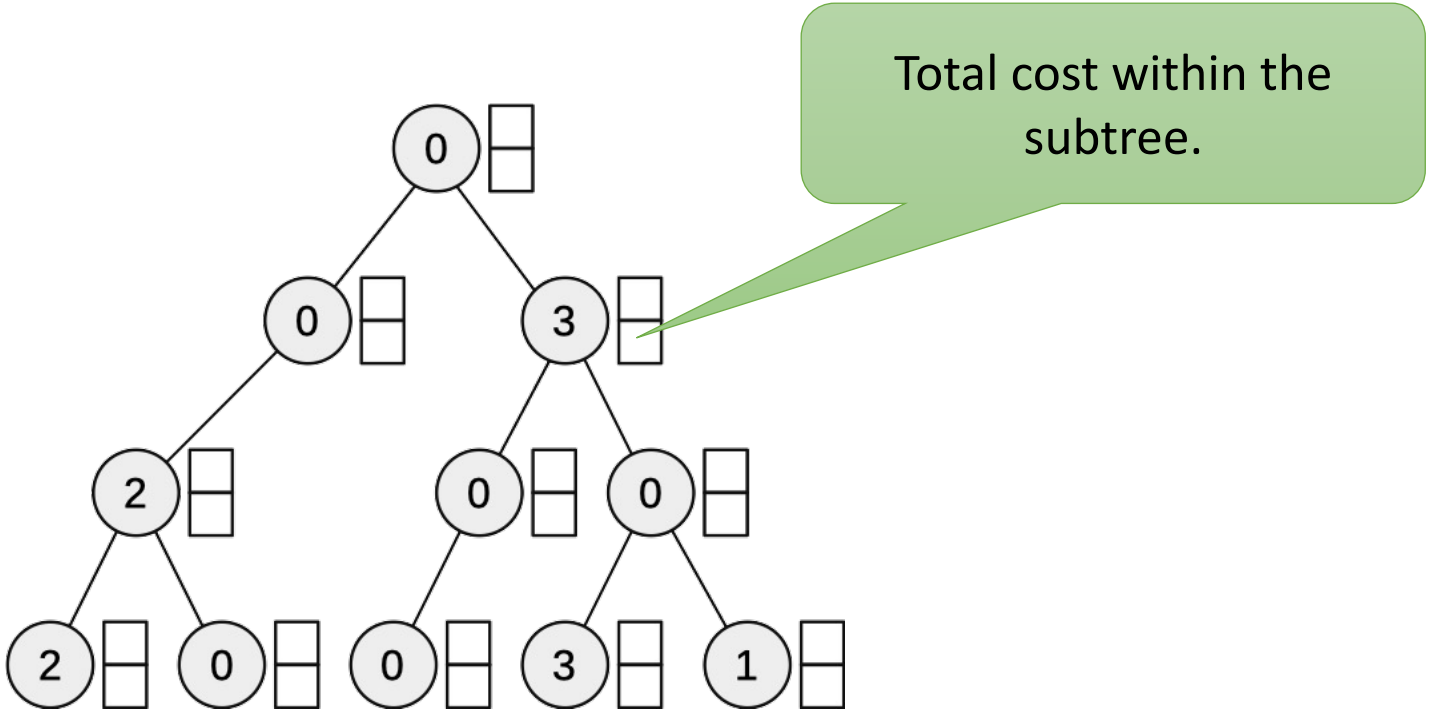
Marbles On A Tree

n boxes are placed on the vertices of a rooted tree, which are numbered from 1 to n, $1 \leq n \leq 10\,000$. Each box is either empty or contains a number of marbles; the total number of marbles is n.

Photo by chefranden on Flickr

The task is to move the marbles such that each box contains exactly one marble. This is to be accomplished be a sequence of moves; each move consists of moving one marble to a box at an adjacent vertex. What is the minimum number of moves required to achieve the goal?

Defining the Subproblems



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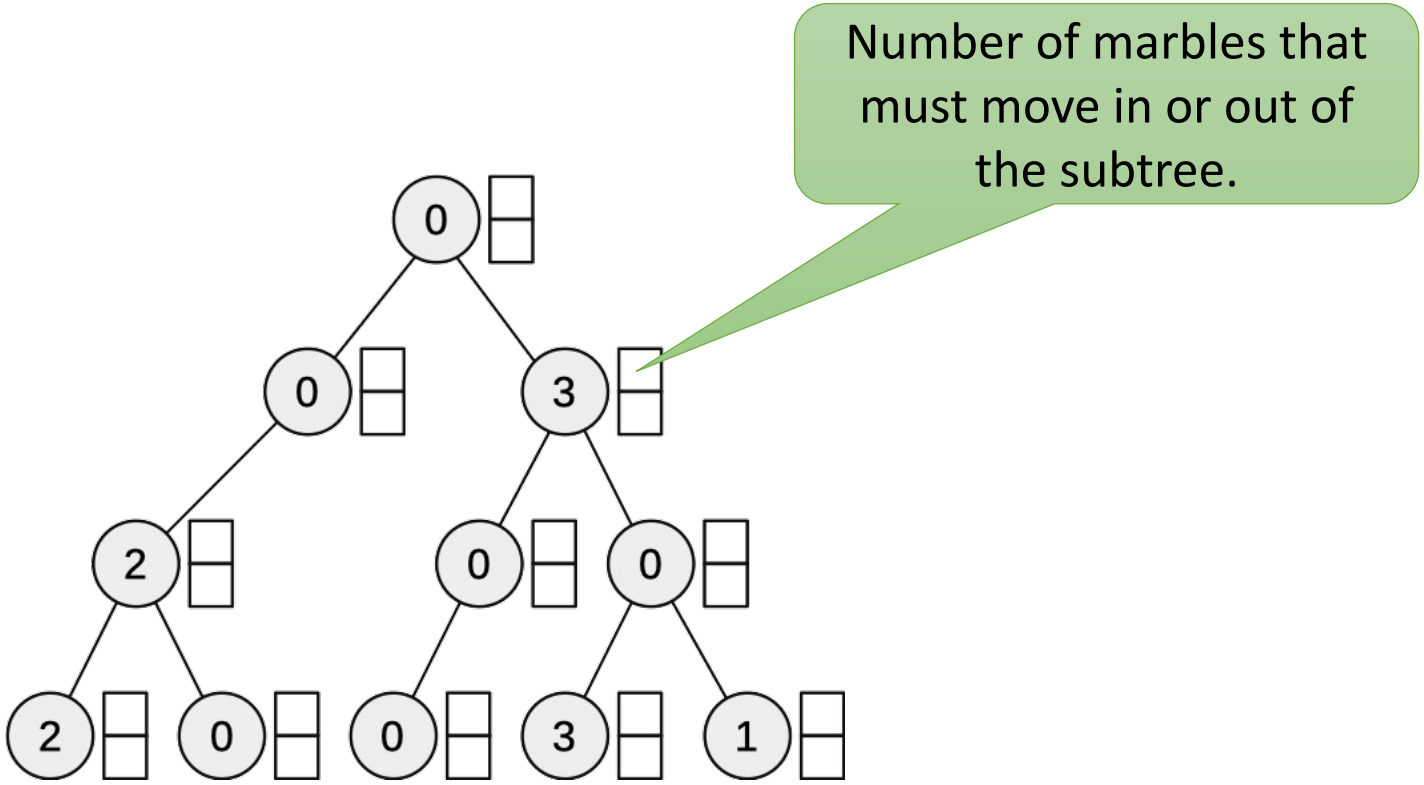
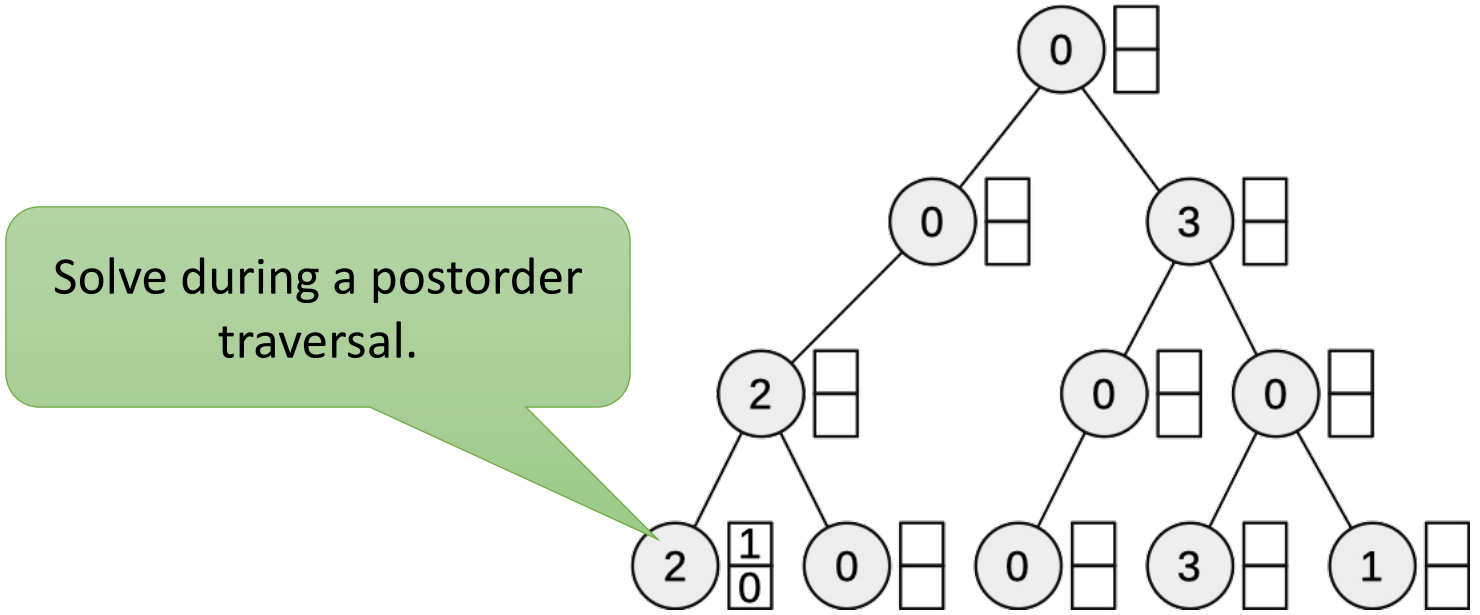


Solving the Subproblems



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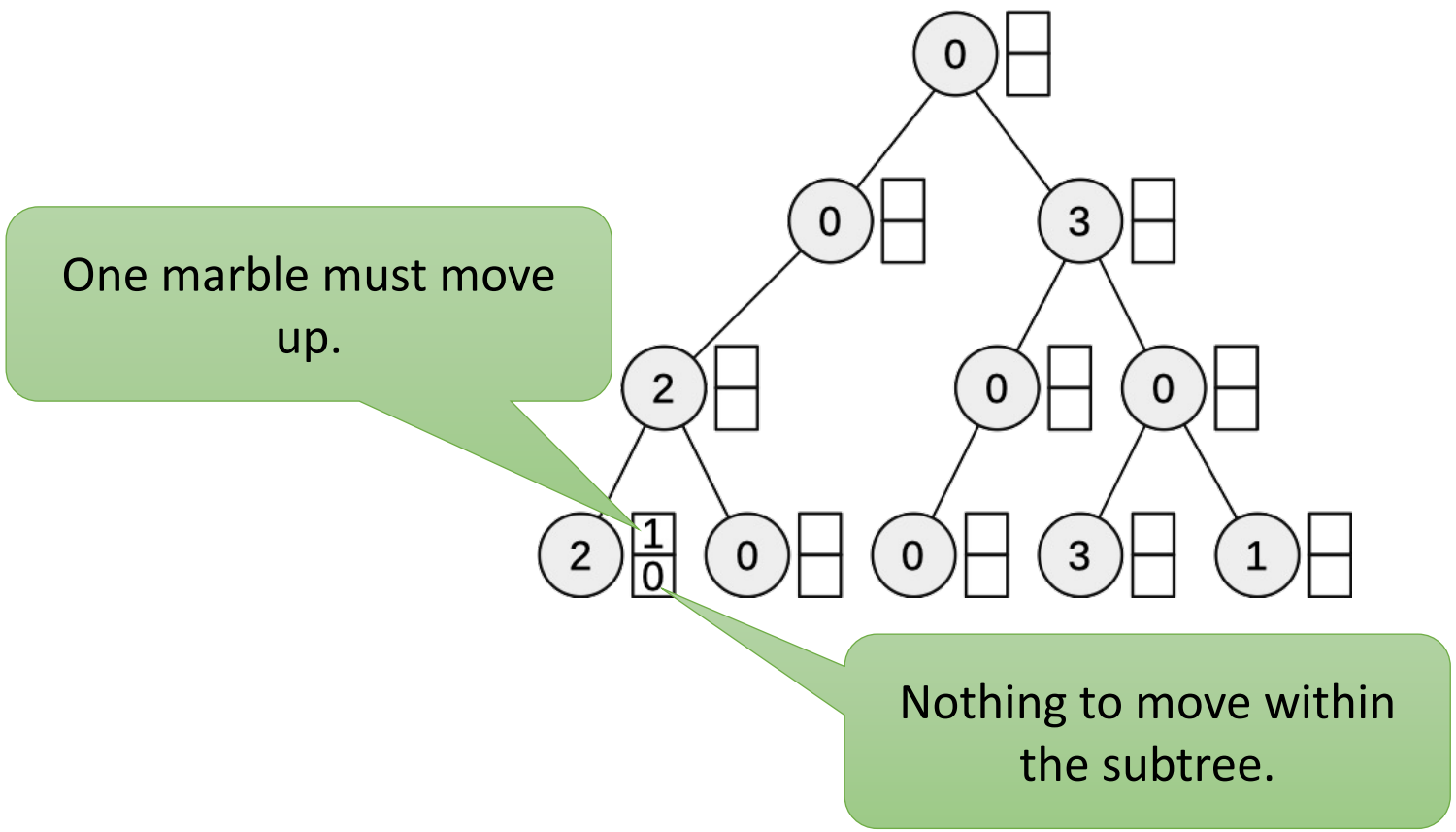

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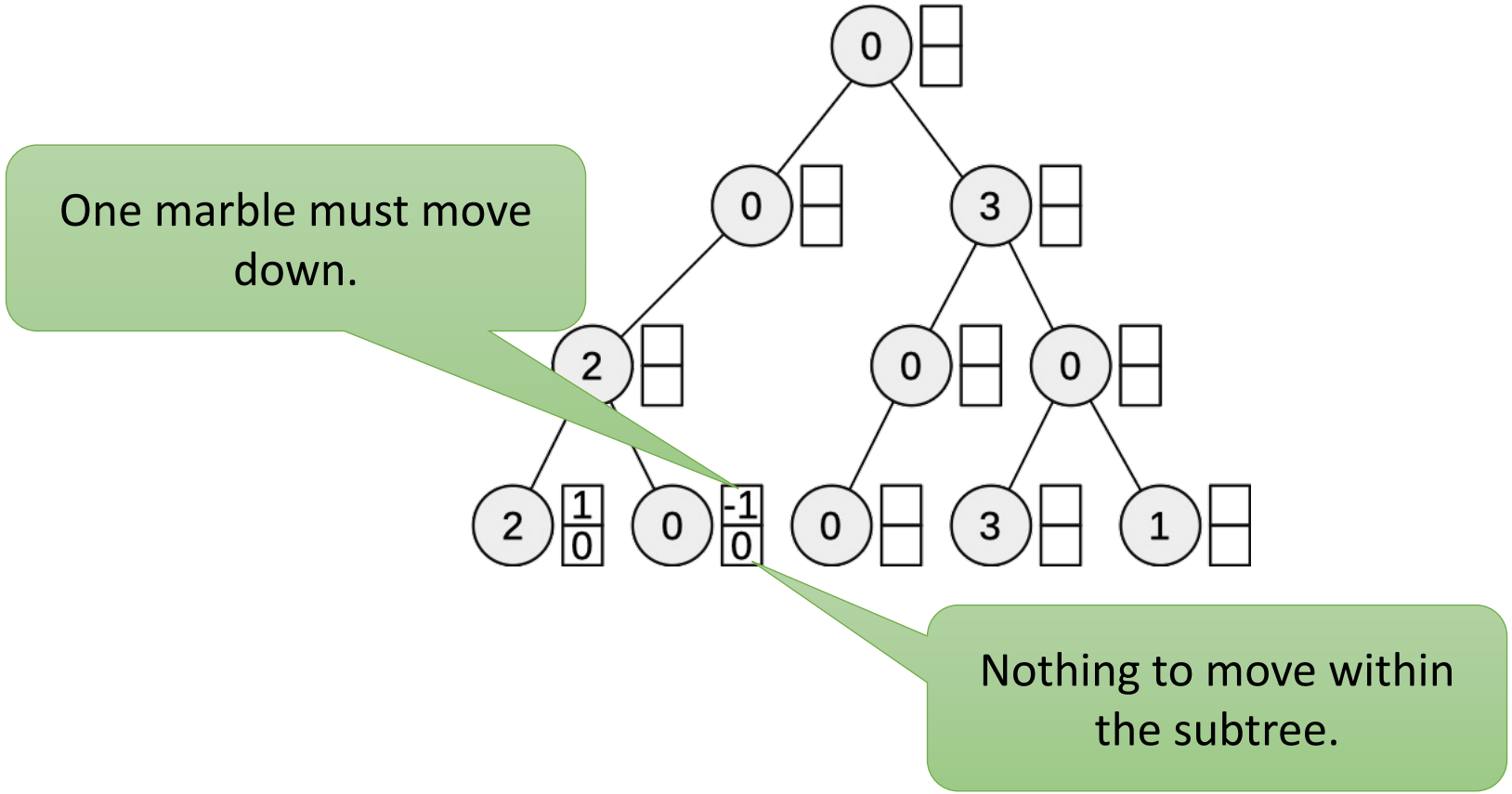

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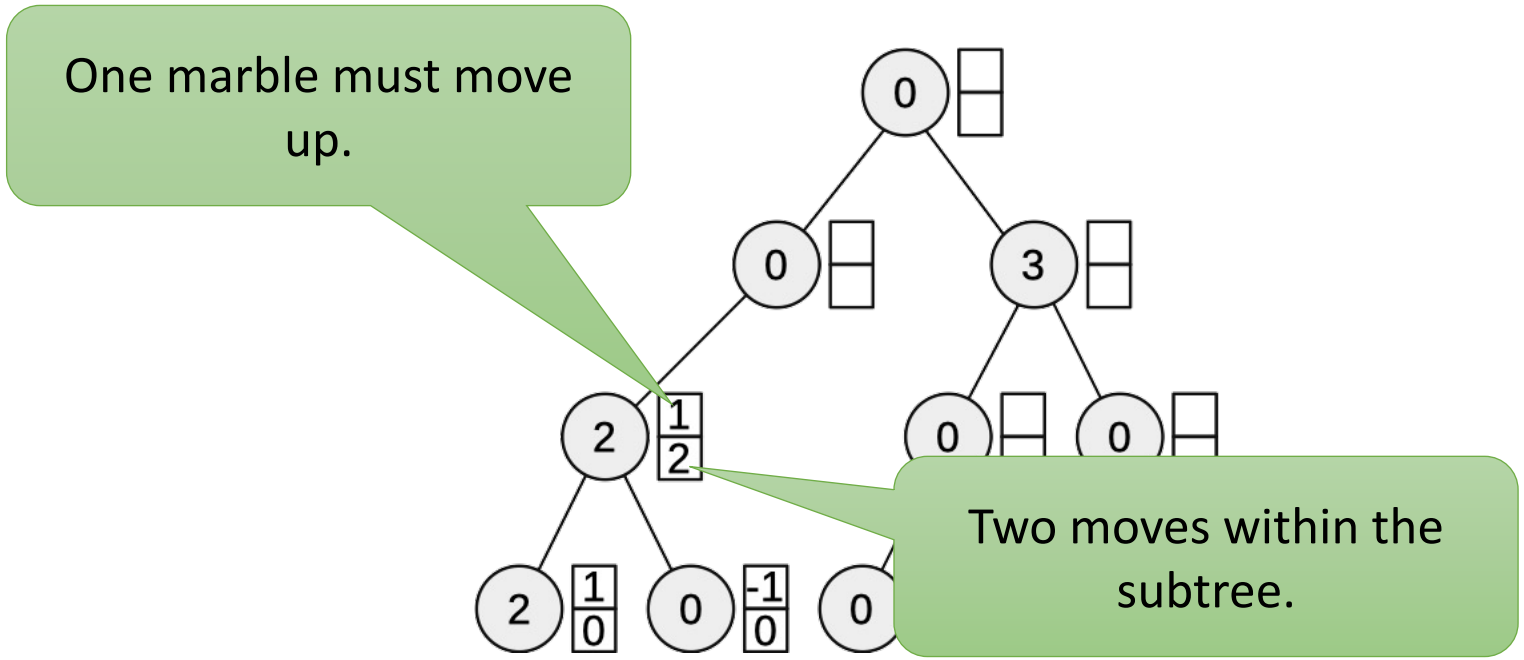

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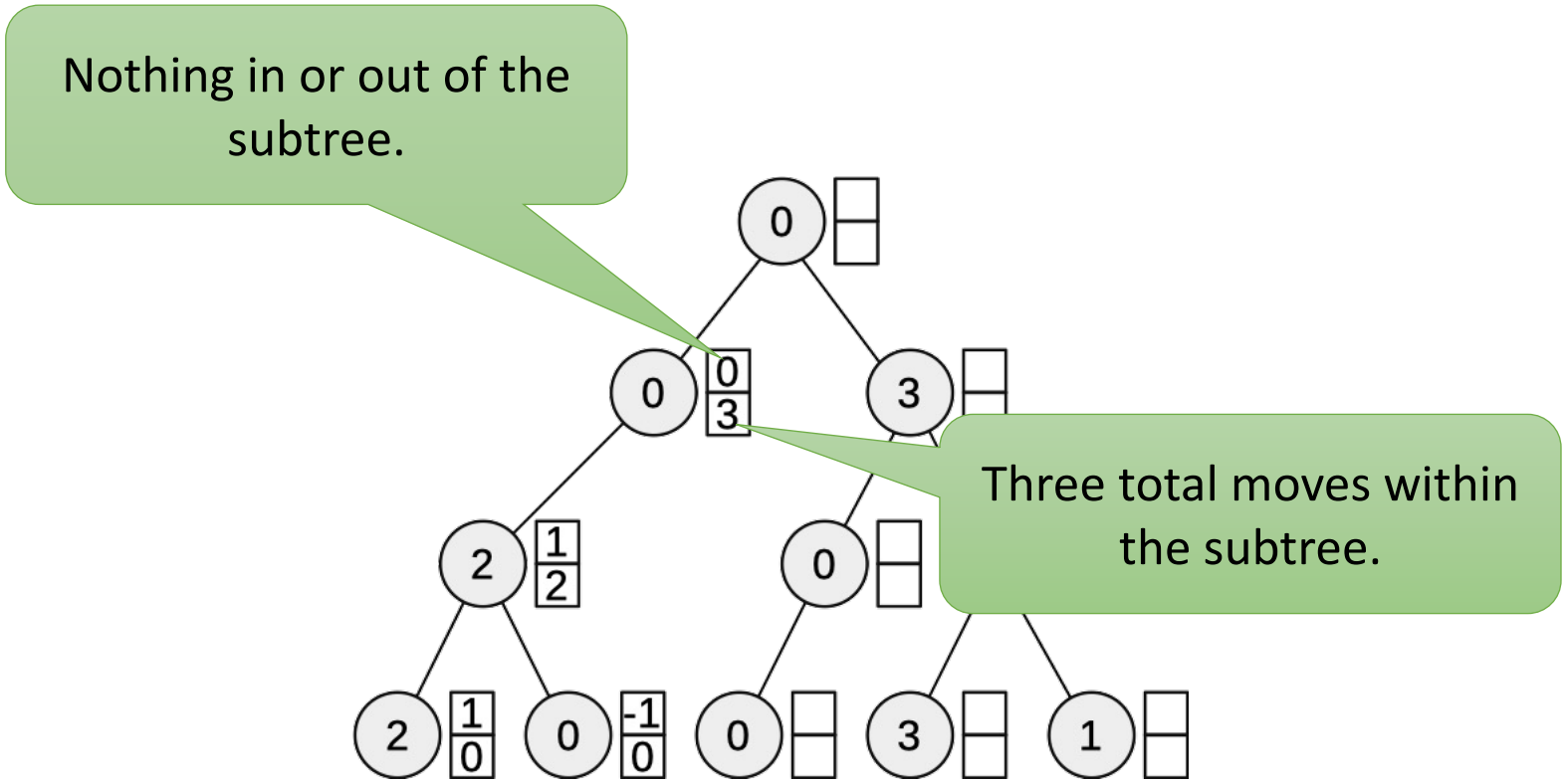

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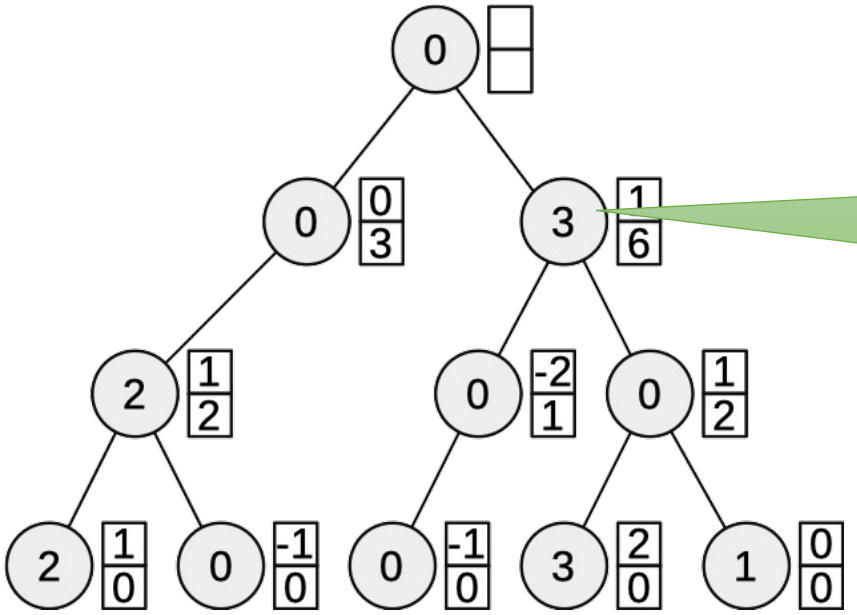

Completing the right subtree.

Solving the Subproblems