Incorporating Inequality Constraints into QAOA

David Bucher

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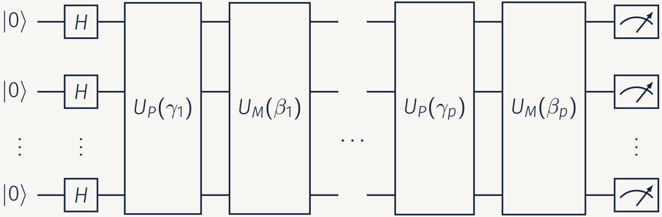
QAOA Brief Introduction

The Quantum Approximate Optimization Ansatz

- Variational circuit for combinatorial optimization, introduced by Farhi et al. [1]
- Trotterization of quantum annealing: $\hat{H}(t) = (1-t)\hat{H}_M + t\hat{H}_P$
- Problem Hamiltonian $\hat{H}_P = -\sum_{i,j} J_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z - \sum_i h_i\hat{\sigma}_i^z$
- Mixer $\hat{H}_M = -\sum_i \hat{\sigma}_i^x$, Initial state $|+\rangle = -\hat{H}_M\,|+\rangle$



The Quantum Alternating Operator Ansatz

- Generalization of the ansatz by Hadfield et al. [2]
  - $U_P(\gamma)$ *phase-separating* unitary that depends on *objective function*
  - $U_M(\beta)$ *mixing* unitary that depends on *domain*

Condition for Mixer

Map between all feasible states

$$\forall \mathbf{x}, \mathbf{y} \in F, \exists \beta : | \langle \mathbf{x} | U_M(\beta) | \mathbf{y} \rangle | > 0$$

Inequality Constraints Before

Constrained Mixer for Inequality Constraints

Experiments

[1] Edward Farhi et al. *A Quantum Approximate Optimization Algorithm*. 2014. arXiv: `1411.4028 [quant-ph]`.

[2] Stuart Hadfield et al. “From the Quantum Approximate Optimization Algorithm to a Quantum Alternating Operator Ansatz”. In: *Algorithms* 12.2 (Feb. 2019), p. 34. ISSN: 1999-4893. DOI: `10.3390/a12020034`. URL: `http://dx.doi.org/10.3390/a12020034`.