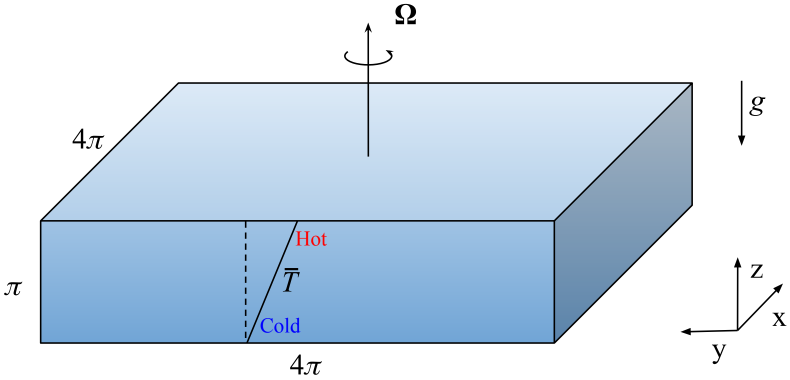
The Effects of Rotation on Stratified Turbulence

UCSC Applied Mathematics

November 25, 2024

Schematic



Governing Equations

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{Ro}(\boldsymbol{e}_z \times \boldsymbol{u}) = -\nabla p + \frac{T}{Fr^2}\boldsymbol{e}_z + \boldsymbol{F} + \frac{1}{Re}\nabla^2 \boldsymbol{u} \quad \text{(mom.)}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T + w = \frac{1}{Pe}\nabla^2 T \quad \text{(temp.)}$$

$$\nabla \cdot \boldsymbol{u} = 0 \quad \text{(cont.)}$$

Forcing Mechanism

We choose our forcing to be purely horizontal and divergence-free:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{u} = 0$$

The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \hat{\boldsymbol{F}} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$
$$G(\boldsymbol{k}_h, t) \in \mathbb{C}$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1.

From Stratified Turbulence to rotationally domainated flow

Typical non-rotating flows, properties of stratified turbulence

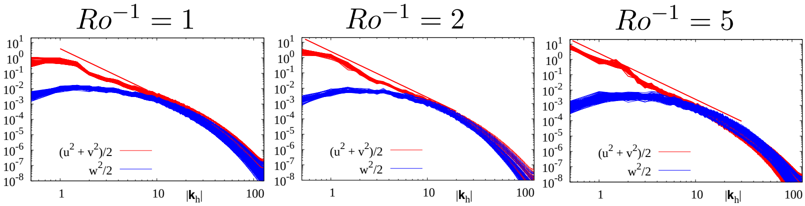
$1/Fr = 1$ $\quad 1/Fr = 3.16$ $\quad 1/Fr = 10$ $\quad 1/Fr = 17.36$

$$\xrightarrow{\hspace{3cm}}$$

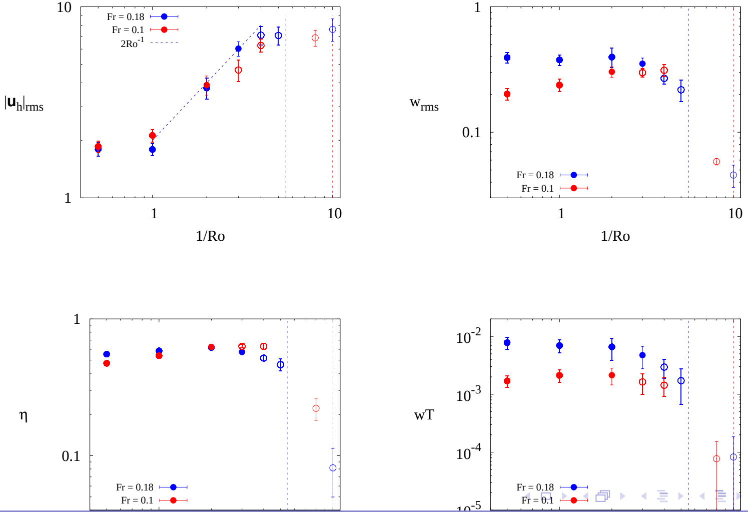
Stratification

Increasing rotation, typical rotating flows

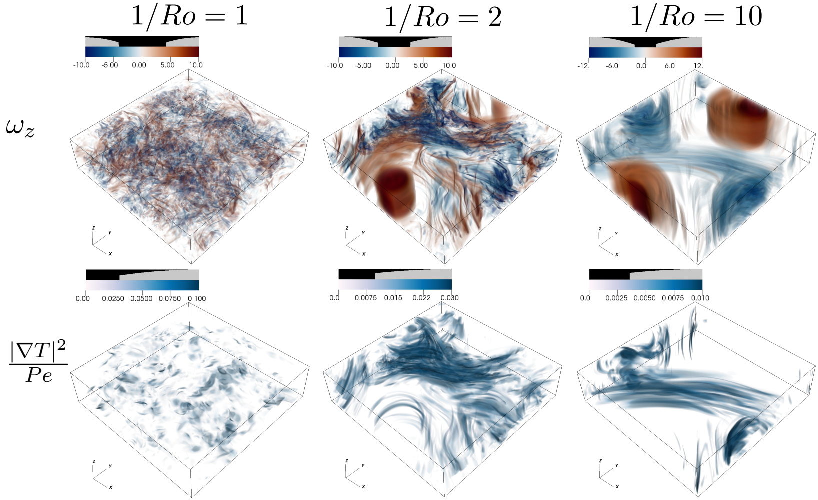
Inverse Energy Cascade



R.M.S. Data


10
1
1
10
1/Ro
$|\mathbf{u}_h|_{rms}$
Fr = 0.18
Fr = 0.1
$2Ro^{-1}$
1
0.1
1
10
1/Ro
w_{rms}
Fr = 0.18
Fr = 0.1
1
0.1
η
Fr = 0.18
Fr = 0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
wT
Fr = 0.18
Fr = 0.1

Mixing and Vertical Vorticity



Conclusion

- Stochastic forcing produces flow with notably different properties compared to steady forcing.
- Method of isolating mean and fluctuation dynamics must be modified.
- Rapid rotation influences the mean flow before it influences the fluctuations. Vertical mixing is only affected when $Ro \to Fr$
- In rapidly rotating flows, mixing is contained in regions of anti-cyclones.
- Very rapid rotation inhibits vertical mixing completely.