The Effects of Rotation on Stratified Turbulence

UCSC Applied Mathematics

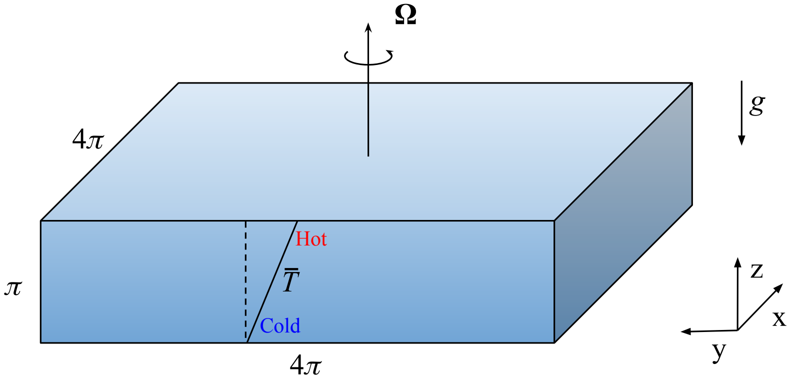
November 25, 2024

Motivation

Must be Geophysically Motivated:
ideas:

- Strat Turb is inpportant for mixing in GAFD
- Dynamics of strat. turb. are inflenced by both Strat. and Rot. in GA flows.
- In the absense of Rotation, Strat. Turb. is dominated by srtongly anisotropic pancake structures with an aspect ratio controlled by the stratification. (show picture of pancake structure).
- By contrast, Rotation barotropic structues wihch are invariant along the axis of rotation. (show pictures of cylinders)
- Using DNS, we will study the competing effects of rotation and stratification on vertical mixing in the flow.

Schematic



Governing Equations

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{Ro}(\boldsymbol{e}_z \times \boldsymbol{u}) &= -\nabla p + \frac{T}{Fr^2}\boldsymbol{e}_z + \boldsymbol{F} + \frac{1}{Re}\nabla^2 \boldsymbol{u} && \text{(mom.)} \\
\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T + w &= \frac{1}{Pe}\nabla^2 T && \text{(temp.)} \\
\nabla \cdot \boldsymbol{u} &= 0 && \text{(cont.)}
\end{aligned}
$$

$$
Re = \frac{UL}{\nu}, \quad Pe = \frac{UL}{\kappa}, \quad Fr = \frac{U}{NL}, \quad Ro = \frac{U}{2\Omega L}
$$

Forcing Mechanism

We choose our forcing to be purely horizontal and divergence-free:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{u} = 0$$

The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \hat{\boldsymbol{F}} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1, where $|\boldsymbol{k}_h| \leq \sqrt{2}$.

Non-rotating Stratified Turbulence

Typical non-rotating flows, properties of stratified turbulence
main idea: show that this forcing produces flows which exhibit stratified turbulence.

$1/Fr = 1$ $\quad$ $1/Fr = 3.16$ $\quad$ $1/Fr = 10$ $\quad$ $1/Fr = 17.36$

$$\xrightarrow{\hspace{3cm}}$$
Stratification

Rotating Stratified Turbulence

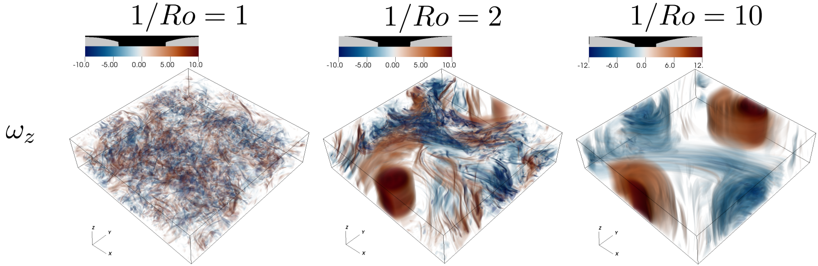
$1/Ro = 1$

Increasing rotation, typical rotating flows

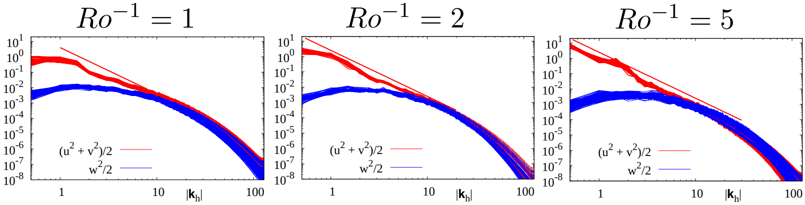
images/XZB1ux.png

$1/Ro = 3.16$ $\quad 1/Fr = 10$ $\quad 1/Fr = 17.36$

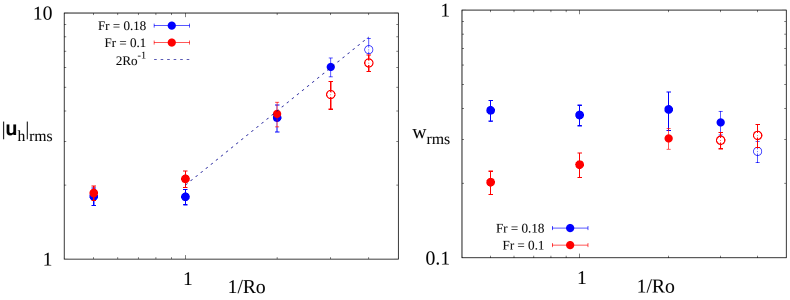
Vertically-invariant Structures in the flow



Inverse Energy Cascade

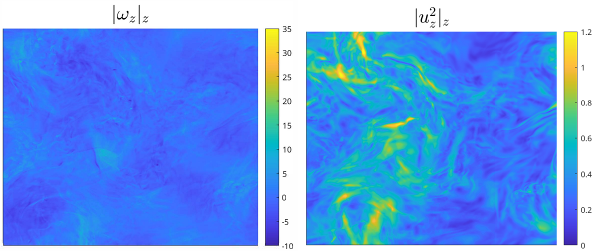


R.M.S. Data

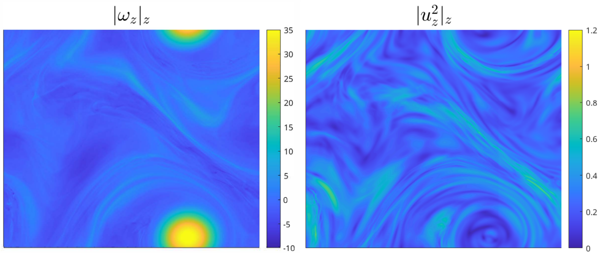


Vertically-Averaged Flow

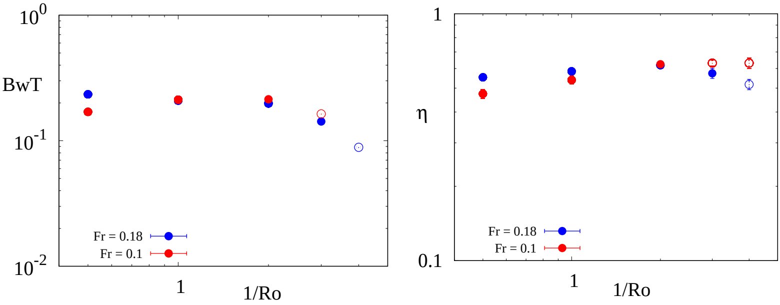
$Ro^{-1} = 1$



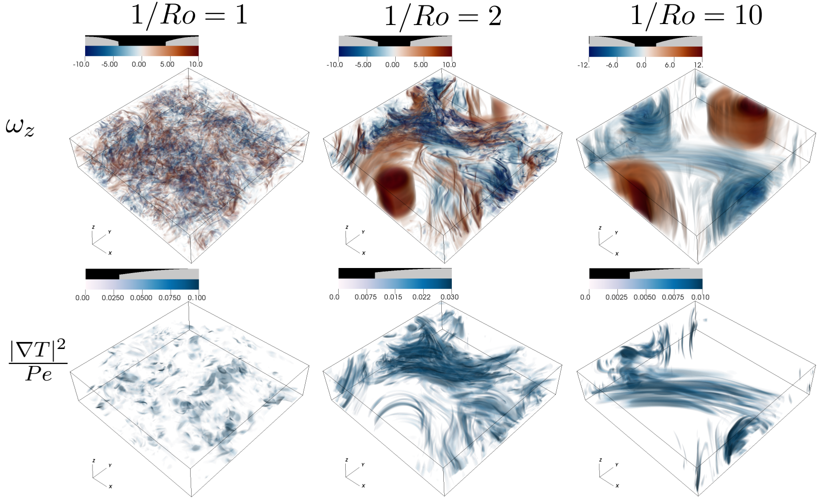
$Ro^{-1} = 3$



Temperature Transport and Mixing in the Flow



Mixing and Vertical Vorticity



Correspondance between Planetary Vorticity and Mixing

Conclusion

- For $Ro > 1$, no significant change from the non-rotating case
- For $1 > Ro > Fr$, horizontal flow becomes increasingly two-dimensional, and vertical mixing is localized in regions of low total vorticity.
- In particular, for low Ro the cyclones are especially stable due to a high total planetary vorticity. Mixing is localized outside of these vortices.
- η is approximately constant for $Ro > Fr$.