The Effects of Rotation on Stratified Turbulence

UCSC Applied Mathematics

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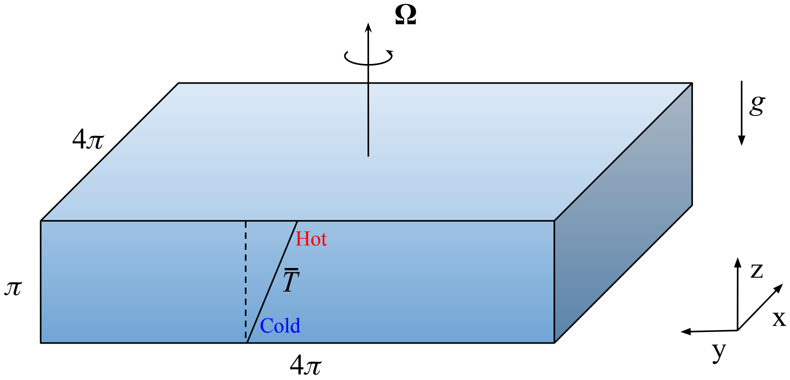
Schematic



Governing Equations

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{Ro}(\boldsymbol{e}_z \times \boldsymbol{u}) = -\nabla p + \frac{T}{Fr^2}\boldsymbol{e}_z + \boldsymbol{F} + \frac{1}{Re}\nabla^2 \boldsymbol{u} \quad \text{(mom.)}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T + w = \frac{1}{Pe}\nabla^2 T \quad \text{(temp.)}$$

$$\nabla \cdot \boldsymbol{u} = 0 \quad \text{(cont.)}$$

Forcing Mechanism

We choose our forcing to be purely horizontal and divergence-free:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{u} = 0$$

The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \hat{\boldsymbol{F}} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$

$$G(\boldsymbol{k}_h, t) \in \mathbb{C}$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1.

From Stratified Turbulence to rotationally domainated flow

This slide might be replaced with a movie (images went below before, maybe both) $1/Fr = 1$ $1/Fr = 3.16$ $1/Fr = 10$ $1/Fr = 17.36$

$$\xrightarrow[\text{Stratification}]{}$$

R.M.S. Data

images/Om0.5Spec.pdf

images/Om1Spec.pdf

Inverse Energy Cascade

$Ro^{-1} = 0.5$	$Ro^{-1} = 1$	$Ro^{-1} = 2$
$Ro^{-1} = 4$	$Ro^{-1} = 5$	$Ro^{-1} = 10$

Mixing Efficiency, η

$$\eta = \frac{\chi}{\chi + \epsilon}$$
$$\chi = \frac{\left\langle |\nabla T|^2 \right\rangle}{Fr^2 Pe}$$
$$\epsilon = \frac{\left\langle |\nabla \boldsymbol{u}|^2 \right\rangle}{Re}$$

Mixing and Vertical Vorticity

(Garaud et al., 2024)

ω_z $|\nabla T|^2/Pe$

Stochastic DNS

Rotating DNS

We studied the effect of rotation by running two series of DNS, each set with a fixed Froude number and varying Rossby number.

Numerical Constraints

- The Courant condition limits the timestep of our DNS ($\Delta t \leq \Delta x / |U|_{\max}$)
- Rapidly rotating DNS require smaller timesteps due to the amplitude of the horizontal velocity.

Columnar Structures

$1/Ro = 0.5$ $1/Ro = 1$ $1/Ro = 2$ $1/Ro = 4$

$\xrightarrow[\text{Rotation}]{}$

Mixing in the Flow

ω_z

$1/Ro = 0.5$ $1/Ro = 2$ $1/Ro = 10$

$\frac{|\nabla T|^2}{Pe}$

Quantities affected by Rotation

Rotation in the Multiscale Theory

We now include the Coriolis term in the multiscale theory by Chini et al. (2022) in order to study the effects of rotation on stratified turbulence.

Weakly rotating regime: $Ro \geq 1$

- Mean flow is weakly influenced by rotation.
- Fluctuations are not rotationally influenced.

$$\frac{\partial \bar{\boldsymbol{u}}_h}{\partial t_s} + \bar{\boldsymbol{u}}_h \cdot \nabla_s \bar{\boldsymbol{u}}_h + \frac{1}{\alpha}\overline{\boldsymbol{u}_h' \cdot \nabla_f \boldsymbol{u}_h'} + \frac{\bar{w}}{\alpha}\frac{\partial \bar{\boldsymbol{u}}_h}{\partial \zeta} + \overline{\frac{w'}{\alpha}\frac{\partial \boldsymbol{u}_h'}{\partial \zeta}} + \frac{1}{Ro}\boldsymbol{e}_z \times \bar{\boldsymbol{u}}$$

$$= -\nabla_s \bar{p} + \bar{\boldsymbol{F}}_h + \frac{1}{Re_b}\frac{\partial^2 \bar{\boldsymbol{u}}_h}{\partial \zeta^2} \qquad \text{(mean)}$$

$$\frac{1}{\alpha}\frac{\partial \boldsymbol{u}_h'}{\partial t_f} + \frac{1}{\alpha}\bar{\boldsymbol{u}_h} \cdot \nabla_f \boldsymbol{u}_h' + \frac{w'}{\alpha}\frac{\partial \bar{\boldsymbol{u}}_h}{\partial \zeta} + \frac{1}{Ro}\boldsymbol{e}_z \times \boldsymbol{u}'$$

$$= -\frac{1}{\alpha}\nabla_f p' + \frac{1}{Re_b}\left(\nabla_f^2 \boldsymbol{u}_h' + \frac{\partial^2 \boldsymbol{u}_h'}{\partial \zeta^2}\right) \qquad \text{(fluctuations)}$$

No change in the scalings obtained by Chini et al. (2022).

Intermediate regime: $\alpha \ll Ro \ll 1$

- Mean flow is strongly influenced by rotation.
- Fluctuations are not influenced by rotation directly.

$$\frac{1}{Ro}\boldsymbol{e}_z \times \bar{\boldsymbol{u}}_h = -\nabla_s \bar{p}, \quad \frac{\partial \bar{p}}{\partial \zeta} = 0 \qquad \text{(mean)}$$

$$\begin{aligned} &\frac{1}{\alpha}\frac{\partial \boldsymbol{u}_h'}{\partial t_f} + \frac{1}{\alpha}\bar{\boldsymbol{u}}_h \cdot \nabla_f \boldsymbol{u}_h' + \frac{w'}{\alpha}\frac{\partial \bar{\boldsymbol{u}}_h}{\partial \zeta} + \frac{1}{Ro}\boldsymbol{e}_z \times \boldsymbol{u}_h' \\ &= -\frac{1}{\alpha}\nabla_f p' + \frac{1}{Re_b}\left(\nabla_f^2 \boldsymbol{u}_h' + \frac{\partial^2 \boldsymbol{u}_h'}{\partial \zeta^2}\right) \end{aligned} \qquad \text{(fluctuations)}$$

- Hydrostatic equilibrium in the fluctuation equations implies $\alpha = Fr$.

Strongly rotating: $Ro \ll \alpha$

- Both mean flow and fluctuations are strongly rotationally influenced.

$$\frac{1}{Ro}\boldsymbol{e}_z \times \bar{\boldsymbol{u}}_h = -\nabla_s \bar{p}, \quad \frac{\partial \bar{p}}{\partial \zeta} = 0 \qquad \text{(mean)}$$

$$\frac{\alpha}{Ro}\boldsymbol{e}_z \times \boldsymbol{u}'_h = -\nabla_f p', \quad \frac{\partial p'}{\partial \zeta} = 0 \qquad \text{(fluctuations)}$$

What is α?

Missing Vertical Length Scale

The appearance of strong columnar vortices in the flow suggests that another vertical length scale is needed in order to create a multiscale theory of rotating stratified turbulence (cf. Sprague et al., 2006; Julien & Knobloch, 2007).

Conclusion

- Stochastic forcing produces flow with notably different properties compared to steady forcing.
- Method of isolating mean and fluctuation dynamics must be modified.
- Rapid rotation influences the mean flow before it influences the fluctuations. Vertical mixing is only affected when $Ro \to Fr$
- In rapidly rotating flows, mixing is contained in regions of anti-cyclones.
- Very rapid rotation inhibits vertical mixing completely.

Future Work

- Additional DNS to extend investigated parameter space $(Fr, Ro, Re, \ldots)$.
- Comprehensive comparison of Steady vs. Stochastic forcing is necessary.
- Develop a multiscale theory of rotating stratified turbulence accounting for multiple vertical length scales.

Chini, Gregory P., Michel, Guillaume, Julien, Keith, Rocha, Cesar B., & Caulfield, Colm-cille P. 2022, Journal of Fluid Mechanics, 933, A22

Garaud, Pascale, P., Chini Gregory, Kaasturi, Shah, & Caulfield, Colm-cille P. 2024, JFM Rapids

Julien, Keith, & Knobloch, Edgar 2007, Journal of Mathematical Physics, 48, 6, 065405

Sprague, Michael, Julien, Keith, Knobloch, Edgar, & Werne, Joseph 2006, Journal of Fluid Mechanics, 551, 141–174