The Effects of Rotation on Stratified Turbulence

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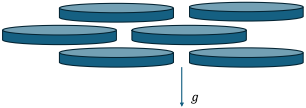
Motivation

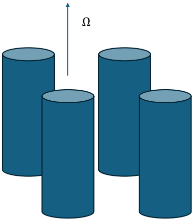
- Stratified turbulence is a crucial phenomenon responsible for mixing and transport in geophysical and astrophysical fluid dynamics.
- In relevant geophysical and astrophysical flows, both stratification and rotation influence dynamics.

Motivation

Nonrotating stratified turbulence is characterized by strongly anisotropic pancake structures within the flow.

Rotation promotes barotropic structures which are invariant along the axis of rotation.

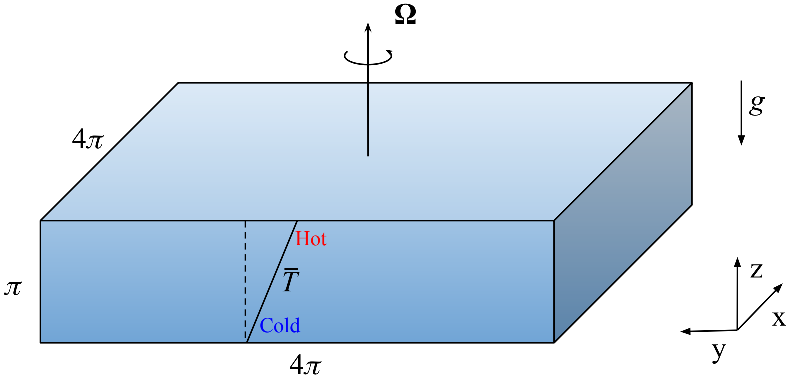




Using DNS, we will study the competing effects of rotation and stratification on vertical mixing in the flow.

Schematic

We condider a tripply periodic domain:



Governing Equations

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{Ro}(\boldsymbol{e}_z \times \boldsymbol{u}) &= -\nabla p + \frac{T}{Fr^2}\boldsymbol{e}_z + \boldsymbol{F} + \frac{1}{Re}\nabla^2 \boldsymbol{u} && \text{(mom.)} \\
\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T + w &= \frac{1}{Pe}\nabla^2 T && \text{(temp.)} \\
\nabla \cdot \boldsymbol{u} &= 0 && \text{(cont.)}
\end{aligned}
$$

$$
Re = \frac{UL}{\nu}, \quad Pe = \frac{UL}{\kappa}, \quad Fr = \frac{U}{NL}, \quad Ro = \frac{U}{2\Omega L}
$$

Forcing Mechanism

We choose the forcing to be purely horizontal and divergence-free stochastic process:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{F} = 0$$

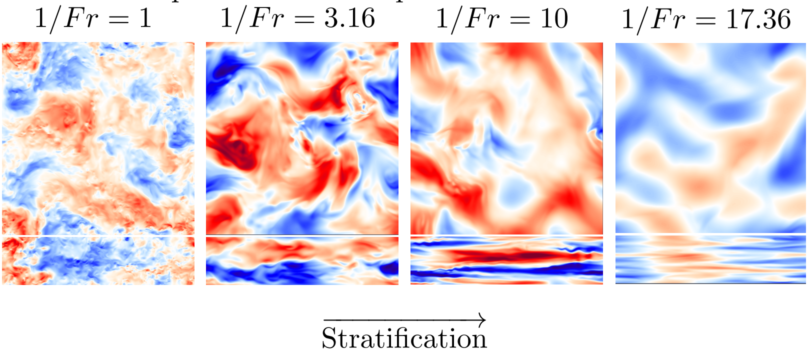
The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \hat{\boldsymbol{F}} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1, and $|\boldsymbol{k}_h| \leq \sqrt{2}$.

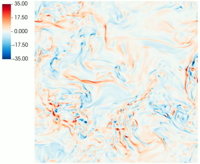
Non-rotating Stratified Turbulence

Numerically integrating these equations in this domain produces anisotropic flow structures.

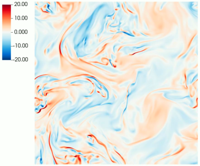


Rotating Stratified Turbulence

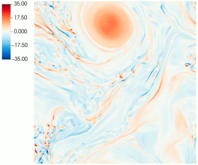
$1/Ro = 0.5$



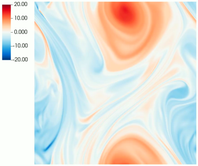
$1/Ro = 1$



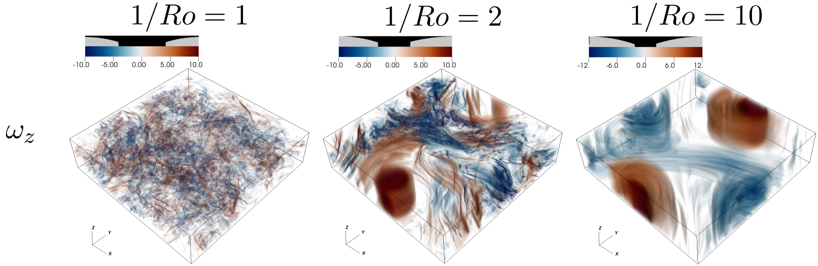
$1/Ro = 2$



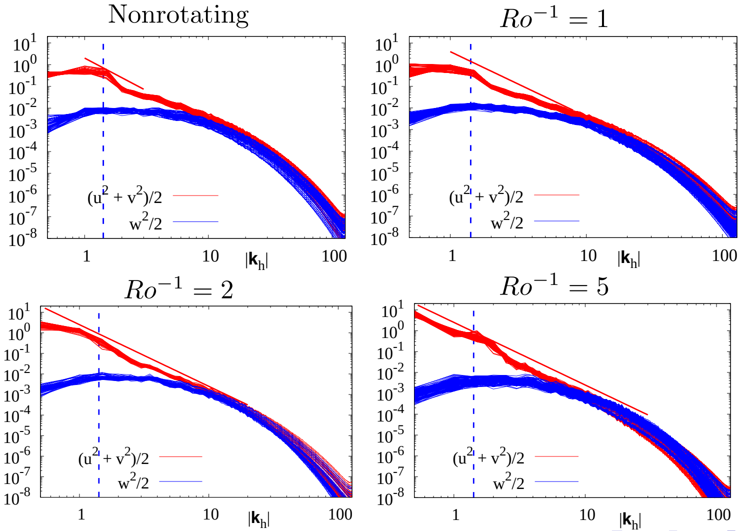
$1/Ro = 5$



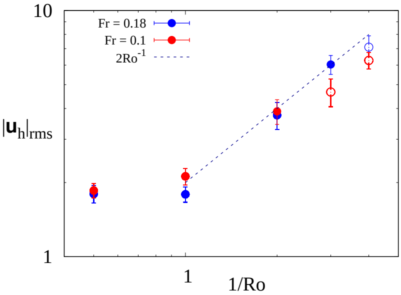
Vertically-invariant Structures in the flow



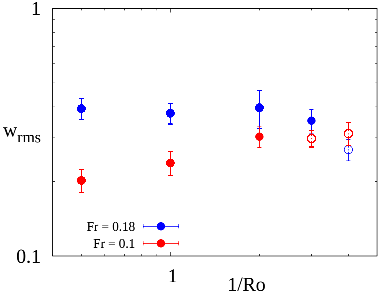
Inverse Energy Cascade



R.M.S. Data

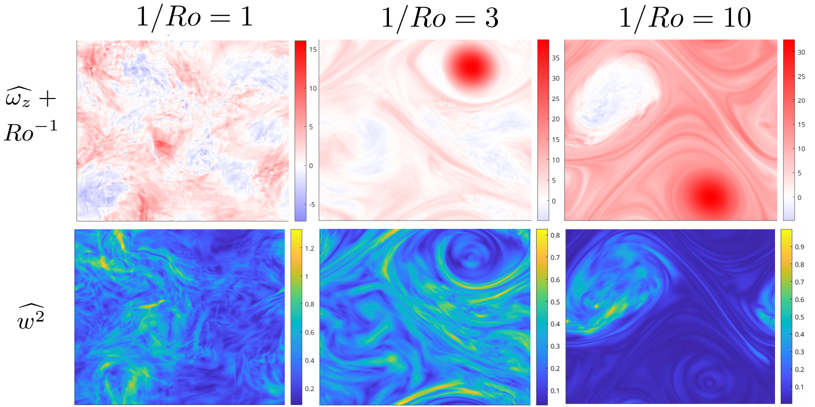


10
1
$|\mathbf{u}_h|_{rms}$
Fr = 0.18
Fr = 0.1
$2Ro^{-1}$
1
1/Ro

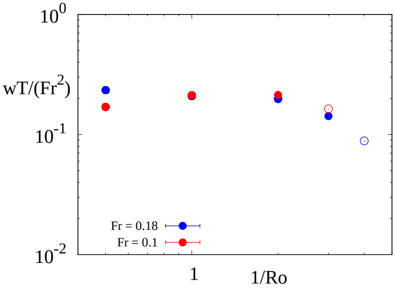


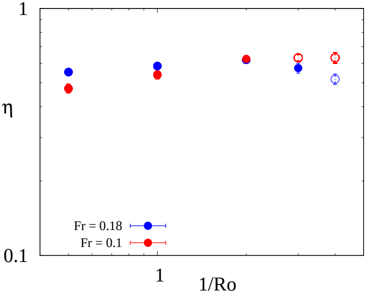
1
0.1
w_{rms}
Fr = 0.18
Fr = 0.1
1
1/Ro

Vertically-Averaged Flow: $\widehat{(\cdot)} \equiv \frac{1}{L_z} \int (\cdot) dz$



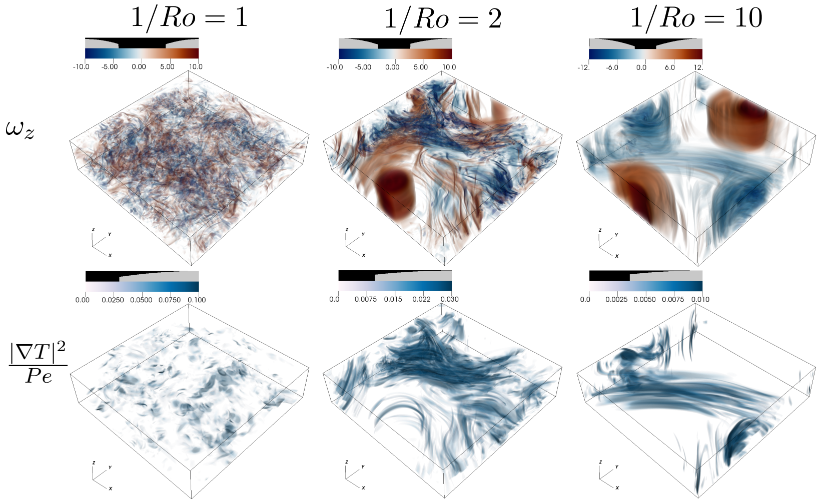
Temperature Transport and Mixing in the Flow



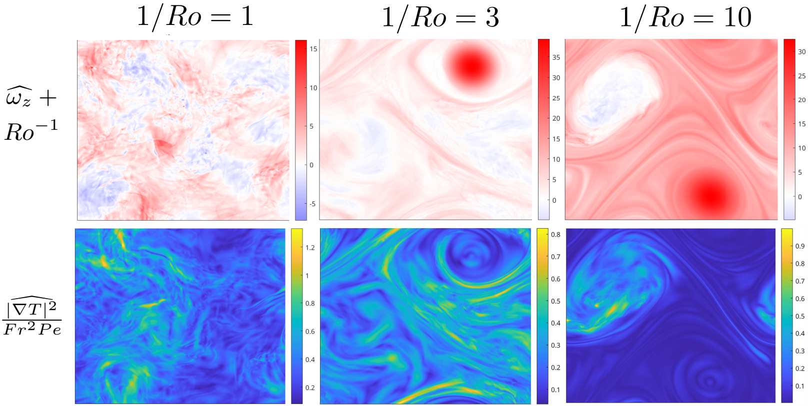


$$\eta = \frac{\frac{|\nabla T|^2}{Fr^2 Pe}}{\frac{|\nabla T|^2}{Fr^2 Pe} + \frac{|\nabla \boldsymbol{u}|^2}{Re}}$$

Mixing and Vertical Vorticity



Correspondance between Planetary Vorticity and Mixing



Conclusion

- For $Ro > 1$, no significant change from the non-rotating case
- For $1 > Ro > Fr$, horizontal flow becomes increasingly two-dimensional, and vertical mixing is localized in regions of low total vorticity.
- In particular, for low Ro the cyclones are especially stable due to a high total planetary vorticity. Mixing is localized outside of these vortices.
- η is approximately constant for $Ro > Fr$.