Midterm 2 - AM 212

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Problem 1

For each of the following 3 ODEs,

a. Plot the numerical solution for $\epsilon = 0.1$, $\epsilon = 0.01$, and $\epsilon = 0.001$

b. Explain in a few words of what method you plan to use the solve this asymptotically and why, based on the numerical solution

c. Find the lowest order uniformly convergent analytical approximation to the solution for small positive ϵ.

d. Compare the numerical and analytical solutions for $\epsilon = 0.01$.

ODE A:

$$\frac{d^2 f}{dt^2} = -f - \epsilon f^2 \left(\frac{df}{dt}\right) \quad \text{with } f(0) = 1, \frac{df}{dt}(0) = 0$$

a. See Figure 1

b. explanation

c. lowest order uniformly converging solution

d. Compare the numerical and analytical solutions for $\epsilon = 0.01$.

ODE B:

$$\frac{d^2 f}{dt^2} = -f - \epsilon f \left(\frac{df}{dt}\right)^4 \quad \text{with } f(0) = 1, \frac{df}{dt}(0) = 0$$

a. See Figure 2

b. explanation

c. lowest order uniformly converging solution

d. Compare the numerical and analytical solutions for $\epsilon = 0.01$.

ODE C:

$$\epsilon \frac{d^2 f}{dt^2} + \frac{df}{dt} + (t+1) f = 0 \quad \text{with } f(0) = 1, f(1) = 2$$

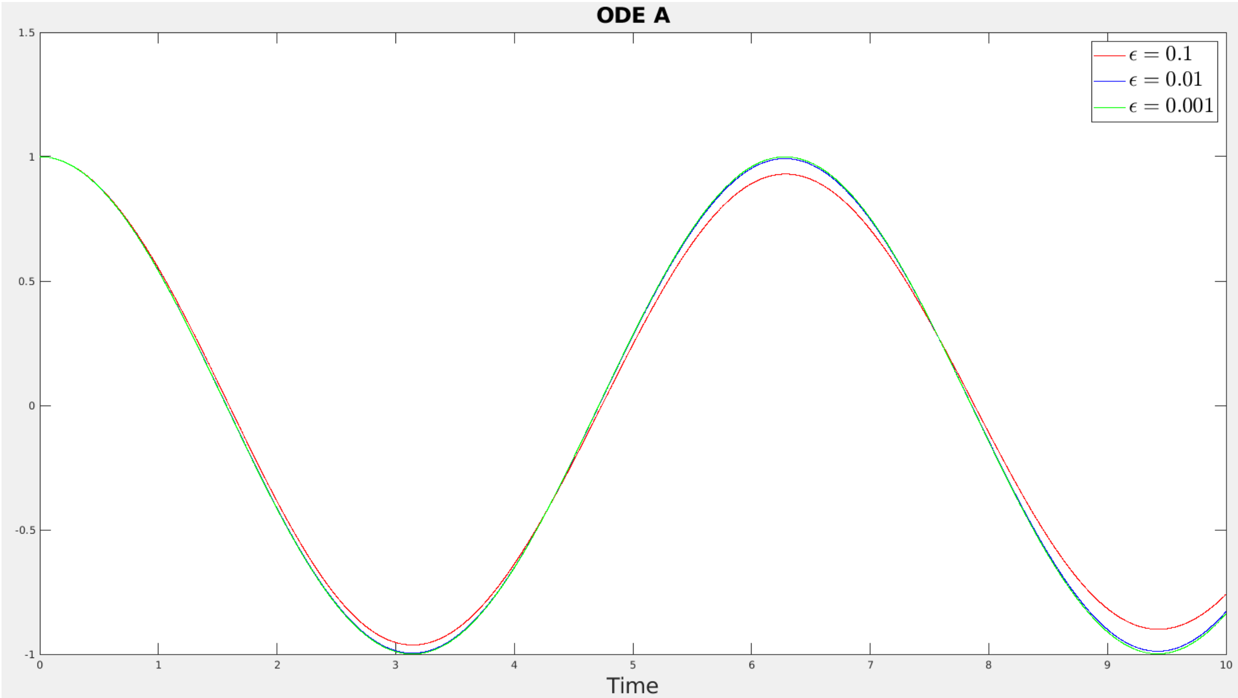

Figure 1: Numerical Solution for differing ϵ using 'ode45'

a. See Figure 3

b. explanation

c. lowest order uniformly converging solution

d. Compare the numerical and analytical solutions for $\epsilon = 0.01$.

Problem 2

Find the eigenvalues and eigenfunctions of this eigenvalue problem, in the limite where the eigenvalue λ is very large and positive.

$$\frac{d^2 f}{dt^2} + \lambda(x+1)^2 f = 0 \quad \text{with } f(1) = 0, f(2) = 0$$

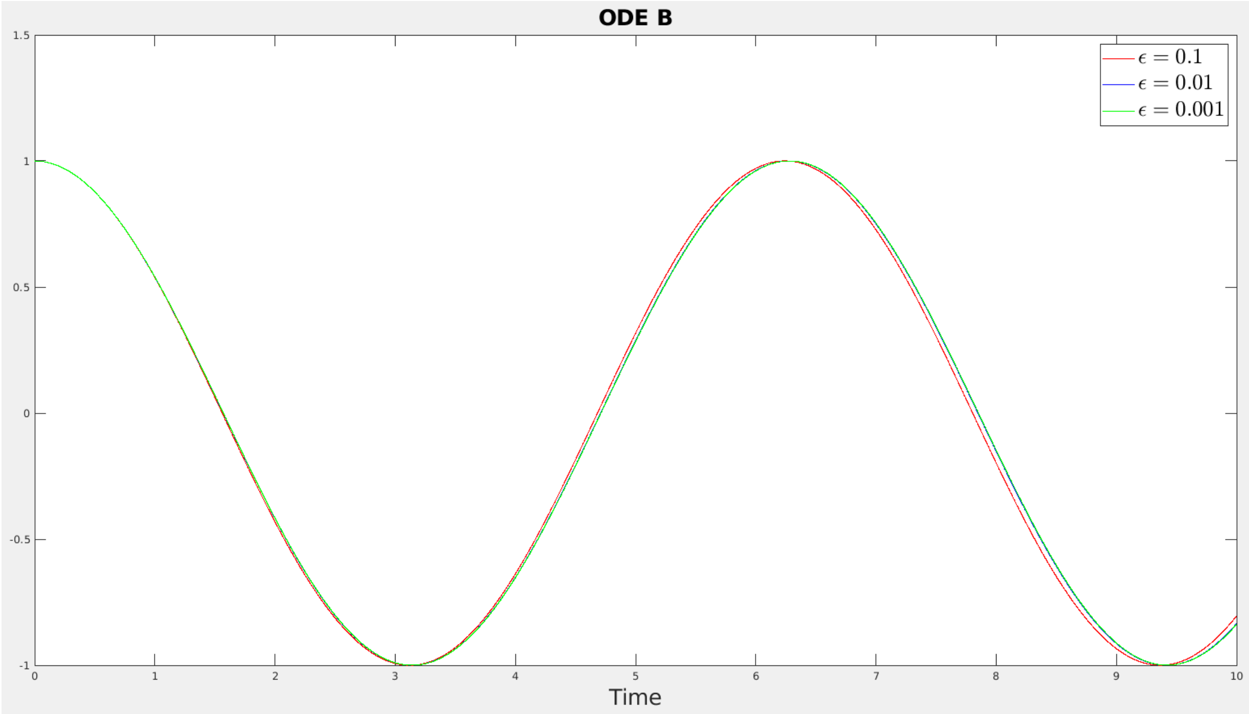

Figure 2: Numerical Solution for differing ϵ using 'ode45'

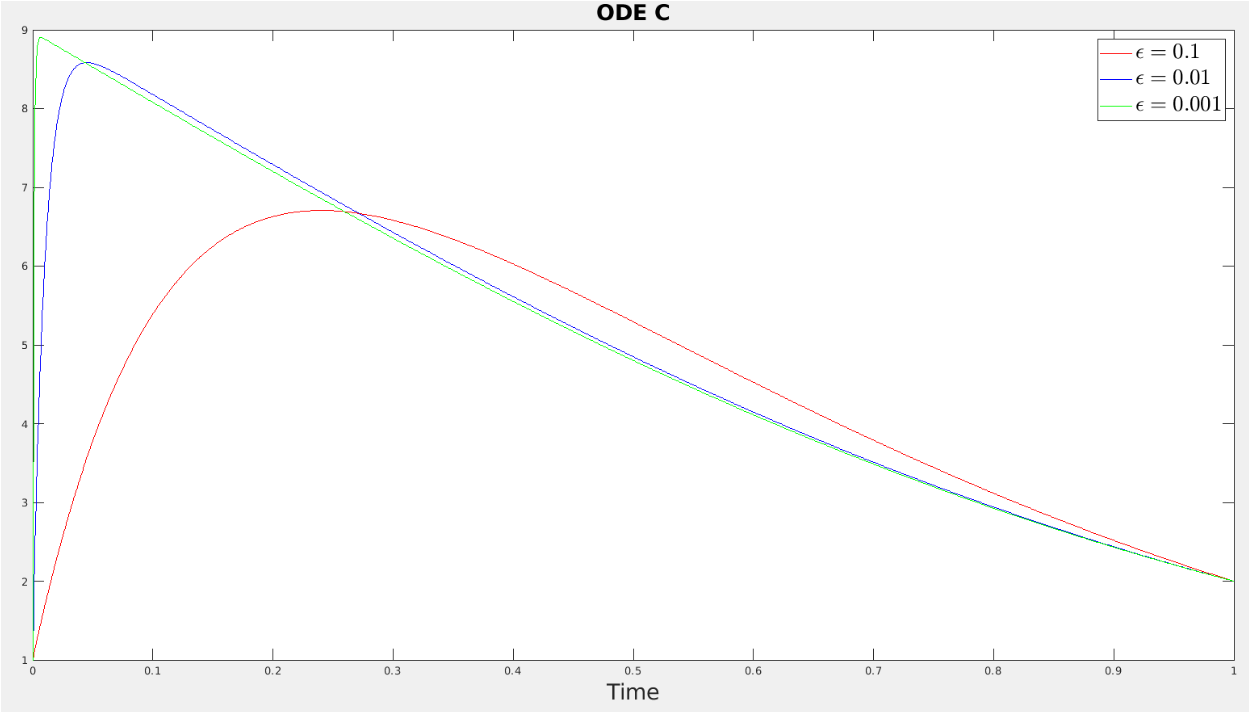

Figure 3: Numerical Solution for differing ϵ obtained using the shooting method and 'ode45'