The Effects of Rotation on Stratified Turbulence

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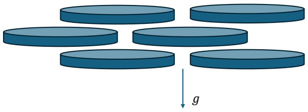
Motivation

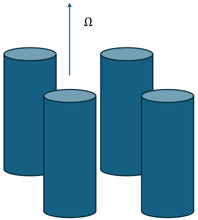
- Stratified turbulence is a crucial phenomenon responsible for mixing and transport in geophysical and astrophysical fluid dynamics.
- In relevant geophysical and astrophysical flows, both stratification and rotation influence dynamics.

Motivation

Nonrotating stratified turbulence is characterized by strongly anisotropic pancake structures within the flow.



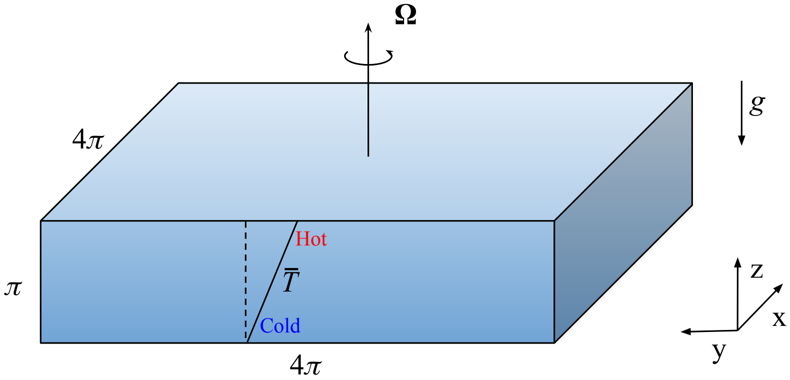
Rotation promotes barotropic structures which are invariant along the axis of rotation.



Using DNS, we will study the competing effects of rotation and stratification on vertical mixing in the flow.

Schematic

We consider a triply periodic domain:



Governing Equations

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{Ro}(\boldsymbol{e}_z \times \boldsymbol{u}) &= -\nabla p + \frac{T}{Fr^2}\boldsymbol{e}_z + \boldsymbol{F} + \frac{1}{Re}\nabla^2 \boldsymbol{u} && \text{(mom.)} \\
\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T + w &= \frac{1}{Pe}\nabla^2 T && \text{(temp.)} \\
\nabla \cdot \boldsymbol{u} &= 0 && \text{(cont.)}
\end{aligned}
$$

$$
Re = \frac{UL}{\nu}, \quad Pe = \frac{UL}{\kappa}, \quad Fr = \frac{U}{NL}, \quad Ro = \frac{U}{2\Omega L}
$$

In what follows, simulations will be conducted with $Re = 600$ and $Pe = 60$ ($Pr = 0.1$).

Forcing Mechanism

We choose the forcing to be purely horizontal and divergence-free stochastic process:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{F} = 0$$

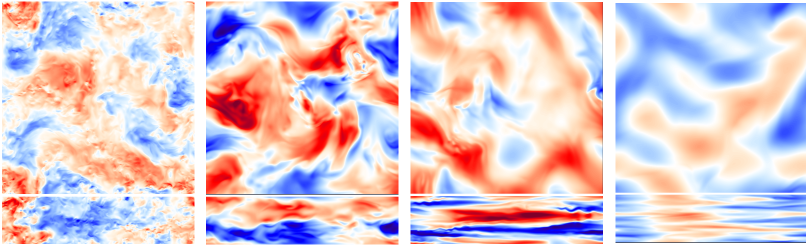
The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \hat{\boldsymbol{F}} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1, and $|\boldsymbol{k}_h| \leq \sqrt{2}$.

Non-Rotating Stratified Turbulence

Decreasing the Froude number creates increasingly anisotropic structures in u.



$1/Fr = 1$ $1/Fr = 3.16$ $1/Fr = 10$ $1/Fr = 17.36$

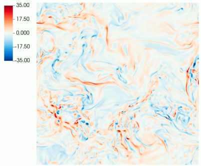
$\xrightarrow{\hspace{3cm}}$

Stratification

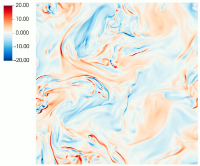
Rotating Stratified Turbulence at fixed $Fr = 0.18$

Evolution of ω_z

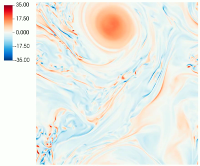
$1/Ro = 0.5$



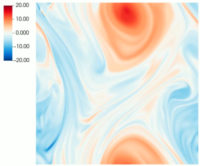
$1/Ro = 1$



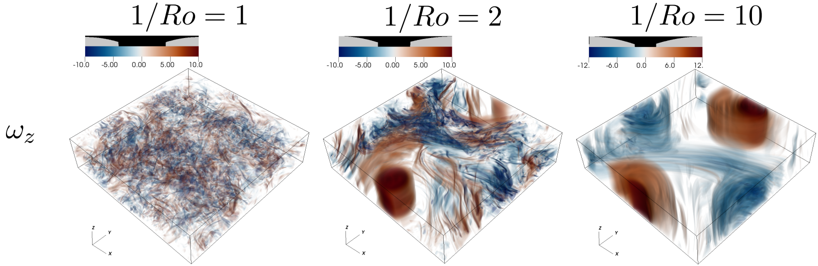
$1/Ro = 2$



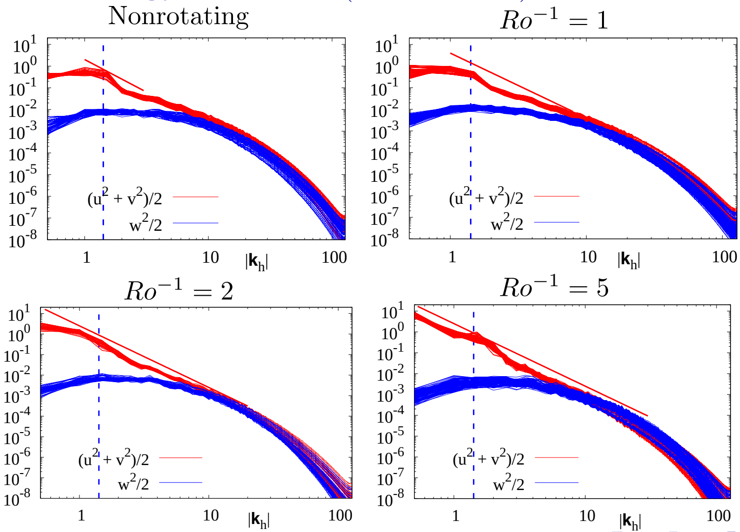
$1/Ro = 5$



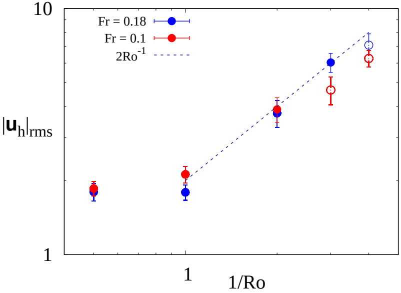
Vertically-Invariant Structures in the flow ($Fr = 0.18$)

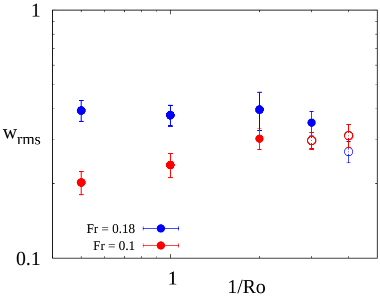


Inverse Energy Cascade ($Fr = 0.18$)

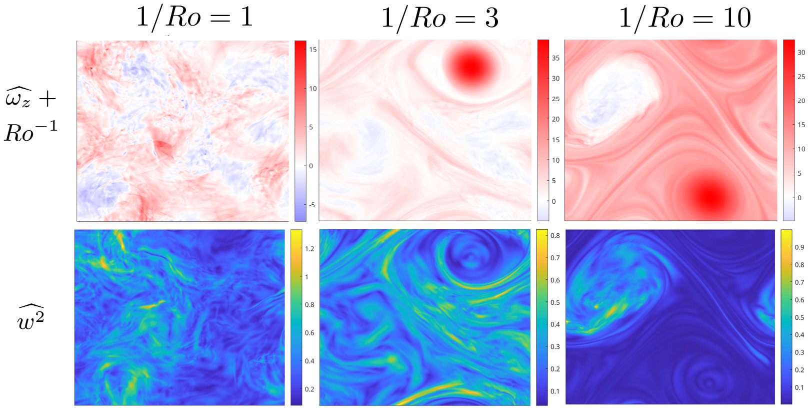


R.M.S. Data

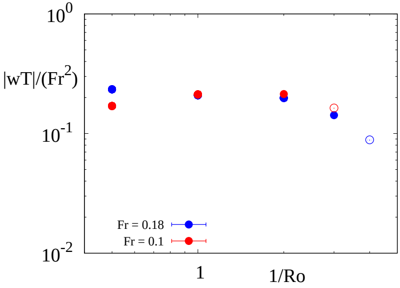


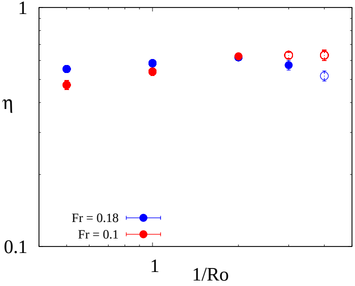


Vertically-Averaged Flow: $\widehat{(\cdot)} \equiv \frac{1}{L_z}\int(\cdot)dz$



Temperature Transport and Mixing in the Flow



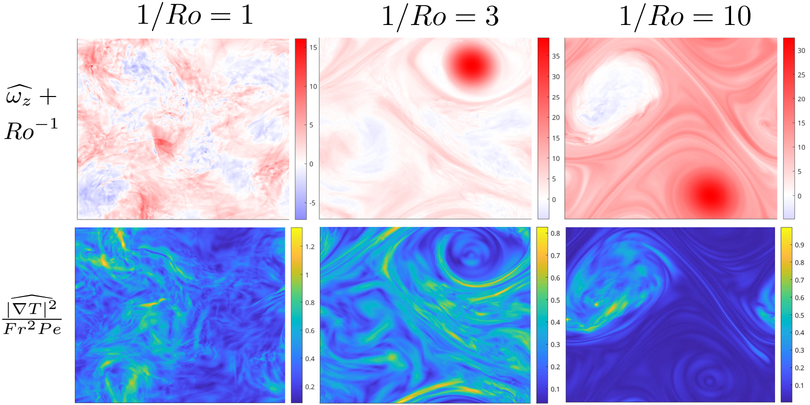


$$\langle \boldsymbol{F} \cdot \boldsymbol{u} \rangle = -\frac{\langle wT \rangle}{Fr^2} + \frac{\langle |\nabla \boldsymbol{u}|^2 \rangle}{Re}$$

$$-\langle wT \rangle = \frac{\langle |\nabla T|^2 \rangle}{Pe}$$

$$\eta = \frac{\frac{\langle |\nabla T|^2 \rangle}{Fr^2 Pe}}{\frac{\langle |\nabla T|^2 \rangle}{Fr^2 Pe} + \frac{\langle |\nabla \boldsymbol{u}|^2 \rangle}{Re}}$$

Correspondance between Total Vorticity and Mixing



Summary

- For $Ro > 1$, no significant change from the non-rotating case
- For $1 > Ro > Fr$, horizontal flow becomes increasingly two-dimensional, and vertical mixing is localized in regions of low total vorticity.
- In particular, for low Ro the cyclones are especially stable due to a high total planetary vorticity. Mixing is localized outside of these vortices.
- η is approximately constant for $Ro > Fr$.