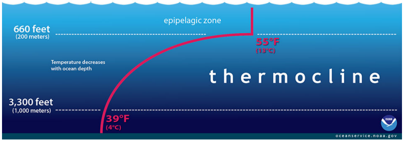
The Effects of Rotation on Stratified Turbulence

UCSC Applied Mathematics

November 25, 2024

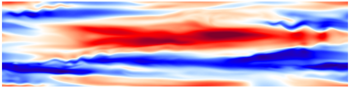
Motivation

- Stratified turbulence is a crucial phenomenon responsible for mixing and transport in geophysical and astrophysical fluid dynamics.
- In relevant geophysical and astrophysical flows, both stratification and rotation influence dynamics.



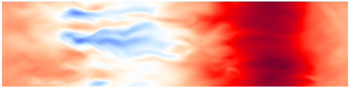
Motivation

Nonrotating stratified turbulence is characterized by strongly anisotropic pancake structures within the flow.



Vertical profile of horizontal velocity

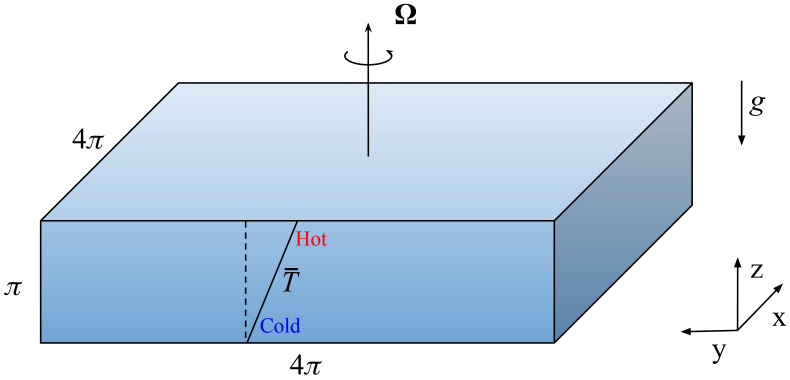
Rotation promotes barotropic structures which are invaraint along the axis of rotation.



Vertical profile of horizontal velocity

Using DNS, we will study the competing effects of rotation and stratification on vertical mixing in the flow.

Schematic



Governing Equations

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \frac{1}{Ro}(\boldsymbol{e}_z \times \boldsymbol{u}) &= -\nabla p + \frac{T}{Fr^2}\boldsymbol{e}_z + \boldsymbol{F} + \frac{1}{Re}\nabla^2\boldsymbol{u} && \text{(mom.)}\\
\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T + w &= \frac{1}{Pe}\nabla^2 T && \text{(temp.)}\\
\nabla\cdot\boldsymbol{u} &= 0 && \text{(cont.)}
\end{aligned}
$$

$$
Re = \frac{UL}{\nu}, \quad Pe = \frac{UL}{\kappa}, \quad Fr = \frac{U}{NL}, \quad Ro = \frac{U}{2\Omega L}
$$

Forcing Mechanism

We choose our forcing to be purely horizontal and divergence-free:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{F} = 0$$

The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \hat{\boldsymbol{F}} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1, and $|\boldsymbol{k}_h| \leq \sqrt{2}$.

Non-rotating Stratified Turbulence

Typical non-rotating flows, properties of stratified turbulence
main idea: show that this forcing produces flows which exhibit stratified turbulence.

$1/Fr = 1 \qquad 1/Fr = 3.16 \qquad 1/Fr = 10 \qquad 1/Fr = 17.36$

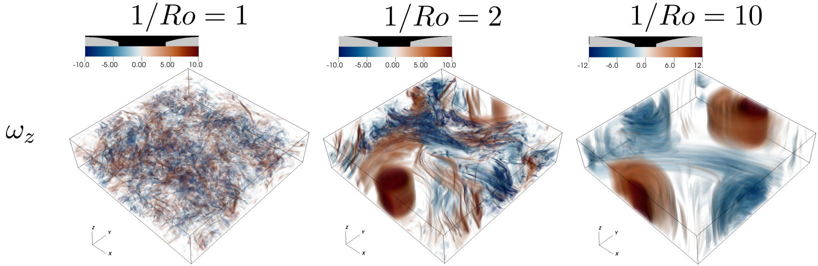
$$\xrightarrow[\text{Stratification}]{\hspace{4cm}}$$

Rotating Stratified Turbulence

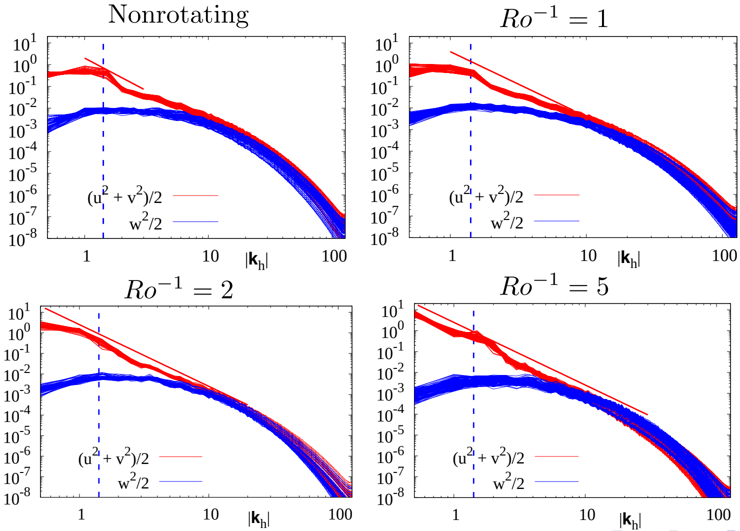
Increasing rotation, typical rotating flows

$1/Ro = 1$ $1/Ro = 3.16$ $1/Fr = 10$ $1/Fr = 17.36$

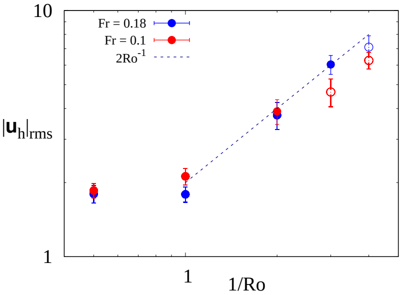
Vertically-invariant Structures in the flow

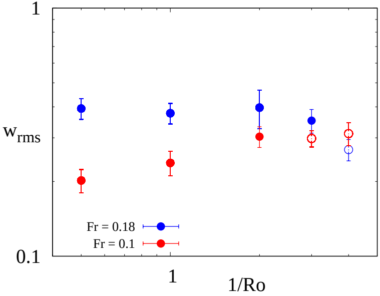


Inverse Energy Cascade

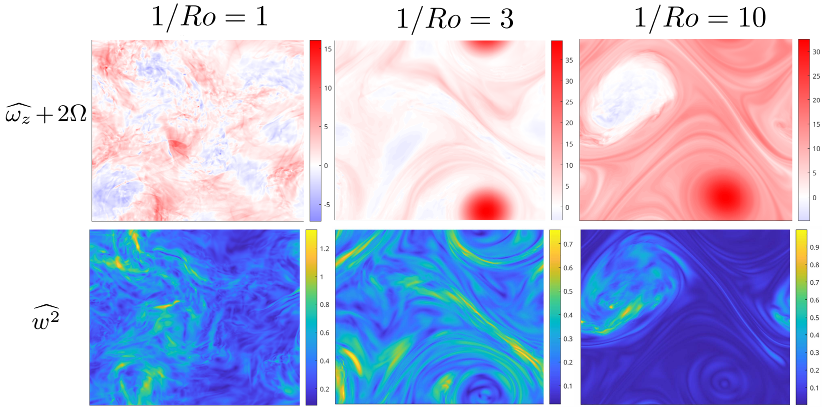


R.M.S. Data

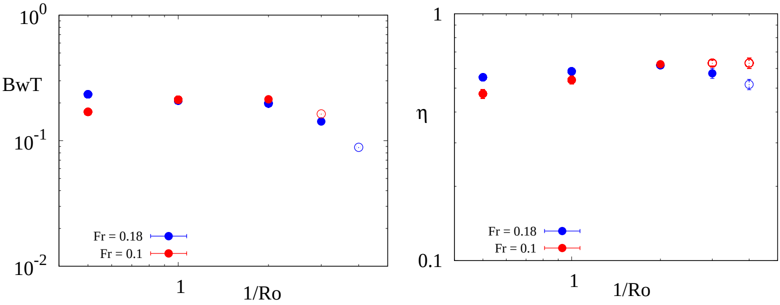




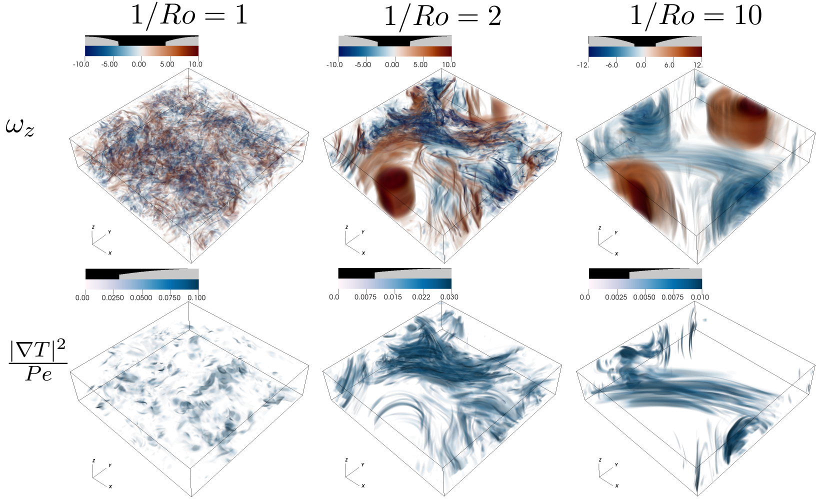
Vertically-Averaged Flow: $\widehat{(\cdot)} \equiv \frac{1}{L_z} \int (\cdot) dz$



Temperature Transport and Mixing in the Flow



Mixing and Vertical Vorticity



Correspondance between Planetary Vorticity and Mixing

Conclusion

- For $Ro > 1$, no significant change from the non-rotating case
- For $1 > Ro > Fr$, horizontal flow becomes increasingly two-dimensional, and vertical mixing is localized in regions of low total vorticity.
- In particular, for low Ro the cyclones are especially stable due to a high total planetary vorticity. Mixing is localized outside of these vortices.
- η is approximately constant for $Ro > Fr$.