The Effects of Rotation on Stratified Turbulence

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November 25, 2024

1 Intro Slide

2 Motivation 1

- We are studying rotating stratified turbulence, as stratified turbulence plays a crucial role in mixing and vertical transport in many geophysical and astrophysical fliud dynamics.
- In many geophysical flows, both stratification and rotation influence dynamics. Some popular examples are the oceans' thermocline, and the Solar Tachocline.

3 Motivation 2

- This problem is particularly interesting because of the competing nature of stratification and rotation.
- Stably stratified flows are typically characterized by strong anisotropy, where pancake structures are formed with an aspect ratio controlled by the Froude number.
- Rotating flows, by contrast, often form tall cylindrical vorticies along the axis of rotation.
- Using DNS, we attempt to study these competing dynamics and their affect on mixing in the flow.

4 Schematic

- Our DNS will use a tripply periodic domain with horizontal lengths of 4π in x and y, and a vertical length of π.
- This domain will have a linear background Temperature profile which is hot at the top and cool at the bottom providing stable stratification.
- The rotation axis will be aligned with the z-axis and gravity which effectively means we are studying the effect of rotation at the poles.

5 Governing Equations

- Our governing equations are the standard incompressible Boussinesque equations which have been nondimensionalized using a characteristic velocity U, a large-eddy horizontal length scale L, a buoyancy frequency N, planetary angular velocity Ω, and viscous and thermal diffusivity coefficients ν and κ respectively.
- We are able to define the Reynolds, Peclet, Froude, and Rossby numbers according to this nondimensionalization. And in the DNS that follow, we have fixed the Reynolds number to 600 and Peclet number to 60, which implies a Prandtl number of 0.1.

6 Forcing Mechanism

- Finally, we employ a stochastic forcing mechanism which is purely horizontal and divergence free.
- This forcing will be employeed in spectral space and will therefore be dependent on the wavenumber. Specifically, we chose only to force horizontal wavenumbers which are in absolute value less than or equal to $\sqrt{2}$, which implies a minimum forcing lengthscale of a little less than half of the domain.
- The stochastic process used is a Gaussian process prescribed to be of amplitude 1 and correlation timescale 1.
- (Optional) similar forcing mechanisms have been used by several studies of stratified turbulence in the past c.f. Waite and Bartello (2004)/(2006)

7 Non-rotating Stratified Turbulence

- Before studing the effect of rotation, we conducted DNS in order to validate this forcing mechanism as we have employeed it.
- The images on this plot show the x-component of the velocity field along the top of the domain and the front of the domain in the top and bottom rows respectively. Note that the strength of the stratification increases from left to right.
- Consistent with prior studies of stratified turbulence, we see that the flow becomes increasingly anisotropic as the Froude number decreases.
- Now that we have confirmed that this forcing mechanism produces stratified turbulence we are ready to study the effect of rotation.

8 Rotating Stratified Turbulence at Fixed $Fr = 0.18$

- Here you see the vertical component of the vorticity field along the top of the domain for simulations with varying rotation rate. Note that the larger the inverse Rossby number, the more rapidly rotating the flow is.
- Notice that in the more weakly rotating simulations (top row), there do not appear to be any stable structures which retain their form. For the more rapidly rotating simulations (bottom row), a stable vortex has appeared corresponding to the areas of strong vertical vortcity shown in red.

9 Vertically-Invariant structures in the flow ($Fr = 0.18$)

- Further investigation of these vortices reveals that they are indeed vertically invariant, that is the vortex penetrates the entire vertical extent of the domain.
- Here you see volume renderings of the vertical vorticity shaded according to a gaussian transfer function visible in the colorbar on each plot.
- Notice that in the weakly rotating simulation (left) there does not appear to be any vortex in the flow, in the moderately rotating simulation (middle), the vortex appears to be vertically invariant, but the regions outside of the vortex are seemingly still turbulent and have non-uniform vertical structures. Finally, in the rapidly rotating simulation (right), the region outside of the main cyclonic vortex seems to have formed into an anti-cyclone which exhibits weak vertical variance.
- What this seems to indicate is that as the rotation rate increases, more energy is being put into large-scale horizontal modes of the velocity.

10 Inverse energy Cascade ($Fr = 0.18$)

- This suspicion is confirmed by the energy spectra for the horizontal wavenumbers of the flow.
- Each plot has the absolute horizontal wavenumber along the x-axis and the energy along the y-axis. Furthemore, the horizontal energy is shown in red, and the vertical energy is shown in blue. Each plot has a red line which corresponds to a $|\boldsymbol{k}_h|^{-5/3}$ energy spectrum, and a blue dashed line which corresponds to the smallest horizontal wavenumber which is forced in each simulation.
- Inspecting these spectra, we deduce that the simulation with inverse Rossby of 1, has an energy spectrum very similar to the non-rotating case and this confirms that the effect of rotation on this simulation is minimal.
- For the two spectra presented from higher inverse Rossby number, we see an increase of energy in the smallest horizontal wavenumbers, indicative of an inverse energy cascade. Furthermore, we see that for the simulation with the highest inverse rossby number shown the horizontal energy spectrum seems to follow the $|\boldsymbol{k}_h|^{-5/3}$ very nicely (with the exception of the forcing also providing some additional energy).

11 R.M.S. Data

- transition sentence
- These plots depict time-averaged rms quantities of the total horizontal velocity (left) and the vertical velocity (right) from different simulations as they vary with inverse Rossby number. The series of blue points correspond to the simulations with $Fr = 0.18$ and the points in red correspond to the simulations with $Fr = 0.1$. Finally, the full circles represent data taken from simulations which are statistically stationary, and the hollow circles represent data taken from simulations which had not yet reached a statistically stationary state.
- As expected due to the inverse energy cascade, the horizontal rms velocity seems to increase proportionally to the inverse rossby number and doesn't seem to depend on the Froude number.
- The vertical velocity which for small inverse Rossby number, is dependent on the Froude number, seems to become less dependent on the Froude number as the inverse Rossby number inreases. The data suggests that for moderate rotation rates the vertical rms velocity is roughly constant and may decrease for strong rotation rate.
- This is a bit odd at first, since as the rotation rate increases we expect there to be less vertical energy in the flow.

12 Vertically-Averaged Flow

- In order to investigate this further, we used a vertical average to understand how the vortices affect the flow.
- Here we show the vertically averaged planetary vorticity (top row) and vertical velocity squared (bottom row) from simulations of varying rotation rates.
- For the weakly rotating simulation (left) there doesn't appear to be any correlation between the planetary vorticity and squared vertical velocity. For the moderately rotating simulation (middle), in the vortex, where the planetary vorticity is strongest, there seems to be a void in the squared vertical velocity. And in the stronger rotating simulation (right), vertical motion seems to be entirely restricted to the anti-cyclone which coresponds to the region of near zero planetary vorticity.
- This gives us a much better understanding of where vertical motions can take place in the flow.

13 Temperature Transport and Mixing in the Flow

- Next we see if the Temperature transport and mixing are affected by the vortex in the same way.
- Similar to the plot of the vertical rms velocity shown earlier, both the Temperature Flux (left), and mixing efficiency (right) seem to remain constant for weak and moderate rotation rates. This is likely due to the fact that the volume fraction of the domain which inhibit vertical motions is still relatively small.
- I should note that the mixing efficiency for these simulations, is rather high, and thats simply because we are in the Low Prandtl Number limit (i.e. the flow is very thermally diffusive)

14 Correspondance between Planetary Vorticity and Mixing

- To confirm the suspicion that mixing is inhibitted by the vortex core and vertically-invariant structures within the flow, we compare the vertically averaged planetary vorticity to the thermal dissipation within the flow.
- Similar to the plots of the squared vertical velocity, we see that thermal dissipation is inhibitted within the vortex core for the moderately rotating simulation (middle) and is strictly limited to the anti-cyclone for the strongly rotating simulation (right).

15 Summary

- To conclude, what we have learned from this work is the following:
- For $Ro \leq 1$, there is no significant change from the non-rotating DNS.
- For $1 > Ro > Fr$, the flow becomes increasingly two dimensional and vertical mixing is localized to regions of low planetary vorticity.
- For particularly low Rossby numbers, the cyclones are especially stable, and mixing is exclusively restricted to the anti-cyclones within the flow.
- For $Ro > Fr$, the mixing efficiency is approximately constant.