The Effects of Rotation on Stratified Turbulence

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1 Intro Slide

- Mention advisor/collaborator

2 Motivation 1

- This talk is motivated by the phenomenon of stratified turbulence, which plays a crucial role in mixing and vertical transport in many geophysical and astrophysical flows.
- In many such flows, both stratification and rotation influence dynamics. Some popular examples are the oceans' thermocline, and the Solar Tachocline.

3 Motivation 2

- This problem is particularly interesting because of the competing nature of stratification and rotation.
- Stably stratified flows are typically characterized by strong anisotropy, where pancake structures are formed with an aspect ratio controlled by the Froude number.
- Rotating flows, by contrast, often form tall cylindrical vorticies along the axis of rotation.
- Using DNS, we attempt to study these competing dynamics and their affect on mixing in the flow.

4 Schematic

- Our DNS will use a tripply periodic domain with horizontal lengths of 4π in x and y, and a vertical length of π.
- This domain will have a linear background Temperature profile which is hot at the top and cool at the bottom providing stable stratification.
- The rotation axis will be aligned with the z-axis and gravity which effectively means we are studying the effect of rotation at the poles.

5 Governing Equations

- Our governing equations are the standard incompressible Boussinesque equations which have been nondimensionalized using a characteristic velocity U, a large-eddy horizontal length scale L, a buoyancy frequency N, planetary angular velocity Ω, and viscous and thermal diffusivity coefficients ν and κ respectively.
- We are able to define the Reynolds, Peclet, Froude, and Rossby numbers according to this nondimensionalization. And in the DNS that follow, we have fixed the Reynolds number to 600 and Peclet number to 60, which implies a Prandtl number of 0.1.

6 Forcing Mechanism

- Finally, we employ a stochastic forcing mechanism which is purely horizontal and divergence free.
- This forcing will be employeed in spectral space and will therefore be dependent on the wavenumber. Specifically, we chose only to force horizontal wavenumbers which are in absolute value less than or equal to $\sqrt{2}$, which implies a minimum forcing lengthscale of a little less than half of the domain.
- The stochastic process used is a Gaussian process prescribed to be of amplitude 1 and correlation timescale 1.
- Similar forcing mechanisms have been used by prior studies of stratified turbulence in the past c.f. Waite and Bartello (2004)/(2006)

7 Non-rotating Stratified Turbulence

- Before studing the effect of rotation, we conducted DNS in order to validate this forcing mechanism as we have employeed it.
- The images on this plot show a horizontal component of the velocity field along the top of the domain (top row) and the front of the domain (bottom row). Note that the strength of the stratification increases from left to right.
- Consistent with prior studies of stratified turbulence, we see that the aspect ratio of the flow decreases with the Froude number.
- Now that we have confirmed that this forcing mechanism produces stratified turbulence we are ready to study the effect of rotation.

8 Rotating Stratified Turbulence at Fixed $Fr = 0.18$

- Here you see the vertical component of the vorticity field along the top of the domain for simulations with varying rotation rate. Note that the larger the inverse Rossby number, the more rapidly rotating the flow is.
- Notice that in the more weakly rotating simulations, there do not appear to be any stable structures. In the more rapidly rotating simulations, a stable vortex with a large horizontal lengthscale has formed and seems to become domain filling.

9 Vertically-Invariant structures in the flow ($Fr = 0.18$)

- Volume renderings of the vertical vorticity reveal that these are vertically invariant cyclones which take up the entire vertical extent of the domain.
- Notice that in the slowly rotating simulation (left) there does not appear to be any cyclone in the flow, in the moderately rotating simulation (middle), the cyclone appears to be vertically invariant, but the regions outside are still turbulent and have non-uniform vertical structure. Finally, in the rapidly rotating simulation (right), the region outside of the main cyclonic vortex seems to have formed into an anti-cyclone which exhibits decreased vertical variance.
- This reveals that more energy is being put into large-scale horizontal modes as the rotation rate increases.

10 Inverse energy Cascade ($Fr = 0.18$)

- This suspicion is confirmed by plots of the energy spectra of the flow.
- These are plots of the horizontal and vertical energies against the total horizontal wavenumber showon in red and blue respectively. In each plot there is a red $|\boldsymbol{k}_h|^{-3}$ spectrum overplotted and a dashed blue line signifying the smallest forced wavenu,ber.
- There are two essential takeaways from these plots, first that the slowly rotating flow seems to be negligably affected, and that the rapidly rotating flows experience an inverse energy cascade, gathering energy at the smallest horizontal wavenumbers.

11 R.M.S. Data

- Looking at some of the more quantitative data, we also compare the total horizontal and vertical rms velocities for different Froude and Rossby numbers. The solid circles on these plots represent time-averaged data from a statistically-stationary state, while the open circles are taken from non-stationary states. Higher inverse Rossby number data is not included on this plot as we do not believe it is close to a stationary state yet.
- The total horizontal velocity seems to increase proportionally to the inverse rossby number starting at $Ro = 1$ for both Froude numbers.
- The vertical velocity remains roughly constant, $\rightarrow$ interesting because its dependence on the Froude number becomes less dominant as the inverse Rossby number increases, and the appearance of cyclones suggest there should be less vertical movement in the flow.

12 Vertically-Averaged Flow

- To investigate this further, we used a vertical average to understand how the cyclones affect mixing and vertical transport in the flow.
- Here we show the vertically averaged total vorticity (top row) and squared vertical velocity (bottom row) from simulations of varying rotation rates.
- The weakly rotating simulation (left) doesn't appear to have any interesting correlation between vorticity and vert velocity which is what we expect as there is no stable cyclone in the flow. The moderately rotating simulation (middle), demonstrates a void in the vertical velocity precisely where the cyclone is concentrated. Finally, the rapidly rotating simulation appears to have little to no vertical motions except where the total vorticity is near zero. This corresponds to an anti-cyclone which forms in the rapidly rotating simulations.

13 Thermal Dissipation and Mixing in the Flow

- The next item to investigate if thermal dissipation and mixing are affected in the same way. Similar to vertical velocity, the rms Temperature Flux (left), and mixing efficiency (right) seem to remain constant for weak and moderate rotation rates.
- I should note that the mixing efficiency for these simulations, is rather high, and thats simply because we are in the Low Prandtl Number limit (i.e. the flow is very thermally diffusive)

14 Correspondance between Total Vorticity and Mixing

- Similar to the plots of the squared vertical velocity, we see that thermal dissipation is inhibitted within the vortex core for the moderately rotating simulation (middle) and is strictly limited to the anti-cyclone for the strongly rotating simulation (right).

15 Summary

- To conclude, what we have learned from this work is the following:
- For $Ro \leq 1$, there is no significant change from the non-rotating DNS.
- For $1 > Ro > Fr$, the flow becomes increasingly two dimensional and vertical mixing is localized to regions of low planetary vorticity.
- For particularly low Rossby numbers, the cyclones are especially stable, and mixing is exclusively restricted to the anti-cyclones within the flow.
- For $Ro > Fr$, the mixing efficiency is approximately constant.