AM 216 - Stochastic Differential Equations: Assignment

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Problem 1: Time inversion property of the Wiener process

Problem 2: Integrating the Wiener process

Problem 3: Reflection principle of the Wiener process

i) Show that $\Pr(M_T \geq a) = 2\Pr(W(T) \geq a)$.

Proof. The core point in this proof is showing that the conditional probability for $\Pr(W(T) \geq a | W(\tau) = a, \tau \in [0, T]) = 0.5$. This follows because the rest of the series $t \in [\tau, T]$ is distributed symmatrically about $W(\tau) = a$. Notice that if $W(\tau) = a$ and we have $M_T \geq a$. Thus we have by bayes theorem,

$$\Pr(W(T) \geq a | M(T) \geq a) = 0.5 = \frac{\Pr(M(T) \geq a | W(T) \geq a) \Pr(W(T) \geq a)}{\Pr(M_T \geq a)}$$
$$\Pr(M_T \geq a) = 2\Pr(W(T) \geq a)$$

□

ii) Find the PDF of M_T.

Proof.

$$stuff = notstuff$$

□

Problem 4: Lambda-chain rule for stochastic integrals

Problem 5: SDE for American Stocks

Problem 6: Another SDE for American Stocks