AM 216 - Stochastic Differential Equations: Assignment

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Problem 1: Problem 1

Roll a set of two fair 6-sided dice, one colored red the other white. Assume that the two dice roll independently. Record the two numbers facing up as X_1 and X_2.

i) Mathematically describe the format of outcome. Describe the sample space.

Proof. The two quantities X_1 and X_2 are samples from a given discrete probability distribution. The sample space has six equally likely outcomes $1, 2, 3, 4, 5, 6$ with an expected value of $E(X) = 3.5$. The two samples X_1 and X_2 are independent and identically distributed samples. □

ii) Let X be the absolute difference between X_1 and X_2. Is X a random variable?

Proof. Yes, the sum/difference of two random variables is also a random variable, albeit with a different sample space and expected value. We can exam the sample space to find that there are 6 possibilities of different likelihood: $S = 0, 1, 2, 3, 4, 5$, with 5 being the least likely (only two outcomes where the result is 5) and 1 being the most likely (there are ten outcomes with a result of 1). □

iii) Find the PMF of X, $E(X)$

Proof. Specifically,

k	P(X = k)
0	0.166...
1	0.277...
2	0.222...
3	0.166...
4	0.111...
5	0.055...

and, therefore,

$$E(X) = 1(0.277...) + 2(0.222...) + 3(0.166...) + 4(0.111...) + 5(0.055...) = 1.944...$$

□