DNS of Stratified Turbuluence with Rotation and Stochastic Forcing

Dante Buhl

October 2023

1 Previous Work

2 Current Work

3 Gaussian Processes

My current job is to design a stochastic forcing structure using the Gaussian random process. Gaussian Processes are a way of generating a regression from current data, fitting a line almost if you will. We are using gaussian processes to use the current data to inform a new point going forward in the code.

The concept of the Gaussian Process is not a novel idea. Its purpose is to generate new points which fit onto an informed window of uncertainty around a given set of initial data. Ultimately, the process samples a gaussian distribution whose mean and covariance matrices are created through the use of precise linear algebra and a kernel chosen to optimize on the desired properties of the gaussian regression.

The purpose of the Gaussian Process in the context of this work is to create a statistically stationary stochastic forcing in which to perturb and drive eddies in a stable manner as done in (Waite 2004) **SOURCE**. In our Spectral Code, the Gaussian Forcing was enforced on low horizontal wavenumbers as to affect the mean background flow, without directly interacting with the turbulence strcutures.

$$\vec{G}(k,t) = < G_x(k,t), G_y(k,t) >$$
$$\vec{G}(k,t) \cdot \vec{k_h} = 0$$

3.1 Dealing with Finite Precision

The procedure in which a Gaussian Process is generated is usually not a very complex Linear Algebra structure. The formulation is as below.

$$f_* \sim \mathcal{N}(\mu_*, \Sigma_*)$$
$$\vec{\mu_*} = K_*^T \times K^{-1} \times f$$
$$\Sigma_* = K_{**} - K_*^T \times K^{-1} \times K_*$$

Where,

$$K = \mathcal{K}(\vec{x}, \vec{x})$$
$$K_* = \mathcal{K}(\vec{x}, \vec{x_*})$$
$$K_{**} = \mathcal{K}(\vec{x_*}, \vec{x_*})$$

4 Code Design and Algorithm Structure