Numerical Methods for the Solution of Differential Equations (AMS 213B)
Homework 2 - Due Sunday April 28

Instructions

Please submit to CANVAS one PDF file (your solution to the assignment), and one .zip file that includes any computer code you develop for the assignment. The PDF file must be a document compiled from Latex source code (mandatory for PhD students), or a PDF created using any other other word processor (MS and SciCAM students). No handwritten work should be submitted.

Question	points
1	30
2	20
3	30
4	20

Question 1 (30 points). Consider the three-dimensional linear dynamical system

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y}, \qquad \boldsymbol{y}(0) = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}^T, \tag{1}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 10 & -10 \\ -100 & -1 & 0 \\ 0 & 10 & -100 \end{bmatrix}. \tag{2}$$

a) (10 points) Determine the largest value of Δt, for which the three-step Adams-Bashforth method (AB3)

$$\boldsymbol{u}_{k+3} = \boldsymbol{u}_{k+2} + \frac{\Delta t}{12}\left(23\boldsymbol{f}_{k+2} - 16\boldsymbol{f}_{k+1} + 5\boldsymbol{f}_k\right) \tag{3}$$

is absolutely stable when applied to (1). You are allowed to use a numerical solver to compute the eigenvalues of $\boldsymbol{A}$.

b) (10 points) Plot the region of absolute stability of AB3.

c) (10 points) Plot the numerical solution of (1) (all three components in one figure) you obtain with AB3 in the time interval $[0, 10]$ for $\Delta t = 10^{-4}$, $\Delta t = \Delta t^* + 0.0001$ and $\Delta t = \Delta t^* - 0.0001$ (three different figures), where Δt^* is the critical value of Δt you determined in part a). Comment on your numerical results.

Question 2 (20 points). Consider the BDF3 scheme

$$\boldsymbol{u}_{k+3} - \frac{18}{11}\boldsymbol{u}_{k+2} + \frac{9}{11}\boldsymbol{u}_{k+1} - \frac{2}{11}\boldsymbol{u}_k = \frac{6}{11}\Delta t\boldsymbol{f}_{k+3} \tag{4}$$

a) (10 points) Show that scheme is convergent with order 3.

b) (10 points) Plot the boundary of the absolute stability region for BDF3. Is BDF3 A-stable? Justify your answer.

Question 3 (30 points). Consider the following explicit linear multistep scheme

$$\boldsymbol{u}_{k+2} - 4\boldsymbol{u}_{k+1} + 3\boldsymbol{u}_k = -2\Delta t \boldsymbol{f}(\boldsymbol{u}_k, t_k). \tag{5}$$

a) (10 points) Show that (5) is consistent and compute the order of consistency.

b) (10 points) Show that (5) is not zero-stable. Is the scheme convergent? Justify your answer.

c) (10 points) Show that (5) is unconditionally absolutely unstable.

Question 4 (20 points) Consider the RK method corresponding to the following Butcher array

0	0	0	0
1/2	1/2	0	0
3/4	0	3/4	0
	2/9	1/3	4/9

a) (10 points) Write down the given RK method and show that the method is convergent.

b) (10 points) Plot the region of absolute stability of RK method defined above. Is the method A-stable? Justify your answer.