Lecture 1

Introduction

- Administrative stuff.
  - A brief tour of Canvas.
- Academic stuff.
  - Study habits and learning.
- Antiderivatives and the indefinite integral

Canvas Highlights

1. General information
2. Lecture schedule

} syllabus

3. Quizzes and Exams.

4. Extra Credit

(i) Contributions to weekly canvas discussions

(ii) TA discussion section attendance.

5. Attendance ...

(i) "mandatory" for lectures.

5% of your grade.

(ii) Optional for TA sections,

But: Extra Credit

(iii) Lecture attendance will be recorded using Canvas Quizzes

Interesting Data: AM 11B W24

146 students

(*) 15 available "engagement" pt.s

(*) 75 available exam pt.s

(*) 135 passed

Engage. pt.s	< 5.5	9 ≤
# of students	59	57
Avg. Exam Pt.s	52	62
Passed	48	57
A- to A+	14	34

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Definition:

An antiderivative of $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$

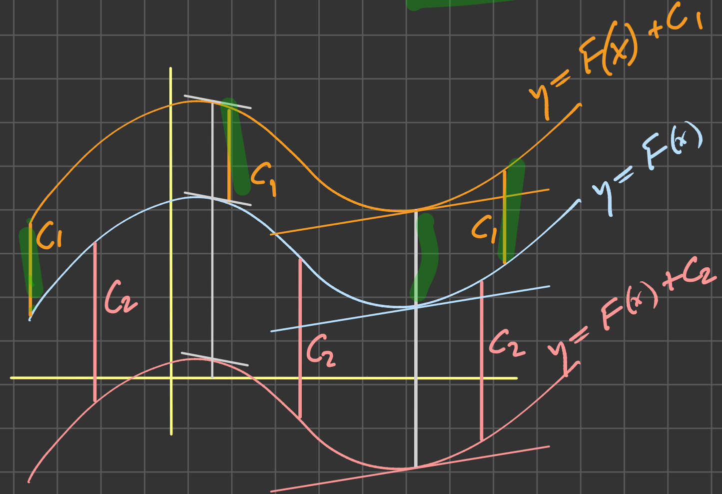
Examples

1. $f(x) = 2x \longleftrightarrow F(x) = x^2 + 1$
2. $g(x) = 3 \longleftrightarrow G(x) = 3x - \sqrt{2}$
3. $h(x) = \frac{1}{x^2} \longleftrightarrow H(x) = -\frac{1}{x} + \pi^2$

2/ Observations:

1. If $G(x) = F(x) + C$, then $G'(x) = F'(x)$

$\Longrightarrow \begin{cases} \text{If } f(x) \text{ has one antiderivative} \\ \text{then } f(x) \text{ has infinitely many} \end{cases}$



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2. If $G'(x) = F'(x)$, then

$$G(x) = F(x) + C,$$

for some constant C

<u>Because</u>

$$\frac{d}{dx}\left(G(x) - F(x)\right)$$

$$= G'(x) - F'(x) = 0$$

mean value theorem

$$G(x) - F(x) = C \text{ (constant)}$$

4)

Definition :

The indefinite integral of $f(x)$ is the set of all antiderivatives of $f(x)$:

$$\{F(x) : F'(x) = f(x)\} = \int f(x)\,dx$$

notation

5)

"Shorthand"

integral sign

$$\int f(x)\,dx = F(x) + C$$

integrand

variable of integration

A particular antiderivative of $f(x)$: $F'(x) = f(x)$

the "constant of integration" → nonspecific

($f(x)\,dx$ → a differential)

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First principle:

to find antiderivatives, we (try to) reverse the rules of differentiation.

Examples

$$(e^x)' = e^x \longrightarrow \int e^x\,dx = e^x + C$$

$$(x)' = 1 \longrightarrow \int 1\,dx = \int dx = x + C$$

$$(x^2)' = 2x \longrightarrow \int 2x\,dx = x^2 + C$$

7/ Basic Rule #1

$$\frac{d}{dx}\left(F(x) \pm G(x)\right) = F'(x) \pm G'(x)$$

$$= f(x) \pm g(x)$$

$$\Longrightarrow \int f(x) \pm g(x)\, dx = F(x) \pm G(x) + C$$

$$+\left\{\begin{array}{l} \int f(x)\, dx = F(x) + C_1 \\ \int g(x)\, dx = G(x) + C_2 \end{array}\right\} + \to \begin{array}{l} C_1 \pm C_2 = C \\ (F(x) + C_1) \\ \pm (G(x) + C_2) \end{array}$$

$$= \int f(x)\, dx \pm \int g(x)\, dx$$

7/ Basic Rule #1

$$\frac{d}{dx}\left(F(x) \pm G(x)\right) = F'(x) \pm G'(x)$$

$$= f(x) \pm g(x)$$

$$\Longrightarrow \int f(x) \pm g(x)\, dx = F(x) \pm G(x) + C$$

$$\begin{cases} \int f(x)\, dx = F(x) + C_1 \\ \int g(x)\, dx = G(x) + C_2 \end{cases}$$

$$= \int f(x)\, dx \pm \int g(x)\, dx$$

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Basic Rule #2:

constant

$$\frac{d}{dx}\left(a \cdot F(x)\right) = a \cdot F'(x) = a \cdot f(x)$$

$$\Longrightarrow \int a \cdot f(x)\, dx = aF(x) + C$$

$$= a\left(F(x) + \frac{C}{a}\right)$$

$a \neq 0$

$$= a \cdot \int f(x)\, dx$$

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Examples

1. $$\int 4x+3\,dx = \int 4x\,dx + \int 3\,dx$$

$$= 2\int 2x\,dx + 3\int dx$$

$$= 2x^2 + 3x + C$$

2. $$\int 1x\,dx = \frac{1}{2}\int 2x\,dx = \frac{1}{2}x^2 + C$$

$$\hookrightarrow \int \frac{1}{2}\cdot 2x\,dx$$

Generalizing Example 2:

$$\frac{d}{dx}\left(x^{k+1}\right) = (k+1)x^{k} \qquad (\text{any } \underline{\underline{k}})$$

$$\int (k+1)\, x^{k}\, dx = x^{k+1} + C$$

$$\boxed{k \neq -1} \longrightarrow \qquad (k+1)\int x^{k}\, dx = x^{k+1} + C$$

$$\boxed{\int x^{k}\, dx = \frac{x^{k+1}}{k+1} + C}$$

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More examples

3. $\int 2x^3 - 4x + 1 \, dx$

$= 2\int x^3 dx - 4\int x \, dx + \int dx$

$= 2 \cdot \frac{1}{4}x^4 - 4 \cdot \frac{1}{2}x^2 + x + C$

$= \frac{1}{2}x^4 - 2x^2 + x + C$

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5. $$\int \frac{3x^3+4x-11}{2x^2}\,dx$$

$$= \int \frac{3x^3}{2x^2} + \frac{4x}{2x^2} - \frac{11}{2x^2}\,dx$$

$$= \int \frac{3}{2}\cdot x + 2\cdot x^{-1} - \frac{11}{2}x^{-2}\,dx$$

$$= \frac{3}{2}\int x\,dx + 2\cdot\int x^{-1}dx - \frac{11}{2}\int x^{-2}dx$$

$$= \frac{3}{4}x^2 + \frac{11}{2}x^{-1} + 2\int x^{-1}dx \quad ? \quad \ldots ?$$

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$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\longrightarrow \int \frac{1}{x}\,dx \overset{?}{=} \ln x + C$$

<u>Problem:</u>

$\frac{1}{x}$ is defined for all $x \neq 0$,

but $\ln x$ is only defined

for $x > 0$ ...

15/ Solution ?

Try $F(x) = \ln|x| = \begin{cases} (\ln x) & : x > 0 \\ (\ln(-x)) & : x < 0 \end{cases}$

$$\frac{d}{dx} \ln|x| = \begin{cases} (\ln x)' & : x > 0 \\ (\ln(-x))' & : x < 0 \end{cases}$$

$$= \begin{cases} \frac{1}{x} & : x > 0 \\ \frac{1}{-x} \cdot (-1) & : x < 0 \end{cases}$$

chain rule

$$= \frac{1}{x} \quad : x \neq 0$$

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First rules:

1. $\int f(x) \pm g(x)\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

2. $a \neq 0 \to \int a f(x)\, dx = a \int f(x)\, dx$

3. $k \neq -1 \to \int x^k\, dx = \frac{x^{k+1}}{k+1} + C$

4. $\int x^{-1}\, dx = \ln|x| + C$