

Case Study 1: Atmospheric Model

Conservation of momentum:

$$\frac{\partial u}{\partial t} - \left(f + u \frac{\tan \phi}{a} \right) v = - \frac{1}{\rho a \cos \phi} \frac{\partial p}{\partial \lambda} + F_\lambda$$

$$\frac{\partial v}{\partial t} + \left(f + u \frac{\tan \phi}{a} \right) v = - \frac{1}{\rho a} \frac{\partial p}{\partial \phi} + F_\phi$$

Hydrostatic approximation:

$$g = - \frac{1}{\rho} \frac{\partial p}{\partial z}$$

Conservation of mass:

$$\frac{\partial \rho}{\partial t} = - \frac{1}{a \cos \phi} \left(\frac{\partial(\rho u)}{\partial \lambda} + \frac{\partial(\rho v \cos \phi)}{\partial \phi} \right) - \frac{\partial(\rho w)}{\partial z}$$

Conservation of energy:

$$C_p \frac{\partial T}{\partial t} - \frac{1}{\rho} \frac{\partial p}{\partial t} = Q$$

State equation (atmosphere):

$$p = R \rho T$$

Solution in two horizontal directions, latitude and longitude

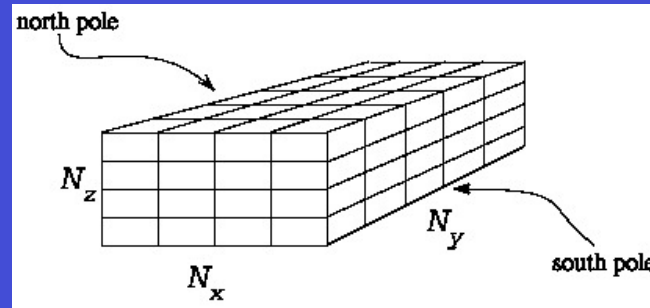
Coupling in vertical direction

Horizontal and vertical gradients

Connections between variables

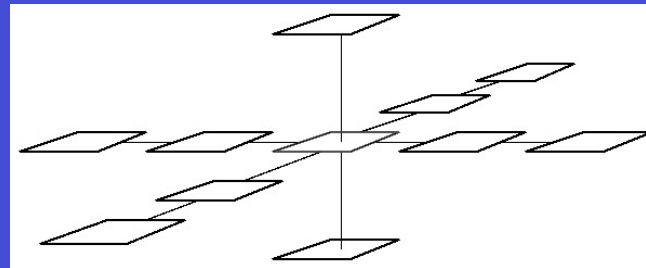
Case Study 1: Atmospheric Model (cont)

“Atmosphere in a box”:



Derivatives grid stencil:

- ✓ 9 points in horizontal
- ✓ 3 points in vertical



Other necessary operations:

$$\text{Total mass} = \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \sum_{k=0}^{N_z-1} M_{i,j,k}$$

Other processes (“extra physics”=precipitation, clouds, radiation): SAY, LOTS OF THEM AND LOTS OF COMPUTATION Complicated accumulations of all z levels only, no x dependence

$$\begin{aligned} TCS_k &= \prod_{i=1}^k (1 - cld_i) TCS_1 \\ &= TCS_{k-1} (1 - cld_k)_1 \end{aligned}$$

Student's task:

Go through algorithmic design process (PCAM) for this problem ...

Case Study 2: Chip Real Estate

Arranging electronic components on a chip.

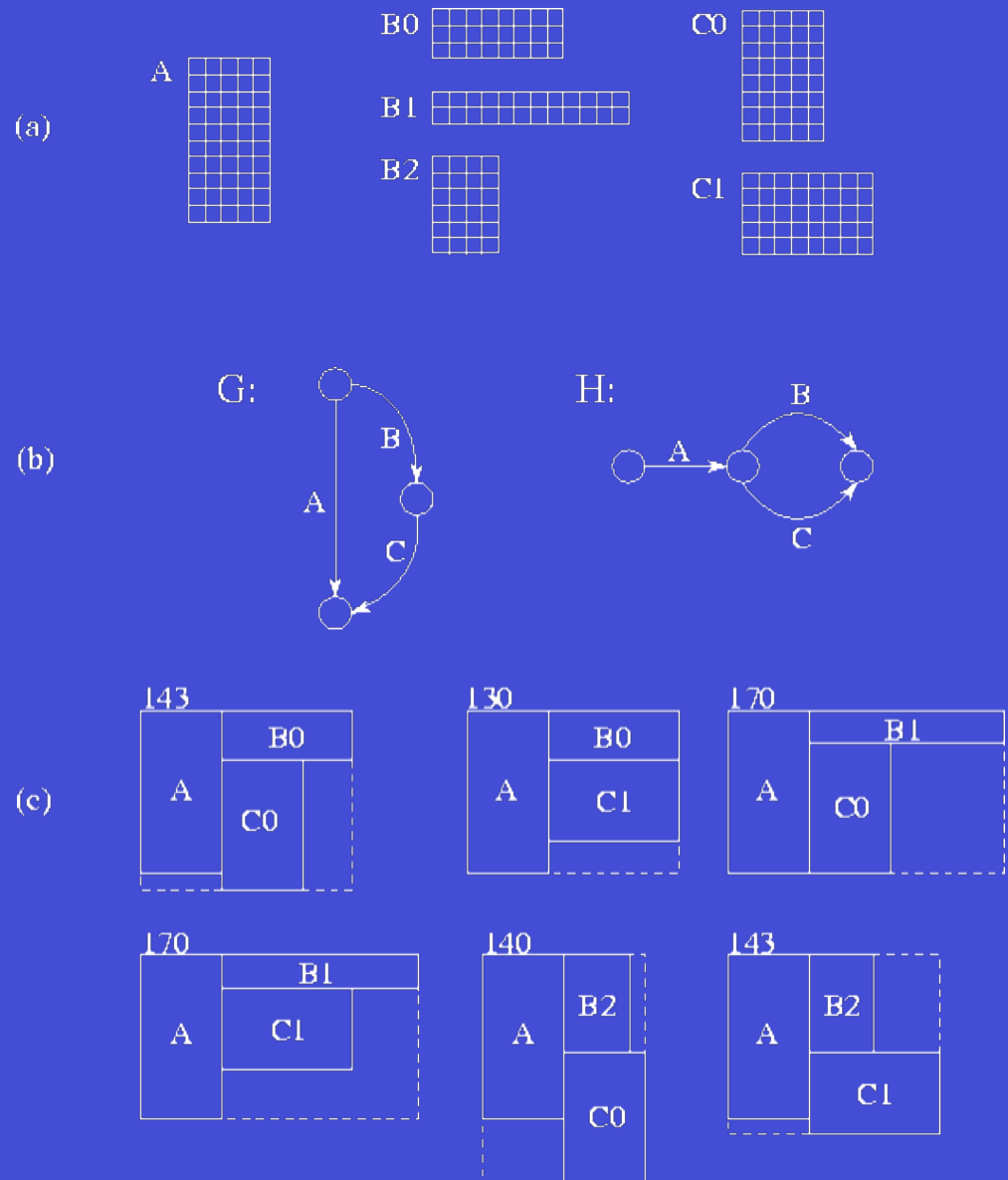
Real estate is a big issue=> Minimise total area

Components have to be in certain order (to connect, heat etc; e.g. above and to left of ...)

But might be able to have various configurations

Treat as “cells” of specific volume, various allowable configurations, order specified

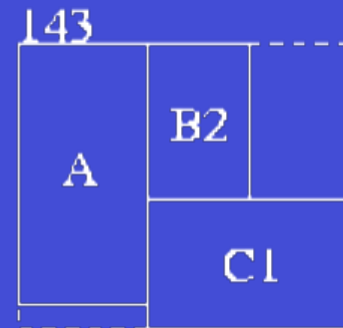
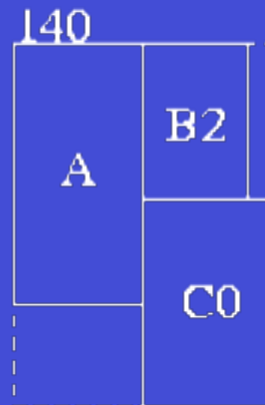
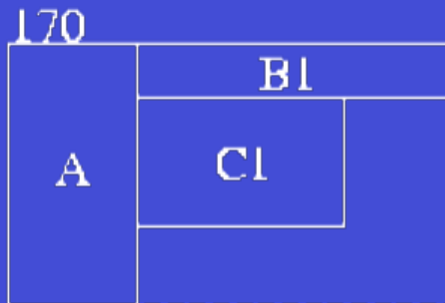
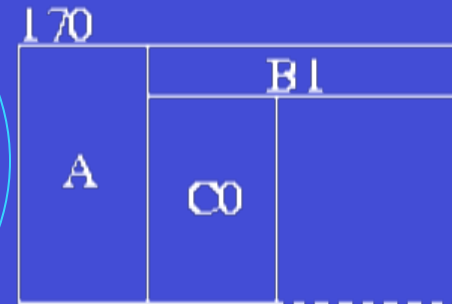
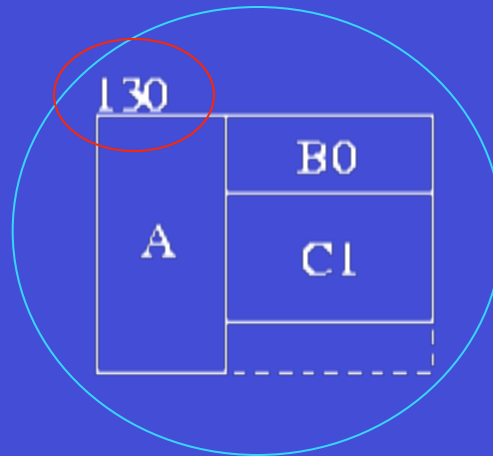
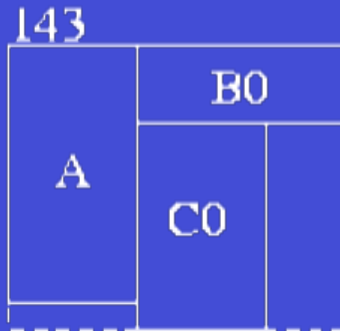
=> Search through set of all possible solutions



Case Study 2: Chip Real Estate (cont)

Solution:

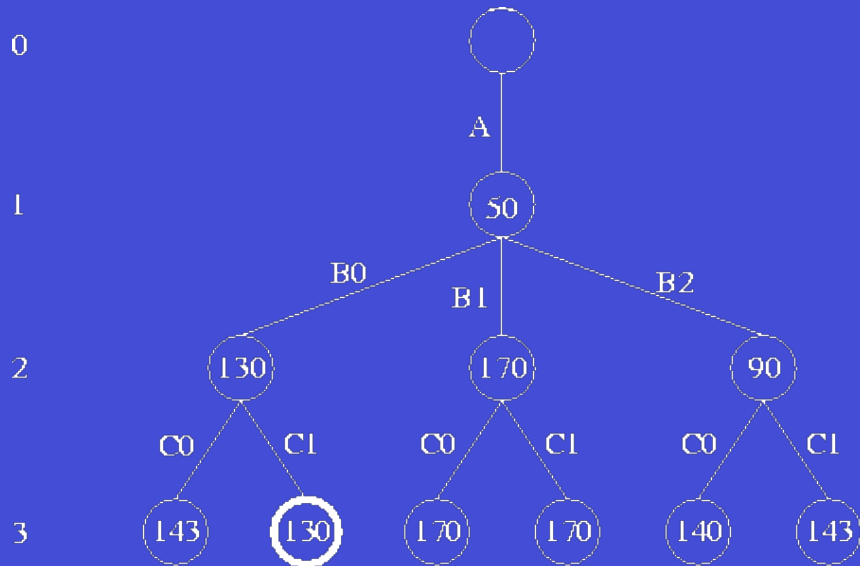
(c)



Case Study 2: Chip Real Estate (cont)

How did you get there?:

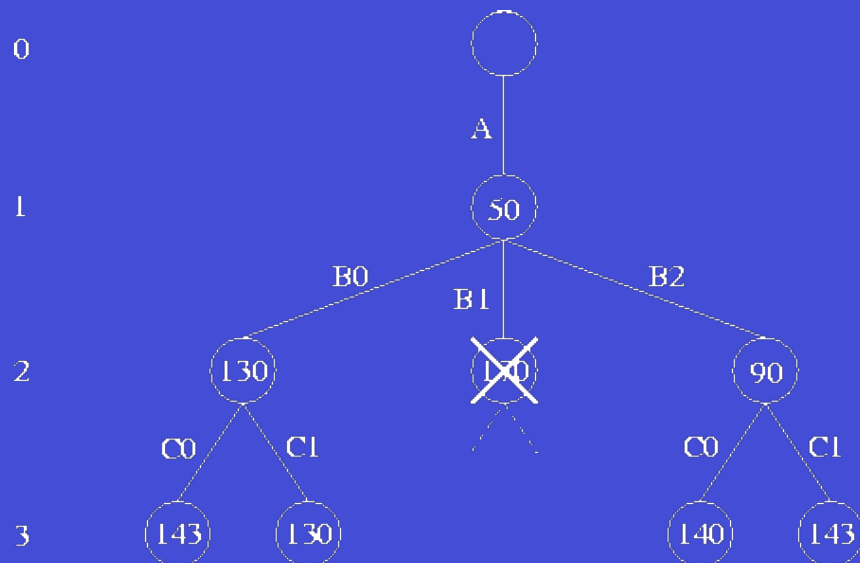
Depth-first exhaustive search
(we already did logic for this)



Faster: Branch-and-bound

Incorporates “pruning”:

If area already greater than best known solution, quit this branch



Case Study 2: Chip Real Estate (cont)

Logic:

```
Procedure bnb_search(A)
Begin
  A_min = infinity
  bnb_search_1(A)
End

Procedure bnb_search_1(A)
Begin
  score=eval(A)
  if (score < A_min) then
    if (leaf(A)) then
      A_min=score
      report soln,score
    else
      foreach (child A_i of A)
        bnb_search_1(A_i)
      endfor
    endif
  endif
end
```

Students: Discuss PCAM for this problem ...