

Numerical Methods for the Solution of Differential Equations (AM 213B)

Homework 1 - Grading rubric

Question 1 (30 points):

(a) (10 points)

- (5 points) For calculating the local polynomial interpolant in terms of Lagrange characteristic polynomials.
- (5 points) For arriving at the correct finite-difference backward differentiation formula at x_j .

(b) (10 points)

- (5 points) For the Taylor series expansion of the local truncation error of the derivative.
- (5 points) For using Taylor series to show that (1) converges with order 3 in Δx .

(c) (10 points)

- (5 points) For plotting the derivative of the function (2) and the finite difference approximation (1) for $n = 20$ and $n = 60$ (two different figures)
- (5 points) For plotting the maximum pointwise error between the analytical and numerical derivatives and show that that the pointwise error (4) decays as n^{-3} , i.e. that the BDF formula (1) is of order 3 in Δx .

Question 2 (70 points):

(a) (15 points)

- (10 points) For computing the analytical solution of (5)-(6). Students need to show some work that leads to the analytical solution (at least the main steps). If no work is shown, then -5 points.
- (5 points) For plotting $y_1(t)$ versus t , $y_2(t)$ versus t , and $y_1(t)$ versus $y_2(t)$

(b) (20 points)

- (10 points) For writing a computer code to compute the numerical solution of the initial value problem by the RK3 method
- (10 points) For writing a computer code to compute the numerical solution of the initial value problem by the AM3 method

(c) (5 points) For providing the formulations of the RK3 and AM3 methods for linear systems $\mathbf{f}(\mathbf{x}, t) = \mathbf{A}\mathbf{x}$.

(d) (20 points)

- (10 points) For running simulations for different values of Δt , i.e., $\Delta t = 0.1, 0.05, 0.005, 0.0005$ and plotting the error in logarithmic scale versus time for each case for the RK3 method
- (10 points) For running simulations for different values of Δt , i.e., $\Delta t = 0.1, 0.05, 0.005, 0.0005$ and plotting the error in logarithmic scale versus time for each case for the AM3 method

(e) (10 points)

- (5 points) For plotting the error (8) at final time in logarithmic scale versus Δt for the RK3 method and showing it converge with order 3
- (5 points) For plotting the error (8) at final time in logarithmic scale versus Δt for the AM3 method and showing it converge with order 4