👉 **To fully prepare for the final, you should study the problems below (on the most recent material) *and* the review problems for midterms 2 and 3.**

1. A monopolistic firm sells one product in two markets, A and B. The daily demand equations for the firm's product in these markets are given by

$$Q_A = 100 - 0.4P_A \qquad \text{and} \qquad Q_B = 120 - 0.5P_B,$$

where Q_A and Q_B are the demands and P_A and P_B are the prices for the firm's product in markets A and B, respectively. The firm's constant marginal cost is \$40 and the its daily fixed cost is \$2500.

(a) Find the prices that the firm should charge in each market to maximize its daily profit. Use the second derivative test to verify that the prices you found yield the *absolute* maximum profit.

(b) Use the *envelope theorem* (and linear approximation) to estimate the change in the firm's max profit if the marginal cost of their product increases to \$40.75.

2. Use the method of Lagrange multipliers to find the constrained critical points and critical values of the objective functions below, with the given constraints.

(a) Objective function: $f(x, y) = x^2 + 3xy + 4y^2$; Constraint: $7x + 2y = 30$.

(b) Objective function: $g(u, v, w) = 10u^{0.3}v^{0.5}w^{0.2}$; Constraint: $6u + 8v + 3w = 14400$

(c) Objective function: $h(x, y, z) = 3\ln x + 4\ln y + 8\ln z$; Constraint: $2x + 5y + 10z = 280$.

3. Jack's (gustatory) utility function is

$$U(x, y, z) = 5\ln x + 7\ln y + 18\ln z,$$

where x is the number of fast-food meals Jack consumes in a month; y is the number of 'diner' meals he consumes in a month; and z is the number of 'fancy restaurant' meals he consumes in a month.

The average price of a fast-food meal is $p_x = \$4.00$; the average price of a 'diner' meal is $p_y = \$8.00$; and the average price of a 'fancy restaurant' meal is $p_z = \$30.00$.

(a) How many meals of each type should Jack consumer per month to **maximize his utility**, if his monthly budget for these meals is $\beta = \$1200.00$?

(b) By approximately how much will Jack's utility increase if his budget increases by \$50.00? Explain/justify your answer.

(c) By approximately how much will Jack need to increase his gustatory budget (from part (a)) if he wants to increase his (max) utility by 2 *utils*? Explain your answer.

4. A firm's production function is given by

$$Q = 10K^{0.4}L^{0.7},$$

where Q is the firm's annual output, K is the annual capital input, and L is the annual labor input. The cost per unit of capital is \$1000, and the cost per unit of labor is \$4000.

(a) Find the levels of labor and capital inputs that **minimize the cost** of producing an output of $Q = 20,000$ units.

(b) Find the levels of labor and capital inputs that **minimize the cost** of producing an output of $Q = q$ units. Express your answer in terms of q.

5. The annual output for a luxury hotel chain is given by $Q = 30K^{2/5}L^{1/2}R^{1/4}$, where K, L and R are the capital, labor and real estate inputs, all measured in \$1,000,000 s, and Q is the average number of rooms rented per day.

The hotel chain's annual budget is $B = \$69$ million.

(a) How should they allocate this budget to the three inputs in order to **maximize their annual output**? What is the maximum output?

(b) What is the *approximate* change in the firm's maximum output if the *capital elasticity of output* increases from $2/5 = 0.4$ to 0.45? Explain your answer in terms of linear approximation and the envelope theorem.