Homework 3: Report

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Question 1: BVP for 2D Poisson's Equation

a) Write Code to solve (1)

$$\begin{cases} \nabla^2 U(x,y) = f(x,y) & (x,y) \in \Omega \\ U(x,y) = g(x,y) & (x,y) \in \partial\Omega, \\ f(x,y) = -20 + 3x^2 + 4y^2 & \\ g(x,y) = 2 - x^2 + 2\sin(\pi y^2) & \end{cases} \tag{1}$$

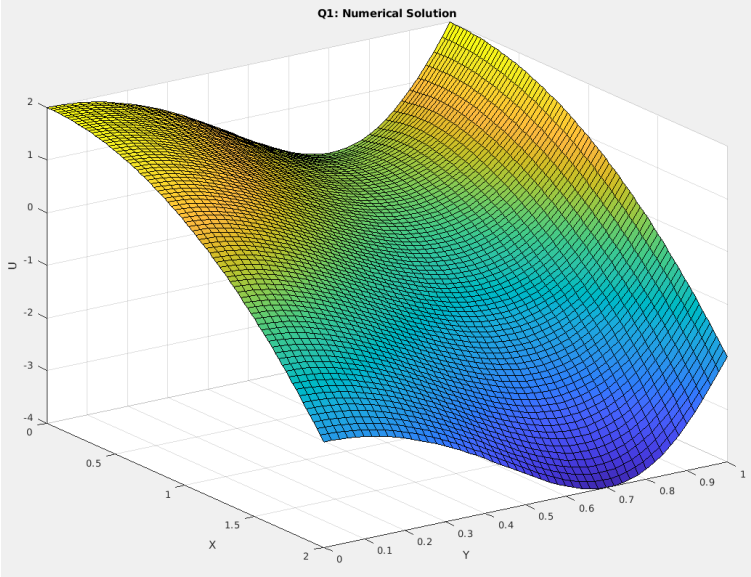

Figure 1: Numerical Solution to (1) with $N = 81, M = 51$

b)

c)

Question 2: IBVP for 1D Heat Equation

a) Determine the analytical solution for (2)

$$\begin{cases} U_t = U_{xx} \qquad x \in [-1,1], \quad t \geq 0 \\ U(x,0) = (3+x) + 5(1-x^2)^2 \\ U(-1,t) = 2, \quad U(1,t) = 4 \end{cases} \tag{2}$$

$$g(x) = 3 + x + 5 - 10x^2 + 5x^4$$

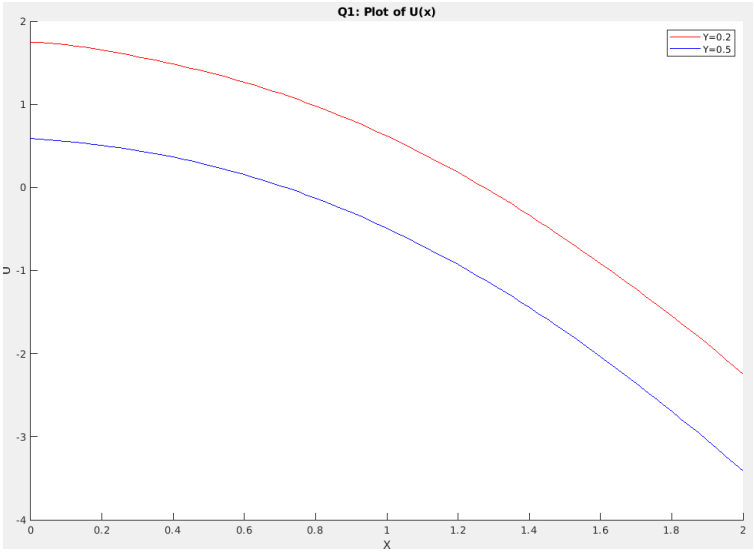

Figure 2: Numerical Solution to (1) at $Y = 0.2, 0.5$

Proof. To begin we look at the general solution for the Heat Equation with Homogeneous BC.

$$\begin{cases} \eta = U - g(x) = U - (3 + x) + 5(1 - x^2)^2 \\ \eta_t = \eta_{xx} - 20 + 60x^2 \qquad x \in [-1, 1], \quad t \geq 0 \\ \eta(x, 0) = 0 \\ \eta(-1, t) = 0, \quad \eta(1, t) = 0 \end{cases}$$

□

b) Plot the analytical solution as a surface plot over [x, t]

c) Write code and integrate using second-order finite differences and CN

d) Wrote code and integrate using Gauss-Chebyshev-Lobatto coallocation method

e) plot maximum pointwise error on a log scale plot between analytical and numerical solutions

Question 3: Extra Credit

a) Write code to compute the nuemrical solution using secondorder finite diff, and AB2.

b) plot the numerical solution as a surface plot

c) plot the numerical sollution at t = 62 as a function of x.