AM 260 - Computational Fluid Dynamics: Lecture Notes

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Contents

Lecture 2:

2.1 Deriving Integral Quantities using FCV

Using a finite control volume we can easily obtain the following equality for the divergence of the flow field in a compressible limit.

$$\nabla \cdot \boldsymbol{u} = \frac{1}{\delta V} \frac{D\delta V}{Dt}$$

Next we look at the continuity equation (density equation) using the FCV (fixed in space). We consider some arbitrary FCV V with surface S. We say, the net mass flow "out" of V through S is equal to the time rate of change of the "decrease" of mass inside V. That is,

$$-\int_S \rho \boldsymbol{u} \cdot \boldsymbol{dS} = \frac{\partial}{\partial t} \left(\int_V \rho dV \right)$$
$$\int_V \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) dV = 0$$

Note that here, the Reynolds transport theorem (adjustment to the density by transport of the FCV) is not necessary due to the volume begin fixed in space. The same can be done with an FCV moving with the fluid although the derivation initially gives the equations in non-conservative form (which is more of a matter of notation than anything)

2.2 Deriving Quantities using IFE

Looking at the fixed in space scenario using an infinitesimal fluid element. We consider a small infinitesimal volume element and the inflow/outflow from that element.

$$\text{Inflow: } \rho \boldsymbol{u} dy dx$$

$$\text{Outflow: } \rho \boldsymbol{u} + \nabla(\rho \boldsymbol{u}) \cdot \boldsymbol{dV}$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

Using an IFE moving with the fluid, we have that the equation becomes,

$$\frac{D\rho dV}{Dt} + \rho \nabla \cdot \boldsymbol{u} = 0$$

2.3 Hyperbolic evolution equations

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$