AM 260 - Computational Fluid Dynamis: Homework 2

Dante Buhl

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Problem 1: Show equivalency between derivations F1-F4

Problem 2: Solve the Burgers' equation for the following IC

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = 0$$
$$u(x,0) = \begin{cases} 2, & |x| < 1/2 \\ -1, & |x| > 1/2 \end{cases}$$

We solve this using the method of characteristics.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \tau} = 0 \tag{1}$$
$$\frac{\partial t}{\partial \tau} = 1, \quad \frac{\partial x}{\partial \tau} = u \tag{2}$$
$$t = \tau, \quad x = u\tau + s \tag{3}$$

We find that u is constant in τ or rather time and that the slope of each characteristic also does not change in time. We now implement the initial condition upon the characteristic solution. We find,

$$x(s,\tau) = \begin{cases} 2\tau + s, & |s| < 1/2 \\ -\tau + s, & |s| > 1/2 \end{cases} \tag{4}$$

The characteristics can be sketched as shown in Figure 1

Problem 3: Solve the scalar conservation law with subsequent IC

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{e^u}{2}\right) = 0$$
$$u(x,0) = \begin{cases} 2, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Problem 4: Weak solutions of the conservation laws

Problem 5: Review on WENO and Numerical Methodology (no response required)

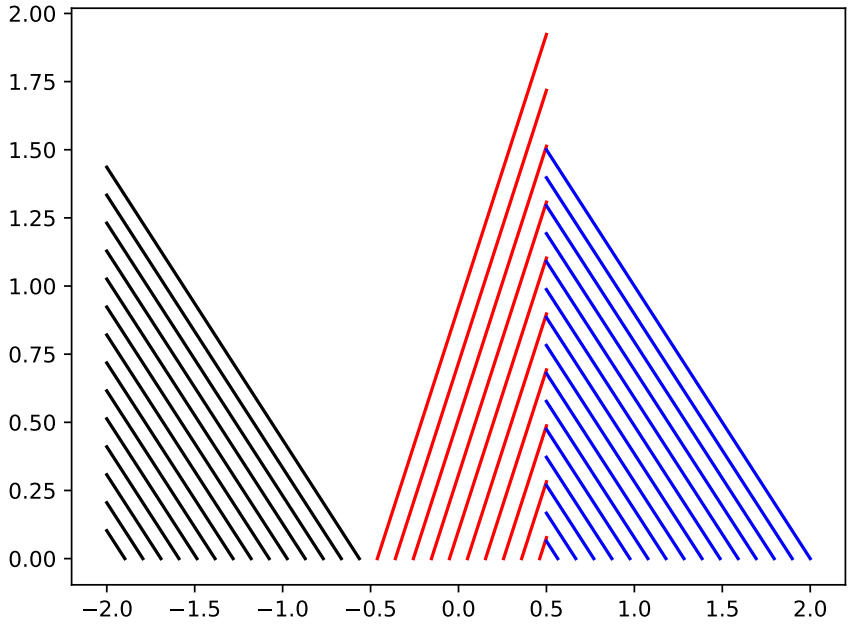

Figure 1: Plot of characteristics ($t \leq t_b$)