AM 260 - Computational Fluid Dynamics: Homework 4

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Problem 1: Part A

I have ran all 11 initial conditions using each of the 7 different integration schemes, and have saved, and plotted all of them. Only the FOG and 3 TVD slope limiter methods are shown in Figures 1 and 2.

Problem 2: Part B

I did two experiments using the data. First, I increased the resolution of the solutions to $Nx = 100$. Then I ran a simulation with a safety factor of 1.1 to see if the solution blew up faster. The $Nx = 100$ results are shown in Figures 3 and 4, while the results for the CFL number being 1.1 are shown in 5 and 6.
Some interesting things that I noticed when running the high resolution ($Nx = 100$) simulations is that most of the shock fronts become steeper due to the higher resolution. That is, a large part of the reason that they appear to not be a vertical front is because of discretization size. If we had an infinitely filled grid, we would probably not see a slope at all! Some other things to notice is that the numerical errors and oscillations around the discontinuities appear to change between the low and high resolution runs. They may still be present but their presentation has changed.
This is contrasted by the "unsafe" (high cfl number) simulations where the predominant thing to notice is that there are rougher patches visible in the simulations. For example, in the rarefraction wave initial condition (IC 4) the edges of the rarefraction fan appear to be jagged. A similar phenomenon can be seen in the FOG version of IC 7, where on the left of the shock front there appears to be another jagged bump. Besides this, there are not many differences between the original cfl = 0.9 and unsafe cfl = 1.1 simulations.

Problem 3: Part C

My solution for part C can be best visualized in Figure 7. The shock front developes as it does in IC 11 from the previous examples, but we also see the shock front advect to the right as the velocity is positive on both sides of the shock front. In order to design an initial condition which develops a shock front and propagates to the left we would need to invert the initial condition (by flipping the sin wave upside down) and then prescribe an initial velocity which is everywhere negative (by subtracting instead of adding 1.5 to the sin wave). In these conditions, a shock front would develop and propagate to the left!

Problem 4: Part D

Sorry Dongwook, I haven't done this part yet :'(.

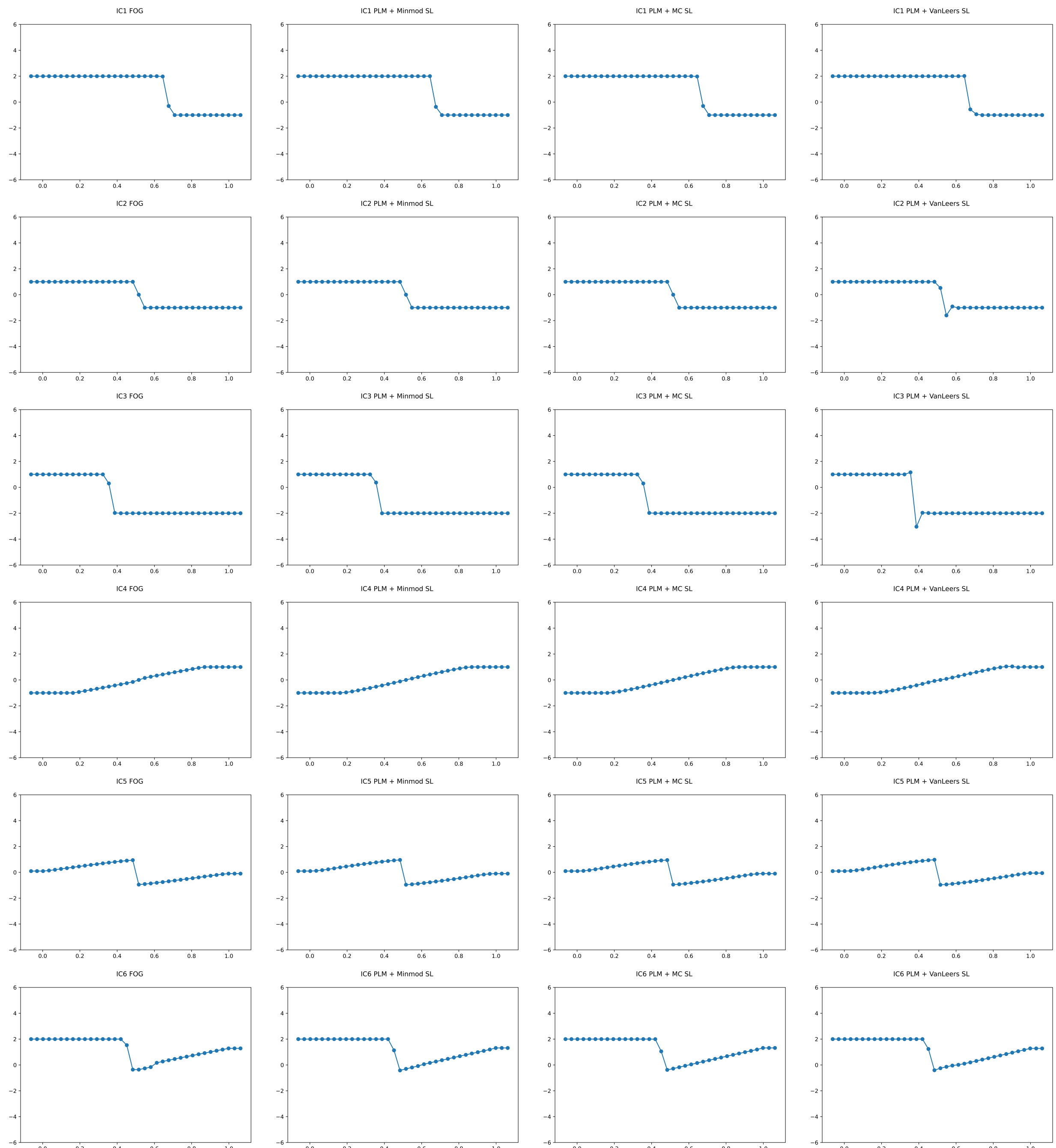

Figure 1: Initial Conditions 1-6 (rows) at t=0.3 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.

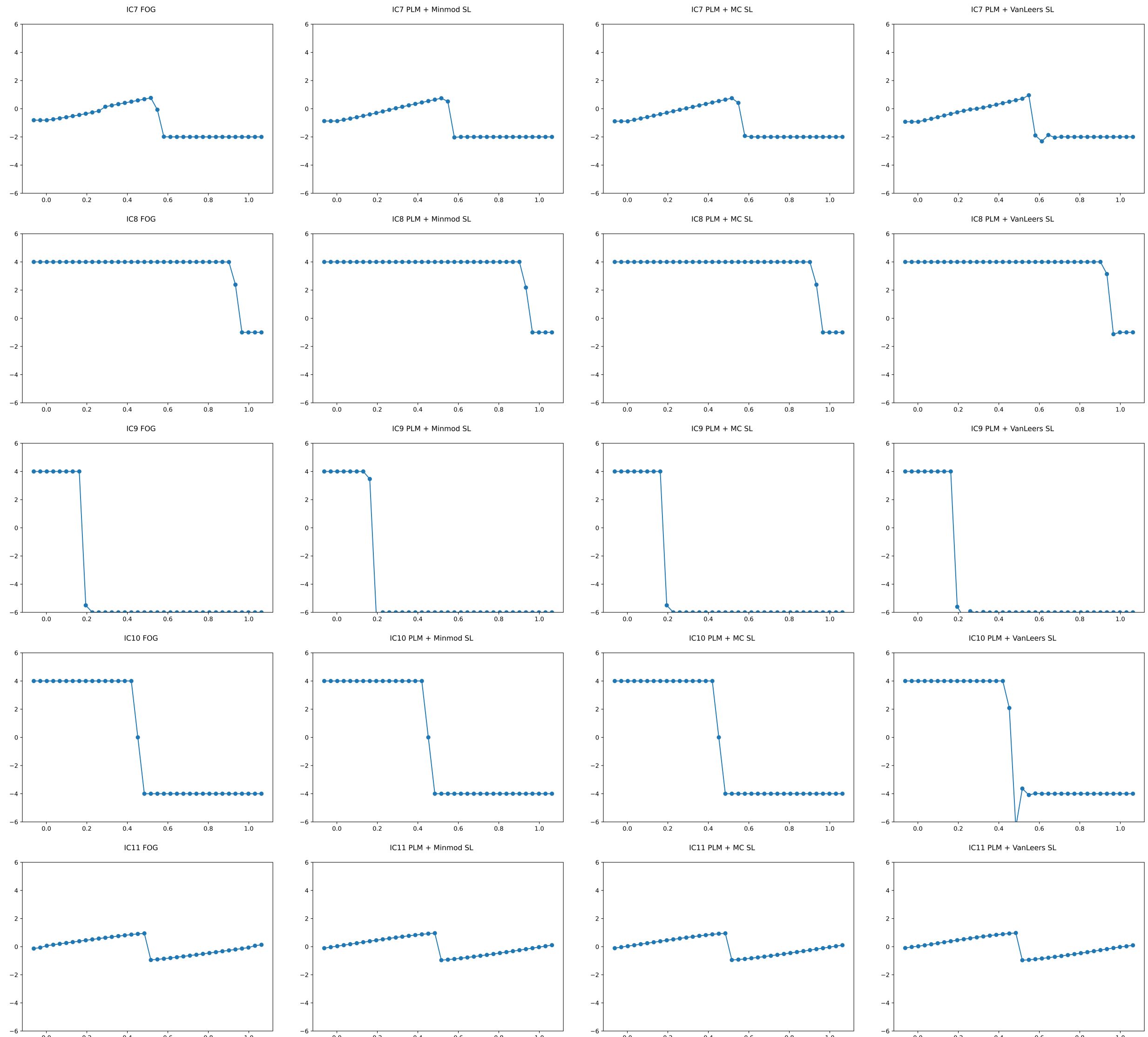

Figure 2: Initial Conditions 7-11 (rows) at t=0.3 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.

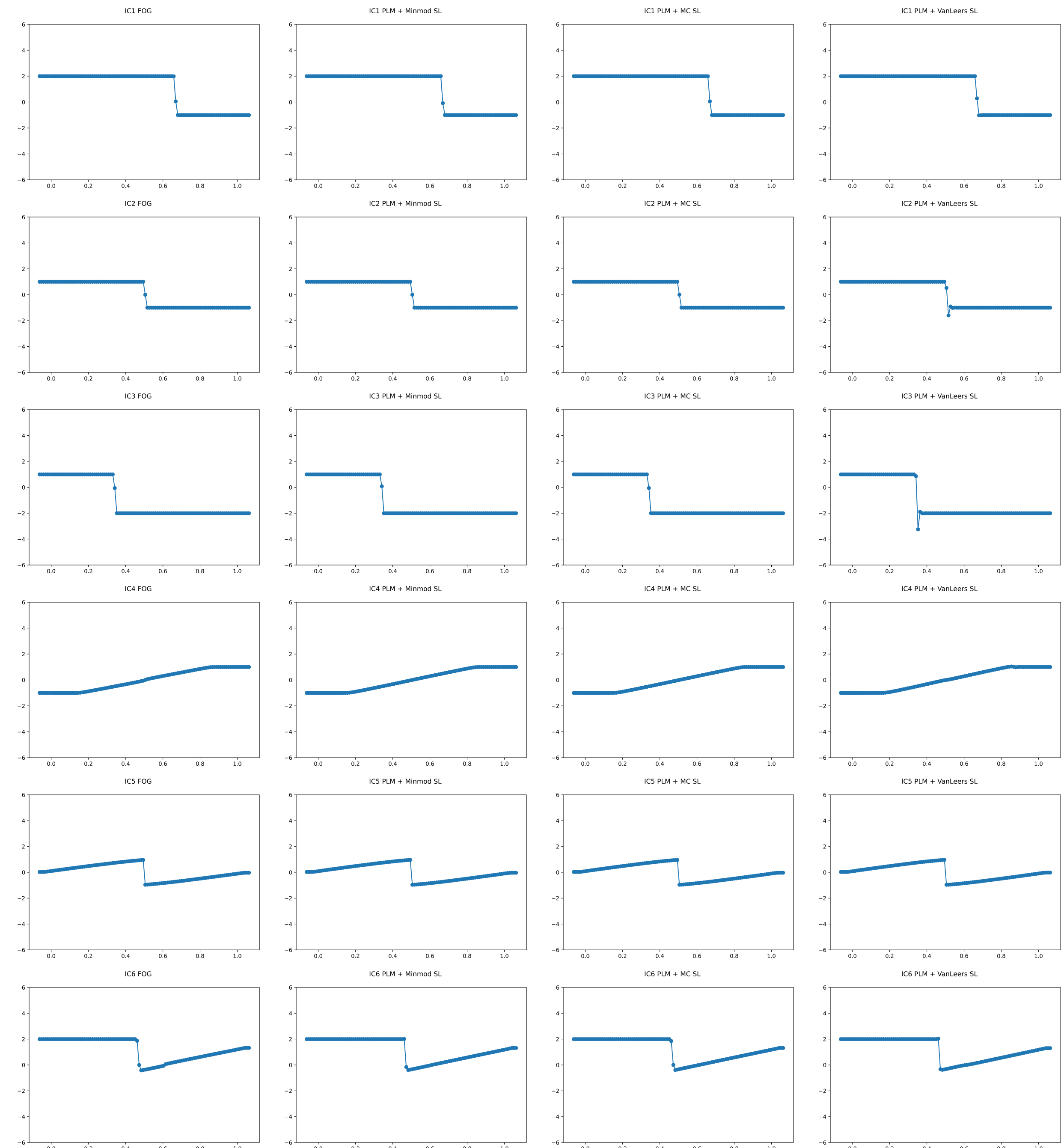

Figure 3: Nx = 100, Initial Conditions 1-6 (rows) at t=0.3 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.

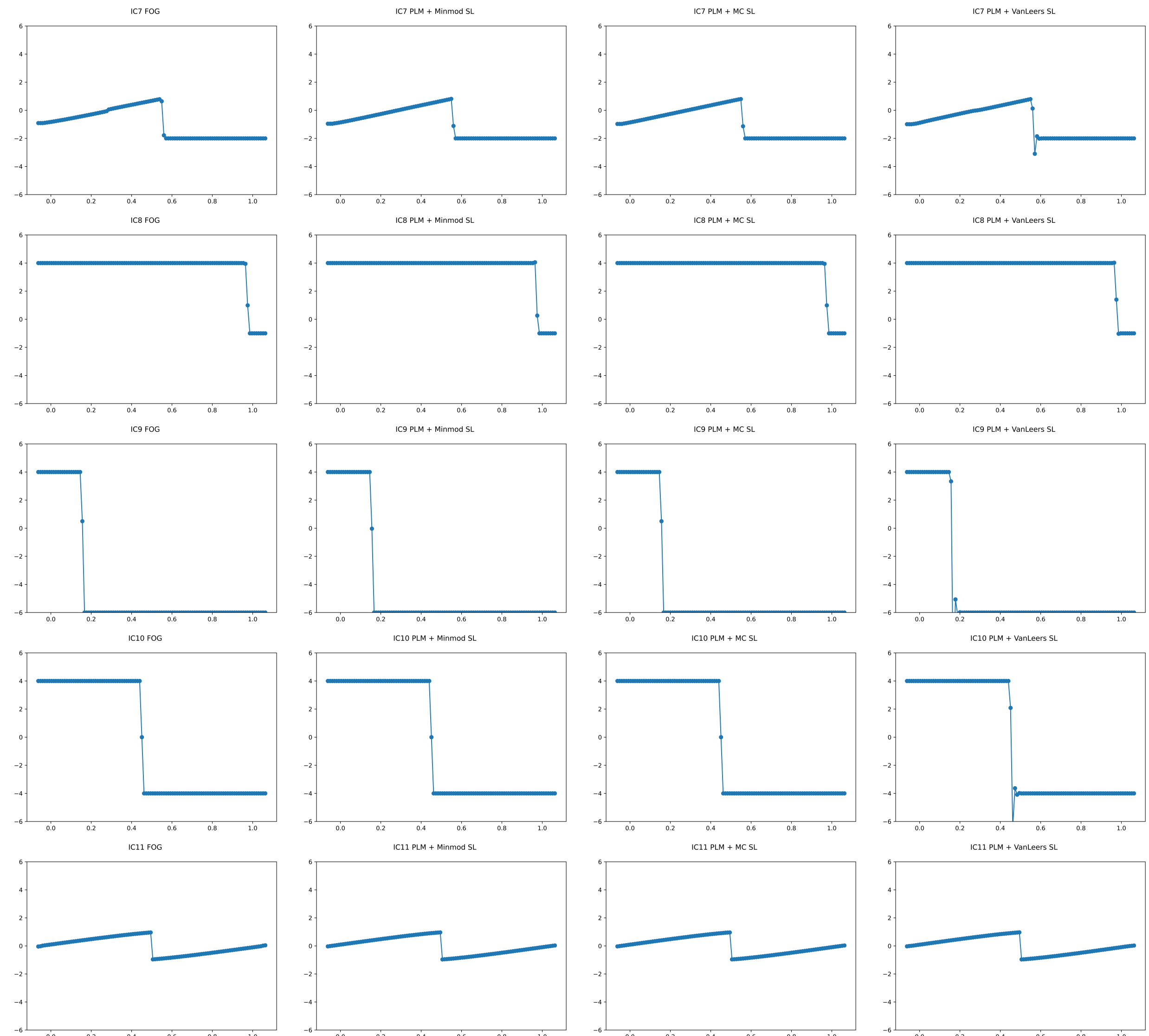

Figure 4: Nx = 100, Initial Conditions 7-11 (rows) at t=0.3 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.

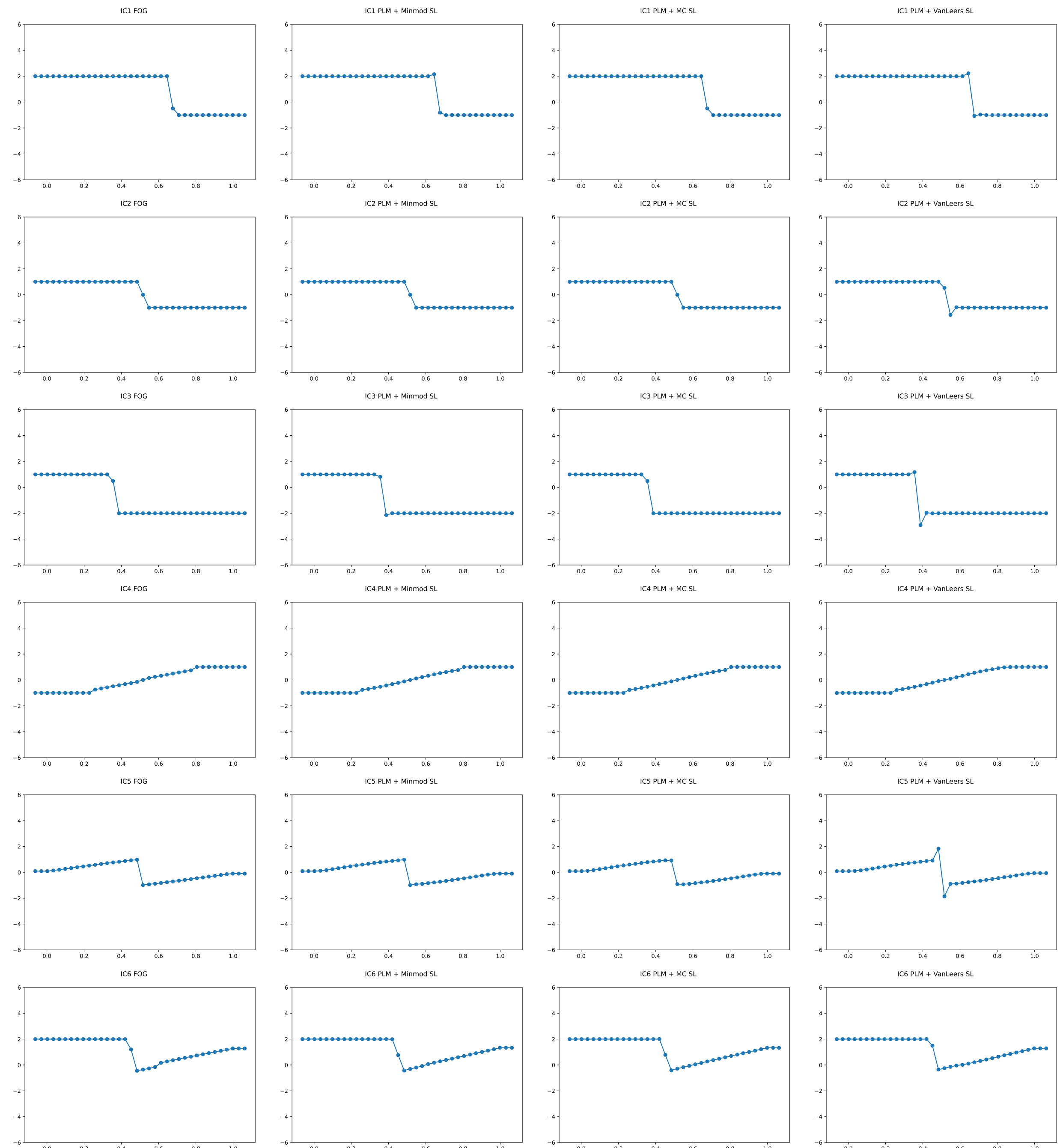

Figure 5: CFL = 1.1, Initial Conditions 1-6 (rows) at t=0.3 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.

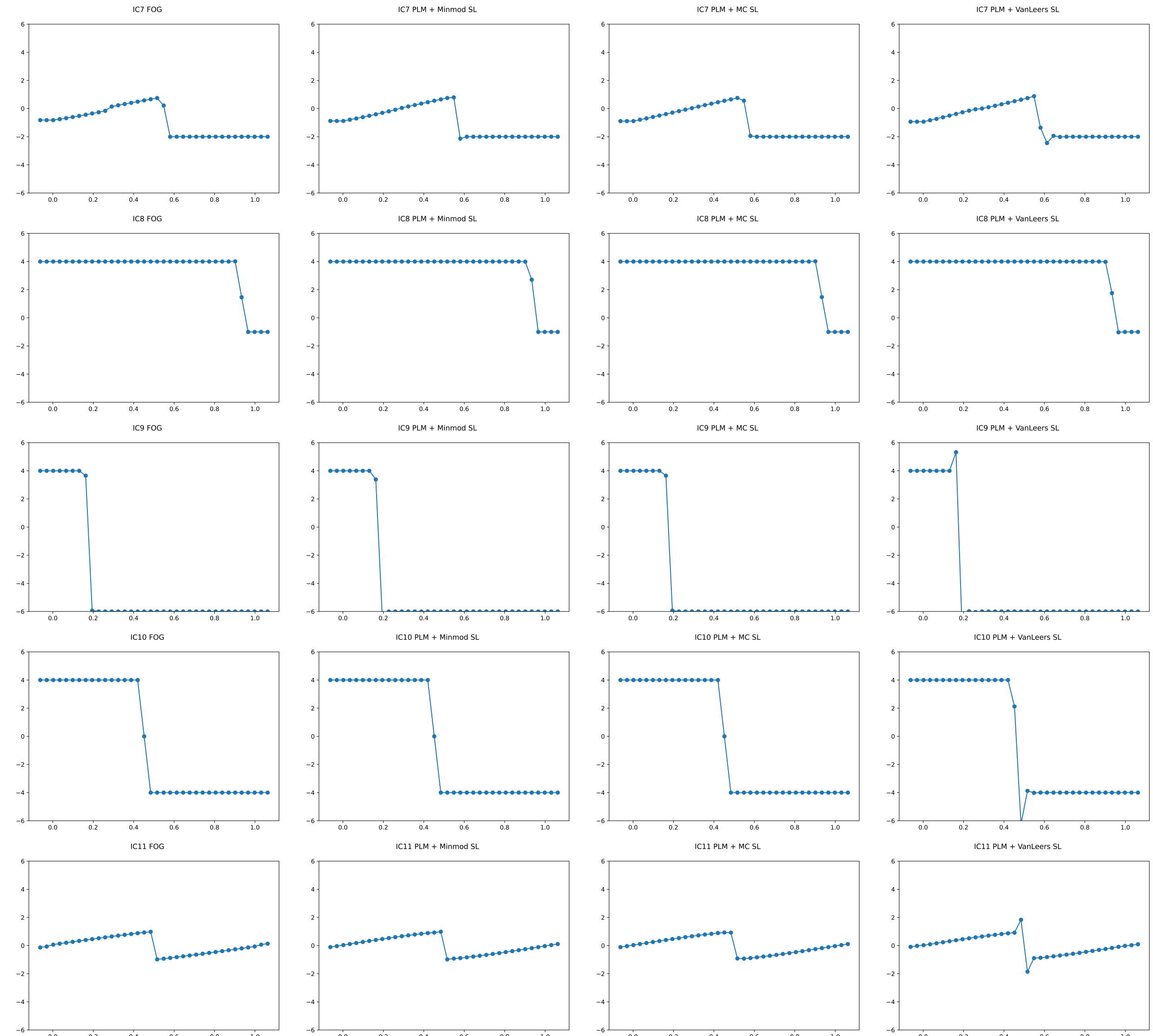

Figure 6: CFL = 1.1, Initial Conditions 7-11 (rows) at t=0.3 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.

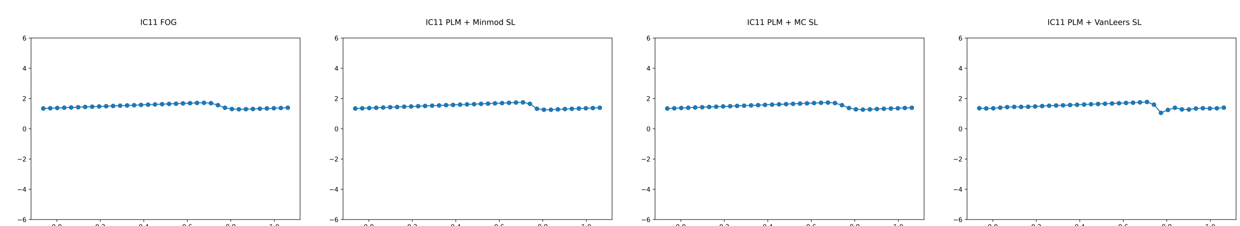

Figure 7: Modified Initial Condition 11 at t=1.5 (or as close as possible before the solver crashed for each run). The columns are from left to right, the FOG, PLM + Minmod, PLM + MC, and PLM + VanLeers solvers.