Rotating Stratified Turbulence

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Abstract

Recent interest in the dynamics of stratified turbulence has led to the development of new models for quantifying vertical transport of momentum and buoyancy (Chini *et al* 2022, Shah *et al* 2024). These models are still incomplete as they do not yet include all of the relevant dynamics often present in real physical settings such as rotation and magnetic fields. Here we expand on prior work by adding rotation. We conduct 3D direct numerical simulations of rotating, stochastically forced, strongly stratified turbulence ($Fr \ll 1$), and vary the Rossby number. We find that rotation gradually suppresses small-scale 3D motions and therefore inhibits vertical transport as Ro decreases towards Fr. The effect is particularly pronounced within the cores of emergent cyclonic vortices. For sufficiently strong rotation, vertical motions are entirely suppressed.

Motivation

Stratified turbulence is a crucial phenomenon for mixing and tranport in many stably-stratified rotating geophysical fluid systems such as Earth's oceans and atmosphere, Jupiter's stratophere, and the solar tachocline. Rotation and stratification have competing influence on the dynamics and flow structure of geophysical systems.

Nonrotating stratified turbulence is characterized by strongly anisotropic pancake structures within the flow.

Rotation promotes barotropic structures which are invariant along the axis of rotation.

The Equations

We consider a triply periodic domain stably stratified by a mean temperature profile $\bar{T}$ in the vertical direciton.

$$\frac{D\boldsymbol{u}}{Dt} + \frac{1}{Ro}\boldsymbol{e}_z \times \boldsymbol{u} = -\nabla p + \boldsymbol{F} + \frac{1}{Fr^2}\tilde{T}\boldsymbol{e}_z + \frac{1}{Re}\nabla^2 \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = 0, \quad T = \bar{T}(z) + \tilde{T}, \quad \frac{D\tilde{T}}{Dt} + w = \frac{1}{Pe}\nabla^2 \tilde{T}$$

$$Re = \frac{UL}{\nu}, \quad Pe = \frac{UL}{\kappa_T}, \quad Fr = \frac{U}{NL}, \quad Ro = \frac{U}{2\Omega L}$$

Stochastic Forcing

We choose the forcing to be a vertically invariant divergence-free stochastic process:

$$\boldsymbol{F} = F_x \boldsymbol{e}_x + F_y \boldsymbol{e}_y, \quad \nabla \cdot \boldsymbol{F} = 0$$

The forcing is applied in spectral space and satisfies $\boldsymbol{k} \cdot \boldsymbol{F} = 0$:

$$\hat{F}_x = \frac{k_y}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t), \quad \hat{F}_y = \frac{-k_x}{|\boldsymbol{k}_h|} G(\boldsymbol{k}_h, t)$$

where $G(\boldsymbol{k}_h, t)$ is a Gaussian process of amplitude 1 and correlation timescale 1, and $|\boldsymbol{k}_h| \leq \sqrt{2}$. The Gaussian process timeseries are autoregressively generated in spectral space using a pseudo-inverse algorithm. The Gaussian kernel is generated using the exponential squared kernel function

$$K(t, t') = \exp\left(\frac{-(t-t')^2}{2\tau^2}\right)$$

in order to improve smoothness and differentiability of the forcing in time. The image on the right shows an example of the forcing for a single wavenumber.

c.f. Waite and Bartello (2004)

Simulation Visualization

Two series of simulations have been conducted with $Re = 600, Pe = 60$ at two different Froude numbers (moderately stratified case $Fr = 0.18$, strongly stratified case $Fr = 0.1$). The Rossby number varies in $[0.1, 2]$. The figures below show results for $Fr = 0.18$, and increasing rotation rate from left to right. The top row shows volume renderings of the vertical component of vorticity. The middle row shows the temperature dissipation. The bottom row shows kinetic energy spectra for the horizontal and vertical flows. The vorticity renderings illustrates the formation of cyclones and anticyclones which exhibit increasing vertical invariance as the rotation rate increases. Associated with the emergence of vertically cohearent vortices, we also note the appearance of an inverse energy cascade characterized by a $\boldsymbol{k}_h^{-3}$ spectrum for the horizontal flow. Finally, we see that the thermal dissipation becomes negligible inside the cyclones.

RMS Data

From simulations which have reached a statistically stationary state (filled circles) we extract time and volume averages of quantities of interest (open circles have not reached a statistically stationary state yet, but are shown for comparison).

$$|\boldsymbol{u}_h|_{\rm rms} = \langle u^2 + v^2 \rangle^{1/2}$$

$$\chi = \langle |\nabla T|^2 \rangle$$

$$\eta = \frac{\chi/(PeFr^2)}{\chi/(PeFr^2) + \langle |\nabla \boldsymbol{u}|^2 \rangle / Re}$$

They are shown on the right as a function of the inverse Rossby number for both moderate and strong stratification. The horizontal velocities increase proportionally to the rotation rate owing to the inverse cascade. For very large rotation rates, we see a decrease in the rms vertical velocity as well as the temperature dissipation χ associated with the increasing volume fraction of the domain occupied by the vortices.

Vertically Averaged Quantities

We aim to characterize the properties of the vortices. We define an effective Rossby number,

$$Ro_{\rm eff}^{-1} = |\widehat{\omega}_z + Ro^{-1}|, \quad \text{where } \widehat{(\cdot)} = \frac{1}{L_z}\int (\cdot) dz$$

The following figures compare 2D maps of $Ro_{\rm eff}^{-1}$, $\widehat{w^2}^{1/2}$, $\widehat{|\nabla T|^2}$, $\widehat{wT}$ for 4 different simulations at moderate and strong stratification and moderate and strong rotation. We see that when $Ro_{\rm eff}^{-1}$ exceeds a critical threshold, which appears to depend on the Froude number, thermal dissipation and vertical velocity are suppressed.

References & Acknowledgements

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References

Michael L. Waite and Peter Bartello. Stratified turbulence dominated by vertical motion. Journal of Fluid Mechanics, 517:281–308, 2004. doi: 10.1017/S0022112004000977.

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