Homework 1

Gomez - Math 19B

Due: Jan 19th, 2024

All exercises are taken from Section 5.1 in the textbook

1. Exercise 8: Let $f(x) = x^2 + x - 2$.

    - Calculate R_3 and L_3 over [2, 5]. NOTE: R_3 and L_3 denote the right and left endpoint approximations of the Area under $f(x)$, each made of three rectangles/intervals.
    - Sketch the graph of f and the rectangles that make up each approximation. Is the area under the graph larger or smaller than R_3? Than L_3 ?

2. Derive the Midpoint Approximation Formula, M_n, for a function, $f(x)$, on the interval $[a, b]$ through the process of this problem.

    - Find the base width for each rectangle, Δx, in terms of n.
    - Find the height of the i-th rectangle, h_i, in terms of f and x_i, where x_i denotes the midpoint of each interval.
    - Find the midpoint of the i-th subinterval, x_i, in terms of $i, \Delta x$.
    - Find the area of the i-th rectangle, A_i, in terms of i, f, and Δx using the equation for the area of a rectangle.
    - Express the midpoint approximation, M_n, as the sum of the areas of all rectangles in the partition in terms of $i, \Delta x$, and f.

Formulas (3)-(5)

$$\sum_{i=1}^{N} i = 1 + 2 + \cdots + N = \frac{N(N+1)}{2} = \frac{N^2}{2} + \frac{N}{2} \tag{3}$$

$$\sum_{i=1}^{N} i^2 = 1 + 4 + \cdots + N^2 = \frac{N(N+1)(2N+1)}{6} = \frac{N^3}{3} + \frac{N^2}{2} + \frac{N}{6} \tag{4}$$

$$\sum_{i=1}^{N} i^3 = 1 + 8 + \cdots + N^3 = \frac{N^2(N+1)^2}{4} = \frac{N^4}{4} + \frac{N^3}{2} + \frac{N^2}{4} \tag{5}$$

3. Exercise 44: Compute the sum using linearity and the formulas (3)-(5) for power sums covered in the textbook (pg. 304)

$$\sum_{i=1}^{10}(i^3 - 2i^2)$$

4. Exercise 52: use linearity and formulas (3)–(5) to evaluate the limit.

$$\lim_{N\to\infty} \sum_{i=1}^{N} \frac{i}{N^2}$$