Lecture 14 Activity Results for Test Student

Score for this attempt: **1** out of 1
Submitted Feb 22 at 9:15am
This attempt took 1 minute.

Question 1

1 / 1 pts

The fundamental solution $\Phi(x,t)$ of the 1D heat equation is governed by the IVP

$$\begin{cases} u_t = ku_{xx}, \ x \in (-\infty,+\infty), \ t>0 \\ u(x,0) = \delta(x), \ x \in (-\infty,+\infty) \end{cases}.$$

Recall that in an IBVP, we use the eigenfunction expansion to reduce a PDE to an ODE. In our study of solving for $\Phi(x,t)$, which tool did we use to reduce the PDE to an ODE?

- Fourier series
- Generalized Fourier series (eigenfunction expansion)
- Laplace transform
- Correct! Fourier transform
- A non-linear scaling of (*x*, *t*).

Additional Comments:

Question 2

0 / 0 pts

Consider the IVP $\begin{cases} u_t = ku_{xx}, \ x \in (-\infty,+\infty), \ t>0 \\ u(x,0) = f(x), \ x \in (-\infty,+\infty) \end{cases}$ for *u*(*x*, *t*).

We are given that the fundamental solution of the 1D heat equation is

$$\Phi(x,t) \equiv \frac{1}{\sqrt{4\pi kt}} e^{\frac{-x^2}{4kt}}$$

Which statement below is true regarding the solution *u*(*x*, *t*) for a general *f*(*x*)?

- $u(x,t) = \int_0^L f(x')\Phi(x-x',t)dx'$
- $u(x,t) = \int_{-\infty}^{x} f(x')\Phi(x-x',t)dx'$
- Correct! $u(x,t) = \int_{-\infty}^{+\infty} f(x')\Phi(x-x',t)dx'$
- $u(x,t) = \int_0^t \int_{-\infty}^{+\infty} f(x')\Phi(x-x',t')dx'dt'$
- There is not enough information to determine *u*(*x*, *t*).

Additional Comments:

Question 3

0 / 0 pts

Consider the 2D Laplace equation with axial symmetry

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r>0.$$

We derived that a general solution is

$$u(r) = c_0 + c_1 \ln r, \ \ r>0.$$

Consider the BVP $\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = \boxed{??} \end{cases}.$

Which of the following is a well-posed BC at $r=\infty$? **Select all that apply.**

- $u(\infty) = 1$
- $u(\infty) = 0$
- No condition needs to be imposed on $u(\infty)$.
- Correct! $u(\infty) = \text{ finite}$
- $\lim_{r\to\infty} u'(r) = 0$

Additional Comments:

Question 4

0 / 0 pts

Consider the 3D Laplace equation with spherical symmetry

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r>0.$$

We derived that a general solution is

$$u(r) = c_0 + c_1(1-\frac{1}{r}), \ \ r>0.$$

Consider the BVP $\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = \boxed{??} \end{cases}.$

Which of the following is a well-posed BC at $r=\infty$? **Select all that apply.**

- Correct! $u(\infty) = 1$
- Correct! $u(\infty) = 0$
- No condition needs to be imposed on $u(\infty)$.
- $u(\infty) = \text{ finite}$
- $\lim_{r\to\infty} u'(r) = 0$

Additional Comments:

Question 5

0 / 0 pts

Consider the BVP of 2D Laplace equation with axial symmetry in an annulus

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,R) \\ u(1)=3, \ \ u(R) = u_R \end{cases}.$$

In our mathematical abstraction in the case of large *R*, we want to replace the BVP for $r \in (1,R)$ with one for $r \in (1,\infty)$. Which one below should we use?

- $\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = u_R \end{cases}$
- $\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = 0 \end{cases}$
- Correct! $\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = \text{finite} \end{cases}$
- $\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = 1 \end{cases}$
- None of the BVPs listed.

Additional Comments:

Question 6

0 / 0 pts

Consider the BVP of 3D Laplace equation with spherical symmetry in a spherical shell

$$\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,R) \\ u(1)=3, \ \ u(R) = u_R \end{cases}.$$

In our mathematical abstraction in the case of large *R*, we want to replace the BVP for $r \in (1,R)$ with one for $r \in (1,\infty)$. Which one below should we use?

- Correct! $\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = u_R \end{cases}$
- $\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = 0 \end{cases}$
- $\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = \text{finite} \end{cases}$
- $\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = 0, \ \ r \in (1,\infty) \\ u(1)=3, \ \ u(\infty) = 1 \end{cases}$
- None of the BVPs listed.

Additional Comments:

Question 7

0 / 0 pts

Let $D = \{\mathbf{x} | \mathbf{x} = (x,y,z) \in \mathbb{R}^3, z>0\}$ (the upper half of R^3).

The solution of $\begin{cases} \nabla^2 u = -\delta(\mathbf{x}-\mathbf{x}'), \ \mathbf{x} \in D, \ \mathbf{x}' \in D \\ u(\mathbf{x})\Big|_{\mathbf{x}=(x,y,0)} = 0 \\ u(\infty) = 0 \end{cases}$ has the form

$$u(\mathbf{x}) = \frac{1}{4\pi|\mathbf{x}-\mathbf{x}'|} - \frac{1}{4\pi|\mathbf{x}-\mathbf{x}^{(2)}|}.$$

For $\mathbf{x}' = (1,-3,2)$, what is $\mathbf{x}^{(2)}$?

- $\mathbf{x}^{(2)} = (1,3,2)$
- $\mathbf{x}^{(2)} = (-1,-3,2)$
- Correct! $\mathbf{x}^{(2)} = (1,-3,-2)$
- $\mathbf{x}^{(2)} = (-1,-3,-2)$
- $\mathbf{x}^{(2)} = (-1,3,-2)$

Additional Comments:

Question 8

0 / 0 pts

Let $D = B(0,R) = \{\mathbf{x} | \mathbf{x} \in \mathbb{R}^2, |\mathbf{x}| < R\}$ (a disk in R^2).

The solution of $\begin{cases} \nabla^2 u = -\delta(\mathbf{x}-\mathbf{x}'), \ \mathbf{x} \in D, \ \mathbf{x}' \in D \\ u(\mathbf{x})\Big|_{|\mathbf{x}|=R} = 0 \end{cases}$ has the form

$$u(\mathbf{x}) = \frac{-1}{2\pi}\ln|\mathbf{x}-\mathbf{x}'| + \frac{1}{2\pi}\ln|\mathbf{x}-\mathbf{x}^{(2)}| + c_0.$$

For $R=3, \ \mathbf{x}'=(-2,1)$, what is $\mathbf{x}^{(2)}$?

- $\mathbf{x}^{(2)} = \frac{9}{\sqrt{5}}(-2,1)$
- $\mathbf{x}^{(2)} = \frac{5}{9}(-2,1)$
- $\mathbf{x}^{(2)} = \frac{3}{\sqrt{5}}(-2,1)$
- $\mathbf{x}^{(2)} = \frac{3}{5}(-2,1)$
- Correct! $\mathbf{x}^{(2)} = \frac{9}{5}(-2,1)$

Additional Comments:

Fudge Points: --

You can manually adjust the score by adding positive or negative points to this box.

Final Score: 1 out of 1

Update Scores

Here's the latest quiz results for Test Student. You can modify the points for any question and add more comments, then click "Update Scores" at the bottom of the page.

Quiz Submissions

Attempt 1: 1

Test Student has 1 attempt left

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