Section 5

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UCSC Math-19B

February 13, 2024

Plan for Today

Topics to Cover

- Integration by Parts
- Trig Substitution

Section Activity 5

- 4 questions

Upcoming Assignments

- Homework 5 (Due Fri, Feb. 16^{th})
- Project 1 (Due Tues, Feb 20^{th}).

Learning Outcomes

- Understanding and applying the concept of Integration by Parts
- Understanding how to use Trig Substitution

Integration By Parts

Integration by parts, similar to u-substition, is motivated by a rule for differentiation. Take a guess. Its the product rule!!!

Recall:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

We can therefore impose that if we are given an integral of the form, $\int f'(x)g(x)dx$.

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

Note: people sometimes write this in an alternate form

$$\int udv = uv - \int vdu$$

Integration by Parts

An example application of this practice can be seen here.

$$\int \ln(x)dx$$

$$f'(x) = 1, \quad g(x) = \ln(x)$$

$$f(x) = x, \quad g'(x) = \frac{1}{x}$$

$$\int \ln(x)dx = x\ln(x) - \int 1dx$$

$$= x\ln(x) - x + C$$

Now do a couple problems yourself

$$\int \cos^2(x)dx$$

Note: this can be done with the trig identity

$$\cos^2(x) + \sin^2(x) = 1$$

$$\int x\ln(x)dx$$

Trigonometric Integrals

Some weird trig integrals have reduction formulas obtained by Integration by Parts. An example of one is below.

$$\int \sin^m(x)\cos^n(x)dx = \frac{\sin^{m+1}(x)\cos^{n-1}(x)}{n} + \frac{n-1}{n}\int \sin^m(x)\cos^{n-2}(x)dx$$

These can be truly ugly. Hint hint, wink wink, you might be asked to prove one or two on your homework :)

Discussion Section Activity 5

Woah look, the TA is about to write the code on the board!