Lecture 6 Activity Results for Test Student

Score for this attempt: **1** out of 1

Submitted Jan 25 at 3:28pm

This attempt took less than 1 minute.

Question 1

1 / 1 pts

Let $L[\bullet] \equiv \big(p(x)(\bullet)_x\big)_x + q(x)(\bullet)$.

Consider the Sturm-Liouville problem $\begin{cases} L[u] = -\lambda r(x) u \\ u(a) = 0,\ u(b) = 0 \end{cases}$, the function space $C_{BC}[a,b] \equiv \left\{ u(x) \middle| u(a) = 0, u(b) = 0 \right\}$, the inner product $\langle u(x), v(x) \rangle \equiv \int_a^b u(x)v(x)r(x)dx$, and the Rayleigh quotient $R(u) \equiv \dfrac{-\langle u, \frac{1}{r(x)} L[u] \rangle}{\langle u, u \rangle}$.

Let $\{\lambda_n, \phi_n(x),\ n = 0, 1, 2, \ldots\}$ be the sequence of eigenvalues and eigenfunctions.

Which statement below is true?

- $\min\limits_{u \in C_{BC}[a,b], u \neq 0} R(u) = 0 \text{ and } R(\phi_0) = 0$
- $\min\limits_{u \in C_{BC}[a,b], u \neq 0} R(u) > \lambda_0 \text{ and } R(\phi_0) > \lambda_0$
- Correct! $\min\limits_{u \in C_{BC}[a,b], u \neq 0} R(u) = \lambda_0,\ R(\phi_0) = \lambda_0$
- $\min\limits_{u \in C_{BC}[a,b], u \neq 0} R(u) > \lambda_0 \text{ and } R(\phi_0) = \lambda_0$

$\min_{u \in C_{BC}[a,b], u \neq 0} R(u) < \lambda_0 \text{ and } R(\phi_0) = \lambda_0$

Additional Comments:

Question 2

0 / 0 pts

Let $L[\bullet] \equiv \left(p(x)(\bullet)_x\right)_x + q(x)(\bullet)$.

Consider the Sturm-Liouville problem $\begin{cases} L[u] = -\lambda r(x) u \\ u(a) = 0, \ u(b) = 0 \end{cases}$, the function space $C_{BC}[a,b] \equiv \left\{ u(x) \middle| u(a) = 0, u(b) = 0 \right\}$, the inner product $\langle u(x), v(x) \rangle \equiv \int_a^b u(x) v(x) r(x) dx$, and the Rayleigh quotient $R(u) \equiv \dfrac{-\langle u, \frac{1}{r(x)} L[u] \rangle}{\langle u, u \rangle}$.

Let $\{\lambda_n, \phi_n(x), \ n = 0, 1, 2, \ldots\}$ be the sequence of eigenvalues and eigenfunctions.

Suppose $u^{(p)}(x)$ is a particular function in $C_{BC}[a,b]$. Which statement below is true?

- $\lambda_0 = R(u^{(p)})$
- $\lambda_0 \geq R(u^{(p)})$

Correct!

- $\lambda_0 \leq R(u^{(p)})$

- There is not enough information to conclude $\lambda_0 \leq R(u^{(p)})$ or $\lambda_0 \geq R(u^{(p)})$.

Additional Comments:

Question 3

0 / 0 pts

Consider the ODE eigenvalue problem $\begin{cases} u_{xx} + cu_x = -\lambda u \\ u(a) = 0, \; u(b) = 0 \end{cases}$. We write it in the SL form

$$\begin{cases} \left(e^{cx} u_x\right)_x = -\lambda e^{cx} u \\ u(a) = 0, \; u(b) = 0 \end{cases}.$$

With respect to which inner product, the eigenfunctions are orthogonal?

- $\langle u(x), v(x) \rangle \equiv \int_a^b u(x)v(x)dx$

- $\langle u(x), v(x) \rangle \equiv \int_a^b u(x)v(x)e^{cx/2}dx$

Correct!

- $\langle u(x), v(x)\rangle \equiv \int_a^b u(x)v(x)e^{cx}\,dx$
- $\langle u(x), v(x)\rangle \equiv \int_a^b u(x)v(x)e^{-cx/2}\,dx$
- $\langle u(x), v(x)\rangle \equiv \int_a^b u(x)v(x)e^{-cx}\,dx$

Additional Comments:

Question 4

0 / 0 pts

Two differential equations below are equivalent.

(DE1): $u_{xx} + cu_x = -\lambda u$

(DE2): $\left(e^{cx}u_x\right)_x = -\lambda e^{cx}u$

Which one do we use when solving for a general solution?

Correct!

- (DE1) for all values of c.
- (DE2) for all values of c.
- (DE1) for c > 0.

(DE2) for c > 0.

Additional Comments:

Question 5

0 / 0 pts

Consider the Cauchy-Euler equation $a_2 x^2 u'' + a_1 x u' + a_0 u = 0$.

Which of the following expressions may be a solution? **Select all that apply.**

Correct!

- [x] $u(x) = x^\alpha$

Correct!

- [x] $u(x) = (\ln x) x^\alpha$

Correct!

- [x] $u(x) = x^r \cos(\beta \ln x)$
- [ ] $u(x) = x^r \cos(\beta x)$
- [ ] $u(x) = e^{\alpha x}$

Additional Comments:

Question 6

0 / 0 pts

Consider the Cauchy-Euler equation $x^2 u'' - 2u = 0$. We try solutions of the form $u(x) = x^\alpha$.

What is the quadratic equation for α?

- $\alpha^2 - 2 = 0$
- $\alpha(\alpha + 1) - 2 = 0$

Correct!

- $\alpha(\alpha - 1) - 2 = 0$
- $\alpha(\alpha - 1) - 2\alpha = 0$
- $\alpha(\alpha - 1) - 2\alpha^2 = 0$

Additional Comments:

Question 7

0 / 0 pts

For a general function of polar coordinates $u(r, \theta)$, the Laplace operator is $\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$.

Suppose we are given that function *u* has the axial symmetry. What is $\nabla^2 u$?

- $\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2}$
- $\nabla^2 u = \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$
- Correct! $\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$
- $\nabla^2 u = \frac{\partial^2 u}{\partial r^2}$
- $\nabla^2 u = 0$

Additional Comments:

Question 8

0 / 0 pts

Consider the differential equation $\left(x u_x\right)_x = -\lambda x u$. We rewrite it as $x u_{xx} + u_x + \lambda x u = 0$.

$$\implies \quad x^2 u_{xx} + x u_x + \lambda x^2 u = 0$$

Is this a Cauchy-Euler equation?

True

Correct!

False

Additional Comments:

Fudge Points:

You can manually adjust the score by adding positive or negative points to this box.

Final Score: 1 out of 1

Update Scores