Thus.
$$CG$$
 algoriths:

 $X^{(0)} = 0$
 $X^{(0)} = 0$

$$(C) \quad e^{(k)} = X^* - X^{(k)}$$

$$\|e^{(k)}\|_{A} = \inf \|X^* - \chi\|_{A}$$

$$u \in \mathcal{V}^{(k)}$$

Runk.
$$\|e^{(k+1)}\|_A \leq \|e^{dsy}\|_A$$
 Note

$$A > b > 0$$

Runk. Rate of convergence of G

$$\|e^{(n)}\|_A \leq \frac{2}{(\textcircled{D})^n + (\textcircled{D})^n} \|e^{(0)}\|_A$$

$$(\textcircled{X}) = \frac{\sqrt{k+1}}{\sqrt{k}-1} ||e^{(0)}\|_A$$

Calculate (Irl)
$$\beta = -\frac{r^T y}{p^T y}$$

$$p = r + \beta p$$

$$- ond do$$

$$Ax = b$$

$$M^{-1}AX = M^{-1}b$$

Ruk: Mis called a preconditioner

$$(A) = A \qquad (A'Ax = A'b)$$

$$(A) = k(I) = 1 \qquad Ix = A'b$$

$$(A) = k(I) = 1 \qquad Ix = A'b$$

$$(A) = k(I) = 1 \qquad Ix = A'b$$

$$(A) = k(I) = 1 \qquad (easy!)$$

$$(A) = I \qquad (easy!)$$

$$(A) = I \qquad (aseless!)$$

$$(A$$

(Jacobi preconditiner) This works well for A: diagnolly dominant Ruk Not all M= diag(A) works all $A : (aii = D), \forall i$ $aij = l, \forall i \neq j$ Consider For example A ==

$$\rightarrow$$
 cond $(M'A) = cond(\int A) = cond(A)$

M = diag (A) would work!

$$M = C^{-T}C^{-1}$$

$$\rightarrow$$
 $M^{\dagger}AX = \sqrt{C^{-1}AX}$

$$M^{\dagger}b = \sqrt{T}c^{\dagger}b$$

$$\Rightarrow (c^{\dagger}Ax = c^{\dagger}b)$$

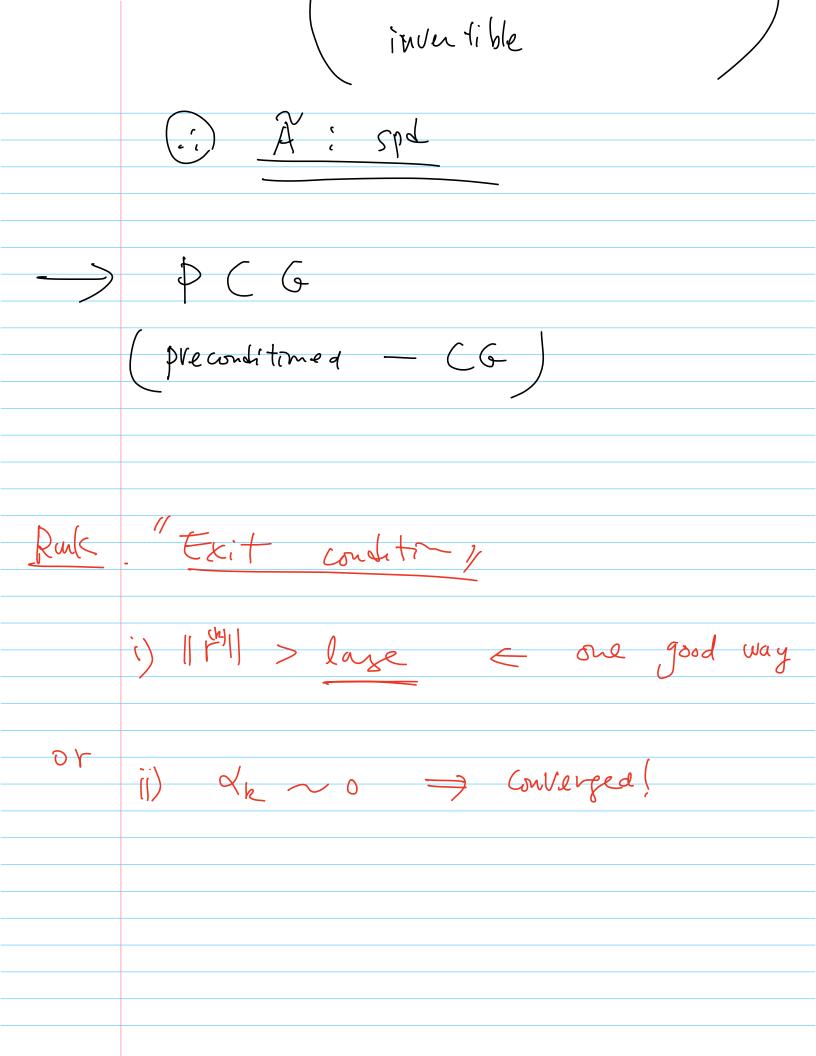
$$= \tilde{A}$$

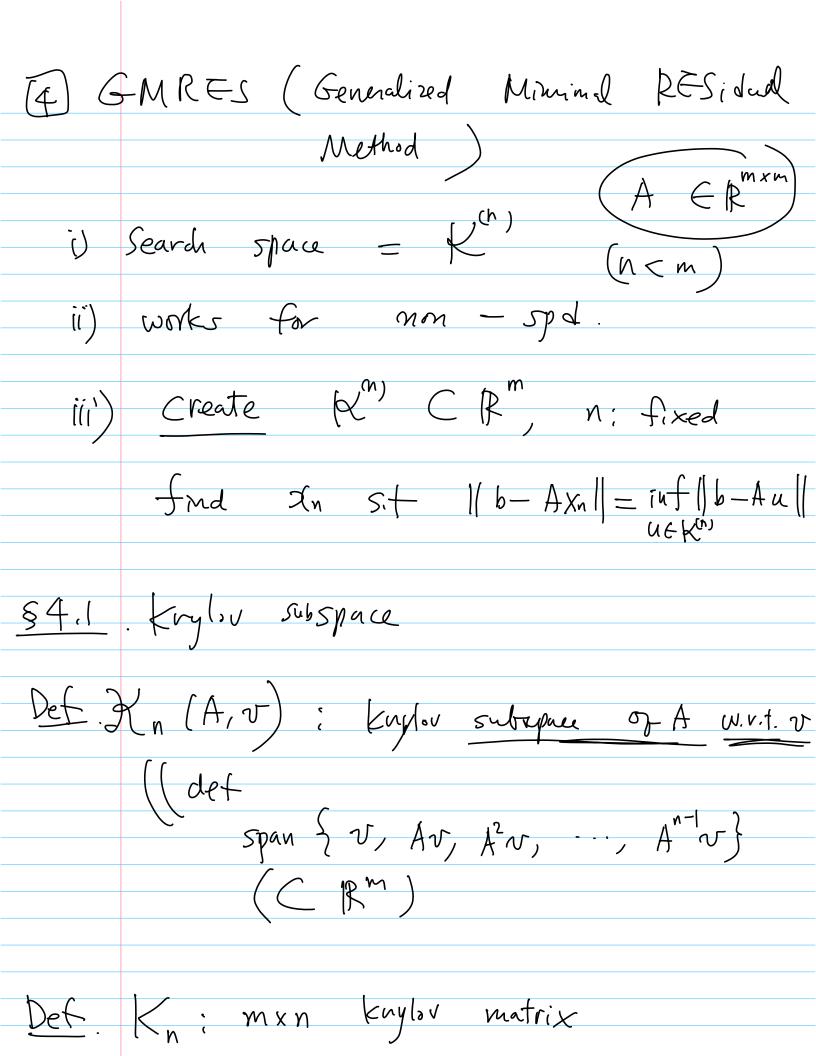
$$= \tilde{A}$$

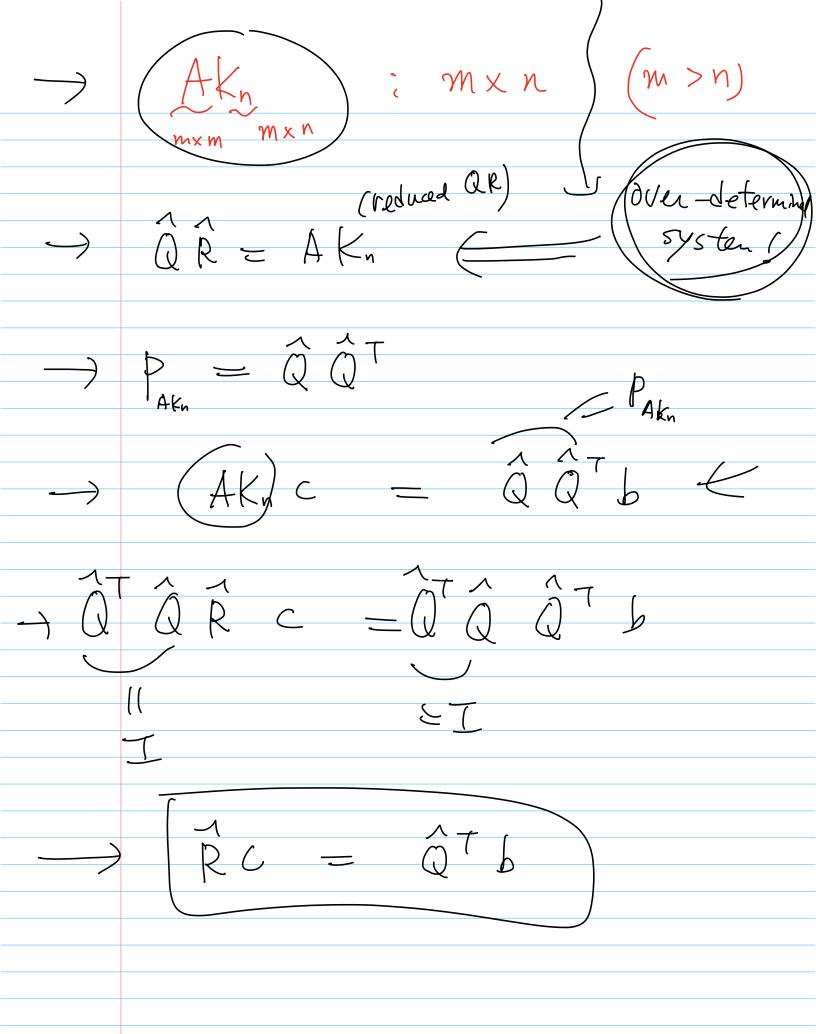
$$\tilde{X} = \tilde{b}$$

$$\Rightarrow (c^{\dagger}Ac^{\dagger}) (c^{\dagger}x) = c^{\dagger}b$$

$$\Rightarrow \tilde{A} \tilde{x} = \tilde{b}$$







Def A; mxn -) A minimal poly of A is "P" Uniquely defined as a monic possible (dn=1)

with the least degree satisfyry P(A) v=0, Vv CR i.e. $P(A)v = \left(\sum_{i=0}^{\infty} d_i A^i\right)v$ $= \left(\langle \langle A \rangle \rangle \right) + \langle \langle A \rangle \rangle + \langle \langle A \rangle$ Ruk Why Kn (A, b) is a good Choice? $(A) = (A) b = (X_0 I + \cdots + A^n) b$ $T = -\frac{1}{\alpha_0} \left(\sum_{i=1}^{n} \alpha_i A \right) b$

$$\frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$$