Lecture 8 Activity Results for Test Student

Score for this attempt: **1** out of 1

Submitted Feb 1 at 4:48pm

This attempt took 1 minute.

Question 1

1 / 1 pts

For a general function of spherical coordinates $u(r, \theta, \phi)$, the Laplace operator is

$$\nabla^2 u = \frac{1}{r^2} \frac{\partial}{\partial r^2} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \phi^2}$$

.

Suppose we are given that function *u* has the spherical symmetry. What is $\nabla^2 u$?

- $\nabla^2 u = \dfrac{\partial^2 u}{\partial r^2} + \dfrac{\partial^2 u}{\partial \theta^2} + \dfrac{\partial^2 u}{\partial \phi^2}$
- $\nabla^2 u = \dfrac{1}{r^2} \dfrac{\partial}{\partial r^2} \left(r^2 \dfrac{\partial u}{\partial r} \right) + \dfrac{1}{r^2 \sin\theta} \dfrac{\partial}{\partial \theta} \left(\sin\theta \dfrac{\partial u}{\partial \theta} \right)$

Correct!

- $\nabla^2 u = \dfrac{1}{r^2} \dfrac{\partial}{\partial r^2} \left(r^2 \dfrac{\partial u}{\partial r} \right)$
- $\nabla^2 u = \dfrac{\partial^2 u}{\partial r^2}$
- $\nabla^2 u = \dfrac{1}{r^2 \sin\theta} \dfrac{\partial}{\partial \theta} \left(\sin\theta \dfrac{\partial u}{\partial \theta} \right) + \dfrac{1}{r^2 \sin^2\theta} \dfrac{\partial^2 u}{\partial \phi^2}$

Additional Comments:

Question 2

0 / 0 pts

In our study, we solved several ODE eigenvalue problems

(ODE1): $x^2 u_{xx} + x u_x + \lambda u = 0,\ \lambda > 0.$

(ODE2): $x^2 u_{xx} + x u_x + \lambda x u = 0,\ \lambda > 0.$

(ODE3): $x^2 u_{xx} + x u_x + \lambda x^2 u = 0,\ \lambda > 0.$

(ODE4): $x^2 u_{xx} + 2x u_x + \lambda x^2 u = 0,\ \lambda > 0.$

Match each application problem below to the corresponding ODE eigenvalue problem.

Correct!	**Axially symmetric vibration of a circular drum.**	ODE3
Correct!	**Small oscillation of a hanging chain**	ODE2
Correct!	**1D heat equation with variable coefficients**	ODE1
Correct!	**Spherically symmetric temperature evolution**	ODE4

Additional Comments:

Question 3

0 / 0 pts

In our study, which of the following did we transform to a Bessel equation? **Select all that apply.**

Correct!

- [x] $x^2u_{xx} + xu_x + \lambda x^2 u = 0, \quad \lambda > 0$
- [ ] $x^2u_{xx} + 2xu_x + \lambda x^2 u = 0, \quad \lambda > 0$

Correct!

- [x] $x^2u_{xx} + xu_x + \lambda xu = 0, \quad \lambda > 0$
- [ ] $x^2u_{xx} + xu_x - x^2 u = 0$
- [ ] $x^2u_{xx} + xu_x + \lambda u = 0, \quad \lambda > 0$

Additional Comments:

Question 4

0 / 0 pts

In our study, how did we solve $x^2 u_{xx} + x u_x + \lambda u = 0,\ \lambda > 0$ for a general solution?

- We transformed it to a Bessel equation via a linear scaling $z = \beta x$.
- We transformed it to a Bessel equation via a non-linear scaling $z = (\beta x)^\eta$.

Correct!

- (selected) We solved it as a Cauchy-Euler equation.
- We transformed it to $y'' + \lambda y = 0$ via a transformation $u(x) = x^\eta y(x)$.
- We transformed it to a Bessel equation via a non-linear scaling $z = e^{\beta x}$.

Additional Comments:

Question 5

0 / 0 pts

In our study, how did we solve $x^2 u_{xx} + 2x u_x + \lambda x^2 u = 0,\ \lambda > 0$ for a general solution?

We transformed it to a Bessel equation via a linear scaling $z = \beta x$.

We transformed it to a Bessel equation via a non-linear scaling $z = (\beta x)^{\eta}$.

We solved it as a Cauchy-Euler equation.

Correct!

We transformed it to $y'' + \lambda y = 0$ via a transformation $u(x) = x^{\eta} y(x)$.

We transformed it to a Bessel equation via a non-linear scaling $z = e^{\beta x}$.

Additional Comments:

Question 6

0 / 0 pts

Consider the BVP of the 2D Laplace equation in polar coordinates in the annulus sector with $R_1 \le r \le R_2$ and $0 \le \theta \le \alpha$.

$$\begin{cases} \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial u}{\partial \theta^2} = 0 \\ u(r,0) = g_1(r),\ u(r,\alpha) = g_2(r) \\ u(R_1,\theta) = 0,\ u(R_2,\theta) = 0 \end{cases}$$

Note that homogeneous BCs are imposed on two arcs (not on the two straight sides). Which statement below is true regarding separation of variables $u(r,\theta) = A(r)u(\theta)$?

- Separation of variables leads to an eigenvalue problem for $u(\theta)$.

Correct!

- Separation of variables leads to an eigenvalue problem for $A(r)$.

- In separation of variables, we can choose to have an eigenvalue problem for $u(\theta)$ or an eigenvalue problem for $A(r)$ according to our preference.

- Separation of variables is not applicable to this BVP.

Additional Comments:

Question 7

0 / 0 pts

Consider the Cauchy-Euler equation
$r^2 A''(r) + rA'(r) - \beta^2 A(r) = 0.$

For β = 0, which one is a general solution?

- $A(r) = c_1 r^{\beta} + c_2 r^{-\beta}$

Correct!

- $A(r) = c_1 + c_2 \ln r$
- It does not have a general solution for β = 0.
- $A(r) = c_1 + c_2 r$

Additional Comments:

Question 8

0 / 0 pts

Consider the Cauchy-Euler equation
$r^2 A''(r) + rA'(r) - \beta^2 A(r) = 0.$

For β > 0, which one is a general solution?

Correct!

- $A(r) = c_1 r^{\beta} + c_2 r^{-\beta}$

$A(r) = c_1 + c_2 \ln r$

It does not have a general solution for β > 0.

$A(r) = c_1 + c_2 r$

Additional Comments:

Fudge Points: --

You can manually adjust the score by adding positive or negative points to this box.

Final Score: 1 out of 1

Update Scores