Instructions for the preparation test

1. This is an open response exam using the venue of canvas quiz. The exam problems are given below. Complete the problems by yourself. You can use books and notes. But do not seek any help from others.

   Clearly mark each problem and box the final answer as shown below.

   Problem X: Find a general solution of $u''(x) = 2u(x)$.

   ...

   $$\boxed{\text{Final answer: } u(x) = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}}$$

2. You have ONE attempt with a time limit of 100 minutes (including the time for scanning and submission). **DO NOT click the "Take the Quiz" button until you are ready and have the next 100 minutes dedicated to take the exam.**

3. Complete the problems on paper. Scan all pages of your exam paper into ONE pdf and name the pdf using your own name. **Do NOT name it "Prep_Test"** (this is to prevent you from accidentally uploading the list of exam problems as your answer!). Upload the scanned pdf as your answer in canvas.

4. After you finish submission, **upload the scanned pdf again as an attachment in the comment of your exam.**

List of exam problems

Problem 1: Find a general solution of $u''(x) = -2u(x)$.

Problem 2: Find a general solution of $u''(x) + 2u'(x) + u(x) = 0$.

Problem 3: Consider the initial value problem

$$\begin{cases} u'(t) = -2u(t) + 2\cos(t) - \sin(t) \\ u(0) = 0 \end{cases} \qquad \text{(IVP-P3)}$$

It is straightforward to verify that $u_p(t) = \cos(t)$ is a particular solution of the differential equation. Use the pinciple of superposition to solve (IVP-P3). The final answer should be the solution of (IVP-P3).

Problem 4: Find the Fourier series of $f(x) = (1 - \frac{x}{L})$, $x \in [0, L]$.

Problem 5: Find the Fourier COSINE series of $f(x) = (1 - \frac{x}{L})$, $x \in [0, L]$.

Problem 6: Find the Fourier SINE series of $f(x) = (1 - \frac{x}{L})$, $x \in [0, L]$.

Problem 7: Find the Fourier SINE series of $f(x) = 1$, $x \in [0, L]$.

Problem 8: Use L'Hospital's rule to find $\lim_{\alpha \to 3} \dfrac{e^{\alpha x} - e^{3x}}{\alpha - 3}$

Problem 9: Use complex exponential to calculate $\sin(\frac{n\pi}{2})$. Discuss the cases of $n = 2k$ and $n = 2k+1$. The final answer should be in the form of

$$\sin(\frac{n\pi}{2}) = \begin{cases} \square, & n = 2k \\ \square, & n = 2k+1 \end{cases}$$

Problem 10: Consider function $u(x)$. We introduce a new variable:

$$z(x) = x^{\eta} \quad \Rightarrow \quad x(z) = z^{1/\eta}$$

We introduce a new function of variable z:

$$w(z) \equiv u\big(x(z)\big) \quad \Rightarrow \quad u(x) = w\big(z(x)\big)$$

Use the chain rule to express $\frac{du}{dx}$ in terms of $\frac{dw}{dz}$ and z. All instances of x should be replaced with $z^{1/\eta}$. The final answer for $\frac{du}{dx}$ should contain only $\frac{dw}{dz}$ and z, not x.