Section 6

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UCSC Math-19B

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Plan for Today

Topics to Cover

- Partial Fraction Decomposition

Section Activity 6

- 1 question

Upcoming Assignments

- Homework 6 (Due Fri, Feb. 16^{th})
- Midterm (On Mon, Feb 26^{th})

Learning Outcomes

- Revisiting the notion of substitution and trigonometric substitution.
- Applying the methods of Partial Fraction Decomposition.

Trig Substitution

This substitution method is used when a trigonometric identity can reduce the complexity of an integral. Lets look at an example.

$$\int \sqrt{1 - x^2} dx$$

$$x = \sin(\theta), \quad dx = \cos(\theta) d\theta$$

$$\int \sqrt{\cos^2(\theta)} \cos(\theta) d\theta$$

$$= \int \cos^2(\theta) d\theta$$

The Usual Suspects:

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\cot^2(x) + 1 = \csc^2(x)$$

Partial Fraction Decomposition

Partial Fraction Decomposition expands rational function into a sum/difference of smaller rational functions. i.e.

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

Notice that the two terms on the right are easier to integrate than the one on the left. They integrate into natural logarithms.

Partial Fraction Decomposition (Continued)

Partial Fraction Decomposition coefficients are found using a series of equations.

$$1 = A(x-3) + B(x-2)$$

Note: it helps to isolate the equations by orders of x. One equation for x^0, one for x^1, one for x^2, and so on as needed.

$$A + B = 0, \quad -3A - 2B = 1$$

$$A = -B, \quad -3A + 2A = 1$$

$$A = -1, \quad B = 1$$

The slightly more complicated case

We also have the case where the polynomial in the denominator is not completely factorable.

$$\int \frac{dx}{x^3 + 4x}$$

$$x^3 + 4x = x(x^2 + 4)$$

Notice that $x^2 + 4$ can't be factored without introducing complex numbers. We therefore need the following form of Partial Fraction Decomposition.

$$\frac{1}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

$$1 = A(x^2 + 4) + (Bx + C)x$$

$$1 = 4A, \quad 0 = A + B, \quad 0 = C$$

$$A = \frac{1}{4}, \quad B = -\frac{1}{4}, \quad C = 0$$

Warm Up - Partial Fraction Decomposition

$$\int \frac{1}{x^2 - 2x} dx$$

When your group is done working on this problem, review polyonomial long division and then let me know (I'll come give you the section activity code).