Section 8

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UCSC Math-19B

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Plan for Today

Topics to Cover

- Infinite Sequences and Series
- Convergence
- Taylor Polynomials

Section Activity 8

- 5 question

Upcoming Assignments

- Homework 8 (Due Mon, Mar. 8^{th})
- Project 2 (Due Fri, Mar. 15^{th})

Learning Outcomes

- Understanding the notion of sequences and series
- Understanding what different convergence tests imply about a sequence or series
- Applying the concept of Taylor Polynomials to complex functions.

Sequences and Series

Sequences are collections of terms which are identified by an index. For example, the following sequence S is a collection of terms a_i identified by the index in the subscript, i.

$$S = \{a_1, \cdots, a_n\}, a_i = f(n)$$

An infinite series, is an infinite sum of terms. We can take for instance the sum of a sequence.

$$\text{Infinite Series } = \sum_{i=1}^{\infty} a_i$$

Convergence

For many sequences and series, the notion of convergence is often relevant or interesting. For example as we take N to infinity, is a_N finite? Does the sum $\sum_{i=1}^{N} a_n$ approach a value? To answer these questions we use some analytical tools:

1. Integral Test
2. Comparison Test
3. Limit Comparison Test
4. Ratio Test

Convergence Tests

Integral Test:

$$a_n = f(n), \quad \sum_{i=1}^{\infty} a_i \leq \int_1^{\infty} f(x)dx$$

Comparison Test:

$$0 \leq a_n \leq b_n, \forall n > 0, \quad \sum_{i=1}^{\infty} a_i \leq \sum_{i=1}^{\infty} b_i$$

$$\text{if } \sum_{i=1}^{\infty} b_i < \infty \implies, \sum_{i=1}^{\infty} a_i < \infty$$

$$\text{if } \sum_{i=1}^{\infty} a_i \text{ diverges } \implies \sum_{i=1}^{\infty} b_i \text{ diverges}$$

Convergence Tests

Ratio Test:

$$\rho = \lim_{n\to\infty} = \left|\frac{a_{n+1}}{a_n}\right|$$

$$\text{if } \rho < 1, \implies \sum a_n < \infty$$

$$\text{if } \rho > 1, \implies \sum a_n \text{ diverges}$$

$$\text{if } \rho = 1, \ \text{inconclusive}$$

Taylor Polynomials

Taylor Polynomials (also known as Taylor Expansions) are very useful mathematical tools when you need to simplify a complex term in a local area. Say we want to fit a polynomial to $\sin(x)$ at $x = 1$. We use taylor expansion.

$$T_n(x) = \sum_{i=0}^{n} \frac{(x-a)^i}{i!} f^{(i)}(a)$$

$$T_n(x) = f(a) + (x-a)f'(a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$T_3(x) = \sin(1) + (x-1)\cos(1) - \frac{\sin(1)}{2}(x-1)^2 - \frac{\cos(1)}{3!}(x-1)^3$$