

Homework 4

Gomez - Math 19B

Due: Feb 9th, 2024

1 U-Substitution

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| 1. | 2. |
| 3. | 4. |
| 5. | 6. |
| 7. | 8. |
| 9. | |

2 Disk Method

- 10.
- 11.

3 Cylindrical Shell Method

- 12.
- 13.

4 Application Problem

- 14.

Equations on next page

5 Equations

5.1 U-Substitution:

$$\int_a^b f'(g(x))g'(x)dx \implies \int_{u(a)}^{u(b)} f'(u)du = F(u(a)) - F(u(b)) \quad (1)$$

$$u = g(x) \rightarrow du = g'(x)dx \quad (2)$$

5.2 Disk, Washer, Cylindrical Shell Methods:

Disk Method

$$V = \pi \int_a^b R^2 dx = \pi \int_a^b (f(x))^2 dx \quad (3)$$

Washer Method

$$V = \pi \int_a^b (R^2 - r^2) dx = \pi \int_a^b ((f(x))^2 - (g(x))^2) dx \quad (4)$$

Shell Method

$$V = 2\pi \int_a^b Rh dx = 2\pi \int_a^b x f(x) dx \quad (5)$$

5.3 Properties of Integrals:

$$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx \quad (6)$$

$$\int C f(x) dx = C \int f(x) dx, \quad C = \text{const.} \quad (7)$$

Additivity Theorem for Adjacent Integrals

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, \quad a < c < b \quad (8)$$

5.4 Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a) \quad (9)$$

$$F(x) = \int_a^x f(s)ds \quad (10)$$

$$\frac{d}{dx}F(x) = \frac{d}{dx} \int_c^x f(s)ds = f(x) \quad (11)$$