Lecture 11 Activity Results for Test Student

Score for this attempt: **1** out of 1

Submitted Feb 13 at 10:57am

This attempt took 4 minutes.

Question 1

1 / 1 pts

Consider the IBVP $\begin{cases} u_t = ku_{xx}, \quad t > 0 \\ u(0,t) = g_1(t), \quad u(L,t) = g_2(t) \\ u(x,0) = f(x) \end{cases}$.

How do we deal with the time dependent inhomogeneous BCs?

- Apply the separation of variables directly to the IBVP with inhomogeneous BCs.

- Find the steady state solution $u_\infty(x)$ satisfying $u''_\infty(x) = 0,\ u_\infty(0) = g_1(t),\ u_\infty(L) = g_2(t)$.

  Use $u_\infty(x)$ to get rid of inhomogeneous BCs. Then apply the separation of variables to the resulting IBVP with homogeneous BCs.

Correct!

- Find a particular function $u^{(b)}(x,t)$ satisfying $u^{(b)}(0,t) = g_1(t),\ u^{(b)}(L,t) = g_2(t)$.

  Use $u^{(b)}(x,t)$ to get rid of inhomogeneous BCs. Then apply the separation of variables to the resulting IBVP with homogeneous BCs.

- Use $u^{(L)}(x,t) = g_1(t)$ to get rid of inhomogeneous BCs. Then apply the separation of variables to the resulting IBVP with homogeneous BCs.

Use $u^{(R)}(x,t) = g_2(t)$ to get rid of inhomogeneous BCs. Then apply the separation of variables to the resulting IBVP with homogeneous BCs.

Additional Comments:

Question 2

0 / 0 pts

Consider the IBVP

$$\begin{cases} u_t = ku_{xx} + F(x,t), \quad t > 0 \\ u(0,t) = 0, \quad u(L,t) = 0 \\ u(x,0) = 0 \end{cases}.$$

How do we deal with the forcing term $F(x,t)$?

Correct!

Apply the separation of variables directly to the IBVP with forcing term $F(x,t)$.

Find the steady state solution $u_\infty(x)$ satisfying

$u''_\infty(x) = \frac{-1}{k}F(x,t),\ u_\infty(0) = 0,\ u_\infty(L) = 0.$

Use $u_\infty(x)$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Find a particular function $u^{(f)}(x,t)$ satisfying $u_t^{(f)} = ku_{xx}^{(f)} + F(x,t)$.

Use $u^{(f)}(x,t)$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Use $u^{(t)}(x,t) = \int F(x,t)dt$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Use $u^{(x)}(x,t) = \frac{-1}{k}\int\left(\int F(x,t)dx\right)dx$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Additional Comments:

Question 3

0 / 0 pts

Sometimes it is more clear and concise to work with complex exponential.

Consider the IVP $\begin{cases} y'(t) = -\alpha y(t) + e^{i\omega t} \\ y(0) = y_0 \end{cases}$.

We rewrite it as

$$\begin{cases} \left(y(t) - \frac{1}{\alpha+i\omega}e^{i\omega t}\right)' = -\alpha\left(y(t) - \frac{1}{\alpha+i\omega}e^{i\omega t}\right) \\ \left(y(t) - \frac{1}{\alpha+i\omega}e^{i\omega t}\right)\Big|_{t=0} = y_0 - \frac{1}{\alpha+i\omega} \end{cases}.$$

What is $y(t)$?

$y(t) = \dfrac{1}{\alpha+i\omega}e^{i\omega t}$

$y(t) = \left(y_0 - \dfrac{1}{\alpha+i\omega}\right)e^{-\alpha t}$

Correct!

$y(t) = \dfrac{1}{\alpha + i\omega} e^{i\omega t} + \left(y_0 - \dfrac{1}{\alpha + i\omega}\right) e^{-\alpha t}$

$y(t) = \dfrac{1}{\alpha + i\omega} e^{i\omega t} + y_0 e^{-\alpha t}$

$y(t) = \dfrac{1}{\alpha + i\omega} e^{i\omega t} - \dfrac{1}{\alpha + i\omega} e^{-\alpha t}$

The solution is always bounded regardless of the forcing frequency.

Additional Comments:

Question 4

0 / 0 pts

Consider the IBVP

$$\begin{cases} u_{tt} = c^2 u_{xx} + F(x,t), \quad t > 0 \\ u(0,t) = 0, \quad u(L,t) = 0 \\ u(x,0) = 0, \quad u_t(x,0) = 0 \end{cases}.$$

How do we deal with the forcing term $F(x,t)$?

Correct!

Apply the separation of variables directly to the IBVP with forcing term $F(x,t)$.

Find the steady state solution $u_\infty(x)$ satisfying
$u''_\infty(x) = \frac{-1}{c^2}F(x,t),\ u_\infty(0) = 0,\ u_\infty(L) = 0.$

Use $u_\infty(x)$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Find a particular function $u^{(f)}(x,t)$ satisfying $u^{(f)}_{tt} = c^2 u^{(f)}_{xx} + F(x,t)$.

Use $u^{(f)}(x,t)$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Use $u^{(t)}(x,t) = \int \Big(\int F(x,t)dt \Big) dt$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Use $u^{(x)}(x,t) = \frac{-1}{c^2} \int \Big(\int F(x,t)dx \Big) dx$ to get rid of the forcing term $F(x,t)$. Then apply the separation of variables to the resulting IBVP with zero forcing.

Additional Comments:

Question 5

0 / 0 pts

Recall that for $\omega^2 \neq \alpha^2$, the solution of $\begin{cases} y''(t) = -\alpha^2 y(t) + \cos(\omega t) \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$

is $y(t) = \dfrac{\cos(\omega t) - \cos(\alpha t)}{\alpha^2 - \omega^2}$.

What is the solution when $\omega^2 = \alpha^2$?

- $y(t) = \dfrac{\cos(\omega t) - \cos(\alpha t)}{\alpha^2 - \omega^2}$
- $y(t) = \dfrac{1 - \cos(\alpha t)}{\alpha^2}$
- $y(t) = \dfrac{\cos(\alpha t)}{\alpha}$

Correct!

- **$y(t) = \dfrac{t\sin(\alpha t)}{2\alpha}$**
- $y(t) = \dfrac{\sin(\alpha t)}{\alpha}$

Additional Comments:

Question 6

0 / 0 pts

We are given that $y(t) = \dfrac{\sin(\alpha t)}{\alpha}$ is the solution of

$$\begin{cases} y''(t) = -\alpha^2 y(t) + f(t) \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$$

What is the forcing $f(t)$?

- $f(t) = \sin(\alpha t)$

- $f(t) = \cos(\alpha t)$

Correct!

- $f(t) = \delta(t - 0_+)$
- $f(t) = \dfrac{\sin(\alpha t)}{\alpha}$
- $f(t) = \dfrac{\cos(\alpha t)}{\alpha}$

Additional Comments:

Question 7

0 / 0 pts

Consider the forced oscillation governed by

$$\begin{cases} y''(t) = -\alpha^2 y(t) + a_f \cos(\omega t) \\ y(0) = 0, \quad y'(0) = 0 \end{cases}.$$

What value of forcing frequency ω can excite an oscillation of the form $y(t) = a_w(\cos(2\alpha t) - 1)$ (twice the intrinsic frequency)?

- $\omega = \alpha$
- $\omega = 2\alpha$
- $\omega = 3\alpha$

Correct!

- It is impossible to excite $y(t) = a_w(\cos(2\alpha t) - 1)$ from equilibrium with a periodic forcing.

$\omega = 2$

We can excite $y(t) = a_w(\cos(2\alpha t) - 1)$ from equilibrium with a well controlled time profile $f(t)$ of forcing.

Additional Comments:

Question 8

0 / 0 pts

The "beating" refers to the situation where in the time dimension, the amplitude develops a slow oscillating envelop (i.e., the sound volume goes up and down slowly).

The "beating" occurs when

- a mode interacts with another mode of exactly the same frequency.
- a mode interacts with another mode of twice the frequency.
- a mode interacts with another mode of a much lower frequency.
- Correct! a mode interacts with another mode of almost the same frequency.
- a mode interacts with another mode of π times the frequency.

Additional Comments:

Fudge Points: --

You can manually adjust the score by adding positive or negative points to this box.

Final Score: 1 out of 1

Update Scores