Lecture 10 Activity

Due Feb 8 at 3:10pm **Points** 1 **Questions** 8
Available Feb 8 at 1:30pm - Feb 8 at 3:10pm 1 hour and 40 minutes
Time Limit 60 Minutes **Allowed Attempts** 2

Instructions

Be sure to select the correct answer in Q1 to get the participation credit.

This quiz was locked Feb 8 at 3:10pm.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	3 minutes	1 out of 1

Score for this attempt: **1** out of 1
Submitted Feb 8 at 4:33pm
This attempt took 3 minutes.

Question 1

1 / 1 pts

Consider the BVP of the 3D Laplace equation in a ball in spherical coordinates (r, θ, ϕ) with $u(R, \theta, \phi) = f(\theta, \phi)$ imposed on its boundary sphere.

What are the boundary conditions in the θ-dimension?

- $u(r, 0, \phi) = 0, \quad u(r, \pi, \phi) = 0$
- $u(r, 0, \phi) = 0, \quad u(r, 2\pi, \phi) = 0$

Correct!

- $u(r, 0, \phi) = \text{finite}, \quad u(r, \pi, \phi) = \text{finite}$
- $u(r, 0, \phi) = \text{finite}, \quad u(r, 2\pi, \phi) = \text{finite}$
- $u(r, \theta, \phi)$ is periodic in θ with period $= \pi$
- $u(r, \theta, \phi)$ is periodic in θ with period $= 2\pi$

Question 2

0 / 0 pts

Consider the BVP of the 3D Laplace equation in a ball in spherical coordinates (r, θ, ϕ) with $u(R, \theta, \phi) = f(\theta, \phi)$ imposed on its boundary sphere.

What are the boundary conditions in the ϕ-dimension?

- $u(r, \theta, 0) = 0, \quad u(r, \theta, \pi) = 0$
- $u(r, \theta, 0) = 0, \quad u(r, \theta, 2\pi) = 0$
- $u(r, \theta, 0) = \text{finite} \;\; u(r, \theta, \pi) = \text{finite}$
- $u(r, \theta, 0) = \text{finite} \;\; u(r, \theta, 2\pi) = \text{finite}$
- $u(r, \theta, \phi)$ is periodic in ϕ with period $= \pi$

Correct!

- $u(r, \theta, \phi)$ is periodic in ϕ with period $= 2\pi$

Question 3

0 / 0 pts

Consider the BVP of the 3D Laplace equation in a ball in spherical coordinates (r, θ, ϕ) with $u(R, \theta, \phi) = f(\theta, \phi)$ imposed on its boundary sphere.

We apply separation of variables to solve this BVP. Which statement below is true?

- We will get a 2D eigenvalue problem in (r, θ) and an ODE in ϕ.

Correct!

- (selected) We will get a 2D eigenvalue problem in (θ, ϕ) and an ODE in *r*.
- We will get a 2D eigenvalue problem in (r, ϕ) and an ODE in θ.
- Separation of variables is not applicable to this BVP.

Question 4

0 / 0 pts

Consider the 2D Sturm-Liouville problem on the unit sphere.

$$\begin{cases} \dfrac{\partial}{\partial \theta}\left(\sin\theta \dfrac{\partial u}{\partial \theta}\right) + \dfrac{1}{\sin\theta}\dfrac{\partial^2 u}{\partial \phi^2} = -\lambda(\sin\theta)u \\ u(\theta, \phi) \text{ is periodic in } \phi \text{ with period} = 2\pi \\ u(0, \phi) = \text{finite},\ u(\pi, \phi) = \text{finite} \end{cases}$$

Which statement below is true?

It is not a proper eigenvalue problem because its boundary conditions are not homogeneous.

It is a proper eigenvalue problem. However, we cannot apply separation of variable to this eigenvalue problem because its boundary conditions are not separable.

Correct!

We can apply separation of variable to this eigenvalue problem but the two 1D eigenvalue problems have to be solved sequentially.

We can apply separation of variable to this eigenvalue problem and the two 1D eigenvalue problems can be solved separately, independent of each other.

Question 5

0 / 0 pts

Consider the 2D Sturm-Liouville problem on the unit sphere.

$$\begin{cases} \dfrac{\partial}{\partial \theta}\left(\sin\theta \dfrac{\partial u}{\partial \theta}\right) + \dfrac{1}{\sin\theta}\dfrac{\partial^2 u}{\partial \phi^2} = -\lambda(\sin\theta)u \\ u(\theta,\phi) \text{ is periodic in } \phi \text{ with period} = 2\pi \\ u(0,\phi) = \text{finite},\ u(\pi,\phi) = \text{finite} \end{cases}$$

Separation of variables $u(\theta,\phi) = a(\theta)b(\phi)$ leads to

$$\frac{b''(\phi)}{b(\phi)} = \frac{-\sin\theta\big((\sin\theta)a'(\theta)\big)'}{a(\theta)} - \lambda\sin^2\theta = -\mu$$

independent of (θ,ϕ).

Which statement below is true?

The two 1D eigenvalue problems can be solved separately, independent of each other.

We first solve the eigenvalue problem for $a(\theta)$. Then we solve the eigenvalue problem for $b(\phi)$.

Correct!

We first solve the eigenvalue problem for $b(\phi)$. Then we solve the eigenvalue problem for $a(\theta)$.

We solve the eigenvalue problem for $b(\phi)$ partially and solve the eigenvalue problem for $a(\theta)$ partially. Then we repeat the process iteratively.

Separation of variables is not applicable to this 2D eigenvalue problem.

Question 6

0 / 0 pts

Consider the 2D Sturm-Liouville problem on the unit sphere.

$$\begin{cases} \dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial u}{\partial\theta}\right)+\dfrac{1}{\sin\theta}\dfrac{\partial^2 u}{\partial\phi^2}=-\lambda(\sin\theta)u \\ u(\theta,\phi) \text{ is periodic in } \phi \text{ with period} = 2\pi \\ u(0,\phi)=\text{finite},\ u(\pi,\phi)=\text{finite} \end{cases}$$

Separation of variables $u(\theta,\phi)=a(\theta)b(\phi)$ leads to

$$\frac{b''(\phi)}{b(\phi)}=\frac{-\sin\theta\big((\sin\theta)a'(\theta)\big)'}{a(\theta)}-\lambda\sin^2\theta=-\mu$$

independent of (θ,ϕ).

For $\mu=m^2$, the differential equation for $a(\theta)$ is

$$\big((\sin\theta)a'(\theta)\big)'-\frac{m^2}{\sin\theta}a(\theta)=-\lambda(\sin\theta)a(\theta).$$

How do we solve this differential equation?

- ( ) We solve it directly since it is a Bessel equation.
- ( ) We solve it directly since it is a Cauchy-Euler equation.
- ( ) We use a scaling to convert it to a Bessel equation.
- ( ) We use a scaling to convert it to a Cauchy-Euler equation.

Correct!

- (•) We use a change of variables to convert it to a general Legendre equation.

Question 7

0 / 0 pts

What are the spherical harmonics?

- The spherical harmonics are the eigenfunctions in the 2D eigenvalue problem for $u(x, y)$ in a rectangle.
- The spherical harmonics are the eigenfunctions in the 2D eigenvalue problem for $u(r, \theta)$ in a disk.

Correct!

- The spherical harmonics are the eigenfunctions in the 2D eigenvalue problem for $u(\theta, \phi)$ on the unit sphere.
- The spherical harmonics are the eigenfunctions in the 3D eigenvalue problem for $u(r, \theta, \phi)$ in a ball
- The spherical harmonics are $\left\{ e^{im\phi} \sin(n\theta), \quad \begin{matrix} m = \text{integer} \in (-\infty, +\infty) \\ n = \text{integer} > 0 \end{matrix} \right\}$

Question 8

0 / 0 pts

Recall that the spherical harmonics are

$$\left\{Y_\ell^m(\theta,\phi)=\sigma_{m,\ell}P_\ell^{|m|}(\cos\theta)e^{im\phi},\ \ -\ell<m<\ell,\ \ell=0,1,2,..\right.$$

where $P_\ell^{|m|}(x)$ is the associated Legend polynomial of degree ℓ and order $|m|$; $\sigma_{\ell,m}$ is the normalizing coefficient to make $Y_\ell^m(\theta,\phi)$ have norm 1.

What is the utility of spherical harmonics?

- ○ The spherical harmonics are for expanding function $u(x,y)$ in a rectangle.
- ○ The spherical harmonics are for expanding function $u(r,\theta)$ in a disk.

Correct!

- ◉ The spherical harmonics are for expanding function $u(\theta,\phi)$ on the unit sphere.
- ○ The spherical harmonics are not orthogonal but they can be used for expanding function $u(r,\theta,\phi)$ in a ball.
- ○ The spherical harmonics are orthogonal and they are good for expanding function $u(r,\theta,\phi)$ in a ball.

Quiz Score: **1** out of 1