Lecture 9 Activity Results for Test Student

Score for this attempt: **1** out of 1

Submitted Feb 6 at 3:45pm

This attempt took less than 1 minute.

Question 1

1 / 1 pts

Consider the 2D Sturm-Liouville problem in $D = [0, L_1] \times [0, L_2]$.

$$\begin{cases} u_{xx} + u_{yy} = -\lambda u, \quad (x, y) \in D \\ u(0, y) = 0, \quad u(L_1, y) = 0 \\ u(x, 0) = 0, \quad \begin{cases} u(x, L_2) = 0, \; x \in [0, L_1/2] \\ u_y(x, L_2) = 0, \quad x \in [L_1/2, L_1] \end{cases} \end{cases}$$

Which statement below is true?

- It is not a proper eigenvalue problem because its boundary conditions are not homogeneous.

Correct!

- It is a proper eigenvalue problem. However, we cannot apply separation of variable to this eigenvalue problem because its boundary conditions are not separable.

- We can apply separation of variable to this eigenvalue problem but the two 1D eigenvalue problems have to be solved sequentially.

- We can apply separation of variable to this eigenvalue problem and the two 1D eigenvalue problems can be solved separately, independent of each other.

Additional Comments:

Question 2

0 / 0 pts

Consider the 2D Sturm-Liouville problem in $D = [0, L_1] \times [0, L_2]$.

$$\begin{cases} u_{xx} + u_{yy} = -\lambda u, \quad (x, y) \in D \\ u(0, y) = 0, \quad u(L_1, y) = 0 \\ u(x, 0) = 0, \quad u_y(x, L_2) = 0 \end{cases}$$

Which statement below is true?

- It is not a proper eigenvalue problem because its boundary conditions are not homogeneous.

- It is a proper eigenvalue problem. However, we cannot apply separation of variable to this eigenvalue problem because its boundary conditions are not separable.

- We can apply separation of variable to this eigenvalue problem but the two 1D eigenvalue problems have to be solved sequentially.

Correct!

We can apply separation of variable to this eigenvalue problem and the two 1D eigenvalue problems can be solved separately, independent of each other.

Additional Comments:

Question 3

0 / 0 pts

Consider the 2D Sturm-Liouville problem in the disk *D*=disk(0, *R*).

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r}\dfrac{\partial^2 u}{\partial \theta^2} = -\lambda r u, \quad (x,y)\in D \\ u(r,\theta) \text{ is periodic in } \theta \text{ with period} = 2\pi \\ u(0,\theta) = \text{finite}, \ u(R,\theta)=0 \end{cases}$$

Which statement below is true?

It is not a proper eigenvalue problem because its boundary conditions are not homogeneous.

It is a proper eigenvalue problem. However, we cannot apply separation of variable to this eigenvalue problem because its boundary conditions are not separable.

Correct!

We can apply separation of variable to this eigenvalue problem but the two 1D eigenvalue problems have to be solved sequentially.

We can apply separation of variable to this eigenvalue problem and the two 1D eigenvalue problems can be solved separately, independent of each other.

Additional Comments:

Question 4

0 / 0 pts

Consider the 2D Sturm-Liouville problem in the disk D=disk(0, R).

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r}\dfrac{\partial^2 u}{\partial \theta^2}=-\lambda r u, \quad (x,y)\in D \\ u(r,\theta) \text{ is periodic in } \theta \text{ with period} = 2\pi \\ u(0,\theta)=\text{finite}, \;\; u(R,\theta)=0 \end{cases}$$

Separation of variables $u(r,\theta)=a(r)b(\theta)$ leads to

$$\frac{b''(\theta)}{b(\theta)}=\frac{-r(ra'(r))'}{a(r)}-\lambda r^2=-\mu \;\text{ independent of } (r,\theta)$$

Which statement below is true?

The two 1D eigenvalue problems can be solved separately, independent of each other.

We first solve the eigenvalue problem for $a(r)$. Then we solve the eigenvalue problem for $b(\theta)$.

Correct!

We first solve the eigenvalue problem for $b(\theta)$. Then we solve the eigenvalue problem for $a(r)$.

We solve the eigenvalue problem for $b(\theta)$ partially and solve the eigenvalue problem for $a(r)$ partially. Then we repeat the process iteratively.

Separation of variables is not applicable to this 2D eigenvalue problem.

Additional Comments:

Question 5

0 / 0 pts

Consider the 2D Sturm-Liouville problem in the disk *D*=disk(0, *R*).

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r}\dfrac{\partial^2 u}{\partial \theta^2} = -\lambda r u, \quad (x,y) \in D \\ u(r,\theta) \text{ is periodic in } \theta \text{ with period} = 2\pi \\ u(0,\theta) = \text{finite}, \ u(R,\theta) = 0 \end{cases}$$

Separation of variables $u(r,\theta) = a(r)b(\theta)$ leads to

$$\frac{b''(\theta)}{b(\theta)} = \frac{-r(ra'(r))'}{a(r)} - \lambda r^2 = -\mu \ \text{ independent of } (r,\theta)$$

For $\mu = n^2$, the differential equation for $a(r)$ is

$r^2 a'' + r a' + (\lambda r^2 - n^2) a = 0$.

How do we solve this differential equation?

- We solve it directly since it is a Bessel equation.
- We solve it directly since it is a Cauchy-Euler equation.

Correct!

- (selected) We use a scaling to convert it to a Bessel equation.
- We use a scaling to convert it to a Cauchy-Euler equation.
- We use a transformation to convert it to $y'' + \lambda y = 0$.

Additional Comments:

Question 6

0 / 0 pts

Consider the 2D Sturm-Liouville problem in $D = [0, L_1] \times [0, L_2]$.

$$\begin{cases} u_{xx} + u_{yy} = -\lambda u, \quad (x, y) \in D \\ u(0, y) = 0, \quad u_x(L_1, y) = 0 \\ u(x, 0) = 0, \quad u(x, L_2) = 0 \end{cases}$$

Separation of variables $u(x, y) = a(x)b(y)$ leads to

$$\frac{a''(x)}{a(x)} = \frac{-b''(y)}{b(y)} - \lambda = -\lambda^{(1)} \text{ independent of } (x, y)$$

What is the solution of the eigenvalue problem for $a(x)$?

- $\lambda_n^{(1)} = (\frac{n\pi}{L})^2, \quad a_n(x) = \sin(\frac{n\pi}{L}x), \quad n = 1, 2, \ldots$
- $\lambda_n^{(1)} = (\frac{n\pi}{L})^2, \quad a_n(x) = \cos(\frac{n\pi}{L}x), \quad n = 0, 1, 2, \ldots$
- $\lambda_n^{(1)} = (\frac{(n-\frac{1}{2})\pi}{L})^2, \quad a_n(x) = \cos(\frac{(n-\frac{1}{2})\pi}{L}x), \quad n = 1, 2, \ldots$
- $\lambda_n^{(1)} = (\frac{(n-\frac{1}{2})\pi}{L})^2, \quad a_n(x) = \sin(\frac{(n-\frac{1}{2})\pi}{L}x), \quad n = 0, 1, 2, \ldots$

Correct!

- $\lambda_n^{(1)} = (\frac{(n-\frac{1}{2})\pi}{L})^2, \quad a_n(x) = \sin(\frac{(n-\frac{1}{2})\pi}{L}x), \quad n = 1, 2, \ldots$

Additional Comments:

Question 7

0 / 0 pts

Consider the 1D eigenvalue problem

$$\begin{cases} b''(\theta) = -\mu b(\theta) \\ u(r,\theta) \text{ is periodic in } \theta \text{ with period} = 2\pi \end{cases}$$

Which of the following is a complete set of eigenvalues and eigenfunctions? **Select all that apply.**

- [ ] $\mu_n = n^2, \quad b_n(\theta) = \sin(nx), \quad n = 1, 2, \ldots$
- [ ] $\mu_n = n^2, \quad b_n(\theta) = \cos(nx), \quad n = 0, 1, \ldots$

Correct!

- [x] $\mu_n = n^2,\ n = 0, 1, \ldots \begin{cases} b_{n,c}(\theta) = \cos(nx),\ n = 0, 1, \ldots \\ b_{n,s}(\theta) = \sin(nx),\ n = 1, 2, \ldots \end{cases}$
- [ ] $\mu_n = n^2,\ n = 1, 2, \ldots \begin{cases} b_{n,c}(\theta) = \cos(nx),\ n = 1, 2, \ldots \\ b_{n,s}(\theta) = \sin(nx),\ n = 1, 2, \ldots \end{cases}$

Correct!

- [x] $\mu_n = n^2, \quad b_n(\theta) = e^{in\theta}, \quad -\infty < n < +\infty$

Additional Comments:

Question 8

0 / 0 pts

Let $J_n(z)$ and $Y_n(z)$ be the bessel functions of respectively the first kind and the second kind of order *n* for $n \geq 0$.

Which of the following is true? **Select all that apply.**

- [ ] $J_n(z)$ has no zero for $z \in (0, +\infty)$.
- [ ] $J_n(z)$ has exactly *n* zeros for $z \in (0, +\infty)$.

Correct!

- [x] $J_n(z)$ has an infinite sequence of zeros for $z \in (0, +\infty)$.
- [ ] $\lim_{z \to 0} J_n(z) = \text{finite}, \quad \lim_{z \to 0} Y_n(z) = \text{finite}$

Correct!

- [x] $\lim_{z \to 0} J_n(z) = \text{finite}, \quad \lim_{z \to 0} Y_n(z) = -\infty$

Additional Comments:

Fudge Points:

You can manually adjust the score by adding positive or negative points to this box.

Final Score: 1 out of 1

Update Scores