Magnetohydrodynamics AMS 275

HW 3

1 Consider a magnetic field in a Cartesian (x, y, z) domain as follows:

$$\mathbf{B} = (B_x(z), B_y(z), 0)$$

(a) Derive (and solve) the condition between the two components of the field such that it is force-free.
(b) Further impose the Beltrami condition (with constant α) i.e. $\nabla \times \mathbf{B} = \alpha \mathbf{B}$ to derive explicit expressions for the field.
(c) Confirm that this field (obviously) solves the Beltrami condition expressed in the Helmholtz form.

2 A flow $\mathbf{u} = U_0(-x, y, 0)$ interacts with a magnetic field $\mathbf{B} = B(x)\hat{\mathbf{y}}$. Neglecting the effect of the magnetic field on the flow, show that the induction equation has the *steady* solution, symmetric about the y-axis

$$B(x) = B_0 \exp\left[-\left(U_0 x^2/2\eta\right)\right],$$

where B_0 is the maximum field strength. [Such a field would be the end result of the action of $\mathbf{u}$ on a uniform field $B\hat{\mathbf{y}}$. The flow acts so as concentrate the field in a region of converging flow. This mechanism may be important for the very strong localised fields found on the surface of the Sun.]

3 A magneto hydrostatic model of a solar prominence balances pressure and gravity against the Lorentz force and is given by

$$0 = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g}.$$

Consider a simple model in which $\mathbf{B} = (B_x, 0, B(x))$, $p = p(x)$, $p = \alpha\rho$, $\mathbf{g} = -g\hat{\mathbf{z}}$, and B_x, μ_0, α and g are all constants. Show that if $B(0) = 0$ and $\rho(0) = \rho_0$, the vertical component of the field is given by

$$B(x) = (2\mu_0 \alpha \rho_0)^{1/2} \tanh\left(\left(\frac{\mu_0 \rho}{2\alpha}\right)^{\frac{1}{2}} \left(\frac{gx}{B_x}\right)\right),$$

and sketch the field lines