AM160 Homework 2

1 Problem 1 (5+5+5 points)

Consider the Lorenz' 96 system that we solved in the class with 8 X variables, 64 Y variables, and 512 Z variables.

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\begin{aligned}
\frac{dX_k}{dt} &= X_{k-1}\left(X_{k+1} - X_{k-2}\right) - X_k + F - \frac{hc}{b}\Sigma_j Y_{j,k} && (1)\\
\frac{dY_{j,k}}{dt} &= -cbY_{j+1,k}\left(Y_{j+2,k} - Y_{j-1,k}\right) - cY_{j,k} + \frac{hc}{b}X_k - \frac{he}{d}\Sigma_i Z_{i,j,k} && (2)\\
\frac{dZ_{i,j,k}}{dt} &= edZ_{i-1,j,k}\left(Z_{i+1,j,k} - Z_{i-2,j,k}\right) - geZ_{i,j,k} + \frac{he}{d}Y_{j,k} && (3)
\end{aligned}
$$

where $i, j, k = 1, 2, \ldots 8$, i.e., there are 8 equations for X, and 64 and 512 equations for Y and Z, respectively. Ignore the Z variables completely. Keep all the paramters the same as was done in class and write code for the following problems:

(a) Follow the procedure in class, and train a model that can predict $Y(t)$ as a function of $X(t)$. Show how accurate the value of Y is on an unknown test set as well as the value of $\Sigma_j Y_{j,k}$ Call this model, M_1.

(b) Now, simulate the system again with $\mathbf{F = 24}$. Using M_1 as the model, but new X values as input and show how well the new values of Y as well as $\Sigma_j Y_{j,k}$ is predicted. Now, fine-tune the model with the new data to make it work on this new system.

(c) For (a), couple the model, M_1 to Eq (1) and simulate the system (*Go back to the video where I explain the hybrid physics engine + AI engine*). This is tricky. You need to write the numerical solver in such a way that the $\Sigma_j Y_{j,k}$ term is now obtained from M_1. Can you stabilize this system?