Pre-candidacy notes:

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Contents

1 Billant & Chomaz Papers

1.1 Experimental evidence for a new instability of a vertical columnar vortex pair in a strongly stratified fluid (2000)

- The first paper in a series of papers by Billant and Chomaz describing and investigating the properties of the so called "zigzag" instability present in the Lamb-Chaplygin vortex pair (a counterrotating vortex dipole).
- This paper demonstrated the existence of such an instability from experimental findings at sufficient stratification. For insufficient stratification $Fr \geq 0.2 \pm 0.01$, the ellipitcal instability appears to be the dominant instability and after its gravitational collapse, the vortex pair appears to irregularly zigzag into layer formation.
- From what can be observed from the zigzag instability is that it doesn't perturb the horizontal cross-section structure of the vortex column, only its vertical structure. It is positted that this phenomenon may be responsible for the layering phenomenon demonstrated in many stratified flows.
- Over a long enough time frame the original vortex pair column ends up divided into pancake dipole segments in the vertical direction, obtaining what is usually described as pancake eddies in the flow.

1.2 Self-similarity of strongly stratified inviscid flows (2001)

- Posits the scaling of an intrinsic vertical length scale of strongly stratified flows, $l_z \propto U/N$.
- Third paper which describes the "Zig-Zag" instability. Two previous papers conducted linear stability analysis of the instability.
- Zig-zag instability is self-similar with respect to $k_z U/N$ which implies that the dominant vertical wavenumber of the flow is proportional to Fr.

1.3 Three-dimensional stability of vertical columnar vortex pair in a stratified fluid

- This paper conducted a numerical stability analysis on the linearized equations using mean-pertubation flow separation. They found for flows with sufficient stratification that the primary instability of the counterrotating vortex pair was the "zig-zag" instability in which the entire vortex column was destabilized and oscillationed ide to side with a typical scale height, later found to be proportional to the froude number. For insufficient stratification, the elliptical instability was the dominant instabtility.
- Among their findings is the approximate scaling that the root mean squared $u_z' \propto 1/Fr$ and $p' \propto Fr$ (normalized by the rms horizontal velocity). Furthermore, these numerical findings for the growth rate of the zig-zag instability concur with the experimental results within reasonable error.
- Their nondimensionalization involved

$$Fr = \frac{U_{\text{prop}}}{NR}$$

where U_{prop} is the propagation speed of the vortex pair, and R is given by the dipole radius. This is similar to the non-dimensionalization from Chini et al, in which the unit velocity and lengthscale are given by the typical horizontal flow (i.e. horizontal forcing which is order 1 in both U and L).

2 Hattori & Hirota Papers

2.1 Stability of two-dimensional Taylor-Green vortices in rotating stratified fluids (2023)

- Conducted a local stability analysis as well as DNS and analyzed the data using modal stability analysis.

- Linear Stability analysis is conducted on a linearized and inviscid version of the governing equations.
- Both the DNS and LSA begin with a base flow composed of Taylor-Green vortices, which are arranged in a grid lattice.
- 5 instabilities are identified from the LSA, each with a different mechanism and different instability/resonance conditions.
- Linear Stability analysis found that the pure hyperbolic instability is often the fastest growing istability as also the most realizable. Variation of the input rossby and froude numbers reveals characteristics of other secondary instabilities which vary with vertical wavenumber, and radius from vortex centers (as well as input parameters).

2.2 Modal stability analysis of arrays of stably stratified vortices (2021)

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3 Miyazaki and Fukumoto

3.1 Three-dimensional instability of strained vortices in a stably stratified fluid (1992)

- This paper conducts a linear stability analysis of "unbounded strained vortices". The linear stability analysis is derived analytically and then solved numerically by a floquet problem. The primary investigation is into the elliptical instability of "Pierrehumbertt type". Two other instability modes are noted which depend distinctly on the buoyancy frequency N.

3.2 Elliptical instability in a stably stratified rotating fluid

- stuff

4 Linden Papers

4.1 The stability of vortices in a rotating, stratified fluid

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5 Herring and Metias

5.1

- stuff

6 Waite and Bartello

6.1

- stuff

7 GFD Group (Garaud, Chini, Shah, Caulfield . . .)

7.1 Exploiting self-organized criticality in strongly stratified turbulence (2021)

- Developed a multiscale model for strongly stratified flows wherein an aspect ratio α is used to describe scale separation of horizontal and vertical motions recovering that $l_z \propto Fr$ as posited by

7.2 Cope et al. 2020

7.3 Shah et al. 2023

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