signals from this unit are fed into a magnetic recording head which is in contact with a magnetic tape driven at a speed of 7·5 in/sec. The signals recorded on the magnetic tape are received by a reproducing head and the distance between it and the recording head can be varied continuously, so that a known time delay is introduced in the signal. It is found that the maximum separation required does not exceed 10 in., corresponding to a delay of about $1\frac{1}{2}$ sec. The signal from the reproducing head is fed into an amplifier and demodulator stage, the output of which is a true replica of the original signals but with a time delay. The outputs from the demodulator and the gated integrator II are adjusted to be equal and are fed into a double triode balanced cathode follower amplifier between the cathodes of which is placed an a.c. milliammeter. The deflection in the detector, due to the a.c. voltage between the cathodes, is a function of the time delay between the almost similar input signals. When the time delay between the signals is adjusted on the average to be zero, the deflection in the a.c. milliammeter will be a minimum. By manually varying the reproducing head position to give minimum output in the a.c. detector, and noting the distance between the record and reproduce heads and the speed of the tape, the time delay T is obtained. This minimum is observed only if the signal output of receiver 2 is delayed with respect to that of receiver 1. In cases in which the time delay is the other way, the receivers connected to the gated integrators are inter-changed by a double-pole double-throw switch S_2 so that the signal whose phase is ahead is fed into the tape delay system following the gated integrator I.

The main advantage of this method is the simplicity in obtaining average time delay without having recourse to taking fading records. The accuracy of delay control setting for minimum in the detector is not high, but the overall accuracy can be improved by taking the average of several observations. The recorder has the limitation that it can record a minimum time delay of 0·07 sec. A detailed paper giving all the technical details will shortly be published elsewhere. We are indebted to the Council of Scientific and Industrial Research for financial support of this research project.

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The anisotropy of turbulence at the meteor level

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Recently it has been realized that the irregular motions that have been observed at the meteor level of the ionosphere (say 90 km in height) cannot be regarded as "turbulence" of the kind studied in the laboratory; nor can the theory of isotropic turbulence be applied. Both the observational and theoretical aspects were discussed at length in the International Symposium on Fluid Mechanics in the Ionosphere (*Journal of Geophysical Research*, 1959, Vol. 64, pp. 2037–2238).

Greenhow and Neufeld (1959a, b) reported "large eddies" with a vertical scale of 6 km and a horizontal scale of the order of 150 km. Greenhow (1959) also mentions that estimates of the parameters describing the small eddies (i.e. the eddies in which most of the turbulent dissipation is taking place) can be obtained from the rate of expansion of a meteor trail during the early stages of its life. BOOKER (1959) has pointed out that if the latter data are used to estimate the rate of turbulent dissipation per unit mass, ε, the value obtained is quite inconsistent with the data for the large eddies, if the ordinary formulae are applied. BOLGIANO (1959, 1960) has outlined a theory in which the smaller eddies (the dissipation range and part of the inertial range if it exists) are isotropic, while larger eddies are anisotropic because of the buoyancy forces.

The purpose of this note is to present a simple way of estimating very roughly the size at which anisotropy should begin (the change does not take place abruptly, of course) and discuss the numerical results so obtained, using GREENHOW'S data. The basic idea was given by STEWART (1959) during a discussion at the International Symposium mentioned above (see p. 2070). One supposes, for reasons which will be apparent shortly, that at sufficiently small sclaes, i.e. sufficiently large wave-number, k, the ordinary theory of isotropic turbulence holds and buoyancy effects are negligible. Thus in the inertial range the energy spectrum function (per unit mass per unit interval of wave number) is

$$E(k) \propto \varepsilon^{2/3}k^{-5/3} \tag{1}$$

where a numerical factor of order unity is omitted. The energy in a band of width k (say from $\frac{1}{2}k$ to $\frac{3}{2}k$) is therefore of order

$$(\varepsilon/k)^{2/3}. \tag{2}$$

This is a rough measure of the turbulent kinetic energy per unit mass due to motions at a scale $L = k^{-1}$.

This must be compared with the potential energy associated with such motions, which is simply the work done in raising unit mass adiabatically through a height L, namely

$$\frac{1}{2}\frac{g}{\theta_0}\frac{d\theta_0}{dz}L^2 \tag{3}$$

(BRUNT, 1944, Chap. II). Here $\theta_0(z)$ is the potential temperature in the unperturbed atmosphere, defined by

$$\frac{1}{\theta_0}\frac{d\theta_0}{dz} = \frac{1}{T}\left(\frac{dT}{dz} + \Gamma\right) \tag{4}$$

$T(z)$ is the actual temperature and $\Gamma = g/c_p$ is the adiabatic lapse rate. So, neglecting the factor $\frac{1}{2}$, we define a Richardson number for an eddy, with respect to the unperturbed density distribution, by the ratio of (2) to (3), namely

$$R_i(k) = \frac{g}{\theta_0}\frac{d\theta_0}{dz}\varepsilon^{-2/3}k^{-4/3} = \left(\frac{k}{k_0}\right)^{-4/3} \tag{5}$$

where k_0 is a critical wave number

$$k_0 = \varepsilon^{-1/2}\left(\frac{g}{\theta_0}\frac{d\theta_0}{dz}\right)^{3/4}. \tag{6}$$

As anticipated (5) shows that for very large k the buoyancy forces are negligible, since R_i is small. In addition, (6) provides an estimate of the wave number at which buoyancy cannot be neglected. Because of the resulting tendency to supress vertical motion the spectrum becomes anisotropic at about this point, and (1) presumably ceases to hold.

All this assumes that $L_0 = k_0^{-1}$ is greater than the scale of the smallest eddies,

$$L_2 = (\nu^3/\varepsilon)^{1/4} \tag{7}$$

where ν is the kinematic viscosity. If, on the other hand, $L_2 > L_0$, there is no inertial range in the usual sense.

Bolgiano (1959, 1960) proposes that the inertial range be divided into a small-scale part, in which (1) applies, and a large-scale part which he calls the "buoyancy subrange". For the latter, a dimensional argument leads to a $k^{-11/5}$ law to replace (1). The two sub-ranges are separated by a wave number k_B, and it may well be asked why k_B differs from the k_0 as defined in (6). The reason for this lies in the choice of parameters assumed to be relevant in the dimensional argument. Bolgiano chooses a "rate of production of mean square density fluctuations" as his parameter. He shows that at $k = k_B$, the Richardson number characterizing an eddy is unity, provided this Richardson number is calculated with respect to the typical gradient in θ arising from the fluctuations at all scales greater than the eddy itself, rather than with respect to the unperturbed gradient, as at (5). Since the turbulent gradients are greater than the mean gradient, we expect $k_B > k_0$, i.e. $L_B < L_0$, where $L_B = k_B^{-1}$.

It seems to the present writer that the difference between these two results is less important than might at first be thought. It is true that in the case of mixing of a passive scalar, the root mean square gradients developed may be greatly in excess of the original gradient (Batchelor, 1959). For our purpose, however, it is only the *vertical* gradient of θ that is relevant, and the suppression of vertical motion at scales greater than that at which anisotropy becomes appreciable suggests that enhanced vertical gradients at such scales are not built up. If this is so, L_0 will not exceed L_B by a very large factor. Clearly both arguments involve unknown numerical factors and can do no more than suggest order-of-magnitude results. In addition, the use of dimensional methods can be criticized in the present situation, on the grounds that a simple power law may be inadequate to represent fields whose horizontal and vertical structures may depend on different parameters.

It is of great interest to consider the numerical consequences of these ideas. To calculate the critical size L_0 we need only Greenhow's (1959) estimate $\varepsilon \approx 70\ \mathrm{cm^2\,sec^{-3}}$ at a height of 90 km, together with $T \approx 200°\mathrm{K}$, $\Gamma \approx 10°\mathrm{km}^{-1}$ and $dT/dz \approx -2°\mathrm{km}^{-1}$. Then (6) yields

$$L_0 = k_0^{-1} = 30\ \mathrm{m}.$$

Taking $\nu = 4 \times 10^4\ \mathrm{cm^2\,sec^{-1}}$, (7) yields

$$L_2 = 10\ \mathrm{m}.$$

Thus the isotropic inertial range barely exists at all. Presumably any reasonable estimate of Bolgiano's $L_B = k_B^{-1}$ would merely strengthen this conclusion. For comparison, we recall that the energy-containing motions have vertical scale

$$L_1 \approx 6\ \mathrm{km}$$

and horizontal scale still bigger. Clearly only a trivial proportion of the total kinetic energy resides in the isotropic part of the spectrum.

This leaves unanswered the question of what *is* the nature of the large-scale energy containing "eddies" observed by Greenhow and Neufeld, and others. All we have shown is that they cannot be regarded as ordinary turbulence with buoyancy effects as a mere perturbation. In particular, the formula $\varepsilon \approx V_1^3/L_1$, where $V_1 \approx 25\ \mathrm{m\,sec^{-1}}$ is a representative velocity, grossly overestimates ε, indicating that the flow has a more ordered structure than it would have if it were an ordinary turbulent one. I have remarked elsewhere (Dougherty, 1960) that by regarding the "large eddies" as internal gravity waves of the kind considered by Hines (1960) and restricting the ideas of turbulence to the small eddies, a satisfactory qualitative picture of the whole motion emerges.

At heights greater than 90 km, the rapid increase in ν, which is unlikely to be matched by a corresponding increase in ε, means that

$$\frac{L_0}{L_2} = \left(\frac{\nu}{\varepsilon}\frac{g}{\theta_0}\frac{d\theta_0}{dz}\right)^{-3/4}$$

decreases still further. When its value falls below about unity, no inertial range exists at all. STEWART (1959, p. 2084) has mentioned that considerations of Reynold's and Richardson's numbers indicate that turbulence should cease altogether at a height of the order of 120 km. These various arguments suggest that the temptation to invoke turbulence as the cause of many of the ionospheric scattering phenomena (spread-F, radio star scintillation, etc.) should be avoided.

HOWELLS (1959, 1960) and DOUGHERTY (1960) have considered the production and propagation of irregularities in the electron density in the presence of a turbulent flow of neutral air, and a magnetic field. The main point of interest at that time was the effect of the magnetic field, and for simplicity isotropic turbulence was assumed. The considerations of the present note show that the results so obtained are relevant only over a rather small range of wave number; however, scattering theory indicates that this may well be the important range for fading experiments since the half-wavelength for reflection from the E-layer is around 50 m.

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The λ2763 Å (0,9) band of the O_2 Schumann-Runge system

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THE SCHUMANN–RUNGE band system of O_2 and its associated photo-dissociation continuum are important constituents of the absorption spectrum of the atmosphere (GOLDBERG, 1954). The continuum ($X^3\Sigma_g^-$, $(v = 0) \rightarrow B^3\Sigma_u^-$ above dissociation limit) lies between 1300 and 1750 Å while bands of the $X^3\Sigma_g^-$, $v = 0$ progression lie between 1750·8 and 2025·2 Å. They are the absorption spectrum in this range of cold oxygen. Bands of higher wavelengths occur in the absorption spectrum of hot oxygen and in the emission spectrum.