AM 160 - SciML:

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Problem 2:

1. ### Perform SINDy for the noisy Lorenz 63 system.

   I performed the SINDy algorithm for the lorenz system. I added noise to the derivative at each point according to the gaussian normal distribution with mean 0 and variance given by the problem set. In order to learn these coefficients, I merely used an rk4 integrator to find a trajectory and then computed the derivatives are each point along that trajectory. Then using a least squares solver, an solution for the coefficients is found. Finally, we iterate the least squares solution in order to neglect terms whose found coefficients are low (this algorithm doesn't take into account the relative weight of each column vector in the feature matrix which would be a vast improvement to the algorithm). Testing for the number of iterations used in the sindy algorithm can be seen by the plot shown in Figure 1.

2. ### Perform SINDy on the Lorenz 63 system using derivative schemes

   We can see from Figure 2 that more accurate derivative schemes enable SINDy to learn the coefficients better. Derivative schemes intrinsically contain errors bounded by the grid size and discretization. Notice that the higher order derivative scheme BDF3 allows the SINDy algorithm to better approximate the coefficients for the lorentz system compared to the first order centered finite difference scheme. I presume that with higher order derivative schemes and smaller grid sizes would decrease the error in the computed derivatives that SINDy learns from (which follows from any standard numerical methods course). The choice in numerical method to prepare data for SINDy is, therefore, very important for bounding the error of the learned coefficients. We notice that the number of iterations doesn't appear to have a large effect on the learned coefficients either. For each derivative scheme, five iteration counts are used and show no visible change in the error on the plot as a function of number of iterations.

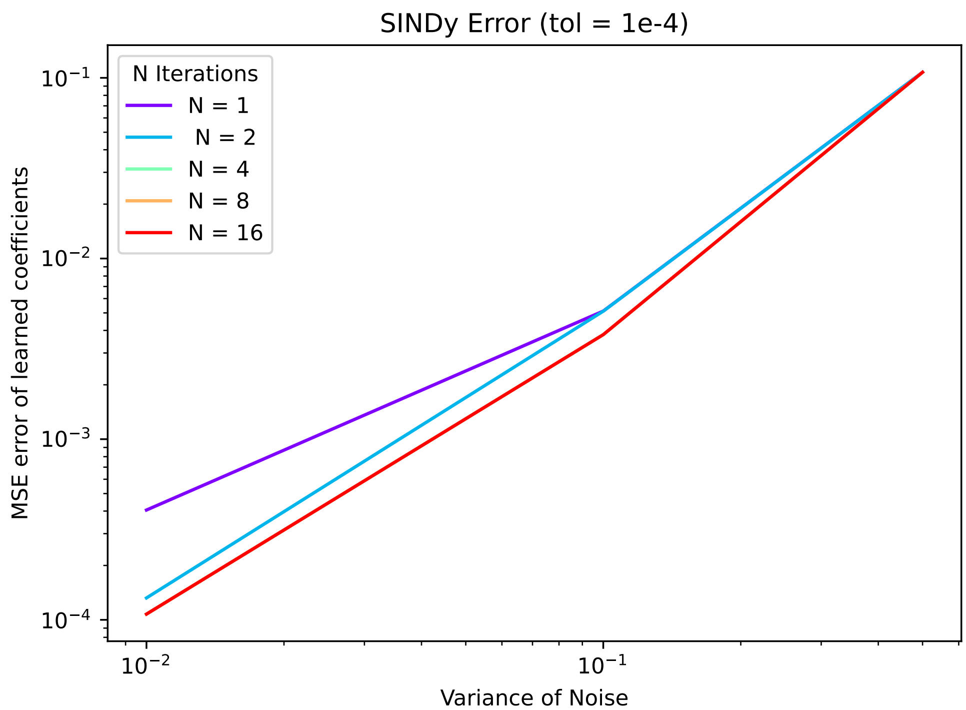

Figure 1: Plot of the MSE error for the learned coefficients ($\sqrt{(\theta_{\text{act}} - \theta_{\text{sindy}})^2}$) for the SINDy algorithm with a cutoff tolerance of 10^{-4}. A different number of iterations was used and this is depicted in color. There seems to be no distinction between the learned coefficients after about 2 iterations as $N = 4, 8, 16$ overlap exactly.

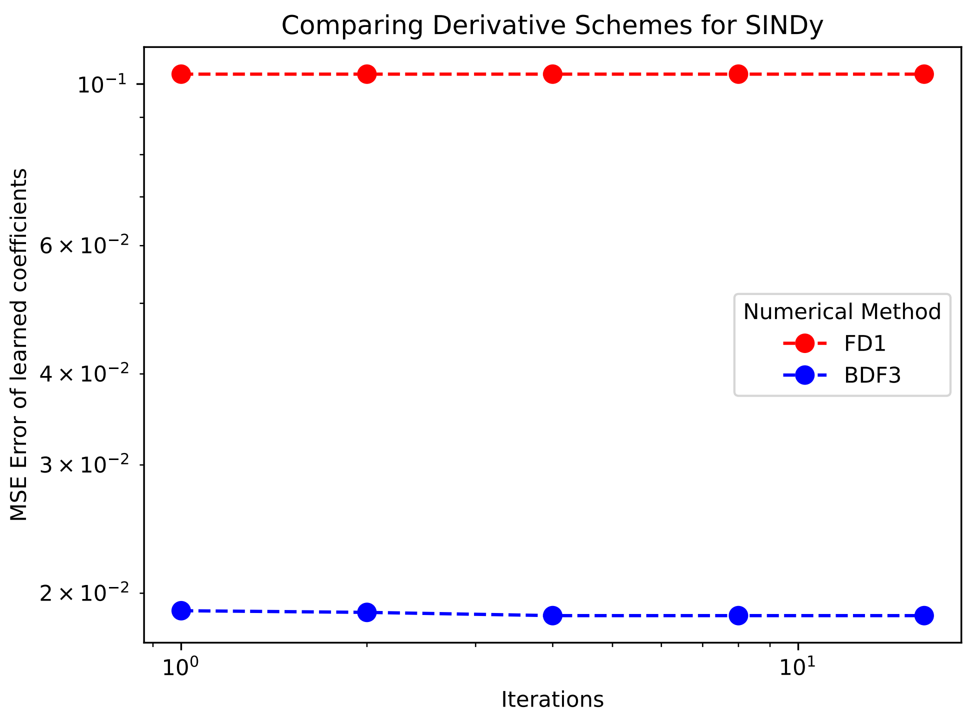

Figure 2: Plot of the MSE error for the learned coefficients ($\sqrt{(\theta_{\text{act}} - \theta_{\text{sindy}})^2}$) for the SINDy algorithm with the same number and cutoff tolerance as in Figure 1, but this time instead of sets of noisy data, we perform sindy on numerically computed derivative schemes and compare the error