Magnetohydrodynamics AMS 275

HW 4

1 Alfven Waves in Rotating Flows

When measurements are made from a rotating frame of reference, rotating with uniform angular velocity $\mathbf{\Omega}$, the momentum equation then contains an extra term $2\rho\mathbf{\Omega} \times \mathbf{u}$ on the left hand side. Show that for this case the Alfven wave problem gets modified such that the dispersion relation for an incompressible, ideal, inviscid fluid becomes

$$\omega^2 \pm 2(\mathbf{k} \cdot \mathbf{\Omega})\omega/k - (\mathbf{k} \cdot \mathbf{B}_0)^2/\mu_0\rho = 0$$

Show that when $|\mathbf{k} \cdot \mathbf{B}_0/\sqrt{\mu_0\rho}| \ll |(\mathbf{k} \cdot \mathbf{\Omega})/k|$, the roots of this dispersion relation are approximately

$$\omega \approx \pm 2\left(\mathbf{k} \cdot \mathbf{\Omega}\right)/k = \omega_I$$

and

$$\omega \approx \pm((\mathbf{k} \cdot \mathbf{B}_0)^2/\mu_0\rho_0)/((\mathbf{k} \cdot \mathbf{\Omega})/k) = \omega_B/\omega_I$$

showing that the solutions are either inertial waves (with ω_I) or Alfven waves modified by an inertial factor, making them propagate more slowly.

(Big hint: Taking the curl ($\mathbf{k} \times ()$) and the curl curl (doing it again) of the momentum equation gets you down to two equations in two "variables".)
(PS. This is not completely easy. Try your best!)