Magnetohydrodynamics AMS 275

HW 2

1 Prove from the induction equation that, in the perfectly conducting limit, the magnetic helicity, H_m, is conserved, where

$$H_m = \int_V \mathbf{A} \cdot \mathbf{B} \, dV$$

where $\mathbf{B} = \nabla \times \mathbf{B}$

2 Use the Cauchy solution to solve the perfectly conducting induction equation for $\mathbf{B}(x,t)$ for the cases:

(a) $\mathbf{u} = (\sin z, \cos z, 0)$, $\mathbf{B}(\mathbf{x}, 0) = (y, z, x)$.

(b) $\mathbf{u} = (\sin z, \cos z, 1)$, $\mathbf{B}(\mathbf{x}, 0) = (1, 1, 1)$.

(c) $\mathbf{u} = (\sin z, \cos z, 1)$, $\mathbf{B}(\mathbf{x}, 0) = (x, y, -2z)$.

(d) $\mathbf{u} = (y, -x, 0)$, $\mathbf{B}(\mathbf{x}, 0) = (x, -y, 0)$.