Banner appropriate to article type will appear here in typeset article

Precandidacy Exam: Report

Dante Buhl[1] †

[1]Department of Applied Mathematics, Baskin School of Engineering, Univeristy of California Santa Cruz, 1156 High Street, Santa Cruz, CA, 95064 US

(Received xx; revised xx; accepted xx)

1. Introduction

In the recent history of fluid dynamics, few studies have completely acclimated the dynamics of both stratification and rotation in a fully comprehensive manner. By contrast, many achievements have been made in studying isolated dynamics, whereby the effects of other physical mechanisms are ignored in order to better understand the instabilities, flow structures, and other properties which arise under specific conditions. This has led to many discoveries about stratified flows (CITE PAPERS HERE) and rotating flows (CITE PAPERS HERE). Solving problems in fluid dynamics becomes more difficult when multiple dynamics are included in the governing equations. This is often due to the nonlinearity implicit to the Navier-Stokes equation which prevents the superposition of solutions. For this reason, there are fewer studies which involve both rotation and stratification, and among those, few that are able to make general and comprehensive statements about rotating stratified flows.

Despite the lack of analytical solutions of the Navier-Stokes equation (coupled with the equations for mass conservation, an equation of state, and other quantities),

2. Numerical methods for high resolution fluid dynamics

3. Multiscale theory for rotating and/or stratified flows

4. Instabilities and turbulence in rotating and/or stratified flows

The topic of instabilities and turbulence in fluid dynamics is perhaps one of the oldest problems.

talk about early investigation into instabilities

For non-stratified flows, typical exhibitions of turbulence are isotropic and generally follow the scaling laws posited by Kolmogorov A. (1991*b*, 1941, 1991*a*) which were obtained using dimensional arguments. These scaling laws include the Kolmogorov velocity u_η, time τ_η and length η scales, i.e. the unit velocity, time, and length scales at which viscous diffusion occurs. These quantities are given by $u_\eta = (\nu\varepsilon)^{1/4}$, $\tau_\eta = \sqrt{\nu/\varepsilon}$, and $\eta = (\nu^3/\varepsilon)^{1/4}$, where ε is the average rate of turbulent energy dissipation and ν is the kinematic viscosity. A Reynolds number can be defined using these values $Re = u_\eta\eta/\nu$ and is equal to one at the kolmogorov scale, implying that viscous diffusion is of $O(1)$ in the Navier-Stokes equation. This foundation of isotropic turbulence has been paramount in the development of models

† Email address for correspondence: dbuhl@ucsc.edu

Abstract must not spill onto p.2

for viscous diffusion. These scaling laws are valid only when the turbulence is isotropic. When, for example, buoyancy forces create pancake vorticies in the flow as is typical in stratified flows, the nature of these eddies is anisotropic. Ozmidov (1965) used a dimensional argument to show that there is a length scale $l_O = \sqrt{\varepsilon/N^3}$ below which isotropic turbulence is expected (where N is the buoyancy frequency of the flow). Futhermore, there is a transition in the energy spectrum once the wavenumber of the flow increases past l_O^{-1}. Typical isotropic turbulence exhibits an energy spectrum proportional to $k^{-5/3}$. In a stratified fluid, this energy spectrum above the Ozmidov scale exhibits an energy spectrum proportional to k_h^{-3} and then transitions to the kolmogorov scaling ($k^{-5/3}$) once the flow becomes isotropic.

4.1. *Non-rotating Stratified flows*

The zigzag instability first studied by Billant & Chomaz (2000*a*,*b*,*c*, 2001) is perhaps one of the most important instabilities in stratified flows. It has been shown by Billant & Chomaz (2000*c*); Hattori *et al.* (2021); Guo *et al.* (2024) see if other papers talk about the growth rate of the zigzag instability that the zigzag instability (also referenced as the mixed-hyperbolic instability) is consistently the fastest growing instability mode for a stratified vortex column (or dipole) for sufficient stratification. The most notable characteristic of this instability is that it is a reliable mechanism for vertically invariant base flows to gain vertical structure. Most notably, several studies have shown that the vertical length scale of the zigzag instability scales with the Froude number and therefore $k_z \propto Fr^{-1}$. A multiscale asymototic analysis of the governing equations for strongly stratified flows recovers this scaling for the vertical length scale and additionally finds that the turbulent horizontal velocities scale with $Fr^{-1/2}$ and the turbulent vertical velocity scales with Fr^{-1} (Chini *et al.* 2022). This result is valid in the thermally-nondiffusive regime, i.e. when the Péclet number is greater than $O(1)$, and alternate scaling laws are valid in the thermally diffusive regime shown by Shah *et al.* (2023).

4.2. *Rotating Non-stratified flows*

4.3. *Rotating stratified flows*

5. Research Proposal

- Numerically simulate rotating stratified flows with stochastic forcing
- Quantity mixing via modal analysis and instability structures
- Eventually propose multiscale asymptotic model for rotating stratified turbulence (find scaling laws for vertical velocity)
- Try different domains

REFERENCES

A., Kolmogorov 1941 On degeneration of isotropic turbulence in an incompressible viscous liquid. *Proc. Acad. Sci. USSR* pp. 538–540.

A., Kolmogorov 1991*a* Dissipation of Energy in the locally isotropic turbulence. *Proceedings: Mathematical and Physical Sciences* (1890), 15–17.

A., Kolmogorov 1991*b* The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers. *Proceedings: Mathematical and Physical Sciences* (1890), 9–13.

Billant, Paul & Chomaz, Jean-Marc 2000*a* Experimental evidence for a new instability of a vertical columnar vortex pair in a strongly stratified fluid. *Journal of Fluid Mechanics* **418** (1), 167–188.

Billant, Paul & Chomaz, Jean-Marc 2000*b* Theoretical analysis of the zigzag instability of a vertical columnar vortex pair in a strongly stratified fluid. *Journal of Fluid Mechanics* **419**, 29–63.

Billant, Paul & Chomaz, Jean-Marc 2000*c* Three-dimensional stability of a vertical columnar vortex pair in a stratified fluid. *Journal of Fluid Mechanics* **419**, 65–91.

Billant, Paul & Chomaz, Jean-Marc 2001 Self-similarity of strongly stratified inviscid flows. *Physics of Fluids* **13** (6), 1645–1651, arXiv: https://pubs.aip.org/aip/pof/article-pdf/13/6/1645/19135376/1645_1_online.pdf.

Chini, Gregory P., Michel, Guillaume, Julien, Keith, Rocha, Cesar B. & Caulfield, Colm-cille P. 2022 Exploiting self-organized criticality in strongly stratified turbulence. *Journal of Fluid Mechanics* **933**, A22, arXiv: 2111.08806.

Guo, Junwei, Taylor, John R. & Zhou, Qi 2024 Zigzag instability of columnar taylor–green vortices in a strongly stratified fluid. *Journal of Fluid Mechanics* **997**, A34.

Hattori, Yuji, Suzuki, Shota, Hirota, Makoto & Khandelwal, Manish 2021 Modal stability analysis of arrays of stably stratified vortices. *Journal of Fluid Mechanics* **909**, A4.

Ozmidov, R. V. 1965 Certain features of energy spectrum of oceanic turbulence. *Dokl. Akad. Nauk SSSR* **161**, 828–831.

Shah, Kasturi, Chini, Gregory P., Caulfield, Colm-cille P. & Garaud, Pascale 2023 Regimes of stratified turbulence at low Prandtl number. *arXiv e-prints* p. arXiv:2311.06424, arXiv: 2311.06424.