AM112 Exam 1

Important note:

- The problems below may have some similarities. **But they have different types of boundary conditions and different functions. Be careful!**
- You may re-use, without re-deriving them, any results we already obtained in lectures, in homework assignment(s), or in exam(s).

Problem 1 (4 points):

Consider the expansion $f(x)=\sum_{n=1}^{\infty} c_n \cos(\frac{(n-\frac{1}{2})\pi}{L}x), \quad x\in[0,L]$.

a) Write out the formula for calculating coefficient c_n.

b) Evaluate coefficient c_n for function $f(x) \equiv 1$, $x \in [0, L]$.

Problem 2 (4 points):

Consider the IBVP below.

$$\begin{cases} u_t = k u_{xx} \\ u(x,0)=\cos(x) \\ u(0,t)=-3\,,\ u_x(L,t)=2 \end{cases} \qquad \text{(IBVP-P2)}$$

a) Find the steady state $u_\infty(x)$.

b) Let $v(x,t)\equiv u(x,t)-u_\infty(x)$. Write out the IBVP for $v(x, t)$.

Do NOT solve the IBVP.

→ Go to the next page for more exam problems.

Problem 3 (4 points):

Consider the IBVP below.

$$\begin{cases} u_t = k u_{xx} \\ u(x,0) = e^x \sin(x) \\ u_x(0,t) = 0 \ , \ u(\pi,t) = 0 \end{cases} \qquad \text{(IBVP-P1)}$$

a) Write out the eigenvalue problem in separation of variables and the solution of the eigenvalue problem (eigenvalues and eigenfunctions).

b) Write out a general solution of $u_t = k u_{xx}$, $u_x(0,t) = 0$, $u(\pi,t) = 0$.

Do NOT enforce the IC.

Problem 4 (8 points):

Solve the IBVP below.

$$\begin{cases} u_t = k u_{xx} \\ u(x,0) = 1 \\ u(0,t) = 1 \ , \ u(L,t) = 4 \end{cases} \qquad \text{(IBVP-P4)}$$

Be sure to add back $u_\infty(x)$ in your final answer.