Magnetohydrodynamics AMS 275

HW 1

1 Work through the result that we used in deriving a more meaningful kinetic energy (KE) equation (sometimes I call it the mechanical energy equation). That is, using the constitutive equation for a Newtonian fluid

$$\tau_{ij} = -p\delta_{ij} + 2\mu(e_{ij} - \frac{1}{3}e_{kk}\delta_{ij}),$$

where

$$e_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$$

is the deformation tensor, we can ultimately derive that

$$u_i\frac{\partial \tau_{ij}}{\partial x_j} = \frac{\partial (u_i\tau_{ij})}{\partial x_j} + pe_{kk} - 2\mu[e_{i,j} - \frac{1}{3}e_{kk}\delta_{ij}]^2$$

(where the left hand side (LHS) is the term that pops up in the KE equation, the first term on the RHS is the total rate of work by stresses, p is the pressure, μ is the dynamic viscosity),

2 Consider an ideal *incompressible* electrically conducting fluid with velocity $\mathbf{u}$ and magnetic field $\mathbf{B}$. Show that the induction equation can be rewritten as

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{B} = (\mathbf{B}\cdot\nabla)\,\mathbf{u}$$

Now consider the case where the ideal fluid is compressible satisfying the continuity equation. In this case show that the induction equation can be written as

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{B}}{\rho}\right) + (\mathbf{u}\cdot\nabla)\left(\frac{\mathbf{B}}{\rho}\right) = \left(\frac{\mathbf{B}}{\rho}\cdot\nabla\right)\mathbf{u}$$

3 Starting from Maxwell's Equations in the MHD approximation and Ohm's Law for a moving conductor, rework the derivation of the induction equation given in lectures for the case where the conductivity σ is **not constant**.

4 Show from *Maxwell's equations* that if initial conditions are chosen such that $\nabla\cdot\mathbf{B} = 0$ at $t = 0$ then $\nabla\cdot\mathbf{B} = 0$ at all times.
Show from *the induction equation* that if initial conditions are chosen such that $\nabla\cdot\mathbf{B} = 0$ at $t = 0$ then $\nabla\cdot\mathbf{B} = 0$ at all times.