

Theoretical Computer Science

Exercise 3 - Reductions

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Show that the language:

$X = \{\langle M, w \rangle \mid M \text{ is a 1-band TM and } M \text{ does not modify the part of the band where the input } w \text{ is}\}$ is not decidable.

Let us assume that X is decidable and let R be the Turing machine that decides X . Now, we can create a Turing machine S that decides A_{TM} ¹. The construction of S is the following:

- The input is $\langle M, w \rangle$, M being a Turing Machine and w being a string.
- Now, we create a new Turing Machine M' using the following steps:
 - The input is code y
 - Run M with input w , without editing the input y
 - If M accepts, M' accepts
 - Otherwise delete the input string y and write 1 and reject
- Now run R with M' as input
- S accepts if R accepts, S rejects if R rejects

If M accepts w , M' will not modify the input (which means it will be accepted by R). If M does not accept w , M' will modify the input (it will be rejected by R). This means that R will accept M' if and only if M accepts w . Consequently, S decides A_{TM} .

However, since we know that A_{TM} is undecidable, S cannot exist. Due to this contradiction R cannot exist either, so X must be undecidable.

¹ $A_{TM} = \{\langle T, w \rangle \mid T \text{ is a TM and } T \text{ accepts } w\}$