Theoretical Computer Science: Exercise 3 - Reductions

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Show that the language:
$X = \{\langle M, w\rangle |$ M is a 1-band TM and M does not modify the part of the band where the input w is$\}$ is not decidable.

Let us assume that X is decidable and let R be the Turing machine that decides X. Now, we can create a Turing machine S that decides A_{TM}[1]. The construction of S is the following:

- The input is $\langle M, w\rangle$, M being a Turing Machine and w being a string.
- Now, we create a new Turing Machine M' using the following steps:
  - The input is code y
  - Run M with input w, without editing the input y
  - If M accepts, M' accepts
  - Otherwise delete the input string y and write 1 and reject
- Now run R with M' as input
- S accepts if R accepts, S rejects if R rejects

If M accepts w, M' will not modify the input (which means it will be accepted by R). If M does not accept w, M' will modify the input (it will be rejected by R). This means that R will accept M' if and only if M accepts w. Consequently, S decides A_{TM}.

However, since we know that A_{TM} is undecidable, S cannot exists. Due to this contradiction R cannot exist either, so X must be undecidable.

[1] $A_{TM} = \{\langle T, w\rangle |$ T is a TM and T accepts w$\}$