Theoretical Computer Science: Exercise 2

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Using the Pumping Lemma, show that: the language $L = 0^i 1^j 0^{ij} | i, j \in \mathbb{N}$ $for \sum = 0, 1$ is not regular.

To solve this problem I will use the proof by contradiction. First, we assume that L is regular and let p be the pumping length of L given by the Pumping Lemma. Now we can set a string:

$$x = 0^p 1 0^p$$

It is clear that x is a member of L, where $i = p, j = 1$. Since x is longer than pumping length p, according to the Pumping Lemma we can split x into three parts $x = uvw$ where:

$$|v| \geq 1 \tag{1}$$

$$|uv| \leq p \tag{2}$$

$$uv^n w \in L, \forall n \in \mathbb{N} \tag{3}$$

From (1) and (2) we can conclude that $1 \leq |v| \leq p$. Since $x = 0^p 1 0^p$ the first p symbols of x are 0s. Therefore, u and v must consist entirely of 0s.
First possibility is that u is an empty string, $v = 0^p$ and $w = 10^p$. However this case does not satisy (3). For example, if $n = 2$, then we can see that the string: $uv^2 w = 0^p 0^p 1 0^p$ clearly is not a member of L.

Second possibility is that $|u| \geq 1$ and $|v| < p$. Similarly, this case also does not satisy (3). If $n = p$, then the string: $uv^p w$ would have a far greater number of 0s in the first part before the 1, regardless of the distribution of 0s between u and v, therefore it also cannot be a member of L.

Since for all possible segmentations $x = uvw, \exists n$ so that $uv^n w$ is not a member of L. Therefore, we can conclude that L is not regular, as per the Pumping Lemma.