Computer Vision

Course 7

Adaptive, Local Noise Reduction Filter

$g(x,y)$ – noisy image, S_{xy} – neighborhood, $\boldsymbol{\sigma}_{\eta}^{2}$ - the variance of the noise, m_{xy} – local avergage intensities in S_{xy}, $\boldsymbol{\sigma}_{xy}^{2}$ - local variance in S_{xy}

Properties of the desired filter:

1. If $\boldsymbol{\sigma}_{\eta}^{2} = \mathbf{0}$, the filter return $g(x, y)$ (no noise).
2. If the local variance $\boldsymbol{\sigma}_{xy}^{2}$ is high relative to $\boldsymbol{\sigma}_{\eta}^{2}$ the filter should return a value close to $g(x, y)$. A high local variance

typically is associated with edges, and these should be preserved.

3. If $\sigma_{xy}^2 \approx \sigma_\eta^2$ the filter returns m_{xy}. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced by averaging.

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_{xy}^2}\left(g(x,y) - m_{xy}\right)$$

Morphological Image Processing

Morphology deals with form and structure. *Mathematical morphology* is a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull. In this chapter, the inputs are binary images but the outputs are attributes extracted from these images.

$$A = \{(i, j);\ I_{i,j} = 1(\text{white})\} \subset \mathbb{Z}^2$$

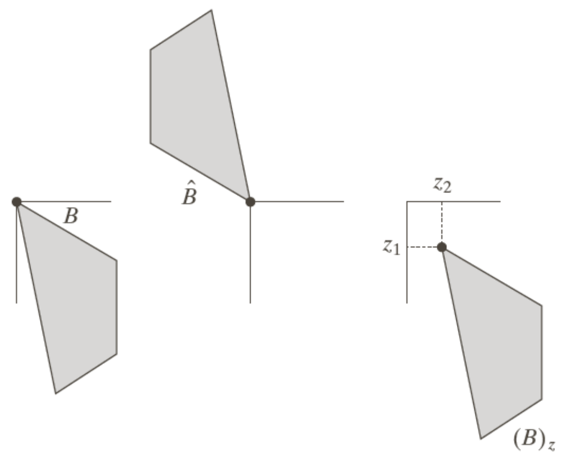
Preliminaries

The *reflection* of a set $\boldsymbol{B}$, denoted $\bar{\boldsymbol{B}}$ is defined as

$$\bar{B} = \{ w \; ; \; w = -b \; , \text{ for } \; b \in B \; \}$$

The *translation* of a set $\boldsymbol{B}$ by point $\boldsymbol{z = (z_1, z_2)}$, denoted $\boldsymbol{(B)_z}$ is defined as

$$(B)_z = \{ c \, ; \, c = z + b \text{ for } b \in B \; \}$$



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

Set reflection and translation are used in morphology to formulate operations based on so-called *structuring elements* (SE): small sets or subimages used to probe an image under study for properties of interest. In addition to a definition of which elements are members of the SE, the origin of a structuring element also must be specified. The origin of the SE is usually indicated by a black dot. When the SE is symmetric and no dot is shown, the assumption is that the origin is at the center of symmetry.

When working with images, it is required that structuring elements to be rectangular arrays. This is accomplished by appending the smallest possible number of background elements necessary to form a rectangular array.

Course 7

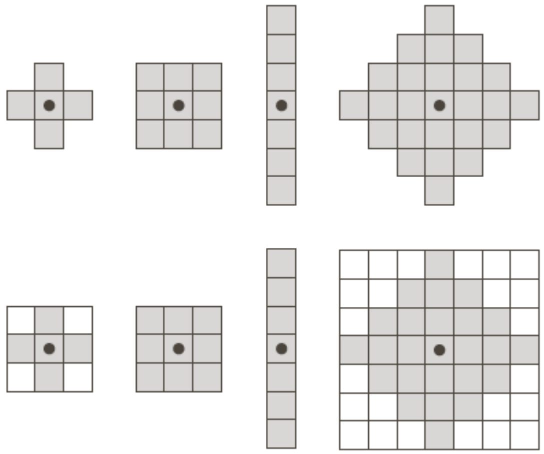


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Erosion and Dilation

Many of the morphological algorithms are based on these two primitive operations: *erosion* and *dilation*.

Erosion

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be to sets from $\mathbb{Z}^2$. The *erosion* of $\boldsymbol{A}$ by $\boldsymbol{B}$, denoted $\boldsymbol{A} \odot \boldsymbol{B}$ is defined as:

$$\boldsymbol{A} \odot \boldsymbol{B} = \{ \boldsymbol{z} \,;\, (\boldsymbol{B})_{\boldsymbol{z}} \subseteq \boldsymbol{A} \}$$

This definition indicates that the erosion of $\boldsymbol{A}$ by $\boldsymbol{B}$ is the set of all points $\boldsymbol{z}$ such that $\boldsymbol{B}$, translated by $\boldsymbol{z}$, is contained in $\boldsymbol{A}$. In the following, set $\boldsymbol{B}$ is assumed to be a structuring element. Because the statement that ***B*** has to be contained in ***A*** is equivalent to ***B*** not shearing any common elements with the background, erosion can be expressed equivalently:

$$\boldsymbol{A \odot B = \{\, z \,;\, (B)_z \cap A^c = \varnothing \,\}}$$

Course 7

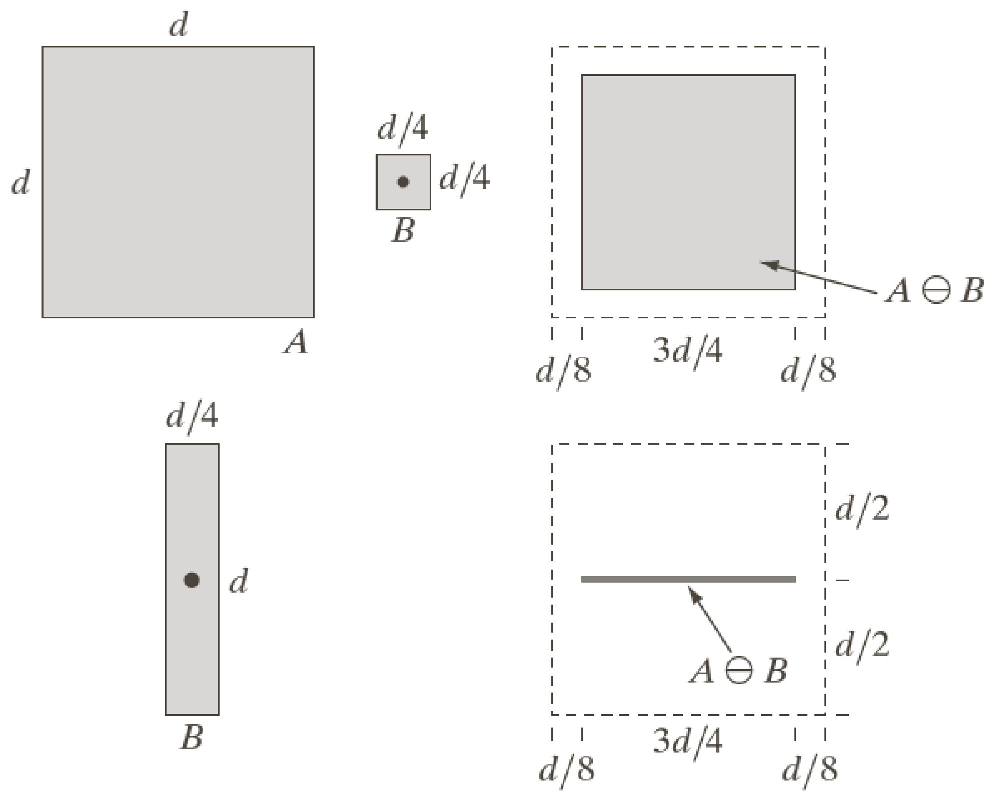


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Equivalent definitions of erosion:

$$\boldsymbol{A} \odot \boldsymbol{B} = \{\, \boldsymbol{w} \in \mathbb{Z}^2 ; \boldsymbol{w} + \boldsymbol{b} \in \boldsymbol{A} \quad \text{for every } \ \boldsymbol{b} \in \boldsymbol{B} \,\}$$

$$\boldsymbol{A} \odot \boldsymbol{B} = \bigcap_{\boldsymbol{b} \in \boldsymbol{B}} (\boldsymbol{A})_{-\boldsymbol{b}}$$

Erosion shrinks or thins objects in a binary image. We can view erosion as a *morphological filtering* operation in which image details smaller than the structuring element are filtered (removed) from the image.

Dilation

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be to sets in $\mathbb{Z}^2$. The *dilation* of $\boldsymbol{A}$ by $\boldsymbol{B}$, denoted $\boldsymbol{A} \oplus \boldsymbol{B}$ is defined as:

$$\boldsymbol{A} \oplus \boldsymbol{B} = \{\, \boldsymbol{z} \,;\; (\bar{\boldsymbol{B}})_{\boldsymbol{z}} \cap \boldsymbol{A} \neq \varnothing \,\}$$

The dilation of $\boldsymbol{A}$ by $\boldsymbol{B}$ is the set of all displacements $\boldsymbol{z}$, such that $\bar{\boldsymbol{B}}$ and $\boldsymbol{A}$ overlap by at least one element.

We assume that $\boldsymbol{B}$ is a structuring element.

Equivalent definitions of dilation:

$$\boldsymbol{A} \oplus \boldsymbol{B} = \{\, \boldsymbol{w} \in \mathbb{Z}^2 \,;\, \boldsymbol{w} = \boldsymbol{a} + \boldsymbol{b} \,,\ \text{for some } \boldsymbol{a} \in \boldsymbol{A} \text{ and } \boldsymbol{b} \in \boldsymbol{B} \,\}$$

$$A \oplus B = \bigcup_{b \in B} (A)_b$$

The basic process of rotating B about its origin and then successively displacing it so that it slides over set (image) A is analogous to spatial convolution. Dilation being based on set operations is a nonlinear operation, whereas convolution is a linear operation.

Unlike erosion which is a shrinking or thinning operation, dilation “grows” or “thickens” objects in a binary image. The specific manner and the extent of this thickening are controlled by the shape of the structuring element used.

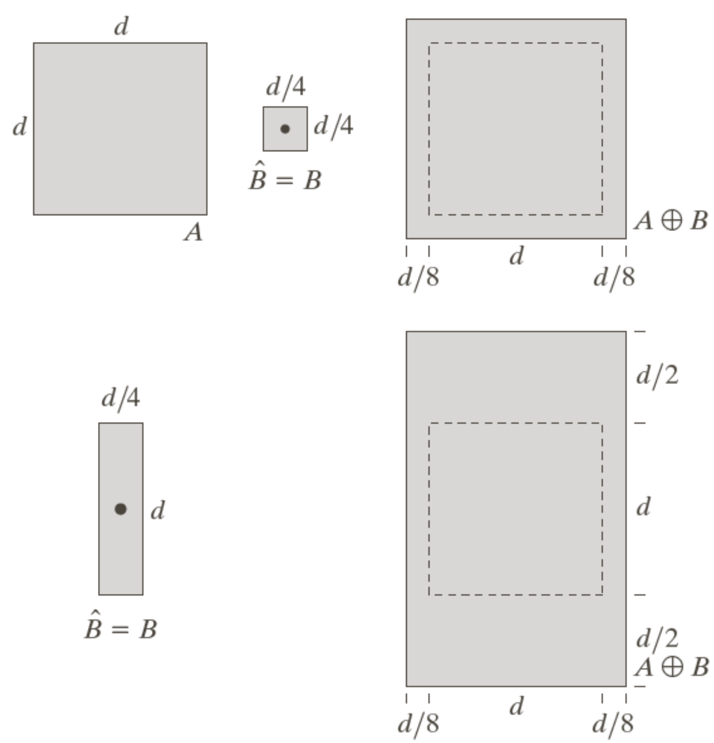
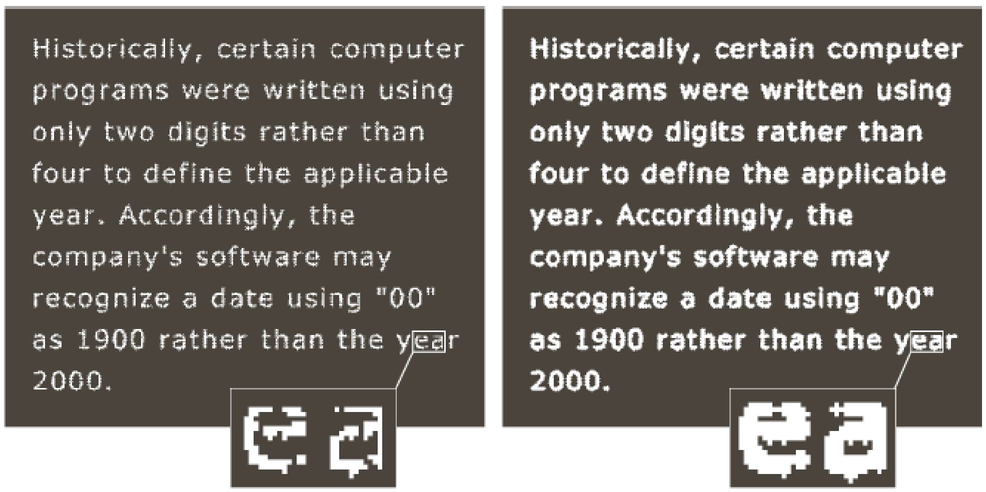


FIGURE 9.6 (a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Course 7

One of the simplest applications of dilation is for bridging gaps.



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection:

$$\left(A \odot B\right)^c = A^c \oplus \bar{B}$$
$$\left(A \oplus B\right)^c = A^c \odot \bar{B}$$

The duality property is useful particularly when the structuring element is symmetric with respect to its origin, so that $B = \bar{B}$. Then, we can obtain the erosion of an image by B simply by dilating its background (i.e. dilating A^c) with the same structuring element and complementing the result.

Opening and Closing

The *Opening* operation generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

Closing also tends to smooth section of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

The *opening* of set ***A*** by structuring element ***B*** is defined as:

$$\boldsymbol{A} \circ \boldsymbol{B} = \left(\boldsymbol{A} \odot \boldsymbol{B}\right) \oplus \boldsymbol{B}$$

Thus, the opening of ***A*** by ***B*** is the erosion of ***A*** by ***B***, followed by a dilation of the result by ***B***.

Similarly, the *closing* of set $\boldsymbol{A}$ by structuring element $\boldsymbol{B}$ is defined as:

$$\boldsymbol{A} \bullet \boldsymbol{B} = (\boldsymbol{A} \oplus \boldsymbol{B}) \odot \boldsymbol{B}$$

which says that the closing of $\boldsymbol{A}$ by $\boldsymbol{B}$ is the dilation of $\boldsymbol{A}$ by $\boldsymbol{B}$, followed by an erosion of the result by $\boldsymbol{B}$.

The opening operation has a simple geometric interpretation. Suppose that we view the structuring element $\boldsymbol{B}$ as a (flat) "rolling ball". The *boundary* of $\boldsymbol{A} \circ \boldsymbol{B}$ is then established by the points in $\boldsymbol{B}$ that reach the farthest into the boundary of A as $\boldsymbol{B}$ is rolled around the inside of this boundary. The opening of $\boldsymbol{A}$ by $\boldsymbol{B}$ is obtained by taking the union of all translates of $\boldsymbol{B}$ that fit into $\boldsymbol{A}$.

$$A \circ B = \bigcup \{ \ (B)_z \, , z \in A; (B)_z \subseteq A \ \}$$

Closing has a similar geometric interpretation, except that now we roll $\boldsymbol{B}$ on the outside of the boundary.

Course 7

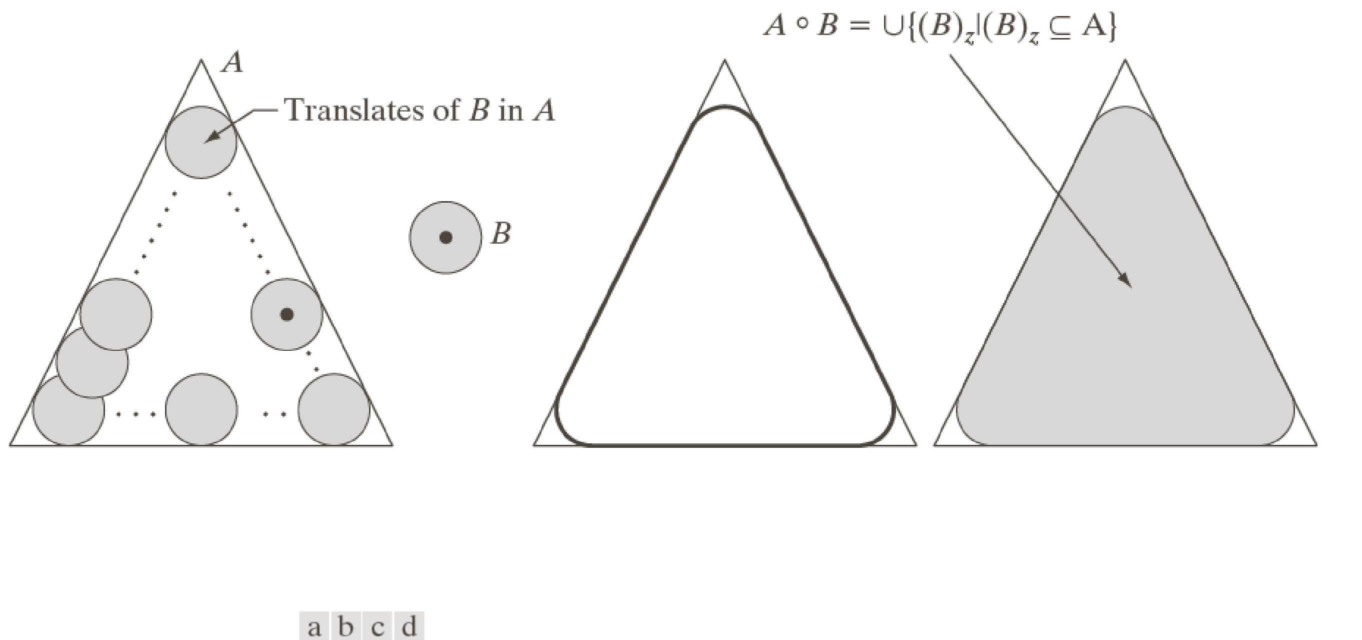
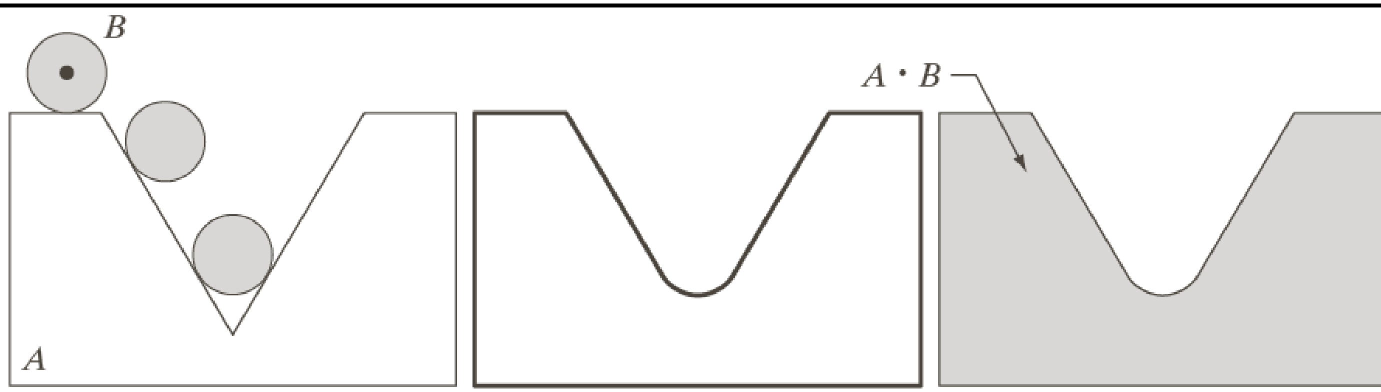


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

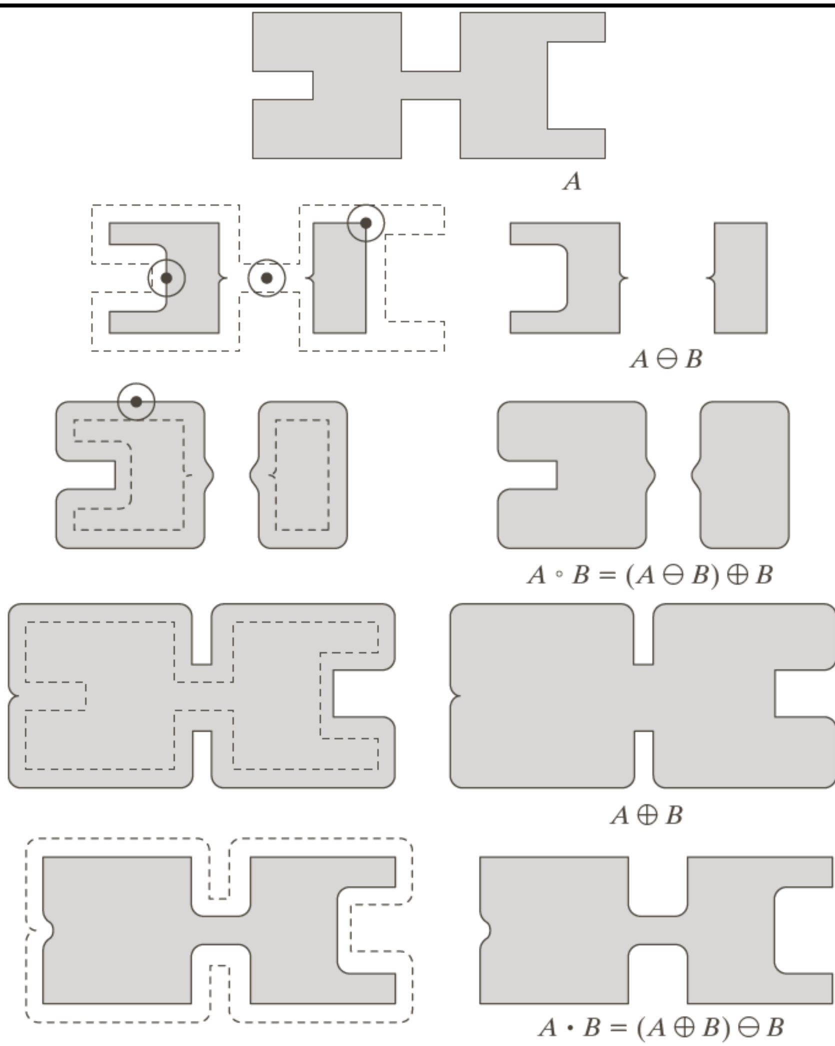
Course 7



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Course 7



a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Opening and closing are dual of each other with respect to set complementation and reflection:

$$\left(A \bullet B\right)^c = A^c \circ \bar{B}$$

$$\left(A \circ B\right)^c = A^c \bullet \bar{B}$$

The opening operation satisfies the following properties:

1. $A \circ B \subseteq A$
2. if $C \subseteq D$ then $C \circ B \subseteq D \circ B$
3. $\left(A \circ B\right) \circ B = A \circ B$

Similarly, the closing operation satisfies the following properties:

1) $A \subseteq A \bullet B$

2) if $C \subseteq D$ then $C \bullet B \subseteq D \bullet B$

3) $(A \bullet B) \bullet B = A \bullet B$

Condition 3 in both cases states that multiple openings or closings of a set have no effect after the operator has been applied once.

Course 7

a b
d c
e f

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Opening of A. (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

The Hit-or-Miss Transformation

The morphological hit-or-miss transformation is a basic tool for shape detection. Consider the set ***A*** from Figure 9.12 consisting of three shapes (subsets) denoted ***C, D,*** and ***E***. The objective is to locate one of the shapes, say, ***D***.

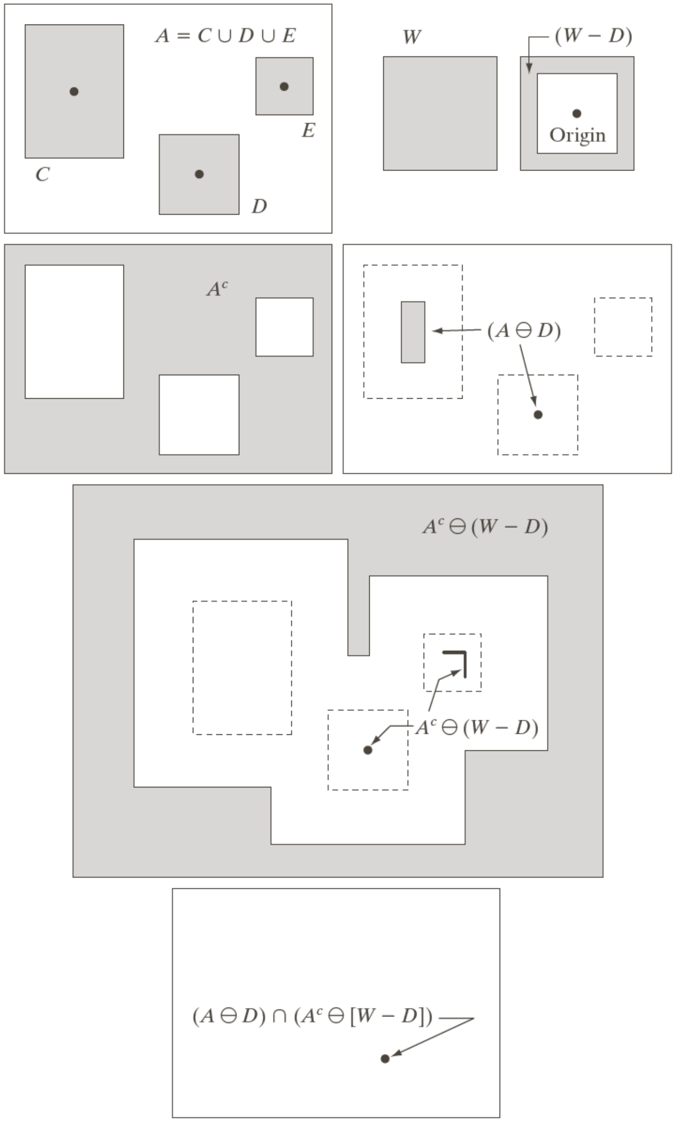
Let the origin of each shape be located at its center of gravity. Let ***D*** be enclosed by a small window, ***W***. The *local background* of ***D*** with respect to ***W*** is defined as the set difference ***(W-D)*** (Figure 9.12(b)). Figure 9.12(c) shows the complement of ***A***. Fig. 9.12(d) shows the erosion of $\boldsymbol{A}$ by $\boldsymbol{D}$. Figure 9.12(e) shows the erosion of the complement of $\boldsymbol{A}$ by the local background set $\boldsymbol{(W\text{-}D)}$.

From Figures 9.12(d) and (e) we can see that the set of location for which $\boldsymbol{D}$ exactly fits inside $\boldsymbol{A}$ is the intersection of the erosion of $\boldsymbol{A}$ by $\boldsymbol{D}$ and the erosion of $\boldsymbol{A^c}$ by $\boldsymbol{(W\text{-}D)}$ as shown in Figure 9.12(f).

If $\boldsymbol{B}$ denotes the set composed of $\boldsymbol{D}$ and its background, the match (or the set of matches) of $\boldsymbol{B}$ in $\boldsymbol{A}$, denoted $\boldsymbol{A \Upsilon B}$ is:

$$\boldsymbol{A \Upsilon B = (A \odot D) \cap \left[A^c \odot (W - D) \right]}$$

Course 7



a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

We can generalize the notation by letting $\boldsymbol{B = (B_1, B_2)}$, where $\boldsymbol{B_1}$ is the set formed from elements of $\boldsymbol{B}$ associated with an object and $\boldsymbol{B_2}$ is the set of elements of $\boldsymbol{B}$ associated with the corresponding background ($\boldsymbol{B_1=D}$, $\boldsymbol{B_2=W-D}$) in the preceding example).

$$\boldsymbol{A \Upsilon B = (A \odot B_1) \cap \left[A^c \odot B_2 \right]}$$

The set $\boldsymbol{A \Upsilon B}$ contains all the (origin) points at which, simultaneously, $\boldsymbol{B_1}$ found a match ("hit") in $\boldsymbol{A}$ and $\boldsymbol{B_2}$ found a match in $\boldsymbol{A^c}$. Taking into account the definition and properties of erosion we can rewrite the above relation as:

$$A \Upsilon B = (A \odot B_1) - (A \oplus \bar{B}_2)$$

The above three equations for $A \Upsilon B$ are referred as the *morphological hit-or-miss transform.*

Some Basic Morphological Algorithms

When dealing with binary images, one of the principal applications of morphology is in extracting image components that are useful in the representation and the description of shape. We consider morphological algorithms for extracting boundaries, connected components, the convex hull, and the skeleton of a region.

The images are shown graphically with 1s shaded and 0s in white.

Boundary Extraction

The boundary of a set $\boldsymbol{A}$, denoted $\boldsymbol{\beta(A)}$, can be obtained by first eroding $\boldsymbol{A}$ by $\boldsymbol{B}$ and then performing the set difference between $\boldsymbol{A}$ and its erosion.

$$\beta(A) = A - \left(A \odot B\right)$$

where $\boldsymbol{B}$ is a suitable structuring element.

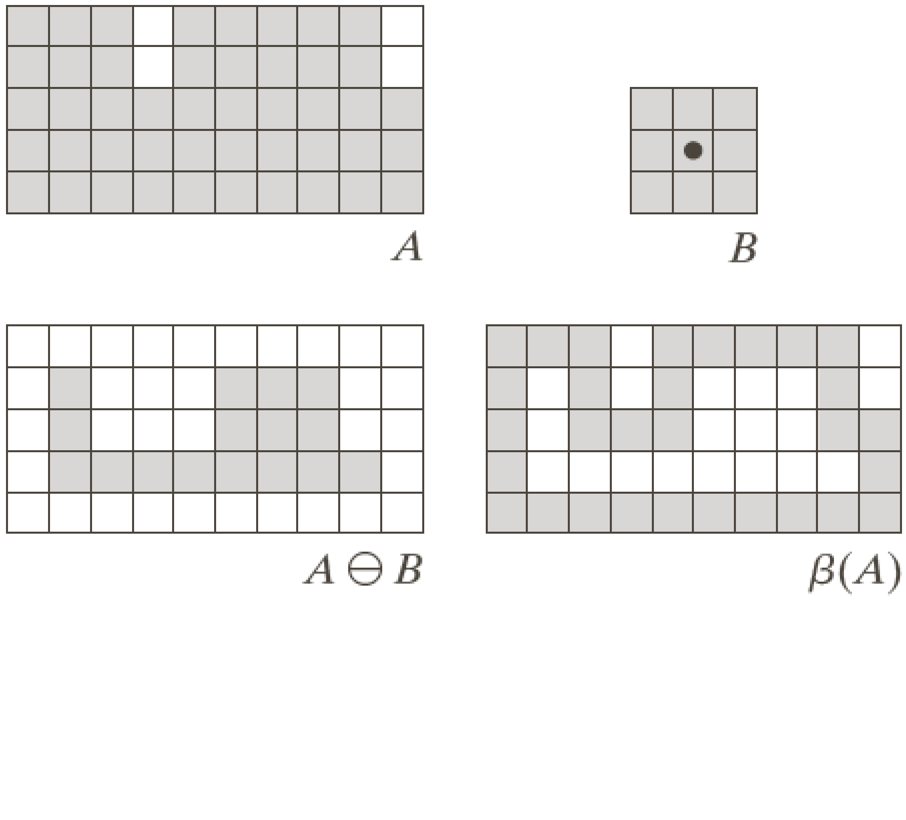


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Course 7

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Filling Holes

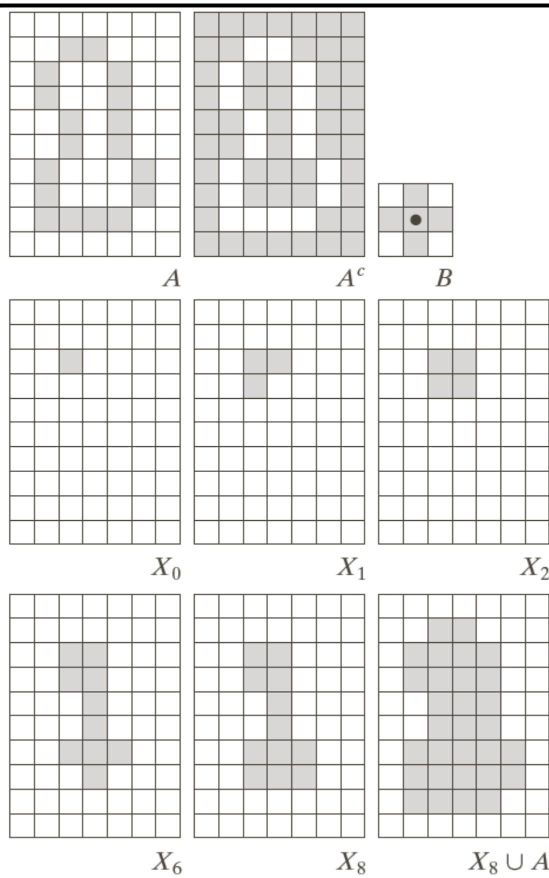
A *hole* may be defined as a background region surrounded by a connected border of foreground pixels. We present an algorithm based on set dilation, complementation, and intersection for filling holes in an image.

Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e. a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

We form an array, X_0, of 0s (the same size as the array containing A), except at the location in X_0 corresponding to the given point in each hole, which is set to ***1***. The following procedure fills all the holes with ***1***s:

$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c \quad , \quad k = 1, 2, 3, \ldots$$

where ***B*** is the symmetric structuring element in Figure 9.15(c). The algorithm terminates at iteration step ***k*** if $X_k = X_{k-1}$. The set X_k then contains all the filled holes. The set union of X_k and ***A*** contains all the filled holes and their boundaries.



a b c
d e f
g h i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Extraction of Connected Components

Extraction of connected components from binary images is important in many automated image analysis applications.

Let $\boldsymbol{A}$ be a set containing one or more connected components. Form an array $\boldsymbol{X_0}$ (of the same size as the array containing $\boldsymbol{A}$) whose elements are $\boldsymbol{0}$s (background values), except at each location known to correspond to a point in each connected component in $\boldsymbol{A}$, which we set to $\boldsymbol{1}$ (foreground value). The objective is to start with $\boldsymbol{X_0}$ and find all the connected components. The procedure that accomplishes this task is the following:

$$X_k = (X_{k-1} \oplus B) \cap A \quad , \quad k = 1, 2, 3, \ldots$$

where $\boldsymbol{B}$ is a suitable structuring element. The procedure terminates when $\boldsymbol{X_k} = \boldsymbol{X_{k-1}}$, with $\boldsymbol{X_k}$ containing all connected components of the input image.

Course 7

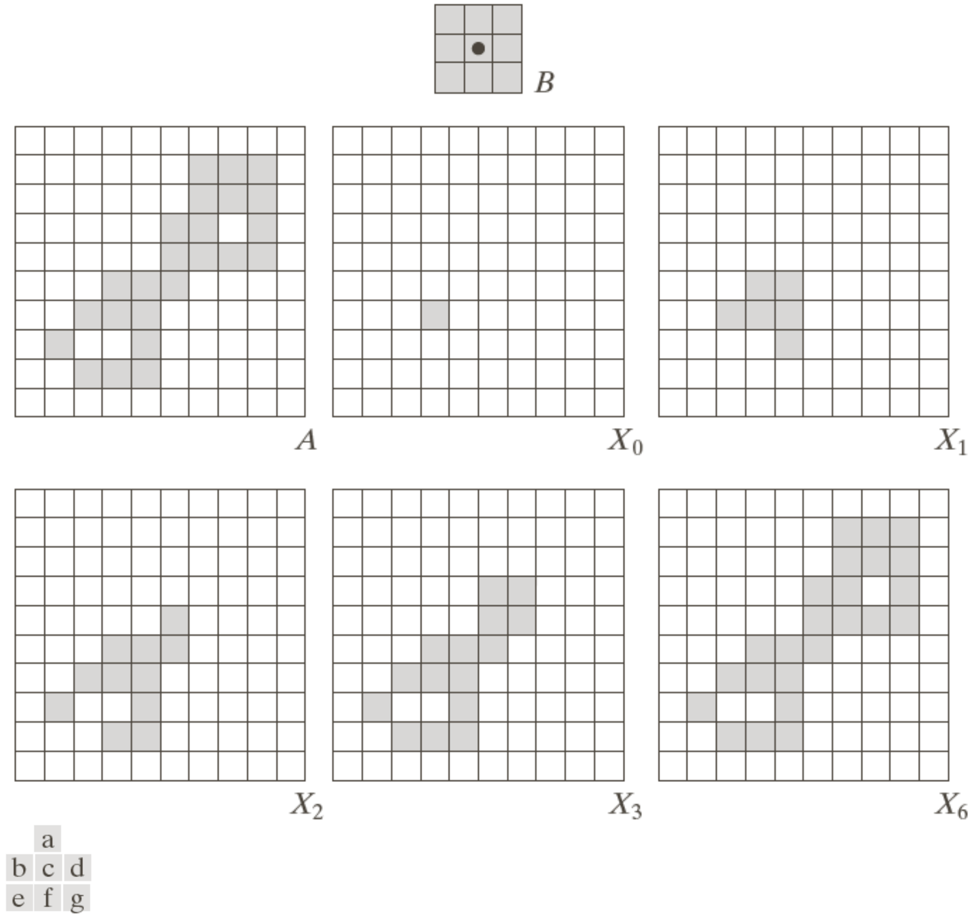


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Figure 9.18(a) shows an ***X***-ray image of a chicken breast that contains bone fragment. It is of considerable interest to be able to detect such objects in processed food befor packing and/or shiping. In this case, the density of the bones is such that their normal intensity values are different from the background. This makes extraction of the bones from the background a simple matter by using a single threshold. The result is the binary image in Figure 9.18(b). We can erode the thresholded image so that only objects of „significant" size remain. In this example, we define as significant any object that remains after erosion with a ***5×5*** structuring elemnt of ***1***s. The result of erosion is shown in Figure 9.18(c). The next step is to analyse the objects that remain. We identify these objects by extracting the connected components in the image. There are a total of ***15*** connected components, with four of them being of dominant size. This is enough to determine that significant undesirable objects are containd in the original image.