

$fraccap, fc$		fractional capability variable
$expression, x, a, b$		expression variable
$terminals$	$::=$	
		λ
		\otimes
		\multimap
		\vdash
		\in
		\forall
		Cap
		Type
f	$::=$	fractional capability
		fc variable
		Zero zero
		Succ f successor
t	$::=$	linear type
		1 unit
		$t \otimes t'$ pair
		$t \multimap t'$ arrow
		$\forall fc. t$ bind fc in t fractional capability abstraction
		Arr $[f]$ array
		$t\{f/fc\}$ M substitution
p	$::=$	primitive
		share share array
		combine unshare array
		free free array
		copy copy array
		swap swap array
		asum $\sum_i x_i $
		axpy $x := \alpha x + y$
		dot $x \cdot y$
		nrm2 $\ x\ ^2$
		rot plane rotation
		rotg Givens rotation
		rotm modified givens rotation
		rotmg generate modified Givens rotation
		scal $x := \alpha x$
		iamax index of maximum absolute value
		iamin index of minimum absolute value (Intel only)
e	$::=$	expression
		x variable
		$()$ unit introduction
		let $() = e$ in e' unit elimination
		(e, e') pair introduction
		let $(a, b) = e$ in e' bind $a \cup b$ in e' pair elimination

	$\lambda x : t. e$ $e e'$ $\mathbf{Array}(e)$ $\mathbf{let } x = e \mathbf{ in } e'$ p $\forall fc. e$ $e[f]$	bind x in e bind x in e' Level 1 BLAS routine primitives frac cap abstraction frac cap specialisation
Θ	$::=$ \cdot Θ, fc	fractional capability environment
Γ, Δ	$::=$ \cdot $\Gamma, x : t$ Γ, Δ	linear types environment
<i>formula</i>	$::=$ $judgement$ $x : t \in \Gamma$ $fc \in \Theta$	
<i>Well_Formed</i>	$::=$ $\Theta \vdash f \mathbf{Cap}$ $\Theta \vdash t \mathbf{Type}$	Valid fractional capabilities Valid types
<i>Types</i>	$::=$ $\Theta; \Gamma \vdash e : t$	Tying rules for expressions
<i>judgement</i>	$::=$ $Well_Formed$ $Types$	
<i>user_syntax</i>	$::=$ $fraccap$ $expression$ $terminals$ f t p e Θ Γ $formula$	

$\boxed{\Theta \vdash f \mathbf{Cap}}$ Valid fractional capabilities

$$\frac{fc \in \Theta}{\Theta \vdash fc \mathbf{Cap}} \quad \text{WF_CAP_VAR}$$

$$\frac{}{\Theta \vdash \mathbf{Zero} \mathbf{Cap}} \quad \text{WF_CAP_ZERO}$$

$$\frac{\Theta \vdash f \text{ Cap}}{\Theta \vdash \mathbf{Succ} f \text{ Cap}} \quad \text{WF_CAP_SUCC}$$

$\boxed{\Theta \vdash t \text{ Type}}$ Valid types

$$\frac{}{\Theta \vdash 1 \text{ Type}} \quad \text{WF_TYPE_UNIT}$$

$$\frac{\Theta \vdash t \text{ Type} \quad \Theta \vdash t' \text{ Type}}{\Theta \vdash t \otimes t' \text{ Type}} \quad \text{WF_TYPE_PAIR}$$

$$\frac{\Theta \vdash t \text{ Type} \quad \Theta \vdash t' \text{ Type}}{\Theta \vdash t \multimap t' \text{ Type}} \quad \text{WF_TYPE_LOLLIPOP}$$

$$\frac{\Theta \vdash f \text{ Cap}}{\Theta \vdash \mathbf{Arr} [f] \text{ Type}} \quad \text{WF_TYPE_ARRAY}$$

$$\frac{\Theta, fc \vdash t \text{ Type}}{\Theta \vdash \forall fc. t \text{ Type}} \quad \text{WF_TYPE_FORALL}$$

$\boxed{\Theta; \Gamma \vdash e : t}$ Tying rules for expressions

$$\frac{}{\Theta; \cdot, x : t \vdash x : t} \quad \text{TY_VAR}$$

$$\frac{}{\Theta; \cdot \vdash () : 1} \quad \text{TY_UNIT_INTRO}$$

$$\frac{\Theta; \Gamma \vdash e : 1 \quad \Theta; \Delta, x : 1 \vdash e' : t}{\Theta; \Gamma, \Delta \vdash \mathbf{let} x = e \mathbf{in} e' : t} \quad \text{TY_UNIT_ELIM}$$

$$\frac{\Theta; \Gamma \vdash e : t \quad \Theta; \Delta \vdash e' : t'}{\Theta; \Gamma, \Delta \vdash (e, e') : t \otimes t'} \quad \text{TY_PAIR_INTRO}$$

$$\frac{\Theta; \Gamma \vdash e_{12} : t_1 \otimes t_2 \quad \Theta; \Delta, a : t_1, b : t_2 \vdash e : t}{\Theta; \Gamma, \Delta \vdash \mathbf{let} (a, b) = e_{12} \mathbf{in} e : t} \quad \text{TY_PAIR_ELIM}$$

$$\frac{\Theta; \Gamma, x : t' \vdash e : t \quad \Theta \vdash t \text{ Type}}{\Theta; \Gamma \vdash \lambda x : t'. e : t' \multimap t} \quad \text{TY_LAMBDA}$$

$$\frac{\Theta; \Gamma \vdash e : t' \multimap t \quad \Theta; \Delta \vdash e' : t'}{\Theta; \Gamma, \Delta \vdash e e' : t} \quad \text{TY_APP}$$

$$\frac{\Theta; \Gamma \vdash e : \mathbf{Arr} [f]}{\Theta; \Gamma \vdash \mathbf{Array} (e) : \mathbf{Arr} [\mathbf{Zero}]} \quad \text{TY_ARRAY_INTRO}$$

$$\frac{\Theta; \Gamma \vdash e : \mathbf{Arr} [f] \quad \Theta; \Delta, x : \mathbf{Arr} [f] \vdash e' : t'}{\Theta; \Gamma, \Delta \vdash \mathbf{let} x = e \mathbf{in} e' : t'} \quad \text{TY_ARRAY_ELIM}$$

$$\frac{\Theta, fc; \Gamma \vdash e : t \quad \Theta, fc \vdash t \text{ Type}}{\Theta; \Gamma \vdash \forall fc. e : \forall fc. t} \quad \text{TY_FORALL_FRAC_CAP}$$

$$\frac{\Theta; \Gamma \vdash e : \forall fc. t}{\Theta; \Gamma \vdash e[f] : t\{f/fc\}} \quad \text{TY_SPECIALISE_FRAC_CAP}$$

Definition rules:	19 good	0 bad
Definition rule clauses:	43 good	0 bad