| | | | |
|---|---|---|---|
| $fc$ | fractional capability variable | | |
| $x$, $g$, $a$, $b$ | expression variable | | |
| $k$ | integer variable | | |
| $el$ | array-element variable | | |
| $symb$ | ::= | | |
| | $\lambda$ | | |
| | $\otimes$ | | |
| | $\multimap$ | | |
| | $\vdash$ | | |
| | $\in$ | | |
| | $\forall$ | | |
| | Cap | | |
| | Type | | |
| | ! | | |
| | $\rightarrow$ | | |
| | **value** | | |

| | | | | |
|---|---|---|---|---|
| $f$ | ::= | | | fractional capability |
| | $fc$ | | | variable |
| | **Z** | | | zero |
| | **S** $f$ | | | successor |

| | | | | |
|---|---|---|---|---|
| $t$ | ::= | | | linear type |
| | **unit** | | | unit |
| | **bool** | | | boolean (true/false) |
| | **int** | | | 63-bit integers |
| | **elt** | | | array element |
| | $f$ **arr** | | | arrays |
| | $f$ **mat** | | | matrices |
| | $!t$ | | | multiple-use type |
| | $\forall fc.t$ | bind $fc$ in $t$ | | frac. cap. generalisation |
| | $t \otimes t'$ | | | pair |
| | $t \multimap t'$ | | | linear function |
| | $t\{f/fc\}$ | M | | substitution |
| | $(t)$ | S | | parentheses |

| | | | | |
|---|---|---|---|---|
| $e$ | ::= | | | expression |
| | $p$ | | | primitives (arithmetic, L1 BLAS, Owl) |
| | $x$ | | | variable |
| | () | | | unit introduction |
| | **true** | | | true (boolean introduction) |
| | **false** | | | false (boolean introduction) |
| | **if** $e$ **then** $e_1$ **else** $e_2$ | | | if (boolean elimination) |
| | $k$ | | | integer |
| | $el$ | | | array element |
| | **many** $e$ | | | packing-up a non-linear value |
| | **let many** $x = e$ **in** $e'$ | | | using a non-linear value |
| | **fun** $fc \rightarrow e$ | | | frac. cap. abstraction |
| | $e[f]$ | | | frac. cap. specialisation |
| | $(e, e')$ | | | pair introduction |
| | **let** $(a, b) = e$ **in** $e'$ | bind $a \cup b$ in $e'$ | | pair elimination |

| | | | |
|---|---|---|---|
| | $\mid$ | $\textbf{fun}\, x : t \to e$ | $\mathsf{bind}\ x\ \mathsf{in}\ e$ | abstraction |
| | $\mid$ | $e\ e'$ | | application |
| | $\mid$ | $\textbf{fix}\, (g, x : t, e : t')$ | $\mathsf{bind}\ g \cup x\ \mathsf{in}\ e$ | fixpoint |

| | | | |
|---|---|---|---|
| $p$ | $::=$ | | primitive |
| | $\mid$ | **set** | array index assignment |
| | $\mid$ | **get** | array indexing |
| | $\mid$ | $(+)$ | integer addition |
| | $\mid$ | $(-)$ | integer subtraction |
| | $\mid$ | $(*)$ | integer multiplication |
| | $\mid$ | $(/)$ | integer division |
| | $\mid$ | $(=)$ | integer equality |
| | $\mid$ | $(<)$ | integer less-than |
| | $\mid$ | $(+.)$ | element addition |
| | $\mid$ | $(-.)$ | element subtraction |
| | $\mid$ | $(*.)$ | element multiplication |
| | $\mid$ | $(/.)$ | element division |
| | $\mid$ | $(= .)$ | element equality |
| | $\mid$ | $(< .)$ | element comparsion (less-than) |
| | $\mid$ | (&&) | boolean conjuction |
| | $\mid$ | (\|\|) | boolean disjunction |
| | $\mid$ | **not** | boolean negation |
| | $\mid$ | **share** | share array |
| | $\mid$ | **unshare** | unshare array |
| | $\mid$ | **free** | free arrary |
| | $\mid$ | **array** | Owl: make array |
| | $\mid$ | **copy** | Owl: copy array |
| | $\mid$ | **sin** | Owl: sine of all elements in array |
| | $\mid$ | **hypot** | Owl: $x_i := \sqrt{x_i^2 + y_i^2}$ |
| | $\mid$ | **asum** | BLAS: $\sum_i \lvert x_i \rvert$ |
| | $\mid$ | **axpy** | BLAS: $x := \alpha x + y$ |
| | $\mid$ | **dot** | BLAS: $x \cdot y$ |
| | $\mid$ | **rotmg** | BLAS: gen. mod. Givens rotation |
| | $\mid$ | **scal** | BLAS: $x := \alpha x$ |
| | $\mid$ | **amax** | BLAS: index of maximum absolute value |
| | $\mid$ | **setM** | matrix index assignment |
| | $\mid$ | **getM** | matrix indexing |
| | $\mid$ | **shareM** | share matrix |
| | $\mid$ | **unshareM** | unshare matrix |
| | $\mid$ | **freeM** | free matrix |
| | $\mid$ | **matrix** | Owl: make matrix |
| | $\mid$ | **copyM** | Owl: copy matrix |
| | $\mid$ | **gemv** | BLAS: $y := \alpha A^{T?} x + \beta y$ |
| | $\mid$ | **symv** | BLAS: $y := \alpha A_{\text{sym}} x + \beta y$ |
| | $\mid$ | **trmv** | BLAS: $x := A^{T?} * x$ |
| | $\mid$ | **trsv** | BLAS: $x := A^{-1 \cdot T?} * x$ |

| | | |
|---|---|---|
| | **ger** | BLAS: $A := \alpha * x * y^T + A$ |
| | **gemm** | BLAS: $C := \alpha * A^{T?} * B^{T?} + \beta C$ |
| | **trmm** | BLAS: $B := \alpha * A^{T?} * B$ and swapped |
| | **trsm** | BLAS: $B := \alpha * A^{-1 \cdot T?} * B$ and swapped |

| | | | |
|---|---|---|---|
| $\Theta$ | ::= | | fractional capability environment |
| | \| | . | |
| | \| | $\Theta, fc$ | |

| | | | |
|---|---|---|---|
| $\Gamma$ | ::= | | linear types environment |
| | \| | . | |
| | \| | $\Gamma, x : t$ | |
| | \| | $\Gamma, \Gamma'$ | |

| | | | |
|---|---|---|---|
| $\Delta$ | ::= | | linear types environment |
| | \| | . | |
| | \| | $\Delta, x : t$ | |

| | | |
|---|---|---|
| *formula* | ::= | |
| | \| | *judgement* |
| | \| | $x : t \in \Delta$ |
| | \| | $x : t \in \Gamma$ |
| | \| | $fc \in \Theta$ |
| | \| | **value** $(e)$ |

| | | | |
|---|---|---|---|
| *Well_Formed* | ::= | | |
| | \| | $\Theta \vdash f$ Cap | Valid fractional capabilities |
| | \| | $\Theta \vdash t$ Type | Valid types |

| | | | |
|---|---|---|---|
| *Values* | ::= | | |
| | \| | **value** $(e)$ | Value restriction for !-introduction |

| | | | |
|---|---|---|---|
| *Types* | ::= | | |
| | \| | $\Theta; \Delta; \Gamma \vdash e : t$ | Typing rules for expressions (no primitives yet) |

| | | |
|---|---|---|
| *judgement* | ::= | |
| | \| | *Well_Formed* |
| | \| | *Values* |
| | \| | *Types* |

| | | |
|---|---|---|
| *user_syntax* | ::= | |
| | \| | *fc* |
| | \| | *x* |
| | \| | *k* |
| | \| | *el* |
| | \| | *symb* |
| | \| | *f* |

| $t$
| $e$
| $p$
| $\Theta$
| $\Gamma$
| $\Delta$
| *formula*

$\boxed{\Theta \vdash f\ \mathsf{Cap}}$ Valid fractional capabilities

$$\frac{fc \in \Theta}{\Theta \vdash fc\ \mathsf{Cap}}\ \textsc{WF_Cap_Var}$$

$$\frac{}{\Theta \vdash \mathbf{Z}\ \mathsf{Cap}}\ \textsc{WF_Cap_Zero}$$

$$\frac{\Theta \vdash f\ \mathsf{Cap}}{\Theta \vdash \mathbf{S}\, f\ \mathsf{Cap}}\ \textsc{WF_Cap_Succ}$$

$\boxed{\Theta \vdash t\ \mathsf{Type}}$ Valid types

$$\frac{}{\Theta \vdash \mathbf{unit}\ \mathsf{Type}}\ \textsc{WF_Type_Unit}$$

$$\frac{}{\Theta \vdash \mathbf{bool}\ \mathsf{Type}}\ \textsc{WF_Type_Bool}$$

$$\frac{}{\Theta \vdash \mathbf{int}\ \mathsf{Type}}\ \textsc{WF_Type_Int}$$

$$\frac{}{\Theta \vdash \mathbf{elt}\ \mathsf{Type}}\ \textsc{WF_Type_Elt}$$

$$\frac{\Theta \vdash f\ \mathsf{Cap}}{\Theta \vdash f\ \mathbf{arr}\ \mathsf{Type}}\ \textsc{WF_Type_Array}$$

$$\frac{\Theta \vdash t\ \mathsf{Type}}{\Theta \vdash !t\ \mathsf{Type}}\ \textsc{WF_Type_Bang}$$

$$\frac{\Theta, fc \vdash t\ \mathsf{Type}}{\Theta \vdash \forall\, fc.t\ \mathsf{Type}}\ \textsc{WF_Type_Gen}$$

$$\frac{\Theta \vdash t\ \mathsf{Type} \quad \Theta \vdash t'\ \mathsf{Type}}{\Theta \vdash t \otimes t'\ \mathsf{Type}}\ \textsc{WF_Type_Pair}$$

$$\frac{\Theta \vdash t\ \mathsf{Type} \quad \Theta \vdash t'\ \mathsf{Type}}{\Theta \vdash t \multimap t'\ \mathsf{Type}}\ \textsc{WF_Type_Lolly}$$

$\boxed{\mathbf{value}\,(e)}$ Value restriction for !-introduction

$$\frac{}{\mathbf{value}\,(p)}\ \textsc{Val_Prim}$$

$$\frac{}{\mathbf{value}\,(())}\ \textsc{Val_Unit_Intro}$$

$$\frac{}{\mathbf{value}\,(\mathbf{true})}\ \textsc{Val_Bool_True}$$

$$\frac{}{\mathbf{value}\,(\mathbf{false})}\ \textsc{Val_Bool_False}$$

$$\frac{}{\mathbf{value}\,(k)}\ \textsc{Val\_Int\_Intro}$$

$$\frac{}{\mathbf{value}\,(el)}\ \textsc{Val\_Elt\_Intro}$$

$$\frac{}{\mathbf{value}\,(x)}\ \textsc{Val\_Var}$$

$$\frac{}{\mathbf{value}\,(\mathbf{fix}\,(g, x : t, e : t'))}\ \textsc{Val\_Fix}$$

$$\frac{}{\mathbf{value}\,(\mathbf{fun}\,x : t \to e)}\ \textsc{Val\_Lambda}$$

$$\frac{\mathbf{value}\,(e)}{\mathbf{value}\,(\mathbf{fun}\,fc \to e)}\ \textsc{Val\_Gen}$$

$$\frac{\mathbf{value}\,(e)}{\mathbf{value}\,(e[fc])}\ \textsc{Val\_Spc}$$

$$\frac{\mathbf{value}\,(e)}{\mathbf{value}\,(\mathbf{many}\,e)}\ \textsc{Val\_Bang\_Intro}$$

$$\frac{\mathbf{value}\,(e_1)\quad \mathbf{value}\,(e_2)}{\mathbf{value}\,((e_1, e_2))}\ \textsc{Val\_Pair\_Intro}$$

$\boxed{\Theta; \Delta; \Gamma \vdash e : t}$ Typing rules for expressions (no primitives yet)

$$\frac{}{\Theta; \Delta; \cdot, x : t \vdash x : t}\ \textsc{Ty\_Var\_Lin}$$

$$\frac{x : t \in \Delta}{\Theta; \Delta; \cdot \vdash x : t}\ \textsc{Ty\_Var}$$

$$\frac{}{\Theta; \Delta; \cdot \vdash () : !\mathbf{unit}}\ \textsc{Ty\_Unit\_Intro}$$

$$\frac{}{\Theta; \Delta; \cdot \vdash \mathbf{true} : !\mathbf{bool}}\ \textsc{Ty\_Bool\_True}$$

$$\frac{}{\Theta; \Delta; \cdot \vdash \mathbf{false} : !\mathbf{bool}}\ \textsc{Ty\_Bool\_False}$$

$$\frac{\Theta; \Delta; \Gamma \vdash e : \mathbf{bool}\quad \Theta; \Delta; \Gamma' \vdash e_1 : t'\quad \Theta; \Delta; \Gamma' \vdash e_2 : t'}{\Theta; \Delta; \Gamma, \Gamma' \vdash \mathbf{if}\,e\,\mathbf{then}\,e_1\,\mathbf{else}\,e_2 : t}\ \textsc{Ty\_Bool\_Elim}$$

$$\frac{}{\Theta; \Delta; \cdot \vdash k : !\mathbf{int}}\ \textsc{Ty\_Int\_Intro}$$

$$\frac{}{\Theta; \Delta; \cdot \vdash el : !\mathbf{elt}}\ \textsc{Ty\_Elt\_Intro}$$

$$\frac{\mathbf{value}\,(e)\quad \Theta; \Delta; \cdot \vdash e : t}{\Theta; \Delta; \cdot \vdash \mathbf{many}\,e : !t}\ \textsc{Ty\_Bang\_Intro}$$

$$\frac{\Theta; \Delta; \Gamma \vdash e : !t\quad \Theta; \Delta, x : t; \Gamma' \vdash e' : t'}{\Theta; \Delta; \Gamma, \Gamma' \vdash \mathbf{let\,many}\,x = e\,\mathbf{in}\,e' : t'}\ \textsc{Ty\_Bang\_Elim}$$

$$\frac{\Theta; \Delta; \Gamma \vdash e : t\quad \Theta; \Delta; \Gamma' \vdash e' : t'}{\Theta; \Delta; \Gamma, \Gamma' \vdash (e, e') : t \otimes t'}\ \textsc{Ty\_Pair\_Intro}$$

$$\frac{\Theta;\Delta;\Gamma \vdash e_{12} : t_1 \otimes t_2 \quad \Theta;\Delta;\Gamma', a : t_1, b : t_2 \vdash e : t}{\Theta;\Delta;\Gamma,\Gamma' \vdash \mathbf{let}\,(a,b) = e_{12}\,\mathbf{in}\,e : t}\ \textsc{Ty\_Pair\_Elim}$$

$$\frac{\Theta \vdash t' \;\mathsf{Type} \quad \Theta;\Delta;\Gamma, x : t' \vdash e : t}{\Theta;\Delta;\Gamma \vdash \mathbf{fun}\,x : t' \to e : t' \multimap t}\ \textsc{Ty\_Lambda}$$

$$\frac{\Theta;\Delta;\Gamma \vdash e : t' \multimap t \quad \Theta;\Delta;\Gamma' \vdash e' : t'}{\Theta;\Delta;\Gamma,\Gamma' \vdash e\;e' : t}\ \textsc{Ty\_App}$$

$$\frac{\Theta, fc;\Delta;\Gamma \vdash e : t}{\Theta;\Delta;\Gamma \vdash \mathbf{fun}\,fc \to e : \forall\, fc.t}\ \textsc{Ty\_Gen}$$

$$\frac{\Theta \vdash f \;\mathsf{Cap} \quad \Theta;\Delta;\Gamma \vdash e : \forall\, fc.t}{\Theta;\Delta;\Gamma \vdash e[f] : t\{f/fc\}}\ \textsc{Ty\_Spc}$$

$$\frac{\Theta;\Delta, g : t \multimap t';\cdot, x : t \vdash e : t'}{\Theta;\Delta;\cdot \vdash \mathbf{fix}\,(g, x : t, e : t') : !(t \multimap t')}\ \textsc{Ty\_Fix}$$

```
Definition rules:         19 good    0 bad
Definition rule clauses: 43 good    0 bad
```