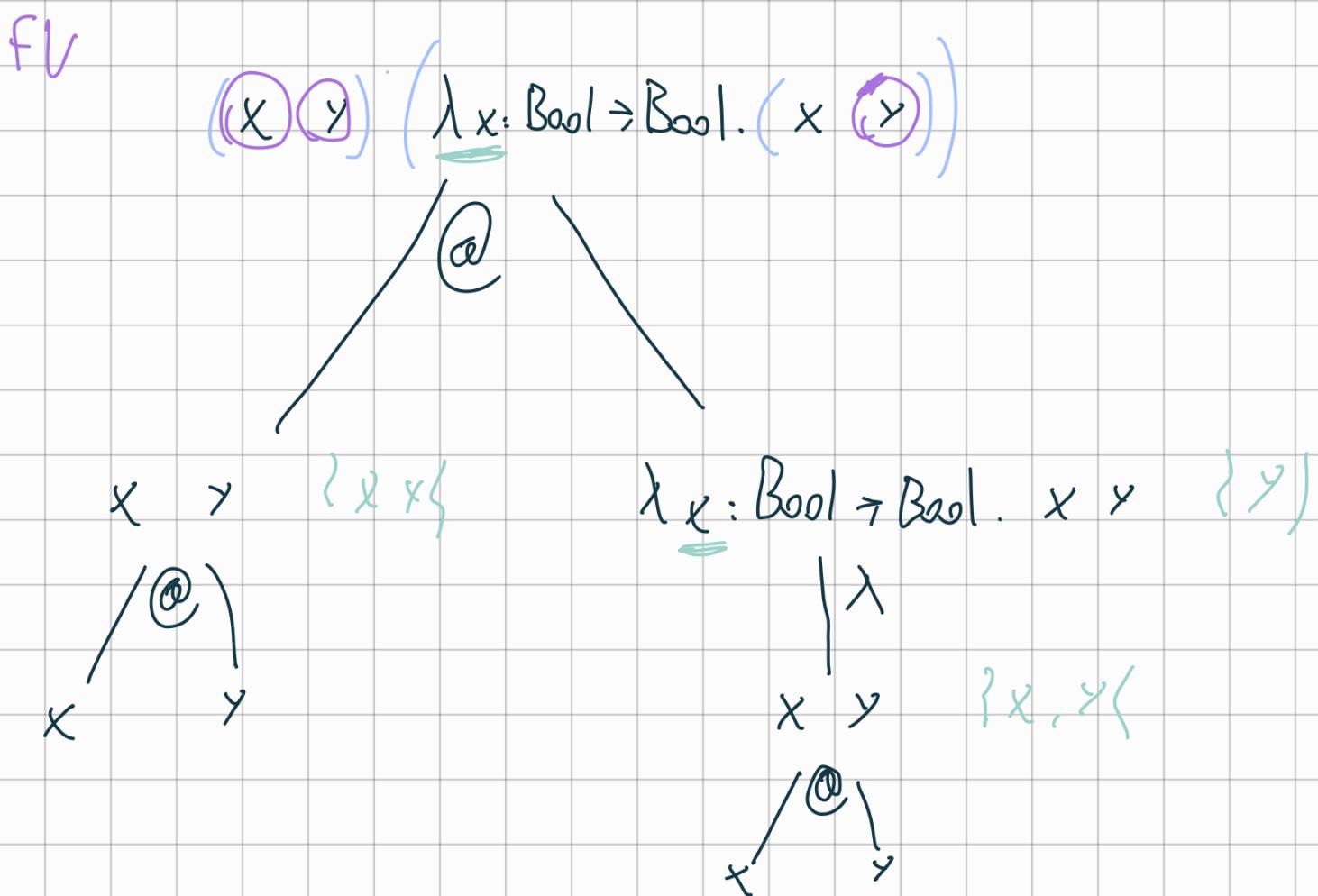
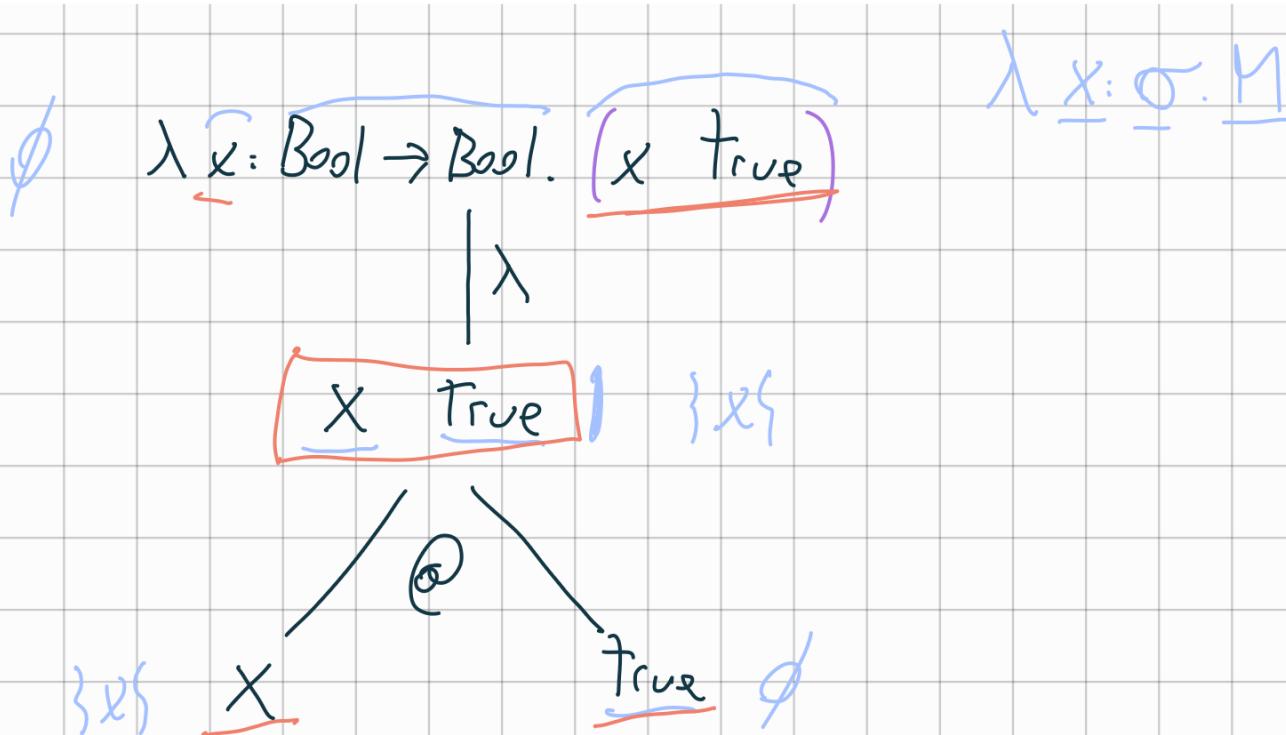


Ejercicio: ¿Cuáles de las siguientes expresiones son términos del cálculo lambda? En los casos que sí lo sean, dibujar su árbol sintáctico y marcar las ocurrencias libres de las variables.

- a) $\lambda x : \text{Bool} \rightarrow \text{Bool}. x \text{ true}$ ✓
- b) $x y \lambda x : \text{Bool} \rightarrow \text{Bool}. x y$ ✓
- c) $(\lambda x : \text{Bool} \rightarrow \text{Bool}. x y)(\lambda y : \text{Bool}. x)$ ✓
- d) $\lambda x : \text{Bool}$ X
- e) $\lambda x. x$ X

- f) $\text{if } x \text{ then } y \text{ else } \lambda z : \text{Bool}. z$
- g) $\lambda y : \sigma. y$ X
- h) true false
- i) $x M$ X
- j) $\text{if } x \text{ then } \lambda x : \text{Bool}. x$



$$(\lambda x: \text{Bool} \rightarrow \text{Bool}). x(y) (\lambda y: \text{Bool}. x)$$

$$\begin{array}{ccc} & @ & \\ \lambda x: \text{Bool} \rightarrow \text{Bool}. x y & & \lambda y: \text{Bool}. x \\ | \lambda & & | \lambda \\ x y & & x \\ x @ y & & \end{array}$$

$$\Gamma = x: \text{Bool}, y: \text{Bool}$$

$$\frac{\Gamma \vdash x: \text{Bool}}{x: \text{Bool}, y: \text{Bool} \vdash \text{if } x \text{ then true else } y: \text{Bool}} \text{-Var} \quad \frac{\Gamma \vdash \text{True}: \text{Bool}}{\Gamma \vdash \text{True}: \text{Bool}} \text{-True} \quad \frac{\Gamma \vdash y: \text{Bool}}{\Gamma \vdash y: \text{Bool}} \text{-Var} \quad \frac{x: \text{Bool} / \vdash \lambda y: \text{Bool}. \text{if } x \text{ then True else } y: \text{Bool} \rightarrow \text{Bool}}{\vdash \lambda x: \text{Bool}. \lambda y: \text{Bool}. \text{if } x \text{ then True else } y: \text{Bool} \rightarrow \text{Bool}} \text{-Abs} \quad \frac{\Gamma \vdash \text{False}: \text{Bool}}{\vdash \text{False}: \text{Bool}} \text{-False} \quad \frac{}{\vdash (\lambda x: \text{Bool}. \lambda y: \text{Bool}. \text{if } x \text{ then true else } y) \text{ false}: \text{Bool} \rightarrow \text{Bool}} \text{-App}$$

$$\frac{\text{---} \quad \Gamma \vdash x : \text{Bool} \quad \Gamma \vdash x : \text{Bool}}{\Gamma \vdash \text{if } x \text{ then } x \text{ else } (\lambda y : \text{Bool}.y) : \text{Bool} \rightarrow \text{Bool}}$$

T-Var

$$\frac{\Gamma \vdash x : \text{Bool} \quad \Gamma \vdash x : \text{Bool} \rightarrow \text{Bool}}{\Gamma \vdash \text{if } x \text{ then } x \text{ else } (\lambda y : \text{Bool}.y) : \text{Bool} \rightarrow \text{Bool}}$$

T-if

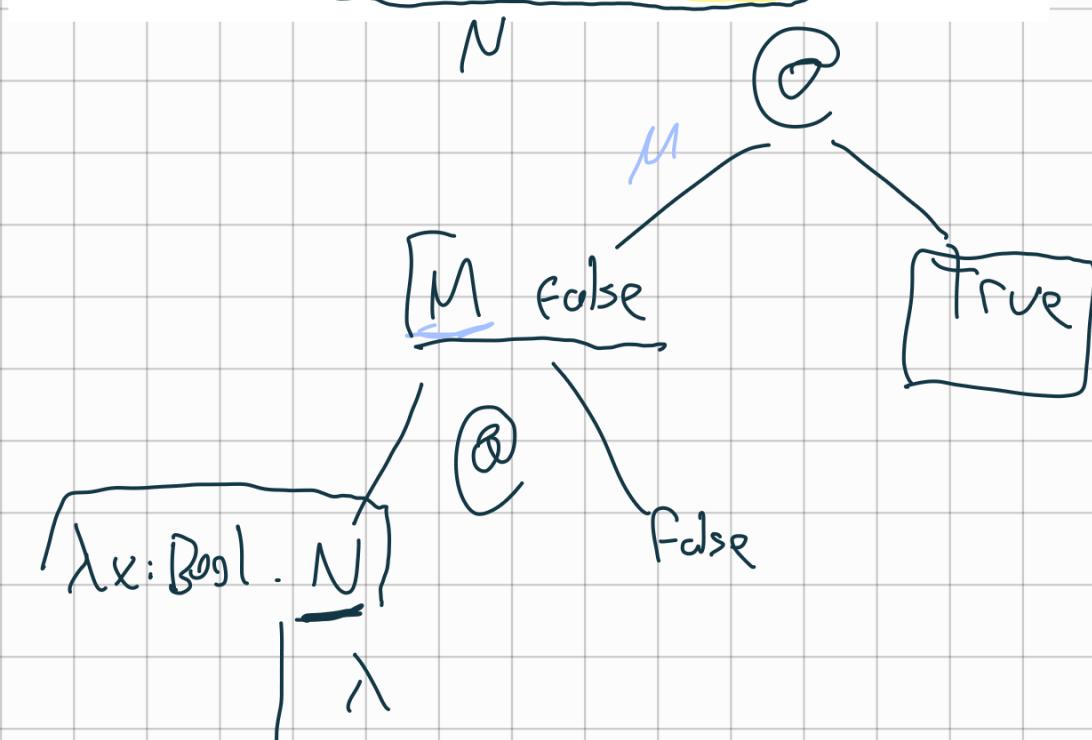
$$\left. \begin{array}{l} \rho = \textcolor{blue}{\boxed{\downarrow}} \rightarrow \textcolor{yellow}{\boxed{\downarrow}} \rightarrow \textcolor{red}{\boxed{\downarrow}} \\ \sigma = \textcolor{red}{\boxed{\downarrow}} \rightarrow \textcolor{teal}{\boxed{\downarrow}} \\ \tau = \textcolor{yellow}{\boxed{\downarrow}} \\ \Omega = \textcolor{blue}{\boxed{\downarrow}} \end{array} \right\} \quad \left. \begin{array}{l} \rho = \sigma \rightarrow (\tau \rightarrow \Omega) \\ \rho = \Omega \rightarrow (\tau \rightarrow \sigma) \end{array} \right.$$

$$\frac{}{\Gamma \vdash x : \textcolor{blue}{\boxed{\downarrow}} \rightarrow \textcolor{yellow}{\boxed{\downarrow}} \rightarrow \textcolor{red}{\boxed{\downarrow}}} \quad \frac{}{\Gamma \vdash y : \textcolor{blue}{\boxed{\downarrow}}} \quad \frac{}{\Gamma \vdash z : \textcolor{yellow}{\boxed{\downarrow}}}$$

$\frac{}{\Gamma \vdash x : \textcolor{red}{\boxed{\downarrow}} \rightarrow \textcolor{teal}{\boxed{\downarrow}}} \quad \frac{\Gamma \vdash x : \textcolor{blue}{\boxed{\downarrow}} \rightarrow \textcolor{red}{\boxed{\downarrow}} \quad \Gamma \vdash y : \textcolor{blue}{\boxed{\downarrow}}}{\Gamma \vdash x y : \textcolor{yellow}{\boxed{\downarrow}} \rightarrow \textcolor{red}{\boxed{\downarrow}}} \quad \frac{\Gamma \vdash z : \textcolor{yellow}{\boxed{\downarrow}} \quad \Gamma \vdash x y z : \textcolor{teal}{\boxed{\downarrow}}}{\Gamma \vdash x y z : \textcolor{teal}{\boxed{\downarrow}}}$

$\frac{\Gamma = x : \rho, y : \sigma, z : \tau \vdash x (x y z) : \textcolor{teal}{\boxed{\downarrow}}}{x : \rho, y : \sigma, z : \tau \vdash \lambda z : \tau. x (x y z) : \textcolor{teal}{\boxed{\downarrow}} \rightarrow \textcolor{teal}{\boxed{\downarrow}}} \quad \frac{x : \rho, y : \sigma \vdash \lambda y : \sigma. \lambda z : \tau. x (x y z) : \textcolor{purple}{\sigma \rightarrow \boxed{\downarrow}}}{x : \rho \vdash \lambda x : \rho. \lambda y : \sigma. \lambda z : \tau. x (x y z) : \rho \rightarrow \textcolor{violet}{\boxed{\downarrow}}}$

M
(($\lambda x : \text{Bool}.$ $\lambda y : \text{Bool}.$ if x then true else y) false) true



$\lambda y : \text{Bool}. \text{if } x \text{ then true else } y$

$N \{ x := \text{false} \}$

$\text{if } x \text{ then true else } y$

x True y

(($\lambda x : \text{Bool}.$ $\lambda y : \text{Bool}.$ if x then true else y) false) true

$\mu, \beta \rightarrow (\lambda y : \text{Bool}. \text{if false then true else } y) \text{ true}$

$\beta \rightarrow \text{if false then true else true}$

$\text{if-f} \rightarrow \text{true}$

$(\lambda x : \text{Bool}.\lambda y : \text{Bool} \rightarrow \text{Bool}.y(yx))((\lambda z : \text{Bool}.\text{true}) \text{ false})(\lambda w : \text{Bool}.w)$

M

u

@

N

O

M(N false)

O

u

M

(N false)

/ @

N False

$(\lambda x : \text{Bool}.\lambda y : \text{Bool} \rightarrow \text{Bool}.y(yx))((\lambda z : \text{Bool}.\text{true}) \text{ false})(\lambda w : \text{Bool}.w)$

$\xrightarrow{\mu, u, \beta} (M \text{ true}) O$

$\xrightarrow{\mu, \beta} (\lambda y : \text{Bool} \rightarrow \text{Bool}. y(y \text{ true})) (\lambda w : \text{Bool}. w)$

$\xrightarrow{\beta} (\lambda w : \text{Bool}. w) ((\lambda w : \text{Bool}. w) \text{ true})$

$\xrightarrow{\mu, \beta} (\lambda w : \text{Bool}. w) \text{ true}$

$\xrightarrow{\beta} \text{ true}$

$$\frac{\frac{x:\text{Nat} \vdash x : \text{Nat}}{x:\text{Nat} \vdash \text{succ}(x) : \text{Nat}} \text{-Var} \quad \frac{}{\vdash \lambda x:\text{Nat}. \text{succ}(x) : \text{Nat} \rightarrow \text{Nat}} \text{-Suc} \quad \frac{}{\vdash \text{Zero} : \text{Nat}} \text{-Abs} \quad \frac{}{\vdash (\lambda x:\text{Nat}. \text{succ}(x)) \text{Zero} : \text{Nat}} \text{-Lam}}{\vdash (\lambda x:\text{Nat}. \text{succ}(x)) \text{Zero} : \text{Nat}} \text{-Opp}$$

Zero
 $\text{isZero}(\text{succ}(\text{pred}(\text{succ}(\text{zero}))))$

$\longrightarrow \text{isZero}(\text{succ}(\text{Zero})) \longrightarrow \text{False}$

E-isZero
E-Succ
E-PredSucc

$\text{isZero}(\text{succ}(\text{pred}(\text{Zero})))$

\rightarrow
E-isZero

Caso Base

- $M \xrightarrow{\beta} M_1, N \xrightarrow{c} M_2$ que $M_2 = M_1$

$$M = (\lambda x : \sigma . N) \vee$$

- $M \xrightarrow{\text{if } t} M_1, N \xrightarrow{c} M_2$ que $M_2 = M_1$

- \hookrightarrow mismo con $M \xrightarrow{\text{FF}} M_1$

Caso inducción:

H.I: $M' \xrightarrow{} M'_1, M' \xrightarrow{} M'_2 \Rightarrow \overbrace{M'_1 = M'_2}$

Caso μ

$M \xrightarrow{\mu} M_1 \circ M = M' N, M_1 = M'_1, N$

donde $N' \xrightarrow{} M'_1$

$M \xrightarrow{} M_2$

$$M'N \rightarrow M'_2 N \quad ; \quad M' \rightarrow M'_2$$

$$M' \rightarrow M'_1 , \quad M' \rightarrow M'_2 \Rightarrow \underline{M'_1 = M'_2}$$

$$\underbrace{M'_2 N}_{M_2} = \underbrace{M'_1 N}_{M_1}$$

$$(\lambda x: \text{Bool} \rightarrow \text{Bool}). X) \text{ True}$$