```
On[Assert]
$Path = Union[Append[$Path, NotebookDirectory[]]];
<< SmolyakQuadrature
(* Plot the number of accurate digits of Gaussian integration of Cos[x] *)
ListPlot[
 With [{prec = 100},
  Table \left[-\text{Log}\left[10, N\left[Abs\left[gint\left[Cos, k, prec\right] - Sin\left[1\right]\right], prec\right]\right], \{k, 20\}\right]\right]
 , PlotLabel → "Gauss-Legendre quadrature of cos(x): accurate digits",
 AxesLabel → {"order", "digits"}]
     Gauss-Legendre quadrature of cos(x): accurate digits
digits
70
60
50
40
30
20
10
checkAccuracyOrder[gw, 10, 100, 2 # - 1 &];
(* Check that KP abscissas are nested *)
Do [Assert [Length [kpAbscissas [n, 20]] ==
     Length [Intersection [kpAbscissas[n, 20], kpAbscissas[n+1, 20]]]], \{n, 4\}] 
Assert[accuracyOrder[kp, 1, 20] == 1]
Assert accuracyOrder kp, 2, 20 = 5
Assert[accuracyOrder[kp, 3, 20] == 11]
Assert[accuracyOrder[kp, 4, 20] == 23]
Assert[accuracyOrder[kp, 5, 20] == 47]
Assert [Min Table ]
     accuracyOrder[kpDelay[delayFull], n, 20] - (2 n - 1), \{n, Length[delayFull]\}]] \ge 0]
(* Check that the quadrature formula is
 the same as gw for legendre polynomial roots *)
Assert[roundPrec[quadrature[(legendreRoots[10]+1)/2, 40], 20]-
    roundPrec[gw[10, 40], 20] == 0]
```

```
(* CC formulas *)
ClearAll [ccAbscissas, ccM, cc]
\operatorname{ccM}[1] := 1
CCM[n_] := 2^{n-1} + 1
ccAbscissas[1, prec_] := \{0\}
cc[n_, prec_] :=
 cc[n, prec] = iteratePrec[quadrature[(ccAbscissas[n, prec] + 1) / 2, #] &, prec]
(* Check that CC abscissas are nested *)
Do [Assert [Length [ccAbscissas [n, 20]] ==
     Length \big[ Intersection \big[ ccAbscissas \big[ n, 20 \big], \ ccAbscissas \big[ n+1, \ 20 \big] \big] \big] \big], \ \big\{ n, \ 4 \big\} \big] 
(* Check the accuracy order of the CC formulas *)
checkAccuracyOrder[cc, 6, 30, ccM[#] &];
(* check the 4d rule *)
{\tt Assert} \big[ {\tt Length} \big[ {\tt ecfRule} \big[ {\tt rule4d9k145} \big] \big] = {\tt 145} \big]
Assert[Precision[to01cube4d[ecfRule[rule4d9k145]]] > 29]
(* check the tetrahedral rule *)
Assert [Length [ecfRule [simplex3d8k43]] == 43]
Assert Abs \begin{bmatrix} 6 \text{ Total} \\ \text{ecfRule} \\ \text{simplex3d8k43} \\ \end{bmatrix} \begin{bmatrix} \begin{bmatrix} All, 1 \end{bmatrix} \end{bmatrix} - 1 \end{bmatrix} < 10^{-30} \end{bmatrix}
(* check the triangle rule *)
Assert [Length [ecfRule[simplex2d10k25a]] == 25]
Assert [Abs[2 Total[ecfRule[simplex2d10k25a][[All, 1]]] - 1] < 10^{-30}]
 \textbf{Assert} \left[ \textbf{Max} \left[ \textbf{Abs} \left[ \textbf{Total} \left[ \textbf{ecfRule} \left[ \textbf{simplex2d10k25a} \right] \left[ \left[ \textbf{All, 2;;} \right] \right], \left\{ 2 \right\} \right] - 1 \right] \right] < 10^{-30} \right] 
intTet[1&, {{0, 0, 0}, {1, 0, 0}, {0, 1, 0}, {0, 0, 1}}];
intTri[1 &, {{0, 0, 0}, {1, 0, 0}, {0, 1, 0}}];
```