

$$\begin{aligned}
& \int_0^{r_c} r^2 \phi^*(r) \left(-\frac{1}{2} \frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} - \frac{2}{r} \right) \phi(r) dr = E \int_0^{r_c} r^2 \phi^*(r) \phi(r) dr \\
& \int_0^{r_c} r^2 \phi^*(r) \left(-\frac{1}{2} \frac{d^2}{dr^2} \right) \phi(r) dr = -\frac{1}{2} \int_0^{r_c} r^2 \phi^*(r) \frac{d^2 \phi(r)}{dr^2} \\
& = -\frac{1}{2} \int_0^{r_c} r^2 \phi^*(r) \frac{d}{dr} \left[\frac{d\phi(r)}{dr} \right] dr \\
& = -\frac{r^2}{2} \phi^*(r) \frac{d\phi(r)}{dr} \Big|_0^{r_c} + \frac{1}{2} \int_0^{r_c} \frac{d}{dr} [r^2 \phi^*(r)] \frac{d\phi(r)}{dr} dr \\
& = \frac{1}{2} \int_0^{r_c} \frac{d}{dr} [r^2 \phi^*(r)] \frac{d\phi(r)}{dr} dr \quad \because \phi^*(r_c) = 0 \\
& = \frac{1}{2} \int_0^{r_c} \left[2r \phi^*(r) + r^2 \frac{d\phi^*(r)}{dr} \right] \frac{d\phi(r)}{dr} dr \\
& = \int_0^{r_c} r \phi^*(r) \frac{d\phi(r)}{dr} dr + \int_0^{r_c} \frac{r^2}{2} \frac{d\phi^*(r)}{dr} \frac{d\phi(r)}{dr} dr \\
& \int_0^{r_c} r^2 \phi^*(r) \left(-\frac{1}{r} \frac{d}{dr} \right) \phi(r) dr = -\int_0^{r_c} r \phi^*(r) \frac{d\phi(r)}{dr} dr \\
& \int_0^{r_c} r^2 \phi^*(r) \left(-\frac{1}{2} \frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} - \frac{2}{r} \right) \phi(r) dr = \int_0^{r_c} r \phi^*(r) \frac{d\phi(r)}{dr} dr + \int_0^{r_c} \frac{r^2}{2} \frac{d\phi^*(r)}{dr} \frac{d\phi(r)}{dr} \\
& \quad - \int_0^{r_c} r \phi^*(r) \frac{d\phi(r)}{dr} dr - 2 \int_0^{r_c} r \phi^*(r) \phi(r) dr \\
& \int_0^{r_c} r^2 \phi^*(r) \left(-\frac{1}{2} \frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} - \frac{1}{r} \right) \phi(r) dr = \int_0^{r_c} \frac{r^2}{2} \frac{d\phi^*(r)}{dr} \frac{d\phi(r)}{dr} dr - 2 \int_0^{r_c} r \phi^*(r) \phi(r) dr \\
& \therefore \int_0^{r_c} \left[\frac{r^2}{2} \frac{d\phi^*(r)}{dr} \frac{d\phi(r)}{dr} - 2r \phi^*(r) \phi(r) \right] dr = E \int_0^{r_c} r^2 \phi^*(r) \phi(r) dr
\end{aligned}$$