$$\frac{d^2U(r)}{dr^2} = -rn(r)$$

$$\int_0^{r_c} \frac{d^2U(r)}{dr^2} U(r)dr = -\int_0^{r_c} rn(r)U(r)dr$$

$$\int_0^{r_c} \frac{d^2U(r)}{dr^2} U(r) = \int_0^{r_c} \frac{d}{dr}\left[\frac{dU(r)}{dr}\right] U(r)$$

$$\int_0^{r_c} \frac{d}{dr}\left[\frac{dU(r)}{dr}\right] U(r) = \frac{dU(r)}{dr} U(r)\Big|_0^{r_c} - \int_0^{r_c} \frac{dU(r)}{dr}\frac{dU(r)}{dr}dr$$

$$\int_0^{r_c} \frac{d}{dr}\left[\frac{dU(r)}{dr}\right] U(r) = -\int_0^{r_c} \frac{dU(r)}{dr}\frac{dU(r)}{dr}dr, \because\ U(0) = \frac{dU(r)}{dr}\Big|_{r=r_c} = 0$$

$$\int_0^{r_c} \frac{dU(r)}{dr}\frac{dU(r)}{dr}dr = \int_0^{r_c} rn(r)U(r)dr$$