$$\left[-\frac{1}{2}\nabla_1^2 - \frac{2}{r_1} + \int d\vec{r}_2 |\phi(\vec{r}_2)|^2 \frac{1}{|\vec{r}_1 - \vec{r}_2|}\right]\phi(\vec{r}_1) = E'\phi(\vec{r}_1)$$

$$E' = E - \int d\vec{r}_2 \phi^*(\vec{r}_2)\left(-\frac{1}{2}\nabla_2^2 - \frac{2}{r_2}\right)\phi(\vec{r}_2)$$

$$E' = E - \int d\vec{r}_1 \phi^*(\vec{r}_1)\left(-\frac{1}{2}\nabla_1^2 - \frac{2}{r_1}\right)\phi(\vec{r}_1)$$

$$E = E' + \int d\vec{r}_1 \phi^*(\vec{r}_1)\left(-\frac{1}{2}\nabla_1^2 - \frac{2}{r_1}\right)\phi(\vec{r}_1)$$

$$E = \phi^*(\vec{r}_1)\left[-\frac{1}{2}\nabla_1^2 - \frac{2}{r_1} + \int d\vec{r}_2 |\phi(\vec{r}_2)|^2 \frac{1}{|\vec{r}_1 - \vec{r}_2|}\right]\phi(\vec{r}_1) + \int d\vec{r}_1 \phi^*(\vec{r}_1)\left(-\frac{1}{2}\nabla_1^2 - \frac{2}{r_1}\right)\phi(\vec{r}_1)$$

$$E = 2\phi^*(\vec{r}_1)\left[-\frac{1}{2}\nabla_1^2 - \frac{2}{r_1} + \int d\vec{r}_2 |\phi(\vec{r}_2)|^2 \frac{1}{|\vec{r}_1 - \vec{r}_2|}\right]\phi(\vec{r}_1) - \int\int d\vec{r}_1 d\vec{r}_2 \frac{|\phi(\vec{r}_1)|^2 |\phi(\vec{r}_2)|^2}{|\vec{r}_1 - \vec{r}_2|}$$

$$E = 2E' - \int\int d\vec{r}_1 d\vec{r}_2 \frac{|\phi(\vec{r}_1)|^2 |\phi(\vec{r}_2)|^2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\therefore\ E = 2E' - \int_0^{r_c} rU(r)n(r)dr$$