$$\int_0^{r_c} r^2\psi^*(r)\left(-\frac{1}{2}\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr} - \frac{1}{r}\right)\psi(r)dr = E\int_0^{r_c} r^2\psi^*(r)\psi(r)dr$$

$$\int_0^{r_c} r^2\psi^*(r)\left(-\frac{1}{2}\frac{d^2}{dr^2}\right)\psi(r)dr = -\frac{1}{2}\int_0^{r_c} r^2\psi^*(r)\frac{d^2\psi(r)}{dr^2}$$

$$= -\frac{1}{2}\int_0^{r_c} r^2\psi^*(r)\frac{d}{dr}\left[\frac{d\psi(r)}{dr}\right]dr$$

$$= -\frac{r^2}{2}\psi^*(r)\frac{d\psi(r)}{dr}\bigg|_0^{r_c} + \frac{1}{2}\int_0^{r_c}\frac{d}{dr}[r^2\psi^*(r)]\frac{d\psi(r)}{dr}dr$$

$$= \frac{1}{2}\int_0^{r_c}\frac{d}{dr}[r^2\psi^*(r)]\frac{d\psi(r)}{dr}dr \quad \because \quad \psi^*(r_c) = 0$$

$$= \frac{1}{2}\int_0^{r_c}\left[2r\psi^*(r) + r^2\frac{d\psi^*(r)}{dr}\right]\frac{d\psi(r)}{dr}dr$$

$$= \int_0^{r_c} r\psi^*(r)\frac{d\psi(r)}{dr}dr + \int_0^{r_c}\frac{r^2}{2}\frac{d\psi^*(r)}{dr}\frac{d\psi(r)}{dr}dr$$

$$\int_0^{r_c} r^2\psi^*(r)\left(-\frac{1}{r}\frac{d}{dr}\right)\psi(r)dr = -\int_0^{r_c} r\psi^*(r)\frac{d\psi(r)}{dr}dr$$

$$\begin{aligned}\int_0^{r_c} r^2\psi^*(r)\left(-\frac{1}{2}\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr} - \frac{1}{r}\right)\psi(r)dr &= \int_0^{r_c} r\psi^*(r)\frac{d\psi(r)}{dr}dr + \int_0^{r_c}\frac{r^2}{2}\frac{d\psi^*(r)}{dr}\frac{d\psi(r)}{dr} \\ &\quad - \int_0^{r_c} r\psi^*(r)\frac{d\psi(r)}{dr}dr - \int_0^{r_c} r\psi^*(r)\psi(r)dr\end{aligned}$$

$$\int_0^{r_c} r^2\psi^*(r)\left(-\frac{1}{2}\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr} - \frac{1}{r}\right)\psi(r)dr = \int_0^{r_c}\frac{r^2}{2}\frac{d\psi^*(r)}{dr}\frac{d\psi(r)}{dr}dr - \int_0^{r_c} r\psi^*(r)\psi(r)$$

$$\therefore \int_0^{r_c}\left[\frac{r^2}{2}\frac{d\psi^*(r)}{dr}\frac{d\psi(r)}{dr} - r\psi^*(r)\psi(r)\right]dr = E\int_0^{r_c} r^2\psi^*(r)\psi(r)dr$$