

変数変換

$$r = el^e + yl^e$$

$$\frac{dy}{dr} = \frac{1}{l^e}$$

$$dr = l^e dy$$

|     |                             |
|-----|-----------------------------|
| $r$ | $nl^e \rightarrow (n+1)l^e$ |
| $y$ | $0 \rightarrow 1$           |

$$\psi^{(e)}(y) = \sum_{i=0}^1 u_i^{(e)} \phi_i(y) \quad (0 \leq y \leq 1)$$

$$\begin{cases} \phi_0(y) = 1 - y \\ \phi_1(y) = y \end{cases}$$

ローカル行列 $\mathbf{A}$ の要素 $A_{\alpha\beta}^{(e)}$ の計算

•  $A_{00}^{(e)}$ の計算

$$\begin{aligned} & \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{dy}{dr} \left[ \frac{\phi_0(y)}{dy} \right] \frac{dy}{dr} \left[ \frac{\phi_1(y)}{dy} \right] (l^e) dy \\ &= \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{1}{l^e} dy \\ &= \int_0^1 \frac{e^2(l^e)^2 + 2e(l^e)^2 y + y^2(l^e)^2}{2} \frac{1}{l^e} dy \\ &= \frac{l^e}{2} \int_0^1 (e^2 + 2ey + y^2) dy \\ &= \frac{l^e}{2} \left[ e^2 y + ey^2 + \frac{1}{3} y^3 \right]_0^1 \\ &= \frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) \\ &- \int_0^1 (el^e + yl^e)(1-y)^2 l^e dy \\ &= -(l^e)^2 \int_0^1 (e+y)(1-y)^2 dy \\ &= -(l^e)^2 \int_0^1 [e(1-y)^2 + y(1-y)^2] dy \\ &= -(l^e)^2 \left\{ \int_0^1 [e(1-y)^2] dy + \int_0^1 y(1-y)^2 dy \right\} \\ &= -(l^e)^2 \left[ -\frac{1}{3} e(1-y)^3 \Big|_0^1 + -\frac{1}{3} y(1-y)^3 \Big|_0^1 + \frac{1}{3} \int_0^1 (1-y)^3 dy \right] \\ &= -(l^e)^2 \left[ \frac{e}{3} - \frac{1}{12} (1-y)^4 \Big|_0^1 \right] \\ &= -(l^e)^2 \left( \frac{e}{3} + \frac{1}{12} \right) \\ A_{00}^{(e)} &= \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{dy}{dr} \left[ \frac{\phi_0(y)}{dy} \right] \frac{dy}{dr} \left[ \frac{\phi_1(y)}{dy} \right] (l^e) dy - \int_0^1 (el^e + yl^e)(1-y)^2 l^e dy \\ &= \frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) - (l^e)^2 \left( \frac{e}{3} + \frac{1}{12} \right) \end{aligned}$$

•  $A_{01}^{(e)} = A_{10}^{(e)}$ の計算

$$\begin{aligned}
& \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{dy}{dr} \left[ \frac{\phi_0(y)}{dy} \right] \frac{dy}{dr} \left[ \frac{\phi_1(y)}{dy} \right] l^e dy \\
&= -\frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) \\
&- \int_0^1 (el^e + yl^e) y(1-y) l^e dy \\
&= -(l^e)^2 \int_0^1 (e+y) y(1-y) dy \\
&= -(l^e)^2 \left[ \int_0^1 ey(1-y) dy + \int_0^1 y^2(1-y) dy \right] \\
&= -(l^e)^2 \left[ \frac{e}{2} y^2(1-y) \Big|_0^1 + \int_0^1 \frac{e}{2} y^2 dy + \int_0^1 y^2(1-y) dy \right] \\
&= -(l^e)^2 \left[ \frac{e}{6} y^3 \Big|_0^1 + \int_0^1 y^2(1-y) dy \right] \\
&= -(l^e)^2 \left[ \frac{e}{6} + \int_0^1 y^2(1-y) dy \right] \\
&= -(l^e)^2 \left[ \frac{e}{6} + \frac{1}{3} y^3(1-y) \Big|_0^1 + \frac{1}{3} \int_0^1 y^3 dy \right] \\
&= -(l^e)^2 \left( \frac{e}{6} + \frac{1}{12} \right) \\
A_{01}^{e(e)} &= A_{10}^{(e)} = \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{dy}{dr} \left[ \frac{\phi_0(y)}{dy} \right] \frac{dy}{dr} \left[ \frac{\phi_1(y)}{dy} \right] l^e dy - \int_0^1 (el^e + yl^e) y(1-y) l^e dy \\
&= -\frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) - (l^e)^2 \left( \frac{e}{6} + \frac{1}{12} \right)
\end{aligned}$$

$$\begin{aligned}
& \bullet A_{11}^{(e)} = A_{11}^{(e)} \text{ の計算} \\
& \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{dy}{dr} \left[ \frac{\phi_0(y)}{dy} \right] \frac{dy}{dr} \left[ \frac{\phi_1(y)}{dy} \right] l^e dy \\
&= \frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) \\
&- \int_0^1 (el^e + yl^e) y^2 l^e dy \\
&= -(l^e)^2 \int_0^1 (e+y) y^2 dy \\
&= -(l^e)^2 \left[ \int_0^1 ey^2 dy + \int_0^1 y^3 dy \right] \\
&= -(l^e)^2 \left( \frac{e}{3} + \frac{1}{4} \right) \\
A_{11}^{(e)} &= \int_0^1 \frac{(el^e + yl^e)^2}{2} \frac{dy}{dr} \left[ \frac{\phi_0(y)}{dy} \right] \frac{dy}{dr} \left[ \frac{\phi_1(y)}{dy} \right] l^e dy - \int_0^1 (el^e + yl^e) y^2 l^e dy \\
&= \frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) - (l^e)^2 \left( \frac{e}{3} + \frac{1}{4} \right)
\end{aligned}$$

ローカル行列  $\mathbf{B}$  の要素  $B_{\alpha\beta}^{(e)}$  の計算

$$\begin{aligned}
& \bullet B_{00}^{(e)} = B_{00}^{(e)} \text{ の計算} \\
B_{00}^{(e)} &= \int_0^1 (el^e + yl^e)^2 \phi_0(y) \phi_0(y) l^e dy \\
&= \int_0^1 (el^e + yl^e)^2 (1-y)^2 l^e dy
\end{aligned}$$

$$\begin{aligned}
&= \int_0^1 (el^e + yl^e)^2 (1-y)^2 l^e dy \\
&= \int_0^1 [e^2(l^e)^2 + 2e(l^e)^2 y + y^2(l^e)^2] (1-y)^2 l^e dy \\
&= (l^e)^3 \int_0^1 [e(l^e)^2 + 2e(l^e)^2 y + y^2(l^e)^2] (1-y)^2 dy \\
&= (l^e)^3 \int_0^1 e^2 (1-y)^2 dy + (l^e)^3 \int_0^1 2ey(1-y)^2 dy + (l^e)^3 \int_0^1 y^2 (1-y)^2 dy \\
&= \frac{(l^e)^3}{3} e^2 + \frac{(l^e)^3}{6} e + \frac{1}{30} (l^e)^3 \\
&= (l^e)^3 \left( \frac{e^2}{3} + \frac{e}{6} + \frac{1}{30} \right)
\end{aligned}$$

•  $B_{01}^{(e)} = B_{10}^{(e)}$  の計算

$$\begin{aligned}
B_{01}^{(e)} &= B_{10}^{(e)} = \int_0^1 (el^e + yl^e)^2 \phi_0(y) \phi_1(y) l^e dy \\
&= (l^e)^3 \int_0^1 (e^2 + 2ey + y^2) y (1-y) dy \\
&= \frac{(l^e)^3}{6} e^2 + \frac{(l^e)^3}{6} e + \frac{(l^e)^3}{20} \\
&= (l^e)^3 \left( \frac{e^2}{6} + \frac{e}{6} + \frac{1}{20} \right)
\end{aligned}$$

•  $B_{11}^{(e)} = B_{11}^{(e)}$  の計算

$$\begin{aligned}
B_{11}^{(e)} &= \int_0^1 (el^e + yl^e)^2 \phi_1(y) \phi_1(y) l^e dy \\
&= \int_0^1 (el^e + yl^e)^2 y^2 l^e dy \\
&= (l^e)^3 \int_0^1 (e^2 + 2ey + y^2) y^2 dy \\
&= (l^e)^3 \left( \frac{e^2}{3} + \frac{e}{2} + \frac{1}{5} \right)
\end{aligned}$$

結果

$$\begin{aligned}
\langle \psi_\alpha^{(e)} | A_{\alpha\beta}^{(e)} | \psi_\beta^{(e)} \rangle &= \langle \psi_\alpha^{(e)} | B_{\alpha\beta}^{(e)} | \psi_\beta^{(e)} \rangle \\
\left\{ \begin{array}{l} A_{00}^{(e)} = \frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) - (l^e)^2 \left( \frac{e}{3} + \frac{1}{12} \right) \\ A_{01}^{(e)} = A_{10}^{(e)} = -\frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) - (l^e)^2 \left( \frac{e}{6} + \frac{1}{12} \right) \\ A_{11}^{(e)} = \frac{l^e}{2} \left( e^2 + e + \frac{1}{3} \right) - (l^e)^2 \left( \frac{e}{3} + \frac{1}{4} \right) \end{array} \right. \\
\left\{ \begin{array}{l} B_{00}^{(e)} = (l^e)^3 \left( \frac{e^2}{3} + \frac{e}{6} + \frac{1}{30} \right) \\ B_{01}^{(e)} = B_{10}^{(e)} = (l^e)^3 \left( \frac{e^2}{6} + \frac{e}{6} + \frac{1}{20} \right) \\ B_{11}^{(e)} = (l^e)^3 \left( \frac{e^2}{3} + \frac{e}{2} + \frac{1}{5} \right) \end{array} \right.
\end{aligned}$$