

$$\phi_1(x) = \frac{x_2 - x}{l}$$

$$\phi_2(x) = \frac{x - x_1}{l}$$

$$u(x) = u_1\phi_1(x) + u_2\phi_2(x)$$

$$\omega(x) = \omega_1\phi_1(x) + \omega_2\phi_2(x)$$

$$\frac{d^2u(x)}{dx^2} = 1$$

$$\begin{aligned} & \int_{x_1}^{x_2} \frac{d}{dx} \left[\frac{du(x)}{dx} \right] \omega(x) \\ &= \int_{x_1}^{x_2} \frac{d}{dx} \left\{ \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \right\} [\omega_1\phi_1(x) + \omega_2\phi_2(x)] dx \\ &= \left\{ \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \right\} [\omega_1\phi_1(x) + \omega_2\phi_2(x)] \Big|_{x_1}^{x_2} \\ &\quad - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx} dx \\ &= \left\{ \frac{d}{dx} \left[u_1 \frac{x_2 - x}{l} + u_2 \frac{x - x_1}{l} \right] \right\} \left(\omega_1 \frac{x_2 - x}{l} + \omega_2 \frac{x - x_1}{l} \right) \Big|_{x_1}^{x_2} \\ &\quad - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx} dx \\ &= \left(\frac{-u_1 + u_2}{l} \right) \left(\omega_1 \frac{x_2 - x}{l} + \omega_2 \frac{x - x_1}{l} \right) \Big|_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx} dx \\ &= \left(\frac{-u_1 + u_2}{l} \right) \omega_2 - \left(\frac{-u_1 + u_2}{l} \right) \omega_1 - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx} dx \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx} \frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx} dx \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \int_{x_1}^{x_2} \frac{d}{dx} \left[u_1 \frac{x_2 - x}{l} + u_2 \frac{x - x_1}{l} \right] \frac{d}{dx} \left[\omega_1 \frac{x_2 - x}{l} + \omega_2 \frac{x - x_1}{l} \right] dx \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \int_{x_1}^{x_2} \left(\frac{-u_1 + u_2}{l} \right) \left(\frac{-\omega_1 + \omega_2}{l} \right) dx \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \left(\frac{-u_1 + u_2}{l} \right) \left(\frac{-\omega_1 + \omega_2}{l} \right) x \Big|_{x_1}^{x_2} \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \left(\frac{-u_1 + u_2}{l} \right) \left(\frac{-\omega_1 + \omega_2}{l} \right) (x_2 - x_1) \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) \\ &= \left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) - \left(\frac{u_1}{l} \omega_1 - \frac{u_2}{l} \omega_1 - \frac{u_1}{l} \omega_2 + \frac{u_2}{l} \omega_2 \right) \end{aligned}$$

$$\left(\frac{u_1 - u_2}{l} \right) (\omega_1 - \omega_2) \text{の項を0としていいらしい?}$$

$$-\begin{pmatrix} \omega_1 & \omega_2 \end{pmatrix} \begin{pmatrix} \frac{1}{l} & -\frac{1}{l} \\ -\frac{1}{l} & \frac{1}{l} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{aligned} & \int_{x_1}^{x_2} \omega(x) dx \\ &= \int_{x_1}^{x_2} [\omega_1\phi_1(x) + \omega_2\phi_2(x)] dx \\ &= \int_{x_1}^{x_2} \left(\omega_1 \frac{x_2 - x}{l} + \omega_2 \frac{x - x_1}{l} \right) dx \end{aligned}$$

$$\begin{aligned}
&= \int_{x_1}^{x_2} \left(\omega_1 \frac{x_2 - x}{l} \right) dx + \int_{x_1}^{x_2} \left(\omega_2 \frac{x - x_1}{l} \right) dx \\
&= \omega_1 \left(\frac{x_2 x}{l} - \frac{x^2}{2l} \right) \Big|_{x_1}^{x_2} + \omega_2 \left(\frac{x^2}{2l} - \frac{x_1 x}{l} \right) \Big|_{x_1}^{x_2} \\
&= \omega_1 \left(\frac{x_2^2}{l} - \frac{x_2^2}{2l} \right) - \omega_1 \left(\frac{x_1 x_2}{l} - \frac{x_1^2}{2l} \right) + \omega_2 \left(\frac{x_2^2}{2l} - \frac{x_1 x_2}{l} \right) - \omega_2 \left(\frac{x_1^2}{2l} - \frac{x_1^2}{l} \right) \\
&= \omega_1 \left(\frac{2x_2^2}{2l} - \frac{x_2^2}{2l} \right) - \omega_1 \left(\frac{2x_1 x_2}{2l} - \frac{x_1^2}{2l} \right) + \omega_2 \left(\frac{x_2^2}{2l} - \frac{2x_1 x_2}{2l} \right) - \omega_2 \left(\frac{x_1^2}{2l} - \frac{2x_1^2}{2l} \right) \\
&= \omega_1 \left(\frac{x_2^2}{2l} - \frac{2x_1 x_2}{2l} + \frac{x_1^2}{2l} \right) + \omega_2 \left(\frac{x_2^2}{2l} - \frac{2x_1 x_2}{2l} + \frac{x_1^2}{2l} \right) \\
&= \omega_1 \frac{(x_1 - x_2)^2}{2l} + \omega_2 \frac{(x_1 - x_2)^2}{2l} \\
&= \omega_1 \frac{l}{2} + \omega_2 \frac{l}{2} \\
&\begin{pmatrix} \omega_1 & \omega_2 \end{pmatrix} \begin{pmatrix} \frac{l}{2} \\ \frac{l}{2} \end{pmatrix} \\
&= \begin{pmatrix} \omega_1 & \omega_2 \end{pmatrix} \begin{pmatrix} \frac{1}{l} & -\frac{1}{l} \\ -\frac{1}{l} & \frac{1}{l} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \omega_1 & \omega_2 \end{pmatrix} \begin{pmatrix} \frac{l}{2} \\ \frac{l}{2} \end{pmatrix} \\
&\begin{pmatrix} \frac{1}{l} & -\frac{1}{l} \\ -\frac{1}{l} & \frac{1}{l} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\frac{l}{2} \\ -\frac{l}{2} \end{pmatrix}
\end{aligned}$$