$$\begin{cases} \phi_1(x) &= \dfrac{x_2 - x}{l} \\ \phi_2(x) &= \dfrac{x - x_1}{l} \end{cases}$$

$$u(x) = u_1\phi_1(x) + u_2\phi_2(x)$$
$$\omega(x) = \omega_1\phi_1(x) + \omega_2\phi_2(x)$$
$$\frac{d^2u(x)}{dx^2} = 1$$
$$\int_{x_1}^{x_2} \frac{d}{dx}\left[\frac{du(x)}{dx}\right]\omega(x)$$
$$= \int_{x_1}^{x_2} \frac{d}{dx}\left\{\frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\right\}[\omega_1\phi_1(x) + \omega_2\phi_2(x)]dx$$
$$= \left\{\frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\right\}[\omega_1\phi_1(x) + \omega_2\phi_2(x)]\Big|_{x_1}^{x_2}$$
$$-\int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx}dx$$
$$= \left\{\frac{d}{dx}\left[u_1\frac{x_2 - x}{l} + u_2\frac{x - x_1}{l}\right]\right\}\left(\omega_1\frac{x_2 - x}{l} + \omega_2\frac{x - x_1}{l}\right)\Big|_{x_1}^{x_2}$$
$$-\int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx}dx$$
$$= \left(\frac{-u_1 + u_2}{l}\right)\left(\omega_1\frac{x_2 - x}{l} + \omega_2\frac{x - x_1}{l}\right)\Big|_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx}$$
$$= \left(\frac{-u_1 + u_2}{l}\right)\omega_2 - \left(\frac{-u_1 + u_2}{l}\right)\omega_1 - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx}$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \int_{x_1}^{x_2} \frac{d[u_1\phi_1(x) + u_2\phi_2(x)]}{dx}\frac{d[\omega_1\phi_1(x) + \omega_2\phi_2(x)]}{dx}dx$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \int_{x_1}^{x_2} \frac{d}{dx}\left[u_1\frac{x_2 - x}{l} + u_2\frac{x - x_1}{l}\right]\frac{d}{dx}\left[\omega_1\frac{x_2 - x}{l} + \omega_2\frac{x - x_1}{l}\right]$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \int_{x_1}^{x_2}\left(\frac{-u_1 + u_2}{l}\right)\left(\frac{-\omega_1 + \omega_2}{l}\right)dx$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \left(\frac{-u_1 + u_2}{l}\right)\left(\frac{-\omega_1 + \omega_2}{l}\right)x\Big|_{x_1}^{x_2}$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \left(\frac{-u_1 + u_2}{l}\right)\left(\frac{-\omega_1 + \omega_2}{l}\right)(x_2 - x_1)$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2)$$
$$= \left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2) - \left(\frac{u_1}{l}\omega_1 - \frac{u_2}{l}\omega_1 - \frac{u_1}{l}\omega_2 + \frac{u_2}{l}\omega_2\right)$$

$\left(\frac{u_1 - u_2}{l}\right)(\omega_1 - \omega_2)$の項は最終的に消える.
行列形式で書き直すと,

$$-\begin{pmatrix} \omega_1 & \omega_2 \end{pmatrix}\begin{pmatrix} \frac{1}{l} & -\frac{1}{l} \\ -\frac{1}{l} & \frac{1}{l} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

と書ける.

$$\int_{x_1}^{x_2} \omega(x)dx$$
$$= \int_{x_1}^{x_2}[\omega_1\phi_1(x) + \omega_2\phi_2(x)]dx$$

$$= \int_{x_1}^{x_2}\left(\omega_1\frac{x_2-x}{l}+\omega_2\frac{x-x_1}{l}\right)dx$$
$$= \int_{x_1}^{x_2}\left(\omega_1\frac{x_2-x}{l}\right)dx+\int_{x_1}^{x_2}\left(\omega_2\frac{x-x_1}{l}\right)dx$$
$$= \omega_1\left(\frac{x_2x}{l}-\frac{x^2}{2l}\right)\Big|_{x_1}^{x_2}+\omega_2\left(\frac{x^2}{2l}-\frac{x_1x}{l}\right)\Big|_{x_1}^{x_2}$$
$$= \omega_1\left(\frac{x_2^2}{l}-\frac{x_2^2}{2l}\right)-\omega_1\left(\frac{x_1x_2}{l}-\frac{x_1^2}{2l}\right)+\omega_2\left(\frac{x_2^2}{2l}-\frac{x_1x_2}{l}\right)-\omega_2\left(\frac{x_1^2}{2l}-\frac{x_1^2}{l}\right)$$
$$= \omega_1\left(\frac{2x_2^2}{2l}-\frac{x_2^2}{2l}\right)-\omega_1\left(\frac{2x_1x_2}{2l}-\frac{x_1^2}{2l}\right)+\omega_2\left(\frac{x_2^2}{2l}-\frac{2x_1x_2}{2l}\right)-\omega_2\left(\frac{x_1^2}{2l}-\frac{2x_1^2}{2l}\right)$$
$$= \omega_1\left(\frac{x_2^2}{2l}-\frac{2x_1x_2}{2l}+\frac{x_1^2}{2l}\right)+\omega_2\left(\frac{x_2^2}{2l}-\frac{2x_1x_2}{2l}+\frac{x_1^2}{2l}\right)$$
$$= \omega_1\frac{(x_1-x_2)^2}{2l}+\omega_2\frac{(x_1-x_2)^2}{2l}$$
$$= \omega_1\frac{l}{2}+\omega_2\frac{l}{2}$$

ベクトル形式で書き直すと,

$$\begin{pmatrix}\omega_1 & \omega_2\end{pmatrix}\begin{pmatrix}\frac{l}{2}\\ \frac{l}{2}\end{pmatrix}$$

と書ける.

両辺を等号で結んで,

$$-\begin{pmatrix}\omega_1 & \omega_2\end{pmatrix}\begin{pmatrix}\frac{1}{l} & -\frac{1}{l}\\ -\frac{1}{l} & \frac{1}{l}\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}\omega_1 & \omega_2\end{pmatrix}\begin{pmatrix}\frac{l}{2}\\ \frac{l}{2}\end{pmatrix}$$

となる. 従って,

$$\begin{pmatrix}\frac{1}{l} & -\frac{1}{l}\\ -\frac{1}{l} & \frac{1}{l}\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}-\frac{l}{2}\\ -\frac{l}{2}\end{pmatrix}$$

である.