

Area of the Rectangle:

If you start with the area of the square (or rectangle formed by reflecting the triangle across its hypotenuse):

$$\text{Area of Rectangle} = \sin(\beta) \cos(\beta)$$

Now, recall the double-angle identity for sine:

$$\sin(2\beta) = 2 \sin(\beta) \cos(\beta)$$

So, the area of the rectangle can indeed be written as:

$$\text{Area of Rectangle} = \frac{1}{2} \sin(2\beta)$$

Area of the Triangle:

Given that the rectangle is formed by reflecting the triangle, the area of the original triangle is half the area of the rectangle:

$$\text{Area of Triangle} = \frac{1}{2} \times \frac{1}{2} \sin(2\beta) = \frac{1}{4} \sin(2\beta)$$

So, you're absolutely correct: the area of the rectangle is $\frac{1}{2} \sin(2\beta)$, and the area of the triangle is $\frac{1}{4} \sin(2\beta)$. This shows a nice connection between the triangle, its reflection, and trigonometric identities.