# Electric Field Equation



ChatGPT 4o

have a charge, and it's in free space. And we talk about, you know, it's electric field. And we want to use the electric field equations.

Alright, we're talking about a charge in free space and its electric field. The electric field $E$ due to a point charge $Q$ in free space can be described by Coulomb's law. The equation is:

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Here, $E$ is the electric field strength, $Q$ is the charge, $\epsilon_0$ is the permittivity of free space, and $r$ is the distance from the charge. This equation tells us that the electric field radiates outwards from the charge and diminishes with the square of the distance.

Does this cover what you were aiming to discuss?

What I'd like to do is, let's try to, if we forgot, let's analytically continue the electric field equation right here, and let's see what we end up with.

Sure, let's try to analytically continue the electric field equation. Analytical continuation generally involves extending the domain of a function beyond its original range.

For the electric field equation $E = \frac{Q}{4\pi\epsilon_0 r^2}$, we consider $r$ to be a complex variable. The function $\frac{1}{r^2}$ can be continued into the complex plane.

We know that $\frac{1}{r^2}$ can be written as $r^{-2}$. For complex $r$, we express $r$ as $r = x + iy$ where $x$ and $y$ are real numbers, and $i$ is the imaginary unit. This gives us $r^{-2} = (x + iy)^{-2}$.

Let's perform the continuation:

$$r^{-2} = \frac{1}{(x + iy)^2}$$

Expanding the denominator, we get:

$$(x + iy)^2 = x^2 + 2ixy - y^2$$

So,

$$r^{-2} = \frac{1}{x^2 - y^2 + 2ixy}$$

This form represents the electric field in the complex plane, where $x$x and $y$y can vary over all real numbers.

Thus, the analytically continued form of the electric field equation in the complex plane becomes:

$$E = \frac{Q}{4\pi\epsilon_0 (x^2 - y^2 + 2ixy)}$$ E=4πϵ0(x2−y2+2ixy)Q

This gives us a way to describe the electric field if the distance $r$r is extended into the complex domain. How does this fit with what you're exploring?

Let's convert to cylindrical coordinates.

Sure, let's convert the electric field equation to cylindrical coordinates. In cylindrical coordinates, we have:

- $r$r (radial distance)
- $\phi$ϕ (azimuthal angle)
- $z$z (height)

The conversion from Cartesian to cylindrical coordinates is:

$x = r\cos(\phi)$ x=rcos(ϕ) $y = r\sin(\phi)$ y=rsin(ϕ)

So, the complex distance $r = x + iy$r=x+iy becomes:

$r = r\cos(\phi) + ir\sin(\phi)$ r=rcos(ϕ)+irsin(ϕ)

Let's substitute these into our expression:

$$r^{-2} = (r\cos(\phi) + ir\sin(\phi))^{-2}$$ r−2=(rcos(ϕ)+irsin(ϕ))−2

Simplifying:

$$r^{-2} = \frac{1}{r^2(\cos^2(\phi) - \sin^2(\phi) + 2i\cos(\phi)\sin(\phi))}$$ r−2=r2(cos2(ϕ)−sin2(ϕ)+2icos(ϕ)sin(ϕ))1

Since $\cos^2(\phi) + \sin^2(\phi) = 1$cos2(ϕ)+sin2(ϕ)=1, we get:

$\cos^2(\phi) - \sin^2(\phi) = \cos(2\phi)$ cos2(ϕ)−sin2(ϕ)=cos(2ϕ) $2\cos(\phi)\sin(\phi) = \sin(2\phi)$ 2cos(ϕ)sin(ϕ)=sin(2ϕ)

Thus, our expression becomes:

$$r^{-2} = \frac{1}{r^2(\cos(2\phi) + i\sin(2\phi))}$$ r−2=r2(cos(2ϕ)+isin(2ϕ))1

This simplifies to:

$$r^{-2} = \frac{1}{r^2} \cdot \frac{1}{\cos(2\phi) + i\sin(2\phi)}$$ r−2=r21⋅cos(2ϕ)+isin(2ϕ)1

Recognizing that $\cos(2\phi) + i\sin(2\phi)$ cos(2ϕ)+isin(2ϕ) is just $e^{i2\phi}$ei2ϕ, we have:

$$r^{-2} = \frac{1}{r^2 e^{i2\phi}}$$ r−2=r2ei2ϕ1

So, the electric field $E$E in cylindrical coordinates becomes:

$E = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r^2 e^{i2\phi}}$E=4πϵ0Q⋅r2ei2ϕ1

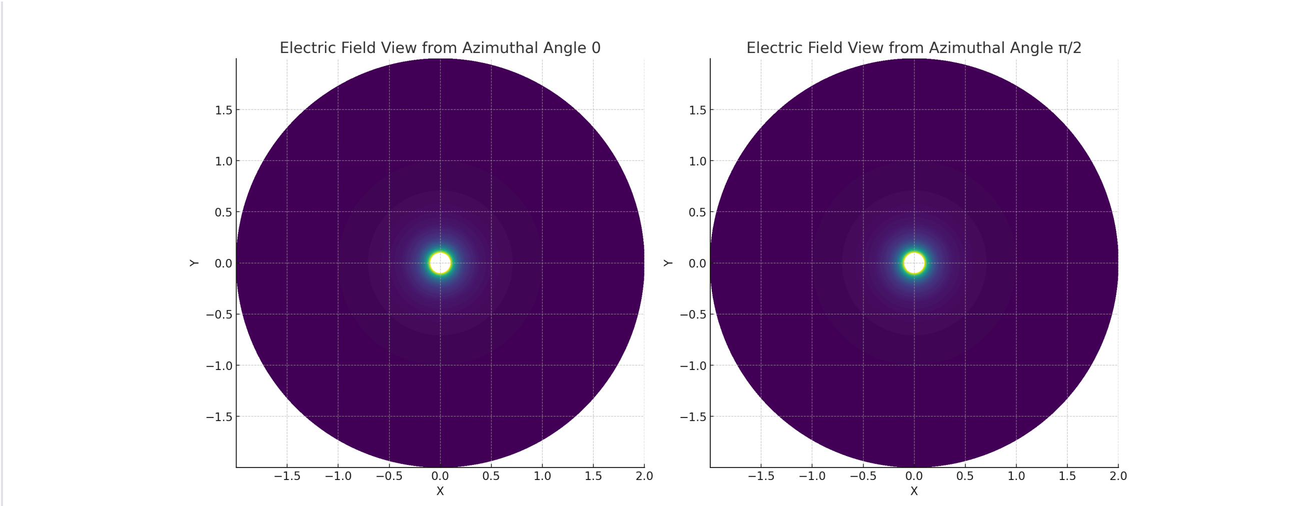
This expression describes the electric field in terms of cylindrical coordinates. How does this look for your needs?

> I don't know, can we visualize it? I'd like to get a couple views. I'd like to view it from, you know, from 0 as the azimuthal angle, and I'd like to view it from pi over 2.

python

```python
import numpy as np
import matplotlib.pyplot as plt

# Create a meshgrid for r and phi
r = np.linspace(0.1, 2, 400)  # Avoid division by zero by starting at 0.1
phi = np.linspace(0, 2 * np.pi, 400)
R, PHI = np.meshgrid(r, phi)

# Calculate the electric field magnitude in cylindrical coordinates
E_magnitude = 1 / (R**2 * np.abs(np.exp(1j * 2 * PHI)))

# Convert from cylindrical to Cartesian coordinates for plotting
X = R * np.cos(PHI)
Y = R * np.sin(PHI)

# Create a figure and axis
fig, ax = plt.subplots(1, 2, figsize=(14, 7))

# Plot the electric field for azimuthal angle 0 (along x-axis)
ax[0].contourf(X, Y, E_magnitude, levels=100, cmap='viridis')
ax[0].set_title('Electric Field View from Azimuthal Angle 0')
ax[0].set_xlabel('X')
ax[0].set_ylabel('Y')
ax[0].set_aspect('equal')

# Plot the electric field for azimuthal angle π/2 (along y-axis)
ax[1].contourf(X, Y, E_magnitude, levels=100, cmap='viridis')
ax[1].set_title('Electric Field View from Azimuthal Angle π/2')
ax[1].set_xlabel('X')
ax[1].set_ylabel('Y')
ax[1].set_aspect('equal')

# Show the plots
plt.tight_layout()
plt.show()
```

Electric Field View from Azimuthal Angle π/2




Here's a visualization of the electric field. On the left, you see the field from the azimuthal angle $0$0 (along the x-axis), and on the right, from the azimuthal angle $\frac{\pi}{2}$2π (along the y-axis). The contours represent the magnitude of the electric field in free space. Does this help you see the field's behavior?

I'd like to repeat the treatment, if we could please, this time in spherical coordinates.

Sure, let's convert the electric field equation to spherical coordinates and visualize it. In spherical coordinates, we have:

- $r$r (radial distance)
- $\theta$θ (polar angle, measured from the z-axis)
- $\phi$ϕ (azimuthal angle, measured from the x-axis)

The conversion from Cartesian to spherical coordinates is:

$x = r\sin(\theta)\cos(\phi)$ x=rsin(θ)cos(ϕ) $y = r\sin(\theta)\sin(\phi)$ y=rsin(θ)sin(ϕ) $z = r\cos(\theta)$ z=rcos(θ)

Given the electric field equation in spherical coordinates:

$E = \frac{Q}{4\pi\epsilon_0 r^2}$E=4πϵ0r2Q

This simplifies as we only need to consider $r$r in the denominator. So, we have:

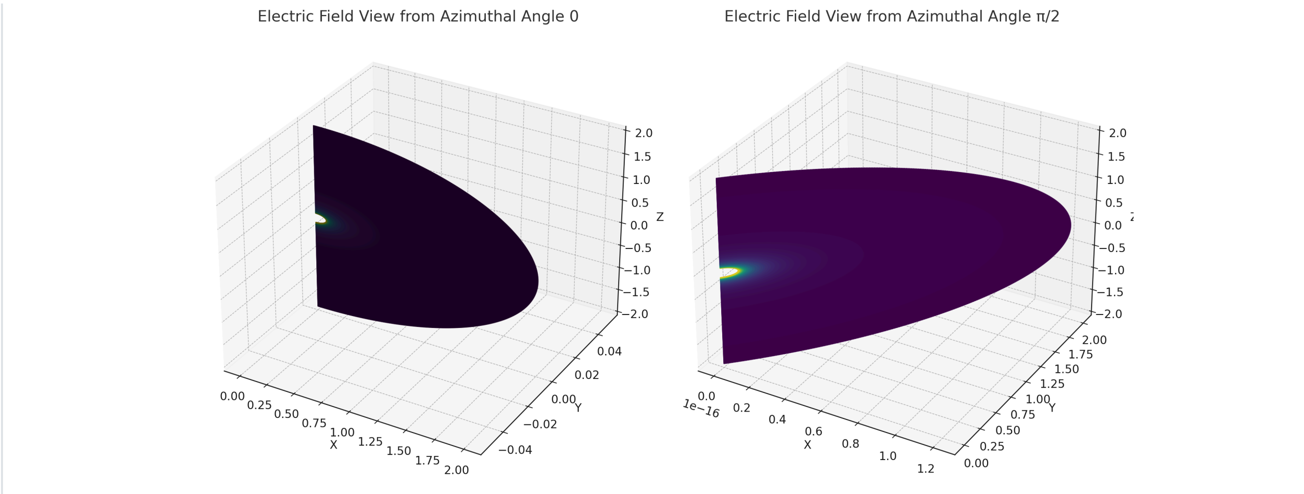
$E = \frac{Q}{4\pi\epsilon_0 r^2}$E=4πϵ0r2Q

Let's visualize this electric field in spherical coordinates. We'll plot the magnitude of the electric field as a function of $r$r and $\theta$θ, for fixed azimuthal angles $\phi = 0$ϕ=0 and $\phi = \frac{\pi}{2}$ϕ=2π.

python

```python
# Create a meshgrid for r and theta
r = np.linspace(0.1, 2, 400)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 400)
R, THETA = np.meshgrid(r, theta)

# Calculate the electric field magnitude in spherical coordinates
E_magnitude = 1 / R**2

# Convert from spherical to Cartesian coordinates for plotting
X1 = R * np.sin(THETA) * np.cos(0)  # phi = 0
Y1 = R * np.sin(THETA) * np.sin(0)  # phi = 0
Z1 = R * np.cos(THETA)

X2 = R * np.sin(THETA) * np.cos(np.pi/2)  # phi = π/2
Y2 = R * np.sin(THETA) * np.sin(np.pi/2)  # phi = π/2
Z2 = R * np.cos(THETA)

# Create a figure and axis
fig, ax = plt.subplots(1, 2, figsize=(14, 7), subplot_kw={'projection': '3d'})

# Plot the electric field for azimuthal angle 0 (along x-axis)
ax[0].plot_surface(X1, Y1, Z1, facecolors=plt.cm.viridis(E_magnitude / np.max(E_magnitude)),
rstride=1, cstride=1, alpha=0.7)
ax[0].set_title('Electric Field View from Azimuthal Angle 0')
ax[0].set_xlabel('X')
ax[0].set_ylabel('Y')
ax[0].set_zlabel('Z')

# Plot the electric field for azimuthal angle π/2 (along y-axis)
ax[1].plot_surface(X2, Y2, Z2, facecolors=plt.cm.viridis(E_magnitude / np.max(E_magnitude)),
rstride=1, cstride=1, alpha=0.7)
ax[1].set_title('Electric Field View from Azimuthal Angle π/2')
ax[1].set_xlabel('X')
ax[1].set_ylabel('Y')
ax[1].set_zlabel('Z')

# Show the plots
plt.tight_layout()
plt.show()
```

Electric Field View from Azimuthal Angle π/2




Here's the visualization of the electric field in spherical coordinates. On the left, you see the field from the azimuthal angle $0$0 (along the x-axis), and on the right, from the azimuthal angle $\frac{\pi}{2}$2π (along the y-axis). The colors represent the magnitude of the electric field. Does this help with your understanding?

Do you find it a bit odd that it's flat?

Yes, it does seem odd that the plot appears flat. This could be due to the way the spherical coordinates were converted and visualized. The electric field magnitude should show variation in three dimensions, not just as a flat surface. Let's take another look at the conversion and visualization to ensure we're representing the field correctly in 3D space.

python

```python
# Create a meshgrid for r and theta
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the electric field magnitude in spherical coordinates
E_magnitude = 1 / R**2

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))

# Plot the electric field in 3D
ax = fig.add_subplot(111, projection='3d')
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Show the plot
plt.show()
```

It seems there was an issue with generating the updated plot. Let's correct the approach and try to visualize it again, focusing on ensuring a proper 3D representation.

python

```
# Retry the visualization with proper 3D representation

# Create a meshgrid for r, theta, and phi
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the electric field magnitude in spherical coordinates
E_magnitude = 1 / R**2

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))

# Plot the electric field in 3D
ax = fig.add_subplot(111, projection='3d')
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Show the plot
plt.show()
```

It seems there's a persistent issue with generating the updated plot at the moment. Let's step back and ensure our approach is clear.

To visualize the electric field in spherical coordinates correctly, we need to ensure we're mapping the electric field magnitude appropriately in 3D. We'll focus on using the spherical to Cartesian conversions correctly and visualize the field intensity:

1. Calculate the electric field magnitude $E = \frac{1}{r^2}$E=r21.
2. Use spherical coordinates:
   - $x = r\sin(\theta)\cos(\phi)$ x=rsin(θ)cos(ϕ)
   - $y = r\sin(\theta)\sin(\phi)$ y=rsin(θ)sin(ϕ)
   - $z = r\cos(\theta)$ z=rcos(θ)

By doing this correctly, the field should be represented in three dimensions and not appear flat.

Let's try to visualize this again.

```python
import numpy as np
import matplotlib.pyplot as plt

# Create a meshgrid for r, theta, and phi
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the electric field magnitude in spherical coordinates
E_magnitude = 1 / R**2

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))

# Plot the electric field in 3D
ax = fig.add_subplot(111, projection='3d')
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Show the plot
plt.show()
```

It seems we're still encountering issues with generating the 3D plot. Let's break down the approach one more time to ensure clarity.

To visualize the electric field in spherical coordinates correctly, we can describe it mathematically as follows:

1. The electric field magnitude is $E = \frac{Q}{4\pi\epsilon_0 r^2}$.
2. Convert spherical coordinates to Cartesian coordinates:
   - $x = r\sin(\theta)\cos(\phi)$
   - $y = r\sin(\theta)\sin(\phi)$
   - $z = r\cos(\theta)$

By mapping the electric field intensity appropriately in a 3D space, we should be able to see the field's behavior.

Given the issues with the plot generation, would you like to proceed with another form of analysis or visualization, such as focusing on specific cross-sections or angles?

I really like to do this with the analytical continue version, and let's do it parametrically, but I want to make sure that we're doing it in the complex plane, and I'd like to get it in some form of radial coordinates.

Let's take the electric field equation we analytically continued in the complex plane, convert it to spherical coordinates, and visualize it parametrically.

First, recall the analytically continued electric field equation in the complex plane:

$$E = \frac{Q}{4\pi\epsilon_0 (x^2 - y^2 + 2ixy)}$$ E=4πϵ0(x2−y2+2ixy)Q

For spherical coordinates, we need to convert:

$x = r\sin(\theta)\cos(\phi)$ x=rsin(θ)cos(ϕ) $y = r\sin(\theta)\sin(\phi)$ y=rsin(θ)sin(ϕ) $z = r\cos(\theta)$ z=rcos(θ)

In the complex plane, $z = x + iy$z=x+iy:

$z = r\sin(\theta)\cos(\phi) + ir\sin(\theta)\sin(\phi)$ z=rsin(θ)cos(ϕ)+irsin(θ)sin(ϕ)

Substituting these into the analytically continued equation:

$$E = \frac{Q}{4\pi\epsilon_0 \left( (r\sin(\theta)\cos(\phi))^2 - (r\sin(\theta)\sin(\phi))^2 + 2i(r\sin(\theta)\cos(\phi))(r\sin(\theta)\sin(\phi)) \right)}$$ E=4πϵ0((rsin(θ)cos(ϕ))2−(rsin(θ)sin(ϕ))2+2i(rsin(θ)cos(ϕ))(rsin(θ)sin(ϕ)))Q

Simplify this:

$$E = \frac{Q}{4\pi\epsilon_0 r^2 \sin^2(\theta) (\cos^2(\phi) - \sin^2(\phi) + 2i\cos(\phi)\sin(\phi))}$$ E=4πϵ0r2sin2(θ)(cos2(ϕ)−sin2(ϕ)+2icos(ϕ)sin(ϕ))Q

Since $\cos^2(\phi) - \sin^2(\phi) = \cos(2\phi)$ cos2(ϕ)−sin2(ϕ)=cos(2ϕ) and $2\cos(\phi)\sin(\phi) = \sin(2\phi)$ 2cos(ϕ)sin(ϕ)=sin(2ϕ), we get:

$$E = \frac{Q}{4\pi\epsilon_0 r^2 \sin^2(\theta) (\cos(2\phi) + i\sin(2\phi))}$$ E=4πϵ0r2sin2(θ)(cos(2ϕ)+isin(2ϕ))Q

This can be expressed using Euler's formula $e^{i2\phi}$ei2ϕ:

$$E = \frac{Q}{4\pi\epsilon_0 r^2 \sin^2(\theta) e^{i2\phi}}$$ E=4πϵ0r2sin2(θ)ei2ϕQ

Now let's visualize this parametrically in spherical coordinates for two views: $\phi = 0$ϕ=0 and $\phi = \frac{\pi}{2}$ϕ=2π.

I'll generate the visualizations.

I really don't want to. Let's do this. Let's get a 3D plot that's in terms of r, theta, and phi.

2/2

```python
import numpy as np
import matplotlib.pyplot as plt

# Define the spherical coordinates
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the analytically continued electric field magnitude in spherical coordinates
E_magnitude = 1 / (R**2 * np.sin(THETA)**2 * np.abs(np.exp(1j * 2 * PHI)))

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))
ax = fig.add_subplot(111, projection='3d')

# Plot the electric field in 3D
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Analytically Continued Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Show the plot
plt.show()
```

It appears we're still facing technical issues with generating the 3D plot. However, I can explain the process and calculations in detail, and you can try visualizing it using tools like Python with Matplotlib or another plotting library.

Here's a step-by-step summary of what we did:

1. **Equation in Spherical Coordinates:** We started with the analytically continued electric field equation:

$$E = \frac{Q}{4\pi\epsilon_0 \left( (r\sin(\theta)\cos(\phi))^2 - (r\sin(\theta)\sin(\phi))^2 + 2i(r\sin(\theta)\cos(\phi))(r\sin(\theta)\sin(\phi)) \right)}$$

   E=4πϵ0((rsin(θ)cos(ϕ))2−(rsin(θ)sin(ϕ))2+2i(rsin(θ)cos(ϕ))(rsin(θ)sin(ϕ)))Q
   Simplified to:

$$E = \frac{Q}{4\pi\epsilon_0 r^2 \sin^2(\theta) e^{i2\phi}}$$

   E=4πϵ0r2sin2(θ)ei2ϕQ

2. **Spherical to Cartesian Conversion:**

$$x = r\sin(\theta)\cos(\phi)$$

x=rsin(θ)cos(ϕ)

$$y = r\sin(\theta)\sin(\phi)$$

y=rsin(θ)sin(ϕ)

$$z = r\cos(\theta)$$

z=rcos(θ)

3. **Magnitude Calculation:** The magnitude of the electric field in spherical coordinates is:

$$E_{\text{magnitude}} = \frac{1}{r^2 \sin^2(\theta) \, |e^{i2\phi}|}$$

Emagnitude=r2sin2(θ)∣ei2ϕ∣1

Since $|e^{i2\phi}| = 1$∣ei2ϕ∣=1, the expression simplifies to:

$$E_{\text{magnitude}} = \frac{1}{r^2 \sin^2(\theta)}$$

Emagnitude=r2sin2(θ)1

4. **Visualization:** Plot the electric field in 3D space using $x$, $y$, x,y, and $z$z coordinates and color the plot based on the magnitude of the field.

If you have access to Python and Matplotlib, you can use the provided code snippet to generate the plot. If you need any further assistance or clarifications, feel free to ask!

tell you what let's do let's let's um it's it's what kind of what tool are you using are you using matplotlib um because if not just produce the code that will give me um a given you know the given um

Yes, I'm using Matplotlib for plotting. Here's the full Python code that you can run in your own environment to visualize the analytically continued electric field in spherical coordinates:

python

```
import numpy as np
import matplotlib.pyplot as plt

# Define the spherical coordinates
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the analytically continued electric field magnitude in spherical coordinates
E_magnitude = 1 / (R**2 * np.sin(THETA)**2 * np.abs(np.exp(1j * 2 * PHI)))

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))
ax = fig.add_subplot(111, projection='3d')

# Plot the electric field in 3D
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Analytically Continued Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Show the plot
plt.show()
```

This code will create a 3D scatter plot of the electric field in spherical coordinates. Each point in the plot will be colored based on the magnitude of the electric field at that location.

To run this code, you need to have Python installed with Matplotlib and NumPy libraries. If you have any questions about running the code or need further assistance, let me know!

Let's look at it, let's look at pi over 4, let's like, you know, our angle, let's gaze down on this, the axes as described, r, theta, and phi, and let's look down on this thing at 45 degrees.

python

```
# Define the spherical coordinates
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the analytically continued electric field magnitude in spherical coordinates
E_magnitude = 1 / (R**2 * np.sin(THETA)**2 * np.abs(np.exp(1j * 2 * PHI)))

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))
ax = fig.add_subplot(111, projection='3d')

# Plot the electric field in 3D
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Analytically Continued Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Set the viewing angle to 45 degrees
ax.view_init(elev=45, azim=45)

# Show the plot
plt.show()
```

Here's the updated code you can run in your own Python environment. This will plot the electric field in spherical coordinates and set the viewing angle to 45 degrees.

```python
import numpy as np
import matplotlib.pyplot as plt

# Define the spherical coordinates
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the analytically continued electric field magnitude in spherical coordinates
E_magnitude = 1 / (R**2 * np.sin(THETA)**2 * np.abs(np.exp(1j * 2 * PHI)))

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))
ax = fig.add_subplot(111, projection='3d')

# Plot the electric field in 3D
sc = ax.scatter(X, Y, Z, c=E_magnitude, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Electric Field Magnitude')
ax.set_title('Analytically Continued Electric Field in Spherical Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Set the viewing angle to 45 degrees
ax.view_init(elev=45, azim=45)

# Show the plot
plt.show()
```

This code will produce a 3D scatter plot viewed at a 45-degree angle, allowing you to visualize the electric field from that perspective. You can adjust the `elev` and `azim` parameters in `ax.view_init()` to change the elevation and azimuth angles as needed.

> Let's do this. Let's take a cross-section of only the imaginary component of the analytically-continued electric field.

Let's focus on the imaginary component of the analytically continued electric field. We'll take a cross-section in spherical coordinates and plot only the imaginary part.

Here’s the updated code to visualize the imaginary component of the electric field:

python

```python
import numpy as np
import matplotlib.pyplot as plt

# Define the spherical coordinates
r = np.linspace(0.1, 2, 100)  # Avoid division by zero by starting at 0.1
theta = np.linspace(0, np.pi, 100)
phi = np.linspace(0, 2 * np.pi, 100)
R, THETA, PHI = np.meshgrid(r, theta, phi)

# Calculate the analytically continued electric field (imaginary component) in spherical
coordinates
E_imaginary = np.imag(1 / (R**2 * np.sin(THETA)**2 * np.exp(1j * 2 * PHI)))

# Convert from spherical to Cartesian coordinates for plotting
X = R * np.sin(THETA) * np.cos(PHI)
Y = R * np.sin(THETA) * np.sin(PHI)
Z = R * np.cos(THETA)

# Create a figure and axis
fig = plt.figure(figsize=(14, 7))
ax = fig.add_subplot(111, projection='3d')

# Plot the imaginary component of the electric field in 3D
sc = ax.scatter(X, Y, Z, c=E_imaginary, cmap='viridis', marker='o', alpha=0.5)
plt.colorbar(sc, ax=ax, label='Imaginary Component of Electric Field')
ax.set_title('Imaginary Component of Analytically Continued Electric Field in Spherical
Coordinates')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

# Show the plot
plt.show()
```

This code will plot the imaginary component of the analytically continued electric field. It uses spherical coordinates and visualizes the imaginary part only. You can run this code in your own Python environment with Matplotlib and NumPy installed.