1. Ricci Tensor

The Ricci tensor is a contraction of the Riemann curvature tensor and provides a more global description of how the space curves. While the Riemann tensor gives us detailed curvature information in all directions, the Ricci tensor simplifies this by summarizing the curvature in a more compact form.

For a 1D system like this (in terms of \theta ), the Ricci tensor is very straightforward because in 1D there’s only one direction to contract. The Ricci tensor is essentially a trace of the Riemann tensor, and in this case, it reduces directly to a single scalar component in 1D.

Ricci Tensor Formula (in 1D):

R\_{\theta \theta} = R^\theta\_{\theta \theta \theta}

So, in our case, since we already have R^\theta\_{\theta \theta \theta} , the Ricci tensor is simply:

R\_{\theta \theta} = R^\theta\_{\theta \theta \theta}

This means that the Riemann curvature tensor you computed earlier already serves as the Ricci tensor in this 1D system. So the expression:

R\_{\theta \theta} = \frac{(-128 \sin^4(\theta) + 128 \sin^2(\theta) - 16)(\omega^2 + 16 \sin^4(\theta) - 16 \sin^2(\theta) + 4)^2 + (-768 \omega^2 - 12288 \sin^4(\theta) + 12288 \sin^2(\theta) - 3072) \sin^2(\theta) \cos^2(\theta) \cos^2(2\theta)}{(\omega^2 + 16 \sin^4(\theta) - 16 \sin^2(\theta) + 4)^3}

is not only the Riemann tensor but also the Ricci tensor in this 1D case.

2. Scalar Curvature

The scalar curvature R is a single number that gives a global measure of the curvature of the entire manifold. It’s computed by contracting the Ricci tensor with the inverse of the metric tensor:

In our case, g^{\theta \theta} is the inverse of the metric component g\_{\theta \theta} , which we already computed:

Thus, the scalar curvature is:

Substituting in the expressions for R\_{\theta \theta} and g\_{\theta \theta} , we get:

R = \frac{R\_{\theta \theta}}{\omega^2 + 16 \sin^4(\theta) - 16 \sin^2(\theta) + 4}

This gives us the scalar curvature, which will indicate the overall curvature of the system at each point \theta . Let’s compute this using SymPy to see how it behaves: