3

Practice Problems

Practice Problem 1: Noncomposite 60 ft Steel Beam Bridge for Limit States Strength I, Fatigue II, and Service

Situation

L	bridge span	60 ft
LL	live load	HL-93
FWS	future wearing surface (bituminous concrete)	3 in
f'_c	concrete strength	5.0 ksi
ADTT	average daily truck traffic	250
	fatigue design life	75 years
	New Jersey barrier	0.45 kips/ft each
w	roadway width	43 ft
w_c	dead load of concrete	150 lbf/ft^3
w_w	dead load of wearing surface (bituminous concrete)	140 lbf/ft^3

Please see Figure 3.1.

Requirements

Review:

Flexural and shear resistance for Strength I Limit State

Fatigue II Limit State

Service Limit State

- Elastic deflection
- Flexure requirement to prevent permanent deflections

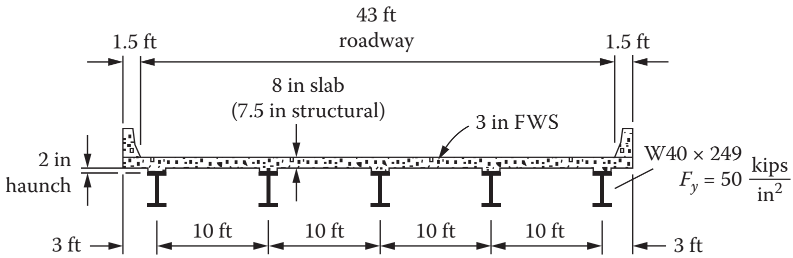

FIGURE 3.1
Cross section of noncomposite steel beam bridge.

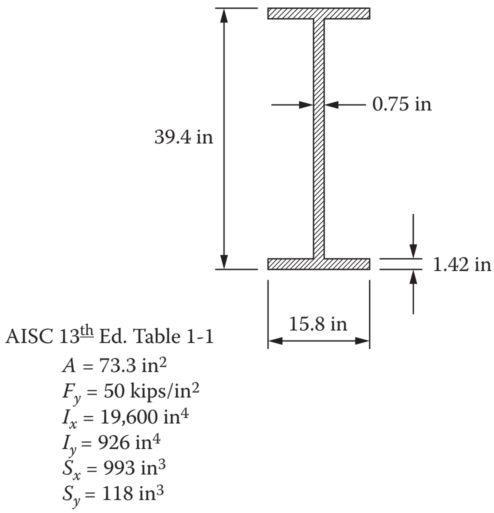

FIGURE 3.2
W40 × 249 properties.

Solution

Step 1: Review Section, W40 × 249

AISC (13th ed.) Tbl. 1-1

Please see Figure 3.2.

Deck slab, 8 in (7.5 in structural)

Effective Flange Width of a concrete deck slab be taken as the tributary width perpendicular to the axis of the member.

A Art. 4.6.2.6.1

t_s	slab thickness	8 in
b_i	effective flange width for interior beams	in
b_e	effective flange width for exterior beams	in
b_w	web width	0.75 in
b_f	flange width of steel section	15.8 in
S	average spacing of adjacent beams	10 ft

The effective flange width for interior beams is

$$b_i = S = 10 \text{ ft} = 120 \text{ in}$$

The effective flange width for exterior beams is

$$b_e = (3 \text{ ft})\left(\frac{12 \text{ in}}{\text{ft}}\right) + \left(\frac{10 \text{ ft}}{2}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 96 \text{ in}$$

Step 2: Evaluate Dead Loads

The structural components dead load, DC_1, per interior girder is calculated as follows and as shown in Figure 3.3.

$$DC_{slab} = \left(\frac{120 \text{ in}}{12\dfrac{\text{in}}{\text{ft}}}\right)\left(\frac{8 \text{ in}}{12\dfrac{\text{in}}{\text{ft}}}\right)\left(0.150\frac{\text{kips}}{\text{ft}^3}\right) = 1.0 \text{ kips/ft}$$

$$DC_{haunch} = \left(\frac{2 \text{ in}}{12\dfrac{\text{in}}{\text{ft}}}\right)\left(\frac{15.8 \text{ in}}{12\dfrac{\text{in}}{\text{ft}}}\right)\left(0.150\frac{\text{kips}}{\text{ft}^3}\right) = 0.033 \text{ kips/ft}$$

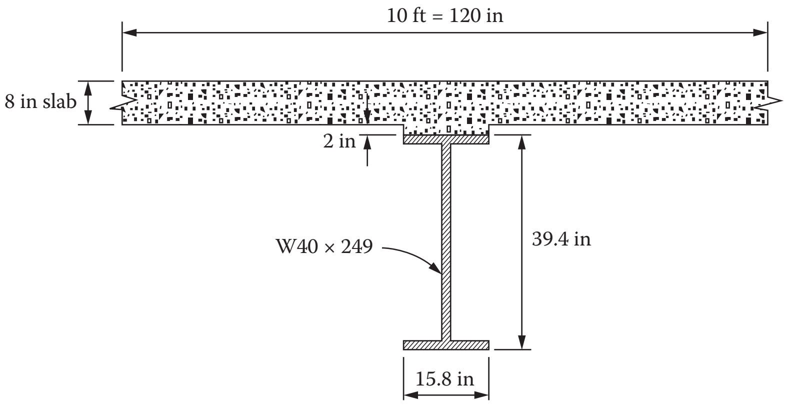

FIGURE 3.3
Dead loads for interior girder.

Assuming 5% of steel weight for diaphragms, stiffeners, and so on,

$$DC_{slab} = 0.249\,\frac{kips}{ft} + (0.05)\left(0.249\,\frac{kips}{ft}\right) = 0.261\ kips/ft$$

$$DC_{stay\text{-}in\text{-}place\ forms} = \left(7\,\frac{lbf}{ft^2}\right)\left(\frac{\text{roadway width}}{\text{no. of girders}}\right) = \left(0.007\,\frac{kips}{ft^2}\right)\left(\frac{43\ ft}{5}\right)$$

$$= 0.06\ kips/ft$$

The structural components dead load is

$$DC_1 = DC_{slab} + DC_{haunch} + DC_{steel} + DC_{SIP\ forms}$$

$$= 1.0\ kips/ft + 0.033\ kip/ft + 0.261\ kips/ft + 0.06\ kips/ft = 1.354\ kips/ft$$

The shear for DC_1 is

$$V_{DC_1} = \frac{wL}{2} = \frac{\left(1.354\,\frac{kips}{ft}\right)(60\ ft)}{2} = 40.62\ kips$$

The moment for DC_1 is

$$M_{DC_1} = \frac{wL^2}{8} = \frac{\left(1.354\,\frac{kips}{ft}\right)(60\ ft)^2}{8} = 609.3\ ft\text{-}kips$$

The nonstructural dead load, DC_2, per interior girder is calculated as

$$DC_{barrier} = \frac{\left(0.45\,\frac{kips}{ft}\right)(2\ barriers)}{5\ girders} = 0.18\ kips/ft$$

The shear for DC_2 is

$$V_{DC_2} = \frac{wL}{2} = \frac{\left(0.18\,\frac{kips}{ft}\right)(60\ ft)}{2} = 5.4\ kips$$

The moment for DC_2 is

$$M_{DC_2} = \frac{wL^2}{8} = \frac{\left(0.18\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})^2}{8} = 81.0\text{ ft-kips}$$

Total structural and nonstructural dead load, DC_{tot}, for interior girders is

$$V_{DC,tot} = V_{DC1} + V_{DC2} = 40.62\text{ kips} + 5.4\text{ kips} = 46.02\ (46\text{ kips})$$

$$M_{DC,tot} = M_{DC1} + M_{DC2} = 609.3\text{ ft-kips} + 81.0\text{ ft-kips} = 690.3\text{ ft-kips}$$

The wearing surface dead load, DW, for interior girders is

$$DW_{FWS} = \frac{(FWS)w_w L}{\text{no. of beams}} = \frac{(3\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(140\,\frac{\text{lbf}}{\text{ft}^3}\right)(43\text{ ft})}{5\text{ beams}}$$

$$= 300\text{ lbf/ft} = 0.3\text{ kips/ft}$$

The shear for DW is

$$V_{DW} = \frac{wL}{2} = \frac{\left(0.3\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})}{2} = 9.0\text{ kips}$$

The moment for DW is

$$M_{DW} = \frac{wL^2}{8} = \frac{\left(0.3\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})^2}{8} = 135\text{ ft-kips}$$

Find the structural components dead load, DC_1, per exterior girder (see Figure 3.4).

$$DC_{slab} = (8\text{ ft})\left(\frac{8\text{ in}}{12\,\frac{\text{in}}{\text{ft}}}\right)\left(0.150\,\frac{\text{kips}}{\text{ft}^3}\right) = 0.80\text{ kips/ft}$$

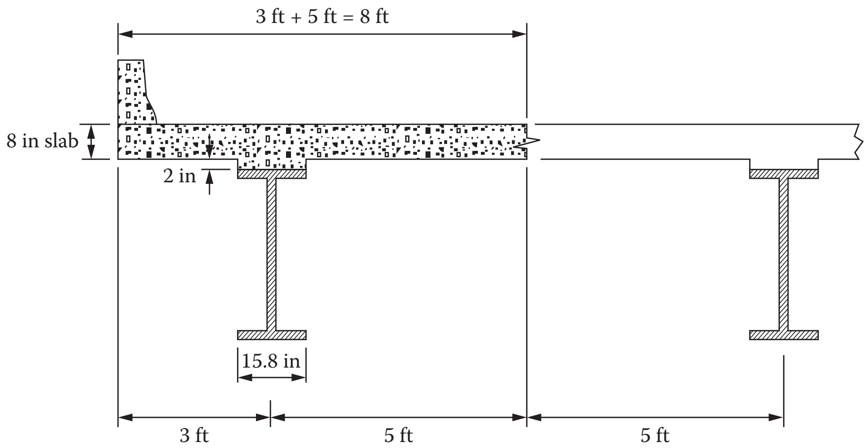

FIGURE 3.4
Dead loads for exterior girder.

$$DC_{haunch} = \left(\frac{2\text{ in}}{12\dfrac{\text{in}}{\text{ft}}}\right)(15.8\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(0.150\frac{\text{kips}}{\text{ft}^3}\right) = 0.033\text{ kips/ft}$$

$$DC_{steel} = \left(0.249\frac{\text{kips}}{\text{ft}}\right) + (0.05)\left(0.249\frac{\text{kips}}{\text{ft}}\right) = 0.261\text{ kips/ft}$$

$$DC_{stay\text{-}in\text{-}place\ forms} = \left(7\frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{\text{roadway width}}{\text{no. of girders}}\right)\left(0.007\frac{\text{kips}}{\text{ft}^2}\right)(43\text{ ft})\left(\frac{1}{5}\right)$$

$$= 0.06\text{ kips/ft}$$

The structural components dead load, DC_1, per exterior girder is

$$DC_1 = DC_{slab} + DC_{haunch} + DC_{steel} + DC_{SIP\ forms}$$

$$= 0.80\text{ kips/ft} + 0.033\text{ kip/ft} + 0.261\text{ kips/ft} + 0.06\text{ kips/ft} = 1.154\text{ kips/ft}$$

The shear for DC_1, per exterior girder is

$$V_{DC_1} = \frac{wL}{2} = \frac{\left(1.154\dfrac{\text{kips}}{\text{ft}}\right)(60\text{ ft})}{2} = 34.62\text{ kips}$$

The moment for DC_1, per exterior girder is

$$M_{DC_1} = \frac{wL^2}{8} = \frac{\left(1.154\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})^2}{8} = 519.3\text{ ft-kips}$$

The nonstructural dead load, DC_2, per exterior girder is

$$DC_{\text{barrier}} = \frac{\left(0.45\,\frac{\text{kips}}{\text{ft}}\right)(2\text{ barriers})}{5\text{ girders}} = 0.18\text{ kips/ft}$$

The shear for DC_2, per exterior girder is

$$V_{DC_2} = \frac{wL}{2} = \frac{\left(0.18\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})}{2} = 5.4\text{ kips}$$

The moment for DC_2, per exterior girder is

$$M_{DC_2} = \frac{wL^2}{8} = \frac{\left(0.18\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})^2}{8} = 81\text{ ft-kips}$$

The shear for the structural and nonstructural dead load, DC_{tot}, for exterior girder is

$$V_{DC,tot} = V_{DC1} + V_{DC2} = 34.62\text{ kips} + 5.4\text{ kips} = 40.0\text{ kips}$$

The moment for the structural and nonstructural dead load for exterior girder is

$$M_{DC,tot} = M_{DC1} + M_{DC2} = 519.3\text{ ft-kips} + 81\text{ ft-kips} = 600.3\text{ ft-kips}$$

The shear for the wearing surface dead load, DW, for exterior girders is

$$V_{DW} = \frac{wL}{2} = \frac{\left(0.3\,\frac{\text{kips}}{\text{ft}}\right)(60\text{ ft})}{2} = 9.0\text{ kips}$$

The moment for the wearing surface dead load, DW, for exterior girders is

TABLE 3.1

Dead Load Summary of Unfactored Shears and Moments for Interior and Exterior Girders

Girder Location	V_{DC} (kips)	M_{DC} (ft-kips)	V_{DW} (kips)	M_{DW} (ft-kips)
Interior	46.0	690.3	9.0	135.0
Exterior	40.0	600.3	9.0	135.0

$$M_{DW} = \frac{wL^2}{8} = \frac{\left(0.3\,\frac{\text{kips}}{\text{ft}}\right)(60\ \text{ft})^2}{8} = 135\ \text{ft-kips}$$

Please see Table 3.1.

Step 3: Evaluate Live Loads

The number of lanes (with fractional parts discounted) is

$$N_L = \frac{w}{12} = \frac{43\ \text{ft}}{12\,\frac{\text{ft}}{\text{lane}}} = 3.58\ \text{lanes}\ \ (3\ \text{lanes})$$

Find the longitudinal stiffness parameter, K_g.

A Art. 4.6.2.2.1

The modulus of elasticity for the concrete deck is

A Eq. 5.4.2.4-1

$$E_{deck} = E_c = 33{,}000\,w_c^{1.5}\sqrt{f'_c} = (33{,}000)\left(0.15\,\frac{\text{kips}}{\text{ft}^3}\right)\sqrt{5\ \text{ksi}} = 4286.8\ \text{kips/in}^2$$

The modulus of elasticity for the beam, E_{beam}, is 29,000 kips/in^2. The modular ratio between steel and concrete is

$$n = \frac{E_{beam}}{E_{deck}}\,\frac{29{,}000\ \text{ksi}}{4286.8\ \text{ksi}} = 6.76$$

The distance between the centers of gravity of the deck and beam is (see Figure 3.5)

$$e_g = 19.7\ \text{in} + 2\ \text{in} + 4\ \text{in} = 25.7\ \text{in}$$

The moment of inertia for the beam, I (steel section only), is 19,600 in^4.

The area, A, is 73.3 in^2.

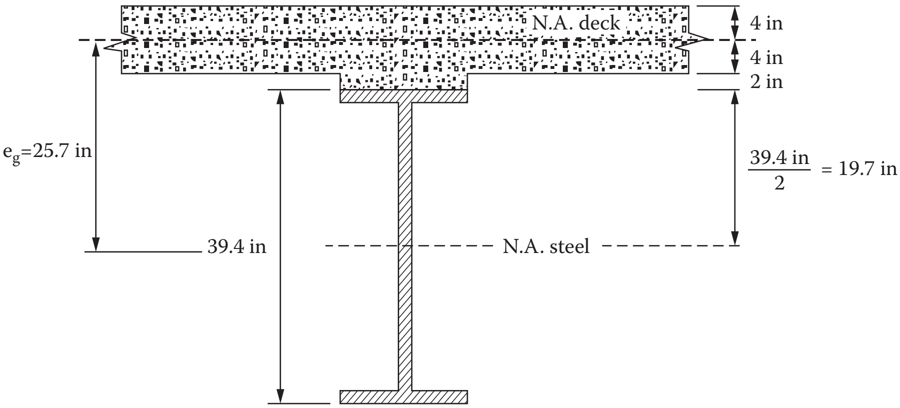

FIGURE 3.5
Section for longitudinal stiffness parameter, K_g.

The longitudinal stiffness parameter is

$$K_g = n(I + Ae_g^2) = (6.76)(19{,}600\ \text{in}^4 + (73.3\ \text{in}^3)(25.7\ \text{in})^2)$$

$$= 459.77\ \text{in}^4$$

A Art. 4.6.2.2.1; A Eq. 4.6.2.2.1-1

The multiple presence factors have been included in the approximate equations for the live distribution factors. These factors must be applied where the lever rule is specified.

A Tbl. 3.6.1.1.2-1; Comm. C3.6.1.1.2

The distribution factor for moments for interior girders of cross-section type (a) with one design lane loaded is

A Tbl. 4.6.2.2.2b-1 or Appendix A

$$\text{DFM}_{si} = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12\,Lt_s^3}\right)^{0.1}$$

$$= 0.06 + \left(\frac{10\ \text{ft}}{14}\right)^{0.4}\left(\frac{10\ \text{ft}}{60\ \text{ft}}\right)^{0.3}\left(\frac{459{,}774\ \text{in}^4}{12(60\ \text{ft})(7.5\ \text{in})^3}\right)^{0.1}$$

$$= 0.59$$

where:
S = spacing of beams (ft)
L = span length (ft)
t_s = depth of concrete slab (in)

The distribution factor for moments for interior girders with two or more design lanes loaded is

$$DFM_{mi} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12\,Lt_s^3}\right)^{0.1}$$

$$= 0.075 + \left(\frac{10\text{ ft}}{9.5}\right)^{0.6}\left(\frac{10\text{ ft}}{60\text{ ft}}\right)^{0.2}\left(\frac{459{,}774\text{ in}^4}{12(60\text{ ft})(7.5\text{ in})^3}\right)^{0.1}$$

$$= 0.82\text{ [controls]}$$

The distribution factor for shear for interior girders of cross-section type (a) with one design lane loaded is

A Tbl. 4.6.2.2.3a-1 or Appendix C

$$DFV_{si} = 0.36 + \frac{S}{25} = 0.36 + \frac{10\text{ ft}}{25} = 0.76$$

The distribution factor for shear for interior girders with two or more design lanes loaded is

$$DFV_{mi} = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^2 = 0.2 + \frac{10\text{ ft}}{12} - \left(\frac{10\text{ ft}}{35}\right)^2 = 0.952\ \text{[controls]}$$

The distribution factors for exterior girders of typical cross section (a).

A Tbl. 4.6.2.2.1-1, 4.6.2.2.2d-1

Use the lever rule to find the distribution factor for moments for exterior girders with one design lane loaded. See Figure 3.6.

A Art. 3.6.1.3.1

$$\Sigma M_{@hinge} = 0$$

$$0 = \frac{P}{2}(9.5\text{ ft}) + \frac{P}{2}(3.5\text{ ft}) - R(10\text{ ft})$$

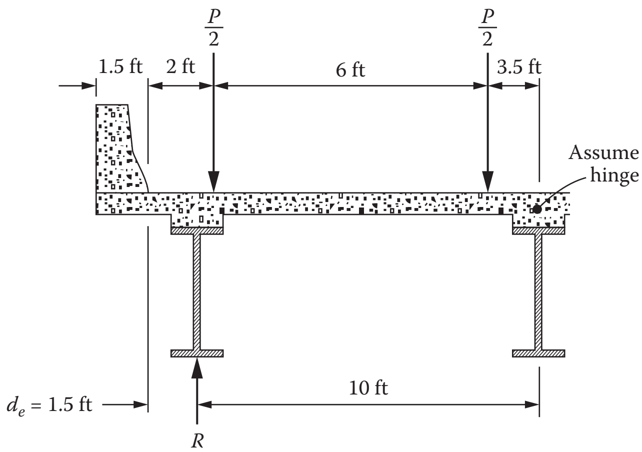

FIGURE 3.6
Lever rule for the distribution factor for moments for exterior girder.

$$R = 0.65\ P$$

$$R = 0.65$$

The distribution factor for moments for exterior girders with one design lane loaded is

A Tbl. 3.6.1.1.2-1

$$\text{DFM}_{se} = (\text{R})(\text{multiple presence factor for one lane loaded, m})$$

$$= (0.65)(1.2) = 0.78\ [\text{controls}]$$

The distribution factor for moments for exterior girders with two or more design lanes loaded, g, is

A Tbl. 4.6.2.2.2d-1 or Appendix B

$$g = eg_{interior}$$

where:

$g_{interior}$ = distribution factor for interior girder = DFM_{mi}
g = DFM_{me} = e(DFM_{mi})

The distance from the center of the exterior web of exterior beam to the interior edge of the barrier, d_e, is 1.5 ft.

A Tbl. 4.6.2.2.2d-1

The correction factor for distribution is

$$e = 0.77 + \frac{d_e}{9.1} = 0.77 + \frac{1.5\text{ ft}}{9.1} = 0.935$$

The distribution factor for moments for exterior girders with two or more design lanes loaded is

$$DFM_{me} = e(DFM_{mi}) = (0.935)(0.826) = 0.77$$

Use the lever rule to find the distribution factor for shear for exterior girder with one design lane loaded.

A Tbl. 4.6.2.2.3b-1 or Appendix D

This is the same as DFM_{se} for exterior girders with one design lane loaded.

$$DFV_{se} = DFM_{se} = 0.78 \text{ [controls]}$$

Find the distribution factor for shear for exterior girders with two or more design lanes loaded using g:

$$g = (e)(g_{int})$$

$$g = DFV_{me} = (e)(DFV_{mi})$$

where:
g_{int} = distribution factor for interior girder = DFV_{mi}

The correction factor for distribution is

A Tbl. 4.6.2.2.3b-1 or Appendix D

$$e = 0.6 + \frac{d_e}{10} = 0.6 + \frac{1.5\text{ ft}}{10} = 0.75$$

The distribution factor for shear for exterior girders with two or more design lanes loaded is

$$DFV_{me} = (0.75)(0.952) = 0.714$$

The HL-93 live load is made of the design lane load plus either the design truck (HS-20) or the design tandem load (whichever is larger).

A Art. 3.6.1.2

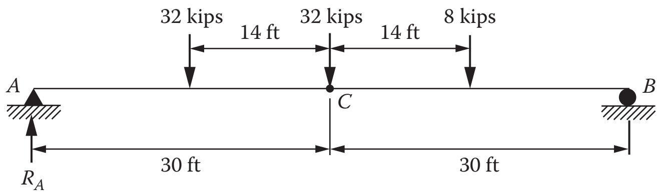

FIGURE 3.7
Design truck (HS-20) load moment at midspan.

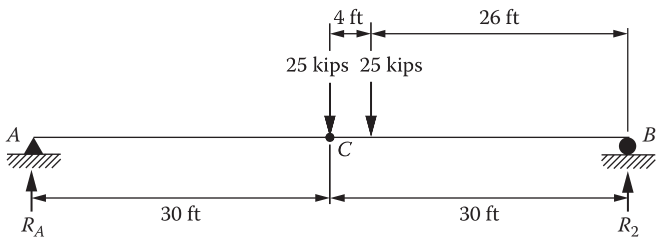

FIGURE 3.8
Design tandem load moment at midspan.

Find the design truck (HS-20) moment due to live load. Please see Figure 3.7.

$$\Sigma M_{@B} = 0$$

The reaction at A is

$$R_A(60\text{ ft}) = (32\text{ kips})(30\text{ ft} + 14\text{ ft}) + (32\text{ kips})(30\text{ ft}) + (8\text{ kips})(30\text{ ft} - 14\text{ ft})$$

$$R_A = 41.6\text{ kips}$$

The design truck moment due to live load is

$$M_{tr} = (41.6\text{ kips})(30\text{ ft}) - (32\text{ kips})(14\text{ ft}) = 800\text{ ft-kips [controls]}$$

Find the design tandem moment. See Figure 3.8.

$$\Sigma M_{@B} = 0$$

The reaction at A is

$$R_A(60\text{ ft}) = (25\text{ kips})(30\text{ ft}) + (25\text{ kips})(26\text{ ft})$$

$$R_A = 23.33\text{ kips}$$

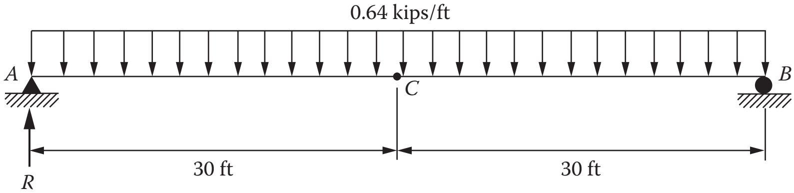

FIGURE 3.9
Design lane load moment.

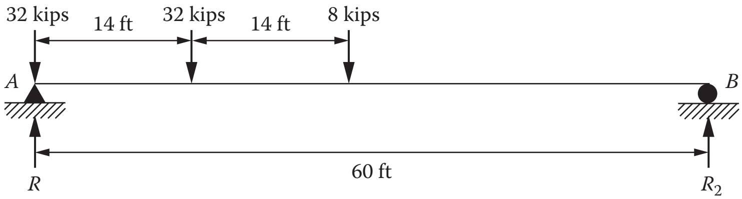

FIGURE 3.10
Design truck (HS-20) shear at support.

The design tandem moment is

$$M_{tandem} = (23.33 \text{ kips})(30 \text{ ft}) = 699.9 \text{ ft-kips } (700 \text{ ft-kips})$$

Find the design lane moment. Please see Figure 3.9.

$$M_c = M_{lane} = \frac{wL^2}{8} = \frac{\left(0.64\,\frac{\text{kips}}{\text{ft}}\right)(60 \text{ ft})^2}{8} = 288 \text{ ft-kips}$$

The dynamic load allowance, IM, is 33% (applied to design truck or design tandem only, not to the design lane load).

A Art. 3.6.2.1

The unfactored total live load moment per lane is

$$M_{LL+IM} = M_{tr}(1 + IM) + M_{ln}$$

$$= (800 \text{ ft-kips})(1 + 0.33) + 288 \text{ ft-kips} = 1352 \text{ ft-kips per lane}$$

Find the design truck shear. See Figure 3.10.

$$\Sigma M_{@B} = 0$$

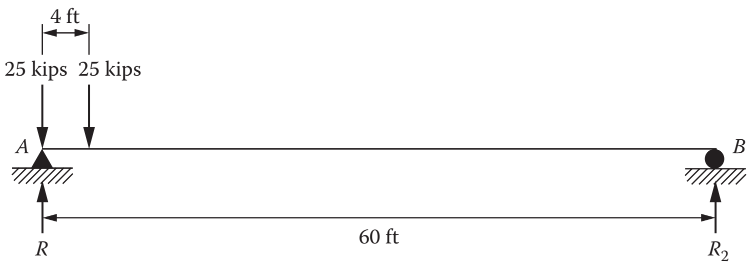

FIGURE 3.11
Design tandem load shear at support.

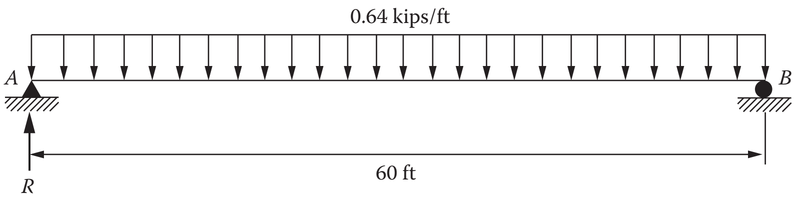

FIGURE 3.12
Design lane load shear.

The reaction at A is

$$R_A(60\ \text{ft}) = (32\ \text{kips})(60\ \text{ft}) + (32\ \text{kips})(46\ \text{ft}) + (8\ \text{kips})(32\ \text{ft})$$

$$R_A = 60.8\ \text{kips}$$

The design truck shear, V_{tr}, is 60.8 kips [controls].
Find the design tandem shear. See Figure 3.11.

$$\Sigma M_{@B} = 0$$

The reaction at A is

$$R_A(60\ \text{ft}) = (25\ \text{kips})(60\ \text{ft}) + (25\ \text{kips})(56\ \text{ft})$$

$$R_A = 48.33\ \text{kips}$$

The design tandem shear, V_{tandem}, is 48.33 kips.
Find the design lane shear. See Figure 3.12.

$$V_{ln} = \frac{wL}{2} = \frac{\left(0.64\,\frac{\text{kips}}{\text{ft}}\right)(60\ \text{ft})}{2} = 19.2\ \text{kips}$$

TABLE 3.2 (**A Art. 4.6.2.2.2**)

Summary of Live Load Effects*

Girder Location	No. of Lanes Loaded	Unfactored M_{LL+IM} (ft-kips per Lane)	DFM	Unfactored M_{LL+IM} (ft-kips per Girder)	Unfactored V_{LL+IM} (kips per Lane)	DFV	Unfactored V_{LL+IM} (kips per Girder)
Interior	1	1352.0	0.596		100.1	0.76	—
	2 or more	1352.0	0.82	1108.6	100.1	0.952	95.3
Exterior	1	1352.0	0.78	1054.6	100.1	0.78	78.1
	2 or more	1352.0	0.77		100.1	0.714	—

* The live load moments and shears per girder are determined by applying the distribution factors, DF, to those per lane.

The dynamic load allowance, IM, is 33% (applied to design truck or design tandem only, not to the design lane load).

A Art. 3.6.2.1

The unfactored total live load shear is

$$V_{LL+IM} = V_{tr}(1 + IM) + V_{ln}$$

$$= (60.8 \text{ kips})(1 + 0.33) + 19.2 \text{ kips} = 100.1 \text{ kips}$$

Please see Table 3.2.

Total live load moment and shear for interior girders

$$M_{LL+IM} = 1108.6 \text{ ft-kips}$$

$$V_{LL+IM} = 95.3 \text{ kips per girder}$$

Total live load moment and shear for exterior girders

$$M_{LL+IM} = 1054.6 \text{ ft-kips}$$

$$V_{LL+IM} = 78.1 \text{ kips}$$

Step 4: Check the Strength I Limit State.

A Arts. 3.3.2; 3.4; Tbls. 3.4.1-1, 3.4.1-2

$$U = 1.25(DC) + 1.5(DW) + 1.75(LL + IM)$$

The factored moment for interior girders is

$$M_u = 1.25\ M_{DC} + 1.5\ M_{DW} + 1.75\ M_{LL+IM}$$

$$= (1.25)(690.3 \text{ ft-kips}) + (1.5)(135.0 \text{ ft-kips}) + (1.75)(1108.6 \text{ ft-kips})$$

$$= 3005.4 \text{ ft-kips}$$

The factored shear for interior girders is

$$\begin{aligned} V_u &= 1.25\ V_{DC} + 1.5\ V_{DW} + 1.75\ V_{LL+IM} \\ &= (1.25)(46\text{ kips}) + (1.5)(9.0\text{ kips}) + (1.75)(95.3\text{ kips}) \\ &= 237.8\text{ kips } (238.0\text{ kips}) \end{aligned}$$

The factored moment for exterior girders is

$$\begin{aligned} M_u &= (1.25)(600.3\text{ ft-kips}) + (1.5)(135\text{ ft-kips}) + (1.75)(1054.6\text{ ft-kips}) \\ &= 2798.4\text{ ft-kips } (2800\text{ ft-kips}) \end{aligned}$$

The factored shear for exterior girders is

$$\begin{aligned} V_u &= (1.25)(40\text{ kips}) + (1.5)(9.0\text{ kips}) + (1.75)(78.1\text{ kips}) \\ &= 200.2\text{ kips } (200\text{ kips}) \end{aligned}$$

Check the Flexural Resistance.

Find the flange stress, f_{bu}, for noncomposite sections with continuously braced flanges in tension or compression. The required flange stress without the flange lateral bending, f_{bu}, must satisfy the following.

A Art. 6.10.8.1.3

$$f_{bu} \leq \Phi_f R_h F_{yf}$$

A Eq. 6.10.8.1.3-1

R_h	hybrid factor	1.0 (for rolled shapes) **A Art. 6.10.1.10.1**
Φ_f	resistance factor for flexure	1.0 **A Art. 6.5.4.2**
F_{yf}	minimum yield strength of a flange	50 ksi

The flange stress for interior girders is

$$f_{bu} = \frac{M_u}{S_x} = \frac{(3005.4\text{ ft-kips})\left(12\,\frac{\text{in}}{\text{ft}}\right)}{992\text{ in}^3}$$

$$= 36.4\text{ kips/in}^2 < \Phi_f R_h F_{yf} = 50\text{ kips/in}^2\ \text{[OK]}$$

Check the shear resistance in the web due to the factored loads, V_u, where:

A Arts. 6.10.9.1, 6.10.9.2

V_p = plastic shear force
C = ratio of the shear–buckling resistance to the shear yield strength

A Art. 6.10.9.3.2

$$V_u \leq \Phi V_n \text{ and } V_n = CV_p$$

Shear in the web, V_u, must satisfy the following.

$$V_u \leq \Phi_v V_n$$

A Eq. 6.10.9.1-1

Nominal shear resistance,

A Eq. 6.10.9.2-1

$V_n = CV_p$		
F_{yw}	specified minimum yield strength of a web	50 ksi
t_w	web thickness	0.75 in
E	modulus of elasticity of steel	29,000 ksi
Φ_v	resistance factor for shear	1.00

A Art. 6.5.4.2

D	depth of steel beam	39.4 in

The shear yielding of the web is

A Eq. 6.10.9.2-2

$$V_p = 0.58\, F_{yw} D t_w$$

$$= (0.58)(50\text{ ksi})(39.4\text{ in})(0.75\text{ in})$$

$$= 856.95\text{ kips }(857\text{ kips})$$

Find the ratio of the shear buckling resistance to the shear yield strength, C.

C is determined from any of these AASHTO equations: Eq. 6.10.9.3.2-4, 6.10.9.3.2-5, 6.10.9.3.2-6.

A Art. 6.10.9.3.2

Try AASHTO Eq. 6.10.9.3.2-4.

$$\text{If } \frac{D}{t_w} \leq 1.12\sqrt{\frac{Ek}{F_{yw}}}\text{, then } C = 1.0.$$

The shear buckling coefficient, k, is 5 for unstiffened web panels.

A Com. 6.10.9.2

$$\frac{D}{t_w} = \frac{39.4 \text{ in}}{0.75 \text{ in}} = 52.5$$

$$1.12\sqrt{\frac{Ek}{F_{yw}}} = 1.12\sqrt{\frac{(29{,}000 \text{ ksi})(5)}{50 \text{ ksi}}} = 60.3 > \frac{D}{t_w} = 52.5 \text{ [OK]}$$

Therefore, C is 1.0.

The nominal shear resistance for unstiffened webs is

A Eq. 6.10.9.2-1

$$V_n = CV_p = (1.0)(857 \text{ kips}) = 857 \text{ kips [OK]}$$

The shear resistance for interior girders is greater than the factored shear for interior girder.

$$\Phi_v V_n = (1.0)(857 \text{ kips}) = 857 \text{ kips} > V_u = 238 \text{ kips [OK]}$$

A Art. 6.10.9

Step 5: Check the Fatigue Limit State

Details shall be investigated for fatigue as specified in AASHTO Art. 6.6.1. The fatigue load combination in AASHTO Tbl. 3.4.1-1 and the fatigue live load specified in AASHTO Art. 3.6.1.4 shall apply.

A Art. 6.10.5.1

For the load induced fatigue, each detail must satisfy the following:

A Art. 6.6.1.2, A Eq. 6.6.1.2.2-1

$$\gamma(\Delta f) \le (\Delta F)_n$$

γ	load factor	
(Δf)	live load stress range due to fatigue load	ksi
$(\Delta F)_n$	nominal fatigue resistance as specified in Art. 6.6.1.2.5	ksi

The fatigue load is one design truck with a constant spacing of 30 ft between 32 kip axles. Find the moment due to fatigue load.

A Art. 3.6.1.4

Find the center of gravity of design truck, x, from the left 32 kip axle

$$x = \frac{(32 \text{ kips})(30 \text{ ft}) + (8 \text{ kips})(30 \text{ ft} + 14 \text{ ft})}{32 \text{ kips} + 32 \text{ kips} + 8 \text{ kips}}$$

$$= 18.22 \text{ ft from left 32 kip axle}$$

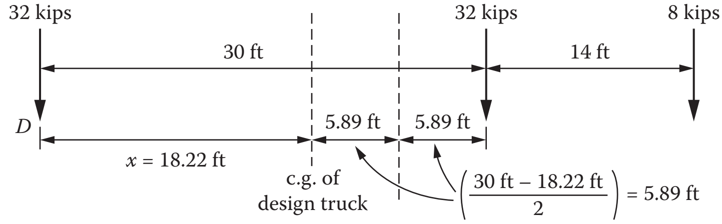

FIGURE 3.13
Center of gravity of design truck loading (HS-20).

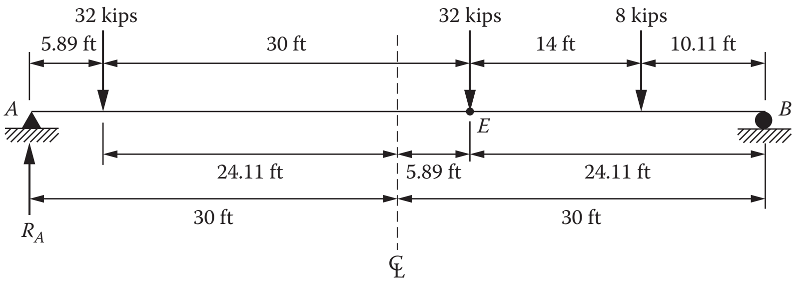

FIGURE 3.14
Fatigue load position for maximum moment.

Please see Figures 3.13 and 3.14.

$$\Sigma M_{@B} = 0$$

The reaction at A is

$$R_A(60 \text{ ft}) = (32 \text{ kips})(54.11 \text{ ft}) + (32 \text{ kips})(24.11 \text{ ft}) + (8 \text{ kips})(10.11 \text{ ft})$$

$$R_A = 43.07 \text{ kips}$$

The maximum moment at point E,

$$M_E = (43.07 \text{ kips})(35.89 \text{ ft}) - (32 \text{ kips})(30 \text{ ft}) = 585.8 \text{ ft-kips [controls]}$$

The maximum moment due to fatigue load is 585.8 ft-kips.

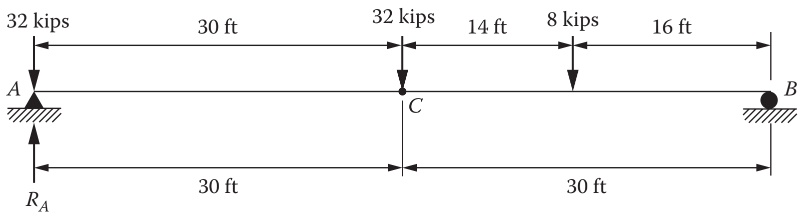

FIGURE 3.15
Fatigue load position for maximum shear.

Find the shear due to the Fatigue Load, V_{fat}. See Figure 3.15.

$$\Sigma M_{@B} = 0$$

The reaction at A is

$$R_A = (32\text{ kips}) + \left(\frac{32\text{ kips}}{2}\right) + (8\text{ kips})\left(\frac{16\text{ ft}}{60\text{ ft}}\right)$$

$$= 50.13\text{ kips}$$

$$V_{fat} = 50.13\text{ kips}$$

The dynamic load allowance, IM, is 15%.

A Tbl. 3.6.2.1-1

Find the Load Distribution for Fatigue, DFM_{fat}.

The distribution factor for one traffic lane shall be used. For single lane, approximate distribution factors in AASHTO Art. 4.6.2.2 are used. The force effects shall be divided by 1.20.

A Art. 3.6.1.4.3b, 3.6.1.1.2

The distribution factor for fatigue moments in interior girders with one lane is

$$DFM_{fat,int} = \frac{DFM_{int}}{1.2} = \frac{0.59}{1.2} = 0.49$$

The distribution factor for the fatigue moments in exterior girders with one loaded lane is

$$DFM_{fat,ext} = \frac{DFM_{ext}}{1.2} = \frac{0.78}{1.2} = 0.65$$

The unfactored distributed moment for interior girders is

$$M_{fat,int} = M_{fatigue}(DFM_{fat,int})(1 + IM)$$

$$= (585.8 \text{ ft-kips})(0.49)(1 + 0.15) = 330.1 \text{ ft-kips } (330 \text{ ft-kips})$$

The unfactored distributed moment for exterior girders is

$$M_{fat,ext} = M_{fatigue}(DFM_{fat,ext})(1 + IM)$$

$$= (585.8 \text{ ft-kips})(0.65)(1 + 0.15) = 437.9 \text{ ft-kips } (438 \text{ ft-kips})$$

Live load stress due to fatigue load for interior girders is

$$\left(\Delta f_{int}\right) = \frac{M_{fat,int}}{S_x} = \frac{330.0 \text{ ft-kips}\left(12\,\frac{\text{in}}{\text{ft}}\right)}{992 \text{ in}^3} = 3.99 \text{ kips/in}^2$$

Live load stress due to fatigue load for exterior girders is

$$\left(\Delta f_{ext}\right) = \frac{M_{fat,ext}}{S_x} = \frac{438 \text{ ft-kips}\left(12\,\frac{\text{in}}{\text{ft}}\right)}{992 \text{ in}^3} = 5.30 \text{ kips/in}^2$$

For load-induced fatigue, each detail shall satisfy:

A Art. 6.6.1.2.2

$$\gamma(\Delta f) \le (\Delta F)_n$$

Find the nominal fatigue resistance, $(\Delta F)_n$, for Fatigue II load combination for finite life.

A Art. 6.6.1.2.5, A Eq. 6.6.1.2.5-2

$$\left(\Delta F\right)_n = \left(\frac{A}{N}\right)^{1/3}$$

The girder is in Detail Category A, because it is a plain rolled member.

A Tbl. 6.6.1.2.3-1

A = constant taken for Detail Category A $\quad 250 \times 10^8$ kips/in^2

A Tbl. 6.6.1.2.5-1

n = number of stress range cycles per truck passage — 1.0 (for simple span girders with span greater than 40 ft)

A Tbl. 6.6.1.2.5-2

p = the fraction of truck traffic in a single lane — 0.80 (because N_L is 3 lanes)

A Tbl. 3.6.1.4.2-1

$(\Delta F)_{TH}$ = the constant amplitude threshold for Detail Category A — 24.0 kips/in^2

A Tbl. 6.6.1.2.5-3

$ADTT_{SL}$ = the number of trucks per day in a single lane over the design life

ADTT = the number of trucks per day in one direction over the design life

$$ADTT_{SL} = (p)(ADTT) = (0.80)(250) = 200 \text{ trucks daily}$$

A Art. 3.6.1.4.2

The number of cycles of stress range N is

$$N = (365 \text{ days})(75 \text{ years})n(ADTT)_{SL} = (365 \text{ days})(75 \text{ years})(1.0)(200 \text{ trucks})$$

$$= 5{,}475{,}000 \text{ cycles}$$

A Eq. 6.6.1.2.5-3

The nominal fatigue resistance is

A Eq. 6.6.1.2.5-2

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{1/3} = \left(\frac{250\times 10^8 \dfrac{\text{kips}}{\text{in}^2}}{5{,}475{,}000 \text{ cycles}}\right)^{1/3} = 16.6 \text{ kips/in}^2$$

Find the Load Factor, γ, and confirm that the following are satisfied.

A Eq. 6.6.1.2.2-1

$$\gamma(\Delta f) \le (\Delta F)_n$$

γ = 0.75 for Fatigue II Limit State

A Tbl. 3.4.1-1

The factored live load stress due to the fatigue load for interior girders is

$$\gamma(\Delta f_{int}) = (0.75)(3.99 \text{ ksi}) = 2.99 \text{ ksi} < 16.6 \text{ ksi [OK]}$$

The factored live load stress due to the fatigue load for exterior girders is

$$\gamma(\Delta f_{ext}) = (0.75)(5.30\ \text{ksi}) = 3.98\ \text{ksi} < 16.6\ \text{ksi [OK]}$$

A Art. 6.10.5.3

Special Fatigue Requirement for Webs

Find the shear in the web due to unfactored permanent load plus factored fatigue shear load, V_u, for interior and exterior girders, satisfying the following provision to control web-buckling and elastic flexing of the web. V_{cr} is the shear buckling resistance (Eq. 6.10.9.3.3-1). The use of the Fatigue I load provisions for the Fatigue II load combination will be considered conservative.

A Eq. 6.10.9.3.3-1

The distribution factor for fatigue shear in interior girders with one design lane loaded is

$$DFV_{fat,int} = \frac{DFV_{int}}{m} = \frac{0.76}{1.2} = 0.63$$

The distribution factor for fatigue shear in exterior girders with one design lane loaded is

$$DFV_{fat,ext} = \frac{DFV_{ext}}{m} = \frac{0.78}{1.2} = 0.65$$

Unfactored fatigue shear load per interior girder is

$$(DFV_{fat,int})(V_{fat}) = (0.63)(50.13\ \text{kips}) = 31.58\ \text{kips}$$

Unfactored fatigue shear load per exterior girder is

$$(DFV_{fat,ext})(V_{fat}) = (0.65)(50.13\ \text{kips}) = 32.58\ \text{kips}$$

$$V_u \le V_{cr}$$

A Eq. 6.10.5.3-1

$$V_u = V_{DL} + \gamma(V_{fat})(1 + IM)$$

The shear buckling resistance for unstiffened webs, V_{cr}, for W40 × 249 with F_y equal to 50 ksi is

A Art. 6.10.9.2; Eq. 6.10.9.2-1

$$V_{cr} = CV_p$$

$$C = 1.0$$

A Eq. 6.10.9.3.2-4

$$V_p = 0.58\ F_{yw}Dt_w = 857\text{ kips}$$

A Eq. 6.10.9.2-2

$$V_{cr} = CV_p = (1.0)(857\text{ kips}) = 857\text{ kips}$$

The distortion-induced fatigue for interior girders is

$$V_{DL} = V_{DC} + V_{DW}$$

$$V_u = V_{DL} + \gamma V_{fat}(1 + IM) = (46\text{ kips} + 9\text{ kips}) + (0.75)(31.58\text{ kips})(1.15)$$

$$= 82.23\text{ kips} < V_{cr} = 857\text{ kips[OK]}$$

The distortion induced fatigue for exterior girders is

$$V_u = V_{DL} + \gamma V_{fat}(1 + IM) = (40\text{ kips} + 9\text{ kips}) + (0.75)(32.58\text{ kips})(1.15)$$

$$= 77.1\text{ kips} < V_{cr} = 857\text{ kips[OK]}$$

Step 6: Check the Service Limit States

Deflection Limit

A Art. 2.5.2.6

The maximum deflection for vehicular load, δ_{max}, where L is the span, is

A Art. 2.5.2.6.2

$$\delta_{max} = \frac{L}{800}$$

Deflection will be the larger of

A Art. 3.6.1.3.2

1. Deflection resulting from the design truck (HS-20) alone
2. Deflection resulting from 25% of design truck plus the design lane load

Find the deflection resulting from the design truck alone (see Figure 3.16)

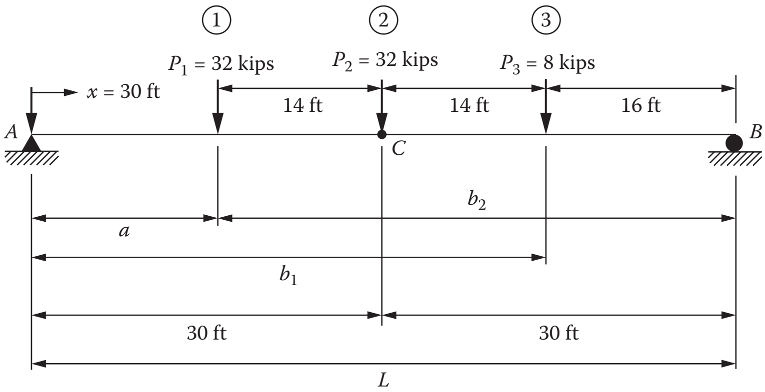

FIGURE 3.16
Design truck loading for maximum deflection at midspan.

The deflections at midspan, δ_1, δ_2, δ_3, are

$$\delta_1 = \frac{P_1 b_2 x}{6\ EIL}\left(L^2 - b_2^2 - x^2\right)$$

$$= \frac{(32\ \text{kips})(30\ \text{ft} + 14\ \text{ft})(30\ \text{ft})\left(12\frac{\text{in}}{\text{ft}}\right)^2}{(6)(29{,}000\ \text{ksi})(19{,}600\ \text{in}^4)(60\ \text{ft})\left(12\frac{\text{in}}{\text{ft}}\right)}$$

$$\left(\left(60\ \text{ft}\left(12\frac{\text{in}}{\text{ft}}\right)\right)^2 - \left((30\ \text{ft} + 14\ \text{ft})\left(12\frac{\text{in}}{\text{ft}}\right)\right)^2 - \left(30\ \text{ft}\left(12\frac{\text{in}}{\text{ft}}\right)\right)^2\right)$$

$$= 0.273\ \text{in}$$

$$\delta_2 = \frac{P_2 L^3}{48\ EI} = \frac{(32\ \text{kips})(60\ \text{ft})^3\left(12\frac{\text{in}}{\text{ft}}\right)^3}{(48)(29{,}000\ \text{ksi})(19{,}600\ \text{in}^4)}$$

$$= 0.438\ \text{in}$$

$$\delta_3 = \frac{P_3 b_1 x}{6\,EIL}\left(L^2 - b_1^2 - x^2\right)$$

$$= \frac{(8\text{ kips})(30\text{ ft} + 14\text{ ft})(30\text{ ft})\left(12\,\frac{\text{in}}{\text{ft}}\right)^2}{(6)(29{,}000\text{ ksi})(19{,}600\text{ in}^4)(60\text{ ft})\left(12\,\frac{\text{in}}{\text{ft}}\right)}$$

$$\left(\left(60\text{ ft}\left(12\,\frac{\text{in}}{\text{ft}}\right)\right)^2 - \left((30\text{ ft} + 14\text{ ft})\left(12\,\frac{\text{in}}{\text{ft}}\right)\right)^2 - \left(30\text{ ft}\left(12\,\frac{\text{in}}{\text{ft}}\right)\right)^2\right)$$

$$= 0.068\text{ in}$$

$$\delta_{LL} = \delta_1 + \delta_2 + \delta_3 = 0.273\text{ in} + 0.438\text{ in} + 0.068\text{ in} = 0.779\text{ in per lane}$$

m multiple presence factor for three loaded lanes 0.85

A Tbl. 3.6.1.1.2-1

N_L number of lanes 3

N_g number of girders 5

The dynamic load allowance, IM, is 33%.

A Art. 3.6.2.1

Distribution factor for deflection is equal to the number of lanes, N_L, divided by the number of beams, N_g.

A Comm. 2.5.2.6.2

$$DF = (m)\left(\frac{N_L}{N_g}\right)$$

A Tbl. 3.6.2.1-1

$$DF = (0.85)\left(\frac{3}{5}\right) = 0.51$$

$$IM = 33\%$$

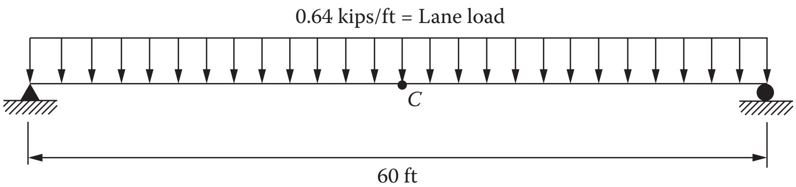

FIGURE 3.17
Design lane loading for maximum deflection at midspan.

The distributed midspan deflection per girder due to the design truck alone is

$$\delta_{LL+IM} = (0.779 \text{ in per lane})(0.51)(1 + 0.33) = 0.53 \text{ in per girder [controls]}$$

A Art. 3.6.1.3.2

Find the deflection resulting from 25% of design truck plus the lane load.

A Art. 3.6.1.3.2

The deflection at midspan due to the lane load is (see Figure 3.17)

$$\delta_{ln} = \frac{5\ wL^4}{384\ EI} = \frac{(5)\left(0.64\dfrac{\text{kips}}{\text{ft}}\right)\left(\dfrac{\text{ft}}{12\text{ in}}\right)(60\text{ft})\left(\dfrac{12\text{ in}}{\text{ft}}\right)^4}{(384)(29{,}000\text{ ksi})(19{,}600\text{ in}^4)} = 0.33\text{ in per lane}$$

The midspan deflection per lane is

$$\delta_{LL25\%+ln} = 25\% \text{ of deflection due to design truck} + \delta_{ln}$$

$$= (0.25)(\delta_{LL}) + \delta_{ln}$$

$$= (0.25)(0.779\text{ in}) + 0.33\text{ in} = 0.525\text{ in per lane [controls]}$$

The distributed midspan deflection per girder is

$$\delta_{LL+IM} = (\delta_{LL25\%+ln})(DF)(1 + IM) = (0.525\text{ in per lane})(0.51)(1 + 0.33)$$

$$= 0.356\text{ in per girder}$$

Confirm that $\delta_{LL+IM} < \delta_{max}$.

$$\delta_{max} = \frac{L}{800} = \frac{(60\ \text{ft})\left(12\,\frac{\text{in}}{\text{ft}}\right)}{800} = 0.90\ \text{in}$$

$$\delta_{LL+IM} = 0.525\ \text{in} < \delta_{max} = 0.90\ \text{in [OK]}$$

A Art. 6.10.4.2

Permanent deformations

For inelastic deformation limitations the Service II Limit State load combination shall apply and both steel flanges of noncomposite sections shall satisfy the following.

Flexure (to prevent objectionable permanent deflections)

A Art. 6.10.4.2.2

Both steel flanges of noncomposite sections shall satisfy the following, where f_f is the flange stress due to Service II loads.

A Eq. 6.10.4.2.2-3

$$f_f + \frac{f_l}{2} \le 0.80\ R_h F_{yf}$$

f_l	flange lateral bending stress due to Service II loads	0 (for continually braced flanges)
R_h	hybrid factor	1.0 (for rolled shapes)

A Art. 6.10.1.10.1

Service II Limit State: U = 1.0 DC + 1.0 DW + 1.3(LL + IM)

A Tbl. 3.4.1-1

The factored design moment for interior girders is

$$M_u = 1.0\ M_{DC} + 1.0\ M_{DW} + 1.3\ M_{LL+IM}$$

$$= (1.0)(690.3\ \text{ft-kips}) + (1.0)(135\ \text{ft-kips} + (1.3)(1108.6\ \text{ft-kips}))$$

$$= 2266.5\ \text{ft-kips}$$

The flange stress due to Service II loads for interior girders is

$$f_f = \frac{M_u}{S_x} = \frac{2266.5\ \text{ft-kips}\left(12\,\frac{\text{in}}{\text{ft}}\right)}{992\ \text{in}^3} = 27.4\ \text{kips/in}^2$$

For steel flanges of noncomposite sections in interior girders, check,

A Art. 6.10.4.2.2, A Eq. 6.10.4.2.2-3

$$f_f + \frac{f_l}{2} \le 0.80\,R_h F_{yf}$$

$$27.4\text{ ksi} + \frac{0}{2} = 27.4\text{ ksi} < (0.8)(1.0)(50\text{ ksi}) = 40\text{ ksi}\quad \text{[OK]}$$

The factored design moment for exterior girders is

M_u = (1.0)(600.3 ft-kips) + (1.0)(135 ft-kips) + (1.3)(1054.6 ft-kips)

= 2106.8 ft-kips

The flange stress due to Service II loads for exterior girders is

$$f_f = \frac{M_u}{S_x} = \frac{(2106.8\text{ ft-kips})\left(12\,\frac{\text{in}}{\text{ft}}\right)}{992\text{ in}^3} = 25.5\text{ ksi}$$

For steel flanges of noncomposite section in exterior girders, check,

$$f_f + \frac{f_l}{2} \le 0.80\,R_h F_{yf}$$

$$25.5\text{ ksi} + \frac{0}{2} = 25.5\text{ ksi} < (0.8)(1.0)(50\text{ ksi}) = 40\text{ ksi}\quad \text{[OK]}$$

Practice Problem 2: 161 ft Steel I-Beam Bridge with Concrete Slab

Situation

L	span length	161 ft
LL	live load model	HL-93
S	beam spacing	13 ft
t_s	slab thickness	
	(including 0.5 in integral wearing surface)	10.5 in
	self-weight of steel girder	335 lbf/ft

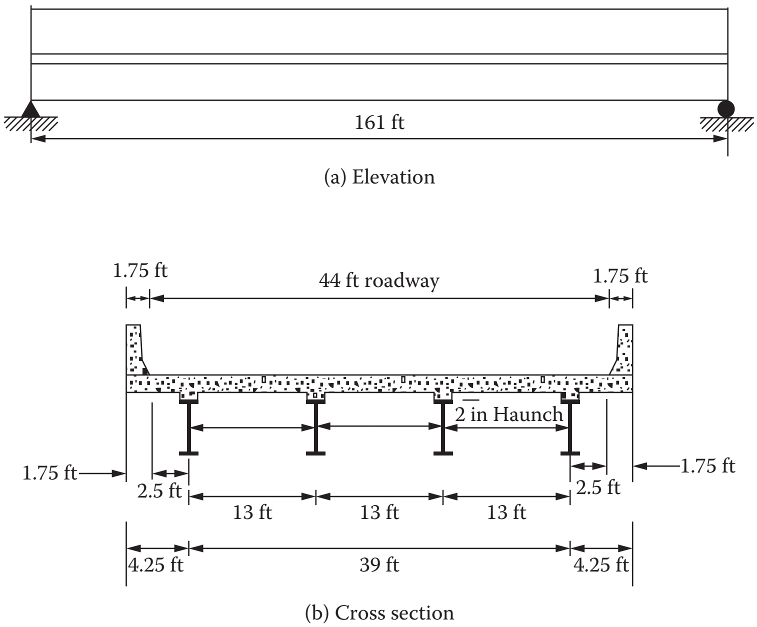

FIGURE 3.18
Steel I-beam with concrete slab.

FWS	future wearing surface	25 lbf/ft^2
	weight of stiffeners and bracings	10% of girder weight
	dead load of the curb/parapet	0.505 kips/ft
	self-weight of stay-in-place (SIP) forms	7 lbf/ft^2
f'_c	concrete strength	5.0 $kips/in^2$
w_c	concrete unit weight	145 lbf/ft^3

The steel girders satisfy the provisions of AASHTO Art. 6.10.1 and 6.10.2 for the cross-section properties′ proportion limits. See Figure 3.18.

Solution

Steel Girder Section Review. See Figure 3.19

A	area of girder	106.0 in^2
y_b	distance from bottom fiber to centroid of section	30.88 in
y_t	distance from top fiber to centroid of section	40.12 in
I	moment of inertia about steel section centroid	99,734.0 in^4

Please see Figure 3.20.

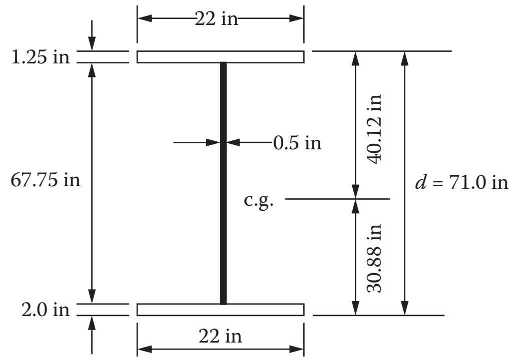

FIGURE 3.19
I-beam properties.

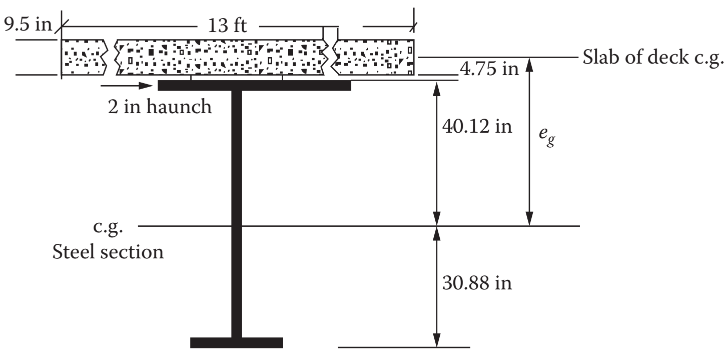

FIGURE 3.20
Cross-section properties for shears and moments due to dead loads.

Step 1: Determine Dead Load Shears and Moments for Interior and Exterior Girders

Dead Loads DC: Components and Attachments

1.1 Interior Beam

1.1.a Noncomposite Dead Loads, DC_1 The steel beam weight plus 10% for stiffeners and bracings is,

$$DC_{\text{beam}} = \left(0.335\frac{\text{kips}}{\text{ft}}\right)(1.10)$$

$$DC_{\text{beam}} = 0.369\frac{\text{kips}}{\text{ft}}$$

The slab dead load is

$$DC_{slab} = (10.5\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)(13\text{ ft})\left(0.15\frac{\text{kips}}{\text{ft}^3}\right)$$

$$DC_{slab} = 1.70\frac{\text{kips}}{\text{ft}}$$

The concrete haunch dead load is

$$DC_{haunch} = (2\text{ in})(22\text{ in})\left(\frac{1\text{ ft}^2}{144\text{ in}^2}\right)\left(0.15\frac{\text{kips}}{\text{ft}}\right)$$

$$DC_{haunch} = 0.046\frac{\text{kips}}{\text{ft}}$$

The stay-in-place (SIP) forms dead load is

$$DC_{SIP} = \left(0.007\frac{\text{kips}}{\text{ft}^2}\right)(44.0\text{ ft})\left(\frac{1}{4\text{ girders}}\right)$$

$$DC_{SIP} = 0.077\frac{\text{kips}}{\text{ft}}$$

$$DC_1 = DC_{beam} + DC_{slab} + DC_{haunch} + DC_{SIP}$$

$$DC_1 = 0.369\frac{\text{kips}}{\text{ft}} + 1.70\frac{\text{kips}}{\text{ft}} + 0.046\frac{\text{kips}}{\text{ft}} + 0.077\frac{\text{kips}}{\text{ft}}$$

$$DC_1 = 2.19\frac{\text{kips}}{\text{ft}}$$

Shear

$$V_{DC_1} = \frac{wL}{2} = \frac{\left(2.19\frac{\text{kips}}{\text{ft}}\right)(161\text{ ft})}{2}$$

$$V_{DC_1} = 176.3\text{ kips}$$

Moment

$$M_{DC_1} = \frac{wL^2}{8} = \frac{\left(2.19\,\frac{\text{kips}}{\text{ft}}\right)(161\text{ ft})^2}{8}$$

$$M_{DC_1} = 7{,}095.9\text{ kip-ft}$$

1.1.b Composite Dead Loads Due to Curb/Parapet, DC_2

$$DC_2 = \left(0.505\,\frac{\text{kips}}{\text{ft}}\right)(2)\left(\frac{1}{4\text{ girders}}\right)$$

$$DC_2 = 0.253\,\frac{\text{kips}}{\text{ft}}$$

Shear

$$V_{DC_2} = \frac{wL}{2} = \frac{\left(0.253\,\frac{\text{kips}}{\text{ft}}\right)(161\text{ ft})}{2}$$

$$V_{DC_2} = 20.4\text{ kips}$$

Moment

$$M_{DC_2} = \frac{wL^2}{8} = \frac{\left(0.253\,\frac{\text{kips}}{\text{ft}}\right)(161\text{ ft})^2}{8}$$

$$M_{DC_2} = 819.8\text{ kip-ft}$$

1.1.c Total Dead Load Effects Due to $DC = DC_1 + DC_2$

Shear

$$V_{DC} = V_{DC_1} + V_{DC_2}$$

$$V_{DC} = 176.3\text{ kips} + 20.4\text{ kips}$$

$$V_{DC} = 196.7\text{ kips}$$

Moment

$$M_{DC} = M_{DC_1} + M_{DC_2}$$

$$M_{DC} = 7.095.9 \text{ kip-ft} + 819.8 \text{ kip-ft}$$

$$M_{DC} = 7,915.7 \text{ kip-ft}$$

1.1.d Dead Loads, DW; Wearing Surfaces

For future wearing surface of 25 lbf/ft^2

$$DW = \left(25 \frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right)(44 \text{ ft})\left(\frac{1}{4 \text{ girders}}\right)$$

$$DW = 0.275 \frac{\text{kips}}{\text{ft}}$$

Shear

$$V_{DW} = \frac{wL}{2} = \frac{\left(0.275 \frac{\text{kips}}{\text{ft}}\right)(161 \text{ ft})}{2}$$

$$V_{DW} = 22.14 \text{ kips}$$

Moment

$$M_{DW} = \frac{wL^2}{8} = \frac{\left(0.275 \frac{\text{kips}}{\text{ft}}\right)(161 \text{ ft})^2}{8}$$

$$M_{DW} = 891.0 \text{ kip-ft}$$

Please see Table 3.3 for summary in interior beams.

1.2 Exterior Beams

The exterior beam has a slab overhang of 4.25 ft. Therefore, the interior beam dead loads control for design.

TABLE 3.3
Summary of Dead Load Shears and Moments in Interior Beams

	DC			
	DC_1	DC_2	$DC_1 + DC_2$	DW
Shear, kips	176.3	20.4	196.7	22.1
Moment, kip-ft	7095.9	819.8	7915.7	891.0

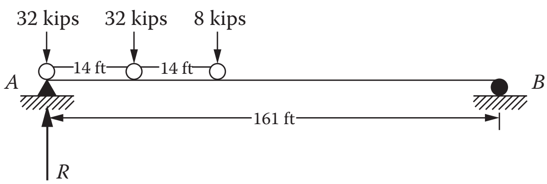

FIGURE 3.21
Design truck load (HS-20) position for maximum shear.

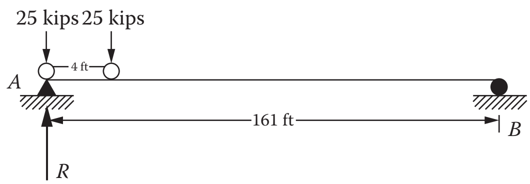

FIGURE 3.22
Design tandem load position for maximum shear.

Step 2: Determine Live Load Shears and Moments Using the AASHTO HL-93 Load.

2.1 Determine Live Load Shears Due to HL-93 Loads

Please see Figure 3.21.

$$\Sigma M_B = 0$$

$$(32\text{ kips})(161\text{ ft}) + (32\text{ kips})(161\text{ ft} - 14\text{ ft}) + (8\text{ kips})(161\text{ ft} - 28\text{ ft}) - R(161\text{ ft}) = 0$$

$$R = 67.8\text{ kips}$$

$$V_{truck} = 67.8\text{ kips [controls]}$$

Please see Figure 3.22.

$$\Sigma M_B = 0$$

$$(25\text{ kips})(161\text{ ft}) + (25\text{ kips})(161\text{ ft} - 4\text{ ft}) - R(161\text{ ft}) = 0$$

$$R = 49.4\text{ kips}$$

$$V_{tandem} = 49.4\text{ kips}$$

Please see Figure 3.23.

$$\Sigma M_B = 0$$

$$(0.640\text{ kips/ft})(161\text{ ft})(1/2) - R(161\text{ ft}) = 0$$

$$R = 67.8\text{ kips}$$

$$V_{lane} = 51.5\text{ kips}$$

2.2 Determine Live Load Moments Due to HL-93 Loads

Please see Figure 3.24 for design truck moment.

$$\Sigma M_B = 0$$

$$(32\text{ kips})(80.5\text{ ft} + 14\text{ ft}) + (32\text{ kips})(80.5\text{ ft}) + (8\text{ kips})(80.5\text{ ft} - 14\text{ ft}) - R(161\text{ ft}) = 0$$

$$R = 38.09\text{ kips}$$

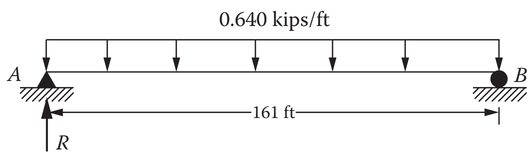

FIGURE 3.23
Design lane load position for maximum shear.

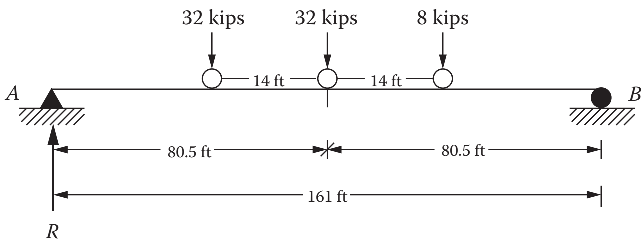

FIGURE 3.24
Design truck load (HS-20) position for maximum moment.

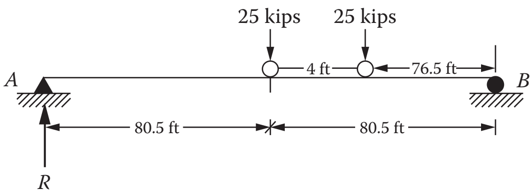

FIGURE 3.25
Design tandem load position for maximum moment.

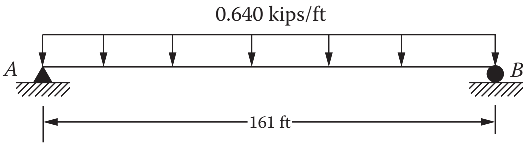

FIGURE 3.26
Design lane load position for maximum moment.

$$M_{truck} = (38.09 \text{ kips})(80.5 \text{ ft}) - (32 \text{ kips})(14 \text{ ft})$$

$$M_{truck} = 2618.2 \text{ kip-ft [controls]}$$

Please see Figure 3.25 for design tandem load moment.

$$\Sigma M_B = 0$$

$$(25 \text{ kips})(80.5 \text{ ft}) + (25 \text{ kips})(80.5 \text{ ft} - 4 \text{ ft}) - R(161 \text{ ft}) = 0$$

$$R = 24.38 \text{ kips}$$

$$M_{tandem} = (24.38 \text{ kips})(80.5 \text{ ft})$$

$$M_{tandem} = 1962.6 \text{ kip-ft}$$

Please see Figure 3.26 for design lane moment.

$$M_{lane} = \frac{wL^2}{8} = \frac{\left(0.640 \frac{\text{kips}}{\text{ft}}\right)(161 \text{ ft})^2}{8}$$

$$M_{lane} = 2073.7 \text{ kip-ft}$$

Please see Table 3.4, summary of live load shears and moments.

TABLE 3.4
Summary of Live Load Shears and Moments

	Truck Load Effect	Tandem Load Effect	Lane Load Effect
Shear, kips	67.8	49.4	51.5
Moment, kip-ft	2618.2	1962.6	2073.7

Step 3: Determine Live Load Distribution Factors

A Art. 4.6.2.2.2; Tbl. 4.6.2.2.1-1

3.1 For Moment in Interior Beam, DFM

A Art. 4.6.2.2.2b

Longitudinal stiffness parameter K_g is,

A Eq. 4.6.2.2.1-1

$$K_g = n\left(I + A e_g^2\right)$$

where:

$$n = \frac{E_s}{E_c}$$

E_s = modulus of elasticity of steel = 29 x 10^3 kips/in^2
E_c = modulus of elasticity of deck concrete material

A Comm., Eq. 5.4.2.4-1

$$E_c = 1820\sqrt{f_c'} \quad \text{for } w_c = 0.145 \text{ kips/ft}^3$$

$$= 1820\sqrt{5\frac{\text{kips}}{\text{in}^2}}$$

$$= 4069.6\frac{\text{kips}}{\text{in}^2}$$

e_g = distance between the centers of gravity of the beam and deck
e_g = 40.12 in + 2 in + 4.75 in
e_g = 46.87 in

$$n = \frac{29\times10^3}{4.0696\times10^3} = 7.13$$

Use n = 7.0.

A = area of basic beam section
A = 106.0 in^2 (given)
I = moment of inertia of basic beam section
I = 99,734.0 in^4 (given)
K_g = 7.0 [99734.0 in^4 + (106 in^2)(46.87 in)^2]
= 2.33 x 10^6 in^4

Distribution of live loads per lane for moment in interior beams with cross-section type (a).

A Tbl. 4.6.2.2.1-1, 4.6.2.2.2b-1 or Appendix A

One design lane loaded:

$$(m)(DFM_{si}) = DFM_{si} = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12\,Lt_s{}^3}\right)^{0.1}$$

where:

m = multiple presence factors have been included in the approximate equations for distribution factors. They are applied where the lever rule is used.

A Art. 3.6.1.1.2; Tbl. 3.6.1.1.2-1

si = single lane loaded, interior girder
DFM = moment distribution factor
S = spacing of beams (ft)
L = span length of beam (ft)
t_s = depth of concrete slab (in)

$$DFM_{si} = 0.06 + \left(\frac{13\text{ ft}}{14}\right)^{0.4}\left(\frac{13\text{ ft}}{161\text{ ft}}\right)^{0.3}\left(\frac{2.33\times10^6\text{ in}^4}{12(161\text{ ft})(9.5\text{ in})^3}\right)^{0.1}$$

$$= 0.532$$

Two or more lanes loaded:

$$DFM_{mi} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12\,Lt_s^3}\right)^{0.1}$$

where:
mi = multiple lanes loaded, interior girder

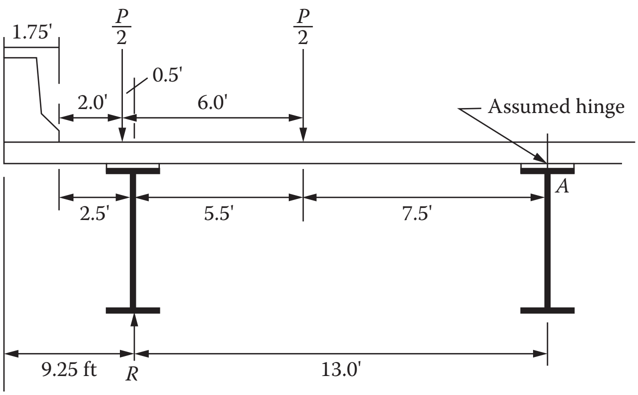

FIGURE 3.27
Lever rule for the distribution factor for moments for exterior girder.

$$DFM_{mi} = 0.075 + \left(\frac{13\text{ ft}}{9.5}\right)^{0.6}\left(\frac{13\text{ ft}}{161\text{ ft}}\right)^{0.2}\left(\frac{2.33\times10^{6}\text{ in}^{4}}{12(161\text{ ft})(9.5\text{ in})^{3}}\right)^{0.1}$$

$$= 0.83$$

3.2 Distribution of Live Load per Lane for Moment in Exterior Beams with Cross-Section Type (a)

A Tbl. 4.6.2.2.2d-1 or Appendix B

One design lane loaded:

Use lever rule. See Figure 3.27.

A Fig. C4.6.2.2.1-1

$$\Sigma M_A = 0$$

$$(P/2)(13.5\text{ ft}/2) + (P/2)(7.5\text{ ft}) - R(13\text{ ft}) = 0$$

$$R = 0.808\text{ P}$$

$$DFM_{se} = (m)(0.808)$$

$$R = (1.2)(0.808) = 0.970$$

where:
se = single lane loaded, exterior girder
m = multiple presence factor for one lane loaded = 1.2

Two or More Design Lanes Loaded:

$g = (e)(g_{interior})$ or
$DFM_{me} = (e)DFM_{mi}$

$$e = 0.77 + \frac{d_e}{9.1}$$

where:
g = DFM = distribution factors for moment
$g_{interior}$ = DFM_{mi} = distribution factor for interior girder
= 0.83
d_e = distance from the centerline of the exterior beam web to the interior edge of curb (ft)
= 2.5 ft

$$e = 0.77 + \frac{2.5 \text{ ft}}{9.1} = 1.045 \text{ ft}$$

$$DFM_{me} = (1.045 \text{ ft})(0.83)$$

$$= 0.867 \text{ or}$$

$$DFM_{me} = g = 0.867$$

3.3 Distribution of Live Load per Lane for Shear in Interior Beams.

A Tbl. 4.6.2.2.3a-1 or Appendix C

One design lane loaded:

$$DFV_{si} = 0.36 + \frac{S}{25.0}$$

$$= 0.36 + \frac{13 \text{ ft}}{25.0}$$

$$= 0.88$$

where:
DFV = shear distribution factor

Two or more lanes loaded:

$$DFV_{mi} = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$$

$$= \left(0.2 + \frac{13\text{ ft}}{12} - \left(\frac{13\text{ ft}}{35}\right)^{2.0}\right)$$

$$= 1.145$$

3.4 Distribution of Live Load per Lane for Shear in Exterior Beams

A Tbl. 4.6.2.2.3b-1 or Appendix D

One design lane loaded:

Use lever rule (same as the distribution factors for moment)

$$DFV_{se} = DFM_{se}$$

$$= 0.970$$

Two or more design lanes loaded:

$$DFV_{se} = (e)(g_{interior})$$

$$g_{interior} = DFV_{mi}$$

$$DFV_{me} = (e)(DFV_{mi})$$

where:

$$e = 0.6 + \frac{d_e}{10}$$

$$e = 0.6 + \frac{2.5\text{ ft}}{10}$$

$$e = 0.85$$

$$DFV_{me} = (0.85)(1.145)$$

$$= 0.973$$

TABLE 3.5
Summary of Live Load Distribution Factors

		Moment		Shear	
	Distribution Factor Equation	Interior Beam	Exterior Beam	Interior Beam	Exterior Beam
One Lane Loaded	Approximate	0.532	–	0.88	—
	Lever Rule	—	0.970	—	0.970
Two or More Lanes Loaded	Approximate	0.830	0.867	1.145	—
	Lever Rule	—	—	—	0.973
Controlling Value		0.830	0.970	1.145	0.973

Please see Table 3.5.

Step 4: Unfactored Distributed Live Load per Beam with Impact

Dynamic load allowance, IM, is equal to

33% for design truck or tandem
0% for design lane
15% for Fatigue and Fracture Limit State

A Art. 3.6.2.1

4.1 Shear

$$V_{LL+IM} = DFV\left[\left(V_{truck} \text{ or } V_{tandem}\right)\left(1+IM\right)+V_{lane}\right]$$

4.1.a Interior Beam

$$V_{LL+IM} = (1.145)\left[(67.8 \text{ kips})(1.33)+51.5 \text{ kips}\right]$$

$$V_{LL+IM} = 162.2 \text{ kips}$$

4.1.b Exterior Beam

$$V_{LL+IM} = (0.973)\left[(67.8 \text{ kips})(1.33)+51.5 \text{ kips}\right]$$

$$V_{LL+IM} = 137.8 \text{ kips}$$

TABLE 3.6

Summary of Unfactored Distributed Live Load Effects per Beam

Beams	V_{LL+IM}, kips	M_{LL+IM}, kip-ft
Interior	162.2	4611.4
Exterior	137.8	5389.2

4.2 Moment

$$M_{LL+IM} = DFM\left[\left(M_{truck} \text{ or } M_{tandem}\right)\left(1+IM\right)+M_{lane}\right]$$

4.2.a Interior Beam

$$M_{LL+IM} = 0.83\left[\left(2618.2 \text{ kip-ft}\right)\left(1.33\right)+2073.7 \text{ kip-ft}\right]$$

$$M_{LL+IM} = 4611.4 \text{ kip-ft}$$

4.2.b Exterior Beam

$$M_{LL+IM} = 0.97\left[\left(2618.2 \text{ kip-ft}\right)\left(1.33\right)+2073.7 \text{ kip-ft}\right]$$

$$M_{LL+IM} = 5389.2 \text{ kip-ft}$$

Please see Table 3.6.

Step 5: Factored Design Loads for Limit State Strength I

A Tbl. 3.4.1-1, 3.4.1-2

5.1 Shear, V_u

$$V_u = 1.25V_{DC} + 1.5V_{DW} + 1.75V_{LL+IM}$$

5.1.a Interior Beam

$$V_u = 1.25\left(196.7 \text{ kips}\right)+1.5\left(22.1 \text{ kips}\right)+1.75\left(162.2 \text{ kips}\right)$$

$$V_u = 562.9 \text{ kips}$$

5.1.b Exterior Beam

$$V_u = 1.25(196.7 \text{ kips}) + 1.5(22.1 \text{ kips}) + 1.75(137.8 \text{ kips})$$

$$V_u = 520.2 \text{ kips}$$

5.2 Moment, M_u

$$M_u = 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL+IM}$$

5.2.a Interior Beam

$$M_u = 1.25(7915.7 \text{ kip-ft}) + 1.5(891.0 \text{ kip-ft}) + 1.75(4611.4 \text{ kip-ft})$$

$$M_u = 19301.0 \text{ kip-ft}$$

5.2.b Exterior Beam

$$M_u = 1.25(7915.7 \text{ kip-ft}) + 1.5(891.0 \text{ kip-ft}) + 1.75(5389.2 \text{ kip-ft})$$

$$M_u = 20662.2 \text{ kip-ft}$$

Step 6: Fatigue II Limit State for Finite Load-Induced Fatigue Life

A Arts. 3.6.1.4, 3.6.1.1.2, 3.6.2.1

6.1 Fatigue Load

Fatigue load shall be one design truck or axles, but with a constant spacing of 30 ft between the 32 kip axles. Please see Figure 3.28.

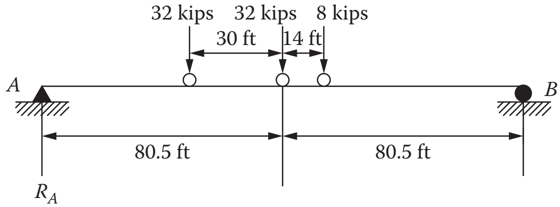

FIGURE 3.28
Fatigue load position for maximum moment at midspan.

$$\Sigma M_B = 0$$

$$(32\text{ kips})(80.5\text{ ft} + 30\text{ ft}) + (32\text{ kips})(80.5\text{ ft}) + (8\text{ kips})(80.5\text{ ft} - 14\text{ ft}) - R_A(161.0\text{ ft}) = 0$$

$$R_A = 41.27\text{ kips}$$

The maximum fatigue load moment, M_{fat}, is,

$$M_{fat} = (41.27\text{ kips})(80.5\text{ ft}) - (32\text{ kips})(30.0\text{ ft})$$

$$M_{fat} = 2362.0\text{ kip-ft}$$

The multiple presence factors have been included in the approximate equations for distribution factors in AASHTO [4.6.2.2 and 4.6.2.3], both for single and multiple lanes loaded. Where the lever rule (a sketch is required to determine load distributions) is used, the multiple presence factors must be included. Therefore, for fatigue investigations in which fatigue truck is placed in a single lane, the factor 1.2 which has been included in the approximate equations should be removed.

A Comm. 3.6.1.1.2

6.2 Fatigue Live Load Distribution Factors per Lane for Moment

The dynamic allowance for Fatigue Limit State, IM, is 15%.

A Tbl. 3.6.2.1-1

Determine distribution factors for moments, DFM_{fat},

6.2.a For Interior Beam

$$DFM^{I}_{fat} = (DFM_{si})\left(\frac{1}{m}\right)$$

$$DFM^{I}_{fat} = (0.532)\left(\frac{1}{1.2}\right)$$

$$DFM^{I}_{fat} = 0.44$$

6.2.b For Exterior Beam

$$DFM^{E}_{fat} = (DFM_{se})\left(\frac{1}{m}\right)$$

$$DFM^{E}_{fat} = (0.97)\left(\frac{1}{1.2}\right)$$

$$DFM^{E}_{fat} = 0.81$$

6.3 Unfactored Distributed Fatigue Live Load Moment per Beam with Impact

$$M_{fat+IM} = (DFM_{fat})(M_{fat})(1 + IM)$$

6.3.a Interior Beam

$$M^{I}_{fat+IM} = (DFM^{I}_{fat})(M_{fat})(1 + IM)$$

$$M^{I}_{fat+IM} = (0.44)(2362.0 \text{ kip-ft})(1 + 0.15)$$

$$M^{I}_{fat+IM} = 1195.2 \text{ kip-ft}$$

6.3.b Exterior Beam

$$M^{E}_{fat+IM} = (DFM^{E}_{fat})(M_{fat})(1 + IM)$$

$$M^{E}_{fat+IM} = (0.81)(2362.0 \text{ kip-ft})(1 + 0.15)$$

$$M^{I}_{fat+IM} = 2200.2 \text{ kip-ft}$$

6.4 Factored Fatigue II Design Live Load Moment per Beam, M_Q

A Tbl. 3.4.1-1

$$Q = 0.75\ M_{LL+IM}$$

6.4.a Interior Beam

$$M^{I}_{Q,fat} = 0.75(M^{I}_{fat+IM})$$

$$M^{I}_{Q,fat} = 0.75(1195.2 \text{ kip-ft})$$

$$M^{I}_{Q,fat} = 896.4 \text{ kip-ft}$$

6.4.b Exterior Beam

$$M^{E}_{Q,fat} = 0.75\left(M^{E}_{fat+IM}\right)$$

$$M^{I}_{Q,fat} = 0.75\left(2200.2 \text{ kip-ft}\right)$$

$$M^{I}_{Q,fat} = 1650.2 \text{ kip-ft}$$

Practice Problem 3: Interior Prestressed Concrete I-Beam

Situation

The following design specifications apply to a bridge in western Pennsylvania.

L	bridge span	80 ft
f'_{ci}	compressive strength of concrete at time of initial prestress	5.5 ksi
f'_{cg}	compressive strength of concrete at 28 days for prestressed I-beams	6.5 ksi
	number of roadway prestressed 24 × 54 in I-beams	6 beams
S	beam spacing	8 ft
t_s	roadway slab thickness	7.5 in
	integral wearing surface of slab	0.50 in
	clear roadway width	44 ft 6 in
f'_{cs}	compressive strength of roadway slab concrete at 28 days	4.5 ksi
w_{FWS}	future wearing surface dead load	0.030 kips/ft^2
w_s	= $w_{C\&P}$ curb and parapet dead load	0.506 kips/ft
A_{ps}	area of prestressing steel (0.5 in diameter low-relaxation strand; seven wire)	0.153 in^2
E_p	modulus of elasticity of prestressing steel	28,500 ksi
f_{pu}	ultimate stress of prestressing steel (stress relieved)	270 ksi

A composite deck and a standard curb and parapet are used. Assume no haunch for composite section properties and 150 lbf/ft^3 concrete for all components. The bridge cross section is shown. See Figures 3.29, 3.30, and 3.31.

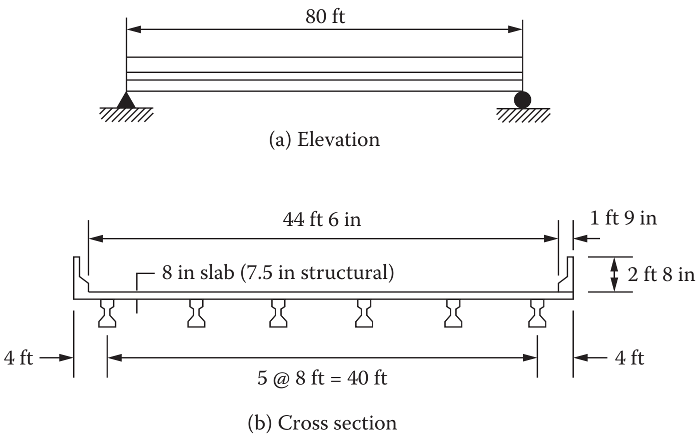

FIGURE 3.29
Prestressed concrete I-beam.

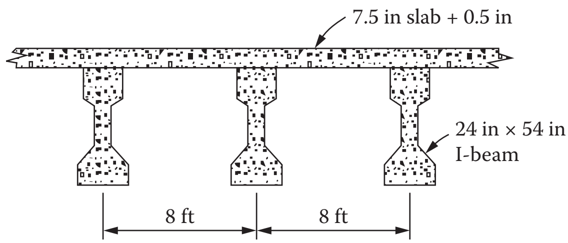

FIGURE 3.30
Deck and I-beam.

Requirement

Review the interior bonded prestressed concrete I-beam for the Load Combination Limit State Strength I.

where:

Q_i = force effect
η = load modifier
γ = load factor
$\eta_i = \eta_D \eta_R \eta_I = 1.0$

Solution

Strength I Limit State: Q = 1.0(1.25 DC + 1.50 DW + 1.75(LL + IM)). Please see Table 3.7.

A Tbl. 3.4.1-1, A Art. 1.3.3, 1.3.4, 1.3.5, 1.3.2.1

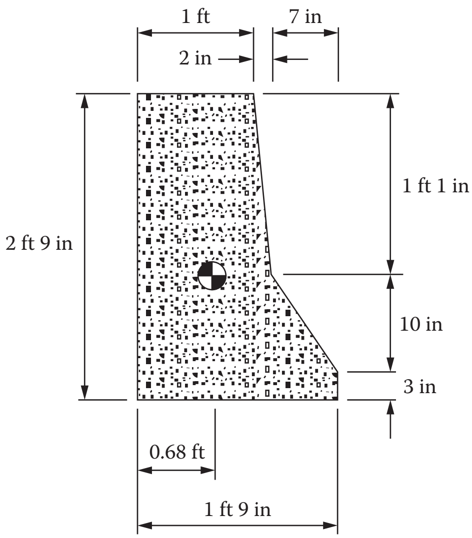

FIGURE 3.31
Curb and parapet.

TABLE 3.7
Load Modifiers

	Strength	Service	Fatigue
Ductility, η_d	1.0	1.0	1.0
Redundancy, η_r	1.0	1.0	1.0
Importance, η_i	1.0	N/A	N/A
$\eta = \eta_D \eta_R \eta_I \geq 0.95$	1.0	1.0	1.0

Step 1: Determine the Cross-Sectional Properties for a Typical Interior Beam

The properties of the basic beam section are given as follows:

A_g	area of basic beam	816 in^2
y_b	center of gravity of the basic beam from the bottom of the basic beam	25.31 in
y_t	distance from the center of gravity of the basic beam to the top of the basic beam	28.69 in
h	basic beam depth	54.0 in
I_g	moment of inertia of the basic beam section about centroidal axis, neglecting reinforcement	255,194 in^4

The section moduli for the extreme fiber of the noncomposite section are

$$S_{ncb} = \frac{I_g}{y_b} = \frac{255,194 \text{ in}^4}{25.31 \text{ in}} = 10,083 \text{ in}^3$$

$$S_{nct} = \frac{I_g}{h - y_b} = \frac{255,194 \text{ in}^4}{54.0 \text{ in} - 25.31 \text{ in}} = 8895 \text{ in}^3$$

The unit weight of concrete, w_c, is 0.150 kips/ft^3.

A Tbl. 3.5.1-1

The modulus of elasticity of concrete at transfer is

A Eq. 5.4.2.4-1

$$E_{ci} = 33,000\, w_c^{1.5}\sqrt{f'_{ci}} = (33,000)\left(0.150 \frac{\text{kips}}{\text{ft}^3}\right)^{1.5}\sqrt{5.5 \text{ ksi}} = 4496 \text{ ksi}$$

The modulus of elasticity for the prestressed I-beam concrete at 28 days is

$$E_{cg} = 33,000\, w_c^{1.5}\sqrt{f'_{cg}} = (33,000)\left(0.150 \frac{\text{kips}}{\text{ft}^3}\right)^{1.5}\sqrt{6.5 \text{ ksi}} = 4888 \text{ ksi}$$

The modulus of elasticity of the roadway slab concrete at 28 days is

$$E_{cs} = 33,000\, w_c^{1.5}\sqrt{f'_{cs}} = (33,000)\left(0.150 \frac{\text{kips}}{\text{ft}^3}\right)^{1.5}\sqrt{4.5 \text{ ksi}} = 4067 \text{ ksi}$$

Please see Figure 3.32.

b_w	web width	8 in
b_{top}	top flange width	18 in

The effective flange width for interior beams is

A Art. 4.6.2.6.1

$$b_i = S = 8 \text{ ft} = 96 \text{ in}$$

The modular ratio of the slab and beam is

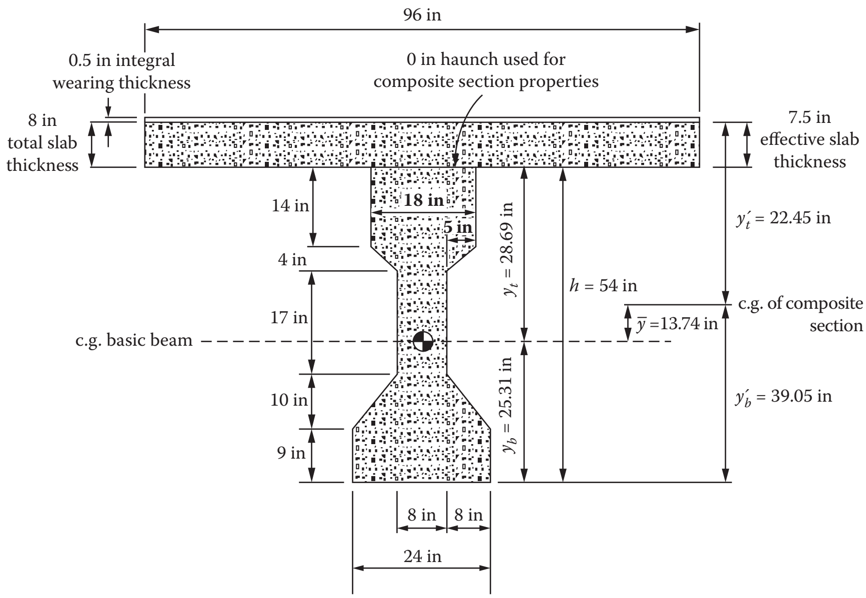

FIGURE 3.32
Composite section.

$$n = \frac{E_{cs}}{E_{cg}} = \frac{4067 \text{ ksi}}{4888 \text{ ksi}} = 0.832$$

Therefore, the transformed flange width is

$$(b_i)(n) = (96 \text{ in})(0.832) = 79.9 \text{ in}$$

Transformed flange area is

$$(b_i n)(t_s) = (79.9 \text{ in})(7.5 \text{ in}) = 599.25 \text{ in } (600.0 \text{ in}^2)$$

The area of transformed gross composite section, A_{gc}, is

$$A_{gc} = A_g + (b_i)(n)(t_s)$$

$$= 816 \text{ in}^2 + 600.0 \text{ in}^2$$

$$= 1416 \text{ in}^2$$

Use the transformed flange area to compute composite section properties by summing moments of areas about the centroid of the basic beam section.

$$A_{gc}\left(\bar{y}\right) = \left(1416\ \text{in}^2\right)\bar{y} = \left(600\ \text{in}^2\right)\left(y_t + \frac{7.5\ \text{in}}{2}\right)$$

$$= \left(600\ \text{in}^2\right)\left(28.69\ \text{in} + 3.75\ \text{in}\right)$$

$$= 19{,}464\ \text{in}^3$$

$\bar{y}$ = 13.74 in from the centroid of the basic beam.

Thus, for the composite section,

$$y'_b = y_b + \bar{y} = 25.31\ \text{in} + 13.74\ \text{in} = 39.05\ \text{in}$$

$$y'_t = \left(h + t_s\right) - y'_b = \left(54.0\ \text{in} + 7.5\ \text{in}\right) - 39.05\ \text{in}$$

$$= 22.45\ \text{in}$$

The composite moment of inertia about the centroid of the composite section is

$$I_c = I_g + A_g\bar{y}^2 + \frac{b_i n t_s^3}{12} + \left(600\ \text{in}^2\right)\left(y'_t - \frac{t_s}{2}\right)^2$$

$$= 255{,}194\ \text{in}^4 + \left(816\ \text{in}^2\right)\left(13.74\ \text{in}\right)^2 + \frac{\left(96\ \text{in}\right)\left(0.832\right)\left(7.5\ \text{in}\right)^3}{12}$$

$$+ \left(600\ \text{in}^2\right)\left(22.45\ \text{in} - \frac{7.5\ \text{in}}{2}\right)^2$$

$$= 621{,}867\ \text{in}^4$$

The section modulus of the composite section for the bottom extreme fiber is

$$S_{cb} = \frac{I_c}{y'_b} = \frac{621{,}867\ \text{in}^4}{39.05\ \text{in}} = 15{,}925\ \text{in}^3$$

The section modulus of the composite section for the top extreme fiber is

$$S_{ct} = \frac{I_c}{y'_t} = \frac{621{,}867\ \text{in}^4}{22.45\ \text{in}} = 27{,}700\ \text{in}^3$$

Step 2: Perform the Dead Load Analysis

The beam weight is

$$w = \left(816\ \text{in}^2\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)^2\left(0.15\frac{\text{kips}}{\text{ft}^3}\right) = 0.85\ \text{kips/ft}$$

The dead weight of the slab is

$$w_D = \left(8\ \text{ft}\right)\left(0.15\frac{\text{kips}}{\text{ft}^3}\right)\left(7.5\ \text{in}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = 0.75\ \text{kips/ft}$$

$$w_{DC1} = w + w_D$$

$$= 0.85\ \text{kips/ft} + 0.75\ \text{kips/ft} = 1.60\ \text{kips/ft}$$

The superimposed dead load, w_s, consists of the parapet and curb loads, distributed equally to the 6 beams.

$$w_s = \left(\frac{\left(0.506\frac{\text{kips}}{\text{ft}}\right)(2)}{6\ \text{beams}}\right) = 0.169\ \text{kips/ft}$$

$$w_s = w_{DC2} = 0.169\ \text{kips/ft}$$

$$w_{DC} = 1.60\ \text{kips/ft} + 0.169\ \text{kips/ft} = 1.769\ \text{kips/ft}$$

The future wearing surface dead load, w_{FWS}, is assumed to be equally distributed to each girder.

$$w_{FWS} = \left(8\ \text{ft}\right)\left(0.03\frac{\text{kips}}{\text{ft}}\right) = 0.240\ \text{kips/ft}$$

$$w_{FWS} = w_{DW} = 0.240\ \text{kips/ft}$$

The maximum dead load moments and shears are

$$M_{max} = \frac{wL^2}{8} = \frac{w\left(80\ \text{ft}\right)^2}{8} = 800\ w\ \text{ft-kips}$$

$$V_{max} = \frac{wL}{2} = \frac{w\left(80\ \text{ft}\right)}{2} = 40\ w\ \text{ft-kips}$$

TABLE 3.8

Dynamic Load Allowance, IM

Component	IM (%)
Deck joints, all limit states	75
Fatigue and fracture limit state	15
All other limit states	33

Step 3: Calculate the Live Load Force Effects for Moment and Shear.

Where w is the clear roadway width between curbs and/or barriers. The number of lanes is

A Art. 3.6.1.1.1

$$N_L = \frac{w}{12} = \frac{48\text{ ft}}{12} = 4 \ \ (4\text{ lanes})$$

A Tbl. 3.6.2.1-1

Please see Table 3.8.

The distance between the centers of gravity of the basic beam and the deck is

$$e_g = y_t + \frac{t_s}{2} = 28.69\text{ in} + \frac{7.5\text{ in}}{2} = 32.44\text{ in}$$

The area of the basic beam, A, is 816 in^2.

The moment of inertia of the basic beam, I_g, is 255,194 in^4.

The modular ratio between the beam and slab is

$$n = \frac{E_{cg}}{E_{cs}} = \frac{4888\text{ ksi}}{4067\text{ ksi}} = 1.2$$

The longitudinal stiffness parameter is

A Eq. 4.6.2.2.1-1

$$K_g = n(I_g + Ae_g^2) = (1.2)(255{,}194\text{ in}^4 + (816\text{ in}^2)(32.44\text{ in})^2)$$

$$= 1{,}336{,}697.4$$

Distribution factor for moments per lane in interior beams with concrete deck,

A Art. 4.2.2.2b

$$\frac{K_g}{12\,Lt_s^3} = \frac{1{,}336{,}697.4}{12(80\text{ ft})(7.5\text{ in})^3} = 3.30$$

Common deck type (j)

A Tbl. 4.6.2.2.1-1

The multiple presence factors apply to the lever rule case only. They have been included in the approximate equations for distribution factors in Arts. 4.6.2.2 and 4.6.2.3 for both single and multiple lanes loaded.

A Comm. 3.6.1.1.2

The distribution factor for moment for interior beams per lane with one design lane loaded is

A Tbl. 4.6.2.2.2b-1 or Appendix A

$$\text{DFM}_{si} = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12\,Lt_s^3}\right)^{0.1}$$

$$= 0.06 + \left(\frac{8\text{ ft}}{14}\right)^{0.4}\left(\frac{8\text{ ft}}{80\text{ ft}}\right)^{0.3}(3.30)^{0.1}$$

$$= 0.51 \text{ lanes per beam}$$

The distribution factor for moment for interior beams with two or more design lanes loaded is

$$\text{DFM}_{mi} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12\,Lt_s^3}\right)^{0.1}$$

$$= 0.075 + \left(\frac{8\text{ ft}}{9.5}\right)^{0.6}\left(\frac{8\text{ ft}}{80\text{ ft}}\right)^{0.2}(3.30)^{0.1}$$

$$= 0.716 \text{ lanes per beam [controls]}$$

The distribution factor for shear for interior beams (one design lane loaded) is

A Art. 4.6.2.2.3; Tbl. 4.6.2.2.3a-1 or Appendix C

$$DFV_{si} = 0.36 + \left(\frac{S}{25}\right) = 0.36 + \left(\frac{8.0\text{ ft}}{25}\right)$$

$$= 0.68 \text{ lanes per beam}$$

The distribution factor for shear for interior beams (two or more design lanes loaded) is

$$DFV_{mi} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^2 = 0.2 + \left(\frac{8.0\text{ ft}}{12}\right) - \left(\frac{8\text{ ft}}{35}\right)^2$$

$$= 0.814 \text{ lanes per beam [controls]}$$

Step 4: Determine the Maximum Live Load Moments and Shears

Approximate the maximum bending moment at midspan due to the HL-93 loading. Please see Figures 3.33 through 3.36.

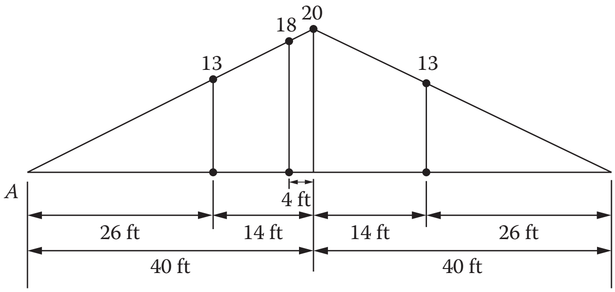

FIGURE 3.33
Influence line diagram for maximum moment at midspan.

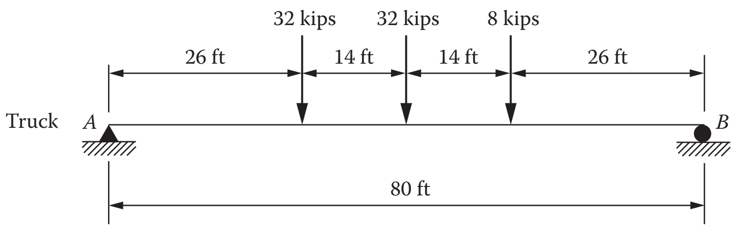

FIGURE 3.34
Design truck (HS-20) position for moment at midspan.

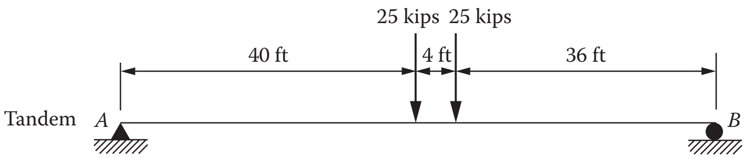

FIGURE 3.35
Design tandem load position for moment at midspan.

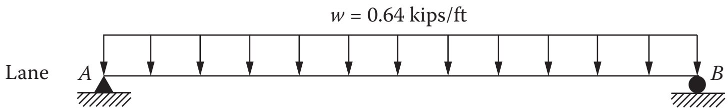

FIGURE 3.36
Design lane load for moment at midspan.

Using the influence line diagram,

$$M_{tr} = (32 \text{ kips})(20 \text{ ft}) + (32 \text{ kips} + 8 \text{ kips})(13 \text{ ft})$$

$$= 1160 \text{ ft-kips [controls]}$$

$$M_{tandem} = (25 \text{ kips})(20 \text{ ft} + 18 \text{ ft}) = 950 \text{ ft-kips}$$

$$M_{ln} = \frac{\left(0.64\,\frac{\text{kips}}{\text{ft}}\right)(80 \text{ ft})^2}{8} = 512 \text{ ft-kips}$$

The maximum live load plus impact moment is defined by the following equation.

$$M_{LL+IM} = DFM_{mi}\left(\left(M_{tr} \text{ or } M_{tandem}\right)\left(1+\frac{IM}{100}\right)+M_{ln}\right)$$

$$= (0.716)((1160 \text{ ft-kips})(1 + 0.33) + 512 \text{ ft-kips})$$

$$= 1471.2 \text{ ft-kips}$$

For the approximate maximum shear due to HL-93 loading, please see Figures 3.37 through 3.40, and use the influence line diagram.

$$V_{tr} = (32 \text{ kips})(1 + 0.825) + (8 \text{ kips})(0.65) = 63.60 \text{ kips [controls]}$$

$$V_{tandem} = (25 \text{ kips})(1 + 0.95) = 48.75 \text{ kips}$$

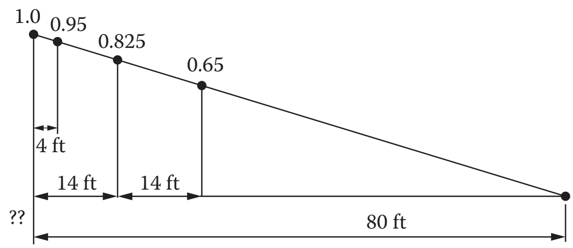

FIGURE 3.37
Influence line diagram for maximum shear at support.

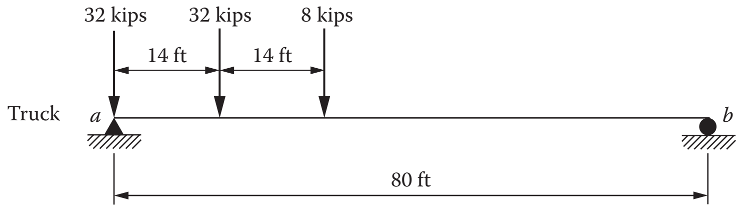

FIGURE 3.38
Design truck position for shear at support.

FIGURE 3.39
Design tandem load position for shear at support.

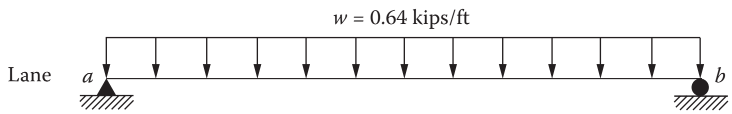

FIGURE 3.40
Design lane load for shear at support.

$$V_{ln} = \frac{\left(0.64\,\dfrac{\text{kips}}{\text{ft}}\right)(80\text{ ft})}{2} = 25.6\text{ kips}$$

The maximum live load plus impact shear is defined by the following equation:

$$V_{LL+IM} = DFV\left(\left(V_{tr} \text{ or } V_{tan\,dem}\right)\left(1+\frac{IM}{100}\right)+V_{ln}\right)$$

$$= (0.814)((63.6 \text{ kips})(1 + 0.33) + 25.6 \text{ kips}) = 89.69 \text{ kips}$$

Interior Beam Summary (using the maximum dead and live/impact load, shear, and moment equations)

Note that $DC = DC_1 + DC_2$ or $w_{DC} = w_{DC1} + w_{DC2}$

$$V_{DC} = V_{max} = 40\ w_{DC} = (40)(1.769 \text{ kips/ft}) = 70.76 \text{ kips}$$

$$M_{DC} = M_{max} = 800\ w_{DC} = (800)(1.769 \text{ kips/ft}) = 1415.2 \text{ ft-kips}$$

Due to the slab and the beam weight,

$$V_{DC1} = 40\ w_{DC1} = (40)(1.60 \text{ kips/ft}) = 64.0 \text{ kips}$$

$$M_{DC1} = 800\ w_{DC1} = (800)(1.60 \text{ kips/ft}) = 1280 \text{ ft-kips}$$

Due to superimposed dead loads – parapet and curb loads,

$$V_{DC2} = 40\ w_{DC2} = (40)(0.169 \text{ kips/ft}) = 6.76 \text{ kips}$$

where:
DC_1 = beam weight + slab weight = 0.85 kips/ft + 0.75 kips/ft = 1.60 kips/ft
DC_2 = superimposed parapet and curb loads = 0.169 kips/ft
$M_{DC2} = 800\ w_{DC2} = (800)(0.169 \text{ kips/ft}) = 135.2$ ft-kips

Due to the wearing surface weight,

$$V_{DW} = 40\ w_{DW} = (40)(0.240 \text{ kips/ft}) = 9.60 \text{ kips}$$

$$M_{DW} = 800\ w_{DW} = (800)(0.240 \text{ kips/ft}) = 192.0 \text{ ft-kips}$$

Please see Table 3.9.
Recall that:

w = beam weight = 0.85 kips/ft
w_D = slab weight = 0.75 kips/ft
$w_{DC1} = w + w_D$ = weights of beam and slab = 1.60 kips/ft
M_{DC1} = 1280.0 ft-kips

TABLE 3.9

Summary of Dead Load Moments and Shears

Load Type	w (kips/ft)	M (ft-kips)	V (kips)
DC	1.769	1,415.2	70.76
DC_1	1.60	1,280.0	64.0
DC_2	0.169	135.2	6.76
DW	0.240	192.0	9.60
LL + IM	N/A	1,471.2	89.69

V_{DC1} = 64.0 kips

w_s = w_{DC2} = superimposed dead loads (parapet and curb) = 0.169 kips/ft

M_{DC2} = 135.2 ft-kips

V_{DC2} = 6.76 kips

w_{DC} = $w_{DC1} + w_{DC2}$ = weight of beam and slab plus parapet and curb = 1.769 kips/ft

M_{DC} = 1415.2 ft-kips

V_{DC} = 70.76 kips

w_{DW} = w_{FWS} = future wearing surface = 0.24 kips/ft

M_{DW} = 192.0 ft-kips

V_{DW} = 9.6 kips

S_{ncb} = section modulus of noncomposite section for the bottom extreme fiber

S_{nct} = section modulus of noncomposite section for the top extreme fiber

S_{cb} = section modulus of composite section for the bottom extreme fiber

S_{ct} = section modulus of composite section for the top extreme fiber

Step 5: Estimate the Required Prestress

Calculate the bottom tensile stress in prestressed concrete, f_{gb}, noncomposite, and composite properties.

Service III Limit State governs for longitudinal analysis relating to tension in prestressed concrete superstructures with the objective of crack control and to principal tension in the webs of segmental concrete girders.

A Art. 3.4.1

Service III Limit State

A Tbl. 3.4.1-1

$$Q = \eta((1.0)(DC + DW) + (0.8)(LL + IM))$$

$$\eta = 1.0$$

A Art. 1.3.2

$$f_{gb} = \frac{M_{DC1}}{S_{ncb}} + \frac{M_{DC2} + M_{DW} + 0.8(M_{LL+IM})}{S_{cb}}$$

$$= \frac{(1280.0 \text{ ft-kips})\left(\dfrac{12 \text{ in}}{\text{ft}}\right)}{10,083 \text{ in}^3}$$

$$+ \frac{135.2 \text{ ft-kips} + 192 \text{ ft-kips} + 0.8(1471.2 \text{ ft-kips})}{15,925 \text{ in}^3}$$

$$= 2.66 \ (2.7 \text{ ksi})$$

Tensile stress limit at service limit state after losses, fully pretensioned is

A Art. 5.9.4.2.2; Tbl. 5.9.4.2.2-1

$$0.19\sqrt{f'_{cg}} = 0.19\sqrt{6.5 \text{ ksi}} = 0.484 \text{ ksi}$$

The excess tension in the bottom fiber due to applied loads is

$$f_t = f_{gb} - 0.19\sqrt{f'_{cg}} = 2.66 \text{ ksi} - 0.484 \text{ ksi} = 2.18 \text{ ksi}$$

The location of the center of gravity of the strands at midspan usually ranges from 5 to 15% of the beam depth. In this example, first assume 10%.

The distance between the center of gravity of the bottom strands to the bottom fiber is assumed to be

$$y_{bs} = (0.1)(54 \text{ in}) = 5.4 \text{ in}$$

The strand eccentricity at midspan is

$$e_c = y_b - y_{bs} = 25.31 \text{ in} - 5.4 \text{ in} = 19.91 \text{ in}$$

To find the initial required prestress force, P_i,

$$f_t = \frac{P_i}{A_g} + \frac{P_i e_c}{S_{ncb}}$$

$$2.18\text{ ksi} = \frac{P_i}{816\text{ in}^2} + \frac{P_i(19.91\text{ in})}{10{,}083\text{ in}^3}$$

$$P_i = 681.2\text{ kips}$$

The stress in prestressing tendon immediately prior to transfer, f_{pbt}, is

A Art. 5.9.3

$$f_{pbt} = 0.75\, f_{pu} = (0.75)(270\text{ ksi}) = 202.5\text{ ksi}$$

A Tbl. 5.9.3-1

Assuming a 25% prestress loss, prestress force per ½ in strand after losses, f_{ps}, is

$$f_{ps} = A_{ps}f_{pbt}(1 - 0.25) = (0.153\text{ in}^2)(202.5\text{ ksi})(1 - 0.25) = 23.2\text{ kips}$$

Number of strands required is

$$\frac{P_i}{f_{ps}} = \frac{681.2\text{ kips}}{23.2\text{ kips}} = 29.4$$

Try 30, ½ in strands.

Check the assumption of bottom strand center of gravity using the following configuration. Please see Figure 3.41.

The distance between the center of gravity of the 30 strands to the bottom fiber is

$$y_{bs} = \frac{(8)(2\text{ in})+(8)(4\text{ in})+(6)(6\text{ in})+(4)(8\text{ in})+(2)(10\text{ in})+(2)(12\text{ in})}{30\text{ strands}}$$

$$= 5.33\text{ in}$$

Inasmuch as the assumed value for y_{bs} was 5.4 in, another iteration is not required.

The final strand eccentricity value is

$$e_c = y_b - y_{bs} = 25.31\text{ in} - 5.33\text{ in} = 19.98\text{ in}$$

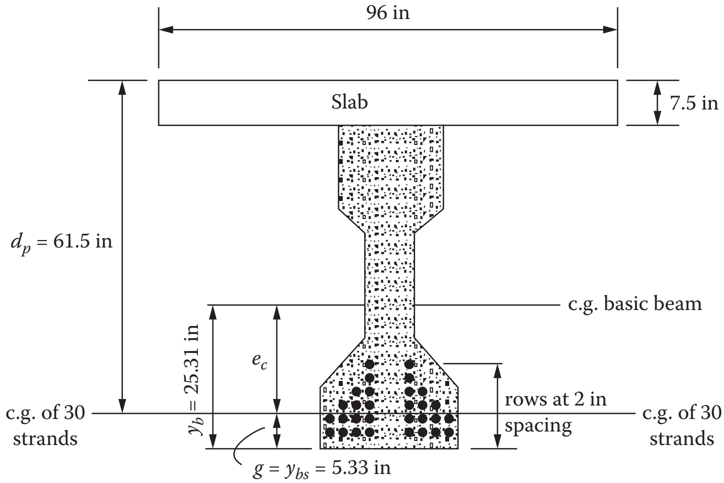

FIGURE 3.41
Prestressed I-beam with 30, ½ in strands.

Revised initial required prestress force using 30 strands as opposed to 29.4 strands.

$$\text{number of strands} = \frac{P_i}{f_{ps}}$$

$$30 \text{ strands} = \frac{P_i}{23.2 \text{ kips}}$$

$$P_i = 696 \text{ kips}$$

Area of prestressing steel is

$$(30 \text{ strands})(0.153 \text{ in}^2) = 4.59 \text{ in}^2$$

Step 6: Calculate the Prestress Losses

In pretensioned members, $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$

A Eq. 5.9.5.1-1

Δf_{pT}	total loss	kips/in²
Δf_{pES}	sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads	
Δf_{pLT}	losses due to long-term shrinkage and creep of concrete, and relaxation of the steel	kips/in²

Temporary allowable concrete stresses before losses due to creep and shrinkage are as follows (i.e., at time of initial prestress).
Find the loss due to elastic shortening, Δf_{pES}.

A Eq. 5.9.5.2.3a-1

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

f_{cgp} is the concrete stress at the center of gravity of prestressing tendons due to prestressing force immediately after transfer and self-weight of member at the section of maximum moment.

f_{cgp} is found using an iterative process. Alternatively, the following calculation can be used to find Δf_{pES} for loss in prestressing steel due to elastic shortening.

A Eq. C5.9.5.2.3a-1

$$\Delta f_{pES} = \frac{A_{ps} f_{pbt}\left(I_g + e_m^2 A_g\right) - e_m M_g A_g}{A_{ps}\left(I_g + e_m^2 A_g\right) + \dfrac{A_g I_g E_{ci}}{E_p}}$$

A_{ps}	area of prestressing steel	4.59 in^2
A_g	gross area of section	816 in^2
E_{ci}	modulus of elasticity of concrete at transfer	4496 ksi
E_p	modulus of elasticity of prestressing tendons	28,500 ksi
$e_m = e_c$	average prestressing steel eccentricity at midspan	19.98 in
I_g	moment of inertia of the basic beam section	255,194 in^4

The midspan moment due to beam self weight is

$$M_g = \frac{wL^2}{8} = \frac{\left(0.85 \dfrac{\text{kips}}{\text{ft}}\right)(80\ \text{ft})^2}{8}$$

$$= 680\ \text{ft-kips}$$

The loss due to elastic shortening is

A Eq. C5.9.5.2.3a-1

$$\Delta f_{pES} = \frac{A_{ps} f_{pbt}\left(I_g + e_m^2 A_g\right) - e_m M_g A_g}{A_{ps}\left(I_g + e_m^2 A_g\right) + \dfrac{A_g I_g E_{ci}}{E_p}}$$

$$= \frac{\left(4.59\text{ in}^2\right)\left(202.5\text{ ksi}\right)\left(255{,}194\text{ in}^4 + \left(19.98\text{ in}\right)^2\left(816\text{ in}^2\right)\right) - \left(19.98\text{ in}\right)\left(680\text{ ft-kips}\right)\left(\dfrac{12\text{ in}}{1\text{ ft}}\right)\left(816\text{ in}^2\right)}{\left(4.59\text{ in}^2\right)\left(255{,}194\text{ in}^4 + \left(19.98\text{ in}\right)^2\left(816\text{ in}^2\right)\right) + \dfrac{\left(816\text{ in}^2\right)\left(255{,}194\text{ in}^4\right)\left(4496\text{ ksi}\right)}{28{,}500\text{ ksi}}}$$

$$= 11.46\text{ ksi}$$

Calculate the long-term prestress loss due to creep of concrete shrinkage of concrete, and relaxation of steel, Δf_{pLT}.

A Art. 5.9.5.3; Eq. 5.9.5.3-1

Where:

f_{pi} = f_{pbt} = stress in prestressing steel immediately prior to transfer
Δf_{pR} = relaxation loss

Δf_{pR} is an estimate of relaxation loss taken as 2.4 ksi for low relaxation strand, 10.0 ksi for stress relieved strand, and in accordance with manufacturer's recommendation for other types of strand.

A Art. 5.9.5.3

γ_h = correction factor for relative humidity

$\gamma_h = 1.7 - 0.01\text{ H} = 1.7 - (0.01)(0.72) = 1.69$

A Eq. 5.9.5.3-2

The average annual ambient relative humidity, H, in western Pennsylvania is approximately 72%.

A Fig. 5.4.2.3.3-1

The correction factor for specified concrete strength at the time of the prestress transfer to the concrete is,

A Eq. 5.9.5.3-3

$$\gamma_{st} = \frac{5}{\left(1+f'_{ci}\right)} = \frac{5}{\left(1+5.5\right)}$$

$$= 0.77$$

$$\Delta f_{pLT} = \left(10.0\right)\frac{f_{pi}A_{ps}}{A_g}\gamma_h\gamma_{st} + \left(12.0\right)\gamma_h\gamma_{st} + \Delta f_{pR}$$

A Eq. 5.9.5.3-1

$$= \left(10.0\right)\left(\frac{\left(202.5 \text{ ksi}\right)\left(4.59 \text{ in}^2\right)}{816 \text{ in}^2}\right)\left(1.69\right)\left(0.77\right) + \left(12\right)\left(1.69\right)\left(0.77\right) + 2.4 \text{ ksi}$$

$$= 32.84 \text{ ksi}$$

The total prestress losses are

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} = 11.46 \text{ ksi} + 32.84 \text{ ksi} = 44.30 \text{ ksi}$$

A Eq. 5.9.5.1-1

Summary of Prestress Forces

The prestress stress per strand before transfer f_{pbt} is

A Tbl. 5.9.3-1

$$f_{pbt} = 0.75\, f_{pu}$$

$$= \left(0.75\right)\left(270\frac{\text{kips}}{\text{in}^2}\right) = 202.5\frac{\text{kips}}{\text{in}^2}$$

The prestress force per strand before transfer is

$$P_{pi} = (f_{pbt})A_{ps}$$

$$= (202.5 \text{ ksi})(0.153 \text{ in}^2) = 31 \text{ kips}$$

The prestress force per strand after all losses is

$$P_{pe} = (202.5 \text{ ksi} - 44.30 \text{ ksi})(0.153 \text{ in}^2) = 24.20 \text{ kips}$$

Check assumption of 25% prestress losses.

$$\%\text{loss} = 1.0 - \frac{\text{prestress force after all losses}\left(= P_{pe}\right)}{\text{prestress force before transfer}\left(= P_{pi}\right)} = 1 - \frac{24.20\text{ kips}}{31\text{ kips}} = 21.9\%$$

Assumption of 25% was conservative because 21.9% < 25% [OK].

Step 7: Check Tensile and Compressive Concrete Stresses at Transfer

The compressive stress of concrete at time of prestressing before losses is

A Art. 5.9.4.1.1

$$f_{ci} = 0.60\, f'_{ci} = (0.6)(5.5\text{ ksi}) = 3.3\text{ ksi}$$

The tensile stress in prestressed concrete before losses is

A Art. 5.9.4.1.2; Tbl. 5.9.4.1.2-1

$$f_{ti} = 0.24\sqrt{f'_{ci}} = 0.24\sqrt{5.5\text{ ksi}} = 0.563\text{ ksi}$$

Stress at bottom of girder (compressive stress)

$$f_{bot} = -\frac{P_i}{A_g} - \frac{P_i e_c}{S_{ncb}} + \frac{M_g}{S_{ncb}}$$

$$= -\frac{696\text{ kips}}{816\text{ in}^2} - \frac{\left(696\text{ kips}\right)\left(19.98\text{ in}\right)}{10{,}083\text{ in}^3} + \frac{680\text{ ft-kips}}{10{,}083\text{ in}^3}\left(\frac{12\text{ in}}{\text{ft}}\right)$$

$$= -1.42\text{ ksi} < -3.3\text{ ksi [OK]}$$

Stress at top of girder (tensile stress)

$$f_{bot} = -\frac{P_i}{A_g} + \frac{P_i e_c}{S_{nct}} - \frac{M_g}{S_{nct}}$$

$$= -\frac{696\text{ kips}}{816\text{ in}^2} + \frac{\left(696\text{ kips}\right)\left(19.98\text{ in}\right)}{8895.0\text{ in}^3} - \frac{680\text{ ft-kips}}{8895.0\text{ in}^3}\left(\frac{12\text{ in}}{\text{ft}}\right)$$

$$= -0.207\text{ ksi} < -0.563\text{ ksi [OK]}$$

Step 8: Determine the Flexural Resistance Using the Strength I Limit State

Strength I Limit State, U = 1.25 DC + 1.50 DW + 1.75(LL + IM).

A Tbl. 3.4.1-1

Recall that DC is the weight due to the deck slab, the basic beam, and the parapet curb, and that DW is the weight due to the future wearing surface.

For these values, the Strength I Limit State can be written as follows:

$$M_u = (1.25)(1415.2\text{ ft-kips}) + (1.50)(192\text{ ft-kips}) + (1.75)(1471.2\text{ ft-kips})$$

$$= 4631.6\text{ ft-kips}$$

Find the average stress in prestressing steel assuming rectangular behavior.

A Eq. 5.7.3.1.1-1

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

where:

d_p = distance from extreme compression fiber to the centroid of the prestressing tendons

f_{pu} = 270 ksi

k = 0.38

A Tbl. C5.7.3.1.1-1

d_p = $(h - y_{bs}$ (= g) + $t_s)$

= (54 in – 5.33 in) + 7.5 in

= 56.17 in

$$c = \frac{(A_{ps})(f_{pu}) + A_s f_s - A'_s f'_s}{0.85\, f'_c\, \beta_1 b + k(A_{ps})\left(\dfrac{f_{pu}}{d_p}\right)}$$

A Eq. 5.7.3.1.1-4

where:

$$\beta_1 = 0.85 - 0.05(f'_{cg} - 4\text{ ksi})$$

A Art. 5.7.2.2

$$= 0.85 - 0.05(6.5\text{ ksi} - 4\text{ ksi})$$

$$= 0.725$$

$$c = \frac{\left(4.59\ \text{in}^2\right)\left(270\ \text{ksi}\right)}{0.85\left(6.5\ \text{ksi}\right)\left(0.725\right)\left(96\ \text{in}\right)+\left(0.38\right)\left(4.59\ \text{in}^2\right)\left(\dfrac{270\ \text{ksi}}{56.17\ \text{in}}\right)}$$

$$= 3.16\ \text{in} < t_s = 7.5\ \text{in}$$

so the assumption is OK.

The average stress in prestressing steel when the nominal resistance of member is required, f_{ps},

$$f_{ps} = \left(270\ \text{ksi}\right)\left(1-\left(0.38\right)\frac{3.16\ \text{in}}{56.17\ \text{in}}\right) = 264.2\ \text{ksi}$$

Find the factored flexural resistance for flanged section:

A Art. 5.7.3.2.2

$$a = \beta_1 c = (0.725)(3.16\ \text{in})$$

$$= 2.29\ \text{in}$$

The factored resistance M_r shall be taken as:

A Eq. 5.7.3.2.2

$$M_r = \Phi M_n$$

where:
M_n = nominal resistance
Φ = resistance factor = 1.0

A Art. 5.5.4.2

$M_r = (1.0)(M_n) = M_n$

A Eq. 5.7.3.2.2-1

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$M_r = M_n$$

$$M_r = \left(4.59\ \text{in}^2\right)\left(264.2\ \text{ksi}\right)\left(56.17\ \text{in} - \frac{2.29\ \text{in}}{2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$M_r = 5560\ \text{ft-kips} > M_u = 4631.6\ \text{ft-kips [OK]}$$