

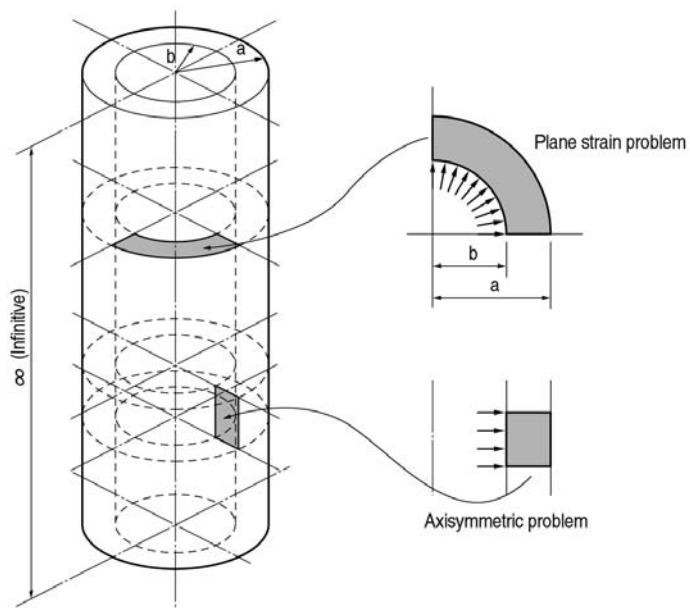
Static-25

Title

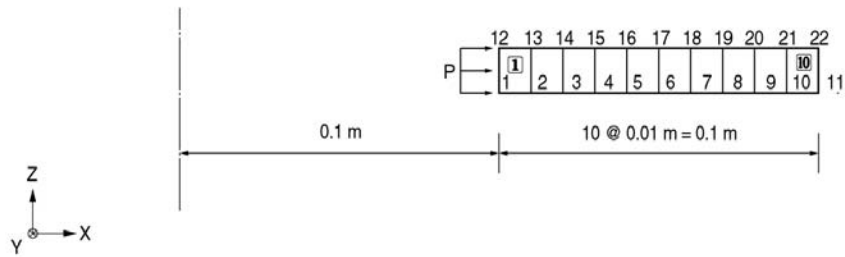
Thick cylinder subjected to an internal pressure load

Description

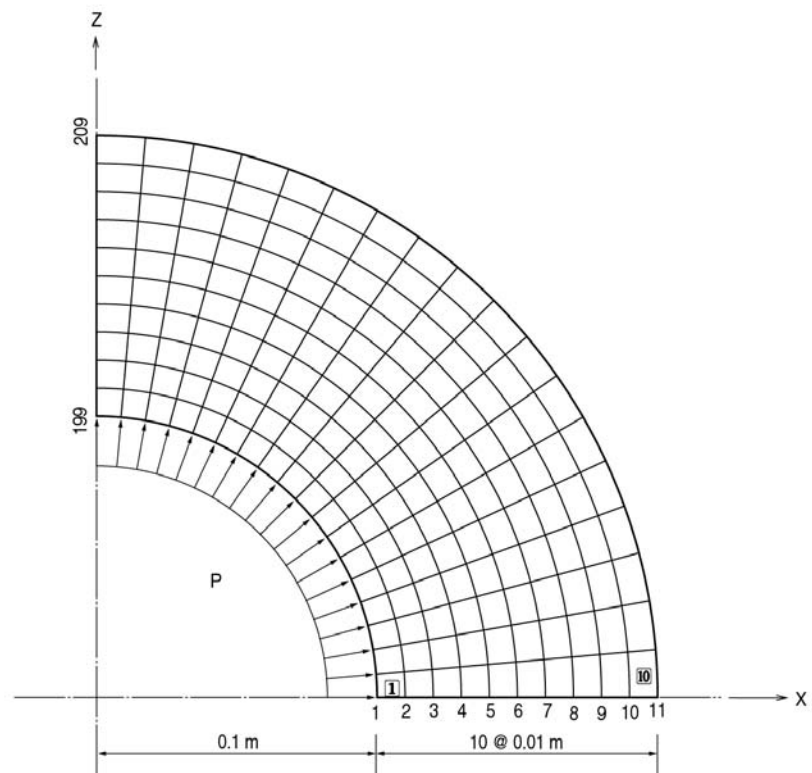
Compute the displacements and stresses of a cylinder modeled with plane strain elements and axisymmetric elements due to an internal pressure load.



Structural geometry



Structural analysis model consisted of axisymmetric elements (Static-25-1)



Structural analysis model consisted of plane strain elements (Static-25-2)

MODEL

Analysis Type

2-D static analysis (X-Z plane)

Unit System

Input : m, N Output : mm, N

Dimension

Outer radius $a = 0.2\text{m}$ Inner radius $b = 0.1\text{m}$

Element

Axisymmetric element and plane strain element

Material

Modulus of elasticity $E = 2.1 \times 10^{11} \text{ Pa}$

Characteristics of element

Possible to model only in X-Z plane, and cannot be used in conjunction with other elements.

Axisymmetric elements ; Analysis per unit radian (1 Radian) of the cylinder

Plane Strain elements ; Analysis per unit thickness

Boundary Condition

Axisymmetric elements ; Nodes 1 ~ 22 : Constrain Dz.

Plain Strain elements ; Nodes 1 ~ 11 : Constrain Dz.

Nodes 199 ~ 209 : Constrain Dx.

Load Case

Axisymmetric elements ; Pressure load, $1.0 \times 10^8 \text{ Pa}$ is applied to the element **1**.

Plane Strain elements ; Pressure load, $1.0 \times 10^8 \text{ Pa}$ is applied to the inner edge of the innermost elements.

Results

Displacements (Axisymmetric elements) : Static-25-1

	Node	Load	DX (mm)	DY (mm)	DZ (mm)	RX ([rad])	RY ([rad])	RZ ([rad])
▶	1	CASE1	0.079299	0.000000	0.000000	0.000000	0.000000	0.000000
	2	CASE1	0.075123	0.000000	0.000000	0.000000	0.000000	0.000000
	3	CASE1	0.071907	0.000000	0.000000	0.000000	0.000000	0.000000
	4	CASE1	0.069429	0.000000	0.000000	0.000000	0.000000	0.000000
	5	CASE1	0.067531	0.000000	0.000000	0.000000	0.000000	0.000000
	6	CASE1	0.066098	0.000000	0.000000	0.000000	0.000000	0.000000
	7	CASE1	0.065042	0.000000	0.000000	0.000000	0.000000	0.000000
	8	CASE1	0.064297	0.000000	0.000000	0.000000	0.000000	0.000000
	9	CASE1	0.063810	0.000000	0.000000	0.000000	0.000000	0.000000
	10	CASE1	0.063542	0.000000	0.000000	0.000000	0.000000	0.000000
	11	CASE1	0.063459	0.000000	0.000000	0.000000	0.000000	0.000000

Stresses (Axisymmetric elements) : Static-25-1

	Elem	Load	Node	Sig-xx (N/mm ²)	Sig-yy (N/mm ²)	Sig-zz (N/mm ²)	Sig-xy (N/mm ²)	Sig-P1 (N/mm ²)	Sig-P2 (N/mm ²)	Sig-P3 (N/mm ²)	Max-Shear (N/mm ²)	Sig-EFF (N/mm ²)	Sig-OCT (N/mm ²)
▶	1	CASE1	Cent	-8.769e+001	0.000e+000	1.544e+002	0.000e+000	1.544e+002	0.000e+000	-8.769e+001	1.211e+002	2.123e+002	1.001e+002
	1	CASE1	1	-8.769e+001	0.000e+000	1.661e+002	0.000e+000	1.661e+002	0.000e+000	-8.769e+001	1.269e+002	2.233e+002	1.053e+002
	1	CASE1	2	-8.769e+001	0.000e+000	1.431e+002	0.000e+000	1.431e+002	0.000e+000	-8.769e+001	1.154e+002	2.018e+002	9.511e+001
	1	CASE1	13	-8.769e+001	0.000e+000	1.431e+002	0.000e+000	1.431e+002	0.000e+000	-8.769e+001	1.154e+002	2.018e+002	9.511e+001
	1	CASE1	12	-8.769e+001	0.000e+000	1.661e+002	0.000e+000	1.661e+002	0.000e+000	-8.769e+001	1.269e+002	2.233e+002	1.053e+002
	10	CASE1	Cent	-1.738e+000	0.000e+000	6.839e+001	0.000e+000	6.839e+001	0.000e+000	-1.738e+000	3.506e+001	6.927e+001	3.265e+001
	10	CASE1	10	-1.738e+000	0.000e+000	7.020e+001	0.000e+000	7.020e+001	0.000e+000	-1.738e+000	3.597e+001	7.108e+001	3.351e+001
	10	CASE1	11	-1.738e+000	0.000e+000	6.660e+001	0.000e+000	6.660e+001	0.000e+000	-1.738e+000	3.417e+001	6.749e+001	3.181e+001
	10	CASE1	22	-1.738e+000	0.000e+000	6.660e+001	0.000e+000	6.660e+001	0.000e+000	-1.738e+000	3.417e+001	6.749e+001	3.181e+001
	10	CASE1	21	-1.738e+000	0.000e+000	7.020e+001	0.000e+000	7.020e+001	0.000e+000	-1.738e+000	3.597e+001	7.108e+001	3.351e+001

Displacements (Plane strain elements) : Static-25-2

	Node	Load	DX (mm)	DY (mm)	DZ (mm)	RX ([rad])	RY ([rad])	RZ ([rad])
▶	1	CASE1	0,079299	0,000000	0,000000	0,000000	0,000000	0,000000
	2	CASE1	0,075123	0,000000	0,000000	0,000000	0,000000	0,000000
	3	CASE1	0,071907	0,000000	0,000000	0,000000	0,000000	0,000000
	4	CASE1	0,069429	0,000000	0,000000	0,000000	0,000000	0,000000
	5	CASE1	0,067531	0,000000	0,000000	0,000000	0,000000	0,000000
	6	CASE1	0,066098	0,000000	0,000000	0,000000	0,000000	0,000000
	7	CASE1	0,065042	0,000000	0,000000	0,000000	0,000000	0,000000
	8	CASE1	0,064296	0,000000	0,000000	0,000000	0,000000	0,000000
	9	CASE1	0,063810	0,000000	0,000000	0,000000	0,000000	0,000000
	10	CASE1	0,063542	0,000000	0,000000	0,000000	0,000000	0,000000
	11	CASE1	0,063459	0,000000	0,000000	0,000000	0,000000	0,000000

Stresses (Plane strain elements) : Static-25-2

	Elem	Load	Node	Sig-xx (N/mm²)	Sig-yy (N/mm²)	Sig-xy (N/mm²)	Sig-P1 (N/mm²)	Sig-P3 (N/mm²)	Max-Shear (N/mm²)	Sig-EFF (N/mm²)	Sig-OCT (N/mm²)
▶	1	CASE1	Cent	-87,693	154,423	0,000	154,423	-87,693	121,058	212,317	100,087
	1	CASE1	1	-87,693	166,144	5,542	166,265	-87,814	127,039	223,507	105,362
	1	CASE1	2	-87,693	143,069	5,038	143,179	-87,803	115,491	201,943	95,197
	1	CASE1	13	-87,694	143,068	-5,037	143,178	-87,803	115,491	201,943	95,197
	1	CASE1	12	-87,694	166,144	-5,541	166,265	-87,814	127,040	223,508	105,363
	10	CASE1	Cent	-1,738	68,386	0,000	68,386	-1,738	35,062	69,271	32,655
	10	CASE1	10	-1,738	70,199	1,570	70,233	-1,772	36,003	71,136	33,534
	10	CASE1	11	-1,738	66,603	1,491	66,636	-1,771	34,203	67,538	31,838
	10	CASE1	22	-1,739	66,603	-1,491	66,636	-1,771	34,203	67,539	31,838
	10	CASE1	21	-1,739	70,199	-1,570	70,233	-1,773	36,003	71,136	33,534

Comparison of Results

		Unit : mm, N/mm ²		
	Division	Theoretical	NISA II	MIDAS/Civil
Inner radial displacement	Axisymmetric element	0.079365	0.079299	0.079299
	Plain strain element	0.079365	0.079259	0.079259
Outer radial displacement	Axisymmetric element	0.063492	0.063459	0.063459
	Plain strain element	0.063492	0.063439	0.063459
Circumferential stress (r=0.105)	Axisymmetric element	154.271	154.423	154.423
	Plain strain element	154.271	154.346	154.423
Circumferential stress (r=0.195)	Axisymmetric element	68.397	68.385	68.385
	Plain strain element	68.397	68.363	68.386

※ Theoretical equations :

Displacement ; Inner radial displacement $\delta = \frac{pb}{E} \left(\frac{a^2 + b^2}{a^2 - b^2} + \nu \right)$

Outer radial displacement $\delta = \frac{p}{E} \left(\frac{2ab^2}{a^2 - b^2} \right)$

Stress : Circumferential stress $\sigma = \frac{pb^2(a^2 + r^2)}{r^2(a^2 - b^2)}$

Radial stress $\sigma = -\frac{pb^2(a^2 - r^2)}{r^2(a^2 - b^2)}$

Where, p : Inner pressure a : Outer radius
 b : Inner radius r : Distance from the center

References

Warren C. Young “Roark's Formulas for Stress & Strain. 6 Edition” McGraw-Hill

“NISA II, Verification Manual Problem 2.5 ”, Version 90.0