This shift in co-ordinates can be effected by substituting in Equation (15), $(y + b)$ for y and $\left(x + \frac{l}{2}\right)$ for x. Then,

$$M_r = M' + V_L\, x - H\, y$$

and Equations (16) become:

$$\Delta_r\, x = \int_L^R (M'\, y + V_L\, x\, y - H\, y^2)\, d\, w$$

$$\Delta_r\, y = \int_L^R (M'\, x + V_L\, x^2 - H\, x\, y)\, d\, w$$

$$\Delta_r\, \phi = \int_L^R (M' + V_L\, x - H\, y)\, d\, w$$

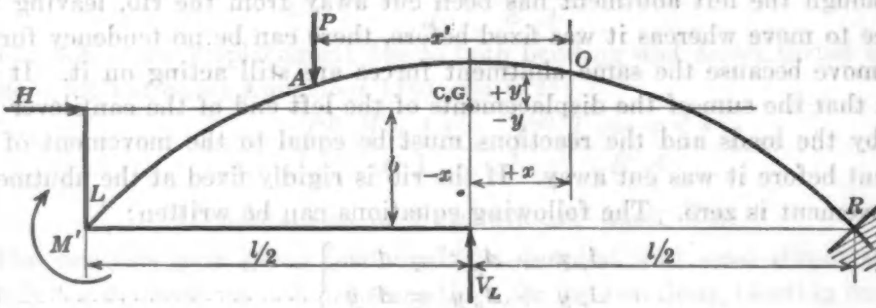

FIG. 9.

Then, as,

$$\int_L^R x\, d\, w = \int_L^R y\, d\, w = \int_L^R x\, y\, d\, w = 0$$

these equations reduce to the following:

$$\Delta_r\, x = -\int_L^R H\, y^2\, d\, w$$

$$\Delta_r\, y = \int_L^R V_L\, x^2\, d\, w$$

$$\Delta_r\, \phi = \int_L^R M'\, d\, w$$

giving the displacements produced by bending only without considering the effect of direct thrust, which in the case of the vertical reaction is so slight as to be negligible. For flat arches, however, the effect of the horizontal thrust should be considered. The values of $\Delta_r\, x$, $\Delta_p\, x$, and $\Delta'\, x$, are considered positive when the span is lengthened and negative when it is shortened.

From Equation (12) for direct thrust,

$$\Delta\, x = -\int \frac{T \cos\phi\, d\, s}{E\, A}$$

As $T = H \cos\phi$, and $d\, s = \dfrac{d\, x}{\cos\phi}$,

$$\therefore\ \Delta\, x = -\int \frac{H \cos\phi\, d\, x}{E\, A}$$