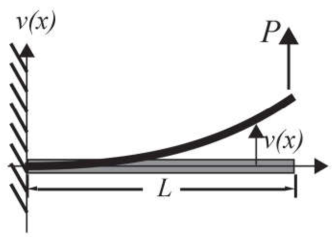
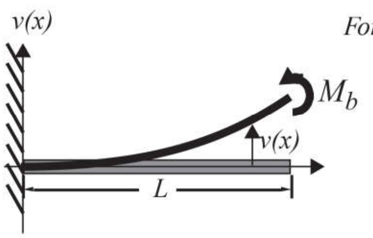
End-loaded Cantilever



For $0<x<L$ $\quad v(x) = [\,PL^3/(6EI)][3(x/L)^2-(x/L)^3]$

$v|_{max} = PL^3/(3EI)$ $\quad$ *at* $x=L$

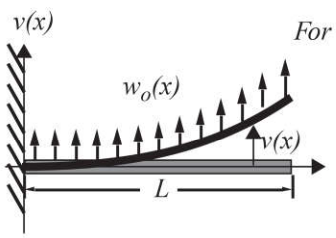
Couple, End-loaded Cantilever



For $0 < x < L$ $\quad v(x) = [\,ML^2/(2EI)]\,(x/L)^2$

$v|_{max} = ML^2/(2EI)$ $\quad$ *at* $x=L$

Uniformly Loaded Cantilever

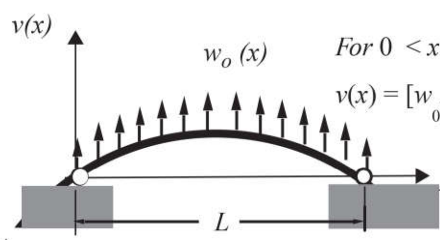


For $0 < x < L$

$v(x) = [\,w_o L^4/(24EI)]\,(x/L)^2[(x/L)^2 - 4(x/L) + 6]$

$v|_{max} = w_o L^4/(8EI)$ $\quad$ *at* $x=L$

Uniformly Loaded Simply-Supported Beam

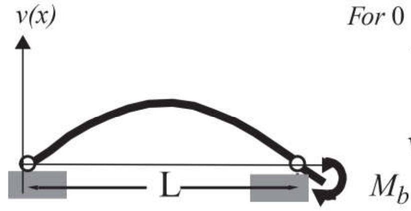


For $0 < x < L$

$v(x) = [w_0 L^4/(24EI)]\,(x/L)\,[1 - 2(x/L)^2 + (x/L)^3]$

$v|_{max} = [5w_o L^4/(384EI)]$ $\quad$ *at* $x=L/2$

Couple, End-loaded Simply-Supported Beam

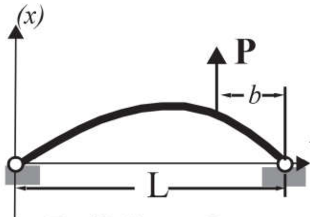


For $0 < x < L$

$$v(x) = - [ML^2/(6EI)]\,(x/L)\,[1 - x^2/L^2]$$

$$v|_{max} = [\,ML^2/(9\sqrt{3}\,EI)] \qquad at\ x=L/\sqrt{3}$$

Point Load, Simply-Supported Beam



For $0<x<(L-b)$

$$v(x) = [PL^3/(6EI)]\,(b/L)\,[\,-(x/L)^3 + (1-b^2/L^2)(x/L)\,]$$

$$v|_{max} = PL^3/[9\sqrt{3}\,EI)]\,(b/L)[1- b^2/L^2]^{3/2} \quad at\ x=(L/\sqrt{3})\sqrt{(1-b^2/L^2)}$$

For $(L-b) < x < L$

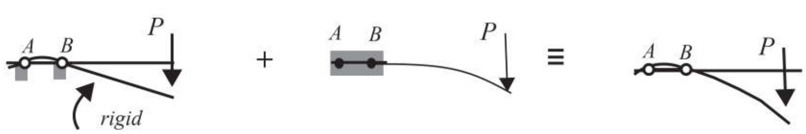
$$v(x) = [\,PL^3/(6EI)]\,(b/L)\,\{\,(L/b)\,[(x/L)- (1-b/L)]^3 - (x/L)^3+(1-b^2/L^2)(x/L)\}$$

With these few relationships we can construct the deflected shapes of beams subjected to more complex loadings and different boundary conditions. We do this by *superimposing* the solutions to more simple loading cases, as represented, for example by the cases cited above.

Exercise 10.2

Show that the expression obtained for the tip deflection as a function of end load in the previous exercise can be obtained by superimposing the displacement fields of two of the cases presented above.

We will consider the beam deflection at the tip to be the sum of two parts: One part will be the deflection due to the beam acting as if it were cantilevered to a wall at the support point *B*, the middle figure below, and a second part due to the rotation of the beam at this imagined *root* of the cantilever at *B* —the figure left below.



We first determine the rotation of the beam at this point, at the support *B*. To do this we must imagine the effect of the load *P* applied at the tip upon the deflected shape back within the region $0 < x < L/4$. This effect can be represented as an equivalent force system at *B* acting internally to the beam. That is, we cut