

# INDEX

## A

Addition operations, matrix algebra, 27–28, 45  
Analysis module, 129, 140–160, 233–245, 322–333  
  beam computer analysis, 233–245  
  joint displacements **d**, 150–152, 238, 329  
  joint loads **P**, 148–150, 234–238, 324–329  
  member end force calculations, 152–159, 239–243, 329–333  
  plane frame computer analysis, 322–333  
  plane truss computer analysis, 140–160  
  structure coordinate number assignment, 54, 140–143, 233–234, 322–324  
  structure stiffness matrix **S**, 143–148, 234–237, 324–329  
  support reactions **R**, 135–136, 152–159, 239–243, 329–333  
Analytical models, 10–11, 22, 49–57, 59, 163–171, 250–255, 350–361, 366–385, 417–498, 530–532  
beams, 163–171, 350–361  
degrees of freedom, 51–57, 166–168, 252–255  
global coordinate system, 49–50, 165–166, 250–252  
grids, 433–455  
hinged joints, 350–361  
inclined roller supports, 530–532  
joint displacement **d**, 53, 252–255  
joint load **P**, 52–56, 168–171  
line diagrams for, 11  
local coordinate system, 50–52, 166, 251–252  
matrix structures and, 10–11, 22  
member releases, 350–361, 366–385

plane frames, 250–255, 350–361  
plane trusses, 49–57, 59, 530–532  
restrained coordinates, 54–55, 252–255  
space frames, 456–494  
space trusses, 418–433  
structure coordinates, 51–57  
support displacements, 366–385  
support reactions **R**, 56, 171  
three-dimensional framed structures, 417–498

Angle of roll  $\psi$ , 479–483

Angle of twist  $\phi$ , 440–444

Approximate matrix analysis, 506–514

Arrow notation, 54–55, 168, 254, 435–436

Axial forces, member stiffness, 61–63

## B

Banded structure stiffness, 559–568  
Beams, 6, 162–248, 348–362, 376–381  
  analysis modules, 233–245  
  analytical model, 163–171  
  bending moment  $M$  and, 175–178  
  bending moment  $M$ , 175–178  
  code number technique for, 202–206  
  computer analysis, 224–245, 361–363  
  defined, 6, 163  
  deflection and, 175–184  
  degrees of freedom, 166–168  
  finite-element formulation, 185–191  
  fixed-end forces, 174–175, 191–197, 206–214  
  global coordinate system, 165–166  
  hinged joint analysis, 350–362  
  hinges, 348–362  
  input (data) modules, 224–233  
  joint load **P**, 168, 206–214  
  load  $W$  and, 191–197  
  local coordinate system, 166  
  local member stiffness matrix **k**, 175–184, 188–191, 202–206

Beams (*contd.*)

- matrix structure classification, 6
  - member releases, 348–362
  - member stiffness, 171–184
  - procedure for analysis, 214–224, 350
  - sign convention for, 175–176, 195
  - stiffness relations, 348–350
  - structure coordinates, 167–168, 233–234
  - structure fixed-joint forces, 206–214
  - structure stiffness, 197–206
  - support displacement analysis, 376–381
  - support reactions **R**, 169
  - virtual work used for, 185–191
- Bending moment *M*, beams, 175–178
- Building frames, approximate matrix analysis of, 506–514

**C**

- Code number technique, 97–102, 202–206, 209, 293–299
- beams, 202–206, 209
  - local member stiffness matrix **k**, 202–206
  - plane frames, 293–299
  - plane trusses, 97–102
  - structure fixed-joint forces **P<sub>f</sub>**, 206–214, 293–299
  - structure stiffness matrix **S**, 97–101, 202–206, 293–299
  - support reaction **R**, 101–102
- Column matrix, 25
- Compatibility conditions, 13–14
- Compatibility equations, 89, 91–92, 197–198
- beams, 197–198
  - plane trusses, 89, 91–92
  - structure stiffness, 89, 91–92, 197–198
- Computer analysis, 21, 128–161, 224–245, 317–334, 361–362, 385–390, 532, 603–604. *See also*
- Analysis module; Data input analysis module, 129, 140–160, 233–245, 322–333
  - beams, 224–245, 361–362
  - coordinate number assignment, 54, 140–143, 233–234
  - cross-sectional property data, 137, 229, 320
  - hinged joints, 361–362
  - inclined roller supports, 532

- input (data) module, 129–139, 224–233, 318–322, 385–386
  - joint data, 132–135, 224–227, 318
  - joint displacement **d**, 150–152, 238, 329
  - joint load **P**, 148–150, 230–231, 234–238, 321, 324–329
  - load data, 138–139, 230–232, 321–322
  - material property data, 136–137, 228, 320
  - member data, 137, 229–232, 321–322
  - member force (stiffness) calculations, 152–159, 239–243, 329–333, 389–390
  - member releases, 361–362, 385–390
  - plane frames, 317–334, 316–362
  - plane trusses, 128–161
  - results, 603–604
  - software, 21, 603–604
  - structure coordinate number assignment, 54, 140–143, 322–324
  - structure stiffness matrix **S**, 143–148, 234–237, 324–329
  - support displacements, 385–390
  - support reactions **R**, 135–136, 152–159, 227–228, 239–243, 318–321, 329–333, 389
- Condensation of degrees of freedom, 514–522
- Connections, *see* Joints
- Constitutive relations, 14–15
- Coordinate transformations, 76–85, 268–275, 422–425, 446–448, 467–485
- end displacements **u** and forces **Q**, 268–275, 448, 467–472
  - global to local system, 77–79
  - grids, 446–448
  - local to global system, 79–80
  - member rotation matrix **r**, 472–485
  - plane frames, 268–275
  - plane trusses, 76–85
  - space frames, 467–485
  - space trusses, 422–425
  - transformation matrix **T**, 78–80, 471–472
- Cross-sectional properties, 137, 229, 320
- beam computer analysis, 229
  - input (data) module, 137, 229, 320
  - plane frame computer analysis, 320
  - plane truss computer analysis, 137

**D**

Data input, 129–139, 224–233, 318–322, 385–386, 603–604

- beam computer analysis, 224–233
- cross-sectional properties, 137, 229, 320
- joints, 132–135, 224–227, 230–231, 318, 321
- loads, 138–139, 230–232, 321–322
- material properties, 136–137, 228, 320
- members, 137, 229–232, 321–322
- plane frame computer analysis, 318–322
- plane truss computer analysis, 129–139
- software procedure, 603–604
- support displacements, 385–386
- support reactions **R**, 135–136, 152–159, 227–228, 318–321
- unit consistency and, 130

Decomposition method, 559–568

Deflection, beams, 175–184

Deformable bodies, principle of virtual work for, 17–20

Deformations, 175–184, 436–437, 440, 443–445, 541–545, 588–589

- beams, 175–184
- bending moment  $M$ , 175–178
- deflection, 175–184
- grids, 436–437, 440, 443–445
- nonlinear structural analysis, 588–589
- shape factor, 542–543
- shear, 541–545
- structure load-deformation relations, 588–589
- torsional moment  $M_T$ , 440, 443–445
- warping, 436–437

Degrees of freedom, 51–57, 166–168, 251–255, 418–419, 435–436, 456–458, 514–522

- arrow notation for, 54–55, 168, 254, 435–436
- beams, 166–168
- condensation of, 514–522
- defined, 52
- external, 515–516
- free coordinates, 54
- free joint, 52
- grids, 435–436
- internal, 515
- joint displacement vector, 53

- joint load vector, 55–56
- kinematic indeterminacy, 53
- plane frames, 251–255
- plane trusses, 51–57
- reaction vector, 56
- restrained coordinates and, 54–55, 252–255
- space frames, 456–458
- space trusses, 418–419
- structure coordinates, 54

Diagonal matrix, 26

Differential operator matrix **D**, 73

Differentiation, matrix algebra, 34, 45

Direct integration method, 191–194

Direct stiffness method, 5, 93–94, 197–202

- beams, 197–202
- plane trusses, 93–94
- structure stiffness by, 93–94, 197–202

Displacement, *see* End displacements **u** and forces **Q**; Support displacements

Displacement functions, 67–72, 185–187

- beams, 185–187
- finite-element formulation, 67–72, 185–187
- plane trusses, 67–72

*Do Loop* command, 132–134, 144–148, 235–238

- input data use of, 132–134
- structure stiffness matrix **S** generation, 144–148, 235–238

**E**

Elements of a matrix, 24

End displacements **u** and forces **Q**, 58–61, 63, 152–158, 171–175, 191–202, 256–259, 268–275, 329–333, 389–390, 437–440, 448, 458–463, 467–472, 533–541

- beams, 171–175, 191–202
- computer calculation of, 152–159, 329–333, 389–390
- coordinate transformations and, 268–275, 448, 467–472
- fixed-end forces, 174–175, 191–197
- global coordinates, 157–158
- grids, 437–440, 448
- local coordinates, 60–61, 63, 155–157, 256–259, 437–440

- End displacements  $\mathbf{u}$  and forces  $\mathbf{Q}$  (*contd.*)  
 member stiffness and, 60–61, 63, 152–158, 171–175, 256–259  
 offset connections, 533–537  
 plane frames, 256–259, 268–275, 329–333  
 plane trusses, 60–61, 63, 152–158  
 semirigid connections, 537–541  
 space frames, 458–463, 467–472  
 structure stiffness and, 197–202  
 support displacements and, 389–390
- Equality operations, matrix algebra, 27, 45
- Equilibrium equations, 12–13, 91, 197  
 beams, 197  
 matrix structures and, 12–13  
 plane trusses, 91  
 structure stiffness and, 91, 197
- Equilibrium matrix  $\mathbf{b}$ , 609–610
- Equivalent joint loads, 210–214, 291–293, 324–329, 362–365, 386–389  
 beams, 210–214  
 computer evaluation of, 324–329, 386–389  
 plane frames, 291–293, 324–329  
 structure fixed-joint forces  $\mathbf{P}_f$  and, 210–214, 291–293, 362–365  
 support displacements and, 362–365, 386–389
- F**
- Fabrication errors, member fixed-end forces due to, 394–410
- Finite-element methods, 4, 67–76, 185–191  
 beams, 185–191  
 displacement functions, 67–72, 185–187  
 formulation, 67–76, 185–191  
 matrix methods compared to, 4  
 member stiffness matrix  $\mathbf{k}$ , 74–76, 188–191  
 plane trusses, 67–76  
 shape functions, 72–73, 187–188  
 strain–displacement relationship, 73–74, 188  
 stress–displacement relationship, 74, 188  
 symmetry and, 76  
 virtual work used for, 67–76, 185–191
- Fixed-end forces, 174–175, 191–197, 206–214, 260–268, 281–284, 390–410, 463–467  
 beam members, 174–175, 191–197  
 direct integration method for, 191–194  
 fabrication errors and, 394–410  
 global coordinate system  $\mathbf{F}_f$ , 281–284  
 load  $W$  and, 191–197  
 local coordinate system, 260–268, 463–467  
 member forces  $\mathbf{Q}_f$ , 175, 195–197, 206–207, 209, 260–268, 463–467  
 member releases, 390–410  
 member stiffness and, 174–175, 260–268, 281–284  
 plane frames, 260–268, 281–284  
 space frames, 463–467  
 structure fixed-joint forces  $\mathbf{P}_f$ , 206–214  
 temperature changes and, 390–394
- Fixed-joint forces, *see* Structure fixed-joint forces  $\mathbf{P}_f$
- Flexibility method, 4–5, 21, 605–611  
 equilibrium matrix  $\mathbf{b}$ , 609–610  
 flexibility coefficient, 608  
 redundants, 605  
 static determinacy, 605  
 stiffness method compared to, 4–5, 21, 605  
 structure flexibility matrix  $\mathbf{f}$ , 608–611  
 truss analysis, 605–611
- Forces, *see* End displacements  $\mathbf{u}$  and forces  $\mathbf{Q}$ ; Joint load  $\mathbf{P}$ ; Member forces  $\mathbf{Q}_f$ ; Structure fixed-joint forces  $\mathbf{P}_f$
- Framed structures, 5–10, 22, 417–198  
 defined, 22  
 matrix structure classification, 5–10  
 three-dimensional, 417–498
- Free coordinates, 54
- Free joint, 52
- G**
- Gauss–Jordan elimination method, 38–45, 150–152  
 joint displacement  $\mathbf{d}$ , solution of by, 150–152  
 matrix inversion by, 43–45  
 simultaneous equations, solutions of by, 38–43

Global coordinate system, 49–50, 76–85,  
85–89, 157–158, 165–166,  
250–252, 268–284, 418, 422–425,  
435, 446–448, 457, 467–485  
analytical models for, 49–50, 250–252  
beams, 165–166  
computer calculations in, 157–158  
coordinate transformations, 76–85,  
268–275, 422–425, 446–448,  
467–485  
fixed-end forces  $\mathbf{Q}_f$  in, 281–284  
grids, 435, 446–448  
member end forces and reactions,  
157–158  
member stiffness matrix  $\mathbf{K}$  in, 85–89,  
157–158, 276–281  
member stiffness relations in, 85–89,  
276–284, 425  
plane frames, 250–252, 268–284  
plane trusses, 49–50, 76–85, 85–89,  
157–158  
space frames, 457, 467–485  
space trusses, 418, 422–425  
symmetry and, 86  
transformation from local system,  
79–80  
transformation to local system, 77–79  
Grids, 7, 9, 433–455  
analytical models, 433–455  
angle of twist  $\phi$ , 440–444  
arrow notation for, 435–436  
beginning and end joints, 435–436  
coordinate transformations, 446–448  
defined, 433  
degrees of freedom, 435–436  
end displacements  $\mathbf{u}$  and forces  
 $\mathbf{Q}$ , 437–440  
global coordinate system, 435,  
446–448  
local coordinate system, 435–448  
matrix structure classification, 7, 9  
member releases, 444–445  
member stiffness relations, 436–445  
procedure for analysis, 449–455  
Saint-Venant's torsion constant  $J$  for,  
440–441  
symmetric cross sections of, 433–435  
torsional moment  $M_T$ , 440, 443–445  
torsional stiffness coefficients,  
440–442  
warping, 436–437

**H**

Half-bandwidth of structure stiffness matrix  $\mathbf{S}$ , 553–559  
Hinges, 342–362  
analytical models, 350–361  
beam members, 348–362  
beginning (MT = 1), 343–345, 348  
both ends (MT = 3), 346–347,  
349–350  
computer implementation, 361–363  
end (MT = 2), 345–346, 348–349  
hinged joint analysis, 350–362  
member type (MT), 342  
plane frame members, 343–348,  
350–362  
procedure for analysis, 350  
stiffness relations, 343–350

**I**

Identity matrix, 27  
Inclined roller supports, 530–532  
Input module, 129–139, 224–233,  
318–322, 385–386. *See also* Data  
input  
beams, 224–233  
plane frames, 318–322  
plane trusses, 129–139  
support displacements, 385–386  
Integration, matrix algebra, 34–35, 45  
Inverse of a matrix, 35–36, 43–45  
Gauss-Jordan elimination method,  
43–45  
square matrix, 35–36  
symmetric matrix, 36

**J**

Joint coordinate matrix, computer  
analysis, 132  
Joint displacement  $\mathbf{d}$ , 53, 89–91,  
150–152, 197, 238, 252–255, 329,  
547–576  
beams, 197, 238  
computer solution for, 150–152, 238,  
329  
degrees of freedom and, 53, 252–255  
Gauss-Jordan elimination method  
for, 150–152  
nonlinear structural analysis  $\delta$ , 547–576  
plane frames, 252–255, 329  
plane trusses, 53, 89–91, 150–152  
structure stiffness relations, 89–91, 197

- Joint load  $\mathbf{P}$ , 55–56, 89–91, 148–150, 197, 210–214, 230–231, 234–238, 291–293, 321, 324–329, 362–365, 386–389  
 analysis module, 148–150, 234–238, 324–329  
 beams, 168, 197, 230–231, 234–238  
 computer generation of, 148–150, 234–237, 324–329  
 degrees of freedom, 52–56  
 equivalent, 210–214, 291–293, 324–329, 362–365, 386–389  
 input (data) module, 230–231, 321  
 member releases, 362–365, 386–389  
 plane frames, 291–293, 321, 324–329  
 plane trusses, 55–56, 89–91, 148–150  
 storage of in structure load, 237–238, 329  
 structure fixed-joint forces  $\mathbf{P}_f$  and, 291–293  
 structure stiffness relations, 89–91, 197, 291–293  
 support displacements and, 362–365, 386–389
- Joints, 52–56, 89–91, 132–135, 148–152, 197, 206–214, 224–227, 230–231, 252–255, 291–299, 318, 321, 350–362, 419–420, 435–436, 531, 533–541. *See also* Fixed-joint forces  
 beams, 206–214, 224–227, 230–231, 350–362  
 beginning and end, 52, 419–420, 435–436  
 code number technique for, 209  
 computer analysis and, 132–135, 148–152, 224–227, 230–231, 318, 321, 361–362  
 degrees of freedom, 52–56  
 displacement  $\mathbf{d}$ , 53, 89–91, 152–153, 252–255  
 free, 52  
 grids, 435–436  
 hinged, 350–362  
 input (data) modules, 132–135, 224–227, 230–231, 318, 312  
 load  $\mathbf{P}$ , 55–56, 89–91, 148–150, 197, 230–231, 321  
 local coordinate system, 52, 531  
 offset connections, 533–537  
 plane frames, 252–255, 291–299, 350–362  
 plane trusses, 52–56, 89–91, 132–135, 148–152, 197  
 semirigid connections, 537–541  
 space trusses, 419–420  
 structure fixed-joint forces  $\mathbf{P}_f$ , 206–214, 291–299  
 structure stiffness relations, 89–91
- K**  
 Kinematic indeterminacy, degree of, 53
- L**  
 Line diagrams, 11  
 Linear structural analysis, 20–22, 573, 576  
 Load data, 138–139, 230–232, 321–322  
 beam computer analysis, 230–232  
 input (data) module, 138–139, 230–232, 321–322  
 joints, 230–231, 321  
 matrix, 138–139  
 members, 231–232, 321–322  
 plane frame computer analysis, 321–322  
 plane truss computer analysis, 138–139  
 Load  $W$ , beam fixed-end forces due to, 191–197  
 Local coordinate system, 50–52, 58–67, 76–85, 155–157, 166, 251–252, 256–275, 419–425, 435–448, 458–485, 531  
 beams, 166  
 beginning and end joints, 52, 419–420, 435–436  
 computer calculations in, 155–157  
 coordinate transformations, 76–85, 268–275, 422–425, 446–448, 467–485  
 fixed-end forces  $\mathbf{Q}_f$  in, 260–268, 463–467  
 force-displacement relationships, 50–52  
 grids, 435–448  
 joints, 52, 531  
 member end forces and reactions, 155–157  
 member stiffness matrix  $\mathbf{k}$  in, 60–67, 259–260, 463–466

- member stiffness relations in, 58–67, 256–268, 420–422, 436–445, 458–467
  - plane frames, 251–252, 256–275
  - plane trusses, 50–52, 58–67, 76–85, 155–157
  - space frames, 458–485
  - space trusses, 419–425
  - transformation from global system, 77–79
  - transformation to global system, 79–80
- M**
- Material properties, 136–137, 228, 320
    - beam computer analysis, 228
    - input (data) module, 136–137, 228, 320
    - plane truss computer analysis, 320
    - plane truss computer analysis, 136–137
  - Matrix, defined, 24, 45
  - Matrix algebra, 23–47
    - addition operations, 27–28, 45
    - column matrix, 25
    - diagonal matrix, 26
    - differentiation, 34, 45
    - elements of, 24
    - equality operations, 27, 45
    - Gauss–Jordan elimination method, 38–45
    - identity matrix, 27
    - integration, 34–35, 45
    - inverse of a matrix, 35–36, 43–45
    - multiplication by a scalar, 28
    - multiplication of matrices, 28–32, 45
    - null matrix, 27
    - order of, 24–25
    - orthogonal matrix, 36–37
    - partitioning, 37–38
    - row matrix, 25
    - square matrix, 25–26, 35–36
    - subtraction operations, 27–28, 45
    - symmetric matrix, 26, 33, 36
    - transpose of a matrix, 32–33, 45
    - triangular matrices (upper and lower), 26
    - unit matrix, 27
    - vectors, 25
  - Matrix structures, 1–22
    - analytical models, 10–11, 22
    - beams, 6
    - classical methods compared to, 3–4
    - compatibility equations, 13–14
    - constitutive relations, 14–15
    - deformable bodies, principle of virtual work for, 17–20
    - direct stiffness method, 5
    - equilibrium equations, 12–13
    - finite element methods compared to, 4
    - flexibility method, 4–5, 21
    - framed structures, 5–10
    - grids, 7, 9
    - history of, 2–3
    - line diagrams, 11
    - linear analysis, 20–22
    - nonlinear analysis, 21
    - plane frames, 6–8
    - plane trusses, 5–6
    - principle of virtual work, 15–20
    - rigid bodies, principle of virtual work for, 15–17
    - software for, 21
    - space frames, 10
    - space trusses, 7
    - stiffness method, 4–5, 21
    - structural analysis, 12–22
  - MDISPG* (member global displacement)
    - subroutine, 152, 154, 330, 390
  - MDISPL* (member local displacement)
    - subroutine, 155, 239, 241, 330, 390
  - Mechanics of materials, principles of for member stiffness, 61–62
  - Member code numbers, 97–102. *See also* Code number technique
  - Member data, 137, 229–232, 321–322
    - beam computer analysis, 229–232
    - input (data) module, 137, 229–232, 321–322
    - load data, 231–232, 322
    - matrix, 137, 231, 321
    - plane frame computer analysis, 321–322
    - plane truss computer analysis, 137
  - Member displacement function, 72–73
  - Member fixed-end forces  $\mathbf{Q}_f$ , 175, 195–197, 206–207, 209, 260–268, 463–467
    - beams, 175, 195–197, 206–207, 209
    - plane frames, 260–268, 281–284
    - space frames, 463–467
  - Member force-displacement relations, 580–586

- Member forces, *see* End displacements and forces; Fixed-end forces; Member stiffness
- Member releases, 340–416, 444–445, 465–467, 530–532
  - analytical models, 350–361
  - beams, 348–362, 376–381
  - computer analysis, 361–363, 385–390
  - fabrication errors and, 394–410
  - fixed-end forces, 390–410
  - grids, 444–445
  - hinged joints, 350–362
  - hinges, 342–362
  - inclined roller supports, 530–532
  - plane frames, 343–347, 350–362, 381–385
  - plane trusses, 371–376
  - procedures for analysis, 350, 370–371, 397
  - space frames, 465–467
  - stiffness relations, 343–350
  - structure fixed-joint forces  $\mathbf{P}_f$ , 362–385
  - support displacements, 362–390
  - temperature changes and, 390–394
  - types (MT), 341–343
- Member rotation matrix  $\mathbf{r}$ , 472–485
  - angle of roll  $\psi$ , 472–479
  - plane frame analysis, 472–485
  - reference point, 479–483
- Member shape function matrix  $\mathbf{N}$ , 72–73
- Member stiffness, 58–67, 74–76, 85–89, 92–93, 152–159, 171–184, 188–191, 202–206, 239–242, 256–268, 276–284, 343–350, 420–422, 425, 436–445, 458–467, 541–545
  - axial forces, 61–63
  - beams, 171–184, 188–191, 202–206, 239–242, 348–350
  - bending moment  $M$  and, 175–178
  - computer calculation of, 152–159, 239–242
  - deflection and, 175–184
  - end displacements  $\mathbf{u}$  and forces  $\mathbf{Q}$ , 58–61, 63, 152–158, 171–175, 256–259, 268–275, 458–463
  - finite-element formulation and, 74–76, 188–191
  - fixed-end forces, 174–175, 260–268, 281–284
  - force calculations, 152–159, 239–242
  - global coordinate system, 85–89, 276–284, 425
  - global matrix  $\mathbf{K}$ , 85–89, 157–158, 276–281
  - grids, 436–445
  - hinges and, 343–350
  - local coordinate system, 58–67, 155–157, 256–268, 420–422, 436–445
  - local matrix  $\mathbf{k}$ , 60–67, 74–76, 175–184, 188–191, 202–206, 259–260, 463–466
  - mechanics of materials, principles of, 61–62
  - member releases and, 343–350, 444–445, 465–467
  - plane frames, 256–268, 276–284, 343–347
  - plane trusses, 58–67, 74–76, 85–89, 92–93, 152–159
  - Saint-Venant's torsion constant  $J$  for, 440–441
  - shear deformations and, 541–545
  - space trusses, 420–422, 425
  - structure stiffness matrix  $\mathbf{S}$  and, 58, 202–206
  - symmetry and, 76, 86
  - torsional moment  $M_T$ , 440, 443–445
  - torsional stiffness coefficients, 440–442
  - transformation matrix  $\mathbf{T}$ , 152–155
  - warping, 436–437
- Member strain–displacement matrix  $\mathbf{B}$ , 72–73
- Member tangent stiffness matrix  $\mathbf{K}_t$ , 586–588
- Member types (MT), 341–343
- MFEDSD* (member global fixed-end displacement from support displacement) subroutine, 387–388, 390
- MFEFG* (member global fixed-end) subroutine, 328–329
- MFEFL* (member local fixed-end force) subroutine, 235, 237, 239, 328, 330, 361, 363
- MFEFSD* (member global fixed-end force from support displacement) subroutine, 388–389
- MFORCEG* (member global force) subroutine, 157, 330



*MFORCEL* (member local force) subroutine, 156–157, 241–242, 330, 390  
*MSTIFFG* (member global stiffness matrix) subroutine, 326–327, 387  
*MSTIFFL* (member local stiffness matrix) subroutine, 155–156, 235, 239, 326, 330, 361–362, 387  
*MTRANS* (member transformation matrix) subroutine, 152, 154–155, 326  
 Multiplication of a matrix by a scalar, 28  
 Multiplication of matrices, 28–32, 45

## N

Newton-Raphson computational method, 589–600  
 Nonlinear structural analysis, 21, 572–602  
   geometric concepts of, 574–579  
   joint displacements  $\delta$ , 547–576  
   linear analysis compared to, 573, 567  
   member force-displacement relations, 580–586  
   member tangent stiffness matrix  $\mathbf{K}_t$ , 586–588  
   Newton-Raphson computational method, 589–600  
   plane trusses, 579–600  
   snap-through instability, 576–579  
   stability and, 576–579  
   structure tangent stiffness matrix  $\mathbf{S}_t$ , 589  
 Nonprismatic members, 545–552  
 Null matrix, 27

## O

Offset connections, 533–537  
 Order of a matrix, 24–25  
 Orthogonal matrix, 36–37

## P

Partitioning, matrix algebra, 37–38  
 Plane frames, 6–8, 249–339, 343–347, 350–362, 381–385  
   analysis module, 322–333  
   analytical model, 250–255  
   computer analysis, 317–334, 361–362  
   coordinate transformations, 268–275  
   defined, 250  
   degrees of freedom, 251–255  
   global coordinate system, 250–252, 268–284

hinged joint analysis, 350–362  
 hinges, 343–347, 350–362  
 input (data) module, 318–322  
 joint displacements  $\mathbf{d}$ , 252–255, 284–285  
 joint load  $\mathbf{P}$ , 284–287  
 local coordinate system, 251–252, 256–275  
 matrix structure classification, 6–8  
 member releases, 343–347, 350–362  
 member stiffness, 256–268, 276–284  
 procedure for analysis, 299–317, 350  
 restrained coordinates, 252–255  
 stiffness relations, 343–347  
 structure stiffness, 284–299  
 support displacement analysis, 381–385

Plane trusses, 5–6, 48–127, 128–161, 371–376, 530–532, 579–600  
   analysis modules, 140–160  
   analytical model, 49–57, 59, 530–532  
   arrow notation for, 54–55  
   code number technique for, 97–102  
   computer analysis, 128–161  
   coordinate transformations, 76–85  
   defined, 49  
   degrees of freedom, 51–57  
   finite-element formulation, 67–76  
   global coordinate system, 49–50, 76–85, 85–89  
   inclined roller supports, 530–532  
   input (data) modules, 129–139  
   joints, 52–56, 89–91, 132–135, 531  
   local coordinate system, 50–52, 58–67, 76–85  
   matrix structure classification, 5–6  
   member force-displacement relations, 580–586  
   member stiffness, 58–67, 85–89, 92–93  
   member tangent stiffness matrix  $\mathbf{K}_t$ , 586–588  
   Newton-Raphson computation method, 589–600  
   nonlinear analysis of, 579–600  
   procedure for analysis, 105–123  
   structure load-deformation relations, 588–589  
   structure stiffness, 89–105  
   structure tangent stiffness matrix  $\mathbf{S}_t$ , 589

- Plane trusses (*contd.*)  
 support displacement analysis, 371–376  
 support reaction **R**, 101–102, 135–136, 152–159  
 virtual work used for, 67–76  
 Primary structure, 605  
 Principle of virtual work, 15–20  
 deformable bodies, 17–20  
 rigid bodies, 15–17  
 Procedure for analysis, plane trusses, 105–123
- R**  
 Reaction vector **R**, 56. *See also* Support reactions **R**  
 Redundants, 605  
 Reference point, member rotation matrix **R** in terms of, 479–483  
 Restrained coordinates, 54–55, 252–255, 500–506  
 degrees of freedom and, 54–55, 252–255  
 numbering of, 54–55  
 plane frames, 252–255  
 plane trusses, 54–55  
 structure stiffness matrix **S** and, 500–506  
 Restraint codes, 135–136, 227, 230, 321  
 beam computer analysis, 227, 230  
 plane frame computer analysis, 321  
 plane truss computer analysis, 135–136  
 Rigid bodies, principle of virtual work for, 15–17  
 Roller supports, 530–532  
 Row matrix, 25
- S**  
 Saint-Venant's torsion constant *J*, 440–441  
 Semirigid connections, 537–541  
 Shape factor (shear), 542–543  
 Shape functions, 72–73, 187  
 beams, 187  
 finite-element formulation, 72–73, 187  
 plane trusses, 72–73  
 Shear deformations, 541–545  
 Sign conventions, 175–176, 195  
 Snap-through instability, 576–579  
 Software, 21, 603–604. *See also* Computer analysis
- Space frames, 10, 456–494  
 analytical models, 456–494  
 coordinate transformations, 467–485  
 degrees of freedom, 456–458  
 end displacements **u** and forces **Q**, 458–463, 467–472  
 global coordinate system, 457, 467–485  
 local coordinate system, 458–485  
 local stiffness matrix **k**, 463–466  
 matrix structure classification, 10  
 member fixed-end forces **Q<sub>f</sub>**, 463–467  
 member releases, 465–467  
 member rotation matrix **r**, 472–485  
 member stiffness relations, 458–467  
 procedure for analysis, 485–494  
 symmetric cross sections of, 456  
 transformation matrix **T**, 471–472  
 Space trusses, 7, 418–433  
 analytical models, 418–433  
 beginning and end joints, 52, 419–420  
 coordinate transformations, 422–425  
 defined, 418  
 degrees of freedom, 418–419  
 global coordinate system, 418, 422–425  
 local coordinate system, 419–425  
 matrix structure classification, 7  
 member stiffness relations, 420–422, 425  
 procedure for analysis, 425–433  
 Square matrix, 25–26, 35–36  
 diagonal elements of, 25–26  
 inverse of, 35–36  
 Stability, structural analysis and, 576–579  
 Static determinacy, 605  
 Stiffness, *see* Member stiffness; Structure stiffness  
 Stiffness method, 4–5, 21  
*STOREPF* (store member global fixed-end force) subroutine, 237–238, 328–330, 389  
*STORER* (store member forces in support reactions) subroutine, 158, 241–243  
*STORES* (store structure stiffness matrix **S**) subroutine, 146–148, 235–236, 327–328  
 Strain–displacement relationship, 73–74, 188  
 beams, 188

- finite-element formulation, 73–74, 188
- plane trusses, 73–74
- Stress–displacement relationship, 74, 188
  - beams, 188
  - finite-element formulation, 74, 188
  - plane trusses, 74
- Structural analysis, 12–22, 506–530, 545–568, 572–602
  - approximate matrix analysis, 506–514
  - building frames, 506–514
  - comparison of linear and nonlinear, 573, 576
  - compatibility equations, 13–14
  - condensation of degrees of freedom, 514–522
  - constitutive relations, 14–15
  - deformable bodies, 17–20
  - equilibrium equations, 12–13
  - large systems, 553–568
  - linear, 20–22, 573, 576
  - nonlinear, 21, 572–602
  - nonprismatic members, 545–552
  - principle of virtual work, 15–20
  - rigid bodies, 15–17
  - snap-through instability, 576–579
  - software for, 21
  - stability and, 576–579
  - substructure analysis, 514, 522–530
- Structure coordinates, 51–57, 140–143, 167–168, 233–234, 252–255, 322–324
  - beams, 167–168, 233–234
  - computer assignment of, 140–143, 233–234, 322–324
  - defined, 54
  - degrees of freedom and, 51–57, 252–255
  - numbering of, 54–55
  - plane frames, 252–255, 322–324
  - plane trusses, 51–57, 140–143
  - restrained coordinates, 54–55, 252–255
- Structure fixed-joint forces  $\mathbf{P}_f$ , 206–214, 291–299, 362–385
  - assembly of, 293–299
  - beams, 206–214
  - code number technique for, 209, 293–299
  - equivalent joint loads and, 210–214, 291–293, 362–365
  - evaluation of due to support displacements, 366–369
  - member fixed-end forces  $\mathbf{Q}_f$  and, 206–207, 209
  - member releases and, 362–385
  - physical interpretation of, 206–209
  - plane frames, 291–299
  - structure stiffness and, 291–299
  - support displacements and, 362–385
- Structure flexibility matrix  $\mathbf{f}$ , 608–611
- Structure load, storage of joint load in, 237–238, 329
- Structure load-deformation relations, 588–589
- Structure stiffness, 58, 89–105, 197–206, 284–299
  - beams, 197–206
  - code number technique for, 97–102, 202–206
  - compatibility equations for, 89, 91–92, 197–198
  - direct stiffness method, 93–94, 197–202
  - end displacements  $\mathbf{u}$  and forces  $\mathbf{Q}$ , 197–202
  - equilibrium equations for, 91, 197
  - joint displacement  $\mathbf{d}$ , 89–91, 197, 284–285
  - joint load  $\mathbf{P}$ , 89–91, 197, 284–287, 291–293
  - matrix  $\mathbf{S}$ , 58, 94–101, 202–206, 287–291, 293–299
  - member stiffness relations, 92–93
  - plane frames, 284–299
  - plane trusses, 58, 89–105
  - structure fixed-joint forces  $\mathbf{P}_f$  and, 291–299
  - support reaction  $\mathbf{R}$ , 101–102
- Structure stiffness matrix  $\mathbf{S}$ , 58, 94–101, 143–148, 202–206, 234–238, 287–291, 293–299, 324–329, 500–506, 553–568
  - analysis module, 143–148, 234–237, 324–329
  - assembly of, 94–101, 202–206, 293–299
  - banded structures, 559–568
  - beams, 202–206, 234–238
  - code number technique for, 97–101, 202–206, 293–299
  - computer generation of, 143–148, 234–238, 324–329
  - decomposition method using, 559–568

- Structure stiffness matrix  $\mathbf{S}$  (*contd.*)
    - Do Loop* commands for, 144–148, 235–238
    - equivalent joint load and, 324–329
    - half-bandwidth of, 553–559
    - large system solutions of, 553–568
    - local coordinate system, 58
    - local member stiffness matrix  $\mathbf{k}$  and, 202–206
    - physical interpretation of, 94–97
    - plane frames, 287–291, 293–299, 324–329
    - plane trusses, 58, 94–101, 143–148
    - restrained coordinates and, 500–506
    - structure fixed-joint forces  $\mathbf{P}_f$  and, 293–299
  - Structure tangent stiffness
    - matrix  $\mathbf{S}_t$ , 589
  - Submatrices, 37
  - Substructure analysis, 514, 522–530
  - Subtraction operations, matrix algebra, 27–28, 45
  - Support displacements, 362–390, 501
    - analytical models, 366–385
    - beam analysis, 376–381
    - computer analysis for, 385–390
    - equivalent joint loads, 362–365, 386–389
    - input (data) module, 385–386
    - member forces, 389–390
    - plane frame analysis, 381–385
    - plane truss analysis, 371–376
    - procedures for analysis, 370–371
    - restrained coordinates  $\mathbf{d}_R$ , 501
    - structure fixed-joint forces  $\mathbf{P}_f$  due to, 362–385
    - support reactions and, 389–390
  - Support reaction  $\mathbf{R}$ , 56, 101–102, 135–136, 152–159, 169, 227–228, 230, 239–242, 318–321, 329–333, 389–390
    - analysis module, 152–159, 239–242, 329–333
    - beams, 169, 227–228, 230, 239–242
    - computer analysis, 135–136, 152–159, 227–228, 230, 239–242, 318–321, 329–333, 389–390
    - calculation of, 152–159, 239–242, 329–333
    - input (data) module, 135–136, 227–228, 230, 318–321
    - plane frames, 318–321, 329–333
    - plane trusses, 56, 101–102, 135–136, 152–159
    - restraint codes for, 135–136, 227, 230, 321
    - structure stiffness and, 101–102
    - support displacement and, 389–390
  - Symmetric matrix, 26, 33, 36
  - Symmetry, 76, 86, 433–435, 456
    - finite-element formulation and, 76
    - grid cross sections, 433–435
    - member stiffness matrices, 76, 86
    - space frame cross sections of, 456
- T**
- Temperature changes, member
    - fixed-end forces due to, 390–394
  - Three-dimensional framed structures, 417–498
    - grids, 433–455
    - procedures for analysis, 425–433, 449–455, 485–494
    - space frames, 456–494
    - space trusses, 418–433
  - Torsion constant  $J$ , 440–441
  - Torsional moment  $M_T$ , 440, 443–445
  - Torsional stiffness coefficients, 440–442
  - Transformation matrix  $\mathbf{T}$ , 78–80, 471–472
  - Transpose of a matrix, 32–33, 45
  - Triangular matrices (upper and lower), 26
  - Trusses, 5–7, 48–127, 128–161, 418–433, 530–532, 579–600, 605–611
    - computer analysis, 128–161
    - defined, 5
    - finite-element formulation, 67–76
    - flexibility method for, 605–611
    - inclined roller supports, 530–532
    - member stiffness relations, 58–67, 85–89, 92–93
    - nonlinear analysis of, 579–600
    - plane, 5–6, 48–127, 128–161, 530–531
    - space, 7, 418–433

**U**

Unassembled flexibility matrix  $\mathbf{f}_M$ ,  
608–611

Unit matrix, 27

**V**

Vectors, column matrices as, 25

Virtual work, 67–76. *See also* Finite  
element method

beams, 185–191

finite-element method using, 67–76,  
185–191

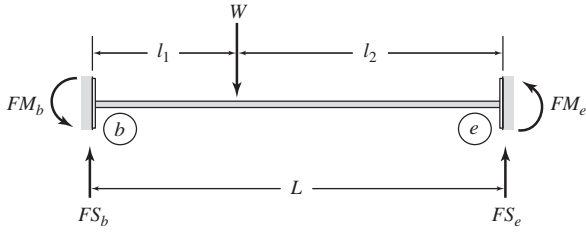
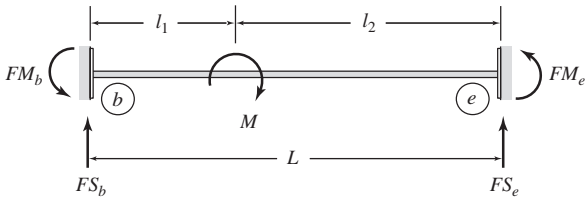
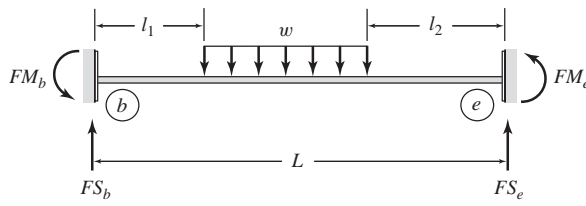
plane trusses, 67–76

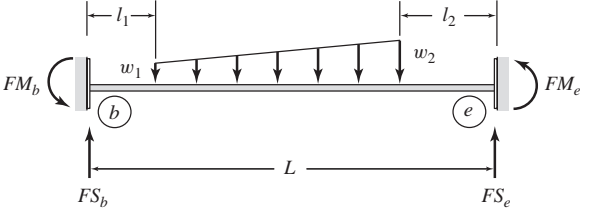
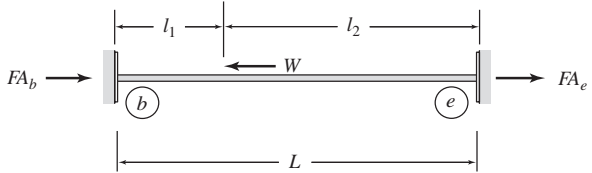
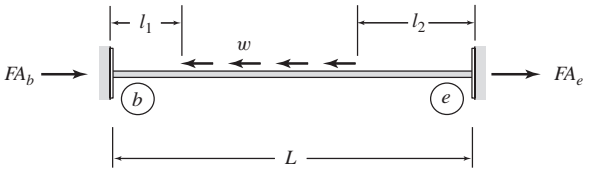
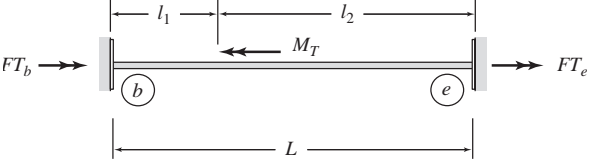
**W**

Warping, grids, 436–437

## FIXED-END FORCE EXPRESSIONS

*Fixed-End Moments, Shears and Axial Forces for Various Loading Conditions*

No.	Loading	Equations for Fixed-End Moments, Shears, and Axial Forces
1.		$FS_b = \frac{Wl_2^2}{L^3}(3l_1 + l_2)$ $FM_b = \frac{Wl_1l_2^2}{L^2}$ $FS_e = \frac{Wl_1^2}{L^3}(l_1 + 3l_2)$ $FM_e = -\frac{Wl_1^2l_2}{L^2}$
2.		$FS_b = -\frac{6Ml_1l_2}{L^3}$ $FM_b = \frac{Ml_2}{L^2}(l_2 - 2l_1)$ $FS_e = \frac{6Ml_1l_2}{L^3}$ $FM_e = \frac{Ml_1}{L^2}(l_1 - 2l_2)$
3.		$FS_b = \frac{wL}{2} \left[ 1 - \frac{l_1}{L^4}(2L^3 - 2l_1^2L + l_1^3) - \frac{l_2^3}{L^4}(2L - l_2) \right]$ $FM_b = \frac{wL^2}{12} \left[ 1 - \frac{l_1^2}{L^4}(6L^2 - 8l_1L + 3l_1^2) - \frac{l_2^3}{L^4}(4L - 3l_2) \right]$ $FS_e = \frac{wL}{2} \left[ 1 - \frac{l_1^3}{L^4}(2L - l_1) - \frac{l_2}{L^4}(2L^3 - 2l_2^2L + l_2^3) \right]$ $FM_e = -\frac{wL^2}{12} \left[ 1 - \frac{l_1^3}{L^4}(4L - 3l_1) - \frac{l_2^2}{L^4}(6L^2 - 8l_2L + 3l_2^2) \right]$

No.	Loading	Equations for Fixed-End Moments, Shears, and Axial Forces
4.		$FS_b = \frac{w_1(L-l_1)^3}{20L^3} \left\{ (7L+8l_1) - \frac{l_2(3L+2l_1)}{(L-l_1)} \right. \\ \left. \times \left[ 1 + \frac{l_2}{L-l_1} + \frac{l_2^2}{(L-l_1)^2} \right] + \frac{2l_2^4}{(L-l_1)^3} \right\} \\ + \frac{w_2(L-l_1)^3}{20L^3} \left\{ (3L+2l_1) \left[ 1 + \frac{l_2}{L-l_1} \right. \right. \\ \left. \left. + \frac{l_2^2}{(L-l_1)^2} \right] - \frac{l_2^3}{(L-l_1)^2} \left[ 2 + \frac{15L-8l_2}{L-l_1} \right] \right\}$ $FM_b = \frac{w_1(L-l_1)^3}{60L^2} \left\{ 3(L+4l_1) - \frac{l_2(2L+3l_1)}{L-l_1} \right. \\ \left. \times \left[ 1 + \frac{l_2}{L-l_1} + \frac{l_2^2}{(L-l_1)^2} \right] + \frac{3l_2^4}{(L-l_1)^3} \right\} \\ + \frac{w_2(L-l_1)^3}{60L^2} \left\{ (2L+3l_1) \left[ 1 + \frac{l_2}{L-l_1} \right. \right. \\ \left. \left. + \frac{l_2^2}{(L-l_1)^2} \right] - \frac{3l_2^3}{(L-l_1)^2} \left[ 1 + \frac{5L-4l_2}{L-l_1} \right] \right\}$ $FS_e = \left( \frac{w_1+w_2}{2} \right) (L-l_1-l_2) - FS_b$ $FM_e = \frac{L-l_1-l_2}{6} [w_1(-2L+2l_1-l_2) \\ -w_2(L-l_1+2l_2)] + FS_b(L) - FM_b$
5.		$FA_b = \frac{Wl_2}{L}$ $FA_e = \frac{Wl_1}{L}$
6.		$FA_b = \frac{w}{2L} (L-l_1-l_2)(L-l_1+l_2)$ $FA_e = \frac{w}{2L} (L-l_1-l_2)(L+l_1-l_2)$
7.		$FT_b = \frac{M_T l_2}{L}$ $FT_e = \frac{M_T l_1}{L}$