

SOME RECENT RESEARCH ON THE BEARING CAPACITY OF FOUNDATIONS

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ABSTRACT

The first part of the paper summarizes the results of recent research on the bearing capacity of spread foundations of various shapes under a central vertical load and outlines the effects of foundation depth, eccentricity and inclination of the load. Simple formulae have been derived for use in practice and their application to the design of rigid and flexible foundations is briefly indicated.

The second part of the paper discusses the bearing capacity of single piles under vertical and inclined loads. The bearing capacity of piled foundations and free-standing pile groups is outlined, and the results of model tests on pile groups under central and eccentric loads are briefly analysed in relation to some problems in practice.

SOMMAIRE

La première partie de cet article présente un résumé des recherches récentes exécutées sur la force portante des fondations superficielles de différentes formes soumises à une charge centrée verticale; elle donne aussi un exposé des effets résultant de la profondeur des fondations, de l'excentricité et de l'obliquité de la charge. Des formules simples y sont dérivées pour l'emploi dans la pratique, et l'application de ces formules au calcul des fondations rigides et flexibles est brièvement expliquée.

La deuxième partie discute de la force portante des pieux simples soumis à des charges verticales et obliques. La force portante des fondations sur pieux et des groupes de pieux isolés y est traitée; les résultats d'essais sur modèle réduit de groupes de pieux soumis à des charges centrées et excentrées sont analysés brièvement en rapport avec quelques problèmes pratiques.

Twenty years have passed since Terzaghi (1943) published his well-known bearing capacity theory, which with his earlier consolidation theory of clays and later settlement analysis of sands led to a better understanding of the fundamentals of foundation behaviour. Under this stimulus the study of the bearing capacity and settlement of foundations in the field amplified by laboratory research has resulted in a more rational approach to foundation design, and some research in the subsequent decade has been discussed previously (Meyerhof, 1953b). During the past decade much further progress has been made in foundation engineering, and within the limits of this paper only some research on the ultimate bearing capacity of foundations will be summarized. An attempt will be made to indicate the relationship of this research to earlier work and its application to some problems in the design of spread and pile foundations.

SPREAD FOUNDATIONS

The ultimate bearing capacity of spread foundations on uniform soils can generally be estimated with sufficient accuracy on the basis of plastic theory, and the theoretical relationship can be represented by (Terzaghi, 1943):

$$(1) \quad q = Q/BL = cN_c + \gamma DN_q + \gamma BN_{\gamma} \cdot \frac{1}{2}$$

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where B = width of foundation, D = depth of foundation, L = length of foundation, c = apparent cohesion of soil, γ = effective unit weight of soil, and N_c , N_q , and N_γ are bearing capacity factors which depend mainly on the angle of internal friction or shearing resistance ϕ of the soil.

The theoretical bearing capacity factors for a shallow horizontal strip foundation under a vertical load are (Prandtl, 1920):

$$(2) \quad \left\{ \begin{array}{ll} N_c = (N_q - 1) \cot \phi \\ N_q = e^{\pi \tan \phi} N_\phi \\ \text{and, approximately, (Meyerhof, 1961b)} \\ N_\gamma = (N_q - 1) \tan (1.4\phi) \\ \text{where } N_\phi = \tan^2 (\frac{1}{4}\pi + \frac{1}{2}\phi). \end{array} \right.$$

The bearing capacity factors for strip foundations and the partly theoretical and partly semi-empirical factors for circular or square footings (Meyerhof, 1951 and 1961a) are shown in Figure 1. Theoretical bearing capacity factors for rectangular foundations with a side ratio of B/L are not yet available, but they can be estimated by interpolating between the factors for strips ($B/L=0$) and circles ($B/L=1$) in direct proportion to the ratio B/L . Alternatively, the bearing capacity factors for rectangles can be obtained by multiplying the factors in equations (2) for strip foundations by the corresponding empirical shape factors

$$(3) \quad \left\{ \begin{array}{ll} s_c = 1 + 0.2N_\phi B/L \\ s_q = s_\gamma = 1, & (\phi = 0) \\ \text{and } s_q = s_\gamma = 1 + 0.1N_\phi B/L, & (\phi > 10^\circ) \end{array} \right.$$

with symbols as before.

It should be noted that for large values of the angle of internal friction ($\phi > 30^\circ$) the experimental factor N_γ for circular or square foundations is smaller than that for strip foundations, whereas the opposite result is expected from bearing capacity theory. As shown previously (Meyerhof, 1961), this difference can be explained by the influence of the intermediate principal stress, which in the plane strain condition under strip foundations raises the experimental bearing capacity for large friction angles considerably above the theoretical value. While triaxial compression tests can, therefore, be used to determine the shear strength parameters for circular or square foundations, plane strain compression tests should be made to determine the corresponding parameters for strip foundations, when good agreement between the estimated and observed bearing capacities will be obtained.

For rectangular foundations the friction angles ϕ can be interpolated between the plane strain and triaxial values in proportion to the side ratio B/L of the foundation. Since the angle of internal friction in plane strain compression tests is roughly 10 per cent greater than in triaxial compression tests (Bishop, 1961; Bjerrum and Kummeneje, 1961) the friction angles for rectangular foundations are roughly given by

$$(4) \quad \phi_r = (1.1 - 0.1B/L)\phi_t$$

where ϕ_t = angle of internal friction or shearing resistance from triaxial compression tests.

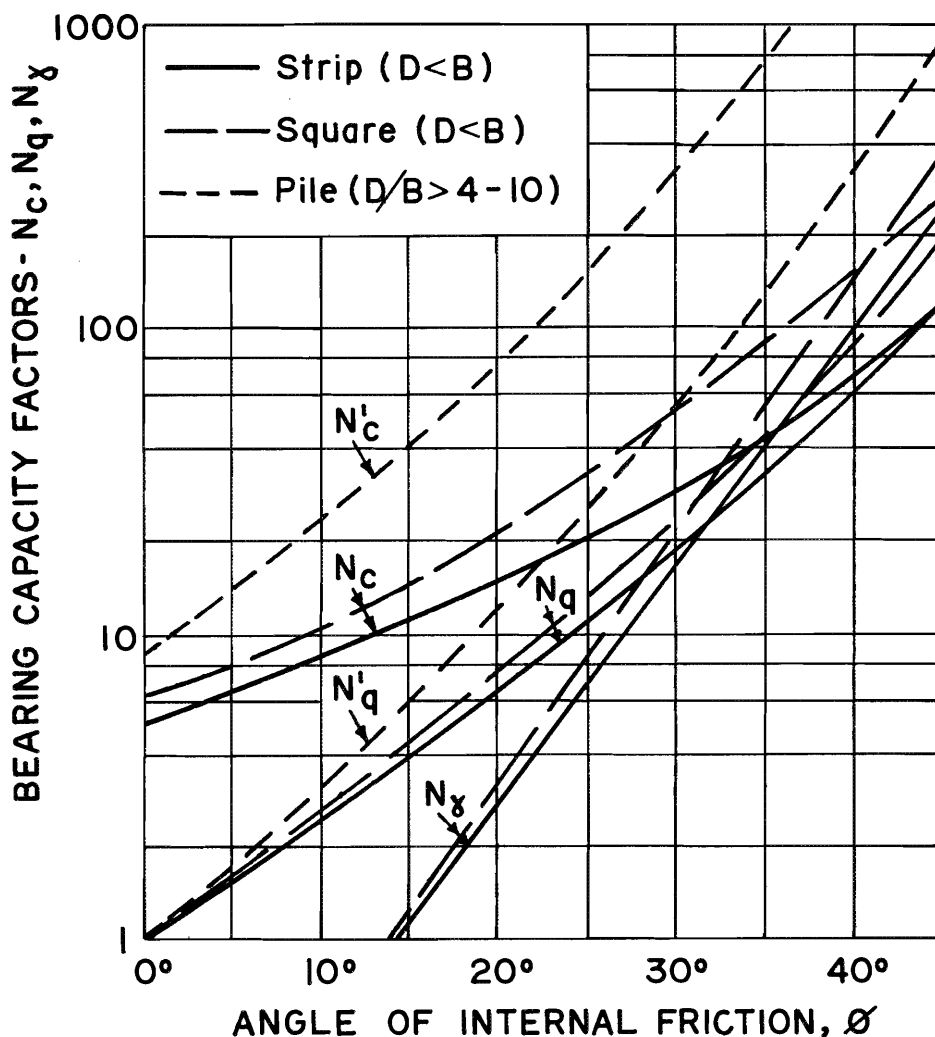


FIGURE 1. Bearing Capacity Factors for Spread and Pile Foundations

If the soil and ground-water conditions vary beneath a foundation, the average values of the effective unit weight, apparent cohesion, and friction angle of the soil within the failure zone should be used. This zone extends from ground level to a distance below the base of about the foundation width for circular or square foundations and up to about twice the foundation width for strip foundations. In the case of stratified materials having widely different soil properties, the bearing capacity can roughly be estimated from the maximum vertical stresses at the interfaces between the layers (Terzaghi and Peck, 1948) since more elaborate recent procedures on the basis of composite failure zones have not yet been sufficiently developed for practical use.

While the simple bearing capacity factors in equations (2) neglect the shearing resistance of the soil above foundation level, the corresponding

increase of the bearing capacity can be estimated from depth factors by which the individual factors have to be multiplied (Skempton, 1951; Hansen, 1955). For practical purposes the following expressions, which correspond to approximate failure surfaces and many test results (Meyerhof, 1951, 1955), are suggested for depths less than the width of the foundation:

$$(5) \quad \left\{ \begin{array}{l} d_c = 1 + 0.2 \sqrt{N_\phi D/B} \\ d_q = d_\gamma = 1, \quad (\phi = 0) \\ \text{and } d_q = d_\gamma = 1 + 0.1 \sqrt{N_\phi D/B}, \quad (\phi > 10^\circ) \end{array} \right.$$

with symbols as before.

As the depth of the foundation increases, the depth factors increase at a decreasing rate and approach a maximum value which can be used for an estimate of the point resistance of piles, as shown below.

Foundations of building frames, retaining walls and similar structures are generally subjected to eccentric and inclined loads, and these loading conditions can substantially reduce the bearing capacity of foundations. In order to allow for the eccentricity e of the resultant load R on the base of a foundation of width B (Figure 2), it was suggested (Meyerhof, 1953a) that an effective

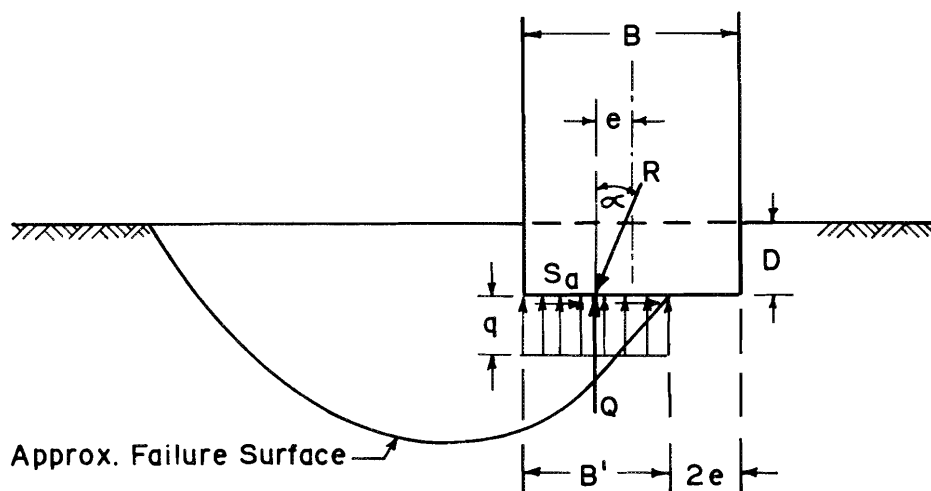


FIGURE 2. Base under Eccentric Inclined Load at Failure

foundation width B' is used in equation (1) and this width is twice the distance of the load from the toe of the foundation, i.e.

$$(6) \quad B' = B - 2e$$

Detailed model tests have shown that this procedure is on the safe side. The experimental effective width is between the above-mentioned value and the width estimated from the conventional contact pressure distribution (trapezium or triangle), which can be explained theoretically by more accurate failure zones (Hansen, 1955). For a double eccentricity of the load an effective contact area can be determined in such a way that its centre of gravity coin-

cides with the resultant of the load. In this connection a straight line of rotation across the base is required as the boundary of the effective contact area, but an approximate rectangular area of an effective width B' and effective length L' based on equation (6) in both directions is sufficiently accurate in practice.

The influence of inclined loads on the bearing capacity of spread foundations can readily be taken into account by using inclination factors, which have been deduced from more exact calculations (Schultze, 1952; Meyerhof, 1953a). In this connection the influence of the lateral soil resistance on foundations below the surface is particularly effective. For rough foundations the vertical component of the bearing capacity under a load inclined at an angle of α with the vertical (Figure 2) can be approximated by the following inclination factors (Meyerhof, 1956b):

$$(7) \quad \left\{ \begin{array}{l} i_c = i_q = (1 - \alpha/90^\circ)^2 \\ \text{and} \quad i_\gamma = (1 - \alpha/\phi)^2. \end{array} \right.$$

The vertical component of the bearing capacity in the general case of eccentric inclined loads is, therefore, approximately,

$$(8) \quad q = Q/B'L' = d_c i_c c N_c + d_q i_q \gamma D N_q + d_\gamma i_\gamma \gamma B' N_\gamma \cdot \frac{1}{2}.$$

Model loading tests have shown that, as the inclination of the load increases, the bearing capacity of square foundations approaches that of strip foundations and the difference is negligible when failure occurs by sliding, as would be expected. Moreover, for a given inclination of the load an inclined foundation with a base normal to the resultant load gives a greater bearing capacity than a horizontal foundation (Meyerhof, 1953a), and inclined bases are therefore preferable for foundations of arch bridges. The theoretical relationships for the bearing capacity of inclined foundations are found to be similar to those for foundations located on the face of a slope or near the top edge of a slope (Meyerhof, 1957). The theory indicates that the bearing capacity of such foundations decreases with greater inclinations of the slope, especially for cohesionless soils, while for foundations on the top of clay slopes the bearing capacity decreases also considerably with greater height of the slope and is frequently governed by overall slope failure.

In order to resist an eccentric inclined load, the foundation will move and tilt sufficiently. At failure the necessary displacement in the direction of the load is approximately 5 to 20 per cent of the foundation width and the angle of rotation is about 1° to 5° according to the relative density or stiffness of the soil and the depth of the foundation. The influence of these relationships on the behaviour of structures is particularly important for statically indeterminate frames, bridges, and the lower storeys of multi-storey buildings. Model and full-scale tests on steel frames have given valuable indications and showed that the customary foundations can generally be considered as hinged bases and for fixity conditions either wide or deep foundations are required (Meyerhof, 1960b).

If the stiffness of the foundation is small in relation to the stiffness of the soil, then the problem of flexible elastic foundation slabs obtains; here calculation is difficult except under simple conditions. In practice one frequently

uses, therefore, a uniform contact pressure distribution or the method based on the modulus of subgrade reaction (Terzaghi, 1943). Recently the ultimate strength design has been extended to this case and a theory has been developed to estimate the load-bearing capacity of pavements and foundation slabs (Meyerhof, 1962a). In this way relatively simple solutions for the ultimate load under various loading conditions and slab sizes have been obtained. The proposed method can be used for both the assumption of a modulus of subgrade reaction and a modulus of deformation of the soil, and is confirmed by the results of loading tests on large slabs and model tests under concentrated loads.

PILE FOUNDATIONS

The bearing capacity of a single pile is the sum of point resistance and skin friction, both of which depend on the properties of the soil and the characteristics and method of installing the pile. Since the point resistance of vertical and inclined piles loaded in the direction of the pile is practically identical, equation (1) can be used if the bearing capacity factors include the limiting values of the depth factors, and the influence of the shape of the base and the compressibility of the soil (Meyerhof, 1959 and 1961a). For axially loaded single piles the ultimate bearing capacity is therefore, approximately,

$$(9) \quad Q = (cN'_c + \gamma DN'_q)A_b + f_s A_s$$

where A_b = cross sectional area of base,

A_s = surface area of shaft,

f_s = unit skin friction, and other symbols as before.

The semi-empirical bearing capacity factors N'_c and N'_q for round or square driven piles with 60° points are shown in Figure 1. These factors apply only if the pile base is embedded in the load-bearing stratum near the base at least to a depth, which is, approximately

$$(10) \quad D = 4\sqrt{N_\phi}B$$

In other cases the bearing capacity factors can be estimated by interpolating between the values for shallow foundations and piles roughly in direct proportion to the depth ratio D/B . If the soil properties vary near the base, the average values between about four times the pile diameter above the base and one pile diameter below the base should be used in cohesionless soils, while a somewhat smaller range is sufficient in clays, as indicated by the corresponding failure zone.

Although the skin friction in cohesionless soils is frequently neglected, it can be estimated for piles driven into sands by using an earth pressure coefficient of about 0.5 to 1.0 according to the relative density of the material (Meyerhof, 1951). The final value of the skin friction in clays after a few months, which governs the bearing capacity in practice, approximates the shearing strength for soft clays but hardly exceeds one-half of the shearing strength for stiff clays (Tomlinson, 1957) with an upper limit of about 1 ton/sq. ft. Detailed laboratory tests have shown (Meyerhof, 1962b) that the ratio of skin friction to shearing strength of soils depends mainly on the

type and roughness of the pile material. and this ratio is about 0.6 to 0.8 for steel, 0.7 to 0.9 for wood (parallel to the grain) and 0.8 to 1.0 for concrete.

For piles driven into cohesionless soils, static cone penetration tests are particularly suitable for an estimate of the bearing capacity, although for large pile diameters the point resistance is more in loose sand and less in dense sand than the cone resistance, which seems to be due to the volume changes associated with the deformation of the soil in the failure zone (Kerisel, 1961). A rough estimate of the bearing capacity of piles in such soils can also be obtained from the results of standard penetration tests (Meyerhof, 1956a).

More difficult is an estimate of the bearing capacity of piles which are not axially loaded. While a pile with transverse load can be designed similarly to a wall but with the additional soil resistance on the sides parallel to the load (Terzaghi, 1943), the bearing capacity of an obliquely loaded pile is between the transverse and longitudinal value. By the inclined loading the lateral earth pressure governing the skin friction is increased and the axial component of the bearing capacity governing the point resistance is reduced, as for a deep foundation. Hence the bearing capacity of batter piles is smaller if the load is more inclined than the batter of the pile, although the bearing capacity can be increased by fixity at the pile head, as shown by the results of tests in sand (Evans, 1954; Tschebotarioff, 1954). For the purpose of estimating the bearing capacity of such piles the relationships can conveniently be represented by polar bearing capacity diagrams (Meyerhof, 1960a).

Recent investigations of the group action of piles and ultimate bearing capacity of pile foundations have shown that two types of pile groups have to be distinguished, namely pile groups in which the caps or foundations rest on load-bearing soil (piled foundations) and pile groups with caps clear of the ground or with piles driven through soft strata into dense soil (free-standing pile groups).

A piled foundation (Figure 3) can be considered as a pier foundation with its base at the depth of the pile points, and the total bearing capacity is practically independent of the pile spacing. Although at present only model tests with a central vertical load on piled foundations in clay are available (Whitaker, 1960), some exploratory loading tests on vertical pile groups with a close pile spacing in sand (Kishida, 1962) indicate that similar conditions probably obtain generally for piled foundations in uniform soils. For a central load on such foundations with vertical piles at customary spacings up to about four or five pile diameters the total bearing capacity is thus the sum of the base resistance of the equivalent pier and the shearing strength of the soil along the perimeter of the group less the weight of the enclosed soil (Terzaghi and Peck, 1948).

For eccentric or inclined load the foundation tilts and the earth pressure on the sides of the group and the base shear should generally be taken into account (Figure 3). Exploratory loading tests on model piled foundations under vertical load with eccentricities up to one-half the width of the group indicate that the reduced vertical base resistance is offset by an increased skin friction. As a result the total bearing capacity of the group is practically unaffected by the eccentricity of the load, within the above experimental limit, and can be estimated as for piled foundations under a central load.

However, if the foundation width is of the same order of magnitude as the pile length, the lateral earth pressure is relatively small and can be neglected. The calculation is then similar to that of a spread foundation in which only compression piles are loaded and tension piles are ignored by using an effective base area (Meyerhof, 1960a). No loading tests to failure appear to have been made on piled foundations with inclined piles under various loading and soil conditions. An estimate of the bearing capacity can be made in a similar way as for groups with vertical piles but allowing for the horizontal component of the pile resistance, and an approximate procedure has been suggested before (Meyerhof, 1960a).

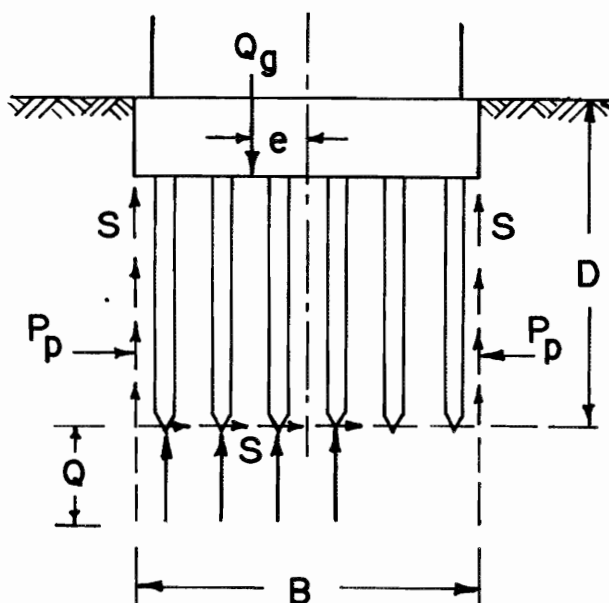


FIGURE 3. Piled Foundation at Failure

The total bearing capacity of a free-standing pile group (Figure 4) is the smaller amount of either the bearing capacity of an equivalent pier foundation or the sum of the bearing capacities of the individual piles. It is found that for a pile spacing of less than about two or three times the pile diameter the individual failure zones in the soil around the piles interact. This overlap produces soil arching between the piles and leads to pier action of the group so that its bearing capacity is the same as that of a piled foundation. For a greater pile spacing the individual action of the piles governs, although the deformation and any volume changes of the soil near the piles have to be considered in estimating the total bearing capacity.

Model tests on free-standing vertical pile groups in clay with central vertical load (Whitaker, 1957; Sowers and Martin, 1961) and with small eccentricities of the load (Saffery and Tate, 1961), which did not affect the total bearing capacity, showed that for a pile spacing of about two pile diameters the group capacity is only about two-thirds of the full bearing capacity reached at a spacing about seven or eight pile diameters (Figure 5). This difference

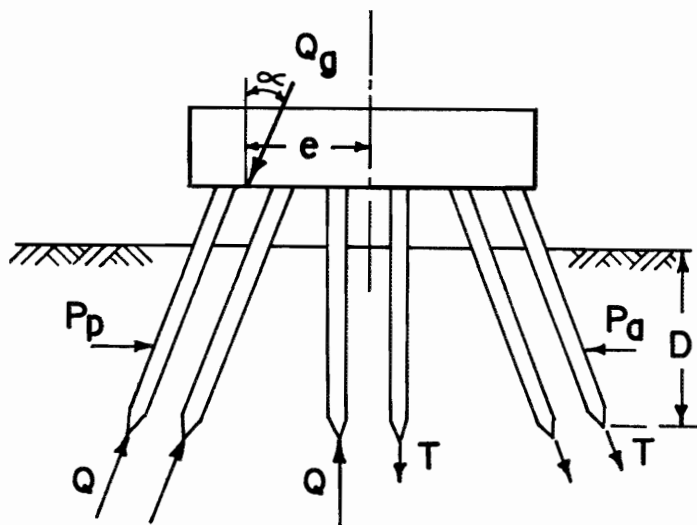


FIGURE 4. Free-Standing Pile Group at Failure

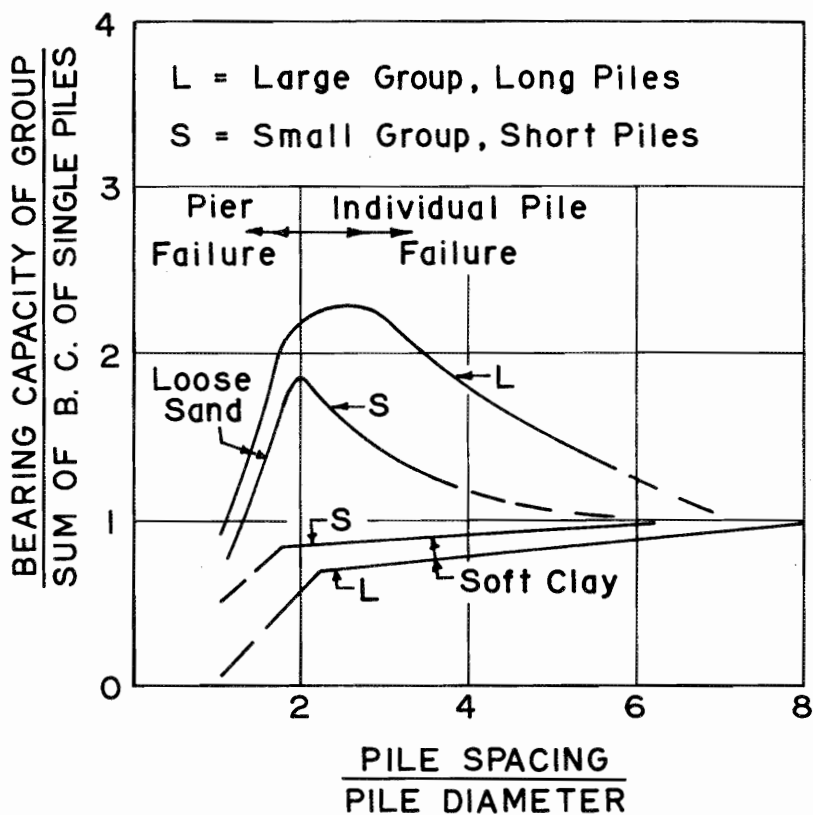


FIGURE 5. Typical Results of Model Tests on Free-Standing Pile Groups

can be explained by the overlapping of the individual zones of shearing deformation in the soil leading to local and progressive shearing failure at customary pile spacings, so that for a given spacing the bearing capacity per pile in a group decreases as the size of the group (number and length of piles) increases.

On the other hand, tests on free-standing model pile groups in loose sand (Kezdi, 1957; Stuart *et al.*, 1960) showed that for a pile spacing of about two pile diameters the total bearing capacity is more than twice the full bearing capacity, which is approached by about six or seven pile diameters (Figure 5). This result is supported by theoretical estimates of the compaction and deformation of loose sand by pile driving to which the effect of an overlapping of the individual soil failure zones has to be added at small pile spacings (Meyerhof, 1959). Thus it was found that for a given pile spacing the bearing capacity per pile in a free-standing group in loose sand increases as the size of the group increases.

In the absence of test results for eccentric and inclined loads the limited data on pile groups summarized above indicate that the bearing capacity of free-standing groups with a small pile spacing can be estimated as for an equivalent pier foundation. For a greater pile spacing, however, the total bearing capacity depends both on the compression and tension piles and on the lateral earth pressure (Figure 4) for which simple methods of analysis have recently been suggested for groups of vertical and inclined piles (Meyerhof, 1960a).

CONCLUSIONS

Although the ultimate bearing capacity of spread foundations can usually be estimated from customary theory and soil tests with sufficient accuracy, recent research has led to a better agreement with observations by improvements of both theory and soil testing. Thus, for strip foundations the theory should be used in conjunction with the shearing strength of the soil obtained from plane strain compression tests, while for circular or square foundations the results of triaxial compression tests apply. The influence of foundation shape and depth, the eccentricity and inclination of the load, the effect of ground-water conditions and sloping ground can readily be taken into account by recent extensions of bearing capacity theory. However, further research is required to deal with variable soil conditions.

The ultimate bearing capacity of single piles can be estimated theoretically for simple cases of loading and uniform soils. Eccentric and inclined loads and variable soil conditions do not yet permit accurate estimates. Moreover, for pile groups only limited information of the ultimate load is available at present from full-scale tests, although the results of model tests give some indication of their behaviour. These tests indicate that piled foundations can generally be treated as piers in practice, but further research is required for eccentric and inclined loads as well as variable soil conditions. More difficult is the problem of free-standing pile groups since the effect of overlapping zones of shearing deformation and volume changes of the soil in the field cannot be deduced from model tests and more full-scale observations are required.

REFERENCES

- BISHOP, A. W., 1961. Discussions on "Soil Properties and their Measurement." *Proc. Fifth Int. Conf. Soil Mech.* 3: 97.
- BJERRUM, L., and KUMMENEJE, O., 1961. "Shearing Resistance of Sand Samples with Circular and Rectangular Cross Sections." *Norwegian Geotechnical Institute Publ. no. 44*: 1.
- EVANS, L. T., 1954. "Bearing Capacity of Piles Subjected to Horizontal Loads." *Am. Soc. Test. Mat., Spec. Tech. Publ. no. 154*: 30.
- HANSEN, J. B., 1955. "Simpel Beregning af Fundamentern Baereevne." *Ingeniøren* 64: 95.
- KERISEL, J. L., 1961. "Deep Foundations in Sands." *Proc. Fifth Int. Conf. Soil Mech.* 2: 73.
- KEZDI, A., 1957. "Bearing Capacity of Piles and Pile Groups." *Proc. Fourth Int. Conf. Soil Mech.* 2: 46.
- KISHIDA, H., 1962. Unpublished Data.
- MEYERHOF, G. G., 1951. "The Ultimate Bearing Capacity of Foundations." *Géotechnique* 2: 301.
- MEYERHOF, G. G., 1953a. "The Bearing Capacity of Foundations under Eccentric and Inclined Loads." *Proc. Third Int. Conf. Soil Mech.* 1: 440.
- MEYERHOF, G. G., 1953b. "Some Recent Foundation Research and Its Application to Design." *Struct. Engineer* 31: 151.
- MEYERHOF, G. G., 1955. "Influence of Roughness of Base and Ground-Water Conditions on the Ultimate Bearing Capacity of Foundations." *Géotechnique* 5: 227.
- MEYERHOF, G. G., 1956a. "Penetration Tests and Bearing Capacity of Cohesionless Soils." *Proc. Am. Soc. Civil Engrs.*, 82, no. SM 1: 866-1.
- MEYERHOF, G. G., 1956b. Discussion on "Rupture Surfaces in Sand under Oblique Loads." *Proc. Am. Soc. Civil Engrs.*, 82, no. SM 3: 1028-15.
- MEYERHOF, G. G., 1957. "The Ultimate Bearing Capacity of Foundations on Slopes." *Proc. Fourth Int. Conf. Soil Mech.* 1: 384.
- MEYERHOF, G. G., 1959. "Compaction of Sands and Bearing Capacity of Piles." *Proc. Am. Soc. Civil Engrs.*, 85, no. SM 6: 1.
- MEYERHOF, G. G., 1960a. "The Design of Franki Piles with Special Reference to Groups in Sands." *Proc. Symp. Pile Found.*, Stockholm: 105.
- MEYERHOF, G. G., 1960b. "Foundation Behaviour and Plastic Design of Frames." *Trans. Eng. Inst. Can.* 4: 58.
- MEYERHOF, G. G., 1961a. "The Ultimate Bearing Capacity of Wedge-shaped Foundations." *Proc. Fifth Int. Conf. Soil Mech.* 2: 105.
- MEYERHOF, G. G., 1961b. Discussion on "Foundations other than Piled Foundations." *Proc. Fifth Int. Conf. Soil Mech.* 3: 193.
- MEYERHOF, G. G., 1962a. "Load-Carrying Capacity of Concrete Pavements." *Proc. Am. Soc. Civil Engrs.*, 88, no. SM 3: 89.
- MEYERHOF, G. G., 1962b. "Some Problems in the Design of Rigid Retaining Walls." *Proc. Fifteenth Can. Soil Mech. Conf.*: 59.
- PRANDTL, L., 1920. "Über die Härte Plastischer Körper." *Proc. Kgl. Ges. Wiss. Göttingen, Math. Phys. Kl.*: 74.
- SAFFERY, M., and TATE, A. P. K., 1961. "Model Tests on Pile Groups in Clay Soils." *Proc. Fifth Int. Conf. Soil Mech.* 2: 129.
- SCHULTZE, E., 1952. "Der Widerstand des Baugrundes gegen Schräge Sohlpressungen." *Bautechnik* 29: 336.
- SKEMPTON, A. W., 1951. "The Bearing Capacity of Clays." *Proc. Build. Res. Congr., London* 1: 180.
- SOWERS, G. F., and MARTIN, C. B., et al., 1961. "The Bearing Capacity of Friction Pile Groups in Homogeneous Clay from Model Studies." *Proc. Fifth Int. Conf. Soil Mech.* 2: 155.
- STUART, J. G., HANNA, T. H., and NAYLOR, A. H., 1960. "Notes on the Behaviour of Model Pile Groups in Sand." *Proc. Symp. Pile Found.*, Stockholm: 97.
- TERZAGHI, K., 1943. *Theoretical Soil Mechanics* (New York: Wiley).
- TERZAGHI, K., and PECK, R. B., 1948. *Soil Mechanics in Engineering Practice* (New York: Wiley).
- TOMLINSON, M. J., 1957. "The Adhesion of Piles Driven in Clay Soils." *Proc. Fourth Int. Conf. Soil Mech.* 2: 66.
- TECHEBOTARIOFF, G. P., 1954. "The Resistance to Lateral Loading of Single Piles and of Pile Groups." *Am. Soc. Test. Mat., Spec. Tech. Publ. no. 154*: 38.
- WHITAKER, T., 1957. "Experiments with Model Piles in Groups." *Géotechnique* 7: 147.
- WHITAKER, T., 1960. "Some Experiments with Model Piled Foundations in Clay." *Proc. Symp. Pile Found.*, Stockholm: 124.