4.6 Rearrangement of Stiffness and Load Arrays. As mentioned previously, the stiffness matrix and the load vector will be rearranged and partitioned so that terms pertaining to the degrees of freedom are separated from those for the support restraints. The beam example in Fig. 4-4a has two degrees of freedom, which are the rotations at points B and C. Figure 4-6 shows revised displacement indexes for the joints of the restrained structure. It is seen that the free displacements are numbered before the support displacements. Otherwise, the sequence of numbering is from left to right, with translations taken before rotations at each joint. These revised displacement indexes may be computed automatically by examining the actual restraint condition for each possible joint displacement. If it is not restrained, the displacement index must be reduced by the cumulative number of restraints encountered up to that point. In such a case, the new index is computed as

$$(j)_{\text{new}} = (j)_{\text{old}} - c_{jo} \qquad \text{(a)}$$

in which c_{jo} represents the cumulative number of restraints for the old index. On the other hand, if the displacement under consideration is actually restrained, the new index is determined by the formula

$$(j)_{\text{new}} = n + c_{jo} \qquad \text{(b)}$$

where n is the number of degrees of freedom. Expressions (a) and (b) will be implemented in a formal way later in this chapter (see Sec. 4.9).

For the present example, a comparison of Fig. 4-6 and Fig. 4-4b shows that the actual degrees of freedom correspond to displacements 4 and 6 of the original numbering system. Therefore, the fourth and sixth rows and columns of the joint stiffness matrix $\mathbf{S}_J$ must be put into the first and second positions. As the first step, rows 4 and 6 of the matrix in Table 4-4 are switched to the first and second rows while all other rows are moved downward without changing their order. Then the array takes the form shown in Table 4-5. Next, the fourth and sixth columns in Table 4-5 are moved to the first two columns, and all other columns are moved to the right without

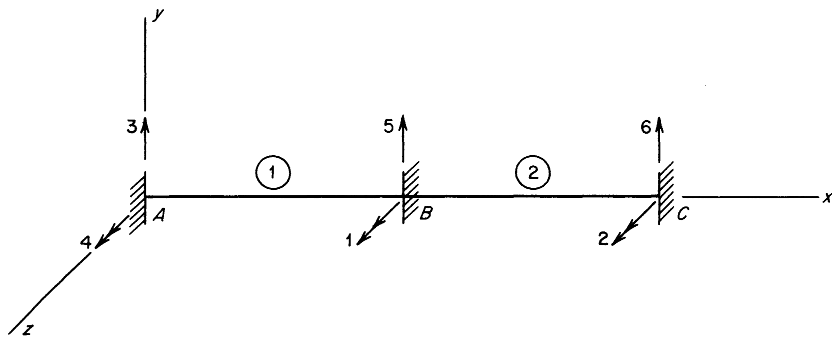

Fig. 4-6. Revised displacement indexes.

Table 4-5
Joint Stiffness Matrix with Rows Rearranged

$$\mathbf{S}_J = \begin{bmatrix} 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \\ 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 0 & 0 & -12 & -6L & 12 & -6L \end{bmatrix} \frac{EI_Z}{L^3}$$

changing their order. This manipulation produces the *rearranged joint stiffness matrix* shown in Table 4-6. Such a matrix is always symmetric and is partitioned in the following manner:

$$\mathbf{S}_J = \begin{bmatrix} \mathbf{S}_{FF} & \mathbf{S}_{FR} \\ \mathbf{S}_{RF} & \mathbf{S}_{RR} \end{bmatrix} \tag{4-8}$$

In this expression the subscripts F and R refer to free and restrained displacements, respectively.

Similarly, the combined load vector $\mathbf{A}_C$ for the example can be rearranged to conform to the numbering system of Fig. 4-6. In this instance the fourth and sixth terms in $\mathbf{A}_C$ are moved to the first and second positions while all others are moved toward the end without changing their order. This rearrangement gives

$$\mathbf{A}_C = \{9PL/8,\ PL/8,\ \vdots\ -P,\ -PL/4,\ -3P/2,\ P/2\}$$

which is partitioned in the form

$$\mathbf{A}_C = \begin{bmatrix} \mathbf{A}_{FC} \\ \mathbf{A}_{RC} \end{bmatrix} \tag{4-9}$$

Thus, the combined loads corresponding to free joint displacements are separated from those corresponding to support restraints.

4.7 Calculation of Results. In the final phase of the analysis, matrices generated in previous steps are used to find unknown joint displacements $\mathbf{D}_F$, support reactions $\mathbf{A}_R$, and member end-actions $\mathbf{A}_{Mi}$ for each

Table 4-6
Joint Stiffness Matrix with Columns Rearranged

$$\mathbf{S}_J = \left[\begin{array}{cc|cccc} 8L^2 & 2L^2 & 6L & 2L^2 & 0 & -6L \\ 2L^2 & 4L^2 & 0 & 0 & 6L & -6L \\ \hline 6L & 0 & 12 & 6L & -12 & 0 \\ 2L^2 & 0 & 6L & 4L^2 & -6L & 0 \\ 0 & 6L & -12 & -6L & 24 & -12 \\ -6L & -6L & 0 & 0 & -12 & 12 \end{array}\right] \frac{EI_Z}{L^3}$$