SECTION 4B
FOUNDATIONS DESIGN

A. SAMER EZELDIN

4B.1 INTRODUCTION

Footings are structural elements that transfer the loads from a structure above ground surface (superstructure) to the underlying soil. The soil-carrying capacity is in general much lower than the high stress intensities carried by the columns and walls in the superstructure. Hence the footings (substructure or foundations) can be considered as interface elements that spread the high-intensity stresses in the supporting elements to much lower stress levels along the weaker soil. This section will be limited to the design of foundations at a shallow depth. Design considerations for foundations in cold regions and in earthquake regions will be also presented.

4B.2 FOOTING TYPES

The most common types of footings are illustrated in Fig. 4B.1.

- Isolated spread footings are used beneath individual columns. They can be square or rectangular in shape. They spread the load of the column to the soil in two perpendicular directions.
- Strip footings or wall footings support bearing walls essentially in a one-dimensional action by cantilevering out on both sides of the wall.
- Combined footings are used to support two or more columns. Usually they have a rectangular or trapezoidal plan. Such footings are often used when a column is close to a property line.
- Pile caps are used to transmit the loads of columns or bearing walls to a series of piles. These piles transfer the loads from the upper poor soil layers to deeper and stronger soil layers.
- A mat or raft foundation is one large footing carrying the loads of all the columns of the structure. This type of foundation is used when weak soil layers are present but piles are not used.

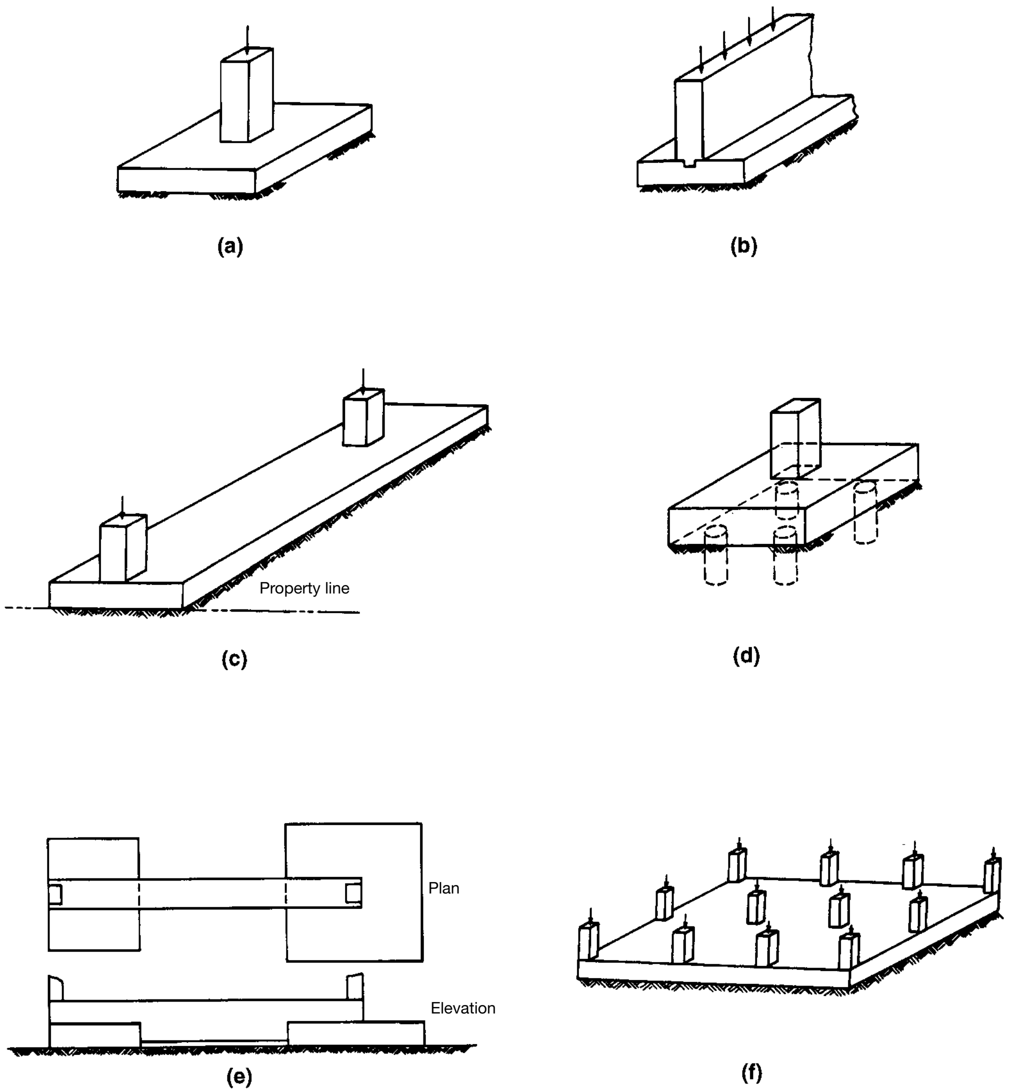


FIGURE 4B.l Types of footings. (*a*) Spread footing. (*b*) Strip or wall footing. (*c*) Combined footing. (*d*) Pile cap. (*e*) Strap footing. (*f*) Mat or raft footing.

4B.3 BEARING CAPACITY OF SOILS UNDER SHALLOW FOUNDATIONS

In order to avoid a bearing failure of the footing, in which the soil beneath the footing moves downward and outward from under the footing, the service load stress under the footing must be limited. This limitation is provided by ensuring that the service load stress q_s is less than or equal to an allowable bearing capacity q_a,

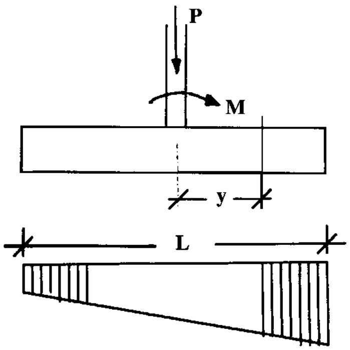


FIGURE 4B.2 Pressure distribution under footing.

$$q_s \leq q_a = \frac{q_{\text{ult}}}{FS} \tag{4B.1}$$

where q_{ult} is the ultimate bearing capacity corresponding to failure of the footing, and FS is a factor of safety, usually taken to be 2.0 to 3.0. Soil mechanics principles are relied upon to establish the ultimate bearing capacity, which depends on the shape of the footing, its depth, the surcharge on top of the footing, the position of the underground water table, and the soil type. The allowable bearing capacity may vary from 15,000 psf for rock to 2000 psf for clay.

The soil beneath a footing is assumed to be subjected to a linearly elastic compression action. The service pressure distribution, shown in Fig. 4B.2, is obtained by the equation

$$q_s = \frac{P}{A} \pm \frac{My}{I} \tag{4B.2}$$

where P = vertical load, positive in compression
A = area of contact surface between soil and footing (length of footing L × width of footing B)
I = moment of inertia of area A
M = moment about centroidal axis of area
y = distance from centroidal axis to point where stress is being calculated

In general tensile stresses are not acceptable underneath concrete footing.

The gross soil pressure is considered the pressure caused by the total load applied on a footing, including dead loads (structure, footing, and surcharge) and live loads. In Fig. 4B.3 the gross soil pressure is

$$q_{\text{gross}} = (h_f - h_c)\gamma_s + h_c\gamma_c + \frac{P}{A} \tag{4B.3}$$

The gross soil pressure must not exceed the allowable bearing capacity q_a in order to avoid failure of the footing.

The net soil pressure is taken as the pressure that will cause internal forces in the footing. Considering Fig. 4B.3, the net soil pressure is

$$q_{\text{net}} = h_c(\gamma_c - \gamma_s) + \frac{P}{A} \tag{4B.4}$$

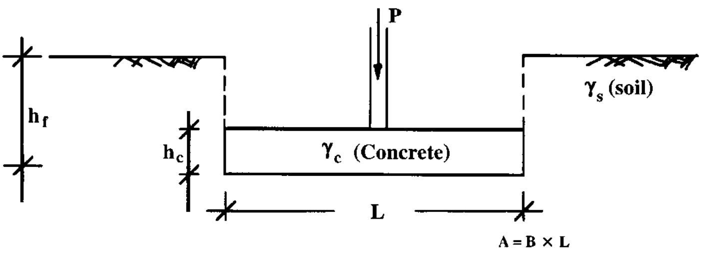


FIGURE 4B.3 Gross and net soil pressures

The net soil pressure is used to calculate the flexural reinforcement and the shear strength of the concrete footing.

4B.4 TYPES OF FAILURE OF FOOTINGS

Three different types of failure may occur in a concrete footing subjected to a concentrated load (Fintel, 1985; Winterkorn and Fang, 1975).

4B.4.1 Diagonal Tension Failure

This type of failure is also referred to as punching shear failure (Fig. 4B.4). The footing fails due to the formation of inclined cracks around the perimeter of the column. Test results have indicated that the critical section can be taken at *d*/2 from the face of the column. To avoid such a failure, the upward ultimate shearing force V_u increased by applying the strength reduction factor ϕ must be lower than the nominal punching shear strength V_c,

$$\frac{V_u}{\phi} \le V_c \tag{4B.5}$$

V_u, acting on the tributary area shown in Fig. 4B.4, is computed with the load factors applied (see Table 3B.l) and ϕ taken as 0.85. V_c is taken as the smallest of

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\,b_0 d \tag{4B.6}$$

$$V_c = \left(\frac{\alpha_s}{b_0/d} + 2\right)\sqrt{f'_c}\,b_0 d \tag{4B.7}$$

$$V_c = 4\sqrt{f'_c}\,b_0 d \tag{4B.8}$$

where b_0 = perimeter of critical section taken at $d/2$ from face of column
d = depth at which tension steel reinforcement is placed
β_c = ratio of long side to short side of column section
α_s = 40 for interior columns, 30 for edge columns, and 20 for corner columns

4B.4.2 One-Way Shear Failure

The footing fails due to the formation of inclined cracks that intercept the bottom of the slab at a distance *d* from the face of the column (Fig. 4B.5). For footings carrying columns with steel base plates, the distance d is measured from a line halfway between the face of the column and the edge of the base plate.

In order to avoid such a failure, Eq. (4B.5) must be satisfied. V_u is the upward ultimate shearing force acting on the tributary area shown in Fig. 4B.5 and ϕ is taken to be 0.85. V_c is taken in accordance with the Ad code as

$$V_c = 2\sqrt{f'_c}\,Bd \tag{4B.9}$$

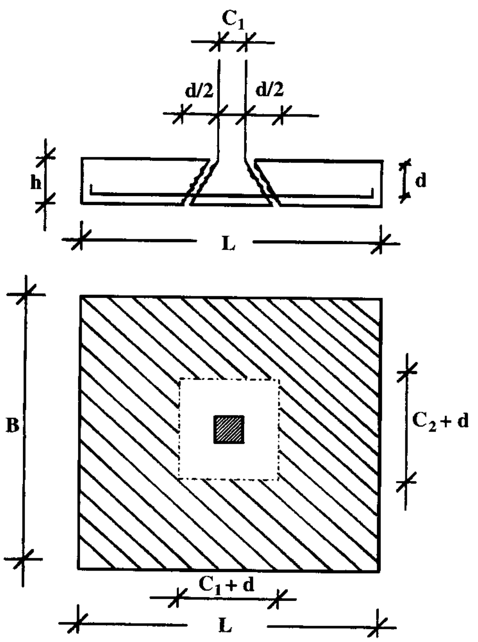


FIGURE 4B.4 Diagonal tension failure.

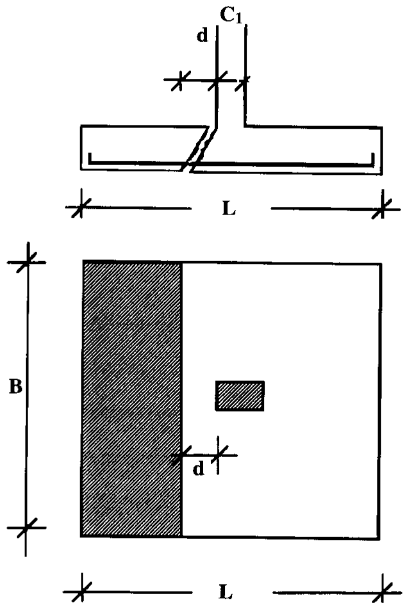


FIGURE 4B.5 One-way shear failure.

4B.4.3 Flexure Failure

A moment M_u/ϕ is acting at the face of column, as shown in Fig. 4B.6, where

$$M_u = (q_{n_u} BX)\frac{X}{2} \tag{4B.10}$$

and $\phi = 0.9$. The factored net soil pressure q_{nu} is obtained by dividing the factored applied loads on the footing by its area.

The moment M_u/ϕ must be lower than or equal to the nominal strength of the concrete section having an effective depth d, a width b, and reinforced with tension steel A_s. Thus

$$\frac{M_u}{\phi} \le M_n \tag{4B.11}$$

where

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \tag{4B.12}$$

$$a = \frac{A_s f_y}{0.85 f'_c B} \tag{4B.13}$$

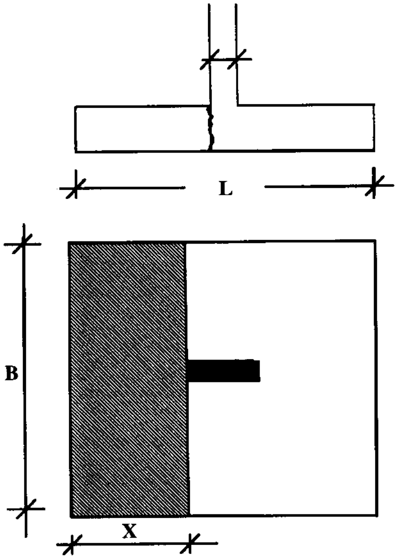


FIGURE 4B.6 Flexure failure.

In a similar manner, the moment M_u/ϕ acting at the perpendicular face of the column must be resisted by the tension reinforcement layer placed orthogonally, resulting in two layers of steel, one in each direction. ACI requires the minimum steel reinforcement placed in structural slabs of uniform thickness to be

$$(A_s)_{\min} = 0.002bh, \quad \text{for } f_y = 40 \text{ or } 50 \text{ ksi } (276 \text{ or } 345 \text{ MPa}) \tag{4B.14}$$

$$(A_s)_{\min} = 0.0018bh, \quad \text{for } f_y = 60 \text{ ksi } (414 \text{ MPa}) \tag{4B.15}$$

4B.4.4 Additional Design Aspects

4B.4.4.1 Development of Reinforcement

The flexural reinforcement is provided in the footing with the assumption that the reinforcement stress reaches the yield stress f_y at the face of the column. In order to ensure that, the reinforcement must be extended beyond the critical section to develop this stress. This implies that a development length l_d must be provided from the critical section. The ACI code development length requirements for different bar diameters were presented in Sec. 3B.6.

4B.4.4.2 Load Transfer from Column to Footing

The ACI code requires that the forces acting on the column be safely transmitted to the footing. Dowels in steel connection are used to transfer any tension forces whereas the compression forces are transferred by bearing.

The bearing capacity of the column is checked by

$$\frac{P_u}{\phi} \le 0.85 f'_c A_1 \tag{4B.16}$$

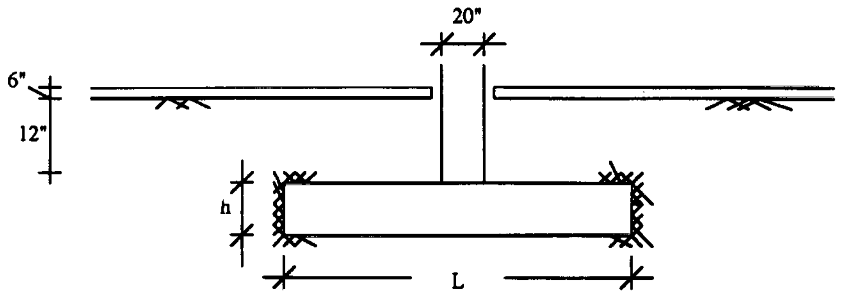
where $\phi = 0.7$
P_u = ultimate load applied on column
A_1 = the column area

The bearing capacity of the concrete footing is checked by

$$\frac{P_u}{\phi} \le 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}} \qquad (4B.17)$$

where A_2 is the maximum area of the supporting surface that is geometrically similar and concentric with A_1. The value of $\sqrt{A_2/A_1}$ should not be greater than 2.

Example 4B.1: Design of Square Spread Footing Design an interior spread footing to carry a service load of 500 kips (2225 kN) and a service live load of 350 kips (1558 kN) from a 20-in (508-mm)-square tied column containing no. 11 bars [1.56 in^2 (960 mm^2)] as the principal column steel. The top of the footing will be covered with l2in (305 mm) of fill having a density of 110 lb/ft^3 (1337 kg/m^3) and a 6-in (152-mm) basement floor. The basement floor loading is 100 psf (4.78 kPa). The allowable bearing pressure on the soil q_a is 7000 psf (335 kPa). Use f'_c = 5000 psi (34.45 MPa) and f_y = 60,000 psi (413.4 MPa).



Solution

1. Estimate the thickness of the footing as between one and two times the width of the column, say h = 36 in (914 mm). The allowable net soil pressure is

$$q_{\text{net}} = 7 \text{ ksf} - (\text{weight of footing} + \text{soil} + \text{floor} + \text{floor load})$$

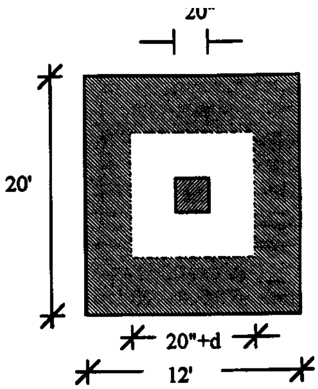
$$= 7 - \left(\frac{36}{12} \times 0.15 + 1 \times 0.11 + 0.5 \times 0.15 + 0.1\right) = 6.265 \text{ ksf } (299.8 \text{ kPa})$$

2. Required area $= \dfrac{P_D + P_L}{q_{\text{net}}} = \dfrac{500 + 350}{6.265} = 135.7 \text{ ft}^2 \ (12.9 \text{ m}^2)$

   Try a 12-fl (3.6-in) square by 36-in (914-mm)-thick footing.

3. The factored net soil pressure is obtained from

$$q_{n_u} = \frac{1.4 \times 500 + 1.7 \times 350}{12^2} = 9 \text{ ksf } (430.6 \text{ kPa})$$



4. The two-way shear check is performed on the critical section at the distance $d/2$ from the face of the column,

$$d = h - \text{concrete cover} - \text{bar diameter}$$

$$= 36 \text{ in} - 3 \text{ in} - 1 \text{ in} = 32 \text{ in (813 mm)}$$

$$V_u = q_{n_u} \text{ (tributary area)} = 9\left[12^2 - \left(\frac{20+32}{12}\right)^2\right]$$

$$= 1127 = \text{kips (5015 kN)}$$

$$\frac{V_u}{\phi} = \frac{1127}{0.85} = 1326 \text{ kips (5900.7 kN)}$$

V_c is the smallest of

$$\left(\frac{2+4}{\beta_c}\right)\sqrt{f'_c}b_0d = \frac{(2+4/1)\sqrt{5000}\times 52\times 4\times 32}{1000} = 2824 \text{ kips (12,567 kN)}$$

$$\left(\frac{\alpha_s}{b_0d}+2\right)\sqrt{f'_c}b_{0d} = \frac{[40/(52\times 4/32)+2]\sqrt{5000}\times 52\times 4\times 32}{1000} = 3836 \text{ kips (17,070 kN)}$$

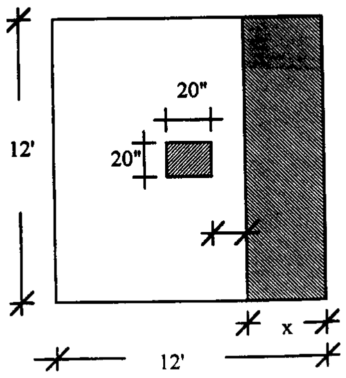
$$4\sqrt{f'_c}b_0d = \frac{4\sqrt{5000}\times 52\times 4\times 32}{1000} = 1883 \text{ kips (8379 kN)} \qquad \therefore \text{Controls design.}$$

Hence

$$V_c = 1883 \text{ kips (8379 kN)} > \frac{V_u}{\phi} = 1326 \text{ kips (5900.7 kN)}$$

The thickness of the footing is adequate to prevent two-way shear failure.

5. The one-way shear is performed on a critical section at the distance d from the face of the column. The width of the tributary area is



$$x = \frac{144-20}{2} - 32 = 30 \text{ in (762 mm)}$$

$$V_u = q_{n_u} \text{ (tributary area)} = 9\left(12\times\frac{30}{12}\right) = 270 \text{ kips (1201 kN)}$$

$$\frac{V_u}{\phi} = \frac{270}{0.85} = 318 \text{ kips (1415 kN)}$$

$$V_c = 2\sqrt{f'_c}bd = \frac{2\sqrt{5000}\times 12\times 12\times 32}{1000}$$

$$= 652 \text{ kips (2901 kN)}$$

Hence

$$V_c = 652 \text{ kips (2901 kN)} > \frac{V_u}{\phi} = 318 \text{ kips (1415 kN)}$$

The thickness of the footing is capable of preventing one-way shear failure.

6. Design for flexure reinforcement. The width of the tributary area is

$$y = \left(\frac{144 - 20}{2}\right) = 62 \text{ in (1575 mm)}$$

$$M_u = 9\left(12 \times \frac{62}{12} \times \frac{62}{2 \times 12}\right) = 1442 \text{ ft} \cdot \text{kip } 1955 \text{ kN} \cdot \text{m)}$$

$$\frac{M_u}{\phi} = \frac{1442}{0.9} = 1602 \text{ ft} \cdot \text{kips (2172.6 kN} \cdot \text{m)}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

Assuming $(d - a/2) = 0.9d$,

$$(A_s)_{\text{req}} = \frac{M_u/\phi}{f_y(0.9d)} = \frac{1602 \times 12{,}000}{60{,}000(0.9 \times 32)} = 11 \text{ in}^2 \text{ (7095 mm}^2\text{)}$$

$$(A_s)_{\min} = 0.0018bh = 0.0018(144 \times 36) = 9.3 \text{ in}^2 \text{ (5998 mm}^2\text{)}$$

Choose 11 no. 9 each way; then $A_s = 11 \text{ in}^2 > (A_s)_{\min}$.

Check the chosen area of the reinforcement:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{11 \times 60{,}000}{0.85 \times 5000 \times 144} = 1.08 \text{ in (27.4 mm)}$$

Then

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 11 \times 60{,}000\left(32 - \frac{1.08}{2}\right) = 20.8 \times 10^6 \text{ in} \cdot \text{lb (2350 kN} \cdot \text{m)}$$

$$= 1730 \text{ ft kips (2350kN} \cdot \text{m)} > \frac{M_u}{\phi} = 1602 \text{ ft} \cdot \text{kips (2172.6 kN} \cdot \text{m)}$$

7. Check the development length:

$$I_{db} = 0.04\frac{A_b f_y}{\sqrt{f_c'}} = 0.04\frac{1.0 \times 60{,}000}{\sqrt{5000}} = 33.94 \text{ in} \cong 34 \text{ in (864 mm)}$$

No increase in the basic development length I_d is needed to account for the effects of bar spacing, cover, stirrup confinement, and reinforcement location.

$$(I_d)_{\min} = 0.03 d_b \frac{f_y}{\sqrt{f'_c}} = 0.03 \times 1.125 \frac{60{,}000}{\sqrt{5000}} = 28.7 \text{ in } (729 \text{ mm})$$

Hence choose $I_d = 34$ in (864 mm).

The bar length available from the location of the maximum moment on each side is

$$y - \text{concrete cover} = 62 - 3 = 59 \text{ in } (1499 \text{ mm}) > 34 \text{ in } (864 \text{ mm})$$

Therefore the development length is provided.

8. Check the bearing at the column-footing interface:

$$P_u = 1.4 \times 500 + 1.7 \times 350 = 1295 \text{ kips } (5763 \text{ kN})$$

$$\frac{P_u}{\phi} = \frac{1295}{0.7} = 1850 \text{ kips } (8233 \text{ kN})$$

Footing capacity:

$$P_n = 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}}$$

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{144 \times 144}{20 \times 20}} = 7.2 \qquad \therefore \text{Use } \sqrt{\frac{A_2}{A_1}} = 2.$$

Then

$$P_n = \frac{0.85 \times 5000 \times 20 \times 20 \times 2}{1000} = 3400 \text{ kips } (15{,}130 \text{ kN})$$

$$P_n = 3400 \text{ kips } (15{,}130 \text{ kN}) > \frac{P_u}{\phi} = 1850 \text{ kips } (8233 \text{ kN}) \qquad \text{O.K.}$$

Column capacity:

$$P_n = 0.85 f'_c A_1 = \frac{0.85 \times 5000 \times 20 \times 20}{1000} = 1700 \text{ kips } (7565 \text{ kN})$$

$$P_n < \frac{P_u}{\phi}$$

Hence dowels are needed to transfer the excess load.

$$\text{Area of dowel required} = \frac{1850 - 1700}{f_y} = \frac{150}{60} = 2.5 \text{ in}^2 \ (1613 \text{ mm}^2)$$

The area of the dowel must be higher than the minimum specified by the ACI code,

$$(\text{Area of dowel})_{\min} = 0.005A_g = 0.005 \times 20 \times 20 = 2 \text{ in}^2 \text{ (1290 mm}^2) < 2.5 \text{ in}^2 \text{ (1613 mm}^2)$$

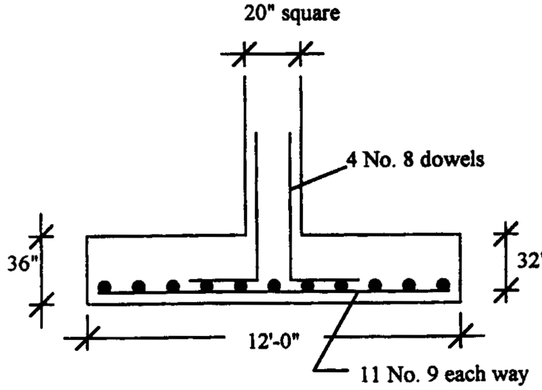
Hence the value of 2.5 in^2 controls. Choose 4 no. 8 bars [3.16 in^2 (2038 mm^2)]. The dowels must extend at least the compression development length of the 8 bar into the footing,

$$I_{db} = 0.02d_b \frac{f_y}{\sqrt{f'_c}} \geq 0.0003d_b f_y$$

$$I_{db} = 15 \text{ in (381 mm)} < 16 \text{ in (406 mm)}$$

Hence extend 4 no. 8 dowels at least 16 in (406 mm) into the footing.

The complete design is detailed here.



4B.5 RECTANGULAR FOOTINGS

Rectangular footings are usually employed as spread footings when the space is inadequate for a square footing. The design procedures for these footings are basically similar to those of square footings, except that the one-way shear and bending moments have to be checked in both principal directions. Also in such footings the flexural reinforcement in the short direction has to be distributed in three regions with more concentration in the region beneath the column (Fig. 4B.7). The total required reinforcement A_s is obtained such that the bending moment at the column face (section A-A) is resisted. The reinforcement in the central region under the column shall be $A_s[2/(\beta + 1)]$, where β is the ratio of the long side of the footing to the short side. The remaining reinforcement is distributed equally between the two outer regions of the footing.

Example 4B.2: Design of Rectangular Footing Redesign the footing of Example 4B.1, given that the maximum width of the footing cannot exceed 10 ft (3 in).

Solution

1. From the solution of Example 4B.1, take $h = 36$ in (914 mm) and $d = 32$ in (813 mm). Then

$$q_{\text{net}} = 6.265 \text{ ksf (299.8 kPa)}$$

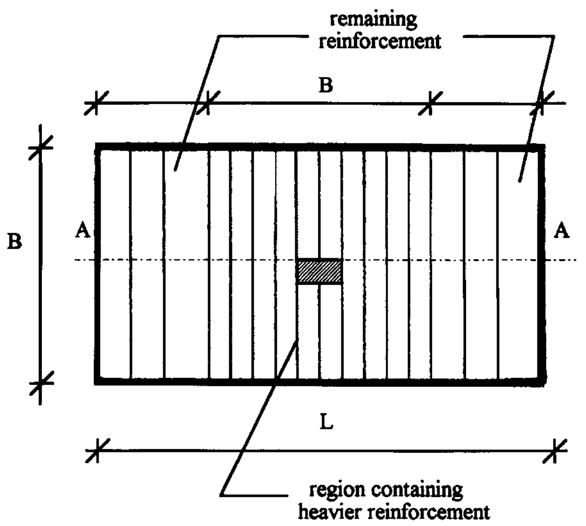


FIGURE 4B.7 Reinforcement in short direction of a rectangular footing.

2.
$$\text{Required area} = \frac{P_D + P_L}{q_{\text{net}}} = \frac{500 + 350}{6.265} = 135.7 \text{ ft}^2 \ (12.9 \text{ m}^2)$$

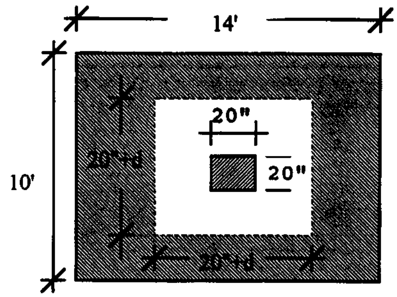
$$\text{Required length} = \frac{135.7}{10} = 13.57 \text{ ft } (4.2 \text{ m}) \qquad \text{Take } L = 14 \text{ ft } (4.25 \text{ m}).$$

Try a footing 10 ft (3 m) wide by 14 ft (4.25 m) long by 36 in (914 mm) thick.

3. The factored net soil pressure is

$$q_{n_u} = \frac{1.4 \times 500 + 1.7 \times 350}{10 \times 14} = 9.25 \text{ ksf } (442.6 \text{ kPa})$$

4. Two-way shear analysis:



$$V_u = q_{n_u}(\text{tributary area}) = 9.25\left[140 - \left(\frac{20+32}{12}\right)^2\right] = 1121 \text{ kips (4988 kN)}$$

$$\frac{V_u}{\phi} = \frac{1121}{0.85} = 1319 \text{ kips (5869 kN)}$$

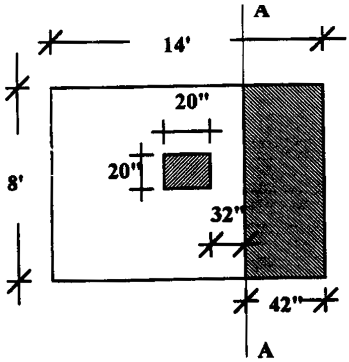
$$V_c = 4\sqrt{f'_c}b_0d = \frac{4\sqrt{5000} \times 52 \times 4 \times 32}{1000} = 1883 \text{ kips (8379 kN)}$$

Hence

$$V_c = 1883 \text{ kips (8379 kN)} > \frac{V_u}{\phi} = 1319 \text{ kips (5869 kN)}$$

The thickness of the footing is adequate to prevent two-way shear failure.

5. The one-way shear is performed along two sections.
   a. Section A-A:



$$V_u = 9.25\left(8 \times \frac{42}{12}\right) = 259 \text{ kips (1153 kN)}$$

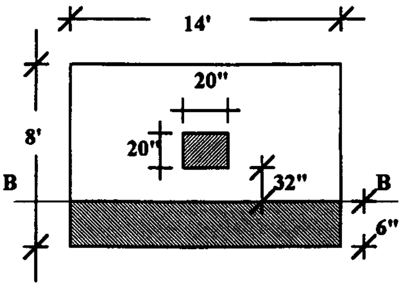
$$\frac{V_u}{\phi} = \frac{259}{0.85} = 305 \text{ kips (1356 kN)}$$

$$V_c = 2\sqrt{f'_c}bd \; \frac{2\sqrt{5000} \times 8 \times 12 \times 32}{1000} = 434 \text{ kips (1931 kN)}$$

Hence

$$V_c = 434 \text{ kips (1931 kN)} > \frac{V_u}{\phi} \; 305 \text{ kips (1356 kN)}$$

b. Section B-B:



$$V_u = 9.25\left(14 \times \frac{6}{12}\right) = 64.75 \text{ kips (289 kN)}$$

$$\frac{V_u}{\phi} = \frac{64.75}{0.85} = 76 \text{ kips (340 kN)}$$

$$V_{uc} = \frac{2\sqrt{5000} \times 14 \times 12 \times 32}{1000} = 760 \text{ kips (3382 kN)}$$

The thickness of the footings is capable of preventing two-way shear failure in both directions.

6. Design for flexure reinforcement
   a. Section A-A (Long direction):

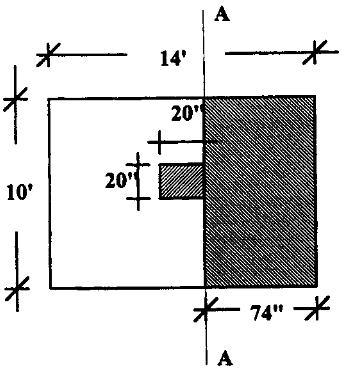
$$M_u = 9.25\left(10 \times \frac{74}{12} \times \frac{74}{2 \times 12}\right) = 1759 \text{ ft} \cdot \text{kips (2385 kN} \cdot \text{m)}$$

$$\frac{M_u}{\phi} = \frac{1759}{0.9} = 1954 \text{ ft} \cdot \text{kips (2650.2 kN} \cdot \text{m)}$$

$$(A_s)_{\text{req}} = 13.6 \text{ in}^2 \text{ (8772 mm}^2\text{)}$$

$$(A_s)_{\text{min}} = 0.0018bh = 0.0018 \times 10 \times 12 \times 36 = 7.8 \text{ in}^2 \text{ (5031 mm}^2\text{)} < (A_s)_{\text{req}}$$

Choose 14 no. 9 [(14 in^2)(9030 mm^2)] in the long direction.



b. Section B-B (short direction):

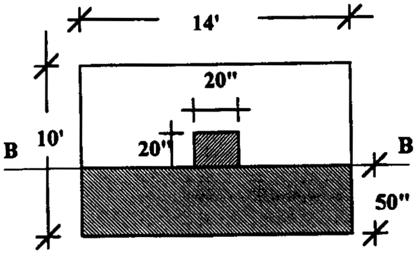
$$M_u = 9.25\left(14 \times \frac{50}{12} \times \frac{50}{2 \times 12}\right) = 1124 \text{ ft} \cdot \text{kip } (1524 \text{ kN} \cdot \text{m})$$

$$\frac{M_u}{\phi} = \frac{1124}{0.9} = 1249 \text{ ft} \cdot \text{kips } (1693.5 \text{ kN} \cdot \text{m})$$

$$(A_s)_{\text{req}} = 8.8 \text{ in}^2 \text{ } (5676 \text{ mm}^2)$$

$$(A_s)_{\min} = 0.0018bh = 0.0018 \times 14 \times 12 \times 36 = 10.9 \text{ in}^2 \text{ } (5031 \text{ mm}^2) \qquad \therefore \text{ Controls.}$$

Choose $A_s = 10.9 \text{ in}^2$ (7031 mm^2) in the short direction.



In the 10-ft inner region provide

$$10.9\left(\frac{2}{\beta + 1}\right) = 10.9\left(\frac{2}{14/10 + 1}\right) = 9.08 \text{ in}^2 \text{ } (5857 \text{ mm}^2) \qquad \text{(12 no. 8)}$$

In the 2-ft outer regions provide

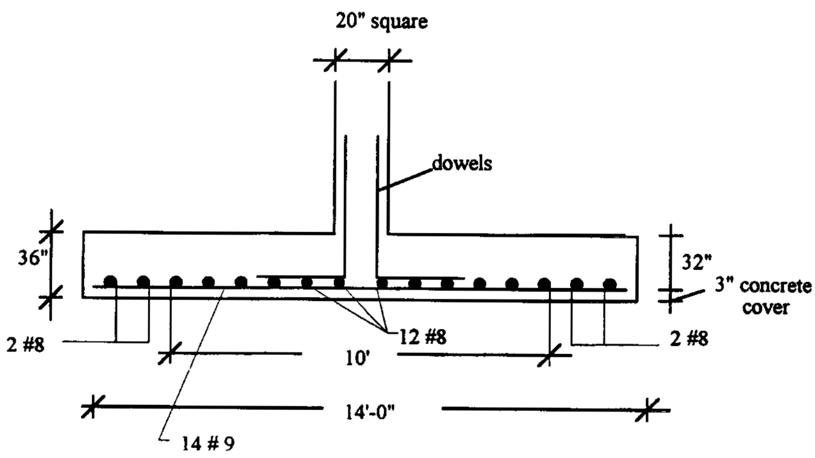
$$\frac{10.9 - 9.08}{2} = 0.91 \text{ in}^2 \text{ } (587 \text{ mm}^2) \text{ on each side} \qquad \text{(2 no. 8)}$$

The checks for the development length and the bearing at the column-footing interface are similar to those in Example 4B.1. The details of the final design are shown at the top of the next page

4B.6 ECCENTRICALLY LOADED SPREAD FOOTINGS

In some cases, due to a moment at the column base or an eccentrically applied load, the bearing pressure beneath the footing will deviate from the uniform distribution shown in Fig. 4B.2. The design of such a footing can be performed in a manner similar to that of a square or rectangular footing with the following conditions satisfied:

1. Tensile stresses are not generated beneath the footing under extreme loading conditions.
2. The difference in compressive stresses between the two edges of the footing is not extremely high in order to avoid tilting settlement of the footing.



3. The designs for one-way shear, two-way shear, and bending moment are performed using the actual pressures under the footing resulting from critical loading conditions that might occur.

4B.7 COMBINED FOOTINGS

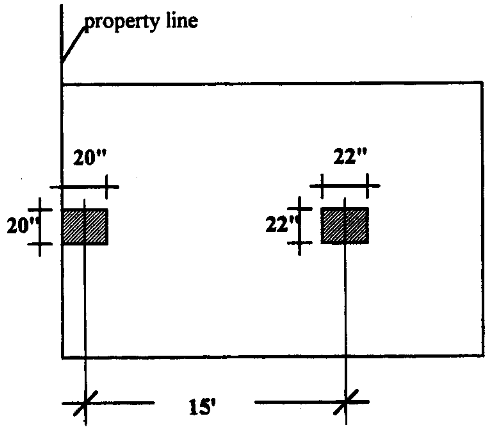
A combined footing 15 usually used when an exterior column is close to a property line, preventing the use of an isolated spread footing (see Fig. 4B.1). Thus a combined footing is used to support the exterior column along with an interior column. The shape of a combined footing is usually rectangular or trapezoidal. That shape is carefully designed in order to have the centroid of the footing coincide with the resultant of the column loads applied to the footing. For cases where the load is lower on the exterior column P_{ext} than on the interior column P_{int} a rectangular combined footing is considered an economical solution. In cases when $0.5 < P_{int}/P_{ext} < 1$ a trapezoidal footing is preferred. However, when $P_{int}/P_{ext} < 0.5$, a strip or cantilever footing should be considered. In a strip or cantilever footing, the overturning of the exterior footing is prevented by connecting it with an adjacent interior footing using a strip beam. The exterior footing is designed for one-way bending whereas the interior footing is designed for two-way bending, as in isolated footings. The strip beam is subjected to a constant shear force and a linearly decreasing negative moment. This behavior is similar to a cantilever beam. It is preferable that all three elements, namely, the exterior footing, the interior footing, and the strip beam, have the same thickness. This thickness is chosen such that the shear requirement for the footings and the shear and flexure requirements for the strip beam are satisfied.

Example 4B.3: Design of a Combined Footing Design a combined rectangular footing to support two columns. The exterior column is 20 in (508 mm) square, carrying service loads of 150 kips (667.5 kN) dead load and 120 kips (534 kN) live load. The interior column is 22 in (559 mm) square carrying service loads of 200 kips (890 kN) dead load and 180 kips (801 kN) live load. The distance between the columns is 15 ft (4.6 in) centerline to centerline. The top of the footing is 3 ft (914 mm) below grade and the fill above the footing is 120 lb/ft^3 (1459 kg/in^3). Use $f'_c = 4000$ psi (27.56 MPa) and $f_y = 60{,}000$ psi (413.4 MPa).

Solution

1. Estimate the depth of the footing to be one to two times the column dimension. Take $h = 36$ in (914 mm). Hence

$$d = h - \text{cover} - \text{bar diameter} = 36 - 3 - 1 = 32 \text{ in (813 mm)}$$



The allowable net soil pressure is

$$q_{\text{net}} = 5 - (\text{weight of footing + soil}) = 5 - (3 \times 0.15) - (3 \times 0.12) = 4.19 \text{ ksf (200.5 kPa)}$$

2. $$\text{Required area} = \frac{P_D + P_L}{q_{\text{net}}} = \frac{(150 + 200) + (120 + 180)}{4.19} = 155 \text{ ft}^2 \text{ (14.73 m}^2\text{)}$$

The distance of the center of gravity of loads from the exterior column is

$$\frac{(150 + 120)0 + (200 + 180)15}{(150 + 120) + (200 + 180)} = 8.77 \text{ ft (2.67 m)}$$

The distance from the property line to the center of gravity is

$$\frac{10 \text{ in}}{12} + 8.77 \text{ ft} = 9.6 \text{ ft (2.93 m)}$$

$$\text{Length of footing} = 2 \times 9.6 \text{ ft} = 19.2 \text{ ft (5.85 m)} \qquad \therefore \text{ Say 19.5 ft (5.9 m).}$$

$$\text{Width of footing} = \frac{155}{19.5} = 7.95 \text{ ft (24 m)} \qquad \therefore \text{ Say 8 ft (2.5 m).}$$

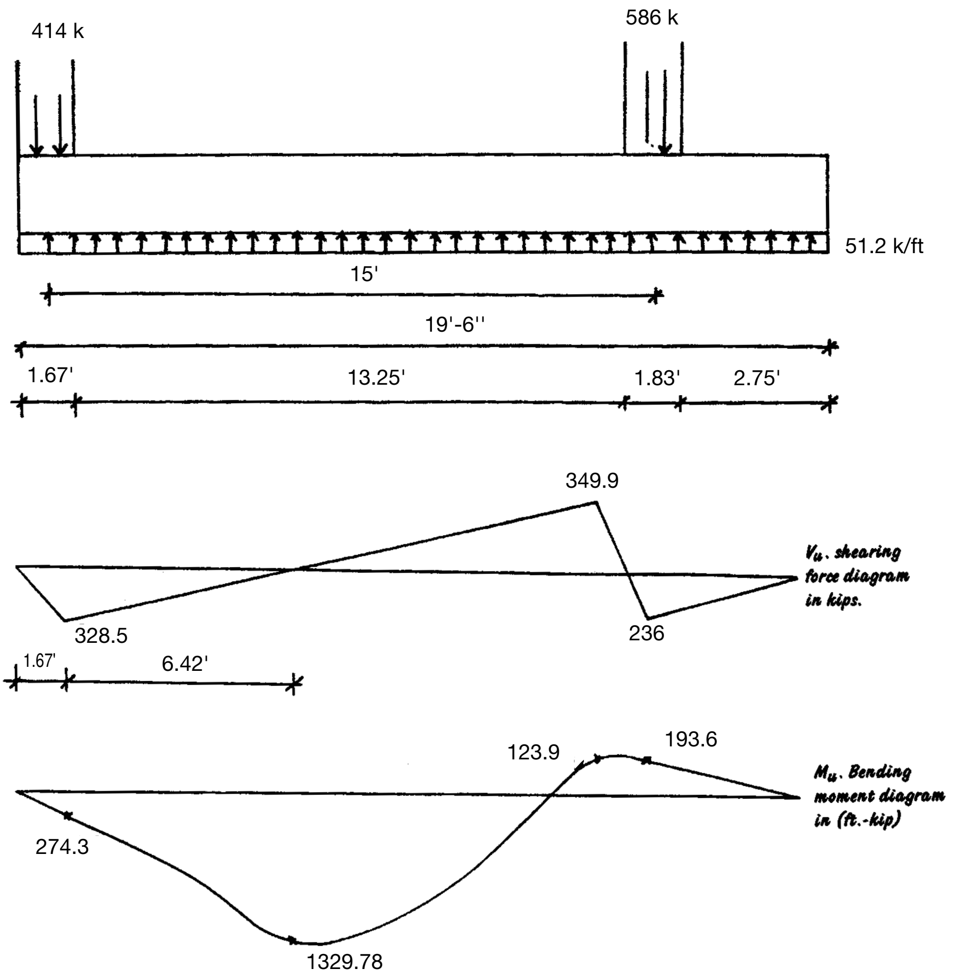
Try a 19.5 × 8 ft (5.9 × 2.5 m) rectangular footing with 36-in (914-mm) thickness.

3. The factored net soil pressure is

$$q_{n_u} = \frac{1.4(150 + 200) + 1.7(120 + 180)}{19.5 \times 8} = 6.4 \text{ ksf (306.25 kPa)}$$

$$= 6.4 \times 8 = 51.2 \text{ kips/ft (7.47 kN/m)}$$

4. Using q_{na}/ft, determine the factored bending moment and shearing force diagrams for the footing. These diagrams are plotted here for the full 8-ft (2.5-in) width of the footing.



5. Check one-way shear: V_u at a distance d from the interior face of the inner column is

$$V_u = 349.9 - \frac{32}{12}51.2 = 213.3 \text{ kips } (949.5 \text{ kN})$$

$$\frac{V_u}{\phi} = \frac{213.37}{0.85} = 251 \text{ kips } (1117 \text{ kN})$$

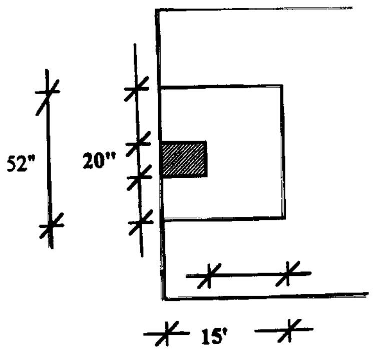
$$V_c = 2\sqrt{f_c'}bd = 2\sqrt{4000} \times 8 \times 12 \times 32 = 388.6 \text{ kips } (1729.3 \text{ kN})$$

Hence,

$$V_c = 388.6 \text{ kips } (1729.3 \text{ kN}) > \frac{V_u}{\phi} = 251 \text{ kips } (1117 \text{ kN})$$

The thickness of the footing is adequate to prevent one-way shear failure.

6. Check of two-way shear
   a. Exterior column:



$$V_u = 414 - q_{na}\left(\frac{52}{12}\right)\left(\frac{36}{12}\right) = 414 - 6.4 \times 4.33 \times 3 = 330.9 \text{ kips (1472.5 kN)}$$

$$\frac{V_u}{\phi} = \frac{330.9}{0.85} = 389.3 \text{ kips (1732 kN)}$$

$$b_0 = 2 \times 36 + 52 = 124 \text{ in (3150 mm)}$$

V_c is the smallest of

$$\frac{(2+4)\sqrt{4000} \times 124 \times 32}{1000} = 1506 \text{ kips (6702 kN)}$$

$$\left(\frac{30}{124/32} + 2\right)\sqrt{4000} \times 124 \times 32 = 2444 \text{ kips (10876 kN)}$$

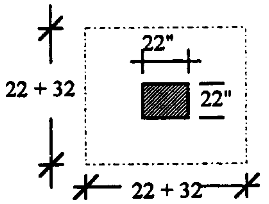
$$4\sqrt{4000} \times 124 \times 32 = 1003 \text{ kips (4463 kN)} \qquad \therefore \text{ Controls design.}$$

Hence

$$\frac{V_u}{\phi} = 389.3 \text{ kips (1732 kN)} < 1003 \text{ kips (4463 kN)}$$

The thickness is adequate for the exterior column.

b. Interior column:



$$V_u = 586 - 6.4\left(\frac{54}{12}\right)\left(\frac{54}{12}\right) = 456.4 \text{ kips (2031 kN)}$$

$$\frac{V_u}{\phi} = \frac{456.4}{0.85} = 537 \text{ kips (2389 kN)}$$

$$b_0 = 4 \times 54 = 216 \text{ in (5486 mm)}$$

$$V_c = 4\sqrt{4000} \times 216 \times 32 = 1749 \text{ kips (7783 kN)}$$

Hence

$$\frac{V_u}{\phi} = 537 \text{ kips } (2389 \text{ kN}) < 1749 \text{ kips } (7783 \text{ kN})$$

The thickness is adequate for the interior column.

7. Design for flexure reinforcement
   a. Midspan negative moment:

$$\frac{M_u}{\phi} = \frac{1329.78}{0.9} = 1477.5 \text{ ft} \cdot \text{kips } (2003.5 \text{ kN} \cdot \text{m})$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

Assuming $(d - a/2) = 0.9d$,

$$(A_s)_{\text{req}} = \frac{M_u/\phi}{f_y(0.9d)} = \frac{1477.5 \times 12{,}000}{60{,}000 \times 0.9 \times 32} = 10.26 \text{ in}^2 \text{ } (6618 \text{ mm}^2)$$

$$(A_s)_{\min} = 0.0018bh = 0.0018 \times 8 \times 12 \times 36 = 6.22 \text{ in}^2 \text{ } (4012 \text{ mm}^2)$$

Choose 11 no.9 $\quad \therefore A_s = 11 \text{ in}^2 \text{ } (7095 \text{ mm}^2) > (A_s)_{\min}$
Checking the area of the reinforcement,

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{11 \times 60{,}000}{0.85 \times 4000 \times 8 \times 12} = 2.0 \text{ in } (50.8 \text{ mm})$$

$$M_n = 11 \times 60{,}000 \left(32 - \frac{2}{2} \right) = 20.45 \times 10^6 \text{ in} \cdot \text{lb}$$

$$= 1704 \text{ ft} \cdot \text{kips } (2311 \text{ kN} \cdot \text{m}) > 1477.5 \text{ ft} \cdot \text{kips } (2003.49 \text{ kN} \cdot \text{m})$$

Use 11 no. 9 top bars at midspan.

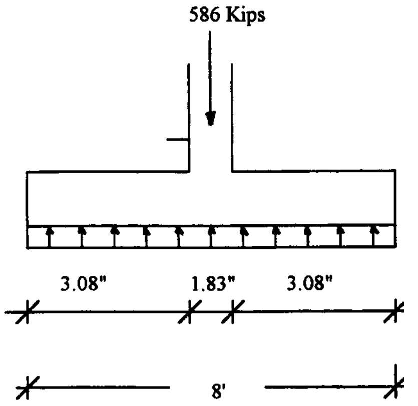
   b. Interior column positive moment:

$$\frac{M_u}{\phi} = \frac{193.6}{0.9} = 215 \text{ ft} \cdot \text{kips } (291.54 \text{ kN} \cdot \text{m})$$

This would require $A_s = 2 \text{ in}^2$ (1290 mm²), which is less than $(A_s)_{\min} = 6.22 \text{ in}^2$ (4012 mm²). Use 7 no. 9 bottom bars for the interior column.

8. Design for transverse beams under columns. It is assumed that transverse beams under each column transmit the load from the longitudinal direction into the columns. The width of the transverse beam is taken to be the width of the column plus an extension d/2 on each side of the column.
   a. Transverse steel under interior column:

$$\text{Beam width} = 22 + 2\left(\frac{32}{2} \right) = 54 \text{ in } (1372 \text{ mm})$$



$$q_{n_u} = \frac{586}{8} = 73.25 \text{ kips/ft (10.68 kN/m)}$$

$$M_u = 73.25\frac{3.08^2}{2} = 347.44 \text{ ft} \cdot \text{kips (471.1 kN} \cdot \text{m)}$$

$$\frac{M_u}{\phi} = \frac{347.44}{0.9} = 386.04 \text{ ft} \cdot \text{kips (523.47 kN} \cdot \text{m)}$$

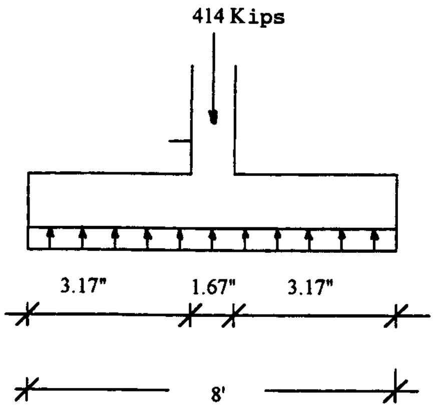
This would require $A_s = 2.5$ in^2 (1613 mm^2).

$$(A_s)_{\min} = 0.0018bh = 0.0018 \times 54 \times 36 = 3.5 \text{ in}^2 \text{ (2258 mm}^2\text{)} \qquad \therefore \text{Controls.}$$

Select 6 no. 7 [3.6 in^2 (2322 mm^2)]

b. Transverse steel under exterior column:

$$\text{Beam width} = 20 + \frac{32}{2} = 36 \text{ in (914.4 mm)}$$



$$q_{n_u} = \frac{414}{8} = 51.75 \text{ kips/ft (7.55 kN/m)}$$

$$M_u = 51.75 \frac{3.17^2}{2} = 260 \text{ ft} \cdot \text{kips (352.6 kN} \cdot \text{m)}$$

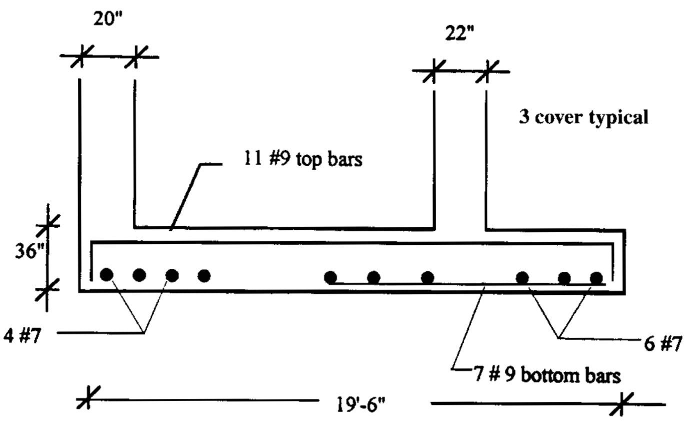
$$\frac{M_u}{\phi} = 289 \text{ ft} \cdot \text{kips (392 kN} \cdot \text{m)}$$

This would require $A_s = 1.9 \text{ in}^2$ (1226 mm^2).

$$(A_s)_{\min} = 0.0018bh = 0.0018 \times 36 \times 36 = 2.33 \text{ in}^2 \text{ (1503 mm}^2\text{)}$$

Select 4 no. 7 [2.4 in^2 (1548 mm^2)].

The checks for the development length and the bearing at the column-footing interface are similar to those in Example 4B.1 and will not be repeated here. The final details of the design are shown here.



4B.8 MAT FOUNDATIONS

4B.8.1 Introduction

A mat foundation consists of a large concrete slab that supports the column of the entire structure (see Fig. 4B.1). It is generally used when the underlying soil has a low bearing capacity. The advantages of using mat foundations are (1) the applied pressure on the supporting soil is reduced because a larger area is used and (2) the bearing capacity of the supporting soil is increased because of the larger foundation depth. Mat foundations can also be used on rock exhibiting irregular compositions, creating weak regions. To overcome the differential settlements that could result from such nonhomogeneous behavior, the mat foundation presents a practical solution. Mat foundations also present an attractive solution to support structures and machinery sensitive to differential settlements.

4B.8.2 Types of Mat Foundations

The most common type of mat foundation is a flat concrete slab of uniform thickness (see Fig. 4B.1). This type provides an economical solution for structures with moderate column loads and uniform and small column spacings. For large column loads the slab thickness is increased beneath the columns to resist resulting shear stresses. If the column spacing becomes large, thickened beams may be used along the column lines. For structures requiring foundations with large flexural rigidity, box structures made of rigid frames or cellular construction are used.

4B.8.3 Design Methods

Different methods for designing mat foundations can be used, depending on the assumptions pertaining to the structure.

4B.8.3.1 Rigid Method

If the mat is rigid enough compared to the subsoil, flexural deflections of the mat will not vary the contact pressure. Hence the contact pressure can be assumed to vary linearly. The line of action of the resultant for the column loads coincides with the centroid of the contact pressure. This assumption is justifiable when the following conditions apply:

1. The column load does not vary by more than 20% compared to adjacent columns.
2. The column spacing is less than $1.75/\lambda$. The coefficient × is defined as

$$\lambda = \sqrt[4]{\frac{K_b b}{4E_c I}} \tag{4B.18}$$

where K_b = coefficient of subgrade reaction
b = width of a strip of mat between centers of adjacent bays
E_c = modulus of elasticity of concrete
I = moment of inertia of strip of width b

On soft soils the actual contact pressure distribution is close to being linear. Hence it is commonly acceptable to design a mat on soft clay or organic soils using the rigid method.

The resultant force of all column loads and its location are first determined. Then the contact pressure q can be calculated using the principles of the strength of materials [Fig. 4B.8(*a*)],

$$q = \frac{\Sigma Q}{A} \pm \frac{(\Sigma Q \cdot e_y)x}{I_y} \pm \frac{(\Sigma Q \cdot e_x)y}{I_x} \tag{4B.19}$$

where ΣQ = resultant force of all column loads
A = total area of mat
e_x, e_y = coordinates determining location of resultant force
x, y = coordinates for a given point under mat
I_x, I_y = moments of inertia of mat with regard to × andy axes

The mat could then be analyzed in each of the two perpendicular directions. As an example, the total shear force acting on section *a-a* is equal to the algebraic sum of the column loads P_1, P_2, and P_3 and the contact pressure reaction on the tributary area $R_{a\text{-}a}$ [Fig. 4B.8(*b*)],

$$V = P_1 + P_2 + P_3 - R_{a\text{-}a} \tag{4B.20}$$

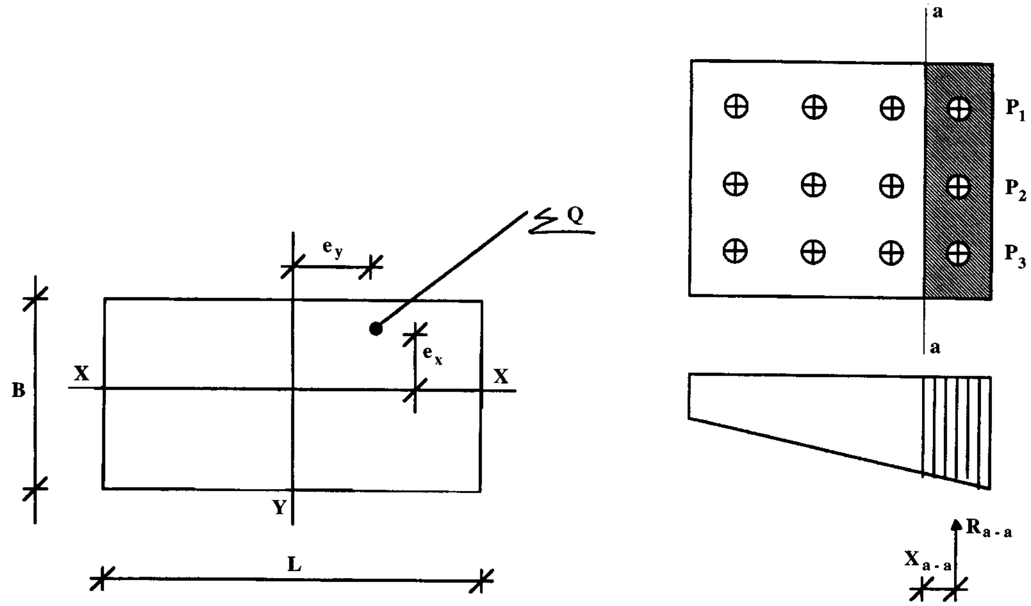


FIGURE 4B.8 Analysis of mat foundations using the rigid method

Similarly the total bending moment acting on section a-a is equal to

$$M = \sum_{i=1}^{n} P_i x_i - (R_{a\text{-}a} x_{a\text{-}a}) \tag{4B.21}$$

The rigid mat should be checked for shear and bending failures. The designer must calculate the shear force at each section and the punching shear under each column and provide an adequate mat thickness. Flexural reinforcement is provided on the top and bottom of the raft foundation in order to guarantee adequate resistance to applied moments.

4B.8.3.2 Elastic Method

This method is based on the theory of plates on elastic foundations (Hetenyi, 1946). For a typical mat on stiff or compact soils it has been found that the effect of a concentrated load damped out quickly. By determining the effect of a column load on the surrounding area, and by superimposing the effects of all the column loads within the influence area, the total effect at any point can be determined. The influence area is usually considered no more than two bays in all directions. The use of polar coordinates is necessary when applying this method since the effect of loads is transferred through the mat to the soil in a radial direction. The application of this method involves extensive mathematical manipulations. Tables have been developed to speed up the solutions. However, for cases involving variable moments of inertia of the mat and possibly variable coefficients of subgrade reaction, the work to be performed remains tedious.

ACI Committee 436 (1966), based on the theory of plates on elastic foundations, recommends the following procedure to design mat foundations of constant moment of inertia and constant coefficient of subgrade reaction.

1. The mat thickness h is chosen such that shear at critical sections is adequately resisted.
2. The coefficient K of subgrade reaction is determined.
3. The flexural rigidity of the mat foundation is calculated using

$$D = \frac{Eh^3}{12(1-\mu^2)} \tag{4B.22}$$

where E = modulus of elasticity of concrete
μ = Poisson's ratio of concrete

4. The radius of effective stiffness l is determined using

$$l = \sqrt[4]{\frac{D}{K_b}} \tag{4B.23}$$

where K_b is the coefficient of subgrade reaction adjusted for mat size.

5. Radial moment M_r, tangential moment M_t, and deflection Δ at any point are calculated:

$$M_r = -\frac{P}{4}\left[Z_4\left(\frac{r}{l}\right) - (1-\mu)\frac{Z_3'(r/l)}{(r/l)}\right] \tag{4B.24}$$

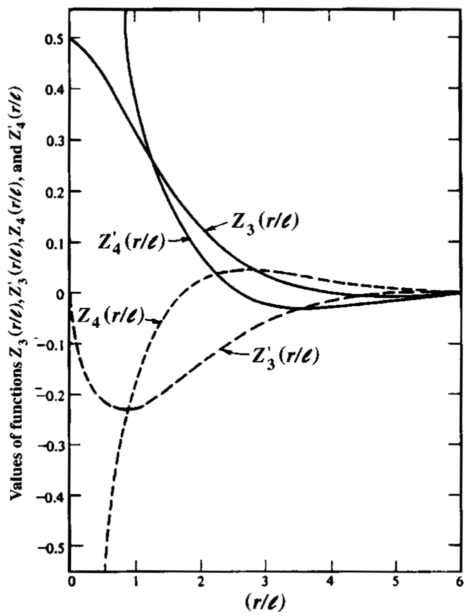


FIGURE 4B.9 Functions for mat foundation design using the elastic method. (*From Hetenyi, 1946.*)

$$M_t = -\frac{P}{4}\left[\mu Z_4\left(\frac{r}{l}\right) - (1-\mu)\frac{Z_3'(r/l)}{(r/l)}\right] \tag{4B.25}$$

$$\Delta = -\frac{Pl^2}{4D} Z_3\left(\frac{r}{l}\right) \tag{4B.26}$$

where P = column load

r = distance of point of interest from column load along radius l

$Z_3\left(\frac{r}{l}\right), Z_3'\left(\frac{r}{l}\right), Z_4\left(\frac{r}{l}\right)$ = functions for moments and deflections (Fig. 4B.9)

6. The radial and tangential moments are transformed to rectangular coordinates,

$$M_x = M_r \cos^2\phi + M_t \sin^2\phi \tag{4B.27}$$

$$M_y = M_r \sin^2\phi + M_t \cos^2\phi \tag{4B.28}$$

where $\tan \phi = y/x$.

7. The shear force Q for a unit width of the mat foundation is determined by

$$Q = -\frac{P}{4l} Z_4'\left(\frac{l}{r}\right) \tag{4B.29}$$

where $Z'_4(l/r)$ is the function for shear (Fig. 4B.10).

8. The moments and shear forces computed for each column are superimposed to obtain the total moment on shear design values.

4B.8.3.3 Numerical Methods

With the increasing use of computers in design applications, numerical methods capable of handling cases of variable moments of inertia and variable coefficients of subgrade reaction are becoming more and more attractive. The methods of finite difference and of finite elements are among the mostly used numerical techniques.

The finite-difference method is based on the assumption that the effect of the underlying soil can be represented by uniformly distributed elastic springs. These springs have an elastic constant K equal to the subgrade reaction. The differential equation of such a mat foundation is

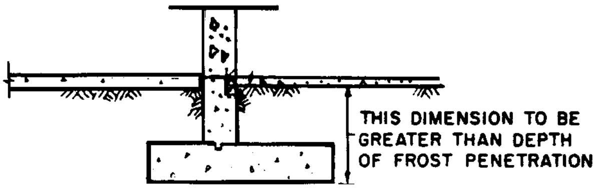


FIGURE 4B.10 Depth of frost penetration.

$$\frac{\delta^4\Delta}{\delta^4 x} + \frac{2\delta^4\Delta}{\delta^4 x\delta^4 y} + \frac{\delta^4\Delta}{\delta^4 y} = \frac{q - Kw}{D} \tag{4B.30}$$

where q = subgrade reaction per unit area of mat
K = coefficient of subgrade reaction
w = deflection
D = rigidity of mat defined in Eq. (4B.22)

The deflection of any point can be related to the deflection at the adjacent points to the right, left, top, and bottom using a numerical difference equation. The mat foundation is divided into a network of points. The difference equations for these points are formulated and rapidly solved for the deflections with a programmed computer. With the knowledge of deflections, the bending moments and shear forces can be determined from the theory of elasticity. The accuracy of the results obtained with the finite-difference method largely depends on the size and number of networks used.

The finite-element method uses the concept of matrix structural analysis to address the problem of plates on elastic foundations. The mat foundation is idealized as a mesh of plates (finite elements) interconnected only at the nodes, where isolated springs are used to model the soil reactions. A more detailed discussion of this method and its applications for foundation design can be found in Weaver and Johnston (1984).

4B.9 FOUNDATIONS IN COLD REGIONS

A locality, city, or state that spends a large amount of its financial resources to maintain a program for continuous social and economical operations under cold weather conditions and snow storms is considered located in a cold region. Seasonal and permanently frozen grounds are characteristics of cold regions and require special attention from the foundations designer.

In areas of seasonal frost during winter months, the foundation depth is carefully taken below the frost line (Fig. 4B.10). This is a necessary measure to prevent heaving of the structure due to freezing of the underlying soil. Heave is a phenomenon caused by the formation and growth of ice particles in the soil. If a foundation is placed at or above the frost line, it will move upward as the underlying soil freezes and expands. Later it will suddenly settle when thawing occurs. An additional problem is encountered in the case of fine-grained soils, namely, the decrease in the soil shear strength when it thaws after being frozen. This loss of strength is due to thawing, liberating moisture that had been soaked up by the soil particles during freezing. Thus the moisture content of the soil is increased compared to conditions prior to freezing. Such a loss of shear strength could result in a foundation failure.

An estimate of the depth of the frost line in different regions can be obtained from data supplied by the U.S. Weather Bureau (see Fig. 4B.11). The depth values obtained from such charts are only approximate. They should be corrected to account for several factors, such as susceptibility of the soil type to frost, location of the footing (interior versus exterior), and local experience (local regulations and adjacent buildings).

For frost action to occur, the following conditions must apply:

1. *Presence of frost-susceptible soil*. These are soils with enough fine pores to initiate and enhance the mechanism of ice formation and growth. Several criteria have been proposed, based on the particle-size distribution of the soil. One of the most widely known of these criteria was proposed by Casagrande (1932):

   > Under natural conditions and with sufficient water supply, one should expect considerable ice segregation in uniform soils containing more than 3% of grains smaller than 0.2 mm and in very uniform soils containing more than 10% smaller than 0.02 mm. No ice segregation was observed in soils containing less than 1% of grains smaller than 0.02 mm, even if the groundwater level was as high as the frost line.

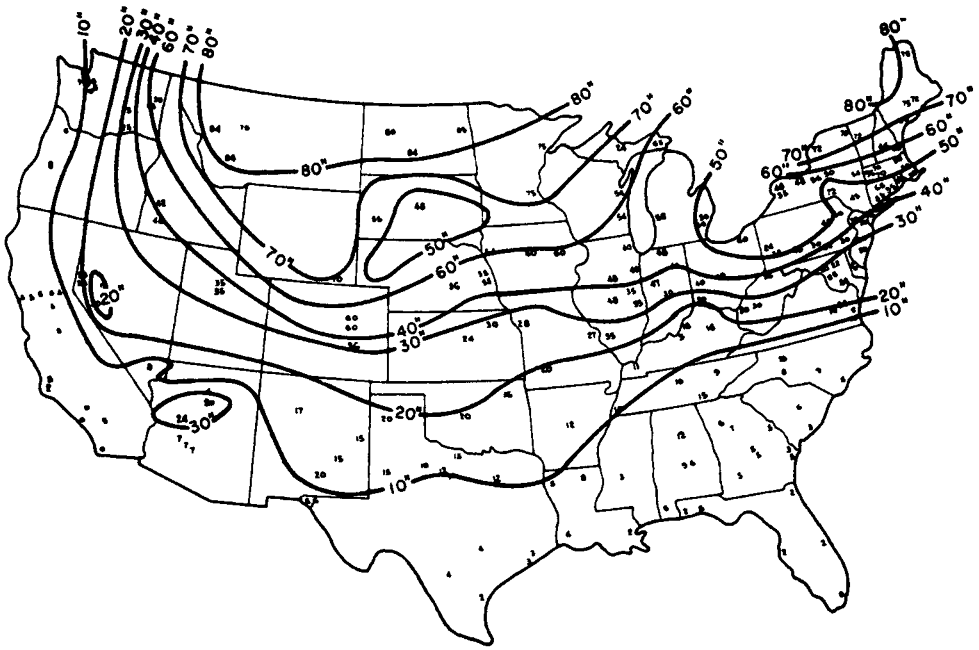


FIGURE 4B.11 Frost penetration map.

A definite distinction between soils that are frost-susceptible and those that are not is not available. Thus soils that are borderline should be used with caution.

2. *Availability of water*. For the ice particles to grow, water in the liquid phase must move in the soil to the frost line. This movement is carried by the capillary action and by suction due to supercooling at the frost front.
3. *Freezing conditions*. These conditions are determined by air temperature, solar radiation, snow cover, and exposure to wind.

In permanently frozen regions (permafrost areas), the loads of the structure are transmitted to the frozen soil with utmost attention to maintaining the frozen state. This is usually performed by insulating and ventilating between the building and the frozen ground such that the presence of the building will not alter the temperature of the ground. Another possible solution is to excavate the soil down to foundation depth and then replace it with soil that is not susceptible to frost action. Thus the foundations will not be affected by the freezing and thawing cycles. In some cases foundations are allowed to bear on frozen ground with a source of artificial refrigeration provided to keep the soil under the footings permanently frozen. This approach is, however, used rarely because of its high cost.

In general the same foundation types used in moderate regions can be used in cold regions, such as spread footings, mat foundations, piles, and caissons. The selection of a specific type of foundation will depend on the particular site conditions, particularly soil type, temperature characteristics, and structural loads. Detailed discussions of the mechanical properties of frozen soil and its bearing capacity are presented in the *Canadian Foundation Engineering Manual*, Andersland and Anderson (1978), and Sodhi (1991). The design of foundations in cold regions rarely requires higher-strength materials to resist the stresses induced from the frost-susceptible soils. What is necessary instead are techniques to avoid problems of frost heaving.

4B.10 FOUNDATIONS IN EARTHQUAKE REGIONS

4B.10.1 General

Earthquakes can produce extensive damage to foundations and structures supported on them. This damage could be related to a gross instability of the soil or to ground movement developing high-intensity stress on the structural systems. Instability of the soil can occur in loose dry sand deposits which are compacted by the ground vibrations of earthquakes, leading to large settlements and differential settlements of the ground surface. The settlements are larger for sands with smaller relative density. In cases where the soil consists of saturated loose sand, the compaction by ground vibrations could increase the hydrostatic pressure to a sufficient magnitude to cause "liquefaction" of the soil. Liquefaction is a phenomenon whereby saturated loose granular soil loses its shear strength due to the earthquake motion. Reports on many earthquakes refer to such liquefaction causing large settlements, tilting, and overturning of structures. Sudden increases in pore water pressures due to ground vibration in deposits of soft clay and sands have been the cause for major landslides in earthquake regions.

Liquefaction is likely to occur under the following soil conditions (Oshaki, 1970):

1. The sand layer is within 45 to 60 ft (15 to 20 m) of the ground level and is not subjected to high overburden pressure.
2. The sand deposits consist of uniform medium-size particles and are below the groundwater level (saturated).
3. The standard penetration test is below a certain value.

To reduce the possibility of liquefaction, several measures can be taken:

1. Increasing the sand relative density by compaction
2. Replacing the sand with another soil having better characteristics to withstand liquefaction
3. Lowering the ground water level or installing drainage equipment

4B.10.2 Dynamic Properties of Soils

In order to perform a seismic design for foundations in an earthquake region, the dynamic soil characteristics must be determined. The following is a brief description of tests used to obtain such data (Wakabayashi, 1986).

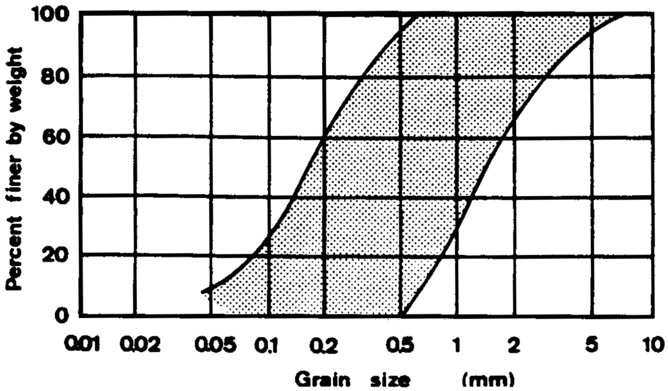


FIGURE 4B.12 Zone of liquefaction potential for cohesionless soils. (*From Ohsaki, 1970.*)

4B.10.2.1 Particle Size Distribution

The soil particle size distribution is related to the liquefaction of saturated cohesionless soils. Figure 4B.12 indicates a liquefaction potential zone based on the performance of cohesionless soils in previous earthquakes.

4B.10.2.2 Relative Density Test

This test indicates the degree of soil compaction. It gives helpful information in determining the possibility of excessive settlement for dry sands and the potential of liquefaction for saturated sands in earthquake regions. The relative density is obtained from one of the equations

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

$$D_r = \frac{\rho_{\max}(\rho - \rho_{\min})}{\rho(\rho_{\max} - \rho_{\min})} \tag{4B.31}$$

where $e_{\max}$, $e_{\min}$ = maximum and minimum void ratios
$\rho_{\max}$, $\rho_{\min}$ = maximum and minimum unit mass
e = in situ void ratio
ρ = in situ unit mass

4B.10.2.3 Cyclic Triaxial Test

This test is performed to determine the shear modulus and damping of cohesive and cohesionless soils. The shear modulus can be obtained from the compressive modulus of elasticity E using

$$G = \frac{E}{2(1 + v)} \tag{4B.32}$$

where v is Poisson's ratio.

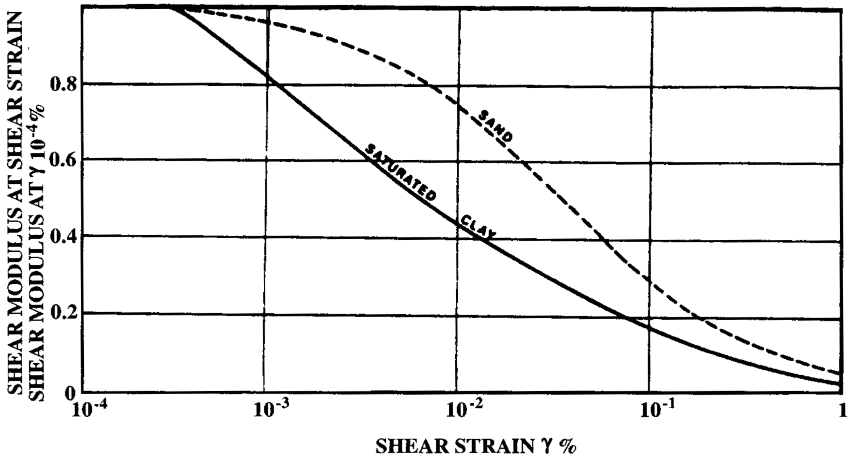


FIGURE 4B.13 Average relationships of shear modulus to strain. (*From Seed and Idrisa, 1970.*)

It can also be obtained directly by performing cyclic shear tests to obtain stress-strain relationships. The shear modulus is strain-dependent. Hence the level at which G is determined must be defined. Average relationships of shear modulus to strain for clay and sand are shown in Fig. 4B.13. During earthquakes, developed shear strains may range from 10^{-3} to 10^{-1}%, with a different maximum strain at each cycle. For this reason it has been suggested to use a value of two-thirds the shear modulus measured at the maximum strain developed for earthquake design purposes. In the field, the shear modulus of soil can be estimated from a shear wave velocity test. An explosive charge or a vibration source is used to initiate waves in the soil. The velocity of these waves is measured and the following relationship is used to determine the shear modulus of elasticity:

$$G = \rho v_s^2 \tag{4B.33}$$

where ρ = mass density of soil
v_s = shear wave velocity

The second chief dynamic parameter for soils is damping. Two different damping phenomena are related to soils—material damping and radiation damping. Material damping takes place when any vibration wave travels through the soil. It is related to the loss of vibration energy resulting from hysteresis in the soil. Damping is generally expressed as a fraction of critical damping and thus referred to as damping ratio. The damping ratio is expressed as (Fig. 4B.14)

$$\varepsilon = \frac{W}{4\pi \Delta W} \tag{4B.34}$$

where W = energy loss per cycle (area of hysteresis loop)
ΔW = strain energy stored in equivalent elastic material (area OAB in Fig. 4B.14)

Typical material damping ratios, representing average values of laboratory test results on sands and saturated clays, are give in Fig. 4B.15.

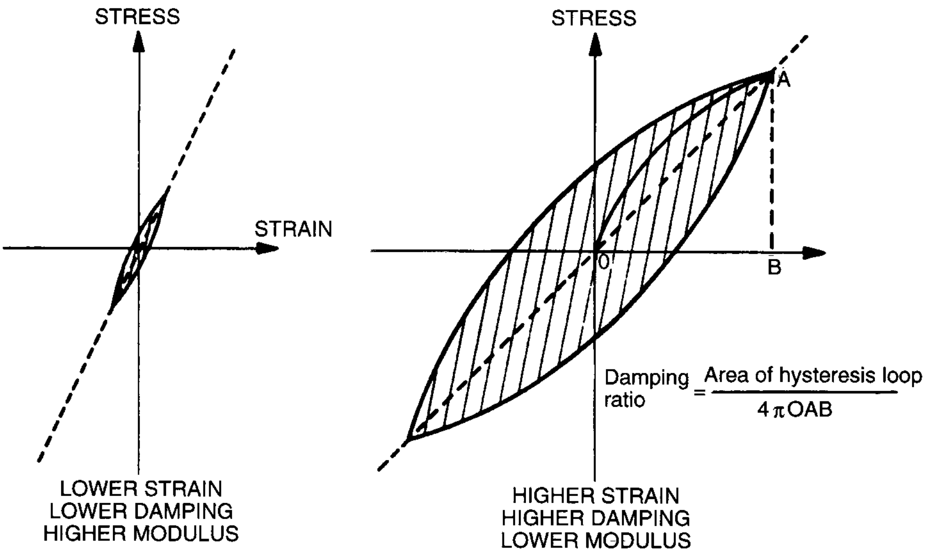


FIGURE 4B.14 Calculation of material damping ratio. (*From Seed and Idriss, 1970.*)

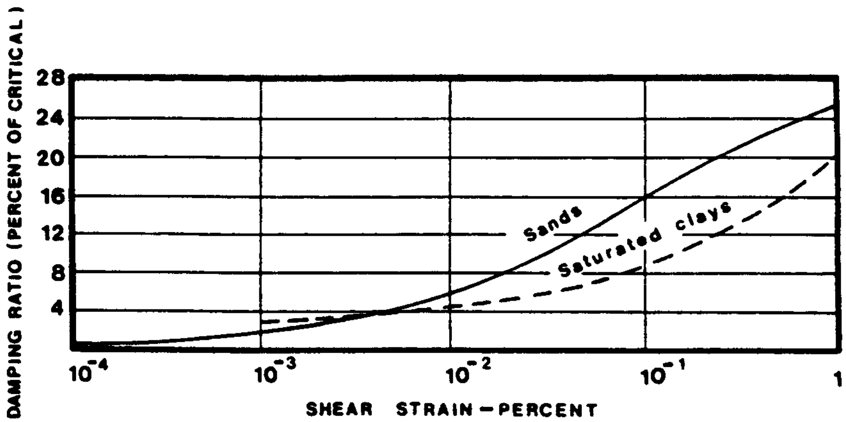


FIGURE 4B.15 Typical material damping ratios. (*From Seed and Idriss, 1970*.)

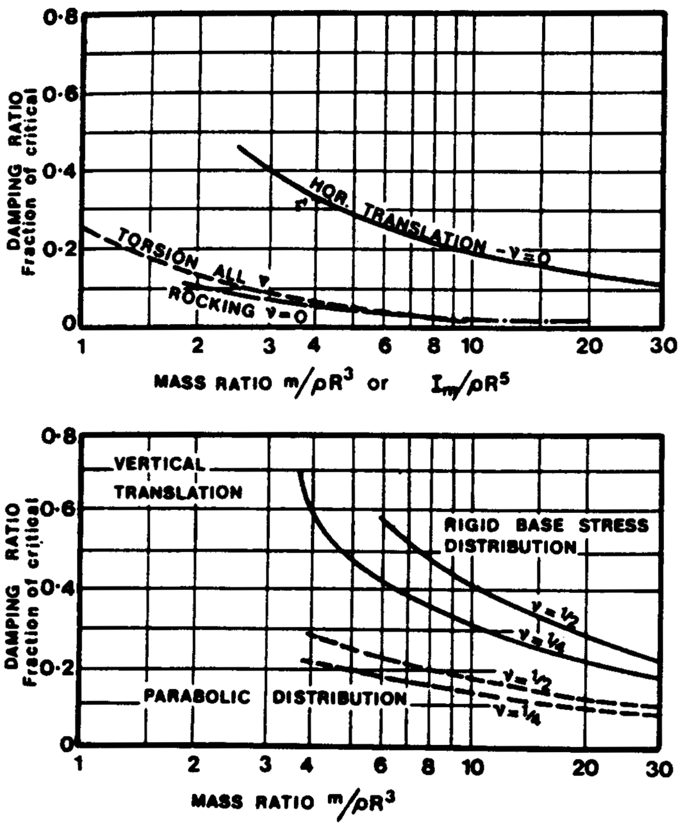


FIGURE 4B.16 Radiation damping values. (*From Whitman and Richart, 1967*.)

Radiation damping is a measure of the loss of energy through the radiation of waves from the structure. It is related to the geometrical properties of the foundation. The theory for the elastic half-space has been used to provide estimates for radiation damping. Figure 4B.16 shows values of radiation damping for circular footings of machinery obtained by Whitman and Richart (1967).

4B.10.3 Design Considerations

Additional design considerations for foundations in earthquake regions include (1) transmission of horizontal base shear forces from the structure to the soil, (2) resisting the earthquake-induced overturing moments, and (3) differential settlements and liquefaction of the subsoil (Wakabayashi, 1986; Dorwick, 1987).

Generally in earthquake design practice two separate stress systems are considered—seismic vertical stress resulting from overturning moments and seismic horizontal stresses caused by the base shear on the structure. Unless it is very slender, overturuing moments are not a design problem for the structure as a whole. However, they can drastically impact on individual footings. Hence the foundation should be proportioned so as to maintain the maximum bearing pressures caused by the overturning moments and gravity loads within the allowable seismic bearing capacity of the present soil. Safe seismic bearing pressures vary from one location to another, and local codes should be used for guidelines. In general most soils are capable of resisting higher short-term loads than long-term loads. Some sensitive clays that lose strength under dynamic loading are an exception.

With shallow foundations the base shear is assumed to be resisted by friction on the bottom surfaces of footings. The total resistance to horizontal displacement of a structure is taken to be equal to the product of the dead load of the structure and the coefficient of friction between soil and footings. Some codes recommend the use of 75% of the standard friction coefficients. Additional horizontal resistance can be obtained from the passive soil pressures developed against footing surfaces. However, if this resistance is to be relied upon, reducing the computed total resistance becomes necessary. This can be done by reducing either the frictional force or the passive resistance by about 50%. Also, careful compaction of the backfill against the sides of the footing must be performed in order to rely on the passive restraint of the soil.

To avoid or minimize damage to the foundation structure in earthquake regions due to differential settlements, it is recommended to provide ties or beams between column footings. These ties should be designed to withstand a prescribed differential movement between the connected footings.

4B.11 REFERENCES

ACI Committee 436, "Suggested Design Procedures for Combined Footings and Mats," *ACI J.*, Oct. 1966. Andersland, 0. B., and D. M. Anderson (Eds.), *Geotechnical Engineering for Cold Regions*, McGraw-Hill, New York, 1978.

Canadian Foundation Engineering Manual, 2d ed., Canadian Geotechnical Society, 1985.

Casagrande, A., "A New Theory on Frost Heaving," *Highway Res. Board Proc.*, no. 11, pp. 168–172, 1932.

Dorwick, D. J., *Earthquake Resistant Design for Engineers and Architects*, 2d ed., Wiley, New York, 1987.

Fintel, M. (Ed.), *Handbook of Concrete Engineering*, 2d ed., Van Nostrand Reinhold, Princeton, N.J., 1985.

Hetenyi, M., *Beams on Elastic Foundation*, University of Michigan Press, Ann Arbor, Mich., 1946.

Ohsaki, Y., "Effects of Sand Compaction on Liquefaction during the Tokachi-Oki Earthquake," *Soil Foundations*, vol. 10, no. 2, pp. 112–128, 1970.

Seed, H. B., and I. M. Idriss, "Soil Moduli and Damping Factors for Dynamic Response Analysis," Report EERC 70-10, Earthquake Engineering Research Center, University of California, Berkeley, Calif., 1970.

Sodhi, D. S. (Ed.), *Cold Regions Engineering, Proc. 6th mt. Specially Conf.*, American Society of Civil Engineers, West Lebanon, N.H., 1991.

Wakahayashi, M., *Design of Earthquake-Resistant Buildings*, McGraw-Hill, New York, 1986.
Weaver, W., Jr., and P. R. Johnston, *Finite Elements for Structural Analysis*, Prentice-Hall, Englewood Cliffs, N.J., 1984.
Whitman, R. V., and F. E. Richart, "Design Procedures for Dynamicalty Loaded Foundations," *J. Soil Mech. Found. Div., ASCE*, vol. 93, no. 5M6, pp. 169–91, 1967.
Winterkorn, H. E., and H. Y. Fang (Eds.), *Foundation Engineering Handbook*, Van Nostrand Reinhold, Princeton, N.J., 1975.