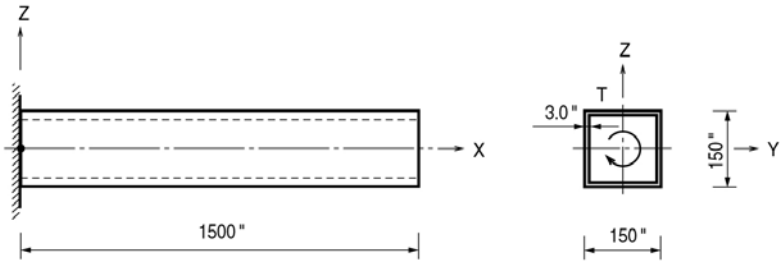
Static-15

Title

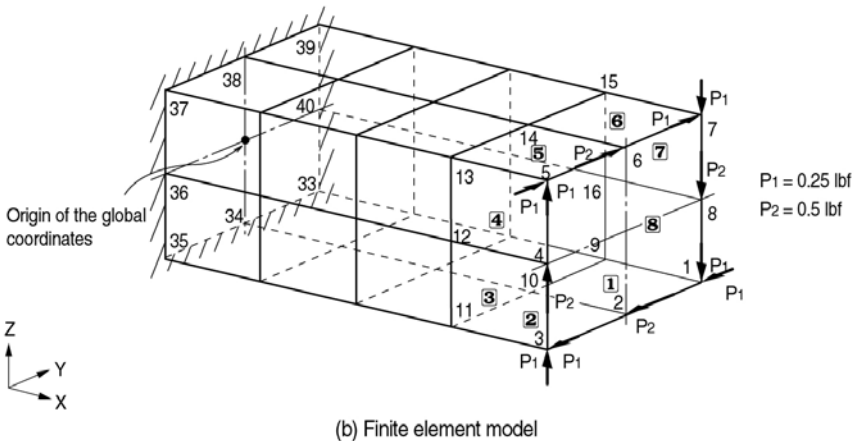
Closed section beam under a torsional moment

Description

Find the shear stresses and the angle of twist for a square box cantilever beam subjected to a torsional moment at the free end.



(a) Cantilever beam under a torsional moment



(b) Finite element model

Structural geometry and analysis model

Model

Analysis Type

3-D static analysis

Unit System

in, lbf

Dimension

Length 1500 in Width 150 in Depth 150 in

Element

Plane stress element

Material

Modulus of elasticity $E = 7.5$ psi

Poisson's ratio $\nu = 0.3$

Element Property

Size $a \times b = 375 \text{ in} \times 75 \text{ in}$

Thickness $t = 3$ in

Boundary Condition

Nodes 33~40 ; Constrain all DOFs.

Load Case

Torsional moment is applied to the free end, expressed in terms of equivalent couples.

Torsional moment = 300.0 lbf-in

Equivalent loads, $P_1 = 0.25$ lbf and $P_2 = 0.5$ lbf (Refer to the figure shown above)

Results

Stresses

	Elem	Load	Node	Sig-XX (lbf/in²)	Sig-YY (lbf/in²)	Sig-ZZ (lbf/in²)	Sig-XY (lbf/in²)	Sig-YZ (lbf/in²)	Sig-XZ (lbf/in²)	Sig-Max (lbf/in²)	Sig-Min (lbf/in²)	Angle ([deg])	Sig-EFF (lbf/in²)
▶	1	CASE1	Cent	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	1	CASE1	1	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	1	CASE1	2	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	1	CASE1	10	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	1	CASE1	9	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	2	CASE1	Cent	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	2	CASE1	2	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	2	CASE1	3	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	2	CASE1	11	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	2	CASE1	10	0,00e+000	0,00e+000	0,00e+000	-2,22e-003	0,00e+000	0,00e+000	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	3	CASE1	Cent	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	3	CASE1	3	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	3	CASE1	4	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	3	CASE1	12	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	3	CASE1	11	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	4	CASE1	Cent	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	4	CASE1	4	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	4	CASE1	5	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	4	CASE1	13	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003
	4	CASE1	12	0,00e+000	0,00e+000	0,00e+000	0,00e+000	0,00e+000	2,22e-003	2,22e-003	-2,22e-003	-45,0000	3,85e-003

Displacements

	Node	Load	DX (in)	DY (in)	DZ (in)	RX ([rad])	RY ([rad])	RZ ([rad])
	1	CASE1	0,000000	-1,155556	-1,155556	0,000000	0,000000	0,000000
	2	CASE1	0,000000	-1,155556	0,000000	0,000000	0,000000	0,000000
	3	CASE1	0,000000	-1,155556	1,155556	0,000000	0,000000	0,000000
	4	CASE1	0,000000	0,000000	1,155556	0,000000	0,000000	0,000000
	5	CASE1	0,000000	1,155556	1,155556	0,000000	0,000000	0,000000
	6	CASE1	0,000000	1,155556	0,000000	0,000000	0,000000	0,000000
	7	CASE1	0,000000	1,155556	-1,155556	0,000000	0,000000	0,000000
	8	CASE1	0,000000	0,000000	-1,155556	0,000000	0,000000	0,000000

Comparison of Results

Theoretical calculation

Shear stress (τ_{xy}) = $\frac{T}{2a^2t}$ = 0.002222 psi

Angle of twist (ϕ) = $\frac{TL}{ta^3G}$ = 0.0154074 rad

Where, T : Torsional moment (300 lbf-in)
L : Length of the cantilever beam (1500 in)
a : Section dimension (150 in)
t : Thickness of the box (3 in)
G : Shear modulus of elasticity (E/2(1+ν)=7.5/2(1+0.3)=2.8846)

FEM analysis

Shear stress (τ_{xy}) = 0.0022222 psi (all elements)
Angle of twist (ϕ) = (nodal displacement)/(distance from the center to the node)
= ($\sqrt{1.155556^2 + 1.155556^2}$)/($\sqrt{2} \cdot 150 / 2$)
= 0.0154074 rad
※ Angle of twist is calculated using the displacement at the node 1.

Units : psi, rad

Result	Theoretical	MIDAS/Civil
Shear stress (τxy)	0.0022222	0.0022222
Angle of twist (ϕ)	0.0154074	0.0154074

References

Timoshenko, S. and Goodier, J. N., *"Theory of Elasticity",* McGraw-Hill, New York, 951, p. 299.