Buckling-3

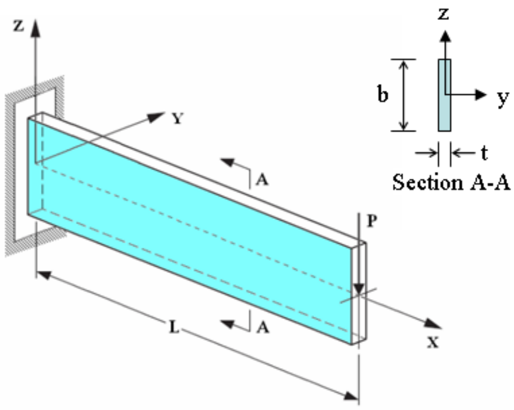
Title

Lateral buckling of a rectangular cantilever beam subjected to a load at the tip.

Description

A cantilever beam with a narrow rectangular section is loaded by a vertical load P applied at the centroid of the free end. The buckling loads are determined for the following three cases in which the cantilever beam is divided into 10, 20 and 40 elements in the length direction. All the three structures are constituted with plate elements. The computed buckling loads are then compared with the analytical exact solutions.

Case 1: The cantilever is evenly divided into 10 beam and 10 plate elements separately.
Case 2: The cantilever is evenly divided into 20 beam and 20 plate elements separately.
Case 3: The cantilever is evenly divided into 40 beam and 40 plate elements separately.



Structural geometry and boundary conditions

Model

Analysis Type

Lateral torsional buckling

Unit System

in, lb

Dimension

Length 20 in

Element

Beam element and plate element (thick type without drilling dof)

Material

Young's modulus of elasticity	$E = 10^8$ lb/in^2
Poission's ratio	$\nu = 2/3$

Section Property

Beam element : solid rectangle 0.05 × 1.0 in
Plate element : thickness 0.05in, width 1.0 in

Boundary Condition

Left end is fixed, and right end is free.

Load

P = 1.0 lbf

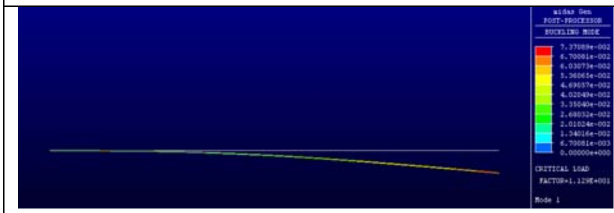
Results

Buckling Analysis Results

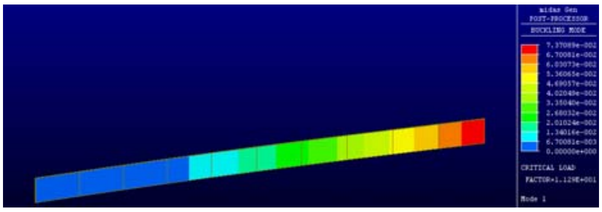
Case 1: Both beam and plate elements (10 elements)

Beam element model

1st mode



Top view



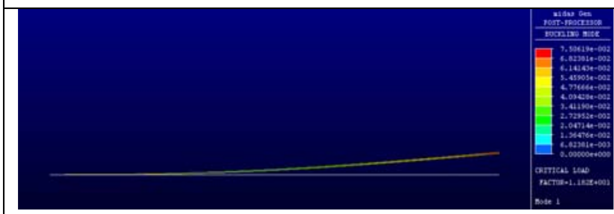
Isometric view

Buckling load

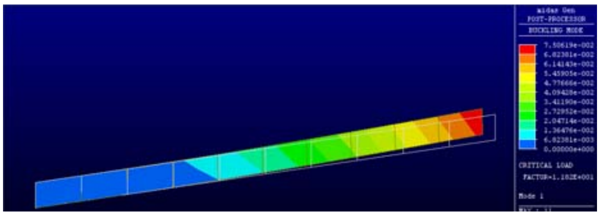
Mode	Eigenvalue	Tolerance
1	11,292842	0,0000e+000
2	29,354526	0,0000e+000
3	48,844145	0,0000e+000
4	70,253132	0,0000e+000

Plate element model

1st mode



Top view



Isometric view

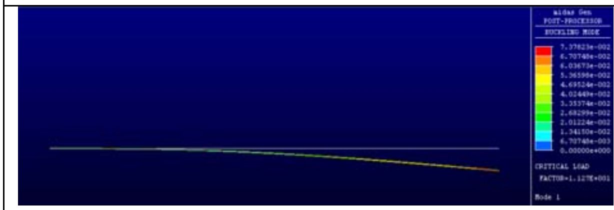
Buckling load

Mode	Eigenvalue	Tolerance
1	11,815303	2,9929e-019
2	30,938193	2,6468e-017
3	52,064170	4,4602e-016
4	75,894429	1,9663e-014

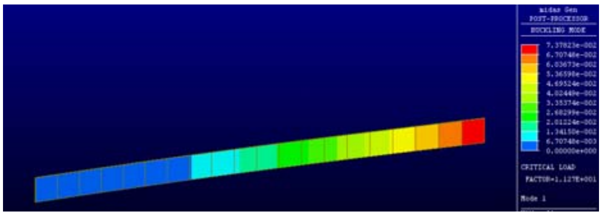
Case 2: Both beam and plate elements (20 elements)

Beam element model

1st mode



Top view



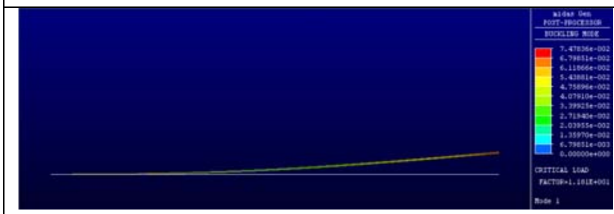
Isometric view

Buckling load

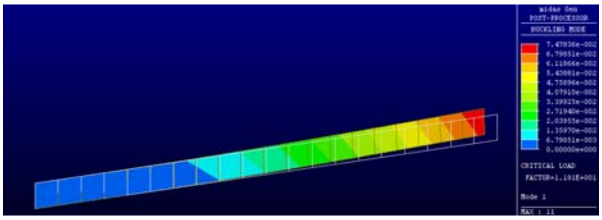
Mode	UX	UY
BUCKLING ANALYSIS		
Mode	Eigenvalue	Tolerance
1	11,271942	0,0000e+000
2	28,912925	0,0000e+000
3	47,000246	0,0000e+000
4	65,653255	0,0000e+000

Plate element model

1st mode



Top view



Isometric view

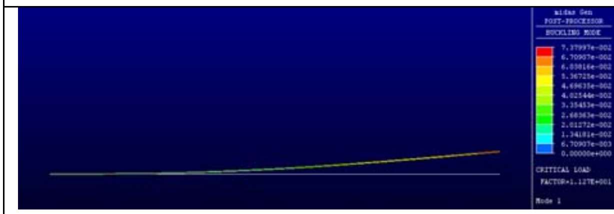
Buckling load

Mode	UX	UY
BUCKLING ANALYSIS		
Mode	Eigenvalue	Tolerance
1	11,808427	2,8185e-016
2	30,455012	1,4190e-014
3	49,995805	7,7538e-015
4	70,810646	1,8557e-013

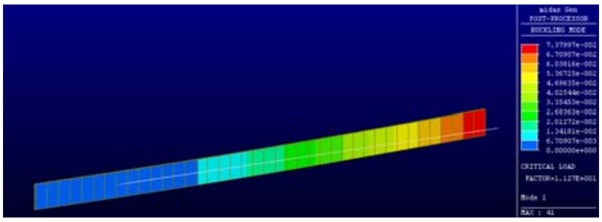
Case 3: Both beam and plate elements (40 elements)

Beam element model

1st mode



Top view



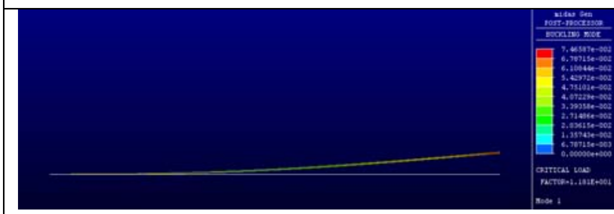
Isometric view

Buckling load

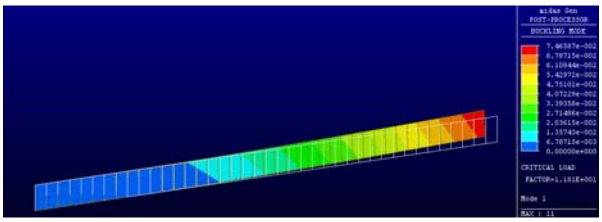
Mode	UX	UY
BUCKLING	ANALYSIS	
Mode	Eigenvalue	Tolerance
1	11,266701	0,0000e+000
2	28,801019	0,0000e+000
3	46,521514	7,5171e-081
4	64,398133	1,4966e-067

Plate element model

1st mode



Top view



Isometric view

Buckling load

Mode	UX	UY
BUCKLING	ANALYSIS	
Mode	Eigenvalue	Tolerance
1	11,808609	1,3573e-013
2	30,323720	3,2689e-013
3	49,392802	2,0228e-011
4	69,193236	3,2941e-012

Comparison of Results

Unit: lbf

Case	Critical load for 1^{st} buckling		
	Theoretical solution	MIDAS	
		Beam element (error)	Plate element (error)
1	11.266	11.293 (0.24%)	11.815 (4.87%)
2		11.272 (0.05%)	11.808 (4.81%)
3		11.267 (0.01%)	11.809 (4.82%)

From the theory of elastic stability (Timoshenko and Gere [1]), the analytical solution for the tip cirtical load P_{cr} is defined by the following expression:

$$P_{cr} = \frac{4.013}{L^2}\sqrt{EI_z GI_{xx}} = \frac{4.013}{L^2}E\sqrt{\frac{I_z I_{xx}}{2(1+\nu)}}$$

where,

L = length of the cantilever beam

E = Young's modulus of elasticity

G = shear modulus of elasticity

ν = poisson's ratio

I_z = moment of inertia about local z-axis

I_{xx} = torsional moment of inertia

Substituting the material and sectional properties into the above equation gives the following result:

$$P_{cr} = \frac{4.013}{L^2}E\sqrt{\frac{I_z I_{xx}}{2(1+\nu)}} = \frac{4.013}{20^2}\times 10^8 \times \sqrt{\frac{(1.041667\times 10^{-5})\times(4.035417\times 10^{-5})}{2(1+2/3)}}$$

$$= 11.266 \text{ lbf}$$

Reference

1. Timoshenko, S.P., and Gere, J.M., (1961). *Theory of Elastic Stability*, McGraw-Hill, New York.