Buckling-5

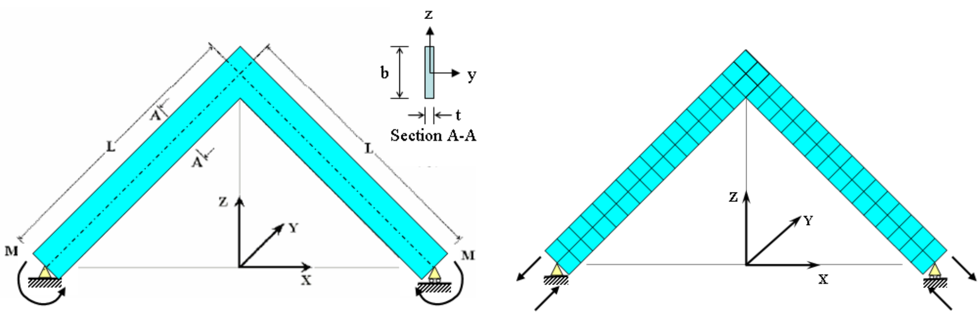
Title

Lateral buckling of a simply supported right-angle frame subjected to bending memonts at both ends

Description

A simply supported right-angle frame is subjected to bending moments **M** applied at the centroids of its ends. The buckling loads are determined for the four cases as described below (Case 1 ~ Case 4). The computed buckling loads are then compared with the analytical solution [1] and the results from the prominent papers [2, 3].

Case 1: Beam element (total 8 elements: 4 elements for each leg)
Case 2: Beam element (total 16 elements: 8 elements for each leg)
Case 3: Beam element (total 20 elements: 10 elements for each leg)
Case 4: Plate element (total 64 elements)



(a) Beam element model *(b) Plate element model*

Structural geometry and boundary conditions

Model

Analysis Type

Lateral torsional buckling

Unit System

N, mm

Dimension

Length 240 mm

Element

Beam element and plate element (thick type without drilling dof)

Material

Young's modulus of elasticity $E = 71,240\ N/mm^2$

Poission's ratio $\nu = 0.31$

Section Property

Beam element : solid rectangle 0.6×30 mm

Plate element : thickness 0.6mm, width 15mm, height 15mm

Boundary Condition

Left end is pinned, and right end is roller.

Load

$M = 1.0\ N \cdot mm$

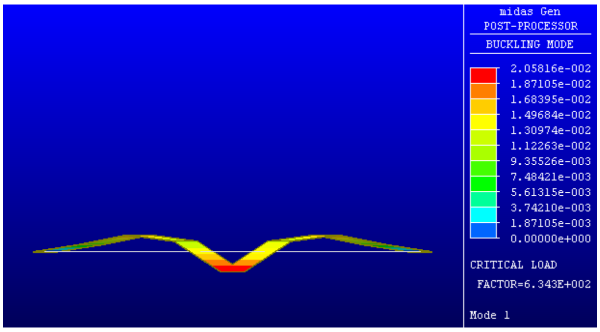
$P = 1/30\ N$

Results

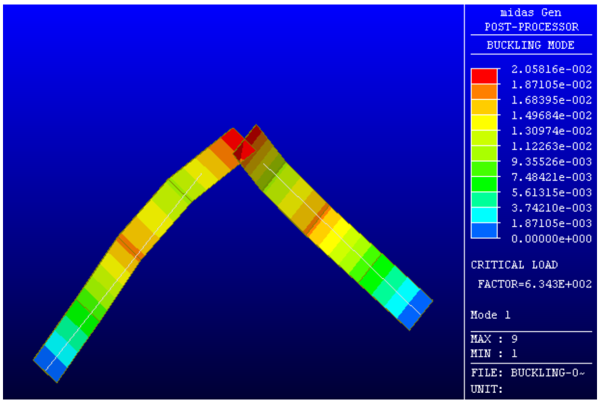
Buckling Analysis Results

Case 1: Beam element (total 8 elements)

1^{st} mode	Buckling load



Top view

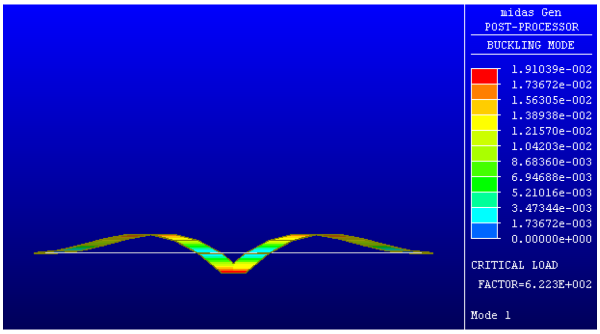


Isometric view

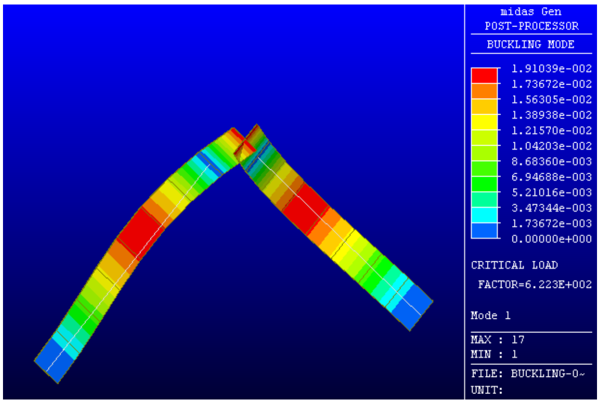
Mode	UX	UY
BUCKLING	ANALYSIS	
Mode	Eigenvalue	Tolerance
1	634,258402	0,0000e+000
2	1199,816145	0,0000e+000
3	1363,439564	0,0000e+000
4	1892,881973	0,0000e+000

Case 2: Beam element (total 16 elements)

1^{st} mode	Buckling load



Top view

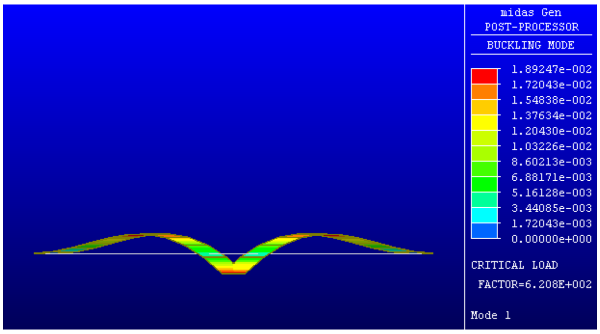


Isometric view

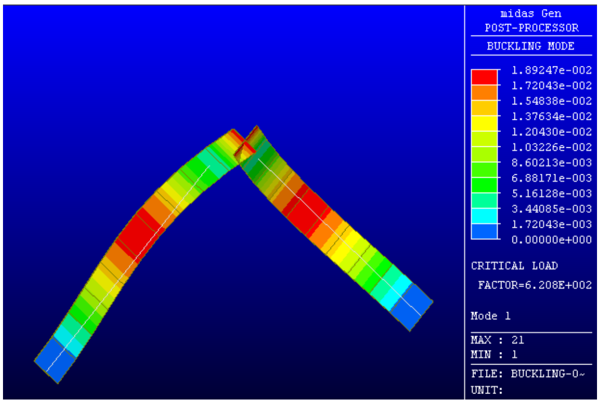
Mode	UX	UY
BUCKLING ANALYSIS		
Mode	Eigenvalue	Tolerance
1	622,250685	0,0000e+000
2	1140,575172	0,0000e+000
3	1268,484442	0,0000e+000
4	1733,343303	0,0000e+000

Case 3: Beam element (total 20 elements)

1st mode	Buckling load



Top view

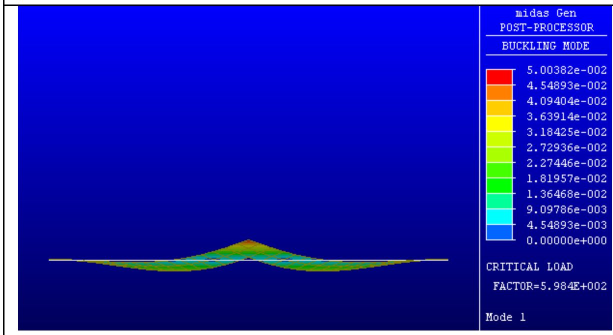


Isometric view

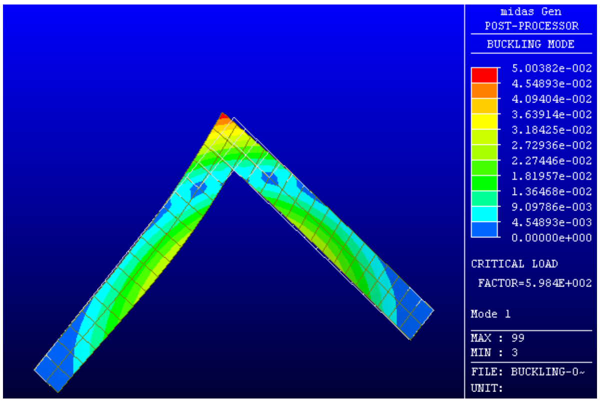
Mode	UX	UY
BUCKLING ANALYSIS		
Mode	Eigenvalue	Tolerance
1	620,816288	0,0000e+000
2	1133,011591	0,0000e+000
3	1256,937373	0,0000e+000
4	1707,718173	0,0000e+000

Case 4: Plate element (total 64 elements)

1^{st} mode	Buckling load



Top view



Isometric view

Mode	UX	UY
BUCKLING ANALYSIS		
Mode	Eigenvalue	Tolerance
1	598,401880	1,0133e-056
2	1135,145470	1,1210e-039
3	1219,541940	4,9435e-037
4	1711,384718	4,6942e-028

Comparison of Results

Unit: N·mm

Case	Type of element	No. of total elements	Critical load for 1st buckling
Timoshenko and Gere [1]	Beam element		649.19
Argyris et al [2]	Triangular shell	86	624.36
Saleeb et al [3]	Beam element	20	627.37
MIDAS Case 1	Beam element	8	634.26
MIDAS Case 2	Beam element	16	622.25
MIDAS Case 3	Beam element	20	620.82
MIDAS Case 4	Plate element	64	598.40

Timoshenko and Gere [1] provided the analytical solution using the theory of elastic stability for the tip cirtical load M_{cr} . Also a number of prominent authors have given approximate solutions obtained from their geometrically nonlinear ananyses [2, 3]. The analytical solution from Timoshenko and Gere is as follows:

$$M_{cr} = \frac{1.05 \cdot \pi}{L} \sqrt{EI_z GI_{xx}} = \frac{1.05 \cdot \pi}{L} E \sqrt{\frac{I_z I_{xx}}{2(1+\nu)}}$$

where,

L = length of the edge of the square plate

E = Young's modulus of elasticity

G = shear modulus of elasticity

ν = poisson's ratio

I_z = moment of inertia about local z-axis

I_{xx} = torsional moment of inertia

Substituting the material and sectional properties into the above equation gives the following result:

$$M_{cr} = \frac{1.05 \cdot \pi}{L} E \sqrt{\frac{I_z I_{xx}}{2(1+\nu)}}$$
$$= \frac{1.05 \cdot \pi}{240} \times 71,240 \times \sqrt{\frac{0.54 \times 2.1328}{2(1+0.31)}}$$
$$= 649.19 \text{ N} \cdot \text{mm}$$

References

1. Timoshenko, S.P., and Gere, J.M., (1961). *Theory of Elastic Stability*, McGraw-Hill, New York.
2. Argyris, J.H., Hilpert, O., Malejannakis, G.A., Sharpf, D.W., (1979). “On the geometrical stiffnesses of a beam in space – a consistent V.W. approach,” *Comp. Meth. Appl. Mech. Eng.,* Vol. 20, 105–31.
3. Saleeb, A.F, Chang T.Y.P, Gendy A.S., (1992). “Effective modeling of spatial buckling of beam assemblages, accounting for warping constraints and rotation-dependency of moments,” *Int. J. Num. Meth. Eng.*, Vol. 33, 469–502.