

## ESTIMATION OF OVERALL SETTLEMENT OF PILED RAFTS

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### ABSTRACT

Estimation of overall (or average) settlement and differential settlement are the two most critical issues in the design of moderate to large sized raft and piled foundations. Recent studies have shown that a central pile group can largely eliminate differential settlements, but it is still necessary to estimate the average settlement of the combined pile group and raft foundation.

This paper presents a simple method of estimating the overall stiffness of piled rafts in a non-homogeneous soil with finite depth, based on the method of Clancy and Randolph (1996). Although the method enables the estimation of the piled raft's stiffness even when piles are installed beneath the full raft area, particular emphasis is placed on the situation where the pile support is limited to the central region of the raft. The method combines the so called 'equivalent pier method' (Poulos and Davis, 1980) and the 'flexibility matrix method' (Randolph, 1983). In the presented method, a piled raft is replaced by a 'capped pier'.

Firstly, a simple method of estimating the overall stiffness of stubby piers in a homogeneous or a non-homogeneous soil is presented. The estimated stiffnesses are compared with those calculated by a finite difference approach (FLAC, Itasca Corporation, Version 3.22). The comparison shows that the simple method gives approximate overall stiffness of piers with a wide range of slenderness ratios (length/radius) and pile-soil stiffness ratios.

Secondly, the applicability of the equivalent pier method to pile group analysis is examined for homogeneous soil conditions. The calculated stiffnesses are compared with those obtained by FLAC, and the so called 'hybrid' method (HyPR: Clancy, 1993) where the existence of individual piles is considered. The applicability of the 'capped pier' method to piled raft analysis is then examined. The results show the presented simple method gives satisfactorily accurate stiffness of pile groups and piled rafts without any long computations.

Finally, a short example of the application is presented for a centrifuge model piled raft. It was found that the presented method gave close overall stiffness to those obtained by observations, and by alternative rigorous calculations.

**Key words:** design, pile group, (piled raft), (raft), settlement, uniform load (IGC: E2)

### INTRODUCTION

The design of large raft or piled foundations is often dominated by limits on allowable differential settlements, in order to avoid damage to the superstructure. In recent years, there has been a growing trend towards mixed piled-raft foundations, in particular seeking to optimize the position and geometry of the pile support so as to minimize differential settlements (Horikoshi and Randolph, 1997b). The resulting foundation, with pile support over the central region of the raft, is more complex to analyze than conventional raft foundations, where a number of useful design charts are available (e.g. Brown, 1969a, b; Fraser and Wardle, 1976), or even pile groups that are uniform over the full foundation area.

New techniques are needed in order to estimate (a)

overall, or average, settlement, and (b) differential settlements. The present paper addresses the first of these issues, estimation of average settlement, focusing on rafts that are supported by a central pile group. The aim is to establish a simple approach whereby simple hand calculations are sufficient to estimate the average settlement, avoiding the complex computations required for a rigorous analysis of piled raft systems.

Before embarking on the proposed simple models of the piled raft system, it is helpful to review the current state of numerical approaches for analysis of piled rafts. Over the last five years, a number of different approaches have been described, with varying degrees of simplification introduced in order to render the problem manageable from a computational viewpoint. In the simplest approaches, the piles are represented by interacting linear

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or non-linear springs beneath the raft; a boundary element approach is used for the raft-soil system; and the flexural response of the raft is analyzed using standard finite element or finite difference approaches (Poulos, 1994; Russo, 1998).

The next level of sophistication, and one that offers a good compromise between rigour and computational efficiency, is the so-called hybrid approach (O'Neill et al., 1977), where a load transfer approach is used for the piles, while boundary element techniques are used to compute the raft-soil response and all interactive effects (Chow and Teh., 1991; Clancy and Randolph, 1993).

The most rigorous and versatile treatment of piled rafts is that of Ta and Small (1996), which is based on finite layer analysis and thus can allow for layered soil properties. The rigour comes at the expense of high computational effort, but the method provides an excellent yardstick to validate simplified approaches.

Any computational approach to estimation of settlement is inevitably limited by the accuracy with which the modulus profile of the soil can be assessed, and the appropriate strain level at which to evaluate the modulus. A most useful contribution in this area has been presented recently by Mandolini and Viggiani (1997), who have back-analyzed a large number of pile group foundations where settlement measurements were available, and found that the measured performance can be predicted accurately using the small strain, or maximum, shear modulus measured by seismic methods. This observation is important in establishing a methodology for estimating foundation response, and supports the growing use of shear-wave velocity measurements in site investigation, using cross-hole, or seismic cone techniques.

Horikoshi and Randolph (1997b) have presented a framework for optimizing the differential settlement of piled raft foundations, with pile support introduced in the central area of the raft. The framework was based on parametric studies using the HyPR program (Hybrid Piled Raft analysis: Clancy, 1993; Clancy and Randolph, 1993). They showed that optimum performance was obtained when the stiffness,  $k_p$ , of the pile group by itself was approximately equal to the stiffness,  $k_r$ , of the raft alone. An important step in assessing piled raft response is therefore to be able to assess the stiffness of the pile group easily, and this may be achieved using the equivalent pier concept (Poulos and Davis, 1980), whereby the pile group is replaced by a cylindrical region of pile reinforced soil with appropriately increased modulus.

In this paper, a simple method of estimating the overall stiffness of piled raft foundations in a non-homogeneous soil with finite depth is described, which is based on the method by Clancy and Randolph (1996). They combined the equivalent pier approach with the flexibility matrix for piled rafts proposed by Randolph (1983) to allow estimation of the overall stiffness of the combined piled raft. The accuracy of the equivalent pier treatment, and estimation of its stiffness by treating it as a stubby pile, is assessed using the finite difference code, FLAC (Itasca Corporation, Version 3.22) and the HyPR pro-

gram. It is shown that the approach allows accurate estimation of the overall stiffness of piled rafts, without the need for extensive computations.

## SIMPLIFICATION OF PILE GROUP

Poulos and Davis (1980) proposed the equivalent pier method for estimating the settlement of a pile group. In this paper, a number of piles are replaced by a single 'equivalent pier' as shown in Fig. 1. In the figure,  $L_p$  is the pile length,  $E_s$ ,  $E_p$  and  $E_{eq}$  are Young's modulus of the soil, piles and the equivalent pier respectively,  $d_{eq}$  is the diameter of the pier, and  $A_g$  is the plan area of the pile group as a block.

Randolph and Clancy (1993) discussed the applicability of the equivalent pier method and proposed an appropriate parameter to categorize pile groups as:

$$R = \sqrt{ns/L_p} \quad (1)$$

where  $n$  is the number of piles and  $s$  is the pile spacing. For values of  $R$  smaller than 4, and certainly for values less than 2, they showed that the equivalent pier approach was suitable.

Once the pile group has been replaced by a 'stubby' pier, the elastic solution for a single compressible pile, such as proposed by Randolph and Wroth (1978) or Poulos and Davis (1980), can be applied to estimate the stiffness of the pier, avoiding a full numerical analysis of the group.

Randolph (1994) suggested that the diameter of the equivalent pier,  $d_{eq}$ , might be approximated as:

$$d_{eq} = \frac{2}{\sqrt{\pi}} \sqrt{A_g} \quad (2)$$

for both friction piles and end-bearing piles.

In the present paper, Eq. (2) is used to calculate the diameter of the equivalent pier. Young's modulus of the equivalent pier,  $E_{eq}$ , is then calculated as:

$$E_{eq} = E_s + (E_p - E_s) \frac{A_{tp}}{A_g} \quad (3)$$

where  $A_{tp}$  is the total cross-sectional area of the piles in the group. For a non-homogeneous soil described below, the averaged soil modulus along the pile length is used.

## STIFFNESS OF PIER

Randolph and Wroth (1978) derived an approximate

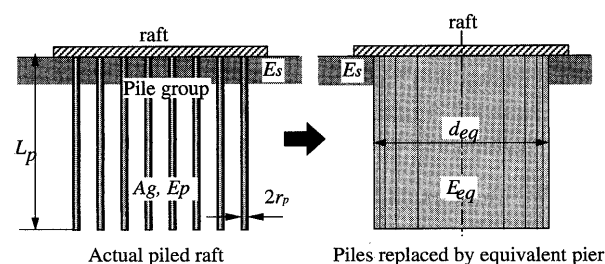


Fig. 1. Concept of equivalent pier method

method of estimating the pile stiffness as:

$$\frac{P_t}{G_l r_p w_t} = \frac{\frac{4\eta}{(1-v_s)\xi} + \rho \frac{2\pi \tanh \mu L_p L_p}{\zeta \frac{\mu L_p}{r_p}}}{1 + \frac{1}{\pi \lambda (1-v_s)\xi} \frac{4\eta}{\mu L_p \frac{L_p}{r_p}}} \quad (4)$$

where

$P_t$	pier top load
$G_l$	soil modulus at a depth of pier length
$r_p$	pier radius
$w_t$	pier top settlement
$L_p$	pier length
$\eta = r_b / r_p$	ratio of under-ream for under-reamed piers
$r_b$	radius of pier base
$v_s$	Poisson's ratio of soil
$\xi = G_l / G_b$	ratio of end-bearing for end bearing piers
$G_b$	shear modulus of soil below the level of pier base
$\rho = G_{ave} / G_l$	variation of soil modulus with depth
$G_{ave}$	average shear modulus of soil along pier length
$\mu L_p = \sqrt{(2/\zeta\lambda)} L_p / r_p$	measure of pier compressibility
$\lambda = E_p / G_l$	pier-soil stiffness ratio
$\zeta = \ln(r_m / r_p)$	measure of radius of influence of pier
$r_m$	maximum radius of influence.

This equation gives the approximate stiffness of a single pier which is installed in a deep soil deposit. Variation of soil modulus with depth can be taken into account in terms of the factor  $\rho$ . Randolph and Wroth (1978), and Fleming et al. (1992) suggested that the following relationships for  $\zeta$  gave accurate solutions for slender piers:

$$\zeta = \ln [2.5 \rho (1 - v_s) L_p / r_p] \quad (\xi = 1, \text{ friction pier}) \quad (5)$$

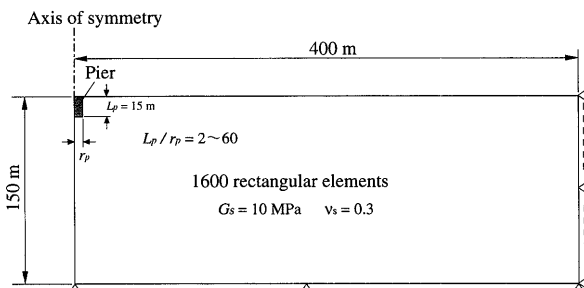


Fig. 2. Axi-symmetric model used for FLAC pier analyses

$$\zeta = \ln \{ [0.25 + 2.5 \rho (1 - v_s) - 0.25] \xi \} L_p / r_p \quad (\xi < 1, \text{ end-bearing pier}). \quad (6)$$

Randolph and Wroth (1978) discussed the accuracy of Eqs. (4) and (5), and showed that the method did not appear suitable for very long compressible piers such as  $(L_p / r_p)^2 / \lambda > 20$ , or for piers of very low aspect ratio.

#### Piers in Deep Homogeneous Soil

Randolph (1994) reported that, in order to improve the accuracy of Eq. (4) for relatively stubby piers ( $L_p / r_p \leq 5$ ), the maximum radius of influence,  $r_m$ , should be empirically be increased giving a revised equation for  $\zeta$  of:

$$\zeta = \ln [A + 2.5(1 - v_s) L_p / r_p] \quad (A = 5, \text{ for small } L_p / r_p) \quad (7)$$

The constant 'A' in the equation has little effect on the stiffness of slender piers. Since the accuracy of Eq. (7) has not been well examined, its applicability in estimating the pier stiffness is explored by comparing the stiffnesses with those calculated by the finite difference code (FLAC). The analytical model and parameters used for the study are shown in Fig. 2 and Table 1. The soil depth was set at 10 times deeper than that of the pier length as shown in Fig. 2, which was considered to represent an infinitely deep layer. The slenderness ratio of piers, i.e.  $L_p / r_p$ , was chosen as a variable.

The comparison is shown in Fig. 3 for the condition  $\lambda = 3000$  in terms of the constant 'A'. This figure indicates that Eq. (7) with  $A = 5$  gives an approximate stiffness of the piers for a wide range of slenderness ratio within an error of about 5%.

It should be noted that, in the equivalent pier approach, the pier modulus and the soil modulus are averaged as shown in Eq. (3) and the resulting equivalent pier-soil stiffness ratio is generally much lower than the corresponding pier-soil stiffness ratio. The same compari-

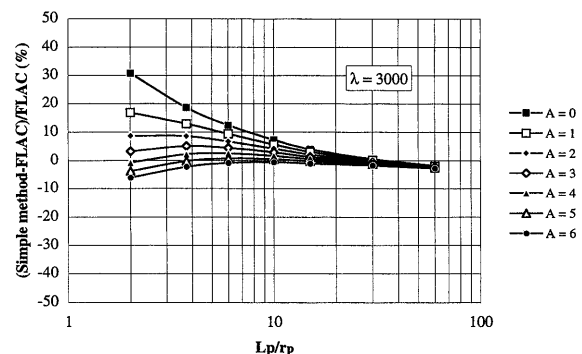


Fig. 3. Difference in stiffness between FLAC and simple approach ( $\rho = 1$ ,  $\lambda = 3000$ )

Table 1. Parameters used for pier analyses

Young's modulus of pier, $E_p$	30 GPa	Pier length, $L_p$	15 m
Shear modulus of soil, $G_s$	10 MPa	Poisson's ratio of soil, $v_s$	0.3
Slenderness ratio, $L_p / r_p$	2, 3.75, 6, 10, 15, 30, 60		

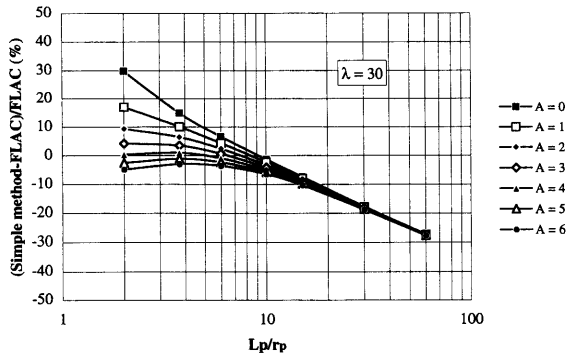


Fig. 4. Difference in stiffness between FLAC and simple approach ( $\rho=1$ ,  $\lambda=30$ )

son is therefore made in Fig. 4 for the condition  $\lambda=30$ . This figure shows that Eq. (7) with  $A=5$  still gives approximate stiffness of the piers with the slenderness ratio of  $L_p/r_p < 10$ , within the error of 10%.

#### Piers in Deep Non-homogeneous Soil

Horikoshi (1995) discussed the applicability of Eq. (7) to piers in deep non-homogeneous soil where the soil modulus increases linearly with depth. He found that for piers installed in non-homogeneous soil, the following equation is suitable:

$$\zeta = \ln \{ 5 + [0.25 + (2.5 \rho(1 - \nu_s) - 0.25) \xi] L_p / r_p \} \quad (\text{for small } L_p / r_p) \quad (8)$$

where  $\xi$  is  $G_l / G'_b$ , and  $G'_b$  is the soil modulus at a depth of  $L_p + d^* d_{eq}$  ( $d^*$  is a depth factor, see Fig. 5). Although the distribution of the soil modulus is continuous, it may be appropriate to adopt a value for  $G'_b$  which is greater than  $G_l$ , so that the increase in shear modulus below the level of the pier base can be accounted for in Eq. (4).

The most appropriate depth (in terms of a depth ratio,  $d^* = d / d_{eq}$ ) from which the representative modulus  $G'_b$  should be taken, is shown in Fig. 6 for the conditions  $\lambda=3000$  and  $\rho=0.5$  (Gibson's soil). Only the soil modulus was changed from the conditions shown in Table 1 to  $G_s = 0.67z$  (MPa), where  $z$  is the depth from the soil surface in  $m$ . This figure shows that  $d^*=1$  gives slightly underestimated stiffness of the stubby piers with a maximum error of about 15%. Note that the slight underestimate will compensate for the tendency of the equivalent pier approach to overpredict the stiffness of the actual pile group.

For the conditions  $\lambda=30$ ,  $\rho=0.5$  and  $d^*=1$ , the accuracy of Eqs. (4) and (8) is examined in Fig. 7 for  $\nu_s=0.3$  and 0.5. This figure shows that the effects of  $\nu_s$  on the accuracy of the method are small, and that the presented simple method still gives an approximate stiffness of stubby piers with  $L_p/r_p < 15$  within an error of 10%.

#### STIFFNESS OF PILE GROUP AND PILED RAFT

Since an approximate method of estimating pier stiff-

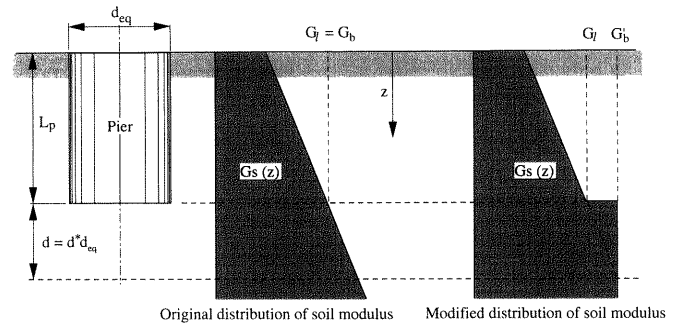


Fig. 5. Consideration of soil in-homogeneity for simple approach

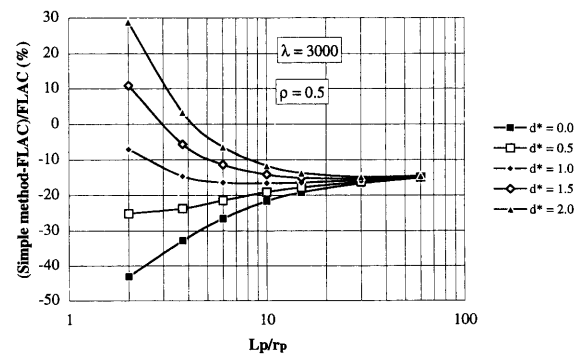


Fig. 6. Appropriate  $d^*$  value for non-homogeneous soil condition ( $\rho=0.5$ ,  $\lambda=3000$ )

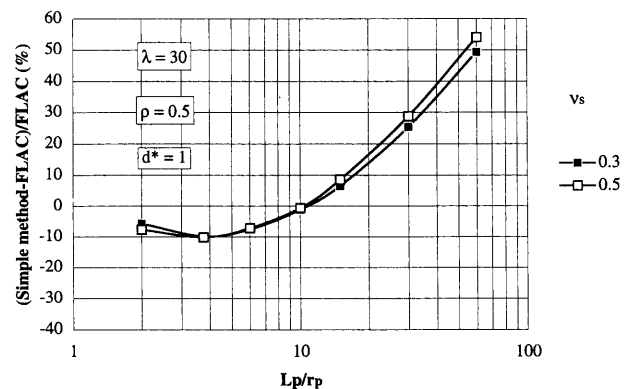


Fig. 7. Difference in stiffness between FLAC and simple approach ( $\rho=0.5$ ,  $\lambda=30$ )

ness has been presented, the applicability of the simple method to estimating the overall stiffness of pile group and piled raft is examined. To this end, a limited parametric study was performed for free standing rigidly capped pile groups and piled rafts.

In the equivalent pier analyses, the overall stiffness of the pier was analyzed by FLAC, HyPR and the simple method described above, in which the piles are replaced by a single stubby pier. It should be noted that homogeneous soil was assumed in the study in order to be able to compare the calculated stiffness with that from a full HyPR analysis of the actual pile group, where the existence of the individual piles, and full interactions are con-

Table 2. Parameters used for analyses of free standing pile groups

Young's modulus of piles, $E_p$	30 GPa	Pile length, $L_p$	35 m
Shear modulus of soil, $G_s$ (MPa)	3, 10, 100	Pile diameter, $d_p$	1.0 m
Poisson's ratio of soil, $\nu_s$	0.5	Pile spacing, $s$ (m)	2.5, 5.0, 7.5
Pile and soil stiffness ratio, $\lambda$	300, 3000, 10000		
Pile spacing ratio, $s/d_p$	2.5, 5.0, 7.5		
Depth of soil, $h/L_p$	1.5, 2, 3, 6, 12, $\infty$		

 Table 3. Equivalent pier and soil stiffness ratio,  $\lambda' = E_{eq}/G_s$ 

		$\lambda = E_p/G_s$		
		300	3000	10000
$s/d_p$	2.5	61	591	1966
	5	20	178	587
	7.5	11	86	279

sidered.

In the study, the thickness of the soil layer was also chosen as one of the variables. Randolph and Wroth assumed that the pile base is represented as a rigid punch acting at the surface of a soil medium ignoring the pile shaft and the surrounding soil as shown below:

$$w_b = \frac{P_b(1 - \nu_s)}{4G_b r_p} \quad (9)$$

Lee (1991) discussed the effects of the soil layer thickness on the settlement of the pile base by considering a rigid circular surface footing on soil of finite depth, and presented the following revised expression:

$$w_b = \frac{P_b(1 - \nu_s)}{4G_b r_p} \left\{ 1 - \exp\left(-\frac{h^*}{d}\right) \right\} \quad (10)$$

where  $d$  is the pile diameter ( $=2r_p$ ) and  $h^*$  is the soil depth from the loading plate, i.e. the depth from the pile tip in this paper. He showed that Eq. (10) was in reasonable agreement with results from finite element analysis for soil layer thickness of up to 3 times the pile diameter,  $d$ .

For the pile shaft settlement, Lee (1991) added the following factor,  $\chi$ , in Eq. (5) as shown below:

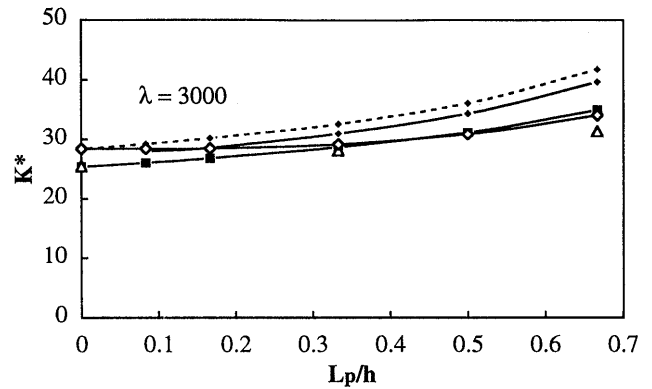
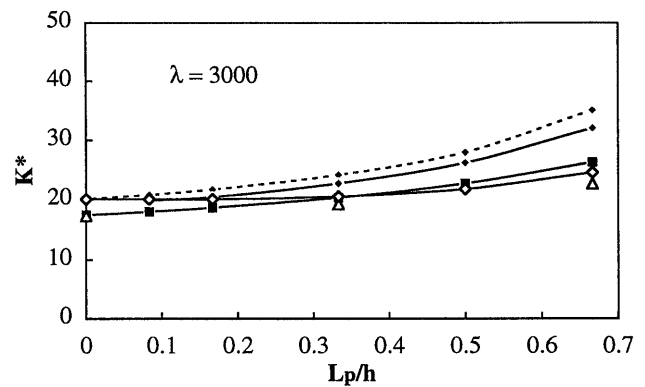
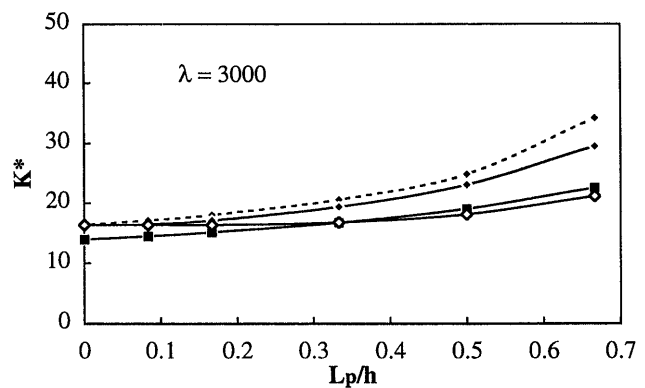
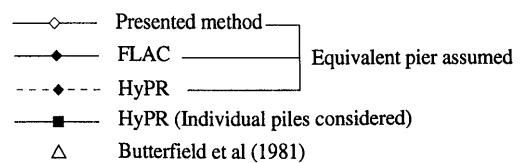
$$\chi = 1 - \exp\left(1 - \frac{h}{L_p}\right) \quad (11)$$

$$\zeta = \ln[2.5\rho(1 - \nu_s)\chi L_p/r_p]. \quad (12)$$

In the presented method, the Eqs. (10) and (12) were considered in the calculation of stiffness of pile group (or pier) installed in finite layer soils.

### Free Standing Pile Group

A free standing rigidly capped group of  $3 \times 3$  piles was chosen as an analytical model. The combinations of parameters used are listed in Table 2. Pile-soil stiffness ratio,  $\lambda$ , soil depth,  $h$ , and the pile spacing,  $s$  were chosen as variables in the study. The equivalent pier-soil stiffness ratio,  $\lambda'$ , is summarized in Table 3, which is much lower than the pile-soil stiffness ratio,  $\lambda$ , as discussed in the previous section.


 (a)  $s/d_p = 2.5$ 

 (b)  $s/d_p = 5.0$ 

 (c)  $s/d_p = 7.5$ 

 Fig. 8. Calculated stiffness of free standing pile group ( $\lambda = 3000$ , effect of pile spacing)

The calculated results are compared in Fig. 8 for different pile spacings with the results shown by Butterfield and Douglas (1981). In the figures, the overall stiffness is normalized by using the expression:

$$K^* = \frac{2k_p}{G_s d_{eq}} \quad \text{or} \quad \frac{2k_{pr}}{G_s d_{eq}} \quad (13)$$

where  $k_p$  and  $k_{pr}$  are the overall stiffness of the pile group or piled raft (as appropriate), and is plotted against the depth ratio  $L_p/h$ . From the figures shown, it is found that the results from full HyPR analyses, plotted by black squares, are in excellent agreement with the results by Butterfield and Douglas (1981). When the soil is deep, i.e.  $L_p/h$  is close to zero, all the calculated stiffnesses are very close although the equivalent pier approach gives slightly overestimated stiffness. For large  $L_p/h$  conditions, the equivalent pier analyses by FLAC show increasingly overestimated overall stiffness. It was considered that this trend is actually consistent with the original method proposed by Poulos and Davis (1980), i.e. if an equivalent pier of the same circumscribed area as the group is assumed, it is necessary for an accurate estimation to assume an equivalent pier length which is shorter than the original pile.

Figure 9 shows the calculated stiffness for other pile/soil stiffness ratios. This figure shows that the presented simple method gives accurate stiffness for a wide range of pile/soil stiffness ratio.

#### Piled Rafts

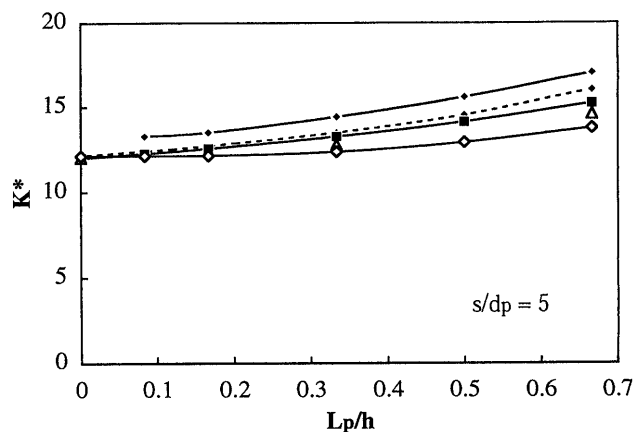
Randolph (1983) proposed a simple method of piled raft analysis allowing for interactions between the pile group and the raft in terms of the flexibility matrix. Randolph derived the following equation to estimate the overall stiffness of the piled raft,  $k_{pr}$ :

$$k_{pr} = \frac{(P_p + P_r)}{w_{pr}} = \frac{k_p + k_r(1 - 2\alpha_{rp})}{1 - (k_r/k_p)\alpha_{rp}^2} \quad (14)$$

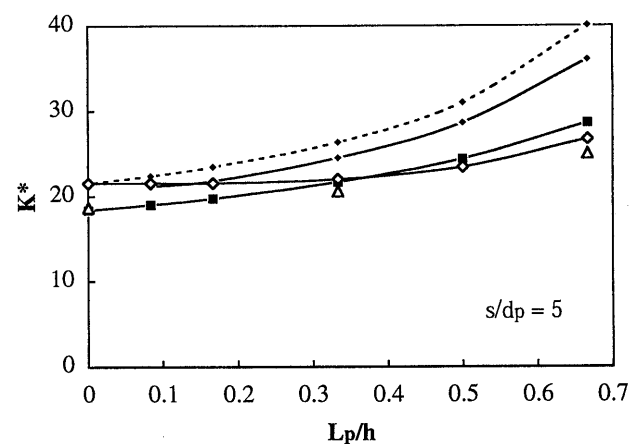
where  $w_{pr}$  is the average settlement of the piled raft, and  $P_p$  and  $P_r$  are the loads carried by the piles and the raft respectively. The interaction factor  $\alpha_{rp}$  can be approximated by the following expression:

$$\alpha_{rp} = 1 - \frac{\ln(r_r/r_p)}{\ln(r_m/r_p)} \quad (15)$$

where  $r_r$  is the radius of the raft,  $r_p$  is the pile (pier) radius, and  $r_m$  is the maximum radius of influence of an individual pile. In this paper,  $r_m$  in the above equation is estimated by taking  $r_m = r_{eq} \exp(\zeta)$  with  $\zeta$  given by Eq. (7), and Eq. (8) with  $d^* = 1$  for homogeneous and non-



(a)  $\lambda = 300$



(b)  $\lambda = 10000$

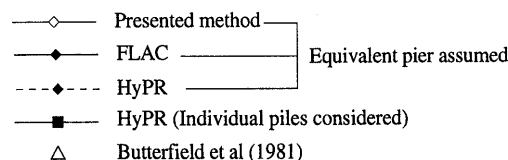


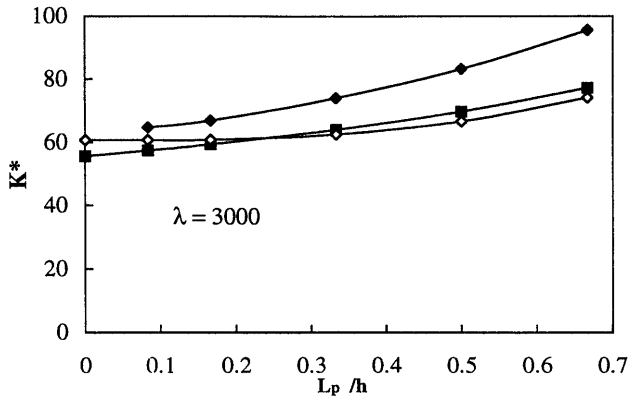
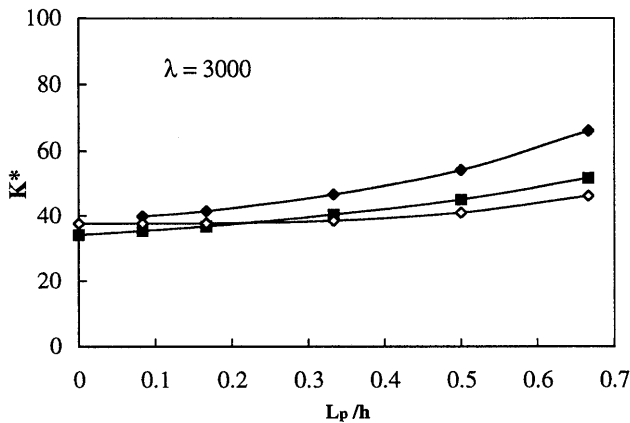
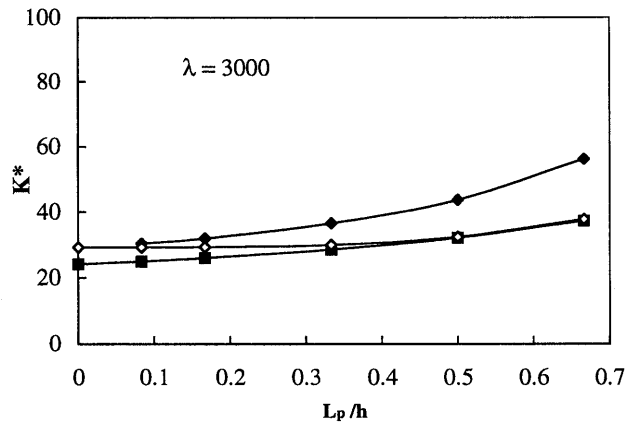
Fig. 9. Calculated stiffness of free standing pile group ( $s/d_p = 5$ , effect of pile-soil stiffness ratio)

homogeneous soil conditions respectively.

In this paper,  $k_p$  in Eq. (14) is estimated by the equivalent pier approach. The raft stiffness,  $k_r$ , can be estimated by assuming a fully rigid raft or the average settlement beneath a fully flexible raft (Poulos and Davis, 1974). When the soil is relatively shallow, the effects of the soil depth on the overall stiffness of each component should be taken into account in calculating  $k_{pr}$ , for example by

Table 4. Parameters used for analyses of piled rafts

Young's modulus of raft, $E_r$	30 GPa	Square raft length, $L (=B)$	20 m
Pile and soil stiffness ratio, $\lambda$	3000	Poisson's ratio of raft, $\nu_r$	0.16
Raft thickness, $t_r$ (m)	0.5714		
Raft soil stiffness ratio, $K_{rs}$	0.1		
Pile spacing ratio, $s/d_p$	2.5, 5.0, 7.5		
Depth of soil, $h/L_p$	1.5, 2, 3, 6, 12, $\infty$		


 (a)  $s/d_p = 2.5$ 

 (b)  $s/d_p = 5.0$ 

 (c)  $s/d_p = 7.5$ 

—◇— Presented method  
 —●— FLAC  
 —■— HyPR (Individual piles considered)

 Fig. 10. Calculated stiffness of piled rafts ( $K_{rs}=0.1$ )

the method introduced by Poulos and Davis (1974).

The accuracy of the piled raft stiffness estimated by the simple method is examined by comparing the results from full HyPR analyses. The basic parameters used for analyses are the same as shown in Table 2. Further analytical parameters are shown in Table 4. In the table,  $K_{rs}$  denotes the raft-soil stiffness ratio for rectangular rafts,

which was defined by Horikoshi and Randolph (1997a) as:

$$K_{rs} = 5.57 \frac{E_r(1-\nu_s^2)}{E_s(1-\nu_r^2)} (B/L)^{0.5} (t_r/L)^3 \quad (16)$$

where  $L$  and  $B$  are the length and the breadth of the rectangular raft ( $L \geq B$ ), and  $t_r$  is the raft thickness. Horikoshi and Randolph (1996, 1998) demonstrated that a  $K_{rs}$  value of about 0.1 may be suitable in terms of minimizing the differential settlement for moderate sized rafts.

The results of the piled raft analyses are summarized in Fig. 10 for  $K_{rs}=0.1$ . It should be noted that the overall stiffness of the piled raft is essentially independent of  $K_{rs}$ , if other conditions are the same. In the simple method, individual HyPR raft analyses were conducted with the effect of the soil depth taken into account. The figure shows that the FLAC equivalent pier analyses give somewhat overestimated stiffness when the soil is shallow. However, the presented simple method gives very close stiffness to that calculated by HyPR full analyses.

#### Design Example

As another case study for the proposed method, the building reported by Cooke et al. (1981), a 16-storey block of flats at Stonebridge Park in London, was chosen. This building was constructed with 351 bored piles of diameter 0.45 m and length 13 m, distributed uniformly over the whole area of the raft. A concrete raft with a thickness of 0.9 m, and size of  $B=20.1$  m by  $L=43.3$  m was constructed over the piles, in direct contact with the ground.

Padfield and Sharrock (1983) reported computational results for re-designed foundations for the building, using 40 central 'settlement reducing' piles of the same size, although they did not attempt to develop a framework for optimum foundation design. Horikoshi (1995) presented a framework for minimizing the differential settlement and demonstrated an optimum foundation design of the same building, using only 18 piles with a diameter of 0.50 m and length of 28 m.

In this section, the accuracy of the proposed method is examined through the reports by Padfield and Sharrock (1983) and Fleming et al. (1992). The ground conditions were assumed with reference to Fleming et al. (1992). The profile of shear strength with depth was approximated by  $s_u$  (kPa) =  $100 + 7.2z$  (m), where  $z$  is the depth below foundation level (2.5 m below ground level). The shear modulus of the soil was assumed as  $G_s = 200s_u$ . Poisson's ratio of the soil was taken as  $\nu_s = 0.1$ . The total load ( $P_t = 156.6$  MN) was assumed to be distributed uniformly over the raft.

The foundation model suggested by Padfield and Sharrock (1983) is shown in Fig. 11. The raft thickness was 0.9 m. The stiffness of the raft alone was first calculated as  $k_r = 3157$  MN/m, by assuming a fully flexible raft (Giroud, 1968) and averaging the settlements. In the calculation of raft stiffness, a uniform modulus at a

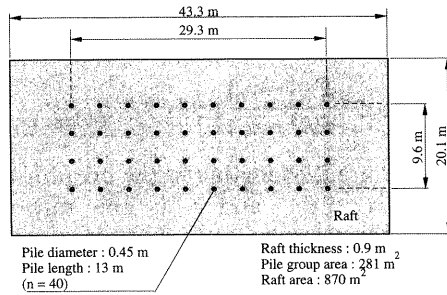


Fig. 11. Alternative design by Padfield and Sharrock (1983)

depth of one equivalent circular raft radius ( $a_{eq} = \sqrt{(BL/\pi)} = 16.6$  m) was used.

The stiffness of the equivalent pier,  $k_{eqp}$ , was then calculated. In the calculation, soil non-homogeneity was considered by applying the proposed depth factor,  $d^*$ . The modulus and the diameter of the equivalent pier were then calculated as 380 MPa and 19 m respectively. The stiffness of the equivalent pier,  $k_p$ , was calculated as  $k_p = 3018$  MN/m.

The interaction factor between the raft and the equivalent pier is approximated as  $\alpha_{rp} = 0.70$  by Eq. (15). This leads to the overall stiffness of the piled rafts of  $k_{pr} = 3604$  MN/m and the average settlement of 43 mm under the total applied load of 156.6 MN. The estimated settlement compares with the settlement of 45 mm reported by Fleming et al. (1992) and of 49 mm calculated by Padfield and Sharrock (1983). These results show that the proposed method gives satisfactorily accurate overall stiffness of piled rafts.

## COMPARISON WITH CENTRIFUGE TEST RESULTS

The accuracy of the presented method is examined through the results of centrifuge tests reported by Horikoshi and Randolph (1996). The calculated overall stiffness of the piled raft is compared with the experimental results. The schematic figure of the centrifuge model is shown in Fig. 12. Piled raft and unpiled raft models were loaded at the same time in the same package. Load was added to the model foundations by supplying and draining the water in the tank. Note that, as shown by Horikoshi and Randolph (1996), minimal water was initially poured into the tank to immerse the pore water pressure transducers. This led to the application of an initial load of about 6.5 MN to the tank. The cyclic loading tests were then conducted after the self weight consolidation due to the above initial load.

The piled raft models used in the centrifuge test are shown in Fig. 13 at prototype scale. Note that prototype scale is used in the following discussion and the figures in this section. In the piled raft model with 9 piles, piles were installed only beneath the central area of the raft to minimize the differential settlement, based on the concept of Randolph (1994). A fully flexible circular raft was modeled, with a diameter of 14 m and thickness equivalent

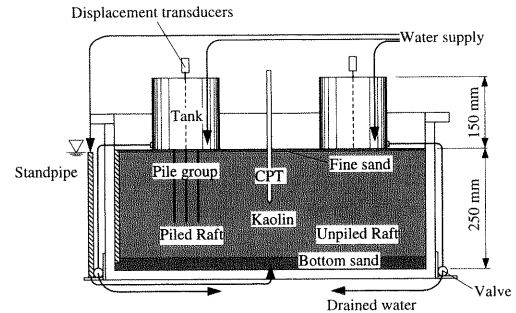


Fig. 12. General section of centrifuge package

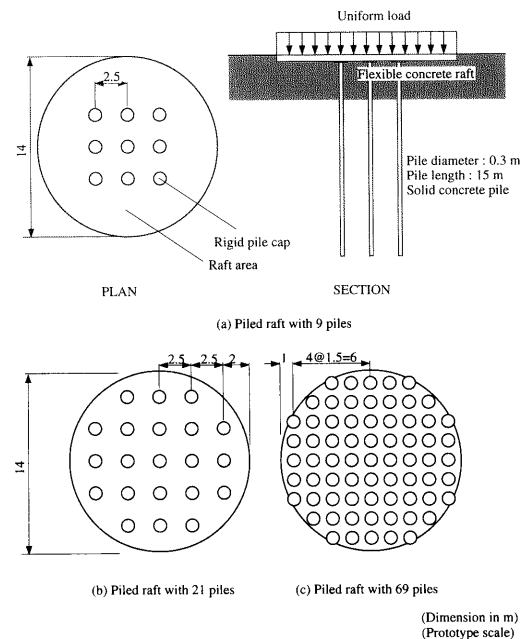


Fig. 13. Piled raft models used for centrifuge tests

lent to only about 50 mm of concrete. (The actual model raft comprised a thin metal plate, chosen deliberately as thin as possible to ensure minimal stiffness of the raft.) A small rigid pile cap with a diameter of 1 m was attached to each pile top. A uniform design pressure of 78 kPa, i.e. load of 12 MN, was applied to the surface of the raft. These conditions were set to satisfy the optimum conditions in terms of the differential settlement as reported by Horikoshi and Randolph (1998). Other piled raft models used for comparison are shown in Fig. 13(b) and (c). Profiles of differential settlement for the piled raft models were described in detail by Horikoshi and Randolph (1996).

The average settlement of the piled rafts was then estimated. The soil modulus in the centrifuge model was approximated by  $s_u = 33 + 1.2z$  kPa ( $z$  is the depth below the foundation in m) from the results of the soil investigation reported by Horikoshi and Randolph (1996).

In calculating the overall stiffness,  $k_r$ , of the raft alone, a uniform soil modulus of  $G_s = 6$  MPa was adopted, corresponding to  $G_s = 145s_u$  and taking the modulus at a



depth of one raft radius (7 m). A rather low value of the soil modulus was used, since the settlement behavior of the model included high non-linearity. Poisson's ratio of the soil was assumed to be  $\nu_s=0.4$  according to Horikoshi (1995). The overall stiffness of the raft alone was then calculated as 343 MN/m, using the code HyPR, where the soil depth has been limited to 25 m, corresponding to the depth of soil in the centrifuge model.

The equivalent pier analysis gives the overall stiffness of the pile group as  $k_p=440$  MN/m with due allowance for the effect of the soil depth and soil non-homogeneity, by using Eqs. (8), (10) and (12). The overall stiffness of the piled raft is estimated as  $k_{pr}=531$  MN/m with an interaction factor  $\alpha_{pr}$  of 0.55. This leads to an average settlement of 23 mm under the design load of 12 MN.

This compares with the measured settlement of 22 mm (at prototype scale) for the model tests. The estimated settlement is compared with the load-settlement curves reported by Horikoshi and Randolph (1998) in Fig. 14. In the figure, the curve noted 'HyPR' denotes the HyPR full non-linear analyses conducted by Horikoshi and Randolph (1998), where the existence of individual piles

and the non-linear behavior of each pile response were taken into account. Note that the initial points of the experimental load settlement curves have been constrained to fall on the curves predicted using HyPR, since piles were initially loaded by the initial minimal applied load of 6.5 MN. (This is necessary since the initial displacements cannot be separated out from background re-consolidation of the soil, as discussed by Horikoshi and Randolph, 1996.) The figure shows the experimental settlement of 22 mm at prototype scale, and a good agreement between the measured and calculated stiffness of the piled raft.

The average settlements for the other piled raft models (with 21 piles and 69 piles) are compared in Fig. 15. In the estimation, the soil modulus was set at the same conditions as used for the piled raft with 9 piles. This figure shows that the presented method gives slightly higher average settlements than those observed in the experiments. This was considered to be due to the soil modulus used in the calculation, i.e., since many piles were installed beneath the raft, the actual soil modulus might be slightly higher for these piled rafts, compared with the piled raft with 9 piles.

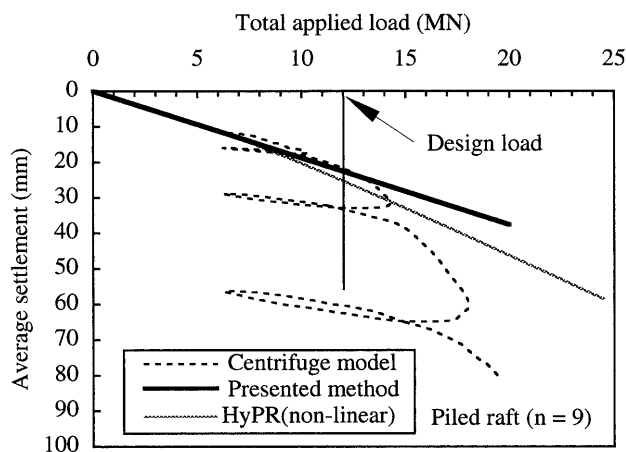
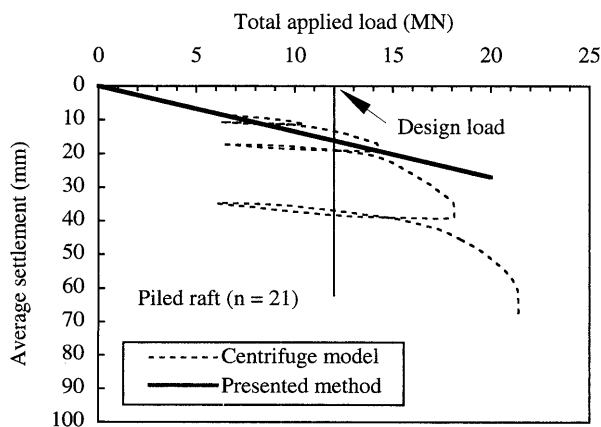


Fig. 14. Calculated average settlement for centrifuge model

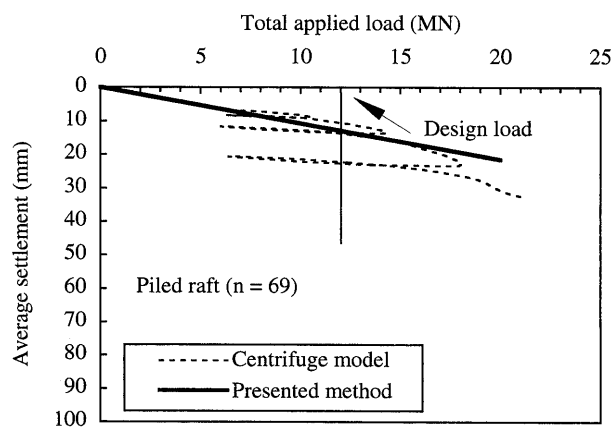
## CONCLUSIONS

A simple method for estimating the overall stiffness of stubby piers, pile groups and piled rafts in homogeneous or non-homogeneous soil has been described, based on the method by Clancy and Randolph (1996). The method simply combines the equivalent pier method for pile groups and the flexibility matrix method for piled rafts. The work of Clancy and Randolph (1996) has been extended to deal with non-homogeneous stiffness profiles in the soil, and also improved expressions for the equivalent pier stiffness have been developed, allowing for the low aspect ratio of the pier and the finite depth of soil.

The accuracy of the method was examined through a number of parametric studies and case studies, which showed that the approach gives satisfactorily accurate



(a) Piled raft with 21 piles



(b) Piled raft with 69 piles

Fig. 15. Comparison of average settlement for other piled raft models

overall stiffness of pile groups and piled rafts, avoiding the need for numerically intensive computations.

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