

PDelta-1

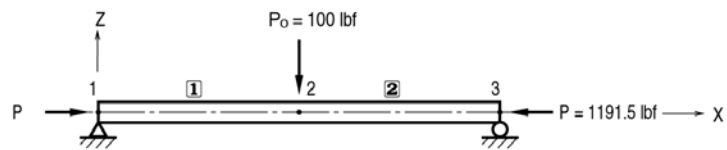
Title

P-Delta effect analysis of a beam

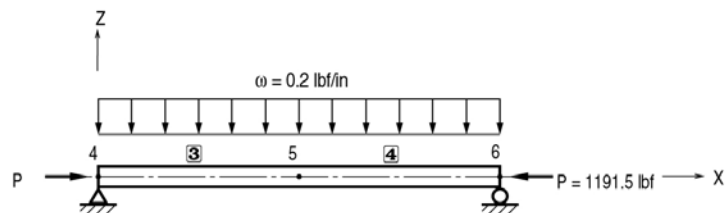
Description

A simply supported beam is subjected to three different types of transverse loads in conjunction with axial forces (mid-span point load, uniform load and two equal end moments)

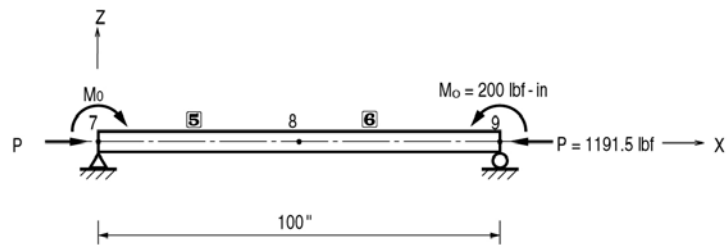
Perform a P-Delta effect analysis and find the displacements and internal bending moments for each load case.



(a) A concentrated load and axial forces



(b) A uniform load and axial forces



(c) End bending moments and axial forces

Structural geometry and analysis model

Model

Analysis Type

2-D, P-Delta effect analysis

Unit System

in, lbf

Dimension

Length $L = 100.0$ in
 Critical load $P_{cr} = 2383.0$ lbf
 Relative displacement tolerance 0.001

Element

Beam Element

Material

Modulus of elasticity $E = 29 \times 10^6$ psi

Section Property

Area $A = 1.0$ in²
 Moment of inertia $I_{yy} = 1/12$ in⁴

Boundary Condition

Nodes 1, 4 and 7 ; Constrain Dx and Dz. (Hinge supports)
 Nodes 3, 6 and 9 ; Constrain Dz. (Roller supports)

Analysis Case

In order to consider the P-Delta effect assume P is the 50 percent of the P_{cr} .

$P = 2383.0/2$ lbf.

Load 1 – A concentrated load is applied at the mid-span of the beam in the -Z direction, $P_o = 100.0$ lbf

Load 2 – A uniform load is distributed over the entire span of the beam in the -Z direction, $\omega = 0.2$ lbf/in

Load 3 – Bending moments are applied to both ends about the Y-axis ,
 $M_o = \pm 200$ lbf-in Maximum number of iteration for the P-Delta effect analysis = 5

Results

Displacements

	Node	Load	DX (in)	DY (in)	DZ (in)	RX ([rad])	RY ([rad])	RZ ([rad])
►	1	CASE1	0.000000	0.000000	0.000000	0.000000	0.052242	0.000000
	2	CASE1	-0.002054	0.000000	-1.704890	0.000000	0.000000	0.000000
	3	CASE1	-0.004109	0.000000	0.000000	0.000000	-0.052242	0.000000
	4	CASE1	0.000000	0.000000	0.000000	0.000000	0.006783	0.000000
	5	CASE1	-0.002054	0.000000	-0.214024	0.000000	0.000000	0.000000
	6	CASE1	-0.004109	0.000000	0.000000	0.000000	-0.006783	0.000000
	7	CASE1	0.000000	0.000000	0.000000	0.000000	0.007482	0.000000
	8	CASE1	-0.002054	0.000000	-0.208970	0.000000	0.000000	0.000000
	9	CASE1	-0.004109	0.000000	0.000000	0.000000	-0.007482	0.000000

Member Forces

	Elem	Load	Part	Axial (lbf)	Shear-y (lbf)	Shear-z (lbf)	Torsion (lbf-in)	Moment-y (lbf-in)	Moment-z (lbf-in)
►	1	CASE1	i	-1191.50	0.00	-50.00	0.00	0.00	0.00
	1	CASE1	1/4	-1191.50	0.00	-50.00	0.00	1132.84	0.00
	1	CASE1	2/4	-1191.50	0.00	-50.00	0.00	2265.69	0.00
	1	CASE1	3/4	-1191.50	0.00	-50.00	0.00	3398.53	0.00
	1	CASE1	j	-1191.50	0.00	-50.00	0.00	4531.38	0.00
	2	CASE1	i	-1191.50	0.00	50.00	0.00	4531.38	0.00
	2	CASE1	1/4	-1191.50	0.00	50.00	0.00	3398.53	0.00
	2	CASE1	2/4	-1191.50	0.00	50.00	0.00	2265.69	0.00
	2	CASE1	3/4	-1191.50	0.00	50.00	0.00	1132.84	0.00
	2	CASE1	j	-1191.50	0.00	50.00	0.00	0.00	0.00
	3	CASE1	i	-1191.50	0.00	-10.00	0.00	0.00	0.00
	3	CASE1	1/4	-1191.50	0.00	-7.50	0.00	173.13	0.00
	3	CASE1	2/4	-1191.50	0.00	-5.00	0.00	315.01	0.00
	3	CASE1	3/4	-1191.50	0.00	-2.50	0.00	425.63	0.00
	3	CASE1	j	-1191.50	0.00	0.00	0.00	505.01	0.00
	4	CASE1	i	-1191.50	0.00	0.00	0.00	505.01	0.00
	4	CASE1	1/4	-1191.50	0.00	2.50	0.00	425.63	0.00
	4	CASE1	2/4	-1191.50	0.00	5.00	0.00	315.01	0.00
	4	CASE1	3/4	-1191.50	0.00	7.50	0.00	173.13	0.00
	4	CASE1	j	-1191.50	0.00	10.00	0.00	0.00	0.00
	5	CASE1	i	-1191.50	0.00	0.00	0.00	200.00	0.00
	5	CASE1	1/4	-1191.50	0.00	0.00	0.00	262.25	0.00
	5	CASE1	2/4	-1191.50	0.00	0.00	0.00	324.49	0.00
	5	CASE1	3/4	-1191.50	0.00	0.00	0.00	386.74	0.00
	5	CASE1	j	-1191.50	0.00	0.00	0.00	448.99	0.00
	6	CASE1	i	-1191.50	0.00	0.00	0.00	448.99	0.00
	6	CASE1	1/4	-1191.50	0.00	0.00	0.00	386.74	0.00
	6	CASE1	2/4	-1191.50	0.00	0.00	0.00	324.49	0.00
	6	CASE1	3/4	-1191.50	0.00	0.00	0.00	262.25	0.00
	6	CASE1	j	-1191.50	0.00	0.00	0.00	200.00	0.00

Comparison of Results

Load type	Result	Unit : in, lbf-in	
		SAP2000	MIDAS/Civil
Concentrated load	Vertical displacement	-1.7058	-1.7049
	Bending moment	4532.428	4531.377
Uniform load	Vertical displacement	-0.2141	-0.2140
	Bending moment	505.127	505.010
End moments	Vertical displacement	-0.2090	-0.2090
	Bending moment	449.045	448.988

References

Livesley, R. K., and Chandler, D. B., “*Stability Functions for Structural Frameworks*”, Manchester University Press, UK, 1956.

“*SAP90, A Series of Computer Programs for the Finite Element Analysis of Structures, Structural Analysis Verification Manual*”, Computer and Structures, Inc., 1992, Example 25.