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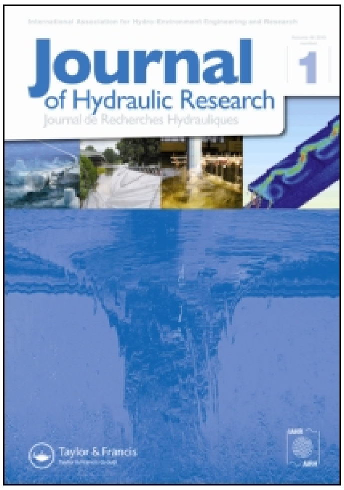

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Energy loss at drops

Pertes d'énergie dans des chutes d'eau

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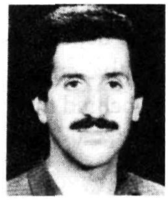

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SUMMARY

This paper presents a critical analysis of the energy loss at drops. Using the results of an experimental study on three drops, several assumptions made by White and Gill in developing an equation for the energy loss at drops have been examined. This paper also presents some interesting measurements on the velocity distribution in the falling jet as well as in the sliding jet on the side of the pool. This work has also indicated that the loss at a drop is mainly due to the mixing of the jet with the pool behind the jet. Further, a method has been developed to predict the energy loss at a drop.

RÉSUMÉ

Cet article présente une étude critique de la perte d'énergie dans des chutes d'eau. A partir des résultats d'une étude expérimentale sur 3 chutes, plusieurs hypothèses, élaborées par White et Gill pour développer une relation donnant la perte d'énergie dans des chutes, ont été analysées. Cet article présente également quelques mesures intéressantes concernant la distribution de vitesse dans un jet en chute libre ainsi que dans un jet glissant sur le bord d'un matelas d'eau. Ce travail a également montré que la perte d'énergie dans une chute est due principalement au mélange du jet dans le bassin qui le reçoit. Enfin, une méthode est présentée pour prédire la perte d'énergie dans une chute d'eau.

Introduction

A vertical drop or free overfall is a common feature in both natural and artificial channels. Natural drops are formed by river erosion while drop structures are built in irrigation systems to reduce the channel slope. A basic contribution to the hydraulics of drops was made by Moore (1943) followed by the discussion of White (1943). These studies were mainly on the energy loss at the base of drop. In his discussion, White (1943) presented a theoretical solution for energy loss that was later modified by Gill (1979). Rand (1955) developed empirical equations for some of the characteristics of the flow in terms of a dimensionless parameter, known as the Drop Number, D. It appears that the mechanism of energy dissipation and velocity distribution of the falling jet have not been studied so far.

A laboratory study was carried out in the T. Blench Hydraulics Laboratory at the University of Alberta to study the characteristics of the flow at a drop. More emphasis has been placed on the falling jet and the pool. The mechanism of energy dissipation is carefully studied with the aid of measurements and observations. The impinging inclined jet is also studied because of its use in the analysis of drops.

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Literature review

Consider a drop located in a rectangular channel as shown in Fig. 1, where H is the height of the drop; the flow approaching the drop is subcritical and the flow immediately downstream of the drop is supercritical. Let Q be the discharge and Y_c the critical depth. Moore (1943) performed an experimental study with drops of two heights and found that the energy loss at the drop could be significant depending upon the relative height of the drop H/Y_c. As H/Y_c increases from 1 to 12, the relative energy loss increases from about zero to 0.53. Moore also found that the depth of the pool Y_p behind the falling jet was predicted well by the equation

$$\frac{Y_p}{Y_c} = \sqrt{\left(\frac{Y_1}{Y_c}\right)^2 + 2\left(\frac{Y_c}{Y_1}\right) - 3} \tag{1}$$

which was obtained from momentum considerations where Y_1 is the depth of the supercritical stream immediately downstream of the drop.

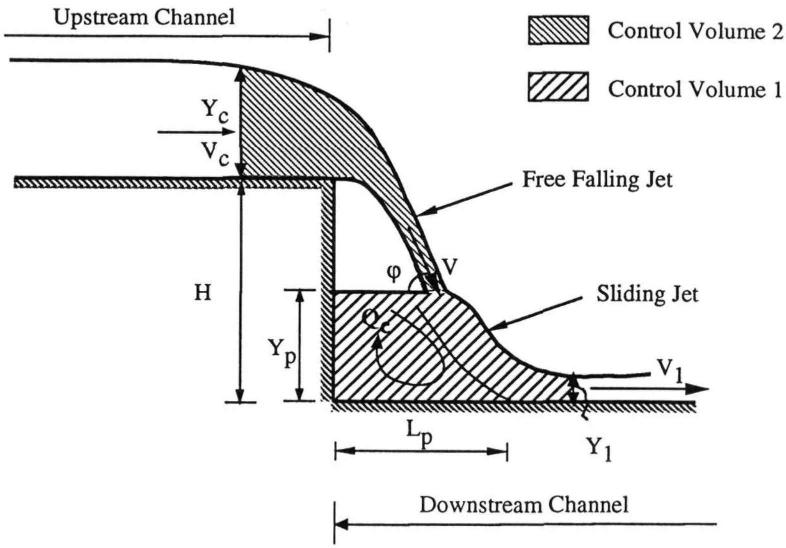


Fig. 1. Definition sketch of a drop.

In a discussion of Moore's paper, White developed a method to predict the energy loss at the drop based on a number of assumptions, which have been questioned by several investigators including Moore. The first assumption is that the circulating flow Q_c in the pool at the bottom of the drop is the same as the backward flow Q_b in an impinging jet of the same angle β (shown in Fig. 2(a) & (b)) and is given by the equation

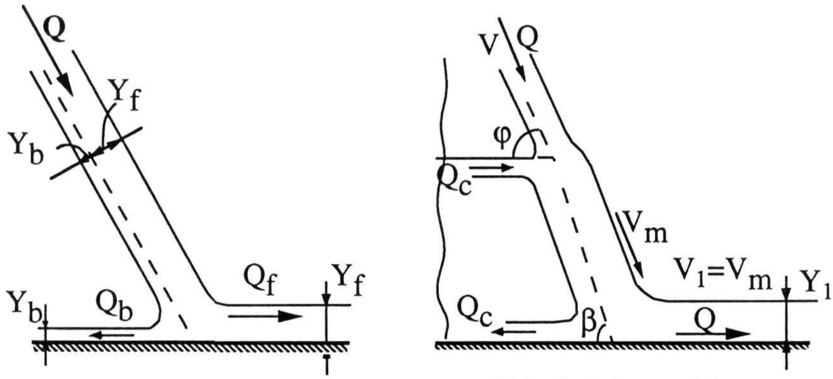
$$\frac{Q_c}{Q} = \frac{1 - \cos\beta}{1 + \cos\beta} \tag{2}$$

which was obtained from momentum and continuity considerations.

The second assumption is that the velocity of the supercritical stream immediately downstream of the drop V_1 is the same as the uniform velocity V_m in a thicker stream at the side of the pool (Fig. 2(b)), and V_m was obtained by assuming that the momentum flux of the jet is not affected by the mixing (with the pool) and is given as

$$V_m = V_1 = \frac{V}{2}(1 + \cos\beta) \tag{3}$$

It was also assumed (third assumption) that the angle of the falling jet was not affected by the presence of the pool.



(a) Inclined jet (b) Jet including pool flow

Fig. 2. Sketches of inclined jet and drop.

Further, White assumed that the energy loss at the drop was due to the mixing with the pool, which appears to be correct (fourth assumption). White also neglected the presence of the pool in calculating the velocity of the jet entering the pool (fifth assumption) in obtaining an expression for V as

$$V = \sqrt{2g(H + 1.5Y_c)} \tag{4}$$

and this has been corrected by Gill (1979). The horizontal component V_x of the velocity V of the jet just before entering the pool was obtained as

$$V_x = V\cos\beta = \frac{3}{2}\sqrt{gY_c} \tag{5}$$

by applying the momentum equation between the critical flow section upstream of the drop and the jet (sixth assumption) and this is valid. Using all these considerations, the specific energy E_1 at the base of the drop was given by the equation

$$\frac{E_1}{Y_c} = \frac{\sqrt{2}}{1.06 + \sqrt{\dfrac{H}{Y_c} + 1.5}} + \frac{1}{4}\left(1.06 + \sqrt{\frac{H_Y}{Y_c} + 1.5}\right)^2 \tag{6}$$

Since the energy upstream of the drop E_0 can be written as

$$E_0 = H + 1.5Y_c \tag{7}$$

and $\Delta E = E_0 - E_1$, it can be shown that $\Delta E/E_0$ is a function of only H/Y_c.

Gill (1979) attempted to modify the theory of White by refining his assumptions. Gill adopted the first assumption of White. Gill modified the second assumption of White to claim that V_m occurs in the thin layer below the pool level and the velocity is not constant in the mixing zone. The fifth assumption of White is modified as

$$V = \sqrt{2g\,(H + 1.5Y_c - Y_p)} \tag{8}$$

allowing for the presence of the pool. Gill modified the sixth assumption as

$$V_x = V_m \cos\varphi \tag{9}$$

where φ is the angle of the jet where it hits the pool and further φ was assumed to be equal to β (third assumption of White), which is not generally valid. Gill further modified the fourth assumption of White and wrote

$$V_1 = \sqrt{V_m^2 + 2g\,(Y_p - Y_1)} \tag{10}$$

This assumption of Gill neglects the energy loss in the pool which is not correct. Then Gill obtained the following equation for the depth below the drop Y_1 as

$$\frac{Y_1}{Y_c} = \frac{1}{\sqrt{\frac{(1+\cos\beta)^2}{2}\left(\frac{H}{Y_c} + 1.5 - \frac{Y_p}{Y_c}\right) + 2\left(\frac{Y_p - Y_1}{Y_c}\right)}} \tag{11}$$

from which E_1 can be obtained using the equation

$$\frac{E_1}{Y_c} = \frac{Y_1}{Y_c} + \frac{1}{2}\left(\frac{Y_c}{Y_1}\right)^2 \tag{12}$$

It should also be mentioned that the modification by Gill to the sixth assumption of White is not entirely reasonable. Gill also performed experiments on four drops with heights of 48.3, 74.0, 99.4 and 176.5 mm and measured primarily the pool depths and the angle of the impinging jet. Gill found that his method predicted values of E_1/Y_c somewhat larger than those predicted by the theory of White. Rand (1955) performed an experimental study on drops and developed empirical equations for the dimensionless characteristic lengths in terms of the Drop number D, equal to $(Y_c/H)^3$. Our objective in this study was to perform experiments on drops and make some basic measurements to check the validity of the first (and most important) assumption of White and the other assumptions and to obtain more information on the energy loss at drops.

Experimental arrangement and experiments

Two ventilated drops of heights of 0.62 and 0.25 m were built in a rectangular channel of width of 0.46 m, length of 6.55 m and depth of 0.91 m. The channel sides were constructed of plexiglas except at the water inlet where they were made of steel. The stilling tank was installed at the inlet to calm the high velocity flow from the supply tank so that the flow approached the drop with minimum disturbance. To avoid air entrainment in the flow at high discharge, it was necessary to keep the supply reservoir full. The discharge was measured with a Magnetic Flow Meter installed in the supply line feeding the head-tank. The second drop of height of 0.25 m was obtained by installing a step below the first drop. For the third drop of the same height, a slot was cut in the back wall, so that no pool was formed behind the jet, thereby simulating an inclined jet.

Water depths were measured with precision point gauges and time-averaged point velocities were measured with Prandtl probes of external diameter of 3.2 mm. In the mixing zone, a thinner Prandtl tube of external diameter of 2.6 mm was used. Flow visualization was performed with dye injection and video photography, to observe the development of flow in the mixing zone and the oscillation of the free falling jet. A total of ten discharges was studied for the two drops, providing a range of 0.06 to 0.35 for Y_c/H. (Two more partial experiments were also performed, in which the upstream flow was supercritical and these results are not generally discussed herein.) For the third drop simulating an inclined impinging jet, four experiments were performed, with Y_c/H varying from 0.2 to 0.35. All the measurements were made in the centreplane.

Experimental results

Fig. 3 and 4(a-c) show the observations made in a typical experiment, which included the water surface profile upstream of the drop, the profile of the falling jet, some (centerplane) velocity profiles in the flow upstream of the drop as well as in the falling jet, and in the supercritical flow downstream of the drop. Observations were also made in the pool behind the falling jet, including the fluctuating pool water surface profiles (see Fig. 5) and the diffusion of the circulating jet. Detailed velocity profiles similar to those shown in Fig. 3 for several other experiments are shown in the thesis of the second author. The falling jet was found to oscillate and the frequency was found to be about 7.5 Hz.

Considering the falling jet, the velocity distribution across the jet was essentially uniform until the jet reached the pool. The use of dye injection revealed that there was mixing between the sliding jet and the circulating flow, aided by the oscillation of the falling jet. The mixing between the jet and the pool was found to be intensive and the velocity profiles look like those in submerged turbulent jets (or more precisely shear layers). Using these jet-like velocity profiles, it was possible to calculate the circulating flow Q_c in the pool. If $Q_r = Q_c/Q$, the variation of Q_r with Y_c/H is shown in Fig. 6. From Fig. 6 it is seen that Q_r decreases from about 0.6 for $Y_c/H = 0.05$ to about 0.17 for $Y_c/H = 0.35$. Fig. 6 also shows the backflow rate Q_b for the impinging jet in terms of Q, which is termed

Q_{br}, and Q_b was calculated using the velocity profiles in the forward flow at the base of the drop. It is seen that Q_{br} is almost constant over the range of Y_c/H from 0.2 to 0.35 and equal to 0.27. From Fig. 6 it appears that Q_r and Q_{br} cannot be assumed to be equal as assumed by White. The present observations have also established that the velocity cannot be assumed to be uniform across the jet when it is sliding over the pool, which also invalidates the equal kinetic energy content assumption of White for the forward and circulating streams at the base of the drop.

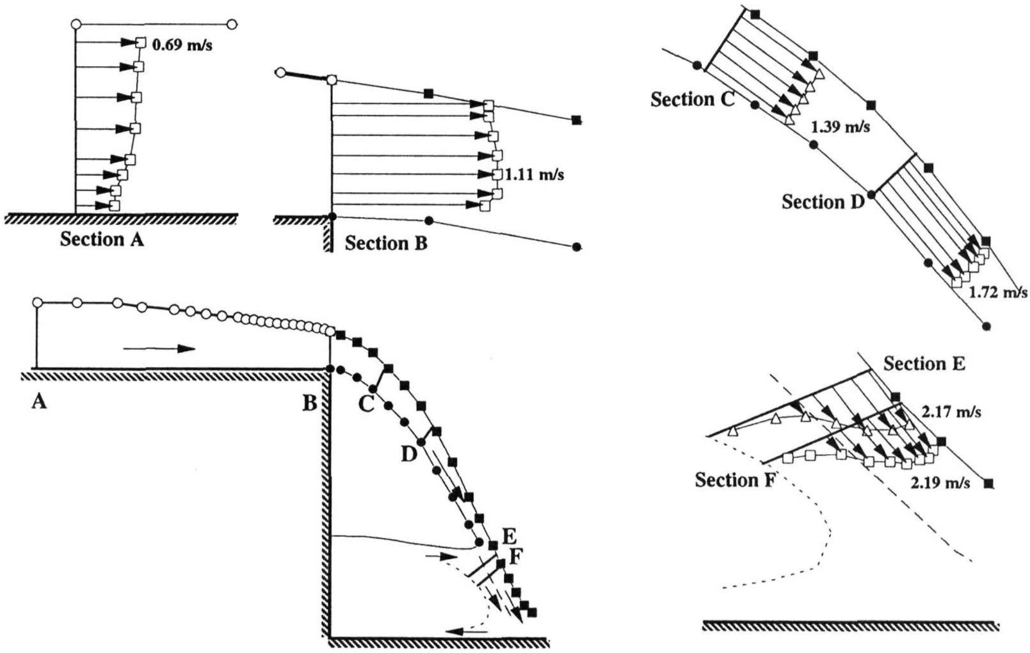


Fig. 3. Flow and velocity profiles at various locations for $Y_c/H = 0.20$.

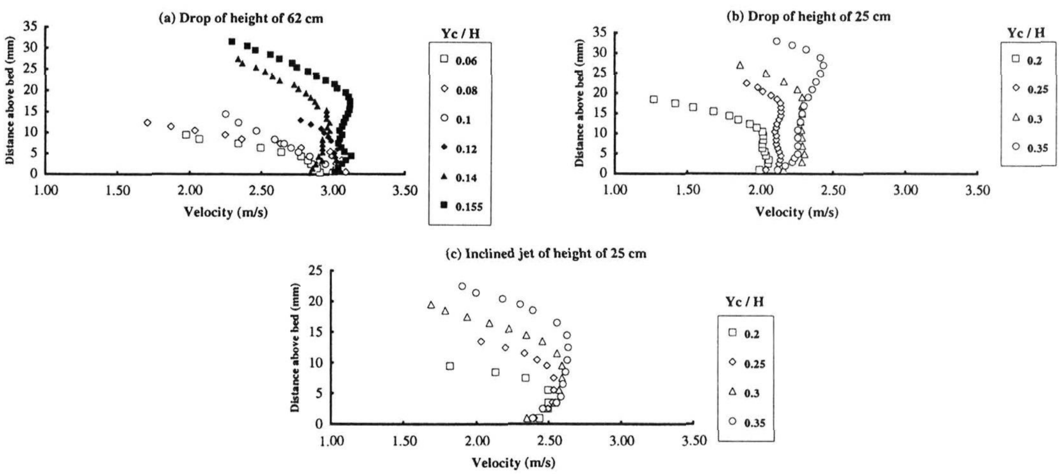


Fig. 4a–c. Velocity profiles at the base of the models.

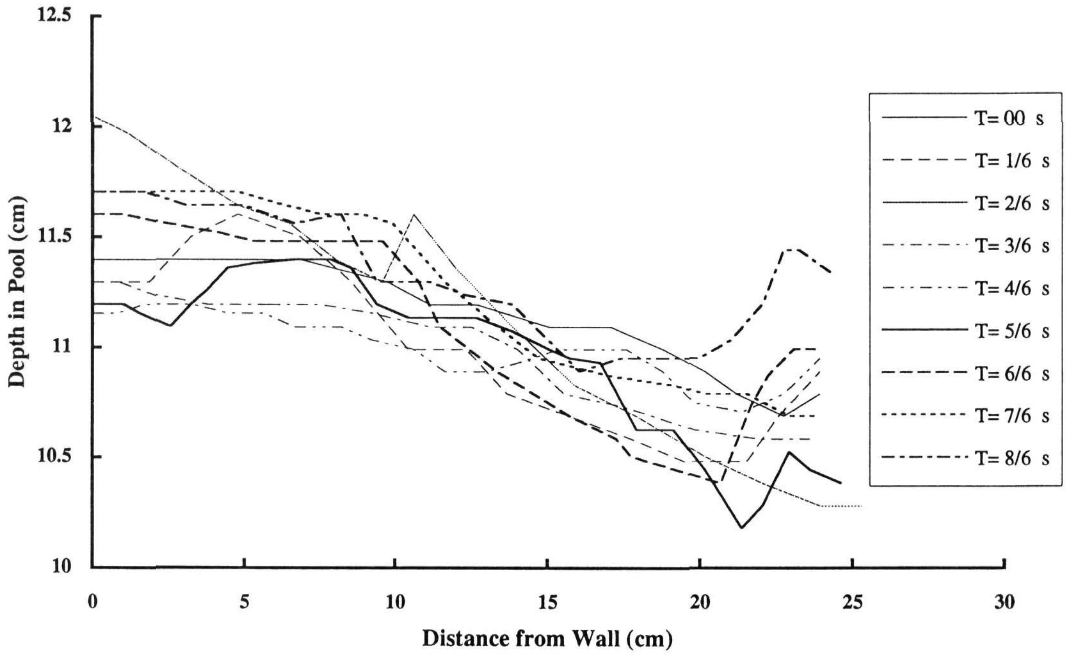


Fig. 5. Water surface profile in the pool at different times.

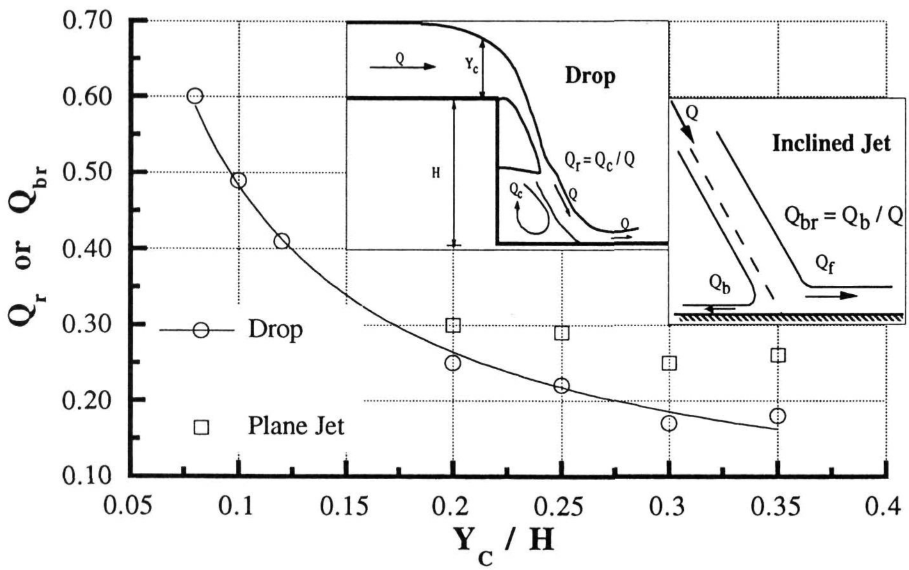


Fig. 6. Variation of the circulation flow and backflow ratios with Y_c/H.

Next, the variation of the relative energy loss $\Delta E/E_0$ with Y_c/H is studied. The thin flow at the base of the drops made the accurate measurement of the flow depth difficult. To obtain the relative energy loss, E_0 and E_1 were obtained as the sum of the flow depth and the average velocity head using the measured velocity profiles at the respective sections. Fig. 7 shows the variation of $\Delta E/E_0$ with Y_c/H from the present work along with the observations of Moore which agree well with the present results and help to define the general variation. The results of Rand are somewhat lower and this might be due to the crude way of calculating the velocity head, using an approximate velocity derived from depth measurements. The relative energy loss decreases from about 0.6 for $Y_c/H = 0.1$ to about 0.1 for $Y_c/H = 1.0$. The relative energy loss for the inclined jet model is about 0.11 which also lends further support to the concept that most of the energy dissipation occurs due to the mixing in the pool. The mean curve shown in Fig. 7 is well described by the equation (correlation coefficient is 0.970)

$$\frac{\Delta E}{E_0} = 0.896\left(\frac{Y_c}{H}\right)^{-0.766} \tag{13}$$

Coming back to the pool behind the jet, the water surface profile was fluctuating (see Fig. 5) with generally a downward slope towards the jet. The primary circulation in the pool did not occupy the full length of the pool and a secondary cell of much reduced circulation appeared to exist. Experimental results on the variation of the depth Y_p/H with Y_c/H are shown in Fig. 8 along with the empirical equation of Rand for the relative depth of the pool. Fig. 8 also shows some observations for the relative length of the pool.

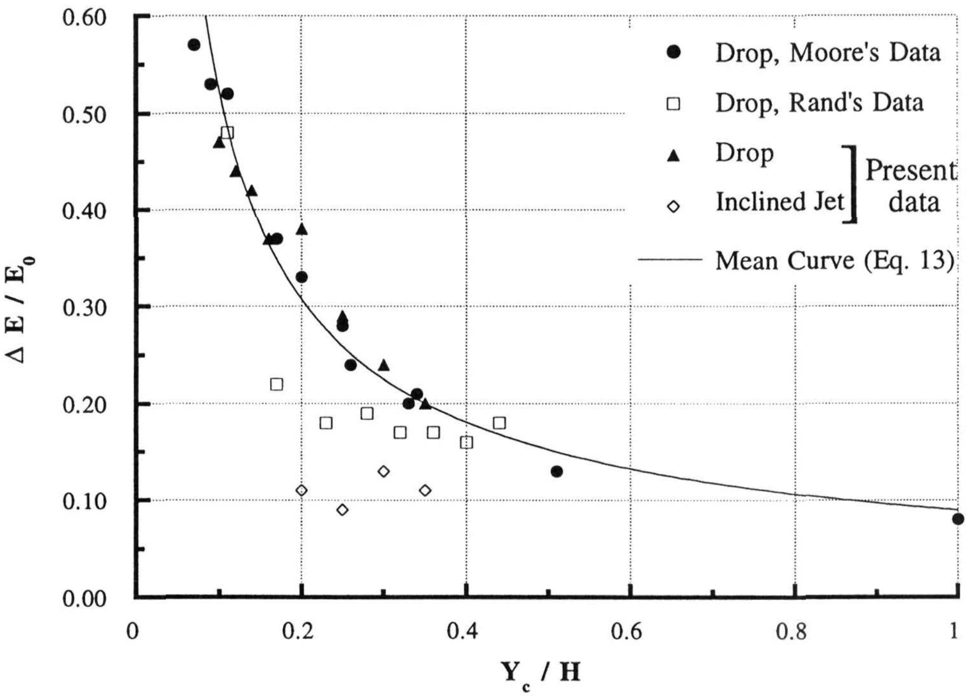


Fig. 7. Variation of the relative energy loss for the drop and the inclined jet.

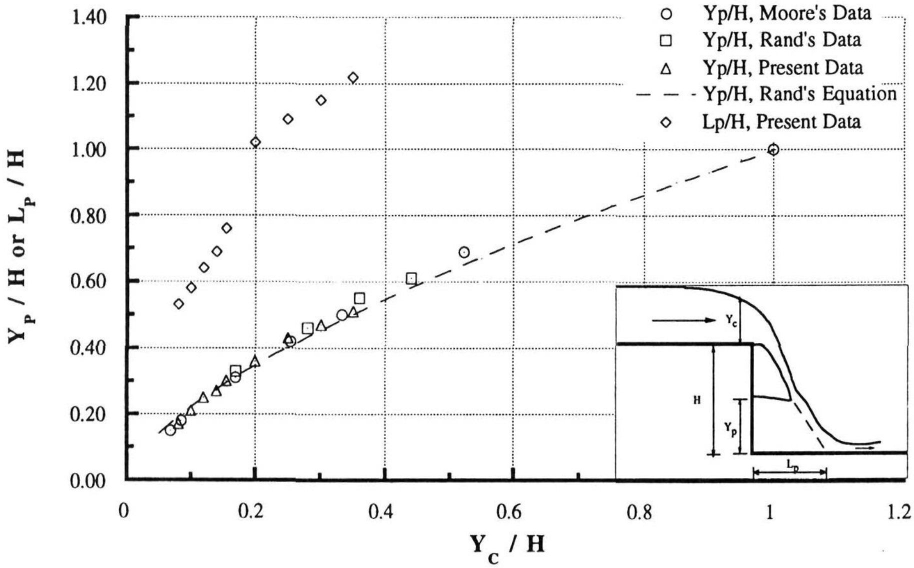


Fig. 8. Variation of the relative depth and length of the pool with Y_c/H.

A method to predict drop characteristics

Having studied the hydraulics of the drops, let us attempt to develop a method to predict the main flow features of drops. The following analysis is based on a number of assumptions to simplify the problem. The effect of air entrainment on flow characteristics is ignored. With reference to Fig. 1, applying the momentum equation to the control volume 1, we obtain:

$$\rho q V \cos\varphi + \frac{1}{2}\gamma Y_p^2 = \rho q V_1 + \frac{1}{2}\gamma Y_1^2 \tag{14}$$

where ρ and γ are respectively the mass density and specific weight of the fluid. It is assumed that the shear force at the bed is negligible. For the subcritical flow that approaches the drop structure at critical depth, the momentum and energy equations for the control volume 2 reduce to

$$\frac{1}{2}\gamma Y_c^2 + \rho q V_c = \rho q V \cos\varphi \tag{15}$$

and

$$H + \frac{3}{2}Y_c = \frac{V^2}{2g} + Y_p \tag{16}$$

The approaching flow is assumed to have a uniform velocity distribution and hydrostatic pressure distribution at the critical section. In addition, the energy loss in the control volume and the upstream bed shear force are ignored. The last relation is the continuity equation which is reduced to

$$q = Y_c\sqrt{gY_c} \tag{17}$$

There are only four equations for five unknowns V, φ, Y_p, V_1 and Y_1. It is convenient to use an empirical equation for the relative depth of the pool which fits the experimental data in Fig. 8 as following:

$$\frac{Y_p}{H} = 1.107\left(\frac{Y_c}{H}\right)^{0.719} \tag{18}$$

These equations were solved and the computed results are compared with the present experimental results in Table 1 for φ, Y_1/H, and V_1 for a range of Y_c/H from 0.06 to 0.35. A study of Table 1 shows that the agreement between the predicted and the experimental values of ϕ is good. The observed values for Y_1/H are mostly higher than the predicted ones and most of the observed values fall within about 10% of the predicted values and the differences decrease for higher discharges. The proposed method consistently over predicts V_1, thereby predicting lower values for the energy loss. The pressure distribution in the pool was assumed to be hydrostatic and it is possible that the circulation in the pool could cause departures from the hydrostatic distribution (Subramanya (1967) and Robinson (1992)). The shear stress along the bed of the pool was neglected and this might also contıibute to the differences between the predicted and measured values. A comparison between the experimental results and the predictions for the relative energy loss at the drop is shown in Fig. 9. In Fig. 9, the computed values of the relative energy loss by the present method are very close to the predictions of Gill. The experimental results deviate from the predictions of White as Y_c/H increases. The experimental results fall between the equations of White on top and closer to those of Gill and the present method on the bottom.

Table 1. Comparison between predicted and experimental values

	φ (deg.)		Y_1/H		$V_1 (m/s)$	
Y_c/H	Measured Values	Predicted Values	Measured Values	Predicted Values	Measured Values	Predicted Values
0.06	-	74.5	0.017	0.013	2.45	2.69
0.08	72	72.0	0.023	0.020	2.47	2.79
0.1	71	69.8	0.026	0.027	2.55	2.88
0.12	69	67.7	0.038	0.035	2.63	2.96
0.14	67	65.9	0.047	0.042	2.75	3.04
0.155	66	64.6	0.053	0.048	2.88	3.10
0.165	64	63.7	0.057	0.053	2.82	3.14
0.2	60	60.9	0.078	0.068	1.86	2.07
0.25	57	57.4	0.094	0.090	2.09	2.17
0.3	53	54.2	0.116	0.114	2.20	2.26
0.35	51.5	51.3	0.140	0.139	2.29	2.34

Because the mixing process in the pool is responsible for the energy loss, it is interesting to see as to whether any correlation exists between Q_r and the relative energy loss. Fig. 10 shows the variation of Q_r and $\Delta E/E_0$ with Y_c/H for Y_c/H varying from 0.06 to 0.35. A study of Fig 10 shows that values of $\Delta E/E_0$ are reasonably close to those of Q_r, indicating perhaps the significance of the circulating flow in the energy dissipation at the drop.

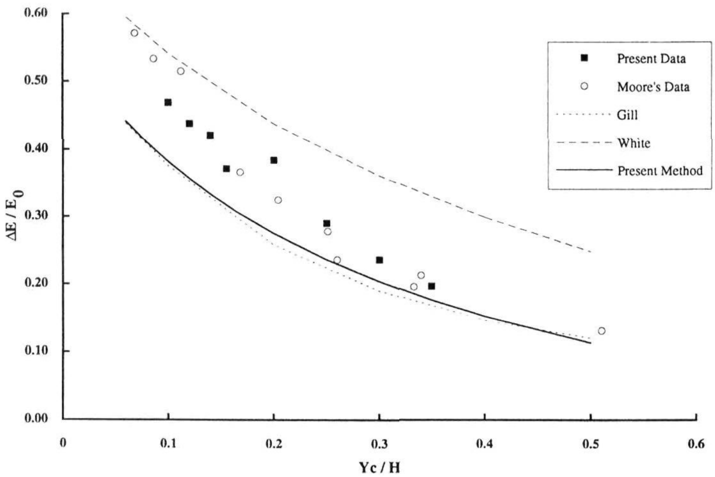


Fig. 9. Variation of the relative energy loss at the base of the drop.

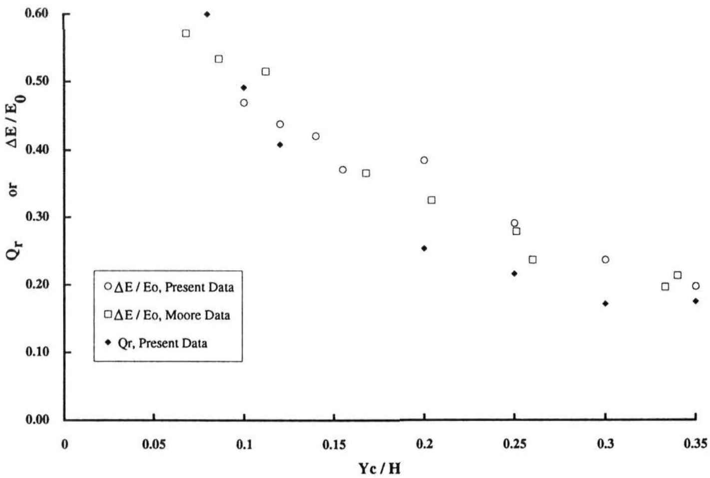


Fig. 10. Relationships established for the relative energy loss based on the circulating discharge ratio.

Conclusions

This paper presents a critical look at the energy loss of drops. In particular, it looks critically at the several assumptions of White and Gill, who developed theoretical equations to predict the energy loss at a drop. The present study also presents some observations on the characteristics of the flow like velocity profiles in the falling jet, as well as in the jet bounded by the pool. Using these observations, the validity of some of the main assumptions has been evaluated. Also a new method has been developed to predict the energy loss at a drop. An empirical equation has also been presented for the relative energy loss at drops.

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Notations

D	Drop number
E_0	Energy at the upstream of a drop or an inclined jet
E_1	Specific energy at the base of a drop or an inclined jet
g	Gravitational acceleration
H	Height of a drop
L_p	Pool length
Q	Discharge
Q_c	Circulating discharge
Q_b	Backflow discharge in an inclined jet
Q_{br}	Backflow discharge ratio
Q_f	Forward discharge in an inclined jet
Q_r	Circulating discharge ratio
q	Discharge per unit width
V	Mean velocity of the free falling jet just above the pool level
V_1	Mean velocity at the base of a drop or an inclined jet
V_m	Mean velocity of the sliding jet
V_x	Horizontal mean velocity component of the free falling jet
Y_1	Depth of flow at the base of a drop
Y_b	Depth of backflow in an inclined jet
Y_f	Depth of forward flow in an inclined jet
Y_c	Critical depth
Y_p	Pool depth
β	Jet inclination at the bed
γ	Specific weight of fluid
ΔE	Energy loss
ρ	Mass density of fluid
φ	Jet inclination at the pool level

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