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STRUCTURAL ANALYSIS AND EVALUATION

Commentary is opposite the text it annotates.

4.1—SCOPE

This section describes methods of analysis suitable for the design and evaluation of bridges, and is limited to the modeling of structures and the determination of force effects.

Other methods of analysis that are based on documented material characteristics and that satisfy equilibrium and compatibility may also be used.

In general, bridge structures are to be analyzed elastically. However, this section permits the inelastic analysis or redistribution of force effects in some continuous beam superstructures. It specifies inelastic analysis for compressive members behaving inelastically and as an alternative for extreme event limit states.

C4.1

This section identifies and promotes the application of methods of structural analysis that are suitable for bridges. The selected method of analysis may vary from the approximate to the very sophisticated, depending on the size, complexity, and priority of the structure. The primary objective in the use of more sophisticated methods of analysis is to obtain a better understanding of structural behavior. Such improved understanding may often, but not always, lead to the potential for saving material.

The methods of analysis outlined herein, which are suitable for the determination of deformations and force effects in bridge structures, have been successfully demonstrated, and most have been used for years. Although many methods will require a computer for practical implementation, simpler methods that are amenable to hand calculation and/or the use of existing computer programs based on line-structure analysis are also provided. Comparison with hand calculations should always be encouraged and basic equilibrium checks should be standard practice.

With rapidly improving computing technology, the more refined and complex methods of analysis are expected to become commonplace. Hence, this section addresses the assumptions and limitations of such methods. It is important that the user understand the method employed and its associated limitations.

In general, the suggested methods of analysis are based on linear material models. This does not mean that cross-sectional resistance is limited to the linear range. This presents an obvious inconsistency in that the analysis is based on material linearity and the resistance model may be based on inelastic behavior for the strength limit states. This same inconsistency existed, however, in the load factor design method of the AASHTO *Standard Specifications for Highway Bridges*, and is present in design codes of other nations using a factored design approach.

The loads and load factors, defined in Section 3, and the resistance factors specified throughout these Specifications were developed using probabilistic principles combined with analyses based on linear material models. Hence, analysis methods based on material nonlinearities to obtain force effects that are more realistic at the strength limit states and subsequent economics that may be derived are permitted only where explicitly outlined herein.

Some nonlinear behavioral effects are addressed in both the analysis and resistance sections. For example, long column behavior may be modeled via geometric nonlinear methods and may also be modeled using approximate formulae in Sections 5, 6, 7, and 8. Either method may be used, but the more refined formulations are recommended.

4.2—DEFINITIONS

Accepted Method of Analysis—A method of analysis that requires no further verification and that has become a regular part of structural engineering practice.

Arc Span—Distance between centers of adjacent bearings or other points of support, measured horizontally along the centerline of a horizontally curved member.

Aspect Ratio—Ratio of the length to the width of a rectangle.

Boundary Conditions—Structural restraint characteristics regarding the support for and/or the continuity between structural models.

Bounding—Taking two or more extreme values of parameters to envelop the response with a view to obtaining a conservative design.

Central Angle—The angle included between two points along the centerline of a curved bridge measured from the center of the curve, as shown in Figure 4.6.1.2.3-1.

Classical Deformation Method—A method of analysis in which the structure is subdivided into components whose stiffness can be independently calculated. Equilibrium and compatibility among the components is restored by determining the deformations at the interfaces.

Classical Force Method—A method of analysis in which the structure is subdivided into statically determinate components. Compatibility among the components is restored by determining the interface forces.

Closed-Box Section—A cross-section composed of two vertical or inclined webs which has at least one completely enclosed cell. A closed-section member is effective in resisting applied torsion by developing shear flow in the webs and flanges.

Closed-Form Solution—One or more equations, including those based on convergent series, that permit calculation of force effects by the direct introduction of loads and structural parameters.

Compatibility—The geometrical equality of movement at the interface of joined components.

Component—A structural unit requiring separate design consideration; synonymous with member.

Condensation—Relating the variables to be eliminated from the analysis to those being kept to reduce the number of equations to be solved.

Core Width—The width of the superstructure of monolithic construction minus the deck overhangs.

Cross-Section Distortion—Change in shape of the cross-section profile due to torsional loading.

Curved Girder—An I-, closed-box, or tub girder that is curved in a horizontal plane.

Damper—A device that transfers and reduces forces between superstructure elements and/or superstructure and substructure elements, while permitting thermal movements. The device provides damping by dissipating energy under seismic, braking, or other dynamic loads.

Deck—A component, with or without wearing surface, directly supporting wheel loads.

Deck System—A superstructure in which the deck is integral with its supporting components or in which the effects or deformation of supporting components on the behavior of the deck is significant.

Deformation—A change in structural geometry due to force effects, including axial displacement, shear displacement, and/or rotations.

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Degree-of-Freedom—One of a number of translations or rotations required to define the movement of a node. The displaced shape of components and/or the entire structure may be defined by a number of degrees-of-freedom.

Design—Proportioning and detailing the components and connections of a bridge to satisfy the requirements of these Specifications.

Dynamic Degree-of-Freedom—A degree-of-freedom with which mass or mass effects have been associated.

Elastic—A structural material behavior in which the ratio of stress to strain is constant; the material returns to its original unloaded state upon load removal.

Element—A part of a component or member consisting of one material.

End Zone—Region of structures where normal beam theory does not apply due to structural discontinuity and/or distribution of concentrated loads.

Equilibrium—A state where the sum of forces and moments about any point in space is 0.0.

Equivalent Beam—A single straight or curved beam resisting both flexural and torsional effects.

Equivalent Strip—An artificial linear element, isolated from a deck for the purpose of analysis, in which extreme force effects calculated for a line of wheel loads, transverse or longitudinal, will approximate those actually taking place in the deck.

Finite Difference Method—A method of analysis in which the governing differential equation is satisfied at discrete points on the structure.

Finite Element Method—A method of analysis in which a structure is discretized into elements connected at nodes, the shape of the element displacement field is assumed, partial or complete compatibility is maintained among the element interfaces, and nodal displacements are determined by using energy variational principles or equilibrium methods.

Finite Strip Method—A method of analysis in which the structure is discretized into parallel strips. The shape of the strip displacement field is assumed and partial compatibility is maintained among the element interfaces. Model displacement parameters are determined by using energy variational principles or equilibrium methods.

First-Order Analysis—Analysis in which equilibrium conditions are formulated on the undeformed structure; that is, the effect of deflections is not considered in writing equations of equilibrium.

Flange Lateral Bending—Bending of a flange about an axis perpendicular to the flange plane due to lateral loads applied to the flange and/or nonuniform torsion in the member.

Flange Lateral Bending Stress—The normal stress caused by flange lateral bending.

Folded Plate Method—A method of analysis in which the structure is subdivided into plate components, and both equilibrium and compatibility requirements are satisfied at the component interfaces.

Footprint—The contact area between wheel and roadway surface.

Force Effect—A deformation, stress, or stress resultant, i.e., axial force, shear force, or flexural or torsional moment, caused by applied loads, imposed deformations, or volumetric changes.

Foundation—A supporting element that derives its resistance by transferring its load to the soil or rock supporting the bridge.

Frame Action—Transverse continuity between the deck and the webs of cellular cross-section or between the deck and primary components in large bridges.

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Frame Action for Wind—Transverse flexure of the beam web and that of framed stiffeners, if present, by which lateral wind load is partially or completely transmitted to the deck.

Girder Radius—The radius of the circumferential centerline of a segment of a curved girder.

Global Analysis—Analysis of a structure as a whole.

Governing Position—The location and orientation of transient load to cause extreme force effects.

Grid Method—A grid method of analysis of girder bridges in which the longitudinal girders are modeled individually using beam elements and the cross-frames are typically modeled as equivalent beam elements. For composite girders, a tributary deck width is considered in the calculation of individual girder cross-section properties.

Inelastic—Any structural behavior in which the ratio of stress and strain is not constant, and part of the deformation remains after load removal.

Lane Live Load—The combination of tandem axle and uniformly distributed loads or the combination of the design truck and design uniformly distributed load.

Large Deflection Theory—Any method of analysis in which the effects of deformation upon force effects is taken into account.

Lever Rule—The statical summation of moments about one point to calculate the reaction at a second point.

Linear Response—Structural behavior in which deflections are directly proportional to loads.

Local Analysis—An in-depth study of strains and stresses in or among components using force effects obtained from a more global analysis.

Local Structural Stress—The stress at a welded detail including all stress-raising effects of a structural detail but excluding all stress concentrations due to the local weld profile itself.

Member—Same as *Component*.

Method of Analysis—A mathematical process by which structural deformations, forces, and stresses are determined.

Model—A mathematical or physical idealization of a structure or component used for analysis.

Monolithic Construction—Single-cell steel and/or concrete box bridges, solid or cellular cast-in-place concrete deck systems, and decks consisting of precast, solid, or cellular longitudinal elements effectively tied together by transverse post-tensioning.

M/R Method—An approximate method for the analysis of curved box girders in which the curved girder is treated as an equivalent straight girder to calculate flexural effects and as a corresponding straight conjugate beam to calculate the concomitant St. Venant torsional moments due to curvature.

Multibeam Decks—Bridges with superstructure members consisting of adjacent precast sections with the top flange as a complete full-depth integral deck or a structural deck section placed as an overlay. Sections can be closed cell boxes or open stemmed.

Negative Moment—Moment producing tension at the top of a flexural element.

Node—A point where finite elements or grid components meet; in conjunction with finite differences, a point where the governing differential equations are satisfied.

Nonlinear Response—Structural behavior in which the deflections are not directly proportional to the loads due to stresses in the inelastic range, or deflections causing significant changes in force effects, or a combination thereof.

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Nonuniform Torsion—An internal resisting torsion in thin-walled sections, also known as warping torsion, producing shear stress and normal stresses, and under which cross-sections do not remain plane. Members resist the externally applied torsion by warping torsion and St. Venant torsion. Each of these components of internal resisting torsion varies along the member length, although the externally applied concentrated torque may be uniform along the member between two adjacent points of torsional restraint. Warping torsion is dominant over St. Venant torsion in members having open cross-sections, whereas St. Venant torsion is dominant over warping torsion in members having closed cross-sections.

Open Section—A cross-section which has no enclosed cell. An open-section member resists torsion primarily by nonuniform torsion, which causes normal stresses at the flange tips.

Orthotropic—Perpendicular to each other, having physical properties that differ in two or more orthogonal directions.

Panel Point—The point where centerlines of members meet, usually in trusses, arches, or cable-stayed and suspension bridges.

Pin Connection—A connection among members by a notionally frictionless pin at a point.

Pinned End—A boundary condition permitting free rotation but not translation in the plane of action.

Plate and Eccentric Beam Method—A method of analysis of composite girder bridges in which the bridge deck is modeled using shell finite elements, the longitudinal girders are modeled using beam elements, and the cross-frames are typically modeled as equivalent beam elements, The girder and cross-frame elements are offset from the deck elements to account for the structural depth of these components relative to the deck.

Point of Contraflexure—The point where the sense of the flexural moment changes; synonymous with point of inflection.

Positive Moment—Moment producing tension at the bottom of a flexural element.

Primary Member—A member designed to carry the loads applied to the structure as determined from an analysis.

Rating Vehicle—A sequence of axles used as a common basis for expressing bridge resistance.

Refined Methods of Analysis—Methods of structural analysis that consider the entire superstructure as an integral unit and provide the required deflections and actions.

Restrainers—A system of high-strength cables or rods that transfers forces between superstructure elements and/or superstructure and substructure elements under seismic or other dynamic loads after an initial slack is taken up, while permitting thermal movements.

Rigidity—Force effect caused by a corresponding unit deformation per unit length of a component.

Secondary Member—A member in which stress is not normally evaluated in the analysis.

Second-Order Analysis—Analysis in which equilibrium conditions are formulated on the deformed structure; that is, in which the deflected position of the structure is used in writing the equations of equilibrium.

Series or Harmonic Method—A method of analysis in which the load model is subdivided into suitable parts, allowing each part to correspond to one term of a convergent infinite series by which structural deformations are described.

Shear Flow—Shear force per unit width acting parallel to the edge of a plate element.

Shear Lag—Nonlinear distribution of normal stress across a component due to shear distortions.

Shock Transmission Unit (STU)—A device that provides a temporary rigid link between superstructure elements and/or superstructure and substructure elements under seismic, braking, or other dynamic loads, while permitting thermal movements.

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Skew Angle—Angle between the centerline of a support and a line normal to the roadway centerline.

Small Deflection Theory—A basis for methods of analysis where the effects of deformation upon force effects in the structure is neglected.

Spacing of Beams—The center-to-center distance between lines of support.

Spine Beam Model—An analytical model of a bridge in which the superstructure is represented by a single beam element or a series of straight, chorded beam elements located along the centerline of the bridge.

Spread Beams—Beams not in physical contact with one another, carrying a cast-in-place concrete deck.

Stiffness—Force effect resulting from a unit deformation.

Strain—Elongation per unit length.

Stress Range—The algebraic difference between extreme stresses.

St. Venant Torsion—That portion of the internal resisting torsion in a member producing only pure shear stresses on a cross-section; also referred to as pure torsion or uniform torsion.

Submodel—A constituent part of the global structural model.

Superimposed Deformation—Effect of settlement, creep, and change in temperature and/or moisture content.

Superposition—The situation where the force effect due to one loading can be added to the force effect due to another loading. Use of superposition is only valid when the stress–strain relationship is linearly elastic and the small deflection theory is used.

Tandem—Two closely spaced and mechanically interconnected axles of equal weight.

Through-Thickness Stress—Bending stress in a web or box flange induced by distortion of the cross-section.

Torsional Shear Stress—Shear stress induced by St. Venant torsion.

Tub Section—An open-topped section which is composed of a bottom flange, two inclined or vertical webs, and top flanges.

Uncracked Section—A section in which the concrete is assumed to be fully effective in tension and compression.

V-Load Method—An approximate method for the analysis of curved I-girder bridges in which the curved girders are represented by equivalent straight girders and the effects of curvature are represented by vertical and lateral forces applied at cross-frame locations. Lateral flange bending at brace points due to curvature is estimated.

Warping Stress—Normal stress induced in the cross-section by warping torsion and/or by distortion of the cross-section.

Wheel Load—One-half of a specified design axle load.

Yield Line—A plastic hinge line.

Yield Line Method—A method of analysis in which a number of possible yield line patterns are examined in order to determine load-carrying capacity.

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4.3—NOTATION

A	=	area of a stringer, beam, or component (in.^{2}) (4.6.2.2.1) (C4.6.2.2.1)
A_b	=	cross-sectional area of barrier (in.^{2}) (4.6.2.6.1)
A_c	=	cross-section area—transformed for steel beams (in.^{2}) (C4.6.6)
A_o	=	area enclosed by centerlines of elements (in.^{2}) (C4.6.2.2.1)
$(AE)_{eq}$	=	equivalent axial rigidity of single-angle members and flange-connected T-section cross-frame members that accounts for bending effects due to end connection eccentricities (kip) (C4.6.3.3.4)
A_s	=	total area of stiffeners (in.^{2}) (4.6.2.6.4)
a	=	length of transition region for effective flange width of a concrete box beam (in.); longitudinal stiffener, spacing, or rib width in an orthotropic steel deck (in.) (4.6.2.6.2) (4.6.2.6.4)
B	=	spacing of transverse beams (in.) (4.6.2.6.4)
b	=	tire length (in.); width of a beam (in.); width of plate element (in.); flange width each side of the web (in.) (4.6.2.1.8) (4.6.2.2.1) (C4.6.2.2.1) (4.6.2.6.2)
b_e	=	effective flange width corresponding to the particular position of the section of interest in the span as specified in Figure 4.6.2.6.2-1 (in.) (4.6.2.6.2)
b_m	=	effective flange width for interior portions of a span as determined from Figure 4.6.2.6.2-2; a special case of b_e (in.) (4.6.2.6.2)
b_n	=	effective flange width for normal forces acting at anchorage zones (in.) (4.6.2.6.2)
b_o	=	width of web projected to midplane of deck (in.) (4.6.2.6.2)
b_{od}	=	effective width of orthotropic deck (in.) (4.6.2.6.4)
b_s	=	effective flange width at interior support or for cantilever arm as determined from Figure 4.6.2.6.2-2; a special case of b_e (in.) (4.6.2.6.2) (C4.6.2.6.2)
C	=	continuity factor; stiffness parameter (4.6.2.1.8) (4.6.2.2.1)
C_m	=	moment gradient coefficient (4.5.3.2.2b)
C_{sm}	=	the dimensionless elastic seismic response coefficient (4.7.4.3.2b)
c_1	=	parameter for skewed supports (4.6.2.2.2e)
C_w	=	girder warping constant (in.^{6}) (C4.6.3.3.2)
D	=	web depth of a horizontally curved girder (ft); D_x/D_y; width of distribution per lane (ft) (C4.6.1.2.4b) (4.6.2.1.8) (4.6.2.2.1)
D_x	=	flexural rigidity in direction of main bars (kip- ft^2/ft) (4.6.2.1.8)
D_y	=	flexural rigidity perpendicular to the main bars (kip-ft^2/ft) (4.6.2.1.8)
d	=	depth of a beam or stringer (in.); depth of member (ft) (4.6.2.2.1) (C4.6.2.7.1)
d_e	=	horizontal distance from the centerline of the exterior web of exterior beam at the deck level to the interior edge of curb or traffic barrier (ft) (4.6.2.2.1)
d_o	=	depth of superstructure (in.) (4.6.2.6.2)
E	=	modulus of elasticity (ksi); equivalent width (in.); equivalent distribution width perpendicular to span (in.) (4.5.3.2.2b) (4.6.2.3) (4.6.2.10.2) (C4.6.6)
E_B	=	modulus of elasticity of beam material (ksi) (4.6.2.2.1)
E_c	=	modulus of elasticity of column (ksi) (C4.6.2.5)
E_D	=	modulus of elasticity of deck material (ksi) (4.6.2.2.1)
E_g	=	modulus of elasticity of beam or other restraining member (ksi) (C4.6.2.5)
E_{MOD}	=	cable modulus of elasticity, modified for nonlinear effects (ksi) (4.6.3.7)
E_{span}	=	equivalent distribution length parallel to span (in.) (4.6.2.10.2)
e	=	correction factor for distribution; eccentricity of a lane from the center of gravity of the pattern of girders (ft); rib spacing in orthotropic steel deck (in.) (4.6.2.2.1) (C4.6.2.2.2d) (4.6.2.6.4)
e_g	=	distance between the centers of gravity of the beam and deck (in.) (4.6.2.2.1)
f_c	=	factored stress, corrected to account for second-order effects (ksi) (4.5.3.2.2b)
f_{2b}	=	stress corresponding to M_{2b} (ksi) (4.5.3.2.2b)
f_{2s}	=	stress corresponding to M_{2s} (ksi) (4.5.3.2.2b)
G	=	final force effect applied to a girder (kip or kip-ft); shear modulus (ksi) (4.6.2.2.5) (C4.6.3.3)
G_a	=	ratio of stiffness of column to stiffness of members resisting column bending at "*a*" end (C4.6.2.5)
G_b	=	ratio of stiffness of column to stiffness of members resisting column bending at "*b*" end (C4.6.2.5)
G_D	=	force effect due to design loads (kip or kip-ft) (4.6.2.2.5)
G_p	=	force effect due to overload truck (kip or kip-ft) (4.6.2.2.5)
g	=	live load distribution factor representing the number of design lanes; acceleration of gravity (ft/sec.^{2}) (4.6.2.2.1) (C4.7.4.3.2)
g_m	=	multiple-lane live load distribution factor (4.6.2.2.5)
g_1	=	single-lane live load distribution factor (4.6.2.2.5)

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H = depth of fill from top of culvert to top of pavement (in.); average height of substructure supporting the seat under consideration (ft) (4.6.2.10.2) (4.7.4.4)
H, H_1, H_2 = horizontal component of cable force (kip) (4.6.3.7)
h = depth of deck (in.) (4.6.2.1.3)
I = moment of inertia (in.^{4}) (4.5.3.2.2b)
I_C = I-girder bridge connectivity index (4.6.3.3.2)
I_c = moment of inertia of column (in.^{4}); inertia of cross-section—transformed for steel beams (in.^{4}) (C4.6.2.5) (C4.6.6)
I_g = moment of inertia of member acting to restrain column bending (in.^{4}) (C4.6.2.5)
IM = dynamic load allowance (C4.7.2.1)
I_p = polar moment of inertia (in.^{4}) (4.6.2.2.1) (C4.6.2.2.1)
I_s = moment of inertia of equivalent strip (in.^{4}) (4.6.2.1.5)
I_s = bridge skew index, taken equal to the maximum of the values of Eq. 4.6.3.3.2-2 determined for each span of the bridge (4.6.3.3.2)
J = St. Venant torsional inertia (in.^{4}) (C4.6.2.2.1)
K = effective length factor for columns and arch ribs; constant for different types of construction; effective length factor for columns in the plane of bending (4.5.3.2.2b) (4.5.3.2.2c) (4.6.2.2.1) (4.6.2.5)
K_g = longitudinal stiffness parameter (in.^{4}) (4.6.2.2.1)
k = factor used in calculation of distribution factor for multibeam bridges (4.6.2.2.1)
k_s = strip stiffness factor (kip/in.) (4.6.2.1.5)
L = span length of deck (ft); span length (ft); span length of beam (ft); length of bridge deck (ft) (4.6.2.1.3) (4.6.2.1.8) (4.6.2.2.1) (C4.6.2.7.1) (4.7.4.4)
L_{as} = effective arc span of a horizontally curved girder (ft) (4.6.1.2.4b)
L_b = spacing of cross-frames or diaphragms (ft) (C4.6.2.7.1)
L_c = unbraced length of column (in.) (C4.6.2.5)
L_g = unsupported length of beam or other restraining member (in.) (C4.6.2.5)
L_s = span length at the centerline (ft) (4.6.3.3.2)
$LLDF$ = factor for distribution of live load with depth of fill, 1.15 or 1.00, as specified in Article 3.6.1.2.6 (4.6.2.10.2)
L_T = length of tire contact area parallel to span, as specified in Article 3.6.1.2.5 (in.) (4.6.2.10.2)
L_1 = modified span length taken to be equal to the lesser of the actual span or 60.0 (ft); distance between points of inflection of the transverse beam (in.) (4.6.2.3)
L_2 = distances between points of inflection of the transverse beam (in.) (4.6.2.6.4) (C4.6.6)
ℓ = unbraced length of a horizontally curved girder (ft) (C4.6.1.2.4b)
ℓ_i = a notional span length (ft) (4.6.2.6.2)
ℓ_u = unsupported length of a compression member (in.); one-half of the length of the arch rib (ft) (4.5.3.2.2b) (4.5.3.2.2c)
M = major-axis bending moment in a horizontally curved girder (kip-ft); moment due to live load in filled or partially filled grid deck (kip-in./ft) (C4.6.1.2.4b) (4.6.2.1.8)
M_c = factored moment, corrected to account for second-order effects (kip-ft); moment required to restrain uplift caused by thermal effects (kip-in.) (4.5.3.2.2b) (C4.6.6)
M_{lat} = flange lateral bending moment due to curvature (kip-ft) (C4.6.1.2.4b)
MM = multimode elastic method (4.7.4.3.1)
M_n = nominal flexural strength (4.7.4.5)
M_w = maximum lateral moment in the flange due to the factored wind loading (kip-ft) (C4.6.2.7.1)
M_{1b} = smaller end moment on compression member due to factored gravity loads that result in no appreciable sidesway; positive if member is bent in single curvature, negative if bent in double curvature (kip-ft) (4.5.3.2.2b)
M_{2b} = moment on compression member due to factored gravity loads that result in no appreciable sidesway calculated by conventional first-order elastic frame analysis; always positive (kip-ft) (4.5.3.2.2b)
M_{2s} = moment on compression member due to factored lateral or gravity loads that result in sidesway, Δ, greater than $\ell_u/1500$, calculated by conventional first-order elastic frame analysis; always positive (kip-ft) (4.5.3.2.2b)
m = bridge type constant, equal to 1 for simple-span bridges or bridge units, and equal to 2 for continuous-span bridges or bridge units, determined at the construction stage being evaluated (4.6.3.3.2)
N = constant for determining the lateral flange bending moment in I-girder flanges due to curvature, taken as 10 or 12 in past practice; axial force (kip); minimum support length (in.) (C4.6.1.2.4b) (C4.6.6) (4.7.4.4)
N_b = number of beams, stringers, or girders (4.6.2.2.1) (C4.6.2.2.2c)
N_c = number of cells in a concrete box girder (4.6.2.2.1)
N_L = number of design lanes (4.6.2.2.1) (C4.6.2.2.1)

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n	=	modular ratio between beam and deck (4.6.2.2.1)
n_{cf}	=	minimum number of intermediate cross-frames or diaphragms within the individual spans of the bridge or bridge unit at the stage of construction being evaluated (4.6.3.3.2)
P	=	axle load (kip) (4.6.2.1.3)
P_D	=	design horizontal wind pressure (ksf) (C4.6.2.7.1)
P_e	=	Euler buckling load (kip) (4.5.3.2.2b)
P_u	=	factored axial load (kip) (4.5.3.2.2b) (4.7.4.5)
P_w	=	lateral wind force applied to the brace point (kips) (C4.6.2.7.1)
p	=	tire pressure (ksi) (4.6.2.1.8)
p_e	=	equivalent uniform static seismic loading per unit length of bridge that is applied to represent the primary mode of vibration (kip/ft) (C4.7.4.3.2c)
$p_e(x)$	=	the intensity of the equivalent static seismic loading that is applied to represent the primary mode of vibration (kip/ft) (C4.7.4.3.2b)
p_o	=	a uniform load arbitrarily set equal to 1.0 (kip/ft) (C4.7.4.3.2b)
R	=	girder radius (ft); load distribution to exterior beam in terms of lanes; minimum radius of curvature at the centerline of the bridge cross-section throughout the length of the bridge or bridge unit at the construction stage and/or loading condition being evaluated (ft); radius of curvature; R-factor for calculation of seismic design forces due to inelastic action (C4.6.1.2.4b) (C4.6.2.2.2d) (4.6.3.3.2) (C4.6.6) (4.7.4.5)
R_d	=	R_d-factor for calculation of seismic displacements due to inelastic action (4.7.4.5)
r	=	reduction factor for longitudinal force effect in skewed bridges (4.6.2.3)
S	=	spacing of supporting components (ft); spacing of beams or webs (ft); clear span (ft); skew of support measured from line normal to span (degrees) (4.6.2.1.3) (4.6.2.2.1) (4.6.2.10.2) (4.7.4.4)
S_b	=	spacing of grid bars (in.) (4.6.2.1.3)
SM	=	single-mode elastic method (4.7.4.3.1)
s	=	length of a side element (in.) (C4.6.2.2.1)
T	=	period of fundamental mode of vibration (sec.) (4.7.4.5)
T_G	=	temperature gradient (Δ°F) (C4.6.6)
TH	=	time-history method (4.7.4.3.1)
T_m	=	period of mth mode of vibration (sec.) (C4.7.4.3.2b)
T_S	=	reference period used to define shape of seismic response spectrum (sec.) (4.7.4.5)
T_u	=	uniform specified temperature (°F) (C4.6.6)
T_{UG}	=	temperature averaged across the cross-section (°F) (C4.6.6)
t	=	thickness of plate-like element (in.); thickness of flange plate in orthotropic steel deck (in.) (C4.6.2.2.1)
t_g	=	depth of steel grid or corrugated steel plank including integral concrete overlay or structural concrete component, less a provision for grinding, grooving, or wear (in.) (4.6.2.2.1)
t_o	=	depth of structural overlay (in.) (4.6.2.2.1)
t_s	=	depth of concrete slab (in.) (4.6.2.2.1) (4.6.2.6.1)
V_{LD}	=	maximum vertical shear at $3d$ or $L/4$ due to wheel loads distributed laterally as specified herein (kips) (4.6.2.2.2a)
V_{LL}	=	distributed live load vertical shear (kips) (4.6.2.2.2a)
V_{LU}	=	maximum vertical shear at $3d$ or $L/4$ due to undistributed wheel loads (kips) (4.6.2.2.2a)
$v_s(x)$	=	deformation corresponding to p_o (ft) (C4.7.4.3.2b)
$v_{s,MAX}$	=	maximum value of $v_s(x)$ (ft) (C4.7.4.3.2c)
W	=	edge-to-edge width of bridge (ft); factored wind force per unit length (kip/ft); total weight of cable (kip); total weight of bridge (kip) (4.6.2.2.1) (C4.6.2.7.1) (4.6.3.7) (C4.7.4.3.2c)
W_e	=	half the web spacing, plus the total overhang (ft) (4.6.2.2.1)
W_1	=	modified edge-to-edge width of bridge taken to be equal to the lesser of the actual width or 60.0 for multilane loading, or 30.0 for single-lane loading (ft) (4.6.2.3)
w	=	width of clear roadway (ft); width of element in cross-section (in.) (4.6.2.2.2b) (C4.6.6)
$w(x)$	=	nominal, unfactored dead load of the bridge superstructure and tributary substructure (kip/ft) (C4.7.4.3.2) (4.7.4.3.2c)
w_p	=	plank width (in.) (4.6.2.1.3)
w_g	=	maximum width between the girders on the outside of the bridge cross-section at the completion of the construction or at an intermediate stage of the steel erection (ft) (4.6.3.3.2)
X	=	distance from load to point of support (ft) (4.6.2.1.3)
X_{ext}	=	horizontal distance from the center of gravity of the pattern of girders to the exterior girder (ft) (C4.6.2.2.2d)
x	=	horizontal distance from the center of gravity of the pattern of girders to each girder (ft) (C4.6.2.2.2d)

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Z = a factor taken as 1.20 where the lever rule was not utilized, and 1.0 where the lever rule was used for a single lane live load distribution factor (4.6.2.2.4)
z = vertical distance from center of gravity of cross-section (in.) (C4.6.6)
α = angle between cable and horizontal (degrees); coefficient of thermal expansion (in./in./°F); generalized flexibility (4.6.3.7) (C4.6.6) (C4.7.4.3.2b)
β = generalized participation (C4.7.4.3.2b)
γ = load factor; generalized mass (C4.6.2.7.1) (C4.7.4.3.2b)
Δ = displacement of point of contraflexure in column or pier relative to point of fixity for the foundation (in.) (4.7.4.5)
Δ_e = displacement calculated from elastic seismic analysis (in.) (4.7.4.5)
Δw = overhang width extension (in.) (C4.6.2.6.1)
δ_b = moment or stress magnifier for braced mode deflection (4.5.3.2.2b)
δ_s = moment or stress magnifier for unbraced mode deflection (4.5.3.2.2b)
ε_u = uniform axial strain due to axial thermal expansion (in./in.) (C4.6.6)
η_i = load modifier relating to ductility, redundancy, and operational importance as specified in Article 1.3.2.1 (C4.6.2.7.1)
θ = skew angle (degrees); maximum skew angle of the bearing lines at the end of a given span, measured from a line taken perpendicular to the span centerline (degrees) (4.6.2.2.1) (4.6.3.3.2)
μ = Poisson's ratio (4.6.2.2.1)
σ_E = internal stress due to thermal effects (ksi) (C4.6.6)
ϕ = rotation per unit length; flexural resistance factor (C4.6.6) (4.7.4.5)
ϕ_K = stiffness reduction factor = 0.75 for concrete members and 1.0 for steel and aluminum members (4.5.3.2.2b)

4.4—ACCEPTABLE METHODS OF STRUCTURAL ANALYSIS

Any method of analysis that satisfies the requirements of equilibrium and compatibility and utilizes stress–strain relationships for the proposed materials may be used, including, but not limited to:

- classical force and displacement methods,
- finite difference method,
- finite element method,
- folded plate method,
- finite strip method,
- grid analogy method,
- series or other harmonic methods,
- methods based on the formation of plastic hinges, and
- yield line method.

The Designer shall be responsible for the implementation of computer programs used to facilitate structural analysis and for the interpretation and use of results.

The name, version, and release date of software used should be indicated in the contract documents.

C4.4

Many computer programs are available for bridge analysis. Various methods of analysis, ranging from simple formulae to detailed finite element procedures, are implemented in such programs. Many computer programs have specific engineering assumptions embedded in their code, which may or may not be applicable to each specific case.

When using a computer program, the Designer should clearly understand the basic assumptions of the program and the methodology that is implemented.

A computer program is only a tool, and the user is responsible for the generated results. Accordingly, all output should be verified to the extent possible.

Computer programs should be verified against the results of:

- universally accepted closed-form solutions,
- other previously verified computer programs, or
- physical testing.

The purpose of identifying software is to establish code compliance and to provide a means of locating bridges designed with software that may later be found deficient.

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4.5—MATHEMATICAL MODELING

4.5.1—General

Mathematical models shall include loads, geometry, and material behavior of the structure, and, where appropriate, response characteristics of the foundation. The choice of model shall be based on the limit states investigated, the force effect being quantified, and the accuracy required.

Unless otherwise permitted, consideration of continuous composite barriers shall be limited to service and fatigue limit states and to structural evaluation.

The stiffness of structurally discontinuous railings, curbs, elevated medians, and barriers shall not be considered in structural analysis.

For the purpose of this section, an appropriate representation of the soil and/or rock that supports the bridge shall be included in the mathematical model of the foundation.

In the case of seismic design, gross soil movement and liquefaction should also be considered.

If lift-off is indicated at a bearing, the analysis shall recognize the vertical freedom of the girder at that bearing.

C4.5.1

Service and fatigue limit states should be analyzed as fully elastic, as should strength limit states, except in case of certain continuous girders where inelastic analysis is specifically permitted, such as inelastic redistribution of negative bending moment and stability investigations. The extreme event limit states may require collapse investigation based entirely on inelastic modeling.

Very flexible bridges, e.g., suspension and cable-stayed bridges, should be analyzed using nonlinear elastic methods, such as the large deflection theory.

The need for sophisticated modeling of foundations is a function of the sensitivity of the structure to foundation movements.

In some cases, the foundation model may be as simple as unyielding supports. In other cases, an estimate of settlement may be acceptable. Where the structural response is particularly sensitive to the boundary conditions, such as in a fixed-end arch or in computing natural frequencies, rigorous modeling of the foundation should be made to account for the conditions present. In lieu of rigorous modeling, the boundary conditions may be varied to extreme bounds, such as fixed or free of restraint, and envelopes of force effects considered.

Where lift-off restraints are provided in the contract documents, the construction stage at which the restraints are to be installed should be clearly indicated. The analysis should recognize the vertical freedom of the girder consistent with the construction sequence shown in the contract documents.

4.5.2—Structural Material Behavior

4.5.2.1—Elastic Versus Inelastic Behavior

For the purpose of analysis, structural materials shall be considered to behave linearly up to an elastic limit and inelastically thereafter.

Actions at the extreme event limit state may be accommodated in both the inelastic and elastic ranges.

4.5.2.2—Elastic Behavior

Elastic material properties and characteristics shall be in accordance with the provisions of Sections 5, 6, 7, and 8. Changes in these values due to maturity of concrete and environmental effects should be included in the model where appropriate.

The stiffness properties of concrete and composite members shall be based upon cracked and/or uncracked sections consistent with the anticipated behavior. Stiffness characteristics of beam-slab-type bridges may be based on full participation of concrete decks.

C4.5.2.2

Tests indicate that in the elastic range of structural behavior, cracking of concrete seems to have little effect on the global behavior of bridge structures. This effect can, therefore, be safely neglected by modeling the concrete as uncracked for the purposes of structural analysis (King et al., 1975; Yen et al., 1995).

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4.5.2.3—Inelastic Behavior

Sections of components that may undergo inelastic deformation shall be shown to be ductile or made ductile by confinement or other means. Where inelastic analysis is used, a preferred design failure mechanism and its attendant hinge locations shall be determined. It shall be ascertained in the analysis that shear, buckling, and bond failures in the structural components do not precede the formation of a flexural inelastic mechanism. Unintended overstrength of a component in which hinging is expected should be considered. Deterioration of geometrical integrity of the structure due to large deformations shall be taken into account.

The inelastic model shall be based either upon the results of physical tests or upon a representation of load-deformation behavior that is validated by tests. Where inelastic behavior is expected to be achieved by confinement, test specimens shall include the elements that provide such confinement. Where extreme force effects are anticipated to be repetitive, the tests shall reflect their cyclic nature.

Except where noted, stresses and deformations shall be based on a linear distribution of strains in the cross-section of prismatic components. Shear deformation of deep components shall be considered. Limits on concrete strain, as specified in Section 5, shall not be exceeded.

The inelastic behavior of compressive components shall be taken into account, wherever applicable.

C4.5.2.3

Where technically possible, the preferred failure mechanism should be based on a response that has generally been observed to provide for large deformations as a means of warning of structural distress.

The selected mechanism should be used to estimate the extreme force effect that can be applied adjacent to a hinge.

Unintended overstrength of a component may result in an adverse formation of a plastic hinge at an undesirable location, forming a different mechanism.

4.5.3—Geometry

4.5.3.1—Small Deflection Theory

If the deformation of the structure does not result in a significant change in force effects due to an increase in the eccentricity of compressive or tensile forces, such secondary force effects may be ignored.

C4.5.3.1

Small deflection theory is usually adequate for the analysis of beam-type bridges. Bridges which resist loads primarily through a couple whose tensile and compressive forces remain in essentially fixed positions relative to each other while the bridge deflects, such as in trusses and tied arches, are generally insensitive to deformations. Columns and structures in which the flexural moments are increased or decreased by deflection tend to be sensitive to deflection considerations. Such structures include suspension bridges, very flexible cable-stayed bridges, and some arches other than tied arches and frames.

In many cases, the degree of sensitivity can be assessed and evaluated by a single-step approximate method, such as the moment magnification factor method. In the remaining cases, a complete second-order analysis may be necessary.

The past traditional boundary between small- and large-deflection theory becomes less distinct as bridges and bridge components become more flexible due to advances in material technology, the change from mandatory to optional deflection limits, and the trend toward more accurate and optimized design. The Engineer needs to consider these aspects in the choice of an analysis method.

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Small-deflection elastic behavior permits the use of the principle of superposition and efficient analytical solutions. These assumptions are typically used in bridge analysis for this reason. The behavior of the members assumed in these provisions is generally consistent with this type of analysis.

Superposition does not apply for the analysis of construction processes that include changes in the stiffness of the structure.

Moments from noncomposite and composite analyses may not be added for the purpose of computing stresses. The addition of stresses and deflections due to noncomposite and composite actions computed from separate analyses is appropriate.

4.5.3.2—Large Deflection Theory

4.5.3.2.1—General

If the deformation of the structure results in a significant change in force effects, the effects of deformation shall be considered in the equations of equilibrium.

The effect of deformation and out-of-straightness of components shall be included in stability analyses and large deflection analyses.

For slender concrete compressive components, those time- and stress-dependent material characteristics that cause significant changes in structural geometry shall be considered in the analysis.

The interaction effects of tensile and compressive axial forces in adjacent components should be considered in the analysis of frames and trusses.

Only factored loads shall be used and no superposition of force effects shall be applied in the nonlinear range. The order of load application in nonlinear analysis shall be consistent with that on the actual bridge.

C4.5.3.2.1

A properly formulated large deflection analysis is one that provides all the force effects necessary for the design. Further application of moment magnification factors is neither required nor appropriate. The presence of compressive axial forces amplifies both out-of-straightness of a component and the deformation due to nontangential loads acting thereon, thereby increasing the eccentricity of the axial force with respect to the centerline of the component. The synergistic effect of this interaction is the apparent softening of the component, i.e., a loss of stiffness. This is commonly referred to as a second-order effect. The converse is true for tension. As axial compressive stress becomes a higher percentage of the so-called Euler buckling stress, this effect becomes increasingly more significant.

The second-order effect arises from the translation of applied load creating increased eccentricity. It is considered as geometric nonlinearity and is typically addressed by iteratively solving the equilibrium equations or by using geometric stiffness terms in the elastic range (Przemieniecki, 1968). The analyst should be aware of the characteristics of the elements employed, the assumptions upon which they are based, and the numerical procedures used in the computer code. Discussions on the subject are given by White and Hajjar (1991) and Galambos (1998). Both references are related to metal structures, but the theory and applications are generally usable. Both contain numerous additional references that summarize the state-of-the-art in this area.

Because large deflection analysis is inherently nonlinear, the loads are not proportional to the displacements, and superposition cannot be used. This includes force effects due to changes in time-dependent properties, such as creep and shrinkage of concrete. Therefore, the order of load application can be important and traditional approaches, such as influence functions, are not directly applicable. The loads should be applied in the order experienced by the structure, i.e., dead load stages followed by live load stages, etc. If the structure undergoes nonlinear deformation, the loads should be applied incrementally with consideration for the changes in stiffness after each increment.

In conducting nonlinear analysis, it is prudent to perform a linear analysis for a baseline and to use the procedures employed on the problem at hand on a simple structure that can be analyzed by hand, such as a cantilever beam. This permits the analyst to observe behavior and develop insight into behavior that is not easily gained from more complex models.

4.5.3.2.2—Approximate Methods

4.5.3.2.2a—General

Where permitted in Sections 5, 6, and 7, the effects of deflection on force effects on beam-columns and arches which meet the provisions of these Specifications may be approximated by the single-step adjustment method known as moment magnification.

C4.5.3.2.2a

The moment magnification procedure outlined herein is one of several variations of the approximate process and was selected as a compromise between accuracy and ease of use. It is believed to be conservative. An alternative procedure thought to be more accurate than the one specified herein may be found in AISC (1993). This alternative procedure will require supplementary calculations not commonly made in bridge design using modern computational methods.

In some cases, the magnitude of movement implied by the moment magnification process cannot be physically attained. For example, the actual movement of a pier may be limited to the distance between the end of longitudinal beams and the backwall of the abutment. In cases where movement is limited, the moment magnification factors of elements so limited may be reduced accordingly.

4.5.3.2.2b—Moment Magnification—Beam Columns

The factored moments or stresses may be increased to reflect effects of deformations as follows:

$$M_c = \delta_b M_{2b} + \delta_s M_{2s} \quad (4.5.3.2.2b\text{-}1)$$

$$f_c = \delta_b f_{2b} + \delta_s f_{2s} \quad (4.5.3.2.2b\text{-}2)$$

in which:

$$\delta_b = \frac{C_m}{1 - \dfrac{P_u}{\phi_K P_e}} \geq 1.0 \quad (4.5.3.2.2b\text{-}3)$$

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_u}{\phi_K \Sigma P_e}} \quad (4.5.3.2.2b\text{-}4)$$

where:

C4.5.3.2.2b

M_{2b} = moment on compression member due to factored gravity loads that result in no appreciable sidesway calculated by conventional first-order elastic frame analysis; always positive (kip-ft)
M_{2s} = moment on compression member due to factored lateral or gravity loads that result in sidesway, Δ, greater than $\ell_u/1500$, calculated by conventional first-order elastic frame analysis; always positive (kip-ft)
f_{2b} = stress corresponding to M_{2b} (ksi)
f_{2s} = stress corresponding to M_{2s} (ksi)
P_u = factored axial load (kip)
ϕ_K = stiffness reduction factor; 0.75 for concrete members and 1.0 for steel, concrete-filled steel tubes, and aluminum members
P_e = Euler buckling load (kip)

For steel/concrete composite columns, the Euler buckling load, P_e, shall be determined as specified in Article 6.9.5.1. For all other cases, P_e shall be taken as:

$$P_e = \frac{\pi^2 EI}{(K\ell_u)^2} \qquad (4.5.3.2.2b\text{-}5)$$

where:

E = modulus of elasticity (ksi)
I = moment of inertia about axis under consideration (in.^{4})
K = effective length factor in the plane of bending as specified in Article 4.6.2.5. For calculation of δ_b, P_e shall be based on the K-factor for braced frames; for calculation of δ_s, P_e shall be based on the K-factor for unbraced frames
ℓ_u = unsupported length of a compression member (in.)

For concrete compression members, the provisions of Article 5.6.4.3 also apply.

For members braced against sidesway, δ_s shall be taken as 1.0 unless analysis indicates that a lower value may be used. For members not braced against sidesway, δ_b shall be determined as for a braced member and δ_s for an unbraced member.

For members braced against sidesway and without transverse loads between supports, C_m may be taken as:

$$C_m = 0.6 + 0.4\frac{M_{1b}}{M_{2b}} \qquad (4.5.3.2.2b\text{-}6)$$

where:

M_{1b} = smaller factored end moment
M_{2b} = larger factored end moment

The ratio M_{1b}/M_{2b} is considered positive if the component is bent in single curvature and negative if it is bent in double curvature.

The previous limit $C_m \geq 0.4$ has been shown to be unnecessary in AISC (1994), Chapter C commentary.

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For all other cases, C_m shall be taken as 1.0.

In structures that are not braced against sidesway, the flexural members and foundation units framing into the compression member shall be designed for the sum of end moments of the compression member at the joint.

Where compression members are subject to flexure about both principal axes, the moment about each axis shall be magnified by δ, determined from the corresponding conditions of restraint about that axis.

Where a group of compression members on one level comprise a bent, or where they are connected integrally to the same superstructure, and collectively resist the sidesway of the structure, the value of δ_s shall be computed for the member group with ΣP_u and ΣP_e equal to the summations for all columns in the group.

4.5.3.2.2c—Moment Magnification—Arches

Live load and impact moments from a small deflection analysis shall be increased by the moment magnification factor, δ_b, as specified in Article 4.5.3.2.2b, with the following definitions:

ℓ_u = one-half of the length of the arch rib (ft)
K = effective length factor specified in Table 4.5.3.2.2c-1
C_m = 1.0

Table 4.5.3.2.2c-1—*K* Values for Effective Length of Arch Ribs

Rise-to-Span Ratio	3-Hinged Arch	2-Hinged Arch	Fixed Arch
0.1–0.2	1.16	1.04	0.70
0.2–0.3	1.13	1.10	0.70
0.3–0.4	1.16	1.16	0.72

4.5.3.2.3—Refined Methods

Refined methods of analysis shall be based upon the concept of forces satisfying equilibrium in a deformed position.

C4.5.3.2.3

Flexural equilibrium in a deformed position may be iteratively satisfied by solving a set of simultaneous equations, or by evaluating a closed-form solution formulated using the displaced shape.

4.5.4—Modeling Boundary Conditions

Boundary conditions shall represent actual characteristics of support and continuity.

Foundation conditions shall be modeled in such a manner as to represent the soil properties underlying the bridge, the soil–pile interaction, and the elastic properties of piles.

C4.5.4

If the accurate assessment of boundary conditions cannot be made, their effects may be bounded.

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4.5.5—Equivalent Members

Nonprismatic components may be modeled by discretizing the components into a number of frame elements with stiffness properties representative of the actual structure at the location of the element.

Components or groups of components of bridges with or without variable cross-sections may be modeled as a single equivalent component provided that it represents all the stiffness properties of the components or group of components. The equivalent stiffness properties may be obtained by closed-form solutions, numerical integration, submodel analysis, and series and parallel analogies.

C4.5.5

Standard frame elements in available analysis programs may be used. The number of elements required to model the nonprismatic variation is dependent on the type of behavior being modeled, e.g., static, dynamic, or stability analysis. Typically, eight elements per span will give sufficient accuracy for actions in a beam loaded statically with cross-sectional properties that vary smoothly. Fewer elements are required to model for deflection and frequency analyses.

Alternatively, elements may be used that are based on the assumed tapers and cross-sections. Karabalis (1983) provides a comprehensive examination of this issue. Explicit forms of stiffness coefficients are given for linearly tapered rectangular, flanged, and box sections. Aristizabal (1987) presents similar equations in a simple format that can be readily implemented into stiffness-based computer programs. Significant bibliographies are given in Karabalis (1983) and Aristizabal (1987).

4.6—STATIC ANALYSIS

4.6.1—Influence of Plan Geometry

4.6.1.1—Plan Aspect Ratio

If the span length of a superstructure with torsionally stiff closed cross-sections exceeds 2.5 times its width, the superstructure may be idealized as a single-spine beam. The following dimensional definitions shall be used to apply this criterion:

- Width—the core width of a monolithic deck or the average distance between the outside faces of exterior web.
- Length for rectangular simply supported bridges—the distance between deck joints.
- Length for continuous and/or skewed bridges—the length of the longest side of the rectangle that can be drawn within the plan view of the width of the smallest span, as defined herein.
- The length-to-width restriction specified above does not apply to concrete box girder bridges.

C4.6.1.1

Where transverse distortion of a superstructure is small in comparison with longitudinal deformation, the former does not significantly affect load distribution, hence, an equivalent beam idealization is appropriate. The relative transverse distortion is a function of the ratio between structural width and height; the latter, in turn, depending on the length. Hence, the limits of such idealization are determined in terms of the width-to-effective length ratio.

Simultaneous torsion, moment, shear, and reaction forces and the attendant stresses are to be superimposed as appropriate. The equivalent beam idealization does not alleviate the need to investigate warping effects in steel structures. In all equivalent beam idealizations, the eccentricity of loads should be taken with respect to the centerline of the equivalent beam. Asymmetrical sections need to consider the relative location of the shear center and center of gravity.

4.6.1.2—Structures Curved in Plan

4.6.1.2.1—General

The moments, shears, and other force effects required to proportion the superstructure components shall be based on a rational analysis of the entire superstructure. Analysis of sections with no axis of symmetry should consider the relative locations of the

C4.6.1.2.1

Since equilibrium of horizontally curved I-girders is developed by the transfer of load between the girders, the analysis must recognize the integrated behavior of all structural components. Equilibrium of curved box girders may be less dependent on the interaction center of gravity and the shear center. The substructure shall also be considered in the case of integral abutments, piers, or bents.

The entire superstructure, including bearings, shall be considered as an integral structural unit. Boundary conditions shall represent the articulations provided by the bearings, integral connections used in the design, or both. Analyses may be based on elastic small-deflection theory, unless more rigorous approaches are deemed necessary by the Engineer.

Analyses shall consider bearing orientation and restraint of bearings afforded by the substructure. These load effects shall be considered in designing bearings, cross-frames, diaphragms, bracing, and the deck.

Distortion of the cross-section need not be considered in the structural analysis.

Centrifugal force effects shall be considered in accordance with Article 3.6.3.

between girders. Bracing members are considered primary members in curved bridges since they transmit forces necessary to provide equilibrium.

The deck acts in flexure, vertical shear, and horizontal shear. Torsion increases the horizontal deck shear, particularly in curved box girders. The lateral restraint of the bearings may also cause horizontal shear in the deck.

Small-deflection theory is adequate for the analysis of most curved-girder bridges. However, curved I-girders are prone to deflect laterally when the girders are insufficiently braced during erection. This behavior may not be well recognized by small-deflection theory.

Classical methods of analysis are usually based on strength-of-materials assumptions that do not recognize cross-section deformation. Finite element analyses that model the actual cross-section shape of the I- or box girders can recognize cross-section distortion and its effect on structural behavior. Cross-section deformation of steel box girders may have a significant effect on torsional behavior, but this effect is limited by the provision of sufficient internal cross bracing.

4.6.1.2.2—Single-Girder Torsionally Stiff Superstructures

Except for concrete box girder bridges, a horizontally curved, torsionally stiff single-girder superstructure meeting the requirements of Article 4.6.1.1 may be analyzed for global force effects as a curved spine beam.

The location of the centerline of such a beam shall be taken at the center of gravity of the cross-section, and the eccentricity of dead loads shall be established by volumetric consideration.

C4.6.1.2.2

In order to apply the aspect ratio provisions of Article 4.6.1.1, as specified, the plan needs to be hypothetically straightened. Force effects should be calculated on the basis of the actual curved layout.

With symmetrical cross-sections, the center of gravity of permanent loads falls outside the center of gravity. Shear center of the cross-section and the resulting eccentricity need to be investigated.

4.6.1.2.3—Concrete Box Girder Bridges

Horizontally curved concrete box girders may be designed with straight segments, for central angles up to 12 degrees within one span, unless concerns about other force effects dictate otherwise.

Horizontally curved nonsegmental concrete box girder bridge superstructures may be analyzed and designed for global force effects as single-spine beams with straight segments for central angles up to 34 degrees within one span as shown in Figure 4.6.1.2.3-1, unless concerns about local force effects dictate otherwise. The location of the centerline of such a beam shall be taken at the center of gravity of the cross-section and the eccentricity of dead loads shall be established by volumetric consideration. Where the substructure is integral with the superstructure, the substructure elements shall be included in the model and allowance made for prestress friction loss due to horizontal curvature or tendon deviation.

C4.6.1.2.3

Concrete box girders generally behave as a single-girder multi-web torsionally stiff superstructure. A parameter study conducted by Song, Chai, and Hida (2003) indicated that the distribution factors from the LRFD formulae compared well with the distribution factors from grid analyses when using straight segments on spans with central angles up to 34 degrees in one span.

Nutt, Redfield, and Valentine (2008) studied the limits of applicability for various methods of analyzing horizontally curved concrete box girder bridges. The focus of this study was on local as well as global force effects and provided the basis for revisions in 2010. They identified three approaches for the analysis of concrete box girder bridges as follows:

- The first method allows bridges with a central angle within one span of less than 12 degrees to be analyzed as if they were straight because curvature has a minor

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effect on response. This is typically done with a plane frame analysis.

- The second method involves a spine beam analysis in which the superstructure is idealized as a series of straight beam chorded segments of limited central angle located along the bridge centerline. Where the substructure is integral with the superstructure, a space frame analysis is required. Whole-width design as described in Article 4.6.2.2.1 was found to yield conservative results when space frame analysis was used. It is acceptable to reduce the number of live load lanes applied to the whole-width model to those that can fit on the bridge when global response such as torsion or transverse bending is being considered.
- Bridges with high curvatures or unusual plan geometry require a third method of analysis that utilizes sophisticated three-dimensional computer models. Unusual plan geometry includes, but is not limited to, bridges with variable widths or with unconventional orientation of skewed supports.

The range of applicability using approximate methods herein is expected to yield results within five percent of the most detailed type of analysis. Analysis of force effects in curved tendons is also addressed in Article 5.9.5.4.3.

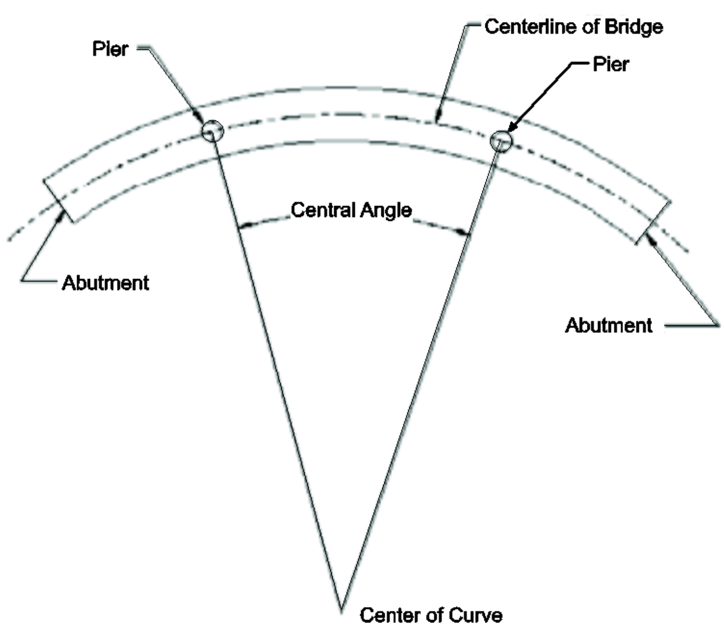

Figure 4.6.1.2.3-1—Definition of Central Angle

Horizontally curved segmental concrete box girder superstructures meeting the requirements of Article 4.6.1.1, and whose central angle within one span is between 12 degrees and 34 degrees, may be analyzed as a single-spine beam comprised of straight segments provided no segment has a central angle greater than 3.5 degrees as shown in Figure 4.6.1.2.3-2. For integral substructures, an appropriate three-dimensional model of the structure shall be used. Redistribution of forces due to the time-dependent properties of concrete shall be accounted for.

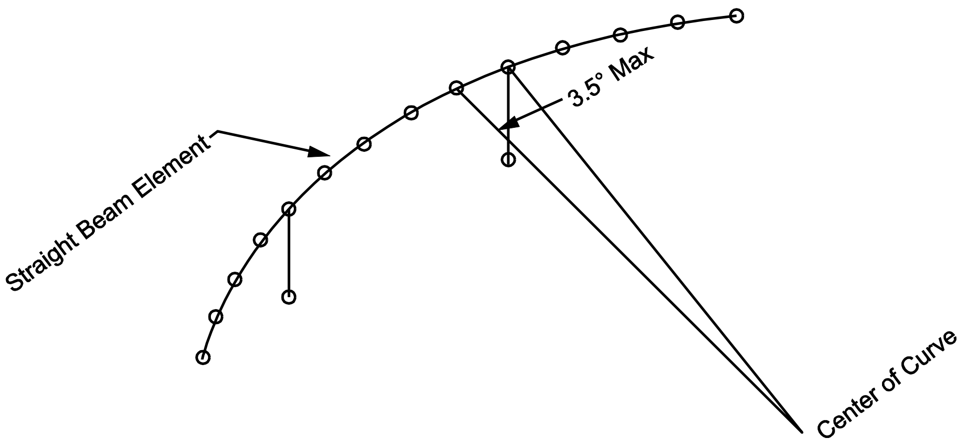

Figure 4.6.1.2.3-2—Three-Dimensional Spine Model of Curved Concrete Box Girder Bridge

For both segmental and nonsegmental box girder bridges with central angles exceeding 34 degrees within any one span or for bridges with a maximum central angle in excess of 12 degrees with unusual plan geometry, the bridge shall be analyzed using six degrees of freedom in a proven three-dimensional analysis method.

4.6.1.2.4—Steel Multiple-Beam Superstructures

4.6.1.2.4a—General

Horizontally curved superstructures may be analyzed as grids or continuums in which the segments of the longitudinal beams are assumed to be straight between nodes. The actual eccentricity of the segment between the nodes shall not exceed 2.5 percent of the length of the segment.

C4.6.1.2.4a

An eccentricity of 2.5 percent of the length of the segment corresponds to a central angle subtended by a curved segment of about 12 degrees.

This Article applies only to major-axis bending moment and does not apply to lateral flange bending or torsion, which should always be examined with respect to curvature.

Bridges with even slight curvature may develop large radial forces at the abutment bearings. Therefore, thermal analysis of all curved bridges is recommended.

4.6.1.2.4b—I-Girders

The effect of curvature on stability shall be considered for all curved I-girders.

Where I-girder bridges meet the following four conditions, the effects of curvature may be ignored in the analysis for determining the major-axis bending moments and bending shears:

C4.6.1.2.4b

The requirement for similar stiffness among the girders is intended to avoid large and irregular changes in stiffness which could alter transverse distribution of load. Under such conditions, a refined analysis would be appropriate. Noncomposite dead load preferably is to be distributed uniformly to the girders since the cross-

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- Girders are concentric;
- Bearing lines are not skewed more than 10 degrees from radial;
- The stiffnesses of the girders are similar;
- For all spans, the arc span divided by the girder radius in feet is less than 0.06 radians where the arc span, L_{as}, shall be taken as follows:

For simple spans:

L_{as} = arc length of the girder (ft)

For end spans of continuous members:

L_{as} = 0.9 times the arc length of the girder (ft)

For interior spans of continuous members:

L_{as} = 0.8 times the arc length of the girder (ft)

An I-girder in a bridge satisfying these criteria may be analyzed as an individual straight girder with span length equal to the arc length. Cross-frame or diaphragm spacing shall be set to limit flange lateral bending effects in the girder, which may be determined from an appropriate approximation. The cross-frame or diaphragm spacing shall also satisfy Eq. 6.7.4.2-1. Cross-frames or diaphragms and their connections shall be designed in accordance with the applicable provisions of Articles 6.7.4.2 and 6.13. At a minimum, cross-frame or diaphragms shall be designed to transfer wind loads according to the provisions of Article 4.6.2.7 and shall meet all applicable slenderness requirements specified in Articles 6.8.4 or 6.9.3.

frames provide restoring forces that prevent the girders from deflecting independently. Certain dead loads applied to the composite bridge may be distributed uniformly to the girders as provided in Article 4.6.2.2.1. However, heavier concentrated line loads such as parapets, sidewalks, barriers, or sound walls should not be distributed equally to the girders. Engineering judgment must be used in determining the distribution of these loads. Often the largest portion of the load on an overhang is assigned to the exterior girder, or to the exterior girder and the first interior girder. The exterior girder on the outside of the curve is often critical in curved girder bridges.

The effect of curvature on the stability of a girder must be considered regardless of the amount of curvature since stability of curved girders is different from that of straight girders (NCHRP, 1999).

In lieu of a refined analysis, Eq. C4.6.1.2.4b-1 may be appropriate for determining the lateral bending moment in I-girder flanges due to curvature (Richardson, Gordon, and Associates, 1976; United States Steel, 1984).

$$M_{lat} = \frac{M\ell^2}{NRD} \qquad \text{(C4.6.1.2.4b-1)}$$

where:

M_{lat} = flange lateral bending moment (kip-ft)
M = major-axis bending moment (kip-ft)
ℓ = unbraced length (ft)
R = girder radius (ft)
D = web depth (ft)
N = a constant taken as 10 or 12 in past practice

Although the depth to be used in computing the flange lateral moment from Eq. C4.6.1.2.4b-1 is theoretically equal to the depth, h, between the midthickness of the top and bottom flanges, for simplicity, the web depth, D, is conservatively used in Eq. C4.6.1.2.4b-1. The Engineer may substitute the depth, h, for D in Eq. C4.6.1.2.4b-1, if desired. Eq. C4.6.1.2.4b-1 assumes the presence of a cross-frame at the point under investigation, that the cross-frame spacing is relatively uniform, and that the major-axis

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bending moment, M, is constant between brace points. Therefore, at points not actually located at cross-frames, flange lateral moments from Eq. C4.6.1.2.4b-1 may not be strictly correct. The constant, N, in Eq. C4.6.1.2.4b-1 has been taken as either 10 or 12 in past practice and either value is considered acceptable depending on the level of conservatism that is desired.

Other conditions that produce torsion, such as skew, should be dealt with by other analytical means which generally involve a refined analysis.

4.6.1.2.4c—Closed Box and Tub Girders

The effect of curvature on strength and stability shall be considered for all curved box girders.

Where box girder bridges meet the following three conditions, the effect of curvature may be ignored in the analysis for determination of the major-axis bending moments and bending shears:

- girders are concentric;
- bearings are not skewed; and
- for all spans, the arc span divided by the girder radius is less than 0.3 radians, and the girder depth is less than the width of the box at mid-depth where the arc span, L_{as}, shall be taken as defined in Article 4.6.1.2.4b.

A box girder in a bridge satisfying these criteria may be analyzed as an individual straight girder with span length equal to the arc length. Internal cross-frame or diaphragm spacing shall be set to limit flange lateral bending effects in the top flanges of a tub girder, which may be determined from an appropriate approximation (Fan and Helwig, 1999), before the deck hardens or is made composite. The spacing of internal cross-frames or diaphragms shall not exceed 40.0 ft. Transverse bending stresses and longitudinal warping stresses due to cross-section distortion may be neglected. Cross-frames and diaphragms and their connections shall be designed in accordance with the applicable provisions of Articles 6.7.4.3 and 6.13. Cross-frame members shall meet all applicable slenderness requirements specified in Articles 6.8.4 or 6.9.3.

Lateral bracing members shall be designed in accordance with Articles 6.7.5 and 6.13 for forces computed by rational means.

C4.6.1.2.4c

Although box-shaped girders have not been examined as carefully as I-girders with regard to approximate methods, bending moments in closed girders are less affected by curvature than are I-girders (Tung and Fountain, 1970). However, in a box shape, torsion is much greater than in an open shape so that web shears are affected by torsion due to curvature, skew, or loads applied away from the shear center of the box. Double bearings resist significant torque compared to a box-centered single bearing.

If the box is haunched or tapered, the shallowest girder depth should be used in conjunction with the narrowest width of the box at middepth in determining whether the effects of curvature may be ignored in calculating the major axis bending moments and bending shears.

Fan and Helwig (1999) provide an approach for approximating top flange lateral bracing forces in tub girders in lieu of a refined analysis.

4.6.2—Approximate Methods of Analysis

4.6.2.1—Decks

4.6.2.1.1—General

An approximate method of analysis in which the deck is subdivided into strips perpendicular to the supporting components shall be considered acceptable for decks other than:

C4.6.2.1.1

In determining the strip widths, the effects of flexure in the secondary direction and of torsion on the distribution of internal force effects are accounted for to obtain flexural force effects approximating those that would be provided by a more refined method of analysis.

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- fully filled and partially filled grids for which the provisions of Article 4.6.2.1.8 shall apply, and
- top slabs of segmental concrete box girders for which the provisions of Article 4.6.2.9.4 shall apply.

Where the strip method is used, the extreme positive moment in any deck panel between girders shall be taken to apply to all positive moment regions. Similarly, the extreme negative moment over any beam or girder shall be taken to apply to all negative moment regions.

Depending on the type of deck, modeling and design in the secondary direction may utilize one of the following approximations:

- secondary strip designed in a manner like the primary strip, with all the limit states applicable;
- resistance requirements in the secondary direction determined as a percentage of those in the primary one as specified in Article 9.7.3.2 (i.e., the traditional approach for reinforced concrete slab in the AASHTO *Standard Specifications for Highway Bridges*); or
- minimum structural and/or geometry requirements specified for the secondary direction independent of actual force effects, as is the case for most wood decks.

The approximate strip model for decks is based on rectangular layouts. Currently about two-thirds of all bridges nationwide are skewed. While skew generally tends to decrease extreme force effects, it produces negative moments at corners, torsional moments in the end zones, substantial redistribution of reaction forces, and a number of other structural phenomena that should be considered in the design.

4.6.2.1.2—Applicability

The use of design aids for decks containing prefabricated elements may be permitted in lieu of analysis if the performance of the deck is documented and supported by sufficient technical evidence. The Engineer shall be responsible for the accuracy and implementation of any design aids used.

For slab bridges and concrete slabs spanning more than 15.0 ft and which span primarily in the direction parallel to traffic, the provisions of Article 4.6.2.3 shall apply.

4.6.2.1.3—Width of Equivalent Interior Strips

The width of the equivalent strip of a deck may be taken as specified in Table 4.6.2.1.3-1. Where decks span primarily in the direction parallel to traffic, strips supporting an axle load shall not be taken to be greater than 40.0 in. for open grids and not greater than 144 in. for all other decks where multilane loading is being investigated. For deck overhangs, where applicable, the provisions of Article 3.6.1.3.4 may be used in lieu of the strip width specified in Table 4.6.2.1.3-1 for deck overhangs. The equivalent strips for decks that span primarily in the transverse direction shall not be subject

C4.6.2.1.3

Values provided for equivalent strip widths and strength requirements in the secondary direction are based on past experience. Practical experience and future research work may lead to refinement.

To get the load per unit width of the equivalent strip, divide the total load on one design traffic lane by the calculated strip width.

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to width limits. The following notation shall apply to Table 4.6.2.1.3-1:

S = spacing of supporting components (ft)
h = depth of deck (in.)
L = span length of deck (ft)
P = axle load (kip)
S_b = spacing of grid bars (in.)
$+M$ = positive moment
$-M$ = negative moment
X = distance from load to point of support (ft)

Table 4.6.2.1.3-1—Equivalent Strips

Type of Deck	Direction of Primary Strip Relative to Traffic	Width of Primary Strip (in.)
Concrete:		
• Cast-in-place	Overhang	$45.0 + 10.0X$
	Either Parallel or Perpendicular	$+M$: $26.0 + 6.6S$ $-M$: $48.0 + 3.0S$
• Cast-in-place with stay-in-place concrete formwork	Either Parallel or Perpendicular	$+M$: $26.0 + 6.6S$ $-M$: $48.0 + 3.0S$
• Precast, post-tensioned	Either Parallel or Perpendicular	$+M$: $26.0 + 6.6S$ $-M$: $48.0 + 3.0S$
Steel:		
• Open grid	Main Bars	$1.25P + 4.0S_b$
• Filled or partially filled grid	Main Bars	Article 4.6.2.1.8 applies
• Unfilled, composite grids	Main Bars	Article 4.6.2.1.8 applies
Wood:		
• Prefabricated glulam		
○ Noninterconnected	Parallel Perpendicular	$2.0h + 30.0$ $2.0h + 40.0$
○ Interconnected	Parallel Perpendicular	$90.0 + 0.84L$ $4.0h + 30.0$
• Stress-laminated	Parallel Perpendicular	$0.8S + 108.0$ $10.0S + 24.0$
• Spike-laminated		
○ Continuous decks or interconnected panels	Parallel Perpendicular	$2.0h + 30.0$ $4.0h + 40.0$
○ Noninterconnected panels	Parallel Perpendicular	$2.0h + 30.0$ $2.0h + 40.0$

Wood plank decks shall be designed for the wheel load of the design truck distributed over the tire contact

Only the wheel load is specified for plank decks. Addition of lane load will cause a negligible increase in area. For transverse planks, i.e., planks perpendicular to traffic direction:

- If $w_p \geq 10.0$ in., the full plank width shall be assumed to carry the wheel load.
- If $w_p < 10.0$ in., the portion of the wheel load carried by a plank shall be determined as the ratio of w_p and 10.0 in.

For longitudinal planks:

- If $w_p \geq 20.0$ in., the full plank width shall be assumed to carry the wheel load.
- If $w_p < 20.0$ in., the portion of the wheel load carried by a plank shall be determined as the ratio of w_p and 20.0 in.

where:

w_p = plank width (in.)

force effects; however, it may be added for uniformity of the design live load in these Specifications.

4.6.2.1.4—Width of Equivalent Strips at Edges of Slabs

4.6.2.1.4a—General

For the purpose of design, the notional edge beam shall be taken as a reduced deck strip width specified herein. Any additional integral local thickening or similar protrusion acting as a stiffener to the deck that is located within the reduced deck strip width can be assumed to act with the reduced deck strip width as the notional edge beam.

4.6.2.1.4b—Longitudinal Edges

Edge beams shall be assumed to support one line of wheels and, where appropriate, a tributary portion of the design lane load.

Where decks span primarily in the direction of traffic, the effective width of a strip, with or without an edge beam, may be taken as the sum of the distance between the edge of the deck and the inside face of the barrier, plus 12.0 in., plus one-quarter of the strip width, specified in either Article 4.6.2.1.3, Article 4.6.2.3, or Article 4.6.2.10, as appropriate, but not exceeding either one-half the full strip width or 72.0 in.

4.6.2.1.4c—Transverse Edges

Transverse edge beams shall be assumed to support one axle of the design truck in one or more design lanes, positioned to produce maximum load effects. Multiple presence factors and the dynamic load allowance shall apply.

The effective width of a strip, with or without an edge beam, may be taken as the sum of the distance between the transverse edge of the deck and the centerline of the first line of support for the deck, usually taken as a girder web, plus one-half of the width of strip as specified in Article 4.6.2.1.3. The effective

C4.6.2.1.4c

For decks covered by Table A4-1, the total moment acting on the edge beam, including the multiple presence factor and the dynamic load allowance, may be calculated by multiplying the moment per unit width, taken from Table A4-1, by the corresponding full strip width specified in Article 4.6.2.1.3.

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width shall not exceed the full strip width specified in Article 4.6.2.1.3.

4.6.2.1.5—Distribution of Wheel Loads

If the spacing of supporting components in the secondary direction exceeds 1.5 times the spacing in the primary direction, all of the wheel loads shall be considered to be applied to the primary strip and the provisions of Article 9.7.3.2 may be applied to the secondary direction.

If the spacing of supporting components in the secondary direction is less than or equal to 1.5 times the spacing in the primary direction, the deck shall be modeled as a system of intersecting strips.

The width of the equivalent strips in both directions may be taken as specified in Table 4.6.2.1.3-1. Each wheel load shall be distributed between two intersecting strips. The distribution shall be determined as the ratio between the stiffness of the strip and the sum of stiffnesses of the intersecting strips. In the absence of more precise calculations, the strip stiffness, k_s, may be estimated as:

$$k_s = \frac{EI_s}{S^3} \qquad (4.6.2.1.5\text{-}1)$$

where:

I_s = moment of inertia of the equivalent strip (in.^{4})
S = spacing of supporting components (in.)

C4.6.2.1.5

This Article attempts to clarify the application of the traditional AASHTO approach with respect to continuous decks.

4.6.2.1.6—Calculation of Force Effects

The strips shall be treated as continuous beams or simply supported beams as appropriate. Span length shall be taken as the center-to-center distance between the supporting components. For the purpose of determining force effects in the strip, the supporting components shall be assumed to be infinitely rigid.

The wheel loads may be modeled as concentrated loads or as patch loads whose length along the span shall be the length of the tire contact area, as specified in Article 3.6.1.2.5, plus the depth of the deck. The strips should be analyzed by classical beam theory.

The design section for negative moments and shear forces, where investigated, may be taken as follows:

- For monolithic construction, closed steel boxes, closed concrete boxes, open concrete boxes without top flanges, and stemmed precast beams, i.e., Cross-sections (b), (c), (d), (e), (f), (g), (h), (i), and (j) from Table 4.6.2.2.1-1, at the face of the supporting component,
- For steel I-beams and steel tub girders, i.e., Cross-sections (a) and (c) from

C4.6.2.1.6

This is a deviation from the traditional approach based on a continuity correction applied to results obtained for analysis of simply supported spans. In lieu of more precise calculations, the unfactored design live load moments for many practical concrete deck slabs can be found in Table A4-1.

For short spans, the force effects calculated using the footprint could be significantly lower, and more realistic, than force effects calculated using concentrated loads.

Reduction in negative moment and shear replaces the effect of reduced span length in the current code. The design sections indicated may be applied to deck overhangs and to portions of decks between stringers or similar lines of support.

Past practice has been to not check shear in typical decks. A design section for shear is provided for use in nontraditional situations. It is not the intent to investigate shear in every deck.

Table 4.6.2.2.1-1, one-quarter the flange width from the centerline of support,

- For precast I-shaped concrete beams and open concrete boxes with top flanges, i.e., Cross-sections (c) and (k) from Table 4.6.2.2.1-1, one-third the flange width, but not exceeding 15.0 in., from the centerline of support,
- For wood beams, i.e., Cross-section (l) from Table 4.6.2.2.1-1, one-fourth the top beam width from centerline of beam.

For open box beams, each web shall be considered as a separate supporting component for the deck. The distance from the centerline of each web and the adjacent design sections for negative moment shall be determined based on the type of construction of the box and the shape of the top of the web using the requirements outlined above.

4.6.2.1.7—Cross-Sectional Frame Action

Where decks are an integral part of box or cellular cross-sections, flexural and/or torsional stiffnesses of supporting components of the cross-section, i.e., the webs and bottom flange, are likely to cause significant force effects in the deck. Those components shall be included in the analysis of the deck.

If the length of a frame segment is modeled as the width of an equivalent strip, provisions of Articles 4.6.2.1.3, 4.6.2.1.5, and 4.6.2.1.6 may be used.

C4.6.2.1.7

The model used is essentially a transverse segmental strip, in which flexural continuity provided by the webs and bottom flange is included. Such modeling is restricted to closed cross-sections only. In open-framed structures, a degree of transverse frame action also exists, but it can be determined only by complex, refined analysis.

In normal beam-slab superstructures, cross-sectional frame action may safely be neglected. If the slab is supported by box beams or is integrated into a cellular cross-section, the effects of frame action could be considerable. Such action usually decreases positive moments, but may increase negative moments resulting in cracking of the deck. For larger structures, a three-dimensional analysis may be appropriate. For smaller structures, the analysis could be restricted to a segment of the bridge whose length is the width of an equivalent strip.

Extreme force effects may be calculated by combining the:

- Longitudinal response of the superstructure approximated by classical beam theory, and
- Transverse flexural response modeled as a cross-sectional frame.

4.6.2.1.8—Live Load Force Effects for Fully and Partially Filled Grids and for Unfilled Grid Decks Composite with Reinforced Concrete Slabs

Moments in kip-in./in. of deck due to live load may be determined as:

- Main bars perpendicular to traffic:

C4.6.2.1.8

The moment equations are based on orthotropic plate theory considering vehicular live loads specified in Article 3.6. The equations take into account relevant factored load combinations, including truck and tandem

For $L \leq 120$ in.

$$M_{transverse} = 1.28 D^{0.197} L^{0.459} C \tag{4.6.2.1.8-1}$$

For $L > 120$ in.

$$M_{transverse} = \frac{D^{0.188}\left(3.7L^{1.35} - 956.3\right)}{L}(C) \tag{4.6.2.1.8-2}$$

- Main bars parallel to traffic:

For $L \leq 120$ in.

$$M_{parallel} = 0.73 D^{0.123} L^{0.64} C \tag{4.6.2.1.8-3}$$

For $L > 120$ in.

$$M_{parallel} = \frac{D^{0.138}\left(3.1L^{1.429} - 1088.5\right)}{L}(C) \tag{4.6.2.1.8-4}$$

where:

L = span length from center to center of supports (in.)
C = continuity factor; 1.0 for simply supported and 0.8 for continuous spans
D = D_x/D_y
D_x = flexural rigidity of deck in main bar direction (kip-in.^{2}/in.)
D_y = flexural rigidity of deck perpendicular to main bar direction (kip-in.^{2}/in.)

loads. The moment equations also account for dynamic load allowance, multiple presence factors, and load positioning on the deck surface to produce the largest possible moment.

Negative moment can be determined as maximum simple span positive moment times the continuity factor, C.

The reduction factor of 1.5 given in Article 4.6.2.1.8 accounts for smaller dynamic load allowance (15 percent vs. 33 percent), smaller load factor (1.50 vs. 1.75) and no multiple presence (1.0 vs. 1.2) when considering the Fatigue I limit state. Use of Eqs. 4.6.2.1.8-1 and 4.6.2.1.8-3 for all spans is appropriate as Eqs. 4.6.2.1.8-1 and 4.6.2.1.8-3 reflect an individual design truck on short span lengths while Eqs. 4.6.2.1.8-2 and 4.6.2.1.8-4 reflect the influence of multiple design tandems that control moment envelope on longer span lengths. The approximation produces reasonable estimates of fatigue moments, however, improved estimates can be determined using fatigue truck patch loads in the infinite series formula provided by Higgins (2003).

For grid decks, D_x and D_y should be calculated as EI_x and EI_y where E is the modulus of elasticity and I_x and I_y are the moment of inertia per unit width of deck, considering the section as cracked and using the transformed area method to calculate the flexural rigidity, EI, in each direction..

Moments for fatigue assessment may be estimated for all span lengths by reducing Eq. 4.6.2.1.8-1 for main bars perpendicular to traffic, or Eq. 4.6.2.1.8-3 for main bars parallel to traffic by a factor of 1.5.

Deflection in units of in. due to vehicular live load may be determined as:

- Main bars perpendicular to traffic:

$$\Delta_{transverse} = \frac{0.0052 D^{0.19} L^3}{D_x} \tag{4.6.2.1.8-5}$$

- Main bars parallel to traffic:

$$\Delta_{parallel} = \frac{0.0072 D^{0.11} L^3}{D_x} \tag{4.6.2.1.8-6}$$

Actual D_x and D_y values can vary considerably depending on the specific deck design, and using assumed values based only on the general type of deck can lead to unconservative design moments. Flexural rigidity in each direction should be calculated analytically as EI considering the section as cracked and using the transformed area method.

The deflection equations permit calculation of the midspan displacement for a deck under service load. The equations are based on orthotropic plate theory and consider both truck and tandem loads on a simply supported deck.

Deflection may be reduced for decks continuous over three or more supports. A reduction factor of 0.8 is conservative.

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4.6.2.1.9—Inelastic Analysis

Inelastic finite element analysis or yield line analysis may be permitted by the Owner.

4.6.2.2—Beam-Slab Bridges

4.6.2.2.1—Application

The provisions of this Article may be applied to straight girder bridges and horizontally curved concrete bridges, as well as horizontally curved steel girder bridges complying with the provisions of Article 4.6.1.2.4. The provisions of this Article may also be used to determine a starting point for some methods of analysis to determine force effects in curved girders of any degree of curvature in plan.

Except as specified in Article 4.6.2.2.5, the provisions of this Article shall be taken to apply to bridges being analyzed for:

- a single lane of loading, or
- multiple lanes of live load yielding approximately the same force effect per lane.

If one lane is loaded with a special vehicle or evaluation permit vehicle, the design force effect per girder resulting from the mixed traffic may be determined as specified in Article 4.6.2.2.5.

For beam spacing exceeding the range of applicability as specified in tables in Articles 4.6.2.2.2 and 4.6.2.2.3, the live load on each beam shall be the reaction of the loaded lanes based on the lever rule unless specified otherwise herein.

The provisions of Article 3.6.1.1.2 specify that multiple presence factors shall not be used with the approximate load assignment methods other than statical moment or lever arm methods because these factors are already incorporated in the distribution factors.

Bridges not meeting the requirements of this Article shall be analyzed as specified in Article 4.6.3.

The distribution of live load, specified in Articles 4.6.2.2.2 and 4.6.2.2.3, may be used for girders, beams, and stringers, other than multiple steel box beams with concrete decks that meet the following conditions and any other conditions identified in tables of distribution factors as specified herein:

- Width of deck is constant;
- Unless otherwise specified, the number of beams is not less than four;

C4.6.2.2.1

The V-load method is one example of a method of curved bridge analysis which starts with straight girder distribution factors (United States Steel, 1984).

The lever rule involves summing moments about one support to find the reaction at another support by assuming that the supported component is hinged at interior supports.

When using the lever rule on a three-girder bridge, the notional model should be taken as shown in Figure C4.6.2.2.1-1. Moments should be taken about the assumed, or notional, hinge in the deck over the middle girder to find the reaction on the exterior girder.

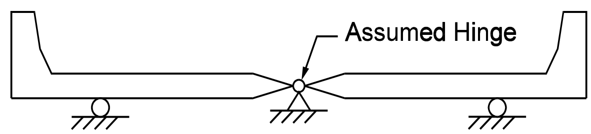

Figure C4.6.2.2.1-1—Notional Model for Applying Lever Rule to Three-girder Bridges

Provisions in Articles 4.6.2.2.2 and 4.6.2.2.3 that do not appear in earlier editions of the Standard Specifications come primarily from Zokaie et al. (1991). Correction factors for continuity have been deleted for two reasons:

- Correction factors dealing with five percent adjustments were thought to imply misleading levels of accuracy in an approximate method, and
- Analyses of many continuous beam-slab-type bridges indicate that the distribution coefficients for negative moments exceed those obtained for positive moments by approximately ten percent. On the other hand, it has been observed that stresses at or near an internal bearing are reduced due to the fanning of the reaction force. This reduction is about the same magnitude as the increase in distribution factors, hence the two tend to cancel each other out, and thus are omitted from these Specifications.

- Beams are parallel and have approximately the same stiffness;
- Unless otherwise specified, the roadway part of the overhang, d_e, does not exceed 3.0 ft;
- Curvature in plan is less than the limit specified in Article 4.6.1.2.4, or where distribution factors are required in order to implement an acceptable approximate or refined analysis method satisfying the requirements of Article 4.4 for bridges of any degree of curvature in plan; and
- Cross-section is consistent with one of the cross-sections shown in Table 4.6.2.2.1-1.

In Strength Load Combination II, applying a distribution factor procedure to a loading involving a heavy permit load can be overly conservative unless lane-by-lane distribution factors are available. Use of a refined method of analysis will circumvent this situation.

A rational approach may be used to extend the provisions of this Article to bridges with splayed girders. The distribution factor for live load at any point along the span may be calculated by setting the girder spacing in the equations of this Article equal to half the sum of the center-to-center distance between the girder under consideration and the two girders to either side. This will result in a variable distribution factor along the length of the girder. While the variable distribution factor is theoretically correct, it is not compatible with existing line girder computer programs that only allow constant distribution factors. Further simplifications may be used to allow the use of such computer programs. One such simplification involves running the computer program a number of times equal to the number of spans in the bridge. For each run, the girder spacing is set equal to the maximum girder spacing in one span and the results from this run are applied to this span. This approach is guaranteed to result in conservative design. In the past, some jurisdictions applied the latter approach, but used the girder spacing at the 2/3 or 3/4 points of the span; which will also be an acceptable approximation.

Where moderate deviations from a constant deck width or parallel beams exist, the distribution factor may either be varied at selected locations along the span or else a single distribution factor may be used in conjunction with a suitable value for beam spacing.

Most of the equations for distribution factors were derived for constant deck width and parallel beams. Past designs with moderate exceptions to these two assumptions have performed well when the *S/D* distribution factors were used. While the distribution factors specified herein are more representative of actual bridge behavior, common sense indicates that some exceptions are still possible, especially if the parameter *S* is chosen with prudent judgment, or if the factors are appropriately varied at selected locations along the span.

Cast-in-place multicell concrete box girder bridge types may be designed as whole-width structures. Such cross-sections shall be designed for the live load distribution factors in Articles 4.6.2.2.2 and 4.6.2.2.3 for interior girders, multiplied by the number of girders, i.e., webs.

Whole-width design is appropriate for torsionally-stiff cross-sections where load-sharing between girders is extremely high and torsional loads are hard to estimate. Prestressing force should be evenly distributed between girders. Cell width-to-height ratios should be approximately 2:1.

Additional requirements for multiple steel box girders with concrete decks shall be as specified in Article 4.6.2.2.2b.

Where bridges meet the conditions specified herein, permanent loads of and on the deck may be distributed uniformly among the beams and/or stringers.

Live load distribution factors, specified herein, may be used for permit and rating vehicles whose overall width is comparable to the width of the design truck.

The following notation shall apply to tables in Articles 4.6.2.2.2 and 4.6.2.2.3:

A = area of stringer, beam or girder (in.^{2})
b = width of beam (in.)

In lieu of more refined information, the St. Venant torsional inertia, J, may be determined as:

- For thin-walled open beam:

$$J = \frac{1}{3}\sum bt^3 \qquad \text{(C4.6.2.2.1-1)}$$

- For stocky open sections, e.g., prestressed I-beams, T-beams, etc., and solid sections:

$$J = \frac{A^4}{40.0 I_p} \qquad \text{(C4.6.2.2.1-2)}$$

C = stiffness parameter
D = width of distribution per lane (ft)
d = depth of beam or stringer (in.)
d_e = horizontal distance from the centerline of the exterior web of exterior beam at deck level to the interior edge of curb or traffic barrier (ft)
e = correction factor
g = live load distribution factor representing the number of design lanes
I_p = polar moment of inertia (in.^{4})
J = St. Venant's torsional inertia (in.^{4})
k = factor used in calculation of distribution factor for girder system bridges
K = constant for different types of construction
K_g = longitudinal stiffness parameter (in.^{4})
L = span of beam (ft)
N_b = number of beams, stringers, or girders
N_c = number of cells in a concrete box girder
N_L = number of design lanes as specified in Article 3.6.1.1.1
S = spacing of beams or webs (ft)
t_g = depth of steel grid or corrugated steel plank including integral concrete overlay or structural concrete component, less a provision for grinding, grooving, or wear (in.)
t_o = depth of structural overlay (in.)
t_s = depth of concrete slab (in.)
W = edge-to-edge width of bridge (ft)
W_e = half the web spacing, plus the total overhang (ft)
θ = skew angle (degrees)
μ = Poisson's ratio

Unless otherwise stated, the stiffness parameters for area, moments of inertia, and torsional stiffness used herein and in Articles 4.6.2.2.2 and 4.6.2.2.3 shall be taken as those of the cross-section to which traffic will be applied, i.e., usually the composite section.

The term L (length) shall be determined for use in the live load distribution factor equations given in Articles 4.6.2.2.2 and 4.6.2.2.3 as shown in Table 4.6.2.2.1-2.

- For closed thin-walled shapes:

$$J = \frac{4A_o^{\,2}}{\sum \frac{s}{t}} \qquad \text{(C4.6.2.2.1-3)}$$

where:

b = width of plate element (in.)
t = thickness of plate-like element (in.)
A = area of cross-section (in.^{2})
I_p = polar moment of inertia (in.^{4})
A_o = area enclosed by centerlines of elements (in.^{2})
s = length of a side element (in.)

Eq. C4.6.2.2.1-2 has been shown to substantially underestimate the torsional stiffness of some concrete I-beams and a more accurate, but more complex, approximation can be found in Eby et al. (1973).

The transverse post-tensioning shown for some cross-sections herein is intended to make the units act together. A minimum 0.25 ksi prestress is recommended.

For beams with variable moment of inertia, K_g may be based on average properties.

For superstructures using bridge type b in Table 4.6.2.2.1-1 consisting of simple span precast concrete spread slab beams with precast stay-in-place forms and a cast-in-place concrete slab, live load distribution factors provided in Articles 4.6.2.2.2 and 4.6.2.2.3 may yield results that do not represent in-service behavior. In lieu of a detailed analysis of shear and moment live load distribution factors for this superstructure type, refer to Report Number FHWA/TX-15/0-6722-1, April 2015. Load distribution factors are only valid for a superstructure that is within the specified applicable range listed in the technical report.

For bridge types "f," "g," "h," "i," and "j," longitudinal joints between precast units of the cross-section are shown in Table 4.6.2.2.1-1. This type of construction acts as a monolithic unit if sufficiently interconnected. In Article 5.12.2.3.3f, a fully interconnected joint is identified as a flexural shear joint. This type of interconnection is enhanced by either transverse post-tensioning of an intensity specified above or by a reinforced structural overlay, which is also specified in Article 5.12.2.3.3f, or both. The use of transverse mild steel rods secured by nuts or similar unstressed dowels should not be considered sufficient to achieve full transverse flexural continuity unless demonstrated by testing or experience. Generally, post-tensioning is thought to be more effective than a structural overlay if the intensity specified above is achieved.

In some cases, the lower limit of deck slab thickness, t_s, shown in the range of applicability column in the tables in Articles 4.6.2.2.2 and 4.6.2.2.3, is less than 7.0 in. The research used to develop the equations in those tables reflects the range of slab thickness

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shown. Article 9.7.1.1 indicates that concrete decks less than 7.0 in. in thickness should not be used unless approved by the Owner. Lesser values shown in tables in Articles 4.6.2.2.2 and 4.6.2.2.3 are not intended to override Article 9.7.1.1.

The load distribution factor equations for bridge type "d", cast-in-place multicell concrete box girders, were derived by first positioning the vehicle longitudinally, and then transversely, using an I-section of the box. While it would be more appropriate to develop an algorithm to find the peak of an influence surface, using the present factor for the interior girders multiplied by the number of girders is conservative in most cases.

The value of L to be used for positive and negative moment distribution factors will differ within spans of continuous girder bridges as will the distribution factors for positive and negative flexure.

The longitudinal stiffness parameter, K_g, shall be taken as:

$$K_g = n\left(I + Ae_g^{\,2}\right) \qquad (4.6.2.2.1\text{-}1)$$

in which:

$$n = \frac{E_B}{E_D} \qquad (4.6.2.2.1\text{-}2)$$

where:

E_B = modulus of elasticity of beam material (ksi)
E_D = modulus of elasticity of deck material (ksi)
I = moment of inertia of beam (in.^{4})
e_g = distance between the centers of gravity of the basic beam and deck (in.)

The parameters A and I in Eq. 4.6.2.2.1-1 shall be taken as those of the noncomposite beam.

The bridge types indicated in tables in Articles4.6.2.2.2 and 4.6.2.2.3, with reference to Table 4.6.2.2.1-1, may be taken as representative of the type of bridge to which each approximate equation applies.

Except as permitted by Article 2.5.2.7.1, regardless of the method of analysis used, i.e., approximate or refined, exterior girders of girder system bridges shall not have less resistance than an interior beam.

In the rare occasion when the continuous span arrangement is such that an interior span does not have any positive uniform load moment (i.e., no uniform load points of contraflexure), the region of negative moment near the interior supports would be increased to the centerline of the span, and the L used in determining the live load distribution factors would be the average of the two adjacent spans.

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Table 4.6.2.2.1-1—Common Deck Superstructures Covered in Articles 4.6.2.2.2 and 4.6.2.2.3

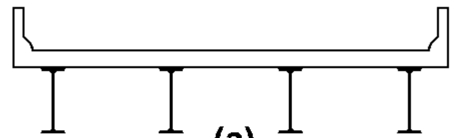
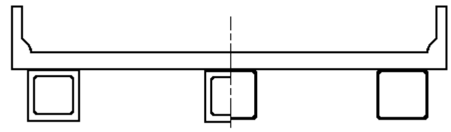
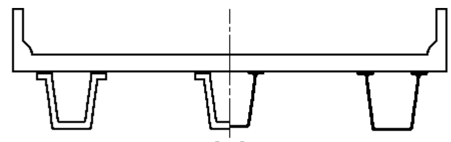
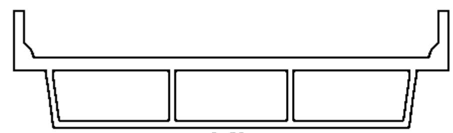
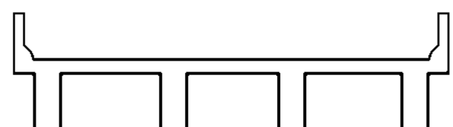
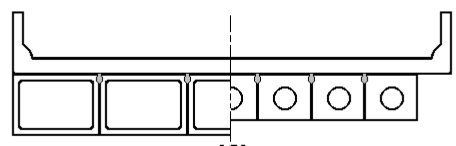
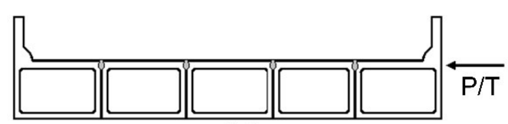
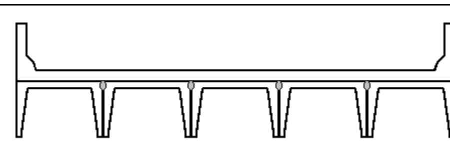
Supporting Components	Type of Deck	Typical Cross-Section
Steel Beam	Cast-in-place concrete slab, precast concrete slab, steel grid, glued/spiked panels, stressed wood	 (a)
Closed Steel or Precast Concrete Boxes	Cast-in-place concrete slab	 (b)
Open Steel or Precast Concrete Boxes	Cast-in-place concrete slab, precast concrete deck slab	 (c)
Cast-in-Place Concrete Multicell Box	Monolithic concrete	 (d)
Cast-in-Place Concrete T-Beam	Monolithic concrete	 (e)
Precast Solid, Voided, or Cellular Concrete Boxes with Shear Keys	Cast-in-place concrete overlay	 (f)
Precast Solid, Voided, or Cellular Concrete Box with Shear Keys and with or without Transverse Post-tensioning	Integral concrete	 (g)
Precast Concrete Channel Sections with Shear Keys	Cast-in-place concrete overlay	 (h)

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Table 4.6.2.2.1-1 (continued)—Common Deck Superstructures Covered in Articles 4.6.2.2.2 and 4.6.2.2.3

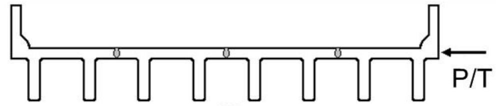
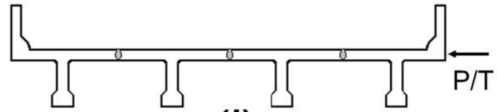
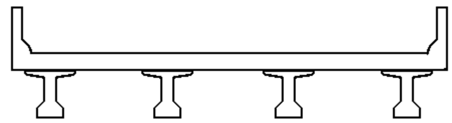
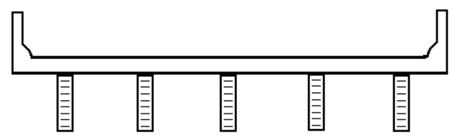
Supporting Components	Type of Deck	Typical Cross-Section
Precast Concrete Double T-Section with Shear Keys and with or without Transverse Post-tensioning	Integral concrete	 (i)
Precast Concrete T-Section with Shear Keys and with or without Transverse Post-tensioning	Integral concrete	 (j)
Precast Concrete I- or Bulb-T Sections	Cast-in-place concrete, precast concrete	 (k)
Wood Beams	Cast-in-place concrete or plank, glued/spiked panels or stressed wood	 (l)

Table 4.6.2.2.1-2—*L* for Use in Live Load Distribution Factor Equations

Force Effect	*L* (ft)
Positive Moment	The length of the span for which moment is being calculated
Negative Moment—Near interior supports of continuous spans from point of contraflexure to point of contraflexure under a uniform load on all spans	The average length of the two adjacent spans
Negative Moment—Other than near interior supports of continuous spans	The length of the span for which moment is being calculated
Shear	The length of the span for which shear is being calculated
Exterior Reaction	The length of the exterior span
Interior Reaction of Continuous Span	The average length of the two adjacent spans

For cast-in-place concrete multicell box shown as cross-section Type "d" in Table 4.6.2.2.1-1, the distribution factors in Article 4.6.2.2.2 and 4.6.2.2.3 shall be taken to apply to a notional shape consisting of a web, overhangs of an exterior web, and the associated half flanges between a web under consideration and the next adjacent web or webs.

With the owner's concurrence, the simplifications provided in Table 4.6.2.2.1-3 may be used:

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Table 4.6.2.2.1-3—Constant Values for Articles 4.6.2.2.2 and 4.6.2.2.3

Equation Parameters	Table Reference	Simplified Value			
		a	e	k	f,g,i,j
$\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$	4.6.2.2.2b-1	1.02	1.05	1.09	—
$\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.25}$	4.6.2.2.2e-1	1.03	1.07	1.15	—
$\left(\frac{12.0Lt_s^3}{K_g}\right)^{0.3}$	4.6.2.2.3c-1	0.97	0.93	0.85	—
$\frac{I}{J}$	4.6.2.2.2b-1, 4.6.2.2.3a-1	—	—	—	$0.54\left(\frac{d}{b}\right)+0.16$

4.6.2.2.2—Distribution Factor Method for Moment and Shear

4.6.2.2.2a—Interior Beams with Wood Decks

The live load flexural moment and shear for interior beams with transverse wood decks may be determined by applying the live load distribution factor, *g*, specified in Table 4.6.2.2.2a-1 and Eq. 4.6.2.2.2a-1.

When investigation of shear parallel to the grain in wood components is required, the distributed live load shear shall be determined by the following expression:

$$V_{LL} = 0.50\left[\left(0.60V_{LU}\right)+V_{LD}\right] \tag{4.6.2.2.2a-1}$$

where:

V_{LL} = distributed live load vertical shear (kips)

V_{LU} = maximum vertical shear at $3d$ or $L/4$ due to undistributed wheel loads (kips)

V_{LD} = maximum vertical shear at $3d$ or $L/4$ due to wheel loads distributed laterally as specified herein (kips)

For undistributed wheel loads, one line of wheels is assumed to be carried by one bending member.

Table 4.6.2.2.2a-1—Live Load Distribution Factor for Moment and Shear in Interior Beams with Wood Decks

Type of Deck	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Plank	a, l	$S/6.7$	$S/7.5$	$S \leq 5.0$
Stressed Laminated	a, l	$S/9.2$	$S/9.0$	$S \leq 6.0$
Spike Laminated	a, l	$S/8.3$	$S/8.5$	$S \leq 6.0$
Glued Laminated Panels on Glued Laminated Stringers	a, l	$S/10.0$	$S/10.0$	$S \leq 6.0$
Glue Laminated Panels on Steel Stringers	a, l	$S/8.8$	$S/9.0$	$S \leq 6.0$

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4.6.2.2.2b—Interior Beams with Concrete Decks

The live load flexural moment for interior beams with concrete decks may be determined by applying the live load distribution factor, g, specified in Table 4.6.2.2.2b-1.

For the concrete beams, other than box beams, used in multibeam decks with shear keys:

- Deep, rigid end diaphragms shall be provided to ensure proper load distribution; and
- If the stem spacing of stemmed beams is less than 4.0 ft or more than 10.0 ft, a refined analysis complying with Article 4.6.3 shall be used.

For multiple steel box girders with a concrete deck in bridges satisfying the requirements of Article 6.11.2.3, the live load flexural moment may be determined using the appropriate distribution factor specified in Table 4.6.2.2.2b-1.

Where the spacing of the box girders varies along the length of the bridge, the distribution factor may either be varied at selected locations along the span or else a single distribution factor may be used in conjunction with a suitable value of N_L. In either case, the value of N_L shall be determined as specified in Article 3.6.1.1.1, using the width, w, taken at the section under consideration.

C4.6.2.2.2b

The results of analytical and model studies of simple span multiple box section bridges, reported in Johnston and Mattock (1967), showed that folded plate theory could be used to analyze the behavior of bridges of this type. The folded plate theory was used to obtain the maximum load per girder, produced by various critical combinations of loading on 31 bridges having various spans, numbers of box girders, and numbers of traffic lanes.

Multiple presence factors, specified in Table 3.6.1.1.2-1, are not applied because the multiple factors in past editions of the Standard Specifications were considered in the development of the equation in Table 4.6.2.2.2b-1 for multiple steel box girders.

The lateral load distribution obtained for simple spans is also considered applicable to continuous structures.

The bridges considered in the development of the equations had interior end diaphragms only, i.e., no interior diaphragms within the spans, and no exterior diaphragms anywhere between boxes. If interior or exterior diaphragms are provided within the span, the transverse load distribution characteristics of the bridge will be improved to some degree. This improvement can be evaluated, if desired, using the analysis methods identified in Article 4.4.

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Table 4.6.2.2.2b-1—Live Load Distribution Factor for Moment in Interior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Distribution Factors	Range of Applicability
Wood Deck on Wood or Steel Beams	a, l	See Table 4.6.2.2.2a-1	
Concrete Deck on Wood Beams	l	One Design Lane Loaded: $S/12.0$ Two or More Design Lanes Loaded: $S/10.0$	$S \leq 6.0$
Concrete Deck or Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	One Design Lane Loaded: $0.06+\left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$ Two or More Design Lanes Loaded: $0.075+\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0\,Lt_s^3}\right)^{0.1}$	$3.5 \leq S \leq 16.0$ $4.5 \leq t_s \leq 12.0$ $20 \leq L \leq 240$ $N_b \geq 4$ $10{,}000 \leq K_g \leq 7{,}000{,}000$
		use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule	$N_b = 3$
Cast-in-Place Concrete Multicell Box	d	One Design Lane Loaded: $\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}$ Two or More Design Lanes Loaded: $\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}$	$7.0 \leq S \leq 13.0$ $60 \leq L \leq 240$ $N_c \geq 3$ If $N_c > 8$ use $N_c = 8$
Concrete Deck on Concrete Spread Box Beams	b, c	One Design Lane Loaded: $\left(\frac{S}{3.0}\right)^{0.35}\left(\frac{Sd}{12.0L^2}\right)^{0.25}$ Two or More Design Lanes Loaded: $\left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125}$	$6.0 \leq S \leq 18.0$ $20 \leq L \leq 140$ $18 \leq d \leq 65$ $N_b \geq 3$
		Use Lever Rule	$S > 18.0$
Concrete Beams used in Multibeam Decks	f, g	One Design Lane Loaded: $k\left(\frac{b}{33.3L}\right)^{0.5}\left(\frac{I}{J}\right)^{0.25}$ where: $k = 2.5(N_b)^{-0.2} \geq 1.5$ Two or More Design Lanes Loaded: $k\left(\frac{b}{305}\right)^{0.6}\left(\frac{b}{12.0L}\right)^{0.2}\left(\frac{I}{J}\right)^{0.06}$	$35 \leq b \leq 60$ $20 \leq L \leq 120$ $5 \leq N_b \leq 20$

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Table 4.6.2.2.2b-1 (continued)—Distribution of Live Loads for Moment in Interior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Distribution Factors	Range of Applicability
	h, also i, j if connected only enough to prevent relative vertical displacement at the interface	Regardless of Number of Loaded Lanes: S/D where: $C = K(W/L) \leq K$ $D = 11.5 - N_L + 1.4N_L(1 - 0.2C)^2$ when C ≤ 5 $D = 11.5 - N_L$ when $C > 5$ $K = \sqrt{\frac{(1+\mu)I}{J}}$ for preliminary design, the following values of K may be used: **Beam Type** — ***K*** Nonvoided rectangular beams — 0.7 Rectangular beams with circular voids: — 0.8 Channel beams — 2.2 T-beam — 2.0 Double T-beam — 2.0	Skew ≤ 45° $N_L \leq 6$
Open Steel Grid Deck on Steel Beams	a	One Design Lane Loaded: $S/7.5$ If $t_g < 4.0$ $S/10.0$ If $t_g \geq 4.0$ Two or More Design Lanes Loaded: $S/8.0$ If $t_g < 4.0$ $S/10.0$ If $t_g \geq 4.0$	$S \leq 6.0$ $S \leq 10.5$
Concrete Deck on Multiple Steel Box Girders	b, c	Regardless of Number of Loaded Lanes: $0.05 + 0.85\frac{N_L}{N_b} + \frac{0.425}{N_L}$	$0.5 \leq \frac{N_L}{N_b} \leq 1.5$

4.6.2.2.2c—Interior Beams with Corrugated Steel Decks

The live load flexural moment for interior beams with corrugated steel plank deck may be determined by applying the live load distribution factor, g, specified in Table 4.6.2.2.2c-1.

Table 4.6.2.2.2c-1—Live Load Distribution Factor for Moment in Interior Beams with Corrugated Steel Plank Decks

One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
$S/9.2$	$S/9.0$	$S \leq 5.5$ $t_g \geq 2.0$

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4.6.2.2.2d—Exterior Beams

The live load flexural moment for exterior beams may be determined by applying the live load distribution factor, g, specified in Table 4.6.2.2.2d-1. However, if the girders are not equally spaced and g for the exterior girder is a function of g_{interior}, g_{interior} should be based on the spacing between the exterior and first-interior girder.

The distance, d_e, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier. However, if a negative value for d_e falls outside the range of applicability as shown in Table 4.6.2.2.2.d-1 d_e should be limited to -1.0.

In steel beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

C4.6.2.2.2d

The distribution factor for girders in a multigirder cross-section, Types "a," "e," and "k" in Table 4.6.2.2.1-1, was determined without consideration of diaphragm or cross-frames, or parapets. Some research shows a minimal contribution to load transfer from diaphragms or cross-bracing and resultant increase in force effects in external girders. However, reactions may be calculated using a procedure similar to the conventional approximation for loads on piles as shown below.

$$R = \frac{N_L}{N_b} + \frac{X_{ext}\sum^{N_L} e}{\sum^{N_b} x^2} \qquad \text{(C4.6.2.2.2d-1)}$$

where:

R = reaction on exterior beam in terms of lanes
N_L = number of loaded lanes under consideration
e = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft)
x = horizontal distance from the center of gravity of the pattern of girders to each girder (ft)
X_{ext} = horizontal distance from the center of gravity of the pattern of girders to the exterior girder (ft)
N_b = number of beams or girders

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Table 4.6.2.2.2d-1—Live Load Distribution Factor for Moment in Exterior Longitudinal Beams

<table>
<tr><th>Type of Superstructure</th><th>Applicable Cross-Section from Table 4.6.2.2.1-1</th><th>One Design Lane Loaded</th><th>Two or More Design Lanes Loaded</th><th>Range of Applicability</th></tr>
<tr><td>Wood Deck on Wood or Steel Beams</td><td>a, l</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td>Concrete Deck on Wood Beams</td><td>l</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td rowspan="2">Concrete Deck or Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections</td><td rowspan="2">a, e, k and also i, j if sufficiently connected to act as a unit</td><td rowspan="2">Lever Rule</td><td>$g = e\, g_{interior}$
$e = 0.77 + \frac{d_e}{9.1}$</td><td>$-1.0 \leq d_e \leq 5.5$</td></tr>
<tr><td>use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule</td><td>$N_b = 3$</td></tr>
<tr><td rowspan="2">Cast-in-Place Concrete Multicell Box</td><td rowspan="2">d</td><td>$g = \frac{W_e}{14}$</td><td>$g = \frac{W_e}{14}$</td><td rowspan="2">$W_e \leq S$</td></tr>
<tr><td colspan="2">or the provisions for a whole-width design specified in Article 4.6.2.2.1</td></tr>
<tr><td rowspan="2">Concrete Deck on Concrete Spread Box Beams</td><td rowspan="2">b, c</td><td rowspan="2">Lever Rule</td><td>$g = e\, g_{interior}$
$e = 0.97 + \frac{d_e}{28.5}$</td><td>$0 \leq d_e \leq 4.5$
$6.0 < S \leq 18.0$</td></tr>
<tr><td>Use Lever Rule</td><td>$S > 18.0$</td></tr>
<tr><td>Concrete Box Beams Used in Multibeam Decks</td><td>f, g</td><td>$g = e\, g_{interior}$
$e = 1.125 + \frac{d_e}{30} \geq 1.0$</td><td>$g = e\, g_{interior}$
$e = 1.04 + \frac{d_e}{25} \geq 1.0$</td><td>$d_e \leq 2.0$</td></tr>
<tr><td>Concrete Beams Other than Box Beams Used in Multibeam Decks</td><td>h, also i, j if connected only enough to prevent relative vertical displacement at the interface</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td>Open Steel Grid Deck on Steel Beams</td><td>a</td><td>Lever Rule</td><td>Lever Rule</td><td>N/A</td></tr>
<tr><td>Concrete Deck on Multiple Steel Box Girders</td><td>b, c</td><td colspan="3">As specified in Table 4.6.2.2.2b-1</td></tr>
</table>

4.6.2.2.2e—Skewed Bridges

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10 degrees, the bending moment in the beams may be reduced in accordance with Table 4.6.2.2.2e-1.

C4.6.2.2.2e

Accepted reduction factors are not currently available for cases not covered in Table 4.6.2.2.2e-1.

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Table 4.6.2.2.2e-1—Reduction of Live Load Distribution Factors for Moment in Longitudinal Beams on Skewed Supports

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Any Number of Design Lanes Loaded	Range of Applicability
Concrete Deck or Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$1-c_1\left(\tan\theta\right)^{1.5}$ $c_1 = 0.25\left(\frac{K_g}{12.0Lt_s^{\,3}}\right)^{0.25}\left(\frac{S}{L}\right)^{0.5}$ If $\theta < 30^\circ$ then $c_1 = 0.0$ If $\theta > 60^\circ$ use $\theta = 60^\circ$	$30^\circ \leq \theta \leq 60^\circ$ $3.5 \leq S \leq 16.0$ $20 \leq L \leq 240$ $N_b \geq 4$
Concrete Deck on Concrete Spread Box Beams, Cast-in-Place Multicell Box, Concrete Beams Used in Multibeam Decks	b, c, d, f, g, h, also i and j if sufficiently connected to prevent vertical displacement at the interface	$1.05 - 0.25 \tan\theta \leq 1.0$ If $\theta > 60^\circ$ use $\theta = 60^\circ$	$0^\circ \leq \theta \leq 60^\circ$

4.6.2.2.2f—Flexural Moments and Shear in Transverse Floorbeams

If the deck is supported directly by transverse floorbeams, the floorbeams may be designed for loads determined in accordance with Table 4.6.2.2.2f-1.

The live load distribution factor, g, provided in Table 4.6.2.2.2f-1 shall be used in conjunction with the 32.0-kip design axle load alone. For spacings of floorbeams outside the given ranges of applicability, all of the design live loads shall be considered, and the lever rule may be used.

Table 4.6.2.2.2f-1—Live Load Distribution Factor for Transverse Beams for Moment and Shear

Type of Deck	Live Load Distribution Factors for Each Floorbeam	Range of Applicability
Plank	$\frac{S}{4}$	N/A
Laminated Wood Deck	$\frac{S}{5}$	$S \leq 5.0$
Concrete	$\frac{S}{6}$	$S \leq 6.0$
Steel Grid and Unfilled Grid Deck Composite with Reinforced Concrete Slab	$\frac{S}{4.5}$	$t_g \leq 4.0$ $S \leq 5.0$
Steel Grid and Unfilled Grid Deck Composite with Reinforced Concrete Slab	$\frac{S}{6}$	$t_g > 4.0$ $S \leq 6.0$
Steel Bridge Corrugated Plank	$\frac{S}{5.5}$	$t_g \geq 2.0$

4.6.2.2.3—Distribution Factor Method for Shear

4.6.2.2.3a—Interior Beams

The live load shear for interior beams may be determined by applying the live load distribution factor, g, specified in Table 4.6.2.2.3a-1. For interior beam types not listed in Table 4.6.2.2.3a-1, lateral distribution of the wheel or axle adjacent to the end of span shall be that produced by use of the lever rule.

For concrete box beams used in multibeam decks, if the values of I or J do not comply with the limitations in Table 4.6.2.2.3a-1, the distribution factor for shear may be taken as that for moment.

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Table 4.6.2.2.3a-1—Live Load Distribution Factor for Shear in Interior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Wood Deck on Wood or Steel Beams	a, l	See Table 4.6.2.2.2a-1		
Concrete Deck on Wood Beams	l	Lever Rule	Lever Rule	N/A
Concrete Deck or Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$0.36+\frac{S}{25.0}$	$0.2+\frac{S}{12}-\left(\frac{S}{35}\right)^{2.0}$	$3.5 \le S \le 16.0$ $20 \le L \le 240$ $4.5 \le t_s \le 12.0$ $N_b \ge 4$
		Lever Rule	Lever Rule	$N_b = 3$
Cast-in-Place Concrete Multicell Box	d	$\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$	$\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1}$	$6.0 \le S \le 13.0$ $20 \le L \le 240$ $35 \le d \le 110$ $N_c \ge 3$
Concrete Deck on Concrete Spread Box Beams	b, c	$\left(\frac{S}{10}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$	$\left(\frac{S}{7.4}\right)^{0.8}\left(\frac{d}{12.0L}\right)^{0.1}$	$6.0 \le S \le 18.0$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$
		Lever Rule	Lever Rule	$S > 18.0$
Concrete Box Beams Used in Multibeam Decks	f, g	$\left(\frac{b}{130L}\right)^{0.15}\left(\frac{I}{J}\right)^{0.05}$	$\left(\frac{b}{156}\right)^{0.4}\left(\frac{b}{12.0L}\right)^{0.1}\left(\frac{I}{J}\right)^{0.05}\left(\frac{b}{48}\right)$ $\frac{b}{48} \ge 1.0$	$35 \le b \le 60$ $20 \le L \le 120$ $5 \le N_b \le 20$ $25{,}000 \le J \le 610{,}000$ $40{,}000 \le I \le 610{,}000$
Concrete Beams Other Than Box Beams Used in Multibeam Decks	h, also i, j if connected only enough to prevent relative vertical displacement at the interface	Lever Rule	Lever Rule	N/A
Open Steel Grid Deck on Steel Beams	a	Lever Rule	Lever Rule	N/A
Concrete Deck on Multiple Steel Box Beams	b, c	As specified in Table 4.6.2.2.2b-1		

4.6.2.2.3b—Exterior Beams

The live load shear for exterior beams may be determined by applying the live load distribution factor, g, specified in Table 4.6.2.2.3b-1. For cases not addressed in Table 4.6.2.2.3a-1 and Table 4.6.2.2.3b-1, the live load distribution to exterior beams shall be determined by using the lever rule.

The parameter d_e shall be taken as positive if the exterior web is inboard of the curb or traffic barrier and negative if it is outboard.

The additional provisions for exterior beams in beam-slab bridges with cross-frames or diaphragms, specified in Article 4.6.2.2.2d, shall apply.

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Table 4.6.2.2.3b-1—Live Load Distribution Factor for Shear in Exterior Beams

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Wood Deck on Wood or Steel Beams	a, l	Lever Rule	Lever Rule	N/A
Concrete Deck on Wood Beams	l	Lever Rule	Lever Rule	N/A
Concrete Deck or Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Beams	a, e, k and also i, j if sufficiently connected to act as a unit	Lever Rule	$g = e\, g_{interior}$ $e = 0.6 + \frac{d_e}{10}$	$-1.0 \le d_e \le 5.5$
			Lever Rule	$N_b = 3$
Cast-in-Place Concrete Multicell Box	d	Lever Rule	$g = e\, g_{interior}$ $e = 0.64 + \frac{d_e}{12.5}$	$-2.0 \le d_e \le 5.0$
		or the provisions for a whole-width design specified in Article 4.6.2.2.1		
Concrete Deck on Concrete Spread Box Beams	b, c	Lever Rule	$g = e\, g_{interior}$ $e = 0.8 + \frac{d_e}{10}$	$0 \le d_e \le 4.5$
			Lever Rule	$S > 18.0$
Concrete Box Beams Used in Multibeam Decks	f, g	$g = e\, g_{interior}$ $e = 1.25 + \frac{d_e}{20} \ge 1.0$	$g = e\, g_{interior}\left(\frac{48}{b}\right)$ $\frac{48}{b} \le 1.0$ $e = 1 + \left(\frac{d_e + \frac{b}{12} - 2.0}{40}\right)^{0.5} \ge 1.0$	$d_e \le 2.0$ $35 \le b \le 60$
Concrete Beams Other Than Box Beams Used in Multibeam Decks	h, also i, j if connected only enough to prevent relative vertical displacement at the interface	Lever Rule	Lever Rule	N/A
Open Steel Grid Deck on Steel Beams	a	Lever Rule	Lever Rule	N/A
Concrete Deck on Multiple Steel Box Beams	b, c	As specified in Table 4.6.2.2.2b-1		

4.6.2.2.3c—Skewed Bridges

Shear in bridge girders shall be adjusted when the line of support is skewed. The value of the correction factor shall be obtained from Table 4.6.2.2.3c-1 and applied to the live load distribution factors, g, specified in Table 4.6.2.2.3b-1 for exterior beams at the obtuse corner of the span, and in Table 4.6.2.2.3a-1 for interior beams. If the beams are well connected and behave as a unit, only the exterior and first interior beam need to be adjusted. The shear correction factors should be applied between the point of support at the obtuse corner and mid-span, and may be decreased linearly to a value of 1.0 at mid-span, regardless of end condition. This factor should not be applied in addition to modeling skewed supports.

In determining the end shear in deck system bridges, the skew correction at the obtuse corner shall be applied to all the beams.

C4.6.2.2.3c

Verifiable correction factors are not available for cases not covered in Table 4.6.2.2.3c-1, including large skews and skews in combination with curved bridge alignments. When torsional force effects due to skew become significant, load distribution factors are inappropriate.

The equal treatment of all beams in a multibeam bridge (box beams and deck girders) is conservative regarding positive reaction and shear. The contribution from transverse post-tensioning is conservatively ignored. However, it is not necessarily conservative regarding uplift in the case of large skew and short exterior spans of continuous beams. A supplementary investigation of uplift should be considered using the correction factor from Table 4.6.2.2.3c-1, i.e., the terms other than 1.0, taken as negative for the exterior beam on the acute corner.

Table 4.6.2.2.3c-1—Correction Factors for Live Load Distribution Factors for Support Shear of the Obtuse Corner

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Correction Factor	Range of Applicability
Concrete Deck or Filled Grid, Partially Filled Grid, or Unfilled Grid Deck Composite with Reinforced Concrete Slab on Steel or Concrete Beams; Concrete T-Beams, T- and Double T-Sections	a, e, k and also i, j if sufficiently connected to act as a unit	$1.0+0.20\left(\frac{12.0Lt_s^3}{K_g}\right)^{0.3}\tan\theta$	$0° \le \theta \le 60°$ $3.5 \le S \le 16.0$ $20 \le L \le 240$ $N_b \ge 4$
Cast-in-Place Concrete Multicell Box	d	For exterior girder: $1.0+\left(0.25+\frac{12.0L}{70d}\right)\tan\theta$ For first interior girder: $1.0+\left(0.042+\frac{12.0L}{420d}\right)\tan\theta$	$0° < \theta \le 60°$ $6.0 < S \le 13.0$ $20 \le L \le 240$ $35 \le d \le 110$ $N_c \ge 3$
Concrete Deck on Spread Concrete Box Beams	b, c	$1.0+\frac{\sqrt{\frac{Ld}{12.0}}}{6S}\tan\theta$	$0° < \theta \le 60°$ $6.0 \le S \le 11.5$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$
Concrete Box Beams Used in Multibeam Decks	f, g	$1.0+\frac{12.0L}{90d}\sqrt{\tan\theta}$	$0° < \theta \le 60°$ $20 \le L \le 120$ $17 \le d \le 60$ $35 \le b \le 60$ $5 \le N_b \le 20$

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4.6.2.2.4—Curved Steel Bridges

Approximate analysis methods may be used for analysis of curved steel bridges. The Engineer shall ascertain that the approximate analysis method used is appropriate by confirming that the method satisfies the requirements stated in Article 4.4.

In curved systems, consideration should be given to placing parapets, sidewalks, barriers, and other heavy line loads at their actual location on the bridge. Wearing surface and other distributed loads may be assumed to be uniformly distributed to each girder in the cross-section.

C4.6.2.2.4

The V-load method (United States Steel, 1984) has been a widely used approximate method for analyzing horizontally curved steel I-girder bridges. The method assumes that the internal torsional load on the bridge—resulting solely from the curvature—is resisted by self-equilibrating sets of shears between adjacent girders. The V-load method does not directly account for sources of torque other than curvature and the method does not account for the horizontal shear stiffness of the concrete deck. The method is only valid for loads such as normal highway loadings. For exceptional loadings, a more refined analysis is required. The method assumes a linear distribution of girder shears across the bridge section; thus, the girders at a given cross-section should have approximately the same vertical stiffness. The V-load method is also not directly applicable to structures with reverse curvature or to a closed-framed system with horizontal lateral bracing near, or in the plane of one or both flanges. The V-load method does not directly account for girder twist; thus, lateral deflections, which become important on bridges with large spans and/or sharp skews and vertical deflections, may be significantly underestimated. In certain situations, the V-load method may not detect uplift at end bearings. The method is best suited for preliminary design, but may also be suitable for final design of structures with radial supports or supports skewed less than approximately 10 degrees.

The M/R method provides a means to account for the effect of curvature in curved box girder bridges. The method and suggested limitations on its use are discussed by Tung and Fountain (1970).

Vertical reactions at interior supports on the concave side of continuous-span bridges may be significantly underestimated by both the V-load and M/R methods.

Live load distribution factors for use with the V-load and M/R methods may be determined using the appropriate provisions of Article 4.6.2.2.

Strict rules and limitations on the applicability of both of these approximate methods do not exist. The Engineer must determine when approximate methods of analysis are appropriate.

4.6.2.2.5—Special Loads with Other Traffic

Except as specified herein, the provisions of this Article may be applied where the approximate methods of analysis for the analysis of beam-slab bridges specified in Article 4.6.2.2 and slab-type bridges specified in Article 4.6.2.3 are used. The provisions of this Article shall not be applied where either:

- the lever rule has been specified for both single lane and multiple lane loadings; or
- the special requirement for exterior girders of beam-slab bridge cross-sections with

C4.6.2.2.5

Because the number of loaded lanes used to determine the multiple lane live load distribution factor, g_m, is not known, the multiple lane multiple presence factor, m, is implicitly set equal to 1.0 in this equation, which assumes only two lanes are loaded, resulting in a conservative final force effect over using the multiple presence factors for three or more lanes loaded.

The factor Z is used to distinguish between situations where the single lane live load distribution factor was determined from a specified algebraic equation and situations where the lever rule was specified for the determination of the single lane live diaphragms specified in Article 4.6.2.2.2d has been utilized for simplified analysis.

Force effects resulting from heavy vehicles in one lane with routine traffic in adjacent lanes, such as might be considered with Load Combination Strength II in Table 3.4.1-1, may be determined as:

$$G = G_p\left(\frac{g_1}{Z}\right) + G_D\left(g_m - \frac{g_1}{Z}\right) \qquad (4.6.2.2.5\text{-}1)$$

where:

G = final force effect applied to a girder (kip or kip-ft)
G_p = force effect due to overload truck (kip or kip-ft)
g_1 = single lane live load distribution factor
G_D = force effect due to design loads (kip or kip-ft)
g_m = multiple lane live load distribution factor
Z = a factor taken as 1.20 where the lever rule was not utilized, and 1.0 where the lever rule was used for a single lane live load distribution factor

load distribution factor. In the situation where an algebraic equation was specified, the multiple presence factor of 1.20 for a single lane loaded has been included in the algebraic equation and must be removed by using Z = 1.20 in Eq. 4.6.2.2.5-1 so that the distribution factor can be utilized in Eq. 4.6.2.2.5-1 to determine the force effect resulting from a multiple lane loading.

This formula was developed from a similar formula presented without investigation by Modjeski and Masters, Inc. (1994) in a report to the Pennsylvania Department of Transportation in 1994, as was examined in Zokaie (1998).

4.6.2.3—Equivalent Strip Widths for Slab-Type Bridges

This Article shall be applied to the types of cross-sections shown schematically in Table 4.6.2.3-1. For the purpose of this Article, cast-in-place voided slab bridges may be considered as slab bridges.

The equivalent width of longitudinal strips per lane for both shear and moment with one lane, i.e., two lines of wheels, loaded may be determined as:

$$E = 10.0 + 5.0\sqrt{L_1 W_1} \qquad (4.6.2.3\text{-}1)$$

The equivalent width of longitudinal strips per lane for both shear and moment with more than one lane loaded may be determined as:

$$E = 84.0 + 1.44\sqrt{L_1 W_1} \leq \frac{12.0W}{N_L} \qquad (4.6.2.3\text{-}2)$$

where:

E = equivalent width (in.)
L_1 = modified span length taken equal to the lesser of the actual span or 60.0 (ft)
W_1 = modified edge-to-edge width of bridge taken to be equal to the lesser of the actual width or 60.0 for multilane loading, or 30.0 for single-lane loading (ft)
W = physical edge-to-edge width of bridge (ft)
N_L = number of design lanes as specified in Article 3.6.1.1.1

C4.6.2.3

In Eq. 4.6.2.3-1, the strip width has been divided by 1.20 to account for the multiple presence effect.

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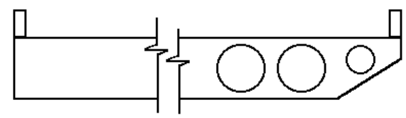
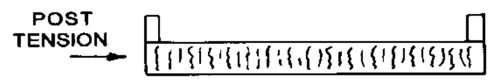
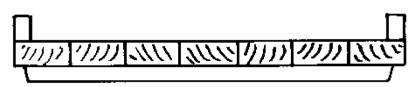
For skewed bridges, the longitudinal force effects may be reduced by the factor, r:

$$r = 1.05 - 0.25\tan\theta \leq 1.00 \qquad (4.6.2.3\text{-}3)$$

where:

θ = skew angle (degrees)

Table 4.6.2.3-1—Typical Schematic Cross-Section

Supporting Components	Type of Deck	Typical Cross-Section
Cast-in-Place Concrete Slab or Voided Slab	Monolithic	 (a)
Stressed Wood Deck	Integral Wood	 (b)
Glued/Spiked Wood Panels with Spreader Beam	Integral Wood	 (c)

4.6.2.4—Truss and Arch Bridges

The lever rule may be used for the distribution of gravity loads in trusses and arches when analyzed as planar structures. If a space analysis is used, either the lever rule or direct loading through the deck or deck system may be used.

Where loads, other than the self-weight of the members and wind loads thereon, are transmitted to the truss at the panel points, the truss may be analyzed as a pin-connected assembly.

4.6.2.5—Effective Length Factor, *K*

Physical column lengths shall be multiplied by an effective length factor, K, to compensate for rotational and translational boundary conditions other than pinned ends.

In the absence of a more refined analysis, where lateral stability is provided by diagonal bracing or other suitable means, the effective length factor in the braced plane, K, for the compression members in triangulated trusses, trusses, and frames may be taken as:

- For bolted or welded end connections at both ends: $K = 0.750$
- For pinned connections at both ends: $K = 0.875$
- For single angles, regardless of end connection: $K = 1.0$

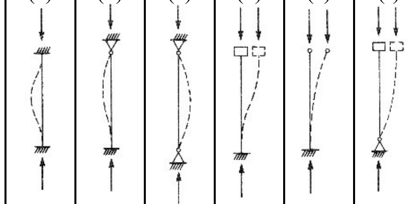
C4.6.2.5

Equations for the compressive resistance of columns and moment magnification factors for beam-columns include a factor, K, which is used to modify the length according to the restraint at the ends of the column against rotation and translation.

K is the ratio of the effective length of an idealized pin-end column to the actual length of a column with various other end conditions. KL represents the length between inflection points of a buckled column influenced by the restraint against rotation and translation of column ends. Theoretical values of K, as provided by the Structural Stability Research Council, are given in Table C4.6.2.5-1 for some idealized column end conditions.

Vierendeel trusses shall be treated as unbraced frames.

Table C4.6.2.5-1—Effective Length Factors, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Design value of *K* when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.1	2.0
End condition code		Rotation fixed Translation fixed Rotation free Translation fixed Rotation fixed Translation free Rotation free Translation free				

Because actual column end conditions seldom comply fully with idealized restraint conditions against rotation and translation, the design values suggested by the Structural Stability Research Council are higher than the idealized values.

Lateral stability of columns in continuous frames, unbraced by attachment to shear walls, diagonal bracing, or adjacent structures, depends on the flexural stiffness of the rigidly connected beams. Therefore, the effective length factor, *K*, is a function of the total flexural restraint provided by the beams at the ends of the column. If the stiffness of the beams is small in relation to that of the column, the value of *K* could exceed 2.0.

Single angles are loaded through one leg and are subject to eccentricity and twist, which is often not recognized. *K* is set equal to 1.0 for these members to more closely match the strength provided in the Guide for Design of Steel Transmission Towers (ASCE Manual No. 52, 1971).

Assuming that only elastic action occurs and that all columns buckle simultaneously, it can be shown that (Chen and Liu, 1991; ASCE Task Committee on Effective Length, 1997):

For braced frames:

$$\frac{G_aG_b}{4}\left(\frac{\pi}{K}\right)^2+\frac{G_a+G_b}{2}\left(1-\frac{\frac{\pi}{K}}{tan\left(\frac{\pi}{K}\right)}\right)+\frac{2\tan\left(\frac{\pi}{2K}\right)}{\frac{\pi}{K}}=1$$

(C4.6.2.5-1)

For unbraced frames:

$$\frac{G_a G_b \left(\frac{\pi}{K}\right)^2 - 36}{6\left(G_a + G_b\right)} = \frac{\frac{\pi}{K}}{\tan\left(\frac{\pi}{K}\right)} \qquad \text{(C4.6.2.5-2)}$$

where subscripts a and b refer to the two ends of the column under consideration

in which:

$$G = \frac{\Sigma\left(\frac{E_c I_c}{L_c}\right)}{\Sigma\left(\frac{E_g I_g}{L_g}\right)} \qquad \text{(C4.6.2.5-3)}$$

where:

Σ = summation of the properties of components rigidly connected to an end of the column in the plane of flexure
E_c = modulus of elasticity of column (ksi)
I_c = moment of inertia of column (in.^{4})
L_c = unbraced length of column (in.)
E_g = modulus of elasticity of beam or other restraining member (ksi)
I_g = moment of inertia of beam or other restraining member (in.^{4})
L_g = unsupported length of beam or other restraining member (in.)
K = effective length factor for the column under consideration

Figures C4.6.2.5-1 and C4.6.2.5-2 are graphical representations of the relationship among K, G_a, and G_b for Eqs. C4.6.2.5-1 and C4.6.2.5-2, respectively. The figures can be used to obtain values of K directly.

Eqs. C4.6.2.5-1, C4.6.2.5-2, and the alignment charts in Figures C4.6.2.5-1 and C4.6.2.5-2 are based on assumptions of idealized conditions. The development of the chart and formula can be found in textbooks such as Salmon and Johnson (1990) and Chen and Lui (1991). When actual conditions differ significantly from these idealized assumptions, unrealistic designs may result. Galambos (1988), Yura (1971), Disque (1973), Duan and Chen (1988), and AISC (1993) may be used to evaluate end conditions more accurately.

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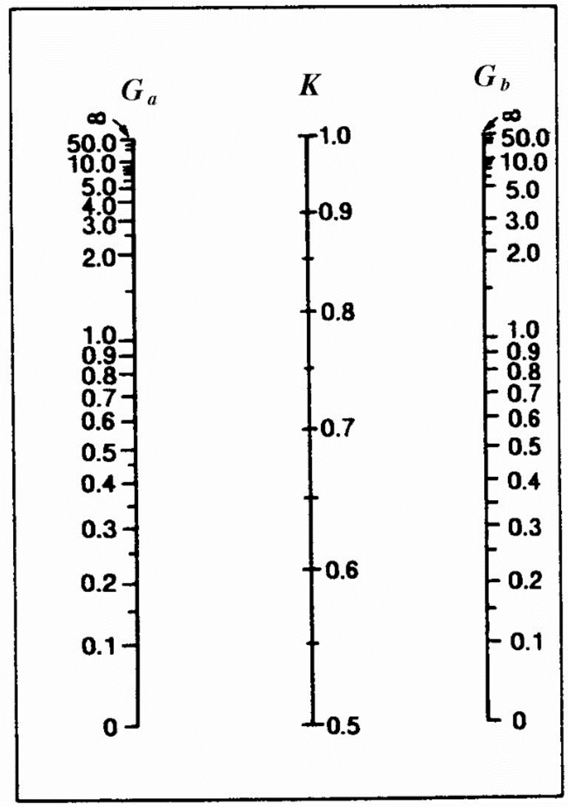

Figure C4.6.2.5-1—Alignment Chart for Determining Effective Length Factor, *K*, for Braced Frames

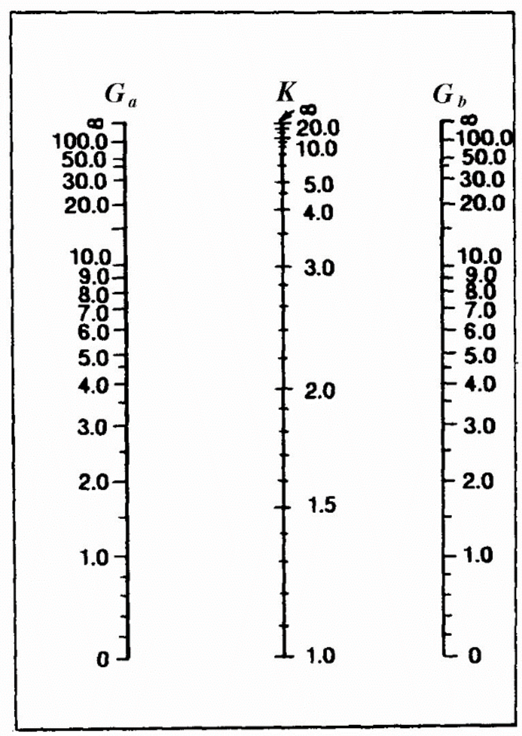

Figure C4.6.2.5-2—Alignment Chart for Determining Effective Length Factor, *K*, for Unbraced Frames

The following applies to the use of Figures C4.6.2.5-1 and C4.6.2.5-2:

- For column ends supported by but not rigidly connected to a footing or foundation, G is theoretically equal to infinity, but unless actually designed as a true frictionless pin, may be taken equal to 10 for practical design. If the column end is rigidly attached to a properly designed footing, G may be taken equal to 1.0. Smaller values may be taken if justified by analysis.
- In computing effective length factors for members with monolithic connections, it is important to properly evaluate the degree of fixity in the foundation using engineering judgment. In absence of a more refined analysis, the following values can be used:

Condition	G
Footing anchored on rock	1.5
Footing not anchored on rock	3.0
Footing on soil	5.0
Footing on multiple rows of end bearing piles	1.0

In lieu of the alignment charts, the following alternative K-factor equations (Duan, King, and Chen, 1993) may be used.

For braced frames:

$$K = 1 - \frac{1}{5+9G_a} - \frac{1}{5+9G_b} - \frac{1}{10+G_aG_b} \qquad \text{(C4.6.2.5-4)}$$

For unbraced frames:

- For $K < 2$

$$K = 4 - \frac{1}{1+0.2G_a} - \frac{1}{1+0.2G_b} - \frac{1}{1+0.01G_aG_b} \qquad \text{(C4.6.2.5-5)}$$

- For $K \geq 2$

$$K = \frac{2\pi a}{0.9+\sqrt{0.81+4ab}} \qquad \text{(C4.6.2.5-6)}$$

in which:

$$a = \frac{G_aG_b}{G_a+G_b} + 3 \qquad \text{(C4.6.2.5-7)}$$

$$b = \frac{36}{G_a + G_b} + 6 \qquad \text{(C4.6.2.5-8)}$$

Eq. C4.6.2.5-5 is used first. If the value of K calculated by Eq. C4.6.2.5-5 is greater than 2, Eq. C4.6.2.5-6 is used. The values for K calculated using Eqs. C4.6.2.5-5 and C4.6.2.5-6 are a good fit with results from the alignment chart Eqs. C4.6.2.5-1, C4.6.2.5-2, C4.6.2.5-3, and allow an Engineer to perform a direct noniterative solution for K.

4.6.2.6—Effective Flange Width

4.6.2.6.1—General

Unless specified otherwise in this Article or in Articles 4.6.2.6.2, 4.6.2.6.3, or 4.6.2.6.5, the effective flange width of a concrete deck slab in composite or monolithic construction may be taken as the tributary width perpendicular to the axis of the member for determining cross-section stiffnesses for analysis and for determining flexural resistances. The effective flange width of orthotropic steel decks shall be as specified in Article 4.6.2.6.4. For the calculation of live load deflections, where required, the provisions of Article 2.5.2.6.2 shall apply.

Where a structurally continuous concrete barrier is present and is included in the structural analysis as permitted in Article 4.5.1, the deck slab overhang width used for the analysis as well as for checking the composite girder resistance may be extended by:

$$\Delta w = \frac{A_b}{2t_s} \qquad \text{(4.6.2.6.1-1)}$$

where:

A_b = cross-sectional area of the barrier (in.^{2})
t_s = thickness of deck slab (in.)

The slab effective flange width in composite girder and/or stringer systems or in the chords of composite deck trusses may be taken as one-half the distance to the adjacent stringer or girder on each side of the component, or one-half the distance to the adjacent stringer or girder plus the full overhang width. Otherwise, the slab effective flange width should be determined by a refined analysis when:

- the composite or monolithic member cross-section is subjected to significant combined axial force and bending, with the exception that forces induced by restraint of thermal expansion may be determined in beam-slab systems using the slab tributary width;

C4.6.2.6.1

Longitudinal stresses are distributed across the deck of composite and monolithic flexural members by in-plane shear stresses. Due to the corresponding shear deformations, plane sections do not remain plane and the longitudinal stresses across the deck are not uniform. This phenomenon is referred to as shear lag. The effective flange width is the width of the deck over which the assumed uniformly distributed longitudinal stresses result in approximately the same deck force and member moments calculated from elementary beam theory assuming plane sections remain plane, as are produced by the nonuniform stress distribution.

The provisions of this Article apply to all longitudinal flexural members composite or monolithic with a deck slab, including girders and stringers. They are based on finite element studies of various bridge types and configurations, corroborated by experimental tests and sensitivity analysis of various candidate regression equations (Chen et al., 2005). Chen et al. (2005) found that bridges with larger L/S (ratio of span length to girder spacing) consistently exhibited an effective width, b_e, equal to the tributary width, b. Nonskewed bridges with $L/S = 3.1$, the smallest value of L/S considered in the Chen et al. (2005) study, exhibited $b_e = b$ in the maximum positive bending regions and approximately $b_e = 0.9b$ in the maximum negative bending regions under service limit state conditions. However, they exhibited $b_e = b$ in these regions in all cases at the strength limit state. Bridges with large skew angles often exhibited $b_e < b$ in both the maximum positive and negative moment regions, particularly in cases with small L/S. However, when various potential provisions were assessed using the Rating Factor (RF) as a measure of impact, the influence of using full width ($b_e = b$) was found to be minimal. Therefore, the use of the tributary width is justified in all cases within the limits specified in this Article. The Chen et al. (2005) study demonstrated that there is no significant relationship between the slab effective width and the slab thickness.

These provisions are considered applicable for skew angles less than or equal to 75 degrees, L/S greater than

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- the largest skew angle, θ, in the bridge system is greater than 75 degrees, where θ is the angle of a bearing line measured relative to a normal to the centerline of a longitudinal component;
- the slab spans longitudinally between transverse floorbeams; or
- the slab is designed for two-way action.

or equal to 2.0 and overhang widths less than or equal to 0.5*S*. In unusual cases where these limits are violated, a refined analysis should be used to determine the slab effective width. Furthermore, these provisions are considered applicable for slab-beam bridges with unequal skew angles of the bearing lines, splayed girders, horizontally curved girders, cantilever spans, and various unequal span lengths of continuous spans, although these parameters have not been investigated extensively in studies to date. These recommendations are based on the fact that the participation of the slab in these broader parametric cases is fundamentally similar to the participation of the slab in the specific parametric cases that have been studied.

The use of one-half the distance to the adjacent stringer or girder in calculating the effective width of the main girders in composite girder and/or stringer systems or the truss chords in composite deck trusses is a conservative assumption for the main structural components, since typically a larger width of the slab can be expected to participate with the main girders or truss chords. However, this tributary width assumption may lead to an underestimation of the shear connector requirements and a lack of consideration of axial forces and bending moments in the composite stringers or girders due to the global effects. To utilize a larger slab width for the main girders or truss chords, a refined analysis should be considered.

The specific cases in which a refined analysis is recommended are so listed because they are significantly beyond the conventional application of the concept of a slab effective width. These cases include tied arches where the deck slab is designed to contribute to the resistance of the tie girders and cable stayed bridges with a composite deck slab. Chen et al. (2005) provides a few case study results for simplified lower-bound slab effective widths in composite deck systems of cable stayed bridges with certain specific characteristics.

4.6.2.6.2—Segmental Concrete Box Beams and Single-Cell, Cast-in-Place Box Beams

The effective flange width may be assumed equal to the physical flange width if:

- $b \leq 0.1\ l_i$
- $b \leq 0.3\ d_o$

Otherwise, the effective width of outstanding flanges may be taken as specified in Figures 4.6.2.6.2-1 through 4.6.2.6.2-4, where:

d_o = depth of superstructure (in.)
b = physical flange width on each side of the web, e.g., b_1, b_2, and b_3, as shown in Figure 4.6.2.6.2-3 (in.)

C4.6.2.6.2

One possible alternative to the procedure specified in this Article is contained in Clause 3-10.2 of the 1991 Ontario Highway Bridge Design Code, which provides an equation for determining the effective flange width for use in calculating flexural resistances and stresses.

Superposition of local two-way slab flexural stresses due to wheel loads and the primary longitudinal flexural stresses is not normally required.

The effective flange widths, b_m and b_s, are determined as the product of the coefficient in Figure 4.6.2.6.2-2 and the physical distance, b, as indicated in Figure 4.6.2.6.2-3.

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b_e = effective flange width corresponding to the particular position of the section of interest in the span as specified in Figure 4.6.2.6.2-1 (in.)
b_m = effective flange width for interior portions of a span as determined from Figure 4.6.2.6.2-2; a special case of b_e (in.)
b_s = effective flange width at interior support or for cantilever arm as determined from Figure 4.6.2.6.2-2; a special case of b_e (in.)
a = portion of span subject to a transition in effective flange width taken as the lesser of the physical flange width on each side of the web shown in Figure 4.6.2.6.2-3 or one quarter of the span length (in.)
ℓ_i = a notional span length specified in Figure 4.6.2.6.2-1 for the purpose of determining effective flange widths using Figure 4.6.2.6.2-2

The following interpretations apply:

- In any event, the effective flange width shall not be taken as greater than the physical width.
- The effects of unsymmetrical loading on the effective flange width may be disregarded.
- The value of b_s shall be determined using the greater of the effective span lengths adjacent to the support.
- If b_m is less than b_s in a span, the pattern of the effective width within the span may be determined by the connecting line of the effective widths, b_s, at adjoining support points.

For the superposition of local and global force effects, the distribution of stresses due to the global force effects may be assumed to have a straight line pattern in accordance with Figure 4.6.2.6.2-3c. The linear stress distribution should be determined from the constant stress distribution using the conditions that the flange force remains unchanged and that the maximum width of the linear stress distribution on each side of a web is 2.0 times the effective flange width.

The section properties for normal forces may be based on the pattern according to Figure 4.6.2.6.2-4 or determined by more rigorous analysis.

If the linear stress distributions intersect a free edge or each other before reaching the maximum width, the linear stress distribution is a trapezoid; otherwise, it is a triangle. This is shown in Figure 4.6.2.6.2-3c.

Figure 4.6.2.6.2-4 is intended only for calculation of resistance due to anchorage of post-tensioning tendons and other concentrated forces and may be disregarded in the general analysis to determine force effects.

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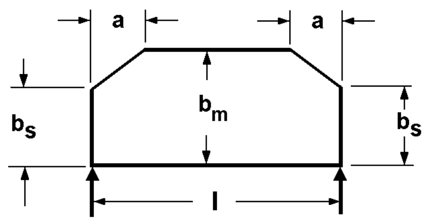
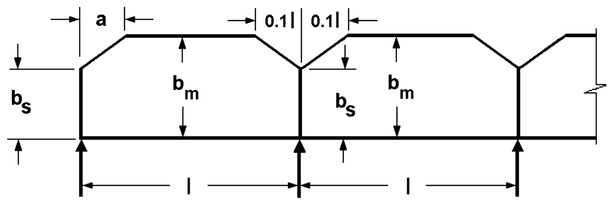
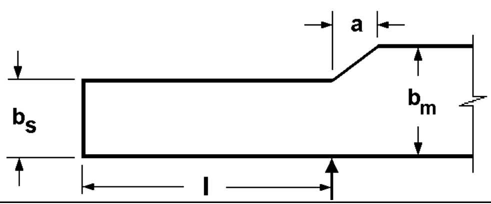
<table>
<tr><th colspan="2">System</th><th>Pattern of b_m/b</th></tr>
<tr><td colspan="2">Single-Span Girder
$\ell_i = 1.0\ell$</td><td>
</td></tr>
<tr><td rowspan="2">Continuous Girder</td><td>End Span
$\ell_i = 0.8\ell$</td><td rowspan="2">
</td></tr>
<tr><td>Interior Span
$\ell_i = 0.6\ell$</td></tr>
<tr><td colspan="2">Cantilever Arm
$\ell_i = 1.5\ell$</td><td>
</td></tr>
</table>

Figure 4.6.2.6.2-1—Pattern of Effective Flange Width, b_e, b_m, and b_s

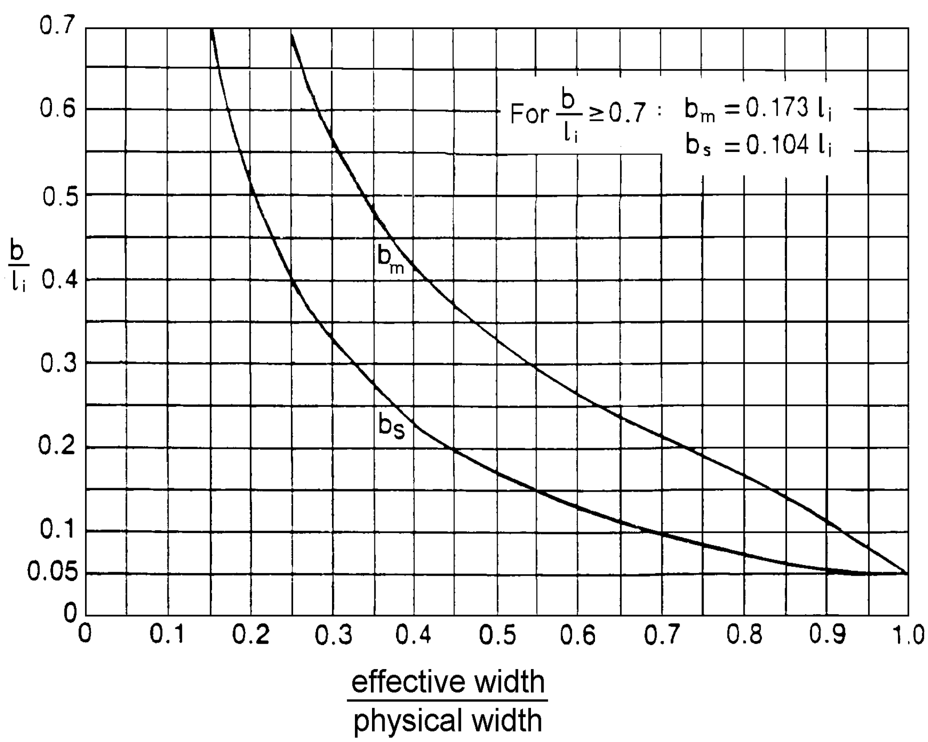


Figure 4.6.2.6.2-2—Values of the Effective Flange Width Coefficients for b_m and b_s for the Given Values of b/ℓ_i

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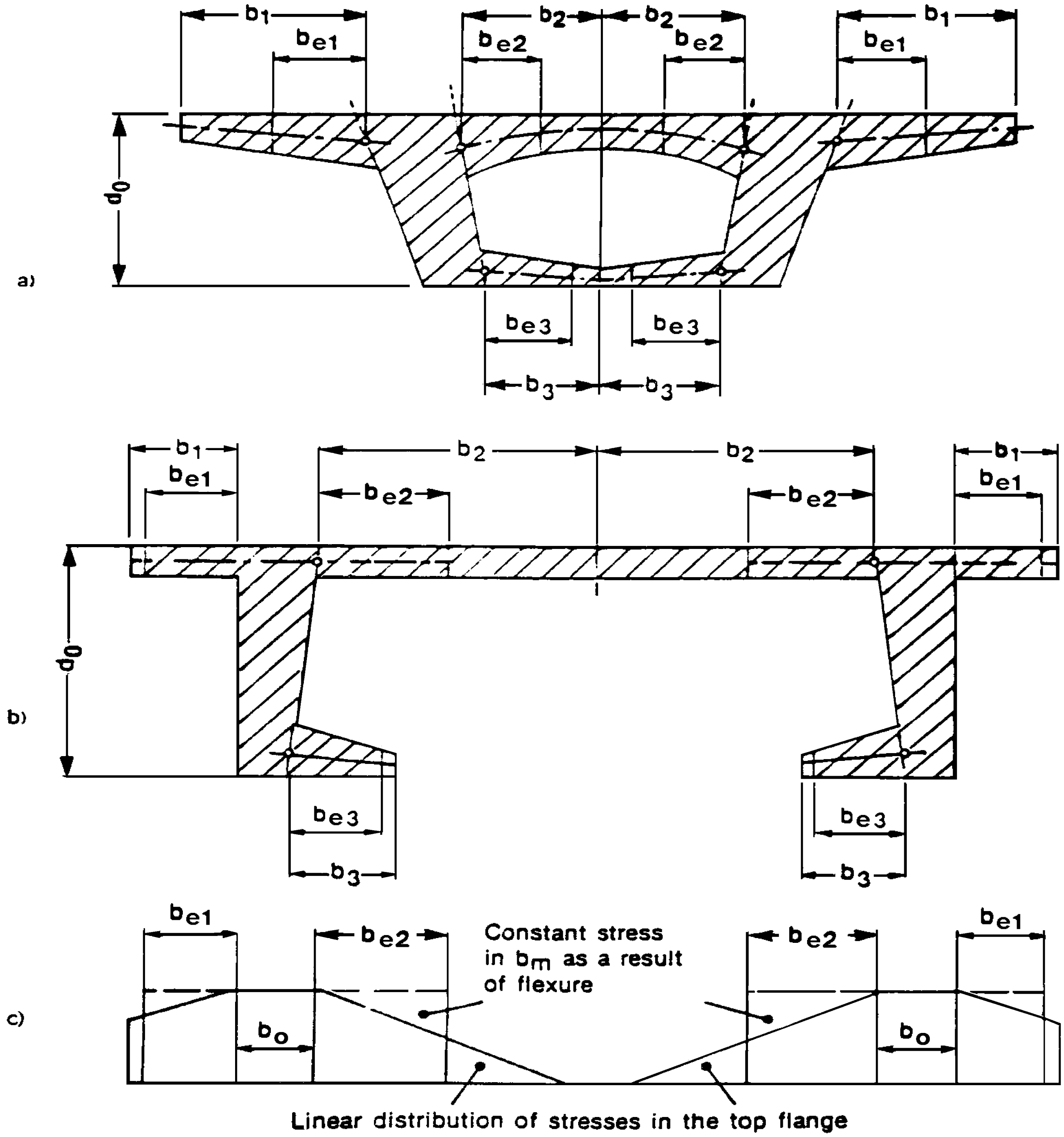

Figure 4.6.2.6.2-3—Cross-Sections and Corresponding Effective Flange Widths, b_e, for Flexure and Shear

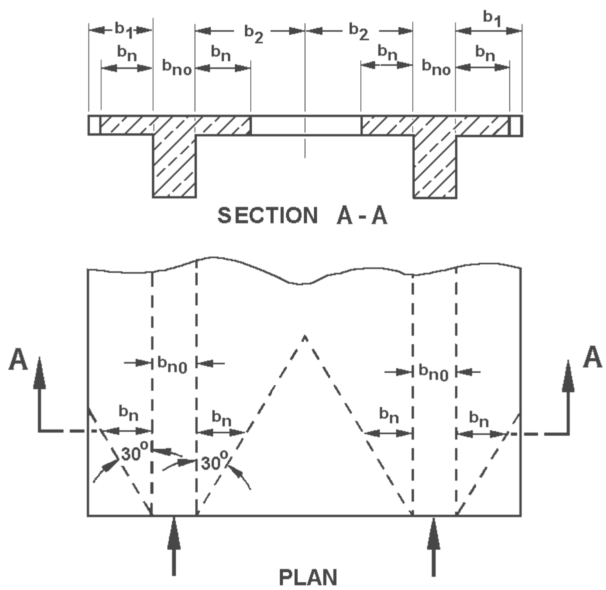

Figure 4.6.2.6.2-4—Effective Flange Widths, *b_n*, for Normal Forces

4.6.2.6.3—Cast-in-Place Multicell Superstructures

The effective width for cast-in-place multiweb cellular superstructures may be taken to be as specified in Article 4.6.2.6.1, with each web taken to be a beam, or it may be taken to be the full width of the deck slab. In the latter case, the effects of shear lag in the end zones shall be investigated.

4.6.2.6.4—Orthotropic Steel Decks

The effective width need not be determined when using refined analysis as specified in Article 4.6.3.2.4. For simplified analysis, the effective width of the deck, including the deck plate and ribs, acting as the top flange of a longitudinal superstructure component or a transverse beam may be taken as:

- $L/B \geq 5$: fully effective
- $L/B < 5$: $b_{od} = \frac{1}{5}L$

where:

L = span length of the orthotropic girder or transverse beam (in.)
B = spacing between orthotropic girder web plates or transverse beams (in.)
b_{od} = effective width of orthotropic deck (in.)

for strength limit states for positive and negative flexure. For service and fatigue limit states in regions of high shear, the effective deck width can be determined by refined analysis or other accepted approximate methods.

C4.6.2.6.4

Consideration of effective width of the deck plate can be avoided by application of refined analysis methods.

The procedures in *Design Manual for Orthotropic Steel Plate Deck Bridges* (AISC, 1963) may be used as an acceptable means of simplified analysis; however, it has been demonstrated that using this procedure can result in rib effective widths exceeding the rib spacing, which may be unconservative.

Tests (Dowling et al., 1977) have shown that for most practical cases, shear lag can be ignored in calculating the ultimate compressive strength of stiffened or unstiffened girder flanges (Lamas and Dowling, 1980; Burgan and Dowling, 1985; Jetteur et al., 1984; and Hindi, 1991). Thus, a flange may normally be considered to be loaded uniformly across its width. It is necessary to consider the flange effectiveness in greater detail only in the case of flanges with particularly large aspect ratios ($L/B < 5$) or particularly slender edge panels or stiffeners (Burgan and Dowling, 1985 and Hindi, 1991).

Consideration of inelastic behavior can increase the effective width as compared to elastic analysis. At ultimate loading, the region of the flange plate above the web can yield and spread the plasticity (and distribute stress) outward if the plate maintains local stability. Results from studies by Chen et al. (2005) on composite steel girders, which included several tub-girder bridges, indicate that the full slab width may be considered effective in both positive and negative moment regions.

Thus, orthotropic plates acting as flanges are considered fully effective for strength limit state evaluations from positive and negative flexure when the L/B ratio is at least 5. For the case of $L/B < 5$, only a width of one-fifth of the effective span should be considered effective. For service and fatigue limit states in regions of high shear, a special investigation into shear lag should be done.

4.6.2.6.5—Transverse Floorbeams and Integral Bent Caps

For transverse floorbeams and for integral bent caps designed with a composite concrete deck slab, the effective flange width overhanging each side of the transverse floorbeam or bent cap web shall not exceed six times the least slab thickness or one-tenth of the span length. For cantilevered transverse floorbeams or integral bent caps, the span length shall be taken as two times the length of the cantilever span.

C4.6.2.6.5

The provisions for the effective flange width for transverse floorbeams and integral bent caps are based on past successful practice, specified by Article 8.10.1.4 of the 2002 *AASHTO Standard Specifications for Highway Bridges*.

4.6.2.7—Lateral Wind Load Distribution in Girder System Bridges

4.6.2.7.1—I-Sections

In bridges with composite decks, noncomposite decks with concrete haunches, and other decks that can provide horizontal diaphragm action, wind load on the upper half of the outside beam, the deck, vehicles, barriers, and appurtenances shall be assumed to be directly transmitted to the deck, acting as a lateral diaphragm carrying this load to supports. Wind load on the lower half of the outside beam shall be assumed to be applied laterally to the lower flange.

For bridges with decks that cannot provide horizontal diaphragm action, the lever rule shall apply for distribution of the wind load to the top and bottom flanges.

Bottom and top flanges subjected to lateral wind load shall be assumed to carry that load to adjacent brace points by flexural action. Such brace points occur at wind bracing nodes or at cross-frames and diaphragm locations.

The lateral forces applied at brace points by the flanges shall be transmitted to the supports by one of the following load paths:

- Truss action of horizontal wind bracing in the plane of the flange;
- Frame action of the cross-frames or diaphragms transmitting the forces into the deck or the wind bracing in the plane of the other flange, and then by diaphragm action

C4.6.2.7.1

Precast concrete plank decks and timber decks are not solid diaphragms and should not be assumed to provide horizontal diaphragm action unless evidence is available to show otherwise.

Unless a more refined analysis is made, the wind force, wind moment, horizontal force to be transmitted by diaphragms and cross-frames, and horizontal force to be transmitted by lateral bracing may be calculated as indicated below. This procedure is presented for beam bridges but may be adapted for other types of bridges.

The wind force, W, may be applied to the flanges of exterior members. For composite members and noncomposite members with cast-in-place concrete or orthotropic steel decks, W need not be applied to the top flange.

$$W = \frac{\eta_i \gamma P_D d}{2} \quad \text{(C4.6.2.7.1-1)}$$

where:

W = factored wind force per unit length applied to the flange (kip/ft)

P_D = design horizontal wind pressure specified in Article 3.8.1 (ksf)

d = depth of the member (ft)

of the deck, or truss action of the wind bracing, to the supports; or

- Lateral bending of the flange subjected to the lateral forces and all other flanges in the same plane, transmitting the forces to the ends of the span; for example, where the deck cannot provide horizontal diaphragm action, and there is no wind bracing in the plane of either flange.

γ = load factor specified in Table 3.4.1-1 for the particular group loading combination
η_i = load modifier relating to ductility, redundancy, and operational importance as specified in Article 1.3.2.1

For the first two load paths, the maximum wind moment on the loaded flange may be determined as:

$$M_w = \frac{WL_b^{\,2}}{10} \qquad \text{(C4.6.2.7.1-2)}$$

where:

M_w = maximum lateral moment in the flange due to the factored wind loading (kip-ft)
W = factored wind force per unit length applied to the flange (kip/ft)
L_b = spacing of brace points (ft)

For the third load path, the maximum wind moment on the loaded flange may be computed as:

$$M_w = \frac{WL_b^{\,2}}{10} + \frac{WL^2}{8N_b} \qquad \text{(C4.6.2.7.1-3)}$$

where:

M_w = total lateral moment in the flange due to the factored wind loading (kip-ft)
W = factored wind force per unit length applied to the flange (kip/ft)
L_b = spacing of cross-frames or diaphragms (ft)
N_b = number of longitudinal members
L = span length (ft)

Eq. C4.6.2.7.1-3 is based on the assumption that cross-frames and diaphragms act as struts in distributing the wind force on the exterior flange to adjacent flanges. If there are no cross-frames or diaphragms, the first term should be taken as 0.0, and N_b should be taken as 1.0.

The horizontal wind force applied to each brace point may be calculated as:

$$P_w = WL_b \qquad \text{(C4.6.2.7.1-4)}$$

where:

P_w = lateral wind force applied to the brace point (kips)
W = wind force per unit length from Eq. C4.6.2.7.1-1 (kip/ft)
L_b = spacing of diaphragms or cross-frames (ft)

Lateral bracing systems required to support both flanges due to transfer of wind loading through diaphragms or cross-frames shall be designed for a horizontal force of $2P_w$ at each brace point.

4.6.2.7.2—Box Sections

One-quarter of the wind force on a box section shall be applied to the bottom flange of the exterior box beam. The section assumed to resist the wind force shall consist of the bottom flange and a part of the web as determined in Sections 5 and 6. The other three quarters of the wind force on a box section, plus the wind force on vehicles, barriers, and appurtenances, shall be assumed to be transmitted to the supports by diaphragm action of the deck.

Interbox lateral bracing shall be provided if the section assumed to resist the wind force is not adequate.

4.6.2.7.3—Construction

The need for temporary wind bracing during construction shall be investigated for I- and box-section bridges.

4.6.2.8—Seismic Lateral Load Distribution

4.6.2.8.1—Applicability

These provisions shall apply to diaphragms, cross-frames, and lateral bracing, which are part of the seismic lateral force resisting system in common slab-on-girder bridges in Seismic Zones 2, 3, and 4. The provisions of Article 3.10.9.2 shall apply to Seismic Zone 1.

4.6.2.8.2—Design Criteria

The Engineer shall demonstrate that a clear, straightforward load path to the substructure exists and that all components and connections are capable of resisting the imposed load effects consistent with the chosen load path.

The flow of forces in the assumed load path must be accommodated through all affected components and details including, but not limited to, flanges and webs of main beams or girders, cross-frames, connections, slab-to-girder interfaces, and all components of the bearing assembly from top flange interface through the confinement of anchor bolts or similar devices in the substructure.

The analysis and design of end diaphragms and cross-frames shall consider horizontal supports at an appropriate number of bearings. Slenderness and connection requirements of bracing members that are part of the lateral force resisting system shall comply with applicable provisions specified for main member design.

Members of diaphragms and cross-frames identified by the Designer as part of the load path carrying seismic forces from the superstructure to the bearings shall be designed and detailed to remain elastic, based on the applicable gross area criteria, under all design earthquakes, regardless of the type of bearings used. The applicable provisions for the design of main members shall apply.

C4.6.2.8.2

Diaphragms, cross-frames, lateral bracing, bearings, and substructure elements are part of a seismic load resisting system in which the lateral loads and performance of each element are affected by the strength and stiffness characteristics of the other elements. Past earthquakes have shown that when one of these elements responded in a ductile manner or allowed some movement, damage was limited. In the strategy taken herein, it is assumed that ductile plastic hinging in substructure is the primary source of energy dissipation. Alternative design strategies may be considered if approved by the Owner.

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4.6.2.8.3—Load Distribution

A viable load path shall be established to transmit lateral loads to the foundation based on the stiffness characteristics of the deck, diaphragms, cross-frames, and lateral bracing. Unless a more refined analysis is made, an approximate load path shall be assumed as noted below.

- In bridges with:
  - a concrete deck that can provide horizontal diaphragm action; or
  - a horizontal bracing system in the plane of the top flange,

  the lateral loads applied to the deck shall be assumed to be transmitted directly to the bearings through end diaphragms or cross-frames. The development and analysis of the load path through the deck or through the top lateral bracing, if present, shall utilize assumed structural actions analogous to those used for the analysis of wind loadings.

- In bridges that have:
  - decks that cannot provide horizontal diaphragm action and
  - no lateral bracing in the plane of the top flange,

  the lateral loads applied to the deck shall be distributed through the intermediate diaphragms and cross-frames to the bottom lateral bracing or the bottom flange, and then to the bearings, and through the end diaphragms and cross-frames, in proportion to their relative rigidity and the respective tributary mass of the deck.
- If a bottom lateral bracing system is not present, and the bottom flange is not adequate to carry the imposed force effects, the first procedure shall be used, and the deck shall be designed and detailed to provide the necessary horizontal diaphragm action.

C4.6.2.8.3

A continuous path is necessary for the transmission of the superstructure inertia forces to the foundation. Concrete decks have significant rigidity in their horizontal plane, and in short to medium slab-on-girder spans, their response approaches a rigid body motion. Therefore, the lateral loading of the intermediate diaphragms and cross-frames is minimal.

Bearings do not usually resist load simultaneously, and damage to only some of the bearings at one end of a span is not uncommon. When this occurs, high load concentrations can result at the location of the other bearings, which should be taken into account in the design of the end cross-frames or diaphragms. Also, a significant change in the load distribution among end cross-frame members may occur. Although studies of cyclic load behavior of bracing systems have shown that with adequate details, bracing systems can allow for ductile behavior, these design provisions require elastic behavior in end diaphragms (Astaneh-Asl and Goel, 1984; Astaneh-Asl et al., 1985; Haroun and Sheperd, 1986; Goel and El-Tayem, 1986).

Because the end diaphragm is required to remain elastic as part of the identified load path, stressing of intermediate cross-frames need not be considered.

4.6.2.9—Analysis of Segmental Concrete Bridges

4.6.2.9.1—General

Elastic analysis and beam theory may be used to determine design moments, shears, and deflections. The effects of creep, shrinkage, and temperature differentials shall be considered as well as the effects of shear lag. Shear lag shall be considered in accordance with the provisions of Article 4.6.2.9.3.

For spans in excess of 250 ft, results of elastic analyses should be evaluated with consideration of possible variations in the modulus of elasticity of the concrete, variations in the concrete creep and shrinkage

C4.6.2.9.1

Analysis of concrete segmental bridges requires consideration of variation of design parameters with time as well as a specific construction schedule and method of erection. This, in turn, requires the use of a computer program developed to trace the time-dependent response of segmentally erected, prestressed concrete bridges through construction and under service loads. Among the many programs developed for this purpose, several are in the public domain and may be purchased for a nominal amount, e.g., (Ketchum, 1986; Shushkewich, 1986; Danon and Gamble, 1977).

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properties, and the impact of variations in the construction schedule on these and other design parameters.

4.6.2.9.2—Strut-and-Tie Models

Strut-and-tie models may be used for analysis in areas of load or geometrical discontinuity.

C4.6.2.9.2

See references for background on transverse analysis of concrete box girder bridges.

4.6.2.9.3—Effective Flange Width

Effective flange width for service load stress calculations may be determined by the provisions of Article 4.6.2.6.2.

The section properties for normal forces may be based on Figure 4.6.2.6.2-4 or determined by more rigorous analysis.

Bending, shear, and normal forces may be evaluated by using the corresponding factored resistances.

The capacity of a cross-section at the strength limit state may be determined by considering the full compression flange width effect.

4.6.2.9.4—Transverse Analysis

The transverse design of box girder segments for flexure shall consider the segment as a rigid box frame. Flanges shall be analyzed as variable depth sections, considering the fillets between the flanges and webs. Wheel loads shall be positioned to provide maximum moments, and elastic analysis shall be used to determine the effective longitudinal distribution of wheel loads for each load location. Consideration shall be given to the increase in web shear and other effects on the cross-section resulting from eccentric loading or unsymmetrical structure geometry.

The provisions of Articles 4.6.2.1 and 4.6.3.2, influence surfaces such as those by Homberg (1968) and Pucher (1964), or other elastic analysis procedures may be used to evaluate live load plus impact moment effects in the top flange of the box section.

Transverse elastic and creep shortening due to prestressing and shrinkage shall be considered in the transverse analysis.

The effect of secondary moments due to prestressing shall be included in stress calculations at the service limit state and construction evaluation. At the strength limit state, the secondary force effects induced by prestressing, with a load factor of 1.0, shall be added algebraically to the force effects due to factored dead and live loads and other applicable loads.

4.6.2.9.5—Longitudinal Analysis

4.6.2.9.5a—General

Longitudinal analysis of segmental concrete bridges shall consider a specific construction method and construction schedule as well as the time-related effects of concrete creep, shrinkage, and prestress losses.

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The effect of secondary moments due to prestressing shall be included in stress calculations at the service limit state. At the strength limit state, the secondary force effects induced by prestressing, with a load factor of 1.0, shall be added algebraically to other applicable factored loads.

4.6.2.9.5b—Erection Analysis

Analysis of the structure during any construction stage shall consider the construction load combinations, stresses, and stability considerations specified in Article 5.12.5.3.

4.6.2.9.5c—Analysis of the Final Structural System

The provisions of Article 5.12.5.2.3 shall apply.

4.6.2.10—Equivalent Strip Widths for Box Culverts

4.6.2.10.1—General

This Article shall be applied to box culverts with depths of fill less than 2.0 ft.

C4.6.2.10.1

Design for depths of fill of 2.0 ft or greater are covered in Article 3.6.1.2.6.

4.6.2.10.2—Case 1: Traffic Travels Parallel to Span

When traffic travels primarily parallel to the span, culverts shall be analyzed for a single loaded lane with the single lane multiple presence factor.

The axle load shall be distributed to the top slab for determining moment, thrust, and shear as follows:

Perpendicular to the span:

$$E = 96 + 1.44S \tag{4.6.2.10.2-1}$$

Parallel to the span:

$$E_{span} = L_T + LLDF(H) \tag{4.6.2.10.2-2}$$

C4.6.2.10.2

Culverts are designed under the provisions of Section 12. Box culverts are normally analyzed as two-dimensional frames. Equivalent strip widths are used to simplify the analysis of the three-dimensional response to live loads. Eqs. 4.6.2.10.2-1 and 4.6.2.10.2-2 are based on research (McGrath et al., 2004) that investigated the forces in box culverts with spans up to 24.0 ft.

The distribution widths are based on distribution of shear forces. Distribution widths for positive and negative moments are wider; however, using the narrower width in combination with a single lane multiple presence factor provides designs adequate for multiple loaded lanes for all force effects.

Although past practice has been to ignore the distribution of live load with depth of fill, consideration of this effect, as presented in Eq. 4.6.2.10.2-2, produces a more accurate model of the changes in design forces with increasing depth of fill. The increased load length parallel to the span, as allowed by Eq. 4.6.2.10.2-2, may be conservatively neglected in design.

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where:

E = equivalent distribution width perpendicular to span (in.)
S = clear span (ft)
E_{span} = equivalent distribution length parallel to span (in.)
L_T = length of tire contact area parallel to span, as specified in Article 3.6.1.2.5 (in.)
$LLDF$ = factor for distribution of live load with depth of fill as specified in Article 3.6.1.2.6
H = depth of fill from top of culvert to top of pavement (in.)

4.6.2.10.3—Case 2: Traffic Travels Perpendicular to Span

When traffic travels perpendicular to the span, live load shall be distributed to the top slab using the equations specified in Article 4.6.2.1 for concrete decks with primary strips perpendicular to the direction of traffic.

C4.6.2.10.3

Culverts with traffic traveling perpendicular to the span can have two or more trucks on the same design strip at the same time. This must be considered, with the appropriate multiple presence factor, in analysis of the culvert structural response.

4.6.2.10.4—Precast Box Culverts

For precast box culverts with top slabs having span-to-thickness ratios (*s*/*t*) of 18 or less and segment lengths greater than or equal to 4 ft in length, shear transfer across the joint need not be provided.

For precast box culverts not satisfying the requirements noted above, the design shall incorporate one of the following:

- Provide the culvert with a means of shear transfer between the adjacent sections. Shear transfer may be provided by pavement, soil fill, or a physical connection between adjacent sections.
- Design the section ends as edge beams in accordance with the provisions of Article 4.6.2.1.4b using the distribution width computed from Eq. 4.6.2.10.2-1. The distribution width shall not exceed the length between two adjacent joints.

C4.6.2.10.4

Precast box culverts manufactured in accordance with AASHTO M 273 are often installed with joints that do not provide a means of direct shear transfer across the joints of adjacent sections under service load conditions. This practice is based on research (James, 1984; Frederick, et al., 1988) which indicated significant shear transfer may not be necessary under service loading. The response of the sections tested was typified by small deflections and strains indicating that cracking did not occur under service wheel loads with no earth cover and that the demand on the section was lower than predicted by the design, which was based conservatively on a cracked section. While there are no known service issues with installation of standard box sections without means of shear transfer across joints, analysis (McGrath et al., 2004) shows that stresses are substantially higher when a box culvert is subjected to a live load at a free edge than when loaded away from a free edge.

However, research performed on precast box culverts that were loaded at the edge of the section (Garg et al., 2007; Abolmaali and Garg, 2008a; Abolmaali and Garg, 2008b) has shown that no means of load transfer across the joint is required when the live load is distributed per Articles 4.6.2.10.2 and 4.6.2.10.3 and the top slab of the box culvert is designed in accordance with Article 5.7.3. The tested boxes were shown to have significantly more shear strength than predicted by Article 5.7.3.

For box culverts outside of the normal ASTM/AASHTO dimensional requirements, some fill or pavement will likely provide sufficient shear transfer to distribute live load to adjacent box sections without shear keys to avoid higher stresses due to edge loading. Otherwise, for box culverts outside of ASTM/AASHTO dimensional requirements with zero depth of cover, and no pavement, soil, or other means of shear transfer such as shear keys, designers should design the culvert section for the specified reduced distribution widths lacking a more rigorous design method.

4.6.3—Refined Methods of Analysis

4.6.3.1—General

Refined methods, listed in Article 4.4, may be used for the analysis of bridges. In such analyses, consideration shall be given to aspect ratios of elements, positioning and number of nodes, and other features of topology that may affect the accuracy of the analytical solution.

A structurally continuous railing, barrier, or median, acting compositely with the supporting components, may be considered to be structurally active at service and fatigue limit states.

When a refined method of analysis is used, a table of live load distribution coefficients for extreme force effects in each span shall be provided in the contract documents to aid in permit issuance and rating of the bridge.

C4.6.3.1

The number of possible locations for positioning the design vehicular live load will be large when determining the extreme force effect in an element using a refined method of analysis. The following are variable:

- The location of the design lanes when the available deck width contains a fraction of a design lane width;
- Which of the design lanes are actually used;
- The longitudinal location of the design vehicular live load in each lane;
- The longitudinal axle spacing of the design vehicular live load; and
- The transverse location of the design vehicular live load in each lane.

This provision reflects the experimentally observed response of bridges. This source of stiffness has traditionally been neglected but exists and may be included, provided that full composite behavior is assured.

These live load distribution coefficients should be provided for each combination of component and lane.

4.6.3.2—Decks

4.6.3.2.1—General

Unless otherwise specified, flexural and torsional deformation of the deck shall be considered in the analysis but vertical shear deformation may be neglected.

Locations of flexural discontinuity through which shear may be transmitted should be modeled as hinges.

In the analysis of decks that may crack and/or separate along element boundaries when loaded, Poisson's ratio may be neglected. The wheel loads shall

C4.6.3.2.1

In many solid decks, the wheel load carrying contribution of torsion is comparable to that of flexure. Large torsional moments exist in the end zones of skewed girder bridges due to differential deflection. In most deck types, shear stresses are rather low, and their contribution to vertical deflection is not significant. In-plane shear deformations, which gave rise to the concept of effective width for composite bridge decks, should not be neglected.

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be modeled as patch loads distributed over an area, as specified in Article 3.6.1.2.5, taken at the contact surface. This area may be extended by the thickness of the wearing surface, integral or nonintegral, on all four sides. When such extension is utilized, the thickness of the wearing surface shall be reduced for any possible wear at the time of interest. Other extended patch areas may be utilized with the permission of the Owner provided that such extended area is consistent with the assumptions in, and application of, a particular refined method of analysis.

4.6.3.2.2—Isotropic Plate Model

For the purpose of this section, bridge decks that are solid, have uniform or close to uniform depth, and whose stiffness is close to equal in every in-plane direction shall be considered isotropic.

C4.6.3.2.2

Analysis is rather insensitive to small deviations in constant depth, such as those due to superelevation, crown, and haunches. In slightly cracked concrete slabs, even a large difference in the reinforcement ratio will not cause significant changes in load distribution.

The torsional stiffness of the deck may be estimated using Eq. C4.6.2.2.1-1 with b equal to 1.0.

4.6.3.2.3—Orthotropic Plate Model

In orthotropic plate modeling, the flexural rigidity of the elements may be uniformly distributed along the cross-section of the deck. Where the torsional stiffness of the deck is not contributed solely by a solid plate of uniform thickness, the torsional rigidity should be established by physical testing, three-dimensional analysis, or generally accepted and verified approximations.

C4.6.3.2.3

The accuracy of the orthotropic plate analysis is sharply reduced for systems consisting of a small number of elements subjected to concentrated loads.

4.6.3.2.4—Refined Orthotropic Deck Model

Refined analysis of orthotropic deck structures subjected to direct wheel loads should be accomplished using a detailed three-dimensional shell or solid finite element structural model. The structural model should include all components and connections and consider local structural stress at fatigue prone details as shown in Table 6.6.1.2.3-1. Structural modeling techniques that utilize the following simplifying assumptions may be applied:

- linear elastic material behavior;
- small deflection theory;
- plane sections remain plane;
- neglect residual stresses; and
- neglect imperfections and weld geometry.

Meshing shall be sufficiently detailed to calculate local stresses at weld toes and to resolve the wheel patch pressure loading with reasonable accuracy.

C4.6.3.2.4

Further guidance on evaluating local structural stresses using finite element modeling is provided in *Manual for Design, Construction, and Maintenance of Orthotropic Steel Bridges* (FHWA, 2012).

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4.6.3.3—Beam-Slab Bridges

4.6.3.3.1—General

The aspect ratio of finite elements and grid panels should not exceed 5.0. Abrupt changes in size and/or shape of finite elements and grid panels should be avoided.

Nodal loads shall be statically equivalent to the actual loads being applied.

C4.6.3.3.1

More restrictive limits for aspect ratio may be specified for the software used.

In the absence of other information, the following guidelines may be used at the discretion of the Engineer:

- A minimum of five, and preferably nine, nodes per beam span should be used.
- For finite element analyses involving plate and beam elements, it is preferable to maintain the relative vertical distances between various elements. If this is not possible, longitudinal and transverse elements may be positioned at the midthickness of the plate-bending elements, provided that the eccentricities are included in the equivalent properties of those sections that are composite.
- For grid analysis or finite element and finite difference analyses of live load, the slab shall be assumed to be effective for stiffness in both positive and negative flexure. In a filled or partially filled grid system, composite section properties should be used.
- In finite element analysis, an element should have membrane capability with discretization sufficient to properly account for shear lag. The force effects so computed should be applied to the appropriate composite or noncomposite section for computing resistance.
- For longitudinal composite members in grid analyses, stiffness should be computed by assuming a width of the slab to be effective, but it need not be less than that specified in Article 4.6.2.6.
- The St. Venant torsional inertia may be determined using the appropriate equation from Article C4.6.2.2.1. Transformation of concrete and steel to a common material should be on the basis of shear modulus, G, which can be taken as $G = 0.5E/(1+\mu)$. It is recommended that the St. Venant rigidity of composite sections utilize only one-half of the effective width of the flexural section, as described above, before transformation. For the analysis of composite loading conditions using plate and eccentric beam structural analysis models, the St. Venant torsional inertia of steel I-girders should be calculated using Eq. C4.6.2.2.1-1 without the consideration of any torsional interaction with the composite deck.

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4.6.3.3.2—Grid and Plate and Eccentric Beam Analyses of Curved and/or Skewed Steel I-Girder Bridges

For the analysis of curved and/or skewed steel I-girder bridges where either $I_C > 1$ or $I_S > 0.3$, the warping rigidity of the I-girders shall be considered in grid and in plate and eccentric beam methods of structural analysis,

in which:

$$I_C = \frac{15{,}000}{R(n_{cf}+1)m} \quad (4.6.3.3.2\text{-}1)$$

$$I_S = \frac{w_g \tan\theta}{L_s} \quad (4.6.3.3.2\text{-}2)$$

where:

I_C = I-girder bridge connectivity index

m = bridge type constant, equal to 1 for simple-span bridges or bridge units, and equal to 2 for continuous-span bridges or bridge units, determined at the loading condition under consideration

n_{cf} = minimum number of intermediate cross-frames or diaphragms within the individual spans of the bridge or bridge unit at the loading condition under consideration

R = minimum radius of curvature at the centerline of the bridge cross-section throughout the length of the bridge or bridge unit at the loading condition under consideration

I_S = bridge skew index, taken equal to the maximum of the values of Eq. 4.6.3.3.2-2 determined for each span of the bridge

L_s = span length at the centerline (ft)

w_g = maximum width between the girders on the outside of the bridge cross-section at the completion of the construction or at an intermediate stage of the steel erection (ft)

θ = maximum skew angle of the bearing lines at the end of a given span, measured from a line taken perpendicular to the span centerline (degrees)

C4.6.3.3.2

Unless otherwise stated, this Article applies to curved and/or skewed steel I-girder bridges analyzed by grid or plate and eccentric beam analysis. In a grid analysis or a plate and eccentric beam analysis of a steel I-girder bridge, the use of only the St. Venant torsional stiffness, GJ/L_b, can result in a substantial underestimation of the girder torsional stiffness. This is due to neglect of the contribution of warping rigidity to the overall girder torsional stiffness. When the contribution from the girder warping rigidity is not accounted for in the analysis, the vertical deflections in curved I-girder systems can be substantially overestimated due to the coupling between the girder torsional and flexural response where $I_C > 1$. Furthermore, the cross-frame forces can be substantially underestimated in straight or curved skewed I-girder bridges due to the underestimation of the torsional stiffness provided by the girders where $I_S > 0.3$.

White et al. (2012) present an approximate method of considering the girder warping rigidity, applicable for I-girder bridges or bridge units in their final constructed condition as well as for intermediate noncomposite conditions during steel erection. For the analysis of composite loading conditions using plate and eccentric beam structural analysis models, it is sufficient to calculate the warping rigidity of the I-girders, EC_w, using solely the steel cross-section with Eq. C6.9.4.1.3-1 and without the consideration of any composite torsional interaction with the composite deck.

Other methods of considering the warping rigidity of steel I-girders include the explicit use of open-section thin-walled beam theory or the use of a general-purpose 3D finite element analysis in which the I-girder is modeled as described previously. Additional information on the modeling of torsion in I-girder bridges may be found in AASHTO/NSBA (2014).

4.6.3.3.3—Curved Steel Bridges

Refined analysis methods should be used for the analysis of curved steel bridges unless the Engineer ascertains that approximate analysis methods are appropriate according to the provisions of Article 4.6.2.2.4.

C4.6.3.3.3

Refined analysis methods, identified in Article 4.4, are generally computer-based. The finite strip and finite element methods have been the most common. The finite strip method is less rigorous than the finite element method and has fallen into disuse with the advent of more powerful computers. Finite element programs may provide grid analyses using a series of beam elements connected in a plane. Refinements of the grid model may include offset elements. Frequently, the torsional warping degree of freedom is not available in beam elements. The finite element method may be applied to a three-dimensional model of the superstructure. A variety of elements may be used in this type of model. The three-dimensional model may be made capable of recognizing warping torsion by modeling each girder cross-section with a series of elements.

The stiffness of supports, including lateral restraint such as integral abutments or integral piers, should be recognized in the analysis. Since bearing restraint is offset from the neutral axis of the girders, large lateral forces at the bearings often occur and may create significant bending in the girders, which may lead to lower girder moments than would be computed if the restraints were not present. The Engineer should ascertain that any such benefit recognized in the design will be present throughout the useful life of the bridge.

Loads may be applied directly to the structural model, or applied to influence lines or influence surfaces. Only where small-deflection elastic solutions are used are influence surfaces or influence lines appropriate. The Engineer should ascertain that dead loads are applied as accurately as possible.

4.6.3.3.4—Cross-Frames and Diaphragms

When modeling a cross-frame with a single line of equivalent beam elements, both the cross-frame flexure and shear deformation shall be considered in determining the equivalent beam element stiffness.

The influence of end connection eccentricities shall be considered in the calculation of the equivalent axial stiffness of single-angle and flange-connected T-section cross-frame members.

C4.6.3.3.4

Due to their predominant action as trusses, cross-frames generally exhibit substantial beam shear deformations when modeled using equivalent beam elements in a structural analysis. The modeling of cross-frames using Euler-Bernoulli beam elements, which neglect beam shear deformation, typically results in substantial misrepresentation of their physical stiffness properties. Timoshenko beam elements, or other types of beam elements that include explicit modeling of beam shear deformations, provide a significantly improved approximation of the cross-frame stiffnesses (White et al., 2012).

In addition, the axial rigidity of single-angle members and flange-connected T-section cross-frame members is reduced due to end connection eccentricities (Wang et al., 2012). In lieu of a more accurate analysis, $(AE)_{eq}$ of equal leg single angles, unequal leg single angles connected to the long leg, and flange-connected T-section members may be taken as $0.65AE$.

For bridges with widely spaced cross-frames or diaphragms, it may be desirable to use notional transverse beam members to model the deck when using grid analysis methods. The number of such beams is to some extent discretionary. The significance of shear lag in the transverse beam-slab width as it relates to lateral load distribution can be evaluated qualitatively by varying the stiffness of the beam-slab elements within reasonable limits and observing the results. Such a sensitivity study often shows this effect is not significant.

Live load force effects in cross-frames and diaphragms should be calculated by grid or finite

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element analysis. The easiest way to establish extreme force effects is by using influence surfaces analogous to those developed for the main longitudinal members.

For bridges with widely spaced diaphragms, it may be desirable to use notional transverse beam members to model the deck. The number of such beams is to some extent discretionary. The significance of shear lag in the transverse beam-slab width as it relates to lateral load distribution can be evaluated qualitatively by varying the stiffness of the beam-slab elements within reasonable limits and observing the results. Such a sensitivity study often shows that this effect is not significant.

Live load force effects in diaphragms should be calculated by grid or finite element analysis. The easiest way to establish extreme force effects is by using influence surfaces analogous to those developed for the main longitudinal members.

4.6.3.4—Cellular and Box Bridges

A refined analysis of cellular bridges may be made by any of the analytic methods specified in Article 4.4, except the yield line method, which accounts for the two dimensions seen in plan view and for the modeling of boundary conditions. Models intended to quantify torsional warping and/or transverse frame action should be fully three-dimensional.

For single box cross-sections, the superstructure may be analyzed as a spine beam for both flexural and torsional effects. A steel box should not be considered to be torsionally rigid unless internal bracing is provided to maintain the box cross-section. The transverse position of bearings shall be modeled.

4.6.3.5—Truss Bridges

A refined plane frame or space frame analysis shall include consideration for the following:

- composite action with the deck or deck system;
- continuity among the components;
- force effects due to self-weight of components, change in geometry due to deformation, and axial offset at panel points; and
- in-plane and out-of-plane buckling of components including original out-of-straightness, continuity among the components, and the effect axial forces present in those components.

Out-of-plane buckling of the upper chords of pony truss bridges shall be investigated. If the truss derives its lateral stability from transverse frames, of which the floorbeams are a part, the deformation of the floorbeams due to vehicular loading shall be considered.

C4.6.3.5

Load applied to deck or floorbeams instead of to truss joints will yield results that more completely quantify out-of-plane actions.

Experience has shown that dead load force effects calculated using either plane frame or space frame analysis in a truss with properly cambered primary and secondary members and detailed to minimize eccentricity at joints, will be quite close to those calculated by the conventional approximations. In many cases, a complete three-dimensional frame analysis may be the only way to accurately calculate forces in secondary members, particularly live load force effects.

4.6.3.6—Arch Bridges

The provisions of Article 4.6.3.5 shall apply where applicable.

The effect of the extension of cable hangers shall be considered in the analysis of an arch tie.

Where not controlled through proper detailing, rib shortening should be investigated.

The use of large deflection analysis of arches of longer spans should be considered in lieu of the moment magnification correction as specified in Article 4.5.3.2.2c.

When the distribution of stresses between the top and bottom chords of trussed arches is dependent on the manner of erection, the manner of erection shall be indicated in the contract documents.

C4.6.3.6

Rib shortening and arch design and construction are discussed by Nettleton (1977).

Any single-step correction factor cannot be expected to accurately model deflection effects over a wide range of stiffnesses.

If a hinge is provided at the crown of the rib in addition to hinges at the abutment, the arch becomes statically determinate, and stresses due to change of temperature and rib shortening are essentially eliminated.

Arches may be analyzed, designed, and constructed as hinged under dead load or portions of dead load and as fixed at some hinged locations for the remaining design loads.

In trussed arches, considerable latitude is available in design for distribution of stresses between the top and bottom chords dependent on the manner of erection. In such cases, the manner of erection should be indicated in the contract documents.

4.6.3.7—Cable-Stayed Bridges

The distribution of force effects to the components of a cable-stayed bridge may be determined by either spatial or planar structural analysis if justified by consideration of tower geometry, number of planes of stays, and the torsional stiffness of the deck superstructure.

Cable-stayed bridges shall be investigated for nonlinear effects that may result from:

- the change in cable sag at all limit states,
- deformation of deck superstructure and towers at all limit states, and
- material nonlinearity at the extreme event limit states.

Cable sag may be investigated using an equivalent member modeled as a chord with modified modulus of elasticity given by Eq. 4.6.3.7-1 for instantaneous stiffness and Eq. 4.6.3.7-2, applied iteratively, for changing cable loads.

$$E_{MOD} = E\left[1+\frac{EAW^2(\cos\alpha)^5}{12H^3}\right]^{-1} \tag{4.6.3.7-1}$$

$$E_{MOD} = E\left[1+\frac{(H_1+H_2)EAW^2\left(\cos\alpha\right)^5}{24H_1^2H_2^2}\right]^{-1} \tag{4.6.3.7-2}$$

where:

C4.6.3.7

Nonlinear effects on cable-stayed bridges are treated in several texts, e.g., (Podolny and Scalzi, 1986; Troitsky, 1977), and a report by the ASCE Committee on Cable Suspended Bridges (ASCE, 1991), from which the particular forms of Eqs. 4.6.3.7-1 and 4.6.3.7-2 were taken.

E = modulus of elasticity of the cable (ksi)
W = total weight of cable (kip)
A = cross-sectional area of cable (in.^{2})
α = angle between cable and horizontal (degrees)
H, H_1, H_2 = horizontal component of cable force (kip)

The change in force effects due to deflection may be investigated using any method that satisfies the provisions of Article 4.5.3.2.1 and accounts for the change in orientation of the ends of cable stays.

Cable-stayed bridges shall be investigated for the loss of any one cable stay.

4.6.3.8—Suspension Bridges

Force effects in suspension bridges shall be analyzed by the large deflection theory for vertical loads. The effects of wind loads shall be analyzed, with consideration of the tension stiffening of the cables. The torsional rigidity of the deck may be neglected in assigning forces to cables, suspenders, and components of stiffening trusses.

C4.6.3.8

In the past, short suspension bridges have been analyzed by conventional small deflection theories. Correction factor methods have been used on short- to moderate-span bridges to account for the effect of deflection, which is especially significant for calculating deck system moments. Any contemporary suspension bridge would have a span such that the large deflection theory should be used. Suitable computer programs are commercially available. Therefore, there is little rationale to use anything other than the large deflection solution.

For the same economic reasons, the span would probably be long enough that the influence of the torsional rigidity of the deck, combined with the relatively small effect of live load compared to dead load, will make the simple sum-of-moments technique suitable to assign loads to the cables and suspenders and usually even to the deck system, e.g., a stiffening truss.

4.6.4—Redistribution of Negative Moments in Continuous Beam Bridges

4.6.4.1—General

The Owner may permit the redistribution of force effects in multispan, multibeam, or girder superstructures. Inelastic behavior shall be restricted to the flexure of beams or girders, and inelastic behavior due to shear and/or uncontrolled buckling shall not be permitted. Redistribution of loads shall not be considered in the transverse direction.

The reduction of negative moments over the internal supports due to the redistribution shall be accompanied by a commensurate increase in the positive moments in the spans.

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4.6.4.2—Refined Method

The negative moments over the support, as established by linear elastic analysis, may be decreased by a redistribution process considering the moment–rotation characteristics of the cross-section or by a recognized mechanism method. The moment–rotation relationship shall be established using material characteristics, as specified herein, and/or verified by physical testing.

4.6.4.3—Approximate Procedure

In lieu of the analysis described in Article 4.6.4.2, simplified redistribution procedures for concrete and steel beams, as specified in Sections 5 and 6, respectively, may be used.

4.6.5—Stability

The investigation of stability shall utilize the large deflection theory.

4.6.6—Analysis for Temperature Gradient

Where determination of force effects due to vertical temperature gradient is required, the analysis should consider axial extension, flexural deformation, and internal stresses.

Gradients shall be as specified in Article 3.12.3.

C4.6.6

The response of a structure to a temperature gradient can be divided into three effects as follows:

- *Axial Expansion*—This is due to the uniform component of the temperature distribution that should be considered simultaneously with the uniform temperature specified in Article 3.12.2. It may be calculated as:

$$T_{UG} = \frac{1}{A_c} \iint T_G \, dw \, dz \tag{C4.6.6-1}$$

The corresponding uniform axial strain is:

$$\varepsilon_u = \alpha \left(T_{UG} + T_u \right) \tag{C4.6.6-2}$$

- *Flexural Deformation*—Because plane sections remain plane, a curvature is imposed on the superstructure to accommodate the linearly variable component of the temperature gradient. The rotation per unit length corresponding to this curvature may be determined as:

$$\phi = \frac{\alpha}{I_c} \iint T_G z \, dw \, dz = \frac{1}{R} \tag{C4.6.6-3}$$

If the structure is externally unrestrained, i.e., simply supported or cantilevered, no external force effects are developed due to this superimposed deformation.

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The axial strain and curvature may be used in both flexibility and stiffness formulations. In the former, ε_u may be used in place of P/AE, and ϕ may be used in place of M/EI in traditional displacement calculations. In the latter, the fixed-end force effects for a prismatic frame element may be determined as:

$$N = EA_c\varepsilon_u \qquad \text{(C4.6.6-4)}$$

$$M = EI_c\phi \qquad \text{(C4.6.6-5)}$$

An expanded discussion with examples may be found in Ghali and Neville (1989). Strains induced by other effects, such as shrinkage and creep, may be treated in a similar manner.

- *Internal Stress*—Using the sign convention that compression is positive, internal stresses in addition to those corresponding to the restrained axial expansion and/or rotation may be calculated as:

$$\sigma_E = E\left[\alpha T_G - \alpha T_{UG} - \phi z\right] \qquad \text{(C4.6.6-6)}$$

where:

T_G = temperature gradient (Δ°F)
T_{UG} = temperature averaged across the cross-section (°F)
T_u = uniform specified temperature (°F)
A_c = cross-section area—transformed for steel beams (in.^{2})
I_c = inertia of cross-section—transformed for steel beams (in.^{4})
α = coefficient of thermal expansion (in./in./°F)
E = modulus of elasticity (ksi)
R = radius of curvature (ft)
w = width of element in cross-section (in.)
z = vertical distance from center of gravity of cross-section (in.)

For example, the flexural deformation part of the gradient flexes a prismatic superstructure into a segment of a circle in the vertical plane. For a two-span structure with span length, L, in ft, the unrestrained beam would lift off from the central support by $\Delta = 6\ L^2/R$ in in. Forcing the beam down to eliminate Δ would develop a moment whose value at the pier would be:

$$M_c = \frac{3}{2}EI_c\phi \qquad \text{(C4.6.6-7)}$$

Therefore, the moment is a function of the beam rigidity and imposed flexure. As rigidity approaches 0.0 at the strength limit state, M_c tends to disappear. This behavior also indicates the need for ductility to ensure structural integrity as rigidity decreases.

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4.7—DYNAMIC ANALYSIS

4.7.1—Basic Requirements of Structural Dynamics

4.7.1.1—General

For analysis of the dynamic behavior of bridges, the stiffness, mass, and damping characteristics of the structural components shall be modeled.

The minimum number of degrees-of-freedom included in the analysis shall be based upon the number of natural frequencies to be obtained and the reliability of the assumed mode shapes. The model shall be compatible with the accuracy of the solution method. Dynamic models shall include relevant aspects of the structure and the excitation. The relevant aspects of the structure may include the:

- distribution of mass;
- distribution of stiffness; and
- damping characteristics.

The relevant aspects of excitation may include the:

- frequency of the forcing function;
- duration of application; and
- direction of application.

C4.7.1.1

Typically, analysis for vehicle- and wind-induced vibrations is not to be considered in bridge design. Although a vehicle crossing a bridge is not a static situation, the bridge is analyzed by statically placing the vehicle at various locations along the bridge and applying a dynamic load allowance, as specified in Article 3.6.2, to account for the dynamic responses caused by the moving vehicle. However, in flexible bridges and long slender components of bridges that may be excited by bridge movement, dynamic force effects may exceed the allowance for impact given in Article 3.6.2. In most observed bridge vibration problems, the natural structural damping has been very low. Flexible continuous bridges may be especially susceptible to vibrations. These cases may require analysis for moving live load.

If the number of degrees-of-freedom in the model exceeds the number of dynamic degrees-of-freedom used, a standard condensation procedure may be employed.

Condensation procedures may be used to reduce the number of degrees-of-freedom prior to the dynamic analysis. Accuracy of the higher modes can be compromised with condensation. Thus if higher modes are required, such procedures should be used with caution.

The number of frequencies and mode shapes necessary to complete a dynamic analysis should be estimated in advance or determined as an early step in a multistep approach. Having determined that number, the model should be developed to have a larger number of applicable degrees-of-freedom.

Sufficient degrees-of-freedom should be included to represent the mode shapes relevant to the response sought. One rule-of-thumb is that there should be twice as many degrees-of-freedom as required frequencies.

The number of degrees-of-freedom and the associated masses should be selected in a manner that approximates the actual distributive nature of mass. The number of required frequencies also depends on the frequency content of the forcing function.

4.7.1.2—Distribution of Masses

The modeling of mass shall be made with consideration of the degree of discretization in the model and the anticipated motions.

C4.7.1.2

The distribution of stiffness and mass should be modeled in a dynamic analysis. The discretization of the model should account for geometric and material variation in stiffness and mass.

The selection of the consistent or lump mass formulation is a function of the system and the response sought and is difficult to generalize. For distributive mass systems modeled with polynomial shape functions in which the mass is associated with distributive stiffness, such as a beam, a consistent mass formulation

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is recommended (Paz, 1985). In lieu of a consistent formulation, lumped masses may be associated at the translational degrees-of-freedom, a manner that approximates the distributive nature of the mass (Clough and Penzian, 1975).

For systems with distributive mass associated with larger stiffness, such as in-plane stiffness of a bridge deck, the mass may be properly modeled as lumped. The rotational inertia effects should be included where significant.

4.7.1.3—Stiffness

The bridge shall be modeled to be consistent with the degrees-of-freedom chosen to represent the natural modes and frequencies of vibration. The stiffness of the elements of the model shall be defined to be consistent with the bridge being modeled.

C4.7.1.3

In seismic analysis, nonlinear effects which decrease stiffness, such as inelastic deformation and cracking, should be considered.

Reinforced concrete columns and walls in Seismic Zones 2, 3, and 4 should be analyzed using cracked section properties. For this purpose, a moment of inertia equal to one-half that of the uncracked section may be used.

4.7.1.4—Damping

Equivalent viscous damping may be used to represent energy dissipation.

C4.7.1.4

Damping may be neglected in the calculation of natural frequencies and associated nodal displacements. The effects of damping should be considered where a transient response is sought.

Suitable damping values may be obtained from field measurement of induced free vibration or by forced vibration tests. In lieu of measurements, the following values may be used for the equivalent viscous damping ratio:

- Concrete construction: two percent
- Welded and bolted steel construction: one percent
- Timber: five percent

4.7.1.5—Natural Frequencies

For the purpose of Article 4.7.2, and unless otherwise specified by the Owner, elastic undamped natural modes and frequencies of vibration shall be used. For the purpose of Articles 4.7.4 and 4.7.5, all relevant damped modes and frequencies shall be considered.

4.7.2—Elastic Dynamic Responses

4.7.2.1—Vehicle-Induced Vibration

When an analysis for dynamic interaction between a bridge and the live load is required, the Owner shall specify and/or approve surface roughness, speed, and dynamic characteristics of the vehicles to be employed for the analysis. Impact shall be derived as a ratio of the extreme dynamic force effect to the corresponding static force effect.

C4.7.2.1

In no case shall the dynamic load allowance used in design be less than 50 percent of the dynamic load allowance specified in Table 3.6.2.1-1, except that no reduction shall be allowed for deck joints.

The limitation on the dynamic load allowance reflects the fact that deck surface roughness is a major factor in vehicle/bridge interaction and that it is difficult to estimate long-term deck deterioration effects thereof at the design stage.

The proper application of the provision for reducing the dynamic load allowance is:

$$IM_{CALC} \geq 0.5 IM_{Table\ 3\text{-}6} \qquad \text{(C4.7.2.1-1)}$$

not:

$$\left(1+\frac{IM}{100}\right)_{CALC} \geq 0.5\left(1+\frac{IM}{100}\right) \qquad \text{(C4.7.2.1-2)}$$

4.7.2.2—Wind-Induced Vibration

4.7.2.2.1—Wind Velocities

For critical or essential structures which may be expected to be sensitive to wind effects, the location and magnitude of extreme pressure and suction values shall be established by simulated wind tunnel tests.

4.7.2.2.2—Dynamic Effects

Wind-sensitive structures shall be analyzed for dynamic effects, such as buffeting by turbulent or gusting winds, and unstable wind-structure interaction, such as galloping and flutter. Slender or torsionally flexible structures shall be analyzed for lateral buckling, excessive thrust, and divergence.

4.7.2.2.3—Design Considerations

Oscillatory deformations under wind that may lead to excessive stress levels, structural fatigue, and user inconvenience or discomfort shall be avoided. Bridge decks, cable stays, and hanger cables shall be protected against excessive vortex and wind-rain-induced oscillations. Where practical, the employment of dampers shall be considered to control excessive dynamic responses. Where dampers or shape modification are not practical, the structural system shall be changed to achieve such control.

C4.7.2.2.3

Additional information on design for wind may be found in AASHTO (1985); Scanlan (1975); Simiu and Scanlan (1978); Basu and Chi (1981a); Basu and Chi (1981b); ASCE (1961); and ASCE (1991).

4.7.3—Inelastic Dynamic Responses

4.7.3.1—General

During a major earthquake or ship collision, energy may be dissipated by one or more of the following mechanisms:

- elastic and inelastic deformation of the object that may collide with the structure;
- inelastic deformation of the structure and its attachments;

- permanent displacement of the masses of the structure and its attachments; and
- inelastic deformation of special-purpose mechanical energy dissipators.

4.7.3.2—Plastic Hinges and Yield Lines

For the purpose of analysis, energy absorbed by inelastic deformation in a structural component may be assumed to be concentrated in plastic hinges and yield lines. The location of these sections may be established by successive approximation to obtain a lower bound solution for the energy absorbed. For these sections, moment–rotation hysteresis curves may be determined by using verified analytic material models.

4.7.4—Analysis for Earthquake Loads

4.7.4.1—General

Minimum analysis requirements for seismic effects shall be as specified in Table 4.7.4.3.1-1.

For the modal methods of analysis, specified in Articles 4.7.4.3.2 and 4.7.4.3.3, the design response spectrum specified in Figure 3.10.4.1-1 and Eqs. 3.10.4.2-1, 3.10.4.2-3, and 3.10.4.2-4 shall be used.

Bridges in Seismic Zone 1 need not be analyzed for seismic loads, regardless of their operational classification and geometry. However, the minimum requirements specified in Articles 4.7.4.4 and 3.10.9 shall apply.

4.7.4.2—Single-Span Bridges

Seismic analysis is not required for single-span bridges, regardless of seismic zone.

Connections between the bridge superstructure and the abutments shall be designed for the minimum force requirements as specified in Article 3.10.9.

Minimum support length requirements shall be satisfied at each abutment as specified in Article 4.7.4.4.

C4.7.4.2

A single-span bridge is comprised of a superstructure unit supported by two abutments with no intermediate piers.

4.7.4.3—Multispan Bridges

4.7.4.3.1—Selection of Method

For multispan structures, the minimum analysis requirements shall be as specified in Table 4.7.4.3.1-1 in which:

*	=	no seismic analysis required
UL	=	uniform load elastic method
SM	=	single-mode elastic method
MM	=	multimode elastic method
TH	=	time-history method

C4.7.4.3.1

The selection of the method of analysis depends on seismic zone, regularity, and operational classification of the bridge.

Regularity is a function of the number of spans and the distribution of weight and stiffness. Regular bridges have less than seven spans; no abrupt or unusual changes in weight, stiffness, or geometry; and no large changes in these parameters from span to span or support to support, abutments excluded. A more rigorous analysis procedure may be used in lieu of the recommended minimum.

Table 4.7.4.3.1-1—Minimum Analysis Requirements for Seismic Effects

Seismic Zone	Single-Span Bridges	Multispan Bridges					
		Other Bridges		Essential Bridges		Critical Bridges	
		regular	irregular	regular	irregular	regular	irregular
1	No seismic analysis required	*	*	*	*	*	*
2		SM/UL	SM	SM/UL	MM	MM	MM
3		SM/UL	MM	MM	MM	MM	TH
4		SM/UL	MM	MM	MM	TH	TH

Except as specified below, bridges satisfying the requirements of Table 4.7.4.3.1-2 may be taken as "regular" bridges. Bridges not satisfying the requirements of Table 4.7.4.3.1-2 shall be taken as "irregular" bridges.

Table 4.7.4.3.1-2—Regular Bridge Requirements

Parameter	Value				
Number of spans	2	3	4	5	6
Maximum subtended angle for a curved bridge	90°	90°	90°	90°	90°
Maximum span length ratio from span to span	3	2	2	1.5	1.5
Maximum bent/pier stiffness ratio from span to span, excluding abutments	—	4	4	3	2

Curved bridges comprised of multiple simple spans shall be considered to be "irregular" if the subtended angle in plan is greater than 20 degrees. Such bridges shall be analyzed by either the multimode elastic method or the time-history method.

A curved continuous-girder bridge may be analyzed as if it were straight, provided all of the following requirements are satisfied:

- The bridge is "regular" as defined in Table 4.7.4.3.1-2, except that for a two-span bridge the maximum span length ratio from span to span must not exceed 2;
- The subtended angle in plan is not greater than 90 degrees; and
- The span lengths of the equivalent straight bridge are equal to the arc lengths of the curved bridge.

If these requirements are not satisfied, then curved continuous-girder bridges must be analyzed using the actual curved geometry.

4.7.4.3.2—Single-Mode Methods of Analysis

4.7.4.3.2a—General

Either of the two single-mode methods of analysis specified herein may be used where appropriate.

4.7.4.3.2b—Single-Mode Spectral Method

The single-mode method of spectral analysis shall be based on the fundamental mode of vibration in either

C4.7.4.3.2b

The single-mode spectral analysis method described in the following steps may be used for both transverse the longitudinal or transverse direction. For regular bridges, the fundamental modes of vibration in the horizontal plane coincide with the longitudinal and transverse axes of the bridge structure. This mode shape may be found by applying a uniform horizontal load to the structure and calculating the corresponding deformed shape. The natural period may be calculated by equating the maximum potential and kinetic energies associated with the fundamental mode shape. The amplitude of the displaced shape may be found from the elastic seismic response coefficient, C_{sm}, specified in Article 3.10.4.2, and the corresponding spectral displacement. This amplitude shall be used to determine force effects.

and longitudinal earthquake motions. Examples illustrating its application are given in AASHTO (1983) and ATC (1981).

- Calculate the static displacements, $v_s(x)$, due to an assumed uniform loading, p_o, as shown in Figure C4.7.4.3.2b-1:

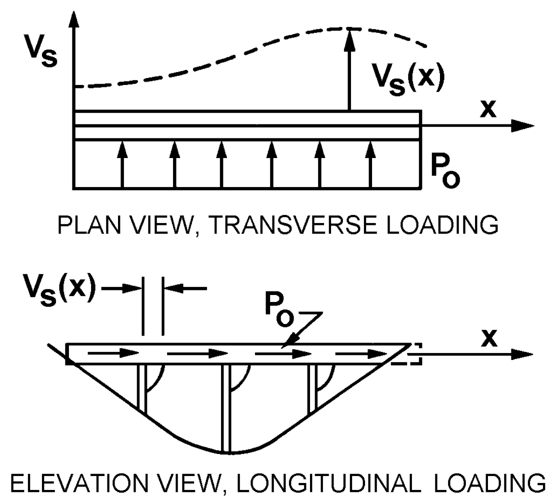

Figure C4.7.4.3.2b-1—Bridge Deck Subjected to Assumed Transverse and Longitudinal Loading

- Calculate factors α, β, and γ as:

$$\alpha = \int v_s(x)\,dx \qquad \text{(C4.7.4.3.2b-1)}$$

$$\beta = \int w(x)\,v_s(x)\,dx \qquad \text{(C4.7.4.3.2b-2)}$$

$$\gamma = \int w(x)\,v_s^2(x)\,dx \qquad \text{(C4.7.4.3.2b-3)}$$

where:

p_o = a uniform load arbitrarily set equal to 1.0 (kip/ft)
$v_s(x)$ = deformation corresponding to p_o (ft)
$w(x)$ = nominal, unfactored dead load of the bridge superstructure and tributary substructure (kip/ft)

The computed factors, α, β, and γ, have units of (ft^2), (kip-ft), and (kip-ft^2), respectively.

- Calculate the period of the bridge as:

$$T_m = 2\pi\sqrt{\frac{\gamma}{p_o g \alpha}} \quad \text{(C4.7.4.3.2b-4)}$$

where:

g = acceleration of gravity (ft/sec.^{2})

- Using T_m and Eqs. 3.10.4.2-1, 3.10.4.2-4, or 3.10.4.2-5, calculate C_{sm}.
- Calculate the equivalent static earthquake loading $p_e(x)$ as:

$$p_e(x) = \frac{\beta C_{sm}}{\gamma} w(x) v_s(x) \quad \text{(C4.7.4.3.2b-5)}$$

where:

C_{sm} = the dimensionless elastic seismic response coefficient given by Eqs. 3.10.4.2-1, 3.10.4.2-4, or 3.10.4.2-5

$p_e(x)$ = the intensity of the equivalent static seismic loading applied to represent the primary mode of vibration (kip/ft)

- Apply loading $p_e(x)$ to the structure, and determine the resulting member force effects.

4.7.4.3.2c—Uniform Load Method

The uniform load method shall be based on the fundamental mode of vibration in either the longitudinal or transverse direction of the base structure. The period of this mode of vibration shall be taken as that of an equivalent single mass-spring oscillator. The stiffness of this equivalent spring shall be calculated using the maximum displacement that occurs when an arbitrary uniform lateral load is applied to the bridge. The elastic seismic response coefficient, C_{sm}, specified in Article 3.10.4.2 shall be used to calculate the equivalent uniform seismic load from which seismic force effects are found.

C4.7.4.3.2c

The uniform load method, described in the following steps, may be used for both transverse and longitudinal earthquake motions. It is essentially an equivalent static method of analysis that uses a uniform lateral load to approximate the effect of seismic loads. The method is suitable for regular bridges that respond principally in their fundamental mode of vibration. Whereas all displacements and most member forces are calculated with good accuracy, the method is known to overestimate the transverse shears at the abutments by up to 100 percent. If such conservatism is undesirable, then the single-mode spectral analysis method specified in Article 4.7.4.3.2b is recommended.

- Calculate the static displacements, $v_s(x)$, due to an assumed uniform load, p_o, as shown in Figure C4.7.4.3.2b-1. The uniform loading, p_o, is applied over the length of the bridge; it has units of force per unit length and may be arbitrarily set equal to 1.0. The static displacement, $v_s(x)$, has units of length.
- Calculate the bridge lateral stiffness, K, and total weight, W, from the following expressions:

$$K = \frac{p_o L}{v_{s,MAX}} \quad \text{(C4.7.4.3.2c-1)}$$

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$$W = \int w(x)dx \qquad \text{(C4.7.4.3.2c-2)}$$

where:

L = total length of the bridge (ft)
$v_{s,MAX}$ = maximum value of $v_s(x)$ (ft)
$w(x)$ = nominal, unfactored dead load of the bridge superstructure and tributary substructure (kip/ft)

The weight should take into account structural elements and other relevant loads including, but not limited to, pier caps, abutments, columns, and footings. Other loads, such as live loads, may be included. Generally, the inertia effects of live loads are not included in the analysis; however, the probability of a large live load being on the bridge during an earthquake should be considered when designing bridges with high live-to-dead load ratios that are located in metropolitan areas where traffic congestion is likely to occur.

- Calculate the period of the bridge, T_m, using the expression:

$$T_m = 2\pi\sqrt{\frac{W}{gK}} \qquad \text{(C4.7.4.3.2c-3)}$$

where:

g = acceleration of gravity (ft/sec.^{2})

- Calculate the equivalent static earthquake loading, p_e, from the expression:

$$p_e = \frac{C_{sm}W}{L} \qquad \text{(C4.7.4.3.2c-4)}$$

where:

C_{sm} = the dimensionless elastic seismic response coefficient given by Eqs. 3.10.4.2-1, 3.10.4.2-4, or 3.10.4.2-5
p_e = equivalent uniform static seismic loading per unit length of bridge applied to represent the primary mode of vibration (kip/ft)

- Calculate the displacements and member forces for use in design either by applying p_e to the structure and performing a second static analysis or by scaling the results of the first step above by the ratio p_e/p_o.

4.7.4.3.3—Multimode Spectral Method

The multimode spectral analysis method shall be used for bridges in which coupling occurs in more than one of the three coordinate directions within each mode

C4.7.4.3.3

of vibration. As a minimum, linear dynamic analysis using a three-dimensional model shall be used to represent the structure.

The number of modes included in the analysis should be at least three times the number of spans in the model. The design seismic response spectrum as specified in Article 3.10.4 shall be used for each mode.

The member forces and displacements may be estimated by combining the respective response quantities (moment, force, displacement, or relative displacement) from the individual modes by the Complete Quadratic Combination (CQC) method.

Member forces and displacements obtained using the CQC combination method are generally adequate for most bridge systems (Wilson et al., 1981).

If the CQC method is not readily available, alternative methods include the square root of the sum of the squares method (SRSS), but this method is best suited for combining responses from well-separated modes. For closely spaced modes, the absolute sum of the modal responses should be used.

4.7.4.3.4—Time-History Method

4.7.4.3.4a—General

Any step-by-step time-history method of analysis used for either elastic or inelastic analysis shall satisfy the requirements of Article 4.7.

The sensitivity of the numerical solution to the size of the time step used for the analysis shall be determined. A sensitivity study shall also be carried out to investigate the effects of variations in assumed material hysteretic properties.

The time histories of input acceleration used to describe the earthquake loads shall be selected in accordance with Article 4.7.4.3.4b.

C4.7.4.3.4

C4.7.4.3.4a

Rigorous methods of analysis are required for critical structures, which are defined in Article 3.10.3, and/or those that are geometrically complex or close to active earthquake faults. Time-history methods of analysis are recommended for this purpose, provided care is taken with both the modeling of the structure and the selection of the input time histories of ground acceleration.

4.7.4.3.4b—Acceleration Time Histories

Developed time histories shall have characteristics that are representative of the seismic environment of the site and the local site conditions.

Response-spectrum-compatible time histories shall be used as developed from representative recorded motions. Analytical techniques used for spectrum matching shall be demonstrated to be capable of achieving seismologically realistic time series that are similar to the time series of the initial time histories selected for spectrum matching.

Where recorded time histories are used, they shall be scaled to the approximate level of the design response spectrum in the period range of significance. Each time history shall be modified to be response-spectrum-compatible using the time-domain procedure.

At least three response-spectrum-compatible time histories shall be used for each component of motion in representing the design earthquake (ground motions having seven percent probability of exceedance in 75 years). All three orthogonal components (x, y, and z) of design motion shall be input simultaneously when

C4.7.4.3.4b

Characteristics of the seismic environment to be considered in selecting time histories include:

- tectonic environment (e.g., subduction zone; shallow crustal faults);
- earthquake magnitude;
- type of faulting (e.g., strike-slip; reverse; normal);
- seismic-source-to-site distance;
- local site conditions; and
- design or expected ground-motion characteristics (e.g., design response spectrum, duration of strong shaking, and special ground motion characteristics such as near-fault characteristics).

Dominant earthquake magnitudes and distances, which contribute principally to the probabilistic design response spectra at a site, as determined from national ground motion maps, can be obtained from deaggregation information on the USGS website: http://geohazards.cr.usgs.gov.

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conducting a nonlinear time-history analysis. The design actions shall be taken as the maximum response calculated for the three ground motions in each principal direction.

If a minimum of seven time histories are used for each component of motion, the design actions may be taken as the mean response calculated for each principal direction.

For near-field sites (D < 6 mi), the recorded horizontal components of motion that are selected should represent a near-field condition and should be transformed into principal components before making them response-spectrum-compatible. The major principal component should then be used to represent motion in the fault-normal direction and the minor principal component should be used to represent motion in the fault-parallel direction.

It is desirable to select time histories that have been recorded under conditions similar to the seismic conditions at the site as listed above, but compromises are usually required because of the multiple attributes of the seismic environment and the limited data bank of recorded time histories. Selection of time histories having similar earthquake magnitudes and distances, within reasonable ranges, are especially important parameters because they have a strong influence on response spectral content, response spectral shape, duration of strong shaking, and near-source ground-motion characteristics. It is desirable that selected recorded motions be somewhat similar in overall ground motion level and spectral shape to the design spectrum to avoid using very large scaling factors with recorded motions and very large changes in spectral content in the spectrum-matching approach. If the site is located within 6 mi of an active fault, then intermediate-to-long-period ground-motion pulses that are characteristic of near-source time histories should be included if these types of ground motion characteristics could significantly influence structural response. Similarly, the high short-period spectral content of near-source vertical ground motions should be considered.

Ground motion modeling methods of strong motion seismology are being increasingly used to supplement the recorded ground motion database. These methods are especially useful for seismic settings for which relatively few actual strong motion recordings are available, such as in the central and eastern United States. Through analytical simulation of the earthquake rupture and wave propagation process, these methods can produce seismologically reasonable time series.

Response spectrum matching approaches include methods in which time series adjustments are made in the time domain (Lilhanand and Tseng, 1988; Abrahamson, 1992) and those in which the adjustments are made in the frequency domain (Gasparini and Vanmarcke, 1976; Silva and Lee, 1987; Bolt and Gregor, 1993). Both of these approaches can be used to modify existing time histories to achieve a close match to the design response spectrum while maintaining fairly well the basic time domain character of the recorded or simulated time histories. To minimize changes to the time domain characteristics, it is desirable that the overall shape of the spectrum of the recorded time history not be greatly different from the shape of the design response spectrum, and that the time history initially be scaled so that its spectrum is at the approximate level of the design spectrum before spectrum matching.

Where three-component sets of time histories are developed by simple scaling rather than spectrum matching, it is difficult to achieve a comparable aggregate match to the design spectra for each component of motion when using a single scaling factor for each time history set. It is desirable, however, to use a single scaling factor to preserve the relationship between the components. Approaches for dealing with this scaling issue include:

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- use of a higher scaling factor to meet the minimum aggregate match requirement for one component while exceeding it for the other two;
- use of a scaling factor to meet the aggregate match for the most critical component with the match somewhat deficient for other components; and
- compromising on the scaling by using different factors as required for different components of a time-history set.

While the second approach is acceptable, it requires careful examination and interpretation of the results and possibly dual analyses for application of the higher horizontal component in each principal horizontal direction.

The requirements for the number of time histories to be used in nonlinear inelastic dynamic analysis and for the interpretation of the results take into account the dependence of response on the time domain character of the time histories (duration, pulse shape, pulse sequencing) in addition to their response spectral content.

Additional guidance on developing acceleration time histories for dynamic analysis may be found in publications by the Caltrans Seismic Advisory Board Adhoc Committee (CSABAC) on Soil-Foundation-Structure Interaction (1999) and the U.S. Army Corps of Engineers (2000). CSABAC (1999) also provides detailed guidance on modeling the spatial variation of ground motion between bridge piers and the conduct of seismic soil-foundation-structure interaction (SFSI) analyses. Both spatial variations of ground motion and SFSI may significantly affect bridge response. Spatial variations include differences between seismic wave arrival times at bridge piers (wave passage effect), ground motion incoherence due to seismic wave scattering, and differential site response due to different soil profiles at different bridge piers. For long bridges, all forms of spatial variations may be important. For short bridges, limited information appears to indicate that wave passage effects and incoherence are, in general, relatively unimportant in comparison to effects of differential site response (Shinozuka et al., 1999; Martin, 1998). Somerville et al. (1999) provide guidance on the characteristics of pulses of ground motion that occur in time histories in the near-fault region.

4.7.4.4—Minimum Support Length Requirements

Support lengths at expansion bearings without restrainers, STUs, or dampers shall either accommodate the greater of the maximum displacement calculated in accordance with the provisions of Article 4.7.4.3, except for bridges in Zone 1, or a percentage of the empirical support length, N, specified by Eq. 4.7.4.4-1. Otherwise, longitudinal restrainers complying with Article 3.10.9.5 shall be provided. Bearings restrained for longitudinal

C4.7.4.4

Support lengths are equal to the length of the overlap between the girder and the seat as shown in Figure C4.7.4.4-1. To satisfy the minimum values for N in this Article, the overall seat width will be larger than N by an amount equal to movements due to prestress shortening, creep, shrinkage, and thermal expansion/contraction. The minimum value for N given in Eq. 4.7.4.4-1 includes an arbitrary allowance for cover concrete at the end of the

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movement shall be designed in compliance with Article 3.10.9. The percentages of N, applicable to each seismic zone, shall be as specified in Table 4.7.4.4-1.

The empirical support length shall be taken as:

$$N = (8 + 0.02L + 0.08H)(1 + 0.000125S^2) \tag{4.7.4.4-1}$$

where:

N = minimum support length measured normal to the centerline of bearing (in.)

L = length of the bridge deck to the adjacent expansion joint, or to the end of the bridge deck; for hinges within a span, L shall be the sum of the distances to either side of the hinge; for single-span bridges, L equals the length of the bridge deck (ft)

H = for abutments, average height of columns supporting the bridge deck from the abutment to the next expansion joint (ft)

for columns and/or piers, column or pier height (ft)

for hinges within a span, average height of the adjacent two columns or piers (ft)

0.0 for single-span bridges (ft)

S = skew of support measured from line normal to span (degrees)

Table 4.7.4.4-1—Percentage *N* by Zone and Acceleration Coefficient A_S, Specified in Eq. 3.10.4.2-2

Zone	Acceleration Coefficient, A_S	Percent, N
1	<0.05	≥75
1	≥0.05	100
2	All Applicable	150
3	All Applicable	150
4	All Applicable	150

girder and face of the seat. If above average cover is used at these locations, N should be increased accordingly.

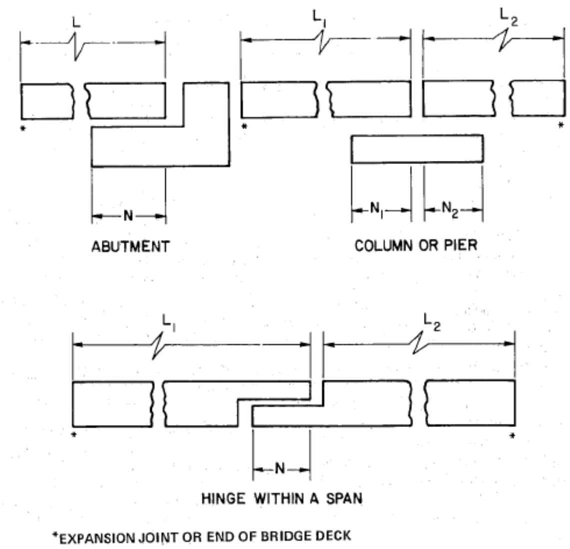

Figure C4.7.4.4-1—Support Length, *N*

4.7.4.5—*P*-Δ Requirements

The displacement of any column or pier in the longitudinal or transverse direction shall satisfy:

$$\Delta P_u < 0.25\phi M_n \tag{4.7.4.5-1}$$

in which:

$$\Delta = R_d \Delta_e \tag{4.7.4.5-2}$$

- If $T < 1.25T_s$, then:

C4.7.4.5

Bridges subject to earthquake ground motion may be susceptible to instability due to P-Δ effects. Inadequate strength can result in ratcheting of structural displacements to larger and larger values causing excessive ductility demand on plastic hinges in the columns, large residual deformations, and possibly collapse. The maximum value for Δ given in this Article is intended to limit the displacements such that P-Δ effects will not significantly affect the response of the bridge during an earthquake.

P-Δ effects lead to a loss in strength once yielding occurs in the columns of a bridge. In severe cases, this

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$$R_d = \left(1 - \frac{1}{R}\right)\frac{1.25T_s}{T} + \frac{1}{R} \qquad (4.7.4.5\text{-}3)$$

- If $T \geq 1.25T_s$, then:

$$R_d = 1 \qquad (4.7.4.5\text{-}4)$$

where:

Δ = displacement of the point of contraflexure in the column or pier relative to the point of fixity for the foundation (ft)
Δ_e = displacement calculated from elastic seismic analysis (in.)
T = period of fundamental mode of vibration (sec.)
T_S = corner period specified in Article 3.10.4.2 (sec.)
R = R-factor specified in Article 3.10.7
P_u = axial load on column or pier (kip)
ϕ = flexural resistance factor for column specified in Article 5.11.4.1.2
M_n = nominal flexural strength of column or pier calculated at the axial load on the column or pier (kip-ft)

can result in the force–displacement relationship having a negative slope once yield is fully developed. The value for Δ given by Eq. 4.7.4.5-1 is such that this reduction in strength is limited to 25 percent of the yield strength of the pier or bent.

4.7.5—Analysis for Collision Loads

Where permitted by the provisions of Section 3, dynamic analysis for ship collision may be replaced by an equivalent static elastic analysis. Where an inelastic analysis is specified, the effect of other loads that may also be present shall be considered.

4.7.6—Analysis of Blast Effects

As a minimum, bridge components analyzed for blast forces should be designed for the dynamic effects resulting from the blast pressure on the structure. The results of an equivalent static analysis shall not be used for this purpose.

C4.7.6

Localized spall and breach damage should be accounted for when designing bridge components for blast forces. Data available at the time these provisions were developed (winter 2010) are not sufficient to develop expressions for estimating the extent of spall/breach in concrete columns; however, spall and breach damage can be estimated for other types of components using guidelines found in Department of Defense (2008a).

The highly impulsive nature of blast loads warrants the consideration of inertial effects during the analysis of a structural component. Past research has demonstrated that, in general, an equivalent static analysis is not acceptable for the design of any structural member subjected to blast loads (Department of Defense, 2008a; Department of Defense, 2002; Bounds, 1998; ASCE, 1997). Information on designing structures to resist blast loads may be found in AASHTO's *Bridge Security Guidelines* (2011), ASCE (1997), Department of Defense (2008a), Conrath, et al. (1999), Biggs (1964), and Bounds (1998).

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4.8—ANALYSIS BY PHYSICAL MODELS

4.8.1—Scale Model Testing

To establish and/or to verify structural behavior, the Owner may require the testing of scale models of structures and/or parts thereof. The dimensional and material properties of the structure, as well as its boundary conditions and loads, shall be modeled as accurately as possible. For dynamic analysis, inertial scaling, load/excitation, and damping functions shall be applied as appropriate. For strength limit state tests, factored dead load shall be simulated. The instrumentation shall not significantly influence the response of the model.

4.8.2—Bridge Testing

Existing bridges may be instrumented and results obtained under various conditions of traffic and/or environmental loads or load tested with special purpose vehicles to establish force effects and/or the load-carrying capacity of the bridge.

C4.8.2

These measured force effects may be used to project fatigue life, to serve as a basis for similar designs, to establish permissible weight limits, to aid in issuing permits, or to establish a basis of prioritizing rehabilitation or retrofit.

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APPENDIX A4—DECK SLAB DESIGN TABLE

Table A4-1 may be used in determining the design moments for different girder arrangements. The following assumptions and limitations were used in developing this table and should be considered when using the listed values for design:

- The moments are calculated using the equivalent strip method as applied to concrete slabs supported on parallel girders.
- Multiple presence factors and the dynamic load allowance are included in the tabulated values.
- See Article 4.6.2.1.6 for the distance between the center of the girders to the location of the design sections for negative moments in the deck. Interpolation between the listed values may be used for distances other than those listed in Table A4-1.
- The moments are applicable for decks supported on at least three girders and having a width of not less than 14.0 ft between the centerlines of the exterior girders.
- The moments represent the upper bound for the moments in the interior regions of the slab and, for any specific girder spacing, were taken as the maximum value calculated, assuming different number of girders in the bridge cross-section. For each combination of girder spacing and number of girders, the following two cases of overhang width were considered:

  (a) Minimum total overhang width of 21.0 in. measured from the center of the exterior girder, and
  (b) Maximum total overhang width equal to the smaller of 0.625 times the girder spacing and 6.0 ft.

  A railing system width of 21.0 in. was used to determine the clear overhang width. For other widths of railing systems, the difference in the moments in the interior regions of the deck is expected to be within the acceptable limits for practical design.
- The moments do not apply to the deck overhangs and the adjacent regions of the deck that need to be designed taking into account the provisions of Article A13.4.1.
- It was found that the effect of two 25^k axles of the tandem, placed at 4.0 ft from each other, produced maximum effects under each of the tires approximately equal to the effect of the 32^k truck axle. The tandem produces a larger total moment, but this moment is spread over a larger width. It was concluded that repeating calculations with a different strip width for the tandem would not result in a significant difference.

Table A4-1—Maximum Live Load Moments per Unit Width, kip-ft/ft

S		Positive Moment	Negative Moment: Distance from CL of Girder to Design Section for Negative Moment						
			0.0 in.	3 in.	6 in.	9 in.	12 in.	18 in.	24 in.
4 ft	–0 in.	4.68	2.68	2.07	1.74	1.60	1.50	1.34	1.25
4 ft	–3 in.	4.66	2.73	2.25	1.95	1.74	1.57	1.33	1.20
4 ft	–6 in.	4.63	3.00	2.58	2.19	1.90	1.65	1.32	1.18
4 ft	–9 in.	4.64	3.38	2.90	2.43	2.07	1.74	1.29	1.20
5 ft	–0 in.	4.65	3.74	3.20	2.66	2.24	1.83	1.26	1.12
5 ft	–3 in.	4.67	4.06	3.47	2.89	2.41	1.95	1.28	0.98
5 ft	–6 in.	4.71	4.36	3.73	3.11	2.58	2.07	1.30	0.99
5 ft	–9 in.	4.77	4.63	3.97	3.31	2.73	2.19	1.32	1.02
6 ft	–0 in.	4.83	4.88	4.19	3.50	2.88	2.31	1.39	1.07
6 ft	–3 in.	4.91	5.10	4.39	3.68	3.02	2.42	1.45	1.13
6 ft	–6 in.	5.00	5.31	4.57	3.84	3.15	2.53	1.50	1.20
6 ft	–9 in.	5.10	5.50	4.74	3.99	3.27	2.64	1.58	1.28
7 ft	–0 in.	5.21	5.98	5.17	4.36	3.56	2.84	1.63	1.37
7 ft	–3 in.	5.32	6.13	5.31	4.49	3.68	2.96	1.65	1.51
7 ft	–6 in.	5.44	6.26	5.43	4.61	3.78	3.15	1.88	1.72
7 ft	–9 in.	5.56	6.38	5.54	4.71	3.88	3.30	2.21	1.94
8 ft	–0 in.	5.69	6.48	5.65	4.81	3.98	3.43	2.49	2.16
8 ft	–3 in.	5.83	6.58	5.74	4.90	4.06	3.53	2.74	2.37
8 ft	–6 in.	5.99	6.66	5.82	4.98	4.14	3.61	2.96	2.58
8 ft	–9 in.	6.14	6.74	5.90	5.06	4.22	3.67	3.15	2.79
9 ft	–0 in.	6.29	6.81	5.97	5.13	4.28	3.71	3.31	3.00
9 ft	–3 in.	6.44	6.87	6.03	5.19	4.40	3.82	3.47	3.20
9 ft	–6 in.	6.59	7.15	6.31	5.46	4.66	4.04	3.68	3.39
9 ft	–9 in.	6.74	7.51	6.65	5.80	4.94	4.21	3.89	3.58
10 ft	–0 in.	6.89	7.85	6.99	6.13	5.26	4.41	4.09	3.77
10 ft	–3 in.	7.03	8.19	7.32	6.45	5.58	4.71	4.29	3.96
10 ft	–6 in.	7.17	8.52	7.64	6.77	5.89	5.02	4.48	4.15
10 ft	–9 in.	7.32	8.83	7.95	7.08	6.20	5.32	4.68	4.34
11 ft	–0 in.	7.46	9.14	8.26	7.38	6.50	5.62	4.86	4.52
11 ft	–3 in.	7.60	9.44	8.55	7.67	6.79	5.91	5.04	4.70
11 ft	–6 in.	7.74	9.72	8.84	7.96	7.07	6.19	5.22	4.87
11 ft	–9 in.	7.88	10.01	9.12	8.24	7.36	6.47	5.40	5.05
12 ft	–0 in.	8.01	10.28	9.40	8.51	7.63	6.74	5.56	5.21
12 ft	–3 in.	8.15	10.55	9.67	8.78	7.90	7.02	5.75	5.38
12 ft	–6 in.	8.28	10.81	9.93	9.04	8.16	7.28	5.97	5.54
12 ft	–9 in.	8.41	11.06	10.18	9.30	8.42	7.54	6.18	5.70
13 ft	–0 in.	8.54	11.31	10.43	9.55	8.67	7.79	6.38	5.86
13 ft	–3 in.	8.66	11.55	10.67	9.80	8.92	8.04	6.59	6.01
13 ft	–6 in.	8.78	11.79	10.91	10.03	9.16	8.28	6.79	6.16
13 ft	–9 in.	8.90	12.02	11.14	10.27	9.40	8.52	6.99	6.30
14 ft	–0 in.	9.02	12.24	11.37	10.50	9.63	8.76	7.18	6.45
14 ft	–3 in.	9.14	12.46	11.59	10.72	9.85	8.99	7.38	6.58
14 ft	–6 in.	9.25	12.67	11.81	10.94	10.08	9.21	7.57	6.72
14 ft	–9 in.	9.36	12.88	12.02	11.16	10.30	9.44	7.76	6.86
15 ft	–0 in.	9.47	13.09	12.23	11.37	10.51	9.65	7.94	7.02

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