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Analyses and settlement study of a group of two, three and four partially stiffened floating granular piles

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ABSTRACT

Use of granular pile as a ground improvement technique in case of soft soils is one of the reliable and economic options. Analysis of a partially stiffened group of granular floating piles has been numerically assessed and presented here. Partial stiffening simply means that instead of using conventional material for making the granular pile (GP) in its full length, top region is replaced partially by some suitable material, having better mechanical properties, i.e. higher deformation modulus. Various normalised parameters like settlement influence factor for top of GP, settlement interaction factor, settlement reduction factor, percentage load shared by the base and shear stress distribution along the length of the granular pile are evaluated. The settlement influence factor for top of GP is found to decrease with the increase in the values of the stiffening parameters. The interfacial shear stresses get redistributed along the length of the granular pile.

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KEYWORDS
Relative stiffness of granular pile; settlement influence factor for top of GP; settlement interaction factor; settlement reduction factor; stiffening factor

1.0 Introduction

In the current scenario of the present world where the land availability is reducing day by day, we are forced to move for construction in the vertical direction rather than in horizontal expansion. The philosophy of multi-storey building constructions, leads to development of heavy loads on the foundation, and hence, this leads to the demand for higher load-bearing capacity. The present paper focuses on increasing the load-bearing capacity either by increasing the stiffening factor or by increasing the percentage length of stiffening from the top of GP. Stone column is an effective ground treatment method because it acts as a stiffer medium and hence increases the load-bearing capacity and improves the stiffness of the ground due to its higher frictional strength compared to the surrounding soils. Most of the GP fails because of of bulging occurring in the top portion of GP due to the lesser confinement at top of the soil. Bulging usually takes place up to a depth of *4d* (*d* = diameter of granular column) and maximum bulging usually occurs at a depth of *2d* (Black *et al.* 2006, Hughes and Withers 1974, McKelvey *et al.* 2004). This fact leads to the idea of stiffening the top portion of the GP by upto 40% of its length. The failure mechanism of partially stiffened GP can be ascertained by the conventional pile load test. The failure mechanism may be expected below the region of the stiffened portion of the GP because of the occurrence of bulging at higher loads. The effectiveness of stiffening is studied in the linear stress–strain range for performance improvement and enhancement in load-carrying capacity of the GP. The present study provides the solution to the bulging problem occurring at the top of the GP by stiffening the top of GP. The analysis has been done based on the elastic continuum approach. A new granular pile displacement matrix is developed for a group of two, three and four partially stiffened floating granular piles. Vertical displacement of the granular pile incorporating the stiffening factor and percentage stiffening length from top has been evaluated. New soil displacement matrix is generated by using equations developed in past (Mindlin 1936, 1937).

Major assumptions made during the analysis are follows:

(1) Soil is considered to be homogeneous, isotropic and linearly elastic.
(2) The base of the stone column/granular pile is assumed to be smooth across which the load is uniformly distributed (Madhav *et al.* 2006).
(3) The disturbance effects in the in-situ soil due to the installation of granular piles are ignored, and even consolidation due to self-weight with passage of time leading to change in elastic deformation modulus of GP as well as soil, is neglected.
(4) The present study has been done by assuming no slip or yield condition.

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2.0 Literature review

Experimental, numerical and analytical approaches for studying behaviour of granular piles have been used by many investigators in the past. Mattes and Poulos (1969) studied the settlement analysis of single compressible pile in non-homogeneous soils. Poulos and Mattes (1971) presented the settlement analysis of pile groups. Preibe (1976) proposed the reduced stress method for the estimation of reduction in settlement of ground improved with stone columns. For improving the in-situ ground conditions, the inclusion of stone columnar is considered as one of the most versatile and cost-effective ground improvement techniques (Alamgir 1996). As studied by Madhav and Nagpure (1995), granular piles allow the treatment of a wide range of soils, ranging from loose sand to soft clays by forming reinforcing elements of low compressibility and high shear strength. Madhav *et al.* (1998) studied the analysis of floating granular pile. Madhav *et al.* (2006) carried out the studies for settlement of the pile taking into considerations the non-homogeneities. Jamsawang *et al.* (2008) studied the performance of stiffened deep mixed pile. Murugesan and Rajagopal (2010) investigated the performance of encased stone columns through 1-g laboratory tests consisting of column loading of granite chips. Najjar and Skeini (2015) studied that for a given reinforcement condition, the percentage improvement in load-carrying capacity was found to be higher for undrained conditions than for drained conditions. Marto *et al.* (2013), Zhang and Zhao (2015) and Hong *et al.* (2016) studied analytically as well as experimentally the effects of encasement stiffness and strength on the response of individual geo-textile-encased granular columns embedded in soft soil. Grover *et al.* (2015) studied the effect of stiffening on a single granular pile for both types of piles, viz., floating and end bearing. Gupta and Sharma (2017) carried out the analysis of a non-homogeneous granular pile with non-linear deformation modulus. Garg and Sharma (2018) analysed analytically settlement of single and group of two partially stiffened floating granular piles for the response of settlement. Garg *et al.* (2018) analysed the role of stiffening for a single partially stiffened floating granular pile for settlement and shear.

3.0 Problem definition and analysis

Present problem is shown in Figure 1(a) and (b), showing a group of two, floating, partially stiffened GPs each of length 'L' and diameter 'd' = ($2a$), such that each granular pile is subjected to an axial load 'P' . The two granular piles are assumed to be placed 's' spacing apart centre to centre of the granular piles. As observed by a number of researchers, it is found that the top portion of the GP tends to bulge out, and so the stiffening effect is provided at the top of the GP. In Figure 2(a) and (b) a group of three, partially stiffened, floating GPs each of length 'L' and diameter 'd' = ($2a$) is shown, with each granular pile subjected to an axial load 'P' . Similarly in Figure 3(a) and (b) a group of four, partially stiffened, symmetrically placed, floating GPs is shown in which each GP is of length 'L' and diameter 'd' = ($2a$), and is subjected to an axial load 'P'. The top portion of each granular pile is stiffened up-to some, definite length 'L_s', $\eta = (L_s/L) \times 100$, percentage length of stiffening from top of GP. The deformation modulus of the granular pile in the un-stiffened portion is taken as 'E_{gp}'. The surrounding soft soil is represented by the deformation modulus 'E_s' and the Poisson's ratio of soil, 'ν_s'. The relative stiffness of GP is described as $K_{gp} = E_{gp}/E_s$, i.e. the ratio of deformation modulus of granular pile to that of the soil. A parameter, 'χ' whose value is considered as greater than 1 is defined to take into account the stiffening effect, i.e. 'χ' is the factor by which the 'K_{gp}' of the un-stiffened portion of each granular pile in the granular pile group is multiplied to get the 'K_{gp}' of the stiffened portion of the granular pile. Continuity of displacement is maintained at the interface of stiffened and un-stiffened part of the pile.

3.1 Soil displacements

The vertical soil displacements of GP are evaluated from the Mindlin's expressions for vertical displacements due to vertical point loads acting within the soil mass using numerical integration scheme. Settlement nodes are taken at the inner side of GP in order to have the maximum effect of the interaction of interfacial shear stresses base pressures for the group. The settlements for cylindrical elements are calculated at the nodes on the GP–soil interface, while for the base, the node is considered at the centre of GP.

The normalised vertical soil displacement equations due to shaft and base stresses for a single granular pile are expressed as

$$\{\rho^s\} = \left\{\frac{S^s}{d}\right\} = [IF^s]\left\{\frac{\tau}{E_s}\right\} \tag{1}$$

where $\{S^s\}$ and $\{\rho^s\}$ are the vertical and normalised vertical soil displacement column vectors of size $(n + 1)$, respectively. $\{\tau\}$ is GP–soil interface shear stress vector inclusive of the base pressure, p_b, of size $(n + 1)$. $[IF^s]$ is a square matrix of size '$(n + 1)$' for soil displacement influence coefficients evaluated with the numerical integration scheme.

The above analysis is now applied for a group of two, three and four partially stiffened floating granular piles as discussed in the following section.

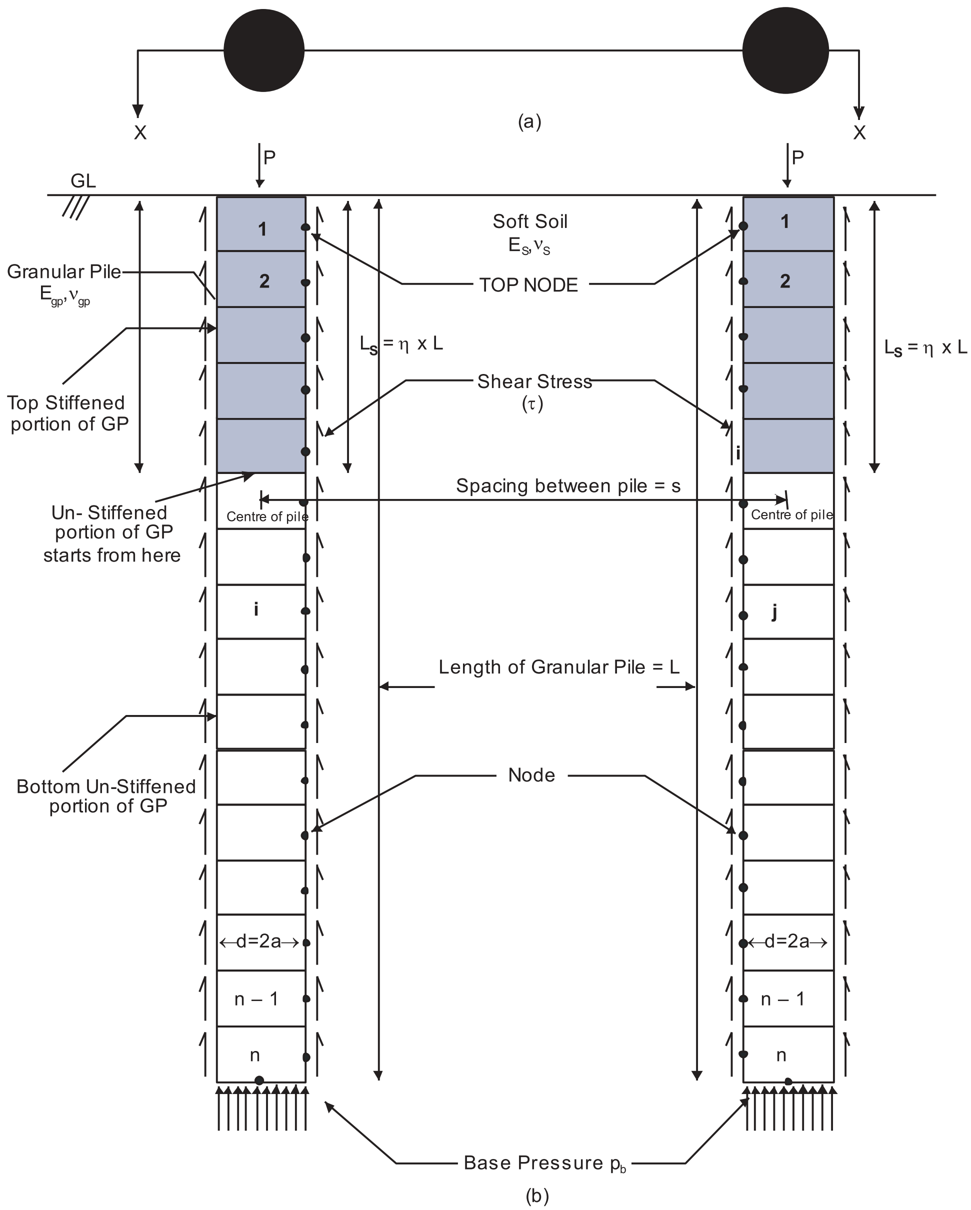

Figure 1. **(a)** Plan of two granular piles partially stiffened floating GPs. **(b)** Section at X-X showing group of two partially stiffened floating granular piles (problem definition sketch).

For a group of two floating GPs,

$$\{\rho^s\} = \left\{\frac{S^s}{d}\right\} = [\ [_1IF^s] + [_2IF^s]\ ] \left\{\frac{\tau}{E_s}\right\} \tag{2}$$

where $[_1IF^s]$and $[_2IF^s]$ are soil displacement square matrices of size $(n + 1)$ eachdue to influence of elemental shear stresses of own and adjacent GP for two GP group as shown in Figure 1(b). Similarly, $\{S^s\}$ and $\{\rho^s\}$ are soil displacement and normalised soil displacement vectors of size '$(n + 1)$', respectively; $\{\tau\}$ is a column vector of size, '$(n + 1)$', for GP–soil interface shear stresses including the base pressure.

For a group of three floating granular piles,

$$\{\rho^s\} = \left\{\frac{S^s}{d}\right\} = [\ [_1IF^s] + [_2IF^s] + [_3IF^s]\ ] \left\{\frac{\tau}{E_s}\right\} \tag{3}$$

where $[_1IF^s]$, $[_2IF^s]$ and $[_3IF^s]$ are soil displacement square matrices of size '$(n + 1)$' each due to influence of elemental shear stresses of self and correspondingly adjacent two symmetric GPs in three pile group as shown in Figure 2(b). Due to symmetry of positions of granular piles second and third, $[_3IF^s] = [_2IF^s]$ and thus soil displacement equations are

$$\{\rho^s\} = \left\{\frac{S^s}{d}\right\} = [\ [_1IF^s] + 2 \times [_2IF^s]\ ] \left\{\frac{\tau}{E_s}\right\} \tag{4}$$

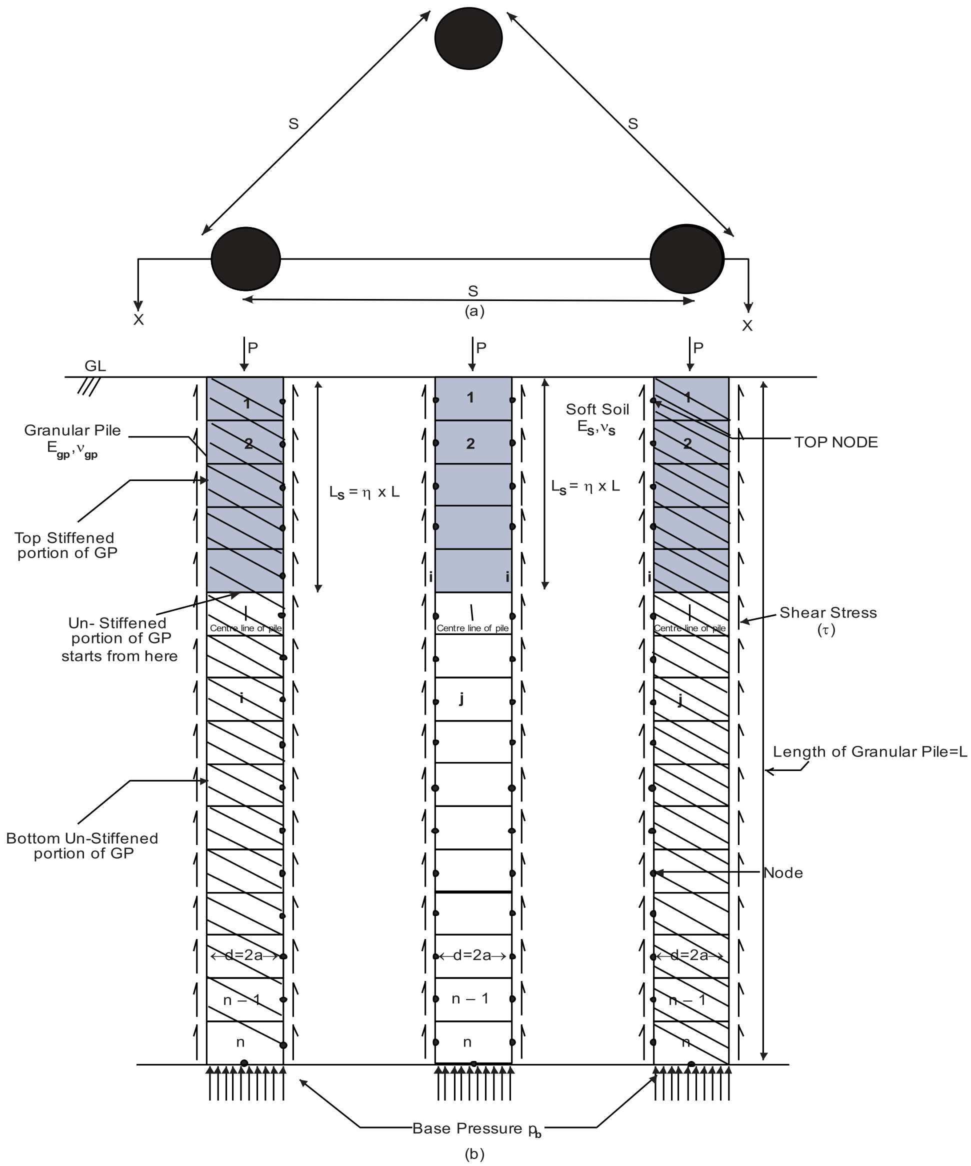

Figure 2. **(a)** Plan of three symmetrically placed floating, partially stiffened GPs.(b) Sectional elevation at X-X of a group of three symmetrically placed floating partially stiffened floating GPs.

where $\{S^s\}$ and $\{\rho^s\}$ are soil displacement and normalised soil displacement vectors of size *'(n + 1)'*, respectively; $\{\tau\}$ is a column vector of size, '*(n + 1)*', for GP–soil interface shear stresses including the base pressure; $[[_1IF^s]+2\text{x}[_2IF^s]]$ is a combined square matrix for soil displacement influence coefficients of size '*(n + 1)*', for a floating granular pile.

Similarly, for the group of four symmetrically placed floating GPs,

$$\{\rho^s\} = \left\{\frac{s^s}{d}\right\} = [\ [_1IF^s] + [_2IF^s] + [_3IF^s] + [_4IF^s]\ ]\left\{\frac{\tau}{E_s}\right\} \quad (5)$$

where $[_1IF^s]$, $[_2IF^s]$, $[_3IF^s]$ and $[_4IF^s]$ are soil displacement square matrices of size *(n + 1)* each due to influence of elemental shear stresses of own (first), second (at spacing *s*), third (at spacing *s*) and fourth GP (at spacing $\sqrt{2}s$) respectively, as shown in Figure 3(a) and (b). All the other terms in Equation (5) are already defined. Due to symmetry of positions of granular piles second and third, $[_3IF^s]$ = $[_2IF^s]$ and thus soil displacement equations are

$$\{\rho^s\} = \left\{\frac{S^s}{d}\right\} = [\ [_1IF^s] + 2 \times [_2IF^s] + [_4IF^s]\ ]\left\{\frac{\tau}{E_s}\right\} \quad (6)$$

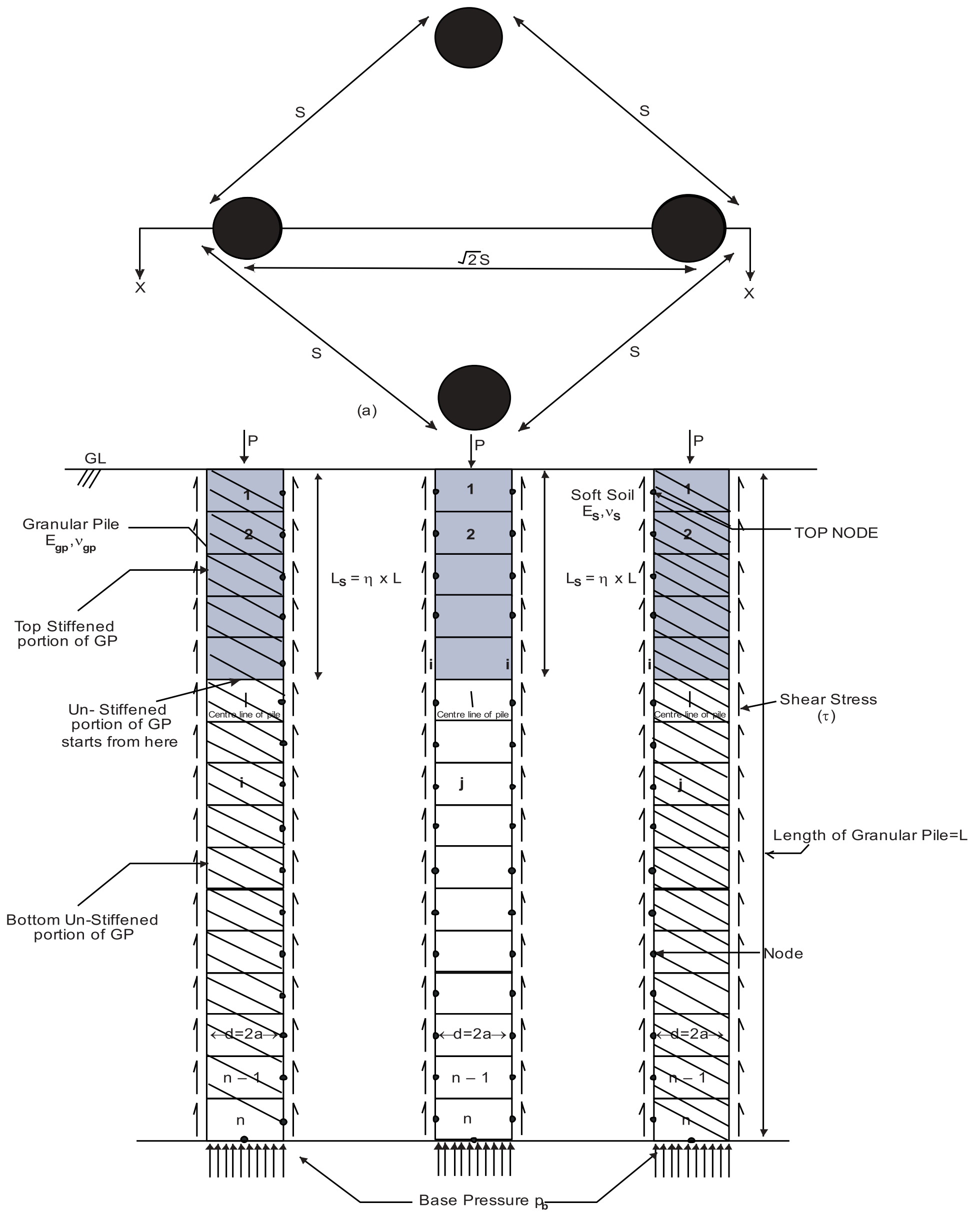

Figure 3. **(a)** Plan of four symmetrically placed floating, partially stiffened GPs. **(b)** Sectional elevation at X-X of a group of four symmetrically placed floating partially stiffened floating GPs.

3.2 Granular pile displacements

Based on a generalised stress–strain relationship, vertical displacements of elements of a single GP are evaluated as

$$\varepsilon_v = \frac{\sigma_v}{E_{gp}} \tag{7}$$

where ε_v is the axial strain of an element, σ_v is the axial stresses on the element, respectively. E_{gp} is the deformation modulus of the granular pile. The above expression in terms of shear stresses and base pressures of the GP is established in the following section.

3.2.1 Relationship between axial and shear stresses of GP

The total load 'P' on GP is related to the shear stresses, τ and base pressure 'p_b' on a single granular pile is given by

$$P = \sum_{j=1}^{j=n} \frac{\tau_j \pi dL}{n} + p_b \frac{\pi d^2}{4} \tag{8}$$

where 'n' is the total number of elements of GP. Considering the element 'i', the axial forces on the top and bottom faces of an element are given by

$$P_{it} = P - \sum_{j=1}^{j=(i-1)} \frac{\tau_j \pi dL}{n} \quad and \quad P_{ib} = P - \sum_{j=1}^{j=i} \frac{\tau_j \pi dL}{n} \tag{9}$$

Combining Equations (8) and (9)

$$P_{it} = \sum_{j=i}^{j=n} \frac{\tau_j \pi dL}{n} + p_b \frac{\pi d^2}{4} \; and \; P_{ib} = \sum_{j=(i+1)}^{j=n} \frac{\tau_j \pi dL}{n} + p_b \frac{\pi d^2}{4} \tag{10}$$

The axial stresses on top and bottom faces of the element '*i*' are given by

$$\sigma_{it} = p_b + \sum_{j=i}^{j=n} \frac{4(L/d)\tau_j}{n} \; and \; \sigma_{ib} = p_b + \sum_{j=(i+1)}^{j=n} \frac{4(L/d)\tau_j}{n} \tag{11}$$

The average axial stress on the element '*i*' is equal to

$$\sigma_{vi} = \frac{\sigma_{it} + \sigma_{ib}}{2} = p_b + \sum_{j=(i+1)}^{j=n} \frac{4(L/d)\tau_j}{n} + \frac{2(L/d)\tau_i}{n} \tag{12}$$

The above equation relates the GP shear stresses to axial stresses of the elements and is expressed in matrix form as

$$\{\sigma_v\} = [MA]\,\{\tau\} \tag{13}$$

where $\{\tau\}$ and $\{\sigma_v\}$ are, respectively, the column vectors of shear stresses on shaft including normalised stress on base and axial stresses of the elements, both of size *(n + 1)*. Matrix $[MA]$ is an upper triangular square matrix of size *(n + 1)* which relates the axial and shear stresses as

$$[MA] = \begin{bmatrix} \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & \frac{4(L/d)}{n} & \frac{(L/d)}{n} & - & - & - & - & 1 \\ 0 & \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & \frac{4(L/d)}{n} & - & - & - & - & 1 \\ 0 & 0 & \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & - & - & - & - & 1 \\ & & & & & & & & \\ - & - & - & - & - & 0 & \frac{2(L/d)}{n} & \frac{4(L/d)}{n} & 1 \\ - & - & - & - & - & - & 0 & \frac{2(L/d)}{n} & 1 \\ - & - & - & - & - & - & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

3.3 Vertical GP displacements

The vertical displacement of the granular pile at each node is evaluated from the displacement of the top of pile ρ_t in the downward direction, by subtracting the displacement of all previous elements up to that node and half of the displacement of the corresponding element from top ρ_t.

Thus, the normalised settlement of the first element of GP is

$$\rho_1^p = \frac{S_1^p}{d} = \rho_t - \varepsilon_{v1} \frac{\Delta z}{2d} \tag{15}$$

where ε_{v1} is the axial strain of the first element of GP and $\Delta z = (L/n)$ is element length. S_1^p and ρ_1^p are the displacement and normalised displacements of the first node, respectively.

Similarly, the normalised displacement of any element '*i*' is obtained as

$$\rho_i^p = \rho_t - \sum_{j=1}^{j=(i-1)} \varepsilon_{vj} \frac{\Delta z}{d} - \varepsilon_{vi} \frac{\Delta z}{2d} \tag{16}$$

where ε_{vi} and ε_{vj} are the axial strains of i^{th} and j^{th} elements, respectively. In order to evaluate the settlement of the base of the granular pile, the strain at the base is

$$\varepsilon_b = -\frac{dS^p}{dz} = \frac{p_b}{E_{gp}} \tag{17}$$

By applying the finite difference scheme with three points of unequal intervals of spacing, the above equation becomes (Garg *et al.* 2018)

$$\frac{4S_{n-1}^p - 36S_n^p + 32S_{n+1}^p}{12(\Delta z/d)} = -\frac{p_b}{E_{gp}} \tag{18}$$

where S_{n-1}^p, S_n^p and S_{n+1}^p are the displacements of elements *n-1*, *n* and *n + 1*, respectively. Expressing Equation (18) in the normalised form, it reduces to

$$4\rho_{n-1}^p - 36\rho_n^p + 32\rho_{n+1}^p = -\frac{p_b}{E_{gp}} \frac{12(L/d)}{n} \tag{19}$$

Substituting the values of ρ_{n-1}^p and ρ_n^p from Equation (16) and rearranging the terms, one gets

$$\rho_{n+1}^p = \rho_t - \sum_{j=1}^{j=(n-2)} \varepsilon_{vj} \frac{\Delta z}{d} - \frac{34}{32} \varepsilon_{v(n-1)} \frac{\Delta z}{d} - \frac{18}{32} \varepsilon_{vn} \frac{\Delta z}{d} - \frac{6}{32} \frac{(L/d)}{nK_{gp}} \frac{p_b}{E_s} \tag{20}$$

Combining Equations (16) and (20), the normalised vertical displacements of the granular pile are

$$\{\rho^p\} = \rho_t\{1\} + [MB] \left\{ \frac{\sigma_v}{E_s} \right\} \tag{21}$$

where $\{1\}$ is a unit column matrix and *[MB]* is a lower triangular matrices of sizes *(n + 1)* ×*(n + 1)* incorporating the stiffening factor, χ and percentage length of stiffening from top, η and is given by

Due consideration is given in the above matrix to maintain the continuity of displacement at the interface of stiffened and un-stiffened portion of GP. Using the

$$[MB] = \frac{(L/d)}{nK_{gp}} \begin{bmatrix} \frac{-0.5}{\chi} & 0 & 0 & 0 & - & - & - & - & 0 \\ \frac{-1}{\chi} & \frac{-0.5}{\chi} & 0 & 0 & - & - & - & - & 0 \\ \frac{-1}{\chi} & \frac{-1}{\chi} & \frac{-0.5}{\chi} & 0 & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ & & & & & & & & \\ & & & & & & & & \\ -1 & -1 & - & - & - & - & - & -0.5 & 0 \\ -1 & -1 & - & - & - & - & -\frac{34}{32} & -\frac{18}{32} & -\frac{6}{32} \end{bmatrix} \begin{bmatrix} \textit{Here Kgp willbe} \\ \textit{replacedby}\chi\textit{Kgp} \\ \textit{fortopelements} \\ \textit{ofGPtotakeinto} \\ \textit{considerationof} \\ \textit{effectof} \\ \textit{stiffeningupto} \\ \textit{adepth}\eta L/d \end{bmatrix} \quad (22)$$

elemental shear and axial stresses (Equation 13), the normalised vertical displacements of GP nodes in terms of shaft shear stresses are

$$\{\rho^p\} = \rho_t\{1\} + [MD]\left\{\frac{\tau}{E_s}\right\} \quad (23)$$

where $\{1\}$ is a unit column matrix and $[MD]$ is a square matrix of size $(n + 1)$ = $[MB]$ $[MA]$.

4.0 Compatibility of displacements of soil and GP

Interaction between stiffened and un-stiffened parts of GP and surrounding soil is represented in terms of interfacial shear stresses and base pressure. The displacements of the element of stiffened and un-stiffened portions of the GP are calculated based on elastic analysis while soil displacements of corresponding nodes of the element are evaluated based on the Mindlin equations (1936, 1937) for load acting inside the semi-infinite soil mass. The compatibility of displacements at each node of elements for soil and granular pile for stiffened and un-stiffened portions is maintained for having interaction between soil and GP to evaluate interfacial shear stresses. For obtaining the solutions in terms of interface shear stresses and base pressure, apply the compatibility condition of displacements of the granular pile and the soil as described as follows.

For a single granular pile (Equations 1 and 23),

$$\{\rho^s\} = \{\rho^p\} \quad \text{or}\, [AMA]\left\{\frac{\tau}{E_s}\right\} = \rho_t\{1\} \quad (24)$$

where $[AMA] = [IF^s] - [MD]$, of size $(n + 1) \times (n + 1)$.

For a group of two granular piles group (Equations 2 and 23),

$$\{\rho^s\} = \{\rho^p\} \quad \text{or}\, [AMB]\left\{\frac{\tau}{E_s}\right\} = \rho_t\{1\} \quad (25)$$

where $[AMB] = [\ [{}_1IF^s] + [{}_2IF^s]\ ] - [MD]$, of size $(n + 1) \times (n + 1)$.

For a group of three granular piles group (Equations 4 and 23),

$$\{\rho^s\} = \{\rho^p\} \quad \text{or}\, [AMC]\left\{\frac{\tau}{E_s}\right\} = \rho_t\{1\} \quad (26)$$

where $[AMC] = [\ [{}_1IF^s] + 2[{}_2IF^s]\ ] - [MD]$, of size $(n + 1) \times (n + 1)$.

For a group of four granular piles group (Equations 6 and 23),

$$\{\rho^s\} = \{\rho^p\} \quad \text{or}\, [AMD]\left\{\frac{\tau}{E_s}\right\} = \rho_t\{1\} \quad (27)$$

where $[AMD] = [\,[\,[{}_1IF^s] + 2 \times [{}_2IF^s] + [{}_4IF^s]\,]\,] - [MD]$, of size $(n + 1) \times (n + 1)$.

The above equations are solved for normalised shear stresses and base load. Total load of a GP is then evaluated. Finally, the settlement influence factor for top of GP, I_{sp}, is obtained as defined as follows.

The top vertical displacement 'S' of a single granular pile subjected to axial load P and for a group of two, three and four granular piles each subjected to equal axial load 'P' is given by

$$S = \frac{P}{E_s d} I_{sp} \quad (28)$$

Here, I_{sp} is a non-dimensional parameter known as the settlement influence factor for top of GP, which in turn depends on the various parameters such as geometrical parameters and material characteristics related to soil and granular pile. Two normalised interaction parameters α, β are described later and results are evaluated in terms of settlement influence factor for top I_{sp}, α, β, normalised shear distributions along the GP–soil interface and the percentage of load transferred to the base. Parameters affecting the results are (i) length to diameter ratio, i.e. relative length L/d, of GP; (ii) normalised spacing between the granular piles in the granular pile group, s/d; (iii) relative stiffness, K_{gp}, of granular pile with respect to soil; (iv) percentage of length of stiffening from top of GP, $\eta = (L_s/L) \times 100$; (v) the Poisson's ratio of soil, ν_s; (vi) stiffening factor χ by which the relative stiffness K_{gp} of the un-stiffened portion of granular pile is multiplied, to get the relative stiffness of the top stiffened portion of granular pile.

In the work done by Poulos and Mattes (1971), the parameter α (settlement interaction factor) is defined as

$$\alpha_{2\mathrm{F}} = \frac{\text{Settlement of a GP in a group of two partially stiffened GP} - \text{settlement of asingle partially stiffened GP}}{\text{Settlement of asingle partially stiffened GP}}$$

For a group of three partially stiffened, floating granular pile,

$$\alpha_{3\mathrm{F}} = \frac{\text{Settlement of a GP in a group of three partially stiffened GP} - \text{settlement of asingle partially stiffened GP}}{\text{Settlement of a single partially stiffened GP}}$$

For a group of four partially stiffened, floating granular pile,

$$\alpha_{4\mathrm{F}} = \frac{\text{Settlement of a GP in a group of four partially stiffened GP group} - \text{settlement of single partially stiffened GP}}{\text{Settlement by asingle partially stiffened GP}}$$

Parameter β_f (settlement reduction factor) is described to compare the settlement of a granular pile in a group of two/three/four partially stiffened, floating granular piles to that of settlement of group of two/three/four un-stiffened floating granular pile,

$$\beta_{\mathrm{F}} = \frac{\text{Settlement of a GP in a group oftwo/three/four partially stiffened floating granular pile}}{\text{Settlement of a GP in a group of two/three/four unstiffened floating granular pile}}$$

Results obtained by applying the above analysis were termed as rigorous results.

Settlement interaction factors for a group of three partially stiffened GPs and a group of four partially stiffened GPs are also obtained from the principle of superposition as given as follows by Equations (29) and (30), respectively

$$\alpha_{3FS} = 2 \times \alpha_{2F}\,(\text{For a group of three GPs}) \tag{29}$$

$$\begin{aligned}\alpha_{4FS} = 2 \times \alpha_{2F}\ &(\text{for spacing, s})\\ + \alpha_{2F}\ &(\text{for spacing}, \sqrt{2}\text{s})\\ &\text{For a group of four GPs}\end{aligned} \tag{30}$$

where α_{2F} is settlement interaction factor for a group of two partially stiffened floating granular pile. The settlement interaction factors and settlement influence factor for top of GP are evaluated by rigorous method and superposition principle and compared with the variations of various parameters already listed.

5.0 Results and discussion

The results found by the above analysis are in close agreement with that of Poulos and Mattes (1971). For a group of two floating GPs in un-stiffened condition for L/d = 10, ν_s = 0.5, s/d = 2, K_{gp} = 10, the value of α

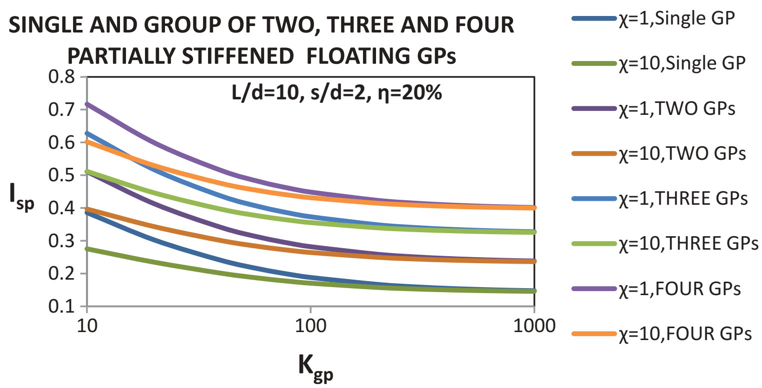

Figure 4. Variation of settlement influence factor for top of GP, $I_{sp,}$ with relative stiffness of GP, K_{gp}– effect of stiffening factor, χ, and number of piles, on a GP, in a single, group of two, three and four partially stiffened floating GPs (L/d = 10, s/d = 2, η = 20%).

obtained is 0.319 while it is 0.32 for Poulos and Mattes (1971). For L/d = 10, ν_s = 0.5, s/d = 3, K_{gp} = 10, the value of α obtained is 0.228 while it is 0.23 for Poulos and Mattes (1971). Similarly, for L/d = 10, ν_s = 0.5, s/d = 3, K_{gp} = 100, the value of α obtained is 0.379 while it is 0.38 for Poulos and Mattes (1971).

Results are obtained for the following range of parameters:

K_{gp} = 10–1000, η = 10–40%, χ = 1–10, L/d = 10–40, ν_s = 0.5, s/d = 2–5

Although the realistic normal range of 'K_{gp}' is 10–100, the study has been done up-to 'K_{gp}' = 1000 for better understanding and to explore later behaviour also. All the results described in the following are obtained from rigorous analysis unless otherwise stated.

Figure 4 depicts the variation of settlement influence factor for top of GP, $I_{sp,}$ with relative stiffness of GP, $K_{gp.}$ The graph reveals that as relative stiffness of GP, $K_{gp,}$ increases the value of settlement influence factor for top of GP, $I_{sp,}$ decreases mainly in the range of K_{gp} = 1–100. This occurs due to the fact that increase in relative stiffness of GP, K_{gp}, gives rise to better load-bearing conditions of the GP. It can well be observed that with increase in the stiffening factor, χ, there is a decrease in the value of settlement influence factor for top of GP, $I_{sp,}$ thus producing the beneficial effect of stiffening, as studied earlier (Garg and Sharma (2018)). It can also be seen from the graph that as the number of GPs increases in the group, the value of settlement influence factor for top of GP, $I_{sp,}$ increases because the stress interaction between the GP increases. It may be noted from the graph that for L/d = 10, s/d = 2, η = 20% and χ = 1, the values of $I_{sp,}$ at K_{gp} = 60, for single GP and a GP in a group of two, three and four GPs are, respectively, 0.213, 0.309, 0.402 and 0.479, while for L/d = 10, s/d = 2, η = 20% and χ = 10, the values of $I_{sp,}$ at K_{gp} = 60, are, respectively, 0.185, 0.281, 0.374 and 0.451 for single GP and a GP in a group of two, three and four partially stiffened GPs, thus showing a percentage decrease of 13, 9, 7 and 6, respectively.

Figure 5 shows the similar variation trends as shown in Figure 4 but for a different value of L/d = 20. Although the variation trends are same, due to increase in the

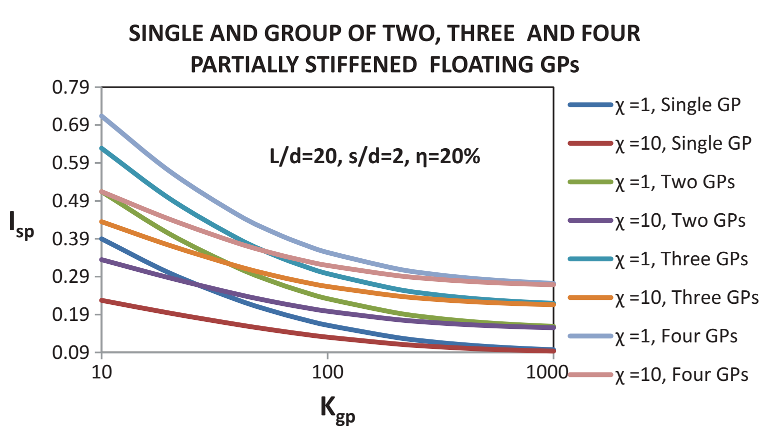

Figure 5. Variation of settlement influence factor for top of GP, I_{sp}, with relative stiffness of GP, K_{gp} – effect of stiffening factor, χ, and number of piles, on a GP, in a single, group of two, three and four partially stiffened floating GPs (L/d = 20, s/d = 2, η = 20%).

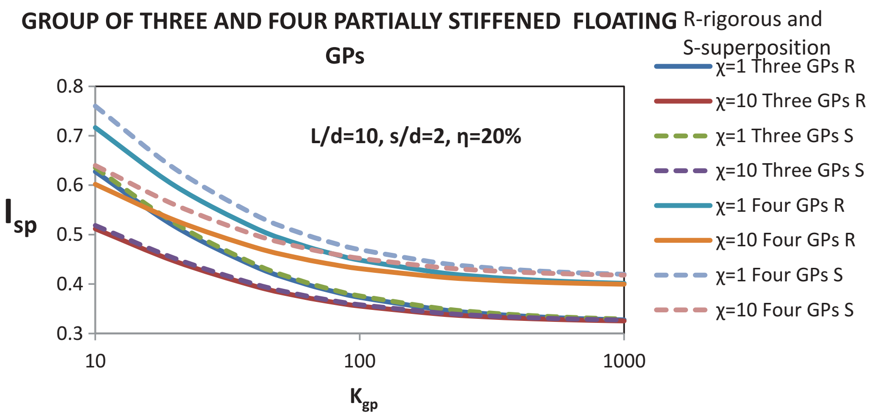

Figure 6. Variation of settlement influence factor for top of GP, I_{sp}, with relative stiffness of GP, K_{gp}– effect of stiffening factor, χ, and number of piles, on a GP, in a single, group of two, three and four partially stiffened floating GPs (L/d = 10, s/d = 2, η = 20%) Rigorous and Superposition analysis.

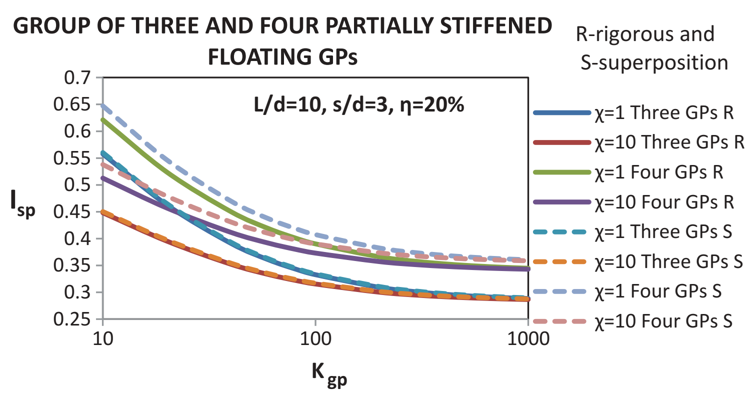

Figure 7. Variation of settlement influence factor for top of GP, I_{sp}, with relative stiffness of GP, K_{gp}– effect of stiffening factor, χ, and number of piles, on a GP, in a single, group of two, three and four partially stiffened floating GPs (L/d = 10, s/d = 3, η = 20%) Rigorous and Superposition analysis.

length of the GP, the values of settlement influence factor for top of GP, I_{sp}, decrease, as more length implies more shear stresses, hence reducing the top settlement. It may well be noted that for $L/d = 20$, $s/d = 2$, $\eta = 20\%$ and $\chi = 1$, the values of I_{sp}, at $K_{gp} = 60$, for single GP and a GP in a group of two, three and four GPs are, respectively, 0.195, 0.272, 0.343 and 0.402, while for $L/d = 20$, $s/d = 2$, $\eta = 20\%$ and $\chi = 10$, the values of I_{sp}, at $K_{gp} = 60$, are, respectively, 0.147, 0.221, 0.290 and 0.348 for single GP and a GP in a group of two, three and four partially stiffened GPs, thus showing a percentage decrease of 24, 18, 15 and 13, respectively.

In Figure 6, the comparison for values of settlement influence factor for top of GP, I_{sp}, is shown for rigorous and superposition analysis carried out for a group of three and four GPs. It can be seen that the two analyses are in close agreement with each other with superposition analysis providing values on the slightly higher side as compared to the rigorous analysis. It can well be observed that for $L/d = 10$, $s/d = 2$, $\eta = 20\%$ and $\chi = 1$, the values of I_{sp}, at $K_{gp} = 60$, for a GP in a group of three and four partially stiffened GPs are 0.402 and 0.479, respectively, as obtained by rigorous analysis, while these values are 0.406 and 0.503, respectively, by superposition analysis showing a change of 0.9% and 5%, respectively. If the same values are read out at $\chi = 10$, they are 0.374 and 0.451, respectively, by rigorous analysis while 0.374 and 0.474, respectively, for superposition analysis; hence, a minor change of 0% and 5% respectively occurs.

Figure 7 depicts the similar variation trends as shown in Figure 6 but with an increased value of spacing between the GPs. It can be observed from the graph that due to increase in the spacing, the stress interaction between the GPs decreases, so the values of settlement influence factor for top of GP, I_{sp}, decrease in contrast to Figure 6. For instance, it can well be observed that for $L/d = 10$, $s/d = 3$, $\eta = 20\%$ and $\chi = 1$, the values of I_{sp}, at $K_{gp} = 60$, for a GP in a group of three and four GPs are 0.360 and 0.418, respectively, as obtained by rigorous analysis, while these values are 0.362 and 0.437, respectively, by superposition analysis showing a change of 0.5% and 4%, respectively. If the same values are read out at $\chi = 10$, they are 0.333 and 0.391, respectively, by rigorous analysis while 0.334 and 0.409, respectively, for superposition analysis; hence, a minor change of 0.3% and 4% respectively occurs.

Variation of settlement influence factor for top of GP, I_{sp}, with stiffening factor, χ with effect of percentage length of stiffening from top of GP, η, on a single GP and a GP in a group of two, three and four is depicted in Figure 8. Figure 8 reveals that as stiffening factor, χ, and percentage length of stiffening from top of GP, η, are increased, the value of settlement influence factor for top of GP, I_{sp}, decreases, showing the very purpose of stiffening. Simultaneously, it may be realised that with increase in the number of GP in the group, the value of settlement influence factor for top of GP, I_{sp}, increases because of increased stress interaction in the group. The graph shows that for $L/d = 10$, $s/d = 2$, $\eta = 10\%$ and $\chi = 3$, the values of I_{sp}, at $K_{gp} = 50$, for single GP and a GP in a group of two, three and four GPs are, respectively, 0.210, 0.308, 0.402 and 0.480, while for $L/d = 10$, $s/d = 2$, $\eta = 40\%$ and $\chi = 3$, the values of I_{sp}, at $K_{gp} = 50$, are, respectively, 0.187, 0.283, 0.376 and 0.452 for single GP and a GP in a group of two, three and four partially stiffened GPs, thus showing a percentage decrease of 11, 8, 6 and 6, respectively. Similarly, it may also be observed that for $L/d = 10$, $s/d = 2$, $\eta = 10\%$ and $\chi = 3$, the values of I_{sp}, at $K_{gp} = 50$, for single GP and a GP in a group of two, three and four GPs are, respectively, 0.210, 0.308, 0.402 and 0.480, while for $L/d = 10$, $s/d = 2$, $\eta = 10\%$ and $\chi = 1$(un-stiffened), the values of I_{sp}, at $K_{gp} = 50$,are, respectively, 0.224, 0.322, 0.416 and 0.493 for single GP and a GP in a group of two, three and four partially stiffened GPs, thus showing a percentage decrease of 6, 4, 3 and 2, respectively. Further, it can be well noticed from the graph that stiffening factor is mainly effective in the range of 1–3 for reduction in I_{sp}.

The effect of changing the relative spacing, s/d, on settlement influence factor for top of GP, I_{sp}, is well depicted in Figure 9. It can well be seen from the graph that with increase in relative spacing, the values of I_{sp} decrease because increase in the relative spacing causes the stress interaction between GP to be reduced. For

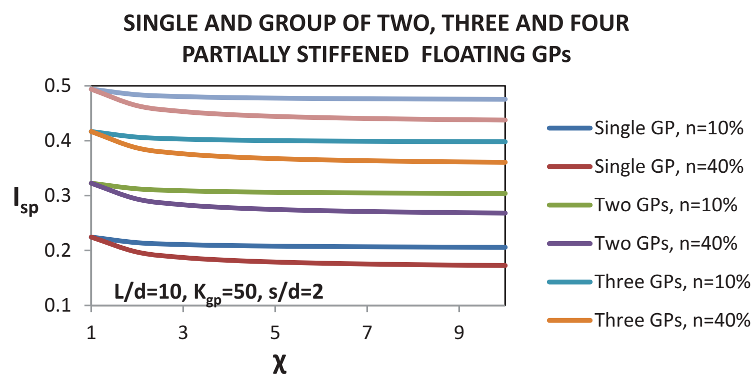

Figure 8. Variation of settlement influence factor for top of GP, I_{sp}, with stiffening factor, χ – effect of percentage length of stiffening from top of GP, η, on a GP, in a single, group of two partially stiffened floating GPs ($L/d = 10$, $K_{gp} = 50$, $s/d = 2$).

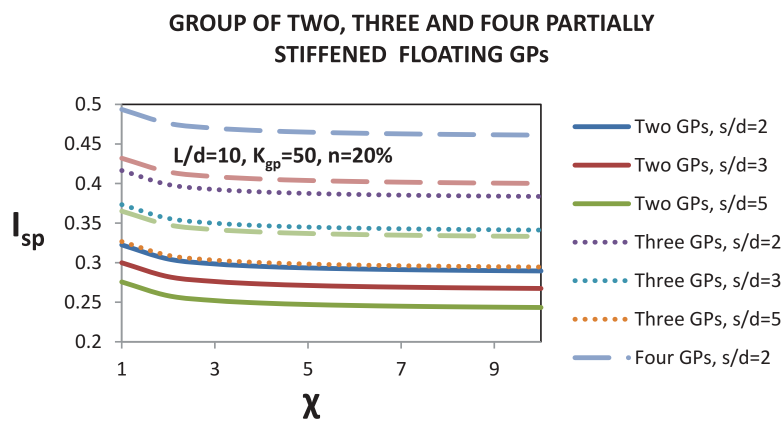

Figure 9. Variation of settlement influence factor for top of GP, I_{sp}, with stiffening factor, χ – effect of normalised spacing, s/d, on a GP in a group of two, three and four partially stiffened floating granular piles ($L/d = 10$, $K_{gp} = 50$, $\eta = 20\%$).

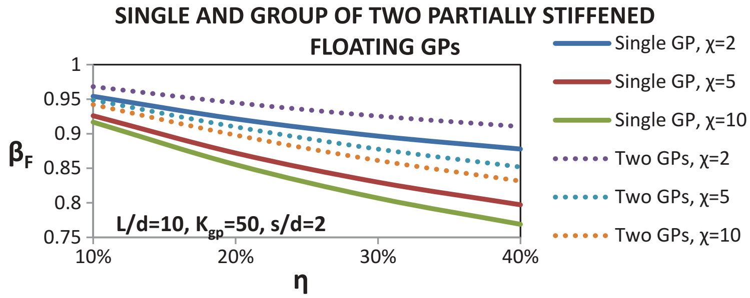

Figure 10. Variation of settlement reduction factor, β_F, with percentage length of stiffening from top, η – effect of number of GP, and stiffening factor, χ, on a single GP and GP in a group of two partially stiffened floating granular piles (L/d = 10, K_{gp} = 50, s/d = 2).

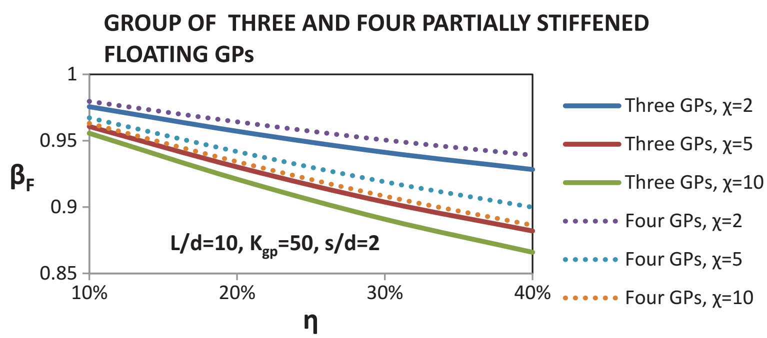

Figure 11. Variation of settlement reduction factor, β_F, with percentage length of stiffening from top of GP, η – effect of number of GP, and stiffening factor, χ, on a GP in a group of three and four partially stiffened floating granular piles (L/d = 10, K_{gp} = 50, s/d = 2).

example, for L/d = 10, s/d = 2, η = 20% and χ = 3, the values of I_{sp}, at K_{gp} = 50, for a GP in a group of two, three and four GPs are, respectively, 0.298, 0.392 and 0.469, while for L/d = 10, s/d = 5, η = 20% and χ = 3, the values of I_{sp}, at K_{gp} = 50, are, respectively, 0.252, 0.303 and 0.341 for a GP in a group of two, three and four partially stiffened GPs, thus showing a percentage decrease of 15, 22 and 27, respectively.

Figure 10 depicts the variation of next parameter, i.e. settlement reduction factor, $\boldsymbol{\beta}_{\mathbf{F}}$, with percentage length of stiffening from top, η with effect of number of GP, and stiffening factor, χ, on a single GP and GP in a group of two GPs. It is observed from the graph that with increase in the percentage length of stiffening from top, η and the stiffening factor, the value of β_F decreases. This value of β_F increases with number of GP in the group. For instance, it can be noticed from the graph that for L/d = 10, s/d = 2, η = 10% and χ = 2, the values of β_F, at K_{gp} = 50, for a single GP and a GP in a group of two GPs are, respectively, 0.954 and 0.968, while for L/d = 10, s/d = 2, η = 40% and χ = 2, the values of β_F, at K_{gp} = 50, are, respectively, 0.877 and 0.910 for a single GP and a GP in a group of two partially stiffened GPs, thus showing a percentage decrease of 8 and 6, respectively.

Figure 11 depicts the similar linear variation trends as shown in Figure 10 with the change that these are for a GP in a group of three and four GPs. Here, it can be noticed from the graph that for L/d = 10, s/d = 2, η = 10% and χ = 2, the values of β_F, at K_{gp} = 50, for a GP in a group of three and four GPs are, respectively, 0.975 and 0.979, while for L/d = 10, s/d = 2, η = 40% and χ = 2, the values of β_F, at K_{gp} = 50, are, respectively, 0.928 and 0.939 for a GP in a group of three and four partially stiffened GPs, thus showing a percentage decrease of 5 and 4, respectively.

Figure 12 shows the curved variation of settlement reduction factor, β_F, with stiffening factor, χ, with effect of number of GP and normalised length, L/d, on a single GP and GP in a group of two GPs. It is observed from the graph that with increase in the stiffening factor as well as normalised length, the value of β_F decreases. For instance, it can be noticed from the graph that for L/d = 10, s/d = 2, η = 20% and χ = 2, the values of β_F, at K_{gp} = 50, for a single GP and a GP in a group of two partially stiffened GPs are, respectively, 0.921 and 0.944, while for L/d = 40, s/d = 2, η = 20% and χ = 2, the values of β_F, at K_{gp} = 50, are, respectively, 0.801 and 0.830 for a single GP and a GP in a group of two GPs, thus showing a percentage decrease of 13 and 12, respectively.

Figure 13 depicts the similar curved variation trends as shown in Figure 12 although these are for a GP in a group of three and four GPs. Here, it can be noticed from the graph that for L/d = 10, s/d = 2, η = 20% and χ

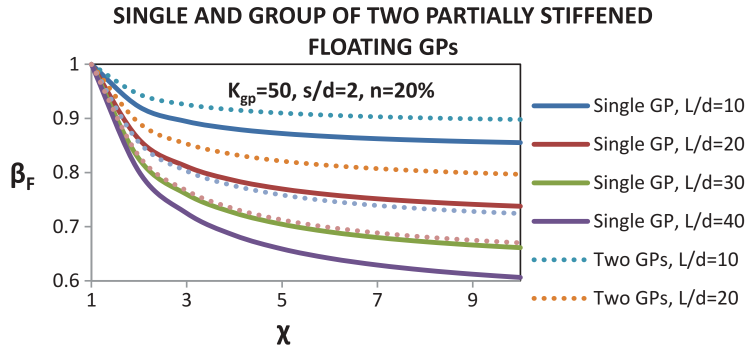

Figure 12. Variation of settlement reduction factor, β_F, with stiffening factor, χ – effect of number of GP, and normalised length, *L/d,* on a single GP and GP in a group of two partially stiffened floating granular piles (K_{gp} = 50, s/d = 2, η = 20%).

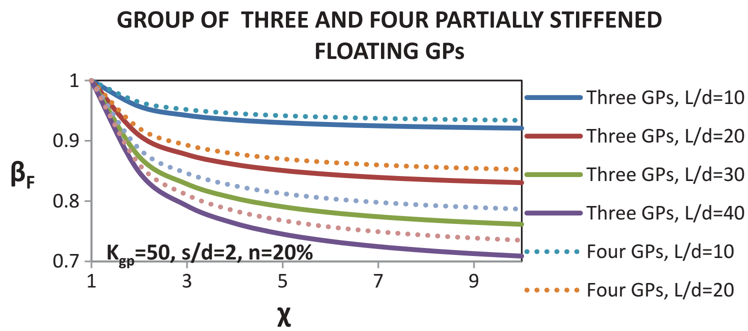

Figure 13. Variation of settlement reduction factor, β_F, with stiffening factor, χ – effect of number of GP, and stiffening factor, χ, on a GP in a group of three and four partially stiffened floating granular piles (K_{gp} = 50, s/d = 2, η = 20%).

= 2, the values of β_F, at K_{gp} = 50, for a GP in a group of three and four GPs are, respectively, 0.957 and 0.964, while for L/d = 40, s/d = 2, η = 20% and χ = 2, the values of β_F, at K_{gp} = 50, are, respectively, 0.848 and 0.861 for a GP in a group of three and four partially stiffened GPs, thus showing a percentage decrease of 11 and 10, respectively.

Figure 14 depicts the variation of settlement interaction factor, α, with relative stiffness of GP, $K_{gp,}$ by both rigorous and superposition analyses. The two analyses are found to be in close agreement. For the un-stiffened (χ = 1) condition of GP, the validation has been shown for the present work with that of Poulos and Mattes (1971). It may be noted from the graph that for L/d = 10, s/d = 3, χ = 8, η = 30%, K_{gp} = 60, the values of α for a GP in a group of three and four partially stiffened GPs are, respectively, 0.832 and 1.16 as obtained by rigorous analysis while for superposition

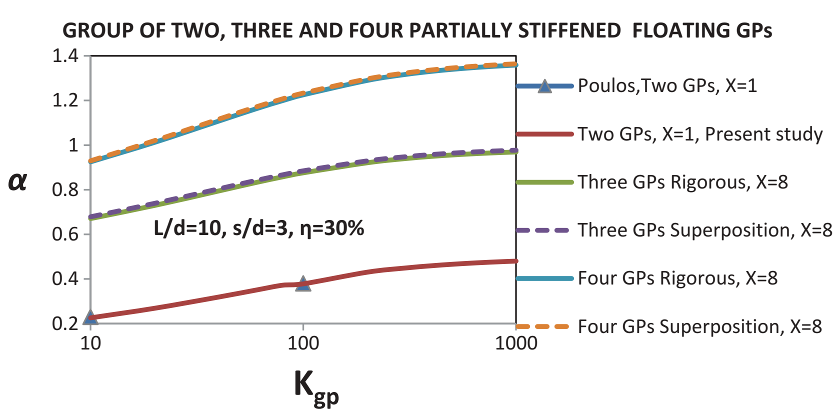

Figure 14. Variation of settlement interaction factor, α, with relative stiffness of GP, K_{gp}– effect of stiffening factor, χ, and number of piles, on a GP in a group of two, three and four partially stiffened floating GPs (L/d = 10, s/d = 3, η = 30%) Rigorous and Superposition analysis and validation with Poulos and Mattes (1971).

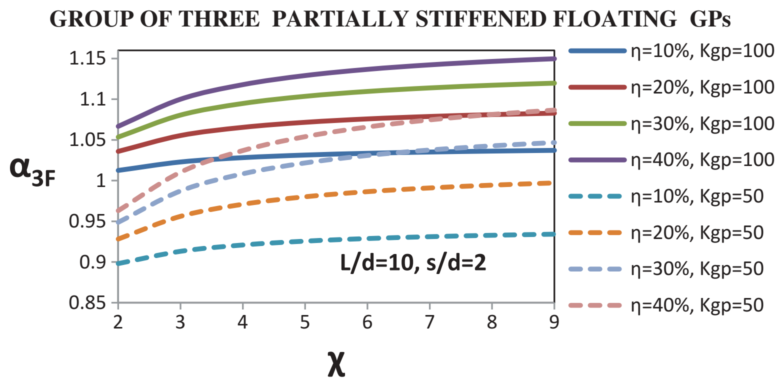

Figure 15. Variation of settlement interaction factor, α_{3F}, with stiffening factor, χ – effect of relative stiffness of GP, K_{gp} and percentage length of stiffening from top, η, on a GP in a group of three partially stiffened floating GPs (L/d = 10, s/d = 2).

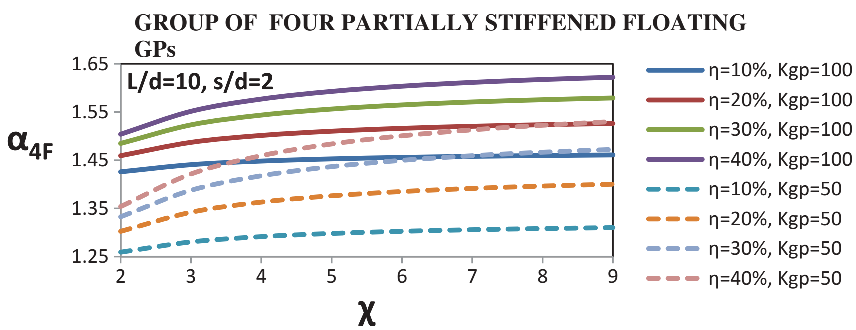

Figure 16. Variation of settlement interaction factor, α_{4F}, with stiffening factor, χ – effect of relative stiffness of GP, K_{gp} and percentage length of stiffening from top, η, on a GP in a group of four partially stiffened floating granular piles (L/d = 10, s/d = 2).

the values are obtained as 0.841 and1.17, showing almost no variation in two analyses.

In Figure 15, the dual effect of relative stiffness of GP, $K_{gp,}$ and percentage length of stiffening from top of GP, η, is shown. As seen, the values of settlement interaction factor, $\alpha_{3F,}$ increase with stiffening factor, χ, and percentage length of stiffening from top of GP, η, because as the stiffening parameters increase, the top displacement of a GP in a group of GPs in both granular pile groups, i.e. three and single granular pile, decreases, but the decrease in the single is relatively more as compared to that in the three granular pile group, hence the effect. In this case for a GP in a group of three partially stiffened GPs with L/d = 10, s/d = 2, χ = 2 with, K_{gp} = 100 and η = 10%, 20%, 30% and 40%, the values of settlement interaction factor, α_{3F}, are, 1.01, 1.03, 1.05 and 1.06, respectively, and for L/d = 10, s/d = 2, χ = 2 K_{gp} = 50 and η = 10%, 20%, 30% and 40%, the values of settlement interaction factor, α_{3F}, are 0.89, 0.92, 0.94 and 0.96, respectively.

Figure 16 shows the variations trends of settlement interaction factor, α_{4F}, with stiffening factor, χ for a GP in a group of four partially stiffened GPs. It can be well noticed that for L/d = 10, s/d = 2, χ = 2 with, K_{gp} = 100 and η = 10%, 20%, 30% and 40%, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.42, 1.45, 1.48 and 1.50, respectively, and for L/d = 10, s/d = 2, χ = 2 with, K_{gp} = 50 and η = 10%, 20%, 30% and 40%, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.25, 1.30, 1.33 and 1.35, respectively. At the same time, it may also be observed that for L/d = 10, s/d = 2, χ = 4, with K_{gp} = 100 and η = 10%, 20%, 30% and 40%, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.44, 1.50, 1.54 and 1.57, respectively, and for K_{gp} = 50 and η = 10%, 20%, 30% and 40%, the values of α_{4F} are 1.29, 1.36, 1.41 and 1.45, respectively.

In Figure 17, the effect of normalised spacing, s/d, and relative stiffness of GP, $K_{gp,}$ are shown simultaneously on settlement interaction factor, $\alpha_{3F,}$ for a GP in a group of three partially stiffened GPs. It is found that as normalised spacing, s/d, increases, the values of settlement interaction factor, $\alpha_{3F,}$ decrease because the interaction of one granular pile with the other decreases as the spacing increases. The effect of relative stiffness of GP, K_{gp} and percentage length of stiffening from top, η, is already explained. In this case, it can be seen that for χ = 3, L/d = 10, η = 10% with K_{gp} = 100 and s/d = 2, 3, 4 and 5, the values of settlement interaction factor, $\alpha_{3F,}$ are 1.02, 0.79, 0.65 and 0.55, respectively, while for χ = 3, L/d = 10, η = 10%, K_{gp} = 50 and s/d = 2, 3, 4 and 5,

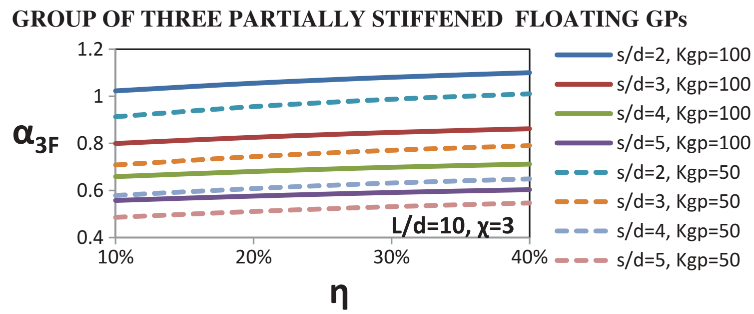

Figure 17. Variation of settlement interaction factor, α_{3F}, with percentage length of stiffening from top of GP, η – effect of relative stiffness of GP, K_{gp} and normalised spacing, s/d, on a GP in a group of three partially stiffened floating granular piles (L/d = 10, χ = 3).

the values of settlement interaction factor, $\alpha_{3F,}$ are 0.91, 0.70, 0.57 and 0.48, respectively.

It can be seen from Figure 18 that for a GP in a group of four partially stiffened GPs with L/d = 10, χ = 3, η = 10 with K_{gp} = 100 and s/d = 2, 3, 4 and 5, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.44, 1.11, 0.91 and 0.76, respectively, while for L/d = 10, χ = 2, η = 10 with K_{gp} = 50 and s/d = 2, 3, 4 and 5, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.28, 0.98, 0.80 and 0.66, respectively.

With increase in values of relative length, L/d, the values of settlement interaction factor, $\alpha_{3F,}$ are decreasing because more lengthy granular pile implies more shear stresses development, hence less top displacement for a GP in a group of three partially stiffened GPs, so lesser value of settlement interaction factor, α_{3F}, as depicted in Figure 19. It is noticeable that for a GP in a group of three partially stiffened GPs with s/d = 2, η = 20%, χ = 2 with, K_{gp} = 100 and L/d = 10, 20, 30 and 40, the values of settlement interaction factor, $\alpha_{3F,}$ are 1.03, 0.92, 0.85 and 0.82, respectively, and for s/d = 2, η = 20%, χ = 2 K_{gp} = 50, and L/d = 10, 20, 30 and 40, the values of settlement interaction factor, $\alpha_{3F,}$ are 0.92, 0.83, 0.79 and 0.78, respectively. Rest trends are the same.

From Figure 20, the variation of settlement interaction factor, α_{4F}, with stiffening factor χ is observable for a GP in a group of four partially stiffened GPs. It can

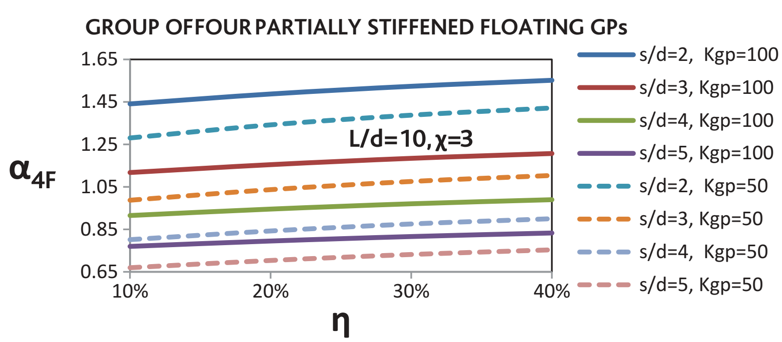

Figure 18. Variation of settlement interaction factor, α_{4F}, with percentage length of stiffening from top of GP, η – effect of relative stiffness of GP, K_{gp} and normalised spacing, s/d, on a GP in a group of four partially stiffened floating granular piles (L/d = 10, χ = 3).

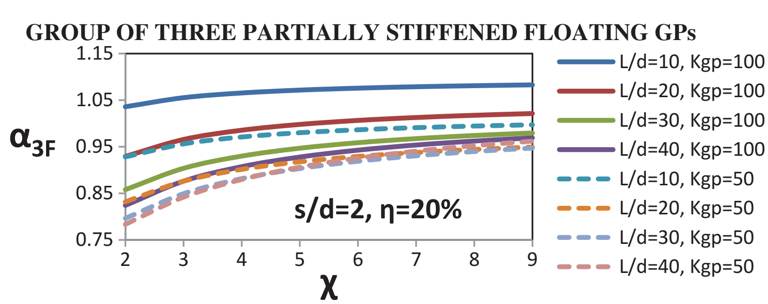

Figure 19. Variation of settlement interaction factor, α_{3F}, with stiffening factor, χ – effect of relative stiffness of GP, K_{gp} and relative length, L/d, on a GP in a group of three partially stiffened floating granular piles (s/d = 2, η = 20%).

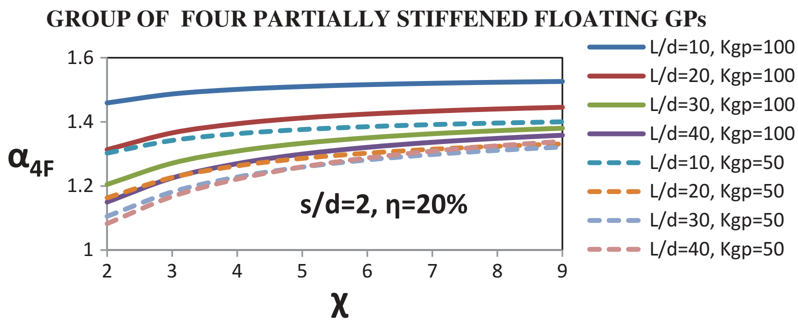

Figure 20. Variation of settlement interaction factor, α_{4F}, with stiffening factor, χ – effect of relative stiffness of GP, K_{gp} and relative length, L/d, on a GP in a group of four partially stiffened floating granular piles (s/d = 2, η = 20%).

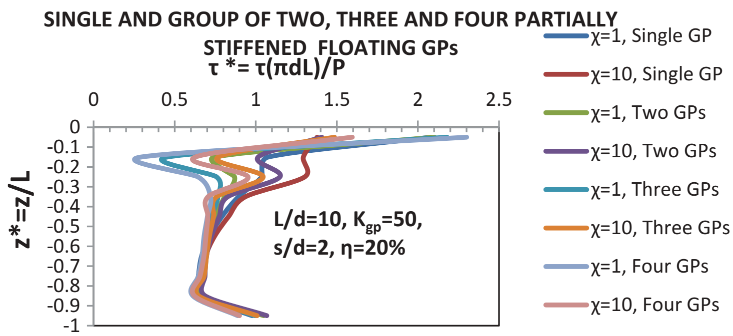

Figure 21. Variation of normalised shear stresses, $\tau^* = \tau(\pi dl)/P$, with normalised depth, $z^* = z/L$–effect of stiffening factor, χ, on a GP for a single and group of two, three and four group of partially stiffened floating granular piles (L/d = 10, K_{gp} = 50, s/d = 2, η = 20%).

be seen from the graph that for $s/d = 2$, $\eta = 20\%$, $\chi = 2$ with, $K_{gp} = 100$ and, L/d = 10, 20, 30 and 40, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.45, 1.31, 1.20 and 1.15, respectively, while in case of $s/d = 2$, $\eta = 20\%$, $\chi = 2$ with, $K_{gp} = 50$ and L/d = 10, 20, 30 and 40, the values of settlement interaction factor, $\alpha_{4F,}$ are 1.30, 1.16, 1.10 and 1.08, respectively.

In Figure 21, the variations of normalised shear stresses, $\tau^* = \tau(\pi dl)/P$, are shown with normalised depth, $z^* = z/L$, for a single GP and a GP in a group of two, three and four group of partially stiffened floating granular pile for stiffened and un-stiffened conditions. In general, shear stresses in case of a GP in a group of two, three and four GPs are more in comparison to single GP alone, due to the effect of adjacent GPs on a GP. With the stiffening of GPs, the shear stresses get redistributed and transferred to a lower portion of GP and base. The pile displacements of nodes of top stiffened portion of GP are less; thus, in order to maintain the compatibility of displacement of nodes for soil and pile, the interfacial shear stresses get reduced in top portion and transferred to lower part of GP. It has been observed that for $L/d = 10$, $K_{gp} = 50$, $s/d = 2$, $\eta = 20\%$, $z/L = -0.05$ with, $\chi = 1$ and a GP in single and a group of two, three and four partially stiffened GPs, the values of $\tau^* = \tau(\pi dL)/P$ are 2.08, 2.10, 2.17 and 2.30, respectively, while for $L/d = 10$, $K_{gp} = 50$, s/d = 2, $\eta = 20\%$, z/L = –0.05 with $\chi = 10$ and a GP in single and a group of two, three and four partially stiffened GPs, the values of $\tau^* = \tau(\pi dL)/P$ are 1.37, 1.40, 1.48 and 1.59, respectively, thus causing a percentage reduction of 34, 33, 32 and 31, respectively.

From Figure 22, the effect of percentage length of stiffening from top, η, is well visible on normalised shear stresses $\tau^* = \tau(\pi dl)/P$. As seen from graph, it may be observed that for $L/d = 10$, $K_{gp} = 100$, $s/d = 2$, $\chi = 2$, $z/L = -0.05$ with $\eta = 20\%$ and a GP in a group of two, three and four partially stiffened GPs, the values of normalised shear stresses, $\tau^* = \tau(\pi dL)/P$, are 1.38, 1.49 and 1.61, respectively.

In Figure 23, the effect of stiffening factor, χ, is visible on the next parameter, i.e. the percentage load transferred to the base for a GP in a group of three GPs. It may be noticed from the graph that as stiffening factor, χ, increases, the percentage load transferred to the base for a GP in a group of three GPs increases marginally owing to more stiffness, a similar effect is also seen with the parameter percentage length of stiffening from top, η. The graph reveals that for $L/d = 10$, $s/d = 2$, $\chi = 2$, with, $K_{gp} = 100$ and η = 10%, 20%, 30% and 40%, the values of percentage load shared by base, $(P_b/P)_{3F} \times 100$, are 16.42, 16.46, 16.53 and 16.61, respectively, and for $K_{gp} = 50$ and η = 10%, 20%, 30% and 40%, the values of percentage load shared by base, $(P_b/P)_{3F} \times 100$, are 14.36, 14.43, 14.53 and 14.67, respectively.

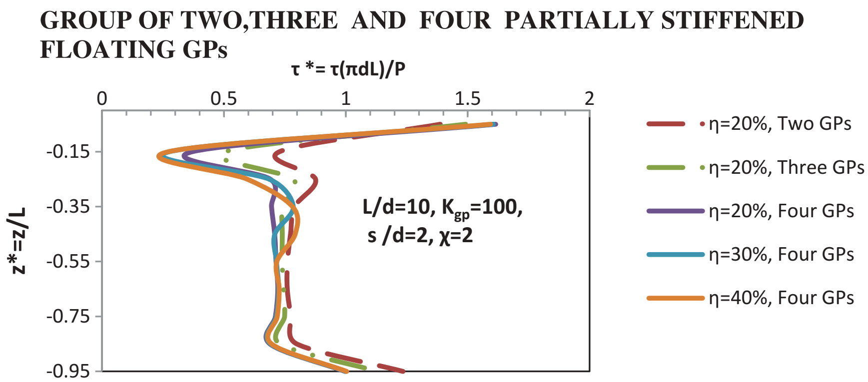

Figure 22. Variation of normalised shear stresses, $\tau^* = \tau(\pi dl)/P$, with normalised depth, $z^* = z/L$ – effect of percentage length of stiffening from top, η, on a GP in a group of two, three and four group of partially stiffened floating granular piles (L/d = 10, K_{gp} = 100, s/d = 2, χ = 2).

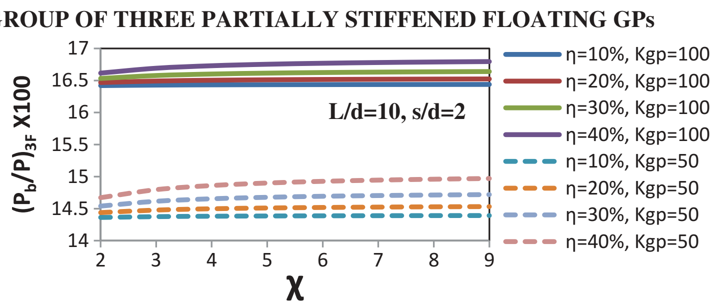

Figure 23. Variation of percentage load shared by base, $(P_b/P)_{3F}$ ×100, with stiffening factor, χ – effect of relative stiffness of GP, K_{gp} and percentage length of stiffening from top, η, on a GP in a group of three partially stiffened floating granular piles (L/d = 10, s/d = 2).

Figure 24 shows the similar variation trends as shown in Figure 23, although it is for a GP in a group of four GPs. It is clearly observable from the graph that as stiffening factor χ increases, the values of percentage load transferred to the base, $(P_b/P)_{4F}$ ×100, for a GP in a group of four GPs increase, clearly showing the effect of stiffening. It has been observed that for L/d = 10, s/d = 2, χ = 2 with K_{gp} = 100 and η = 10%, 20%, 30% and 40%, the values of percentage load shared by base, $(P_b/P)_{4F}$ ×100, are 20.82, 20.88, 20.94 and 21.03, respectively, while for L/d = 10, s/d = 2, χ = 2 with K_{gp} = 50 and η = 10%, 20%, 30% and 40%, the values of percentage load shared by base, $(P_b/P)_{4F}$ ×100, are 18.47, 18.55, 18.66 and 18.81, respectively.

In Figure 25, the variation of percentage load shared by base, $(P_b/P)_{3F}$ ×100, with percentage length of stiffening from top, η, is shown for a GP in a group of three partially stiffened GPs. It can be seen that with increase in values of

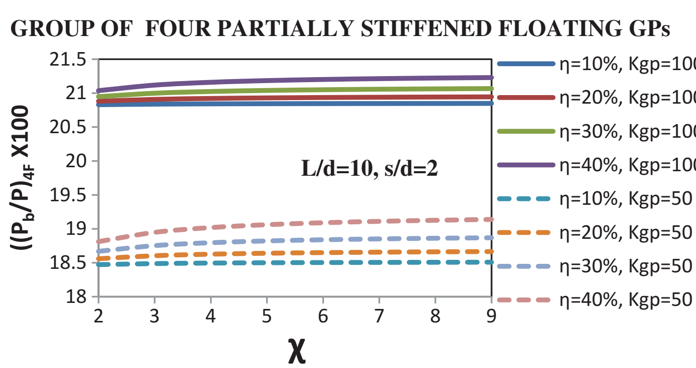

Figure 24. Variation of percentage load shared by base, $(P_b/P)_{4F}$ ×100, with stiffening factor, χ – effect of relative stiffness of GP, K_{gp}, and percentage length of stiffening from top, η, on a GP in a group of four partially stiffened floating GPs (L/d = 10, s/d = 2).

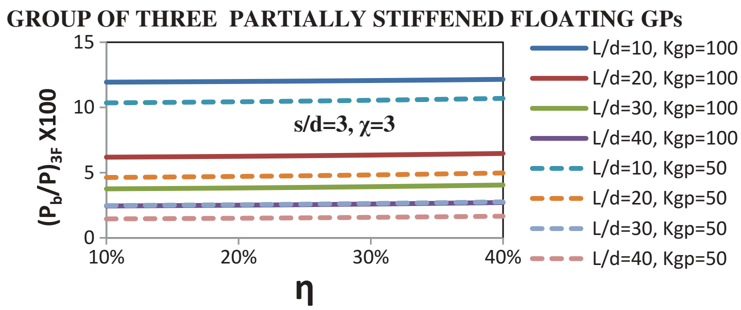

Figure 25. Variation of percentage load shared by base, $(P_b/P)_{3F}$ ×100 with percentage length of stiffening from top, η – effect of relative stiffness of GP, K_{gp} and relative length, L/d, on a GP in a group of three partially stiffened floating granular piles (s/d = 3, χ = 3).

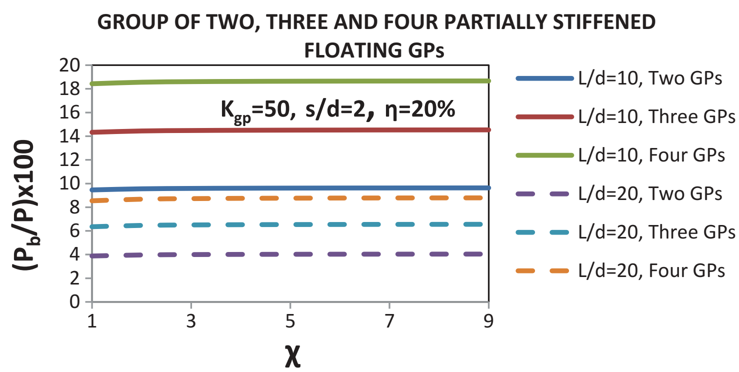

Figure 26. Variation of percentage load shared by base, (P_b/P) ×100, with stiffening factor, χ–effect of relative length, L/d, and group of two, three and four partially stiffened floating granular piles (K_{gp} = 50, s/d = 2, η = 20%).

relative length, L/d, the values of percentage load transferred to the base decrease. It can well be noticed that in this case for $s/d = 3$, $\chi = 3$, $\eta = 10\%$, with $K_{gp} = 100$ and L/d = 10, 20, 30 and 40, the values of percentage load shared by base, $(P_b/P)_{3F}$ ×100, are 11.93, 6.18, 3.75 and 2.44, respectively, and for $s/d = 3$, $\chi = 3$, $\eta = 10\%$, $K_{gp} = 50$ and L/d = 10, 20, 30 and 40, the values of percentage load shared by base, $(P_b/P)_{3F}$ ×100, are 10.35, 4.62, 2.47 and 1.45, respectively.

Figure 26 compares the effect of stiffening factor, χ, as well as normalised length L/d, on the percentage load shared by base for a GP in the group of two, three and four partially stiffened GPs. It has been observed that for $K_{gp} = 50$, $s/d = 2$, $\chi = 2$, $\eta = 10$ with $L/d = 10$ and a GP in a group of two, three and four partially stiffened GPs, the values of percentage load shared by base, (P_b/P) ×100, are 9.55, 14.43 and 18.55, respectively. The value of load shared by the base increases as the number of piles increases in the group because of more interference. Although it can be observed that there is a very minor change in the value of percentage load shared by the base with increase in the stiffening factor, χ.

In Figure 27, the effect of stiffening factor, χ, is depicted, as already described but the effect of normalised spacing, s/d, can be well seen that as the value of normalised spacing, s/d, increases, the interaction of one granular pile with other decreases leading to reduction in percentage load transferred to the base of the granular pile. It can be seen that for $K_{gp} = 50$, $L/d = 20$, $\eta = 20\%$, $\chi = 2$ with $s/d = 2$ and a GP in a group of two, three and four partially stiffened GPs, the values of percentage load shared by base, (P_b/P) ×100, are 3.96, 6.46 and 8.67, respectively, while for $K_{gp} = 50$, $L/d = 20$, η = 20%, $\chi = 2$ with $s/d = 3$ and a GP in a group of two, three and four partially stiffened GPs, the values of percentage load shared by base, (P_b/P) ×100, are 3.19, 4.67 and 5.92, respectively.

6.0 Conclusions

Mathematical formulation is developed using Mindlin equation to incorporate the stiffening parameter for a group of two, three and four partially stiffened floating granular pile. A new pile displacement matrix is formulated in order to incorporate the stiffening parameters, i.e. stiffening factor, χ, and percentage length of stiffening from top, η. The analysis is carried out using finite difference technique and is based on elastic continuum approach. It can be concluded that

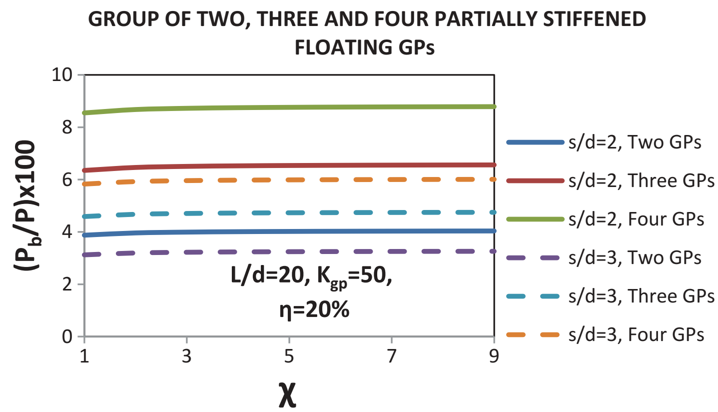

Figure 27. Variation of percentage load shared by base, $(P_b/P) \times 100$, with stiffening factor, χ – effect of normalised spacing, s/d, and group of two, three and four partially stiffened floating granular piles (L/d = 20, K_{gp} = 50, η = 20%).

(1) In case of group of three partially stiffened floating granular piles, it was found that as the stiffening factor, χ, increases from 1 to 8 for the relative stiffness of GP, K_{gp} = 100, relative length, L/d = 10, normalised spacing, s/d = 2, percentage length of stiffening from top, η = 20%. There is a reduction in settlement influence factor for top of GP, $I_{sp,}$ by 7.5%.

(2) With increase in percentage length of stiffening from top of GP, η from 10% to40%, relative length, L/d = 10, normalised spacing, s/d = 2, the relative stiffness of GP, K_{gp} = 60, stiffening factor, χ = 2 settlement influence factor for top of GP, I_{sp}, for a group of three partially stiffened floating GPs decreases by 5%.

(3) The percentage load transferred to the base of a GP in a group of three partially stiffened floating GPs increases by 0.6% by increasing stiffening factor, χ from 2 to 5 at relative stiffness of GP, K_{gp} = 100, relative length, L/d = 10, normalised spacing, s/d = 2, percentage stiffening length of the pile, η = 10% and with percentage length of stiffening from top of GP, η varying between 10% and 40% at relative stiffness of GP, K_{gp} = 60, relative length, L/d = 20, normalised spacing, s/d = 2, stiffening factor, χ = 2 percentage load transferred to the base of a GP in a group of three GPs increases by 4.5% and also found to increase with relative stiffness of GP and decreases with relative length, normalised spacing.

(4) The percentage load transferred to the base of a GP in a group of four partially stiffened floating GPs increases by 0.2% by increasing stiffening factor, χ from 2 to 5 at relative stiffness of GP, K_{gp} = 100, relative length, L/d = 10, normalised spacing, s/d = 2, percentage length of stiffening from top of GP, η = 10% and with percentage length of stiffening from top, η varying between 10% and 40% at relative stiffness of GP, K_{gp} = 60, relative length, L/d = 20, normalised spacing, s/d = 2, stiffening factor, χ = 2 percentage load transferred to the base of a GP in a group of four GPs increases by 3% and found to increase with relative stiffness of GP and decreases with relative length, normalised spacing.

(5) The normalised shear stresses are reducing at the top by 22% in case of group of three partially stiffened floating GPs by increasing the stiffening factor, χ, from 1 to 8 while by 20% for both groups of three/four partially stiffened GPs at relative stiffness of GP, K_{gp} = 100, relative length, L/d = 10, normalised spacing, s/d = 2, percentage length of stiffening from top of GP, η = 30%, i.e. in nutshell, there is a redistribution of normalised shear stresses by effect of stiffening and normalised shear stresses are shifting toward the base of the GP.

(6) The settlement interaction factor, $\alpha_{3F,}$ is found to decrease by increasing the value of relative length, L/d, and with increasing value of normalised spacing, s/d, while it increases with increase in relative stiffness of granular pile, $K_{gp.}$ For an increase of stiffening factor, χ, from 1 to 8 at relative length, L/d = 10, normalised spacing, s/d = 3, percentage length of stiffening from top of GP, η = 30%, relative stiffness of granular pile, K_{gp} = 60, the settlement interaction factor, $\alpha_{3F,}$ increases by 20%, and with increase in percentage length of stiffening, η from 10% to 40% for relative length, L/d = 10, normalised spacing, s/d = 2, relative stiffness of granular pile, K_{gp} = 60, stiffening factor, χ = 2, the settlement interaction factor, $\alpha_{3F,}$ increases by 6%.

(7) The settlement interaction factor, $\alpha_{4F,}$ is found to decrease by increasing the value of relative length, L/d and with increasing value of normalised spacing, s/d, while increases with increase in relative stiffness of granular pile, $K_{gp,.}$ For an increase of stiffening factor, χ, from 1 to 8 at relative length, L/d = 10, normalised spacing, s/d = 3, percentage length of stiffening from top of GP, η = 30%, relative stiffness of granular pile, K_{gp} = 60, the settlement interaction factor, $\alpha_{4F,}$ increases by 21%, and with increase in percentage length of stiffening, η from 10% to 40% for relative length, L/d = 20, normalised spacing, s/d = 2, relative stiffness of granular pile, K_{gp} = 60, stiffening factor, χ = 2, the settlement interaction factor, $\alpha_{4F,}$ increases by 7%.

(8) For a GP in a group of three partially stiffened floating GPs, the settlement reduction factor, $\beta_{F,}$ decreases with increasing relative length, L/d, normalised spacing, s/d. As percentage length of stiffening from top, η, increases from 10% to 40% with relative length, L/d = 20, normalised spacing, s/d = 2, relative stiffness of granular pile, K_{gp} = 60, stiffening factor, χ = 2, the settlement reduction factor, $\beta_{F,}$ reduces by 8% and with an increase in stiffening factor, χ, from 1 to 8 with relative length, L/d = 10, normalised spacing, s/d = 3, relative stiffness of granular pile, K_{gp} = 60, percentage length of stiffening from top of GP, η = 30%, the settlement reduction factor, $\beta_{F,}$ reduces by 11%.

(9) For a GP in a group of four partially stiffened floating GPs, the settlement reduction factor, $\beta_{F,}$ decreases with increasing relative length, L/d, normalised spacing, s/d. As percentage length of stiffening from top of GP, η increases from 10% to 40% with relative length, L/d = 10, normalised spacing, s/d = 2, relative stiffness of granular pile, K_{gp} = 60, stiffening factor, χ = 2, the settlement reduction factor, $\beta_{F,}$ reduces by 7% and with increase in stiffening factor, χ, from 1 to 8 with relative length, L/d = 10, normalised spacing, s/d = 3, relative stiffness of granular pile, K_{gp} = 60, percentage length of stiffening from top, η = 30% the settlement reduction factor, $\beta_{F,}$ reduces by 9%.

(10) In general, the values of settlement influence factor for top are found to increase with increase in number of piles in the group for partially stiffened floating GPs.

(11) The superimposed values are found to be in close proximity with those obtained by rigorous analysis.

(12) In general, the stiffening factor is found to be effective in the range of 1–3.

(13) The effectiveness of stiffening is mainly observed in the working range of values of K_{gp}, i.e. 10–100 for GP. The effectiveness of percentage length of stiffening from top of GP is observed mainly up-to 40%.

(14) The designs charts are prepared in terms of non-dimensional parameters, which can be used for design purpose by engineers with the application of stiffening.

Disclosure statement

No potential conflict of interest was reported by the authors.

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