2

Design Examples

Design Example 1: Reinforced Concrete T-Beam Bridge

Problem Statement

A bridge will be designed with a span length of 50 ft. The superstructure consists of five beams spaced at 10 ft with a concrete deck slab of 9 in. The overall width of the bridge is 48 ft and the clear (roadway) width is 44 ft, 6 in. Design the superstructure of a reinforced, cast-in-place concrete T-beam bridge using the following design specifications.

The three Load Combination Limit States considered are Strength I, Fatigue II, and Service I.

C&P	Curb and parapet cross section	3.37 ft^2
E_c	Modulus of elasticity of concrete	4×10^3 $kips/in^2$
f'_c	Specified compressive strength of concrete	4.5 $kips/in^2$
f_y	Specified yield strength of epoxy-coated reinforcing bars	60 $kips/in^2$
w_c	Self-weight of concrete	0.15 $kips/ft^3$
w_{FWS}	Future wearing surface load	0.03 $kips/ft^2$

Figure 2.1 shows the elevation view, section view, and overhang detail of the reinforced concrete T-beam bridge described

Solution

Step 1: Design T-Beam Using Strength I Limit State

The factored load, Q, is calculated using the load factors given in AASHTO Tables 3.4.1-1 and 3.4.1-2.

$$Q = 1.25\ DC + 1.5\ DW + 1.75(TL+LN)$$

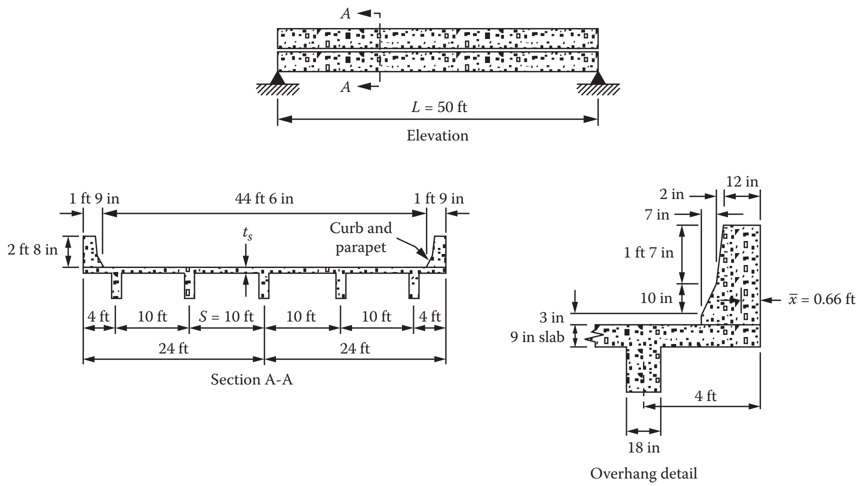

FIGURE 2.1
T-beam design example.

Find the flange width and web thickness.

A Art. 5.14.1.5.1

For the top flange of T-beams serving as deck slabs, the minimum concrete deck must be greater than or equal to 7 in. First, try using a slab with a thickness, t_s, of 9 in.

A Arts. 5.14.1.5.1a, 9.7.1.1

The minimum web thickness is 8 in.

A Art. 5.14.1.5.1c; Com. 5.14.1.5.1c

Minimum concrete cover for main epoxy-coated bars shall be 1 in. Use 1.5 in cover for main and stirrup bars.

A Tbl. 5.12.3-1; Art. 5.12.4

Find the width of T-beam stem (web thickness), b.

A Art. 5.10.3.1.1

The minimum width, b_{min}, is found as follows:

Assume two layers of no. 11 bars for positive reinforcement.

d_b for a no. 11 bar = 1.41 in

Four no. 11 bars in a row and no. 4 stirrup bars require a width of

$$b_{min} = 2\ (2.0 \text{ in cover and no. 4 stirrup}) + 4\ d_b + 3\ (1.5\ d_b)$$

$$= 2\ (2.0 \text{ in}) + 4\ (1.41 \text{ in}) + 3\ (1.5 \times 1.41 \text{ in}) = 16 \text{ in}$$

Try b_w = width of stem = 18 in.

Find the beam depth including deck.

A Tbl. 2.5.2.6.3-1

$$h_{min} = 0.070\ L = 0.070\ (50\ ft \times 12\ in) = 42\ in.$$

Try h = beam width = 44 in.

Find the effective flange width.

A Art. 4.6.2.6.1

Find the effective flange width, where

b_e	effective flange for exterior beams	in
b_i	effective flange width for interior beams	in
b_w	web width	18 in
L	effective span length (actual span length)	50 ft
S	average spacing of adjacent beams	10 ft
t_s	slab thickness	9 in

The effective flange width for interior beams is equal to one-half the distance to the adjacent girder on each side of the girder,

$$b_i = S = 10\ ft \times 12\ in = 120\ in$$

The effective flange width for exterior beams is equal to half of one-half the distance to the adjacent girder plus the full overhang width,

$$b_e = ½\ (10\ ft \times 12\ in) + (4\ ft \times 12\ in) = 108\ in$$

Find the interior T-beam section. Please see Figure 2.2.

The section properties of the preceding interior T-beam are as follows.

The area of the T-beam is

$$A = (9\ in)\ (120\ in) + (35\ in)\ (18\ in) = 1710\ in^2$$

The center of gravity from the extreme tension fiber is

$$y_t = \frac{\sum y_t A}{\sum A} = \frac{(9\ in)(120\ in)(35\ in + 4.5\ in) + (35\ in)(18\ in)(17.5\ in)}{1710.0\ in^2}$$
$$= 31.4\ in$$

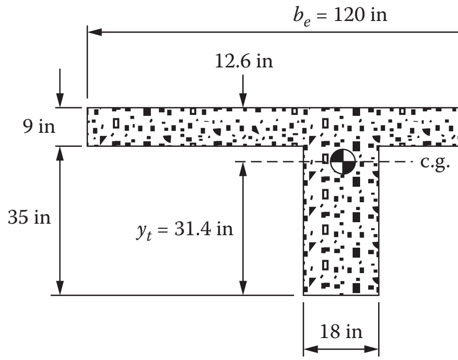

FIGURE 2.2
Interior T-beam section.

The moment of inertia (the gross concrete section) about the center of gravity is

$$I_g = I = \Sigma\left(\bar{I} + Ad^2\right)$$

$$= \frac{(120\text{ in})(9\text{ in})^3}{12} + (120\text{ in})(9\text{ in})(12.5\text{ in} - 4.5\text{ in})^2$$

$$+ \frac{(18\text{ in})(35\text{ in})^3}{12} + (18\text{ in})(35\text{ in})(31.4\text{ in} - 17.5\text{ in})^2$$

$$I_g = 264{,}183.6\text{ in}^4$$

The T-beam stem is

$$(35\text{ in})(18\text{ in}) = 630\text{ in}^2$$

The section modulus at bottom fiber is

$$S = \frac{I_g}{y_t} = \frac{264{,}183.6\text{ in}^4}{31.4\text{ in}} = 8413.5\text{ in}^3$$

Find the number of design lanes.

A Art. 3.6.1.1.1

The number of design lanes, N_L, is the integer portion of the ratio of the clear road width divided by a 12 ft traffic lane width.

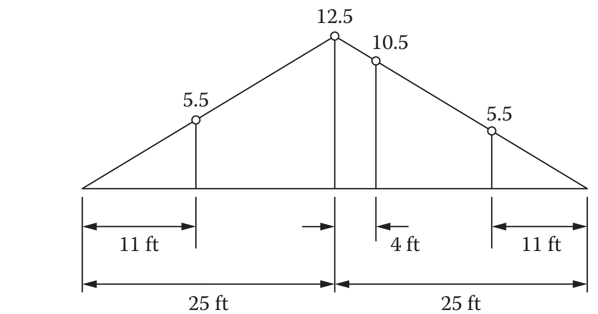

FIGURE 2.3a
Influence lines for moment at midspan.

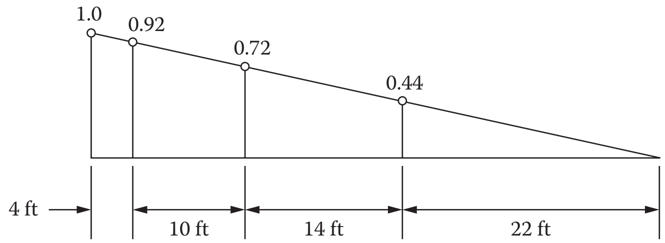

FIGURE 2.3b
Influence lines for shear at support.

$$N_L = \frac{w}{12\,\frac{ft}{lane}} = \frac{44.5\text{ ft}}{12\text{ ft}}$$

$$N_L = 3.7\text{ (3 lanes)}$$

Find the truck load moments and shears.

Design truck load (HS-20) for moment at midspan (Figure 2.3a).
Design tandem load for moment at midspan (Figure 2.3b).
The design truck load (HS-20) is shown in Figure 2.4. Figures 2.5 and 2.6 show the design tandem load position for moment at midspan and the design truck load (HS-20) position for shear at support, respectively. The design tandem load position for shear at support is shown in Figure 2.7.

NOTE: The maximum design truck or design tandem moment is generally used with the HL-93 center axle at midspan.

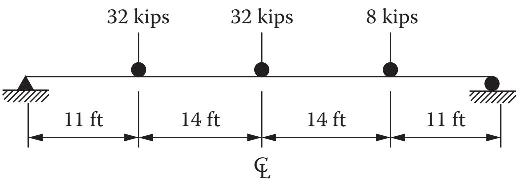

FIGURE 2.4
Design truck (HS-20) position for moment at midspan.

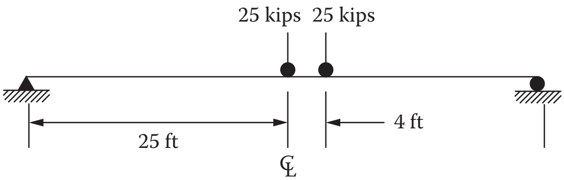

FIGURE 2.5
Design tandem load position for moment at midspan.

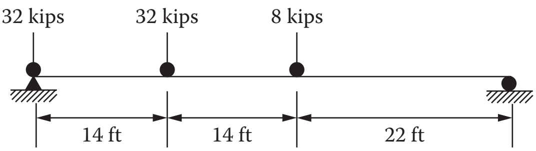

FIGURE 2.6
Design truck (HS-20) position for shear at support.

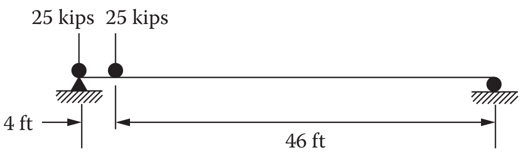

FIGURE 2.7
Design tandem load position for shear at support.

The truck load moment per lane is

$$M_{tr} = 32 \text{ kips } (12.5 \text{ ft} + 5.5 \text{ ft}) + 8 \text{ kips } (5.5 \text{ ft})$$

$$= 620 \text{ kip-ft}$$

The tandem load moment per lane is

$$M_{tandem} = 25 \text{ kips } (12.5 \text{ ft} + 10.5 \text{ ft})$$

$$= 575 \text{ kip-ft}$$

The truck load shear per lane is

$$V_{tr} = 32 \text{ kips } (1 + 0.72) + 8 \text{ kips } (0.44)$$

$$= 58.6 \text{ kips}$$

The tandem shear per lane is

$$V_{tandem} = 25 \text{ kips } (1 + 0.92)$$

$$= 48 \text{ kips}$$

The lane load per lane is

$$M_{ln} = \frac{wL^2}{8} = \frac{\left(0.64 \dfrac{\text{kips}}{\text{ft}}\right)(50 \text{ ft})^2}{8} = 200 \text{ kip-ft}$$

The lane load shear per lane is

$$V_{ln} = \frac{wL}{2} = \frac{\left(0.64 \dfrac{\text{kips}}{\text{ft}}\right)(50 \text{ ft})}{2} = 16 \text{ kips}$$

Find the live load distribution factors for moments, DFM.

For cast-in-place concrete T-beam, the deck type is (e).

A Tbl. 4.6.2.2.1-1

For interior beams,

A Tbl. 4.6.2.2.2; A Art. 4.6.2.2.2b; Tbl. 4.6.2.2.2b-1; Appendix A*

K_g	longitudinal stiffness		in^4
L	span length of beams		50 ft
N_b	number of beams		5
S	spacing of beams		10 ft
t_s	slab thickness		9 in
$3.5 \text{ ft} \le S \le 16 \text{ ft}$		$S = 10$ ft	*OK*
$4.5 \text{ in} \le t_s \le 12 \text{ in}$		$t_s = 9$ in	*OK*
$20 \text{ ft} \le L \le 240 \text{ ft}$		$L = 50$ ft	*OK*
$N_b \ge 4$		$N_b = 5$	*OK*

* Refer to the Appendices at the end of the book.

The modular ratio between beam and deck material, n, is 1.0.

A Eq. 4.6.2.2.1-2

A	area of beam or T-beam	1710 in^2
L	span length	50 ft
t_s	slab thickness	9 in
n	modular ratio between beam and deck material	1

The moment of inertia of the basic beam (portion of beam below deck) is

$$I = \frac{(18\ \text{in})(35\ \text{in})^3}{12} = 64{,}312.5\ \text{in}^4$$

The distance between the centers of gravity of the basic beam and deck is

$$e_g = 17.5\ \text{in} + 4.5\ \text{in} = 22\ \text{in}$$

The longitudinal stiffness parameter is

A Eq. 4.6.2.2.1-1

$$K_g = n\,[I + (A)(e_g^{\,2})] = 1[64312.5\ \text{in}^4 + (1{,}710\ \text{in}^2)(22\ \text{in})^2]$$

$$K_g = 891{,}953.0\ \text{in}^4$$

A simplified value may be considered.

A Tbl. 4.6.2.2.1-2

$$\left[\frac{K_g}{12\,Lt_s^3}\right]^{0.1} = 1.05$$

Multiple presence factors, m, shall not be applied in conjunction with approximate load distribution factors specified in Art. 4.6.2.2 and 4.6.2.3, except where the lever rule is used.

A Art. 3.6.1.1.2

The distribution factor for moment for interior beams with one design lane, where *si* is the single lane loaded in interior beams is found as follows.

A Tbl. 4.6.2.2.2b-1 or Appendix A

The multiple presence factor, m, is applicable only when the lever rule is used for the distribution factors. Therefore, m = 1.0 where DFM is the distribution factor for moment.

$$DFM_{si} = m\left[0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12Lt_s{}^3}\right)^{0.1}\right]$$

$$= (1.0)\left[0.06 + \left(\frac{10\text{ ft}}{14}\right)^{0.4}\left(\frac{10\text{ ft}}{50\text{ ft}}\right)^{0.3}(1.05)\right]$$

$$= 0.629\text{ lane/girder}$$

The distribution factor for moment for interior beams with two or more design lanes loaded, where *mi* is the multiple lanes loaded in interior beams is

A Tbl. 4.6.2.2.2b-1 or Appendix A

$$DFM_{mi} = \left[0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12\,Lt_s^3}\right)^{0.1}\right]$$

$$= \left[0.075 + \left(\frac{10\text{ ft}}{9.5}\right)^{0.6}\left(\frac{10\text{ ft}}{50\text{ ft}}\right)^{0.2}(1.05)\right]$$

$$= 0.859\text{ lane/girder}\left[\text{governs for interior beams}\right]$$

For the distribution of moment for exterior beams with one design lane, use the lever rule. See Figure 2.8.

A Art. 3.6.1.3.1, A Art. 4.6.2.2.2d; Tbl. 4.6.2.2.2d-1 or Appendix B

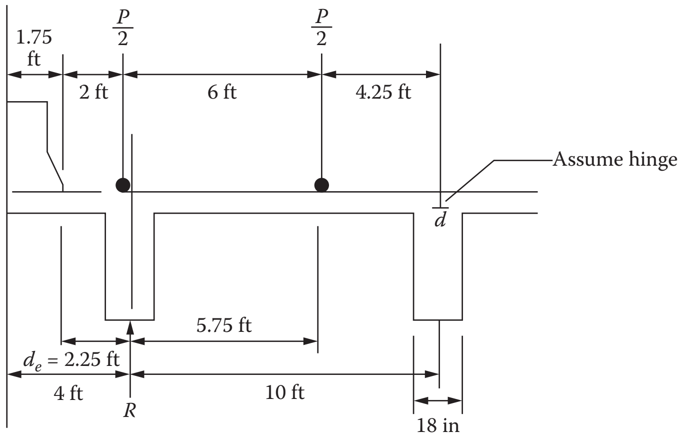

FIGURE 2.8
Lever rule for determination of distribution factor for moment in exterior beam, one lane loaded.

Σ Moment at "a" = 0

$$0 = -R(10\text{ ft}) + \frac{P}{2}(10.25\text{ ft}) + \frac{P}{2}(4.25\text{ ft})$$

$$R = 0.725\text{ P}$$

The multiple presence factor for one design lane loaded, m, is 1.20.

A Tbl. 3.6.1.1.2-1

The distribution factor for moment for exterior beams for one design lane loaded is, where $_{se}$ is the designation for single lane loaded in the extreme beam,

$$DFM_{se} = m[0.725] = 1.20\,[0.725]$$

$$= 0.87\text{ lane/girder [governs for exterior beam]}$$

The exterior web of the exterior beam to the interior edge of the curb, d_e, is 2.25 ft, which is OK for the $-1.0\text{ ft} \le d_e \le 5.5\text{ ft}$ range.

A Tbl. 4.6.2.2.2d-1 or Appendix B

The distribution factor for the exterior beam is

$$(e)(g_{interior}) = (e)(DFM_{mi})$$

$$e = 0.77 + \frac{d_e}{9.1}$$

$$e = 0.77 + \frac{2.25}{9.1}$$

$$e = 1.017$$

Use e = 1.0.

The distribution factor for moment for exterior beams with two or more design lanes loaded, where me is the designation for multiple lanes loaded in the exterior beam, is

A Tbl. 3.6.1.1.2-1

$$DFM_{me} = (m)(e)(g_{interior}) = (m)(e)(DFM_{mi}) = (0.85)(1.0)(0.859)$$

$$= 0.730\text{ lane/girder}$$

Find the distributed live load moments.

The governing distribution factors are:

Interior beam DFM_{mi} = 0.859 lane/girder
Exterior beam DFM_{se} = 0.870 lane/girder

IM = 33%

A Tbl. 3.6.2.1-1

The unfactored live load moment per beam for interior beams due to truck load is

$$M_{TL} = M_{tr}(DFM)(1 + IM) = (620\text{ ft-kips})(0.859)(1+0.33)$$

$$= 708.33\text{ ft-kips}$$

The unfactored live load moment per beam for interior beams due to lane load is

$$M_{LN} = M_{ln}\,(DFM) = (200\text{ ft-kips})(0.859)$$

$$= 171.8\text{ ft-kips}$$

The unfactored live load moment per beam for exterior beams due to truck load is

$$M_{TL} = M_{tr}(DFM)(1 + IM) = (620\text{ ft-kips})(0.87)(1 + 0.33)$$

$$= 717.40\text{ ft-kips}$$

The unfactored live load moment per beam for exterior beams due to lane load is

$$M_{LN} = M_{ln}\,(DFM) = (200\text{ ft-kips})(0.87)$$

$$= 174.0\text{ ft-kips}$$

Find the distribution factors for shears, DFV.

A Art. 4.6.2.2.3

For a cast-in-place concrete T-beam, the deck type is (e).

A Tbl. 4.6.2.2.1-1

The distribution factor for shear for interior beams with one design lane loaded is

A Tbl. 4.6.2.2.3a-1 or Appendix C

$$DFV_{si} = \left[0.36 + \frac{S}{25}\right] = \left[0.36 + \frac{10\text{ ft}}{25}\right]$$

$$= 0.76\text{ lanes}$$

The distribution factor for shear for interior beams with two or more design lanes loaded is

$$DFV_{mi} = \left[0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}\right] = \left[0.2 + \frac{10\text{ ft}}{12} - \left(\frac{10\text{ ft}}{35}\right)^{2.0}\right]$$

DFV_{mi} = 0.95 lane/girder [controls for interior beams]

Using the lever rule for moment, the distribution factor for shear for exterior beams with one design lane loaded is

A Tbl. 4.6.2.2.3b-1

$DFV_{se} = DFM_{se}$ = 0.87 lane/girder [controls for exterior beams]

The distribution factor for shear for exterior beams with two or more design lanes loaded is

$$g = (e)g_{interior} = (e)(DFV_{mi})$$

$$e = 0.6 + \frac{d_e}{10} = 0.6 + \frac{2.25\text{ ft}}{10} = 0.825$$

$DFV_{me} = mg = (m)(e)(g_{interior})$, where $g_{interior} = DFV_{mi}$

A Tbl. 3.6.1.1.2-1

= (1.0)(0.825)(0.95)

= 0.784 lane/girder

Find the distributed live load shears for Strength I.

The governing distribution factors are:

Interior beam DFV_{mi} = 0.95 lane/girder
Exterior beam DFV_{se} = 0.87 lane/girder

The unfactored live load shears per beam due to truck load for interior beams is

$$V_{TL} = V_{tr}(DFV)(1 + IM)$$
$$= (58.6 \text{ kips})(0.95)(1.33)$$
$$= 74.04 \text{ kips}$$

The unfactored live load shear per beam due to lane load for interior beams is

$$V_{LN} = V_{ln}(DFV)$$
$$= (16.0 \text{ kips})(0.95)$$
$$= 15.2 \text{ kips}$$

The unfactored live load shears per beam for exterior beams are

$$V_{TL} = V_{tr}(DFV)(1 + IM)$$
$$= (58.6 \text{ kips})(0.87)(1.33)$$
$$= 67.80 \text{ kips}$$
$$V_{LN} = V_{ln}(DFV)$$
$$= (16.0 \text{ kips})(0.87)$$
$$= 13.92 \text{ kips}$$

Find the dead load force effects.

The self-weights of the T-beam, the deck, and the curb and parapet for interior beams are represented by the variable DC.

$$DC_{T-beam} = \left(1710 \text{ in}^2\right)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)\left(0.15 \frac{\text{kips}}{\text{ft}^3}\right)$$

$$= 1.78 \frac{\text{kips}}{\text{ft}}$$

$$DC_{C\&P} = 2\left(3.37 \text{ ft}^2\right)\left(0.15 \frac{\text{kips}}{\text{ft}^3}\right)\left(\frac{1}{5 \text{ beams}}\right)$$

$$= 0.202 \frac{\text{kips}}{\text{ft}}$$

$$w_{DC} = DC_{T\text{-beam}} + DC_{C\&P}$$

$$= 1.78\,\frac{\text{kips}}{\text{ft}} + 0.202\,\frac{\text{kips}}{\text{ft}}$$

$$= 1.98\,\frac{\text{kips}}{\text{ft}}$$

The corresponding shear is

$$V_{DC} = \frac{wL}{2} = \frac{\left(1.98\,\frac{\text{kips}}{\text{ft}}\right)(50\text{ ft})}{2}$$

$$= 49.5\text{ kips}$$

The corresponding moment is

$$M_{DC} = \frac{wL^2}{8} = \frac{\left(1.98\,\frac{\text{kips}}{\text{ft}}\right)(50\text{ ft})^2}{8}$$

$$= 618.75\text{ kip-ft}$$

The future wearing surface load, DW, per beam is

$$w_{DW} = \left(0.03\,\frac{\text{kips}}{\text{ft}^2}\right)\left(10\,\frac{\text{ft}}{\text{beam}}\right)$$

$$= 0.3\,\frac{\text{kips}}{\text{ft}}$$

The corresponding shear is

$$V_{DW} = \frac{w_{DW}L}{2} = \frac{\left(0.3\,\frac{\text{kips}}{\text{ft}}\right)(50\text{ ft})}{2}$$

$$= 7.5\text{ kips}$$

TABLE 2.1
Distributed Live Load and Dead Load Effects for Interior Beam for Reinforced Concrete T-Beam Bridge

	Moment M (ft-kips)	**Shear V (kips)**
DC	618.75	49.5
DW	93.75	7.5
TL	708.33	74.04
LN	171.8	15.2

The corresponding moment is

$$M_{DW} = \frac{w_{DW}L^2}{8} = \frac{\left(0.3\,\frac{\text{kips}}{\text{ft}}\right)(50\text{ ft})^2}{8}$$

$$= 93.75\,\text{kip-ft}$$

The unfactored interior beam moments and shears due to the dead loads plus the live loads are given in Table 2.1. Note that dead loads consist of the exterior T-beam stem, deck slab, curb/parapet, and wearing surface.

For exterior girders, the deck slab load is,

$$w_s = (9\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(0.15\,\frac{\text{kips}}{\text{ft}^3}\right)$$

$$= 0.113\,\frac{\text{kips}}{\text{ft}^2}$$

For exterior girders, the wearing surface load is given as

$$w_{DW} = 0.03\,\frac{\text{kips}}{\text{ft}^2}$$

The total load is

$$w = w_S + w_{DW}$$

$$= 0.113\,\frac{\text{kips}}{\text{ft}^2} + 0.03\,\frac{\text{kips}}{\text{ft}^2}$$

$$= 0.143\,\frac{\text{kips}}{\text{ft}^2}$$

See Figure 2.9.

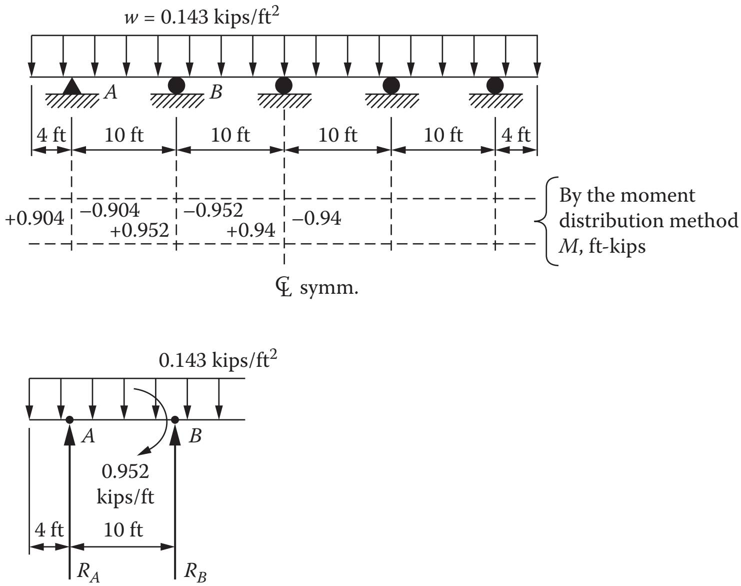

FIGURE 2.9
Moment distribution for deck slab and wearing surface loads.

Σ moments about B = 0

$$0 = -0.952 \text{ ft-kips} - R_A(10 \text{ ft}) + (0.143 \text{ kips/ft}^2)(14 \text{ ft})(7 \text{ ft}).$$

R_A = 1.31 kips per foot of exterior beam due to the deck slab and wearing surface dead loads

The load for the curb and parapet for exterior girders is,

$$w_{C\&P} = \left(3.37 \text{ ft}^2\right)\left(0.15 \frac{\text{kips}}{\text{ft}^3}\right)$$

$$= 0.51 \frac{\text{kips}}{\text{ft}}$$

See Figure 2.10.

Σ moments about B = 0

$$0 = 0.476 \text{ ft-kips} - R_A(10 \text{ ft}) + (0.51 \text{ kips})(13.34 \text{ ft})$$

R_A = 0.728 kips per foot of exterior beam due to the curb and parapet dead loads

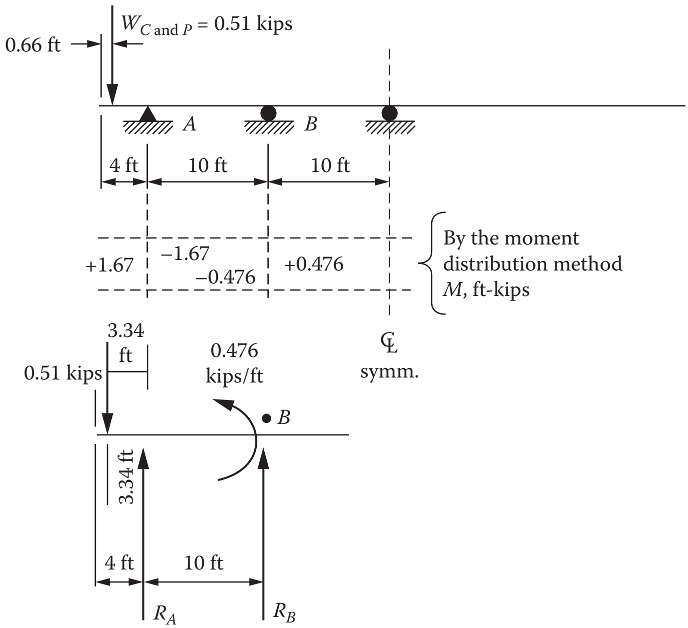

FIGURE 2.10
Moment distribution for curb and parapet loads for exterior girder.

The reactions at exterior beam due to the dead loads are as follows

$$\text{DC deck slab} = \left(1.31\,\frac{\text{kips}}{\text{ft}}\right)\left(\frac{0.113\,\frac{\text{kips}}{\text{ft}^2}}{0.143\,\frac{\text{kips}}{\text{ft}^2}}\right) = 1.04\,\frac{\text{kips}}{\text{ft}}$$

Curb and parapet overhang

$$w_{C\&P} = 0.728\,\frac{\text{kips}}{\text{ft}}$$

$$\text{girder stem} = \left(630\text{in}^2\right)\left(\frac{1\text{ ft}^2}{144\text{ in}^2}\right)\left(0.15\,\frac{\text{kips}}{\text{ft}^3}\right) = 0.656\,\frac{\text{kips}}{\text{ft}}$$

$$w_{DC} = 1.04\,\frac{\text{kips}}{\text{ft}} + 0.728\,\frac{\text{kips}}{\text{ft}} + 0.656\,\frac{\text{kips}}{\text{ft}} = 2.42\,\frac{\text{kips}}{\text{ft}}$$

Future wearing surface

$$w_{DW} = \left(1.31\,\frac{\text{kips}}{\text{ft}}\right)\left(\frac{0.03\,\dfrac{\text{kips}}{\text{ft}^2}}{0.143\,\dfrac{\text{kips}}{\text{ft}^2}}\right) = 0.27\,\frac{\text{kips}}{\text{ft}}$$

The unfactored exterior girder moments and shears due to the dead loads plus the live loads are as follows.

$$M_{DC} = \frac{w_{DC}L^2}{8} = \frac{\left(2.42\,\dfrac{\text{kips}}{\text{ft}}\right)(50\ \text{ft})^2}{8} = 756.3\ \text{kip-ft}$$

$$V_{DC} = \frac{w_{DC}L}{2} = \frac{\left(2.42\,\dfrac{\text{kips}}{\text{ft}}\right)(50\ \text{ft})}{2} = 60.5\ \text{kips}$$

$$M_{DW} = \frac{w_{DW}L^2}{8} = \frac{\left(0.27\,\dfrac{\text{kips}}{\text{ft}}\right)(50\ \text{ft})^2}{8} = 84.4\ \text{kip-ft}$$

$$V_{DW} = \frac{w_{DW}L}{2} = \frac{\left(0.27\,\dfrac{\text{kips}}{\text{ft}}\right)(50\ \text{ft})}{2} = 6.75\ \text{kips}$$

The unfactored beam moments and shears due to the dead loads plus the live loads are given in Table 2.2.

TABLE 2.2

Unfactored Beam Moments and Shears Due to Dead Loads and Live Loads for Reinforced Concrete T-Beam Bridge

	Interior Beam		**Exterior Beam**	
	Moment M (ft-kips)	**Shear V (kips)**	**Moment M (ft-kips)**	**Shear V (kips)**
DC	618.75	49.5	756.3	60.5
DW	93.75	7.5	84.4	6.75
TL	708.33	74.04	717.4	67.8
LN	171.8	15.2	174.0	13.92

Find the factored moments and shears for Strength I.

A Tbls. 3.4.1-1; 3.4.1-2]

$$Q = 1.25\ DC + 1.5\ DW + 1.75(TL + LN)$$

For interior girders, the unfactored moment is

$$M_u = 1.25\ M_{DC} + 1.5\ M_{DW} + 1.75\ (M_{TL} + M_{LN})$$

$$= 1.25\ (618.75\ \text{kip-ft}) + 1.5\ (93.75\ \text{kip-ft}) + 1.75\ (708.33\ \text{kip-ft} + 171.8\ \text{kip-ft})$$

$$= 2454.29\ \text{kip-ft}$$

For interior girders, the factored shear is

$$V_u = 1.25\ V_{DC} + 1.5\ V_{DW} + 1.75\ (V_{TL} + V_{LN})$$

$$= 1.25\ (49.5\ \text{kips}) + 1.5\ (7.5\ \text{kips}) + 1.75\ (74.04\ \text{kips} + 15.2\ \text{kips})$$

$$= 229.2\ \text{kips [controls]}$$

For exterior girders, the factored moment is

$$M_u = 1.25\ M_{DC} + 1.5\ M_{DW} + 1.75\ (M_{TL} + M_{LN})$$

$$= 1.25\ (756.3\ \text{kip-ft}) + 1.5\ (84.4\ \text{kip-ft}) + 1.75\ (717.4\ \text{kip-ft} + 174.0\ \text{kip-ft})$$

$$= 2631.9\ \text{kip-ft [controls]}$$

For exterior girders, the factored shear is

$$V_u = 1.25\ V_{DC} + 1.5\ V_{DW} + 1.75\ (V_{TL} + V_{LN})$$

$$= 1.25\ (60.5\ \text{kips}) + 1.5\ (6.75\ \text{kips}) + 1.75\ (67.8\ \text{kips} + 13.92\ \text{kips})$$

$$= 228.8\ \text{kips}$$

Find the design flexural reinforcements, neglecting compression reinforcement, and note the exterior girder moment and interior girder shear control.

$$M_u = 2631.9\ \text{kip-ft}$$

$$V_u = 229.2\ \text{kips}$$

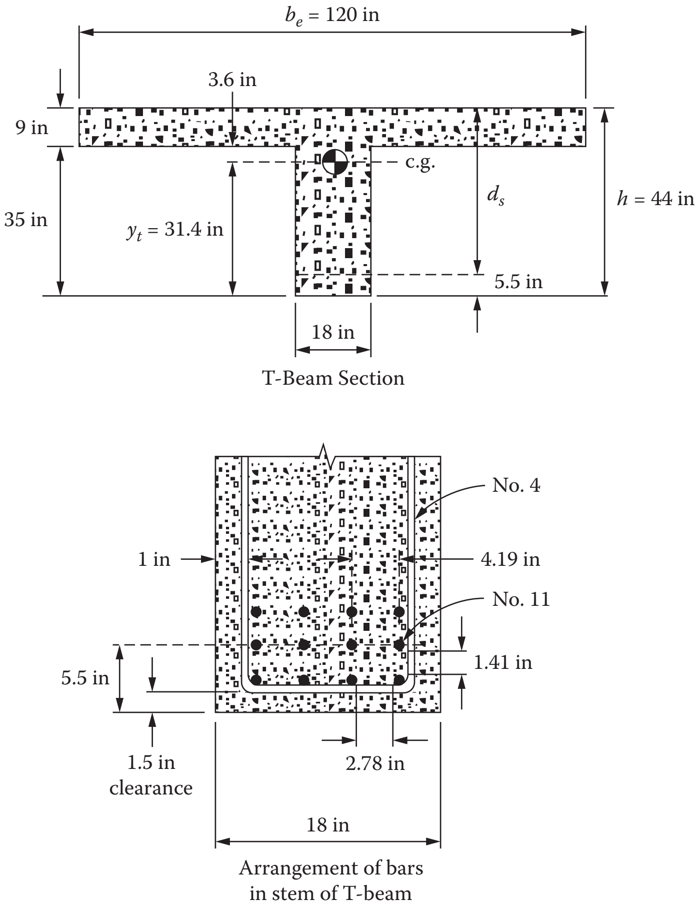

FIGURE 2.11
T-beam section and reinforcement in T-beam stem.

For stem reinforcement, try 12 no. 11 bars. See Figure 2.11.
Web thickness, b_w, is,

A Art. 5.14.1.5.1c

$$b_w = 2(1\text{ in}) + 2(0.5\text{ in}) + 4\ d_b + 3(1.5\ d_b)$$

$$= 2\text{ in} + 1\text{ in} + 4(1.4\text{ in}) + 3(1.5 \text{ x } 1.4\text{ in})$$

$$= 14.9\text{ in}$$

To give a little extra room, use $b_w = 18$ in

The distance from extreme compression fiber to the centroid of tensile reinforcement is

$$d_s = 44 \text{ in} - 5.5 \text{ in}$$

$$= 38.5 \text{ in}$$

$$d_b = 1.41 \text{ in [no. 11 bar]}$$

For 12 no. 11 bars, $A_s = 18.72$ in^2.

The minimum clearance between bars in a layer must not be less than

A Art. 5.10.3.1.1

$$1.5\, d_b = 1.5\,(1.41 \text{ in}) = 2.1 \text{ in}$$

$$2.1 \text{ in} < \text{provided} = 2.78 \text{ in [OK]}$$

The clear distance between layers shall not be less than 1 in or d_b.

A Art. 5.10.3.1.3

$$d_b = 1.41 \text{ in [OK]}$$

Concrete cover for epoxy-coated main reinforcing bars is 1.0 in, provided = 1.5 in [OK]

A Art. 5.12.3

$$\Phi = 0.90$$

A Art. 5.5.4.2

$$A_s = 18.75 \text{ in}^2$$

$$f_y = 60.0 \text{ ksi}$$

$$d_s = 38.5 \text{ in}$$

The thickness of the deck slab, t_s, is 9 in and the width of compression face, b, is 120 in.

The factor for concrete strength β_1 is

A Art. 5.7.2.2

$$\beta_1 = 0.85 - \left(\frac{f'_c - 4000\,\frac{\text{kips}}{\text{in}^2}}{1000\,\frac{\text{kips}}{\text{in}^2}} \right)(0.05)$$

$$= 0.85 - \left(\frac{4500\,\frac{\text{kips}}{\text{in}^2} - 4000\,\frac{\text{kips}}{\text{in}^2}}{1000\,\frac{\text{kips}}{\text{in}^2}} \right)(0.05)$$

$$= 0.825$$

The distance from the extreme compression fiber to the neutral axis, c, is

A Eq. 5.7.3.1.1-4

$$c = \frac{A_s f_y}{0.85\, f'_c\, \beta_1\, b}$$

$$= \frac{(18.75\ \text{in}^2)\left(60\,\frac{\text{kips}}{\text{in}^2}\right)}{0.85\left(4.5\,\frac{\text{kips}}{\text{in}^2}\right)(0.825)(120\ \text{in})}$$

$$c = 2.97\ \text{in}$$

$$a = c\, \beta_1$$

$$= (2.97\ \text{in})(0.825)$$

$$= 2.45\ \text{in} < t_s = 9\ \text{in [OK]}$$

The nominal resisting moment, M_n, is

A Art. 5.7.3.2.2

$$M_n = A_s f_y \left(d_s - \frac{a}{2} \right)$$

$$= (18.75\ \text{in}^2)\left(60\,\frac{\text{kips}}{\text{in}^2}\right)\left(38.5\ \text{in} - \frac{2.45\ \text{in}}{2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 3488.9\ \text{in}^2$$

The factored resisting moment, M_r, is

A Art. 5.7.3.2.1

$$M_r = \Phi M_n$$

$$= 0.90(3488.9 \text{ ft-kips})$$

$$= 3140.0 \text{ ft-kips} > M_u = 2631.9 \text{ ft-kips [OK]}$$

Check the reinforcement requirements.

A Art. 5.7.3.3.1; 5.7.3.3.2

This provision for finding the maximum reinforcement was deleted from AASHTO in 2005. Therefore, check for the minimum reinforcement. The minimum reinforcement requirement is satisfied if M_r is at least equal to the lesser of:

The minimum reinforcement requirement is satisfied if ΦM_n (= M_r) is at least equal to the lesser of:

- $M_r \geq 1.2\, M_{cr}$
- $M_r \geq 1.33$ times the factored moment required by the applicable strength load combination specified in AASHTO Table 3.4.1-1

where:
M_r = factored flexural resistance
M_{cr} = cracking moment
f_r = modulus of rupture

$$M_{cr} = S_c f_r$$

A Eq. 5.7.3.3.2-1

The section modulus is

$$S_c = \frac{I_g}{y_b}$$

The modulus of rupture of concrete is

A Art. 5.4.2.6

$$f_r = 0.37\sqrt{f'_c}$$

$$= 0.37\sqrt{4.5\frac{\text{kips}}{\text{in}^2}}$$

$$= 0.7849\frac{\text{kips}}{\text{in}^2}$$

$$M_{cr} = \left(\frac{I_g}{y_t}\right)(f_r)$$

$$= \left(\frac{264{,}183.6\ \text{in}^4}{31.4\ \text{in}}\right)\left(0.7849\,\frac{\text{kips}}{\text{in}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 550.3\ \text{kip-ft}$$

$$1.2\ M_{cr} = (1.2)(550.3\ \text{kip-ft})$$

$$= 660.4\ \text{kip-ft [controls]}$$

$$1.33\ M_u = (1.33)(2631.9\ \text{kip-ft})$$

$$= 3500.4\ \text{kip-ft}$$

$$M_r = \Phi M_n = 0.9(3488.9\ \text{kip-ft}) = 3140.0\ \text{kip-ft} > 1.2\ M_{cr} = 660.4\ \text{kip-ft [OK]}$$

Design for shear.

The effective shear depth, d_v, taken as the distance between the resultants of the tensile and compressive forces due to flexure

A Art. 5.8.2.9

$$d_v = d_s - \frac{a}{2} = 38.5\ \text{in} - \frac{2.45\ \text{in}}{2} = 37.3\ \text{in}$$

where:
d_s = distance from the extreme compression fiber to the centroid of tensile reinforcement

The effective shear depth, d_v, need not be less than the greater of:

A Art. 5.8.2.9

- $0.9\ d_e = 0.9(38.5\ \text{in}) = 34.6\ \text{in} < 37.3\ \text{in}$
- $0.72\ h = 0.72(44\ \text{in}) = 31.7\ \text{in} < 37.3\ \text{in}$

The critical section for shear is taken as d_v from the internal face of support. The distance from the center of bearing support, x, is calculated as

A Art. 5.8.3.2

$$x = 37.3\ \text{in} + 5.7\ \text{in} = 43\ \text{in}\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = 3.58\ \text{ft}$$

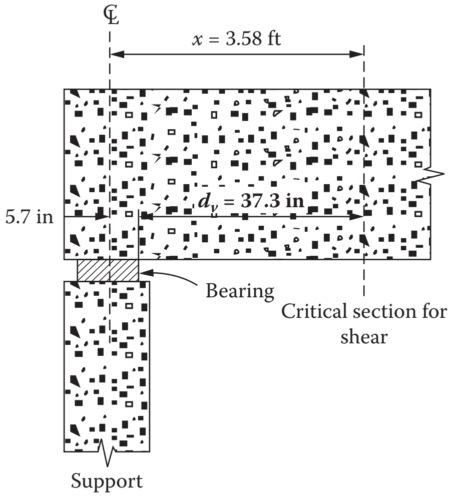

FIGURE 2.12
Critical shear section at support.

See Figure 2.12.

Find the unfactored shear forces and concurrent moments due to dead load.

Note that the exterior girder controls for moment.

The shear at the critical section for shear due to the DC load [interior girder controls] is

$$w_{DC} = 1.98 \text{ kips/ft}$$

$$V_{DC} = \left(\frac{1.98 \frac{\text{kips}}{\text{ft}}(50 \text{ ft})}{2}\right) - \left(1.98 \frac{\text{kips}}{\text{ft}}\right)(3.58 \text{ ft})$$

$$V_{DC} = 42.2 \text{ kips}$$

The concurrent moment at the critical section for shear for the interior girder due to the DC load is

$$M_{DC} = \left(\frac{50 \text{ ft}}{2}\right)\left(1.98 \frac{\text{kips}}{\text{ft}}\right)(3.58 \text{ ft}) - \left(1.98 \frac{\text{kips}}{\text{ft}}\right)(3.58 \text{ ft})^2\left(\frac{1}{2}\right)$$

$$= 189.9 \text{ kip-ft}$$

The shear at the critical section for shear due to the DW load is

$$w_{DW} = 0.30\text{ kips/ft}$$

$$V_{DW} = \frac{\left(0.30\,\frac{\text{kips}}{\text{ft}}\right)(50\text{ ft})}{2} - \left(0.30\,\frac{\text{kips}}{\text{ft}}\right)(3.58\text{ ft})$$

$$V_{DW} = 6.43\text{ kips}$$

The concurrent moment at the critical section for shear due to the DW load is

$$M_{DW} = \frac{(50\text{ ft})\left(0.30\,\frac{\text{kips}}{\text{ft}}\right)}{2}(3.58\text{ ft}) - \left(0.30\,\frac{\text{kips}}{\text{ft}}\right)(3.58\text{ ft})^2\left(\frac{1}{2}\right)$$

$$M_{DW} = 24.93\text{ kip-ft}$$

Find the shear at the critical section for shear for the interior girder due to the design truck (HS-20).

MBE-2 App Tbl. H6B*

Let x = L – x; multiply by 2 for two wheel lines.

$$V = \frac{36\text{ kips}(x - 9.33)}{L}(2)$$

$$V_{tr,\,HS\text{-}20} = \frac{72\text{ kips}(L - x - 9.33)}{L}$$

$$= \frac{72\text{ kips}(50\text{ ft} - 3.58\text{ ft} - 9.33)}{50\text{ ft}}$$

$$= 53.41\text{ kips per lane}$$

$$V_{TL} = V_{tr}(DFV)(1 + IM)$$

$$= (53.71\text{ kips})(0.95)(1.33)$$

$$= 67.48\text{ kips per beam}$$

* MBE-2 refers to the *Manual for Bridge Evaluation* (second edition), 2011, American Association of State Highway and Transportation Officials (AASHTO)

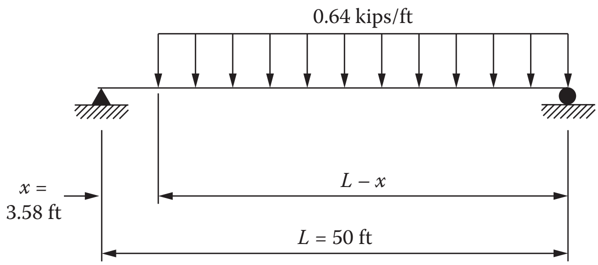

FIGURE 2.13
Lane load position for maximum shear at critical shear section.

Find the concurrent maximum moment per lane interior girder at the critical section for shear due to the design truck (HS-20).

MBE-2 App. Tbl. J6B

Let x = L – x; multiply by 2 for two wheel lines.

$$M = \frac{36\text{ kips}(L-x)(x-9.33)}{L}(2)$$

$$M_{tr,\,HS\text{-}20} = \frac{72\text{ kips}(x)(L-x-9.33)}{L}$$

$$= \frac{72\text{ kips}(3.58\text{ ft})(50\text{ ft}-3.58\text{ ft}-9.33)}{50\text{ ft}}$$

$$= 191.2\text{ ft-kips per lane}$$

$$M_{TL} = M_{tr}(DFM)(1 + IM)$$

$$= (191.2\text{ ft-kips})(0.822)(1.33)$$

$$= 209.03\text{ ft-kips per beam}$$

Find the shear at the critical section for interior girder due to lane load. See Figure 2.13.

$$V_{ln} = \frac{0.32(L-x)^2}{L}$$

$$= \frac{0.32(50\text{ ft}-3.58\text{ ft})^2}{50\text{ ft}}$$

$$= 13.79\text{ kips per lane}$$

$$V_{LN} = V_{ln}(DFV) = (13.79 \text{ kips})(0.95)$$

$$= 13.1 \text{ kips per beam}$$

Find the concurrent moment at the critical section for shear for interior girder due to lane load, where

$$R = \frac{0.32(L-x)^2}{L}$$

$$M_{ln} = Rx$$

$$= \frac{0.32(L-x)^2(x)}{L}$$

$$= \frac{0.32(50 \text{ ft} - 3.58 \text{ ft})^2(3.58 \text{ ft})}{50 \text{ ft}}$$

$$= 49.37 \text{ ft-kips per beam}$$

$$M_{LN} = M_{ln}(DFM) = (49.37 \text{ ft-kips})(0.822)$$

$$= 40.58 \text{ ft-kips per beam}$$

The factored shear at the critical section is,

A Tbls. 3.4.1-1; 3.4.1-2

$$V_u = 1.25\ V_{DC} + 1.5\ V_{DW} + 1.75(V_{TL} + V_{LN})$$

$$= 1.25(42.2 \text{ kips}) + 1.5(6.43 \text{ kips}) + 1.75(67.48 \text{ kips} + 13.1 \text{ kips})$$

$$V_u = 203.4 \text{ kips}$$

The factored moment at the critical section is

$$M_u = 1.25\ M_{DC} + 1.5\ M_{DW} + 1.75(M_{TL} + M_{LN})$$

$$= 1.25(189.9 \text{ kip-ft}) + 1.5(24.93 \text{ kip-ft}) + 1.75(209.03 \text{ kip-ft} + 40.58 \text{ kip-ft})$$

$$M_u = 711.59 \text{ kip-ft}$$

Find the shear stress on the concrete.

A Eq. 5.8.2.9-1

The effective web width, b_v, is 18 in.
The resistance factor for shear specified for reinforced concrete is 0.9.

A Art. 5.5.4.2

The distance from extreme compression fiber to the centroid of tensile reinforcement, d_s, is 38.5 in (d_s is equivalent to d_e).

A Art. 5.8.2.9

NOTE: d_e is also the distance from the centerline of the exterior web of the exterior beam to the interior edge of curb or traffic barrier (Art. 4.6.2.2.1).

The factored shear stress on the concrete at d_v = 37.3 in is,

A Eqs. 5.8.2.9-1, 5.5.4.2.1

$$v_u = \frac{V_u}{\Phi b_v d_v} = \frac{203.4\ \text{kips}}{(0.9)(18\ \text{in})(37.3\ \text{in})} = 0.34\frac{\text{kips}}{\text{in}^2}$$

Find the tensile strain in the transverse reinforcement for sections where the transverse reinforcement is found using Eq. 5.8.3.4.2-1 and has the following characteristics.

A Art. 5.8.3.4.2

$$\theta = 30°$$

$$\cot\theta = 1.732$$

$$E_s = 29{,}000\ \text{kips/in}^2$$

$$A_s = 18.75\ \text{in}^2$$

$$d_v = 37.3\ \text{in}$$

$$\varepsilon_x = \frac{\frac{|M_u|}{d_v} + 0.5\,V_u\cot\theta}{2E_sA_s}$$

A App. B5 Eq. B5.2-1

$$= \frac{\left(\frac{711.59\ \text{ft-kips}}{37.3\ \text{in}}\right)\left(12\frac{\text{in}}{\text{ft}}\right) + 0.5(203.4\ \text{kips})(1.732)}{2\left(29000\frac{\text{kips}}{\text{in}^2}\right)(18.75\ \text{in}^2)}$$

$$= 0.00037 < 0.001\ [\text{OK}]$$

Transverse reinforcement shall be provided where:

A Eq. 5.8.2.4-1; Arts. 5.8.3.4.1

$$V_u \geq 0.5\ \Phi V_c$$

$\beta = 2$ and $\theta = 45°$ may be used.

The nominal shear resistance, V_n, shall be as the lesser of:

A Art. 5.8.3.3; Eq. 5.8.3.3-1; Eq. 5.8.3.3-2

$$V_n = V_c + V_s$$

$$V_n = 0.25\ f'_c b_v d_v$$

The nominal shear resistance by the concrete is

$$V_c = 0.0316\,\beta\sqrt{f'_c}\ b_v d_v$$

$$= 0.0316(2)\sqrt{4.5\frac{\text{kips}}{\text{in}^2}}(18\text{ in})(37.3\text{ in})$$

$$= 90.1\text{ kips}$$

Transverse reinforcement shall be provided if $V_u \geq 0.5\ \Phi V_c$

A Art. 5.8.2.4

$$0.5\ \Phi V_c = (0.5)(0.9)(90.1\text{ kips})$$

$$= 40.5\text{ kips} < V_u = 203.4\text{ kips}$$

Therefore, the transverse reinforcement is provided at the critical section (x = 3.58 ft) for shear.

The nominal shear resistance of a transversely reinforced section is $V_n = V_c + V_s$. V_s is the shear resistance by reinforcement.

A Eq. 5.8.3.3-1

The nominal shear resistance of the section is

A Eq. 5.8.3.3-2

$$V_n = 0.25\ f'_c b_v d_v = (0.25)\left(4.5\frac{\text{kips}}{\text{in}^2}\right)(18\text{ in})(37.3\text{ in})$$

$$= 755.3\text{ kips}$$

The factored shear resistance, V_r, is

A Eq. 5.8.2.1-2

$$V_r = \Phi V_n = (0.9)(755.3 \text{ kips})$$

$$= 679.8 \text{ kips} > V_u = 203.4 \text{ kips [OK]}$$

The factored shear force does not exceed the maximum factored shear resistance, therefore, the section size is good for shear.

Find the transverse reinforcement requirements.

The maximum shear resistance provided by shear reinforcement is

A Art. 5.8.3.3

$$V_s = V_n - V_c = 0.25\, f_c'\, b_v d_v - 0.0316\beta\sqrt{f_c'}\, b_v d_v$$

$$= 755.3 \text{ kips} - 90.1 \text{ kips}$$

$$= 665.2 \text{ kips}$$

Shear required by shear reinforcement at the critical section is determined by letting the nominal shear resistance, V_n, equal to the factored shear forces, V_u, divided by Φ.

$$V_{s.required} = V_n - V_c = \frac{V_u}{\Phi} - V_c$$

$$= \frac{203.4 \text{ kips}}{0.9} - 90.1 \text{ kips}$$

$$= 136.0 \text{ kips} < 665.2 \text{ kips [OK]}$$

θ = angle of inclination of diagonal compressive stress = 45°

α = angle of inclination of transverse reinforcement to longitudinal axis = 90°

A Comm. 5.8.3.3, A Eq. 5.8.3.3-4

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s}$$

$$= \frac{A_v f_y d_v \left(\cot 45° + \cot 90°\right)\left(\sin 90°\right)}{s} = \frac{A_v f_y d_v \left(1.0 + 0\right)\left(1.0\right)}{s}$$

Try two legs of no. 4 bar stirrups at the critical section, $A_v = 0.4\ in^2$.

$$s = \frac{A_v f_y d_v}{V_s}$$

$$= \frac{(0.4\ in^2)\left(60\ \frac{kips}{in^2}\right)(37.3\ in)}{136.0\ kips}$$

$$= 6.58\ in$$

Use s = 6.5 in

Minimum transverse reinforcement (where b_v is the width of the web) shall be

A Eq. 5.8.2.5-1

$$A_{v,\,min} \geq 0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y}$$

$$A_{v,\,min} = 0.0316\sqrt{4.5\ \frac{kips}{in^2}}\ \frac{(18\ in)(6.5\ in)}{\left(60\ \frac{kips}{in^2}\right)}$$

$$= 0.13\ in^2 < 0.4\ in^2 \text{ at 6.5 in spacing provided [OK]}$$

At the critical section (x = 3.58 ft) from the bearing of the left support, the transverse reinforcement will be two no. 4 bar stirrups at 6.5 in spacing.

Determine the maximum spacing for transverse reinforcement required.

A Art. 5.8.2.7

v_u is the average factored shear stress in the concrete.

A Eq. 5.8.2.9-1

$$v_u = \frac{V_u}{\Phi b_v d_v}$$

$$= \frac{203.4\ kips}{(0.9)(18\ in)(37.3\ in)}$$

$$= 0.337\ \frac{kips}{in^2}$$

If $v_u < 0.125\ f'_c$

$$s_{max} = 0.8\ d_v < 24\ \text{in}$$

A Eq. 5.8.2.7-1

$$v_u = 0.337\frac{\text{kips}}{\text{in}^2} < 0.125\left(4.5\frac{\text{kips}}{\text{in}^2}\right) = 0.563\frac{\text{kips}}{\text{in}^2}$$

Thus, $s_{max} = 0.8(37.3\ \text{in}) = 29.8\ \text{in} \geq 24\ \text{in}$
$s_{provided} = 6.5\ \text{in} \leq 24\ \text{in}$ [OK]

Check tensile capacity of longitudinal reinforcement.

A Art. 5.8.3.5

Shear causes tension in the longitudinal reinforcement in addition to shear caused by the moment.

A Art. 5.8.3.5

At the critical section for shear, x is 3.58 ft.

M_u	factored moment	711.59 ft-kips
V_u	factored shear	203.4 kips
Φ_f, Φ_v	resistance factors for moment and shear	0.90

A Art. 5.5.4.2.1

For two no. 4 stirrups at 6.5 in spacing and cot 45° = 1.0, the shear resistance provided by shear reinforcement is,

A Eq. 5.8.3.3-4

$$V_s = \frac{A_v f_y d_v}{s} = \frac{\left(0.4\ \text{in}^2\right)\left(60\frac{\text{kips}}{\text{in}^2}\right)\left(37.3\ \text{in}\right)}{6.5\ \text{in}}$$

$$= 137.7\ \text{kips}$$

The required tensile capacity of the reinforcement on the flexural tensile side shall satisfy:

A Eq. 5.8.3.5-1

$$A_s f_y = \frac{M_u}{\varphi_f d_v} + \left(\frac{V_u}{\varphi_v} - 0.5V_s\right)\cot\theta$$

$$= \frac{711.59\ \text{ft-kips}}{(0.9)(37.3\ \text{in})}\left(12\frac{\text{in}}{\text{ft}}\right) + \left(\frac{203.4\ \text{kips}}{0.9} - 0.5(137.7\ \text{kips})\right)(1.0)$$

$$= 411.5\ \text{kips}$$

The available tension capacity of longitudinal reinforcement is

$$A_s f_y = \left(18.75\ \text{in}^2\right)\left(60\,\frac{\text{kips}}{\text{in}^2}\right)$$

$$= 1123.2\ \text{kips} > 411.5\ \text{kips [OK]}$$

Step 2: Check the Fatigue Limit State

A Art. 5.5.3

The fatigue strength of the bridge is related to the range of live load stress and the number of stress cycles under service load conditions. The fatigue load combination specified in AASHTO Table 3.4.1-1 is used to determine the allowable fatigue stress range, f_f (= constant-amplitude fatigue threshold, $(\Delta F)_{TH}$) (AASHTO Art. 5.5.3.1). The minimum live-load stress level, f_{min}, is determined by combining the fatigue load with the permanent loads. f_{min} will be positive if it is in tension.

The factored load, Q, is calculated using the load factors given in AASHTO Tables 3.4.1-1 and 3.4.1-2.

For a simple span bridge with no prestressing, there are no compressive stresses in the bottom of the beam under typical dead load conditions. Therefore, fatigue must be considered.

A Art. 5.5.3.1

Fatigue Limit State II (related to finite load-induced fatigue life) Q = 0.75(LL + IM), where LL is the vehicular live load and IM is the dynamic load allowance.

A Tbl. 3.4.1-1

The following information will be used to determine the fatigue load.
One design truck that has a constant spacing of 30 ft between 32 kip axles.

A Art. 3.6.1.4.1

Dynamic load allowance, IM, is 15%.

A Tbl. 3.6.2.1-1

A distribution factor, DFM, for one traffic lane shall be used.

A Art. 3.6.1.4.3b

The multiple presence factor of 1.2 shall be removed.

A Comm. 3.6.1.1.2

The allowable fatigue stress range, f_f (= $(\Delta F)_{TH}$), shall be equal to (24 – 0.33 f_{min}).

A Eq. 5.5.3.2-1

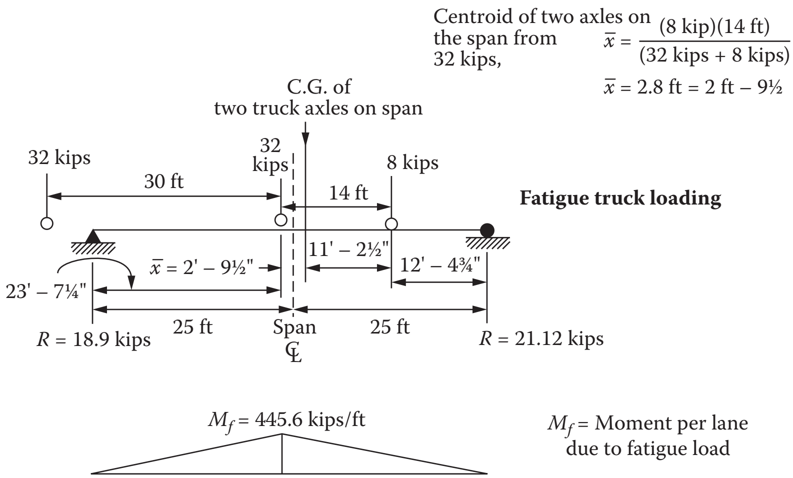

FIGURE 2.14
Fatigue truck loading and maximum moment at 32 kips position per lane due to fatigue loading.

f_f Allowable fatigue stress range, kips/in^2

f_{min} Minimum live-load stress resulting from the fatigue load combined with the permanent loads; positive if tension, kips/in^2

See Figure 2.14.

The moment per lane due to fatigue load, M_f, is 445.6 ft-kips. For one traffic lane loaded,

$$DFM_{si} = 0.629 \text{ lanes for interior girders}$$

$$DFM_{se} = 0.87 \text{ lanes for exterior girders}$$

The multiple presence factor of 1.2 must be removed. Therefore, the distribution factor for fatigue load using DFM_{se} is,

A Art. 3.6.1.1.2

$$DFM_{fatigue} = \frac{0.87}{1.2} = 0.725 \text{ lanes}$$

Find the unfactored fatigue load moment per beam.

The moment due to fatigue load per beam, $M_{fatigue}$, is

$$M_{fatigue} = M_f(DFM)(1 + IM)$$

$$= (445.6 \text{ kip-ft})(0.725)(1 + 0.15)$$

$$= 371.5 \text{ kip-ft}$$

Fatigue Investigations.

It is noted that provisions used for Fatigue I are conservative for Fatigue II load.

A Art. 5.5.3.1

For fatigue investigations, the section properties shall be based on cracked sections when the sum of the stresses, due to unfactored permanent loads and Fatigue I load combination, is tensile and exceeds $0.095\sqrt{f_c'}$. Referring to the unfactored exterior girder moments previously found,

$$M_{DC} + M_{DW} + M_{fatigue} = 756.3 \text{ kip-ft} + 84.4 \text{ kip-ft} + 371.5 \text{ kip-ft}$$

$$= 1212.2 \text{ kip-ft [for exterior girders]}$$

Find the tensile stress at the bottom fiber of gross section of interior girders using the stresses for exterior girders to be conservative.

A Art. 5.5.3.1

$$S_g = \frac{I_g}{y_t}$$

$$= \frac{264{,}183.6 \text{ in}^4}{31.4 \text{ in}}$$

$$= 8{,}413.5 \text{ in}^3$$

The tensile stress at the bottom fiber f_t is

$$f_t = \frac{M}{S_g}$$

$$= \frac{1212.2 \text{ kip-ft}\left(\dfrac{12 \text{ in}}{1 \text{ ft}}\right)}{8413.5 \text{ in}^3}$$

$$= 1.73 \text{ kips/in}^2$$

$$0.095\sqrt{f_c'} = 0.095\sqrt{4.5\frac{\text{kips}}{\text{in}^2}}$$

$$= 0.20 \text{ kips/in}^2 < f_t = 1.73 \text{ kip/in}^2$$

A Art. 5.5.3.1

Therefore, cracked section analysis should be used for fatigue investigation. The modulus of elasticity for concrete with w_c = 0.15 kips/in² is

$$E_c = 33{,}000(w_c)^{1.5}\sqrt{f'_c}$$

A Eq. 5.4.2.4-1

$$= 33{,}000\left(0.15\frac{\text{kips}}{\text{ft}^3}\right)^{1.5}\sqrt{4.5\frac{\text{kips}}{\text{in}^2}}$$

$$E_c = 4{,}066.8 \text{ kips/in}^2$$

The modulus of elasticity for steel, E_s, is 29,000 kips/in²
The modular ratio between steel and concrete is

$$n = \frac{E_s}{E_c}$$

$$= 7.13, \text{ use } n = 7$$

Determine the transformed area.

$$nA_s = 7(18.75 \text{ in}^2) = 131.25 \text{ in}^2$$

Find the factored fatigue moment per beam.

The factored load for Fatigue II load combination, Q, is

A Table 3.4.1-1

$$Q = 0.75(LL + IM)$$

$$M_{F,\,fatigue} = 0.75(M_{fatigue}) = 0.75(371.5 \text{ kip-ft}) = 278.6 \text{ kip-ft}$$

See Figure 2.15 for the cracked section analysis.

Find the distance, x, from the top fiber to the neutral axis. Taking the moment of areas about the neutral axis,

$$(120 \text{ in})(x)\left(\frac{x}{2}\right) = (131.25 \text{ in}^2)(38.5 \text{ in} - x)$$

$$x = 8.15 \text{ in}$$

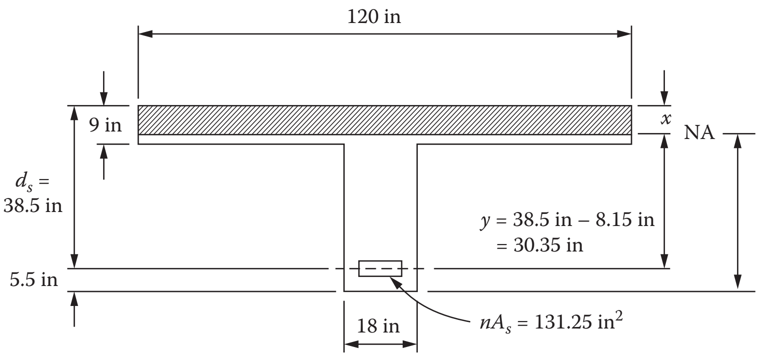

FIGURE 2.15
Cracked section determination of T-beam.

$$I_{NA} = \frac{1}{3}bx^3 + Ay^2$$

$$= \frac{1}{3}(120\ \text{in})(8.15\ \text{in})^3 + (131.25\ \text{in}^2)(38.5\ \text{in} - 8.15\ \text{in})^2$$

$$= 142{,}551.0\ \text{in}^4$$

Find the stress in the reinforcement.

The stress in the reinforcement due to the factored fatigue live load f_s, is,

$$f_s = n\left(\frac{(M)(y)}{I}\right)$$

$$= n\left(\frac{M_{F\ fatigue}}{I_{NA}}\right)(y)$$

$$= (7)\left(\frac{(278.6\ \text{kip-ft})\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)}{142{,}551\ \text{in}^4}\right)(38.5\ \text{in} - 8.15\ \text{in})$$

$$f_s = 4.98\ \text{kips/in}^2$$

Find the fatigue stress range.

The permissible stress range in the reinforcing bars resulting from the fatigue load combination, f_f, must not exceed the stress in the reinforcement, f_s.

Using the previously calculated unfactored exterior girder moments due to the dead loads, the total moment due to the dead load is

$$M_{\text{dead load}} = M_{DC} + M_{DW} = 756.3 \text{ ft-kips} + 84.4 \text{ ft-kips}$$

$$= 840.7 \text{ ft-kips}$$

The minimum live load stress resulting from the fatigue load is

$$f_{min} = f_{s,\text{ dead load}} = (n)\left(\frac{M_{\text{deadload}}y}{I}\right)$$

$$= (7)\left(\frac{(840.7 \text{ ft-kips})\left(12\frac{\text{ft}}{\text{in}}\right)(38.5 \text{ in} - 8.15 \text{ in})}{142551 \text{ in}^4}\right)$$

$$= 15.0 \text{ kips/in}^2$$

The allowable fatigue stress range f_f, is,

A Eq. 5.5.3.2-1

$$f_f = 24 - 0.33\, f_{min}$$

$$= 24 - (0.33)(15.0 \text{ kips/in}^2)$$

$$= 19.05 \text{ kips/in}^2$$

f_s = stress in the reinforcement due to the factored fatigue live load

$= 4.98 \text{ kips/in}^2 < f_f = 19.05 \text{ kips/in}^2$ [OK]

The stress in the reinforcement due to the fatigue live load is less than the allowable fatigue stress range, so it is ok.

Step 3: Check the Service I Limit State

The factored load, Q, is calculated using the load factors given in AASHTO Table 3.4.1-1.

The load combination relating to the normal operational use of the bridge with all loads taken at their nominal values (with no impact allowance, no load factors, and so on), is similar to that for the allowable stress methods. This combination is for deflection control and crack width control in reinforced concrete.

$$Q = 1.0(DC) + 1.0(DW) + 1.0(TL + LN)$$

A Tbl. 3.4.1-1

Although deflection and depth limitations are optional in AASHTO, bridges should be designed to avoid undesirable structural or psychological effects due to deformations.

A Art. 2.5.2.6.1

Criteria for deflection in this section shall be considered optional. However, in the absence of other criteria, the following limits may be considered.

A Art. 2.5.2.6.2

For steel, aluminum, concrete vehicular bridges,	
vehicular load general	span/800
For wood vehicular bridges,	
vehicular and pedestrian loads	span/425
vehicular load on wood planks and panels	
(extreme relative deflection between adjacent edges)	0.10 in.

Using the previously calculated exterior girder moments due to the dead and live loads, the total service load moment is

$$\begin{aligned} M_{service} &= 1.0\ M_{DC} + 1.0\ M_{DW} + 1.0(M_{TL} + M_{LN}) \\ &= (1.0)(756.3\ \text{ft-kips}) + (1.0)(84.4\ \text{ft-kips}) \\ &\quad + (1.0)(717.4\ \text{ft-kips} + 174.0\ \text{ft-kips}) \\ &= 1732.1\ \text{ft-kips} \end{aligned}$$

Find the control of cracking by distribution of reinforcement.

The maximum spacing of tension reinforcement applies if the tension in the cross section exceeds 80% of the modulus of rupture, f_r, at the Service I Limit State.

A Art. 5.7.3.4

The modulus of rupture is

A Art. 5.4.2.6

$$f_r = 0.24\sqrt{f'_c}$$

$$= 0.24\sqrt{4.5\frac{\text{kips}}{\text{in}^2}}$$

$$= 0.51 \text{ kips/in}^2$$

The section modulus at the bottom fiber for the gross cross section (where y_t is equivalent to y_b) is

$$S_g = \frac{I_g}{y_t}$$

where:

y_t = distance from the neutral axis to the extreme tension fiber
  = y + 5.5 in = 35.85 in

$$= \frac{264{,}183.6 \text{ in}^4}{35.85 \text{ in}}$$

$$= 7369.1 \text{ in}^3$$

The tensile stress at the bottom fiber is

$$f_t = \frac{M_{service}}{S_g} = \frac{(1732.1 \text{ kip-ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)}{7369.1 \text{ in}^3}$$

$$= 2.82 \text{ kips/in}^2 > 0.8\, f_r = 0.8(0.51 \text{ kips/in}^2) = 0.41 \text{ kips/in}^2$$

Thus, flexural cracking is controlled by limiting the spacing, s, in the tension reinforcement. Therefore, the maximum spacing, s, of the tension reinforcement should satisfy the following.

A Eq. 5.7.3.4-1

$$s \le \frac{700\,\gamma_e}{\beta_s f_{ss}} - 2\,d_c$$

The exposure factor for a class 2 exposure condition for concrete, γ_e, is 0.75. The overall thickness, h, is 44 in. The concrete cover measured from extreme tension fiber to the center of the flexural reinforcement located closest thereto is

A Art. 5.7.3.4

$$d_c = 1.5 \text{ in} + 0.5 \text{ in} + 1.41 \text{ in}/2$$

$$= 2.7 \text{ in (see Figure 2.11)}$$

The modular ratio between the steel and concrete, n, was previously calculated as 7. The tensile stress in steel reinforcement at the Service I Limit State is

A Art. 5.7.3.4

$$f_{ss} = n\left(\frac{M_{service}}{S_g}\right),$$

where:

$$S_g = \frac{I_g}{y} = \frac{264{,}183.6 \text{ in}^4}{30.35 \text{ in}} = 8704.6 \text{ in}^3$$

$$= 7\left(\frac{(1732.1 \text{ kip-ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)}{8{,}413.5 \text{ in}^3}\right)$$

$$= 16.7 \text{ kips/in}^2$$

The ratio of the flexural strain at extreme tension face to the strain at the centroid of reinforcement layer nearest to the tension face is

A Art. 5.7.3.4

$$\beta_s = 1 + \left(\frac{d_c}{0.7(h - d_c)}\right)$$

$$= 1 + \left(\frac{2.7 \text{ in}}{0.7(44 \text{ in} - 2.7 \text{ in})}\right)$$

$$= 1.09$$

Therefore, the maximum spacing, s, of the tension reinforcement shall satisfy the following.

$$s \le \frac{700\,\gamma_e}{\beta_s f_{ss}} - 2\,d_c$$

$$\le \frac{700(0.75)}{(1.09)\left(16.7\,\frac{\text{kips}}{\text{in}^2}\right)} - 2(2.7 \text{ in})$$

$$\le 24.3 \text{ in}$$

Provided bar spacing, s, is 4.19 in, which is less than the maximum spacing allowed so it is ok (see Figure 2.11).

Step 4: Design the Deck Slab

In AASHTO, concrete decks can be designed either by the empirical method or the traditional method.

A Art. 9.7

The empirical design method may be used for concrete deck slabs supported by longitudinal components if the following conditions are satisfied.

A Arts. 9.7.2, 9.7.2.4

a. The supporting components are made of steel and/or concrete.
b. The deck is fully cast-in-place and water cured.
c. The deck is of uniform depth, except for haunches at girder flanges and other local thickening.
d. The ratio of effective length to design depth does not exceed 18 and is not less than 6.0.

The effective length is face-to-face of the beams monolithic with slab; therefore,

$$\text{ratio} = \frac{\text{effective length}}{\text{design depth}} = \frac{10\text{ ft} - 1.5\text{ ft}}{(9\text{ in})\left(\dfrac{1\text{ ft}}{12\text{ in}}\right)}$$

$$= 11.33 < 18 > 6\ [\text{OK}]$$

e. Core depth of the slab (out-to-out of reinforcement) is not less than 4 in.

Assuming clear cover of 2.5 in for top bars and 1.5 in for bottom bars,

$$9\text{ in} - 2.5\text{ in} - 1.5\text{ in} = 5\text{ in} > 4\text{ in}\ [\text{OK}]$$

f. The effective length does not exceed 13.5 ft.

$$8.5\text{ ft} < 13.5\text{ ft}\ [\text{OK}]$$

g. The minimum depth of slab is not less than 7 in.

$$9 \text{ in} > 7 \text{ in [OK]}$$

h. There is an overhang beyond the centerline of the outside girder of at least 5 times the depth of slab. This condition is satisfied if the overhang is at least 3 times the depth of slab and a structurally continuous concrete barrier is made composite with the overhang.

Overhang = (4.0 ft)(12 in/ft).

Check requirements.

$$(5)(9 \text{ in}) = 45 \text{ in} < 48 \text{ in [OK]}$$

$$(3)(9 \text{ in}) = 27 \text{ in} < 48 \text{ in [OK]}$$

i. The specified 28-day strength of the deck concrete, f'_c, is not less than 4 kips/in^2.

$$f'_c = 4.5 \text{ kips/in}^2 > 4 \text{ kips/in}^2 \text{ [OK].}$$

j. The deck is made composite with the supporting structural elements.

Extending the beam stem stirrups into deck will satisfy this requirement [OK].

The empirical method may be used because all of the above conditions are met.

Four layers of isotropic reinforcement shall be provided. The minimum amount of reinforcement in each direction shall be 0.27 in^2/ft for bottom steel, and 0.18 in^2/ft for top steel. Spacing of steel shall not exceed 18 in.

A Art. 9.7.2.5

Bottom reinforcement: no. 5 bars at 12 in; A_s = 0.31 in^2 > 0.18 in^2 [OK].

Top reinforcement: no. 5 bars at 18 in; A_s = 0.20 in^2 > 0.18 in^2 [OK].

The outermost layers shall be placed in the direction of the slab length.

Alternatively, the traditional design method may be used.

A Art. 9.7.3

If the conditions for the empirical design are not met, or the designer chooses not to use the empirical method, the LRFD method allows the use of the traditional design method.

Concrete slab shall have four layers of reinforcement, two in each direction and clear cover shall comply with the following AASHTO Art. 9.7.1.1 provisions.

Top bars have 2.5 in and bottom bars have 1.5 in clear covers.

The approximate method of analysis for decks divides the deck into strips perpendicular to the support elements. The width of the strip is calculated according to AASHTO Art. 4.6.2.1.3.

A Art. 4.6.2.1

The width of the primary strip is calculated for a cast-in-place concrete deck using the following calculations from AASHTO Table 4.6.2.1.3-1.

Overhang: 45 + 10 X

+M: 26 + 6.6 S

–M: 48 + 3.0 S

The spacing of supporting elements, S, is measured in feet. The distance from load to point of support, X, is measured in feet.

A deck slab may be considered as a one-way slab system. The strip model of the slab consists of the continuous beam, and the design truck is positioned transverse for the most critical actions with a pair of 16 kip axles spaced 6 ft apart.

Design Example 2: Load Rating of Reinforced Concrete T-Beam by the Load and Resistance Factor Rating (LRFR) Method

Problem Statement

Use the *Manual for Bridge Evaluation* (*MBE-2*), Second Edition 2011, Section 6: Load Rating, Part A Load and Resistance Factor Rating (LRFR). Determine the load rating of the reinforced-concrete T-beam bridge interior beam using the bridge data given for the Limit State Combination Strength I. Also refer to *FHWA July 2009 Bridge Inspection System*, Condition and Appraisal rating guidelines.

L	span length	50 ft
s	beam spacing	10 ft
f'_c	concrete strength	4.5 kips/in^2
f_y	specified minimum yield strength of steel	60 kips/in^2
DW	future wearing surface load	0.03 kips/ft^2
ADTT	average daily truck traffic in one direction	1900
	skew	0°
Year Built:		1960