Appendix B

Units, Conversion Factors and Unity Brackets

The S.I. system of units has been in use for some years now. Inspite of this, it is often necessary to refer back to the imperial unit system when dealing with old drawings, books or for some other purpose. For this reason, units relevant to soil mechanics, and their conversion from one system to the other are listed in this Appendix. Also, the unity bracket method of conversion, initiated by A.C. Walshaw, is introduced.

Basic units

The S.I. and the imperial systems are based on the following three units:

Table B1 (a)

	S.I.	Imperial
Length (*L*)	Metre (m)	Foot (ft)
Mass (*M*)	kilogramme or kilogram (kg)	Pound (lb)
Time (*T*)	Seconds (s)	Seconds (s)

Note: The unit of force (*F*), in general, is F Newton.

The unit of gravitational force is $W=Mg$ Newton. The imperial equivalent is the pound force (lbf) at $g=32.2$ ft/s^2.

Table B1 (b)

Force (*F*)	Newton (N)	Pound force (lbf)
Weight (*W*)	kg ↓ 9.81 m/s^2 → N	lb ↓ 32.2 ft/s^2 → lbf

Introduction to Soil Mechanics, First Edition. Béla Bodó and Colin Jones.

Multiplier prefixes

The basic units may be made larger or smaller by multiplying them by ± powers of ten. The standard multipliers, their symbols and names are:

Table B.1 (c)

Name	Symbol	Multiplier	Conversion factors
tera	T	10^{12}	Not normally used in soil mechanics
giga	G	10^{9}	
mega	M	10^{6}	Mega gram: 1 Mg = 10^{6} g
kilo	k	10^{3}	Kilometre: 1 km = 10^{3} m
hecto	h	10^{2}	Hectometre: 1 hm = 10^{2} m
deca	da	10	Decametre: 1 dam = 10 m
deci	d	10^{-1}	Decimetre: 1 dm = 10^{-1} m
centi	c	10^{-2}	Centimetre: 1 cm = 10^{-2} m
milli	m	10^{-3}	Millimetre: 1 mm = 10^{-3} m
micro	μ	10^{-6}	Micrometre: 1 μm = 10^{-6} m
nano	n	10^{-9}	Not normally used in soil mechanics
pico	p	10^{-12}	
femto	f	10^{-15}	
atto	a	10^{-18}	

Note: M in this table, must not be confused with mass!

Unity bracket

Conversion factors may be transformed into unity by transferring the quantity from one side of the equality sign to the other and enclose it in a square bracket, as illustrated by the following simple example:

From Table B.1: Conversion factor: 1 cm = 0.01 m. The unity bracket may be written in two ways, as the reciprocal of unity is still unity.

Either $$1 = \left[\frac{0.01\ \text{m}}{\text{cm}}\right] \quad \text{or} \quad 1 = \left[\frac{\text{cm}}{0.01\ \text{m}}\right]$$

The expressions in brackets are therefore equal to one. The significance of this will be explained later.

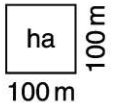
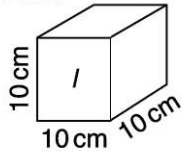
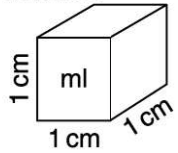
Application of the unity brackets

The procedure is based on the two simple facts:

1. Unity multiplied by unity remains unity.
2. Unity raised to any power remains unity.

Because each bracket equals unity, they may be multiplied together or exponentiated and the final result is equated to unity. New conversion factors can be formulated in this way.

Table B.2 Conversion Factors

	Unit System				
Quantity	**S.I.**		**Imperial**		**Conversion factors**
Length	Metre	m	Foot	ft	1 m = 3.281 ft 1 ft = 0.3048 m
			Inch	in	1 ft = 12 in 1 in = 25.4 mm
	Millimetre	mm			
	Kilometre	km	Mile (statute)	mile	1 mile = 5280 ft 1 mile = 1.609 km
			Yard	yd	1 yd = 3 ft 1 yd = 0.9144 m
Area	Area = 10 m^2	are			1 are = 100 m^2
	Hectare	ha			1 ha = 10^4 m^2
			Acre	acre	1 Acre = 4840 yd^2 1 Acre = 0.4047 ha
	sq. kmetre	km^2	sq. mile	$mile^2$	1 $mile^2$ = 2.59 km^2
	sq. metre	m^2	sq. feet	ft^2	1 m^2 = 10.764 ft^2
	sq. cm	cm^2	sq. inch	in^2	1 cm^2 = 0.155 in^2
	sq. mm	mm^2			1 in^2 = 645.2 mm^2
Volume and capacity	Litre	*l*	UK gallon	gal	1 gal = 4.546 *l*
			US gallon		1 gal = 3.785 *l*
					1 *l* = 1000 cm^3
			cu. feet	ft^3	1 ft^3 = 28.32 *l*
	Millilitre	ml			ml = 1 cm^3
			cu. inch	in^3	1 in^3 = 16.39 ml
		m^3		ft^3	1 m^3 = 35.32 ft^3
					1 m^3 = 1000 *l*
				yd^3	1 yd^3 = 0.7646 m^3
		ml	Fluid ounce	fl. oz	1 ml = 0.0352 fl. oz
		l	Pint	Pint	1 *l* = 1.76 pint

(continued)

Table B.2 (continued)

Quantity	Unit System S.I.		Imperial		Conversion factors
Mass	gramme	g	Ounce	oz	1 oz = 28.349 g
	kilogramme	kg	Pound	lb	1 lb = 0.4536 kg
	tonne (1. Mg)	tonne	ton	ton	1 tonne = 0.9842 ton
					1 tonne = 1000 kg
			Hundredweight	cwt	1 cwt = 50.802 kg
			Stone	Stone	1 stone = 14 lb
		kg		Stone	1 stone = 6.35 kg
Force	Newton	N	Pound force	lbf	1 lbf = 4.45 N
			Ton force	tonf	tonf = 2240 lbf
	kilonewton	kN			1 kN = 1000 N
				tonf	1 tonf = 9.968 kN
Pressure and stress	Newton/m^2	N/m^2	Poundforce / ft^2	lbf / ft^2	lbf / ft^2 = 47.88 N/m^2
	pascal	Pa			1 Pa = 1 N/m^2
	bar	b			1 b = 10^5 N/m^2
	millibar	mb			1 mb = 10^2 N/m^2
	kilonewton/m^2	kN/m^2	Tonforce / ft^2	tonf / ft^2	1 tonf / ft^2 = 107.25 kN/m^2
	Newton/mm^2	N/mm^2	Poundforce / in^2	lbf / in^2	N/mm^2 = 145.14 lbf / in^2
		N/mm^2	Tonforce / in^2	tonf /in^2	tonf / in^2 = 15.44 N/mm^2
	meganewton	MN			1 MN = 10^6 N
		mm of water			1 mm H_2O = 9.81 N/m^2
		mm of mercury			1 mm Hg = 133.3 N/m^2
		Average atmospheric pressure at sea level			1 atm = 101.33 Pa 1 atm = 14.7 lbf / in^2
Density (*P*)	kilogram / m^3	kg/m^3	Pound /ft^3	lb / ft^3	1 lb / ft^3 = 16.02 kg/m^3
	kilogram / litre	kg / *l*	Pound /in^3	lb / in^3	1 lb / in^3 = 27.68 Kg / *l*
	gram / m^3	g/m^3			1 lb / in^3 = 10^6 × 27.68 g/m^3
Unit Weight (γ)	kilonewton / m^3	kN/m^3	Poundforce / ft^3	lb / ft^3	1 kN/m^3 = 6.363 lb / ft^3
			Poundforce / in^3	lb / in^3	1 kN/m^3 = 3.683 lb / m^3

For water: $\gamma_w = 9.81\,kN/m^3 = 62.44\,lb_f/ft^3$

$\rho_w = 1\,kg/m^3$

Example B1

Prove that: a) $1\,\text{tonf} = 9.968\,\text{kN}$

b) $1\,\text{N/mm}^2 = 145.14\,\text{lbf/in}^2$

c) $\gamma_w = 9.81\,\text{kN/m}^3 = 62.44\,\text{lbf/ft}^3$

a) Step 1

Start from another known conversion factor containing tonf. Choose say: 1 tonf = 2240 lbf to get the bracket:

$$1 = \left[\frac{2240\,\text{lbf}}{\text{tonf}}\right]$$

Another factor had to be found, which contains 'lbf' in order to eliminate it from the first bracket. Choose, say:

$$1\,\text{lbf} = 4.45\,\text{N}$$

Hence,
$$1 = \left[\frac{4.45\,\text{N}}{\text{lbf}}\right]$$

Step 2

Multiply the brackets together and cancel lbf:

$$1 = \left[\frac{2240\,\text{lbf}}{\text{tonf}}\right]\left[\frac{4.45\,\text{N}}{\text{lbf}}\right] = \frac{2240 \times 4.45\,\text{N}}{\text{tonf}}$$

But, the result should be in terms of kN, so N has to be eliminated by:

$$1\,\text{kN} = 1000\,\text{N}$$
$$\text{or } 1 = \left[\frac{\text{kN}}{1000\,\text{N}}\right]$$

Step 3

Multiply the brackets together and cancel N.

$$1 = \left[\frac{2240 \times 4.45\,\text{N}}{\text{tonf}}\right]\left[\frac{\text{kN}}{1000\,\text{N}}\right] = \left[\frac{2240 \times 4.45\,\text{kN}}{1000\,\text{tonf}}\right]$$
$$= \left[\frac{9.968\,\text{kN}}{\text{tonf}}\right]$$

Converting it to factor: tonf = 9.968 kN ∴ true

Alternatively, any other initial choice may be made, say,

$$1\,\text{tonf/ft}^2 = 107.25\,\text{kN/m}^2$$
$$\text{or}$$
$$1 = \left[\frac{107.25\,\text{kN} \times \text{ft}^2}{\text{tonf} \times \text{in}^2}\right]$$

Now, m^2 and ft^2 have to be eliminated.

Choose, $1\,\text{m} = 3.281\,\text{ft}$

And $$1 = \left[\frac{\text{m}}{3.281\,\text{ft}}\right]$$

Squaring the bracket $$1 = \left[\frac{\text{m}}{3.281\,\text{ft}}\right]^2 = \left[\frac{\text{m}^2}{10.765\,\text{ft}^2}\right]$$

Multiply the brackets and cancel m^2 and ft^2,

$$1 = \left[\frac{107.25\,\text{kN}\cancel{\text{ft}^2}}{\text{tonf} \times \cancel{\text{m}^2}}\right] \times \left[\frac{\cancel{\text{m}^2}}{10.765\,\cancel{\text{ft}^2}}\right] = \left[\frac{9.963\,\text{kN}}{\text{tonf}}\right]$$

b) Choose: $1\,\text{lbf} = 4.45\,\text{N}$ $\therefore$ $$1 = \left[\frac{4.45\,\text{N}}{\text{lbf}}\right]$$

Also, $1\,\text{in} = 25.4\,\text{mm}$ $\therefore$ $$1 = \left[\frac{25.4\,\text{mm}}{\text{in}}\right]$$

Square the second bracket to get in^2 and mm^2

$$1 = \left[\frac{25.4\,\text{mm}}{\text{in}}\right]^2$$

Multiply the brackets: $$1 = \left[\frac{4.45\,\text{N}}{\text{lbf}}\right] \times \left[\frac{\text{in}}{25.4\,\text{mm}}\right]^2$$

$$1 = \left[\frac{4.45\,\text{N}\,\text{in}^2}{645.16\,\text{mm}^2 \times \text{lbf}}\right] = \left[\frac{\text{N}\,\text{in}^2}{145.\ \text{mm}^2 \times \text{lbf}}\right]$$

From which, $$1\frac{\text{N}}{\text{mm}^2} = 145\frac{\text{lbf}}{\text{in}^2}$$

NB: Any discrepancies is due to cumulative arithmetic errors.

c) $\gamma_w = 9.81\frac{\text{kN}}{\text{m}^3} = 9.81\left[\frac{\text{kN}}{\text{m}^3}\right]$

Choose: $1\,\text{lbf} = 4.45\,\text{N}$ $$1 = \left[\frac{4.45\,\text{N}}{\text{lbf}}\right]$$

And: $1\,\text{kN} = 1000\,\text{N}$ $$1 = \left[\frac{1000\,\text{N}}{\text{kN}}\right]$$

And: $1\,\text{m} = 3.281\,\text{ft}$ $$1 = \left[\frac{3.281\,\text{ft}}{\text{m}}\right]$$

Multiply the brackets and eliminate kN, m^3 and N

$$\gamma_w = 9.81\left[\frac{\text{kN}}{\text{m}^3}\right] \times \left[\frac{1000\,\text{N}}{\text{kN}}\right] \times \left[\frac{\text{lb}_f}{4.45\,\text{N}}\right] \times \left[\frac{\text{m}}{3.281\,\text{ft}}\right]^3$$

$$= 9.81\left[\frac{1000 \times \text{lbf}}{4.45\,\text{m}^3}\right] \times \left[\frac{\text{m}^3}{35.32\,\text{ft}^3}\right]$$

$$= 9.81\left[\frac{6.3624\,\text{lbf}}{\text{ft}^3}\right] = 62.41\,\text{lbf/ft}^3 \qquad \text{True}$$

Note: Any other conversion factor not present in Tables B.1 and B.2 may be derived in this manner.