Markov Renewal Model for Maximum Bridge Loading

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Abstract: The prediction of maximum vehicle loadings on a bridge is studied. The stationary distribution of the static response of highway bridges under random truck loading is obtained using a Markov Renewal Model. This model is a generalization of Markov chains and renewal processes and can be used to model the arrival of trucks on a multilane bridge. The model also accounts for random truck characteristics such as axle weights, axle spacings, speed, and the headway distribution between trucks. The stationary distribution of the response is obtained assuming that the bridge (represented by its influence line) acts as a filter to the truck arrival process. The maximum lifetime response is obtained from the stationary distribution using an approximation to Rice's upcrossing rate formula. The results are then compared to a simulation program and acceptable agreement is reported.

Introduction

The theory of stochastic processes provides a useful tool for analyzing civil engineering structures subjected to random loadings. One such problem is the static response of highway bridges under random truck loading. This problem is characterized by the occurrence of millions of load events and the need to find the distribution of the maximum loading. The parameters entering the modeling include the distribution and characterization of truck vehicles, which includes axle and gross weights, axle spacings, and the headway or spacing of different vehicles in the same and adjacent lanes. Bridge lives typically exceed 50 years, although load predictions for shorter periods may be needed in evaluating existing bridges. For long spans the trucks may be considered as point loadings, but for typical short and medium spans (less than 150 m), the vehicle length and loading distribution is important.

Several studies were concerned with solving this problem, most of which were either based on simulation techniques or else simply dealt with one lane loadings. For example, in one of the earliest studies on the subject, Tung (19) idealized the vehicles as point loads arriving according to a Poisson process. The bridge acts as a filter and the stationary distribution of the bridge's stochastic response is calculated. Ditlevsen (5) assumes the trucks to have random weights distributed over fixed lengths, such that the spacings between trucks are a mixture of random and fixed distances. Larrabee (9), in his review of bridge loading analyses, investigates more of these studies and reports on generalizations to account for multilane loadings.

This paper presents a multi-dimensional stochastic process approach

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for studying the multilane loading of highway bridges. The technique used is based on the Markov Renewal theory used by several researchers to solve shot noise problems such as the noise at a given point due to vehicles traveling on a nearby highway (11,18). The Markov renewal model provides a technique by which the probability density function of the stationary bridge response can be evaluated. The distribution of the response can then be used to estimate the probability distribution of the maximum response over the lifetime of the bridge structure. One advantage of the Markov renewal model is in the fact that it can be adapted to both measured or idealized truck arrival models (headway distributions) and truck properties (weights and axle spacing distributions), as long as the basic Markov and renewal assumptions are preserved.

Truck Loading Model

Trucks arrive at a bridge in a random sequence, which is often idealized as a renewal process (15,19). The assumption in such an idealization is that the time duration between consecutive truck arrivals are independent and identically distributed. The most frequently used renewal model to represent truck traffic patterns such as arrival of trucks on a bridge is the simple Poisson process. Most models are one-dimensional in the sense that they represent arrivals in only one lane. In addition, the restrictive Poisson model (interoccurrence times follow an exponential distribution) has been verified for general car traffic (2,12) and may not necessarily be suitable for truck traffic. A more general model suggested herein, is the Markov Renewal process (see Appendix I). Such a model can represent a multistate system, such as truck arrivals in several lanes, and can be adapted to any measured or idealized arrival model. Restrictions on this model include the fact that truck properties such as gross weights, truck type, etc., are independent of their arrival sequence.

Observing truck traffic arriving at a given point, such as the beginning of a bridge on a two lane highway, three different lane position states or truck arrival events can be distinguished as shown in Fig. 1:

1. A truck is in the right lane (this state will be referred to as lane position 0 in the rest of the text).
2. A truck is in the left lane (lane position 1).
3. Two trucks are side by side (lane position 2).

Case 3 is considered as a separate state since a renewal process does not permit an interoccurrence time equal to zero, i.e., no simultaneous occurrences are possible in a renewal model. Fig. 1 gives a sample path of the truck arrival process showing the different lane position states considered.

The Markov renewal process conditions the time of a new arrival on the time of arrival and the lane position of the previous truck. From Fig. 1, it is obvious that under certain cases, truck arrival #5 could overlap truck #3 unless the time of arrival of truck 5 is conditioned on the time of arrival of trucks 4 and 3. So, as a modification to the Markov renewal

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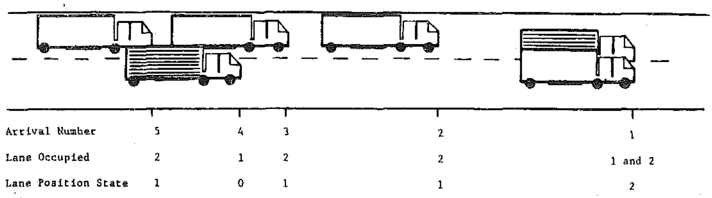

FIG. 1.—Sample Path of Truck Arrival Process Showing Lane Position States

process, each new truck arrival is herein conditioned on the two previous arrivals as will be seen below.

Moment Response of Bridge System.—The structural response of a bridge system subjected to a moving load is described by an influence line. If truck arrivals follow a Markov Renewal process, the bridge acts as a filter represented by its moment influence line at a given location, and the moment response $M(t)$ follows a filtered Markov Renewal process. If L is the influence line representation of a bridge structure, the response at time t, at a point on the bridge due to a single truck n is

$$M_n(t) = L(t - T_n, X_n = i, \mathbf{Z}_n) \quad \text{(1)}$$

in which $t - T_n$ describes the location of truck n with respect to the time, t, at which the response is being measured. This truck location is calculated from the time of arrival of the truck, T_n, and the truck's speed, assuming that the speed is constant while the truck is on the bridge. X_n describes the truck lane position, which will be denoted by i. The term i can take any of the values 0, 1, or 2, as shown in Fig. 1. $\mathbf{Z}_n$ = a vector representing the properties of the truck event, n. These properties, which are random variables, are speed, gross weight, number of axles, axle spacings, and axle weights. It should be noted that, if by time t, the truck has left the bridge then the influence line function automatically assigns a value of zero to $M_n(t)$.

Given the time of occurrence of a truck event, the bridge response due to this event is a random variable since the trucks' properties are random. The characteristic function of a single event's response is given by

$$\Gamma_i(t,\omega) = \int_{-\infty}^{\infty} e^{\sqrt{-1}\omega L(t-T_n, X_n=i, \mathbf{Z}_n)} dH(\mathbf{Z}_n) \quad \text{(2)}$$

in which H = the cumulative distribution of vector, $\mathbf{Z}_n$.

The total response on the bridge due to N truck events between a reference time $T_1 = 0$ (arrival of the first truck and t, the time at which the response is measured) is given by

$$M(t) = \sum_{n=1}^{N} L(t - T_n, X_n, \mathbf{Z}_n) \quad \text{(3)}$$

Since the arrival of a new truck event is conditioned on the previous two truck arrivals, one can study the whole renewal process by examining the arrival of the first three trucks on the bridge. If N is greater than or equal to 3, then a minimum of three trucks arrived at the bridge before time t and the response can be expanded:

$$M(t) = L(t - T_1, X_1 = i, \mathbf{Z}_1) + L(t - T_2, X_2 = j, \mathbf{Z}_2)$$

$$+ L(t - T_3, X_3 = k, \mathbf{Z}_3) + \sum_{n=4}^{N} L(t - T_n, X_n, \mathbf{Z}_n) \quad \text{(4)}$$

If t (time at which the moment is being evaluated) is far enough from the time of arrival of the first truck ($T_1 = 0$), then by time t the first truck has cleared the bridge and its effect on the total response is null. To study the response on the bridge a "long" time after the initiation of the process (i.e., the stationary response), one can then neglect the moment effect of the first truck. However, since the second arrival is conditioned on the lane position state of the first truck, the stationary response of all the trucks is then conditioned on that lane position state. The lane position states of the first, second, and third truck arrival event are respectively indicated here by the indices i, j, and k. If the first truck was in lane position i, then $M_i(t)$ is defined as the response of the rest of the trucks at time t. That is, the total response at t can be divided into two parts, the response due to the first truck, which is equal to $L(t - T_1, X_1 = i, \mathbf{Z}_1)$, and the response of all the rest of the trucks that arrive later than T_1 and which are conditioned on i (i = the lane position of the first truck), and their response is denoted by $M_i(t)$. This will give the following equations:

$$M_i(t) = M(t) - L(t - T_1, X_1, \mathbf{Z}_1)$$

$$M_i(t) = L(t - T_2, X_2 = j, \mathbf{Z}_2) + L(t - T_3, X_3 = k, \mathbf{Z}_3) + \sum_{n=4}^{N} L(t - T_n, X_n, \mathbf{Z}_n)$$

$$M_i(t) = L(t - T_2, X_2 = j, \mathbf{Z}_2) + L(t - T_3, X_3 = k, \mathbf{Z}_3) + M_k(t - T_3) \quad \text{(5)}$$

in which $M_k(t - T_3)$ = the response at time t of the trucks that arrive after time T_3, given that the truck that arrived at T_3 is in lane position k, which can = 0, 1, or 2 as defined earlier.

The characteristic function of $M_i(t)$ is denoted by $\phi_i(t, \omega)$. Since the characteristic function of the sum of independent random variables is the product of their characteristic functions, then from Eq. 5

$$\phi_i(t, \omega) = \Gamma_j(t - T_2, \omega)\Gamma_k(t - T_3, \omega)\phi_k(t - T_3, \omega) \quad \text{(6)}$$

in which $\Gamma_j(t - T_2, \omega)$ = the characteristic function of $L(t - T_2, X_2 = j, \mathbf{Z}_2)$; $\Gamma_k(t - T_3, \omega)$ = the characteristic function of $L(t - T_3, X_3 = k, \mathbf{Z}_3)$; and $\Gamma_k(t - T_3, \omega)$ = the characteristic function of $M_k(t - T_3)$.

If T_2 is less than t but T_3 is greater than t, then $\Gamma_k(t - T_3, \omega) = 1$ and $\phi_k(t - T_3, \omega) = 1$, so

$$\phi_i(t, \omega) = \Gamma_j(t - T_2, \omega) \quad \text{(7)}$$

If both T_2 and T_3 are greater than t then also $\Gamma_j(t - T_2, \omega) = 1$ and

$$\phi_i(t, \omega) = 1 \quad \text{(8)}$$

The three cases considered in Eqs. 6–8 correspond to specific arrival times of the first three trucks T_1, T_2, and T_3 with specific lane position indices and have corresponding probabilities. The first step is to condition on the first truck arrival time T_1 used as reference ($T_1 = 0$) and the lane position of the first arrival event i, to include all possible combinations of lane positions and arrival times of the rest of the trucks. This is accomplished by summing j and k over the three truck event lane positions (0,1,2) and integrating for all possible T_2 and T_3. Using Eqs. 6–8 gives the expression

$$\phi_i(t,\omega) = \sum_{j=0}^{2} \sum_{k=0}^{2} \left[\int_{T_2=0}^{t} \int_{T_3=T_2}^{t} \Gamma_j(t - T_2, \omega)\Gamma_k(t - T_3, \omega) \right.$$

$$\phi_k(t - T_3, \omega)P(i,j,k; T_2, T_3)dT_3dT_2$$

$$\left. + \int_{T_2=0}^{t} \Gamma_j(t - T_2, \omega)P'(i,j,k; T_2, t)dT_2 + P''(i,j,k; t) \right] \quad (9)$$

in which $P(i,j,k; T_2, T_3)$ = the probability that, given a first truck in lane position i, there is a second truck in lane position j at time T_2, and a third truck in lane position k at time T_3.

$P'(i,j,k; T_2, t)$ = the probability that, given a first truck in lane position i, there is a second truck in lane position j at time T_2, and a third truck in lane position k at *a* any time beyond t.

$P''(i,j,k; t)$ = the probability that, given a first truck in lane position i, the two following trucks in lane positions j and k respectively arrive later than t.

Since i can have any one of the three lane position states 0, 1, or 2, Eq. 9 represents a set of simultaneous integral equations to be solved for the functions $\phi_i(t,\omega)$. A numerical method to directly solve these equations has been presented in Ref. 6.

If t is large enough that the effect of the first truck is zero, then $\phi_i(t,\omega)$ becomes the characteristic function of the total response conditional on the lane position of the first arrival. In order to obtain the unconditional characteristic function, a summation is executed over all possible lane positions of the first truck with their corresponding probabilities:

$$\phi(t,\omega) = \sum_{i=0}^{2} \phi_i(t,\omega)P(i) \quad (10)$$

in which i = 0, 1, or 2 denotes the lane position of the first truck and $P(i)$ = the probability of having a truck in lane position i. The probability density function of $M(t)$ can be obtained from $\phi(t,\omega)$ by inverting it as illustrated by Racicot (16).

Calculating Probability Density Function of Response.—Racicot (16) suggested the use of Fourier series expansions to invert the characteristic function. A density function can be expressed by its Fourier coefficients such that

$$f(x) = \sum_{k=-\infty}^{\infty} C_k e^{-\sqrt{-1}\, k\pi x/L} \quad (11)$$

$$\text{in which} \quad C_k = \frac{1}{2L} \int_{-L}^{L} f(x) e^{\sqrt{-1}\, k\pi x/L} dx \quad (12)$$

In this case, $f(x)$ = the stationary probability density function of the bridge response. The infinite limit ∞ is substituted by L such that $f(x)$ goes to zero as x goes to L. The characteristic function of $f(x)$ is defined as

$$\phi(t,\omega) = \int_{-\infty}^{\infty} f(x)e^{\sqrt{-1}\omega x}dx \quad \text{(13)}$$

So for $\omega = k\pi/L$, C_k becomes

$$C_k = \frac{1}{2L}\phi\left(t, \frac{k\pi}{L}\right) \quad \text{(14)}$$

The term $f(x)$ can also be written in the trigonometric form:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\frac{k\pi x}{L} + b_k \sin\frac{k\pi x}{L} \quad \text{(15)}$$

in which $a_0 = 2C_0$; $a_k = C_k + C_0$; and $b_k = \sqrt{-1}\,(C_k - C_{-k})$.

Due to the nature of truck loading on short and medium span bridges, it is known that for most of the time, the bridge does not experience any significant load. Light automobile traffic and periods of no traffic dominate the stationary load. It is for only a short portion of the time (less than 10%) that the bridge experiences heavy truck loading. This fact indicates that the probability density of the bridge response is dominated by a high spike at the level of zero response. Since the Fourier transform procedure is a smoothing technique, then the simple application of the formula will emphasize the spike at zero and give less accurate results for the rest of the responses that interest us most. For this reason, a constant is subtracted from the characteristic function. The Fourier transform of a constant is a delta function centered at the origin and in this case, corresponds to the probability of no activity on the bridge. The probability distribution for the active loading period is then obtained from the Fourier transform of the calculated characteristic function (Eq. 10) modified by subtracting a constant, ϕ_{min}. In this calculation algorithm, the constant is arbitrarily chosen as the minimum characteristic value.

The overall solution needed is the distribution of maximum response in the lifetime of the bridge structure, which is usually around 50 yr. Thus, very good accuracy for the probability density of the stationary response is needed. This is especially true at the high moment levels or the tail of the distribution. Theoretically in order to obtain high accuracy, one needs good accuracy in the calculation of the characteristic function in addition to large number of terms in the Fourier series. Using a large number of terms in the series is very costly as each term involves the solution of three simultaneous integral equations. The use of fast convergence techniques is mandatory. In this study, fast convergence is achieved through the use of an attenuation factor σ as suggested by Lanczos (8):

$$\sigma_k = \frac{\sin\dfrac{\pi k}{m}}{\dfrac{\pi k}{m}} \quad \text{(16)}$$

in which m = the maximum number of terms in the series. In these calculations 100 points are used.

Despite the increased accuracy obtained through the use of σ, this was still not sufficient for 50-yr projections. For this reason, other methods of statistical projections are used, as will be explained in the next section.

As an example of the accuracy of the proposed procedure, the described techniques are applied to a hypothetical Poisson arrival model, and the results are compared to the available exact answers. Trucks in this example are idealized as point loads with a uniform weight distribution between 40 and 80 kips. They are assumed to arrive at a simple span bridge of 30 ft, according to a Poisson model with an arrival rate of $\lambda = 0.025$ trucks/sec and a speed of 100 ft/sec. The probability density of the stationary response at the midspan is calculated. The mean stationary response obtained is 1,683 kip-ft and the second moment is 536.1 (kip-ft)^2. Exact results for this problem are available and produce a mean response of 1,688 kip-ft and a second moment of 527.85 (kip-ft)^2. This represents errors of 0.3% and 1.6%, respectively. These results illustrate the possibility of using the aforementioned techniques for accurate estimation of the distribution of the stationary response.

Barrier Upcrossing Rate and Probability Distribution of Maximum Lifetime Response.—As given by Rice (17), the mean upcrossing rate of a certain level, m, can be obtained using Eq. 17:

$$\nu_m = \int_0^\infty m' f(m, m') dm' \quad \text{(17)}$$

in which $f(m, m')$ = the bivariate distribution function of $M(t)$ and its derivative with respect to time $M'(t)$ which are uncorrelated (3). If in addition, $M'(t)$ is assumed to be independent of $M(t)$ given that there is a vehicle on the bridge ($m \neq 0$), then $f(m, m') = f(m) \cdot f(m'|m \neq 0)$, and ν_m becomes

$$\nu_m = f(m) \int_0^\infty m' f(m'|m \neq 0) dm' \quad \text{(18)}$$

The assumption of independence between m and m' is an idealization of the process that is approached as the span length increases and the number of axles simultaneously on the bridges increases, and the stationary distribution approaches a Gaussian process.

The calculation of $f(m')$ is executed in the same manner that $f(m)$ is executed, however the influence line of the $M'(t)$ effect is the derivative with respect to time of the influence line of the $M(t)$ effect. The barrier upcrossing rate is obtained from the average of the positive part of the distribution of the $M'(t)$ response and is conditional on having a response $m \neq 0$. The calculation of $f(m')$ as explained earlier includes the derivative of the response when no vehicles are on the bridge. So, to calculate $f(m'|m \neq 0)$, $f(m')$ should be renormalized by truncating the part of $f(m')$ having a zero response.

To calculate the maximum response in a period T, Larrabee (10) suggests the use of a Poisson model for the arrival of peaks such that

$$G_m(T) = e^{-\nu_m T} \quad \text{(19)}$$

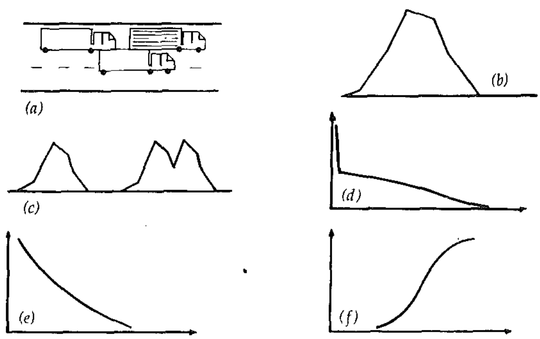

FIG. 2.—Steps Involved in Calculation of Maximum Lifetime Response of Highway Bridges: (*a*) Truck Arrivals; (*b*) One Truck's Response; (*c*) Response Realizations; (*d*) Probability Density Function of Stationary Response; (*e*) Upcrossing Rates; (*f*) Probability Distribution of Maximum Lifetime Response

in which $G_m(T)$ = the probability distribution function of the maximum response in a period, T.

However, when T is extremely large (up to 50 yr for bridge structures) in order to obtain an accurate median value on the maximum response, ν_m should be on the order of 10^{-10}. If $f(m'|m \neq 0)dm'$ is on the order of 10^2, then $f(m)$ should be on the order of 10^{-12}. Such a low probability is difficult to obtain despite the fast convergence techniques. To supplement the suggested techniques, a statistical projection approach can be used to extrapolate the results. Two methods are suggested:

1. Plotting $G_m(T')$ for some short period, T', e.g., one day or a fraction of a day, on a probability paper to determine the type of that distribution and calculate its properties; then obtaining $G_m(T)$ by raising $G_m(T')$ to the proper power. This method proved accurate for the longer spans.
2. Data fitting used on the tail of the distribution of the stationary response, $f(m)$, and projecting to the desired limits.

The entire process of calculation of maximum load effect is summarized in Figs. 2(*a*–*f*).

Results

As a verification of the accuracy of the techniques developed in the previous sections, the distribution of the maximum response over two different periods of time (one day and 50 yr) are compared to the results of a simulation program developed by Nyman (14).

Both the stochastic model and the simulation program require numerical solutions that involve round off errors and extrapolation due to the small probabilities and the large projection periods in this problem. The main aim of the comparison between the two techniques is to compare the median of the maximum expected response. Exact solutions for this problem are not available, and most reported studies on the subject use very short projection periods and much longer spans.

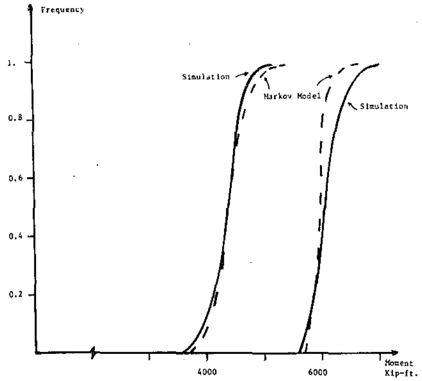

FIG 3.—Comparison of Simulation to Markov Renewal Model, Method 2, 150 ft Span

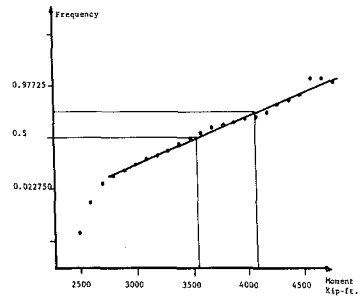

FIG. 4.—Plot of 0.1 Day Maximum Probabilities on Normal Probability Curve

The calculations here are executed for the maximum moment at the midspan of (150 ft) simply supported bridge. The input data for the headway histogram and gross weights were obtained from weigh-in-motion field measurement studies at CWRU (13). Fig. 3 compares the results of the simulation program to the results of the Markov renewal program using method 2. The projection in the Markov renewal program was obtained using an exponential fit on the stationary distribution function of the type b exp (ax^4).

Another method used to obtain the maximum load distribution for long projection periods was described earlier as method 1. The results

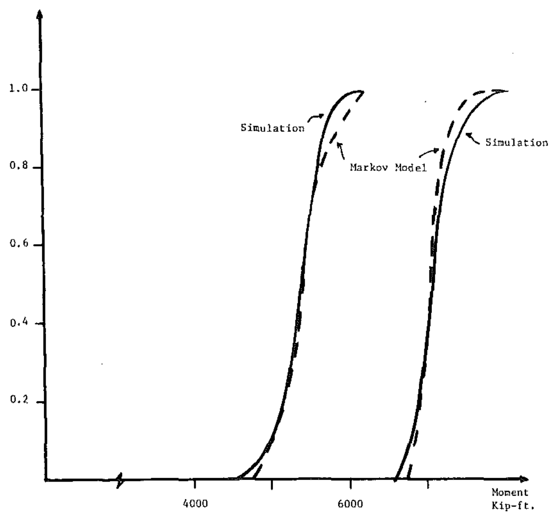

FIG. 5.—Comparison of Simulation to Markov Renewal Model, Method 1, 150 ft Span

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of the Markov Renewal program are obtained from Eq. 19 for a short projection period; 0.1 day is used. The 0.1 day projection is mapped into a normal distribution. Fig. 4 gives a plot of the 0.1 day maximum distribution on normal probability paper for the 150-ft span. As can be observed from Fig. 4, the tail of the distribution is very close to a normal distribution with a mean of 3,552 (kip-ft) and a standard deviation of 537 (kip-ft). The maximum distribution for a 50-yr period is then obtained by raising the cumulative normal distribution with the statistics given above to a power of 182,500 ($10 \times 365 \times 50$). Fig. 5 compares the results of the simulation with the Markov renewal program when this method is used. Again, acceptable agreement between medians of the maximum response from the simulation and the Markov Renewal model can be seen. This agreement was also observed in other comparisons reported in Ref. 6.

Conclusions

1. A basis for the calculation of the distribution of the maximum bridge response over the lifetime of the structure has been presented.
2. The assumptions of the model include the following:
   a. Trucks are assumed to arrive at a bridge following a Markov renewal process.
   b. The bridge acts as a filter represented by its influence line, and the bridge response is a filtered Markov renewal process.
3. The theory of Markov renewal processes provides a set of equations that can be solved to calculate the characteristic function of the response. The characteristic function can then be inverted to calculate the probability density function of the stationary bridge response.
4. The mean upcrossing rate is calculated according to Rice's formula.
5. The distribution of the lifetime maximum is calculated from the mean upcrossing rates assuming a Poisson arrival of the peaks. Good agreement between the median of the maximum response for the Markov model and the simulation have been found. The Markov Model is also capable of handling a variety of field measured distributions for vehicle characteristics, headway spacings, and other site specific bridge and traffic data.

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Appendix I.—Theoretical Background

Characteristic Functions.—A characteristic function of a continuous random variable X with a differentiable distribution function is defined as (7):

$$\Gamma(\omega) = \int_{-\infty}^{\infty} e^{\sqrt{-1}\omega x} dF_X(x) = \int_{-\infty}^{\infty} e^{\sqrt{-1}\omega x} f(x)dx \quad (20)$$

The characteristic function is basically the Fourier transform of the density distribution function. The relationship between the distribution function and the characteristic function of a random variable is 1:1. Further, if $X_1 \ldots X_n$ = independent random variables, then the characteristic function of their sum is the product of their characteristic functions. This property makes the characteristic functions extremely useful when dealing with the sum of random variables.

Markov Renewal Processes.—The stochastic process $(X,T) = \{X_n, T_n, n \in N\}$ is said to be a Markov Renewal process with state space E if (4)

$$\Pr\{X_{n+1} = j, T_{n+1} - T_n < t | X_1, \ldots X_n, T_1, \ldots T_n\}$$

$$= \Pr\{X_{n+1} = j, T_{n+1} - T_n < t | X_n, T_n\} \quad (21)$$

for all $n \in N$ and $j \in E$ and $t \in T$, in which n = the number of the occurrence being observed; X_n = the state of the nth occurrence, which can also be denoted by an integer j; and T_n = the time of the nth occurrence.

From this definition, it is observed that a Markov Renewal process is a stochastic process moving from state to state in the time domain, t, such that the states visited form a Markov chain. The distribution of interoccurrence time depends on the present state and the one to be visited next, and is independent of the previous states or the interoccurrence time between any two of them. Markov Renewal processes are then a generalization of both Markov chains and renewal processes.

Filtered Markov Renewal Processes.—A filtered Markov Renewal process is a process $M(t)$ arising by means of defined linear operations L on a Markov renewal process (11,18). The process describes the response of a system subjected to an input that follows a Markov Renewal process. It can be described by an equation of the form:

$$M(t) = \sum_{n=1}^{N} L(t - T_n, X_n, \mathbf{Z}_n) \quad (22)$$

in which T_n denotes the time of occurrence of the nth event; X_n = the state reached at the nth transition; and $\mathbf{Z}_n$ = a vector giving the properties of the nth event.

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