

The following notation will be used:

$M_r$  = moment of reactions at any section;

$M_p$  = moment of loads at any section;

$\Delta_r x$  = horizontal displacement of the left end due to the reactions;

$\Delta_r y$  = vertical displacement of the left end due to the reactions;

$\Delta_r \phi$  = angular displacement of the left end due to the reactions;

$\Delta_p x$  = horizontal displacement of the left end due to the loads;

$\Delta_p y$  = vertical displacement of the left end due to the loads;

$\Delta_p \phi$  = angular displacement of the left end due to the loads;

$\Delta' x$  = horizontal movement of the left abutment;

$\Delta' y$  = vertical movement of the left abutment; and,

$\Delta' \phi$  = angular movement of the left abutment.

Although the left abutment has been cut away from the rib, leaving that end free to move whereas it was fixed before, there can be no tendency for the rib to move because the same abutment forces are still acting on it. It will be seen that the sum of the displacements of the left end of the cantilever produced by the loads and the reactions must be equal to the movement of the abutment before it was cut away. If the rib is rigidly fixed at the abutments, this movement is zero. The following equations can be written:

$$\left. \begin{aligned} \Delta_r x + \Delta_p x &= \Delta' x \\ \Delta_r y + \Delta_p y &= \Delta' y \\ \Delta_r \phi + \Delta_p \phi &= \Delta' \phi \end{aligned} \right\} \dots \quad (14)$$

Using the left end of the rib as the origin of co-ordinates, the moment,  $M_r$ , at any point,  $A$ , due to the reactions is:

$$M_r = M_L + V_L x - H y \dots \quad (15)$$

Substituting this value in Equations (8), (9), and (10) from Section I,

$$\left. \begin{aligned} \Delta_r x &= \int_L^R \frac{M_r y \, ds}{E I} = \int_L^R (M_L y + V_L x y - H y^2) \, dw \\ \Delta_r y &= \int_L^R \frac{M_r x \, ds}{E I} = \int_L^R (M_L x + V_L x^2 - H x y) \, dw \\ \Delta_r \phi &= \int_L^R \frac{M_r \, ds}{E I} = \int_L^R (M_L + V_L x - H y) \, dw \end{aligned} \right\} \dots \quad (16)$$

$$\text{As before, } ds = \frac{d s}{E I}$$

If the center of co-ordinates is now shifted to the center of gravity of the values of  $\frac{ds}{E I}$ , the terms,  $\int_L^R \frac{y \, ds}{E I}$ ,  $\int_L^R \frac{x \, ds}{E I}$ , and  $\int_L^R \frac{xy \, ds}{E I}$ , will be equal to zero. This condition is shown in Fig. 9. The reactions,  $H$  and  $V_L$ , are considered to be acting on rigid cantilever arms attached to the left end so that they are acting through the "elastic center of gravity." In order that the moment at the left end shall be kept the same as before, it is necessary to replace  $M_L$  with the moment,  $M'$ , which must be equal to  $M_L - Hb + \frac{V_L l}{2}$ , or,

$$M_L = M' + H b - \frac{V_L l}{2} \dots \quad (17)$$