Buckling-6

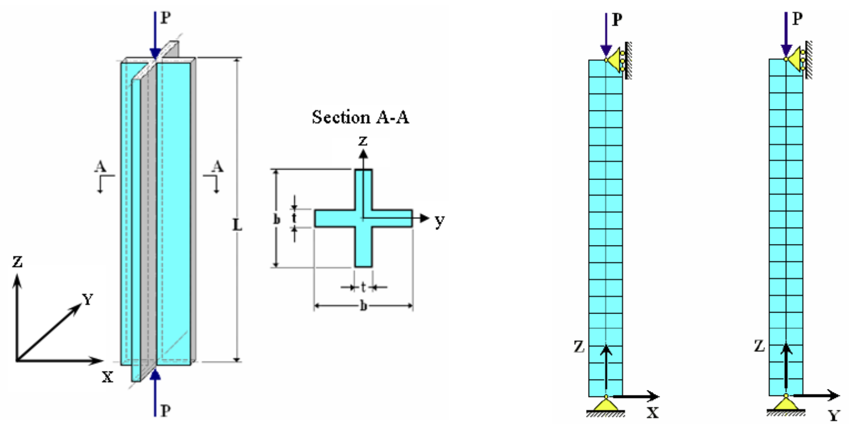
Title

Lateral buckling of a simply supported cruciform column subjected to a concentric axial load

Description

A simply supported cruciform column consisted of narrow rectangular fins undergoes a vertical load P applied at the centroid of the top end. The buckling loads are determined for the three cases in which the column is divided into 10 and 20 beam elements and 80 plate elements. The computed buckling loads are then compared with the analytical exact solution.

Case 1: 10 Beam elements evenly divided vertically
Case 2: 20 Beam elements evenly divided vertically
Case 3: 80 Plate elements evenly divided into 20 segments vertically



(a) Beam element model *(b) Plate element model*

Structural geometry and boundary conditions

Model

Analysis Type

Lateral torsional buckling

Unit System

kN, mm

Dimension

Length 3000mm

Element

Beam element and plate element (thick type without drilling dof)

Material

Young's modulus of elasticity $E = 200kN/mm^2$

Poission's ratio $\nu = 0.25$

Section Property

Beam element : combined section in a cruciform shape - thickness 6mm, width 300mm

Plate element : thickness 6mm, width 150mm, height 150mm

Boundary Condition

Bottom end is pinned, and top end is roller.

Load

P = 1.0 kN

Results

Buckling Analysis Results

Case 1: Beam elements (total 10 elements)

Buckling load

Mode	UX	UY
BUCKLING	ANALYSIS	
Mode	Eigenvalue	Tolerance
1	458,085966	0,0000e+000

Case 2: Beam element (total 20 elements)

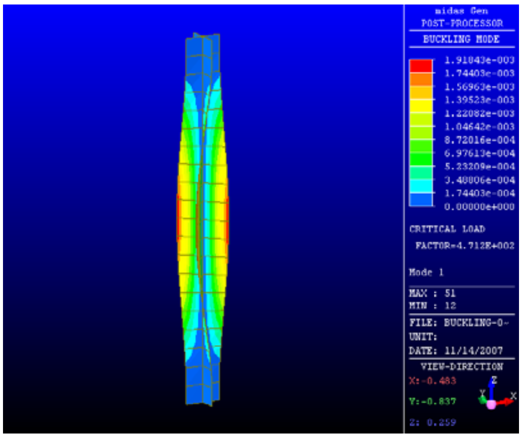
Buckling load

Mode	UX	UY
BUCKLING	ANALYSIS	
Mode	Eigenvalue	Tolerance
1	458,085966	0,0000e+000

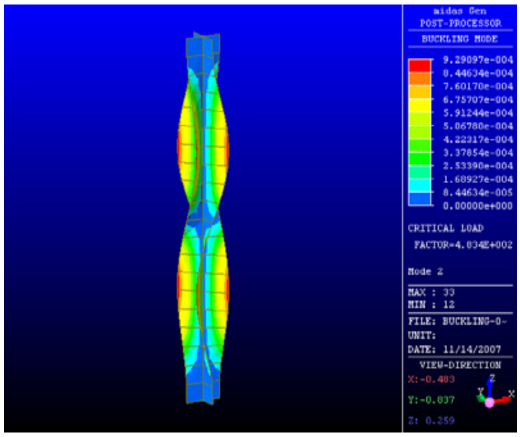
Case 3: Plate element (total 80 elements)

Buckling load

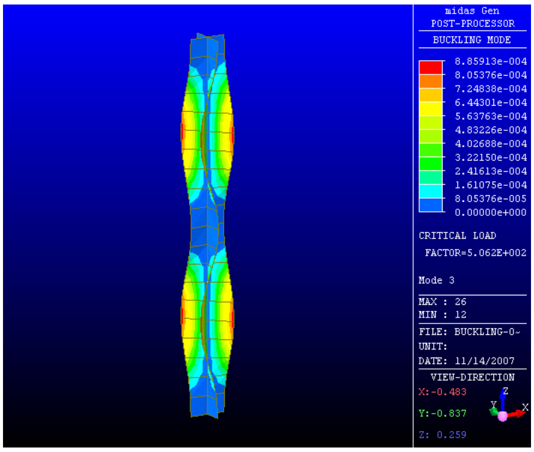
Mode	UX	UY
BUCKLING	ANALYSIS	
Mode	Eigenvalue	Tolerance
1	471,201398	4,4933e-010
2	483,422443	6,6010e-009
3	506,200573	8,0022e-009
4	532,748321	5,2235e-009



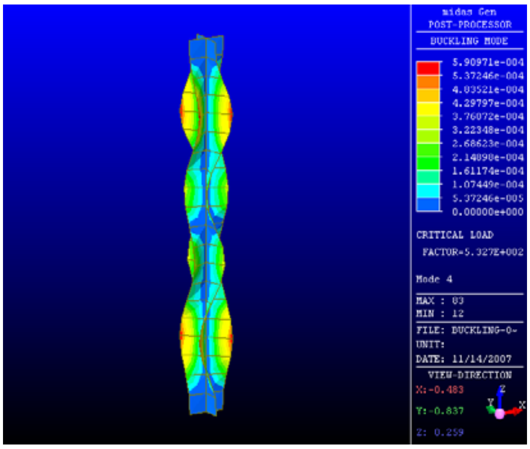
1st mode



2nd mode

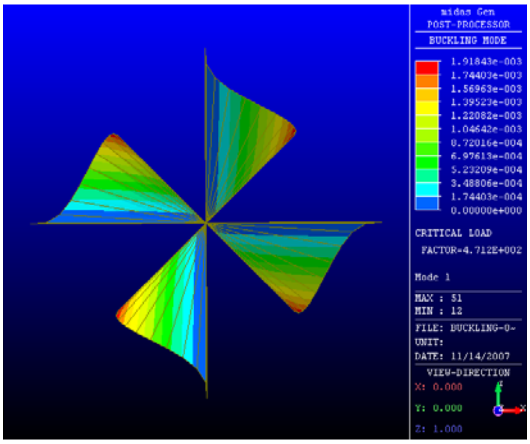


3rd mode

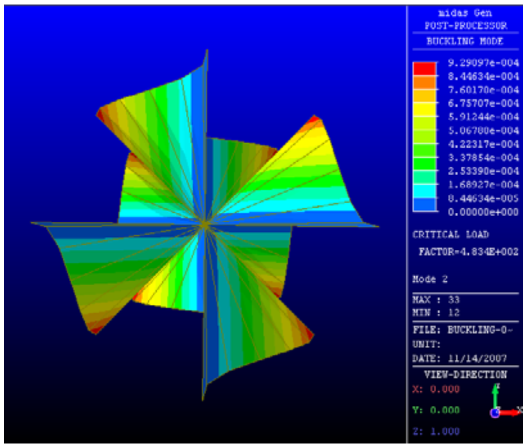


4th mode

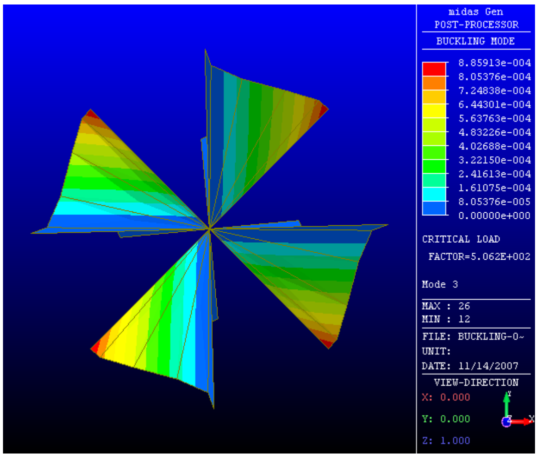
Isometric view of buckling modes of Case 3



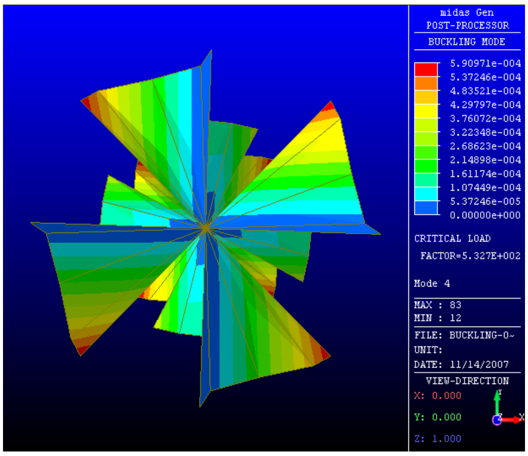
1st mode



2nd mode



3rd mode



4th mode

Perspective Top view of buckling modes of Case 3

Comparison of Results

Unit: kN

Case	Theoretical solution	Type of element	No. of total elements	Critical load for 1st buckling	Error
1		Beam element	10	458.086	0.00%
2	458.086	Beam element	20	458.086	0.00%
3		Plate element	80	471.201	2.86%

From the theory of elastic stability (Timoshenko and Gere [1]), the analytical solution for the tip cirtical load P_{cr} is defined by the following expression:

$$P_{cr} = \frac{GI_{xx}A}{I_y + I_z} = \frac{I_{xx}A}{I_y + I_z} \times \frac{E}{2(1+\nu)}$$

where,
E = Young's modulus of elasticity
G = shear modulus of elasticity
ν = poisson's ratio
I_y = moment of inertia about local y-axis
I_z = moment of inertia about local z-axis
I_{xx} = torsional moment of inertia

Substituting the material and sectional properties into the above equation gives the following result:

$$P_{cr} = \frac{I_{xx}A}{I_y + I_z} \times \frac{E}{2(1+\nu)} = \frac{4.339636 \times 10^4 \times 3.564 \times 10^3}{2 \times 1.350529 \times 10^7} \times \frac{200}{2(1+0.25)}$$
$$= 458.086 \text{ kN}$$

Reference

1. Timoshenko, S.P., and Gere, J.M., (1961). *Theory of Elastic Stability*, McGraw-Hill, New York.