An Embankment on Soft Clays with Sand Drains Numerical Characterization of the Parameters from In-situ Measurements

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SYNOPSIS The deformation parameters of a soft lacustrine deposit, with vertical sand drains, are evaluated by means of field measurements obtained during and after the construction of a railway embankment. The geotechnical system, modelled as linearly elastic and in plane strain, is analyzed by means of the finite element method and the estimation problem is solved adopting a Bayesian approach. The experimental data, the "a priori" estimation of the parameters and their uncertainties are considered in the back-analysis. The results provide the "optimal" values of the parameters, a measure of their uncertainties and, consequently, an index of the effectiveness of the field measurement program.

INTRODUCTION

The field measurements performed during construction or excavation works (e.g. displacements, settlements, pore pressure values, etc.) can be used to improve the knowledge on some aspects of the geotechnical medium. For instance, they can be adopted to evaluate (or refine) the values of the soil mass properties when a suitable mathematical model is chosen for describing the behaviour of the geotechnical medium (back-analysis or calibration problem).

With respect to this problem the following points have to be mentioned:

a) The results of the back-analysis depend on the number of in-situ measurements, on the instrument accuracy and on the location of the measurement points, (see e.g. Cividini et al., 1981; Shimizu and Sakurai, 1983).
b) Previous experience, the engineering judgement and preliminary in-situ or laboratory tests could provide an initial estimate of the soil mass properties. This valuable information should be introduced in the numerical (back) analysis.
c) An evaluation of the uncertainties of the back-analyzed parameters appears desirable. In fact this information can be taken into account in the analysis of further engineering works in the same area, on the basis of the same mathematical model.
d) The effectiveness of the measurement program during construction is related to the improvement of the parameter accuracy with respect to the accuracy provided by the initial information (point b).

The so-called Bayesian approach is a powerful tool which allows to include the mentioned factors in a numerical back-analysis (see e.g. Beliveau, 1974; Asaoka, 1979; Cividini et al., 1983). This approach is used in the present work for defining the average elastic moduli and the permeability coefficients of a soft soil deposit.

The case here considered concerns an embankment, 6 to 8 m high, of the new railroad under construction between Rome and Florence (Central Italy), which crosses a large area of soft lacustrine deposits. These deposits consist of silty clays, which are normally consolidated except for the surficial (O.C.) exsiccated crust, overlying sandy silts. Vertical bored sand drains were installed in order to accelerate the rate of settlement under the embankment. The effectiveness of the sand drains was controlled by measuring displacements and pore pressures at selected points of the soft deposit during time.

In the following, the site and soil conditions, the criteria adopted for the embankment design and the characteristics of the measurement program are first described. Then, the basic aspects of the numerical back-analysis approach are outlined and, finally, the results of calculations leading to the "optimal" values of the soil mass parameters are discussed.

SITE AND SOIL CONDITIONS

The construction area is located in "Val di Chiana", an alluvial plain (more than 50 km long and 5 to 15 km wide), surrounded by gently sloping hills. During Late Pliocene and Pleistocene the basin was occupied by a lake, whose water flowed to River Tevere; more recently (Late Pleistocene and Holocene), the lake reduced progressively to swamp and went to extinction: two small lakes, Montepulciano and Chiusi, are the only remains. The lacustrine and swamp deposits are underlain by blue, overconsolidated marine clayey silts (Lower Pliocene), outcropping on the hills where they are sometimes covered with yellow fine sand. The phreatic level is about 2 m below the ground surface; however, the water table can rise up to the ground surface during wet seasons and sometimes the plain itself is flooded.

Prior to the embankment construction the following field explorations were carried out:
- two bore-holes, 35 m deep, down to the marine

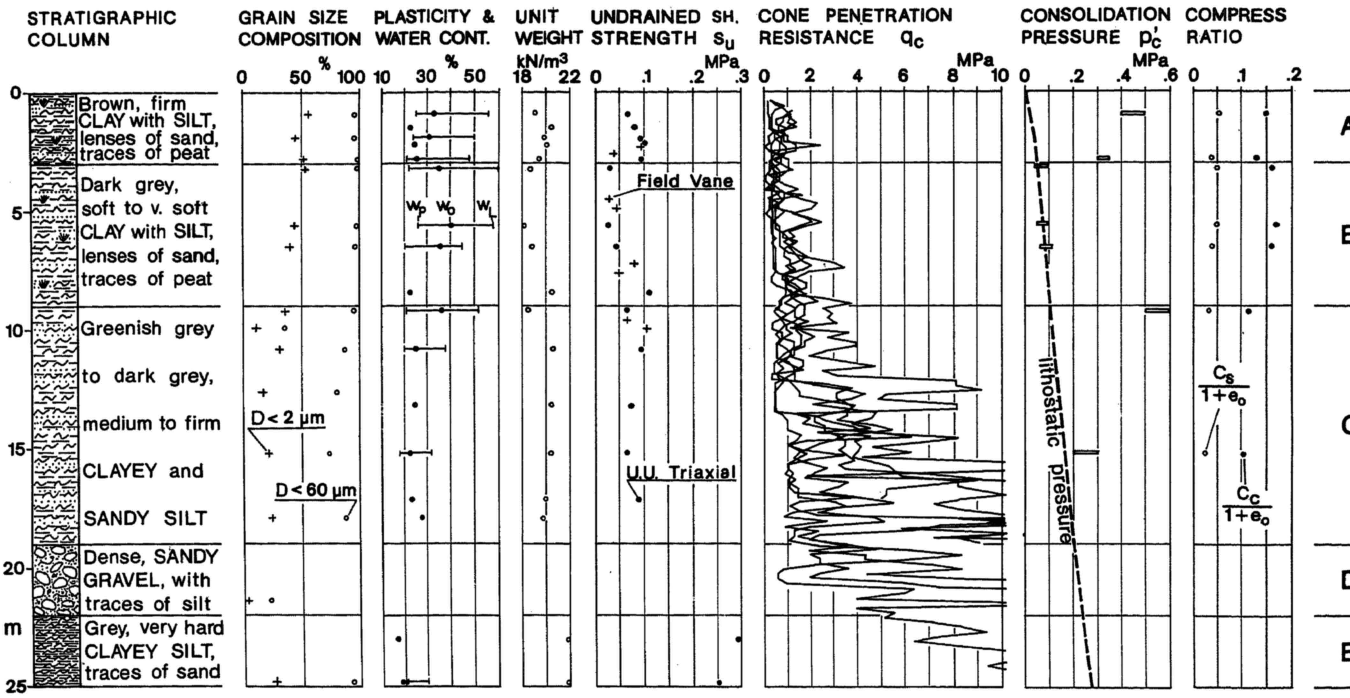

Fig. 1. Soil profile (from superposition of 2 borings and 6 cone penetration tests).

deposits; as boring proceeded, vane-tests and standard penetration tests (SPT) were performed in bore-hole base;

- six cone penetration tests (CPT), to a depth of about 20÷25 m, performed by means of mechanical 200 kN Gouda penetrometer, with friction jacket cone; these tests permit to improve the stratigraphic correlation, according to the criteria proposed by Begemann (1965) and Schmertmann (1969).

The conventional laboratory tests, on specimens from undisturbed soil samples, included:

- identification and classification tests;
- unconsolidated-undrained (UU) and isotropically consolidated-undrained (CIU) triaxial tests with pore pressure measurement;
- one-dimensional consolidation tests (some of them on specimens with internal sand drain).

Site and laboratory investigations showed that the ground conditions are reasonably uniform all over the area: the typical soil stratigraphy is depicted in fig. 1, together with the summary of the soil properties from field and laboratory tests.

The layers from A to D represent the sequence of Quaternary continental deposits and layer E corresponds to the Pliocene marine substratum. Both layers A and B consist of medium to high plasticity clays (CL-CH). The upper one, approximately 3 m thick, is characterized by randomly distributed high values of the undrained shear strength s_u, and appears overconsolidated by surficial desiccation. In layer B, the undrained shear strength and the cone resistance q_c increase almost linearly with depth; however, the ratio of s_u to the lithostatic pressure p'_o, ranging from 0.5 to 0.8, and the ratio of the apparent preconsolidation pressure p'_c to p'_o, ranging from 1 to 2, permit to define soil B as "normally consolidated aged deposit" (Bjerrum, 1973). Layer C is characterized by frequent interbedding of sand lenses and the silty fraction is, in general, predominant; however, this soils can be classified as low plasticity clays (CL group). Layer C is over consolidated (p'_c/p'_o = 2÷5), likely by surface desiccation alternate to deposition of new alluvial material. The gravelly soil D is a continuous layer, 2÷4 m thick, marking the bottom of the lacustrine basin. The marine clayey substratum is characterized by an overconsolidation ratio p'_c/p'_o greater than 5 and by high undrained shear strength.

DESIGN CRITERIA

Conventional criteria were adopted in the design of the embankment, the geometrical characteristics of which are (cfr. fig. 2):

- length: about 400 m;
- height: varying from 6 to 7.7 m (including 0.9 m of ballast);
- side slope: 2/3;
- width: 11.5 m at the top, 33÷35 m at the base (including two lateral berms, 2 m high).

Table I summarizes the values of the geotechnical parameters adopted in stability and settlement analyses.

The maximum height to ensure construction stability was determined on the basis of the classical bearing capacity formula (strip foundation and undrained conditions) and neglecting the shear resistance of the surficial desiccated crust (layer A). Adopting a factor of safety equal to 2 (in order to avoid local shear failu-

TABLE I. Summary of the geotechnical parameters

Soil Layer	Unit weight γ (kN/m³)	Undrained cohesion s_u (MPa)	Oedometric modulus E_{oed} (MPa)	Coefficient of consolidation c_v (m²/s)	c_h (m²/s)
A	20.	.050 ÷ .100	5.5 ÷ 6.0	$3.\ 10^{-8}$	$9.\ 10^{-8}$
B	18. ÷ 19.	.025 ÷ .080	1.2 ÷ 1.6	$3.\ 10^{-8}$	$9.\ 10^{-8}$
C	20.	.070 ÷ .110	12.0 ÷ 14.0	$3.\ 10^{-7}$	$9.\ 10^{-7}$
D	21.	---	---	---	---
E	22.	> .250	---	---	---

re), a value of 4.50 m was obtained and, consequently, the embankment construction had to proceed by steps, allowing for the increase in shear strength due to consolidation under the preceding load increment.

The design prediction of the settlement, at the embankment centerline, gave the following results:

$\rho_i \simeq$ 42 mm (immediate settlement; elastic theory: deformable layer on rigid base);

$\rho_c \simeq$ 411 mm (consolidation settlement; one-dimensional theory).

The consolidation settlement ρ_c is due to the contributions of layer A (99 mm), layer B (281 mm) and layer C (31 mm).

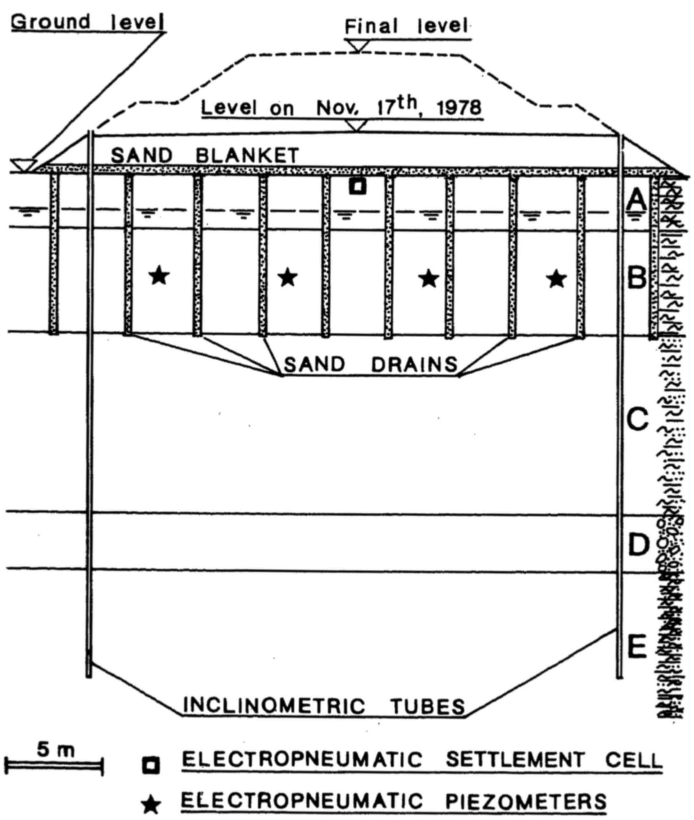

Fig. 2. Embankment cross section and instrument location.

As to the settlement rate, it was estimated that 90% of settlement would occur within 8 years. Since the time required was excessive compared to the 2 years requested in the construction contract, it was decided to accelerate consolidation by installing vertical drains; the length of the drains was limited to 9 m since the settlement is mainly due to the primary consolidation of layer B.
Economical considerations implied the selection of bored sand drains. Adopting a nominal diameter of 0.4 m, the drain spacing (triangular grid) was computed (Barron, 1948; Navdocks, 1962), so to reach 90% of combined horizontal and vertical consolidation within 2 years. In the final design the computed spacing (4.0 m) was reduced to 3.5÷3.8 m, in order to take into account the so-called "smear effect" (Navdocks, 1962; Johnson, 1970).

INSTRUMENTATION

In order to check the effectiveness of the sand drains and to record the overall behaviour of the treated foundation soil, three transversal sections (distant about 150 m one from another) were instrumented with the following devices, (cfr. fig. 2):

a) one electropneumatic cell (measurement range -0.5÷3.5 m; precision 5 mm; sensitivity 2 mm), at the embankment centerline under the continuous, draining sand blanket;
b) four electropneumatic piezometers (range 0÷1 MPa; precision 0.1%; sensitivity 0.05%), in the middle of layer B (5÷6 m deep);
c) two vertical inclinometer tubes (measurement range of the inclinometer ±30°, with precision and sensitivity of 10"), driven from each lateral berm down to 30 m.

The embankment construction started in October 1977, and the height of 2 m was reached in two months, while the installation of instruments was possible only one year later (zero reading on November 17th, 1978). However, the following reasons made it possible to disregard the effect of the first load increment in the numerical analysis:

- the settlement under the first (small) load increment was limited by the natural aging of the deposit;
- the computed degree of consolidation after 1 year was about 75÷80%;

- the piezometers indicated complete excess pore pressure dissipation (within the limits of their accuracy).

Since December 1978, the embankment height was gradually increased by steps; the embankment was completed on September 1979, and the final ballast was placed during October-November, 1980.

The measurements were performed weekly during the first year and every other week until 1980. Two additional readings were made in 1983, giving a proof of the long-term performance of all instruments.

CALIBRATION PROBLEM

The calibration or back-analysis problem concerns the determination of the "equivalent" soil parameters $\underline{P}$ that adopted in the numerical stress analysis of the geotechnical system (soft deposits under the embankment load) lead to displacements and pore pressures (grouped in vector $\underline{x}^m$) as close as possible to the corresponding experimental data $\underline{x}^M$.
Note that vector $\underline{x}^m$ collects all the mesurements performed at various times during the construction process.

Two aspects of the problem under examination have to be considered in the choise of the material model for the calibration analysis. On one hand, the intrinsic non-homogeneity of the natural deposit, and the one introduced by the installation of sand drains, suggest the use of a relatively large number of mechanical parameters for describing the overall behaviour of the soil mass. On the other hand, the limited amount of experimental data represents a constraint to the number of problem unknowns.
In order to reach a reasonable compromise between these opposite requirements, the back-analysis was performed by subdividing the soil mass into zones having different mechanical properties, but considering as unknowns only the permeability coefficients (in the vertical and horizontal directions) and the elastic modulus for each zone.
The finite element method was adopted for developing the mathematical model of the geotechnical system.

Two basic approaches can be used for the mathematical modelling of the consolidation problem, which will be referred to in the following as "coupled" and "uncoupled" approaches, (see e.g. Gatti and Gioda, 1983).
The first one (two phase analysis) leads to the solution of the coupled problems of seepage and deformation of the porous medium, while in the second case the stress redistribution within the soil mass during consolidation is ignored and an ordinary diffusion problem is solved (equivalent to a heat transfer analysis), see e.g. Davies and Poulos (1972).
The finite element program used for the coupled analysis is based on the formulation presented by Sandhu (1968) and adopts 8 node quadrilateral subparametric elements for the discretization of the displacement field, to which 4 node isoparametric elements are superposed for the discretization of the pore pressure distribution. The

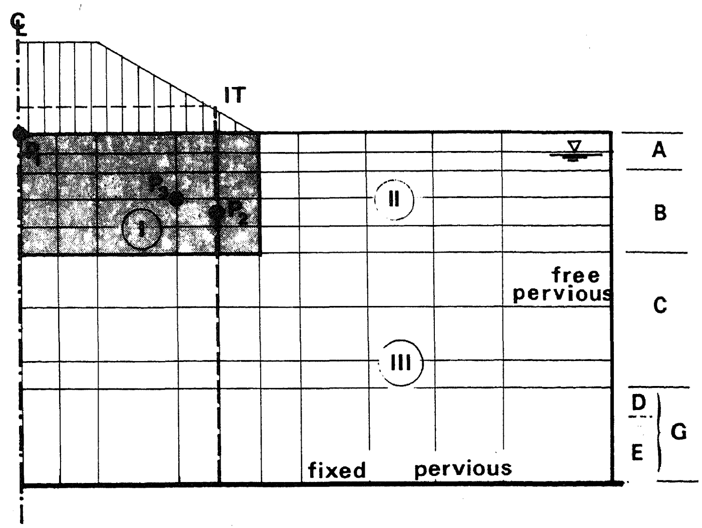

Fig. 3. Finite element mesh.

finite element mesh and the boundary conditions for the problem at hand are shown in fig. 3, where a dark area represents the zone with vertical sand drains.
The uncoupled analyses were carried out by discretizing the medium with triangular elements (two for each rectangle in fig. 3). Note that, since in this case the nodal pore pressures are the only free variables, the numerical solution of the uncoupled problem is remarkably simpler, but of course less accurate, than that of the coupled one.

In order to reduce the time and the cost of calculations, the calibration problem was solved in four subsequent steps:

1) The elastic moduli of the zones shown in fig. 3 are determined, on the basis of the more recently acquired experimental data, by assuming that the excess pore pressure due to consolidation is completely dissipated (long-term conditions) and, hence, using a one-phase, linear elastic finite element model.
2) The coupled approach is used to estimate the initial pore pressure distribution under undrained conditions, which depends on the previously determined elastic moduli.
3) The optimal values of the permeability coefficients are defined by adopting the uncoupled approach and the initial values of the pore pressures determined at second step.
4) On the basis of the back-calculated values of elastic moduli and permeability coefficients a coupled analysis is performed from the initial (undrained) situation up to the "complete" excess pore pressure dissipation. This allows to assess the agreement between the behaviour of the "calibrated" numerical model and that of the real geotechnical system.

As previously mentioned, a Bayesian approach has been adopted for the solution of the calibration problem. This procedure, which can be reduced under some simplifying assumptions to two popular parameter estimation methods (i.e. the least-squares and the minimum variance methods), is particularly adequate for problems involving a relatively large number of unknowns and a

sparsity of data. The final parameter estimate depends not only on the experimental data $\underline{x}^M$, and on the "a priori" information $\underline{P}_o$, but also on their "uncertainties" represented, respectively, by the covariance matrices $\underline{C}_u$ and $\underline{C}_p^o$. In addition to the optimal parameter values, the method provides also a measure of the "uncertainties" affecting them.
Since the dependence of the response vector $\underline{x}^m(\underline{P})$ on the parameter $\underline{P}$ is, in general, non linear, an iterative procedure based on the linearization of such relation can be adopted for the numerical solution (Cividini et al., 1983). The basic steps of this procedure are:

1) The responce vector $\underline{x}^m(\underline{P})$ is computed, on the basis of the initial parameter estimate $\underline{P}_o$, by means of an analysis of the geotechnical problem.
2) The sensitivity (or partial derivative) matrix $\underline{L}(\underline{P})$ is evaluated numerically by a finite difference approximation. This requires the solution of an analysis problem for each parameter perturbation and the evaluation of the ratios between response vector increments and relevant parameter increments.
3) The above information allows one to calculate a new estimate $\underline{P}'$ of the unknowns; then a convergency test is performed. If this test is not satisfied the calculations are repeated from step 1 adopting the new estimated parameters $\underline{P}'$.

When the final parameter vector $\underline{P}_f$ has been determined, it is possible to calculate the "a posteriori" covariance matrix $\underline{C}_p^f$. This requires the evaluation of the "information" matrix (Beliveau, 1974; Cividini et al., 1983)

$$\underline{Q} = [\underline{L}^T(\underline{P}_f)\ \underline{C}_u^{-1}\ \underline{L}(\underline{P}_f)]$$

which in turns depends on the uncertainties of the experimental data (matrix $\underline{C}_u$) and on the sensitivity of the responce vector $\underline{x}^m(\underline{P}_f)$ to parameter variations, i.e. on matrix $\underline{L}(\underline{P}_f)$. If the entries of the "information" matrix are, in the average, negligible with respect to those of the inverse of the initial covariance matrix, this implies that the measurements do not provide an useful information for the parameter back-analysis.
The inverse of the "information" matrix permits to assess the effectiveness of the experimental program with respect to:

a) the instrument accuracy,
b) the number of measurement points and
c) the location of measurement points.

It is easy to show the the evaluation of the "information" matrix provides a useful tool in planning, or "designing" the in-situ measurement program (Cividini et al., 1983). In fact, the following procedure is applicable as a possible alternative to the simulation procedure used by Cividini et al.(1981):

1) A suitable set of parameters $\underline{P}_t$ is adopted in the analysis of the geotechnical problem.
2) The results of this analysis are used to generate sets of fictitious experimental data. Each set corresponds to a possible measurement program. Some of these sets contain the same number of measurements, but refer to different locations, while others have a different number of data.
3) For each set of data several back-analyses are carried out by varying the instrument accuracy, i.e. matrix $\underline{C}_u$. Clearly the starting parameter vector of the iterative procedure is different from the one used to generate the fictitious experimental data.
4) The comparison between the back-analyzed parameters and the "true" parameters $\underline{P}_t$ provides a first measure of the effectiveness of each possible measurement program.
5) As previously observed, the comparison between the information matrix and the inverse of the initial covariance matrix permits to assess the effectiveness of each mesurement program. Specifically, it can be assessed whether the experimental program provides a significant information on the values of the sought parameters.
6) If large (co)-variances are assumed in the initial estimates for the fictitious back-analyses, the diagonal elements in the inverse of the information matrix coincide with the variances of the estimated parameters. Then, they can be used to set up diagrams relating the scattering of the estimated parameters to the aforementioned factors (a), (b), (c), and consequently, to assess the effectiveness of the possible experimental programs.

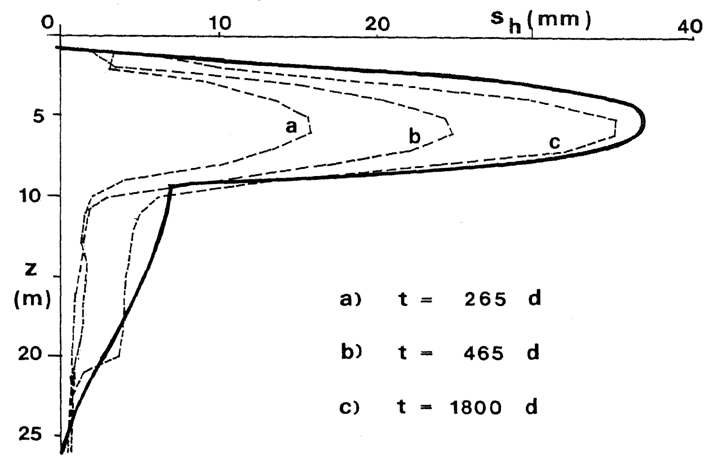

Fig. 4. Measured vertical distributions of lateral movements (dashed lines) and long-term analysis results (solid line).

RESULTS OF THE ANALYSES

The meaurements obtained in the three instrumented sections of the embankment (cfr. fig. 2) have similar trend and values: this confirms the horizontal homogeneity of the deposit and makes it possible to consider only one set of data in the back-analysis.
For sake of simplicity, the covariance matrix of the meaurements errors $\underline{C}_u$ is assumed to be diagonal, and its non vanishing terms are defined taking into account:

1) the instrument accuracy;
2) the lack of information on the seasonal variation of the water table level;
3) the approximation introduced by assuming the horizontal measurements to be uncorrelated.

Assuming a reasonable estimation of Poisson's ratio (ν=0.2 for layer B and 0.4 for the other soil layers) the characterizaction of the elastic moduli (step 1 of the back-analysis), on the basis of vertical and horizontal displacement measurements, lead to the following results:

E_A = 14.0 MPa; E_B = 1.7 MPa;
E_C = 40.0 MPa; E_G = 79.0 MPa.

The corresponding lateral movements at the inclinometer location (cfr. fig. 4) are in good agreement with the experimental data. The uncertainties of the elastic moduli of layers B and G (expressed as non dimensional ratios of the variances to the square of the corresponding parameters) are lower, and hence their estimated values are more reliable, than those of layers A and C. Note that the displacement boundary conditions on the bottom and right sides of the mesh have a non negligible influence on the final value of E_G and on its variance. The attempt to back-analyze separately the elastic coefficients of zones I and II of layers A and B did not lead to meaningful results. In fact, the difference between the initial and final values of the elastic parameter variance for the zone without sand drains turned out to be very small, thus denoting a limited (negligible) influence of this parameters on the response vector $\underline{x}^m$.
The input data for the third step of the back-analysis, aiming to define the permeability coefficients, were the degree of consolidation settlement $U_1(t)$ at the embankment centerline, and the pore pressures, measured at various times. It has to be considered however that the measured pore pressure values are barely reliable, because they are influenced by the seasonal variation of the water table level, which is not known with acceptable accuracy. In order to overcome this deficiency, a further information was added to the input data, namely the degree of lateral consolidation movement $U_2(t)$ (at point P_2 in fig. 3). The permeability coefficients of zones I, II and III (cfr. fig. 3) were back calculated assuming for each zone isotropic (case I) and anisotropic (case A) permeabilities. The results of these analyses are summarized in Table II, where h and v denote, respectively, the horizontal and vertical directions.

TABLE II. Results of permeability back-analysis

	zone I k_h	zone I k_v	zone II k_h	zone II k_v	zone III k_h	zone III k_v
P_f/P_o (I)		2.53		1.01		0.94
P_f/P_o (A)	0.94	2.10	1.00	1.00	0.73	0.95
C_p^f/C_p^o (I)		.4480		1.000		.8812
C_p^f/C_p^o (A)	.9810	.4420	.9999	.9999	.9461	.8775

P_f/P_o = ratio of final to initial parameters

C_p^f/C_p^o = ratio of final to initial variances

(I) : isotropic case; (A) : anisotropic case.

The above results show that the permeability of zone II cannot be evaluated with sufficient ac-

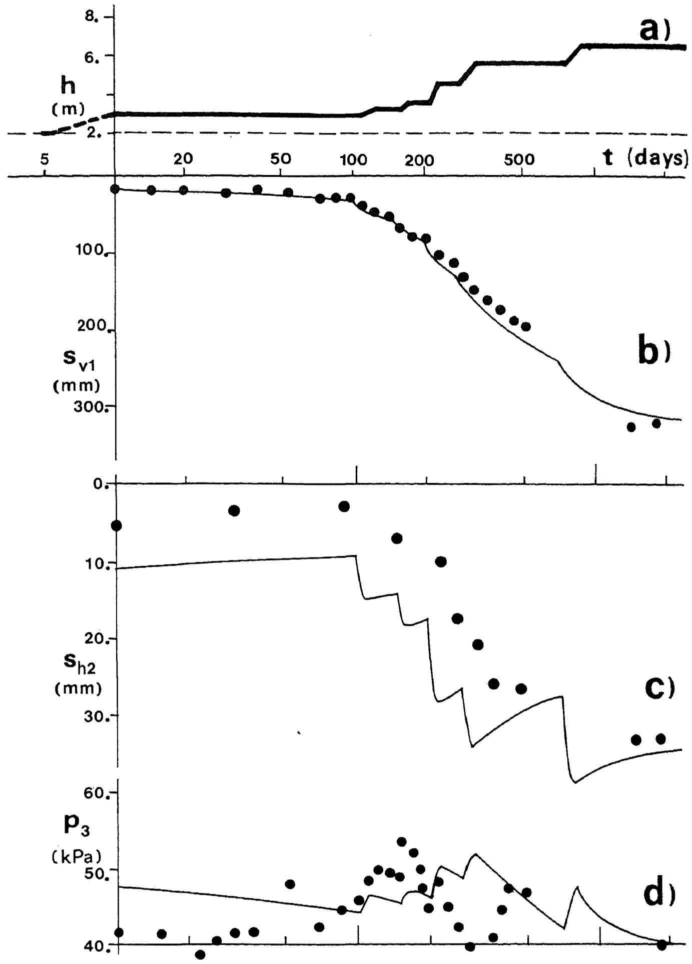

Fig. 5. Coupled consolidation analysis:
a) embankment height;
b) surface settlement (at P_1 in fig.3);
c) lateral movement (at P_2 in fig.3);
d) pore water pressure (at P_3 in fig.3);
(black dots: experimental data).

curacy on the basis of the available data (in fact the in-situ measurements are mainly influenced by the vertical permeability of zone I) and that the determination of the vertical permeabilities is more accurate than that of the horizontal ones.
Even if the permeability coefficients were determined by an "uncoupled" back-analysis (in which the variation of the total stresses with time is neglected), the final parameters introduced into a "coupled" analysis of the complete consolidation process led to results (fig. 5) in reasonable agreement with the in-situ measurements (but for the pore pressures affected by the above mentioned uncertainty on the variation of the water table). Note that the anisotropic permeability coefficients were introduced in the final consolidation analysis.

CONCLUSIONS

The application of a Bayesian approach has been illustrated concerning the numerical calibration of a geotechnical system (soft deposit with sand drains under a railway embankment) on the basis of a series of in-situ measurements.

With respect to the measurements performed during and after the construction, and to the criteria adopted in the embankment design, it can be observed that:

- the long-term performance of the instruments proved to be satisfactory, even after 5 years from their installation;
- the effectiveness of the sand drains in accelerating the consolidation process was adequate;
- a reasonable agreement exists between the settlements and the settlement rates predicted by the design calculations and those actually measured in the field, in spite of some modification of the planned load increment program.

Two calibration analyses were carried out separately, both of them based on the finite element discretization of the geotechnical system.
The first one led to the optimal values of the elastic moduli of the various zones in which the soft deposit was subdivided (under the assumption of linear elastic and isotropic material behaviour for each zone), while the second one concerned the determination of the permeability coefficients, both in the isotropic and in the anisotropic cases. Subsequently, the back-calculated parameters were adopted in a coupled (two phase) analysis simulating the construction process of the embankment, and a satisfactory agreement was obtained between the numerical results and the corresponding in-situ measurements, but for the pore pressures. In fact, the data from the installed piezometers were affected by the seasonal variation of the water table level the value of which is not known with sufficient precision.

With respect to the Bayesian approach adopted for the solution of the calibration problem, and on the basis of the results of the analyses, the following considerations can be made:

- The back-analysis approach is capable to take into account the different accuracies of the various instruments used in the field and to consider the "a priori" information on the parameters to be estimated.
- In addition to the optimal values of the parameters the approach is capable to evaluate their degree of uncertainties. This last information shows that, for instance, the vertical permeabilities are determined with an accuracy larger than that of the horizontal ones.
- In general, the Bayesian procedure can be efficiently adopted in order to "design" and to optimize the program of in-situ measurements with respect to the type of parameters to be evaluated and to the accuracy of the chosen instruments.

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