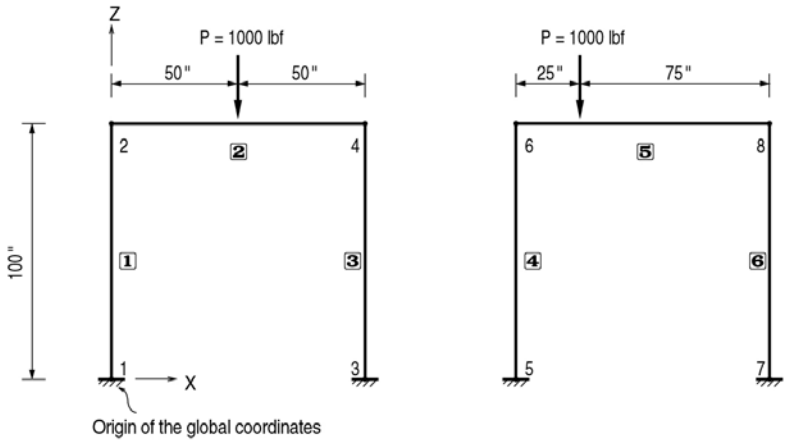
PDelta-3

Title

P-Delta effect analysis of a portal frame.

Description

Shown below is a 2-D, one story, one bay, portal frame supported at the base subjected to a symmetric load and an eccentric load separately.
Compare the displacements, shear forces and bending moments between the results based on a P-Delta effect analysis and a conventional frame analysis.



Structural geometry and analysis model

Model

Analysis Type

2-D P-Delta analysis

Unit System

in, lbf

Dimension

Height H = 100.0 in

Width B = 100.0 in

Relative displacement tolerance 0.001

Element

Beam element

Material

Modulus of elasticity $E = 29 \times 10^6$ psi

Section Property

Area $A = 1.0$ in^2

Moment of inertia $I_{yy} = 1/12$ in^4

Boundary Condition

Nodes 1, 3, 5 and 7 ; Constrain all DOFs.

Analysis Case

Case 1 ; A symmetric load, 1000 lbf is applied to the mid-point of the element 2 in the -Z direction.

Case 2 ; An eccentric load ,1000 lbf is applied to the first quarter point of the element 5 in the -Z direction.

Maximum number of iteration for the P-Delta effect analysis= 5

Results

Displacements (P-Delta effect analysis: PDelta-3-1)

	Node	Load	DX (in)	DY (in)	DZ (in)	RX ([rad])	RY ([rad])	RZ ([rad])
▶	1	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	2	CASE1	0.000222	0.000000	-0.001724	0.000000	0.091785	0.000000
	3	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	4	CASE1	-0.000222	0.000000	-0.001724	0.000000	-0.091785	0.000000
	5	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	6	CASE1	1.893201	0.000000	-0.002588	0.000000	0.101391	0.000000
	7	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	8	CASE1	1.892851	0.000000	-0.000860	0.000000	-0.036721	0.000000

Member Forces (P-Delta effect analysis: PDelta-3-1)

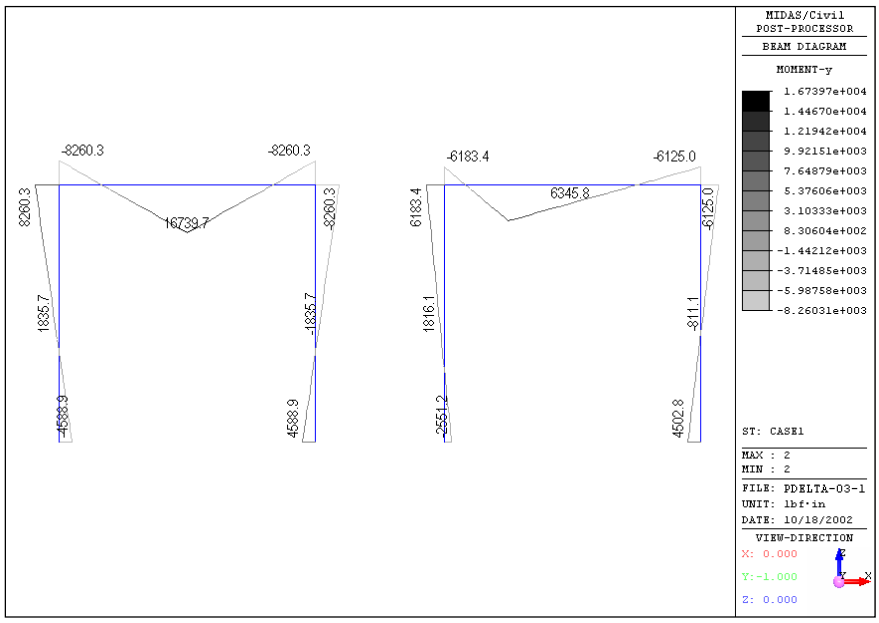
	Elem	Load	Part	Axial (lbf)	Shear-y (lbf)	Shear-z (lbf)	Torsion (lbf-in)	Moment-y (lbf-in)	Moment-z (lbf-in)
▶	1	CASE1	i	-500.00	0.00	-128.49	0.00	-4588.92	0.00
	1	CASE1	1/4	-500.00	0.00	-128.49	0.00	-1376.61	0.00
	1	CASE1	2/4	-500.00	0.00	-128.49	0.00	1835.69	0.00
	1	CASE1	3/4	-500.00	0.00	-128.49	0.00	5048.00	0.00
	1	CASE1	j	-500.00	0.00	-128.49	0.00	8260.31	0.00
	2	CASE1	i	-128.49	0.00	-500.00	0.00	-8260.31	0.00
	2	CASE1	1/4	-128.49	0.00	-500.00	0.00	4239.69	0.00
	2	CASE1	2/4	-128.49	0.00	500.00	0.00	16739.69	0.00
	2	CASE1	3/4	-128.49	0.00	500.00	0.00	4239.69	0.00
	2	CASE1	j	-128.49	0.00	500.00	0.00	-8260.31	0.00
	3	CASE1	i	-500.00	0.00	128.49	0.00	4588.92	0.00
	3	CASE1	1/4	-500.00	0.00	128.49	0.00	1376.61	0.00
	3	CASE1	2/4	-500.00	0.00	128.49	0.00	-1835.69	0.00
	3	CASE1	3/4	-500.00	0.00	128.49	0.00	-5048.00	0.00
	3	CASE1	j	-500.00	0.00	128.49	0.00	-8260.31	0.00
	4	CASE1	i	-750.59	0.00	-101.56	0.00	-2551.21	0.00
	4	CASE1	1/4	-750.59	0.00	-101.56	0.00	-367.56	0.00
	4	CASE1	2/4	-750.59	0.00	-101.56	0.00	1816.10	0.00
	4	CASE1	3/4	-750.59	0.00	-101.56	0.00	3999.75	0.00
	4	CASE1	j	-750.59	0.00	-101.56	0.00	6183.41	0.00
	5	CASE1	i	-101.56	0.00	-750.59	0.00	-6183.41	0.00
	5	CASE1	1/4	-101.56	0.00	249.41	0.00	12581.20	0.00
	5	CASE1	2/4	-101.56	0.00	249.41	0.00	6345.82	0.00
	5	CASE1	3/4	-101.56	0.00	249.41	0.00	110.43	0.00
	5	CASE1	j	-101.56	0.00	249.41	0.00	-6124.96	0.00
	6	CASE1	i	-249.41	0.00	101.56	0.00	4502.78	0.00
	6	CASE1	1/4	-249.41	0.00	101.56	0.00	1845.84	0.00
	6	CASE1	2/4	-249.41	0.00	101.56	0.00	-811.09	0.00
	6	CASE1	3/4	-249.41	0.00	101.56	0.00	-3468.02	0.00
	6	CASE1	j	-249.41	0.00	101.56	0.00	-6124.96	0.00

Displacements (Conventional analysis: PDelta-3-2)

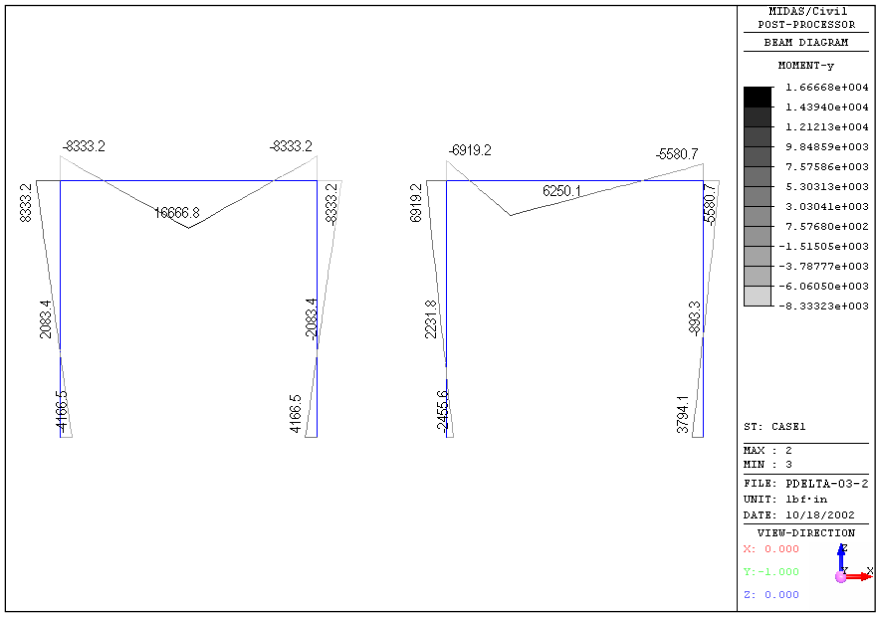
	Node	Load	DX (in)	DY (in)	DZ (in)	RX ([rad])	RY ([rad])	RZ ([rad])
▶	1	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	2	CASE1	0.000216	0.000000	-0.001724	0.000000	0.086209	0.000000
	3	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	4	CASE1	-0.000216	0.000000	-0.001724	0.000000	-0.086209	0.000000
	5	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	6	CASE1	1.384852	0.000000	-0.002632	0.000000	0.092351	0.000000
	7	CASE1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	8	CASE1	1.384528	0.000000	-0.000816	0.000000	-0.036963	0.000000

Member Forces (Conventional analysis: PDelta-3-2)

	Elem	Load	Part	Axial (lbf)	Shear-y (lbf)	Shear-z (lbf)	Torsion (lbf-in)	Moment-y (lbf-in)	Moment-z (lbf-in)
▶	1	CASE1	i	-500.00	0.00	-125.00	0.00	-4166.46	0.00
	1	CASE1	1/4	-500.00	0.00	-125.00	0.00	-1041.54	0.00
	1	CASE1	2/4	-500.00	0.00	-125.00	0.00	2083.39	0.00
	1	CASE1	3/4	-500.00	0.00	-125.00	0.00	5208.31	0.00
	1	CASE1	j	-500.00	0.00	-125.00	0.00	8333.23	0.00
	2	CASE1	i	-125.00	0.00	-500.00	0.00	-8333.23	0.00
	2	CASE1	1/4	-125.00	0.00	-500.00	0.00	4166.77	0.00
	2	CASE1	2/4	-125.00	0.00	500.00	0.00	16666.77	0.00
	2	CASE1	3/4	-125.00	0.00	500.00	0.00	4166.77	0.00
	2	CASE1	j	-125.00	0.00	500.00	0.00	-8333.23	0.00
	3	CASE1	i	-500.00	0.00	125.00	0.00	4166.46	0.00
	3	CASE1	1/4	-500.00	0.00	125.00	0.00	1041.54	0.00
	3	CASE1	2/4	-500.00	0.00	125.00	0.00	-2083.39	0.00
	3	CASE1	3/4	-500.00	0.00	125.00	0.00	-5208.31	0.00
	3	CASE1	j	-500.00	0.00	125.00	0.00	-8333.23	0.00
	4	CASE1	i	-763.39	0.00	-93.75	0.00	-2455.58	0.00
	4	CASE1	1/4	-763.39	0.00	-93.75	0.00	-111.89	0.00
	4	CASE1	2/4	-763.39	0.00	-93.75	0.00	2231.81	0.00
	4	CASE1	3/4	-763.39	0.00	-93.75	0.00	4575.50	0.00
	4	CASE1	j	-763.39	0.00	-93.75	0.00	6919.19	0.00
	5	CASE1	i	-93.75	0.00	-763.39	0.00	-6919.19	0.00
	5	CASE1	1/4	-93.75	0.00	236.61	0.00	12165.44	0.00
	5	CASE1	2/4	-93.75	0.00	236.61	0.00	6250.08	0.00
	5	CASE1	3/4	-93.75	0.00	236.61	0.00	334.71	0.00
	5	CASE1	j	-93.75	0.00	236.61	0.00	-5580.66	0.00
	6	CASE1	i	-236.61	0.00	93.75	0.00	3794.11	0.00
	6	CASE1	1/4	-236.61	0.00	93.75	0.00	1450.42	0.00
	6	CASE1	2/4	-236.61	0.00	93.75	0.00	-893.27	0.00
	6	CASE1	3/4	-236.61	0.00	93.75	0.00	-3236.96	0.00
	6	CASE1	j	-236.61	0.00	93.75	0.00	-5580.66	0.00



Bending moment diagram of the structure (P-Delta effect analysis)



Bending moment diagram of the structure (Conventional analysis)

Comparison of Results

Symmetric Load Case

Unit : rad, lbf-in, lbf

Result	P-Delta effect analysis			Conventional analysis		
	Theoretical	SAP2000	MIDAS /Civil	Theoretical	SAP2000	MIDAS /Civil
Rotational displacement(θ_2)	0.09192	0.09178	0.09179	0.08620	0.08620	0.08621
Moment (M_{12})	4606.6	4589.1	4588.9	4166.7	4166.7	4166.5
Moment (M_{21})	8254.0	8260.4	8260.3	8333.3	8333.3	8333.2
Shear force (F)	128.6	128.5	128.5	125.0	125.0	125.0

Eccentric Load Case

Unit : rad, lbf-in, lbf

Result	P-Delta effect analysis			Conventional analysis		
	Theoretical	SAP2000	MIDAS /Civil	Theoretical	SAP2000	MIDAS /Civil
Lateral displacement (δ)	1.893	1.894	1.893	1.385	1.385	1.385
Rotational displacement (θ_6)	0.1013	0.1014	0.1014	0.0924	0.0924	0.0924
Rotational displacement (θ_8)	0.0367	0.0367	0.0367	0.0369	0.0369	0.0370
Moment (M_{56})	2544.9	2550.9	2551.2	2455.4	2455.4	2455.6
Moment (M_{65})	6088.6	6183.6	6183.4	6919.6	6919.6	6919.2
Moment (M_{78})	4456.9	4503.5	4502.8	3794.6	3794.6	3794.1
Moment (M_{87})	6153.0	6124.9	6125.0	5580.4	5580.4	5580.7
Shear force (F)	101.4	101.6	101.6	93.75	93.75	93.75

References

Livesley, R. K., and Chandler, D. B., "*Stability Functions for Structural Frameworks*", Manchester University Press, UK, 1956.

"*SAP90, A Series of Computer Programs for the Finite Element Analysis of Structures, Structural Analysis Verification Manual*", Computer and Structures, Inc., 1992, Example 27.