P • A • R • T • 4

REINFORCED CONCRETE FOUNDATIONS

SECTION 4A

FUNDAMENTALS OF REINFORCED CONCRETE

A. SAMER EZELDIN

NOTATIONS

a = depth of equivalent rectangular stress block as defined in Sec. 4A.2.4. 1

A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral

A_g = gross area of concrete section g

A_s = area of tension steel reinforcement
area of steel per unit width of slab or plate

A'_s = area of compression steel reinforcement

A_{st} = total area of longitudinal reinforcement

b = width of cross section

b_w = web of cross section

c = distance from extreme compression fiber to neutral axis

d = distance from extreme compression fiber to centroid of tension reinforcement

d' = distance from extreme compression fiber to centroid of compression reinforcement
d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto
E_c = modulus of elasticity of concrete
E_s = Young's modulus of steel
EI = flexural stiffness of compression member
f_c = compressive strength in concrete
f'_c = cylinder compressive strength of concrete
f_r = modulus of rupture of concrete
f_s = calculated stress in reinforcement at service loads
f'_t = tensile splitting strength of concrete
f_y = specified yield strength of nonprestressed reinforcement
h = overall thickness of member
I_{cr} = moment of inertia of cracked section
I_e = effective moment of inertia for computation of deflection
I_g = gross moment of inertia
k = effective length factor for compression members
L = clear long span length
l = effective beam span
l_d = development length
l_e = equivalent embedded length of a hook
M = positive moment
M' = negative moment
M_a = moment of maximum service load in span
M_{cr} = cracking moment
M_n = nominal moment strength at section
P_b = nominal axial load strength at balanced strain conditions
P_c = critical load
P_n = nominal axial load strength at given eccentricity
P_0 = nominal axial load strength at zero eccentricity
P_u = factored axial load at given eccentricity; $\leq \phi P_n$, ϕ being a reduction factor
s = spacing of stirrups or ties
V_c = nominal shear strength provided by concrete
V_s = nominal shear strength provided by steel reinforcement
W = total uniform load per unit area
x = shorter overall dimension of rectangular part of cross section
x_1 = shorter center-to-center dimension of closed rectangular stirrup
y = longer overall dimension of rectangular part of cross section
y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension
y_1 = longer center-to-center dimension of closed rectangular stirrup

β_1 = factor, varying from 0.85 for f'_c = 4000 p5i 100.65 minimum; it decreases at a rate of 0.05 per 1000-psi strength above 4000 psi (27.6 MPa)

δ = moment magnification factor

ε_s = unit strain in reinforcement steel

ν = Poisson's ratio

ρ = ratio of nonprestressed tension reinforcement; A_s/bd for beam and $A_s/12d$ for slab

$\bar{\rho}_b$ = reinforcement ratio producing balanced strain conditions

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of spirally reinforced compressive member

ρ_t = active steel ratio; = A_s/A_t

ω = reinforcement index; = $\rho f_y/f'_c$

Σ^0 = sum of circumferences of reinforcing elements

$\Sigma x^2 y$ = torsional section properties

4A.1 INTRODUCTION

Concrete is obtained by mixing cement, water, fine aggregate, coarse aggregate, and frequently other additives in specified proportions. The hardened concrete is strong in compression but weak in tension, making it vulnerable to cracking. Also, concrete is brittle and fails without warning. In order to overcome the negative implications of these two main weaknesses, steel bars are added to reinforce the concrete. Hence reinforced concrete, when designed properly, can be used as an economically strong and ductile construction material.

In many civil engineering applications, reinforced concrete is used extensively as a construction material for structures and foundations. This chapter provides a basic knowledge of reinforced concrete elements (beams, columns, and slabs) subjected to flexure, shear, and torsion. Once the response of an individual element is understood, the designer will have the necessary background to analyze and design reinforced concrete systems composed of these elements, such as foundations and buildings.

Two popular methods are available for analyzing and designing the strength of reinforced concrete members. The first is referred to as the working stress design method. This method is based on limiting the computed stresses in members as they are subjected to service loads up to the allowable stresses. The second design method, called the strength method, is based on predicting the maximum resistance of a member rather than predicting stresses under service loads.

The strength method is the design method recommended by the current edition of the ACI code.1 Members are designed for factored loads that are greater than the service loads. The factored loads are obtained by multiplying the service load by load factors greater than 1. Table 4A.gives the load factors for various types of load. Using factored loads, the designer performs an elastic analysis to obtain the required strength of the members. Members are designed so that their design strength is equal to or greater than the required strength,[2–4]

$$\text{Required strength} \le \text{design strength} \tag{4A. 1}$$

The design strength is used to express the nominal capacity of a member reduced by a strength reduction factor ϕ. The nominal capacity is evaluated in accordance with provisions and assumptions specified by the ACI code. Reduction factors for different loading conditions are presented in Table 4A.2. The design criteria just presented provide for a margin of safety in two ways. First, the required strength is computed by increasing service loads by load factors. Also, the design strength

TABLE 4A.1 Load Factors for Ultimate-Strength Design Method

Condition	U
1. Dead load D + live load L	$1.4D + 1.7L$
2. Dead + live + wind load W when additive	$0.75(1.4D + 7L + 1.7W)$
3. Same as item 2 when gravity counteracts wind-load effects	$0.9D + 1.3W$
4. In structures designed for earthquake loads or forces E, replace W by $1.1E$ in items 2 and 3	
5. When lateral earth pressure H acts in addition to gravity forces when effects are additive	$1.4D + 1.7L + 1.7H$
6. Same as item 5 when gravity counteracts earth-pressure effects	$0.9D + 1.7H$
7. When lateral liquid pressure F acts in addition to gravity loads, replace $1.7H$ by $1.4F$ in items 5 and 6	
8. Vertical liquid pressures shall be considered as dead loads D	
9. Impact effects, if any, shall be included in live loads L	
10. When effects T of settlement, creep, shrinkage, or temperature change are significant and additive	$0.75(1.4D + 1.4T + 1.7L)$
11. Same as item 10 when gravity counteracts T	$1.4(D + T)$
12. In no case shall U be less than given by item 1	

Source: From ACI.[1]

TABLE 4A.2 Reduction Factors for Ultimate-Strength Design Method

Kind of strength	Strength reduction factor ϕ
Flexural, with or without axial tension	0.90
Axial tension	0.90
Axial compression, with or without flexure:	
Members with spiral reinforcement	0.75
Other reinforced members	0.70
Exception: for low values of axial load, 4 may be increased in accordance with the following:	
For members in which f_y does not exceed 60,000 psi, with symmetrical reinforcement, and with $(h - d' - d_s)/h$ not less than 0.70, ϕ may be increased linearly to 0.90 as ϕP_n decreases from $0.10f'_c A_s$ to zero.	
For other reinforced members, ϕ may be increased linearly to 0.90 as ϕP_n decreases from $0.10f'_c A_g$ or ϕP_{nb}, whichever is smaller, to zero.	
Shear and torsion	0.85
Bearing on concrete	0.70
Flexure in plain concrete	0.65

Source: From ACI.[1]

is computed by reducing the nominal strength by a strength reduction factor. This design criterion applies to all possible states of stress, namely, bending, shear, torsion, and axial stresses.

4A.2 FLEXURE BEHAVIOR

Four basic assumptions are made when deriving a general theory for flexure behavior of reinforced concrete members.

1. Plane sections before bending remain plane after bending.
2. The stress-strain relationships for the steel reinforcement and the concrete are known.
3. The tensile strength of the concrete is neglected after cracking.
4. Perfect bonding exists between the concrete and the steel reinforcement.

The simple reinforced concrete beam shown in Fig. 4A.1(*a*) exhibits the basic characteristics of flexural behavior. Depending on the magnitude of the bending moment, the beam's response will be in the elastic range, either uncracked or cracked, or in the inelastic range and cracked.

4A.2.1 Uncracked, Elastic Range

When the bending moment is smaller than the cracking moment, the strain and stress distributions at the maximum moment section are as shown in Fig. 4A.l(*b*). Stresses are related to strain by the modulus of elasticity of concrete E_c. The reinforcement in concrete makes only a minor contribution at this stage, because its strain is too small to achieve an appreciable resisting stress. The internal force couple formed by the tension and compression forces in concrete provides the required resistance to the externally applied moment. The principle of elastic theory and the transformed area concept are employed for computing stresses in concrete and in steel reinforcement.

The stress in concrete with the extreme fiber in tension is given by

$$f_t + \frac{M(h-c)}{I_t} \tag{4A.2}$$

and the stress in the steel reinforcement by

$$f_s + \frac{M(dh-c)}{I_t} n \tag{4A.3}$$

where $$n = \frac{E_s}{E_c}$$

and I_t is the moment of inertia of the transformed section.

4A.2.2 Cracked, Elastic Range

As the applied load is increased, the maximum moment reaches higher values, resulting in the maximum tension in concrete approaching the modulus of rupture. At this level, the tension cracks start forming on the tension face of the concrete section. The cracking moment can be computed using the equation

$$M_{cr} = \frac{f_r I_t}{y_t} \tag{4A.4}$$

Here I_t can be replaced by the gross moment of inertia I_g, neglecting the contribution of the steel reinforcement without appreciable error.

Under increasing loads and progressive cracking, the neutral axis shifts upward. Since concrete cannot resist the developed tensile stresses, the reinforcing steel is called upon to resist the entire

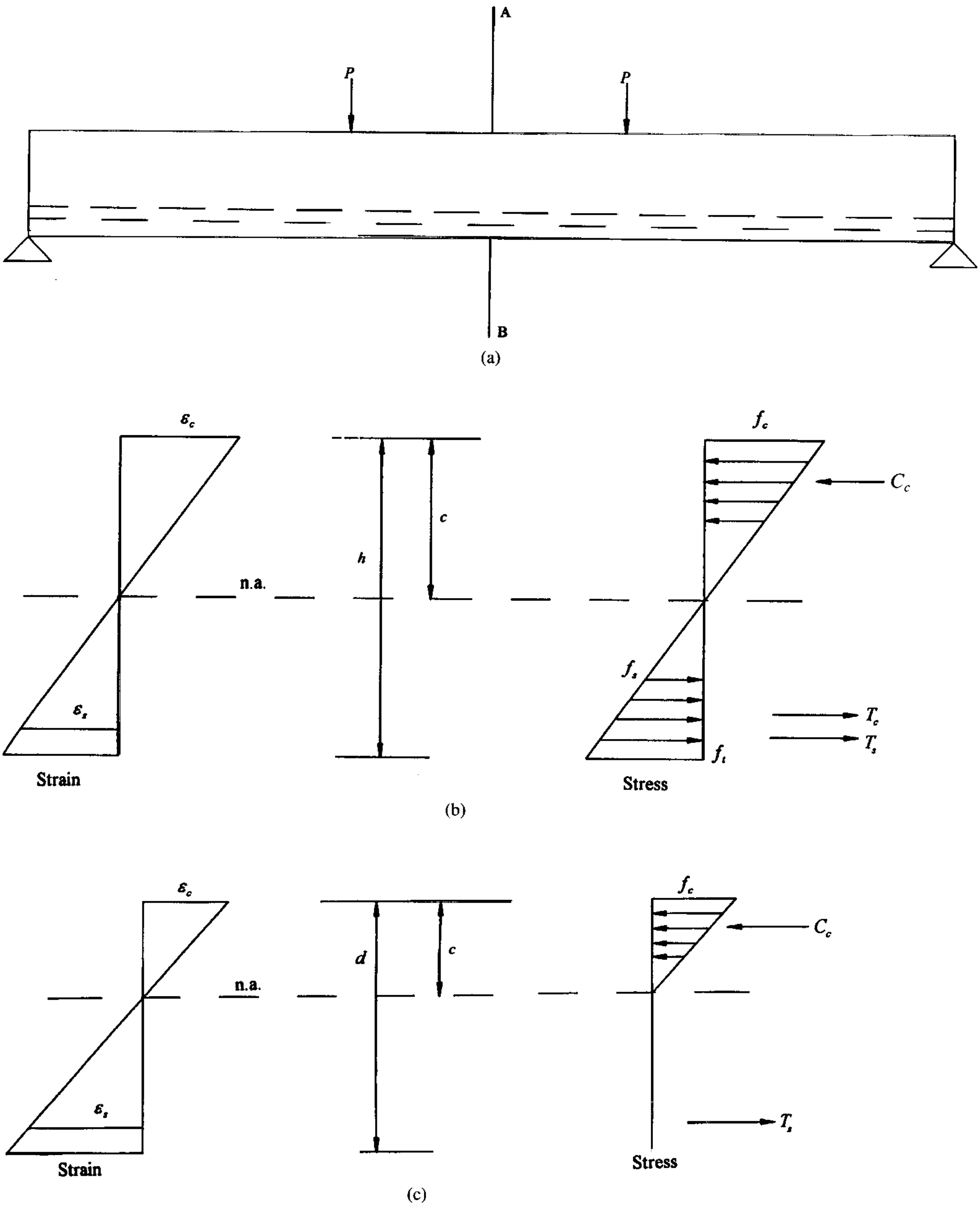


FIGURE 4A.1 Flexural behavior of simple reinforced concrete beam. (*a*) Reinforced concrete beam. (*b)* Uncracked, elastic stage. (*c)* Cracked, elastic stage.

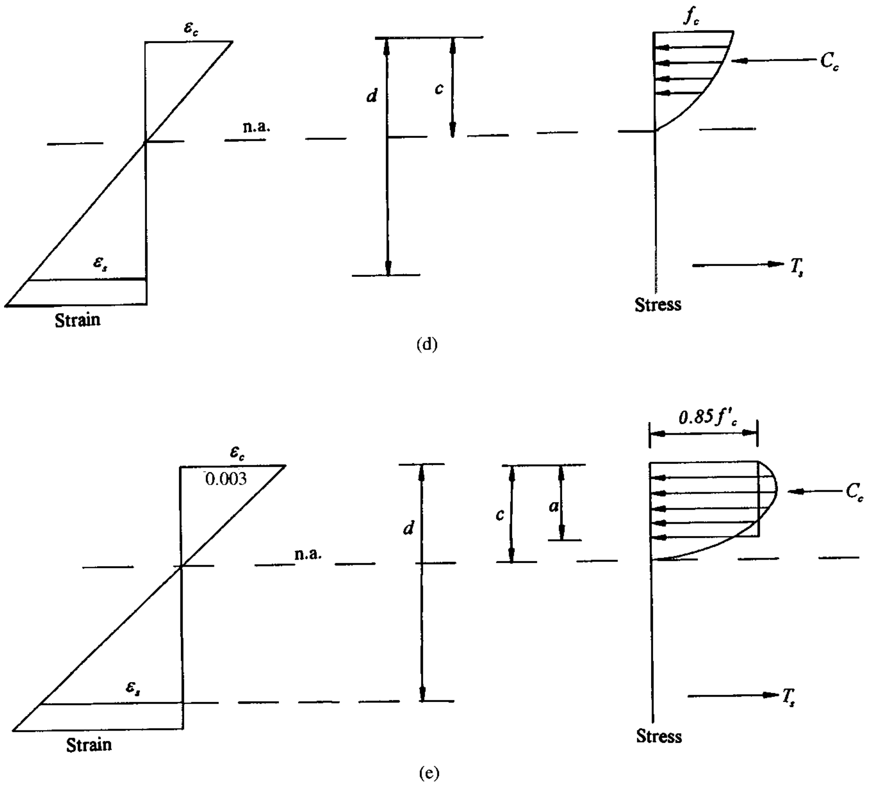


FIGURE 4A.1 (*Continued*) (*d*) Cracked, inelastic stage. (*e*) Flexural strength.

tension force. Up to a compressive stress of about $0.5f'_c$, the linear relationship is still valid. Figure 4A.l(*c*) shows the response of a reinforced concrete section at this stage. The internal resisting couple is provided by a concrete compressive force C_c and a steel tensile force T_s. The moment equilibrium equation is given by

$$M = T_s\left(d - \frac{c}{3}\right) = C_c\left(d - \frac{c}{3}\right) \tag{4A.5}$$

The concept of transformed section can still be used to compute the stress in concrete and in steel reinforcement. However, at this stage the cracked concrete is assumed to make no contribution to I_t.

Example 4A.1 Calculate the bending stresses in the beam shown in Fig. X.1, using the transformed area method.

Given:

$f'_c = 4000$ psi (27.6 MPa)

$M = 900{,}000$ in · lb(l0l.7k N · m)

$E_s = 29{,}000{,}000$ psi (199,810 MPa)

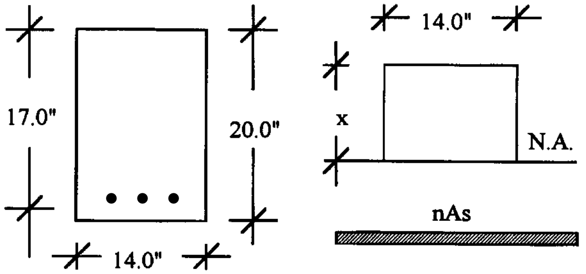


FIGURE X.1

$b = 14.0$ in (356 mm)

$h = 20.0$ in (508 mm)

$d = 17.0$ in (432 mm)

$A_s = 3$ no. 9 bars [3 in^2 (1935 mm^2)]

Solution

$$M_{cr} = \frac{f_r I_t}{y_t}$$

$$= \frac{(7.5\sqrt{4000})(14 \times 20^3/12)}{10} = 442{,}717 \text{ in} \cdot \text{lb} < 900{,}000 \text{ in} \cdot \text{lb} \qquad \therefore \text{Section has cracked.}$$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57{,}000\sqrt{f'_c}} = \frac{29 \times 10^6}{4 \times 10^6} = 7.2$$

Taking moments about the neutral axis,

$$14x\left(\frac{x}{2}\right) = nA_s(d - x)$$

$$7x^2 = 367.2 - 21.6x$$

Solving the quadratic equation we get

$$x = \frac{-21.6 \pm \sqrt{21.6^2 + 4 \times 7 \times 367.2}}{2 \times 7} = 5.86 \text{ in (148.8 mm)}$$

The moment of inertia of the transformed section is

$$I_t = \frac{bx^3}{3} + nA_s(d - x)^2$$

$$= \frac{14 \times 5.86^3}{3} + 7.2 \times 3(17 - 5.86)^2 = 3620 \text{ in}^4 \; (1506 \times 10^6 \text{ mm}^4)$$

The concrete top-fiber compression stress is

$$f_c = \frac{Mc}{I_t} = \frac{900{,}000 \times 5.86}{3620} = 1457 \text{ psi (10 MPa)}$$

The steel tension stress is

$$f_t = n\frac{Mc}{I_t} = \frac{7.2 \times 900{,}000(17 - 5.86)}{3620} = 19{,}941 \text{ psi (137.4 MPa)}$$

4A.2.3 Cracked, Inelastic Range, and Flexural Strength

With increasing loads the stresses in concrete exceed the $0.5f'_c$ value. The proportionality of stresses and strains ceases to exist, and nonlinear characteristics are observed, as shown in Fig. 4A.1(*d*).

It is a common practice to utilize an elastic-plastic idealization for the stress-strain relationship of steel. This implies that stresses and strains are related with the steel modulus of elasticity E_s up to the yield stress and its corresponding strain ε_y. At higher strain values, stresses in steel are taken equal to the yield stress f_y irrespective of the strain magnitude. However, the inelastic performance of concrete under load will result in a parabolic stress distribution. The diagram of the area of stress as well as the line of action of the resulting force have to be obtained in order to compute the stresses acting on the section.

The section reaches its flexural strength (nominal strength) when the extreme compressive fiber of concrete reaches its maximum usable strain. The ACI code recommends a maximum usable strain value of 0.003. If the properties of the compressive stress block just prior to failure are defined, the flexural strength can be computed [Fig. 4A.1(*e*)]. The ACI code allows the use of a simplified equivalent rectangular stress block to represent the stress distribution of the concrete. The rectangular block has a mean stress of $0.85f'_c$ and a depth a, where $a/c = \beta_1 = 0.85$ for $f'_c \leq 4000$ psi. β_1 is reduced incrementally by 0.05 for each 1000 psi of strength in excess of 4000 psi (27.6 MPa), provided that it does not go below 0.65. The reduction of β_1 is mainly due to less favorable properties of the stress-strain relationship for higher-strength concrete.

When the ultimate flexural capacity is reached, two types of failure can occur, depending on the amount of steel reinforcement. If a relatively low percentage of steel is used, the strain on the tension face will be beyond the yield strain ε_y. This triggers excessive deflections and wide cracks, providing a warning of imminent failure. This type of ductile failure is a desirable mode for flexural members. On the other hand, if a relatively high percentage of steel reinforcement is incorporated in the section, failure will occur prior to yielding of the reinforcement. Hence violent concrete crushing occurs in a sudden brittle manner. Because of the nonductile behavior of this mode of failure, it should be avoided. When the extreme compressive fiber of concrete reaches its maximum usable strain simultaneously with the steel reaching its yielding strain, the section is defined as a balanced section. The balanced section is used as a datum to identify ductile and nonductile reinforced concrete sections.

4A.2.4 Rectangular Sections with Tension Reinforcement Only

4A.2.4.1 Moment Capacity

From Fig. 4A.1(*e*), equating the horizontal forces C and T and solving for the depth of the compression block a, we obtain

$$0.85f'_c ba = A_s f_s \tag{4A.6}$$

$$a = \beta_1 c = \frac{A_s f_s}{0.85 f_c' b} \tag{4A.7}$$

Hence the nominal flexural strength is

$$M_n = A_s f_s \left(d - \frac{a}{2}\right) \tag{4A.8}$$

4A.2.4.2 Balanced Condition

From Fig. 4A.l(e), the linear strain distribution gives

$$\frac{c}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} = \frac{0.003}{0.003 + f_y/29 \times 10^6} = \frac{87{,}000}{87{,}000 + f_y} \tag{4A.9}$$

From Eqs. (4A.7) and (4A.9), we can write

$$c = \frac{a}{\beta_1} = \left(\frac{87{,}000}{87{,}000 + f_y}\right) d = \frac{A_s f_y}{0.85 f_c' b \beta_1}$$

Defining $\rho_b = A_s/bd$ we obtain

$$\rho_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right)\left(\frac{0.85 f_c'}{f_y}\right)\beta_1 \tag{4A.10}$$

Equation (4A.10) gives the balanced section reinforcement ratio. To ensure ductile failure, the ACI code limits the maximum tension reinforcement ratio to 75% of ρ_b. Reinforcement ratios higher than ρ_b produce nonductile failure, with steel reinforcement not yielding prior to the crushing of concrete.

4A.2.4.3 Ductile Tension Failure

For this condition Eqs. (4A.7) and (4A.8) can be written as follows:

$$a = \frac{A_s f_y}{0.85 f_c' ba} \tag{4A.11}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \tag{4A.12}$$

4A.2.4.4 Nonductile Failure

The nominal flexural strength is obtained from

$$M_n = A_s f_s \left(d - \frac{a}{2}\right) \tag{4A.13}$$

where f_s is obtained from the following quadratic equation:

$$A_s(f_s)^2 + (87{,}000A_s)f_s - (0.85f'_c b) \times (87{,}000\beta_1 d) = 0 \tag{4A.14}$$

$$a = \frac{A_s f_s}{0.85f'_c b} \tag{4A.15}$$

4A.2.4.5 Minimum Percentage of Steel

If the nominal flexural strength of the section is less than its cracking moment, the section will fail immediately when a crack occurs. This type of failure occurs in very lightly reinforced beams without warning.

In order to avoid such a failure, the ACI code requires a minimum steel percentage $\rho_{\min}$ equal to

$$\rho_{\min} = \frac{200}{f_y} \tag{4A.16}$$

Example 4A.2 Determine the nominal flexural strength M_n, of the rectangular section shown in Fig. X.2.

Given:

$f'_c = 4000$ psi (27.6 Mpa)
$f_y = 60{,}000$ psi (413.4 MPa)
$b = 14.0$ in (356 mm)
$h = 24.0$ in (610 mm)
$d = 21.5$ in (546 mm)
$A_s = 4$ no. 10 bars [5.08 in^2 (3277 mm^2)]

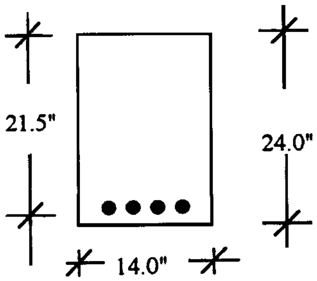


FIGURE X.2

Solution

$$\rho_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right)\left(\frac{0.85f'_c}{f_y}\right)\beta_1 = (87{,}000\backslash 87{,}000 + 60{,}000)(0.85 \times 4000\backslash 60{,}000)0.85 = 0.0285$$

$$\rho_{\max} = 0.75\rho_b = 0.75 \times 0.0285 = 0.0213$$

$$\rho = \frac{A_s}{bd} = \frac{5.08}{14 \times 21.5} = 0.0169 < 0.0213 \qquad \therefore \text{ Ductile failure.}$$

$$a = \frac{A_s f_y}{0.85f'_c b} = \frac{5.08 \times 60{,}000}{0.85 \times 4000 \times 14} = 6.4 \text{ in (16.3 mm)}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$= 5.08 \times 60{,}000\left(21.5 - \frac{6.4}{2}\right) = 5.58 \times 10^6 \text{in} \cdot \text{lb (630.63 kN} \cdot \text{m)}$$

Example 4A.3 Determine the flexural strength of the rectangular section shown in Fig. X.3.

Given:

$f'_c = 5000$ psi (34.45 MPa)

$f_y = 60{,}000$ psi (413.4 MPa)

$b = 14.0$ in (356 mm)

$d = 20.0$ in (508 mm)

$h = 24.0$ in (610 mm)

$A_s = 8$ no. 10 bars [10.16 in^2 (6553.2 mm^2)]

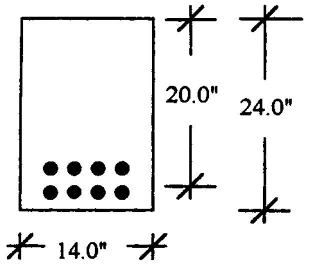


FIGURE X.3

Solution

$$\rho_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right)\left(\frac{0.85 f'_c}{f_y}\right)\beta_1 = \left(\frac{87{,}000}{87{,}000 + 60{,}000}\right)\left(\frac{0.85 \times 5000}{60{,}000}\right)0.8 = 0.0335$$

$$\rho = \frac{A_s}{bd} = \frac{10.16}{14 \times 20.0} = 0.036 > \rho_b \qquad \therefore \text{ Nonductile failure}$$

(The section does not satisfy ACI code requirements for ductility.)

Hence $f_s < f_y$. To compute f_s,

$$(A_s)f_s^2 + (87{,}000 A_s)f_s - (0.85 f'_c b \times 87{,}000 \beta_1 d) = 0$$

$$10.16 f_s^2 + (87{,}000 \times 10.16)f_s - (0.85 \times 5000 \times 14 \times 87{,}000 \times 0.8 \times 20) = 0$$

$$10.16 f_s^2 + 883{,}920 f_s - 8.28 \times 10^{10} = 0$$

Hence

$$f_s = \frac{-883{,}920 \pm \sqrt{883{,}920^2 + 4 \times 10.16 \times 8.28 \times 10^{10}}}{2 \times 10.16}$$

$$= 56{,}720 \text{ psi (390 MPa)} < 60{,}000 \text{ psi (413.4 MPa)}$$

$$a = \frac{A_s f_s}{0.85 f'_c b} = \frac{10.16 \times 56{,}720}{0.85 \times 5000 \times 14} = 9.68 \text{ in (24.59 mm)}$$

$$M_n = A_s f_s\left(d - \frac{a}{2}\right)$$

$$= 10.16 \times 56{,}720\left(20 - \frac{9.68}{2}\right) = 8.73 \times 10^6 \text{ in} \cdot \text{lb (986.63 kN} \cdot \text{m)}$$

4A.2.5 Doubly Reinforced Sections

4A.2.5.1 Moment Capacity

Doubly reinforced sections contain reinforcement on the tension side as well as on the compression side of the cross section. These sections become necessary when the size of the rectangular section is restricted due to architectural or mechanical limitations such that the required moment is larger than the resisting design moment of singly reinforced sections.

The analysis of a doubly reinforced section is carried out by theoretically dividing the cross section into two parts, as shown in Fig. 4A.2. Beam 1 is comprised of compression reinforcement at the top and sufficient steel at the bottom to have $T_1 = C_1$. Beam 2 consists of the concrete web and the remaining tensile reinforcement.

The nominal strength of part 1 can be obtained by taking the moment about the tension steel,

$$M_{n1} = A_{s1} f_y (d - d') A'_s f_y (d - d') \tag{4A.17}$$

The nominal strength of part 2 is obtained by taking the moment about the compression force,

$$M_{n2} = (A_s - A_{s1}) f_y \left(d - \frac{a}{2} \right) = (A_s - A'_s) f_y \left(d - \frac{a}{2} \right) \tag{4A.18}$$

where
$$a = \frac{(A_s - A_s 1) f_y}{0.85 f'_c b} \tag{4A.19}$$

Hence the nominal strength is

$$M_n = M_{n1} + M_{n2}$$

or
$$M_n = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2} \right) \tag{4A.20}$$

This equation is only valid when A'_s reaches the yield stress prior to concrete crushing. This condition is satisfied if

$$\rho - \rho' \geq \left(\frac{0.85 \beta_1 f'_c d'}{f_y d} \right) \left(\frac{87{,}00}{87{,}000 - f_y} \right) \tag{4A.21}$$

Otherwise the nominal strength equation is written as

$$M_n = A'_s f'_s (d - d') + (A_s f_y - A'_s f_s) \left(d - \frac{a}{2} \right) \tag{4A.22}$$

where
$$a = \frac{A_s f_y - A'_s f'_s}{f'_c b} \tag{4A.23}$$

and $f'_s < f_y$.

The following iterative procedure can be followed to obtain f'_s:

1. For the first trial assume

$$f'_s = 87{,}000 \left[1 - \frac{0.85 \beta_1 f'_c}{(\rho - \rho') f_y} \frac{d'}{d} \right]$$

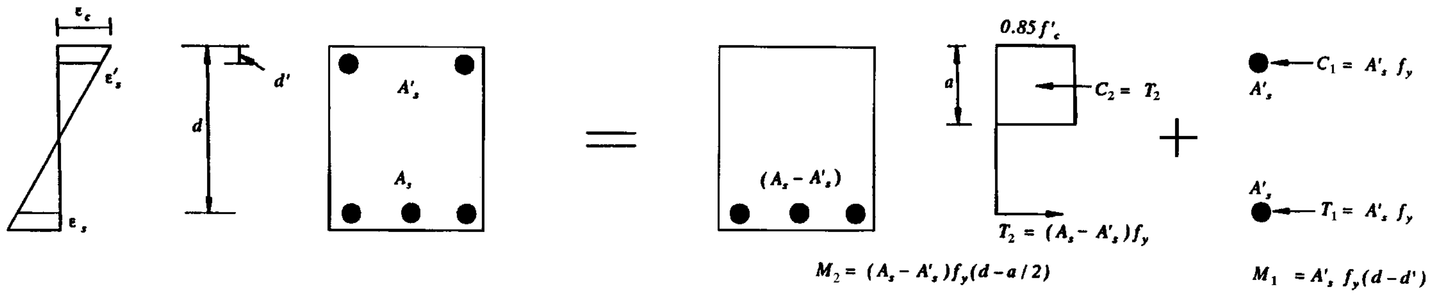


FIGURE 4A.2 Flexural behavior of a doubly reinforced concrete section.

2. Obtain the depth of the compression block,

$$a = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b}$$

3. Calculate the depth of the neutral axis, $c = a/\beta_1$.
4. Using similar triangles as in Fig. 4A.2, compute the strain ε'_s and then the adjusted compression stress of the steel, $f'_s = \varepsilon'_s E_s$.
5. Repeat steps 2 to 4 until an acceptable convergence is reached. Usually one trial will give acceptable results.

4A.2.5.2 Ductile Failure

To ensure ductile failure, the tension steel reinforcement ρ should be limited to

$$\rho \le 0.75\bar{\rho}_b + \rho'_s \frac{f'_s}{f_y} \tag{4A.24}$$

where $\bar{\rho}_b$ is the balanced steel ratio for a singly reinforced beam with a tension steel area of $A_{s1} = A_s - A'_s$.

Example 4A.4 Determine the nominal strength of the rectangular section shown in Fig. X.4.

Given:

f'_c = 5000 psi (34.45 MPa)
f_y = 60,000 psi (413.4 MPa)
d = 26.0 in (660 mm)
b = 14.0 in (356 mm)
h = 30.0 in (762 mm)
d = 26.0 in (660 mm)
d' = 2.0 in (51 mm)
A_s = 8 no. 9 bars [8 in^2 (5160 mm^2)]
A'_s = 2 no. 8 bars [1.58 in^2 (1019 mm^2)]

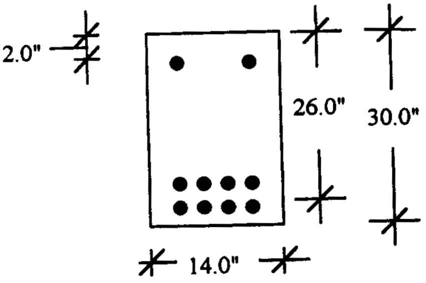


FIGURE X.4

Solution

$$\rho = \frac{A_s}{bd} = \frac{8.0}{14 \times 26.0} = 0.022$$

$$\rho' = \frac{A'_s}{bd} = \frac{1.58}{14 \times 26.0} = 0.0043$$

$$\bar{\rho}_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right)\left(\frac{f'_c}{f_y}\right)\beta_1 = \left(\frac{87{,}000}{87{,}000 + 60{,}000}\right)\left(\frac{0.85 \times 5000}{60{,}000}\right)0.8 = 0.0336$$

Check for yielding of steel in compression:

$$\rho - \rho_b = 0.022 - 0.0043 = 0.0177$$

$$\left(\frac{0.85\beta_1 f'_c d'}{f_y d}\right)\left(\frac{87{,}000}{87{,}000 - f_y}\right) = \left(\frac{0.85 \times 0.8 \times 5000 \times 2}{60{,}000 \times 26}\right)\left(\frac{87{,}000}{87{,}000 - 60{,}000}\right) = 0.014$$

$$\rho - \rho_b = 0.0177 > 0.0\ 14 \qquad \therefore \text{ Steel is yielding in compression.}$$

$$f'_s = f_y = 60{,}000 \text{ psi } (413.4 \text{ MPa})$$

Check for yielding of steel in tension:

$$0.75\bar{\rho}_b + \rho'\frac{f'_s}{f_y} = 0.75 \times 0.0336 + 0.0043 = 0.0295$$

$$\rho = 0.022 < 0.0295 \qquad \therefore \text{ Steel is yielding in tension.}$$

$$a = \frac{(A_s - A'_s)f_y}{0.85 f'_c b} = \frac{(8 - 1.58)60{,}000}{0.85 \times 5000 \times 14} = 6.47 \text{ in } (16.4 \text{ mm})$$

$$M_{n1} = A'_s f_y (d - d') = 1.58 \times 60{,}000(26 - 2) = 2.27 \times 10^6 \text{ in} \cdot \text{lb } (256.5 \text{ kN} \cdot \text{m})$$

$$M_{n2} = (A_s - A'_s)f_y\left(d - \frac{a}{2}\right)$$

$$= (8 - 1.58) \times 60{,}000\left(26 - \frac{6.47}{2}\right) = 8.76 \times 10^6 \text{ in} \cdot \text{lb } (990.02 \text{ kN} \cdot \text{m})$$

$$M_n = (2.27 + 8.76)10^6 = 11 \times 10^{\,6} \text{ in} \cdot \text{lb } (1246.52 \text{ kN} \cdot \text{m})$$

Example 4A.5 Determine the nominal strength M5 of the rectangular section shown in Fig. X.5.

Given:

f'_c = 5000 psi (34.45 MPa)

f_y = 60,000 psi (413.4 MPa)

b = 14.0 in (356 mm)

h = 24.0 in (610 mm)

d = 21.0 in (S33 mm)

d' = 2.0 in (S1 mm)

A_s = 4 no. 10 bars [S08 in^2 (3277 mm^2]

A'_s = 3 no.7 bars [1.8 in^2 (1161 mm^2]

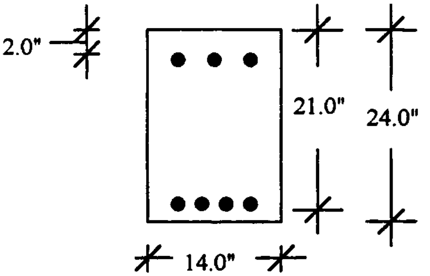


FIGURE X.5

Solution

$$\rho = \frac{A_s}{bd} = \frac{5.08}{14 \times 21.0} = 0.0173$$

$$\rho' = \frac{A'_s}{bd} = \frac{1.8}{14 \times 21.0} = 0.0061$$

$$\bar{\rho}_b = 0.0336$$

Check for yielding of steel in compression:

$$\rho - \rho' = 0.0173 - 0.0061 = 0.0112 < 0.0173 \qquad \therefore \text{ Steel did not yield in compression.}$$

Assume

$$f_s' = 87{,}000\left[1 - \frac{0.85\beta_1 f_c' d'}{(\rho - \rho')f_y d}\right]$$

$$= 87{,}000\left[1 - \frac{0.85 \times 0.8 \times 5000 \times 2}{0.0112 \times 60{,}000 \times 21}\right] = 45{,}000 \text{ psi (310 MPa)}$$

Start the iteration cycle,

$$a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b} = \frac{5.08 \times 60{,}000 - 1.8 \times 45{,}000}{0.85 \times 5000 \times 14} = 3.76 \text{ in (95.5 mm)}$$

$$(f_s') \text{ adjusted} = 29 \times 10^6 \times 0.003\left(\frac{3.76/0.8 - 2}{3.76/0.8}\right) = 50{,}000 \text{ psi (344.5 Mpa)}$$

Take $f_s' = 50{,}000$ psi,

$$a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b} = \frac{5.08 \times 60{,}000 - 1.8 \times 50{,}000}{0.85 \times 5000 \times 14} = 3.6 \text{ in (91.4 mm)}$$

$$M_n = (A_s f_y - A_s' f_s)\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d')$$

$$= (5.08 \times 60{,}000 - 1.8 \times 50{,}000)\left(21 - \frac{3.6}{2}\right) + 1.8 \times 50{,}000(21 - 2)$$

$$= 5.83 \times 10^6 \text{ in} \cdot \text{lb}(658.88 \text{ kN} \cdot \text{m})$$

4A.2.6 Flanged Section

4A.2.6.1 Moment Capacity

Because rectangular beams are generally cast monolithically with concrete slabs, a full composite action between the slab and the beam is obtained. In the positive moment diagram, the slab is in compression and, hence, contributes to the moment strength of the section. The effective cross section of the beam has a T shape or an L shape, consisting of the rectangular beam as the web and a portion of the slab as the flange (Fig. 4A.3). The effective width of the slab contributing to the section strength has to satisfy the following requirements.

For the T shape (interior beam),

$$\begin{aligned} b &\le 16h_f + b_w \\ &\le b_w + l_c \\ &\le 1_n/4 \end{aligned} \tag{4A.25}$$

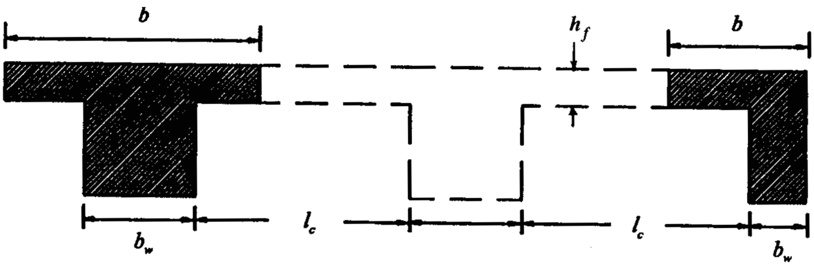


FIGURE 4A.3 Effective cross section of a flanged concrete beam. I_n = beam span length.

For the L shape (edge beam),

$$\begin{aligned} b &\le 6h_f + b_w \\ &\le b_w + l_c/2 \\ &\le 1_n/12 \end{aligned} \tag{4A.26}$$

Flanged beams possess a high compression capacity because of the large contribution of the concrete on the compression face. Hence the neutral axis generally lies in the flange. When this situation occurs, the section behaves as a rectangular singly reinforced section having a width b equal to the effective width of the slab. The flexural strength of this section is

$$M_n = A_s f_y \left(\frac{d - a}{2} \right) \tag{4A.27}$$

where $$a = \frac{A_s f_y}{0.85 f_c' b}$$

The neutral axis will fall below the flange if the tension force $A_s f_y$ is greater than the compression force capacity of the flange $0.85 f_c' b h_f$,

$$A_s f_y > 0.85 f_c' b h_f$$

Hence

$$a = \frac{A_s f_y}{0.85 f_c' b} > h_f \tag{4A.28}$$

In this case the analysis can be conducted by considering the resistance provided by the overhanging flanges and that provided by the remaining rectangular beam, as shown in Fig. 4A.4. Beam 1 consists of the overhanging flange area A_f, stressed to $0.85 f_c'$, giving a compressive force C_f, which acts at the centroid of the area of the overhanging flanges. To maintain equilibrium, beam 1 has a tensile steel area A_{sf} chosen such that

$$A_{sf} f_y = A_f (0.85 f_c')$$

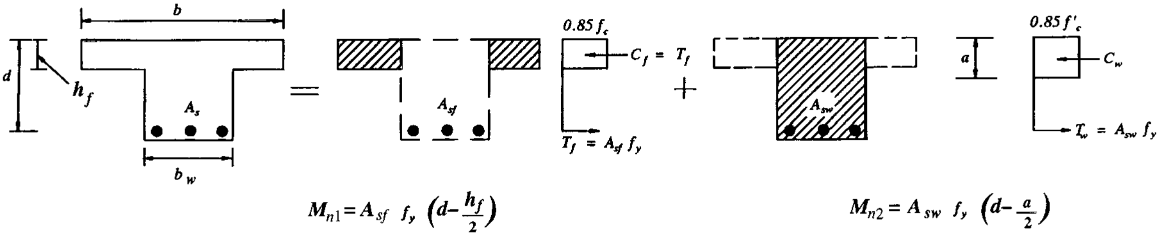


FIGURE 4A.4 Flexural behavior of concrete beam.

This steel area A_{sf} is a portion of the total area A_f and is assumed to be at the same centroid. The moment capacity of beam 1 is obtained by taking the moment about the tensile steel area A_{sf},

$$M_{n1} = 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2}\right) \tag{4A.29}$$

Beam 2 is a rectangular beam having a width b_w. The compressive force of the beam, $C_2 = 0.85 f'_c b_w a$, acts through the centroid of the compression area. Equilibrium is maintained by utilizing the remaining tensile steel area $A_s - A_{sf} = A_{sw}$. The moment capacity is obtained by taking the moment about the compression force C_2,

$$M_{n2} = A_{sw} f_y \left(d - \frac{a}{2}\right) \tag{4A.30}$$

where

$$A_{sw} = A_s - A_{sf} = A_s - \frac{0.85 f'_c (b - b_w) h_f}{f_y} \tag{4A.31}$$

and

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} \tag{4A.32}$$

Hence the total nominal strength is

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) \tag{4A.33}$$

4A.2.6.2 Ductile Failure

To ensure ductile failure, the tension steel reinforcement ratio ρ should be limited to

$$\rho \le 0.75 \frac{b_w}{b} (\bar{\rho}_b + \rho_f) \tag{4A.34}$$

where $\rho = A_s / bd$

ρ_b = balanced steel ratio for a rectangular section (b_w and h) with tension reinforcement, $A_{sw} = A_s - A_{sf}$

$$\rho_f = \frac{0.85 f'_c (b - b_w) h_f}{f_y b_w d} = \frac{A_{sf}}{b_w d}$$

Example 4A.6 Determine the nominal flexural strength M_n, of the precast T beam shown in Fig. X.6.

Given:

$f'_c = 5000$ psi (34.45 MPa)

$f_y = 60{,}000$ psi (413.4 MPa)

$b = 36.0$ in(914 mm)

$bw = 12.0$ in (305 mm)

$h = 20.0$ in (508 mm)

$h_f = 2.0$ in (51 mm)

$d = 17.0$ in (432 mm)

$A_s = 6$ no. 9 bars [6 in^2 (3870 mm^2)]

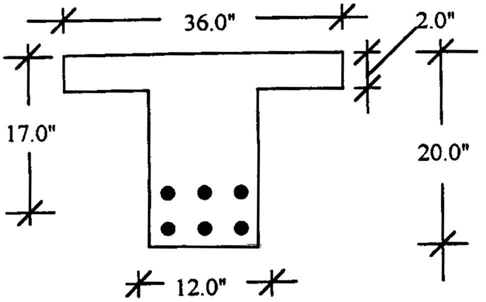


FIGURE X.6

Solution Check whether the tension force is greater than the compressive force,

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6 \times 60{,}000}{0.85 \times 5000 \times 36} = 2.35 \text{ in (59.7 mm)} > h_f = 2 \text{ in (51 mm)}$$

Hence the section will act as a T beam.

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} = \frac{0.85 \times 5000(36 - 12)2}{60{,}000} = 3.4 \text{ in}^2 \text{ (2193 mm}^2\text{)}$$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} = \frac{(6 - 3.4)600{,}000}{0.85 \times 5000 \times 12} = 3.06 \text{ in (77.7 mm)}$$

$$M_{n1} = 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2} \right)$$

$$= 0.85 \times 5000(36 - 12)2 \left(17 - \frac{2}{2} \right) = 3.26 \times 10^6 \text{ in} \cdot \text{lb (368.43 kN} \cdot \text{m)}$$

$$M_{n2} = A_{sw} f_y \left(d - \frac{a}{2} \right)$$

$$= (6 - 3.4)60{,}000 \left(17 - \frac{3.06}{2} \right) = 2.4 \times 10^6 \text{ in} \cdot \text{lb (271.24 kN} \cdot \text{m)}$$

Hence the total nominal strength is

$$M_n = M_{n1} + M_{n2} = (3.26 + 2.41)10^6 = 5.67 \times 10^6 \text{ in} \cdot \text{lb (639.67 kN} \cdot \text{m)}$$

4A.3 SHEAR BEHAVIOR

4A.3.1 Plain Concrete

Because structural members are usually subjected to shear stresses combined with axial, flexure, and tension forces rather than to pure shear stresses, the behavior of concrete under pure shear forces is not of major importance. Furthermore, even if pure shear is encountered in a member, a principal stress of equal magnitude will be produced on another inclined plane, leading to the failure of concrete in tension before its shearing strength can be reached.

Consider a small element A at the neutral axis of the beam presented in Fig. 4A.5(*a*). It can be shown that the case of pure shear [Fig. 4A.5(*b*)] is equivalent to a set of normal tension and compression stresses σ_1 and σ_2, respectively. Cracking of concrete will occur if the tension stress referred to as diagonal tension exceeds the tension strength of the concrete. As stated earlier, shear stresses are usually combined with other stresses. Considering a small element located below the neutral axis of the beam in Fig. 4A.5(*a*), two types of stresses occur, bending stresses and shear stresses. If the beam is behaving in the elastic range, these stresses can be obtained as follows:

$$\sigma = \frac{Mc}{I} \tag{4A.35}$$

$$\tau = \frac{VQ}{Ib} \tag{4A.36}$$

This element can be rotated at an angle ϕ to obtain the principal normal stresses σ_1 and σ_2. The magnitude of the principal stresses and their orientations [Fig. 4A.5(*c*)] are determined from the following expressions:

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \tag{4A.37}$$

$$\sigma_2 = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \tag{4A.38}$$

$$\tan 2\theta = \frac{\tau}{\sigma/2} \tag{4A.39}$$

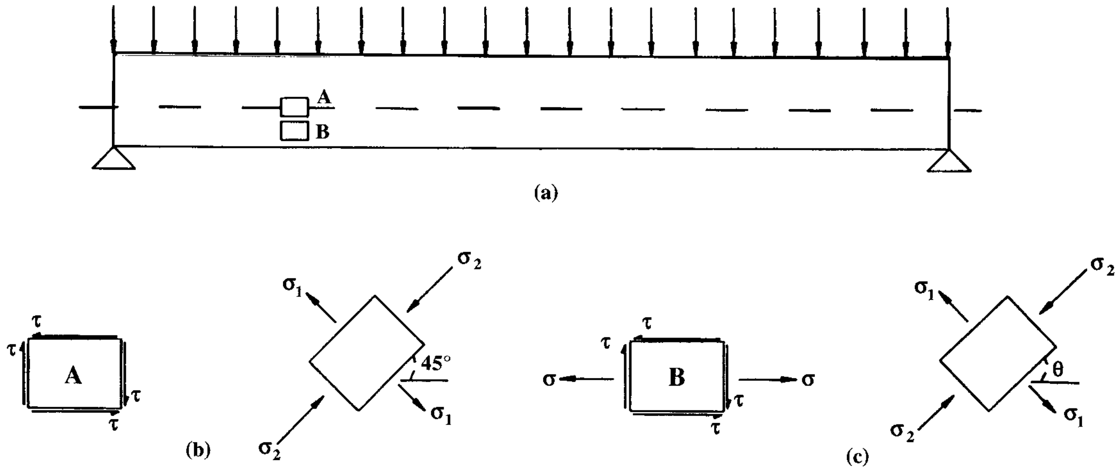


FIGURE 4A.5 (a) Shear behavior of concrete beam. (b) Pure shear. (c) Combined shear and bending.

It should be clear at this stage that, depending on the relative values of the bending moments and shear forces, the magnitudes of the principal stresses and their orientations will vary. When bending moments are relatively predominant compared to shearing forces, flexure cracks are observed. However, if the shear stresses are sufficiently higher than the bending stresses, inclined cracks of the web propagating from the neutral axis are to be expected.

4A.3.2 Reinforced Concrete Beams

The shear strength of concrete beams reinforced with steel bars resisting flexural loading is denoted by V_c. The failure plane shown in Fig. 4A.6 indicates that V_c is mainly provided by three sources—shear resistance of the uncracked concrete V_{uc} resistance by aggregate interlock V_a and dowel action provided by the longitudinal flexural steel reinforcements V_d. These three terms cannot be determined individually. Their combined total effect is evaluated empirically based on a large number of available test results. The ACT code suggests the following conservative equation to determine V_c:

$$V_c = 2\sqrt{f'_c}b_w d \tag{4A.40}$$

A less conservative expression, which takes into account the effects of the longitudinal reinforcement and the moment-to-shear ratio, may also be used:

$$V_c = \left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right)b_w d \leq 3.5\sqrt{f'_c}b_w d \tag{4A.41}$$

In this expression $V_u d/M_u$ may not be taken as greater than 1.0. In spite of the fact that Eq. (4A.41) is less conservative, its complex form makes its use justifiable in cases of large numbers of similar members. If the shear strength is to be determined for lightweight concrete members, the term $\sqrt{f'_c}$ should be replaced with $f_{ct}/6.7 \leq f'_c$, where f_{ct} is the split cylinder strength of concrete. If the f_{ct} value is not available, then the term $\sqrt{f'_c}$ is to be multiplied by 0.75 for all lightweight concrete and by 0.85 for sand lightweight concrete.

When axial compression exists, the ACT code permits the use of the following equation:

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f'_c}b_w d \tag{4A.42}$$

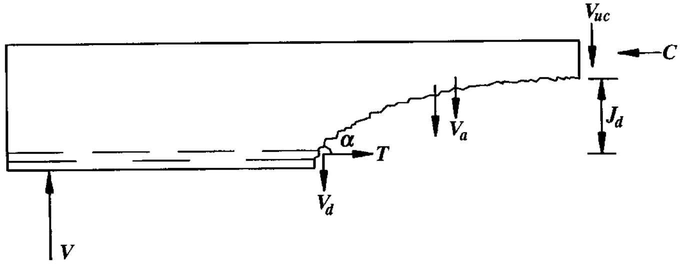


FIGURE 4A.6 Shear-resisting mechanism in reinforced concrete beams.

A more conservative equation is adopted by the ACI code for the case of axial tension,

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f'_c}b_w d \tag{4A.43}$$

where N_u is negative for tension and the ratio N_u/A_g is expressed in pounds per square inch.

When the factored shear force V_u is relatively higher compared to V_c, an additional web reinforcement is provided, as shown in Fig. 4A.7. Several theories have been presented to explain the behavior of web reinforcement. The truss analogy theory is being used widely to illustrate the contribution of web reinforcement to the shear strength of the reinforced concrete beams. According to this theory, the behavior of a reinforced concrete beam with shear reinforcement is analogous to that of a statically determinate paralleled chord truss with pinned joints.

If it is conservatively assumed that the horizontal projection of the crack is equal to the effective depth of the section d, it can be shown that the shear contribution of the vertical stirrups is

$$V_s = \frac{A_v f_y d}{s} \tag{4A.44}$$

If inclined stirrups are used, their contribution to the section shear strength is

$$V_s = \frac{A_v f_y d}{s}(\sin\alpha + \cos\alpha) \tag{4A.45}$$

where α is the angle between the stirrups and the longitudinal axis of the member.

The total nominal shear strength of a section is therefore

$$V_n = V_c + V_s \tag{4A.46}$$

The ACI code limits the maximum vertical spacing to $d/2 \leq 24$ in (305 mm). If the shear resistance V_s of the web reinforcement exceeds $4\sqrt{f'_c}b_w d$, the maximum spacing limit is reduced by one-half to $d/4 \leq 12$ in (305 mm). A minimum practical spacing that could be adopted is approximately 3 to 4 in (75 to 100 mm).

The code also provides maximum and minimum limits for the area of shear reinforcement. To avoid concrete crushing prior to the yielding of shear reinforcement, a maximum limit is set. This is provided by limiting the contribution of V_s to the shear resistance to

$$V_s \leq 8\sqrt{f'_c}b_w d \tag{4A.47}$$

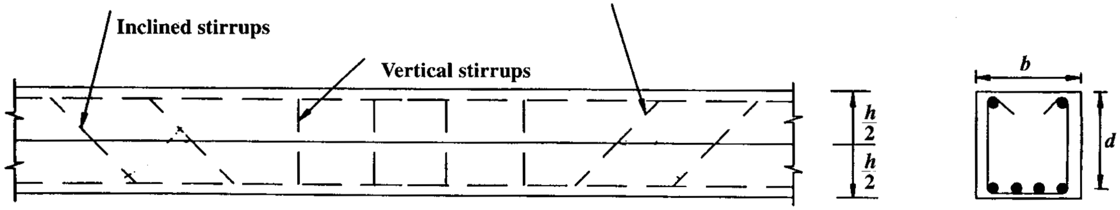


FIGURE 4A.7 Web reinforcement for shear resistance.

If a safe design calls for a higher V_s contribution than the limit set in Eq. (4A.47), the concrete section needs to be enlarged to increase the contribution of concrete to the shear resistance V_c.

The ACI code also requires a minimum shear reinforcement $(A_v)_{\min}$ if V_u exceeds $\phi V_c/2$ such that

$$(A_v)_{\min} = \frac{50 b_w s}{f_y} \tag{4A.48}$$

This requirement is necessary to avoid possible brittle failure after the formation of early-stage diagonal cracking. The yield strength of stirrups is limited by the code to 60,000 psi (413.4 MPa) to control the crack width and provide for aggregate interlock availability.

Example 4A.7 The rectangular cross section shown in Fig. X.7 is subjected to the following factored loads:

$$V_u = 70{,}000 \text{ lb } (311.5 \text{ kN})$$

$$M_u = 1 \times 10^6 \text{ in} \cdot \text{lb}(113 \text{ kN} \cdot \text{m})$$

Given:

$f'_c = 4000$ psi (27.S6 MPa)

f_y(stirrups) = 60,000 psi (413.4 MPa)

$b = b_w = 12.0$ in (305 mm)

$h = 28.0$ in (711 mm)

$d = 25.0$ in (635 mm)

$A_s = 3$ no. 9 bars (3 in^2 (1935 mm^2)]

Determine the required shear reinforcement.

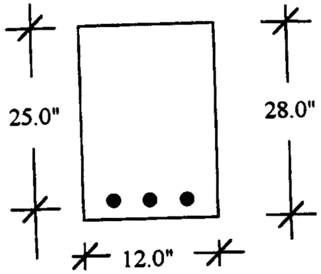


FIGURE X.7

Solution Determine the shear resistance V_c provided by concrete:

1. Using the simplified expression in Eq. (4A.40),

$$V_c = 2 f'_c b_w d = 2\sqrt{4000} \times 12 \times 25 = 37{,}947 \text{ lb } (168.9 \text{ kN})$$

2. Using the detailed expression in Eq. (4A.41),

$$V_c = \left(1.9 f'_c + 2500 \rho_w \frac{V_d d}{M_u}\right) b_w d \le 3.5 f'_c b_w d$$

where
$$\rho_w = \frac{A_s}{b_w d} = \frac{3}{12 \times 25} = 0.01$$

$$\frac{V_d d}{M_u} = \frac{70{,}000 \times 25}{10^6} = 1.75 > 1.0 \qquad \therefore \text{ Use } 1.0.$$

$$V_c = (1.9\sqrt{4000} + 2500 \times 0.01 \times 1)12 \times 25 \le 3.5\sqrt{4000} \times 12 \times 25$$

$$= 43{,}550 \text{ lb } (193.8 \text{ kN}) \le 66{,}407 \text{ lb } (295.5 \text{ kN})$$

Thus V_c = 43,550 lb (193.8 kN), compared to 37,947 lb (168.9 kN) obtained using the simplified expression. Use V_c = 43,550 lb (193.8 kN) for the rest of the example.

$$V_s = V_n - V_c = \frac{V_u}{\phi} - V_c = \frac{70{,}000}{0.85} - 43550 = 38{,}803 \text{ 1b (172.7kN)}$$

$$V_s = 38{,}803 \text{ lb (172.7 kN)} \le 8\sqrt{f_c'}b_w d = 151{,}788 \text{ lb (675.5 kN)} \qquad \therefore \text{No need to enlarge section.}$$

$$V_s = 38{,}803 \text{ lb (172.7 kN)} \le 4\sqrt{f_c'}b_w d = 75{,}894 \text{ lb (337.7 kN)}$$

The maximum stirrup spacing is

$$s = \frac{d}{2} = \frac{25}{2} = 125 \text{ in (318 mm)} \le 24 \text{ in (610 mm)}$$

Hence a maximum spacing of 12.S in (318 mm) should be used.
Use a U-shape no. 3 bars for stirrups. Hence $A_v = 0.11 \times 2 = 0.22$ in^2 (142 mm^2). The required spacing is

$$s = \frac{A_v f_y d}{V_s} = \frac{0.22 \times 60{,}000 \times 25}{38{,}803} = 8.5 \text{ in (216 mm)} < 12.\text{S in (318 mm)} \qquad \text{O.K.}$$

Check the minimum reinforcement:

$$(A_v)_{\min} = \frac{50 b_w s}{f_y} = \frac{50 \times 12 \times 8.5}{60{,}000} = 0.085 \text{ in}^2 \text{ (55 mm}^2) < 0.22 \text{ in}^2 \text{ (142 mm}^2) \qquad \text{O.K.}$$

Therefore use U-shape no. 3 bars with spacing s = 8.0 in (200 mm).

Example 4A.8 Provide shear reinforcement for the beam shown in Fig. X.8.

Given:

$b_w = b$ = 14 in (3S6 mm)

d = 23 in (584 mm)

h = 26 in (660 mm)

f_c' = 5000 psi (34.45 MPa)

f_y = 60,000 psi (413.4 MPa)

w_D = 3000 lb/ft (437.S N/in)

w_L = 5000 lb/ft (729.2 N/in)

Solution Critical section:

$$W_u = 1.4 \times 3000 + 1.7 \times 5000 = 12{,}700 \text{ lb/ft (4710 N/in)}$$

$$V_u \text{ at face of support} = 12{,}700\left(\frac{15}{2}\right) = 95{,}250 \text{ lb (423.9 kN)}$$

$$V_u \text{ at distance d} = 23 \text{ in from face of support } 95{,}250\left(\frac{90-23}{90}\right) = 70{,}908 \text{ lb (315.5 kN)}$$

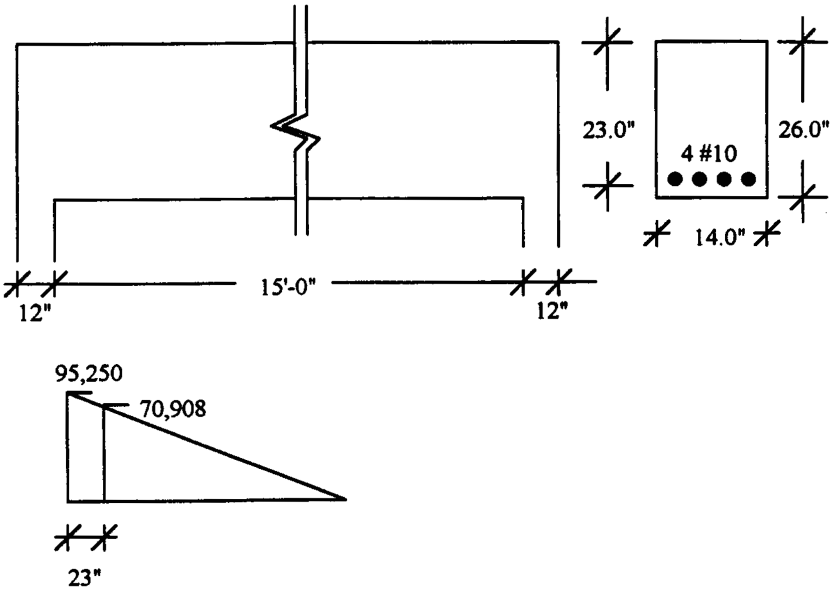


FIGURE X.8

$$V_c \text{ using simplified expression} = 2\sqrt{f'_c}b_w d = 2\sqrt{5000} \times 14 \times 23 = 45{,}S37 \text{ lb } (202.6 \text{ kN})$$

$$V_s \text{ required} = \frac{V_u}{\phi} - V_c = \frac{70{,}908}{0.85} - 45{,}537 = 37{,}884 \text{ lb } (168.6 \text{ kN})$$

$$V_s = 37{,}884 \leq 8\sqrt{f'_c}b_w d = 182{,}148 \text{ lb } (810.6 \text{ kN}) \qquad \therefore \text{No need to enlarge section.}$$

$$V_s = 37{,}884 \leq 4\sqrt{f'_c}b_w d = 91{,}075 \text{ lb } (405.3 \text{ kN})$$

The maximum stirrup spacing is

$$s = \frac{d}{2} = \frac{23}{2} = 11.5 \text{ in } (292 \text{ mm}) \leq 24 \text{ in } (610 \text{ mm})$$

Hence a maximum spacing of 11.5 in (292 mm) should be used.

Use U-shape no. 3 bars for stirrups, and we get

$$s = \frac{A_s f_y d}{V_s} = \frac{0.22 \times 60{,}000 \times 23}{37{,}884} = 8.01 \text{ in } (203 \text{ mm})$$

Therefore uses $s = 8$ in (200 mm) $< s_{max} = 11.5$ in (292 mm). O.K.

Check the minimum reinforcement:

$$(A_v)_{min} = \frac{50 b_w s}{f_y} = \frac{50 \times 14 \times 8}{60{,}000} = 0.093 \text{ in}^2 \ (60 \text{ mm}^2) < 0.22 \text{ in}^2 \ (142 \text{ mm}^2) \qquad \text{O.K.}$$

Section at which s_{max} can be used:

$$V_s \text{ with } s = 11.5 \text{ in} = \frac{A_v f_y d}{s} = \frac{0.22 \times 60{,}000 \times 23}{11.5} = 26{,}400 \text{ lb } (117.5 \text{ kN})$$

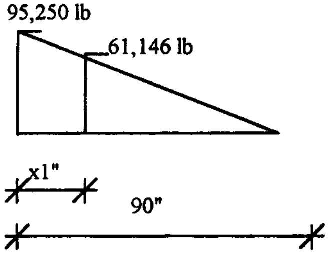
$$V_n = \frac{V_u}{\phi} = V_s + V_c = 26{,}400 + 45{,}537 = 71{,}937 \text{ lb } (320.1 \text{ kN})$$

$$V_u = 71{,}937 \times 0.85 = 61{,}146 \text{ lb } (272.1 \text{ kN})$$

This value is obtained at distance x_1 from the face of the support, such that

$$90 - x_1 = 90\left(\frac{61{,}146}{95{,}250}\right)$$

$$x_1 = 32 \text{ in } (813 \text{ mm})$$



Section at which stirrups can be omitted: In order not to use stirrups, the ACI code requires that

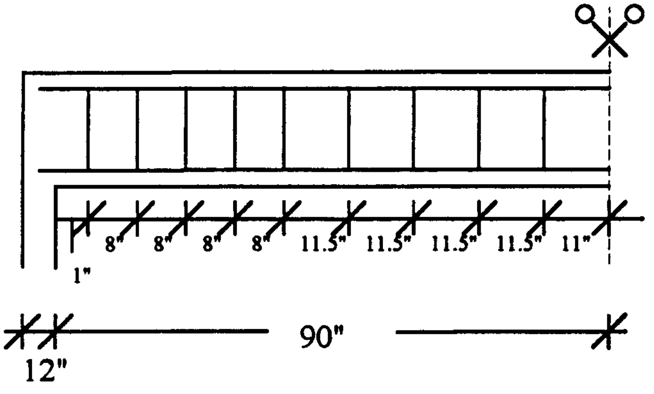
$$V_u \leq \frac{\phi V_c}{2} = \frac{0.85 \times 45{,}537}{2} = 19{,}353 \text{ lb } (86.1 \text{ kN})$$

This value is obtained at distance x_2 from the face of the support,

$$90 - x_2 = 90\left(\frac{19{,}353}{95{,}250}\right)$$

$$X_2 = 72 \text{ in } (1829 \text{ mm})$$

Shear reinforcement summary: This is illustrated by the following figure.



4A.4 TORSION BEHAVIOR

4A.4.1 Torsion Strength of Reinforced Concrete Beams

Torsion occurs in many structures, such as main girders of bridges, edge beams in buildings, spiral stairways, and balcony girders. Torsion, however, usually occurs in combination with shear and bending.

If a plain concrete member is subjected to pure torsion, it will first crack, then fail along 45° spiral lines. This failure pattern is due to the diagonal tension corresponding to the torsional stresses, which have opposite signs on both sides of the member (Fig. 4A.8).

The maximum torsional stresses $v_{\max}$ caused in a rectangular cross section can be calculated theoretically using the following expression:

$$v_{\max} = \alpha_1 \frac{T_{cr}}{x^2 y} \tag{4A.49}$$

where α is a constant varying from 3 to 5, according to the y/x ratio.

Using the lowest value of a (namely, 3 and equating the maximum torsional stress that concrete can resist without cracking to $6\sqrt{f_c'}$,the torsional moment at which diagonal tension cracking occurs can be estimated using the following expression:

$$T_{cr} = 2\sqrt{f_c'}x^2 y \tag{4A.50}$$

If the member cross section is T- or L-shaped, the following expression can be used satisfactorily:

$$T_{cr} = 2\sqrt{f_c'}\Sigma x^2 y \tag{4A.51}$$

In this case the section is divided into a set of rectangles, each resisting part of the twisting moment in proportion to its torsional rigidity. The ACT code limits the length of the overhanging flange to be considered effective in torsional rigidity computations to three times its thickness.

Due to the combined effect of bending and torsion, a portion of the diagonal cracks will fail in the compression zone on one side of the beam. As a result, even though diagonal torsion cracks have developed on part of the beam, the other part continues to resist some torsion. The ACI conservatively limits the torsional resistance of the cracked section to 40% of the uncracked section. Hence,

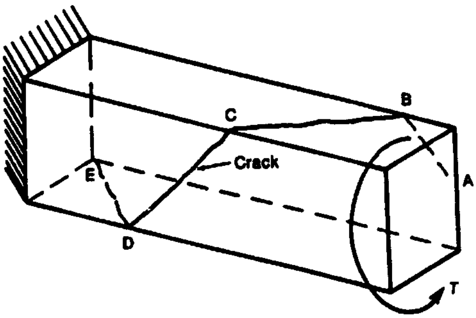


FIGURE 4A.8 Cracking pattern due to torsion.

$$T_c = 2 \times 0.4\sqrt{f'_c}\Sigma x^2 y = 0.8\sqrt{f'_c}\Sigma x^2 y \tag{4A.52}$$

If the factored torsional moment T_u exceeds the resistance of the reinforced concrete section without web reinforcement, additional torsional reinforcement needs to be provided such that

$$T_u \leq \phi(T_c + T_s) \tag{4A.53}$$

The torsion reinforcement is provided using closed vertical stirrups to form a continuous loop and additional longitudinal reinforcing bars, as shown in Fig. 4A.9. The longitudinal bars should be no smaller than no. 3, and they should be 12 in (0.3 m) apart.

According to the ACI code, the total twisting moment T_s resisted by the vertical closed stirrups and the longitudinal reinforcing bars can be estimated using the following expression:

$$T_s = \frac{A_t \alpha_1 x_1 y_1 f_y}{s} \tag{4A.54}$$

where A_t = area of one leg of a closed stirrup spaced at a distance s
x_1 = shorter dimension of stirrup
y_1 = longer dimension of stirrup
α_1 = empirical coefficient; $= 0.66 + 0.33y_1/x_1 \leq 1.5$.

The area of the longitudinal bars required for torsional resistance is obtained as the larger of the following two equations:

$$A_l = 2A_t\left(\frac{x_1 + y_1}{s}\right) \tag{4A.55}$$

or

$$A_l = \left[\frac{400b_w s}{f_y}\left(\frac{T_u}{T_u + V_u/3C_t}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right) \tag{4A.56}$$

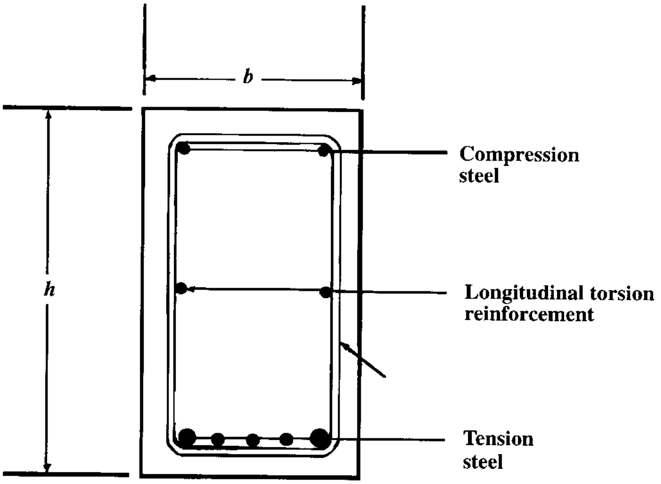


FIGURE 4A.9 Reinforcement for torsion resistance.

where $$C_t = \frac{b_w d}{\Sigma x^2 y}$$

In Eq. (4A.56) the value of A_l need not exceed the value obtained if $50b_w s/f_y$ is substituted in place of $2A_t$.

The spacing of the closed stirrups selected to provide torsion resistance should not be greater than $(x_1 + y_1)/4$ or 12 in (0.3 in). Also the code limits the amount of torsion reinforcement in order to ensure ductility. Hence the torsion resistance obtained for steel reinforcement is limited to

$$T_s \leq 4T_c \tag{4A.57}$$

Furthermore, the steel reinforcement yield stress is limited to 60,000 psi (413.4 MPa).

4A.4.2 Combined Shear, Bending, and Torsion

Due to the combined action of shear, bending, and torsion the following modified expressions are provided in the ACI code to obtain V_c and T_c, respectively:

$$V_c = \frac{2\sqrt{f'_c}\,b_w d}{\sqrt{1 + (2.5C_t T_u/V_u)^2}} \tag{4A.58}$$

$$T_c = \frac{0.8\sqrt{f'_c}\,\Sigma x^2 y}{\sqrt{1 + (0.4V_u/C_t T_u)^2}} \tag{4A.59}$$

Equations (4A.44) and (4A.54) could be used to provide shear and torsion reinforcement, respectively.

For reasons of ductility, the code limits the reinforcement contribution as follows:

$$V_t \leq 8\sqrt{f'_c}\,b_w d \tag{4A.60}$$

$$T_s \leq 4T_c \tag{4A.61}$$

The minimum area of steel reinforcement may not be less than $50b_w s/f_y$. Hence

$$A_v + 2A_t \leq \frac{50b_w s}{f_y} \tag{4A.62}$$

The minimum reinforcement needs to be provided if T_u exceeds $0.5\sqrt{f'_c}\,\Sigma x^2 y$. Otherwise torsional effects can be neglected.

Example 4A.9 A 6-in (152-mm) slab cantilevers 6 ft (1829 mm) from the face of a 14 × 24 in (356 × 610 mm) simple beam, as shown in Fig. X.9. The beam spans 30 ft (9144 mm). It carries a uniform service live load of 25 psf (1.02 kPa) on the cantilever.

Given:

$f'_c = 4000$ psi (27.56 MPa)

$f_y = 60{,}000$ psi (413.4 MPa)

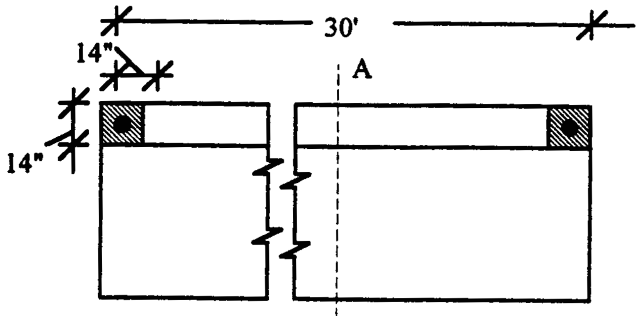
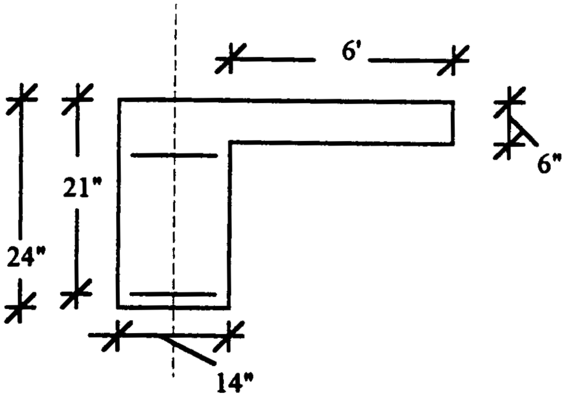


Figure X.9



SECTION A-A

$A_s = 3 \text{ in}^2$ (1935 mm^2)

Column dimensions = 14 × 14 in (356 × 356 mm)

Determine the required shear and torsion reinforcement at the critical section.

Solution

$$w_D = \frac{6}{12} \times 150 = 75 \text{ psf (3.59 kPa)}$$

$$w_u = 1.4 \times 75 + 1.7 \times 25 = 148 \text{ psf (7.08 kPa)}$$

At the centerline of the column,

$$V_u = \frac{148 \times 7.17 \times 30}{2} + 1.4\left[\frac{14(24-6)}{144} \times 150\right]\frac{30}{2} = 21{,}430 \text{ lb (95.36 kN)}$$

$$T_u = \frac{148 \times 30 \times 6}{2}\left(3 + \frac{7}{12}\right) = 47{,}730 \text{ ft} \cdot \text{lb (64 72 kN} \cdot \text{m)}$$

At a distance $d = 21$ in (533 mm) from the face of the support (critical section),

$$V_u = 21{,}430\left[\frac{15 - \left(\dfrac{7+21}{12}\right)}{15}\right] = 18{,}096 \text{ lb (80.53 kN)}$$

$$T_u = 47{,}730\left[\frac{15 - \left(\dfrac{7+21}{12}\right)}{15}\right] = 40{,}305 \text{ ft}\cdot\text{lb (54.65 kN}\cdot\text{m)}$$

$$\Sigma x^2 y = 14^2 \times 24 + 6^2 \times 24 = 5352 \text{ in}^3\ (8.8 \times 10^7 \text{ mm}^3)$$

$$\phi(0.5\sqrt{f'_c}\,\Sigma x^2 y) = 0.85(0.5\sqrt{4000} \times 5352) = 143{,}858 \text{ in}\cdot\text{lb (16.26 kN}\cdot\text{m)}$$

$$< T_u = 483{,}660 \text{ in}\cdot\text{lb (54.65 kN}\cdot\text{m)} \qquad \therefore \text{Torsional effects need to be considered.}$$

$$C_t = \frac{b_w d}{\Sigma x^2 y} = \frac{14 \times 21}{5332} = 0.055$$

$$V_c = \frac{2\sqrt{f'_c}\,b_w d}{\sqrt{1 + \left(\dfrac{2.5 C_t T_u}{V_u}\right)^2}} = \frac{2\sqrt{4000} \times 14 \times 21}{\sqrt{1 + \left(\dfrac{2.5 \times 0.055 \times 483{,}660}{18{,}096}\right)^2}} = 9763 \text{ lb (43.4 kN)}$$

$$T_c = \frac{0.8\sqrt{f'_c}\,\Sigma x^2 y}{\sqrt{1 + \left(\dfrac{0.4 V_u}{C_t T_u}\right)^2}} = \frac{0.8\sqrt{4000} \times 5352}{\sqrt{1 + \left(\dfrac{0.4 \times 18{,}096}{0.055 \times 483{,}660}\right)^2}} = 261{,}291 \text{ in}\cdot\text{lb (29.53 kN}\cdot\text{m)}$$

$$V_s = \frac{V_u}{\phi} - V_c = \frac{18{,}906}{0.85} - 9763 = 11{,}526 \text{ lb (51.29 kN)} < 8\,f'_c b_w d = 148{,}753 \text{ (662 kN)}$$

$\therefore$ No need to enlarge section.

$$T_s = \frac{T_u}{\phi} - T_c = \frac{483{,}660}{0.85} - 261{,}291 = 307{,}721 \text{ in}\cdot\text{lb (34.77 kN}\cdot\text{m)} < 4T_c = 1{,}045 \text{ in}\cdot\text{lb}$$

(118.1 kN · m) $\qquad \therefore$ No need to enlarge section.

$$\frac{A_s}{s} = \frac{V_s}{f_y d} = \frac{11{,}526}{60{,}000 \times 21} = 0.0091 \text{ in}^2\text{/in spacing for two legs}$$

Assume 1.5 in (38 mm) clear cover and no. 4 closed stirrup,

$$x_1 = 14 - 2(1.5 + 0.25) = 10.5 \text{ in (267 mm)}$$

$$y_1 = 24 - 2(1.5 + 0.25) = 20.5 \text{ in (521 mm)}$$

$$\alpha_1 = 0.66 + 0.3\left(\frac{y_1}{x_1}\right) = 0.66 + 0.33\left(\frac{20.5}{10.5}\right) = 1.3 \leq 1.5$$

Hence

$$\frac{A_t}{s} = \frac{T_s}{f_y \alpha_1 x_1 y_1} = \frac{307{,}721}{60{,}000 \times 1.3 \times 10.5 \times 20.5} = 0.0183 \text{ in}^2/\text{in spacing for two legs}$$

Stirrup for combined shear and torsion:

$$\frac{A_v}{s} = \frac{2A_t}{s} = 0.0091 + 2 \times 0.0183 = 0.046 \text{ in}^2/\text{in spacing for two legs}$$

$$\frac{50b_w}{f_y} = \frac{50 \times 14}{60{,}000} = 0.0117 < 0.046 \qquad \text{O.K.}$$

Using no. 4 and area = 2 × 0.2 = 0.4 in^2 (258 mm^2), we get

$$s = \frac{\text{area}}{A_v/s + 2A_t/s} = \frac{0.4}{0.046} = 8.7 \text{ in (221mm)}$$

The maximum allowable spacing is

$$s_{\max} = \frac{x_1 + y_1}{4} = \frac{10.5 + 20.5}{4} = 7.75 \text{ in (197 mm)}$$

Therefore use no. 4 closed stirrups at spacing s = 7.75 in (197 mm).
Longitudinal torsional steel:

$$A_t = 2A_t\left(\frac{x_1 + y_1}{s}\right) = 2 \times 0.0183(10.5 + 20.5) = 1.13 \text{ in}^2 \text{ (728.9mm}^2\text{)}$$

or

$$A_t = \frac{400xs}{f_y}\left[\left(\frac{T_u}{T_u + V_u/3C_t}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right)$$

Substituting $50b_w d/f_y$ for $2A_s$ if it is larger than $2A_t$, we get

$$\frac{50b_w d}{f_y} = \frac{50 \times 14 \times 7.5}{60{,}000} = 0.875 < 2A_t = 2 \times 0.0183 \times 7.5 = 2.745$$

Hence we use $2A_t$ = 0.2745,

$$A_t = \frac{400 \times 14 \times 7.5}{60{,}000}\left[\left(\frac{40{,}305}{40{,}305 + 18{,}096/(3 \times 0.055)}\right) - 0.2745\right]\left(\frac{10.5 + 20.5}{7.5}\right)$$

$$= -0.017 \text{ in}^2 \text{ } (-10.96 \text{ mm}^2)$$

Hence we use A_t = 1.13 in^2 (728.9 mm^2), and add $A_t/4$ = 0.3 in^2 (193.5 mm^2) on each face of the cross section. Thus we use 2 no. 4 bars on each vertical side of the cross section and on the top face [= 0.4 in^2 (258 mm^2)]. The reinforcement on the bottom face becomes A_s = 3 in^2 + 0.3 in^2 = 3.3 in^2, and we use 2 no. 9 + 2 no. 8 bars [= 3.56 in^2 (2296 mm^2)]. The final design is shown in Fig. X.l0.

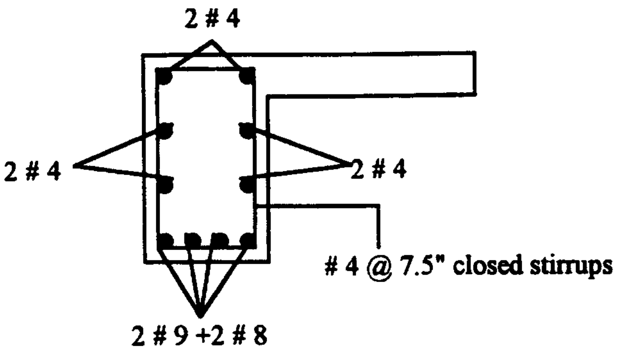


FIGURE X.10

4A.5 COLUMNS

4A.5.1 General

Columns are vertical members subjected to axial loads or a combination of axial loads and bending moments. They can be divided into three categories, depending on their structural behavior. Short compression blocks or pedestals are members with a height less than three times the least lateral dimensions. They may be designed with plain concrete with a maximum stress of $0.85\phi f'_c$, where $\phi = 0.7$. For higher stresses the pedestal should be designed with reinforced concrete. Short reinforced concrete columns have low slenderness ratios, resulting in transverse deformations that will not affect the ultimate strength. Slender reinforced concrete columns have slenderness ratios that exceed the limits given for short columns. For this case the secondary moments due to transverse deformations reduce the ultimate strength of the member.

Sections of reinforced concrete columns are usually square or rectangular shapes for lower construction costs. Longitudinal steel bars are added to increase the load-carrying capacity. A substantial strength increase is obtained by providing lateral bracing for the longitudinal bars. If bracing is provided with separate closed ties, the column is referred to as a tied column. If a continuous helical spiral is used to contain the longitudinal bars, the columns are referred to as spiral columns. Commonly, a circular shaped cross section is used for spiral columns. Composite columns consist of structural steel shapes encased in concrete. The concrete may or may not be reinforced with longitudinal steel bars.

4A.5.2 Axially Loaded Columns

The theoretical nominal strength of an axially loaded short column can be determined by the following expression:

$$P_n = 0.85f'_c(A_g - A_{st}) + f_y A_{st} \tag{4A.63}$$

where A_g = gross concrete area
A_{st} = total cross-sectional area of longitudinal reinforcement

In actual construction situations there are no perfect axially loaded columns. Minimum moments occur even though no calculated moments are present. To account for these minimum moments, the ACT code requires that the theoretical nominal strength obtained from Eq. (4A.63) be multiplied by a reduction factor α. This reduction factor is equal to 0.85 for spiral columns and 0.80 for tied columns. The use of Eq. (4A.63) and of the reduction factor α is applicable for small moments where the eccentricity e is less than $0.1h$ for tied columns and less than $0.05h$ for spiral columns. For higher eccentricity values the procedures presented in the next section should be used.

4A.5.3 Uniaxial Bending and Compression

The ultimate-strength behavior of sections under combined bending and axial compression is presented in Fig. 4A.10. Depending on the magnitude of the strain in the tension steel, the section would fail either in tension or in compression. Tension failure is characterized by initial yielding of steel preceding crushing of concrete. Compression failure is due to concrete crushing before yielding of steel. If the tension steel yields at the same time that the concrete crushes, this condition is termed balanced condition and is defined by the following expressions:

$$\varepsilon_s = \varepsilon_s' = \varepsilon_y = \frac{f_y}{E_s} \tag{4A.64}$$

$$c_b = d\left(\frac{0.003}{0.003 + \varepsilon_y}\right) \tag{4A.65}$$

$$a_b = B_1 c_b \tag{4A.66}$$

$$P_{nb} = 0.85 f_c' b a_b + A_s' f_y - A_s f_y \tag{4A.67}$$

$$M_{nb} = 0.85 f_c' b a_b\left(\bar{y} - \frac{a}{2}\right) + A_s' f_y(\bar{y} - d) + A_s f_y(d - \bar{y}) \tag{4A.68}$$

$$e_b = \frac{M_{nb}}{P_{nb}} \tag{4A.69}$$

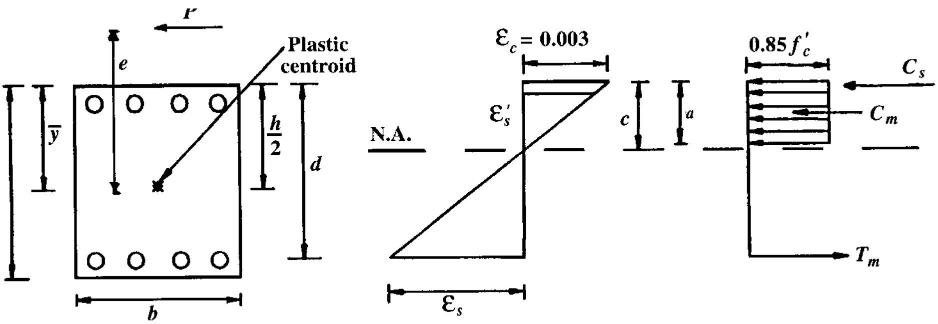


FIGURE 4A.10 Behavior of reinforced concrete columns.

Tension failure is obtained if $P_n < P_b$ or $e > e_b$, whereas compression failure is obtained if $P_n > P_b$ or $e < e_b$. For both types of failure the strain compatibility and equilibrium relationships have to be maintained.

The following procedure is used to obtain P_n and M for a given section and a known eccentricity e:

1. Assume a depth for neutral axis c. Then obtain $a = c\beta_1$.
2. Compute the compression strain of steel ε'_s and the tension strain of steel ε_s,

$$\varepsilon'_s = 0.003\left(\frac{c - d'}{c}\right) \tag{4A.70}$$

$$\varepsilon_s = 0.003\left(\frac{d - c}{d}\right) \tag{4A.71}$$

3. Compute the compression stress of steel f'_s and the tension stress of steel f_s,

$$f'_s = \varepsilon'_s E_s \le f_y \tag{4A.72}$$

$$f_s = \varepsilon_s E_s \le f_y \tag{4A.73}$$

4. Compute the value of P_n and M_n,

$$P_n = 0.85 f'_c ba + A'_s f'_s - A_s f_s \tag{4A.74}$$

$$M_n = 0.85 f'_c ba\left(\bar{y} - \frac{a}{2}\right) + A'_s f'_s(\bar{y} - a) + A_s f_s(d - \bar{y}) \tag{4A.75}$$

5. Compute $e^* = M_n/P_n$.
6. Compare e^* with the known eccentricity e. If equal, the values of P_n and M_n represent the nominal strength of the cross section. If different, repeat steps 1 to 6 with a different c value.

The procedure just presented, which ensures strain compatibility and equilibrium, converges rapidly, particularly if a computer program is used. It could be applied to a circular cross section with minor changes in Eqs. (4A.70) to (4A.75).

Example 4A.10 A 12 × 20 in (305 × 508 mm) tied column is carrying a vertical load with an eccentricity $e = 8$ in (203 mm), as illustrated in Fig. X.11.

Given:

$f'_c = 4000$ psi (27.56 MPa)

$f_y = 60{,}000$ psi (413.4 MPa)

$A_s = A'_s = 4$ no. 7 bars $= 2.4$ in^2 (1548 mm^2)

Find the nominal load P_n

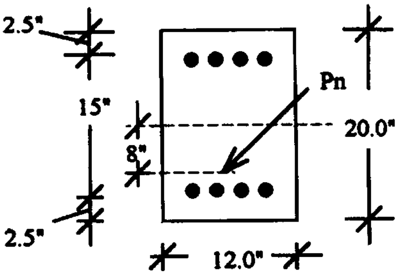


FIGURE X.11

Solution To obtain the balanced condition,

$$\varepsilon_s = \varepsilon'_s = \frac{60{,}000}{29 \times 10^6} = 0.0021$$

$$c_b = d\left(\frac{0.003}{0.003 + \varepsilon_y}\right) = 17.5\left(\frac{0.003}{0.003 + 0.0021}\right) = 10.29 \text{ in } 261.4 \text{ mm)}$$

$$a_b = \beta_1 c_b = 085 \times 10.29 = 8.75 \text{ in } (222.3 \text{ mm})$$

$$P_{nb} = 0.85 f'_c b a_b + A'_s f_y - A_s f_y = 0.85 \times 4000 \times 12 \times 8.75 = 357{,}000 \text{ lb } (1588.7 \text{ kN})$$

$$M_{nb} = 0.85 f'_c b a_b\left(\bar{y} - \frac{a}{2}\right) + A'_s f_y(\bar{y} - d) + A_s f_y(d - \bar{y})$$

$$= 0.85 \times 4000 \times 12 \times 8.75\left(10 - \frac{8.75}{2}\right) + 2.4 \times 60{,}000(10 - 2.5) + 2.4 \times 60{,}000(17.5 - 10)$$

$$= 4.17 \times 10^6 \text{ in} \cdot \text{lb } (471.21 \text{ kN} \cdot \text{m})$$

$$e_b = \frac{M_{nb}}{P_{nb}} = \frac{4.17 \times 10^6}{357{,}000} = 11.7 \text{ in } (297.2 \text{ mm})$$

$$e = 8\ in\ (203 \text{ mm}) < e_b = 11.7\text{in } (297.2 \text{ mm}) \qquad \therefore \text{Column will fail in compression.}$$

First trial

1. Assume $c = 15$ in (381 mm).

$$a = 15 \times 0.85 = 12.75 \text{ in } (323.9 \text{ mm})$$

2. $$\varepsilon'_s = 0.003\left(\frac{c - d'}{c}\right) = 0.003\left(\frac{15 - 2.5}{15}\right) = 0.0025$$

$$\varepsilon_s = 0.003\left(\frac{d - c}{c}\right) = 0.003\left(\frac{17.5 - 15}{15}\right) = 0.0005$$

3. $f'_s = 0.0025 \times 29 \times 10^6 = 72{,}500$; take $f'_s = 60{,}000$ psi (413.4 MPa)

$$f_s = 0.0005 \times 29 \times 10^6 = 14{,}500 \text{ psi } (99.9 \text{ MPa})$$

4. $P_n = 0.85 f'_c ba + A'f_s - A_s f_s$

$$= 0.85 \times 4000 \times 12 \times 12.75 + 2.4 \times 60{,}000 - 2.4 \times 14{,}500 = 629{,}400 \text{ lb } (2800.8 \text{ kN})$$

$$M_n = 0.85 f'_c ba\left(\bar{y} - \frac{a}{2}\right) + A'_s s_y(\bar{y} - d) + A_s s_y(d - \bar{y})$$

$$= 0.85 \times 4000 \times 12 \times 12.75\left(10 - \frac{12.75}{2}\right) + 2.4 \times 60{,}000(10 - 2.5) + 2.4 \times 14{,}500(17.5 - 10)$$

$$= 3.2 \times 10^6 \text{ in} \cdot \text{lb}(361.6 \text{ kN} \cdot \text{m})$$

5. $e^* = \dfrac{M_n}{P_n} = \dfrac{3.2 \times 10^6}{629{,}400} = 5.12$ in (130.05 mm)

Second trial
Assume $c = 11$ in (279.4 mm), and by repeating the same procedure we get

$$P_n = 465{,}000 \text{ lb } (2069.3 \text{ kN})$$

$$M_n = 3.8 \times 10^6 \text{ in} \cdot \text{lb } (429.4 \text{ kN} \cdot \text{m})$$

$$e^* = \frac{3.8 \times 10^6}{465{,}000} = 8.1 \text{ in} \cong 8 \text{ in given } 465{,}000$$

Hence the nominal force P_n that can be applied on this section with an eccentricity $e = 8$ in (203 mm) is 465,000 lb (2069.3 kN).

4A.5.4 Load-Moment Interaction Diagrams

The strength of a concrete section subjected to combined axial and bending loads can be conveniently determined with the help of interaction diagrams. An interaction diagram gives a relationship between the nominal axial load P_n and the nominal moment capacity M_n of a given section, as shown in Fig. 4A.11. Each point on the diagram represents one possible combination of nominal axial load and nominal axial moment. The interaction diagram is divided into the tension control region and the compression control region by the balanced condition point (P_{nb}, M_{nb}). These diagrams are available as design aids for different column sections with different reinforcement percentages and arrangements. Typical diagrams are presented in Fig. 4A.12.

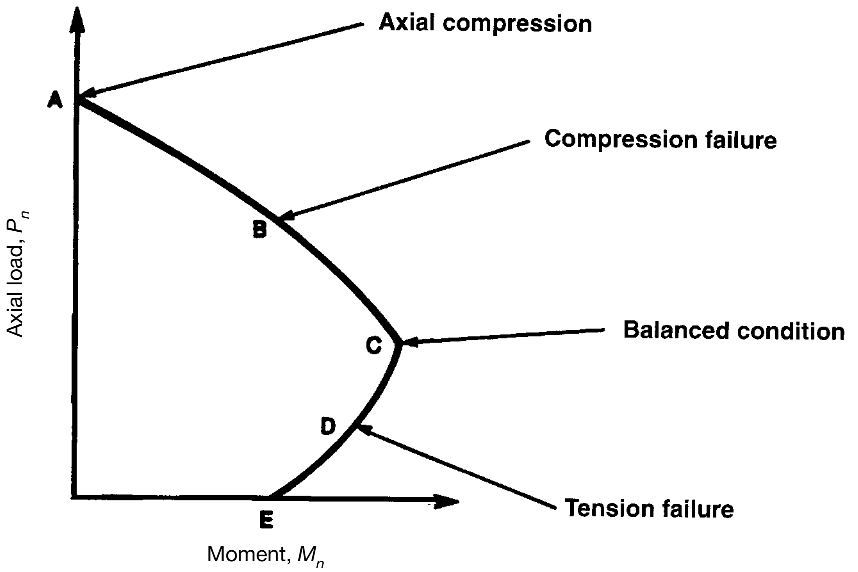
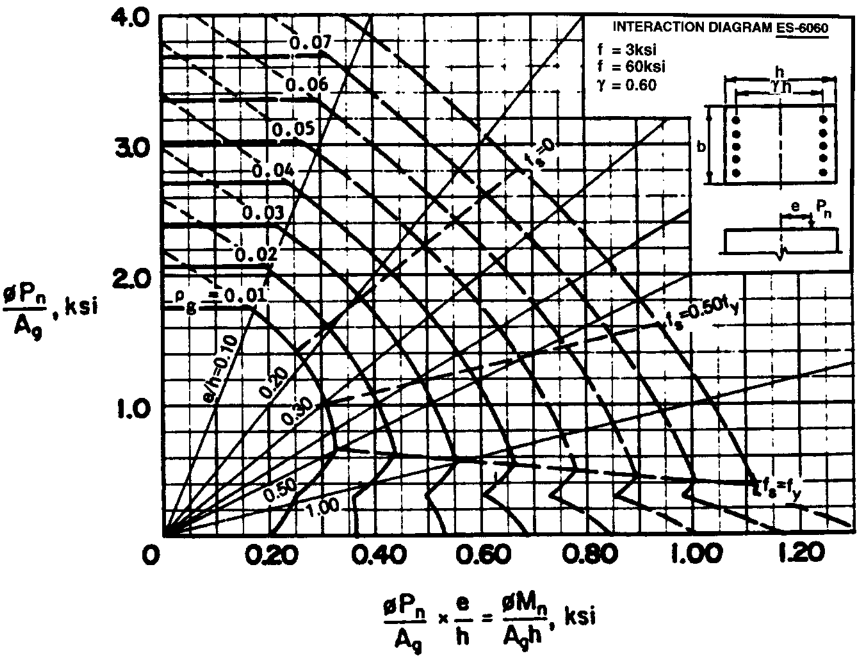


FIGURE 4A.11 Load-moment interaction diagrams for columns.



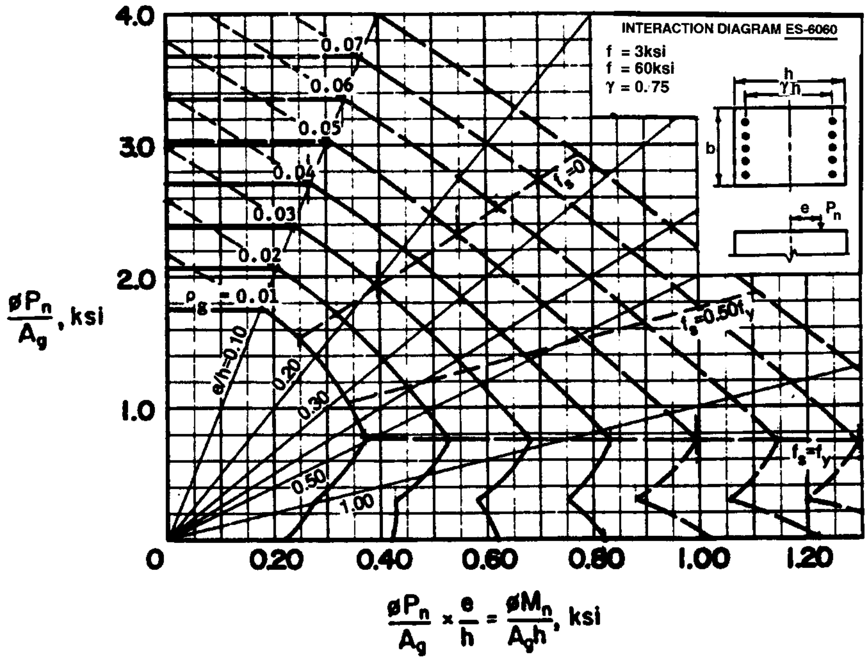
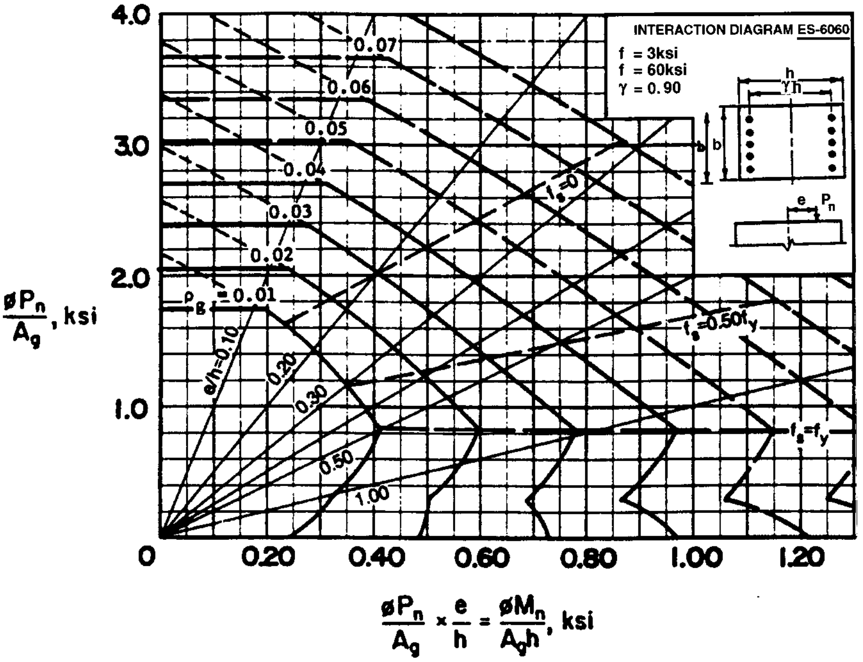


FIGURE 4A.12 Typical load-moment interaction diagrams.



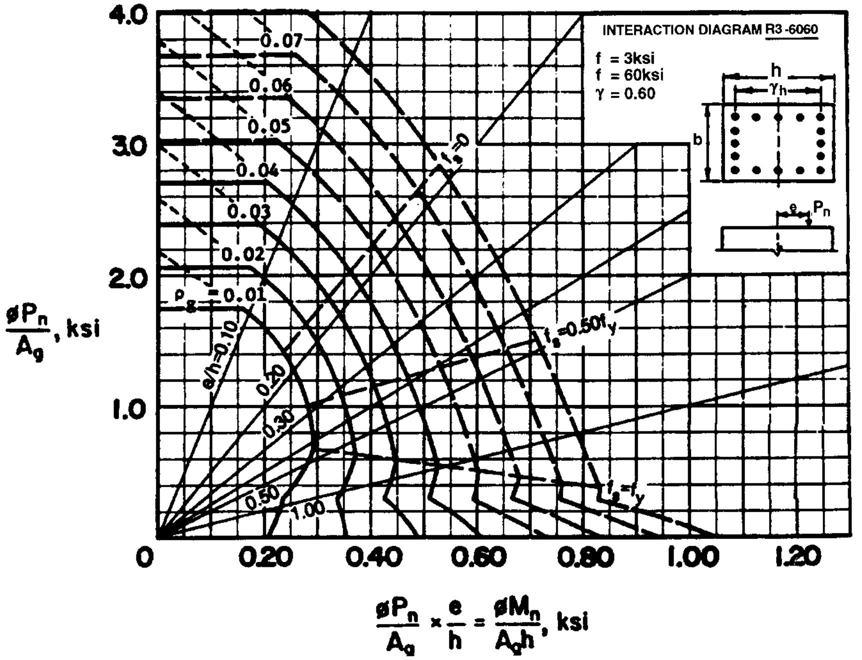
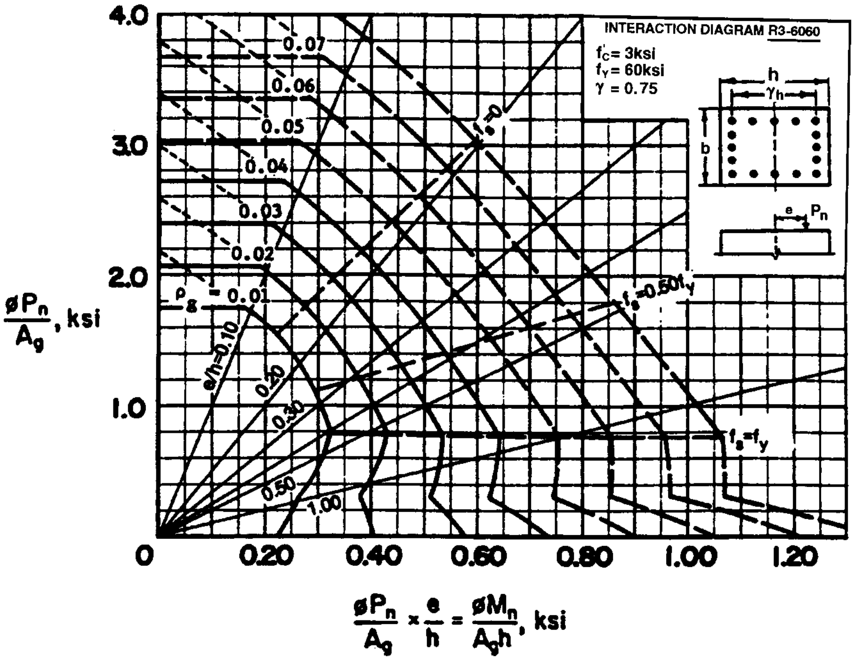


FIGURE 4A.12 (*Continued*)



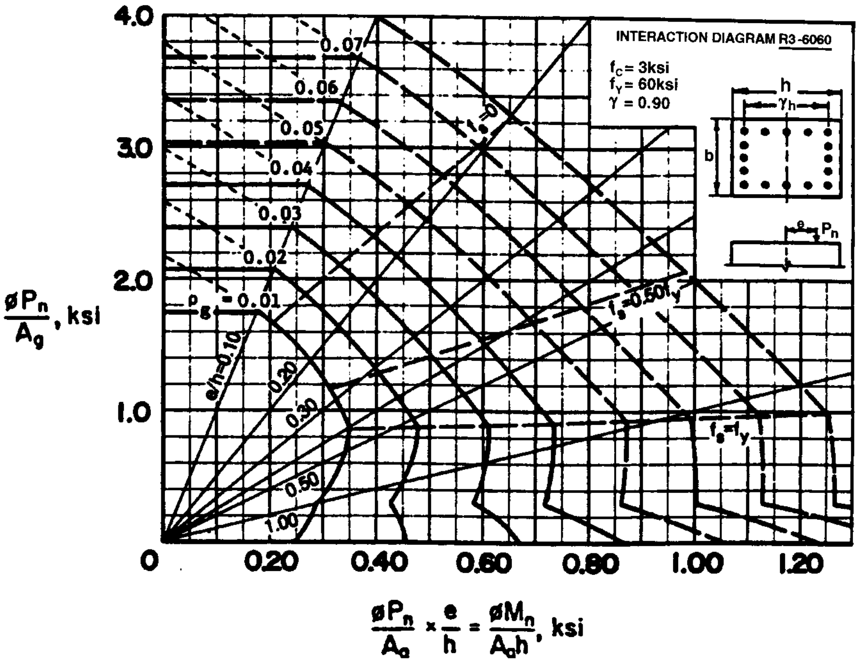
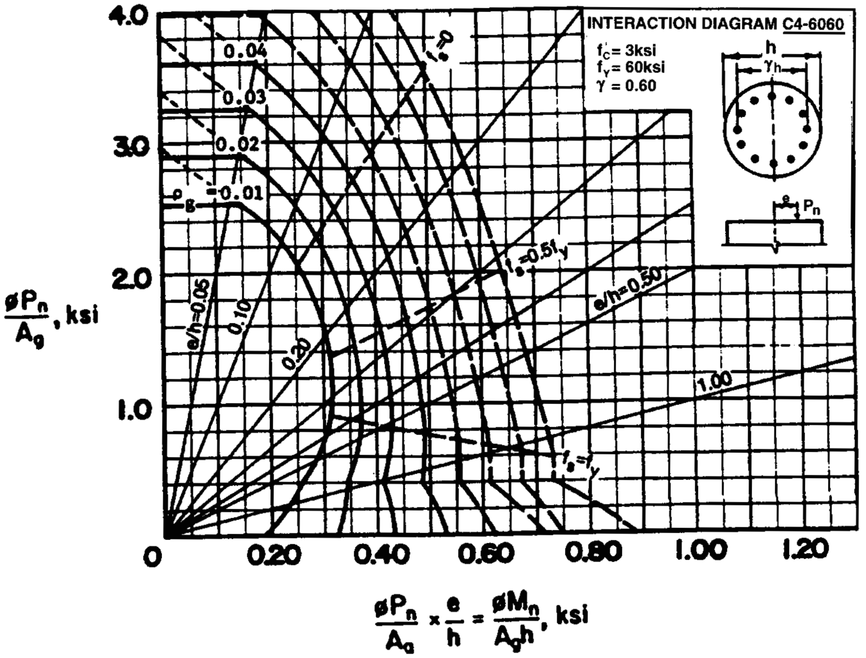


FIGURE 4A.12 (*Continued*)



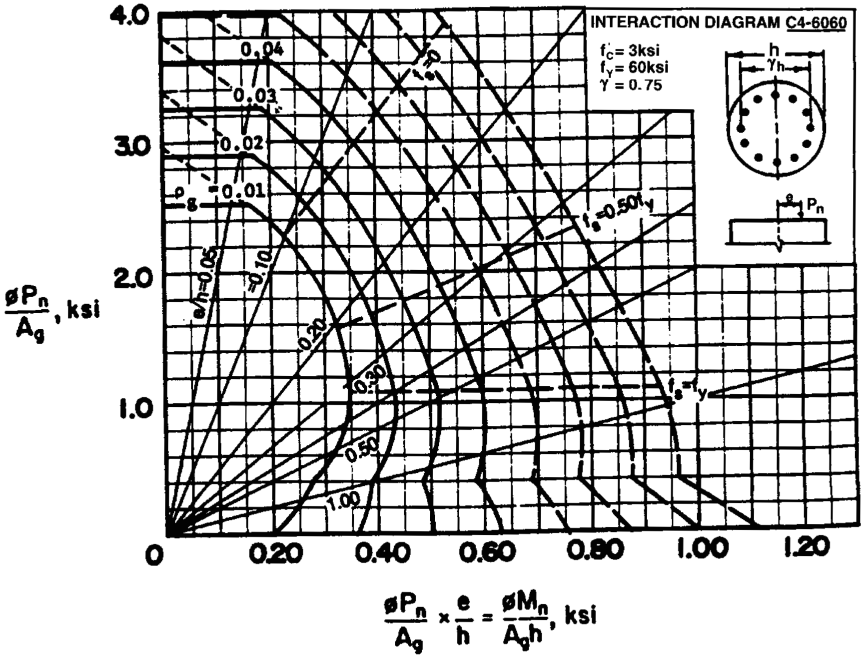


FIGURE 4A.12 (*Continued*)

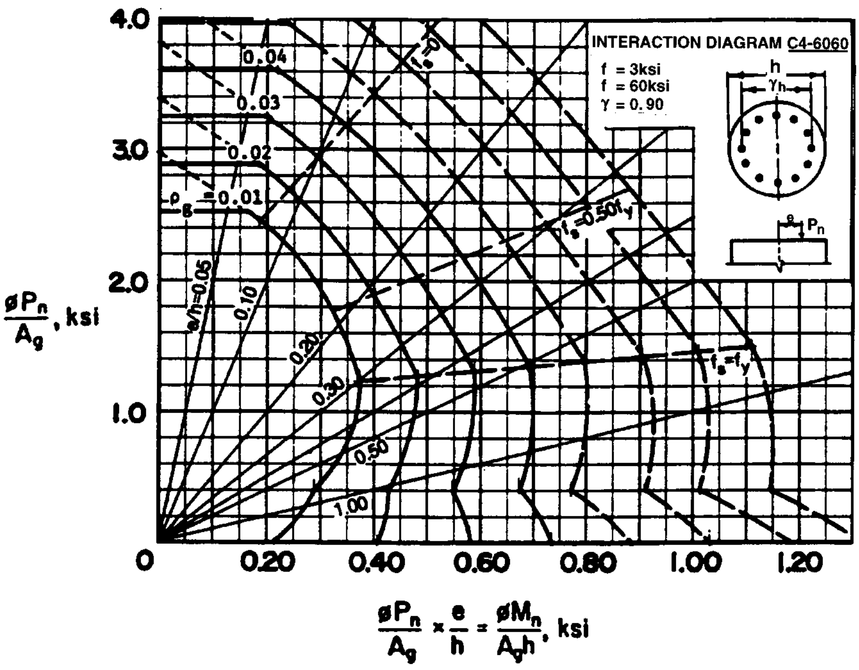


FIGURE 4A.12 (*Continued*)

Example 4A.11 Solve Example 4A. 10 using the moment interaction diagrams.

Solution

$$\gamma_h = 20 - 2.5 = 15 \text{ in (381 mm)}$$

$$\gamma = \frac{\gamma_h}{h} = \frac{15}{20} = 0.75$$

$$\rho_{\text{total}} = \frac{2 \times 2.4}{20 \times 12} = 0.02$$

$$\frac{e}{h} = \frac{8}{20} = 0.4$$

From load-moment interaction diagrams,

$$\frac{\phi P_n}{A_g} = 1.35$$

Hence

$$P_n = \frac{1.35 \times 20 \times 12}{0.7} = 463{,}000 \text{ lb (2060.3 kN)}$$

4A.5.5 Slender Columns (Buckling Effect)

The strength of slender columns is affected by the secondary stresses due to the slenderness effect. The slenderness is usually expressed in terms of the slenderness ratio KL_u/r, where K depends on the column's end conditions, L_u is the column length, and r is the radius of gyration (= I/A). The lower limits for neglecting the slenderness effect are

$$\text{Braced frames} \qquad \frac{KL_u}{r} < 34 - 12\frac{M_1}{M_2} \tag{4A.76}$$

$$\text{Unbraced frames} \qquad \frac{KL_u}{r} < 22 \tag{4A.77}$$

where M_1 and M_2 are the smaller and larger moments at the opposite ends of the compression member, respectively. This ratio is positive if the column is bent in single curvature, negative if it is bent in double curvature.

Two methods can be used to analyze slender columns.

1. The moment magnification method, where the member is designed for a magnified moment δM, with $\delta \leq 1.0$ and M the nominal moment based on an analysis neglecting the slenderness effect. This method is presented in the ACI code and is applicable to compression members with slenderness ratios lower than 100.
2. The second-order analysis takes into consideration the effect of deflection, change in stiffness, sustained load effects, and stability. This type of analysis is usually done with the aid of computers and is only required by the code for slenderness ratios greater than 100. It should he noted that the majority of reinforced concrete columns does not require such an analysis because in most cases the slenderness ratio is below 100.

In the moment magnification method, the magnified moment M_c can be determined using the following equation:

$$M_c = \delta_b M_{2b} + \delta_s M_{2s} \tag{4A.78}$$

where the subscripts b and s refer to the moments due to gravity loads and lateral loads, respectively. Also,

$$\delta_b = \frac{C_m}{1 - P_u/\phi P_c} \geq 1.0 \tag{4A.79}$$

$$\delta_s = \frac{1}{1 - \Sigma P_u/\phi \Sigma P_c} \geq 1.0 \tag{4A.80}$$

$$P_c = \frac{\pi^2 EI}{(KL_u)^2} \tag{4A.81}$$

and $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$ for columns braced against side sway and not exposed to transverse loads between supports. For all other cases $C_m = 1.0$. EI in Eq. (4A.81) must account for the effects of cracking, creep, and nonlinearity of concrete. The ACI code provides the following two equations. For a heavily reinforced member,

$$EI = \frac{E_c I_g/5 + E_s I_s}{1 + \beta_d} \tag{4A.82}$$

and for a lightly reinforced member,

$$EI = \frac{E_c I_g/2.5}{1 + \beta_d} \tag{4A.83}$$

where $\beta_d = \dfrac{\text{design dead-load moment}}{\text{design total moment}}$

In practical situations, columns have end conditions that are partially restricted by adjoining members. Therefore the factor K will vary with the ratio of column stiffness to flexure member stiffness, which provides restraint at the column ends. The value of K can be determined from charts given in Fig. 4A.13. The Ad code recommends the use of $0.5I_g$ for flexural members and I_g for compression members to compute the stiffness parameter ψ in 14g. 4A.13.

4A.5.6 Biaxial Bending

Many reinforced concrete columns are subjected to biaxial bending, or bending about both axes. Corner columns in buildings where beams and girders frame into the columns from both directions

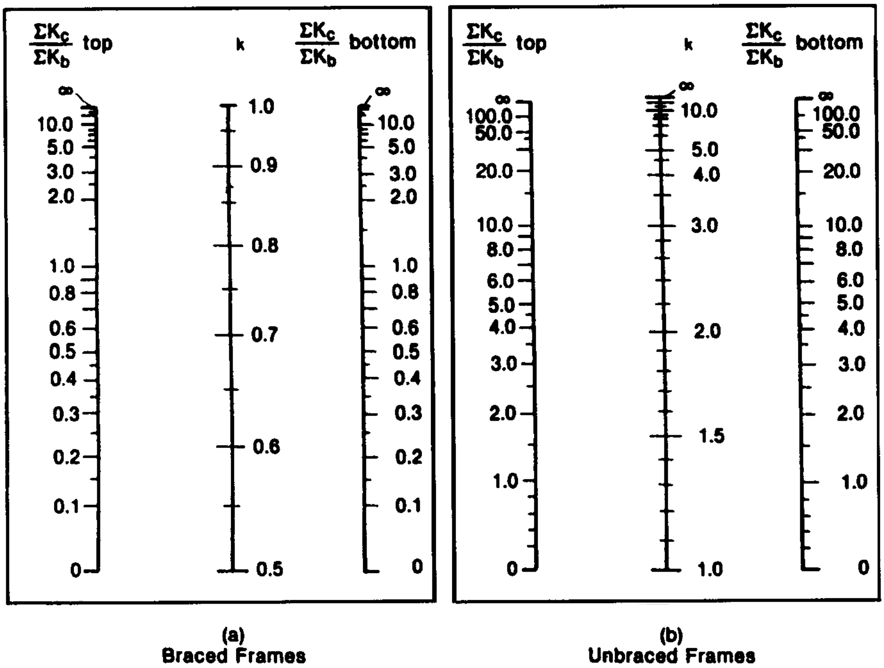


FIGURE 4A.13 Column stiffness factors iii. (a) Braced frames. (b) Unbraced frames.

are typical examples of biaxial bending. An approximate procedure developed by Bresler[5] has been found to provide satisfactory results. For a given column section, the nominal axial capacity under biaxial bending P_n can be computed using the following equation:

$$\frac{1}{P_n} = \frac{1}{P_{n_x}} + \frac{1}{P_{ny}} - \frac{1}{P_{n0}} \tag{4A.84}$$

where P_{n_x} = axial load capacity when load is placed at eccentricity e_x with $e_y = 0$
P_{ny} = axial load capacity when load is placed at eccentricity e_y with $e_x = 0$
P_{n0} = capacity for axially loaded case

Bresler' s equation produces reliable results if the axial load P_n is larger than 0.1 P_{n0}. For lower P_n values it is satisfactory to neglect the axial force and design the section as a member subjected to biaxial bending.

4A.5.7 ACI Code Requirements

The ACT code requires the following limitations on dimensions, reinforcing steel, and lateral restraint:

1. The percentage of longitudinal reinforcement should not be less than 1% nor greater than 8% of the gross cross-sectional area of the column. Usually for practical considerations the percentage of reinforcement does not exceed 4%.
2. Ties provided shall not be less than no. 3 for longitudinal bars no. 10 or smaller. They shall not be less than no. 4 for longitudinal bars larger than no. 10 and for bar bundles.
3. Tie spacing is restricted to the least of the following three values:
   a. Least lateral column dimension
   b. 16 times the diameter of the longitudinal bars
   c. 48 times the diameter of the tie
4. The spiral reinforcement size is determined from

$$\rho_s = \frac{\text{volume of spiral in one loop}}{\text{volume of concrete core for pitch}} = \frac{a_s \pi (D_c - d_b)}{\frac{1}{4}\pi \Delta_s^2 s}$$

where a_s = cross section of spiral bar
D_c = diameter of concrete core
d_b = diameter of spiral

$$(\rho_s)_{\min} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y}$$

where A_g = cross-sectional area
A_c = core cross section

4A.6 DEVELOPMENT OF REINFORCING BARS

4A.6.1 General

Reinforced concrete is a composite material where the compressive stresses are resisted by concrete and the tensile stresses are resisted by steel reinforcement. For this mechanism to work properly, a bond or a force transfer must exist between the two materials. The bond strength is controlled by several factors: (1) adhesion between concrete and steel reinforcement, (2) frictional resistance between steel and surrounding concrete, (3) shear interlock between bar deformations and concrete, (4) concrete strength in tension and compression, and (5) the geometrical characteristics of the steel bar—diameter, deformation spacing, and deformation height. The actual bond stress along the length of a steel bar embedded in concrete and subjected to tension varies with its location and the crack pattern. For this reason the ACI code uses the concept of development length rather than bond stress to ensure an adequate anchorage. Development length is defined as the minimum length of a bar in which the bar stress can increase from zero to the yield stress f_y. Hence a shorter embedded length will result in the bar pulling out of the concrete. The ACI code specifies different development length for steel bars in tension and in compression.

4A.6.2 Tension Development Length

A basic development length I_{db} is determined according to the following ACI code equations. Its value, however, should not be less than 12 in (0.3 in).

- No. 11 bars and smaller and deformed wire,

$$I_{db} = \frac{0.04A_b f_y}{\sqrt{f'_c}} \tag{4A.85}$$

- No. 14 bars,

$$I_{db} = \frac{0.085f_y}{\sqrt{f'_c}} \tag{4A.86}$$

- No. 18 bars,

$$I_{db} = \frac{0.125f_y}{\sqrt{f'_c}} \tag{4A.87}$$

where A_b is the cross-sectional area of the bar, and $\sqrt{f'_c}$ shall not be taken greater than 100. Values of I_{db} are given in Table 4A.3.[6]

The basic development length is multiplied by a series of multipliers given in the ACI code, secs. 12.2.3, 12.2.4, and 12.2.5, to obtain the necessary development length I_d. These multipliers account for bar spacing, amount of cover, transverse reinforcement, reinforcement location, concrete unit weight, coating of steel reinforcement, and amount of reinforcement.

For development length multipliers

1. Compute $I_d = \lambda_d I_{db}$, where λ_d is from *a, b,* or *c:*
   a. $\lambda_d = 1$ if (1) clear spacing $s = 3d_b$ or more and stirrups are used with minimum ACI code cover requirements, or (2) bars in inner layer of slab or wall with clear spacing $s \geq 3d_b$ or (3)bars with cover $\geq 2d_b$ and clear spacing $\geq 3d_b$.
   b. λ_d 2 for bars with cover $<d_b$ or clear spacing $<2d_b$.
   c. $\lambda_d = 1.4$ for bars not covered by *a* or *b*.
2. Reduce the multiplier λ_d by multiplying it by spacing
   a. 0.8 for no. 11 bars and smaller if clear spacing $\geq 5d_b$ and cover $\geq 2.5d_b$ and
   b. 0.75 for reinforcement enclosed within spiral reinforcement of diameter ≥¼ in and pitch ≤4 in, or ties of no. 4 or more with spacing ≤4 in.

TABLE 4A.3 Basic Tension Development Length*

Bar no.	f'_c = 3000 psi (20.7 MPa) Bottom bar	Top bar	Lower limit†	f'_c = 3750 psi (25.9 MPa) Bottom bar	Top bar	Lower limit†	f'_c = 4000 psi (27.6 MPa) Bottom bar	Top bar	Lower limit†	f'_c = 5000 psi (34.5 MPa) Bottom bar	Top bar	Lower limit†	f'_c = 6000 psi (41.4 MPa) Bottom bar	Top bar	Lower limit†
f'_y = 60,000 psi, normal = weight concrete															
3	4.8	6.3	12.3	4.3	5.6	11	4.2	5.4	11	3.7	4.9	9.5	3.4	4.4	8.7
4	8.8	11.4	16.4	7.8	10.2	15	7.6	9.9	14	6.8	8.8	13	6.2	8.1	12
5	14	18	21	12	16	18	12	15.5	18	10.5	14	16	9.6	12.5	15
6	19	25	25	17	22	22	18	23	21	15	20	19	14	18	18
7	26	34	29	24	31	26	23	30	25	20	27	22	19	24	20
8	35	45	33	31	40	29	30	39	29	27	35	25	25	32	23
9	44	57	37	39	51	33	38	49	32	34	44	29	31	40	26
10	S6	72	42	50	65	37	48	62	36	43	56	32	39	51	30
11	68	89	46	61	80	46	59	77	44	53	69	36	48	63	33
14	93	121	S6	83	108	50	81	105	48	72	94	43	66	86	39
18	137	178	74	122	159	66	119	154	64	106	138	S7	97	126	52
f_y = 40,000 psi, normal-weight concrete															
3	3.2	4.2	8.2	2.9	3.7	7.4	2.8	3.6	7.1	2.S	3.2	6.4	2.3	3	5.8
4	5.8	7.6	11	5.2	6.8	9.8	5.1	6.6	9.5	4.5	55.9	8.5	4.1	5.4	7.7
5	9	12	14	8.1	10.5	12	7.8	10.2	12	7.0	9.1	10.6	6.4	8.3	9.7
6	13	17	17	11.5	15	15	11.1	14.S	14.5	10	13	13	9.1	11.8	12

*$I_d = I_{db}$ × factors in ACI Sec. 12.2.3 but not less than lower limit × factors in Secs. 12.2.4 and 12.2.5 but not less than 12 in (0.3 m).
Source: From MacGregor.[6]

3. The resulting development length as modified in 1 and 2 should not be taken less than $0.03 d_b f_y/\sqrt{f'_c}$, with the value of $\sqrt{f'_c}$ not to exceed 100.
4. The following additional multipliers λ_{dd} are applied for special conditions to obtain the development length $I_d = \lambda_d \lambda_{dd} I_{db}$:
   a. $\lambda_{dd} = 1.3$ for top reinforcement, that is, horizontal reinforcement with more than 12 in of concrete cast below the bars.
   b. $\lambda_{dd} = 1.3$ for lightweight concrete. When f_{ct} is specified, use $6.7\sqrt{f'_c}/f_{ct}$, where f_{ct} is the splitting tensile strength of concrete.
   c. For epoxy-coated reinforcement (1) when cover $<3d_b$ or clear spacing between bars $<6d_b$, use $\lambda_{dd} = 1.5$; (2) for other conditions use $\lambda_{dd} = 1.2$.
   d. Multiply by the excess reinforcement ratio,

$$\lambda_{dd} = \frac{A_s(\text{required})}{A_s(\text{provided})}$$

4A.6.3 Compression Development Length

The basic development length I_{db} is computed according to the Ad code from

$$I_{db} = \frac{0.02 d_b f_y}{\sqrt{f'_c}} \geq 0.0003 d_b f_y \tag{4A.85}$$

Then the following multipliers are applied to obtain $I_d = \lambda_d I_{db}$:

1. For excess reinforcement, $\lambda_d = A_s(\text{required})/A_s(\text{provided})$.
2. For spirally enclosed reinforcement, $\lambda_d = 0.75$.

The minimum total development length should be greater than 8 in (200 in).

4A.7 REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete," ACI 318-89; also "Commentary on Building Code Requirements for Reinforced Concrete," American Concrete Institute, Detroit, Mich., 1989.
2. J. C. McCormac, *Design of Reinforced Concrete*, 3d ed., HarperCollins, New York, 1993.
3. E. G. Nawy, *Reinforced Concrete: a Fundamental Approach*, 2d ed., Prentice-Hall, Englewood Cliffs N.J., 1990.
4. C. K. Wang and C. G. Salmon, *Reinforced Concrete Design*, 5th ed., HarperCollins, New York, 1992.
5. B. Bresler, "Design Criteria for Reinforced Concrete Columns under Axial Load and Biaxial Bending," *Journal of the Americon Concrete Institute*, vol. 57, pp 481–490, November 1960.
6. J. G. MacGregor, *Reinforced Concrete: Mechanics and Design*, 2d ed., Prentice-Hall, Englewood Cliffs, N.J., 1992.