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Vertical Sand Drains

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Vertical Sand Drains

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Contents

Checking up on Vertical Sand Drains

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● VERTICAL sand drains, or sand piles as they are sometimes called, have been more and more widely used in recent years. They represent an attempt to augment the consolidation of unstable saturated soils by providing vertical drainage outlets for the water presumed to be squeezed from the soil by the weight of the surcharge or fill. The objective of consolidation by such artificial means is to develop increased soil resistance and support for superimposed loads, usually consisting of earth fills in highway or airport construction.

Whether or not sand drains are actually effective and do produce the results represented by the theory has been a debated question. The writer has always been skeptical of the theory of consolidation in principle and little impressed with its attempted application in practice. The consolidation theory conceives that settlement is caused by squeezing water out of the voids of a saturated soil under the applied pressure. This theory postulates that the movement of moisture is caused by pore-water pressure or a differential hydrostatic pressure as distinguished from the pressure components acting on the soil mass as a whole which results in shearing stress. It ignores the fact that adsorbed moisture in fine-grained soils frequently exceeds the solids in volume and that the same molecular forces which must be overcome in order to force out the water are the source of cohesion or shearing resistance of the soil. Under the situation most representative of saturated soils in which the solid particles are dispersed in a semifluid medium, it has always been difficult for the writer to understand how the water could be forced out without carrying the solid particles with it. In other words, in such soil masses as encountered in practice without artificial confinement, it would seem that displacement or shearing failure would take precedence over consolidation.

The consolidation theory as formulated cannot be applied to unsaturated soils in which settlement, as observed in practice, differs only in magnitude from that taking place in saturated soils. Some of the uncertainty involved in its application to saturated clays are discussed by its foremost proponents who recognize some of its shortcomings (1). In discussing an apparent lag in the reaction of clay to applied loads characteristic of plastic solids, it is stated in the reference that:

> These delays in the reaction of clay to a change in stress, like the secondary time effect and the influence on c_v (coefficient of consolidation) of the magnitude of the load increment, cannot be explained by means of the simple mechanical concept on which the theory of consolidation is based. Their characteristics and conditions for occurrence can be investigated only by observation.

Its inaccuracy in application to practical conditions is likewise recognized as in the following statement:

> It is obvious that the results of a settlement computation are not even approximately correct unless the assumed hydraulic boundary conditions are in accordance with the drainage conditions in the field. Every continuous sand or silt seam located within a bed of clay acts like a drainage layer and accelerates the consolidation of the clay, whereas lenses of sand and silt have no effect. If the test boring records indicate that a bed of clay contains partings of sand and silt, the engineer is commonly unable to find out whether or not these partings are continuous. In such instances the theory of consolidation can be used only for determining an upper and lower limiting value for the rate of settlement. The real rate remains unknown until it is observed.

The extension and elaboration of this uncertain theoretical concept to the design of a roadway fill or airport runway involving sand drains includes the calculation of several definite quantitative components of the completed facility. These include the spacing of sand drains, the height of surcharge required to produce a required degree of improvement, a specified consolidation period, and finally, a completed fill with a definite safe load. To the uninitiated client or to the contractor who bids on the job, the precise evaluation of these items in plans and specifications as quantitative requirements or objectives to be accomplished not only implies but gives assurance that the whole subject of sand drains has been established beyond any reasonable doubt. Critical examination of such evidence as has been presented indicates that application of the principles involved has scarcely progressed beyond an experimental stage and that the claims of successful application are seldom supported by more than fragmentary and inconclusive evidence.

Occasionally one hears of failures in connection with sand-drain installations, and if these failures are exhaustively investigated, they are seldom, if ever, so reported for public consumption. Accounts of supposedly successful installations which are more numerous usually presume that the lack of complete subsidence or failure has been accomplished by the sand drains. Generally, these claims are not supported by a factual stability analysis to demonstrate that the original bearing capacity was clearly deficient or that there was a definite improvement due to consolidation. Settlement is classified as consolidation settlement, but no clear demonstration is given as to how much was displacement and how much consolidation. The writer has yet to see in a published report a comprehensive set of reliable factual data on moisture content, soil density, or shearing resistance before and after consolidation from an actual full-scale sand-drain installation.

In the few instances in which the writer has had the opportunity to investigate the effectiveness of sand drains on an actual construction project, the results have been definitely negative. Perhaps these were unfortunate coincidences, but this experience only reinforced some rather strong convictions expressed in this discussion with regard to the theoretical background of consolidation. It is possible that there are materials and stratified soil deposits in which vertical sand drains are of material benefit. The writer has not encountered such conditions and suspects that if they do exist, they are the exception rather than the rule.

Having a deep interest in the practical value of sand drains, the opportunity to conduct a comprehensive investigation of the results produced by an installation of sand drains on a major project came as a distinct opportunity. The project with which this investigation was concerned was the extension of a runway into a marshy area at the Norfolk Naval Air Station. The investigation was made for the Contractors, Ralph E. Mills Company of Salem, Virginia, and Blythe Brothers Company of Charlotte, North Carolina, and was made with the permission of the representatives of the Bureau of Yards and Docks of the U. S. Navy. The objective of the investigation was to determine the reason for a number of difficulties which had been encountered during the construction. This determination boiled down to whether or not these difficulties were the result of improper construction methods or were inherent in the design as presented by the

Figure 1. Fill settlement and slide planes at north side runway Stations 45-47.

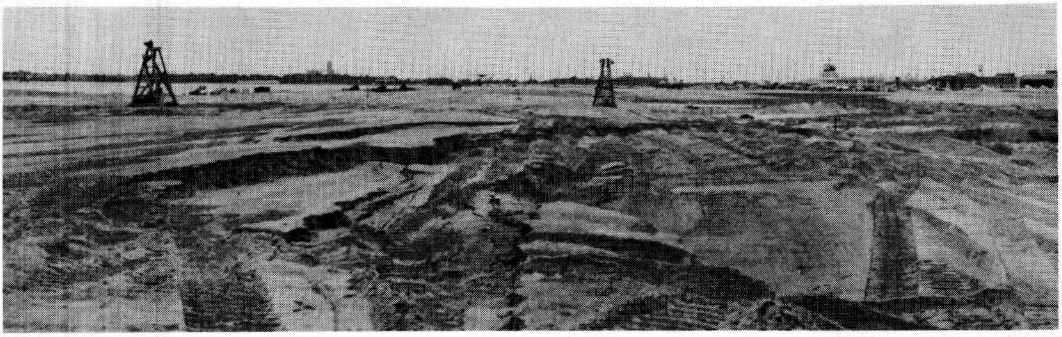

Figure 2. Fill settlement and slide planes at north side runway Stations 45-47.

plans and specifications.

The difficulties referred to which presented some serious problems to both the Navy and the contractor may be described briefly as follows:

First, due to slides and displacement of the softer strata near the surface, the amount of fill material being required was mounting up to an excessive overrun involving increased cost beyond the funds which had been provided for this purpose.

Secondly, the slides and displacement destroyed the settlement platforms which had been placed for the purpose of determining the final volume of fill and made it necessary to establish some other method of determining the final pay quantity. In other areas, the displacement of the soft materials took place as fast as the sand fill was placed, and it had been impossible to establish a working platform from which the settlement platforms could be placed.

Third, in other, more-stable areas, the contractor had been able to place the working platform, drive the sand drains, and place part and sometimes all of the surcharge called for by the plans and specifications. However, in several of these areas failures developed producing the most-extensive slides and mud waves that were encountered in the project. It should be noted in connection with this problem that the plans called for the full surcharge to be placed immediately after the sand drains

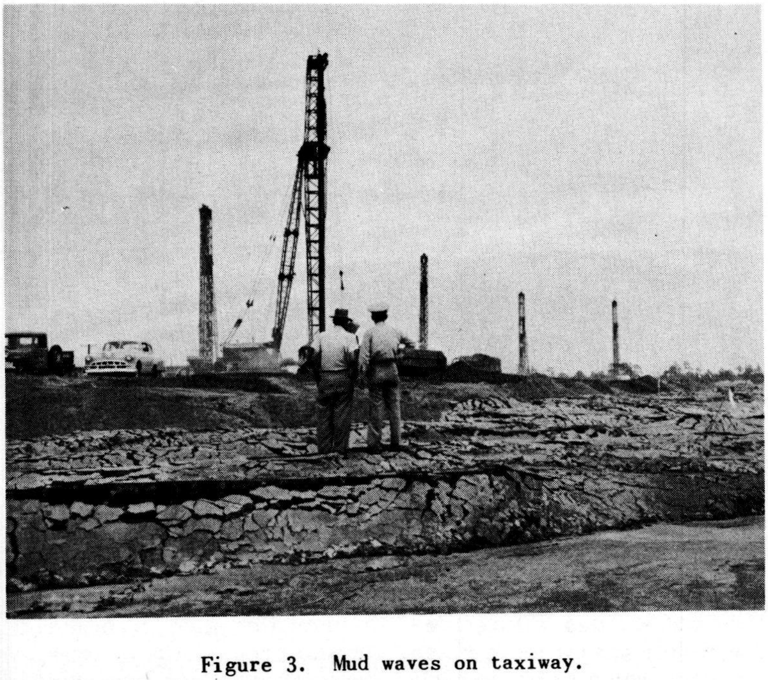

Figure 3. Mud waves on taxiway.

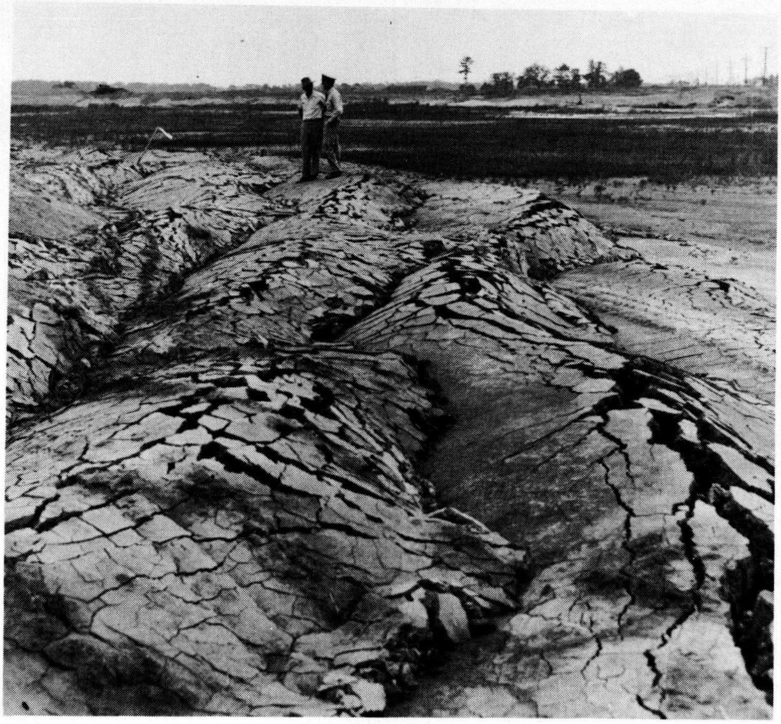

Figure 4. Mud waves on taxiway.

LOCATION PLAN

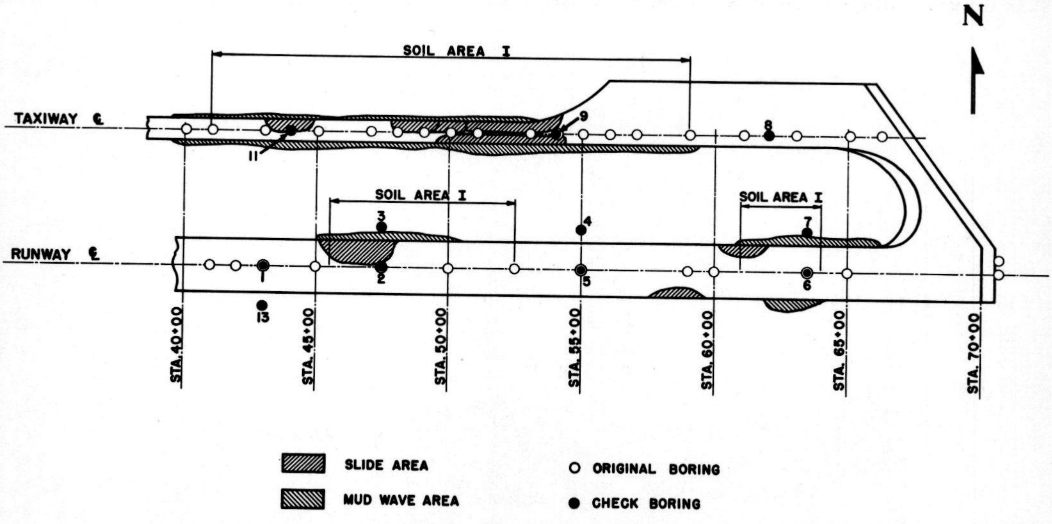

Figure 5.

had been completed and provided for a 90-day consolidation period. In such areas where it had been impossible to place the surcharge fill without serious failures developing, the contractor was forced to place the fill in small lifts which were then left in place until it was presumed that the consolidation had been completed, after which the rest of the surcharge was placed.

Fourth, aside from the delays and increased cost which somebody would have to bear, the continued excessive displacement raised a serious question as to whether there was any consolidation being accomplished by the surcharge fill and whether a useable runway could ever be produced under the procedure called for by the plans and specifications.

Fifth, in certain areas after a consolidation period of from 2 to 5 months, it was presumed that the objectives of the design had been accomplished and the contractor was instructed to remove the surcharge and spread it in flat slopes on either side of the runway and taxiway. However, after the surcharge had been removed, it was found that the settlement continued in the runway at such a sufficiently high rate that there was some question as to whether the contractor would be able to maintain final grade for the paving operation within the tolerances laid down by the specifications.

Sixth, during the entire operation the contractor was required to remove the mud waves which had piled up on either side of the runway and taxiway and which were furnishing a counterweight tending to stabilize the fill. This removal was being required in order to provide temporary drainage to carry away the water which was supposed to be coming out of the porous blanket on top of the sand piles. This removal was also required in the final operation when the sand surcharge was to be removed and spread on either side of the finished runway and taxiway to form the final cross section. It had been the contractor's experience that the continuous removal of this mud wave in order to maintain temporary ditches continued to promote the excessive fill settlement and displacement. The question which this raised was whether or not this counterweight could be removed at any time without setting off a new series of slides.

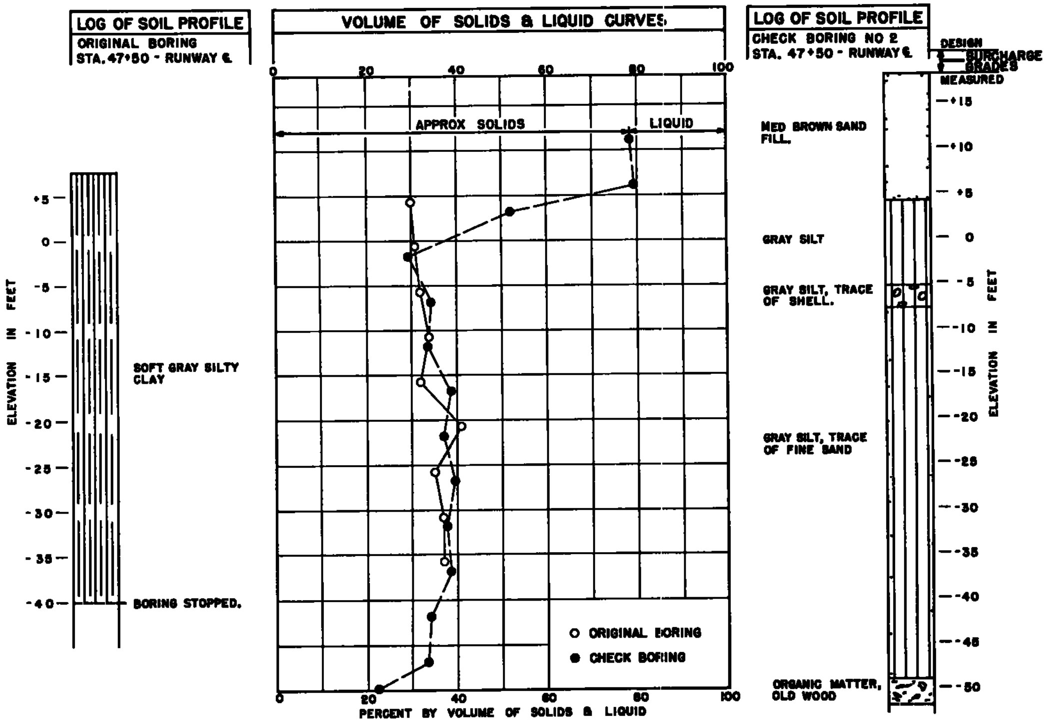

Figure 6.

To give some idea of the character and magnitude of the fill settlement and mud waves, there are presented the first four figures, photographs taken in the field in the early part of September 1951. Figure 1 and Figure 2 show the failure areas which were first encountered toward the west end of the runway. Figure 3 and Figure 4 show very clearly the mud waves caused by displacement of the soft clay during the placing of the working platform and the driving of the sand piles along the taxiway.

Obviously the most-essential part of an investigation of the enumerated problems was a comprehensive soil investigation,

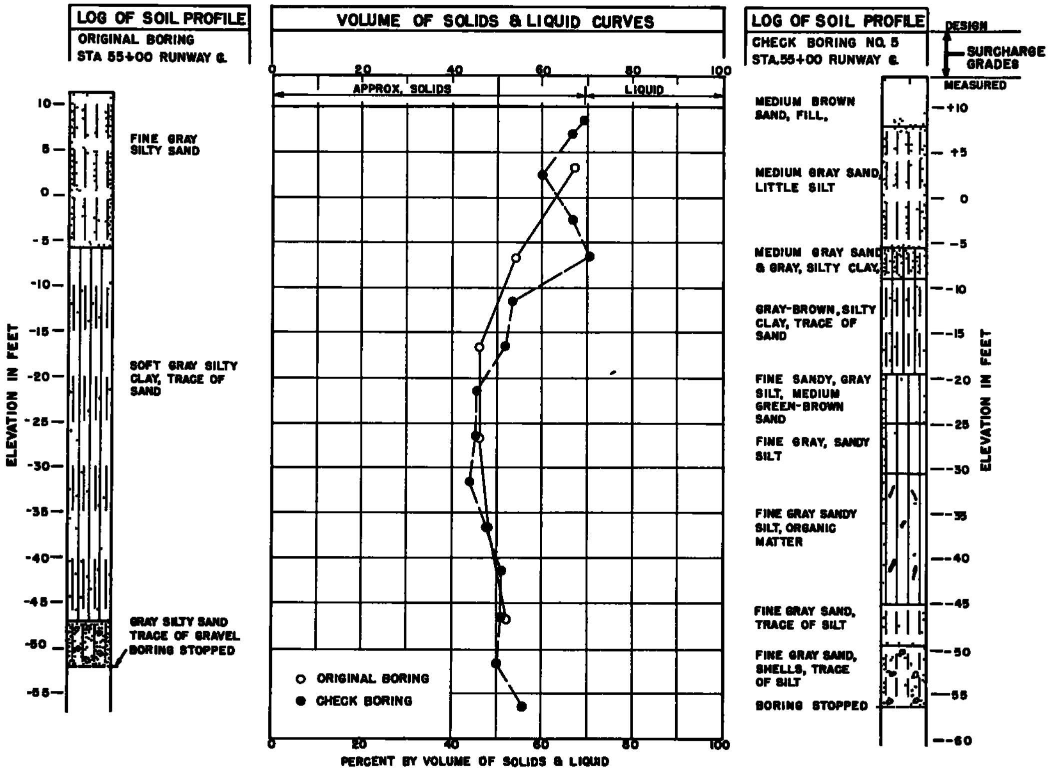

Figure 7.

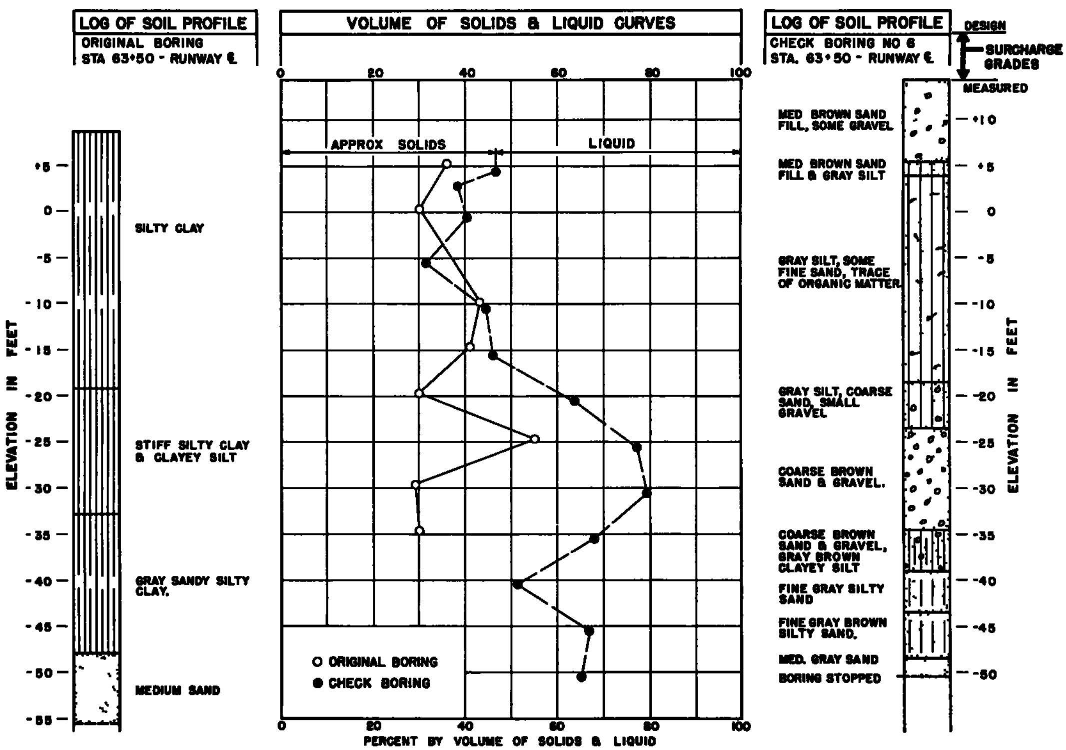

Figure 8.

including borings and all of the laboratory tests necessary to fully describe existing soil conditions. The original boring logs, which served as a basis for preparation of the plans and specifications, were available to the contractor being incorporated in the plans. Available data included moisture content in percent by dry weight, the dry weight per cubic foot in a general description of the soil. The predominant material was described as a gray silty clay, but there were several areas with sand or sandy soil both near the surface and at greater depths.

As a part of the contractor's investigation, there was a total of 45 check borings made, 11 of which were deep borings with undisturbed samples being taken for testing at the Soil Mechanics Laboratory at the University of Michigan. The remaining 34 borings were made primarily to determine the depth of penetration of the sand fill. The borings and undisturbed sampling were done by the Raymond Concrete Pile Company under the Michigan specifications requiring core samples 1⅞ inches in diameter and approximately 7 inches long. A complete description of the sampling procedure is available elsewhere and will not be repeated in this discussion (2).

Figure 5 is a location plan which shows the general layout of the runway and taxiway extension and the location of borings and critical soil areas, which will be discussed further. The open circles along the centerline of the runway and taxiway are used to designate the original borings made prior to preparation of the plans and specifications. The solid circles designate the 11 check borings made as part of the contractor's investigation from which undisturbed samples were obtained. Borings 1, 2, 5, and 6, along the centerline of the runway, and Borings 8, 9, and 11, along the centerline of the taxiway, were taken in areas where the surcharge fill of varying height had been in place for consolidation periods varying from 2 to 5 months. Undisturbed samples of the saturated clay in these areas are presumed to represent the condition of the soil after it has been subjected to the prescribed consolidation periods. Borings 3, 4, and 7, in between the taxiway and runway in the open marsh area, were taken for the purpose of obtaining representative samples of the original soil in its unconsolidated state. Boring 13 was taken outside of the runway area to compare with Boring 1 at a point where the soil conditions varied substantially from those over the greater portion of the runway and taxiway extension.

Selection of the location of the check borings was predicated on an analysis of the soil conditions as portrayed by the original borings. From these original data, areas which were presumed to represent the more-critical soil conditions were selected and have been designated on Figure 5 as soil area 1. In this area, the soil test data from the original borings indicated that there was gray silty clay of high moisture content from the surface of the ground to depths varying from 50 to 60 feet. This soil was originally described as very soft to soft silty clay with moisture contents varying from 65 to 95 percent by dry weight and 60 to 80 percent by volume. Tests performed as a part of the contractor's soil investigation indicated that the predominant soil was a silty clay containing 30 to 35 percent clay and colloids, 30 to 50 percent silt and 20 to 30 percent sand. The plastic limit of this material varied from 20 to 30, the liquid limit from 50 to 60, with an average plasticity index of approximately 30. In general, the natural moisture content of this material varied from 10 to 20 percent above the liquid limit, indicating that the soil should be classified as a flocculated clay.

In the vicinity of Borings 1 and 13, sand, substantially granular in character, was found from the surface to a depth of 20 to 25 feet, followed by a relatively thin body of the very soft silty clay approximately 15 feet in depth, underlain again by a body of sand which continued for the full depth of the boring. In the vicinity of Boring 5, similar conditions existed, except that the depth of the soft silty clay between the surface sand and underlying sand was approximately 35 feet, somewhat more critical than in the case of the other sand area but presumably more stable than the critical areas designated as Area 1. Practically the entire length of the taxiway has been designated as a critical soil area with the soft silty clay extending from the surface to a depth of 60 to 70 feet. Near the east end of the widened taxiway in the vicinity of Boring 8, the same classification of soil is found in the original borings although the natural

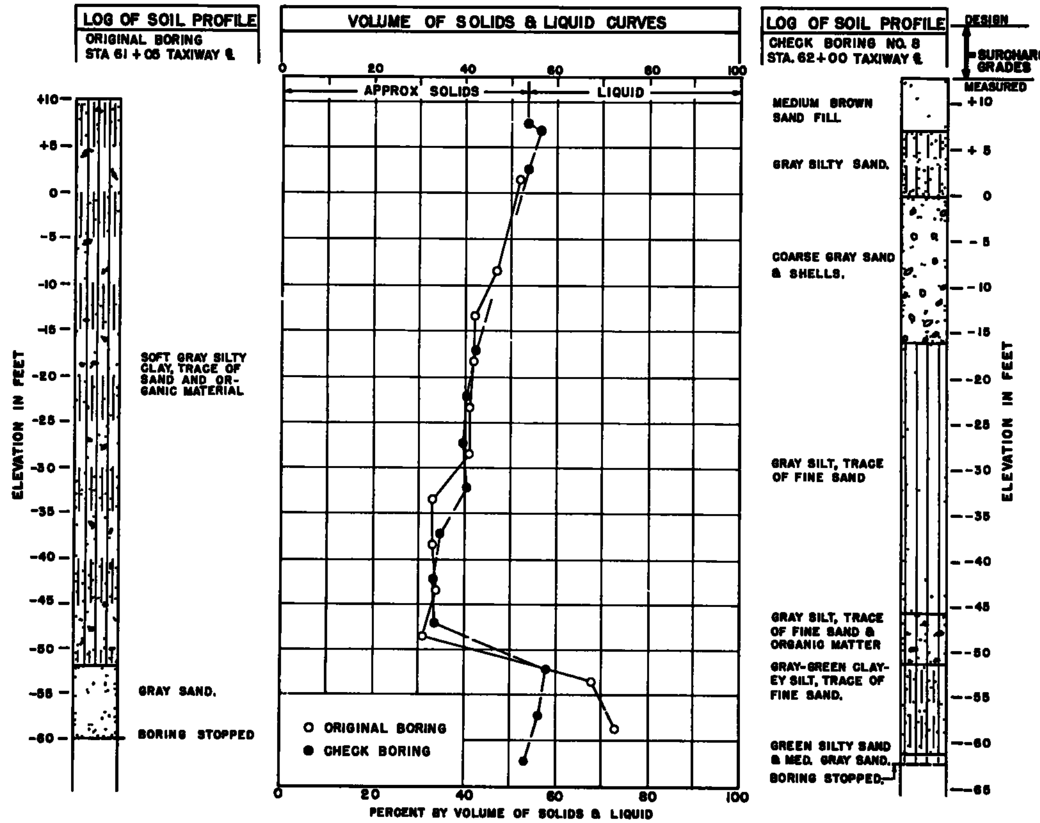

Figure 9.

moisture content is substantially less than in the highly saturated clay indicating a more stable soil.

There is also shown (Fig. 5) the areas in which the major slides and displacements were encountered. In general, these areas agree with the classification of critical soil areas based upon the original soil test data and classification. This includes a major slide area in the vicinity of Boring 2, another similar area in the vicinity of Boring 6 and a continuous area of subsidence and displacement along the major portion of the taxiway from near Boring 9 to past Boring 11.

As has been previously indicated in this discussion, the primary objective of the contractor's investigation was to obtain comprehensive, factual data on the moisture content and shearing resistance of the saturated clay soil in the critical soil areas before and after consolidation. This information was required in the first place to determine whether or not the unsatisfactory performance was inherent in the existing soil conditions and the procedures called for by the plans and specifications and whether it was physically possible for the contractor to accomplish the objective set forth in these plans and specifications. Having accepted a contract to produce a completed facility planned by others, a contractor is frequently in the unfortunate position of having to accept the burden of proof when faced with the possible contention that the failure to achieve the planned objective is a matter of construction methods and his legal responsibility. Furthermore, from the standpoint of everybody concerned, the situation had reached a stage in which it was highly essential to establish whether it was physically possible to provide a completed runway which could be safely used by the Navy over a reasonable period of years.

The results of the tests to determine the degree of consolidation which had been produced by the sand piles and the surcharge fill which had been in place over periods varying from 2 to 5 months are shown on a series of graphs presented by Figures 6 to 12 inclusive. Figures 6 through 11 present the moisture content before and after consolidation for individual borings. In Figure 6, the boring log from the original boring is shown on the left-hand side of the figure and the log of the check boring is shown on the right-hand side. At the top of the right-hand boring log is shown the surcharge grade, both as designed and as measured at the time the boring was made. The thickness and bottom elevation of the sand fill on the check boring can be compared with the ground-surface elevation on the original boring to indicate the depth to which the sand fill has settled. In Figure 6, the indicated settlement is approximately 4 feet.

The primary purpose of Figure 6 through 11 is to present the moisture content measurements in the central chart. Moisture contents and solids are shown in percent of the total volume with the moisture or liquid volume being shown on the right as the complementary percentage of the volume of solids which is shown on the left-hand side of the chart. Results from the original test data are shown by the open circle, the results from the check boring by the solid circle. At the location of Boring 2 in Figure 6 in the original soil below the sand fill, moisture contents vary from 60 to 70 percent with the volume of solids varying from 30 to 40 percent. The moisture content in the sand fill is approximately 20 percent, corresponding to the percentage of voids in such a granular material when compacted. Particular attention is directed to the fact that there is no measurable change in moisture content for the entire depth of the original soft silty clay under discussion.

It may be noted, however, that the textural classification or description of this soil does vary from the original description of a soft, gray, silty clay to the textural classification of gray silt shown in the check boring. In the latter case, the descriptions of soil type are from visual examination by the boring crew superintendent. It is the practice in the Michigan laboratory not to alter the soil

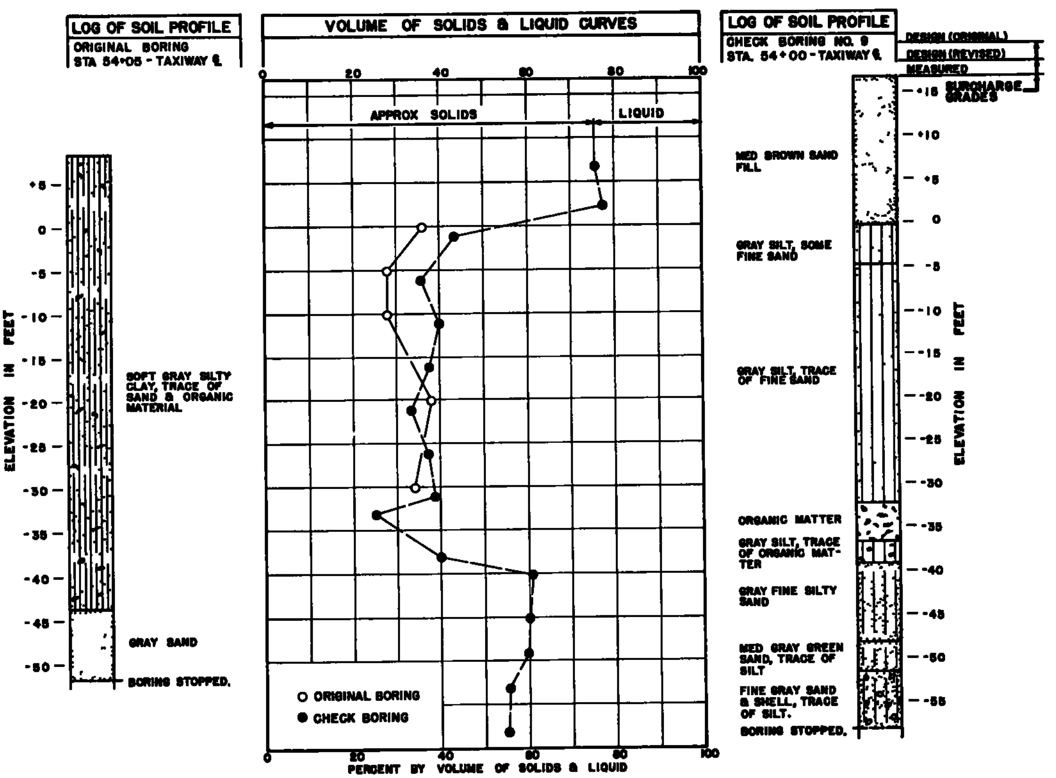

Figure 10.

description given by the boring contractor but to make a separate classification based on the mechanical analysis where such classification is required. As previously noted, the laboratory classification of the predominant saturated soil was a silty clay in substantial agreement with the original classification. All through the apparent decrease in moisture content after the period of consolidation in the region from Elevation 0 to Elevation -10. However, this apparent decrease in moisture content is clearly related to the textural differences which are shown due to the subsidence of the surface layer of sand under the surcharge fill.

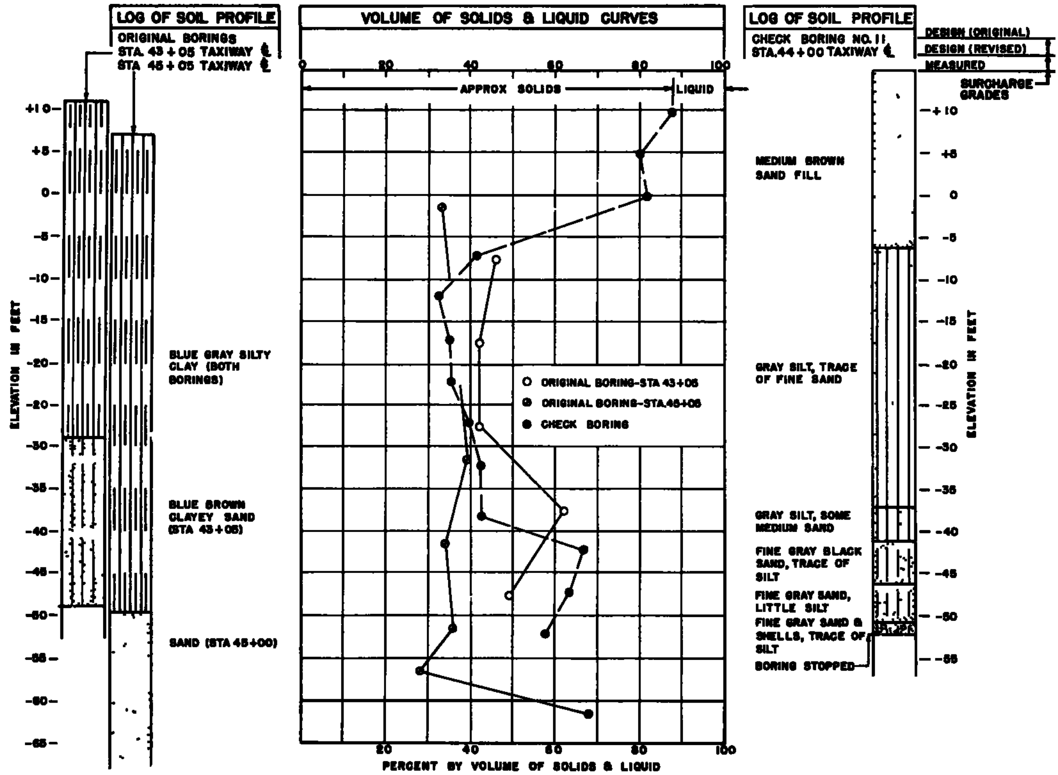

Figure 11.

comparative tests in the six figures under discussion, similar textural variation will be in evidence and where these variations are of any particular significance, special comment will be made.

Figure 7 is a similar chart for results from Boring 5. This area has not been classified as soil Area 1 although as previously noted the thickness of soft silty clay beneath the surface stratum of sand is sufficient to raise some question as to the ultimate stability of this area. It may be noted that the height of surcharge as planned and measured is much less than in the previous case and the amount of settlement is somewhat less being only a little over 3 feet. Again there is no measurable change in moisture content in the silty clay at greater depth but there is an

Boring 6 shown in Figure 8 is in another critical area where original soil conditions indicated more than 50 feet of unstable soil starting at the surface. There is, in this case, a marked difference in moisture content before and after consolidation, but there is also a marked textural change in the check boring. The reasons for this are quite clear, as the coarse, granular material encountered at the greater depth is obviously that used in the sand piles. Thus, the check boring happened to strike such a pile for part of its depth. At the only depths below the penetration of granular fill where the original soil appeared to be present, moisture contents are quite comparable before and after consolidation.

Boring 8 in Figure 9 is in the middle

portion of the widened taxiway and was designated as less critical based on original soil test data. In the check boring, the textural classification is even more definitely granular near the surface. This particular boring was beyond the general failure area along the taxiway and the check boring indicates a smaller fill settlement. Moisture contents before and after consolidation are remarkably close together for the entire depth of the borings, even though the soil is more

from above by subsidence of the fill. On this boring between Elevation 0 and Elevation -15 is the only evidence of decreased moisture content after consolidation which is not clearly accounted for by a difference in soil texture. However, such a result could very easily be produced by the vertical displacement from above of soil having lower moisture content. Unfortunately, no tests were reported from the original borings above Elevation 0, although the trend toward lower moisture

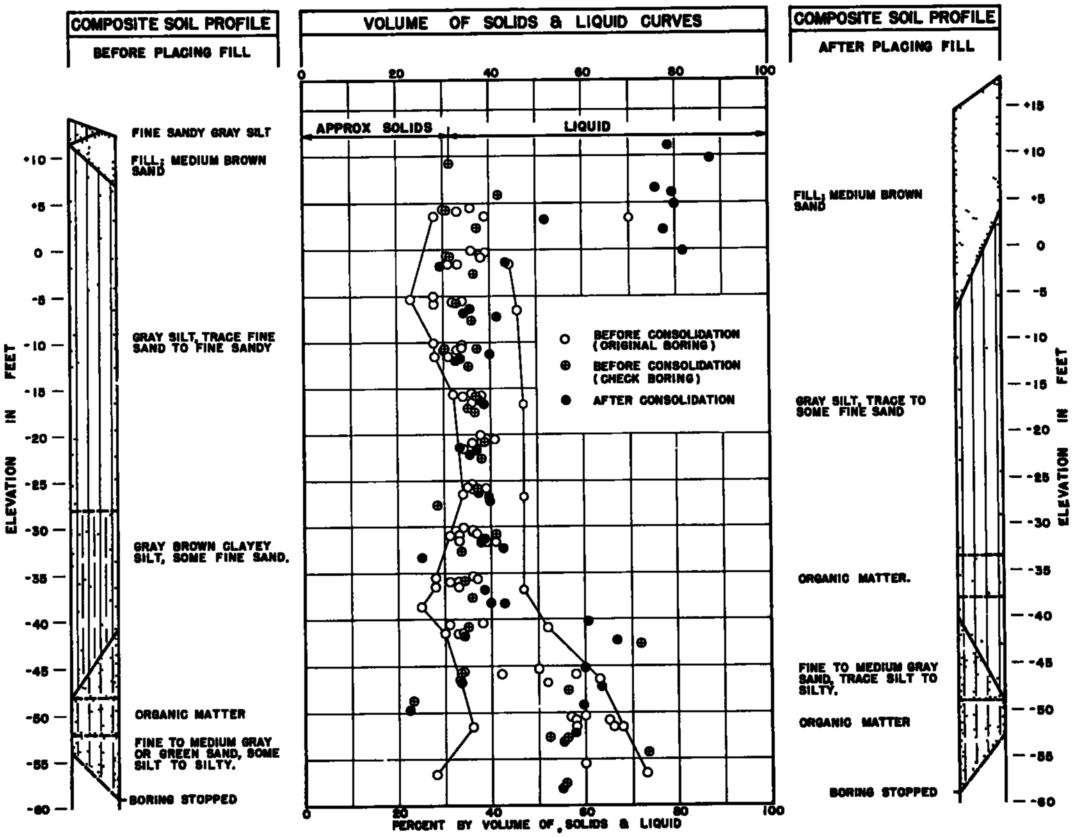

Figure 12.

granular and presumably more permeable.

Boring 9 in Figure 10 is at the point of transition from the wide to the narrow taxiway and is an area designated as critical and where fill failures were general. In such areas, borings made along the centerline were generally in a portion of the soil mass and fill between the slide planes from either side of the taxiway. Thus, they do not represent the deepest penetration of the fill in the actual slides but may be an area in which a mixture of original soil and fill has been carried down

contents near the surface is indicated. Below Elevation -15 for the balance of the depth in the saturated clay, moisture contents are the same before and after consolidation.

Boring 11 in Figure 11 is midway between two of the original borings in one of the more-critical areas along the taxiway. Fill settlement is approximately 15 feet. Below the fill in the depth of soft saturated soil above the underlying sand, moisture contents are in the same range before and after consolidation, although it must be kept in mind that the original borings are somewhat removed from the check boring. Any differences in moisture contents at the greater depths are definitely related to textural changes, with the underlying sand being encountered at varying elevations in the three borings compared.

Figure 12 is a composite of the data obtained from the individual borings in Figure 6 to 11 presented to represent the same comparison of moisture content before and after consolidation on the project as a whole. Moisture contents are shown for the original borings by open circles. Moisture contents from check borings made outside the filled area are shown by a circle with a cross and from check borings made within the filled area after the period of consolidation are designated by a solid circle. An envelope has been drawn indicating the range of moisture content in the original borings. With the exception of the sand fill and in the granular materials underlying the soft, clayey silt, moisture contents after consolidation fall within this envelope with no indication that there has been any significant change in moisture content before and after consolidation.

Even more than moisture content, the shearing resistance of the soil before and after consolidation should provide direct evidence of the degree of consolidation produced by the sand drains and surcharge fill. If shearing resistance tests were conducted in connection with the original borings, these data were not made available to the contractor. Consequently, as previously noted, Borings 3, 4, and 7 were made in the open marsh area between the runway and taxiway. Shearing resistance tests were conducted in the laboratory on samples from borings used to represent the original soil condition and from the borings taken from the filled area to represent shearing resistance after consolidation.

Following the customary practice at the Michigan laboratory, two types of shear test were conducted on all samples: (1) the ring-shear test developed by the University of Michigan Soil Mechanics Laboratory and (2) the unconfined-compression test used by a number of other soil mechanics laboratories.

The ring-shear test is a measure of what may be called the static yield value or shear stress greater than which the soil will suffer progressive deformation. In these tests, observations are made of the rate of shearing deformation for each load increment applied. From this may be determined, by extrapolation, the actual load at which progressive deformation occurs. The final results are thus independent of dynamic resistance and represent that applied stress which may be sustained in static equilibrium.

The shearing resistance, as determined by the unconfined-compression test conducted in accordance with generally accepted procedures, has not been corrected for dynamic effects, so the test may be termed a rapid shear test. The load is applied at a continuous rate until failure is produced and in a much-shorter period of time (5-minute loading period). Over several thousands of parallel tests, it has been found that shear values obtained from these unconfined-compression tests are close to four times those obtained from the ring-shear tests in the case of plastic clays of the truly cohesive type or in the case of saturated clays. It has also been found that for stiff clays and for materials which have granular characteristics the ratio of the shear values obtained from these two tests is quite erratic and, in general, higher than in the case of plastic clays. In presenting the results from the unconfined compression, it has been the practice to plot these shear values to a scale four times greater than is used for the ring-shear test, which brings the divergent results into close focus in most cases. In reporting the shearing resistance from unconfined compression, it has also become common practice to report the equivalent shearing resistance to the reduced scale or a quarter of the measured value in the rapid shear test.

In connection with the shearing resistance test on this soft, silty clay, one other observation should be made. In the borings outside the filled area, the equivalent shearing resistance from the unconfined-compression test runs generally lower than the yield value determined by the ring-shear test, a characteristic which has been found common to flocculated clays of high moisture content that have not been disturbed or remolded. While the writer can offer no well-supported explanation of this phenomenon, it appears to be related to the sensitivity of the clay and the fact that, after remolding, the highly sensitive clay seldom, if ever, regains its original shearing resistance. While this departure from the usual ratio between shear values from these two tests is apparent on most of the samples from those borings, it does not hold for the samples from borings within the filled area, where the results from in place for periods of time varying from 2 to 5 months. Shear profiles of these two groups of borings have been combined to form a composite shear profile from which average shear values may be determined as

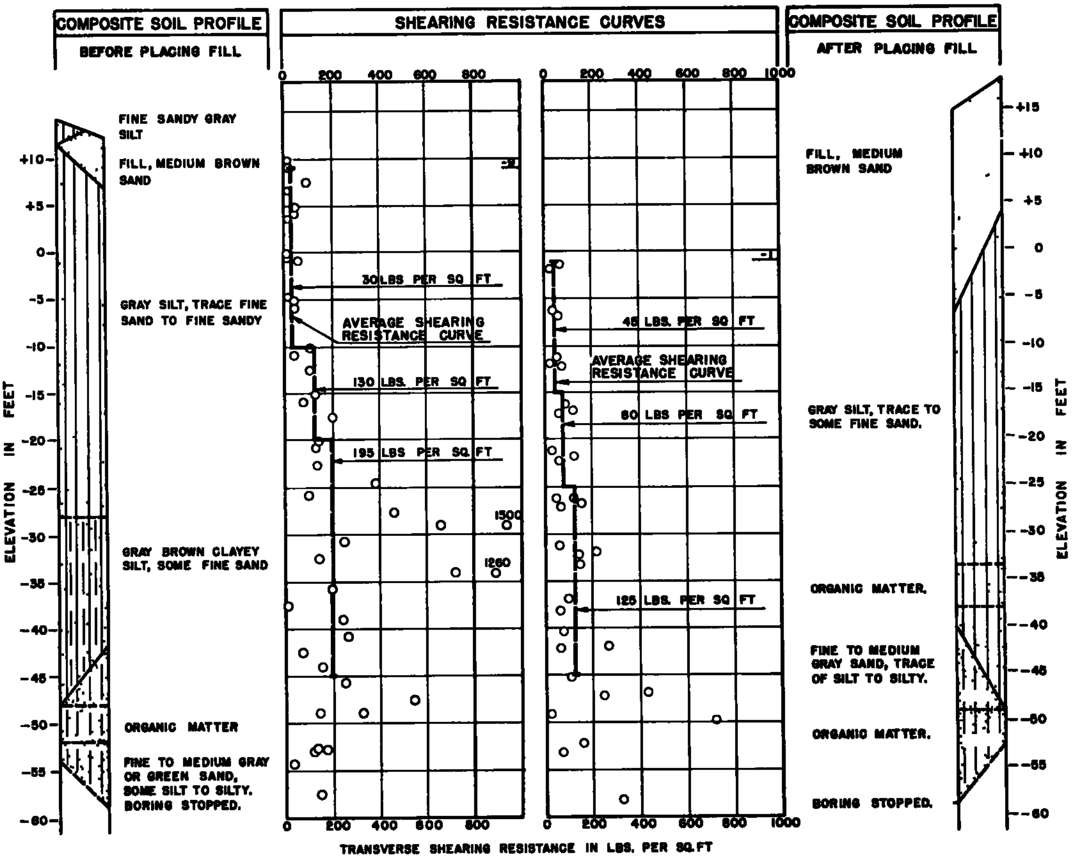

Figure 13.

these two tests is close to the usual ratio of 4 to 1. In this connection, it may be noted that in borings under the runways the soft, silty clay may have been remolded by pile driving and by displacement under the weight of the overload fill, from which it might be presumed that the unusual result obtained in the case of the undisturbed flocculated clay is a function of a flocculated structure.

In connection with the contractor's investigation, Borings 3, 4, and 7 have been selected as representative of soil area 1 in its original condition before it has been altered by construction operations. Borings 2, 9, and 11 have been selected as representative of the same soil area under the runway and taxiway after the sand piles have been driven and the overload has been representative of the soft silty clay under discussion.

The comparison of shearing resistance before and after consolidation in terms of the transverse shearing resistance or ring-shear test is shown in Figure 13. Composite boring logs are indicated on the left for Borings 3, 4, and 7 and on the right for Borings 2, 9, and 11. The composite shear profile for the original soil in Area 1 is shown on the left-hand portion of the central chart and the composite shear profile after consolidation is shown on the right-hand portion of the chart. Average shear values at several different depths have been indicated by the heavy dashed line. These shear values are shown to the nearest 5 lb. per sq. ft. and are based on the arithmetic average of shear test results for the silty clay only. All shear test results are shown on the charts including those from samples of sand and in the case of Boring 7, a more-highly consolidated chalky silt found only in that boring around Elevation -30. Such unrepresentative and exceptional test results were not included in the average shear curve shown by the heavy dashed line.

For the purposes of the present discussion, a comparison of shear values must be limited to the original soil below the depth to which the sand fill has penetrated and which has not been displaced by the superimposed load. It should be noted in Borings 3, 4, and 7 on the left-hand portion of the central chart that the uppermost layer of clay down to approximately Elevation -10 has almost negligible shearing resistance and has, in all probability, been almost completely displaced by the sand fill. This soft, silty clay sometimes would not even support the weight of the small steel ring in the ring-shear test (which produces a shearing stress of approximately 4 lb. per sq. ft.) and this is the value that has been reported for this material in a number of the tests. In the case of the unconfined-compression test on this softest clay when the 3-inch samples were extruded from the liner, the small cylinder slumped under its own weight and it was impossible to apply any compression load.

Omitting comparisons in the softest clay strata above Elevation -10, which has largely been displaced by sand fill, it may be seen that the average transverse shearing resistance in those borings representing the original soil conditions varies from 130 to 195 lb. per sq. ft. within the depths at which these tests were performed. The range in the average shearing resistance for these same depths from the borings taken in the filled area after consolidation is from 45 to 125 lb. per sq. ft. indicating a measurable loss in shearing resistance, due presumably to the disturbance caused by pile driving or other construction operations and not recovered during the consolidation period.

Figure 14 presents a similar comparison based upon the results from the unconfined-compression test. In this case, the unconfined-compression test on samples from the borings representing original soil conditions and undisturbed clay gave an equivalent shearing resistance from 85 to 160 lb. per sq. ft. In those borings from the filled area after the consolidation period, the equivalent shearing resistance from unconfined compression varied from 55 to 110 lb. per sq. ft., again indicating a definite loss in shearing resistance, presumably due to remolding or other disturbance during the construction period which was not recovered by consolidation. It should also be noted that the equivalent shearing resistance from unconfined compression is measurably less than the transverse shearing resistance in Borings 3, 4, and 7, representing the original soil conditions and undisturbed clay, but this variation was not established by a similar comparison for the remolded soil.

While the direct comparison between moisture contents and shearing resistance before and after consolidation were the primary objectives of that part of the investigation being presented in this paper, there were several other subjects studied in the contractor's investigation which are of passing interest in the present discussion. A stability analysis was made to determine the overload ratios imposed on the original soil mass by the proposed heights of surcharge called for by the plans and specifications. This analysis indicated that the overload ratios with respect to the uppermost layer of softest clay were so high that it was quite obvious that complete displacement could be anticipated before the full surcharge could be placed. Overload ratios were computed only for Area 1 which had been judged to be the most critical from preliminary investigations. However, such critical classification covered the major portion of the length of the taxiway, and a considerable portion of the length of the runway. In these areas, the marked correlation between high overload ratios, failure areas, and points of maximum settlement left no doubt as to the source of excessive settlement and subsidence.

CONCLUSION

In summarizing the results of the soil investigation made by the contractors, the following conclusions are presented:

1. Laboratory tests on undisturbed samples from borings in the critical failure area, designated as Area 1, identified the soft, silty clay extending from the surface to a depth of approximately 60 feet as a flocculated clay of high sensitivity which

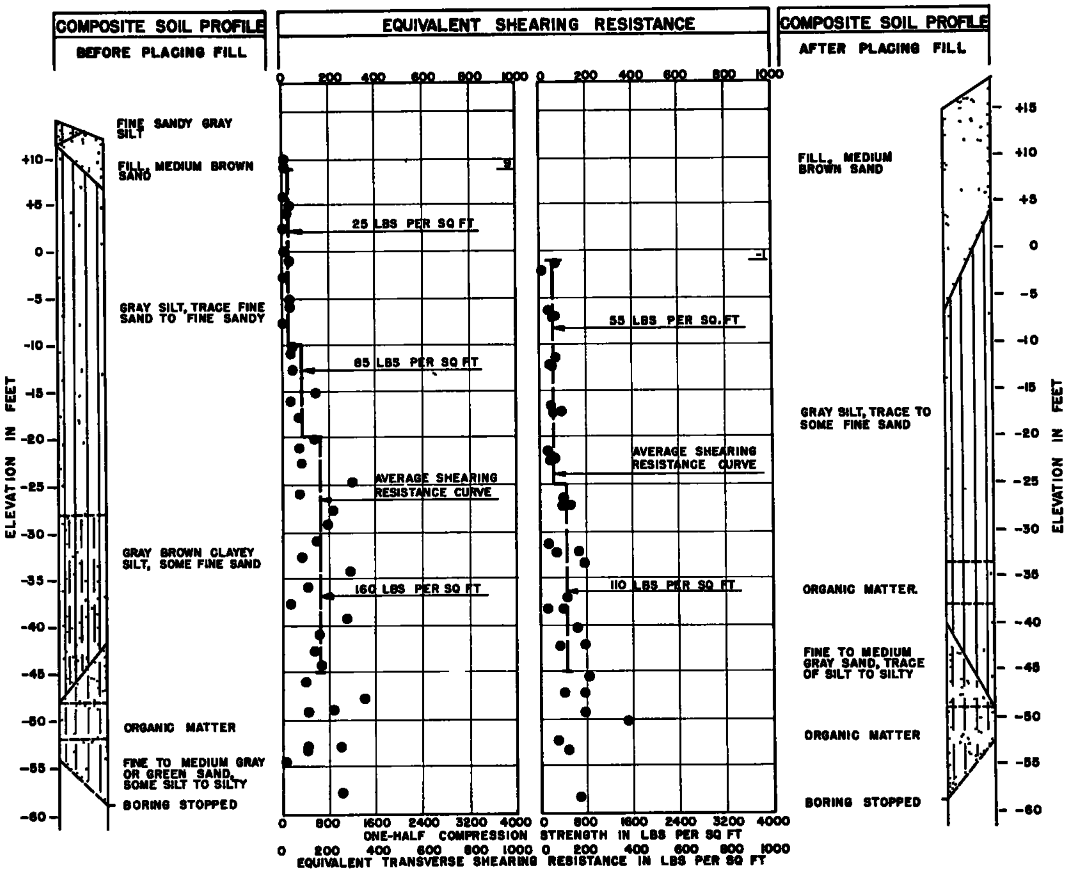

Figure 14.

has been found to suffer a loss in shearing resistance upon being disturbed or remolded.

2. Shearing-resistance tests on undisturbed samples from borings outside the runway area representing the original soil indicated that the yield value or static shearing resistance based on the average of three borings varied from 30 to 195 lb. per sq. ft. in ring shear and from 25 to 160 lb. per sq. ft. in the unconfined-compression test. The flocculated clay on top in several cases was so soft that shear values were not measurable.

3. In the group of three borings taken along the centerline of the runway and taxiway in soil Area 1, the softest clay on top had been largely displaced by the sand fill, and the shearing resistance in the lower strata, which had been disturbed and remolded by driving of the sand piles, was definitely less than in the original state, even after having been subjected to periods of presumed consolidation by the overload fill in place for periods of 2 to 5 months. In this case, the static shearing resistance, or yield value, from the ring-shear test, based on the average of three borings, varied from 45 to 125 lb. per sq. ft., as compared to 130 to 195 lb. per sq. ft. in the undisturbed or original condition.

4. Comparison of moisture contents from the present investigation between the undisturbed soil and that which had presumably been consolidated for 2 to 5 months under the overload fill indicated that there had been no measurable change in moisture content due to consolidation. This conclusion was also confirmed by the comparison of moisture contents from the original borings, in which there was striking agreement for comparable samples between moisture contents before and after consolidation.

5. The stability analysis made as part of this investigation showed that overload ratios in the critical soil area investigated were far in excess of those certain to cause

soil displacement. The coincidence between high overload ratios, failure areas where subsidence and mudwaves were observed, and points of maximum settlement leave little question that excessive fill settlement was due to exceeding the supporting capacity of the soft subsoil.

6. Settlement of the fill was excessive following the consolidation period until the final paving was completed in the summer of 1952. This excessive settlement has continued and in approximately a year has amounted to a maximum of 1.5 feet in the more-critical areas which coincide with those designated by the contractor's investigation. As a final testimony to the ineffectiveness of the sand-drain piles under the soil conditions of the Norfolk project and impracticability under those conditions of a design predicated on the general theory of consolidation, it may be noted that the Navy, during the past fall, found it necessary to arrange for the restoration of the runway extension which has been the subject of this discussion. Those most familiar with the present status of the project anticipate that the maintenance of the runway will be a serious problem for a long time to come.

REFERENCES

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2. Procedures for Testing Soils - American Society for Testing Materials, 1950.

Discussion

L. A. PALMER, Engineering Consultant, Soil Mechanics and Paving, Bureau of Yards and Docks, Department of the Navy — There have been instances of success in deep soil stabilization by vertical sand drains. A description of a notable instance of such success in Navy experience is found in the 1949 Transactions of ASCE in a paper; "Failure of Quay Wall at Mare Island," by L. C. Coxe. There have been other instances of only partial success and in a few instances, the installation of vertical sand drains has been entirely futile in accomplishing any noticeable degree or extent of deep soil stabilization.

The instance described by Housel tends to fall in the second category. The easterly end of the runway extension at NAS Norfolk has sunk at least 16 inches, following completion of construction, and indications are that the settlement will continue.

This writer believes that mud waving sheared off a considerable number of the vertical sand drains in this case and that installation of the drains should have been accomplished after placing the excess surcharge over the site and not prior to this operation. The sequence of operations should have included first the placing of the excess fill to drive the water out of the mud. This would cause mud waves which should have been allowed to reach an equilibrium condition and without removal of the displaced mud to displace such equilibrium. Thereafter, the vertical sand drains could have been constructed without being sheared off by displacement of the soft, organic silt under pressure of the surcharge.

Any of us can be misled by technical publications which indicate that the speed of consolidation of the soft soil by the sand drains can be predicted with a fair degree of accuracy. Since estimating the coefficient of permeability in the horizontal direction is mostly guesswork, such a prediction is not possible. After there is full realization of the fact that one usually cannot obtain a reliable estimate of the rate of settlement after sand-drain installation, one tends to proceed with more caution and prefers, whenever possible, to install the drains after and not before the surcharge loads are applied.

If it were possible to now locate the points where sand drains were installed and covered over with fill at NAS Norfolk, one could take borings down through the sand-drain locations. If this were done, it would likely develop that the drains were broken off and displaced in shear by the flowing mud.

Usually, the purpose of sand drains is to accelerate the settlement and have the bulk of it completed prior to construction. Where the soft soil is deeply seated and located at appreciable depths below the proposed elevations of footings of a structure, this is the primary consideration. But when it is necessary to build up bearing capacity in the soft soil as well as to complete its consolidation before construction, then one must make a careful study of the remolding characteristics of the soil to be stabilized by sand drains. The disturbance, caused by the installation of sand drains at 8 to 12 feet, on centers, may weaken a soil much more than its subsequent drainage under load into sand drains can strengthen it. This condition was observed by this writer at a proposed warehouse site in the Norfolk area that is about 3 to 4 miles from the air station. The deep layer of organic silt at this site proved to be sensitive to remolding, and for this reason, sand drains were not recommended.

Neither are sand drains efficacious in peat soil. This writer made an extensive study of the settlements of concrete barricades at Port Chicago, California. Here sand drains had been installed and overloaded with extra surcharge which, after several weeks, was cut back to finished grade with subsequent construction of the barricades. These have settled from 4 to 9 feet or more since 1944. The soil in this case is disintegrated (not fibrous) peat with an organic content of more than 30 percent. In consolidation tests, the rate of strain was independent of the thickness of the sample. In other words, the rate of consolidation of peat (not peaty or peat-like soils) is not determined by the coefficient of permeability, and hence it is independent of the distance of flow of the escaping water.

A thorough field investigation of sand drains under controlled conditions is long overdue. It is not a problem that can be worked out altogether in the laboratory. Much more basic and practical information should be made available before one can have a reasonable degree of confidence in the use of vertical sand drains.

Basically, the theory of soil consolidation is probably sound. However, there is much wrong with its use and application. One must be an irresponsible optimist to believe in the possibility of precise estimates of the subsidence of millions of cubic yards of soil on the basis of the observed laboratory behavior of a few pounds of the stuff. It is just as detrimental to the interest of the public to oversell the utility of soil mechanics as it is to destroy all confidence in its use.

Water held by physiochemical forces in fine-grained soil, referred to by Housel as absorbed water, is probably never lost to any appreciable extent by normal conditions of loading. Also, the bulk of such adsorbed water is retained in the soil at the boiling temperature of water. But free water, lost on air drying, can and is released by the soil under external load application. The release of this mechanically held water into pervious layers that bound the layers of fine-grained soil is adequately described by the theory of consolidation, the mathematics of which is about identical to that for the unidirectional flow of heat in an insulated metal rod or the diffusion of a salt into a solvent. Difficulties aries when one idealizes certain conditions, such as "previous boundaries," the continuity of stresses (due to external load) through multiple layers of dissimilar types of soil, and the assumed homogeneity of large masses of fine-grained soil with respect to permeability. The near-ideal conditions do sometimes exist, and they lead to fairly accurate settlement predictions which receive altogether too-much publicity.

WILLIAM S. HOUSEL, Closure — The written discussion of this paper by Palmer was welcome, inasmuch as he had first-hand knowledge of the particular sand-drain installation under discussion. The author was glad to have his somewhat-qualified confirmation of the conclusion that the sand drains were "entirely futile in accomplishing any noticeable degree or extent of deep soil stabilization." There were also a number of other points brought up by Palmer on which the author wishes to comment. The question of shearing off the sand drains by fill displacement is an example of only one of the important uncertainties about sand-drain installation which are all too frequently taken for granted without any real effort made to determine the final result achieved. The continuity of the vertical sand drains is largely a matter of speculation, as it is difficult to determine by direct observation. It is quite probable that they have been sheared off where there has been excessive fill settlement. However, there are other areas where fill settlement has not been excessive and, in fact, substantially less than anticipated from assumed consolidation. In these areas, there is no reason to expect abnormal disturbance of the sand drains from subsequent fill settlement. In locating the check borings, the attempt was made to obtain data in some of these relatively undisturbed areas. The moisture content and shearing resistance data presented in the paper show the same lack of consolidation effects in all areas which is the essential consideration under discussion.

Shearing off of the sand drains is only one phase of soil disturbance in sand-drain installations. Disturbance due to placing of the sand drains is another reality which has been passed over too lightly. The steel pipe mandrel used in placing the vertical sand drains at Norfolk was 14 inches in diameter and, in a depth of 60 feet, involves a volume displacement of approximately 2.5 cu. yd. Years of pile driving experience has shown that piles driven in soft, saturated clay soils result in practically complete volume displacement. Specifications for heave and alignments of all piles recognize the serious nature of this volume displacement. Occasional collapse and damage to steel shells of cast-in-place concrete piles furnish further evidence of this phenomenon. In the light of all this accumulated experience, it is something more than optimism not to recognize the probable damage to unsupported columns of sand without any structural continuity whatsoever. In the author's opinion, it is extremely problematical that the vertical alignment of sand drains is sufficiently consistent to produce effective vertical drainage aside from all other questions raised as to their effectiveness.

Another aspect of soil disturbance due to installation of the sand drains which does not have to be left to speculation is the loss of shearing resistance due to the remolding effect on flocculated clays of high sensitivity. This point was brought up in the original paper and also emphasized by Palmer. The variation in shearing resistance before and after consolidation clearly established the detrimental effects and more permanent character of remolding under the soil conditions at the Norfolk site. This led Palmer to place such soils in the classification in which vertical sand drains would not be practical. He also adds peat to the soils not benefited by vertical sand drains which would appear to the author to leave only a restricted range of rather exceptional soil conditions for their effective use.

Palmer cites two sand drain projects in his discussion; the quay wall at Mare Island, California and fill construction at Port Chicago, California, both in the San Francisco Bay area. By strange coincidence, the author was asked to investigate both of these projects during the period in which he was stationed in this area while serving with the Navy. Both projects were in some difficulty and the assignment was to make an evaluation of them and submit any suggestions which resulted to the officer in charge.

It has always troubled the author to see Port Chicago included among the examples of sand-drain installations which are presumably cited as successful applications.[1] His evaluation of it at the time of construction was not much different from that of the present Norfolk project, except that the fill displacements were not as extensive. No change in procedure was suggested as the project was then nearing completion. As now recalled, the author summarized this project as an unsuccessful attempt to make the soil behave in accordance with a theory which did not represent a realistic concept of soil behavior. The results of the study which Palmer made some years later is rather surprising confirmation of this previous evaluation.

The quay wall at Mare Island was only one of several perplexing problems in a difficult area that was then under development. The author made no special study of it, although it was included under a suggested program of investigation which included undisturbed samples, shearing resistance tests, and complete stability analyses. The author had no further contact with these problems or knowledge of the quay wall until the final results were reported by Commander L. C. Coxe as part of the "Symposium on Lateral Earth Pressures on Flexible Retaining Walls" in the 1949 Transactions of the American Society of Civil Engineers. The essential phases of this project may be summarized as fol-

[1] "Vertical Sand Drains Speed Consolidation of Soft Water-Bearing Subsoil", Engineering News Record, 1947, Vol. 138 p. 386.

lows: The quay wall and dock first failed under a superimposed load which exceeded the supporting capacity of the underlying "plastic clay mud." It was then rebuilt with certain changes in design which included both an anchorage system and vertical sand drains. As described in the article, "Concrete anchors supported by wood, vertical and batter piles and connected to the quay walls by steel tie rods 2 inches and 2.5 inches in diameter were not included in the original design but were installed after failure of the wall." After the tie rods were connected, there was still "appreciable movement" and sand drains were installed. The exact sequence of these several steps is not entirely clear and the "appreciable movement" was also influenced by the load of a sinking tug moored to the dock.

At any rate after the installation of both anchorage system and sand drains was complete, the movement was reduced to a more-acceptable magnitude. Data given in the paper up to 1946 indicate a maximum of additional vertical settlement of 0.43 feet or about 5 inches in 42 months prior to that date. Graphs indicate a continued progressive settlement at some points of approximately a uniform rate of 1.5 inches per year. There are several interesting statements in this connection which deserve special attention and may be quoted as follows:

> The action of the sand drains and surcharge was very effective although no actual data as to the reduction in pore pressures are available. This method of stabilization combined with the anchored tie rods completely checked any further motion of the quay wall and effective stabilization was realized in December 1942, approximately two months after the sand drain installation was completed.

The statement that the movement of the quay wall was "completely checked" is not borne out by subsequent data taken up until 1946 and noted in the previous paragraph. That the relative improvement was due to reduction in so-called pore pressure was an assumption unsupported by factual data. Without going into the hypothetical ramifications of pore pressure, it might have been just as reasonable to suppose that the anchorage system that was added was the more responsible factor in the reduction in movement of the wall.

In the paper presented by the author, the statement was made with respect to sand drains "that the claims of successful application are seldom supported by more than fragmentary and inconclusive evidence." After careful consideration of the description of the Mare Island quay wall, it would seem that it might serve as a fair example of the fragmentary and inconclusive evidence referred to and in the author's opinion, this would be a very generous evaluation of that project.

Aside from these areas of substantial agreement, there are several points brought up by Palmer which require either further clarification or represent points of some difference. Understanding that the primary purpose of vertical sand drains is to permit the gradual consolidation of soft unstable soil deposits without displacement, it would seem that placing of the surcharge fill first would defeat the declared purpose of the sand drains. If as stated in the discussion the displacement under the surcharge fill was "allowed to reach equilibrium," there would no longer be any point in having the vertical sand drains. In connection with this same discussion, it would appear to the author that Palmer does not clearly differentiate between "placing the excess fill to drive the water out of the mud" which is consolidation and allowing the mud waves to reach an equilibrium which is clearly a function of shearing displacement.

In the original paper, the author definitely questioned the validity of the theory of consolidation and would continue to hold that viewpoint. Actually, the difference in viewpoint is not too great when Palmer says, "Basically, the theory of consolidation is probably sound" and the author would say, "Basically, the theory of consolidation is probably not sound." Both positions leave room for an adjustment of ideas as additional evidence is presented. In supporting the negative position, the author would fall back on the definition of theory as a description of the results of observation and experiment and the contention that when theory and practice disagree, the theory must give way. It is certainly in this area that the theory is being questioned and the experimental data presented in the original paper are capable of standing on their own feet.

In the closing part of his discussion, Palmer brings up the important subject of adsorbed water and the conditions and forces involved in its removal from the soil. His differentiation between free and adsorbed water by evaporation at the boiling point would seem to be an over-simplification not liable to clarify the confusion and misconceptions which may arise in a discussion of these phenomena. The author, on the other hand, would regard all moisture in a soft, saturated clay as being adsorbed to a varying degree, depending upon the magnitude of the forces of molecular attraction with which it is held. Much of the water in such physical combination, sometimes called the water of placticity, may be subject to evaporation at the boiling point of water. The most-closely held water sometimes considered as chemically combined would be far beyond the effects of ordinary physical pressure and temperature less than the boiling point of water.

In the original paper, the attempt was made to be quite specific on the probable range of consolidation effects involving applied pressure and its relation to shearing resistance which controls mass displacement. The only amplification of the view presented which appears to be required at this point is to acknowledge that pressure concentration which may be applied within the limit of shearing displacement does represent a range of actual consolidation which might be achieved under practical conditions. To the author, this represents the only range within which the consolidation theory may be considered valid. It is felt that within this range, consolidation is too long and too little to be of more than negligible value under practical conditions in the types of soil under discussion.

Hawaii's Experience with Vertical Sand Drains

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Settlement observations taken on two sand-drain projects were plotted and compared with the estimated time rate of settlement. Settlement calculations were made by combining the separate percent consolidations due to the vertical drainage and the radial drainage of the pore water. The coefficient of consolidation for the vertical drainage of the pore water was determined by means of a laboratory consolidation test. The coefficient for the radial drainage of water was computed from the results of field permeability tests and from observations on the rate of settlement shortly after the installation of sand drains.

● VERTICAL sand drains to accelerate the settlement of highway embankments have been used on two recent projects in the Territory of Hawaii: (1) Federal-Aid Project No. F 15(3) at the crossing of Kalanianaole Highway over Kahanaiki Swamp, and (2) Federal-Aid Secondary Project No. S 223(2), Haleiwa Cutoff Road, across the Kiikii Swamps.

The author is indebted to the pioneer work of O. J. Porter (1) and others in regard to design and construction methods involving vertical sand drains. Since these aspects of the subject have been competently and adequately covered by others and data are readily available, no extended discussion concerning them will be attempted here. Instead, this paper will attempt to deal with the more-technical phases of the subject about which there appears to be a paucity of information in the literature.

Both swamps referred to above were formed of alluvial sediments washed in from higher ground mixed with some organic material and occur over areas which, in the geologic past, were probably under sea level or, at least, close to the shore line, so that they are underlain by sandy material of marine origin. By marine origin is meant that the sand was either part of an ocean beach, or a sand dune in case the shore line was some distance out. In the case of Kahanaiki Swamp on Project F 15(3) the entire material over the sand bottom was soft, so there was a drainage face at the bottom of the swamp. In the case of the Kiikii Swamps on Project S 223(2), part of the alluvial deposits over the sand had consolidated or hardened sufficiently so that, apparently, it was uncompressible and relatively impervious to the flow of moisture. Vertical drainage was considered to be only in an upward direction. The existence of this hard layer was discovered during the soil-profile investigations prior to construction. A cylindrical steel mandrel was used to place the sand drains, and during actual construction it was possible, by means of the heavy pile driving hammer used, to drive the steel mandrel through this hard layer, although with considerable effort.[1] The subsequent settlement data appeared to show that this hard layer is not consolidating under the weight of the embankment. Thus, the driving of the sand piles through this hard layer, although it has done no harm, has not affected the settlement of the embankment, which is due entirely to the consolidation of the upper, softer layer.

WORKING TABLE, CONSTRUCTION METHODS, SETTLEMENT DATA

A working table measuring approximately 2 feet 6 inches in thickness was first laid over the surface of the swamp so as to provide a relatively stable surface over which equipment could be operated. Immediately after the working table was leveled off, holes were dug through it, settlement platforms installed over the top of the soft swamp layer, and the holes backfilled.

The settlement platforms consisted of a base 3 feet by 3 feet built of heavy 2-inch planks with a length of ½-inch galvanized pipe attached at the center. As the embankment was built up, additional lengths of ½-inch pipe were added as needed. Elevations were taken on the pipes as the work progressed and the results plotted to show the settlement with time. This will be discussed later (see Figs. 5 and 6).

A paper on the subject was presented by the author at the June 1952 convention of the

[1] That is relative to the effort required to penetrate the upper softer layer.

Western Association of State Highway Officials at Seattle, Washington, at which time one of the projects here discussed was still under construction. As of April 6, 1953,[2] Project F 15(3) has been completed and in service for 19 months. Project S 223(2) has been completed and in service for 7⅓ months. Levels were taken on the finished pavement recently and compared to levels at the time of completion. The additional settlements under service thus observed are shown on the time-settlement curves (Figs. 5 and 6).

Placing of sand drains was begun as soon as the working table was ready to receive the necessary equipment.

Using a power auger, holes were dug through the working table to the top of the swamp. A steel mandrel was then driven through the soft, compressible layer to stable (noncompressible) material below. The mandrel was driven, without leads, by means of a regular pile driving hammer. It was a simple matter to stand up the mandrel by letting it fall of its own weight into the soft layer. Guy wires tied onto truck winches then kept the mandrel plumb. For the lengths of mandrels used on the two projects here reported, up to approximately 40 feet, the above method of driving without leads proved practicable. (Driving with leads was tried and found to be much slower.)

The mandrels on the two projects were both fabricated by the contractors from heavy 18-inch-diameter steel pipe. An orange-peel arrangement, which resulted in a conical point when closed by means of an inside catch, was used for the driving end on Project F 15(3). After driving to the necessary depth, the mandrel was filled with sand. A "fish line" extending down the inside of the mandrel to the point was pulled to open the orange peel. This allowed the sand to be deposited in the hole as the mandrel was slowly withdrawn. On the other project, the contractor used a solid, conical shoe hinged on one side for his driving point. In driving, the resistance of the ground kept the shoe in the closed position. In raising the mandrel, the shoe dropped of its own weight to the open position, thus allowing the sand to run out.

One phenomenon that has to be guarded against is that of arching of the sand in the mandrel. To overcome this and to insure proper density of the sand in the hole, the mandrel was provided with a tight cover and compressed air was applied to the top of the sand when withdrawing the mandrel from the hole. The pressure of the air and the rate of withdrawal of the mandrel have to be carefully regulated, otherwise there is danger of blowing and dispersing the sand into the swamp muck or of having a partially empty hole.

[2] Date of this paper.

Cost data for the two Hawaii projects have been analyzed in Appendix A.

SOIL TESTS

Undisturbed samples were taken and consolidation tests were run. Due to a lack of deep sampling tools, the samples were obtained only from relatively near the surface.

From the consolidation tests the following data were obtained: (1) the voids-ratio versus applied pressure relationship and (2) time-settlement relationship for the sample. From the time-settlement relationship for the sample, the coefficient of consolidation was calculated by the square root of time (Gilboy's) method.

Since the laboratory procedures in carrying out the above tests and the various empirical rules for adjusting the laboratory data are well known, the subject will not be considered further here.

The pressures on the soft swamp material at various stations along the centerline profile, due to the highway embankment, were calculated and the corresponding voids-ratios estimated from the voids-ratio versus pressure relationship developed by the consolidation tests. The total settlement was then estimated by means of the following formula:

$$S = \frac{e_1 - e_0}{1 - e_0} D \qquad (1)$$

where S = total settlement
e_1 = ultimate voids-ratio due to applied loads
e_0 = voids-ratio prior to application of loads
D = thickness of compressible layer

The voids-ratios e_0 and e_1 were taken as the average of the voids-ratios at the top and at the bottom of the compressible layer. The pressure at the bottom of the compressible layer due to the embankment load was calculated by means of the following formula:

$$P = \frac{q}{\Pi}(B + \sin B) \qquad (2)$$

where P = pressure at bottom of compressible layer at a point directly below the center of the embankment section
q = intensity of surface loading
Π = 3.1416
B = angle subtended by the width of the embankment at the point where P is estimated.

METHOD OF COMPUTING TIME-SETTLEMENT

In ordinary consolidation, the drainage of the pore water is in the vertical direction and takes place according to the following differential equation, due to Terzaghi (2).

$$\frac{\partial P}{\partial t} = \frac{c_v \partial^2 P}{\partial z^2} \qquad (3)$$

$$c_v = \frac{k_v(1 + e)}{a\gamma} \qquad (4)$$

where P = hydrodynamic excess pressure of the pore water at any time, subsequent to t = 0, due to the embankment load
z = depth below surface at which the pressure P is measured.
t = time
c_v = coefficient of consolidation for vertical drainage of the pore water
k_v = coefficient of permeability of the swamp material for vertical flow of the pore water
a = coefficient of compressibility of the swamp material
e = voids ratio
γ = density of the pore water

The boundary conditions appropriate to the present problem were taken as,

$$\left.\begin{aligned} P &= P_1 \text{ when } t = 0 \\ P &= 0 \text{ for } z = 0 \\ \frac{\partial P}{\partial z} &= 0 \text{ for } z = H \end{aligned}\right\} \qquad (5)$$

where P_1= pressure due to the weight of the embankment, assumed to be uniform throughout the whole depth of the swamp
H = half-depth of soft-compressible layer for Project F 15(3) and the whole depth of soft material in the case of Project S 223 (2)

The basic conception of the theory of consolidation is that when the hydrodynamic excess pressure P in the pore water reduces to zero, the consolidation is complete and grain-to-grain contact of the soil results. Hence, $(P_1 - P)/P_1$ is a measure of the state of consolidation and

$$\text{Percent Consolidation} = U\% = \frac{P_1 - P}{P_1} \times 100 \qquad (6)$$

The differential equation (Equation 3) has been completely solved for various boundary conditions and various tables and curves are available in the literature. For the boundary conditions here specified, the curve (Fig. 1) shows the relationship between the percent of consolidation and the nondimensional time factor T_v,

$$T_v = \frac{c_v}{H^2} t \qquad (7)$$

If we know the values of c_v and H for the swamp, we can compute by Equation 7 the time factor T_v corresponding to any time t subsequent to the application of the embankment load onto the swamp, and thus find the corresponding percent of consolidation (U%) from the curve of Figure 1. This percent of consolidation applied to the total ultimate settlement computed by Equation 1 is then the estimated settlement up to the time t.

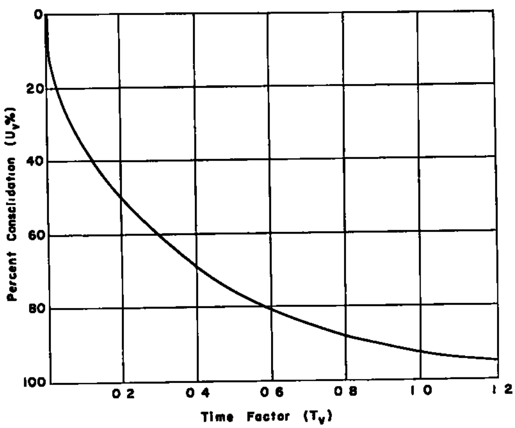


Figure 1. Chart for determining percent of consolidation due to vertical drainage.

With the installation of sand drains the drainage of the pore water can take place in horizontal radial directions as well as vertical. For this three-dimensional consolidation, the appropriate differential equation is

$$\frac{\partial P}{\partial t} = c\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial y^2}\right) \quad (8)$$

Transforming to cylindrical coordinates, and making use of axial symmetry, Equation 8 becomes

$$\frac{\partial P}{\partial t} = c\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial z^2}\right) \quad (9)$$

Equation 9 is based on the assumption that the coefficients of consolidation in the vertical and radial (horizontal) directions are the same. In general this is not the case so that instead of Equation 9 we will have,

$$\frac{\partial P}{\partial t} = c_v \frac{\partial^2 P}{\partial z^2} + c_r\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right] \quad (10)$$

where $c_v = \frac{k_v(1+e)}{a\gamma}$ = coefficient of consolidation for the vertical drainage of pore-water

k_v = coefficient of permeability for the vertical flow of pore-water

$c_r = \frac{k_r(1+e)}{a\gamma}$ = coefficient of consolidation for the radial (horizontal) drainage of pore-water (10a)

k_r = coefficient of permeability for the radial (horizontal) flow of pore-water

Equation 10 can be solved as a whole or, more conveniently, it can be split into two parts,

$$\frac{\partial P}{\partial t} = c_v \frac{\partial^2 P}{\partial z^2} \quad (3)$$

$$\frac{\partial P}{\partial t} = c_r\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right] \quad (11)$$

Each part with appropriate boundary conditions is then solved separately and the corresponding percent of consolidation at a given time t computed for each. It can be shown that these partial percent consolidations can be combined according to the following formula[3] to give the combined percent of consolidation applicable to equation 10.

$$100 - U\% = (100 - U_v\%)(100 - U_r\%)(1/100) \quad (12)$$

[3]See Reference 3.

where $U\%$ = combined percent consolidation

$U_v\%$ = percent consolidation due to the vertical drainage of water

$U_r\%$ = percent consolidation due to the radial (horizontal) drainage of water

The part $U_v\%$ can be estimated easily by means of the curve in Figure 1. To calculate $U_r\%$ is not so easy, since published data on the subject are comparatively meager. The method of computing the percent of consolidation due to radial drainage, corresponding to Equation 11 is given in Appendix B. Numerous curves suitable for engineering use in connection with consolidation problems is given in a work by Barron (4).

Unlike the vertical percent of consolidation, the radial percent of consolidation depends not only on a time factor T_r but also on a parameter b/a, where b is the radius of the area, assumed to be circular, drained by each sand drain and a is the radius of the sand drain itself. The sand drains were arranged in a hexagonal pattern as shown in Figure 2 and the values of b/a were 14.2 and 11.9 for Project F 15(3) and S 223(2), respectively.

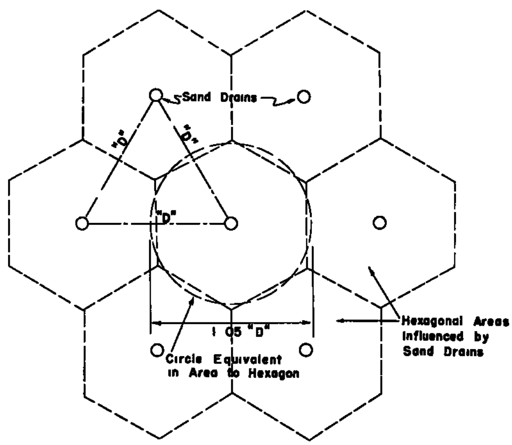


Figure 2.

Figure 3 shows the relationship, computed as shown in Appendix B, of $U_r\%$ to the time factor T_r where

$$T_r = \frac{c_r t}{4b^2} \quad (13)$$

where T_r = time factor for radial drainage of pore-water

t = time

c_r = coefficient of consolidation for radial drainage of pore-water
b = radius of circular area influenced by each sand drain

If curves such as Figures 1 and 3 or corresponding tabular data are available, the problem reduces to the following steps: (1) For any time t, subsequent to the instant when the embankment load is applied, compute the corresponding time factors by Equations 7 and 13; (2) From the curves of Figures 1 and 3 find the corresponding percents of consolidation U_v% and U_r%; (3) Combine the above according to Equation 12; and (4) the combined percent of consolidation applied to the ultimate settlement as calculated by means of Equation 1 will give the settlement up to the time t.

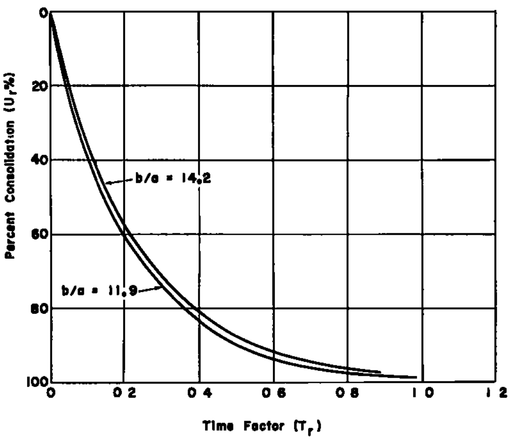


Figure 3. Percent of consolidation for radial (horizontal) drainage.

In the theory, it is assumed that the total consolidating pressure is applied suddently at time t = 0. Physically this is impossible. The working table was laid in one lift, but the embankment was built up in layers and the work took many days to complete. Hence, the embankment load was actually applied gradually and not suddenly as assumed in the theory. Because of this fact, the settlement was computed by the method of superposition, which is valid because the differential equations are linear. The time when each convenient increment of load was applied was taken as a new zero of time in computing the settlement due to that particular increment. These partial settlements for corresponding times on the time scales of Figures 5 and 6 were added to give the total calculated settlements. The results were plotted as calculated time-settlement curves.

The consolidation due to vertical drainage begins as soon as work on the working table starts, but consolidation due to radial drainage does not begin until some time later when work on the sand drains begins. This introduces an error. However, the error is significant for small values of t only. For large values of t, that is for long time effects, the error is negligible.

In the determination of the time factors the only unknown quantities are the two coefficients of consolidation. The first, c_v, can be computed from the laboratory consolidation test. Various techniques have been devised with this object in view and have been thoroughly discussed in the literature so that additional comment at this time seems unnecessary.

The direct laboratory determination of c_r, the coefficient of consolidation for radial drainage of pore water, is probably not possible. In the case of alluvial sediments, as in swamps, the deposit is stratified. For use in the consolidation problem, the value of the coefficient of consolidation that is required is not that of the individual layers but of the deposit as a whole.

Referring to Equations 4 and 10(a), we see that c_v and c_r are proportional to the respective coefficients of permeability. The ratio of the permeabilities is therefore also the ratio of the coefficients of consolidation.

DETERMINATION OF THE PERMEABILITY RATIO

In theory we can obtain samples from various strata and determine the coefficients of permeability of the various strata. For vertical flow, the permeabilities are in series and for horizontal flow they are in parallel. Then following the electrical analogy of conductances in series and in parallel, it can be seen that the permeability of the deposit as a whole in the horizontal direction is always greater than the permeability in the vertical direction. In practice, however, the method is cumbersome and uncertain due to the presence of poorly defined strata and lenses of different materials.

In the analysis of the settlement for Project F 15(3), the coefficient of consolidation for horizontal drainage was estimated from the difference in rate of settlement before and shortly after the installation of the sand drains. In the case of Project S 223(2), an independent check of the ratios of the coefficients of consolidation was made by conducting field permeability tests.

The field permeability tests were made using the tube method and the auger-hole method.

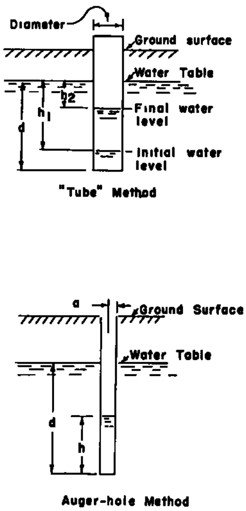
In the tube method, a tube known diameter is placed tightly in a hole of the same size to a known depth below a water table as shown in Figure 4. The water is then pumped out of the tube down to some known elevation below the water table and above the bottom of the tube. Water from the surrounding area is allowed to flow into the tube from the bottom. The time it takes for the water to rise in the tube a given distance is measured. The permeability is computed by means of the following formula:

$$k = \frac{\Pi R^2 \ln (h_1/h_2)}{A\,t} \qquad (14)$$

where k = coefficient of permeability
R = radius of tube
$\ln$ = natural logarithms
h_1, h_2 = initial and final water levels in tube (see Fig. 4)
t = time required for water to rise in tube from h_1 to h_2
A = a coefficient determined by use of an electric analogue

Since the flow of water is upward into the tube, the coefficient of permeability thus measured is that in the vertical direction.

In the auger-hole method suggested by Kirkham and Van Bavel (6, 7), an auger hole of known diameter is dug down to a known depth. The water in the hole is pumped out and its rate of rise noted. If the auger hole is dug all the way down to an impermeable layer, the problem is subject to exact mathematical analysis. If the hole is not carried down to an impermeable layer, the problem can be solved by means of an electrical analogy. In both cases, the equipotential and stream lines can be drawn and from a study of these curves it is seen that the flow into the auger hole is predominantly horizontal. Hence, the



Permeability by "tube" method

a Determine time t for water level in tube to rise from h_1 to h_2
b From depth-diameter ratio find value of A to use in formula (14) See reference (5) pg 436 and reference (9) pg 137
c Substitute values in eq (14)

Permeability by auger-hole method

a Record values of the time t corresponding to various levels h of the water in the hole
b Plot common logarithms of (d-h) as ordinate against t as abscissa
c If observations were started from h/d=0.2 or nearly so, the first part of the graph will be straight Calculate the slope of this straight part and call it tan B
d Find the value of A from the graph on pg 93 of reference 7
e Calculate the permeability using the formula,

$$k = -\frac{2.303\ \pi\ a^2 \tan B}{A}$$

where π = 3.1416

a = radius of auger hole

Figure 4.

permeability as computed by this auger hole method is predominantly horizontal.

Spangler in his "Soil Engineering" describes this auger-hole method (9) and gives what appears to be a simplified formula for computing the coefficient of permeability. In our experience the formula gave values of the permeability that was much too high. Hence, in our work we followed strictly the method of Van Bavel and Kirkham (7).

The auger-hole method requires several hours to perform. Since the very act of boring an auger hole puddles the soil, it is necessary to fill and empty the hole several times in order to de-puddle the soil pores. Hence, a single determination takes at least a day. The tube method takes much longer. With the thought that it might be of interest, a brief description of both methods is given in connection with Figure 4.

Having thus determined the permeability in the vertical and radial (horizontal) directions, we can use their ratio to compute the value of c_r/c_v. In this way, for Project S 223(2), the value of c_r was estimated to be 0.0146 or 0.015 sq. ft. per hour based on a value of c_v of 0.002 sq. ft. per hour, as determined by the laboratory consolidation test.

The above values of c_v and c_r were used to calculate the time-settlement curves for Stations 25+10 and 25+75 on Project S 223(2). The actual long-term rate of settlement appeared to point to a value of c_r somewhat lower than given in the above and so a ratio of c_r/c_v of 6 was used to calculate the settlements at Stations 47+43 and 48+20.[4] settlement is shown in heavy lines and the calculated settlement in light lines. The observed settlement curves in both cases show a sharp drop at the time of installation of the sand drains. On both projects

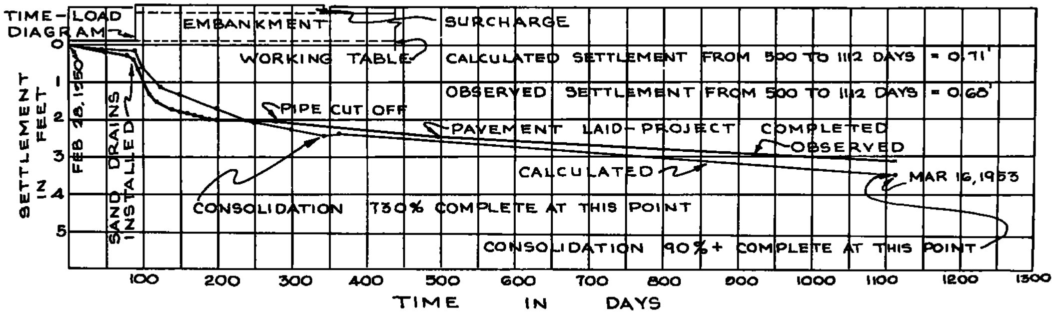


Figure 5. Comparison of calculated versus observed settlements for vertical sand drains over the Kahanaiki swamp, Project F 15 (3), Station 269 + 25.

It is interesting to compare the estimated combined percent of consolidation as of a recent date, April 6, 1953, with the hypothetical percent of consolidation as of the same date calculated on the assumption of vertical drainage only (no sand drains).

TABLE 1
COMBINED PERCENT CONSOLIDATION COMPARED WITH PERCENT CONSOLIDATION DUE TO VERTICAL DRAINAGE ONLY
PROJECT S 223(2)

Station	Date Started	Date Sand Drains Installed	Date Last Data Obtained	Ratio c_r/c_v	Calculated: Consolidation without Sand Drains	Calculated: Consolidation with Sand Drains
					%	%
25+10	10-27-51	1-23-52	4-6-53	7 ½	28 4	91 4
25+75	10-27-51	1-30-52	4-6-53	7 ½	39 6	92 1
47+43	10-25-51	12-11-51	4-6-53	6	25 5	88 8
48+20	10-25-51	12-11-51	4-6-53	6	22 9	88 4

A glance at the last two columns shows that the sand drains were effective in speeding up the settlement. It seems safe to assert that future settlement, if any, will be slight; whereas if sand drains had not been installed, further progressive settlement could be expected resulting in high maintenance costs. The depths at the four stations listed in Table 1 are comparable and from the last column of the table, the combined percent of consolidation does not appear to be greatly affected by a change in the ratio of c_r/c_v from 7½ to 6.

SETTLEMENT CURVES

The observed settlement and the calculated combined settlement were plotted for a number of stations on both projects. An example from each project is presented (see Figs. 5 and 6). The observed

[4] A recheck of our field-permeability test data gave a value of c_r/c_v of slightly less than 6, or a value of c_v of 0.012 sq. ft. per hr.

some lateral plastic displacement was observed, indicating too fast a rate of loading, which probably accounts for the greater rate of observed settlement at the beginning. It can be seen that, if a constant displacement of about ½ foot be added in Figure 6, beginning shortly after the time the sand drains were installed, the calculated and observed settlement curves will agree closely and the above-mentioned plastic displacement would be accounted for. On Project F 15(3) a longitudinal plastic displacement under a bridge abutment appears to have taken place, although actually the problem is more complicated because erosion damage took place during a rainstorm that occurred shortly after the project was completed.

During March and April of 1953, after both projects had been completed and in service for some time, levels were taken and the actual settlements at various points since completion were compared with the previously computed values. Such a comparison is given in Table 2.

Considering the fact that c_r and c_v vary from station to station and in view of the many other uncertainties involved, the observed settlement curves are considered to be in good agreement with the calculated.

CONCLUSION

Based on our experience on the two projects mentioned, vertical sand drains are an effective and satisfactory method of accelerating the consolidation of soft, com-

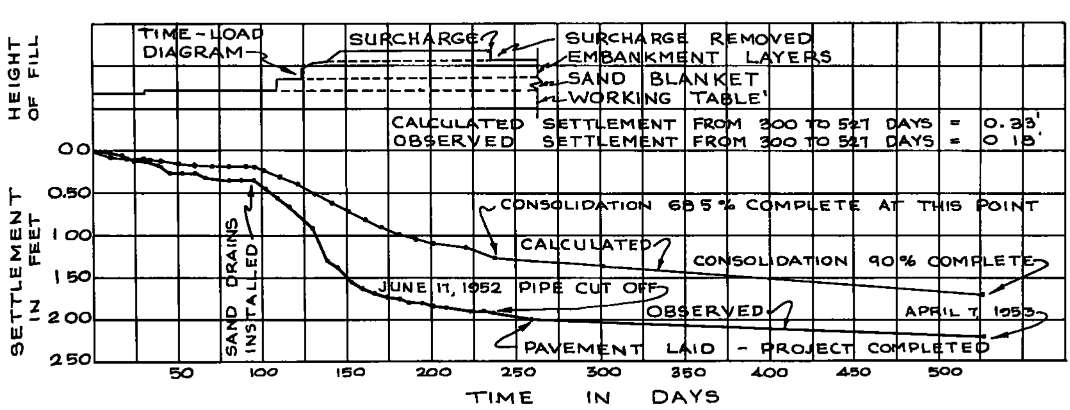


Figure 6. Comparison of calculated versus observed settlements for vertical sand drains over Kiikii swamps, Project S 223 (2) Station 25+75.

TABLE 2
OBSERVED AND CALCULATED SETTLEMENTS SINCE COMPLETION OF SAND DRAIN PROJECTS

Project	Station	Observed Settlement	Calculated Settlement
		ft.	ft.
F 15(3)	268+00	0.05	0.81
F 15(3)	269+25	.68	.71
F 15(3)	271+50	.47	.99
F 15(3)	273+50	.44	.37
S 223(2)	25+10	.17	.49
S 223(2)	25+75	.18	.33
S 223(2)	47+43	.09	.54
S 223(2)	48+20	.22	.57

pressible foundations. Too small a diameter of sand drains is probably not advisable, because of possible difficulties in filling the mandrel with sand and of the arching of the latter due to friction of the sides. The 18-inch-diameter sand drains used on the two projects were of satisfactory size in these respects. In general, compressed air must be used to force the sand out of the mandrel, and both the pressure of the air and rate of withdrawal of the mandrel must be carefully regulated. The spacing of the sand drains can be varied so that reasonably complete consolidation will have taken place by the time the project is completed.

The coefficient of consolidation due to radial drainage of the pore water can be estimated by means of field permeability tests.

In computing the settlement due to vertical drainage of the pore water, it was assumed that the pressure distribution throughout the compressible layer was rectangular, whereas actually it was somewhat trapezoidal. However, because of the relatively small part it plays in the total combined consolidation, it is not believed the error is serious.

The discrepancy between the estimated settlements and the observed settlements for the periods since the completion of the two projects could be due to errors in the values of the coefficients of consolidation, to errors in the estimated ultimate settlements, or to other indeterminate factors.

In the field permeability tests, comparatively large "samples" are involved compared to the usual laboratory samples. Moreover, what is measured is the rate of seepage of the swamp water itself and not distilled water, which may have different seepage characteristics (8). Hence, the field permeability tests lead to more-representative values for the swamp material as a whole, at least theoretically.

ACKNOWLEDGMENTS

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APPENDIX A

Cost Data on Vertical Sand Drains

Kalanianaole Highway, Federal Aid Project F 15 (3)

Number of sand drains	249
Total length	8521 lin. ft.
Average length of drain	34 ft.
Diameter of sand drains	18 in.
Volume of sand required	646 cu. yd.

Labor Cost

Foreman	296 hr. at	$1.75	$518.00	
Riggers	684 " "	1.40	957.60	
Crane Operator	296 " "	1.80	532.80	
Pitman	296 " "	1.00	296.00	
Laborers	780 " "	1.10	858.00	
Truck Drivers*	86 " "	1.25	107.50	
				$3,269.90

Equipment Costs

Crane, leads, and hammer	296 hr. at	$12.00	$3,552.00
Compressors	592 " "	3.50	2,072.00
Air tanks	592 " "	.50	296.00
Post hole digger	110 " "	3.50	385.00
6 c. y. Trucks	86 " "	2.00	172.00
Cost of mandrel (estimated)			1,000.00
			$ 7,477.00
Total cost - labor and material			$10,746.90
Cost per lin. ft. exclusive of sand, taxes, overhead, and profit			1.26

NOTE: On this project, the contractor had free access to a sand pit, so that the cost of sand is not included in the above costs. Haul from sand pit to project approximately 4 miles.

* Hauling Sand from Sand Pit to Project.

Haleiwa Road Cut-off, Federal Aid Project S223(2)

No. of sand drains	390
Total length	17,023 lin. ft.
Average length of drain	43 lin. ft.
Diameter of sand drains	18 inches
Volume of sand required (including spillage)	1,400 c. y.
No. of working days required to complete work	39 days

Equipment Cost (Average Cost Per Day)

1 - P & H Crane	8 hrs. at \$9.80 =	\$ 78.40
1 - Trencher	8 " " 6.15 =	49.20
1 - Hough Loader	8 " " 2.80 =	22.40
1 - 5000# Hammer	8 " " 2.50 =	20.00
1 - 500 cfm Compressor	8 " " 7.00 =	56.00
Total Cost Equipment		\$226.00

Labor Cost (Average Cost Per Day)

1 - Superintendent		\$25.00
2 - Operators	16 hr. at \$2.00 =	32.00
3 - Operators	24 " " 1.85 =	44.40
1 - Mechanic	8 " " 1.80 =	14.40
1 - Laborer	8 " " 1.45 =	11.60
2 - Laborers	16 " " 1.25 =	20.00
1 - Laborer	8 " " 1.10 =	8.80
Total Cost Equipment		\$156.20
W.C. & P.L. Insurance - 2.906%		4.53
		\$160.73

Material Cost

1.400 c. y. at \$1.25 = \$1,750.00

Following is Cost Per Lin. Ft.

Mandrel \$1,000.00 / 17,023 = \$0.0587 per lin. ft.

Equipment Cost 39 days at \$226.00 = \$8,814.00
\$8,814.00 / 17.023 = \$0.5178 per lin. ft.

Labor Cost 39 days at \$160.73 = \$6,268.47
\$6,268.47 / 17.023 = \$0.3682 per lin. ft.

Material Cost
\$1,750.00 / 17,023 = \$0.1028 per lin. ft.
TOTAL \$1.0475 per lin. ft.

SUMMARY:
Cost per lin. ft. of driving sand drains - including vertical holes and sand, back-fill, but exclusive of taxes, overhead, and profit \$1.05.

APPENDIX B

Consolidation Due to Horizontal Flow

The differential equation of consolidation due to radial, horizontal flow is (see eq. (4), Page 291 of "Theoretical Soil Mechanic" by Terzaghi).

$$\frac{\partial P}{\partial t} = c\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right) \quad (1)$$

Where P = hydrodynamic pressure within the pores of the compressible material at any time t

t = time

r = radial distance from the center of a vertical sand drain to the point where the pressure P is measured

c = coefficient of consolidation for flow in the horizontal direction

Boundary Conditions: Let the radii of a sand drain and that of the equivalent circular area drained by it be a and b respectively. Then if the applied load is P_1 per unit area the boundary conditions are

$$P = 0 \text{ for } r = a \quad (2)$$

$$\frac{\partial P}{\partial r} = 0 \text{ for } r = b \quad (3)$$

$$P = P_1 \text{ for } t = 0 \quad (4)$$

The boundary condition (Equation 2) requires that the pressure drop abruptly to zero at the boundary between the sand drain and the compressible material. This condition can be met if the sand drains are free-draining. The boundary condition (Equation 3) requires that there be no flow across the outer radius b of the area drained. This means that the flow at any point must be radially toward the nearest sand drain. Thus the requirement is a reasonable one.

Solution of Equation (1). The solution of Equation (1) subject to the given boundary conditions is

$$P = P_1 \sum_{k=1}^{k=\infty} e^{-u_k^2 ct} A_k B_0 (u_k r) \quad (5)$$

Where

$$B_0 (u_k r) = J_0 (u_k r) - \frac{J_0 (u_k a)}{N_0 (u_k a)} N_0 (u_k r) \quad (6)$$

e = base of Naperian logarithms

$J_0 (u_k r)$ = Bessel function of first kind of zero order

$N_0 (u_k r)$ = Bessel function of the second kind of zero order (Neumann function)

A_k and u_k are constant; to be determined in the manner set forth below.

Determination of u_k: In differentiating Equation (5) with respect to r as called for by the boundary (Equation 3), we note that $B_0 (u_k r)$ is the only factor containing r. Hence the u's must be determined in such a way that

$$\frac{\partial B(ur)}{\partial r} = 0 \quad \text{for } r = b \quad (7)$$

Making use of the relations, -

$$\frac{\partial J_0(ur)}{\partial r} = -uJ_1(ur) \qquad (8)$$

and

$$\frac{\partial N_0(ur)}{\partial r} = -uN_1(ur) \qquad (9)$$

we find that Equation (7) is equivalent to

$$J_1(ub) - \frac{J_0(ua)}{N_0(ua)} N_1(ub) = 0 \qquad (10)$$

Where $J_1(ur)$ = a Bessel function of the first kind of the first order

$N_1(ur)$ = a Bessel function of the second kind of the first order

Equation (10) has an infinite number of roots u_1b, u_2b, u_3b, u_4b, etc. These may be found by a process of successive approximations and the values of u_1, u_2, u_3, u_4, etc., determined.

Determination of A_k: The coefficients A_k are determined by the following equation:

$$A_k = \frac{\frac{2a}{u_k} B_1(u_k a)}{b^2\left[B_0(u_k b)\right]^2 - a^2\left[B_1(u_k a)\right]^2} \qquad (11)$$

Where $B_1(u_k a) = \frac{J_0(u_k a)}{N_0(u_k a)} N_1(u_k a) - J_1(u_k a) \qquad (12)$

For t = 0, Equation (5) reduces to

$$P = P_1 \sum_{r=1}^{r=\infty} A_k B_0(u_k r) \qquad (13)$$

and the purpose of Equation (11) is to determine the coefficients A_k in such a way so that

$$A_1B_0(u_1r) + A_2 B_0(u_2r) + A_3B_0(u_3r) + \text{etc.} = 1 \qquad (14)$$

for all values of r. Then

$P = P_1$ for $t = 0$ for all values of r and the boundary condition Equation (4) will hold true.

Percent Consolidation: We can compute the hydrodynamic pressure for any given point subsequent to $t = 0$ by means of Equation (5). The difference $P_1 - P$ is the loss of pressure and is a measure of the amount of consolidation. Thus the percentage consolidation at a point is,

$$U_p\% = \frac{P_1 - P}{P_1} \times 100 \qquad (15)$$

Where $U_p\%$ = percent consolidation at a given point

To find the average percentage consolidation over the entire area influenced by a sand drain, we need to find the average loss of pressure over the area. Thus,

$$U\% = 100 \times \frac{\Pi(b^2-a^2)P_1 - 2\,\Pi\int_a^b Prdr}{\Pi(b^2-a^2)P_1} \qquad (16)$$

Where U% = average percent consolidation over the entire area influenced by a sand drain.

Evaluating the integral in Equation (16) after first substituting for P from Equation (5), we find

$$U\% = 100 \left[1 - \frac{\Sigma^{2} \frac{a}{u_k} A_k B_1 (u_k a) e^{-u_k^2 ct}}{b^2 - a^2} \right] \quad (17)$$

(The summation in the above is with respect to the successive values of k)

By a suitable transformation, it can be easily shown that the value of U% in Equation (17) depends on a parameter b/a.

By means of Equation (17), the percentage consolidation corresponding to various values of time can be computed and a time-consolidation curve plotted. In general, if we are interested mainly in long-time effects, say consolidation greater than 30%, one term of the series under the summation sign in Equation (17) will give sufficiently accurate results. Many times the coefficient of consolidation c is unknown and it is desirable to plot a general curve from which a time-consolidation curve can be plotted later when the value of c is determined. We can do this by introducing a zero-dimensioned time factor,

$$T = \frac{ct}{4b^2} \quad (18)$$

The exponential factor in Equation (17) then becomes,

$$e^{-u_k^2 ct} = e^{-4b^2 u_k^2 T} \quad (19)$$

By plotting T against U%, we obtain a time-factor, consolidation curve such as Figure 3.

SUMMARY

The steps in computing a time-consolidation curve are as follows:

1. From the spacing of the sand drains determine the radius of the equivalent circular area drained by each. Call this radius b. Also decide on the radius a of the sand drains.
2. By means of tables such as Jahnke and Emde's "Tables of Functions," solve the Equation (10) for values of u. Call these roots (more properly eigen values) in order of magnitude u_1, u_2, u_3, u_4, etc.
3. Having determined the u's, we find from Equation (11) the coefficients A_1, A_2, A_3, A_4, etc., corresponding to the u's.
4. Finally by means of Equation (17), we can compute the percentage consolidation corresponding to various assumed values of t (or T) and thus obtain a time-consolidation curve (or time factor, consolidation curve).

EXAMPLE

Given sand drains 18 inches in diameter (a = 0.75 feet) spaced 17 feet center to center in a geometrical pattern as shown in Figure 6. ($b = \frac{1.05 \times 17}{2} = 8.93$ ft.).

The parameter b/a = 11.9. Compute the percent consolidation due to radial, horizontal flow for a time-factor T = .0542

Solution

1. Values of b and a are as given above.
2. Second step. Compute the values of the u's satisfying Equation (10). This is equivalent to finding the values of u for which the graph of the function crosses the

horizontal zero axis. As a preliminary, assume various values of u, compute the values of the function and note their signs, whether plus or minus. In this way we find that the function changes signs as follows:

from - 0.1072 for u = 0.1 to + 0.3996 for u = 0.2
from + 0.2382 for u = 0.5 to - 0.3013 for u = 0.6
from - 0.8391 for u = 0.8 to + 0.8037 for u = 1.0
from + 0.0637 for u = 1.7 to - 0.0588 for u = 1.73

Therefore the first four roots u_1, u_2, u_3, and u_4, lie between the above values. The first root u_1 lies between 0.1 and 0.2. The algebraic difference between the two values of the function is the change in value of the function as u varies from 0.1 to 0.2. Dividing the change in the function by the change in the value of u gives the derivative or slope of the secant line.

$$\text{Slope} = \frac{0.3996 - (-0.1072)}{0.2 - 0.1} = 5.068$$

Thus the function changes 5.068 units for a unit change in u. But for u = 0.1, the function is -0.1072 and is increasing algebraically as u increases. In order for the function to increase from 0.1072 to zero, the value of u must therefore increase by

$$\frac{0.1072}{0.5068} = 0.0211$$

Hence u = 0.1 + 0.0211=0.1211 to a first approximation. Substituting this value of u in Equation (10), we find that the function does not become zero but has a value of +0.0192.

We next repeat the above approximation for the shorter interval u = 0.1 to u = 0.1211. This second approximation gives us a value of u = 0.1178. For this value of u the function on the left of Equation (10) has the value + 0.0015. By a third approximation, we find u_1 = 0.1175 and for this value of u, the value of the function in Equation (10) is zero for practical purposes. In this way the first four roots of Equation (10) are found to be

$$u_1 = 0.1175$$
$$u_2 = 0.5455$$
$$u_3 = 0.9390$$
$$u_4 = 1.7151$$

3. Third step. Having found the values u_1, u_2, u_3, etc., the next step is to find the values of the coefficients A_k by means of Equation (11). To compute A_1, we need the values of the following:

$$B_1(u_1a) = B_1(0.088)$$

$$B_0(u_1b) = B_0(1.049)$$

Looking up the tabular values of the various functions involved we find,

$$A_1 = \frac{\dfrac{2x.75}{0.1175}\left[(-0.6163)\ (-7.2332) - 0.0440\right]}{8.93^2\left[0.7432 - (-0.6163)\ (0.1255)\right]^2 - 0.75^2\left[-0.6163)\ (-7.2332) - (0.0440)\right]^2}$$
$$= 1.3191$$

In this way the coefficients are found to be

$$A_1 = 1.3191$$
$$A_2 = 0.2168$$
$$A_3 = 0.0612$$
$$A_4 = -0.0849$$

4. Fourth step. In computing the percent consolidation by means of Equation (17), we split the part under the summation sign into two parts as follows:

u_k	$\frac{a}{u_k} B_1(u_k a)$	2A	$2A_k \frac{a}{u_k} B_1(u_k a)$
u_1	28.1731	2.6382	74.3263
u_2	3.6272	0.4336	1.5728
u_3	3.8625	0.1224	0.4728
u_4	-0.7765	-0.1698	+0.1318

and

u_k	$e^{-4b^2 u_k^2 T}$
u_1 =0.1175	0.799
u_2 =0.5455	0.006
u_3 =0.9390	0.0000(-)

The third term is less than the fourth place of decimals. Hence we need to take into account only the first two terms at the most.

Thus U for T = 0.0542 we find is

$$U\% = 100 \times \left[\frac{1 - 74.33 \times 0.799 + 1.573 \times .006}{8.93^2 - .75^2}\right] = 25.01\%$$

Repeating the calculations of Step 4 for various values of T, enough points can be found to enable one to plot a curve such as those of Figure 5 for the particular value of b/a involved.

The above example is not an hypothetical one. It and the curves of Figure 5 were computed for actual projects.

APPENDIX C

Figures A and B are centerline profiles through the sand-drain portions of the two projects. The profiles show: (a) original ground surface prior to construction; (b) location of the water table; (c) variation in thickness along the centerline of the compressible swamp material; (d) finished centerline profile of the pavement as of the respective dates of completion; (e) settlement of the pavement surface since completion and up to the respective dates given; (f) bottom of fill, as of the respective dates of completion, as determined by settlement observations; and (g) bottom of fill, as of the respective dates of completion, as calculated by means of theoretical formulas.

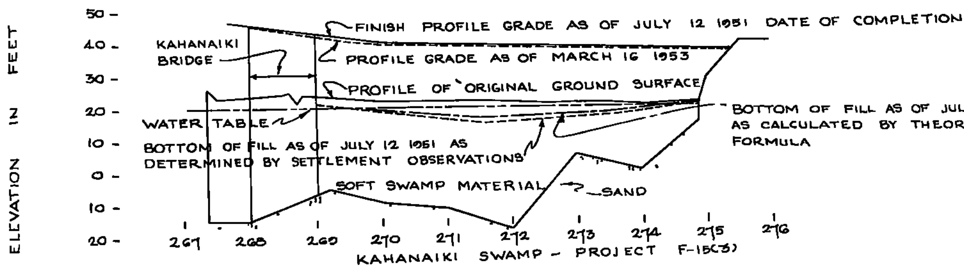

Figure A.

It is to be noted that, as of the respective dates of completion, the calculated percent consolidation over the deeper portions was as low as 70 percent. Additional settlement was expected for at least the following 2 years. But as shown in Table 2 the actual additional settlement proved in most cases to be much less than the calculated. The expected additional settlements are all fractions of a foot and, in view of the relatively crude nature of soil testing and the many uncertainties inherent in the problem, it is not reasonable to expect very close agreement between the calculated and observed values.

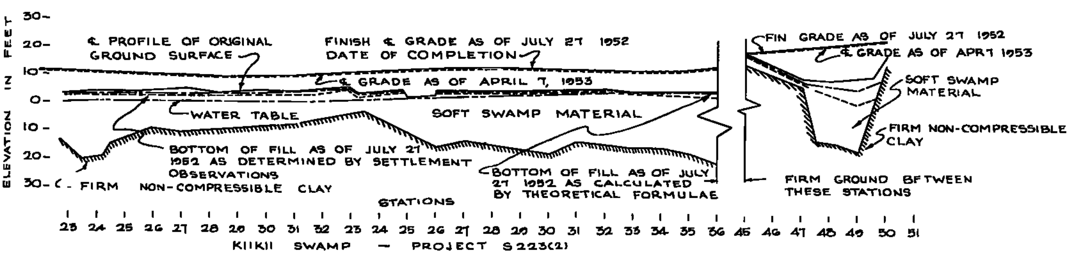

Figure B.

From a practical point of view, what one wishes to know is whether additional large settlements should be expected in the future. Both theory and actual observations up to the present time indicate that, in the case of these two projects, consolidation is now over 90 percent complete, so the chances are that little, if any, additional settlements will take place in the future.

Both swamps are located in low, flat areas and are composed of alluvial materials with a considerable admixture of organic matter. The latter ranges from 12.9 percent to 14.6 percent in the case of the Kahanaiki swamp and from 17.5 percent to 19.0 percent in the case of the Kiikii swamp.

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