Ejercicios propuestos: Parte III

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Ejercicio 1

Función masa de probabilidad:

X/Y	0	2
2	1/16	1/16
4	1/8	1/4
6	0	1/2

Se pide calcular la función generatriz de momentos y, a partir de ella, la matriz de covarianzas.

f.g.m.:

$$M(t_1,t_2)=E[e^{t_1x+t_2y}]=\frac{1}{16}e^{2t_1}+\frac{1}{16}e^{2t_1+2t_2}+\frac{1}{8}e^{4t_1}+\frac{1}{4}e^{4t_1+2t_2}+\frac{1}{2}e^{6t_1+2t_2}=\frac{1}{16}(e^{2t_1}+e^{2t_1+2t_2}+2e^{4t_1}+4e^{4t_1+2t_2}+8e^{6t_1+2t_2})$$

Queremos calcular la **matriz de covarianzas**

$$\begin{pmatrix}\mathrm{Var}(X) & \mathrm{Cov}(X,Y)\\ \mathrm{Cov}(X,Y) & \mathrm{Var}(Y)\end{pmatrix}=\begin{pmatrix}\mu_{20} & \mu_{11}\\ \mu_{11} & \mu_{02}\end{pmatrix}$$

Para hallar los momentos centrados, podemos hallar primero los no centrados. Usaremos la función generatriz de momentos para hallarlos.

$$m_{10}=\frac{\partial M}{\partial t_1}(t_1,t_2)\Big|_{t_1=t_2=0}=\frac{1}{16}(2e^{2t_1}+2e^{2t_1+2t_2}+8e^{4t_1}+16e^{4t_1+2t_2}+48e^{6t_1+2t_2})\Big|_{t_1=t_2=0}=\frac{2+2+8+16+48}{16}=\frac{19}{4}$$

$$m_{01}=\frac{\partial M}{\partial t_2}(t_1,t_2)\Big|_{t_1=t_2=0}=\frac{1}{16}(2e^{2t_1+2t_2}+8e^{4t_1+2t_2}+16e^{6t_1+2t_2})\Big|_{t_1=t_2=0}=\frac{2+8+16}{16}=\frac{13}{8}$$

$$m_{20}=\frac{\partial^2 M}{\partial t_1^2}(t_1,t_2)\Big|_{t_1=t_2=0}=\frac{1}{16}(4e^{2t_1}+4e^{2t_1+2t_2}+32e^{4t_1}+64e^{4t_1+2t_2}+288e^{6t_1+2t_2})\Big|_{t_1=t_2=0}=\frac{4+4+32+64+288}{16}=\frac{49}{2}$$

$$m_{02}=\frac{\partial^2 M}{\partial t_2^2}(t_1,t_2)\Big|_{t_1=t_2=0}=\frac{1}{16}(4e^{2t_1+2t_2}+16e^{4t_1+2t_2}+32e^{6t_1+2t_2})\Big|_{t_1=t_2=0}=\frac{4+16+32}{16}=\frac{13}{4}$$

$$m_{11}=\frac{\partial^2 M}{\partial t_1 t_2}(t_1,t_2)\Big|_{t_1=t_2=0}=\frac{1}{16}(4e^{2t_1+2t_2}+32e^{4t_1+2t_2}+96e^{6t_1+2t_2})\Big|_{t_1=t_2=0}=\frac{4+32+96}{16}=\frac{33}{4}$$

Ya podemos hallar los momentos centrados:

$$\mu_{20}=m_{20}-m_{10}^2=\frac{49}{2}-\left(\frac{19}{4}\right)^2$$
$$\mu_{11}=m_{11}-m_{10}m_{01}$$
$$\mu_{02}=m_{02}-m_{01}^2$$