

# Ejercicios propuestos: Parte III

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## Ejercicio 1

Función masa de probabilidad:

$X/Y$	0	2
2	1/16	1/16
4	1/8	1/4
6	0	1/2

Se pide calcular la función generatriz de momentos y, a partir de ella, la matriz de covarianzas.

**f.g.m.:**

$$M(t_1, t_2) = E[e^{t_1 x + t_2 y}] = \frac{1}{16}e^{2t_1} + \frac{1}{16}e^{2t_1+2t_2} + \frac{1}{8}e^{4t_1} + \frac{1}{4}e^{4t_1+2t_2} + \frac{1}{2}e^{6t_1+2t_2} = \frac{1}{16}(e^{2t_1} + e^{2t_1+2t_2} + 2e^{4t_1} + 4e^{4t_1+2t_2} + 8e^{6t_1+2t_2})$$

Queremos calcular la **matriz de covarianzas**

$$\begin{pmatrix} \text{Var}(X) & \text{Cov}(X, Y) \\ \text{Cov}(X, Y) & \text{Var}(Y) \end{pmatrix} = \begin{pmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{pmatrix}$$

Para hallar los momentos centrados, podemos hallar primero los no centrados. Usaremos la función generatriz de momentos para hallarlos.

$$m_{10} = \frac{\partial M}{\partial t_1}(t_1, t_2) \Big|_{t_1=t_2=0} = \frac{1}{16}(2e^{2t_1} + 2e^{2t_1+2t_2} + 8e^{4t_1} + 16e^{4t_1+2t_2} + 48e^{6t_1+2t_2}) \Big|_{t_1=t_2=0} = \frac{2+2+8+16+48}{16} = \frac{19}{4}$$

$$m_{01} = \frac{\partial M}{\partial t_2}(t_1, t_2) \Big|_{t_1=t_2=0} = \frac{1}{16}(2e^{2t_1+2t_2} + 8e^{4t_1+2t_2} + 16e^{6t_1+2t_2}) \Big|_{t_1=t_2=0} = \frac{2+8+16}{16} = \frac{13}{8}$$

$$m_{20} = \frac{\partial^2 M}{\partial t_1^2}(t_1, t_2) \Big|_{t_1=t_2=0} = \frac{1}{16}(4e^{2t_1} + 4e^{2t_1+2t_2} + 32e^{4t_1} + 64e^{4t_1+2t_2} + 288e^{6t_1+2t_2}) \Big|_{t_1=t_2=0} = \frac{4+4+32+64+288}{16} = \frac{49}{2}$$

$$m_{02} = \frac{\partial^2 M}{\partial t_2^2}(t_1, t_2) \Big|_{t_1=t_2=0} = \frac{1}{16}(4e^{2t_1+2t_2} + 16e^{4t_1+2t_2} + 32e^{6t_1+2t_2}) \Big|_{t_1=t_2=0} = \frac{4+16+32}{16} = \frac{13}{4}$$

$$m_{11} = \frac{\partial^2 M}{\partial t_1 \partial t_2}(t_1, t_2) \Big|_{t_1=t_2=0} = \frac{1}{16}(4e^{2t_1+2t_2} + 32e^{4t_1+2t_2} + 96e^{6t_1+2t_2}) \Big|_{t_1=t_2=0} = \frac{4+32+96}{16} = \frac{33}{4}$$

Ya podemos hallar los momentos centrados:

$$\begin{aligned} \mu_{20} &= m_{20} - m_{10}^2 = \frac{49}{2} - \left(\frac{19}{4}\right)^2 = \frac{31}{16} \\ \mu_{11} &= m_{11} - m_{10}m_{01} = \frac{33}{4} - \frac{19}{4} \frac{13}{8} = \frac{17}{32} \\ \mu_{02} &= m_{02} - m_{01}^2 = \frac{13}{4} - \left(\frac{13}{8}\right)^2 = \frac{39}{64} \end{aligned}$$

Por tanto la matriz de covarianzas es

$$\begin{pmatrix} \frac{31}{16} & \frac{17}{32} \\ \frac{17}{32} & \frac{39}{64} \end{pmatrix}$$