5) Consider the pure birth process:

$$\sum_{n=0}^{\infty} P_n(t) = 1 \quad \forall t \quad \Leftrightarrow \quad \sum_{n=0}^{\infty} \lambda_n P_n = \infty$$

we prove this inductively:

let $\sum_n$ be the minimum exponential holding time $\sum_n \sim Exp(\lambda_n)$

$\therefore \sum_n \sim Exp(\lambda_n) \Rightarrow \{ \{ \{