

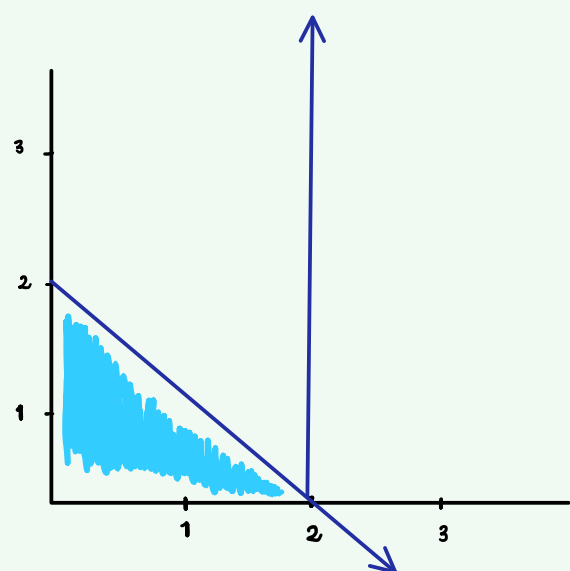
# Taller 8

martes, 29 de marzo de 2022

10:37 a.m.

1.  
a)

$$f_{x,y}(x,y) = \frac{3}{4} y \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 2-x$$



$$\begin{aligned} x &= 0 \\ x &= 2 \\ y &= 0 \\ y &= 2-x \end{aligned}$$

b)

$$f_x(x) = \int_0^{2-x} \frac{3}{4} y \, dy = \left[ \frac{3}{4} \cdot \frac{y^2}{2} \right]_0^{2-x} = \frac{3}{8} (2-x)^2 \quad \text{para } 0 \leq x \leq 2$$

$$f_y(y) = \int_0^{2-y} \frac{3}{4} y \, dx = \left[ \frac{3}{4} y x \right]_0^{2-y} = \frac{3}{4} y (2-y) \quad \text{para } 0 \leq y \leq 2$$

c)  $f_{x|y}(x,y) = \frac{\frac{3}{4} y}{\frac{3}{4} y (2-y)} = \frac{1}{2-y}$

$E_{x|y=1.6} = \int_0^2 \frac{x}{2-y} \, dx = 2.5 \left( \frac{x^2}{2} \right)_0^2 = 5$ . El valor promedio de la función de densidad conjunta es 5.

d)  $E(x,y) = \int_0^2 \int_0^{2-x} x \cdot y \cdot \frac{3}{4} y \, dy \, dx = \int_0^2 \int_0^{2-x} \frac{3}{4} x \cdot y^2 \, dy \, dx = \int_0^2 \left[ \frac{3}{4} x \cdot \frac{y^3}{3} \right]_0^{2-x} \, dx = \int_0^2 \frac{3}{4} x \cdot \frac{(2-x)^3}{3} \, dx$

$$\frac{3}{4} \int_0^2 \frac{8x - 12x^2 + 6x^3 - x^4}{3} \, dx = 0.4$$

e)  $E(x) = \int_0^2 \int_0^{2-x} x \cdot \frac{3}{4} y \, dy \, dx = \int_0^2 \frac{3x^3 - 12x^2 + 12x}{8} \, dx = \frac{1}{2} = 0.5$

$$E(y) = \int_0^2 \int_0^{2-x} y \cdot \frac{3}{4} y \, dy \, dx = \int_0^2 \frac{-x^3 + 6x^2 - 12x + 8}{4} \, dx = 1$$

$$\text{Cov}(x,y) = 0.4 - 0.5 \cdot 1 = -0.1$$

f)  $\rho_{xy} = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y} = \frac{-0.1}{0.387 \cdot 0.447} = -0.5780$

2.

a)  $E_{x|y=1} = (0 \cdot 0.1389) + (1 \cdot 0.4167) + (2 \cdot 0.3056) + (3 \cdot 0.1389) = 1.49$

$$g_{x|y} = \frac{g_{xy}(x_i, y_i)}{g_y(y_i)} = \frac{g_{xy}(x_i, 1)}{0.36} \begin{cases} x=0 & g_{xy}: 0.1389 \\ x=1 & g_{xy}: 0.4167 \\ x=2 & g_{xy}: 0.3056 \\ x=3 & g_{xy}: 0.1389 \end{cases}$$

b)  $\text{Cov}(x,y)$

x	0	1	2	3
g(x)	0.9	0.34	0.29	0.28

$$E(x) = \sum x \cdot g(x)$$

$$= 0 \cdot 0.9 + 1 \cdot 0.34 + 2 \cdot 0.29 + 3 \cdot 0.28$$

$$E(x) = 1.76$$

y	0	1	2	3
g(y)	0.27	0.36	0.25	0.12

$$E(y) = 0 \cdot 0.27 + 1 \cdot 0.36 + 2 \cdot 0.25 + 3 \cdot 0.12 = 1.22$$

$$E(x \cdot y) = \sum x \cdot y \cdot f(x,y) = 2.38$$

$$\text{Cov}(x,y) = 2.38 - 1.76 \cdot 1.22 = 0.2328$$

3.

Valor esperado planta 1.

$$E(x) = \frac{a+b+c}{3} = \frac{15+85+48}{3} = 49.33 \text{ kg de cuero desperdiciado}$$

Valor esperado planta 2

$$E(x) = \frac{a+b+c}{3} = \frac{8+115+72}{3} = 65 \text{ kg de cuero desperdiciado}$$

Valor esperado planta 3

$$E(x) = \frac{a+b}{2} = \frac{5+75}{2} = 40 \text{ kg de cuero desperdiciado}$$

Valor esperado de los valores esperados

$$E(x) = 0.20 \cdot 49.33 + 0.35 \cdot 65 + 0.45 \cdot 40 = 50.616 \text{ kg de cuero desperdiciado por cada lote de zapatos producido.}$$