TALLER 8

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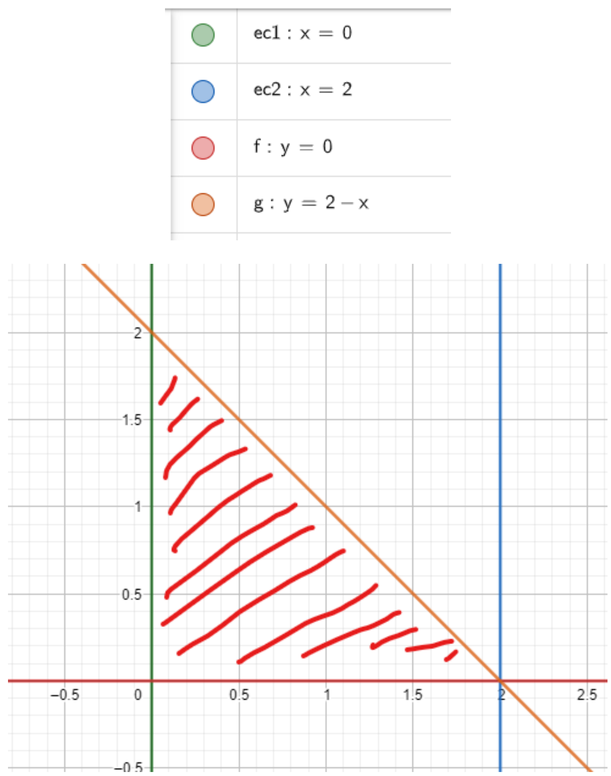
PRIMER PUNTO:

a)



b)

$$f_Y(y) = \int_0^{2-y} \frac{3}{4} y dx = \frac{3}{4} y(x)|_0^{2-y} = \frac{3}{4} y(2-y)$$

$$0 < y < 2$$

$$f_X(x) = \int_0^{2-x} \frac{3}{4} y dy = \frac{3}{4} \left(\frac{y^2}{2}\right)_0^{2-x} = \frac{3}{8}(2-x)^2$$

$$0 < x < 2$$

c)

$$f_{X|Y} = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{\frac{3}{4} y}{\frac{3}{4} y(2-y)} = \frac{1}{2-y}$$

$$E_{X|Y=1.6} = \int_0^2 x \cdot \frac{1}{2-1.6} dx = 2.5 \left(\frac{x^2}{2}\right)_0^2 = 5$$

d)

$$E(XY) = \int_0^2 \int_0^{2-x} xy \cdot \frac{3}{4} y dy\, dx = \frac{3}{4}\int_0^2 x \int_0^{2-x} y^2 dy\, dx = \frac{3}{4}\int_0^2 x \cdot \frac{(2-x)^3}{3} dx$$

$$= \frac{3}{4}\int_0^2 \frac{8x - 12x^2 + 6x^3 - x^4}{3} dx = \frac{3}{4} \cdot \frac{8}{15} = \frac{2}{5} = 0.4$$

e)

$$E(X) = \int_0^2 \int_0^{2-x} x \cdot \frac{3}{4} y dy\, dx = \int_0^2 \frac{3x^3 - 12x^2 + 12x}{8} dx = \frac{1}{2} = 0.5$$

$$E(Y) = \int_0^2 \int_0^{2-x} y \cdot \frac{3}{4} y dy\, dx = \int_0^2 \frac{-x^3 + 6x^2 - 12x + 8}{4} dx = 1$$

$$cov(X,Y) = E[XY] - E[X] \cdot E[Y] = 0.4 - 0.5 \cdot 1 = -0.1$$

f)

$$\sigma_x = 0.387$$

$$\sigma_y = 0.447$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_x \sigma_y} = -\frac{0.1}{(0.387)(0.447)} = -0.578$$

SEGUNDO PUNTO:

a)

Primero hallamos la tabla de probabilidades de x dado y:

g(x|y)		y			
		0	1	2	3
x	0	0	0.13888889	0.12	0.08333333
	1	0.51851852	0.41666667	0.16	0.08333333
	2	0.2962963	0.30555556	0.28	0.25
	3	0.18518519	0.13888889	0.44	0.58333333
TOTAL		1	1	1	1

$$E_{X|Y}(x, y=1) = (0 \cdot 0.1389) + (1 \cdot 0.4167) + (2 \cdot 0.3056) + (3 \cdot 0.1389) = 1.44$$

b)

$$\mu_x = \sum_{x=0}^{3} x \cdot g_x = 0 + 0.34 + 0.58 + 0.84 = 1.76$$

$$\mu_y = \sum_{y=0}^{3} y \cdot g_y = 0 + 0.36 + 0.5 + 0.36 = 1.22$$

$$E(XY) = \sum_{i=0}^{3} \sum_{j=0}^{3} x_i y_j \cdot g_{xy}(x_i, y_j) = 2.38$$

Sale de:

g(x,y)		y				
		0	1	2	3	TOTAL
x	0	0	0.05	0.03	0.01	0.09
	1	0.14	0.15	0.04	0.01	0.34
	2	0.08	0.11	0.07	0.03	0.29
	3	0.05	0.05	0.11	0.07	0.28
TOTAL		0.27	0.36	0.25	0.12	1

x*y		y				
		0	1	2	3	TOTAL
x	0	0	0	0	0	
	1	0	1	2	3	
	2	0	2	4	6	
	3	0	3	6	9	

g(x,y)*xy		y				
		0	1	2	3	TOTAL
x	0	0	0	0	0	0
	1	0	0.15	0.08	0.03	0.26
	2	0	0.22	0.28	0.18	0.68
	3	0	0.15	0.66	0.63	1.44
TOTAL		0	0.52	1.02	0.84	2.38

$$cov(x, y) = E(XY) - E(X) \cdot E(Y) = 2.38 - 1.76 \cdot 1.22 = 0.2328$$

PUNTO TRES:

Las funciones de probabilidad están dadas por:

$$g_{p1} = \begin{cases} \dfrac{2(x-15)}{(85-15)(48-15)} & 15 \leq x \leq 48 \\ \dfrac{2(85-x)}{(85-15)(85-48)} & 48 < x \leq 85 \\ 0\ d.l.c \end{cases}$$

$$g_{p1} = \begin{cases} \dfrac{2(x-8)}{(115-8)(72-8)} & 8 \leq x \leq 72 \\ \dfrac{2(115-x)}{(115-8)(115-72)} & 72 < x \leq 115 \\ 0\ d.l.c \end{cases}$$

$$g_{p1} = \begin{cases} \dfrac{1}{75-5} & 5 \leq x \leq 75 \\ 0\ d.l.c \end{cases}$$

De forma simplificada:

$$g_{p1} = \begin{cases} \dfrac{(x-15)}{1155} & 15 \leq x \leq 48 \\ \dfrac{(85-x)}{1295} & 48 < x \leq 85 \\ 0\ d.l.c \end{cases}$$

$$g_{p1} = \begin{cases} \dfrac{(x-8)}{3424} & 8 \leq x \leq 72 \\ \dfrac{2(115-x)}{4601} & 72 < x \leq 115 \\ 0\ d.l.c \end{cases}$$

$$g_{p1} = \begin{cases} \dfrac{1}{70} & 5 \leq x \leq 75 \\ 0\ d.l.c \end{cases}$$

$$E(p_1) = \int_{15}^{48} x \cdot \frac{(x-15)}{1155} dx + \int_{48}^{85} x \cdot \frac{(85-x)}{1295} dx = \frac{1221}{70} + \frac{6697}{210} = 49.3$$

$$E(p_2) = \int_{8}^{72} x \cdot \frac{(x-8)}{3424} dx + \int_{72}^{115} x \cdot \frac{2(115-x)}{4601} dx = \frac{9728}{321} + 34.6947 = 65$$

$$E(p_3) = \int_{5}^{75} x \cdot \frac{1}{70} dx = 40$$

$$\mu(x) = 49.3 \cdot 0.2 + 65 \cdot 0.35 + 40 \cdot 0.45 = 50.61$$