# Transient Heat Conduction and Convection Lumped and Non-Lumped Analysis of Copper and Stainless Steel Spheres

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ME419: Heat Transfer

## Background Information

To model the temperature of a solid body, analysis must be done to predict its reaction to a change in its thermal environment. There are two main methods to analyze the temperature within a solid during a thermal transition. The first, the "lumped capacitance method," assumes that the thermal gradient within the solid is negligible and simplifies the analysis of its reaction to thermal changes. For more complex systems, this method can give inaccurate results and that is where the non-lumped analysis must be used. In order to determine when the simplified lumped analysis can be used to solve transient heat transfer problems, the Biot number must be calculated.

The Biot number is the comparison of the thermal resistance associated with heat conduction versus the thermal resistance associated with heat convection of the solid. For lumped analysis, the Biot number must be less than 0.1 which signifies that the resistance due to conduction is much larger than the resistance due to convection. In simple terms, the effects of conduction occur so rapidly with relatively little resistance, that the entire solid body can be considered one temperature. A Biot number that is greater than 0.1 means that each of the solid body's thermal layers be taken into account, deeming it a non-lumped system.

This experiment measures the transient thermal response of two spheres when moved from a hot water bath to a cold water bath, and vice-versa. Copper (high thermal conductivity) and Stainless Steel (lower thermal conductivity) spheres are used and a thermal couple measures both the center and surface temperature of each sample. Through analysis of the Biot number, it can be predicted whether lumped analysis for each sample may be used and this experiment verifies the validity of the Biot number prediction by comparing the surface and center temperatures.

$$Biot\ Number = \frac{R_{conduction}}{R_{convection}}$$

$$R_{conduction} = \frac{l}{k}\ R_{convection} = \frac{1}{h}$$

where $l\ (m)$ is the length scale of the object, $k\left(\frac{W}{m*°C}\right)$ is the thermal conductivity coefficient and $h\left(\frac{W}{m^2*°C}\right)$ is the convection coefficient.

$$k_{Copper} = 401\left(\frac{W}{m*°C}\right),\ k_{Stainless\ Steel} = 13.4$$

## Hypothesis

Due to the high conductivity of Copper, it was predicted that the center and surface of the copper sphere will be the same and a lumped analysis may be used for the sample. For the Stainless Steel, with a lower conductivity, it is unclear whether a lumped analysis could be used because although metals are typically good thermal conductors, but it may not be conductive enough to be considered a lumped system. In both cases, the system can be modeled as a non-lumped system because the lumped analysis is derived from the non-lumped analysis by simplifying it with certain assumptions.

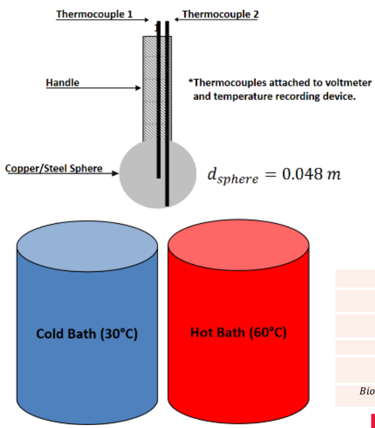
## Measurement Methods

Two water baths of equal size and volume are filled with 30°C and 60°C water and four trials will measure the temperature within a sphere moving from one bath to another.

| Trial | Sphere | Transition |
|---|---|---|
| 1 | Copper | Cold to Hot |
| 2 | Copper | Hot to Cold |
| 3 | Stainless Steel | Cold to Hot |
| 4 | Stainless Steel | Hot to Cold |

The Software in LabView records the temperature of both the surface and the center of each sphere sample for each trial. Each trial is ran until the system reaches equilibrium, meaning no more changes in both the surface and center temperatures. This procedure determines whether or not the sample's surface and center temperature are the same throughout the entire experiment and can be analyzed using a lumped analysis. If the surface and center temperature converge only over time, then a non-lumped analysis must be used.

Copper and 316 Stainless Steel are ideal materials to use for this testing because of their differences in thermal conductivity ($k$) values. Copper has a high conductivity which leads to the prediction that a lumped system analysis may be used. Stainless Steel has a much lower thermal conductivity value.




Ideal measurement set up with cold and hot baths being 30°C and 60°C, respectively. The spheres are Copper and 316 Stainless Steel.

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## Analysis/Results

Figure 1: Temperature vs. Time, Cooling

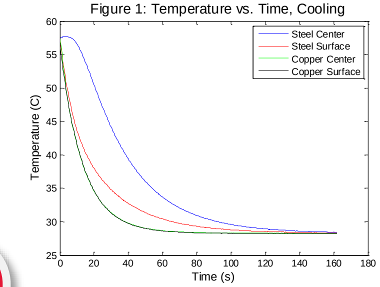
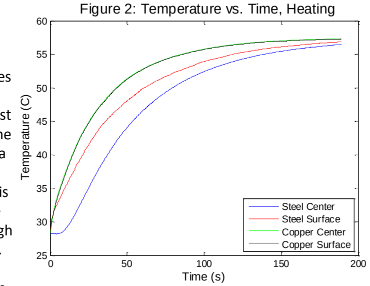



Figure 2: Temperature vs. Time, Heating




From figures 1 and 2, it is evident that the temperature of Stainless Steel's center temperature lagged the surface temperatures when heating or cool. As for Copper, the surface and center temperatures were almost exactly the same. Therefore, this supports the hypothesis that Copper can be analyzed as a lumped system the surface and center temperatures since were the same. Also, this concludes the question whether or not the Stainless Steel's thermal conductivity was high enough to be considered a lumped system. Since the center temperature lagged the surface temperature in both cases, Stainless Steel can only be analyzed using a non-lumped analysis.

Figure 3: Temperature Difference vs. Time, Cooling

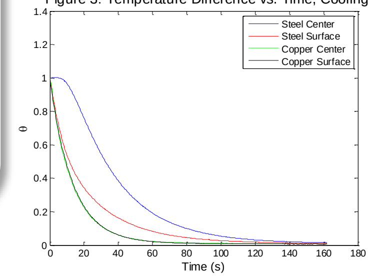
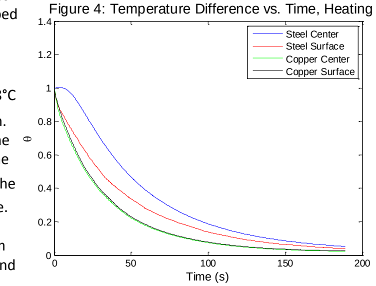



Figure 4: Temperature Difference vs. Time, Heating




Figures 3 and 4 represent the equation, $\theta = \frac{T-T_\infty}{T_i-T_\infty}$, with $T_\infty$ being approximately 28°C for the cold bath and 58°C for the hot bath. The quicker the line goes to zero means, the faster the effects of convection effected the sphere. Since $\theta = \exp\left(-\frac{t}{\tau}\right)$ and $\tau = \frac{\rho cV}{hA}$, the smaller $\tau$ is, the faster $\theta$ changes over time. Therefore, for materials with higher convection coefficient values (Copper from figures 3 and 4), the faster $\theta$ went to zero and the temperature reached $T_\infty$.

Figure 5: ln(θ) vs. Time, Cooling

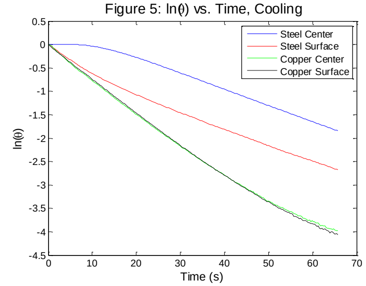
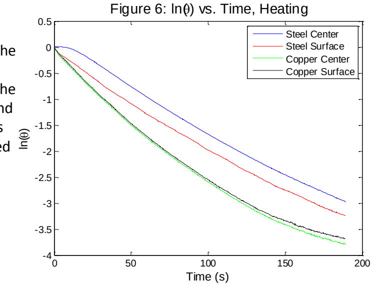



Figure 6: ln(θ) vs. Time, Heating




Similar to figure 3 and 4, figures 5 and 6 display the steeper the slope of the line is, the faster the sphere approaches $T_\infty$. The following equations and table summarizes the calculation of the convection coefficients and Biot numbers for the Copper and Stainless Steel spheres using lumped and non-lumped analysis:

Lumped Analysis

$$h_{lumped} = -\frac{\ln(\theta)}{t}\frac{(\rho c_p V)}{A_s}$$

$$Biot\ Number = \frac{hr_0}{k}$$

| | Copper | Stainless Steel |
|---|---|---|
| $\ln(\theta)/t(\frac{1}{s})$ (heating) | -0.0196 | -0.01685 |
| $h\ (lumped)\left(\frac{W}{m^2*°C}\right)$ | 539.7 | 519.7 |
| $h\ (non-lumped)$ | 551.4 | 675.6 |
| $\Delta h$ (% difference) | 11.7 (2%) | 155.9 (23%) |
| $Biot\ Number\ (center, non-\ lumped)$ | 0.033 | 1.210 |

Non-lumped Analysis (assuming A=1)

$$\frac{\ln(\theta)}{t} = -\frac{\lambda^2\alpha}{r_0^2},\quad \alpha = \frac{k}{\rho c_p}$$

$$\lambda = \sqrt{-\frac{\ln(\theta)}{t}\frac{r_0^2}{\alpha}}$$

A one term approximate solution of transient one-dimensional heat conduction chart is used to calculate the Biot number from $\lambda$ and the convection coefficient can be found using the Biot number equation above.

## Conclusion

This experiment determined whether Copper and Stainless Steel spheres can be analyzed as a lumped system. The hypothesis predicted that Copper could be considered as both a lumped and non-lumped system, which was true as Copper's surface and center temperature were the same. But, the hypothesis was not sure whether or not Stainless Steel could be considered a lumped system. The results showed that Stainless Steel's center temperature lagged its surface temperature proving that it must be analyzed only as a non-lumped system. Furthermore, when solving for the convection coefficient, it was evident that the values for both the lumped and non-lumped analysis for Copper were very similar. Conversely, the values for the Stainless Steel varied by 23 percent. This verifies that non-lumped and lumped analysis can both be used accurately for materials with a small enough Biot number (0.033 for Copper).