BU College of Engineering

ME 303 Fluid Mechanics

Laboratory Exercise No. 2 – Instruction Manual

# Friction Loss in Pipes

Spring 2009 Revision

# Revision History

| Name | Date | Notes |
|---|---|---|
| Prof. Bala Bharadvaj | 1984 | Original Document |
| Ronal Garrigues and Prof. Isaacson | 1986 | New apparatus build |
| Mike Ravicz | 1988 | Apparatus modified |
| Steven P. Weibel and Sam Bogan | 1989 | Instructions revised |
| R. Adam Rugg | 1991 | Revised |
| Joe Estano, David Campbell and Prof. Isaacson | 2000 | New apparatus built |
| Mikhail Dvilyanskiy, David Kelly and Mitchell Raymond | | Instructions revised |
| Profs. Wroblewski and Isaacson | 2001 | |
| Prof. Barba and Simon Layton | 2009 | Transferred to LaTeX by Simon Layton |

# 1 Purpose

In this experiment you will be measuring the pressure drop as a function of the flow rate for the flow of water in uniform, circular cross section pipes. Pipes of different diameters, lengths, and entry lengths will be tested. The data will then be analyzed to yield the values of the dimensionless friction factor, $f$ (representing the pressure drop), as a function of the Reynolds number, $Re$ (representing the flow rate). In the laminar regime, for fully developed flow in all straight, circular cross section pipes, the friction factor is predicted theoretically to be a single curve given by

$$f = \frac{64}{Re}, \tag{1}$$

independent of pipe wall roughness, diameter and length. In the turbulent regime, for fully developed flow, only experimental data are available to compare to, since no universally valid theory exists. In this regime, walls of different roughness give rise to different curves of friction factor vs. Reynolds number. The *Moody chart* summarizes these relationships between friction factor and Reynolds number for pipes of different relative roughness. In this experiment, you will verify the behavior of the friction factor as a function of the Reynolds number, as given in the Moody chart in both the laminar and turbulent regimes.

A second objective of the experiment is to investigate the effect of the "hydrodynamic entry length" on the behavior of the friction factor as a function of the Reynolds number, and thus the applicability of the Moody chart. When a fluid first flows from a larger vessel into the inlet of a pipe, the flow velocity is nearly the same at every point across a given cross section of the pipe. As the fluid flows down the pipe, friction along the pipe wall slows down fluid near the wall. The farther the fluid flows down the pipe, the farther into the center of the pipe this friction effect penetrates. Therefore, the velocity at different points across the cross section (i.e., at different radial distances out from the center line of the pipe) varies as a function of both radial distance from the center line and as a function of distance along the pipe from the inlet. At some distance along the pipe from the pipe inlet the friction effect completely penetrates to the centerline of the pipe. From this point on, along the pipe, the fluid velocity no longer varies as a function of distance along the pipe, but only as a function of radial position out from the center line. This is called *fully developed* flow. The distance along the pipe from the inlet to this location is called the "hydrodynamic entry length." The hydrodynamic entry length depends on whether the flow is laminar or turbulent. (If there is very little disturbance in a flow, it can be laminar at the same Reynolds number for which it would normally be turbulent). The hydrodynamic entry length is longer for laminar flow than for turbulent flow at the same Reynolds number. In addition, the pipe that is used in this experiment has a very smooth inlet which results in very little disturbance to the flow at the entry point to the pipe. Also the pipe itself is very smooth, and the use of a constant head tank further reduces flow disturbances. Under such conditions, even with high enough Reynolds numbers for turbulence to occur in the pipe, the flow is initially laminar in the section immediately following the inlet and develops into a fully turbulent flow only a certain distance after the inlet. In this case, the hydrodynamic entry length can be approximated roughly as $500,000(D/Re)$[1], where $D$ is the diameter of the pipe and $Re$ is the Reynolds number. Note that for purely laminar flow with no turbulence occurring anywhere in the pipe, the hydrodynamic entry length is approximated by $0.03(D)(Re)$[2]. The Moody chart is strictly valid only for pipes in which the flow is fully developed. In practical applications, it can be used from end to end of a pipe, as long as the overall length of the pipe is much longer than the hydrodynamic entry length. In this experiment measurements from both upstream (near entrance) and downstream (farther from entrance) sections of the pipe will be taken and compared to observe the effect of the entry length phenomenon on the applicability of the Moody chart.

# 2 Theory

In fluid mechanics, the available mechanical energy in a flow is called the total head, and consists of a gravitational term, a pressure term, and a velocity term. Total head is defined as:

$$H = z + \frac{p}{\rho g} + \frac{V^2}{2g}. \tag{2}$$

For an ideal fluid, which is considered to be inviscid, there is no energy loss due to friction, and flow energy is conserved. For a real fluid flowing through a pipe, however, the wall of the pipe is in contact with the fluid, and a frictional force (due to fluid viscosity) between the moving fluid and the fluid next to the wall tends to retard its motion. The energy dissipated in this viscous interaction between the fluid and the wall manifests itself as a decrease in the total head, the "friction head loss," $h_f$. Since $h_f$ is merely the difference in head between two points in a flow,$h_f$ may be written as

[1]Ludwig Prandtl, Essentials of Fluid Dynamics, Blackie & Son Limited, London, 1967 edition, p. 167. Translated from the original German edition published in 1949.

[2]Note, even if $z_1 \neq z_2$, it can be shown that Equation (4) is still valid if $p_1 - p_2$ is determined from measurements using a differential water manometer.

$$-h_f = (H_2 - H_1) = (z_2 - z_1) + \frac{1}{\rho g}(p_2 - p_1) + \frac{1}{2g}(V_2^2 - V_1^2), \tag{3}$$

in which subscript "2" refers to downstream, subscript "1" refers to upstream, and $h_f$ is considered a positive quantity.

If the pipe is horizontal, $z_1 = z_2$, and if the pipe has a uniform diameter, $V_1 = V_2$, so the previous relationship for $h_f$ simplifies to

$$h_f = \frac{(p_1 - p_2)}{\rho g}, \tag{4}$$

or, solving for the pressure drop associated with the friction head loss,

$$\Delta p = \rho g h_f, \tag{5}$$

in which $\Delta p = p_1 - p_2$.

The pressure drop due to friction in a circular pipe, $\Delta p$, is actually a function of six variables: pipe length $L$, pipe diameter $D$, surface roughness of the pipe $\epsilon$, fluid density $\rho$, fluid viscosity $\mu$, and average fluid velocity $V$. Using methods from dimensional analysis, it is possible to show that

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = funct\left[\left(\frac{L}{D}\right), \left(\frac{\epsilon}{D}\right), \left(\frac{\rho V D}{\mu}\right)\right]. \tag{6}$$

The term $\frac{\rho V D}{\mu}$ is readily recognized as $Re$. Also, it is an experimental fact that $\frac{\Delta p}{\rho V^2}$ is linearly dependent on $\frac{L}{D}$, so the previous relation may be rewritten

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \left(\frac{L}{D}\right) funct\left[\left(\frac{\epsilon}{D}\right), Re\right]. \tag{7}$$

Remembering that $h_f = \frac{\Delta p}{\rho g}$, finally

$$\frac{h_f}{\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)} = funct\left[\left(\frac{\epsilon}{D}\right), Re\right] = f, \tag{8}$$

where $f$ is called the friction factor and is the dimensionless head loss due to friction. Therefore, from equations (7) and (8), the relationship between pressure drop and friction factor is:

$$f = \frac{\Delta p}{\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho V^2\right)}. \qquad \text{(experimental)} \tag{9}$$

In fully developed laminar flow, the head loss due to friction may be derived from laminar flow theory to be

$$h_f = 32\frac{\mu}{\rho g}\frac{L}{D^2}V. \qquad \text{(theoretical)} \tag{10}$$

This may be non-dimensionalized to yield the theoretical friction factor for laminar flow

$$f = \frac{64}{Re}. \qquad \text{(theoretical)} \tag{11}$$

In turbulent flow, energy is dissipated by turbulent mixing as well as by viscous interaction with the wall. Consequently, both the head loss and friction factor become impossible to predict theoretically. However, if the variables on the right hand side of equation (9) are determined from experimental measurements, experimental values of $f$ can be calculated from equation (9) for both turbulent and laminar flow. The accepted values of $f$ for fully developed laminar and turbulent flows are tabulated in the Moody chart, attached at the back of this manual.

# 3 Prelab

1. Use dimensional analysis to derive equation (6), i.e. reduce the relationship among seven dimensioned variables ($\Delta p = funct[L, D, \epsilon, \rho, \mu, V]$) to a relationship among four dimensionless variables. **You may use the Buckingham Pi Theorem.**

2. Derive equation (11) from equation (10).

3. For the small pipe used in the experiment, calculate the approximate distance downstream of the inlet that corresponds to the hydrodynamic entry length for Reynolds numbers of $3,000$ - $5,000$ (see "Purpose" section). Based on this, what conclusion can you make with respect to the applicability of the Moody chart to predict the behavior of the friction factor vs. Reynolds number in the upstream (between $1^{st}$ and $2^{nd}$ taps) and downstream (between $2^{nd}$ and $3^{rd}$ taps) sections of the small pipe?

4. Using the calibration data attached at the end of this manual, complete the values for flow velocity ($V = \frac{\dot{Q}}{A}$) and Reynolds number corresponding to each setting of the flowmeters in tables 1 and 2. See sample calculation, attached.

# 4 Procedure

1. The teaching fellow will set up the pipe flow apparatus for you. Make sure to check the system for air bubbles (even tiny ones) which can cause errors in readings. Be especially careful to check the manifold, valves, and the tubing lines connected to it. Squeeze each tubing line where it is connected to the manifold several times to see if there are any air bubbles trapped in the connection.

2. Open the stopcock valves leading to the incline manometer on both high and low pressure manifolds. Then, on the high pressure manifold open the stopcock valve indicated "Small Pipe, front," and on the low pressure manifold open the valve indicated "Small Pipe, end." With this configuration, the pressure difference is being read over the *entire length* of the pipe.

3. Set the flow level to zero in all three flowmeters. Check that the incline manometer is zeroed properly by reading off the values for high and low pressure that it indicates. They should be the same at zero flow, if the manometer is zeroed properly. If they are not the same (within one half of a millimeter) recheck the system for trapped air bubbles and check the inclination of the manometer. Record the values in Table 1. (*Tip*: to accurately take a reading on an incline manometer use a straight edge to read off values. Hold it perpendicularly across the scale aligned with scale marks on both edges of the scale and align it with the bottom of the meniscus). Close the "Small Pipe, end" valve on the low pressure manifold and open the "Small Pipe, middle" valve on the same manifold. The pressure difference is now being read over the *upstream section* of the pipe. Check again that the manometer is zeroed by making sure that two readings on the manometer are the same and recording them. Finally, close the "Small Pipe, front" valve on the high pressure side and open "Small Pipe, middle" valve on the same manifold; then close the "Small Pipe, middle" valve on the low pressure side and open the "Small Pipe, end" valve on the same manifold to take the reading of pressure difference over the *downstream section* of the pipe. The values on the manometer should still be the same. Record these values in Table 1.

4. (a) Set up the system so that you are reading pressure difference over the entire length of the small pipe. Direct the flow through the small diameter pipe by opening the yellow control valve #11. Increase the flow level to $10\%$ on the small flowmeter. (Note: the water to the small flowmeter should be going through the constant head tank. Make sure there is a little bit of overflow from the constant head tank to the drain line this insures that the water level in the tank is constant.)

Allow the system to reach equilibrium by waiting for about $2$ minutes before taking measurements. Record the manometer readings for high and low pressure from the incline manometer in Table 1.
(b) Then take readings for the upstream and downstream configurations (half pipe lengths) at the same flow rate (check that the flow rate remains constant). Again, allow $2$ minutes each time before taking a reading.

5. Continue taking readings for all three configurations of the small pipe at each flow rate indicated in Table 1. When you need to switch to the vertical manometer (when you run out of range on the incline manometer), close the incline manometer stopcock valves and open the vertical manometer valves. Set the flow to zero and check that the manometer is properly zeroed by making sure both high and low pressure sides read the same values. Record these values in Table 1. If they are different, ask the GTF to purge the lines of air bubbles and re-zero the manometer. Then continue with your measurements.

6. Close the yellow control valve to the small pipe, labeled "11", and open the control valve to the large pipe, labeled "13" (second pipe from the top). For this pipe, only take measurements over the entire length of the pipe.

7. Take measurements for the large pipe for the flow rates indicated in Table 2, using the incline manometer. Check that the manometer is properly zeroed before taking measurements. If it isnt, ask the GTF to purge the lines of air bubbles and re-zero the manometer.

8. Include photo copies of your data tables in the data section of your laboratory report.

# 5 Analysis and Results

1. Calculate and record in tables 1 and 2 the actual pressure differences for each flow level for both pipes. Note that for values from the incline manometer you will have to divide the indicated pressure difference (difference in manometer scale readings) by $2$ to obtain the actual result, because the manometer is at a $30$ degree angle ($\sin 30° = 0.5$). Convert and record the values in units of $Pa$. See "Sample Calculation" attached.

2. Calculate the experimental friction factors for all flow levels (both laminar and turbulent regimes) from equation (9). Assume that $\rho = 997.1 \frac{kg}{m^3}$.

3. Read off the values for $f$ from the Moody chart for the Reynolds numbers at which the experiment was run. Assume smooth pipes. Note: In the laminar range, the Moody chart values come from equation (11), which gives a straight line on the log-log plot (Moody chart). In the turbulent range the values must be read from the chart.

4. Include in your laboratory report a summary of your calculations in the form of a table with columns for flowmeter setting, $\dot{Q}$, $V$, $Re$, $p_1$, $p_2$, $\Delta p_{\text{actual}}$, $f_{\text{experimental}}$ and $f_{\text{moody}}$ (see attached example). This can be done most easily using a spreadsheet program such as Excel. You may just copy the Excel table into your laboratory report, but be sure to have column labels on your table. Using Excel also makes it easy to plot the results graph. Also include in your laboratory report a sample calculation for your first data point, similar to the attached example, but at a different flowmeter setting.

5. Plot on separate log-log graphs (one for each pipe configuration) the Moody chart values for $f$ along with the experimental values. Use symbols without lines for the experimental values and lines with no symbols for the Moody chart values. See the example plot attached. Include these plots in your laboratory report.

6. Plot on a single graph (log-log) the Moody chart values for $f$ along with experimental values for all configurations for both pipes. Use different symbols for experimental values for different configurations as well as for different pipes, and lines without symbols for the Moody chart values. Include this plot in your laboratory report. How many Moody charts do you have? Reflect on the meaning of the dimensionless groups.

# 6 Questions

1. Comment on the behavior of your experimental data in the turbulent transition region, (Reynolds numbers roughly from $2,000$ to $5,000$), especially comparing the two plots from the upstream and downstream sections of the small pipe. What does this say about the accuracy and usefulness of the Moody chart with respect to the hydrodynamic entry length?

2. Outside of the transition region did all of your data collapse to a single curve as predicted by dimensional analysis?

3. Comment on the validity of laminar flow theory in the laminar range. How close did the experimental results come to the theoretical values? Did they lie along

a straight line on the log-log plot? How reliable is your data point at the lowest value of Re? If the manometer column lengths were off by $\pm 1mm$ at the lowest value of $Re$, would this be enough to reconcile the theoretical and experimental values? Are the results as "sensitive" to small errors in the pressure readings at higher values of $Re$?

4. Are the pipes in the pipe apparatus relatively smooth? How do you know?

5. Describe some sources that could introduce error in this experiment. Be precise in your description.

*Sample Calculation - Small Pipe, full length, Flow rate = 30% (incline manometer)*

Small pipe dimensions:

Distance between inlet and first tap = $36in$

$D = 0.344in = 0.0087376m$

$L = 96in = 2.4384m$

$A = 5.996 \times 10^{-5}m$

| Setting (%) | High ($cm$ of water) | Low ($cm$ of water) |
|---|---|---|
| 0 | 27.9 | 27.9 |
| 30 | 30.5 | 25.65 |

$$\begin{aligned} p_1 - p_2 &= (p_1 - p_{1,0}) - (p_2 - p_{2,0}) = (30.5 - 27.9) - (25.65 - 27.9) \\ &= 30.5 - 25.65 = 4.85cm = 48.5mm \end{aligned}$$

$$\Delta p_{\text{actual}} = \frac{p_1 - p_2}{2} = \frac{48.5}{2} = 24.25mm = 237.81(Pa) \tag{12}$$

$$\dot{Q}(30\%) = 0.228gal/min = 1.438 \times 10^{-5}m^3/s \tag{13}$$

$$V = \frac{Q}{A} = \frac{1.438 \times 10^{-5}}{5.996 \times 10^{-5}} = 0.2398m/s \tag{14}$$

$$Re = \frac{VD}{\nu} = \frac{0.2398m/s \cdot 0.0087376m}{0.989^{-6}m^2/s} = 2333.27 \tag{15}$$

$$f_{\text{experimental}} = \frac{2 \cdot \Delta p \cdot D}{L \cdot \rho \cdot V^2} = \frac{2 \cdot (237.81Pa) \cdot 0.0087376in}{2.4384m \cdot 997.1kg/m^3 \cdot 0.2398^2 m^2/s^2} = 0.0297 \tag{16}$$

$$f_{\text{theoretical}} = \frac{64}{Re} = \frac{64}{2333.27} = 0.0274 \tag{17}$$

**Sample Calculation Table**

| Setting | $\dot{Q}$ (gal/min) | V (m/s) | Re | $p_1$ (cm) | $p_2$ (cm) | $\Delta p_{actual}$ (Pa) | $f_{exp.}$ | $f_{moody}$ |
|---|---|---|---|---|---|---|---|---|
| **Small pipe – full length – small flowmeter:** | | | | | | | | |
| 10 | | | | | | | | |
| ……. | | | | | | | | |
| 30 | 0.228 | 0.2398 | 2333.24 | 30.5 | 25.65 | 237.811 | 0.0297 | 0.0274 |
| ……. | | | | | | | | |
| ……. | | | | | | | | |
| **Small pipe – full length – medium flowmeter:** | | | | | | | | |
| …… | | | | | | | | |
| …… | | | | | | | | |
| **Small pipe –upstream half length – small flowmeter:** | | | | | | | | |
| …… | | | | | | | | |
| …… | | | | | | | | |
| **Small pipe – upstream half length – medium flowmeter:** | | | | | | | | |
| …… | | | | | | | | |
| …… | | | | | | | | |
| **Small pipe – downstream half length – small flowmeter:** | | | | | | | | |
| …… | | | | | | | | |
| …… | | | | | | | | |

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Table 1: Small Pipe

| Setting | Flow Velocity $m/s$ | Reynolds Number | Full Length High $(cm)$ | Low $(cm)$ | $\Delta p$ $(Pa)$ | Upstream High $(cm)$ | Upstream Low $(cm)$ | Upstream $\Delta p$ $(Pa)$ | Downstream High $(cm)$ | Downstream Low $(cm)$ | Downstream $\Delta p$ $(Pa)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Small Flowmeter | | | | | | | | | | | |
| 0 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |
| 16 | | | | | | | | | | | |
| 20 | | | | | | | | | | | |
| 26 | | | | | | | | | | | |
| 30 | | | | | | | | | | | |
| 40 | | | | | | | | | | | |
| 50 | | | | | | | | | | | |
| 70 | | | | | | | | | | | |
| 90 | | | | | | | | | | | |
| Medium Flowmeter | | | | | | | | | | | |
| 0 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |
| 20 | | | | | | | | | | | |
| 40 | | | | | | | | | | | |
| 60 | | | | | | | | | | | |
| 80 | | | | | | | | | | | |

Table 2: Large Pipe

| Setting | Flow Velocity $(m/s)$ | Reynolds Number | High $(cm)$ | Low $(cm)$ | $\Delta p$ $(Pa)$ |
|---|---|---|---|---|---|
| Medium Flowmeter | | | | | |
| 0 | | | | | |
| 10 | | | | | |
| 20 | | | | | |
| 30 | | | | | |
| 40 | | | | | |
| 50 | | | | | |
| 70 | | | | | |
| 90 | | | | | |
| Large Flowmeter | | | | | |
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 4 | | | | | |
| 6 | | | | | |
| 8 | | | | | |

# 7 Pipe Flow Apparatus

Table 3: Pipe dimensions

| | Diameter $(in)$ |
|---|---:|
| Small pipe | 0.344 |
| Large pipe | 0.605 |

Table 4: Small Flow Meter Calibration (7/11/2001)

| Flow Meter Scale $(\%)$ | Actual Flow $(gal/min)$ At $25°C$ |
|---|---|
| 10 | 0.085 |
| 16 | 0.128 |
| 20 | 0.162 |
| 26 | 0.196 |
| 30 | 0.228 |
| 40 | 0.298 |
| 50 | 0.371 |
| 60 | 0.447 |
| 70 | 0.523 |
| 80 | 0.605 |
| 90 | 0.698 |
| 100 | 0.775 |

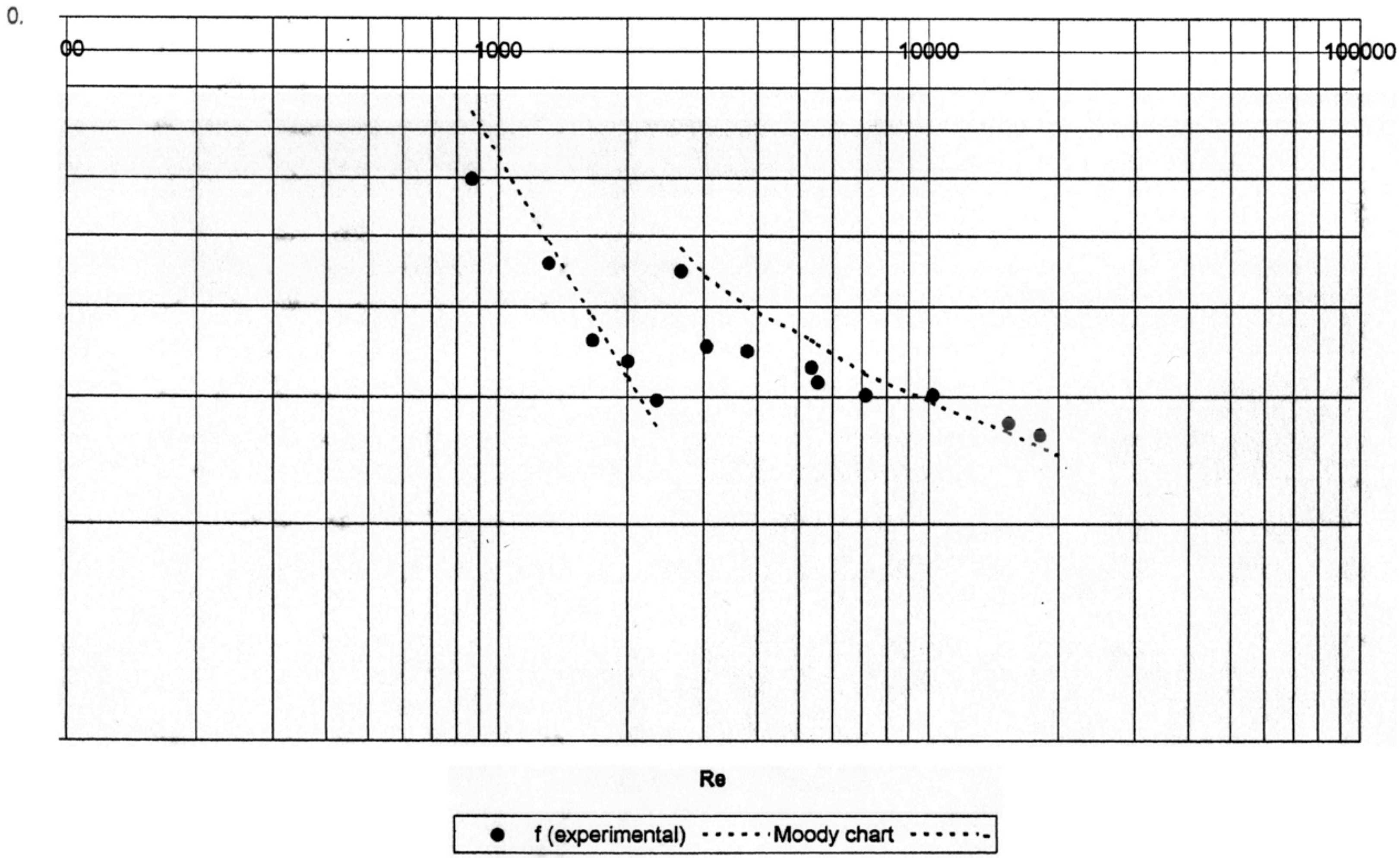
Table 5: Medium Flow Meter Calibration (1990-1995)

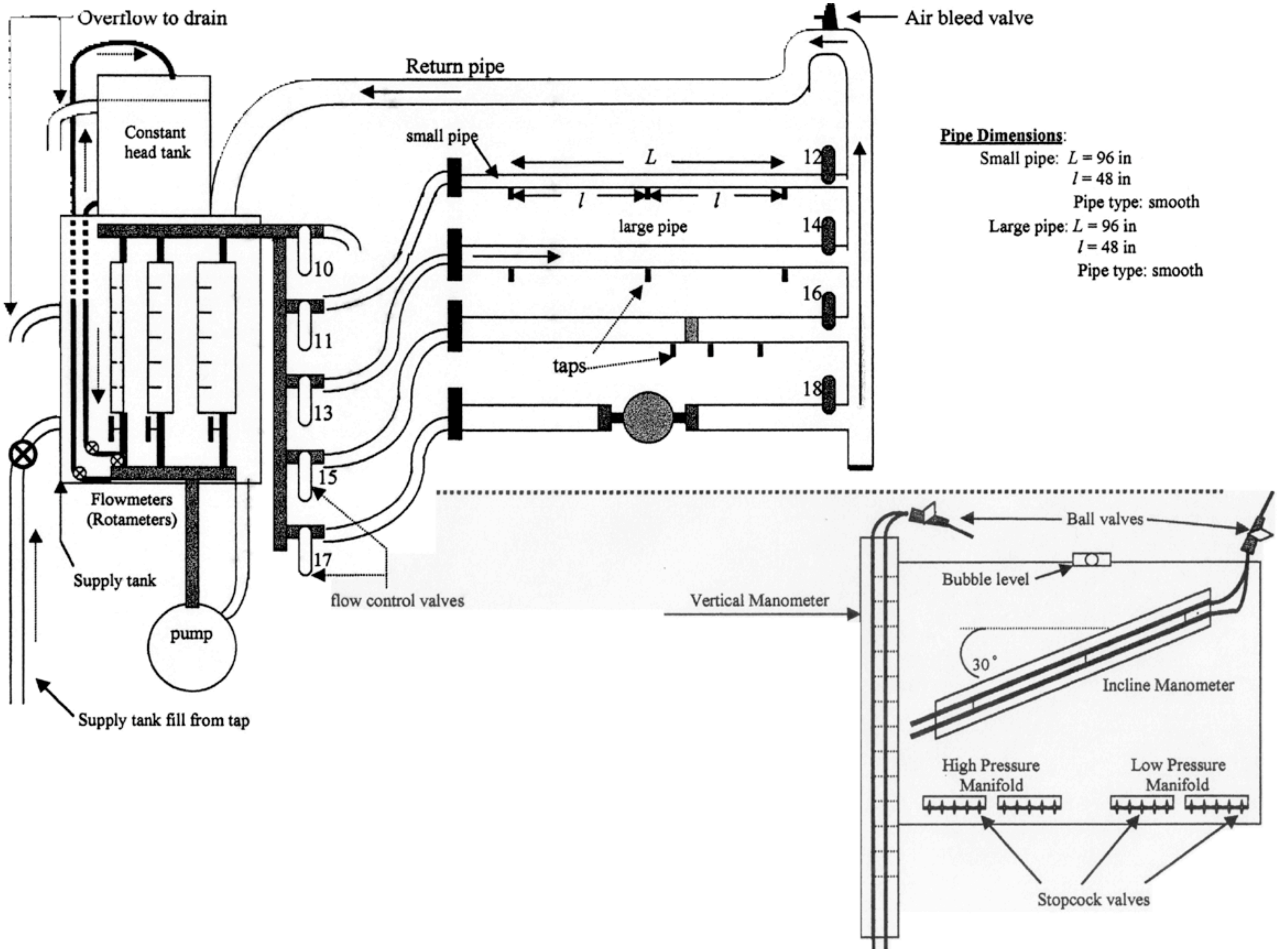
| Flow Meter Scale $(\%)$ | Actual Flow $(gal/min)$ |
|---|---|
| 10 | 0.26 |
| 20 | 0.54 |
| 30 | 0.75 |
| 40 | 1.00 |
| 50 | 1.26 |
| 60 | 1.50 |
| 70 | 1.77 |
| 80 | 2.03 |
| 90 | 2.31 |
| 100 | 2.61 |

Table 6: large Flow Meter Calibration (1986)

| Flow Meter Scale $(gal/min)$ | Actual Flow $(gal/min)$ At $19°C$ |
|---|---|
| 1 | 1.06 |
| 2 | 1.93 |
| 3 | 3.01 |
| 4 | 4.08 |
| 5 | 5.09 |
| 6 | 5.98 |
| 7 | 7.36 |
| 8 | 8.21 |
| 9 | 9.32 |

**f vs. Re, ;mall Pipe, F Length. Sample Data.**
**8/13/2001**





Overflow to drain
Air bleed valve
Return pipe
Constant head tank
small pipe
L
l
l
large pipe
12
14
16
18
10
11
13
15
17
taps
Flowmeters (Rotameters)
Supply tank
pump
flow control valves
Supply tank fill from tap
Pipe Dimensions:
Small pipe: L = 96 in
l = 48 in
Pipe type: smooth
Large pipe: L = 96 in
l = 48 in
Pipe type: smooth
Ball valves
Bubble level
Vertical Manometer
30°
Incline Manometer
High Pressure Manifold
Low Pressure Manifold
Stopcock valves