BU College of Engineering

ME 303 Fluid Mechanics

Laboratory Exercise No. 1 – Instruction Manual

Continuity Equation & Bernoulli's Principle

Fall 2011 Revision

Revision History

Name	**Date**	**Notes**
Prof. Bala Bharadvaj	1984	Original Document
Blake Smith	1985	
Steven P. Weibel, Sam Bogan, and Prof. Mort Isaacson	1989	
Revised by Steven P. Weibel	1990	
R. Adam Rugg	1991	
John Collier	1999-2000	For Hampden Wind Tunnel
Mike Canney, Ivan Ma, and Prof. Isaacson	2000	
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Prof. Grace	2011	Updated text

1 Prelab

Read the laboratory handout completely before proceeding to prelab questions. Answer all questions in metric units.

1. Refer to the converging/diverging test section in Figure 3. Assuming the air flows from left to right, where will the velocity be greatest? Least? Where will the pressure be greatest? Least?

2. Referring again to Figure 3, calculate the cross-sectional area of the duct at each of the numbered ports. Hint: Channel height varies linearly between inlet or outlet and throat.

3. Suppose the following data has been taken for the pressure difference between the Kiel probe and a static port in the wind tunnel using the microtector and pressure transducer:

   Measurement 1 (fan off):

   Micrometer reading = -0.010 inches

   Transducer reading = 3.000 volts

   Measurement 2 (fan on):

   Micrometer reading = +0.500 inches

   Transducer reading = 3.586 volts

   Assuming the relationship between pressure and voltage is linear, calculate an equation which relates the transducer voltage (in volts) to the pressure difference (in Pa). (Refer to Appendix A for information on performing a two-point calibration.)

4. Suppose that the difference between stagnation and static pressure at some vertical location at Port #8 in the test section is displayed as 3.450 volts on the meter. Use your equation from Question 3 to convert this value to a pressure differential in Pascal. Calculate the air velocity at this point. (Use a tabulated value for the density of air assume the air temperature is 20°C.)

5. If the room temperature is 20°C, what is the speed of sound?

6. Assume that the average velocity through the test section at Port #8 is equal to the point velocity you found in Question 4. What is the Mach number of the flow? Can this be considered an incompressible flow?

7. Assume that the average velocity through the test section at Port #8 is equal to the point velocity you found in Question 4. Apply continuity to determine the average velocity at Port #2.

2 Introduction

The concept of continuity and the relationship between pressure and velocity in fluid flows are among the most fundamental governing principles of fluid dynamics. This experiment is designed to illustrate these basic concepts for a quasi one-dimensional flow.

3 Theory

A velocity profile exists for viscous flow in a duct because of the formation of boundary layers. Two examples of possible profiles are shown in Figure 1. The profile on the left shows the boundary layers extending to the center of the duct. This profile occurs for low-speed, laminar, flow. For higher-speed, turbulent, flows the boundary layers can be very thin and thus lead to a profile like that shown on the right. For the turbulent profile, the flow speed is u_{max} at most radial locations and the viscous effects only affect the velocity profile in a region very close to the wall.

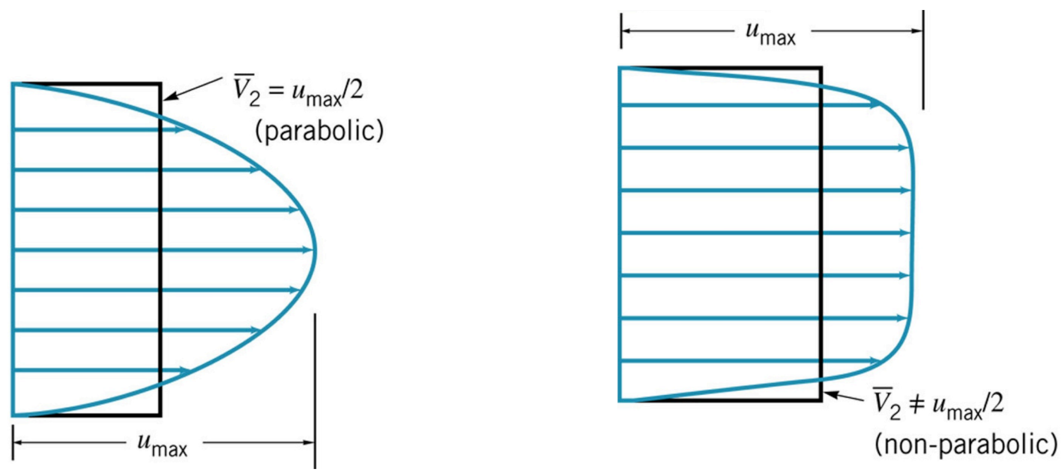


Figure 1: Possible 2D profiles for flow in a duct.

3.1 Continuity

The continuity equation in fluid dynamics is an expression of the principle of conservation of mass. The control volume form of conservation of mass for a steady flow is

$$0 = \int\int_{CS} \rho(\vec{v} \cdot \hat{n}) dA \tag{1}$$

where ρ is the density, $\vec{v}$ is the velocity, $\hat{n}$ is normal to the control volume surface and the integral is performed over the surface of the control volume. When the flow is incompressible (i.e. a flow of a liquid or a flow of a gas with Mach number less than 0.3) and the inflow (1) and outflow (2) are perpendicular to the control volume surface, this simplifies to

$$\dot{m}_1 = \rho \int V_1 \mathrm{d}A_1 = \dot{m}_2 = \rho \int V_2 \mathrm{d}A_2. \tag{2}$$

where $V_{1,2}$ are the speeds of the inflow and outflow. Integrating, we obtain

$$\rho V_{ave,1} A_1 = \rho V_{ave,2} A_2. \tag{3}$$

For an incompressible flow, this simplifies to

$$\boxed{V_{ave,1} A_1 = V_{ave,2} A_2,} \tag{4}$$

where V_{ave} indicates the spatial average of the flow speed over the cross section.

3.2 Bernoulli

In class, we derived the equation that governs the pressure difference in an inviscid moving fluid by applying Newton's second law along a streamline. It was shown that

$$\frac{\partial P_s}{\partial s} = \rho \left(V \frac{\partial V}{\partial s} \right) + \rho g \frac{\partial z}{\partial s} \tag{5}$$

where s refers to the distance along the streamline from a fixed location, z is the vertical height, and P_s is the static pressure.

For steady, incompressible flow, the second equation can be integrated along a streamline to obtain Bernoulli's equation, Eq. (6);

$$\boxed{P_s + \frac{1}{2}\rho V^2 + \rho g z = \text{constant}} \tag{6}$$

where the constant on the right side of the equation is fixed along a streamline. If Bernoulli's is applied between two points at the same elevation but with the flow speed at one point being zero and at the other being V, we have

$$P_t = P_s + \frac{1}{2}\rho V^2 \tag{7}$$

where P_t is called the total or stagnation pressure. If one brings a flow to rest at a point and measures the pressure at that point, one measures the value of the stagnation pressure. If at the same location one could ascertain or measure the static pressure, then the speed of the flow at that location can be calculated. This is true at any point in the flow, whether it is in a viscous region or not. It is this principle that will be used in the lab.

4 Experiment

In the experiment, airflow is created through an inlet into a converging/diverging duct. The velocity of air will be measured at a number of vertical positions across the flow passage at a number of different cross sections along the duct, including the narrowest part called he throat; also, the static pressure will be obtained at ten horizontal locations along the duct. The measured data will be compared with the simple theory for one-dimensional flow of a frictionless, incompressible fluid.

The flow is created by means of an axial fan driven by an AC motor with a frequency driver. The device is set up in a suction configuration; that is, the fan is positioned after the test section and pulls air though the tunnel (see Figures 2 and 3). Flow in the system runs from left to right as one faces the bench top. Any swirling motion introduced by the fan is minimized by the flow straightener, which is placed immediately before the test section.

The measurement of velocity in a moving fluid is often based on the measurement of two separate pressures: static and stagnation. For an inviscid flow (with no other dissipative mechanisms and no heat transfer), the stagnation pressure remains constant throughout the flow. Therefore for flows with profiles such as that on the right in Figure 1, the stagnation pressure should remain constant across most of a cross section of the duct. Within the boundary layer though, where viscous effects are present, the stagnation pressure varies. To understand how the static pressure acts across the cross-section of the duct, one can turn to the Navier-Stokes equations. Given a coordinate system appropriate for a rectangular duct in which x is the axial or flow direction and y is perpendicular to the flow direction, the form of the Navier-Stokes equation that is valid for steady, incompressible, viscous flow (where the viscosity is constant) is:

$$\vec{v} \cdot \nabla v_x = -\frac{1}{\rho}\frac{\partial P_s}{\partial x} + \mu \nabla^2 v_x \tag{8}$$

$$\vec{v} \cdot \nabla v_y = -\frac{1}{\rho}\frac{\partial P_s}{\partial y} - gz + \mu \nabla^2 v_y \tag{9}$$

$$\tag{10}$$

When we note that in the duct flow $v_y << v_x$, and that ρg for a gas is very small such that body forces (which lead to the gz terms above) can be neglected, we find that the second equation simplifies to

$$\frac{\partial P_s}{\partial y} \sim 0 \tag{11}$$

which means that there is no variation of the static pressure across the duct at any given cross section. However, from the first equation it is obvious that the static pressure can vary in the axial direction.

Along the length of the duct there are ten locations where pressure can be measured. The static pressure (P_s) is measured using the static pressure holes ("taps") along the bottom of the section. The taps are all connected to a manifold, which can be adjusted to isolate the pressure from any one tap. Each static pressure tap has a corresponding hole at the top of the duct. By lowering a Kiel probe into the hole, the stagnation (also called "total") pressure (P_t) can be measured at any vertical position across the cross section of the duct as well. These pressures are related by the Bernoulli equation as shown in Equation (7) Upon solving for the velocity V, one finds

$$\boxed{V = \sqrt{\frac{2(P_t - P_s)}{\rho}}.} \tag{12}$$

no In this experiment where the working fluid is air flow ρ in the above equation is the density of air.

In the experiment, this differential pressure ($P_t - P_s$) is measured with a $\pm 2.5''$ H_2O electronic differential pressure transducer. This is a device which takes a differential pressure as an input signal and converts it into a corresponding voltage as an output signal. The transducer output ranges linearly from 1 to 5 volts (nominally 1 volt at $2.5''$ H_2O differential pressure and 5 volts at $+2.5''$ H_2O, with a linear relationship between voltage and pressure in between). In order to use the transducer, you need to experimentally determine an accurate linear relation between output voltage and differential pressure input. This is called "calibrating" the pressure transducer. The calibration process consists of providing known pressure input to the transducer and reading the corresponding voltage outputs on a voltmeter. In order to determine the actual values of the input pressures used for calibration, the pressure lines from the wind tunnel will be hooked up to a microtector gauge as well as the electronic pressure transducer. The microtector consists of three components: a simple U-tube manometer, a micrometer, and an ammeter. The U-tube manometer senses the difference between the two pressures to be compared through the variation in the level of the liquid (distilled water) in the U-tube. The micrometer is employed to accurately measure the difference in liquid levels of the two columns, with the ammeter aiding in the determination of the exact location of the liquid surface. When the tip of the micrometer touches the liquid surface it completes an electric circuit, which causes a current to flow, which causes the ammeter needle to deflect. Since capillary action causes the liquid to "stick" to the micrometers point as it is removed from the liquid, an accurate indication of the liquid surface level can only be obtained as the micrometers point makes contact, not breaks contact. The micrometer on the microtector reads in thousandths of an inch. Since the change in liquid level is measured only in one of the two columns of the U-tube, the reading indicated by the micrometer is only one-half of the height difference that should be used in the hydrostatic equation. When you begin the lab, the teaching fellow will give you more information on the use of this device.

5 Procedure

In order to perform the experiment, follow the procedure outlined below. The duct is referred to as the test section throughout this section.

1. Turn on the main power using the switch on the right side of the manometer board.

2. Attach the Kiel probe to the traverse over the test section and position it horizontally over Port #8. Insert it into the test section and adjust it vertically so that the scale reads 100 mm when the tip is just touching the bottom of the test section, and ensure that the tip is parallel to the direction of flow. Then move the probe about half way up the test section. Also be sure that the hole is sealed around the probe and all other holes are sealed (including the end of the valve manifold).

3. Adjust the valves so that the Kiel probe and static pressure manifold are connected to both the microtector and pressure transducer. Adjust the static pressure manifold so that Port #8 is open and the rest are closed.

4. Before starting the fan, record the readings on both the transducer meter and the microtector. These are the "zero readings" for voltage and column height when $(P_t - P_s) = 0$

5. Start the motor by pressing FWD on the frequency driver and pressing the UP button until the display reads 40.00 Hz. Record microtector reading and the voltage reading.

6. Adjust the valves so that the Kiel probe and static pressure manifold are connected only to the transducer, i.e., shut off both pressure lines to the microtector.

7. Move the tip of the Kiel probe to the vertical position 66 mm (about 5 mm from the top) and record the reading from the transducer. Then, move the probe vertically across the test section, in 5 mm increments, recording the voltage at each location until you reach vertical position 96 mm (about 5 mm from the bottom of the test section). This will result in a pressure (hence velocity) profile of the throat of the test section.

8. Raise the Kiel probe out of the test section, and repeat Step 7 to obtain a pressure profile for the test section at Port #1, Port #5 and Port #10 (these can be done in 10 mm increments). Start at vertical position 8 mm and end at vertical position 158 mm for Port #1, start at vertical position 46 mm and end at vertical position 116 mm for Port #5, and start at vertical position 30 mm and end at vertical position 130 mm for Port #10. Be sure to adjust the static pressure manifold and seal all the holes at the top of the test section each time the probe is moved to a new port. During the measurements for Port #1, at each vertical position, along with the average value of the voltage, estimate and record the possible error in your estimate for the average voltage due to the fluctuations in the voltage.

9. Remove the Kiel probe completely from the test section, and seal all the holes along the top. Open Port #1 on the static pressure manifold (close all others), and record the reading on the transducer. Repeat this process for each of the other numbered ports.

10. Look at the streamers that are taped to the top of the converging and diverging sections. Note any differences you see between the motion of the two sets of streamers and the two sides (left and right facing down the tunnel) of the test section. From the side of the tunnel by the test section, look at the internal structure of the tunnel just upstream of the test section. Note obstacles in the flow path just before the air stream enters the test section (e.g., the shape of the flow

straightener). From the inlet end of the tunnel, look down the inside of the tunnel all the way to the outlet end of the tunnel. Note any obstacles to the flow that might cause either up-down or left-right asymmetries in the flow through a tunnel cross section.

11. Press STOP on the frequency driver to shut down the motor, and turn off the main power. Record the temperature and barometric pressure of the air in the room.

6 Analysis

The plots that you produce in this section should all have labels on their axes. When the figures are inserted in the text they should be numbered and have captions (which take the place of figure titles that would appear at the top of the figure.)

1. Assuming a linear relationship between transducer input pressure and output voltage, perform a two-point calibration of the transducer. In Procedure Steps 3-5 above, you obtained data for differential pressures in terms of the microtector height changes [2x(reading with fan on reading with fan off)] and corresponding voltages at two states (one at no flow resulting in no microtector height change, but still a voltage reading, and one at 40 Hz). From the microtector height change you can calculate the differential pressure (Hint: use the hydrostatic equation $\Delta P = \rho g h$). Plot on a graph the voltage vs. differential pressure, and determine an equation for the straight line that connects the two points, the "calibration curve". Then, for each of the voltage readings collected in the lab, calculate the corresponding differential pressure using the inverse of the equation for the calibration curve. See Appendix A for more information on constructing and using the calibration curve.)

2. From the data collected in Step 7 of the Procedure, calculate the air velocity at each point and make a graph of velocity vs. vertical position for this cross section. Be extremely careful in the choice of the range. [The vertical position should be plotted on the vertical axis, and the velocity should be plotted on the horizontal axis. The velocity scale should start at zero and the highest velocity should lie near the right hand end of the graph. You should just plot symbols to show your experimental values. Do not include a trend line or curve through them.] From this data determine the experimentally measured average air velocity across the cross-section at Port #8 and the experimentally determined total volume flow

rate at Port #8's cross section by numerical integration. See Appendix C. Since the total pressure could not be accurately measured near the wall, the velocity near the wall cannot be determined exactly. For calculating the average velocity, this is not a significant problem, but for the total volume flow rate through a cross section, we will have to assume the average velocity holds near the wall. Determine the average velocity by numerical integration only between the top and bottom measurement points (using all values between them) and divide by the distance between the top and bottom measurement points. Use the boxed formula in Appendix C. **To obtain the flow rate through the entire cross section, multiply the average velocity by average of the cross section.** (Note: Use the measured air temperature and barometric pressure to determine the density of air.)

3. Calculate the Reynolds number based on this average velocity and the geometry of the cross-section (refer to Appendix B).

4. From the data collected in Step 8 of the Procedure, calculate the velocity at each measurement point for the three cross sections and make a single graph of velocity vs. vertical position for all four cross sections where measurements were made. Plot the data for all four cross sections on the same graph. [Use the same vertical position scale and same velocity scale for all cross sections. The vertical position should be plotted on the vertical axis, and the velocity should be plotted on the horizontal axis. The velocity scale should start at zero and the highest velocity should lie near the right hand end of the graph. You should just plot symbols to show your experimental values. Do not include trend lines or curves through them. Use different symbols for different cross sections, and include a legend to label which symbol represents which cross section.] Note: for the cross section at Port #10, there may be back flow at the top and bottom due to flow separation. Positions with dynamic pressures below static (voltages at or below the "zero reading" which was obtained when the same pressure was applied to both sides of the differential pressure transducer with the tunnel shut off) should be ignored.

5. Calculate the experimental average velocities and experimental total volume flow rates at Port #1, Port #5, and Port #10 by numerically integrating the velocity profiles plotted in the previous step. Follow the same procedure as in Analysis Step 2, above, but for Port #10's cross section, assume the velocity is zero at positions with dynamic pressures below static and do not include them in your numerical integration. Graph the experimental volume flow rate as a function of position

along the test section (including Port #8). [Use symbols only, i.e. no lines.]

6. From the data in Step 9 of the Procedure, determine the static pressure at each port and plot these experimental static pressures vs. horizontal distance along the test section (starting at the inlet of the converging section). (Hint: Remember that the stagnation probe remained outside the test section for this part of the lab.) [Use symbols only, i.e. no lines.]

7. For each port where static pressure measurements were made, calculate the theoretical mean velocity using the continuity equation and the volume flow rate for Port #8's cross section, as calculated in Analysis Step 2, above. This assumes the velocity profile is uniform at every cross section and the volume flow rate is the same for all cross-sections. Graph the theoretical mean velocities just obtained as functions of position along the test section. Also put on this graph the experimental mean velocities determined in Analysis Steps 2 and 5. **Use symbols without lines for the experimental data, and lines with no smoothing and without symbols for the theoretical ones.**

8. Using the theoretical mean velocities just obtained, calculate the theoretical static pressure at each port using Bernoulli's equation. To determine the Bernoulli constant $(P_s + \frac{1}{2}\rho V^2)$, match the theoretical and experimental static pressures at Port #1. Plot these theoretical pressures along with the experimentally measured pressures on a single graph similar to the graph made for only the experimentally measured pressures in Analysis Step 6, above. **Use symbols without lines for the experimental data, and lines with no smoothing and without symbols for the theoretical ones.**

9. On a single graph, different from that in the Step 8, above, plot both the theoretical mean velocity and theoretical static pressure vs. horizontal distance along the test section. **Note that you need different scales** and plotting ranges for the velocity and pressure, so that both curves take up most of the graph. This way, increasing and decreasing trends can be easily compared. **Use lines with no smoothing and without symbols for the theoretical values.** Use different line types for the two different sets of values. Include a legend to identify the lines.

7 Discussion

1. Look at your graph of velocity vs. vertical position for the four cross sections you plotted in Step 4 of the Analysis. Comment on the jaggedness of the velocity

profile at Port #1's cross section and if it can be explained by your estimates of uncertainty in your voltage readings. If not, how can you explain it? (Hint: What did you observe in Step 10 of the Procedure?) How is the shape of the data different for the cross section at Port #10? How do you account for this difference? Explain how this relates to the motion of the streamers that you observed in Step 10 of the Procedure.

2. Comment on the experimental total volume flow rate (that you determined through numerical integration of velocity determined from pressure measurements at the individual cross sections) as a function of position. What does this say about the validity of the continuity equation for incompressible flow in this case? If the total volume flow rate at Port #10 is significantly different from that at the other ports, explain the discrepancy.

3. Compare the experimental and theoretical values of average velocity one determined by integration of local velocity determined from pressure measurements (from Steps 2 and 4 of the Analysis) and the other determined using the continuity equation (from Step 6) at Ports #1, #5, #8, and #10. Are there significant difference? How do you account for them?

4. Compare the measured and predicted static pressures along the test section. How well do they compare? Why might there be differences? Do discrepancies between measured and predicated pressures relate to whether the test section is converging or diverging? If so, how do you account for this?

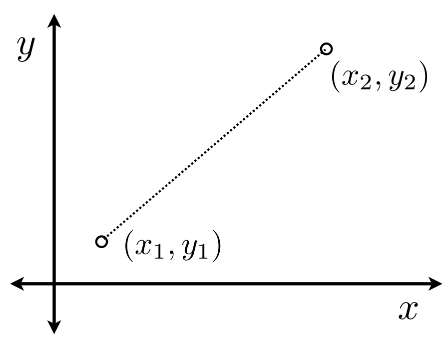
5. Comment on your observations in Procedure Step 10. Suggest ways in which the design of the wind tunnel could be improved.

Appendix

A Equation of a line between two points - Construction of a linear calibration curve

Many times it is necessary to determine the relationship between two measured parameters in an experiment. This normally involves calculating an equation that accurately relates the two parameters. For large sets of data, this equation can be determined using methods such as least-squares regression. When the relationship is known to be linear (i.e., follows a straight line) and the linearity is not something you are trying to prove, a simpler analysis can be used. That is what will be done in this experiment.

Suppose that two data points, (x_1, y_1) and (x_2, y_2), are collected in an experiment. It is necessary to determine how the dependent variable (y) is affected by the independent variable (x), by calculating an equation for the line that connects the two points.



The desired form of the equation is

$$y = mx + b, \tag{13}$$

where m is the slope and b is the y-intercept.

The slope m is the ratio between the change in the dependent variable and the change in the independent variable.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_2}{x_2 - x_1} \tag{14}$$

After the slope is determined, the final equation is found by substituting either of the known points (x_0, y_0) into the equation, and solving for b:

$$b = y_0 - mx_0 \tag{15}$$

In our case, if x is to be the pressure difference across the pressure transducer (Δp) in Pascal and y is to be the voltage reading of the transducer (V), then

$$x_1 = 2(h_1 - h_0)\rho g \tag{16}$$

$$x_2 = 2(h_2 - h_0)\rho g \tag{17}$$

$$v_1 = V_1 \tag{18}$$

$$y_2 = V_2, \tag{19}$$

where:

- h_1 and h_2 are the microtector readings at states 1 and 2, respectively.
- h_0 is the microtector reading when there is no flow in the tunnel and both sides of the pressure transducer are open to the atmosphere. NOTE: If this reading is below 0.000 inches, be sure to use the correct *negative* distance.
- ρ is the density of the manometer liquid (water, in this case), and g is the local gravitational acceleration.
- V_1 and V_2 are the transducer voltages at states 1 and 2, respectively.

This accounts for the fact you are measuring only one half the change in the manometer column height between the two sides of the differential (U-tube) manometer, and also compensates for any zero offset on the manometer (i.e., a reading other than "zero" when no pressure difference is being applied). Also, in this experiment, since state 1 is with the fan off (no pressure difference applied to the differential pressure transducer), $x_1 = 2(h_1 h_0)\rho g = 0$. Now, having the calibration equation (curve):

$$y = mx + b, \tag{20}$$

or, in this case

$$V = m(\Delta p) + b, \tag{21}$$

to find Δp knowing V, one inverts the equation to get

$$\Delta p = (V - b)/m. \tag{22}$$

B Reynolds Number

The Reynolds number, Re, for flow in a circular pipe is a dimensionless quantity defined by

$$Re = \frac{\rho V_{ave} D}{\mu}, \tag{23}$$

where ρ is the fluid density, V_{ave} is the mean velocity, D is the pipe diameter, and μ is the dynamic viscosity. Physically, the Reynolds number represents a ratio of inertial to viscous forces in a given flow.

For flow in noncircular ducts, the Reynolds number is given by

$$Re = \frac{\rho V_{ave} D_h}{\mu}, \tag{24}$$

where D_h is the hydraulic diameter, defined as

$$D_h = \frac{4A}{P} \tag{25}$$

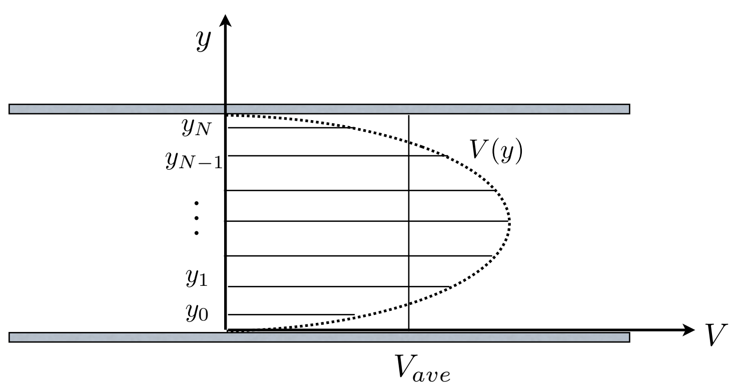
and A and P are the area and perimeter, respectively, of the cross-section.

The Reynolds number is useful for determining if two flows are dynamically similar. You will learn more about this in your lecture, but the general requirement for the similarity of two flows is that their Reynolds numbers be equal. (The flows must be of the same type, e.g., pipe flow.) The value of the Reynolds number to dynamic stability makes it an important tool in the design of experiments. Most importantly, the numerical value of the Reynolds number for pipe flow determines whether the flow is laminar, turbulent, or in transition between the two. In particular, if $Re < 2100$ (approximately), the laminar flow state is stable and even if it is disturbed, it returns to its laminar state. In fully developed laminar flow, the velocity profile in a pipe is parabolic, with the maximum at the center and zero at the walls. For Reynolds numbers which are greater than that value, the flow could be partially or fully turbulent. In a fully developed, fully turbulent flow, the time-averaged velocity profile is much flatter, with high gradients near the wall where the velocity goes from its nearly uniform value in the core to zero at the wall. In this experiment, since the cross sectional area of the duct changes with distance along the duct, the flow is not fully developed. However, comparing the velocity profile at Port #1's cross section to that at Port #5's cross section to that at Port #10's cross section shows that converging flow tends to smoothen out flow disturbances and make the velocity profile more uniform, while diverging flow tends to increase flow disturbances and decrease the velocity profiles uniformity.

C Numerical Integration and Average Velocity

Many situations in pipe and duct flow involve velocity profiles that are not constant across the cross-section. For example, the shape of the velocity profile for *fully developed* laminar flow in a straight duct is well known to be parabolic. In many cases, however, the flow is not fully developed laminar flow and the profile will have some shape other than parabolic. It is often useful to calculate an average (or mean) velocity, which is considered constant ("uniform") across the cross section of the duct or pipe.

The figure below shows a typical parabolic velocity profile, defined by a number of measured points, and its associated average velocity. Note that since both profiles share the same cross-sectional area, they must have the same volume flow rate. This fact can be used to determine the magnitude of the average velocity.



In this example, the velocity profile $V(y)$ is made up of $N+1$ measured points. The velocity is known for each y_j, where $j = 0...N$. To estimate the area under the curve defined by these points, the trapezoidal method can be applied, which divides the area into N trapezoids (by connecting the data points with straight lines, in this case). The total area is found by calculating the area of each trapezoid and summing them up.

Mathematically, if one assumes that the distance between points (h) is the same for each pair of adjacent points, then the average of $V(y)$ from top to bottom of the channel can be obtained from

$$\int V(y)\mathrm{d}y = \frac{h}{2}\sum_{j=0}^{N-1}\left[V(y_j) + V(y_{j+1})\right], \tag{26}$$

where h is the spacing between the y_j points and (assuming $y_0 = 0$) is defined as (total height divided by total number of points):

$$h = \frac{y_N - y_0}{N} = \frac{y_N}{N} \tag{27}$$

The final calculations for a rectangular cross-section are quite simple. Once the area under the $V(y)$ curve is known just multiply by the depth of the duct (W) to obtain the volumetric flow rate.

Since $Q = AV_{ave}$ and $A = WH = Wy_N$, the final expression for average velocity becomes

$$V_{ave} = \frac{h}{2}\frac{W}{Wy_N}\sum_{j=0}^{N-1}\left[V(y_j) + V(y_{j+1})\right] = \boxed{\frac{1}{2N}\sum_{j=0}^{N-1}\left[V(y_j) + V(y_{j+1})\right]} \tag{28}$$

Figure 2: Schematic of Hampden Wind Tunnel

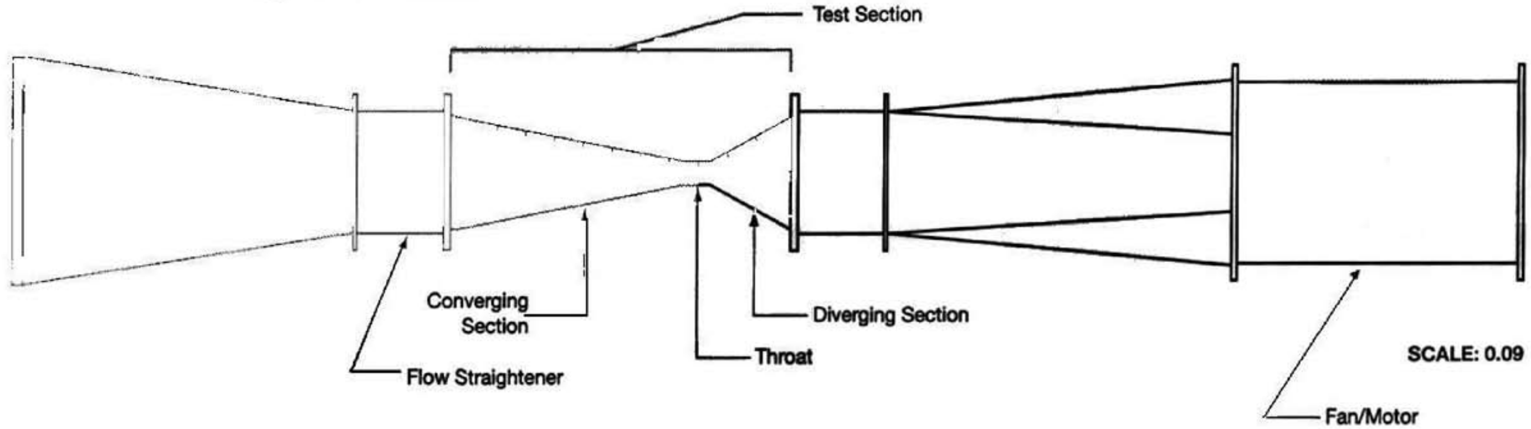
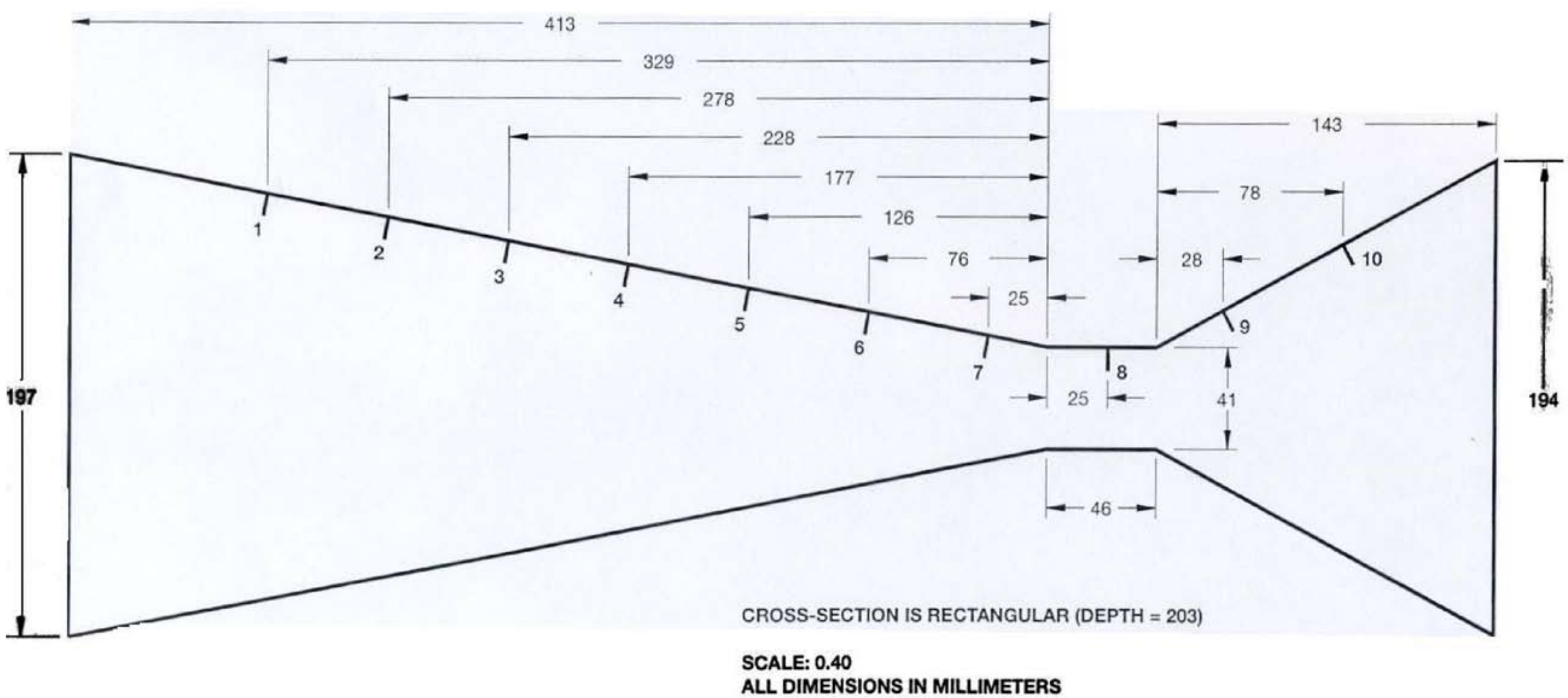


Figure 3: Converging/Diverging Test Section



Data Sheet 1

NAME: **DATE:**

PRESSURE & VELOCITY PROFILE - PORT #8

Vertical Position (mm)	Transducer Reading (volts)	Differential Pressure (Pa)	Velocity (m/s)

PRESSURE & VELOCITY PROFILE - PORT #1

Vertical Position (mm)	Transducer Reading (volts)	Differential Pressure (Pa)	Velocity (m/s)

PRESSURE & VELOCITY PROFILE - PORT #5

Vertical Position (mm)	Transducer Reading (volts)	Differential Pressure (Pa)	Velocity (m/s)

PRESSURE & VELOCITY PROFILE - PORT #10

Vertical Position (mm)	Transducer Reading (volts)	Differential Pressure (Pa)	Velocity (m/s)

Data Sheet 2

NAME: **DATE:**

STATIC PRESSURE ALONG TEST SECTION

			Experimental Values			Theoretical Values	
Port Number	Horizontal Distance from Inlet (mm)	Area of Cross-Section (mm^2)	Transducer Reading (volts)	Differential Pressure (Pa)	Static Pressure (Pa)	Velocity (m/s)	Static Pressure (Pa)
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							