Pipe Flow

$0 < Re_D < \sim 2300$ Laminar flow

$$f = \frac{64}{Re_D}$$

$Re_D > 2300$ Turbulent flow

Variables:

$\Delta p,\ \rho,\ V,\ \mu,\ L,\ D\ \boxed{\epsilon}$

New!
Pipe Roughness

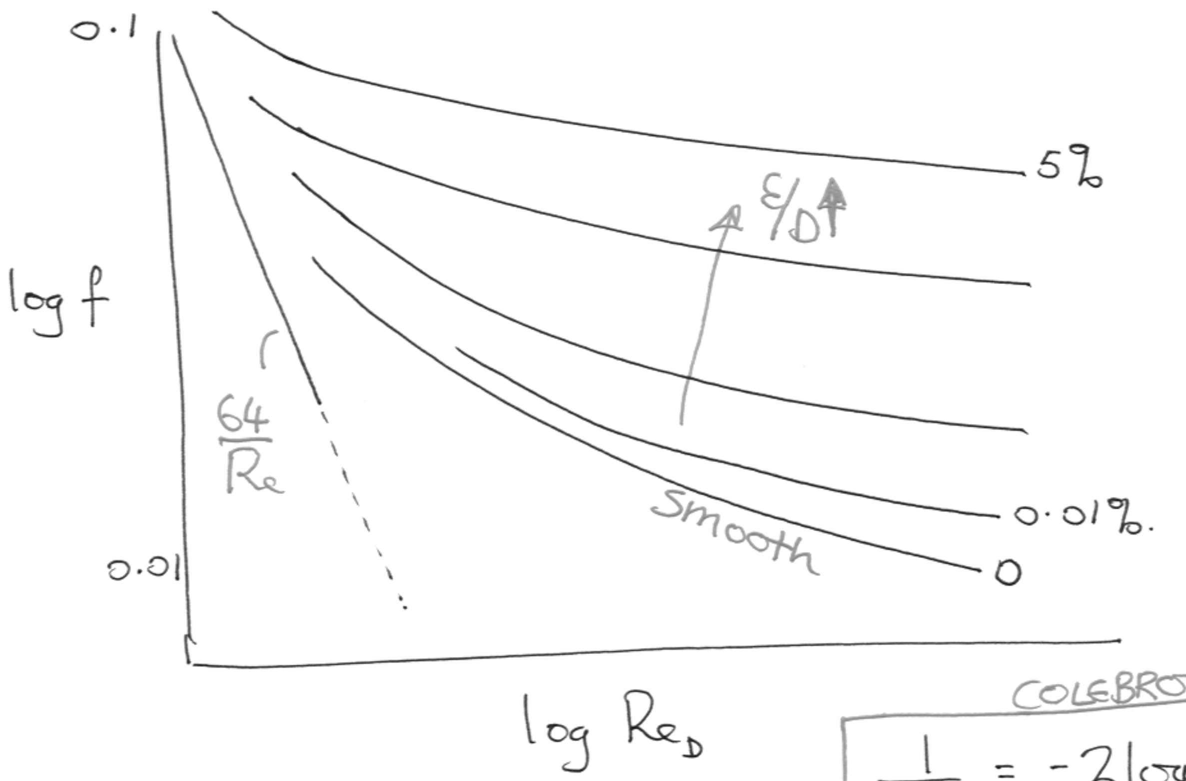
Π_i - analysis:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2},\quad \frac{D}{L},\quad Re \quad \frac{\epsilon}{D}$$

$$\underbrace{\frac{\Delta p}{\frac{1}{2}\rho V^2}\cdot\frac{D}{L}}_{\text{friction factor}} = f\left(Re, \frac{\epsilon}{D}\right)$$

laminar flow

$f = \frac{64}{Re}$, no dep. on ϵ/D



COLEBROOK Formula.

$$\boxed{\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)}$$

Features

- as $\varepsilon/D \uparrow$
  - transition is at lower Re
  - friction factor is higher

- small roughness makes a big difference

	ε (mm)
Plastic/Glass	0
Concrete	~0.3 - 3
Steel	0.045

(p492 Munson)

Energy Analysis

Recall:

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + Z_2 + h_L$$

head Loss

Modify:

α = factor to account for changes in profile $\begin{cases} \text{laminar} \\ \text{turbulent} \\ \text{not fully developed} \end{cases}$

In a fully developed pipe

$$\alpha V_1^2 = \alpha V_2^2 \quad ; P_1 - P_2 = \Delta p$$

$\Rightarrow$ if $z_1 = z_2$ (no change in elevation:

$$P_1 - P_2 = \gamma h_L = f \cdot \frac{L}{D} \cdot \frac{\rho V^2}{2}$$

$$\boxed{\Rightarrow h_L = f \frac{L}{D} \frac{V^2}{2g}}$$

$$\Rightarrow P_1 - P_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f\frac{L}{D}\frac{\rho V^2}{2}$$

Example 8.9 p515

Oil @ 140°F

$\gamma = 53.7\ lb/ft^3$

$\mu = 8 \times 10^{-5}\ lb \cdot s/ft^2$

pumped 800 miles through a 4 ft pipe (steel)

$Q = 2.4 \times 10^6$ Barrels/day

$= 117\ ft^3/sec$

$V = 9.31\ ft/sec.$

Find horsepower needed

Answer

Assume $Z_1 = Z_2$

$\frac{L}{D} = \frac{\cancel{7}800 \text{ miles}}{4 ft} \cong 1 \times 10^6$

$h_L = f \frac{L}{D} \frac{V^2}{2g}$ → need f :

$Re = \frac{\rho V D}{\mu} = 7.8 \times 10^5$

$\frac{\varepsilon}{D} = \frac{0.045 mm}{4 ft}$

$= 0.0000375$

$\Rightarrow f = 0.0125$

$$h_L = 0.0125\,(1.05\times10^6)\,\frac{(9.31)^2}{2(32.2)} = \underline{17{,}000\,\text{ft.}}$$

$$\text{Power} = \gamma Q h_L$$

$$\cong \underline{200{,}000\,\text{hp.}}$$

Other Losses

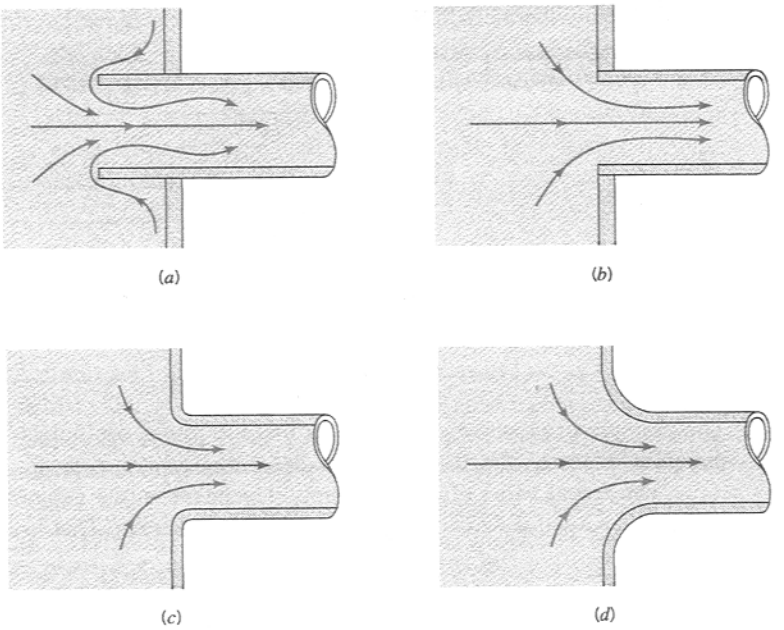
- Valves, Elbows, T-junctions etc.

Define Loss coefficient

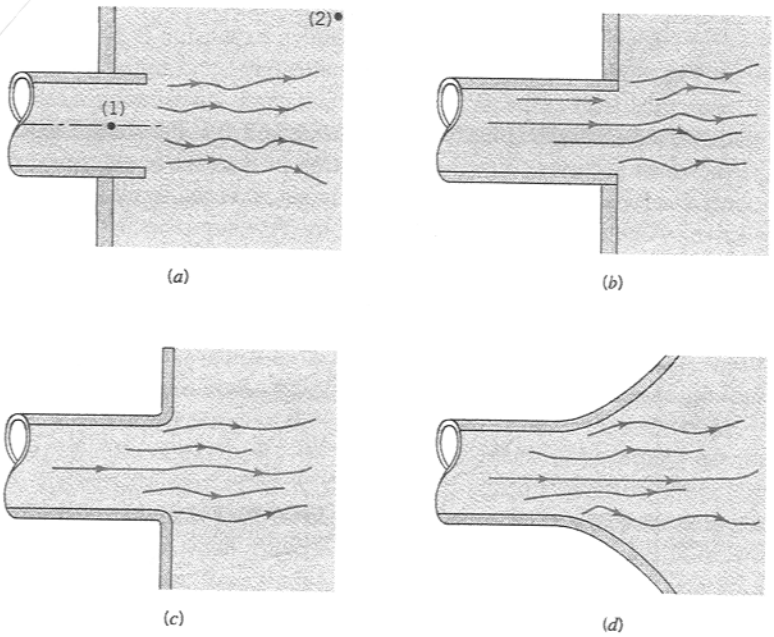
$$K_L = \frac{\Delta p}{\frac{1}{2}\rho V^2} \quad \text{or} \quad \frac{h_L}{V^2/2g}.$$

f(geometry, Re)

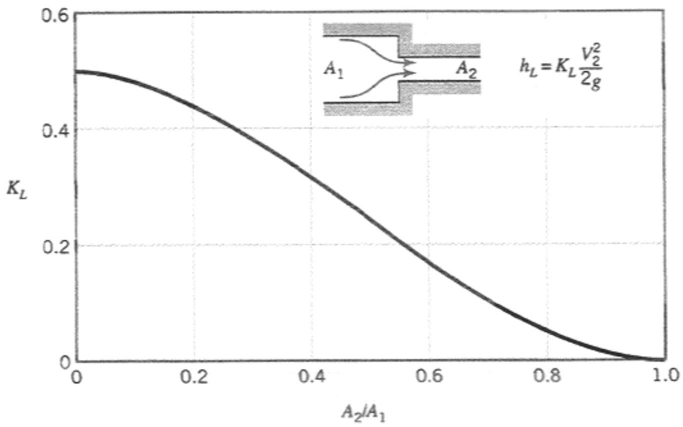
Examples.



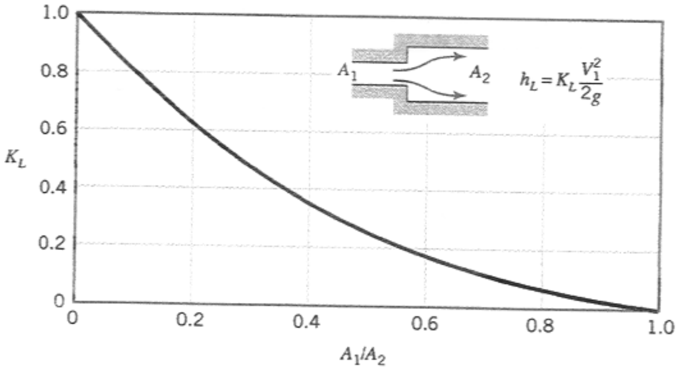
■ FIGURE 8.22 **Entrance flow conditions and loss coefficient (Refs. 28, 29). (*a*) Reentrant, $K_L = 0.8$, (*b*) sharp-edged, $K_L = 0.5$, (*c*) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (*d*) well-rounded, $K_L = 0.04$ (see Fig. 8.24).**



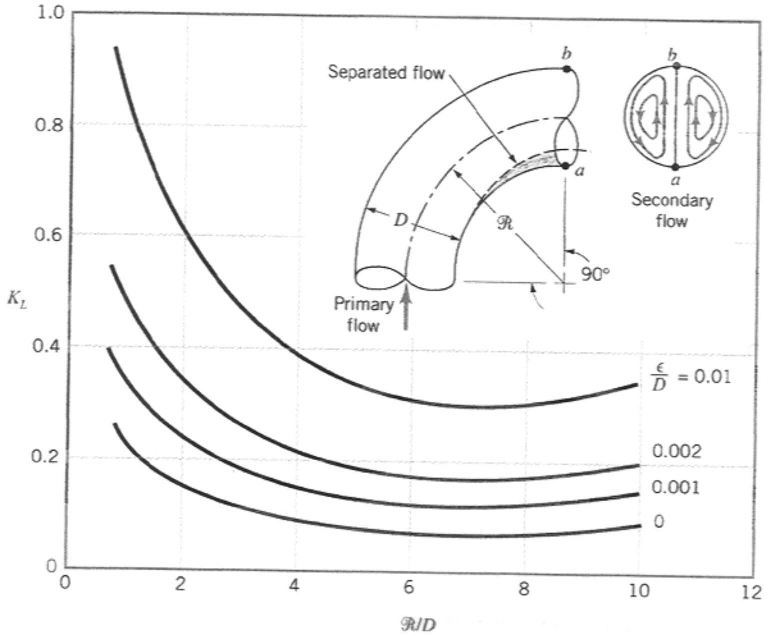
■ FIGURE 8.25 **Exit flow conditions and loss coefficient. (*a*) Reentrant, $K_L = 1.0$, (*b*) sharp-edged, $K_L = 1.0$, (*c*) slightly rounded, $K_L = 1.0$, (*d*) well-rounded, $K_L = 1.0$.**



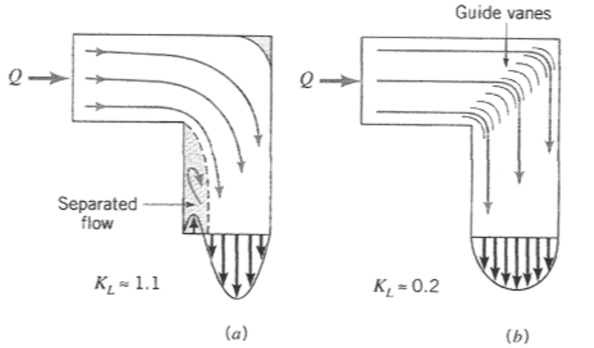
■ FIGURE 8.26 **Loss coefficient for a sudden contraction (Ref. 10).**



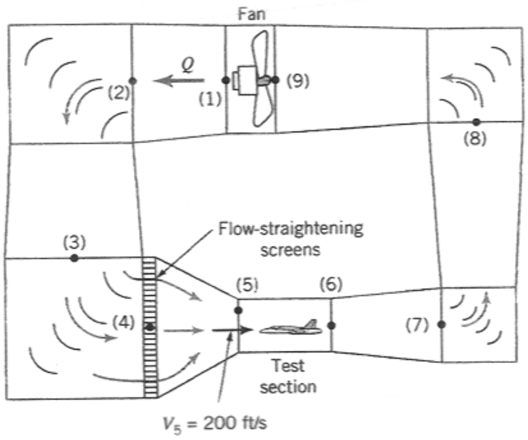
■ FIGURE 8.27 **Loss coefficient for a sudden expansion (Ref. 10).**



■ FIGURE 8.30 **Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).**



■ FIGURE 8.31 **Character of the flow in a 90° mitered bend and the associated loss coefficient: (a) without guide vanes, (b) with guide vanes.**



■ FIGURE E8.6