TORSION LAB

RULES AND EXPECTATIONS:

1. It is expected that you should complete any pre-lab exercises *before* attending the lab session. These shall be turned in at the beginning of the lab period.
2. It is expected that you will discuss the lab assignment with classmates and teachers in order to improve your understanding of the material.
3. Everything written down that is turned in for credit should be from your own work based on your understanding of the material.
4. Any input from others should be cited in your assignment.
5. A perfect lab report should (a) be legible and well organized, (b) demonstrate a clear, reproducible, and extensible thought process, and (c) be correct. Your work will be evaluated on how nearly it meets this ideal.

1 Introduction

1.1 Objectives

1. To review Coulomb torsion theory and observe twisting of uniform shafts.
2. To test the validity of Coulomb torsion theory in two ways:
   (a) Test prediction of linearity.
   (b) Validate Coulomb torsion by using it to measure material shear modulus and comparing to tabulated values.

1.2 Theory and Background

Coulomb torsion theory is applicable to the twisting of straight shafts with circular cross section. The material comprising the shaft is assumed to be linear elastic. The presentation here will be restricted to shafts that are homogenous and uniform.
Twisting of elastic shafts with circular cross sections is described by the following system of differential equations:

$$\frac{dT}{dx} = t(x) \qquad \text{equilibrium} \qquad (1)$$

$$T(x) = JG\theta(x) \qquad \text{constitutive eqn} \qquad (2)$$

$$\frac{d\phi}{dx} = \theta(x) \qquad \text{kinematics} \qquad (3)$$

In the above equations:

- $t(x)$ [force $\times$ distance/distance] is the externally applied torque intensity;
- $T(x)$ [force $\times$ distance] is the internal torque on the cross section at location x;
- G [force/area] is shear modulus, or modulus of rigidity of material comprising the shaft;
- J [distance^4] is the polar moment of area of the cross section about its center; for the circular cross section considered here, $J = \pi d^4/32$, where $d =$ diameter of the circular cross section;
- $\theta(x)$ [radians/distance] is the "twist" (rate of rotation) of the cross section (at location x);
- $\phi(x)$ [radians] is the total rotation of the cross section at location x.

In deriving these equations, it was assumed that the material responds in its linear range, and that the quantity $\alpha = \phi\, d/L \ll 1$. More precisely, we neglected quantities of $O(\alpha^2)$ relative to 1. Therefore, we should expect the predictions of these equations to be accurate to within a factor of approximately α^2.
Equations (1) - (3) may be combined to obtain one differential equation for the shaft rotation:

$$\frac{d}{dx}\left(GJ\frac{d\phi}{dx}\right) = t(x) \qquad (4)$$

1.3 Twisting of a uniform shaft

For a shaft with constant diameter along its length, then $J(x)$ =constant. If the shaft is comprised of the same material throughout, then G, the shear modulus of the material, has the same value at every point in the shaft. Hence, G =constant. For a uniform shaft, therefore, we may assume GJ = constant. If we further restrict our current consideration to loading only at the end of the shaft, then $t(x) = 0$. Thus (4) simplifies to:

$$GJ\frac{d^2\phi}{dx^2} = 0 \tag{5}$$

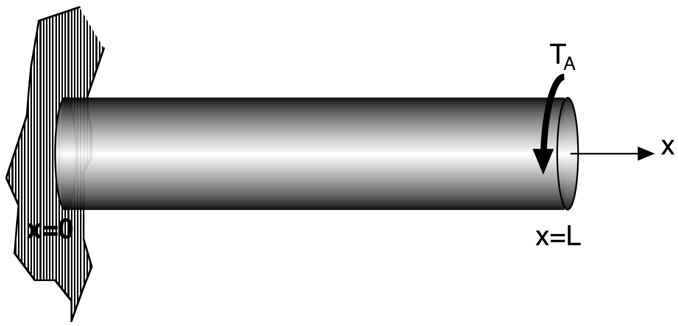

Figure 1: Schematic of circular shaft anchored at left end ($x = 0$), with applied torque T_A at right end ($x = L$).

In the configuration shown in figure 1, the left end, located at $x = 0$, is anchored to a wall. At the right end, located at $x = L$, there is an applied torque T_A. These conditions give us the two boundary conditions:

$$\phi(0) = 0; \qquad GJ\frac{d\phi}{dx}(L) = T(L) = T_A \tag{6}$$

Integrating equation (5) twice with respect to x and using the two boundary conditions (6) gives us the rotation as a function of position in the shaft:

$$\phi(x) = \frac{T_A}{GJ}x. \tag{7}$$

Evaluating (7) at the right end gives the overall rotation of the shaft:

$$\phi(L) = \frac{T_A\,L}{G\,J}. \tag{8}$$

Where no confusion is to arise, we often simply refer to $\phi(L)$ as ϕ, and T_A as T; thus (8) would read:

$$\phi = \frac{T\,L}{G\,J}. \tag{9}$$

Our goal is to test the predictions of equation (9). First we will test the prediction that overall torque is linearly related to overall rotation. Secondly, we will test the material property dependence of (9) by evaluating G for several materials and comparing the measured values to tabulated values.

1.3.1 Torional spring constants

Equation (9) can be recast as:

$$T = k_{torsion}\,\phi \qquad \text{torsional spring} \tag{10}$$

$$k_{torsion} = \frac{GJ}{L} \qquad \text{torsional spring constant} \tag{11}$$

$$\tag{12}$$

Here, $k_{torsion}$ is to be interpreted as a torsional constant due to shaft twisting. Likewise, it depends on the material properties, (e.g. through G), and the geometry (e.g. through L and J).

1.3.2 Measuring shear modulus

Equation (11) can be rearranged to determine G in terms of the other quantities:

$$G = \frac{T\,L}{\phi\,J} = \frac{k_{torsion}\;L}{J}. \tag{13}$$

In this lab, we will measure independently the four quantities on the right hand side of (13), and thus determine shear modulus G for various materials.

1.4 Pre-lab Assignment

1. Look up the modulus of rigidity, G, and yield strength σ_{yield}, of 2011-T3 Aluminum, 360 Brass, and 304 Stainless Steel, in psi. Record the values. Save a copy for the lab analysis later.
2. Consider an 18 in long circular shaft, with circular cross section of $1/8$ in diameter. Compute the torque required to yield the shaft; assume $\tau_{yield} = \frac{1}{2}\sigma_{yield}$.
3. For each of the materials under consideration, compute the total rotation ϕ for the shaft at yielding. Convert to degrees. (In the lab, we want to ensure our rotation stays well below these values.)
4. Determine the force P which, if applied on a lever of length 6 in, gives the yield torques found above.
5. Determine the mass, in grams, whose weight on Earth gives the loads P computed above.

2 Lab procedure

The experiment will be conducted using the Megazord [1] apparatus, shown in figure 2. The apparatus supports 3 different experimental configurations; here we will use the torsion configuration. The apparatus should be positioned on the table so that the torque wheel is just slightly hanging over the edge of the table.
In the torsion configuration, one end of the sample will be fixed in place, while the other end will be clamped and twisted via incremental applied torques. A digital angle gauge (DLAG) will be used to measure the shaft rotation. The materials used in this lab will be 2011-T3 Aluminum, 360 Brass, and 304 Stainless Steel.
Apparatus parameters: The torque wheel diameter is 11.875 in. The distance between the apparatus inner walls is 23.75 in.

2.1 Balancing the apparatus

The apparatus is supported by three feet, two under the torsion wheel wall, and a central adjustable foot under the opposite wall that features the U-shaped cutout. Use a level to verify that the apparatus is plumb along its main axis. Use the adjustable foot to adjust level if needed. The GTFs may have already completed this step for you.

2.2 Verify physical length measurements

Verify each of the following nominal dimensions of your system:

1. The torque wheel diameter is 11.875 in.
2. The distance between the apparatus inner walls (length of the shaft) is 23.75 in.
3. Shaft diameter is 0.125 in. (You'll want a micrometer for this. Check at three locations along length of the shaft. Record all measurements.)

2.3 Torsion testing procedure

Nota Bene: Due to the size of the apparatus and range of loads involved, the samples are necessarily thin. Accordingly, they are very sensitive to vibration and touch, so be careful when you attach loads to the torsion wheel. Similarly, especially outside of the testing apparatus when they are not otherwise supported, handle the samples with care so they are not permanently bent.

Steps:

1. Select one (nominally) $1/8$" diameter cylindrical test sample. Make a note of the specimen identification markings and the material type.
2. Insert the sample into the chuck opening through the back of the wall support, hex end first, so that it reaches the full length of the apparatus. The sample should pass through the back of the chuck before its end is fully inserted into the hex-shaped opening in the torque mount (attached to the right support in Fig. 2). Note that guiding the sample through and into both hex fittings on each support takes some finesse and careful handling. Be sure that the digital level angle gauge (DLAG) is oriented (more or less) parallel to the ground and is right side up. Secure the sample by clamping the cam lever on the back wall.
3. Turn on the DLAG.

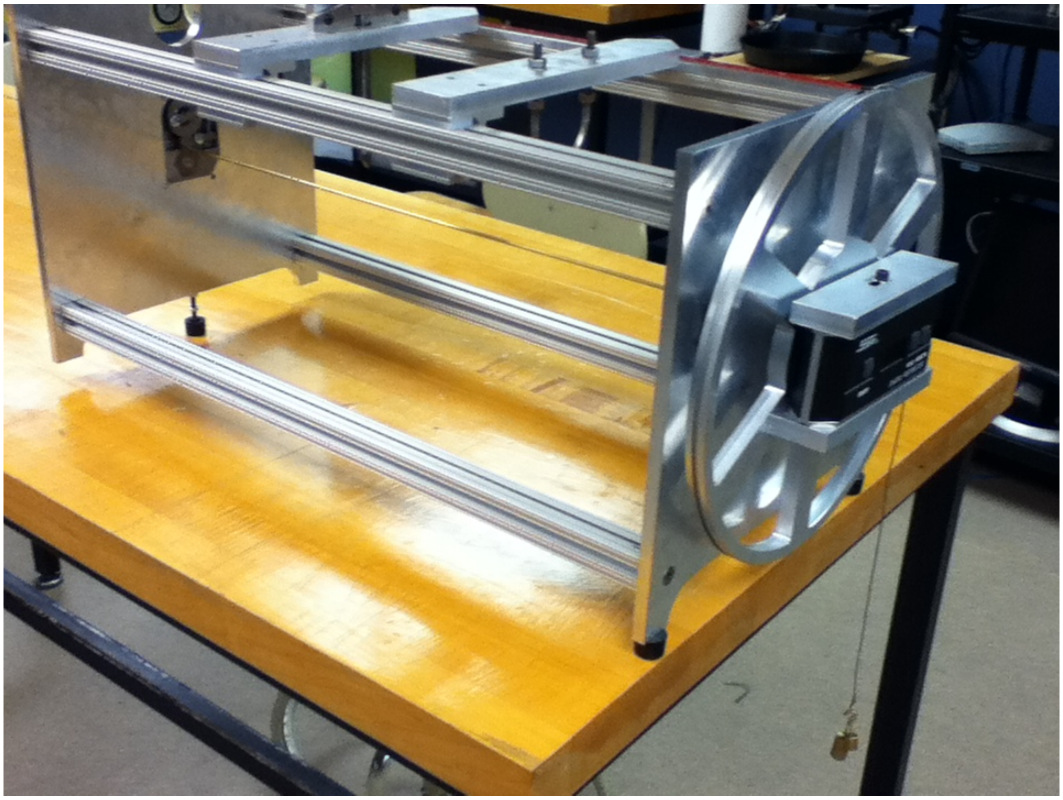

Figure 2: Torsion testing.

.

4. Add a $10g$ weight to the load-wire on the torque wheel. This weight is to remove any "slack" in the system, close any gaps in the fittings, etc. It won't count in our "load total". Wait for the wheel to stop moving, if necessary. After the wheel has come to rest, check the wheel settling position by giving it a *slight* push and allowing it to resettle.
5. After the wheel has come to rest, zero the DLAG. Record the "zero" reading.
6. Add a $20g$ weight to the load-wire on the torque wheel. After the wheel has come to rest, check the wheel settling position by giving it a slight push and allowing it to resettle. Once you're satisfied that the reading is accurate, record the reading displayed by the DLAG.
7. Repeat step 6, adding weight in $20g$ increments up to a total load of $120g$. (At the final load, there should be $130g$ on the wire: $10g$ preload plus five increments of $20g$.)
8. Turn off the DLAG.
9. Select another material sample and repeat steps 2 - 8, for each material being tested.

3 Analysis

3.1 Validation measures

We will validate Coulomb torsion theory in two ways. First, we will test the prediction of equation (10): that torque is a linear function of rotation. Secondly, we will validate Coulomb torsion by verifying that it produces

material property measurements that agree with with tabulated values.
For all requested plots, use some analysis software package (e.g. Matlab). When plotting "A vs. B", the convention is to plot A on the y-axis and B on the x-axis; i.e. we plot y vs. x.

3.2 Testing linearity of response

1. Compute net rotations, ϕ_j: Subtract the zero rotation reading from all rotation measurements; convert to radians.

2. The DLAG reports rotations to within $\pm 0.1°$ Assume an error in each of your rotation measurements of $\pm 0.05°$ (consistent with round-off error.) Compute $\Delta\phi$ in radians, the resulting uncertainty in ϕ_j.

3. Compute net applied torque, T_j: Compute torque ($\mathrm{lb} - \mathrm{in}$) applied to the shaft for each added mass increment. (Determine gravitational force exerted on mass; convert units of force to radians.) Do not count the $10g$ preload.

4. Plot T versus ϕ for all samples on the same full-page plot.

5. According to equation (10), the torsional stiffness of the shaft, $k_{torsion}$, may be defined as the slope of the line that best describes the data you just plotted. *Note that $T = k_{torsion}\ \phi$ implies that $T = 0$ at $\phi = 0$. Make sure your fit satisfies this requirement.* Thus determine $k_{torsion}$ for each of the shafts measured.

6. Add (unconnected) points to the plot representing the error range of the measured angles. That is, for each data point (T_j, ϕ_j), add to the plot the unconnected points: $(T_j, \phi_j - \Delta\phi)$, and $(T_j, \phi_j + \Delta\phi)$.

3.3 Validating Coulomb torsion by measuring shear modulus G

7. For each sample tested, compute J using the measured diameter d, and

$$J = \frac{\pi d^4}{32}. \tag{14}$$

8. For each material tested, compute G from $k_{torsion}$ and equation (13):

$$G = \frac{k_{torsion}\ L}{J} \tag{15}$$

9. Create a table that lists G_{meas}, $G_{tabulated}$ (from Prelab assignment), and their difference, for each sample measured.

References

[1] Megazord based on the original design from *Robust Revisionaries Senior Design Project*, M Taddonio, P DiTullio, M Brooks, M Sylvia, C Lee, ENG ME414, Mechanical Engineering, Boston University, 2012.