1 Cramer-Rao bound

Let us consider a single antenna transmitter and receiver that communicates through a channel that is composed by a single tap. The transmitted signal is defined as $s(t)$ and the received signal is $r(t)$. Assuming the receiver samples the received signal $r(t)$, the system model is written as

$$r(nT_s) = \beta s(nT_s - \tau)\mathrm{e}^{-2\pi \jmath f_D T_s n} + z(nT_s), \tag{1}$$

where β is the propagation loss, f_D is the doppler deviation, τ is the delay, and T_s is the sampling period. The pdf of the received signal is give by

$$g(f_D, \tau) = \frac{1}{\sqrt{2\sigma^2}}\mathrm{e}^{\Pi_{n=1}^{N} \frac{\left(r(nT_s) - \beta s(nT_s - \tau)\mathrm{e}^{-2\pi \jmath f_D T_s n}\right)^2}{2\sigma^2}} \tag{2}$$

$$\begin{aligned}
\frac{\partial \ln\{g(f_D, \tau)\}}{\partial f_D} &= \frac{\partial}{\partial f_D}\sum_{n=1}^{N} \frac{\left(r(nT_s) - \beta s(nT_s - \tau)\mathrm{e}^{-2\pi \jmath f_D T_s n}\right)^2}{2\sigma^2} && (3)\\
&= \sum_{n=1}^{N} \frac{\left(r(nT_s) - \beta s(nT_s - \tau)\mathrm{e}^{-2\pi \jmath f_D T_s n}\right)}{\sigma^2} \jmath 2\pi\beta T_s n s(nT_s - \tau)\mathrm{e}^{-\jmath 2\pi f_D T_s n} && (4)\\
&= \sum_{n=1}^{N} \frac{\left(r(nT_s) - \beta s(nT_s - \tau)\mathrm{e}^{-\jmath 2\pi f_D T_s n}\right)}{\sigma^2} 2\pi\beta T_s n s(nT_s - \tau)\mathrm{e}^{-\jmath 2\pi f_D T_s n + \pi/2} && (5)\\
& && (6)
\end{aligned}$$