1 System Model

Consider a $N_r \times N_t$ channel matrix between transmitter and receiver that is frequency selective with N_c delay taps. Each tap is denoted as $\mathbf{H}_d$, $d = \{0, 1, \ldots, N_c - 1\}$. The variable ρ denotes the average received power and $\mathbf{z}_n \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$ the circularly symmetric complex Gaussian distributed noise vector, therefore, the received signal is expressed as

$$\mathbf{r}_{l,n} = \sqrt{\rho} \sum_{d=0}^{N_c-1} \mathbf{H}_d \mathbf{f}_l s_{n-d} + \mathbf{z}_{l,n}, \tag{1}$$

where s_n is the nth non-zero of the training frame of length $N + N_c - 1$

$$\mathbf{s} = [0, \ldots, 0, s_1, s_2, \ldots, s_N]. \tag{2}$$

At the receiver, and a RF combiner $\mathbf{w}_k$ is applied over the training frame, so that the combiner output is expressed as

$$y_{k,l,n} = \sqrt{\rho} \mathbf{w}_k^H \sum_{d=0}^{N_c-1} \mathbf{H}_d \mathbf{f}_l s_{n-d} + z_{k,l,n}, \tag{3}$$

where $z_{k,l,n} = w_k^H \mathbf{z}_{l,n}$ The output signal can described in term of tensor notation

$$y_{k,l,n} = \mathcal{H} \times_1 \mathbf{w}_k^H \times_2 \mathbf{f}_l \times \mathbf{s}_n, \tag{4}$$

where $\mathcal{H}$ is generated from the concatenation in the third dimension of the N_c delay taps. Assuming a collection of transmitter and receiver beams, i.e. $l = \{1, \ldots, L\}$ and $k = \{1, \ldots, K\}$, respectively, and the N time instants within the training frame, we can express the signal model as the third-order tensor

$$\mathcal{Y} = \mathcal{H} \times_1 \mathbf{W} \times_2 \mathbf{F} \times_3 \mathbf{S} + \mathcal{Z} \times_1 W, \tag{5}$$

where $\mathbf{S}$ is toeplitz because there is a convolution operation over the third dimension.