

# Channel Estimation of Frequency Selective Channels Using Sparse PARAFAC Model

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**Abstract**—The abstract goes here.

## I. INTRODUCTION

*Notation:* A scalar is denoted in italic, e.g.  $a$ . A column vector is a bold lowercase letter, e.g.  $\mathbf{a}$  whose  $i$ th entry is  $\mathbf{a}(i)$ . A matrix is denoted by a bold uppercase letter, e.g.  $\mathbf{A}$  with  $(i, j)$ th entry  $\mathbf{A}(i, j)$ ;  $\mathbf{A}(:, j)$  is the  $j$ th column of  $\mathbf{A}$ . A third-order tensor is a caligraphic uppercase letter, e.g.  $\mathcal{A}$  with  $(i, j, k)$ th entry  $\mathcal{A}(i, j, k)$ . The  $\text{vec}\{\cdot\}$  operator stacks the columns of its argument into one big column vector;  $\otimes$  stands for the Kronecker product and  $\odot$  is the Khatri-Rao product (column-wise Kronecker product) [1], [2].

## II. SYSTEM MODEL

Consider a single user multiple-input multiple-output (MIMO) system with transmitter and receiver equipped with  $N_t$  and  $N_r$  antennas, respectively. Assume that both transmitter and receiver have a hybrid beamforming structure with  $M_t$  and  $M_r$  chains.

Consider a  $N_r \times N_t$  channel matrix between transmitter and receiver composed by a summation of  $N_c$  delay tap matrices  $\mathbf{H}_d$ ,  $d = \{0, 1, \dots, N_c - 1\}$ . The variable  $\rho$  denotes the average received power and  $\mathbf{z}_n \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$  the circularly symmetric complex Gaussian distributed noise vector, therefore, the received signal is expressed as

$$\mathbf{r}_{i,n} = \sqrt{\rho} \sum_{d=0}^{N_c-1} \mathbf{H}_d \mathbf{f}_i s_{n-d} + \mathbf{z}_{i,n}, \quad (1)$$

where  $s_n$  is the  $n$ th non-zero of the training frame of length  $N + N_c - 1$

$$\mathbf{s} = [0, \dots, 0, s_1, s_2, \dots, s_N]. \quad (2)$$

At the receiver, and a *radiofrequency*(RF) combiner  $\mathbf{w}_k$  is applied over the training frame, so that the combiner output is expressed as

$$y_{k,i,n} = \sqrt{\rho} \mathbf{w}_k^T \sum_{d=0}^{N_c-1} \mathbf{H}_d \mathbf{f}_i s_{n-d} + z_{k,i,n}, \quad (3)$$

where  $z_{k,i,n} = \mathbf{w}_k^H \mathbf{z}_{i,n}$ . The output signal can be described in term of tensor notation

$$y_{k,i,n} = \mathcal{H} \times_1 \mathbf{w}_k \times_2 \mathbf{f}_i \times_3 \mathbf{s}_n, \quad (4)$$

where  $\mathcal{H}$  is generated from the concatenation in the third dimension of the  $N_c$  delay taps. Assuming a collection of transmitter and receiver beams, i.e.  $i = \{1, \dots, I\}$  and  $k = \{1, \dots, K\}$ , respectively, and the  $N$  time instants within the training frame, we can express the signal model as the third-order tensor [1], [3]

$$\mathcal{Y} = \mathcal{H} \times_1 \mathbf{W} \times_2 \mathbf{F} \times_3 \mathbf{S} + \mathcal{Z} \times_1 \mathbf{W}, \quad (5)$$

where  $\mathbf{S}$  is toeplitz because there is a convolution operation over the third dimension.

## III. CONCLUSION

The conclusion goes here.

## ACKNOWLEDGMENT

## REFERENCES

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