Appendix B. Proof of Total Unimodularity of *A*

A square integer matrix is *Unimodular* if it has a determinant of 1 or -1, and a matrix A is *Totally Unimodular* if and only if every *non-singular* square submatrix of A is *Unimodular*, or equivalently, every submatrix has a determinant equal to 0, 1 or -1 . So checking *Total Unimodularity* is a complicated task as the required condition needs to be verified for every submatrix, making it a combinatorial procedure at its core; however, some additional theorems state sufficient, though not necessary conditions that are easier to verify. One of those says that a matrix A which only has 0, 1, or -1 entries (like the system matrices presented in this paper) is *Totally Unimodular* if it contains at most two non-zero entries in each column. In our matrices A, considering each column as a variable, we can't apply this result directly as some columns have three or four non-zero entries because some variables are present in more than two constraints, more specifically for thermal generation because of the ramping constraints, as shown in B.1. In B.1 the system is represented with only three periods instead of 8736, this is valid because we don't have a constraint that requires more than two different time periods.

$$
\begin{array}{r}
\\
1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\\19\\20\\21\\22\\23\\24\\25\\26\\27\\28
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc}
P_{w,1,i} & P_{w,2,i} & P_{w,3,i} & P_{t,1,j} & P_{t,2,j} & P_{t,3,j} & f_{1,i,j} & f_{2,i,j} & f_{3,i,j} & f_{1,j,i} & f_{2,j,i} & f_{3,j,i}
\end{array}\\
\left(\begin{array}{cccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1\\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{array}
\begin{array}{r}
\\
Bal._{i,k=1}\\Bal._{i,k=2}\\Bal._{i,k=3}\\Bal._{j,k=1}\\Bal._{j,k=2}\\Bal._{j,k=3}\\
Cap._{i,j,k=1}\\Cap._{i,j,k=2}\\Cap._{i,j,k=3}\\Cap._{j,i,k=1}\\Cap._{j,i,k=2}\\Cap._{j,i,k=3}\\
Max._{w,k=1}\\Max._{w,k=2}\\Max._{w,k=3}\\Max._{t,k=1}\\Max._{t,k=2}\\Max._{t,k=3}\\
Min._{w,k=1}\\Min._{w,k=2}\\Min._{w,k=3}\\Min._{t,k=1}\\Min._{t,k=2}\\Min._{t,k=3}\\
RU_{k=2}\\RU_{k=3}\\RD_{k=1}\\RD_{k=2}
\end{array}
\tag{B.1}
$$

From this matrix, we take only the linearly independent rows (as including any linearly dependent one would make the determinant zero, thus satisfying the *Total Unimodularity* criterion) and we end up with the matrix shown in B.2 in which the original row numbers were left for clarity.

$$
\begin{array}{ccc}
 & \begin{array}{cccccccccccc} P_{w,1,i} & P_{w,2,i} & P_{w,3,i} & P_{t,1,j} & P_{t,2,j} & P_{t,3,j} & f_{1,i,j} & f_{2,i,j} & f_{3,i,j} & f_{1,j,i} & f_{2,j,i} & f_{3,j,i} \end{array} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 8 \\ 9 \\ 10 \\ 16 \\ 25 \\ 26 \end{array} &
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} &
\begin{array}{r} Bal._{i,k=1} \\ Bal._{i,k=2} \\ Bal._{i,k=3} \\ Bal._{j,k=1} \\ Bal._{j,k=2} \\ Bal._{j,k=3} \\ Cap._{i,j,k=2} \\ Cap._{i,j,k=3} \\ Cap._{j,i,k=1} \\ Max._{t,k=1} \\ RU_{k=2} \\ RU_{k=3} \end{array}
\end{array}
\tag{B.2}
$$

Then, using a brute force approach, we checked that every square submatrix has determinant 1, 0 or -1; therefore the original matrix A is *Total Unimodular* $\square$.