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An event-triggered control approach for the leader-tracking problem with heterogeneous agents

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ABSTRACT
This paper presents an event-triggered control and communication framework for the cooperative leader-tracking problem with communication constraints. Continuous communication among agents is not assumed in this work and decentralised event-based strategies are proposed for agents with heterogeneous linear dynamics. Also, the leader dynamics are unknown and only intermittent measurements of its states are obtained by a subset of the followers. The event-based method not only represents a way to restrict communication among agents, but it also provides a decentralised scheme for scheduling information broadcasts. Notably, each agent is able to determine its own broadcasting instants independently of any other agent in the network. In an extension, the case where transmission of information is affected by time-varying communication delays is addressed. Finally, positive lower-bounds on the inter-event time intervals are obtained in order to show that Zeno behaviour does not exist and, therefore, continuous exchange of information is never needed in this framework.

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1. Introduction

Control and coordination of groups of agents represents a rapidly growing research area with immediate academic, commercial, and military applications. Design and analysis of cooperative strategies and algorithms for groups of vehicles is a challenging problem. Several factors including communication constraints, agent dynamics, and heterogeneity of the group need to be addressed by the agent's cooperative scheme. An important problem in multi-agent systems is to design and implement decentralised algorithms for control and communication of agents. It is well understood that each agent should be able to determine its own control laws independently and based only on local information. This has been an important research topic (Ren, Beard, & Atkins, 2007). These papers consider agents with continuous-time dynamics, and it is assumed that agents can have continuous access to the states of their neighbours. However, continuous communication is not possible in many scenarios, and it becomes important to discern how frequently the agents should communicate in order to preserve the properties inherent in the corresponding control algorithms with continuous information exchange.

The sampled-data approach is commonly used to estimate the sampling periods (Cao & Ren, 2010; Qin & Gao, 2012). However approaches based on periodic transmission are not, in general, efficient with respect to bandwidth constraints and event-based methods for control and communication have provided an attractive choice to design and determine the broadcasting time instants at each node. Event-based controllers also provide a decentralised method to determine the transmission sequence for each agent. Contrary to sampled-data approaches, event-based controllers do not require synchronisation between the agents in order to implement the same sampling period and the same sampling instants.

Consensus problems where all agents are described by identical linear models have been addressed by Li, Duan, and Chen (2011) and Ma and Zhang (2010). Consensus and leader-following tracking involving agents with linear dynamics and switching topology was studied in Su and Huang (2012). More recently, the reference (Li, Duan, & Lewis, 2014) proposed an adaptive consensus protocol for agents with identical nominal dynamics but each agent is subject to different uncertainties. In Bidram, Lewis, and Davoudi (2014), the authors use input–output feedback linearisation in order to synchronise heterogeneous nonlinear systems. Output synchronisation of agents with double integrator dynamics and parameter uncertainties was analysed in Seyboth, Dimarogonas, Johansson, Frasca, and Allgower (2015), in which the

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synchronisation problem of networks of non-identical harmonic oscillators was also addressed.

All the previous references concerning linear systems assume that continuous communication between agents is possible. In this paper, apart from considering agents with heterogeneous linear dynamics, we study the consensus problem with a dynamic leader and with limited communication by means of event-triggered control strategies. Through the use of event-triggered methods for control and communication, we are able to disregard the periodic approach in favour of the decentralised design of transmissions sequences based on the occurrence of certain events at the local node or agent. In event-triggered broadcasting (Anta & Tabuada, 2010; Astrom & Bernhardson, 2002; Donkers & Heemels, 2010; Garcia & Antsaklis, 2013; Tabuada, 2007; Wang & Lemmon, 2011), a subsystem sends its local state to the network only when it is necessary, more precisely, only when a measure of the local subsystem state error is above a specified threshold. The state error is usually defined as $e(t) = x(t_k) - x(t)$, that is, the last measured state minus the current state of the system, where the measurement received at the controller node is held constant until a new measurement arrives. When this happens, the error is set to zero and starts growing until it triggers a new measurement update.

In the references (Dimarogonas, Frazzoli, & Johansson, 2012; Garcia, Cao, Yu, Antsaklis, & Casbeer, 2013; Nowzari & Cortes, 2014; Seyboth, Dimarogonas, & Johansson, 2013), the consensus problem with single integrator dynamics has been addressed using event-triggered control methods. Event-triggered control provides a more robust and efficient use of network bandwidth. Its implementation in networks of multiple systems also provides a highly decentralised way to schedule transmission instants which does not require synchronisation compared to periodic sampled-data approaches. Different authors have extended this approach, for instance, Chen and Hao (2012) studied event-triggered consensus for discrete-time integrators. The authors of Guo and Dimarogonas (2013) considered event-triggered consensus of single integrator systems using nonlinear consensus protocols.

Event-triggered consensus of identical linear systems has been recently addressed in Demir and Lunze (2012), Garcia et al. (2014a) and Liu, Hill, and Liu, 2012. In the references Garcia et al. (2014a) and Garcia, Cao, and Casbeer (2017), the event-triggered consensus of agents with linear dynamics which are connected by means of undirected graphs was considered. In those references, every agent is restricted to have the same dynamics (homogeneous agents were considered in those papers). In the present paper, we consider a more general and challenging case where agents have different dynamics (heterogeneous agents). We also address the problem of tracking a dynamic leader which has unknown nonlinear dynamics, in general. In addition, the leader also implements an event-based broadcasting strategy. Also, we study the case where each agent's dynamics are unknown to neighbours and the communication graph is directed as opposed to the more restrictive case of undirected graphs which were considered in Garcia et al. (2014a) and Garcia et al. (2017). Finally, we show that when the leader has known linear dynamics, the followers achieve asymptotic tracking of the leader. This is in contrast to the results in Garcia et al. (2014a) and Garcia et al. (2017) where only bounded convergence is obtained, even in the case when agents have the same dynamics.

The remainder of this paper is organised as follows. Section 2 states the problem. Event-based dynamic leader-tracking is studied in Section 3. The specific case of a leader with linear dynamics is considered in Section 4. Section 5 addresses the problem of communication delays. Examples are shown in Section 6 and conclusions are drawn in Section 7.

2. Preliminaries

2.1 Graph theory

For a team of N agents, the communication among them can be described by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, \dots, N\}$ denotes the agent set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge set. An edge (i, j) in the set $\mathcal{E}$ denotes that agent j can obtain information from agent i, but not necessarily *vice versa*. For an edge $(i, j) \in \mathcal{E}$, agent i is a neighbour of agent j. The set $\mathcal{N}_j$ is called the set of neighbours of agent j, and N_j is its cardinality. A directed path from agent i to agent j is a sequence of edges in a directed graph of the form $(i, p_1), (p_1, p_2), \dots, (p_{\kappa-1}, p_\kappa)(p_\kappa, j)$, where $p_\ell \in \mathcal{V}, \forall \ell = 1, \dots, \kappa$. A directed graph is *strongly connected* if there is a directed path from every agent to every other agent. A directed graph has *a directed spanning tree* if there exists at least one agent with directed paths to all other agents.

The adjacency matrix $\mathcal{A} \in \mathbb{R}^{N\times N}$ of a directed graph $\mathcal{G}$ is defined by $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D}$ represents the degree matrix which is a diagonal matrix with entries $d_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$. Define $\bar{d}_i = \max_i d_{ii}$. If a directed graph has a directed spanning tree, then the corresponding Laplacian matrix has only one eigenvalue equal to zero, $\lambda_1 = 0$, and the following holds for the remaining eigenvalues: $Re\{\lambda_i\} > 0$, for $i = 2, \dots, N$.

Lemma 2.1 (Ren & Cao, 2011)**:** *Consider $N - 1$ followers labelled as $i = 2, \dots, N$ and a leader $i = 1$. Let $\mathcal{G}$ be the* *directed graph of the $N-1$ followers and let $\mathcal{L}$ be the corresponding Laplacian matrix. Define $\hat{\mathcal{L}} = \mathcal{L} + diag\{a_{i1}\}$ for $i = 2, \dots, N$, where $a_{i1} = 1$ if the leader is a neighbour of agent i and $a_{i1} = 0$ otherwise. Then, all eigenvalues of $\hat{\mathcal{L}}$ have positive real parts if and only if the leader has directed paths to all followers.*

2.2 Problem statement

Consider a leader x_1 with dynamics given by

$$\dot{x}_1 = f_1(x_1, u_1, \omega) \tag{1}$$

where $x_1 \in \mathbb{R}^n$, $u_1 \in \mathbb{R}^{m_1}$, and $\omega \in \mathbb{R}^p$ are the state, the control input, and the external disturbance of the leader system dynamics. In this paper, we consider the general case where the leader dynamics are unknown and only a bound on the maximum rate of change of its state is given. Also consider a group of $N-1$ heterogeneous agents, labelled as followers, whose dynamics can be described by the following:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t), x_i(0) = x_{i_0}, i = 2, \dots, N \tag{2}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$. The objective of the leader-tracking problem is for the $N-1$ followers to track the state of the leader where only a subset of the followers is able to receive intermittent measurements of the leader's state.

In this paper, we consider the cooperative leader-tracking problem using event-triggered communication. This means that agents do not have continuous access to the states of their neighbours and every agent should design its own event-based rules in order to transmit its current information and update other agents in the network. Additionally, we do not assume that agents have identical dynamics, that is, we consider heterogeneous linear systems. Also, each agent's dynamics are unknown to their neighbours. Bounds on the tracking error are obtained. In the particular case where the leader has known linear dynamics, it is possible for the agents with heterogeneous dynamics to track the leader asymptotically. In addition, we consider the case where communication delays exist. Delays are time-varying and non-consistent in general. By non-consistent delays we refer to the general case where the delay associated to a transmitted state can be different to every receiving agent. Finally, in every case, we derive positive lower-bounds on the inter-event time intervals in order to exclude Zeno behaviour.

3. Multi-agent leader tracking

In this section, we consider the case where the function $f_1(x_1, u_1, \omega)$ in Equation (1) is unknown to the followers and only a subset of the followers is able to receive intermittent measurements of the state x_1. The measurements $x_1(t_{k_1})$ are transmitted by the leader according to its own local event-based policy.

The leader broadcasts its state measurement at time instants t_{k_1} which are defined by

$$t_{k_1+1} = \min\left\{t > t_{k_1} \middle| \, \|e_1(t)\|_\infty \geq \delta\right\} \tag{3}$$

for $\delta > 0$, where $e_1(t) = x_1(t_{k_1}) - x_1(t)$. In this paper we use infinity norms, but similar results can be obtained for other traditional vector and matrix norms. The subscript ∞ will not be explicitly shown next to the norm operators from this point forward unless it is necessary.

The event-triggered control method employed in this paper is based on dynamic controllers which are implemented by every follower. The state of this dynamic controller evolves according to

$$\begin{aligned}\dot{\phi}_i(t) = &-\sum_{j=2}^{N} a_{ij}(\phi_i(t) - \phi_j(t_{k_j})) - a_{i1}(\phi_i(t) \\ &- x_1(t_{k_1})), \quad \phi_i(0) = \phi_{i_0}\end{aligned} \tag{4}$$

where $\phi_i \in \mathbb{R}^n$ is the state of the dynamic controller implemented by follower i, for $i = 2, \dots, N$. When an event is triggered at node i, at time t_{k_i}, the variable $\phi_i(t_{k_i})$ is transmitted to agents j, such that $a_{ji} = 1$. The index t_{k_i} represents the sequence of time instants at which agent i generates its own events. Thus, the information that is exchanged by the followers is $\phi_i(t_{k_i})$, instead of $x_i(t_{k_i})$ as it is usual in other event-triggered consensus approaches. The goal is to make the variables ϕ_i track the leader's state while a local control input, $u_i(t) = g_i(x_i(t), \phi_i(t))$, which has continuous access to the local dynamic controller state ϕ_i, is designed such that the state x_i tracks or follows ϕ_i. Figure 1 shows the local controllers and event detector that are implemented by each follower. The discontinuous arrows outside the large block represent the agents' capabilities of receiving and transmitting discontinuous signals (generated by events). The figure shows what specific signals are received and transmitted by each agent.

The leader-tracking method presented in this paper consists on the separate design of dynamic controllers (with state represented by ϕ_i) that follow the leader's response and the design of local control inputs (given by u_i) that track the state of the local dynamic controller.

Note that $x_1(t_{k_1})$ appears in Equation (4) because the leader does not implement a dynamic controller and it

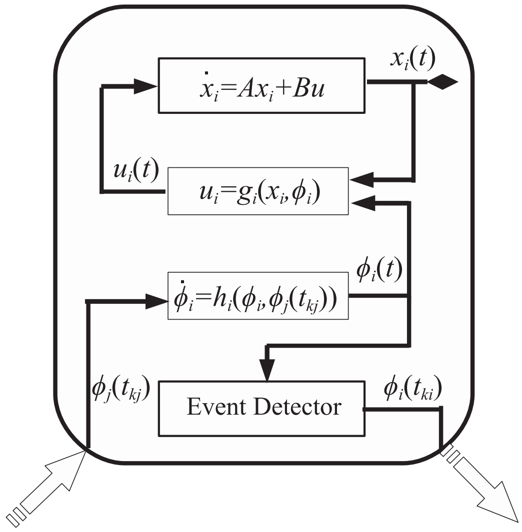

Figure 1. Agent controller and event detector blocks.

broadcasts its state x_1 at time instants t_{k_1}. Note that, by Lemma 2.1, when the leader has directed paths to all followers there exist $\hat{\alpha}, \hat{\beta} > 0$ such that $\|\mathbf{e}^{-\hat{\mathcal{L}}t}\| \leq \hat{\beta}\mathbf{e}^{-\hat{\alpha}t}$.

Let us define the dynamic controller state error as $e_i(t) = \phi_i(t_{k_i}) - \phi_i(t)$ for $i = 2, \ldots, N$. This error will be used to detect events and to decide when to broadcast the dynamic controller state ϕ_i. Also, define $e(t) = [e_2(t) \ldots e_N(t)]^T$.

Follower i's local control input u_i is designed according to the tracking controller in Castillo, Di Genaro, Monaco, and Normand-Cyrot (1997) and (Francis, 1977). For our particular problem we have that u_i is given by

$$u_i(t) = \Gamma_i\phi_i(t) + K_i(x_i(t) - \phi_i(t)) \tag{5}$$

where K_i is such that $A_i + B_iK_i$ is Hurwitz, and Γ_i satisfies the following equations:

$$\Lambda = A_i + B_i\Gamma_i \tag{6}$$

where $\Lambda = 0_{n\times n}$ in the case where the leader dynamics are unknown and $\Lambda = A_1$ in the case where the leader has known linear dynamics. The latter case will be discussed in Section 4. Let α' and β' be two positive constants such that $\|\mathbf{e}^{(A_i+B_iK_i)t}\| \leq \beta'\mathbf{e}^{-\alpha' t}$ for $i = 2, \ldots, N$.

Define the error ξ_i as the difference between the state of the dynamic controller of agent i and the leader's state, that is, $\xi_i(t) = \phi_i(t) - x_1(t)$, for $i = 2, \ldots, N$. Let $\xi = [\xi_2^T, \ldots \xi_N^T]^T$ and $\bar{\xi} = \|\xi(0)\|$. Define $\zeta_i(t) = x_i(t) - x_1(t)$ which is the tracking error of follower i with respect to the leader. Also, $\zeta = [\zeta_2^T, \ldots \zeta_N^T]^T$. The following theorem provides bounds on the tracking error and a positive lower-bound on inter-event time intervals when the leader dynamics are unknown to the followers.

Theorem 3.1: *Assume that the leader has directed paths to all followers and that its state rate of change is uniformly bounded, i.e.*

$$\|\dot{x}_1(t)\| \leq \bar{x}_1 \tag{7}$$

for $t \geq 0$. Then, the norm of the tracking error ζ is ultimately bounded by

$$(\gamma\bar{d}_i + \delta + \bar{x}_1)\left(\frac{\hat{\beta}}{\hat{\alpha}} + \frac{\tilde{\beta}}{\tilde{\alpha}}\right) \tag{8}$$

if the events of agent i, for $i = 2, \ldots, N$, are triggered according to the following condition:

$$t_{k_i+1} = \min\left\{t > t_{k_i} \mid \|e_i(t)\| \geq \beta\mathbf{e}^{-\alpha t} + \gamma\right\} \tag{9}$$

where $\beta > 0$, $\gamma > 0$, $0 < \alpha < \hat{\alpha}$, $\tilde{\beta} > 0$ and $0 < \tilde{\alpha} \leq \min\{\alpha', \hat{\alpha}\}$. Additionally, the agents do not exhibit Zeno behaviour and the inter-event times $t_{k_i+1} - t_{k_i}$ for every follower $i = 2, \ldots, N$ are bounded below by the positive time $\check{\tau}$, that is

$$\check{\tau} \leq t_{k_i+1} - t_{k_i} \tag{10}$$

where $\check{\tau} = \min\{\tau_1, \tau_2\}$,

$$\tau_1 = \frac{1}{\hat{\alpha}}\ln\left(1 + \frac{\beta}{H}\right)$$
$$\tau_2 = \frac{\hat{\alpha}\gamma}{\hat{\beta}(2N_i + a_{i1})(\gamma\bar{d}_i + \delta + \bar{x}_1) + \hat{\alpha}(N_i\gamma + a_{i1}\delta)} \tag{11}$$

and $H = \frac{\beta}{\alpha}N_i + \frac{\beta\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha}(2N_i + a_{i1})(\frac{1}{\alpha} - \frac{\mathbf{e}^{(\alpha-\hat{\alpha})t_{k_i}}}{\hat{\alpha}}) + \frac{\hat{\beta}\bar{\xi}}{\hat{\alpha}}(2N_i + a_{i1})\mathbf{e}^{(\alpha-\hat{\alpha})t_{k_i}}$.

Proof: We start with a proof of Equation (8), and then move to the lower-bound on the inter-event time intervals. Let us first note that at the time when an event is triggered by agent i the error e_i is reset to zero, that is, $e_i(t_{k_i}) = 0$. Thus, from Equation (9), the error e_i satisfies $\|e_i(t)\| \leq \beta\mathbf{e}^{-\alpha t} + \gamma$, for $i = 2, \ldots, N$. Hence, we have that $\|e(t)\|_\infty \leq \beta\mathbf{e}^{-\alpha t} + \gamma$.

The dynamics of the error between the leader's state and the state of the dynamic controller of agent i can be written as follows:

$$\dot{\xi}_i(t) = -\sum_{j=2}^{N} a_{ij}(\phi_i(t) - \phi_j(t_{k_j})) - a_{i1}(\phi_i - x_1(t_{k_1})) - \dot{x}_1(t)$$

$$= -\sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t)) - a_{i1}\xi_i(t) + \sum_{j=2}^{N} a_{ij}e_j(t) + a_{i1}e_1(t) - \dot{x}_1(t) \quad (12)$$

for $i = 2, \dots ,N$. Equation (12) can be written in compact form:

$$\dot{\xi} = -\bar{\mathcal{L}}\xi + \bar{\mathcal{A}}e + \bar{a}_1 e_1 - (1_{N-1} \otimes I_n)\dot{x}_1 \quad (13)$$

where $\bar{\mathcal{L}} = \hat{\mathcal{L}} \otimes I_n$, $\bar{\mathcal{A}} = \mathcal{A} \otimes I_n$, $\bar{a}_1 = a_1 \otimes I_n$, and $a_1 = [a_{21} \dots a_{N1}]^T$. The matrix $\hat{\mathcal{L}}$ was defined in Lemma 2.1. The response of the error ξ can be bounded as follows:

$$\begin{aligned}\|\xi(t)\| &= \left\| \mathbf{e}^{-\hat{\mathcal{L}}t}\xi(0) + \int_0^t \mathbf{e}^{-\bar{\mathcal{L}}(t-s)}(\bar{\mathcal{A}}e(s) + \bar{a}_1 e_1(s) - \dot{x}_1(s))ds \right\| \\ &\le \hat{\beta}\bar{\xi}\mathbf{e}^{-\hat{\alpha}t} + \beta\hat{\beta}\bar{d}_i \int_0^t \mathbf{e}^{-\hat{\alpha}(t-s)}\mathbf{e}^{-\alpha s}ds + \hat{\beta}(\gamma\bar{d}_i + \delta + \bar{x}_1)\int_0^t \mathbf{e}^{-\hat{\alpha}(t-s)}ds \\ &\le \hat{\beta}\bar{\xi}\mathbf{e}^{-\hat{\alpha}t} + \frac{\beta\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha}(\mathbf{e}^{-\alpha t} - \mathbf{e}^{-\hat{\alpha}t}) + \frac{\hat{\beta}}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1)(1 - \mathbf{e}^{-\hat{\alpha}t}) \end{aligned} \quad (14)$$

and as t goes to infinity we have that

$$\lim_{t\to\infty} \|\xi(t)\| \le \frac{\hat{\beta}}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1).$$

So far we have shown that the error between the leader's state and each follower's dynamic controller state is bounded. In order to complete this proof we need to show that the error between each follower state, x_i, and the leader state, x_1, is bounded for all time $t \ge 0$. Let us define $\psi_i = x_i - \phi_i$ for $i = 2, \dots ,N$, which is the difference between each follower's state and the state of its local dynamic controller. We have that

$$\dot{\psi}_i = A_i x_i(t) + B_i u_i(t) + \sum_{j=2}^{N} a_{ij}(\phi_i(t) - \phi_j(t_{k_j})) + a_{i1}(\phi_i(t) - x_1(t_{k_1})). \quad (15)$$

Substituting Equation (5) into Equation (15) we obtain

$$\dot{\psi}_i = (A_i + B_i K_i)\psi_i(t) + \sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t)) + a_{i1}\xi_i(t) - \sum_{j=2}^{N} a_{ij}e_j(t) - a_{i1}e_1(t). \quad (16)$$

We can write the dynamics of the errors ψ and ξ as

$$\begin{bmatrix}\dot{\psi}\\ \dot{\xi}\end{bmatrix} = \begin{bmatrix} I_{N-1}\otimes(A_i + B_i K_i) & \bar{\mathcal{L}} \\ 0_{n(N-1)\times n(N-1)} & -\bar{\mathcal{L}}\end{bmatrix}\begin{bmatrix}\psi\\ \xi\end{bmatrix} + \begin{bmatrix}-\bar{\mathcal{A}}\\ \bar{\mathcal{A}}\end{bmatrix}e + \begin{bmatrix}-\bar{a}_1\\ \bar{a}_1\end{bmatrix}e_1 - \begin{bmatrix}0_{n(N-1)}\\ (1_{N-1}\otimes I_n)\dot{x}_1\end{bmatrix} \quad (17)$$

where $\|\mathbf{e}^{(I_{N-1}\otimes(A_i+B_iK_i))t}\| \le \beta'\mathbf{e}^{-\alpha' t}$. Let $\Psi = [\psi^T \xi^T]^T$ and $\bar{\Psi} = \|\Psi(0)\|$, then, we can write the following:

$$\begin{aligned}\|\Psi(t)\| &\le \tilde{\beta}\mathbf{e}^{-\tilde{\alpha}t}\bar{\Psi} + \beta\tilde{\beta}\bar{d}_i\int_0^t \mathbf{e}^{-\tilde{\alpha}(t-s)}\mathbf{e}^{-\alpha s}ds + \tilde{\beta}(\gamma\bar{d}_i + \delta + \bar{x}_1)\int_0^t \mathbf{e}^{-\tilde{\alpha}(t-s)}ds \\ &\le \tilde{\beta}\mathbf{e}^{-\tilde{\alpha}t}\bar{\Psi} + \frac{\beta\tilde{\beta}\bar{d}_i}{\tilde{\alpha}-\alpha}(\mathbf{e}^{-\alpha t} - \mathbf{e}^{-\tilde{\alpha}t}) + \frac{\tilde{\beta}}{\tilde{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1)(1 - \mathbf{e}^{-\tilde{\alpha}t}).\end{aligned} \quad (18)$$

Additionally, we can write $\zeta_i = x_i - x_1 = \psi_i + \xi_i$. Hence, the error $\zeta = \psi + \xi$ satisfies

$$\|\zeta(t)\| \le \|\psi(t)\| + \|\xi(t)\| \le \|\Psi(t)\| + \|\xi(t)\|. \quad (19)$$

So we have that the tracking error ζ is ultimately bounded by Equation (8).

We will now prove that the inter-event time intervals of any agent are lower-bounded by a positive scalar. This shows that Zeno behaviour does not occur at any node. Let us consider the dynamics of the errors $e_i(t)$, for $i = 2, \dots ,N$. This error is used to compute and trigger the events at each node as stated in Equation (9). Because $e_i(t) = \phi(t_{k_i}) - \phi_i(t)$ we have that $\dot{e}_i(t) = -\dot{\phi}_i(t) = \sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t)) + a_{i1}\xi_i(t) - \sum_{j=2}^{N} a_{ij}e_j(t) - a_{i1}e_1(t)$, for $t \in [t_{k_i}, t_{k_i+1})$. Also,

$$\begin{aligned}\frac{d}{dt}\|e_i(t)\| &\le (2N_i + a_{i1})\|\xi(t)\| + N_i(\beta\mathbf{e}^{-\alpha t} + \gamma) + a_{i1}\delta \\ &\le (2N_i + a_{i1})[\hat{\beta}\mathbf{e}^{-\hat{\alpha}t}\bar{\xi} + \frac{\beta\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha}(\mathbf{e}^{-\alpha t} - \mathbf{e}^{-\hat{\alpha}t}) + \frac{\hat{\beta}}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1)(1 - \mathbf{e}^{-\hat{\alpha}t})] \\ &\quad + N_i(\beta\mathbf{e}^{-\alpha t} + \gamma) + a_{i1}\delta \end{aligned} \quad (20)$$

for $t \in [t_{k_i}, t_{k_i+1})$. We can bound the response of the error during the time interval $t \in [t_{k_i}, t_{k_i+1})$ as follows:

$$\|e_i(t)\| \leq (2N_i + a_{i1}) \int_{t_{k_i}}^{t} \left(\hat{\beta}\bar{\xi} - \frac{\beta\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{\hat{\beta}}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1) \right) \mathbf{e}^{-\hat{\alpha}s} ds + \int_{t_{k_i}}^{t} \left((2N_i + a_{i1}) \frac{\beta\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i\beta \right) \mathbf{e}^{-\alpha s} ds + \left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma\bar{d}_i + \delta + \bar{x}_1) + N_i\gamma + a_{i1}\delta \right)(t - t_{k_i}). \tag{21}$$

Evaluating the integrals in Equation (21) and letting $\tau = t - t_{k_i}$ we obtain

$$\|e_i(t)\| \leq \frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \left(\bar{\xi} - \frac{\beta\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1) \right) \times (\mathbf{e}^{-\hat{\alpha}t_{k_i}} - \mathbf{e}^{-\hat{\alpha}(\tau + t_{k_i})}) + \frac{\beta}{\alpha}\left((2N_i + a_{i1}) \times \frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i \right)(\mathbf{e}^{-\alpha t_{k_i}} - \mathbf{e}^{-\alpha(\tau + t_{k_i})}) + \left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma\bar{d}_i + \delta + \bar{x}_1) + N_i\gamma + a_{i1}\delta \right)\tau. \tag{22}$$

At this point we realise that the time $\tau > 0$ that it takes for the last expression in Equation (22) to grow from zero, at time t_{k_i}, to reach the threshold $\beta\mathbf{e}^{-\alpha t} + \gamma = \beta\mathbf{e}^{-\alpha(t_{k_i}+\tau)} + \gamma$ is less or equal than the time it takes the error $\|e_i(t)\|$ to grow from zero, at time t_{k_i}, to reach the same threshold and generate the following event at time t_{k_i+1}, that is, $0 < \tau \leq t_{k_i+1} - t_{k_i}$. Thus, we wish to find a lower-bound $\tau > 0$ such that the following holds

$$\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \left(\bar{\xi} - \frac{\beta\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1) \right) \times (1 - \mathbf{e}^{-\hat{\alpha}\tau})\mathbf{e}^{-\hat{\alpha}t_{k_i}} + \frac{\beta}{\alpha}\left((2N_i + a_{i1}) \frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i \right) \times (1 - \mathbf{e}^{-\alpha\tau})\mathbf{e}^{-\alpha t_{k_i}} + \left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma\bar{d}_i + \delta + \bar{x}_1) + N_i\gamma + a_{i1}\delta \right)\tau \leq \beta\mathbf{e}^{-\alpha(t_{k_i}+\tau)} + \gamma. \tag{23}$$

Here, we strive to obtain the largest value of τ, which we denote as $\breve{\tau}$, such that Equation (23) holds. This is achieved by replacing the less than or equal sign in Equation (23) with an equality sign and solving the resulting equation numerically to obtain the lower-bound on the inter-event time intervals $\breve{\tau}$.

An explicit but conservative solution for $\breve{\tau}$ can be obtained by noting that if we can guarantee that both

$$\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \left(\bar{\xi} - \frac{\beta\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1) \right) \times (1 - \mathbf{e}^{-\hat{\alpha}\tau})\mathbf{e}^{-\hat{\alpha}t_{k_i}} + \frac{\beta}{\alpha}\left((2N_i + a_{i1}) \frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i \right) \times (1 - \mathbf{e}^{-\alpha\tau})\mathbf{e}^{-\alpha t_{k_i}} \leq \beta\mathbf{e}^{-\alpha(t_{k_i}+\tau)} \tag{24}$$

and

$$\left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma\bar{d}_i + \delta + \bar{x}_1) + N_i\gamma + a_{i1}\delta \right)\tau \leq \gamma \tag{25}$$

hold, then, inequality Equation (23) holds. From Equation (24) we can write

$$\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \left(\bar{\xi} - \frac{\beta\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma\bar{d}_i + \delta + \bar{x}_1) \right) \times (1 - \mathbf{e}^{-\hat{\alpha}\tau})\mathbf{e}^{-\hat{\alpha}t_{k_i}} + \frac{\beta}{\alpha}\left((2N_i + a_{i1}) \frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i \right) \times (1 - \mathbf{e}^{-\alpha\tau})\mathbf{e}^{-\alpha t_{k_i}} \leq \frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \left(\bar{\xi} - \frac{\beta\bar{d}_i}{\hat{\alpha}-\alpha} \right)(1 - \mathbf{e}^{-\hat{\alpha}\tau})\mathbf{e}^{-\hat{\alpha}t_{k_i}} + \frac{\beta}{\alpha}\left((2N_i + a_{i1}) \frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i \right)(1 - \mathbf{e}^{-\hat{\alpha}\tau})\mathbf{e}^{-\alpha t_{k_i}} \tag{26}$$

since $\mathbf{e}^{-\hat{\alpha}\tau} \leq \mathbf{e}^{-\alpha\tau}$ for any $\tau \geq 0$. Thus, by solving

$$\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \left(\bar{\xi} - \frac{\beta\bar{d}_i}{\hat{\alpha}-\alpha} \right)(1 - \mathbf{e}^{-\hat{\alpha}\tau_1})\mathbf{e}^{-\hat{\alpha}t_{k_i}} + \frac{\beta}{\alpha}\left((2N_i + a_{i1}) \frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i \right)(1 - \mathbf{e}^{-\hat{\alpha}\tau_1})\mathbf{e}^{-\alpha t_{k_i}} = \beta\mathbf{e}^{-\hat{\alpha}\tau_1}\mathbf{e}^{-\alpha t_{k_i}} \tag{27}$$

we guarantee that Equation (24) holds. Such solution is given by τ_1 in Equation (11).

In order to obtain the solution τ_2 in Equation (11), we simply solve the following equation:

$$\left(\tfrac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma\bar{d}_i + \delta + \bar{x}_1) + N_i\gamma + a_{i1}\delta\right)\tau_2 = \gamma.$$

Then, we choose $\breve{\tau} = \min\{\tau_1, \tau_2\}$ in order to satisfy both Equations (24) and (25). Note that $\tau_1 > 0$ and $\tau_2 > 0$ which guarantees that Zeno behaviour does not occur at any follower node.

Finally, the exclusion of Zeno behaviour at the leader node is obtained as follows. Recall that the leader also implements an event-triggered strategy, shown in Equation (3), in order to transmit its state to a subset of followers where the leader error is given by $e_1(t) = x_1(t_{k_1}) - x_1(t)$, $e_1(t_{k_1}) = 0$. Hence, $\dot{e}_1(t) = -\dot{x}_1(t)$ and $\frac{d}{dt}||e_1(t)|| \leq \bar{x}_1$ by Equation (7). The response of the error e_1 during the time interval $t \in [t_{k_1}, t_{k_1+1})$ is bounded by $||e_1(t)|| \leq \bar{x}_1(t - t_{k_1})$. Therefore, the error $||e_1(t)||$ will grow at a slower pace than the linear term $\bar{x}_1(t - t_{k_1})$ and its inter-event time intervals satisfy $t_{k_1+1} - t_{k_1} \geq \bar{\tau}$ where $\bar{\tau}$ is the solution of $\bar{x}_1\bar{\tau} = \delta$. Hence,

$$t_{k_1+1} - t_{k_1} \geq \frac{\delta}{\bar{x}_1} > 0 \tag{28}$$

and Zeno behaviour is excluded at the leader node. ■

4. Leader with linear dynamics

In this section, we address the particular case where the leader has known linear dynamics:

$$\dot{x}_1 = A_1 x_1. \tag{29}$$

where $x_1 \in \mathbb{R}^n$. Let the constant c satisfy $c \geq 1/Re(\lambda_1(\hat{\mathcal{L}}))$ where $\lambda_1(\hat{\mathcal{L}})$ is the eigenvalue of $\hat{\mathcal{L}}$ with smallest real part (recall that all eigenvalues of $\hat{\mathcal{L}}$ have positive real part). Also, let F be a symmetric and positive definite, matrix such that $A_1 - F$ is Hurwitz. Assume that the leader has directed paths to each follower. Then, the matrix $\bar{A}_1 - \bar{B}$ is a Hurwitz matrix, where $\bar{A}_1 = I_{N-1} \otimes A_1$ and $\bar{B} = c\hat{\mathcal{L}} \otimes F$ and there exists positive constants α_1 and β_1 such that $\left\|\mathbf{e}^{(\bar{A}_1 - \bar{B})t}\right\| \leq \beta_1 \mathbf{e}^{-\alpha_1 t}$.

The previous statement can be proved by noting that a matrix F can always be obtained by finding a stabilising control gain for the generic linear system $\dot{x} = A_1 x + Bu$. where $B = I_n$. Also note that the pair (A_1, I_n) is controllable for any A_1.

In addition, there exists a similarity transformation S such that $\hat{\mathcal{L}}_J = S^{-1}\hat{\mathcal{L}}S$ is in Jordan canonical form. Define $\bar{S} = S \otimes I_n$ and calculate the following:

$$\begin{aligned}\bar{S}^{-1}(\bar{A}_1 - \bar{B})\bar{S} &= \bar{S}^{-1}\bar{A}_1\bar{S} - \bar{S}^{-1}(c\hat{\mathcal{L}} \otimes F)\bar{S} \\ &= I_{N-1} \otimes A_1 - c\hat{\mathcal{L}}_J \otimes F.\end{aligned} \tag{30}$$

By applying the similarity transformation we obtain that the eigenvalues of $\bar{A}_1 - \bar{B}$ are given by the eigenvalues of $A_1 - c\lambda_i F$, where $\lambda_i = \lambda_i(\hat{\mathcal{L}})$.

Because the matrix $A_1 - F$ is Hurwitz, for $F > 0$, then it satisfies $(A_1 - F)^T F + F(A_1 - F) = A_1^T F + FA_1 - 2FF < 0$. Also, we have that $(A_1 - c\lambda_i F)^T F + F(A_1 - c\lambda_i F) = A_1^T F + FA_1 - 2c\lambda_i FF \leq A_1^T F + FA_1 - 2FF < 0$ because $c\lambda_i \geq 1$ for any $\lambda_i(\hat{\mathcal{L}})$. Therefore, all matrices $A_1 - c\lambda_i F$ are Hurwitz and so is the matrix $\bar{A}_1 - \bar{B}$.

We emphasise that the leader system (29) is an autonomous system, that is, it cannot be controlled. The design of the matrix $F > 0$ is used by the followers in order to track the leader with dynamics (29) where A_1 is known by each follower.

In this case we implement dynamic controllers of the following form:

$$\begin{aligned}\dot{\phi}_i(t) = A_1\phi_i - cF\Bigg(&\sum_{j=2}^{N} a_{ij}(\phi_i(t) - \hat{\phi}_j(t)) \\ &+ a_{i1}(\phi_i(t) - \hat{x}_1(t))\Bigg)\end{aligned} \tag{31}$$

for $i = 2, \ldots, N$ where the variables $\hat{\phi}_j$ represent decoupled models or estimates of the dynamic controller states which are updated using the corresponding states ϕ_j, that is

$$\dot{\hat{\phi}}_j(t) = A_1\hat{\phi}_j(t), \quad \hat{\phi}_j(t_{k_j}) = \phi_j(t_{k_j}) \tag{32}$$

for $t \in [t_{k_j}, t_{k_j+1})$ and for j such that $a_{ij} = 1$. Similarly,

$$\dot{\hat{x}}_1(t) = A_1\hat{x}_1(t), \quad \hat{x}_1(t_{k_1}) = x_1(t_{k_1}). \tag{33}$$

for $t \in [t_{k_1}, t_{k_1+1})$ if $a_{i1} = 1$. Note that the followers use the matrix A_1 to implement both the dynamic controllers and the models of neighbours' dynamic controllers. However, the real system dynamics for each agent are still given by Equation (2).

The method used in this section is the model-based approach (Garcia et al., 2014a, 2014b) where each agent implements models of neighbour states. Comparing Equation (31) to Equation (4) we note that an estimate of the controller variables $\phi_j(t)$, denoted by $\hat{\phi}_j(t)$, is used instead of simply using $\phi_j(t_{k_j})$. In other words, in the model-based approach the agents do not implement the received information from the neighbours, $\phi_j(t_{k_j})$, but they use this information to estimate the controller state $\phi_j(t)$ for the time interval $t \in [t_{k_j}, t_{k_j+1})$ as it is shown by Equation (32).

In this section, the dynamic controller state errors are given by $e_i(t) = \hat{\phi}_i(t) - \phi_i(t)$. Define $A_\infty = \|A_1\|_\infty$ and $F_\infty = \|F\|_\infty$. The following theorem establishes asymptotic tracking when the leader state matrix A_1 is known to the followers.

Theorem 4.1: *Assume that the leader* (29) *has directed paths to all followers. The events of the leader are given by*

Equation (3). Then, the followers track the leader asymptotically if the events of agent i are triggered according to the following condition:

$$t_{k_i+1} = \min\left\{t > t_{k_i} \mid \|e_i(t)\| = \beta \mathbf{e}^{-\alpha t}\right\} \tag{34}$$

where $\beta > 0$ and $0 < \alpha < \alpha_1$. Additionally, the agents do not exhibit Zeno behaviour and the inter-event times $t_{k_i+1} - t_{k_i}$ for every follower $i = 2, \ldots, N$ are bounded below by the positive time $\breve{\tau}$ as in Equation (10) where

$$\breve{\tau} = \frac{1}{A_\infty + \alpha_1} \ln\left(1 + \frac{\beta}{H_L}\right) \tag{35}$$

and $H_L = \frac{\beta}{A_\infty+\alpha}((2N_i+a_{i1})\frac{c\beta_1\bar{d}_i F_\infty}{\alpha_1-\alpha}+N_i)+\frac{\beta_1(2N_i+a_{i1})}{A_\infty+\alpha_1}(\bar{\xi}-\frac{c\beta\bar{d}_i F_\infty}{\alpha_1-\alpha})\mathbf{e}^{-\alpha_1 t_{k_i}}$.

Proof: Let $\xi_i = \phi_i - x_1$ be the error between the leader's state and each followers' dynamic controller state. Then, we have that

$$\begin{aligned}\dot{\xi}_i(t) &= A_1\phi_i - cF\left(\sum_{j=2}^{N} a_{ij}(\phi_i(t) - \hat{\phi}_j(t))\right.\\ &\quad \left. + a_{i1}(\phi_i(t) - \hat{x}_1(t))\right) - A_1 x_1\\ &= A_1\xi_i - cF\left(\sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t)) + a_{i1}\xi_i(t)\right.\\ &\quad \left. - \sum_{j=2}^{N} a_{ij}e_j(t) - a_{i1}e_1(t)\right).\end{aligned} \tag{36}$$

Note that, given an appropriate δ, due to the known linear dynamics of the leader (29) and the models of the leader (33) implemented by agents i such that $a_{i1} = 1$, the error $e_1(t) = 0$ for $t \geq t_{k_1}$ for $k_1 = 0$. Thus, we can write the dynamics of the errors ξ_i in compact form:

$$\dot{\xi}(t) = (\bar{A}_1 - \bar{B})\xi(t) + \bar{B}_{\mathcal{A}} e(t). \tag{37}$$

where $\bar{B}_{\mathcal{A}} = c\mathcal{A} \otimes F$.

Now, we analyse the tracking properties of the state of each follower. The dynamics of each follower are given by Equation (2) with local control inputs given by Equations (5) and (6) for $\Lambda = A_1$. Let $\zeta_i = x_i - x_1$ and $\psi_i = x_i - \phi_i$ for $i = 2, \ldots, N$. We have that

$$\begin{aligned}\dot{\psi}_i &= (A_i + B_i K_i)\psi_i(t) + cF\left(\sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t))\right.\\ &\quad \left. + a_{i1}\xi_i(t) - \sum_{j=2}^{N} a_{ij}e_j(t)\right).\end{aligned} \tag{38}$$

We can write the dynamics of the errors ψ and ξ as follows:

$$\begin{aligned}\begin{bmatrix}\dot{\psi}\\ \dot{\xi}\end{bmatrix} &= \begin{bmatrix} I_{N-1} \otimes (A_i + B_i K_i) & \bar{B}\\ 0_{N-1} & \bar{A}_1 - \bar{B}\end{bmatrix}\begin{bmatrix}\psi\\ \xi\end{bmatrix}\\ &\quad + \begin{bmatrix}-\bar{B}_{\mathcal{A}}\\ \bar{B}_{\mathcal{A}}\end{bmatrix} e\end{aligned} \tag{39}$$

Since both matrices in the main diagonal of Equation (39) are Hurwitz we can show that

$$\lim_{t\to\infty} \Psi(t) = 0 \tag{40}$$

where $\Psi = [\psi^T \xi^T]^T$. Thus, the states of the followers, x_i, and the states of the followers' dynamic controllers ϕ_i track the leader's state, x_1, asymptotically, since $\zeta = \psi + \xi$.

In order to show that no follower exhibits Zeno behaviour we analyse the dynamics of the error $e_i(t)$, for $i = 2, \ldots, N$. We have that $\dot{e}_i(t) = A_1 e_i + cF(\sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t)) + a_{i1}\xi_i(t) - \sum_{j=2}^{N} a_{ij}e_j(t))$. Thus, the error dynamics can be bounded as follows:

$$\frac{d}{dt}\|e_i(t)\| \leq A_\infty \|e_i(t)\| + (2N_i + a_{i1})\|\xi(t)\| + \beta N_i \mathbf{e}^{-\alpha t}$$

where

$$\|\xi(t)\| \leq \beta_1\bar{\xi}\mathbf{e}^{-\alpha_1 t} + \frac{c\beta\beta_1\bar{d}_i F_\infty}{\alpha_1 - \alpha}(\mathbf{e}^{-\alpha t} - \mathbf{e}^{-\alpha_1 t}).$$

Letting $\tau = t - t_{k_i}$, the response of the error e_i can then be bounded by

$$\begin{aligned}\|e_i(t)\| &\leq \frac{\beta_1(2N_i + a_{i1})}{A_\infty + \alpha_1}\left(\bar{\xi} - \frac{c\beta\bar{d}_i F_\infty}{\alpha_1 - \alpha}\right)\\ &\quad \times (\mathbf{e}^{A_\infty\tau} - \mathbf{e}^{-\alpha_1\tau})\mathbf{e}^{-\alpha_1 t_{k_i}}\\ &\quad + \frac{\beta}{A_\infty + \alpha}\left(\frac{(2N_i + a_{i1})c\beta_1\bar{d}_i F_\infty}{\alpha_1 - \alpha} + N_i\right)\\ &\quad \times (\mathbf{e}^{A_\infty\tau} - \mathbf{e}^{-\alpha\tau})\mathbf{e}^{-\alpha t_{k_i}}\end{aligned} \tag{41}$$

Similar to Theorem 3.1, the time $\tau > 0$ that it takes for the last expression in Equation (41) to grow from zero, at time t_{k_i}, to reach the threshold $\beta\mathbf{e}^{-\alpha(t_{k_i}+\tau)}$ is less or equal than the time it takes the error $\|e_i(t)\|$ to grow from zero, at time t_{k_i}, to reach the same threshold and generate the following event at time t_{k_i+1}, that is, $0 < \tau \leq t_{k_i+1} - t_{k_i}$. Thus, we wish to find a strictly positive lower-bound τ such that the following holds:

$$\frac{\beta_1(2N_i + a_{i1})}{A_\infty + \alpha_1}\left(\bar{\xi} - \frac{c\beta\bar{d}_i F_\infty}{\alpha_1 - \alpha}\right)(\mathbf{e}^{A_\infty\tau} - \mathbf{e}^{-\alpha_1\tau})\mathbf{e}^{(\alpha-\alpha_1)t_{k_i}}$$

$$+\frac{\beta}{A_\infty+\alpha}\left(\frac{(2N_i+a_{i1})c\beta_1\bar{d}_iF_\infty}{\alpha_1-\alpha}+N_i\right)(\mathbf{e}^{A_\infty\tau}-\mathbf{e}^{-\alpha\tau}) \\ \leq \beta\mathbf{e}^{-\alpha\tau} \tag{42}$$

An explicit solution can be obtained by noting that the left-hand side of Equation (42) is less than or equal to $H_L(\mathbf{e}^{A_\infty\tau}-\mathbf{e}^{-\alpha_1\tau})$ and also noting that $\beta\mathbf{e}^{-\alpha\tau} \geq \beta\mathbf{e}^{-\alpha_1\tau}$ for $\tau \geq 0$. These two relations hold since $\alpha < \alpha_1$. Thus, by solving the equation

$$H_L(\mathbf{e}^{A_\infty\breve{\tau}}-\mathbf{e}^{-\alpha_1\breve{\tau}}) = \beta\mathbf{e}^{-\alpha_1\breve{\tau}} \tag{43}$$

we guarantee that Equation (42) holds. Such solution is given by Equation (35). Note that $\breve{\tau} > 0$ which guarantees that Zeno behaviour does not occur at any node and the proof is complete. ∎

5. Event-based leader tracking with communication delays

We consider the general case where $\dot{x}_1 = f_1(x_1, u_1, \omega)$ and the function $f_1(\cdot,\cdot,\cdot)$ is unknown. This is similar to the case shown in Section 3. In this section, we also consider time-varying and non-consistent communication delays. Non-consistent means that delays $\sigma_{ij}(t_{k_j})$ associated to the same transmitted message can be different for every receiving agent.

The state of the dynamic controller evolves according to

$$\dot{\phi}_i(t) = -\sum_{j=2}^{N} a_{ij}(\phi_i(t)-\phi_{ij}(t)) - a_{i1}(\phi_i(t)-x_{i1}(t)) \tag{44}$$

where $\phi_{ij}(t) = \phi_j(t_{k_j})$ for $t \in [t_{k_j}+\sigma_{ij}(t_{k_j}), t_{k_j+1}+\sigma_{ij}(t_{k_j+1}))$ and $\sigma_{ij}(t_{k_j})$ represents the communication delay from agent j to agent i associated to agent j triggering time t_{k_j}.

Similarly, $x_{i1}(t) = x_1(t_{k_1})$ for $t \in [t_{k_1}+\sigma_{i1}(t_{k_1}), t_{k_1+1}+\sigma_{i1}(t_{k_1+1}))$. The leader generates events according to condition (3), for $\delta > 0$. Time-varying and non-consistent communication delays affecting information transmitted by the leader are denoted by $\sigma_{i1}(t_{k_1})$. Let us assume that $\sigma_{i1}(t_{k_1}) \leq \sigma_1$ for some $\sigma_1 > 0$, for any i such that $a_{i1} = 1$, and for $k_1 = 0, 1, \ldots$ Then, we have that $\|e_{i1}(t_{k_1})\| \leq \delta_d$ for any i such that $a_{i1} = 1$ and for $k_1 = 0, 1, \ldots$, where $\delta_d = \delta + \sigma_1\bar{x}_1$.

Define the errors associated to delayed information:

$$e_{ij}(t) = \phi_{ij}(t) - \phi_j(t) \\ e_{i1}(t) = x_{i1}(t) - x_1(t). \tag{45}$$

Define $H_d = \frac{\beta_d}{\alpha}N_i + \frac{\beta_d\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha}(2N_i+a_{i1})(\frac{1}{\alpha}-\frac{\mathbf{e}^{(\alpha-\hat{\alpha})t_{k_i}}}{\hat{\alpha}}) + \frac{\hat{\beta}\bar{\xi}}{\hat{\alpha}}(2N_i+a_{i1})\mathbf{e}^{(\alpha-\hat{\alpha})t_{k_i}}$.

In the following theorem we consider again a leader with unknown dynamics and provide bounds on the tracking error and the inter-event time intervals. We also provide an estimate of the admissible delays, that is, what is the maximum allowable delay in the system such that the followers are guaranteed to track the leader with the prescribed performance.

Theorem 5.1: *Assume that the leader has directed paths to all followers and that it satisfies Equation (7). Then, for communication delays $\sigma_{ij} \in [0, \breve{\sigma}]$ the tracking error ζ is ultimately bounded by*

$$(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1)\left(\frac{\hat{\beta}}{\hat{\alpha}} + \frac{\tilde{\beta}}{\tilde{\alpha}}\right). \tag{46}$$

if the events of agent i are triggered according to condition (9), for $i = 2, \ldots, N$, where $\beta > 0$, $\gamma > 0$, $0 < \alpha < \hat{\alpha}$, $\tilde{\beta} \geq 0$ and $0 < \tilde{\alpha} \leq \min\{\alpha', \hat{\alpha}\}$. The maximum admissible delay is given by $\breve{\sigma} = \min\{\sigma_1, \sigma_2\}$, where

$$\sigma_1 = \frac{1}{\hat{\alpha}}\ln\left(\frac{\beta_d + H_d}{\beta + H_d}\right) \\ \sigma_2 = \frac{\hat{\alpha}(\gamma_d - \gamma)}{\hat{\beta}(2N_i+a_{i1})(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1) + \hat{\alpha}(N_i\gamma_d + a_{i1}\delta_d)} \tag{47}$$

$\beta_d > \beta$, $\gamma_d > \gamma$, and $H_d = \frac{\beta_d}{\alpha}N_i + \frac{\beta_d\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha}(2N_i+a_{i1})(\frac{1}{\alpha}-\frac{\mathbf{e}^{(\alpha-\hat{\alpha})t_{k_i}}}{\hat{\alpha}}) + \frac{\hat{\beta}\bar{\xi}}{\hat{\alpha}}(2N_i+a_{i1})\mathbf{e}^{(\alpha-\hat{\alpha})t_{k_i}}$.

Additionally, the agents do not exhibit Zeno behaviour and the inter-event times $t_{k_i+1} - t_{k_i}$ for every follower $i = 2, \ldots, N$ are bounded below by the positive time $\breve{\tau}$ as in Equation (10) where $\breve{\tau} = \min\{\tau_1, \tau_2\}$

$$\tau_1 = \frac{1}{\hat{\alpha}}\ln\left(1 + \frac{\beta}{H_d}\right) \\ \tau_2 = \frac{\hat{\alpha}\gamma}{\hat{\beta}(2N_i+a_{i1})(\gamma_d\bar{d}_i+\delta_d+\bar{x}_1)+\hat{\alpha}(N_i\gamma_d+a_{i1}\delta_d)}. \tag{48}$$

Proof: The dynamics of the error between the leader's state and the state of the dynamic controller of agent i can be written as follows:

$$\dot{\xi}_i(t) = -\sum_{j=2}^{N} a_{ij}(\xi_i(t)-\xi_j(t)) - a_{i1}\xi_i(t) \\ + \sum_{j=2}^{N} a_{ij}e_{ij}(t) + a_{i1}e_{i1}(t) - \dot{x}_1(t) \tag{49}$$

for $i = 2, \dots, N$. Equation (49) can be written in compact form:

$$\dot{\xi} = -\bar{\mathcal{L}}\xi + e_d + e_{1_d} - (1_{N-1} \otimes I_n)\dot{x}_1 \tag{50}$$

where

$$e_d = \begin{bmatrix} \sum_{j=2}^{N} a_{2j}e_{2j} \\ \vdots \\ \sum_{j=2}^{N} a_{Nj}e_{Nj} \end{bmatrix}, \quad e_{1_d} = \begin{bmatrix} a_{21}e_{21} \\ \vdots \\ a_{N1}e_{N1} \end{bmatrix}$$

Because $\beta_d > \beta$ and $\gamma_d > \gamma$, there exists some $\breve{\sigma} > 0$, which will be determined in this proof, such that $\|e_{ij}(t)\| \le \beta_d \mathbf{e}^{-\alpha t} + \gamma_d$, for $t \in [t_{k_i}, t_{k_i} + \breve{\sigma})$ and for $i = 2, \dots, N$.

The response of the error ξ can be bounded as follows:

$$\begin{aligned} \|\xi(t)\| &\le \hat{\beta}\bar{\xi}\mathbf{e}^{-\hat{\alpha}t} + \beta_d\hat{\beta}\bar{d}_i \int_0^t \mathbf{e}^{-\hat{\alpha}(t-s)}\mathbf{e}^{-\alpha s} ds \\ &\quad + \hat{\beta}(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1)\int_0^t \mathbf{e}^{-\hat{\alpha}(t-s)} ds \\ &\le \hat{\beta}\bar{\xi}\mathbf{e}^{-\hat{\alpha}t} + \frac{\beta_d\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha}(\mathbf{e}^{-\alpha t} - \mathbf{e}^{-\hat{\alpha}t}) \\ &\quad + \frac{\hat{\beta}}{\hat{\alpha}}(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1)(1 - \mathbf{e}^{-\hat{\alpha}t}). \end{aligned} \tag{51}$$

In the limit as t goes to infinity we have that

$$\lim_{t\to\infty} \|\xi(t)\| \le \frac{\hat{\beta}}{\hat{\alpha}}(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1).$$

The main part of the proof consists in estimating the maximum admissible delay. In order to obtain an estimate of the admissible delay denoted by $\breve{\sigma}$ we consider the dynamics of $e_{ji}(t)$, for $i = 2, \dots, N$, and j such that $a_{ji} = 1$ during the time interval $t \in [t_{k_i}, t_{k_i} + \sigma_{ji}(t_{k_i}))$. We have that $\dot{e}_{ji}(t) = -\dot{\phi}_i(t) = \sum_{j=2}^{N} a_{ij}(\xi_i(t) - \xi_j(t)) + a_{i1}\xi_i(t) - \sum_{j=2}^{N} a_{ij}e_{ij}(t) - a_{i1}e_{i1}(t)$. Following similar steps as in Equations (20) and (21) we obtain

$$\begin{aligned} \|e_{ji}(t)\| &\le \|e_{ji}(t_{k_i})\| + \frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \\ &\quad \times \left(\bar{\xi} - \frac{\beta_d\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1)\right) \\ &\quad \times (\mathbf{e}^{-\hat{\alpha}t_{k_i}} - \mathbf{e}^{-\hat{\alpha}t}) + \frac{\beta_d}{\alpha}\left((2N_i + a_{i1})\frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i\right) \\ &\quad \times (\mathbf{e}^{-\alpha t_{k_i}} - \mathbf{e}^{-\alpha t}) \\ &\quad + \left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1) + N_i\gamma_d + a_{i1}\delta_d\right)\sigma. \end{aligned} \tag{52}$$

where $\sigma = t - t_{k_i}$. Similarly, the time it takes for the last expression in Equation (52) to grow from $\|e_{ji}(t_{k_i})\| = \beta\mathbf{e}^{-\alpha t_{k_i}} + \gamma$, at time t_{k_i}, to reach the threshold $\beta_d\mathbf{e}^{-\alpha t} + \gamma_d = \beta_d\mathbf{e}^{-\alpha(t_{k_i}+\tau)} + \gamma_d$ is less or equal than the time it takes the error $\|e_{ji}(t)\|$ to grow from $\beta\mathbf{e}^{-\alpha t_{k_i}} + \gamma$, at time t_{k_i}, to reach the value $\beta_d\mathbf{e}^{-\alpha t} + \gamma_d$. Thus, we wish to find a lower-bound $\sigma > 0$ such that the following holds:

$$\begin{aligned} &\beta\mathbf{e}^{-\alpha t_{k_i}} + \gamma + \frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \\ &\times \left(\bar{\xi} - \frac{\beta_d\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1)\right)(1 - \mathbf{e}^{-\hat{\alpha}\sigma})\mathbf{e}^{-\hat{\alpha}t_{k_i}} \\ &+ \frac{\beta_d}{\alpha}\left((2N_i + a_{i1})\frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i\right)(1 - \mathbf{e}^{-\alpha\sigma})\mathbf{e}^{-\alpha t_{k_i}} \\ &+ \left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1) + N_i\gamma_d + a_{i1}\delta_d\right)\sigma \\ &\le \beta_d\mathbf{e}^{-\alpha(t_{k_i}+\sigma)} + \gamma_d \end{aligned} \tag{53}$$

which guarantees that $\|e_{ji}(t)\| \le \beta_d\mathbf{e}^{-\alpha t} + \gamma_d$ for $t \in [t_{k_i}, t_{k_i} + \sigma)$. We can numerically solve the corresponding equation to Equation (53) to determine $\breve{\sigma}$, that is, solve for $\breve{\sigma}$ in

$$\begin{aligned} &\beta\mathbf{e}^{-\alpha t_{k_i}} + \gamma + \frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1}) \\ &\times \left(\bar{\xi} - \frac{\beta_d\bar{d}_i}{\hat{\alpha}-\alpha} - \frac{1}{\hat{\alpha}}(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1)\right)(1 - \mathbf{e}^{-\hat{\alpha}\breve{\sigma}})\mathbf{e}^{-\hat{\alpha}t_{k_i}} \\ &+ \frac{\beta_d}{\alpha}((2N_i + a_{i1})\frac{\hat{\beta}\bar{d}_i}{\hat{\alpha}-\alpha} + N_i)(1 - \mathbf{e}^{-\alpha\breve{\sigma}})\mathbf{e}^{-\alpha t_{k_i}} \\ &+ \left(\frac{\hat{\beta}}{\hat{\alpha}}(2N_i + a_{i1})(\gamma_d\bar{d}_i + \delta_d + \bar{x}_1) + N_i\gamma_d + a_{i1}\delta_d\right)\breve{\sigma} \\ &= \beta_d\mathbf{e}^{-\alpha(t_{k_i}+\breve{\sigma})} + \gamma_d \end{aligned} \tag{54}$$

We can also apply similar steps as in Equations (24)–(27) to obtain an explicit but conservative solution of the delay $\breve{\sigma} > 0$. Such solution is given by Equation (47).

Similarly, using the dynamics $\dot{e}_i = -\dot{\phi}_i$, the condition $\|e_i(t_{k_i})\| = 0$, and the relation $\|e_{ji}(t)\| \le \beta_d\mathbf{e}^{-\alpha t} + \gamma_d$ we can show in a similar way that $\breve{\tau} = \min\{\tau_1, \tau_2\}$, where τ_1 and τ_2 are given by Equation (48).

Finally, since the delays only affect the communication among agents, the local tracking of the dynamic controller by the real system can be analysed in the same way as in Theorem 3.1, Equations (15)–(19), in order to obtain Equation (46). ■

6. Example

Consider a dynamic leader and five followers. The leader represents a mobile vehicle with dynamics given by: $\dot{x}_{1_1} = v\cos x_{1_3} + w$, $\dot{x}_{1_2} = v\sin x_{1_3} + w$, $\dot{x}_{1_3} = u_1$, where $v = 0.5$, $|u_1| \le 1$, and w represents an external disturbance that satisfies $|w| \le 0.1$. However, the leader dynamics are unknown to the followers and only follower 2 receives intermittent measurements of the leader position, $(x_{1_1}(t_{k_1}), x_{1_2}(t_{k_1}))$, according to rule (3) where $\delta = 0.2$. The non-zero elements of the adjacency matrix are given by: $a_{21} = a_{23} = a_{32} = a_{43} = a_{56} = a_{62} = a_{64} = 1$. The followers implement two-dimensional dynamic controllers (44); that is, the goal is to track the position of the leader with absolutely no information about the state x_{1_3}. The followers' dynamics are given by

$$A_2 = A_4 = \begin{bmatrix} 1.4 & 0.1 \\ -0.5 & -1 \end{bmatrix}, \quad B_2 = B_4 = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$
$$A_3 = A_6 = \begin{bmatrix} -0.8 & 0.5 \\ 2 & -1.2 \end{bmatrix}, \quad B_3 = B_6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$A_5 = \begin{bmatrix} 0.2 & 0.4 \\ 0 & -1 \end{bmatrix}, \quad B_5 = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The threshold parameters are selected to be $\alpha = 0.221$, $\beta = 2$, and $\gamma = 0.1$. The delay parameters are $\beta_d = 5$ and $\gamma_d = 1.4$. The leader delay bound is $\sigma_1 = 0.01$. Note that $\bar{x}_1 = 0.6$.

The lower-bound on the inter-event time intervals and the maximum admissible delay are functions of the triggering times t_{k_i}. Figure 2 shows the numerical solution $\breve{\sigma}(t_{k_2})$ of Equation (54) for different values of t_{k_2}. The bottom of the figure shows similar numerical solutions that correspond to the lower-bound $\tau(t_{k_2})$. Here, we only show the admissible delays and the lower-bounds on inter-event times for follower 2, similar graphs are obtained for all other followers but they are not shown due to space constraints. The smallest admissible delay by any agent and at any possible event time t_{k_i} is $\breve{\sigma} = 0.0103$.

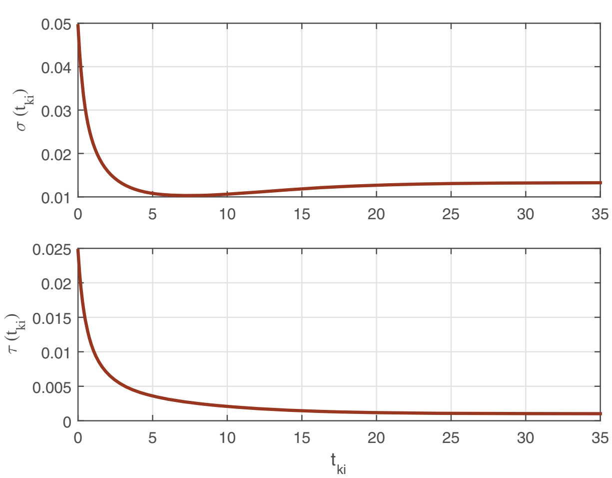

Figure 2. Admissible delays and lower-bound on the inter-event time intervals for follower 2.

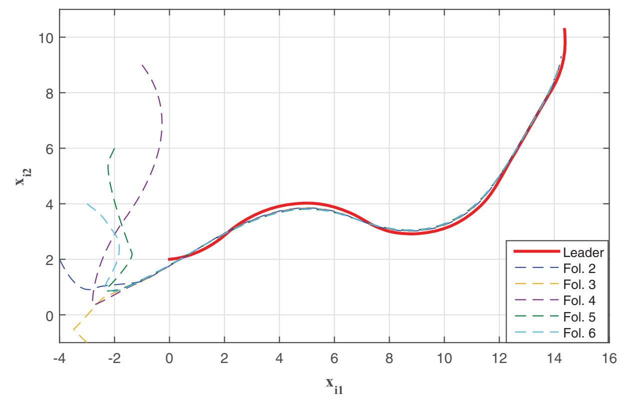

Figure 3. Trajectories of the leader x_1 (bold) and followers x_i, $i = 2, 3, 4, 5, 6$, (dashed).

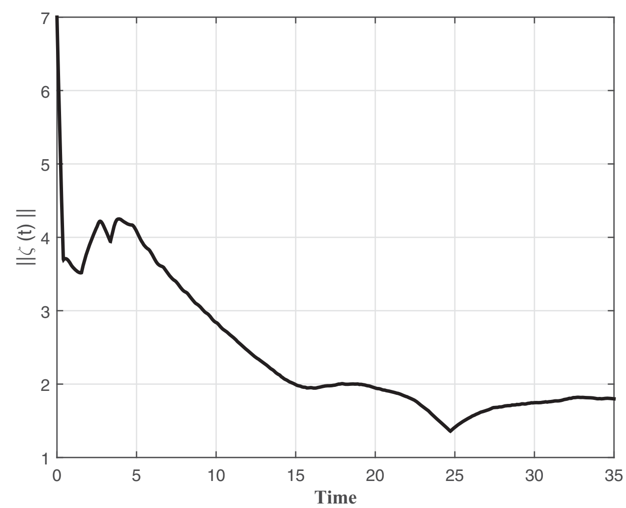

Figure 4. Maximum tracking error.

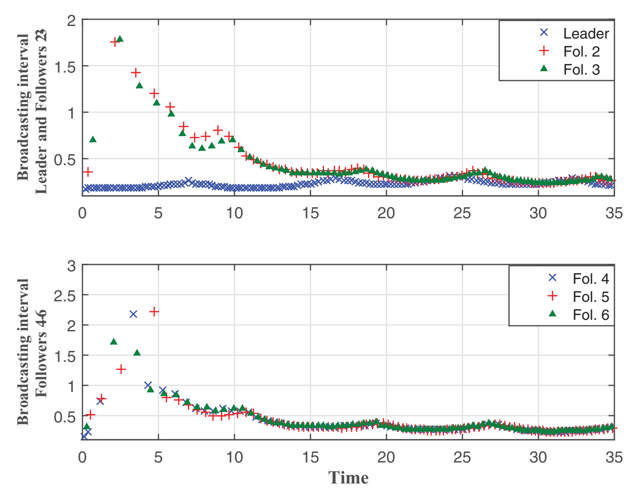

Figure 5. Broadcasting time intervals.

Results of simulation to the initial conditions given by $x_1(0) = [0\ 2]^T, x_2(0) = [-4\ 2]^T, x_3(0) = [-3-1]^T, x_4(0) = [-1\ 9]^T, x_4(0) = [-2\ 6]^T, x_4(0) = [-3\ 4]^T$ are shown in Figure 3 for communication delays $\sigma_{ij} \in [0.005, 0.01]$. The norm of the tracking error, ζ, is shown in Figure 4 and the broadcasting time intervals by the leader and by each follower are shown in Figure 5.

7. Conclusions

The event-based cooperative leader–follower problem has been studied in this paper. In this scenario, a group of heterogeneous systems is tasked to track a dynamic leader with unknown dynamics and only using intermittent measurements of the leader states. The communication among followers is represented by a directed graph; however, existing communication links are not capable of transmitting continuous information. An event-based communication strategy has been devised to tackle this problem and also to provide a decentralised communication scheme where agents decide their broadcasting instants in a completely independent way, that is, there is no need for agents to agree on sample periods or sample time instants.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Anta, A., & Tabuada, P. (2010). To sample or not to sample: Self-triggered control for nonlinear systems. *IEEE Transactions on Automatic Control*, *55*(9), 2030–2042.

Astrom, K. J., & Bernhardson, B. M. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *41st IEEE Conference on Decision and Control* (pp. 2011–2016). Las Vegas, NV: IEEE.

Bidram, A., Lewis, F., & Davoudi, A. (2014). Synchronization of nonlinear heterogeneous cooperative systems using input-output feedback linearization. *Automatica*, *50*(10), 2578–2585.

Cao, Y., & Ren, W. (2010). Multi-vehicle coordination for double integrator dynamics under fixed undirected/directed interaction in a sampled-data setting. *International Journal of Robust and Nonlinear Control*, *20*, 987–1000.

Castillo, B., Di Genaro, S., Monaco, S., & Normand-Cyrot, D. (1997). On regulation under sampling. *IEEE Transactions on Automatic Control*, *42*(6), 864–868.

Chen, X., & Hao, F. (2012). Event-triggered average consensus control for discrete-time multi-agent systems. *IET Control Theory and Applications*, *6*(16), 2493–2498.

Demir, O., & Lunze, J. (2012). Event-based synchronisation of multi-agent systems. In *IFAC Conference on Analysis and Design of Hybrid Systems* (pp. 1–6). Eindhoven, The Netherlands: Elsevier.

Dimarogonas, D. V., Frazzoli, E., & Johansson, K. H. (2012). Distributed event-triggered control for multi-agent systems. *IEEE Transactions on Automatic Control*, *57*(5), 1291–1297.

Donkers, M. C. F., & Heemels, W. P. M. H. (2010). Output-based event-triggered control with guaranteed linfty-gain and improved event-triggering. In *49th IEEE Conference on Decision and Control* (pp. 3246–3251). Atlanta, GA: IEEE.

Francis, B. (1977). The linear multivariable regulator problem. *SIAM Journal of Control and Optimization*, *15*(3), 486–505.

Garcia, E., & Antsaklis, P. J. (2013). Model-based event-triggered control for systems with quantization and time-varying network delays. *IEEE Transactions on Automatic Control*, *58*(2), 422–434.

Garcia, E., Cao, Y., & Casbeer, D. W. (2014a). Decentralized event-triggered consensus with general linear dynamics. *Automatica*, *50*(10), 2633–2640.

Garcia, E., Cao, Y., & Casbeer, D. W. (2014b). Event-triggered cooperative control with general linear dynamics and communication delays. In *53rd IEEE Conference on Decision and Control* (pp. 2914–2919). Los Angeles, CA: IEEE.

Garcia, E., Cao, Y., &Casbeer, D. W. (2017). Periodic event-triggered synchronization of linear multi-agent systems with communication delays. *IEEE Transactions on Automatic Control*, *62*(1), 366–371.

Garcia, E., Cao, Y., Yu, H., Antsaklis, P. J., &Casbeer, D. W. (2013). Decentralized event-triggered cooperative control with limited communication. *International Journal of Control*, *86*(9), 1479–1488.

Guo, M., &Dimarogonas, D. V. (2013). Nonlinear consensus via continuous, sampled, and aperiodic updates. *International Journal of Control*, *86*(4), 567–578.

Li, Z., Duan, Z., &Chen, G. (2011). Dynamic consensus of linear multi-agent systems. *Control Theory & Applications, IET*, *5*(1), 19–28.

Li, Z., Duan, Z., &Lewis, F. (2011). Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties. *Automatica*, *50*(3), 883–889.

Liu, T., Hill, D. J., &Liu, B. (2012). Synchronization of dynamical networks with distributed event-based communication. In *51st IEEE Conference on Decision and Control* (pp. 7199–7204). Maui, HI: IEEE.

Ma, C. Q., &Zhang, J. F. (2010). Necessary and sufficient conditions for consensusability of linear multi-agent systems. *IEEE Transactions on Automatic Control*, *55*(5), 1263–1268.

Nowzari, C., &Cortes, J. (2014). Zeno-free, distributed event-triggered communication and control for multi-agent average consensus. In *American Control Conference* (pp. 2148–2153). Portland, OR: IEEE.

Qin, J., &Gao, H. (2012). A sufficient condition for convergence of sampled-data consensus for double integrator dynamics with nonuniform and time-varying communication delays. *IEEE Transactions on Automatic Control*, *57*(9), 2417–2422.

Ren, W., Beard, R. W., &Atkins, E. M. (2007). Information consensus in multivehicle cooperative control. *IEEE Control Systems Magazine*, *27*(2), 71–82.

Ren, W., & Cao, Y. (2011). Distributed coordination of multi-agent networks: Emergent problems, models, and issues. London: Springer-Verlag.

Seyboth, G. S., Dimarogonas, D. V., &Johansson, K. H. (2013). Event-based broadcasting for multi-agent average consensus. *Automatica*, *49*(1), 245–252.
Seyboth, G. S., Dimarogonas, D. V., Johansson, K. H., Frasca, P., &Allgower, F. (2015). On robust synchronization of heterogeneous linear multi-agent systems with static couplings. *Automatica*, *53*, 392–399.
Su, Y., &Huang, J. (2012). Stability of a class of linear switching systems with applications to two consensus problems. *IEEE Transactions on Automatic Control*, *57*(6), 1420–1430.
Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, *52*(9), 1680–1685.
Wang, X., &Lemmon, M. (2011). Event-triggering in distributed networked control systems. *IEEE Transactions on Automatic Control*, *56*(3), 586–601.