# Notes: fitting met data

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## 1 Gaussian Process

### 1.1 Radial Basis Function

$$\phi(\|\mathbf{x_1} - \mathbf{x_2}\|) = (\sigma)^2 e^{-\frac{\|\mathbf{x_1}-\mathbf{x_2}\|^2}{2l^2}}$$

- Variance can be 1
- Create covariance matrix, $x \rightarrow \mathbf{X}$
- Distance l2 norm
- $l$ is length scale

### 1.2 Fitting and Predicting

$$\mathbf{K} = \phi(\mathbf{X_1}, \mathbf{X_2}) + \sigma^2_{noise}\mathbf{1}$$
$$\mathbf{K} = \mathbf{L}\mathbf{L}^T$$

- L is Cholesky decomposition

$$\mathbf{L^T}.\alpha = \beta$$
$$\mathbf{L}.\beta = y_{train}$$

- So solve for $\beta$ then solve for $\alpha$
- More formally
$$\mu = \mathbf{K}.\mathbf{K}^{-1}.y = \mathbf{K}.\alpha$$
- ... allowing for $x$ to be taken from appropriate sets of training or test data
- Variance is
$$var = \mathbf{K} - \mathbf{K}.\mathbf{K}^{-1}.\mathbf{K}^T$$
- again allow dimensions of K as appropriate, and code has a way around inverse

- "Learning" optimises log likelihood:

$$log(p|l, x) = -1/2(y^T.K^{-1}.y + logdet(K)) + C)$$

- Have 'dumb' model with $\sigma, l = 1$ and an ML model where it is optimised (note that the optimised one is v expensive - see why NN popular)

# 2 Neural Network

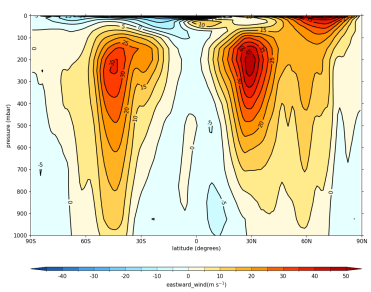
- Code generated by (presumably) a NN :)
- More standard pytorch procedure
- 3 hidden layers + 1 output - 10 nodes

# 3 Results

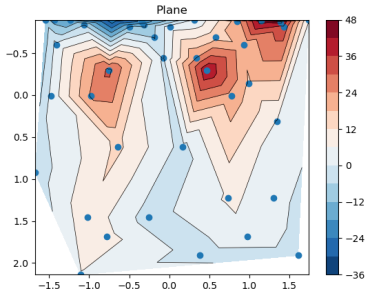
- NN and GP-learnt have picked up features
- Note model data in standardized space
- might need more test points and better chosen distributions to characterise low pressure regions

# 4 Group meeting

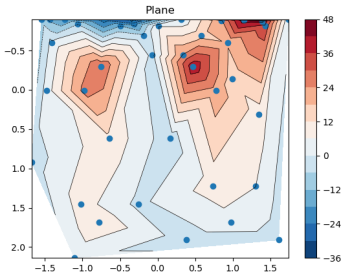
- Show code, gpu implementation, profiling
- Implications for CF? Example scripts? Common data transformations?
- Sets of data - set properties - standardization - projection onto PCAs - if these are common, could functionality be provided?
- Use this for general linear algebra?



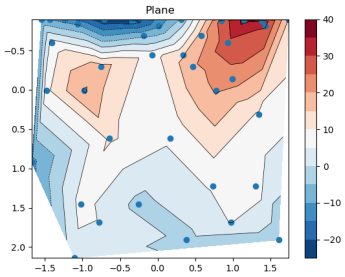
(a) Training + Test Contour plot (CF)




(b) 4 layer NN (Tanh)




(c) GP learnt 50 epochs




(d) Dumb GP

Figure 1: Total data vs Test data for different models (blue points are test posns - small number and not best distribution (bottom half) hinders plotting as well as issues with fit - for eyeball test only