Verified Extraction from Coq to OCaml

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One of the central claims of fame of the Coq proof assistant is extraction, *i.e.,* the ability to obtain efficient programs in industrial programming languages such as OCaml, Haskell, or Scheme from programs written in Coq's expressive dependent type theory. Extraction is of great practical usefulness, used crucially *e.g.,* in the CompCert project. However, for such executables obtained by extraction, the extraction process is part of the trusted code base (TCB), as are Coq's kernel and the compiler used to compile the extracted code. The extraction process contains intricate semantic transformation of programs that rely on subtle operational features of both the source and target language. Its code has also evolved since the last theoretical exposition in the seminal PhD thesis of Pierre Letouzey. Furthermore, while the exact correctness statements for the execution of extracted code are described clearly in academic literature, the interoperability with unverified code has never been investigated formally, and yet is used in virtually every project relying on extraction. In this paper, we describe the development of a novel extraction pipeline from Coq to OCaml, implemented and verified in Coq itself, with a clear correctness theorem and guarantees for safe interoperability. We build our work on the MetaCoq project, which aims at decreasing the TCB of Coq's kernel by re-implementing it in Coq itself and proving it correct w.r.t. a formal specification of Coq's type theory in Coq. Since OCaml does not have a formal specification, we make use of the Malfunction project specifying the semantics of the intermediate language of the OCaml compiler. Our work fills some gaps in the literature and highlights important differences between the operational semantics of Coq programs and their extraction. In particular, we focus on the guarantees that can be provided for interoperability with unverified code, and prove that extracted programs of first-order data type are correct and can safely interoperate, whereas for higher-order programs already simple interoperations can lead to incorrect behaviour and even outright segfaults.

CCS Concepts: • **Software and its engineering** → **Compilers**; **Functional languages**; **Formal software verification**; • **Theory of computation** → **Type theory**.

Additional Key Words and Phrases: Coq, verified compilation, extraction, functional programming

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1 INTRODUCTION

Extraction of programs written with a proof assistant based on type theory can be seen as a compilation process from a high-level language to a more low-level one (*e.g.,* OCaml, Haskell, or C). This is the standard way to get a certified executable code from a Coq formalisation [Letouzey 2004], that is used for instance by the CompCert compiler [Leroy 2006], the CertiCoq project [Anand et al. 2017], the verified Javascript reference interpreter JSCert [Bodin et al. 2014], or the Velús verified compiler for Lustre [Bourke et al. 2017]. From this point of view, extraction is part of the *trusted code base* (TCB) of the proof assistant and thus should come with strong guarantees.

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Since the behavior of the extraction process is crucial for the trust in formally verified software, it seems natural to have clear and formally verifiable guarantees about this process. Precisely, one would expect two guarantees: First, preservation of the operational semantics, that is if a program evaluates to a value in the proof assistant, its extracted version evaluates to a corresponding value in the target language. Second, preservation of typing, that is any extracted program is well-typed in the target language. Unfortunately, there are severe limitations to both properties. For the preservation of operational semantics, one needs to first capture the operational semantics of the target language, and notions such as "corresponding value". For the case of OCaml, there is no agreed upon formal semantics, and defining correspondence relations is intricate. Of course, on values of first-order data type such as natural numbers or booleans, the algebraic datatypes definition are almost identical and straightforward, but users of extraction are usually interested in extracting programs that take arguments as inputs and produce values. This means that we need to define what it means for a function written in the low-level target language to be equivalent to a program written in a proof assistant to give any interesting operational specification of extraction.

The situation on the typing side is even less satisfactory, when the target language is typed[1]. For extraction, the type system of the source language usually features general polymorphism or even dependent types, and is thus much more expressive that the one of the target language. In practice, the extracted code thus needs to use unsafe typing features to produce a piece of code that may be accepted by the typechecker of the target language. For instance, the extraction from Coq to OCaml quickly makes use of the unsafe type cast operation `Obj.magic`. If one wants to formally specify the behaviour of such programs, one quickly encounters Xavier Leroy's famous motto:[2]

"*Repeat after me: "*`Obj.magic` *is not part of the OCaml language".*"

Indeed, as soon as one starts using such unsafe features, one looses guarantees from the type checker, and efforts to make the rest of the extracted code well-typed seem futile. However, the user of extracted code usually only needs it to behave correctly with respect to an interface. It hence suffices for the extracted code to be *semantically* well-typed at the interface boundary.

Letouzey's seminal PhD thesis [Letouzey 2004] presents the current extraction mechanism of the Coq proof assistant and discusses correctness guarantees. The mathematical development is mainly centered around an intermediate language called $\lambda_\square$, which has a formal semantics that is similar to Coq's operational semantics. The translation from $\lambda_\square$ to languages such as OCaml is then left unspecified, due to a lack of formal semantics for OCaml. However, the discrepancy between provided guarantees and uses of extraction in practice can already be explained in terms of $\lambda_\square$. Letouzey proves two central theorems: First, that the operational behaviour of erased $\lambda_\square$ terms and Coq terms is in agreement through a (non-deterministic) erasure relation [Letouzey 2004, Thm. 6, pg. 60]. Secondly, that higher order functions, *e.g.*, of type $(\mathbb{N} \to \mathbb{N}) \to \mathbb{N}$, behave correctly when they are called on $\lambda_\square$ functions that extensionally agree with a Coq function of the same type [Letouzey 2004, Thm. 9, pg. 74]. The second result is proved using a logical relation, and, provided a formal semantics of OCaml and a proof of the first result regarding OCaml, could be extended to OCaml as well. However, this means that the only safe interoperability of extracted Coq functions is, in practice, with other extracted Coq functions.

We address three shortcomings of the state of the art in the present paper:

(1) Due to a lack of formal semantics for OCaml, the extraction process is not and cannot be formally specified and thus not proved correct.

[1] Coq's Extraction also supports Scheme for example, for which this discussion does not apply.
[2] Message on the OCaml mailing list: https://sympa.inria.fr/sympa/arc/caml-list/2009-10/msg00181.html

(2) Interoperation of higher-order functions with (potentially) nonterminating or effectful OCaml programs is not captured by the correctness guarantees, because they do not extensionally agree with a Coq program.
(3) Even though a type interface is provided by extraction, the use of `Obj.magic` means that there are no guarantees by OCaml's type checker.

We propose to address those issues in the particular setting of extraction from programs written in the Coq proof assistant to the OCaml programming language. For other proof assistants such as Lean or Agda, and other target languages such as Haskell, similar but subtly different issues arise. We however hope that by the exposition of this special case, our techniques can be transported and reused for other source and target languages.

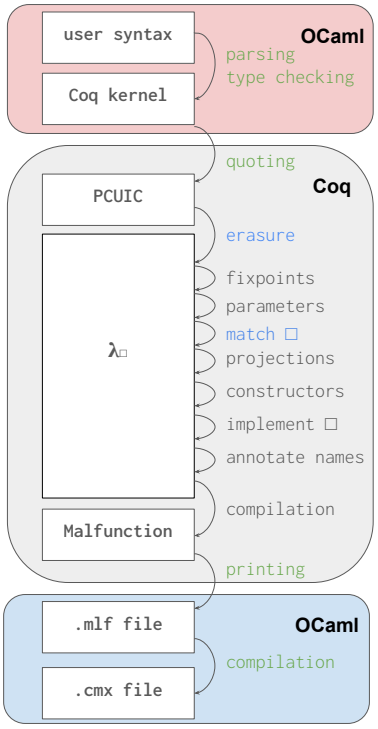
To solve the issue of trust regarding operational correctness (1), we contribute a formal semantics of the Malfunction language by Dolan [2016]. Malfunction has three central design goals: First, it is "a thin wrapper around OCaml's [...] intermediate representation", and thus trivial to compile to it,[3], Secondly, its "syntax is simplified, to allow easy code generation", Thirdly, its semantics is more restrictive in most cases than that of the OCaml intermediate representation, to be robust to future changes to OCaml. We then give a machine-checked proof of preservation of evaluation behaviour, connecting the formal semantics of Coq to the formal semantics of Malfunction by a proof in Coq. The proof is split into a long pipeline of intermediate passes to make the proof engineering overhead manageable. The central theorem talks about terms of *first-order data type*, *i.e.,* of non-propositional inductive type were all constructor arguments are of first-order data type.

To solve the issue of interoperability (2), we prove that for *first-order functions*, *i.e.,* functions from first-order data types to first-order data types, *any* interoperation with OCaml code is safe, even if the Coq code internally uses higher-order features and proofs. In the next section, we provide a counter-example that show that outside this setting, it is possible to get extracted code that produces a segmentation fault when called with a well-typed OCaml argument. This is because most of the target languages of extraction can perform effects such as writing to reference cells, and the strong guarantees provided by the proof assistant only apply to pure functions.

To solve the typing issue (3), we advocate for a drastic change of perspective on extracted code: Instead of hoping that the extracted code is well-typed in the target language and correcting problems heuristically using unsafe casts, we advocate for just exposing safe first-order function interfaces as discussed in the previous paragraph, and using untyped but verified code for the internal, higher-order parts of the code. Running the typechecker of the target language is superfluous anyways: the generated code is already type checked by the proof assistant and it is folklore that OCaml does not run type-based optimisations. Thus what we contribute in this paper is a definition of a semantic typing judgment [Dreyer 2018] for the target language à la realisability, and a formal proof that the extracted code from a Coq program with a simple enough type realizes the corresponding erased type in OCaml. The type system of OCaml is one way to statically check that a program realises a type, but Coq's type system plus the extraction mechanism should be seen as another way to guarantee realisability, eliminating the need to use the OCaml type checker for our purposes. Targeting Malfunction is in line with these considerations, since it is untyped but supports exposing a typed interface.

Overall, our compilation chain is depicted in Fig. 1. It is available at https://github.com/yforster/coq-verified-extraction/tree/v1.0+8.19 and can be summarised as follows. A program is typechecked by the Coq kernel (red area). It is then quoted using a quotation mechanism that is essentially the identity function into MetaCoq's term representation of Coq itself [?], with the assumption that the quotation yields a program that is well-typed according to MetaCoq's typing

[3]The OCaml compiler's intermediate language is called `Lambda`. To avoid confusion with $\lambda_{\square}$, we do not use the name.




```
From VerifiedExtraction
 Require Import Loader.

Definition function_or_ℕ
 : ∀ (b:𝔹), if b then 𝔹 → 𝔹 else ℕ :=
 fun b ⇒
  match b with
  | true ⇒ fun x ⇒ x
  | false ⇒ S O
  end.

Definition function := function_or_ℕ true.

MetaCoq Verified Extraction
 function.
MetaCoq Run Print mli function.

(* Prints:
   type 𝔹 = True | False
   val function : 𝔹 → 𝔹 *)
```

Output in file `out.mlf`:

```
(module
  ($def_function_or_nat
    (lambda ($b)
      (let ($discr $b)
        (switch $discr<
          ((0 0) (lambda ($x) $x))
          ((1 1) (block (tag 0) 0))))))
  ($def_function
    (apply $def_arity_function 0))
  ($function $def_function)
(export $function))
```

Fig. 1. Left: Overview of the extraction process. Red indicates parts of the Coq proof assistant implementation in OCaml, grey parts implemented and verified in Coq, blue parts of the Malfunction compiler implementation in OCaml. Green font means the implementations are part of the TCB. Blue font means that we are using work by Sozeau et al. [2019]. Right: Example invocation of the verified extraction plugin and its output.

judgment. The theory of the kernel of Coq formalised in MetaCoq is called PCUIC. We perform verified type and proof erasure [Sozeau et al. 2019], targeting the untyped λ-calculus $\lambda_{\square}$. We then perform several transformation steps on $\lambda_{\square}$, namely to change the representation of fixpoints, to remove parameters, to remove case analysis on proofs, to implement native projections, to treat constructors as blocks, to implement erasure residues as functions, and to annotate names. All this is in the grey area. The final transformation is a translatation to representation of Malfunction in Coq. To get an executable (blue area), we finally print the s-expression representation of the resulting Malfunction program to a file and use the Malfunction compiler to compile to OCaml compilation units (`.cmx`). Note that the Malfunction compiler shares a lot of its codebase with the regular OCaml compiler.

Before diving into the technical details of our formal setting, let us look into examples that illustrate our claims and the current limitations of Coq's official extraction mechanism to OCaml.

1.1 Scope and Limitations of Extraction to OCaml

We here discuss general limitations of any extraction to OCaml, to which any extraction process – and thus Coq's current extraction process as well as our verified replacement – are subject to.

First, a situation where OCaml's type system is too weak arrives as soon as one uses higher-order dependent types and exposes this function to be called from unverified OCaml code. We are going to develop a function `assumes_purity` : (`unit` → 𝔹) → 𝔹 where applying it to an (effectful)

OCaml function `impure` leads to a segmentation fault. We here consider the example of a function `function_or_𝔹` that expects a boolean value `b` and returns a function from $\mathbb{B}$ to $\mathbb{B}$ if `b` is `true` and a natural number otherwise, inspired by Letouzey's discussion [2004, §3.1.3, 3.2.1.].

```
Definition function_or_𝔹 : ∀ (b:𝔹), if b then 𝔹 → 𝔹 else ℕ :=
 fun b ⇒ match b with true ⇒ fun x ⇒ x | false ⇒ S O end.
```

Since OCaml's type system does not support type-on-term dependency, there is no way to give a type to the extraction `function_or_𝔹` and thus the extraction implemented in Coq as of today is:

```
(** val function_or_𝔹 : 𝔹 → Obj.t **)
let function_or_𝔹 = function | True → Obj.magic (fun x → x) | False → Obj.magic (S O)
```

The type `Obj.t` is actually a synonym to the universal type, which can be used to type any term using `Obj.magic`, meaning that `function_or_𝔹` cannot be typed in OCaml without unsafe casts. The use of `Obj.magic`, besides circumventing the type system, also pollutes the rest of the code, because any use of `function_or_𝔹` must then be enclosed in an `Obj.magic` invocation to cast the return type to $\mathbb{B} \to \mathbb{B}$ or $\mathbb{N}$ (see `apply_function_or_N` below). In practice, the extraction tries to avoid the greedy use of `Obj.magic` which has the even less satisfactory consequence of producing ill-typed terms in some complex situations [Sozeau et al. 2019, §3.6].

This taken aside, let us continue our example and define a function that takes a boolean value `b` and, as input, again either function from $\mathbb{B}$ to $\mathbb{B}$ or a natural number (depending on `b`) and applies the function to an argument in the first case while just returning a default value in the second:

```
Definition apply_function_or_𝔹 : ∀ b : 𝔹, (if b then 𝔹 → 𝔹 else ℕ) → 𝔹 :=
 fun b ⇒ match b with true ⇒ fun f ⇒ f true | false ⇒ fun _ ⇒ false end.
```

Again, the OCaml extraction has to use unsafe casts:

```
(** val apply_function_or_𝔹 : 𝔹 → __ → 𝔹 **)
let apply_function_or_𝔹 b f = match b with | True → Obj.magic f True | False → False
```

We now use `apply_function_or_𝔹` with `function_or_𝔹` as argument, while letting the boolean be determined by a function from `unit` to $\mathbb{B}$ applied to the only element `tt` of `unit`:

```
Definition assumes_purity : (unit → 𝔹) → 𝔹 :=
 fun f ⇒ apply_function_or_𝔹 (f tt) (function_or_𝔹 (f tt)).
```

This function can be extracted to an OCaml function whose type is a perfectly valid OCaml type:

```
(** val assumes_purity : (unit → 𝔹) → 𝔹 **)
let assumes_purity f = apply_function_or_𝔹 (f ()) (function_or_𝔹 (f ()))
```

Now on the Coq side the function `assumes_purity` is well-typed under the invariant that `f tt` always returns the same value. Thus, nothing will go wrong in OCaml as long as `assumes_purity` is always called with an OCaml function of type `unit → 𝔹` that always returns the same value.

If this invariant however is broken, the internal unsafe casts `Obj.magic` become invalid, and all guarantees are lost. And indeed, it is easy to break this invariant in OCaml by using a reference:

```
let impure : unit → 𝔹 = let x : 𝔹 ref = ref False in
 fun _ → match !x with False → (x := True; False) | True → True
```

The function `impure` returns `False` the first time it is called and `True` afterwards. When applied to `assume_purity`, the execution is essentially equivalent to `apply_function_or_𝔹 True (function_or_𝔹 False)`, meaning `apply_function_or_𝔹` expects a function as argument, but gets a natural number. Thus, the program tries to evaluate `Obj.magic False True` which induces a segmentation fault:

```
assumes_purity impure
(** Segmentation fault: 11 **)
```

This examples illustrates the fact that even if an extracted program has a valid OCaml interface, this does not mean that its interaction with well-typed OCaml pieces of code will not go wrong.

1.2 What Guarantees Can We Expect From Extracted Code?

The situation looks pretty bad. On the one side, the target type system is fundamentally too weak to support extraction of dependently typed programs. On the other side, even on the common part of the type system, the semantics associated to the type is very different between the two systems. In particular, because CIC is a pure language, it satisfies strong invariants, such as the fact that a function always returns the same output on the same input. But by using dependent types, those invariants at the computational level can be lifted at the type level. This is the case for instance for the `pure_apply` function. Thus, one can wonder whether we can expect any guarantee at all for the interaction between an extracted program and OCaml pieces of code.

This paper provides the first positive answer in two steps. First, we do not expect extraction to produce a well-typed OCaml program, but rather we prove that it produces an untyped low-level functional code that operationally behaves the same as Coq's initial program. Then, by defining a realisability semantics for low-level pieces of functional code over the OCaml type system, we can show that for first order types (*i.e.,* functions whose domain and codomain are first-order data types) the extracted code actually meets its specification and can thus interact safely with any pieces of OCaml code. Note that this result is significantly less restrictive than it might seem, *e.g.,* the benchmarks used for evaluation of compilation times in CertiCoq readily satisfy the constraints.

Major programs relying on extraction as of today might not all be in this shape, but for reasons related to convenience and thus they could be brought to this shape. For instance for CompCert, the main compilation function[4] would satisfy this condition if not for a representation of integers as unbounded numbers with an explicit proof of a bound (which could easily be re-organised into a syntax type without this bounding constraint and a first translation phase that establishes it). For our own extraction procedure, the only hinderance is representing non-empty lists (for universe instances) via `X * list X` rather than a pair of a list and a non-emptiness proofs.

1.3 A General Plan to Support Certified Extraction Towards Several Languages

In the process of proving operational correctness and realisability properties of extraction, there is a sizable part that is independent of the target language and can be defined commonly. To make this explicit, $\lambda_\square$ has been introduced by Letouzey [2004] as a common intermediate untyped language in which an original Coq term can be erased. Sozeau et al. [2019] have mechanised a weak call-by-value variant of $\lambda_\square$ in Coq based on the MetaCoq project. In this context, weak means that function bodies are not evaluated, and call-by-value that function arguments are evaluated before the function is called. They give a machine-checked proof of type and proof erasure, removing type information and proof terms, and replace everything with a canonical $\square$ term.

In this paper, we show how to define a compilation pipeline that gradually transform a $\lambda_\square$ term to a term that computes the same values, but is operationally closer to the semantics of the target language. These different phases correspond to compilation phases usually provided by compilers and are not specific to extraction to OCaml. They consist for instance in removing the parameters of constructors of inductive types that are required in Coq for type inference but have no computational content and thus can be discarded in a simply typed or untyped setting. There are

[4] https://compcert.org/doc/html/compcert.driver.Compiler.html#transf_c_program

also other phases such as η-expansion of constructors of inductive types that are not fully syntactic and require typing to be defined. Those phases need thus to be performed before erasure to $\lambda_\square$.

As of today, this pipeline based on $\lambda_\square$ is already use in the CertiCoq project [Anand et al. 2017] targeting CompCert C and in the ConCert project [Annenkov et al. 2020] targeting smart-contract languages. It could be used *to target* other call-by-value languages such as Scheme or Rust.

It could also be used *as a target* for other proof assistants. Indeed, it seems feasible to get extraction from the Agda or Lean proof assistants to OCaml via Malfunction by implementing type and proof erasure from Agda or Lean to $\lambda_\square$.

1.4 Malfunction vs. Other Target Languages in the OCaml compiler

We here discuss what target language to choose inside the OCaml eco-system, which is necessary for our tool to replace some use cases of Coq's current extraction to OCaml.

In order to give a machine-checked implementation of extraction, we require a specification of the target language in Coq. This specification will remain part of the TCB: it has to be correct for the verification to be meaningful. Thus, it is desirable to have a *clear*, easy-to-review specification.

One path would be to specify the operational semantics of OCaml in Coq. Such a semantics could be obtained by translating the semantics of OCaml given in prose, or by reverse-engineering a specification from a compiler, interpreter, or an abstract machine for OCaml. However, the semantics of OCaml is complex, and it would require a major amount of trust to believe that the specification indeed is correct. Furthermore, several parts of OCaml are intentionally unspecified, *e.g.,* the evaluation order of function call arguments.

Another path would be to directly specify the operational semantics of Lambda, the intermediate language of the OCaml compiler. However, the semantics of the OCaml intermediate representation is intentionally unspecified (even the `-dlambda` option of the compiler to generate the intermediate representation from OCaml code is undocumented), and changes between releases.

Lastly, it is also desirable that the chosen semantics be *economical*, in the sense that it can be used in a verification project without generating an overhead which lets even easy tasks take months.

Thus, to have a clear and economical specification, it seems natural to choose Malfunction as target language for extraction: Malfunction has a clear semantics provided by an interpreter, it has been around the (scientific part of the) OCaml community for several years, and it can serve as a stable interface to deal with changes of the semantics of Lambda. Furthermore, Malfunction is fully specified: *e.g.,* evaluation order of operators is clearly specified to be left-to-right. Note that this is achieved by explicit `let`-binding when translating to Lambda, meaning this strict semantics is fully compatible with OCaml's unspecified evaluation order and the observed right-to-left order in OCaml's current compilers.

1.5 Plan of the Paper

In Section 2, we present the statement and main idea of the proof of the main correctness theorem. Then we proceed in Section 3 to a phase-by-phase description of each necessary transformation outlined in Fig. 1. We then discuss in Section 4 our contribution as a practical tool to replace current Coq's extraction to OCaml. Section 5 explains how we bootstrap our extraction mechanism to obtain a self-hosted compiler. We compare the performance of our extraction to Coq's extraction and CertiCoq in Section 6. Finally, we discuss related (Section 7) and future work (Section 8).

Links to the available online formalisation are provided using the syntax "`myNotion` [myFile.v]". The links are voluntarily obfuscated for double blind but they are explicit enough so that they can be mapped to the anonymous supplementary material provided with this paper.

```
Inductive term :=
| tRel (n : ℕ)
| tSort (s : Universe.t)
| tCast (t : term) (kind : cast_kind) (v : term)
| tProd (na : aname) (ty : term) (body : term)
| tLambda (na : aname) (ty : term) (body : term)
| tLetIn (na : aname) (def : term) (def_ty : term) (body : term)
| tApp (f : term) (args : list term)
| tConst (c : kername) (u : Instance.t)
| tInd (ind : inductive) (u : Instance.t)
| tConstruct (ind : inductive) (idx : ℕ) (u : Instance.t)
| tCase (ci : case_info) (type_info:predicate term)
        (discr:term) (branches : list (branch term))
| tProj (proj : projection) (t : term)
| tFix (mfix : mfixpoint term) (idx : ℕ)
| tCoFix (mfix : mfixpoint term) (idx : ℕ)
| tInt (i : PrimInt63.int)
| tFloat (f : PrimFloat.float).
```

```
Inductive t :=
| Mvar of ℕ
| Mlambda of Ident.t list * t
| Mapply of t * t list
| Mlet of binding list * t
| Mnum of numconst
| Mstring of string
| Mglobal of Longident.t
| Mswitch of t * (case list * t) list
(* Numbers *)
| Mnumop1 of unary_num_op * numtype * t
| Mnumop2 of binary_num_op * numtype * t * t
| Mconvert of numtype * numtype * t
(* Vectors *)
| Mvecnew of vector_type * t * t
| Mvecget of vector_type * t * t
| Mvecset of vector_type * t * t * t
| Mveclen of vector_type * t
(* Lazy *)
| Mlazy of t
| Mforce of t
(* Blocks *)
| Mblock of int * t list
| Mfield of int * t
with binding :=
| Unnamed of t | Named of Ident.t * t
| Recursive of (Ident.t * t) list.
```

Fig. 2. Syntax of Coq (`term` [Ast.v]) and Malfunction (`t` [Malfunction.v]) formalised in Coq

2 THE CORRECTNESS THEOREMS

In this section, we technically explain the *statement* of our two central theorems, providing key elements of their proofs. From a bird's-eye view, the first central theorem is a simple simulation theorem for a compiler, saying that if a Coq program t of first-order data type is convertible to an irreducible term v, then its translation to Malfunction evaluates to the value translation of v. A first-order data type (`firstorder_ind` [PCUICFirstOrder.v]) is a mutual inductive type whose constructors only have arguments of first-order data type. The value translation is a simple function translating a constructor application to:

- an integer if the constructor has only parameters and no real arguments
- otherwise, an applied tagged block, stripping the parameters from the argument list

The second central theorem deals with first-order functions, that is functions whose arguments and return value are of first-order data types. It states that the Malfunction program obtained by extracting a Coq first-order function can be applied safely to *any* pieces of OCaml (compiled to Malfunction) that has the right corresponding type. In particular, we prove that the kind of counter-examples such as `assumes_purity` of Section 1.1 cannot be produced for first-order functions. So interoperability with unverified OCaml programs is safe for this fragment.

2.1 Malfunction and Coq

The internal language of the kernel of Coq (Fig. 2, top) is formalised in MetaCoq [**?**] as the inductive type `term` [Ast.v]. On the computational level, it features basic constructs of functional language such as (DeBruijn) variables (`tRel`), lambda abstraction (`tLambda`), (n-ary) application (`tApp`) and a `tLetIn` construct. It also features inductive and co-inductive features, `tConstruct` for constructors of (co-)inductive types, `tCase` for pattern-matching, fixpoints (`tFix`) and co-fixpoints (`tCoFix`), and a notion of projections (`tProj`) which corresponds to a negative presentation of record types. Coq also features constructions that are very specific to dependent type theory. In particular, types appear in the syntax of terms and can be manipulated as any other terms. Thus, it features terms `tSort` for universe, `tInd` for inductive types (it takes as arguments on `inductive` definition and a universe `Instance.t`) and `tProd` for dependent function types and a `tCast` function to explicitly cast a term to a given type. Finally, Coq features a primitive notion of integers (`tInt`) and floats (`tFloat`).

The syntax comes with a notion one-step reduction `red1` [PCUICReduction.v] , noted $\Sigma ; \Gamma \vdash t \rightsquigarrow t'$, of a term `t` into `t'` in the `global_env_ext` [PCUICAst.v] Σ and in the local context Γ. The reflexive, transitive closure of one-step reduction is noted $\Sigma ; \Gamma \vdash t \rightsquigarrow^* t'$. It also comes with a specification of `typing` [PCUICTyping.v], $\Sigma ; \Gamma \vdash t : T$. All of these definitions are quite complex in detail, but a deep understanding is not required to understand the rest of this paper and we refer the interested reader to Sozeau et al. [2019] for further explanations.

Malfunction (Fig. 2, bottom) specifies the following syntactic constructs relevant for extraction: named variables, *n*-ary abstraction and application, mutual recursion via `Mlet` rec, tagged *n*-ary `Mblock`, an operation to project out the *i*-th field of a block, and a `Mswitch` statement branching over the tag of a block. Besides, Malfunction features laziness, native integers, native floats and vectors that are not relevant to our formal treatment of extraction from Coq. We do carry primitive integer, float and array literals from Coq to Malfunction but do not verify anything about it yet. We also implement an unverified translation of co-fixpoints to fixpoints + `lazy` and `force`, see 8 for a discussion of these extra features.

Operationally, Malfunction and Coq are relatively simple. However, since Coq is a dependent type theory, it has first-class types and proofs that can play roles in computations and have no counterpart in Malfunction, *i.e.,* they have to be erased. The further semantic differences are subtle. We take care of them one-by-one, to avoid combinatorial explosion in proofs.

Still, combinatorial explosion lurks behind every corner. After all, we are dealing with the real semantics of Coq rather than an idealised type theory. We thus put focus on finding economical proofs, often proving a slightly weaker property than what could be proven which is however still strong enough to establish the wanted theorem. Standard techniques like using observational equivalence of programs are for instance not economical: An induction over the derivation of observational equivalence would have to deal with far more than 50 cases. Instead, our results are all stated w.r.t. a big-step evaluation relation of programs.

The reasons we restrict to values of first-order data types then become apparent: First, those are the only values where one can meaningfully talk about their normal form w.r.t. evaluation, because they are just constructor applications. Second, it becomes possible to prove that a weak call-by-value evaluation relation computes the same values—rather than just observationally equivalent ones—as Coq's unrestricted conversion. Third, type and proof erasure is only complete in this fragment, because detecting whether a certain term is erasable sometimes requires evaluating it fully, and full evaluation being too inefficient is a central reason extraction processes are needed.

2.2 Operational Semantics of Malfunction in Coq

Malfunction [Dolan 2016] is a "thin wrapper" around the untyped intermediate language of the OCaml compiler. It comes with an interpreter written in OCaml, and a compiler hooking into the OCaml compiler's pipeline to either produce bytecode or native `cmx` files. Thus, it can make use of OCaml's optimisations. Both interpreter and compiler expect its input in s-expression format. As such, Malfunction is supposed to be easy to generate rather than easy to read, and thus fits our purpose perfectly. Explicitly, whenever the interpreter computes a value, the compiled code is expected to produce the same value. Thus, we can derive Malfunction's semantics in Coq purely from the interpreter, and as long as the derived semantics indeed matches, we have a guarantee that the compiled code behaves correctly by adding the Malfunction compiler to the intermediate representation (which is essentially the identity function) and the OCaml compiler to our TCB.

The interpreter is defined in less than 300 lines of (impure) OCaml code and serves as the specification. It uses an environment for variables and a higher-order abstract syntax (HOAS) representation [Pfenning and Elliott 1988] of values for function thunks, *i.e.*, reuses OCaml functions to represent Malfunction function thunks. Most other features of Malfunction are specified

using the corresponding feature in OCaml and mutable references: The operational semantics of the `Mlazy` constructor is defined using the corresponding `lazy` primitive in OCaml, mutable vectors are defined using mutable references and for `let rec` constructs, the mutual recursive knot is tied via references.

To define a specification for Malfunction in Coq, we need to define a semantics matching the interpreter in Coq, and for this, we cannot rely on impure features of the language. This means that we need to make explicit the notion of `heap` [SemanticsSpec.v] in the operational semantics. To make sure that our notion of semantics is closely related to the initial version in OCaml, we proceed in two steps.

First, to disentangle the (non-terminating) interpreter from our formal proof, we provide an inductive, weak call-by-value evaluation relation `eval` [SemanticsSpec.v] in Coq, closely following the OCaml interpreter, but using the heap explicitly instead of relying on imperative feature of the language. This relation requires a local environment `locals` to be defined on open terms. We simply specify it using a function in Coq:

```
Inductive eval (locals : Ident.t → value) : heap → t → heap → value → Prop := ...
```

We also provide a functional interpretation of `let rec` instead of relying on the imperative one. It depends on an explicit strictly-positive representation of values.

Second, as a sanity check, we define a Coq function acting exactly as the interpreter for Malfunction implemented in OCaml, with the difference that we use an explicit heap and provide a machine-checked proof that whenever a program evaluates to a value for this predicate, then the interpreter computes this value. This is only possible by switching off termination checking in Coq locally because the evaluated term may be non terminating so there is no guarantee that the interpreter will terminate in Coq.

Then, to trust that the resulting semantics matches the intended semantics as specified in Malfunction (i) the reference interpreter of Malfunction written in OCaml has to be correct w.r.t. the compiler, (ii) our pure Coq implementation of the interpreter has to be correct, which can be validated by extracting it to OCaml and unit testing against the reference interpreter.

The unsafe proof that our relational semantics relates to the Malfunction interpreter is just a sanity check that our formal semantics relates to the proposal of [Dolan 2016] and is not required in our correctness theorems. Therefore, the fact that it switches off the termination checker is not crucial. However, it should be noticed that by making this proof, we actually found a bug in the bound check for vectors of Malfunction reference interpreter. The interpreter was performing an out of bounds vector access and subsequently crashing rather than reporting the access. We also discovered a miscompilation of pattern matchings in the Malfunction compiler due to a change in OCaml 4.14+`flambda` (compiled programs were computing the wrong value or even segfaulting).

2.3 Extraction Theorem For First-Order Data Types

We now explain the exact statement of the first correctness theorem about terms of first-order data type detailing the exact pre-conditions.

We use Coq syntax to ease the task of ensuring that the theorem is exactly as claimed also for readers with less experience reading Coq code of the theorem `verified_malfunction_pipeline_theorem` [Pipeline.v].

We assume that we are working in a well-formed, axiom-free global environment containing declarations of definitions and inductive types, where all definitions are η-expanded. We explain the need for η-expanded terms in the next section.

```
Variable Σ : global_env_ext.
```

```
Variables (ΣH : wf_ext Σ) (axfree : axiom_free Σ) (Σexp : expanded_global_env Σ).
```

We then fix a Coq-term `t`, which is also η-expanded and whose type in this environment is a first-order data type `i` at some universe level `u` with parameters and arguments `args`:

```
Variable t : term.
Variable expt : expanded Σ [] t.
Variables (i : inductive) (u : Instance.t) (args : list PCUICAst.term).
Variable fo : firstorder_ind Σ (firstorder_env Σ) i.
Variable typing : Σ ; [] ⊢ t : mkApps (tInd i u) args.
```

To simplify the proof slightly, we also assume that all inductives in the environment have no parameters. This assumption could be removed at the cost of a slightly more tedious proof.[5]

```
Variable noParam : ∀ i mdecl, lookup_env Σ i = Some (InductiveDecl mdecl) → ind_npars mdecl = 0.
```

We then assume that `t` reduces using Coq's reduction to a term `v` that is irreducible. This is a direct consequence of the strong normalization property we assume of PCUIC (as `t` is well-typed) but could be obtained by simpler means. Note that Coq's reduction relations is maximally general and thus agnostic towards the specific reduction order used.

```
Variable v : term.
Variable v_red : Σ ; [] ⊢ t ⇝* v.
Variable v_irred : ∀ t', ( Σ ; [] ⊢ v ⇝ t' ) → False.
```

We then define the environment Σ' as the result of applying the middle-end of the extraction pipeline to the environment Σ and assume that there is a Malfunction environment containing exactly the values of every definition of Σ' compiled to Malfunction.

Lastly, we assume that after erasure, only extractable inductive types remain in the global environment. Concretely, this enforces some numerical maximums on number of constructor and number of arguments of constructors and that the environment contains no axioms. This numerical checks are here because for instance Coq's inductive types can have as many constructors as needed, whereas in OCaml and Malfunction there are explicit bounds on their numbers. For technical reasons, we prefer to deal with it as an axiom that is invoked exactly once in the proof of the erasure pipeline. This approach allows us to still get an (unsafe) extraction procedure for Coq's terms that do not meet those restriction. See Section 3 for more details.

We can then prove that in this environment, the compilation of `t` evaluates to the value translation of `v` in Malfunction, leaving the heap `h` it is run in unchanged:

```
Theorem verified_malfunction_pipeline_theorem : ∀ (h:heap),
 eval Σ' empty_locals h (compile_malfunction_pipeline expt typing) h (compile_value_mf Σ v).
```

In the above, `compile_malfunction_pipeline` is the function compiling a Coq term to a Malfunction term and `compile_value_mf` is the function compiling a Coq first-order value (*i.e.,* a value in a first-order data type) to a Malfunction value. The function `empty_locals` is used to encode the empty local environment and returns an error on every identifier.

For instance, the irreducible Coq term `@cons 𝔹 true (@cons 𝔹 false (@nil 𝔹))` (*i.e.,* the list `[true, false]`) is translated to `block (tag 0) 0 (block (tag 0) 1) 0`, because `cons` is the first constructor of `list` with arguments (thus `tag 0`), `true` is the first constructor of `𝔹` not taking arguments (thus `0`), `false` is the second constructor of `𝔹` not taking arguments (thus `1`), and `nil` is the first constructor of `list` not taking arguments (thus `0`). This semantics follows OCaml's internal representation of values in order to enable interoperability.

[5]Note that the restriction however does not really inhibit the expressivity of the theorem, since instead of parameters all inductives could simply declare their parameters as indices.

2.4 Extraction Theorem For First-Order Functions

Even though the extraction theorem holds for arbitrary functions, it does not state anything about the shape of values outside the first-order data type fragment because there is no control on the translation of the body of functions. We can however characterise the compilation of first-order functions in terms of interoperabilty—or to phrase it in a more foundational manner, in terms of realisability semantics. To this end, we define a notion of values realising in Malfunction a first-order data type coming from Coq. Concretely, we define a notion of OCaml type, `camlType` [RealizabilitySemantics.v], that only contains function types (`Arrow`) and algebraic data types (`Adt`).

```
Inductive camlType : Set :=
  Arrow : camlType → camlType → camlType
| Rel : ℕ → camlType
| Adt : kername → ℕ → list camlType → camlType.
```

The type `Rel n` refers to the n-th mutually defined data type in an `Adt`. We can construct a function `CoqType_to_camlType` [Firstorder.v] that builds an `Adt` from any first-order data type of Coq. Finally, to state the correctness theorem for firstorder function, we need to define what it means for a value `v` to be a realiser of `adt` at number `ind`, noted `v ⊩ (adt , ind)`. The number specifies which type of the potentially mutually defined `Adt` is realised.

Then, with similar hypothesis as in Section 2.3, we can prove the following interoperability theorem (`interoperability_firstorder_function` [Firstorder.v][6]) for a first-order function `f`. To reduce technical details, we assume a function that maps one type of a mutual first-order inductive type `mind` to another—the generalisation to arbitrary first-order functions is mathematically straightforward, but contains a lot of tedious technical manipulations.

```
Theorem interoperability_firstorder_function :
 ∀ (typing : Σ ; [] ⊢ f : tProd na (tInd (mkInd kn ind) []) (tInd (mkInd kn ind') [])),
 let adt := CoqType_to_camlType mind fo in
 let compile_f := compile_malfunction_pipeline expt typing in
 ∀ arg : t,
   (∀ (h h' : heap) (v : value), eval [] empty_locals h arg h' v → v ⊩ (adt , ind)) →
    ∀ (h h' : heap) (v : value), eval [] empty_locals h (Mapply (compile_f, [arg])) h' v → v ⊩ (adt , ind').
```

The crux of the proof lies in three main ingredients. First, we show that compilation to Malfunction is modular in the sense that a non-erasable application (*e.g.,* of firstorder type) in Coq is compiled to the application of the compiled function to its compiled argument in Malfunction. Second, it uses the fact that *any* value realising an `Adt` coming from a first-order data type can be obtained by the compilation of a Coq term (`camlValue_to_CoqValue` [Firstorder.v]), *i.e.,* we prove a decompilation theorem from Malfunction values to Coq values. Lastly, it crucially uses the fact that the evaluation of compiled terms does not change the heap; *i.e.,* compilation to Malfunction produces *pure* terms.

Note that the second aspect is not valid outside the first-order fragment, because the function `impure` defined in Section 1.1 does not correspond to the compilation of any Coq function.

3 THE MAN IN THE MIDDLE: $\lambda_\square$, AN INTERMEDIATE PLATFORM FOR EXTRACTION

In this section, we describe the middle-end of the extraction process. The language $\lambda_\square$ was introduced by Letouzey [2004] as an operational model of Coq after computationally irrelevant parts—*i.e.,*

[6] At the time of writing, the proof of this theorem is done over an abstract notion of compilation that satisfies the properties mentioned in the text. The formal connection with our concrete pipeline properties—in particular dealing with the concrete pre- and post-conditions of the pipeline—requires still few days of work.

```
Definition program env term := env * term.

Context {env env' env'': Type}.
Context {term term' term'': Type}.
Context {value value' value'': Type}.
Context {eval : program env term → value → Prop}.
Context {eval' : program env' term' → value' → Prop}.
Context {eval'' : program env'' term'' → value'' → Prop}.

Record Transform.t :=
{ name : string;
  pre : program env term → Prop;
  post : program' env' term' → Prop;
  transform : ∀ p : program env term, pre p → program' env' term';
  correctness : ∀ input (p : pre input), post (transform input p);
  obseq : program env term → program' env' term' →
          value → value' → Prop;
  preservation : ∀ p v (pr : pre p), eval p v →
    let p' := transform p pr in
    ∃ v', eval' p' v' ∧ obseq p pr p' v v' }.

Definition Transform.compose :
    (o : Transform.t env env' term term' value value' eval eval')
    (o' : Transform.t env' env'' term' term'' value' value'' eval' eval''),
    (∀ p : program env' term', post o p → pre o' p) →
    Transform.t env env'' term term'' value value'' eval eval''.

Notation " o ▹ o' " := (Transform.compose o o' _)
```

```
Program Definition erasure_pipeline : Transform.t :=
(* Build an efficient lookup table *)
build_template_program_env ▹
(* Eta-expand constructors and fixpoint *)
eta_expand ▹
(* Casts are removed, application is binary,
   case annotations are inferred *)
template_to_pcuic_transform ▹
(* Erasure of proofs terms in Prop and types *)
erase_transform ▹
(* Simulation of the guarded fixpoint rules
   with a single unguarded one *)
guarded_to_unguarded_fix ▹
(* Remove all constructor parameters *)
remove_params_optimization ▹
(* Rebuild the efficient lookup table *)
rebuild_wf_env_transform ▹
(* Remove cases/projections on propositions *)
optimize_prop_discr_optimization ▹
(* Rebuild the efficient lookup table *)
rebuild_wf_env_transform ▹
(* Inline projections to cases *)
inline_projections_optimization ▹
(* Rebuild the efficient lookup table *)
rebuild_wf_env_transform ▹
(* First-order constructor representation *)
constructors_as_blocks_transformation.
```

Fig. 3. The erasure pipeline

types and proofs—are all erased to the same particular value □. It was first mechanised by Sozeau et al. [2019] in their correctness proof of type and proof erasure, with a focus on weak call-by-value semantics. We integrate this erasure procedure into a pipeline with multiple translation steps to finally extract to OCaml.

Those multiple steps are necessary because, like Coq, $\lambda_\square$ has higher-order constructors, structural fixpoints with principal arguments that have to reduce to a constructor for the fixpoint to unfold, and no dedicated evaluation strategy. On the other hand, constructors in OCaml are blocks (*e.g.,* `cons` by itself is not well-typed, only `cons(x,l)` is), `let rec` does not indicate a special principal argument, and evaluation is a variant of weak call-by-value reduction. These subtle differences pose challenges for machine-checked reasoning, since they are traditionally ignored in proofs on paper. We parametrise the evaluation relation of $\lambda_\square$ in various flags which can be used to alter it to avoid code duplication, and give separate correctness proofs while gradually translating the differences between the languages away. The complete strung-together phases of the pipeline are described in Fig. 3.

Each phase is an instance of `Transform.t`, parameterised by a type of input and output programs (`program` and `program'`), values (`value` and `value'`) and their evaluation relations (`eval` and `eval'`). Each phase is a `transform` taking input programs respecting the precondition `pre` and producing output programs that validate the postcondition `post`, as the `correctness` theorem states. Additionally, programs that respect the precondition and evaluate to a value `v` get transformed into programs that also evaluate to an observationally equivalent value `v'`. In most cases, the observational equivalence of values will be syntactic equality between a value and a translated value, *i.e.,* `transform v = v'`. Two transform phases `o` and `o'` can be composed (noted `o ▹ o'`) when the postcondition of `o` implies the precondition of `o'`.

The erasure pipeline as a whole transforms a program from TemplateCoq (an environment and term directly quoted from Coq) into an erased program, with term asts representing the value types and evaluation from TemplateCoq (weak call-by-value evaluation of Gallina terms) being simulated by evaluation of erased programs, where evaluation has no rule for pattern-matching on □ and uses an unguarded fixpoint expansion rule (i.e., one argument is enough to trigger fixpoint unfolding) and a representation of constructors as blocks. This pipeline mainly grew in a by-need fashion, depending on the requirements of the target language.

In this section, we concisely outline the different phases, focusing on the pre-conditions for their correctness, the post-conditions they establish, as well as the evaluation relations and observational equivalence on values they use when it is not simply syntactic equality.[7] For all these phases, the main proof effort was the simulation proof and/or type preservation, while preserving the wellformedness and η-expandedness invariants was usually comparatively easier.

Eta-expand. This phase η-expands all fixpoints and constructors. For instance, the term `map cons`, which is a well-typed function in Coq but not in OCaml, is η-expanded to the convertible term `map (fun x l ⇒ cons x l)` which is also well-typed in OCaml. Because MetaCoq does not include η-expansions in its theory yet, for reasons discussed by Lennon-Bertrand [Lennon-Bertrand 2022], the correctness proof of this phase is just assumed together with the correctness of the quotation phase. The pre-condition is well-typedness of `t` in the global environment Σ, written `∃ T, Σ ; [] ⊢ t : T` in the Coq formalisation. The post-condition is well-typedness of the return term `t'` and η-expandedness of all constant definition in Σ and `t'`. Both evaluation relations are weak call-by-value evaluation of the formalisation of Coq in Coq.

Translation to PCUIC. PCUIC is the idealised calculus underlying Coq used in MetaCoq. Technically, this translation removes casts and makes application unary to ease meta-theoretical proofs. The translation has no pre-condition, and the post-condition is that the translated term and environment do not contain casts. The main property of this phase is that it preserves well-typedness. The evaluation relation of the target is weak call-by-value evaluation of PCUIC.

Lets in constructor types. Constructors in Coq can be defined as `C (x1 : X1) (x2 := t2) (x3 : X3)` where `X3` can use both `x1` and `x2`, and dually, pattern-matching can bind these defined arguments, while these let-bindings are not actually stored in constructor applications. We translate away such bindings via substitution to ease several verifications later on. As far as we know, Coq is the only language supporting let-bindings in constructor types, which is a useful feature when specifying inductive relations. This translation brings the language closer to usual presentations of algebraic datatypes. The phase has no pre-condition and ensures that in the translated term, constructor types are functional and pattern-matching reduction only involves a simple substitution. It preserves well-typedness and η-expandedness of fixpoints and constructors and the evaluation relation is simplified. This phase is the least economical in terms of proof-engineering as we must show a type preservation theorem for the whole theory of PCUIC: the proofs amount to 10kLoC.

Type and proof erasure. This phase goes from PCUIC to $\lambda_{\square}$. We use the verified type and proof erasure procedure due to Sozeau et al. [2019]. The pre-condition of this phase requires well-typedness of the term. The post-condition cannot be well-typedness anymore because $\lambda_{\square}$ is untyped and erasure breaks typing. However, to still control the shape of the output term, the post-condition is a well-formedness criterion: all terms in the environment are well-scoped (for the de Bruijn local variables) and all global references are properly declared. Additionally, well-formedness is parameterized by flags that control which constructors can appear in an extracted term. Well-formedness is carried through all the subsequent phases, with varying flags depending on the available constructors at each phase. This phase preserves η-expand-edness of constructors

[7] A more detailed overview of the passes is given in the extended version of this article.

and fixpoints and the target evaluation relation is the weak call-by-value relation of $\lambda_\square$, with structural fixpoints, `match` on □, and higher-order constructors. The observational equivalence on values for this phase is more complex and makes use of an erasure relation, which only agrees with the syntactic equality on first-order values. This weaker notion of equivalence is however enough to get our final theorem because it will ultimately be restricted to first-order values.

Unary fix translation. The evaluation of fixpoints in Coq and up-to this point of the pipeline is performed by evaluating its structural argument to a constructor application and then unfolding the fixpoint. The structural argument is the one guaranteeing termination and thus other unfolding strategies are not safe. However, in OCaml, evaluation of fixpoints is performed in a similar way as β-reduction, by *always* evaluating its first argument to a value and then unfolding the fixpoint. This rule is called the "unary fixpoint rule". So the purpose of this phase, which is the identity on programs, is to justify that enabling the unary fixpoint rule in the target preserves evaluation of programs. It crucially relies on fixpoints being η-expanded so that when they get unfolded in an evaluation path, we have a guarantee that their structural argument already evaluates to a constructor.

Parameter stripping. This phase removes parameters from inductive types in the global context and from constructor applications. It requires constructors to be η-expanded. As an output, it guarantees that programs are well-formed, when the use of parameters is disallowed and that constructors are fully applied.

Remove match on □. This phase removes case analysis on propositional content, the residue of erased computational case analysis on proofs, by replacing terms of shape `match x in I _ with C a1 ... an ⇒ t end` where `I` is a propositional inductive type and `x` must be □ by $t[\square/a_1, .., \square/a_n]$. This is not done by the erasure phase to simplify its own proof, but could be fused with it. It is necessary to do this, because we are going to implement □ as a recursive function in Malfunction, so no case analysis on □ should remain. This phase has no pre- and post-conditions as it is purely syntactic. It preserves η-expandedness of constructors. It allows disabling the evaluation rule for `match` on □ in the target.

Inline projections. Primitive projections correspond to a negative treatment of pattern-matching when the inductive type under consideration is a record type. Primitive projections in Coq have a more compact representation than the ones defined by pattern-matching and they enjoy η-conversion. This transformation phase inlines primitive projections as (trivially exhaustive) `match` constructs, which is supported by more target languages, it has no pre-condition and guarantees syntactically that no projections are left in the translated program.

Constructors as blocks. Because constructors of inductive types are fully applied in OCaml and Malfunction, this phase translates any application of a constructor $C\ n\ [\,]$ (where n is the name and $[\,]$ is the a priori empty list of fields of the constructor) to a full list of arguments $a_1, \ldots, a_n$ to $C\ n\ [a_1, \ldots, a_n]$. This transformation requires η-expanded constructors and has no specific post-condition. It enables the rules treating constructors as blocks in the evaluation of a program and disables the application congruence rule for constructors, which further simplifies the evaluation relation.

Enforce extractability. For extraction in Malfunction to be correct, we need to check that

(1) inductives have at most as many constructors as fit into OCaml's integer type,
(2) inductives have at most 200 non-constant constructors,
(3) constructors have at most as many arguments as fit into OCaml's array type,
(4) no axioms occur in the global environment,
(5) no □, named or existential variables, projections, or co-fixpoints remain in the term or context.

Note that we explicitly assume these conditions are fulfilled for the proof using an `Axiom` and do not abort extraction if they are not fulfilled. This is in order to ensure that also programs that do not fall in the scope of the correctness theorem can be extracted, especially ones which have axioms.

Implement □. This phase implements □ as a fixpoint that consumes its argument, *i.e.,* as `fix f x := f`. It requires well-formedness as pre- and post-conditions and ensures that □ does not appear anymore in the translated program.

Linearise case. Let-binds the discriminee of cases, *i.e.,* replaces `match d with ... end` by `let x := d in match x with ... end`. This makes the translation to `switch` and `field` later in Malfunction preserve complexity of programs. The reason we perform this translation at this stage is that it is easier to verify with variables represented as de Bruijn indices, but harder once variables are names as it will be the case in next phase.

Named variables and environment semantics. This phase translates terms with de Bruijn indices to non-capturing, named terms. For named terms, an environment semantics with a dedicated value type (containing amongst others closures and recursive closures) is used. We use two unfolding relations, unfolding respectively a named term or a value in an environment to a term with de Bruijn variables. This phase has no pre-condition and ensures as post-condtion that there exists a well-formed term s such that t (with names) unfolds to s. After this phase, evaluation is switched to a named environment semantics.

Global environments. Several of the passes need to do lookups on the global environment, *e.g.,* to compute the number of parameters of an inductive type. Global environments are represented as an association list of unique identifiers to declarations of constants and inductive types in MetaCoq. While this allows a straightforward formalisation of the well-formedness predicate on global contexts naturally representing dependencies between declarations, this is a very suboptimal datastructure to use when transforming programs, as they generally need to lookup information in the global environment, which can contain thousands of declarations. This problem is well-known in the literature about compiler programming in Coq, with specialists resorting to highly optimised tree datastructures [Appel and Leroy 2023] to avoid performance issues inside Coq and after extraction. Hence we intersperse transformations that turn a well-formed environment represented as a list into a lookup table based on AVL trees that can be used for fast lookups. Some of our transformations, *e.g.,* erasure itself, are nested, so they need to follow the order of declarations given by the association list and likewise produce association lists as outputs, while relying on the efficient environment representation for lookups. A subtletly that appears during formalisation is that we generally need to show global environment weakening lemmas on our transformations so that we don't need to extend our efficient global environment structures after handling each entry in the environment but can leave it untouched for a whole pass. This solely rests on the fact that identifiers are globally unique in a well-formed environment, with this well-formedness condition being carried around throughout the pre- and post-conditions of all our transformations.

Compilation from $\lambda_{\square}$ to Malfunction. The last pass in our extraction pipeline replaces the case analysis `match x with C1 a1 ... an ⇒ t1 | ... end` by `switch x with C1 ⇒ (fun a1 ... an ⇒ t1) (field 1 x) ... (field n x) | ... end`, translates constructors without arguments to integers, constructors with arguments to `block (tag n) ...`, and mutual fixpoints to mutual `let rec` blocks. Its precondition is that the term is extractable, *i.e.,* the five conditions explained above for the enforce extractability phase. The post-condition ensures that the resulting Malfunction program is well-formed, meaning there are no free variables, all constants are declared, blocks are tagged with an integer less than 200, and every occurring `switch` does not have an empty list of branches.

4 VERIFIED EXTRACTION AS A DROP-IN REPLACEMENT

In this section, we discuss our contribution from the point of view of a practical tool, neglecting its formal guarantees, and thereby comparing it to Coq's current extraction to OCaml [Letouzey 2004] which is not formally verified. Note that even though formal guarantees can only be given for programs of first-order type, our extraction process can be run on every Coq program, just as Coq's current extraction method.

Coq's current extraction has several features that we have not discussed until now:

(1) It allows the user to enable optimisations of the generated code via `Set Extraction Optimize`. Optimisations are *e.g.,* commutation of function application and `match`, simplifying `lets` where the bound term only occurs once in the body, etc.
(2) It generates a `.mli` interface by re-inferring types of the generated OCaml code using the type inference algorithm $\mathcal{M}$ [Lee and Yi 1998]. Because the code might be ill-typed in OCaml's type system, `Obj.magic` is heuristically inserted, so the types in the inferfaces of extracted code can contain uninformative `Obj.t` occurrences.
(3) It supports separate extraction to `.ml` files matching the module structure of the corresponding Coq code.
(4) It allows fine-tuning of the extracted code using `Extract Constant` and `Extract Inductive` directives. This is for instance used to map Coq's $\mathbb{B}$ type to the one of OCaml (which has an exactly swapped memory representation).
(5) It allows fine-tuning of the extracted code using `Extract Inline` directives, to perform selective constant inlining.
(6) It extracts terms of type `{x : A | P x}` where `P` is a propositional predicate to terms of type corresponding to `A` in OCaml, *i.e.,* unboxing of singleton constructors.

Regarding (1), a central goal of the optimisations is to obtain readable and idiomatic code. This aspect is not relevant to our work because we provide low level code with formal guarantees instead. The microbenchmarks in Section 6 seem to indicate that in general, optimisations are not necessary to obtain efficient code. We plan to investigate which projects benefit from the optimisations and then implement them on demand in future work.

Regarding (2), since we bypass OCaml's type checker, we only have to generate `.mli` files for the parts of the extracted code that can and should be exposed, *i.e.,* for first-order functions. We provide an `.mli` printer for such functions, which has a simple implementation since it has to solve a significantly simpler problem than the type reinference algorithm. Nevertheless, we support extracting higher-order code, but require the user to annotate the types in the `.mli` file manually.

Regarding (3), separate extraction is mainly a concern for readability of code, which we have not considered. However, we could easily implement it if users demand for it.

Regarding (4), we plan on providing such directives soon. However, we want to remark that of the four frequent use cases, not all are actually safe.

Firstly, to direct extraction to extract Coq's $\mathbb{B}$ to OCaml's `bool`, which is a subtly different type: Coq's `Inductive` $\mathbb{B}$ `:= true | false` has `true` as first constructor, which will thus be represented as integer `0` in memory, whereas OCaml's `type bool = false | true` has `false` as first constructor, thus `true` being represented as integer `1`. Here, we advocate for using such type conversion explicitly when calling and receiving arguments of an extracted Coq function explicitly, *i.e.,* by converting between the two types. We can also implement a verified reordering pass in the future, see Section 8.

Secondly, to direct extraction to extract Coq's unbounded $\mathbb{N}$ to OCaml's `int`. However, this extraction is plain wrong, because `int` is not unbounded. It is however still correct on small enough numbers. Here, we thus advocate for proving this property for small enough numbers explicitly on the Coq side. This is possible because Coq features a primitive integer type, see below.

Thirdly, to direct extraction to extract Coq's unbounded $\mathbb{N}$ to OCaml's `BigInt`. This extraction should preserve correctness, however it is a complex transformation to implement: constructors and pattern-matchings need to be bound to the `BigInt` interface and a formal proof of correspondence should be given, requiring some kind of program logic for the target language. While this may be doable in the future, at this time we rather advocate for using semi-automatic transfer tools like [Cohen et al. 2024] inside Coq.

Fourthly, to direct extraction to extract Coq's primitive integer, float and array types to the corresponding types in OCaml. This use case is already supported since Coq's primitive types are declared as axioms, and our extraction just maps all axioms to a call to a module called `Axioms` that has to be provided by the user. We however lack a correctness proof, see Section 8.

Regarding (5), inlining definitions during extraction is mainly concerned with readability of code and we leave it to future work. Should inlining be performance-critical, we will implement and verify a configurable inlining pass. We do not expect this to be a difficult proof.

Regarding (6), unboxing such types would potentially even be beneficial for performance, see Section 8 on future work.

Thus, we estimate that our verified re-implementation of Coq's extraction is almost ready to replace Coq's current extraction mechanism on safe use-cases.

5 BOOTSTRAPPING A SELF-HOSTING COMPILER

A compiler is called *self-hosting* if it is implemented in the same language it is a compiler for. This is the case for us: Our extraction process is implemented in Coq, and translates Coq to Malfunction. The process to obtain a first executable (*e.g.,* by using a different) compiler is called *bootstrapping*.

Since Coq is a dependent type theory, already its type checker comes with the ability to execute programs. Thus, in principle, two ways of bootstrapping are conceivable:

First, using the existing, unverified extraction from Coq to OCaml (implemented in OCaml). This means that in the bootstrapping process, this unverified extraction algorithm becomes part of the TCB.

Secondly, by executing the extraction pipeline live in Coq using Coq's execution mechanisms once on itself. Unfortunately, execution in Coq is not very performant (which, of course, is the main reason why extraction is needed). In this case, we could not manage to make the process terminate, because Coq is exhausting its stack. Even setting `ulimit -s unlimited` does not help on our working machines—but it would be an interesting experiment to let it run longer on a more performant server. The compiler pipeline can however still be executed on smaller examples than the compilation pipeline itself.

Our bootstrapping method via the existing extraction to OCaml works. However, we have to patch the generated OCaml files to circumvent some bugs of the current extraction mechanism. Patching is fine because we have no guarantee on this phase anyway, but it illustrates the limitations of current extraction mechanism. By statically adding the resulting `.mlf` file to the repository, and using it to re-extract new future versions of the extraction pipeline, the use of the existing extraction can be limited to the initial bootstrapping once and for all, meaning it is not crucial that it is kept a part of Coq in the future.

Note that our correctness theorem does not yet apply to the bootstrapped extraction pipeline. Concretely, we would have to (i) slightly change the input format to be a first-order data type by modelling annotated universe instances with a datatype of non-empty lists instead of provably non-empty lists. (ii) cover primitive operations for integers and floats in the theorem, because they are needed for printing integers and floats to strings in the output.

Especially the second part requires a global change to MetaCoq, most crucially to its operational semantics and typing judgment, so we leave it for future work.

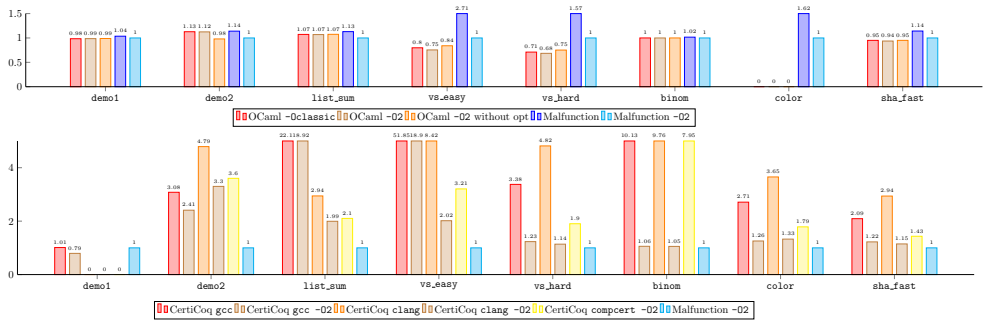


Fig. 4. Runtime for microbenchmarks, normalised against Malfunction -O2. Graphs cut off at factors 1.5 / 5.

6 BENCHMARKS

We compare the performance of extracted code comparing Coq's current extraction to OCaml, CertiCoq's extraction to C, and our verified extraction to Malfunction. All benchmarks are run on a 11th Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz and Coq version `8.17.1`. We have ensured that no thermal throttling occurs in the processor and the frequency of 2.80GHz is maintained.

We use the same benchmark set as in the evaluation of CertiCoq by Paraskevopoulou [2020]:

- `demo1`, appending two lists of booleans of length 500 and 300.
- `demo2`, mapping negation on a list of 100 booleans (using the higher-order function `map`).
- `list_sum`, computing the sum of a list containing 100 times the number 1.
- `vs_easy`, running the VeriStar [Stewart et al. 2012] validity prover for a fragment of separation logic on an easy entailment.
- `vs_hard`, same as `vs_easy`, but with a harder entailment.
- `binom`, constructing two binomial queues, merging them, and finding the maximum element, using a verified implementation of binomial queues [Appel 2023; Vuillemin 1978].
- `color`, coloring a graph using a formally verified implementation of the Kempe/Chaitin algorithm [Appel 2023; Chaitin et al. 1981]. Note that this benchmark is not executed with OCaml extraction, because the generated code contains type errors.
- `sha`, computing the SHA-256 hash of a string consisting of 484 characters.

For OCaml we use version 4.14.0 with `flambda` enabled. We only use the native-code compiler, with different optimisation options. We furthermore set and unset the (heuristic, unproved) optimisations Coq's current extraction performs on the code. All benchmarks are performed using `Core_bench` [Jane Street 2024]. Error margins are below 0.5 percent and thus not reported in the graph. For CertiCoq, we use commit version `79eae7` from March 2024. We compile the resulting code with `gcc` (version `13.2.1`, using both `-O0` and `-O2`), `clang` (version `16.0.6`, using both `-O0` and `-O2`), and CompCert version `3.14` with `-O2`. For verified extraction to Malfunction, we compare performance by disabling optimisations when calling the OCaml compiler pipeline, or calling it with the default of two rounds of optimisations.

In total, we compile code by 11 different means (the results are depicted in Fig. 4): By Coq's current extraction to OCaml, with `Set Extraction Optimize` and using the `ocamlopt` native-code compiler with `-Oclassic` optimisation (1st column) or the `ocamlopt` native-code compiler with `-O2` optimisation (2nd column). By Coq's current extraction to OCaml, with `Unset Extraction Optimize` and using the `ocamlopt` native-code compiler with `-O2` optimisation (3rd column). By verified extraction to Malfunction, using `malfunction cmx -O0` compilation (4th column) or `malfunction cmx -O2` compilation (5th column). By CertiCoq, using `gcc -O0` (1st column in the

2nd graphics), `gcc -O2` (2nd column), `clang -O0` (3rd column), `clang -O2` (4th column), `ccomp -O2` (5th column), and again for comparison `malfunction cmx -O2` compilation (6th column).

We observe that compiling our code with the default optimisations has performance very similar to using the OCaml native-code compiler `ocamlopt -O2`. On some benchmarks our extraction produces slightly faster code (`list_sum`), whereas on others it produces slightly less performant code. The performance difference can be explained for instance by differences in pattern matching compilation, where the OCaml compiler tries to optimise heavily and Malfunction performs some mild optimisations. In all cases, we outperform CertiCoq with `gcc`.

Unrelated to the contributions of this paper, we observe that `Set Extraction Optimize` has a somewhat unpredictable effect on the performance: Sometimes it leads to a slight performance gain (*e.g.,* for the `vs_hard` benchmark), sometimes it leads to a slight slowdown (*e.g.,* for the `demo2` benchmark). In general, this might hint at the fact that the current optimisations are mainly of cosmetical nature, *i.e.,* are not concerned with obtaining more efficient code. The notable exception could be unboxing dependent pair types `{x : A | P x}` with a propositional predicate `P` (which can be seen as refinement or subset types) such that elements of this type are treated as elements of type `A`, rather than as pairs of an element of type `A` and a □. In future work, we would like to do a more in depth analysis of the impact of different optimisations for larger projects such as CompCert.

7 RELATED WORK

There are several ongoing projects on obtaining correct programs in industrial programming languages like OCaml, Haskell, or C from programs in richly typed dependent programming languages like Coq, Lean, Idris, and Agda, or the simple type theories underlying Isabelle/HOL, HOL light, or HOL4. We here discuss our project in the context of the ConCert project [Annenkov et al. 2021, 2020], Œuf [Mullen et al. 2018], Rupicola [Pit-Claudel et al. 2022], and CertiCoq, which cover verified extraction / compilation from Coq. Furthermore, we discuss verified extraction from HOL systems such as Isabelle HOL and HOL4 into CakeML [Abrahamsson et al. 2020; Hupel and Nipkow 2018], and the unverified compilation mechanisms of Lean and Isabelle.

Our project is closely related to all of these, but has a different motivation. We want to replace the current extraction process of Coq to OCaml [Letouzey 2004] with a verified process, without causing maintenance issues in the projects using extraction. In particular, we want our extraction to be applicable to all of Coq, not just a subset, and to require no modification of existing code. Conversely, to achieve this generality, we are accepting performance differences with extraction mechanisms tailored to efficiently compile a subset of Coq programs.

There are two wide-spread techniques to verify extraction: via a proof-producing translation or via a verified translation coming with a simulation proof. In a proof-producing translation, the user obtains both a program in the target language *and* a proof in the proof assistant showing that the program correctly implements the function that was extracted. Proof-producing translations can be implemented in an external programming language. They can be and often are incomplete. In a verified translation coming with a simulation proof, one talks about the semantics of the proof assistant in the proof assistant itself, and verifies the correctness once and for all. Usually, such a verified translation is defined in the proof assistant itself to be applicable for verification.

Verified extraction from Coq. The CertiCoq project [Anand et al. 2017; Appel et al. 2022] aims at extracting Coq code to the C light interface of CompCert. Most but not all following phases of CertiCoq are verified, meaning it still lacks an end-to-end theorem. CertiCoq relies on the $\lambda_{\square}$ pipeline presented in and contributed by this paper. After running this pipeline, we have three additional steps: implementing □ as a fixpoint, switching to named syntax, and converting to Malfunction. In contrast, CertiCoq runs a multitude of additional steps and includes a verified garbage collector to extract to C code.

The ConCert project [Annenkov et al. 2021, 2020] is a platform to extract Coq code to ML-like and blockchain languages. Similarly to CertiCoq, ConCert also relies on the $\lambda_{\square}$ pipeline. Since most target languages they consider are not specified and their implementation is under constant evolution, the final phase of their extraction process is trusted pretty-printing.

Œuf [Mullen et al. 2018] formalises a subset of Coq and provides a verified compiler from this language to Assembly code. The number of datatypes Œuf supports is fixed and not extensible. Furthermore, functions need to be defined using recursors rather than using `fix` and `match`, meaning compatibility of Coq developments needs to be manually ensured.

The Rupicola project [Pit-Claudel et al. 2022] offers a proof-producing translation of functional models specified in Coq into low-level code via relational compilation. Rupicola targets a user-extensible fragment of Coq and compiles this to Bedrock2, an imperative language. If the input program falls into a simple fragment that can be captured by loops, the output is both verified and comparable in performance to hand-written, optimised code.

CakeML. The most feature-complete project related to ours is automatic extraction from ML-like functions from HOL4 to CakeML by Myreen and Owens [2014]. CakeML [Kumar et al. 2014] formalises a substantial subset of Standard ML. It was devised as an ML-like language to facilitate translation from higher-order logic systems, in a wider sense comprising, *e.g.,* dependent type theory. CakeML comes with a verified compiler backend, compiling to ARMv6, ARMv8, x86-64, MIPS-64, RISC-V, and Silver RSA architectures. It furthermore comes with two frontends. Frontend 2 has a parser and performs type inference. Frontend 1 can extract ML-like HOL4 functions directly to CakeML and prove a correspondence between the generated AST w.r.t the CakeML semantics and the HOL4 function. Since Frontend 2 together with the backend is an ML-like HOL4 function, Frontend 1 can be used to bootstrap a CakeML compiler in HOL4 without a leap of trust.

Since CakeML compiles down to processor architectures via a multitude of intermediate languages, there is a true end-to-end theorem, whereas our work only encompasses the first step and then relies on the unverified OCaml compiler pipeline. In future work, it would both be interesting to increase the trust in the OCaml compiler by verifying parts of it, and to target CakeML.

As a language, (untyped) CakeML is very similar to Malfunction, with the notable difference that in source-level CakeML constructors are represented directly rather than via their memory representation as blocks and integers, and case analysis is a native construct, rather than implementable using `switch` and `proj`. Conceptually, extraction from HOL4 to CakeML and extraction from Coq to Malfunction thus are a technically similar goal. And indeed, the subset of HOL4 that is supported [Myreen and Owens 2012], comprising total recursive functions, type variables, functions as first-class values, and nested pattern matching and user-defined datatypes can be seen as, operationally, something close to Coq. The extraction process furthermore supports partially specified functions, *e.g.,* those with missing pattern match cases, which can only be expressed in Coq if the missing case can be proved to be logically impossible, and functions using monads for various effects [Abrahamsson et al. 2020].

Overall, the way the two projects achieve this goal is different however: While HOL4 extraction to CakeML is implemented in Standard ML and proof-producing, our extraction from Coq to Malfunction is implemented in Coq itself via MetaCoq and comes with a simulation proof. The advantage of a proof-producing translation is that the correctness of the resulting program can be reasoned with *inside* HOL4. The advantage of the simulation technique is that it cannot fail: There is a proof that our extraction is *always* correct. Proof-production in HOL4 can in principle fail. Note that this is no trust issue, rather it forms a potential incompleteness where extraction does not work even though it should.

Extraction from HOL4 to CakeML has been used in the Candle project [Abrahamsson et al. 2022], providing a verified implementation of HOL light in HOL4, subsequently extracted to CakeML and

compiled to assembly. Hupel and Nipkow [2018] report on a proof-producing reification of Isabelle functions into a model of Isabelle in Isabelle, and then give a simulation-based proof for extraction from this deeply embedded model to CakeML. A proof checker of Isabelle has been verified in Isabelle based on this [Nipkow and Roßkopf 2021].

Other compilation from proof assistants. Besides verified extraction to CakeML, Isabelle/HOL can also generate LLVM code for definitions using a proof-producing translation [Lammich 2019]. The Lean 4 proof assistant comes with unverified compilation to C [Moura and Ullrich 2021]. Idris 2 comes with a self-hosting compiler [Brady 2021], bootstrapped via Chez Scheme [Dybvig 2006]. A subset of F* can be extracted to C via the KaRaMeL compiler. The process was proved correct on paper [Protzenko et al. 2017].

8 FUTURE WORK

Our current extraction pipeline makes the axiomatic assumption that η-expanding fixpoints and constructors does not change the values of first-order data types w.r.t. weak call-by-value evaluation. The easiest way to prove this is to extend MetaCoq's reduction relation with a suitable notion of η-conversion. The difficulties of this are discussed by Lennon-Bertrand [2022].

Regarding features of Coq we support, we are missing two central syntactic constructs: Primitive operations on integers, floats, and arrays and co-fixpoints. For the former, one would have to extend the reduction semantics of MetaCoq with primitive operations, and thread the corresponding correctness proofs through the pipeline. In the target, one then has to prove that these functions can be correctly implemented using certain Malfunction programs. Most likely, a program logic for Malfunction would be helpful here, or alternatively a program logic for an envisioned variant of OCaml with formal semantics that can be compiled to Malfunction. For the latter, there are different options: One is to implement co-fixpoints without sharing as thunked fixpoints, but this does not allow sharing, used for example for memoization. A second would be to use Malfunction's `lazy` construct, at the cost of letting extraction produce impure programs, and hence requiring to reason on observationally pure but internally effectful programs.

Regarding an extension of our correctness theorem, we would like to work in the direction of proving a theorem of interoperability with an arbitrary higher-order fragment as long as the only effect used by the unverified OCaml program is nontermination, *i.e.,* requiring that the heap remains unchanged. However, such properties cannot be enforced by OCaml's type-checker automatically, meaning they would have to be manually verified by the user.

Regarding optimisations implemented in Coq's current extraction mechanism, we want to investigate their exact impact on larger projects such as CompCert, and implement and verify them as needed. In the short term, we plan on implementing and verifying an inliner (*e.g.,* 'Extract Inline') and the unboxing of singleton inductive types.

Regarding proof production, we want to investigate whether the type and erasure phase of our pipeline can be proof-producing rather than verified using a simulation proof.

Regarding effects, it would be interesting to investigate whether the techniques used in HOL4 for extraction of monadic functions [Abrahamsson et al. 2020] can be translated to Coq.

Lastly, the middle-end of our extraction pipeline based on $\lambda_\square$ could be used as a more general platform for verified extraction. It is conceivable to implement type and proof erasure from other type theory-based proof assistants such as Lean or Agda to $\lambda_\square$, and it is conceivable to implement back ends to other programming languages than OCaml/ Malfunction. The CertiCoq project in its current stage can already be seen as such a back end to C. In particular, we would like to implement a back-end for CakeML, allowing for an end-to-end theorem with no parts of the compilation chain that need to be trusted.

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