

ECO2100 Part 2 Homework 1

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a)

Social welfare is given by:

$$S = \overbrace{f(k)(1-u)}^{\text{Production of firm in match}} - \underbrace{vk}_{\text{total cost of vacancies}}$$

\downarrow \downarrow
 Number of matches

With the constraint:

$$u = \frac{\delta}{\delta + \frac{m(\lambda)}{\lambda}}$$

Which can be re-written

$$u = \frac{\lambda\delta}{\lambda\delta + m(\lambda)}$$

Also note that

$$u = v \frac{u}{v} = v\lambda \iff v = \frac{u}{\lambda}$$

Thus the Planner's problem becomes

$$\max_{\lambda, k} f(k) \left(1 - \frac{\lambda\delta}{\lambda\delta + m(\lambda)}\right) - k \frac{\delta}{\lambda\delta + m(\lambda)}$$

b)

First, we can simplify the above function to

$$\begin{aligned} & f(k) \left(\frac{m(\lambda)}{\lambda\delta + m(\lambda)} \right) - k \frac{\delta}{\lambda\delta + m(\lambda)} \\ &= \frac{f(k)m(\lambda) - k\delta}{\lambda\delta + m(\lambda)} \end{aligned}$$

Taking the first order condition with respect to capital yields:

$$\begin{aligned} \frac{m(\lambda)f'(k) - \delta}{\lambda\delta + m(\lambda)} &= 0 \\ \iff m(\lambda)f'(k) &= \delta \end{aligned}$$

The first order condition with respect to λ

$$\begin{aligned} & \frac{m'(\lambda)f(k)(\lambda\delta + m(\lambda)) - (\delta + m'(\lambda))(f(k)m(\lambda) - k\delta)}{(\lambda\delta + m(\lambda))^2} = 0 \\ \iff m'(\lambda)f(k)(\lambda\delta + m(\lambda)) - (\delta + m'(\lambda))(f(k)m(\lambda) - k\delta) &= 0 \end{aligned}$$