Investigations in Home Equity Position Markets

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Abstract

Description here.

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1. Introduction

As participants in the Budapest Semesters in Mathematics program in Fall 2013, Emily proposed an Elective Undergraduate Research project based on an internship during the preceeding summer. The internship was with a startup company called PRIMARQ in San Francisco. The company hopes to create a market for the trade of equity positions in owner-occupied, residential real estate. As such a market has never existed before and will differ from the known stock and real estate markets, Emily hoped to model the growth and behavior of such a market and Dan offered to work with her in this research.

In the next few pages, we will present the structure of the overall market we are trying to model, the methods and techniques we used in specific aspects of the model, and our results. We will conclude with open questions and aspects to be explored further in the next iterations of the model.

2. Structure of the Market

To begin, we would like to present the overall structure of a PRIMARQ transaction, as that forms the basis of the our model. In broad terms, the model will track the creation and behavior of Home Equity Positions (hearafter referred to as HEPs). These HEPS are created as a portions of the equity in a home, sold to interested investors. These HEPs are later traded on the Secondary Market in the manner of normal stocks. The life cycle of a HEP can be briefly explained as follows:

1. A homeowner triggers the creation of a HEP.
2. The new HEP, purchased for $10,000, is auctioned to the lowest (in equity percentage) bidder among the investors. The equity percentage is now fixed for the life of the HEP.
3. The Investor owning the HEP decides to sell the position and makes the HEP available on the secondary market.
4. The secondary market auction takes place, the HEP is sold to the highest (in dollars) bidder.
5. Repeat steps 3, 4.
6. The homeowner sells the underlying property and the HEP claim "matures".
7. The investor(s) is (are) paid the final claim value $C_f = S_f - M_0$ (S_f: the final sale price of the home; M_0 the initial mortgage amount). We use M_0 because the homeowner is reimbursed all principal payments from the sale proceeds before the remainder is divided between the homeowner and the investor.

We will explain in detail with an example. Homebuyer A decides to buy a home in San Francisco but asks PRIMARQ for an additional $10,000 to help with the down payment, in exchange for a percentage stake in the equity of the home. Homebuyer A decides that she is willing to offer 8% of the equity in her home in exchange for the $10,000. So, she decides to offer this equity on the Exchange.

Prospective investors registered with PRIMARQ have access to the Exchange website. There, they see that there is a position available in a home in San Francisco, that Homebuyer A is asking for the fixed $10,000 and that she is opening the equity position bidding at 8%. The investors also see some financial information about Homebuyer A, including her credit score, etc. The investors evaluate this position (including the home in question, the neighborhood and city in which is it located, etc) and investors B, C, and D decide to place bids to purchase the HEP.

Following their decisions to place a bid, investor B begins the process. He offers Homeowner A the $10,000 in exchange for the 8% equity position, as she originally offered. Investor C, thinking that the home will probably increase considerably in value in the next few years, offers to give Homebuyer A the $10,000 but for only a 7.5% position in the equity of the home. Investor D is more hopeful still and offers the $10,000 for only a 7% equity stake in Homebuyer A's new home. Investors B and C do not want to bid any lower, and Homebuyer A likes Investor D's offer, so Homebuyer A and Investor D enter into the transaction together. The equity percentage of 7% is now fixed for the entire lifetime of the HEP.

Homebuyer (now Homeowner) A and Investor D now own the home as Tenants in Common[1], and this arrangement will continue until one of two things happens: either Homeowner A decides

[1]Note: This is different in legal structure than a Joint Tenancy, specifically because Joint Tenancy includes Right of Survivorship, wherein upon the death of Homeowner A, the ownership of the home would pass to Investor D. This is *not* the case in a Tenancy in Common. (US Department of Housing and Urban Development)

to sell the home, or Investor D decides to sell his equity position[2].

As soon as a home equity position has been created in a home, it can be sold and traded as a stock on PRIMARQ's Secondary Market. If Investor D decides he wishes to sell his HEP, he can post his asking price on the Exchange for other registered investors to see. The HEP can change hands without any change happenening to the homeowner's claim to the home. The amount of equity assigned to a HEP does not change when the position is sold - the price may fluctuate depending on the valuation of investors, but the underlying claim to the house remains fixed at the percentage assigned at the time of the HEP's creation.

If the homeowner decides to sell the home, then after all closing costs are paid and the homeowner is reimbursed for all prinicpal mortgage payments, the remaining profit is divided according to the originally decided equity split between the homeowner and whoever owns the HEP at the time, and the HEP position is terminated. If the new owner of the home decides to create a new HEP, then the process begins again.

We now make a note of a few important rules in these transactions:

- More than one HEP can be in place in one home. This means that the homeowner may be on the title of the home with the Trust, and the trust may have multiple benficiaries.
- The homeowner must always own the majority (51%) of the equity in the home.

This is the market that PRIMARQ is on its way to creating. Our job this semester was to try and see how this market might develop, using the research that has already gone in to modeling traditional equity and real estate markets. We broke the above process down and examined it away from the lense of a business model. Instead, we scrutinized each section of the transaction and tried to figure out which areas of mathematics would help us most in determining the behavior of such a theoretical market.

We began by reviewing topics in Stochastic Calculus, which is an area frequently used in the analysis of financial markets. In particular, our study of Brownian Motions and other stochastic processes helped us to model the growth of a home's value in relation to the neighborhood around it.

We also spent a good deal of time discussing Portfolio Theory, which is a branch of financial mathematics that deals with the behavior of risk and return when it comes to different bundles (or portfolios) of assets. As most of the investors in this model will have portfolios of their own, we wanted to make sure we had an understanding of the effects portfolio strategy and analysis can have on investor behavior, in particular the correlations between real estate related assets and the other asset classes

Finally, we reveiwed some literature on Prospect Theory, which is at the intersection of financial mathematics, behavioral economics, and psychology. Prospect Theory seeks to come up with models that fit the anomalies in the decision-making process that investors exhibit, leading to (what was traditionally considered as) irrational behavior. Again, as investors are a key part of the model we sought to build, we thought it vital to have at least a ground level understanding of the current research in investor behavior.

[2]Specifically, the home is put in the name of the Homeowner and a Trust as Tenants In Common, and he beneficiary of that Trust is (are) the investor(s). In this manner, if the investors trade their positions, it doesn't not affect the homeowner's claim to the home or the equity split. The beneficiary of the fixed trust simply changes, which prevents the equity split having to be reevaluated with every sale of the HEP on he secondary market. For full details of the legal structure of a PRIMARQ transaction, see www.primarq.com.

3. Challenges

Two challenges in particular presented themselves in the structuring of this model. The first was the design of the of the primary market, and the second of the secondary market. Both involve modeling investor behavior (which, in turn, relies heavily on which types of investors are involved in the market - trend-followers, strongly risk-averse investors, individuals with specific preferences when it comes to geographic areas, etc), which is known to be a complicated task. To help us in that area, we spent a great deal of time researching both Portfolio Theory and Prospect Theory, to better understand the possible motives of individuals in an equity exchange.

3.1. The Primary Market

In the primary HEP market, investors bid down the percentage of the equity in a new home that they are willing to accept in exchange for the price offered. The main question was how to model this bidding process - an investor might be actually willing to pay $50,000 for only an 8% equity stake in a home, but if he/she is not forced to go to 8% by other bidders, he/she might gladly pay that $50,000 in exhange for a 12% stake. Modeling this competition and driving down of prices posed a challenge.

Eventually, we decided to use some research on the theory of auctions to help in this process.

3.2. The Secondary Market

The secondary market revolves around the sale of HEPs once they have become pieces of equity on the PRIMARQ exchange. In this part of the model, we needed to be able to formulate the trade of the HEPs - including when an investor might want to sell a HEP that he/she owns, how much they would accept as an offer for that HEP, and then what might trigger other investors to actually bid on it. Further, we needed to model cases in which the owner of the underlying home of a HEP might sell that position.

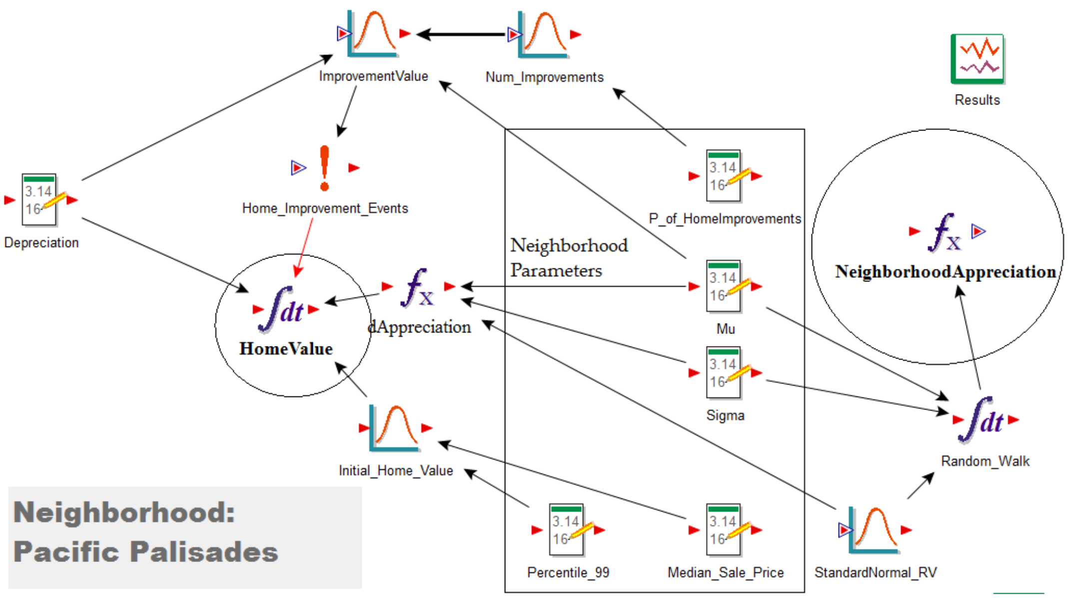
4. The Model

Our model was built up in several layers. The very ground layer was the structure of the neighborhoods in question. We wanted to model specifically five cities(San Francisco, Los Angeles, San Diego, Washington, DC and New York City)[3] and seven active[4] neighborhoods within each of these cities. Within each neighbohood, there were a certain number of HEPs available at the start. The number of homes available grows within the model depending on how each neighborhood is appreciating, along with other factors that might spark investor interest.

Within each neighborhood, the given homes each have a certain value, determined by several factors. These factors include the home's depreciation due to age, the neighborhood appreciation, and the value added or subtracted by home improvement projects and disasters, respectively. The picture below depicts one neighborhood in Los Angeles: Pacific Palisades. Here, neighborhood-specific data is used to calculate the average home value, neighborhood appreciation, etc. Every home in each neighborhood will have is value calculated in such a way.

[3] These cities are based on the decided-upon launch cities for the company PRIMARQ.

[4] As time passes in the model, more neighborhoods become active, depending on the average HEP value in the rest of the city and other factors.



The above picture shows how we visualized the process of a home valuation originally in the GoldSim©[5] model. It is easy to see the various probability distributions from which we got the frequency of home improvements, for example, in addition to their value added to the home (or subtracted from!). From this simple model, we could visualize the growth of a home's value in relation to that of the neighborhood around it.

After constructing the process for one home there, we used Python to build the structure that could model the creation of multiple HEPs in multiple neighborhoods and track each of their values.

5. Results

We have two separate sections of results we would like to present. In order to show more clearly how the model works, our first results will be from the simulation of just one home in one neighborhood.

6. Discussion: Next Steps

There are several aspects of the model that, as we progressed, found were too complex to tackle completely in one semester. Many of these things could either be simulated randomly for the first iteration of the model, or could be simply put on hold until a later version.

6.1. Investor Behavior

We hoped to use both Portfolio Theory and Prospect Theory to have a better understanding of investor behavior. However, as time grew to a close in the semester, we decided that modeling the auction process in the primary market and the trading in the secondary market was more important than having the complex investor profiles that we originally had planned. The model is structured in such a way that given more time, it will be easy to insert a more complex investor profile. As it stands, events like the timing of an investor's bid in the primary market are triggered

[5]Copyright 2013 GoldSim Technology Group.

by values chosen at random from a distribution. This and other similar situations in the model can be more accurately triggered if more of the information from Prospect Theory was built in.

In addition, given more time and communication with PRIMARQ, a more accurate idea of the registered investors portfolios, and thus a better view of how much they would be likely to invest in the HEP market.

6.2. Neighborhood Behavior

Emily spent a great deal of time during her internship starting to model the process of neighborhood appreciation. As investors are most interested in investing in areas they believe will appreciate in the future, coming up with a predictive index for neighborhood appreciation is very valuable. Through the course of her time with PRIMARQ, she researched and discovered upwards of twenty different factors that influence the appreciation of a neighborhood. However, at the end of her internship, Emily was not finished with this index. More research is needed to complete the task, and the time required for that project wasn't available in addition to the time needed for the task at hand. So, as in the section above, Dan and Emily researched which kinds of distributions are used that most accurately reflect neighborhood appreciation and used that in the model for now. Given the time to finish this index, the research behind it would certainly be a useful aspect to add into the model, and as with the investor behavior, there is room for it to be encorporated.

7. Reviewed Works

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8. Appendix

8.1. Notes About Software

The choice of software was a complicated one for this project. For several reasons, the program GoldSim© was optimal. Not only is the software designed for large model implementations (specifically using Monte Carlo simulations for things like Risk Analysis and Decision Analysis), but also Emily had had some experience with it over the summer. In addition, GoldSim© is a very visual environment that can display results and show the model design in explanatory ways, and link dynamically with other programs for data (such as Excel). Finally, there were several pieces of data that were available to Emily and Dan only via this software (from employees at PRIMARQ), and so the decision was made to use GoldSim© as the primary software.

However, several difficulties arose. The first was that the modeling of HEP behavior in the market (tracking the speed of growth in value, tracking which owners own the HEP at what time, and for how long) is something that falls very easily into the object-oriented programming paradigm, which is not supported by GoldSim©. Dan had a lot of experience in Python, Emily in Java, and working to overcome this block in GoldSim© was often very discouraging. In addition, the amount of calculations that GoldSim© performs during the Monte Carlo simulation process proved to be just a bit too much data for our laptops to handle, at least at the level of complexity that we wanted it to work.

In the end, we decided it was best to plan our model conceptually in GoldSim© (as it provides a nice, visual overview, with a clear picture of what elements depend on what in the model). We used that software as our drawing board, but build the bulk of the model in Python, and then used GoldSim©'s many features to help display our results from the simulations.

For the next iteration of this project, we think use of GoldSim certainly could be helpful, but without the computer power necessary, we think the same issues would arise.

8.2. Python Code