Progress:

Preliminary Investigations of Home Equity Position Markets

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Outline

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Introduction

A few weeks ago, we began our research into home equity position markets. To start, we reviewed some relevant topics in stochastic calculus, financial analysis, and economics.

Each week, we reviewed certain literature and also began to build very basic models that incorporated what we were learning, so each week, the models grew more complex.

Reviewed Topics:

- Stochastic Calculus
    - SDEs
    - Stochastic Processes and Option Pricing
- Modern Portfolio Theory
- Prospect Theory

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Stochastic Calculus

Stochastic Calculus is frequently used to model movements in markets. It relies on the evaluation of *stochastic processes*, which are processes that involve random behavior.

A stochastic (or random) process models the evolution of a system over time. Unlike in deterministic situations, in a stochastic process even if the initial condition is known, there are several directions in which the process can evolve.

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Stochastic Process

Given a probability space (Ω, F, P) and a measurable space (S, Σ), an S-valued stochastic process is a collection of S-valued random variables on Ω, indexed by T (time). So, a stochastic or random process X is a collection:

$$\{X_t : t \in T\}$$

Note that there are both discrete and continuous stochastic processes.

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Brownian Motion

In particular, we studied a kind of process called a Brownian Motion or a Wiener Process, which is a very specific kind of random walk.

One key aspect of a Brownian Motion W_t (t representing time) is the following:

For $s \leq t$, $(W_t - W_s) \sim N(0, t - s)$

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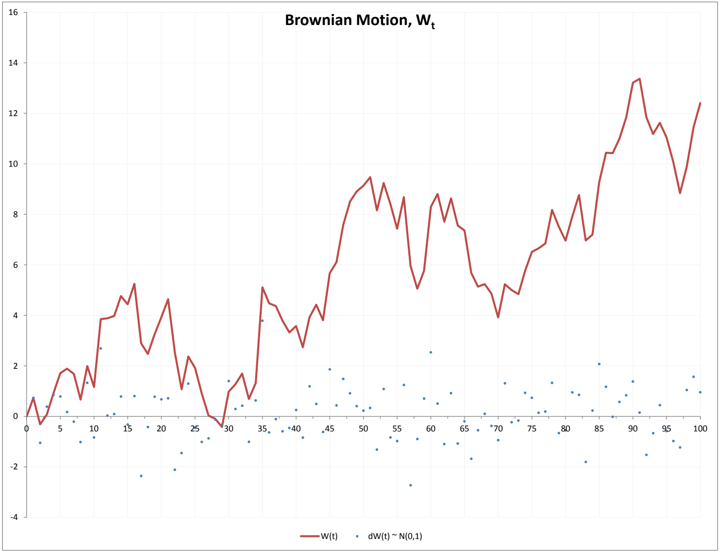
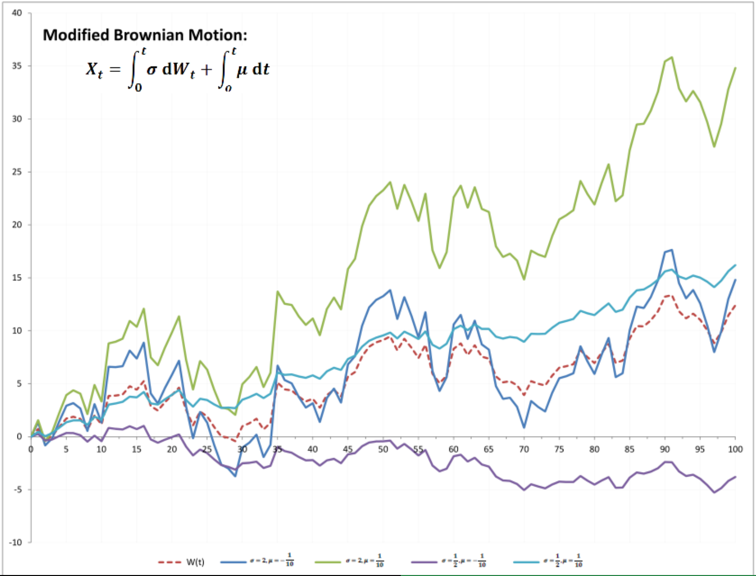

Brownian Motion, W_t
16
14
12
10
8
6
4
2
0
-2
-4
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
W(t)
dW(t) ~ N(0,1)

Stochastic Equations

An example of a stochastic process X_t, with an underlying Brownian Motion W_t, volatility σ and drift μ is:

$$X_t = X_0 + \int_0^t \sigma dW_s + \int_0^t \mu ds$$


Modified Brownian Motion:
$X_t = \int_0^t \sigma \, dW_t + \int_0^t \mu \, dt$
40
35
30
25
20
15
10
5
0
-5
-10
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
W(t)
$\sigma = 2, \mu = -\frac{1}{10}$
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Applications of Stochastic Processes

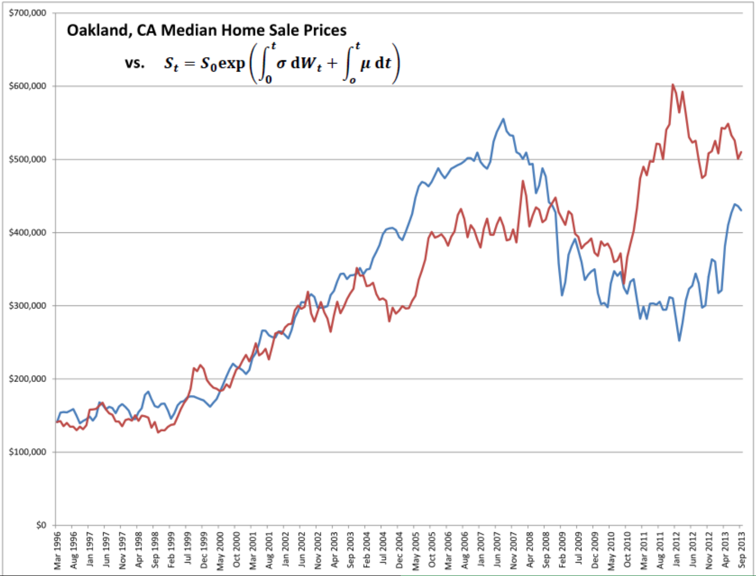
- One application of such processes is to model home prices over a given time period.
- We used the SDE presented before to model logreturns of home prices:

$$r_t = r_0 + \int_0^t \sigma dW_s + \int_0^t \mu ds$$

- We then applied

$$S_t = S_0 e^{(r_t)}$$

  It is known that the lognormal distribution is very accurate for modeling such prices.


Oakland, CA Median Home Sale Prices
vs. $S_t = S_0 \exp\left(\int_0^t \sigma\, dW_t + \int_o^t \mu\, dt\right)$
$700,000
$600,000
$500,000
$400,000
$300,000
$200,000
$100,000
$0
Mar 1996
Aug 1996
Jan 1997
Jun 1997
Nov 1997
Apr 1998
Sep 1998
Feb 1999
Jul 1999
Dec 1999
May 2000
Oct 2000
Mar 2001
Aug 2001
Jan 2002
Jun 2002
Nov 2002
Apr 2003
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Feb 2004
Jul 2004
Dec 2004
May 2005
Oct 2005
Mar 2006
Aug 2006
Jan 2007
Jun 2007
Nov 2007
Apr 2008
Sep 2008
Feb 2009
Jul 2009
Dec 2009
May 2010
Oct 2010
Mar 2011
Aug 2011
Jan 2012
Jun 2012
Nov 2012
Apr 2013
Sep 2013

Applications to the Model

These and other topics in stochastic calulus helped us to model home appreciation in certain neighborhoods, given that area's volitility, etc.

We used this to track the growth of a home's value in relation to the growth of the neighborhood appreciation in our first model.

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Portfolio Theory

Classic and Modern Approaches

Portfolio Theory attempts to organize and quantify exactly what the optimal 'portfolio' or bundle of assets for any given investor will be.

In Portfolio Theory, the goal is to correctly structure a group of investments to optimize gain while avoiding risk.

- Correlation
- Diversification

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Portfolio Theory, cont'd

Tenants of Classic Portfolio Theory:

- Investors are rational and always risk-averse.
- Each investor will tolerate a certain amount of risk in exchange for a return above the *risk-free* return.
- Through these preferences, we can construct a function to determine the optimal portfolio of an investor in terms of risk and return.

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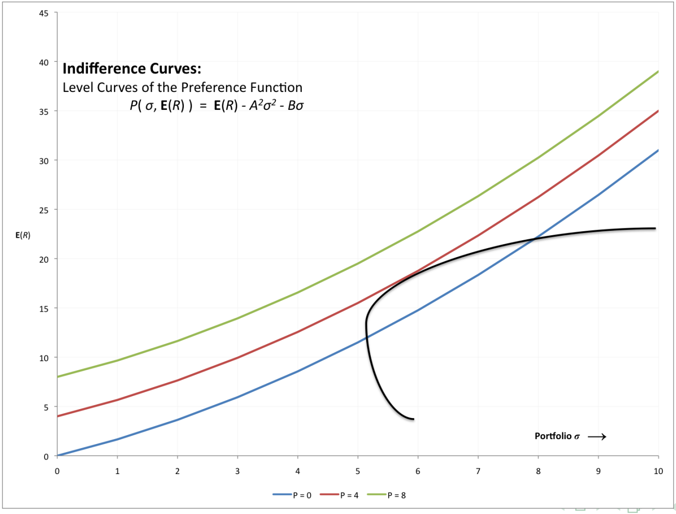
Indifference Curves



Correlation and Diversification

If R_p is the return of a given portfolio, R_i the return on a specific asset and w_i the weight of asset i (i.e. the proportion of asset i in the portfolio), we have

$$E(R_p) = \sum_i w_i E(R_i)$$

and the return variance of the portfolio σ_p^2 where

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{i \neq j} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Where ρ_{ij} is the correlation coefficient between assets i and j.

Applications to the Model

Though all home equity positions are part of the same market, each individual home equity position will have its own volatility and trends depending on where the underlying home is located.

How do local city economies affect other local economies?

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1. Introduction
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   - Portfolio Theory
   - Prospect Theory
3. Model
4. Next Steps
5. Conclusion

Prospect Theory

The drawbacks of classic portfolio theory are obvious even to an outsider. Prospect theory attempts to uncover and mathematically describe some of the *irrational* behaviors exhibited by investors.

Prospect Theory suggests that when examining a risky prospect, there is an *editing process* investors go through to evaluate the decision at hand.

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Risk assessment by individuals must play a role in our models:

- How do investors evaluate a home before they purchase a home equity position?
- How do investors evaluate potential trades of home equity positions?
- How do homebuyers evaluate the outcomes related to selling an equity position in their home?

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- First Model: one neighborhood, one home
- Second Model: three neighborhoods, three homes
- Third Model: three neighborhoods, three homes, one investor

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- Begin to model the secondary market.
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- How do we model the relationship between investor preferences and home prices?
- What kind of bubbles can we expect? How can we accurately model those?

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Questions?

Thank you for your time and attention.

We appreciate the opportunity to pursue this research at BSM.
We are grateful in particular to our project advisors Dezső and Rozi Miklos.

Reviewed Works

- Baxter M. / Rennie A. *Financial Calculus: An Introduction to Derivative Pricing*. Cambridge, UK: Cambridge University Press. 1996.
- Hull, J. *Options, Futures, and Other Derivatives*. Upper Saddle River, NJ: Pearson Education, Inc. 2009.
- Miklos, R. "Prospect Theory and its Financial Applications". Universiteit van Amsterdam. 2011.
- Shreve, S. *Stochastic Calculus for Finance II :Continuous-Time Models*. Pittsburgh, PA: Springer. 2008.