Category Theory I

Week 12, Spring 2023

Chenchao Ding, Apr 2

The Spirit of Structuralism

in general…

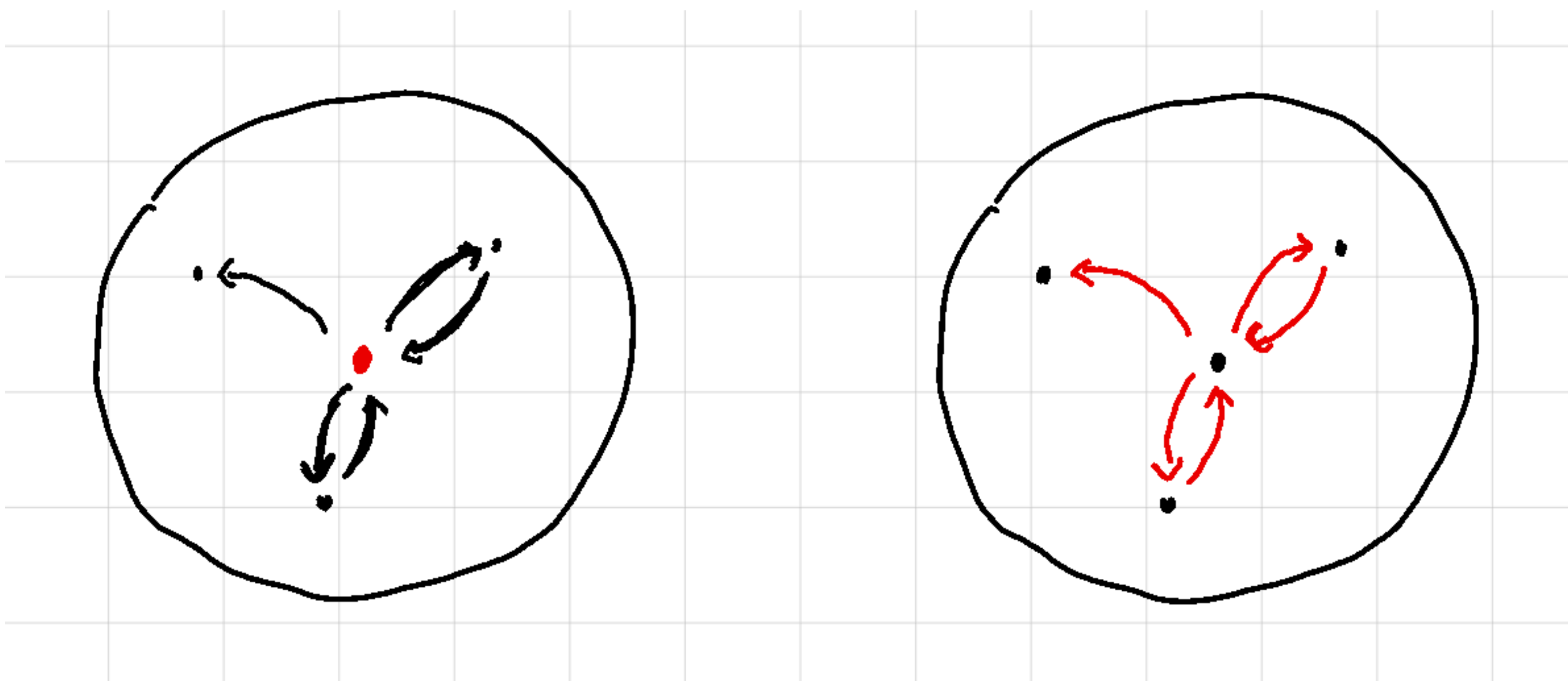
Structuralism is a mode of knowledge that is interested in relations (among objects) rather than individual objects.

Objects are defined by the set of relationships of which they are part, and not by the “internal qualities” possessed by them.

The Spirit of Structuralism

in category theory

Objects are defined by the set of morphisms they are “involved”.



The Spirit of Structuralism

in category theory

Objects are defined by the set of morphisms they are "involved".

for each object A, it's defined by

- $\mathrm{Hom}_{\mathcal{C}}(-, A)$
- $\mathrm{Hom}_{\mathcal{C}}(A, -)$

$\mathcal{C} \to (\mathcal{C}^{op} \to \mathrm{Set})$

$\mathcal{C} \to (\mathcal{C} \to \mathrm{Set})$

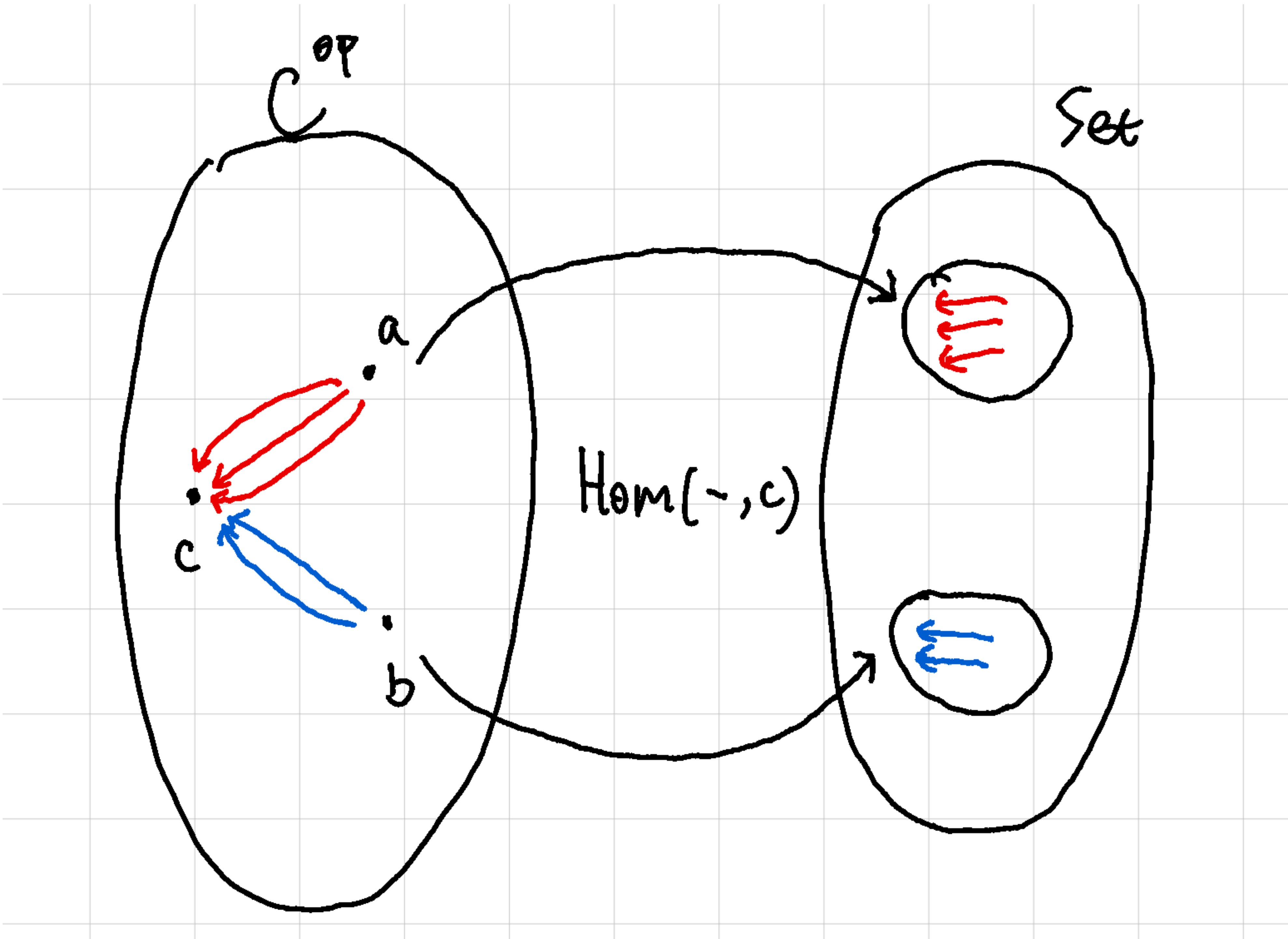
two maps

$A \mapsto \mathrm{Hom}_{\mathcal{C}}(-, A)$

$A \mapsto \mathrm{Hom}_{\mathcal{C}}(A, -)$

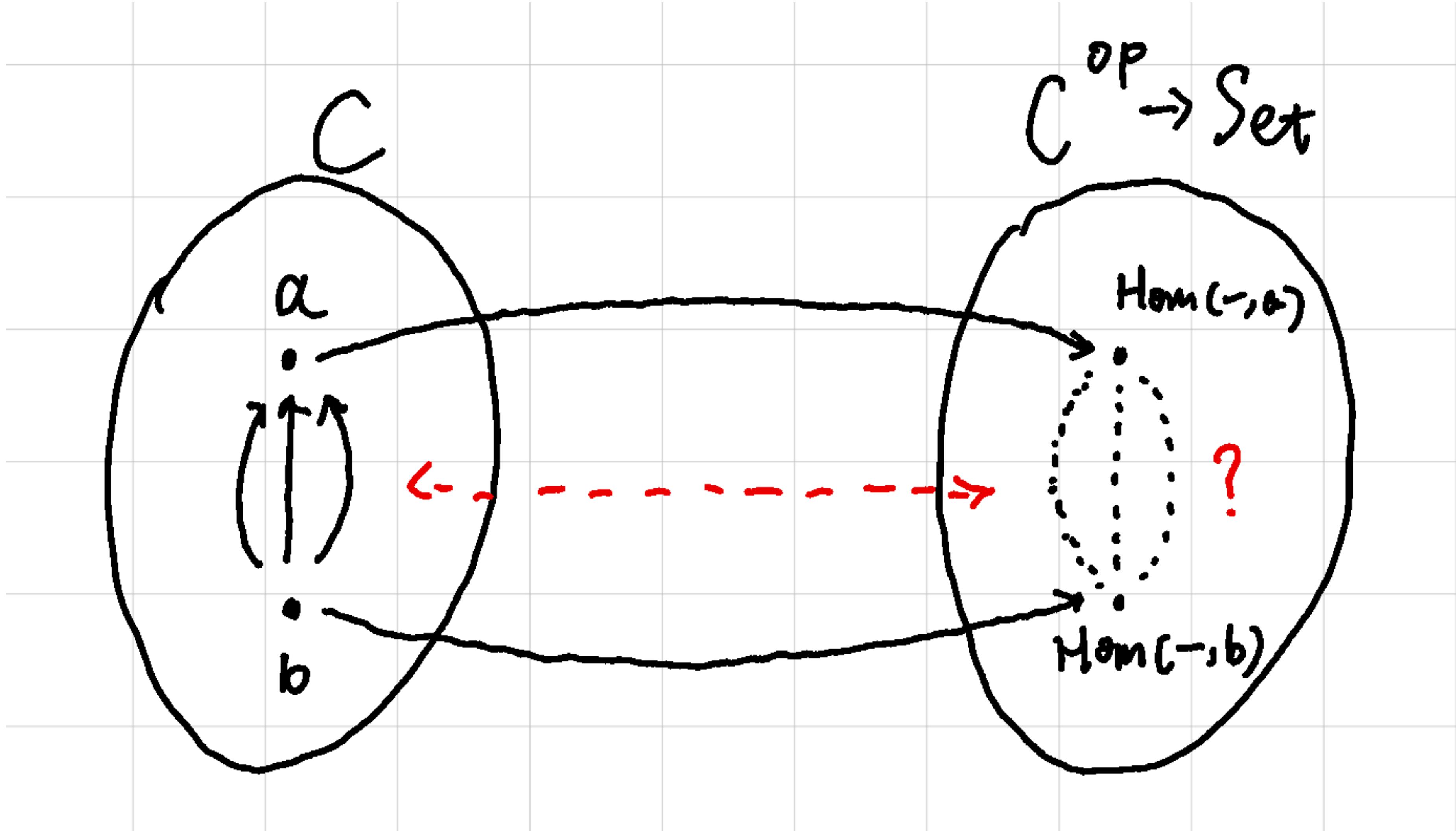
The Spirit of Structuralism

in category theory



The Spirit of Structuralism

in category theory



Category of Cubes

one representation

Category of cubes ()

- objects : $[0], [1], \ldots, [n]$ $(n \in \mathbb{N})$
- morphisms : face maps , degeneracies , $(\ldots)$

$$\begin{tikzcd}
{[0]} \arrow[r, bend left] \arrow[r, bend right] & {[1]} \arrow[r, bend left] \arrow[r, bend right] & {[2]}
\end{tikzcd}$$

face maps $\Rightarrow$ $f : [n] \to [n+1]$

degeneracies $\Rightarrow$ $f : [n+1] \to [n]$

identity $\Rightarrow$ $1_n : [n] \to [n]$

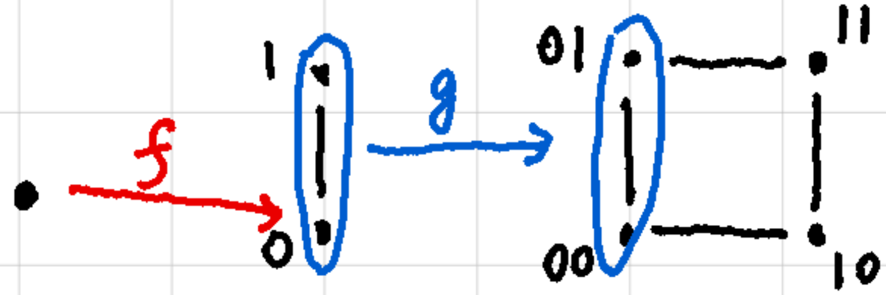
Category of Cubes

one representation

* composition of morphisms

$f : [0] \to [1]$

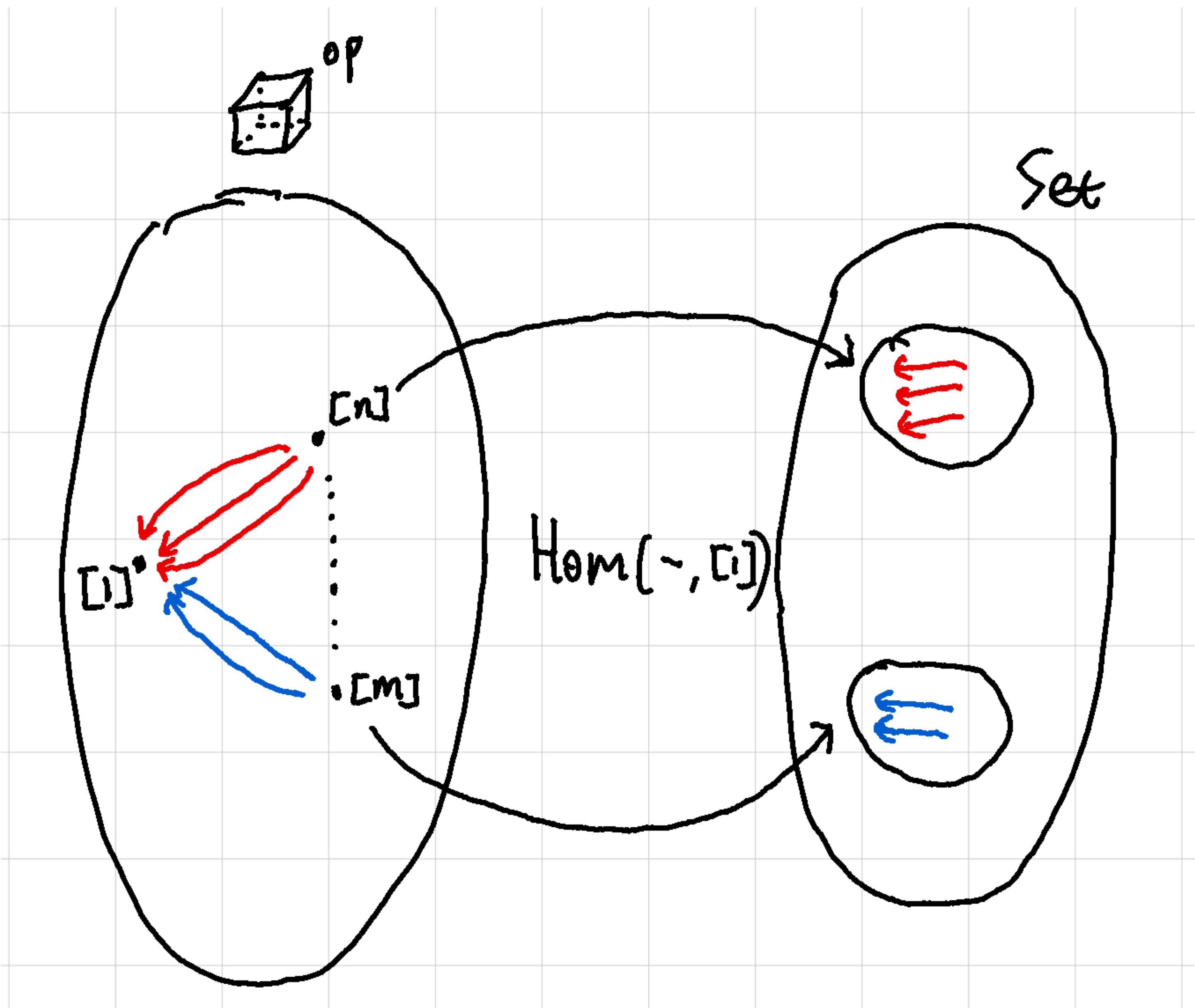
$g : [1] \to [2]$



$g \circ f : [0] \to [2]$

Cubical Sets

as presheaves of category of cubes



Cubical Sets

as presheaves over category of cubes