Cubical Agda Explore

Week 6, Spring 2023

Chenchao Ding, Feb 19

Path

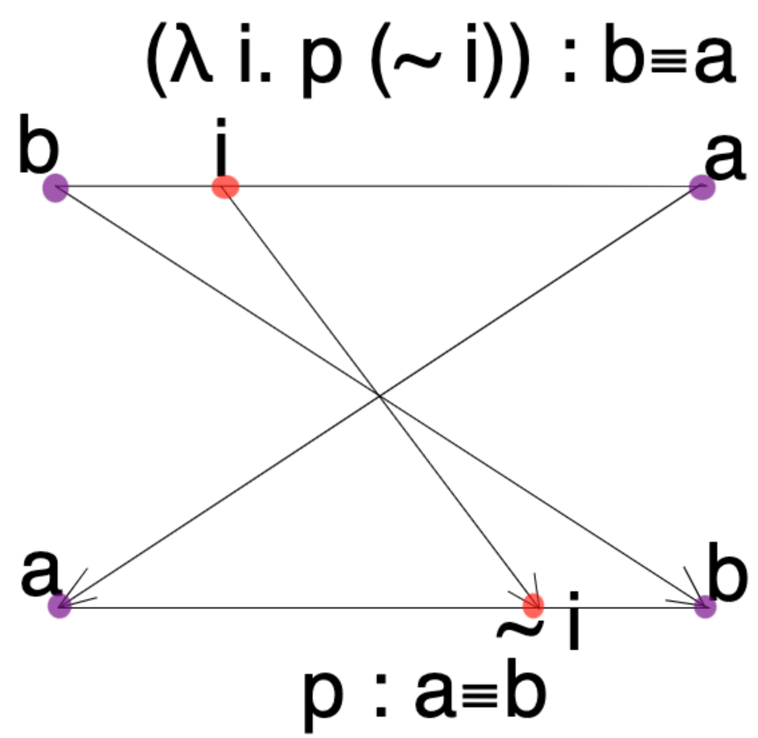
Reflexivity

```
refl' : (a : A) → Path A a a
refl' a = λ _ → a
```

Path

Symmetricity

```
!'_ : ∀ {ℓ} {A : Type ℓ} {a b : A} → a ≡ b → b ≡ a
!'_ {ℓ}{A}{a}{b} p = λ i → p (~ i)
```



Path

Symmetricity

```
!_ : ∀ {ℓ} {A : Type ℓ} {a b : A} → a ≡ b → b ≡ a
!_ {ℓ}{A}{a}{b} p i = hcomp walls a
  where
    walls : ∀ (j : I) → Partial (~ i ∨ i) A
    walls j (i = i0) = p j
    walls j (i = i1) = a
```

Path

Symmetricity

```
coe0→1 : ∀ {ℓ} (A : I → Type ℓ) → A i0 → A i1
coe0→1 A a = transp (λ i → A i) i0 a

!''_ : ∀ {ℓ} {A : Type ℓ} {a b : A} → a ≡ b → b ≡ a
!''_ {ℓ}{A}{a}{b} p = coe0→1 (λ i → p i ≡ a) refl
```

Path

Transitivity

```
compPath : ∀ {ℓ} {A : Type ℓ} {a b c : A} → a ≡ b → b ≡ c → a ≡ c
compPath {ℓ}{A}{a}{b}{c} p q i = hcomp walls (p i)
  where
    walls : ∀ (j : I) → Partial (~ i ∨ i) A
    walls j (i = i0) = a
    walls j (i = i1) = q j
```

Path

Transitivity

```
_••_••_ : ∀ {ℓ} {A : Type ℓ} {x y z w : A} → x ≡ y → y ≡ z → z ≡ w → x ≡ w
_••_••_ {ℓ}{A}{x}{y}{z}{w} p q r i = hcomp walls (q i)
  where
    walls : ∀ (j : I) → Partial (~ i ∨ i) A
    walls j (i = i0) = (! p) j  -- or p (~ j)
    walls j (i = i1) = r j
```

Path

Transitivity

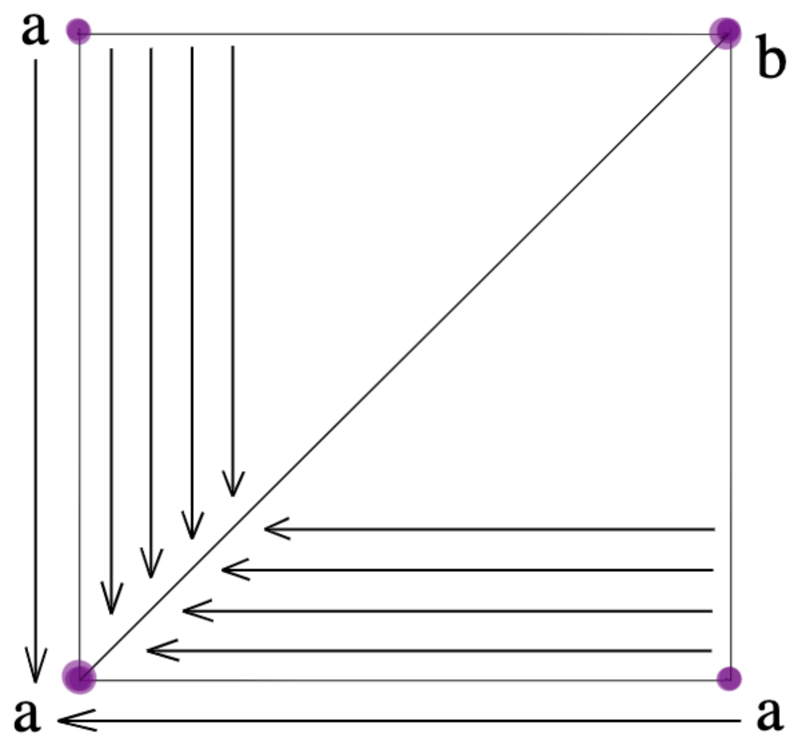
```
compPath1 : ∀ {ℓ} {A : Type ℓ} {a b c : A} → a ≡ b → b ≡ c → a ≡ c
compPath1 {ℓ}{A}{a}{b}{c} p q = refl ∙∙ p ∙∙ q

compPath2 : ∀ {ℓ} {A : Type ℓ} {a b c : A} → a ≡ b → b ≡ c → a ≡ c
compPath2 {ℓ}{A}{a}{b}{c} p q = p ∙∙ refl ∙∙ q

compPath3 : ∀ {ℓ} {A : Type ℓ} {a b c : A} → a ≡ b → b ≡ c → a ≡ c
compPath3 {ℓ}{A}{a}{b}{c} p q = p ∙∙ q ∙∙ refl
```

Weak connections

Meet



Weak connections

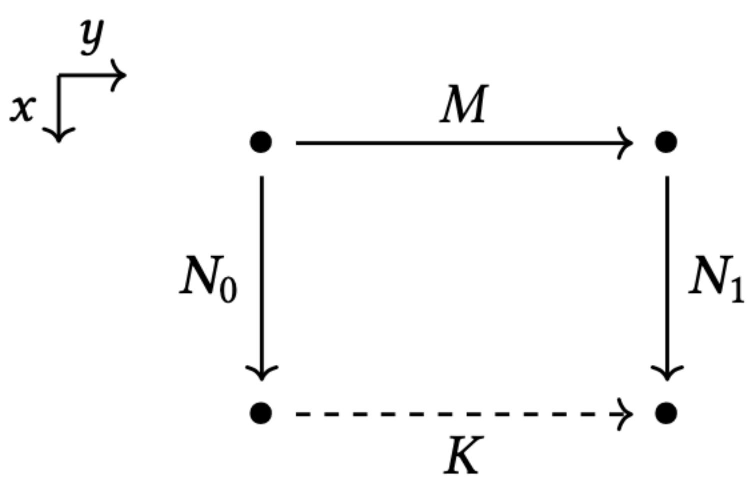
Join

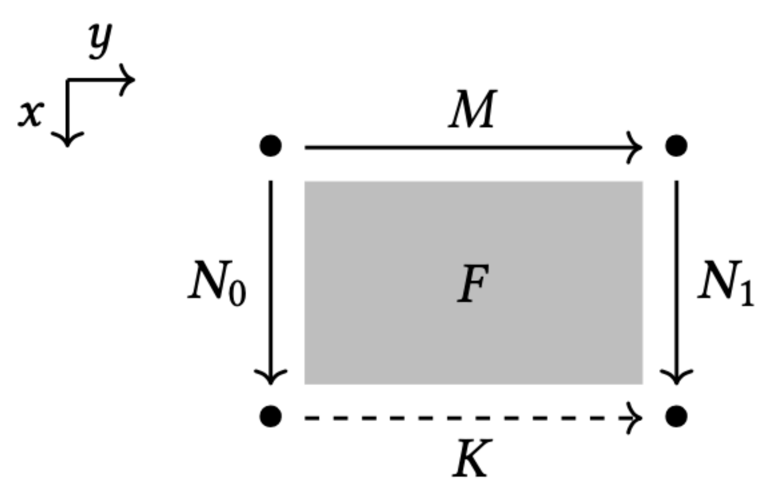
The groupoid laws

Right unit

The groupoid laws

hcomp and hfill





$$y : \mathbb{I} \gg K \coloneqq \mathsf{hcom}_A^{0 \to 1}(M; y \equiv 0 \hookrightarrow x.N_0, y \equiv 1 \hookrightarrow x.N_1) \in A$$

$$x : \mathbb{I}, y : \mathbb{I} \gg F \coloneqq \mathsf{hcom}_A^{0 \to x}(M; y \equiv 0 \hookrightarrow x.N_0, y \equiv 1 \hookrightarrow x.N_1) \in A$$

The groupoid laws

Left unit

The groupoid laws

Right cancel

The groupoid laws

Left cancel

The groupoid laws

Involution

The groupoid laws

Associativity

The groupoid laws…

For path of types