

ThesisTitle

by

Daniel C. Cole

Other Degrees

A thesis submitted to the
Faculty of the Graduate School of the
University of Colorado in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Physics Physics
2018

This thesis entitled:
ThesisTitle
written by Daniel C. Cole
has been approved for the Physics Physics

Reader1

Reader2

Date _____

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Cole, Daniel C. (Ph.D., Physics)

ThesisTitle

Thesis directed by Dr. Scott A. Diddams

Optical frequency combs have revolutionized precision metrology by enabling measurements of optical frequencies, with implications both for fundamental scientific questions and for applications such as fast, broadband spectroscopy. In this thesis, I describe the development of comb generation platforms with smaller footprints and higher repetition rates, with the ultimate goal of bringing frequency combs to new applications in a chip-integrated package. I present two new types of frequency combs: electro-optic modulation (EOM) combs and Kerr-microresonator-based frequency combs (microcombs). First I describe the EOM comb scheme and, in particular, techniques for mitigating noise in the comb generation process, and I present the results of a proof-of-principle metrology experiment and some possible applications. Then I discuss developments in microcomb technology. I present novel soliton crystal states, which have highly structured fingerprint optical spectra that correspond to ordered pulse trains exhibiting crystallographic defects. These pulse trains arise through interaction of the solitons with avoided mode-crossings in the resonator spectrum. Next, I describe the direct and deterministic generation of single microresonator solitons using a phase-modulated pump laser. This technique removes the dependence on initial conditions that was formerly a universal feature of these experiments, presenting a solution to a significant technical barrier to the practical application of microcombs. I also discuss generation of Kerr combs in the Fabry-Perot (FP) geometry. I introduce a nonlinear partial differential equation describing dynamics in an FP cavity and discuss the differences between the FP geometry and the ring cavity, which is the geometry used in previous Kerr-comb experiments. Finally, I discuss a technique for reducing the repetition rate of a high-repetition-rate frequency comb, which will be a necessary post-processing step for some applications. I conclude with a discussion of avenues for future research, including the chip-integration of Fabry-Perot Kerr resonators and the use of band-engineered photonic crystal cavities to further simplify soliton generation.

Acknowledgements

The work in this thesis would not have been possible...

- Acknowledgement line 1
- Acknowledgement line 2

Contents

1	Microresonators	1
1.1	Nonlinear optics in microresonators	3

Figures

Chapter 1

Microresonators

This chapter introduces the basic physics of Kerr-nonlinear optical ring resonators, and the two subsequent chapters describe results obtained in these systems.

correct?

An optical ring resonator, shown schematically in Fig. ??, guides light around a closed path in a dielectric medium by total internal reflection, similar to the mechanism that guides light in an optical fiber. A ring resonator supports propagating guided optical *modes* of electromagnetic radiation that occur at (vacuum) wavelengths that evenly divide the optical round-trip path length: $\lambda_m = n_{eff}(\lambda_m)L/m$, with associated resonance frequencies $\nu_m = c/\lambda_m = mc/n_{eff}(\nu_m)L$, leading to constructive interference from round-trip to round-trip. Here L is the physical round-trip length of the resonator, m is the azimuthal mode number, and $n_{eff}(\lambda_m)$ is an effective index of refraction that depends on the resonator geometry and the mode's transverse mode profile (see e.g. [REFHERE] for more information). The free-spectral range f_{FSR} of a resonator is the *local* frequency spacing between modes, calculated via:

$$f_{FSR} \approx \nu_{m+1} - \nu_m \approx \nu_m - \nu_{m-1}, \quad (1.1)$$

$$= \frac{\partial \nu_m}{\partial m}, \quad (1.2)$$

$$= \frac{c}{n_{eff}(\nu)L} - \frac{mc}{n_{eff}^2(\nu)L} \frac{\partial n}{\partial \nu} \frac{\partial \nu}{\partial m}, \quad (1.3)$$

$$\Rightarrow f_{FSR} = \frac{c/L}{\left(n + \frac{\nu}{n} \frac{\partial n}{\partial \nu}\right)} = \frac{c}{n_g L} = 1/T_{RT}, \quad (1.4)$$

where $n_g = n + \frac{\nu}{n} \frac{\partial n}{\partial \nu}$ is the group velocity of the mode and T_{RT} is the mode's round-trip time.

Unless special efforts are made, ring resonators are typically multi-mode, meaning that many

different transverse mode profiles are supported. To calculate the frequency-dependent effective index $n_{eff}(\nu)$, thereby enabling calculation of the resonance frequencies and wavelengths, one must solve Maxwell's equations for the resonator geometry. Except in special cases of high symmetry [**microsphereresonators**], this is typically done numerically using finite-element modeling tools like COMSOL. The modes of an optical resonator, both within a mode family defined by a transverse mode profile and between mode families, must be orthogonal.

The timescale over which circulating photons are dissipated in a resonator is fundamental to its fitness for applications. This is quantified by the basic relation for the number of circulating photons $N(t) = N_o e^{-t/\tau_\gamma}$ in the presence of solely linear loss, which defines the photon lifetime τ_γ . Two commonly used practical quantities are linked to the photon lifetime: the resonator finesse $\mathcal{F} = 2\pi\tau_\gamma/T_{RT}$, which for a ring resonator can be interpreted literally as the azimuthal resonator angle traced out by a typical photon over its lifetime; and the resonator quality factor $Q = \omega_c\tau_\gamma$, the phase over which the optical field evolves during the photon lifetime. The lifetime of a photon at a particular frequency is related to the cavity's full-width at half-maximum (FWHM) linewidth as we can calculate through a Fourier transform of the field $E(t) \propto \sqrt{N(t)}$ with angular carrier frequency ω_c :

$$\mathcal{F}\{E\}(\omega) \propto \int_0^\infty dt e^{-\left(\frac{1}{2\tau_\gamma} + i(\omega_c - \omega)\right)t}, \quad (1.5)$$

which immediately yields the Lorentzian lineshape

$$|\mathcal{F}\{E\}|^2 \propto \frac{1}{(\omega - \omega_c)^2 + \frac{1}{4\tau_\gamma^2}}, \quad (1.6)$$

with FWHM linewidth $\Delta\omega = 1/\tau_\gamma$. With this relationship, the finesse and quality factor can be rewritten as simple ratios of the relevant frequencies: $\mathcal{F} = f_{FSR}/\Delta\nu$; $Q = \nu_c/\Delta\nu$, where $\Delta\nu = \Delta\omega/2\pi$.

is there a
good cita-
tion here?

1.1 Nonlinear optics in microresonators

Resonators with long photon lifetimes can support very high circulating optical intensities, and this fact finds immediate application in the use of microresonators for nonlinear optics. The experiments described in this thesis are conducted in silica microresonators. Silica falls into a broader class of materials that exhibit both centro-symmetry, which dictates that the second-order nonlinear susceptibility $\chi^{(2)}$ must vanish, and a significant third-order susceptibility $\chi^{(3)}$. The n^{th} -order susceptibility is a term in the Taylor expansion describing the response of the medium's polarization to an external electric field: $P = P_0 + \epsilon_0\chi^{(1)}E + \epsilon_0\chi^{(2)}E^2 + \epsilon_0\chi^{(3)}E^3 + \dots$. The effect of $\chi^{(3)}$ can be described in a straightforward way as a dependence of the refractive index on the local intensity, $n = n_0 + n_2I$ [somethingelse], where $n_2 = \frac{3\chi^{(3)}}{4n_0^2\epsilon_0c}$ [CosoAndSolis, alsoLLbook?]. The intensity-dependence of the refractive index is referred to as the optical Kerr effect.

The combination of the Kerr effect and the high circulating intensities that are accessible in high-finesse cavities provides a powerful platform for nonlinear optics. Specifically, the Kerr effect (or third-order susceptibility $\chi^{(3)}$) enables self-phase modulation, , and four-wave mixing. others

that exhibit the nonlinearity, which can be the dominant source of nonlinearity in centro-symmetric materials in which the second-order nonlinearity $\chi^{(2)}$ must vanish. The $\chi^{(3)}$ nonlinearity, or Kerr nonlinearity, arises from the term in the Taylor expansion for the polarization response of the medium that scales with the optical intensity: $P_{NL} = \dots + \epsilon_0\chi^{(3)}E^3 + \dots$ [something]. This can equivalently and usefully be described as a dependence of the refractive index on the intensity via .

The third-order

The resonator quality factor is an important figure of merit for the use of optical resonators as platforms for nonlinear optics. This is discussed extensively below; typically the threshold power for nonlinear optics scales as Q^{-2} , meaning that in the design of practical platforms targeting high Q is an important consideration. Fabrication of ultrahigh- Q resonators has been achieved with a variety of designs and materials, including ...

These resonators can be constructed with extremely high quality factors, upwards of 10^8 , which facilitates high circulating intensity.

Combs using optical ring resonators leverage this confinement and the resulting long photon lifetimes in high-quality factor resonators