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Optical frequency combs have revolutionized precision metrology by enabling measurements of optical frequencies, with implications both for fundamental scientific questions and for applications such as fast, broadband spectroscopy. In this thesis, I describe the development of comb generation platforms with smaller footprints and higher repetition rates, with the ultimate goal of bringing frequency combs to new applications in a chip-integrated package. I present two new types of frequency combs: electro-optic modulation (EOM) combs and Kerr-microresonator-based frequency combs (microcombs). First I describe the EOM comb scheme and, in particular, techniques for mitigating noise in the comb generation process, and I present the results of a proof-of-principle metrology experiment and some possible applications. Then I discuss developments in microcomb technology. I present novel soliton crystal states, which have highly structured fingerprint optical spectra that correspond to ordered pulse trains exhibiting crystallographic defects. These pulse trains arise through interaction of the solitons with avoided mode-crossings in the resonator spectrum. Next, I describe the direct and deterministic generation of single microresonator solitons using a phase-modulated pump laser. This technique removes the dependence on initial conditions that was formerly a universal feature of these experiments, presenting a solution to a significant technical barrier to the practical application of microcombs. I also discuss generation of Kerr combs in the Fabry-Perot (FP) geometry. I introduce a nonlinear partial differential equation describing dynamics in an FP cavity and discuss the differences between the FP geometry and the ring cavity, which is the geometry used in previous Kerr-comb experiments. Finally, I discuss a technique for reducing the repetition rate of a high-repetition-rate frequency comb, which will be a necessary post-processing step for some applications. I conclude with a discussion of avenues for future research, including the chip-integration of Fabry-Perot Kerr resonators and the use of band-engineered photonic crystal cavities to further simplify soliton generation.

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Chapter 1

PM Pumping

This chapter discusses the direct generation and control of single solitons in optical microring resonators using a pump-laser phase modulated at a frequency near the resonator's free spectral range. Based on a proposal by Taheri, Eftekhar, and Adibi in 2015 [42], these experimental results represent a promising new method for simple and deterministic generation of single solitons.

To illustrate the utility of using a phase-modulated pump laser, we first present theoretical investigations into the effect of this phase modulation (PM), and then we present experimental results on the generation and control of single solitons.

1.1 Theoretical investigation of soliton generation with a phase-modulated pump laser

To theoretically explore the physics of soliton generation with PM pumping, we use the LLE with a modified driving term that incorporates the effect of phase modulation [42]:

$$\frac{\partial \psi}{\partial \tau} = -(1+i\alpha)\psi + i|\psi|^2\psi - i\frac{\beta}{2}\frac{\partial^2 \psi}{\partial \theta^2} + Fe^{i\delta_{PM}\cos\theta}. \tag{1.1}$$

Here δ_{PM} represents the phase-modulation index, where the resonator is driven by a field $E_{PM} = E_0 e^{i\delta_{PM}\cos(2\pi f_{PM} t)}$; $f_{PM} \sim f_{FSR}$ is the frequency of the applied phase modulation.

Simulations of Eq.1.1 reveal that PM transforms the resonator excitation spectrum from a series of $N = 0, 1, 2, ...$ up to N_{max} solitons to a single level $N = 1$ near threshold, eliminating degeneracy between these states as shown in Fig. **??**. This occurs due to amplitude variations resulting from the phase modulation, with dispersion and nonlinearity providing PM-to-AM conversion. We can gain some insight into the origin of this effect by inserting the ansatz $\psi(\theta,\tau) = \phi(\theta,\tau)e^{i\delta_{PM}\cos(\theta)}$ into Eq. 1.1 [40]. By expanding the second-derivative term and setting derivatives of ϕ to zero[1] we arrive at an equation for the quasi-CW background in the PM-pumped resonator:

$$F = (\gamma(\theta) + i\eta(\theta))\,\phi + i|\phi|^2\phi, \tag{1.2}$$

where effective local loss and detuning terms have been defined as:

$$\gamma(\theta) = 1 + \frac{\beta_2}{2}\delta_{PM}\cos\theta, \tag{1.3}$$

$$\eta(\theta) = \alpha - \frac{\beta_2}{2}\delta_{PM}^2\sin^2\theta. \tag{1.4}$$

This equation immediately yields an approximation for the stationary solution ψ_s:

$$\psi_s = \frac{Fe^{i\delta_{PM}\cos\theta}}{\gamma(\theta) + i\left(\eta(\theta) - \rho(\theta)\right)}, \tag{1.5}$$

where $\rho(\theta) = |\phi(\theta)|^2$ is the (smallest real) solution to the cubic polynomial that results from taking the modulus-square of Eq. 1.2:

$$F^2 = \left[\gamma(\theta)^2 + \left(\eta(\theta) - \rho(\theta)\right)^2\right]\rho(\theta). \tag{1.6}$$

In neglecting spatial derivatives of ϕ but retaining the derivatives of the phase term $e^{i\delta_{PM}\cos\theta}$ we have made the approximation that the dominant effect of dispersion comes from its action on the existing broadband phase-modulation spectrum. This model reveals that amplitude variations in the quasi-CW background can be expected as a result of the spatially-varying effective loss and detuning terms that arise from the periodically-chirped pump laser.

Fig. **??** compares the predictions of numerical LLE simulations (color) with the analytical model (black). The two agree quantitatively at small modulation depth ($\delta_{PM} = \pi/2$, blue) and qualitatively at larger depth ($\delta_{PM} = 4\pi$, green). Both the simulations and the approximate analytic solution indicate that the background has two peaks per round trip in the presence of phase modulation, which suggests a mechanism for spontaneous single-soliton generation: At threshold the larger peak

[1] We note the contribution of Miro Erkintalo in suggesting this approximation.

becomes locally unstable, and a soliton is formed by local modulation instability [**Ceoldo2016**, 41]. Moreover, it is known that if solitons exist elsewhere they are pushed to the larger peak by the backgrounds modulated phase [40]. This makes superpositions of $N > 1$ solitons unstable and practically forbidden. Generation of single solitons then simply requires tuning the pump power and frequency to appropriate values, regardless of initial conditions.

The detuning for soliton generation can be estimated using Eq. 1.2 by calculating the value of α where $\rho(\theta = 0) = 1$. This comes with a further approximation, as simulations reveal that the critical detuning for soliton formation is near but not necessarily at $\rho = 1$ because the spatial interval over which threshold is exceeded must have some minimum width. However, this approach quantitatively captures the behavior shown in Fig. **??**, predicting soliton generation at $\alpha = 2.737$.

1.2 Spontaneous generation of single solitons using a phase-modulated pump laser

We demonstrate deterministic generation of single solitons without condensation from an extended pattern in a 22-GHz FSR silica ring resonator with $\Delta\nu \sim 1.5$ MHz linewidth [**Lee2012**]. NPhot 6, 369
We generate a frequency-agile laser for pumping the resonator by passing a CW laser through a single-sideband modulator that is driven by a voltage-controlled oscillator [**Stone2017**]; the seed laser is extinguished in the modulator and the resulting sideband can be swept by . The pump laser is phase-modulated with index $\delta_{PM} \sim \pi$ and amplified to normalized power F^2 between 2 and 6. rate and description here
We are able to measure and control the pump-laser detuning in real time using an AOM-shifted probe beam as shown in Fig. **??**, which allows thermal instabilities associated with the red detuning that is required for soliton generation to be overcome. To generate single solitons, we begin with large red detuning $\nu_0 - \nu_{pump}$ =40 MHz and decrease the detuning until a soliton is generated near $\nu_0 - \nu_{pump}$ ~5 MHz detuning, where this value depends on the pump power and coupling condition. We measure the power converted by the Kerr nonlinearity to new frequencies by passing a portion of the resonator's output through an optical band-reject filter; this 'comb power' measurement reveals a step upon soliton formation, as shown in Fig. **??**. After soliton generation, we observe that the

soliton can be preserved while the detuning is increased again up to a maximum value near , consistent with Fig. **??**. Additionally, we observe that it is possible to turn off the phase modulation without loss of the soliton, in agreement with the simulations presented in Ref. [42].

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Automating soliton generation by repeatedly scanning the laser into resonance ($\nu_0 - \nu_{pump}$ ∼5 MHz) and back out again ($\nu_0 - \nu_{pump}$ ∼20 MHz, far enough that the soliton is lost) has enabled reversible generation of 1000 solitons in 1000 trials over 100 seconds, with a measured 100 % success rate. Our probe beam allows measurement of the detuning at which soliton generation occurs, which changes little from run to run. We present a histogram of detuning measurements for the generation of 160 solitons in Fig. **??**.

1.3 Soliton control using a phase-modulated pump laser

In addition to enabling deterministic generation of single solitons, phase modulation of the pump laser also facilitates timing and repetition-rate control of the resulting pulse train. Results demonstrating this control are presenting in Fig. **??**. In our experiments, the repetition rate of the out-coupled pulse train (f_{rep}) remains locked to f_{PM} over a bandwidth of ±40 kHz. This observation is consistent with an estimate of the locking range $\delta_{PM} \times D_2/2\pi$ ∼44 kHz that is presented in Ref. [40], where we have used the approximate value D_2 =14 kHz/mode. Fig. **??** shows the measured repetition rate as f_{PM} is swept sinusoidally through a range of ±50 kHz around the soliton's natural repetition rate; the repetition rate follows the PM except for glitches near the peaks of the sweep. In the inset of Fig. **??** we overlay the results of LLE simulations that qualitatively match the observed behavior. These simulations are conducted by introducing the term $+\beta_1 \frac{\partial \psi}{\partial \theta}$ to the right-hand side of Eq. 1.1, where $\beta_1 = -2(f_{FSR} - f_{PM})/\Delta\nu$ incorporates a difference between the modulation frequency and the FSR of the resonator into the model; β_1 may be varied in time to simulate the sweep of f_{PM}. These simulations indicate that the periodic nature of the glitches is due to the residual pulling of the phase modulation on the soliton when the latter periodically cycles through the pumps phase maximum.

To evaluate the utility of phase modulation for fast control of the soliton's properties, we measure the repetition rate of the pulse train as f_{PM} is rapidly switched ± 40 kHz, within the soliton's locking range. This measurement is conducted by photodetecting the pulse train after removing the central spectral lines corresponding to the spectrum of the pump laser using an optical band-reject filter. In order to obtain a measurement trace of the repetition rate as a function of time, the photodetected signal is split and one path is sent through a reactive circuit element that induces a frequency-dependent phase shift. By comparing the phase between the two paths as a function of time, the time-dependent repetition rate can be determined.

We construct eye-diagrams out of the resulting data; these are shown in Fig. **??**. In Fig. **??**,m f_{PM} is switched with 200 μs period and 10 μs transition time; in Fig. **??** it is switched with 100 μs period and 60 ns transition time.

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