# Beyond modelocking: High repetition-rate frequency combs derived from a continuous-wave laser

by

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Optical frequency combs based on modelocked lasers have revolutionized precision metrology by facilitating measurements of optical frequencies, with implications both for fundamental scientific questions and for applications such as fast, broadband spectroscopy. In this thesis, I describe advances in the generation of frequency combs without modelocking in platforms with smaller footprints and higher repetition rates, with the ultimate goal of bringing frequency combs to new applications in a chip-integrated package. I discuss two approaches for comb generation: parametric frequency conversion in Kerr microresonators and active electro-optic modulation of a continuous-wave laser. After introducing microresonator-based frequency combs (microcombs), I discuss two specific developments in microcomb technology: First, I describe a new, extremely reliable method for generation of soliton pulses through the use of a phase-modulated pump laser. This technique eliminates the dependence on initial conditions that was formerly a universal feature of these experiments, presenting a solution to a significant technical barrier to the practical application of microcombs. Second, I present observations of *soliton crystal* states with highly structured 'fingerprint' optical spectra that correspond to ordered pulse trains exhibiting crystallographic defects. These pulse trains arise through interaction of solitons with avoided mode-crossings in the resonator spectrum. I also discuss generation of Kerr soliton combs in the Fabry-Perot (FP) geometry, with a focus on the differences between the FP geometry and the ring geometry that has been the choice of most experimenters to date. Next, I discuss combs based on electro-optic modulation. I introduce the operational principle, and then describe the first self-referencing of a frequency comb of this kind and a proof-of-principle application experiment. Finally, I discuss a technique for reducing the repetition rate of a high repetition-rate frequency comb, which will be a necessary post-processing step for some applications.

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# Appendix A

## Derivation of the Lugiato-Lefever equation from the nonlinear Schrodinger equation

Here we show how the Lugiato-Lefever equation can be obtained by modeling propagation in a high-finesse ring cavity with the nonlinear Schrodinger equation and periodically applying an operator that implements in-coupling and out-coupling, including the effects of the round-trip phase shift associated with the detuning of the pump laser from a cavity mode. To my knowledge, the derivation given here was first performed by Haelterman, Trillo, and Wabnitz [57]. We use the NLSE for a pulse of restricted bandwidth such that higher-order nonlinearities are unimportant, but this derivation may also be carried out using a generalized nonlinear Schrodinger equation to include higher-order effects (e.g. Raman and self-steepening) in the LLE. Our equation is:

$$\frac{\partial A}{\partial z} = -\frac{\alpha_\ell}{2}A + i\gamma|A|^2A - i\frac{k''}{2}\frac{\partial^2 A}{\partial T^2}. \tag{A.1}$$

This equation is ubiquitous in the study of pulse propagation in Kerr-nonlinear media, and a derivation of it is provided, for example, in Ref. [39]. As discussed in Sec. **??**, it describes the evolution of a pulse envelope $A$ in a 'fast-time' reference frame parametrized by $T$ as it propagates in a Kerr-nonlinear medium, where the propagation distance is parametrized by the variable $z$. ere $\gamma = \frac{2\pi}{\lambda}\frac{n_2}{A_{eff}}$ is the nonlinear coefficient of the medium, where $n_2$ is the Kerr index, $A_{eff}$ is the effective nonlinear mode area, $\lambda$ the wavelength of the carrier wave, and $k'' = \frac{\partial^2}{\partial\omega^2}\frac{n_{eff}(\omega)\omega}{c}$ is the GVD parameter. Propagation loss described by the coefficient $\alpha_\ell$ has been included in Eq. A.1, with $\alpha_\ell$ the loss coefficient in power (i.e. $\partial P/\partial z = -\alpha_\ell P$, where $P = |A|^2$).

The dynamics in a ring resonator constructed of a Kerr medium can be described by evolving the field envelope $A$ over a round trip of length $L$ and then applying an operator that accounts for out-coupling of the circulating field $A$ and in-coupling of a pump field $A_{in}$, as well as a round-trip phase shift $\phi_{RT}$ associated with the detuning of the carrier frequency from a cavity mode. This allows us to advance the field at the end of the $n^{\text{th}}$ round trip $A_n(L,T)$ to the field $A_{n+1}(0,T)$ at the beginning of the $n+1^{\text{th}}$ as:

$$A_{n+1}(0,T) = e^{i\phi_{RT}} \left(1 - \frac{T_{RT}}{2\tau_{ext}}\right) A_n(L,T) + \sqrt{\frac{T_{RT}}{\tau_{ext}}} A_{in}, \tag{A.2}$$

where $\tau_{ext}$ describes in- and out-coupling as explained in Sec. **??** and $T_{RT} = L/v_g$ is the round-trip time. If we define an operator $G_L(A)$ that advances the field over a distance $L$ according to Eq. A.1 as $A(z+L,T) = G_L(A)A(z,T)$, then we have:

$$A_{n+1}(0,T) = e^{i\phi_{RT}} \left(1 - \frac{T_{RT}}{2\tau_{ext}}\right) G_L\left[A_n(0,T)\right] A_n(0,T) + \sqrt{\frac{T_{RT}}{\tau_{ext}}} A_{in}. \tag{A.3}$$

The description of the field envelope in a Kerr-nonlinear ring cavity according to Eq. A.3 through iterated evolution according to the NLSE and then application of the in- and out-coupling operator is referred to as an Ikeda map [148]. We obtain the LLE by assuming that the operator $G_L(A) \approx 1$, that is, that the field does not evolve much over the round-trip length. This is equivalent to the assumption that the cavity length $L$ is much less than the loss, nonlinear, and dispersion length scales $L_\ell = 1/\alpha_\ell$, $L_{NL} = 1/\gamma P_0$, and $L_D = T_0^2/|k''|$ over which the terms on the right-hand side of Eq. A.1 lead to appreciable evolution of the pulse envelope [39]. Here $P_0$ and $T_0$ are the peak power and temporal width, respectively, of a localized excitation in the pulse envelope $A$.

We can write the operator $G_L$ explicitly as:

$$G_L(A) = \left[1 + L\left(-\alpha_\ell/2 + i\gamma|A|^2 - i\frac{k''}{2}\frac{\partial^2}{\partial T^2}\right)\right]. \tag{A.4}$$

We assume that each term in this operator besides the identity term is small. If we note that the round-trip phase shift $\phi_{RT}$ must also be small in a high-finesse cavity for appreciable build-up to occur, then we can expand the first term on the right-hand side of Eq. A.3 and retain only first order terms to find:

$$A_{n+1}(0,T) = \left(1 - \frac{T_{RT}}{2\tau_{ext}} + i\phi_{RT} - \frac{L\alpha_\ell}{2} + iL\gamma|A|^2 - iL\frac{k''}{2}\frac{\partial^2}{\partial T^2}\right) A_n(0,T) + \sqrt{\frac{T_{RT}}{\tau_{ext}}} A_{in}. \quad \text{(A.5)}$$

By replacing $n$ with the slow time $t = nT_{RT}$ and allowing $t$ to vary continuously we arrive at a Lugiato-Lefever equation, albeit in a different form from the one presented in Eq. **??**:

$$T_{RT}\frac{\partial A}{\partial t} = \left(-T_{RT}/2\tau_{ext} + i\phi_{RT} - L\alpha_\ell/2 + iL\gamma|A|^2 - iL\frac{k''}{2}\frac{\partial^2}{\partial T^2}\right) A + \sqrt{\frac{T_{RT}}{\tau_{ext}}} A_{in}. \quad \text{(A.6)}$$

To recast this equation in the standard form used in the body of this thesis, we first pass to the normalized temporal and spatial variables $\tau$ and $\theta$ and the parameters $\alpha$ and $\beta_2$. We note that $L\alpha_\ell/2 = T_{RT}/2\tau_{int}$ (as each describes the intrinsic loss over one round trip), and we define $\theta = 2\pi T/T_{RT}$, so that $\frac{\partial^2}{\partial T^2} = \left(\frac{2\pi}{T_{RT}}\right)^2 \frac{\partial^2}{\partial \theta^2}$. We divide by $T_{RT}$ and obtain:

$$\frac{\partial A}{\partial t} = -\frac{\Delta\omega}{2}A + \frac{1}{T_{RT}}\left(i\phi_{RT} + iL\gamma|A|^2\right) A - i\frac{L}{T_{RT}}\left(\frac{2\pi}{T_{RT}}\right)^2 \frac{k''}{2}\frac{\partial^2 A}{\partial\theta^2} + A_{in}/\sqrt{T_{RT}\tau_{ext}} \quad \text{(A.7)}$$

Dividing this by $\Delta\omega/2$ brings us to the normalized temporal variable $\tau = t/2\tau_{ph} = \Delta\omega t/2$. The quantity $\phi_{RT}/T_{RT}$ is exactly the frequency detuning $\sigma = \omega_p - \omega_0$ between the pump laser and the cavity resonance frequency, so that the quantity $2\phi_{RT}/T_{RT}\Delta\omega$ that results from division by $\Delta\omega/2$ is simply equal to the normalized detuning term $-\alpha = 2\sigma/\Delta\omega$. Further, recalling from Sec. **??** that $D_1 = 2\pi/T_{RT} = 2\pi v_g/L$ and $D_2 = -D_1^2 v_g k''$, we have:

$$\frac{L}{T_{RT}}\left(\frac{2\pi}{T_{RT}}\right)^2 \frac{k''}{2} = -D_2/2. \quad \text{(A.8)}$$

Using the definition of the normalized dispersion for the LLE $\beta_2 = -2D_2/\Delta\omega$ and combining these relations, we have:

$$\frac{\partial A}{\partial \tau} = -(1+i\alpha)A + i\frac{2L\gamma}{T_{RT}\Delta\omega}|A|^2 A - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial\theta^2} + \sqrt{\frac{4\Delta\omega_{ext}}{T_{RT}\Delta\omega^2}} A_{in}, \quad \text{(A.9)}$$

where we recall the definition $\Delta\omega_{ext} = 1/\tau_{ext}$. By defining $\psi = \sqrt{\frac{2L\gamma}{T_{RT}\Delta\omega}} A$, we arrive at the LLE as presented in Eq. **??**:

$$\frac{\partial\psi}{\partial\tau} = -(1+i\alpha)\psi + i|\psi|^2\psi - i\frac{\beta_2}{2}\frac{\partial^2\psi}{\partial\theta^2} + F. \quad \text{(A.10)}$$

Here the pump term has been normalized as:

$$F = A_{in}\sqrt{\frac{8L\gamma\Delta\omega_{ext}}{T_{RT}^2\Delta\omega^3}} \tag{A.11}$$

$$= A_{in}\sqrt{\frac{8g_0\Delta\omega_{ext}}{\Delta\omega^3}\frac{1}{\hbar\omega_p}}, \tag{A.12}$$

with

$$g_0 = n_2 c\hbar\omega_p^2/n_g^2 V_0 \tag{A.13}$$

as defined in Sec. **??**, where $V_0 = LA_{eff}$. Assuming that $A_{in}$ is real simply fixes the phase of $\psi$, which is otherwise arbitrary. With this assumption we have $A_{in} = \sqrt{P_{in}}$, and we recover the normalization from Sec. **??**:

$$F = \sqrt{\frac{8g_0\Delta\omega_{ext}}{\Delta\omega^3}\frac{P_{in}}{\hbar\omega_p}}. \tag{A.14}$$

## A.1 ***A posteriori*** confirmation that $L_D \gg L$ and $L_{NL} \gg L$ for LLE solitons

The analytical approximation to the soliton solution of the LLE presented in Eq. **??** has $\theta$-width $\sqrt{\frac{-\beta}{2\alpha}}$, and therefore temporal width $T_0 = \frac{T_{RT}}{2\pi}\sqrt{\frac{-\beta}{2\alpha}}$, and has approximate peak power $2\alpha$. Using the expressions presented above, one can calculate that for an LLE soliton the nonlinear length $L_{NL} = 1/\gamma P_0$ and the dispersion length $L_D = T_0/|k''|$ are the same, and depend on the detuning $\alpha$:

$$L_{NL} = L_D = \frac{v_g\tau_{ph}}{\alpha}. \tag{A.15}$$

Using the definition of the cavity finesse $\mathcal{F} = 2\pi\tau_{ph}/T_{RT}$, we find that the ratio of these characteristic lengths to the cavity length is:

$$\frac{L_{NL}}{L} = \frac{L_D}{L} = \frac{\mathcal{F}}{2\pi\alpha}, \tag{A.16}$$

and therefore that the assumption $G_L \approx 1$ clearly holds for LLE solitons, since $\mathcal{F} \gg 1$ for a high-finesse cavity and $\alpha$ is of order 1 or 10.

## A.2 Normalization of the Ikeda map

For direct comparison of numerical results between the Ikeda map and the LLE, and also possibly for future work involving simulations of Kerr-combs outside of the high-finesse limit, it is useful to normalize the Ikeda map given in Eq. A.3 in the same way that the LLE is normalized. To conduct an Ikeda map calculation that corresponds to the LLE with parameters $\alpha$, $\beta_2$, and $F$, it is natural to describe the Ikeda-map system by specifying two additional parameters: the resonator finesse $\mathcal{F}$ and the coupling ratio $\eta = \Delta\omega_{ext}/\Delta\omega$. Then the following representations of the operators in Eq. A.3 can be used:

$$1 - \frac{T_{RT}}{2\tau_{ext}} = 1 - \frac{\pi\eta}{\mathcal{F}}, \tag{A.17}$$

$$\sqrt{\frac{T_{RT}}{\tau_{ext}}} = \sqrt{\frac{2\pi\eta}{\mathcal{F}}}, \tag{A.18}$$

$$e^{i\phi_{RT}} = e^{-i\frac{\pi}{\mathcal{F}}\alpha}, \tag{A.19}$$

$$\frac{L\alpha_\ell}{2} = \frac{T_{RT}}{2\tau_{int}} = \frac{\pi(1-\eta)}{\mathcal{F}}. \tag{A.20}$$

Using a normalized distance $s = z \cdot \pi/\mathcal{F}L$ we define an operator $G_1$ that advances the field $\psi(s,\theta)$ a distance $\Delta s = \pi/\mathcal{F}$ as $G_1\psi(s,\theta) = \psi(s + \pi/\mathcal{F}, \theta)$ according to a nonlinear Schrodinger equation of the form:

$$\frac{\partial\psi}{\partial s} = -(1-\eta)\psi + i|\psi|^2\psi - i\frac{\beta_2}{2}\frac{\partial^2\psi}{\partial\theta^2}, \tag{A.21}$$

where in general the operation is now performed continuously and not approximated as a single step as in the operator $G_L$ above. The Ikeda map that is equivalent to the LLE with parameters $\alpha$, $\beta_2$ and $F$ in the limit of high finesse is:

$$\psi_{n+1}(0,\theta) = e^{-i\frac{\pi}{\mathcal{F}}\alpha}\left(1 - \frac{\pi\eta}{\mathcal{F}}\right) G_1\left[\psi_n(0,\theta)\right]\psi_n(0,\theta) + \sqrt{\frac{\pi}{2\mathcal{F}\eta}}F. \tag{A.22}$$

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