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Optical frequency combs have revolutionized precision metrology by enabling measurements of optical frequencies, with implications both for fundamental scientific questions and for applications such as fast, broadband spectroscopy. In this thesis, I describe the development of comb generation platforms with smaller footprints and higher repetition rates, with the ultimate goal of bringing frequency combs to new applications in a chip-integrated package. I present two new types of frequency combs: electro-optic modulation (EOM) combs and Kerr-microresonator-based frequency combs (microcombs). First I describe the EOM comb scheme and, in particular, techniques for mitigating noise in the comb generation process, and I present the results of a proof-of-principle metrology experiment and some possible applications. Then I discuss developments in microcomb technology. I present novel soliton crystal states, which have highly structured fingerprint optical spectra that correspond to ordered pulse trains exhibiting crystallographic defects. These pulse trains arise through interaction of the solitons with avoided mode-crossings in the resonator spectrum. Next, I describe the direct and deterministic generation of single microresonator solitons using a phase-modulated pump laser. This technique removes the dependence on initial conditions that was formerly a universal feature of these experiments, presenting a solution to a significant technical barrier to the practical application of microcombs. I also discuss generation of Kerr combs in the Fabry-Perot (FP) geometry. I introduce a nonlinear partial differential equation describing dynamics in an FP cavity and discuss the differences between the FP geometry and the ring cavity, which is the geometry used in previous Kerr-comb experiments. Finally, I discuss a technique for reducing the repetition rate of a high-repetition-rate frequency comb, which will be a necessary post-processing step for some applications. I conclude with a discussion of avenues for future research, including the chip-integration of Fabry-Perot Kerr resonators and the use of band-engineered photonic crystal cavities to further simplify soliton generation.

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Chapter 1

Microresonators

This chapter introduces the basic physics of Kerr-nonlinear optical ring resonators, and the two subsequent chapters describe results obtained in these systems. correct?

An optical ring resonator, shown schematically in Fig. **??**, guides light around a closed path in a dielectric medium by total internal reflection, similar to the mechanism that guides light in an optical fiber. The utility of these resonators lies in the fact that light can circulate for many roundtrips before it is coupled out or dissipated, which means that very high circulating intensities can be achieved.

A ring resonator supports propagating guided optical *modes* of electromagnetic radiation that occur at (vacuum) wavelengths that evenly divide the optical round-trip path length: $\lambda_m = n_{eff}(\lambda_m)L/m$, with associated resonance frequencies $\nu_m = c/\lambda_m = mc/n_{eff}(\nu_m)L$, leading to constructive interference from round-trip to round-trip. Here L is the physical round-trip length of the resonator, m is the azimuthal mode number, and $n_{eff}(\lambda_m)$ is an effective index of refraction that depends on the resonator geometry and the mode's transverse mode profile (see e.g. [**REFHERE**] for more information). The free-spectral range f_{FSR} of a resonator is the *local* frequency spacing between modes, calculated via:

$$f_{FSR} \approx \nu_{m+1} - \nu_m \approx \nu_m - \nu_{m-1}, \tag{1.1}$$

$$= \frac{\partial \nu_m}{\partial m}, \tag{1.2}$$

$$= \frac{c}{n_{eff}(\nu)L} - \frac{mc}{n_{eff}^2(\nu)L}\frac{\partial n_{eff}}{\partial \nu}\frac{\partial \nu}{\partial m}, \tag{1.3}$$

$$\Rightarrow f_{FSR} = \frac{c/L}{\left(n_{eff} + \frac{\nu}{n_{eff}}\frac{\partial n_{eff}}{\partial \nu}\right)} = \frac{c}{n_g L} = 1/T_{RT}, \tag{1.4}$$

where $n_g = n_{eff} + \frac{\nu}{n_{eff}}\frac{\partial n_{eff}}{\partial \nu}$ is the group velocity of the mode and T_{RT} is the mode's round-trip time. Importantly, both intrinsic material dispersion and geometric dispersion resulting from, e.g., different sampling of core versus cladding material properties for different transverse mode profiles, lead to a frequency dependence for each of the parameters n_{eff}, n_g, and f_{FSR}, and a resulting non-uniform spacing in the cavity modes in frequency despite the linearity of ν_m in m.

Unless special efforts are made, ring resonators are typically multi-mode, meaning that many different transverse mode profiles are supported. To calculate the frequency-dependent effective index $n_{eff}(\nu)$, thereby enabling calculation of the resonance frequencies and wavelengths, one must solve Maxwell's equations for the resonator geometry. Except in special cases of high symmetry [**microsphereresonators**], this is typically done numerically using finite-element modeling tools like COMSOL. The modes of an optical resonator, both within a mode family defined by a transverse mode profile and between mode families, must be orthogonal.

is there a good citation here?

The timescale over which circulating photons are dissipated in a resonator is fundamental to its fitness for applications. This is quantified by the basic relation for the number of circulating photons $N(t) = N_o e^{-t/\tau_\gamma}$ in the presence of solely linear loss, which defines the photon lifetime τ_γ. Two commonly used practical quantities are linked to the photon lifetime: the resonator finesse $\mathcal{F} = 2\pi\tau_\gamma/T_{RT}$, which for a ring resonator can be interpreted literally as the azimuthal resonator angle traced out by a typical photon over its lifetime; and the resonator quality factor $Q = \omega_c\tau_\gamma$, the phase over which the optical field evolves during the photon lifetime. The lifetime of a photon at a particular frequency is related to the cavity's full-width at half-maximum (FWHM) linewidth as we can calculate through a Fourier transform of the field $E(t) \propto \sqrt{N(t)}$ with angular carrier frequency ω_c:

$$\mathcal{F}\{E\}(\omega) \propto \int_0^\infty dt\, e^{-\left(\frac{1}{2\tau_\gamma}+i(\omega_c-\omega)\right)t}, \tag{1.5}$$

which immediately yields the Lorentzian lineshape

$$|\mathcal{F}\{E\}|^2 \propto \frac{1}{(\omega-\omega_c)^2+\frac{1}{4\tau_\gamma^2}}, \tag{1.6}$$

with FWHM linewidth $\Delta\omega = 1/\tau_\gamma$. With this relationship, the finesse and quality factor can be rewritten as simple ratios of the relevant frequencies: $\mathcal{F} = f_{FSR}/\Delta\nu$; $Q = \nu_c/\Delta\nu$, where $\Delta\nu = \Delta\omega/2\pi$.

1.1 Basic experiments

In a typical microresonator frequency-comb experiment, a frequency-tunable pump laser is coupled evanescently into and out of the resonator using a tapered optical fiber (for e.g. free-standing silica disc resonators) or a bus waveguide (for chip-integrated resonators). The frequencies of the cavity modes can be identified by recording the transmission of the pump laser past the resonator as the pump frequency is varied, provided that sufficient overlap between the fiber field and the resonator field is achieved. In order to couple light into the microresonator so that substantial build-up of the intracavity field occurs, the pump laser must be held at a frequency near a cavity resonance. However, the thermal response of the microresonator introduces complexities into this process; in particular, in materials with positive thermo-optic coefficient $\partial n/\partial T$ the cavity resonance wavelength increases with increased circulating intensity, as intrinsic material absorption leads to heating of the resonator. This basic fact leads to so-called thermal 'bi-stability' behavior [**Carmon2004**] — it is possible to maintain 'blue' laser detuning $\nu_{laser} - \nu_{resonance} > 0$ indefinitely due to naturally occurring thermal self-locking, while 'red' laser detuning $\nu_{laser} - \nu_{resonance} < 0$ is thermally unstable. A second effect of the thermal shift of the resonance frequencies is that, when any substantial pump-laser power is coupled into the resonator, the observed cavity lineshape is no longer Lorentzian, and the symmetry between increasing- and decreasing-pump-frequency scans is broken, as shown in Fig. **??**.

1.2 Microring resonator Kerr frequency combs

The high circulating optical intensities accessible in resonators with long photon lifetimes find immediate application in the use of microresonators for nonlinear optics. The experiments described in this thesis are conducted in silica microresonators. Silica falls into a broader class of materials that exhibit both centro-symmetry, which dictates that the second-order nonlinear susceptibility $\chi^{(2)}$ must vanish, and a significant third-order susceptibility $\chi^{(3)}$. The n^{th}-order susceptibility is a term in the Taylor expansion describing the response of the medium's polarization to an external electric field: $P = P_0 + \epsilon_0\chi^{(1)}E + \epsilon_0\chi^{(2)}E^2 + \epsilon_0\chi^{(3)}E^3 + ...$ The effect of $\chi^{(3)}$ can be described in a straightforward way as a dependence of the refractive index on the local intensity, $n = n_0 + n_2 I$[**somethingelse**], where $n_2 = \frac{3\chi^{(3)}}{4n_0^2\epsilon^0 c}$[**CosoAndSolis**, **alsoLLbook?**]. The intensity-dependence of the refractive index is referred to as the optical Kerr effect.

The combination of the Kerr effect and the high circulating intensities that are accessible in high-finesse cavities provides a powerful platform for nonlinear optics. Specifically, the Kerr effect (or third-order susceptibility $\chi^{(3)}$) enables self-phase modulation, cross-phase modulation, and four-wave mixing, the last of which is depicted schematically in Fig. **??**.

In 2007, a remarkable result heralded the beginning of a new era for frequency comb research. Del'Haye et al reported *cascaded four-wave mixing* in toroidal silica microcavities on silicon chips, the result of which was a series of many co-circulating optical fields that were uniformly spaced by f_{rep} ranging from 375 GHz to ~750 GHz (depending on the platform)[**DelHaye2007**]. Measurements indicated that the frequency spacing was uniform to a precision of 7.3×10^{-18}, thereby establishing that the output of the system was a frequency comb. This result built on previous demonstrations of few-mode parametric oscillations in microresonators [**Kippenberg2004**, **Savchenkov2004**, **Agha2007**], and showed that the non-uniform distribution of cavity resonance frequencies could be overcome to generate an output with equidistant frequency modes.

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