ThesisTitle

by

Daniel C. Cole

Other Degrees

A thesis submitted to the

Faculty of the Graduate School of the

University of Colorado in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

Physics Physics

2018

This thesis entitled:
ThesisTitle
written by Daniel C. Cole
has been approved for the Physics Physics

Reader1

Reader2

Date

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Cole, Daniel C. (Ph.D., Physics)

ThesisTitle

Thesis directed by Dr. Scott A. Diddams

Nunc sed pede. Praesent vitae lectus. Praesent neque justo, vehicula eget, interdum id, facilisis et, nibh. Phasellus at purus et libero lacinia dictum. Fusce aliquet. Nulla eu ante placerat leo semper dictum. Mauris metus. Curabitur lobortis. Curabitur sollicitudin hendrerit nunc. Donec ultrices lacus id ipsum.

Acknowledgements

The work in this thesis would not have been possible...

- Acknowledgement line 1
- Acknowledgement line 2

Contents

Figures

Chapter 1

Introduction

The invention of the optical frequency comb two decades ago provided a revolution in precision measurement by dramatically improving the resolution with which we can measure time. This revolution came about through the development of a simple scheme (that required markedly *less* simple advancements in capabilities in nonlinear optics) by which the terahertz-scale optical frequencies of a mode-locked laser could be effectively measured by electronics operating much more slowly, with bandwidth limitations on the gigahertz scale. The new frequency comb technology immediately permitted measurement of fundamental properties of matter, for example the electronic transition frequency in hydrogen, with unprecedented levels of precision. Since those first demonstrations, optical frequency combs have played an integral part in myriad contexts, including record-setting optical clocks, systems for ultra-low-noise microwave synthesis, broadband spectroscopy applications, and stable long-term calibration of astronomical spectrographs for exoplanet detection. Further development of the technology beyond the first stabilization of the Ti:sapphire laser that heralded the frequency comb's arrival enabled these applications and others, and combs are now versatile tools for measurement in many contexts across many wavelength bands. The technology is reaching maturity, and frequency combs have been commercially available for some time.

In the last decade, methods for generating optical frequency combs that go beyond the mode-locked laser have suggested the possibility of bringing their capabilities to a wide set of applications outside the controlled environment of the research laboratory. These new frequency combs come with higher repetition rates and lower size, weight, and power (SWAP) requirements, making them particularly appropriate for applications like arbitrary microwave and optical waveform generation, telecommunications, and broadband, fast-acquisition-time spectroscopy. Moreover, low-SWAP combs bring the features that make mode-locked laser-based combs attractive to the field, enabling e.g. direct optical frequency synthesis on a chip [**Spencer2018**].

This thesis focuses on this second generation of optical frequency combs. The bulk of the thesis covers microresonator-based frequency combs, and especially the nonlinear dynamics involved in the parametric generation of these frequency combs based on the Kerr nonlinearity. The penultimate chapter presents a second method for generating a high-repetition-rate frequency comb without modelocking that is based on active modulation of a seed c.w. laser and subsequent nonlinear spectral broadening. In the final chapter, I present experimental and theoretical investigations of a technique for repetition-rate reduction of frequency combs, which may prove useful for adapting low-SWAP combs and their intrinsically high repetition rates to some applications as the technology continues to develop.

In the remainder of this chapter, I discuss the basics of optical frequency comb technology.

1.1 Optical frequency combs

An optical frequency comb is obtained by fully stabilizing the spectrum of an optical pulse train. The first frequency combs came about through full frequency-stabilization of modelocked lasers; this thesis focuses on frequency combs with pulse trains generated through other means.

1.1.1 Optical pulse trains and their spectra

In the time domain, a frequency comb consists of a train of uniformly spaced optical pulses arriving at the pulse train's repetition rate f_r. These pulses are typically very short compared to their repetition period $T = 1/f_r$. In the frequency domain, the comb consists of a set of modes that are spaced by f_r in frequency and that have amplitudes determined by an overall spectral envelope centered at the optical carrier frequency, with bandwidth inversely related to the temporal duration of the pulses. The usual description of a frequency comb, which is natural for modelocked-laser-based combs that are not derived from a c.w. laser, gives the frequencies of these modes as

$$\nu_n = nf_r + f_0, \tag{1.1}$$

where $n \sim f_{carrier}/f_r \gg 0$ for the optical modes that make up the comb and f_0 is the carrier-envelope offset frequency. The offset frequency results from the pulse-to-pulse evolution of the carrier wave underneath the temporal intensity envelope of the pulses due to a difference in group and phase velocities. An equivalent representation of the frequencies of the comb that is more natural for frequency combs directly derived from a c.w. laser, as described in this thesis, is

$$\nu_\mu = \nu_c + \mu f_r, \tag{1.2}$$

where ν_c is the frequency of the c.w. laser, the 'pump' or 'seed' laser, from which the frequency comb is derived and μ is a pump-referenced mode number, in contrast with the zero-referenced mode number of Eq. 1.1. Fig. **??** depicts the properties of a frequency comb in the time domain and the frequency domain.

It is useful to consider a mathematical treatment of an optical pulse train to understand the relationships presented above. In the time domain, the electric field $E(t)$ of the pulse train consists of periodically-recurring optical pulses with baseband (centered at zero frequency) field envelope $A(t)$ multiplying the carrier wave of angular frequency ω_c:

$$E(t) = \sum_{k=-\infty}^{\infty} A(t - kT)e^{i\omega_c t}. \tag{1.3}$$

Here, T is the repetition period of the pulse train. Eq. 1.3 can be viewed as describing a laser of angular frequency ω_c with a time-varying amplitude. This temporal modulation leads to a broadband spectrum for E. Intuitively, the spectrum of the comb is the spectrum of the periodic baseband field envelope $\Sigma_k A(t - kT)$, shifted by the multiplication with $e^{i\omega_c t}$ so that it is centered around the optical carrier. More formally, we can calculate the spectrum $|\mathcal{F}\{E\}|^2$ by calculating

$$\mathcal{F}\{E\}(\omega) \sim \left(\sum_{k=-\infty}^{\infty} \mathcal{F}\{A(t - kT)\}\right) * \delta(\omega - \omega_c), \tag{1.4}$$

which results from the convolution (denoted by $*$) theorem for Fourier transforms. We use the Fourier transform's property that a temporal translation results in a linear spectral phase shift to obtain:

$$\mathcal{F}\{E\} \sim \left(\mathcal{F}\{A\} \times \sum_{k=-\infty}^{\infty} e^{-i\omega kT}\right) * \delta(\omega - \omega_c). \tag{1.5}$$

The quantity $\Sigma_k e^{-i\omega kT}$ is the Fourier-series representation of the series of δ-functions $\Sigma_\mu \delta(\omega - 2\pi\mu/T)$, so we get

$$\mathcal{F}\{E\} \sim \left(\mathcal{F}\{A\} \times \sum_{\mu=-\infty}^{\infty} \delta\left(\omega - 2\pi\mu/T\right)\right) * \delta(\omega - \omega_c), \tag{1.6}$$

and performing the convolution leads to the replacement of ω with $\omega - \omega_c$, leading to:

$$\mathcal{F}\{E\} \sim \sum_{\mu=-\infty}^{\infty} \delta\left(\omega - \omega_c - \mu\omega_r\right) \mathcal{F}\{A\}\left(\omega - \omega_c\right). \tag{1.7}$$

This expression indicates that the spectrum of the comb has frequency content at modes $\nu_\mu = \nu_c + \mu f_r$, and that their amplitudes are determined by the spectrum of the baseband field envelope, shifted up to the optical carrier frequency ν_c. This is the natural formulation in the case of a comb derived from a c.w. laser, but it hides the carrier-envelope offset frequency in the difference between ν_c and the nearest multiple of the repetition rate, so that f_0 is the remainder of $\nu_c \div f_r$. In practice, if f_r is known, then a measurement of f_0 is equivalent to a measurement of the frequency of the input c.w. laser.

1.1.2 Frequency stabilization of optical pulse trains

The scientific need for a method to measure optical frequencies motivated the development of optical frequency combs. While the measurement bandwidth of electronic frequency counters has improved since 1999, it remains limited to frequencies roughly one *million* times lower than the frequency of, e.g., visible red light. Frequency combs present a method for measurement of the unknown frequency f_{opt} of an optical signal through heterodyne with a frequency comb — if f_{opt} falls within the bandwidth of the frequency comb, then the frequency of the heterodyne between the comb and the signal is guaranteed to be less than $f_r/2$, which is typically a frequency that can be measured electronically, at least for modelocked-laser-based combs. Therefore, if the frequencies of the comb are known, measurement of the heterodyne of the comb with the signal reveals the frequency of the signal, provided that the comb mode number n, as defined by Eq. 1.1, can be determined. This can be done via a wavelength measurement if sufficient precision is available, or by measuring the change $\partial f_b/\partial f_r = \pm n$, where f_b is the measured frequency of the beat.

The unique utility of the optical frequency comb lies in the fact that measurement of two microwave frequencies, f_0 and f_0, is sufficient to determine the optical frequencies of all of the modes of the comb, thereby enabling frequency measurement of optical signals. Measurement of the repetition rates of optical pulse trains was possible for many years before the realization of optical frequency comb technology, as this can be done by simply impinging the pulse train on a photodetector. It was the confluence of several technological developments around the turn of the twenty-first century that allowed detection and measurement of the carrier-envelope offset frequency , thereby enabling creation of fully-stabilized modelocked-laser pulse trains: optical frequency combs.

have I made clear that these are microwave frequencies

cite

The carrier-envelope offset frequency of a pulse train is challenging to measure because it describes evolution of the optical carrier wave underneath the intensity envelope, and therefore cannot be measured through straightforward detection of the intensity of the pulse train. Presently, the most straightforward way to measure f_0 is $f-2f$ *self-referencing*, which is illustrated in Fig.**??**. This can be performed only with a pulse train whose spectrum spans an octave — a factor of two in frequency. Given such an octave-spanning supercontinuum spectrum, a group of modes near mode number N is frequency-doubled in a medium with the $\chi^(2)$nonlinearity. This frequency-doubled light is heterodyned with the native light in the supercontinuum with mode number near $2N$. The frequency of the resulting beat is f_0:

make fig

$$f_b = f_{doubled} - f_{native} \tag{1.8}$$

$$= 2(Nf_r + f_0) - (2Nf_r + f_0) \tag{1.9}$$

$$= f_0. \tag{1.10}$$

Generating the necessary octave-spanning supercontinuum spectrum typically requires nonlinear spectral broadening of the pulse train after its initial generation, except for in specific, carefully engineered cases. Achieving the required degree of spectral broadening while preserving the coherence properties of the pulse train is a significant challenge — typically this requires launching a train

cite Tara, others

of high energy ($\sim$1 nJ), temporally short ($\leq$ 100 fs) pulses into the spectral-broadening stage, and meeting these requirements is one of the important engineering considerations in designing optical frequency comb systems; this point is relevant to chapters **??** and **??**.

References

[1] A. Turing. The Chemical Basis of Morphogenesis. *Philosophical Transactions of the Royal Society B*, 237, **1952**, 37–72.

[2] T. Kobayashi, T. Sueta, Y. Cho, and Y. Matsuo. High-repetition-rate optical pulse generator using a Fabry-Perot electro-optic modulator. *Applied Physics Letters*, 21 (8), **1972**, 341–343. DOI: 10.1063/1.1654403.

[3] M. Kourogi, K. Nakagawa, and M. Ohtsu. Wide-Span Optical Frequency Comb Generator for. 29 (10), **1993**.

[4] H. Murata, A. Morimoto, T. Kobayashi, and S. Yamamoto. Optical pulse generation by electrooptic-modulation method and its application to integrated ultrashort pulse generators. *IEEE Journal of Selected Topics in Quantum Electronics*, 6 (6), **2000**, 1325–1331. DOI: 10.1109/2944.902186.

[5] T. Sakamoto, T. Kawanishi, and M. Izutsu. frequency comb generation using a Mach Zehnder. *Optics Letters*, 32 (11), **2007**, 1515–1517.

[6] I. Morohashi, T. Sakamoto, H. Sotobayashi, T. Kawanishi, I. Hosako, and M. Tsuchiya. Widely repetition-tunable 200 fs pulse source using a Mach-Zehnder-modulator-based flat comb generator and dispersion-flattened dispersion-decreasing fiber. *Optics letters*, 33 (11), **2008**, 1192–1194. DOI: 10.1364/OL.33.001192.

[7] A. Ishizawa, T. Nishikawa, A. Mizutori, H. Takara, S. Aozasa, A. Mori, H. Nakano, A. Takada, and M. Koga. Octave-spanning frequency comb generated by 250 fs pulse train emitted from 25 GHz externally phase-modulated laser diode for carrier-envelope-offset-locking. *Electronics Letters*, 46 (19), **2010**, 1343. DOI: 10.1049/el.2010.2228.

[8] R. Wu, V. R. Supradeepa, C. M. Long, D. E. Leaird, and A. M. Weiner. Generation of very flat optical frequency combs from continuous-wave lasers using cascaded intensity and phase modulators driven by tailored radio frequency waveforms. *Optics Letters*, 35 (19), **2010**, 3234. DOI: 10.1364/OL.35.003234. arXiv: 1005.5373.

[9] V. R. Supradeepa and A. M. Weiner. Bandwidth scaling and spectral flatness enhancement of optical frequency combs from phase-modulated continuous-wave lasers using cascaded four-wave mixing. *Optics Letters*, 37 (15), **2012**, 3066. DOI: 10.1364/OL.37.003066.

[10] A. J. Metcalf, V. Torres-company, D. E. Leaird, S. Member, A. M. Weiner, and A. Broadband. High-Power Broadly Tunable Electrooptic Frequency Comb Generator. *IEEE Journal of Selected Topics in Quantum Electronics*, 19 (6), **2013**, 3500306. URL: `http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=06553388`.

[11] R. Wu, V. Torres-company, D. E. Leaird, and A. M. Weiner. Optical Frequency Comb Generation. 21 (5), **2013**, 6045–6052. DOI: `10.1364/OE.21.006045`.

[12] J. M. Dudley, G. G. Genty, and S. Coen. Supercontinuum generation in photonic crystal fiber. *Reviews of Modern Physics*, 78 (4), **2006**, 1135–1184. DOI: `10.1103/RevModPhys.78.1135`.

[13] G. P. Agrawal. **Nonlinear Fiber Optics**. 4th. Burlington, MA: Elsevier, 2007.

[14] A. M. Weiner. Femtosecond pulse shaping using spatial light modulators. *Review of Scientific Instruments*, 71 (5), **2000**, 1929–1960. DOI: `10.1063/1.1150614`.

[15] K. Beha, D. C. Cole, P. Del'Haye, A. Coillet, S. A. Diddams, and S. B. Papp. Electronic synthesis of light. *Optica*, 4 (4), **2017**, 406–411. DOI: `10.1364/OPTICA.4.000406`.

[16] a. a. Amorim, M. V. Tognetti, P Oliveira, J. L. Silva, L. M. Bernardo, F. X. Kärtner, and H. M. Crespo. Sub-two-cycle pulses by soliton self-compression in highly nonlinear photonic crystal fibers. *Optics letters*, 34 (24), **2009**, 3851–3853. DOI: `10.1364/OL.34.003851`.

[17] S. Backus, C. G. Durfee, M. M. Murnane, and H. C. Kapteyn. High power ultrafast lasers. *Review of Scientific Instruments*, 69 (3), **1998**, 1207. DOI: `10.1063/1.1148795`.

[18] A. Baltuska, M. Uiberacker, E. Goulielmakis, R. Kienberger, V. S. Yakovlev, T. Udem, T. W. Hänsch, and F. Krausz. Phase-Controlled Amplification of Few-Cycle Laser Pulses. *IEEE Journal of Selected Topics in Quantum Electronics*, 9 (4), **2003**, 972–989.

[19] C. Gohle, J. Rauschenberger, T. Fuji, T. Udem, A. Apolonski, F. Krausz, and T. W. Hänsch. Carrier envelope phase noise in stabilized amplifier systems. 30 (18), **2005**, 2487–2489.

[20] J. Rauschenberger, T. Fuji, M. Hentschel, A.-J. Verhoef, T. Udem, C. Gohle, T. W. Hänsch, and F. Krausz. Carrier-envelope phase-stabilized amplifier system. *Laser Physics Letters*, 3 (1), **2006**, 37–42. DOI: `10.1002/lapl.200510053`.

[21] M. Hirano, T. Nakanishi, T. Okuno, and M. Onishi. Silica-Based Highly Nonlinear Fibers and Their Application. *Sel. Top. Quantum Electron.*, 15 (1), **2009**, 103–113. DOI: `10.1109/JSTQE.2008.2010241`.

[22] D. Mandridis, I. Ozdur, F. Quinlan, M. Akbulut, J. J. Plant, P. W. Juodawlkis, and P. J. Delfyett. Low-noise, low repetition rate, semiconductor-based mode-locked laser source suitable for high bandwidth photonic analog digital conversion. *Applied Optics*, 49 (15), **2010**, 2850–2857.

[23] D. C. Cole, K. M. Beha, S. A. Diddams, and S. B. Papp. Octave-spanning supercontinuum generation via microwave frequency multiplication. *Proceedings of the 8th Sympsion on Frequency Standards and Metrology 2015, Journal of Physics: Conference Series*, 723, **2016**, 012035. DOI: `10.1088/1742-6596/723/1/012035`.

[24] A. M. Heidt. Efficient Adaptive Step Size Method for the Simulation of Supercontinuum Generation in Optical Fibers. *Journal of Lightwave Technology*, 27 (18), **2009**, 3984–3991.

[25] J. Hult. A Fourth-Order Runge-Kutta in the Interaction Picture Method for Simulating Supercontinuum Generation in Optical Fibers. *Journal of Lightwave Technology*, 25 (12), **2007**, 3770–3775. DOI: 10.1109/JLT.2007.909373.

[26] R. Driad, J. Rosenzweig, R. E. Makon, R. Lösch, V. Hurm, H. Walcher, and M. Schlechtweg. InP DHBT-Based IC Technology for 100-Gb / s Ethernet. *IEEE Trans. on Electron. Devices*, 58 (8), **2011**, 2604–2609.

[27] D. Ferenci, M. Grozing, M. Berroth, R. Makon, R. Driad, and J. Rosenzweig. A 25 GHz Analog Demultiplexer with a Novel Track and Hold Circuit for a 50 GS / s A / D-Conversion System in InP DHBT Technology. In: **Microwave Symposium Digest**. 2012, pp. 1–3.