$$\gamma = \frac{2\pi}{\lambda}\frac{n_2}{A_{eff}} \qquad F = 2\pi\tau_{ph}/T_{RT}$$

$$F = \sqrt{\frac{8g_0\Delta\omega_{ext}}{\Delta\omega^3}\frac{P_{in}}{\hbar\omega}} \qquad g_0 = \frac{n_2 c\hbar\omega_p^2}{n_g^2 V_0} \qquad \tau = t/2\tau_{ph}$$

$$\beta_2 = -\frac{2D_2}{\Delta\omega} \qquad D_2 = -D_1^2\frac{k''}{k'} = -\left(\frac{2\pi}{T_{RT}}\right)^2 k'' v_g$$

$$\frac{\partial^2}{\partial T^2} = \frac{\partial^2}{\partial\theta^2}\frac{\partial\theta^2}{\partial T^2} \qquad \theta = \frac{T}{T_{RT}}2\pi \Rightarrow \frac{\partial^2}{\partial T^2} = \left(\frac{2\pi}{T_{RT}}\right)^2$$

$$D_2 = -\frac{\Delta\omega\beta}{2} \qquad L_{NL} = \frac{1}{\gamma P_0} \qquad L_D = \frac{T_0^2}{k''}$$

Calculate F

$$\sqrt{\frac{8L\gamma\,\Delta\omega_{ext}}{T_{RT}^2\,\Delta\omega^3}} = F$$

$$\frac{L\gamma}{T_{RT}^2} = L\,\frac{2\pi}{\lambda}\,\frac{n_2}{A_{eff}}\,\frac{v_g^2}{L^2}$$

$$= \frac{\omega_p}{c}\,\frac{n_2}{V_0}\,v_g^2$$

$$= \frac{1}{\hbar\omega}\,\frac{\hbar\omega_p^2 c\, n_2}{V_0 n_g^2} = \frac{1}{\hbar\omega}\,g_0$$

$$F = \sqrt{\frac{8 g_0\,\Delta\omega_{ext}}{\Delta\omega^3}\,\frac{P}{\hbar\omega}}$$

Calculate dispersion

$$\frac{L}{T_{RT}} \left(\frac{2\pi}{T_{RT}}\right)^2 \frac{k''}{2}$$

$$k'' = \frac{k' D_2}{-D_1{}^2} = \frac{-1}{v_g} D_2 \left(\frac{T_{RT}}{2\pi}\right)^2$$

$$\frac{L}{T_{RT}} \cancel{\left(\frac{2\pi}{T_{RT}}\right)^2 \left(\frac{T_{RT}}{2\pi}\right)^2} \left(-\frac{1}{v_g}\right) \frac{D_2}{2}$$

$$= -\frac{D_2}{2}$$

$$\sqrt{\frac{2L\gamma}{T_{RT}\,\Delta\omega} \frac{4\,\Delta\omega_{ext}}{T_{RT}\,\Delta\omega^2}}\, A_{in} = F$$

$$\sqrt{\frac{8L\gamma\,\Delta\omega_{ext}}{T_{RT}{}^2\,\Delta\omega^3}}$$

$L_{NL}, L_D$

$$\Delta\Theta = \sqrt{\frac{-\beta}{2\alpha}}$$

$$\Delta T = T_0 = \sqrt{\frac{-\beta}{2\alpha}}\ \frac{T_{RT}}{2\pi}$$

$$L_D = \frac{T_0^2}{|k''|} = \frac{-\beta}{2\alpha}\left(\frac{T_{RT}}{2\pi}\right)^2$$

$$k'' = -\frac{1}{v_g} D_2 \left(\frac{T_{RT}}{2\pi}\right)^2$$

$$L_D = \frac{v_g \frac{\beta}{2\alpha}}{D_2}$$

$$\beta = -\frac{2D_2}{\Delta\omega}$$

$$L_D = \frac{v_g}{\Delta\omega\,\alpha} = v_g \tau_{ph}/\alpha$$

$$L_{NL} = \frac{1}{\gamma P_0}$$

$$P_0 = 2\alpha\ \frac{T_{RT}\,\Delta\omega}{2L\gamma}$$

$$L_{NL} = \frac{1}{\gamma}\ \frac{2L\gamma}{2\alpha T_{RT}\,\Delta\omega}$$

$$= \frac{L}{\alpha T_{RT}\,\Delta\omega}$$

$$= v_g \tau_{ph}/\alpha$$

$$\frac{v_g \tau_{ph}/\alpha}{L} = \frac{\tau_{ph}}{\alpha T_{RT}}$$

$$= \frac{2\pi\,\tau_{ph}/\alpha T_{RT}}{2\pi}$$

$$= \frac{F}{2\pi\alpha}$$

Normalization of Ikeda map

$$T_{RT}\Delta\omega = \frac{T_{RT}}{\tau_{ph}} = 2\pi/F \quad (F = 2\pi\,\tau_{ph}/T_{RT})$$

$$\eta = \frac{\Delta\omega_{ext}}{\Delta\omega}$$

$$1 - \frac{T_{RT}}{2\tau_{ext}} = 1 - \frac{T_{RT}\,\Delta\omega}{2}\eta$$

$$= 1 - \frac{\pi\eta}{F}$$

$$\sqrt{\frac{T_{RT}}{\tau_{ext}}} = \sqrt{T_{RT}\,\Delta\omega\,\eta}$$

$$= \sqrt{\frac{2\pi\eta}{F}}$$

$$\alpha_L$$

$$\phi_{RT}/T_{RT} = \omega_p - \omega_0 = +\sigma$$

$$L\frac{\alpha_L}{2} = \frac{T_{RT}}{2\tau_{int}}$$

$$\alpha = -2\,\phi_{RT}/T_{RT}/\Delta\omega$$

$$= \frac{T_{RT}\,\Delta\omega(1-\eta)}{2}$$

$$\phi_{RT} = -\frac{\Delta\omega\,T_{RT}\,\alpha}{2}$$

$$= \frac{2\pi}{2F}(1-\eta)$$

$$= -\frac{\pi\,\alpha}{F}$$

$$= \frac{\pi}{F}(1-\eta)$$

# Normalization of $A_{in}$ for Ikeda map

$$\Psi_{in} = \sqrt{\frac{2L\gamma}{T_{RT}\,\Delta\omega}}\,A_{in}$$

$$F = \sqrt{\frac{8L\gamma\,\Delta\omega_{ext}}{T_{RT}^2\,\Delta\omega^3}}\,A_{in}$$

$$\Psi_{in} = \sqrt{\frac{2L\gamma}{T_{RT}\,\Delta\omega}\,\frac{T_{RT}^2\,\Delta\omega^3}{8L\gamma\,\Delta\omega_{ext}}}\;F$$

$$= \sqrt{\frac{T_{RT}\,\Delta\omega^2}{4\,\eta}}\;F$$

$$= \sqrt{\frac{\pi}{2\eta\mathcal{F}}}\;F$$

Normalization of NLSE for Ikeda map

$$\frac{\partial \psi}{\partial z} = -\frac{\alpha_e}{2}\psi + i\gamma|A|^2\psi - i\frac{k''}{2}\frac{\partial^2\psi}{\partial T^2}$$

$$\frac{\partial^2}{\partial T^2} = \frac{\partial^2}{\partial\theta^2}\left(\frac{2\pi}{T_{RT}}\right)^2$$

$$\frac{k''}{2}\left(\frac{2\pi}{T_{RT}}\right)^2 = -\frac{1}{V_g}\frac{D_2}{2} = -\frac{1}{V_g}\frac{1}{2}\frac{-\Delta\omega\beta}{2}$$

$$= \frac{T_{RT}\Delta\omega}{L}\frac{1}{2}\frac{\beta}{2}$$

$$= \frac{2\pi}{2FL}\frac{\beta}{2} = \frac{\pi}{FL}\frac{\beta}{2}$$

$s = z/L$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial s}\frac{\partial s}{\partial z} = \frac{\partial}{\partial s}\frac{1}{L} \qquad s = \frac{z}{L}\frac{\pi}{F} \qquad \frac{\partial}{\partial z} = \frac{\partial}{\partial s}\frac{\pi}{LF}$$

$$\frac{\partial\psi}{\partial s} = -\frac{\pi}{F}(1-\eta)\psi + iL\gamma|A|^2\psi - i\frac{\pi}{F}\frac{\beta}{2}\frac{\partial^2\psi}{\partial\theta^2}$$

$$\psi = \sqrt{\frac{2L\gamma}{T_{RT}\Delta\omega}}\,A$$

$$\frac{\partial\psi}{\partial s} = -\frac{\pi}{F}(1-\eta)\psi + \underbrace{i\frac{T_{RT}\Delta\omega}{2}|\psi|^2\psi}_{= i\frac{\pi}{F}|\psi|^2\psi} - i\frac{\pi}{F}\frac{\beta}{2}\frac{\partial^2\psi}{\partial\theta^2}$$