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Nunc sed pede. Praesent vitae lectus. Praesent neque justo, vehicula eget, interdum id, facilisis et, nibh. Phasellus at purus et libero lacinia dictum. Fusce aliquet. Nulla eu ante placerat leo semper dictum. Mauris metus. Curabitur lobortis. Curabitur sollicitudin hendrerit nunc. Donec ultrices lacus id ipsum.

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The work in this thesis would not have been possible...

- Acknowledgement line 1
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Chapter 1

Introduction

The invention of the optical frequency comb two decades ago provided a revolution in precision measurement by dramatically improving the resolution with which we can measure time. This revolution came about through the development of a simple scheme (that required markedly *less* simple advancements in capabilities in nonlinear optics) by which the terahertz-scale optical frequencies of a mode-locked laser could be effectively measured by electronics operating much more slowly, with bandwidth limitations on the gigahertz scale. The new frequency comb technology immediately permitted measurement of fundamental properties of matter, for example the electronic transition frequency in hydrogen, with unprecedented levels of precision. Since those first demonstrations, optical frequency combs have played an integral part in myriad contexts, including record-setting optical clocks, systems for ultra-low-noise microwave synthesis, broadband spectroscopy applications, and stable long-term calibration of astronomical spectrographs for exoplanet detection. Further development of the technology beyond the first stabilization of the Ti:sapphire laser that heralded the frequency comb's arrival enabled these applications and others, and combs are now versatile tools for measurement in many contexts across many wavelength bands. The technology is reaching maturity, and frequency combs have been commercially available for some time.

In the last decade, methods for generating optical frequency combs that go beyond the mode-locked laser have suggested the possibility of bringing their capabilities to a wide set of applications outside the controlled environment of the research laboratory. These new frequency combs come with higher repetition rates and lower size, weight, and power (SWAP) requirements, making them particularly appropriate for applications like arbitrary microwave and optical waveform generation, telecommunications, and broadband, fast-acquisition-time spectroscopy. Moreover, low-SWAP combs bring the features that make mode-locked laser-based combs attractive to the field, enabling e.g. direct optical frequency synthesis on a chip [**Spencer2018**].

This thesis focuses on this second generation of optical frequency combs. The bulk of the thesis covers microresonator-based frequency combs, and especially the nonlinear dynamics involved in the parametric generation of these frequency combs based on the Kerr nonlinearity. The penultimate chapter presents a second method for generating a high-repetition-rate frequency comb without modelocking that is based on active modulation of a seed c.w. laser and subsequent nonlinear spectral broadening. In the final chapter, I present experimental and theoretical investigations of a technique for repetition-rate reduction of frequency combs, which may prove useful for adapting low-SWAP combs and their intrinsically high repetition rates to some applications as the technology continues to develop.

In the remainder of this chapter, I discuss the basics of optical frequency comb technology.

1.1 Optical frequency combs

An optical frequency comb is obtained by fully stabilizing the spectrum of an optical pulse train. The first frequency combs came about through full frequency-stabilization of modelocked lasers; this thesis focuses on frequency combs with pulse trains generated through other means.

1.1.1 Optical pulse trains and their spectra

In the time domain, a frequency comb consists of a train of uniformly spaced optical pulses arriving at the pulse train's repetition rate f_r. These pulses are typically very short compared to their repetition period $T = 1/f_r$. In the frequency domain, the comb consists of a set of modes that are spaced by f_r in frequency and that have amplitudes determined by an overall spectral envelope centered at the optical carrier frequency, with bandwidth inversely related to the temporal duration of the pulses. The usual description of a frequency comb, which is natural for modelocked-laser-based combs that are not derived from a c.w. laser, gives the frequencies of these modes as

$$\nu_n = nf_r + f_0, \tag{1.1}$$

where $n \sim f_{carrier}/f_r \gg 0$ for the optical modes that make up the comb and f_0 is the carrier-envelope offset frequency. The offset frequency results from the pulse-to-pulse evolution of the carrier wave underneath the temporal intensity envelope of the pulses due to a difference in group and phase velocities. An equivalent representation of the frequencies of the comb that is more natural for frequency combs directly derived from a c.w. laser, as described in this thesis, is

$$\nu_\mu = \nu_c + \mu f_r, \tag{1.2}$$

where ν_c is the frequency of the c.w. laser, the 'pump' or 'seed' laser, from which the frequency comb is derived and μ is a pump-referenced mode number, in contrast with the zero-referenced mode number of Eq. 1.1. Fig. **??** depicts the properties of a frequency comb in the time domain and the frequency domain.

It is useful to consider a mathematical treatment of an optical pulse train to understand the relationships presented above. In the time domain, the electric field $E(t)$ of the pulse train consists of periodically-recurring optical pulses with baseband (centered at zero frequency) field envelope $A(t)$ multiplying the carrier wave of angular frequency ω_c:

$$E(t) = \sum_{k=-\infty}^{\infty} A(t - kT)e^{i\omega_c t}. \tag{1.3}$$

Here, T is the repetition period of the pulse train. Eq. 1.3 can be viewed as describing a laser of angular frequency ω_c with a time-varying amplitude. This temporal modulation leads to a broadband spectrum for E. Intuitively, the spectrum of the comb is the spectrum of the periodic baseband field envelope $\Sigma_k A(t - kT)$, shifted by the multiplication with $e^{i\omega_c t}$ so that it is centered around the optical carrier. More formally, we can calculate the spectrum $|\mathcal{F}\{E\}|^2$ by calculating

$$\mathcal{F}\{E\}(\omega) \sim \left(\sum_{k=-\infty}^{\infty} \mathcal{F}\{A(t - kT)\} \right) * \delta(\omega - \omega_c), \tag{1.4}$$

which results from the convolution (denoted by $*$) theorem for Fourier transforms. We use the Fourier transform's property that a temporal translation results in a linear spectral phase shift to obtain:

$$\mathcal{F}\{E\} \sim \left(\mathcal{F}\{A\} \times \sum_{k=-\infty}^{\infty} e^{-i\omega kT}\right) * \delta(\omega - \omega_c). \tag{1.5}$$

The quantity $\Sigma_k e^{-i\omega kT}$ is the Fourier-series representation of the series of δ-functions $\Sigma_\mu \delta(\omega - 2\pi\mu/T)$, so we get

$$\mathcal{F}\{E\} \sim \left(\mathcal{F}\{A\} \times \sum_{\mu=-\infty}^{\infty} \delta\left(\omega - 2\pi\mu/T\right)\right) * \delta(\omega - \omega_c), \tag{1.6}$$

and performing the convolution leads to the replacement of ω with $\omega - \omega_c$, leading to:

$$\mathcal{F}\{E\} \sim \sum_{\mu=-\infty}^{\infty} \delta\left(\omega - \omega_c - \mu\omega_r\right) \mathcal{F}\{A\}\left(\omega - \omega_c\right). \tag{1.7}$$

This expression indicates that the spectrum of the comb has frequency content at modes $\nu_\mu = \nu_c + \mu f_r$, and that their amplitudes are determined by the spectrum of the baseband field envelope, shifted up to the optical carrier frequency ν_c. This is the natural formulation in the case of a comb derived from a c.w. laser, but it hides the carrier-envelope offset frequency in the difference between ν_c and the nearest multiple of the repetition rate, so that f_0 is the remainder of $\nu_c \div f_r$. In practice, if f_r is known, then a measurement of f_0 is equivalent to a measurement of the frequency of the input c.w. laser.

1.1.2 Frequency measurements with an optical pulse train

The application that motivated the development of optical frequency combs was measurement of optical frequencies. While the measurement bandwidth of electronic frequency counters has improved since 1999, it remains limited to frequencies roughly one *million* times lower than the frequency of, e.g., visible red light. Frequency combs present a method for measurement of the unknown frequency f_{opt} of an optical signal through heterodyne with a frequency comb - if f_{opt} falls within the bandwidth of the frequency comb, then the frequency of the heterodyne between the comb and the signal is guaranteed to be less than $f_r/2$. Therefore, if the frequencies of the comb

modes at the set of frequencies

Equivalently, if the function $A(t)$ is localized to a small interval around zero relative to the period T, this equation be written to emphasize the phase-shift between the carrier wave and the intensity envelope:

$$E(t) = \sum_{n=-\infty}^{\infty} A(t - nT) e^{i\omega_c (t-nT)} e^{in\phi_{CE}}, \tag{1.8}$$

where here the field $A(t)e^{i\omega_c t}$ is repeated every period, and is multiplied by a phase increasing incrementally by $\phi_{CE} = \omega_c T$. Eq.

The spectrum of the frequency comb consists of a set of uniformly spaced optical modes at frequencies , multiplied by an overall spectral envelope centered at the optical carrier frequency and corresponding to the temporal intensity envelope of the pulses. The optical frequencies ν_n are spaced by f_r. The carrier-envelope offset frequency f_0 represents the offset of the zero$^{\text{th}}$ comb mode from zero frequency, and therefore the offset of each mode ν_n from the closest harmonic of the repetition rate. This offset arises from the pulse-to-pulse evolution of the carrier wave under the pulse train's intensity envelope.

In the above, we have used the linearity and convolution properties of the Fourier transform, with convolution denoted by $*$. We have also used the Fourier transform for the Dirac comb . Eq. 1.7 directly reveals the connection between the spectrum of the electric field of a pulse train and the baseband pulse envelope $A(t)$, the carrier frequency $f_c = \omega_c/2\pi$, and the repetition rate $f_r = 1/T = \omega_r/2\pi$.

The first optical frequency combs came about through full frequency-stabilization of optical pulse trains generated in modelocked lasers. A laser cavity with broadband gain can support many oscillating frequency modes; this number is on the order of ten thousand for a typical telecommunications-band fiber laser, and can be hundreds of thousands for a Ti:sapphire laser cavity. Without a mechanism to enforce a fixed relationship between the modes, the modes oscillate independently and the laser output is uncontrolled. A modelocked laser is obtained through the introduction of a modelocking mechanism that provides for lower cavity losses in pulsed operation, in which the modes oscillate together and periodically constructively interfere, relative to unsynchronized multi-mode operation. Common modelocking mechanisms are saturable-absorber mirrors, Kerr-lens modelocking, and nonlinear polarization-rotation modelocking.

The pulse train generated in a modelocked laser can be described equally well in either the time domain or the frequency domain.

The spectrum of the pulse train consists of a set of equidistant modes with optical frequencies described by $\nu_n = nf_r + f_0$. Here ν_n is the frequency of the n^{th} mode, referenced to zero frequency; f_r is the pulse train's repetition rate, and is the separation between adjacent modes in the frequency domain, and f_0 is the 'carrier-envelope-offset frequency,' denoting

modelocked laser consists of many oscillating modes, supported by broadband laser gain, that have a fixed phase relationship imposed by a modelocking mechanism. In a typical 250 MHz repetition-rate optical pulse train in erbium-doped fiber, this number is on the order of ten thousand. These pulse trains result from synchronization of many oscillating laser modes in a cavity with broadband gain through the introduction of a modelocking mechanism. A modelocked laser consists of a laser cavity having 1. broadband gain and 2. a modelocking mechanism. generate an optical pulse train when it has broadband gain in the presence of a modelocking mechanism. If a laser is made to oscillate under these conditions without a modelocking mechanism, these modes can 'lase' independently. This generates an output waveform that appears random, resulting from the superposition of these thousands of modes and their individual frequency fluctuations. If, on the other hand, modelocking is enforced so that the modes oscillate in a coherent fashion, their electric field frequency components periodically constructively interfere, yielding a train of pulses. This is depicted schematically in Fig. [**fig:MLnoMLpulsetrains**]. To induce modelocking, a mechanism that favors higher peak-intensity pulsed operation is introduced into the cavity. Two common modelocking mechanisms are Kerr-lens based modelocking, in which the spatial Kerr effect focuses a beam through a small aperture in the laser cavity more effectively at higher power, and semiconductor saturable absorber mirrors that have higher reflectivity at higher incident intensities. Modelocking can be used to generate laser pulses that are on the order of hundreds of femtoseconds long, or even shorter.

References

[1] A. Turing. The Chemical Basis of Morphogenesis. *Philosophical Transactions of the Royal Society B*, 237, **1952**, 37–72.

[2] T. Kobayashi, T. Sueta, Y. Cho, and Y. Matsuo. High-repetition-rate optical pulse generator using a Fabry-Perot electro-optic modulator. *Applied Physics Letters*, 21 (8), **1972**, 341–343. DOI: 10.1063/1.1654403.

[3] M. Kourogi, K. Nakagawa, and M. Ohtsu. Wide-Span Optical Frequency Comb Generator for. 29 (10), **1993**.

[4] H. Murata, A. Morimoto, T. Kobayashi, and S. Yamamoto. Optical pulse generation by electrooptic-modulation method and its application to integrated ultrashort pulse generators. *IEEE Journal of Selected Topics in Quantum Electronics*, 6 (6), **2000**, 1325–1331. DOI: 10.1109/2944.902186.

[5] T. Sakamoto, T. Kawanishi, and M. Izutsu. frequency comb generation using a Mach – Zehnder. *Optics Letters*, 32 (11), **2007**, 1515–1517.

[6] I. Morohashi, T. Sakamoto, H. Sotobayashi, T. Kawanishi, I. Hosako, and M. Tsuchiya. Widely repetition-tunable 200 fs pulse source using a Mach-Zehnder-modulator-based flat comb generator and dispersion-flattened dispersion-decreasing fiber. *Optics letters*, 33 (11), **2008**, 1192–1194. DOI: 10.1364/OL.33.001192.

[7] A. Ishizawa, T. Nishikawa, A. Mizutori, H. Takara, S. Aozasa, A. Mori, H. Nakano, A. Takada, and M. Koga. Octave-spanning frequency comb generated by 250 fs pulse train emitted from 25 GHz externally phase-modulated laser diode for carrier-envelope-offset-locking. *Electronics Letters*, 46 (19), **2010**, 1343. DOI: 10.1049/el.2010.2228.

[8] R. Wu, V. R. Supradeepa, C. M. Long, D. E. Leaird, and A. M. Weiner. Generation of very flat optical frequency combs from continuous-wave lasers using cascaded intensity and phase modulators driven by tailored radio frequency waveforms. *Optics Letters*, 35 (19), **2010**, 3234. DOI: 10.1364/OL.35.003234. arXiv: 1005.5373.

[9] V. R. Supradeepa and A. M. Weiner. Bandwidth scaling and spectral flatness enhancement of optical frequency combs from phase-modulated continuous-wave lasers using cascaded four-wave mixing. *Optics Letters*, 37 (15), **2012**, 3066. DOI: 10.1364/OL.37.003066.

[10] A. J. Metcalf, V. Torres-company, D. E. Leaird, S. Member, A. M. Weiner, and A. Broadband. High-Power Broadly Tunable Electrooptic Frequency Comb Generator. *IEEE Journal of Selected Topics in Quantum Electronics*, 19 (6), **2013**, 3500306. URL: `http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=06553388`.

[11] R. Wu, V. Torres-company, D. E. Leaird, and A. M. Weiner. Optical Frequency Comb Generation. 21 (5), **2013**, 6045–6052. DOI: `10.1364/OE.21.006045`.

[12] J. M. Dudley, G. G. Genty, and S. Coen. Supercontinuum generation in photonic crystal fiber. *Reviews of Modern Physics*, 78 (4), **2006**, 1135–1184. DOI: `10.1103/RevModPhys.78.1135`.

[13] G. P. Agrawal. **Nonlinear Fiber Optics**. 4th. Burlington, MA: Elsevier, 2007.

[14] A. M. Weiner. Femtosecond pulse shaping using spatial light modulators. *Review of Scientific Instruments*, 71 (5), **2000**, 1929–1960. DOI: `10.1063/1.1150614`.

[15] K. Beha, D. C. Cole, P. Del'Haye, A. Coillet, S. A. Diddams, and S. B. Papp. Electronic synthesis of light. *Optica*, 4 (4), **2017**, 406–411. DOI: `10.1364/OPTICA.4.000406`.

[16] a. a. Amorim, M. V. Tognetti, P Oliveira, J. L. Silva, L. M. Bernardo, F. X. Kärtner, and H. M. Crespo. Sub-two-cycle pulses by soliton self-compression in highly nonlinear photonic crystal fibers. *Optics letters*, 34 (24), **2009**, 3851–3853. DOI: `10.1364/OL.34.003851`.

[17] S. Backus, C. G. Durfee, M. M. Murnane, and H. C. Kapteyn. High power ultrafast lasers. *Review of Scientific Instruments*, 69 (3), **1998**, 1207. DOI: `10.1063/1.1148795`.

[18] A. Baltuska, M. Uiberacker, E. Goulielmakis, R. Kienberger, V. S. Yakovlev, T. Udem, T. W. Hänsch, and F. Krausz. Phase-Controlled Amplification of Few-Cycle Laser Pulses. *IEEE Journal of Selected Topics in Quantum Electronics*, 9 (4), **2003**, 972–989.

[19] C. Gohle, J. Rauschenberger, T. Fuji, T. Udem, A. Apolonski, F. Krausz, and T. W. Hänsch. Carrier envelope phase noise in stabilized amplifier systems. 30 (18), **2005**, 2487–2489.

[20] J. Rauschenberger, T. Fuji, M. Hentschel, A.-J. Verhoef, T. Udem, C. Gohle, T. W. Hänsch, and F. Krausz. Carrier-envelope phase-stabilized amplifier system. *Laser Physics Letters*, 3 (1), **2006**, 37–42. DOI: `10.1002/lapl.200510053`.

[21] M. Hirano, T. Nakanishi, T. Okuno, and M. Onishi. Silica-Based Highly Nonlinear Fibers and Their Application. *Sel. Top. Quantum Electron.*, 15 (1), **2009**, 103–113. DOI: `10.1109/JSTQE.2008.2010241`.

[22] D. Mandridis, I. Ozdur, F. Quinlan, M. Akbulut, J. J. Plant, P. W. Juodawlkis, and P. J. Delfyett. Low-noise, low repetition rate, semiconductor-based mode-locked laser source suitable for high bandwidth photonic analog – digital conversion. *Applied Optics*, 49 (15), **2010**, 2850–2857.

[23] D. C. Cole, K. M. Beha, S. A. Diddams, and S. B. Papp. Octave-spanning supercontinuum generation via microwave frequency multiplication. *Proceedings of the 8th Sympsion on Frequency Standards and Metrology 2015, Journal of Physics: Conference Series*, 723, **2016**, 012035. DOI: `10.1088/1742-6596/723/1/012035`.

[24] A. M. Heidt. Efficient Adaptive Step Size Method for the Simulation of Supercontinuum Generation in Optical Fibers. *Journal of Lightwave Technology*, 27 (18), **2009**, 3984–3991.

[25] J. Hult. A Fourth-Order Runge-Kutta in the Interaction Picture Method for Simulating Supercontinuum Generation in Optical Fibers. *Journal of Lightwave Technology*, 25 (12), **2007**, 3770–3775. DOI: `10.1109/JLT.2007.909373`.

[26] R. Driad, J. Rosenzweig, R. E. Makon, R. Lösch, V. Hurm, H. Walcher, and M. Schlechtweg. InP DHBT-Based IC Technology for 100-Gb / s Ethernet. *IEEE Trans. on Electron. Devices*, 58 (8), **2011**, 2604–2609.

[27] D. Ferenci, M. Grozing, M. Berroth, R. Makon, R. Driad, and J. Rosenzweig. A 25 GHz Analog Demultiplexer with a Novel Track and Hold Circuit for a 50 GS / s A / D-Conversion System in InP DHBT Technology. In: **Microwave Symposium Digest**. 2012, pp. 1–3.