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Optical frequency combs have revolutionized precision metrology by enabling measurements of optical frequencies, with implications both for fundamental scientific questions and for applications such as fast, broadband spectroscopy. In this thesis, I describe the development of comb generation platforms with smaller footprints and higher repetition rates, with the ultimate goal of bringing frequency combs to new applications in a chip-integrated package. I present two new types of frequency combs: electro-optic modulation (EOM) combs and Kerr-microresonator-based frequency combs (microcombs). First I describe the EOM comb scheme and, in particular, techniques for mitigating noise in the comb generation process, and I present the results of a proof-of-principle metrology experiment and some possible applications. Then I discuss developments in microcomb technology. I present novel 'soliton crystal' states, which have highly structured 'fingerprint' optical spectra that correspond to ordered pulse trains exhibiting crystallographic defects. These pulse trains arise through interaction of the solitons with avoided mode-crossings in the resonator spectrum. Next, I describe the direct and deterministic generation of single microresonator solitons using a phase-modulated pump laser. This technique removes the dependence on initial conditions that was formerly a universal feature of these experiments, presenting a solution to a significant technical barrier to the practical application of microcombs. I also discuss generation of Kerr combs in the Fabry-Perot (FP) geometry. I introduce a nonlinear partial differential equation describing dynamics in an FP cavity and discuss the differences between the FP geometry and the ring cavity, which is the geometry used in previous Kerr-comb experiments. Finally, I discuss a technique for reducing the repetition rate of a high-repetition-rate frequency comb, which will be a necessary post-processing step for some applications. I conclude with a discussion of avenues for future research, including the chip-integration of Fabry-Perot Kerr resonators and the use of band-engineered photonic crystal cavities to further simplify soliton generation.

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Chapter 1

Microresonators

This chapter introduces the basic physics of Kerr-nonlinear optical ring resonators, and the two subsequent chapters describe results obtained in these systems. correct?

An optical ring resonator, shown schematically in Fig. **??**, guides light around a closed path in a dielectric medium by total internal reflection, similar to the mechanism that guides light in an optical fiber. A ring resonator supports propagating guided optical *modes* of electromagnetic radiation that occur at (vacuum) wavelengths that evenly divide the optical round-trip path length: $\lambda_m = n_{eff}(\lambda_m)L/m$, with associated resonance frequencies $\nu_m = c/\lambda_m = mc/n_{eff}(\nu_m)L$, leading to constructive interference from round-trip to round-trip. Here L is the physical round-trip length of the resonator, m is the azimuthal mode number, and $n_{eff}(\lambda_m)$ is an effective index of refraction that depends on the resonator geometry and the mode's transverse mode profile (see e.g. [**REFHERE**] for more information). The free-spectral range f_{FSR} of a resonator is the *local* frequency spacing between modes, calculated via:

$$f_{FSR} \approx \nu_{m+1} - \nu_m \approx \nu_m - \nu_{m-1}, \tag{1.1}$$

$$= \frac{\partial \nu_m}{\partial m}, \tag{1.2}$$

$$= \frac{c}{n_{eff}(\nu)L} - \frac{mc}{n_{eff}^2(\nu)L}\frac{\partial n}{\partial \nu}\frac{\partial \nu}{\partial m}, \tag{1.3}$$

$$\Rightarrow f_{FSR} = \frac{c/L}{\left(n + \frac{\nu}{n}\frac{\partial n}{\partial \nu}\right)} = \frac{c}{n_g L} = 1/T_{RT}, \tag{1.4}$$

where $n_g = n + \frac{\nu}{n}\frac{\partial n}{\partial \nu}$ is the group velocity of the mode and T_{RT} is the mode's round-trip time.

Unless special efforts are made, ring resonators are typically multi-mode, meaning that many different transverse mode profiles are supported — this is important to the results discussed in chapter **??**. To calculate the frequency-dependent effective index $n_{eff}(\nu)$, thereby enabling calculation of the resonance frequencies and wavelengths, one must solve Maxwell's equations for the resonator geometry. Except in special cases of high symmetry, this is typically done numerically using finite-element modeling tools like COMSOL. The modes of an optical resonator, both within a mode family defined by a transverse mode profile and between mode families, must be orthogonal.

An important parameter describing an optical resonator is the timescale over which circulating photons are dissipated — the basic relation for the number of circulating photons $N(t) = N_o e^{-t/\tau_\gamma}$ in the presence of solely linear loss defines the photon lifetime τ_γ. Two commonly used practical quantities are linked to the photon lifetime: the resonator finesse $\mathcal{F} = 2\pi\tau_\gamma/T_{RT}$, which for a ring resonator can be interpreted literally as the azimuthal resonator angle traced out by a typical photon over its lifetime; and the resonator quality factor $Q = \omega_c\tau_\gamma$, the phase over which the optical field evolves during the photon lifetime. The lifetime of a photon at a particular frequency is related to the cavity's full-width at half-maximum (FWHM) linewidth as we can calculate through a Fourier transform of the field $E(t) \propto \sqrt{N(t)}$ with angular carrier frequency ω_c:

$$\mathcal{F}\{E\}(\omega) \propto \int_0^\infty dt\, e^{-\left(\frac{1}{2\tau_\gamma}+i(\omega_c-\omega)\right)t}, \tag{1.5}$$

which immediately yields the Lorentzian lineshape

$$|\mathcal{F}\{E\}|^2 \propto \frac{1}{(\omega-\omega_c)^2+\frac{1}{4\tau_\gamma^2}}, \tag{1.6}$$

with FWHM linewidth $\Delta\omega = 1/\tau_\gamma$. With this relationship, the finesse and quality factor can be rewritten as simple ratios of the relevant frequencies: $\mathcal{F} = f_{FSR}/\Delta\nu$; $Q = \nu_c/\Delta\nu$, where $\Delta\nu = \Delta\omega/2\pi$.

1.1 Nonlinear optics in microresonators

The resonator quality factor is an important figure of merit for the use of optical resonators as platforms for nonlinear optics. This is discussed extensively below; typically the threshold power for nonlinear optics scales as Q^{-2}, meaning that in the design of practical platforms targeting high Q is an important consideration. Fabrication of ultrahigh-Q resonators has been achieved with a variety of designs and materials, including ...

These resonators can be constructed with extremely high quality factors, upwards of 10^8, which facilitates high circulating intensity.

Combs using optical ring resonators leverage this confinement and the resulting long photon lifetimes in high-quality factor resonators

References

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