Technology-Driven Unemployment

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(Newest Version)

Abstract

To examine the relationship between technological progress and unemployment, I study a model that features putty-clay production, directed technical change, and labor market bargaining. I have two primary goals. First, I aim to understand the forces that deliver a constant steady state unemployment rate in the presence of labor-saving technical change. The interaction between directed technical change and wage bargaining plays a crucial role in this process. Labor-saving technical change increases unemployment, which lowers wages and creates incentives for future investment in labor-using technologies. In the long run, this interaction generates a balanced growth path that is observationally equivalent to that of the standard neoclassical growth model, except that is also incorporates a steady state level of unemployment. Second, I investigate the effect of 'technological breakthroughs' that permanently lower the cost of creating new labor-saving technologies. Breakthroughs lead to faster growth in output per worker and wages but also yield higher long-run unemployment and a lower labor share of income. Surprisingly, they also lower output in the short-run, because additional investment in labor-saving technologies is inefficient when workers are not scarce, as in the presence of technology-driven unemployment.

Keywords Labor-Saving Technical Change, Unemployment, Directed Technical Change, Growth

JEL Codes E24, O33, O40

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1 Introduction

From the Luddites to Keynes (1930) to Silicon Valley, there has long been fear that labor-saving technical change would lead to increased unemployment and even the obsolescence of labor in production. While the most extreme versions of these predictions have not come to fruition, it is certainly the case that new technologies have eliminated jobs in particular industries at particular times. The lack of ever-increasing unemployment suggests that past job losses caused by labor-saving technologies have been offset by other economic forces. Nonetheless, there remains great concern that a coming technological revolution will fundamentally alter the labor market in developed countries (Akst, 2013; Autor, 2014, 2015).

The neoclassical growth model assumes that an unlimited number of workers can each productively use any fixed quantity of specific capital goods. While convenient for many applications, this unrealistic assumption precludes any relationship between labor-saving technical change and the saturation of labor demand. To study technology-driven unemployment, therefore, I focus on the case of putty-clay production, which growth theorists have long considered more realistic (e.g., Johansen, 1959; Samuelson, 1962; Cass and Stiglitz, 1969). Embodied technological characteristics determine the input requirements of creating and operating each capital good. Substitution between inputs occurs via the choice of technology (Solow, 1962; Calvo, 1976; Jones, 2005). For a given set of technologies and quantity of installed capital, the economy can only support a finite number of workers with positive marginal product. As a result, insufficient labor demand can generate unemployment (Johansen, 1959; Solow et al., 1966; Akerlof and Stiglitz, 1969).[1,2]

I build on the existing literature on putty-clay production by incorporating two new elements, directed technical change and labor-market frictions. The standard labor-augmenting technology lowers labor input requirements and, holding all else constant, reduces employment. This type of technology is labor-saving. A second type of technology lowers the cost of producing new capital goods. This type of technology is capital-augmenting, labor-using, and capital-saving. The two types of technology are embodied in capital goods and evolve over time according to profit maximizing research and development activity (Acemoglu, 1998, 2002). Frictions in the labor market ensure that the wage lies between the marginal product of workers and their opportunity cost of working (Pissarides, 2000; Rogerson et al., 2005).

I have two primary goals. First, I aim to understand the forces that deliver constant unemployment in the presence of labor-saving technical change. Second, I investigate how fundamental

[1] In his seminal contribution that demonstrated how to 'synthesize' the fixed-factor and aggregate approaches, Johansen (1959) addressed the importance of considering technological change in this context: "[i]n conclusion it is suggested that the proposed hypothesis would be particularly appropriate in studying the introduction of new techniques and the relationship between population growth, the rate of saving and 'structural' unemployment." This is an apt description of the current paper, which examines how the evolution of cutting-edge technologies interacts with labor force growth and capital accumulation to determine long-run, technology-driven unemployment.

[2] Solow et al. (1966) note the possibility of this outcome, but do not provide any formal analysis, arguing that it would not be a sustainable outcome in a market economy. In this sense, a key contribution of the current paper is to show how adding labor-market frictions and directed technical change creates the possibility for this type of technology-driven unemployment to exist in equilibrium.

changes to the structure of the economy, such as a 'third industrial revolution' or 'second machine age,' will affect labor market outcomes (Brynjolfsson and McAfee, 2012, 2014). I first perform a theoretical analysis that highlights the forces of balanced growth, as well as the consequences of changes in fundamental parameters. I then calibrate the model to aggregate macroeconomic data from the U.S. and perform computational experiments to better understand the transition dynamics. Throughout the analysis, I pay special attention to trade-offs between equity and efficiency and between short- and long-run outcomes.

In the model, a balanced growth path with a constant unemployment rate emerges from the interaction between directed technical change and labor market bargaining. Given a set of technologies and amount of installed capital, the economy can only support a finite number of workers. When labor becomes more efficient, fewer workers can be profitably employed in the short-run, holding all else constant. This tendency towards increased unemployment is offset by economic expansion. In particular, the accumulation of capital goods – via increased investment or technological progress that lowers the cost of capital goods – increases the number of jobs in the economy. The long-run unemployment rate depends on the relative growth rates of the labor-saving and labor-using forces. Improvements in labor-saving technologies lead to higher unemployment, which lowers wages and, consequently, increases future investment in labor-using technologies. Thus, a balanced growth path exists with investment in both types of technology, as well as capital accumulation and population growth. Indeed, the balanced growth path of the putty-clay model with directed technical change is observationally equivalent to that of the standard neoclassical growth model,[3] except that is also incorporates a steady state level of unemployment, endogenous research allocations, and capital-augmenting technical change.[4]

I then consider the impact of a 'technological breakthrough' that permanently lowers the cost of developing new labor-saving technologies. After the breakthrough, labor-saving technology grows at a faster rate and unemployment increases. The increase in unemployment lowers wages. Although cheaper, the development of new technologies is still costly. As a result, the fall in wages leads capital good producers to invest in labor-using technologies. The economy eventually converges to a new long-run equilibrium with a higher steady state rate of unemployment. Thus, even fundamental changes to the structure of the economy do not lead to ever-increasing unemployment and 'the end of work' as predicted by many economists and public intellectuals.[5]

[3]The standard neoclassical growth model was a response to the earlier models of Harrod (1948) and Domar (1946), which used production functions with fixed inputs coefficients to achieve balanced growth (Solow, 1994). These models required a knife-edge condition to yield constant unemployment, a shortcoming emphasized by Solow (1956, 1994). In this paper, the knife-edge case generating constant unemployment is an endogenous outcome.

[4]The Uzawa steady state theorem implies that, with a standard neoclassical production function, all technological progress must labor-augmenting on a balanced growth path (Uzawa, 1961; Schlicht, 2006; Jones and Scrimgeour, 2008). With only one type of technology, it is not possible to have both labor-saving an labor-using technology in the long-run. Following Casey and Horii (2017), the new model generates balanced growth with capital-augmenting technical change by incorporating land into the production function. In this way, the current paper is related to the broader literature examining the Uzawa theorem (e.g., Acemoglu, 2003; Jones, 2005; Grossman et al., 2017).

[5]See, for example, references in Akst (2013), Brynjolfsson and McAfee (2014), and Autor (2014, 2015).

Even though improvements in technology do not lead to the irrelevance of work, faster technological progress is not unambiguously positive for workers. When labor efficiency improves, fewer workers are needed to a given amount of output. This has two countervailing effects. Productivity is higher, which pushes up wages. At the same time, unemployment increases, which lowers worker bargaining power. In the long-run, breakthroughs increase both the growth rate of wages and unemployment, helping some workers but hurting others. Also, the growth rate of output per capita increases, but the labor share decreases. Thus, there are equity-efficiency trade-offs between different groups of workers and between different factors of production.

Perhaps more surprisingly, technological breakthroughs lower output in the short-run. In the presence of technology-driven unemployment, investment in labor-saving technology is wasteful, because it saves a resources that is not scarce. Total output could be increased by substituting unemployed workers for investment in labor-saving technology and employing the unused R&D inputs in the development of new capital-saving technologies. Technological breakthroughs exacerbate this inefficiency by causing a reallocation of R&D inputs towards labor-saving technologies. This increases the long-run growth rate of the economy, but decreases output in the short-run.[6]

The paper proceeds as follows. Section 2 discusses how the current study fits into the existing literature. Section 3 presents the model, while Section 4 presents the calibration and outcome of the quantitative exercises. Section 5 concludes.

2 Relation to Existing Literature

Recently, there has been considerable interest in the study of labor-saving technical change (e.g., Graetz and Michaels, 2017; Acemoglu and Restrepo, 2017a,b). This paper complements the existing theoretical literature by focusing the relationship between labor-saving technical change and involuntary unemployment (Benzell et al., 2015; Acemoglu and Restrepo, 2016; Hémous and Olsen, 2016). As explained above, unemployment persists in the model because of putty-clay production. Standard neoclassical production functions allow *ex post* substitution between capital and labor. While this is convenient for many applications, it hampers the study of unemployment by assuming that an unlimited number of workers can each productively use a finite set of capital goods.

This paper also complements the existing theoretical literature by examining the role of factor-augmenting technologies. In standard neoclassical settings, it is rarely the case that factor-augmenting technologies are labor-saving (Acemoglu, 2010; Acemoglu and Autor, 2011). As a result, the existing literature has focused on other types of technological progress. For example, Peretto and Seater (2013) consider a growth model where innovation changes the exponents in a Cobb-Douglas

[6] While the main goal of this paper is to examine the relationship between technology and unemployment, I also consider the impact of changes in worker bargaining power and labor force growth. Changes in bargaining power introduce a set of equity-efficiency trade-offs that differ from those associated with technological breakthroughs. In particular, reductions in worker bargaining power increase output and employment, but lower the labor share of income. Changes in bargaining power have no long-run effect on technological progress. Slower labor force growth creates scarcity, leading to faster wage growth, a lower unemployment rate, and a higher labor share of income.

production function.[7] Relatedly, Acemoglu and Restrepo (2016) and Hémous and Olsen (2016) build on the tasked-based framework of Acemoglu and Autor (2011) and Zeira (1998). By considering the role of putty-clay production with labor market bargaining, the current model shows how labor-augmenting technology can cause unemployment – lowering the wages of some workers to zero – and even lower wages for employed workers. In this way, the model demonstrates that labor-augmenting technology can have a complicated and time-varying effect on wages and on worker welfare more generally.

Of these existing papers, the current study is most closely related to the work of Acemoglu and Restrepo (2016), who also focus on a model with a homogeneous set of workers.[8] As noted above, I build on their work by incorporating involuntary unemployment and by studying a different form of technological change. Despite the significant differences between the two models, they yield some overlapping insights, reinforcing the findings of both models.

This paper is also related to the broader literature on directed technical change (e.g., Acemoglu, 1998, 2002). In particular, the model shows how directed technical change is important in generating a steady state level of unemployment. With exogenous technical change, constant unemployment would only exists in a knife-edge case. The model developed in this paper differs from the seminal approach of Acemoglu (1998, 2002) by considering innovation in different characteristics of capital goods, rather than innovation in different intermediate sectors. This new approach maintains tractability in the putty-clay framework and leads to different research incentives when compared to the standard model.[9]

In related work, Stiglitz (2014) also considers the role of innovation in generating constant unemployment in a setting with putty-clay production. As in this paper, the long-run direction of technical change depends on wages, which in turn depend on the rate of unemployment.[10] The current paper differs from his work in three important ways. First, in Stiglitz (2014), all innovation is directed towards labor-augmenting technology in the steady state.[11] This is inconsistent with evidence on the declining price of investment goods (Greenwood et al., 1997; Krusell et al., 2000; Grossman et al., 2017). Second, while there is a trade-off between different types of technology in his framework, innovation is costless. In this sense, his model captures the notion of induced innovation (e.g., Kennedy, 1964; Samuelson, 1965), rather than endogenous and directed technical

[7]The authors focus on the role of labor-saving technical change in offsetting diminishing returns and generating endogenous growth, rather than effect of automation on labor market outcomes. See also Seater (2005) and Zuleta (2008), who focus on social planner solutions.

[8]Hémous and Olsen (2016) use the task-based framework to consider the effects of automation on inequality between workers with different skill levels, while abstracting from issues of employment. For more on automation and inequality between different groups of workers, see Autor and Dorn (2013) and Michaels et al. (2014).

[9]In this way, the current paper is related to the work of Hassler et al. (2012, 2016) and Casey (2017), who use putty-clay models of directed technical change to investigate energy efficiency. The latter paper uses a decentralized model with similarities to the current paper.

[10]Relatedly, Akerlof and Stiglitz (1969) consider how savings behavior can generate constant unemployment in a model with putty-clay production. In particular, they focus on the case where the marginal propensity to save out of wages is lower than the marginal propensity to save out of rental income. While the mechanism is different, much of the same intuition is at play. When unemployment increases, wages decrease and rental income increases. This leads to greater savings and capital formation, which lowers the rate of unemployment.

[11]See Acemoglu (2003) for a related result that abstracts from unemployment.

change (e.g., Romer, 1990; Aghion and Howitt, 1992; Acemoglu, 1998, 2002). Third, I examine the impact of a technological breakthrough, which is a primary concern in public policy discussions (e.g., Brynjolfsson and McAfee, 2012, 2014).

By investigating a separate mechanism, this paper also complements work studying unemployment due to search and matching frictions (Pissarides, 2000; Rogerson et al., 2005). The putty-clay model focuses on unemployment caused by the difference between the number of jobs – both filled and vacant – and the size of the workforce, rather than the process of job separation and matching. To highlight the important growth mechanisms and maintain tractability, the model incorporates matching and bargaining in a simple manner abstracts from search frictions. Much can be gained from future work that integrates the technology-driven unemployment with other models of equilibrium unemployment.[12] The most natural synthesis is likely to occur with matching models in which purchases of capital create vacancies, implying that the underlying production function is Leontief (e.g., Acemoglu and Shimer, 1999; Acemoglu, 2001). To this set-up, the putty-clay model examines two types of technologies, a labor-augmenting technology that lowers the labor input requirement for each capital good and a capital-augmenting technology that expands economic production and increases the demand for labor.

3 Model

3.1 Environment

3.1.1 Production

Final output is produced under perfect competition. As is standard in the endogenous growth literature, there are a continuum of capital goods. These capital goods are combined with land and labor to produce final output. The aggregate production function is given by

$$Y_t = \int_0^1 \min\left[X_t(i)^\alpha M^{1-\alpha}, A_{L,t}(i)L_t(i)\right]di, \tag{1}$$

where $X_t(i)$ is the quantity of capital good i, $M^{1-\alpha}$ is natural capital (e.g., land), and $L_t(i)$ is labor hired to work with capital good i. Labor-augmenting technical change, $A_{L,t}(i)$, determines the labor requirement of capital good i. Technology is embodied in the capital goods. The minimum function captures the rigid 'clay' properties of installed capital. Specifically, there is no substitution between capital and labor after capital goods are installed. As will be discussed below, substitution occurs via the choice of technology.

[12] Aghion and Howitt (1994) and Mortensen (2005) examine the relationship between creative destruction and unemployment in models with endogenous, but undirected, growth and search and matching frictions. The current model abstracts from these Schumpeterian forces.

Each type of capital good is produced by a single monopolist. The efficiency of capital good production is given by $A_{K,t}(i)$. Thus, market clearing for capital is given by

$$\int_0^1 \frac{X_t(i)}{A_{K,t}(i)}\, di \leq K_t, \tag{2}$$

where K_t is aggregate capital (i.e., saved output). This type of technology is capital-augmenting and capital-saving.[13] As will be discussed in the next section, capital good producers can improve technological characteristics through R&D investment.

Aggregate employment is given by

$$L_t \equiv \int_0^1 L_t(i) di \leq N_t, \tag{3}$$

where N_t is the (exogenous) size of the labor force at time t. The putty-clay production structure creates the possibility of unemployment due to saturation of labor demand. Once a firm has installed a fixed quantity of capital goods with associated technological characteristics, it can only support a finite number of workers with positive marginal product. This logic extends to the entire economy, implying that the nature of technology and level of capital accumulation create an upper bound on employment. Formally, we can capture this intuition by specifying the constraint $A_{L,t}(i)L_t(i) \leq X_t(i)^\alpha M^{1-\alpha}\ \forall i$.[14]

Standard neoclassical production functions differ from the putty-clay approach by allowing substitution between capital and labor after capital goods are installed. While this is a useful and convenient assumption for most economic analyses, it is an impediment to the study of technology-driven unemployment. In particular, the standard model assumes that an unlimited number of workers can each use a fixed set of specific capital goods. This is an unrealistic assumption that shuts down an important potential source unemployment that is closely related to labor-saving technical change. Specifically, equations (1) and (3) imply that, holding all else equal, advances in labor-saving (i.e., labor-augmenting) technology will increase unemployment. On a balanced growth path, these reductions in potential employment are offset by economic expansion, which occurs via the accumulation of capital goods.

3.1.2 Research and Development

Monopolists can hire R&D inputs to improve either capital- or labor-saving technology. In either case, technology evolves according to

$$A_{J,t}(i) \quad = \quad \big(1 + \eta_J R_J(i)\big) A_{J,t-1}, \tag{4}$$

[13] A long literature examines the existence and implications of this investment-specific technical change (e.g., Greenwood et al., 1997; Krusell et al., 2000; Cummins and Violante, 2002; Grossman et al., 2017). The model presented here is isomorphic to one in which $A_{K,t}(i)$ appears in the final good production function and the efficiency of capital good production is constant.

[14] Given the structure of wage bargaining in the model, this constraint is technically redundant.

where $J = K, L$, $R_{J,t}(i)$ are research inputs hired by firm i to improve technology characteristic J, and $A_{J,t-1} = \int_0^1 A_{J,t-1}(i)\ di$.[15] Patents last for one period, after which technology flows freely between firms. Thus, the properties of new technologies depend on the amount of research inputs hired, as well as the aggregate state of technology.[16]

There are a unit mass of research inputs in all periods. As a result, market clearing is given by

$$\int_0^1 R_{L,t}(i)di + \int_0^1 R_{K,t}(i)di = 1. \tag{5}$$

I also define $R_{J,t} \equiv \int_0^1 R_{J,t}(i)di,\ J = K, L$ for simplicity.[17]

3.1.3 Individuals

The consumer side of the problem is standard. The representative household chooses

$$\{C_t\}_{t=0}^{\infty} = \operatorname{argmax} \sum_{t=0}^{\infty} \beta^t N_t \frac{\tilde{c}_t^{1-\xi}}{1-\xi}, \tag{6}$$

where $\tilde{c}_t = C_t/N_t$ is consumption per person. I focus on the decentralized equilibrium where the household takes prices and technology levels as given. For simplicity, I also focus on the case of full depreciation. The simulation exercises use a period length of ten years. The relevant budget constraint is given by

$$C_t + K_{t+1} = w_t L_t + r_t K_t + p_{R,t} + \Pi_t + p_{M,t} M = Y_t, \tag{7}$$

where w_t is wages, r_t is the rental rate on capital, $p_{R,t}$ is the rental rate for R&D inputs, $p_{M,t}$ is the rental rate for land, and Π_t is total profits. The labor force grows at a constant and exogenous rate

$$N_{t+1} = (1+n)N_t. \tag{8}$$

3.1.4 Wage Determination

Firms install capital before they hire workers. Each worker has marginal product $A_{L,t}(i)$, and they bargain with the final good producer to determine how much of that marginal product they keep in wages. Workers receive higher wages when they (a) have more bargaining power, (b) are more

[15] R&D inputs could be scientists, research labs, etc.

[16] A convenient alternative would be for firms to build on the best available technology. If firms only maximize single period profits, these two specifications are isomorphic.

[17] The assumption of a fixed set of research inputs is commonly made in the directed technical change literature (e.g., Acemoglu, 2003; Acemoglu et al., 2012; Hassler et al., 2016). It is a stand in for two offsetting forces, an increase in aggregate research inputs and an increase in the cost of generating a given aggregate growth rate (Jones, 1995b, 2002; Bloom et al., 2016). For the expositional purposes of this paper, this simplifying assumption is quite useful. To extend the current model to quantitative analysis of specific policy interventions, it would be profitable to include either aspects of semi-endogenous growth (e.g., Jones, 1995a; Kortum, 1997; Segerstrom, 1998) or allow the entry of new capital goods (Peretto, 1998; Dinopoulos and Thompson, 1998; Howitt, 1999).

productive, or (c) have better outside options. Outside options improve when labor markers are tighter (i.e., unemployment is lower) or average productivity in other jobs is higher.

There are many possible bargaining technologies that could determine the actual wage schedule. To be consistent with balanced growth, I assume that the bargaining process yields wages according to

$$w_t(i) = \chi A_{L,t}(i) + \chi \Gamma(u_t) A_{L,t}, \tag{9}$$

where $u_t = 1 - \frac{L_t}{N_t}$ and $\Gamma'(u_t) < 0$. In words, χ captures the bargaining power of the worker, and $\Gamma(u_t)$ captures the difficulty of getting another job if an agreement cannot be reached. Appendix Section A.4 presents a simple microfoundation using Nash bargaining.

Consider the determination of wages in the standard continuous time search and matching model of unemployment with Nash bargaining (e.g., Pissarides, 2000). In that model, workers are paid a fraction of their marginal product plus their reservation wage, which is determined in part by the value of unemployment. The value of unemployment, in turn, depends on how difficult it is to get a new job and the potential wage that could be negotiated in that job. Expression (9) captures this intuition. The important difference with the continuous time model is that there is an explicit expression for wages in terms of productivity and unemployment. This allows for a tractable investigation of endogenous and directed technical change.

3.2 Optimization

As demonstrated in Appendix Section A.1.1, the inverse demand for physical capital has the familiar iso-elastic form,

$$p_{X,t}(i) \quad = \quad \alpha[1 - \frac{w_t(i)}{A_{L,t}(i)}] X_t(i)^{\alpha-1} M^{1-\alpha}. \tag{10}$$

Intuitively, this occurs because physical capital is combined with natural capital in a Cobb-Douglas manner. The overall demand for capital maintains this functional form, but is adjusted for payments to labor. The final good producer must employ labor to run each unit of the capital good. Thus, the demand for capital depends on the operation cost.

The iso-elastic demand yields convenient analytic expressions. Capital good producers set prices as a constant markup over unit costs:

$$p_{X,t}(i) \quad = \quad \frac{1}{\alpha} \frac{r_t}{A_{K,t}(i)}. \tag{11}$$

Appendix Section A.1.2 derives the behavior of capital good producers. Profits from capital good production are given by

$$\bar{\pi}_{X,t}(i) = (1 - \frac{1}{\alpha}) \alpha^{\frac{2}{1-\alpha}} A_{K,t}(i)^{\frac{\alpha}{1-\alpha}} r_t^{\frac{-\alpha}{1-\alpha}} [1 - \chi - \chi \frac{\Gamma(u_t) A_{L,t}}{A_{L,t}(i)}]^{\frac{1}{1-\alpha}} M. \tag{12}$$

This is again a standard expression adjusted for payments to labor. Capital good producers choose research inputs to maximize these profits, subject to the research production function given in equation (4).

Importantly, the two types of technology enter the profit function with different functional forms. Capital good producers benefit from sales of the capital goods, but not payments to labor. They only care about labor input costs to the degree that they affect the demand for capital goods. In particular, improvements in labor-saving technical change have a negative and convex effect on the input cost. This fact plays an important role in guaranteeing the existence of balanced growth path.

The research arbitrage equation is given by

$$\frac{p^L_{R,t}}{p^X_{R,t}} = \frac{\chi\Gamma(u_t)A_{L,t}A_{K,t}(i)}{[1-\chi-\chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]A_{L,t}(i)^2}\frac{\eta_L A_{L,t-1}}{\eta_K A_{K,t-1}}, \tag{13}$$

where $p^J_{R,t}$ is the rental rate paid to R&D inputs used to improve technology J. Given that R&D inputs are mobile across sectors, factor payments are equal in equilibrium.

In some dimensions, the research arbitrage equation resembles the standard directed technical change model (Acemoglu, 1998, 2002). As expected, the relative value of improving labor-saving technical change, $A_{L,t}(i)$, is increasing in the wage, the relative efficiency of R&D production, and in the value of the other technology, $A_{K,t}(i)$. Moreover, $\chi + \chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}$ is the labor cost per unit of output, and higher labor cost increase incentives for labor-saving technical change. In other dimensions, the research incentives differ from those in the standard model. In particular, the relative cost of inputs also enters through $\chi\Gamma(u_t)A_{L,t}$ in the numerator and the square of labor-efficiency in the denominator. This asymmetry is driven by the fact that technology is embodied in the capital good. Also, there are no *market size effects*. Factor accumulation only matters via the unemployment rate, which affect wages. The task-based model of Acemoglu and Restrepo (2016) also features a research arbitrage equation that is driven by price effects, but not market size effects.

The consumer maximization problem yields the following Euler equation

$$\frac{\tilde{c}_{t+1}}{\tilde{c}_t} = \beta r_{t+1}, \tag{14}$$

which is standard.

3.3 Substitution between Capital and Labor

In this section, I take a brief detour to discuss two important features of the model: factor substitution and the technology menu. Together, these features highlight the close relationship between the putty-clay model and more standard approaches. Despite the *ex post* Leontief production function, the putty-clay model still features substitution between capital and labor. This substitution occurs via the choice of technology. Figure 1 highlights this process.

It is helpful to note that all capital good producers face an identical problem and will make identical decisions. Since there are a unit mass of capital good producers, $R_{J,t}(i) = R_{J,t}$ and $A_{J,t}(i) = A_{J,t}\ \forall J, t, i$. Thus, I will drop the i subscripts for everything that follows. Market clearing implies that $Y_t = \min[(A_{K,t}K_t)^{\alpha} M^{1-\alpha}, A_{L,t}L_t]$. Since $\delta = 1$, the minimum function will be satisfied with equality in all periods.

After capital goods are installed, technology parameters are fixed. There is a minimum amount of labor and capital needed to produce a given amount of output. The isoquant for producing a given amount of output, $\bar{Y}$, is given by the following correspondence:

$$\bar{K} = \begin{cases} [\frac{\bar{Y}^{\frac{1}{\alpha}}}{A_{K,t}}, \infty] & \text{if } L_t = \frac{\bar{Y}}{A_{L,t}} \\ \frac{\bar{Y}^{\frac{1}{\alpha}}}{A_{K,t}} & \text{if } L_t > \frac{\bar{Y}}{A_{L,t}} \\ \emptyset & \text{otherwise,} \end{cases} \tag{15}$$

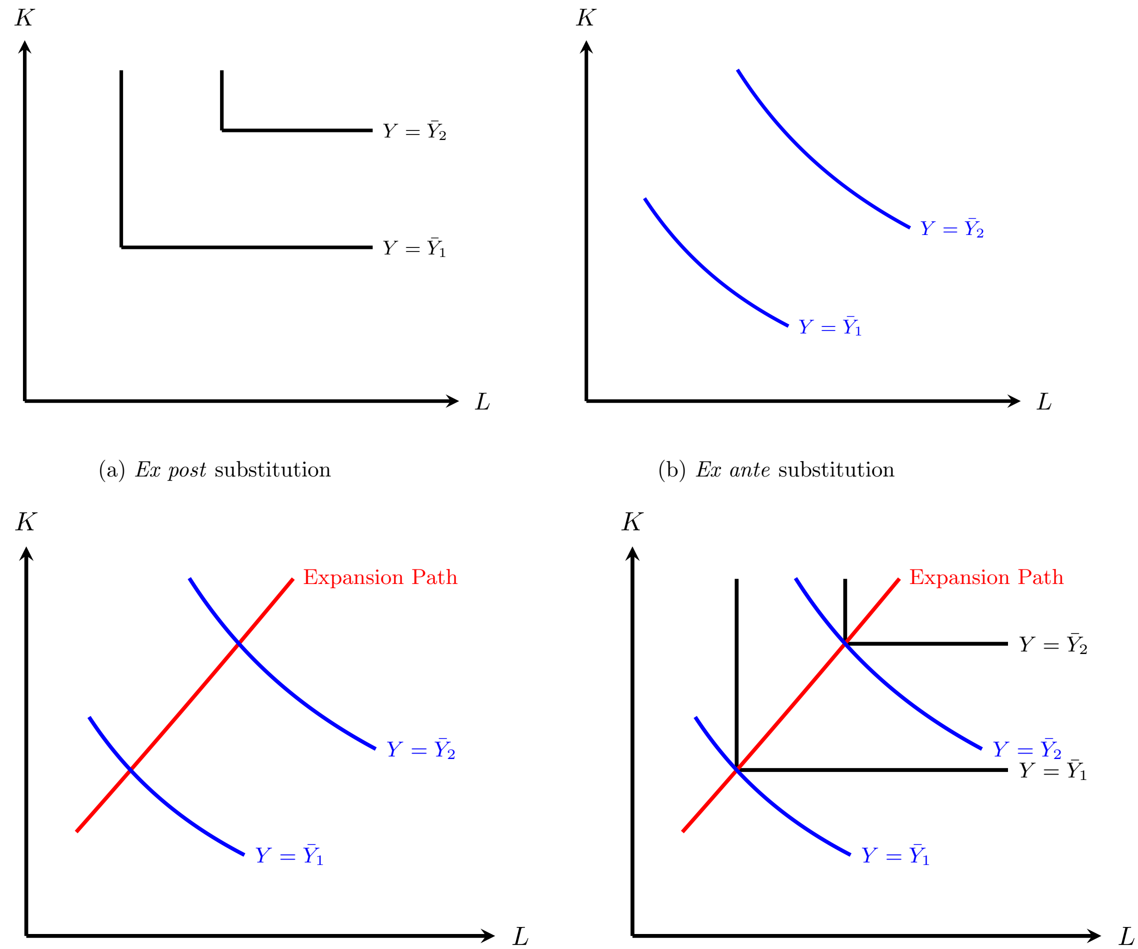
where I have normalized M to one for convenience. I will maintain this assumption through this section. Panel (a) plots the isoquants in (K, L) space. It demonstrates the defining property of Leontief production: there is no *ex post* substitution between capital and labor. The presence of land implies that the corners of the isoquants to not follow a straight line out from the origin.

Prior to the installation of capital goods, there exists a menu of potential technology pairs. In particular, the resource constraint for R&D inputs implies that $1 = R_{L,t} + R_{K,t}$. Together with the law of motion of technology, equation (4), this can be rewritten as

$$\tilde{\eta} A_{K,t-1} = A_{K,t} + \frac{A_{K,t-1}}{A_{L,t-1}} \frac{\eta_K}{\eta_L} A_{L,t}, \tag{16}$$

where $\tilde{\eta} = \eta_K(1 + \frac{1}{\eta_K} + \frac{1}{\eta_L})$ is constant. Capital good producers take lagged technology levels as given. Thus, the technology menu captures the feasibility constraint in allocating R&D resources. Given that technology must be strictly increasing, there is an additional constraint that $A_{J,t} \geq A_{J,t-1}\ \forall t, J$. In each period, $A_{K,t-1}$ increases, which demonstrates the expansion of the production possibility frontier for technologies. In the long-run, the two technologies will not grow at the same rate, implying that the trade-off between technologies changes over time.

Equation (16) highlights how the putty-clay models resembles earlier literature with a distribution of different types of capital goods (e.g., Samuelson, 1962; Jones, 2005; Caselli and Coleman, 2006). Unlike the existing literature, the entire menu of cutting-edge technologies is not freely available. Instead, capital good producers must hire R&D inputs to create the new technologies. As a result, only one technology pair will actually materialize. Since R&D inputs are distinct from production inputs, it will always be profitable to use a cutting-edge technology.



(a) *Ex post* substitution

(b) *Ex ante* substitution

(c) Expansion path

(d) *Ex post* and *ex ante* substitution

Figure 1: This figures demonstrates that, despite the Leontief production function, there is still substitution between capital and labor. Panel (a) presents the isoquants for the final good producer, who takes technology as given. Panel (b) presents the isoquants when considering the ability of the capital good producers to choose from the technology menu. The expansion path, presented in panel (c), is determined by the profit maximizing behavior of the capital good producer. Finally, panel (d) combines the results.

The technology menu demonstrates the possibility for *ex ante* substitution between capital and labor. When considering the choice of technology, the relationship given in (15) becomes

$$\bar{K} = \begin{cases} \frac{\bar{Y}^{\frac{1}{\alpha}}}{\tilde{\eta} A_{K,t-1} - \frac{A_{K,t-1}}{A_{L,t-1}} \frac{\bar{Y}}{L_t}} & \text{if } L_t \in [\frac{\bar{Y}}{(1+\eta_L) A_{L,t-1}}, \frac{\bar{Y}}{A_{L,t-1}}] \\ \emptyset & \text{otherwise,} \end{cases} \tag{17}$$

which defines the smooth curves given in panel (b). Thus, the putty-clay model allows for smooth substitution between capital and labor, much like the standard neoclassical growth model. This substitution, however, occurs via the choice of technology, and it is not possible to substitute factors after capital goods are installed.

Capital good producers choose technology pairs to maximize profits. The optimal decision is given in equation (13).[18] Once these technological parameters are set, the expansion path is given by $K_t = \frac{A_{L,t}^{\frac{1}{\alpha}}}{A_{K,t}} L_t^{\frac{1}{\alpha}}$, which follows from the fact that $Y_t = A_{L,t}L_t = \left(A_{K,t}K_t\right)^{\alpha} M^{1-\alpha}$ $\forall t$. This is shown in panel (c). Once capital good producers have chosen a point on the technology menu, they sell specific capital goods to the final good producer. The final good producers only have access to the Leontief technology, as depicted in panel (d).

3.4 Dynamics

In this subsection, I analyze the dynamics of the economy. Compared to the standard neoclassical set-up, the putty-clay model of directed technical change adds unemployment and endogenous technological progress, while removing the standard *ex post* substitution between labor and installed capital. Despite these changes, the analysis uses familiar steps, which allows the new model to remain tractable and transparent.

Noting that all monopolists make identical decisions, market clearing condition (2) implies that the aggregate production function for the economy can be written as

$$Y_t = \min[(A_{K,t}K_t)^{\alpha} M^{1-\alpha}, A_{L,t}L_t]. \tag{18}$$

The case of excess savings is ruled out by the assumption of perfect depreciation.[19] Thus, $Y_t = \left(A_{K,t}K_t\right)^{\alpha} M^{1-\alpha}$ $\forall t$. Of course, this formulation looks quite similar to the standard neoclassical model, except that land is replacing labor and capital-augmenting technology is replacing labor-augmenting technology. As a result, the usual intensive-form approach to solving the model will be useful.

Define $z_t = \frac{Z_t}{A_{K,t}^{\frac{\alpha}{1-\alpha}} M}$ for any variable Z_t. Now,

$$y_t \quad = \quad k_t^{\alpha}, \tag{19}$$

as in the standard neoclassical growth model. Moreover, equation (7) implies that the law of motion for capital is given by $K_{t+1} = Y_t - C_t$. In intensive form,

$$k_{t+1} \quad = \quad \frac{y_t - c_t}{(1 + g_{K,t+1})^{\frac{\alpha}{1-\alpha}}}. \tag{20}$$

Similarly, the Euler equation, (14), gives

$$\frac{c_{t+1}}{c_t} \quad = \quad \frac{\beta r_{t+1}(1+n)^{\xi}}{(1 + g_{K,t+1})^{\frac{\xi\alpha}{1-\alpha}}}. \tag{21}$$

[18]This decision is also captured in equation (24), which is discussed in the next section.
[19]This is a sufficient, but not necessary, condition.

Both of these expressions are relatively standard, except that population growth appears in an unusual way. This occurs because the intensive form is not normalized by population. Moreover, capital-augmenting, rather than labor-augmenting, technology determines the evolution of the intensive form.

The results thus far highlight the benefit of including land in the production function. In addition to being realistic, including land gives the model a familiar set of dynamic equations. This occurs because there are diminishing returns to physical capital, even within the first argument of the production function.

To close the model, it is necessary to bring in the newer elements, unemployment and technological progress. To start, consider the real interest rate. Note that in equilibrium $\frac{w_t}{A_{L,t}(i)} = \chi + \chi\Gamma(u_t)\ \forall i$. Thus, the real interest rate is given by

$$r_t = \alpha^2[1 - \chi - \chi\Gamma(u_t)]k_t^{\alpha-1}, \tag{22}$$

which is again similar to the standard neoclassical growth model with monopolistic competition. The difference is that the real interest rate must be adjusted for payments to labor, which depend on the unemployment rate. Higher unemployment leads to lower wages, increasing the value of capital from the perspective of the final good producer.

Now, I establish the more unique portions of the model, the dynamics of technology and unemployment. Considering (18) and noting that the minimum function is met with equality, $1 - u_t = \frac{L_t}{N_t} = \frac{(A_{K,t}K_t)^\alpha M^{1-\alpha}}{A_{L,t}N_t}$. This yields

$$\frac{1-u_{t+1}}{1-u_t} = \frac{(1+g_{K,t+1})^{\frac{\alpha}{1-\alpha}}}{(1+g_{L,t+1})(1+n)} \cdot \big(\frac{k_{t+1}}{k_t}\big)^\alpha. \tag{23}$$

This equation captures the division between labor-saving and labor-using forces discussed in Subsection 3.1.1. Technology-driven unemployment exists because of the finite quantity of capital and specific labor-input requirements. A finite number of workers have positive marginal product, leading to unemployment when bargaining ensures a positive wage. Unemployment decreases when economic production expands via the accumulation of capital goods, which results from saving or capital-saving technological progress. Conversely, labor-saving technology increases the unemployment, as does population growth.

Now, I consider the dynamics of technology. Let $g_{J,t} \equiv \frac{A_{J,t}}{A_{J,t-1}} - 1$, $J = K, L$. Again noting that all monopolists make identical decisions, equation (4) yields

$$g_{L,t} \quad = \quad \frac{1}{1+\tilde{\Gamma}(u_t)}\bigg(\frac{\eta_L}{\eta_K} + \eta_L - \tilde{\Gamma}(u_t)\bigg), \tag{24}$$

where $\tilde{\Gamma}(u_t) = \frac{\alpha\big(1-\chi-\chi\Gamma(u_t)\big)}{\chi\Gamma(u_t)}$, for any interior solution.[20] Since $\tilde{\Gamma}(u_t)$ is strictly increasing in u_t, the growth rate of labor-saving technological change is decreasing in the unemployment rate. When

[20] See Appendix Section A.1.3 for a formal derivation.

unemployment falls, wages rise, generating greater incentives for producers to economize on labor inputs. Finally, applying the market clearing condition (5) yields

$$g_{K,t} \quad = \quad \eta_K - \frac{\eta_K}{\eta_L} g_{L,t}, \tag{25}$$

which captures the trade-off between the two types of technological progress.

Proposition 1. *The dynamics of the economy are given by a sequence* $\{y_t, k_t, c_t, r_t, u_t, g_{L,t}, g_{K,t}\}_{t=0}^{\infty}$ *that solves equations (19) – (25) for all periods* $t = 0, 1, 2\ldots,$ *together with initial conditions* u_{-1} *and* k_0*.*

Proof. The proposition follows from the preceding discussion. □

3.5 Balanced Growth Path

3.5.1 Characterization

In this subsection, I characterize the balanced growth path and highlight the key forces in the model. Throughout the remainder of the paper, I use asterisks to denote balanced growth levels.

Definition 1. *A* balanced growth path (BGP) *occurs when output, consumption, capital and technology all grow at constant rates.*

One of the primary goals of this paper is to understand the forces that generate constant unemployment in the presence of labor-saving technical change.

Remark. *On any BGP, the rate of unemployment is constant.*

Proof. The remark follows from equation (24) and the definition of a BGP. □

If the direction of technological progress is determined by profit-maximizing firms, investment in labor-saving technology will increase when labor costs are high. Labor costs, in turn, depend on unemployment. On a BGP, relative incentives for R&D in the two types of technology must be constant, implying that the unemployment rate must also be constant. Incentives for investment are captured by equation (24), which is derived from the research arbitrage equation (13).

With constant unemployment and technological growth rates, equations (19) – (22) are essentially identical to the standard neoclassical growth model with monopolistic competition and constant population. As a result, the steady state of this system will also exhibit the usual properties.

Remark. *On any BGP, the intensive-form representation of the economy must be in a steady state. This implies that aggregate variables* K_t*,* Y_t*, and* C_t *grow at factor* $(1 + g_K^*)^{\frac{\alpha}{1-\alpha}}$ *and that the real interest rate is constant.*

Proof. Equation (24) implies that the unemployment rate must be constant on a BGP. By definition, the technology growth rates are also constant. By equation (23), therefore, k_t must also be constant. This directly implies that y_t must also be constant by equation (19). With both k_t and u_t constant, r_t must also be constant by equation (22), which implies that c_t must be constant by equation (21). □

As noted in the previous subsection, these results highlight the importance of including land in the production function. The diminishing returns to effective capital imply that the dynamical system has all of the usual properties, even before considering the role of labor inputs.

I now turn to examing the link between the two types of technical change.

Remark. *On any BGP,*

$$(1+n)(1+g_L^*) = (1+g_K^*)^{\frac{\alpha}{1-\alpha}}, \tag{26}$$

which implies that aggregate variables K_t, Y_t, and C_t grow at factor $(1+n)(1+g_L^*)$ as in the standard neoclassical growth model.

Proof. The result follows from equation (23) and the preceding remark. □

The putty-clay model of directed technical change converges to the usual BGP and explains the standard stylized macroeconomic facts (Solow, 1994; Jones, 2016). In particular, the capital-output ratio, savings rate, and interest rate will all be constant. Moreover, output per worker and capital per worker will grow at the same constant rate.

This result further highlights the importance of directed technical change. The BGP relationship between the two types of technical change is driven by the requirement that unemployment be constant, which is guaranteed by research arbitrage. Thus, directed technical change is essential to explaining both the constant unemployment rate and the standard BGP stylized facts.

Combing equations (25) and (26) yields

$$(1+\eta_K - \frac{\eta_K}{\eta_L} g_{L,t}^*)^{\frac{\alpha}{1-\alpha}} = (1+g_L^*)(1+n), \tag{27}$$

which immediately implies that the BGP growth rate of labor-augmenting technology is unique. For the remainder of the analysis, I impose

$$(1+n) > (1+\eta_K)^{\frac{\alpha}{1-\alpha}}, \tag{A1}$$

such that $g_L^* > 0$, which is the empirically relevant case.

Now, equation (23) implies that the BGP unemployment rate is unique. In particular,

$$u^* = \arg\text{solve}\left\{ g_L^* - \frac{1}{1+\tilde{\Gamma}(u^*)}\left(\frac{\eta_L}{\eta_K} + \eta_L - \tilde{\Gamma}(u^*)\right) = 0 \right\}, \tag{28}$$

where g_L^* is given by the implicit solution to equation (27). Together, equations (27) and (28) demonstrate how the steady state unemployment rate is 'driven' by technology.[21] With unique rates of unemployment and technological progress, equations (19) – (22), the more standard aspects of the dynamical system, also have a unique steady-state.

Remark. *The balanced growth path is unique.*

Proof. The remark follows from the preceding discussion. □

To complete the characterization of the BGP, I now turn to analyzing factor shares, which I denote with $\kappa_{Q,t}$ for any factor Q. At all times, $Y_t = A_{L,t}L_t$. Together with the fact that all capital good producers make identical decisions, this yields

$$\kappa_{L,t} = \frac{w_t}{A_{L,t}} = \chi + \chi\Gamma(u_t), \tag{29}$$

which is constant on the BGP. Intuitively, the faction of output paid to workers is constant when the worker bargaining position is constant. This occurs when unemployment reaches its steady state value. The income shares of capital and land are given by

$$\kappa_{K,t} = \alpha^2[1 - \chi - \chi\Gamma(u_t)], \tag{30}$$

$$\kappa_{M,t} = (1-\alpha)[1 - \chi - \chi\Gamma(u_t)], \tag{31}$$

which are both constant on the BGP. As shown in Appendix Section A.1.2, the income share of R&D inputs is given by

$$\kappa_{R,t} = \alpha^2[1 - \chi - \chi\Gamma(u_t)]\frac{\eta_K}{1 + g_{K,t}}, \tag{32}$$

which is again constant on the BGP. All remaining income is paid out as profits to the capital good producers.

All of the results discussed above are summarized in Proposition 2.

Proposition 2. *There exists a* unique balanced growth path*, where each of the following holds true:*

1. *The relationship between technological growth rates is given by* $(1+n)(1+g_L^*) = (1+g_K^*)^{\frac{\alpha}{1-\alpha}}$.

2. *The unemployment rate is constant, which implies that aggregate employment grows at factor* $(1+n)$.

[21] The model focuses on this type of technology-driven unemployment and demonstrates how the forces of technical change and factor accumulation balance to yield a constant rate of unemployment. Of course, there are other factors, especially search and matching frictions, that contribute to unemployment. Existing work already demonstrates how these factors balance to yield a steady state rate of unemployment in the absence of labor-saving technical change (Pissarides, 2000; Rogerson et al., 2005). The current paper abstracts from matching frictions to focus on the role of technology. Integrating these two approaches to unemployment presents a particularly interesting avenue for future work.

3. *The real interest rate is constant.*

4. K_t, Y_t, *and* C_t *grow at factor* $(1+n)(1+g_L^*)$, *which implies that the capital-output ratio is fixed and that capital per worker and output per worker grow at constant rate* g_L^*.

5. *Factor shares are constant.*

Proof. The proposition follows from the preceding discussion. □

3.5.2 Comparative Statics

In this section, I discuss the effects of an increase in η_L, which captures the notion of a technological breakthrough. I am interested in three outcomes: the growth rate of labor productivity, the unemployment rate, and the labor share of income. The formal analysis is presented in Appendix Section A.3. In the appendix, I also discuss the comparative statics following from changes in χ and n. These results are discussed in Section 4.

Building on the law of motion for unemployment and market clearing for R&D resources, equation (27) describes a necessary condition for constant unemployment that is independent of incentives for R&D expenditure. From this equation, it is immediate that a technological breakthrough will increase the growth rate of labor productivity, g_L^*. Breakthroughs also affect the capital good producers' incentives for R&D investment, as demonstrated in equation (28). The unemployment rate affects the incentives for R&D investment, but does not appear in equation (27). The overall effect of a technological breakthrough on unemployment, therefore, depends on the magnitude of the change in incentives, as captured by equation (28), relative to the relationship given by equation (27).

In the most intuitive case, the unemployment rate increases following a breakthrough. A sufficient condition for this outcome is given by

$$\left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\tilde{\Gamma}(u^*)} > 1. \tag{A2}$$

The left hand side captures the change in incentives. Appendix Section A.3 shows that $\frac{dg_L^*}{d\eta_L} < 1$. A key goal of the calibration in Section 4.1 is to show that this condition is satisfied in the data. As demonstrated in equation (29), technological breakthroughs affect the labor share of income only via unemployment. When unemployment rises, the bargaining position of workers deteriorates, leading to a fall in the labor share.

Proposition 3. *A technological breakthrough leads to faster growth in labor-productivity, i.e.* $\frac{dg_L^*}{d\eta_L} > 0$. *Also, if Assumption (A2) holds, then* $\frac{du^*}{d\eta_L} > 0$ *and* $\frac{d\kappa_L^*}{d\eta_L} < 0$.

Proof. See Appendix Section A.3. □

3.6 Efficiency and the Transition Path

In this paper, I examine the welfare-relevant trade-offs induced by labor-saving technical change. It is beyond the scope of this paper to provide of theory of how to aggregate, say, simultaneous increases in unemployment and output per capita into a social welfare function. Despite these reservations, there is a fundamental sense in which investment in labor-saving technical change is socially inefficient in the presence of technology-induced unemployment. Labor-saving technical change is wasteful because it saves a resources, labor, that is not scarce. Policy could increase total output by redirecting R&D inputs towards labor-using technology and employing a greater number of workers.

Remark. *Consider an economy at some point t where K_t, N_t, and $A_{J,t-1}$, $J = K, L$ are given. If $L_t < N_t$ and $R_{L,t} > 0$, then it is possible to increase total output and decrease unemployment by shifting some R&D resources to $R_{K,t}$.*

Proof. It will always be the case that $\big(A_{K,t}(i)X_t(i)\big)^{\alpha} M^{1-\alpha} = A_{L,t}(i)L_t(i)\ \forall i$. Let $\epsilon > 0$ be arbitrarily small. Decreasing $R_{L,t}(i)$ by ϵ and adding these resources to $R_{K,t}(i)$ increases potential output by $\alpha A_{K,t}(i)^{\alpha} X_t(i)^{\alpha-1} M^{1-\alpha} \eta_K A_{K,t-1}\epsilon > 0$. Then, to increase actual output, it is necessary to employ more workers to ensure that the arguments of the production function are equal. Since $L_t < N_t$, this must be feasible for an arbitrarily small ϵ. □

The task-based model of Acemoglu and Restrepo (2016) also yields inefficiently high investment in labor-saving technologies. In that alternate setting, it is the labor supply decision that drives the inefficiency, and technologies are not factor-augmenting. The fact that two different approaches yield the same qualitative result reinforces the generality of this finding.

The inefficiency of labor-saving R&D investment plays an important role in the transition dynamics of the putty-clay model. In particular, technological breakthroughs create incentives for capital good producers to invest more heavily in labor-saving technologies. This will boost the long-run growth rate of the economy, but exacerbate short-run inefficiencies.

Proposition 4. *Consider an economy on a BGP at some point t, where K_t, N_t, and $A_{J,t-1}$ $J = K, L$ are given. An exogenous increase in η_L lowers output and increases unemployment in the short-run, relative to a baseline scenario without this exogenous shock. It also increases the growth rate of labor productivity and decreases the labor share of income.*

Proof. Research allocations are interior on the BGP. Thus, equation (24) must hold before the shock. This implies that an increase in η_L leads to an increase in both u_t and $g_{L,t}$ (it is not possible to have one without the other). By the resource constraint for R&D inputs, this leads to a decrease in $g_{K,t}$, as demonstrated in equation (25). Since $Y_t = \big(A_{K,t}K_t\big)^{\alpha} M^{1-\alpha}$, output decreases in the short run.[22] □

[22] The proof relies on the assumption that the amount of R&D inputs is fixed. This assumption is particularly realistic in the short run. As discussed in footnote 17, over longer horizons, fixed research inputs are a convenient stand in for the offsetting forces of diminishing R&D productivity and increasing research inputs.

4 Simulation

In this section, I present the calibration and simulation of the model. The calibration makes it possible to confirm that Assumption (A2) is satisfied in the data. The simulation allows for a more thorough investigation of the transition path when compared to the theoretical results presented in Section 3.6. The main analysis focuses on the effects of technological breakthrough, but I also explore the consequences of changes in bargaining institutions, captured by χ, and population growth.

It is necessary to assume a function form for $\Gamma(u_t)$. I assume

$$w_t(i) = \chi A_{L,t}(i) + \chi\nu(1-u_t)A_{L,t}. \tag{33}$$

Here, ν captures the relative importance of outside options in the bargaining process. The term $1-u_t$ captures the notion that increase in unemployment lead to lower wages.

The calibration procedure leaves one free parameter. Thus, I set the ratio $\frac{\eta_K}{\eta_L}$ exogenously. To separately identify these two parameters, it would be necessary to observe how R&D inputs within firms are divided between labor-saving and capital-saving technical change. For the baseline analysis, I assume that $\eta_K = \eta_L$. I also show that none of the key results are driven by this assumption. In this sense, the purpose of the calibration is to constrain, rather than estimate, the parameters of the model.

4.1 Calibration

As demonstrated in Section 3.5, the BGP of the putty-clay model closely resembles the standard neoclassical growth model. To calibrate the model, I use aggregate data from the United States. I take the period length to be ten years. Three parameters can be determined exogenously. I start by assuming log preferences, i.e., $\xi = 1$. I take $\beta = .860$ from Golosov et al. (2014), which uses an identical period length. Using data on the size of the labor force from the Bureau of Labor Statistics (BLS), I take $n = 0.15$ (1.4% per year). Details on data sources can be found in Appendix Section A.5.

Together with the exogenous ratio $\frac{\eta_K}{\eta_L}$, a simple calibration procedure uses four equations to calibrate the remaining five unknown parameters, $\{\eta_K, \eta_L, \chi, \nu, \alpha\}$. The steps of the calibration closely follow the discussion on the uniqueness of the BGP in Section 3.5.1. As noted above, I assume that the research productivities are equal in the main analysis and demonstrate that all results are robust to alternate values.

As usual, α can be identified from factor shares. Combining equations (29) and (31) yields

$$\kappa_M^* = (1-\alpha)(1-\kappa_L^*). \tag{34}$$

Taking $\kappa_M^* = 5\%$ and $\kappa_L^* = 67\%$ from Valentinyi and Herrendorf (2008) gives $\alpha = 0.85$. The structure of the model yields a strict relationship between the technological growth rates on the

Table 1: Calibrated Parameters

Parameter	Value	Description	Target	Source
α	0.85	Physical capital share (excl. labor)	κ^*_M	Valentinyi and Herrendorf (2008)
η	0.31	R&D efficiency	g^*_L	BLS
$\chi\nu$	0.41	Bargaining power	u^*	BLS
ν	1.43	Bargaining power	κ^*_L	Valentinyi and Herrendorf (2008)

BGP given in equation (27), which is reproduced below for convenience:

$$(1+\eta_K - \frac{\eta_K}{\eta_L} g^*_L)^{\frac{\alpha}{1-\alpha}} = (1+n)(1+g^*_L).$$

This relationship captures the fact that labor-saving and labor-using technical change exactly balance in the long-run, resulting in constant unemployment. Taking $g^*_L = 0.24$ (2.2% per year) from BLS data on labor productivity yields $\eta_K = \eta_L = 0.31$.

The remaining parameters, χ and ν, capture worker bargaining power and the importance of outside options in the bargaining process. Their product can be identified from the relationship between the growth rate of labor-saving technical change and unemployment. I re-write equations (28), (29), (33) as

$$g^*_L = \frac{1}{1+\frac{\alpha(1-\kappa^*_L)}{\chi\nu(1-u^*)}}\left(\frac{\eta_L}{\eta_K} + \eta_L - \frac{\alpha(1-\kappa^*_L)}{\chi\nu(1-u^*)}\right). \tag{35}$$

Combining the previous results with an estimate of $u^* = 5.5\%$ from the BLS data implies that $\chi\nu = 0.34$. Finally, rearranging (29) with the functional form given by (33) yields

$$\chi = \kappa^*_L - \chi\nu(1-u^*), \tag{36}$$

which implies that $\chi = 0.34$ and $\nu = 1.43$.

Table 1 summarizes the results. It is important to note that $\left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\tilde{\Gamma}(u^*)} = 2.28 > 1$, which implies that $\frac{du^*}{d\eta_L} > 0$ and $\frac{d\kappa^*_L}{d\eta_L} < 0$. In words, technological breakthroughs increase unemployment and decrease the labor share of income.

For robustness, I also consider alternate assumptions about research productivity. The model and calibration procedure place an upper bound on the $\frac{\eta_K}{\eta_L}$. For example, Assumption (A1) describes a limit the size of η_K that must hold for the model to yield interior research allocations on the BGP. This condition is not binding. Equation (36) places an upper bound on $\chi\nu$, which is inversely related to η_L as shown in equation (35). Thus, I show that all results are robust to setting $\chi\nu \approx \frac{\kappa^*_L}{1-u^*}$. This yields $\eta_L = 0.28$, $\eta_K = 0.56$ and $\nu = 1.01$. Also, $\left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\tilde{\Gamma}(u^*)} = 1.95 > 1$. There is no lower bound on the ratio of research productivities. I present results with $\frac{\eta_K}{\eta_L} = 0.5$ to demonstrate the qualitative effect that low η_L has on the transition path.[23]

[23] In this case, $\eta_K = 0.19,\ \eta_L = .37,\ \chi = 0.51,$ and $\nu = 0.34$. Also, $\left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\tilde{\Gamma}(u^*)} = 2.34 > 1$.

4.2 Technological Breakthrough

I start by tracing out the dynamic impacts of a 10% increase in η_L. This represents a technological breakthrough that permanently decreases the cost of creating new labor-saving technologies. The results are presented in Figure 2. Given the difficulty of quantifying a technological breakthrough, the primary goal of this analysis is to understand the qualitative features of the transition path and steady state.

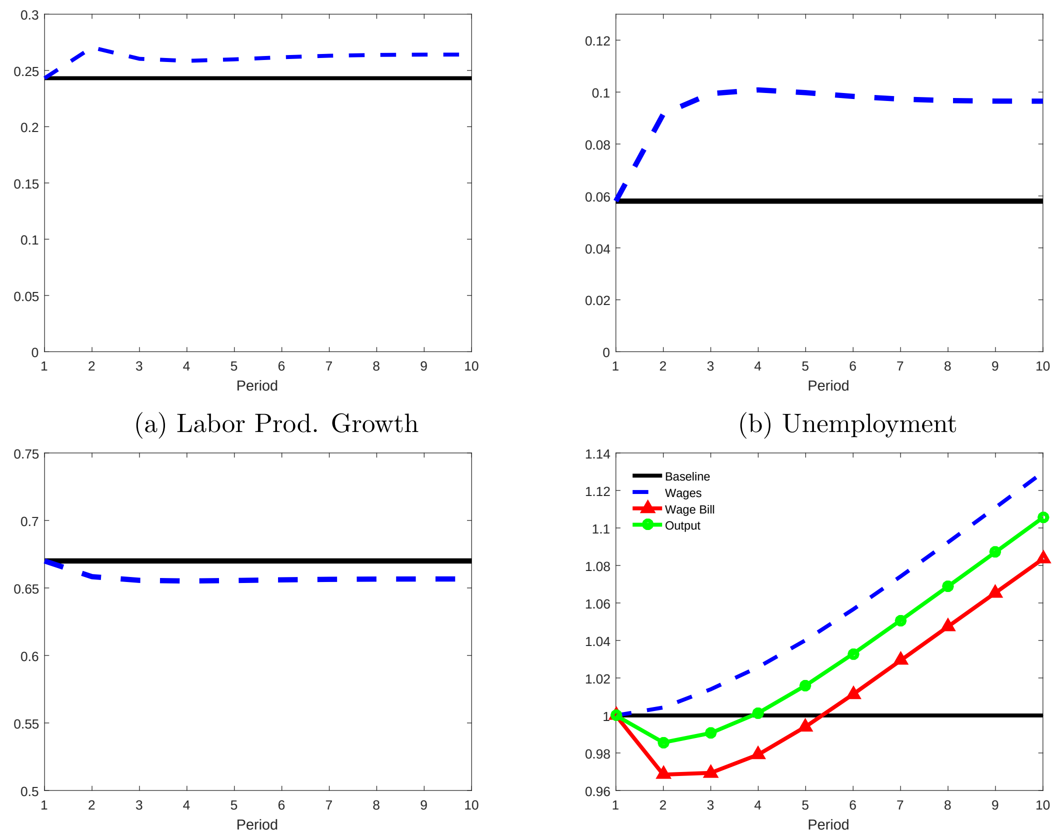
An increase in η_L leads to greater investment in labor-saving technologies. Since labor-saving technology also makes workers more productive, the growth rate of worker productivity increases, as demonstrated in panel (a). An increase in the rate of labor-saving technical change also leads to higher unemployment, as shown in panel (b). This is the fundamental trade-off associated with labor-saving technical change: when worker productivity increases, fewer workers are needed to operate the existing capital stock, holding all else equal. In the long-run, employed workers see faster wage growth, but a greater fraction of workers cannot find a job. Thus, there is a strong equity-efficiency trade-off associated with a technological breakthrough, even when looking within a homogeneous group of workers.

Panel (c) demonstrates the impact of a technological breakthrough on the labor share of income. Since higher unemployment lowers the bargaining power of workers, the labor share of income decreases. In this way, labor-saving technical change also induces an equity-efficiency trade-off when considering the distribution of income between factors of production.

Panel (d) shows results from three important aggregate variables that are relevant to the well-being of workers: total output, wages, and the total wage bill. As demonstrated in Section 3.6, an unexpected technological breakthrough leads to a short-run decrease in output. The formal proof took advantage of the fact that capital accumulation and lagged technologies were held constant in the very short run (i.e., within a period). The simulation demonstrates that it may take several periods before the economy catches up to where it would have been without the breakthrough. The breakthrough, however, permanently increases the growth rate of labor productivity. Thus, output eventually grows faster than in the original steady state.

The short-run reductions are even larger for the wage bill, since both output and the labor share decrease. The effect on wages, however, is more complicated. The increase in productivity pushes up wages, but the increase in unemployment undermines worker bargaining power, pushing them down. The net effect cannot be determined from the theoretical analysis. It is the only qualitative outcome that is sensitive the the exogenously determined ratio of research efficiencies. In the main analysis, where the research efficiencies are equal prior to the breakthrough, wages are essentially unchanged in the short-run, implying that these two forces offset. When the ratio $\frac{\eta_K}{\eta_L}$ is low, wages increase rapidly following a shock (Figure A.1). When $\frac{\eta_K}{\eta_L}$ is high, wages decrease following the shock (Figure A.4). In all cases, the wage bill decreases sharply and remains below baseline for many periods.[24]

[24] These results also show how factor-augmenting technologies can effectively capture the notion of labor-saving technical change when putty-clay production is taken into account. With a standard neoclassical production function



(a) Labor Prod. Growth (b) Unemployment

(c) Labor Share (d) Output and Wages

Figure 2: This figures traces the impact of a technological breakthrough that increases η_L by 10%. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.

4.3 Bargaining Institutions

The putty-clay model of directed technical change also presents a natural environment in which to study the effect of changes in bargaining institutions. In particular, I examine the effect of a 3.5% decrease in χ, which is almost sufficient to eliminate technology-driven unemployment. The results are presented in Figure 3.

Equation (27) demonstrates that a change in bargaining institutions has no long-run effect on the growth rate of labor productivity.[25] The BGP condition for relative rates of technological progress is unrelated to incentives for R&D investment. Panel (a) suggests that changes in bargaining institutions, even when large enough to almost eliminate technology-driven unemployment, also do

where all factors are paid their marginal products, factor-augmenting technologies almost always increase the marginal product of all factors (Acemoglu, 2010; Acemoglu and Autor, 2011). This fact has motivated the existing literature to focus on task-based models of automation (Acemoglu and Restrepo, 2016; Hémous and Olsen, 2016). In the putty-clay framework, the wages of some workers go to zero as labor-augmenting technology improves. This implies that labor-augmenting technological change is also labor-saving, a fact which drives the trade-offs central to the this model. Moreover, wages for employed workers can also decrease following a shock, but this result depends on the calibrated parameters.

[25]See Appendix Section A.3 for a formal analysis of the comparative statics.

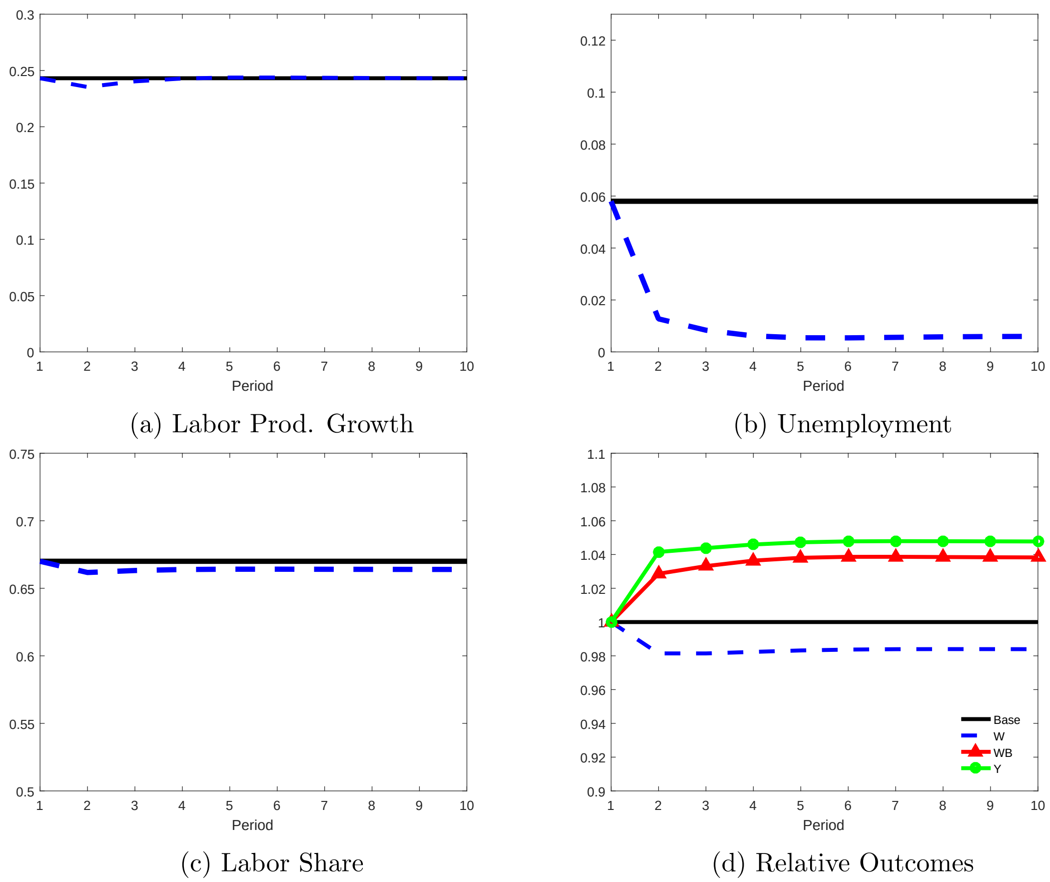


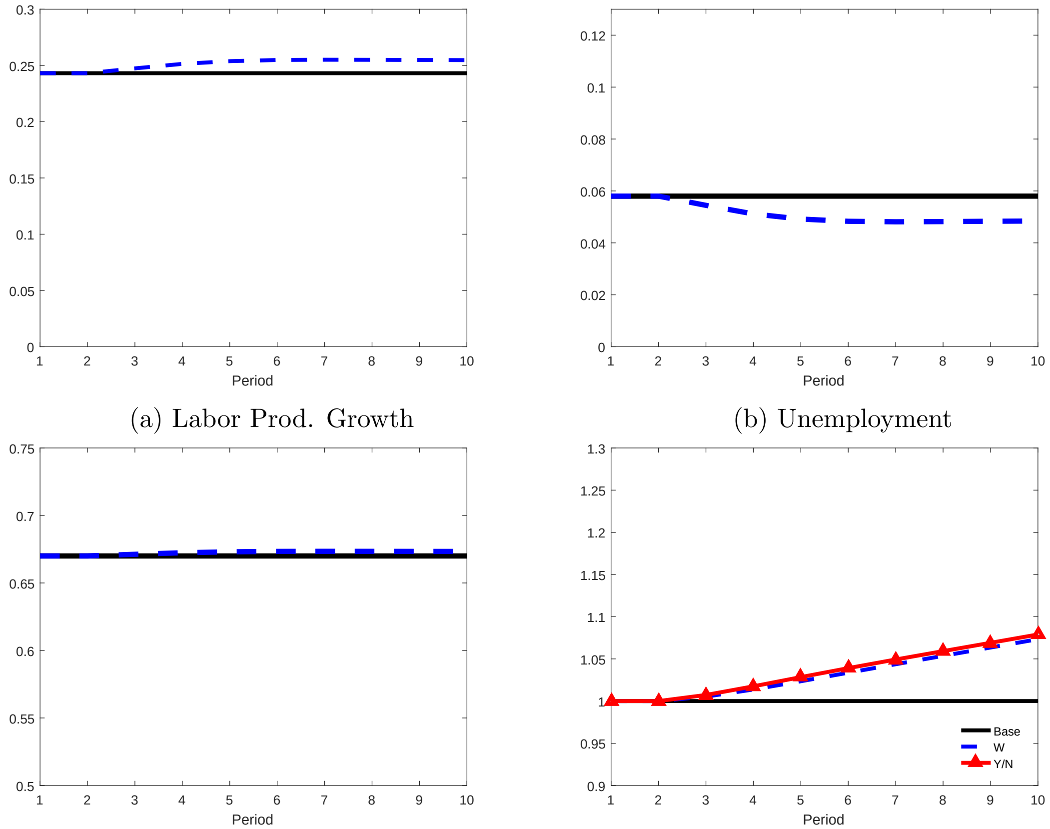
Figure 3: This figures traces the impact of a change in bargaining institutions that decreases χ by 3.5%, which nearly eliminates technology-driven unemployment. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.

not necessarily have a large impact on productivity growth on the transition path. Indeed, the economy reverts to the BGP quickly after the change in bargaining institutions.

While bargaining institutions do not affect long-run productivity growth rates, they do alter wages and incentives for R&D investment, as shown in equation (28). A reduction in χ, therefore, induces changes in unemployment such that long-run R&D incentives are unchanged. Specifically, a decrease in χ leads to a decrease in u^*, as shown in panel (b).

The fall in χ lowers the labor share of income. The fall in u^* has the opposite effect. Panel (c) demonstrates that the former effect is stronger for the calibrated model. Figures A.2 and A.5 present the robustness exercises. All results are qualitatively similar.

The inefficiencies in the model depend on labor market bargaining. Since there is surplus labor and no opportunity cost to working, wages would be zero in a perfectly competitive labor market with technology-driven unemployment. With zero wages, there would be no incentive to invest in inefficient labor-saving technologies. Decreases in the exogenous component of worker bargaining power lessen the degree of inefficiency, lowering unemployment and increasing output. As in the previous section, panel (d) examines output, wages, and the wage bill. Since productivity and the



(a) Labor Prod. Growth

(b) Unemployment

(c) Labor Share

(d) Relative Outcomes

Figure 4: This figures traces the impact of a decrease in population growth from 1.4%/year to .7%/year. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.

labor share both decrease, wages fall. Since output and the labor share move in opposite directions, however, the impact on the wage bill is ambiguous. Panel (c) shows that the decline in the labor share is small. As a result, the wage bill increases.

4.4 Population Growth

Finally, I examine the effect of changes in population growth on labor market outcomes. Unlike technological breakthroughs or changes in labor market institutions, exogenous changes in the population growth rate do not induce welfare-relevant trade-offs.[26] Figure 4 presents the results.[27]

At all times, effective capital and effective labor grow at the same rate. A decrease in the labor force growth rate, therefore, must lead to a reallocation of R&D inputs towards labor-saving (i.e., labor-augmenting) technology. This result is shown in panel (a). In a competitive equilibrium, this also implies that the incentive for such R&D is higher. In other words, unemployment decreases,

[26]Of course, this abstracts from the reason for a change in the population growth rate, which may itself have welfare consequences.

[27]Appendix Section A.3 presents a formal analysis of the comparative statics.

as shown in panel (b). The decline in unemployment increases the labor share, as demonstrated in panel (c). Panel (d) presents the results for wages and income per worker. Both productivity and the labor share of income increase, implying that wages and income per person also increase. Figures A.3 and A.6 present the results from the the alternate calibration outcomes, which are qualitatively similar.

5 Conclusion

There is a vigorous public debate about the effects of labor-saving technical change on unemployment and other labor market outcomes, as well as the appropriate response of government policy. These questions involves dynamic, general equilibrium outcomes. Growth theory should have much to add. Most modern growth theory, however, relies on the assumption of an aggregate production function with smooth *ex post* substitution between capital and labor. While convenient for many applications, this unrealistic assumption precludes the possibility that labor-saving technical change can cause structural unemployment via the saturation of labor demand. This limits the ability of economic theory to contribute to the public debate.

In this paper, I examine the long-run relationship between technological change and unemployment. To allow for a direct relationship between technology and unemployment, I consider the role of putty-clay production. In this setting, it is possible that labor demand will be insufficient to generate full employment at any positive wage. Economists have long considered models of putty-clay production to be more realistic than aggregate production functions (e.g., Johansen, 1959; Samuelson, 1962; Akerlof and Stiglitz, 1969). I use the model to ask two primary questions. First, I examine the forces that lead to a constant long-run rate of unemployment in the presence of labor-saving technical change. Together with labor market bargaining, endogenous and directed research activity plays a pivotal role in this regard. Second, I examine the consequences of a technological breakthrough that permanently reduces the cost of developing new labor-saving technologies. Breakthroughs do not cause labor to become obsolete, but they do introduce welfare-relevant trade-offs. In particular, they increase the long-run growth rate of wages and output, but increase unemployment and lower the labor share of income. They also exacerbate inefficiencies, leading to lower output in the short run.

I examine a simple, expository model in order to study the key mechanisms driving economic outcomes. Future work can further this line of inquiry by developing more elaborate, quantitative models to determine the precise consequences of policy intervention. One fruitful direction would be to combine the model developed here, which focuses on factor-augmenting technical change, with the task-based approach of Acemoglu and Autor (2011) and Acemoglu and Restrepo (2016). The hybrid model would be a natural environment in which to study the effects of labor-saving technology on workers with different types of skills and education (Hémous and Olsen, 2016). It would also be interesting to broaden the forces generating unemployment. I consider a very simple representation of labor market bargaining in order to focus on the role of technology. This is the opposite of most research on unemployment, which examines search frictions, but abstracts from labor-saving technical change (Pissarides, 2000; Rogerson et al., 2005). Finally, future work could shed further light on the long-run relationship between technical change and unemployment by considering endogenous human capital accumulation that allows workers to become scientists capable of inventing new technologies.

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A Appendix

A.1 Derivations

A.1.1 Final Good Producer

Let $\mathcal{L}_{FG}$ be the Lagrangian for the final good producer's maximization problem. Since the final good producer would never higher excess workers, it is immediate that $A_{L,t}(i)L_t(i) < X_t(i)^\alpha M^{1-\alpha}\ \forall i$. Thus,

$$\mathcal{L}_{FG} = \int_0^1 A_{L,t}(i)L_t(i)di - \int_0^1 w_t(i)L_t(i)di - \int_0^1 p_{X,t}(i)X_t(i)di - p_{M,t}M \\ - \int_0^1 \lambda_t(i)\big[A_{L,t}(i)L_t(i) - X_t(i)^\alpha M^{1-\alpha}\big]di. \quad \text{(A.1)}$$

With $\delta = 1$, there will never be unused capital. I focus on this case and ignore the complementary slackness condition. The first order conditions are given by

$$A_{L,t}(i) = w_t(i) + \lambda_t(i)A_{L,t}(i), \quad \text{(A.2)}$$

$$p_{X,t}(i) = \alpha\lambda_t(i)X_t(i)^{\alpha-1}M^{1-\alpha}, \quad \text{(A.3)}$$

$$p_{M,t} = (1-\alpha)\int_0^1 \lambda_t(i)X_t(i)^\alpha M^{-\alpha}di. \quad \text{(A.4)}$$

Rearranging (A.2) and plugging in to (A.3) and (A.4) yields

$$p_{X,t}(i) = \alpha[1 - \frac{w_t(i)}{A_{L,t}(i)}]X_t(i)^{\alpha-1}M^{1-\alpha}, \quad \text{(A.5)}$$

$$p_{M,t} = (1-\alpha)\int_0^1 [1 - \frac{w_t(i)}{A_{L,t}(i)}]X_t(i)^\alpha M^{-\alpha}di. \quad \text{(A.6)}$$

A.1.2 Capital Good Producers

Capital good producers choose prices, production quantities, and technological characteristics to maximize profits, subject to inverse demand, research productivity, and investment price constraints. Let $\mathcal{L}_{CG}$ be the Lagrangian for the capital good producer problem. Then,

$$\mathcal{L}_{CG} = p_{X,t}(i)X_t(i) - \frac{r_t}{A_{K,t}(i)}X_t(i) - p_{R,t}\big(R_{X,t}(i)+R_{L,t}(i)\big) - \sum_{J=L,K}\kappa_J[A_{J,t}(i) - \big(1+\eta_J R_{J,t}(i)\big)A_{J,t-1}] \\ - \upsilon_t(i)\left[p_{X,t}(i) - \alpha[1-\chi-\chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]X_t(i)^{\alpha-1}M^{1-\alpha}\right], \quad \text{(A.7)}$$

which applies the fact that $w_t(i) = \chi A_{L,t}(i) + \chi\Gamma(u_t)A_{L,t}$. The first order conditions are

$$\begin{aligned}
\upsilon_t(i) &= X_t(i), && \text{(A.8)}\\
p_{X,t}(i) &= \frac{r_t}{A_{K,t}(i)} - \upsilon_t(i)\alpha(\alpha-1)[1-\chi-\chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]A_{K,t}(i)^{\alpha}X_t(i)^{\alpha-2}M^{1-\alpha}, && \text{(A.9)}\\
\kappa_K &= r_t A_{K,t}(i)^{-2}X_t(i), && \text{(A.10)}\\
\kappa_L &= \upsilon_t(i)\alpha\Gamma(u_t)A_{L,t}A_{L,t}(i)^{-2}X_t(i)^{\alpha-1}M^{1-\alpha}, && \text{(A.11)}\\
p_{R,t} &= \kappa_K\eta_K A_{K,t-1}, && \text{(A.12)}\\
p_{R,t} &= \kappa_L\eta_L A_{L,t-1}. && \text{(A.13)}
\end{aligned}$$

Plugging (A.8) into (A.9) and applying (A.5) yields,

$$\begin{aligned}
p_{X,t}(i) &= \frac{1}{\alpha}\frac{r_t}{A_{K,t}(i)}, && \text{(A.14)}\\
X_t(i) &= \alpha^{\frac{2}{1-\alpha}}A_{K,t}^{\frac{1}{1-\alpha}}r_t^{\frac{-1}{1-\alpha}}[1-\chi-\frac{\chi\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]^{\frac{1}{1-\alpha}}M. && \text{(A.15)}
\end{aligned}$$

Putting these together yields

$$\bar{\pi}_{X,t}(i) = (1-\frac{1}{\alpha})\alpha^{\frac{2}{1-\alpha}}A_{K,t}^{\frac{\alpha}{1-\alpha}}r_t^{\frac{-\alpha}{1-\alpha}}[1-\chi-\frac{\chi\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]^{\frac{1}{1-\alpha}}M, \tag{A.16}$$

where $\bar{\pi}_{X,t}(i)$ is profits before considering payments to R&D inputs.

Taking the ratio of (A.11) and (A.10) yields

$$\frac{\kappa_L}{\kappa_K} = \frac{\alpha\chi\Gamma(u_t)A_{L,t}X_t(i)^{\alpha-1}M^{1-\alpha}A_{K,t}(i)}{\frac{r_t}{A_{K,t}(i)}A_{L,t}(i)^2} \tag{A.17}$$

Applying (A.8), (A.14), and (A.5) gives

$$\frac{\kappa_L}{\kappa_K} = \frac{\chi\Gamma(u_t)A_{L,t}A_{K,t}(i)}{\alpha A_{L,t}(i)^2[1-\chi-\chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]}. \tag{A.18}$$

Similarly, taking the ratio of (A.13) and (A.12) yields

$$1 = \frac{\kappa_L\eta_L A_{L,t-1}}{\kappa_K\eta_K A_{K,t-1}}. \tag{A.19}$$

Combining these expressions yields

$$1 = \frac{\chi\Gamma(u_t)A_{L,t}A_{K,t}(i)}{\alpha[1-\chi-\chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]A_{L,t}(i)^2}\frac{\eta_L A_{L,t-1}}{\eta_K A_{K,t-1}}, \tag{A.20}$$

which is the research arbitrage equation given in the main text. This expression can also be derived by maximizing (A.16) subject to law of motion for technology, equation (4).

A.1.3 Research Allocations

In this section, I derive the equilibrium research allocations. It is immediate from equation (A.20) that all capital good producers make identical decisions, which implies that $A_{J,t}(i) = A_{J,t}\ \forall i, t, J$. Since there is a unit mass of research inputs, this also implies that $R_{J,t}(i) = R_{J,t}\ \forall i, t, J$. Applying these results to (A.20) and rearranging yields

$$\tilde{\Gamma}(u_t)\frac{A_{L,t}}{A_{L,t-1}} = \frac{A_{K,t}}{A_{K,t-1}}\frac{\eta_L}{\eta_K}, \tag{A.21}$$

where $\tilde{\Gamma}(u_t) = \frac{\alpha\left(1-\chi-\chi\Gamma(u_t)\right)}{\chi\Gamma(u_t)}$. Applying (4) and (5) yields

$$\tilde{\Gamma}(u_t)\left(1 + \eta_L R_{L,t}\right) = \left(1 + \eta_K(1 - R_{L,t})\right)\frac{\eta_L}{\eta_K}. \tag{A.22}$$

Rearranging yields

$$\eta_L R_{L,t} = \frac{1}{1 + \tilde{\Gamma}(u_t)}\left(\frac{\eta_L}{\eta_K} + \eta_L - \tilde{\Gamma}(u_t)\right), \tag{A.23}$$

and noting that $\eta_L R_{L,t} = g_{L,t}$ gives (24) in the main text.

A.2 Factor Shares

Throughout this section, I take advantage of the fact that all capital good producers make identical decisions. As described in the main text

$$\frac{w_t L_t}{Y_t} = \frac{w_t}{A_{,t}} = \chi + \chi\Gamma(u_t). \tag{A.24}$$

Using equations (2) and (A.14), $r_t K_t = \alpha A_{K,t} p_{X,t} K_t = \alpha^2[1 - \chi - \chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]\left(A_{K,t}K_t\right)^{\alpha} M^{1-\alpha}$. So,

$$\frac{r_t K_t}{Y_t} = \alpha^2[1 - \chi - \chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]. \tag{A.25}$$

Similarly, equation (A.6) implies

$$\frac{p_{M,t}M_t}{Y_t} = (1-\alpha)[1 - \chi - \chi\frac{\Gamma(u_t)A_{L,t}}{A_{L,t}(i)}]. \tag{A.26}$$

Next, I solve for the rental rate paid to R&D inputs. Combining equations (A.8), (A.10), and (A.12) with market clearing condition (2) gives

$$p_{R,t} = \alpha^2[1 - \chi - \chi\Gamma(u_t)]\left(A_{K,t}K_t\right)^{\alpha} M^{1-\alpha}\eta_K\frac{1}{(1 + g_{X,t})}. \tag{A.27}$$

Noting that there is a unit mass of R&D inputs, this yields

$$\frac{p_{R,t}}{Y_t} = \alpha^2[1 - \chi - \chi\Gamma(u_t)]\eta_K \frac{1}{(1+g_{X,t})}. \tag{A.28}$$

The remainder of output is paid to capital good producers as profits.

A.3 Comparative Statics

In this section, I proof the results presented in Section 3.5.2. I also investigate the comparative statics for changes in χ and n. These results inform the analysis presented in Sections 4.3 and 4.4.

I start by considering the determinants of the growth rate of labor productivity, g_L^*. Using equation (27), I define

$$G(g_L^*, \eta_L, \eta_K, n, \chi) \equiv (1 + \eta_K - \frac{\eta_K}{\eta_L} g_L^*)^{\frac{\alpha}{1-\alpha}} - (1+n)(1+g_L^*) = 0. \tag{A.29}$$

By the implicit function theorem,

$$\frac{\partial g_L^*}{\partial n} = \frac{-G_n}{G_{g_L^*}} = \frac{(1+g_L^*)}{-\psi\frac{\eta_K}{\eta_L} - (1+n)} < 0, \tag{A.30}$$

where $\psi = \frac{\alpha}{1-\alpha}(1 + \eta_K - \frac{\eta_K}{\eta_L} g_L^*)^{\frac{2\alpha-1}{1-\alpha}}$. Assumption (A1) implies that $\psi > 0$. Moreover, since all of the arguments of G, other than g_L^*, are exogenous parameters, $\frac{\partial g_L^*}{\partial n} = \frac{dg_L^*}{dn}$. This will be true for all subsequent analyses as well.

Similarly,

$$\frac{dg_L^*}{d\chi} = \frac{-G_\chi}{G_{g_L^*}} = \frac{0}{-\psi\frac{\eta_K}{\eta_L} - (1+n)} = 0, \tag{A.31}$$

and

$$\frac{dg_L^*}{d\eta_L} = \frac{-G_{\eta_L}}{G_{g_L^*}} = \frac{-\psi\eta_K g_L^* \eta_L^{-2}}{-\psi\frac{\eta_K}{\eta_L} - (1+n)} \tag{A.32}$$

$$= \frac{\psi\eta_K g_L^*}{\psi\eta_K\eta_L + (1+n)\eta_L^2} \in (0,1). \tag{A.33}$$

The upper bound follows from the fact that $g_L^* \leq \eta_L$. The inequality is strict when research allocations are interior, which is guaranteed by Assumption (A1).

Next, I investigate the determinants of the steady state rate of unemployment, u^*. I use equation (28) to define

$$H\big(u^*, \eta_L, \eta_K, n, \chi\big) \equiv g_L^*(\eta_L, \eta_K, n, \chi) - \frac{1}{1+\tilde{\Gamma}(u^*, \chi)}\left(\frac{\eta_L}{\eta_K} + \eta_L - \tilde{\Gamma}(u^*, \chi)\right) = 0, \tag{A.34}$$

where $\tilde{\Gamma}(u^*,\chi) = \frac{\alpha\left(1-\chi-\chi\Gamma(u_t^*)\right)}{\chi\Gamma(u_t^*)}$. The function $g_L^*(\eta_L,\eta_K,n,\chi)$ is implicitly defined above. It is helpful to note that $H_{\tilde{\Gamma}(u^*)} > 0$ and $\tilde{\Gamma}_{u^*} > 0$. Thus, $H_{u^*} > 0$. Moreover, $\tilde{\Gamma}_\chi < 0$, implying that $H_\chi < 0$. By the implicit function theorem,

$$\frac{du^*}{dn} = \frac{-H_n}{H_{u^*}} = \frac{\frac{-dg_L^*}{dn}}{H_{u^*}} > 0, \tag{A.35}$$

and

$$\frac{du^*}{d\chi} = \frac{-H_\chi}{H_{u^*}} > 0. \tag{A.36}$$

Also,

$$\frac{du^*}{d\eta_L} = \frac{-H_{\eta_L}}{H_{u^*}} = \frac{\frac{-dg_L^*}{d\eta_L} - \left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\tilde{\Gamma}(u^*,\chi)}}{H_{u^*}}. \tag{A.37}$$

As a result,

$$\frac{du^*}{d\eta_L} > 0 \iff \tag{A.38}$$

$$\frac{dg_L^*}{d\eta_L} < \left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\tilde{\Gamma}(u^*,\chi)}, \tag{A.39}$$

where $\frac{dg_L^*}{d\eta_L} < 1$, as shown in the previous section.

Finally, I examine the determinants of the labor share of income. I define

$$I(\kappa_L^*,\eta_L,\eta_K,n,\chi) \equiv \kappa_L^* - \chi - \chi\Gamma(u^*) = 0. \tag{A.40}$$

By the implicit function theorem,

$$\frac{d\kappa_L^*}{d\eta_L} = \frac{-I_{\eta_L}}{I_{\kappa_L^*}} = \frac{-\chi\Gamma'(u^*)\frac{du^*}{d\eta_L}}{1}. \tag{A.41}$$

Since $\Gamma'(u^*) < 0$, $\frac{d\kappa_L^*}{d\eta_L} > 0 \iff \frac{du^*}{d\eta_L} > 0$. Similarly,

$$\frac{d\kappa_L^*}{dn} = \frac{-I_n}{I_{\kappa_L^*}} = \frac{-\chi\Gamma'(u^*)\frac{du^*}{dn}}{1} < 0 \tag{A.42}$$

and

$$\frac{d\kappa_L^*}{d\chi} = \frac{-I_n}{I_{\kappa_L^*}} = \frac{-(1+\Gamma'(u^*)\frac{du^*}{d\chi})}{1}. \tag{A.43}$$

Noting that $\Gamma'(u^*)\frac{du^*}{d\chi}) < 0$, $\frac{d\kappa_L^*}{d\chi} > 0 \iff \left|\Gamma'(u^*)\frac{du^*}{d\chi}\right| > 1$.

A.4 Microfounded Wage Determination

As explained in the main text, the expression for wages given in equation (9) captures the main features of wage determination in a search and matching model of bargaining. At the same time, it is also convenient for the modeling of endogenous and directed technical change. In this section, I present a very simple model of bargaining that yields the expression for wages given in the text. Rather than providing a comprehensive and realistic model of the markets, the purpose of this exercise is to show that the main equation is indeed capturing the major forces behind bargaining models.

For simplicity, I consider a matching and bargaining model in which all vacancies are filled instantaneously. I divide each period into two sub-period of equal duration. Between the two sub-periods, there is a shock that separates a fraction λ of workers form their jobs. At the beginning of the period, a continuum of size $L_t = \frac{A_{K,t}}{A_{L,t}} K_t^{\alpha} M^{1-\alpha}$ atomistic workers are each matched to a random job opening. They engage in Nash bargaining with the representative firm via a risk-neutral intermediary (e.g., a labor union). Then, between the two sub-periods, that shock occurs and a continuum λL_t of these jobs become vacant. Once again, the jobs are instantaneously matched to workers. The pool of workers includes those recently separated from their first sub-period jobs, a continuum of size λL_t, and those who were unemployed in the first sub-period, a continuum of size $N_t - L_t$. Again, wages are determined via Nash bargaining. All agents have perfect information and foresight when bargaining. Neither workers nor the firm can break contracts once they have been set.

A.4.1 Second sub-period bargaining

To start, I consider the bargaining process that occurs when workers meet firms at the beginning of the second sub-period. As noted in the main text, the firm rents capital before bargaining with workers. The rental costs, therefore, are sunk and not relevant to the bargaining surplus. All agents take the outcome of second sub-period bargaining as given at the beginning of the first sub-period.

There is no dis-utility from working and the worker bargains through a risk neutral intermediary. So, the worker gets payoff $\frac{1}{2} w_{t,2}(i)$ if they reach an agreement and 0 otherwise. Here, $w_{t,2}(i)$ is the wage that is proposed – and eventually paid – in the second period. Similarly, the payoff to the firm is $\frac{1}{2}\big(A_L(i) - w_{t,2}(i)\big)$ if they reach an agreement and 0 otherwise.

Nash bargaining maximizes the surplus. As usual, therefore, the negotiated wage is given by

$$w_{t,2}(i) = \text{argmax}\Big\{\beta \ln\big[A_L(i) - w_{t,2}(i)\big] + (1-\beta)\ln\big[w_{t,2}(i)\big]\Big\}, \tag{A.44}$$

where β is the bargaining power of the firm. This yields

$$w_{t,2}(i) = (1-\beta) A_{L,t}(i). \tag{A.45}$$

All workers and firms make this deal, implying that no job is left unfilled. Both workers and the representative firm take this outcome as given when they consider undertake bargaining at the beginning of the first period. Since workers do not know the job to which they will be matched in the second sub-period, they will take into account the average wage, $w_{2,t} = (1-\beta)A_{L,t}$, when bargaining, where $A_{L,t} = \int_0^1 A_{L,t}(i)di$ as in the main text.

At the beginning of the second sub-period, there are two groups of workers looking for jobs: those who were unemployed during the first sub-period and those that experience the job separation shock. Each worker has an equal probability of being matched to a job at the beginning of the second sub-period. The probability that a searching worker finds a match, therefore, is given by

$$\frac{\text{\# of openings}}{\text{\# of searching workers}} = \frac{\lambda L_t}{(\lambda-1)L_t + N_t} \tag{A.46}$$

$$= \left[\frac{\lambda-1}{\lambda} + \frac{1}{\lambda}\frac{1}{1-u_t}\right]^{-1} \tag{A.47}$$

$$= \left[\frac{\lambda(1-u_t)}{1-(1-\lambda)(1-u_t)}\right] \tag{A.48}$$

$$\equiv \hat{\Gamma}(u_t), \tag{A.49}$$

where $u_t = 1 - \frac{L_t}{N_t}$. It is helpful to note that $\hat{\Gamma}(u_t) \in (0,1)$ and $\hat{\Gamma}'(u_t) < 0$.

A.4.2 First sub-period bargaining

Now, I consider the bargaining that occurs at the beginning of the first sub-period. Noting that there is no down time and a job is filled with certainty in the second sub-period, the payoff to the representative firm if it reaches a first sub-period deal is $\frac{1}{2}(2-\lambda)\big[A_{L,t}(i) - w_{t,2}\big] + \lambda\frac{1}{2}\beta A_{L,t}(i)$. The firm's payoff if a deal is not reached is $\frac{1}{2}\beta A_{L,t}(i)$. Thus, the surplus from reaching a deal is $\frac{1}{2}\left[\big((2-\lambda)+\beta(\lambda-1)\big)A_{L,t}(i) - (2-\lambda)w_{1,t}(i)\right]$.

Meanwhile, the payoff to the worker of reaching a deal is $(1-\lambda\frac{1}{2})w_{t,1}(i) + \lambda\frac{1}{2}\hat{\Gamma}(u_t)(1-\beta)A_{L,t}$. The payoff if a deal is not reach is $\frac{1}{2}\hat{\Gamma}(u)A_{L,t}$. Thus, the surplus from reaching a deal in the first sub-period is $\frac{1}{2}\left[(2-\lambda)w_{t,1}(i) + (\lambda-1)\hat{\Gamma}(u_t)A_{L,t}\right]$.

Now, Nash bargaining in the first sub-period yields

$$\begin{aligned} w_{1,t}(i) = \operatorname{argmax}\Big\{&\beta \ln\big[\big((2-\lambda)+\beta(\lambda-1)\big)A_{L,t}(i) - (2-\lambda)w_{1,t}(i)\big] \\ &+ (1-\beta)\ln\big[(2-\lambda)w_{t,1}(i) + (\lambda-1)\hat{\Gamma}(u_t)A_{L,t}\big]\Big\}, \end{aligned} \tag{A.50}$$

which yields

$$w_{t,1}(i) = \frac{(1-\beta)}{2-\lambda}\big[(2-\lambda)+\beta(\lambda-1)\big]A_{L,t}(i) + \frac{\beta(1-\lambda)}{2-\lambda}\hat{\Gamma}(u_t)A_{L,t}. \tag{A.51}$$

A.4.3 Average wages

When firms demand capital, they care about the associated expected wage bill. Similarly, the labor share of income is determined by total wages. In both cases, the relevant wage is given by

$$w_t(i) \quad = \quad (1 + \frac{\lambda}{2})w_{t,1}(i) + \frac{\lambda}{2}w_{t,2}(i). \tag{A.52}$$

Substituting (A.45) and (A.51) into (A.52) yields

$$w_t(i) = \frac{1}{2}(1-\beta)\big[2 - \beta(1-\lambda)\big]A_{L,t}(i) + \frac{1}{2}\beta(1-\lambda)\hat{\Gamma}(u_t)A_{L,t}, \tag{A.53}$$

an expression with the same form as equation (9). In particular, we can define $\chi = \frac{1}{2}(1-\beta)\big[2 - \beta(1-\lambda)\big]$ and $\Gamma(u_t) = \frac{\beta(1-\lambda)}{(1-\beta)\big[2-\beta(1-\lambda)\big]}\hat{\Gamma}(u_t)$. The central goal of this paper is to evaluate the qualitative behavior of the dynamical system. Thus, I use the alternate formulation from equation (**??**) for the quantitative analysis and comparative statics, because this yields simpler and more intuitive expressions.

A.5 Calibration Data

Income Share of Labor. Definition: Payments to labor as a share of GDP. Value in model: κ_L^*. Value in data: 67%. Source: Table 2 of Valentinyi and Herrendorf (2008).

Income Share of Land. Definition: Payments to land as a share of GDP. Value in model: κ_M^*. Value in data: 5%. Source: Table 2 of Valentinyi and Herrendorf (2008).

Unemployment Rate. Definition: Fraction of labor force not employed. Value in model: u^*. Value in data: 5.8%, annual average of civilian unemployment rate, 1948 – 2016. Source: Bureau of Labor Statistics (BLS) via FRED database.

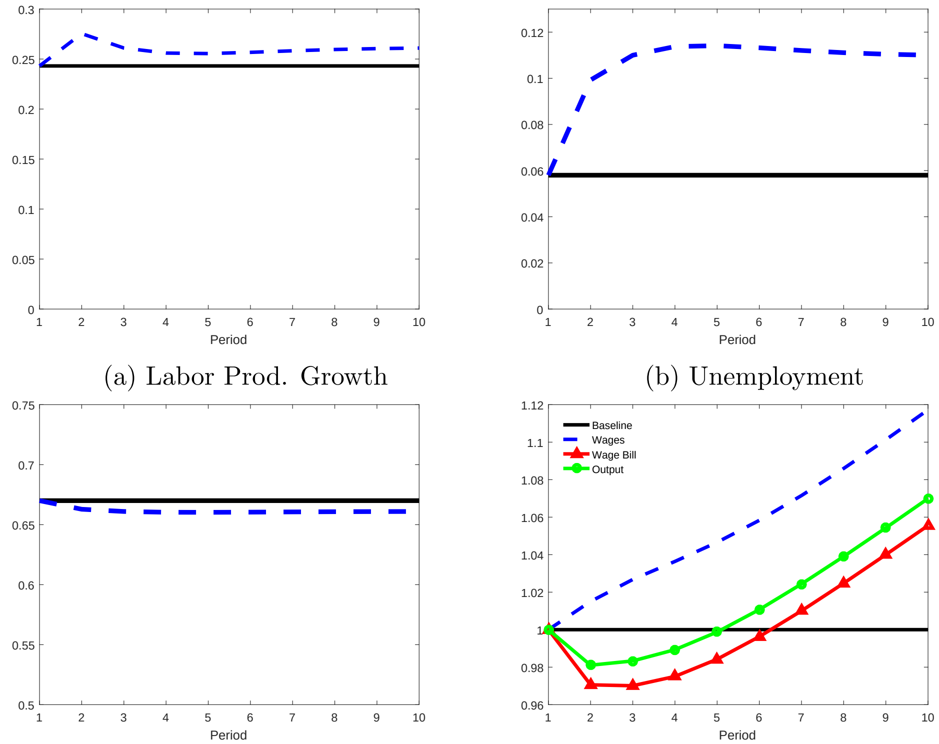
Labor Productivity Growth. Definition: Growth rate of output per worker. Value in model: g_L^*. Value in data: 2.2%/year, geometric average of growth in real output per person in the nonfarm business sector, 1948 – 2016. Source: BLS via FRED.

Labor Force Growth. Definition: Growth rate of the labor force. Value in model: n. Value in data: 1.4%/year, geometric average of growth in civilian labor force, 1948 – 2016. Source: BLS via FRED.

A.6 Figures for Robustness Exercises

A.6.1 High η_L

This section presents the results when assuming that $\frac{\eta_K}{\eta_L} = 0.5$. The qualitative results are the same as the baseline case. Figure A.2 shows the impact of a 2.5% decrease in χ.



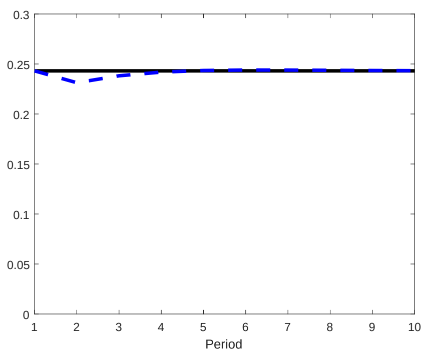
(a) Labor Prod. Growth

(b) Unemployment

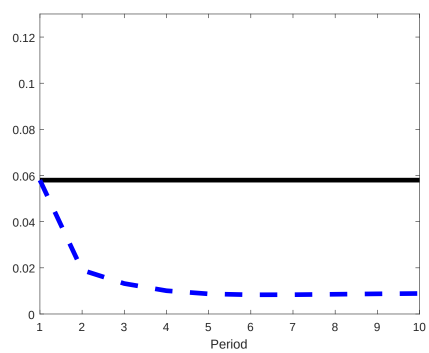
(c) Labor Share

(d) Output and Wages

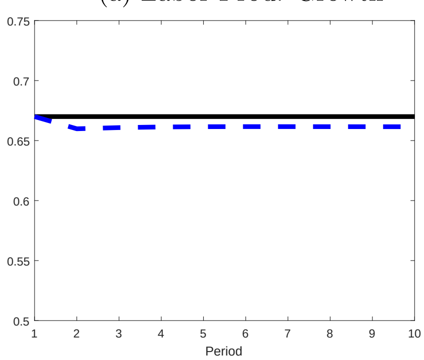
Figure A.1: This figures traces the impact of a technological breakthrough that decreases η_L by 10%. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.



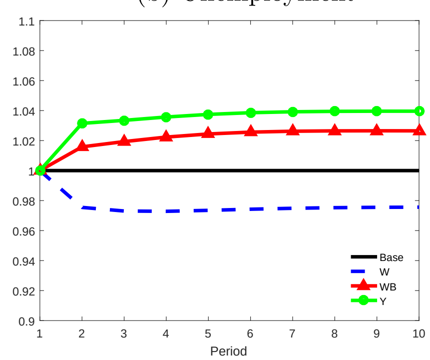
(a) Labor Prod. Growth



(b) Unemployment



(c) Labor Share



(d) Relative Outcomes

Figure A.2: This figures traces the impact of a change in bargaining institutions that decreases χ by 2.5%, which nearly eliminates technology-driven unemployment. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.

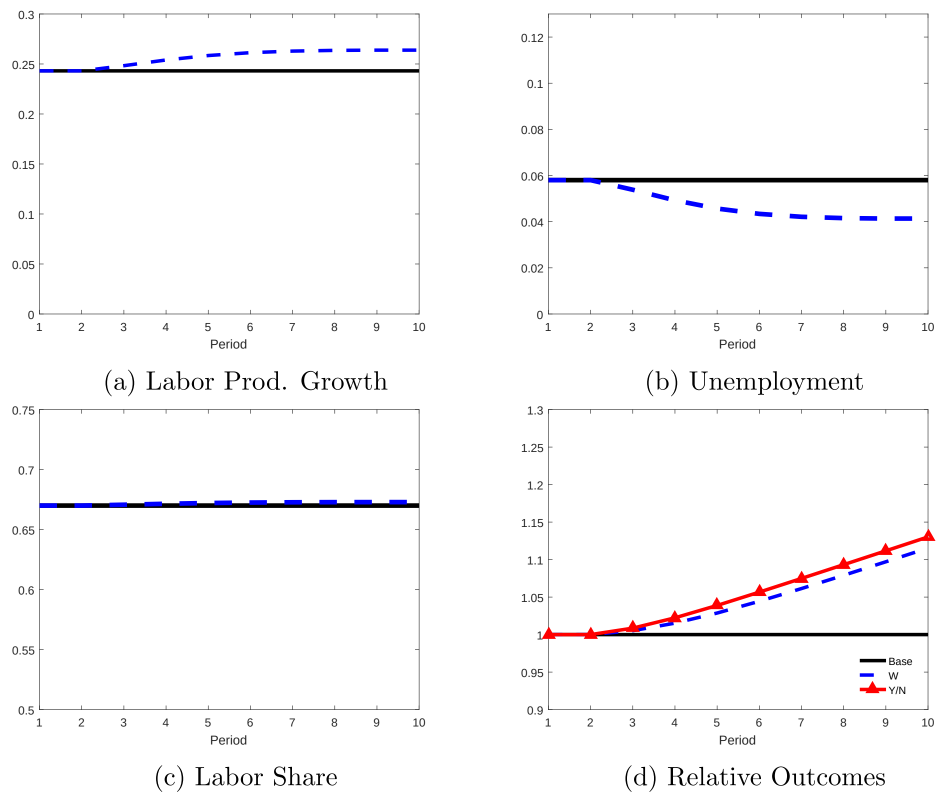
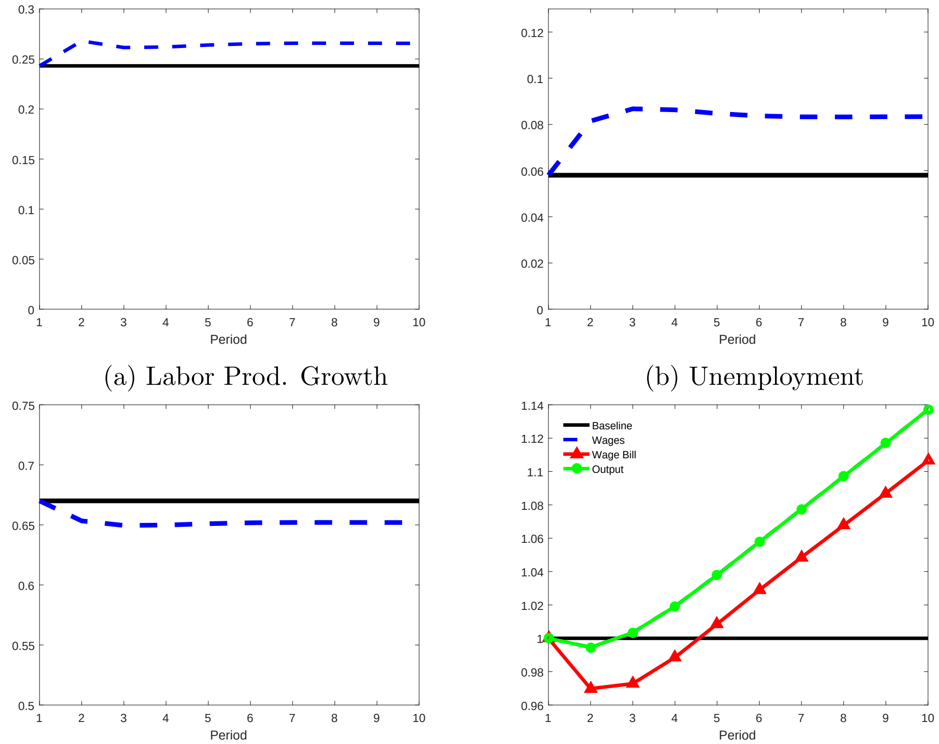


Figure A.3: This figures traces the impact of a decrease in population growth from 1.4%/year to .7%/year. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.

A.6.2 Low η_L

This section presents the results when assuming that $\frac{\eta_K}{\eta_L} = 2$. The qualitative results are the same as the baseline case. Figure A.2 shows the impact of a 4.2% decrease in χ.



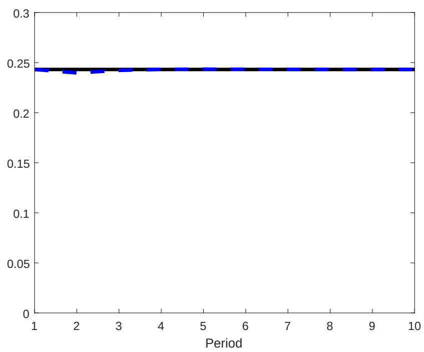
(a) Labor Prod. Growth

(b) Unemployment

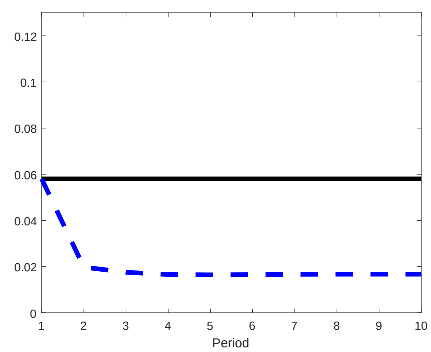
(c) Labor Share

(d) Output and Wages

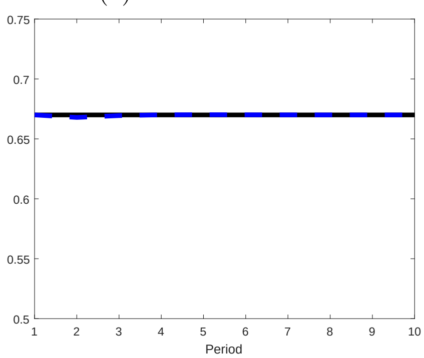
Figure A.4: This figures traces the impact of a technological breakthrough that decreases η_L by 10%. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.



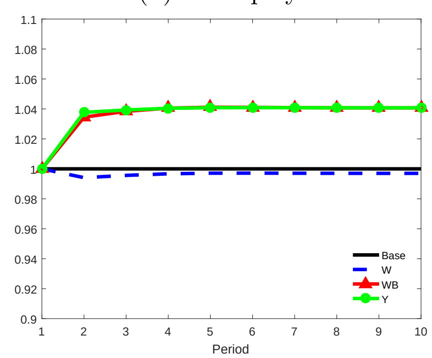
(a) Labor Prod. Growth



(b) Unemployment

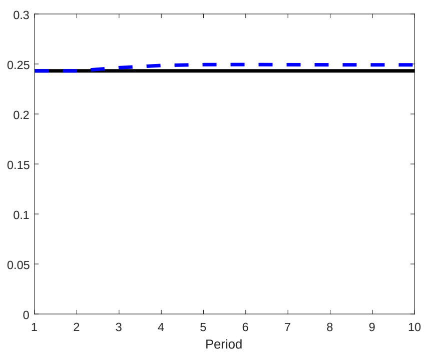


(c) Labor Share

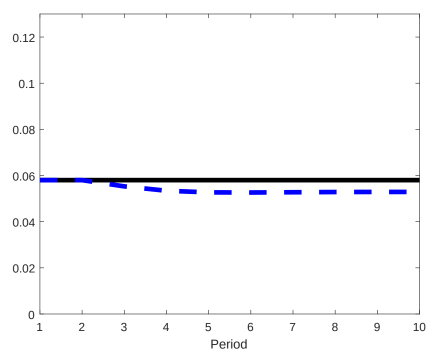


(d) Relative Outcomes

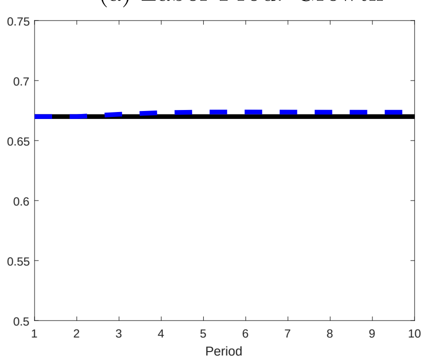
Figure A.5: This figures traces the impact of a change in bargaining institutions that decreases χ by 4%, which nearly eliminates technology-driven unemployment. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.



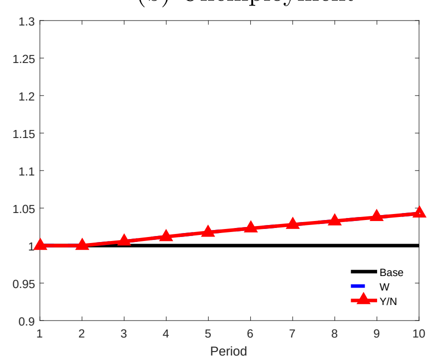
(a) Labor Prod. Growth



(b) Unemployment



(c) Labor Share



(d) Relative Outcomes

Figure A.6: This figures traces the impact of a decrease in population growth from 1.4%/year to .7%/year. All results are presented relative to a baseline scenario without a breakthrough. The economy starts on a balanced growth path.