

$$2(c) \ CE(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_i \left( y_i \log(\hat{y}_i) \right)$$

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| $\delta_1 = \frac{\partial CE}{\partial \mathbf{z}_2} =$ | $-\frac{\partial}{\partial \mathbf{z}_2} \sum_i \left( y_i \log(\hat{y}_i) \right)$                 | (1) |
| =  | $\hat{\mathbf{y}} - \mathbf{y}$ from 2(b), by substituting $\mathbf{z}_2$ for $\boldsymbol{\theta}$ | (2) |

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| $\delta_2 = \frac{\partial CE}{\partial \mathbf{h}} =$ | $\frac{\partial CE}{\partial \mathbf{z}_2} \frac{\partial \mathbf{z}_2}{\partial \mathbf{h}}$ | (3) |
| =  | $\delta_1 \frac{\partial (\mathbf{h} \mathbf{W}_2 + \mathbf{b}_2)}{\partial \mathbf{h}}$      | (4) |
| =  | $\delta_1 \mathbf{W}_2^T$   | (5) |

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| $\delta_3 = \frac{\partial CE}{\partial \mathbf{z}_1} =$ | $\frac{\partial CE}{\partial \mathbf{h}} \frac{\partial \mathbf{h}}{\partial \mathbf{z}_1}$                | (6) |
| =  | $\delta_1 \frac{\partial (\text{sigmoid}(\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1))}{\partial \mathbf{z}_1}$ | (7) |
| =  | $\delta_1 \frac{\partial (\sigma(\mathbf{z}_1))}{\partial \mathbf{z}_1}$                                   | (8) |
| =  | $\delta_1 \sigma'(\mathbf{z}_1)$   | (9) |

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| $\frac{\partial CE}{\partial \mathbf{x}} =$ | $\frac{\partial CE}{\partial \mathbf{z}_1} \frac{\partial \mathbf{z}_1}{\partial \mathbf{x}}$ | (10) |
| =   | $\delta_3 \frac{\partial (\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1)}{\partial \mathbf{x}}$      | (11) |
| =   | $\delta_3 \mathbf{W}_1^T$   |      |

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| $\frac{\partial CE}{\partial \mathbf{W}_2} =$ | $\frac{\partial CE}{\partial \mathbf{z}_2} \frac{\partial \mathbf{z}_2}{\partial \mathbf{W}_2}$ | (12) |
| $=$   | $\delta_1 \frac{\partial (\mathbf{h} \mathbf{W}_2 + \mathbf{b}_2)}{\partial \mathbf{W}_2}$      | (13) |
| $=$   | $\mathbf{h}^T \delta_1$   | (14) |

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| $\frac{\partial CE}{\partial \mathbf{b}_2} =$ | $\frac{\partial CE}{\partial \mathbf{z}_2} \frac{\partial \mathbf{z}_2}{\partial \mathbf{b}_2}$ | (15) |
| $=$   | $\delta_1 \frac{\partial (\mathbf{h} \mathbf{W}_2 + \mathbf{b}_2)}{\partial \mathbf{b}_2}$      | (16) |
| $=$   | $\mathbf{1}^T \delta_1$   | (17) |

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| $\frac{\partial CE}{\partial \mathbf{W}_1} =$ | $\frac{\partial CE}{\partial \mathbf{h}} \frac{\partial \mathbf{h}}{\partial \mathbf{z}_1} \frac{\partial \mathbf{z}_1}{\partial \mathbf{W}_1}$ | (18) |
| $=$   | $\delta_1 \mathbf{W}_2^T \sigma'(\mathbf{z}_1) \frac{\partial (\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1)}{\partial \mathbf{W}_1}$                 | (19) |
| $=$   | $\mathbf{x}^T \delta_1 \mathbf{W}_2^T \sigma'(\mathbf{z}_1)$  | (20) |

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| $\frac{\partial CE}{\partial \mathbf{b}_1} =$ | $\frac{\partial CE}{\partial \mathbf{h}} \frac{\partial \mathbf{h}}{\partial \mathbf{z}_1} \frac{\partial \mathbf{z}_1}{\partial \mathbf{b}_1}$ | (21) |
| $=$   | $\delta_1 \mathbf{W}_2^T \sigma'(\mathbf{z}_1) \frac{\partial (\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1)}{\partial \mathbf{b}_1}$                 | (22) |
| $=$   | $\mathbf{1}^T \delta_1 \mathbf{W}_2^T \sigma'(\mathbf{z}_1)$  | (23) |