

$$3(a) \frac{\partial(CE(\mathbf{y}, \hat{\mathbf{y}}))}{\partial \mathbf{v}_c}$$

=	$-\frac{\partial}{\partial \mathbf{v}_c} \sum_{w=1}^W \left(y_w \log(\hat{y}_w) \right)$	(1)
=	$-\frac{\partial}{\partial \mathbf{v}_c} \left(\left(y_1 \log \hat{y}_1 \right) + \left(y_2 \log \hat{y}_2 \right) + \dots + \left(y_o \log \hat{y}_o \right) + \dots + \left(y_W \log \hat{y}_W \right) \right)$	(2)
=	$-\frac{\partial}{\partial \mathbf{v}_c} \left(\left(y_1 \log \left(\frac{\exp(\mathbf{u}_1^T \mathbf{v}_c)}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right) \right) + \dots + \left(y_o \log \left(\frac{\exp(\mathbf{u}_o^T \mathbf{v}_c)}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right) \right) + \dots + \left(y_W \log \left(\frac{\exp(\mathbf{u}_W^T \mathbf{v}_c)}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right) \right) \right)$	(3)

If we look at the i^{th} term of Equation (3), it is of the form below:

=	$-\frac{\partial}{\partial \mathbf{v}_c} \left(y_i \log \left(\frac{\exp(\mathbf{u}_i^T \mathbf{v}_c)}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right) \right)$	(4)
=	$-\frac{\partial}{\partial \mathbf{v}_c} \left(y_i \left(\log(\exp(\mathbf{u}_i^T \mathbf{v}_c)) - \log \sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c) \right) \right)$	(5)
=	$-y_i \left(\frac{\partial(\mathbf{u}_i^T \mathbf{v}_c)}{\partial \mathbf{v}_c} - \frac{\partial}{\partial \mathbf{v}_c} \left(\log \sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c) \right) \right)$	(6)
=	$-y_i \left(\mathbf{u}_i - \left(\frac{1}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right) \frac{\partial}{\partial \mathbf{v}_c} \left(\sum_{x=1}^{x=W} \exp(\mathbf{u}_x^T \mathbf{v}_c) \right) \right)$	(7)
=	$-y_i \left(\mathbf{u}_i - \sum_{x=1}^{x=W} \frac{\frac{\partial}{\partial \mathbf{v}_c} (\exp(\mathbf{u}_x^T \mathbf{v}_c))}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right)$	(8)
=	$-y_i \left(\mathbf{u}_i - \sum_{x=1}^{x=W} \frac{\exp(\mathbf{u}_x^T \mathbf{v}_c) \mathbf{u}_x}{\sum_{m=1}^{m=W} \exp(\mathbf{u}_m^T \mathbf{v}_c)} \right)$	(9)

=	$-y_i \left(\mathbf{u}_i - \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right)$	(10)
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Substituting in Equation (3):

$\frac{\partial(CE(\mathbf{y}, \hat{\mathbf{y}}))}{\partial \mathbf{v}_c}$	$= -y_l \left(\mathbf{u}_l - \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right) - \dots - y_o \left(\mathbf{u}_o - \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right) - \dots - y_W \left(\mathbf{u}_W - \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right)$	(11)
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=	$y_l \left(-\mathbf{u}_l + \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right) + \dots + y_o \left(-\mathbf{u}_o + \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right) + \dots + y_W \left(-\mathbf{u}_W + \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x \right)$	(12)
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Since only **o** is the expected word (only $y_o = 1$), Equation (12) becomes:

=	$-\mathbf{u}_o + \sum_{x=1}^{x=W} \hat{y}_x \mathbf{u}_x$	(13)
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