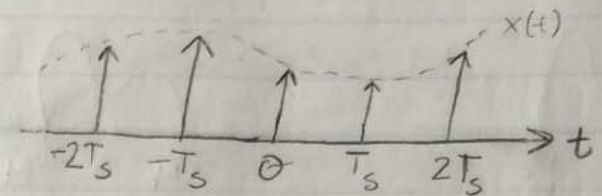
PS08
3/26/2015
Deniz Celik

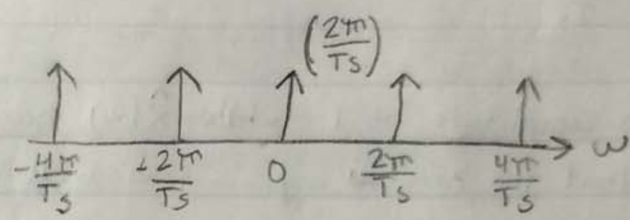
PS08 : Sig Sys

①

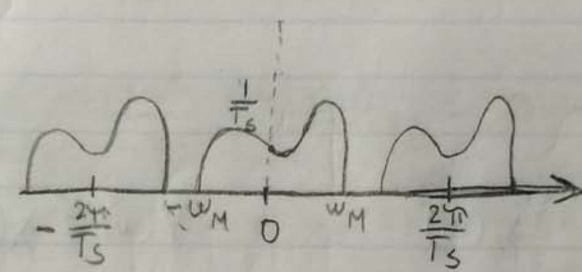
(a) $x_p(t)$



(b) $P(\omega)$



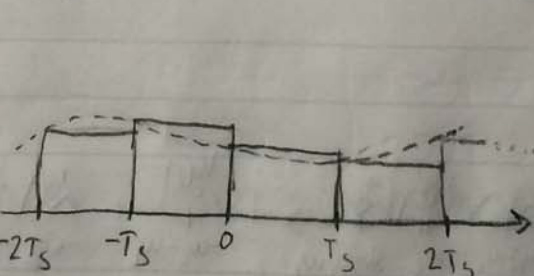
(c) $X_p(\omega)$



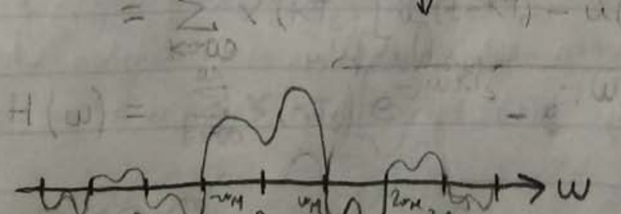
(D) $\frac{2\pi}{T_s} > 2\omega_M \rightarrow \omega_0 > 2\omega_M \rightarrow$ Nyquist frequency

(E) Multiply it by a new $H(\omega)$ that has value T_s from $-\omega_M$ to ω_M and is periodic, Then apply ideal low pass filter to $X_p(\omega)H(\omega)$ to recover $x(t)$

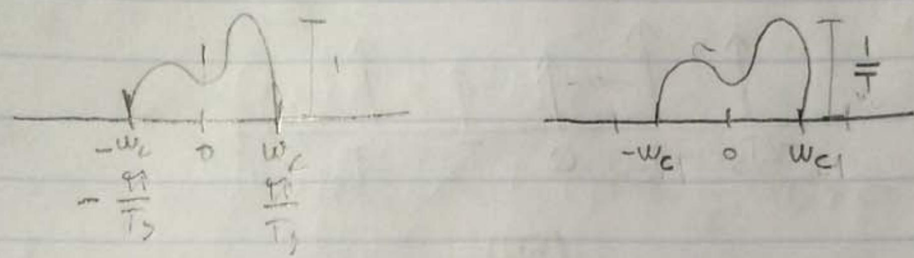
(G) $x_z(t)$



(H) $X_z(\omega) = X_p(\omega)\, Z(\omega) \rightarrow$ $z(t)$ is just half of a rectangular pulse shifted. So we know the transform of a rect pulse is $\text{sinc}(\omega \frac{T_s}{2})$. We then shift it: $e^{-j\omega\frac{T_s}{2}} \text{sinc}(\omega \frac{T_s}{2})$



(I) $\bar{X}(\omega) = X_2(\omega)H(\omega)$ $\qquad$ $\hat{X}(\omega) = X_p(\omega)H(\omega)$

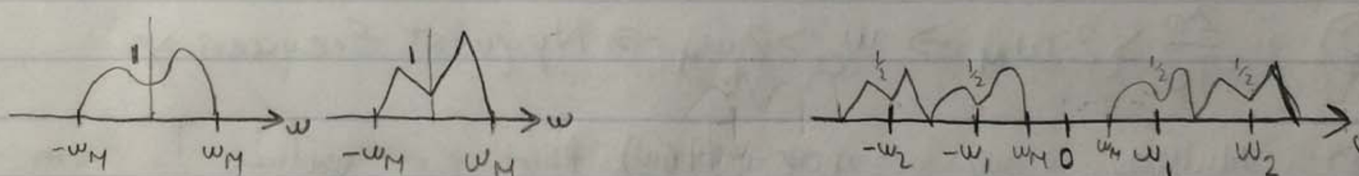


(j) $\bar{X}(\omega)$ has an amplitude of 1 while $\hat{X}(\omega)$ has an amplitude of $\frac{1}{T}$. It also happens that the zerohold approximation will not be an identical copy.

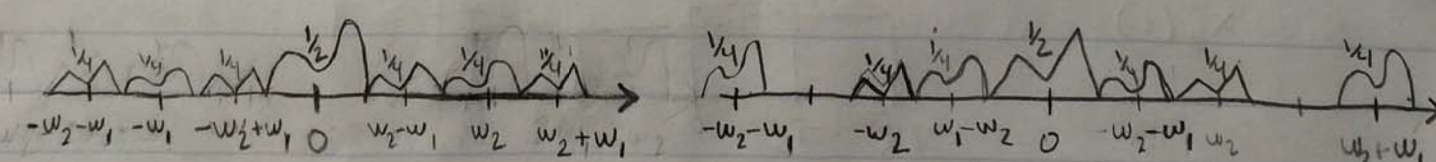
(k) $\frac{\bar{X}(\omega_M)}{\hat{X}(\omega_M)} = 1$

(2) $X_1(\omega)$ $\qquad$ $X_2(\omega)$ $\qquad$ $Y(\omega) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

(A)



(B) $\mathcal{F}\{y(t)\cos(\omega_1 t)\}$ $\qquad$ $\mathcal{F}\{y(t)\cos(\omega_2 t)\}$



(C) We sample at a high enough frequency ($>2\omega_c$) that we pick up the signal completely, we then apply an ideal low pass filter with amplitude 2. This filter has a frequency spectrum between $-\omega_M$ and ω_M. depending on which one we want to pickup we multiply $y(t)$ by either $\cos(\omega_1 t)$ or $\cos(\omega_2 t)$ respectively and apply the filter.

(3) $V_{in} = V_R(t) + V_L(t) + V_{out}(t) = R\,i(t) + L\frac{d}{dt}i(t) + V_{out} =$

(A) $= RC\frac{d}{dt}V_{out}(t) + LC\frac{d}{dt}\frac{d}{dt}V_{out}(t) + V_{out}$

$V_{in} = LC\frac{d^2}{dt^2}V_{out}(t) + RC\frac{d}{dt}V_{out}(t) + V_{out}$

(B) $V_{in}(w) = (jw)^2 LC\,V_{out}(w) + jwRC\,V_{out}(w) + V_{out}(w) = (1 + jwRC + j^2w^2LC)\,V_{out}(w)$

$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = \frac{1}{1+jwRC+j^2w^2LC} = \frac{1}{(1-w^2LC)+jwRC}$

$w \to 0 \quad |H(w)| \to 1$

$w \to \infty \quad |H(w)| \to 0$

Low Pass.

$|H(w)| = \frac{1}{\sqrt{(1-w^2LC)^2+(jwRC)^2}} = \frac{1}{\sqrt{1+w^2R^2C^2+C^2L^2w^4-2CLw^2}}$

(D) $\frac{d}{dw}|H(w)| = -\frac{Cw(2CL^2w^2+CR^2-2L)}{(C^2w^2(L^2w^2+R^2)-2CLw^2+1)^{3/2}} = 0 \rightarrow$

$\rightarrow \underset{w=0}{\underset{\uparrow}{\cancel{C}w}}(2CL^2w^2+CR^2-2L) = 0 \rightarrow 2CL^2w^2+CR^2-2L=0 \rightarrow w^2 = \frac{2L-CR^2}{2CL^2} \rightarrow$

$\rightarrow w = \sqrt{\frac{2L-CR^2}{2CL^2}}$ or 0