

$x(t)$  has amplitude  $A$ , period  $T$ , and  $\omega_0 = \frac{2\pi}{T}$ ,

$$\begin{aligned} \textcircled{3} \textcircled{A} C_0 &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{j k \omega_0 t} dt = \frac{1}{T} \int_{-T/2}^{T/2} A dt = \frac{A}{T} \left( \frac{T}{2} - \left( -\frac{T}{2} \right) \right) = \frac{A}{T} (T) = A \\ C_k &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j k \omega_0 t} dt = \frac{1}{T} \int_{-T/2}^{T/2} A e^{-j k \omega_0 t} dt = \frac{A}{T} \left( \frac{1}{-j k \omega_0} \right) e^{-j k \omega_0 t} \Big|_{-T/2}^{T/2} \\ &= \frac{A}{T} \left( \frac{1}{-j k \omega_0} \right) \left[ e^{-j k \omega_0 \frac{T}{2}} - e^{-j k \omega_0 \left( -\frac{T}{2} \right)} \right] = \frac{A}{j k 2\pi} \left( e^{j k \pi} - e^{-j k \pi} \right) = \end{aligned}$$

$$= \frac{A}{j k 2\pi} \left( e^{j k \pi} - e^{-j k \pi} \right) = \frac{A}{k\pi} \left( \frac{1}{2j} e^{j k \pi} - \frac{1}{2j} e^{-j k \pi} \right) \text{ Let's say } \theta = k\pi. \text{ Then,}$$

$$= \frac{A}{k\pi} \sin(\theta) = A \left( \frac{\sin(k\pi)}{k\pi} \right) = A \operatorname{sinc}(k)$$

$$C_k = A \operatorname{sinc}(k) \quad \therefore \text{Series} = C_0 + C_1 + C_2 + \dots$$

See Attached Graphs:

However this is only for odd:

$$C_{1,3,5,\dots} = A \operatorname{sinc} k$$

$$C_{2,4,6,\dots} = 0$$

© The value overshoots before coming back down to 0. This is due to the inability of the series to represent discontinuities.

(10) is the notes only works for signals that are continuous.

$$\textcircled{4} \textcircled{A} C_{ky} = \frac{1}{T} \int_{-T/2}^{T/2} x(t-T_1) e^{-j \frac{2\pi}{T} k t} dt$$

$$\text{Let's say } u = t - T_1; \quad t = u + T_1$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(u) e^{-j \frac{2\pi}{T} k (u+T_1)} du$$

$$= \left( e^{-j \frac{2\pi}{T} k T_1} \right) \frac{1}{T} \int_{-T/2}^{T/2} x(u) e^{-j \frac{2\pi}{T} k u} du = e^{-j \frac{2\pi}{T} k T_1} (C_{kx})$$

$$\boxed{C_{ky} = e^{-j \frac{2\pi}{T} k T_1} C_{kx}}$$