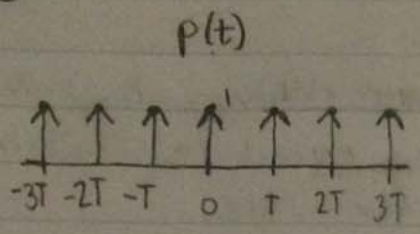
PS07 · Signals and Systems

Deniz Celik

1 (A)



(B)

Series $= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ where $\omega_0 = \frac{2\pi}{T}$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t) e^{-jk\omega_0 t} = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, dt = \frac{1}{T}$$

Series $\rightarrow \boxed{\sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk\omega_0 t}}$

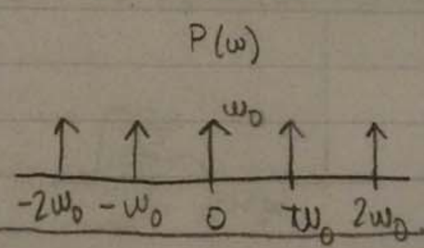
(C) $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 k t}$ where $\omega_0 = \frac{2\pi}{T}$

Transform of $e^{j\omega_0 k t} = 2\pi\delta(\omega - k\omega_0)$

$$X(\omega) = \sum_{k=-\infty}^{\infty} c_k 2\pi\delta(\omega - \omega_0) = \boxed{2\pi \sum_{k=-\infty}^{\infty} c_k \delta(\omega - k\omega_0)}$$

(D) $P(\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{T}\right)(2\pi\delta(\omega - \omega_0)) = \frac{2\pi}{T}\sum_{k-\infty}^{\infty} \delta(\omega - k\omega_0) = \boxed{\omega_0 p(\omega)}$

(E)

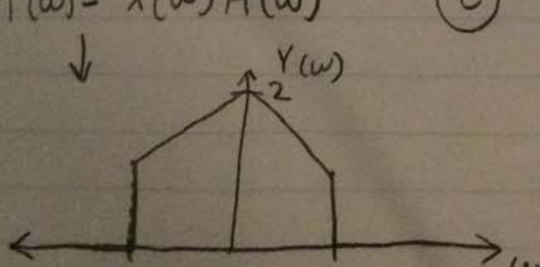


2 (A) $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi}\left(\int_{-\infty}^{-\omega_c} 0 e^{j\omega t} d\omega + \int_{\omega_c}^{\infty} 0 e^{j\omega t} d\omega + \int_{-\omega_c}^{\omega_c} 1 e^{j\omega t} d\omega\right) =$

$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t} d\omega = \frac{1}{2\pi}\left(+\frac{1}{jt} e^{j\omega_c t} - \frac{1}{jt} e^{-j\omega_c t}\right) = \frac{1}{\pi t}\left(\frac{1}{2j} e^{j\omega_c t} - \frac{1}{2j} e^{-j\omega_c t}\right) =$

$= \frac{1}{\pi t}\sin(\omega_c t) = \frac{\sin(\omega_c t)}{\pi t}$

(B) $Y(\omega) = X(\omega) H(\omega)$

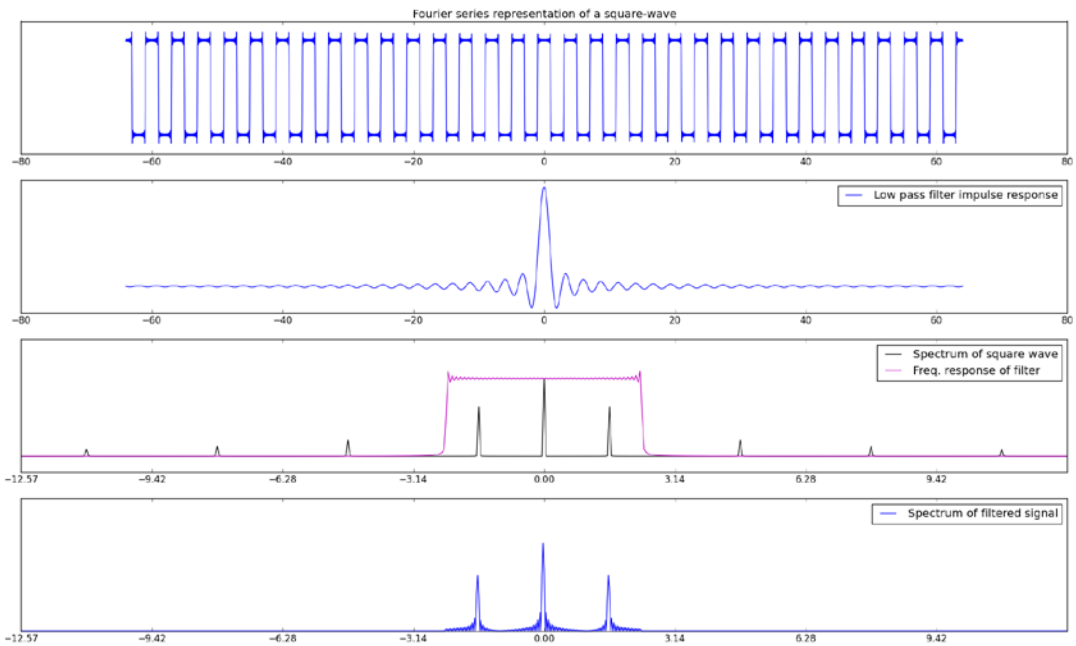


(C) This LTI system removes anything above ω_c and anything below $-\omega_c$ since $Y(\omega)$ is just multiplying $X(\omega)$ and $H(\omega)$. This results in the signal being preserved only between $-\omega_c$ and ω_c.

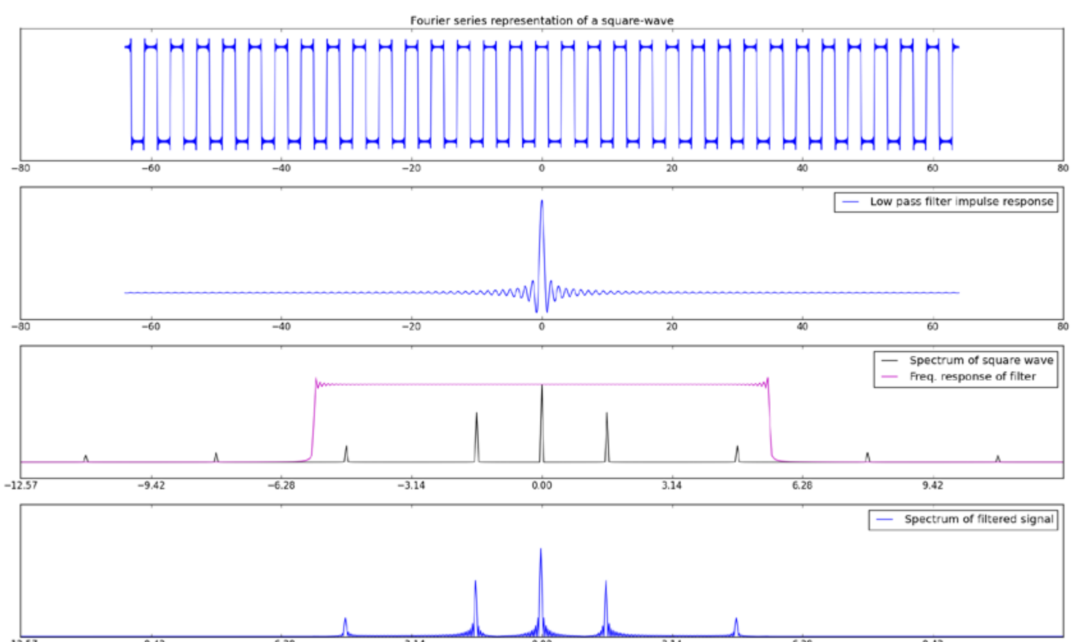
(D) See attached Graphs

Question 2:

ω_C = 0.75*π



ω_C = 1.75*π



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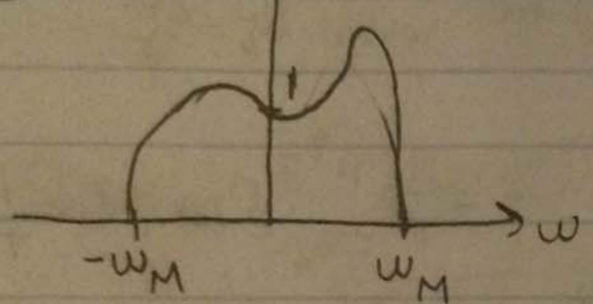
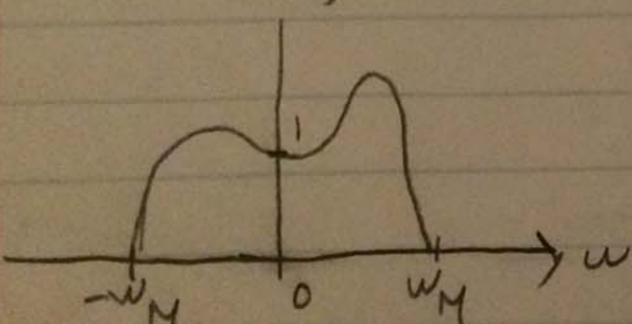


$y(t) = x(t)\cos(\omega_c t)$

We know that multiplying a signal in the time domain is equal to shifting in the frequency domain. ie.

$$e^{j\omega_0 t}x(t) \Longrightarrow X(\omega-\omega_0)$$

$$y(t) = \cos(\omega_c t)\,x(t)$$
$$= \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)x(t) = \frac{1}{2}\left(e^{j\omega_c t}x(t) + e^{-j\omega_c t}x(t)\right) \Longrightarrow$$
$$\Longrightarrow \frac{1}{2}\left(X(\omega-\omega_c) + X(\omega+\omega_c)\right) = \frac{1}{2}X(\omega-\omega_c) + \frac{1}{2}X(\omega+\omega_c)$$



$\Longrightarrow$

$Y(\omega) = \frac{1}{2}X(\omega-\omega_c) + \frac{1}{2}X(\omega+\omega_c)$