Signals and Systems: PS 06

Deniz Celik

(1) The shooting range acts as the impulse response, meaning that if we input some signal $x(t)$, in this the case, the output from the shooting range is some signal $y(t)$. Because the impulse (violin sound) must be modified by the impulse response $h(t)$ in the frequency domain, we know that in the time domain that is equivalent to convolution. i.e. $\underset{\text{output}}{y(t)} = \underset{\text{impulse}}{x(t)} * \underset{\text{impulse response}}{h(t)}$

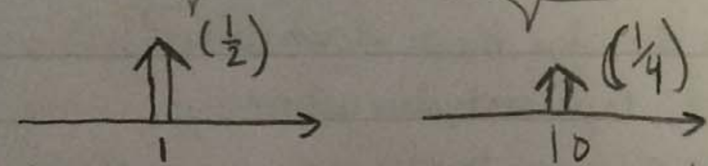
(2) We know that $y(t) = x(t) * h(t)$ so,

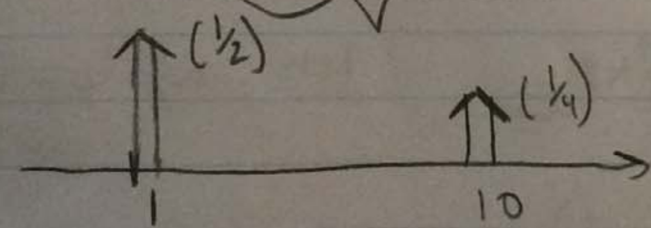
$$y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)$$

$$= \frac{1}{2}\left(\text{impulse at } t=1\right) + \frac{1}{4}\left(\text{impulse at } t=10\right)$$

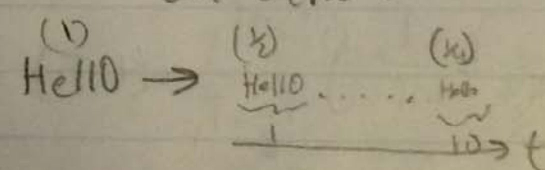
$$= \underbrace{\frac{1}{2}\delta(t-1)}_{} + \underbrace{\frac{1}{4}\delta(t-10)}_{}$$

=





This model can reasonably be called an echo channel since it returns a response 1 second later at half the amplitude and the 9 seconds after that it responds at a quarter of the amplitude. Such it would sound like an echo:



$x(t)$ has amplitude A, period T, and $\omega_0 = \frac{2\pi}{T}$,

3 A $C_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} A\, dt = \frac{A}{T}\left(\frac{T}{2} - \left(-\frac{T}{2}\right)\right) = \frac{A}{T}(T) = A$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} A e^{-jk\omega_0 t}\, dt = \frac{A}{T}\left(\frac{1}{-jk\omega_0}\right) e^{-jk\omega_0 t}\Big|_{-T/2}^{T/2} =$$

$$= \frac{A}{\cancel{T}}\left(\frac{1}{-jk\omega_0}\right)\left[e^{-jk\omega_0 \frac{T}{2}} - e^{-jk\omega_0\left(-\frac{T}{2}\right)}\right] = \frac{A}{jk2\pi}\left(e^{jk\pi} - e^{-jk\pi}\right) =$$

$$= \frac{A}{jk2\pi}\left(e^{jk\pi} - e^{-jk\pi}\right) = \frac{A}{k\pi}\left(\frac{1}{2j}e^{jk\pi} - \frac{1}{2j}e^{-jk\pi}\right) =$$ lets say $\theta = k\pi$. Then,

$$= \frac{A}{k\pi}\sin(\theta) = A\left(\frac{\sin(k\pi)}{k\pi}\right) = A\,\mathrm{sinc}(k) \therefore$$

$C_k = A\,\mathrm{Sinc}(k)$ $\therefore$ Series $= C_0 + C_1 + C_2 \dots$

However this is only for odds:

$C_{1,3,5,\dots} = A\,\mathrm{Sinc}\,k$

$C_{2,4,6,\dots} = 0$

B See Attached Graphs:

C The value overshoots 1 before coming back down to 1. This is due to the inability of the series to represent discontinuitys. (10) is the notes only works for signals that are continuous.

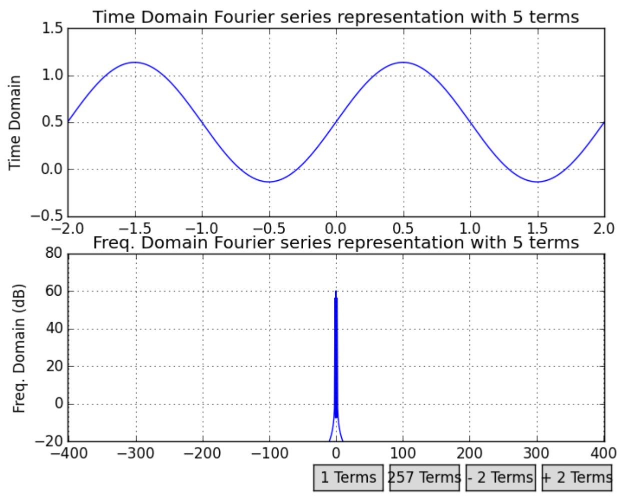
4 A $C_{ky} = \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(t-T_1)\, e^{-j\frac{2\pi}{T}kt}\, dt$ lets say $u = t - T_1$: $t = u + T_1$

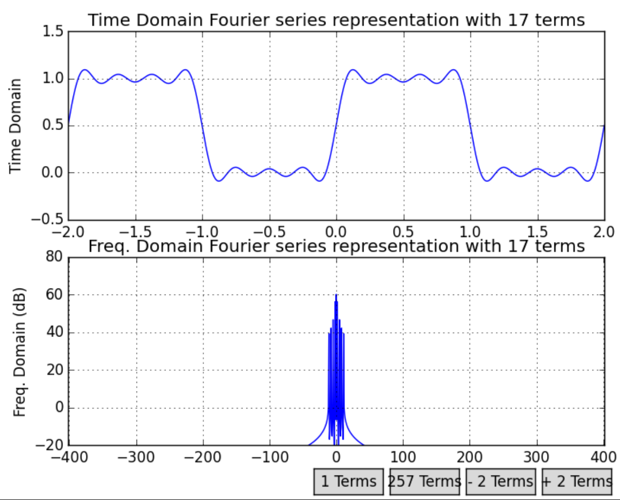
$$= \frac{1}{T}\int_{-T/2+(u-t)}^{T/2+(u-t)} x(u)\, e^{-j\frac{2\pi}{T}k(u+T_1)}\, du$$

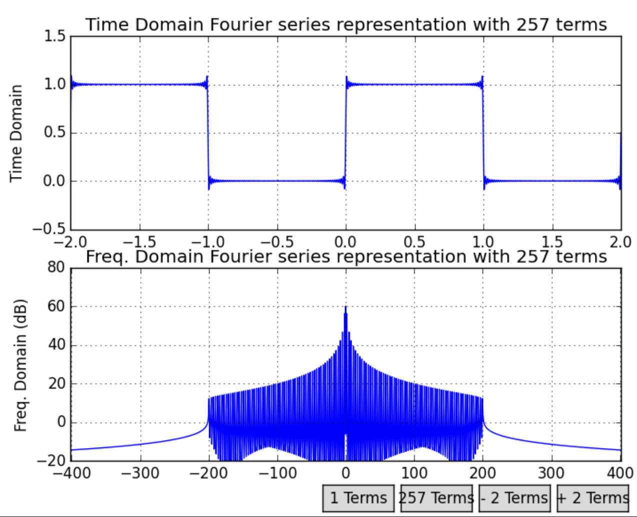
$$= \left(e^{-j\frac{2\pi}{T}kT_1}\right)\frac{1}{T}\int_{-T/2+(u-t)}^{T/2+(u-t)} x(u)\, e^{-j\frac{2\pi}{T}k(u)}\, du = e^{-j\frac{2\pi}{T}kT_1}\left(C_{kx}\right)$$

$$\boxed{C_{ky} = e^{-j\frac{2\pi}{T}kT_1} C_{kx}}$$

Question 3B:

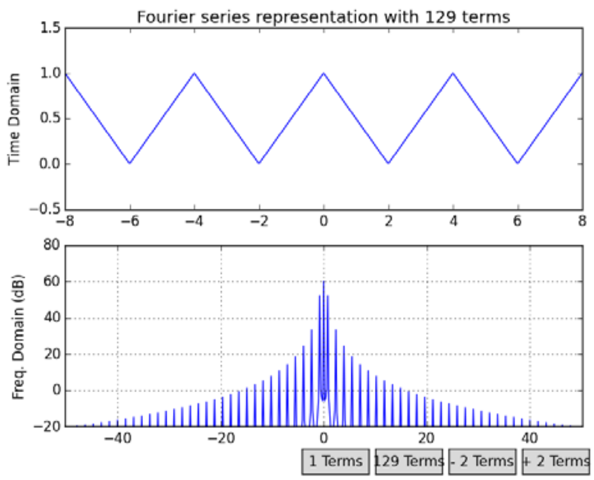

Time Domain Fourier series representation with 5 terms
Time Domain
Freq. Domain Fourier series representation with 5 terms
Freq. Domain (dB)
1 Terms
257 Terms
- 2 Terms
+ 2 Terms


Time Domain Fourier series representation with 17 terms
Time Domain
Freq. Domain Fourier series representation with 17 terms
Freq. Domain (dB)
1 Terms
257 Terms
- 2 Terms
+ 2 Terms


Time Domain Fourier series representation with 257 terms
Time Domain
Freq. Domain Fourier series representation with 257 terms
Freq. Domain (dB)
1 Terms
257 Terms
- 2 Terms
+ 2 Terms

Question 4B:

Graphs:



Code:

```
def fs_triangleDENIZ(ts, M=3, T=4):
    # computes a fourier series representation of a triangle wave
    # with M terms in the Fourier series approximation
    # if M is odd, terms -(M-1)/2 -> (M-1)/2 are used
    # if M is even terms -M/2 -> M/2-1 are used

    # create an array to store the signal
    x = np.zeros(len(ts))

    # if M is even
    if np.mod(M,2) ==0:
        for k in range(-int(M/2), int(M/2)):
            # if n is odd compute the coefficients
            if np.mod(k, 2)==1:
                Coeff = -2/((np.pi)**2*(k**2))
            if np.mod(k,2)==0:
                Coeff = 0
            if k == 0:
                Coeff = 0.5
            x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)*np.exp(-1j*np.pi*k)

    # if M is odd
    if np.mod(M,2) == 1:
        for k in range(-int((M-1)/2), int((M-1)/2)+1):
           # if n is odd compute the coefficients
            if np.mod(k, 2)==1:
                Coeff = -2/((np.pi)**2*(k**2))
            if np.mod(k,2)==0:
                Coeff = 0
            if k == 0:
                Coeff = 0.5
            x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)*np.exp(-1j*np.pi*k)

    return x
```

4) B

The wave from the notes is shifted by $T/2$. therefore our new coeff is:

$$C_{new} = e^{-j\frac{2\pi}{T}k\frac{T}{2}} C_k = \boxed{e^{-j\pi k} C_k}$$

See attached code and graph: