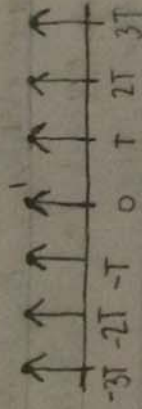


PS07: Signals and Systems

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① A

$p(t)$



③

Series = $\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ where $\omega_0 = \frac{2\pi}{T}$

$$c_k = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) dt = \frac{1}{T}$$

Series $\rightarrow \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk\omega_0 t}$

③ $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ where $\omega_0 = \frac{2\pi}{T}$

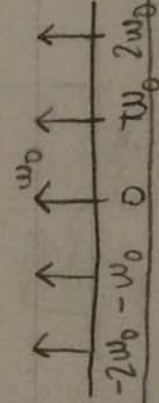
Transform of $e^{jk\omega_0 t} = 2\pi \delta(\omega - k\omega_0)$

$$X(\omega) = \sum_{k=-\infty}^{\infty} c_k 2\pi \delta(\omega - k\omega_0) = 2\pi \sum_{k=-\infty}^{\infty} c_k \delta(\omega - k\omega_0)$$

④ $P(\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{T}\right) (2\pi \delta(\omega - k\omega_0)) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0) = \boxed{\omega_0 \rho(\omega)}$

⑤

$P(\omega)$



②

④ $h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left(\int_{-\omega_c}^{\omega_c} \delta(\omega) e^{j\omega t} d\omega + \int_{\omega_c}^{\infty} 0 e^{j\omega t} d\omega + \int_{-\infty}^{-\omega_c} 0 e^{j\omega t} d\omega \right) =$

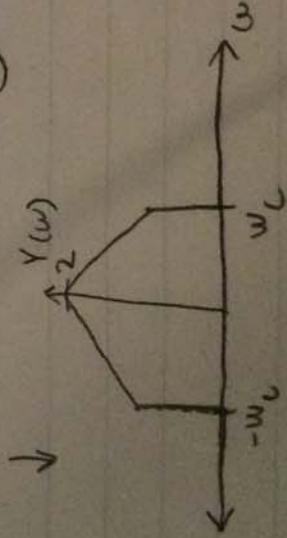
$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega t} d\omega = \frac{1}{2\pi} \left(\frac{1}{jt} e^{j\omega t} \Big|_{-\omega_c}^{\omega_c} \right) = \frac{1}{\pi t} \left(\frac{1}{2j} e^{j\omega_c t} - \frac{1}{2j} e^{-j\omega_c t} \right) =$$

$$= \frac{1}{\pi t} \sin(\omega_c t) = \frac{\text{sinc}(\omega_c t)}{\omega_c}$$

⑤ $Y(\omega) = X(\omega) H(\omega)$

③

This LTI system removes anything above ω_c and anything below $-\omega_c$ since $Y(\omega)$ is just multiplying $X(\omega)$ and $H(\omega)$. This results in the signal being preserved only between $-\omega_c$ and ω_c .



⑥ See attached graphs