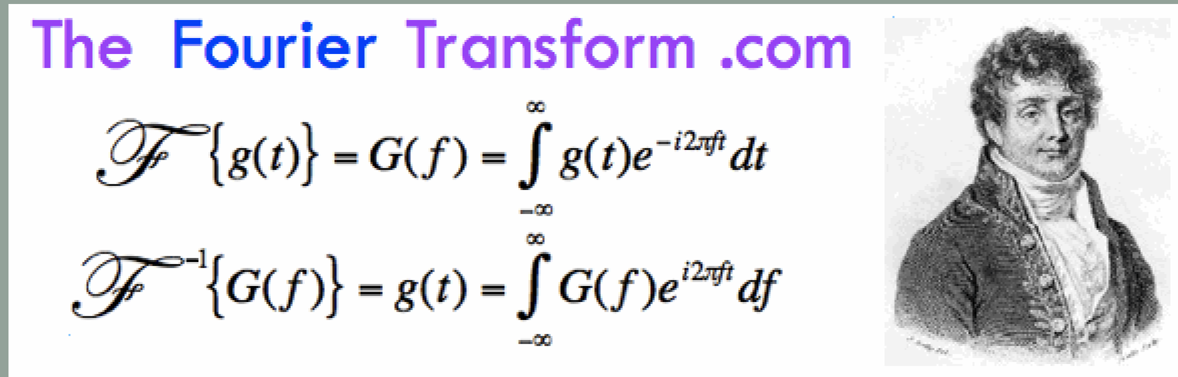

Fast Fourier Transforms

By Jacob Riedel and Deniz Celik

What is a Fourier Transform?

- Converts a signal between the time domain and frequency domain.

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx$$

- The Fourier Transform is a result of the study of a Fourier Series.

Fourier Series

- The Fourier Series is an approximation of complex signals by summing simpler sine and cosine functions

- The transform is produced when these approximations are allowed to continue until time goes to infinity.

Discrete Fourier Transform

- A Discrete Fourier Transform take a finite number of points from a complex sinusoid and transforms it using summation, rather then integration.

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i 2\pi k n / N}, k \in \mathbb{Z}$$

Fast Fourier Transform

- A Fast Fourier Transform is an Algorithm which makes computing a Discrete Fourier Transform computationally easier.
- A FFT will produce the same values as a DFT in less time.
- FFT can actually be more accurate than DFT in the presence of rounding errors.

FFT Algorithms

- There are many different algorithms used in computing FFT.
- Cooley-Tukey Algorithm is a divide-and-conquer method, using recursion and breaking down DFTs.
- Their algorithm is inspired by I. J. Good’s Prime Factorization Algorithm.
- Their formula was actually a rediscovery of Carl Friedrich Gauss’s algorithm

Code Demo

- Slow:
  - Uses Matrix Multiplication
  - $\vec{X} = M \cdot \vec{x}, where\, M_{k,n} = e^{-i2\pi kn/N}$
- Fast:
  - Uses Cooley-Tukey Algorithm
  - Symmetry of Fourier Transform – Odd is Even???

- Numpy:
  - Uses Optimized Cooley-Tukey Algorithm
  - Tweaked and Written in FORTRAN

Applications

- Solving Differential Equations
- Signal Processing
- Image Processing
- Nuclear magnetic resonance and MRI's
- Tomography – Delay in time is phase shift
- Convolution: $(f*g)(t)=\int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$
  - Is multiplication in Frequency

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