Interpolation Notes

Daniel Celis Garza

February 4, 2016

1 Neville's Algorithm

For $n = 4$

$$
\begin{array}{llccccc}
x_1: & y_1 = & P_{1,1} & & & & \\
 & & & P_{1,2} & & & \\
x_2: & y_2 = & P_{2,2} & & P_{1,2,3} & & \\
 & & & P_{2,3} & & & P_{1,2,3,4} \\
x_3: & y_3 = & P_{3,3} & & P_{2,3,4} & & \\
 & & & P_{3,4} & & & \\
x_1: & y_4 = & P_{4,4} & & & &
\end{array}
$$

$$
P_{i,i} = y_i \ , \qquad 0 \leq i \leq n \tag{1a}
$$

$$
P_{i,j} = \frac{(x_j - x)P_{i,j-1} + (x - x_i)P_{i+1,j}}{x_j - x_i} \ , \qquad 0 \leq i < j \leq n \tag{1b}
$$

The original formula is:

$$
{}^{n}P(x) = \sum_{i=0}^{n} \left(\prod_{\substack{0 \leq j \leq n \\ j \neq i}} \frac{x - x_j}{x_i - x_j} \right) y_i \tag{2}
$$

For $n = 2$ eq. (1) is

$$
P_{1,2} = \frac{(x_2 - x)P_{1,1} + (x - x_1)P_{2,2}}{x_2 - x_1}
$$

$$
P_{1,2} = \frac{(x_2 - x)y_1 + (x - x_1)y_2}{x_2 - x_1}
$$

and eq. (2) is

$$
\begin{aligned}
{}^{n}P &= \frac{x - x_2}{x_1 - x_2} y_1 + \frac{x - x_1}{x_2 - x_1} y_2 \\
{}^{n}P &= \frac{-(x_2 - x)}{-(x_2 - x_1)} y_1 + \frac{x - x_1}{x_2 - x_1} y_2
\end{aligned}
$$