Static charge susceptibility $\chi_{ch}[G_H, U](\mathbf{q})$ to $\mathcal{O}(U^n)$

