Static charge susceptibility  $\chi_{ch}[G_H, U](\mathbf{q})$  to  $\mathcal{O}(U^n)$ 

