MATH 1300: Calculus I, Spring 2008
FINAL EXAM

May 7, 2008

YOUR NAME: IRA B – SOLUTIONS

001	N. Flores . . . . . . (8AM)	**009**	R. Krieger . . . . . . (2PM)
002	A. Angel . . . . . . (9AM)	**011**	R. Grover . . . . . . (10AM)
003	D. Ernst . . . . . . (9AM)	**012**	I. Mishev . . . . . . (12PM)
004	M. Formichella . . . . . . (10AM)	**013**	R. Chestnut . . . . . . (1PM)
005	I. Becker . . . . . . (11AM)	**014**	I. Becker . . . . . . (1PM)
006	D. Vernerey . . . . . . (11AM)	**015**	D. McCarl . . . . . . (3PM)
007	J. Harper . . . . . . (12PM)	**017**	N. Flores . . . . . . (10AM)
008	L. Harris . . . . . . (2PM)		

Total Points +1 =

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

SIGNATURE:

INSTRUCTIONS: Answer each of the following multiple choice questions by circling the correct answer. Each question is worth 3 points and no partial credit will be given. No calculators, no books, no notes are allowed on this exam.

1. Find the area of the region bounded by the graphs $f(x) = 2 - x^2$ and $g(x) = x$.

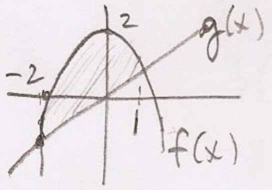
   (a) 9

   (b) $\frac{3}{4}$

   (c) $\frac{9}{2}$

   (d) 0

   (e) none of the above



$$2 - x^2 = x$$
$$x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$
$$x = -2 \quad | \quad x = 1$$

$$A = \int_{-2}^{1} \left[(2-x^2) - x\right] dx$$
$$= \left(2x - \frac{x^3}{3} - \frac{x^2}{2}\right)\Bigg|_{-2}^{1}$$
$$= \left[2 - \frac{1}{3} - \frac{1}{2}\right] - \left[-4 + \frac{8}{3} - 2\right]$$
$$= 8 - \frac{1}{3} - \frac{1}{2} - \frac{8}{3} = 8 - \frac{9}{3} - \frac{1}{2} = 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2}$$
$$= 4\tfrac{1}{2} = 9/2$$

2. Evaluate the following limit.

$$\lim_{x \to +\infty} \frac{x^2}{e^x} \overset{\text{L'HOP}}{=} \lim_{x \to \infty} \frac{2x}{e^x}$$

$$\overset{\text{L'Hop}}{=} \lim_{x \to \infty} \frac{2}{e^x} = 0$$

$e^x \to \infty$

   (a) $+\infty$

   (b) $-\infty$

   (c) 0

   (d) 1

   (e) does not exist

3. The absolute minimum value attained by $f(x) = x^3 - 3x + 3$ on $[-3, 2]$ is

(a) 1

(b) -15

(c) 5

(d) -2

(e) there is no absolute minimum

$f'(x) = 3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = 1 \mid x = -1$

$f(-3) = -27 + 9 + 3 = -15 \leftarrow$ ABS MIN

$f(2) = 8 - 6 + 3 = 5$

$f(1) = 1 - 3 + 3 = 1$

$f(-1) = -1 + 3 + 3 = 5$

4. Find $\frac{d}{dx}(\cos(\cos(x)))$.

(a) $\sin^2(x)$

(b) $-\sin^2(x)$

(c) $-\sin(\cos x)\sin x$

(d) $\sin(\cos x)\sin x$

(e) none of the above

$\frac{d}{dx}\left[\cos(\cos x)\right]$

$= -\sin(\cos x) \cdot \frac{d}{dx}[\cos x]$

$= -\sin(\cos x)(-\sin x)$

$= \sin(\cos x) \cdot \sin x$

$= dV/dt$

5. Grain pouring from a chute at the rate of 10 ft^3/min forms a conical pile whose height is always half the radius of the base of the cone. How fast is the height of the pile increasing at the instant when the pile is 5 ft high? (Hint: the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base of the cone and h is its height.)

(a) $\frac{4}{5\pi}$

(b) $\frac{1}{10\pi}$

(c) $\frac{4}{5}$

(d) $\frac{1}{80\pi}$

(e) $\sqrt{6/\pi}$

HEIGHT IS ALWAYS HALF THE BASE RADIUS

$$h = \tfrac{1}{2} r \Rightarrow r = 2h$$

$$\text{So } V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi (2h)^2 h = \tfrac{4}{3}\pi h^3$$

$$\text{So } \frac{dV}{dt} = \frac{d}{dt}\left[\tfrac{4}{3}\pi h^3\right] = \tfrac{4}{3}\pi\left(3h^2 \frac{dh}{dt}\right) = 4\pi h^2 \frac{dh}{dt}$$

$$\text{So } \frac{dh}{dt} = \left.\frac{dV/dt}{4\pi h^2}\right|_{h=5\text{ ft}} = \frac{10\ \text{ft}^3/\text{min}}{4\pi(5\text{ ft})^2} = \frac{10}{4\pi(25)}\ \text{ft/min}$$

$$= \frac{10}{100\pi}\ \text{ft/min}$$

$$= \frac{1}{10\pi}\ \text{ft/min}$$

6. Which limit of Riemann sums is equal to $\int_0^2 x^2\, dx$?

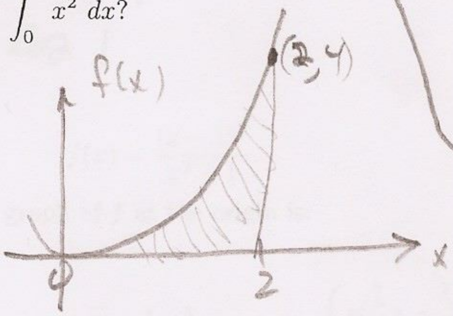
(a) $\lim_{n\to\infty} \sum_{k=1}^{n} k\left(\frac{2}{n}\right)^2 \frac{1}{n}$

(b) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(k\frac{2}{n}\right)^2 \frac{1}{n}$

(c) $\lim_{n\to\infty} \sum_{k=1}^{n} k^2\left(\frac{2}{n}\right) \frac{2}{n}$

(d) $\lim_{n\to\infty} \sum_{k=1}^{n} k^2\left(\frac{2}{n}\right)^2 \frac{2}{n}$

(e) none of the above



WIDTH of RECT $= \Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$ WHERE $n = \#$ of RECTS

$$x_k = a + k\cdot\Delta x = 0 + k\cdot\frac{2}{n} = \frac{2k}{n}$$

HEIGHT OF k^{th} RECT $= f(x_k) = f\left(\frac{2k}{n}\right) = \left(\frac{2k}{n}\right)^2 = k^2\left(\frac{2}{n}\right)^2$

AREA of k^{th} rect $= f(x_k)\cdot\Delta x = k^2\left(\frac{2}{n}\right)^2\cdot\frac{2}{n}$

RIEMANN SUM $= \sum_{k=1}^{n} k^2\left(\frac{2}{n}\right)^2\left(\frac{2}{n}\right)$

7. Evaluate the following definite integral.

$$\int_0^{\frac{\pi}{2}} \sin(x)\sin(\cos(x))dx \quad = \int_0^{\pi/2} \sin(\underbrace{\cos x}_{u})\,\underbrace{\sin x\, dx}_{-du}$$

(a) $1-\cos\left(\frac{\sqrt{2}}{2}\right)$

(b) $1-\cos(1)$

(c) $\cos(1)-\cos\left(\frac{\sqrt{2}}{2}\right)$

(d) $\cos(1)$

(e) none of the above

Let $u = \cos x$

Then $\frac{du}{dx} = -\sin x$

$-du = \sin x\, dx$

$$= -\int \sin u\, du = -(-\cos u) = \cos u = \cos(\cos x)\Big|_0^{\pi/2}$$

$$= \cos(\cos \pi/2) - \cos(\cos 0)$$

$$= \cos 0 \quad - \cos 1$$

$$= 1 \quad - \cos 1$$

8. Suppose

$$f(x) = \frac{(x-1)^2}{x^2+1}.$$

Then the slope of the tangent line to the graph of f at the origin is:

(a) 0

(b) -2

(c) 2

(d) 1

(e) -1

$$f'(x) = \frac{(x^2+1)(2)(x-1) - (x-1)^2(2x)}{(x^2+1)^2}$$

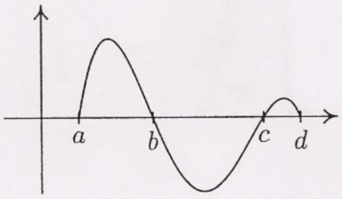
$$f'(0) = \frac{(0^2+1)(2)(0-1) - (0-1)^2(2)(0)}{(0^2+1)^2}$$

$$= \frac{-2 \quad -0}{1}$$

$$= -2$$

~~GRAPH DOES NOT PASS THRU THE ORIGIN, SO THERE ... LINE AT THE ORIGIN!~~

9. Using the figure and information below, find $\int_a^d |f(x)|\, dx$.



Graph of f

$$\int_a^b f(x)\, dx = 4, \quad \int_b^c f(x)\, dx = -4.5, \quad \int_c^d f(x)\, dx = 1$$

(a) .5

(b) −.5

(c) 9.5

(d) 1.5

(e) none of the above

$$\int_a^d |f(x)|\, dx = \int_a^b |f(x)|\, dx + \int_b^c |f(x)|\, dx + \int_c^d |f(x)|\, dx$$

$$= \int_a^b f(x)\, dx - \int_b^c f(x)\, dx + \int_c^d f(x)\, dx$$

$$= 4 - (-4.5) + 1$$

$$= 9.5$$

10. Use implicit differentiation to find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$.

(a) 0

(b) $\frac{10y + \cos y}{2x}$

(c) $\frac{2y}{10x + \cos x}$

(d) $\frac{2x}{10y + \cos y}$

(e) $\frac{10y + \cos y}{2y}$

$$\frac{d}{dx}\left[5y^2 + \sin y\right] = \frac{d}{dx}\left[x^2\right]$$

$$10y\frac{dy}{dx} + (\cos y)\,dy/dx = 2x$$

$$\frac{dy}{dx}\left[10y + \cos y\right] = 2x$$

$$\frac{dy}{dx} = \frac{2x}{10y + \cos y}$$

11. Let $F(x) = \int_0^x \sin(t^2)\, dt$. Then $F'(x)$ is

(a) $2x \cos(x^2)$

(b) $\cos(x^2)$

(c) $\sin(x^2)$

(d) $2x \sin(x^2)$

(e) none of the above

$$F'(x) = \frac{d}{dx}\left[\int_0^x \sin(t^2)\,dt\right]$$

$$= \sin(x^2)$$

FTC PART 2

12. The function $f(x) = x^3 - 9x$ is increasing on which interval(s)?

(a) $(-3, 0)$ and $(3, \infty)$

(b) $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$

(c) $(-\sqrt{3}, \sqrt{3})$

(d) $(-3, 3)$

(e) $(-\infty, \infty)$

$$f'(x) = 3x^2 - 9 = 3(x^2 - 3)$$

$$= 3(x - \sqrt{3})(x + \sqrt{3}) = 0$$

	$x-\sqrt{3}$	$x+\sqrt{3}$	$f'(x)$	$f(x)$
$(-\infty, -\sqrt{3})$	$-$	$-$	$+$	INCR
$(-\sqrt{3}, \sqrt{3})$	$-$	$+$	$-$	DECR
$(\sqrt{3}, \infty)$	$+$	$+$	$+$	INCR

$x = \sqrt{3}$ | $x = -\sqrt{3}$

(STATIONARY) CRITICAL PTS

13. Which of the following is an inflection point of $g(x) = x - \cos x$?

(a) $(0, -1)$

(b) $(\pi/3, \pi/3 - 1/2)$

(c) $(\pi/2, \pi/2)$

(d) $(\pi, \pi + 1)$

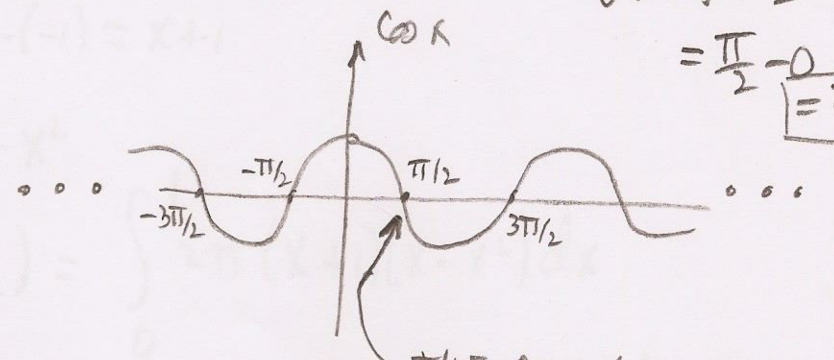
(e) $(\pi/4, \pi/4 - \sqrt{2}/2)$

$$g'(x) = 1 - (-\sin x) = 1 + \sin x$$

$$g''(x) = \cos x = 0 \text{ when } \boxed{x = \pi/2}$$

$$g(\pi/2) = \frac{\pi}{2} - \cos\frac{\pi}{2}$$

$$= \frac{\pi}{2} - 0 \; \boxed{= \pi/2}$$



THE ONLY X AMONG THE FIVE CHOICES FOR WHICH $g''(x) = 0$

14. Consider the region in the first quadrant bounded by the curves $y = \sqrt{x}$ and $y = x^3$. If this region is revolved around the x-axis, the resulting volume of revolution is

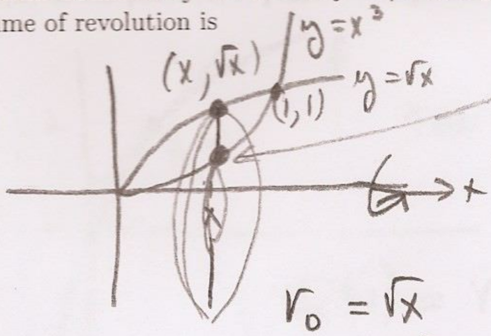
(a) $5\pi/12$

(b) $5\pi/14$

(c) $6\pi/13$

(d) $7\pi/12$

(e) $4\pi/15$



(x, x^3)

WASHER

$r_O = \sqrt{x}$

$r_I = x^3$

$$V = \int_0^1 \pi\left[(\sqrt{x})^2 - (x^3)^2\right]dx = \pi\int_0^1 (x - x^6)\,dx$$

$$= \pi\left(x^2/2 - \frac{x^7}{7}\right)\Big|_0^1 = \pi\left(\frac{1}{2} - \frac{1}{7}\right) = \pi\left(\frac{7}{14} - \frac{2}{14}\right) = \frac{5\pi}{14}$$

15. Consider the region enclosed by the curves $y = x$ and $y = x^2$. If this region is revolved about the line $x = -1$, then the resulting volume of revolution is

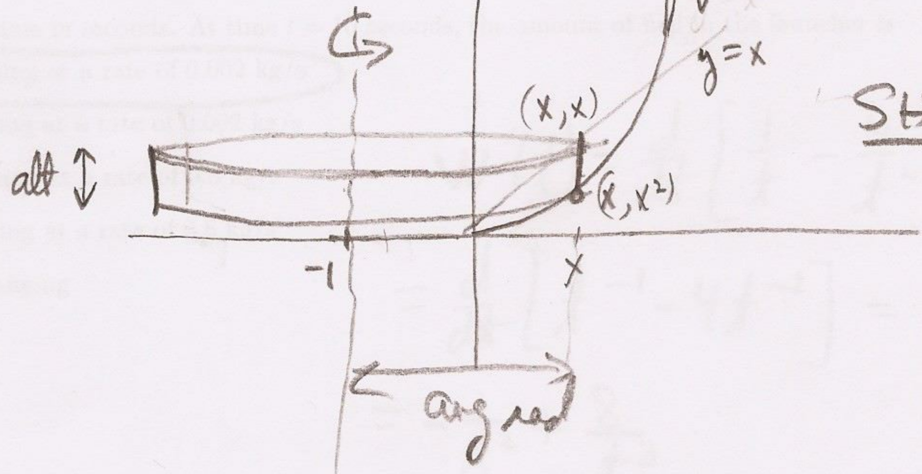
(a) $\frac{5\pi}{14}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

(e) $\frac{7\pi}{30}$



SHELL

avg rad $= x_{RT} - x_{LEFT} = x - (-1) = x + 1$

alt $= y_{TOP} - y_{BOT} = x - x^2$

$$V = \int 2\pi\,(\text{avg rad})(\text{alt})(\text{th}) = \int_0^1 2\pi\,(x+1)(x - x^2)\,dx$$

$$= 2\pi\int_0^1 (x^2 + x - x^3 - x^2)\,dx = 2\pi\int_0^1 (x - x^3)\,dx = 2\pi\left(x^2/2 - \frac{x^4}{4}\right)\Big|_0^1 = 2\pi\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= 2\pi(1/4) = \pi/2$$

16. Given that the tangent line to $y = f(x)$ at the point $(1, 2)$ passes through the point $(-1, -1)$, find $f'(1)$.

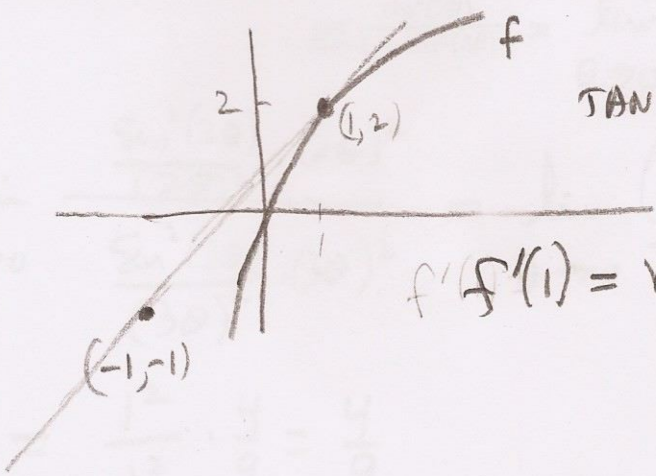
(a) 1/2

(b) 2/3

(c) 3/2

(d) −2/3

(e) −3/2



TAN LINE PASSES THRU (1,2) AND (-1,-1)

$$f'(1) = m_{TAN} = \frac{\Delta y}{\Delta x} = \frac{2-(-1)}{1-(-1)} = \frac{3}{2}$$

17. The weight (in kg.) of rocket fuel in a rocket launcher is given by

$$W(t) = \frac{1}{t} - \frac{4}{t^2},$$

where t is time in seconds. At time $t = 10$ seconds, the amount of fuel in the launcher is

(a) decreasing at a rate of 0.002 kg/s

(b) increasing at a rate of 0.002 kg/s

(c) decreasing at a rate of 0.8 kg/s

(d) increasing at a rate of 0.8 kg/s

(e) not changing

$$W'(t) = \frac{d}{dt}\left[\frac{1}{t} - \frac{4}{t^2}\right]$$

$$= \frac{d}{dt}\left[t^{-1} - 4t^{-2}\right] = -1t^{-2} - 4(-2t^{-3})$$

$$= -\frac{1}{t^2} + \frac{8}{t^3}$$

$$W'(10) = -\frac{1}{10^2} + \frac{8}{10^3} = -\frac{1}{100} + \frac{8}{1000}$$

$$= \frac{-10}{1000} + \frac{8}{1000} = -\frac{2}{1000}$$

$$= -.002$$

DECREASING

18. Evaluate the following limit.

$$\lim_{\theta\to 0}\frac{\sin^2(2\theta)}{1-\cos^2(3\theta)} = \lim_{\theta\to 0}\frac{\sin^2(2\theta)}{\sin^2(3\theta)}$$

(a) 0

(b) 1

(c) $\frac{2}{3}$

(d) $\frac{4}{9}$

(e) does not exist

$$= \lim_{\theta\to 0}\frac{\frac{\sin^2(2\theta)}{(2\theta)^2}\cdot(2\theta)^2}{\frac{\sin^2(3\theta)}{(3\theta)^2}\cdot(3\theta)^2} = \lim_{\theta\to 0}\frac{\left(\frac{\sin(2\theta)}{2\theta}\right)^2\cdot 4\cancel{\theta^2}}{\left(\frac{\sin(3\theta)}{3\theta}\right)^2\cdot 9\cancel{\theta^2}}$$

$$= \frac{1^2}{1^2}\cdot\frac{4}{9} = \frac{4}{9}$$

19. A polynomial $f(x)$ has a relative maximum at $(-2,4)$, a relative minimum at $(1,1)$, a relative maximum at $(5,7)$, and no other critical points. How many real zeros (x-intercepts) does $f(x)$ have?

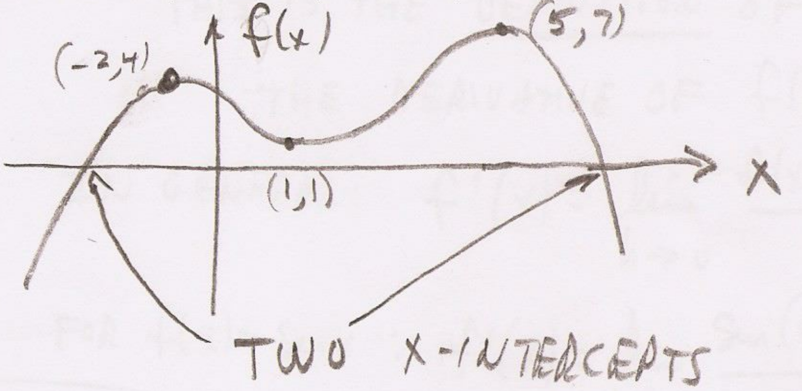
(a) one

(b) two

(c) three

(d) four

(e) five



20. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$, then c equals

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{4}{3}$

(e) 2

$f'(x) = 3x^2 - 4x$

$f'(c) = 3c^2 - 4c$

$f(0) = 0$

$f(2) = 2^3 - 2\cdot 2^2 = 0$

SO SLOPE OF MEAN LINE IS $\frac{\Delta y}{\Delta x} = \frac{0-0}{2-0} = 0$

MVT $\Rightarrow 3c^2 - 4c = 0$

BUT MVT ASSERTS THAT c MUST BE IN THE OPEN INTERVAL $(0,2)$, SO $c=0$ IS EXCLUDED

$c(3c-4) = 0$

$c = 0$ | $3c - 4 = 0$, $c = 4/3$

21. Let $f(x) = e^x$. Find $\ln[f'(2)]$.

(a) 2

(b) 0

(c) $\frac{1}{e^2}$

(d) $2e$

(e) e^2

$f'(x) = e^x$

so $f'(2) = e^2$

$$\ln[f'(2)] = \ln(e^2) = 2 \ln e = 2 \cdot 1 = 2$$

22. Evaluate the following limit.

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} = \cos x$$

(a) 0

(b) 1

(c) $\sin x$

(d) $\cos x$

(e) does not exist

THIS IS THE DEFINITION OF THE DERIVATIVE OF $f(x) = \sin x$

IN GENERAL: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

FOR $f(x) = \sin x$: $f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

AND $f'(x) = \frac{d}{dx}[\sin x] = \cos x$

23. If $f(x) = \ln(\sqrt{x})$, then $f''(x)$ is equal to

(a) $-\frac{2}{x^2}$

(b) $-\frac{1}{2x^2}$

(c) $\frac{1}{2x}$

(d) $-\frac{1}{2x^{3/2}}$

(e) $\frac{2}{x^2}$

$$f'(x) = \frac{d}{dx}[\ln \sqrt{x}] = \frac{d}{dx}[\ln x^{1/2}] = \frac{d}{dx}\left[\frac{1}{2}\ln x\right] = \frac{1}{2} \cdot \frac{1}{x}$$

$$f''(x) = \frac{d}{dx}\left[\frac{1}{2} \cdot \frac{1}{x}\right] = \frac{1}{2} \cdot \frac{d}{dx}\left[\frac{1}{x}\right] = \frac{1}{2} \cdot \left(-\frac{1}{x^2}\right) = -\frac{1}{2x^2}$$

24. If $\int_1^3 f(x)\,dx = 6$, what is the value of $\int_4^2 f(5-x)\,dx$?

(a) 6

(b) 3

(c) 0

(d) -1

(e) -6

Let $u = 5 - x$

Then $\frac{du}{dx} = -1$

$du = -dx$

$-du = dx$

If $x = 2$, then $u = 3$

If $x = 4$, then $u = 1$

THEN $\int_4^2 f(\underbrace{5-x}_{u}) \underbrace{dx}_{-du} = \int_1^3 f(u)(-du) = -\int_1^3 f(u)\,du$

$= -\int_1^3 f(x)\,dx = -6$

SINCE u & x ARE BOTH DUMMY VARS

25. The range of the function $f(x) = \sin^{-1}(x)$ is

(a) $\cos^{-1}(x)$

(b) all real numbers

(c) the interval $[-1, 1]$

(d) the interval $[-\pi/2, \pi/2]$

(e) $[0, 2\pi]$

DOM sin $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ RESTRICTED DOMAIN SO THAT $\sin^{-1}$ EXISTS

BUT RAN $\sin^{-1}$ = DOM sin

$= [-\pi/2, \pi/2]$

26. Find the slope of the secant line to $f(x) = \tan^{-1}(x)$ through the points on the graph with x-coordinates $x = 0$ and $x = 1$.

(a) $\frac{\pi}{4}$

(b) 0

(c) 1

(d) $\frac{1}{x^2+1}$

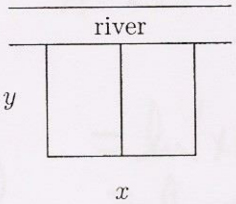
(e) $\frac{\pi}{2}$

$\tan^{-1}(0) = 0$ SINCE $\tan 0 = 0$

$\tan^{-1}(1) = \pi/4$ SINCE $\tan \pi/4 = 1$

SO $m_{SEC} = \frac{\Delta y}{\Delta x} = \frac{\pi/4 - 0}{1 - 0} = \pi/4$

27. A farmer has 1200 feet of fencing with which to enclose a pasture for grazing nuggets. The farmer only needs to enclose 3 sides of the pasture since the remaining side is bounded by a river (no, nuggets can't swim). In addition, some of the nuggets don't get along with some of the other nuggets. He plans to separate the troublesome nuggets by forming two adjacent corrals (see figure). Determine the dimensions that would yield the maximum area for the pasture.



$1200\text{ ft} = x + 3y$

SO $x = 1200 - 3y$

(a) $x = 600$ and $y = 600$

(b) $x = 1199$ and $y = 1$

(c) $x = 200$ and $y = 600$

(d) $x = 600$ and $y = 200$

(e) $x = 300$ and $y = 400$

$A = xy = (1200 - 3y)y = 1200y - 3y^2$

$A(y) = 1200y - 3y^2$

$A'(y) = 1200 - 6y = 6(200 - y) = 0$

WHEN $y = 200$

THEN $x = 1200 - 3(200)$

$= 1200 - 600$

$= 600$

$A''(y) = -6 < 0$

SO A IS CD

SO $y = 200$ PRODUCES A MAX FOR A

SO $\boxed{\begin{array}{l} x = 600 \\ y = 200 \end{array}}$

28. Suppose $f'(x) = x\sin(x)$. Find the derivative of the function $g(x) = 2f(2x)$.

(a) $\sin^2(x)$

(b) $f'(2x)$

(c) $2\sin(2x)$

(d) $8x\sin(2x)$

(e) $4x\sin(2x)$

IF $f'(x) = x \sin(x)$

THEN $f'(\sqcup) = \sqcup \cdot \sin \sqcup$

SO $f'(2x) = 2x \cdot \sin(2x)$

$$g'(x) = 2 \cdot \frac{d}{dx}\left[f(2x)\right] = 2 \cdot f'(2x) \cdot \frac{d}{dx}[2x] = 2 \cdot f'(2x) \cdot 2 = 4f'(2x) = 4 \cdot 2x \cdot \sin 2x = 8x \sin 2x$$

29. Let $A(x)$ denote the area under the curve $f(t) = \frac{1}{t}$ from 1 to x, where x in in the interval $(1, \infty)$. What familiar function could $A(x)$ be equal to?

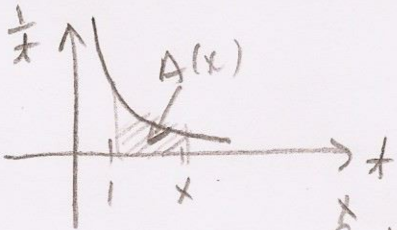
(a) e^x

(b) $\frac{1}{t}$

(c) $\frac{1}{x}$

(d) $\ln x$

(e) $\frac{-1}{t^2}$



$$A(x) = \int_1^x \frac{1}{t}\,dt = \ln|t|\Big|_1^x = \ln|x| - \ln|1| = \ln x - \ln 1 = \ln x - 0 = \ln x$$

$x > 0$
SINCE x IN $(1, \infty)$
SO $|x| = x$

30. On a 90 mile trip Daffy drives 45 miles with an average velocity of 45 mph then 45 miles with an average velocity of 30 mph. What was Daffy's average velocity for the entire trip?

(a) $\frac{420}{13}$ mph

(b) 36 mph

(c) $\frac{154}{5}$ mph

(d) 37.5 mph

(e) 45 mph

$d = rt$ so $t = d/r$

FIRST 45 miles: $t = \frac{d}{r} = \frac{45 \text{ miles}}{45 \text{ miles/hr}} = 1$ hour

SECOND 45 miles: $t = \frac{d}{r} = \frac{45 \text{ miles}}{30 \text{ miles/hr}} = \frac{3}{2}$ hours

SO ENTIRE TRIP TAKES $1 + \frac{3}{2}$ hours $= 5/2$ hours

$d = rt$ so $r = \frac{d}{t} = \frac{90 \text{ miles}}{5/2 \text{ hours}} = \frac{180}{5}$ mph $= 36$ mph

WHOLE TRIP

31. Let $\Gamma(f)$ denote the set of all ordered pairs on the graph of the function $f(x)$. That is, $\Gamma(f) = \{(x, y) : y = f(x)\}$. Assume that f is one-to-one, so that f has an inverse function. If f^{-1} denotes the inverse function of f, which of the following sets correctly describes $\Gamma(f^{-1})$?

(a) $\{(x, y) : y = f^{-1}(x)\}$

(b) $\{(y, x) : y = f^{-1}(x)\}$

(c) $\{(y, x) : x = f(y)\}$

(d) $\{(x, y) : y = f(x)\}$

(e) both (b) and (c)

$\Gamma(f^{-1})$ = set of all pts (x,y) WHERE $y = f^{-1}(x)$

$= \{(x,y) : y = f^{-1}(x)\}$

32. Let $g(x)$ be defined via

$$g(x) = \begin{cases} \sin x & x < 0 \\ x^2 & 0 \le x < 1 \\ 2 - x & 1 \le x < 2 \\ x - 3 & x \ge 2 \end{cases}.$$

For what values of x is g NOT continuous?

(a) 0

(b) 1

(c) 2

(d) π

(e) g is continuous everywhere

EACH OF $\sin x$, x^2, $2-x$, AND $x-3$ ARE CONT EVERYWHERE

SO THE ONLY POSSIBLE PTS OF DISCONTINUITY ARE AT THE CROSS-OVER PTS $x = 0, 1, 2$

AT $x=0$: $\lim\limits_{x\to 0} g(x) = \lim\limits_{x\to 0^-}(\sin x) = \lim\limits_{x\to 0^+}(x^2) = 0$ so g is cont at $x=0$

AT $x=1$: $\lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1^-}(x^2) = \lim\limits_{x\to 1^+}(2-x) = 1$ so g is cont at $x=1$

BUT AT $x=2$: $\lim\limits_{x\to 2^-} g(x) = \lim\limits_{x\to 2^-} 2-x = 2-2 = 0$

and $\lim\limits_{x\to 2^+} g(x) = \lim\limits_{x\to 2^+} x-3 = 2-3 = -1$

NOT EQUAL so $\lim\limits_{x\to 2} g(x) = \text{DNE}$

so g NOT cont at 2

33. If f is differentiable at $x = a$, which of the following could be FALSE?

(a) f is continuous at $x = a$

(b) $\lim_{x \to a} f(x)$ exists

(c) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists

(d) $f'(a)$ is defined

(e) $f''(a)$ is defined

(a) DIFF $\Rightarrow$ CONT SO (a) IS TRUE

(b) SINCE f IS CONT at 'a', $\lim_{x \to a} f(x) = f(a)$ EXISTS

SO (b) IS TRUE

(c) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ IS THE DEFINITION OF $f'(a)$

AND SINCE f IS GIVEN TO BE DIFFERENTIABLE AT 'a',

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ MUST EXIST

SO (c) IS TRUE

(d) SINCE f IS GIVEN TO BE DIFFERENTIABLE at 'a', THIS MEAN $f'(a)$ IS DEFINED

SO (d) IS TRUE

(e) MIGHT NOT BE TRUE

i.e. COULD BE FALSE