Thesis Outline

1. Introduction
   (a) Define Coxeter system, braid relation, commutation, bond strength, Coxeter graph, types ($A_n \cong S_{n+1}$ via $s_i \mapsto (i\ i+1)$); examples.
   (b) Define word, reduced expression, length, Coxeter element, fully commutative, cyclically reduced, cyclically equivalent, commutation equivalent, cyclically fully commutative, heaps; examples.
   (c) FC if and only if no reduced expression has braid relations, observation; example.
   (d) Matsumoto's theorem, observation; example.
   (e) Heap correspondence with commutation classes, FCC if and only if unique heap; example.
   (f) Define kappa-equivalence; example.
   (g) Cyclic version of Matsumoto's theorem; example.
   (h) Erikssons' theorem, observation; example.
   (i) Reduced expressions box in A_4.
   (j) Note that some of these results are analogous to known results in the symmetric group, but we put them in the Coxeter context.
2. CFC elements in A_n
   (a) Define CFC in A_n, weak and strong block types; examples.
   (b) Switch between heaps and cycles.
   (c) Pattern avoidance with diagrams; examples.
   (d) String diagrams (crossings, choice of crossing implies more than one heap), heaps; examples.
   (e) Example using string diagram, heap, cycle, one line to show CFC, FC, not FC.
   (f) Note that cycle type does not generalize to types other than A_n; same block type implies same cycle type implies conjugate.
3. Conjugacy classes of CFC elements in A_n
   (a) Lemma – CFC is cyclically equivalent to diagonal block type (k), proof; example.
   (b) Define squeeze operation; example.
   (c) Lemma – $m(s,t) = 3$ implies $stst = ts$ (heaps).
   (d) Lemma – translate left and right, proof; example (show exactly by which element you have to conjugate).
   (e) Define boomerang operation; example.
   (f) Lemma – switch between block type (k,m) to (m,k), proof; example (show exactly by which element you have to conjugate; to switch back, conjugate by the inverse).
   (g) Examples.
   (h) If $w \in \mathrm{CFC}(W)$, is $xwx^{-1} \in \mathrm{CFC}(W)$ not on the list? No. Take the "simple" (diagonal, tight) CFC heap of the corresponding block type, then translate and swap.
4. Connection to cycles
5. Open questions