A Multiresolution Stochastic Process Model for Basketball Possession Outcomes

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NBA optical tracking data

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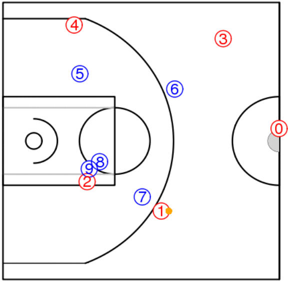

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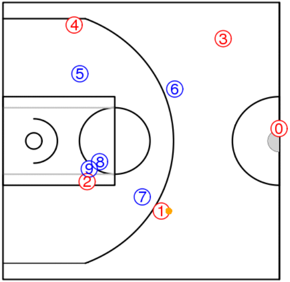
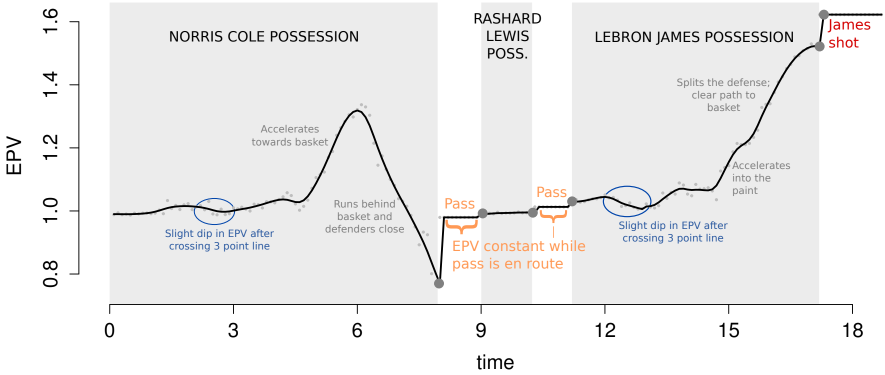


- (x, y) locations for all 10 players (5 on each team) at 25Hz.
- (x, y, z) locations for the ball at 25Hz.
- Event annotations (shots, passes, fouls, etc.).
- 1230 games from 2013-14 NBA, each 48 minutes, featuring 461 players in total.

Expected Possession Value (EPV)



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The *expected possession value* (EPV) at time $t \geq 0$ during a possession is $\nu_t = \mathbb{E}[X|\mathcal{F}_t^{(Z)}]$.

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Definition

The *expected possession value* (EPV) at time $t \geq 0$ during a possession is $\nu_t = \mathbb{E}[X|\mathcal{F}_t^{(Z)}]$.

- EPV provides an instantaneous snapshot of the possession's value, given its full spatiotemporal history.
- ν_t is a Martingale: $\mathbb{E}[\nu_{t+h}|\mathcal{F}_t^{(Z)}] = \nu_t$ for all $h > 0$.

Calculating EPV

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Regression-type prediction methods:

- Data are not traditional input/output pairs.
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Brute force, "God model" for basketball.

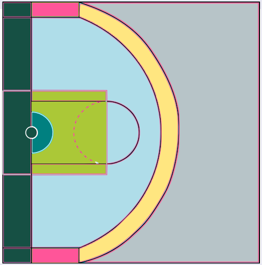
+ Allows Monte Carlo calculation of ν_t by simulating future possession paths.
- Z_t is high dimensional and includes discrete events (passes, shots, turnovers).

A coarsened process

Finite collection of states $\mathcal{C} = \mathcal{C}_{\text{poss}} \cup \mathcal{C}_{\text{end}} \cup \mathcal{C}_{\text{trans}}$.

$\mathcal{C}_{\text{poss}}$: Ball possession states

$\{\text{player}\} \times \{\text{region}\} \times \{\text{defender within 5 feet}\}$



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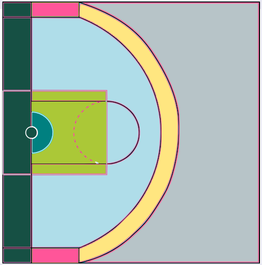

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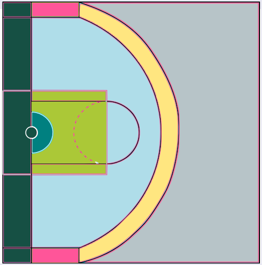

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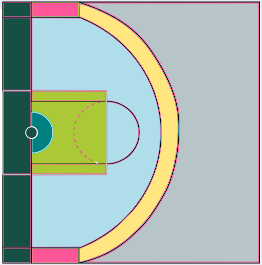

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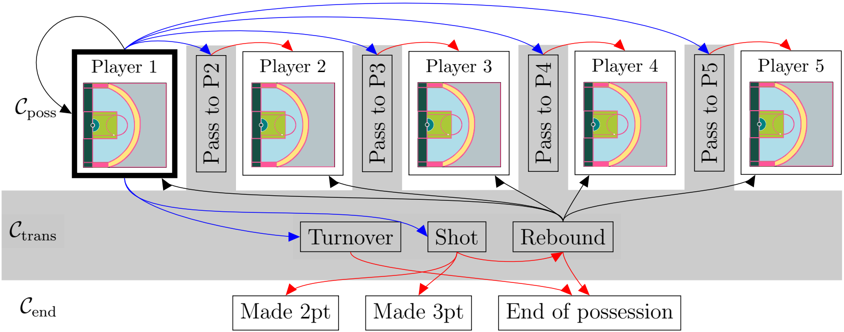
- $C_t \in \mathcal{C}$: state of the possession at time t.
- $C^{(0)}, C^{(1)}, \ldots, C^{(K)}$: discrete sequence of distinct states.

Possible paths for C_t



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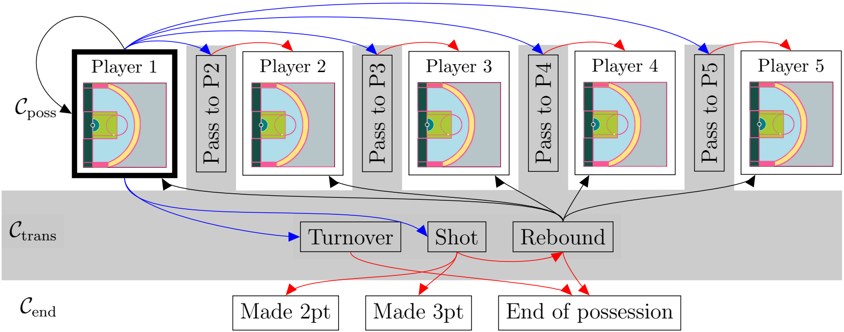

$$\tau_t = \begin{cases} \min\{s : s > t, C_s \in \mathcal{C}_{\text{trans}}\} & \text{if } C_t \in \mathcal{C}_{\text{poss}} \\ t & \text{if } C_t \notin \mathcal{C}_{\text{poss}} \end{cases}$$

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Theorem

Assume (A1)–(A2), then for all $0 \leq t < T$,

$$\nu_t = \sum_{c \in \{\mathcal{C}_{trans} \cup \mathcal{C}_{end}\}} \mathbb{E}[X | C_{\delta_t} = c] \mathbb{P}(C_{\delta_t} = c | \mathcal{F}_t^{(Z)}).$$

Multiresolution models

EPV:

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Let $M(t)$ be the event $\{\tau_t \leq t + \epsilon\}$.

M1 $\mathbb{P}(Z_{t+\epsilon} | M(t)^c, \mathcal{F}_t^{(Z)})$: the *microtransition model*.

M2 $\mathbb{P}(M(t) | \mathcal{F}_t^{(Z)})$: the *macrotransition entry model*.

M3 $\mathbb{P}(C_{\delta_t} | M(t), \mathcal{F}_t^{(Z)})$: the *macrotransition exit model*.

M4 $\mathbf{P}$, with $P_{qr} = \mathbb{P}(C^{(n+1)} = c_r | C^{(n)} = c_q)$: the *Markov transition probability matrix*.

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Monte Carlo computation of ν_t:

- Draw $C_{\delta_t} | \mathcal{F}_t^{(Z)}$ using (M1)–(M3).
- Calculate $\mathbb{E}[X | C_{\delta_t}]$ using (M4).

Microtransition model

Player ℓ's position at time t is $\mathbf{z}^\ell(t) = (x^\ell(t), y^\ell(t))$.

$$x^\ell(t+\epsilon) = x^\ell(t) + \alpha_x^\ell[x^\ell(t) - x^\ell(t-\epsilon)] + \eta_x^\ell(t)$$

- $\eta_x^\ell(t) \sim \mathcal{N}(\mu_x^\ell(\mathbf{z}^\ell(t)), (\sigma_x^\ell)^2)$.
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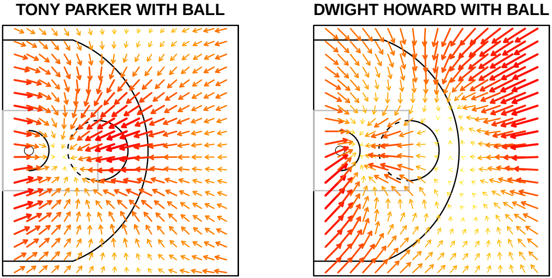

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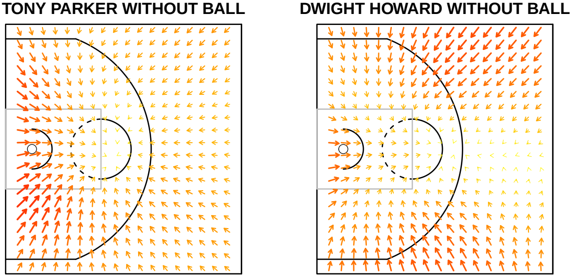

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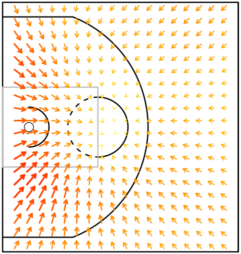
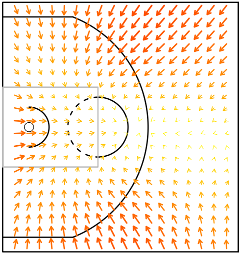
TONY PARKER WITHOUT BALL



DWIGHT HOWARD WITHOUT BALL



- Defensive microtransition model based on defensive matchups [Franks et al., 2015].

Macrotransition entry model

Recall $M(t) = \{\tau_t \leq t + \epsilon\}$:

- Six different "types", based on entry state $C_{\tau_t}, \cup_{j=1}^{6} M_j(t) = M(t)$.
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$$\log(\lambda_j(t)) = [\mathbf{W}_j^{\ell}(t)]' \boldsymbol{\beta}_j^{\ell} + \xi_j^{\ell}\left(\mathbf{z}^{\ell}(t)\right)$$

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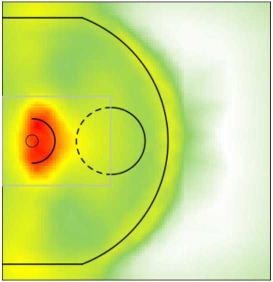

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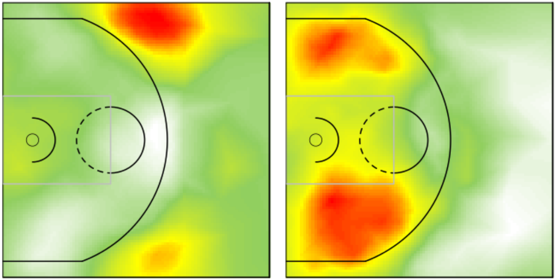
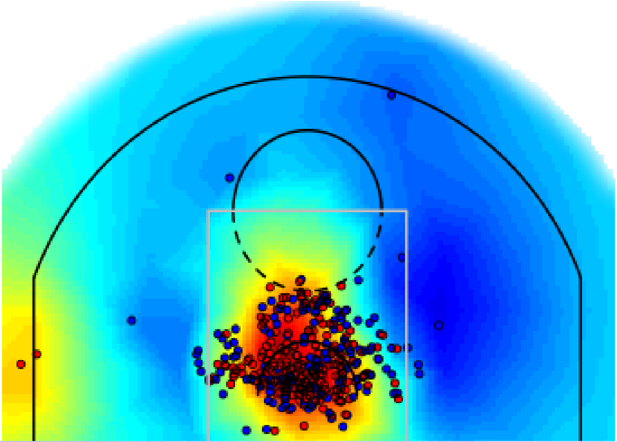


Hierarchical modeling

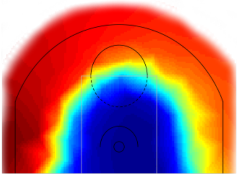
Dwight Howard's shot chart:

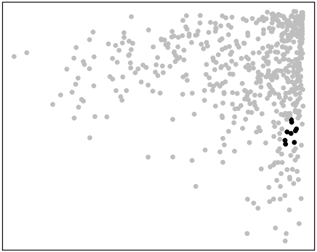


Hierarchical modeling

Shrinkage needed:

- Across space.
- Across different players.





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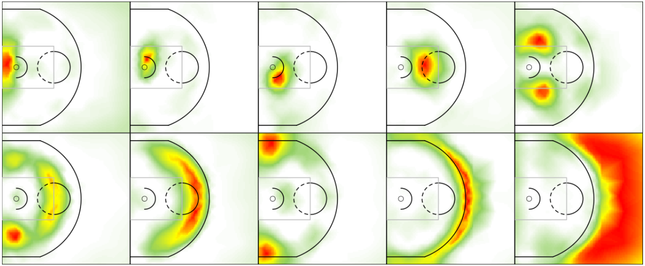
Information sharing

- $\boldsymbol{\phi}_j$ allows for non-stationarity, correlations between disjoint regions [Higdon, 2002].
- $\mathbf{w}_j^\ell$: weights across players follow a CAR model [Besag, 1974] based on player similarity graph $\mathbf{H}$.

Basis representation of spatial effects

Basis functions ϕ_j learned in pre-processing step:



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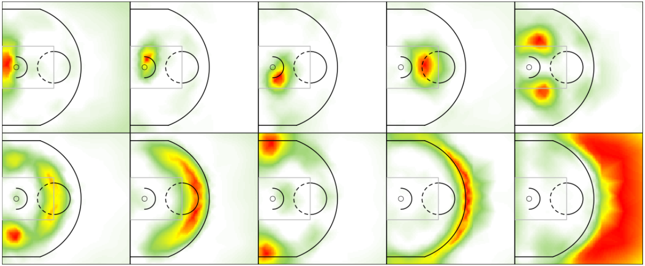
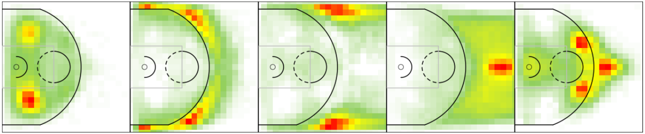


Graph $\mathbf{H}$ learned from players' court occupancy distribution:



Inference

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- Multiresolution transition models provide partial likelihood factorization [Cox, 1975].
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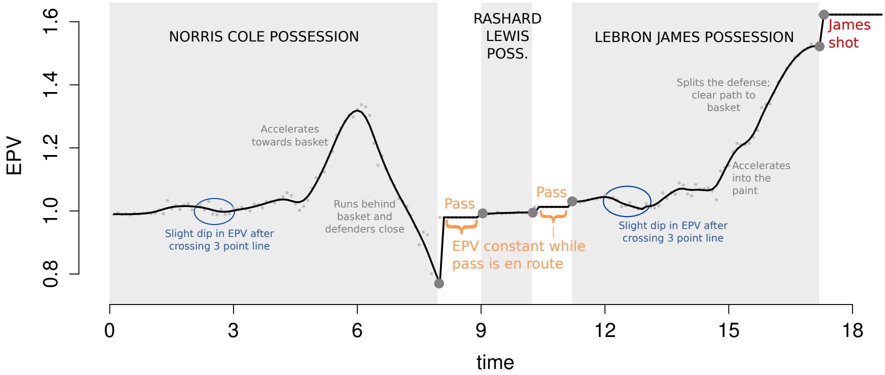
Distributed computing implementation:

- Preprocessing involves low-resource, highly parallelizable tasks.
- Parameter estimation involves several CPU- and memory-intensive tasks.
- Calculating EPV from parameter estimates involves low-resource, highly parallelizable tasks.

New insights from basketball possessions



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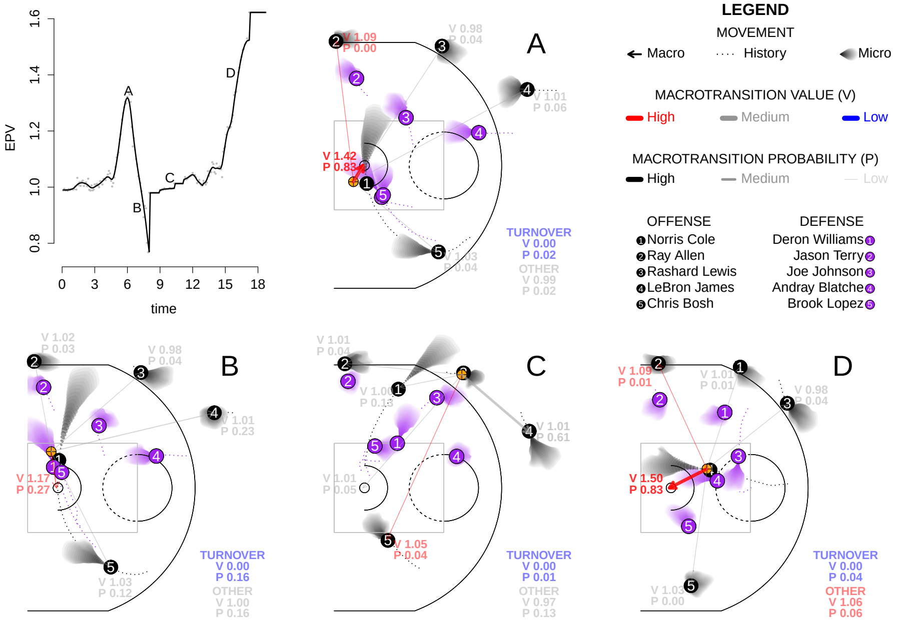

New metrics for player performance

EPV-added:

Rank	Player	EPVA
1	Kevin Durant	3.26
2	LeBron James	2.96
3	Jose Calderon	2.79
4	Dirk Nowitzki	2.69
5	Stephen Curry	2.50
6	Kyle Korver	2.01
7	Serge Ibaka	1.70
8	Channing Frye	1.65
9	Al Horford	1.55
10	Goran Dragic	1.54

Rank	Player	EPVA
277	Zaza Pachulia	-1.55
278	DeMarcus Cousins	-1.59
279	Gordon Hayward	-1.61
280	Jimmy Butler	-1.61
281	Rodney Stuckey	-1.63
282	Ersan Ilyasova	-1.89
283	DeMar DeRozan	-2.03
284	Rajon Rondo	-2.27
285	Ricky Rubio	-2.36
286	Rudy Gay	-2.59

Table : Top 10 and bottom 10 players by EPV-added (EPVA) per game in 2013-14, minimum 500 touches during season.

New metrics for player performance

Shot satisfaction:

Rank	Player	Satis.
1	Mason Plumlee	0.35
2	Pablo Prigioni	0.31
3	Mike Miller	0.27
4	Andre Drummond	0.26
5	Brandan Wright	0.24
6	DeAndre Jordan	0.24
7	Kyle Korver	0.24
8	Jose Calderon	0.22
9	Jodie Meeks	0.22
10	Anthony Tolliver	0.22

Rank	Player	Satis.
277	Garrett Temple	-0.02
278	Kevin Garnett	-0.02
279	Shane Larkin	-0.02
280	Tayshaun Prince	-0.03
281	Dennis Schroder	-0.04
282	LaMarcus Aldridge	-0.04
283	Ricky Rubio	-0.04
284	Roy Hibbert	-0.05
285	Will Bynum	-0.05
286	Darrell Arthur	-0.05

Table : Top 10 and bottom 10 players by shot satisfaction in 2013-14, minimum 500 touches during season.

Acknowledgements and future work

Our EPV framework can be extended to better incorporate unique basketball strategies:

- Additional macrotransitions can be defined, such as pick and rolls, screens, and other set plays.
- Use more information in defensive matchups (only defender locations, not identities, are currently used).
- Summarize and aggregate EPV estimates into useful player- or team-specific metrics.

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